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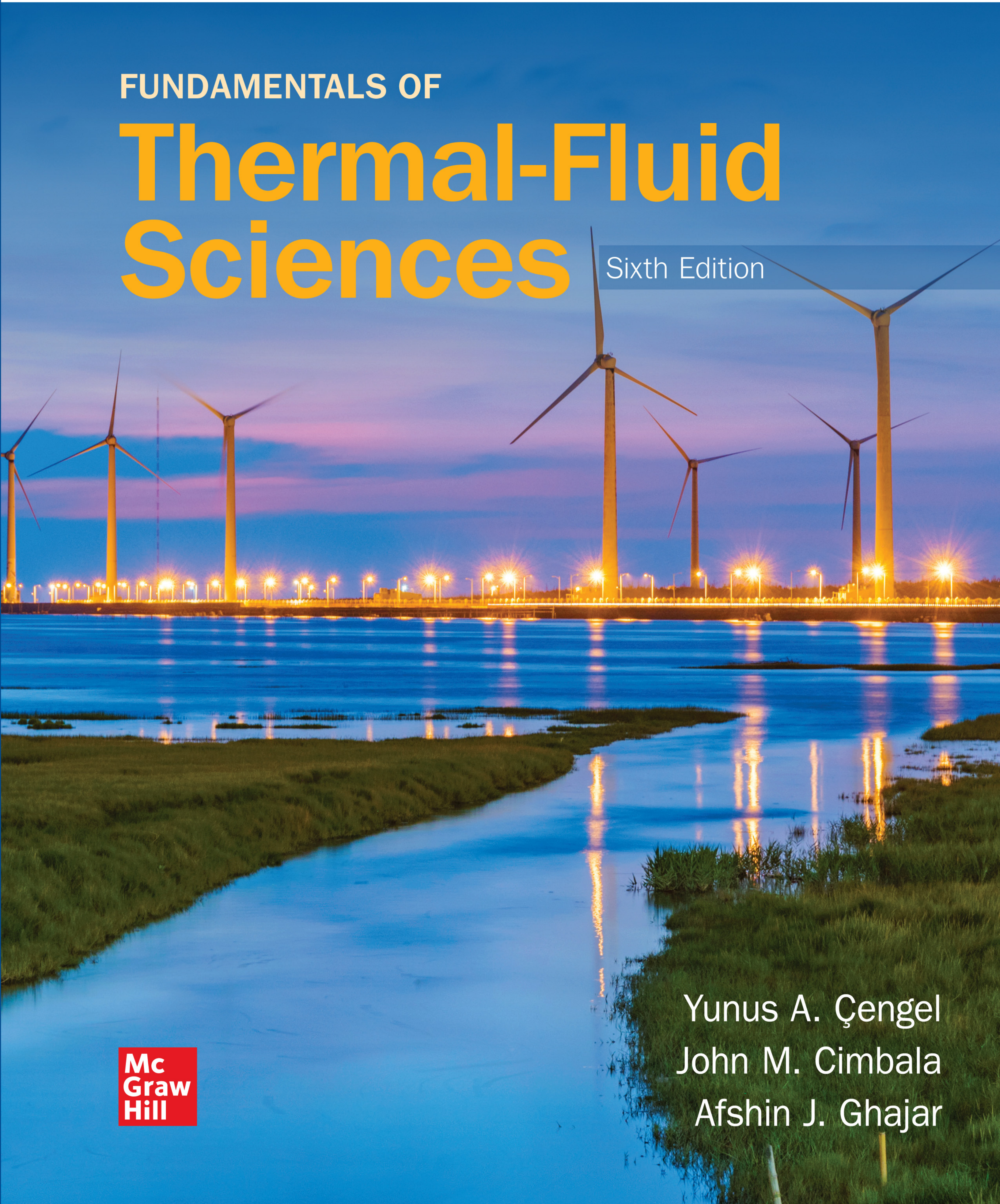
FUNDAMENTALS OF

Thermal-Fluid Sciences

Sixth Edition



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Mc Graw Hill

FUNDAMENTALS OF THERMAL-FLUID SCIENCES

SIXTH EDITION

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Dr. Çengel is also the author or coauthor of the widely adopted textbooks *Thermodynamics: An Engineering Approach* (9th ed., 2019), *Heat and Mass Transfer: Fundamentals and Applications* (6th ed., 2020), *Introduction to Thermodynamics and Heat Transfer* (2nd ed., 2008), *Fluid Mechanics: Fundamentals and Applications* (4th ed., 2018), *Essentials of Fluid Mechanics: Fundamentals and Applications* (1st ed., 2008), *Differential Equations for Engineers and Scientists* (1st ed., 2013), *Fundamentals and Applications of Renewable Energy* (1st ed., 2020), and *Energy Efficiency and Management for Engineers* (1st ed., 2020), all published by McGraw-Hill. Some of his textbooks have been translated into Chinese, Japanese, Korean, Thai, Spanish, Portuguese, Turkish, Italian, Greek, and French.

Dr. Çengel has delivered several keynote and invited lectures at technical conferences and academic institutions. He is the recipient of several outstanding teacher awards, and he has received the ASEE Meriam/Wiley Distinguished Author Award for excellence in authorship twice.

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Dr. Cimbala is the coauthor of *Indoor Air Quality Engineering: Environmental Health and Control of Indoor Pollutants* (2003), published by Marcel-Dekker, Inc.; *Essentials of Fluid Mechanics: Fundamentals and Applications* (2008); *Fluid Mechanics: Fundamentals and Applications*, 4th ed. (2018), and *Fundamentals and Applications of Renewable Energy* (2020), all published by McGraw-Hill. He has also contributed to parts of other books, and is the author or coauthor of dozens of journal and conference papers. He has also recently published a nontechnical novel. More information can be found at www.mne.psu.edu/cimbala.

Professor Cimbala is the recipient of several outstanding teaching awards and views his book writing as an extension of his love of teaching. He is a member of the American Society of Mechanical Engineers (ASME), the American Society for Engineering Education (ASEE), and the American Physical Society (APS).

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He has received several outstanding teaching, research, advising, and service awards from the College of Engineering at Oklahoma State University. His latest significant awards are the 75th Anniversary Medal of the ASME Heat Transfer Division "in recognition of his service to the heat transfer community and contributions to the field," awarded in 2013. He received the ASME ICNMM 2016 Outstanding Leadership Award, which recognizes a person whose service within the ICNMM (International Conference on Nanochannels, Microchannels, and Minichannels) is exemplary. He also received the 2017 Donald Q. Kern Award "in recognition of his outstanding leadership in the field of heat exchangers and two-phase flow, book and archival publications, and service to the academic and industrial profession." Dr. Ghajar is a Fellow of the American Society of Mechanical Engineers (ASME), Heat Transfer Series Editor for CRC Press/Taylor & Francis, and Editor-in-Chief of *Heat Transfer Engineering*, an international journal aimed at practicing engineers and specialists in heat transfer published by Taylor & Francis. Dr. Ghajar is the coauthor of the textbook *Heat and Mass Transfer: Fundamentals and Applications* (6th ed., 2020, McGraw-Hill), and the author of *Two-Phase Gas-Liquid Flow in Pipes with Different Orientations* (2020, Springer).

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PREFACE

BACKGROUND

This text is an abbreviated version of standard thermodynamics, fluid mechanics, and heat transfer texts, covering topics that engineering students are most likely to need in their professional lives. The thermodynamics portion of this text is based on the text *Thermodynamics: An Engineering Approach* by Y. A. Çengel, M. A. Boles, and Mehmet Kanoğlu, the fluid mechanics portion is based on *Fluid Mechanics: Fundamentals and Applications* by Y. A. Çengel and J. M. Cimbala, and the heat transfer portion is based on *Heat and Mass Transfer: Fundamentals and Applications* by Y. A. Çengel and A. J. Ghajar, all published by McGraw-Hill. Most chapters are practically independent of each other and can be covered in any order. The text is well-suited for curriculums that have a common introductory course or a two-course sequence on thermal-fluid sciences.

It is recognized that all topics of thermodynamics, fluid mechanics, and heat transfer cannot be covered adequately in a typical three-semester-hour course, so sacrifices must be made from the depth if not from the breadth of coverage. Selecting the right topics and finding the proper combination of depth and breadth are no small challenge for instructors, and this text is intended to provide the basis for that selection. Students in a combined thermal-fluids course can gain a basic understanding of energy and energy interactions, various mechanisms of heat transfer, and fundamentals of fluid flow. Such a course can also instill in students the confidence and the background to do further reading of their own and to be able to communicate effectively with specialists in thermal-fluid sciences.

OBJECTIVES

This book is intended for use as a textbook in a first course in thermal-fluid sciences for undergraduate engineering students in their sophomore, junior, or senior year, and as a reference book for practicing engineers. Students are assumed to have an adequate background in calculus, physics, and engineering mechanics. The objectives of this text are

- To cover the *basic principles* of thermodynamics, fluid mechanics, and heat transfer.
- To present numerous and diverse real-world *engineering examples* to give students a feel for how thermal-fluid sciences are applied in engineering practice.
- To develop an *intuitive understanding* of thermal-fluid sciences by emphasizing the physics and physical arguments.

The text contains sufficient material to give instructors flexibility and to accommodate their preferences on the right blend of thermodynamics, fluid mechanics, and heat transfer for their students. By careful selection of topics, an instructor can spend one-third, one-half, or two-thirds of the course on thermodynamics and the rest on selected topics of fluid mechanics and heat transfer.

PHILOSOPHY AND GOAL

The philosophy that contributed to the warm reception of the first edition of this book has remained unchanged. Our goal is to offer an engineering textbook that

- Communicates directly to the minds of tomorrow's engineers in a *simple yet precise* manner.

- Leads students toward a clear understanding and firm grasp of the *basic principles* of thermal-fluid sciences.
- Encourages *creative thinking* and development of a *deeper understanding* and *intuitive feel* for thermal-fluid sciences.
- Is *read* by students with *interest* and *enthusiasm* rather than being used as an aid to solve problems.

Special effort has been made to appeal to readers' natural curiosity and to help students explore the exciting subject area of thermal-fluid sciences. The enthusiastic response we received from the users of the previous editions—from small colleges to large universities all over the world—indicates that our objectives have largely been achieved. It is our philosophy that the best way to learn is by practice. Therefore, special effort is made throughout the book to reinforce material that was presented earlier.

Yesterday's engineers spent a major portion of their time substituting values into formulas and obtaining numerical results. Today, formula manipulations and number crunching are being left to computers. Tomorrow's engineers will need to have a clear understanding and a firm grasp of the *basic principles* so that he or she can understand even the most complex problems, formulate them, and interpret the results. A conscious effort is made to emphasize these basic principles while also providing students with a look at how modern tools are used in engineering practice.

NEW IN THIS EDITION

All the popular features of the previous editions have been retained while new ones have been added. The main body of the text remains largely unchanged. Updates and changes for clarity and readability have been made throughout the text.

Recent new definitions of kilogram, mole, ampere, and kelvin in the 26th General Conference on Weights and Measures in France are provided in Chaps. 1 and 2. A new subsection, "Equation Solvers," is also added to Chap. 1.

In Chap. 14, we now highlight the explicit Churchill equation as an alternative to the implicit Colebrook equation. A number of exciting new pictures have been added to fluid mechanics chapters.

In Chap. 18, the graphical representation of the one-dimensional transient heat conduction solutions (Heisler charts) have been eliminated, and the emphasis has been placed on the solutions with more accurate approximate or exact analytical expressions.

A large number of the end-of-chapter problems in the text have been modified, and many problems were replaced by new ones. Also, several of the solved example problems have been replaced.

Video Resources—2D/3D animation videos have been added to the eBook to help clarify challenging concepts such as thermodynamic cycles. In addition to these conceptual video resources, worked example problem videos are included in the eBook to help students apply their conceptual understanding to problem solving.

LEARNING TOOLS

EMPHASIS ON PHYSICS

A distinctive feature of this book is its emphasis on the physical aspects of subject matter in addition to mathematical representations and manipulations. The authors believe that the emphasis in undergraduate education should remain on *developing a sense of underlying physical mechanisms* and a *mastery of solving* *practical problems* that an engineer is likely to face in the real world. Developing an intuitive understanding should also make the course a more motivating and worthwhile experience for students.

EFFECTIVE USE OF ASSOCIATION

An observant mind should have no difficulty understanding engineering sciences. After all, the principles of engineering sciences are based on our *everyday experiences* and *experimental observations.* A more physical, intuitive approach is used throughout this text. Frequently, *parallels are drawn* between the subject matter and students' everyday experiences so that they can relate the subject matter to what they already know.

SELF-INSTRUCTING

The material in the text is introduced at a level that an average student can follow comfortably. It speaks to students, not over them. In fact, it is *self-instructive.* Noting that the principles of science are based on experimental observations, most of the derivations in this text are largely based on physical arguments, and thus they are easy to follow and understand.

EXTENSIVE USE OF ARTWORK

Figures are important learning tools that help students to "get the picture." The text makes effective use of graphics, and it contains a great number of figures and illustrations. Figures attract attention and stimulate curiosity and interest. Some of the figures in this text are intended to serve as a means of emphasizing some key concepts that would otherwise go unnoticed; some serve as page summaries.

CHAPTER OPENERS AND SUMMARIES

Each chapter begins with an overview of the material to be covered and chapter objectives. A *summary* is included at the end of each chapter for a quick review of basic concepts and important relations.

NUMEROUS WORKED-OUT EXAMPLES

Each chapter contains several worked-out *examples* that clarify the material and illustrate the use of the basic principles. An *intuitive* and *systematic* approach is used in the solution of the example problems, with particular attention to the proper use of units.

A WEALTH OF REAL-WORLD END-OF-CHAPTER PROBLEMS

The end-of-chapter problems are grouped under specific topics in the order they are covered to make problem selection easier for both instructors and students. Within each group of problems are *Concept Questions*, indicated by "C" to check the students' level of understanding of basic concepts. The problems under *Review Problems* are more comprehensive in nature and are not directly tied to any specific section of a chapter—in some cases they require review of material learned in previous chapters. The problems under the *Design and Essay Problems* title are intended to encourage students to make engineering judgments, to conduct independent exploration of topics of interest, and to communicate their findings in a professional manner. Several economics- and safety-related problems are incorporated throughout to enhance cost and safety awareness among engineering students. Answers to selected problems are listed immediately following the problem for convenience to students.

A SYSTEMATIC SOLUTION PROCEDURE

A well-structured approach is used in problem solving while maintaining an informal conversational style. The problem is first stated and the objectives are identified, and the assumptions made are stated together with their justifications. The properties needed to solve the problem are listed separately. Numerical values are used together with their units to emphasize that numbers without units are meaningless, and unit manipulations are as important as manipulating the numerical values with a calculator. The significance of the findings is discussed following the solutions. This approach is also used consistently in the solutions presented in the Instructor's Solutions Manual.

RELAXED SIGN CONVENTION

The use of a formal sign convention for heat and work is abandoned as it often becomes counterproductive. A physically meaningful and engaging approach is adopted for interactions instead of a mechanical approach. Subscripts "in" and "out," rather than the plus and minus signs, are used to indicate the directions of interactions.

A CHOICE OF SI ALONE OR SI / ENGLISH UNITS

In recognition of the fact that English units are still widely used in some industries, both SI and English units are used in this text, with an emphasis on SI. The material in this text can be covered using combined SI/English units or SI units alone, depending on the preference of the instructor. The property tables and charts in the appendices are presented in both units, except the ones that involve dimensionless quantities. Problems, tables, and charts in English units are designated by "E" after the number for easy recognition, and they can be ignored easily by the SI users.

CONVERSION FACTORS

Frequently used conversion factors and physical constants are listed after the index for easy reference.

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INSTRUCTOR RESOURCES

A number of supplements are available to instructors through Connect. This includes text images in PowerPoint format, Solutions Manual, and Lecture PowerPoints.

ACKNOWLEDGMENTS

We would like to acknowledge with appreciation the numerous and valuable comments, suggestions, criticisms, and praise of these academic evaluators:

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John M. Cimbala
Afshin J. Ghajar

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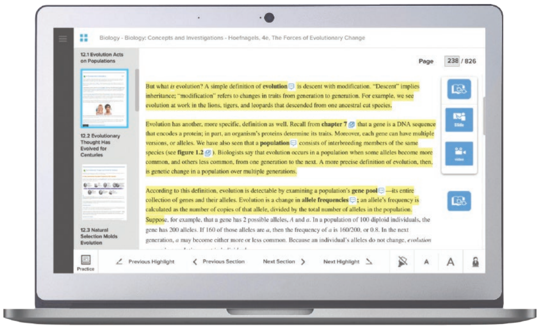

But what *is* evolution? A simple definition of **evolution** is descent with modification. "Descent" implies inheritance; "modification" refers to changes in traits from generation to generation. For example, we see evolution at work in the lions, tigers, and leopards that descended from one ancestral cat species.

Evolution has another, more specific, definition as well. Recall from **chapter 7** that a gene is a DNA sequence that encodes a protein; in part, an organism's proteins determine its traits. Moreover, each gene can have multiple versions, or alleles. We have also seen that a **population** consists of interbreeding members of the same species (see **figure 1.2**). Biologists say that evolution occurs in a population when some alleles become more common, and others less common, from one generation to the next. A more precise definition of evolution, then, is genetic change in a population over multiple generations.

According to this definition, evolution is detectable by examining a population's **gene pool** —its entire collection of genes and their alleles. Evolution is a change in **allele frequencies**; an allele's frequency is calculated as the number of copies of that allele, divided by the total number of alleles in the population. Suppose, for example, that a gene has 2 possible alleles, *A* and *a*. In a population of 100 diploid individuals, the gene has 200 alleles. If 160 of those alleles are *a*, then the frequency of *a* is 160/200, or 0.8. In the next generation, *a* may become either more or less common. Because an individual's alleles do not change, *evolution*

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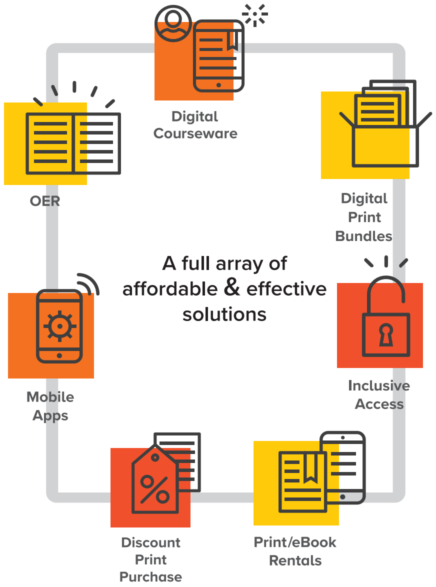

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CHAPTER

1

INTRODUCTION AND OVERVIEW

Many engineering systems involve the transfer, transport, and conversion of energy, and the sciences that deal with these subjects are broadly referred to as *thermal-fluid sciences.* Thermal-fluid sciences are usually studied under the subcategories of *thermodynamics, heat transfer,* and *fluid mechanics.* We start this chapter with an overview of these sciences, and give some historical background. Then we review the unit systems that will be used, and discuss dimensional homogeneity. We then present an intuitive systematic *problem-solving technique* that can be used as a model in solving engineering problems, followed by a discussion of the proper place of software packages in engineering. Finally, we discuss accuracy and significant digits in engineering measurements and calculations.

OBJECTIVES

Objectives of this chapter are to:

- Be acquainted with the engineering sciences thermodynamics, heat transfer, and fluid mechanics, and understand the basic concepts of thermal-fluid sciences.
- Be comfortable with the metric SI and English units commonly used in engineering.
- Develop an intuitive systematic problem-solving technique.
- Learn the proper use of software packages in engineering.
- Develop an understanding of accuracy and significant digits in calculations.

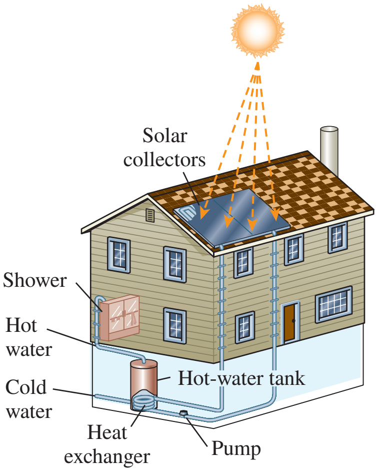


FIGURE 1–1
The design of many engineering systems, such as this solar hot-water system, involves thermal-fluid sciences.

1–1 ▪ INTRODUCTION TO THERMAL-FLUID SCIENCES

The word *thermal* stems from the Greek word *therme,* which means *heat.* Therefore, thermal sciences can loosely be defined as the sciences that deal with heat. The recognition of different forms of energy and its transformations has forced this definition to be broadened. Today, the physical sciences that deal with energy and the transfer, transport, and conversion of energy are usually referred to as **thermal-fluid sciences** or just **thermal sciences**. Traditionally, the thermal-fluid sciences are studied under the subcategories of thermodynamics, heat transfer, and fluid mechanics. In this book, we present the basic principles of these sciences, and apply them to situations that engineers are likely to encounter in their practice.

The design and analysis of most thermal systems such as power plants, automotive engines, and refrigerators involve all categories of thermal-fluid sciences as well as other sciences (Fig. 1–1). For example, designing the radiator of a car involves the determination of the amount of energy transfer from a knowledge of the properties of the coolant using *thermodynamics,* the determination of the size and shape of the inner tubes and the outer fins using *heat transfer,* and the determination of the size and type of the water pump using *fluid mechanics.* Of course, the determination of the materials and the thickness of the tubes requires the use of material science as well as strength of materials. The reason for studying different sciences separately is simply to facilitate learning without being overwhelmed. Once the basic principles are mastered, they can then be synthesized by solving comprehensive real-world practical problems. But first we present an overview of thermal-fluid sciences.

Application Areas of Thermal-Fluid Sciences

All activities in nature involve some interaction between energy and matter; thus, it is hard to imagine an area that does not relate to thermal-fluid sciences in some manner. Therefore, developing a good understanding of basic principles of thermal-fluid sciences has long been an essential part of engineering education.

Thermal-fluid sciences are commonly encountered in many engineering systems and other aspects of life, and one does not need to go very far to see some application areas of them. In fact, one does not need to go anywhere. The heart is constantly pumping blood to all parts of the human body, various energy conversions occur in trillions of body cells, and the body heat generated is constantly rejected to the environment. Human comfort is closely tied to the rate of this metabolic heat rejection. We try to control this heat transfer rate by adjusting our clothing to the environmental conditions. Also, any defect in the heart and the circulatory system is a major cause for alarm.

Other applications of thermal-fluid sciences are right where one lives. An ordinary house is, in some respects, an exhibition hall filled with wonders of thermal-fluid sciences. Many ordinary household utensils and appliances are designed, in whole or in part, by using the principles of thermal-fluid sciences. Some examples include the electric or gas range, heating and air-conditioning systems, refrigerator, humidifier, pressure cooker, water heater, shower, iron, plumbing and sprinkling systems, and even the computer, TV, and DVD player. On a larger scale, thermal-fluid sciences play a major part in the design and analysis of automotive engines, rockets, jet engines, and conventional or nuclear power plants, solar collectors, the transportation of water, crude oil, and natural gas, the water distribution systems in cities, and the design of vehicles from ordinary cars to airplanes (Fig. 1–2). The energy-efficient home that you may be living in, for example, is designed on the basis of minimizing heat

Refrigerator

Jill Braaten/McGraw-Hill Education

Boats

Doug Menuez/Getty Images

Aircraft and spacecraft

PhotoLink/Getty Images

Power plants

Malcolm Fife/Getty Images

Human body

Ryan McVay/Getty Images

Cars

Mark Evans/Getty Images

Wind turbines

F. Schussler/PhotoLink/Getty Images

Food processing

Glow Images

A piping network in an industrial facility

Vithun Khamsong/EyeEm/Getty Images

FIGURE 1–2
Some application areas of thermal-fluid sciences.

loss in winter and heat gain in summer. The size, location, and the power input of the fan of your computer is also selected after a thermodynamic, heat transfer, and fluid flow analysis of the computer.

1–2 ▪ THERMODYNAMICS

Thermodynamics can be defined as the science of *energy*. Although everybody has a feeling of what energy is, it is difficult to give a precise definition for it. Energy can be viewed as the ability to cause changes.

The name *thermodynamics* stems from the Greek words *therme* (heat) and *dynamis* (power), which is most descriptive of the early efforts to convert heat into power. Today the same name is broadly interpreted to include all aspects of energy and energy transformations including power generation, refrigeration, and relationships among the properties of matter.

One of the most fundamental laws of nature is the **conservation of energy principle**. It simply states that during an interaction, energy can change from one form to another but the total amount of energy remains constant. That is, energy cannot be created or destroyed. A rock falling off a cliff, for example, picks up speed as a result of its potential energy being converted to kinetic energy (Fig. 1–3). The conservation of energy principle also forms the backbone of the diet industry: A person who has a greater energy input (food) than energy output (exercise) will gain weight (store energy in the form of fat), and a person who has a smaller energy input than output will lose weight (Fig. 1–4). The change in the energy content of a body or any other system is equal to the difference between the energy input and the energy output, and the energy balance is expressed as $E_{in} - E_{out} = \Delta E$.

The **first law of thermodynamics** is simply an expression of the conservation of energy principle, and it asserts that *energy* is a thermodynamic property. The **second law of thermodynamics** asserts that energy has *quality* as well as *quantity,* and actual processes occur in the direction of decreasing quality of energy. For example, a cup of hot coffee left on a table eventually cools, but a cup of cool coffee in the same room never gets hot by itself. The high-temperature energy of the coffee is degraded (transformed into a less useful form at a lower temperature) once it is transferred to the surrounding air.

Although the principles of thermodynamics have been in existence since the creation of the universe, thermodynamics did not emerge as a science until the construction of the first successful atmospheric steam engines in England by Thomas Savery in 1697 and Thomas Newcomen in 1712. These engines were very slow and inefficient, but they opened the way for the development of a new science.

The first and second laws of thermodynamics emerged simultaneously in the 1850s, primarily out of the works of William Rankine, Rudolph Clausius, and Lord Kelvin (formerly William Thomson). The term *thermodynamics* was first used in a publication by Lord Kelvin in 1849. The first thermodynamics textbook was written in 1859 by William Rankine, a professor at the University of Glasgow.

It is well known that a substance consists of a large number of particles called *molecules.* The properties of the substance naturally depend on the behavior of these particles. For example, the pressure of a gas in a container is the result of momentum transfer between the molecules and the walls of the container. However, one does not need to know the behavior of the gas particles to determine the pressure in the container. It would be sufficient to attach a pressure gage to the container. This macroscopic approach to the study of thermodynamics that does not require a knowledge of the behavior of individual particles is called **classical thermodynamics**. It provides a direct and easy way to solve engineering problems. A more elaborate approach, based on the average behavior of large groups of individual particles, is called **statistical thermodynamics**. This microscopic approach is rather involved and is used in this text only in a supporting role.

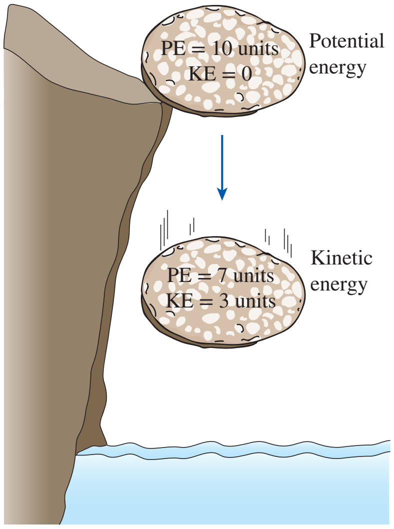


FIGURE 1–3
Energy cannot be created or destroyed; it can only change forms (the first law).

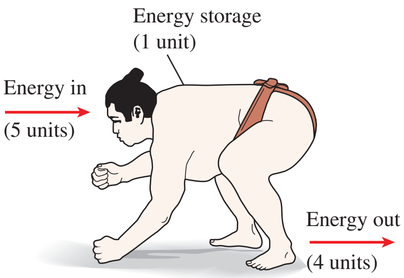


FIGURE 1–4
Conservation of energy principle for the human body.

1–3 ▪ HEAT TRANSFER

We all know from experience that a cold canned drink left in a room warms up and a warm canned drink left in a refrigerator cools down. This is accomplished by the transfer of *energy* from the warm medium to the cold one. The energy transfer is always from the higher temperature medium to the lower temperature one, and the energy transfer stops when the two media reach the same temperature.

Energy exists in various forms. In heat transfer, we are primarily interested in **heat**, which is *the form of energy that can be transferred from one system to another as a result of temperature difference.* The science that deals with the determination of the *rates* of such energy transfers is **heat transfer**.

You may be wondering why we need to undertake a detailed study on heat transfer. After all, we can determine the amount of heat transfer for any system undergoing any process using a thermodynamic analysis alone. The reason is that thermodynamics is concerned with the *amount* of heat transfer as a system undergoes a process from one equilibrium state to another, and it gives no indication about *how long* the process will take. A thermodynamic analysis simply tells us how much heat must be transferred to realize a specified change of state to satisfy the conservation of energy principle.

In practice we are more concerned about the rate of heat transfer (heat transfer per unit time) than we are with the amount of it. For example, we can determine the amount of heat transferred from a thermos bottle as the hot coffee inside cools from 90°C to 80°C by a thermodynamic analysis alone. But a typical user or designer of a thermos bottle is primarily interested in *how long* it will be before the hot coffee inside cools to 80°C, and a thermodynamic analysis cannot answer this question. Determining the rates of heat transfer to or from a system and thus the times of heating or cooling, as well as the variation of the temperature, is the subject of heat transfer (Fig. 1–5).

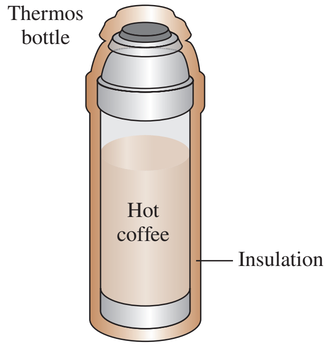


FIGURE 1–5
We are normally interested in how long it takes for the hot coffee in a thermos bottle to cool to a certain temperature, which cannot be determined from a thermodynamic analysis alone.

Thermodynamics deals with equilibrium states and changes from one equilibrium state to another. Heat transfer, on the other hand, deals with systems that lack thermal equilibrium, and thus it is a *nonequilibrium* phenomenon. Therefore, the study of heat transfer cannot be based on the principles of thermodynamics alone. However, the laws of thermodynamics lay the framework for the science of heat transfer. The *first law* requires that the rate of energy transfer into a system be equal to the rate of increase of the energy of that system. The *second law* requires that heat be transferred in the direction of decreasing temperature (Fig. 1–6). This is like saying that a car parked on an inclined road must go downhill in the direction of decreasing elevation when its brakes are released. It is also analogous to the electric current flowing in the direction of decreasing voltage or the fluid flowing in the direction of decreasing total pressure.

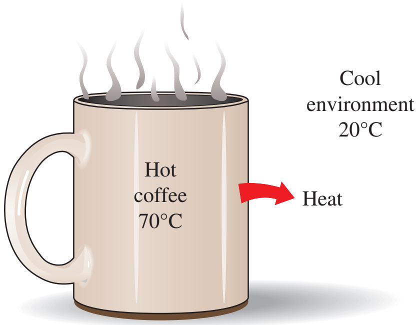


FIGURE 1–6
Heat flows in the direction of decreasing temperature.

The basic requirement for heat transfer is the presence of a *temperature difference*. There can be no net heat transfer between two bodies that are at the same temperature. The temperature difference is the *driving force* for heat transfer, just as the *voltage difference* is the driving force for electric current flow and *pressure difference* is the driving force for fluid flow. The rate of heat transfer in a certain direction depends on the magnitude of the *temperature gradient* (the temperature difference per unit length or the rate of change of temperature) in that direction. The larger the temperature gradient, the higher the rate of heat transfer.

1–4 ▪ FLUID MECHANICS

Mechanics is the oldest physical science that deals with both stationary and moving bodies under the influence of forces. The branch of mechanics that deals with bodies at rest is called **statics**, while the branch that deals with bodies in motion under the action of forces is called **dynamics**. The subcategory **fluid mechanics** is defined as the science that deals with the behavior of fluids at rest (*fluid statics*) or in motion (*fluid dynamics*), and the interaction of fluids with solids or other fluids at the boundaries. Fluid mechanics is also referred to as **fluid dynamics** by considering fluids at rest as a special case of motion with zero velocity (Fig. 1–7).

FIGURE 1–7
Fluid mechanics deals with liquids and gases in motion or at rest.
franckreporter/E+/Getty Images

Fluid mechanics itself is also divided into several categories. The study of the motion of fluids that can be approximated as incompressible (such as liquids, especially water, and gases at low speeds) is usually referred to as **hydrodynamics**. A subcategory of hydrodynamics is **hydraulics**, which deals with liquid flows in pipes and open channels. **Gas dynamics** deals with the flow of fluids that undergo significant density changes, such as the flow of gases through nozzles at high speeds. The category **aerodynamics** deals with the flow of gases (especially air) over bodies such as aircraft, rockets, and automobiles at high or low speeds. Some other specialized categories such as **meteorology**, **oceanography**, and **hydrology** deal with naturally occurring flows.

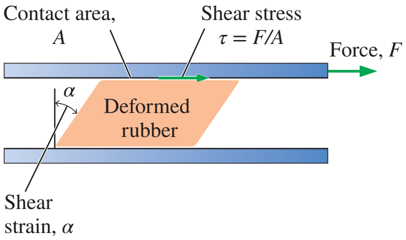
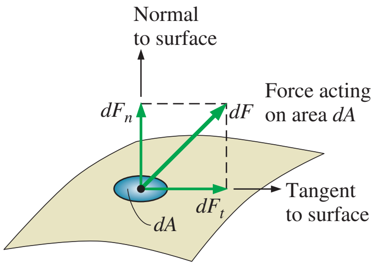


FIGURE 1–8
Deformation of a rubber block placed between two parallel plates under the influence of a shear force. The shear stress shown is that on the rubber—an equal but opposite shear stress acts on the upper plate.

You will recall from physics that a substance exists in three primary phases: solid, liquid, or gas. (At very high temperatures, it also exists as plasma.) A substance in the liquid or gas phase is referred to as a **fluid**. Distinction between a solid and a fluid is made on the basis of the substance's ability to resist an applied shear (or tangential) stress that tends to change its shape. A solid can resist an applied shear stress by deforming, whereas *a fluid deforms continuously under the influence of a shear stress,* no matter how small. In solids, stress is proportional to *strain,* but in fluids, stress is proportional to *strain rate*. When a constant shear force is applied, a solid eventually stops deforming at some fixed strain angle, whereas a fluid never stops deforming and approaches a constant *rate* of strain.

Consider a rectangular rubber block tightly placed between two plates. As the upper plate is pulled with a force F while the lower plate is held fixed, the rubber block deforms, as shown in Fig. 1–8. The angle of deformation α (called the *shear strain* or *angular displacement*) increases in proportion to the applied force F. Assuming there is no slip between the rubber and the plates, the upper surface of the rubber is displaced by an amount equal to the displacement of the upper plate while the lower surface remains stationary. In equilibrium, the net force acting on the upper plate in the horizontal direction must be zero, and thus a force equal and opposite to F must be acting on the plate. This opposing force that develops at the plate–rubber interface due to friction is expressed as $F = \tau A$, where τ is the shear stress and A is the contact area between the upper plate and the rubber. When the force is removed, the rubber returns to its original position. This phenomenon would also be observed with other solids such as a steel block provided that the applied force does not exceed the elastic range. If this experiment were repeated with a fluid (with two large parallel plates placed in a large body of water, for example), the fluid layer in contact with the upper plate would move with the plate continuously at the velocity of the plate no matter how small the force F. The fluid velocity would decrease with depth because of friction between fluid layers, reaching zero at the lower plate.



Normal stress: $\sigma = \dfrac{dF_n}{dA}$

Shear stress: $\tau = \dfrac{dF_t}{dA}$

FIGURE 1–9
The normal stress and shear stress at the surface of a fluid element. For fluids at rest, the shear stress is zero and pressure is the only normal stress.

You will recall from statics that **stress** is defined as force per unit area and is determined by dividing the force by the area upon which it acts. The normal component of a force acting on a surface per unit area is called the **normal stress**, and the tangential component of a force acting on a surface per unit area is called **shear stress** (Fig. 1–9). In a fluid at rest, the normal stress is called **pressure**. A fluid at rest is at a state of zero shear stress. When the walls are removed or a liquid container is tilted, a shear develops as the liquid moves to reestablish a horizontal free surface.

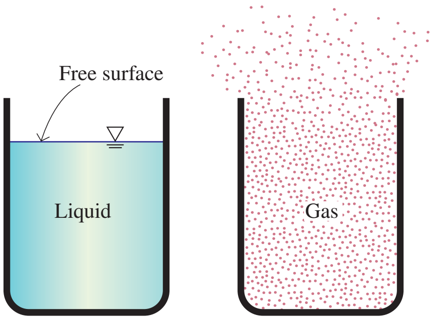


FIGURE 1–10
Unlike a liquid, a gas does not form a free surface, and it expands to fill the entire available space.

In a liquid, groups of molecules can move relative to each other, but the volume remains relatively constant because of the strong cohesive forces between the molecules. As a result, a liquid takes the shape of the container it is in, and it forms a free surface in a larger container in a gravitational field. A gas, on the other hand, expands until it encounters the walls of the container and fills the entire available space. This is because the gas molecules are widely spaced, and the cohesive forces between them are very small. Unlike liquids, a gas in an open container cannot form a free surface (Fig. 1–10).

Although solids and fluids are easily distinguished in most cases, this distinction is not so clear in some borderline cases. For example, *asphalt* appears and behaves as a solid since it resists shear stress for short periods of time. When these forces are exerted over extended periods of time, however, the asphalt deforms slowly, behaving as a fluid. Some plastics, lead, and slurry mixtures exhibit similar behavior. Such borderline cases are beyond the scope of this text. The fluids we deal with in this text will be clearly recognizable as fluids.

1–5 ▪ IMPORTANCE OF DIMENSIONS AND UNITS

Any physical quantity can be characterized by **dimensions**. The magnitudes assigned to the dimensions are called **units**. Some basic dimensions such as mass m, length L, time t, and temperature T are selected as **primary** or **fundamental dimensions**, while others such as velocity V, energy E, and volume $\mathcal{V}$ are expressed in terms of the primary dimensions and are called **secondary dimensions**, or **derived dimensions**.

A number of unit systems have been developed over the years. Despite strong efforts in the scientific and engineering community to unify the world with a single unit system, two sets of units are still in common use today: the **English system**, which is also known as the *United States Customary System* (USCS), and the **metric SI** (from *Le Système International d' Unités*), which is also known as the *International System.* The SI is a simple and logical system based on a decimal relationship between the various units, and it is used for scientific and engineering work in most of the industrialized nations, including England. The English system, however, has no apparent systematic numerical base, and various units in this system are related to each other rather arbitrarily (12 in = 1 ft, 1 mile = 5280 ft, 4 qt = 1 gal, etc.), which makes it confusing and difficult to learn. The United States is the only industrialized country that has not yet fully converted to the metric system.

The systematic efforts to develop a universally acceptable system of units dates back to 1790 when the French National Assembly charged the French Academy of Sciences to come up with such a unit system. An early version of the metric system was soon developed in France, but it did not find universal acceptance until 1875 when *The Metric Convention Treaty* was prepared and signed by 17 nations, including the United States. In this international treaty, meter and gram were established as the metric units for length and mass, respectively, and a *General Conference of Weights and Measures* (CGPM) was established that was to meet every six years. In 1960, the CGPM produced the SI, which was based on six fundamental quantities, and their units were adopted in 1954 at the Tenth General Conference of Weights and Measures: *meter* (m) for length, *kilogram* (kg) for mass, *second* (s) for time, *ampere* (A) for electric current, *degree Kelvin* (°K) for temperature, and *candela* (cd) for luminous intensity (amount of light). In 1971, the CGPM added a seventh fundamental quantity and unit: *mole* (mol) for the amount of matter.

Accurate and universal definitions of fundamental units have been challenging for the scientific community for many years. Recent new definitions of kilogram, mole, ampere, and kelvin are considered to be a historical milestone.

The kilogram unit represents the mass of one liter of pure water at 4°C. Previously, the kilogram was officially defined as the mass of a shiny metal cylinder that has been stored in Paris since 1889. This International Prototype of Kilogram is an alloy of 90 percent platinum and 10 percent iridium, also known as Le Grand K.

On November 26, 2018, representatives from 60 countries gathered for the 26th General Conference on Weights and Measures in Versailles, France, and adopted a resolution to define the unit of mass in terms of the Planck constant h, which has a fixed value of $6.62607015 \times 10^{-34}\ \text{m}^2\cdot\text{kg/s}$. Note that the Planck constant has kg in it, and certain equations in physics relate the Planck constant to one kilogram. Using the above definition of h,

$$1\ \text{kg} = \frac{h}{6.62607015 \times 10^{-34}}\,\frac{\text{s}}{\text{m}^2}$$

where the units of second (s) and meter (m) are themselves defined in terms of two other fundamental constants, c (speed of light in a vacuum) and $\Delta\nu_{\text{Cs}}$

TABLE 1–1

The seven fundamental (or primary) dimensions and their units in SI

Dimension	Unit
Length	meter (m)
Mass	kilogram (kg)
Time	second (s)
Temperature	kelvin (K)
Electric current	ampere (A)
Amount of light	candela (cd)
Amount of matter	mole (mol)

TABLE 1–2

Standard prefixes in SI units

Multiple	Prefix
10^{24}	yotta, Y
10^{21}	zetta, Z
10^{18}	exa, E
10^{15}	peta, P
10^{12}	tera, T
10^{9}	giga, G
10^{6}	mega, M
10^{3}	kilo, k
10^{2}	hecto, h
10^{1}	deka, da
10^{-1}	deci, d
10^{-2}	centi, c
10^{-3}	milli, m
10^{-6}	micro, μ
10^{-9}	nano, n
10^{-12}	pico, p
10^{-15}	femto, f
10^{-18}	atto, a
10^{-21}	zepto, z
10^{-24}	yocto, y

(ground state hyperfine structure transition frequency of the cesium-133 atom). Thus, the standard kilogram is now formally defined as

$$1\text{ kg} = \frac{(299{,}792{,}458)^2}{(6.62607015 \times 10^{-34})(9{,}192{,}631{,}770)}\,\frac{h \cdot \Delta\nu_{\text{Cs}}}{c^2}$$

At the same conference, the approach of using fixed universal constants was also adopted for the new definitions of the mole, the kelvin, and the ampere. The mole (sometimes mol) is related to the value of Avogadro's constant and the ampere to the value of the elementary charge. The kelvin is related to the Boltzmann constant, whose value is fixed at 1.380649×10^{-23} J/K.

The standard meter unit was originally defined as 1/10,000,000 of the distance between the north pole and the equator. This distance was measured as accurately as possible at the time, and in the late 18th century a "master metre" stick of this length was made. All other meters were measured from this stick. Subsequent calculations of the pole–equator distance showed that the original measurement was inaccurate. In 1983, the meter was redefined as the distance traveled by light in a vacuum in 1/299,792,458 of a second.

Based on the notational scheme introduced in 1967, the degree symbol was officially dropped from the absolute temperature unit, and all unit names were to be written without capitalization even if they were derived from proper names (Table 1–1). However, the abbreviation of a unit was to be capitalized if the unit was derived from a proper name. For example, the SI unit of force, which is named after Sir Isaac Newton (1647–1723), is *newton* (not Newton), and it is abbreviated as N. Also, the full name of a unit may be pluralized, but its abbreviation cannot. For example, the length of an object can be 5 m or 5 meters, *not* 5 ms or 5 meter. Finally, no period is to be used in unit abbreviations unless they appear at the end of a sentence. For example, the proper abbreviation of meter is m *(not* m.).

The move toward the metric system in the United States seems to have started in 1968 when Congress, in response to what was happening in the rest of the world, passed a Metric Study Act. Congress continued to promote a voluntary switch to the metric system by passing the Metric Conversion Act in 1975. A trade bill passed by Congress in 1988 set a September 1992 deadline for all federal agencies to convert to the metric system. However, the deadlines were relaxed later with no clear plans for the future.

The industries that are heavily involved in international trade (such as the automotive, soft drink, and liquor industries) have been quick to convert to the metric system for economic reasons (having a single worldwide design, fewer sizes, smaller inventories, etc.). Today, nearly all the cars manufactured in the United States are metric. Most car owners probably do not realize this until they try an English socket wrench on a metric bolt. Most industries, however, resisted the change, thus slowing down the conversion process.

At present the United States is a dual-system society, and it will stay that way until the transition to the metric system is completed. This puts an extra burden on today's engineering students, since they are expected to retain their understanding of the English system while learning, thinking, and working in terms of the SI. Given the position of the engineers in the transition period, both unit systems are used in this text, with particular emphasis on SI units.

As pointed out, the SI is based on a decimal relationship between units. The prefixes used to express the multiples of the various units are listed in Table 1–2. They are standard for all units, and the student is encouraged to memorize them because of their widespread use (Fig. 1–11).

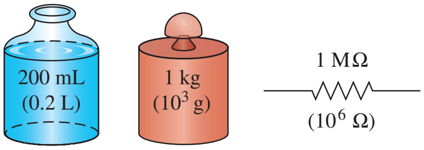


FIGURE 1–11
The SI unit prefixes are used in all branches of engineering.

Some SI and English Units

In SI, the units of mass, length, and time are the kilogram (kg), meter (m), and second (s), respectively. The respective units in the English system are the pound-mass (lbm), foot (ft), and second (s). The pound symbol *lb* is actually the abbreviation of *libra,* which was the ancient Roman unit of weight. The English retained this symbol even after the end of the Roman occupation of Britain in 410. The mass and length units in the two systems are related to each other by

$$1 \text{ lbm} = 0.45356 \text{ kg}$$

$$1 \text{ ft} = 0.3048 \text{ m}$$

In the English system, force is usually considered to be one of the primary dimensions and is assigned a nonderived unit. This is a source of confusion and error that necessitates the use of a dimensional constant (g_c) in many formulas. To avoid this nuisance, we consider force to be a secondary dimension whose unit is derived from Newton's second law, that is,

$$\text{Force} = (\text{Mass})(\text{Acceleration})$$

or

$$F = ma \tag{1–1}$$

In SI, the force unit is the newton (N), and it is defined as the *force required to accelerate a mass of 1 kg at a rate of 1 m/s*^{2}. In the English system, the force unit is the **pound-force** (lbf) and is defined as the *force required to accelerate a mass of 1 slug (32.174 lbm) at a rate of 1 ft/s*^{2} (Fig. 1–12). That is,

$$1 \text{ N} = 1 \text{ kg·m/s}^2$$

$$1 \text{ lbf} = 32.174 \text{ lbm·ft/s}^2$$

A force of 1 N is roughly equivalent to the weight of a small apple $(m = 102 \text{ g})$, whereas a force of 1 lbf is roughly equivalent to the weight of four medium apples ($m_{\text{total}} = 454$ g), as shown in Fig. 1–13. Another force unit in common use in many European countries is the *kilogram-force* (kgf), which is the weight of a 1 kg mass at sea level (1 kgf = 9.807 N).

The term **weight** is often incorrectly used to express mass, particularly by the "weight watchers." Unlike mass, weight W is a *force.* It is the gravitational force applied to a body, and its magnitude is determined from Newton's second law,

$$W = mg \quad \text{(N)} \tag{1–2}$$

where m is the mass of the body, and g is the local gravitational acceleration (g is 9.807 m/s^2 or 32.174 ft/s^2 at sea level and 45° latitude). An ordinary bathroom scale measures the gravitational force acting on a body.

The mass of a body remains the same regardless of its location in the universe. Its weight, however, changes with a change in gravitational acceleration. A body weighs less on top of a mountain since g decreases with altitude. On the surface of the moon, an astronaut weighs about one-sixth of what she or he normally weighs on earth (Fig. 1–14).

At sea level a mass of 1 kg weighs 9.807 N, as illustrated in Fig. 1–15. A mass of 1 lbm, however, weighs 1 lbf, which misleads people into believing that pound-mass and pound-force can be used interchangeably as pound (lb), which is a major source of error in the English system.

It should be noted that the *gravity force* acting on a mass is due to the *attraction* between the masses, and thus it is proportional to the magnitudes of the masses and inversely proportional to the square of the distance between them. Therefore, the gravitational acceleration g at a location depends on *latitude,* the *distance* to the center of the earth, and to a lesser extent, the positions of the moon and the sun. The value of g varies with location from 9.832 m/s^2 at the poles (9.789 at the equator) to 7.322 m/s^2 at 1000 km above sea level. However, at altitudes up

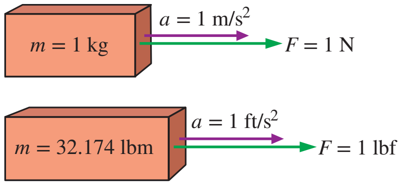


FIGURE 1–12
The definition of the force units.

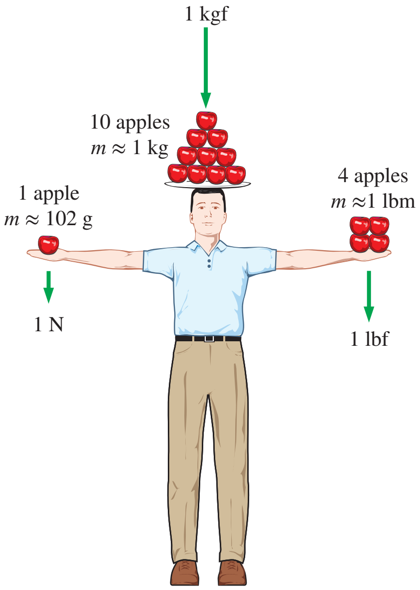


FIGURE 1–13
The relative magnitudes of the force units newton (N), kilogram-force (kgf), and pound-force (lbf).

FIGURE 1–14
A body weighing 150 lbf on earth will weigh only 25 lbf on the moon.

to 30 km, the variation of g from the sea-level value of 9.807 m/s^2 is less than 1 percent. Therefore, for most practical purposes, the gravitational acceleration can be assumed to be *constant* at 9.807 m/s^2, often rounded to 9.81 m/s^2. It is interesting to note that at locations below sea level, the value of g increases with distance from sea level, reaches a maximum at about 4500 m, and then starts decreasing. (What do you think the value of g is at the center of the earth?)

The primary cause of confusion between mass and weight is that mass is usually measured *indirectly* by measuring the *gravity force* it exerts. This approach also assumes that the forces exerted by other effects such as air buoyancy and fluid motion are negligible. This is like measuring the distance to a star by measuring its redshift, or measuring the altitude of an airplane by measuring barometric pressure. Both of these are also indirect measurements. The correct *direct* way of measuring mass is to compare it to a known mass. This is cumbersome, however, and it is mostly used for calibration and measuring precious metals.

Work, which is a form of energy, can simply be defined as force times distance; therefore, it has the unit "newton-meter (N·m)," which is called a **joule** (J). That is,

$$1\ \text{J} = 1\ \text{N·m} \tag{1–3}$$

A more common unit for energy in SI is the kilojoule (1 kJ = 10^3 J). In the English system, the energy unit is the **Btu** (British thermal unit), which is defined as the energy required to raise the temperature of 1 lbm of water at 68°F by 1°F. In the metric system, the amount of energy needed to raise the temperature of 1 g of water at 14.5°C by 1°C is defined as 1 **calorie** (cal), and 1 cal = 4.1868 J. The magnitudes of the kilojoule and Btu are almost identical (1 Btu = 1.0551 kJ). Here is a good way to get a feel for these units: If you light a typical match and let it burn itself out, it yields approximately one Btu (or one kJ) of energy (Fig. 1–16).

The unit for time rate of energy is joule per second (J/s), which is called a **watt** (W). In the case of work, the time rate of energy is called *power.* A commonly used unit of power is horsepower (hp), which is equivalent to 746 W. Electrical energy typically is expressed in the unit kilowatt-hour (kWh), which is equivalent to 3600 kJ. An electric appliance with a rated power of 1 kW consumes 1 kWh of electricity when running continuously for one hour. When dealing with electric power generation, the units kW and kWh are often confused. Note that kW or kJ/s is a unit of power, whereas kWh is a unit of energy. Therefore, statements like "the new wind turbine will generate 50 kW of electricity per year" are meaningless and incorrect. A correct statement should be something like "the new wind turbine with a rated power of 50 kW will generate 120,000 kWh of electricity per year."

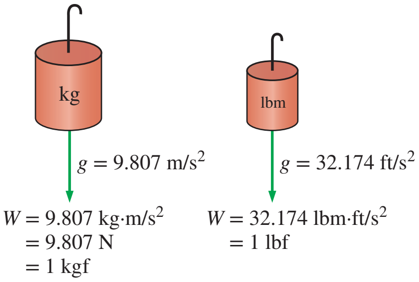


FIGURE 1–15
The weight of a unit mass at sea level.

FIGURE 1–16
A typical match yields about one Btu (or one kJ) of energy if completely burned.
John M. Cimbala

FIGURE 1–17
A wind turbine, as discussed in Example 1–1.
Bear Dancer Studios

Dimensional Homogeneity

We all know that apples and oranges do not add. But we somehow manage to do it (by mistake, of course). In engineering, all equations must be *dimensionally homogeneous.* That is, every term in an equation must have the same unit. If, at some stage of an analysis, we find ourselves in a position to add two quantities that have different units, it is a clear indication that we have made an error at an earlier stage. So checking dimensions can serve as a valuable tool to spot errors.

EXAMPLE 1–1 Electric Power Generation by a Wind Turbine

A school is paying \$0.12/kWh for electric power. To reduce its power bill, the school installs a wind turbine (Fig. 1–17) with a rated power of 30 kW. If the turbine operates 2200 hours per year at the rated power, determine the amount of electric power generated by the wind turbine and the money saved by the school per year.

SOLUTION A wind turbine is installed to generate electricity. The amount of electric energy generated and the money saved per year are to be determined.
Analysis The wind turbine generates electric energy at a rate of 30 kW or 30 kJ/s. Then the total amount of electric energy generated per year becomes

$$\begin{aligned}\text{Total energy} &= (\text{Energy per unit time})(\text{Time interval})\\ &= (30\ \text{kW})(2200\ \text{h})\\ &= \mathbf{66{,}000\ kWh}\end{aligned}$$

The money saved per year is the monetary value of this energy determined as

$$\begin{aligned}\text{Money saved} &= (\text{Total energy})(\text{Unit cost of energy})\\ &= (66{,}000\ \text{kWh})(\$0.12/\text{kWh})\\ &= \mathbf{\$7920}\end{aligned}$$

Discussion The annual electric energy production also could be determined in kJ by unit manipulations as

$$\text{Total energy} = (30\ \text{kW})(2200\ \text{h})\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)\left(\frac{1\ \text{kJ/s}}{1\ \text{kW}}\right) = 2.38 \times 10^8\ \text{kJ}$$

which is equivalent to 66,000 kWh (1 kWh = 3600 kJ).

We all know from experience that units can give terrible headaches if they are not used carefully in solving a problem. However, with some attention and skill, units can be used to our advantage. They can be used to check formulas; sometimes they can even be used to *derive* formulas, as explained in the following example.

EXAMPLE 1–2 Obtaining Formulas from Unit Considerations

A tank is filled with oil whose density is $\rho = 850\ \text{kg/m}^3$. If the volume of the tank is $V = 2\ \text{m}^3$, determine the amount of mass m in the tank.

SOLUTION The volume of an oil tank is given. The mass of oil is to be determined.
Assumptions Oil is a nearly incompressible substance and thus its density is constant.
Analysis A sketch of the system just described is given in Fig. 1–18. Suppose we forgot the formula that relates mass to density and volume. However, we know that mass has the unit of kilograms. That is, whatever calculations we do, we should end up with the unit of kilograms. Putting the given information into perspective, we have

$$\rho = 850\ \text{kg/m}^3 \quad \text{and} \quad V = 2\ \text{m}^3$$

It is obvious that we can eliminate m^3 and end up with kg by multiplying these two quantities. Therefore, the formula we are looking for should be

$$m = \rho V$$

Thus,

$$m = (850\ \text{kg/m}^3)(2\ \text{m}^3) = \mathbf{1700\ kg}$$

Discussion Note that this approach may not work for more complicated formulas. Nondimensional constants also may be present in the formulas, and these cannot be derived from unit considerations alone.

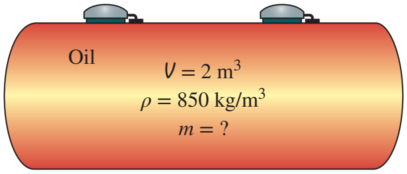


FIGURE 1–18
Schematic for Example 1–2.

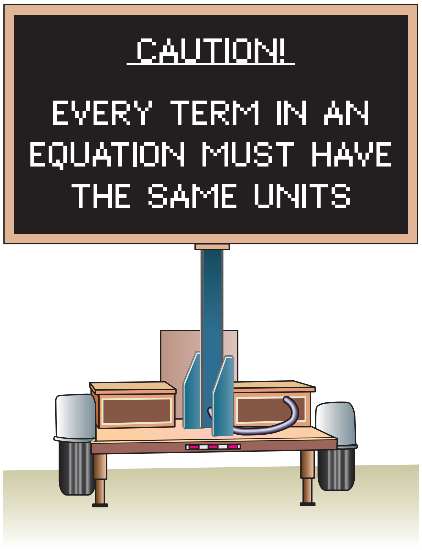


FIGURE 1–19
Always check the units in your calculations.

You should keep in mind that a formula that is not dimensionally homogeneous is definitely wrong (Fig. 1–19), but a dimensionally homogeneous formula is not necessarily right.

Unity Conversion Ratios

Just as all nonprimary dimensions can be formed by suitable combinations of primary dimensions, *all nonprimary units (**secondary units**) can be formed by combinations of primary units.* Force units, for example, can be expressed as

$$1\text{ N} = 1\text{ kg}\frac{\text{m}}{\text{s}^2} \quad \text{and} \quad 1\text{ lbf} = 32.174\text{ lbm}\frac{\text{ft}}{\text{s}^2}$$

They can also be expressed more conveniently as **unity conversion ratios** as

$$\frac{1\text{ N}}{1\text{ kg·m/s}^2} = 1 \quad \text{and} \quad \frac{1\text{ lbf}}{32.174\text{ lbm·ft/s}^2} = 1$$

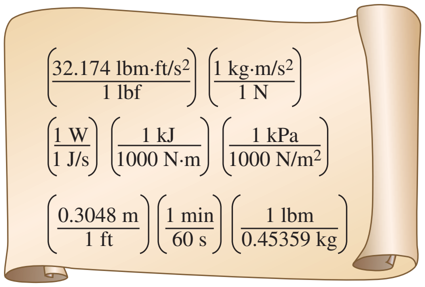


FIGURE 1–20
Every unity conversion ratio (as well as its inverse) is exactly equal to 1. Shown here are a few commonly used unity conversion ratios, each within its own set of parentheses.

Unity conversion ratios are equal to 1 and are unitless, and thus such ratios (or their inverses) can be inserted conveniently into any calculation to properly convert units (Fig. 1–20). You are encouraged to always use unity conversion ratios such as those given here when converting units. Some textbooks insert the archaic gravitational constant g_c defined as $g_c = 32.174\text{ lbm·ft/lbf·s}^2 = 1\text{ kg·m/N·s}^2 = 1$ into equations in order to force units to match. This practice leads to unnecessary confusion and is strongly discouraged by the present authors. We recommend that you instead use unity conversion ratios.

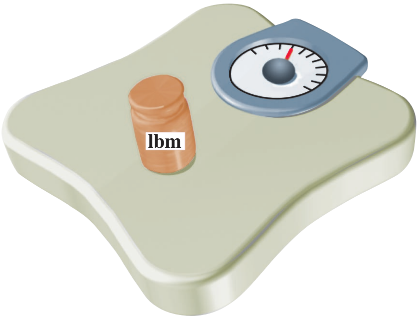


FIGURE 1–21
A mass of 1 lbm weighs 1 lbf on earth.

EXAMPLE 1–3 The Weight of One Pound-Mass

Using unity conversion ratios, show that 1.00 lbm weighs 1.00 lbf on earth (Fig. 1–21).

SOLUTION A mass of 1.00 lbm is subjected to standard earth gravity. Its weight in lbf is to be determined.

Assumptions Standard sea-level conditions are assumed.

Properties The gravitational constant is $g = 32.174\text{ ft/s}^2$.

Analysis We apply Newton's second law to calculate the weight (force) that corresponds to the known mass and acceleration. The weight of any object is equal to its mass times the local value of gravitational acceleration. Thus,

$$W = mg = (1.00\text{ lbm})(32.174\text{ ft/s}^2)\left(\frac{1\text{ lbf}}{32.174\text{ lbm·ft/s}^2}\right) = \mathbf{1.00\text{ lbf}}$$

Discussion The quantity in large parentheses in this equation is a unity conversion ratio. Mass is the same regardless of its location. However, on some other planet with a different value of gravitational acceleration, the weight of 1 lbm would differ from that calculated here.

FIGURE 1–22
A quirk in the metric system of units.

When you buy a box of breakfast cereal, the printing may say "Net weight: One pound (454 grams)." (See Fig. 1–22.) Technically, this means that the cereal inside the box weighs 1.00 lbf on earth and has a *mass* of 453.6 g (0.4536 kg). Using Newton's second law, the actual weight of the cereal on earth is

$$W = mg = (453.6\text{ g})(9.81\text{ m/s}^2)\left(\frac{1\text{ N}}{1\text{ kg·m/s}^2}\right)\left(\frac{1\text{ kg}}{1000\text{ g}}\right) = 4.49\text{ N}$$

1–6 ▪ PROBLEM-SOLVING TECHNIQUE

The first step in learning any science is to grasp the fundamentals and to gain a sound knowledge of them. The next step is to master the fundamentals by testing this knowledge. This is done by solving significant real-world problems. Solving such problems, especially complicated ones, requires a systematic approach.

By using a step-by-step approach, an engineer can reduce the solution of a complicated problem into the solution of a series of simple problems (Fig. 1–23). When you are solving a problem, we recommend that you use the following steps zealously as applicable. This will help you avoid some of the common pitfalls associated with problem solving.

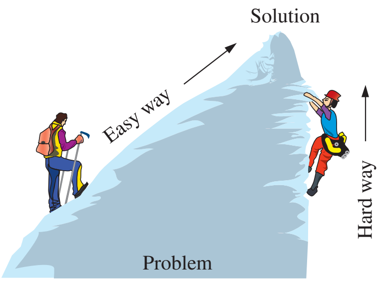


FIGURE 1–23
A step-by-step approach can greatly simplify problem solving.

Step 1: Problem Statement

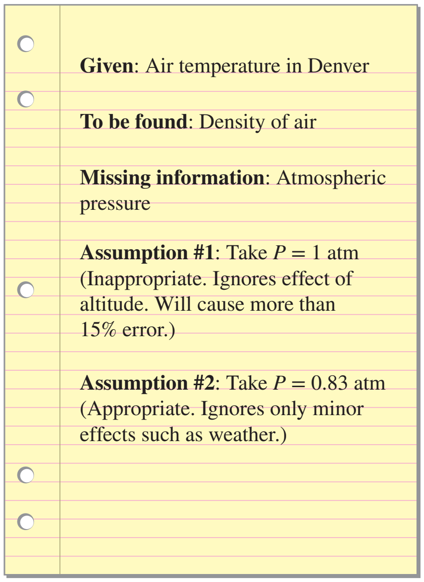
In your own words, briefly state the problem, the key information given, and the quantities to be found. This is to make sure that you understand the problem and the objectives before you try to solve the problem.

Step 2: Schematic

Draw a realistic sketch of the physical system involved, and list the relevant information on the figure. The sketch does not have to be something elaborate, but it should resemble the actual system and show the key features. Indicate any energy and mass interactions with the surroundings. Listing the given information on the sketch helps one to see the entire problem at once. Also, check for properties that remain constant during a process (such as temperature during an isothermal process), and indicate them on the sketch.

Step 3: Assumptions and Approximations

State any appropriate assumptions and approximations made to simplify the problem to make it possible to obtain a solution. Justify the questionable assumptions. Assume reasonable values for missing quantities that are necessary. For example, in the absence of specific data for atmospheric pressure, it can be taken to be 1 atm. However, it should be noted in the analysis that the atmospheric pressure decreases with increasing elevation. For example, it drops to 0.83 atm in Denver (elevation 1610 m) (Fig. 1–24).



Given: Air temperature in Denver

To be found: Density of air

Missing information: Atmospheric pressure

Assumption #1: Take $P = 1$ atm (Inappropriate. Ignores effect of altitude. Will cause more than 15% error.)

Assumption #2: Take $P = 0.83$ atm (Appropriate. Ignores only minor effects such as weather.)

FIGURE 1–24
The assumptions made while solving an engineering problem must be reasonable and justifiable.

Step 4: Physical Laws

Apply all the relevant basic physical laws and principles (such as the conservation of mass), and reduce them to their simplest form by utilizing the assumptions made. However, the region to which a physical law is applied must be clearly identified first. For example, the increase in speed of water flowing through a nozzle is analyzed by applying conservation of mass between the inlet and outlet of the nozzle.

Step 5: Properties

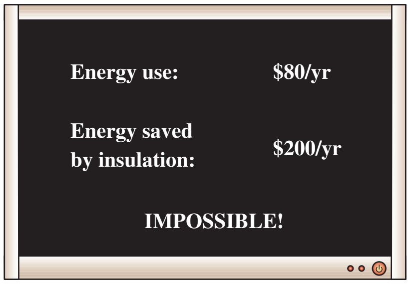
Determine the unknown properties at known states necessary to solve the problem from property relations or tables. List the properties separately, and indicate their source, if applicable.

Step 6: Calculations

Substitute the known quantities into the simplified relations and perform the calculations to determine the unknowns. Pay particular attention to the units and unit cancellations, and remember that a dimensional quantity without a unit is meaningless. Also, don't give a false implication of high precision by copying all the digits from the screen of the calculator—round the results to an appropriate number of significant digits (see the subsection on significant digits at the end of this section).

Step 7: Reasoning, Verification, and Discussion

Check to make sure that the results obtained are reasonable and intuitive, and verify the validity of the questionable assumptions. Repeat the calculations that resulted in unreasonable values. For example, insulating a water heater that uses \$80 worth of natural gas a year cannot result in savings of \$200 a year (Fig. 1–25).



Energy use:	**\$80/yr**
Energy saved by insulation:	**\$200/yr**

IMPOSSIBLE!

FIGURE 1–25
The results obtained from an engineering analysis must be checked for reasonableness.

Also, point out the significance of the results, and discuss their implications. State the conclusions that can be drawn from the results and any recommendations that can be made from them. Emphasize the limitations under which the results are applicable, and caution against any possible misunderstandings and using the results in situations where the underlying assumptions do not apply. For example, if you determined that wrapping a water heater with a $30 insulation jacket will reduce the energy cost by $40 a year, indicate that the insulation will pay for itself from the energy it saves in less than a year. However, also indicate that the analysis does not consider labor costs, and that this will be the case if you install the insulation yourself.

FIGURE 1–26
Neatness and organization are highly valued by employers.

Keep in mind that the solutions you present to your instructors, and any engineering analysis presented to others, is a form of communication. Therefore neatness, organization, completeness, and visual appearance are of utmost importance for maximum effectiveness (Fig. 1–26). Besides, neatness also serves as a great checking tool since it is very easy to spot errors and inconsistencies in neat work. Carelessness and skipping steps to save time often end up costing more time and unnecessary anxiety.

The approach described here is used in the solved example problems without explicitly stating each step, as well as in the Solutions Manual of this text. For some problems, some of the steps may not be applicable or necessary. For example, often it is not practical to list the properties separately. However, we cannot overemphasize the importance of a logical and orderly approach to problem solving. Most difficulties encountered while solving a problem are not due to a lack of knowledge; rather, they are due to a lack of organization. You are strongly encouraged to follow these steps in problem solving until you develop your own approach that works best for you.

Engineering Software Packages

You may be wondering why we are about to undertake an in-depth study of the fundamentals of another engineering science. After all, almost all such problems we are likely to encounter in practice can be solved using one of several sophisticated software packages readily available in the market today. These software packages not only give the desired numerical results but also supply the outputs in colorful graphical form for impressive presentations. It is unthinkable to practice engineering today without using some of these packages. This tremendous computing power available to us at the touch of a button is both a blessing and a curse. It certainly enables engineers to solve problems easily and quickly, but it also opens the door for abuses and misinformation. In the hands of poorly educated people, these software packages are as dangerous as sophisticated, powerful weapons in the hands of poorly trained soldiers.

Thinking that a person who can use the engineering software packages without proper training on fundamentals can practice engineering is like thinking that a person who can use a wrench can work as a car mechanic. If it were true that engineering students do not need fundamental courses because practically everything can be done by computers quickly and easily, then it would also be true that employers would no longer need high-salaried engineers since any person who knows how to use a word-processing program can also learn how to use those software packages. However, the statistics show that the need for engineers is on the rise, not on the decline, despite the availability of these powerful packages.

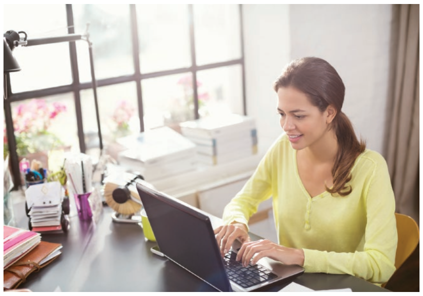


FIGURE 1–27
An excellent word-processing program does not make a person a good writer; it simply makes a good writer a more efficient writer.
Caia Images/Glow Images

We should always remember that all the computing power and the engineering software packages available today are just *tools,* and tools have meaning only in the hands of masters. Having the best word-processing program does not make a person a good writer, but it certainly makes the job of a good writer much easier and makes the writer more productive (Fig. 1–27). Hand calculators did not eliminate the need to teach our children how to add or subtract, and sophisticated medical software packages did not take the place of medical school training. Neither will engineering software packages replace traditional engineering education. They will simply cause a shift in emphasis in the courses from mathematics to physics. That is, more time will be spent in the classroom discussing the physical aspects of the problems in greater detail, and less time on the mechanics of solution procedures.

All these marvelous and powerful tools available today put an extra burden on today's engineers. They must still have a thorough understanding of the fundamentals, develop a "feel" for the physical phenomena, be able to put the data into proper perspective, and make sound engineering judgments, just like their predecessors. However, they must do it much better, and much faster, using more realistic models because of the powerful tools available today. The engineers in the past had to rely on hand calculations, slide rules, and later hand calculators and computers. Today they rely on software packages. The easy access to such power and the possibility of a simple misunderstanding or misinterpretation causing great damage make it more important today than ever to have solid training in the fundamentals of engineering. In this text we make an extra effort to put the emphasis on developing an intuitive and physical understanding of natural phenomena instead of on the mathematical details of solution procedures.

Equation Solvers

You are probably familiar with the equation solving capabilities of spreadsheets such as Microsoft Excel. Despite its simplicity, Excel is commonly used in solving systems of equations in engineering as well as finance. It enables the user to conduct parametric studies, plot the results, and ask "what if" questions. It can also solve simultaneous equations if properly set up. There are also many sophisticated equation solvers that are commonly used in engineering practice, such as the Engineering Equation Solver (EES), which is a program that easily solves systems of linear or nonlinear algebraic or differential equations numerically. It has a large library of built-in thermodynamic property functions as well as mathematical functions, and it allows the user to supply additional property data.

Unlike some software packages, equation solvers do not solve engineering problems; they only solve the equations supplied by the user. Therefore, the user must understand the problem and formulate it by applying any relevant physical laws and relations. Equation solvers save the user considerable time and effort by simply solving the resulting mathematical equations. This makes it possible to attempt significant engineering problems not suitable for hand calculations and to conduct parametric studies quickly and conveniently.

EXAMPLE 1–4 Solving a System of Equations Numerically

The difference of two numbers is 4, and the sum of the squares of these two numbers is equal to the sum of the numbers plus 20. Determine these two numbers.

SOLUTION Relations are given for the difference and the sum of the squares of two numbers. The two numbers are to be determined.

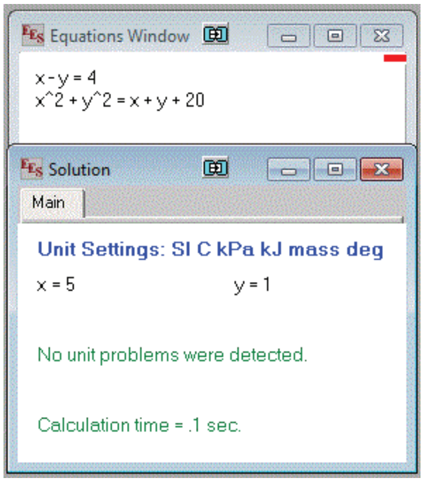
Analysis We first solve the problem using EES. We start the EES program by double-clicking on its icon, open a new file, and type the following on the blank screen that appears:

$$\mathrm{x - y = 4}$$

$$\mathrm{x\hat{}2 + y\hat{}2 = x + y + 20}$$

which is an exact mathematical expression of the problem statement with x and y denoting the unknown numbers. The solution to this system of equations (one linear and one nonlinear) with two unknowns is obtained by a single click on the "calculator" icon on the taskbar. It gives (Fig. 1–28)

$$\mathbf{x = 5} \quad \text{and} \quad \mathbf{y = 1}$$



x-y=4
x^2+y^2=x+y+20

Unit Settings: SI C kPa kJ mass deg

x = 5 y = 1

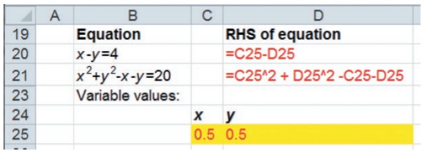
No unit problems were detected.

Calculation time = .1 sec.

FIGURE 1–28
EES screen images for Example 1–4.
Microsoft Corporation

We now solve the same problem using Excel. Start Excel. File/Options/Add-Ins/Solver Add-In/OK, where the underline means to click on that option and the slash separates each sequential option. Choose a cell for x and a cell for y and enter initial guesses there (we chose cells C25 and D25 and guessed 0.5 and 0.5). We must rewrite the two equations so that no variables are on the right-hand side (RHS): $x - y = 4$ and $x^2 + y^2 - x - y = 20$. Choose a cell for the RHS of each equation and enter the formula there (we chose cells D20 and D21; see the equations in Fig. 1–29*a*). Data/Solver. Set the cell for the RHS of the first equation (D20) as the "Objective" with a value of 4, set the cells for x and y (C25:D25) as those subject to constraints, and set the constraint such that the cell for the RHS of the second equation (D21) must equal 20. Solve/OK. The solution iterates to the correct final values of $x = 5$ and $y = 1$, respectively (Fig. 1–29*b*). *Note:* For better convergence, the precision, number of allowed iterations, etc. can be changed in Data/Solver/Options.

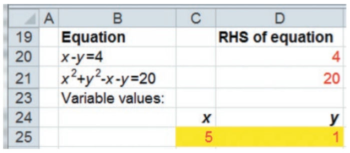
Discussion Note that all we did was formulate the problem as we would on paper; EES or Excel took care of all the mathematical details of the solution. Also note that equations can be linear or nonlinear, and they can be entered in any order with unknowns on either side. Friendly equation solvers such as EES allow the user to concentrate on the physics of the problem without worrying about the mathematical complexities associated with the solution of the resulting system of equations.



	A	B	C	D
19		Equation		RHS of equation
20		x-y=4		=C25-D25
21		x²+y²-x-y=20		=C25^2 + D25^2 -C25-D25
23		Variable values:		
24			x	y
25			0.5	0.5

(*a*)

Microsoft Corporation



	A	B	C	D
19		Equation		RHS of equation
20		x-y=4		4
21		x²+y²-x-y=20		20
23		Variable values:		
24			x	y
25			5	1

(*b*)

Microsoft Corporation

FIGURE 1–29
Excel screen images for Example 1–4. (*a*) Equations, with initial guesses highlighted. (*b*) Final results after using Excel's Solver, with converged values highlighted.

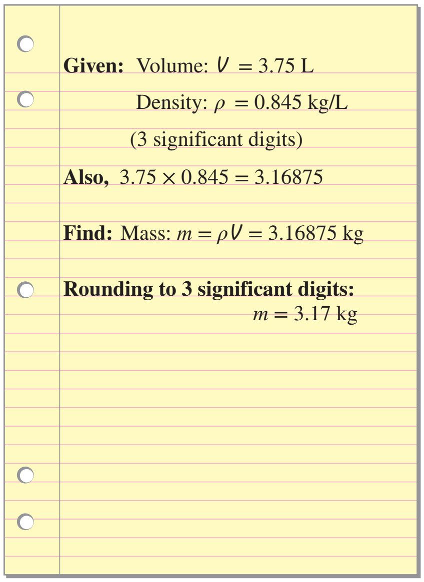
A Remark on Significant Digits

In engineering calculations, the information given is not known to more than a certain number of significant digits, usually three digits. Consequently, the results obtained cannot possibly be accurate to more significant digits. Reporting results in more significant digits falsely implies greater accuracy than exists, and it should be avoided.

For example, consider a 3.75-L container filled with gasoline whose density is 0.845 kg/L, and try to determine its mass. Probably the first thought that comes to your mind is to multiply the volume and density to obtain 3.16875 kg for the mass, which falsely implies that the mass determined is accurate to six significant digits. In reality, however, the mass cannot be more accurate than three significant digits since both the volume and the density are accurate to three significant digits only. Therefore, the result should be rounded to three significant digits, and the mass should be reported to be 3.17 kg instead of what appears in the screen of the calculator. The result 3.16875 kg would be correct only if the volume and density were given to be 3.75000 L and 0.845000 kg/L, respectively. The value 3.75 L implies that we are fairly confident that the volume is accurate within ±0.01 L, and it cannot be 3.74 or 3.76 L. However, the volume can be 3.746, 3.750, 3.753, etc., since they all round to 3.75 L (Fig. 1–30). It is more appropriate to retain all the digits during intermediate calculations and to do the rounding in the final step since this is what a computer will normally do.

When solving problems, we will assume the given information to be accurate to at least three significant digits. Therefore, if the length of a pipe is given to be 40 m, we will assume it to be 40.0 m in order to justify using three significant digits in the final results. You should also keep in mind that all experimentally determined values are subject to measurement errors, and such errors will reflect in the results obtained. For example, if the density of a substance has an uncertainty of 2 percent, then the mass determined using this density value will also have an uncertainty of 2 percent.

You should also be aware that we sometimes knowingly introduce small errors in order to avoid the trouble of searching for more accurate data. For example, when dealing with liquid water, we just use the value of 1000 kg/m^3 for density, which is the density value of pure water at 0°C. Using this value at 75°C will result in an error of 2.5 percent since the density at this temperature is 975 kg/m^3. The minerals and impurities in the water will introduce additional error. This



Given: Volume: $V = 3.75$ L
Density: $\rho = 0.845$ kg/L
(3 significant digits)
Also, $3.75 \times 0.845 = 3.16875$
Find: Mass: $m = \rho V = 3.16875$ kg
Rounding to 3 significant digits:
$m = 3.17$ kg

FIGURE 1–30
A result with more significant digits than that of given data falsely implies more precision.

being the case, you should have no reservation in rounding the final results to a reasonable number of significant digits. Besides, having a few percent uncertainty in the results of engineering analysis is usually the norm, not the exception.

SUMMARY

In this chapter, some basic concepts of thermal-fluid sciences are introduced and discussed. The physical sciences that deal with energy and the transfer, transport, and conversion of energy are referred to as *thermal-fluid sciences,* and they are studied under the subcategories of thermodynamics, heat transfer, and fluid mechanics.

Thermodynamics is the science that primarily deals with energy. The *first law of thermodynamics* is simply an expression of the conservation of energy principle, and it asserts that *energy* is a thermodynamic property. The *second law of thermodynamics* asserts that energy has *quality* as well as *quantity,* and actual processes occur in the direction of decreasing quality of energy. Determining the rates of heat transfer to or from a system and thus the times of cooling or heating, as well as the variation of the temperature, is the subject of *heat transfer.* The basic requirement for heat transfer is the presence of a *temperature difference.* A substance in the liquid or gas phase is referred to as a *fluid. Fluid mechanics* is the science that deals with the behavior of fluids at rest (*fluid statics*) or in motion (*fluid dynamics*), and the interaction of fluids with solids or other fluids at the boundaries.

In engineering calculations, it is important to pay particular attention to the units of the quantities to avoid errors caused by inconsistent units, and to follow a systematic approach. It is also important to recognize that the information given is not known to more than a certain number of significant digits, and the results obtained cannot possibly be accurate to more significant digits.

When solving a problem, it is recommended that a step-by-step approach be used. Such an approach involves stating the problem, drawing a schematic, making appropriate assumptions, applying the physical laws, listing the relevant properties, making the necessary calculations, and making sure that the results are reasonable.

REFERENCES AND SUGGESTED READINGS

1. American Society for Testing and Materials. *Standards for Metric Practice*. ASTM E 380–79, January 1980.
2. Y. A. Çengel and A. J. Ghajar. *Heat and Mass Transfer: Fundamentals and Applications*. 6th ed. New York: McGraw-Hill, 2020.
3. Y. A. Çengel, M. A. Boles and M. Kanoğlu. *Thermodynamics. An Engineering Approach*. 9th ed. New York: McGraw-Hill, 2019.
4. Y. A. Çengel and John M. Cimbala. *Fluid Mechanics: Fundamentals and Applications*. 4th ed. New York: McGraw-Hill, 2018.

PROBLEMS*

Thermodynamics, Heat Transfer, and Fluid Mechanics

1–1C Why does a bicyclist pick up speed on a downhill road even when he is not pedaling? Does this violate the conservation of energy principle?

1–2C An office worker claims that a cup of cold coffee on his table warmed up to 80°C by picking up energy from the surrounding air, which is at 25°C. Is there any truth to his claim? Does this process violate any thermodynamic laws?

1–3C One of the most amusing things a person can experience is that in certain parts of the world, a still car in neutral can go uphill when its brakes are released. Such occurrences are even broadcast on TV. Can this really happen, or is it bad eyesight? How can you verify if a road is really uphill or downhill?

1–4C How does the science of heat transfer differ from the science of thermodynamics?

1–5C What is the driving force for (*a*) heat transfer, (*b*) electric current, and (*c*) fluid flow?

1–6C Why is heat transfer a nonequilibrium phenomenon?

1–7C Can there be any heat transfer between two bodies that are at the same temperature but at different pressures?

1–8C Define stress, normal stress, shear stress, and pressure.

Mass, Force, and Units

1–9C Explain why the light-year has the dimension of length.

1–10C What is the difference between pound-mass and pound-force?

1–11C What is the net force acting on a car cruising at a constant velocity of 70 km/h (*a*) on a level road and (*b*) on an uphill road?

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* Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the icon 💻 are comprehensive in nature and are intended to be solved with appropriate software.

1–12 What is the weight, in N, of an object with a mass of 200 kg at a location where $g = 9.6 \text{ m/s}^2$?

1–13E If the mass of an object is 10 lbm, what is its weight, in lbf, at a location where $g = 32.0 \text{ ft/s}^2$?

1–14 The acceleration of high-speed aircraft is sometimes expressed in g's (in multiples of the standard acceleration of gravity). Determine the upward force, in N, that a 90-kg man would experience in an aircraft whose acceleration is 6 g's.

1–15 The value of the gravitational acceleration g decreases with elevation from 9.807 m/s^2 at sea level to 9.767 m/s^2 at an altitude of 13,000 m, where large passenger planes cruise. Determine the percent reduction in the weight of an airplane cruising at 13,000 m relative to its weight at sea level.

1–16 A 3-kg plastic tank that has a volume of 0.2 m^3 is filled with liquid water. Assuming the density of water is 1000 kg/m^3, determine the weight of the combined system.

1–17 A 2-kg rock is thrown upward with a force of 200 N at a location where the local gravitational acceleration is 9.79 m/s^2. Determine the acceleration of the rock, in m/s^2.

1–18 Solve Prob. 1–17 using appropriate software. Print out the entire solution, including the numerical results with proper units.

1–19 A 4-kW resistance heater in a water heater runs for 3 hours to raise the water temperature to the desired level. Determine the amount of electric energy used in both kWh and kJ.

1–20E A 150-lbm astronaut took his bathroom scale (a spring scale) and a beam scale (compares masses) to the moon, where the local gravity is $g = 5.48 \text{ ft/s}^2$. Determine how much he will weigh (*a*) on the spring scale and (*b*) on the beam scale. *Answers:* (*a*) 25.5 lbf, (*b*) 150 lbf

1–21 The gas tank of a car is filled with a nozzle that discharges gasoline at a constant flow rate. Based on unit considerations of quantities, obtain a relation for the filling time in terms of the volume V of the tank (in L) and the discharge rate of gasoline $\dot{V}$ (in L/s).

Solving Engineering Problems and Equation Solvers

1–22C What is the value of the engineering software packages in (*a*) engineering education and (*b*) engineering practice?

1–23 Determine a positive real root of this equation using appropriate software:

$$2x^3 - 10x^{0.5} - 3x = -3$$

1–24 Solve this system of two equations with two unknowns using appropriate software:

$$x^3 - y^2 = 5.9$$
$$3xy + y = 3.5$$

1–25 Solve this system of three equations with three unknowns using appropriate software:

$$2x - y + z = 7$$
$$3x^2 + 3y = z + 3$$
$$xy + 2z = 4$$

1–26 Solve this system of three equations with three unknowns using appropriate software:

$$x^2y - z = 1$$
$$x - 3y^{0.5} + xz = -2$$
$$x + y - z = 2$$

Review Problems

1–27E The reactive force developed by a jet engine to push an airplane forward is called thrust, and the thrust developed by the engine of a Boeing 777 is about 85,000 lbf. Express this thrust in N and kgf.

1–28 The weight of bodies may change somewhat from one location to another as a result of the variation of the gravitational acceleration g with elevation. Accounting for this variation using the relation $g = a - bz$ where $a = 9.807 \text{ m/s}^2$ and $b = 3.32 \times 10^{-6} \text{ s}^{-2}$, determine the weight of an 80-kg person at sea level ($z = 0$), in Denver ($z = 1610$ m), and on the top of Mount Everest ($z = 8848$ m).

1–29E A man goes to a traditional market to buy a steak for dinner. He finds a 12-oz steak (1 lbm = 16 oz) for \$5.50. He then goes to the adjacent international market and finds a 300-g steak of identical quality for \$5.20. Which steak is the better buy?

1–30E What is the weight of a 1-kg substance in N, kN, kg·m/s^2, kgf, lbm·ft/s^2, and lbf?

1–31 Consider the flow of air through a wind turbine whose blades sweep an area of diameter D (in m). The average air velocity through the swept area is V (in m/s). On the bases of the units of the quantities involved, show that the mass flow rate of air (in kg/s) through the swept area is proportional to air density, the wind velocity, and the square of the diameter of the swept area.

1–32 The drag force exerted on a car by air depends on a dimensionless drag coefficient, the density of air, the car velocity, and the frontal area of the car. That is, $F_D = F_D(C_{\text{drag}}, A_{\text{front}}, \rho, V)$. Based on unit considerations alone, obtain a relation for the drag force.

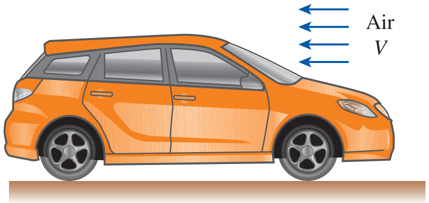


FIGURE P1–32

Design and Essay Problems

1–33 Write an essay on the various mass- and volume-measurement devices used throughout history. Also, explain the development of the modern units for mass and volume.

PART

1

THERMODYNAMICS

CHAPTER

2

BASIC CONCEPTS OF THERMODYNAMICS

Every science has a unique vocabulary associated with it, and thermodynamics is no exception. Precise definition of basic concepts forms a sound foundation for the development of a science and prevents possible misunderstandings. We start this chapter with a discussion of some basic concepts such as *system, state, state postulate, equilibrium, process*, and *cycle*. We discuss intensive and extensive properties of a system and define density, specific gravity, and specific weight. We also discuss *temperature* and *temperature scales*. We then present *pressure,* which is the normal force exerted by a fluid per unit area, and we discuss *absolute* and *gage* pressures, the variation of pressure with depth, and pressure measurement devices, such as manometers and barometers. Careful study of these concepts is essential for a good understanding of the topics in the following chapters.

OBJECTIVES

The objectives of this chapter are to:

- Identify the unique vocabulary associated with thermodynamics through the precise definition of basic concepts to form a sound foundation for the development of the principles of thermodynamics.
- Explain the basic concepts of thermodynamics such as system, state, state postulate, equilibrium, process, and cycle.
- Discuss properties of a system and define density, specific gravity, and specific weight.
- Review concepts of temperature, temperature scales, pressure, and absolute and gage pressure.

2–1 ■ SYSTEMS AND CONTROL VOLUMES

A system is defined as a *quantity of matter or a region in space chosen for study.* The mass or region outside the system is called the **surroundings**. The real or imaginary surface that separates the system from its surroundings is called the **boundary** (Fig. 2–1). The boundary of a system can be *fixed* or *movable.* Note that the boundary is the contact surface shared by both the system and the surroundings. Mathematically speaking, the boundary has zero thickness, and thus it can neither contain any mass nor occupy any volume in space.

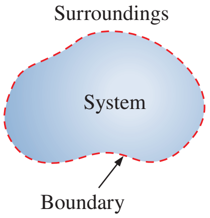


FIGURE 2–1
System, surroundings, and boundary.

Systems may be considered to be *closed* or *open,* depending on whether a fixed mass or a fixed volume in space is chosen for study. A **closed system** (also known as a **control mass** or just *system* when the context makes it clear) consists of a fixed amount of mass, and no mass can cross its boundary. That is, no mass can enter or leave a closed system, as shown in Fig. 2–2. But energy, in the form of heat or work, can cross the boundary; and the volume of a closed system does not have to be fixed. If, as a special case, even energy is not allowed to cross the boundary, that system is called an **isolated system**.

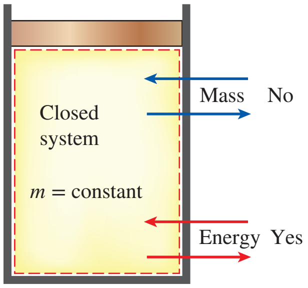


FIGURE 2–2
Mass cannot cross the boundaries of a closed system, but energy can.

Consider the piston-cylinder device shown in Fig. 2–3. Let us say that we would like to find out what happens to the enclosed gas when it is heated. Since we are focusing our attention on the gas, it is our system. The inner surfaces of the piston and the cylinder form the boundary, and since no mass is crossing this boundary, it is a closed system. Notice that energy may cross the boundary, and part of the boundary (the inner surface of the piston, in this case) may move. Everything outside the gas, including the piston and the cylinder, is the surroundings.

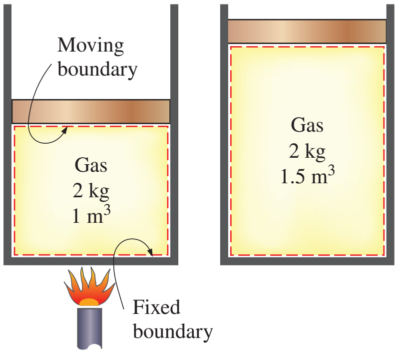
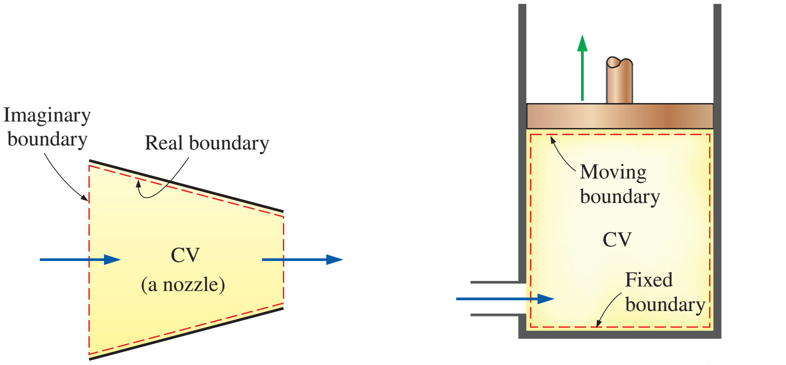


FIGURE 2–3
A closed system with a moving boundary.

An **open system**, or a **control volume**, as it is often called, is a properly selected region in space. It usually encloses a device that involves mass flow such as a compressor, turbine, or nozzle. Flow through these devices is best studied by selecting the region within the device as the control volume. Both mass and energy can cross the boundary of a control volume.

A large number of engineering problems involve mass flow in and out of a system and, therefore, are modeled as *control volumes.* A water heater, a car radiator, a turbine, and a compressor all involve mass flow and should be analyzed as control volumes (open systems) instead of as control masses (closed systems). In general, *any arbitrary region in space* can be selected as a control volume. There are no concrete rules for the selection of control volumes, but the proper choice certainly makes the analysis much easier. If we were to analyze the flow of air through a nozzle, for example, a good choice for the control volume would be the region within the nozzle.

The boundaries of a control volume are called a *control surface,* and they can be real or imaginary. In the case of a nozzle, the inner surface of the nozzle forms the real part of the boundary, and the entrance and exit areas form the imaginary part, since there are no physical surfaces there (Fig. 2–4*a*).



(*a*) A control volume (CV) with real and imaginary boundaries

(*b*) A control volume (CV) with fixed and moving boundaries as well as real and imaginary boundaries

FIGURE 2–4
A control volume can involve fixed, moving, real, and imaginary boundaries.

A control volume can be fixed in size and shape, as in the case of a nozzle, or it may involve a moving boundary, as shown in Fig. 2–4*b*. Most control volumes, however, have fixed boundaries and thus do not involve any moving boundaries. A control volume can also involve heat and work interactions just as a closed system, in addition to mass interaction.

As an example of an open system, consider the water heater shown in Fig. 2–5. Let us say that we would like to determine how much heat we must transfer to the water in the tank in order to supply a steady stream of hot water. Since hot water will leave the tank and be replaced by cold water, it is not convenient to choose a fixed mass as our system for the analysis. Instead, we can concentrate our attention on the volume formed by the interior surfaces of the tank and consider the hot and cold water streams as mass leaving and entering the control volume. The interior surfaces of the tank form the control surface for this case, and mass is crossing the control surface at two locations.

In an engineering analysis, the system under study *must* be defined carefully. In most cases, the system investigated is quite simple and obvious, and defining the system may seem like a tedious and unnecessary task. In other cases, however, the system under study may be rather involved, and a proper choice of the system may greatly simplify the analysis.

FIGURE 2–5
An open system (a control volume) with one inlet and one exit.
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2–2 ▪ PROPERTIES OF A SYSTEM

Any characteristic of a system is called a **property**. Some familiar properties are pressure P, temperature T, volume V, and mass m. The list can be extended to include less familiar ones such as viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistivity, and even velocity and elevation.

Properties are considered to be either *intensive* or *extensive*. **Intensive properties** are those that are independent of the mass of a system, such as temperature, pressure, and density. **Extensive properties** are those whose values depend on the size—or extent—of the system. Total mass, total volume, and total momentum are some examples of extensive properties. An easy way to determine whether a property is intensive or extensive is to divide the system into two equal parts with an imaginary partition, as shown in Fig. 2–6. Each part will have the same value of intensive properties as the original system, but half the value of the extensive properties.

Generally, uppercase letters are used to denote extensive properties (with mass m being a major exception), and lowercase letters are used for intensive properties (with pressure P and temperature T being the obvious exceptions).

Extensive properties per unit mass are called **specific properties**. Some examples of specific properties are specific volume ($v = V/m$) and specific total energy ($e = E/m$).

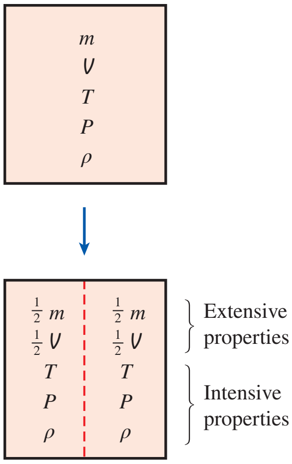


FIGURE 2–6
Criterion to differentiate intensive and extensive properties.

Continuum

Matter is made up of atoms that are widely spaced in the gas phase. Yet it is very convenient to disregard the atomic nature of a substance and view it as a continuous, homogeneous matter with no holes, that is, a **continuum**. The continuum idealization allows us to treat properties as point functions and to assume the properties vary continually in space with no jump discontinuities. This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules. This is the case in practically all problems, except some specialized ones. The continuum idealization is implicit in many statements we make, such as "the density of water in a glass is the same at any point."

To have a sense of the distance involved at the molecular level, consider a container filled with oxygen at atmospheric conditions. The diameter of the oxygen molecule is about 3×10^{-10} m and its mass is 5.3×10^{-26} kg. Also, the *mean free path* of oxygen at 1 atm pressure and 20°C is 6.3×10^{-8} m. That is, an oxygen molecule travels, on average, a distance of 6.3×10^{-8} m (about 200 times its diameter) before it collides with another molecule.

Also, there are about 3×10^{16} molecules of oxygen in the tiny volume of 1 mm^3 at 1 atm pressure and 20°C (Fig. 2–7). The continuum model is applicable as long as the characteristic length of the system (such as its diameter) is much larger than the mean free path of the molecules. At very high vacuums or very high elevations, the mean free path may become large (for example, it is about 0.1 m for atmospheric air at an elevation of 100 km). For such cases the **rarefied gas flow theory** should be used, and the impact of individual molecules should be considered. In this text we will limit our consideration to substances that can each be modeled as a continuum.

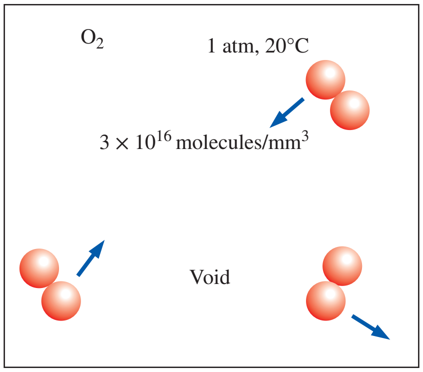


FIGURE 2–7
Despite the relatively large gaps between molecules, a gas can usually be treated as a continuum because of the very large number of molecules even in an extremely small volume.

2–3 ▪ DENSITY AND SPECIFIC GRAVITY

Density is defined as *mass per unit volume* (Fig. 2–8).

Density:
$$\rho = \frac{m}{V} \quad \text{(kg/m}^3\text{)} \tag{2–1}$$

The reciprocal of density is the **specific volume** υ, which is defined as *volume per unit mass.* That is,

$$\upsilon = \frac{V}{m} = \frac{1}{\rho} \tag{2–2}$$

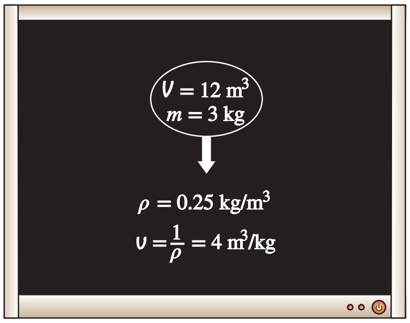


FIGURE 2–8
Density is mass per unit volume; specific volume is volume per unit mass.

For a differential volume element of mass δm and volume δV, density can be expressed as $\rho = \delta m/\delta V$.

The density of a substance, in general, depends on temperature and pressure. The density of most gases is proportional to pressure and inversely proportional to temperature. Liquids and solids, on the other hand, are essentially incompressible substances, and the variation of their density with pressure is usually negligible. At 20°C, for example, the density of water changes from 998 kg/m^3 at 1 atm to 1003 kg/m^3 at 100 atm, a change of just 0.5 percent. The density of liquids and solids depends more strongly on temperature than it does on pressure. At 1 atm, for example, the density of water changes from 998 kg/m^3 at 20°C to 975 kg/m^3 at 75°C, a change of 2.3 percent, which can still be neglected in many engineering analyses.

Sometimes the density of a substance is given relative to the density of a well-known substance. Then it is called **specific gravity**, or **relative density**, and is defined as *the ratio of the density of a substance to the density of some standard substance at a specified temperature* (usually water at 4°C, for which $\rho_{H_2O} = 1000$ kg/m^3). That is,

Specific gravity:
$$\text{SG} = \frac{\rho}{\rho_{H_2O}} \tag{2–3}$$

Note that the specific gravity of a substance is a dimensionless quantity. However, in SI units, the numerical value of the specific gravity of a substance is exactly equal to its density in g/cm^3 or kg/L (or 0.001 times the density in kg/m^3) since the density of water at 4°C is 1 g/cm^3 = 1 kg/L = 1000 kg/m^3. The specific gravity of mercury at 0°C, for example, is 13.6. Therefore, its density at 0°C is 13.6 g/cm^3 = 13.6 kg/L = 13,600 kg/m^3. The specific gravities of some substances at 0°C are given in Table 2–1. Note that substances with specific gravities less than 1 are lighter than water, and thus they would float on water.

TABLE 2–1

Specific gravities of some substances at 0°C

Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl alcohol	0.79
Mercury	13.6
Wood	0.3–0.9
Gold	19.2
Bones	1.7–2.0
Ice	0.92
Air (at 1 atm)	0.0013

The weight of a unit volume of a substance is called **specific weight** and is expressed as

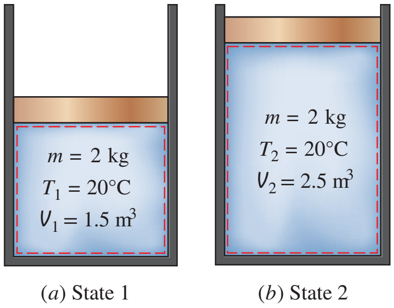
Specific weight:
$$\gamma_s = \rho g \quad \text{(N/m}^3\text{)} \tag{2–4}$$

where g is the gravitational acceleration.

The densities of liquids are essentially constant, and thus they can often be approximated as being incompressible substances during most processes without sacrificing much in accuracy.

2–4 ▪ STATE AND EQUILIBRIUM

Consider a system not undergoing any change. At this point, all the properties can be measured or calculated throughout the entire system, which gives us a set of properties that completely describes the condition, or the **state**, of the system. At a given state, all the properties of a system have fixed values. If the value of even one property changes, the state will change to a different one. In Fig. 2–9 a system is shown at two different states.

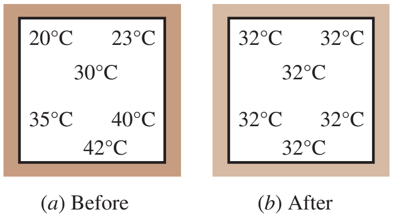


(*a*) State 1 (*b*) State 2

FIGURE 2–9
A system at two different states.

Thermodynamics deals with *equilibrium* states. The word **equilibrium** implies a state of balance. In an equilibrium state there are no unbalanced potentials (or driving forces) within the system. A system in equilibrium experiences no changes when it is isolated from its surroundings.

There are many types of equilibrium, and a system is not in thermodynamic equilibrium unless the conditions of all the relevant types of equilibrium are satisfied. For example, a system is in **thermal equilibrium** if the temperature is the same throughout the entire system, as shown in Fig. 2–10. That is, the system involves no temperature differential, which is the driving force for heat flow. **Mechanical equilibrium** is related to pressure, and a system is in mechanical equilibrium if there is no change in pressure at any point of the system with time. However, the pressure may vary within the system with elevation as a result of gravitational effects.



(*a*) Before (*b*) After

FIGURE 2–10
A closed system reaching thermal equilibrium.

For example, the higher pressure at a bottom layer is balanced by the extra weight it must carry, and, therefore, there is no imbalance of forces. The variation of pressure as a result of gravity in most thermodynamic systems is relatively small and usually disregarded. If a system involves two phases, it is in **phase equilibrium** when the mass of each phase reaches an equilibrium level and stays there. Finally, a system is in **chemical equilibrium** if its chemical composition does not change with time, that is, no chemical reactions occur. A system will not be in equilibrium unless all the relevant equilibrium criteria are satisfied.

The State Postulate

As noted earlier, the state of a system is described by its properties. But we know from experience that we do not need to specify all the properties in order to fix a state. Once a sufficient number of properties are specified, the rest of the properties assume certain values automatically. That is, specifying a certain number of properties is sufficient to fix a state. The number of properties required to fix the state of a system is given by the **state postulate**:

> The state of a simple compressible system is completely specified by two independent, intensive properties.

A system is called a **simple compressible system** in the absence of electrical, magnetic, gravitational, motion, and surface tension effects. These effects are due to external force fields and are negligible for most engineering problems. Otherwise, an additional property needs to be specified for each effect that is significant. If the gravitational effects are to be considered, for example, the elevation z needs to be specified in addition to the two properties necessary to fix the state.

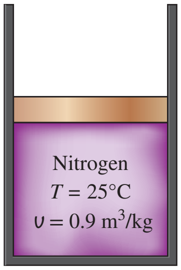


FIGURE 2–11
The state of nitrogen is fixed by two independent, intensive properties.

The state postulate requires that the two properties specified be independent to fix the state. Two properties are **independent** if one property can be varied while the other one is held constant. Temperature and specific volume, for example, are always independent properties, and together they can fix the state of a simple compressible system (Fig. 2–11). Temperature and pressure, however, are independent properties for single-phase systems, but are dependent properties for multiphase systems. At sea level (P = 1 atm), water boils at 100°C, but on a mountaintop where the pressure is lower, water boils at a lower temperature. That is, $T = f(P)$ during a phase-change process; thus, temperature and pressure are not sufficient to fix the state of a two-phase system. Phase-change processes are discussed in detail in Chap. 4.

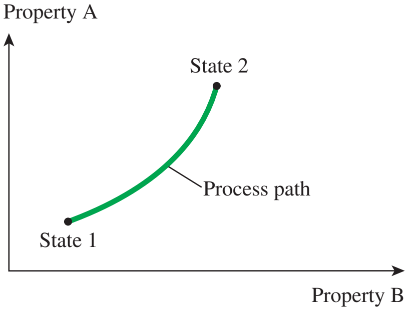
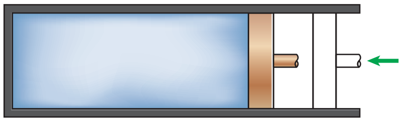


FIGURE 2–12
A process between states 1 and 2 and the process path.

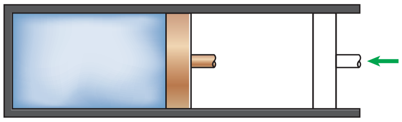
2–5 ▪ PROCESSES AND CYCLES

Any change that a system undergoes from one equilibrium state to another is called a **process**, and the series of states through which a system passes during a process is called the **path** of the process (Fig. 2–12). To describe a process completely, one should specify the initial and final states of the process, as well as the path it follows, and the interactions with the surroundings.

When a process proceeds in such a manner that the system remains infinitesimally close to an equilibrium state at all times, it is called a **quasi-static**, or **quasi-equilibrium**, **process**. A quasi-equilibrium process can be viewed as a sufficiently slow process that allows the system to adjust itself internally so that properties in one part of the system do not change any faster than those at other parts.



(*a*) Slow compression
(quasi-equilibrium)



(*b*) Very fast compression
(nonquasi-equilibrium)

FIGURE 2–13
Quasi-equilibrium and nonquasi-equilibrium compression processes.

This is illustrated in Fig. 2–13. When a gas in a piston-cylinder device is compressed suddenly, the molecules near the face of the piston will not have enough time to escape, and they will have to pile up in a small region in front of the piston, thus creating a high-pressure region there. Because of this pressure difference, the system can no longer be said to be in equilibrium, and this makes the entire process nonquasi-equilibrium. However, if the piston is moved slowly, the molecules will have sufficient time to redistribute, and there will not be a molecule pileup in front of the piston. As a result, the pressure inside the cylinder will always be nearly uniform and will rise at the same rate at all locations. Since equilibrium is maintained at all times, this is a quasi-equilibrium process.

It should be pointed out that a quasi-equilibrium process is an idealized process and is not a true representation of an actual process. But many actual processes closely approximate it, and they can be modeled as quasi-equilibrium with negligible error. Engineers are interested in quasi-equilibrium processes for two reasons. First, they are easy to analyze; second, work-producing devices deliver the most work when they operate on quasi-equilibrium processes. Therefore, quasi-equilibrium processes serve as standards to which actual processes can be compared.

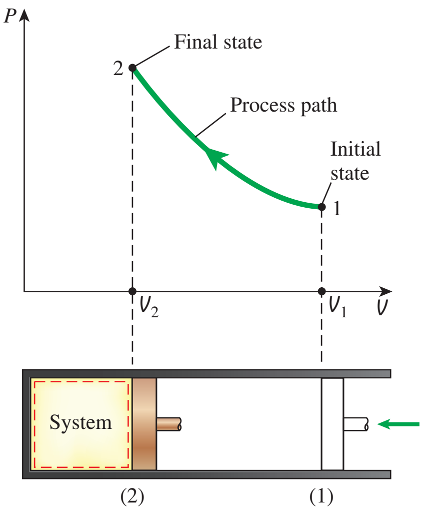


FIGURE 2–14
The P-V diagram of a compression process.

Process diagrams plotted by employing thermodynamic properties as coordinates are very useful in visualizing the processes. Some common properties that are used as coordinates are temperature T, pressure P, and volume V (or specific volume v). Fig. 2–14 shows the P-V diagram of a compression process of a gas.

Note that the process path indicates a series of equilibrium states through which the system passes during a process and has significance for quasi-equilibrium processes only. For nonquasi-equilibrium processes, we cannot characterize the entire system by a single state, and thus we cannot speak of a process path for a system as a whole. A nonquasi-equilibrium process is denoted by a dashed line between the initial and final states instead of a solid line.

The prefix *iso-* is often used to designate a process for which a particular property remains constant. An **isothermal process**, for example, is a process during which the temperature T remains constant; an **isobaric process** is a process during which the pressure P remains constant; and an **isochoric** (or **isometric**) **process** is a process during which the specific volume v remains constant.

A system is said to have undergone a **cycle** if it returns to its initial state at the end of the process. That is, for a cycle the initial and final states are identical.

The Steady-Flow Process

The terms *steady* and *uniform* are used often in engineering, and thus it is important to have a clear understanding of their meanings. The term *steady* implies *no change with time.* The opposite of steady is *unsteady,* or *transient.* The term *uniform,* however, implies *no change with location* over a specified region. These meanings are consistent with their everyday use (steady girlfriend, uniform properties, etc.).

A large number of engineering devices operate for long periods of time under the same conditions, and they are classified as *steady-flow devices.* Processes involving such devices can be represented reasonably well by a somewhat idealized process, called the **steady-flow process**, which can be defined as a *process during which a fluid flows through a control volume steadily* (Fig. 2–15). That is, the fluid properties can change from point to point within the control volume, but at any fixed point they remain the same during the entire process. Therefore, the volume V, the mass m, and the total energy content E of the control volume remain constant during a steady-flow process (Fig. 2–16).

Steady-flow conditions can be closely approximated by devices that are intended for continuous operation such as turbines, pumps, boilers, condensers, and heat exchangers or power plants or refrigeration systems. Some cyclic devices, such as reciprocating engines or compressors, do not satisfy any of the conditions stated above since the flow at the inlets and the exits will be pulsating and not steady. However, the fluid properties vary with time in a periodic manner, and the flow through these devices can still be analyzed as a steady-flow process by using time-averaged values for the properties.

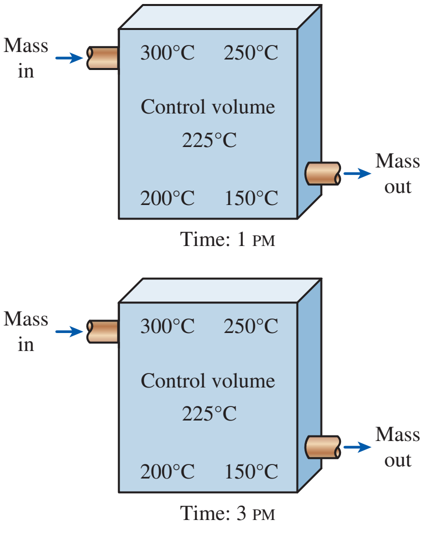


FIGURE 2–15
During a steady-flow process, fluid properties within the control volume may change with position but not with time.

2–6 ▪ TEMPERATURE AND THE ZEROTH LAW OF THERMODYNAMICS

Although we are familiar with temperature as a measure of "hotness" or "coldness," it is not easy to give an exact definition for it. Based on our physiological sensations, we express the level of temperature qualitatively with words like *freezing cold, cold, warm, hot,* and *red-hot.* However, we cannot assign numerical values to temperatures based on our sensations alone. Furthermore, our senses may be misleading. A metal chair, for example, will feel much colder than a wooden one even when both are at the same temperature.

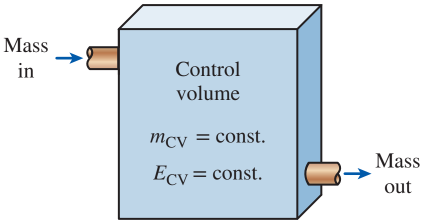


FIGURE 2–16
Under steady-flow conditions, the mass and energy contents of a control volume remain constant.

Fortunately, several properties of materials change with temperature in a *repeatable* and *predictable* way, and this forms the basis for accurate temperature measurement. The commonly used mercury-in-glass thermometer, for example, is based on the expansion of mercury with temperature. Temperature is also measured by using several other temperature-dependent properties.

It is a common experience that a cup of hot coffee left on the table eventually cools off and a cold drink eventually warms up. That is, when a body is brought into contact with another body that is at a different temperature, heat is transferred from the body at higher temperature to the one at lower temperature until both bodies attain the same temperature (Fig. 2–17). At that point, the heat transfer stops, and the two bodies are said to have reached **thermal equilibrium**. The equality of temperature is the only requirement for thermal equilibrium.

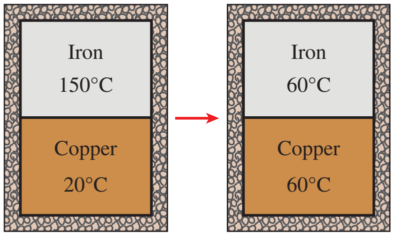


FIGURE 2–17
Two bodies reaching thermal equilibrium after being brought into contact in an isolated enclosure.

The **zeroth law of thermodynamics** states that if two bodies are in thermal equilibrium with a third body, they are also in thermal equilibrium with each other. It may seem silly that such an obvious fact is called one of the basic laws of thermodynamics. However, it cannot be concluded from the other laws of thermodynamics, and it serves as a basis for the validity of temperature measurement. By replacing the third body with a thermometer, the zeroth law can be restated as *two bodies are in thermal equilibrium if both have the same temperature reading even if they are not in contact.*

The zeroth law was first formulated and labeled by R. H. Fowler in 1931. As the name suggests, its value as a fundamental physical principle was recognized more than half a century after the formulation of the first and the second laws of thermodynamics. It was named the zeroth law since it should have preceded the first and the second laws of thermodynamics.

Temperature Scales

Temperature scales enable us to use a common basis for temperature measurements, and several have been introduced throughout history. All temperature scales are based on some easily reproducible states such as the freezing and boiling points of water, which are also called the *ice point* and the *steam point,* respectively. A mixture of ice and water that is in equilibrium with air saturated with vapor at 1 atm pressure is said to be at the ice point, and a mixture of liquid water and water vapor (with no air) in equilibrium at 1 atm pressure is said to be at the steam point.

The temperature scales used in the SI and in the English system today are the **Celsius scale** (formerly called the *centigrade scale;* in 1948 it was renamed after the Swedish astronomer A. Celsius, 1702–1744, who devised it) and the **Fahrenheit scale** (named after the German instrument maker G. Fahrenheit, 1686–1736), respectively. On the Celsius scale, the ice and steam points were originally assigned the values of 0 and 100°C, respectively. The corresponding values on the Fahrenheit scale are 32 and 212°F. These are often referred to as *two-point scales* since temperature values are assigned at two different points.

In thermodynamics, it is very desirable to have a temperature scale that is independent of the properties of any substance or substances. Such a temperature scale is called a **thermodynamic temperature scale**, which is developed later in conjunction with the second law of thermodynamics. The thermodynamic temperature scale in the SI is the **Kelvin scale**, named after Lord Kelvin (1824–1907). The temperature unit on this scale is the **kelvin**, which is designated by K (not °K; the degree symbol was officially dropped from kelvin in 1967). The lowest temperature on the Kelvin scale is absolute zero, or 0 K. Then it follows that only one nonzero reference point needs to be assigned to establish the slope of this linear scale. Using nonconventional refrigeration techniques, scientists have approached absolute zero kelvin (they achieved 0.000000002 K in 1989).

The thermodynamic temperature scale in the English system is the **Rankine scale**, named after William Rankine (1820–1872). The temperature unit on this scale is the **rankine**, which is designated by R.

A temperature scale that turns out to be nearly identical to the Kelvin scale is the **ideal-gas temperature scale**. The temperatures on this scale are measured using a **constant-volume gas thermometer**, which is basically a rigid vessel filled with a gas, usually hydrogen or helium, at low pressure. This thermometer is based on the principle that *at low pressures, the temperature of a gas is proportional to its pressure at constant volume.* That is, the temperature of a gas of fixed volume varies *linearly* with pressure at sufficiently low pressures. Then the relationship between the temperature and the pressure of the gas in the vessel can be expressed as

$$T = a + bP \tag{2–5}$$

where the values of the constants a and b for a gas thermometer are determined experimentally. Once a and b are known, the temperature of a medium can be calculated from this relation by immersing the rigid vessel of the gas thermometer into the medium and measuring the gas pressure when thermal equilibrium is established between the medium and the gas in the vessel whose volume is held constant.

An ideal-gas temperature scale can be developed by measuring the pressures of the gas in the vessel at two reproducible points (such as the ice and the steam points) and assigning suitable values to temperatures at those two points. Considering that only one straight line passes through two fixed points on a plane, these two measurements are sufficient to determine the constants a and b in Eq. 2–5. Then the unknown temperature T of a medium corresponding to a pressure reading P can be determined from that equation by a simple calculation. The values of the constants will be different for each thermometer, depending on the type and the amount of the gas in the vessel, and the temperature values assigned at the two reference points. If the ice and steam points are assigned the values 0°C and 100°C, respectively, then the gas temperature scale will be identical to the Celsius scale. In this case the value of the constant a (which corresponds to an absolute pressure of zero) is determined to be –273.15°C regardless of the type and the amount of the gas in the vessel of the gas thermometer. That is, on a P-T diagram, all the straight lines passing through the data points in this case will intersect the temperature axis at –273.15°C when extrapolated, as shown in Fig. 2–18. This is the lowest temperature that can be obtained by a gas thermometer, and thus we can obtain an *absolute gas temperature scale* by assigning a value of zero to the constant a in Eq. 2–5. In that case, Eq. 2–5 reduces to $T = bP$, and thus we need to specify the temperature at only *one* point to define an absolute gas temperature scale.

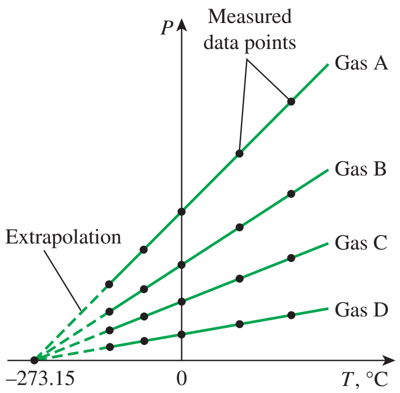


FIGURE 2–18

P versus T plots of the experimental data obtained from a constant-volume gas thermometer using four different gases at different (but low) pressures.

It should be noted that the absolute gas temperature scale is not a thermodynamic temperature scale, since it cannot be used at very low temperatures (due to condensation) and at very high temperatures (due to dissociation and ionization). However, absolute gas temperature is identical to the thermodynamic temperature in the temperature range in which the gas thermometer can be used. Thus, we can view the thermodynamic temperature scale at this point as an absolute gas temperature scale that utilizes an "ideal" or "imaginary" gas that always acts as a low-pressure gas regardless of the temperature. If such a gas thermometer existed, it would read zero kelvin at absolute zero pressure, which corresponds to −273.15°C on the Celsius scale (Fig. 2–19).

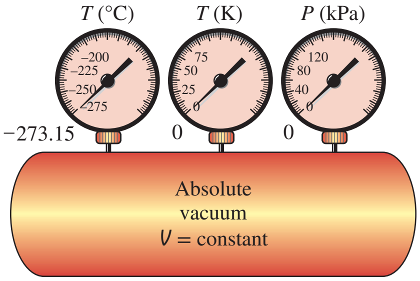


FIGURE 2–19

A constant-volume gas thermometer would read −273.15°C at absolute zero pressure.

The Kelvin scale is related to the Celsius scale by

$$T(\text{K}) = T(^\circ\text{C}) + 273.15 \tag{2–6}$$

The Rankine scale is related to the Fahrenheit scale by

$$T(\text{R}) = T(^\circ\text{F}) + 459.67 \tag{2–7}$$

It is common practice to round the constant in Eq. 2–6 to 273 and that in Eq. 2–7 to 460.

The temperature scales in the two unit systems are related by

$$T(\text{R}) = 1.8T(\text{K}) \tag{2–8}$$

$$T(^\circ\text{F}) = 1.8T(^\circ\text{C}) + 32 \tag{2–9}$$

A comparison of various temperature scales is given in Fig. 2–20.

The reference temperature chosen in the original Kelvin scale was 273.15 K (or 0°C), which is the temperature at which water freezes (or ice melts) and water exists as a solid–liquid mixture in equilibrium under standard atmospheric pressure (the *ice point*). At the Tenth General Conference on Weights and Measures in 1954, the reference point was changed to a much more precisely reproducible point, the *triple point* of water (the state at which all three phases of water coexist in equilibrium), which is assigned the value 273.16 K. The Celsius scale was also redefined at this conference in terms of the ideal-gas temperature scale and a single fixed point, which is again the triple point of water with an assigned value of 0.01°C.

The *International Temperature Scale of 1990,* which supersedes the International Practical Temperature Scale of 1968 (IPTS-68), 1948 (IPTS-48), and 1927

(ITS-27), was adopted by the International Committee of Weights and Measures at its meeting in 1989 at the request of the Eighteenth General Conference on Weights and Measures. The ITS-90 is similar to its predecessors except that it is more refined with updated values of fixed temperatures, has an extended range, and conforms more closely to the thermodynamic temperature scale. On this scale, the unit of thermodynamic temperature T is the kelvin (K), defined as the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. The ice point remains the same at 0°C (273.15 K) in both ITS-90 and IPTS-68, but the steam point is 99.975°C in ITS-90, whereas it was 100.000°C in IPTS-68.

In 2018, at the 26th General Conference on Weights and Measures in Versailles, France, the kilogram, the mole, the ampere, and the kelvin were redefined using fixed values of universal constants. The new official definition of the kelvin is: *The kelvin, symbol K, is the SI unit of thermodynamic temperature; its magnitude is set by fixing the numerical value of the Boltzmann constant to be equal to exactly* 1.380649×10^{-23} *J/K*. The Boltzmann constant relates the amount of thermodynamic energy to its temperature. This new definition provides a universal measurement of temperature that is independent of the properties of any substance. The values for the triple point, the ice point, and the steam point of water as set in ITS-90 are not affected by the new universal definition of the kelvin.

We emphasize that the magnitudes of each division of 1 K and 1°C are identical (Fig. 2–21). Therefore, when we are dealing with temperature differences ΔT, the temperature interval on both scales is the same. Raising the temperature of a substance by 10°C is the same as raising it by 10 K. That is,

$$\Delta T(\text{K}) = \Delta T(^\circ\text{C}) \tag{2–10}$$

$$\Delta T(\text{R}) = \Delta T(^\circ\text{F}) \tag{2–11}$$

Some thermodynamic relations involve the temperature T, and often the question arises whether it is in K or °C. If the relation involves temperature differences (such as $a = b\Delta T$), it makes no difference, and either can be used. However, if the relation involves temperatures only instead of temperature differences (such as $a = bT$), then K must be used. When in doubt, it is always safe to use K because there are virtually no situations in which the use of K is incorrect, but there are many thermodynamic relations that will yield an erroneous result if °C is used.

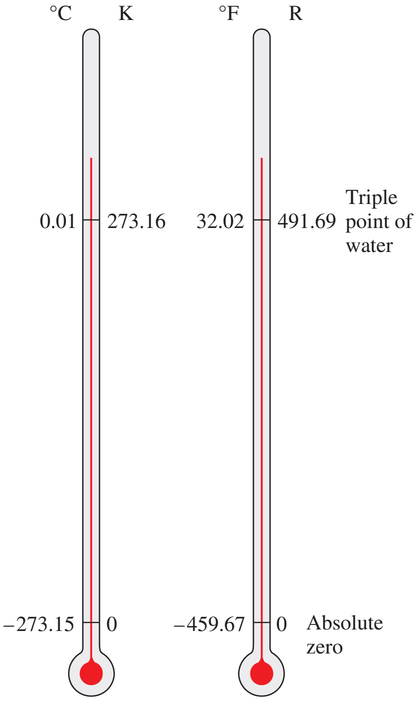


FIGURE 2–20
Comparison of temperature scales.

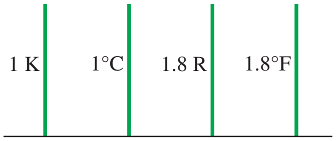


FIGURE 2–21
Comparison of magnitudes of various temperature units.

EXAMPLE 2–1 Expressing Temperatures in Different Units

Humans are most comfortable when the temperature is between 65°F and 75°F. Express these temperature limits in °C. Convert the size of this temperature range (10°F) to K, °C, and R. Is there any difference in the size of this range as measured in relative or absolute units?

SOLUTION A temperature range given in °F is to be converted to °C, and the temperature difference in °F is to be expressed in K, °C, and R.

Analysis The lower and upper limits of comfort in °C are

$$T(^\circ\text{C}) = \frac{T(^\circ\text{F}) - 32}{1.8} = \frac{65 - 32}{1.8} = \mathbf{18.3^\circ C}$$

$$T(^\circ\text{C}) = \frac{T(^\circ\text{F}) - 32}{1.8} = \frac{75 - 32}{1.8} = \mathbf{23.9^\circ C}$$

A temperature change of 10°F in various units is

$$\Delta T(\text{R}) = \Delta T(^\circ\text{F}) = \mathbf{10\ R}$$

$$\Delta T(^\circ\text{C}) = \frac{\Delta T(^\circ\text{F})}{1.8} = \frac{10}{1.8} = \mathbf{5.6^\circ C}$$

$$\Delta T(\text{K}) = \Delta T(^\circ\text{C}) = \mathbf{5.6\ K}$$

Therefore, the units °C and K in the SI system and °F and R in the English system are interchangeable when dealing with temperature differences.

Discussion Students should be careful when making temperature unit conversions. They should identify first whether the conversion involves a temperature value or a temperature change value.

2–7 ▪ PRESSURE

Pressure is defined as *a normal force exerted by a fluid per unit area.* Normally, we speak of pressure when we deal with a gas or a liquid. The counterpart of pressure in solids is *normal stress.* Note, however, that pressure is a scalar quantity while stress is a tensor. Since pressure is defined as force per unit area, it has the unit of newtons per square meter (N/m^2), which is called a **pascal** (Pa). That is,

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

The pressure unit pascal is too small for most pressures encountered in practice. Therefore, its multiples *kilopascal* (1 kPa = 10^3 Pa) and *megapascal* (1 MPa = 10^6 Pa) are commonly used. Three other pressure units commonly used in practice, especially in Europe, are *bar, standard atmosphere,* and *kilogram-force per square centimeter:*

$$\begin{aligned} 1 \text{ bar} &= 10^5 \text{ Pa} = 0.1 \text{ MPa} = 100 \text{ kPa} \\ 1 \text{ atm} &= 101{,}325 \text{ Pa} = 101.325 \text{ kPa} = 1.01325 \text{ bars} \\ 1 \text{ kgf/cm}^2 &= 9.807 \text{ N/cm}^2 = 9.807 \times 10^4 \text{ N/m}^2 = 9.807 \times 10^4 \text{ Pa} \\ &= 0.9807 \text{ bar} \\ &= 0.9679 \text{ atm} \end{aligned}$$

Note the pressure units bar, atm, and kgf/cm^2 are almost equivalent to each other. In the English system, the pressure unit is *pound-force per square inch* (lbf/in^2, or psi), and 1 atm = 14.696 psi. The pressure units kgf/cm^2 and lbf/in^2 are also denoted by kg/cm^2 and lb/in^2, respectively, and they are commonly used in tire gages. It can be shown that 1 kgf/cm^2 = 14.223 psi.

Pressure is also used on solid surfaces as synonymous with *normal stress,* which is the force acting perpendicular to the surface per unit area. For example, a 150-pound person with a total foot imprint area of 50 in^2 exerts a pressure of 150 lbf/50 in^2 = 3.0 psi on the floor (Fig. 2–22). If the person stands on one foot, the pressure doubles. If the person gains excessive weight, he or she is likely to encounter foot discomfort because of the increased pressure on the foot (the size of the bottom of the foot does not change with weight gain). This also explains how a person can walk on fresh snow without sinking by wearing large snowshoes, and how a person cuts with little effort when using a sharp knife.

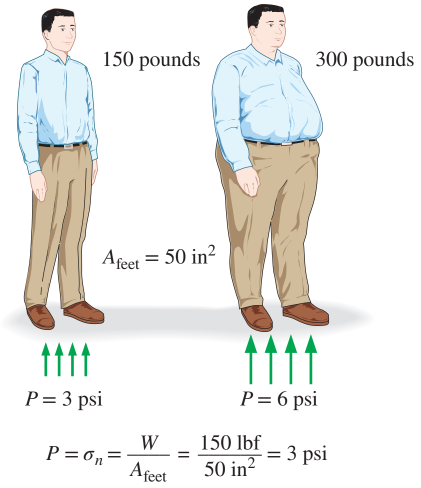


FIGURE 2–22
The normal stress (or "pressure") on the feet of a chubby person is much greater than on the feet of a slim person.

FIGURE 2–23
Some basic pressure gages.
©Ashcroft Inc.

The actual pressure at a given position is called the **absolute pressure**, and it is measured relative to absolute vacuum (i.e., absolute zero pressure). Most pressure-measuring devices, however, are calibrated to read zero in the atmosphere (Fig. 2–23), and so they indicate the difference between the absolute pressure and the local atmospheric pressure. This difference is called the **gage pressure**. P_{gage} can be positive or negative, but pressures below atmospheric pressure are sometimes called **vacuum pressures** and are measured by vacuum gages that indicate the difference between the atmospheric pressure and the absolute pressure. Absolute, gage, and vacuum pressures are related to each other by

$$P_{\text{gage}} = P_{\text{abs}} - P_{\text{atm}} \quad \textbf{(2–12)}$$

$$P_{\text{vac}} = P_{\text{atm}} - P_{\text{abs}} \quad \textbf{(2–13)}$$

This is illustrated in Fig. 2–24.

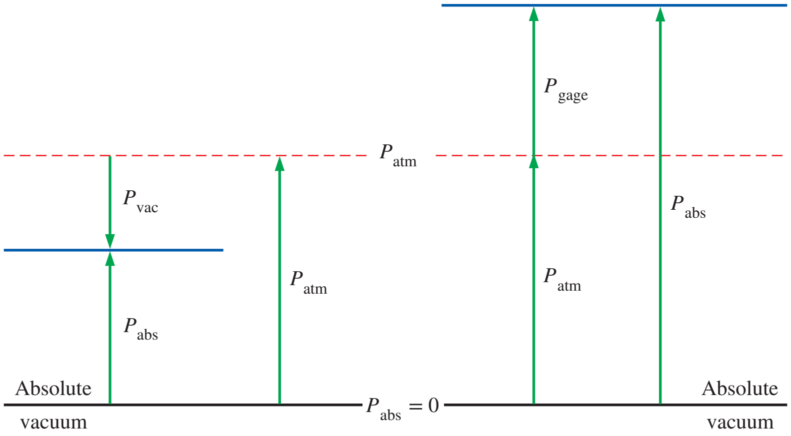


FIGURE 2–24
Absolute, gage, and vacuum pressures.

Like other pressure gages, the gage used to measure the air pressure in an automobile tire reads the gage pressure. Therefore, the common reading of 32.0 psi (2.25 kgf/cm^2) indicates a pressure of 32.0 psi above the atmospheric pressure. At a location where the atmospheric pressure is 14.3 psi, for example, the absolute pressure in the tire is 32.0 + 14.3 = 46.3 psi.

In thermodynamic relations and tables, absolute pressure is almost always used. Throughout this text, the pressure P will denote *absolute pressure* unless specified otherwise. Often the letters "a" (for absolute pressure) and "g" (for gage pressure) are added to pressure units (such as psia and psig) to clarify what is meant.

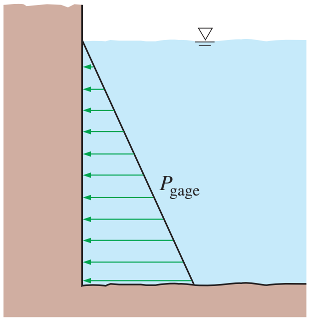


FIGURE 2–25
The pressure of a fluid at rest increases with depth (as a result of added weight).

EXAMPLE 2–2 **Absolute Pressure of a Vacuum Chamber**

A vacuum gage connected to a chamber reads 5.8 psi at a location where the atmospheric pressure is 14.5 psi. Determine the absolute pressure in the chamber.

SOLUTION The gage pressure of a vacuum chamber is given. The absolute pressure in the chamber is to be determined.

Analysis The absolute pressure is easily determined from Eq. 2–3 to be

$$P_{abs} = P_{atm} - P_{vac} = 14.5 - 5.8 = \mathbf{8.7\ psi}$$

Discussion Note that the *local* value of the atmospheric pressure is used when determining the absolute pressure.

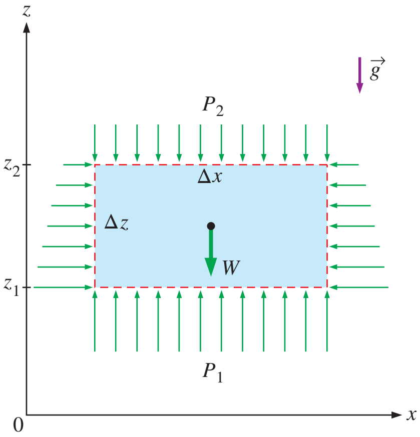


FIGURE 2–26
Free-body diagram of a rectangular fluid element in equilibrium.

Variation of Pressure with Depth

It will come as no surprise to you that pressure in a fluid at rest does not change in the horizontal direction. This can be shown easily by considering a thin horizontal layer of fluid and doing a force balance in any horizontal direction. However, this is not the case in the vertical direction in a gravity field. Pressure in a fluid increases with depth because more fluid rests on deeper layers, and the effect of this "extra weight" on a deeper layer is balanced by an increase in pressure (Fig. 2–25).

To obtain a relation for the variation of pressure with depth, consider a rectangular fluid element of height Δz, length Δx, and unit depth ($\Delta y = 1$ into the page) in equilibrium, as shown in Fig. 2–26. Assuming the density of the fluid ρ to be constant, a force balance in the vertical z-direction gives

$$\sum F_z = ma_z = 0: \qquad P_1 \Delta x \Delta y - P_2 \Delta x \Delta y - \rho g \Delta x \Delta y \Delta z = 0$$

where $W = mg = \rho g\, \Delta x\, \Delta y\, \Delta z$ is the weight of the fluid element and $\Delta z = z_2 - z_1$. Dividing by $\Delta x\, \Delta y$ and rearranging gives

$$\Delta P = P_2 - P_1 = -\rho g\, \Delta z = -\gamma_s\, \Delta z \tag{2–14}$$

where $\gamma_s = \rho g$ is the *specific weight* of the fluid. Thus, we conclude that the pressure difference between two points in a constant density fluid is proportional to the vertical distance Δz between the points and the density ρ of the fluid. Noting the negative sign, *pressure in a static fluid increases linearly with depth.* This is what a diver experiences when diving deeper in a lake.

An easier equation to remember and apply between any two points in the same fluid under hydrostatic conditions is

$$P_{\text{below}} = P_{\text{above}} + \rho g|\Delta z| = P_{\text{above}} + \gamma_s|\Delta z| \tag{2–15}$$

where "below" refers to the point at lower elevation (deeper in the fluid) and "above" refers to the point at higher elevation. If you use this equation consistently, you should avoid sign errors.

For a given fluid, the vertical distance Δz is sometimes used as a measure of pressure, and it is called the *pressure head.*

We also conclude from Eq. 2–14 that for small to moderate distances, the variation of pressure with height is negligible for gases because of their low density. The pressure in a tank containing a gas, for example, can be considered to be uniform since the weight of the gas is too small to make a significant difference. Also, the pressure in a room filled with air can be approximated as a constant (Fig. 2–27).

If we take the "above" point to be at the free surface of a liquid open to the atmosphere (Fig. 2–28), where the pressure is the atmospheric pressure P_{atm}, then from Eq. 2–15 the pressure at a depth h below the free surface becomes

$$P = P_{\text{atm}} + \rho g h \quad \text{or} \quad P_{\text{gage}} = \rho g h \tag{2–16}$$

Liquids are essentially incompressible substances, and thus the variation of density with depth is negligible. This is also the case for gases when the elevation change is not very large. The variation of density of liquids or gases with temperature can be significant, however, and may need to be considered when high accuracy is desired. Also, at great depths such as those encountered in oceans, the change in the density of a liquid can be significant because of the compression by the tremendous amount of liquid weight above.

The gravitational acceleration g varies from 9.807 m/s^2 at sea level to 9.764 m/s^2 at an elevation of 14,000 m where large passenger planes cruise. This is a change of just 0.4 percent in this extreme case. Therefore, g can be approximated as a constant with negligible error.

For fluids whose density changes significantly with elevation, a relation for the variation of pressure with elevation can be obtained by dividing Eq. 2–14 by Δz, and taking the limit as $\Delta z \to 0$. This yields

$$\frac{dP}{dz} = -\rho g \tag{2–17}$$

Note that dP is negative when dz is positive since pressure decreases in an upward direction. When the variation of density with elevation is known, the pressure difference between any two points 1 and 2 can be determined by integration to be

$$\Delta P = P_2 - P_1 = -\int_1^2 \rho g\, dz \tag{2–18}$$

For constant density and constant gravitational acceleration, this relation reduces to Eq. 2–14, as expected.

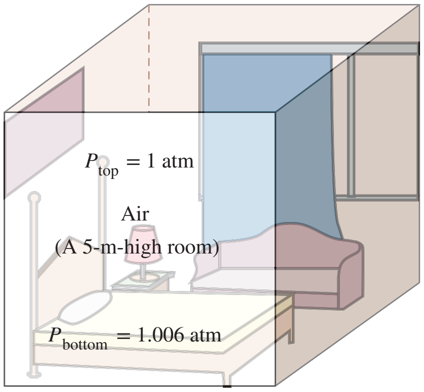


FIGURE 2–27
In a room filled with a gas, the variation of pressure with height is negligible.

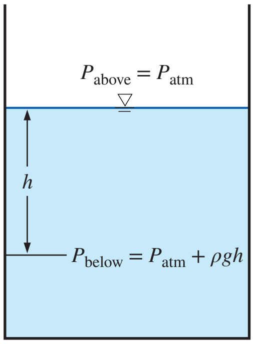


FIGURE 2–28
Pressure in a liquid at rest increases linearly with distance from the free surface.

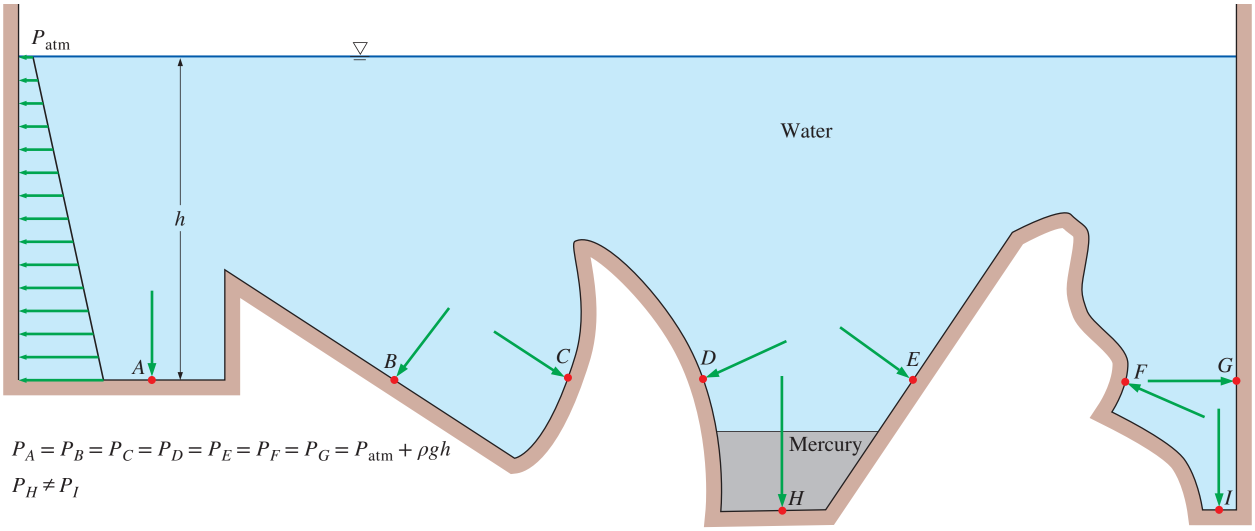


FIGURE 2–29

Under hydrostatic conditions, the pressure is the same at all points on a horizontal plane in a given fluid regardless of geometry, provided that the points are interconnected by the same fluid.

Pressure in a fluid at rest is independent of the shape or cross section of the container. It changes with the vertical distance but remains constant in other directions. Therefore, the pressure is the same at all points on a horizontal plane in a given fluid. The Dutch mathematician Simon Stevin (1548–1620) published in 1586 the principle illustrated in Fig. 2–29. Note that the pressures at points *A, B, C, D, E, F,* and *G* are the same since they are at the same depth, and they are interconnected by the same static fluid. However, the pressures at points *H* and *I* are not the same since these two points cannot be interconnected by the same fluid (i.e., we cannot draw a curve from point *I* to point *H* while remaining in the same fluid at all times), although they are at the same depth. (Can you tell at which point the pressure is higher?) Also notice that the pressure force exerted by the fluid is always normal to the surface at the specified points.

A consequence of the pressure in a fluid remaining constant in the horizontal direction is that *the pressure applied to a confined fluid increases the pressure throughout by the same amount.* This is called **Pascal's law**, after Blaise Pascal (1623–1662). Pascal also knew that the force applied by a fluid is proportional to the surface area. He realized that two hydraulic cylinders of different areas could be connected, and the larger could be used to exert a proportionally greater force than that applied to the smaller. "Pascal's machine" has been the source of many inventions that are a part of our daily lives such as hydraulic brakes and lifts. This is what enables us to lift a car easily with one arm, as shown in Fig. 2–30. Noting that $P_1 = P_2$ since both pistons are at the same level (the effect of small height differences is negligible, especially at high pressures), the ratio of output force to input force is determined to be

$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1} \tag{2–19}$$

The area ratio A_2/A_1 is called the *ideal mechanical advantage* of the hydraulic lift. Using a hydraulic car jack with a piston area ratio of $A_2/A_1 = 100$, for example, a person can lift a 1000-kg car by applying a force of just 10 kgf (= 90.8 N).

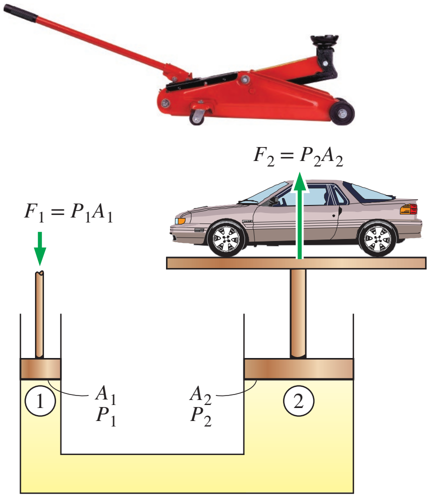


FIGURE 2–30

Lifting of a large weight by a small force by the application of Pascal's law. A common example is a hydraulic jack.

(Top) ©Stockbyte/Getty Images

2–8 ▪ PRESSURE MEASUREMENT DEVICES

The Barometer

Atmospheric pressure is measured by a device called a **barometer**; thus, the atmospheric pressure is often referred to as the *barometric pressure.*

The Italian Evangelista Torricelli (1608–1647) was the first to conclusively prove that the atmospheric pressure can be measured by inverting a mercury-filled tube into a mercury container that is open to the atmosphere, as shown in Fig. 2–31. The pressure at point B is equal to the atmospheric pressure, and the pressure at point C can be taken to be zero since there is only mercury vapor above point C, and the pressure is very low relative to P_{atm} and can be neglected to an excellent approximation. Writing a force balance in the vertical direction gives

$$P_{\text{atm}} = \rho g h \tag{2–20}$$

where ρ is the density of mercury, g is the local gravitational acceleration, and h is the height of the mercury column above the free surface. Note that the length and the cross-sectional area of the tube have no effect on the height of the fluid column of a barometer (Fig. 2–32).

A frequently used pressure unit is the *standard atmosphere,* which is defined as the pressure produced by a column of mercury 760 mm in height at 0°C (ρ_{Hg} = 13,595 kg/m^3) under standard gravitational acceleration (g = 9.807 m/s^2). If water instead of mercury were used to measure the standard atmospheric pressure, a water column of about 10.3 m would be needed. Pressure is sometimes expressed (especially by weather forecasters) in terms of the height of the mercury column. The standard atmospheric pressure, for example, is 760 mmHg (29.92 inHg) at 0°C. The unit mmHg is also called the **torr** in honor of Torricelli. Therefore, 1 atm = 760 torr and 1 torr = 133.3 Pa.

Atmospheric pressure P_{atm} changes from 101.325 kPa at sea level to 89.88, 79.50, 54.05, 26.5, and 5.53 kPa at altitudes of 1000, 2000, 5000, 10,000, and 20,000 meters, respectively. The typical atmospheric pressure in Denver (elevation = 1610 m), for example, is 83.4 kPa. Remember that the atmospheric pressure at a location is simply the weight of the air above that location per unit surface area. Therefore, it changes not only with elevation but also with weather conditions.

The decline of atmospheric pressure with elevation has far-reaching ramifications in daily life. For example, cooking takes longer at high altitudes since water boils at a lower temperature at lower atmospheric pressures. Nose bleeding is a common experience at high altitudes since the difference between the blood pressure and the atmospheric pressure is larger in this case, and the delicate walls of veins in the nose are often unable to withstand this extra stress.

For a given temperature, the density of air is lower at high altitudes, and thus a given volume contains less air and less oxygen. So it is no surprise that we tire more easily and experience breathing problems at high altitudes. To compensate for this effect, people living at higher altitudes develop more efficient lungs. Similarly, a 2.0-L car engine will act like a 1.7-L car engine at 1500 m altitude (unless it is turbocharged) because of the 15 percent drop in pressure and thus 15 percent drop in the density of air (Fig. 2–33). A fan or compressor will displace 15 percent less air at that altitude for the same volume displacement rate. Therefore, larger cooling fans may need to be selected for operation at high altitudes to ensure the specified mass flow rate. The lower pressure and thus lower density also affects lift and drag: airplanes need a longer runway at high altitudes to develop the required lift, and they climb to very high altitudes for cruising in order to reduce drag and thus achieve better fuel efficiency.

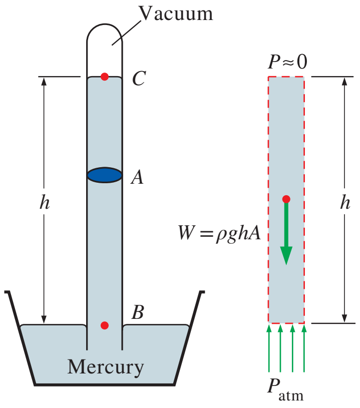


FIGURE 2–31
The basic barometer.

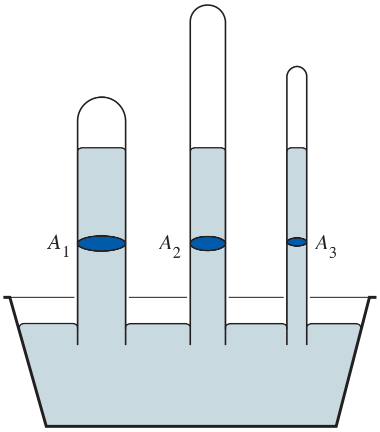


FIGURE 2–32
The length and the cross-sectional area of the tube have no effect on the height of the fluid column of a barometer, provided that the tube diameter is large enough to avoid surface tension (capillary) effects.

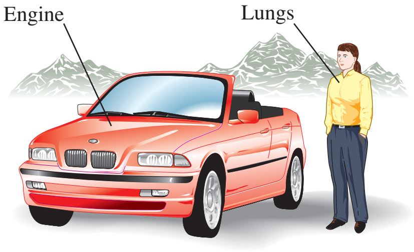


FIGURE 2–33
At high altitudes, a car engine generates less power and a person gets less oxygen because of the lower density of air.

EXAMPLE 2–3 Measuring Atmospheric Pressure with a Barometer

Determine the atmospheric pressure at a location where the barometric reading is 740 mmHg and the gravitational acceleration is $g = 9.805\ \text{m/s}^2$. Assume the temperature of mercury to be 10°C, at which its density is $13{,}570\ \text{kg/m}^3$.

SOLUTION The barometric reading at a location in height of mercury column is given. The atmospheric pressure is to be determined.
Assumptions The temperature of mercury is assumed to be 10°C.
Properties The density of mercury is given to be $13{,}570\ \text{kg/m}^3$.
Analysis From Eq. 2–20, the atmospheric pressure is determined to be

$$\begin{aligned} P_{\text{atm}} &= \rho g h \\ &= (13{,}570\ \text{kg/m}^3)(9.805\ \text{m/s}^2)(0.740\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= \mathbf{98.5\ kPa} \end{aligned}$$

Discussion Note that density changes with temperature, and thus this effect should be considered in calculations.

EXAMPLE 2–4 Gravity-Driven Flow from an IV Bottle

Intravenous infusions usually are driven by gravity by hanging the fluid bottle at sufficient height to counteract the blood pressure in the vein and to force the fluid into the body (Fig. 2–34). The higher the bottle is raised, the higher the flow rate of the fluid will be. (*a*) If it is observed that the fluid and the blood pressures balance each other when the bottle is 1.2 m above the arm level, determine the gage pressure of the blood. (*b*) If the gage pressure of the fluid at the arm level needs to be 20 kPa for sufficient flow rate, determine how high the bottle must be placed. Take the density of the fluid to be $1020\ \text{kg/m}^3$.

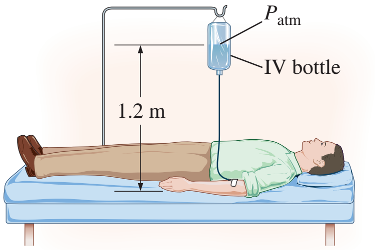


FIGURE 2–34
Schematic for Example 2–4.

SOLUTION It is given that an IV fluid and the blood pressures balance each other when the bottle is at a certain height. The gage pressure of the blood and elevation of the bottle required to maintain flow at the desired rate are to be determined.
Assumptions **1** The IV fluid is incompressible. **2** The IV bottle is open to the atmosphere.
Properties The density of the IV fluid is given to be $\rho = 1020\ \text{kg/m}^3$.
Analysis (*a*) Noting that the IV fluid and the blood pressures balance each other when the bottle is 1.2 m above the arm level, the gage pressure of the blood in the arm is simply equal to the gage pressure of the IV fluid at a depth of 1.2 m,

$$\begin{aligned} P_{\text{gage,arm}} &= P_{\text{abs}} - P_{\text{atm}} = \rho g h_{\text{arm–bottle}} \\ &= (1020\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.20\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) \\ &= \mathbf{12.0\ kPa} \end{aligned}$$

(*b*) To provide a gage pressure of 20 kPa at the arm level, the height of the surface of the IV fluid in the bottle from the arm level is again determined from $P_{\text{gage,arm}} = \rho g h_{\text{arm–bottle}}$ to be

$$\begin{aligned} h_{\text{arm–bottle}} &= \frac{P_{\text{gage,arm}}}{\rho g} \\ &= \frac{20\ \text{kPa}}{(1020\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right)\left(\frac{1\ \text{kN/m}^2}{1\ \text{kPa}}\right) \\ &= \mathbf{2.00\ m} \end{aligned}$$

Discussion Note that the height of the reservoir can be used to control flow rates in gravity-driven flows. When there is flow, the pressure drop in the tube due to frictional effects also should be considered. For a specified flow rate, this requires raising the bottle a little higher to overcome the pressure drop.

EXAMPLE 2–5 Hydrostatic Pressure in a Solar Pond with Variable Density

Solar ponds are small artificial lakes a few meters deep that are used to store solar energy. The rise of heated (and thus less dense) water to the surface is prevented by adding salt at the pond bottom. In a typical salt gradient solar pond, the density of water increases in the gradient zone, as shown in Fig. 2–35, and the density can be expressed as

$$\rho = \rho_0 \sqrt{1 + \tan^2\left(\frac{\pi}{4}\frac{s}{H}\right)}$$

where ρ_0 is the density on the water surface, s is the vertical distance measured downward from the top of the gradient zone ($s = -z$), and H is the thickness of the gradient zone. For $H = 4$ m, $\rho_0 = 1040$ kg/m^3, and a thickness of 0.8 m for the surface zone, calculate the gage pressure at the bottom of the gradient zone.

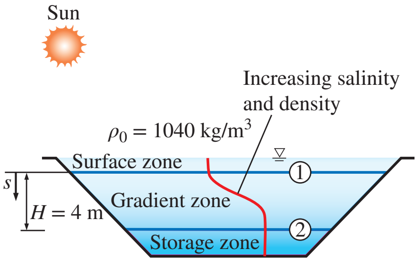


FIGURE 2–35
Schematic for Example 2–5.

SOLUTION The variation of density of saline water in the gradient zone of a solar pond with depth is given. The gage pressure at the bottom of the gradient zone is to be determined.

Assumptions The density in the surface zone of the pond is constant.

Properties The density of brine on the surface is given to be 1040 kg/m^3.

Analysis We label the top and the bottom of the gradient zone as 1 and 2, respectively. Noting that the density of the surface zone is constant, the gage pressure at the bottom of the surface zone (which is the top of the gradient zone) is

$$P_1 = \rho g h_1 = (1040 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.8 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg·m/s}^2}\right) = \mathbf{8.16 \text{ kPa}}$$

since 1 kN/m^2 = 1 kPa. Since $s = -z$, the differential change in hydrostatic pressure across a vertical distance of ds is given by

$$dP = \rho g\, ds$$

Integrating from the top of the gradient zone (point 1 where $s = 0$) to any location s in the gradient zone (no subscript) gives

$$P - P_1 = \int_0^s \rho g\, ds \quad \rightarrow \quad P = P_1 + \int_0^s \rho_0 \sqrt{1 + \tan^2\left(\frac{\pi}{4}\frac{s}{H}\right)}\, g\, ds$$

Performing the integration gives the variation of gage pressure in the gradient zone to be

$$P = P_1 + \rho_0 g \frac{4H}{\pi} \sinh^{-1}\left(\tan\frac{\pi}{4}\frac{s}{H}\right)$$

Then the pressure at the bottom of the gradient zone ($s = H = 4$ m) becomes

$$P_2 = 8.16 \text{ kPa} + (1040 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\frac{4(4 \text{ m})}{\pi}\sinh^{-1}\left(\tan\frac{\pi}{4}\frac{4}{4}\right)\left(\frac{1 \text{ kN}}{1000 \text{ kg·m/s}^2}\right)$$
$$= \mathbf{54.0 \text{ kPa (gage)}}$$

Discussion The variation of gage pressure in the gradient zone with depth is plotted in Fig. 2–36. The dashed line indicates the hydrostatic pressure for the case of constant density at 1040 kg/m^3 and is given for reference. Note that the variation of pressure with depth is not linear when density varies with depth. That is why integration was required.

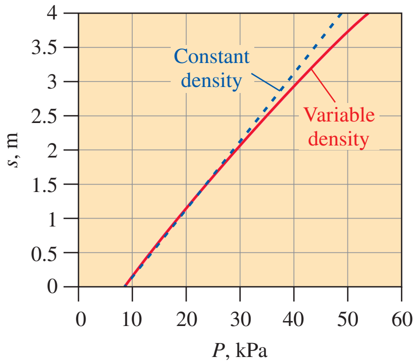


FIGURE 2–36
The variation of gage pressure with depth in the gradient zone of the solar pond.

The Manometer

We notice from Eq. 2–14 that an elevation change of $-\Delta z$ in a fluid at rest corresponds to $\Delta P/\rho g$, which suggests that a fluid column can be used to measure pressure differences. A device based on this principle is called a **manometer**, and it is commonly used to measure small and moderate pressure differences. A manometer consists of a glass or plastic U-tube containing one or more fluids such as mercury, water, alcohol, or oil (Fig. 2–37). To keep the size of the manometer to a manageable level, heavy fluids such as mercury are used if large pressure differences are anticipated.

FIGURE 2–37
A simple U-tube manometer, with high pressure applied to the right side.
©John M. Cimbala

Consider the manometer shown in Fig. 2–38 that is used to measure the pressure in the tank. Since the gravitational effects of gases are negligible, the pressure anywhere in the tank and at position 1 has the same value.

Furthermore, since pressure in a fluid does not vary in the horizontal direction within a fluid, the pressure at point 2 is the same as the pressure at point 1, $P_2 = P_1$.

The differential fluid column of height h is in static equilibrium, and it is open to the atmosphere. Then the pressure at point 2 is determined directly from Eq. 2–15 to be

$$P_2 = P_{\text{atm}} + \rho g h \tag{2–21}$$

where ρ is the density of the manometer fluid in the tube. Note that the cross-sectional area of the tube has no effect on the differential height h and thus on the pressure exerted by the fluid. However, the diameter of the tube should be large enough (more than several millimeters) to ensure that the surface tension effect and thus the capillary rise is negligible.

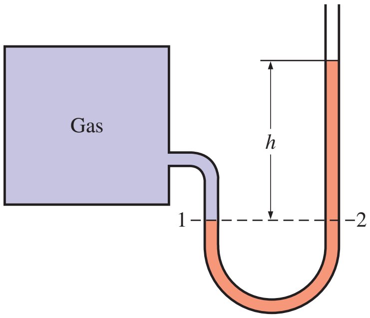


FIGURE 2–38
The basic manometer.

EXAMPLE 2–6 **Measuring Pressure with a Manometer**

A manometer is used to measure the pressure of a gas in a tank. The fluid used has a specific gravity of 0.85, and the manometer column height is 55 cm, as shown in Fig. 2–39. If the local atmospheric pressure is 96 kPa, determine the absolute pressure within the tank.

SOLUTION The reading of a manometer attached to a tank and the atmospheric pressure are given. The absolute pressure in the tank is to be determined.

Assumptions The density of the gas in the tank is much lower than the density of the manometer fluid.

Properties The specific gravity of the manometer fluid is given to be 0.85. We take the standard density of water to be 1000 kg/m^3.

Analysis The density of the fluid is obtained by multiplying its specific gravity by the density of water,

$$\rho = \text{SG}(\rho_{\text{water}}) = (0.85)(1000\ \text{kg/m}^3) = 850\ \text{kg/m}^3$$

Then from Eq. 2–21,

$$\begin{aligned} P &= P_{\text{atm}} + \rho g h \\ &= 96\ \text{kPa} + (850\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.55\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= \mathbf{100.6\ kPa} \end{aligned}$$

Discussion Note that the gage pressure in the tank is 4.6 kPa.

Some manometers use a slanted or inclined tube in order to increase the resolution (precision) when reading the fluid height. Such devices are called *inclined manometers.*

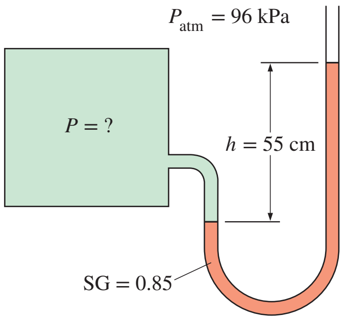


FIGURE 2–39
Schematic for Example 2–6.

Many engineering problems and some manometers involve multiple immiscible fluids of different densities stacked on top of each other. Such systems can be analyzed easily by remembering that (1) the pressure change across a fluid column of height h is $\Delta P = \rho gh$, (2) pressure increases downward in a given fluid and decreases upward (i.e., $P_{bottom} > P_{top}$), and (3) two points at the same elevation in a continuous fluid at rest are at the same pressure.

The last principle, which is a result of *Pascal's law,* allows us to "jump" from one fluid column to the next in manometers without worrying about pressure change as long as we stay in the same continuous fluid and the fluid is at rest. Then the pressure at any point can be determined by starting with a point of known pressure and adding or subtracting ρgh terms as we advance toward the point of interest. For example, the pressure at the bottom of the tank in Fig. 2–40 can be determined by starting at the free surface where the pressure is P_{atm}, moving downward until we reach point 1 at the bottom, and setting the result equal to P_1. It gives

$$P_{atm} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 = P_1$$

In the special case of all fluids having the same density, this relation reduces to $P_{atm} + \rho g(h_1 + h_2 + h_3) = P_1$.

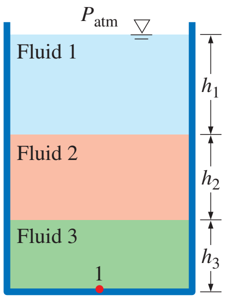


FIGURE 2–40
In stacked-up fluid layers at rest, the pressure change across each fluid layer of density ρ and height h is ρgh.

Manometers are particularly well-suited to measure pressure drops across a horizontal flow section between two specified points due to the presence of a device such as a valve or heat exchanger or any resistance to flow. This is done by connecting the two legs of the manometer to these two points, as shown in Fig. 2–41. The working fluid can be either a gas or a liquid whose density is ρ_1. The density of the manometer fluid is ρ_2, and the differential fluid height is h. The two fluids must be immiscible, and ρ_2 must be greater than ρ_1.

A relation for the pressure difference $P_1 - P_2$ can be obtained by starting at point 1 with P_1, moving along the tube by adding or subtracting the ρgh terms until we reach point 2, and setting the result equal to P_2:

$$P_1 + \rho_1 g(a + h) - \rho_2 gh - \rho_1 ga = P_2 \tag{2–22}$$

Note that we jumped from point A horizontally to point B and ignored the part underneath since the pressure at both points is the same. Simplifying,

$$P_1 - P_2 = (\rho_2 - \rho_1)gh \tag{2–23}$$

Note that the distance a must be included in the analysis even though it has no effect on the result. Also, when the fluid flowing in the pipe is a gas, then $\rho_1 \ll \rho_2$ and the relation in Eq. 2–23 simplifies to $P_1 - P_2 \cong \rho_2 gh$.

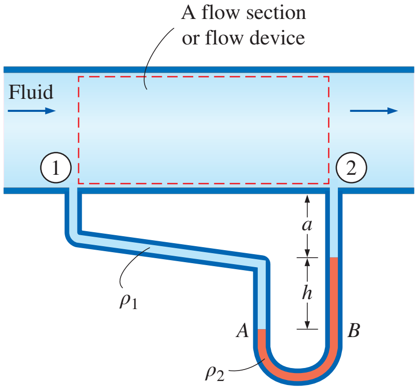


FIGURE 2–41
Measuring the pressure drop across a flow section or a flow device by a differential manometer.

EXAMPLE 2–7 Measuring Pressure with a Multifluid Manometer

The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in Fig. 2–42. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if $h_1 = 0.1$ m, $h_2 = 0.2$ m, and $h_3 = 0.35$ m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.

SOLUTION The pressure in a pressurized water tank is measured by a multifluid manometer. The air pressure in the tank is to be determined.

Assumptions The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus we can determine the pressure at the air–water interface.

Properties The densities of water, oil, and mercury are given to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.

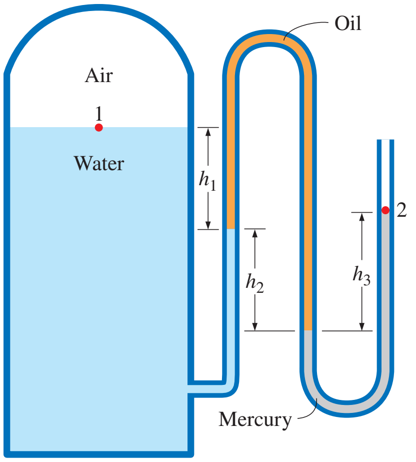


FIGURE 2–42
Schematic for Example 2–7; drawing not to scale.

Analysis Starting with the pressure at point 1 at the air–water interface, moving along the tube by adding or subtracting the ρgh terms until we reach point 2, and setting the result equal to P_{atm} since the tube is open to the atmosphere gives

$$P_1 + \rho_{\text{water}} g h_1 + \rho_{\text{oil}} g h_2 - \rho_{\text{mercury}} g h_3 = P_2 = P_{\text{atm}}$$

Solving for P_1 and substituting,

$$\begin{aligned} P_1 &= P_{\text{atm}} - \rho_{\text{water}} g h_1 - \rho_{\text{oil}} g h_2 + \rho_{\text{mercury}} g h_3 \\ &= P_{\text{atm}} + g(\rho_{\text{mercury}} h_3 - \rho_{\text{water}} h_1 - \rho_{\text{oil}} h_2) \\ &= 85.6 \text{ kPa} + (9.81 \text{ m/s}^2)[(13{,}600 \text{ kg/m}^3)(0.35 \text{ m}) - (1000 \text{ kg/m}^3)(0.1 \text{ m}) \\ &\quad - (850 \text{ kg/m}^3)(0.2 \text{ m})]\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) \\ &= \mathbf{130 \text{ kPa}} \end{aligned}$$

Discussion Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis considerably. Also note that mercury is a toxic fluid, and mercury manometers and thermometers are being replaced by ones with safer fluids because of the risk of exposure to mercury vapor during an accident.

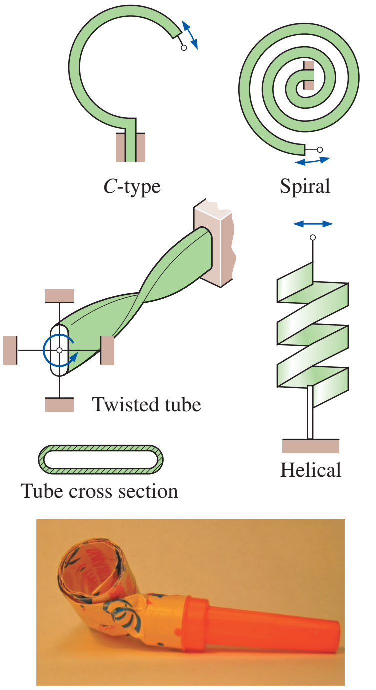


FIGURE 2–43
Various types of Bourdon tubes used to measure pressure. They work on the same principle as party noise-makers (bottom photo) due to the flat tube cross section.
Photo: ©John M. Cimbala

Other Pressure Measurement Devices

Another type of commonly used mechanical pressure measurement device is the **Bourdon tube**, named after the French engineer and inventor Eugene Bourdon (1808–1884), which consists of a bent, coiled, or twisted hollow metal tube whose end is closed and connected to a dial indicator needle (Fig. 2–43). When the tube is open to the atmosphere, the tube is undeflected, and the needle on the dial at this state is calibrated to read zero (gage pressure). When the fluid inside the tube is pressurized, the tube stretches and moves the needle in proportion to the applied pressure.

Electronics have made their way into every aspect of life, including pressure measurement devices. Modern pressure sensors, called **pressure transducers**, use various techniques to convert the pressure effect to an electrical effect such as a change in voltage, resistance, or capacitance. Pressure transducers are smaller and faster, and they can be more sensitive, reliable, and precise than their mechanical counterparts. They can measure pressures from less than a millionth of 1 atm to several thousands of atm.

A wide variety of pressure transducers is available to measure gage, absolute, and differential pressures in a wide range of applications. *Gage pressure transducers* use the atmospheric pressure as a reference by venting the back side of the pressure-sensing diaphragm to the atmosphere, and they give a zero signal output at atmospheric pressure regardless of altitude. *Absolute pressure transducers* are calibrated to have a zero signal output at full vacuum. *Differential pressure transducers* measure the pressure difference between two locations directly instead of using two pressure transducers and taking their difference.

Strain-gage pressure transducers work by having a diaphragm deflect between two chambers open to the pressure inputs. As the diaphragm stretches in response to a change in pressure difference across it, the strain gage stretches and a Wheatstone bridge circuit amplifies the output. A capacitance transducer works similarly, but capacitance change is measured instead of resistance change as the diaphragm stretches.

Piezoelectric transducers, also called solid-state pressure transducers, work on the principle that an electric potential is generated in a crystalline substance when it is subjected to mechanical pressure. This phenomenon, first discovered by brothers Pierre and Jacques Curie in 1880, is called the piezoelectric (or press-electric) effect. Piezoelectric pressure transducers have a much faster frequency response than diaphragm units and are very suitable for high-pressure applications, but they are generally not as sensitive as diaphragm-type transducers, especially at low pressures.

Another type of mechanical pressure gage called a **deadweight tester** is used primarily for *calibration* and can measure extremely high pressures (Fig. 2–44). As its name implies, a deadweight tester measures pressure *directly* through application of a weight that provides a force per unit area—the fundamental definition of pressure. It is constructed with an internal chamber filled with a fluid (usually oil), along with a tight-fitting piston, cylinder, and plunger. Weights are applied to the top of the piston, which exerts a force on the oil in the chamber. The total force F acting on the oil at the piston–oil interface is the sum of the weight of the piston plus the applied weights. Since the piston cross-sectional area A_e is known, the pressure is calculated as $P = F/A_e$. The only significant source of error is that due to static friction along the interface between the piston and cylinder, but even this error is usually negligibly small. The reference pressure port is connected to either an unknown pressure that is to be measured or to a pressure sensor that is to be calibrated.

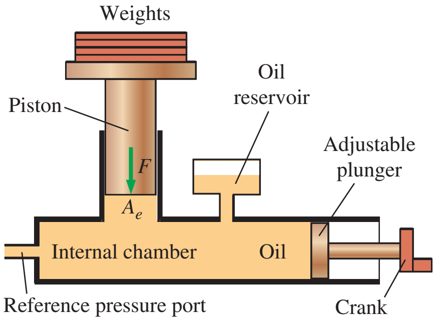


FIGURE 2–44
A deadweight tester can measure extremely high pressures (up to 10,000 psi in some applications).

SUMMARY

In this chapter, the basic concepts of thermodynamics are introduced and discussed. A system of fixed mass is called a *closed system,* or *control mass,* and a system that involves mass transfer across its boundaries is called an *open system,* or *control volume.* The mass-dependent properties of a system are called *extensive properties* and the others *intensive properties. Density* is mass per unit volume, and *specific volume* is volume per unit mass.

A system is said to be in *thermodynamic equilibrium* if it maintains thermal, mechanical, phase, and chemical equilibrium. Any change from one state to another is called a *process.* A process with identical end states is called a *cycle.* During a *quasi-static* or *quasi-equilibrium process,* the system remains practically in equilibrium at all times. The state of a simple, compressible system is completely specified by two independent, intensive properties.

The *zeroth law of thermodynamics* states that two bodies are in thermal equilibrium if both have the same temperature reading even if they are not in contact.

The temperature scales used in the SI and the English systems today are the *Celsius scale* and the *Fahrenheit scale,* respectively. They are related to absolute temperature scales by

$$T(\mathrm{K}) = T(^\circ\mathrm{C}) + 273.15$$

$$T(\mathrm{R}) = T(^\circ\mathrm{F}) + 459.67$$

The magnitudes of each division of 1 K and 1°C are identical, and so are the magnitudes of each division of 1 R and 1°F. Therefore,

$$\Delta T(\mathrm{K}) = \Delta T(^\circ\mathrm{C}) \quad \text{and} \quad \Delta T(\mathrm{R}) = \Delta T(^\circ\mathrm{F})$$

The normal force exerted by a fluid per unit area is called *pressure,* and its unit is the *pascal,* 1 Pa = 1 N/m^2. The pressure relative to absolute vacuum is called the *absolute pressure,* and the difference between the absolute pressure and the local atmospheric pressure is called the *gage pressure.* Pressures below atmospheric pressure are called *vacuum pressures.* The absolute, gage, and vacuum pressures are related by

$$P_{\text{gage}} = P_{\text{abs}} - P_{\text{atm}} \qquad (\text{for pressure above } P_{\text{atm}})$$
$$P_{\text{vac}} = P_{\text{atm}} - P_{\text{abs}} \qquad (\text{for pressure below } P_{\text{atm}})$$

The pressure at a point in a fluid has the same magnitude in all directions. The variation of pressure with elevation is given by

$$\frac{dP}{dz} = -\rho g$$

where the positive z direction is taken to be upward. When the density of the fluid is constant, the pressure difference across a fluid layer of thickness Δz is

$$\Delta P = P_2 - P_1 = \rho g\, \Delta z$$

The absolute and gage pressures in a liquid open to the atmosphere at a depth h from the free surface are

$$P = P_{\text{atm}} + \rho g h \quad \text{or} \quad P_{\text{gage}} = \rho g h$$

Small to moderate pressure differences are measured by a *manometer.* The pressure in a stationary fluid remains constant in the horizontal direction. *Pascal's principle* states that the pressure applied to a confined fluid increases the pressure throughout by the same amount.

The atmospheric pressure is measured by a *barometer* and is given by

$$P_{\text{atm}} = \rho g h$$

where h is the height of the liquid column.

REFERENCES AND SUGGESTED READINGS

1. A. Bejan. *Advanced Engineering Thermodynamics.* 3rd ed. New York: Wiley, 2006.
2. J. A. Schooley. *Thermometry.* Boca Raton, FL: CRC Press, 1986.

PROBLEMS*

Systems, Properties, State, and Processes

2–1C How would you define a system to determine the rate at which an automobile adds carbon dioxide to the atmosphere?

2–2C A large fraction of the thermal energy generated in the engine of a car is rejected to the air by the radiator through the circulating water. Should the radiator be analyzed as a closed system or as an open system? Explain.

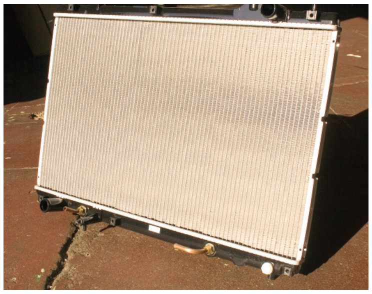


FIGURE P2–2C

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2–3C A can of soft drink at room temperature is put into the refrigerator so that it will cool. Would you model the can of soft drink as a closed system or as an open system? Explain.

2–4C How would you define a system to determine the temperature rise created in a lake when a portion of its water is used to cool a nearby electrical power plant?

2–5C How would you describe the state of the air in the atmosphere? What kind of process does this air undergo from a cool morning to a warm afternoon?

2–6C What is the difference between intensive and extensive properties?

2–7C The specific weight of a system is defined as the weight per unit volume (note that this definition violates the normal specific property-naming convention). Is the specific weight an extensive or intensive property?

2–8C Is the number of moles of a substance contained in a system an extensive or intensive property?

2–9C Is the state of the air in an isolated room completely specified by the temperature and the pressure? Explain.

2–10C What is a quasi-equilibrium process? What is its importance in engineering?

2–11C Define the isothermal, isobaric, and isochoric processes.

2–12C What is specific gravity? How is it related to density?

2–13 💻 The density of atmospheric air varies with elevation, decreasing with increasing altitude. (*a*) Using the data given in the table, obtain a relation for the variation of density with elevation, and calculate the density at an elevation of 7000 m. (*b*) Calculate the mass of the atmosphere using the correlation you obtained. Assume the earth to be a perfect sphere with a radius of 6377 km, and take the thickness of the atmosphere to be 25 km.

z, km	ρ, kg/m^3
6377	1.225
6378	1.112
6379	1.007
6380	0.9093
6381	0.8194
6382	0.7364
6383	0.6601
6385	0.5258
6387	0.4135
6392	0.1948
6397	0.08891
6402	0.04008

Temperature

2–14C What are the ordinary and absolute temperature scales in the SI and the English system?

2–15C Consider an alcohol and a mercury thermometer that read exactly 0°C at the ice point and 100°C at the steam point. The distance between the two points is divided into 100 equal parts in both thermometers. Do you think these thermometers will give exactly the same reading at a temperature of, say, 60°C? Explain.

2–16C Consider two closed systems A and B. System A contains 3000 kJ of thermal energy at 20°C, whereas system B contains 200 kJ of thermal energy at 50°C. Now the systems are brought into contact with each other. Determine the direction of any heat transfer between the two systems.

2–17E Consider a system whose temperature is 18°C. Express this temperature in R, K, and °F.

2–18E Steam enters a heat exchanger at 300 K. What is the temperature of this steam in °F?

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* Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

2–19 The temperature of a system rises by 130°C during a heating process. Express this rise in temperature in kelvins.

2–20E The temperature of a system drops by 45°F during a cooling process. Express this drop in temperature in K, R, and °C.

2–21E The temperature of the lubricating oil in an automobile engine is measured as 150°F. What is the temperature of this oil in °C?

2–22E Heated air is at 150°C. What is the temperature of this air in °F?

Pressure, Manometer, and Barometer

2–23C What is the difference between gage pressure and absolute pressure?

2–24C Explain why some people experience nose bleeding and some others experience shortness of breath at high elevations.

2–25C A health magazine reported that physicians measured 100 adults' blood pressure using two different arm positions: parallel to the body (along the side) and perpendicular to the body (straight out). Readings in the parallel position were up to 10 percent higher than those in the perpendicular position, regardless of whether the patient was standing, sitting, or lying down. Explain the possible cause for the difference.

2–26C Someone claims that the absolute pressure in a liquid of constant density doubles when the depth is doubled. Do you agree? Explain.

2–27C Consider two identical fans, one at sea level and the other on top of a high mountain, running at identical speeds. How would you compare (*a*) the volume flow rates and (*b*) the mass flow rates of these two fans?

2–28E The absolute pressure in a compressed air tank is 200 kPa. What is this pressure in psia?

2–29E A manometer measures a pressure difference as 40 inches of water. What is this pressure difference in pound-force per square inch, psi? *Answer:* 1.44 psi

2–30 A vacuum gage connected to a chamber reads 35 kPa at a location where the atmospheric pressure is 92 kPa. Determine the absolute pressure in the chamber.

2–31E The maximum safe air pressure of a tire is typically written on the tire itself. The label on a tire indicates that the maximum pressure is 35 psi (gage). Express this maximum pressure in kPa.

FIGURE P2–31E

2–32E A pressure gage connected to a tank reads 50 psi at a location where the barometric reading is 29.1 in Hg. Determine the absolute pressure in the tank. Take $\rho_{Hg} = 848.4$ lbm/ft^3. *Answer:* 64.3 psia

2–33 A pressure gage connected to a tank reads 500 kPa at a location where the atmospheric pressure is 94 kPa. Determine the absolute pressure in the tank.

2–34E A 200-pound man has a total foot imprint area of 72 in^2. Determine the pressure this man exerts on the ground if (*a*) he stands on both feet and (*b*) he stands on one foot.

2–35 The gage pressure in a liquid at a depth of 3 m is read to be 42 kPa. Determine the gage pressure in the same liquid at a depth of 9 m.

2–36 The absolute pressure in water at a depth of 9 m is read to be 185 kPa. Determine (*a*) the local atmospheric pressure and (*b*) the absolute pressure at a depth of 5 m in a liquid whose specific gravity is 0.85 at the same location.

2–37 Consider a 1.75-m-tall man standing vertically in water and completely submerged in a pool. Determine the difference between the pressures acting at the head and at the toes of the man, in kPa.

2–38 The barometer of a mountain hiker reads 750 mbars at the beginning of a hiking trip and 650 mbars at the end. Neglecting the effect of altitude on local gravitational acceleration, determine the vertical distance climbed. Assume an average air density of 1.20 kg/m^3. *Answer:* 850 m

2–39 The basic barometer can be used to measure the height of a building. If the barometric readings at the top and at the bottom of a building are 675 and 695 mmHg, respectively, determine the height of the building. Take the densities of air and mercury to be 1.18 kg/m^3 and 13,600 kg/m^3, respectively.

FIGURE P2–39

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2–40 Solve Prob. 2–39 using appropriate software. Print out the entire solution, including the numerical results with proper units.

2–41 A gas is contained in a vertical, frictionless piston–cylinder device. The piston has a mass of 3.2 kg and a cross-sectional area of 35 cm^2. A compressed spring above the piston exerts a force of 150 N on the piston. If the atmospheric pressure is 95 kPa, determine the pressure inside the cylinder. *Answer:* 147 kPa

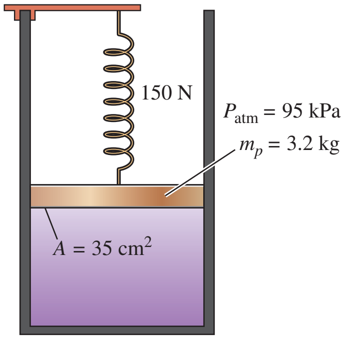

FIGURE P2–41

2–42 Reconsider Prob. 2–41. Using appropriate software, investigate the effect of the spring force in the range of 0 to 500 N on the pressure inside the cylinder. Plot the pressure against the spring force, and discuss the results.

2–43 The piston of a vertical piston–cylinder device containing a gas has a mass of 60 kg and a cross-sectional area of 0.04 m^2, as shown in Fig. P2–43. The local atmospheric pressure is 0.97 bar, and the gravitational acceleration is 9.81 m/s^2. (*a*) Determine the pressure inside the cylinder. (*b*) If some heat is transferred to the gas and its volume is doubled, do you expect the pressure inside the cylinder to change?

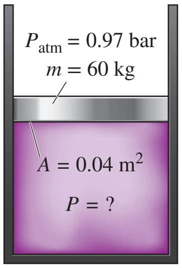

FIGURE P2–43

2–44 Both a gage and a manometer are attached to a gas tank to measure its pressure. If the reading on the pressure gage is 80 kPa, determine the distance between the two fluid levels of the manometer if the fluid is (*a*) mercury ($\rho = 13{,}600\ \text{kg/m}^3$) or (*b*) water ($\rho = 1000\ \text{kg/m}^3$).

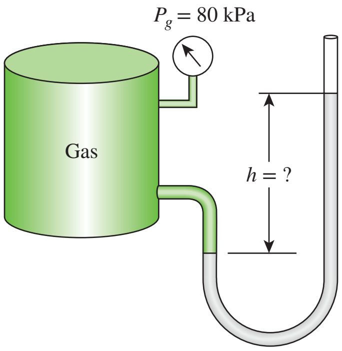

FIGURE P2–44

2–45 Reconsider Prob. 2–44. Using appropriate software, investigate the effect of the manometer fluid density in the range of 800 to 13,000 kg/m^3 on the differential fluid height of the manometer. Plot the differential fluid height against the density, and discuss the results.

2–46 A manometer containing oil ($\rho = 850\ \text{kg/m}^3$) is attached to a tank filled with air. If the oil-level difference between the two columns is 80 cm and the atmospheric pressure is 98 kPa, determine the absolute pressure of the air in the tank. *Answer:* 105 kPa

2–47E A manometer is used to measure the air pressure in a tank. The fluid used has a specific gravity of 1.25, and the differential height between the two arms of the manometer is 28 in. If the local atmospheric pressure is 12.7 psia, determine the absolute pressure in the tank for the cases of the manometer arm with the (*a*) higher and (*b*) lower fluid level being attached to the tank.

2–48 A mercury manometer ($\rho = 13{,}600\ \text{kg/m}^3$) is connected to an air duct to measure the pressure inside. The difference in the manometer levels is 30 mm, and the atmospheric pressure is 100 kPa. (*a*) Judging from Fig. P2–48, determine if the pressure in the duct is above or below the atmospheric pressure. (*b*) Determine the absolute pressure in the duct.

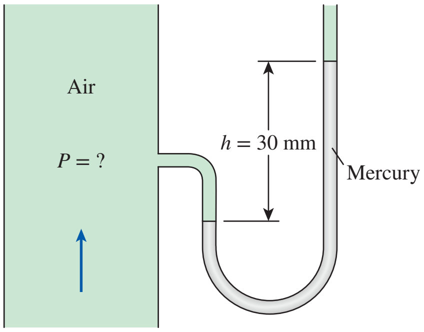

FIGURE P2–48

2–49 Repeat Prob. 2–48 for a differential mercury height of 45 mm.

2–50E The pressure in a natural gas pipeline is measured by the manometer shown in Fig. P2–50E with one of the arms open to the atmosphere where the local atmospheric pressure is 14.2 psia. Determine the absolute pressure in the pipeline.

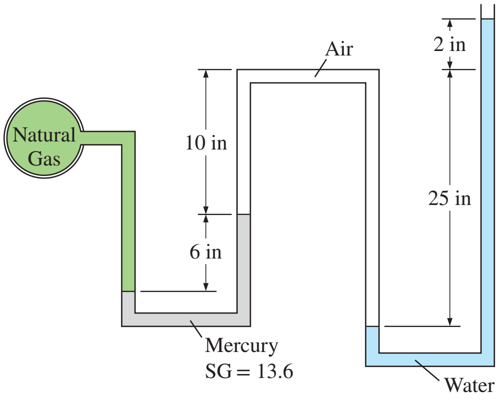

FIGURE P2–50E

2–51E Repeat Prob. 2–50E by replacing air with oil with a specific gravity of 0.69.

2–52E Blood pressure is usually measured by wrapping a closed air-filled jacket equipped with a pressure gage around the upper arm of a person at the level of the heart. Using a mercury manometer and a stethoscope, the systolic pressure (the maximum pressure when the heart is pumping) and the diastolic pressure (the minimum pressure when the heart is resting) are measured in mmHg. The systolic and diastolic pressures of a healthy person are about 120 mmHg and 80 mmHg, respectively, and are indicated as 120/80. Express both of these gage pressures in kPa, psi, and meter water column.

2–53 The maximum blood pressure in the upper arm of a healthy person is about 120 mmHg. If a vertical tube open to the atmosphere is connected to the vein in the arm of the person, determine how high the blood will rise in the tube. Take the density of the blood to be 1050 kg/m^3.

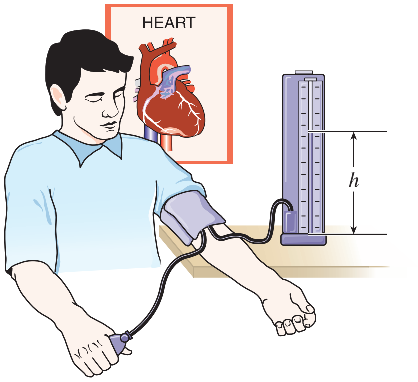

FIGURE P2–53

2–54 Consider a U-tube whose arms are open to the atmosphere. Now water is poured into the U-tube from one arm, and light oil (ρ = 790 kg/m^3) from the other. One arm contains 70-cm-high water, while the other arm contains both fluids with an oil-to-water height ratio of 4. Determine the height of each fluid in that arm.

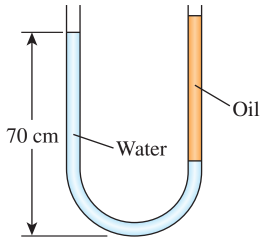

FIGURE P2–54

2–55 Consider a double-fluid manometer attached to an air pipe shown in Fig. P2–55. If the specific gravity of one fluid is 13.55, determine the specific gravity of the other fluid for the indicated absolute pressure of air. Take the atmospheric pressure to be 100 kPa. *Answer:* 1.59

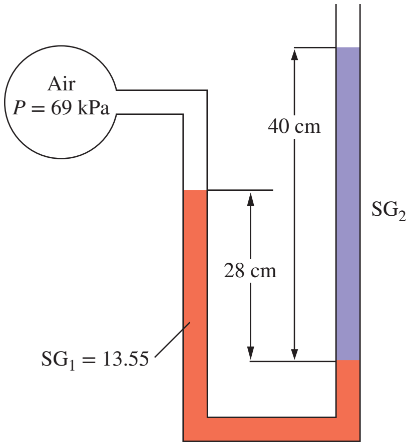

FIGURE P2–55

2–56 The hydraulic lift in a car repair shop has an output diameter of 30 cm and is to lift cars up to 2500 kg. Determine the fluid gage pressure that must be maintained in the reservoir.

2–57 Consider the system shown in Fig. P2–57. If a change of 0.7 kPa in the pressure of air causes the brine–mercury interface in the right column to drop by 5 mm in the brine level in the right column while the pressure in the brine pipe remains constant, determine the ratio of A_2/A_1.

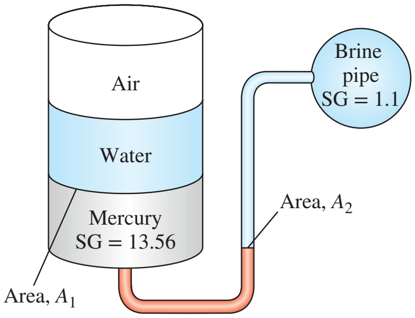

FIGURE P2–57

2–58 The gage pressure of the air in the tank shown in Fig. P2–58 is measured to be 80 kPa. Determine the differential height h of the mercury column.

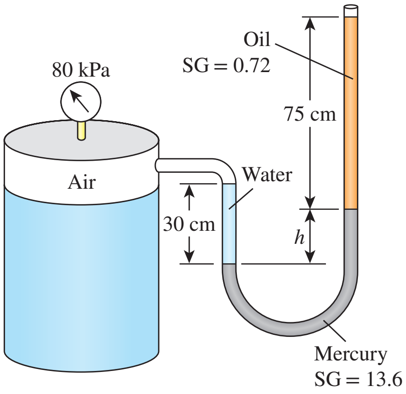

FIGURE P2–58

2–59 Repeat Prob. 2–58 for a gage pressure of 40 kPa.

Review Problems

2–60E The pressure in a steam boiler is given to be 92 kgf/cm^2. Express this pressure in psi, kPa, atm, and bars.

2–61 A hydraulic lift is to be used to lift a 1900-kg weight by putting a weight of 25 kg on a piston with a diameter of 10 cm. Determine the diameter of the piston on which the weight is to be placed.

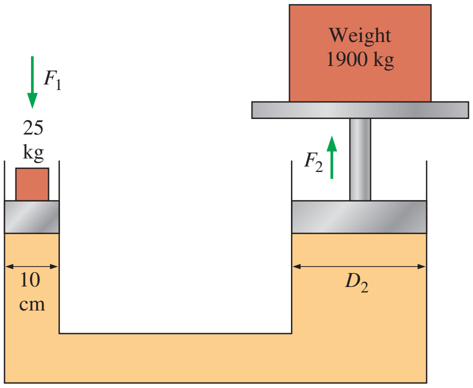

FIGURE P2–61

2–62 The average atmospheric pressure on earth is approximated as a function of altitude by the relation $P_{\text{atm}} = 101.325 (1 - 0.02256z)^{5.256}$, where P_{atm} is the atmospheric pressure in kPa and z is the altitude in km with $z = 0$ at sea level. Determine the approximate atmospheric pressures at Atlanta ($z = 306$ m), Denver ($z = 1610$ m), Mexico City ($z = 2309$ m), and the top of Mount Everest ($z = 8848$ m).

2–63E Hyperthermia of 5°C (i.e., 5°C rise above the normal body temperature) is considered fatal. Express this fatal level of hyperthermia in (*a*) K, (*b*) °F, and (*c*) R.

2–64E The boiling temperature of water decreases by about 3°C for each 1000-m rise in altitude. What is the decrease in the boiling temperature in (*a*) K, (*b*) °F, and (*c*) R for each 1000-m rise in altitude?

2–65E A house is losing heat at a rate of 1800 kJ/h per °C temperature difference between the indoor and the outdoor temperatures. Express the rate of heat loss from this house per (*a*) K, (*b*) °F, and (*c*) R difference between the indoor and the outdoor temperature.

2–66E The average body temperature of a person rises by about 2°C during strenuous exercise. What is the rise in the body temperature in (*a*) K, (*b*) °F, and (*c*) R during strenuous exercise?

2–67 The average temperature of the atmosphere in the world is approximated as a function of altitude by the relation

$$T_{\text{atm}} = 288.15 - 6.5z$$

where T_{atm} is the temperature of the atmosphere in K and z is the altitude in km with $z = 0$ at sea level. Determine the average temperature of the atmosphere outside an airplane that is cruising at an altitude of 12,000 m.

2–68 A vertical, frictionless piston–cylinder device contains a gas at 180 kPa absolute pressure. The atmospheric pressure outside is 100 kPa, and the piston area is 25 cm^2. Determine the mass of the piston.

2–69 A vertical piston–cylinder device contains a gas at a pressure of 100 kPa. The piston has a mass of 10 kg and a diameter of 14 cm. Pressure of the gas is to be increased by placing some weights on the piston. Determine the local atmospheric pressure and the mass of the weights that will double the pressure of the gas inside the cylinder. *Answers:* 93.6 kPa, 157 kg

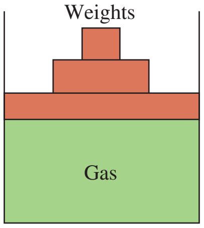

FIGURE P2–69

2–70 The force generated by a spring is given by $F = kx$, where k is the spring constant and x is the deflection of the spring. The spring of Fig. P2–70 has a spring constant of 8 kN/cm. The pressures are $P_1 = 5000$ kPa, $P_2 = 10{,}000$ kPa, and $P_3 = 1000$ kPa. If the piston diameters are $D_1 = 8$ cm and $D_2 = 3$ cm, how far will the spring be deflected? *Answer:* 1.72 cm

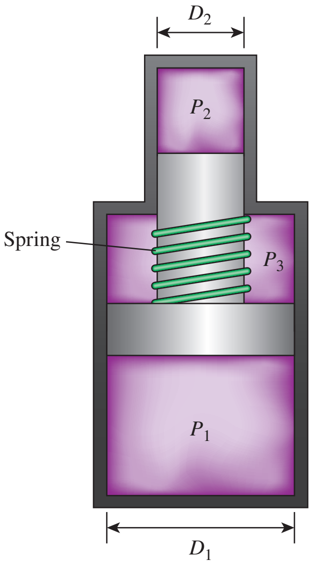

FIGURE P2–70

2–71 An air-conditioning system requires a 35-m-long section of 15-cm-diameter ductwork to be laid underwater. Determine the upward force the water will exert on the duct. Take the densities of air and water to be 1.3 kg/m^3 and 1000 kg/m^3, respectively.

2–72 Balloons are often filled with helium gas because it weighs only about one-seventh of what air weighs under identical conditions. The buoyancy force, which can be expressed as $F_b = \rho_{\text{air}} g V_{\text{balloon}}$, will push the balloon upward. If the balloon has a diameter of 12 m and carries two people, 85 kg each, determine the acceleration of the balloon when it is first released. Assume the density of air is $\rho = 1.16$ kg/m^3, and neglect the weight of the ropes and the cage. *Answer:* 22.4 m/s^2

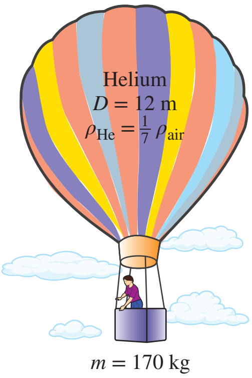

FIGURE P2–72

2–73 Reconsider Prob. 2–72. Using appropriate software, investigate the effect of the number of people carried in the balloon on acceleration. Plot the acceleration against the number of people, and discuss the results.

2–74 Determine the maximum amount of load, in kg, the balloon described in Prob. 2–72 can carry. *Answer:* 900 kg

2–75 The lower half of a 6-m-high cylindrical container is filled with water ($\rho = 1000$ kg/m^3) and the upper half with oil that has a specific gravity of 0.85. Determine the pressure difference between the top and bottom of the cylinder. *Answer:* 54.4 kPa

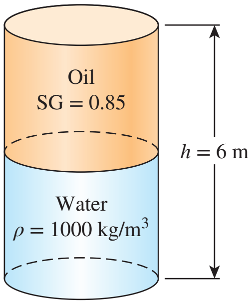

FIGURE P2–75

2–76 A pressure cooker cooks a lot faster than an ordinary pan by maintaining a higher pressure and temperature inside. The lid of a pressure cooker is well sealed, and steam can escape only through an opening in the middle of the lid. A separate metal piece, the petcock, sits on top of this opening and prevents steam from escaping until the pressure force overcomes the weight of the petcock. The periodic escape of the steam in this manner prevents any potentially dangerous pressure buildup and keeps the pressure inside at a constant value. Determine the mass of the petcock of a pressure cooker whose operation pressure is 100 kPa gage and has an opening cross-sectional area of 4 mm^2. Assume an atmospheric pressure of 101 kPa, and draw the free-body diagram of the petcock. *Answer:* 40.8 g

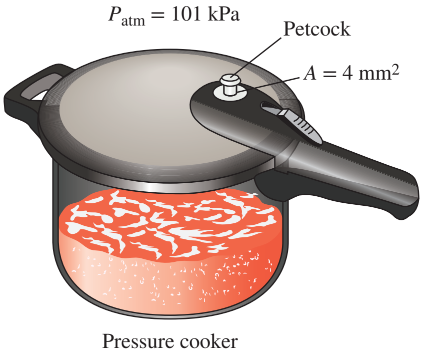

FIGURE P2–76

2–77 The pilot of an airplane reads the altitude 6400 m and the absolute pressure 45 kPa when flying over a city. Calculate the local atmospheric pressure in that city in kPa and in mmHg. Take the densities of air and mercury to be 0.828 kg/m^3 and 13,600 kg/m^3, respectively.

FIGURE P2–77

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2–78 A glass tube is attached to a water pipe, as shown in Fig. P2–78. If the water pressure at the bottom of the tube is 107 kPa and the local atmospheric pressure is 99 kPa, determine how high the water will rise in the tube, in m. Take the density of water to be 1000 kg/m^3.

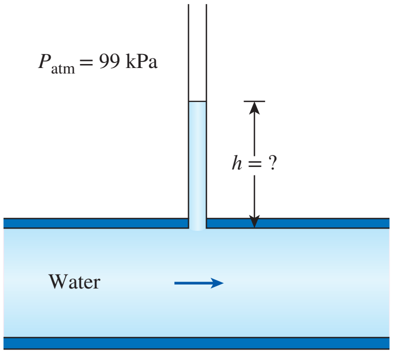


FIGURE P2–78

2–79E Consider a U-tube whose arms are open to the atmosphere. Now equal volumes of water and light oil ($\rho = 49.3$ lbm/ft^3) are poured from different arms. A person blows from the oil side of the U-tube until the contact surface of the two fluids moves to the bottom of the U-tube, and thus the liquid levels in the two arms are the same. If the fluid height in each arm is 30 in, determine the gage pressure the person exerts on the oil by blowing.

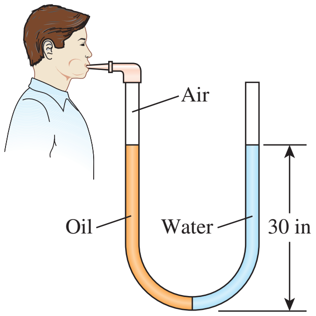


FIGURE P2–79E

2–80E A water pipe is connected to a double-U manometer as shown in Fig. P2–80E at a location where the local atmospheric pressure is 14.2 psia. Determine the absolute pressure at the center of the pipe.

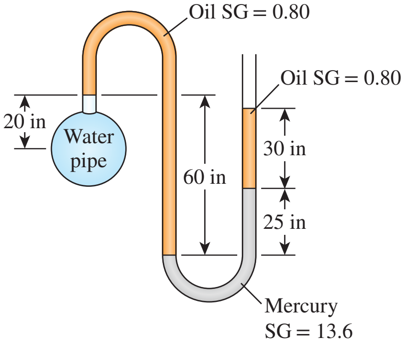


FIGURE P2–80E

2–81 A gasoline line is connected to a pressure gage through a double-U manometer, as shown in Fig. P2–81. If the reading of the pressure gage is 370 kPa, determine the gage pressure of the gasoline line.

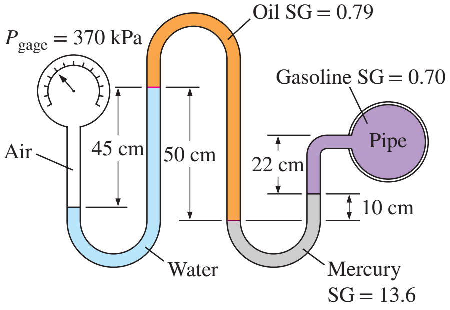


FIGURE P2–81

2–82 Repeat Prob. 2–81 for a pressure gage reading of 180 kPa.

2–83 When measuring small pressure differences with a manometer, often one arm of the manometer is inclined to improve the accuracy of reading. (The pressure difference is still proportional to the *vertical* distance and not the actual length of the fluid along the tube.) The air pressure in a circular duct is to be measured using a manometer whose open arm is inclined 45° from the horizontal, as shown in Fig. P2–83. The density of the liquid in the manometer is 0.81 kg/L, and the vertical distance between the fluid levels in the two arms of the manometer is 12 cm. Determine the gage pressure of air in the duct and the length of the fluid column in the inclined arm above the fluid level in the vertical arm.

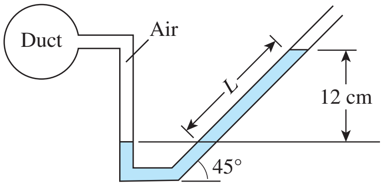


FIGURE P2–83

2–84E It is well known that cold air feels much colder in windy weather than what the thermometer reading indicates because of the "chilling effect" of the wind. This effect is due to the increase in the convection heat transfer coefficient with increasing air velocities. The *equivalent wind chill temperature* in °F is given by [ASHRAE, *Handbook of Fundamentals* (Atlanta, GA, 1993), p. 8.15]

$$T_{\text{equiv}} = 91.4 - (91.4 - T_{\text{ambient}}) \times (0.475 - 0.0203V + 0.304\sqrt{V})$$

where V is the wind velocity in mi/h and T_{ambient} is the ambient air temperature in °F in calm air, which is taken to be air with light winds at speeds up to 4 mi/h. The constant 91.4°F in the given equation is the mean skin temperature of a resting person in a comfortable environment. Windy air at temperature T_{ambient} and velocity V will feel as cold as the calm air at temperature T_{equiv}. Using proper conversion factors, obtain an equivalent relation in SI units where V is the wind velocity in km/h and T_{ambient} is the ambient air temperature in °C.

Answer: $T_{\text{equiv}} = 33.0 - (33.0 - T_{\text{ambient}}) \times (0.475 - 0.0126V + 0.240\sqrt{V})$

2–85E Reconsider Prob. 2–84E. Using appropriate software, plot the equivalent wind chill temperatures in °F as a function of wind velocity in the range of 4 to 40 mph for the ambient temperatures of 20, 40, and 60°F. Discuss the results.

Design and Essay Problems

2–86 Write an essay on different temperature measurement devices. Explain the operational principle of each device, its advantages and disadvantages, its cost, and its range of applicability. Which device would you recommend for use in the following cases: taking the temperatures of patients in a doctor's office, monitoring the variations of temperature of a car engine block at several locations, and monitoring the temperatures in the furnace of a power plant?

CHAPTER

3

ENERGY, ENERGY TRANSFER, AND GENERAL ENERGY ANALYSIS

Whether we realize it or not, energy is an important part of most aspects of daily life. The quality of life, and even its sustenance, depends on the availability of energy. Therefore, it is important to have a good understanding of the sources of energy, the conversion of energy from one form to another, and the ramifications of these conversions.

Energy exists in numerous forms such as thermal, mechanical, electric, chemical, and nuclear. Even mass can be considered a form of energy. Energy can be transferred to or from a closed system (a fixed mass) in two distinct forms: *heat* and *work*. For control volumes, energy can also be transferred by mass flow. An energy transfer to or from a closed system is *heat* if it is caused by a temperature difference. Otherwise it is *work,* and it is caused by a force acting through a distance.

We start this chapter with a discussion of various forms of energy and energy transfer by heat. We then introduce various forms of work and discuss energy transfer by work. We continue with developing a general intuitive expression for the *first law of thermodynamics,* also known as the *conservation of energy principle,* which is one of the most fundamental principles in nature, and we then demonstrate its use. Finally, we discuss the efficiencies of some familiar energy conversion processes. Detailed treatments of the first law of thermodynamics for closed systems and control volumes are given in Chaps. 5 and 6, respectively.

It should be kept in mind that physical laws or the laws of nature, such as the first law of thermodynamics, are universally accepted statements related to observed phenomena. They are conclusions drawn on the basis of numerous scientific experiments and observations over a long period of time. A physical law dictates that a particular phenomenon always occurs when specified conditions are present. Everything in the universe obeys them with no exception, and there can be no violations. As such, physical laws serve as powerful predictive tools, enabling scientists to predict how things will happen in the physical universe before they happen. They remain unchanged since first discovered, and nothing in nature seems to affect the laws of nature.

OBJECTIVES

The objectives of this chapter are to:

- Introduce the concept of energy and define its various forms.
- Discuss the nature of internal energy.
- Define the concept of heat and the terminology associated with energy transfer by heat.
- Define the concept of work, including electrical work and several forms of mechanical work.
- Introduce the first law of thermodynamics, energy balances, and mechanisms of energy transfer to or from a system.
- Determine that a fluid flowing across a control surface of a control volume carries energy across the control surface in addition to any energy transfer across the control surface that may be in the form of heat and/or work.
- Define energy conversion efficiencies.

3–1 ▪ INTRODUCTION

We are familiar with the conservation of energy principle, which is an expression of the first law of thermodynamics, back from our high school years. We are told repeatedly that energy cannot be created or destroyed during a process; it can only change from one form to another. This seems simple enough, but let's test ourselves to see how well we understand and truly believe in this principle.

Consider a room whose door and windows are tightly closed, and whose walls are well-insulated so that heat loss or gain through the walls is negligible. Now let's place a refrigerator in the middle of the room with its door open, and plug it into a wall outlet (Fig. 3–1). You may even use a small fan to circulate the air in order to maintain temperature uniformity in the room. Now, what do you think will happen to the average temperature of air in the room? Will it be increasing or decreasing? Or will it remain constant?

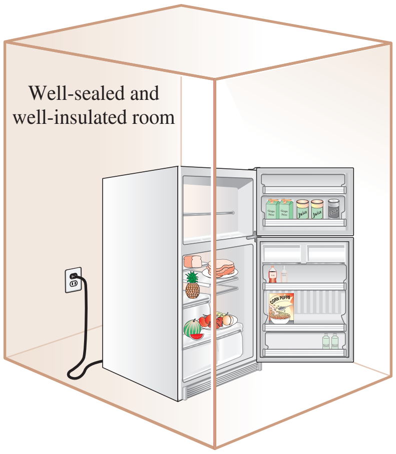


FIGURE 3–1
A refrigerator operating with its door open in a well-sealed and well-insulated room.

Probably the first thought that comes to mind is that the average air temperature in the room will decrease as the warmer room air mixes with the air cooled by the refrigerator. Some may draw our attention to the heat generated by the motor of the refrigerator, and may argue that the average air temperature may rise if this heating effect is greater than the cooling effect. But they will get confused if it is stated that the motor is made of superconducting materials, and thus there is hardly any heat generation in the motor.

Heated discussion may continue with no end in sight until we remember the conservation of energy principle that we take for granted: If we take the entire room—including the air and the refrigerator—as the system, which is an adiabatic closed system since the room is well-sealed and well-insulated, the only energy interaction involved is the electrical energy crossing the system boundary and entering the room. The conservation of energy requires the energy content of the room to increase by an amount equal to the amount of the electrical energy drawn by the refrigerator, which can be measured by an ordinary electric meter. The refrigerator or its motor does not store this energy. Therefore, this energy must now be in the room air, and it will manifest itself as a rise in the air temperature. The temperature rise of air can be calculated on the basis of the conservation of energy principle using the properties of air and the amount of electrical energy consumed. What do you think would happen if we had a window air conditioning unit instead of a refrigerator placed in the middle of this room? What if we operated a fan in this room instead (Fig. 3–2)?

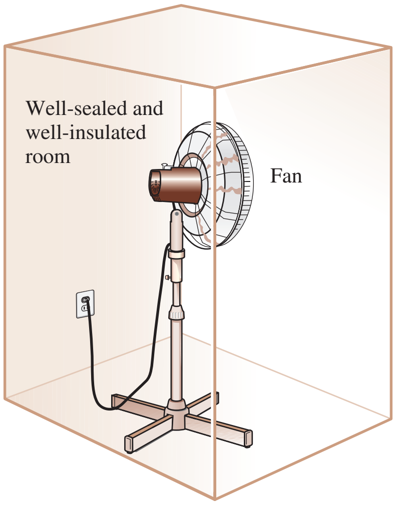


FIGURE 3–2
A fan running in a well-sealed and well-insulated room will raise the temperature of air in the room.

Note that energy is conserved during the process of operating the refrigerator placed in a room—the electrical energy is converted into an equivalent amount of thermal energy stored in the room air. If energy is already conserved, then what are all those speeches about on energy conservation and the measures taken to conserve energy? Actually, by "energy conservation" what is meant is the conservation of the *quality* of energy, not the quantity. Electricity, which is of the highest quality of energy, for example, can always be converted to an equal amount of thermal energy (also called *heat*). But only a fraction of thermal energy, which is the lowest quality of energy, can be converted back to electricity, as we discuss in Chap. 7. Think about the things that you can do with the electrical energy that the refrigerator has consumed, and the air in the room that is now at a higher temperature.

Now if asked to name the energy transformations associated with the operation of a refrigerator, we may still have a hard time answering because all we see is electrical energy entering the refrigerator and heat dissipated from the refrigerator to the room air. Obviously there is need to study the various forms of energy first, and this is exactly what we do next, followed by a study of the mechanisms of energy transfer.

3–2 ▪ FORMS OF ENERGY

Energy can exist in numerous forms such as thermal, mechanical, kinetic, potential, electric, magnetic, chemical, and nuclear (Fig. 3–3), and their sum constitutes the **total energy** E of a system. The total energy of a system on a *unit mass* basis is denoted by e and is expressed as

$$e = \frac{E}{m} \qquad \text{(kJ/kg)} \tag{3–1}$$

Thermodynamics provides no information about the absolute value of the total energy. It deals only with the *change* of the total energy, which is what matters in engineering problems. Thus the total energy of a system can be assigned a value of zero ($E = 0$) at some convenient reference point. The change in total energy of a system is independent of the reference point selected. The decrease in the potential energy of a falling rock, for example, depends on only the elevation difference and not the reference level selected.

In thermodynamic analysis, it is often helpful to consider the various forms of energy that make up the total energy of a system in two groups: *macroscopic* and *microscopic*. The **macroscopic** forms of energy are those a system possesses as a whole with respect to some outside reference frame, such as kinetic and potential energies (Fig. 3–4). The **microscopic** forms of energy are those related to the molecular structure of a system and the degree of the molecular activity, and they are independent of outside reference frames. The sum of all the microscopic forms of energy is called the **internal energy** of a system and is denoted by U.

The term *energy* was coined in 1807 by Thomas Young, and its use in thermodynamics was proposed in 1852 by Lord Kelvin. The term *internal energy* and its symbol U first appeared in the works of Rudolph Clausius and William Rankine in the second half of the 19th century, and it eventually replaced the alternative terms *inner work, internal work,* and *intrinsic energy* commonly used at the time.

The macroscopic energy of a system is related to motion and the influence of some external effects such as gravity, magnetism, electricity, and surface tension. The energy that a system possesses as a result of its motion relative to some reference frame is called **kinetic energy** (KE). When all parts of a system move with the same velocity, the kinetic energy is expressed as

$$\text{KE} = m\frac{V^2}{2} \qquad \text{(kJ)} \tag{3–2}$$

or, on a unit mass basis,

$$\text{ke} = \frac{V^2}{2} \qquad \text{(kJ/kg)} \tag{3–3}$$

where V denotes the velocity of the system relative to some fixed reference frame. The kinetic energy of a rotating solid body is given by $\frac{1}{2}I\omega^2$ where I is the moment of inertia of the body and ω is the angular velocity.

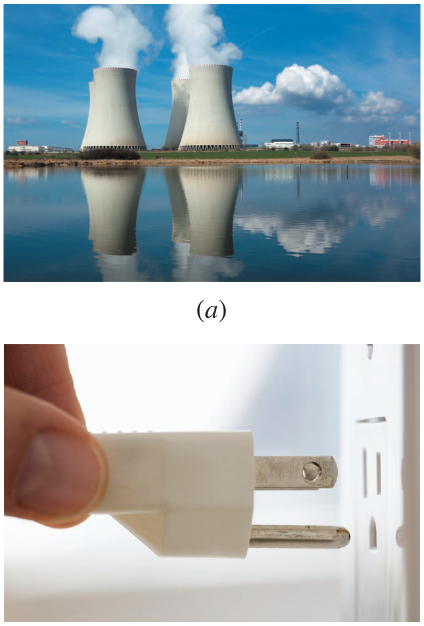
The energy that a system possesses as a result of its elevation in a gravitational field is called **potential energy** (PE) and is expressed as

$$\text{PE} = mgz \qquad \text{(kJ)} \tag{3–4}$$

or, on a unit mass basis,

$$\text{pe} = gz \qquad \text{(kJ/kg)} \tag{3–5}$$

where g is the gravitational acceleration and z is the elevation of the center of gravity of a system relative to some arbitrarily selected reference level.


(*a*)

(*b*)

FIGURE 3–3
At least six different forms of energy are encountered in bringing power from a nuclear plant to your home: nuclear, thermal, mechanical, kinetic, magnetic, and electrical.

(a) hornyak/Shutterstock
(b) Tetra Images/Getty Images

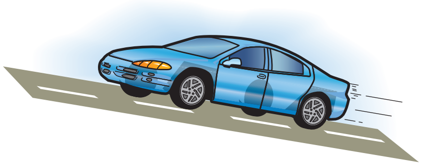


FIGURE 3–4
The macroscopic energy of an object changes with velocity and elevation.

The magnetic, electric, and surface tension effects are significant in some specialized cases only and are usually ignored. In the absence of such effects, the total energy of a system consists of the kinetic, potential, and internal energies and is expressed as

$$E = U + \mathrm{KE} + \mathrm{PE} = U + m\frac{V^2}{2} + mgz \qquad (\mathrm{kJ}) \tag{3–6}$$

or, on a unit mass basis,

$$e = u + \mathrm{ke} + \mathrm{pe} = u + \frac{V^2}{2} + gz \qquad (\mathrm{kJ/kg}) \tag{3–7}$$

Most closed systems remain stationary during a process and thus experience no change in their kinetic and potential energies. Closed systems whose velocity and elevation of the center of gravity remain constant during a process are often referred to as **stationary systems**. The change in the total energy ΔE of a stationary system is identical to the change in its internal energy ΔU. In this text, a closed system is assumed to be stationary unless stated otherwise.

Control volumes typically involve fluid flow for long periods of time, and it is convenient to express the energy flow associated with a fluid stream in the rate form. This is done by incorporating the **mass flow rate** $\dot{m}$, which is *the amount of mass flowing through a cross section per unit time*. It is related to the **volume flow rate** $\dot{V}$, which is the volume of a fluid flowing through a cross section per unit time, by

Mass flow rate: $$\dot{m} = \rho\dot{V} = \rho A_c V_{\mathrm{avg}} \qquad (\mathrm{kg/s}) \tag{3–8}$$

which is analogous to $m = \rho V$. Here ρ is the fluid density, A_c is the cross-sectional area of flow, and V_{avg} is the average flow velocity normal to A_c. The dot over a symbol is used to indicate *time rate* throughout the book. Then the energy flow rate associated with a fluid flowing at a rate of $\dot{m}$ is (Fig. 3–5)

Energy flow rate: $$\dot{E} = \dot{m}e \qquad (\mathrm{kJ/s\ or\ kW}) \tag{3–9}$$

which is analogous to $E = me$.

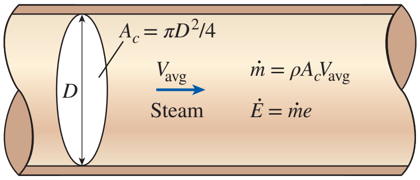


FIGURE 3–5
Mass and energy flow rates associated with the flow of steam in a pipe of inner diameter D with an average velocity of V_{avg}.

Some Physical Insight into Internal Energy

Internal energy was defined earlier as the sum of all the *microscopic* forms of energy of a system. It is related to the *molecular structure* and the degree of *molecular activity* and can be viewed as the sum of the *kinetic* and *potential* energies of the molecules.

To have a better understanding of internal energy, let us examine a system at the molecular level. The molecules of a gas move through space with some velocity, and thus they possess some kinetic energy. This is known as *translational energy*. The atoms of polyatomic molecules rotate about an axis, and the energy associated with this rotation is *rotational kinetic energy*. The atoms of a polyatomic molecule may also vibrate about their common center of mass, and the energy associated with this back-and-forth motion is *vibrational kinetic energy*. For gases, kinetic energy is mostly due to translational and rotational motions, with vibrational motion becoming significant at higher temperatures. The electrons in an atom rotate about the nucleus and thus possess rotational kinetic energy. Electrons at outer orbits have larger kinetic energies. Electrons also spin about their axes, and the energy associated with this motion is *spin energy*. Other particles in the nucleus of an atom also possess spin energy. The portion of the internal energy of a system associated with the kinetic energies of the molecules is called **sensible energy** (Fig. 3–6). The average velocity and the degree of activity of the

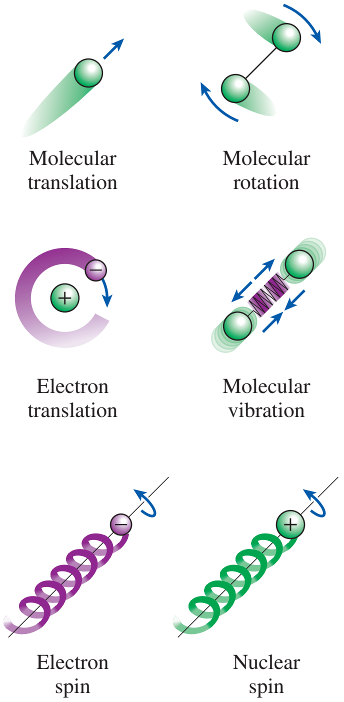

Molecular translation
Molecular rotation
Electron translation
Molecular vibration
Electron spin
Nuclear spin

FIGURE 3–6
The various forms of microscopic energies that make up *sensible* energy.

molecules are proportional to the temperature of the gas. Therefore, at higher temperatures, the molecules possess higher kinetic energies, and as a result the system has a higher internal energy.

Internal energy is also associated with various *binding forces* between the molecules of a substance, between the atoms within a molecule, and between the particles within an atom and its nucleus. The forces that bind the *molecules* to each other are, as one would expect, strongest in solids and weakest in gases. If sufficient energy is added to the molecules of a solid or liquid, the molecules overcome these molecular forces and break away, turning the substance into a gas. This is a phase-change process. Because of this added energy, a system in the gas phase is at a higher internal energy level than it is in the solid or the liquid phase. The internal energy associated with the phase of a system is called **latent energy**. The phase-change process can occur without a change in the chemical composition of a system. Most practical problems fall into this category, and one does not need to pay any attention to the forces binding the atoms in a molecule to each other.

An atom consists of neutrons and positively charged protons bound together by very strong nuclear forces in the nucleus, and negatively charged electrons orbiting around it. The internal energy associated with the atomic bonds in a molecule is called **chemical energy**. During a chemical reaction, such as a combustion process, some chemical bonds are destroyed while others are formed. As a result, the internal energy changes. The nuclear forces are much larger than the forces that bind the electrons to the nucleus. The tremendous amount of energy associated with the strong bonds within the nucleus of the atom itself is called **nuclear energy** (Fig. 3–7). Obviously, we need not be concerned with nuclear energy in thermodynamics unless, of course, we deal with fusion or fission reactions. A chemical reaction involves changes in the structure of the electrons of the atoms, but a nuclear reaction involves changes in the core or nucleus. Therefore, an atom preserves its identity during a chemical reaction but loses it during a nuclear reaction. Atoms may also possess *electric* and *magnetic dipole-moment energies* when subjected to external electric and magnetic fields due to the twisting of the magnetic dipoles produced by the small electric currents associated with the orbiting electrons.

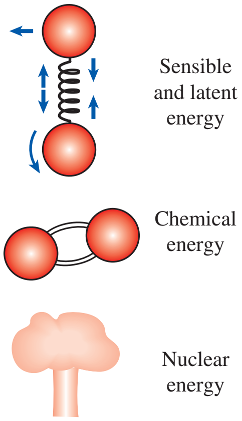


FIGURE 3–7
The internal energy of a system is the sum of all forms of the microscopic energies.

The forms of energy already discussed, which constitute the total energy of a system, can be *contained* or *stored* in a system, and thus can be viewed as the *static* forms of energy. The forms of energy not stored in a system can be viewed as the *dynamic* forms of energy or as *energy interactions.* The dynamic forms of energy are recognized at the system boundary as they cross it, and they represent the energy gained or lost by a system during a process. The only two forms of energy interactions associated with a closed system are **heat transfer** and **work**. An energy interaction is heat transfer if its driving force is a temperature difference. Otherwise it is work, as explained in the next section. A control volume can also exchange energy via mass transfer since any time mass is transferred into or out of a system, the energy content of the mass is also transferred with it.

In daily life, we often refer to the sensible and latent forms of internal energy as *heat,* and we talk about the heat content of bodies. In thermodynamics, however, we usually refer to those forms of energy as **thermal energy** to prevent any confusion with *heat transfer.*

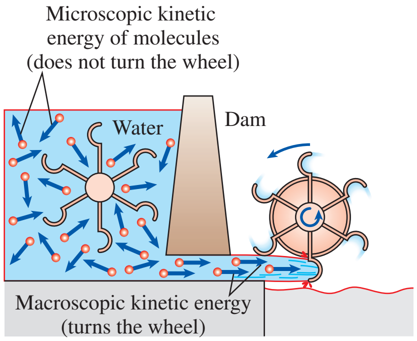


FIGURE 3–8
The *macroscopic* kinetic energy is an organized form of energy and is much more useful than the disorganized *microscopic* kinetic energies of the molecules.

A distinction should be made between the macroscopic kinetic energy of an object as a whole and the microscopic kinetic energies of its molecules that constitute the sensible internal energy of the object (Fig. 3–8). The kinetic energy of an object is an *organized* form of energy associated with the orderly motion of all molecules in one direction in a straight path or around an axis. In contrast, the kinetic energies of the molecules are completely *random* and highly *disorganized.* As you will see in later chapters, the organized energy is much more valuable than the disorganized energy, and a major application area of thermodynamics is the conversion of disorganized energy (heat) into organized energy (work). You will also see that the organized energy can be converted to disorganized energy completely, but only a fraction of disorganized energy can be converted to organized energy by specially built devices called *heat engines* (like car engines and power plants). A similar argument can be given for the macroscopic potential energy of an object as a whole and the microscopic potential energies of the molecules.

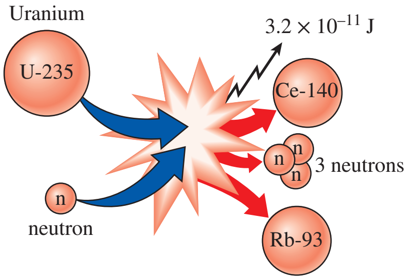
More on Nuclear Energy

The best-known fission reaction involves the splitting of the uranium atom (the U-235 isotope) into other elements. It is commonly used to generate electricity in nuclear power plants (450 reactors in 2016 with 392,000 MW capacity), to power nuclear submarines and aircraft carriers, and even to power spacecraft, in addition to its use in nuclear bombs. The percentage of electricity produced by nuclear power is 76 percent in France, 19 percent in Russia and the U.K., 14 percent in Germany, and 20 percent in the United States.

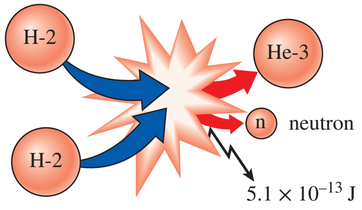
The first nuclear chain reaction was achieved by Enrico Fermi in 1942, and the first large-scale nuclear reactors were built in 1944 for the purpose of producing material for nuclear weapons. When a uranium-235 atom absorbs a neutron and splits during a fission process, it produces a cesium-140 atom, a rubidium-93 atom, three neutrons, and 3.2×10^{-11} J of energy. In practical terms, the complete fission of 1 kg of uranium-235 releases 8.314×10^{10} kJ of heat, which is more than the heat released when 3700 tons of coal are burned. Therefore, for the same amount of fuel, a nuclear fission reaction releases several million times more energy than a chemical reaction. The safe disposal of used nuclear fuel, however, remains a concern.

Nuclear energy by fusion is released when two small nuclei combine into a larger one. The huge amount of energy radiated by the sun and the other stars originates from such a fusion process, which involves the combination of two hydrogen nuclei into a helium nucleus. When two heavy hydrogen (deuterium) nuclei combine during a fusion process, they produce a helium-3 atom, a free neutron, and 5.1×10^{-13} J of energy (Fig. 3–9).

Fusion reactions are much more difficult to achieve in practice because of the strong repulsion between the positively charged nuclei, called the *Coulomb repulsion.* To overcome this repulsive force and to enable the two nuclei to fuse together, the energy level of the nuclei must be raised by heating them to about 100 million °C. But such high temperatures are found only in the center of stars or in exploding atomic bombs (the A-bomb). In fact, the uncontrolled fusion reaction in a hydrogen bomb (the H-bomb) is initiated by a small atomic bomb. The uncontrolled fusion reaction was achieved in the early 1950s, but all the efforts since then to achieve controlled fusion by massive lasers, powerful magnetic fields, and electric currents to generate power have failed.



(*a*) Fission of uranium



(*b*) Fusion of hydrogen

FIGURE 3–9
The fission of uranium and the fusion of hydrogen during nuclear reactions, and the release of nuclear energy.

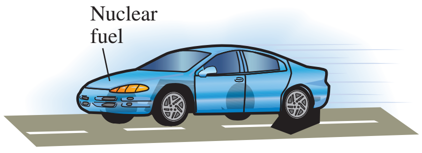


FIGURE 3–10
Schematic for Example 3–1.

EXAMPLE 3–1 A Car Powered by Nuclear Fuel

An average car consumes about 5 L of gasoline a day, and the capacity of the fuel tank of a car is about 50 L. Therefore, a car needs to be refueled once every 10 days. Also, the density of gasoline ranges from 0.68 to 0.78 kg/L, and its lower heating value is about 44,000 kJ/kg (that is, 44,000 kJ of heat is released when 1 kg of gasoline is completely burned). Suppose all the problems associated with the radioactivity and waste disposal of nuclear fuels are resolved, and a car is to be powered by U-235. If a new car comes equipped with 0.1 kg of the nuclear fuel U-235, determine if this car will ever need refueling under average driving conditions (Fig. 3–10).

SOLUTION A car powered by nuclear energy comes equipped with nuclear fuel. It is to be determined if this car will ever need refueling.

Assumptions **1** Gasoline is an incompressible substance with an average density of 0.75 kg/L. **2** Nuclear fuel is completely converted to thermal energy.

Analysis The mass of gasoline used per day by the car is

$$m_{\text{gasoline}} = (\rho V)_{\text{gasoline}} = (0.75\ \text{kg/L})(5\ \text{L/day}) = 3.75\ \text{kg/day}$$

Noting that the heating value of gasoline is 44,000 kJ/kg, the energy supplied to the car per day is

$$\begin{aligned} E &= (m_{\text{gasoline}})(\text{Heating value}) \\ &= (3.75\ \text{kg/day})(44{,}000\ \text{kJ/kg}) = 165{,}000\ \text{kJ/day} \end{aligned}$$

The complete fission of 0.1 kg of uranium-235 releases

$$(8.314 \times 10^{10}\ \text{kJ/kg})(0.1\ \text{kg}) = 8.314 \times 10^{9}\ \text{kJ}$$

of heat, which is sufficient to meet the energy needs of the car for

$$\text{No. of days} = \frac{\text{Energy content of fuel}}{\text{Daily energy use}} = \frac{8.314 \times 10^{9}\ \text{kJ}}{165{,}000\ \text{kJ/day}} = \mathbf{50{,}390\ days}$$

which is equivalent to about 138 years. Considering that no car will last more than 100 years, this car will never need refueling. It appears that nuclear fuel of the size of a cherry is sufficient to power a car during its lifetime.

Discussion Note that this problem is not quite realistic since the necessary critical mass cannot be achieved with such a small amount of fuel. Further, all of the uranium cannot be converted in fission, again because of the critical mass problems after partial conversion.

Mechanical Energy

Many engineering systems are designed to transport a fluid from one location to another at a specified flow rate, velocity, and elevation difference, and the system may generate mechanical work in a turbine or it may consume mechanical work in a pump or fan during this process (Fig. 3–11). These systems do not involve the conversion of nuclear, chemical, or thermal energy to mechanical energy. Also, they do not involve any heat transfer in any significant amount, and they operate essentially at constant temperature. Such systems can be analyzed conveniently by considering the *mechanical forms of energy* only and the frictional effects that cause the mechanical energy to be lost (i.e., to be converted to thermal energy that usually cannot be used for any useful purpose).

Mechanical energy can be defined as *the form of energy that can be converted to mechanical work completely and directly by an ideal mechanical device such as an ideal turbine.* Kinetic and potential energies are the familiar forms of mechanical energy. Thermal energy is not mechanical energy, however, since it cannot be converted to work directly and completely (the second law of thermodynamics).

A pump transfers mechanical energy to a fluid by raising its pressure, and a turbine extracts mechanical energy from a fluid by dropping its pressure. Therefore, the pressure of a flowing fluid is also associated with its mechanical energy. In fact, the pressure unit Pa is equivalent to Pa = N/m^2 = N·m/m^3 = J/m^3, which is energy per unit volume, and the product $P\upsilon$ or its equivalent P/ρ has the unit J/kg, which is energy per unit mass. Note that pressure itself is not a form of energy, but a pressure force acting on a fluid through a distance produces work, called *flow work,* in the amount of P/ρ per unit mass. Flow work is expressed in terms of fluid properties, and it is convenient to view it as part of the energy of a

FIGURE 3–11
Mechanical energy is a useful concept for flows that do not involve significant heat transfer or energy conversion, such as the flow of gasoline from an underground tank into a car.
altrendo images/Getty Images

flowing fluid and call it *flow energy*. Therefore, the mechanical energy of a flowing fluid can be expressed on a unit mass basis as

$$e_{\text{mech}} = \frac{P}{\rho} + \frac{V^2}{2} + gz \tag{3–10}$$

where P/ρ is the *flow energy*, $V^2/2$ is the *kinetic energy*, and gz is the *potential energy* of the fluid, all per unit mass. It can also be expressed in rate form as

$$\dot{E}_{\text{mech}} = \dot{m}e_{\text{mech}} = \dot{m}\left(\frac{P}{\rho} + \frac{V^2}{2} + gz\right) \tag{3–11}$$

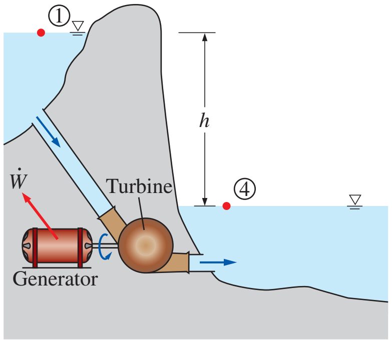
where $\dot{m}$ is the mass flow rate of the fluid. Then the mechanical energy change of a fluid during incompressible (ρ = constant) flow becomes

$$\Delta e_{\text{mech}} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \qquad \text{(kJ/kg)} \tag{3–12}$$

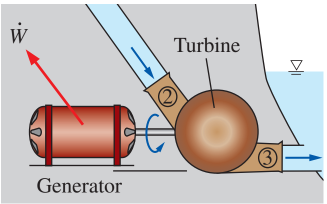
and

$$\Delta\dot{E}_{\text{mech}} = \dot{m}\Delta e_{\text{mech}} = \dot{m}\left(\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right) \qquad \text{(kW)} \tag{3–13}$$

Therefore, the mechanical energy of a fluid does not change during flow if its pressure, density, velocity, and elevation remain constant. In the absence of any irreversible losses, the mechanical energy change represents the mechanical work supplied to the fluid (if $\Delta e_{\text{mech}} > 0$) or extracted from the fluid (if $\Delta e_{\text{mech}} < 0$). The maximum (ideal) power generated by a turbine, for example, is $\dot{W}_{\text{max}} = \dot{m}\Delta e_{\text{mech}}$, as shown in Fig. 3–12.



$$\dot{W}_{\text{max}} = \dot{m}\Delta e_{\text{mech}} = \dot{m}g(z_1 - z_4) = \dot{m}gh$$
since $P_1 \approx P_4 = P_{\text{atm}}$ and $V_1 = V_4 \approx 0$
(*a*)



$$\dot{W}_{\text{max}} = \dot{m}\Delta e_{\text{mech}} = \dot{m}\frac{P_2 - P_3}{\rho} = \dot{m}\frac{\Delta P}{\rho}$$
since $V_1 \approx V_3$ and $z_2 = z_3$
(*b*)

FIGURE 3–12
Mechanical energy is illustrated by an ideal hydraulic turbine coupled with an ideal generator. In the absence of irreversible losses, the maximum produced power is proportional to (*a*) the change in water surface elevation from the upstream to the downstream reservoir or (*b*) (close-up view) the drop in water pressure from just upstream to just downstream of the turbine.

EXAMPLE 3–2 Wind Energy

A site evaluated for a wind farm is observed to have steady winds at a speed of 8.5 m/s (Fig. 3–13). Determine the wind energy (*a*) per unit mass, (*b*) for a mass of 10 kg, and (*c*) for a flow rate of 1154 kg/s for air.

SOLUTION A site with a specified wind speed is considered. Wind energy per unit mass, for a specified mass, and for a given mass flow rate of air are to be determined.
Assumptions Wind flows steadily at the specified speed.
Analysis The only harvestable form of energy of atmospheric air is the kinetic energy, which is captured by a wind turbine.
(*a*) Wind energy per unit mass of air is

$$e = \text{ke} = \frac{V^2}{2} = \frac{(8.5\text{ m/s})^2}{2}\left(\frac{1\text{ J/kg}}{1\text{ m}^2/\text{s}^2}\right) = \mathbf{36.1\ J/kg}$$

(*b*) Wind energy for an air mass of 10 kg is

$$E = me = (10\text{ kg})(36.1\text{ J/kg}) = \mathbf{361\ J}$$

(*c*) Wind energy for a mass flow rate of 1154 kg/s is

$$\dot{E} = \dot{m}e = (1154\text{ kg/s})(36.1\text{ J/kg})\left(\frac{1\text{ kW}}{1000\text{ J/s}}\right) = \mathbf{41.7\ kW}$$

Discussion It can be shown that the specified mass flow rate corresponds to a 12-m-diameter flow section when the air density is 1.2 kg/m^3. Therefore, a wind turbine with a wind span diameter of 12 m has a power generation potential of 41.7 kW. Real wind turbines convert about one-third of this potential to electric power.

FIGURE 3–13
A site for a wind farm as discussed in Example 3–2.
Image Source/Getty Images

3–3 ▪ ENERGY TRANSFER BY HEAT

Energy can cross the boundary of a closed system in two distinct forms: *heat* and *work* (Fig. 3–14). It is important to distinguish between these two forms of energy. Therefore, they will be discussed first, to form a sound basis for the development of the laws of thermodynamics.

We know from experience that a can of cold soda left on a table eventually warms up and that a hot baked potato on the same table cools down. When a body is left in a medium that is at a different temperature, energy transfer takes place between the body and the surrounding medium until thermal equilibrium is established, that is, the body and the medium reach the same temperature. The direction of energy transfer is always from the higher temperature body to the lower temperature one. Once the temperature equality is established, energy transfer stops. In the processes described above, energy is said to be transferred in the form of heat.

Heat is defined as *the form of energy that is transferred between two systems (or a system and its surroundings) by virtue of a temperature difference* (Fig. 3–15). That is, an energy interaction is heat only if it takes place because of a temperature difference. Then it follows that there cannot be any heat transfer between two systems that are at the same temperature.

Several phrases in common use today—such as heat flow, heat addition, heat rejection, heat absorption, heat removal, heat gain, heat loss, heat storage, heat generation, electrical heating, resistance heating, frictional heating, gas heating, heat of reaction, liberation of heat, specific heat, sensible heat, latent heat, waste heat, body heat, process heat, heat sink, and heat source—are not consistent with the strict thermodynamic meaning of the term *heat,* which limits its use to the *transfer* of thermal energy during a process. However, these phrases are deeply rooted in our vocabulary, and they are used by both ordinary people and scientists without causing any misunderstanding since they are usually interpreted properly instead of being taken literally. (Besides, no acceptable alternatives exist for some of these phrases.) For example, the phrase *body heat* is understood to mean *the thermal energy content* of a body. Likewise, *heat flow* is understood to mean *the transfer of thermal energy,* not the flow of a fluidlike substance called heat, although the latter incorrect interpretation, which is based on the caloric theory, is the origin of this phrase. Also, the transfer of heat into a system is often referred to as *heat addition* and the transfer of heat out of a system as *heat rejection.* Perhaps there are thermodynamic reasons for being so reluctant to replace *heat* with *thermal energy:* It takes less time and energy to say, write, and comprehend *heat* than it does *thermal energy.*

Heat is energy in transition. It is recognized only as it crosses the boundary of a system. Consider the hot baked potato one more time. The potato contains energy, but this energy is heat transfer only as it passes through the skin of the potato (the system boundary) to reach the air, as shown in Fig. 3–16. Once in the surroundings, the transferred heat becomes part of the internal energy of the surroundings. Thus, in thermodynamics, the term *heat* simply means *heat transfer.*

A process during which there is no heat transfer is called an **adiabatic process** (Fig. 3–17). The word *adiabatic* comes from the Greek word *adiabatos,* which means *not to be passed.* There are two ways a process can be adiabatic: Either the system is well insulated so that only a negligible amount of heat can pass through the boundary, or both the system and the surroundings are at the same temperature and therefore there is no driving force (temperature difference) for heat transfer. An adiabatic process should not be confused with an isothermal process. Even though there is no heat transfer during an adiabatic process, the energy content and thus the temperature of a system can still be changed by other means such as work.

As a form of energy, heat has energy units, kJ (or Btu) being the most common one. The amount of heat transferred during the process between two states (states

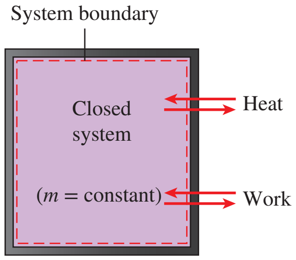


FIGURE 3–14
Energy can cross the boundaries of a closed system in the form of heat and work.

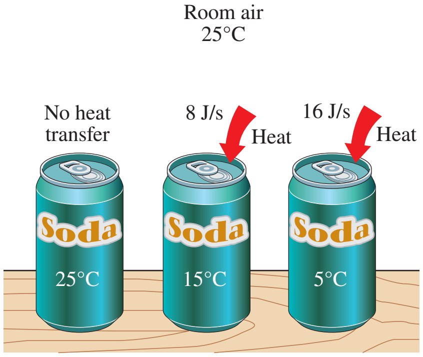


FIGURE 3–15
Temperature difference is the driving force for heat transfer. The larger the temperature difference, the higher is the rate of heat transfer.

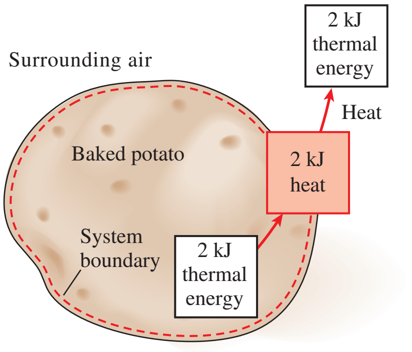


FIGURE 3–16
Energy is recognized as heat transfer only as it crosses the system boundary.

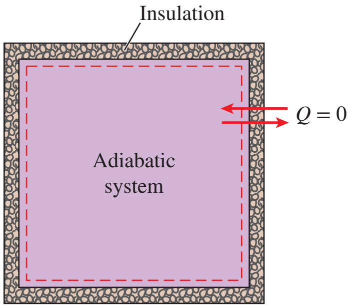


FIGURE 3–17
During an adiabatic process, a system exchanges no heat with its surroundings.

1 and 2) is denoted by Q_{12}, or just Q. Heat transfer *per unit mass* of a system is denoted q and is determined from

$$q = \frac{Q}{m} \qquad \text{(kJ/kg)} \tag{3–14}$$

Sometimes it is desirable to know the *rate of heat transfer* (the amount of heat transferred per unit time) instead of the total heat transferred over some time interval (Fig. 3–18). The heat transfer rate is denoted $\dot{Q}$, where the overdot stands for the time derivative, or "per unit time." The heat transfer rate $\dot{Q}$ has the unit kJ/s, which is equivalent to kW. When $\dot{Q}$ varies with time, the amount of heat transfer during a process is determined by integrating $\dot{Q}$ over the time interval of the process:

$$Q = \int_{t_1}^{t_2} \dot{Q}\, dt \qquad \text{(kJ)} \tag{3–15}$$

When $\dot{Q}$ remains constant during a process, this relation reduces to

$$Q = \dot{Q}\Delta t \qquad \text{(kJ)} \tag{3–16}$$

where $\Delta t = t_2 - t_1$ is the time interval during which the process takes place.

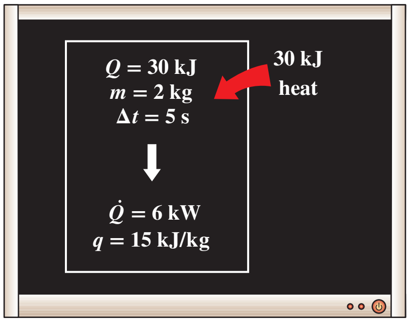


FIGURE 3–18
The relationships among q, Q, and $\dot{Q}$.

Historical Background on Heat

Heat has always been perceived to be something that produces in us a sensation of warmth, and one would think that the nature of heat is one of the first things understood by humankind. However, it was only in the middle of the 19th century that we had a true physical understanding of the nature of heat, thanks to the development at that time of the **kinetic theory**, which treats molecules as tiny balls that are in motion and thus possess kinetic energy. Heat is then defined as the energy associated with the random motion of atoms and molecules. Although it was suggested in the 18th and early 19th centuries that heat is the manifestation of motion at the molecular level (called the *live force*), the prevailing view of heat until the middle of the 19th century was based on the caloric theory proposed by the French chemist Antoine Lavoisier (1744–1794) in 1789. The caloric theory asserts that heat is a fluidlike substance called **caloric** that is a massless, colorless, odorless, and tasteless substance that can be poured from one body into another (Fig. 3–19). When caloric was added to a body, its temperature increased; when caloric was removed from a body, its temperature decreased. When a body could not contain any more caloric, much the same as when a glass of water could not dissolve any more salt or sugar, the body was said to be saturated with caloric. This interpretation gave rise to the terms *saturated liquid* and *saturated vapor* that are still in use today.

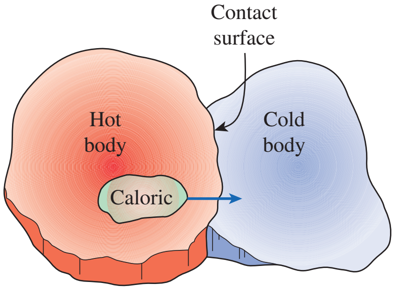


FIGURE 3–19
In the early 19th century, heat was thought to be an invisible fluid called the *caloric* that flowed from warmer bodies to cooler ones.

The caloric theory came under attack soon after its introduction. It maintained that heat was a substance that could not be created or destroyed. Yet it was known that heat could be generated indefinitely by rubbing one's hands together or rubbing two pieces of wood together. In 1798, the American Benjamin Thompson (Count Rumford) (1754–1814) showed in his papers that heat can be generated continuously through friction. The validity of the caloric theory was also challenged by several others. But it was the careful experiments of the Englishman James P. Joule (1818–1889) published in 1843 that finally convinced the skeptics that heat was not a substance after all, and thus put the caloric theory to rest. Although the caloric theory was totally abandoned in the middle of the 19th century, it contributed greatly to the development of thermodynamics and heat transfer.

Heat is transferred by three mechanisms: conduction, convection, and radiation. **Conduction** is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interaction between particles. **Convection** is the transfer of energy between a solid surface and the adjacent fluid that is in motion, and it involves the combined effects of conduction and fluid motion. **Radiation** is the transfer of energy due to the emission of electromagnetic waves (or photons).

3–4 ▪ ENERGY TRANSFER BY WORK

Work, like heat, is an energy interaction between a system and its surroundings. As mentioned earlier, energy can cross the boundary of a closed system in the form of heat or work. Therefore, *if the energy crossing the boundary of a closed system is not heat, it must be work.* Heat is easy to recognize: Its driving force is a temperature difference between the system and its surroundings. Then we can simply say that an energy interaction that is not caused by a temperature difference between a system and its surroundings is work. More specifically, *work is the energy transfer associated with a force acting through a distance.* A rising piston, a rotating shaft, and an electric wire crossing the system boundaries are all associated with work interactions.

Work is also a form of energy transferred like heat and, therefore, has energy units such as kJ. The work done during a process between states 1 and 2 is denoted by W_{12}, or simply W. The work done *per unit mass* of a system is denoted by w and is expressed as

$$w = \frac{W}{m} \qquad \text{(kJ/kg)} \tag{3–17}$$

The work done *per unit time* is called **power** and is denoted $\dot{W}$ (Fig. 3–20). The unit of power is kJ/s, or kW.

Heat and work are *directional quantities,* and thus the complete description of a heat or work interaction requires the specification of both the *magnitude* and *direction.* One way of doing that is to adopt a sign convention. The generally accepted **formal sign convention** for heat and work interactions is as follows: *heat transfer to a system and work done by a system are positive; heat transfer from a system and work done on a system are negative.* Another way is to use the subscripts *in* and *out* to indicate direction (Fig. 3–21). For example, a work input of 5 kJ can be expressed as $W_{in} = 5$ kJ, while a heat loss of 3 kJ can be expressed as $Q_{out} = 3$ kJ. When the direction of a heat or work interaction is not known, we can simply *assume* a direction for the interaction (using the subscript *in* or *out*) and solve for it. A positive result indicates the assumed direction is right. A negative result, on the other hand, indicates that the direction of the interaction is the opposite of the assumed direction. This is just like assuming a direction for an unknown force when solving a statics problem and reversing the direction when a negative result is obtained for the force. We will use this *intuitive approach* in this book as it eliminates the need to adopt a formal sign convention and the need to carefully assign negative values to some interactions.

Note that a quantity that is transferred to or from a system during an interaction is not a property since the amount of such a quantity depends on more than just the state of the system. Heat and work are *energy transfer mechanisms* between a system and its surroundings, and there are many similarities between them:

1. Both are recognized at the boundaries of a system as they cross the boundaries. That is, both heat and work are *boundary* phenomena.
2. Systems possess energy, but not heat or work.
3. Both are associated with a *process,* not a state. Unlike properties, heat or work has no meaning at a state.

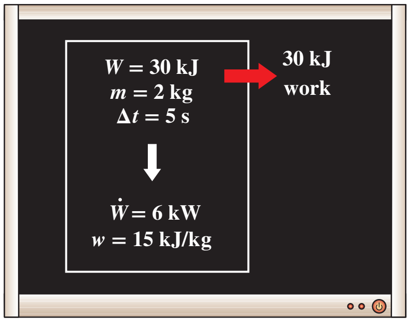


FIGURE 3–20
The relationships among w, W, and $\dot{W}$.

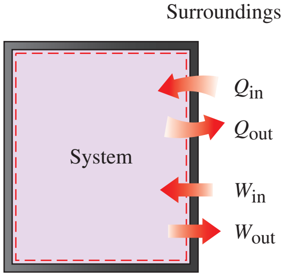


FIGURE 3–21
Specifying the directions of heat and work.

4. Both are *path functions* (i.e., their magnitudes depend on the path followed during a process as well as the end states).

Path functions have **inexact differentials** designated by the symbol δ. Therefore, a differential amount of heat or work is represented by δQ or δW, respectively, instead of dQ or dW. Properties, however, are **point functions** (i.e., they depend on the state only, and not on how a system reaches that state), and they have **exact differentials** designated by the symbol d. A small change in volume, for example, is represented by dV, and the total volume change during a process between states 1 and 2 is

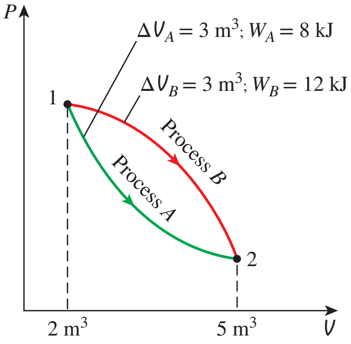


FIGURE 3–22
Properties are point functions; but heat and work are path functions (their magnitudes depend on the path followed).

$$\int_1^2 dV = V_2 - V_1 = \Delta V$$

That is, the volume change during process 1–2 is always the volume at state 2 minus the volume at state 1, regardless of the path followed (Fig. 3–22). The total work done during process 1–2, however, is

$$\int_1^2 \delta W = W_{12} \qquad (\textit{not } \Delta W)$$

That is, the total work is obtained by following the process path and adding the differential amounts of work (δW) done along the way. The integral of δW *is not* $W_2 - W_1$ (i.e., the work at state 2 minus work at state 1), which is meaningless since work is not a property and systems do not possess work at a state.

EXAMPLE 3–3 Burning of a Candle in an Insulated Room

A candle is burning in a well-insulated room. Taking the room (the air plus the candle) as the system, determine (*a*) if there is any heat transfer during this burning process and (*b*) if there is any change in the internal energy of the system.

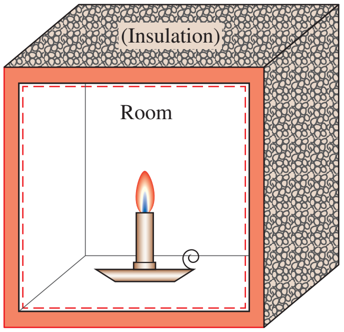


FIGURE 3–23
Schematic for Example 3–3.

SOLUTION A candle burning in a well-insulated room is considered. It is to be determined whether there is any heat transfer and any change in internal energy.

Analysis (*a*) The interior surfaces of the room form the system boundary, as indicated by the dashed lines in Fig. 3–23. As pointed out earlier, heat is recognized as it crosses the boundaries. Since the room is well insulated, we have an adiabatic system, and no heat will pass through the boundaries. Therefore, $Q = 0$ for this process.

(*b*) The internal energy involves energies that exist in various forms (sensible, latent, chemical, nuclear). During the process just described, part of the chemical energy is converted to sensible energy. Since there is no increase or decrease in the total internal energy of the system, $\Delta U = 0$ for this process.

EXAMPLE 3–4 Heating of a Potato in an Oven

A potato initially at room temperature (25°C) is being baked in an oven that is maintained at 200°C, as shown in Fig. 3–24. Is there any heat transfer during this baking process?

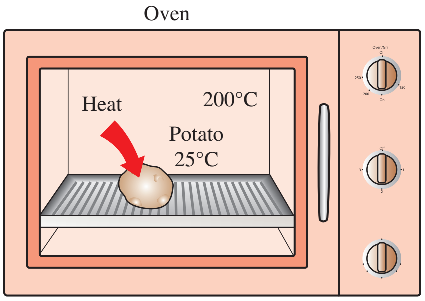


FIGURE 3–24
Schematic for Example 3–4.

SOLUTION A potato is being baked in an oven. It is to be determined whether there is any heat transfer during this process.

Analysis This is not a well-defined problem since the system is not specified. Let us assume that we are observing the potato, which will be our system. Then the outer surface of the skin of the potato can be viewed as the system boundary. Part of the energy in the oven will pass through the skin to the potato. Since the driving force for this energy transfer is a temperature difference, this is a heat transfer process.

EXAMPLE 3–5 Heating of an Oven by Work Transfer

A well-insulated electric oven is being heated through its heating element. If the entire oven, including the heating element, is taken to be the system, determine whether this is a heat or work interaction.

SOLUTION A well-insulated electric oven is being heated by its heating element. It is to be determined whether this is a heat or work interaction.
Analysis For this problem, the interior surfaces of the oven form the system boundary, as shown in Fig. 3–25. The energy content of the oven obviously increases during this process, as evidenced by a rise in temperature. This energy transfer to the oven is not caused by a temperature difference between the oven and the surrounding air. Instead, it is caused by *electrons* crossing the system boundary and thus doing work. Therefore, this is a work interaction.

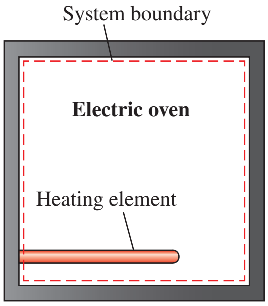


FIGURE 3–25
Schematic for Example 3–5.

EXAMPLE 3–6 Heating of an Oven by Heat Transfer

Answer the question in Example 3–5 if the system is taken as only the air in the oven without the heating element.

SOLUTION The question in Example 3–5 is to be reconsidered by taking the system to be only the air in the oven.
Analysis This time, the system boundary will include the outer surface of the heating element and will not cut through it, as shown in Fig. 3–26. Therefore, no electrons will be crossing the system boundary at any point. Instead, the energy generated in the interior of the heating element will be transferred to the air around it as a result of the temperature difference between the heating element and the air in the oven. Therefore, this is a heat transfer process.
Discussion For both cases, the amount of energy transfer to the air is the same. These two examples show that an energy transfer can be heat or work, depending on how the system is selected.

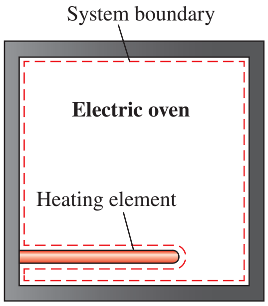


FIGURE 3–26
Schematic for Example 3–6.

Electrical Work

It was pointed out in Example 3–5 that electrons crossing the system boundary do electrical work on the system. In an electric field, electrons in a wire move under the effect of electromotive forces, doing work. When N coulombs of electrical charge move through a potential difference $\mathbf{V}$, the electrical work done is

$$W_e = \mathbf{V}N$$

which can also be expressed in the rate form as

$$\dot{W}_e = \mathbf{V}I \qquad \text{(W)} \tag{3–18}$$

where $\dot{W}_e$ is the **electrical power** and I is the number of electrical charges flowing per unit time, that is, the *current* (Fig. 3–27). In general, both $\mathbf{V}$ and I vary with time, and the electrical work done during a time interval Δt is expressed as

$$W_e = \int_1^2 \mathbf{V}I\,dt \qquad \text{(kJ)} \tag{3–19}$$

When both $\mathbf{V}$ and I remain constant during the time interval Δt, it reduces to

$$W_e = \mathbf{V}I\,\Delta t \qquad \text{(kJ)} \tag{3–20}$$

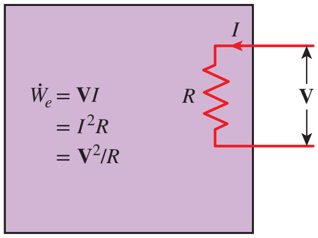


FIGURE 3–27
Electrical power in terms of resistance R, current I, and potential difference $\mathbf{V}$.

3–5 ▪ MECHANICAL FORMS OF WORK

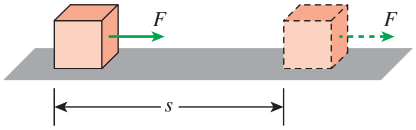


FIGURE 3–28
The work done is proportional to the force applied (F) and the distance traveled (s).

There are several different ways of doing work, each in some way related to a force acting through a distance (Fig. 3–28). In elementary mechanics, the work done by a constant force F on a body displaced a distance s in the direction of the force is given by

$$W = Fs \qquad \text{(kJ)} \tag{3–21}$$

If the force F is not constant, the work done is obtained by adding (i.e., integrating) the differential amounts of work,

$$W = \int_1^2 F\,ds \qquad \text{(kJ)} \tag{3–22}$$

Obviously, one needs to know how the force varies with displacement to perform this integration. Equations 3–21 and 3–22 give only the magnitude of the work. The sign is easily determined from physical considerations: The work done on a system by an external force acting in the direction of motion is negative, and work done by a system against an external force acting in the opposite direction to motion is positive.

There are two requirements for a work interaction between a system and its surroundings to exist: (1) there must be a *force* acting on the boundary, and (2) the boundary must *move*. Therefore, the presence of forces on the boundary without any displacement of the boundary does not constitute a work interaction. Likewise, the displacement of the boundary without any force to oppose or drive this motion (such as the expansion of a gas into an evacuated space) is not a work interaction since no energy is transferred.

In many thermodynamic problems, mechanical work is the only form of work involved. It is associated with the movement of the boundary of a system or with the movement of the entire system as a whole. Some common forms of mechanical work are discussed next.

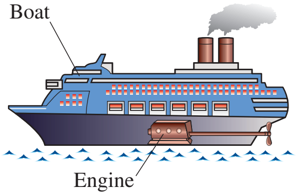


FIGURE 3–29
Energy transmission through rotating shafts is commonly encountered in practice.

Shaft Work

Energy transmission with a rotating shaft is very common in engineering practice (Fig. 3–29). Often the torque T applied to the shaft is constant, which means that the force F applied is also constant. For a specified constant torque, the work done during n revolutions is determined as follows: A force F acting through a moment arm r generates a torque T of (Fig. 3–30)

$$\mathrm{T} = Fr \quad \rightarrow \quad F = \frac{\mathrm{T}}{r} \tag{3–23}$$

This force acts through a distance s, which is related to the radius r by

$$s = (2\pi r)n \tag{3–24}$$

Then the shaft work is determined from

$$W_{\text{sh}} = Fs = \left(\frac{\mathrm{T}}{r}\right)(2\pi rn) = 2\pi n\mathrm{T} \qquad \text{(kJ)} \tag{3–25}$$

The power transmitted through the shaft is the shaft work done per unit time, which can be expressed as

$$\dot{W}_{\text{sh}} = 2\pi\dot{n}\mathrm{T} \qquad \text{(kW)} \tag{3–26}$$

where $\dot{n}$ is the number of revolutions per unit time.

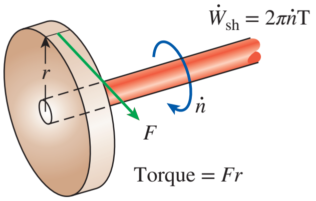


FIGURE 3–30
Shaft work is proportional to the torque applied and the number of revolutions of the shaft.

EXAMPLE 3–7 Power Transmission by the Shaft of a Car

Determine the power transmitted through the shaft of a car when the torque applied is 200 N·m and the shaft rotates at a rate of 4000 revolutions per minute (rpm).

SOLUTION The torque and the rpm for a car engine are given. The power transmitted is to be determined.

Analysis A sketch of the car is given in Fig. 3–31. The shaft power is determined directly from

$$\dot{W}_{sh} = 2\pi \dot{n}T = (2\pi)\left(4000\ \frac{1}{\text{min}}\right)(200\ \text{N·m})\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{1\ \text{kJ}}{1000\ \text{N·m}}\right)$$

$$= \mathbf{83.8\ kW} \qquad (\text{or } 112\ \text{hp})$$

Discussion Note that power transmitted by a shaft is proportional to torque and the rotational speed.

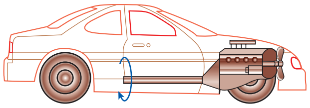


FIGURE 3–31
Schematic for Example 3–7.

Spring Work

It is common knowledge that when a force is applied on a spring, the length of the spring changes (Fig. 3–32). When the length of the spring changes by a differential amount dx under the influence of a force F, the work done is

$$\delta W_{\text{spring}} = F\,dx \qquad \textbf{(3–27)}$$

To determine the total spring work, we need to know a functional relationship between F and x. For linear elastic springs, the displacement x is proportional to the force applied (Fig. 3–33). That is,

$$F = kx \qquad (\text{kN}) \qquad \textbf{(3–28)}$$

where k is the spring constant and has the unit kN/m. The displacement x is measured from the undisturbed position of the spring (that is, $x = 0$ when $F = 0$). Substituting Eq. 3–28 into Eq. 3–27 and integrating yield

$$W_{\text{spring}} = \tfrac{1}{2}k(x_2^2 - x_1^2) \qquad (\text{kJ}) \qquad \textbf{(3–29)}$$

where x_1 and x_2 are the initial and the final displacements of the spring, respectively, measured from the undisturbed position of the spring.

There are many other forms of mechanical work. Next we introduce some of them briefly.

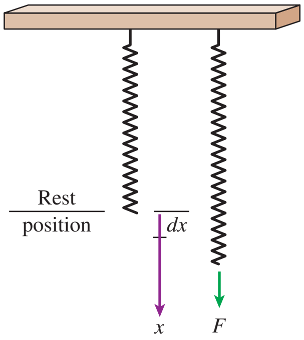


FIGURE 3–32
Elongation of a spring under the influence of a force.

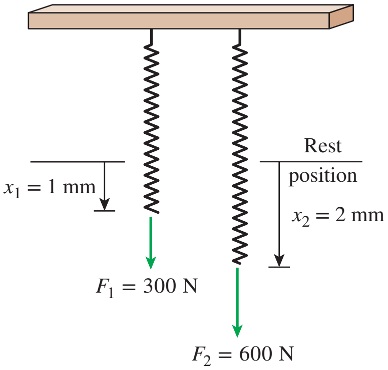


FIGURE 3–33
The displacement of a linear spring doubles when the force is doubled.

Work Done on Elastic Solid Bars

Solids are often modeled as linear springs because under the action of a force they contract or elongate, as shown in Fig. 3–34, and when the force is lifted, they return to their original lengths, like a spring. This is true as long as the force is in the elastic range, that is, not large enough to cause permanent (plastic) deformations. Therefore, the equations given for a linear spring can also be used for elastic solid bars. Alternately, we can determine the work associated with the expansion or contraction of an elastic solid bar by replacing pressure P with its counterpart in solids, *normal stress* $\sigma_n = F/A$, in the work expression:

$$W_{\text{elastic}} = \int_1^2 F\,dx = \int_1^2 \sigma_n A\,dx \qquad (\text{kJ}) \qquad \textbf{(3–30)}$$

where A is the cross-sectional area of the bar. Note that the normal stress has pressure units.

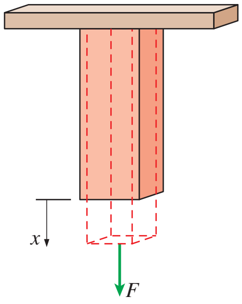


FIGURE 3–34
Solid bars behave as springs under the influence of a force.

Work Associated with the Stretching of a Liquid Film

Consider a liquid film such as soap film suspended on a wire frame (Fig. 3–35). We know from experience that it will take some force to stretch this film by the movable portion of the wire frame. This force is used to overcome the microscopic forces between molecules at the liquid–air interfaces. These microscopic forces are perpendicular to any line in the surface, and the force generated by these forces per unit length is called the **surface tension** σ_s, whose unit is N/m. Therefore, the work associated with the stretching of a film is also called *surface tension work*. It is determined from

$$W_{\text{surface}} = \int_1^2 \sigma_s \, dA \qquad \text{(kJ)} \tag{3–31}$$

where $dA = 2b\,dx$ is the change in the surface area of the film. The factor 2 is due to the fact that the film has two surfaces in contact with air. The force acting on the movable wire as a result of surface tension effects is $F = 2b\sigma_s$ where σ_s is the surface tension force per unit length.

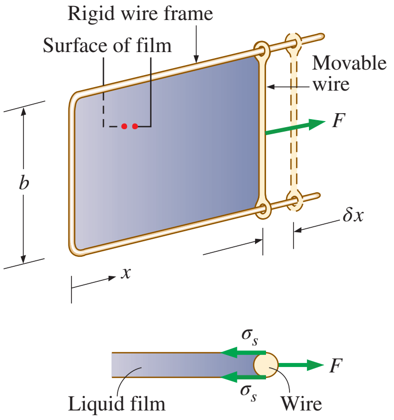


FIGURE 3–35
Stretching a liquid film with a U-shaped wire, and the forces acting on the movable wire of length *b*.

Work Done to Raise or to Accelerate a Body

When a body is raised in a gravitational field, its potential energy increases. Likewise, when a body is accelerated, its kinetic energy increases. The conservation of energy principle requires that an equivalent amount of energy must be transferred to the body being raised or accelerated. Remember that energy can be transferred to a given mass by heat and work, and the energy transferred in this case obviously is not heat since it is not driven by a temperature difference. Therefore, it must be work. Then we conclude that (1) the work transfer needed to raise a body is equal to the change in the potential energy of the body, and (2) the work transfer needed to accelerate a body is equal to the change in the kinetic energy of the body (Fig. 3–36). Similarly, the potential or kinetic energy of a body represents the work that can be obtained from the body as it is lowered to the reference level or decelerated to zero velocity.

This discussion together with the consideration for friction and other losses form the basis for determining the required power rating of motors used to drive devices such as elevators, escalators, conveyor belts, and ski lifts. It also plays a primary role in the design of automotive and aircraft engines, and in the determination of the amount of hydroelectric power that can be produced from a given water reservoir, which is simply the potential energy of the water relative to the location of the hydraulic turbine.

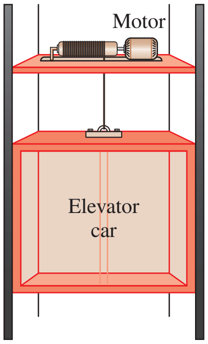


FIGURE 3–36
The energy transferred to a body while being raised is equal to the change in its potential energy.

EXAMPLE 3–8 Power Needs of a Car to Climb a Hill

A man whose mass is 100 kg pushes a cart whose mass, including its contents, is 100 kg up a ramp that is inclined at an angle of 20° from the horizontal (Fig. 3–37). The local gravitational acceleration is 9.8 m/s^2. Determine the work, in kJ, needed to move along this ramp a distance of 100 m considering (*a*) the man and (*b*) the cart and its contents as the system.

SOLUTION A man is pushing a cart with its contents up a ramp that is inclined at an angle of 20° from the horizontal. The work needed to move along this ramp is to be determined considering (*a*) the man and (*b*) the cart and its contents as the system.

Analysis (*a*) Considering the man as the system, letting l be the displacement along the ramp, and letting θ be the inclination angle of the ramp,

$$\begin{aligned} W &= Fl\sin\theta = mgl\sin\theta \\ &= (100 + 100\text{ kg})(9.8\text{ m/s}^2)(100\text{ m})(\sin 20°)\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{67.0\ kJ} \end{aligned}$$

FIGURE 3–37
Schematic for Example 3–8.
Lars A. Niki

This is work that the man must do to raise the weight of the cart and contents, plus his own weight, a distance of $l \sin \theta$.
(*b*) Applying the same logic to the cart and its contents gives

$$W = Fl \sin\theta = mgl \sin\theta$$
$$= (100 \text{ kg})(9.8 \text{ m/s}^2)(100 \text{ m})(\sin 20°)\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = \mathbf{33.5 \text{ kJ}}$$

Discussion The result in part (*a*) is more realistic since the man has to move himself in addition to the cart.

EXAMPLE 3–9 Power Needs of a Car to Accelerate

Determine the power required to accelerate a 900-kg car shown in Fig. 3–38 from rest to a velocity of 80 km/h in 20 s on a level road.

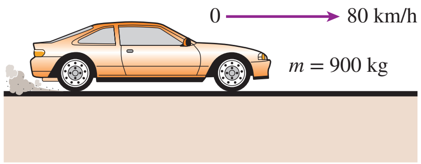


FIGURE 3–38
Schematic for Example 3–9.

SOLUTION The power required to accelerate a car to a specified velocity is to be determined.
Analysis The work needed to accelerate a body is simply the change in the kinetic energy of the body,

$$W_a = \tfrac{1}{2}m(V_2^2 - V_1^2) = \tfrac{1}{2}(900 \text{ kg})\left[\left(\frac{80{,}000 \text{ m}}{3600 \text{ s}}\right)^2 - 0^2\right]\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right)$$
$$= 222 \text{ kJ}$$

The average power is determined from

$$\dot{W}_a = \frac{W_a}{\Delta t} = \frac{222 \text{ kJ}}{20 \text{ s}} = \mathbf{11.1 \text{ kW}} \qquad \text{(or 14.9 hp)}$$

Discussion This is in addition to the power required to overcome friction, rolling resistance, and other imperfections.

Nonmechanical Forms of Work

The treatment in Section 3–5 represents a fairly comprehensive coverage of mechanical forms of work except the *moving boundary work* that is covered in Chap. 5. Some work modes encountered in practice are not mechanical in nature. However, these nonmechanical work modes can be treated in a similar manner by identifying a *generalized force F* acting in the direction of a *generalized displacement x*. Then the work associated with the differential displacement under the influence of this force is determined from $\delta W = F dx$.

Some examples of nonmechanical work modes are **electrical work**, where the generalized force is the *voltage* (the electrical potential) and the generalized displacement is the *electrical charge,* as discussed earlier; **magnetic work**, where the generalized force is the *magnetic field strength* and the generalized displacement is the total *magnetic dipole moment;* and **electrical polarization work**, where the generalized force is the *electric field strength* and the generalized displacement is the *polarization of the medium* (the sum of the electric dipole rotation moments of the molecules). Detailed consideration of these and other nonmechanical work modes can be found in specialized books on these topics.

3–6 ▪ THE FIRST LAW OF THERMODYNAMICS

So far, we have considered various forms of energy such as heat Q, work W, and total energy E individually, and no attempt is made to relate them to each other during a process. The *first law of thermodynamics,* also known as *the conservation of* *energy principle,* provides a sound basis for studying the relationships among the various forms of energy and energy interactions. Based on experimental observations, the first law of thermodynamics states that *energy can be neither created nor destroyed during a process; it can only change forms.* Therefore, every bit of energy should be accounted for during a process.

We all know that a rock at some elevation possesses some potential energy, and part of this potential energy is converted to kinetic energy as the rock falls (Fig. 3–39). Experimental data show that the decrease in potential energy ($mg\ \Delta z$) exactly equals the increase in kinetic energy $[m(V_2^2 - V_1^2)/2]$ when the air resistance is negligible, thus confirming the conservation of energy principle for mechanical energy.

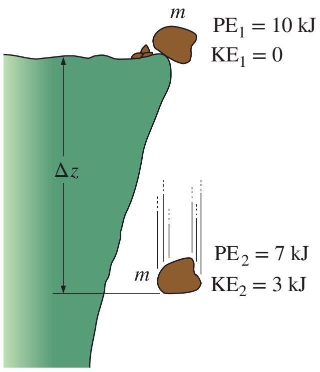


FIGURE 3–39
Energy cannot be created or destroyed; it can only change forms.

Consider a system undergoing a series of *adiabatic* processes from a specified state 1 to another specified state 2. Being adiabatic, these processes obviously cannot involve any heat transfer, but they may involve several kinds of work interactions. Careful measurements during these experiments indicate the following: *For all adiabatic processes between two specified states of a closed system, the net work done is the same regardless of the nature of the closed system and the details of the process.* Considering that there are an infinite number of ways to perform work interactions under adiabatic conditions, this statement appears to be very powerful, with a potential for far-reaching implications. This statement, which is largely based on the experiments of Joule in the first half of the 19th century, cannot be drawn from any other known physical principle and is recognized as a fundamental principle. This principle is called the **first law of thermodynamics** or just the **first law**.

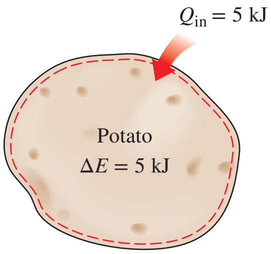


FIGURE 3–40
The increase in the energy of a potato in an oven is equal to the amount of heat transferred to it.

A major consequence of the first law is the existence and the definition of the property *total energy E.* Considering that the net work is the same for all adiabatic processes of a closed system between two specified states, the value of the net work must depend on the end states of the system only, and thus it must correspond to a change in a property of the system. This property is the *total energy.* Note that the first law makes no reference to the value of the total energy of a closed system at a state. It simply states that the *change* in the total energy during an adiabatic process must be equal to the net work done. Therefore, any convenient arbitrary value can be assigned to total energy at a specified state to serve as a reference point.

Implicit in the first law statement is the conservation of energy. Although the essence of the first law is the existence of the property *total energy,* the first law is often viewed as a statement of the *conservation of energy* principle. Next, we develop the first law or the conservation of energy relation with the help of some familiar examples using intuitive arguments.

First, we consider some processes that involve heat transfer but no work interactions. The potato baked in the oven is a good example for this case (Fig. 3–40). As a result of heat transfer to the potato, the energy of the potato will increase. If we disregard any mass transfer (moisture loss from the potato), the increase in the total energy of the potato becomes equal to the amount of heat transfer. That is, if 5 kJ of heat is transferred to the potato, the energy increase of the potato will also be 5 kJ.

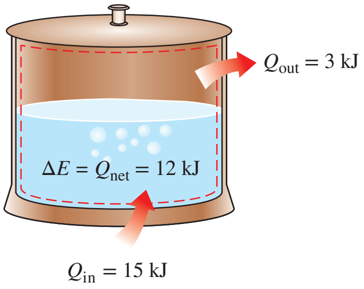


FIGURE 3–41
In the absence of any work interactions, the energy change of a system is equal to the net heat transfer.

As another example, consider the heating of water in a pan on top of a range (Fig. 3–41). If 15 kJ of heat is transferred to the water from the heating element and 3 kJ of it is lost from the water to the surrounding air, the increase in energy of the water will be equal to the net heat transfer to water, which is 12 kJ.

Now consider a well-insulated (i.e., adiabatic) room heated by an electric heater as our system (Fig. 3–42). As a result of electrical work done, the energy of the system will increase. Since the system is adiabatic and cannot have any heat transfer to or from the surroundings ($Q = 0$), the conservation of energy principle dictates that the electrical work done on the system must equal the increase in energy of the system.

Next, let us replace the electric heater with a paddle wheel (Fig. 3–43). As a result of the stirring process, the energy of the system will increase. Again, since there is no heat interaction between the system and its surroundings ($Q = 0$), the shaft work done on the system must show up as an increase in the energy of the system.

Many of you have probably noticed that the temperature of air rises when it is compressed (Fig. 3–44). This is because energy is transferred to the air in the form of boundary work. In the absence of any heat transfer ($Q = 0$), all boundary work will be stored in the air as part of its total energy. The conservation of energy principle again requires that the increase in the energy of the system be equal to the boundary work done on the system.

We can extend these discussions to systems that involve various heat and work interactions simultaneously. For example, if a system gains 12 kJ of heat during a process while 6 kJ of work is done on it, the increase in the energy of the system during that process is 18 kJ (Fig. 3–45). That is, the change in the energy of a system during a process is simply equal to the net energy transfer to (or from) the system.

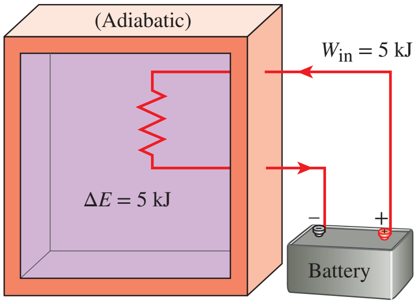


FIGURE 3–42
The work (electrical) done on an adiabatic system is equal to the increase in the energy of the system.

Energy Balance

In the light of the preceding discussions, the conservation of energy principle can be expressed as follows: *The net change (increase or decrease) in the total energy of the system during a process is equal to the difference between the total energy entering and the total energy leaving the system during that process.* That is,

$$\left(\begin{array}{c}\text{Total energy}\\ \text{entering the system}\end{array}\right) - \left(\begin{array}{c}\text{Total energy}\\ \text{leaving the system}\end{array}\right) = \left(\begin{array}{c}\text{Change in the total}\\ \text{energy of the system}\end{array}\right)$$

or

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$

This relation is often referred to as the **energy balance** and is applicable to any kind of system undergoing any kind of process. The successful use of this relation to solve engineering problems depends on understanding the various forms of energy and recognizing the forms of energy transfer.

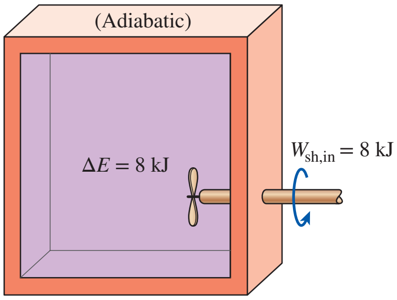


FIGURE 3–43
The work (shaft) done on an adiabatic system is equal to the increase in the energy of the system.

Energy Change of a System, ΔE_{system}

The determination of the energy change of a system during a process involves the evaluation of the energy of the system at the beginning and at the end of the process, and taking their difference. That is,

$$\text{Energy change} = \text{Energy at final state} - \text{Energy at initial state}$$

or

$$\Delta E_{\text{system}} = E_{\text{final}} - E_{\text{initial}} = E_2 - E_1 \qquad \textbf{(3–32)}$$

Note that energy is a property, and the value of a property does not change unless the state of the system changes. Therefore, the energy change of a system is zero if the state of the system does not change during the process. Also, energy can exist in numerous forms such as internal (sensible, latent, chemical, and nuclear), kinetic, potential, electric, and magnetic, and their sum constitutes the *total energy* E of a system. In the absence of electric, magnetic, and surface tension effects (i.e., for simple compressible systems), the change in the total energy of a system during a process is the sum of the changes in its internal, kinetic, and potential energies and can be expressed as

$$\Delta E = \Delta U + \Delta \text{KE} + \Delta \text{PE} \qquad \textbf{(3–33)}$$

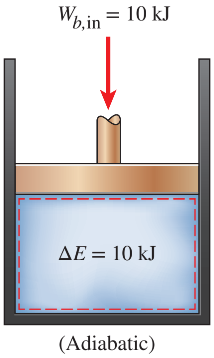


FIGURE 3–44
The work (boundary) done on an adiabatic system is equal to the increase in the energy of the system.

where

$$\Delta U = m(u_2 - u_1)$$
$$\Delta \mathrm{KE} = \tfrac{1}{2}m(V_2^2 - V_1^2)$$
$$\Delta \mathrm{PE} = mg(z_2 - z_1)$$

When the initial and final states are specified, the values of the specific internal energies u_1 and u_2 can be determined directly from the property tables or thermodynamic property relations.

Most systems encountered in practice are stationary, that is, they do not involve any changes in their velocity or elevation during a process (Fig. 3–46).

Thus, for **stationary systems**, the changes in kinetic and potential energies are zero (that is, $\Delta \mathrm{KE} = \Delta \mathrm{PE} = 0$), and the total energy change relation in Eq. 3–33 reduces to $\Delta E = \Delta U$ for such systems. Also, the energy of a system during a process will change even if only one form of its energy changes while the other forms of energy remain unchanged.

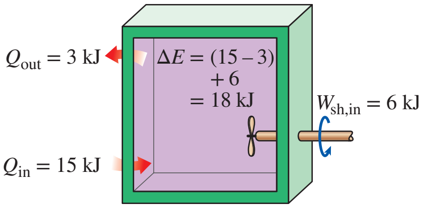


FIGURE 3–45
The energy change of a system during a process is equal to the *net* work and heat transfer between the system and its surroundings.

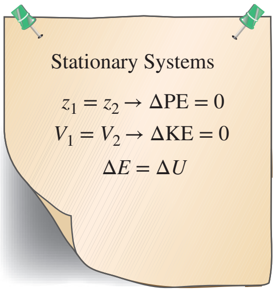


FIGURE 3–46
For stationary systems, $\Delta\mathrm{KE} = \Delta\mathrm{PE} = 0$; thus $\Delta E = \Delta U$.

Mechanisms of Energy Transfer, E_{in} and E_{out}

Energy can be transferred to or from a system in three forms: *heat, work,* and *mass flow.* Energy interactions are recognized at the system boundary as they cross it, and they represent the energy gained or lost by a system during a process. The only two forms of energy interactions associated with a fixed mass or closed system are *heat transfer* and *work.*

1. **Heat Transfer, *Q*** Heat transfer to a system (heat gain) increases the energy of the molecules and thus the internal energy of the system, and heat transfer from a system (heat loss) decreases it since the energy transferred out as heat comes from the energy of the molecules of the system.
2. **Work Transfer, *W*** An energy interaction that is not caused by a temperature difference between a system and its surroundings is work. A rising piston, a rotating shaft, and an electrical wire crossing the system boundaries are all associated with work interactions. Work transfer to a system (i.e., work done on a system) increases the energy of the system, and work transfer from a system (i.e., work done by the system) decreases it since the energy transferred out as work comes from the energy contained in the system. Car engines and hydraulic, steam, or gas turbines produce work, while compressors, pumps, and mixers consume work.
3. **Mass Flow, *m*** Mass flow in and out of the system serves as an additional mechanism of energy transfer. When mass enters a system, the energy of the system increases because mass carries energy with it (in fact, mass is energy). Likewise, when some mass leaves the system, the energy contained within the system decreases because the departing mass takes out some energy with it. For example, when some hot water is taken out of a water heater and is replaced by the same amount of cold water, the energy content of the hot-water tank (the control volume) decreases as a result of this mass interaction (Fig. 3–47).

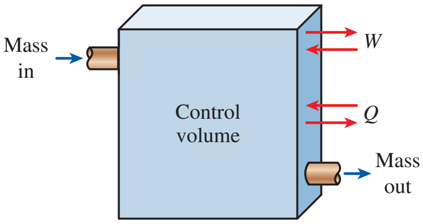


FIGURE 3–47
The energy content of a control volume can be changed by mass flow as well as by heat and work interactions.

Noting that energy can be transferred in the forms of heat, work, and mass, and that the net transfer of a quantity is equal to the difference between the amounts transferred in and out, the energy balance can be written more explicitly as

$$E_{in} - E_{out} = (Q_{in} - Q_{out}) + (W_{in} - W_{out}) + (E_{mass,in} - E_{mass,out}) = \Delta E_{system} \tag{3–34}$$

where the subscripts *in* and *out* denote quantities that enter and leave the system, respectively. All six quantities on the right side of the equation represent "amounts," and thus they are *positive* quantities. The direction of any energy transfer is described by the subscripts *in* and *out.*

The heat transfer Q is zero for adiabatic systems, the work transfer W is zero for systems that involve no work interactions, and the energy transport with mass E_{mass} is zero for systems that involve no mass flow across their boundaries (i.e., closed systems).

Energy balance for any system undergoing any kind of process can be expressed more compactly as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}} \qquad \text{(kJ)} \tag{3–35}$$

or, in the **rate form**, as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal,} \\ \text{kinetic, potential, etc., energies}}} \qquad \text{(kW)} \tag{3–36}$$

For constant rates, the total quantities during a time interval Δt are related to the quantities per unit time as

$$Q = \dot{Q}\Delta t, \quad W = \dot{W}\Delta t, \quad \text{and} \quad \Delta E = (dE/dt)\Delta t \qquad \text{(kJ)} \tag{3–37}$$

The energy balance can be expressed on a **per unit mass** basis as

$$e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}} \qquad \text{(kJ/kg)} \tag{3–38}$$

which is obtained by dividing all the quantities in Eq. 3–35 by the mass m of the system. Energy balance can also be expressed in the differential form as

$$\delta E_{\text{in}} - \delta E_{\text{out}} = dE_{\text{system}} \quad \text{or} \quad \delta e_{\text{in}} - \delta e_{\text{out}} = de_{\text{system}} \tag{3–39}$$

For a closed system undergoing a **cycle**, the initial and final states are identical, and thus $\Delta E_{\text{system}} = E_2 - E_1 = 0$. Then the energy balance for a cycle simplifies to $E_{\text{in}} - E_{\text{out}} = 0$ or $E_{\text{in}} = E_{\text{out}}$. Noting that a closed system does not involve any mass flow across its boundaries, the energy balance for a cycle can be expressed in terms of heat and work interactions as

$$W_{\text{net,out}} = Q_{\text{net,in}} \quad \text{or} \quad \dot{W}_{\text{net,out}} = \dot{Q}_{\text{net,in}} \qquad \text{(for a cycle)} \tag{3–40}$$

That is, the net work output during a cycle is equal to net heat input (Fig. 3–48).

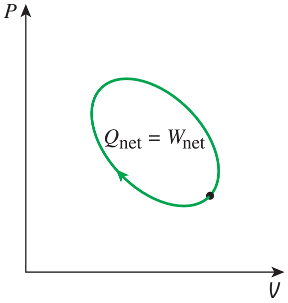


FIGURE 3–48
For a cycle $\Delta E = 0$, thus $Q = W$.

EXAMPLE 3–10 Cooling of a Hot Fluid in a Tank

A rigid tank contains a hot fluid that is cooled while being stirred by a paddle wheel. Initially, the internal energy of the fluid is 800 kJ. During the cooling process, the fluid loses 500 kJ of heat, and the paddle wheel does 100 kJ of work on the fluid. Determine the final internal energy of the fluid. Neglect the energy stored in the paddle wheel.

SOLUTION A fluid in a rigid tank loses heat while being stirred. The final internal energy of the fluid is to be determined.

Assumptions **1** The tank is stationary and thus the kinetic and potential energy changes are zero, $\Delta\text{KE} = \Delta\text{PE} = 0$. Therefore, $\Delta E = \Delta U$ and internal energy is the only form of the system's energy that may change during this process. **2** Energy stored in the paddle wheel is negligible.

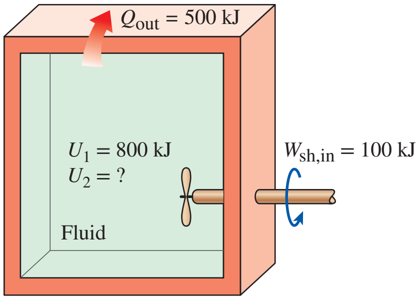


FIGURE 3–49
Schematic for Example 3–10.

Analysis Take the contents of the tank as the system (Fig. 3–49). This is a *closed system* since no mass crosses the boundary during the process.

We observe that the volume of a rigid tank is constant, and thus there is no moving boundary work. Also, heat is lost from the system, and shaft work is done on the system. Applying the energy balance on the system gives

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$W_{sh,in} - Q_{out} = \Delta U = U_2 - U_1$$

$$100\text{ kJ} - 500\text{ kJ} = U_2 - 800\text{ kJ}$$

$$U_2 = \mathbf{400\ kJ}$$

Therefore, the final internal energy of the system is 400 kJ.

EXAMPLE 3–11 Acceleration of Air by a Fan

A fan that consumes 20 W of electric power when operating is claimed to discharge air from a ventilated room at a rate of 1.0 kg/s at a discharge velocity of 8 m/s (Fig. 3–50). Determine if this claim is reasonable.

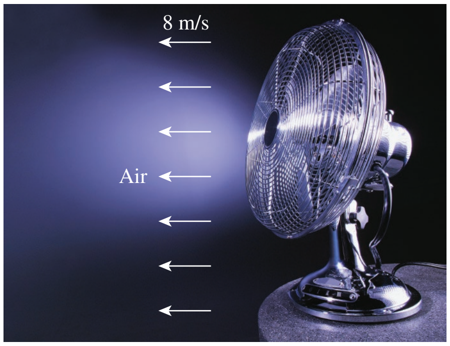


FIGURE 3–50
Schematic for Example 3–11.
Design Pics

SOLUTION A fan is claimed to increase the velocity of air to a specified value while consuming electric power at a specified rate. The validity of this claim is to be investigated.
Assumptions The ventilating room is relatively calm, and air velocity in it is negligible.
Analysis First, let's examine the energy conversions involved: The motor of the fan converts part of the electrical power it consumes to mechanical (shaft) power, which is used to rotate the fan blades in air. The blades are shaped such that they impart a large fraction of the mechanical power of the shaft to air by mobilizing it. In the limiting ideal case of no losses (no conversion of electrical and mechanical energy to thermal energy) in steady operation, the electric power input will be equal to the rate of increase of the kinetic energy of air. Therefore, for a control volume that encloses the fan-motor unit, the energy balance can be written as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{system}/dt^{\nearrow 0\text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}} = 0 \rightarrow \dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{elect,in} = \dot{m}_{air}\text{ke}_{out} = \dot{m}_{air}\frac{V_{out}^2}{2}$$

Solving for V_{out} and substituting gives the maximum air outlet velocity to be

$$V_{out} = \sqrt{\frac{2\dot{W}_{elect,in}}{\dot{m}_{air}}} = \sqrt{\frac{2(20\text{ J/s})}{1.0\text{ kg/s}}\left(\frac{1\text{ m}^2/\text{s}^2}{1\text{ J/kg}}\right)} = 6.3\text{ m/s}$$

which is less than 8 m/s. Therefore, the claim is **false**.
Discussion The conservation of energy principle requires the energy to be preserved as it is converted from one form to another, and it does not allow any energy to be created or destroyed during a process. From the first law point of view, there is nothing wrong with the conversion of all electrical energy into kinetic energy. Therefore, the first law has no objection to air velocity reaching 6.3 m/s—but this is the upper limit. Any claim of higher velocity is in violation of the first law, and thus impossible. In reality, the air velocity will be considerably lower than 6.3 m/s because of the losses associated with the conversion of electrical energy to mechanical shaft energy, and the conversion of mechanical shaft energy to kinetic energy or air.

EXAMPLE 3–12 Heating Effect of a Fan

A room is initially at the outdoor temperature of 25°C. Now a large fan that consumes 200 W of electricity when running is turned on (Fig. 3–51). The heat transfer rate between the room and the outdoor air is given as $\dot{Q} = UA(T_i - T_o)$ where $U = 6$ W/m²·°C is the overall heat transfer coefficient, $A = 30$ m² is the exposed surface area of the room, and T_i and T_o are the indoor and outdoor air temperatures, respectively. Determine the indoor air temperature when steady operating conditions are established.

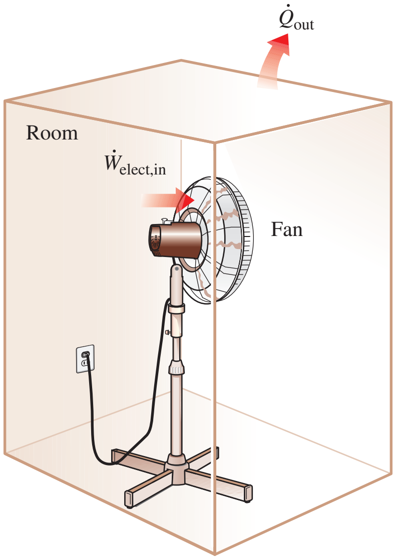


FIGURE 3–51
Schematic for Example 3–12.

SOLUTION A large fan is turned on and kept on in a room that loses heat to the outdoors. The indoor air temperature is to be determined when steady operation is reached.

Assumptions **1** Heat transfer through the floor is negligible. **2** There are no other energy interactions involved.

Analysis The electricity consumed by the fan is energy input for the room, and thus the room gains energy at a rate of 200 W. As a result, the room air temperature tends to rise. But as the room air temperature rises, the rate of heat loss from the room increases until the rate of heat loss equals the electric power consumption. At that point, the temperature of the room air, and thus the energy content of the room, remains constant, and the conservation of energy for the room becomes

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt^{\nearrow 0\,\text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}} = 0 \rightarrow \dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{W}_{\text{elect,in}} = \dot{Q}_{\text{out}} = UA(T_i - T_o)$$

Substituting,

$$200\text{ W} = (6\text{ W/m}^2\cdot{}^\circ\text{C})(30\text{ m}^2)(T_i - 25^\circ\text{C})$$

It gives

$$T_i = \mathbf{26.1^\circ C}$$

Therefore, the room air temperature will remain constant after it reaches 26.1°C.

Discussion Note that a 200-W fan heats a room just like a 200-W resistance heater. In the case of a fan, the motor converts part of the electric energy it draws to mechanical energy in the form of a rotating shaft, while the remaining part is dissipated as heat to the room air because of the motor inefficiency (no motor converts 100 percent of the electric energy it receives to mechanical energy, although some large motors come close with a conversion efficiency of over 97 percent). Part of the mechanical energy of the shaft is converted to kinetic energy of air through the blades, which is then converted to thermal energy as air molecules slow down because of friction. At the end, all electric energy drawn by the fan motor is converted to thermal energy of air, which manifests itself as a rise in temperature.

EXAMPLE 3–13 Annual Lighting Cost of a Classroom

The lighting needs of a classroom are met by 30 fluorescent lamps, each consuming 80 W of electricity (Fig. 3–52). The lights in the classroom are kept on for 12 hours a day and 250 days a year. For a unit electricity cost of 11 cents per kWh, determine the annual energy cost of lighting for this classroom. Also, discuss the effect of lighting on the heating and air-conditioning requirements of the room.

FIGURE 3–52
Fluorescent lamps lighting a classroom as discussed in Example 3–13.
PhotoLink/Getty Images

SOLUTION The lighting of a classroom by fluorescent lamps is considered. The annual electricity cost of lighting for this classroom is to be determined, and the lighting's effect on the heating and air-conditioning requirements is to be discussed.

Assumptions The effect of voltage fluctuations is negligible, so each fluorescent lamp consumes its rated power.

Analysis The electric power consumed by the lamps when all are on and the number of hours they are kept on per year are

$$\begin{aligned}\text{Lighting power} &= (\text{Power consumed per lamp}) \times (\text{No. of lamps})\\ &= (80\ \text{W/lamp})(30\ \text{lamps})\\ &= 2400\ \text{W} = 2.4\ \text{kW}\end{aligned}$$

$$\text{Operating hours} = (12\ \text{h/day})(250\ \text{days/yr}) = 3000\ \text{h/yr}$$

Then the amount and cost of electricity used per year become

$$\begin{aligned}\text{Lighting energy} &= (\text{Lighting power})(\text{Operating hours})\\ &= (2.4\ \text{kW})(3000\ \text{h/yr}) = 7200\ \text{kWh/yr}\end{aligned}$$

$$\begin{aligned}\text{Lighting cost} &= (\text{Lighting energy})(\text{Unit cost})\\ &= (7200\ \text{kWh/yr})(\$0.11/\text{kWh}) = \mathbf{\$792/yr}\end{aligned}$$

Light is absorbed by the surfaces it strikes and is converted to thermal energy. Disregarding the light that escapes through the windows, the entire 2.4 kW of electric power consumed by the lamps eventually becomes part of thermal energy of the classroom. Therefore, the lighting system in this room reduces the heating requirements by 2.4 kW but increases the air-conditioning load by 2.4 kW.

Discussion Note that the annual lighting cost of this classroom alone is close to $800. This shows the importance of energy conservation measures. If incandescent lightbulbs were used instead of fluorescent tubes, the lighting costs would be four times as much since incandescent lamps use four times as much power for the same amount of light produced.

3–7 ▪ ENERGY CONVERSION EFFICIENCIES

Efficiency is one of the most often used terms in thermodynamics, and it indicates how well an energy conversion or transfer process is accomplished. Efficiency is also one of the most often misused terms in thermodynamics and a source of misunderstandings. This is because efficiency is often used without being properly defined first. Next, we will clarify this further and define some efficiencies commonly used in practice.

Efficiency, in general, can be expressed in terms of the desired output and the required input as

$$\text{Efficiency} = \frac{\text{Desired output}}{\text{Required input}} \tag{3–41}$$

If you are shopping for a water heater, a knowledgeable salesperson will tell you that the efficiency of a conventional electric water heater is about 90 percent (Fig. 3–53). You may find this confusing, since the heating elements of electric water heaters are resistance heaters, and the efficiency of all resistance heaters is 100 percent as they convert all the electrical energy they consume into thermal energy. A knowledgeable salesperson will clarify this by explaining that the heat losses from the hot-water tank to the surrounding air amount to 10 percent of the electrical energy consumed, and the **efficiency of a water heater** is defined as the ratio of the *energy delivered to the house by hot water* to the *energy supplied to the water heater.* A clever salesperson may even talk you into buying a more expensive water heater with thicker insulation that has an efficiency of 94 percent. If you are a knowledgeable consumer and have access to natural gas, you will probably purchase a gas water heater whose efficiency is only 55 percent since a gas unit costs about the same as an electric unit to purchase and install, but the annual energy cost of a gas unit will be much less than that of an electric unit.

Water heater

Type	Efficiency
Gas, conventional	55%
Gas, high-efficiency	62%
Electric, conventional	90%
Electric, high-efficiency	94%

FIGURE 3–53

Typical efficiencies of conventional and high-efficiency electric and natural gas water heaters.

Christopher Kerrigan/McGraw-Hill Education

Perhaps you are wondering how the efficiency for a gas water heater is defined, and why it is much lower than the efficiency of an electric heater. As a general rule, the efficiency of equipment that involves the combustion of a fuel is based on the **heating value of the fuel**, which is *the amount of heat released when a unit amount of fuel at room temperature is completely burned and the combustion products are cooled to the room temperature* (Fig. 3–54). Then the performance of combustion equipment can be characterized by **combustion equipment efficiency** $\eta_{\text{comb. equip.}}$, defined as

$$\eta_{\text{comb. equip.}} = \frac{Q_{\text{useful}}}{\text{HV}} = \frac{\text{Useful heat delivered by the combustion equipment}}{\text{Heating value of the fuel burned}} \tag{3–42}$$

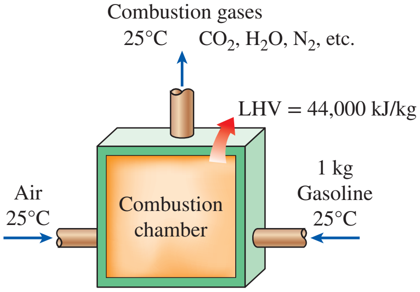


FIGURE 3–54
The definition of the heating value of gasoline.

This efficiency can take different names, depending on the type of the combustion unit such as furnace efficiency η_{furnace}, boiler efficiency η_{boiler}, or heater efficiency η_{heater}. For example, an efficiency of 70 percent for a coal-burning heater used to heat a building in winter indicates that 70 percent of the heating value of the coal is transferred to the building as useful heat while the remaining 30 percent is lost, mostly by the hot stack gases leaving the heater.

Most fuels contain hydrogen, which forms water when burned, and the heating value of a fuel will be different depending on whether the water in combustion products is in the liquid or vapor form. The heating value is called the *lower heating value,* or LHV, when the water leaves as a vapor, and the *higher heating value,* or HHV, when the water in the combustion gases is completely condensed and thus the heat of vaporization is also recovered. The difference between these two heating values is equal to the product of the amount of water and the enthalpy of vaporization of water at room temperature. For example, the lower and higher heating values of gasoline are 44,000 kJ/kg and 47,300 kJ/kg, respectively. An efficiency definition should make it clear whether it is based on the higher or lower heating value of the fuel. Efficiencies of cars and jet engines are normally based on *lower heating values* since water normally leaves as a vapor in the exhaust gases, and it is not practical to try to recover the heat of vaporization. Efficiencies of furnaces, on the other hand, are based on *higher heating values.*

The efficiency of space heating systems of residential and commercial buildings is usually expressed in terms of the **annual fuel utilization efficiency**, or AFUE, which accounts for the combustion equipment efficiency as well as other losses such as heat losses to unheated areas and start-up and cool-down losses. The AFUE of most new heating systems is about 85 percent, although the AFUE of some old heating systems is under 60 percent. The AFUE of some new high-efficiency furnaces exceeds 96 percent, but the high cost of such furnaces cannot be justified for locations with mild to moderate winters. Such high efficiencies are achieved by reclaiming most of the heat in the flue gases, condensing the water vapor, and discharging the flue gases at temperatures as low as 38°C (or 100°F) instead of about 200°C (or 400°F) for the conventional models.

For *car engines,* the work output is understood to be the power delivered by the crankshaft. But for power plants, the work output can be the mechanical power at the turbine exit, or the electrical power output of the generator.

A generator is a device that converts mechanical energy to electrical energy, and the effectiveness of a generator is characterized by the **generator efficiency**, which is the ratio of the *electrical power output* to the *mechanical power input.* The *thermal efficiency* of a power plant, which is of primary interest in thermodynamics, is usually defined as the ratio of the net shaft work output of the turbine to the heat input to the working fluid. The effects of other factors are incorporated by defining an **overall efficiency** for the power plant as the ratio of the *net electrical power output* to the *rate of fuel energy input.* That is,

$$\eta_{\text{overall}} = \eta_{\text{comb. equip.}}\eta_{\text{thermal}}\eta_{\text{generator}} = \frac{\dot{W}_{\text{net,electric}}}{\text{HHV} \times \dot{m}_{\text{fuel}}} \tag{3–43}$$

The overall efficiencies are about 25–30 percent for gasoline automotive engines, 35–40 percent for diesel engines, and up to 60 percent for large power plants.

We are all familiar with the conversion of electrical energy to *light* by incandescent lightbulbs, fluorescent tubes, and high-intensity discharge lamps. The efficiency for the conversion of electricity to light can be defined as the ratio of the energy converted to light to the electrical energy consumed. For example, common incandescent lightbulbs convert about 5 percent of the electrical energy they consume to light; the rest of the energy consumed is dissipated as heat, which adds to the cooling load of the air conditioner in summer. However, it is more common to express the effectiveness of this conversion process by **lighting efficacy**, which is defined as the *amount of light output in lumens per W of electricity consumed.*

The efficacy of different lighting systems is given in Table 3–1. Note that a compact fluorescent lightbulb produces about four times as much light as an incandescent lightbulb per W, and thus a 15-W fluorescent bulb can replace a 60-W incandescent lightbulb (Fig. 3–55). Also, a compact fluorescent bulb lasts about 10,000 h, which is 10 times as long as an incandescent bulb, and it plugs directly into the socket of an incandescent lamp. Therefore, despite their higher initial cost, compact fluorescents reduce the lighting costs considerably through reduced electricity consumption. Sodium-filled high-intensity discharge lamps provide the most efficient lighting, but their use is limited to outdoor use because of their yellowish light.

We can also define efficiency for cooking appliances since they convert electrical or chemical energy to heat for cooking. The **efficiency of a cooking appliance** can be defined as the ratio of the *useful energy transferred to the food to the energy consumed by the appliance* (Fig. 3–56). Electric ranges are more efficient than gas ranges, but it is much cheaper to cook with natural gas than with electricity because of the lower unit cost of natural gas (Table 3–2).

The cooking efficiency depends on user habits as well as the individual appliances. Convection and microwave ovens are inherently more efficient than conventional ovens. On average, convection ovens save about *one-third* and microwave ovens save about *two-thirds* of the energy used by conventional ovens. The cooking efficiency can be increased by using the smallest oven for baking, using a pressure cooker, using an electric slow cooker for stews and soups, using the smallest pan that will do the job, using the smaller heating element for small pans on electric ranges, using flat-bottomed pans on electric burners to assure good contact, keeping burner drip pans clean and shiny, defrosting frozen foods in the refrigerator before cooking, avoiding preheating unless it is necessary, keeping the pans covered during cooking, using timers and thermometers to avoid overcooking, using

TABLE 3–1

The efficacy of different lighting systems

Type of lighting	Efficacy, lumens/W
Combustion	
Candle	0.3
Kerosene lamp	1–2
Incandescent	
Ordinary	6–20
Halogen	15–35
Fluorescent	
Compact	40–87
Tube	60–120
High-intensity discharge	
Mercury vapor	40–60
Metal halide	65–118
High-pressure sodium	85–140
Low-pressure sodium	70–200
Solid-state	
LED	20–160
OLED	15–60
Theoretical limit	300*

*This value depends on the spectral distribution of the assumed ideal light source. For white light sources, the upper limit is about 300 lm/W for metal halide, 350 lm/W for fluorescents, and 400 lm/W for LEDs. Spectral maximum occurs at a wavelength of 555 nm (green) with a light output of 683 lm/W.

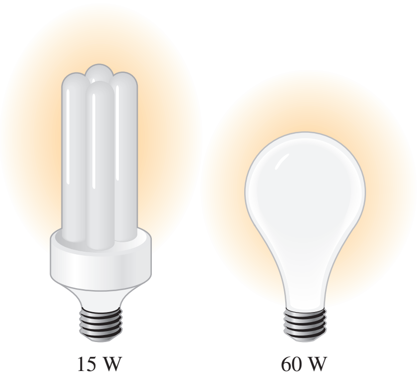


FIGURE 3–55

A 15-W compact fluorescent lamp provides as much light as a 60-W incandescent lamp.

TABLE 3–2

Energy costs of cooking a casserole with different appliances*

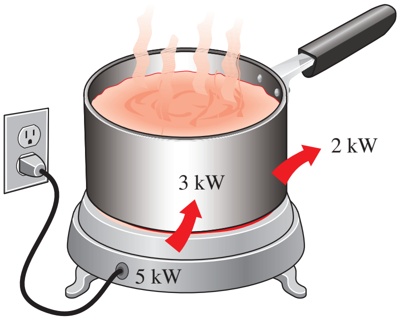
Cooking appliance	Cooking temperature	Cooking time	Energy used	Cost of energy
Electric oven	350°F (177°C)	1 h	2.0 kWh	$0.19
Convection oven (elect.)	325°F (163°C)	45 min	1.39 kWh	$0.13
Gas oven	350°F (177°C)	1 h	0.112 therm	$0.13
Frying pan	420°F (216°C)	1 h	0.9 kWh	$0.09
Toaster oven	425°F (218°C)	50 min	0.95 kWh	$0.09
Crockpot	200°F (93°C)	7 h	0.7 kWh	$0.07
Microwave oven	"High"	15 min	0.36 kWh	$0.03

*Assumes a unit cost of $0.095/kWh for electricity and $1.20/therm for gas.

[From J. T. Amann, A. Wilson, and K. Ackerly, *Consumer Guide to Home Energy Savings,* 9th ed., American Council for an Energy-Efficient Economy, Washington, D.C., 2007, p. 163.]

the self-cleaning feature of ovens right after cooking, and keeping inside surfaces of microwave ovens clean.

Using energy-efficient appliances and practicing energy conservation measures help our pocketbooks by reducing our utility bills. This also helps the **environment** by reducing the amount of pollutants emitted to the atmosphere during the combustion of fuel at home or at the power plants where electricity is generated. The combustion of *each therm of natural gas* produces 6.4 kg of carbon dioxide, which causes global climate change; 4.7 g of nitrogen oxides and 0.54 g of hydrocarbons, which cause smog; 2.0 g of carbon monoxide, which is toxic; and 0.030 g of sulfur dioxide, which causes acid rain. Each therm of natural gas saved eliminates the emission of these pollutants while saving $0.60 for the average consumer in the United States. Each kWh of electricity conserved saves 0.4 kg of coal and 1.0 kg of CO_2 and 15 g of SO_2 from a coal power plant.



$$\text{Efficiency} = \frac{\text{Energy utilized}}{\text{Energy supplied to appliance}}$$

$$= \frac{3\text{ kWh}}{5\text{ kWh}} = 0.60$$

FIGURE 3–56
The efficiency of a cooking appliance represents the fraction of the energy supplied to the appliance that is transferred to the food.

EXAMPLE 3–14 Cost of Cooking with Electric and Gas Ranges

The efficiency of cooking appliances affects the internal heat gain from them since an inefficient appliance consumes a greater amount of energy for the same task, and the excess energy consumed shows up as heat in the living space. The efficiency of open burners is determined to be 73 percent for electric units and 38 percent for gas units (Fig. 3–57). Consider a 2-kW electric burner at a location where the unit costs of electricity and natural gas are $0.12/kWh and $1.20/therm, respectively. Determine the rate of energy consumption by the burner and the unit cost of utilized energy for both electric and gas burners.

SOLUTION The operation of electric and gas ranges is considered. The rate of energy consumption and the unit cost of utilized energy are to be determined.

Analysis The efficiency of the electric heater is given to be 73 percent. Therefore, a burner that consumes 2 kW of electrical energy will supply

$$\dot{Q}_{\text{utilized}} = (\text{Energy input}) \times (\text{Efficiency}) = (2\text{ kW})(0.73) = 1.46\text{ kW}$$

of useful energy. The unit cost of utilized energy is inversely proportional to the efficiency and is determined from

$$\text{Cost of utilized energy} = \frac{\text{Cost of energy input}}{\text{Efficiency}} = \frac{\$0.12/\text{kWh}}{0.73} = \mathbf{\$0.164/kWh}$$

Noting that the efficiency of a gas burner is 38 percent, the energy input to a gas burner that supplies utilized energy at the same rate (1.46 kW) is

$$\dot{Q}_{\text{input,gas}} = \frac{\dot{Q}_{\text{utilized}}}{\text{Efficiency}} = \frac{1.46\text{ kW}}{0.38} = \mathbf{3.84\ kW} \qquad (= 13{,}100\text{ Btu/h})$$

since 1 kW = 3412 Btu/h. Therefore, a gas burner should have a rating of at least 13,100 Btu/h to perform as well as the electric unit.

Noting that 1 therm = 29.3 kWh, the unit cost of utilized energy in the case of a gas burner is determined to be

$$\text{Cost of utilized energy} = \frac{\text{Cost of energy input}}{\text{Efficiency}} = \frac{\$1.20/29.3\text{ kWh}}{0.38}$$
$$= \mathbf{\$0.108/kWh}$$

Discussion The cost of utilized gas is less than that of utilized electricity. Therefore, despite its higher efficiency, cooking with an electric burner will cost about 52 percent more compared to a gas burner in this case. This explains why cost-conscious consumers always ask for gas appliances, and it is not wise to use electricity for heating purposes.

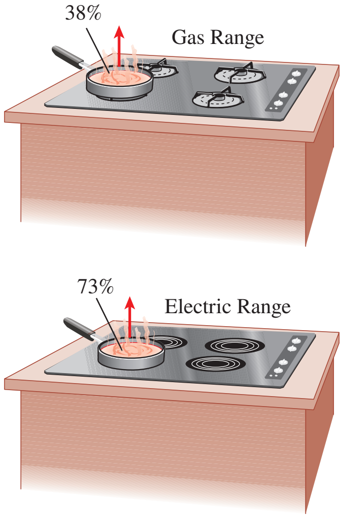
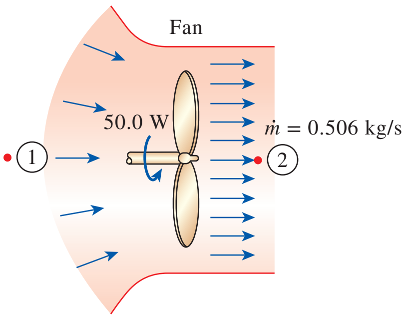


FIGURE 3–57
Schematic of the 73 percent efficient electric heating unit and 38 percent efficient gas burner discussed in Example 3–14.

Efficiencies of Mechanical and Electrical Devices

The transfer of mechanical energy is usually accomplished by a rotating shaft, and thus mechanical work is often referred to as *shaft work*. A pump or a fan receives shaft work (usually from an electric motor) and transfers it to the fluid as mechanical energy (less frictional losses). A turbine, on the other hand, converts the mechanical energy of a fluid to shaft work. In the absence of any irreversibilities such as friction, mechanical energy can be converted entirely from one mechanical form to another, and the **mechanical efficiency** of a device or process can be defined as (Fig. 3–58)



$V_1 \approx 0$, $V_2 = 12.1$ m/s
$z_1 = z_2$
$P_1 \approx P_{\text{atm}}$ and $P_2 \approx P_{\text{atm}}$

$$\eta_{\text{mech,fan}} = \frac{\Delta \dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{shaft,in}}} = \frac{\dot{m}V_2^2/2}{\dot{W}_{\text{shaft,in}}} = \frac{(0.506\text{ kg/s})(12.1\text{ m/s})^2/2}{50.0\text{ W}} = 0.741$$

FIGURE 3–58
The mechanical efficiency of a fan is the ratio of the rate of increase of the mechanical energy of air to the mechanical power input.

$$\eta_{\text{mech}} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy input}} = \frac{E_{\text{mech,out}}}{E_{\text{mech,in}}} = 1 - \frac{E_{\text{mech,loss}}}{E_{\text{mech,in}}} \qquad \textbf{(3–44)}$$

A conversion efficiency of less than 100 percent indicates that conversion is less than perfect and some losses have occurred during conversion. A mechanical efficiency of 97 percent indicates that 3 percent of the mechanical energy input is converted to thermal energy as a result of frictional heating, and this will manifest itself as a slight rise in the temperature of the fluid.

In fluid systems, we are usually interested in increasing the pressure, velocity, and/or elevation of a fluid. This is done by *supplying mechanical energy* to the fluid by a pump, a fan, or a compressor (we will refer to all of them as pumps). Or we are interested in the reverse process of *extracting mechanical energy* from a fluid by a turbine and producing mechanical power in the form of a rotating shaft that can drive a generator or any other rotary device. The degree of perfection of the conversion process between the mechanical work supplied or extracted and the mechanical energy of the fluid is expressed by the **pump efficiency** and **turbine efficiency**, defined as

$$\eta_{\text{pump}} = \frac{\text{Mechanical energy increase of the fluid}}{\text{Mechanical energy input}} = \frac{\Delta \dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{shaft,in}}} = \frac{\dot{W}_{\text{pump},u}}{\dot{W}_{\text{pump}}} \qquad \textbf{(3–45)}$$

where $\Delta \dot{E}_{\text{mech,fluid}} = \dot{E}_{\text{mech,out}} - \dot{E}_{\text{mech,in}}$ is the rate of increase in the mechanical energy of the fluid, which is equivalent to the **useful pumping power** $\dot{W}_{\text{pump},u}$ supplied to the fluid, and

$$\eta_{\text{turbine}} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy decrease of the fluid}} = \frac{\dot{W}_{\text{shaft,out}}}{|\Delta \dot{E}_{\text{mech,fluid}}|} = \frac{\dot{W}_{\text{turbine}}}{\dot{W}_{\text{turbine},e}} \qquad \textbf{(3–46)}$$

where $|\Delta \dot{E}_{\text{mech,fluid}}| = \dot{E}_{\text{mech,in}} - \dot{E}_{\text{mech,out}}$ is the rate of decrease in the mechanical energy of the fluid, which is equivalent to the mechanical power extracted from the fluid by the turbine $\dot{W}_{\text{turbine},e}$, and we use the absolute value sign to avoid negative values for efficiencies. A pump or turbine efficiency of 100 percent indicates perfect conversion between the shaft work and the mechanical energy of the fluid, and this value can be approached (but never attained) as the frictional effects are minimized.

Electrical energy is commonly converted to *rotating mechanical energy* by electric motors to drive fans, compressors, robot arms, car starters, and so forth. The effectiveness of this conversion process is characterized by the *motor efficiency* η_{motor}, which is the ratio of the *mechanical energy output* of the motor to the *electrical energy input.* The full-load motor efficiencies range from about 35 percent for small motors to over 97 percent for large high-efficiency motors. The difference between the electrical energy consumed and the mechanical energy delivered is dissipated as waste heat.

The mechanical efficiency should not be confused with the **motor efficiency** and the **generator efficiency**, which are defined as

Motor:

$$\eta_{\text{motor}} = \frac{\text{Mechanical power output}}{\text{Electric power input}} = \frac{\dot{W}_{\text{shaft,out}}}{\dot{W}_{\text{elect,in}}} \qquad \textbf{(3–47)}$$

and

Generator:

$$\eta_{\text{generator}} = \frac{\text{Electric power output}}{\text{Mechanical power input}} = \frac{\dot{W}_{\text{elect,out}}}{\dot{W}_{\text{shaft,in}}} \tag{3–48}$$

A pump is usually packaged together with its motor, and a turbine with its generator. Therefore, we are usually interested in the **combined** or **overall efficiency** of pump–motor and turbine–generator combinations (Fig. 3–59), which are defined as

$$\eta_{\text{pump–motor}} = \eta_{\text{pump}}\eta_{\text{motor}} = \frac{\dot{W}_{\text{pump},u}}{\dot{W}_{\text{elect,in}}} = \frac{\Delta\dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{elect,in}}} \tag{3–49}$$

and

$$\eta_{\text{turbine–gen}} = \eta_{\text{turbine}}\eta_{\text{generator}} = \frac{\dot{W}_{\text{elect,out}}}{\dot{W}_{\text{turbine},e}} = \frac{\dot{W}_{\text{elect,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|} \tag{3–50}$$

All the efficiencies just defined range between 0 and 100 percent. The lower limit of 0 percent corresponds to the conversion of all mechanical or electric energy input to thermal energy, and the device in this case functions like a resistance heater. The upper limit of 100 percent corresponds to the case of perfect conversion with no friction or other irreversibilities, and thus no conversion of mechanical or electric energy to thermal energy.

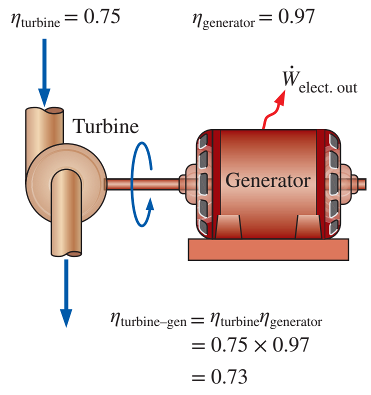


FIGURE 3–59
The overall efficiency of a turbine–generator is the product of the efficiency of the turbine and the efficiency of the generator, and it represents the fraction of the mechanical power of the fluid converted to electrical power.

EXAMPLE 3–15 Power Generation from a Hydroelectric Plant

Electric power is to be generated by installing a hydraulic turbine–generator at a site 70 m below the free surface of a large water reservoir that can supply water at a rate of 1500 kg/s steadily (Fig. 3–60). If the mechanical power output of the turbine is 800 kW and the electric power generation is 750 kW, determine the turbine efficiency and the combined turbine–generator efficiency of this plant. Neglect losses in the pipes.

SOLUTION A hydraulic turbine–generator installed at a large reservoir is to generate electricity. The combined turbine–generator efficiency and the turbine efficiency are to be determined.

Assumptions **1** The water elevation in the reservoir remains constant. **2** The mechanical energy of water at the turbine exit is negligible.

Analysis We take the free surface of water in the reservoir to be point 1 and the turbine exit to be point 2. We also take the turbine exit as the reference level ($z_2 = 0$) so that the potential energies at 1 and 2 are $\text{pe}_1 = gz_1$ and $\text{pe}_2 = 0$. The flow energy P/ρ at both points is zero since both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$). Further, the kinetic energy at both points is zero ($\text{ke}_1 = \text{ke}_2 = 0$) since the water at point 1 is essentially motionless, and the kinetic energy of water at turbine exit is assumed to be negligible. The potential energy of water at point 1 is

$$\text{pe}_1 = gz_1 = (9.81\ \text{m/s}^2)(70\ \text{m})\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 0.687\ \text{kJ/kg}$$

Then the rate at which the mechanical energy of water is supplied to the turbine becomes

$$\begin{aligned} |\Delta\dot{E}_{\text{mech,fluid}}| &= \dot{m}(e_{\text{mech,in}} - e_{\text{mech,out}}) = \dot{m}(\text{pe}_1 - 0) = \dot{m}\text{pe}_1 \\ &= (1500\ \text{kg/s})(0.687\ \text{kJ/kg}) \\ &= 1031\ \text{kW} \end{aligned}$$

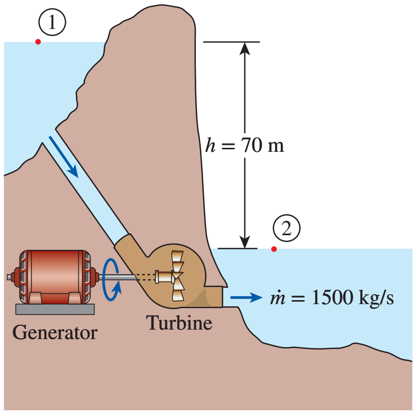


FIGURE 3–60
Schematic for Example 3–15.

The combined turbine–generator and the turbine efficiency are determined from their definitions to be

$$\eta_{\text{turbine - gen}} = \frac{\dot{W}_{\text{elect,out}}}{|\Delta \dot{E}_{\text{mech,fluid}}|} = \frac{750 \text{ kW}}{1031 \text{ kW}} = 0.727 \text{ or } \mathbf{72.7\%}$$

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft,out}}}{|\dot{E}_{\text{mech,fluid}}|} = \frac{800 \text{ kW}}{1031 \text{ kW}} = 0.776 \text{ or } \mathbf{77.6\%}$$

Therefore, the reservoir supplies 1031 kW of mechanical energy to the turbine, which converts 800 kW of it to shaft work that drives the generator, which then generates 750 kW of electric power.

Discussion This problem can also be solved by taking point 1 to be at the turbine inlet and using flow energy instead of potential energy. It would give the same result since the flow energy at the turbine inlet is equal to the potential energy at the free surface of the reservoir.

EXAMPLE 3–16 Cost Savings Associated with High-Efficiency Motors

A 60-hp electric motor (a motor that delivers 60 hp of shaft power at full load) that has an efficiency of 89.0 percent is worn out and is to be replaced by a 93.2 percent efficient high-efficiency motor (Fig. 3–61). The motor operates 3500 hours a year at full load. Taking the unit cost of electricity to be \$0.08/kWh, determine the amount of energy and money saved as a result of installing the high-efficiency motor instead of the standard motor. Also, determine the simple payback period if the purchase prices of the standard and high-efficiency motors are \$4520 and \$5160, respectively.

SOLUTION A worn-out standard motor is to be replaced by a high-efficiency one. The amount of electrical energy and money saved as well as the simple payback period are to be determined.

Assumptions The load factor of the motor remains constant at 1 (full load) when operating.

Analysis The electric power drawn by each motor and their difference can be expressed as

$$\dot{W}_{\text{electric in, standard}} = \dot{W}_{\text{shaft}}/\eta_{\text{st}} = (\text{Rated power})(\text{Load factor})/\eta_{\text{st}}$$

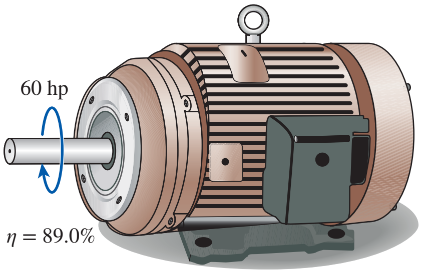
$$\dot{W}_{\text{electric in, efficient}} = \dot{W}_{\text{shaft}}/\eta_{\text{eff}} = (\text{Rated power})(\text{Load factor})/\eta_{\text{eff}}$$

$$\begin{aligned}\text{Power savings} &= \dot{W}_{\text{electric in, standard}} - \dot{W}_{\text{electric in, efficient}}\\ &= (\text{Rated power})(\text{Load factor})(1/\eta_{\text{st}} - 1/\eta_{\text{eff}})\end{aligned}$$

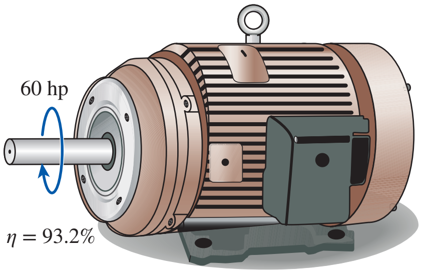
where η_{st} is the efficiency of the standard motor, and η_{eff} is the efficiency of the comparable high-efficiency motor. Then the annual energy and cost savings associated with the installation of the high-efficiency motor become

$$\begin{aligned}\text{Energy savings} &= (\text{Power savings})(\text{Operating hours})\\ &= (\text{Rated power})(\text{Operating hours})(\text{Load factor})(1/\eta_{\text{st}} - 1\eta_{\text{eff}})\\ &= (60 \text{ hp})(0.7457 \text{ kW/hp})(3500 \text{ h/yr})(1)(1/0.89 - 1/0.932)\\ &= \mathbf{7929 \text{ kWh/yr}}\end{aligned}$$

$$\begin{aligned}\text{Cost savings} &= (\text{Energy savings})(\text{Unit cost of energy})\\ &= (7929 \text{ kWh/yr})(\$0.08/\text{kWh})\\ &= \mathbf{\$634/yr}\end{aligned}$$



Standard motor



High-efficiency motor

FIGURE 3–61
Schematic for Example 3–16.

Also,

$$\text{Excess initial cost} = \text{Purchase price differential} = \$5160 - \$4520 = \$640$$

This gives a simple payback period of

$$\text{Simple payback period} = \frac{\text{Excess initial cost}}{\text{Annual cost savings}} = \frac{\$640}{\$634/\text{yr}} = \mathbf{1.01\ yr}$$

Discussion Note that the high-efficiency motor pays for its price differential within about one year from the electrical energy it saves. Considering that the service life of electric motors is several years, the purchase of the higher efficiency motor is definitely indicated in this case.

SUMMARY

The sum of all forms of energy of a system is called *total energy,* which consists of internal, kinetic, and potential energy for simple compressible systems. *Internal energy* represents the molecular energy of a system and may exist in sensible, latent, chemical, and nuclear forms.

Mass flow rate $\dot{m}$ is defined as the amount of mass flowing through a cross section per unit time. It is related to the *volume flow rate* $\dot{V}$, which is the volume of a fluid flowing through a cross section per unit time, by

$$\dot{m} = \rho \dot{V} = \rho A_c V_{\text{avg}}$$

The energy flow rate associated with a fluid flowing at a rate of $\dot{m}$ is

$$\dot{E} = \dot{m} e$$

which is analogous to $E = me$.

The *mechanical energy* is defined as *the form of energy that can be converted to mechanical work completely and directly by a mechanical device such as an ideal turbine.* It is expressed on a unit mass basis and rate form as

$$e_{\text{mech}} = \frac{P}{\rho} + \frac{V^2}{2} + gz$$

and

$$\dot{E}_{\text{mech}} = \dot{m} e_{\text{mech}} = \dot{m}\left(\frac{P}{\rho} + \frac{V^2}{2} + gz\right)$$

where P/ρ is the *flow energy,* $V^2/2$ is the *kinetic energy,* and gz is the *potential energy* of the fluid per unit mass.

Energy can cross the boundaries of a closed system in the form of heat or work. For control volumes, energy can also be transported by mass. If the energy transfer is due to a temperature difference between a closed system and its surroundings, it is *heat;* otherwise, it is *work.*

Work is the energy transferred as a force acts on a system through a distance. Various forms of work are expressed as follows:

Electrical work: $W_e = \mathbf{V} I \Delta t$

Shaft work: $W_{\text{sh}} = 2\pi n \mathrm{T}$

Spring work: $W_{\text{spring}} = \frac{1}{2} k(x_2^2 - x_1^2)$

The *first law of thermodynamics* is essentially an expression of the conservation of energy principle, also called the *energy balance.* The general energy balance for *any system* undergoing *any process* can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}} \qquad \text{(kJ)}$$

It can also be expressed in the *rate form* as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal,}\\ \text{kinetic, potential, etc., energies}}} \qquad \text{(kW)}$$

The efficiencies of various devices are defined as

$$\eta_{\text{pump}} = \frac{\Delta \dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{shaft,in}}} = \frac{\dot{W}_{\text{pump},u}}{\dot{W}_{\text{pump}}}$$

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft,out}}}{\left|\Delta \dot{E}_{\text{mech,fluid}}\right|} = \frac{\dot{W}_{\text{turbine}}}{\dot{W}_{\text{turbine},e}}$$

$$\eta_{\text{motor}} = \frac{\text{Mechanical power output}}{\text{Electric power input}} = \frac{\dot{W}_{\text{shaft,out}}}{\dot{W}_{\text{elect,in}}}$$

$$\eta_{\text{generator}} = \frac{\text{Electric power output}}{\text{Mechanical power input}} = \frac{\dot{W}_{\text{elect,out}}}{\dot{W}_{\text{shaft,in}}}$$

$$\eta_{\text{pump-motor}} = \eta_{\text{pump}}\eta_{\text{motor}} = \frac{\Delta \dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{elect,in}}}$$

$$\eta_{\text{turbine-gen}} = \eta_{\text{turbine}}\eta_{\text{generator}} = \frac{\dot{W}_{\text{elect,out}}}{\left|\Delta \dot{E}_{\text{mech,fluid}}\right|}$$

REFERENCES AND SUGGESTED READINGS

1. ASHRAE *Handbook of Fundamentals.* SI version. Atlanta, GA: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc., 1993.
2. Y. A. Çengel. "An Intuitive and Unified Approach to Teaching Thermodynamics." ASME International Mechanical Engineering Congress and Exposition, Atlanta, Georgia, AES-Vol. 36, pp. 251–260, November 17–22, 1996.

PROBLEMS*

Forms of Energy

3–1C What is the difference between the macroscopic and microscopic forms of energy?

3–2C What is total energy? Identify the different forms of energy that constitute the total energy.

3–3C List the forms of energy that contribute to the internal energy of a system.

3–4C How are heat, internal energy, and thermal energy related to each other?

3–5C What is mechanical energy? How does it differ from thermal energy? What are the forms of mechanical energy of a fluid stream?

3–6C Portable electric heaters are commonly used to heat small rooms. Explain the energy transformation involved during this heating process.

3–7C Natural gas, which is mostly methane CH_4, is a fuel and a major energy source. Can we say the same about hydrogen gas, H_2?

3–8C Consider the falling of a rock off a cliff into seawater, and eventually settling at the bottom of the sea. Starting with the potential energy of the rock, identify the energy transfers and transformations involved during this process.

3–9 Electric power is to be generated by installing a hydraulic turbine–generator at a site 120 m below the free surface of a large water reservoir that can supply water at a rate of 1500 kg/s steadily. Determine the power generation potential.

3–10E The specific kinetic energy of a moving mass is given by $\text{ke} = V^2/2$, where V is the velocity of the mass. Determine the specific kinetic energy of a mass whose velocity is 100 ft/s, in Btu/lbm. *Answer:* 0.2 Btu/lbm

3–11 Determine the specific kinetic energy of a mass whose velocity is 30 m/s, in kJ/kg.

3–12E Calculate the total potential energy, in Btu, of an object that is 20 ft below a datum level at a location where $g = 31.7$ ft/s^2 and which has a mass of 100 lbm.

3–13 Determine the specific potential energy, in kJ/kg, of an object 50 m above a datum in a location where $g = 9.8$ m/s^2.

3–14 A water jet that leaves a nozzle at 60 m/s at a flow rate of 120 kg/s is to be used to generate power by striking the buckets located on the perimeter of a wheel. Determine the power generation potential of this water jet.

3–15 Consider a river flowing toward a lake at an average velocity of 3 m/s at a rate of 500 m^3/s at a location 90 m above the lake surface. Determine the total mechanical energy of the river water per unit mass and the power generation potential of the entire river at that location.

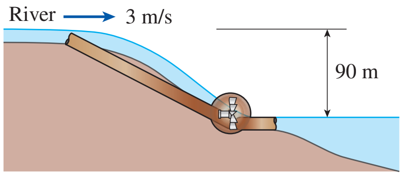


FIGURE P3–15

3–16 At a certain location, wind is blowing steadily at 10 m/s. Determine the mechanical energy of air per unit mass and the power generation potential of a wind turbine with 60-m-diameter blades at that location. Take the air density to be 1.25 kg/m^3.

Energy Transfer by Heat and Work

3–17C What is the caloric theory? When and why was it abandoned?

3–18C In what forms can energy cross the boundaries of a closed system?

3–19C What is an adiabatic process? What is an adiabatic system?

3–20C When is the energy crossing the boundaries of a closed system heat and when is it work?

3–21C Consider an automobile traveling at a constant speed along a road. Determine the direction of the heat and work interactions, taking the following as the system: (*a*) the car radiator, (*b*) the car engine, (*c*) the car wheels, (*d*) the road, and (*e*) the air surrounding the car.

3–22C A room is heated by an iron that is left plugged in. Is this a heat or work interaction? Take the entire room, including the iron, as the system.

3–23C A room is heated as a result of solar radiation coming in through the windows. Is this a heat or work interaction for the room?

3–24C A gas in a piston–cylinder device is compressed, and as a result its temperature rises. Is this a heat or work interaction?

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* Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

3–25 A small electrical motor produces 5 W of mechanical power. What is this power in (*a*) N, m, and s units; and (*b*) kg, m, and s units? *Answers:* (*a*) 5 N·m/s, (*b*) 5 kg·m^2/s^3

Mechanical Forms of Work

3–26C A car is accelerated from rest to 85 km/h in 10 s. Would the energy transferred to the car be different if it were accelerated to the same speed in 5 s?

3–27E A construction crane lifts a prestressed concrete beam weighing 3 short tons from the ground to the top of piers that are 24 ft above the ground. Determine the amount of work done considering (*a*) the beam and (*b*) the crane as the system. Express your answers in both lbf·ft and Btu.

3–28E Determine the torque applied to the shaft of a car that transmits 225 hp and rotates at a rate of 3000 rpm.

3–29 How much work, in kJ, can a spring whose spring constant is 3 kN/cm produce after it has been compressed 3 cm from its unloaded length?

3–30 A ski lift has a one-way length of 1 km and a vertical rise of 200 m. The chairs are spaced 20 m apart, and each chair can seat three people. The lift is operating at a steady speed of 10 km/h. Neglecting friction and air drag and assuming that the average mass of each loaded chair is 250 kg, determine the power required to operate this ski lift. Also estimate the power required to accelerate this ski lift in 5 s to its operating speed when it is first turned on.

3–31 The engine of a 1500-kg automobile has a power rating of 75 kW. Determine the time required to accelerate this car from rest to a speed of 100 km/h at full power on a level road. Is your answer realistic?

3–32 A damaged 1200-kg car is being towed by a truck. Neglecting the friction, air drag, and rolling resistance, determine the extra power required (*a*) for constant velocity on a level road, (*b*) for constant velocity of 50 km/h on a 30° (from horizontal) uphill road, and (*c*) to accelerate on a level road from stop to 90 km/h in 12 s. *Answers:* (*a*) 0, (*b*) 81.7 kW, (*c*) 31.3 kW

3–33 As a spherical ammonia vapor bubble rises in liquid ammonia, its diameter changes from 1 cm to 3 cm. Calculate the amount of work produced by this bubble, in kJ, if the surface tension of ammonia is 0.02 N/m. *Answer:* 5.03×10^{-8} kJ

3–34 A steel rod of 0.5 cm diameter and 10 m length is stretched 3 cm. Young's modulus for this steel is 21 kN/cm^2. How much work, in kJ, is required to stretch this rod?

The First Law of Thermodynamics

3–35C What are the different mechanisms for transferring energy to or from a control volume?

3–36C For a cycle, is the net work necessarily zero? For what kinds of systems will this be the case?

3–37C On a hot summer day, a student turns his fan on when he leaves his room in the morning. When he returns in the evening, will the room be warmer or cooler than the neighboring rooms? Why? Assume all the doors and windows are kept closed.

3–38 An adiabatic closed system is accelerated from 0 m/s to 30 m/s. Determine the specific energy change of this system, in kJ/kg.

3–39 A fan is to accelerate quiescent air to a velocity of 8 m/s at a rate of 9 m^3/s. Determine the minimum power that must be supplied to the fan. Take the density of air to be 1.18 kg/m^3. *Answer:* 340 W

3–40E A vertical piston–cylinder device contains water and is being heated on top of a range. During the process, 65 Btu of heat is transferred to the water, and heat losses from the side walls amount to 8 Btu. The piston rises as a result of evaporation, and 5 Btu of work is done by the vapor. Determine the change in the energy of the water for this process. *Answer:* 52 Btu

3–41E At winter design conditions, a house is projected to lose heat at a rate of 60,000 Btu/h. The internal heat gain from people, lights, and appliances is estimated to be 6000 Btu/h. If this house is to be heated by electric resistance heaters, determine the required rated power of these heaters in kW to maintain the house at constant temperature.

3–42E A water pump increases the water pressure from 15 psia to 70 psia. Determine the power input required, in hp, to pump 0.8 ft^3/s of water. Does the water temperature at the inlet have any significant effect on the required flow power? *Answer:* 11.5 hp

3–43 The lighting needs of a storage room are being met by six fluorescent light fixtures, each fixture containing four lamps rated at 60 W each. All the lamps are on during operating hours of the facility, which are 6 a.m. to 6 p.m. 365 days a year. The storage room is actually used for an average of 3 h a day. If the price of electricity is $0.11/kWh, determine the amount of energy and money that will be saved as a result of installing motion sensors. Also, determine the simple payback period if the purchase price of the sensor is $32 and it takes 1 h to install it at a cost of $40.

3–44 Consider a room that is initially at the outdoor temperature of 20°C. The room contains a 40-W lightbulb, a 110-W TV set, a 300-W refrigerator, and a 1200-W iron. Assuming no heat transfer through the walls, determine the rate of increase of the energy content of the room when all of these electric devices are on.

3–45 An escalator in a shopping center is designed to move 50 people, 75 kg each, at a constant speed of 0.6 m/s at 45° slope. Determine the minimum power input needed to drive this escalator. What would your answer be if the escalator velocity were to be doubled?

3–46 Consider a 2100-kg car cruising at constant speed of 70 km/h. Now the car starts to pass another car by accelerating to 110 km/h in 5 s. Determine the additional power needed to achieve this acceleration. What would your answer be if the total mass of the car were only 700 kg? *Answers:* 117 kW, 38.9 kW

3–47E One way to improve the fuel efficiency of a car is to use tires that have a lower rolling resistance—tires that roll with less resistance. Highway tests at 65 mph showed that tires with the lowest rolling resistance can improve fuel efficiency by nearly 2 mpg (miles per gallon). Consider a car that gets 35 mpg on high-rolling-resistance tires and is driven 15,000 miles per year. For a fuel cost of $3.5/gal, determine how much money will be saved per year by switching to low-rolling-resistance tires.

Energy Conversion Efficiencies

3–48C What is mechanical efficiency? What does a mechanical efficiency of 100 percent mean for a hydraulic turbine?

3–49C How is the combined pump–motor efficiency of a pump and motor system defined? Can the combined pump–motor efficiency be greater than either the pump or the motor efficiency?

3–50C Can the combined turbine–generator efficiency be greater than either the turbine efficiency or the generator efficiency? Explain.

3–51 Consider a 2.4-kW hooded electric open burner in an area where the unit costs of electricity and natural gas are \$0.10/kWh and \$1.20/therm (1 therm = 105,500 kJ), respectively. The efficiency of open burners can be taken to be 73 percent for electric burners and 38 percent for gas burners. Determine the rate of energy consumption and the unit cost of utilized energy for both electric and gas burners.

3–52E The steam requirements of a manufacturing facility are being met by a boiler whose rated heat input is 5.5×10^6 Btu/h. The combustion efficiency of the boiler is measured to be 0.7 by a handheld flue gas analyzer. After tuning up the boiler, the combustion efficiency rises to 0.8. The boiler operates 4200 h a year intermittently. Taking the unit cost of energy to be \$13/10^6 Btu, determine the annual energy and cost savings as a result of tuning up the boiler.

3–53E Reconsider Prob. 3–52E. Using appropriate software, study the effects of the unit cost of energy, the new combustion efficiency on the annual energy, and cost savings. Let the efficiency vary from 0.7 to 0.9, and let the unit cost vary from \$12 to \$14 per million Btu. Plot the annual energy and cost savings against the efficiency for unit costs of \$12, \$13, and \$14 per million Btu, and discuss the results.

3–54 A 75-hp (shaft output) motor that has an efficiency of 91.0 percent is worn out and is replaced by a high-efficiency 75-hp motor that has an efficiency of 95.4 percent. Determine the reduction in the heat gain of the room due to higher efficiency under full-load conditions.

3–55 An exercise room has six weight-lifting machines that have no motors and seven treadmills each equipped with a 2.5-hp (shaft output) motor. The motors operate at an average load factor of 0.7, at which their efficiency is 0.77. During peak evening hours, all 13 pieces of exercising equipment are used continuously, and there are also two people doing light exercises while waiting in line for one piece of the equipment. Assuming the average rate of heat dissipation from people in an exercise room is 600 W, determine the rate of heat gain of the exercise room from people and the equipment at peak load conditions.

3–56 A room is cooled by circulating chilled water through a heat exchanger located in the room. The air is circulated through the heat exchanger by a 0.25-hp (shaft output) fan. Typical efficiency of small electric motors driving 0.25-hp equipment is 60 percent. Determine the rate of heat supply by the fan–motor assembly to the room.

3–57 The water in a large lake is to be used to generate electricity by the installation of a hydraulic turbine–generator at a location where the depth of the water is 50 m. Water is to be supplied at a rate of 5000 kg/s. If the electric power generated is measured to be 1862 kW and the generator efficiency is 95 percent, determine (*a*) the overall efficiency of the turbine–generator, (*b*) the mechanical efficiency of the turbine, and (*c*) the shaft power supplied by the turbine to the generator.

3–58 A 7-hp (shaft) pump is used to raise water to an elevation of 15 m. If the mechanical efficiency of the pump is 82 percent, determine the maximum volume flow rate of water.

3–59 A geothermal pump is used to pump brine whose density is 1050 kg/m^3 at a rate of 0.3 m^3/s from a depth of 200 m. For a pump efficiency of 74 percent, determine the required power input to the pump. Disregard frictional losses in the pipes, and assume the geothermal water at 200 m depth to be exposed to the atmosphere.

3–60 At a certain location, wind is blowing steadily at 7 m/s. Determine the mechanical energy of air per unit mass and the power generation potential of a wind turbine with 80-m-diameter blades at that location. Also determine the actual electric power generation assuming an overall efficiency of 30 percent. Take the air density to be 1.25 kg/m^3.

3–61 Reconsider Prob. 3–60. Using appropriate software, investigate the effect of wind velocity and the blade span diameter on wind power generation. Let the velocity vary from 5 to 20 m/s in increments of 5 m/s, and let the diameter vary from 20 to 120 m in increments of 20 m. Tabulate the results, and discuss their significance.

3–62 Water is pumped from a lower reservoir to a higher reservoir by a pump that provides 20 kW of shaft power. The free surface of the upper reservoir is 45 m higher than that of the lower reservoir. If the flow rate of water is measured to be 0.03 m^3/s, determine mechanical power that is converted to thermal energy during this process due to frictional effects.

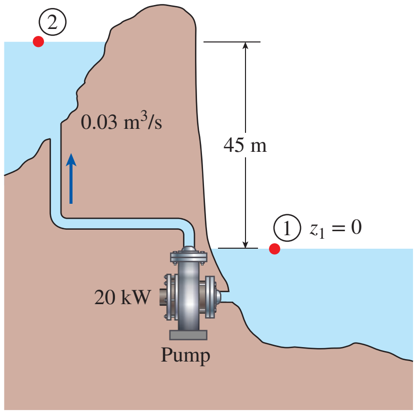


FIGURE P3–62

3–63E An 80-percent-efficient pump with a power input of 20 hp is pumping water from a lake to a nearby pool at a rate of 1.5 ft^3/s through a constant-diameter pipe. The free surface of the pool is 80 ft above that of the lake. Determine the mechanical power used to overcome frictional effects in piping. *Answer:* 2.37 hp

3–64 Water is pumped from a lake to a storage tank 15 m above at a rate of 70 L/s while consuming 15.4 kW of electric power.

Disregarding any frictional losses in the pipes and any changes in kinetic energy, determine (*a*) the overall efficiency of the pump–motor unit and (*b*) the pressure difference between the inlet and the exit of the pump.

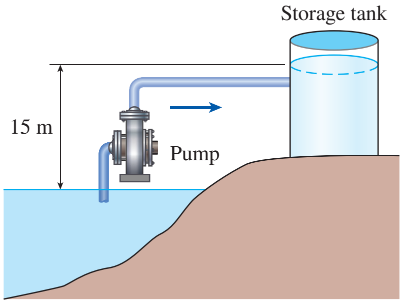


FIGURE P3–64

3–65 Large wind turbines with a power capacity of 8 MW and blade span diameters of over 160 m are available for electric power generation. Consider a wind turbine with a blade span diameter of 100 m installed at a site subjected to steady winds at 8 m/s. Taking the overall efficiency of the wind turbine to be 32 percent and the air density to be 1.25 kg/m^3, determine the electric power generated by this wind turbine. Also, assuming steady winds of 8 m/s during a 24-h period, determine the amount of electric energy and the revenue generated per day for a unit price of \$0.09/kWh for electricity.

3–66 A hydraulic turbine has 85 m of elevation difference available at a flow rate of 0.25 m^3/s, and its overall turbine–generator efficiency is 91 percent. Determine the electric power output of this turbine.

3–67 The water behind Hoover Dam in Nevada is 206 m higher than the Colorado River below it. At what rate must water pass through the hydraulic turbines of this dam to produce 50 MW of power if the turbines are 100 percent efficient?

FIGURE P3–67

Photo by Lynn Betts, USDA Natural Resources Conservation Service

3–68 An oil pump is drawing 44 kW of electric power while pumping oil with $\rho = 860$ kg/m^3 at a rate of 0.1 m^3/s. The inlet and outlet diameters of the pipe are 8 cm and 12 cm, respectively. If the pressure rise of oil in the pump is measured to be 500 kPa and the motor efficiency is 90 percent, determine the mechanical efficiency of the pump.

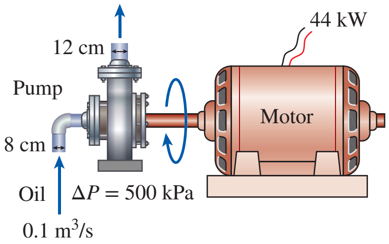


FIGURE P3–68

3–69 A wind turbine is rotating at 15 rpm under steady winds flowing through the turbine at a rate of 42,000 kg/s. The tip velocity of the turbine blade is measured to be 250 km/h. If 180 kW of power is produced by the turbine, determine (*a*) the average velocity of the air and (*b*) the conversion efficiency of the turbine. Take the density of air to be 1.31 kg/m^3.

Review Problems

3–70 Some engineers have developed a device that provides lighting to rural areas with no access to grid electricity. The device is intended for indoor use. It is driven by gravity, and it works as follows: A bag of rock or sand is raised by human power to a higher location. As the bag descends very slowly, it powers a sprocket-wheel which also rotates slowly. A gear train mechanism converts this slow motion to high speed, which drives a DC generator. The electric output from the generator is used to power an LED bulb.

Consider a gravity-driven LED bulb that provides 16 lumens of lighting. The device uses a 10-kg sandbag that is raised by human power to a 2-m height. For continuous lighting, the bag needs to be raised every 20 minutes. Using an efficacy of 150 lumens per watt for the LED bulb, determine (*a*) the velocity of the sandbag as it descends and (*b*) the overall efficiency of the device.

3–71 Consider a classroom for 55 students and one instructor, each generating heat at a rate of 100 W. Lighting is provided by 18 fluorescent lightbulbs, 40 W each, and the ballasts consume an additional 10 percent. Determine the rate of internal heat generation in this classroom when it is fully occupied.

3–72 Consider a homeowner who is replacing his 25-year-old natural gas furnace that has an efficiency of 55 percent. The homeowner is considering a conventional furnace that has an efficiency of 82 percent and costs \$1600 and a high-efficiency furnace that has an efficiency of 95 percent and costs \$2700. The homeowner would like to buy the high-efficiency furnace if the savings from the natural gas pay for the additional cost in less than 8 years. If the homeowner now pays \$1200 a year for heating, determine if he should buy the conventional or the high-efficiency model.

3–73 A homeowner is considering these heating systems for heating his house: Electric resistance heating with \$0.12/kWh and 1 kWh = 3600 kJ, gas heating with \$1.24/therm and 1 therm = 105,500 kJ, and oil heating with \$2.3/gal and 1 gal of oil = 138,500 kJ. Assuming efficiencies of 100 percent for the electric furnace and 87 percent for the gas and oil furnaces, determine the heating system with the lowest energy cost.

3–74 The U.S. Department of Energy estimates that 570,000 barrels of oil would be saved per day if every household in the United States lowered the thermostat setting in winter by 6°F (3.3°C). Assuming the average heating season to be 180 days and the cost of oil to be \$55/barrel, determine how much money would be saved per year.

3–75 A typical household pays about \$1200 a year on energy bills, and the U.S. Department of Energy estimates that 46 percent of this energy is used for heating and cooling, 15 percent for heating water, 15 percent for refrigerating and freezing, and the remaining 24 percent for lighting, cooking, and running other appliances. The heating and cooling costs of a poorly insulated house can be reduced by up to 30 percent by adding adequate insulation. If the cost of insulation is \$200, determine how long it will take for the insulation to pay for itself from the energy it saves.

3–76 The force F required to compress a spring a distance x is given by $F - F_0 = kx$ where k is the spring constant and F_0 is the preload. Determine the work, in kJ, required to compress a spring a distance of 1 cm when its spring constant is 300 N/cm and the spring is initially compressed by a force of 100 N.

3–77 The force required to expand the gas in a gas spring a distance x is given by

$$F = \frac{\text{Constant}}{x^k}$$

where the constant is determined by the geometry of this device and k is determined by the gas used in the device. Such a gas spring is arranged to have a constant of 1000 $\text{N}\cdot\text{m}^{1.3}$ and $k = 1.3$. Determine the work, in kJ, required to compress this spring from 0.1 m to 0.3 m. *Answer:* 1.87 kJ

3–78 Consider a TV set that consumes 120 W of electric power when it is on and is kept on for an average of 6 h per day. For a unit electricity cost of 12 cents per kWh, determine the cost of electricity this TV consumes per month (30 days).

3–79E Water is pumped from a 200-ft-deep well into a 100-ft-high storage tank. Determine the power, in kW, that would be required to pump 200 gal/min.

3–80 Consider a vertical elevator whose cabin has a total mass of 800 kg when fully loaded and 150 kg when empty. The weight of the elevator cabin is partially balanced by a 400-kg counterweight that is connected to the top of the cabin by cables that pass through a pulley located on top of the elevator well. Neglecting the weight of the cables and assuming the guide rails and the pulleys to be frictionless, determine (*a*) the power required while the fully loaded cabin is rising at a constant speed of 1.2 m/s and (*b*) the power required while the empty cabin is descending at a constant speed of 1.2 m/s. What would your answer be to (*a*) if no counterweight were used? What would your answer be to (*b*) if a friction force of 800 N has developed between the cabin and the guide rails?

3–81 A grist mill of the 1800s employed a waterwheel that was 14 m high; 480 L/min of water flowed onto the wheel near the top. How much power, in kW, could this waterwheel have produced? *Answer:* 1.10 kW

3–82 In a hydroelectric power plant, 65 m^3/s of water flows from an elevation of 90 m to a turbine, where electric power is generated. The overall efficiency of the turbine–generator is 84 percent. Disregarding frictional losses in piping, estimate the electric power output of this plant. *Answer:* 48.2 MW

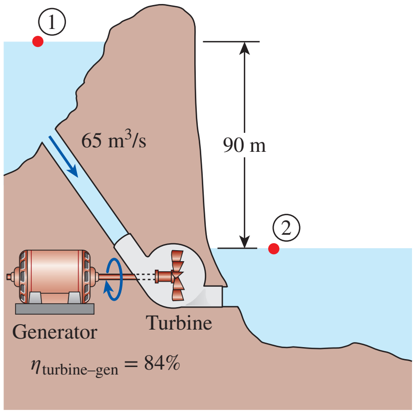


FIGURE P3–82

3–83 The demand for electric power is usually much higher during the day than it is at night, and utility companies often sell power at night at much lower prices to encourage consumers to use the available power generation capacity and to avoid building new, expensive power plants that will be used only a short time during peak periods. Utilities are also willing to purchase power produced during the day from private parties at a high price.

Suppose a utility company is selling electric power for \$0.05/kWh at night and is willing to pay \$0.12/kWh for power produced during the day. To take advantage of this opportunity, an entrepreneur is considering building a large reservoir 40 m above the lake level, pumping water from the lake to the reservoir at night using cheap power, and letting the water flow from the reservoir back to the lake during the day, producing power as the pump–motor operates as a turbine–generator during reverse flow. Preliminary analysis shows that a water flow rate of 2 m^3/s can be used in either direction. The combined pump–motor and turbine–generator efficiencies are expected to be 75 percent each. Disregarding the frictional losses in piping and assuming the system operates for 10 h each in the pump and turbine modes during a typical day, determine the potential revenue this pump–turbine system can generate per year.

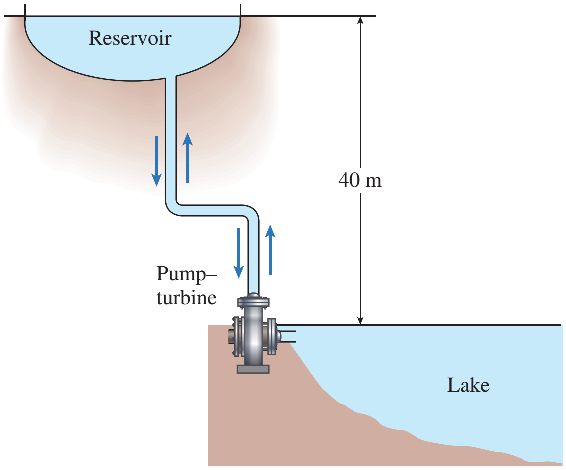


FIGURE P3–83

3–84 The pump of a water distribution system is powered by a 15-kW electric motor whose efficiency is 90 percent. The water flow rate through the pump is 50 L/s. The diameters of the inlet and outlet pipes are the same, and the elevation difference across the pump is negligible. If the pressures at the inlet and outlet of the pump are measured to be 100 kPa and 300 kPa (absolute), respectively, determine the mechanical efficiency of the pump. *Answer:* 74.1 percent

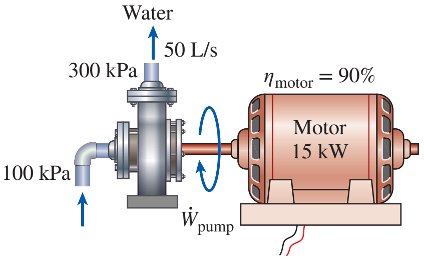


FIGURE P3–84

Design and Essay Problems

3–85 Your neighbor lives in a 2500-square-foot (about 250 m^2) older house heated by natural gas. The current gas heater was installed in the early 1980s and has an efficiency (called the Annual Fuel Utilization Efficiency rating, or AFUE) of 65 percent. It is time to replace the furnace, and the neighbor is trying to decide between a conventional furnace that has an efficiency of 80 percent and costs $1500 and a high-efficiency furnace that has an efficiency of 95 percent and costs $2500. Your neighbor offered to pay you $100 if you help him make the right decision. Considering the weather data, typical heating loads, and the price of natural gas in your area, make a recommendation to your neighbor based on a convincing economic analysis.

3–86 Find out the prices of heating oil, natural gas, and electricity in your area, and determine the cost of each per kWh of energy supplied to the house as heat. Go through your utility bills and determine how much money you spent for heating last January. Also determine how much your January heating bill would be for each of the heating systems if you had the latest and most efficient system installed.

3–87 Prepare a report on the heating systems available in your area for residential buildings. Discuss the advantages and disadvantages of each system and compare their initial and operating costs. What are the important factors in the selection of a heating system? Give some guidelines. Identify the conditions under which each heating system would be the best choice in your area.

3–88 The roofs of many homes in the United States are covered with photovoltaic (PV) solar cells that resemble roof tiles, generating electricity quietly from solar energy. An article stated that over its projected 30-year service life, a 4-kW roof PV system in California will reduce the production of CO_2 that causes global warming by 433,000 lbm, sulfates that cause acid rain by 2900 lbm, and nitrates that cause smog by 1660 lbm. The article also claims that a PV roof will save 253,000 lbm of coal, 21,000 gal of oil, and 27 million ft^3 of natural gas. Making reasonable assumptions for incident solar radiation, efficiency, and emissions, evaluate these claims and make corrections if necessary.

3–89 The performance of a device is defined as the ratio of the desired output to the required input, and this definition can be extended to nontechnical fields. For example, your performance in this course can be viewed as the grade you earn relative to the effort you put in. If you have been investing a lot of time in this course and your grades do not reflect it, you are performing poorly. In that case, perhaps you should try to find out the underlying cause and how to correct the problem. Give three other definitions of performance from nontechnical fields and discuss them.

3–90 Some engineers have suggested that air compressed into tanks can be used to propel personal transportation vehicles. Current compressed-air tank technology permits us to compress and safely hold air at up to 4000 psia. Tanks made of composite materials require about 10 lbm of construction materials for each 1 ft^3 of stored gas. Approximately 0.01 hp is required per pound of vehicle weight to move a vehicle at a speed of 30 miles per hour. What is the maximum range that this vehicle can have? Account for the weight of the tanks only and assume perfect conversion of the energy in the compressed air.

CHAPTER

4

PROPERTIES OF PURE SUBSTANCES

We start this chapter with the introduction of the concept of a *pure substance* and a discussion of the physics of phase-change processes. We then illustrate the various property diagrams and *P*-*υ*-*T* surfaces of pure substances. After demonstrating the use of the property tables, the hypothetical substance *ideal gas* and the *ideal-gas equation of state* are discussed. The *compressibility factor,* which accounts for the deviation of real gases from ideal-gas behavior, is introduced.

OBJECTIVES

The objectives of this chapter are to:

- Introduce the concept of a pure substance.
- Discuss the physics of phase-change processes.
- Illustrate the *P*-*υ*, *T*-*υ*, and *P*-*T* property diagrams and *P*-*υ*-*T* surfaces of pure substances.
- Demonstrate the procedures for determining thermodynamic properties of pure substances from tables of property data.
- Describe the hypothetical substance “ideal gas” and the ideal-gas equation of state.
- Apply the ideal-gas equation of state in the solution of typical problems.
- Introduce the compressibility factor, which accounts for the deviation of real gases from ideal-gas behavior.

4–1 ▪ PURE SUBSTANCE

A substance that has a fixed chemical composition throughout is called a **pure substance**. Water, nitrogen, helium, and carbon dioxide, for example, are all pure substances.

A pure substance does not have to be of a single chemical element or compound, however. A mixture of various chemical elements or compounds also qualifies as a pure substance as long as the mixture is homogeneous. Air, for example, is a mixture of several gases, but it is often considered to be a pure substance because it has a uniform chemical composition (Fig. 4–1). However, a mixture of oil and water is not a pure substance. Since oil is not soluble in water, it will collect on top of the water, forming two chemically dissimilar regions.

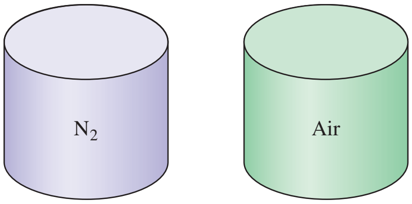
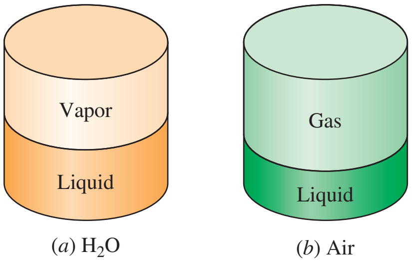


FIGURE 4–1
Nitrogen and gaseous air are pure substances.

A mixture of two or more phases of a pure substance is still a pure substance as long as the chemical composition of all phases is the same (Fig. 4–2). A mixture of ice and liquid water, for example, is a pure substance because both phases have the same chemical composition. A mixture of liquid air and gaseous air, however, is not a pure substance since the composition of liquid air is different from the composition of gaseous air, and thus the mixture is no longer chemically homogeneous. This is due to different components in air condensing at different temperatures at a specified pressure.



(*a*) H_2O (*b*) Air

FIGURE 4–2
A mixture of liquid and gaseous water is a pure substance, but a mixture of liquid and gaseous air is not.

4–2 ▪ PHASES OF A PURE SUBSTANCE

We all know from experience that substances exist in different phases. At room temperature and pressure, copper is a solid, mercury is a liquid, and nitrogen is a gas. Under different conditions, each may appear in a different phase. Even though there are three principal phases—solid, liquid, and gas—a substance may have several phases within a principal phase, each with a different molecular structure. Carbon, for example, may exist as graphite or diamond in the solid phase. Helium has two liquid phases; iron has three solid phases. Ice may exist at seven different phases at high pressures. A phase is identified as having a distinct molecular arrangement that is homogeneous throughout and separated from the others by easily identifiable boundary surfaces. The two phases of H_2O in iced water represent a good example of this.

When studying phases or phase changes in thermodynamics, one does not need to be concerned with the molecular structure and behavior of different phases. However, it is very helpful to have some understanding of the molecular phenomena involved in each phase, and a brief discussion of phase transformations follows.

Intermolecular bonds are strongest in solids and weakest in gases. This is why molecules in solids are closely packed together, whereas in gases they are separated by relatively large distances.

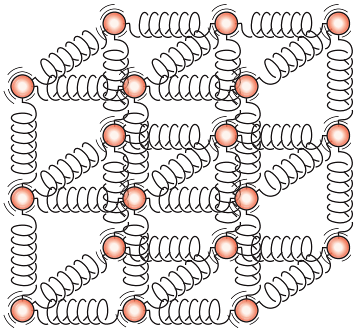


FIGURE 4–3
The molecules in a solid are kept at their positions by the large springlike intermolecular forces.

The molecules in a **solid** are arranged in a three-dimensional pattern (lattice) that is repeated throughout (Fig. 4–3). Because of the small distances between molecules in a solid, the attractive forces of molecules on each other are large and keep the molecules at fixed positions. Note that the attractive forces between molecules turn to repulsive forces as the distance between the molecules approaches zero, thus preventing the molecules from piling up on top of each other. Even though the molecules in a solid cannot move relative to each other, they continually oscillate about their equilibrium positions. The velocity of the molecules during these oscillations depends on the temperature. At sufficiently high temperatures, the velocity (and thus the momentum) of the molecules may reach a point where the intermolecular forces are partially overcome and groups of molecules break away (Fig. 4–4). This is the beginning of the melting process.

The molecular spacing in the **liquid** phase is not much different from that of the solid phase, except the molecules are no longer at fixed positions relative to

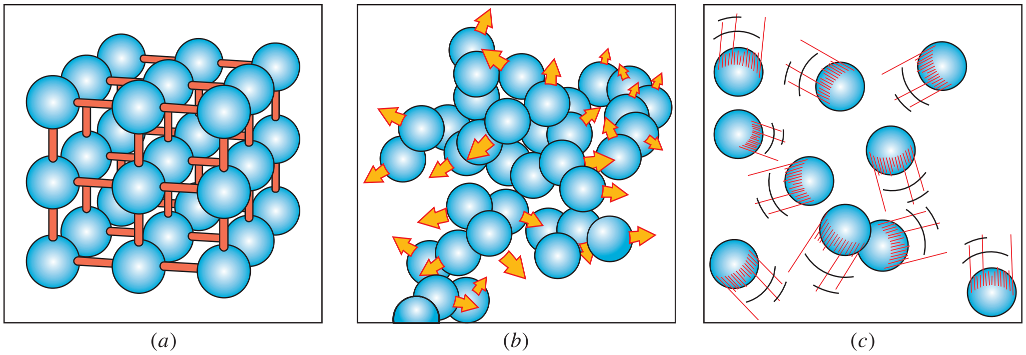

(*a*) (*b*) (*c*)

FIGURE 4–4
The arrangement of atoms in different phases: (*a*) molecules are at relatively fixed positions in a solid, (*b*) groups of molecules move about each other in the liquid phase, and (*c*) molecules move about at random in the gas phase.

each other and they can rotate and translate freely. In a liquid, the intermolecular forces are weaker relative to solids, but still relatively strong compared with gases. The distances between molecules generally experience a slight increase as a solid turns liquid, with water being a notable exception.

In the **gas** phase, the molecules are far apart from each other, and a molecular order is nonexistent. Gas molecules move about at random, continually colliding with each other and the walls of the container they are in. Particularly at low densities, the intermolecular forces are very small, and collisions are the only mode of interaction between the molecules. Molecules in the gas phase are at a considerably higher energy level than they are in the liquid or solid phases. Therefore, the gas must release a large amount of its energy before it can condense or freeze.

4–3 ▪ PHASE-CHANGE PROCESSES OF PURE SUBSTANCES

There are many practical situations where two phases of a pure substance coexist in equilibrium. Water exists as a mixture of liquid and vapor in the boiler and the condenser of a steam power plant. The refrigerant turns from liquid to vapor in the freezer of a refrigerator. Even though many homeowners consider the freezing of water in underground pipes to be the most important phase-change process, attention in this section is focused on the liquid and vapor phases and their mixture. As a familiar substance, water is used to demonstrate the basic principles involved. Remember, however, that all pure substances exhibit the same general behavior.

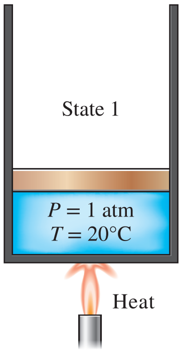


FIGURE 4–5
At 1 atm and 20°C, water exists in the liquid phase (*compressed liquid*).

Compressed Liquid and Saturated Liquid

Consider a piston–cylinder device containing liquid water at 20°C and 1 atm pressure (state 1, Fig. 4–5). Under these conditions, water exists in the liquid phase, and it is called a **compressed liquid**, or a **subcooled liquid**, meaning that it is *not about to vaporize*. Heat is now transferred to the water until its temperature rises to, say, 40°C. As the temperature rises, the liquid water expands slightly, and so its specific volume increases. To accommodate this expansion, the piston moves up slightly. The pressure in the cylinder remains constant at 1 atm during this process since it depends on the outside barometric pressure and the weight of the piston, both of which are constant. Water is still a compressed liquid at this state since it has not started to vaporize.

As more heat is transferred, the temperature keeps rising until it reaches 100°C (state 2, Fig. 4–6). At this point water is still a liquid, but any heat addition will

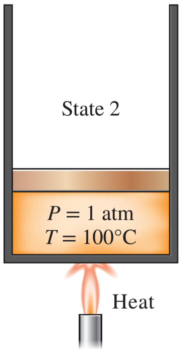


FIGURE 4–6
At 1 atm pressure and 100°C, water exists as a liquid that is ready to vaporize (*saturated liquid*).

cause some of the liquid to vaporize. That is, a phase-change process from liquid to vapor is about to take place. A liquid that is *about to vaporize* is called a **saturated liquid**. Therefore, state 2 is a saturated liquid state.

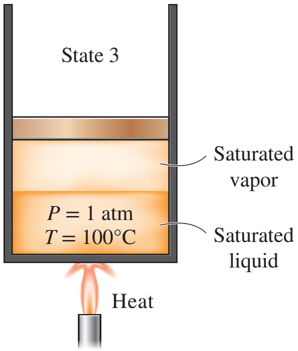


FIGURE 4–7
As more heat is transferred, part of the saturated liquid vaporizes (*saturated liquid–vapor mixture*).

Saturated Vapor and Superheated Vapor

Once boiling starts, the temperature stops rising until the liquid is completely vaporized. That is, the temperature will remain constant during the entire phase-change process if the pressure is held constant. This can easily be verified by placing a thermometer into boiling pure water on top of a stove. At sea level (P = 1 atm), the thermometer will always read 100°C if the pan is uncovered or covered with a light lid. During a boiling process, the only change we will observe is a large increase in the volume and a steady decline in the liquid level as a result of more liquid turning to vapor.

Midway about the vaporization line (state 3, Fig. 4–7), the cylinder contains equal amounts of liquid and vapor. As we continue transferring heat, the vaporization process continues until the last drop of liquid is vaporized (state 4, Fig. 4–8). At this point, the entire cylinder is filled with vapor that is on the borderline of the liquid phase. Any heat loss from this vapor will cause some of the vapor to condense (phase change from vapor to liquid). A vapor that is *about to condense* is called a **saturated vapor**. Therefore, state 4 is a saturated vapor state. A substance at states between 2 and 4 is referred to as a **saturated liquid–vapor mixture** since the *liquid and vapor phases coexist* in equilibrium at these states.

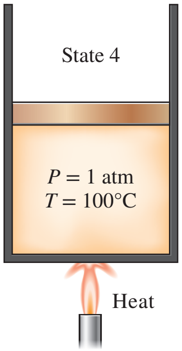


FIGURE 4–8
At 1 atm pressure, the temperature remains constant at 100°C until the last drop of liquid is vaporized (*saturated vapor*).

Once the phase-change process is completed, we are back to a single-phase region again (this time vapor), and further transfer of heat results in an increase in both the temperature and the specific volume (Fig. 4–9). At state 5, the temperature of the vapor is, let us say, 300°C; and if we transfer some heat from the vapor, the temperature may drop somewhat but no condensation will take place as long as the temperature remains above 100°C (for P = 1 atm). A vapor that is *not about to condense* (i.e., not a saturated vapor) is called a **superheated vapor**. Therefore, water at state 5 is a superheated vapor. This constant-pressure phase-change process is illustrated on a T-v diagram in Fig. 4–10.

If the entire process described here is reversed by cooling the water while maintaining the pressure at the same value, the water will go back to state 1, retracing the same path, and in so doing, the amount of heat released will exactly match the amount of heat added during the heating process.

In our daily life, water implies liquid water and steam implies water vapor. In thermodynamics, however, both water and steam usually mean only one thing: H_2O.

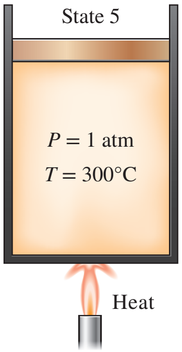


FIGURE 4–9
As more heat is transferred, the temperature of the vapor starts to rise (*superheated vapor*).

Saturation Temperature and Saturation Pressure

It probably came as no surprise to you that water started to boil at 100°C. Strictly speaking, the statement "water boils at 100°C" is incorrect. The correct statement is "water boils at 100°C at 1 atm pressure." The only reason water started boiling at 100°C was because we held the pressure constant at 1 atm (101.325 kPa). If the pressure inside the cylinder were raised to 500 kPa by adding weights on top of the piston, water would start boiling at 151.8°C. That is, *the temperature at which water starts boiling depends on the pressure; therefore, if the pressure is fixed, so is the boiling temperature.*

At a given pressure, the temperature at which a pure substance changes phase is called the **saturation temperature** T_{sat}. Likewise, at a given temperature, the pressure at which a pure substance changes phase is called the **saturation pressure** P_{sat}. At a pressure of 101.325 kPa, T_{sat} is 99.97°C. Conversely, at a temperature of 99.97°C, P_{sat} is 101.325 kPa. (At 100.00°C, P_{sat} is 101.42 kPa in the ITS-90 discussed in Chap. 2.)

Saturation tables that list the saturation pressure against the temperature (or the saturation temperature against the pressure) are available for practically all

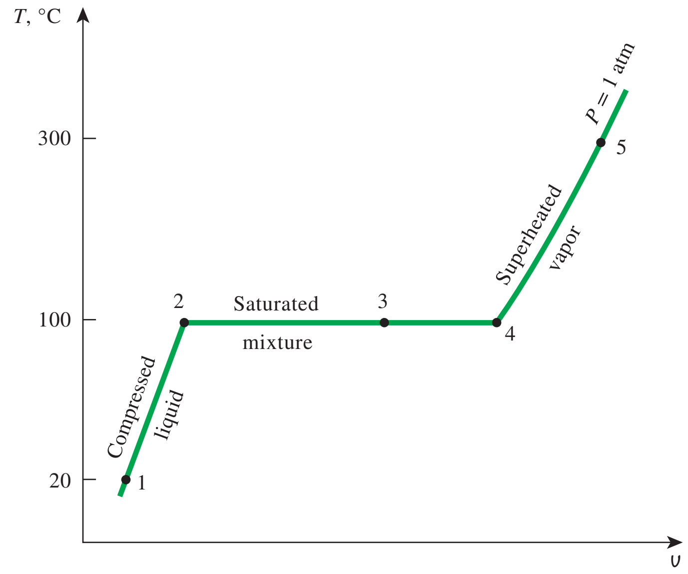


FIGURE 4–10
T-υ diagram for the heating process of water at constant pressure.

substances. A partial listing of such a table is given in Table 4–1 for water. This table indicates that the pressure of water changing phase (boiling or condensing) at 25°C must be 3.17 kPa, and the pressure of water must be maintained at 3976 kPa (about 40 atm) to have it boil at 250°C. Also, water can be frozen by dropping its pressure below 0.61 kPa.

It takes a large amount of energy to melt a solid or vaporize a liquid. The amount of energy absorbed or released during a phase-change process is called the **latent heat**. More specifically, the amount of energy absorbed during melting is called the **latent heat of fusion** and is equivalent to the amount of energy released during freezing. Similarly, the amount of energy absorbed during vaporization is called the **latent heat of vaporization** and is equivalent to the energy released during condensation. The magnitudes of the latent heats depend on the temperature or pressure at which the phase change occurs. At 1 atm pressure, the latent heat of fusion of water is 333.7 kJ/kg and the latent heat of vaporization is 2256.5 kJ/kg.

During a phase-change process, pressure and temperature are obviously dependent properties, and there is a definite relation between them, that is, $P_{sat} = f(T_{sat})$. A plot of P_{sat} versus T_{sat}, such as the one given for water in Fig. 4–11, is called a **liquid–vapor saturation curve**. A curve of this kind is characteristic of all pure substances.

It is clear from Fig. 4–11 that T_{sat} increases with P_{sat}. Thus, a substance at higher pressures boils at higher temperatures. In the kitchen, higher boiling temperatures mean shorter cooking times and energy savings. A beef stew, for example, may take 1 to 2 h to cook in a regular pan that operates at 1 atm pressure, but only 20 min in a pressure cooker operating at 3 atm absolute pressure (corresponding boiling temperature: 134°C).

The atmospheric pressure, and thus the boiling temperature of water, decreases with elevation. Therefore, it takes longer to cook at higher altitudes than it does at sea level (unless a pressure cooker is used). For example, the standard atmospheric pressure at an elevation of 2000 m is 79.50 kPa, which corresponds to a boiling temperature of 93.3°C as opposed to 100°C at sea level (zero elevation). The variation of the boiling temperature of water with altitude at standard atmospheric conditions is given in Table 4–2. For each 1000 m increase in elevation, the boiling temperature drops by a little over 3°C. Note that the atmospheric pressure at a location, and thus the boiling temperature, changes slightly with the

TABLE 4–1

Saturation (or vapor) pressure of water at various temperatures

Temperature T, °C	Saturation pressure P_{sat}, kPa
–10	0.260
–5	0.403
0	0.611
5	0.872
10	1.23
15	1.71
20	2.34
25	3.17
30	4.25
40	7.38
50	12.35
100	101.3 (1 atm)
150	475.8
200	1554
250	3973
300	8581

weather conditions. But the corresponding change in the boiling temperature is no more than about 1°C.

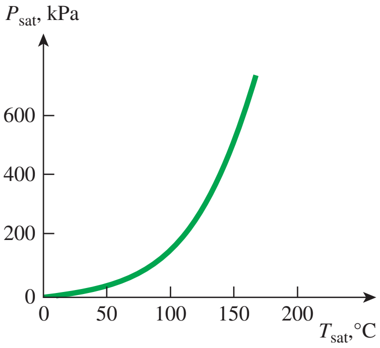


FIGURE 4–11
The liquid–vapor saturation curve of a pure substance (numerical values are for water).

TABLE 4–2

Variation of the standard atmospheric pressure and the boiling (saturation) temperature of water with altitude

Elevation, m	Atmospheric pressure, kPa	Boiling temperature, °C
0	101.33	100.0
1,000	89.55	96.5
2,000	79.50	93.3
5,000	54.05	83.3
10,000	26.50	66.3
20,000	5.53	34.7

Some Consequences of T_{sat} and P_{sat} Dependence

We mentioned earlier that a substance at a specified pressure boils at the saturation temperature corresponding to that pressure. This phenomenon allows us to control the boiling temperature of a substance by simply controlling the pressure, and it has numerous applications in practice. In this section we give some examples. The natural drive to achieve phase equilibrium by allowing some liquid to evaporate is at work behind the scenes.

Consider a sealed can of *liquid refrigerant-134a* in a room at 25°C. If the can has been in the room long enough, the temperature of the refrigerant in the can is also 25°C. Now, if the lid is opened slowly and some refrigerant is allowed to escape, the pressure in the can will start dropping until it reaches the atmospheric pressure. If you are holding the can, you will notice its temperature dropping rapidly, and even ice forming outside the can if the air is humid. A thermometer inserted in the can will register −26°C when the pressure drops to 1 atm, which is the saturation temperature of refrigerant-134a at that pressure. The temperature of the liquid refrigerant will remain at −26°C until the last drop of it vaporizes.

Another aspect of this interesting physical phenomenon is that a liquid cannot vaporize unless it absorbs energy in the amount of the latent heat of vaporization, which is 217 kJ/kg for refrigerant-134a at 1 atm. Therefore, the rate of vaporization of the refrigerant depends on the rate of heat transfer to the can: the larger the rate of heat transfer, the higher the rate of vaporization. The rate of heat transfer to the can and thus the rate of vaporization of the refrigerant can be minimized by insulating the can heavily. In the limiting case of no heat transfer, the refrigerant will remain in the can as a liquid at −26°C indefinitely.

The boiling temperature of *nitrogen* at atmospheric pressure is −196°C (see Table A–3*a*). This means the temperature of liquid nitrogen exposed to the atmosphere must be −196°C since some nitrogen will be evaporating. The temperature of liquid nitrogen remains constant at −196°C until it is depleted. For this reason, nitrogen is commonly used in low-temperature scientific studies (such as superconductivity) and cryogenic applications to maintain a test chamber at a constant temperature of −196°C. This is done by placing the test chamber into a liquid nitrogen bath that is open to the atmosphere. Any heat transfer from the environment to the test section is absorbed by the nitrogen, which evaporates isothermally and keeps the test chamber temperature constant at −196°C (Fig. 4–12). The entire test section must be insulated heavily to minimize heat transfer and thus liquid nitrogen consumption. Liquid nitrogen is also used for medical purposes to burn off unsightly spots on the skin. This is done by soaking a cotton swab in liquid nitrogen and wetting the target area with it. As the nitrogen evaporates, it freezes the affected skin by rapidly absorbing heat from it.

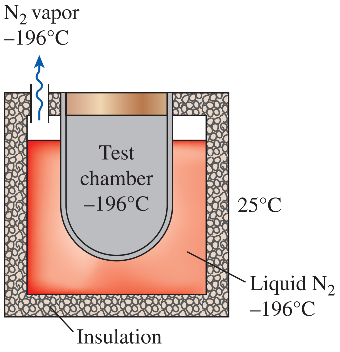


FIGURE 4–12
The temperature of liquid nitrogen exposed to the atmosphere remains constant at −196°C, and thus it maintains the test chamber at −196°C.

A practical way of cooling leafy vegetables is **vacuum cooling**, which is based on *reducing the pressure* of the sealed cooling chamber to the saturation pressure at the desired low temperature and evaporating some water from the products to be cooled. The heat of vaporization during evaporation is absorbed from the products, which lowers the product temperature. The saturation pressure of water at 0°C is 0.61 kPa, and the products can be cooled to 0°C by lowering the pressure to this level. The cooling rate can be increased by lowering the pressure below 0.61 kPa, but this is not desirable because of the danger of freezing and the added cost.

In vacuum cooling, there are two distinct stages. In the first stage, the products at ambient temperature, say at 25°C, are loaded into the chamber, and the operation begins. The temperature in the chamber remains constant until the *saturation pressure* is reached, which is 3.17 kPa at 25°C. In the second stage that follows, saturation conditions are maintained inside at progressively *lower pressures* and the corresponding *lower temperatures* until the desired temperature is reached (Fig. 4–13).

Vacuum cooling is usually more expensive than the conventional refrigerated cooling, and its use is limited to applications that result in much faster cooling. Products with large surface area per unit mass and a high tendency to release moisture such as lettuce and spinach are well suited for vacuum cooling. Products with a low surface area–to–mass ratio are not suitable, especially those that have relatively impervious peels such as tomatoes and cucumbers. Some products such as mushrooms and green peas can be vacuum cooled successfully by wetting them first.

The vacuum cooling just described becomes **vacuum freezing** if the vapor pressure in the vacuum chamber is dropped below 0.61 kPa, the saturation pressure of water at 0°C. The idea of making ice by using a vacuum pump is nothing new. Dr. William Cullen actually made ice in Scotland in 1775 by evacuating the air in a water tank (Fig. 4–14).

Package icing is commonly used in small-scale cooling applications to remove heat and keep the products cool during transit by taking advantage of the large latent heat of fusion of water, but its use is limited to products that are not harmed by contact with ice. Also, ice provides *moisture* as well as *refrigeration.*

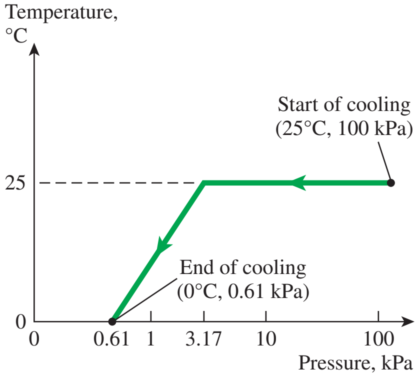


FIGURE 4–13
The variation of the temperature of fruits and vegetables with pressure during vacuum cooling from 25°C to 0°C.

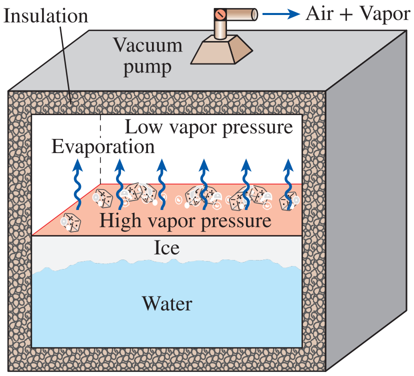


FIGURE 4–14
In 1775, ice was made by evacuating the airspace in a water tank.

4–4 ▪ PROPERTY DIAGRAMS FOR PHASE-CHANGE PROCESSES

The variations of properties during phase-change processes are best studied and understood with the help of property diagrams. Next, we develop and discuss the *T*-v, *P*-v, and *P*-*T* diagrams for pure substances.

1 The *T*-v Diagram

The phase-change process of water at 1 atm pressure was described in detail in the last section and plotted on a *T*-v diagram in Fig. 4–10. Now we repeat this process at different pressures to develop the *T*-v diagram.

Let us add weights on top of the piston until the pressure inside the cylinder reaches 1 MPa. At this pressure, water has a somewhat smaller specific volume than it does at 1 atm pressure. As heat is transferred to the water at this new pressure, the process follows a path that looks very much like the process path at 1 atm pressure, as shown in Fig. 4–15, but there are some noticeable differences. First, water starts boiling at a much higher temperature (179.9°C) at this pressure. Second, the specific volume of the saturated liquid is larger and the specific volume of the saturated vapor is smaller than the corresponding values at 1 atm pressure. That is, the horizontal line that connects the saturated liquid and saturated vapor states is much shorter.

As the pressure is increased further, this saturation line continues to shrink, as shown in Fig. 4–15, and it becomes a point when the pressure reaches 22.06 MPa for the case of water. This point is called the **critical point**, and it is defined as *the point at which the saturated liquid and saturated vapor states are identical.*

The temperature, pressure, and specific volume of a substance at the critical point are called, respectively, the *critical temperature* T_{cr}, *critical pressure* P_{cr}, and *critical specific volume* v_{cr}. The critical-point properties of water are $P_{cr} = 22.06$ MPa, $T_{cr} = 373.95$°C, and $v_{cr} = 0.003106$ m^3/kg. For helium, they are 0.23 MPa, −267.85°C, and 0.01444 m^3/kg. The critical properties for various substances are given in Table A–1 in the appendix.

At pressures above the critical pressure, there is not a distinct phase-change process (Fig. 4–16). Instead, the specific volume of the substance continually increases, and at all times there is only one phase present. Eventually, it resembles

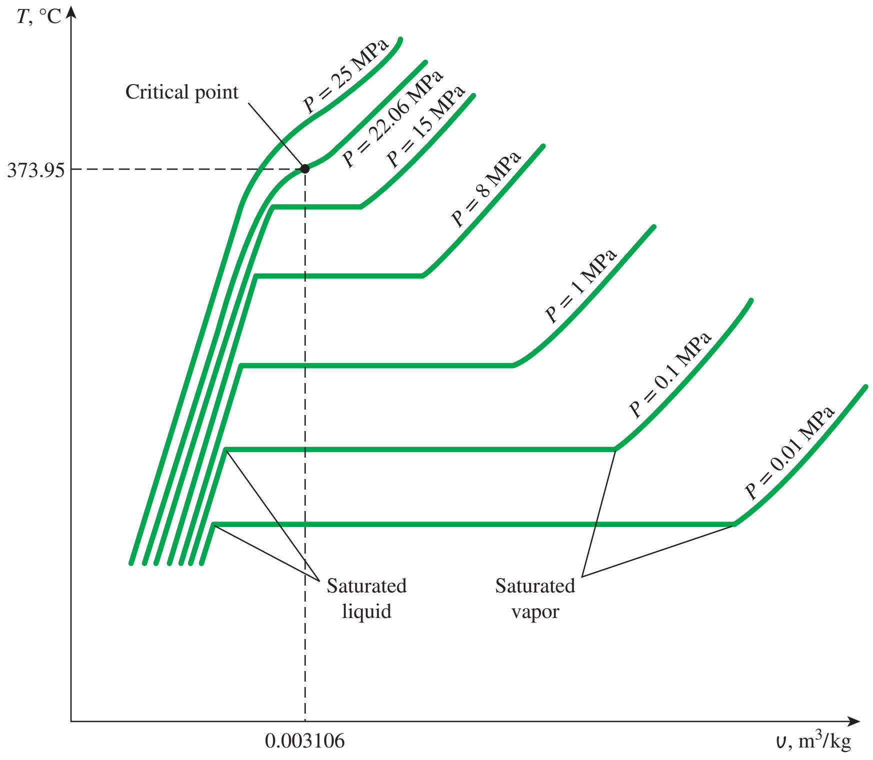


FIGURE 4–15

T-υ diagram of constant-pressure phase-change processes of a pure substance at various pressures (numerical values are for water).

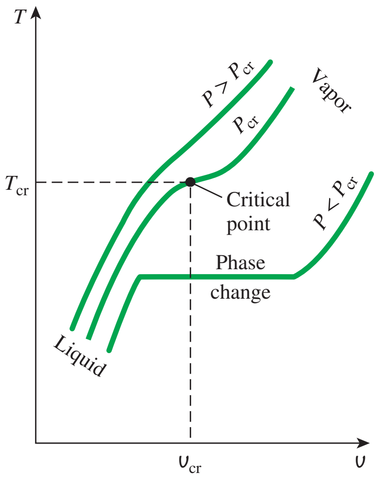


FIGURE 4–16

At supercritical pressures ($P > P_{cr}$), there is no distinct phase-change (boiling) process.

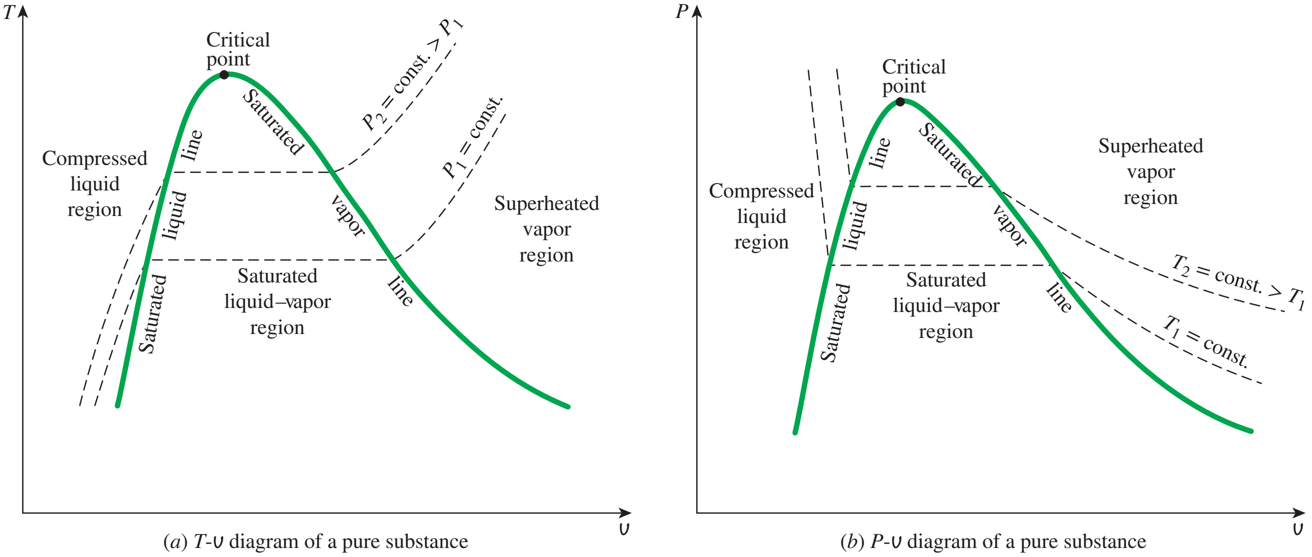
a vapor, but we can never tell when the change has occurred. Above the critical state, there is no line that separates the compressed liquid region and the superheated vapor region. However, it is customary to refer to the substance as superheated vapor at temperatures above the critical temperature and as compressed liquid at temperatures below the critical temperature.

The saturated liquid states in Fig. 4–15 can be connected by a line called the **saturated liquid line**, and saturated vapor states in the same figure can be connected by another line, called the **saturated vapor line**. These two lines meet at the critical point, forming a dome as shown in Fig. 4–17*a*. All the compressed liquid states are located in the region to the left of the saturated liquid line, called the **compressed liquid region**. All the superheated vapor states are located to the right of the saturated vapor line, called the **superheated vapor region**. In these two regions, the substance exists in a single phase, a liquid or a vapor. All the states that involve both phases in equilibrium are located under the dome, called the **saturated liquid–vapor mixture region**, or the **wet region**.

2 The *P*-υ Diagram

The general shape of the *P*-υ diagram of a pure substance is very much like the *T*-υ diagram, but the *T* = constant lines on this diagram have a downward trend, as shown in Fig. 4–17*b*.

Consider again a piston–cylinder device that contains liquid water at 1 MPa and 150°C. Water at this state exists as a compressed liquid. Now the weights on top of the piston are removed one by one so that the pressure inside the cylinder decreases gradually (Fig. 4–18). The water is allowed to exchange heat with the surroundings so its temperature remains constant. As the pressure decreases, the volume of the water increases slightly. When the pressure reaches the saturation-pressure value at the specified temperature (0.4762 MPa), the water starts to boil. During this vaporization process, both the temperature and the pressure remain constant, but the specific volume increases. Once the last drop of liquid is vaporized, further



(*a*) *T*-υ diagram of a pure substance

(*b*) *P*-υ diagram of a pure substance

FIGURE 4–17
Property diagrams of a pure substance.

reduction in pressure results in a further increase in specific volume. Notice that during the phase-change process, we did not remove any weights. Doing so would cause the pressure and therefore the temperature to drop [since $T_{sat} = f(P_{sat})$], and the process would no longer be isothermal.

When the process is repeated for other temperatures, similar paths are obtained for the phase-change processes. Connecting the saturated liquid and the saturated vapor states by a curve, we obtain the *P*-υ diagram of a pure substance, as shown in Fig. 4–17*b*.

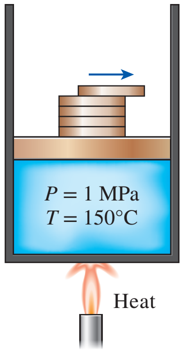


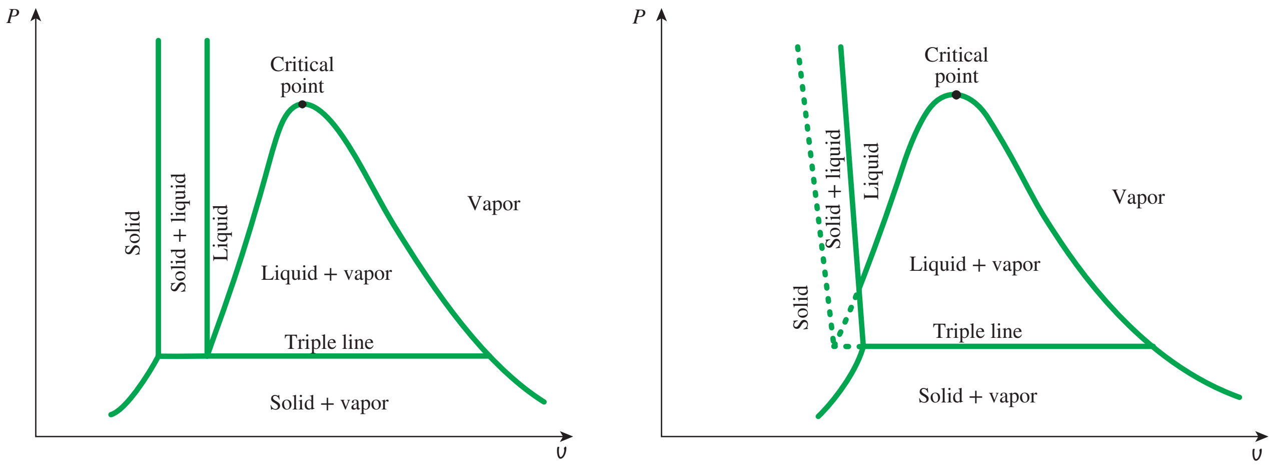
FIGURE 4–18
The pressure in a piston–cylinder device can be reduced by reducing the weight of the piston.

Extending the Diagrams to Include the Solid Phase

The two equilibrium diagrams developed so far represent the equilibrium states involving the liquid and the vapor phases only. However, these diagrams can easily be extended to include the solid phase as well as the solid–liquid and the solid–vapor saturation regions. The basic principles discussed in conjunction with the liquid–vapor phase-change process apply equally to the solid–liquid and solid–vapor phase-change processes. Most substances contract during a solidification (i.e., freezing) process. Others, like water, expand as they freeze. The *P*-υ diagrams for both groups of substances are given in Figs. 4–19*a* and 4–19*b*. These two diagrams differ only in the solid–liquid saturation region. The *T*-υ diagrams look very much like the *P*-υ diagrams, especially for substances that contract on freezing.

The fact that water expands upon freezing has vital consequences in nature. If water contracted on freezing as most other substances do, the ice formed would be heavier than the liquid water, and it would settle to the bottom of rivers, lakes, and oceans instead of floating at the top. The sun's rays would never reach these ice layers, and the bottoms of many rivers, lakes, and oceans would be covered with ice at times, seriously disrupting marine life.

We are all familiar with two phases being in equilibrium, but under some conditions all three phases of a pure substance coexist in equilibrium (Fig. 4–20). On *P*-υ or *T*-υ diagrams, these triple-phase states form a line called the **triple line**. The states on the triple line of a substance have the same pressure



(*a*) *P*-υ diagram of a substance that contracts on freezing

(*b*) *P*-υ diagram of a substance that expands on freezing (such as water)

FIGURE 4–19
P-υ diagrams of different substances.

FIGURE 4–20
At triple-point pressure and temperature, a substance exists in three phases in equilibrium.

and temperature but different specific volumes. The triple line appears as a point on the *P-T* diagrams and, therefore, is often called the **triple point**. The triple-point temperatures and pressures of various substances are given in Table 4–3. For water, the triple-point temperature and pressure are 0.01°C and 0.6117 kPa,

TABLE 4–3

Triple-point temperatures and pressures of various substances

Substance	Formula	T_{tp}, K	P_{tp}, kPa
Acetylene	C_2H_2	192.4	120
Ammonia	NH_3	195.40	6.076
Argon	A	83.81	68.9
Carbon (graphite)	C	3900	10,100
Carbon dioxide	CO_2	216.55	517
Carbon monoxide	CO	68.10	15.37
Deuterium	D_2	18.63	17.1
Ethane	C_2H_6	89.89	8×10^{-4}
Ethylene	C_2H_4	104.0	0.12
Helium 4 (λ point)	He	2.19	5.1
Hydrogen	H_2	13.84	7.04
Hydrogen chloride	HCl	158.96	13.9
Mercury	Hg	234.2	1.65×10^{-7}
Methane	CH_4	90.68	11.7
Neon	Ne	24.57	43.2
Nitric oxide	NO	109.50	21.92
Nitrogen	N_2	63.18	12.6
Nitrous oxide	N_2O	182.34	87.85
Oxygen	O_2	54.36	0.152
Palladium	Pd	1825	3.5×10^{-3}
Platinum	Pt	2045	2.0×10^{-4}
Sulfur dioxide	SO_2	197.69	1.67
Titanium	Ti	1941	5.3×10^{-3}
Uranium hexafluoride	UF_6	337.17	151.7
Water	H_2O	273.16	0.61
Xenon	Xe	161.3	81.5
Zinc	Zn	692.65	0.065

Source: Data from National Bureau of Standards (U.S.) Circ., 500 (1952).

respectively. That is, all three phases of water coexist in equilibrium only if the temperature and pressure have precisely these values. No substance can exist in the liquid phase in stable equilibrium at pressures below the triple-point pressure. The same can be said for temperature for substances that contract on freezing. However, substances at high pressures can exist in the liquid phase at temperatures below the triple-point temperature. For example, water cannot exist in liquid form in equilibrium at atmospheric pressure at temperatures below 0°C, but it can exist as a liquid at −20°C at 200 MPa pressure. Also, ice exists at seven different solid phases at pressures above 100 MPa.

There are two ways a substance can pass from the solid to the vapor phase: either it melts first into a liquid and subsequently evaporates, or it evaporates directly without melting first. The latter occurs at pressures below the triple-point value, since a pure substance cannot exist in the liquid phase at those pressures (Fig. 4–21). Passing from the solid phase directly into the vapor phase is called **sublimation**. For substances that have a triple-point pressure above the atmospheric pressure such as solid CO_2 (dry ice), sublimation is the only way to change from the solid to the vapor phase at atmospheric conditions.

FIGURE 4–21
At low pressures (below the triple-point value), solids evaporate without melting first (*sublimation*).

3 The *P-T* Diagram

Fig. 4–22 shows the *P-T* diagram of a pure substance. This diagram is often called the **phase diagram** since all three phases are separated from each other by three lines. The sublimation line separates the solid and vapor regions, the vaporization line separates the liquid and vapor regions, and the melting (or fusion) line separates the solid and liquid regions. These three lines meet at the triple point, where all three phases coexist in equilibrium. The vaporization line ends at the critical point because no distinction can be made between liquid and vapor phases above the critical point. Substances that expand and contract on freezing differ only in the melting line on the *P-T* diagram.

The *P-υ-T* Surface

The state of a simple compressible substance is fixed by any two independent, intensive properties. Once the two appropriate properties are fixed, all the other properties become dependent properties. Remembering that any equation with

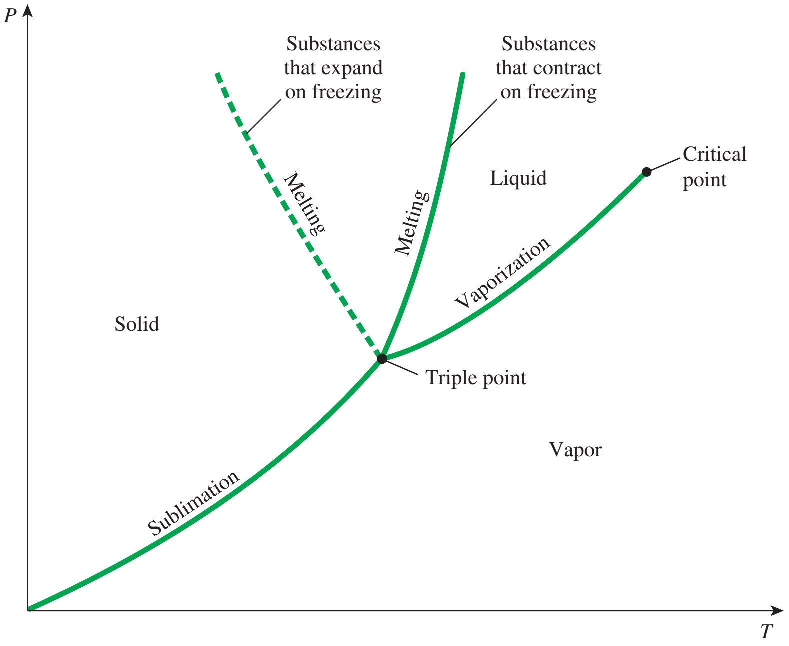


FIGURE 4–22
P-T diagram of pure substances.

two independent variables in the form $z = z(x, y)$ represents a surface in space, we can represent the P-υ-T behavior of a substance as a surface in space, as shown in Figs. 4–23 and 4–24. Here T and υ may be viewed as the independent variables (the base) and P as the dependent variable (the height).

All the points on the surface represent equilibrium states. All states along the path of a quasi-equilibrium process lie on the P-υ-T surface since such a process must pass through equilibrium states. The single-phase regions appear as curved surfaces on the P-υ-T surface, and the two-phase regions as surfaces perpendicular to the P-T plane. This is expected since the projections of two-phase regions on the P-T plane are lines.

All the two-dimensional diagrams we have discussed so far are merely projections of this three-dimensional surface onto the appropriate planes. A P-υ diagram is just a projection of the P-υ-T surface on the P-υ plane, and a T-υ diagram is nothing more than the bird's-eye view of this surface. The P-υ-T surfaces present a great deal of information at once, but in a thermodynamic analysis it is more convenient to work with two-dimensional diagrams, such as the P-υ and T-υ diagrams.

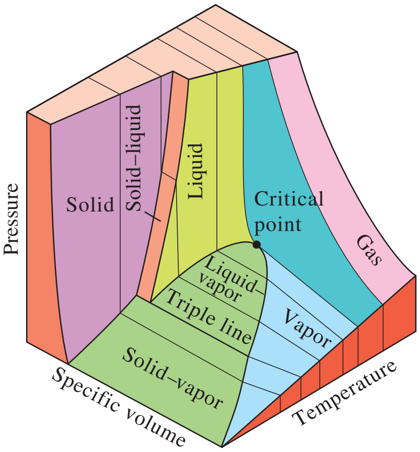


FIGURE 4–23
P-υ-T surface of a substance that *contracts* on freezing.

4–5 ▪ PROPERTY TABLES

For most substances, the relationships among thermodynamic properties are too complex to be expressed by simple equations. Therefore, properties are frequently presented in the form of tables. Some thermodynamic properties can be measured easily, but others cannot, and the latter are calculated by using the relations between them and measurable properties. The results of these measurements and calculations are presented in tables in a convenient format. In the following discussion, the steam tables are used to demonstrate the use of thermodynamic property tables. Property tables of other substances are used in the same manner.

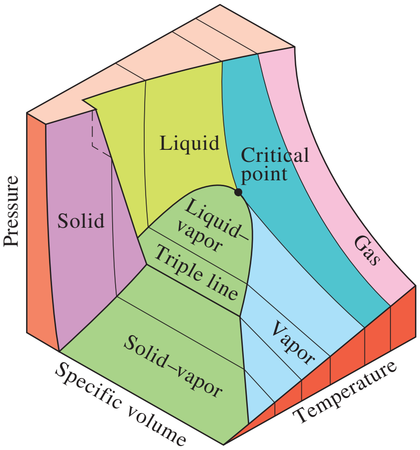


FIGURE 4–24
P-υ-T surface of a substance that expands on freezing (like water).

For each substance, the thermodynamic properties are listed in more than one table. In fact, a separate table is prepared for each region of interest such as the superheated vapor, compressed liquid, and saturated (mixture) regions. Property tables are given in the Appendix in both SI and English units. The tables in English units carry the same number as the corresponding tables in SI, followed by an identifier E. Tables A–6 and A–6E, for example, list properties of superheated water vapor, the former in SI and the latter in English units. Before we get into the discussion of property tables, we define a new property called *enthalpy.*

Enthalpy—A Combination Property

A person looking at the tables will notice two new properties: enthalpy h and entropy s. Entropy is a property associated with the second law of thermodynamics, and we will not use it until it is properly defined in Chap. 8. However, it is appropriate to introduce enthalpy at this point.

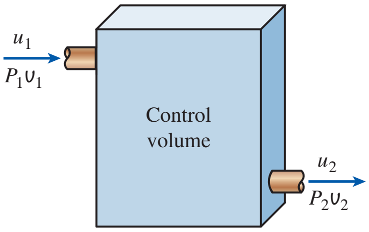


FIGURE 4–25
The combination $u + P\upsilon$ is often encountered in the analysis of control volumes.

In the analysis of certain types of processes, particularly in power generation and refrigeration (Fig. 4–25), we often encounter the combination of properties $u + P\upsilon$. For the sake of simplicity and convenience, this combination is defined as a new property, **enthalpy**, and given the symbol h:

$$h = u + P\upsilon \qquad \text{(kJ/kg)} \tag{4–1}$$

or,

$$H = U + PV \qquad \text{(kJ)} \tag{4–2}$$

Both the total enthalpy H and specific enthalpy h are simply referred to as enthalpy since the context clarifies which one is meant. Notice that the equations given above are dimensionally homogeneous. That is, the unit of the pressure–volume product may differ from the unit of the internal energy by only a factor (Fig. 4–26). For example, it can be easily shown that 1 kPa·m^3 = 1 kJ. In some tables encountered in practice, the internal energy u is frequently not listed, but it can always be determined from $u = h - P\mathrm{v}$.

The widespread use of the property enthalpy is due to Professor Richard Mollier, who recognized the importance of the group $u + P\mathrm{v}$ in the analysis of steam turbines and in the representation of the properties of steam in tabular and graphical form (as in the famous Mollier chart). Mollier referred to the group $u + P\mathrm{v}$ as *heat content* and *total heat*. These terms were not quite consistent with the modern thermodynamic terminology and were replaced in the 1930s by the term *enthalpy* (from the Greek word *enthalpien,* which means *to heat*).

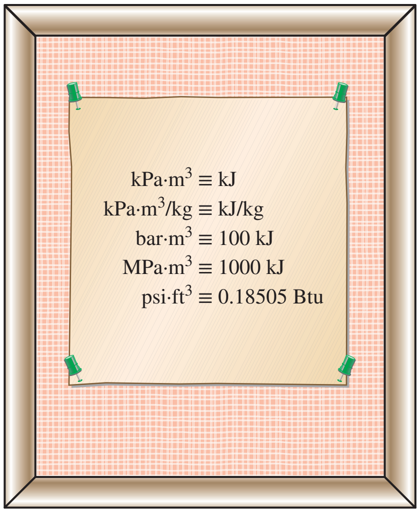


FIGURE 4–26
The product *pressure* × *volume* has energy units.

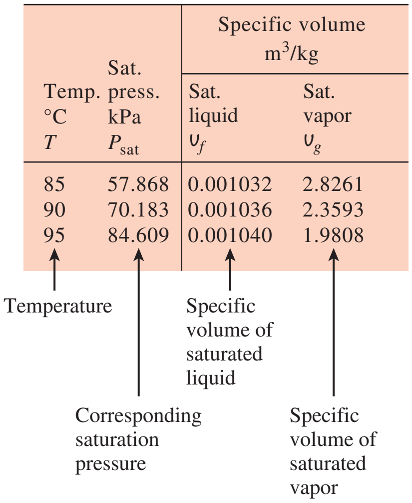
1a Saturated Liquid and Saturated Vapor States

The properties of saturated liquid and saturated vapor for water are listed in Tables A–4 and A–5. Both tables give the same information. The only difference is that in Table A–4 properties are listed under temperature and in Table A–5 under pressure. Therefore, it is more convenient to use Table A–4 when *temperature* is given and Table A–5 when *pressure* is given. The use of Table A–4 is illustrated in Fig. 4–27.

The subscript f is used to denote properties of a saturated liquid and the subscript g to denote the properties of saturated vapor. These symbols are commonly used in thermodynamics and originated from German. Another subscript commonly used is fg, which denotes the difference between the saturated vapor and saturated liquid values of the same property. For example,

v_f = specific volume of saturated liquid
v_g = specific volume of saturated vapor
v_{fg} = difference between v_g and v_f (that is $\mathrm{v}_{fg} = \mathrm{v}_g - \mathrm{v}_f$)

The quantity h_{fg} is called the **enthalpy of vaporization** (or latent heat of vaporization). It represents the amount of energy needed to vaporize a unit mass of saturated liquid at a given temperature or pressure. It decreases as the temperature or pressure increases and becomes zero at the critical point.



Temp. °C T	Sat. press. kPa P_{sat}	Specific volume m^3/kg Sat. liquid v_f	Sat. vapor v_g
85	57.868	0.001032	2.8261
90	70.183	0.001036	2.3593
95	84.609	0.001040	1.9808

FIGURE 4–27
A partial list of Table A–4.

EXAMPLE 4–1 Pressure of Saturated Liquid in a Tank

A rigid tank contains 50 kg of saturated liquid water at 90°C. Determine the pressure in the tank and the volume of the tank.

SOLUTION A rigid tank contains saturated liquid water. The pressure and volume of the tank are to be determined.

Analysis The state of the saturated liquid water is shown on a T-v diagram in Fig. 4–28. Since saturation conditions exist in the tank, the pressure must be the saturation pressure at 90°C:

$$P = P_{\text{sat @ 90°C}} = \mathbf{70.183\ kPa} \qquad \text{(Table A-4)}$$

The specific volume of the saturated liquid at 90°C is

$$\mathrm{v} = \mathrm{v}_{f\,@\,90°\text{C}} = 0.001036\ \text{m}^3/\text{kg} \qquad \text{(Table A-4)}$$

Then the total volume of the tank becomes

$$V = m\mathrm{v} = (50\ \text{kg})(0.001036\ \text{m}^3/\text{kg}) = \mathbf{0.0518\ m^3}$$

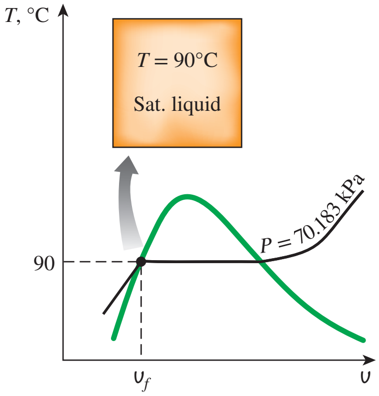


FIGURE 4–28
Schematic and T-v diagram for Example 4–1.

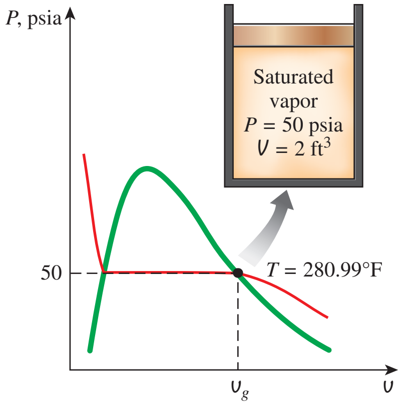


FIGURE 4–29
Schematic and P-υ diagram for Example 4–2.

EXAMPLE 4–2 Temperature of Saturated Vapor in a Cylinder

A piston–cylinder device contains 2 ft^3 of saturated water vapor at 50-psia pressure. Determine the temperature and the mass of the vapor inside the cylinder.

SOLUTION A cylinder contains saturated water vapor. The temperature and the mass of vapor are to be determined.

Analysis The state of the saturated water vapor is shown on a P-υ diagram in Fig. 4–29. Since the cylinder contains saturated vapor at 50 psia, the temperature inside must be the saturation temperature at this pressure:

$$T = T_{\text{sat @ 50 psia}} = \mathbf{280.99^\circ F} \qquad \text{(Table A–5E)}$$

The specific volume of the saturated vapor at 50 psia is

$$\upsilon = \upsilon_{g\text{ @ 50 psia}} = 8.5175\ \text{ft}^3/\text{lbm} \qquad \text{(Table A–5E)}$$

Then the mass of water vapor inside the cylinder becomes

$$m = \frac{V}{\upsilon} = \frac{2\ \text{ft}^3}{8.5175\ \text{ft}^3/\text{lbm}} = \mathbf{0.235\ lbm}$$

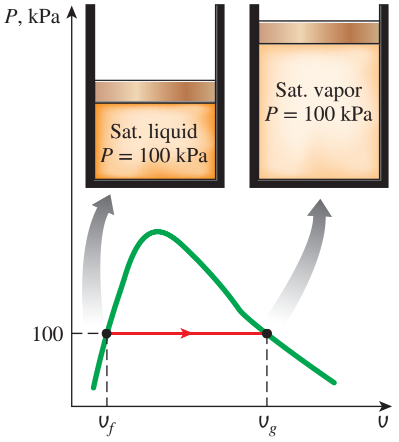


FIGURE 4–30
Schematic and P-υ diagram for Example 4–3.

EXAMPLE 4–3 Volume and Energy Change During Evaporation

A mass of 200 g of saturated liquid water is completely vaporized at a constant pressure of 100 kPa. Determine (*a*) the volume change and (*b*) the amount of energy transferred to the water.

SOLUTION Saturated liquid water is vaporized at constant pressure. The volume change and the energy transferred are to be determined.

Analysis (*a*) The process described is illustrated on a P-υ diagram in Fig. 4–30. The volume change per unit mass during a vaporization process is υ_{fg}, which is the difference between υ_g and υ_f. Reading these values from Table A–5 at 100 kPa and substituting yield

$$\upsilon_{fg} = \upsilon_g - \upsilon_f = 1.6941 - 0.001043 = 1.6931\ \text{m}^3/\text{kg}$$

Thus,

$$\Delta V = m\upsilon_{fg} = (0.2\ \text{kg})(1.6931\ \text{m}^3/\text{kg}) = \mathbf{0.3386\ m^3}$$

(*b*) The amount of energy needed to vaporize a unit mass of a substance at a given pressure is the enthalpy of vaporization at that pressure, which is $h_{fg} = 2257.5$ kJ/kg for water at 100 kPa. Thus, the amount of energy transferred is

$$mh_{fg} = (0.2\ \text{kg})(22575.5\ \text{kJ/kg}) = \mathbf{451.5\ kJ}$$

Discussion Note that we have considered the first four decimal digits of υ_{fg} and disregarded the rest. This is because υ_g has significant numbers to the first four decimal places only, and we do not know the numbers in the other decimal places. Copying all the digits from the calculator would mean that we are assuming $\upsilon_g = 1.694100$, which is not necessarily the case. It could very well be that $\upsilon_g = 1.694138$ since this number, too, would truncate to 1.6941. All the digits in our result (1.6931) are significant. But if we did not truncate the result, we would obtain $\upsilon_{fg} = 1.693057$, which falsely implies that our result is accurate to the sixth decimal place.

1b Saturated Liquid–Vapor Mixture

During a vaporization process, a substance exists as part liquid and part vapor. That is, it is a mixture of saturated liquid and saturated vapor (Fig. 4–31). To analyze this mixture properly, we need to know the proportions of the liquid and vapor phases in the mixture. This is done by defining a new property called the **quality** x as the ratio of the mass of vapor to the total mass of the mixture:

$$x = \frac{m_{\text{vapor}}}{m_{\text{total}}} \tag{4–3}$$

where

$$m_{\text{total}} = m_{\text{liquid}} + m_{\text{vapor}} = m_f + m_g$$

Quality has significance for *saturated mixtures* only. It has no meaning in the compressed liquid or superheated vapor regions. Its value is between 0 and 1. The quality of a system that consists of *saturated liquid* is 0 (or 0 percent), and the quality of a system consisting of *saturated vapor* is 1 (or 100 percent). In saturated mixtures, quality can serve as one of the two independent intensive properties needed to describe a state. Note that *the properties of the saturated liquid are the same whether it exists alone or in a mixture with saturated vapor.* During the vaporization process, only the amount of saturated liquid changes, not its properties. The same can be said about a saturated vapor.

A saturated mixture can be treated as a combination of two subsystems: the saturated liquid and the saturated vapor. However, the amount of mass for each phase is usually not known. Therefore, it is often more convenient to imagine that the two phases are mixed well, forming a homogeneous mixture (Fig. 4–32). Then the properties of this "mixture" will simply be the average properties of the saturated liquid–vapor mixture under consideration. Here is how it is done.

Consider a tank that contains a saturated liquid–vapor mixture. The volume occupied by saturated liquid is V_f, and the volume occupied by saturated vapor is V_g. The total volume V is the sum of the two:

$$V = V_f + V_g$$

$$V = mv \longrightarrow m_t v_{\text{avg}} = m_f v_f + m_g v_g$$

$$m_f = m_t - m_g \longrightarrow m_t v_{\text{avg}} = (m_t - m_g)v_f + m_g v_g$$

Dividing by m_t yields

$$v_{\text{avg}} = (1 - x)v_f + x v_g$$

since $x = m_g/m_t$. This relation can also be expressed as

$$v_{\text{avg}} = v_f + x v_{fg} \qquad (\text{m}^3/\text{kg}) \tag{4–4}$$

where $v_{fg} = v_g - v_f$. Solving for quality, we obtain

$$x = \frac{v_{\text{avg}} - v_f}{v_{fg}} \tag{4–5}$$

Based on this equation, quality can be related to the horizontal distances on a P-v or T-v diagram (Fig. 4–33). At a given temperature or pressure, the numerator of Eq. 4–5 is the distance between the actual state and the saturated liquid state, and the denominator is the length of the horizontal line that connects the saturated liquid and saturated vapor states. A state of 50 percent quality lies in the middle of this horizontal line.

The analysis given above can be repeated for internal energy and enthalpy with the following results:

$$u_{\text{avg}} = u_f + x u_{fg} \qquad (\text{kJ/kg}) \tag{4–6}$$

$$h_{\text{avg}} = h_f + x h_{fg} \qquad (\text{kJ/kg}) \tag{4–7}$$

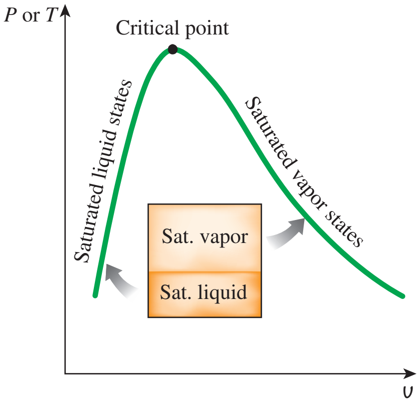


FIGURE 4–31
The relative amounts of liquid and vapor phases in a saturated mixture are specified by the *quality* x.

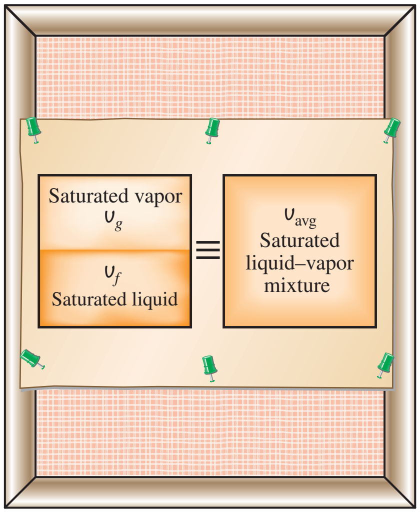


FIGURE 4–32
A two-phase system can be treated as a homogeneous mixture for convenience.

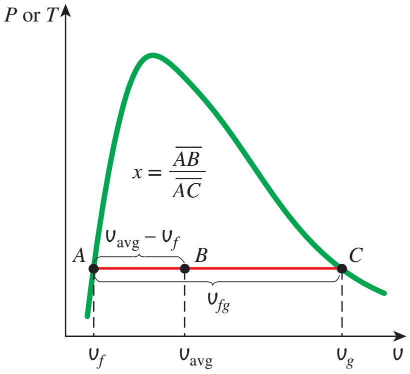


FIGURE 4–33
Quality is related to the horizontal distances on P-v and T-v diagrams.

All the results are of the same format, and they can be summarized in a single equation as

$$y_{\text{avg}} = y_f + x y_{fg}$$

where y is v, u, or h. The subscript "avg" (for "average") is usually dropped for simplicity. The values of the average properties of the mixtures are always *between* the values of the saturated liquid and the saturated vapor properties (Fig. 4–34). That is,

$$y_f \le y_{\text{avg}} \le y_g$$

Finally, all the saturated-mixture states are located under the saturation curve, and to analyze saturated mixtures, all we need are saturated liquid and saturated vapor data (Tables A–4 and A–5 in the case of water).

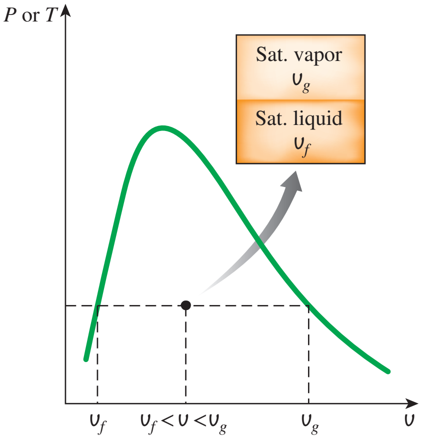


FIGURE 4–34
The v value of a saturated liquid–vapor mixture lies between the v_f and v_g values at the specified T or P.

EXAMPLE 4–4 Pressure and Volume of a Saturated Mixture

A rigid tank contains 10 kg of water at 90°C. If 8 kg of the water is in the liquid form and the rest is in the vapor form, determine (*a*) the pressure in the tank and (*b*) the volume of the tank.

SOLUTION A rigid tank contains saturated mixture. The pressure and the volume of the tank are to be determined.

Analysis (*a*) The state of the saturated liquid–vapor mixture is shown in Fig. 4–35. Since the two phases coexist in equilibrium, we have a saturated mixture, and the pressure must be the saturation pressure at the given temperature:

$$P = P_{\text{sat @ 90°C}} = \mathbf{70.183\ kPa} \qquad \text{(Table A–4)}$$

(*b*) At 90°C, we have $v_f = 0.001036\ \text{m}^3/\text{kg}$ and $v_g = 2.3593\ \text{m}^3/\text{kg}$ (Table A–4). One way of finding the volume of the tank is to determine the volume occupied by each phase and then add them:

$$\begin{aligned} V = V_f + V_g &= m_f v_f + m_g v_g \\ &= (8\ \text{kg})(0.001036\ \text{m}^3/\text{kg}) + (2\ \text{kg})(2.3593\ \text{m}^3/\text{kg}) \\ &= \mathbf{4.73\ m^3} \end{aligned}$$

Another way is to first determine the quality x, then the average specific volume v, and finally the total volume:

$$x = \frac{m_g}{m_t} = \frac{2\ \text{kg}}{10\ \text{kg}} = 0.2$$

$$\begin{aligned} v &= v_f + x v_{fg} \\ &= 0.001036\ \text{m}^3/\text{kg} + (0.2)[(2.3593 - 0.001036)\ \text{m}^3/\text{kg}] \\ &= 0.473\ \text{m}^3/\text{kg} \end{aligned}$$

and

$$V = mv = (10\ \text{kg})(0.473\ \text{m}^3/\text{kg}) = 4.73\ \text{m}^3$$

Discussion The first method appears to be easier in this case since the masses of each phase are given. In most cases, however, the masses of each phase are not available, and the second method becomes more convenient.

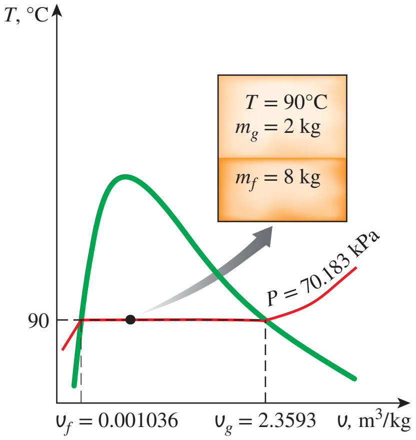


FIGURE 4–35
Schematic and T-v diagram for Example 4–4.

EXAMPLE 4–5 Properties of Saturated Liquid–Vapor Mixture

An 80-L vessel contains 4 kg of refrigerant-134a at a pressure of 160 kPa. Determine (*a*) the temperature, (*b*) the quality, (*c*) the enthalpy of the refrigerant, and (*d*) the volume occupied by the vapor phase.

SOLUTION A vessel is filled with refrigerant-134a. Some properties of the refrigerant are to be determined.

Analysis (*a*) The state of the saturated liquid–vapor mixture is shown in Fig. 4–36. At this point we do not know whether the refrigerant is in the compressed liquid, superheated vapor, or saturated mixture region. This can be determined by comparing a suitable property to the saturated liquid and saturated vapor values. From the information given, we can determine the specific volume:

$$v = \frac{V}{m} = \frac{0.080 \text{ m}^3}{4 \text{ kg}} = 0.02 \text{ m}^3\text{/kg}$$

At 160 kPa, we read

$$v_f = 0.0007435 \text{ m}^3\text{/kg}$$
$$v_g = 0.12355 \text{ m}^3\text{/kg} \qquad \text{(Table A–12)}$$

Obviously, $v_f < v < v_g$, and the refrigerant is in the saturated mixture region. Thus, the temperature must be the saturation temperature at the specified pressure:

$$T = T_{\text{sat @ 160 kPa}} = \mathbf{-15.60°C}$$

(*b*) Quality can be determined from

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.02 - 0.0007435}{0.12355 - 0.0007435} = \mathbf{0.157}$$

(*c*) At 160 kPa, we also read from Table A–12 that $h_f = 31.18$ kJ/kg and $h_{fg} = 209.96$ kJ/kg. Then,

$$\begin{aligned} h &= h_f + xh_{fg} \\ &= 31.18 \text{ kJ/kg} + (0.157)(209.96 \text{ kJ/kg}) \\ &= \mathbf{64.1 \text{ kJ/kg}} \end{aligned}$$

(*d*) The mass of the vapor is

$$m_g = xm_t = (0.157)(4 \text{ kg}) = 0.628 \text{ kg}$$

and the volume occupied by the vapor phase is

$$V_g = m_g v_g = (0.628 \text{ kg})(0.12355 \text{ m}^3\text{/kg}) = \mathbf{0.0776 \text{ m}^3} \text{ (or 77.6 L)}$$

The rest of the volume (2.4 L) is occupied by the liquid.

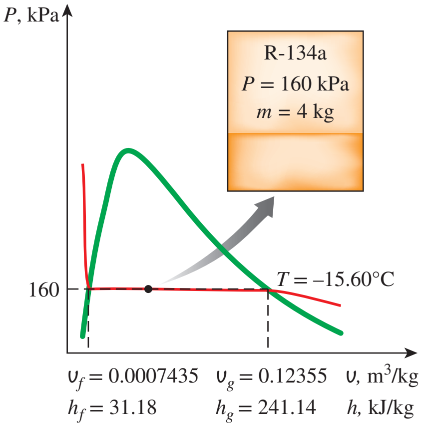


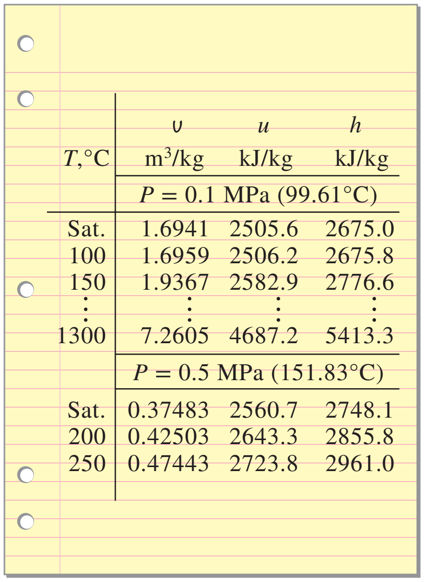
FIGURE 4–36
Schematic and P-v diagram for Example 4–5.

Property tables are also available for saturated solid–vapor mixtures. Properties of saturated ice–water vapor mixtures, for example, are listed in Table A–8. Saturated solid–vapor mixtures can be handled just like saturated liquid–vapor mixtures.

2 Superheated Vapor

In the region to the right of the saturated vapor line and at temperatures above the critical point temperature, a substance exists as superheated vapor. Since the superheated region is a single-phase region (vapor phase only), temperature and pressure are no longer dependent properties, and they can conveniently be used as the two independent properties in the tables. The format of the superheated vapor tables is illustrated in Fig. 4–37.

In these tables, the properties are listed against temperature for selected pressures starting with the saturated vapor data. The saturation temperature is given in parentheses following the pressure value.



T,°C	v m³/kg	u kJ/kg	h kJ/kg
	P = 0.1 MPa (99.61°C)		
Sat.	1.6941	2505.6	2675.0
100	1.6959	2506.2	2675.8
150	1.9367	2582.9	2776.6
⋮	⋮	⋮	⋮
1300	7.2605	4687.2	5413.3
	P = 0.5 MPa (151.83°C)		
Sat.	0.37483	2560.7	2748.1
200	0.42503	2643.3	2855.8
250	0.47443	2723.8	2961.0

FIGURE 4–37
A partial listing of Table A–6.

Compared to saturated vapor, superheated vapor is characterized by

Lower pressures ($P < P_{\text{sat}}$ at a given T)
Higher temperatures ($T > T_{\text{sat}}$ at a given P)
Higher specific volumes ($v > v_g$ at a given P or T)
Higher internal energies ($u > u_g$ at a given P or T)
Higher enthalpies ($h > h_g$ at a given P or T)

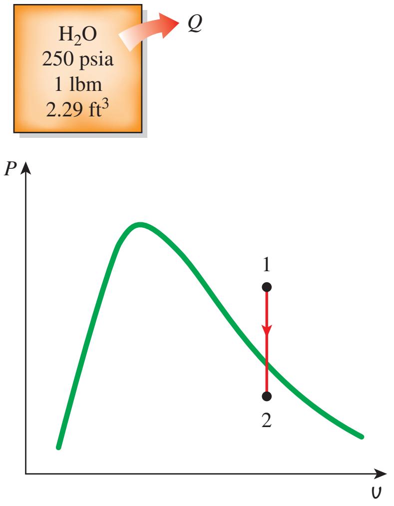


FIGURE 4–38
Schematic and P-v diagram for Example 4–6.

EXAMPLE 4–6 Cooling of Superheated Water Vapor

One pound-mass of water fills a 2.29-ft^3 rigid container at an initial pressure of 250 psia. The container is then cooled to 100°F. Determine the initial temperature and final pressure of the water.

SOLUTION A rigid container that is filled with water is cooled. The initial temperature and final pressure are to be determined.
Analysis The initial specific volume is

$$v_1 = \frac{V}{m} = \frac{2.29\ \text{ft}^3}{1\ \text{lbm}} = 2.29\ \text{ft}^3/\text{lbm}$$

At 250 psia, the specific volume of saturated vapor is $v_g = 1.8440\ \text{ft}^3/\text{lbm}$ (Table A–5E). Since $v_1 > v_g$, the water is initially in the superheated vapor region. The temperature is determined to be

$$\left.\begin{aligned} P_1 &= 250\ \text{psia} \\ v_1 &= 2.29\ \text{ft}^3/\text{lbm} \end{aligned}\right\}\ T_1 = \mathbf{550°F} \qquad \text{(Table A–6E)}$$

This is a constant-volume cooling process ($v = V/m =$ constant), as shown in Fig. 4–38. The final state is saturated mixture, and thus the pressure is the saturation pressure at the final temperature:

$$\left.\begin{aligned} T_2 &= 100°\text{F} \\ v_2 &= v_1 = 2.29\ \text{ft}^3/\text{lbm} \end{aligned}\right\}\ P_2 = P_{\text{sat @ 100°F}} = \mathbf{0.9505\ psia} \qquad \text{(Table A–4E)}$$

Discussion When a substance undergoes a process in a closed rigid tank, the specific volume remains constant, and the process appears as a vertical line in the P-v diagram.

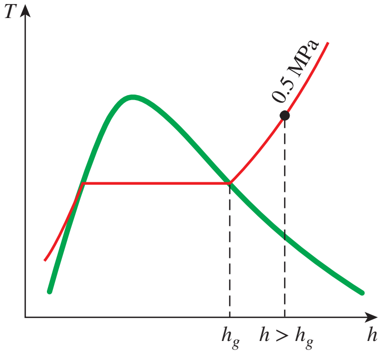


FIGURE 4–39
At a specified P, superheated vapor exists at a higher h than the saturated vapor (Example 4–7).

EXAMPLE 4–7 Temperature of Superheated Vapor

Determine the temperature of water at a state of $P = 0.5$ MPa and $h = 2890$ kJ/kg.

SOLUTION The temperature of water at a specified state is to be determined.
Analysis At 0.5 MPa, the enthalpy of saturated water vapor is $h_g = 2748.1$ kJ/kg. Since $h > h_g$, as shown in Fig. 4–39, we again have superheated vapor. Under 0.5 MPa in Table A–6 we read

T, °C	h, kJ/kg
200	2855.8
250	2961.0

Obviously, the temperature is between 200 and 250°C. By linear interpolation it is determined to be

$$T = \mathbf{216.3°C}$$

3 Compressed Liquid

Compressed liquid tables are not as commonly available, and Table A–7 is the only compressed liquid table in this text. The format of Table A–7 is very much like the format of the superheated vapor tables. One reason for the lack of compressed liquid data is the relative independence of compressed liquid properties from pressure. Variation of properties of compressed liquid with pressure is very mild. Increasing the pressure 100 times often causes properties to change less than 1 percent.

In the absence of compressed liquid data, a general approximation is *to treat compressed liquid as saturated liquid at the given temperature* (Fig. 4–40). This is because the compressed liquid properties depend on temperature much more strongly than they do on pressure. Thus,

$$y \cong y_{f\,@\,T} \tag{4–8}$$

for compressed liquids, where y is v, u, or h. Of these three properties, the property whose value is most sensitive to variations in the pressure is the enthalpy h. Although the preceding approximation results in negligible error in v and u, the error in h may reach undesirable levels. However, the error in h at low to moderate pressures and temperatures can be reduced significantly by evaluating it from

$$h \cong h_{f\,@\,T} + v_{f\,@\,T}(P - P_{\text{sat}\,@\,T}) \tag{4–9}$$

instead of taking it to be just h_f. Note, however, that the approximation in Eq. 4–9 does not yield any significant improvement at moderate to high temperatures and pressures, and it may even backfire and result in greater error due to overcorrection at very high temperatures and pressures (*see* Kostic, 2006).

In general, a compressed liquid is characterized by

Higher pressures ($P > P_{\text{sat}}$ at a given T)
Lower temperatures ($T < T_{\text{sat}}$ at a given P)
Lower specific volumes ($v < v_f$ at a given P or T)
Lower internal energies ($u < u_f$ at a given P or T)
Lower enthalpies ($h < h_f$ at a given P or T)

But unlike superheated vapor, the compressed liquid properties are not much different from the corresponding saturated liquid values.

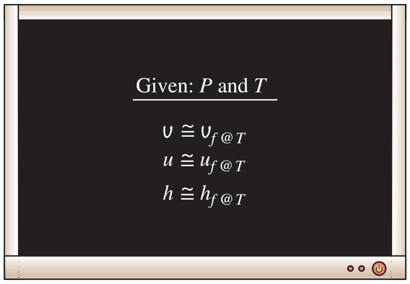


FIGURE 4–40
A compressed liquid may be approximated as a saturated liquid at the given temperature.

EXAMPLE 4–8 Approximating Compressed Liquid as Saturated Liquid

Determine the internal energy of compressed liquid water at 80°C and 5 MPa, using (*a*) data from the compressed liquid table and (*b*) saturated liquid data. What is the error involved in the second case?

SOLUTION The exact and approximate values of the internal energy of liquid water are to be determined.

Analysis At 80°C, the saturation pressure of water is 47.416 kPa, and since 5 MPa > P_{sat}, we obviously have compressed liquid, as shown in Fig. 4–41.

(*a*) From the compressed liquid table (Table A–7)

$$\left.\begin{aligned} P &= 5\text{ MPa} \\ T &= 80^\circ\text{C} \end{aligned}\right\} \quad u = \mathbf{333.82\ kJ/kg}$$

(*b*) From the saturation table (Table A–4), we read

$$u \cong u_{f\,@\,80^\circ\text{C}} = \mathbf{334.97\ kJ/kg}$$

The error involved is

$$\frac{334.97 - 333.82}{333.82} \times 100 = \mathbf{0.34\%}$$

which is less than 1 percent.

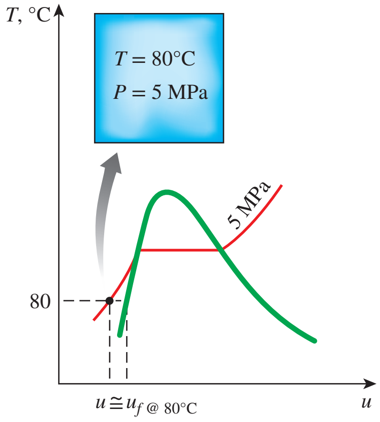


FIGURE 4–41
Schematic and T-u diagram for Example 4–8.

Reference State and Reference Values

The values of u, h, and s cannot be measured directly, and they are calculated from measurable properties using the relations between thermodynamic properties. However, those relations give the *changes* in properties, not the values of properties at specified states. Therefore, we need to choose a convenient *reference state* and assign a value of *zero* for a convenient property or properties at that state. For water, the state of saturated liquid at 0.01°C is taken as the reference state, and the internal energy and entropy are assigned zero values at that state. For refrigerant-134a, the state of saturated liquid at –40°C is taken as the reference state, and the enthalpy and entropy are assigned zero values at that state. Note that some properties may have negative values as a result of the reference state chosen.

It should be mentioned that sometimes different tables list different values for some properties at the same state as a result of using a different reference state. However, in thermodynamics we are concerned with the *changes* in properties, and the reference state chosen is of no consequence in calculations as long as we use values from a single consistent set of tables or charts.

EXAMPLE 4–9 The Use of Steam Tables to Determine Properties

Determine the missing properties and the phase descriptions in the following table for water:

	T, °C	P, kPa	u, kJ/kg	x	Phase description
(*a*)		200		0.6	
(*b*)	125		1600		
(*c*)		1000	2950		
(*d*)	75	500			
(*e*)		850		0.0	

SOLUTION Properties and phase descriptions of water are to be determined at various states.

Analysis (*a*) The quality is given to be $x = 0.6$, which implies that 60 percent of the mass is in the vapor phase and the remaining 40 percent is in the liquid phase. Therefore, we have saturated liquid–vapor mixture at a pressure of 200 kPa. Then the temperature must be the saturation temperature at the given pressure:

$$T = T_{\text{sat @ 200 kPa}} = \mathbf{120.21°C} \qquad \text{(Table A–5)}$$

At 200 kPa, we also read from Table A–5 that $u_f = 504.50$ kJ/kg and $u_{fg} = 2024.6$ kJ/kg. Then the average internal energy of the mixture is

$$\begin{aligned} u &= u_f + x u_{fg} \\ &= 504.50 \text{ kJ/kg} + (0.6)(2024.6 \text{ kJ/kg}) \\ &= \mathbf{1719.26\ kJ/kg} \end{aligned}$$

(*b*) This time the temperature and the internal energy are given, but we do not know which table to use to determine the missing properties because we have no clue as to whether we have saturated mixture, compressed liquid, or superheated vapor. To determine the region we are in, we first go to the saturation table (Table A–4) and determine the u_f and u_g values at the given temperature. At 125°C, we read $u_f = 524.83$ kJ/kg and $u_g = 2534.3$ kJ/kg. Next we compare the given u value to these u_f and u_g values, keeping in mind that

if $u < u_f$ we have *compressed liquid*
if $u_f \le u \le u_g$ we have *saturated mixture*
if $u > u_g$ we have *superheated vapor*

In our case the given u value is 1600 kJ/kg, which falls between the u_f and u_g values at 125°C. Therefore, we have saturated liquid–vapor mixture. Then the pressure must be the saturation pressure at the given temperature:

$$P = P_{\text{sat @ 125°C}} = \mathbf{232.23\ kPa} \qquad \text{(Table A–4)}$$

The quality is determined from

$$x = \frac{u - u_f}{u_{fg}} = \frac{1600 - 524.83}{2009.5} = \mathbf{0.535}$$

The preceding criteria for determining whether we have compressed liquid, saturated mixture, or superheated vapor can also be used when enthalpy h or specific volume v is given instead of internal energy u, or when pressure is given instead of temperature.

(c) This is similar to case (b), except pressure is given instead of temperature. Following the argument given above, we read the u_f and u_g values at the specified pressure. At 1 MPa, we have $u_f = 761.39$ kJ/kg and $u_g = 2582.8$ kJ/kg. The specified u value is 2950 kJ/kg, which is greater than the u_g value at 1 MPa. Therefore, we have superheated vapor, and the temperature at this state is determined from the superheated vapor table by interpolation to be

$$T = \mathbf{395.2°C} \qquad \text{(Table A–6)}$$

We would leave the quality column blank in this case since quality has no meaning for a superheated vapor.

(d) In this case the temperature and pressure are given, but again we cannot tell which table to use to determine the missing properties because we do not know whether we have saturated mixture, compressed liquid, or superheated vapor. To determine the region we are in, we go to the saturation table (Table A–5) and determine the saturation temperature value at the given pressure. At 500 kPa, we have $T_{\text{sat}} = 151.83$°C. We then compare the given T value to this T_{sat} value, keeping in mind that

if $T < T_{\text{sat @ given } P}$ we have *compressed liquid*
if $T = T_{\text{sat @ given } P}$ we have *saturated mixture*
if $T > T_{\text{sat @ given } P}$ we have *superheated vapor*

In our case, the given T value is 75°C, which is less than the T_{sat} value at the specified pressure. Therefore, we have compressed liquid (Fig. 4–42), and normally we would determine the internal energy value from the compressed liquid table. But in this case the given pressure is much lower than the lowest pressure value in the compressed liquid table (which is 5 MPa), and therefore we are justified to treat the compressed liquid as saturated liquid at the given temperature (*not* pressure):

$$u \cong u_{f\text{ @ 75°C}} = \mathbf{313.99\ kJ/kg} \qquad \text{(Table A–4)}$$

We would leave the quality column blank in this case since quality has no meaning in the compressed liquid region.

(e) The quality is given to be $x = 0$, and thus we have saturated liquid at the specified pressure of 850 kPa. Then the temperature must be the saturation temperature at the given pressure, and the internal energy must have the saturated liquid value:

$$T = T_{\text{sat @ 850 kPa}} = \mathbf{172.94°C}$$
$$u = u_{f\text{ @ 850 kPa}} = \mathbf{731.00\ kJ/kg} \qquad \text{(Table A–5)}$$

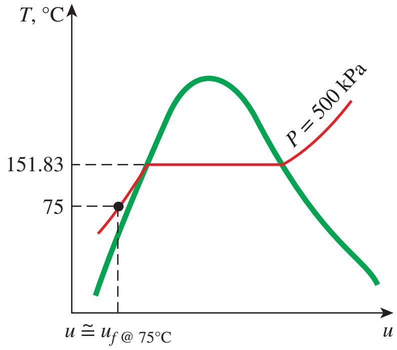


FIGURE 4–42
At a given P and T, a pure substance will exist as a compressed liquid if $T < T_{\text{sat @ } P}$.

4–6 ▪ THE IDEAL-GAS EQUATION OF STATE

Property tables provide very accurate information about the properties, but they are bulky and vulnerable to typographical errors. A more practical and desirable approach would be to have some simple relations among the properties that are sufficiently general and accurate.

Any equation that relates the pressure, temperature, and specific volume of a substance is called an **equation of state**. Property relations that involve other properties of a substance at equilibrium states are also referred to as equations of state. There are several equations of state, some simple and others very complex. The simplest and best-known equation of state for substances in the gas phase is the ideal-gas equation of state. This equation predicts the P-v-T behavior of a gas quite accurately within some properly selected region.

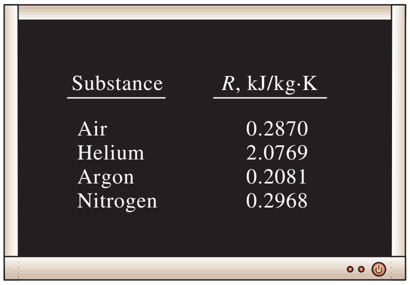
Gas and *vapor* are often used as synonymous words. The vapor phase of a substance is customarily called a *gas* when it is above the critical temperature. *Vapor* usually implies a gas that is not far from a state of condensation.

In 1662, Robert Boyle, an Englishman, observed during his experiments with a vacuum chamber that the pressure of gases is inversely proportional to their volume. In 1802, J. Charles and J. Gay-Lussac, Frenchmen, experimentally determined that at low pressures the volume of a gas is proportional to its temperature. That is,

$$P = R\left(\frac{T}{v}\right)$$

or

$$Pv = RT \qquad \textbf{(4–10)}$$



Substance	R, kJ/kg·K
Air	0.2870
Helium	2.0769
Argon	0.2081
Nitrogen	0.2968

FIGURE 4–43
Different substances have different gas constants.

where the constant of proportionality R is called the **gas constant**. Equation 4–10 is called the **ideal-gas equation of state**, or simply the **ideal-gas relation**, and a gas that obeys this relation is called an **ideal gas**. In this equation, P is the absolute pressure, T is the absolute temperature, and v is the specific volume.

The gas constant R is different for each gas (Fig. 4–43) and is determined from

$$R = \frac{R_u}{M} \qquad (\text{kJ/kg·K or kPa·m}^3\text{/kg·K})$$

where R_u is the **universal gas constant** and M is the molar mass (also called *molecular weight)* of the gas. The constant R_u is the same for all substances, and its value is

$$R_u = \begin{cases} 8.31447 \text{ kJ/kmol·K} \\ 8.31447 \text{ kPa·m}^3\text{/kmol·K} \\ 0.0831447 \text{ bar·m}^3\text{/kmol·K} \\ 1.98588 \text{ Btu/lbmol·R} \\ 10.7316 \text{ psia·ft}^3\text{/lbmol·R} \\ 1545.37 \text{ ft·lbf/lbmol·R} \end{cases} \qquad \textbf{(4–11)}$$

The **molar mass** M can simply be defined as *the mass of one mole* (also called a *gram-mole,* abbreviated gmol) *of a substance in grams,* or *the mass of one kmol* (also called a *kilogram-mole,* abbreviated kgmol) *in kilograms.* In English units, it is the mass of 1 lbmol in lbm. Notice that the molar mass of a substance has the same numerical value in both unit systems because of the way it is defined. When we say the molar mass of nitrogen is 28, it simply means the mass of 1 kmol of nitrogen is 28 kg, or the mass of 1 lbmol of nitrogen is 28 lbm. That is, $M = 28$ kg/kmol $= 28$ lbm/lbmol.

The mass of a system is equal to the product of its molar mass M and the mole number N:

$$m = MN \qquad (\text{kg}) \qquad \textbf{(4–12)}$$

The values of R and M for several substances are given in Table A–1.

The ideal-gas equation of state can be written in several different forms:

$$V = mv \longrightarrow PV = mRT \qquad \textbf{(4–13)}$$

$$mR = (MN)R = NR_u \longrightarrow PV = NR_uT \qquad \textbf{(4–14)}$$

$$V = N\overline{v} \longrightarrow P\overline{v} = R_uT \qquad \textbf{(4–15)}$$

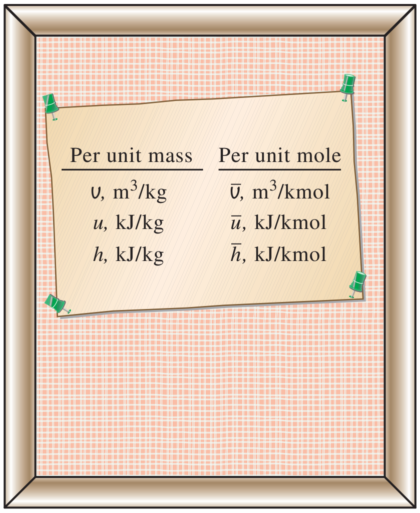
where $\bar{\upsilon}$ is the molar specific volume, that is, the volume per unit mole (in m^3/kmol or ft^3/lbmol). A bar above a property denotes values on a *unit-mole basis* throughout this text (Fig. 4–44).

By writing Eq. 4–13 twice for a fixed mass and simplifying, the properties of an ideal gas at two different states are related to each other by

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \tag{4–16}$$

An ideal gas is an *imaginary* substance that obeys the relation $P\upsilon = RT$. It has been experimentally observed that the ideal-gas relation given closely approximates the P-υ-T behavior of real gases at low densities. At low pressures and high temperatures, the density of a gas decreases, and the gas behaves as an ideal gas under these conditions. What constitutes low pressure and high temperature is explained later.

In the range of practical interest, many familiar gases such as air, nitrogen, oxygen, hydrogen, helium, argon, neon, and carbon dioxide and even heavier gases such as krypton can be treated as ideal gases with negligible error (often less than 1 percent). Dense gases such as water vapor in steam power plants and refrigerant vapor in refrigerators, however, should not be treated as ideal gases. Instead, the property tables should be used for these substances.



Per unit mass	Per unit mole
υ, m^3/kg	$\bar{\upsilon}$, m^3/kmol
u, kJ/kg	$\bar{u}$, kJ/kmol
h, kJ/kg	$\bar{h}$, kJ/kmol

FIGURE 4–44
Properties per unit mole are denoted with a bar on the top.

EXAMPLE 4–10 Temperature Rise of Air in a Tire During a Trip

The gage pressure of an automobile tire is measured to be 210 kPa before a trip and 220 kPa after the trip at a location where the atmospheric pressure is 95 kPa (Fig. 4–45). Assuming the volume of the tire remains constant and the air temperature before the trip is 25°C, determine air temperature in the tire after the trip.

SOLUTION The pressure in an automobile tire is measured before and after a trip. The temperature of air in the tire after the trip is to be determined.

Assumptions **1** The volume of the tire remains constant. **2** Air is an ideal gas.

Properties The local atmospheric pressure is 95 kPa.

Analysis The absolute pressures in the tire before and after the trip are

$$P_1 = P_{\text{gage.1}} + P_{\text{atm}} = 210 + 95 = 305 \text{ kPa}$$
$$P_2 = P_{\text{gage.2}} + P_{\text{atm}} = 220 + 95 = 315 \text{ kPa}$$

Note that air is an ideal gas and the volume is constant. The air temperature after the trip is determined to be

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \longrightarrow T_2 = \frac{P_2}{P_1} T_1 = \frac{315 \text{ kPa}}{305 \text{ kPa}} (25 + 273) \text{ K} = 307.8 \text{ K} = \mathbf{34.8°C}$$

Therefore, the absolute temperature of air in the tire will increase by 3.3 percent during this trip.

Discussion Note that the air temperature has risen nearly 10°C during this trip. This shows the importance of measuring the tire pressures before long trips to avoid errors due to the temperature increase of air in tires. Also note that the unit kelvin is used for temperature in the ideal-gas relation.

FIGURE 4–45
Stockbyte/Getty Images

Is Water Vapor an Ideal Gas?

This question cannot be answered with a simple yes or no. The error involved in treating water vapor as an ideal gas is calculated and plotted in Fig. 4–46. It is clear from this figure that at pressures below 10 kPa, water vapor can be treated as an ideal gas, regardless of its temperature, with negligible error (less than 0.1 percent). At higher pressures, however, the ideal-gas assumption yields unacceptable errors, particularly in the vicinity of the critical point (over 100 percent) and the saturated vapor line. Therefore, in air-conditioning applications, the water vapor in the air

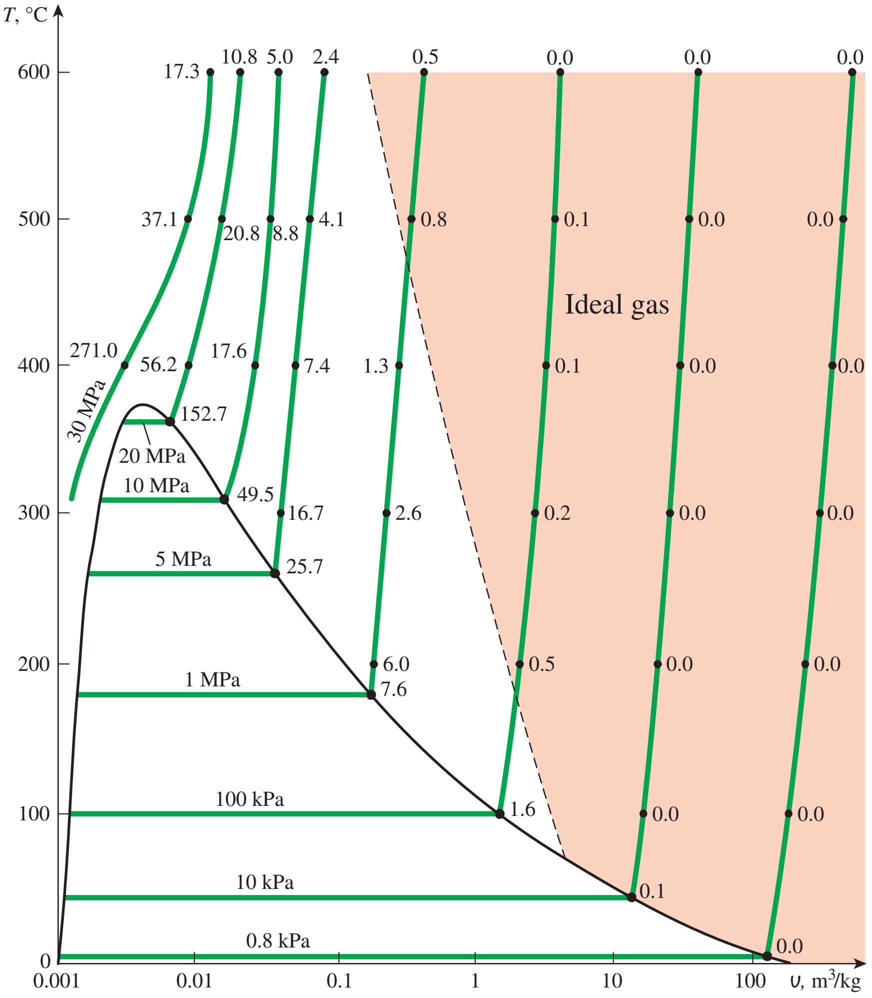


FIGURE 4–46
Percentage of error ($[|\upsilon_{\text{table}} - \upsilon_{\text{ideal}}|/\upsilon_{\text{table}}] \times 100$) involved in assuming steam to be an ideal gas, and the region where steam can be treated as an ideal gas with less than 1 percent error.

can be treated as an ideal gas with essentially no error since the pressure of the water vapor is very low. In steam power plant applications, however, the pressures involved are usually very high; therefore, the ideal-gas relation should not be used.

4–7 ▪ COMPRESSIBILITY FACTOR—A MEASURE OF DEVIATION FROM IDEAL-GAS BEHAVIOR

The ideal-gas equation is very simple and thus very convenient to use. However, as illustrated in Fig. 4–47, gases deviate from ideal-gas behavior significantly at states near the saturation region and the critical point. This deviation from ideal-gas behavior at a given temperature and pressure can accurately be accounted for by the introduction of a correction factor called the **compressibility factor** *Z*, defined as

$$Z = \frac{P\upsilon}{RT} \tag{4–17}$$

or

$$P\upsilon = ZRT \tag{4–18}$$

It can also be expressed as

$$Z = \frac{\upsilon_{\text{actual}}}{\upsilon_{\text{ideal}}} \tag{4–19}$$

where $v_{\text{ideal}} = RT/P$. Obviously, $Z = 1$ for ideal gases. For real gases Z can be greater than or less than unity (Fig. 4–47). The farther away Z is from unity, the more the gas deviates from ideal-gas behavior.

We have said that gases follow the ideal-gas equation closely at low pressures and high temperatures. But what exactly constitutes low pressure or high temperature? Is −100°C a low temperature? It definitely is for most substances but not for air. Air (or nitrogen) can be treated as an ideal gas at this temperature and atmospheric pressure with an error under 1 percent. This is because nitrogen is well over its critical temperature (–147°C) and away from the saturation region. At this temperature and pressure, however, most substances would exist in the solid phase. Therefore, the pressure or temperature of a substance is high or low relative to its critical temperature or pressure.

Gases behave differently at a given temperature and pressure, but they behave very much the same at temperatures and pressures normalized with respect to their critical temperatures and pressures. The normalization is done as

$$P_R = \frac{P}{P_{\text{cr}}} \quad \text{and} \quad T_R = \frac{T}{T_{\text{cr}}} \tag{4–20}$$

Here P_R is called the **reduced pressure** and T_R the **reduced temperature**. The Z factor for all gases is approximately the same at the same reduced pressure and temperature. This is called the **principle of corresponding states**. In Fig. 4–48, the

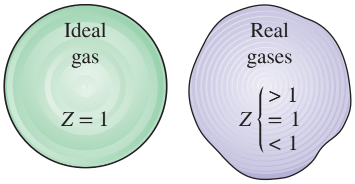


FIGURE 4–47
The compressibility factor is unity for ideal gases.

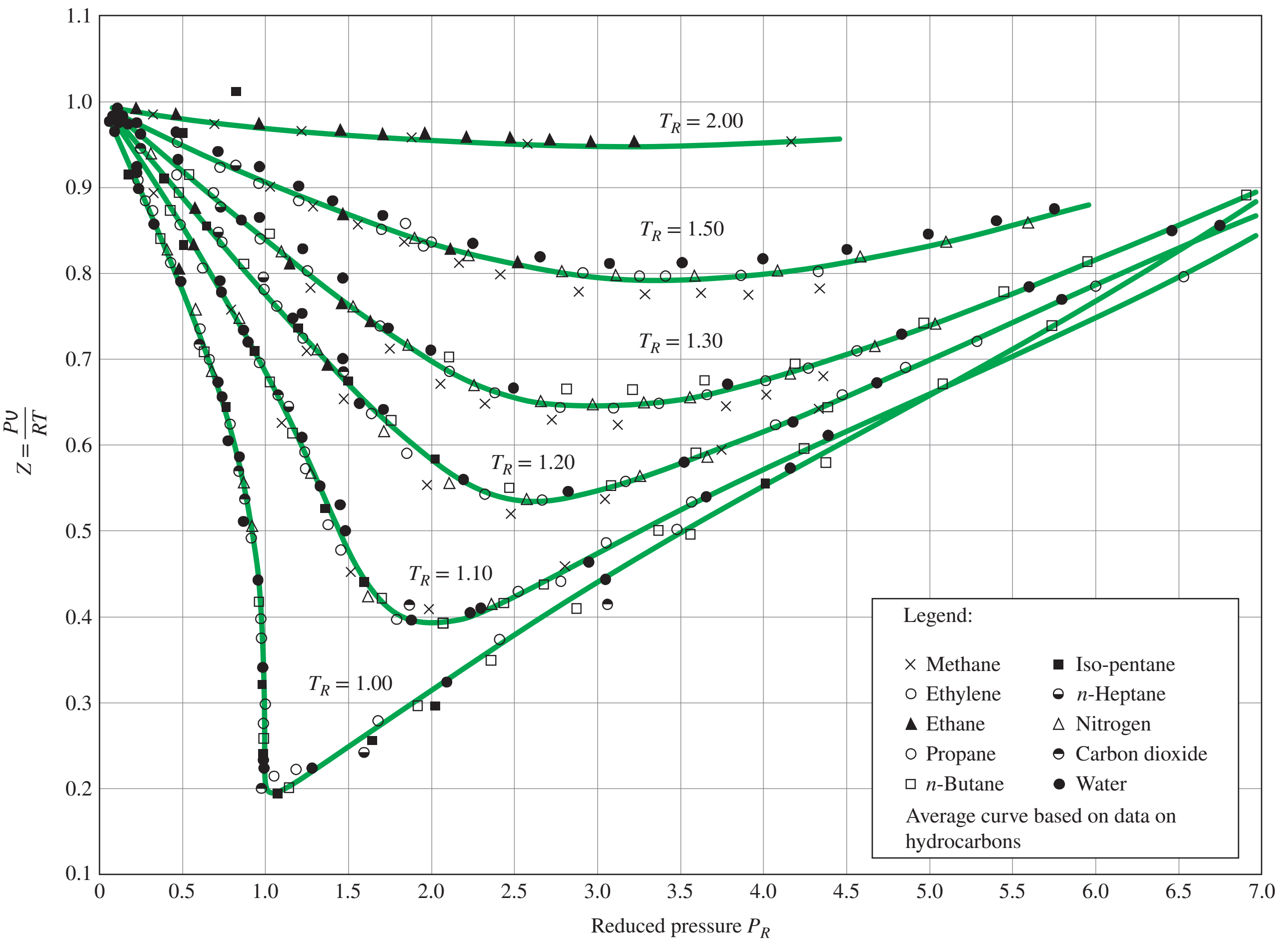


FIGURE 4–48
Comparison of Z factors for various gases.

Source: Gour-Jen Su, "Modified Law of Corresponding States," *Ind. Eng. Chem.* (international ed.) 38 (1946), p. 803.

experimentally determined Z values are plotted against P_R and T_R for several gases. The gases seem to obey the principle of corresponding states reasonably well. By curve-fitting all the data, we obtain the **generalized compressibility chart** that can be used for all gases (Fig. A–28).

The following observations can be made from the generalized compressibility chart:

1. At very low pressures ($P_R << 1$), gases behave as ideal gases regardless of temperature (Fig. 4–49).
2. At high temperatures ($T_R > 2$), ideal-gas behavior can be assumed with good accuracy regardless of pressure (except when $P_R >> 1$).
3. The deviation of a gas from ideal-gas behavior is greatest in the vicinity of the critical point (Fig. 4–50).

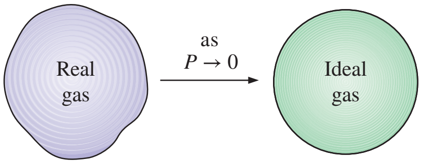


FIGURE 4–49

At very low pressures, all gases approach ideal-gas behavior (regardless of their temperature).

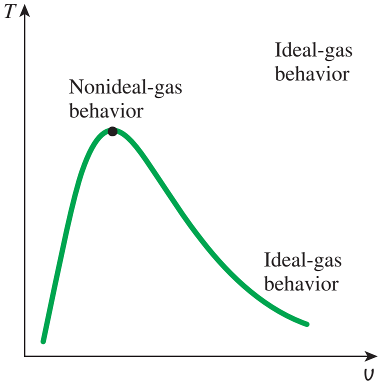


FIGURE 4–50

Gases deviate from the ideal-gas behavior the most in the neighborhood of the critical point.

EXAMPLE 4–11 The Use of Generalized Charts

Determine the specific volume of refrigerant-134a at 1 MPa and 50°C, using (*a*) the ideal-gas equation of state and (*b*) the generalized compressibility chart. Compare the values obtained to the actual value of 0.021796 m^3/kg and determine the error involved in each case.

SOLUTION The specific volume of refrigerant-134a is to be determined assuming ideal- and nonideal-gas behavior.

Analysis The gas constant, the critical pressure, and the critical temperature of refrigerant-134a are determined from Table A–1 to be

$$R = 0.0815 \text{ kPa·m}^3\text{/kg·K}$$
$$P_{cr} = 4.059 \text{ MPa}$$
$$T_{cr} = 374.2 \text{ K}$$

(*a*) The specific volume of refrigerant-134a under the ideal-gas assumption is

$$\upsilon = \frac{RT}{P} = \frac{(0.0815 \text{ kPa·m}^3\text{/kg·K})(323 \text{ K})}{1000 \text{ kPa}} = \mathbf{0.026325\ m^3/kg}$$

Therefore, treating the refrigerant-134a vapor as an ideal gas would result in an error of (0.026325 – 0.021796)/0.021796 = 0.208, or 20.8 percent in this case.

(*b*) To determine the correction factor Z from the compressibility chart, we first need to calculate the reduced pressure and temperature:

$$\left.\begin{aligned} P_R &= \frac{P}{P_{cr}} = \frac{1 \text{ MPa}}{4.059 \text{ MPa}} = 0.246 \\ T_R &= \frac{T}{T_{cr}} = \frac{323 \text{ K}}{374.2 \text{ K}} = 0.863 \end{aligned}\right\} \quad Z = 0.84$$

Thus

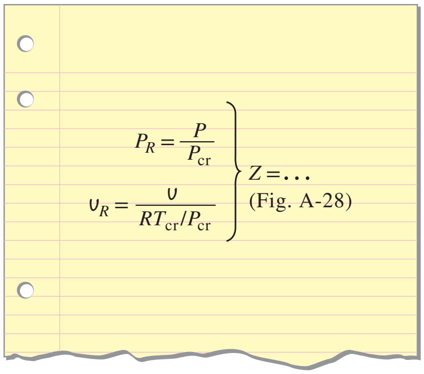
$$\upsilon = Z\upsilon_{ideal} = (0.84)(0.026325 \text{ m}^3\text{/kg}) = \mathbf{0.022113\ m^3/kg}$$

Discussion The error in this result is less than **2 percent**. Therefore, in the absence of tabulated data, the generalized compressibility chart can be used with confidence.

When P and υ, or T and υ, are given instead of P and T, the generalized compressibility chart can still be used to determine the third property, but it would involve tedious trial and error. Therefore, it is necessary to define one more reduced property called the **pseudo-reduced specific volume** υ_R as

$$\upsilon_R = \frac{\upsilon_{actual}}{RT_{cr}/P_{cr}} \qquad \textbf{(4–21)}$$

Note that υ_R is defined differently from P_R and T_R. It is related to T_{cr} and P_{cr} instead of υ_{cr}. Lines of constant υ_R are also added to the compressibility charts, and this enables one to determine T or P without having to resort to time-consuming iterations (Fig. 4–51).



$$\left.\begin{aligned} P_R &= \frac{P}{P_{cr}} \\ \upsilon_R &= \frac{\upsilon}{RT_{cr}/P_{cr}} \end{aligned}\right\} \begin{aligned} &Z = \ldots \\ &\text{(Fig. A-28)} \end{aligned}$$

FIGURE 4–51

The compressibility factor can also be determined from a knowledge of P_R and υ_R.

EXAMPLE 4–12 Using Generalized Charts to Determine Pressure

Determine the pressure of water vapor at 600°F and 0.51431 ft³/lbm, using (*a*) the steam tables, (*b*) the ideal-gas equation, and (*c*) the generalized compressibility chart.

SOLUTION The pressure of water vapor is to be determined in three different ways.

Analysis A sketch of the system is given in Fig. 4–52. The gas constant, the critical pressure, and the critical temperature of steam are determined from Table A–1E to be

$$\begin{aligned} R &= 0.5956 \text{ psia·ft}^3\text{/lbm·R} \\ P_{cr} &= 3200 \text{ psia} \\ T_{cr} &= 1164.8 \text{ R} \end{aligned}$$

(*a*) The pressure at the specified state is determined from Table A–6E to be

$$\left.\begin{aligned} \upsilon &= 0.51431 \text{ ft}^3\text{/lbm} \\ T &= 600°\text{F} \end{aligned}\right\} \quad P = \mathbf{1000 \text{ psia}}$$

This is the experimentally determined value, and thus it is the most accurate.

(*b*) The pressure of steam under the ideal-gas assumption is determined from the ideal-gas relation to be

$$P = \frac{RT}{\upsilon} = \frac{(0.5956 \text{ psia·ft}^3\text{/lbm·R})(1060 \text{ R})}{0.51431 \text{ ft}^3\text{/lbm}} = \mathbf{1228 \text{ psia}}$$

Therefore, treating the steam as an ideal gas would result in an error of (1228 – 1000)/1000 = 0.228, or 22.8 percent in this case.

(*c*) To determine the correction factor Z from the compressibility chart (Fig. A–28), we first need to calculate the pseudo-reduced specific volume and the reduced temperature:

$$\left.\begin{aligned} \upsilon_R &= \frac{\upsilon_{\text{actual}}}{RT_{cr}/P_{cr}} = \frac{(0.51431 \text{ ft}^3\text{/lbm})(3200 \text{ psia})}{(0.5956 \text{ psia·ft}^3\text{/lbm·R})(1164.8 \text{ R})} = 2.372 \\ T_R &= \frac{T}{T_{cr}} = \frac{1060 \text{ R}}{1164.8 \text{ R}} = 0.91 \end{aligned}\right\} \quad P_R = 0.33$$

Thus,

$$P = P_R P_{cr} = (0.33)(3200 \text{ psia}) = \mathbf{1056 \text{ psia}}$$

Discussion Using the compressibility chart reduced the error from 22.8 to 5.6 percent, which is acceptable for most engineering purposes (Fig. 4–53). A bigger chart, of course, would give better resolution and reduce the reading errors. Notice that we did not have to determine Z in this problem since we could read P_R directly from the chart.

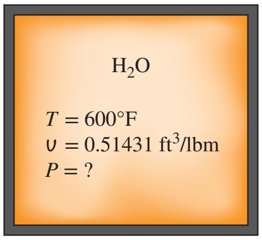
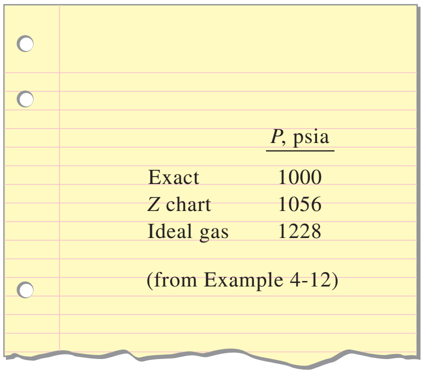


FIGURE 4–52

Schematic for Example 4–12.



	P, psia
Exact	1000
Z chart	1056
Ideal gas	1228

(from Example 4-12)

FIGURE 4–53

Results obtained by using the compressibility chart are usually within a few percent of actual values.

The ideal-gas equation of state is very simple, but its range of applicability is limited. It is desirable to have equations of state that represent the P-υ-T behavior of substances accurately over a larger region with no limitations. Such equations are naturally more complicated. Several equations have been proposed for this purpose including the *van der Waals* equation which is one of the earliest; the *Beattie-Bridgeman* equation of state which is one of the best known and is reasonably accurate; and the *Benedict-Webb-Rubin* equation which is one of the more recent and is very accurate.

SUMMARY

A substance that has a fixed chemical composition throughout is called a *pure substance.* A pure substance exists in different phases depending on its energy level. In the liquid phase, a substance that is not about to vaporize is called a *compressed* or *subcooled liquid.* In the gas phase, a substance that is not about to condense is called a *superheated vapor.* During a phase-change process, the temperature and pressure of a pure substance are dependent properties. At a given pressure, a substance changes phase at a fixed temperature, called the *saturation temperature.* Likewise, at a given temperature, the pressure at which a substance changes phase is called the *saturation pressure.* During a boiling process, both the liquid and the vapor phases coexist in equilibrium, and under this condition the liquid is called *saturated liquid* and the vapor *saturated vapor.*

In a saturated liquid–vapor mixture, the mass fraction of vapor is called the *quality* and is expressed as

$$x = \frac{m_{\text{vapor}}}{m_{\text{total}}}$$

Quality may have values between 0 (saturated liquid) and 1 (saturated vapor). It has no meaning in the compressed liquid or superheated vapor regions. In the saturated mixture region, the average value of any intensive property y is determined from

$$y = y_f + x y_{fg}$$

where f stands for saturated liquid and g for saturated vapor.

In the absence of compressed liquid data, a general approximation is to treat a compressed liquid as a saturated liquid at the given *temperature,*

$$y \cong y_{f\,@\,T}$$

where y stands for υ, u, or h.

The state beyond which there is no distinct vaporization process is called the *critical point.* At supercritical pressures, a substance gradually and uniformly expands from the liquid to vapor phase. All three phases of a substance coexist in equilibrium at states along the *triple line* characterized by triple-line temperature and pressure. The compressed liquid has lower υ, u, and h values than the saturated liquid at the same T or P. Likewise, superheated vapor has higher υ, u, and h values than the saturated vapor at the same T or P.

Any relation among the pressure, temperature, and specific volume of a substance is called an *equation of state.* The simplest and best-known equation of state is the *ideal-gas equation of state,* given as

$$P\upsilon = RT$$

where R is the gas constant. Caution should be exercised in using this relation since an ideal gas is a fictitious substance. Real gases exhibit ideal-gas behavior at relatively low pressures and high temperatures.

The deviation from ideal-gas behavior can be properly accounted for by using the *compressibility factor Z,* defined as

$$Z = \frac{P\upsilon}{RT} \quad \text{or} \quad Z = \frac{\upsilon_{\text{actual}}}{\upsilon_{\text{ideal}}}$$

The Z factor is approximately the same for all gases at the same *reduced temperature* and *reduced pressure,* which are defined as

$$T_R = \frac{T}{T_{\text{cr}}} \quad \text{and} \quad P_R = \frac{P}{P_{\text{cr}}}$$

where P_{cr} and T_{cr} are the critical pressure and temperature, respectively. This is known as the *principle of corresponding states.* When either P or T is unknown, it can be determined from the compressibility chart with the help of the *pseudo-reduced specific volume,* defined as

$$\upsilon_R = \frac{\upsilon_{\text{actual}}}{RT_{\text{cr}}/P_{\text{cr}}}$$

The P-υ-T behavior of substances can be represented more accurately by more complex equations of state.

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PROBLEMS*

Pure Substances, Phase-Change Processes, Property Diagrams

4–1C A propane tank is filled with a mixture of liquid and vapor propane. Can the contents of this tank be considered a pure substance? Explain.

4–2C Is iced water a pure substance? Why?

4–3C What is the difference between saturated vapor and superheated vapor?

4–4C What is the difference between saturated liquid and compressed liquid?

4–5C If the pressure of a substance is increased during a boiling process, will the temperature also increase or will it remain constant? Why?

4–6C Is it true that water boils at higher temperature at higher pressure? Explain.

4–7C What is the difference between the critical point and the triple point?

4–8C A househusband is cooking beef stew for his family in a pan that is (*a*) uncovered, (*b*) covered with a light lid, and (*c*) covered with a heavy lid. For which case will the cooking time be the shortest? Why?

4–9C How does a boiling process at supercritical pressures differ from the boiling process at subcritical pressures?

Property Tables

4–10C What is quality? Does it have any meaning in the superheated vapor region?

4–11C Does the amount of heat absorbed as 1 kg of saturated liquid water boils at 100°C have to be equal to the amount of heat released as 1 kg of saturated water vapor condenses at 100°C?

4–12C Does the reference point selected for the properties of a substance have any effect on thermodynamic analysis? Why?

4–13C What is the physical significance of h_{fg}? Can it be obtained from a knowledge of h_f and h_g? How?

4–14C Does h_{fg} change with pressure? How?

4–15C Is it true that it takes more energy to vaporize 1 kg of saturated liquid water at 100°C than it would at 120°C?

4–16C Which process requires more energy: completely vaporizing 1 kg of saturated liquid water at 1 atm pressure or completely vaporizing 1 kg of saturated liquid water at 8 atm pressure?

4–17C In what kind of pot will a given volume of water boil at a higher temperature: a tall and narrow one or a short and wide one? Explain.

4–18C It is well known that warm air in a cooler environment rises. Now consider a warm mixture of air and gasoline on top of an open gasoline can. Do you think this gas mixture will rise in a cooler environment?

4–19C In the absence of compressed liquid tables, how is the specific volume of a compressed liquid at a given P and T determined?

4–20C A perfectly fitting pot and its lid often stick after cooking, and it becomes very difficult to open the lid when the pot cools down. Explain why this happens and what you would do to open the lid.

4–21 Complete this table for H_2O:

T, °C	P, kPa	υ, m^3/kg	Phase description
140		0.035	
	550		Saturated liquid
125	750		
300		0.140	

4–22E Complete this table for H_2O:

T, °F	P, psia	u, Btu/lbm	Phase description
300		782	
	40		Saturated liquid
500	120		
400	400		

4–23E 💻 Reconsider Prob. 4–22E. Using appropriate software, determine the missing properties of water. Repeat the solution for refrigerant-134a, refrigerant-22, and ammonia.

4–24 Complete this table for H_2O:

T, °C	P, kPa	h, kJ/kg	x	Phase description
	200		0.7	
140		1800		
	950		0.0	
80	500			
	800	3162.2		

4–25E Complete this table for refrigerant-134a:

T, °F	P, psia	h, Btu/lbm	x	Phase description
	80	78		
15			0.6	
10	70			
	180	129.46		
110			1.0	

4–26 Complete this table for refrigerant-134a:

T, °C	P, kPa	u, kJ/kg	Phase description
20		95	
−12			Saturated liquid
	400	300	
8	600		

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*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

4–27 A 1.8-m^3 rigid tank contains steam at 220°C. One-third of the volume is in the liquid phase and the rest is in the vapor form. Determine (*a*) the pressure of the steam, (*b*) the quality of the saturated mixture, and (*c*) the density of the mixture.

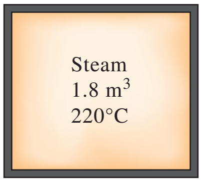


FIGURE P4–27

4–28E One pound-mass of water fills a container whose volume is 2 ft^3. The pressure in the container is 100 psia. Calculate the total internal energy and enthalpy in the container. *Answers:* 661 Btu, 698 Btu

4–29 A piston–cylinder device contains 0.85 kg of refrigerant-134a at –10°C. The piston that is free to move has a mass of 12 kg and a diameter of 25 cm. The local atmospheric pressure is 88 kPa. Now, heat is transferred to refrigerant-134a until the temperature is 15°C. Determine (*a*) the final pressure, (*b*) the change in the volume of the cylinder, and (*c*) the change in the enthalpy of the refrigerant-134a.

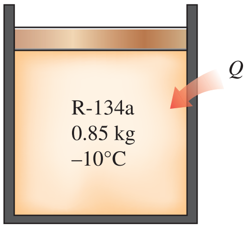


FIGURE P4–29

4–30 10 kg of R-134a fill a 1.115-m^3 rigid container at an initial temperature of –30°C. The container is then heated until the pressure is 200 kPa. Determine the final temperature and the initial pressure. *Answers:* 14.2°C, 84.43 kPa

4–31 What is the specific internal energy of water at 50 kPa and 200°C?

4–32 What is the specific volume of water at 5 MPa and 100°C? What would it be if the incompressible liquid approximation were used? Determine the accuracy of this approximation.

4–33 What is the specific volume of R-134a at 20°C and 700 kPa? What is the internal energy at that state?

4–34 Refrigerant-134a at 200 kPa and 25°C flows through a refrigeration line. Determine its specific volume.

4–35 One kilogram of R-134a fills a 0.14-m^3 weighted piston–cylinder device at a temperature of –26.4°C. The container is now heated until the temperature is 100°C. Determine the final volume of the R-134a. *Answer:* 0.3014 m^3

4–36 One kilogram of water vapor at 200 kPa fills the 1.1989-m^3 left chamber of a partitioned system shown in Fig. P4–36. The right chamber has twice the volume of the left and is initially evacuated. Determine the pressure of the water after the partition has been removed and enough heat has been transferred so that the temperature of the water is 3°C.

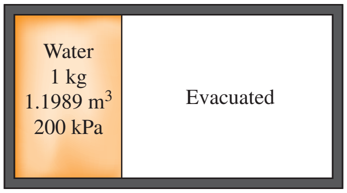


FIGURE P4–36

4–37E The temperature in a pressure cooker during cooking at sea level is measured to be 250°F. Determine the absolute pressure inside the cooker in psia and in atm. Would you modify your answer if the place were at a higher elevation?

FIGURE P4–37E

4–38E How much error would one expect in determining the specific enthalpy by applying the incompressible-liquid approximation to water at 3000 psia and 400°F?

4–39 Water is to be boiled at sea level in a 30-cm-diameter stainless steel pan placed on top of a 3-kW electric burner. If 60 percent of the heat generated by the burner is transferred to the water during boiling, determine the rate of evaporation of water.

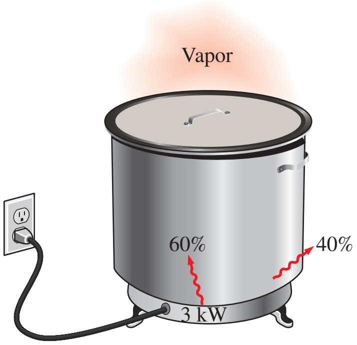


FIGURE P4–39

4–40 Repeat Prob. 4–39 for a location at an elevation of 1500 m where the atmospheric pressure is 84.5 kPa and thus the boiling temperature of water is 95°C.

4–41 10 kg of R-134a at 300 kPa fills a rigid container whose volume is 14 L. Determine the temperature and total enthalpy in the container. The container is now heated until the pressure is

600 kPa. Determine the temperature and total enthalpy when the heating is completed.

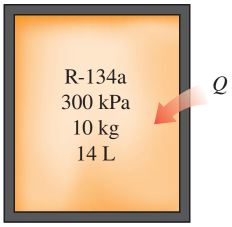


FIGURE P4–41

4–42 100 kg of R-134a at 200 kPa are contained in a piston–cylinder device whose volume is 12.322 m^3. The piston is now moved until the volume is one-half its original size. This is done such that the pressure of the R-134a does not change. Determine the final temperature and the change in the total internal energy of the R-134a.

4–43 Water initially at 200 kPa and 300°C is contained in a piston–cylinder device fitted with stops. The water is allowed to cool at constant pressure until it exists as a saturated vapor and the piston rests on the stops. Then the water continues to cool until the pressure is 100 kPa. On the T-υ diagram, sketch, with respect to the saturation lines, the process curves passing through the initial, intermediate, and final states of the water. Label the T, P, and υ values for end states on the process curves. Find the overall change in internal energy between the initial and final states per unit mass of water.

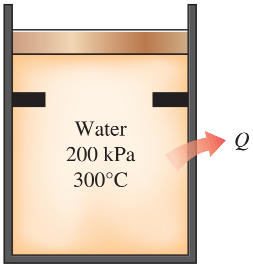


FIGURE P4–43

4–44 Saturated steam coming off the turbine of a steam power plant at 40°C condenses on the outside of a 3-cm-outer-diameter, 35-m-long tube at a rate of 70 kg/h. Determine the rate of heat transfer from the steam to the cooling water flowing through the pipe.

4–45 A person cooks a meal in a 30-cm-diameter pot that is covered with a well-fitting lid and lets the food cool to the room temperature of 20°C. The total mass of the food and the pot is 8 kg. Now the person tries to open the pan by lifting the lid up. Assuming no air has leaked into the pan during cooling, determine if the lid will open or if the pan will move up together with the lid.

4–46 Water is boiled at 1 atm pressure in a 25-cm-internal-diameter stainless steel pan on an electric range. If it is observed that the water level in the pan drops by 10 cm in 45 min, determine the rate of heat transfer to the pan.

4–47 Repeat Prob. 4–46 for a location at 2000-m elevation where the standard atmospheric pressure is 79.5 kPa.

4–48 Water is boiled in a pan covered with a poorly fitting lid at a specified location. Heat is supplied to the pan by a 2-kW resistance heater. The amount of water in the pan is observed to decrease by 1.19 kg in 30 min. If it is estimated that 75 percent of electricity consumed by the heater is transferred to the water as heat, determine the local atmospheric pressure in that location. *Answer:* 85.4 kPa

4–49 A rigid tank with a volume of 1.8 m^3 contains 40 kg of saturated liquid–vapor mixture of water at 90°C. Now the water is slowly heated. Determine the temperature at which the liquid in the tank is completely vaporized. Also, show the process on a T-υ diagram with respect to saturation lines. *Answer:* 256°C

4–50 A piston–cylinder device contains 0.005 m^3 of liquid water and 0.9 m^3 of water vapor in equilibrium at 600 kPa. Heat is transferred at constant pressure until the temperature reaches 200°C.

(*a*) What is the initial temperature of the water?
(*b*) Determine the total mass of the water.
(*c*) Calculate the final volume.
(*d*) Show the process on a P-υ diagram with respect to saturation lines.

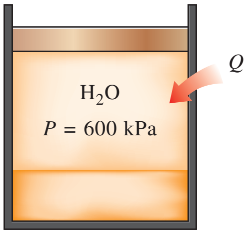


FIGURE P4–50

4–51 Reconsider Prob. 4–50. Using appropriate software, investigate the effect of pressure on the total mass of water in the tank. Let the pressure vary from 0.1 MPa to 1 MPa. Plot the total mass of water against pressure, and discuss the results. Also, show the process in Prob. 4–50 on a P-υ diagram using the property plot feature of the software.

4–52E A 5-ft^3 rigid tank contains a saturated mixture of refrigerant-134a at 50 psia. If the saturated liquid occupies 20 percent of the volume, determine the quality and the total mass of the refrigerant in the tank.

4–53E Superheated water vapor at 180 psia and 500°F is allowed to cool at constant volume until the temperature drops to 250°F. At the final state, determine (*a*) the pressure, (*b*) the quality, and (*c*) the enthalpy. Also, show the process on a T-υ diagram with respect to saturation lines. *Answers:* (*a*) 29.84 psia, (*b*) 0.219, (*c*) 426.0 Btu/lbm

4–54E Reconsider Prob. 4–53E. Using appropriate software, investigate the effect of initial pressure on the quality of water at the final state. Let the pressure vary from 100 psia to 300 psia. Plot the quality against initial pressure, and discuss the results. Also, show the process in Prob. 4–53E on a T-υ diagram using the property plot feature of the software.

4–55 One kilogram of water fills a 150-L rigid container at an initial pressure of 2 MPa. The container is then cooled to 40°C. Determine the initial temperature and the final pressure of the water.

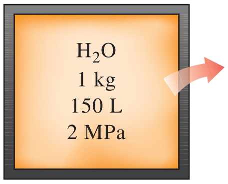


FIGURE P4–55

4–56 10 kg of R-134a fill a 0.7-m^3 weighted piston–cylinder device at a pressure of 200 kPa. The container is now heated until the temperature is 30°C. Determine the initial temperature and final volume of the R-134a.

4–57 A piston–cylinder device contains 0.6 kg of steam at 300°C and 0.5 MPa. Steam is cooled at constant pressure until one-half of the mass condenses.

(*a*) Show the process on a T-v diagram.
(*b*) Find the final temperature.
(*c*) Determine the volume change.

4–58 A piston–cylinder device initially contains 1.4 kg saturated liquid water at 200°C. Now heat is transferred to the water until the volume quadruples and the cylinder contains saturated vapor only. Determine (*a*) the volume of the cylinder, (*b*) the final temperature and pressure, and (*c*) the internal energy change of the water.

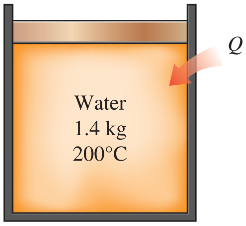


FIGURE P4–58

4–59 Water is being heated in a vertical piston–cylinder device. The piston has a mass of 40 kg and a cross-sectional area of 150 cm^2. If the local atmospheric pressure is 100 kPa, determine the temperature at which the water starts boiling.

4–60 A rigid tank initially contains 1.4 kg saturated liquid water at 200°C. At this state, 25 percent of the volume is occupied by water and the rest by air. Now heat is supplied to the water until the tank contains saturated vapor only. Determine (*a*) the volume of the tank, (*b*) the final temperature and pressure, and (*c*) the internal energy change of the water.

4–61 A piston–cylinder device initially contains 50 L of liquid water at 40°C and 200 kPa. Heat is transferred to the water at constant pressure until all liquid is vaporized.

(*a*) What is the mass of the water?
(*b*) What is the final temperature?
(*c*) Determine the total enthalpy change.
(*d*) Show the process on a T-v diagram with respect to saturation lines.

Answers: (*a*) 49.61 kg, (*b*) 120.21°C, (*c*) 125,950 kJ

4–62 The spring-loaded piston–cylinder device shown in Fig. P4–62 is filled with 0.5 kg of water vapor that is initially at 4 MPa and 400°C. Initially, the spring exerts no force against the piston. The spring constant in the spring force relation $F = kx$ is $k = 0.9$ kN/cm and the piston diameter is $D = 20$ cm. The water now undergoes a process until its volume is one-half of the original volume. Calculate the final temperature and the specific enthalpy of the water. *Answers:* 220°C, 1721 kJ/kg

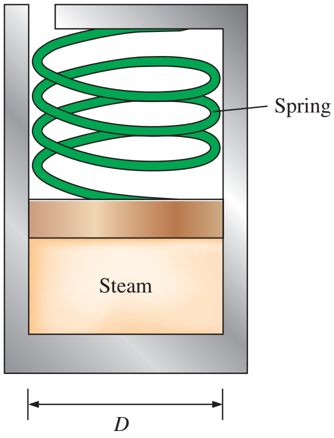


FIGURE P4–62

4–63 A piston–cylinder device initially contains steam at 3.5 MPa, superheated by 5°C. Now, steam loses heat to the surroundings and the piston moves down, hitting a set of stops, at which point the cylinder contains saturated liquid water. The cooling continues until the cylinder contains water at 200°C. Determine (*a*) the initial temperature, (*b*) the enthalpy change per unit mass of the steam by the time the piston first hits the stops, and (*c*) the final pressure and the quality (if mixture).

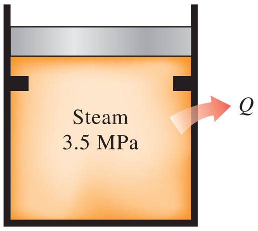


FIGURE P4–63

Ideal Gas

4–64C Under what conditions is the ideal-gas assumption suitable for real gases?

4–65C What is the difference between mass and molar mass? How are these two related?

4–66C Propane and methane are commonly used for heating in winter, and the leakage of these fuels, even for short periods, poses a fire danger for homes. Which gas leakage do you think poses a greater risk for fire? Explain.

4–67E What is the specific volume of oxygen at 25 psia and 80°F?

4–68 A 100-L container is filled with 1 kg of air at a temperature of 27°C. What is the pressure in the container?

4–69E A mass of 1 lbm of argon is maintained at 200 psia and 100°F in a tank. What is the volume of the tank?

4–70 A 400-L rigid tank contains 5 kg of air at 25°C. Determine the reading on the pressure gage if the atmospheric pressure is 97 kPa.

4–71 The pressure gage on a 2.5-m^3 oxygen tank reads 500 kPa. Determine the amount of oxygen in the tank if the temperature is 28°C and the atmospheric pressure is 97 kPa.

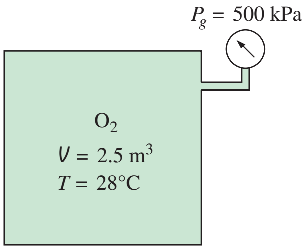


FIGURE P4–71

4–72 A spherical balloon with a diameter of 9 m is filled with helium at 27°C and 200 kPa. Determine the mole number and the mass of the helium in the balloon. *Answers:* 30.6 kmol, 123 kg

4–73 Reconsider Prob. 4–72. Using appropriate software, investigate the effect of the balloon diameter on the mass of helium contained in the balloon for the pressures of (*a*) 100 kPa and (*b*) 200 kPa. Let the diameter vary from 5 m to 15 m. Plot the mass of helium against the diameter for both cases.

4–74 A 1-m^3 tank containing air at 10°C and 350 kPa is connected through a valve to another tank containing 3 kg of air at 35°C and 150 kPa. Now the valve is opened, and the entire system is allowed to reach thermal equilibrium with the surroundings, which are at 20°C. Determine the volume of the second tank and the final equilibrium pressure of air. *Answers:* 1.77 m^3, 222 kPa

4–75 A mass of 10 g of oxygen fill a weighted piston–cylinder device at 20 kPa and 100°C. The device is now cooled until the temperature is 0°C. Determine the change of the volume of the device during this cooling.

4–76 A mass of 0.1 kg of helium fills a 0.2 m^3 rigid vessel at 350 kPa. The vessel is heated until the pressure is 700 kPa. Calculate the temperature change of helium (in °C and K) as a result of this heating.

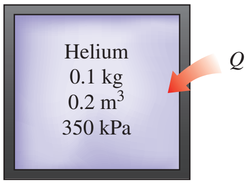


FIGURE P4–76

4–77 A rigid tank whose volume is unknown is divided into two parts by a partition. One side of the tank contains an ideal gas at 927°C. The other side is evacuated and has a volume twice the size of the part containing the gas. The partition is now removed and the gas expands to fill the entire tank. Heat is now transferred to the gas until the pressure equals the initial pressure. Determine the final temperature of the gas. *Answer:* 3327°C

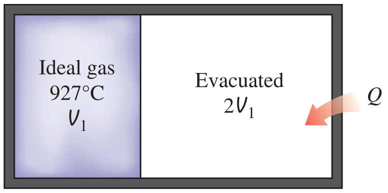


FIGURE P4–77

4–78E A rigid tank contains 20 lbm of air at 20 psia and 70°F. More air is added to the tank until the pressure and temperature rise to 25 psia and 90°F, respectively. Determine the amount of air added to the tank. *Answer:* 4.09 lbm

4–79E In an informative article in a magazine it is stated that tires lose roughly 1 psi of pressure for every 10°F drop in outside temperature. Investigate whether this is a valid statement.

Compressibility Factor

4–80C What is the physical significance of the compressibility factor Z?

4–81 Determine the specific volume of refrigerant-134a vapor at 0.9 MPa and 70°C based on (*a*) the ideal-gas equation, (*b*) the generalized compressibility chart, and (*c*) data from tables. Also, determine the error involved in the first two cases.

4–82E Refrigerant-134a at 400 psia has a specific volume of 0.1384 ft^3/lbm. Determine the temperature of the refrigerant based on (*a*) the ideal-gas equation, (*b*) the generalized compressibility chart, and (*c*) the refrigerant tables.

4–83 Determine the specific volume of superheated water vapor at 15 MPa and 350°C using (*a*) the ideal-gas equation, (*b*) the generalized compressibility chart, and (*c*) the steam tables. Also determine the error involved in the first two cases. *Answers:* (*a*) 0.01917 m^3/kg, 67.0 percent, (*b*) 0.01246 m^3/kg, 8.5 percent, (*c*) 0.01148 m^3/kg

4–84 Reconsider Prob. 4–83. Solve the problem using appropriate software. Compare the specific volume of water for the three cases at 15 MPa over the temperature range of 350 to 600°C in 25°C intervals. Plot the percent error involved in the ideal-gas approximation against temperature, and discuss the results.

4–85 Determine the specific volume of superheated water vapor at 3.5 MPa and 450°C based on (*a*) the ideal-gas equation, (*b*) the generalized compressibility chart, and (*c*) the steam tables. Determine the error involved in the first two cases.

4–86 Determine the specific volume of nitrogen gas at 10 MPa and 150 K based on (*a*) the idea-gas equation and (*b*) the generalized compressibility chart. Compare these results with the experimental value of 0.002388 m^3/kg, and determine the error involved in each case. *Answers:* (*a*) 0.004452 m^3/kg, 86.4 percent, (*b*) 0.002404 m^3/kg, 0.7 percent

4–87 Ethylene is heated at constant pressure from 5 MPa and 20°C to 200°C. Using the compressibility chart, determine the change in the ethylene's specific volume as a result of this heating. *Answer:* 0.0172 m^3/kg

4–88 Carbon dioxide gas enters a pipe at 3 MPa and 500 K at a rate of 2 kg/s. CO_2 is cooled at constant pressure as it flows in the pipe, and the temperature of the CO_2 drops to 450 K at the exit. Determine the volume flow rate and the density of carbon dioxide at the inlet and the volume flow rate at the exit of the pipe using (*a*) the ideal-gas equation and (*b*) the generalized compressibility chart. Also, determine (*c*) the error involved in the first case.

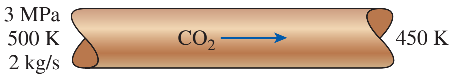


FIGURE P4–88

4–89E Ethane in a rigid vessel is to be heated from 50 psia and 100°F until its temperature is 540°F. What is the final pressure of the ethane as predicted by the compressibility chart?

4–90 A 0.016773-m^3 tank contains 1 kg of refrigerant-134a at 110°C. Determine the pressure of the refrigerant using (*a*) the ideal-gas equation, (*b*) the generalized compressibility chart, and (*c*) the refrigerant tables. *Answers:* (*a*) 1.861 MPa, (*b*) 1.583 MPa, (*c*) 1.6 MPa

4–91E Saturated water vapor at 400°F is heated at constant pressure until its volume has doubled. Determine the final temperature using the ideal-gas equation of state, the compressibility charts, and the steam tables.

4–92 What is the percentage of error involved in treating carbon dioxide at 5 MPa and 25°C as an ideal gas? *Answer:* 45 percent

Review Problems

4–93 Complete the blank cells in the following table of properties of steam. In the last column, describe the condition of steam as compressed liquid, saturated mixture, superheated vapor, or insufficient information, and, if applicable, give the quality.

P, kPa	T, °C	v, m^3/kg	u, kJ/kg	Phase description and quality (if applicable)
200	30			
270.3	130			
	400	1.5493		
300		0.500		
500			3084	

4–94 Complete the blank cells in the following table of properties of refrigerant-134a. In the last column, describe the condition of refrigerant-134a as compressed liquid, saturated mixture, superheated vapor, or insufficient information, and, if applicable, give the quality.

P, kPa	T, °C	v, m^3/kg	u, kJ/kg	Phase description and quality (if applicable)
320	−12			
1000	39.37			
	40	1.17794		
180		0.0700		
200			249	

4–95 A rigid tank contains an ideal gas at 300 kPa and 600 K. Now half of the gas is withdrawn from the tank and the gas is found at 100 kPa at the end of the process. Determine (*a*) the final temperature of the gas and (*b*) the final pressure if no mass was withdrawn from the tank and the same final temperature was reached at the end of the process.

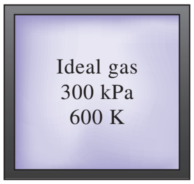


FIGURE P4–95

4–96 Carbon dioxide gas at 3 MPa and 500 K flows steadily in a pipe at a rate of 0.4 kmol/s. Determine (*a*) the volume and mass flow rates and the density of carbon dioxide at this state. If CO_2 is cooled at constant pressure as it flows in the pipe so that the temperature of CO_2 drops to 450 K at the exit of the pipe, determine (*b*) the volume flow rate at the exit of the pipe.

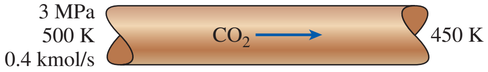


FIGURE P4–96

4–97 The gage pressure of an automobile tire is measured to be 200 kPa before a trip and 220 kPa after the trip at a location where the atmospheric pressure is 90 kPa. Assuming the volume of the tire remains constant at 0.035 m^3, determine the percent increase in the absolute temperature of the air in the tire.

4–98 A tank contains argon at 600°C and 200 kPa gage. The argon is cooled in a process by heat transfer to the surroundings such that the argon reaches a final equilibrium state at 300°C. Determine the final gage pressure of the argon. Assume atmospheric pressure is 100 kPa.

4–99 The combustion in a gasoline engine may be approximated by a constant-volume heat addition process. The cylinder contains the air–fuel mixture before combustion and the combustion gases after it, and both may be approximated as air, an ideal gas. In a gasoline engine, the cylinder conditions are 1.2 MPa and

450°C before the combustion and 1900°C after it. Determine the pressure at the end of the combustion process. *Answer:* 3.61 MPa

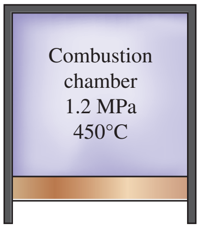


FIGURE P4–99

4–100 A rigid tank contains nitrogen gas at 227°C and 100 kPa gage. The gas is heated until the gage pressure reads 250 kPa. If the atmospheric pressure is 100 kPa, determine the final temperature of the gas, in °C.

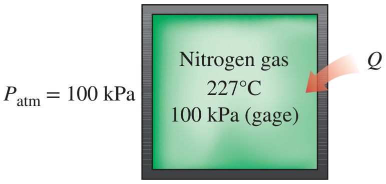


FIGURE P4–100

4–101 One kilogram of R-134a fills a 0.090-m^3 rigid container at an initial temperature of –40°C. The container is then heated until the pressure is 280 kPa. Determine the initial pressure and final temperature. *Answers:* 51.25 kPa, 50°C

4–102 A rigid tank with a volume of 0.117 m^3 contains 1 kg of refrigerant-134a vapor at 240 kPa. The refrigerant is now allowed to cool. Determine the pressure when the refrigerant first starts condensing. Also, show the process on a P-v diagram with respect to saturation lines.

4–103 Water initially at 300 kPa and 250°C is contained in a constant-volume tank. The water is allowed to cool until its pressure is 150 kPa. On the P-v and T-v diagrams, sketch, with respect to the saturation lines, the process curve passing through both the initial and final states of the water. Label the end states on the process curve. Also, on both the P-v and T-v diagrams, sketch the isotherms passing through both states and show their values, in °C, on the isotherms.

4–104 A 9-m^3 tank contains nitrogen at 17°C and 600 kPa. Some nitrogen is allowed to escape until the pressure in the tank drops to 400 kPa. If the temperature at this point is 15°C, determine the amount of nitrogen that has escaped. *Answer:* 20.6 kg

4–105 A 10-kg mass of superheated refrigerant-134a at 1.2 MPa and 70°C is cooled at constant pressure until it exists as a compressed liquid at 20°C.

(*a*) Show the process on a T-v diagram with respect to saturation lines.

(*b*) Determine the change in volume.

(*c*) Find the change in total internal energy.

Answers: (*b*) –0.187 m^3, (*c*) –1984 kJ

4–106 A 4-L rigid tank contains 2 kg of saturated liquid–vapor mixture of water at 50°C. The water is now slowly heated until it exists in a single phase. At the final state, will the water be in the liquid phase or the vapor phase? What would your answer be if the volume of the tank were 400 L instead of 4 L?

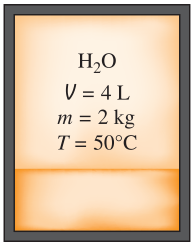


FIGURE P4–106

4–107 A piston–cylinder device initially contains 0.2 kg of steam at 200 kPa and 300°C. Now, the steam is cooled at constant pressure until it is at 150°C. Determine the volume change of the cylinder during this process using the compressibility factor, and compare the result to the actual value.

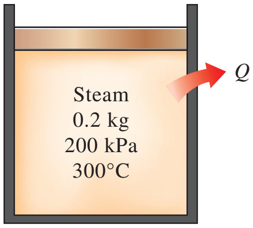


FIGURE P4–107

4–108 A tank whose volume is unknown is divided into two parts by a partition. One side of the tank contains 0.03 m^3 of refrigerant-134a that is a saturated liquid at 0.9 MPa, while the other side is evacuated. The partition is now removed, and the refrigerant fills the entire tank. If the final state of the refrigerant is 20°C and 280 kPa, determine the volume of the tank.

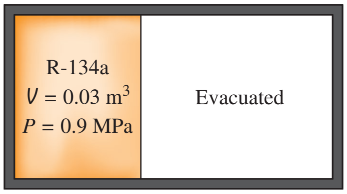


FIGURE P4–108

4–109 Reconsider Prob. 4–108. Using appropriate software, investigate the effect of the initial pressure of refrigerant-134a on the volume of the tank. Let the initial pressure vary from 0.5 to 1.5 MPa. Plot the volume of the tank versus the initial pressure, and discuss the results.

4–110 A tank contains helium at 37°C and 140 kPa gage. The helium is heated in a process by heat transfer from the surroundings such that the helium reaches a final equilibrium state at 200°C. Determine the final gage pressure of the helium. Assume atmospheric pressure is 100 kPa.

4–111 On the property diagrams indicated below, sketch (not to scale) with respect to the saturated liquid and saturated vapor lines and label the following processes and states for refrigerant-134a. Use arrows to indicate the direction of the process, and label the initial and final states:

(*a*) On the P-v diagram, sketch the constant-temperature process through the state $P = 280$ kPa, $v = 0.06$ m^3/kg as pressure changes from $P_1 = 400$ kPa to $P_2 = 200$ kPa. Place the value of the temperature on the process curve on the P-v diagram.

(*b*) On the T-v diagram, sketch the constant-specific-volume process through the state $T = 20°\text{C}$, $v = 0.02$ m^3/kg from $P_1 = 1200$ kPa to $P_2 = 300$ kPa. For this data set, place the temperature values at states 1 and 2 on its axis. Place the value of the specific volume on its axis.

4–112 Ethane at 10 MPa and 100°C is heated at constant pressure until its volume has increased by 60 percent. Determine the final temperature using (*a*) the ideal-gas equation of state and (*b*) the compressibility factor. Which of these two results is the more accurate?

4–113 Steam at 400°C has a specific volume of 0.02 m^3/kg. Determine the pressure of the steam based on (*a*) the ideal-gas equation, (*b*) the generalized compressibility chart, and (*c*) the steam tables. *Answers:* (*a*) 15,529 kPa, (*b*) 12,574 kPa, (*c*) 12,515 kPa

Design and Essay Problems

4–114 A solid normally absorbs heat as it melts, but there is a known exception at temperatures close to absolute zero. Find out which solid it is, and give a physical explanation for it.

4–115 In an article on tire maintenance, it is stated that tires lose air over time, and pressure losses as high as 90 kPa (13 psi) per year are measured. The article recommends checking tire pressure at least once a month to avoid low tire pressure that hurts fuel efficiency and causes uneven thread wear on tires. Taking the beginning tire pressure to be 220 kPa (gage) and the atmospheric pressure to be 100 kPa, determine the fraction of air that can be lost from a tire per year.

4–116 It is well known that water freezes at 0°C at atmospheric pressure. The mixture of liquid water and ice at 0°C is said to be at stable equilibrium since it cannot undergo any changes when it is isolated from its surroundings. However, when water is free of impurities and the inner surfaces of the container are smooth, the temperature of water can be lowered to –2°C or even lower without any formation of ice at atmospheric pressure. But at that state even a small disturbance can initiate the formation of ice abruptly, and the water temperature stabilizes at 0°C following this sudden change. The water at –2°C is said to be in a *metastable state*. Write an essay on metastable states, and discuss how they differ from stable equilibrium states.

CHAPTER 5

ENERGY ANALYSIS OF CLOSED SYSTEMS

In Chap. 3, we considered various forms of energy and energy transfer, and we developed a general relation for the conservation of energy principle or energy balance. Then in Chap. 4, we learned how to determine the thermodynamics properties of substances. In this chapter, we apply the energy balance relation to systems that do not involve any mass flow across their boundaries—that is, closed systems.

We start this chapter with a discussion of the *moving boundary work* or *P d*V *work* commonly encountered in reciprocating devices such as automotive engines and compressors. We continue by applying the *general energy balance* relation, which is simply expressed as $E_{in} - E_{out} = \Delta E_{system}$, to systems that involve pure substance. Then we define *specific heats,* obtain relations for the internal energy and enthalpy of *ideal gases* in terms of specific heats and temperature changes, and perform energy balances on various systems that involve ideal gases. We repeat this for systems that involve solids and liquids, which are approximated as *incompressible substances.*

OBJECTIVES

The objectives of this chapter are to:

- Examine the moving boundary work or $P\,dV$ work commonly encountered in reciprocating devices such as automotive engines and compressors.
- Identify the first law of thermodynamics as simply a statement of the conservation of energy principle for closed (fixed-mass) systems.
- Develop the general energy balance applied to closed systems.
- Define the specific heat at constant volume and the specific heat at constant pressure.
- Relate the specific heats to the calculation of the changes in internal energy and enthalpy of ideal gases.
- Describe incompressible substances and determine the changes in their internal energy and enthalpy.
- Solve energy balance problems for closed (fixed-mass) systems that involve heat and work interactions for general pure substances, ideal gases, and incompressible substances.

5–1 ▪ MOVING BOUNDARY WORK

One form of mechanical work often encountered in practice is associated with the expansion or compression of a gas in a piston–cylinder device. During this process, part of the boundary (the inner face of the piston) moves back and forth. Therefore, the expansion and compression work is often called **moving boundary work**, or simply **boundary work** (Fig. 5–1). Some call it the $P\,dV$ work for reasons explained later. Moving boundary work is the primary form of work involved in *automobile engines.* During their expansion, the combustion gases force the piston to move, which in turn forces the crankshaft to rotate.

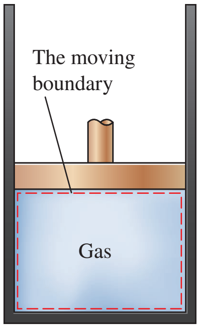


FIGURE 5–1
The work associated with a moving boundary is called *boundary work.*

The moving boundary work associated with real engines or compressors cannot be determined exactly from a thermodynamic analysis alone because the piston usually moves at very high speeds, making it difficult for the gas inside to maintain equilibrium. Then the states through which the system passes during the process cannot be specified, and no process path can be drawn. Work, being a path function, cannot be determined analytically without a knowledge of the path. Therefore, the boundary work in real engines or compressors is determined by direct measurements.

In this section, we analyze the moving boundary work for a *quasi-equilibrium process,* a process during which the system remains nearly in equilibrium at all times. A quasi-equilibrium process, also called a *quasi-static process,* is closely approximated by real engines, especially when the piston moves at low velocities. Under identical conditions, the work output of the engines is found to be a maximum, and the work input to the compressors to be a minimum, when quasi-equilibrium processes are used in place of nonquasi-equilibrium processes. In the following example, the work associated with a moving boundary is evaluated for a quasi-equilibrium process.

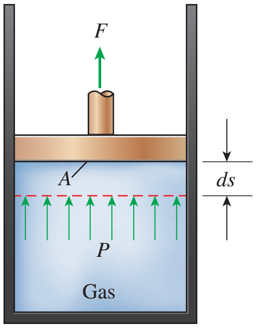


FIGURE 5–2
A gas does a differential amount of work δW_b as it forces the piston to move by a differential amount ds.

Consider the gas enclosed in the piston–cylinder device shown in Fig. 5–2. The initial pressure of the gas is P, the total volume is V, and the cross-sectional area of the piston is A. If the piston is allowed to move a distance ds in a quasi-equilibrium manner, the differential work done during this process is

$$\delta W_b = F\,ds = PA\,ds = P\,dV \tag{5–1}$$

That is, the boundary work in the differential form is equal to the product of the absolute pressure P and the differential change in the volume dV of the system. This expression also explains why the moving boundary work is sometimes called the $P\,dV$ work.

Note in Eq. 5–1 that P is the absolute pressure, which is always positive. However, the volume change dV is positive during an expansion process (volume increasing) and negative during a compression process (volume decreasing). Thus, the boundary work is positive during an expansion process and negative during a compression process. Therefore, Eq. 5–1 can be viewed as an expression for boundary work output, $W_{b,\text{out}}$. A negative result indicates boundary work input (compression).

The total boundary work done during the entire process as the piston moves is obtained by adding all the differential works from the initial state to the final state:

$$W_b = \int_1^2 P\,dV \qquad \text{(kJ)} \tag{5–2}$$

This integral can be evaluated only if we know the functional relationship between P and V during the process. That is, $P = f(V)$ should be available. Note that $P = f(V)$ is simply the equation of the process path on a P-V diagram.

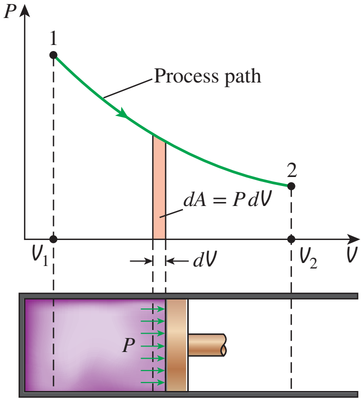


FIGURE 5–3
The area under the process curve on a P-V diagram represents the boundary work.

The quasi-equilibrium expansion process described is shown on a P-V diagram in Fig. 5–3. On this diagram, the differential area dA is equal to $P\,dV$, which is the differential work. The total area A under the process curve 1–2 is obtained by adding these differential areas:

$$\text{Area} = A = \int_1^2 dA = \int_1^2 P\,dV \tag{5–3}$$

A comparison of this equation with Eq. 5–2 reveals that *the area under the process curve on a P-V diagram is equal, in magnitude, to the work done during a quasi-equilibrium expansion or compression process of a closed system.* (On the P-v diagram, it represents the boundary work done per unit mass.)

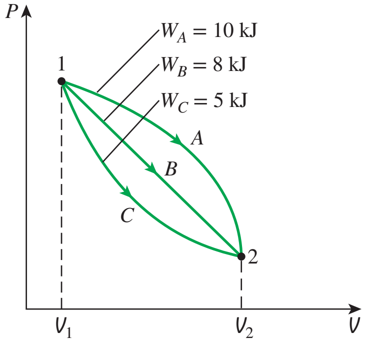


FIGURE 5–4
The boundary work done during a process depends on the path followed as well as the end states.

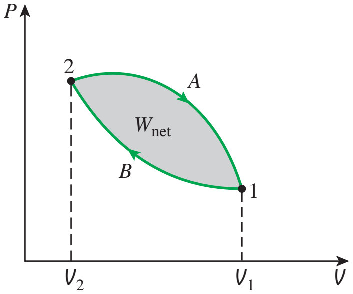


FIGURE 5–5
The net work done during a cycle is the difference between the work done by the system and the work done on the system.

A gas can follow several different paths as it expands from state 1 to state 2. In general, each path will have a different area underneath it, and since this area represents the magnitude of the work, the work done will be different for each process (Fig. 5–4). This is expected, since work is a path function (i.e., it depends on the path followed as well as the end states). If work were not a path function, no cyclic devices (car engines, power plants) could operate as work-producing devices. The work produced by these devices during one part of the cycle would have to be consumed during another part, and there would be no net work output. The cycle shown in Fig. 5–5 produces a net work output because the work done by the system during the expansion process (area under path A) is greater than the work done on the system during the compression part of the cycle (area under path B), and the difference between these two is the net work done during the cycle (the colored area).

If the relationship between P and V during an expansion or a compression process is given in terms of experimental data instead of in a functional form, obviously we cannot perform the integration analytically. We can, however, plot the P-V diagram of the process using these data points and calculate the area underneath graphically to determine the work done.

Strictly speaking, the pressure P in Eq. 5–2 is the pressure at the inner surface of the piston. It becomes equal to the pressure of the gas in the cylinder only if the process is quasi-equilibrium and thus all gas in the cylinder is at the same pressure at any given time. Equation 5–2 can also be used for nonquasi-equilibrium processes provided that the pressure *at the inner face of the piston* is used for P. (Besides, we cannot speak of the pressure of a *system* during a nonquasi-equilibrium process since properties are defined for equilibrium states only.) Therefore, we can generalize the boundary work relation by expressing it as

$$W_b = \int_1^2 P_i\,dV \tag{5–4}$$

where P_i is the pressure at the inner face of the piston.

Note that work is a mechanism for energy interaction between a system and its surroundings, and W_b represents the amount of energy transferred from the system during an expansion process (or to the system during a compression process). Therefore, it has to appear somewhere else, and we must be able to account for it since energy is conserved. In a car engine, for example, the boundary work done by the expanding hot gases is used to overcome friction between the piston and the cylinder, to push atmospheric air out of the way, and to rotate the crankshaft. Therefore,

$$W_b = W_{\text{friction}} + W_{\text{atm}} + W_{\text{crank}} = \int_1^2 (F_{\text{friction}} + P_{\text{atm}}A + F_{\text{crank}})\,dx \tag{5–5}$$

Of course the work used to overcome friction appears as frictional heat and the energy transmitted through the crankshaft is transmitted to other components (such as the wheels) to perform certain functions. But note that the energy transferred by the system as work must equal the energy received by the crankshaft, the atmosphere, and the energy used to overcome friction. The use of the boundary work relation is not limited to the quasi-equilibrium processes of gases only. It can also be used for solids and liquids.

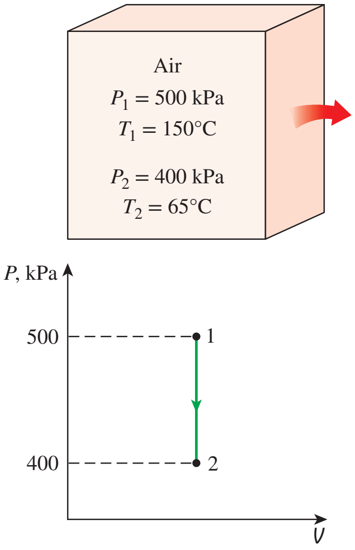


FIGURE 5–6
Schematic and P-V diagram for Example 5–1.

EXAMPLE 5–1 Boundary Work for a Constant-Volume Process

A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.

SOLUTION Air in a rigid tank is cooled, and both the pressure and temperature drop. The boundary work done is to be determined.
Analysis A sketch of the system and the P-V diagram of the process are shown in Fig. 5–6. The boundary work can be determined from Eq. 5–2 to be

$$W_b = \int_1^2 P\, d\overset{\nearrow 0}{V} = \mathbf{0}$$

Discussion This is expected since a rigid tank has a constant volume and $dV = 0$ in this equation. Therefore, there is no boundary work done during this process. That is, the boundary work done during a constant-volume process is always zero. This is also evident from the P-V diagram of the process (the area under the process curve is zero).

EXAMPLE 5–2 Boundary Work for a Constant-Pressure Process

A frictionless piston–cylinder device contains 10 lbm of steam at 60 psia and 320°F. Heat is now transferred to the steam until the temperature reaches 400°F. If the piston is not attached to a shaft and its mass is constant, determine the work done by the steam during this process.

SOLUTION Steam in a piston–cylinder device is heated and the temperature rises at constant pressure. The boundary work done is to be determined.
Assumptions The expansion process is quasi-equilibrium.
Analysis A sketch of the system and the P-v diagram of the process are shown in Fig. 5–7. Even though it is not explicitly stated, the pressure of the steam within the cylinder remains constant during this process since both the atmospheric pressure and the weight of the piston remain constant. Therefore, this is a constant-pressure process, and, from Eq. 5–2

$$W_b = \int_1^2 P\, dV = P_0 \int_1^2 dV = P_0(V_2 - V_1) \tag{5–6}$$

or

$$W_b = mP_0(v_2 - v_1)$$

since $V = mv$. From the superheated vapor table (Table A–6E), the specific volumes are determined to be $v_1 = 7.4863$ ft^3/lbm at state 1 (60 psia, 320°F) and $v_2 = 8.3548$ ft^3/lbm at state 2 (60 psia, 400°F). Substituting these values yields

$$\begin{aligned} W_b &= (10 \text{ lbm})(60 \text{ psia})[(8.3548 - 7.4863)\text{ft}^3/\text{lbm}]\left(\frac{1 \text{ Btu}}{5.404 \text{ psia}\cdot\text{ft}^3}\right) \\ &= \mathbf{96.4\ Btu} \end{aligned}$$

Discussion The positive sign indicates that the work is done by the system. That is, the steam used 96.4 Btu of its energy to do this work. The magnitude of this work could also be determined by calculating the area under the process curve on the P-V diagram, which is simply $P_0\, \Delta V$ for this case.

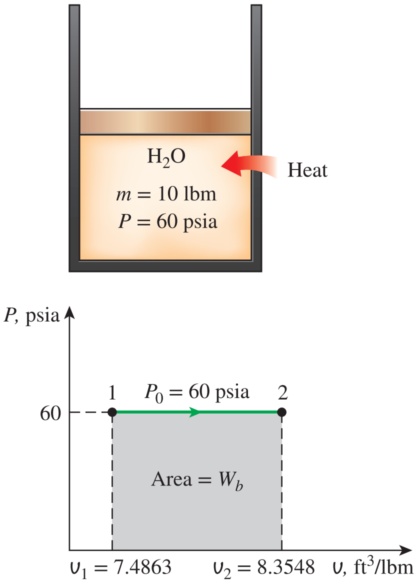


FIGURE 5–7
Schematic and P-v diagram for Example 5–2.

EXAMPLE 5–3 Isothermal Compression of an Ideal Gas

A piston–cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.

SOLUTION Air in a piston–cylinder device is compressed isothermally. The boundary work done is to be determined.

Assumptions **1** The compression process is quasi-equilibrium. **2** At specified conditions, air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values.

Analysis A sketch of the system and the P-V diagram of the process are shown in Fig. 5–8. For an ideal gas at constant temperature T_0,

$$PV = mRT_0 = C \quad \text{or} \quad P = \frac{C}{V}$$

where C is a constant. Substituting this into Eq. 5–2, we have

$$W_b = \int_1^2 P\,dV = \int_1^2 \frac{C}{V}\,dV = C\int_1^2 \frac{dV}{V} = C\ln\frac{V_2}{V_1} = P_1V_1\ln\frac{V_2}{V_1} \tag{5–7}$$

In Eq. 5–7, P_1V_1 can be replaced by P_2V_2 or mRT_0. Also, V_2/V_1 can be replaced by P_1/P_2 for this case since $P_1V_1 = P_2V_2$.

Substituting the numerical values into Eq. 5–7 yields

$$\begin{aligned} W_b &= (100\text{ kPa})(0.4\text{ m}^3)\left(\ln\frac{0.1}{0.4}\right)\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right) \\ &= \mathbf{-55.5\text{ kJ}} \end{aligned}$$

Discussion The negative sign indicates that this work is done on the system (a work input), which is always the case for compression processes.

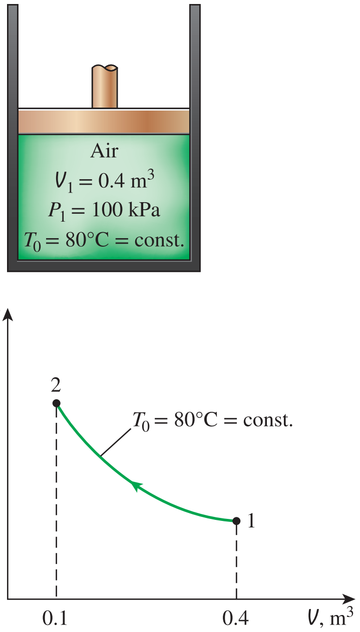


FIGURE 5–8
Schematic and P-V diagram for Example 5–3.

Polytropic Process

During actual expansion and compression processes of gases, pressure and volume are often related by $PV^n = C$, where n and C are constants. A process of this kind is called a **polytropic process** (Fig. 5–9). Next we develop a general expression for the work done during a polytropic process. The pressure for a polytropic process can be expressed as

$$P = CV^{-n} \tag{5–8}$$

Substituting this relation into Eq. 5–2, we obtain

$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-n}\,dV = C\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n} \tag{5–9}$$

since $C = P_1V_1^n = P_2V_2^n$. For an ideal gas ($PV = mRT$), this equation can also be written as

$$W_b = \frac{mR(T_2 - T_1)}{1-n} \qquad n \neq 1 \qquad (\text{kJ}) \tag{5–10}$$

For the special case of $n = 1$ the boundary work becomes

$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-1}\,dV = PV\ln\left(\frac{V_2}{V_1}\right)$$

For an ideal gas this result is equivalent to the isothermal process discussed in the previous example.

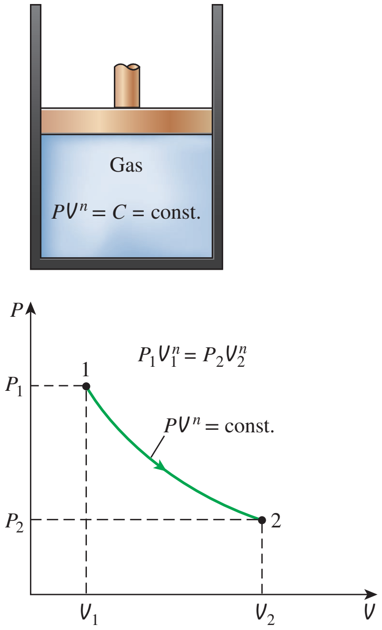


FIGURE 5–9
Schematic and P-V diagram for a polytropic process.

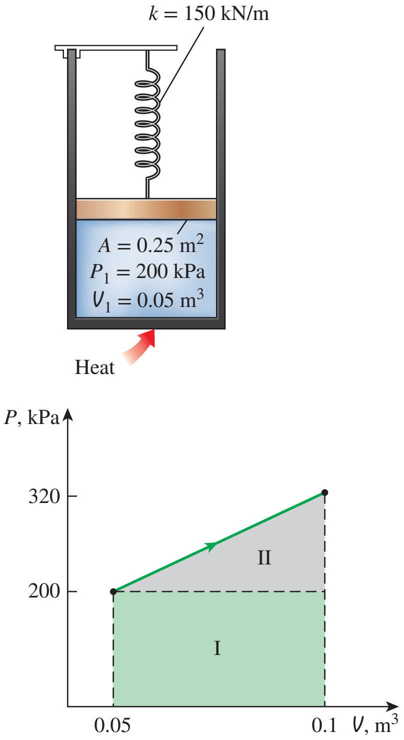


FIGURE 5–10
Schematic and P-V diagram for Example 5–4.

EXAMPLE 5–4 Expansion of a Gas Against a Spring

A piston–cylinder device contains 0.05 m^3 of a gas initially at 200 kPa. At this state, a linear spring that has a spring constant of 150 kN/m is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.25 m^2, determine (*a*) the final pressure inside the cylinder, (*b*) the total work done by the gas, and (*c*) the fraction of this work done against the spring to compress it.

SOLUTION A gas in a piston–cylinder device equipped with a linear spring expands as a result of heating. The final gas pressure, the total work done, and the fraction of the work done to compress the spring are to be determined.

Assumptions **1** The expansion process is quasi-equilibrium. **2** The spring is linear in the range of interest.

Analysis A sketch of the system and the P-V diagram of the process are shown in Fig. 5–10.

(*a*) The enclosed volume at the final state is

$$V_2 = 2V_1 = (2)(0.05\ \text{m}^3) = 0.1\ \text{m}^3$$

Then the displacement of the piston (and of the spring) becomes

$$x = \frac{\Delta V}{A} = \frac{(0.1 - 0.05)\text{m}^3}{0.25\ \text{m}^3} = 0.2\ \text{m}$$

The force applied by the linear spring at the final state is

$$F = kx = (150\ \text{kN/m})(0.2\ \text{m}) = 30\ \text{kN}$$

The additional pressure applied by the spring on the gas at this state is

$$P = \frac{F}{A} = \frac{30\ \text{kN}}{0.25\ \text{m}^2} = 120\ \text{kPa}$$

Without the spring, the pressure of the gas would remain constant at 200 kPa while the piston is rising. But under the effect of the spring, the pressure rises linearly from 200 kPa to

$$200 + 120 = \mathbf{320\ kPa}$$

at the final state.

(*b*) An easy way of finding the work done is to plot the process on a P-V diagram and find the area under the process curve. From Fig. 5–10 the area under the process curve (a trapezoid) is determined to be

$$W = \text{area} = \frac{(200 + 320)\ \text{kPa}}{2}[(0.1 - 0.05)\ \text{m}^3]\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right) = \mathbf{13\ kJ}$$

Note that the work is done by the system.

(*c*) The work represented by the rectangular area (region I) is done against the piston and the atmosphere, and the work represented by the triangular area (region II) is done against the spring. Thus,

$$W_{\text{spring}} = \tfrac{1}{2}[(320 - 200)\ \text{kPa}](0.05\ \text{m}^3)\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right) = \mathbf{3\ kJ}$$

Discussion This result could also be obtained from

$$W_{\text{spring}} = \tfrac{1}{2}k(x_2^2 - x_1^2) = \tfrac{1}{2}(150\ \text{kN/m})[(0.2\ \text{m})^2 - 0^2]\left(\frac{1\ \text{kJ}}{1\ \text{kN}\cdot\text{m}}\right) = 3\ \text{kJ}$$

5–2 ▪ ENERGY BALANCE FOR CLOSED SYSTEMS

Energy balance for any system undergoing any kind of process was expressed as (see Chap. 3)

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\\text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\\text{potential, etc., energies}}} \qquad (\text{kJ}) \tag{5–11}$$

or, in the **rate form**, as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\\text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal,}\\\text{kinetic, potential, etc., energies}}} \qquad (\text{kW}) \tag{5–12}$$

For constant rates, the total quantities during a time interval Δt are related to the quantities per unit time as

$$Q = \dot{Q}\,\Delta t, \qquad W = \dot{W}\,\Delta t, \qquad \text{and} \qquad \Delta E = (dE/dt)\Delta t \qquad (\text{kJ}) \tag{5–13}$$

The energy balance can be expressed on a **per unit mass** basis as

$$e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}} \qquad (\text{kJ/kg}) \tag{5–14}$$

which is obtained by dividing all the quantities in Eq. 5–11 by the mass m of the system. Energy balance can also be expressed in the differential form as

$$\delta E_{\text{in}} - \delta E_{\text{out}} = dE_{\text{system}} \quad \text{or} \quad \delta e_{\text{in}} - \delta e_{\text{out}} = de_{\text{system}} \tag{5–15}$$

For a closed system undergoing a **cycle**, the initial and final states are identical, and thus $\Delta E_{\text{system}} = E_2 - E_1 = 0$. Then, the energy balance for a cycle simplifies to $E_{\text{in}} - E_{\text{out}} = 0$ or $E_{\text{in}} = E_{\text{out}}$. Noting that a closed system does not involve any mass flow across its boundaries, the energy balance for a cycle can be expressed in terms of heat and work interactions as

$$W_{\text{net,out}} = Q_{\text{net,in}} \quad \text{or} \quad \dot{W}_{\text{net,out}} = \dot{Q}_{\text{net,in}} \qquad (\text{for a cycle}) \tag{5–16}$$

That is, the net work output during a cycle is equal to net heat input (Fig. 5–11).

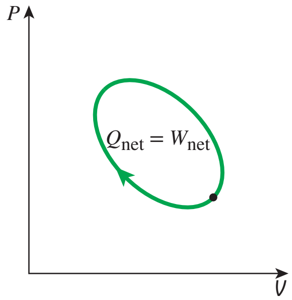
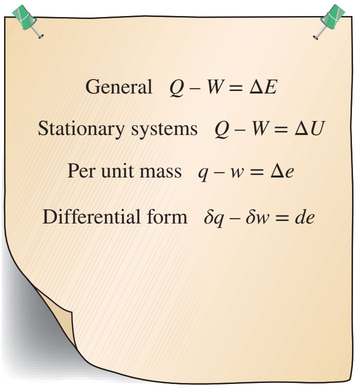


FIGURE 5–11
For a cycle $\Delta E = 0$, thus $Q = W$.

The energy balance (or the first-law) relations already given are intuitive in nature and are easy to use when the magnitudes and directions of heat and work transfers are known. However, when performing a general analytical study or solving a problem that involves an unknown heat or work interaction, we need to assume a direction for the heat or work interactions. In such cases, it is common practice to use the classical thermodynamics sign convention and to assume heat to be transferred *into the system* (heat input) in the amount of Q and work to be done *by the system* (work output) in the amount of W, and then to solve the problem. The energy balance relation in that case for a closed system becomes

$$Q_{\text{net,in}} - W_{\text{net,out}} = \Delta E_{\text{system}} \quad \text{or} \quad Q - W = \Delta E \tag{5–17}$$

where $Q = Q_{\text{net,in}} = Q_{\text{in}} - Q_{\text{out}}$ is the *net heat input* and $W = W_{\text{net,out}} = W_{\text{out}} - W_{\text{in}}$ is the *net work output*. Obtaining a negative quantity for Q or W simply means that the assumed direction for that quantity is wrong and should be reversed. Various forms of this "traditional" first-law relation for closed systems are given in Fig. 5–12.



General $Q - W = \Delta E$

Stationary systems $Q - W = \Delta U$

Per unit mass $q - w = \Delta e$

Differential form $\delta q - \delta w = de$

FIGURE 5–12
Various forms of the first-law relation for closed systems.

The first law cannot be proven mathematically, but no process in nature is known to have violated the first law, and this should be taken as sufficient proof. Note that if it were possible to prove the first law on the basis of other physical principles, the first law then would be a consequence of those principles instead of being a fundamental physical law itself.

As energy quantities, heat and work are not that different, and you probably wonder why we keep distinguishing them. After all, the change in the energy content of a system is equal to the amount of energy that crosses the system boundaries, and it makes no difference whether the energy crosses the boundary as heat or work. It seems as if the first-law relations would be much simpler if we had just one quantity that we could call *energy interaction* to represent both heat and work. Well, from the first-law point of view, heat and work are not different at all. From the second-law point of view, however, heat and work are very different, as is discussed in later chapters.

EXAMPLE 5–5 Electric Heating of a Gas at Constant Pressure

A piston–cylinder device contains 25 g of saturated water vapor that is maintained at a constant pressure of 300 kPa. A resistance heater within the cylinder is turned on and passes a current of 0.2 A for 5 min from a 120-V source. At the same time, a heat loss of 3.7 kJ occurs. (*a*) Show that for a closed system the boundary work W_b and the change in internal energy ΔU in the first-law relation can be combined into one term, ΔH, for a constant-pressure process. (*b*) Determine the final temperature of the steam.

SOLUTION Saturated water vapor in a piston–cylinder device expands at constant pressure as a result of heating. It is to be shown that $\Delta U + W_b = \Delta H$, and the final temperature is to be determined.

Assumptions **1** The cylinder is stationary, and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$. Therefore, $\Delta E = \Delta U$, and internal energy is the only form of energy of the system that may change during this process. **2** Electrical wires constitute a very small part of the system, and thus the energy change of the wires can be neglected.

Analysis We take the contents of the cylinder, including the resistance wires, as the *system* (Fig. 5–13). This is a *closed system* since no mass crosses the system boundary during the process. We observe that a piston–cylinder device typically involves a moving boundary and thus boundary work W_b. The pressure remains constant during the process and thus $P_2 = P_1$. Also, heat is lost from the system and electrical work W_e is done on the system.

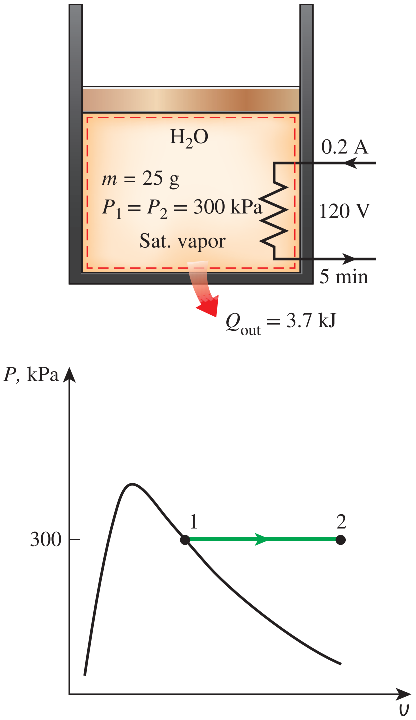


FIGURE 5–13
Schematic and P-V diagram for Example 5–5.

(*a*) This part of the solution involves a general analysis for a closed system undergoing a quasi-equilibrium constant-pressure process, and thus we consider a general closed system. We take the direction of heat transfer Q to be to the system and the work W to be done by the system. We also express the work as the sum of boundary and other forms of work (such as electrical and shaft). Then, the energy balance can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$Q - W = \Delta U + \cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE}$$

$$Q - W_{other} - W_b = U_2 - U_1$$

For a constant-pressure process, the boundary work is given as $W_b = P_0(V_2 - V_1)$. Substituting this into the preceding relation gives

$$Q - W_{other} - P_0(V_2 - V_1) = U_2 - U_1$$

However,

$$P_0 = P_2 = P_1 \rightarrow Q - W_{other} = (U_2 + P_2V_2) - (U_1 + P_1V_1)$$

Also $H = U + PV$, and thus

$$Q - W_{other} = H_2 - H_1 \qquad \text{(kJ)} \tag{5–18}$$

which is the desired relation (Fig. 5–14). This equation is very convenient to use in the analysis of closed systems undergoing a ***constant-pressure quasi-equilibrium process*** since the boundary work is automatically taken care of by the enthalpy terms, and one no longer needs to determine it separately.

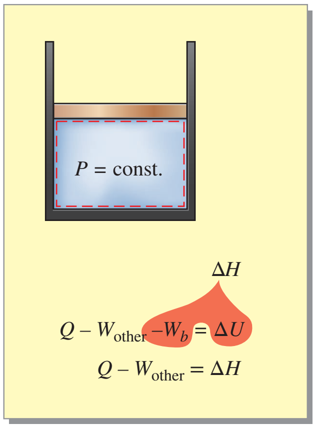


FIGURE 5–14
For a closed system undergoing a quasi-equilibrium, P = constant process, $\Delta U + W_b = \Delta H$. Note that this relation is NOT valid for closed-system processes during which pressure DOES NOT remain constant.

(*b*) The only other form of work in this case is the electrical work, which can be determined from

$$W_e = \mathbf{V}I\Delta t = (120\text{ V})(0.2\text{ A})(300\text{ s})\left(\frac{1\text{ kJ/s}}{1000\text{ VA}}\right) = 7.2\text{ kJ}$$

$$\textit{State 1:}\quad \left.\begin{array}{l} P_1 = 300\text{ kPa} \\ \text{Sat. vapor} \end{array}\right\} h_1 = h_{g\,@\,300\text{ kPa}} = 2724.9\text{ kJ/kg} \quad (\text{Table A–5})$$

The enthalpy at the final state can be determined directly from Eq. 5–18 by expressing heat transfer from the system and work done on the system as negative quantities (since their directions are opposite to the assumed directions). Alternately, we can use the general energy balance relation with the simplification that the boundary work is considered automatically by replacing ΔU with ΔH for a constant-pressure expansion or compression process:

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$W_{e,\text{in}} - Q_{\text{out}} - W_b = \Delta U$$

$$W_{e,\text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1) \quad (\text{since } P = \text{constant})$$

$$7.2\text{ kJ} - 3.7\text{ kJ} = (0.025\text{ kg})(h_2 - 2724.9)\text{ kJ/kg}$$

$$h_2 = 2864.9\text{ kJ/kg}$$

Now the final state is completely specified since we know both the pressure and the enthalpy. The temperature at this state is

$$\textit{State 2:}\quad \left.\begin{array}{l} P_2 = 300\text{ kPa} \\ h_2 = 2864.9\text{ kJ/kg} \end{array}\right\} T_2 = \mathbf{200°C} \quad (\text{Table A–6})$$

Therefore, the steam will be at 200°C at the end of this process.

Discussion Strictly speaking, the potential energy change of the steam is not zero for this process since the center of gravity of the steam rose somewhat. Assuming an elevation change of 1 m (which is rather unlikely), the change in the potential energy of the steam would be 0.0002 kJ, which is very small compared to the other terms in the first-law relation. Therefore, in problems of this kind, the potential energy term is always neglected.

EXAMPLE 5–6 Unrestrained Expansion of Water

A rigid tank is divided into two equal parts by a partition. Initially, one side of the tank contains 5 kg of water at 200 kPa and 25°C, and the other side is evacuated. The partition is then removed, and the water expands into the entire tank. The water is allowed to exchange heat with its surroundings until the temperature in the tank returns to the initial value of 25°C. Determine (*a*) the volume of the tank, (*b*) the final pressure, and (*c*) the heat transfer for this process.

SOLUTION One-half of a rigid tank is filled with liquid water while the other side is evacuated. The partition between the two parts is removed, and water is allowed to expand and fill the entire tank while the temperature is maintained constant. The volume of the tank, the final pressure, and the heat transfer are to be to determined.

Assumptions **1** The system is stationary and thus the kinetic and potential energy changes are zero, $\Delta \text{KE} = \Delta \text{PE} = 0$ and $\Delta E = \Delta U$. **2** The direction of heat transfer is to the system (heat gain, Q_{in}). A negative result for Q_{in} indicates the assumed direction is wrong and thus it is a heat loss. **3** The volume of the rigid tank is constant, and thus there is no energy transfer as boundary work. **4** There is no electrical, shaft, or any other kind of work involved.

Analysis We take the contents of the tank, including the evacuated space, as the *system* (Fig. 5–15). This is a *closed system* since no mass crosses the system boundary during the process. We observe that the water fills the entire tank when the partition is removed (possibly as a liquid–vapor mixture).

(*a*) Initially the water in the tank exists as a compressed liquid since its pressure (200 kPa) is greater than the saturation pressure at 25°C (3.1698 kPa). Approximating the compressed liquid as a saturated liquid at the given temperature, we find

$$v_1 \cong v_{f\,@\,25^\circ\mathrm{C}} = 0.001003\ \mathrm{m^3/kg} \cong 0.001\ \mathrm{m^3/kg} \quad \text{(Table A–4)}$$

Then the initial volume of the water is

$$V_1 = m v_1 = (5\ \mathrm{kg})(0.001\ \mathrm{m^3/kg}) = 0.005\ \mathrm{m^3}$$

The total volume of the tank is twice this amount:

$$V_{\mathrm{tank}} = (2)(0.005\ \mathrm{m^3}) = \mathbf{0.01\ m^3}$$

(*b*) At the final state, the specific volume of the water is

$$v_2 = \frac{V_2}{m} = \frac{0.01\ \mathrm{m^3}}{5\ \mathrm{kg}} = 0.002\ \mathrm{m^3/kg}$$

which is twice the initial value of the specific volume. This result is expected since the volume doubles while the amount of mass remains constant.

$$\text{At } 25^\circ\mathrm{C}\text{:} \quad v_f = 0.001003\ \mathrm{m^3/kg} \quad \text{and} \quad v_g = 43.340\ \mathrm{m^3/kg} \quad \text{(Table A–4)}$$

Since $v_f < v_2 < v_g$, the water is a saturated liquid–vapor mixture at the final state, and thus the pressure is the saturation pressure at 25°C:

$$P_2 = P_{\mathrm{sat}\,@\,25^\circ\mathrm{C}} = \mathbf{3.1698\ kPa} \quad \text{(Table A–4)}$$

(*c*) Under stated assumptions and observations, the energy balance on the system can be expressed as

$$\underbrace{E_{\mathrm{in}} - E_{\mathrm{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\mathrm{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$Q_{\mathrm{in}} = \Delta U = m(u_2 - u_1)$$

Notice that even though the water is expanding during this process, the system chosen involves fixed boundaries only (the dashed lines), and therefore the moving boundary work is zero (Fig. 5–16). Then $W = 0$ since the system does not involve any other forms of work. (Can you reach the same conclusion by choosing the water as our system?) Initially,

$$u_1 \cong u_{f\,@\,25^\circ\mathrm{C}} = 104.83\ \mathrm{kJ/kg}$$

The quality at the final state is determined from the specific volume information:

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.002 - 0.001}{43.34 - 0.001} = 2.3 \times 10^{-5}$$

Then

$$\begin{aligned} u_2 &= u_f + x_2 u_{fg} \\ &= 104.83\ \mathrm{kJ/kg} + (2.3 \times 10^{-5})(2304.3\ \mathrm{kJ/kg}) \\ &= 104.88\ \mathrm{kJ/kg} \end{aligned}$$

Substituting yields

$$Q_{\mathrm{in}} = (5\ \mathrm{kg})[(104.88 - 104.83)\ \mathrm{kJ/kg}] = \mathbf{0.25\ kJ}$$

Discussion The positive sign indicates that the assumed direction is correct, and heat is transferred to the water.

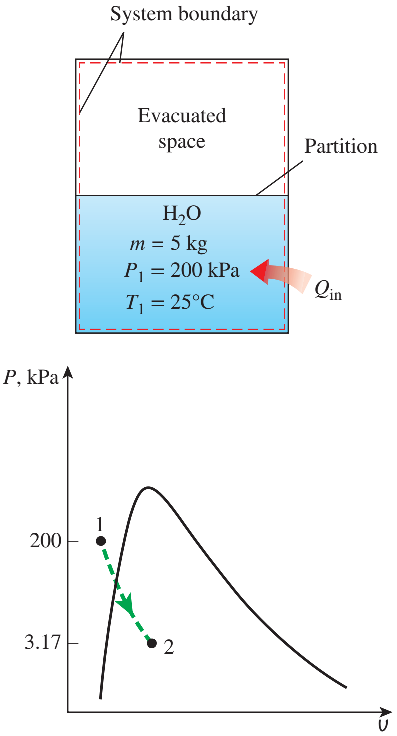


FIGURE 5–15
Schematic and P-v diagram for Example 5–6.

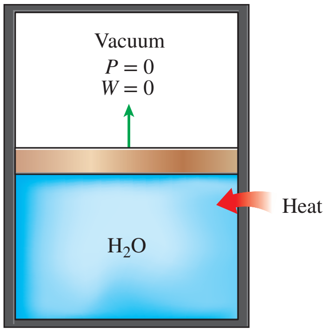


FIGURE 5–16
Expansion against a vacuum involves no work and thus no energy transfer.

5–3 ▪ SPECIFIC HEATS

We know from experience that it takes different amounts of energy to raise the temperature of identical masses of different substances by one degree. For example, we need about 4.5 kJ of energy to raise the temperature of 1 kg of iron from 20 to 30°C, whereas it takes about nine times this much energy (41.8 kJ to be exact) to raise the temperature of 1 kg of liquid water by the same amount (Fig. 5–17). Therefore, it is useful to have a property that will enable us to compare the energy storage capabilities of various substances. This property is the specific heat.

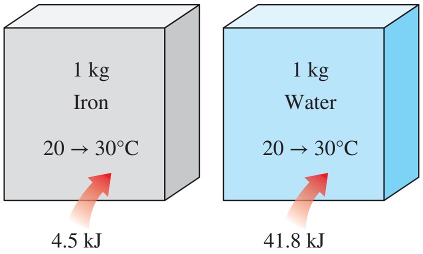


FIGURE 5–17
It takes different amounts of energy to raise the temperature of different substances by the same amount.

The **specific heat** is defined as *the energy required to raise the temperature of a unit mass of a substance by one degree* (Fig. 5–18). In general, this energy depends on how the process is executed. In thermodynamics, we are interested in two kinds of specific heats: **specific heat at constant volume** c_υ and **specific heat at constant pressure** c_p.

Physically, the specific heat at constant volume c_υ can be viewed as *the energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.* The energy required to do the same as the pressure is maintained constant is the specific heat at constant pressure c_p. This is illustrated in Fig. 5–19. The specific heat at constant pressure c_p is always greater than c_υ because at constant pressure the system is allowed to expand, and the energy for this expansion work must also be supplied to the system.

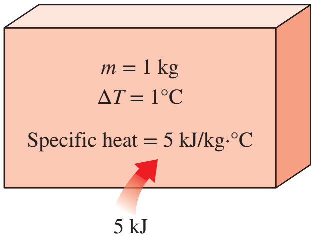


FIGURE 5–18
Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.

Now we try to express the specific heats in terms of other thermodynamic properties. First, consider a fixed mass in a stationary closed system undergoing a constant-volume process (and thus no expansion or compression work is involved). The conservation of energy principle $e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}}$ for this process can be expressed in the differential form as

$$\delta e_{\text{in}} - \delta e_{\text{out}} = du$$

The left-hand side of this equation represents the net amount of energy transferred to the system. From the definition of c_υ, this energy must be equal to $c_\upsilon\, dT$, where dT is the differential change in temperature. Thus,

$$c_\upsilon dT = du \qquad \text{at constant volume}$$

or

$$c_\upsilon = \left(\frac{\partial u}{\partial T}\right)_\upsilon \tag{5–19}$$

Similarly, an expression for the specific heat at constant pressure c_p can be obtained by considering a constant-pressure expansion or compression process. It yields

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p \tag{5–20}$$

Equations 5–19 and 5–20 are the defining equations for c_υ and c_p, and their interpretation is given in Fig. 5–20.

Note that c_υ and c_p are expressed in terms of other properties; thus, they must be properties themselves. Like any other property, the specific heats of a substance depend on the state that, in general, is specified by two independent, intensive properties. That is, the energy required to raise the temperature of a substance by one degree is different at different temperatures and pressures (Fig. 5–21). But this difference is usually not very large.

A few observations can be made from Eqs. 5–19 and 5–20. First, these equations are *property relations* and as such *are independent of the type of process.* They are valid for *any* substance undergoing *any* process. The only relevance c_υ has to a constant-volume process is that c_υ happens to be the energy transferred

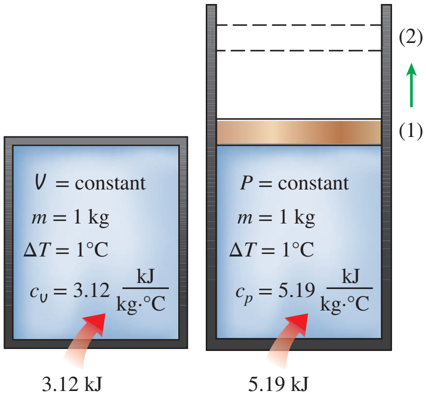


FIGURE 5–19
Constant-volume and constant-pressure specific heats c_υ and c_p (values given are for helium gas).

to a system during a constant-volume process per unit mass, per unit degree rise in temperature. This is how the values of c_υ are determined. This is also how the name *specific heat at constant volume* originated. Likewise, the energy transferred to a system per unit mass per unit temperature rise during a constant-pressure process happens to be equal to c_p. This is how the values of c_p can be determined, and it also explains the origin of the name *specific heat at constant pressure.*

Another observation that can be made from Eqs. 5–19 and 5–20 is that c_υ is related to the changes in *internal energy* and c_p to the changes in *enthalpy.* In fact, it would be more proper to define c_υ as *the change in the internal energy of a substance per unit change in temperature at constant volume.* Likewise, c_p can be defined as *the change in the enthalpy of a substance per unit change in temperature at constant pressure.* In other words, c_υ is a measure of the variation of internal energy of a substance with temperature, and c_p is a measure of the variation of enthalpy of a substance with temperature.

Both the internal energy and enthalpy of a substance can be changed by the transfer of *energy* in any form, with heat being only one of them. Therefore, the term *specific energy* is probably more appropriate than the term *specific heat,* which implies that energy is transferred (and stored) in the form of heat.

A common unit for specific heats is kJ/kg·°C or kJ/kg·K. Notice that these two units are *identical* since $\Delta T(°\text{C}) = \Delta T(\text{K})$, and a 1°C change in temperature is equivalent to a change of 1 K. The specific heats are sometimes given on a *molar basis.* They are then denoted by $\bar{c}_\upsilon$ and $\bar{c}_p$ and have the unit kJ/kmol·°C or kJ/kmol·K.

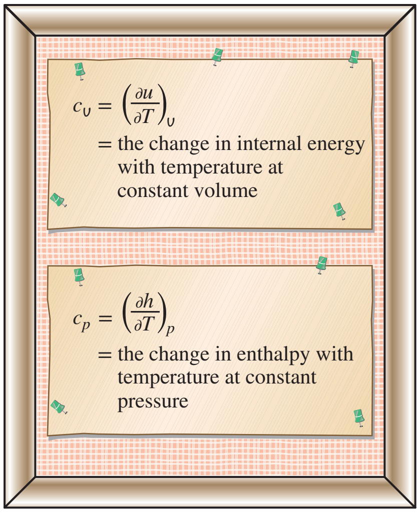


FIGURE 5–20
Formal definitions of c_υ and c_p.

5–4 ▪ INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

We defined an ideal gas as a gas whose temperature, pressure, and specific volume are related by

$$P\upsilon = RT$$

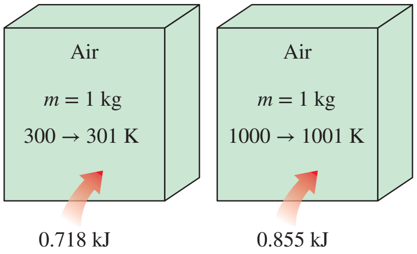


FIGURE 5–21
The specific heat of a substance changes with temperature.

It has been demonstrated mathematically and experimentally (Joule, 1843) that for an ideal gas the internal energy is a function of the temperature only. That is,

$$u = u(T) \tag{5–21}$$

In his classic experiment, Joule submerged two tanks connected with a pipe and a valve in a water bath, as shown in Fig. 5–22. Initially, one tank contained air at a high pressure and the other tank was evacuated. When thermal equilibrium was attained, he opened the valve to let air pass from one tank to the other until the pressures equalized. Joule observed no change in the temperature of the water bath and assumed that no heat was transferred to or from the air. Since there was also no work done, he concluded that the internal energy of the air did not change even though the volume and the pressure changed. Therefore, he reasoned, the internal energy is a function of temperature only and not a function of pressure or specific volume. (Joule later showed that for gases that deviate significantly from ideal-gas behavior, the internal energy is not a function of temperature alone.)

Using the definition of enthalpy and the equation of state of an ideal gas, we have

$$\left.\begin{aligned} h &= u + P\upsilon \\ P\upsilon &= RT \end{aligned}\right\} \quad h = u + RT$$

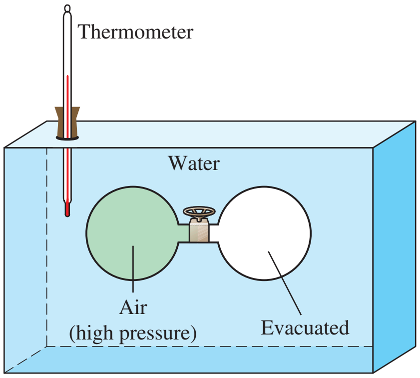


FIGURE 5–22
Schematic of the experimental apparatus used by Joule.

Since R is constant and $u = u(T)$, it follows that the enthalpy of an ideal gas is also a function of temperature only:

$$h = h(T) \tag{5–22}$$

Since u and h depend only on temperature for an ideal gas, the specific heats c_υ and c_p also depend, at most, on temperature only. Therefore, at a given temperature, u, h, c_υ, and c_p of an ideal gas have fixed values regardless of the specific volume or pressure (Fig. 5–23). Thus, for ideal gases, the partial derivatives in Eqs. 5–19 and 5–20 can be replaced by ordinary derivatives. Then, the differential changes in the internal energy and enthalpy of an ideal gas can be expressed as

$$du = c_\upsilon(T)\,dT \tag{5–23}$$

and

$$dh = c_p(T)\,dT \tag{5–24}$$

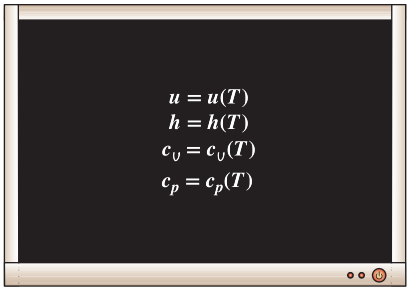


FIGURE 5–23
For ideal gases, u, h, c_υ, and c_p vary with temperature only.

The change in internal energy or enthalpy for an ideal gas during a process from state 1 to state 2 is determined by integrating these equations:

$$\Delta u = u_2 - u_1 = \int_1^2 c_\upsilon(T)\,dT \qquad \text{(kJ/kg)} \tag{5–25}$$

and

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T)\,dT \qquad \text{(kJ/kg)} \tag{5–26}$$

To carry out these integrations, we need to have relations for c_υ and c_p as functions of temperature.

At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only. The specific heats of real gases at low pressures are called *ideal-gas specific heats,* or *zero-pressure specific heats,* and they are often denoted c_{p0} and $c_{\upsilon 0}$. Accurate analytical expressions for ideal-gas specific heats, based on direct measurements or calculations from statistical behavior of molecules, are available and are given as third-degree polynomials in the appendix (Table A–2*c*) for several gases. A plot of $\bar{c}_{p0}(T)$ data for some common gases is given in Fig. 5–24.

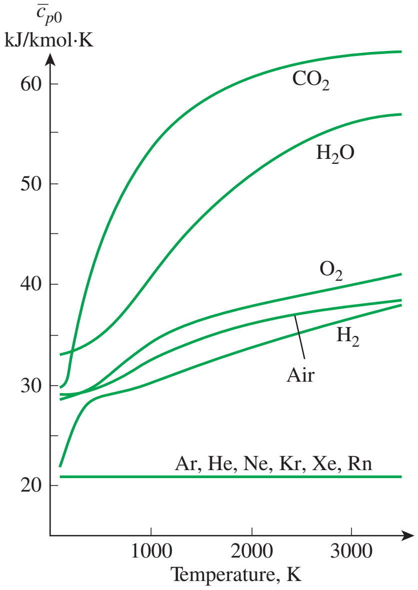
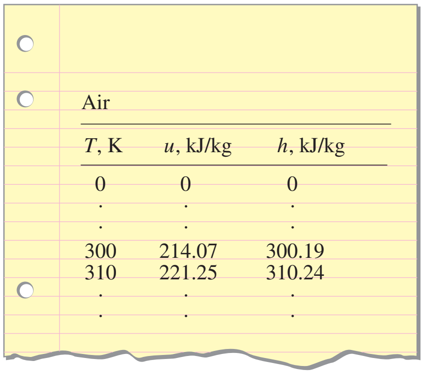


FIGURE 5–24
Ideal-gas constant-pressure specific heats for some gases (see Table A–2*c* for c_p equations).

The use of ideal-gas specific heat data is limited to low pressures, but these data can also be used at moderately high pressures with reasonable accuracy as long as the gas does not deviate from ideal-gas behavior significantly.

The integrations in Eqs. 5–25 and 5–26 are straightforward but rather time-consuming and thus impractical. To avoid these laborious calculations, u and h data for a number of gases have been tabulated over small temperature intervals. These tables are obtained by choosing an arbitrary reference point and performing the integrations in Eqs. 5–25 and 5–26 by treating state 1 as the reference state. In the ideal-gas tables given in the appendix, zero kelvin is chosen as the reference state, and both the enthalpy and the internal energy are assigned zero values at that state (Fig. 5–25). The choice of the reference state has no effect on Δu or Δh calculations. The u and h data are given in kJ/kg for air (Table A–21) and usually in kJ/kmol for other gases. The unit kJ/kmol is very convenient in the thermodynamic analysis of chemical reactions.



Air

T, K	u, kJ/kg	h, kJ/kg
0	0	0
.	.	.
.	.	.
300	214.07	300.19
310	221.25	310.24
.	.	.
.	.	.

FIGURE 5–25
In the preparation of ideal-gas tables, 0 K is chosen as the reference temperature.

Some observations can be made from Fig. 5–24. First, the specific heats of gases with complex molecules (molecules with two or more atoms) are higher and increase with temperature. Also, the variation of specific heats with temperature is smooth and may be approximated as linear over small temperature intervals (a few hundred degrees or less). Therefore, the specific heat functions in Eqs. 5–25 and 5–26 can be replaced by the constant average specific heat values. Then, the integrations in these equations can be performed, yielding

$$u_2 - u_1 = c_{\upsilon,\text{avg}}(T_2 - T_1) \qquad \text{(kJ/kg)} \tag{5–27}$$

and

$$h_2 - h_1 = c_{p,\text{avg}}(T_2 - T_1) \qquad \text{(kJ/kg)} \tag{5–28}$$

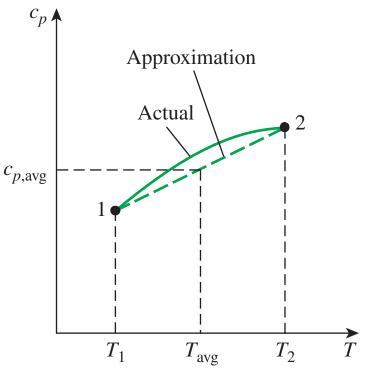


FIGURE 5–26
For small temperature intervals, the specific heats may be assumed to vary linearly with temperature.

The specific heat values for some common gases are listed as a function of temperature in Table A–2*b*. The average specific heats $c_{p,\text{avg}}$ and $c_{v,\text{avg}}$ are evaluated from this table at the average temperature $(T_1 + T_2)/2$, as shown in Fig. 5–26. If the final temperature T_2 is not known, the specific heats may be evaluated at T_1 or at the anticipated average temperature. Then T_2 can be determined by using these specific heat values. The value of T_2 can be refined, if necessary, by evaluating the specific heats at the new average temperature.

Another way of determining the average specific heats is to evaluate them at T_1 and T_2 and then take their average. Usually both methods give reasonably good results, and one is not necessarily better than the other.

Another observation that can be made from Fig. 5–24 is that the ideal-gas specific heats of *monatomic gases* such as argon, neon, and helium remain constant over the entire temperature range. Thus, Δu and Δh of monatomic gases can easily be evaluated from Eqs. 5–27 and 5–28.

Note that the Δu and Δh relations given previously are not restricted to any kind of process. They are valid for all processes. The presence of the constant-volume specific heat c_v in an equation should not lead one to believe that this equation is valid for a constant-volume process only. On the contrary, the relation $\Delta u = c_{v,\text{avg}}\,\Delta T$ is valid for *any* ideal gas undergoing *any* process (Fig. 5–27). A similar argument can be given for c_p and Δh.

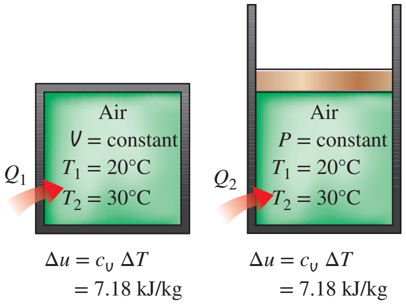


FIGURE 5–27
The relation $\Delta u = c_v\,\Delta T$ is valid for *any* kind of process, constant-volume or not.

To summarize, there are three ways to determine the internal energy and enthalpy changes of ideal gases (Fig. 5–28):

1. By using the tabulated *u* and *h* data. This is the easiest and most accurate way when tables are readily available.
2. By using the c_v or c_p relations as a function of temperature and performing the integrations. This is very inconvenient for hand calculations but quite desirable for computerized calculations. The results obtained are very accurate.
3. By using average specific heats. This is very simple and certainly very convenient when property tables are not available. The results obtained are reasonably accurate if the temperature interval is not very large.

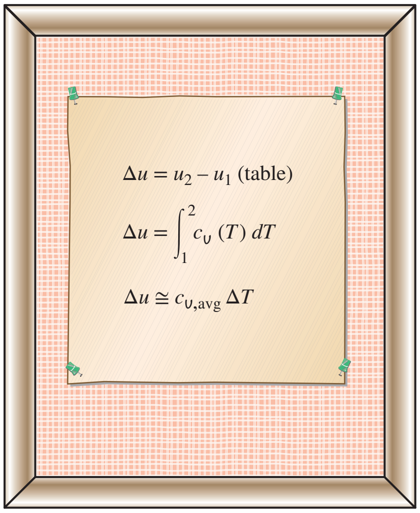


FIGURE 5–28
Three ways of calculating Δu.

Specific Heat Relations of Ideal Gases

A special relationship between c_p and c_v for ideal gases can be obtained by differentiating the relation $h = u + RT$, which yields

$$dh = du + R\,dT$$

Replacing dh with $c_p\,dT$ and du with $c_v\,dT$ and dividing the resulting expression by dT, we obtain

$$c_p = c_v + R \qquad \text{(kJ/kg·K)} \tag{5–29}$$

This is an important relationship for ideal gases since it enables us to determine c_v from a knowledge of c_p and the gas constant R.

When the specific heats are given on a molar basis, R in Eq. 5–29 should be replaced by the universal gas constant R_u (Fig. 5–29).

$$\bar{c}_p = \bar{c}_v + R_u \qquad \text{(kJ/kmol·K)} \tag{5–30}$$

At this point, we introduce another ideal-gas property called the **specific heat ratio** *k*, defined as

$$k = \frac{c_p}{c_v} \tag{5–31}$$

The specific ratio also varies with temperature, but this variation is very mild. For monatomic gases, its value is essentially constant at 1.667. Many diatomic gases, including air, have a specific heat ratio of about 1.4 at room temperature.

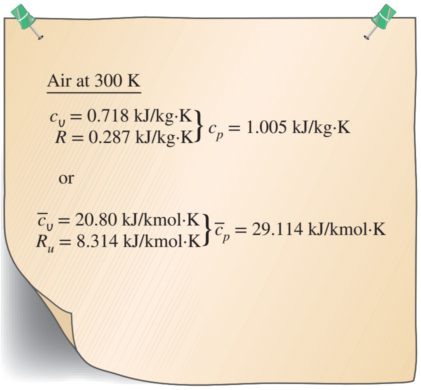


FIGURE 5–29
The c_p of an ideal gas can be determined from a knowledge of c_υ and R.

EXAMPLE 5–7 Evaluation of the Δ*u* of an Ideal Gas

Air at 300 K and 200 kPa is heated at constant pressure to 600 K. Determine the change in internal energy of air per unit mass, using (*a*) data from the air table (Table A–21), (*b*) the functional form of the specific heat (Table A–2*c*), and (*c*) the average specific heat value (Table A–2*b*).

SOLUTION The internal energy change of air is to be determined in three different ways.
Assumptions At specified conditions, air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values.
Analysis The internal energy change Δ*u* of ideal gases depends on the initial and final temperatures only, and not on the type of process. Thus, the following solution is valid for any kind of process.
(*a*) One way of determining the change in internal energy of air is to read the *u* values at T_1 and T_2 from Table A–21 and take the difference:

$$u_1 = u_{@\,300\text{ K}} = 214.07\text{ kJ/kg}$$

$$u_2 = u_{@\,600\text{ K}} = 434.78\text{ kJ/kg}$$

Thus,

$$\Delta u = u_2 - u_1 = (434.78 - 214.07)\text{kJ/kg} = \mathbf{220.71\ kJ/kg}$$

(*b*) The $\bar{c}_p(T)$ of air is given in Table A–2*c* in the form of a third-degree polynomial expressed as

$$\bar{c}_p(T) = a + bT + cT^2 + dT^3$$

where $a = 28.11$, $b = 0.1967 \times 10^{-2}$, $c = 0.4802 \times 10^{-5}$, and $d = -1.966 \times 10^{-9}$. From Eq. 5–30,

$$\bar{c}_\upsilon(T) = \bar{c}_p - R_u = (a - R_u) + bT + cT^2 + dT^3$$

From Eq. 5–25,

$$\Delta\bar{u} = dT\int_1^2 \bar{c}_\upsilon(T)\,dT = \int_{T_1}^{T_2}[(a - R_u) + bT + cT^2 + dT^3]dT$$

Performing the integration and substituting the values, we obtain

$$\Delta\bar{u} = 6447\text{ kJ/kmol}$$

The change in the internal energy on a unit-mass basis is determined by dividing this value by the molar mass of air (Table A–1):

$$\Delta u = \frac{\Delta\bar{u}}{M} = \frac{6447\text{ kJ/kmol}}{28.97\text{ kg/kmol}} = \mathbf{222.5\ kJ/kg}$$

which differs from the tabulated value by 0.8 percent.

(*c*) The average value of the constant-volume specific heat $c_{\upsilon,\text{avg}}$ is determined from Table A–2*b* at the average temperature of $(T_1 + T_2)/2 = 450$ K to be

$$c_{\upsilon,\text{avg}} = c_{\upsilon\ @\,450\text{ K}} = 0.733\text{ kJ/kg·K}$$

Thus,

$$\begin{aligned}\Delta u &= c_{\upsilon,\text{avg}}(T_2 - T_1) = (0.733\text{ kJ/kg·K})[(600 - 300)\text{K}]\\ &= \mathbf{220\ kJ/kg}\end{aligned}$$

Discussion This answer differs from the tabulated value (220.71 kJ/kg) by only 0.4 percent. This close agreement is not surprising since the assumption that c_v varies linearly with temperature is a reasonable one at temperature intervals of only a few hundred degrees. If we had used the c_v value at $T_1 = 300$ K instead of at T_{avg}, the result would be 215.4 kJ/kg, which is in error by about 2 percent. Errors of this magnitude are acceptable for most engineering purposes.

EXAMPLE 5–8 Heating of a Gas in a Tank by Stirring

An insulated rigid tank initially contains 1.5 lbm of helium at 80°F and 50 psia. A paddle wheel with a power rating of 0.02 hp is operated within the tank for 30 min. Determine (*a*) the final temperature and (*b*) the final pressure of the helium gas.

SOLUTION Helium gas in an insulated rigid tank is stirred by a paddle wheel. The final temperature and pressure of helium are to be determined.

Assumptions **1** Helium is an ideal gas since it is at a very high temperature relative to its critical-point value of −451°F. **2** Constant specific heats can be used for helium. **3** The system is stationary, and thus the kinetic and potential energy changes are zero, $\Delta\text{KE} = \Delta\text{PE} = 0$ and $\Delta E = \Delta U$. **4** The volume of the tank is constant, and thus there is no boundary work. **5** The system is adiabatic, and thus there is no heat transfer.

Analysis We take the contents of the tank as the *system* (Fig. 5–30). This is a *closed system* since no mass crosses the system boundary during the process. We observe that there is shaft work done on the system.

(*a*) The amount of paddle-wheel work done on the system is

$$W_{\text{sh}} = \dot{W}_{\text{sh}}\,\Delta t = (0.02\text{ hp})(0.5\text{ h})\left(\frac{2545\text{ Btu/h}}{1\text{ hp}}\right) = 25.45\text{ Btu}$$

Under the stated assumptions and observations, the energy balance on the system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$W_{\text{sh,in}} = \Delta U = m(u_2 - u_1) = mc_{v,\text{avg}}(T_2 - T_1)$$

As we pointed out earlier, the ideal-gas specific heats of monatomic gases (helium being one of them) are constant. The c_v value of helium is determined from Table A–2E*a* to be $c_v = 0.753$ Btu/lbm·°F. Substituting this and other known quantities into the preceding equation, we obtain

$$25.45\text{ Btu} = (1.5\text{ lbm})(0.753\text{ Btu/lbm·°F})(T_2 - 80)\text{°F}$$
$$T_2 = \mathbf{102.5°F}$$

(*b*) The final pressure is determined from the ideal-gas relation

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

where V_1 and V_2 are identical and cancel out. Then the final pressure becomes

$$\frac{50\text{ psia}}{(80 + 460)\text{R}} = \frac{P_2}{(102.5 + 460)\text{R}}$$
$$P_2 = \mathbf{52.1\ psia}$$

Discussion Note that the pressure in the ideal-gas relation is always the absolute pressure.

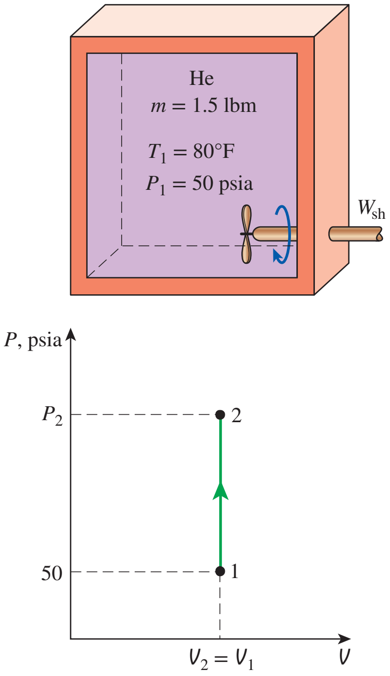


FIGURE 5–30
Schematic and P-V diagram for Example 5–8.

EXAMPLE 5–9 Heating of a Gas by a Resistance Heater

A piston–cylinder device initially contains 0.5 m^3 of nitrogen gas at 400 kPa and 27°C. An electric heater within the device is turned on and is allowed to pass a current of 2 A for 5 min from a 120-V source. Nitrogen expands at constant pressure, and a heat loss of 2800 J occurs during the process. Determine the final temperature of nitrogen.

SOLUTION Nitrogen gas in a piston–cylinder device is heated by an electric resistance heater. Nitrogen expands at constant pressure while some heat is lost. The final temperature of nitrogen is to be determined.

Assumptions **1** Nitrogen is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values of −147°C and 3.39 MPa. **2** The system is stationary, and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$ and $\Delta E = \Delta U$. **3** The pressure remains constant during the process, and thus $P_2 = P_1$. **4** Nitrogen has constant specific heats at room temperature.

Analysis We take the contents of the cylinder as the *system* (Fig. 5–31). This is a *closed system* since no mass crosses the system boundary during the process. We observe that a piston–cylinder device typically involves a moving boundary and thus boundary work, W_b. Also, heat is lost from the system and electrical work W_e is done on the system.

First, let us determine the electrical work done on the nitrogen:

$$W_e = \mathbf{V}I\,\Delta t = (120\text{ V})(2\text{ A})(5 \times 60\text{ s})\left(\frac{1\text{ kJ/s}}{1000\text{ VA}}\right) = 72\text{ kJ}$$

The mass of nitrogen is determined from the ideal-gas relation:

$$m = \frac{P_1 V_1}{RT_1} = \frac{(400\text{ kPa})(0.5\text{ m}^3)}{(0.297\text{ kPa·m}^3\text{/kg·K})(300\text{ K})} = 2.245\text{ kg}$$

Under the stated assumptions and observations, the energy balance on the system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$W_{e,\text{in}} - Q_{\text{out}} - W_{b,\text{out}} = \Delta U$$

$$W_{e,\text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1) = mc_p(T_2 - T_1)$$

since $\Delta U + W_b = \Delta H$ for a closed system undergoing a quasi-equilibrium expansion or compression process at constant pressure. From Table A–2*a*, $c_p = 1.039$ kJ/kg·K for nitrogen at room temperature. The only unknown quantity in the previous equation is T_2, and it is found to be

$$72\text{ kJ} - 2.8\text{ kJ} = (2.245\text{ kg})(1.039\text{ kJ/kg·K})(T_2 - 27)°\text{C}$$

$$T_2 = \mathbf{56.7°C}$$

Discussion Note that we could also solve this problem by determining the boundary work and the internal energy change rather than the enthalpy change.

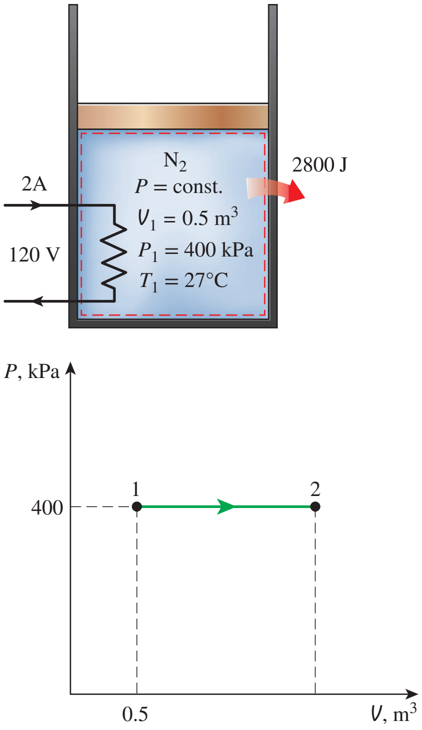


FIGURE 5–31
Schematic and P-V diagram for Example 5–9.

EXAMPLE 5–10 Heating of a Gas at Constant Pressure

A piston–cylinder device initially contains air at 150 kPa and 27°C. At this state, the piston is resting on a pair of stops, as shown in Fig. 5–32, and the enclosed volume is 400 L. The mass of the piston is such that a 350-kPa pressure is required to move it. The air is now heated until its volume has doubled. Determine (*a*) the final temperature, (*b*) the work done by the air, and (*c*) the total heat transferred to the air.

SOLUTION Air in a piston–cylinder device with a set of stops is heated until its volume is doubled. The final temperature, work done, and the total heat transfer are to be determined.

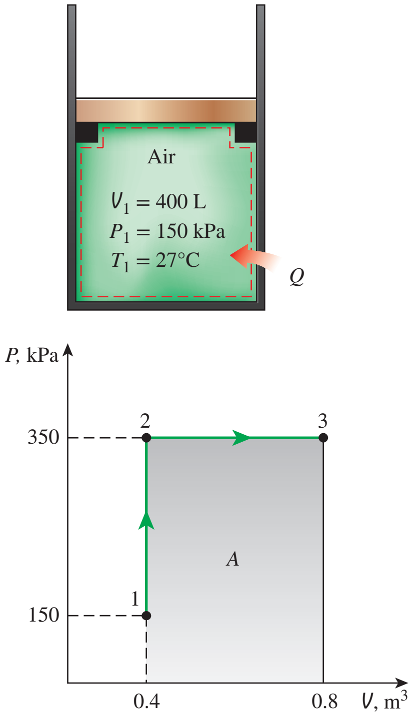


FIGURE 5–32
Schematic and P-V diagram for Example 5–10.

Assumptions **1** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **2** The system is stationary and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$ and $\Delta E = \Delta U$. **3** The volume remains constant until the piston starts moving, and the pressure remains constant afterwards. **4** There are no electrical, shaft, or other forms of work involved.

Analysis We take the contents of the cylinder as the *system* (Fig. 5–32). This is a *closed system* since no mass crosses the system boundary during the process. We observe that a piston–cylinder device typically involves a moving boundary and thus boundary work, W_b. Also, the boundary work is done by the system, and heat is transferred to the system.

(*a*) The final temperature can be determined easily by using the ideal-gas relation between states 1 and 3 in the following form:

$$\frac{P_1V_1}{T_1} = \frac{P_3V_3}{T_3} \longrightarrow \frac{(150\text{ kPa})(V_1)}{300\text{ K}} = \frac{(350\text{ kPa})(2V_1)}{T_3}$$

$$T_3 = \mathbf{1400\ K}$$

(*b*) The work done could be determined by integration, but for this case it is much easier to find it from the area under the process curve on a *P*-*V* diagram, shown in Fig. 5–32:

$$A = (V_3 - V_1)P_2 = (0.4\text{ m}^3)(350\text{ kPa}) = 140\text{ m}^3\cdot\text{kPa}$$

Therefore,

$$W_{13} = \mathbf{140\ kJ}$$

The work is done by the system (to raise the piston and to push the atmospheric air out of the way), and thus it is work output.

(*c*) Under the stated assumptions and observations, the energy balance on the system between the initial and final states (process 1–3) can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$Q_{\text{in}} - W_{b,\text{out}} = \Delta U = m(u_3 - u_1)$$

The mass of the system can be determined from the ideal-gas relation:

$$m = \frac{P_1V_1}{RT_1} = \frac{(150\text{ kPa})(0.4\text{ m}^3)}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{ K})} = 0.697\text{ kg}$$

The internal energies are determined from the air table (Table A–21) to be

$$u_1 = u_{@\,300\text{ K}} = 214.07\text{ kJ/kg}$$

$$u_3 = u_{@\,1400\text{ K}} = 1113.52\text{ kJ/kg}$$

Thus,

$$Q_{\text{in}} - 140\text{ kJ} = (0.697\text{ kg})[(1113.52 - 214.07)\text{ kJ/kg}]$$

$$Q_{\text{in}} = \mathbf{767\ kJ}$$

Discussion The positive sign verifies that heat is transferred to the system.

5–5 ▪ INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLIDS AND LIQUIDS

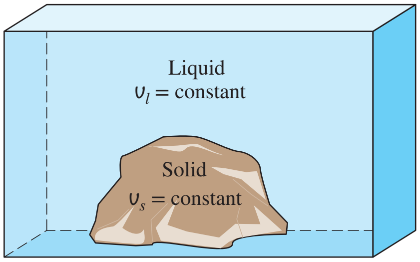


FIGURE 5–33
The specific volumes of incompressible substances remain constant during a process.

A substance whose specific volume (or density) is constant is called an **incompressible substance**. The specific volumes of solids and liquids essentially remain constant during a process (Fig. 5–33). Therefore, liquids and solids can be approximated as incompressible substances without sacrificing much in accuracy. The constant-volume assumption should be taken to imply that the energy associated with the volume change is negligible compared with other forms of energy. Otherwise, this assumption would be ridiculous for studying the thermal stresses in solids (caused by volume change with temperature) or analyzing liquid-in-glass thermometers.

It can be mathematically shown that the constant-volume and constant-pressure specific heats are identical for incompressible substances (Fig. 5–34). Therefore, for solids and liquids, the subscripts on c_p and c_υ can be dropped, and both specific heats can be represented by a single symbol c. That is,

$$c_p = c_\upsilon = c \tag{5–32}$$

This result could also be deduced from the physical definitions of constant-volume and constant-pressure specific heats. Specific heat values for several common liquids and solids are given in Table A–3.

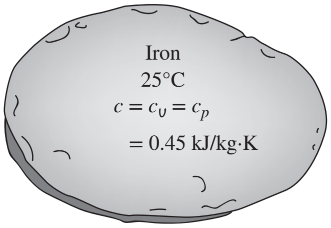


FIGURE 5–34
The c_υ and c_p values of incompressible substances are identical and are denoted by c.

Internal Energy Changes

Like those of ideal gases, the specific heats of incompressible substances depend on temperature only. Thus, the partial differentials in the defining equation of c_υ can be replaced by ordinary differentials, which yield

$$du = c_\upsilon dT = c(T)\, dT \tag{5–33}$$

The change in internal energy between states 1 and 2 is then obtained by integration:

$$\Delta u = u_2 - u_1 = \int_1^2 c(T)\, dT \qquad \text{(kJ/kg)} \tag{5–34}$$

The variation of specific heat c with temperature should be known before this integration can be carried out. For small temperature intervals, a c value at the average temperature can be used and treated as a constant, yielding

$$\Delta u \cong c_{\text{avg}}(T_2 - T_1) \qquad \text{(kJ/kg)} \tag{5–35}$$

Enthalpy Changes

Using the definition of enthalpy $h = u + P\upsilon$ and noting that $\upsilon =$ constant, the differential form of the enthalpy change of incompressible substances can be determined by differentiation to be

$$dh = du + \upsilon\, dP + P\, d\upsilon^{\nearrow 0} = du + \upsilon\, dP \tag{5–36}$$

Integrating,

$$\Delta h = \Delta u + \upsilon\, \Delta P \cong c_{\text{avg}}\, \Delta T + \upsilon\, \Delta P \qquad \text{(kJ/kg)} \tag{5–37}$$

For *solids,* the term $\upsilon\, \Delta P$ is insignificant, and thus $\Delta h \cong \Delta u \cong c_{\text{avg}} \Delta T$. For *liquids,* two special cases are commonly encountered:

1. *Constant-pressure processes,* as in heaters ($\Delta P = 0$): $\Delta h = \Delta u \cong c_{\text{avg}} \Delta T$
2. *Constant-temperature processes,* as in pumps ($\Delta T = 0$): $\Delta h = \upsilon\, \Delta P$

For a process between states 1 and 2, the last relation can be expressed as $h_2 - h_1 = \upsilon(P_2 - P_1)$. By taking state 2 to be the compressed liquid state at a given T and P and state 1 to be the saturated liquid state at the same temperature, the enthalpy of the compressed liquid can be expressed as

$$h_{@\,P,T} \cong h_{f\,@\,T} + \upsilon_{f\,@\,T}(P - P_{\text{sat}\,@\,T}) \tag{5–38}$$

as discussed in Chap. 4. This is an improvement over the assumption that the enthalpy of the compressed liquid could be taken as h_f at the given temperature (that is, $h_{@\,P,T} \cong h_{f\,@\,T}$). However, the contribution of the last term is often very small and is neglected. (Note that at high temperatures and pressures, Eq. 5–38 may overcorrect the enthalpy and result in a larger error than the approximation $h \cong h_{f\,@\,T}$.)

EXAMPLE 5–11 Enthalpy of Compressed Liquid

Determine the enthalpy of liquid water at 100°C and 15 MPa (*a*) by using compressed liquid tables, (*b*) by approximating it as a saturated liquid, and (*c*) by using the correction given by Eq. 5–38.

SOLUTION The enthalpy of liquid water is to be determined exactly and approximately.
Analysis At 100°C, the saturation pressure of water is 101.42 kPa, and since $P > P_{\text{sat}}$, the water exists as a compressed liquid at the specified state.

(*a*) From compressed liquid tables, we read

$$\left.\begin{aligned} P &= 15\text{ MPa} \\ T &= 100^\circ\text{C} \end{aligned}\right\} \; h = \mathbf{430.39\ kJ/kg} \qquad \text{(Table A–7)}$$

This is the exact value.

(*b*) Approximating the compressed liquid as a saturated liquid at 100°C, as is commonly done, we obtain

$$h \cong h_{f\,@\,100^\circ\text{C}} = \mathbf{419.17\ kJ/kg}$$

This value is in error by about 2.6 percent.

(*c*) From Eq. 5–38,

$$\begin{aligned} h_{@\,P,T} &\cong h_{f\,@\,T} + v_{f\,@\,T}(P - P_{\text{sat}\,@\,T}) \\ &= (419.17\text{ kJ/kg}) + (0.001\text{ m}^3\text{/kg})[(15{,}000 - 101.42)\text{ kPa}]\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right) \\ &= \mathbf{434.07\ kJ/kg} \end{aligned}$$

Discussion Note that the correction term reduced the error from 2.6 to about 1 percent in this case. However, this improvement in accuracy is often not worth the extra effort involved.

EXAMPLE 5–12 Cooling of an Iron Block by Water

A 50-kg iron block at 80°C is dropped into an insulated tank that contains 0.5 m^3 of liquid water at 25°C. Determine the temperature when thermal equilibrium is reached.

SOLUTION An iron block is dropped into water in an insulated tank. The final temperature when thermal equilibrium is reached is to be determined.
Assumptions **1** Both water and the iron block are incompressible substances. **2** Constant specific heats at room temperature can be used for water and the iron. **3** The system is stationary, and thus the kinetic and potential energy changes are zero, $\Delta\text{KE} = \Delta\text{PE} = 0$ and $\Delta E = \Delta U$. **4** There are no electrical, shaft, or other forms of work involved. **5** The system is well insulated, and thus there is no heat transfer.
Analysis We take the entire contents of the tank as the *system* (Fig. 5–35). This is a *closed system* since no mass crosses the system boundary during the process. We observe that the volume of a rigid tank is constant, and thus there is no boundary work. The energy balance on the system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$0 = \Delta U$$

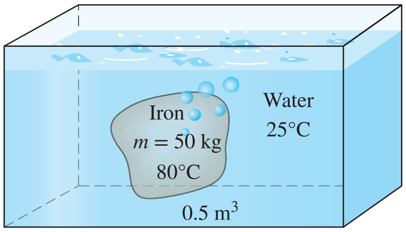


FIGURE 5–35
Schematic for Example 5–12.

The total internal energy U is an extensive property, and therefore it can be expressed as the sum of the internal energies of the parts of the system. Then the total internal energy change of the system becomes

$$\Delta U_{\text{sys}} = \Delta U_{\text{iron}} + \Delta U_{\text{water}} = 0$$
$$[mc(T_2 - T_1)]_{\text{iron}} + [mc(T_2 - T_1)]_{\text{water}} = 0$$

The specific volume of liquid water at or about room temperature can be taken to be 0.001 m³/kg. Then the mass of the water is

$$m_{\text{water}} = \frac{V}{v} = \frac{0.5 \text{ m}^3}{0.001 \text{ m}^3/\text{kg}} = 500 \text{ kg}$$

The specific heats of iron and liquid water are determined from Table A–3 to be $c_{\text{iron}} = 0.45$ kJ/kg·°C and $c_{\text{water}} = 4.18$ kJ/kg·°C. Substituting these values into the energy equation, we obtain

$$(50 \text{ kg})(0.45 \text{ kJ/kg·°C})(T_2 - 80\text{°C}) + (500 \text{ kg})(4.18 \text{ kJ/kg·°C})(T_2 - 25\text{°C}) = 0$$

$$T_2 = \mathbf{25.6°C}$$

Therefore, when thermal equilibrium is established, both the water and iron will be at 25.6°C.

Discussion The small rise in water temperature is due to its large mass and large specific heat.

EXAMPLE 5–13 Cooling of Carbon Steel Balls in Air

Carbon steel balls ($\rho = 7833$ kg/m³ and $c_p = 0.465$ kJ/kg·°C) 8 mm in diameter are annealed by heating them first to 900°C in a furnace, and then allowing them to cool slowly to 100°C in ambient air at 35°C, as shown in Fig. 5–36. If 2500 balls are to be annealed per hour, determine the total rate of heat transfer from the balls to the ambient air.

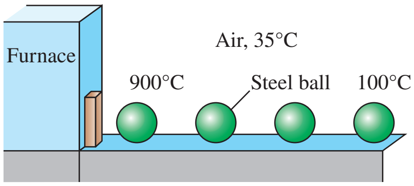


FIGURE 5–36
Schematic for Example 5–13.

SOLUTION Carbon steel balls are to be annealed at a rate of 2500/h by heating them first and then allowing them to cool slowly in ambient air. The total rate of heat transfer from the balls to the ambient air is to be determined.

Assumptions **1** The thermal properties of the balls are constant. **2** There are no changes in kinetic and potential energies. **3** The balls are at a uniform temperature at the end of the process.

Properties The density and specific heat of the balls are given to be $\rho = 7833$ kg/m³ and $c_p = 0.465$ kJ/kg·°C.

Analysis We take a single ball as the system. The energy balance for this closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$
$$-Q_{\text{out}} = \Delta U_{\text{ball}} = m(u_2 - u_1)$$
$$Q_{\text{out}} = mc(T_1 - T_2)$$

The amount of heat transfer from a single ball is

$$m = \rho V = \rho \frac{\pi D^3}{6} = (7833 \text{ kg/m}^3)\frac{\pi (0.008 \text{ m})^3}{6} = 0.00210 \text{ kg}$$

$$Q_{\text{out}} = mc(T_1 - T_2) = (0.00210 \text{ kg})(0.465 \text{ kJ/kg·°C})(900 - 100)\text{°C}$$
$$= 0.781 \text{ kJ (per ball)}$$

Then the total rate of heat transfer from the balls to the ambient air becomes

$$\dot{Q}_{\text{out}} = \dot{n}_{\text{ball}}\, Q_{\text{out}} = (2500 \text{ ball/h})(0.781 \text{ kJ/ball}) = 1953 \text{ kJ/h} = \mathbf{542 \text{ W}}$$

Discussion For solids and liquids, constant-pressure and constant-volume specific heats are identical and can be represented by a single symbol c. However, it is customary to use the symbol c_p for the specific heat of incompressible substances.

SUMMARY

Work is the energy transferred as a force acts on a system through a distance. The most common form of mechanical work is the *boundary work,* which is the work associated with the expansion and compression of substances. On a P-V diagram, the area under the process curve represents the boundary work for a quasi-equilibrium process. Various forms of boundary work are expressed as follows:

1. General $\qquad W_b = \int_1^2 P\, dV$

2. Isobaric process

$$W_b = P_0(V_2 - V_1) \qquad (P_1 = P_2 = P_0 = \text{constant})$$

3. Polytropic process

$$W_b = \frac{P_2V_2 - P_1V_1}{1 - n} \quad (n \neq 1) \qquad (PV^n = \text{constant})$$

4. Isothermal process of an ideal gas

$$W_b = P_1V_1 \ln\frac{V_2}{V_1}$$

$$= mRT_0 \ln\frac{V_2}{V_1} \qquad (PV = mRT_0 = \text{constant})$$

The first law of thermodynamics is essentially an expression of the conservation of energy principle, also called the energy balance. The general energy balance for *any system* undergoing *any process* can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

It can also be expressed in the *rate form* as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal,} \\ \text{kinetic, potential, etc., energies}}}$$

Taking heat transfer *to* the system and work done *by* the system to be positive quantities, the energy balance for a closed system can also be expressed as

$$Q - W = \Delta U + \Delta \text{KE} + \Delta \text{PE}$$

where

$$W = W_{\text{other}} + W_b$$
$$\Delta U = m(u_2 - u_1)$$
$$\Delta \text{KE} = \tfrac{1}{2}m(V_2^2 - V_1^2)$$
$$\Delta \text{PE} = mg(z_2 - z_1)$$

For a *constant-pressure process,* $W_b + \Delta U = \Delta H$. Thus,

$$Q - W_{\text{other}} = \Delta H + \Delta \text{KE} + \Delta \text{PE}$$

Note that the relation above is limited to constant-pressure processes of closed systems and is NOT valid for processes during which pressure varies.

The amount of energy needed to raise the temperature of a unit mass of a substance by one degree is called the *specific heat at constant volume* c_υ for a constant-volume process and the *specific heat at constant pressure* c_p for a constant-pressure process. They are defined as

$$c_\upsilon = \left(\frac{\partial u}{\partial T}\right)_\upsilon \quad \text{and} \quad c_p = \left(\frac{\partial h}{\partial T}\right)_p$$

For ideal gases *u, h,* c_υ, and c_p are functions of temperature alone. The Δu and Δh of ideal gases are expressed as

$$\Delta u = u_2 - u_1 = \int_1^2 c_\upsilon(T)\, dT \cong c_{\upsilon,\text{avg}}(T_2 - T_1)$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T)\, dT \cong c_{p,\text{avg}}(T_2 - T_1)$$

For ideal gases, c_υ and c_p are related by

$$c_p = c_\upsilon + R$$

where R is the gas constant. The *specific heat ratio* k is defined as

$$k = \frac{c_p}{c_\upsilon}$$

For *incompressible substances* (liquids and solids), both the constant-pressure and constant-volume specific heats are identical and denoted by c:

$$c_p = c_\upsilon = c$$

The Δu and Δh of incompressible substances are given by

$$\Delta u = \int_1^2 c(T)\, dT \cong c_{\text{avg}}(T_2 - T_1)$$

$$\Delta h = \Delta u + \upsilon\, \Delta P$$

REFERENCES AND SUGGESTED READINGS

1. ASHRAE *Handbook of Fundamentals.* SI version. Atlanta, GA: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc., 1993.
2. ASHRAE *Handbook of Refrigeration.* SI version. Atlanta, GA: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc., 1994.

PROBLEMS*

Moving Boundary Work

5–1C Is the boundary work associated with constant-volume systems always zero?

5–2C On a P-v diagram, what does the area under the process curve represent?

5–3C An ideal gas at a given state expands to a fixed final volume first at constant pressure and then at constant temperature. For which case is the work done greater?

5–4 Calculate the total work, in kJ, for process 1–3 shown in Fig. P5–4 when the system consists of 2 kg of nitrogen.

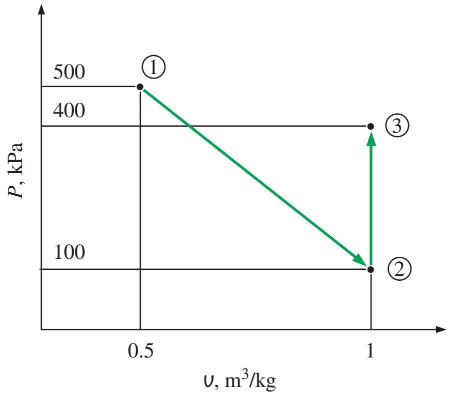


FIGURE P5–4

5–5 Nitrogen at an initial state of 300 K, 150 kPa, and 0.2 m^3 is compressed slowly in an isothermal process to a final pressure of 800 kPa. Determine the work done during this process.

5–6 The volume of 1 kg of helium in a piston–cylinder device is initially 5 m^3. Now helium is compressed to 2 m^3 while its pressure is maintained constant at 130 kPa. Determine the initial and final temperatures of helium as well as the work required to compress it, in kJ.

5–7 A piston–cylinder device with a set of stops initially contains 0.6 kg of steam at 1.0 MPa and 400°C. The location of the stops corresponds to 40 percent of the initial volume. Now the steam is cooled. Determine the compression work if the final state is (*a*) 1.0 MPa and 250°C and (*b*) 500 kPa. (*c*) Also determine the temperature at the final state in part *b*.

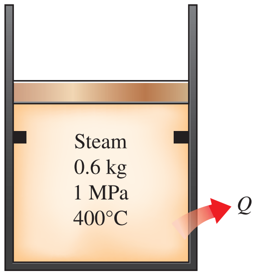


FIGURE P5–7

5–8 A mass of 5 kg of saturated water vapor at 150 kPa is heated at constant pressure until the temperature reaches 200°C. Calculate the work done by the steam during this process.

5–9E A frictionless piston–cylinder device contains 16 lbm of superheated water vapor at 40 psia and 600°F. Steam is now cooled at constant pressure until 70 percent of it, by mass, condenses. Determine the work done during this process.

5–10 1 m^3 of saturated liquid water at 200°C is expanded isothermally in a closed system until its quality is 80 percent. Determine the total work produced by this expansion, in kJ.

5–11 Argon is compressed in a polytropic process with $n = 1.2$ from 120 kPa and 30°C to 1200 kPa in a piston–cylinder device. Determine the final temperature of the argon.

5–12 A gas is compressed from an initial volume of 0.42 m^3 to a final volume of 0.12 m^3. During the quasi-equilibrium process, the pressure changes with volume according to the relation $P = aV + b$, where $a = -1200$ kPa/m^3 and $b = 600$ kPa. Calculate the work done during this process (*a*) by plotting the process on a P-V diagram and finding the area under the process curve and (*b*) by performing the necessary integrations.

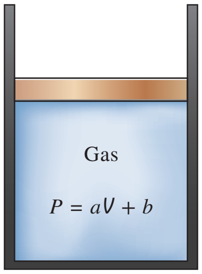


FIGURE P5–12

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* Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

5–13 A mass of 1.5 kg of air at 120 kPa and 24°C is contained in a gas-tight, frictionless piston–cylinder device. The air is now compressed to a final pressure of 600 kPa. During the process, heat is transferred from the air such that the temperature inside the cylinder remains constant. Calculate the work input during this process. *Answer:* 206 kJ

5–14 During some actual expansion and compression processes in piston–cylinder devices, the gases have been observed to satisfy the relationship $PV^n = C$, where n and C are constants. Calculate the work done when a gas expands from 350 kPa and 0.03 m^3 to a final volume of 0.2 m^3 for the case of $n = 1.5$.

5–15 Reconsider Prob. 5–14. Using appropriate software, plot the process described in the problem on a P-V diagram, and investigate the effect of the polytropic exponent n on the boundary work. Let the polytropic exponent vary from 1.1 to 1.6. Plot the boundary work versus the polytropic exponent, and discuss the results.

5–16 A frictionless piston–cylinder device contains 5 kg of nitrogen at 100 kPa and 250 K. Nitrogen is now compressed slowly according to the relation $PV^{1.4}$ = constant until it reaches a final temperature of 450 K. Calculate the work input during this process. *Answer:* 742 kJ

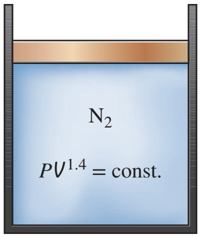


FIGURE P5–16

5–17E During an expansion process, the pressure of a gas changes from 15 to 100 psia according to the relation $P = aV + b$, where a = 5 psia/ft^3 and b is a constant. If the initial volume of the gas is 7 ft^3, calculate the work done during the process. *Answer:* 181 Btu

5–18 A piston–cylinder device initially contains 0.4 kg of nitrogen gas at 160 kPa and 140°C. The nitrogen is now expanded isothermally to a pressure of 100 kPa. Determine the boundary work done during this process. *Answer:* 23.0 kJ

FIGURE P5–18

5–19 A piston–cylinder device contains 0.15 kg of air initially at 2 MPa and 350°C. The air is first expanded isothermally to 500 kPa, then compressed polytropically with a polytropic exponent of 1.2 to the initial pressure, and finally compressed at the constant pressure to the initial state. Determine the boundary work for each process and the net work of the cycle.

5–20 1 kg of water that is initially at 90°C with a quality of 10 percent occupies a spring-loaded piston–cylinder device, such as that in Fig. P5–20. This device is now heated until the pressure rises to 800 kPa and the temperature is 250°C. Determine the total work produced during this process, in kJ. *Answer:* 24.5 kJ

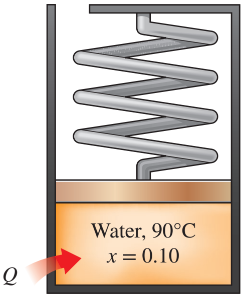


FIGURE P5–20

5–21 An ideal gas undergoes two processes in a piston–cylinder device as follows:

1–2 Polytropic compression from T_1 and P_1 with a polytropic exponent n and a compression ratio of $r = V_1/V_2$.

2–3 Constant pressure expansion at $P_3 = P_2$ until $V_3 = V_1$.

(*a*) Sketch the processes on a single P-V diagram.

(*b*) Obtain an expression for the ratio of the compression-to-expansion work as a function of n and r.

(*c*) Find the value of this ratio for values of $n = 1.4$ and $r = 6$.

Answers: (*b*) $\frac{1}{n-1}\left(\frac{1-r^{1-n}}{r-1}\right)$ (*c*) 0.256

5–22 A piston–cylinder device contains 50 kg of water at 250 kPa and 25°C. The cross-sectional area of the piston is 0.1 m^2. Heat is now transferred to the water, causing part of it to evaporate and expand. When the volume reaches 0.2 m^3, the piston reaches a linear spring whose spring constant is 100 kN/m. More heat is transferred to the water until the piston rises 20 cm more. Determine (*a*) the final pressure and temperature and (*b*) the work done during this process. Also, show the process on a P-V diagram. *Answers:* (*a*) 450 kPa, 147.9°C, (*b*) 44.5 kJ

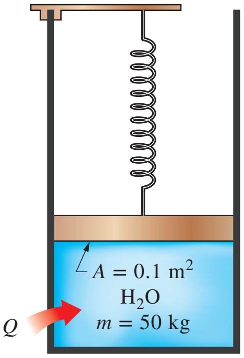


FIGURE P5–22

5–23 Reconsider Prob. 5–22. Using appropriate software, investigate the effect of the spring constant on the final pressure in the cylinder and the boundary work done. Let the spring constant vary from 50 kN/m to 500 kN/m. Plot the final pressure and the boundary work against the spring constant, and discuss the results.

5–24 Carbon dioxide contained in a piston–cylinder device is compressed from 0.3 to 0.1 m^3. During the process, the pressure and volume are related by $P = aV^{-2}$, where $a = 8$ kPa·m^6. Calculate the work done on the carbon dioxide during this process. *Answer:* 53.3 kJ

Closed System Energy Analysis

5–25 A rigid container equipped with a stirring device contains 2.5 kg of motor oil. Determine the rate of specific energy increase when heat is transferred to the oil at a rate of 1 W, and 1.5 W of power is applied to the stirring device.

5–26 Complete each line of the following table on the basis of the conservation of energy principle for a closed system.

Q_{in} kJ	W_{out} kJ	E_1 kJ	E_2 kJ	m kg	$e_2 - e_1$ kJ/kg
280	—	1020	860	3	—
−350	130	550	—	5	—
—	260	300	—	2	−150
300	—	750	500	1	—
—	−200	—	300	2	−100

FIGURE P5–26

5–27 A substance is contained in a well-insulated rigid container that is equipped with a stirring device, as shown in Fig. P5–27. Determine the change in the internal energy of this substance when 15 kJ of work is applied to the stirring device.

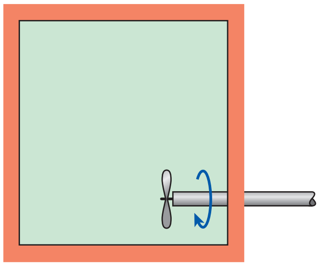


FIGURE P5–27

5–28 A 0.5-m^3 rigid tank contains refrigerant-134a initially at 160 kPa and 40 percent quality. Heat is now transferred to the refrigerant until the pressure reaches 700 kPa. Determine (*a*) the mass of the refrigerant in the tank and (*b*) the amount of heat transferred. Also, show the process on a P-υ diagram with respect to saturation lines.

5–29E A 20-ft^3 rigid tank initially contains saturated refrigerant-134a vapor at 160 psia. As a result of heat transfer from the refrigerant, the pressure drops to 50 psia. Show the process on a P-υ diagram with respect to saturation lines, and determine (*a*) the final temperature, (*b*) the amount of refrigerant that has condensed, and (*c*) the heat transfer.

5–30 A rigid 10-L vessel initially contains a mixture of liquid water and vapor at 100°C with 12.3 percent quality. The mixture is then heated until its temperature is 180°C. Calculate the heat transfer required for this process. *Answer:* 92.5 kJ

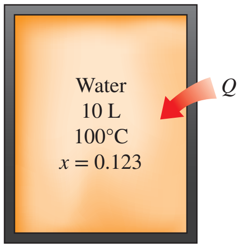


FIGURE P5–30

5–31E A rigid 1-ft^3 vessel contains R-134a originally at –20°F and 27.7 percent quality. The refrigerant is then heated until its temperature is 100°F. Calculate the heat transfer required to do this. *Answer:* 84.7 Btu

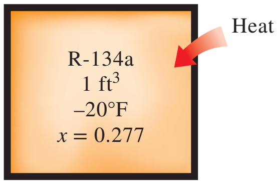


FIGURE P5–31E

5–32 A piston–cylinder device contains 5 kg of refrigerant-134a at 800 kPa and 70°C. The refrigerant is now cooled at constant pressure until it exists as a liquid at 15°C. Determine the amount of heat loss and show the process on a T-υ diagram with respect to saturation lines. *Answer:* 1173 kJ

5–33E A piston–cylinder device contains 0.5 lbm of water initially at 120 psia and 2 ft^3. Now 200 Btu of heat is transferred to the water while its pressure is held constant. Determine the final temperature of the water. Also, show the process on a T-υ diagram with respect to saturation lines.

5–34 2 kg of saturated liquid water at 150°C is heated at constant pressure in a piston–cylinder device until it is saturated vapor. Determine the heat transfer required for this process.

5–35 An insulated piston–cylinder device contains 5 L of saturated liquid water at a constant pressure of 175 kPa. Water is stirred by a paddle wheel while a current of 8 A flows for 45 min through a resistor placed in the water. If one-half of the liquid is evaporated during this constant-pressure process and the paddle-wheel work amounts to 400 kJ, determine the voltage of the source. Also, show the process on a P-υ diagram with respect to saturation lines. *Answer:* 224 V

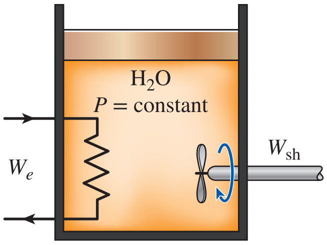


FIGURE P5–35

5–36 A 40-L electrical radiator containing heating oil is placed in a 50-m^3 room. Both the room and the oil in the radiator are initially at 10°C. The radiator with a rating of 2.4 kW is now turned on. At the same time, heat is lost from the room at an average rate of 0.35 kJ/s. After some time, the average temperature is measured to be 20°C for the air in the room, and 50°C for the oil in the radiator. Taking the density and the specific heat of the oil to be 950 kg/m^3 and 2.2 kJ/kg·°C, respectively, determine how long the heater is kept on. Assume the room is well sealed so that there are no air leaks.

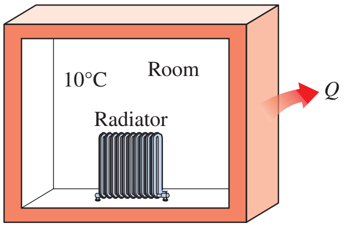


FIGURE P5–36

5–37 Steam at 75 kPa and 8 percent quality is contained in a spring-loaded piston–cylinder device, as shown in Fig. P5–37, with an initial volume of 2 m^3. Steam is now heated until its volume is 5 m^3 and its pressure is 225 kPa. Determine the heat transferred to and the work produced by the steam during this process.

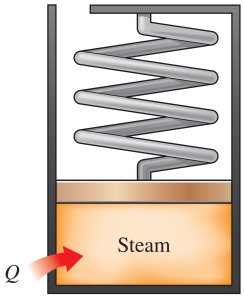


FIGURE P5–37

5–38 A piston–cylinder device initially contains 0.6 m^3 of saturated water vapor at 250 kPa. At this state, the piston is resting on a set of stops, and the mass of the piston is such that a pressure of 300 kPa is required to move it. Heat is now slowly transferred to the steam until the volume doubles. Show the process on a P-υ diagram with respect to saturation lines and determine (*a*) the final temperature, (*b*) the work done during this process, and (*c*) the total heat transfer. *Answers:* (*a*) 662°C, (*b*) 180 kJ, (*c*) 910 kJ

5–39 An insulated tank is divided into two parts by a partition. One part of the tank contains 2.5 kg of compressed liquid water at 60°C and 600 kPa while the other part is evacuated. The partition is now removed, and the water expands to fill the entire tank. Determine the final temperature of the water and the volume of the tank for a final pressure of 10 kPa.

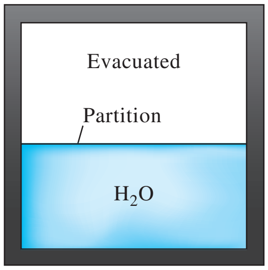


FIGURE P5–39

5–40 Reconsider Prob. 5–39. Using appropriate software, investigate the effect of the initial pressure of water on the final temperature in the tank. Let the initial pressure vary from 100 to 600 kPa. Plot the final temperature against the initial pressure, and discuss the results.

5–41 Two tanks (Tank A and Tank B) are separated by a partition. Initially Tank A contains 2 kg of steam at 1 MPa and 300°C while Tank B contains 3 kg of saturated liquid–vapor mixture at 150°C with a vapor mass fraction of 50 percent. The partition is now removed and the two sides are allowed to mix until mechanical and thermal equilibrium are established. If the pressure at the final state is 300 kPa, determine (*a*) the temperature and quality of the steam (if mixture) at the final state and (*b*) the amount of heat lost from the tanks.

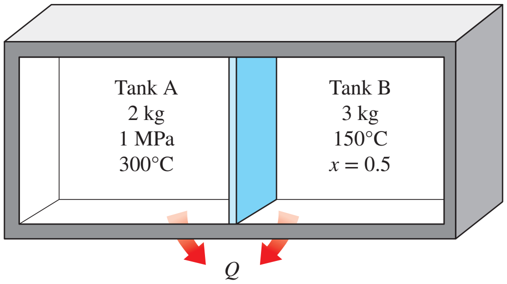


FIGURE P5–41

Specific Heats, Δu, and Δh of Ideal Gases

5–42C Is the energy required to heat air from 295 to 305 K the same as the energy required to heat it from 345 to 355 K? Assume the pressure remains constant in both cases.

5–43C A fixed mass of an ideal gas is heated from 50 to 80°C at a constant pressure of (*a*) 1 atm and (*b*) 3 atm. For which case do you think the energy required will be greater? Why?

5–44C A fixed mass of an ideal gas is heated from 50 to 80°C at a constant volume of (*a*) 1 m^3 and (*b*) 3 m^3. For which case do you think the energy required will be greater? Why?

5–45C A fixed mass of an ideal gas is heated from 50 to 80°C (*a*) at constant volume and (*b*) at constant pressure. For which case do you think the energy required will be greater? Why?

5–46C Is the relation $\Delta u = mc_{\upsilon,\text{avg}}\Delta T$ restricted to constant-volume processes only, or can it be used for any kind of process of an ideal gas?

5–47C Is the relation $\Delta h = mc_{p,\text{avg}}\Delta T$ restricted to constant-pressure processes only, or can it be used for any kind of process of an ideal gas?

5–48E What is the change in the internal energy, in Btu/lbm, of air as its temperature changes from 100 to 200°F? Is there any difference if the temperature were to change from 0 to 100°F?

5–49 Neon is compressed from 100 kPa and 20°C to 500 kPa in an isothermal compressor. Determine the change in the specific volume and specific enthalpy of neon caused by this compression.

5–50 What is the change in the enthalpy, in kJ/kg, of oxygen as its temperature changes from 150 to 250°C? Is there any difference if the temperature change were from 0 to 100°C? Does the pressure at the beginning and end of this process have any effect on the enthalpy change?

5–51 A mass of 10 g of nitrogen is contained in the spring-loaded piston–cylinder device shown in Fig. P5–51. The spring constant is 1 kN/m, and the piston diameter is 10 cm. When the spring exerts no force against the piston, the nitrogen is at 120 kPa and 27°C. The device is now heated until its volume is 10 percent greater than the original volume. Determine the change in the specific internal energy and enthalpy of the nitrogen. *Answers:* 46.8 kJ/kg, 65.5 kJ/kg

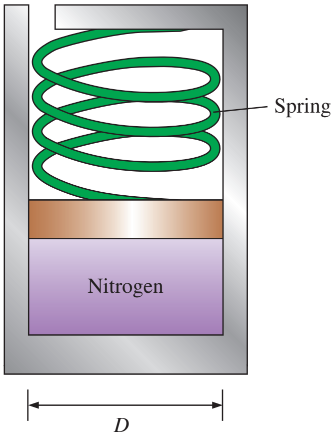


FIGURE P5–51

5–52 Determine the internal energy change Δu of hydrogen, in kJ/kg, as it is heated from 200 to 800 K, using (*a*) the empirical specific heat equation as a function of temperature (Table A–2*c*), (*b*) the c_v value at the average temperature (Table A–2*b*), and (*c*) the c_v value at room temperature (Table A–2*a*)

5–53E Determine the enthalpy change Δh of oxygen, in Btu/lbm, as it is heated from 800 to 1500 R, using (*a*) the empirical specific heat equation as a function of temperature (Table A–2E*c*), (*b*) the c_p value at the average temperature (Table A–2E*b*), and (*c*) the c_p value at room temperature (Table A–2E*a*). *Answers:* (*a*) 170 Btu/lbm, (*b*) 169 Btu/lbm, (*c*) 153 Btu/lbm

Closed-System Energy Analysis: Ideal Gases

5–54C Is it possible to compress an ideal gas isothermally in an adiabatic piston–cylinder device? Explain.

5–55 Nitrogen in a rigid vessel is cooled by rejecting 100 kJ/kg of heat. Determine the internal energy change of the nitrogen, in kJ/kg.

5–56E A piston–cylinder device containing carbon dioxide gas undergoes an isobaric process from 15 psia and 80°F to 200°F. Determine the work and the heat transfer associated with this process, in Btu/lbm. *Answers:* 5.42 Btu/lbm, 24.4 Btu/lbm

5–57 A 3-m^3 rigid tank contains hydrogen at 250 kPa and 550 K. The gas is now cooled until its temperature drops to 350 K. Determine (*a*) the final pressure in the tank and (*b*) the amount of heat transfer.

5–58 1 kg of oxygen is heated from 20 to 120°C. Determine the amount of heat transfer required when this is done during a (*a*) constant-volume process and (*b*) isobaric process.

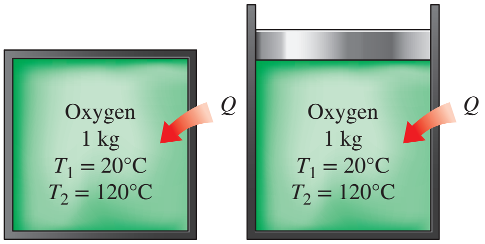


FIGURE P5–58

5–59E A 10-ft^3 tank contains oxygen initially at 14.7 psia and 80°F. A paddle wheel within the tank is rotated until the pressure inside rises to 20 psia. During the process 20 Btu of heat is lost to the surroundings. Determine the paddle-wheel work done. Neglect the energy stored in the paddle wheel.

5–60 A 4-m × 5-m × 7-m room is heated by the radiator of a steam-heating system. The steam radiator transfers heat at a rate of 10,000 kJ/h, and a 100-W fan is used to distribute the warm air in the room. The rate of heat loss from the room is estimated to be about 5000 kJ/h. If the initial temperature of the room air is 10°C, determine how long it will take for the air temperature to rise to 20°C. Assume constant specific heats at room temperature.

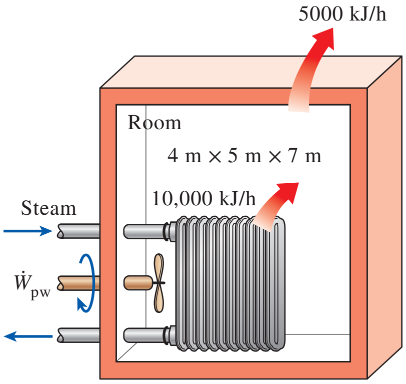


FIGURE P5–60

5–61 An insulated rigid tank is divided into two equal parts by a partition. Initially, one part contains 4 kg of an ideal gas at 800 kPa and 50°C, and the other part is evacuated. The partition is now removed, and the gas expands into the entire tank. Determine the final temperature and pressure in the tank.

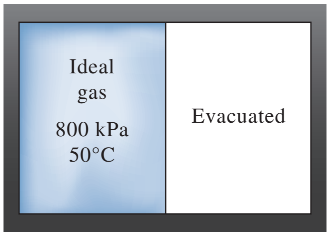


FIGURE P5–61

5–62 An ideal gas contained in a piston–cylinder device undergoes an isothermal compression process which begins with an initial pressure and volume of 100 kPa and 0.6 m^3, respectively. During the process there is a heat transfer of 60 kJ from the ideal gas to the surroundings. Determine the volume and pressure at the end of the process. *Answers:* 0.221 m^3, 272 kPa

5–63 A 4-m × 5-m × 6-m room is to be heated by a baseboard resistance heater. It is desired that the resistance heater be able to raise the air temperature in the room from 5 to 25°C within 17 min. Assuming no heat losses from the room and an atmospheric pressure of 100 kPa, determine the required power of the resistance heater. Assume constant specific heats at room temperature. *Answer:* 2.12 kW

5–64 An insulated piston–cylinder device initially contains 0.3 m^3 of carbon dioxide at 200 kPa and 27°C. An electric switch is turned on, and a 110-V source supplies current to a resistance heater inside the cylinder for a period of 10 min. The pressure is held constant during the process, while the volume is doubled. Determine the current that passes through the resistance heater.

5–65 Argon is compressed in a polytropic process with $n = 1.2$ from 120 kPa and 10°C to 800 kPa in a piston–cylinder device. Determine the work produced and heat transferred during this compression process, in kJ/kg.

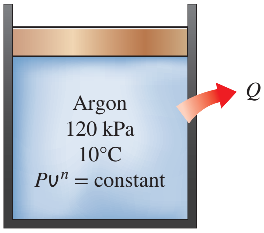


FIGURE P5–65

5–66 An insulated piston–cylinder device contains 100 L of air at 400 kPa and 25°C. A paddle wheel within the cylinder is rotated until 15 kJ of work is done on the air while the pressure is held constant. Determine the final temperature of the air. Neglect the energy stored in the paddle wheel.

5–67 Air is contained in a variable-load piston–cylinder device equipped with a paddle wheel. Initially, air is at 400 kPa and 17°C. The paddle wheel is now turned by an external electric motor until 75 kJ/kg of work has been transferred to air. During this process, heat is transferred to maintain a constant air temperature while allowing the gas volume to triple. Calculate the required amount of heat transfer, in kJ/kg. *Answer:* 16.4 kJ/kg

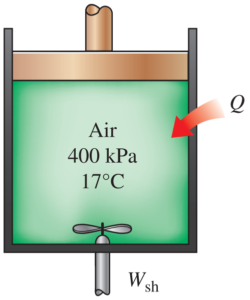


FIGURE P5–67

5–68 A mass of 15 kg of air in a piston–cylinder device is heated from 25 to 95°C by passing current through a resistance heater inside the cylinder. The pressure inside the cylinder is held constant at 300 kPa during the process, and a heat loss of 60 kJ occurs. Determine the electric energy supplied, in kWh. *Answer:* 0.310 kWh

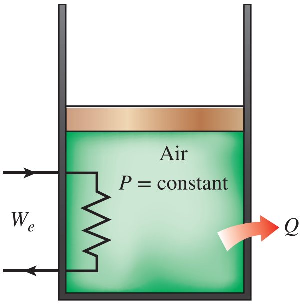


FIGURE P5–68

5–69E A 3-ft^3 adiabatic rigid container is divided into two equal volumes by a thin membrane, as shown in Fig. P5–69E. Initially, one of these chambers is filled with air at 100 psia and 100°F while the other chamber is evacuated. Determine the internal energy change of the air when the membrane is ruptured. Also determine the final air pressure in the container.

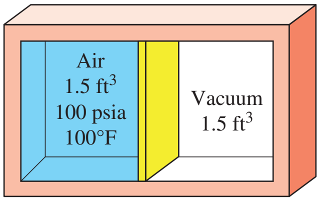


FIGURE P5–69E

5–70 A piston–cylinder device contains 2.2 kg of nitrogen initially at 100 kPa and 25°C. The nitrogen is now compressed slowly in a polytropic process during which $P\mathrm{V}^{1.3}$ = constant until the volume is reduced by one-half. Determine the work done and the heat transfer for this process.

5–71 Reconsider Prob. 5–70. Using appropriate software, plot the process described in the problem on a P-V diagram, and investigate the effect of the polytropic exponent n on the boundary work and heat transfer. Let the polytropic exponent vary from 1.0 to 1.4. Plot the boundary work and the heat transfer versus the polytropic exponent, and discuss the results.

5–72 A piston–cylinder device contains 4 kg of argon at 250 kPa and 35°C. During a quasi-equilibrium, isothermal expansion process, 15 kJ of boundary work is done by the system, and 3 kJ of paddle-wheel work is done on the system. Determine the heat transfer for this process.

5–73 A spring-loaded piston–cylinder device contains 5 kg of helium as the system, as shown in Fig. P5–73. This system is heated from 100 kPa and 20°C to 800 kPa and 160°C. Determine the heat transferred to and the work produced by this system.

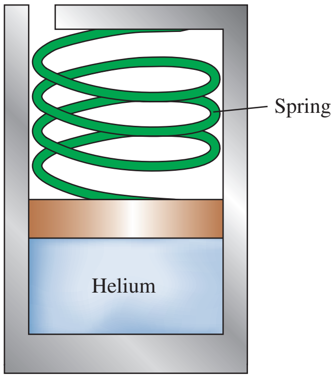


FIGURE P5–73

5–74 A piston–cylinder device whose piston is resting on top of a set of stops initially contains 0.5 kg of helium gas at 100 kPa and 25°C. The mass of the piston is such that 500 kPa of pressure is required to raise it. How much heat must be transferred to the helium before the piston starts rising? *Answer:* 1857 kJ

5–75 A piston–cylinder device, whose piston is resting on a set of stops, initially contains 3 kg of air at 200 kPa and 27°C. The mass of the piston is such that a pressure of 400 kPa is required to move it. Heat is now transferred to the air until its volume doubles. Determine the work done by the air and the total heat transferred to the air during this process. Also show the process on a P-υ diagram. *Answers:* 516 kJ, 2674 kJ

Closed-System Energy Analysis: Solids and Liquids

5–76 A 1-kg block of iron is heated from 25 to 75°C. What is the change in the iron's total internal energy and enthalpy?

5–77E The state of liquid water is changed from 50 psia and 50°F to 2000 psia and 100°F. Determine the change in the internal energy and enthalpy of water on the basis of the (*a*) compressed liquid tables, (*b*) incompressible substance approximation and property tables, and (*c*) specific-heat model.

5–78E During a picnic on a hot summer day, all the cold drinks disappear quickly, and the only available drinks are those at the ambient temperature of 85°F. In an effort to cool a 12-fluid-oz drink in a can, a person grabs the can and starts shaking it in the iced water of the chest at 32°F. Using the properties of water for the drink, determine the mass of ice that will melt by the time the canned drink cools to 37°F.

5–79 An ordinary egg can be approximated as a 5.5-cm-diameter sphere. The egg is initially at a uniform temperature of 8°C and is dropped into boiling water at 97°C. Taking the properties of the egg to be $\rho = 1020$ kg/m^3 and $c_p = 3.32$ kJ/kg·°C, determine how much heat is transferred to the egg by the time the average temperature of the egg rises to 80°C.

5–80 Consider a 1000-W iron whose base plate is made of 0.5-cm-thick aluminum alloy 2024-T6 ($\rho = 2770$ kg/m^3 and $c_p = 875$ J/kg·°C). The base plate has a surface area of 0.03 m^2. Initially, the iron is in thermal equilibrium with the ambient air at 22°C. Assuming 90 percent of the heat generated in the resistance wires is transferred to the plate, determine the minimum time needed for the plate temperature to reach 200°C.

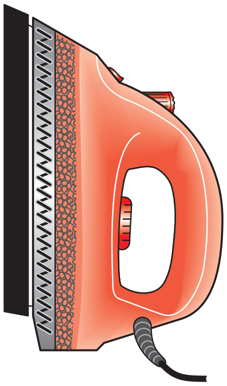


FIGURE P5–80

5–81 Stainless steel ball bearings ($\rho = 8085$ kg/m^3 and $c_p = 0.480$ kJ/kg·°C) having a diameter of 1.2 cm are to be quenched in water at a rate of 800 per minute. The balls leave the oven at a uniform temperature of 900°C and are exposed to air at 25°C for a while before they are dropped into the water. If the temperature of the balls drops to 850°C prior to quenching, determine the rate of heat transfer from the balls to the air.

5–82E In a production facility, 1.6-in-thick 2-ft × 2-ft square brass plates ($\rho = 532.5$ lbm/ft^3 and $c_p = 0.091$ Btu/lbm·°F) that are initially at a uniform temperature of 75°F are heated by passing them through an oven at 1500°F at a rate of 300 per minute. If the plates remain in the oven until their average temperature rises to 900°F, determine the rate of heat transfer to the plates in the furnace.

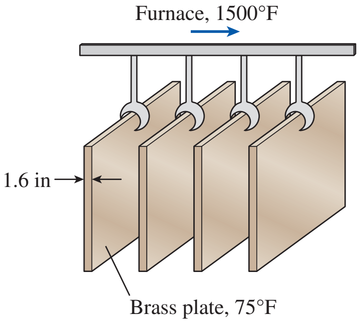


FIGURE P5–82E

5–83 Long cylindrical steel rods ($\rho = 7833$ kg/m^3 and $c_p = 0.465$ kJ/kg·°C) of 8 cm diameter are heat-treated by drawing them at a velocity of 2 m/min through an oven maintained at 900°C. If the rods enter the oven at 30°C and leave at a mean temperature of 500°C, determine the rate of heat transfer to the rods in the oven.

5–84 An electronic device dissipating 25 W has a mass of 20 g and a specific heat of 850 J/kg·°C. The device is lightly used, and it is on for 5 min and then off for several hours, during which it cools to the ambient temperature of 25°C. Determine the highest possible temperature of the device at the end of the 5-min operating period. What would your answer be if the device were attached to a 0.5-kg aluminum heat sink? Assume the device and the heat sink to be nearly isothermal.

5–85 Reconsider Prob. 5–84. Using appropriate software, investigate the effect of the mass of the heat sink on the maximum device temperature. Let the mass of the heat sink vary from 0 to 1 kg. Plot the maximum temperature against the mass of the heat sink, and discuss the results.

5–86 If you ever slapped someone or got slapped yourself, you probably remember the burning sensation. Imagine you had the unfortunate occasion of being slapped by an angry person, which caused the temperature of the affected area of your face to rise by 2.4°C (ouch!). Assuming the slapping hand has a mass of 0.9 kg and that about 0.150 kg of the tissue on the face and the hand is affected by the incident, estimate the velocity of the hand just before impact. Take the specific heat of the tissue to be 3.8 kJ/kg·K.

Review Problems

5–87 Which of two gases—neon or air—requires less work when compressed in a closed system from P_1 to P_2 using a polytropic process with $n = 1.5$?

5–88 Which of two gases—neon or air—produces more work when expanded from P_1 to P_2 in a closed-system polytropic process with $n = 1.2$?

5–89 Consider a classroom that is losing heat to the outdoors at a rate of 12,000 kJ/h. If there are 40 students in class, each dissipating sensible heat at a rate of 84 W, determine if it is necessary to turn the heater in the classroom on to prevent the room temperature from dropping.

5–90 The temperature of air changes from 0 to 10°C while its velocity changes from zero to a final velocity, and its elevation changes from zero to a final elevation. At which values of final air velocity and final elevation will the internal, kinetic, and potential energy changes be equal? *Answers:* 120 m/s, 732 m

5–91 A rigid tank contains a gas mixture with a specific heat of $c_\upsilon = 0.748$ kJ/kg·K. The mixture is cooled from 200 kPa and 200°C until its pressure is 100 kPa. Determine the heat transfer during this process, in kJ/kg.

5–92 Consider a piston–cylinder device that contains 0.5 kg air. Now heat is transferred to the air at constant pressure and the air temperature increases by 5°C. Determine the expansion work done during this process.

5–93 A mass of 0.2 kg of saturated refrigerant-134a is contained in a piston–cylinder device at 200 kPa. Initially, 75 percent of the mass is in the liquid phase. Now heat is transferred to the refrigerant at constant pressure until the cylinder contains vapor only. Show the process on a P-υ diagram with respect to saturation lines. Determine (*a*) the volume occupied by the refrigerant initially, (*b*) the work done, and (*c*) the total heat transfer.

5–94E Air in the amount of 2 lbm is contained in a well-insulated, rigid vessel equipped with a stirring paddle wheel. The initial state of this air is 30 psia and 60°F. How much work, in Btu, must be transferred to the air with the paddle wheel to raise the air pressure to 40 psia? Also, what is the final temperature of the air?

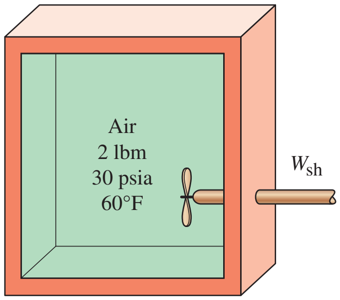


FIGURE P5–94E

5–95 Air is expanded in a polytropic process with $n = 1.2$ from 1 MPa and 400°C to 110 kPa in a piston–cylinder device. Determine the final temperature of the air.

5–96 Nitrogen at 100 kPa and 25°C in a rigid vessel is heated until its pressure is 300 kPa. Calculate the work done and the heat transferred during this process, in kJ/kg.

5–97 A well-insulated, rigid vessel contains 3 kg of saturated liquid water at 40°C. The vessel also contains an electrical resistor that draws 10 A when 50 V are applied. Determine the final temperature in the vessel after the resistor has been operating for 30 min.

5–98 A mass of 3 kg of saturated liquid–vapor mixture of water is contained in a piston–cylinder device at 160 kPa. Initially, 1 kg of the water is in the liquid phase and the rest is in the vapor phase. Heat is now transferred to the water, and the piston, which is resting on a set of stops, starts moving when the pressure inside reaches 500 kPa. Heat transfer continues until the total volume increases by 20 percent. Determine (*a*) the initial and final temperatures, (*b*) the mass of liquid water when the piston first starts moving, and (*c*) the work done during this process. Also, show the process on a P-υ diagram.

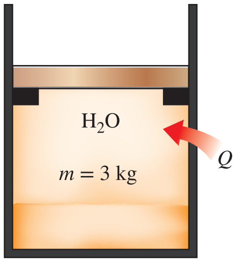


FIGURE P5–98

5–99 A mass of 12 kg of saturated refrigerant-134a vapor is contained in a piston–cylinder device at 240 kPa. Now 300 kJ of heat is transferred to the refrigerant at constant pressure while a 110-V source supplies current to a resistor within the cylinder for 6 min. Determine the current supplied if the final temperature is

70°C. Also, show the process on a T-υ diagram with respect to the saturation lines. *Answer:* 12.8 A

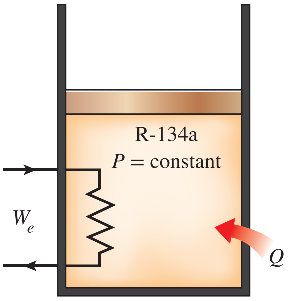


FIGURE P5–99

5–100 Saturated water vapor at 200°C is condensed to a saturated liquid at 50°C in a spring-loaded piston–cylinder device. Determine the heat transfer for this process in kJ/kg.

5–101 A piston–cylinder device contains helium gas initially at 100 kPa, 10°C, and 0.2 m^3. The helium is now compressed in a polytropic process (PV^n = constant) to 700 kPa and 290°C. Determine the heat loss or gain during this process. *Answer:* 6.51 kJ loss

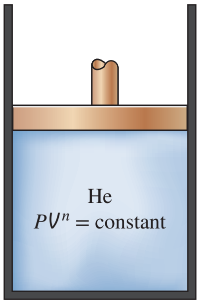


FIGURE P5–101

5–102 A frictionless piston–cylinder device initially contains air at 100 kPa and 0.15 m^3. At this state, a linear spring ($F \propto x$) is touching the piston but exerts no force on it. The air is now heated to a final state of 0.45 m^3 and 800 kPa. Determine (*a*) the total work done by the air and (*b*) the work done against the spring. Also, show the process on a P-V diagram. *Answers:* (*a*) 135 kJ, (*b*) 105 kJ

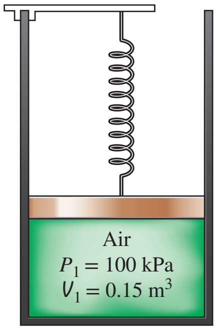


FIGURE P5–102

5–103 A frictionless piston–cylinder device and a rigid tank initially contain 12 kg of an ideal gas each at the same temperature, pressure, and volume. We wish to raise the temperatures of both systems by 15°C. Determine the amount of extra heat that must be supplied to the gas in the cylinder, which is maintained at constant pressure to achieve this result. Assume the molar mass of the gas is 25.

5–104 An insulated piston–cylinder device initially contains 0.01 m^3 of saturated liquid–vapor mixture with a quality of 0.2 at 120°C. Now some ice at 0°C is added to the cylinder. If the cylinder contains saturated liquid at 120°C when thermal equilibrium is established, determine the amount of ice added. The melting temperature and the heat of fusion of ice at atmospheric pressure are 0°C and 333.7 kJ/kg, respectively.

5–105 A passive solar house that is losing heat to the outdoors at an average rate of 50,000 kJ/h is maintained at 22°C at all times during a winter night for 10 h. The house is to be heated by 50 glass containers each containing 20 L of water that is heated to 80°C during the day by absorbing solar energy. A thermostat-controlled 15-kW backup electric resistance heater turns on whenever necessary to keep the house at 22°C. (*a*) How long did the electric heating system run that night? (*b*) How long would the electric heater run that night if the house incorporated no solar heating? *Answers:* (*a*) 4.77 h, (*b*) 9.26 h

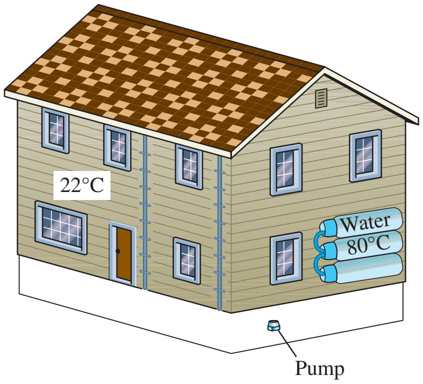


FIGURE P5–105

5–106 Water is boiled at sea level in a coffeemaker equipped with an immersion-type electric heating element. The coffeemaker contains 1 L of water when full. Once boiling starts, it is observed that half of the water in the coffeemaker evaporates in 13 min. Determine the power rating of the electric heating element immersed in water. Also, determine how long it will take for this heater to raise the temperature of 1 L of cold water from 18°C to the boiling temperature.

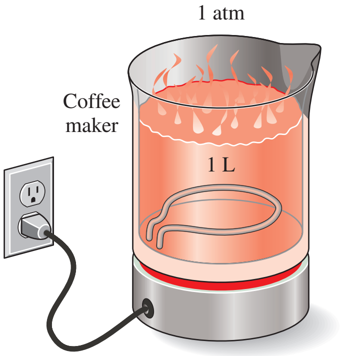


FIGURE P5–106

5–107 The energy content of a certain food is to be determined in a bomb calorimeter that contains 3 kg of water by burning a 2-g sample of it in the presence of 100 g of air in the reaction chamber. If the water temperature rises by 3.2°C when equilibrium is established, determine the energy content of the food, in kJ/kg, by neglecting the thermal energy stored in the reaction chamber and the energy supplied by the mixer. What is a rough estimate of the error involved in neglecting the thermal energy stored in the reaction chamber? *Answer:* 20,060 kJ/kg

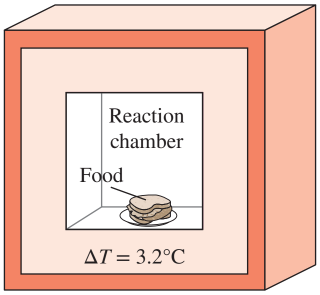


FIGURE P5–107

5–108 A 68-kg man whose average body temperature is 39°C drinks 1 L of cold water at 3°C in an effort to cool down. Taking the average specific heat of the human body to be 3.6 kJ/kg·°C, determine the drop in the average body temperature of this person under the influence of this cold water.

5–109 An insulated piston–cylinder device initially contains 1.8 kg of saturated liquid water at 120°C. Now an electric resistor placed in the cylinder is turned on for 10 min until the volume quadruples. Determine (*a*) the volume of the cylinder, (*b*) the final temperature, and (*c*) the electrical power rating of the resistor. *Answers:* (*a*) 0.00763 m^3, (*b*) 120°C, (*c*) 0.0236 kW

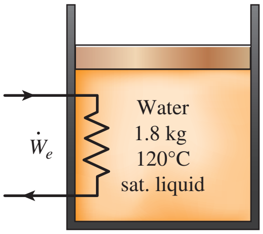


FIGURE P5–109

5–110 An insulated rigid tank initially contains 1.4 kg of saturated liquid water at 200°C and air. At this state, 25 percent of the volume is occupied by liquid water and the rest by air. Now an electric resistor placed in the tank is turned on, and the tank is observed to contain saturated water vapor after 20 min. Determine (*a*) the volume of the tank, (*b*) the final temperature, and (*c*) the electric power rating of the resistor. Neglect energy added to the air. *Answers:* (*a*) 0.00648 m^3, (*b*) 371°C, (*c*) 1.58 kW

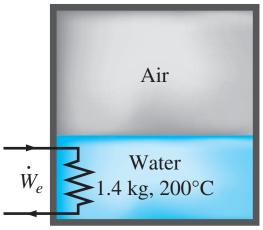


FIGURE P5–110

5–111 In order to cool 1 ton of water at 20°C in an insulated tank, a person pours 130 kg of ice at –5°C into the water. Determine the final equilibrium temperature in the tank. The melting temperature and the heat of fusion of ice at atmospheric pressure are 0°C and 333.7 kJ/kg, respectively. *Answer:* 8.2°C

5–112 A 0.3-L glass of water at 20°C is to be cooled with ice to 5°C. Determine how much ice needs to be added to the water, in grams, if the ice is at (*a*) 0°C and (*b*) –20°C. Also determine how much water would be needed if the cooling is to be done with cold water at 0°C. The melting temperature and the heat of fusion of ice at atmospheric pressure are 0°C and 333.7 kJ/kg, respectively, and the density of water is 1 kg/L.

5–113 Reconsider Prob. 5–112. Using appropriate software, investigate the effect of the initial temperature of the ice on the final mass required. Let the ice temperature vary from –26 to 0°C. Plot the mass of ice against the initial temperature of ice, and discuss the results.

5–114 A well-insulated 3-m × 4-m × 6-m room initially at 7°C is heated by the radiator of a steam heating system. The radiator has a volume of 15 L and is filled with superheated vapor at 200 kPa and 200°C. At this moment both the inlet and the exit valves to the radiator are closed. A 120-W fan is used to distribute the air in the room. The pressure of the steam is observed to drop to 100 kPa after 45 min as a result of heat transfer to the room. Assuming constant specific heats for air at room temperature, determine the average temperature of air in 45 min. Assume the air pressure in the room remains constant at 100 kPa.

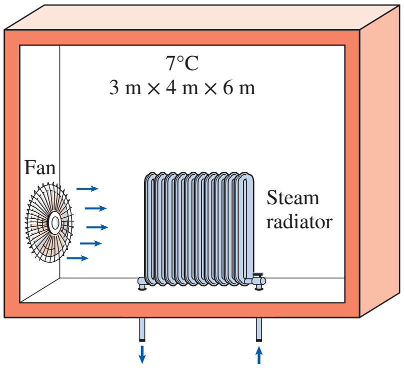


FIGURE P5–114

5–115 Two adiabatic chambers, 2 m^3 each, are interconnected by a valve, as shown in Fig. P5–115, with one chamber containing oxygen at 1000 kPa and 127°C and the other chamber evacuated. The valve is now opened until the oxygen fills both chambers and both tanks have the same pressure. Determine the total internal energy change and the final pressure in the tanks.

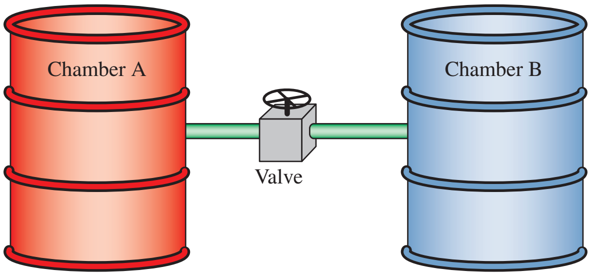


FIGURE P5–115

5–116 A vertical 10-cm-diameter piston–cylinder device contains an ideal gas at the ambient conditions of 1 bar and 24°C. Initially, the inner face of the piston is 20 cm from the base of the cylinder. Now an external shaft connected to the piston exerts a force corresponding to a boundary work input of 0.1 kJ. The temperature of the gas remains constant during the process. Determine (*a*) the amount of heat transfer, (*b*) the final pressure in the cylinder, and (*c*) the distance that the piston is displaced.

5–117 A piston–cylinder device initially contains 0.35 kg of steam at 3.5 MPa, superheated by 7.4°C. Now the steam loses heat to the surroundings and the piston moves down, hitting a set of stops at which point the cylinder contains saturated liquid water. The cooling continues until the cylinder contains water at 200°C. Determine (*a*) the final pressure and the quality (if mixture), (*b*) the boundary work, (*c*) the amount of heat transfer when the piston first hits the stops, and (*d*) the total heat transfer.

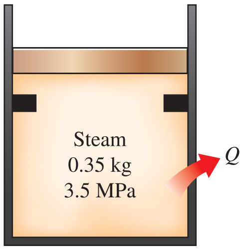


FIGURE P5–117

5–118E Two 10-ft^3 adiabatic tanks are connected by a valve. Initially, one tank contains water at 450 psia with 10 percent quality, while the second contains water at 15 psia with 75 percent quality. The valve is now opened, allowing the water vapor from the high-pressure tank to move to the low-pressure tank until the pressure in the two becomes equal. Determine the final pressure and the final mass in each tank. *Answers:* 313 psia, 41.6 lbm

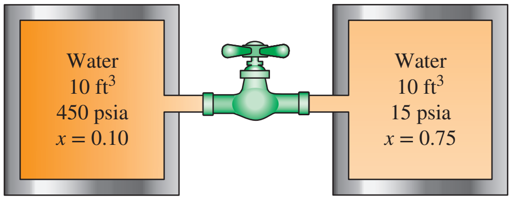


FIGURE P5–118E

5–119 In solar-heated buildings, energy is often stored as sensible heat in rocks, concrete, or water during the day for use at night. To minimize the storage space, it is desirable to use a material that can store a large amount of heat while experiencing a small temperature change. A large amount of heat can be stored essentially at constant temperature during a phase-change process, and thus materials that change phase at about room temperature such as glaubers salt (sodium sulfate decahydrate), which has a melting point of 32°C and a heat of fusion of 329 kJ/L, are very suitable for this purpose. Determine how much heat can be stored in a 5-m^3 storage space using (*a*) glaubers salt undergoing a phase change, (*b*) granite rocks with a heat capacity of 2.32 kJ/kg·°C and a temperature change of 20°C, and (*c*) water with a heat capacity of 4.00 kJ/kg·°C and a temperature change of 20°C.

Design and Essay Problems

5–120 Find out how the specific heats of gases, liquids, and solids are determined in national laboratories. Describe the experimental apparatus and the procedures used.

5–121 You are asked to design a heating system for a swimming pool that is 2 m deep, 25 m long, and 25 m wide. Your client wants the heating system to be large enough to raise the water temperature from 20 to 30°C in 3 h. The rate of heat loss from the water to the air at the outdoor design conditions is determined to be 960 W/m^2, and the heater must also be able to maintain the pool at 30°C at those conditions. Heat losses to the ground are expected to be small and can be disregarded. The heater considered is a natural gas furnace whose efficiency is 80 percent. What heater size (in kW input) would you recommend to your client?

5–122 Compressed gases and phase-changing liquids are used to store energy in rigid containers. What are the advantages and disadvantages of each substance as a means of storing energy?

5–123 Design an experiment complete with instrumentation to determine the specific heats of a liquid using a resistance heater. Discuss how the experiment will be conducted, what measurements need to be taken, and how the specific heats will be determined. What are the sources of error in your system? How can you minimize the experimental error? How would you modify this system to determine the specific heat of a solid?

CHAPTER

6

MASS AND ENERGY ANALYSIS OF CONTROL VOLUMES

In Chap. 5, we applied the general energy balance relation expressed as $E_{in} - E_{out} = \Delta E_{system}$ to closed systems. In this chapter, we extend the energy analysis to systems that involve mass flow across their boundaries, that is, control volumes, with particular emphasis on steady-flow systems.

We start this chapter with the development of the general *conservation of mass* relation for control volumes, and we continue with a discussion of flow work and the energy of fluid streams. We then apply the energy balance to systems that involve *steady-flow processes* and analyze the common steady-flow devices such as nozzles, diffusers, compressors, turbines, throttling valves, mixing chambers, and heat exchangers. Finally, we apply the energy balance to general *unsteady-flow processes* such as the charging and discharging of vessels.

OBJECTIVES

The objectives of this chapter are to:

- Develop the conservation of mass principle.
- Apply the conservation of mass principle to various systems, including steady- and unsteady-flow control volumes.
- Apply the first law of thermodynamics as the statement of the conservation of energy principle to control volumes.
- Identify the energy carried by a fluid stream crossing a control surface as the sum of internal energy, flow work, kinetic energy, and potential energy of the fluid and to relate the combination of the internal energy and the flow work to the property enthalpy.
- Solve energy balance problems for common steady-flow devices such as nozzles, compressors, turbines, throttling valves, mixing chambers, and heat exchangers.
- Apply the energy balance to general unsteady-flow processes with particular emphasis on the uniform-flow process as the model for commonly encountered charging and discharging processes.

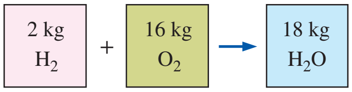


FIGURE 6–1
Mass is conserved even during chemical reactions.

6–1 ▪ CONSERVATION OF MASS

The conservation of mass principle is one of the most fundamental principles in nature. We are all familiar with this principle, and it is not difficult to understand. A person does not have to be a rocket scientist to figure out how much vinegar-and-oil dressing will be obtained by mixing 100 g of oil with 25 g of vinegar. Even chemical equations are balanced on the basis of the conservation of mass principle. When 16 kg of oxygen reacts with 2 kg of hydrogen, 18 kg of water is formed (Fig. 6–1). In an electrolysis process, the water separates back to 2 kg of hydrogen and 16 kg of oxygen.

Technically, mass is not exactly conserved. It turns out that mass m and energy E can be converted to each other according to the well-known formula proposed by Albert Einstein (1879–1955):

$$E = mc^2 \tag{6–1}$$

where c is the speed of light in a vacuum, which is $c = 2.9979 \times 10^8$ m/s. This equation suggests that there is equivalence between mass and energy. All physical and chemical systems exhibit energy interactions with their surroundings, but the amount of energy involved is equivalent to an extremely small mass compared to the system's total mass. For example, when 1 kg of liquid water is formed from oxygen and hydrogen at normal atmospheric conditions, the amount of energy released is 15.8 MJ, which corresponds to a mass of only 1.76×10^{-10} kg. However, even in nuclear reactions, the mass equivalence of the amount of energy interacted is a very small fraction of the total mass involved. Therefore, in most engineering analyses, we consider both mass and energy to be conserved quantities.

For *closed systems,* the conservation of mass principle is implicitly used by requiring that the mass of the system remain constant during a process. For *control volumes,* however, mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.

Mass and Volume Flow Rates

The amount of mass flowing through a cross section per unit time is called the **mass flow rate** and is denoted by $\dot{m}$. The dot over a symbol is used to indicate *time rate of change.*

A fluid flows into or out of a control volume, usually through pipes or ducts. The differential mass flow rate of fluid flowing across a small area element dA_c in a cross section of a pipe is proportional to dA_c itself, the fluid density ρ, and the component of the flow velocity normal to dA_c, which we denote as V_n, and which is expressed as (Fig. 6–2)

$$\delta\dot{m} = \rho V_n dA_c \tag{6–2}$$

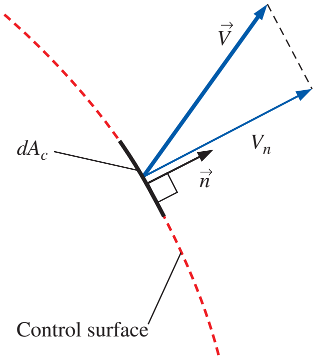


FIGURE 6–2
The normal velocity V_n for a surface is the component of velocity perpendicular to the surface.

Note that both δ and d are used to indicate differential quantities, but δ is typically used for quantities (such as heat, work, and mass transfer) that are *path functions* and have *inexact differentials,* while d is used for quantities (such as properties) that are *point functions* and have *exact differentials.* For flow through an annulus of inner radius r_1 and outer radius r_2, for example, $\int_1^2 dA_c = A_{c2} - A_{c1} = \pi(r_2^2 - r_1^2)$ but $\int_1^2 \delta\dot{m} = \dot{m}_{\text{total}}$ (total mass flow rate through the annulus), not $\dot{m}_2 - \dot{m}_1$. For specified values of r_1 and r_2, the value of the integral of dA_c is fixed (thus the names point function and exact differential), but this is not the case for the integral of $\delta\dot{m}$ (thus the names path function and inexact differential).

The mass flow rate through the entire cross-sectional area of a pipe or duct is obtained by integration:

$$\dot{m} = \int_{A_c} \delta\dot{m} = \int_{A_c} \rho V_n \, dA_c \qquad \text{(kg/s)} \tag{6–3}$$

While Eq. 6–3 is always valid (in fact it is *exact*), it is not always practical for engineering analyses because of the integral. We would like instead to express mass flow rate in terms of average values over a cross section of the pipe. In a general compressible flow, both ρ and V_n vary across the pipe. In many practical applications, however, the density is essentially uniform over the pipe cross section, and we can take ρ outside the integral of Eq. 6–3. Velocity, however, is *never* uniform over a cross section of a pipe because of the no-slip condition at the walls. Rather, the velocity varies from zero at the walls to some maximum value at or near the centerline of the pipe. We define the **average velocity** V_{avg} as the average value of V_n across the entire cross section of the pipe (Fig. 6–3),

Average velocity:

$$V_{\text{avg}} = \frac{1}{A_c}\int_{A_c} V_n \, dA_c \tag{6–4}$$

where A_c is the area of the cross section normal to the flow direction. Note that if the speed were V_{avg} all through the cross section, the mass flow rate would be identical to that obtained by integrating the actual velocity profile. Thus for incompressible flow or even for compressible flow where ρ is approximated as uniform across A_c, Eq. 6–3 becomes

$$\dot{m} = \rho V_{\text{avg}} A_c \qquad \text{(kg/s)} \tag{6–5}$$

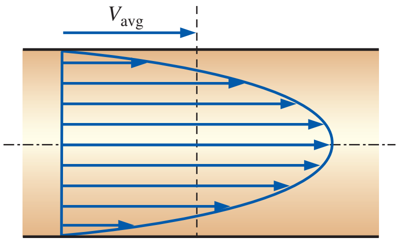


FIGURE 6–3
The average velocity V_{avg} is defined as the average speed through a cross section.

For compressible flow, we can think of ρ as the bulk average density over the cross section, and then Eq. 6–5 can be used as a reasonable approximation. For simplicity, we drop the subscript on the average velocity. Unless otherwise stated, V denotes the average velocity in the flow direction. Also, A_c denotes the cross-sectional area normal to the flow direction.

The volume of the fluid flowing through a cross section per unit time is called the **volume flow rate** $\dot{V}$ (Fig. 6–4) and is given by

$$\dot{V} = \int_{A_c} V_n \, dA_c = V_{\text{avg}} A_c = V A_c \qquad (\text{m}^3/\text{s}) \tag{6–6}$$

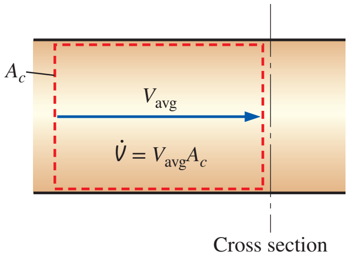


FIGURE 6–4
The volume flow rate is the volume of fluid flowing through a cross section per unit time.

An early form of Eq. 6–6 was published in 1628 by the Italian monk Benedetto Castelli (circa 1577–1644). Note that many fluid mechanics textbooks use Q instead of $\dot{V}$ for volume flow rate. We use $\dot{V}$ to avoid confusion with heat transfer.

The mass and volume flow rates are related by

$$\dot{m} = \rho\dot{V} = \frac{\dot{V}}{v} \tag{6–7}$$

where v is the specific volume. This relation is analogous to $m = \rho V = V/v$, which is the relation between the mass and the volume of a fluid in a container.

Conservation of Mass Principle

The **conservation of mass principle** for a control volume can be expressed as: *The net mass transfer to or from a control volume during a time interval* Δt *is equal to the net change (increase or decrease) of the total mass within the control volume during* Δt. That is,

$$\begin{pmatrix}\text{Total mass entering} \\ \text{the CV during } \Delta t\end{pmatrix} - \begin{pmatrix}\text{Total mass leaving} \\ \text{the CV during } \Delta t\end{pmatrix} = \begin{pmatrix}\text{Net change of mass} \\ \text{within the CV during } \Delta t\end{pmatrix}$$

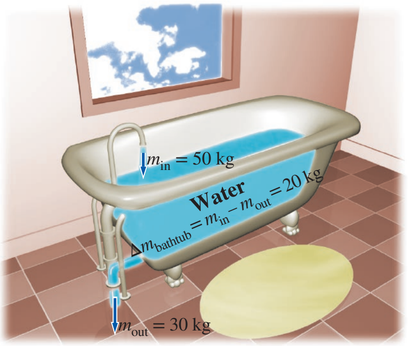


FIGURE 6–5
Conservation of mass principle for an ordinary bathtub.

or

$$m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{CV}} \qquad \text{(kg)} \tag{6–8}$$

where $\Delta m_{\text{CV}} = m_{\text{final}} - m_{\text{initial}}$ is the change in the mass of the control volume during the process (Fig. 6–5). It can also be expressed in *rate form* as

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = dm_{\text{CV}}/dt \qquad \text{(kg/s)} \tag{6–9}$$

where $\dot{m}_{\text{in}}$ and $\dot{m}_{\text{out}}$ are the total rates of mass flow into and out of the control volume, and dm_{CV}/dt is the rate of change of mass within the control volume boundaries. Equations 6–8 and 6–9 are often referred to as the **mass balance** and are applicable to any control volume undergoing any kind of process.

Consider a control volume of arbitrary shape, as shown in Fig. 6–6. The mass of a differential volume $d\mathcal{V}$ within the control volume is $dm = \rho\, d\mathcal{V}$. The total mass within the control volume at any instant in time t is determined by integration to be

Total mass within the CV:
$$m_{\text{cv}} = \int_{\text{CV}} \rho\, d\mathcal{V} \tag{6–10}$$

Then the time rate of change of the amount of mass within the control volume is expressed as

Rate of change of mass within the CV:
$$\frac{dm_{\text{CV}}}{dt} = \frac{d}{dt}\int_{\text{CV}} \rho\, d\mathcal{V} \tag{6–11}$$

For the special case of no mass crossing the control surface (i.e., the control volume is a closed system), the conservation of mass principle reduces to $dm_{\text{CV}}/dt = 0$. This relation is valid whether the control volume is fixed, moving, or deforming.

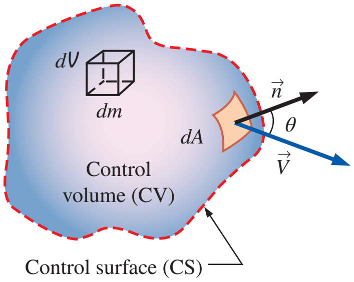


FIGURE 6–6
The differential control volume $d\mathcal{V}$ and the differential control surface dA used in the derivation of the conservation of mass relation.

Now consider mass flow into or out of the control volume through a differential area dA on the control surface of a fixed control volume. Let $\vec{n}$ be the outward unit vector of dA normal to dA and $\vec{V}$ be the flow velocity at dA relative to a fixed coordinate system, as shown in Fig. 6–6. In general, the velocity may cross dA at an angle θ off the normal of dA, and the mass flow rate is proportional to the normal component of velocity $\vec{V}_n = \vec{V}\cos\theta$ ranging from a maximum outflow of $\vec{V}$ for $\theta = 0$ (flow is normal to dA) to a minimum of zero for $\theta = 90°$ (flow is tangent to dA) to a maximum *inflow* of $\vec{V}$ for $\theta = 180°$ (flow is normal to dA but in the opposite direction). Making use of the concept of dot product of two vectors, the magnitude of the normal component of velocity is

Normal component of velocity:
$$V_n = V\cos\theta = \vec{V}\cdot\vec{n} \tag{6–12}$$

The mass flow rate through dA is proportional to the fluid density ρ, normal velocity V_n, and the flow area dA, and is expressed as

Differential mass flow rate:
$$\delta\dot{m} = \rho V_n\, dA = \rho(V\cos\theta)\, dA = \rho(\vec{V}\cdot\vec{n})\, dA \tag{6–13}$$

The net flow rate into or out of the control volume through the entire control surface is obtained by integrating $\delta\dot{m}$ over the entire control surface,

Net mass flow rate:
$$\dot{m}_{\text{net}} = \int_{\text{CS}} \delta\dot{m} = \int_{\text{CS}} \rho V_n\, dA = \int_{\text{CS}} \rho(\vec{V}\cdot\vec{n})\, dA \tag{6–14}$$

Note that $V_n = \vec{V}\cdot\vec{n} = V\cos\theta$ is positive for $\theta < 90°$ (outflow) and negative for $\theta > 90°$ (inflow). Therefore, the direction of flow is automatically accounted for, and the surface integral in Eq. 6–14 directly gives the *net* mass flow rate. A positive value for $\dot{m}_{\text{net}}$ indicates a net outflow of mass and a negative value indicates a net inflow of mass.

Rearranging Eq. 6–9 as $dm_{CV}/dt + \dot m_{out} - \dot m_{in} = 0$, the conservation of mass relation for a fixed control volume is then expressed as

General conservation of mass:
$$\frac{d}{dt}\int_{CV}\rho\, dV + \int_{CS}\rho(\vec{V}\cdot\vec{n})\, dA = 0 \qquad \textbf{(6–15)}$$

It states that *the time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero.*

Splitting the surface integral in Eq. 6–15 into two parts—one for the outgoing flow streams (positive) and one for the incoming flow streams (negative)—the general conservation of mass relation can also be expressed as

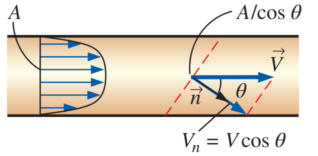
$$\frac{d}{dt}\int_{CV}\rho\, dV + \sum_{out}\rho|V_n|A - \sum_{in}\rho|V_n|A = 0 \qquad \textbf{(6–16)}$$

where A represents the area for an inlet or outlet, and the summation signs are used to emphasize that *all* the inlets and outlets are to be considered. Using the definition of mass flow rate, Eq. 6–16 can also be expressed as

$$\frac{d}{dt}\int_{CV}\rho\, dV = \sum_{in}\dot m - \sum_{out}\dot m \qquad \text{or} \qquad \frac{dm_{CV}}{dt} = \sum_{in}\dot m - \sum_{out}\dot m \qquad \textbf{(6–17)}$$

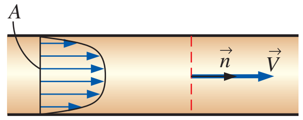
There is considerable flexibility in the selection of a control volume when solving a problem. Many control volume choices are available, but some are more convenient to work with. A control volume should not introduce any unnecessary complications. A wise choice of a control volume can make the solution of a seemingly complicated problem rather easy. A simple rule in selecting a control volume is to make the control surface *normal to the flow* at all locations where it crosses the fluid flow, whenever possible. This way the dot product $\vec{V}\cdot\vec{n}$ simply becomes the magnitude of the velocity, and the integral $\int_A \rho(\vec{V}\cdot\vec{n})\, dA$ becomes simply ρVA (Fig. 6–7).

Equations 6–15 and 6–16 are also valid for moving or deforming control volumes provided that the *absolute velocity* $\vec{V}$ is replaced by the *relative velocity* $\vec{V}_r$, which is the fluid velocity relative to the control surface.



$\dot m = \rho(V\cos\theta)(A/\cos\theta) = \rho VA$

(*a*) Control surface *at an angle* to the flow



$\dot m = \rho VA$

(*b*) Control surface *normal* to the flow

FIGURE 6–7
A control surface should always be selected *normal to the flow* at all locations where it crosses the fluid flow to avoid complications, even though the result is the same.

Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant). Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it. For a garden hose nozzle in steady operation, for example, the amount of water entering the nozzle per unit time is equal to the amount of water leaving it per unit time.

When dealing with steady-flow processes, we are not interested in the amount of mass that flows in or out of a device over time; instead, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate $\dot m$. The conservation of mass principle* for a general steady-flow system with multiple inlets and outlets is expressed in rate form as (Fig. 6–8)

Steady flow:
$$\sum_{in}\dot m = \sum_{out}\dot m \qquad \text{(kg/s)} \qquad \textbf{(6–18)}$$

It states that *the total rate of mass entering a control volume is equal to the total rate of mass leaving it.*

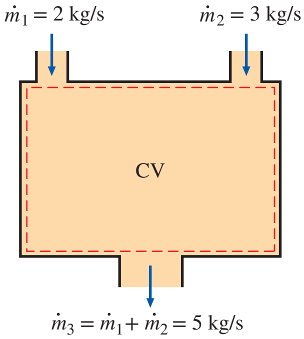


FIGURE 6–8
Conservation of mass principle for a two-inlet–one-outlet steady-flow system.

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet). For these cases, we typically denote the inlet state by the subscript 1 and the outlet state by the subscript 2, and drop the summation signs. Then Eq. 6–18 reduces, for *single-stream steady-flow systems,* to

$$\textit{Steady flow (single stream):}\quad \dot m_1 = \dot m_2 \quad \rightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2 \tag{6–19}$$

Special Case: Incompressible Flow

The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids. Canceling the density from both sides of the general steady-flow relation gives

$$\textit{Steady, incompressible flow:}\qquad \sum_{\text{in}} \dot V = \sum_{\text{out}} \dot V \qquad (\text{m}^3/\text{s}) \tag{6–20}$$

For single-stream steady-flow systems Eq. 6–20 becomes

$$\textit{Steady, incompressible flow (single stream):}\quad \dot V_1 = \dot V_2 \rightarrow V_1 A_1 = V_2 A_2 \tag{6–21}$$

It should always be kept in mind that there is no such thing as a “conservation of volume” principle. Therefore, the volume flow rates into and out of a steady-flow device may be different. The volume flow rate at the outlet of an air compressor is much less than that at the inlet even though the mass flow rate of air through the compressor is constant (Fig. 6–9). This is due to the higher density of air at the compressor exit. For steady flow of liquids, however, the volume flow rates remain nearly constant since liquids are essentially incompressible (constant-density) substances. Water flow through the nozzle of a garden hose is an example of the latter case.

The conservation of mass principle requires every bit of mass to be accounted for during a process. If you can balance your checkbook (by keeping track of deposits and withdrawals, or by simply observing the “conservation of money” principle), you should have no difficulty applying the conservation of mass principle to engineering systems.

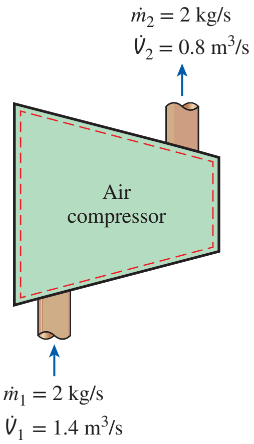


FIGURE 6–9
During a steady-flow process, volume flow rates are not necessarily conserved, although mass flow rates are.

FIGURE 6–10
Schematic for Example 6–1.
John M. Cimbala

EXAMPLE 6–1 Water Flow Through a Garden Hose Nozzle

A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit (Fig. 6–10). If it takes 50 s to fill the bucket with water, determine (*a*) the volume and mass flow rates of water through the hose, and (*b*) the average velocity of water at the nozzle exit.

SOLUTION A garden hose is used to fill a water bucket. The volume and mass flow rates of water and the exit velocity are to be determined.

Assumptions **1** Water is a nearly incompressible substance. **2** Flow through the hose is steady. **3** There is no waste of water by splashing.

Properties We take the density of water to be 1000 kg/m^3 = 1 kg/L.

Analysis (*a*) Noting that 10 gal of water are discharged in 50 s, the volume and mass flow rates of water are

$$\dot V = \frac{V}{\Delta t} = \frac{10\text{ gal}}{50\text{ s}}\left(\frac{3.7854\text{ L}}{1\text{ gal}}\right) = \mathbf{0.757\ L/s}$$

$$\dot m = \rho \dot V = (1\text{ kg/L})(0.757\text{ L/s}) = \mathbf{0.757\ kg/s}$$

(*b*) The cross-sectional area of the nozzle exit is

$$A_e = \pi r_e^2 = \pi(0.4\text{ cm})^2 = 0.5027\text{ cm}^2 = 0.5027 \times 10^{-4}\text{ m}^2$$

The volume flow rate through the hose and the nozzle is constant. Then the average velocity of water at the nozzle exit becomes

$$V_e = \frac{\dot{\mathcal{V}}}{A_e} = \frac{0.757\text{ L/s}}{0.5027 \times 10^{-4}\text{ m}^2}\left(\frac{1\text{ m}^3}{1000\text{ L}}\right) = \mathbf{15.1\ m/s}$$

Discussion It can be shown that the average velocity in the hose is 2.4 m/s. Therefore, the nozzle increases the water velocity by over six times.

EXAMPLE 6–2 Discharge of Water from a Tank

A 4-ft-high, 3-ft-diameter cylindrical water tank whose top is open to the atmosphere is initially filled with water. Now the discharge plug near the bottom of the tank is pulled out, and a water jet whose diameter is 0.5 in streams out (Fig. 6–11). The average velocity of the jet is approximated as $V = \sqrt{2gh}$, where h is the height of water in the tank measured from the center of the hole (a variable) and g is the gravitational acceleration. Determine how long it takes for the water level in the tank to drop to 2 ft from the bottom.

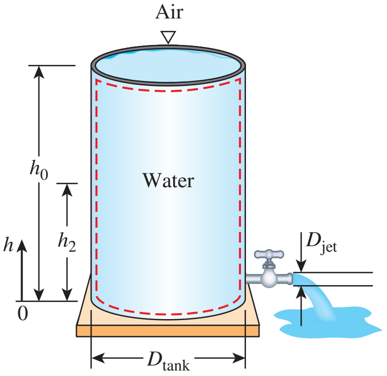


FIGURE 6–11
Schematic for Example 6–2.

SOLUTION The plug near the bottom of a water tank is pulled out. The time it takes for half of the water in the tank to empty is to be determined.
Assumptions **1** Water is a nearly incompressible substance. **2** The distance between the bottom of the tank and the center of the hole is negligible compared to the total water height. **3** The gravitational acceleration is 32.2 ft/s^2.
Analysis We take the volume occupied by water as the control volume. The size of the control volume decreases in this case as the water level drops, and thus this is a variable control volume. (We could also treat this as a fixed control volume that consists of the interior volume of the tank by disregarding the air that replaces the space vacated by the water.) This is obviously an unsteady-flow problem since the properties (such as the amount of mass) within the control volume change with time.

The conservation of mass relation for a control volume undergoing any process is given in rate form as

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{CV}}}{dt} \tag{1}$$

During this process no mass enters the control volume ($\dot{m}_{\text{in}} = 0$), and the mass flow rate of discharged water is

$$\dot{m}_{\text{out}} = (\rho VA)_{\text{out}} = \rho\sqrt{2gh}\,A_{\text{jet}} \tag{2}$$

where $A_{\text{jet}} = \pi D_{\text{jet}}^2/4$ is the cross-sectional area of the jet, which is constant. Noting that the density of water is constant, the mass of water in the tank at any time is

$$m_{\text{CV}} = \rho\mathcal{V} = \rho A_{\text{tank}} h \tag{3}$$

where $A_{\text{tank}} = D_{\text{tank}}^2/4$ is the base area of the cylindrical tank. Substituting Eqs. 2 and 3 into the mass balance relation (Eq. 1) gives

$$-\rho\sqrt{2gh}\,A_{\text{jet}} = \frac{d(\rho A_{\text{tank}} h)}{dt} \rightarrow -\rho\sqrt{2gh}(\pi D_{\text{jet}}^2/4) = \frac{\rho(\pi D_{\text{tank}}^2/4)dh}{dt}$$

Canceling the densities and other common terms and separating the variables give

$$dt = -\frac{D_{\text{tank}}^2}{D_{\text{jet}}^2}\frac{dh}{\sqrt{2gh}}$$

Integrating from $t = 0$ at which $h = h_0$ to $t = t$ at which $h = h_2$ gives

$$\int_0^t dt = -\frac{D_{\text{tank}}^2}{D_{\text{jet}}^2\sqrt{2g}}\int_{h_0}^{h_2}\frac{dh}{\sqrt{h}} \rightarrow t = \frac{\sqrt{h_0}-\sqrt{h_2}}{\sqrt{g/2}}\left(\frac{D_{\text{tank}}}{D_{\text{jet}}}\right)^2$$

Substituting, the time of discharge is determined to be

$$t = \frac{\sqrt{4\text{ ft}}-\sqrt{2\text{ ft}}}{\sqrt{32.2/2\text{ ft/s}^2}}\left(\frac{3\times 12\text{ in}}{0.5\text{ in}}\right)^2 = 757\text{ s} = \mathbf{12.6\text{ min}}$$

Therefore, it takes 12.6 min after the discharge hole is unplugged for half of the tank to be emptied.

Discussion Using the same relation with $h_2 = 0$ gives $t = 43.1$ min for the discharge of the entire amount of water in the tank. Therefore, emptying the bottom half of the tank takes much longer than emptying the top half. This is due to the decrease in the average discharge velocity of water with decreasing h.

6–2 ▪ FLOW WORK AND THE ENERGY OF A FLOWING FLUID

Unlike closed systems, control volumes involve mass flow across their boundaries, and some work is required to push the mass into or out of the control volume. This work is known as the **flow work**, or **flow energy**, and is necessary for maintaining a continuous flow through a control volume.

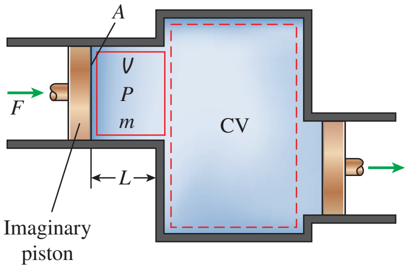


FIGURE 6–12
Schematic for flow work.

To obtain a relation for flow work, consider a fluid element of volume V as shown in Fig. 6–12. The fluid immediately upstream forces this fluid element to enter the control volume; thus, it can be regarded as an imaginary piston. The fluid element can be chosen to be sufficiently small so that it has uniform properties throughout.

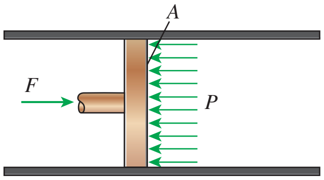


FIGURE 6–13
In the absence of acceleration, the force applied on a fluid by a piston is equal to the force applied on the piston by the fluid.

If the fluid pressure is P and the cross-sectional area of the fluid element is A (Fig. 6–13), the force applied on the fluid element by the imaginary piston is

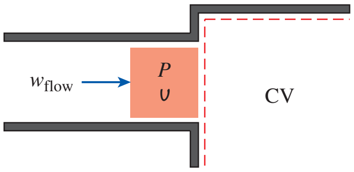
$$F = PA \tag{6–22}$$

To push the entire fluid element into the control volume, this force must act through a distance L. Thus, the work done in pushing the fluid element across the boundary (i.e., the flow work) is

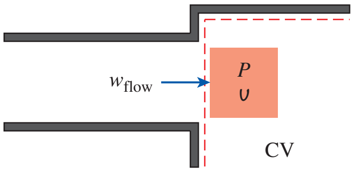
$$W_{\text{flow}} = FL = PAL = PV \qquad \text{(kJ)} \tag{6–23}$$

The flow work per unit mass is obtained by dividing both sides of this equation by the mass of the fluid element:

$$w_{\text{flow}} = Pv \qquad \text{(kJ/kg)} \tag{6–24}$$



(a) Before entering



(b) After entering

FIGURE 6–14
Flow work is the energy needed to push a fluid into or out of a control volume, and it is equal to Pv.

The flow work relation is the same whether the fluid is pushed into or out of the control volume (Fig. 6–14).

It is interesting that unlike other work quantities, flow work is expressed in terms of properties. In fact, it is the product of two properties of the fluid. For that reason, some people view it as a *combination property* (like enthalpy) and refer to it as *flow energy, convected energy,* or *transport energy* instead of flow work. Others, however, argue rightfully that the product PV represents energy for flowing fluids only and does not represent any form of energy for nonflow

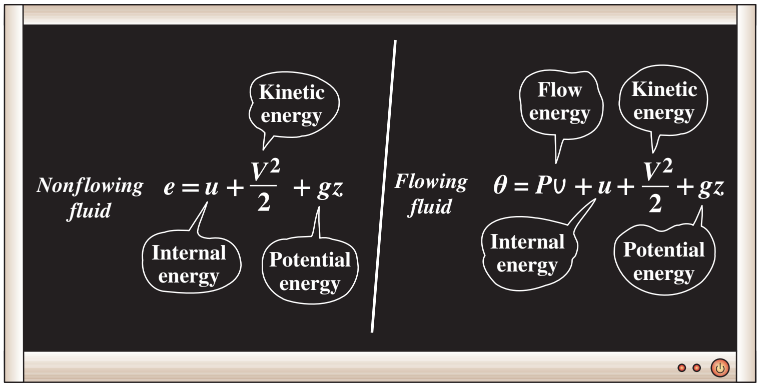


FIGURE 6–15
Total energy consists of three parts for a nonflowing fluid and four parts for a flowing fluid.

(closed) systems. Therefore, it should be treated as work. This controversy is not likely to end, but it is comforting to know that both arguments yield the same result for the energy balance equation. In the discussions that follow, we consider the flow energy to be part of the energy of a flowing fluid, since this greatly simplifies the energy analysis of control volumes.

Total Energy of a Flowing Fluid

As we discussed in Chap. 3, the total energy of a simple compressible system consists of three parts: internal, kinetic, and potential energies (Fig. 6–15). On a unit-mass basis, it is expressed as

$$e = u + \text{ke} + \text{pe} = u + \frac{V^2}{2} + gz \qquad \text{(kJ/kg)} \tag{6–25}$$

where V is the velocity and z is the elevation of the system relative to some external reference point.

The fluid entering or leaving a control volume possesses an additional form of energy—the *flow energy* $P\upsilon$, as already discussed. Then the **total energy of a flowing fluid** on a unit-mass basis (denoted by θ) becomes

$$\theta = P\upsilon + e = P\upsilon + (u + \text{ke} + \text{pe}) \tag{6–26}$$

But the combination $P\upsilon + u$ has been previously defined as the enthalpy h. So the relation in Eq. 6–26 reduces to

$$\theta = h + \text{ke} + \text{pe} = h + \frac{V^2}{2} + gz \qquad \text{(kJ/kg)} \tag{6–27}$$

By using the enthalpy instead of the internal energy to represent the energy of a flowing fluid, one does not need to be concerned about the flow work. The energy associated with pushing the fluid into or out of the control volume is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy. From now on, the energy of a fluid stream flowing into or out of a control volume is represented by Eq. 6–27, and no reference will be made to flow work or flow energy.

Energy Transport by Mass

Noting that θ is total energy per unit mass, the total energy of a flowing fluid of mass m is simply $m\theta$, provided that the properties of the mass m are uniform. Also, when a fluid stream with uniform properties is flowing at a mass flow rate of $\dot{m}$, the rate of energy flow with that stream is $\dot{m}\theta$ (Fig. 6–16). That is,

Amount of energy transport by mass:
$$E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right) \qquad \text{(kJ)} \tag{6–28}$$

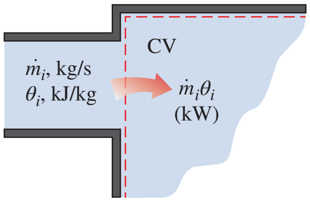


FIGURE 6–16
The product $\dot{m}_i\theta_i$ is the energy transported into control volume by mass per unit time.

Rate of energy transport by mass: $$\dot{E}_{\text{mass}} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right) \qquad \text{(kW)} \tag{6–29}$$

When the kinetic and potential energies of a fluid stream are negligible, as is often the case, these relations simplify to $E_{\text{mass}} = mh$ and $\dot{E}_{\text{mass}} = \dot{m}h$.

In general, the total energy transported by mass into or out of the control volume is not easy to determine since the properties of the mass at each inlet or exit may be changing with time as well as over the cross section. Thus, the only way to determine the energy transport through an opening as a result of mass flow is to consider sufficiently small differential masses δm that have uniform properties and to add their total energies during flow.

Again noting that θ is total energy per unit mass, the total energy of a flowing fluid of mass δm is $\theta\,\delta m$. Then the total energy transported by mass through an inlet or exit ($m_i\theta_i$ and $m_e\theta_e$) is obtained by integration. At an inlet, for example, it becomes

$$E_{\text{in,mass}} = \int_{m_i} \theta_i \,\delta m_i = \int_{m_i} \left(h_i + \frac{V_i^2}{2} + gz_i\right)\delta m_i \tag{6–30}$$

Most flows encountered in practice can be approximated as being steady and one-dimensional, and thus the simple relations in Eqs. 6–28 and 6–29 can be used to represent the energy transported by a fluid stream.

EXAMPLE 6–3 **Energy Transport by Flowing Air**

Air flows steadily in a pipe at 300 kPa, 77°C, and 25 m/s at a rate of 18 kg/min (Fig. 6–17). Determine (*a*) the diameter of the pipe, (*b*) the rate of flow energy, (*c*) the rate of energy transport by mass, and (*d*) the error involved in part *c* if the kinetic energy is neglected.

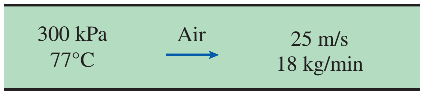


FIGURE 6–17
Schematic for Example 6–3.

SOLUTION Air flows steadily in a pipe at a specified state. The diameter of the pipe, the rate of flow energy, and the rate of energy transport by mass are to be determined. Also, the error involved in the determination of energy transport by mass is to be determined.

Assumptions **1** The flow is steady. **2** The potential energy is negligible.

Properties The properties of air are $R = 0.287$ kJ/kg·K and $c_p = 1.008$ kJ/kg·K (at 350 K from Table A-2*b*).

Analysis (*a*) The diameter is determined as follows:

$$\mathsf{v} = \frac{RT}{P} = \frac{(0.287\ \text{kJ/kg·K})(77 + 273\ \text{K})}{300\ \text{kPa}} = 0.3349\ \text{m}^3/\text{kg}$$

$$A = \frac{\dot{m}\mathsf{v}}{V} = \frac{(18/60\ \text{kg/s})(0.3349\ \text{m}^3/\text{kg})}{25\ \text{m/s}} = 0.004018\ \text{m}^2$$

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.004018\ \text{m}^2)}{\pi}} = \mathbf{0.0715\ m}$$

(*b*) The rate of flow energy is determined from

$$\dot{W}_{\text{flow}} = \dot{m}P\mathsf{v} = (18/60\ \text{kg/s})(300\ \text{kPa})(0.3349\ \text{m}^3/\text{kg}) = \mathbf{30.14\ kW}$$

(*c*) The rate of energy transport by mass is

$$\begin{aligned}\dot{E}_{\text{mass}} &= \dot{m}(h + \text{ke}) = \dot{m}\left(c_pT + \frac{1}{2}V^2\right)\\ &= (18/60\ \text{kg/s})\left[(1.008\ \text{kJ/kg·K})(77 + 273\ \text{K}) + \frac{1}{2}(25\ \text{m/s})^2\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right)\right]\\ &= \mathbf{105.94\ kW}\end{aligned}$$

(*d*) If we neglect kinetic energy in the calculation of energy transport by mass

$$\dot{E}_{\text{mass}} = \dot{m}h = \dot{m}c_pT = (18/60 \text{ kg/s})(1.005 \text{ kJ/kg·K})(77 + 273 \text{ K}) = 105.84 \text{ kW}$$

Therefore, the error involved if we neglect the kinetic energy is only **0.09 percent.**

Discussion The numerical value of the energy transport with air alone does not mean much since this value depends on the reference point selected for enthalpy (it could even be negative). The significant quantity is the difference between the enthalpies of the air in the pipe and the ambient air since it relates directly to the amount of energy supplied to heat air from ambient temperature to 77°C.

6–3 ▪ ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS

A large number of engineering devices such as turbines, compressors, and nozzles operate for long periods of time under the same conditions once the transient start-up period is completed and steady operation is established, and they are classified as *steady-flow devices* (Fig. 6–18). Processes involving such devices can be represented reasonably well by a somewhat idealized process, called the **steady-flow process**, which was defined in Chap. 2 as *a process during which a fluid flows through a control volume steadily.* That is, the fluid properties can change from point to point within the control volume, but at any point, they remain constant during the entire process. (Remember, *steady* means *no change with time.*)

During a steady-flow process, no intensive or extensive properties *within the control volume* change with time. Thus, the volume V, the mass m, and the total energy content E of the control volume remain constant (Fig. 6–19). As a result, the boundary work is zero for steady-flow systems (since V_{CV} = constant), and the total mass or energy entering the control volume must be equal to the total mass or energy leaving it (since m_{CV} = constant and E_{CV} = constant). These observations greatly simplify the analysis.

The fluid properties at an inlet or exit remain constant during a steady-flow process. The properties may, however, be different at different inlets and exits. They may even vary over the cross section of an inlet or an exit. However, all properties, including the velocity and elevation, must remain constant with time at a fixed point at an inlet or exit. It follows that the mass flow rate of the fluid at an opening must remain constant during a steady-flow process (Fig. 6–20). As an added simplification, the fluid properties at an opening are usually considered to be uniform (at some average value) over the cross section. Thus, the fluid properties at an inlet or exit may be specified by the average single values. Also, the *heat* and *work* interactions between a steady-flow system and its surroundings do not change with time. Thus, the power delivered by a system and the rate of heat transfer to or from a system remain constant during a steady-flow process.

The *mass balance* for a general steady-flow system was given in Sec. 6–1 as

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \qquad \text{(kg/s)} \tag{6–31}$$

The mass balance for a single-stream (one-inlet and one-outlet) steady-flow system was given as

$$\dot{m}_1 = \dot{m}_2 \quad \longrightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2 \tag{6–32}$$

where the subscripts 1 and 2 denote the inlet and the exit states, respectively, ρ is density, V is the average flow velocity in the flow direction, and A is the cross-sectional area normal to flow direction.

FIGURE 6–18
Many engineering systems such as power plants operate under steady conditions.
Malcolm Fife/Getty Images

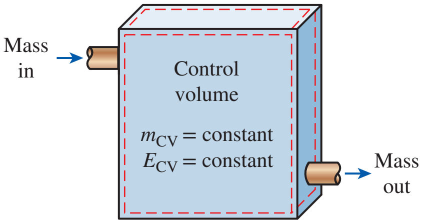


FIGURE 6–19
Under steady-flow conditions, the mass and energy contents of a control volume remain constant.

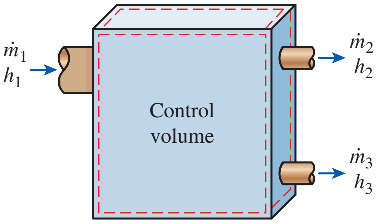


FIGURE 6–20
Under steady-flow conditions, the fluid properties at an inlet or exit remain constant (do not change with time).

During a steady-flow process, the total energy content of a control volume remains constant (E_{CV} = constant), and thus the change in the total energy of the control volume is zero ($\Delta E_{\text{CV}} = 0$). Therefore, the amount of energy entering a control volume in all forms (by heat, work, and mass) must be equal to the amount of energy leaving it. Then the rate form of the general energy balance reduces for a steady-flow process to

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}}\overset{\nearrow 0\text{ (steady)}}{} = 0 \qquad \textbf{(6–33)}$$

or

$$\textit{Energy balance:} \qquad \underbrace{\dot{E}_{\text{in}}}_{\substack{\text{Rate of net energy transfer in}\\ \text{by heat, work, and mass}}} = \underbrace{\dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer out}\\ \text{by heat, work, and mass}}} \qquad \text{(kW)} \qquad \textbf{(6–34)}$$

Noting that energy can be transferred by heat, work, and mass only, the energy balance in Eq. 6–34 for a general steady-flow system can also be written more explicitly as

$$\dot{Q}_{\text{in}} + \dot{W}_{\text{in}} + \sum_{\text{in}} \dot{m}\theta = \dot{Q}_{\text{out}} + \dot{W}_{\text{out}} + \sum_{\text{out}} \dot{m}\theta \qquad \textbf{(6–35)}$$

or

$$\dot{Q}_{\text{in}} + \dot{W}_{\text{in}} + \sum_{\text{in}} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each inlet}} = \dot{Q}_{\text{out}} + \dot{W}_{\text{out}} + \sum_{\text{out}} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each exit}} \qquad \textbf{(6–36)}$$

since the energy of a flowing fluid per unit mass is $\theta = h + \text{ke} + \text{pe} = h + V^2/2 + gz$. The energy balance relation for steady-flow systems first appeared in 1859 in a German thermodynamics book written by Gustav Zeuner.

Consider, for example, an ordinary electric hot-water heater under steady operation, as shown in Fig. 6–21. A cold-water stream with a mass flow rate $\dot{m}$ is continuously flowing into the water heater, and a hot-water stream of the same mass flow rate is continuously flowing out of it. The water heater (the control volume) is losing heat to the surrounding air at a rate of $\dot{Q}_{\text{out}}$, and the electric heating element is supplying electrical work (heating) to the water at a rate of $\dot{W}_{\text{in}}$. On the basis of the conservation of energy principle, we can say that the water stream experiences an increase in its total energy as it flows through the water heater that is equal to the electric energy supplied to the water minus the heat losses.

The energy balance relation just given is intuitive in nature and is easy to use when the magnitudes and directions of heat and work transfers are known. When performing a general analytical study or solving a problem that involves an unknown heat or work interaction, however, we need to assume a direction for the heat or work interactions. In such cases, it is common practice to assume heat to be transferred *into the system* (heat input) at a rate of $\dot{Q}$, and work produced *by the system* (work output) at a rate of $\dot{W}$, and then solve the problem. The first-law or energy balance relation in that case for a general steady-flow system becomes

$$\dot{Q} - \dot{W} = \sum_{\text{out}} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each exit}} - \sum_{\text{in}} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each inlet}} \qquad \textbf{(6–37)}$$

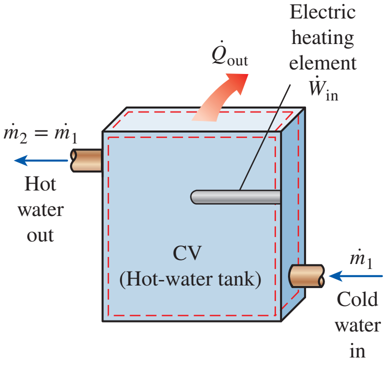


FIGURE 6–21
A water heater in steady operation.

Obtaining a negative quantity for $\dot{Q}$ or $\dot{W}$ simply means that the assumed direction is wrong and should be reversed. For single-stream devices, the steady-flow energy balance equation becomes

$$\dot{Q} - \dot{W} = \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right] \quad \textbf{(6–38)}$$

Dividing Eq. 6–38 by $\dot{m}$ gives the energy balance on a unit-mass basis as

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \quad \textbf{(6–39)}$$

where $q = \dot{Q}/\dot{m}$ and $w = \dot{W}/\dot{m}$ are the heat transfer and work done per unit mass of the working fluid, respectively. When the fluid experiences negligible changes in its kinetic and potential energies (that is, $\Delta\text{ke} \cong 0$, $\Delta\text{pe} \cong 0$), the energy balance equation is reduced further to

$$q - w = h_2 - h_1 \quad \textbf{(6–40)}$$

The various terms appearing in the preceding equations are as follows:

$\dot{Q}$ = **rate of heat transfer between the control volume and its surroundings**. When the control volume is losing heat (as in the case of the water heater), $\dot{Q}$ is negative. If the control volume is well insulated (i.e., adiabatic), then $\dot{Q} = 0$.

$\dot{W}$ = **power**. For steady-flow devices, the control volume is constant; thus, there is no boundary work involved. The work required to push mass into and out of the control volume is also taken care of by using enthalpies for the energy of fluid streams instead of internal energies. Then $\dot{W}$ represents the remaining forms of work done per unit time (Fig. 6–22). Many steady-flow devices, such as turbines, compressors, and pumps, transmit power through a shaft, and $\dot{W}$ simply becomes the shaft power for those devices. If the control surface is crossed by electric wires (as in the case of an electric water heater), $\dot{W}$ represents the electrical work done per unit time. If neither is present, then $\dot{W} = 0$.

$\Delta h = h_2 - h_1$. The enthalpy change of a fluid can easily be determined by reading the enthalpy values at the exit and inlet states from the tables. For ideal gases, it can be approximated by $\Delta h = c_{p,\text{avg}}(T_2 - T_1)$. Note that (kg/s)(kJ/kg) ≡ kW.

$\Delta\text{ke} = (V_2^2 - V_1^2)/2$. The unit of kinetic energy is m^2/s^2, which is equivalent to J/kg (Fig. 6–23). The enthalpy is usually given in kJ/kg. To add these two quantities, the kinetic energy should be expressed in kJ/kg. This is easily accomplished by dividing it by 1000. A velocity of 45 m/s corresponds to a kinetic energy of only 1 kJ/kg, which is a very small value compared with the enthalpy values encountered in practice. Thus, the kinetic energy term at low velocities can be neglected. When a fluid stream enters and leaves a steady-flow device at about the same velocity ($V_1 \cong V_2$), the change in the kinetic energy is close to zero regardless of the velocity. Caution should be exercised at high velocities, however, since small changes in velocities may cause significant changes in kinetic energy (Fig. 6–24).

$\Delta\text{pe} = g(z_2 - z_1)$. A similar argument can be given for the potential energy term. A potential energy change of 1 kJ/kg corresponds to an elevation difference of 102 m. The elevation difference between the inlet and exit of most industrial devices such as turbines and compressors is well below this value, and the potential energy term is always neglected for these devices. The only time the potential energy term is significant is when a process involves pumping a fluid to high elevations and we are interested in the required pumping power.

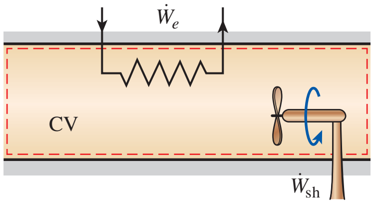
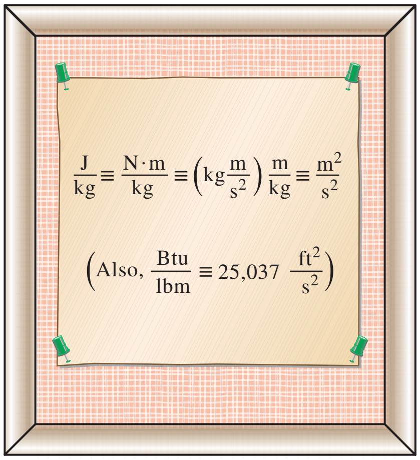


FIGURE 6–22

Under steady operation, shaft work and electrical work are the only forms of work a simple compressible system may involve.

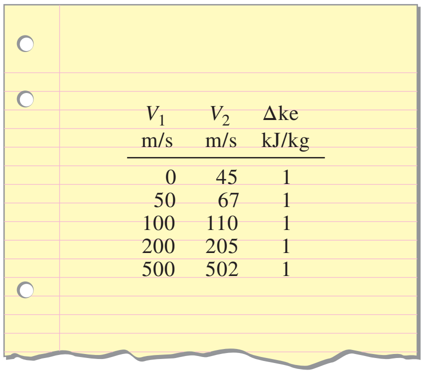


$$\frac{\text{J}}{\text{kg}} \equiv \frac{\text{N}\cdot\text{m}}{\text{kg}} \equiv \left(\text{kg}\frac{\text{m}}{\text{s}^2}\right)\frac{\text{m}}{\text{kg}} \equiv \frac{\text{m}^2}{\text{s}^2}$$

$$\left(\text{Also, } \frac{\text{Btu}}{\text{lbm}} \equiv 25{,}037\ \frac{\text{ft}^2}{\text{s}^2}\right)$$

FIGURE 6–23

The units m^2/s^2 and J/kg are equivalent.



V_1 m/s	V_2 m/s	Δke kJ/kg
0	45	1
50	67	1
100	110	1
200	205	1
500	502	1

FIGURE 6–24

At very high velocities, even small changes in velocities can cause significant changes in the kinetic energy of the fluid.

6–4 ▪ SOME STEADY-FLOW ENGINEERING DEVICES

Many engineering devices operate essentially under the same conditions for long periods of time. The components of a steam power plant (turbines, compressors, heat exchangers, and pumps), for example, operate nonstop for months before the system is shut down for maintenance (Fig. 6–25). Therefore, these devices can be conveniently analyzed as steady-flow devices.

In this section, some common steady-flow devices are described, and the thermodynamic aspects of the flow through them are analyzed. The conservation of mass and the conservation of energy principles for these devices are illustrated with examples.

1 Nozzles and Diffusers

Nozzles and diffusers are commonly utilized in jet engines, rockets, spacecraft, and even garden hoses. A **nozzle** is a device that *increases the velocity of a fluid* at the expense of pressure. A **diffuser** is a device that *increases the pressure of a fluid* by slowing it down. That is, nozzles and diffusers perform opposite tasks. The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.

The rate of heat transfer between the fluid flowing through a nozzle or a diffuser and the surroundings is usually very small ($\dot{Q} \approx 0$) since the fluid has high velocities, and thus it does not spend enough time in the device for any significant heat transfer to take place. Nozzles and diffusers typically involve no work ($\dot{W} = 0$), and any change in potential energy is negligible ($\Delta\text{pe} \cong 0$). But nozzles and diffusers usually involve very high velocities, and as a fluid passes through a nozzle or diffuser, it experiences large changes in its velocity (Fig. 6–26). Therefore, the kinetic energy changes must be accounted for in analyzing the flow through these devices ($\Delta\text{ke} \not\cong 0$).

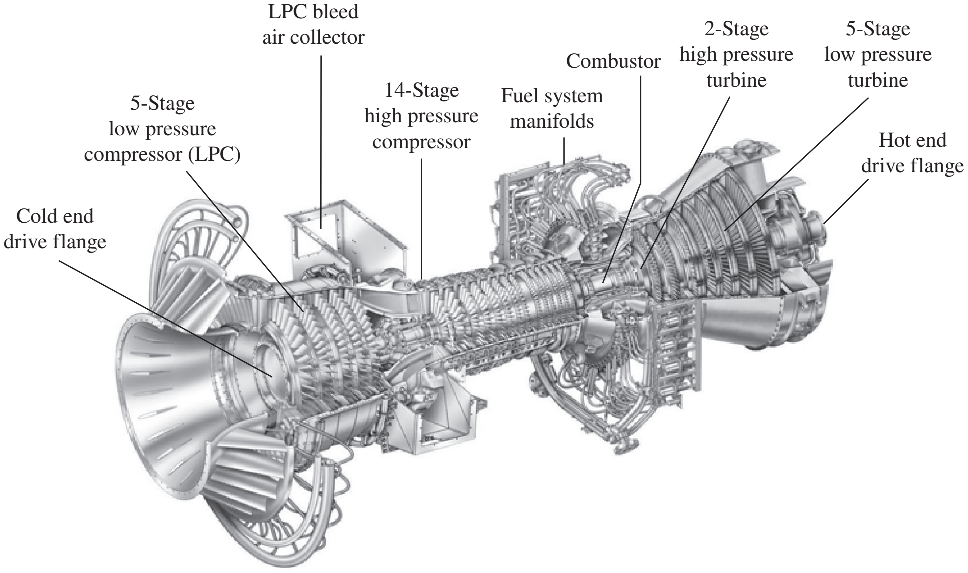


FIGURE 6–25

A modern land-based gas turbine used for electric power production. This is a General Electric LM5000 turbine. It has a length of 6.2 m, weighs 12.5 tons, and produces 55.2 MW at 3600 rpm with steam injection.

Courtesy of GE Power Systems

EXAMPLE 6–4 Deceleration of Air in a Diffuser

Air at 10°C and 80 kPa enters the diffuser of a jet engine steadily with a velocity of 200 m/s. The inlet area of the diffuser is 0.4 m^2. The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. Determine (*a*) the mass flow rate of the air and (*b*) the temperature of the air leaving the diffuser.

SOLUTION Air enters the diffuser of a jet engine steadily at a specified velocity. The mass flow rate of air and the temperature at the diffuser exit are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$. **2** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **3** The potential energy change is zero, $\Delta\text{pe} = 0$. **4** Heat transfer is negligible. **5** Kinetic energy at the diffuser exit is negligible. **6** There are no work interactions.

Analysis We take the *diffuser* as the system (Fig. 6–27). This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$.

(*a*) To determine the mass flow rate, we need to find the specific volume of the air first. This is determined from the ideal-gas relation at the inlet conditions:

$$\upsilon_1 = \frac{RT_1}{P_1} = \frac{(0.287\ \text{kPa·m}^3/\text{kg·K})(283\ \text{K})}{80\ \text{kPa}} = 1.015\ \text{m}^3/\text{kg}$$

Then,

$$\dot{m} = \frac{1}{\upsilon_1}V_1A_1 = \frac{1}{1.015\ \text{m}^3/\text{kg}}(200\ \text{m/s})(0.4\ \text{m}^2) = \mathbf{78.8\ kg/s}$$

Since the flow is steady, the mass flow rate through the entire diffuser remains constant at this value.

(*b*) Under stated assumptions and observations, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}}\nearrow^{0\ (\text{steady})} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) \quad (\text{since } \dot{Q} \cong 0,\ \dot{W} = 0, \text{ and } \Delta\text{pe} \cong 0)$$

$$h_2 = h_1 - \frac{V_2^2 - V_1^2}{2}$$

The exit velocity of a diffuser is usually small compared with the inlet velocity ($V_2 << V_1$); thus, the kinetic energy at the exit can be neglected. The enthalpy of air at the diffuser inlet is determined from the air table (Table A–21) to be

$$h_1 = h_{@\ 283\ \text{K}} = 283.14\ \text{kJ/kg}$$

Substituting, we get

$$h_2 = 283.14\ \text{kJ/kg} - \frac{0 - (200\ \text{m/s})^2}{2}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right)$$
$$= 303.14\ \text{kJ/kg}$$

From Table A–21, the temperature corresponding to this enthalpy value is

$$T_2 = \mathbf{303\ K}$$

Discussion This result shows that the temperature of the air increases by about 20°C as it is slowed down in the diffuser. The temperature rise of the air is mainly due to the conversion of kinetic energy to internal energy.

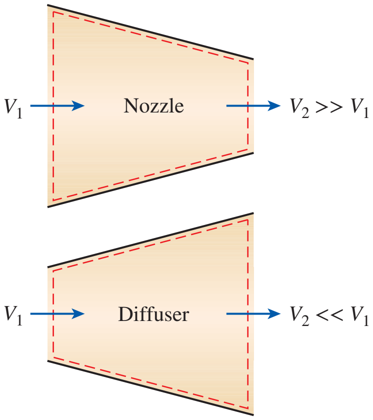


FIGURE 6–26
Nozzles and diffusers are shaped so that they cause large changes in fluid velocities and thus kinetic energies.

FIGURE 6–27
The diffuser of a jet engine discussed in Example 6–4.
Yunus Çengel

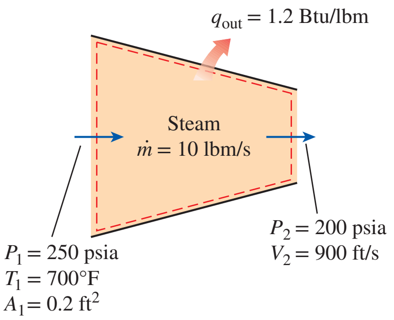


FIGURE 6–28
Schematic for Example 6–5.

EXAMPLE 6–5 Acceleration of Steam in a Nozzle

Steam at 250 psia and 700°F steadily enters a nozzle whose inlet area is 0.2 ft^2. The mass flow rate of steam through the nozzle is 10 lbm/s. Steam leaves the nozzle at 200 psia with a velocity of 900 ft/s. Heat losses from the nozzle per unit mass of the steam are estimated to be 1.2 Btu/lbm. Determine (*a*) the inlet velocity and (*b*) the exit temperature of the steam.

SOLUTION Steam enters a nozzle steadily at a specified flow rate and velocity. The inlet velocity of steam and the exit temperature are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\text{CV}} = 0$ and $\Delta E_{\text{CV}} = 0$. **2** There are no work interactions. **3** The potential energy change is zero, $\Delta\text{pe} = 0$.

Analysis We take the *nozzle* as the system (Fig. 6–28). This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$.

(*a*) The specific volume and enthalpy of steam at the nozzle inlet are

$$\left.\begin{aligned} P_1 &= 250 \text{ psia} \\ T_1 &= 700\text{°F} \end{aligned}\right\} \quad \begin{aligned} v_1 &= 2.6883 \text{ ft}^3/\text{lbm} \\ h_1 &= 1371.4 \text{ Btu/lbm} \end{aligned} \qquad \text{(Table A–6E)}$$

Then,

$$\dot{m} = \frac{1}{v_1} V_1 A_1$$

$$10 \text{ lbm/s} = \frac{1}{2.6883 \text{ ft}^3/\text{lbm}} (V_1)(0.2 \text{ ft}^2)$$

$$V_1 = \mathbf{134.4\ ft/s}$$

(*b*) Under stated assumptions and observations, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\cancelto{0\ \text{(steady)}}{dE_{\text{system}}/dt}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) \quad (\text{since } \dot{Q} \cong 0,\ \dot{W} = 0, \text{ and } \Delta\text{pe} \cong 0)$$

Dividing by the mass flow rate $\dot{m}$ and substituting, h_2 is determined to be

$$\begin{aligned} h_2 &= h_1 - q_{\text{out}} - \frac{V_2^2 - V_1^2}{2} \\ &= (1371.4 - 1.2) \text{ Btu/lbm} - \frac{(900 \text{ ft/s})^2 - (134.4 \text{ ft/s})^2}{2}\left(\frac{1 \text{ Btu/lbm}}{25{,}037 \text{ ft}^2/\text{s}^2}\right) \\ &= 1354.4 \text{ Btu/lbm} \end{aligned}$$

Then,

$$\left.\begin{aligned} P_2 &= 200 \text{ psia} \\ h_2 &= 1354.4 \text{ Btu/lbm} \end{aligned}\right\} \quad T_2 = \mathbf{662.0°F} \qquad \text{(Table A–6E)}$$

Discussion Note that the temperature of steam drops by 38.0°F as it flows through the nozzle. This drop in temperature is mainly due to the conversion of internal energy to kinetic energy. (The heat loss is too small to cause any significant effect in this case.)

2 Turbines and Compressors

In steam, gas, or hydroelectric power plants, the device that drives the electric generator is the turbine. As the fluid passes through the turbine, work is done against the blades, which are attached to the shaft. As a result, the shaft rotates, and the turbine produces work (Fig. 6–29).

Compressors, as well as pumps and fans, are devices used to increase the pressure of a fluid. Work is supplied to these devices from an external source through a rotating shaft. Therefore, compressors involve work inputs. Even though these three devices function similarly, they do differ in the tasks they perform. A *fan* increases the pressure of a gas slightly and is mainly used to mobilize a gas. A *compressor* is capable of compressing the gas to very high pressures. *Pumps* work very much like compressors except that they handle liquids instead of gases.

Note that turbines produce power output whereas compressors, pumps, and fans require power input. Heat transfer from turbines is usually negligible ($\dot{Q} \approx 0$) since they are typically well insulated. Heat transfer is also negligible for compressors unless there is intentional cooling. Potential energy changes are negligible for all of these devices ($\Delta\text{pe} \cong 0$). The velocities involved in these devices, with the exception of turbines and fans, are usually too low to cause any significant change in the kinetic energy ($\Delta\text{ke} \cong 0$). The fluid velocities encountered in most turbines are very high, and the fluid experiences a significant change in its kinetic energy. However, this change is usually very small relative to the change in enthalpy, and thus it is often disregarded.

FIGURE 6–29
Turbine blades attached to the turbine shaft.
Miss Kanithar Aiumla-Or/Shutterstock

EXAMPLE 6–6 Compressing Air with a Compressor

Air at 100 kPa and 280 K is compressed steadily to 600 kPa and 400 K. The mass flow rate of the air is 0.02 kg/s, and a heat loss of 16 kJ/kg occurs during the process. Assuming the changes in kinetic and potential energies are negligible, determine the necessary power input to the compressor.

SOLUTION Air is compressed steadily by a compressor to a specified temperature and pressure. The power input to the compressor is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\text{CV}} = 0$ and $\Delta E_{\text{CV}} = 0$. **2** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **3** The kinetic and potential energy changes are zero, $\Delta\text{ke} = \Delta\text{pe} = 0$.

Analysis We take the *compressor* as the system (Fig. 6–30). This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Also, heat is lost from the system and work is supplied to the system.

Under stated assumptions and observations, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy tranfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}}\overset{\nearrow 0 \text{ (steady)}}{} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{Q}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \Delta\text{ke} = \Delta\text{pe} \cong 0)$$

$$\dot{W}_{\text{in}} = \dot{m}q_{\text{out}} + \dot{m}(h_2 - h_1)$$

The enthalpy of an ideal gas depends on temperature only, and the enthalpies of the air at the specified temperatures are determined from the air table (Table A–21) to be

$$h_1 = h_{@\,280\text{ K}} = 280.13\text{ kJ/kg}$$

$$h_2 = h_{@\,400\text{ K}} = 400.98\text{ kJ/kg}$$

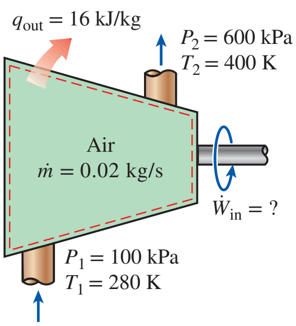


FIGURE 6–30
Schematic for Example 6–6.

Substituting, the power input to the compressor is determined to be

$$\begin{aligned}\dot{W}_{\text{in}} &= (0.02\text{ kg/s})(16\text{ kJ/kg}) + (0.02\text{ kg/s})(400.98 - 280.13)\text{ kJ/kg}\\ &= \mathbf{2.74\ kW}\end{aligned}$$

Discussion Note that the mechanical energy input to the compressor manifests itself as a rise in enthalpy of air and heat loss from the compressor.

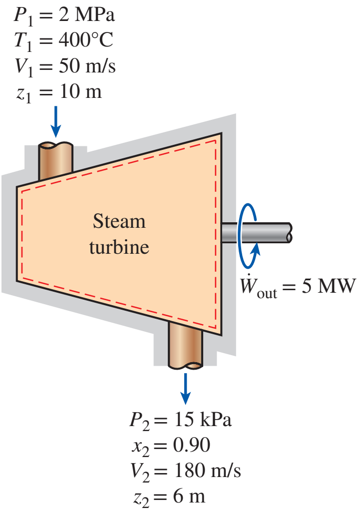


FIGURE 6–31
Schematic for Example 6–7.

EXAMPLE 6–7 Power Generation by a Steam Turbine

The power output of an adiabatic steam turbine is 5 MW, and the inlet and the exit conditions of the steam are as indicated in Fig. 6–31. (*a*) Compare the magnitudes of Δh, Δke, and Δpe. (*b*) Determine the work done per unit mass of the steam flowing through the turbine. (*c*) Calculate the mass flow rate of the steam.

SOLUTION The inlet and exit conditions of a steam turbine and its power output are given. The changes in kinetic energy, potential energy, and enthalpy of steam, as well as the work done per unit mass and the mass flow rate of steam are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\text{CV}} = 0$ and $\Delta E_{\text{CV}} = 0$. **2** The system is adiabatic and thus there is no heat transfer.

Analysis We take the *turbine* as the system. This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Also, work is done by the system. The inlet and exit velocities and elevations are given, and thus the kinetic and potential energies are to be considered.

(*a*) At the inlet, steam is in a superheated vapor state, and its enthalpy is

$$\left.\begin{aligned}P_1 &= 2\text{ MPa}\\ T_1 &= 400°\text{C}\end{aligned}\right\}\ h_1 = 3248.4\text{ kJ/kg} \qquad \text{(Table A–6)}$$

At the turbine exit, we obviously have a saturated liquid–vapor mixture at 15-kPa pressure. The enthalpy at this state is

$$h_2 = h_f + x_2 h_{fg} = [225.94 + (0.9)(2372.3)]\text{ kJ/kg} = 2361.01\text{ kJ/kg}$$

Then

$$\Delta h = h_2 - h_1 = (2361.01 - 3248.4)\text{kJ/kg} = \mathbf{-887.39\ kJ/kg}$$

$$\Delta\text{ke} = \frac{V_2^2 - V_1^2}{2} = \frac{(180\text{ m/s})^2 - (50\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{14.95\ kJ/kg}$$

$$\Delta\text{pe} = g(z_2 - z_1) = (9.81\text{ m/s}^2)[(6 - 10)\text{ m}]\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{-0.04\ kJ/kg}$$

(*b*) The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer in}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}}\nearrow^{0\text{ (steady)}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) = \dot{W}_{\text{out}} + \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right) \qquad (\text{since } \dot{Q} = 0)$$

Dividing by the mass flow rate $\dot{m}$ and substituting, the work done by the turbine per unit mass of the steam is determined to be

$$w_{\text{out}} = -\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right] = -(\Delta h + \Delta \text{ke} + \Delta \text{pe})$$

$$= -[-887.39 + 14.95 - 0.04]\ \text{kJ/kg} = \mathbf{872.48\ kJ/kg}$$

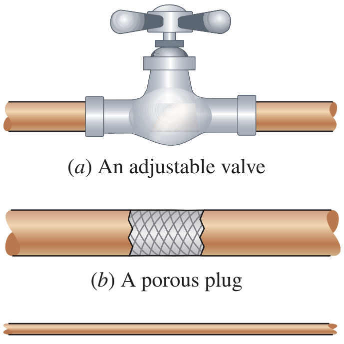
(*c*) The required mass flow rate for a 5-MW power output is

$$\dot{m} = \frac{\dot{W}_{\text{out}}}{w_{\text{out}}} = \frac{5000\ \text{kJ/s}}{872.48\ \text{kJ/kg}} = \mathbf{5.73\ kg/s}$$

Discussion Two observations can be made from these results. First, the change in potential energy is insignificant in comparison to the changes in enthalpy and kinetic energy. This is typical for most engineering devices. Second, as a result of low pressure and thus high specific volume, the steam velocity at the turbine exit can be very high. Yet the change in kinetic energy is a small fraction of the change in enthalpy (less than 2 percent in our case) and is therefore often neglected.

3 Throttling Valves

Throttling valves are *any kind of flow-restricting devices* that cause a significant pressure drop in the fluid. Some familiar examples are ordinary adjustable valves, capillary tubes, and porous plugs (Fig. 6–32). Unlike turbines, they produce a pressure drop without involving any work. The pressure drop in the fluid is often accompanied by a *large drop in temperature,* and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications. The magnitude of the temperature drop (or, sometimes, the temperature rise) during a throttling process is governed by a property called the *Joule-Thomson coefficient*.



(*a*) An adjustable valve

(*b*) A porous plug

(*c*) A capillary tube

FIGURE 6–32

Throttling valves are devices that cause large pressure drops in the fluid.

Throttling valves are usually small devices, and the flow through them may be assumed to be adiabatic ($q \cong 0$) since there is neither sufficient time nor large enough area for any effective heat transfer to take place. Also, there is no work done ($w = 0$), and the change in potential energy, if any, is very small ($\Delta \text{pe} \cong 0$). Even though the exit velocity is often considerably higher than the inlet velocity, in many cases, the increase in kinetic energy is insignificant ($\Delta \text{ke} \cong 0$). Then the conservation of energy equation for this single-stream steady-flow device reduces to

$$h_2 \cong h_1 \qquad \text{(kJ/kg)} \tag{6–41}$$

That is, enthalpy values at the inlet and exit of a throttling valve are the same. For this reason, a throttling valve is sometimes called an *isenthalpic device.* Note, however, that for throttling devices with large exposed surface areas such as capillary tubes, heat transfer may be significant.

To gain some insight into how throttling affects fluid properties, let us express Eq. 6–41 as follows:

$$u_1 + P_1 v_1 = u_2 + P_2 v_2$$

or

$$\text{Internal energy} + \text{Flow energy} = \text{Constant}$$

Thus the final outcome of a throttling process depends on which of the two quantities increases during the process. If the flow energy increases during the process ($P_2 v_2 > P_1 v_1$), it can do so at the expense of the internal energy. As a result, internal energy decreases, which is usually accompanied by a drop in temperature. If the product $P\upsilon$ decreases, the internal energy and the temperature of a fluid will increase during a throttling process. In the case of an ideal gas, $h = h(T)$, and thus the temperature has to remain constant during a throttling process (Fig. 6–33).

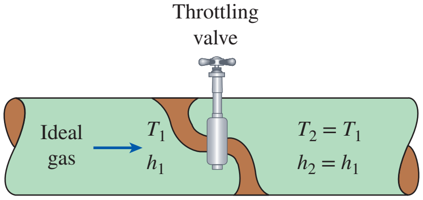


FIGURE 6–33
The temperature of an ideal gas does not change during a throttling (h = constant) process since $h = h(T)$.

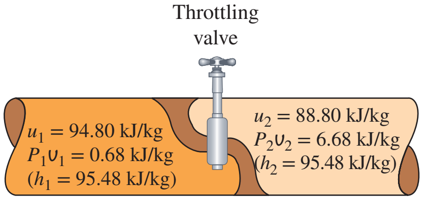


FIGURE 6–34
During a throttling process, the enthalpy (flow energy + internal energy) of a fluid remains constant. But internal and flow energies may be converted to each other.

EXAMPLE 6–8 Expansion of Refrigerant-134a in a Refrigerator

Refrigerant-134a enters the capillary tube of a refrigerator as saturated liquid at 0.8 MPa and is throttled to a pressure of 0.12 MPa. Determine the quality of the refrigerant at the final state and the temperature drop during this process.

SOLUTION Refrigerant-134a that enters a capillary tube as saturated liquid is throttled to a specified pressure. The exit quality of the refrigerant and the temperature drop are to be determined.

Assumptions **1** Heat transfer from the tube is negligible. **2** Kinetic energy change of the refrigerant is negligible.

Analysis A capillary tube is a simple flow-restricting device that is commonly used in refrigeration applications to cause a large pressure drop in the refrigerant. Flow through a capillary tube is a throttling process; thus, the enthalpy of the refrigerant remains constant (Fig. 6–34).

At inlet:
$$\left.\begin{array}{l} P_1 = 0.8\ \text{MPa} \\ \text{sat. liquid} \end{array}\right\} \begin{array}{l} T_1 = T_{\text{sat @ 0.8 MPa}} = 31.31^\circ\text{C} \\ h_1 = h_{f\text{ @ 0.8 MPa}} = 95.48\ \text{kJ/kg} \end{array} \qquad \text{(Table A–12)}$$

At exit:
$$\left.\begin{array}{l} P_2 = 0.12\ \text{MPa} \\ h_2 = h_1 \end{array}\right\} \begin{array}{l} h_f = 22.47\ \text{kJ/kg} \quad T_{\text{sat}} = -22.32^\circ\text{C} \\ h_g = 236.99\ \text{kJ/kg} \end{array}$$

Obviously $h_f < h_2 < h_g$; thus, the refrigerant exists as a saturated mixture at the exit state. The quality at this state is

$$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{95.48 - 22.47}{236.99 - 22.47} = \mathbf{0.340}$$

Since the exit state is a saturated mixture at 0.12 MPa, the exit temperature must be the saturation temperature at this pressure, which is –22.32°C. Then the temperature change for this process becomes

$$\Delta T = T_2 - T_1 = (-22.32 - 31.31)^\circ\text{C} = \mathbf{-53.63^\circ C}$$

Discussion Note that the temperature of the refrigerant drops by 53.63°C during this throttling process. Also note that 34.0 percent of the refrigerant vaporizes during this throttling process, and the energy needed to vaporize this refrigerant is absorbed from the refrigerant itself.

4a Mixing Chambers

In engineering applications, mixing two streams of fluids is not a rare occurrence. The section where the mixing process takes place is commonly referred to as a **mixing chamber**. The mixing chamber does not have to be a distinct "chamber." An ordinary T-elbow or a Y-elbow in a shower, for example, serves as the mixing chamber for the cold- and hot-water streams (Fig. 6–35).

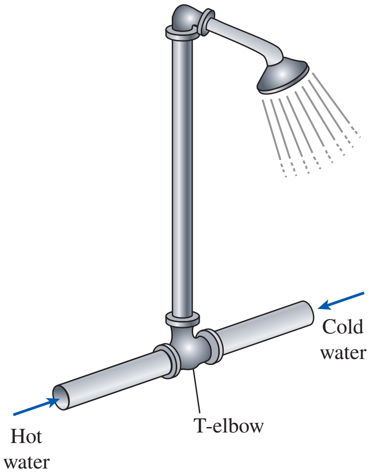


FIGURE 6–35
The T-elbow of an ordinary shower serves as the mixing chamber for the hot- and the cold-water streams.

The conservation of mass principle for a mixing chamber requires that the sum of the incoming mass flow rates equal the mass flow rate of the outgoing mixture.

Mixing chambers are usually well insulated ($q \cong 0$) and usually do not involve any kind of work ($w = 0$). Also, the kinetic and potential energies of the fluid streams are usually negligible (ke $\cong 0$, pe $\cong 0$). Then all there is left in the energy equation is the total energies of the incoming streams and the outgoing mixture. The conservation of energy principle requires that these two equal each other. Therefore, the conservation of energy equation becomes analogous to the conservation of mass equation for this case.

EXAMPLE 6–9 Mixing of Hot and Cold Waters in a Shower

Consider an ordinary shower where hot water at 140°F is mixed with cold water at 50°F. If it is desired that a steady stream of warm water at 110°F be supplied, determine the ratio of the mass flow rates of the hot to cold water. Assume the heat losses from the mixing chamber to be negligible and the mixing to take place at a pressure of 20 psia.

SOLUTION In a shower, cold water is mixed with hot water at a specified temperature. For a specified mixture temperature, the ratio of the mass flow rates of the hot to cold water is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\text{CV}} = 0$ and $\Delta E_{\text{CV}} = 0$. **2** The kinetic and potential energies are negligible, ke ≅ pe ≅ 0. **3** Heat losses from the system are negligible and thus $\dot{Q} \cong 0$. **4** There is no work interaction involved.

Analysis We take the *mixing chamber* as the system (Fig. 6–36). This is a *control volume* since mass crosses the system boundary during the process. We observe that there are two inlets and one exit.

Under the stated assumptions and observations, the mass and energy balances for this steady-flow system can be expressed in the rate form as follows:

Mass balance:

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \cancelto{0\text{ (steady)}}{dm_{\text{system}}/dt} = 0$$

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Energy balance:

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\cancelto{0\text{ (steady)}}{dE_{\text{system}}/dt}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}_1 h_1 + \dot{m}_1 h_2 = \dot{m}_3 h_3 \quad (\text{since } \dot{Q} \cong 0,\ \dot{W} = 0,\ \text{ke} \cong \text{pe} \cong 0)$$

Combining the mass and energy balances,

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$$

Dividing this equation by $\dot{m}_2$ yields

$$y h_1 + h_2 = (y_1 + 1) h_3$$

where $y = \dot{m}_1/\dot{m}_2$ is the desired mass flow rate ratio.

The saturation temperature of water at 20 psia is 227.92°F. Since the temperatures of all three streams are below this value ($T < T_{\text{sat}}$), the water in all three streams exists as a compressed liquid (Fig. 6–37). A compressed liquid can be approximated as a saturated liquid at the given temperature. Thus,

$$h_1 \cong h_{f\,@\,140°\text{F}} = 107.99 \text{ Btu/lbm}$$

$$h_2 \cong h_{f\,@\,50°\text{F}} = 18.07 \text{ Btu/lbm}$$

$$h_3 \cong h_{f\,@\,110°\text{F}} = 78.02 \text{ Btu/lbm}$$

Solving for y and substituting yields

$$y = \frac{h_3 - h_2}{h_1 - h_3} = \frac{78.02 - 18.07}{107.99 - 78.02} = \mathbf{2.0}$$

Discussion Note that the mass flow rate of the hot water must be twice the mass flow rate of the cold water for the mixture to leave at 110°F.

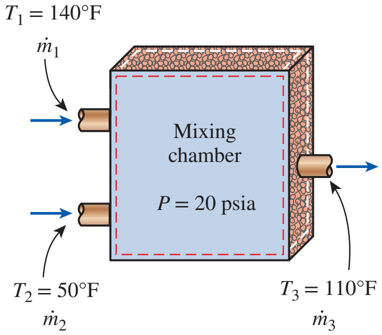


FIGURE 6–36
Schematic for Example 6–9.

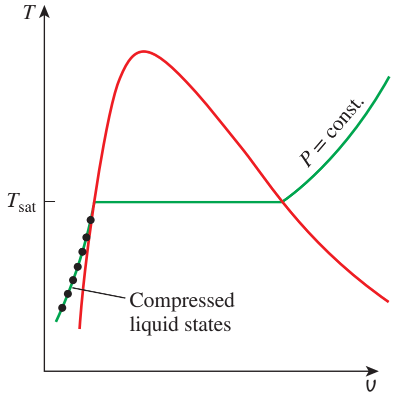


FIGURE 6–37
A substance exists as a compressed liquid at temperatures below the saturation temperatures at the given pressure.

4b Heat Exchangers

As the name implies, **heat exchangers** are devices where two moving fluid streams exchange heat without mixing. Heat exchangers are widely used in various industries, and they come in various designs.

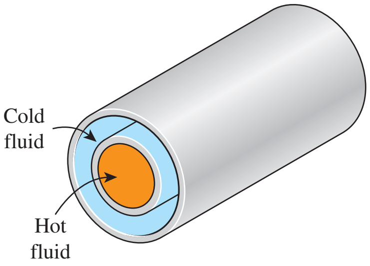


FIGURE 6–38

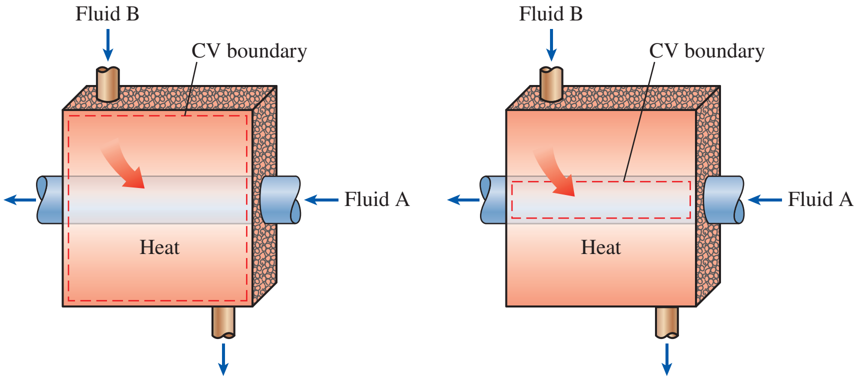
A heat exchanger can be as simple as two concentric pipes.

The simplest form of a heat exchanger is a *double-tube heat exchanger,* shown in Fig. 6–38. It is composed of two concentric pipes of different diameters. One fluid flows in the inner pipe, and the other in the annular space between the two pipes. Heat is transferred from the hot fluid to the cold one through the wall separating them. Sometimes the inner tube makes a couple of turns inside the shell to increase the heat transfer area, and thus the rate of heat transfer. The mixing chambers discussed earlier are sometimes classified as *direct-contact* heat exchangers.

The conservation of mass principle for a heat exchanger in steady operation requires that the sum of the inbound mass flow rates equal the sum of the outbound mass flow rates. This principle can also be expressed as follows: *Under steady operation, the mass flow rate of each fluid stream flowing through a heat exchanger remains constant.*

Heat exchangers typically involve no work interactions ($w = 0$) and negligible kinetic and potential energy changes ($\Delta \text{ke} \cong 0$, $\Delta \text{pe} \cong 0$) for each fluid stream. The heat transfer rate associated with heat exchangers depends on how the control volume is selected. Heat exchangers are intended for heat transfer between two fluids *within* the device, and the outer shell is usually well insulated to prevent any heat loss to the surrounding medium.

When the entire heat exchanger is selected as the control volume, $\dot{Q}$ becomes zero, since the boundary for this case lies just beneath the insulation and little or no heat crosses the boundary (Fig. 6–39). If, however, only one of the fluids is selected as the control volume, then heat will cross this boundary as it flows from one fluid to the other, and $\dot{Q}$ will not be zero. In fact, $\dot{Q}$ in this case will be the rate of heat transfer between the two fluids.



(*a*) System: Entire heat exchanger ($Q_{\text{CV}} = 0$)

(*b*) System: Fluid A ($Q_{\text{CV}} \neq 0$)

FIGURE 6–39

The heat transfer associated with a heat exchanger may be zero or nonzero depending on how the control volume is selected.

EXAMPLE 6–10 Cooling of Refrigerant-134a by Water

Refrigerant-134a is to be cooled by water in a condenser. The refrigerant enters the condenser with a mass flow rate of 6 kg/min at 1 MPa and 70°C and leaves at 35°C. The cooling water enters at 300 kPa and 15°C and leaves at 25°C. Neglecting any pressure drops, determine (*a*) the mass flow rate of the cooling water required and (*b*) the heat transfer rate from the refrigerant to water.

SOLUTION Refrigerant-134a is cooled by water in a condenser. The mass flow rate of the cooling water and the rate of heat transfer from the refrigerant to the water are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$. **2** The kinetic and potential energies are negligible, ke ≅ pe ≅ 0. **3** Heat losses from the system are negligible and thus $\dot{Q} \cong 0$. **4** There is no work interaction.

Analysis We take the *entire heat exchanger* as the system (Fig. 6–40). This is a *control volume* since mass crosses the system boundary during the process. In general, there are several possibilities for selecting the control volume for multiple-stream steady-flow devices, and the proper choice depends on the situation at hand. We observe that there are two fluid streams (and thus two inlets and two exits) but no mixing.

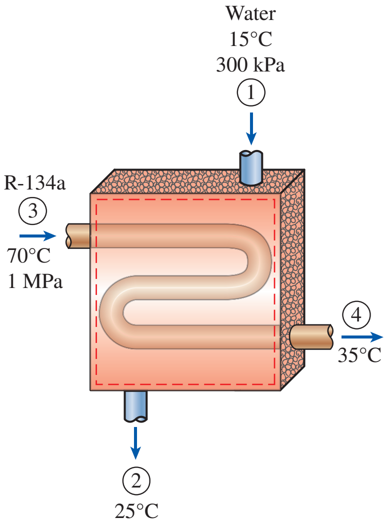


FIGURE 6–40
Schematic for Example 6–10.

(*a*) Under the stated assumptions and observations, the mass and energy balances for this steady-flow system can be expressed in the rate form as follows:

Mass balance: $$\dot{m}_{in} = \dot{m}_{out}$$

for each fluid stream since there is no mixing. Thus,

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_w$$
$$\dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

Energy balance:

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{system}/dt}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}}\overset{\nearrow 0\text{ (steady)}}{} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4 \quad (\text{since } \dot{Q} \cong 0,\ \dot{W} = 0,\ \text{ke} \cong \text{pe} \cong 0)$$

Combining the mass and energy balances and rearranging give

$$\dot{m}_w(h_1 - h_2) = \dot{m}_R(h_4 - h_3)$$

Now we need to determine the enthalpies at all four states. Water exists as a compressed liquid at both the inlet and the exit since the temperatures at both locations are below the saturation temperature of water at 300 kPa (133.52°C). Approximating the compressed liquid as a saturated liquid at the given temperatures, we have

$$\begin{aligned} h_1 &\cong h_{f\,@\,15^\circ\text{C}} = 62.982\ \text{kJ/kg} \\ h_2 &\cong h_{f\,@\,25^\circ\text{C}} = 104.83\ \text{kJ/kg} \end{aligned} \qquad \text{(Table A–4)}$$

The refrigerant enters the condenser as a superheated vapor and leaves as a compressed liquid at 35°C. From refrigerant-134a tables,

$$\left.\begin{aligned} P_3 &= 1\ \text{MPa} \\ T_3 &= 70^\circ\text{C} \end{aligned}\right\} h_3 = 303.87\ \text{kJ/kg} \qquad \text{(Table A–13)}$$

$$\left.\begin{aligned} P_4 &= 1\ \text{MPa} \\ T_4 &= 70^\circ\text{C} \end{aligned}\right\} h_4 \cong h_{f\,@\,35^\circ\text{C}} = 100.88\ \text{kJ/kg} \qquad \text{(Table A–11)}$$

Substituting, we find

$$\dot{m}_w(62.982 - 104.83)\text{kJ/kg} = (6\ \text{kg/min})[(100.88 - 303.87)\ \text{kJ/kg}]$$
$$\dot{m}_w = \mathbf{29.1\ kg/min}$$

(*b*) To determine the heat transfer from the refrigerant to the water, we have to choose a control volume whose boundary lies on the path of heat transfer. We can choose the volume occupied by either fluid as our control volume. For no particular reason, we choose the volume occupied by the water. All the assumptions stated earlier apply, except that the heat transfer is no longer zero. Then assuming heat to be transferred to water, the energy balance for this single-stream steady-flow system reduces to

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}}\overset{\nearrow 0\text{ (steady)}}{} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{Q}_{w,\text{in}} + \dot{m}_w h_1 = \dot{m}_w h_2$$

Rearranging and substituting,

$$\dot{Q}_{w,\text{in}} = \dot{m}_w(h_2 - h_1) = (29.1\text{ kg/min})[(104.83 - 62.982)\text{ kJ/kg}]$$

$$= \mathbf{1218\ kJ/min}$$

Discussion Had we chosen the volume occupied by the refrigerant as the control volume (Fig. 6–41), we would have obtained the same result for $\dot{Q}_{R,\text{out}}$ since the heat gained by the water is equal to the heat lost by the refrigerant.

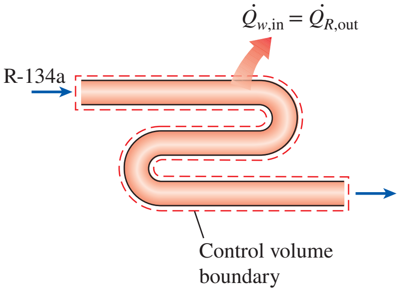


FIGURE 6–41
In a heat exchanger, the heat transfer depends on the choice of the control volume.

5 Pipe and Duct Flow

The transport of liquids or gases in pipes and ducts is of great importance in many engineering applications. Flow through a pipe or a duct usually satisfies the steady-flow conditions and thus can be analyzed as a steady-flow process. This, of course, excludes the transient start-up and shut-down periods. The control volume can be selected to coincide with the interior surfaces of the portion of the pipe or the duct that we are interested in analyzing.

Under normal operating conditions, the amount of heat gained or lost by the fluid may be very significant, particularly if the pipe or duct is long (Fig. 6–42). Sometimes heat transfer is desirable and is the sole purpose of the flow. Water flow through the pipes in the furnace of a power plant, the flow of refrigerant in a freezer, and the flow in heat exchangers are some examples of this case. At other times, heat transfer is undesirable, and the pipes or ducts are insulated to prevent any heat loss or gain, particularly when the temperature difference between the flowing fluid and the surroundings is large. Heat transfer in this case is negligible.

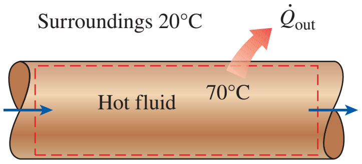


FIGURE 6–42
Heat losses from a hot fluid flowing through an uninsulated pipe or duct to the cooler environment may be very significant.

If the control volume involves a heating section (electric wires), a fan, or a pump (shaft), the work interactions should be considered (Fig. 6–43). Of these, fan work is usually small and often neglected in energy analysis.

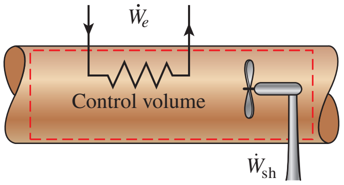


FIGURE 6–43
Pipe or duct flow may involve more than one form of work at the same time.

The velocities involved in pipe and duct flow are relatively low, and the kinetic energy changes are usually insignificant. This is particularly true when the pipe or duct diameter is constant and the heating effects are negligible. Kinetic energy changes may be significant, however, for gas flow in ducts with variable cross-sectional areas especially when the compressibility effects are significant. The potential energy term may also be significant when the fluid undergoes a considerable elevation change as it flows in a pipe or duct.

EXAMPLE 6–11 Electric Heating of Air in a House

The electric heating systems used in many houses consist of a simple duct with resistance heaters. Air is heated as it flows over resistance wires. Consider a 15-kW electric heating system. Air enters the heating section at 100 kPa and 17°C with a volume flow rate of 150 m^3/min. If heat is lost from the air in the duct to the surroundings at a rate of 200 W, determine the exit temperature of air.

SOLUTION The electric heating system of a house is considered. For specified electric power consumption and air flow rate, the air exit temperature is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$. **2** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **3** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **4** Constant specific heats at room temperature can be used for air.

Analysis We take the *heating section portion of the duct* as the system (Fig. 6–44). This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Also, heat is lost from the system and electrical work is supplied to the system.

At temperatures encountered in heating and air-conditioning applications, Δh can be replaced by $c_p \Delta T$ where c_p = 1.005 kJ/kg·°C—the value at room temperature—with negligible error (Fig. 6–45). Then the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{system}/dt}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}} \nearrow^{0\text{ (steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{e,in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{e,in} - \dot{Q}_{out} = \dot{m}c_p(T_2 - T_1)$$

From the ideal-gas relation, the specific volume of air at the inlet of the duct is

$$\upsilon_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa·m}^3/\text{kg·K})(290 \text{ K})}{100 \text{ kPa}} = 0.832 \text{ m}^3/\text{kg}$$

The mass flow rate of the air through the duct is determined from

$$\dot{m} = \frac{\dot{V}_1}{\upsilon_1} = \frac{150 \text{ m}^3/\text{min}}{0.832 \text{ m}^3/\text{kg}}\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 3.0 \text{ kg/s}$$

Substituting the known quantities, the exit temperature of the air is determined to be

$$(15 \text{ kJ/s}) - (0.2 \text{ kJ/s}) = (3 \text{ kg/s})(1.005 \text{ kJ/kg·°C})(T_2 - 17)\text{°C}$$

$$T_2 = \mathbf{21.9°C}$$

Discussion Note that heat loss from the duct reduces the exit temperature of air.

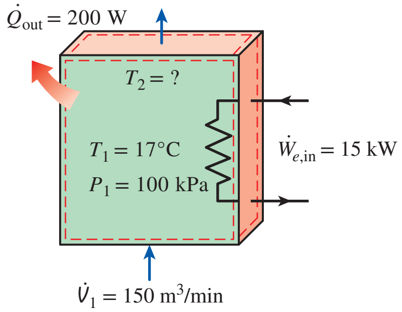


FIGURE 6–44
Schematic for Example 6–11.

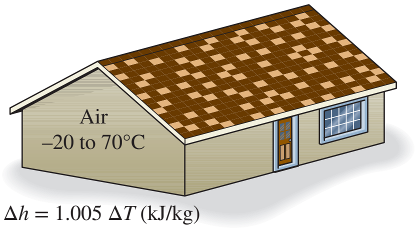


FIGURE 6–45
The error involved in $\Delta h = c_p \Delta T$, where c_p = 1.005 kJ/kg·°C, is less than 0.5 percent for air in the temperature range –20 to 70°C.

6–5 ▪ ENERGY ANALYSIS OF UNSTEADY-FLOW PROCESSES

During a steady-flow process, no changes occur within the control volume; thus, one does not need to be concerned about what is going on within the boundaries. Not having to worry about any changes within the control volume with time greatly simplifies the analysis.

Many processes of interest, however, involve *changes* within the control volume with time. Such processes are called *unsteady-flow,* or *transient-flow,* processes. The steady-flow relations developed earlier are obviously not applicable to these processes. When an unsteady-flow process is analyzed, it is important to keep track of the mass and energy contents of the control volume as well as the energy interactions across the boundary.

Some familiar unsteady-flow processes are the charging of rigid vessels from supply lines (Fig. 6–46), discharging a fluid from a pressurized vessel, driving a

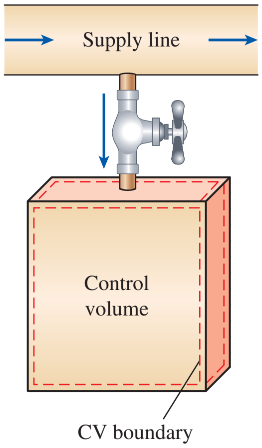


FIGURE 6–46
Charging of a rigid tank from a supply line is an unsteady-flow process since it involves changes within the control volume.

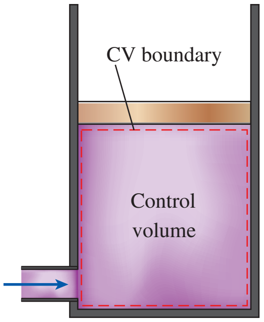


FIGURE 6–47
The shape and size of a control volume may change during an unsteady-flow process.

gas turbine with pressurized air stored in a large container, inflating tires or balloons, and even cooking with an ordinary pressure cooker.

Unlike steady-flow processes, unsteady-flow processes start and end over some finite time period instead of continuing indefinitely. Therefore in this section, we deal with changes that occur over some time interval Δt instead of with the rate of changes (changes per unit time). An unsteady-flow system, in some respects, is similar to a closed system, except that the mass within the system boundaries does not remain constant during a process.

Another difference between steady- and unsteady-flow systems is that steady-flow systems are fixed in space, size, and shape. Unsteady-flow systems, however, are not (Fig. 6–47). They are usually stationary; that is, they are fixed in space, but they may involve moving boundaries and thus boundary work.

The *mass balance* for any system undergoing any process can be expressed as (*see* Sec. 6–1)

$$m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}} \qquad \text{(kg)} \tag{6–42}$$

where $\Delta m_{\text{system}} = m_{\text{final}} - m_{\text{initial}}$ is the change in the mass of the system. For control volumes, it can also be expressed more explicitly as

$$m_i - m_e = (m_2 - m_1)_{\text{CV}} \tag{6–43}$$

where i = inlet, e = exit, 1 = initial state, and 2 = final state of the control volume. Often one or more terms in the equation above are zero. For example, $m_i = 0$ if no mass enters the control volume during the process, $m_e = 0$ if no mass leaves, and $m_1 = 0$ if the control volume is initially evacuated.

The energy content of a control volume changes with time during an unsteady-flow process. The magnitude of change depends on the amount of energy transfer across the system boundaries as heat and work as well as on the amount of energy transported into and out of the control volume by mass during the process. When analyzing an unsteady-flow process, we must keep track of the energy content of the control volume as well as the energies of the incoming and outgoing flow streams.

The general energy balance was given earlier as

$$\textit{Energy balance:} \qquad \underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}} \qquad \text{(kJ)} \tag{6–44}$$

The general unsteady-flow process, in general, is difficult to analyze because the properties of the mass at the inlets and exits may change during a process. Most unsteady-flow processes, however, can be represented reasonably well by the **uniform-flow process**, which involves the following idealization: *The fluid flow at any inlet or exit is uniform and steady, and thus the fluid properties do not change with time or position over the cross section of an inlet or exit. If they do, they are averaged and treated as constants for the entire process.*

Note that unlike the steady-flow systems, the state of an unsteady-flow system may change with time, and the state of the mass leaving the control volume at any instant is the same as the state of the mass in the control volume at that instant. The initial and final properties of the control volume can be determined from the knowledge of the initial and final states, which are completely specified by two independent intensive properties for simple compressible systems.

Then the energy balance for a uniform-flow system can be expressed explicitly as

$$\left(Q_{\text{in}} + W_{\text{in}} + \sum_{\text{in}} m\theta\right) - \left(Q_{\text{out}} + W_{\text{out}} + \sum_{\text{out}} m\theta\right) = (m_2e_2 - m_1e_1)_{\text{system}} \tag{6–45}$$

where $\theta = h + \text{ke} + \text{pe}$ is the energy of a fluid stream at any inlet or exit per unit mass, and $e = u + \text{ke} + \text{pe}$ is the energy of the nonflowing fluid within the control volume per unit mass. When the kinetic and potential energy changes associated with the control volume and fluid streams are negligible, as is usually the case, the energy balance above simplifies to

$$Q - W = \sum_{\text{out}} mh - \sum_{\text{in}} mh + (m_2u_2 - m_1u_1)_{\text{system}} \tag{6–46}$$

where $Q = Q_{\text{net,in}} = Q_{\text{in}} - Q_{\text{out}}$ is the net heat input and $W = W_{\text{net,out}} = W_{\text{out}} - W_{\text{in}}$ is the net work output. Note that if no mass enters or leaves the control volume during a process ($m_i = m_e = 0$, and $m_1 = m_2 = m$), this equation reduces to the energy balance relation for closed systems (Fig. 6–48). Also note that an unsteady-flow system may involve boundary work as well as electrical and shaft work (Fig. 6–49).

Although both the steady-flow and uniform-flow processes are somewhat idealized, many actual processes can be approximated reasonably well by one of these with satisfactory results. The degree of satisfaction depends on the desired accuracy and the degree of validity of the assumptions made.

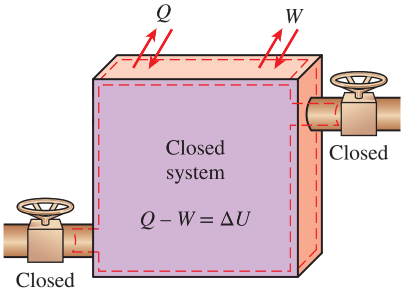


FIGURE 6–48
The energy equation of a uniform-flow system reduces to that of a closed system when all the inlets and exits are closed.

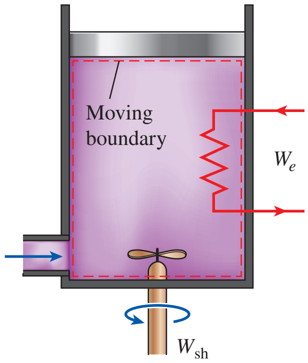


FIGURE 6–49
A uniform-flow system may involve electrical, shaft, and boundary work all at once.

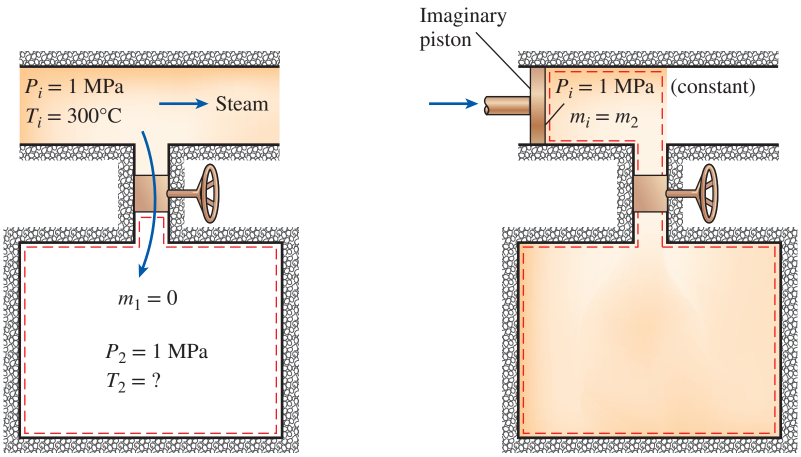
EXAMPLE 6–12 Charging of a Rigid Tank by Steam

A rigid, insulated tank that is initially evacuated is connected through a valve to a supply line that carries steam at 1 MPa and 300°C. Now the valve is opened, and steam is allowed to flow slowly into the tank until the pressure reaches 1 MPa, at which point the valve is closed. Determine the final temperature of the steam in the tank.

SOLUTION A valve connecting an initially evacuated tank to a steam line is opened, and steam flows in until the pressure inside rises to the line level. The final temperature in the tank is to be determined.

Assumptions **1** This process can be analyzed as a *uniform-flow process* since the properties of the steam entering the control volume remain constant during the entire process. **2** The kinetic and potential energies of the streams are negligible, ke $\cong$ pe $\cong$ 0. **3** The tank is stationary and thus its kinetic and potential energy changes are zero; that is, $\Delta\text{KE} = \Delta\text{PE} = 0$ and $\Delta E_{\text{system}} = \Delta U_{\text{system}}$. **4** There are no boundary, electrical, or shaft work interactions involved. **5** The tank is well insulated and thus there is no heat transfer.

Analysis We take the *tank* as the system (Fig. 6–50). This is a *control volume* since mass crosses the system boundary during the process. We observe that this is an unsteady-flow process since changes occur within the control volume. The control volume is initially evacuated and thus $m_1 = 0$ and $m_1u_1 = 0$. Also, there is one inlet and no exits for mass flow.



(*a*) Flow of steam into an evacuated tank

(*b*) The closed-system equivalence

FIGURE 6–50
Schematic for Example 6–12.

Noting that microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}} \rightarrow m_i = m_2 - \cancelto{0}{m_1} = m_2$

Energy balance:

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$m_i h_i = m_2 u_2 \qquad (\text{since } W = Q = 0,\ \text{ke} \cong \text{pe} \cong 0,\ m_1 = 0)$$

Combining the mass and energy balances gives

$$u_2 = h_i$$

That is, the final internal energy of the steam in the tank is equal to the enthalpy of the steam entering the tank. The enthalpy of the steam at the inlet state is

$$\left.\begin{array}{l} P_i = 1 \text{ MPa} \\ T_i = 300^\circ\text{C} \end{array}\right\} h_i = 3051.6 \text{ kJ/kg} \qquad (\text{Table A–6})$$

which is equal to u_2. Since we now know two properties at the final state, it is fixed and the temperature at this state is determined from the same table to be

$$\left.\begin{array}{l} P_2 = 1 \text{ MPa} \\ u_2 = 3051.6 \text{ kJ/kg} \end{array}\right\} T_2 = \mathbf{456.1^\circ C}$$

Discussion Note that the temperature of the steam in the tank has increased by 156.1°C. This result may be surprising at first, and you may be wondering where the energy to raise the temperature of the steam came from. The answer lies in the enthalpy term $h = u + PV$. Part of the energy represented by enthalpy is the flow energy PV, and this flow energy is converted to sensible internal energy once the flow ceases to exist in the control volume, and it shows up as an increase in temperature (Fig. 6–51).

Alternative solution This problem can also be solved by considering the region within the tank and the mass that is destined to enter the tank as a closed system, as shown in Fig. 6–50*b*. Since no mass crosses the boundaries, viewing this as a closed system is appropriate.

During the process, the steam upstream (the imaginary piston) will push the enclosed steam in the supply line into the tank at a constant pressure of 1 MPa. Then the boundary work done during this process is

$$W_{b,\text{in}} = -\int_1^2 P_i\, dV = -P_i(V_2 - V_1) = -P_i[V_{\text{tank}} - (V_{\text{tank}} + V_i)] = P_i V_i$$

where V_i is the volume occupied by the steam before it enters the tank and P_i is the pressure at the moving boundary (the imaginary piston face). The energy balance for the closed system gives

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$\begin{aligned} W_{b,\text{in}} &= \Delta U \\ m_i P_i V_i &= m_2 u_2 - m_i u_i \\ u_2 &= u_i + P_i V_i = h_i \end{aligned}$$

since the initial state of the system is simply the line conditions of the steam. This result is identical to the one obtained with the uniform-flow analysis. Once again, the temperature rise is caused by the so-called flow energy or flow work, which is the energy required to move the fluid during flow.

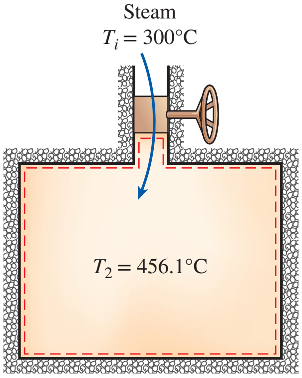


FIGURE 6–51
The temperature of steam rises from 300 to 456.1°C as it enters a tank as a result of flow energy being converted to internal energy.

EXAMPLE 6–13 Discharge of Heated Air at Constant Temperature

An insulated 8-m^3 rigid tank contains air at 600 kPa and 400 K. A valve connected to the tank is now opened, and air is allowed to escape until the pressure inside drops to 200 kPa. The air temperature during the process is maintained constant by an electric resistance heater placed in the tank. Determine the electrical energy supplied to air during this process.

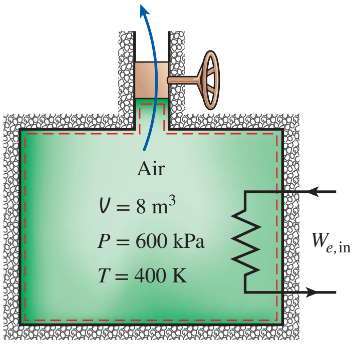


FIGURE 6–52
Schematic for Example 6–13.

SOLUTION Pressurized air in an insulated rigid tank equipped with an electric heater is allowed to escape at constant temperature until the pressure inside drops to a specified value. The amount of electrical energy supplied to air is to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the exit conditions remain constant. **2** Kinetic and potential energies are negligible. **3** The tank is insulated and thus heat transfer is negligible. **4** Air is an ideal gas with variable specific heats.

Analysis We take the contents of the tank as the system, which is a control volume since mass crosses the boundary (Fig. 6–52). Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy *h* and internal energy *u*, respectively, the mass and energy balances for this uniform-flow system can be expressed as

$$\textit{Mass balance:}\quad m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}} \rightarrow m_e = m_1 - m_2$$

$$\textit{Energy balance:}\quad \underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

$$W_{e,\text{in}} - m_e h_e = m_2 u_2 - m_1 u_1 \qquad (\text{since } Q \cong \text{ke} \cong \text{pe} \cong 0)$$

The gas constant of air is $R = 0.287$ kPa·m^3/kg·K (Table A-1). The initial and final masses of air in the tank and the discharged amount are determined from the ideal gas relation to be

$$m_1 = \frac{P_1 V_1}{RT_1} = \frac{(600\text{ kPa})(8\text{ m}^3)}{(0.287\text{ kPa·m}^3/\text{kg·K})(400\text{ K})} = 41.81\text{ kg}$$

$$m_2 = \frac{P_2 V_2}{RT_2} = \frac{(200\text{ kPa})(8\text{ m}^3)}{(0.287\text{ kPa·m}^3/\text{kg·K})(400\text{ K})} = 13.94\text{ kg}$$

$$m_e = m_1 - m_2 = 41.81 - 13.94 = 27.87\text{ kg}$$

The enthalpy and internal energy of air at 400 K are $h_e = 400.98$ kJ/kg and $u_1 = u_2 = 286.16$ kJ/kg (Table A-21). The electrical energy supplied to air is determined from the energy balance to be

$$\begin{aligned} W_{e,\text{in}} &= m_e h_e + m_2 u_2 - m_1 u_1 \\ &= (27.87\text{ kg})(400.98\text{ kJ/kg}) + (13.94\text{ kg})(286.16\text{ kJ/kg}) \\ &\quad - (41.81\text{ kg})(286.16\text{ kJ/kg}) \\ &= 3200\text{ kJ} = \mathbf{0.889\ kWh} \end{aligned}$$

since 1 kWh = 3600 kJ.

Discussion If the temperature of discharged air changes during the process, the problem can be solved with reasonable accuracy by evaluating h_e at the average discharge temperature $T_e = (T_2 + T_1)/2$ and treating it as constant.

SUMMARY

The *conservation of mass principle* states that the net mass transfer to or from a system during a process is equal to the net change (increase or decrease) in the total mass of the system during that process, and it is expressed as

$$m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}} \quad \text{and} \quad \dot{m}_{\text{in}} - \dot{m}_{\text{out}} = dm_{\text{system}}/dt$$

where $\Delta m_{\text{system}} = m_{\text{final}} - m_{\text{initial}}$ is the change in the mass of the system during the process, $\dot{m}_{\text{in}}$ and $\dot{m}_{\text{out}}$ are the total rates of mass flow into and out of the system, and dm_{system}/dt is the rate of change of mass within the system boundaries. These relations are also referred to as the mass balance and are applicable to any system undergoing any kind of process.

The amount of mass flowing through a cross section per unit time is called the *mass flow rate* and is expressed as

$$\dot{m} = \rho V A$$

where ρ = density of fluid, V = average fluid velocity normal to A, and A = cross-sectional area normal to flow direction. The volume of the fluid flowing through a cross section per unit time is called the *volume flow rate* and is expressed as

$$\dot{V} = VA = \dot{m}/\rho$$

The work required to push a unit mass of fluid into or out of a control volume is called *flow work* or *flow energy* and is expressed as $w_{\text{flow}} = P\mathrm{v}$. In the analysis of control volumes, it is convenient to combine the flow energy and internal energy into *enthalpy*. Then the total energy of a flowing fluid is expressed as

$$\theta = h + \text{ke} + \text{pe} = h + \frac{V^2}{2} + gz$$

The total energy transported by a flowing fluid of mass m with uniform properties is $m\theta$. The rate of energy transport by a fluid with a mass flow rate of $\dot{m}$ is $\dot{m}\theta$. When the kinetic and potential energies of a fluid stream are negligible, the amount and rate of energy transport become $E_{\text{mass}} = mh$ and $\dot{E}_{\text{mass}} = \dot{m}h$, respectively.

The *first law of thermodynamics* is essentially an expression of the conservation of energy principle, also called the *energy balance*. The general mass and energy balances for *any system* undergoing *any process* can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

It can also be expressed in the *rate form* as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

Thermodynamic processes involving control volumes can be considered in two groups: steady-flow processes and unsteady-flow processes. During a *steady-flow process,* the fluid flows through the control volume steadily, experiencing no change with time at a fixed position. The mass and energy content of the control volume remain constant during a steady-flow process. Taking heat transfer *to* the system and work done *by* the system to be positive quantities, the conservation of mass and energy equations for steady-flow processes are expressed as

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m}$$

$$\dot{Q} - \dot{W} = \sum_{\text{out}} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each exit}} - \sum_{\text{in}} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each inlet}}$$

These are the most general forms of the equations for steady-flow processes. For single-stream (one-inlet–one-exit) systems such as nozzles, diffusers, turbines, compressors, and pumps, they simplify to

$$\dot{m}_1 = \dot{m}_2 \quad \longrightarrow \quad \frac{1}{\mathrm{v}_1} V_1 A_1 = \frac{1}{\mathrm{v}_2} V_2 A_2$$

$$\dot{Q} - \dot{W} = \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

In these relations, subscripts 1 and 2 denote the inlet and exit states, respectively.

Most unsteady-flow processes can be modeled as a *uniform-flow process,* which requires that the fluid flow at any inlet or exit is uniform and steady, and thus the fluid properties do not change with time or position over the cross section of an inlet or exit. If they do, they are averaged and treated as constants for the entire process. When kinetic and potential energy changes associated with the control volume and the fluid streams are negligible, the mass and energy balance relations for a uniform-flow system are expressed as

$$m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}}$$

$$Q - W = \sum_{\text{out}} mh - \sum_{\text{in}} mh + (m_2 u_2 - m_1 u_1)_{\text{system}}$$

where $Q = Q_{\text{net,in}} = Q_{\text{in}} - Q_{\text{out}}$ is the net heat input and $W = W_{\text{net,out}} = W_{\text{out}} - W_{\text{in}}$ is the net work output.

When solving thermodynamic problems, it is recommended that the general form of the energy balance $E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$ be used for all problems and that we simplify it for the particular problem instead of using the specific relations given here for different processes.

REFERENCES AND SUGGESTED READINGS

1. ASHRAE *Handbook of Fundamentals.* SI version. Atlanta, GA: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc., 1993.
2. ASHRAE *Handbook of Refrigeration.* SI version. Atlanta, GA: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc., 1994.
3. Y. A. Çengel and J. M. Cimbala, *Fluid Mechanics: Fundamentals and Applications,* 4th ed. New York: McGraw-Hill, 2018.

PROBLEMS*

Conservation of Mass

6–1C Name four physical quantities that are conserved and two quantities that are not conserved during a process.

6–2C Define mass and volume flow rates. How are they related to each other?

6–3C Does the amount of mass entering a control volume have to be equal to the amount of mass leaving during an unsteady-flow process?

6–4C Consider a device with one inlet and one outlet. If the volume flow rates at the inlet and at the outlet are the same, is the flow through this device necessarily steady? Why?

6–5 The ventilating fan of the bathroom of a building has a volume flow rate of 30 L/s and runs continuously. If the density of air inside is 1.20 kg/m^3, determine the mass of air vented out in one day.

6–6 Air enters a 16-cm-diameter pipe steadily at 200 kPa and 20°C with a velocity of 5 m/s. Air is heated as it flows, and it leaves the pipe at 180 kPa and 40°C. Determine (*a*) the volume flow rate of air at the inlet, (*b*) the mass flow rate of air, and (*c*) the velocity and volume flow rate at the exit.

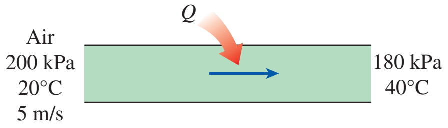


FIGURE P6–6

6–7E A garden hose attached with a nozzle is used to fill a 20-gal bucket. The inner diameter of the hose is 1 in and it reduces to 0.5 in at the nozzle exit. If the average velocity in the hose is 8 ft/s, determine (*a*) the volume and mass flow rates of water through the hose, (*b*) how long it will take to fill the bucket with water, and (*c*) the average velocity of water at the nozzle exit.

6–8E A steady-flow compressor is used to compress helium from 15 psia and 70°F at the inlet to 200 psia and 600°F at the outlet. The outlet area and velocity are 0.01 ft^2 and 100 ft/s, respectively, and the inlet velocity is 50 ft/s. Determine the mass flow rate and the inlet area. *Answers:* 0.0704 lbm/s, 0.133 ft^2

6–9 Air enters the 1-m^2 inlet of an aircraft engine at 100 kPa and 20°C with a velocity of 180 m/s. Determine the volume flow rate, in m^3/s, at the engine's inlet and the mass flow rate, in kg/s, at the engine's exit.

6–10 A 2-m^3 rigid tank initially contains air whose density is 1.18 kg/m^3. The tank is connected to a high-pressure supply line through a valve. The valve is opened, and air is allowed to enter the tank until the density in the tank rises to 5.30 kg/m^3. Determine the mass of air that has entered the tank. *Answer:* 8.24 kg

6–11 Air enters a nozzle steadily at 2.21 kg/m^3 and 40 m/s and leaves at 0.762 kg/m^3 and 180 m/s. If the inlet area of the nozzle is 90 cm^2, determine (*a*) the mass flow rate through the nozzle, and (*b*) the exit area of the nozzle. *Answers:* (*a*) 0.796 kg/s, (*b*) 58.0 cm^2

6–12 Water enters the constant 130-mm inside-diameter tubes of a boiler at 7 MPa and 65°C and leaves the tubes at 6 MPa and 450°C with a velocity of 80 m/s. Calculate the velocity of the water at the tube inlet and the inlet volume flow rate.

6–13 A desktop computer is to be cooled by a fan whose flow rate is 0.34 m^3/min. Determine the mass flow rate of air through the fan at an elevation of 3400 m where the air density is 0.7 kg/m^3. Also, if the average velocity of air is not to exceed 110 m/min, determine the diameter of the casing of the fan. *Answers:* 0.238 kg/min, 6.3 cm

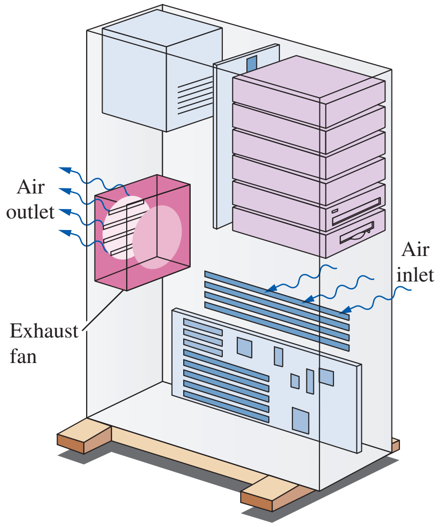


FIGURE P6–13

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*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

6–14 A hair dryer is basically a duct of constant diameter in which a few layers of electric resistors are placed. A small fan pulls the air in and forces it through the resistors where it is heated. If the density of air is 1.20 kg/m^3 at the inlet and 0.95 kg/m^3 at the exit, determine the percent increase in the velocity of air as it flows through the dryer.

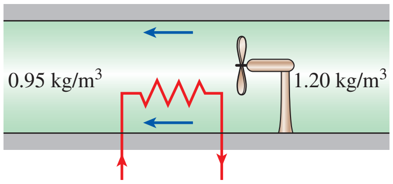


FIGURE P6–14

6–15 Refrigerant-134a enters a 28-cm-diameter pipe steadily at 200 kPa and 20°C with a velocity of 5 m/s. The refrigerant gains heat as it flows and leaves the pipe at 180 kPa and 40°C. Determine (*a*) the volume flow rate of the refrigerant at the inlet, (*b*) the mass flow rate of the refrigerant, and (*c*) the velocity and volume flow rate at the exit.

Flow Work and Energy Transfer by Mass

6–16C What are the different mechanisms for transferring energy to or from a control volume?

6–17C How do the energies of a flowing fluid and a fluid at rest compare? Name the specific forms of energy associated with each case.

6–18 An air compressor compresses 6 L of air at 120 kPa and 20°C to 1000 kPa and 400°C. Determine the flow work, in kJ/kg, required by the compressor. *Answer:* 109 kJ/kg

6–19 A house is maintained at 1 atm and 24°C, and warm air inside a house is forced to leave the house at a rate of 90 m^3/h as a result of outdoor air at 5°C infiltrating into the house through the cracks. Determine the rate of net energy loss of the house due to mass transfer. *Answer:* 0.567 kW

6–20 Refrigerant-134a enters the compressor of a refrigeration system as saturated vapor at 0.14 MPa and leaves as superheated vapor at 0.8 MPa and 60°C at a rate of 0.06 kg/s. Determine the rates of energy transfers by mass into and out of the compressor. Assume the kinetic and potential energies to be negligible.

6–21E Steam is leaving a pressure cooker whose operating pressure is 20 psia. It is observed that the amount of liquid in the cooker has decreased by 0.6 gal in 45 minutes after the steady operating conditions are established, and the cross-sectional area of the exit opening is 0.15 in^2. Determine (*a*) the mass flow rate of the steam and the exit velocity, (*b*) the total and flow energies of the steam per unit mass, and (*c*) the rate at which energy is leaving the cooker by steam.

Steady-Flow Energy Balance: Nozzles and Diffusers

6–22C How is a steady-flow system characterized?

6–23C Can a steady-flow system involve boundary work?

6–24C A diffuser is an adiabatic device that decreases the kinetic energy of the fluid by slowing it down. What happens to this *lost* kinetic energy?

6–25C The kinetic energy of a fluid increases as it is accelerated in an adiabatic nozzle. Where does this energy come from?

6–26E The stators in a gas turbine are designed to increase the kinetic energy of the gas passing through them adiabatically. Air enters a set of these nozzles at 300 psia and 700°F with a velocity of 80 ft/s and exits at 250 psia and 645°F. Calculate the velocity at the exit of the nozzles.

6–27 The diffuser in a jet engine is designed to decrease the kinetic energy of the air entering the engine compressor without any work or heat interactions. Calculate the velocity at the exit of a diffuser when air at 100 kPa and 30°C enters it with a velocity of 350 m/s and the exit state is 200 kPa and 90°C.

FIGURE P6–27

Stockbyte/Getty Images

6–28E Air enters a nozzle steadily at 50 psia, 140°F, and 150 ft/s and leaves at 14.7 psia and 900 ft/s. The heat loss from the nozzle is estimated to be 6.5 Btu/lbm of air flowing. The inlet area of the nozzle is 0.1 ft^2. Determine (*a*) the exit temperature of air and (*b*) the exit area of the nozzle. *Answers:* (*a*) 507 R, (*b*) 0.0480 ft^2

6–29 Air at 600 kPa and 500 K enters an adiabatic nozzle that has an inlet-to-exit area ratio of 2:1 with a velocity of 120 m/s and leaves with a velocity of 380 m/s. Determine (*a*) the exit temperature and (*b*) the exit pressure of the air. *Answers:* (*a*) 437 K, (*b*) 331 kPa

6–30 Carbon dioxide enters an adiabatic nozzle steadily at 1 MPa and 500°C with a mass flow rate of 6000 kg/h and leaves at 100 kPa and 450 m/s. The inlet area of the nozzle is 40 cm^2. Determine (*a*) the inlet velocity and (*b*) the exit temperature.

6–31 Steam enters a nozzle at 400°C and 800 kPa with a velocity of 10 m/s, and leaves at 375°C and 400 kPa while losing heat at a rate of 25 kW. For an inlet area of 800 cm^2, determine the velocity and the volume flow rate of the steam at the nozzle exit. *Answers:* 260 m/s, 1.55 m^3/s

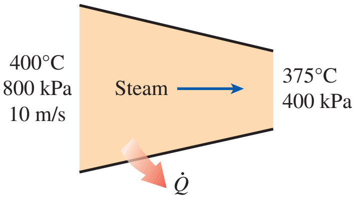


FIGURE P6–31

6–32 Air at 80 kPa and 127°C enters an adiabatic diffuser steadily at a rate of 6000 kg/h and leaves at 100 kPa. The velocity of the airstream is decreased from 230 to 30 m/s as it passes through the diffuser. Find (*a*) the exit temperature of the air and (*b*) the exit area of the diffuser.

6–33E Air at 13 psia and 65°F enters an adiabatic diffuser steadily with a velocity of 750 ft/s and leaves with a low velocity at a pressure of 14.5 psia. The exit area of the diffuser is 3 times the inlet area. Determine (*a*) the exit temperature and (*b*) the exit velocity of the air.

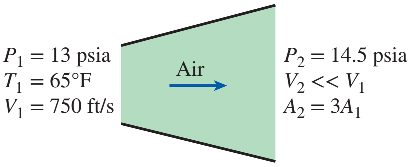


FIGURE P6–33E

6–34 Refrigerant-134a at 700 kPa and 120°C enters an adiabatic nozzle steadily with a velocity of 20 m/s and leaves at 400 kPa and 30°C. Determine (*a*) the exit velocity and (*b*) the ratio of the inlet to exit area A_1/A_2.

6–35 Refrigerant-134a enters a diffuser steadily as saturated vapor at 600 kPa with a velocity of 160 m/s, and it leaves at 700 kPa and 40°C. The refrigerant is gaining heat at a rate of 2 kJ/s as it passes through the diffuser. If the exit area is 80 percent greater than the inlet area, determine (*a*) the exit velocity and (*b*) the mass flow rate of the refrigerant. *Answers:* (*a*) 82.1 m/s, (*b*) 0.298 kg/s

6–36 Air at 80 kPa, 27°C, and 220 m/s enters a diffuser at a rate of 2.5 kg/s and leaves at 42°C. The exit area of the diffuser is 400 cm^2. The air is estimated to lose heat at a rate of 18 kJ/s during this process. Determine (*a*) the exit velocity and (*b*) the exit pressure of the air. *Answer:* (*a*) 62.0 m/s, (*b*) 91.1 kPa

Turbines and Compressors

6–37C Consider an adiabatic turbine operating steadily. Does the work output of the turbine have to be equal to the decrease in the energy of the steam flowing through it?

6–38C Will the temperature of air rise as it is compressed by an adiabatic compressor? Why?

6–39C Somebody proposes the following system to cool a house in the summer: Compress the regular outdoor air, let it cool back to the outdoor temperature, pass it through a turbine, and discharge the cold air leaving the turbine into the house. From a thermodynamic point of view, is the proposed system sound?

6–40 Air is expanded from 1000 kPa and 600°C at the inlet of a steady-flow turbine to 100 kPa and 200°C at the outlet. The inlet area and velocity are 0.1 m^2 and 30 m/s, respectively, and the outlet velocity is 10 m/s. Determine the mass flow rate and outlet area.

6–41E Air enters a gas turbine at 150 psia and 700°F and leaves at 15 psia and 100°F. Determine the inlet and outlet volume flow rates when the mass flow rate through this turbine is 5 lbm/s.

6–42 Refrigerant-134a enters a compressor at 180 kPa as a saturated vapor with a flow rate of 0.35 m^3/min and leaves at 900 kPa. The power supplied to the refrigerant during the compression process is 2.35 kW. What is the temperature of R-134a at the exit of the compressor? *Answer:* 52.5°C

6–43 Steam flows steadily through an adiabatic turbine. The inlet conditions of the steam are 4 MPa, 500°C, and 80 m/s, and the exit conditions are 30 kPa, 92 percent quality, and 50 m/s. The mass flow rate of the steam is 12 kg/s. Determine (*a*) the change in kinetic energy, (*b*) the power output, and (*c*) the turbine inlet area. *Answers:* (*a*) −1.95 kJ/kg, (*b*) 12.1 MW, (*c*) 0.0130 m^2

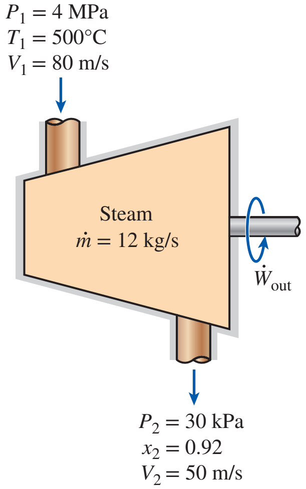


FIGURE P6–43

6–44 Reconsider Prob. 6–43. Using appropriate software, investigate the effect of the turbine exit pressure on the power output of the turbine. Let the exit pressure vary from 10 to 200 kPa. Plot the power output against the exit pressure, and discuss the results.

6–45E Steam flows steadily through a turbine at a rate of 45,000 lbm/h, entering at 1000 psia and 900°F and leaving at 5 psia as saturated vapor. If the power generated by the turbine is 4 MW, determine the rate of heat loss from the steam.

6–46 Steam enters an adiabatic turbine at 8 MPa and 500°C at a rate of 3 kg/s and leaves at 20 kPa. If the power output of the turbine is 2.5 MW, determine the temperature of the steam at the turbine exit. Neglect kinetic energy changes. *Answer:* 60.1°C

6–47 Carbon dioxide enters an adiabatic compressor at 100 kPa and 300 K at a rate of 0.5 kg/s and leaves at 600 kPa and 450 K. Neglecting kinetic energy changes, determine (*a*) the volume flow rate of the carbon dioxide at the compressor inlet and (*b*) the power input to the compressor. *Answers:* (*a*) 0.283 m^3/s, (*b*) 68.8 kW

6–48 Steam flows steadily into a turbine with a mass flow rate of 26 kg/s and a negligible velocity at 6 MPa and 600°C. The steam leaves the turbine at 0.5 MPa and 200°C with a velocity of 180 m/s. The rate of work done by the steam in the turbine is measured to be 20,350 kW. If the elevation change between the turbine inlet and exit is negligible, determine the rate of heat transfer associated with this process. *Answer:* 105 kW

6–49 Air is compressed by an adiabatic compressor from 100 kPa and 20°C to 1.8 MPa and 400°C. Air enters the compressor through a 0.15-m^2 opening with a velocity of 30 m/s. It exits through a 0.08-m^2 opening. Calculate the mass flow rate of air and the required power input.

6–50 Air enters the compressor of a gas-turbine plant at ambient conditions of 100 kPa and 25°C with a low velocity and exits at 1 MPa and 347°C with a velocity of 90 m/s. The compressor is cooled at a rate of 1500 kJ/min, and the power input to the compressor is 250 kW. Determine the mass flow rate of air through the compressor.

6–51 A portion of the steam passing through a steam turbine is sometimes removed for the purposes of feedwater heating as shown in Fig. P6–51. Consider an adiabatic steam turbine with 12.5 MPa and 550°C steam entering at a rate of 20 kg/s. Steam is bled from this turbine at 1000 kPa and 200°C with a mass flow rate of 1 kg/s. The remaining steam leaves the turbine at 100 kPa and 100°C. Determine the power produced by this turbine. *Answer:* 15,860 kW

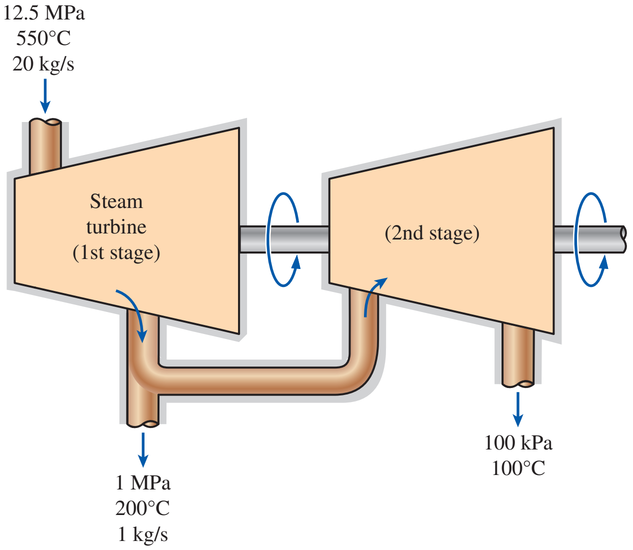


FIGURE P6–51

Throttling Valves

6–52C Why are throttling devices commonly used in refrigeration and air-conditioning applications?

6–53C Would you expect the temperature of air to drop as it undergoes a steady-flow throttling process? Explain.

6–54C During a throttling process, the temperature of a fluid drops from 30 to –20°C. Can this process occur adiabatically?

6–55C Someone claims, based on temperature measurements, that the temperature of a fluid rises during a throttling process in a well-insulated valve with negligible friction. How do you evaluate this claim? Does this process violate any thermodynamic laws?

6–56 Refrigerant-134a is throttled from the saturated liquid state at 700 kPa to a pressure of 160 kPa. Determine the temperature drop during this process and the final specific volume of the refrigerant. *Answers:* 42.3°C, 0.0345 m^3/kg

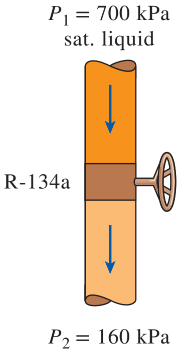


FIGURE P6–56

6–57 A saturated liquid–vapor mixture of water, called wet steam, in a steam line at 1500 kPa is throttled to 50 kPa and 100°C. What is the quality in the steam line? *Answer:* 0.944

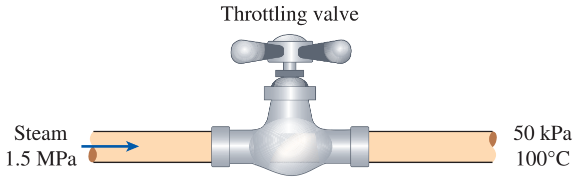


FIGURE P6–57

6–58 An adiabatic capillary tube is used in some refrigeration systems to drop the pressure of the refrigerant from the condenser level to the evaporator level. The R-134a enters the capillary tube as a saturated liquid at 50°C and leaves at –20°C. Determine the quality of the refrigerant at the inlet of the evaporator.

6–59 A well-insulated valve is used to throttle steam from 8 MPa and 350°C to 2 MPa. Determine the final temperature of the steam. *Answer:* 285°C

6–60 Reconsider Prob. 6–59. Using appropriate software, investigate the effect of the exit pressure of steam on the exit temperature after throttling. Let the exit pressure vary from 6 to 1 MPa. Plot the exit temperature of steam against the exit pressure, and discuss the results.

6–61E Refrigerant-134a enters the expansion valve of a refrigeration system at 120 psia as a saturated liquid and leaves at 20 psia. Determine the temperature and internal energy changes across the valve.

Mixing Chambers and Heat Exchangers

6–62C Consider a steady-flow mixing process. Under what conditions will the energy transported into the control volume by the incoming streams be equal to the energy transported out of it by the outgoing stream?

6–63C Consider a steady-flow heat exchanger involving two different fluid streams. Under what conditions will the amount of heat lost by one fluid be equal to the amount of heat gained by the other?

6–64C When two fluid streams are mixed in a mixing chamber, can the mixture temperature be lower than the temperature of both streams? Explain.

6–65 Refrigerant-134a at 700 kPa, 70°C, and 8 kg/min is cooled by water in a condenser until it exists as a saturated liquid at the same pressure. The cooling water enters the condenser at 300 kPa and 15°C and leaves at 25°C at the same pressure. Determine the mass flow rate of the cooling water required to cool the refrigerant. *Answer:* 42.0 kg/min

6–66 Hot and cold streams of a fluid are mixed in a rigid mixing chamber. The hot fluid flows into the chamber at a mass flow rate of 5 kg/s with an energy in the amount of 150 kJ/kg. The cold fluid flows into the chamber with a mass flow rate of 15 kg/s and carries energy in the amount of 50 kJ/kg. There is heat transfer to the surroundings from the mixing chamber in the amount of 5.5 kW. The mixing chamber operates in a steady-flow manner and does not gain or lose energy or mass with time. Determine the energy carried from the mixing chamber by the fluid mixture per unit mass of fluid, in kJ/kg.

6–67 A hot-water stream at 80°C enters a mixing chamber with a mass flow rate of 0.5 kg/s where it is mixed with a stream of cold water at 20°C. If it is desired that the mixture leave the chamber at 42°C, determine the mass flow rate of the cold-water stream. Assume all the streams are at a pressure of 250 kPa. *Answer:* 0.865 kg/s

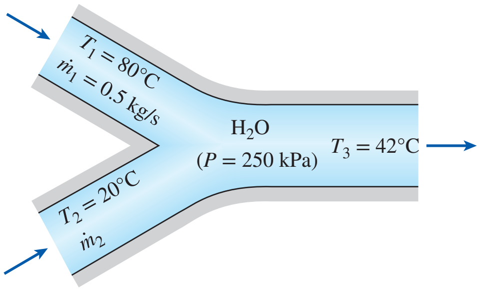


FIGURE P6–67

6–68E Water at 80°F and 20 psia is heated in a chamber by mixing it with saturated water vapor at 20 psia. If both streams enter the mixing chamber at the same mass flow rate, determine the temperature and the quality of the exiting stream. *Answers:* 228°F, 0.423

6–69 Cold water (c_p = 4.18 kJ/kg·°C) leading to a shower enters a thin-walled double-pipe counterflow heat exchanger at 15°C at a rate of 0.60 kg/s and is heated to 45°C by hot water (c_p = 4.19 kJ/kg·°C) that enters at 100°C at a rate of 3 kg/s. Determine the rate of heat transfer in the heat exchanger and the exit temperature of the hot water.

6–70E Steam is to be condensed on the shell side of a heat exchanger at 75°F. Cooling water enters the tubes at 50°F at a rate of 45 lbm/s and leaves at 65°F. Assuming the heat exchanger to be well-insulated, determine the rate of heat transfer in the heat exchanger and the rate of condensation of the steam.

6–71 Air (c_p = 1.005 kJ/kg·°C) is to be preheated by hot exhaust gases in a crossflow heat exchanger before it enters the furnace. Air enters the heat exchanger at 95 kPa and 20°C at a rate of 0.6 m^3/s. The combustion gases (c_p = 1.10 kJ/kg·°C) enter at 160°C at a rate of 0.95 kg/s and leave at 95°C. Determine the rate of heat transfer to the air and its outlet temperature.

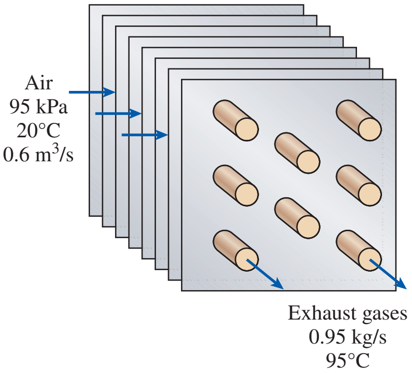


FIGURE P6–71

6–72E An open feedwater heater heats the feedwater by mixing it with hot steam. Consider an electric power plant with an open feedwater heater that mixes 0.1 lbm/s of steam at 10 psia and 200°F with 2.0 lbm/s of feedwater at 10 psia and 100°F to produce 10 psia and 120°F feedwater at the outlet. The diameter of the outlet pipe is 0.5 ft. Determine the mass flow rate and feedwater velocity at the outlet. Would the outlet flow rate and velocity be significantly different if the temperature at the outlet were 180°F?

6–73 Refrigerant-134a at 1 MPa and 90°C is to be cooled to 1 MPa and 30°C in a condenser by air. The air enters at 100 kPa and 27°C with a volume flow rate of 600 m^3/min and leaves at 95 kPa and 60°C. Determine the mass flow rate of the refrigerant. *Answer:* 100 kg/min

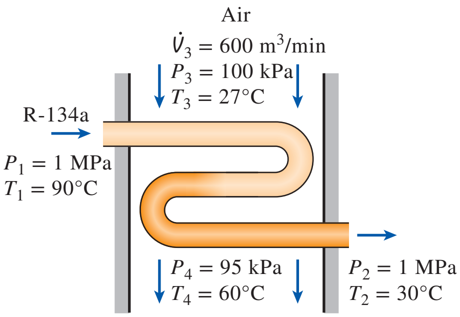


FIGURE P6–73

6–74 The evaporator of a refrigeration cycle is basically a heat exchanger in which a refrigerant is evaporated by absorbing heat from a fluid. Refrigerant-22 enters an evaporator at 200 kPa with a quality of 22 percent and a flow rate of 2.65 L/h. R-22 leaves the evaporator at the same pressure superheated by 5°C. The refrigerant is evaporated by absorbing heat from air whose flow rate is 0.75 kg/s. Determine (*a*) the rate of heat absorbed from the air and (*b*) the temperature change of air. The properties of R-22 at the inlet and exit of the condenser are h_1 = 220.2 kJ/kg, υ_1 = 0.0253 m^3/kg, and h_2 = 398.0 kJ/kg.

6–75 An air-conditioning system involves the mixing of cold air and warm outdoor air before the mixture is routed to the conditioned room in steady operation. Cold air enters the mixing chamber at 7°C and 105 kPa at a rate of 0.55 m^3/s, while warm air enters at 34°C and 105 kPa. The air leaves the room at 24°C. The ratio of the mass flow rates of the hot to cold airstreams is 1.6. Using variable specific heats, determine (*a*) the mixture temperature at the inlet of the room and (*b*) the rate of heat gain of the room.

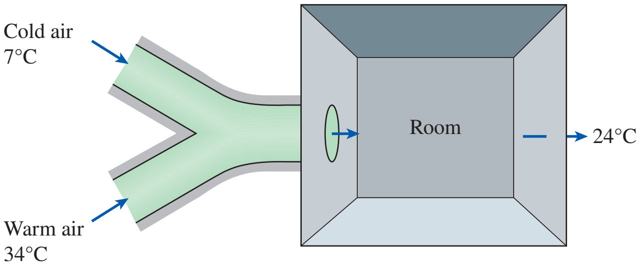


FIGURE P6–75

6–76 Hot exhaust gases of an internal combustion engine are to be used to produce saturated water vapor at 2 MPa pressure. The exhaust gases enter the heat exchanger at 400°C at a rate of 32 kg/min while water enters at 15°C. The heat exchanger is not well insulated, and it is estimated that 10 percent of heat given up by the exhaust gases is lost to the surroundings. If the mass flow rate of the exhaust gases is 15 times that of the water, determine the temperature of the exhaust gases at the heat exchanger exit and the rate of heat transfer to the water. Use the constant specific heat properties of air for the exhaust gases.

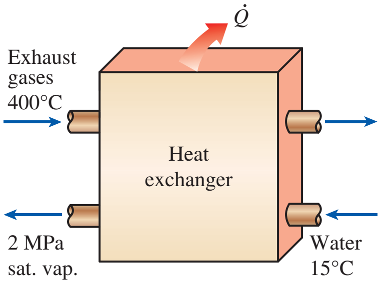


FIGURE P6–76

6–77 A well-insulated shell-and-tube heat exchanger is used to heat water (c_p = 4.18 kJ/kg·°C) in the tubes from 20 to 70°C at a rate of 4.5 kg/s. Heat is supplied by hot oil (c_p = 2.30 kJ/kg·°C) that enters the shell side at 170°C at a rate of 10 kg/s. Determine the rate of heat transfer in the heat exchanger and the exit temperature of oil.

6–78 Steam is to be condensed in the condenser of a steam power plant at a temperature of 50°C with cooling water from a nearby lake, which enters the tubes of the condenser at 18°C at a rate of 101 kg/s and leaves at 27°C. Determine the rate of condensation of the steam in the condenser. *Answer:* 1.60 kg/s

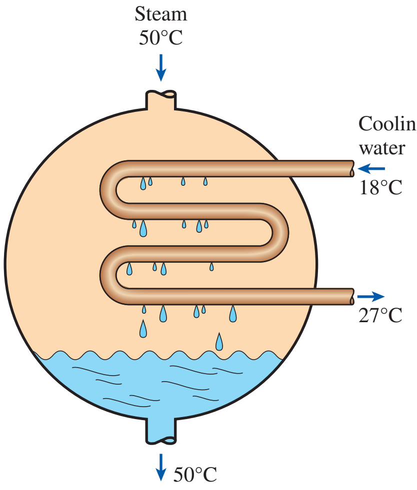


FIGURE P6–78

6–79 Reconsider Prob. 6–78. Using appropriate software, investigate the effect of the inlet temperature of cooling water on the rate of condensation of steam. Let the inlet temperature vary from 10 to 20°C, and assume the exit temperature to remain constant. Plot the rate of condensation of steam against the inlet temperature of the cooling water, and discuss the results.

6–80 Two streams of water are mixed in an insulated container to form a third stream leaving the container. The first stream has a flow rate of 30 kg/s and a temperature of 90°C. The flow rate of the second stream is 200 kg/s, and its temperature is 50°C. What is the temperature of the third stream?

Pipe and Duct Flow

6–81 Water is heated in an insulated, constant-diameter tube by a 7-kW electric resistance heater. If the water enters the heater steadily at 20°C and leaves at 75°C, determine the mass flow rate of water.

6–82 A 110-volt electrical heater is used to warm 0.3 m^3/s of air at 100 kPa and 15°C to 100 kPa and 30°C. How much current in amperes must be supplied to this heater?

6–83 The ducts of an air heating system pass through an unheated area. As a result of heat losses, the temperature of the air in the duct drops by 4°C. If the mass flow rate of air is 120 kg/min, determine the rate of heat loss from the air to the cold environment.

6–84E The fan on a personal computer draws 0.3 ft^3/s of air at 14.7 psia and 70°F through the box containing the CPU and other components. Air leaves at 14.7 psia and 83°F. Calculate the electrical power, in kW, dissipated by the PC components. *Answer:* 0.0740 kW

FIGURE P6–84E

PhotoDisc/Getty Images

6–85 Saturated liquid water is heated in a steady-flow steam boiler at a constant pressure of 2 MPa at a rate of 4 kg/s to an outlet temperature of 250°C. Determine the rate of heat transfer in the boiler.

6–86E Water enters the tubes of a cold plate at 70°F with an average velocity of 40 ft/min and leaves at 105°F. The diameter of the tubes is 0.25 in. Assuming 15 percent of the heat generated is dissipated from the components to the surroundings by convection and radiation, and the remaining 85 percent is removed by the cooling water, determine the amount of heat generated by the electronic devices mounted on the cold plate. *Answer:* 614 W

6–87 Consider a hollow-core printed circuit board 9 cm high and 18 cm long, dissipating a total of 15 W. The width of the air gap in the middle of the PCB is 0.25 cm. If the cooling air enters the 12-cm-wide core at 25°C and 1 atm at a rate of 0.8 L/s, determine the average temperature at which the air leaves the hollow core. *Answer:* 46.0°C

6–88 A computer cooled by a fan contains eight PCBs, each dissipating 10 W power. The height of the PCBs is 12 cm and the length is 18 cm. The cooling air is supplied by a 25-W fan mounted at the inlet. If the temperature rise of air as it flows through the case of the computer is not to exceed 10°C, determine (*a*) the flow rate of the air that the fan needs to deliver and (*b*) the fraction of the temperature rise of air that is due to the heat generated by the fan and its motor. *Answers:* (*a*) 0.0104 kg/s, (*b*) 24 percent

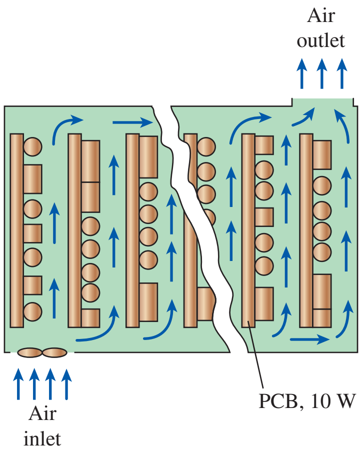


FIGURE P6–88

6–89 A desktop computer is to be cooled by a fan. The electronic components of the computer consume 60 W of power under full-load conditions. The computer is to operate in environments at temperatures up to 45°C and at elevations up to 3400 m where the average atmospheric pressure is 66.63 kPa. The exit temperature of air is not to exceed 60°C to meet the reliability requirements. Also, the average velocity of air is not to exceed 110 m/min at the exit of the computer case where the fan is installed to keep the noise level down. Determine the flow rate of the fan that needs to be installed and the diameter of the casing of the fan.

6–90 Repeat Prob. 6–89 for a computer that consumes 100 W of power.

6–91 A 4-m × 5-m × 6-m room is to be heated by an electric resistance heater placed in a short duct in the room. Initially, the room is at 15°C, and the local atmospheric pressure is 98 kPa. The room is losing heat steadily to the outside at a rate of 150 kJ/min. A 200-W fan circulates the air steadily through the duct and the electric heater at an average mass flow rate of 40 kg/min. The duct can be assumed to be adiabatic, and there is no air leaking in or out of the room. If it takes 25 min for the room air to reach an average temperature of 25°C, find (*a*) the power rating of the electric heater and (*b*) the temperature rise that the air experiences each time it passes through the heater.

6–92 A house has an electric heating system that consists of a 300-W fan and an electric resistance heating element placed in a duct. Air flows steadily through the duct at a rate of 0.6 kg/s and experiences a temperature rise of 7°C. The rate of heat loss from the air in the duct is estimated to be 300 W. Determine the power rating of the electric resistance heating element. *Answer:* 4.22 kW

6–93 A long roll of 2-m-wide and 0.5-cm-thick 1-Mn manganese steel plate ($\rho = 7854$ kg/m^3 and $c_p = 0.434$ kJ/kg·°C) coming off a furnace at 820°C is to be quenched in an oil bath at 45°C to a temperature of 51.1°C. If the metal sheet is moving at a steady velocity of 10 m/min, determine the required rate of heat removal from the oil to keep its temperature constant at 45°C. *Answer:* 4368 kW

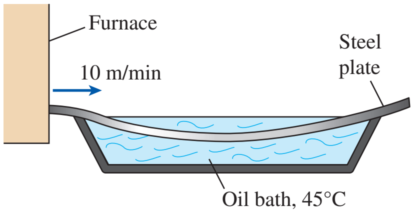


FIGURE P6–93

6–94 Reconsider Prob. 6–93. Using appropriate software, investigate the effect of the moving velocity of the steel plate on the rate of heat transfer from the oil bath. Let the velocity vary from 5 to 50 m/min. Plot the rate of heat transfer against the plate velocity, and discuss the results.

6–95E The hot-water needs of a household are to be met by heating water at 55°F to 180°F with a parabolic solar collector at a rate of 4 lbm/s. Water flows through a 1.25-in-diameter thin aluminum tube whose outer surface is black-anodized in order to maximize its solar absorption ability. The centerline of the tube coincides with the focal line of the collector, and a glass sleeve is placed outside the tube to minimize the heat losses. If solar energy is transferred to water at a net rate of 400 Btu/h per ft length of the tube, determine the required length of the parabolic collector to meet the hot-water requirements of this house.

6–96 Argon steadily flows into a constant-pressure heater at 300 K and 100 kPa with a mass flow rate of 6.24 kg/s. Heat transfer in the rate of 150 kW is supplied to the argon as it flows through the heater. (*a*) Determine the argon temperature at the heater exit, in °C. (*b*) Determine the argon volume flow rate at the heater exit, in m^3/s.

6–97 Steam enters a long, horizontal pipe with an inlet diameter of $D_1 = 16$ cm at 2 MPa and 300°C with a velocity of 2.5 m/s. Farther downstream, the conditions are 1.8 MPa and 250°C, and the diameter is $D_2 = 14$ cm. Determine (*a*) the mass flow rate of the steam and (*b*) the rate of heat transfer. *Answers:* (*a*) 0.401 kg/s, (*b*) 45.1 kJ/s

6–98 Refrigerant-134a enters the condenser of a refrigerator at 900 kPa and 60°C and leaves as a saturated liquid at the same pressure. Determine the heat transfer from the refrigerant per unit mass.

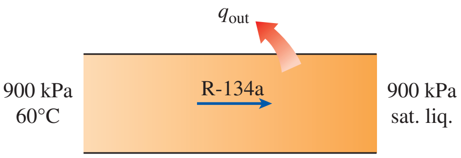


FIGURE P6–98

6–99 A hair dryer is basically a duct in which a few layers of electric resistors are placed. A small fan pulls the air in and forces it through the resistors where it is heated. Air enters a 1200-W hair dryer at 100 kPa and 22°C and leaves at 47°C.

The cross-sectional area of the hair dryer at the exit is 60 cm^2. Neglecting the power consumed by the fan and the heat losses through the walls of the hair dryer, determine (*a*) the volume flow rate of air at the inlet and (*b*) the velocity of the air at the exit. *Answers:* (*a*) 0.0404 m^3/s, (*b*) 7.31 m/s

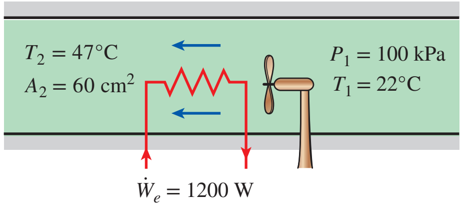


FIGURE P6–99

6–100 Reconsider Prob. 6–99. Using appropriate software, investigate the effect of the exit cross-sectional area of the hair dryer on the exit velocity. Let the exit area vary from 25 to 75 cm^2. Plot the exit velocity against the exit cross-sectional area, and discuss the results. Include the effect of the flow kinetic energy in the analysis.

6–101E Air enters the duct of an air-conditioning system at 15 psia and 50°F at a volume flow rate of 450 ft^3/min. The diameter of the duct is 10 in, and heat is transferred to the air in the duct from the surroundings at a rate of 2 Btu/s. Determine (*a*) the velocity of the air at the duct inlet and (*b*) the temperature of the air at the exit.

Charging and Discharging Processes

6–102 An insulated rigid tank is initially evacuated. A valve is opened, and atmospheric air at 95 kPa and 17°C enters the tank until the pressure in the tank reaches 95 kPa, at which point the valve is closed. Determine the final temperature of the air in the tank. Assume constant specific heats.

6–103 A rigid, insulated tank that is initially evacuated is connected through a valve to a supply line that carries steam at 4 MPa. Now the valve is opened, and steam is allowed to flow into the tank until the pressure reaches 4 MPa, at which point the valve is closed. If the final temperature of the steam in the tank is 550°C, determine the temperature of the steam in the supply line and the flow work per unit mass of the steam.

6–104 A 2-m^3 rigid insulated tank initially containing saturated water vapor at 1 MPa is connected through a valve to a supply line that carries steam at 400°C. Now the valve is opened, and steam is allowed to flow slowly into the tank until the pressure in the tank rises to 2 MPa. At this instant the tank temperature is measured to be 300°C. Determine the mass of the steam that has entered and the pressure of the steam in the supply line.

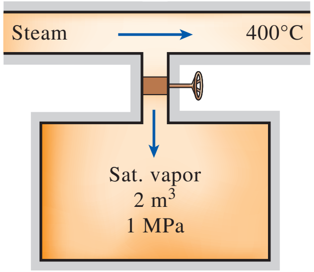


FIGURE P6–104

6–105 Consider a 35-L evacuated rigid bottle that is surrounded by the atmosphere at 100 kPa and 22°C. A valve at the neck of the bottle is now opened and the atmospheric air is allowed to flow into the bottle. The air trapped in the bottle eventually reaches thermal equilibrium with the atmosphere as a result of heat transfer through the wall of the bottle. The valve remains open during the process so that the trapped air also reaches mechanical equilibrium with the atmosphere. Determine the net heat transfer through the wall of the bottle during this filling process. *Answer:* 3.50 kJ

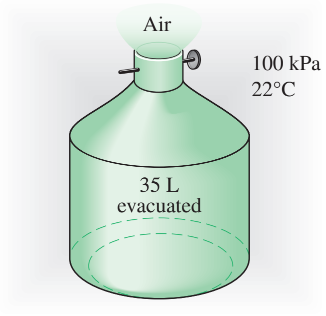


FIGURE P6–105

6–106 A 2-m^3 rigid tank initially contains air at 100 kPa and 22°C. The tank is connected to a supply line through a valve. Air is flowing in the supply line at 600 kPa and 22°C. The valve is opened, and air is allowed to enter the tank until the pressure in the tank reaches the line pressure, at which point the valve is closed. A thermometer placed in the tank indicates that the air temperature at the final state is 77°C. Determine (*a*) the mass of air that has entered the tank and (*b*) the amount of heat transfer. *Answers:* (*a*) 9.58 kg, (*b*) 339 kJ

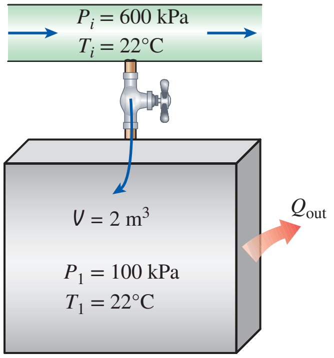


FIGURE P6–106

6–107 A 0.2-m^3 rigid tank equipped with a pressure regulator contains steam at 2 MPa and 300°C. The steam in the tank is now heated. The regulator keeps the steam pressure constant by letting out some steam, but the temperature inside rises. Determine the amount of heat transferred when the steam temperature reaches 500°C.

6–108E A 3-ft^3 rigid tank initially contains saturated water vapor at 300°F. The tank is connected by a valve to a supply line that carries steam at 200 psia and 400°F. Now the valve is opened, and steam is allowed to enter the tank. Heat transfer takes place with the surroundings such that the temperature in the tank remains constant at 300°F at all times. The valve is closed when it is observed that one-half of the volume of the tank is occupied by liquid water. Find (*a*) the final pressure in the tank, (*b*) the amount of steam that has entered the tank, and (*c*) the amount of heat transfer. *Answers:* (*a*) 67.03 psia, (*b*) 85.74 lbm, (*c*) 80,900 Btu

6–109 A 4-L pressure cooker has an operating pressure of 175 kPa. Initially, one-half of the volume is filled with liquid and the other half with vapor. If it is desired that the pressure cooker not run out of liquid water for 75 min, determine the highest rate of heat transfer allowed.

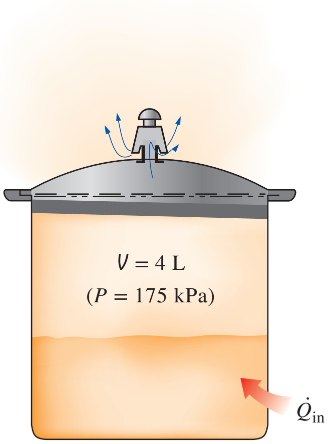


FIGURE P6–109

6–110 An air-conditioning system is to be filled from a rigid container that initially contains 5 kg of liquid R-134a at 24°C. The valve connecting this container to the air-conditioning system is now opened until the mass in the container is 0.25 kg, at which time the valve is closed. During this time, only liquid R-134a flows from the container. Presuming that the process is isothermal while the valve is open, determine the final quality of the R-134a in the container and the total heat transfer. *Answers:* 0.506, 22.6 kJ

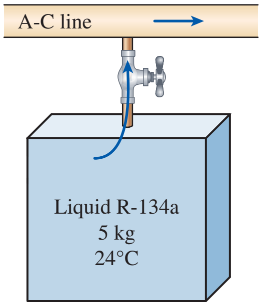


FIGURE P6–110

6–111E Oxygen is supplied to a medical facility from ten 1.5-ft^3 compressed oxygen tanks. Initially, these tanks are at 1500 psia and 80°F. The oxygen is removed from these tanks slowly enough that the temperature in the tanks remains at 80°F. After two weeks, the pressure in the tanks is 300 psia. Determine the mass of oxygen used and the total heat transfer to the tanks.

6–112 A 0.05-m^3 rigid tank initially contains refrigerant-134a at 0.8 MPa and 100 percent quality. The tank is connected by a valve to a supply line that carries refrigerant-134a at 1.2 MPa and 40°C. Now the valve is opened, and the refrigerant is allowed to enter the tank. The valve is closed when it is observed that the tank contains saturated liquid at 1.2 MPa. Determine (*a*) the mass of the refrigerant that has entered the tank and (*b*) the amount of heat transfer. *Answers:* (*a*) 54.0 kg, (*b*) 202 kJ

6–113 A 0.3-m^3 rigid tank is filled with saturated liquid water at 200°C. A valve at the bottom of the tank is opened, and liquid is withdrawn from the tank. Heat is transferred to the water such that the temperature in the tank remains constant. Determine the amount of heat that must be transferred by the time one-half of the total mass has been withdrawn.

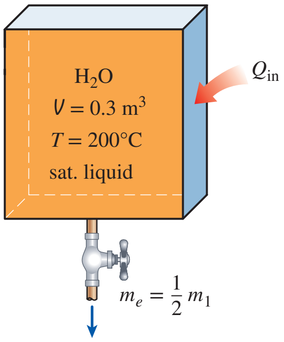


FIGURE P6–113

6–114 The air-release flap on a hot-air balloon is used to release hot air from the balloon when appropriate. On one hot-air balloon, the air release opening has an area of 0.5 m^2, and the filling opening has an area of 1 m^2. During a two-minute adiabatic flight maneuver, hot air enters the balloon at 100 kPa and 35°C with a velocity of 2 m/s; the air in the balloon remains at 100 kPa and 35°C; and air leaves the balloon through the air-release flap at velocity 1 m/s. At the start of this maneuver, the volume of the balloon is 75 m^3. Determine the final volume of the balloon and the work produced by the air inside the balloon as it expands the balloon skin.

FIGURE P6–114

Getty Images

6–115 A balloon initially contains 40 m^3 of helium gas at atmospheric conditions of 100 kPa and 17°C. The balloon is connected by a valve to a large reservoir that supplies helium gas at 125 kPa and 25°C. Now the valve is opened, and helium is allowed to enter the balloon until pressure equilibrium with the helium at the supply line is reached. The material of the balloon is such that its volume increases linearly with pressure. If no heat transfer takes place during this process, determine the final temperature in the balloon. *Answer:* 315 K

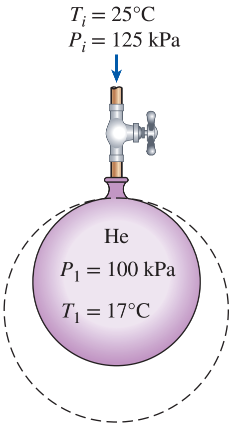


FIGURE P6–115

6–116 An insulated 0.15-m^3 tank contains helium at 3 MPa and 130°C. A valve is now opened, allowing some helium to escape. The valve is closed when one-half of the initial mass has escaped. Determine the final temperature and pressure in the tank. *Answers:* 257 K, 956 kPa

6–117 A vertical piston–cylinder device initially contains 0.2 m^3 of air at 20°C. The mass of the piston is such that it maintains a constant pressure of 300 kPa inside. Now a valve connected to the cylinder is opened, and air is allowed to escape until the volume inside the cylinder is decreased by one-half. Heat transfer takes place during the process so that the temperature of the air in the cylinder remains constant. Determine (*a*) the amount of air that has left the cylinder and (*b*) the amount of heat transfer. *Answers:* (*a*) 0.357 kg, (*b*) 0

6–118 A vertical piston–cylinder device initially contains 0.25 m^3 of air at 600 kPa and 300°C. A valve connected to the cylinder is now opened, and air is allowed to escape until three-quarters of the mass leaves the cylinder, at which point the volume is 0.05 m^3. Determine the final temperature in the cylinder and the boundary work during this process.

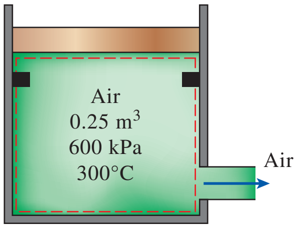


FIGURE P6–118

6–119 A vertical piston–cylinder device initially contains 0.01 m^3 of steam at 200°C. The mass of the frictionless piston is such that it maintains a constant pressure of 500 kPa inside. Now steam at 1 MPa and 350°C is allowed to enter the cylinder from a supply line until the volume inside doubles. Neglecting any heat transfer that may have taken place during the process, determine (*a*) the final temperature of the steam in the cylinder and (*b*) the amount of mass that has entered. *Answers:* (*a*) 261.7°C, (*b*) 0.0176 kg

6–120 A piston–cylinder device initially contains 0.6 kg of steam with a volume of 0.1 m^3. The mass of the piston is such that it maintains a constant pressure of 800 kPa. The cylinder is connected through a valve to a supply line that carries steam at 5 MPa and 500°C. Now the valve is opened and steam is allowed to flow slowly into the cylinder until the volume of the cylinder doubles and the temperature in the cylinder reaches 250°C, at which point the valve is closed. Determine (*a*) the mass of steam that has entered and (*b*) the amount of heat transfer.

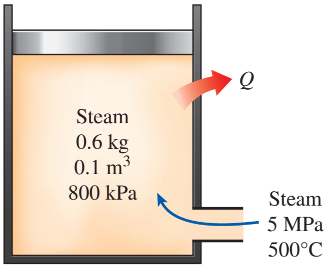


FIGURE P6–120

6–121E The weighted piston of the device shown in Fig. P6–121E maintains the pressure of the piston-cylinder contents at 200 psia. Initially, this system contains no mass. The valve is now opened, and steam from the line flows into the cylinder until the volume is 10 ft^3. This process is adiabatic, and the steam in the line remains at 300 psia and 450°F. Determine the final temperature (and quality if appropriate) of the steam in the cylinder and the total work produced as the device is filled.

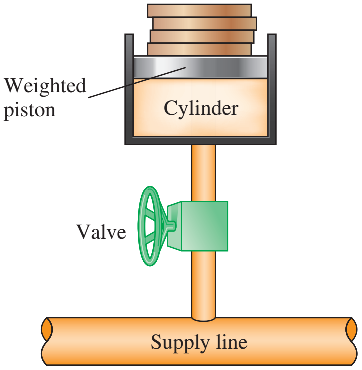


FIGURE P6–121E

6–122E Repeat Prob. 6–121E when the supply line carries oxygen at 300 psia and 450°F. *Answers:* 450°F, 370 Btu

Review Problems

6–123 Underground water is being pumped into a pool whose cross section is 6 m × 9 m while water is discharged through a 7-cm-diameter orifice at a constant average velocity of 4 m/s. If the water level in the pool rises at a rate of 2.5 cm/min, determine the rate at which water is supplied to the pool, in m^3/s.

6–124 A long roll of 1-m-wide and 0.5-cm-thick 1-Mn manganese steel plate ($\rho = 7854$ kg/m^3) coming off a furnace is to be quenched in an oil bath to a specified temperature. If the metal sheet is moving at a steady velocity of 10 m/min, determine the mass flow rate of the steel plate through the oil bath.

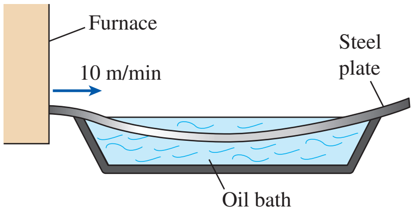


FIGURE P6–124

6–125 Helium steadily enters a pipe with a mass flow rate of 8 kg/s at 427°C and 100 kPa and leaves the pipe at 27°C. The pressure during the process is constant at 100 kPa. (*a*) Determine the heat transfer for the process, in kW. (*b*) Determine the volume flow rate of the helium at the pipe exit, in m^3/s.

6–126 Air at 4.18 kg/m^3 enters a nozzle that has an inlet-to-exit area ratio of 2:1 with a velocity of 120 m/s and leaves with a velocity of 380 m/s. Determine the density of air at the exit. *Answer:* 2.64 kg/m^3

6–127 Water is boiled at 100°C electrically by a 3-kW resistance wire. Determine the rate of evaporation of water.

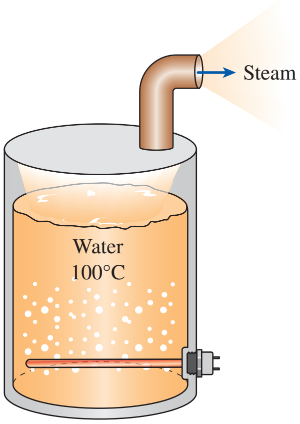


FIGURE P6–127

6–128 An air compressor compresses 15 L/s of air at 120 kPa and 20°C to 800 kPa and 300°C while consuming 6.2 kW of power. How much of this power is being used to increase the pressure of the air versus the power needed to move the fluid through the compressor? *Answers:* 4.48 kW, 1.72 kW

6–129 A steam turbine operates with 1.6 MPa and 350°C steam at its inlet and saturated vapor at 30°C at its exit. The mass flow rate of the steam is 22 kg/s, and the turbine produces 12,350 kW of power. Determine the rate at which heat is lost through the casing of this turbine.

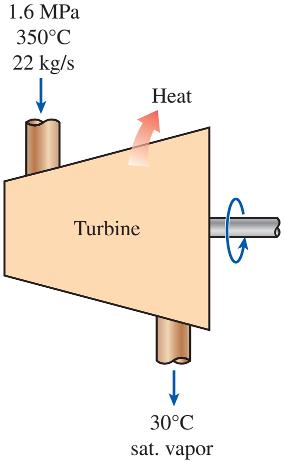


FIGURE P6–129

6–130E Refrigerant-134a enters an adiabatic compressor at 15 psia and 20°F with a volume flow rate of 10 ft^3/s and leaves at a pressure of 100 psia. The power input to the compressor is 45 hp. Find (*a*) the mass flow rate of the refrigerant and (*b*) the exit temperature.

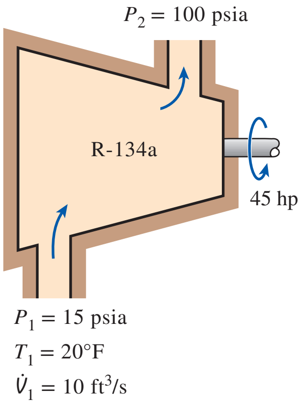


FIGURE P6–130E

6–131E Nitrogen gas flows through a long, constant-diameter adiabatic pipe. It enters at 100 psia and 120°F and leaves at 50 psia and 70°F. Calculate the velocity of the nitrogen at the pipe's inlet and outlet.

6–132 A 110-V electric water heater warms 0.1 L/s of water from 18 to 30°C. Calculate the current in amperes that must be supplied to this heater. *Answer:* 45.6 A

6–133 A fan is powered by a 0.5-hp motor and delivers air at a rate of 85 m^3/min. Determine the highest value for the average velocity of air mobilized by the fan. Take the density of air to be 1.18 kg/m^3.

6–134 Steam enters a long, insulated pipe at 1200 kPa, 250°C, and 4 m/s, and exits at 1000 kPa. The diameter of the pipe is 0.15 m at the inlet, and 0.1 m at the exit. Calculate the mass flow rate of the steam and its speed at the pipe outlet.

6–135 Steam enters a nozzle with a low velocity at 150°C and 200 kPa, and leaves as a saturated vapor at 75 kPa. There is a heat transfer from the nozzle to the surroundings in the amount of 26 kJ for every kilogram of steam flowing through the nozzle. Determine (*a*) the exit velocity of the steam and (*b*) the mass flow rate of the steam at the nozzle entrance if the nozzle exit area is 0.001 m^2.

6–136E Consider a heat exchanger that uses hot air to heat cold water. Air enters this heat exchanger at 20 psia and 200°F at a rate of 100 ft^3/min and leaves at 17 psia and 100°F. Water enters this unit at 20 psia and 50°F at a rate of 0.5 lbm/s and exits at 17 psia and 90°F. Determine the total flow power, in hp, required for this unit and the flow work, in Btu/lbm, for both the air and water streams.

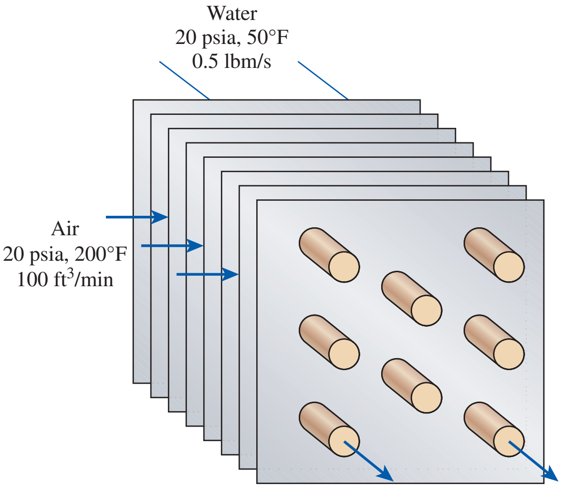


FIGURE P6–136E

6–137 Steam at 40°C condenses on the outside of a 5-m-long, 3-cm-diameter thin horizontal copper tube by cooling water that enters the tube at 25°C at an average velocity of 2 m/s and leaves at 35°C. Determine the rate of condensation of the steam. *Answer:* 0.0245 kg/s

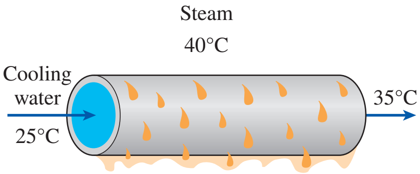


FIGURE P6–137

6–138 In large steam power plants, the feedwater is frequently heated in a closed feedwater heater by using steam extracted from the turbine at some stage. Steam enters the feedwater heater at 1 MPa and 200°C and leaves as saturated liquid at the same pressure. Feedwater enters the heater at 2.5 MPa and 50°C and leaves at 10°C below the exit temperature of the steam. Determine the ratio of the mass flow rates of the extracted steam and the feedwater.

6–139 In large gas-turbine power plants, air is preheated by the exhaust gases in a heat exchanger called the regenerator before it enters the combustion chamber. Air enters the regenerator at 1 MPa and 550 K at a mass flow rate of 800 kg/min. Heat is transferred to the air at a rate of 2700 kJ/s. Exhaust gases enter the regenerator at 140 kPa and 800 K and leave at 130 kPa and 600 K. Treating the exhaust gases as air, determine (*a*) the exit temperature of the air and (*b*) the mass flow rate of exhaust gases. *Answers:* (*a*) 741 K, (*b*) 12.6 kg/s

6–140 Cold water enters a steam generator at 20°C and leaves as saturated vapor at 200°C. Determine the fraction of heat used in the steam generator to preheat the liquid water from 20°C to the saturation temperature of 200°C.

6–141 An ideal gas expands in an adiabatic turbine from 1200 K and 900 kPa to 800 K. Determine the turbine inlet volume flow rate of the gas, in m^3/s, required to produce turbine work output at the rate of 650 kW. The average values of the specific heats for this gas over the temperature range and the gas constant are $c_p = 1.13$ kJ/kg·K, $c_v = 0.83$ kJ/kg·K, and $R = 0.30$ kJ/kg·K.

6–142 Determine the power input for a compressor that compresses helium from 110 kPa and 20°C to 400 kPa and 200°C. Helium enters this compressor through a 0.1-m^2 pipe at a velocity of 7 m/s.

6–143 Chickens with an average mass of 2.2 kg and average specific heat of 3.54 kJ/kg·°C are to be cooled by chilled water that enters a continuous-flow-type immersion chiller at 0.5°C. Chickens are dropped into the chiller at a uniform temperature of 15°C at a rate of 500 chickens per hour and are cooled to an average temperature of 3°C before they are taken out. The chiller gains heat from the surroundings at a rate of 200 kJ/h. Determine (*a*) the rate of heat removal from the chickens, in kW, and (*b*) the mass flow rate of water, in kg/s, if the temperature rise of water is not to exceed 2°C.

6–144 Repeat Prob. 6–143 assuming that the heat gain of the chiller is negligible.

6–145E A refrigeration system is being designed to cool eggs ($\rho = 67.4$ lbm/ft^3 and $c_p = 0.80$ Btu/lbm·°F) with an average mass of 0.14 lbm from an initial temperature of 90°F to a final average temperature of 50°F by air at 34°F at a rate of 3000 eggs per hour. Determine (*a*) the rate of heat removal from the eggs, in Btu/h and (*b*) the required volume flow rate of air, in ft^3/h, if the temperature rise of air is not to exceed 10°F.

6–146 A glass bottle washing facility uses a well-agitated hot-water bath at 50°C that is placed on the ground. The bottles enter at a rate of 450 per minute at an ambient temperature of 20°C and leave at the water temperature. Each bottle has a mass of 150 g and removes 0.2 g of water as it leaves the bath wet. Make-up water is supplied at 15°C. Disregarding any heat losses from the outer surfaces of the bath, determine the rate at which (*a*) water and (*b*) heat must be supplied to maintain steady operation.

6–147 In a dairy plant, milk at 4°C is pasteurized continuously at 72°C at a rate of 20 L/s for 24 h a day and 365 days a year. The milk is heated to the pasteurizing temperature by hot water heated in a natural-gas-fired boiler that has an efficiency of 90 percent. The pasteurized milk is then cooled by cold water at 18°C before it is finally refrigerated back to 4°C. To save energy and money, the plant installs a regenerator that has an effectiveness of 82 percent. If the cost of natural gas is $1.10/therm (1 therm = 105.500 kJ), determine how much energy and money the regenerator will save this company per year.

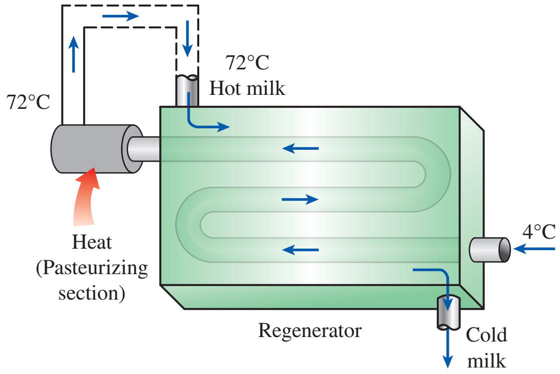


FIGURE P6–147

6–148 Long aluminum wires of diameter 5 mm ($\rho = 2702$ kg/m^3 and $c_p = 0.896$ kJ/kg·°C) are extruded at a temperature of 350°C and are cooled to 50°C in atmospheric air at 25°C. If the wire is extruded at a velocity of 8 m/min, determine the rate of heat transfer from the wire to the extrusion room.

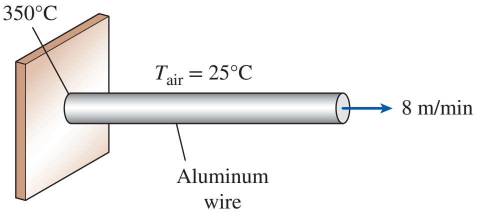


FIGURE P6–148

6–149 Repeat Prob. 6–148 for a copper wire ($\rho = 8950$ kg/m^3 and $c_p = 0.383$ kJ/kg·°C).

6–150E Steam at 80 psia and 400°F is mixed with water at 60°F and 80 psia steadily in an adiabatic device. Steam enters the device at a rate of 0.05 lbm/s, while the water enters at 1 lbm/s. Determine the temperature of the mixture leaving this device when the outlet pressure is 80 psia. *Answer:* 117°F

6–151E It is well established that indoor air quality (IAQ) has a significant effect on general health and productivity of employees at a workplace. A study showed that enhancing IAQ by increasing the building ventilation from 5 cfm (cubic feet per minute) to 20 cfm increased productivity by 0.25 percent, valued at \$90 per person per year, and decreased respiratory illnesses by 10 percent for an average annual savings of \$39 per person while increasing annual energy consumption by \$6 and equipment cost by about \$4 per person per year (*ASHRAE Journal,* December 1998). For a workplace with 120 employees, determine the net monetary benefit of installing an enhanced IAQ system to the employer per year. *Answer:* \$14,280/yr

6–152 The ventilating fan of the bathroom of a building has a volume flow rate of 30 L/s and runs continuously. The building is located in San Francisco, California, where the average winter temperature is 12.2°C, and it is maintained at 22°C at all times. The building is heated by electricity whose unit cost is \$0.12/kWh. Determine the amount and cost of the heat "vented out" per month in winter.

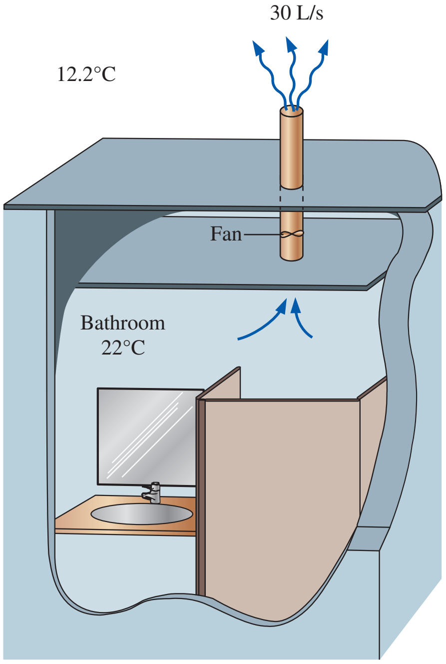


FIGURE P6–152

6–153 Determine the rate of sensible heat loss from a building due to infiltration if the outdoor air at –5°C and 95 kPa enters the building at a rate of 60 L/s when the indoors is maintained at 25°C.

6–154 Consider a large classroom on a hot summer day with 150 students, each dissipating 60 W of sensible heat. All the lights, with 6.0 kW of rated power, are kept on. The room has no external walls, and thus heat gain through the walls and the roof is negligible. Chilled air is available at 15°C, and the temperature of the return air is not to exceed 25°C. Determine the required flow rate of air, in kg/s, that needs to be supplied to the room to keep the average temperature of the room constant. *Answer:* 1.49 kg/s

6–155 An air-conditioning system requires airflow at the main supply duct at a rate of 130 m^3/min. The average velocity of air in the circular duct is not to exceed 8 m/s to avoid excessive vibration and pressure drops. Assuming the fan converts 80 percent of the electrical energy it consumes into kinetic energy of air, determine the size of the electric motor needed to drive the fan and the diameter of the main duct. Take the density of air to be 1.20 kg/m^3.

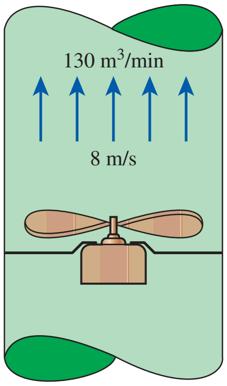


FIGURE P6–155

6–156 A building with an internal volume of 400 m^3 is to be heated by a 30-kW electric resistance heater placed in the duct inside the building. Initially, the air in the building is at 14°C, and the local atmospheric pressure is 95 kPa. The building is losing heat to the surroundings at a steady rate of 450 kJ/min. Air is forced to flow through the duct and the heater steadily by a 250-W fan, and it experiences a temperature rise of 5°C each time it passes through the duct, which may be assumed to be adiabatic.

(*a*) How long will it take for the air inside the building to reach an average temperature of 24°C?

(*b*) Determine the average mass flow rate of air through the duct.

Answers: (*a*) 146 s, (*b*) 6.02 kg/s

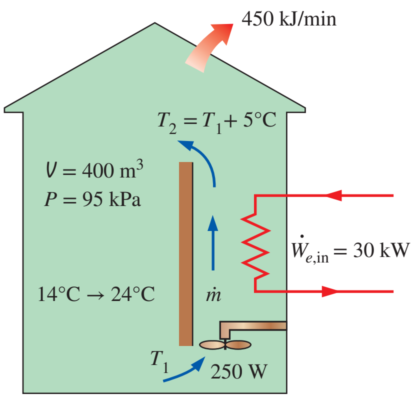


FIGURE P6–156

6–157 The maximum flow rate of standard shower heads is about 3.5 gpm (13.3 L/min) and can be reduced to 2.75 gpm (10.5 L/min) by switching to low-flow shower heads that are equipped with flow controllers. Consider a family of four, with each person taking a 5-min shower every morning. City water at 15°C is heated to 55°C in an electric water heater and tempered to 42°C by cold water at the T-elbow of the shower before being routed to the shower heads. Assuming a constant specific heat of 4.18 kJ/kg·°C for water, determine (*a*) the ratio of the flow rates of the hot and cold water as they enter the T-elbow and (*b*) the amount of electricity that will be saved per year, in kWh, by replacing the standard shower heads with the low-flow ones.

6–158 Reconsider Prob. 6–157. Using appropriate software, investigate the effect of the inlet temperature of cold water on the energy saved by using the low-flow shower head. Let the inlet temperature vary from 10°C to 20°C. Plot the electric energy savings against the water inlet temperature, and discuss the results.

6–159 Steam enters a turbine steadily at 7 MPa and 600°C with a velocity of 60 m/s and leaves at 25 kPa with a quality of 95 percent. A heat loss of 20 kJ/kg occurs during the process. The inlet area of the turbine is 150 cm^2, and the exit area is 1400 cm^2. Determine (*a*) the mass flow rate of the steam, (*b*) the exit velocity, and (*c*) the power output.

6–160 Reconsider Prob. 6–159. Using appropriate software, investigate the effects of turbine exit area and turbine exit pressure on the exit velocity and power output of the turbine. Let the exit pressure vary from 10 to 50 kPa (with the same quality), and let the exit area vary from 1000 to 3000 cm^2. Plot the exit velocity and the power outlet against the exit pressure for the exit areas of 1000, 2000, and 3000 cm^2, and discuss the results.

6–161 It is proposed to have a water heater that consists of an insulated pipe of 7.5-cm diameter and an electric resistor inside. Cold water at 20°C enters the heating section steadily at a rate of 24 L/min. If water is to be heated to 48°C, determine (*a*) the power rating of the resistance heater and (*b*) the average velocity of the water in the pipe.

6–162 A liquid R-134a bottle has an internal volume of 0.0015 m^3. Initially it contains 0.55 kg of R-134a (saturated mixture) at 26°C. A valve is opened and R-134a vapor only (no liquid) is allowed to escape slowly such that temperature remains constant until the mass of R-134a remaining is 0.15 kg. Find the heat transfer with the surroundings that is needed to maintain the temperature and pressure of the R-134a constant.

6–163 A piston–cylinder device initially contains 2 kg of refrigerant-134a at 800 kPa and 80°C. At this state, the piston is touching on a pair of stops at the top. The mass of the piston is such that a 500-kPa pressure is required to move it. A valve at the bottom of the cylinder is opened, and R-134a is withdrawn from the cylinder. After a while, the piston is observed to move and the valve is closed when half of the refrigerant is withdrawn from the cylinder and the temperature in the tank drops to 20°C. Determine (*a*) the work done and (*b*) the heat transfer. *Answers:* (*a*) 11.6 kJ, (*b*) 60.7 kJ

6–164 A piston–cylinder device initially contains 1.2 kg of air at 700 kPa and 200°C. At this state, the piston is touching on a pair of stops. The mass of the piston is such that 600-kPa pressure is required to move it. A valve at the bottom of the tank is opened, and air is withdrawn from the cylinder. The valve is closed when the volume of the cylinder decreases to 80 percent of the initial volume. If it is estimated that 40 kJ of heat is lost from the cylinder, determine (*a*) the final temperature of the air in the cylinder, (*b*) the amount of mass that has escaped from the cylinder, and (*c*) the work done. Use constant specific heats at the average temperature.

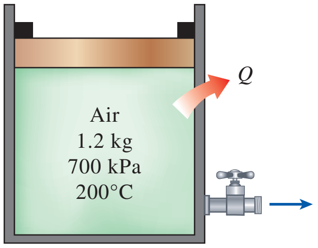


FIGURE P6–164

6–165 A pressure cooker is a pot that cooks food much faster than ordinary pots by maintaining a higher pressure and temperature during cooking. The pressure inside the pot is controlled by a pressure regulator (the petcock) that keeps the pressure at a constant level by periodically allowing some steam to escape, thus preventing any excess pressure buildup. Pressure cookers, in general, maintain a gage pressure of 2 atm (or 3 atm absolute) inside. Therefore, pressure cookers cook at a temperature of about 133°C instead of 100°C, cutting the cooking time by as much as 70 percent while minimizing the loss of nutrients. The newer pressure cookers use a spring valve with several pressure settings rather than a weight on the cover.

A certain pressure cooker has a volume of 6 L and an operating pressure of 75 kPa gage. Initially, it contains 1 kg of water. Heat is supplied to the pressure cooker at a rate of 500 W for 30 min after the operating pressure is reached. Assuming an atmospheric pressure of 100 kPa, determine (*a*) the temperature at which cooking takes place and (*b*) the amount of water left in the pressure cooker at the end of the process. *Answers:* (*a*) 116.04°C, (*b*) 0.6 kg

6–166 In a single-flash geothermal power plant, geothermal water enters the flash chamber (a throttling valve) at 230°C as a saturated liquid at a rate of 50 kg/s. The steam resulting from the flashing process enters a turbine and leaves at 20 kPa with a moisture content of 5 percent. Determine the temperature of the steam after the flashing process and the power output from the turbine if the pressure of the steam at the exit of the flash chamber is (*a*) 1 MPa, (*b*) 500 kPa, (*c*) 100 kPa, (*d*) 50 kPa.

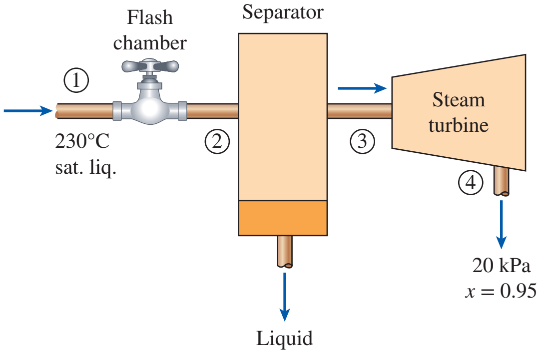


FIGURE P6–166

6–167 An adiabatic air compressor is to be powered by a direct-coupled adiabatic steam turbine that is also driving a generator. Steam enters the turbine at 12.5 MPa and 500°C at a rate of 25 kg/s and exits at 10 kPa and a quality of 0.92. Air enters the compressor at 98 kPa and 295 K at a rate of 10 kg/s and exits at 1 MPa and 620 K. Determine the net power delivered to the generator by the turbine.

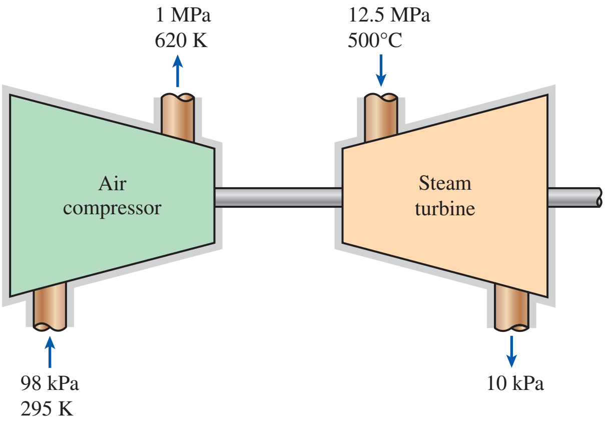


FIGURE P6–167

Design and Essay Problems

6–168 Pneumatic nail drivers used in construction require 0.02 ft^3 of air at 100 psia and 1 Btu of energy to drive a single nail. You have been assigned the task of designing a compressed-air storage tank with enough capacity to drive 500 nails. The pressure in this tank cannot exceed 500 psia, and the temperature cannot exceed that normally found at a construction site. What is the maximum pressure to be used in the tank and what is the tank's volume?

6–169 You have been given the responsibility of picking a steam turbine for an electrical-generation station that is to produce 300 MW of electrical power that will sell for $0.08 per kilowatt-hour. The boiler will produce steam at 700 psia and 700°F, and the condenser is planned to operate at 80°F. The cost of generating and condensing the steam is $0.015 per kilowatt-hour of electricity produced. You have narrowed your selection to the three turbines in the following table. Your criterion for selection is to pay for the equipment as quickly as possible. Which turbine should you choose?

Turbine	Capacity (MW)	η	Cost ($Million)	Operating Cost ($/kWh)
A	50	0.9	5	0.01
B	100	0.92	11	0.01
C	100	0.93	10.5	0.015

6–170 An air cannon uses compressed air to propel a projectile from rest to a final velocity. Consider an air cannon that is to accelerate a 10-gram projectile to a speed of 300 m/s using compressed air, whose temperature cannot exceed 20°C. The volume of the storage tank is not to exceed 0.1 m^3. Select the storage volume size and maximum storage pressure that require the minimum amount of energy to fill the tank.

6–171 Design a 1200-W electric hair dryer such that the air temperature and velocity in the dryer will not exceed 50°C and 3 m/s, respectively.

6–172 To maintain altitude, the temperature of the air inside a hot-air balloon must remain within a 1°C band, while the volume cannot vary by more than 1 percent. At a 300-m altitude, the air in a 1000 m^3 hot-air balloon needs to maintain a 35°C average temperature. This balloon loses heat at a rate of 3 kW through the fabric. When the burner is activated, it adds 30 kg/s of air at 200°C and 100 kPa to the balloon. When the flap that allows air to escape is opened, air leaves the balloon at a rate of 20 kg/s. Design the burner and exhaust-flap control cycles (on time and off time) necessary to maintain the balloon at a 300-m altitude.

CHAPTER 7

THE SECOND LAW OF THERMODYNAMICS

To this point, we have focused our attention on the first law of thermodynamics, which requires that energy be conserved during a process. In this chapter, we introduce the second law of thermodynamics, which asserts that processes occur in a certain direction and that energy has quality as well as quantity. A process cannot take place unless it satisfies both the first and second laws of thermodynamics. In this chapter, the thermal energy reservoirs, reversible and irreversible processes, heat engines, refrigerators, and heat pumps are introduced first. Various statements of the second law are followed by a discussion of the thermodynamic temperature scale. The Carnot cycle is introduced next, and the Carnot principles are discussed. Finally, idealized Carnot heat engines, refrigerators, and heat pumps are examined.

OBJECTIVES

The objectives of this chapter are to:

- Introduce the second law of thermodynamics.
- Identify valid processes as those that satisfy both the first and second laws of thermodynamics.
- Discuss thermal energy reservoirs, reversible and irreversible processes, heat engines, refrigerators, and heat pumps.
- Describe the Kelvin–Planck and Clausius statements of the second law of thermodynamics.
- Apply the second law of thermodynamics to cycles and cyclic devices.
- Apply the second law to develop the absolute thermodynamic temperature scale.
- Describe the Carnot cycle.
- Examine the Carnot principles, idealized Carnot heat engines, refrigerators, and heat pumps.
- Determine the expressions for the thermal efficiencies and coefficients of performance for reversible heat engines, heat pumps, and refrigerators.

7–1 ▪ INTRODUCTION TO THE SECOND LAW

In Chaps. 5 and 6, we applied the *first law of thermodynamics,* or the *conservation of energy principle,* to processes involving closed and open systems. As pointed out repeatedly in those chapters, energy is a conserved property, and no process is known to have taken place in violation of the first law of thermodynamics. Therefore, it is reasonable to conclude that a process must satisfy the first law to occur. However, as explained here, satisfying the first law alone does not ensure that the process will actually take place.

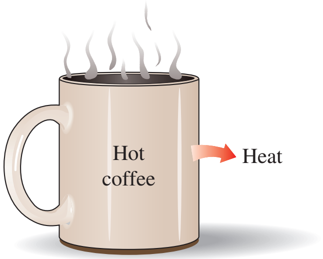


FIGURE 7–1
A cup of hot coffee does not get hotter in a cooler room.

It is common experience that a cup of hot coffee left in a cooler room eventually cools off (Fig. 7–1). This process satisfies the first law of thermodynamics since the amount of energy lost by the coffee is equal to the amount gained by the surrounding air. Now let us consider the reverse process—the hot coffee getting even hotter in a cooler room as a result of heat transfer from the room air. We all know that this process never takes place. Yet, doing so would not violate the first law as long as the amount of energy lost by the air is equal to the amount gained by the coffee.

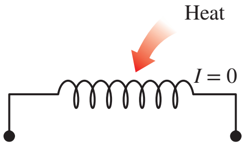


FIGURE 7–2
Transferring heat to a wire will not generate electricity.

As another familiar example, consider the heating of a room by the passage of electric current through a resistor (Fig. 7–2). Again, the first law dictates that the amount of electric energy supplied to the resistance wires be equal to the amount of energy transferred to the room air as heat. Now let us try to reverse this process. It will come as no surprise that transferring some heat to the wires does not cause an equivalent amount of electric energy to be generated in the wires.

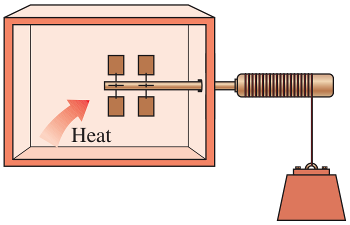


FIGURE 7–3
Transferring heat to a paddle wheel will not cause it to rotate.

Finally, consider a paddle-wheel mechanism that is operated by the fall of a mass (Fig. 7–3). The paddle wheel rotates as the mass falls and stirs a fluid within an insulated container. As a result, the potential energy of the mass decreases, and the internal energy of the fluid increases in accordance with the conservation of energy principle. However, the reverse process, raising the mass by transferring heat from the fluid to the paddle wheel, does not occur in nature, although doing so would not violate the first law of thermodynamics.

FIGURE 7–4
Processes occur in a certain direction, and not in the reverse direction.

It is clear from these arguments that processes proceed in a *certain direction* and not in the reverse direction (Fig. 7–4). The first law places no restriction on the direction of a process, but satisfying the first law does not ensure that the process can actually occur. This inadequacy of the first law to identify whether a process can take place is remedied by introducing another general principle, the *second law of thermodynamics.* We show later in this chapter that the reverse processes we just mentioned violate the second law of thermodynamics. This violation is easily detected with the help of a property, called *entropy,* defined in Chap. 8. A process cannot occur unless it satisfies both the first and the second laws of thermodynamics (Fig. 7–5).

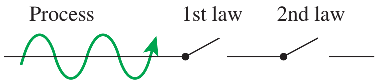


FIGURE 7–5
A process must satisfy both the first and second laws of thermodynamics to proceed.

There are many valid statements of the second law of thermodynamics. Two such statements are presented and discussed later in this chapter in relation to some engineering devices that operate on cycles.

The use of the second law of thermodynamics is not limited to identifying the direction of processes. The second law also asserts that energy has *quality* as well as quantity. The first law is concerned with the quantity of energy and the transformations of energy from one form to another with no regard to its quality. Preserving the quality of energy is a major concern to engineers, and the second law provides the necessary means to determine the quality as well as the degree of degradation of energy during a process. As discussed later in this chapter, more of high-temperature energy can be converted to work, and thus it has a higher quality than the same amount of energy at a lower temperature.

The second law of thermodynamics is also used in determining the *theoretical limits* for the performance of commonly used engineering systems, such as heat engines and refrigerators, as well as predicting the *degree of completion* of chemical reactions. The second law is also closely associated with the concept of *perfection.* In fact, the second law *defines* perfection for thermodynamic processes. It can be used to quantify the level of perfection of a process and to point in the direction to eliminate imperfections effectively.

7–2 ▪ THERMAL ENERGY RESERVOIRS

In the development of the second law of thermodynamics, it is very convenient to have a hypothetical body with a relatively large *thermal energy capacity* (mass × specific heat) that can supply or absorb finite amounts of heat without undergoing any change in temperature. Such a body is called a **thermal energy reservoir**, or just a reservoir. In practice, large bodies of water such as oceans, lakes, and rivers as well as the atmospheric air can be modeled accurately as thermal energy reservoirs because of their large thermal energy storage capabilities or thermal masses (Fig. 7–6). The *atmosphere,* for example, does not warm up as a result of heat losses from residential buildings in winter. Likewise, megajoules of waste energy dumped into large rivers by power plants do not cause any significant change in water temperature.

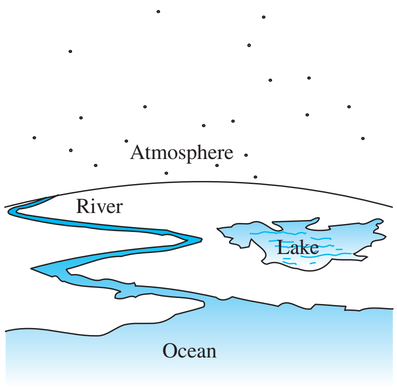


FIGURE 7–6
Bodies with relatively large thermal masses can be modeled as thermal energy reservoirs.

A *two-phase system* can also be modeled as a reservoir since it can absorb and release large quantities of heat while remaining at constant temperature. Another familiar example of a thermal energy reservoir is the *industrial furnace.* The temperatures of most furnaces are carefully controlled, and they can supply large quantities of thermal energy as heat in an essentially isothermal manner. Therefore, they can be modeled as reservoirs.

A body does not actually have to be very large to be considered a reservoir. Any physical body whose thermal energy capacity is large relative to the amount of energy it supplies or absorbs can be modeled as one. The air in a room, for example, can be treated as a reservoir in the analysis of the heat dissipation from a TV set in the room, since the amount of heat transfer from the TV set to the room air is not large enough to have a noticeable effect on the room air temperature.

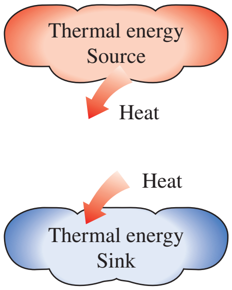


FIGURE 7–7
A source supplies energy in the form of heat, and a sink absorbs it.

A reservoir that supplies energy in the form of heat is called a **source**, and one that absorbs energy in the form of heat is called a **sink** (Fig. 7–7). Thermal energy reservoirs are often referred to as **heat reservoirs** since they supply or absorb energy in the form of heat.

Heat transfer from industrial sources to the environment is of major concern to environmentalists as well as to engineers. Irresponsible management of waste energy can significantly increase the temperature of portions of the environment, causing what is called *thermal pollution.* If it is not carefully controlled, thermal pollution can seriously disrupt marine life in lakes and rivers. However, by careful design and management, the waste energy dumped into large bodies of water can be used to improve the quality of marine life by keeping the local temperature increases within safe and desirable levels.

7–3 ▪ HEAT ENGINES

As pointed out earlier, work can easily be converted to other forms of energy, but converting other forms of energy to work is not that easy. The mechanical work done by the shaft shown in Fig. 7–8, for example, is first converted to the internal energy of the water. This energy may then leave the water as heat. We know from experience that any attempt to reverse this process will fail. That is, transferring heat to the water does not cause the shaft to rotate. From this and other observations, we conclude that work can be converted to heat directly and completely, but converting heat to work requires the use of some special devices. These devices are called **heat engines**.

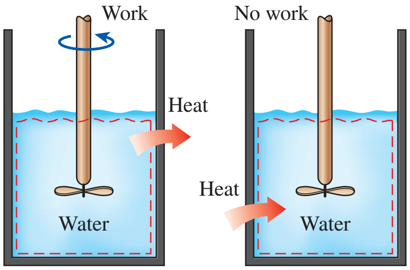


FIGURE 7–8
Work can always be converted to heat directly and completely, but the reverse is not true.

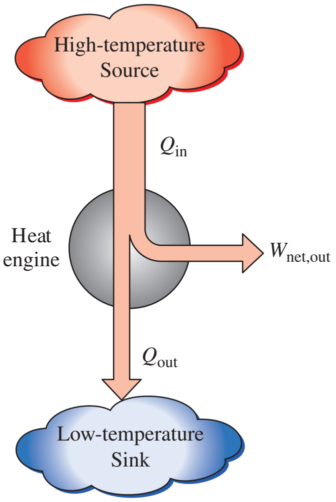


FIGURE 7–9
Part of the heat received by a heat engine is converted to work, while the rest is rejected to a sink.

Heat engines differ considerably from one another, but all can be characterized by the following (Fig. 7–9):

1. They receive heat from a high-temperature source (solar energy, oil furnace, nuclear reactor, etc.).
2. They convert part of this heat to work (usually in the form of a rotating shaft).
3. They reject the remaining waste heat to a low-temperature sink (the atmosphere, rivers, etc.).
4. They operate on a cycle.

Heat engines and other cyclic devices usually involve a fluid to and from which heat is transferred while undergoing a cycle. This fluid is called the **working fluid**.

The term *heat engine* is often used in a broader sense to include work-producing devices that do not operate in a thermodynamic cycle. Engines that involve internal combustion such as gas turbines and car engines fall into this category. These devices operate in a mechanical cycle but not in a thermodynamic cycle since the working fluid (the combustion gases) does not undergo a complete cycle. Instead of being cooled to the initial temperature, the exhaust gases are purged and replaced by fresh air-and-fuel mixture at the end of the cycle.

The work-producing device that best fits into the definition of a heat engine is the *steam power plant,* which is an external-combustion engine. That is, combustion takes place outside the engine, and the thermal energy released during this process is transferred to the steam as heat. The schematic of a basic steam power plant is shown in Fig. 7–10. This is a rather simplified diagram, and the discussion of actual steam power plants is given in later chapters. The various quantities shown on this figure are as follows:

Q_{in} = amount of heat supplied to steam in boiler from a high-temperature source (furnace)
Q_{out} = amount of heat rejected from steam in condenser to a low-temperature sink (the atmosphere, a river, etc.)
W_{out} = amount of work delivered by steam as it expands in turbine
W_{in} = amount of work required to compress water to boiler pressure

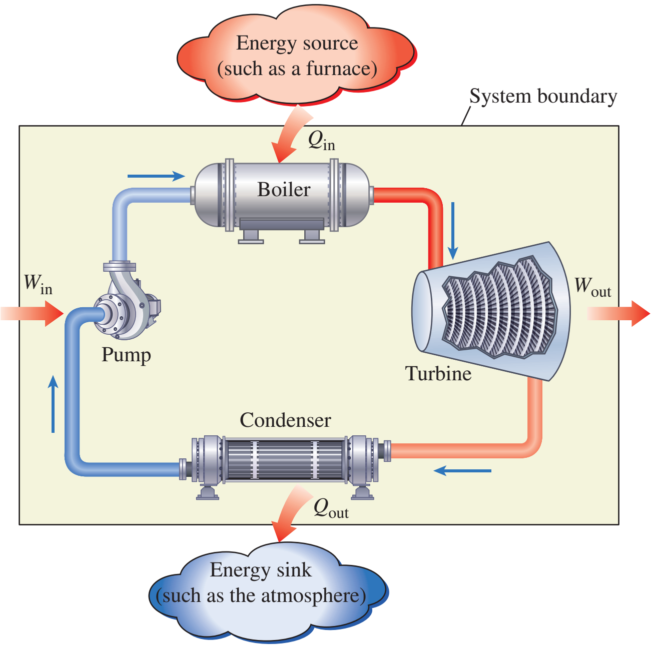


FIGURE 7–10
Schematic of a steam power plant.

Notice that the directions of the heat and work interactions are indicated by the subscripts *in* and *out*. Therefore, all four of the described quantities are always *positive*.

The net work output of this power plant is simply the difference between the total work output of the plant and the total work input (Fig. 7–11):

$$W_{\text{net,out}} = W_{\text{out}} - W_{\text{in}} \qquad \text{(kJ)} \tag{7–1}$$

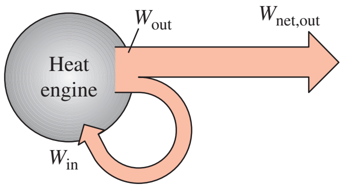


FIGURE 7–11
A portion of the work output of a heat engine is consumed internally to maintain continuous operation.

The net work can also be determined from the heat transfer data alone. The four components of the steam power plant involve mass flow in and out, and therefore should be treated as open systems. These components, together with the connecting pipes, however, always contain the same fluid (not counting the steam that may leak out, of course). No mass enters or leaves this combination system, which is indicated by the shaded area on Fig. 7–10; thus, it can be analyzed as a closed system. Recall that for a closed system undergoing a cycle, the change in internal energy ΔU is zero, and therefore the net work output of the system is also equal to the net heat transfer to the system:

$$W_{\text{net,out}} = Q_{\text{in}} - Q_{\text{out}} \qquad \text{(kJ)} \tag{7–2}$$

Thermal Efficiency

In Eq. 7–2, Q_{out} represents the magnitude of the energy wasted in order to complete the cycle. But Q_{out} is never zero; thus, the net work output of a heat engine is always less than the amount of heat input. That is, only part of the heat transferred to the heat engine is converted to work. The fraction of the heat input that is converted to net work output is a measure of the performance of a heat engine and is called the **thermal efficiency** η_{th} (Fig. 7–12).

For heat engines, the desired output is the net work output, and the required input is the amount of heat supplied to the working fluid. Then the thermal efficiency of a heat engine can be expressed as

$$\text{Thermal efficiency} = \frac{\text{Net work output}}{\text{Total heat input}} \tag{7–3}$$

or

$$\eta_{\text{th}} = \frac{W_{\text{net,out}}}{Q_{\text{in}}} \tag{7–4}$$

It can also be expressed as

$$\eta_{\text{th}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} \tag{7–5}$$

since $W_{\text{net,out}} = Q_{\text{in}} - Q_{\text{out}}$.

Cyclic devices of practical interest such as heat engines, refrigerators, and heat pumps operate between a high-temperature medium (or reservoir) at temperature T_H and a low-temperature medium (or reservoir) at temperature T_L. To bring uniformity to the treatment of heat engines, refrigerators, and heat pumps, we define these two quantities:

Q_H = magnitude of heat transfer between the cyclic device and the high-temperature medium at temperature T_H
Q_L = magnitude of heat transfer between the cyclic device and the low-temperature medium at temperature T_L

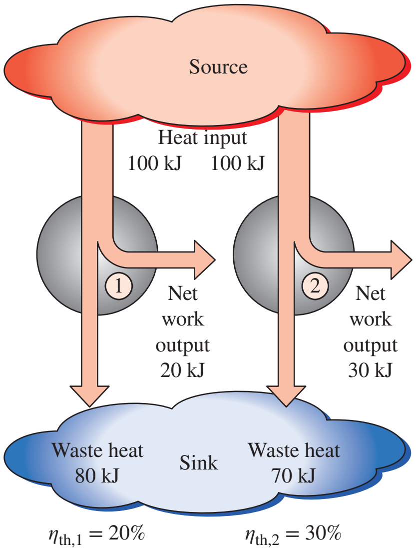


FIGURE 7–12
Some heat engines perform better than others (convert more of the heat they receive to work).

Notice that both Q_L and Q_H are defined as *magnitudes* and therefore are positive quantities. The direction of Q_H and Q_L is easily determined by inspection. Then, the net work output and thermal efficiency relations for any heat engine (shown in Fig. 7–13) can also be expressed as

$$W_{\text{net,out}} = Q_H - Q_L$$

and

$$\eta_{\text{th}} = \frac{W_{\text{net,out}}}{Q_H} \quad \text{or} \quad \eta_{\text{th}} = 1 - \frac{Q_L}{Q_H} \tag{7–6}$$

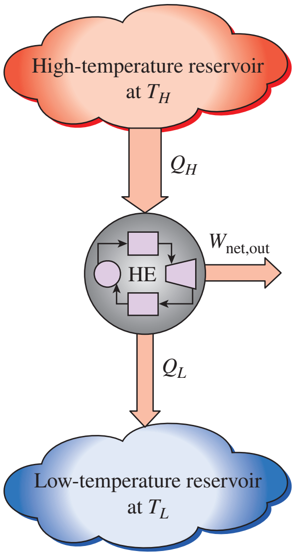


FIGURE 7–13
Schematic of a heat engine.

The thermal efficiency of a heat engine is always less than unity since both Q_L and Q_H are defined as positive quantities.

Thermal efficiency is a measure of how efficiently a heat engine converts the heat that it receives to work. Heat engines are built for the purpose of converting heat to work, and engineers are constantly trying to improve the efficiencies of these devices since increased efficiency means less fuel consumption and thus lower fuel bills and less pollution.

The thermal efficiencies of work-producing devices are relatively low. Ordinary spark-ignition automobile engines have a thermal efficiency of about 25 percent. That is, an automobile engine converts about 25 percent of the chemical energy of the gasoline to mechanical work. This number is as high as 40 percent for diesel engines and large gas-turbine plants and as high as 60 percent for large combined gas-steam power plants. Thus, even with the most efficient heat engines available today, almost one-half of the energy supplied ends up in the rivers, lakes, or the atmosphere as waste or useless energy (Fig. 7–14).

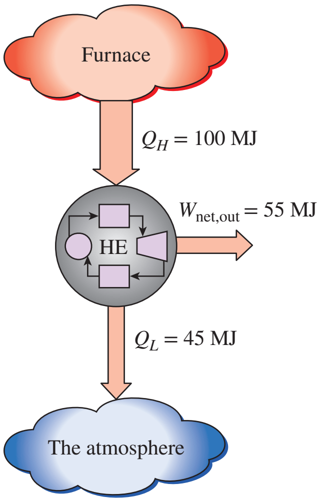


FIGURE 7–14
Even the most efficient heat engines reject almost one-half of the energy they receive as waste heat.

Can We Save Q_{out}?

In a steam power plant, the condenser is the device where large quantities of waste heat are rejected to rivers, lakes, or the atmosphere. Then one may ask, can we not just take the condenser out of the plant and save all that waste energy? The answer to this question is, unfortunately, a firm *no* for the simple reason that without a heat rejection process in a condenser, the cycle cannot be completed. (Cyclic devices such as steam power plants cannot run continuously unless the cycle is completed.) This is demonstrated next with the help of a simple heat engine.

Consider the simple heat engine shown in Fig. 7–15 that is used to lift weights. It consists of a piston–cylinder device with two sets of stops. The working fluid is the gas contained within the cylinder. Initially, the gas temperature is 30°C. The piston, which is loaded with the weights, is resting on top of the lower stops. Now 100 kJ of heat is transferred to the gas in the cylinder from a source at 100°C, causing it to expand and to raise the loaded piston until the piston reaches the upper stops, as shown in the figure. At this point, the load is removed, and the gas temperature is observed to be 90°C.

The work done on the load during this expansion process is equal to the increase in its potential energy, say 15 kJ. Even under ideal conditions (weightless piston, no friction, no heat losses, and quasi-equilibrium expansion), the amount of heat supplied to the gas is greater than the work done since part of the heat supplied is used to raise the temperature of the gas.

Now let us try to answer this question: *Is it possible to transfer the 85 kJ of excess heat at 90°C back to the reservoir at 100°C for later use?* If it is, then we will have a heat engine that can have a thermal efficiency of 100 percent under ideal conditions. The answer to this question is again *no,* for the very simple reason that heat is always transferred from a high-temperature medium to a low-temperature one, and never the other way around. Therefore, we

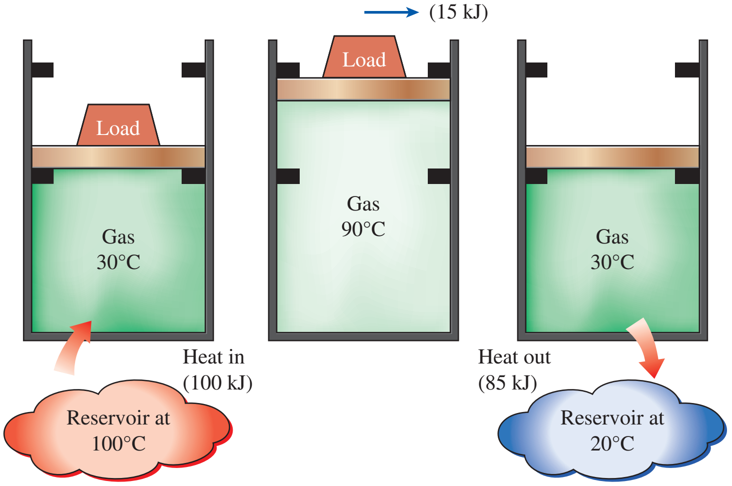


FIGURE 7–15
A heat-engine cycle cannot be completed without rejecting some heat to a low-temperature sink.

cannot cool this gas from 90 to 30°C by transferring heat to a reservoir at 100°C. Instead, we have to bring the system into contact with a low-temperature reservoir, say at 20°C, so that the gas can return to its initial state by rejecting its 85 kJ of excess energy as heat to this reservoir. This energy cannot be recycled, and it is properly called *waste energy.*

We conclude from this discussion that every heat engine must *waste* some energy by transferring it to a low-temperature reservoir in order to complete the cycle, even under idealized conditions. The requirement that a heat engine exchange heat with at least two reservoirs for continuous operation forms the basis for the Kelvin–Planck expression of the second law of thermodynamics discussed later in this section.

EXAMPLE 7–1 Net Power Production of a Heat Engine

Heat is transferred to a heat engine from a furnace at a rate of 80 MW. If the rate of waste heat rejection to a nearby river is 50 MW, determine the net power output and the thermal efficiency for this heat engine.

SOLUTION The rates of heat transfer to and from a heat engine are given. The net power output and the thermal efficiency are to be determined.
Assumptions Heat losses through the pipes and other components are negligible.
Analysis A schematic of the heat engine is given in Fig. 7–16. The furnace serves as the high-temperature reservoir for this heat engine and the river as the low-temperature reservoir. The given quantities can be expressed as

$$\dot{Q}_H = 80 \text{ MW} \quad \text{and} \quad \dot{Q}_L = 50 \text{ MW}$$

The net power output of this heat engine is

$$\dot{W}_{\text{net,out}} = \dot{Q}_H - \dot{Q}_L = (80 - 50) \text{ MW} = \mathbf{30 \text{ MW}}$$

Then the thermal efficiency is easily determined to be

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net,out}}}{\dot{Q}_H} = \frac{30 \text{ MW}}{80 \text{ MW}} = \mathbf{0.375} \text{ (or 37.5 percent)}$$

Discussion Note that the heat engine converts 37.5 percent of the heat it receives to work.

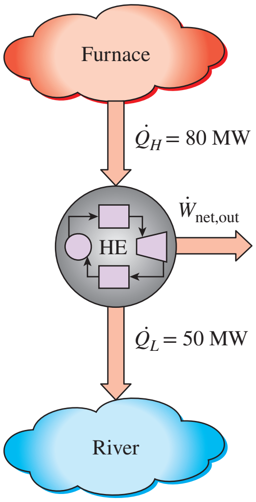


FIGURE 7–16
Schematic for Example 7–1.

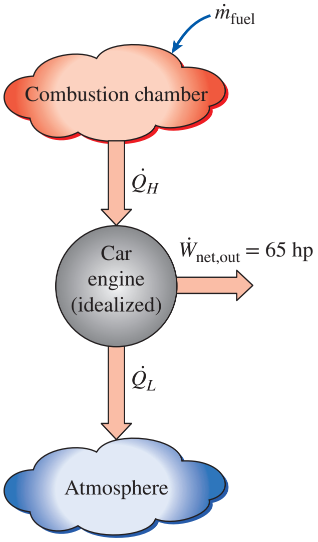


FIGURE 7–17
Schematic for Example 7–2.

EXAMPLE 7–2 Fuel Consumption Rate of a Car

A car engine with a power output of 65 hp has a thermal efficiency of 24 percent. Determine the fuel consumption rate of this car if the fuel has a heating value of 19,000 Btu/lbm (that is, 19,000 Btu of energy is released for each lbm of fuel burned).

SOLUTION The power output and the efficiency of a car engine are given. The rate of fuel consumption of the car is to be determined.
Assumptions The power output of the car is constant.
Analysis A schematic of the car engine is given in Fig. 7–17. The car engine is powered by converting 24 percent of the chemical energy released during the combustion process to work. The amount of energy input required to produce a power output of 65 hp is determined from the definition of thermal efficiency to be

$$\dot{Q}_H = \frac{\dot{W}_{\text{net,out}}}{\eta_{\text{th}}} = \frac{65\text{ hp}}{0.24}\left(\frac{2545\text{ Btu/h}}{1\text{ hp}}\right) = 689{,}270\text{ Btu/h}$$

To supply energy at this rate, the engine must burn fuel at a rate of

$$\dot{m}_{\text{fuel}} = \frac{689{,}270\text{ Btu/h}}{19{,}000\text{ Btu/lbm}} = \mathbf{36.3\ lbm/h}$$

since 19,000 Btu of thermal energy is released for each lbm of fuel burned.
Discussion Note that if the thermal efficiency of the car could be doubled, the rate of fuel consumption would be reduced by half.

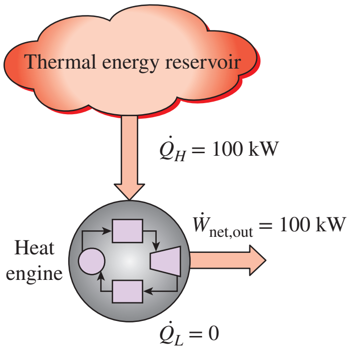


FIGURE 7–18
A heat engine that violates the Kelvin–Planck statement of the second law.

The Second Law of Thermodynamics: Kelvin–Planck Statement

We demonstrated earlier with reference to the heat engine shown in Fig. 7–15 that, even under ideal conditions, a heat engine must reject some heat to a low-temperature reservoir in order to complete the cycle. That is, no heat engine can convert all the heat it receives to useful work. This limitation on the thermal efficiency of heat engines forms the basis for the Kelvin–Planck statement of the second law of thermodynamics, which is expressed as follows:

> It is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work.

That is, a heat engine must exchange heat with a low-temperature sink as well as a high-temperature source to keep operating. The Kelvin–Planck statement can also be expressed as *no heat engine can have a thermal efficiency of 100 percent* (Fig. 7–18), or as *for a power plant to operate, the working fluid must exchange heat with the environment as well as the furnace.*

Note that the impossibility of having a 100 percent efficient heat engine is not due to friction or other dissipative effects. It is a limitation that applies to both the idealized and the actual heat engines. Later in this chapter, we develop a relation for the maximum thermal efficiency of a heat engine. We also demonstrate that this maximum value depends on the reservoir temperatures only.

7–4 ▪ REFRIGERATORS AND HEAT PUMPS

We all know from experience that heat is transferred in the direction of decreasing temperature, that is, from high-temperature mediums to low-temperature ones. This heat transfer process occurs in nature without requiring any devices. The reverse process, however, cannot occur by itself. The transfer of heat from a low-temperature medium to a high-temperature one requires special devices called **refrigerators**.

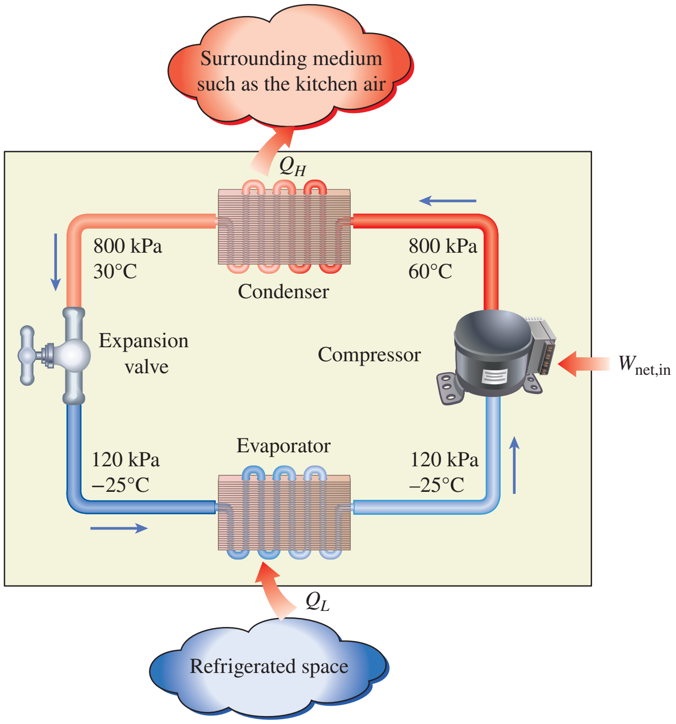


FIGURE 7–19
Basic components of a refrigeration system and typical operating conditions.

Refrigerators, like heat engines, are cyclic devices. The working fluid used in the refrigeration cycle is called a **refrigerant**. The most frequently used refrigeration cycle is the *vapor-compression refrigeration cycle,* which involves four main components: a compressor, a condenser, an expansion valve, and an evaporator, as shown in Fig. 7–19.

The refrigerant enters the compressor as a vapor and is compressed to the condenser pressure. It leaves the compressor at a relatively high temperature and cools down and condenses as it flows through the coils of the condenser by rejecting heat to the surrounding medium. It then enters a capillary tube where its pressure and temperature drop drastically due to the throttling effect. The low-temperature refrigerant then enters the evaporator, where it evaporates by absorbing heat from the refrigerated space. The cycle is completed as the refrigerant leaves the evaporator and reenters the compressor.

In a household refrigerator, the freezer compartment where heat is absorbed by the refrigerant serves as the evaporator, and the coils, usually behind the refrigerator where heat is dissipated to the kitchen air, serve as the condenser.

A refrigerator is shown schematically in Fig. 7–20. Here Q_L is the magnitude of the heat removed from the refrigerated space at temperature T_L, Q_H is the magnitude of the heat rejected to the warm environment at temperature T_H, and $W_{\text{net,in}}$ is the net work input to the refrigerator. As discussed before, Q_L and Q_H represent magnitudes and thus are positive quantities.

Coefficient of Performance

The *efficiency* of a refrigerator is expressed in terms of the **coefficient of performance** (COP), denoted by COP_R. The objective of a refrigerator is to remove heat (Q_L) from the refrigerated space. To accomplish this objective, it requires a work input of $W_{\text{net,in}}$. Then the COP of a refrigerator can be expressed as

$$\text{COP}_\text{R} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_L}{W_{\text{net,in}}} \tag{7–7}$$

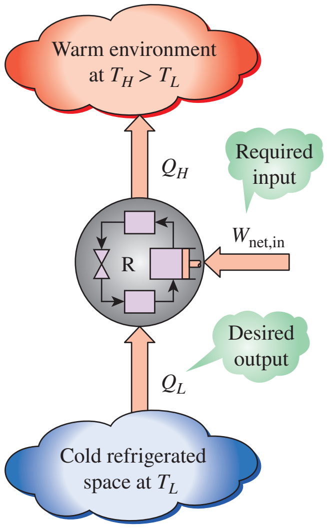


FIGURE 7–20
The objective of a refrigerator is to remove Q_L from the cooled space.

This relation can also be expressed in rate form by replacing Q_L with $\dot{Q}_L$ and $W_{\text{net,in}}$ with $\dot{W}_{\text{net,in}}$.

The conservation of energy principle for a cyclic device requires that

$$W_{\text{net,in}} = Q_H - Q_L \qquad \text{(kJ)} \tag{7–8}$$

Then the COP relation becomes

$$\text{COP}_\text{R} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{Q_H/Q_L - 1} \tag{7–9}$$

Notice that the value of COP_R can be *greater than unity*. That is, the amount of heat removed from the refrigerated space can be greater than the amount of work input. This is in contrast to the thermal efficiency, which can never be greater than 1. In fact, one reason for expressing the efficiency of a refrigerator with another term—the coefficient of performance—is the desire to avoid the oddity of having efficiencies greater than unity.

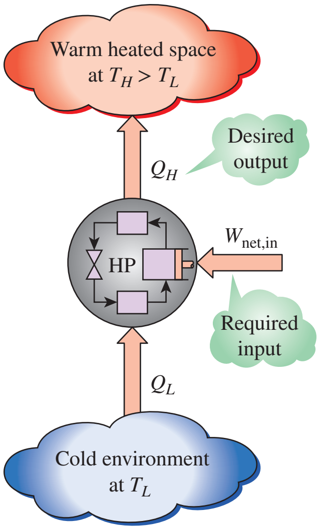


FIGURE 7–21
The objective of a heat pump is to supply heat Q_H into the warmer space.

Heat Pumps

Another device that transfers heat from a low-temperature medium to a high-temperature one is the **heat pump**, shown schematically in Fig. 7–21. Refrigerators and heat pumps operate on the same cycle but differ in their objectives.

The objective of a refrigerator is to maintain the refrigerated space at a low temperature by removing heat from it. Discharging this heat to a higher-temperature medium is merely a necessary part of the operation, not the purpose. The objective of a heat pump, however, is to maintain a heated space at a high temperature. This is accomplished by absorbing heat from a low-temperature source, such as well water or cold outside air in winter, and supplying this heat to the high-temperature medium such as a house (Fig. 7–22).

An ordinary refrigerator that is placed in the window of a house with its door open to the cold outside air in winter will function as a heat pump since it will try to cool the outside by absorbing heat from it and rejecting this heat into the house through the coils behind it.

The measure of performance of a heat pump is also expressed in terms of the **coefficient of performance** COP_HP, defined as

$$\text{COP}_\text{HP} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_H}{W_{\text{net,in}}} \tag{7–10}$$

which can also be expressed as

$$\text{COP}_\text{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - Q_L/Q_H} \tag{7–11}$$

A comparison of Eqs. 7–7 and 7–10 reveals that

$$\text{COP}_\text{HP} = \text{COP}_\text{R} + 1 \tag{7–12}$$

for fixed values of Q_L and Q_H. This relation implies that the coefficient of performance of a heat pump is always greater than unity since COP_R is a positive quantity. That is, a heat pump will function, at worst, as a resistance heater, supplying as much energy to the house as it consumes. In reality, however, part of Q_H is lost to the outside air through piping and other devices, and COP_HP may drop below unity when the outside air temperature is too low. When this happens, the system usually switches to a resistance heating mode. Most heat pumps in operation today have a seasonally averaged COP of 2 to 3.

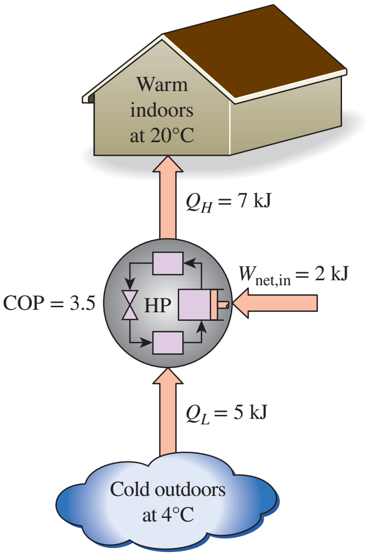


FIGURE 7–22
The work supplied to a heat pump is used to extract energy from the cold outdoors and carry it into the warm indoors.

Most existing heat pumps use the cold outside air as the heat source in winter, and they are referred to as *air-source heat pumps.* The COP of such heat pumps is about 3.0 at design conditions. Air-source heat pumps are not appropriate for cold climates since their efficiency drops considerably when temperatures are below the freezing point. In such cases, geothermal (also called ground-source) heat pumps that use the ground as the heat source can be used. Geothermal heat pumps require the burial of pipes in the ground 1 to 2 m deep. Such heat pumps are more expensive to install, but they are also more efficient (up to 45 percent more efficient than air-source heat pumps). The COP of ground-source heat pumps can be as high as 6 or more.

Air conditioners are basically refrigerators whose refrigerated space is a room or a building instead of the food compartment. A window air-conditioning unit cools a room by absorbing heat from the room air and discharging it to the outside. The same air-conditioning unit can be used as a heat pump in winter by installing it backwards. In this mode, the unit absorbs heat from the cold outside and delivers it to the room. Air-conditioning systems that are equipped with proper controls and a reversing valve operate as air conditioners in summer and as heat pumps in winter.

Performance of Refrigerators, Air Conditioners, and Heat Pumps

The performance of air conditioners and heat pumps is often expressed in terms of the **energy efficiency ratio** (EER) or **seasonal energy efficiency ratio** (SEER) determined by following certain testing standards. SEER is the ratio of the total amount of heat removed by an air conditioner or heat pump during a normal cooling season (in Btu) to the total amount of electricity consumed (in watt-hours, Wh), and it is a measure of seasonal performance of cooling equipment. EER, on the other hand, is a measure of the instantaneous energy efficiency and is defined as the ratio of the rate of heat removal from the cooled space by the cooling equipment to the rate of electricity consumption in steady operation. Therefore, both EER and SEER have the unit Btu/Wh. Considering that 1 kWh = 3412 Btu and thus 1 Wh = 3.412 Btu, a device that removes 1 kWh of heat from the cooled space for each kWh of electricity it consumes (COP = 1) will have an EER of 3.412. Therefore, the relation between EER (or SEER) and COP is

$$\text{EER} = 3.412\ \text{COP}_\text{R}$$

To promote the efficient use of energy, governments worldwide have mandated minimum standards for the performance of energy-consuming equipment. Most air conditioners or heat pumps in the market have SEER values from 13 to 21, which correspond to COP values of 3.8 to 6.2. Best performance is achieved using units equipped with variable-speed drives (also called inverters). Variable-speed compressors and fans allow the unit to operate at maximum efficiency for varying heating/cooling needs and weather conditions as determined by a microprocessor. In the air-conditioning mode, for example, they operate at higher speeds on hot days and at lower speeds on cooler days, enhancing both efficiency and comfort.

The EER or COP of a refrigerator decreases with decreasing refrigeration temperature. Therefore, it is not economical to refrigerate to a lower temperature than needed. The COPs of refrigerators are in the range of 2.6–3.0 for cutting and preparation rooms; 2.3–2.6 for meat, deli, dairy, and produce; 1.2–1.5 for frozen foods; and 1.0–1.2 for ice cream units. Note that the COP of freezers is about half of the COP of meat refrigerators, and thus it costs twice as much to cool the meat products with refrigerated air that is cold enough to cool frozen foods. It is good energy conservation practice to use separate refrigeration systems to meet different refrigeration needs.

EXAMPLE 7–3 Analysis of a Household Refrigerator

A household refrigerator with a COP of 1.2 removes heat from the refrigerated space at a rate of 60 kJ/min (Fig. 7–23). Determine (*a*) the electric power consumed by the refrigerator and (*b*) the rate of heat transfer to the kitchen air.

SOLUTION The COP and the refrigeration rate of a refrigerator are given. The power consumption and the rate of heat rejection are to be determined.

Assumptions The refrigerator operates steadily.

Analysis (*a*) Using the definition of the coefficient of performance, the power input to the refrigerator is determined to be

$$\dot{W}_{\text{net,in}} = \frac{\dot{Q}_L}{\text{COP}_\text{R}} = \frac{60\text{ kJ/min}}{1.2} = 50\text{ kJ/min} = \mathbf{0.833\ kW}$$

(*b*) The heat transfer rate to the kitchen air is determined from the energy balance,

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{\text{net,in}} = 60 + 50 = \mathbf{110\ kJ/min}$$

Discussion Notice that both the energy removed from the refrigerated space as heat and the energy supplied to the refrigerator as electrical work eventually show up in the room air and become part of the internal energy of the air. This demonstrates that energy can change from one form to another and can move from one place to another, but it is never destroyed during a process.

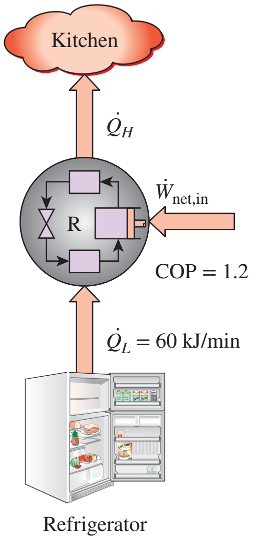


FIGURE 7–23
Schematic for Example 7–3.

EXAMPLE 7–4 Heating a House with a Heat Pump

A heat pump is used to meet the heating requirements of a house and maintain it at 20°C. On a day when the outdoor air temperature drops to –2°C, the house is estimated to lose heat at a rate of 80,000 kJ/h. If the heat pump under these conditions has a COP of 2.5, determine (*a*) the power consumed by the heat pump and (*b*) the rate at which heat is absorbed from the cold outdoor air.

SOLUTION The COP of a heat pump is given. The power consumption and the rate of heat absorption are to be determined.

Assumptions Steady operating conditions exist.

Analysis (*a*) The power consumed by this heat pump, shown in Fig. 7–24, is determined from the definition of the coefficient of performance to be

$$\dot{W}_{\text{net,in}} = \frac{\dot{Q}_H}{\text{COP}_\text{HP}} = \frac{80{,}000\text{ kJ/h}}{2.5} = \mathbf{32{,}000\ kJ/h}\ \text{(or 8.9 kW)}$$

(*b*) The house is losing heat at a rate of 80,000 kJ/h. If the house is to be maintained at a constant temperature of 20°C, the heat pump must deliver heat to the house at the same rate, that is, at a rate of 80,000 kJ/h. Then the rate of heat transfer from the outdoor becomes

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{\text{net,in}} = (80{,}000 - 32{,}000)\text{ kJ/h} = \mathbf{48{,}000\ kJ/h}$$

Discussion Note that 48,000 of the 80,000 kJ/h heat delivered to the house is actually extracted from the cold outdoor air. Therefore, we are paying only for the 32,000 kJ/h energy that is supplied as electrical work to the heat pump. If we were to use an electric resistance heater instead, we would have to supply the entire 80,000 kJ/h to the resistance heater as electric energy. This would mean a heating bill that is 2.5 times higher. This explains the popularity of heat pumps as heating systems and why they are preferred to simple electric resistance heaters despite their considerably higher initial cost.

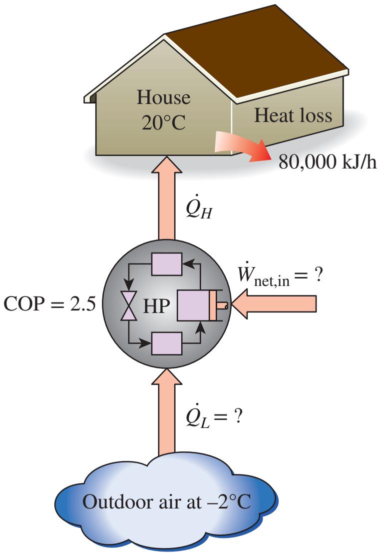


FIGURE 7–24
Schematic for Example 7–4.

The Second Law of Thermodynamics: Clausius Statement

There are two classic statements of the second law—the Kelvin–Planck statement, which is related to heat engines and discussed in the preceding section, and the Clausius statement, which is related to refrigerators or heat pumps. The Clausius statement is expressed as follows:

> It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower-temperature body to a higher-temperature body.

It is common knowledge that heat does not, of its own volition, transfer from a cold medium to a warmer one. The Clausius statement does not imply that a cyclic device that transfers heat from a cold medium to a warmer one is impossible to construct. In fact, this is precisely what a common household refrigerator does. It simply states that a refrigerator cannot operate unless its compressor is driven by an external power source, such as an electric motor (Fig. 7–25). This way, the net effect on the surroundings involves the consumption of some energy in the form of work, in addition to the transfer of heat from a colder body to a warmer one. That is, it leaves a trace in the surroundings. Therefore, a household refrigerator is in complete compliance with the Clausius statement of the second law.

Both the Kelvin–Planck and the Clausius statements of the second law are negative statements, and a negative statement cannot be proved. Like any other physical law, the second law of thermodynamics is based on experimental observations. To date, no experiment has been conducted that contradicts the second law, and this should be taken as sufficient proof of its validity.

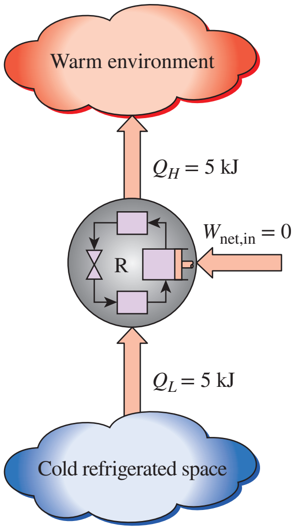
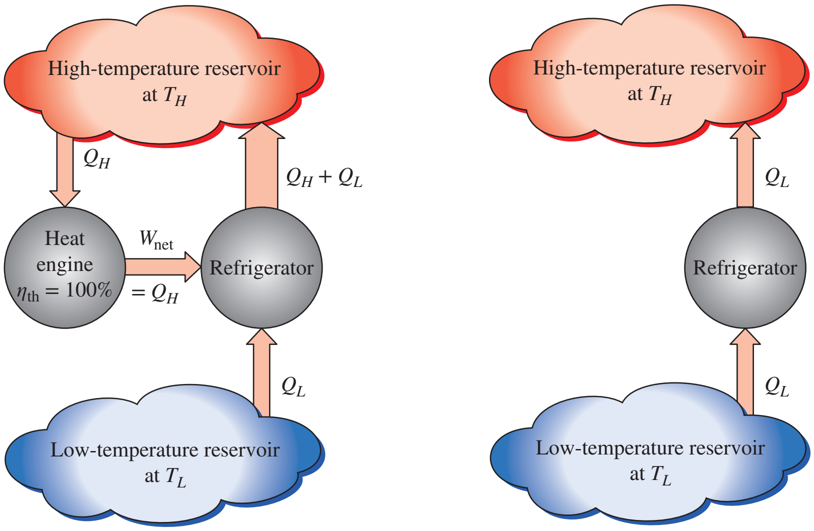


FIGURE 7–25
A refrigerator that violates the Clausius statement of the second law.

Equivalence of the Two Statements

The Kelvin–Planck and the Clausius statements are equivalent in their consequences, and either statement can be used as the expression of the second law of thermodynamics. Any device that violates the Kelvin–Planck statement also violates the Clausius statement, and vice versa. This can be demonstrated as follows.

Consider the heat engine–refrigerator combination shown in Fig. 7–26*a*, operating between the same two reservoirs. The heat engine is assumed to have, in violation of the Kelvin–Planck statement, a thermal efficiency of 100 percent, and therefore it converts all the heat Q_H it receives to work W. This work



(*a*) A refrigerator that is powered by a 100 percent efficient heat engine

(*b*) The equivalent refrigerator

FIGURE 7–26
Proof that the violation of the Kelvin–Planck statement leads to the violation of the Clausius statement.

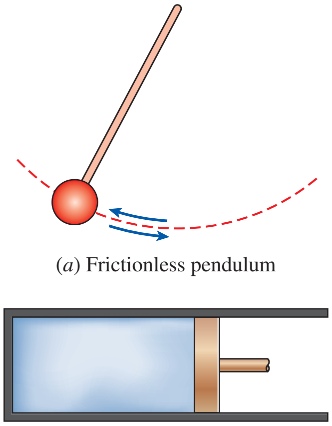
is now supplied to a refrigerator that removes heat in the amount of Q_L from the low-temperature reservoir and rejects heat in the amount of $Q_L + Q_H$ to the high-temperature reservoir. During this process, the high-temperature reservoir receives a net amount of heat Q_L (the difference between $Q_L + Q_H$ and Q_H). Thus, the combination of these two devices can be viewed as a refrigerator, as shown in Fig. 7–26*b*, that transfers heat in an amount of Q_L from a cooler body to a warmer one without requiring any input from outside. This is clearly a violation of the Clausius statement. Therefore, a violation of the Kelvin–Planck statement results in the violation of the Clausius statement.

It can also be shown in a similar manner that a violation of the Clausius statement leads to the violation of the Kelvin–Planck statement. Therefore, the Clausius and the Kelvin–Planck statements are two equivalent expressions of the second law of thermodynamics.

7–5 ▪ REVERSIBLE AND IRREVERSIBLE PROCESSES

The second law of thermodynamics states that no heat engine can have an efficiency of 100 percent. Then one may ask, what is the highest efficiency that a heat engine can possibly have? Before we can answer this question, we need to define an idealized process first, which is called the *reversible process.*

The processes that were discussed at the beginning of this chapter occurred in a certain direction. Once having taken place, these processes cannot reverse themselves spontaneously and restore the system to its initial state. For this reason, they are classified as *irreversible processes.* Once a cup of hot coffee cools, it will not heat up by retrieving the heat it lost from the surroundings. If it could, the surroundings, as well as the system (coffee), would be restored to their original condition, and this would be a reversible process.


(*a*) Frictionless pendulum

(*b*) Quasi-equilibrium expansion and compression of a gas

FIGURE 7–27
Two familiar reversible processes.

A **reversible process** is defined as a *process that can be reversed without leaving any trace on the surroundings* (Fig. 7–27). That is, both the system *and* the surroundings are returned to their initial states at the end of the reverse process. This is possible only if the net heat *and* net work exchange between the system and the surroundings is zero for the combined (original and reverse) process. Processes that are not reversible are called **irreversible processes**.

It should be pointed out that a system can be restored to its initial state following a process, regardless of whether the process is reversible or irreversible. But for reversible processes, this restoration is made without leaving any net change on the surroundings, whereas for irreversible processes, the surroundings usually do some work on the system and therefore do not return to their original state.

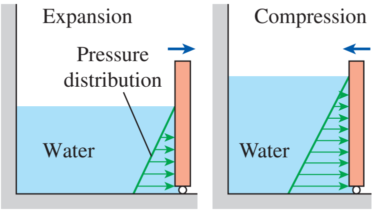
Reversible processes actually do not occur in nature. They are merely *idealizations* of actual processes. Reversible processes can be approximated by actual devices, but they can never be achieved. That is, all the processes occurring in nature are irreversible. You may be wondering, then, *why* we are bothering with such fictitious processes. There are two reasons. First, they are easy to analyze, since a system passes through a series of equilibrium states during a reversible process. Second, they serve as idealized models to which actual processes can be compared.

In daily life, the concepts of Mr. Right and Ms. Right are also idealizations, just like the concept of a reversible (perfect) process. People who insist on finding Mr. or Ms. Right to settle down with are bound to remain Mr. or Ms. Single for the rest of their lives. The possibility of finding the perfect prospective mate is no higher than the possibility of finding a perfect (reversible) process. Likewise, a person who insists on perfection in friends is bound to have no friends.

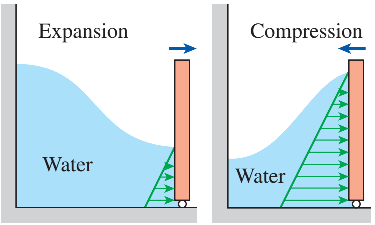
Engineers are interested in reversible processes because work-producing devices such as car engines and gas or steam turbines *deliver the most work,* and work-consuming devices such as compressors, fans, and pumps *consume the least work* when reversible processes are used instead of irreversible ones (Fig. 7–28).

Reversible processes can be viewed as *theoretical limits* for the corresponding irreversible ones. Some processes are more irreversible than others. We may never be able to have a reversible process, but we can certainly approach it. The more closely we approximate a reversible process, the more work delivered by a work-producing device or the less work required by a work-consuming device.

The concept of reversible processes leads to the definition of the **second-law efficiency** for actual processes, which is the degree of approximation to the corresponding reversible processes. This enables us to compare the performance of different devices that are designed to do the same task on the basis of their efficiencies. The better the design, the lower the irreversibilities and the higher the second-law efficiency.



(*a*) Slow (reversible) process



(*b*) Fast (irreversible) process

FIGURE 7–28
Reversible processes deliver the most and consume the least work.

Irreversibilities

The factors that cause a process to be irreversible are called **irreversibilities**. They include friction, unrestrained expansion, mixing of two fluids, heat transfer across a finite temperature difference, electric resistance, inelastic deformation of solids, and chemical reactions. The presence of any of these effects renders a process irreversible. A reversible process involves none of these. Some of the frequently encountered irreversibilities are discussed briefly below.

Friction is a familiar form of irreversibility associated with bodies in motion. When two bodies in contact are forced to move relative to each other (a piston in a cylinder, for example, as shown in Fig. 7–29), a friction force that opposes the motion develops at the interface of these two bodies, and some work is needed to overcome this friction force. The energy supplied as work is eventually converted to heat during the process and is transferred to the bodies in contact, as evidenced by a temperature rise at the interface. When the direction of the motion is reversed, the bodies are restored to their original position, but the interface does not cool, and heat is not converted back to work. Instead, more of the work is converted to heat while overcoming the friction forces that also oppose the reverse motion. Since the system (the moving bodies) and the surroundings cannot be returned to their original states, this process is irreversible. Therefore, any process that involves friction is irreversible. The larger the friction forces involved, the more irreversible the process is.

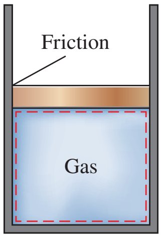
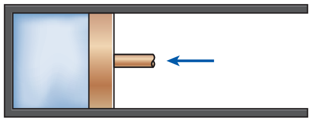


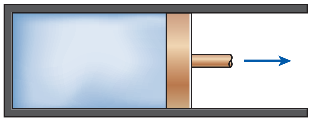
FIGURE 7–29
Friction renders a process irreversible.

Friction does not always involve two solid bodies in contact. It is also encountered between a fluid and a solid and even between the layers of a fluid moving at different velocities. A considerable fraction of the power produced by a car engine is used to overcome the friction (the drag force) between the air and the external surfaces of the car, and it eventually becomes part of the internal energy of the air. It is not possible to reverse this process and recover that lost power, even though doing so would not violate the conservation of energy principle.

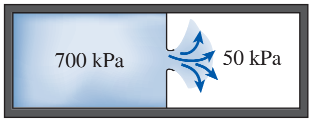
Another example of irreversibility is the **unrestrained expansion of a gas** separated from a vacuum by a membrane, as shown in Fig. 7–30. When the membrane is ruptured, the gas fills the entire tank. The only way to restore the system to its original state is to compress it to its initial volume, while transferring heat from the gas until it reaches its initial temperature. From the conservation of energy considerations, it can easily be shown that the amount of heat transferred from the gas equals the amount of work done on the gas by the surroundings. The restoration of the surroundings involves conversion of this heat completely to work, which would violate the second law. Therefore, unrestrained expansion of a gas is an irreversible process.



(*a*) Fast compression



(*b*) Fast expansion



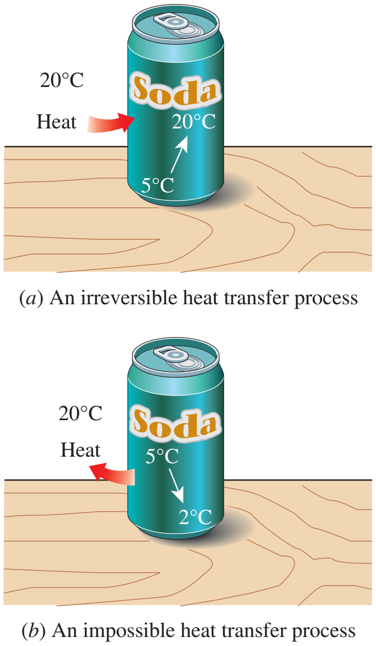
(*c*) Unrestrained expansion

FIGURE 7–30
Irreversible compression and expansion processes.

A third form of irreversibility familiar to us all is **heat transfer** through a finite temperature difference. Consider a can of cold soda left in a warm room (Fig. 7–31). Heat is transferred from the warmer room air to the cooler soda. The only way this process can be reversed and the soda restored to its original temperature is to provide refrigeration, which requires some work input. At the end of the reverse process, the soda will be restored to its initial state, but the surroundings will not be. The internal energy of the surroundings will increase by an amount equal in magnitude to the work supplied to the refrigerator. The restoration of the surroundings to the initial state can be done only by converting this excess internal energy completely to work, which is impossible to do without violating the second law. Since only the system, not both the system and the surroundings, can be restored to its initial condition, heat transfer through a finite temperature difference is an irreversible process.

Heat transfer can occur only when there is a temperature difference between a system and its surroundings. Therefore, it is physically impossible to have a reversible heat transfer process. But a heat transfer process becomes less and less irreversible as the temperature difference between the two bodies approaches zero. Then, heat transfer through a differential temperature difference *dT* can be considered to be reversible. As *dT* approaches zero, the process can be reversed in direction (at least theoretically) without requiring any refrigeration. Notice that reversible heat transfer is a conceptual process and cannot be duplicated in the real world.

The smaller the temperature difference between two bodies, the smaller the heat transfer rate will be. Any significant heat transfer through a small temperature difference requires a very large surface area and a very long time. Therefore, even though approaching reversible heat transfer is desirable from a thermodynamic point of view, it is impractical and not economically feasible.



(*a*) An irreversible heat transfer process

(*b*) An impossible heat transfer process

FIGURE 7–31
(*a*) Heat transfer through a temperature difference is irreversible, and (*b*) the reverse process is impossible.

Internally and Externally Reversible Processes

A typical process involves interactions between a system and its surroundings, and a reversible process involves no irreversibilities associated with either of them.

A process is called **internally reversible** if no irreversibilities occur within the boundaries of the system during the process. During an internally reversible process, a system proceeds through a series of equilibrium states, and when the process is reversed, the system passes through exactly the same equilibrium states while returning to its initial state. That is, the paths of the forward and reverse processes coincide for an internally reversible process. The quasi-equilibrium process is an example of an internally reversible process.

A process is called **externally reversible** if no irreversibilities occur outside the system boundaries during the process. Heat transfer between a reservoir and a system is an externally reversible process if the outer surface of the system is at the temperature of the reservoir.

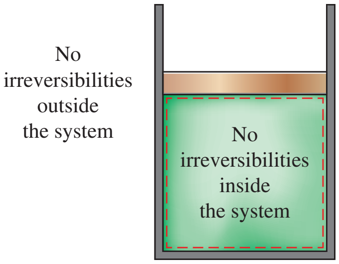


FIGURE 7–32
A reversible process involves no internal and external irreversibilities.

A process is called **totally reversible**, or simply **reversible**, if it involves no irreversibilities within the system or its surroundings (Fig. 7–32). A totally reversible process involves no heat transfer through a finite temperature difference, no nonquasi-equilibrium changes, and no friction or other dissipative effects.

As an example, consider the transfer of heat to two identical systems that are undergoing a constant-pressure (thus constant-temperature) phase-change process, as shown in Fig. 7–33. Both processes are internally reversible, since both take place isothermally and both pass through exactly the same equilibrium states. The first process shown is externally reversible also, since heat transfer for this process takes place through an infinitesimal temperature difference *dT*. The second process, however, is externally irreversible, since it involves heat transfer through a finite temperature difference ΔT.

7–6 ▪ THE CARNOT CYCLE

We mentioned earlier that heat engines are cyclic devices and that the working fluid of a heat engine returns to its initial state at the end of each cycle. Work is done by the working fluid during one part of the cycle and on the working fluid during another part. The difference between these two is the net work delivered by the heat engine. The efficiency of a heat-engine cycle greatly depends on how the individual processes that make up the cycle are executed. The net work, thus the cycle efficiency, can be maximized by using processes that require the least amount of work and deliver the most, that is, by using *reversible processes.* Therefore, it is no surprise that the most efficient cycles are reversible cycles, that is, cycles that consist entirely of reversible processes.

Reversible cycles cannot be achieved in practice because the irreversibilities associated with each process cannot be eliminated. However, reversible cycles provide upper limits on the performance of real cycles. Heat engines and refrigerators that work on reversible cycles serve as models to which actual heat engines and refrigerators can be compared. Reversible cycles also serve as starting points in the development of actual cycles and are modified as needed to meet certain requirements.

Probably the best known reversible cycle is the **Carnot cycle**, first proposed in 1824 by French engineer Sadi Carnot. The theoretical heat engine that operates on the Carnot cycle is called the **Carnot heat engine**. The Carnot cycle is composed of four reversible processes—two isothermal and two adiabatic—and it can be executed either in a closed or a steady-flow system.

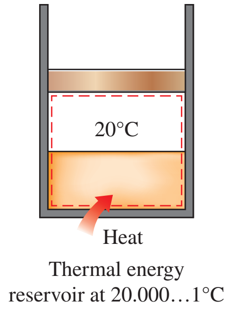
Consider a closed system that consists of a gas contained in an adiabatic piston–cylinder device, as shown in Fig. 7–34. The insulation of the cylinder head is such that it may be removed to bring the cylinder into contact with reservoirs to provide heat transfer. The four reversible processes that make up the Carnot cycle are as follows:

Reversible Isothermal Expansion (process 1-2, T_H = constant). Initially (state 1), the temperature of the gas is T_H and the cylinder head is in close contact with a source at temperature T_H. The gas is allowed to expand slowly, doing work on the surroundings. As the gas expands, the temperature of the gas tends to decrease. But as soon as the temperature drops by an infinitesimal amount *dT,* some heat is transferred from the reservoir into the gas, raising the gas temperature to T_H. Thus, the gas temperature is kept constant at T_H. Since the temperature difference between the gas and the reservoir never exceeds a differential amount *dT,* this is a reversible heat transfer process. It continues until the piston reaches position 2. The amount of total heat transferred to the gas during this process is Q_H.

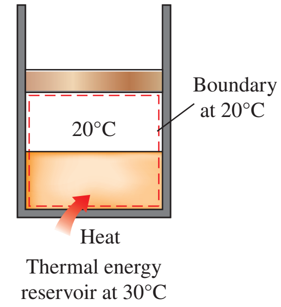
Reversible Adiabatic Expansion (process 2-3, temperature drops from T_H to T_L). At state 2, the reservoir that was in contact with the cylinder head is removed and replaced by insulation so that the system becomes adiabatic. The gas continues to expand slowly, doing work on the surroundings until its temperature drops from T_H to T_L (state 3). The piston is assumed to be frictionless and the process to be quasi-equilibrium, so the process is reversible as well as adiabatic.

Reversible Isothermal Compression (process 3-4, T_L = constant). At state 3, the insulation at the cylinder head is removed, and the cylinder is brought into contact with a sink at temperature T_L. Now the piston is pushed inward by an external force, doing work on the gas. As the gas is compressed, its temperature tends to rise. But as soon as it rises by an infinitesimal amount *dT,* heat is transferred from the gas to the sink, causing the gas temperature to drop to T_L. Thus, the gas temperature remains constant at T_L. Since the temperature difference between the gas and the sink never exceeds a differential amount *dT,* this is a reversible heat transfer process. It continues until the piston reaches state 4. The amount of heat rejected from the gas during this process is Q_L.

Reversible Adiabatic Compression (process 4-1, temperature rises from T_L to T_H). State 4 is such that when the low-temperature reservoir is removed, the

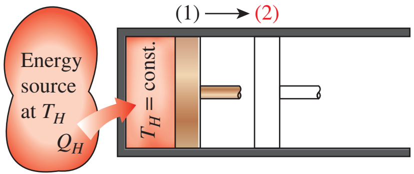


(*a*) Totally reversible

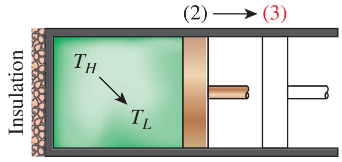


(*b*) Internally reversible

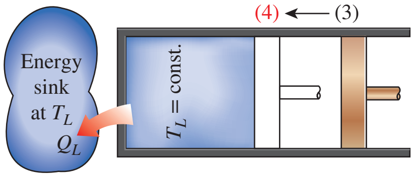
FIGURE 7–33
Totally and internally reversible heat transfer processes.



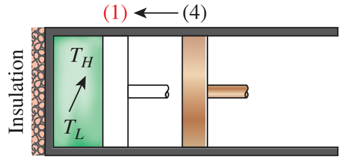
(*a*) Process 1-2



(*b*) Process 2-3



(*c*) Process 3-4



(*d*) Process 4-1

FIGURE 7–34
Execution of the Carnot cycle in a closed system.

insulation is put back on the cylinder head, and the gas is compressed in a reversible manner, so the gas returns to its initial state (state 1). The temperature rises from T_L to T_H during this reversible adiabatic compression process, which completes the cycle.

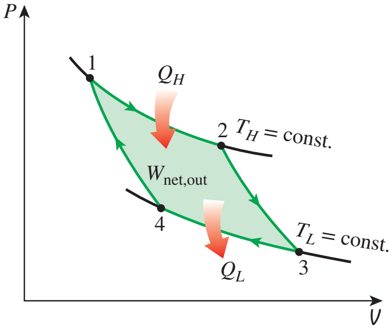


FIGURE 7–35
P-V diagram of the Carnot cycle.

The *P-V* diagram of this cycle is shown in Fig. 7–35. Remembering that on a *P-V* diagram the area under the process curve represents the boundary work for quasi-equilibrium (internally reversible) processes, we see that the area under curve 1-2-3 is the work done by the gas during the expansion part of the cycle, and the area under curve 3-4-1 is the work done on the gas during the compression part of the cycle. The area enclosed by the path of the cycle (area 1-2-3-4-1) is the difference between these two and represents the net work done during the cycle.

Notice that if we acted stingily and compressed the gas at state 3 adiabatically instead of isothermally in an effort to save Q_L, we would end up back at state 2, retracing the process path 3-2. By doing so we would save Q_L, but we would not be able to obtain any net work output from this engine. This illustrates once more the necessity of a heat engine exchanging heat with at least two reservoirs at different temperatures to operate in a cycle and produce a net amount of work.

The Carnot cycle can also be executed in a steady-flow system. It is discussed in later chapters in conjunction with other power cycles.

Being a reversible cycle, the Carnot cycle is the most efficient cycle operating between two specified temperature limits. Even though the Carnot cycle cannot be achieved in reality, the efficiency of actual cycles can be improved by attempting to approximate the Carnot cycle more closely.

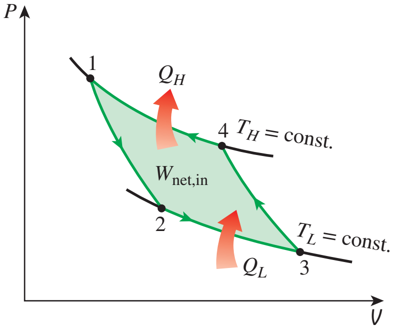


FIGURE 7–36
P-V diagram of the reversed Carnot cycle.

The Reversed Carnot Cycle

The Carnot heat-engine cycle just described is a totally reversible cycle. Therefore, all the processes that comprise it can be *reversed,* in which case it becomes the **Carnot refrigeration cycle**. This time, the cycle remains exactly the same, except that the directions of any heat and work interactions are reversed: Heat in the amount of Q_L is absorbed from the low-temperature reservoir, heat in the amount of Q_H is rejected to a high-temperature reservoir, and a work input of $W_{\text{net,in}}$ is required to accomplish all this.

The *P-V* diagram of the reversed Carnot cycle is the same as the one given for the Carnot cycle, except that the directions of the processes are reversed, as shown in Fig. 7–36.

7–7 ▪ THE CARNOT PRINCIPLES

The second law of thermodynamics puts limits on the operation of cyclic devices as expressed by the Kelvin–Planck and Clausius statements. A heat engine cannot operate by exchanging heat with a single reservoir, and a refrigerator cannot operate without a net energy input from an external source.

We can draw valuable conclusions from these statements. Two conclusions pertain to the thermal efficiency of reversible and irreversible (i.e., actual) heat engines, and they are known as the **Carnot principles** (Fig. 7–37), expressed as follows:

1. The efficiency of an irreversible heat engine is always less than the efficiency of a reversible one operating between the same two reservoirs.
2. The efficiencies of all reversible heat engines operating between the same two reservoirs are the same.

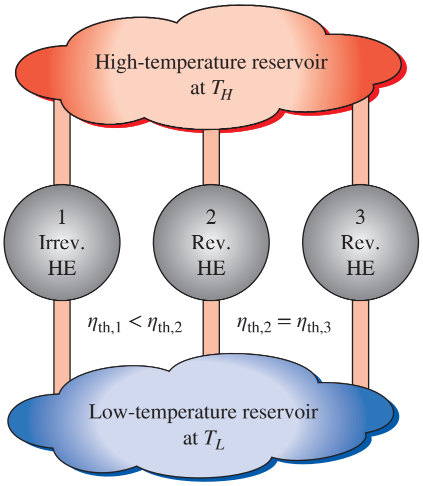
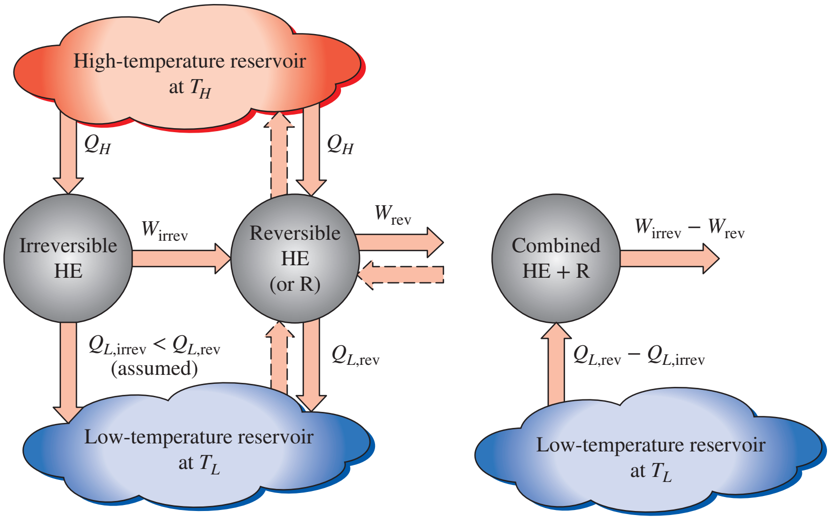


FIGURE 7–37
The Carnot principles.

These two statements can be proved by demonstrating that the violation of either statement results in the violation of the second law of thermodynamics.

To prove the first statement, consider two heat engines operating between the same reservoirs, as shown in Fig. 7–38. One engine is reversible and the other is



(*a*) A reversible and an irreversible heat engine operating between the same two reservoirs (the reversible heat engine is then reversed to run as a refrigerator)

(*b*) The equivalent combined system

FIGURE 7–38
Proof of the first Carnot principle.

irreversible. Now each engine is supplied with the same amount of heat Q_H. The amount of work produced by the reversible heat engine is W_{rev}, and the amount produced by the irreversible one is W_{irrev}.

In violation of the first Carnot principle, we assume that the irreversible heat engine is more efficient than the reversible one (that is, $\eta_{\text{th,irrev}} > \eta_{\text{th,rev}}$) and thus delivers more work than the reversible one. Now let the reversible heat engine be reversed and operate as a refrigerator. This refrigerator will receive a work input of W_{rev} and reject heat to the high-temperature reservoir. Since the refrigerator is rejecting heat in the amount of Q_H to the high-temperature reservoir and the irreversible heat engine is receiving the same amount of heat from this reservoir, the net heat exchange for this reservoir is zero. Thus, it could be eliminated by having the refrigerator discharge Q_H directly into the irreversible heat engine.

Now considering the refrigerator and the irreversible engine together, we have an engine that produces a net work in the amount of $W_{\text{irrev}} - W_{\text{rev}}$ while exchanging heat with a single reservoir—a violation of the Kelvin–Planck statement of the second law. Therefore, our initial assumption that $\eta_{\text{th,irrev}} > \eta_{\text{th,rev}}$ is incorrect. Then we conclude that no heat engine can be more efficient than a reversible heat engine operating between the same reservoirs.

The second Carnot principle can also be proved in a similar manner. This time, let us replace the irreversible engine with another reversible engine that is more efficient and thus delivers more work than the first reversible engine. By following through the same reasoning, we end up having an engine that produces a net amount of work while exchanging heat with a single reservoir, which is a violation of the second law. Therefore, we conclude that no reversible heat engine can be more efficient than a reversible one operating between the same two reservoirs, regardless of how the cycle is completed or the kind of working fluid used.

7–8 ▪ THE THERMODYNAMIC TEMPERATURE SCALE

A temperature scale that is independent of the properties of the substances that are used to measure temperature is called a **thermodynamic temperature scale**. Such a temperature scale offers great conveniences in thermodynamic calculations, and its derivation is given below using some reversible heat engines.

The second Carnot principle discussed in Sec. 7–7 states that all reversible heat engines have the same thermal efficiency when operating between the same two reservoirs (Fig. 7–39). That is, the efficiency of a reversible engine is independent of the working fluid employed and its properties, the way the cycle is executed, or the type of reversible engine used. Since energy reservoirs are characterized by their temperatures, the thermal efficiency of reversible heat engines is a function of the reservoir temperatures only. That is,

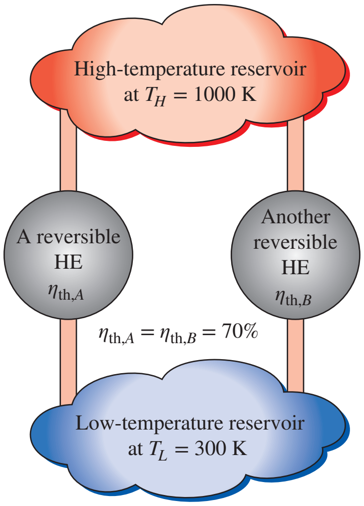


FIGURE 7–39
All reversible heat engines operating between the same two reservoirs have the same efficiency (the second Carnot principle).

$$\eta_{\text{th,rev}} = g(T_H, T_L)$$

or

$$\frac{Q_H}{Q_L} = f(T_H, T_L) \tag{7–13}$$

since $\eta_{\text{th}} = 1 - Q_L/Q_H$. In these relations T_H and T_L are the temperatures of the high- and low-temperature reservoirs, respectively.

The functional form of $f(T_H, T_L)$ can be developed with the help of the three reversible heat engines shown in Fig. 7–40. Engines A and C are supplied with the same amount of heat Q_1 from the high-temperature reservoir at T_1. Engine C rejects Q_3 to the low-temperature reservoir at T_3. Engine B receives the heat Q_2 rejected by engine A at temperature T_2 and rejects heat in the amount of Q_3 to a reservoir at T_3.

The amounts of heat rejected by engines B and C must be the same since engines A and B can be combined into one reversible engine operating between the same reservoirs as engine C and thus the combined engine will have the same efficiency as engine C. Since the heat input to engine C is the same as the heat input to the combined engines A and B, both systems must reject the same amount of heat.

Applying Eq. 7–13 to all three engines separately, we obtain

$$\frac{Q_1}{Q_2} = f(T_1, T_2), \quad \frac{Q_2}{Q_3} = f(T_2, T_3), \quad \text{and} \quad \frac{Q_1}{Q_3} = f(T_1, T_3)$$

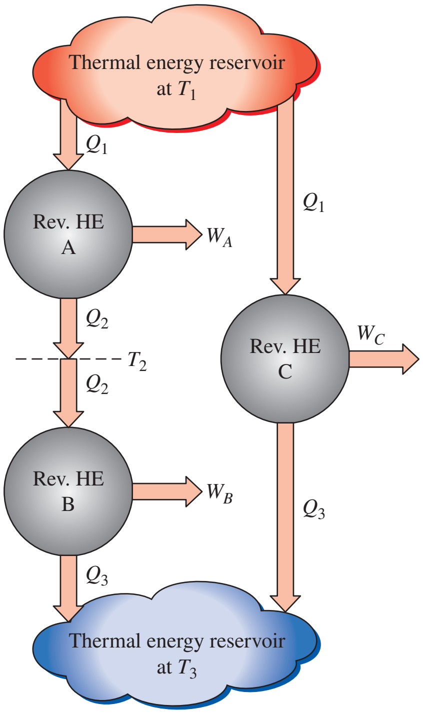


FIGURE 7–40
The arrangement of heat engines used to develop the thermodynamic temperature scale.

Now consider the identity

$$\frac{Q_1}{Q_3} = \frac{Q_1}{Q_2}\frac{Q_2}{Q_3}$$

which corresponds to

$$f(T_1, T_3) = f(T_1, T_2) \cdot f(T_2, T_3)$$

A careful examination of this equation reveals that the left-hand side is a function of T_1 and T_3, and therefore the right-hand side must also be a function of T_1 and T_3 only, and not T_2. That is, the value of the product on the right-hand side of this equation is independent of the value of T_2. This condition will be satisfied only if the function f has the following form:

$$f(T_1, T_2) = \frac{\phi(T_1)}{\phi(T_2)} \quad \text{and} \quad f(T_2, T_3) = \frac{\phi(T_2)}{\phi(T_3)}$$

so that $\phi(T_2)$ will cancel from the product of $f(T_1, T_2)$ and $f(T_2, T_3)$, yielding

$$\frac{Q_1}{Q_2} = f(T_1, T_3) = \frac{\phi(T_1)}{\phi(T_3)} \tag{7–14}$$

This relation is much more specific than Eq. 7–13 for the functional form of Q_1/Q_3 in terms of T_1 and T_3.

For a reversible heat engine operating between two reservoirs at temperatures T_H and T_L, Eq. 7–14 can be written as

$$\frac{Q_H}{Q_L} = \frac{\phi(T_H)}{\phi(T_L)} \tag{7–15}$$

This is the only requirement that the second law places on the ratio of heat transfers to and from the reversible heat engines. Several functions $\phi(T)$ satisfy this equation, and the choice is completely arbitrary. Lord Kelvin first proposed taking $\phi(T) = T$ to define a thermodynamic temperature scale as (Fig. 7–41)

$$\left(\frac{Q_H}{Q_L}\right)_{\text{rev}} = \frac{T_H}{T_L} \tag{7–16}$$

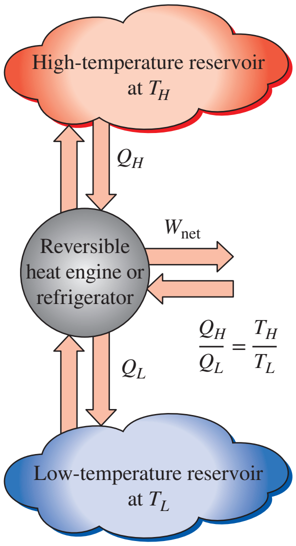


FIGURE 7–41
For reversible cycles, the heat transfer ratio Q_H/Q_L can be replaced by the absolute temperature ratio T_H/T_L.

This temperature scale is called the **Kelvin scale**, and the temperatures on this scale are called **absolute temperatures**. On the Kelvin scale, the temperature ratios depend on the ratios of heat transfer between a reversible heat engine and the reservoirs and are independent of the physical properties of any substance. On this scale, temperatures vary between zero and infinity.

The thermodynamic temperature scale is not completely defined by Eq. 7–16 since it gives us only a ratio of absolute temperatures. We also need to know the magnitude of a kelvin. At the International Conference on Weights and Measures held in 1954, the triple point of water (the state at which all three phases of water exist in equilibrium) was assigned the value 273.16 K (Fig. 7–42). The *magnitude of a kelvin* is defined as 1/273.16 of the temperature interval between absolute zero and the triple-point temperature of water. The magnitudes of temperature units on the Kelvin and Celsius scales are identical (1 K ≡ 1°C). The temperatures on these two scales differ by a constant 273.15:

$$T(^\circ\text{C}) = T(\text{K}) - 273.15 \tag{7–17}$$

Even though the thermodynamic temperature scale is defined with the help of the reversible heat engines, it is not possible, nor is it practical, to actually operate such an engine to determine numerical values on the absolute temperature scale. Absolute temperatures can be measured accurately by other means, such as the constant-volume ideal-gas thermometer together with extrapolation techniques as discussed in Chap. 2. The validity of Eq. 7–16 can be demonstrated from physical considerations for a reversible cycle using an ideal gas as the working fluid.

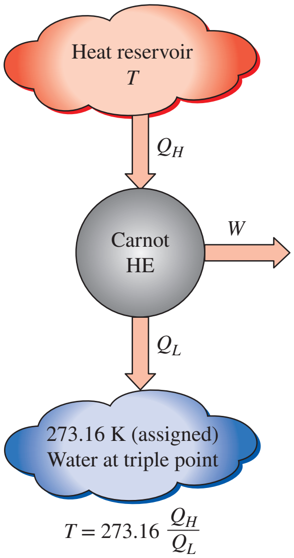


FIGURE 7–42
A conceptual experimental setup to determine thermodynamic temperatures on the Kelvin scale by measuring heat transfers Q_H and Q_L.

7–9 ▪ THE CARNOT HEAT ENGINE

The hypothetical heat engine that operates on the reversible Carnot cycle is called the **Carnot heat engine**. The thermal efficiency of any heat engine, reversible or irreversible, is given by Eq. 7–6 as

$$\eta_{\text{th}} = 1 - \frac{Q_L}{Q_H}$$

where Q_H is heat transferred to the heat engine from a high-temperature reservoir at T_H, and Q_L is heat rejected to a low-temperature reservoir at T_L. For reversible heat engines, the heat transfer ratio in the preceding relation can be replaced by the ratio of the absolute temperatures of the two reservoirs, as given by Eq. 7–16. Then the efficiency of a Carnot engine, or any reversible heat engine, becomes

$$\eta_{\text{th,rev}} = 1 - \frac{T_L}{T_H} \tag{7–18}$$

This relation is often referred to as the **Carnot efficiency**, since the Carnot heat engine is the best known reversible engine. *This is the highest efficiency a heat engine operating between the two thermal energy reservoirs at temperatures T_L and T_H can have* (Fig. 7–43). All irreversible (i.e., actual) heat engines operating between these temperature limits (T_L and T_H) have lower efficiencies. An actual heat engine cannot reach this maximum theoretical efficiency value because it is impossible to completely eliminate all the irreversibilities associated with the actual cycle.

Note that T_L and T_H in Eq. 7–18 are *absolute temperatures.* Using °C or °F for temperatures in this relation gives results grossly in error.

The thermal efficiencies of actual and reversible heat engines operating between the same temperature limits compare as follows (Fig. 7–44):

$$\eta_{\text{th}} \begin{cases} < \ \eta_{\text{th,rev}} & \text{irreversible heat engine} \\ = \ \eta_{\text{th,rev}} & \text{reversible heat engine} \\ > \ \eta_{\text{th,rev}} & \text{impossible heat engine} \end{cases} \tag{7–19}$$

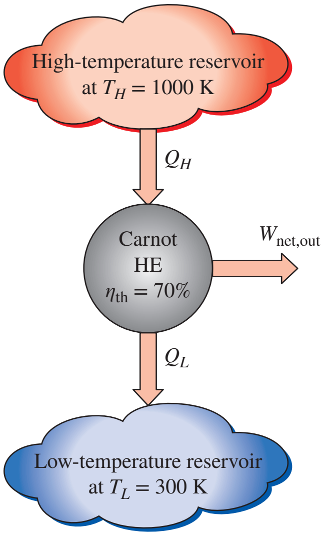


FIGURE 7–43
The Carnot heat engine is the most efficient of all heat engines operating between the same high- and low-temperature reservoirs.

Most work-producing devices (heat engines) in operation today have efficiencies under 40 percent, which appear low relative to 100 percent. However, when the performance of actual heat engines is assessed, the efficiencies should not be compared to 100 percent; instead, they should be compared to the efficiency of a reversible heat engine operating between the same temperature limits—because this is the true theoretical upper limit for the efficiency, not 100 percent.

The maximum efficiency of a steam power plant operating between T_H = 1000 K and T_L = 300 K is 70 percent, as determined from Eq. 7–18. Compared with this value, an actual efficiency of 40 percent does not seem so bad, even though there is still plenty of room for improvement.

It is obvious from Eq. 7–18 that the efficiency of a Carnot heat engine increases as T_H is increased, or as T_L is decreased. This is to be expected since as T_L decreases, so does the amount of heat rejected, and as T_L approaches zero, the Carnot efficiency approaches unity. This is also true for actual heat engines. *The thermal efficiency of actual heat engines can be maximized by supplying heat to the engine at the highest possible temperature* (limited by material strength) *and rejecting heat from the engine at the lowest possible temperature* (limited by the temperature of the cooling medium such as rivers, lakes, or the atmosphere).

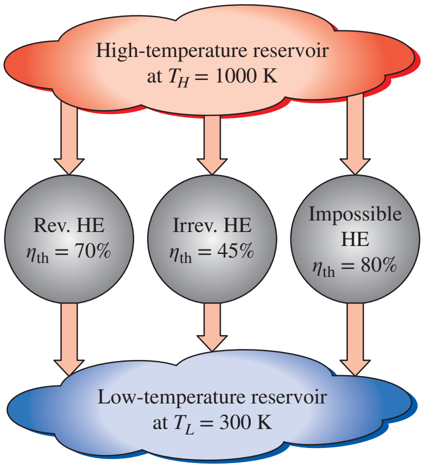


FIGURE 7–44
No heat engine can have a higher efficiency than a reversible heat engine operating between the same high- and low-temperature reservoirs.

EXAMPLE 7–5 Analysis of a Carnot Heat Engine

A Carnot heat engine, shown in Fig. 7–45, receives 500 kJ of heat per cycle from a high-temperature source at 652°C and rejects heat to a low-temperature sink at 30°C. Determine (*a*) the thermal efficiency of this Carnot engine and (*b*) the amount of heat rejected to the sink per cycle.

SOLUTION The heat supplied to a Carnot heat engine is given. The thermal efficiency and the heat rejected are to be determined.

Analysis (*a*) The Carnot heat engine is a reversible heat engine, and so its efficiency can be determined from Eq. 7–18 to be

$$\eta_{\text{th,rev}} = 1 - \frac{T_L}{T_H} = 1 - \frac{(30 + 273)\ \text{K}}{(652 + 273)\ \text{K}} = \mathbf{0.672}$$

That is, this Carnot heat engine converts 67.2 percent of the heat it receives to work.

(*b*) The amount of heat rejected Q_L by this reversible heat engine is easily determined from Eq. 7–16 to be

$$Q_{L,\text{rev}} = \frac{T_L}{T_H} Q_{H,\text{rev}} = \frac{(30 + 273)\text{ K}}{(652 + 273)\text{ K}}(500\text{ kJ}) = \mathbf{164\ kJ}$$

Discussion Note that this Carnot heat engine rejects to a low-temperature sink 164 kJ of the 500 kJ of heat it receives during each cycle.

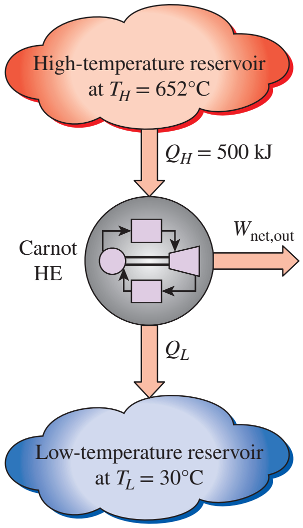


FIGURE 7–45
Schematic for Example 7–5.

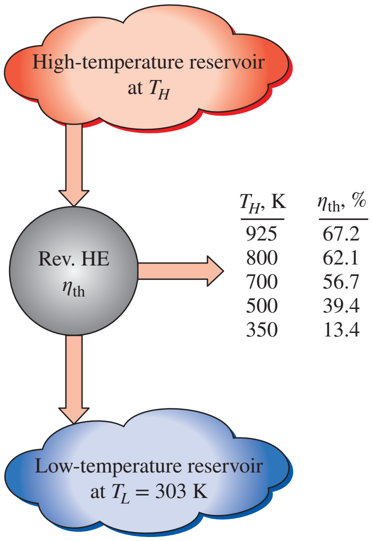
The Quality of Energy

The Carnot heat engine in Example 7–5 receives heat from a source at 925 K and converts 67.2 percent of it to work while rejecting the rest (32.8 percent) to a sink at 303 K. Now let us examine how the thermal efficiency varies with the source temperature when the sink temperature is held constant.

The thermal efficiency of a Carnot heat engine that rejects heat to a sink at 303 K is evaluated at various source temperatures using Eq. 7–18 and is listed in Fig. 7–46. Clearly, the thermal efficiency decreases as the source temperature is lowered. When heat is supplied to the heat engine at 500 instead of 925 K, for example, the thermal efficiency drops from 67.2 to 39.4 percent. That is, the fraction of heat that can be converted to work drops to 39.4 percent when the temperature of the source drops to 500 K. When the source temperature is 350 K, this fraction becomes a mere 13.4 percent.

These efficiency values show that energy has **quality** as well as quantity. It is clear from the thermal efficiency values in Fig. 7–46 that *more of the high-temperature thermal energy can be converted to work. Therefore, the higher the temperature, the higher the quality of the energy* (Fig. 7–47).

Large quantities of solar energy, for example, can be stored in large bodies of water called *solar ponds* at about 350 K. This stored energy can then be supplied to a heat engine to produce work (electricity). However, the efficiency of solar pond power plants is very low (under 5 percent) because of the low quality of the energy stored in the source, and the construction and maintenance costs are relatively high. Therefore, they are not competitive even though the energy supply of such plants is free. The temperature (and thus the quality) of the solar energy stored could be raised by utilizing concentrating collectors, but the equipment cost in that case becomes very high.



T_H, K	η_{th}, %
925	67.2
800	62.1
700	56.7
500	39.4
350	13.4

FIGURE 7–46
The fraction of heat that can be converted to work of source temperature (for T_L = 303 K).

Work is a more valuable form of energy than heat since 100 percent of work can be converted to heat, but only a fraction of heat can be converted to work. When heat is transferred from a high-temperature body to a lower-temperature one, it is degraded since less of it now can be converted to work. For example, if 100 kJ of heat is transferred from a body at 1000 K to a body at 300 K, at the end we will have 100 kJ of thermal energy stored at 300 K, which has no practical value. But if this conversion is made through a heat engine, up to 1 – 300/1000 = 70 percent of it could be converted to work, which is a more valuable form of energy. Thus 70 kJ of work potential is wasted as a result of this heat transfer, and energy is degraded.

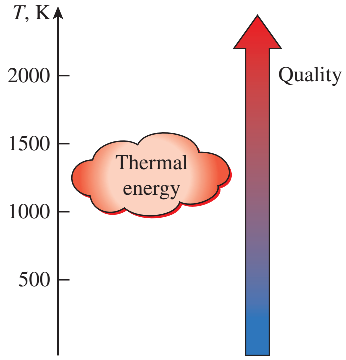


FIGURE 7–47
The higher the temperature of the thermal energy, the higher its quality.

7–10 ▪ THE CARNOT REFRIGERATOR AND HEAT PUMP

A refrigerator or a heat pump that operates on the reversed Carnot cycle is called a **Carnot refrigerator**, or a **Carnot heat pump**. The coefficient of performance of any refrigerator or heat pump, reversible or irreversible, is given by Eqs. 7–9 and 7–11 as

$$\text{COP}_\text{R} = \frac{1}{Q_H/Q_L - 1} \quad \text{and} \quad \text{COP}_\text{HP} = \frac{1}{1 - Q_L/Q_H}$$

where Q_L is the amount of heat absorbed from the low-temperature medium and Q_H is the amount of heat rejected to the high-temperature medium. The COPs of all reversible refrigerators or heat pumps can be determined by replacing the heat transfer ratios in the preceding relations with the ratios of the absolute temperatures of the high- and low-temperature reservoirs, as expressed by Eq. 7–16. Then the COP relations for reversible refrigerators and heat pumps become

$$\text{COP}_{\text{R,rev}} = \frac{1}{T_H/T_L - 1} \tag{7–20}$$

and

$$\text{COP}_{\text{HP,rev}} = \frac{1}{1 - T_L/T_H} \tag{7–21}$$

These are the highest coefficients of performance that a refrigerator or a heat pump operating between the temperature limits of T_L and T_H can have. All actual refrigerators or heat pumps operating between these temperature limits (T_L and T_H) have lower coefficients of performance (Fig. 7–48).

The coefficients of performance of actual and reversible refrigerators operating between the same temperature limits can be compared as follows:

$$\text{COP}_{\text{R}} \begin{cases} < \text{COP}_{\text{R,rev}} & \text{irreversible refrigerator} \\ = \text{COP}_{\text{R,rev}} & \text{reversible refrigerator} \\ > \text{COP}_{\text{R,rev}} & \text{impossible refrigerator} \end{cases} \tag{7–22}$$

A similar relation can be obtained for heat pumps by replacing all COP_{R}'s in Eq. 7–22 with COP_{HP}.

The COP of a reversible refrigerator or heat pump is the maximum theoretical value for the specified temperature limits. Actual refrigerators or heat pumps may approach these values as their designs are improved, but they can never reach them.

As a final note, the COPs of both the refrigerators and the heat pumps decrease as T_L decreases. That is, it requires more work to absorb heat from lower-temperature media. As the temperature of the refrigerated space approaches zero, the amount of work required to produce a finite amount of refrigeration approaches infinity, and COP_{R} approaches zero.

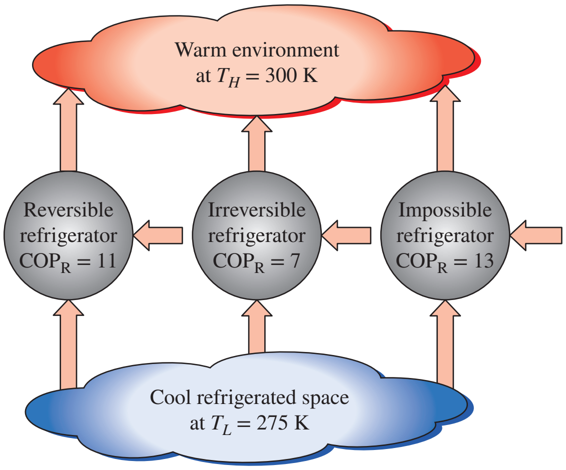


FIGURE 7–48
No refrigerator can have a higher COP than a reversible refrigerator operating between the same temperature limits.

EXAMPLE 7–6 A Carnot Refrigeration Cycle Operating in the Saturation Dome

A Carnot refrigeration cycle is executed in a closed system in the saturated liquid–vapor mixture region using 0.8 kg of refrigerant-134a as the working fluid (Fig. 7–49). The maximum and the minimum temperatures in the cycle are 20 and –8°C, respectively. It is known that the refrigerant is saturated liquid at the end of the heat rejection process, and the net work input to the cycle is 15 kJ. Determine the fraction of the mass of the refrigerant that vaporizes during the heat addition process and the pressure at the end of the heat rejection process.

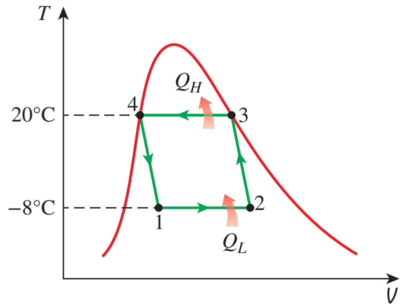


FIGURE 7–49
Schematic for Example 7–6.

SOLUTION A Carnot refrigeration cycle is executed in a closed system. The mass fraction of the refrigerant that vaporizes during the heat addition process and the pressure at the end of the heat rejection process are to be determined.

Assumptions The refrigerator operates on the ideal Carnot cycle.

Analysis Knowing the high and low temperatures, the coefficient of performance of the cycle is

$$\text{COP}_\text{R} = \frac{1}{T_H/T_L - 1} = \frac{1}{(20 + 273\text{ K})/(-8 + 273\text{ K}) - 1} = 9.464$$

The amount of cooling is determined from the definition of the coefficient of performance to be

$$Q_L = \text{COP}_\text{R} \times W_\text{in} = (9.464)(15\text{ kJ}) = 142\text{ kJ}$$

The enthalpy of vaporization of R-134a at –8°C is $h_{fg} = 204.59$ kJ/kg (Table A–11). Then the amount of refrigerant that vaporizes during heat absorption becomes

$$Q_L = m_\text{evap} h_{fg\text{ @ }-8°\text{C}} \rightarrow m_\text{evap} = \frac{142\text{ kJ}}{204.59\text{ kJ/kg}} = 0.694\text{ kg}$$

Therefore, the fraction of mass that vaporized during the heat addition process to the refrigerant is

$$\text{Mass fraction} = \frac{m_\text{evap}}{m_\text{total}} = \frac{0.694\text{ kg}}{0.8\text{ kg}} = \mathbf{0.868}\text{ (or 86.8 percent)}$$

The pressure at the end of the heat rejection process is simply the saturation pressure at heat rejection temperature,

$$P_4 = P_{\text{sat @ }20°\text{C}} = \mathbf{572.1\text{ kPa}}$$

Discussion The reversed Carnot cycle is an idealized refrigeration cycle, thus it cannot be achieved in practice. Practical refrigeration cycles are analyzed in Chap. 9.

EXAMPLE 7–7 Heating a House with a Carnot Heat Pump

A heat pump is to be used to heat a house during the winter, as shown in Fig. 7–50. The house is to be maintained at 21°C at all times. The house is estimated to be losing heat at a rate of 135,000 kJ/h when the outside temperature drops to –5°C. Determine the minimum power required to drive this heat pump.

SOLUTION A heat pump maintains a house at a constant temperature. The required minimum power input to the heat pump is to be determined.

Assumptions Steady operating conditions exist.

Analysis The heat pump must supply heat to the house at a rate of $\dot{Q}_H = 135{,}000$ kJ/h = 37.5 kW. The power requirements are minimum when a reversible heat pump is used to do

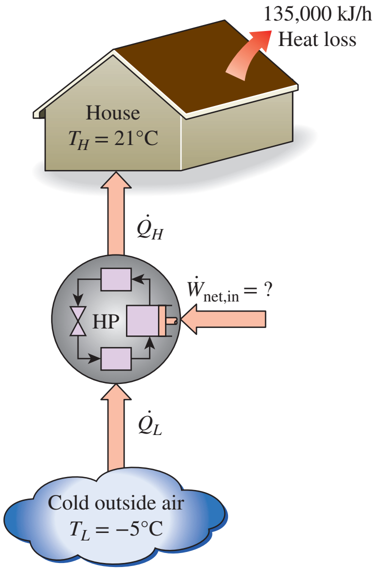


FIGURE 7–50
Schematic for Example 7–7.

the job. The COP of a reversible heat pump operating between the house and the outside air is

$$\text{COP}_{\text{HP,rev}} = \frac{1}{1 - T_L/T_H} = \frac{1}{1 - (-5 + 273\text{ K})/(21 + 273\text{ K})} = 11.3$$

Then, the required power input to this reversible heat pump becomes

$$\dot{W}_{\text{net,in}} = \frac{\dot{Q}_H}{\text{COP}_{\text{HP}}} = \frac{37.5\text{ kW}}{11.3} = \mathbf{3.32\text{ kW}}$$

Discussion This reversible heat pump can meet the heating requirements of this house by consuming electric power at a rate of 3.32 kW only. If this house were to be heated by electric resistance heaters instead, the power consumption would jump up 11.3 times to 37.5 kW. This is because in resistance heaters the electric energy is converted to heat at a one-to-one ratio. With a heat pump, however, energy is absorbed from the outside and carried to the inside using a refrigeration cycle that consumes only 3.32 kW. Notice that the heat pump does not create energy. It merely transports it from one medium (the cold outdoors) to another (the warm indoors).

SUMMARY

The *second law of thermodynamics* states that processes occur in a certain direction, not in any direction. A process does not occur unless it satisfies both the first and the second laws of thermodynamics. Bodies that can absorb or reject finite amounts of heat isothermally are called *thermal energy reservoirs* or *heat reservoirs.*

Work can be converted to heat directly, but heat can be converted to work only by some devices called *heat engines.* The *thermal efficiency* of a heat engine is defined as

$$\eta_{\text{th}} = \frac{W_{\text{net,out}}}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

where $W_{\text{net,out}}$ is the net work output of the heat engine, Q_H is the amount of heat supplied to the engine, and Q_L is the amount of heat rejected by the engine.

Refrigerators and heat pumps are devices that absorb heat from low-temperature media and reject it to higher-temperature ones. The performance of a refrigerator or a heat pump is expressed in terms of the *coefficient of performance,* which is defined as

$$\text{COP}_{\text{R}} = \frac{Q_L}{W_{\text{net,in}}} = \frac{1}{Q_H/Q_L - 1}$$

$$\text{COP}_{\text{HP}} = \frac{Q_H}{W_{\text{net,in}}} = \frac{1}{1 - Q_L/Q_H}$$

The *Kelvin–Planck statement* of the second law of thermodynamics states that no heat engine can produce a net amount of work while exchanging heat with a single reservoir only. The *Clausius statement* of the second law states that no device can transfer heat from a cooler body to a warmer one without leaving an effect on the surroundings.

A process is said to be *reversible* if both the system and the surroundings can be restored to their original conditions. Any other process is *irreversible.* The effects such as friction, non-quasi-equilibrium expansion or compression, and heat transfer through a finite temperature difference render a process irreversible and are called *irreversibilities.*

The *Carnot cycle* is a reversible cycle that is composed of four reversible processes, two isothermal and two adiabatic. The *Carnot principles* state that the thermal efficiencies of all reversible heat engines operating between the same two reservoirs are the same and that no heat engine is more efficient than a reversible one operating between the same two reservoirs. These statements form the basis for establishing a *thermodynamic temperature scale* related to the heat transfers between a reversible device and the high- and low-temperature reservoirs by

$$\left(\frac{Q_H}{Q_L}\right)_{\text{rev}} = \frac{T_H}{T_L}$$

Therefore, the Q_H/Q_L ratio can be replaced by T_H/T_L for reversible devices, where T_H and T_L are the absolute temperatures of the high- and low-temperature reservoirs, respectively.

A heat engine that operates on the reversible Carnot cycle is called a *Carnot heat engine.* The thermal efficiency of a Carnot heat engine, as well as all other reversible heat engines, is given by

$$\eta_{\text{th,rev}} = 1 - \frac{T_L}{T_H}$$

This is the maximum efficiency a heat engine operating between two reservoirs at temperatures T_H and T_L can have.

The COPs of reversible refrigerators and heat pumps are given in a similar manner as

$$\text{COP}_{\text{R,rev}} = \frac{1}{T_H/T_L - 1}$$

and

$$\text{COP}_{\text{HP,rev}} = \frac{1}{1 - T_L/T_H}$$

Again, these are the highest COPs a refrigerator or a heat pump operating between the temperature limits of T_H and T_L can have.

REFERENCES AND SUGGESTED READINGS

1. *ASHRAE Handbook of Refrigeration,* SI version. Atlanta, GA: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc. 1994.

2. D. Stewart. "Wheels Go Round and Round, but Always Run Down." *Smithsonian,* pp. 193–208, November 1986.

PROBLEMS*

Second Law of Thermodynamics and Thermal Energy Reservoirs

7–1C A mechanic claims to have developed a car engine that runs on water instead of gasoline. What is your response to this claim?

7–2C Describe an imaginary process that violates both the first and the second laws of thermodynamics.

7–3C Describe an imaginary process that satisfies the first law but violates the second law of thermodynamics.

7–4C Describe an imaginary process that satisfies the second law but violates the first law of thermodynamics.

7–5C An experimentalist claims to have raised the temperature of a small amount of water to 150°C by transferring heat from high-pressure steam at 120°C. Is this a reasonable claim? Why? Assume no refrigerator or heat pump is used in the process.

7–6C Consider the process of baking potatoes in a conventional oven. Can the hot air in the oven be treated as a thermal energy reservoir? Explain.

Heat Engines and Thermal Efficiency

7–7C What are the characteristics of all heat engines?

7–8C What is the Kelvin–Planck expression of the second law of thermodynamics?

7–9C Is it possible for a heat engine to operate without rejecting any waste heat to a low-temperature reservoir? Explain.

7–10C Does a heat engine that has a thermal efficiency of 100 percent necessarily violate (*a*) the first law and (*b*) the second law of thermodynamics? Explain.

7–11C In the absence of any friction and other irreversibilities, can a heat engine have an efficiency of 100 percent? Explain.

7–12C Are the efficiencies of all the work-producing devices, including the hydroelectric power plants, limited by the Kelvin–Planck statement of the second law? Explain.

7–13C Baseboard heaters are basically electric resistance heaters and are frequently used in space heating. A homeowner claims that her 5-year-old baseboard heaters have a conversion efficiency of 100 percent. Is this claim in violation of any thermodynamic laws? Explain.

7–14C Consider a pan of water being heated (*a*) by placing it on an electric range and (*b*) by placing a heating element in the water. Which method is a more efficient way of heating water? Explain.

7–15 A heat engine has a total heat input of 1.3 kJ and a thermal efficiency of 35 percent. How much work will it produce?

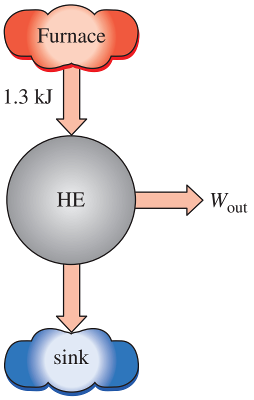


FIGURE P7–15

7–16 A steam power plant receives heat from a furnace at a rate of 280 GJ/h. Heat losses to the surrounding air from the steam as it passes through the pipes and other components are estimated to be about 8 GJ/h. If the waste heat is transferred to the cooling water at a rate of 165 GJ/h, determine (*a*) net power output and (*b*) the thermal efficiency of this power plant. *Answers:* (*a*) 29.7 MW, (*b*) 38.2 percent

7–17E A heat engine has a heat input of 3×10^4 Btu/h and a thermal efficiency of 40 percent. Calculate the power it will produce, in hp.

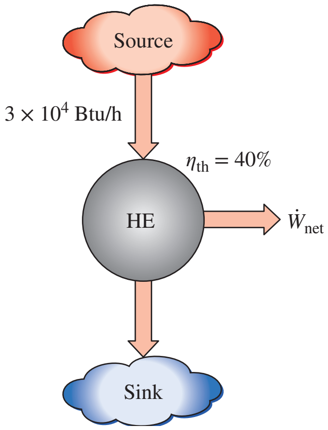


FIGURE P7–17E

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* Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

7–18 A 600-MW steam power plant, which is cooled by a nearby river, has a thermal efficiency of 40 percent. Determine the rate of heat transfer to the river water. Will the actual heat transfer rate be higher or lower than this value? Why?

7–19 A heat engine with a thermal efficiency of 45 percent rejects 500 kJ/kg of heat. How much heat does it receive? *Answer:* 909 kJ/kg

7–20E A heat engine that propels a ship produces 500 Btu/lbm of work while rejecting 300 Btu/lbm of heat. What is its thermal efficiency?

7–21 A steam power plant with a power output of 150 MW consumes coal at a rate of 60 tons/h. If the heating value of the coal is 30,000 kJ/kg, determine the overall efficiency of this plant. *Answer:* 30.0 percent

7–22 An automobile engine consumes fuel at a rate of 22 L/h and delivers 55 kW of power to the wheels. If the fuel has a heating value of 44,000 kJ/kg and a density of 0.8 g/cm^3, determine the efficiency of this engine. *Answer:* 25.6 percent

7–23E Solar energy stored in large bodies of water, called solar ponds, is being used to generate electricity. If such a solar power plant has an efficiency of 3 percent and a net power output of 150 kW, determine the average value of the required solar energy collection rate, in Btu/h.

7–24 A coal-burning steam power plant produces a net power of 300 MW with an overall thermal efficiency of 32 percent. The actual gravimetric air–fuel ratio in the furnace is calculated to be 12 kg air/kg fuel. The heating value of the coal is 28,000 kJ/kg. Determine (*a*) the amount of coal consumed during a 24-hour period and (*b*) the rate of air flowing through the furnace. *Answers:* (*a*) 2.89×10^6 kg, (*b*) 402 kg/s

7–25E An Ocean Thermal Energy Conversion (OTEC) power plant built in Hawaii in 1987 was designed to operate between the temperature limits of 86°F at the ocean surface and 41°F at a depth of 2100 ft. About 13,300 gpm of cold seawater was to be pumped from deep ocean through a 40-in-diameter pipe to serve as the cooling medium or heat sink. If the cooling water experiences a temperature rise of 6°F and the thermal efficiency is 2.5 percent, determine the amount of power generated. Take the density of seawater to be 64 lbm/ft^3.

7–26 A country needs to build new power plants to meet the increasing demand for electric power. One possibility is to build coal-fired power plants, which cost \$1300 per kW to construct and have an efficiency of 40 percent. Another possibility is to build clean-burning Integrated Gasification Combined Cycle (IGCC) plants where the coal is subjected to heat and pressure to gasify it while removing sulfur and particulate matter from it. The gaseous coal is then burned in a gas turbine, and part of the waste heat from the exhaust gases is recovered to generate steam for the steam turbine. The construction of IGCC plants costs about \$1500 per kW, but their efficiency is about 48 percent. The average heating value of the coal is about 28,000,000 kJ per ton (that is, 28,000,000 kJ of heat is released when 1 ton of coal is burned). If the IGCC plant is to recover its cost difference from fuel savings in five years, determine what the price of coal should be in \$ per ton.

7–27 Reconsider Prob. 7–26. Using appropriate software, investigate the price of coal for varying simple payback periods, plant construction costs, and operating efficiency.

7–28 Repeat Prob. 7–26 for a simple payback period of three years instead of five years.

Refrigerators and Heat Pumps

7–29C What is the difference between a refrigerator and a heat pump?

7–30C What is the difference between a refrigerator and an air conditioner?

7–31C Define the coefficient of performance of a refrigerator in words. Can it be greater than unity?

7–32C Define the coefficient of performance of a heat pump in words. Can it be greater than unity?

7–33C A heat pump that is used to heat a house has a COP of 2.5. That is, the heat pump delivers 2.5 kWh of energy to the house for each 1 kWh of electricity it consumes. Is this a violation of the first law of thermodynamics? Explain.

7–34C A refrigerator has a COP of 1.5. That is, the refrigerator removes 1.5 kWh of energy from the refrigerated space for each 1 kWh of electricity it consumes. Is this a violation of the first law of thermodynamics? Explain.

7–35C In a refrigerator, heat is transferred from a lower-temperature medium (the refrigerated space) to a higher-temperature one (the kitchen air). Is this a violation of the second law of thermodynamics? Explain.

7–36C A heat pump is a device that absorbs energy from the cold outdoor air and transfers it to the warmer indoors. Is this a violation of the second law of thermodynamics? Explain.

7–37C What is the Clausius expression of the second law of thermodynamics?

7–38C Show that the Kelvin–Planck and the Clausius expressions of the second law are equivalent.

7–39 A food freezer is to produce a 5-kW cooling effect, and its COP is 1.3. How many kW of power will this refrigerator require for operation?

7–40 An automotive air conditioner produces a 1-kW cooling effect while consuming 0.75 kW of power. What is the rate at which heat is rejected from this air conditioner?

7–41 A food refrigerator is to provide a 15,000-kJ/h cooling effect while rejecting 22,000 kJ/h of heat. Calculate the COP of this refrigerator. *Answer:* 2.14

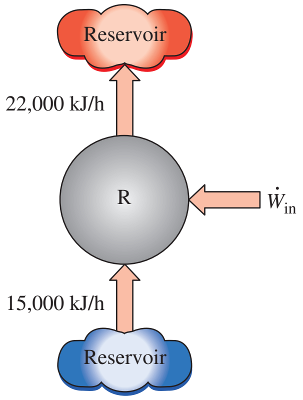

FIGURE P7–41

7–42 Determine the COP of a refrigerator that removes heat from the food compartment at a rate of 5040 kJ/h for each kW of power it consumes. Also, determine the rate of heat rejection to the outside air.

7–43 Determine the COP of a heat pump that supplies energy to a house at a rate of 8000 kJ/h for each kW of electric power it draws. Also, determine the rate of energy absorption from the outdoor air. *Answers:* 2.22, 4400 kJ/h

7–44 A heat pump has a COP of 1.7. Determine the heat transferred to and from this heat pump when 50 kJ of work is supplied.

7–45E A heat pump with a COP of 1.4 is to produce a 100,000 Btu/h heating effect. How much power does this device require, in hp?

7–46 An air conditioner removes heat steadily from a house at a rate of 750 kJ/min while drawing electric power at a rate of 5.25 kW. Determine (*a*) the COP of this air conditioner and (*b*) the rate of heat transfer to the outside air. *Answers:* (*a*) 2.38, (*b*) 1065 kJ/min

7–47 A household refrigerator that has a power input of 450 W and a COP of 1.5 is to cool 5 large watermelons, 10 kg each, to 8°C. If the watermelons are initially at 28°C, determine how long it will take for the refrigerator to cool them. The watermelons can be treated as water whose specific heat is 4.2 kJ/kg·°C. Is your answer realistic or optimistic? Explain. *Answer:* 104 min

7–48 When a man returns to his well-sealed house on a summer day, he finds that the house is at 35°C. He turns on the air conditioner, which cools the entire house to 20°C in 30 min. If the COP of the air-conditioning system is 2.8, determine the power drawn by the air conditioner. Assume all mass within the house is equivalent to 800 kg of air for which $c_\upsilon = 0.72$ kJ/kg·°C and $c_p = 1.0$ kJ/kg·°C.

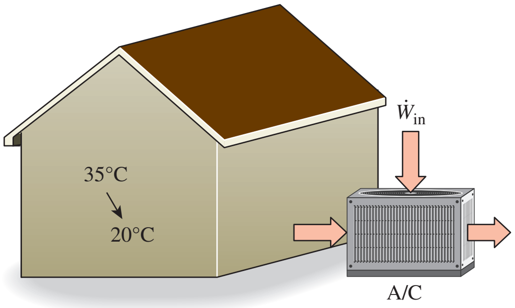

FIGURE P7–48

7–49 Reconsider Prob. 7–48. Using appropriate software, determine the power input required by the air conditioner to cool the house as a function of air-conditioner EER ratings in the range 5 to 15. Discuss your results and include representative costs of air-conditioning units in the EER rating range.

7–50E Water enters an ice machine at 55°F and leaves as ice at 25°F. If the COP of the ice machine is 2.4 during this operation, determine the required power input for an ice production rate of 28 lbm/h. (169 Btu of energy needs to be removed from each lbm of water at 55°F to turn it into ice at 25°F.)

7–51 A refrigerator is used to cool water from 23 to 5°C in a continuous manner. The heat rejected in the condenser is 570 kJ/min and the power is 2.65 kW. Determine the rate at which water is cooled in L/min and the COP of the refrigerator. The specific heat of water is 4.18 kJ/kg·°C and its density is 1 kg/L. *Answers:* 5.46 L/min, 2.58

7–52 A household refrigerator runs one-fourth of the time and removes heat from the food compartment at an average rate of 800 kJ/h. If the COP of the refrigerator is 2.2, determine the power the refrigerator draws when running.

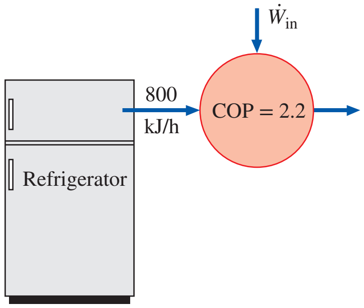

FIGURE P7–52

7–53E Consider an office room that is being cooled adequately by a 12,000 Btu/h window air conditioner. Now it is decided to convert this room into a computer room by installing several computers, terminals, and printers with a total rated power of 8.4 kW. The facility has several 7000 Btu/h air conditioners in storage that can be installed to meet the additional cooling requirements. Assuming a usage factor of 0.4 (i.e., only 40 percent of the rated power will be consumed at any given time) and additional occupancy of seven people, each generating heat at a rate of 100 W, determine how many of these air conditioners need to be installed in the room.

7–54 A house that was heated by electric resistance heaters consumed 1200 kWh of electric energy in a winter month. If this house were heated instead by a heat pump that has an average COP of 2.4, determine how much money the homeowner would have saved that month. Assume a price of $0.12/kWh for electricity.

7–55 Refrigerant-134a enters the condenser of a residential heat pump at 800 kPa and 35°C at a rate of 0.018 kg/s and leaves at 800 kPa as a saturated liquid. If the compressor consumes 1.2 kW of power, determine (*a*) the COP of the heat pump and (*b*) the rate of heat absorption from the outside air.

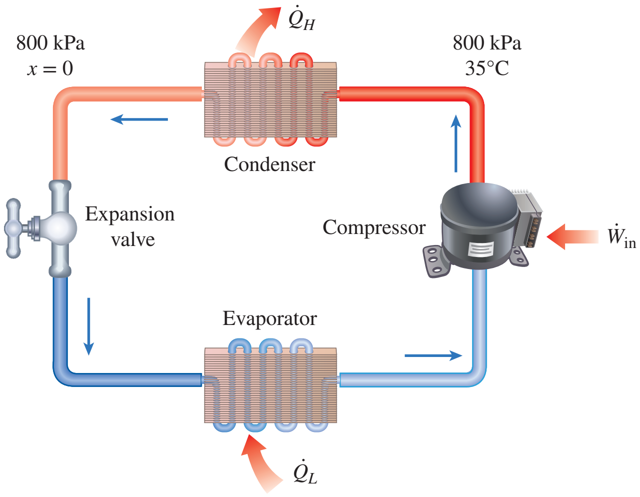

FIGURE P7–55

Reversible and Irreversible Processes

7–56C Why are engineers interested in reversible processes even though they can never be achieved?

7–57C A cold canned drink is left in a warmer room where its temperature rises as a result of heat transfer. Is this a reversible process? Explain.

7–58C A block slides down an inclined plane with friction and no restraining force. Is this process reversible or irreversible? Justify your answer.

7–59C How do you distinguish between internal and external irreversibilities?

7–60C Show that processes involving rapid chemical reactions are irreversible by considering the combustion of a natural gas (e.g., methane) and air mixture in a rigid container.

7–61C Show that processes that use work for mixing are irreversible by considering an adiabatic system whose contents are stirred by turning a paddle wheel inside the system (e.g., stirring a cake mix with an electric mixer).

7–62C Why does a nonquasi-equilibrium compression process require a larger work input than the corresponding quasi-equilibrium one?

7–63C Why does a nonquasi-equilibrium expansion process deliver less work than the corresponding quasi-equilibrium one?

7–64C Is a reversible expansion or compression process necessarily quasi-equilibrium? Is a quasi-equilibrium expansion or compression process necessarily reversible? Explain.

The Carnot Cycle and Carnot Principles

7–65C What are the four processes that make up the Carnot cycle?

7–66C What are the two statements known as the Carnot principles?

7–67C Is it possible to develop (*a*) an actual and (*b*) a reversible heat-engine cycle that is more efficient than a Carnot cycle operating between the same temperature limits? Explain.

7–68C Somebody claims to have developed a new reversible heat-engine cycle that has a higher theoretical efficiency than the Carnot cycle operating between the same temperature limits. How do you evaluate this claim?

7–69C Somebody claims to have developed a new reversible heat-engine cycle that has the same theoretical efficiency as the Carnot cycle operating between the same temperature limits. Is this a reasonable claim?

Carnot Heat Engines

7–70C Is there any way to increase the efficiency of a Carnot heat engine other than by increasing T_H or decreasing T_L?

7–71C Consider two actual power plants operating with solar energy. Energy is supplied to one plant from a solar pond at 80°C and to the other from concentrating collectors that raise the water temperature to 600°C. Which of these power plants will have a higher efficiency? Explain.

7–72 You are an engineer in an electric-generation station. You know that the flames in the boiler reach a temperature of 1200 K and that cooling water at 300 K is available from a nearby river. What is the maximum efficiency your plant will ever achieve?

7–73 Reconsider Prob. 7–72. You also know that the metallurgical temperature limit for the blades in the turbine is 1000 K before they will incur excessive creep. Now what is the maximum efficiency for this plant?

7–74E A thermodynamicist claims to have developed a heat engine with 50 percent thermal efficiency when operating with thermal energy reservoirs at 1260 R and 510 R. Is this claim valid?

7–75E A heat engine is operating on a Carnot cycle and has a thermal efficiency of 47 percent. The waste heat from this engine is rejected to a nearby lake at 60°F at a rate of 800 Btu/min. Determine (*a*) the power output of the engine and (*b*) the temperature of the source. *Answers:* (*a*) 16.7 hp, (*b*) 981 R

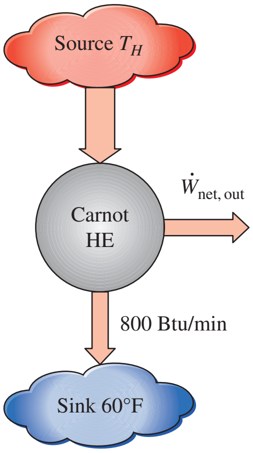

FIGURE P7–75E

7–76E A completely reversible heat engine operates with a source at 1500 R and a sink at 500 R. At what rate must heat be supplied to this engine, in Btu/h, for it to produce 5 hp of power? *Answer:* 19,100 Btu/h

7–77 An inventor claims to have developed a heat engine that receives 700 kJ of heat from a source at 500 K and produces 300 kJ of net work while rejecting the waste heat to a sink at 290 K. Is this a reasonable claim? Why?

7–78 A Carnot heat engine operates between a source at 1000 K and a sink at 300 K. If the heat engine is supplied with heat at a rate of 800 kJ/min, determine (*a*) the thermal efficiency and (*b*) the power output of this heat engine. *Answers:* (*a*) 70 percent, (*b*) 9.33 kW

7–79 A heat engine operates between a source at 477°C and a sink at 25°C. If heat is supplied to the heat engine at a steady rate of 65,000 kJ/min, determine the maximum power output of this heat engine.

7–80 Reconsider Prob. 7–79. Using appropriate software, study the effects of the temperatures of the heat source and the heat sink on the power produced and the cycle thermal efficiency. Let the source temperature vary from 300 to 1000°C, and the sink temperature to vary from 0 to 50°C. Plot the power produced and the cycle efficiency against the source temperature for sink temperatures of 0°C, 25°C, and 50°C, and discuss the results.

7–81E An experimentalist claims that, based on his measurements, a heat engine receives 300 Btu of heat from a source of 900 R, converts 160 Btu of it to work, and rejects the rest as waste heat to a sink at 540 R. Are these measurements reasonable? Why?

7–82 In tropical climates, the water near the surface of the ocean remains warm throughout the year as a result of solar energy absorption. In the deeper parts of the ocean, however, the water remains at a relatively low temperature since the sun's rays cannot penetrate very far. It is proposed to take advantage of this temperature difference and construct a power plant that will absorb heat from the warm water near the surface and reject the waste heat to the cold water a few hundred meters below. Determine the maximum thermal efficiency of such a plant if the water temperatures at the two respective locations are 24 and 3°C.

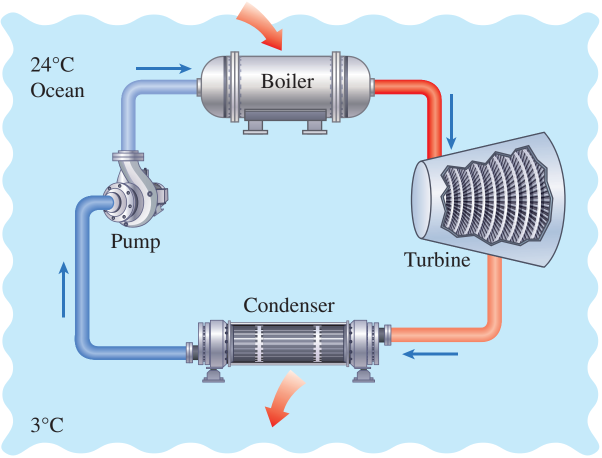

FIGURE P7–82

7–83 It is claimed that the efficiency of a completely reversible heat engine can be doubled by doubling the temperature of the energy source. Justify the validity of this claim.

Carnot Refrigerators and Heat Pumps

7–84C What is the highest COP that a refrigerator operating between temperature levels T_L and T_H can have?

7–85C A homeowner buys a new refrigerator and a new air conditioner. Which one of these devices would you expect to have a higher COP? Why?

7–86C A homeowner buys a new refrigerator with no freezer compartment and a deep freezer for the new kitchen. Which of these devices would you expect to have a lower COP? Why?

7–87C How can we increase the COP of a Carnot refrigerator?

7–88C In an effort to conserve energy in a heat-engine cycle, somebody suggests incorporating a refrigerator that will absorb some of the waste energy Q_L and transfer it to the energy source of the heat engine. Is this a smart idea? Explain.

7–89C It is well established that the thermal efficiency of a heat engine increases as the temperature T_L at which heat is rejected from the heat engine decreases. In an effort to increase the efficiency of a power plant, somebody suggests refrigerating the cooling water before it enters the condenser, where heat rejection takes place. Would you be in favor of this idea? Why?

7–90C It is well known that the thermal efficiency of heat engines increases as the temperature of the energy source increases. In an attempt to improve the efficiency of a power plant, somebody suggests transferring heat from the available energy source to a higher-temperature medium by a heat pump before energy is supplied to the power plant. What do you think of this suggestion? Explain.

7–91 A thermodynamicist claims to have developed a heat pump with a COP of 1.7 when operating with thermal energy reservoirs at 273 K and 293 K. Is this claim valid?

7–92 Determine the minimum work per unit of heat transfer from the source reservoir that is required to drive a heat pump with thermal energy reservoirs at 460 K and 535 K.

7–93 A completely reversible refrigerator is driven by a 10-kW compressor and operates with thermal energy reservoirs at 250 K and 300 K. Calculate the rate of cooling provided by this refrigerator. *Answer:* 50 kW

7–94 An air-conditioning system operating on the reversed Carnot cycle is required to transfer heat from a house at a rate of 750 kJ/min to maintain its temperature at 24°C. If the outdoor air temperature is 35°C, determine the power required to operate this air-conditioning system. *Answer:* 0.463 kW

7–95 A heat pump operates on a Carnot heat pump cycle with a COP of 12.5. It keeps a space at 24°C by consuming 2.15 kW of power. Determine the temperature of the reservoir from which the heat is absorbed and the heating load provided by the heat pump. *Answers:* 273 K, 26.9 kW

7–96E An air-conditioning system is used to maintain a house at 70°F when the temperature outside is 100°F. The house is gaining heat through the walls and the windows at a rate of 800 Btu/min, and the heat generation rate within the house from people, lights, and appliances amounts to 100 Btu/min. Determine the minimum power input required for this air-conditioning system. *Answer:* 1.20 hp

7–97 A Carnot refrigerator absorbs heat from a space at 15°C at a rate of 16,000 kJ/h and rejects heat to a reservoir at 36°C. Determine the COP of the refrigerator, the power input in kW, and the rate of heat rejected to the high-temperature reservoir in kJ/h.

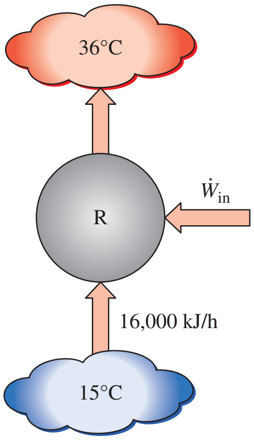

FIGURE P7–97

7–98E A completely reversible refrigerator operates between thermal energy reservoirs at 450 R and 540 R. How many kilowatts of power are required for this device to produce a 15,000-Btu/h cooling effect?

7–99 A Carnot refrigerator operates in a room in which the temperature is 25°C. The refrigerator consumes 500 W of power when operating and has a COP of 4.5. Determine (*a*) the rate of heat removal from the refrigerated space and (*b*) the temperature of the refrigerated space. *Answers*: (*a*) 135 kJ/min, (*b*) –29.2°C

7–100 A heat pump is used to heat a house and maintain it at 24°C. On a winter day when the outdoor air temperature is –5°C, the house is estimated to lose heat at a rate of 80,000 kJ/h. Determine the minimum power required to operate this heat pump.

7–101 A commercial refrigerator with refrigerant-134a as the working fluid is used to keep the refrigerated space at –35°C by rejecting waste heat to cooling water that enters the condenser at 18°C at a rate of 0.25 kg/s and leaves at 26°C. The refrigerant enters the condenser at 1.2 MPa and 50°C and leaves at the same pressure subcooled by 5°C. If the compressor consumes 3.3 kW of power, determine (*a*) the mass flow rate of the refrigerant, (*b*) the refrigeration load, (*c*) the COP, and (*d*) the minimum power input to the compressor for the same refrigeration load.

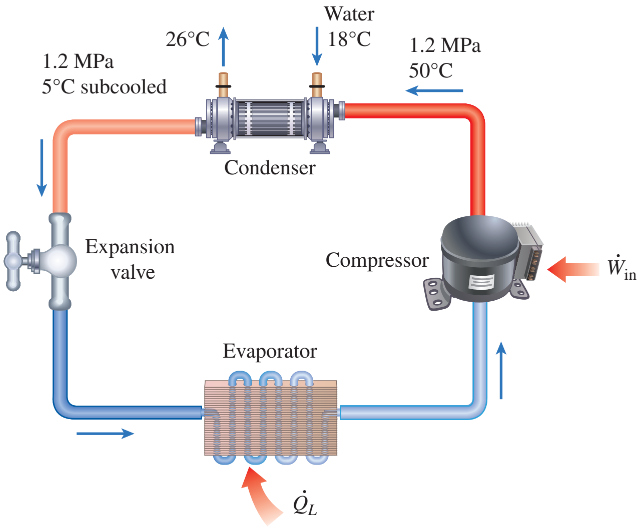

FIGURE P7–101

7–102 The performance of a heat pump degrades (i.e., its COP decreases) as the temperature of the heat source decreases. This makes using heat pumps at locations with severe weather conditions unattractive. Consider a house that is heated and maintained at 20°C by a heat pump during the winter. What is the maximum COP for this heat pump if heat is extracted from the outdoor air at (*a*) 10°C, (*b*) –5°C, and (*c*) –30°C?

7–103E A heat pump is to be used for heating a house in winter. The house is to be maintained at 78°F at all times. When the temperature outdoors drops to 25°F, the heat losses from the house are estimated to be 70,000 Btu/h. Determine the minimum power required to run this heat pump if heat is extracted from (*a*) the outdoor air at 25°F and (*b*) the well water at 50°F.

7–104 A completely reversible heat pump has a COP of 1.6 and a sink temperature of 300 K. Calculate (*a*) the temperature of the source and (*b*) the rate of heat transfer to the sink when 1.5 kW of power is supplied to this heat pump.

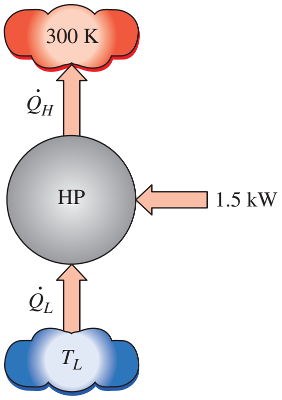

FIGURE P7–104

7–105 A Carnot heat pump is to be used to heat a house and maintain it at 25°C in winter. On a day when the average outdoor temperature remains at about 2°C, the house is estimated to lose heat at a rate of 55,000 kJ/h. If the heat pump consumes 4.8 kW of power while operating, determine (*a*) how long the heat pump ran on that day; (*b*) the total heating costs, assuming an average price of $0.11/kWh for electricity; and (*c*) the heating cost for the same day if resistance heating is used instead of a heat pump. *Answers:* (*a*) 5.90 h, (*b*) $3.11, (*c*) $40.3

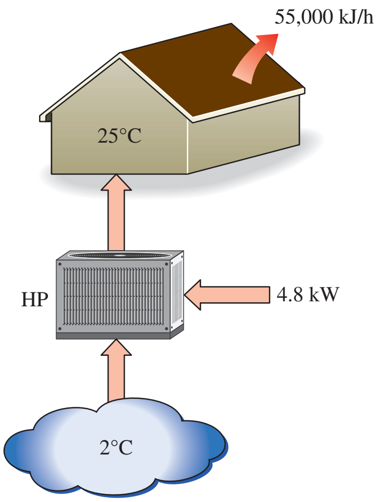

FIGURE P7–105

7–106 A Carnot heat engine receives heat from a reservoir at 900°C at a rate of 800 kJ/min and rejects the waste heat to the ambient air at 27°C. The entire work output of the heat engine is used to drive a refrigerator that removes heat from the refrigerated space at –5°C and transfers it to the same ambient air at 27°C. Determine (*a*) the maximum rate of heat removal from the refrigerated space and (*b*) the total rate of heat rejection to the ambient air. *Answers:* (*a*) 4982 kJ/min, (*b*) 5782 kJ/min

7–107 The structure of a house is such that it loses heat at a rate of 4500 kJ/h per °C difference between the indoors and outdoors. A heat pump that requires a power input of 4 kW is used to maintain this house at 24°C. Determine the lowest outdoor temperature for which the heat pump can meet the heating requirements of this house. *Answer:* –6.8°C

7–108 Derive an expression for the COP of a completely reversible refrigerator in terms of the thermal energy reservoir temperatures, T_L and T_H.

Review Problems

7–109 A manufacturer of ice cream freezers claims that its product has a coefficient of performance of 1.3 while freezing ice cream at 250 K when the surrounding environment is at 300 K. Is this claim valid?

7–110 A heat pump designer claims to have an air-source heat pump whose coefficient of performance is 1.8 when heating a building whose interior temperature is 300 K and when the atmospheric air surrounding the building is at 260 K. Is this claim valid?

7–111 An air-conditioning system is used to maintain a house at a constant temperature of 20°C. The house is gaining heat from outdoors at a rate of 20,000 kJ/h, and the heat generated in the house from the people, lights, and appliances amounts to 8000 kJ/h. For a COP of 2.5, determine the required power input to this air-conditioning system. *Answer:* 3.11 kW

7–112E A Carnot heat pump is used to heat and maintain a residential building at 75°F. An energy analysis of the house reveals that it loses heat at a rate of 2500 Btu/h per °F temperature difference between the indoors and the outdoors. For an outdoor temperature of 35°F, determine (*a*) the coefficient of performance and (*b*) the required power input to the heat pump. *Answers:* (*a*) 13.4, (*b*) 2.93 hp

7–113E A refrigeration system uses a water-cooled condenser for rejecting the waste heat. The system absorbs heat from a space at 25°F at a rate of 21,000 Btu/h. Water enters the condenser at 65°F at a rate of 1.45 lbm/s. The COP of the system is estimated to be 1.9. Determine (*a*) the power input to the system in kW, (*b*) the temperature of the water at the exit of the condenser in °F, and (*c*) the maximum possible COP of the system. The specific heat of water is 1.0 Btu/bm·°F.

7–114 A refrigeration system is to cool bread loaves with an average mass of 350 g from 30 to –10°C at a rate of 1200 loaves per hour with refrigerated air at –30°C. Taking the average specific and latent heats of bread to be 2.93 kJ/kg·°C and 109.3 kJ/kg, respectively, determine (*a*) the rate of heat removal from the breads, in kJ/h; (*b*) the required volume flow rate of air, in m^3/h, if the temperature rise of air is not to exceed 8°C; and (*c*) the size of the compressor of the refrigeration system, in kW, for a COP of 1.2 for the refrigeration system.

7–115 A heat pump with a COP of 2.8 is used to heat an airtight house. When running, the heat pump consumes 5 kW of power. If the temperature in the house is 7°C when the heat pump is turned on, how long will it take for the heat pump to raise the temperature of the house to 22°C? Is this answer realistic or optimistic? Explain. Assume all mass within the house (air, furniture, etc.) is equivalent to 1500 kg of air. *Answer:* 19.2 min

7–116 Consider a Carnot heat-engine cycle executed in a closed system using 0.025 kg of steam as the working fluid. It is known that the maximum absolute temperature in the cycle is twice the minimum absolute temperature, and the net work output of the cycle is 60 kJ. If the steam changes from saturated vapor to saturated liquid during heat rejection, determine the temperature of the steam during the heat rejection process.

7–117 Reconsider Prob. 7–116. Using appropriate software, investigate the effect of the net work output on the required temperature of the steam during the heat rejection process. Let the work output vary from 40 to 60 kJ.

7–118 Consider a Carnot refrigeration cycle executed in a closed system in the saturated liquid–vapor mixture region using 0.96 kg of refrigerant-134a as the working fluid. It is known that the maximum absolute temperature in the cycle is 1.2 times the minimum absolute temperature, and the net work input to the cycle is 22 kJ. If the refrigerant changes from saturated vapor to saturated liquid during the heat rejection process, determine the minimum pressure in the cycle.

7–119 Reconsider Prob. 7–118. Using appropriate software, investigate the effect of the net work input on the minimum pressure. Let the work input vary from 10 to 30 kJ. Plot the minimum pressure in the refrigeration cycle as a function of net work input, and discuss the results.

7–120 Consider a Carnot heat-engine cycle executed in a steady-flow system using steam as the working fluid. The cycle has a thermal efficiency of 30 percent, and steam changes from saturated liquid to saturated vapor at 275°C during the heat addition process. If the mass flow rate of the steam is 3 kg/s, determine the net power output of this engine, in kW.

7–121 Consider two Carnot heat engines operating in series. The first engine receives heat from the reservoir at 1400 K and rejects the waste heat to another reservoir at temperature *T*. The second engine receives this energy rejected by the first one, converts some of it to work, and rejects the rest to a reservoir at 300 K. If the thermal efficiencies of both engines are the same, determine the temperature *T*. *Answer:* 648 K

7–122 A heat engine operates between two reservoirs at 800 and 20°C. One-half of the work output of the heat engine is used to drive a Carnot heat pump that removes heat from the cold surroundings at 2°C and transfers it to a house maintained at 22°C. If the house is losing heat at a rate of 62,000 kJ/h, determine the minimum rate of heat supply to the heat engine required to keep the house at 22°C.

7–123 An old gas turbine has an efficiency of 21 percent and develops a power output of 6000 kW. Determine the fuel consumption rate of this gas turbine, in L/min, if the fuel has a heating value of 42,000 kJ/kg and a density of 0.8 g/cm^3.

7–124 Consider a Carnot heat-pump cycle executed in a steady-flow system in the saturated liquid–vapor mixture region using refrigerant-134a flowing at a rate of 0.18 kg/s as the working fluid. It is known that the maximum absolute temperature in the cycle is 1.2 times the minimum absolute temperature, and the net power input to the cycle is 5 kW. If the refrigerant changes from saturated vapor to saturated liquid during the heat rejection process, determine the ratio of the maximum to minimum pressures in the cycle.

7–125 The cargo space of a refrigerated truck whose inner dimensions are 12 m × 2.3 m × 3.5 m is to be precooled from 25°C to an average temperature of 5°C. The construction of the truck is such that a transmission heat gain occurs at a rate of 120 W/°C. If the ambient temperature is 25°C, determine how long it will take for a system with a refrigeration capacity of 11 kW to precool this truck.

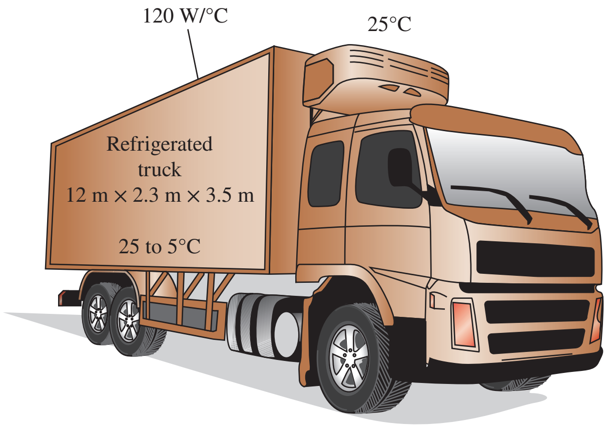

FIGURE P7–125

7–126 The maximum flow rate of a standard shower head is about 3.5 gpm (13.3 L/min) and can be reduced to 2.75 gpm (10.5 L/min) by switching to a low-flow shower head that is equipped with flow controllers. Consider a family of four, with each person taking a 6-minute shower every morning. City water at 15°C is heated to 55°C in an oil-fired water heater whose efficiency is 65 percent and then tempered to 42°C by cold water at the T-elbow of the shower before being routed to the shower head. The price of heating oil is \$2.80/gal and its heating value is 146,300 kJ/gal. Assuming a constant specific heat of 4.18 kJ/kg·°C for water, determine the amount of oil and money saved per year by replacing the standard shower heads with the low-flow ones.

7–127 The drinking water needs of a production facility with 20 employees is to be met by a bubbler-type water fountain. The refrigerated water fountain is to cool water from 22 to 8°C and supply cold water at a rate of 0.4 L per hour per person. Heat is transferred to the reservoir from the surroundings at 25°C at a rate of 45 W. If the COP of the refrigeration system is 2.9, determine the size of the compressor, in W, that will be suitable for the refrigeration system of this water cooler.

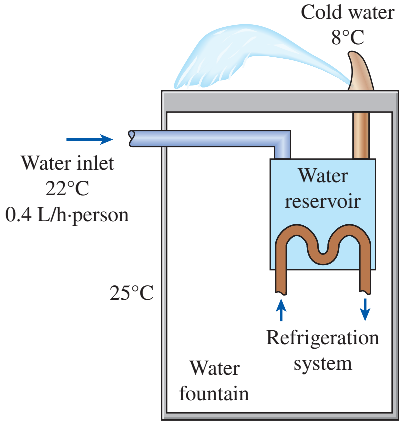

FIGURE P7–127

7–128 A typical electric water heater has an efficiency of 95 percent and costs \$350 a year to operate at a unit cost of electricity of \$0.11/kWh. A typical heat pump–powered water heater has a COP of 3.3 but costs about \$800 more to install. Determine how many years it will take for the heat pump water heater to pay for its cost differential from the energy it saves.

FIGURE P7–128

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7–129 Reconsider Prob. 7–128. Using appropriate software, investigate the effect of the heat pump COP on the yearly operation costs and the number of years required to break even. Let the COP vary from 2 to 5. Plot the payback period against the COP and discuss the results.

7–130 A homeowner is trying to decide between a high-efficiency natural gas furnace with an efficiency of 97 percent and a ground-source heat pump with a COP of 3.5. The unit costs of electricity and natural gas are \$0.115/kWh and \$0.75/therm (1 therm = 105,500 kJ). Determine which system will have a lower energy cost.

7–131 Replacing incandescent lights with energy-efficient fluorescent lights can reduce the lighting energy consumption to one-fourth of what it was before. The energy consumed by the lamps is eventually converted to heat, and thus switching to energy-efficient lighting also reduces the cooling load in summer but increases the heating load in winter. Consider a building that is heated by a natural gas furnace with an efficiency of 80 percent and cooled by an air conditioner with a COP of 3.5. If electricity costs \$0.12/kWh and natural gas costs \$1.40/therm (1 therm = 105,500 kJ), determine if efficient lighting will increase or decrease the total energy cost of the building (*a*) in summer and (*b*) in winter.

7–132 A heat pump supplies heat energy to a house at the rate of 140,000 kJ/h when the house is maintained at 25°C. Over a period of one month, the heat pump operates for 100 hours to transfer energy from a heat source outside the house to inside the house. Consider a heat pump receiving heat from two different outside energy sources. In one application the heat pump receives heat from the outside air at 0°C. In a second application the heat pump receives heat from a lake having a water temperature of 10°C. If electricity costs \$0.12/kWh, determine the maximum amount of money saved by using the lake water rather than the outside air as the outside energy source.

7–133 Cold water at 10°C enters a water heater at the rate of 0.02 m^3/min and leaves the water heater at 50°C. The water heater receives heat from a heat pump that receives heat from a heat source at 0°C.

(*a*) Assuming the water to be an incompressible liquid that does not change phase during heat addition, determine the rate of heat supplied to the water, in kJ/s.

(*b*) Assuming the water heater acts as a heat sink having an average temperature of 30°C, determine the minimum power supplied to the heat pump, in kW.

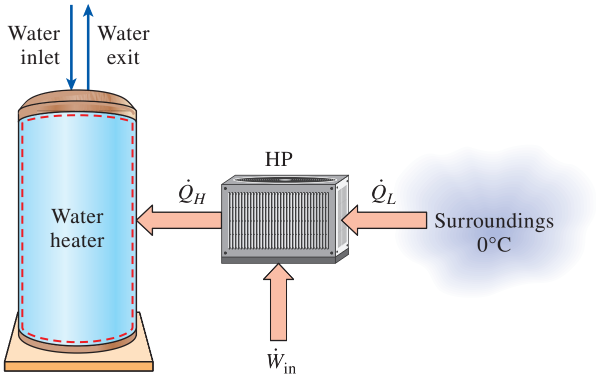


FIGURE P7–133

7–134 A Carnot heat engine is operating between a source at T_H and a sink at T_L. If we wish to double the thermal efficiency of this engine, what should the new source temperature be? Assume the sink temperature is held constant.

7–135E Calculate and plot the thermal efficiency of a completely reversible heat engine as a function of the source temperature up to 2000 R with the sink temperature fixed at 500 R.

7–136 Show that $\text{COP}_{\text{HP}} = \text{COP}_{\text{R}} + 1$ when both the heat pump and the refrigerator have the same Q_L and Q_H values.

Design and Essay Problems

7–137 The sun supplies electromagnetic energy to the earth. It appears to have an effective temperature of approximately 5800 K. On a clear summer day in North America, the energy incident on a surface facing the sun is approximately 0.95 kW/m^2. The electromagnetic solar energy can be converted into thermal energy by being absorbed on a darkened surface. How might you characterize the work potential of the sun's energy when it is to be used to produce work?

7–138 Using a thermometer, measure the temperature of the main food compartment of your refrigerator, and check to see if it is between 1 and 4°C. Also, measure the temperature of the freezer compartment, and check to see if it is at the recommended value of –18°C.

7–139 Using a timer (or watch) and a thermometer, conduct the following experiment to determine the rate of heat gain of your refrigerator. First make sure that the door of the refrigerator is not opened for at least a few hours so that steady operating conditions are established. Start the timer when the refrigerator stops running and measure the time ΔT_1 it stays off before it kicks in. Then, measure the time ΔT_2 it stays on. Noting that the heat removed during ΔT_2 is equal to the heat gain of the refrigerator during $\Delta T_1 + \Delta T_2$ and using the power consumed by the refrigerator when it is running, determine the average rate of heat gain for your refrigerator, in W. Take the COP (coefficient of performance) of your refrigerator to be 1.3 if it is not available.

7–140 Design a hydrocooling unit that can cool fruits and vegetables from 30 to 5°C at a rate of 20,000 kg/h under the following conditions:

The unit will be of flood type, which will cool the products as they are conveyed into the channel filled with water. The products will be dropped into the channel filled with water at one end and be picked up at the other end. The channel can be as wide as 3 m and as high as 90 cm. The water is to be circulated and cooled by the evaporator section of a refrigeration system. The refrigerant temperature inside the coils is to be –2°C, and the water temperature is not to drop below 1°C and not to exceed 6°C.

Assuming reasonable values for the average product density, specific heat, and porosity (the fraction of air volume in a box), recommend reasonable values for (*a*) the water velocity through the channel and (*b*) the refrigeration capacity of the refrigeration system.

7–141 In the search to reduce thermal pollution and take advantage of renewable energy sources, some people have proposed that we take advantage of such sources as discharges from electric power plants, geothermal energy, and ocean thermal energy. Although many of these sources contain an enormous amount of energy, the amount of work they are capable of producing is limited. How might you use the work potential to assign an "energy quality" to these proposed sources? Test your proposed "energy quality" measure by applying it to the ocean thermal source, where the temperature 30 m below the surface is perhaps 5°C lower than at the surface. Apply it also to the geothermal water source, where the temperature 2 to 3 km below the surface is perhaps 150°C hotter than at the surface.

CHAPTER

8

ENTROPY

In Chap. 7, we introduced the second law of thermodynamics and applied it to cycles and cyclic devices. In this chapter, we apply the second law to processes. The first law of thermodynamics deals with the property *energy* and the conservation of it. The second law leads to the definition of a new property called *entropy*. Entropy is a somewhat abstract property, and it is difficult to give a physical description of it without considering the microscopic state of the system. Entropy is best understood and appreciated by studying its uses in commonly encountered engineering processes, and this is what we intend to do.

This chapter starts with a discussion of the Clausius inequality, which forms the basis for the definition of entropy, and continues with the increase of entropy principle. Unlike energy, entropy is a nonconserved property, and there is no such thing as *conservation of entropy*. Next, the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases are discussed, and a special class of idealized processes, called *isentropic processes,* is examined. Then, the reversible steady-flow work and the isentropic efficiencies of various engineering devices such as turbines and compressors are considered. Finally, entropy balance is introduced and applied to various systems.

OBJECTIVES

The objectives of this chapter are to:

- Apply the second law of thermodynamics to processes.
- Define a new property called entropy to quantify the second-law effects.
- Establish the increase of entropy principle.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Examine a special class of idealized processes, called isentropic processes, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.

8–1 ■ ENTROPY

The second law of thermodynamics often leads to expressions that involve inequalities. An irreversible (i.e., actual) heat engine, for example, is less efficient than a reversible one operating between the same two thermal energy reservoirs. Likewise, an irreversible refrigerator or a heat pump has a lower coefficient of performance (COP) than a reversible one operating between the same temperature limits. Another important inequality that has major consequences in thermodynamics is the **Clausius inequality**. It was first stated by the German physicist R. J. E. Clausius (1822–1888), one of the founders of thermodynamics, and is expressed in 1865 as

$$\oint \frac{\delta Q}{T} \leq 0$$

That is, *the cyclic integral of $\delta Q/T$ is always less than or equal to zero.* This inequality is valid for all cycles, reversible or irreversible. The symbol $\oint$ (integral symbol with a circle in the middle) is used to indicate that the integration is to be performed over the entire cycle. Any heat transfer to or from a system can be considered to consist of differential amounts of heat transfer. Then the cyclic integral of $\delta Q/T$ can be viewed as the sum of all these differential amounts of heat transfer divided by the temperature at the boundary.

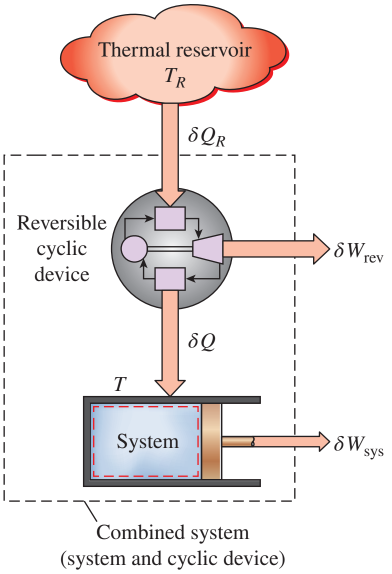


FIGURE 8–1
The system considered in the development of the Clausius inequality.

To demonstrate the validity of the Clausius inequality, consider a system connected to a thermal energy reservoir at a constant thermodynamic (i.e., absolute) temperature of T_R through a *reversible* cyclic device (Fig. 8–1). The cyclic device receives heat δQ_R from the reservoir and supplies heat δQ to the system whose temperature at that part of the boundary is T (a variable) while producing work δW_{rev}. The system produces work δW_{sys} as a result of this heat transfer. Applying the energy balance to the combined system identified by dashed lines yields

$$\delta W_C = \delta Q_R - dE_C$$

where δW_C is the total work of the combined system ($\delta W_{\text{rev}} + \delta W_{\text{sys}}$) and dE_C is the change in the total energy of the combined system. Considering that the cyclic device is a *reversible* one, we have

$$\frac{\delta Q_R}{T_R} = \frac{\delta Q}{T}$$

where the sign of δQ is determined with respect to the system (positive if *to* the system and negative if *from* the system) and the sign of δQ_R is determined with respect to the reversible cyclic device. Eliminating δQ_R from the two preceding relations yields

$$\delta W_C = T_R \frac{\delta Q}{L} - dE_C$$

We now let the system undergo a cycle while the cyclic device undergoes an integral number of cycles. Then the preceding relation becomes

$$W_C = T_R \oint \frac{\delta Q}{T}$$

since the cyclic integral of energy (the net change in the energy, which is a property, during a cycle) is zero. Here W_C is the cyclic integral of δW_C, and it represents the net work for the combined cycle.

It appears that the combined system is exchanging heat with a single thermal energy reservoir while involving (producing or consuming) work W_C during a cycle. On the basis of the Kelvin–Planck statement of the second law, which states that *no system can produce a net amount of work while operating in a cycle and exchanging heat with a single thermal energy reservoir,* we reason that W_C cannot be a work output, and thus it cannot be a positive quantity. Considering that T_R is the thermodynamic temperature and thus a positive quantity, we must have

$$\oint \frac{\delta Q}{T} \leq 0 \qquad \textbf{(8–1)}$$

which is the *Clausius inequality.* This inequality is valid for all thermodynamic cycles, reversible or irreversible, including the refrigeration cycles.

If no irreversibilities occur within the system as well as the reversible cyclic device, then the cycle undergone by the combined system is internally reversible. As such, it can be reversed. In the reversed cycle case, all the quantities have the same magnitude but the opposite sign. Therefore, the work W_C, which could not be a positive quantity in the regular case, cannot be a negative quantity in the reversed case. Then it follows that $W_{C,\text{int rev}} = 0$ since it cannot be a positive or negative quantity, and therefore

$$\oint \left(\frac{\delta Q}{T}\right)_{\text{int rev}} = 0 \qquad \textbf{(8–2)}$$

for internally reversible cycles. Thus, we conclude that *the equality in the Clausius inequality holds for totally or just internally reversible cycles and the inequality for the irreversible ones.*

To develop a relation for the definition of entropy, let us examine Eq. 8–2 more closely. Here we have a quantity whose cyclic integral is zero. Let us think for a moment what kinds of quantities can have this characteristic. We know that the cyclic integral of *work* is not zero. (It is a good thing that it is not. Otherwise, heat engines that work on a cycle such as steam power plants would produce zero net work.) Neither is the cyclic integral of heat.

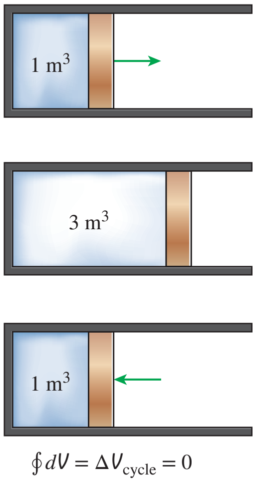
Now consider the volume occupied by a gas in a piston–cylinder device undergoing a cycle, as shown in Fig. 8–2. When the piston returns to its initial position at the end of a cycle, the volume of the gas also returns to its initial value. Thus, the net change in volume during a cycle is zero. This is also expressed as

$$\oint dV = 0 \qquad \textbf{(8–3)}$$

That is, the cyclic integral of volume (or any other property) is zero. Conversely, a quantity whose cyclic integral is zero depends on the *state* only and not the process path, and thus it is a property. Therefore, the quantity $(\delta Q/T)_{\text{int rev}}$ must represent a property in the differential form.

Clausius realized in 1865 that he had discovered a new thermodynamic property, and he chose to name this property **entropy**. It is designated S and is defined as

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \qquad (\text{kJ/K}) \qquad \textbf{(8–4)}$$



$$\oint dV = \Delta V_{\text{cycle}} = 0$$

FIGURE 8–2
The net change in volume (a property) during a cycle is always zero.

Entropy is an extensive property of a system and sometimes is referred to as *total entropy*. Entropy per unit mass, designated *s*, is an intensive property and has the unit kJ/kg·K. The term *entropy* is generally used to refer to both total entropy and entropy per unit mass since the context usually clarifies which one is meant.

The entropy change of a system during a process can be determined by integrating Eq. 8–4 between the initial and the final states:

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \qquad (\text{kJ/K}) \tag{8–5}$$

Notice that we have actually defined the *change* in entropy instead of entropy itself, just as we defined the change in energy instead of the energy itself when we developed the first-law relation. Absolute values of entropy are determined on the basis of the third law of thermodynamics, which is discussed later in this chapter. Engineers are usually concerned with the *changes* in entropy. Therefore, the entropy of a substance can be assigned a zero value at some arbitrarily selected reference state, and the entropy values at other states can be determined from Eq. 8–5 by choosing state 1 to be the reference state ($S = 0$) and state 2 to be the state at which entropy is to be determined.

To perform the integration in Eq. 8–5, one needs to know the relation between Q and T during a process. This relation is often not available, and the integral in Eq. 8–5 can be performed for a few cases only. For the majority of cases we have to rely on tabulated data for entropy.

Note that entropy is a property, and like all other properties, it has fixed values at fixed states. Therefore, the entropy change ΔS between two specified states is the same no matter what path, reversible or irreversible, is followed during a process (Fig. 8–3).

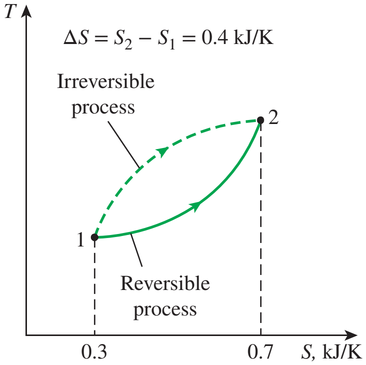


FIGURE 8–3
The entropy change between two specified states is the same whether the process is reversible or irreversible.

Also note that the integral of $\delta Q/T$ gives us the value of entropy change *only if* the integration is carried out along an *internally reversible* path between the two states. The integral of $\delta Q/T$ along an irreversible path is not a property, and in general, different values will be obtained when the integration is carried out along different irreversible paths. Therefore, even for irreversible processes, the entropy change should be determined by carrying out this integration along some convenient *imaginary* internally reversible path between the specified states.

A Special Case: Internally Reversible Isothermal Heat Transfer Processes

Recall that isothermal heat transfer processes are internally reversible. Therefore, the entropy change of a system during an internally reversible isothermal heat transfer process can be determined by performing the integration in Eq. 8–5:

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int rev}} = \int_1^2 \left(\frac{\delta Q}{T_0}\right)_{\text{int rev}} = \frac{1}{T_0}\int_1^2 (\delta Q)_{\text{int rev}}$$

which reduces to

$$\Delta S = \frac{Q}{T_0} \qquad (\text{kJ/K}) \tag{8–6}$$

where T_0 is the constant temperature of the system and Q is the heat transfer for the internally reversible process. Equation 8–6 is particularly useful for determining the entropy changes of thermal energy reservoirs that can absorb or supply heat indefinitely at a constant temperature.

Notice that the entropy change of a system during an internally reversible isothermal process can be positive or negative, depending on the direction of heat transfer. Heat transfer to a system increases the entropy of a system, whereas heat transfer from a system decreases it. In fact, losing heat is the only way the entropy of a system can be decreased.

EXAMPLE 8–1 Entropy Change During an Isothermal Process

A piston–cylinder device contains a liquid–vapor mixture of water at 300 K. During a constant-pressure process, 750 kJ of heat is transferred to the water. As a result, part of the liquid in the cylinder vaporizes. Determine the entropy change of the water during this process.

SOLUTION Heat is transferred to a liquid–vapor mixture of water in a piston–cylinder device at constant pressure. The entropy change of water is to be determined.

Assumptions No irreversibilities occur within the system boundaries during the process.

Analysis We take *all water* (liquid + vapor) in the cylinder as the system (Fig. 8–4). This is a *closed system* since no mass crosses the system boundary during the process. We note that the temperature of the system remains constant at 300 K during this process since the temperature of a pure substance remains constant at the saturation value during a phase-change process at constant pressure.

The system undergoes an internally reversible, isothermal process, and thus its entropy change can be determined directly from Eq. 8–6 to be

$$\Delta S_{\text{sys,isothermal}} = \frac{Q}{T_{\text{sys}}} = \frac{750\text{ kJ}}{300\text{ K}} = \mathbf{2.5\ kJ/K}$$

Discussion Note that the entropy change of the system is positive, as expected, since heat transfer is to the system.

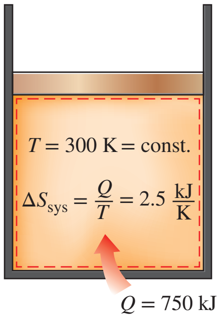


FIGURE 8–4
Schematic for Example 8–1.

8–2 ▪ THE INCREASE OF ENTROPY PRINCIPLE

Consider a cycle that is made up of two processes: process 1-2, which is arbitrary (reversible or irreversible), and process 2-1, which is internally reversible, as shown in Fig. 8–5. From the Clausius inequality,

$$\oint \frac{\delta Q}{T} \leq 0$$

or

$$\int_1^2 \frac{\delta Q}{T} + \int_2^1 \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \leq 0$$

The second integral in the previous relation is recognized as the entropy change $S_1 - S_2$. Therefore,

$$\int_1^2 \frac{\delta Q}{T} + S_1 - S_2 \leq 0$$

which can be rearranged as

$$S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T} \qquad \textbf{(8–7)}$$

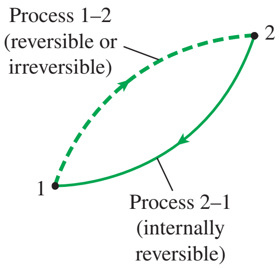


FIGURE 8–5
A cycle composed of a reversible and an irreversible process.

It can also be expressed in differential form as

$$dS \geq \frac{\delta Q}{T} \tag{8–8}$$

where the equality holds for an internally reversible process and the inequality for an irreversible process. We may conclude from these equations that the entropy change of a closed system during an irreversible process is greater than the integral of $\delta Q/T$ evaluated for that process. In the limiting case of a reversible process, these two quantities become equal. We again emphasize that T in these relations is the *thermodynamic temperature* at the *boundary* where the differential heat δQ is transferred between the system and the surroundings.

The quantity $\Delta S = S_2 - S_1$ represents the *entropy change* of the system. For a reversible process, it becomes equal to $\int_1^2 \delta Q/T$, which represents the *entropy transfer* with heat.

The inequality sign in the preceding relations is a constant reminder that the entropy change of a closed system during an irreversible process is always greater than the entropy transfer. That is, some entropy is *generated* or *created* during an irreversible process, and this generation is due entirely to the presence of irreversibilities. The entropy generated during a process is called **entropy generation** and is denoted by S_{gen}. Noting that the difference between the entropy change of a closed system and the entropy transfer is equal to entropy generation, Eq. 8–7 can be rewritten as an equality as

$$\Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}} \tag{8–9}$$

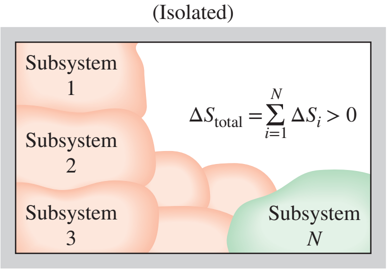


FIGURE 8–6

The entropy change of an isolated system is the sum of the entropy changes of its components and is never less than zero.

Note that the entropy generation S_{gen} is always a *positive* quantity or zero. Its value depends on the process, and thus it is *not* a property of the system. Also, in the absence of any entropy transfer, the entropy change of a system is equal to the entropy generation.

Equation 8–7 has far-reaching implications in thermodynamics. For an isolated system (or simply an adiabatic closed system), the heat transfer is zero, and Eq. 8–7 reduces to

$$\Delta S_{\text{isolated}} \geq 0 \tag{8–10}$$

This equation can be expressed as *the entropy of an isolated system during a process always increases or, in the limiting case of a reversible process, remains constant.* In other words, it *never* decreases. This is known as the **increase of entropy principle**. Note that in the absence of any heat transfer, entropy change is due to irreversibilities only, and their effect is always to increase entropy.

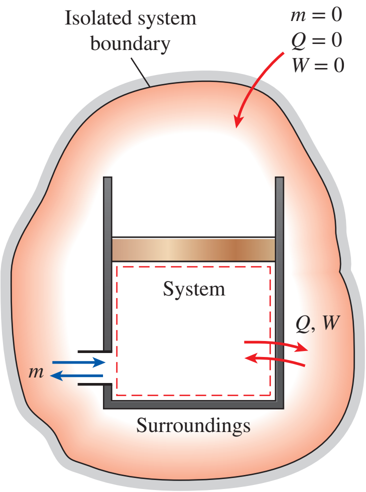


FIGURE 8–7

A system and its surroundings form an isolated system.

Entropy is an extensive property, thus the total entropy of a system is equal to the sum of the entropies of the parts of the system. An isolated system may consist of any number of subsystems (Fig. 8–6). A system and its surroundings, for example, constitute an isolated system since both can be enclosed by a sufficiently large arbitrary boundary across which there is no heat, work, or mass transfer (Fig. 8–7). Therefore, a system and its surroundings can be viewed as the two subsystems of an isolated system, and the entropy change of this isolated system during a process is the sum of the entropy changes of the system and its surroundings, which is equal to the entropy generation since an isolated system involves no entropy transfer. That is,

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0 \tag{8–11}$$

where the equality holds for reversible processes and the inequality for irreversible ones. Note that ΔS_{surr} refers to the change in the entropy of the surroundings as a result of the occurrence of the process under consideration.

Since no actual process is truly reversible, we can conclude that some entropy is generated during a process, and therefore the entropy of the universe, which can be considered to be an isolated system, is continuously increasing. The more irreversible a process, the larger the entropy generated during that process. No entropy is generated during reversible processes ($S_{\text{gen}} = 0$).

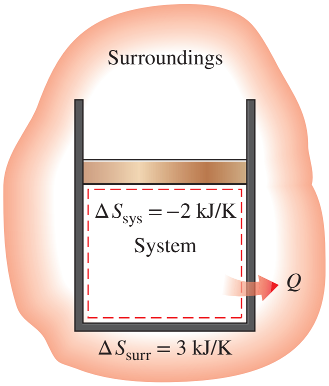
Entropy increase of the universe is a major concern not only to engineers but also to philosophers, theologians, economists, and environmentalists since entropy is viewed as a measure of the disorder (or "mixed-up-ness") in the universe.

The increase of entropy principle does not imply that the entropy of a system cannot decrease. The entropy change of a system *can* be negative during a process (Fig. 8–8), but entropy generation cannot. The increase of entropy principle can be summarized as follows:

$$S_{\text{gen}} \begin{cases} > 0 & \text{irreversible process} \\ = 0 & \text{reversible process} \\ < 0 & \text{impossible process} \end{cases}$$

This relation serves as a criterion in determining whether a process is reversible, irreversible, or impossible.

Things in nature have a tendency to change until they attain a state of equilibrium. The increase of entropy principle dictates that the entropy of an isolated system increases until the entropy of the system reaches a *maximum* value. At that point, the system is said to have reached an equilibrium state since the increase of entropy principle prohibits the system from undergoing any change of state that results in a decrease in entropy.



$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} = 1$ kJ/K

FIGURE 8–8
The entropy change of a system can be negative, but the entropy generation cannot.

Some Remarks About Entropy

In light of the preceding discussions, we draw the following conclusions:

1. Processes can occur in a *certain* direction only, not in *any* direction. A process must proceed in the direction that complies with the increase of entropy principle, that is, $S_{\text{gen}} \geq 0$. A process that violates this principle is impossible. This principle often forces chemical reactions to come to a halt before reaching completion.
2. Entropy is a *nonconserved property,* and there is *no* such thing as the *conservation of entropy principle.* Entropy is conserved during the idealized reversible processes only and increases during *all* actual processes.
3. The performance of engineering systems is degraded by the presence of irreversibilities, and *entropy generation* is a measure of the magnitudes of the irreversibilities present during that process. The greater the extent of irreversibilities, the greater the entropy generation. Therefore, entropy generation can be used as a quantitative measure of irreversibilities associated with a process. It is also used to establish criteria for the performance of engineering devices. This point is illustrated further in Example 8–2.

EXAMPLE 8–2 Entropy Generation During Heat Transfer Processes

A heat source at 800 K loses 2000 kJ of heat to a sink at (*a*) 500 K and (*b*) 750 K. Determine which heat transfer process is more irreversible.

SOLUTION Heat is transferred from a heat source to two heat sinks at different temperatures. The heat transfer process that is more irreversible is to be determined.

Analysis A sketch of the reservoirs is shown in Fig. 8–9. Both cases involve heat transfer through a finite temperature difference, and therefore both are irreversible. The magnitude of the irreversibility associated with each process can be determined by calculating

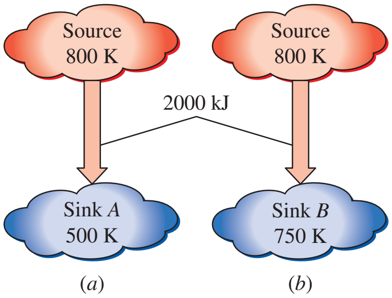


FIGURE 8–9
Schematic for Example 8–2.

the total entropy change for each case. The total entropy change for a heat transfer process involving two reservoirs (a source and a sink) is the sum of the entropy changes of each reservoir since the two reservoirs form an adiabatic system.

Or do they? The problem statement gives the impression that the two reservoirs are in direct contact during the heat transfer process. But this cannot be the case since the temperature at a point can have only one value, and thus it cannot be 800 K on one side of the point of contact and 500 K on the other side. In other words, the temperature function cannot have a jump discontinuity. Therefore, it is reasonable to assume that the two reservoirs are separated by a partition through which the temperature drops from 800 K on one side to 500 K (or 750 K) on the other. In that case, the entropy change of the partition should also be considered when evaluating the total entropy change for this process. However, considering that entropy is a property and the values of properties depend on the state of a system, we can argue that the entropy change of the partition is zero since the partition appears to have undergone a *steady* process and thus experienced no change in its properties at any point. We base this argument on the fact that the temperature on both sides of the partition and thus throughout remains constant during this process. Therefore, we are justified to assume that $\Delta S_{\text{partition}} = 0$ since the entropy (as well as the energy) content of the partition remains constant during this process.

The entropy change for each reservoir can be determined from Eq. 8–6 since each reservoir undergoes an internally reversible, isothermal process.

(*a*) For the heat transfer process to a sink at 500 K:

$$\Delta S_{\text{source}} = \frac{Q_{\text{source}}}{T_{\text{source}}} = \frac{-2000\text{ kJ}}{800\text{ K}} = -2.5\text{ kJ/K}$$

$$\Delta S_{\text{sink}} = \frac{Q_{\text{sink}}}{T_{\text{sink}}} = \frac{2000\text{ kJ}}{500\text{ K}} = 4.0\text{ kJ/K}$$

and

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{source}} + \Delta S_{\text{sink}} = (-2.5 + 4.0)\text{ kJ/K} = \mathbf{1.5\text{ kJ/K}}$$

Therefore, 1.5 kJ/K of entropy is generated during this process. Noting that both reservoirs have undergone internally reversible processes, the entire entropy generation took place in the partition.

(*b*) Repeating the calculations in part (*a*) for a sink temperature of 750 K, we obtain

$$\Delta S_{\text{source}} = -2.5\text{ kJ/K}$$
$$\Delta S_{\text{sink}} = 2.7\text{ kJ/K}$$

and

$$S_{\text{gen}} = \Delta S_{\text{total}} = (-2.5 + 2.7)\text{kJ/K} = \mathbf{0.2\text{ kJ/K}}$$

The total entropy change for the process in part (*b*) is smaller, and therefore it is less irreversible. This is expected since the process in (*b*) involves a smaller temperature difference and thus a smaller irreversibility.

Discussion The irreversibilities associated with both processes could be eliminated by operating a Carnot heat engine between the source and the sink. For this case it can be shown that $\Delta S_{\text{total}} = 0$.

8–3 ▪ ENTROPY CHANGE OF PURE SUBSTANCES

Entropy is a property, and thus the value of entropy of a system is fixed once the state of the system is fixed. Specifying two intensive independent properties fixes the state of a simple compressible system, and thus the value of entropy and of other properties at that state. Starting with its defining relation, the entropy change of a substance can be expressed in terms of other properties (see Sec. 8–7). But in general, these relations are too complicated and are not practical to use for hand calculations. Therefore, using a suitable reference state, the entropies of substances are evaluated from measurable property data following rather involved computations, and the results are tabulated in the same manner as the other properties such as υ, u, and h (Fig. 8–10).

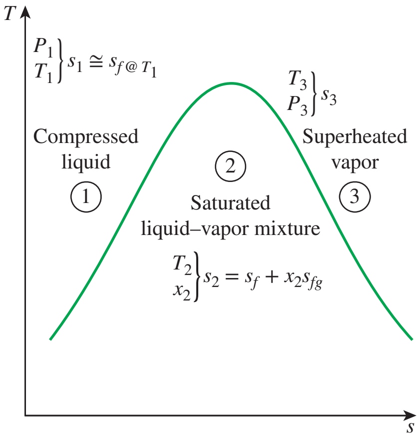


FIGURE 8–10
The entropy of a pure substance is determined from the tables (like other properties).

The entropy values in the property tables are given relative to an arbitrary reference state. In steam tables the entropy of saturated liquid s_f at 0.01°C is assigned the value of zero. For refrigerant-134a, the zero value is assigned to saturated liquid at –40°C. The entropy values become negative at temperatures below the reference value.

The value of entropy at a specified state is determined just like any other property. In the compressed liquid and superheated vapor regions, it can be obtained directly from the tables at the specified state. In the saturated mixture region, it is determined from

$$s = s_f + xs_{fg} \qquad \text{(kJ/kg·K)}$$

where x is the quality and s_f and s_{fg} values are listed in the saturation tables. In the absence of compressed liquid data, the entropy of the compressed liquid can be approximated by the entropy of the saturated liquid at the given temperature:

$$s_{@\,T,P} \cong s_{f\,@\,T} \qquad \text{(kJ/kg·K)}$$

The entropy change of a specified mass m (a closed system) during a process is simply

$$\Delta S = m\Delta s = m(s_2 - s_1) \qquad \text{(kJ/K)} \qquad \textbf{(8–12)}$$

which is the difference between the entropy values at the final and initial states.

When studying the second-law aspects of processes, entropy is commonly used as a coordinate on diagrams such as the T-s and h-s diagrams. The general characteristics of the T-s diagram of pure substances are shown in Fig. 8–11 using data for water. Notice from this diagram that the constant-volume lines are steeper than the constant-pressure lines and the constant-pressure lines are parallel to the constant-temperature lines in the saturated liquid–vapor mixture region. Also, the constant-pressure lines almost coincide with the saturated liquid line in the compressed liquid region.

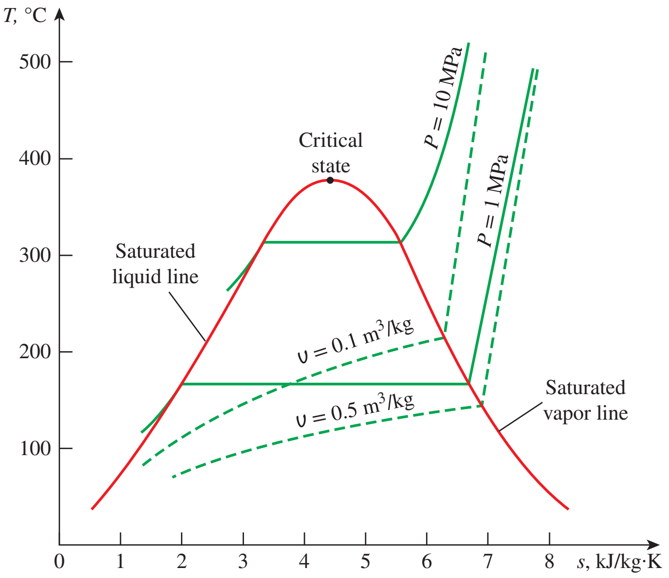


FIGURE 8–11
Schematic of the T-s diagram for water.

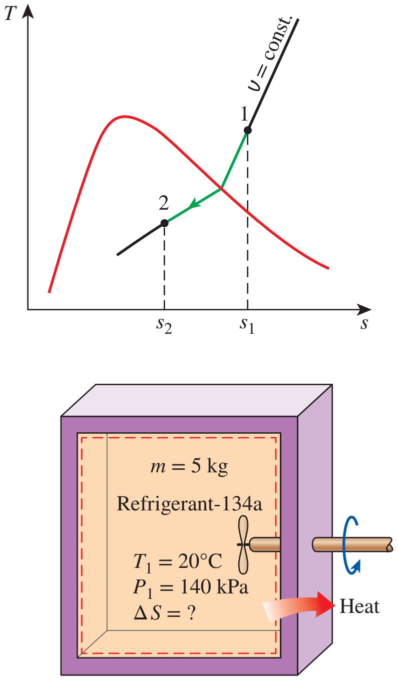


FIGURE 8–12

Schematic and *T*-*s* diagram for Example 8–3.

EXAMPLE 8–3 Entropy Change of a Substance in a Tank

A rigid tank contains 5 kg of refrigerant-134a initially at 20°C and 140 kPa. The refrigerant is now cooled while being stirred until its pressure drops to 100 kPa. Determine the entropy change of the refrigerant during this process.

SOLUTION The refrigerant in a rigid tank is cooled while being stirred. The entropy change of the refrigerant is to be determined.

Assumptions The volume of the tank is constant and thus $\mathrm{v}_2 = \mathrm{v}_1$.

Analysis We take the refrigerant in the tank as the system (Fig. 8–12). This is a *closed system* since no mass crosses the system boundary during the process. We note that the change in entropy of a substance during a process is simply the difference between the entropy values at the final and initial states. The initial state of the refrigerant is completely specified.

Recognizing that the specific volume remains constant during this process, the properties of the refrigerant at both states are

State 1:

$$\left.\begin{aligned} P_1 &= 140 \text{ kPa} \\ T_1 &= 20°\text{C} \end{aligned}\right\} \begin{aligned} s_1 &= 1.0625 \text{ kJ/kg·K} \\ \mathrm{v}_1 &= 0.16544 \text{ m}^3\text{/kg} \end{aligned}$$

State 2:

$$\left.\begin{aligned} P_2 &= 100 \text{ kPa} \\ \mathrm{v}_2 &= \mathrm{v}_1 \end{aligned}\right\} \begin{aligned} \mathrm{v}_f &= 0.0007258 \text{ m}^3\text{/kg} \\ \mathrm{v}_g &= 0.19255 \text{ m}^3\text{/kg} \end{aligned}$$

The refrigerant is a saturated liquid–vapor mixture at the final state since $\mathrm{v}_f < \mathrm{v}_2 < \mathrm{v}_g$ at 100 kPa pressure. Therefore, we need to determine the quality first:

$$x_2 = \frac{\mathrm{v}_2 - \mathrm{v}_f}{\mathrm{v}_{fg}} = \frac{0.16544 - 0.0007258}{0.19255 - 0.0007258} = 0.859$$

Thus,

$$s_2 = s_f + x_2 s_{fg} = 0.07182 + (0.859)(0.88008) = 0.8278 \text{ kJ/kg·K}$$

Then, the entropy change of the refrigerant during this process is

$$\begin{aligned} \Delta S &= m(s_2 - s_1) = (5 \text{ kg})(0.8278 - 1.0625) \text{ kJ/kg·K} \\ &= \mathbf{-1.173 \text{ kJ/K}} \end{aligned}$$

Discussion The negative sign indicates that the entropy of the system is decreasing during this process. This is not a violation of the second law, however, since it is the *entropy generation* S_{gen} that cannot be negative.

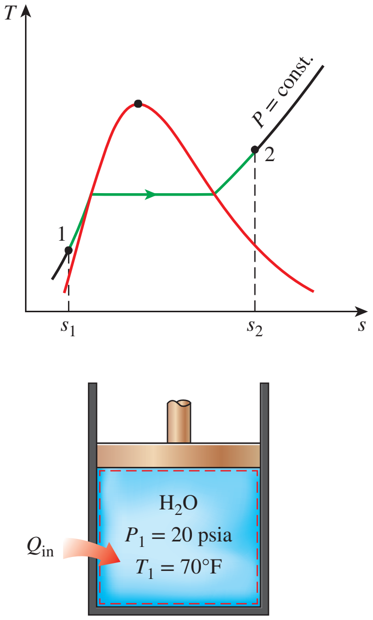


FIGURE 8–13

Schematic and *T*-*s* diagram for Example 8–4.

EXAMPLE 8–4 Entropy Change During a Constant-Pressure Process

A piston–cylinder device initially contains 3 lbm of liquid water at 20 psia and 70°F. The water is now heated at constant pressure by the addition of 3450 Btu of heat. Determine the entropy change of the water during this process.

SOLUTION Liquid water in a piston–cylinder device is heated at constant pressure. The entropy change of water is to be determined.

Assumptions **1** The tank is stationary and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$. **2** The process is quasi-equilibrium. **3** The pressure remains constant during the process and thus $P_2 = P_1$.

Analysis We take the water in the cylinder as the *system* (Fig. 8–13). This is a *closed system* since no mass crosses the system boundary during the process. We note that a piston–cylinder device typically involves a moving boundary and thus boundary work W_b. Also, heat is transferred to the system.

Water exists as a compressed liquid at the initial state since its pressure is greater than the saturation pressure of 0.3632 psia at 70°F. By approximating the compressed liquid as a saturated liquid at the given temperature, the properties at the initial state are

$$\text{State 1:}\quad \left.\begin{array}{l} P_1 = 20\text{ psia} \\ T_1 = 70°\text{F} \end{array}\right\} \begin{array}{l} s_1 \cong s_{f\,@\,70°\text{F}} = 0.07459\text{ Btu/lbm·R} \\ h_1 \cong h_{f\,@\,70°\text{F}} = 38.08\text{ Btu/lbm} \end{array}$$

At the final state, the pressure is still 20 psia, but we need one more property to fix the state. This property is determined from the energy balance,

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$Q_{\text{in}} - W_b = \Delta U$$

$$Q_{\text{in}} = \Delta H = m(h_2 - h_1)$$

$$3450\text{ Btu} = (3\text{ lbm})(h_2 - 38.08\text{ Btu/lbm})$$

$$h_2 = 1188.1\text{ Btu/lbm}$$

since $\Delta U + W_b = \Delta H$ for a constant-pressure quasi-equilibrium process. Then,

$$\text{State 2:}\quad \left.\begin{array}{l} P_2 = 20\text{ psia} \\ h_2 = 1188.1\text{ Btu/lbm} \end{array}\right\} \begin{array}{l} s_2 = 1.7761\text{ Btu/lbm·R} \\ \text{(Table A-6E, interpolation)} \end{array}$$

Therefore, the entropy change of water during this process is

$$\Delta S = m(s_2 - s_1) = (3\text{ lbm})(1.7761 - 0.07459)\text{ Btu/lbm·R}$$
$$= \mathbf{5.105\ Btu/R}$$

8–4 ▪ ISENTROPIC PROCESSES

We mentioned earlier that the entropy of a fixed mass can be changed by (1) heat transfer and (2) irreversibilities. Then it follows that the entropy of a fixed mass does not change during a process that is *internally reversible* and *adiabatic* (Fig. 8–14). A process during which the entropy remains constant is called an **isentropic process**. It is characterized by

$$\textit{Isentropic process:}\qquad \Delta s = 0 \quad \text{or} \quad s_2 = s_1 \qquad \text{(kJ/kg·K)} \qquad \textbf{(8–13)}$$

That is, a substance will have the same entropy value at the end of the process as it does at the beginning if the process is carried out in an isentropic manner.

Many engineering systems or devices such as pumps, turbines, nozzles, and diffusers are essentially adiabatic in their operation, and they perform best when the irreversibilities, such as the friction associated with the process, are minimized. Therefore, an isentropic process can serve as an appropriate model for actual processes. Also, isentropic processes enable us to define efficiencies for processes to compare the actual performance of these devices to the performance under idealized conditions.

It should be recognized that a *reversible adiabatic* process is necessarily isentropic ($s_2 = s_1$), but an *isentropic* process is not necessarily a reversible adiabatic process. (The entropy increase of a substance during a process as a result of irreversibilities may be offset by a decrease in entropy as a result of heat losses, for example.) However, the term *isentropic process* is customarily used in thermodynamics to imply an *internally reversible, adiabatic process.*

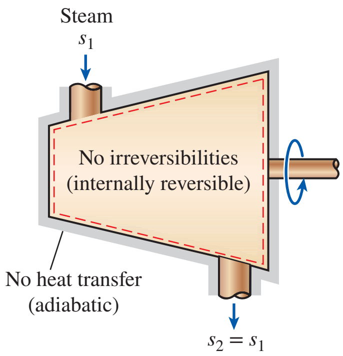


FIGURE 8–14
During an internally reversible, adiabatic (isentropic) process, the entropy remains constant.

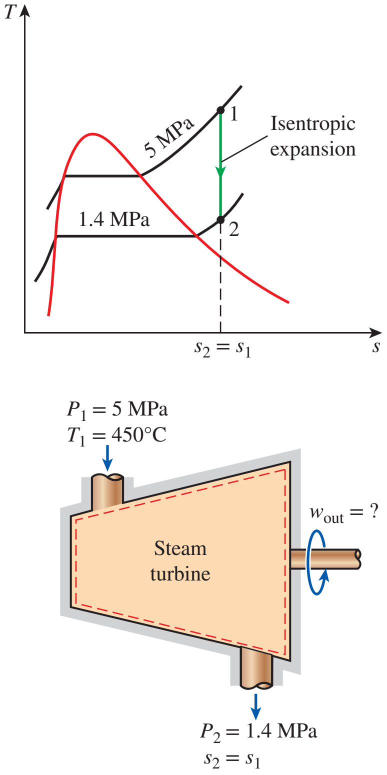


FIGURE 8–15
Schematic and *T-s* diagram for Example 8–5.

EXAMPLE 8–5 Isentropic Expansion of Steam in a Turbine

Steam enters an adiabatic turbine at 5 MPa and 450°C and leaves at a pressure of 1.4 MPa. Determine the work output of the turbine per unit mass of steam if the process is reversible.

SOLUTION Steam is expanded in an adiabatic turbine to a specified pressure in a reversible manner. The work output of the turbine is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$, $\Delta E_{CV} = 0$, and $\Delta S_{CV} = 0$. **2** The process is reversible. **3** Kinetic and potential energies are negligible. **4** The turbine is adiabatic and thus there is no heat transfer.

Analysis We take the *turbine* as the system (Fig. 8–15). This is a *control volume* since mass crosses the system boundary during the process. We note that there is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$.

The power output of the turbine is determined from the rate form of the energy balance,

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{system}/dt}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}} \nearrow^{0\ \text{(steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{W}_{out} + \dot{m}h_2 \quad (\text{since } \dot{Q} = 0,\ \text{ke} \cong \text{pe} \cong 0)$$

$$\dot{W}_{out} = \dot{m}(h_1 - h_2)$$

The inlet state is completely specified since two properties are given. But only one property (pressure) is given at the final state, and we need one more property to fix it. The second property comes from the observation that the process is reversible and adiabatic, and thus isentropic. Therefore, $s_2 = s_1$, and

$$\text{State 1:} \quad \left.\begin{array}{l} P_1 = 5 \text{ MPa} \\ T_1 = 450°\text{C} \end{array}\right\} \begin{array}{l} h_1 = 3317.2 \text{ kJ/kg} \\ s_1 = 6.8210 \text{ kJ/kg·K} \end{array}$$

$$\text{State 2:} \quad \left.\begin{array}{l} P_2 = 1.4 \text{ MPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 2967.4 \text{ kJ/kg}$$

Then, the work output of the turbine per unit mass of the steam becomes

$$w_{out} = h_1 - h_2 = 3317.2 - 2967.4 = \mathbf{349.8\ kJ/kg}$$

8–5 ▪ PROPERTY DIAGRAMS INVOLVING ENTROPY

Property diagrams serve as great visual aids in the thermodynamic analysis of processes. We have used *P-v* and *T-v* diagrams extensively in previous chapters in conjunction with the first law of thermodynamics. In the second-law analysis, it is very helpful to plot the processes on diagrams for which one of the coordinates is entropy. The two diagrams commonly used in the second-law analysis are the *temperature-entropy* and the *enthalpy-entropy* diagrams.

Consider the defining equation of entropy (Eq. 8–4). It can be rearranged as

$$\delta Q_{int\ rev} = T\,dS \qquad \text{(kJ)} \tag{8–14}$$

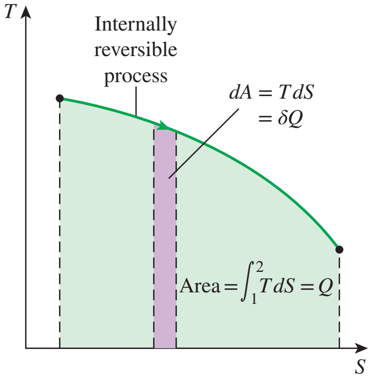


FIGURE 8–16
On a *T-S* diagram, the area under the process curve represents the heat transfer for internally reversible processes.

As shown in Fig. 8–16, $\delta Q_{int\ rev}$ corresponds to a differential area on a *T-S* diagram. The total heat transfer during an internally reversible process is determined by integration to be

$$Q_{int\ rev} = \int_1^2 T\,dS \qquad \text{(kJ)} \tag{8–15}$$

which corresponds to the area under the process curve on a T-S diagram. Therefore, we conclude that *the area under the process curve on a T-S diagram represents heat transfer during an internally reversible process.* This is somewhat analogous to reversible boundary work being represented by the area under the process curve on a P-V diagram. Note that the area under the process curve represents heat transfer for processes that are internally (or totally) reversible. The area has no meaning for irreversible processes.

Equations 8–14 and 8–15 can also be expressed on a unit-mass basis as

$$\delta q_{\text{int rev}} = T\,ds \qquad (\text{kJ/kg}) \tag{8–16}$$

and

$$q_{\text{int rev}} = \int_1^2 T\,ds \qquad (\text{kJ/kg}) \tag{8–17}$$

To perform the integrations in Eqs. 8–15 and 8–17, one needs to know the relationship between T and s during a process. One special case for which these integrations can be performed easily is the *internally reversible isothermal process.* It yields

$$Q_{\text{int rev}} = T_0\,\Delta S \qquad (\text{kJ}) \tag{8–18}$$

or

$$q_{\text{int rev}} = T_0\,\Delta s \qquad (\text{kJ/kg}) \tag{8–19}$$

where T_0 is the constant temperature and ΔS is the entropy change of the system during the process.

An isentropic process on a T-s diagram is easily recognized as a *vertical-line segment.* This is expected since an isentropic process involves no heat transfer, and therefore the area under the process path must be zero (Fig. 8–17). The T-s diagrams serve as valuable tools for visualizing the second-law aspects of processes and cycles, and thus they are frequently used in thermodynamics. The T-s diagram of water is given in the appendix in Fig. A–9.

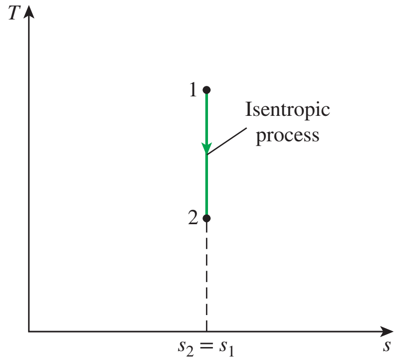


FIGURE 8–17
The isentropic process appears as a vertical line segment on a T-s diagram.

Another diagram commonly used in engineering is the enthalpy-entropy diagram, which is quite valuable in the analysis of steady-flow devices such as turbines, compressors, and nozzles. The coordinates of an h-s diagram represent two properties of major interest: enthalpy, which is a primary property in the first-law analysis of the steady-flow devices, and entropy, which is the property that accounts for irreversibilities during adiabatic processes. In analyzing the steady flow of steam through an adiabatic turbine, for example, the vertical distance between the inlet and the exit states Δh is a measure of the work output of the turbine, and the horizontal distance Δs is a measure of the irreversibilities associated with the process (Fig. 8–18).

The h-s diagram is also called a **Mollier diagram** after the German scientist R. Mollier (1863–1935). An h-s diagram is given in the appendix for steam in Fig. A–10.

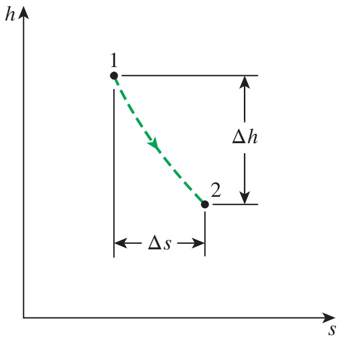


FIGURE 8–18
For adiabatic steady-flow devices, the vertical distance Δh on an h-s diagram is a measure of work, and the horizontal distance Δs is a measure of irreversibilities.

EXAMPLE 8–6 The *T-S* Diagram of the Carnot Cycle

Show the Carnot cycle on a T-S diagram and indicate the areas that represent the heat supplied Q_H, heat rejected Q_L, and the net work output $W_{\text{net,out}}$ on this diagram.

SOLUTION The Carnot cycle is to be shown on a T-S diagram, and the areas that represent Q_H, Q_L, and $W_{\text{net,out}}$ are to be indicated.

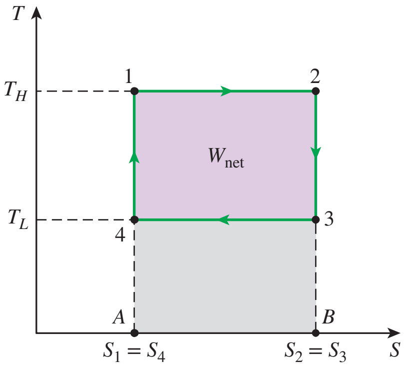


FIGURE 8–19
The *T-S* diagram of a Carnot cycle (Example 8–6).

Analysis Recall that the Carnot cycle is made up of two reversible isothermal (T = constant) processes and two isentropic (s = constant) processes. These four processes form a rectangle on a *T-S* diagram, as shown in Fig. 8–19.

On a *T-S* diagram, the area under the process curve represents the heat transfer for that process. Thus the area $A12B$ represents Q_H, the area $A43B$ represents Q_L, and the difference between these two (the area in color) represents the net work since

$$W_{\text{net,out}} = Q_H - Q_L$$

Therefore, the area enclosed by the path of a cycle (area 1234) on a *T-S* diagram represents the net work. Recall that the area enclosed by the path of a cycle also represents the net work on a *P-V* diagram.

8–6 ▪ WHAT IS ENTROPY?

It is clear from the previous discussion that entropy is a useful property and serves as a valuable tool in the second-law analysis of engineering devices. But this does not mean that we know and understand entropy well. Because we do not. In fact, we cannot even give an adequate answer to the question, what is entropy? Not being able to describe entropy fully, however, does not take anything away from its usefulness. We could not define *energy* either, but it did not interfere with our understanding of energy transformations and the conservation of energy principle. Granted, entropy is not a household word like energy. But with continued use, our understanding of entropy will deepen, and our appreciation of it will grow. The next discussion should shed some light on the physical meaning of entropy by considering the microscopic nature of matter.

Entropy can be viewed as a measure of *molecular disorder,* or *molecular randomness.* As a system becomes more disordered, the positions of the molecules become less predictable and the entropy increases. Thus, it is not surprising that the entropy of a substance is lowest in the solid phase and highest in the gas phase (Fig. 8–20). In the solid phase, the molecules of a substance continually oscillate about their equilibrium positions, but they cannot move relative to each other, and their position at any instant can be predicted with good certainty. In the gas phase, however, the molecules move about at random, collide with each other, and change direction, making it extremely difficult to predict accurately the microscopic state of a system at any instant. Associated with this molecular chaos is a high value of entropy.

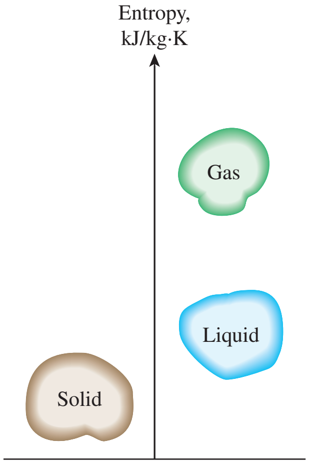


FIGURE 8–20
The level of molecular disorder (entropy) of a substance increases as it melts or evaporates.

When viewed microscopically from a statistical thermodynamics point of view, an isolated system that appears to be at a state of equilibrium actually exhibits a high level of activity because of the continual motion of the molecules. To each state of macroscopic equilibrium, there corresponds a large number of molecular microscopic states or molecular configurations. Boltzmann first hypothesized that the entropy of a system at a specified macrostate is related to the total number of possible relevant microstates of that system, W (from *Wahrscheinlichkeit,* the German word for probability). This thought was formulated later by Planck using a constant k with the entropy unit of J/K named after Boltzmann (and inscribed on Boltzmann's tombstone) as

$$S = k \ln W \tag{8–20a}$$

which is known as the **Boltzmann relation**. The thermal motion randomness or disorder as related to entropy was later generalized by Gibbs as a measure of the sum of all microstates' uncertainties, i.e., probabilities, as

$$S = -k\sum p_i \log p_i \tag{8–20b}$$

Gibbs' formulation is more general since it allows for nonuniform probability, p_i, of microstates. With an increase of particle momenta or thermal disorder and volume occupied, more information is required for the characterization of the system, relative to more ordered systems. Gibbs' formulation reduces to the Boltzmann relation for the equi-probable, uniform probability of all W microstates since $p_i = 1/W = \text{constant} \ll 1$.

From a microscopic point of view, the entropy of a system increases whenever the thermal randomness or disorder (i.e., the number of possible relevant molecular microstates corresponding to a given bulk macrostate) of a system increases. Thus, entropy can be viewed as a measure of thermal randomness or molecular disorder, which increases any time an isolated system undergoes a process.

As mentioned earlier, the molecules of a substance in solid phase continually oscillate, creating an uncertainty about their position. These oscillations, however, fade as the temperature is decreased, and the molecules supposedly become motionless at absolute zero. This represents a state of ultimate molecular order (and minimum energy). Therefore, *the entropy of a pure crystalline substance at absolute zero temperature is zero* since there is no uncertainty about the state of the molecules at that instant (Fig. 8–21). This statement is known as the **third law of thermodynamics**. The third law of thermodynamics provides an absolute reference point for the determination of entropy. The entropy determined relative to this point is called **absolute entropy**, and it is extremely useful in the thermodynamic analysis of chemical reactions. Notice that the entropy of a substance that is not pure crystalline (such as a solid solution) is not zero at absolute zero temperature. This is because more than one molecular configuration exists for such substances, which introduces some uncertainty about the microscopic state of the substance.

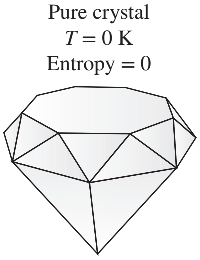


FIGURE 8–21
A pure crystalline substance at absolute zero temperature is in perfect order, and its entropy is zero (the third law of thermodynamics).

Molecules in the gas phase possess a considerable amount of kinetic energy. However, we know that no matter how large their kinetic energies are, the gas molecules do not rotate a paddle wheel inserted into the container and produce work. This is because the gas molecules, and the energy they possess, are disorganized. Probably the number of molecules trying to rotate the wheel in one direction at any instant is equal to the number of molecules that are trying to rotate it in the opposite direction, causing the wheel to remain motionless. Therefore, we cannot extract any useful work directly from disorganized energy (Fig. 8–22).

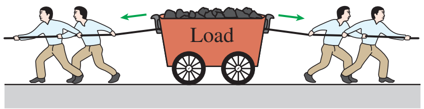


FIGURE 8–22
Disorganized energy does not create much useful effect, no matter how large it is.

Now consider a rotating shaft shown in Fig. 8–23. This time the energy of the molecules is completely organized since the molecules of the shaft are rotating in the same direction together. This organized energy can readily be used to perform useful tasks such as raising a weight or generating electricity. Being an organized form of energy, work is free of disorder or randomness and thus free of entropy. *There is no entropy transfer associated with energy transfer as work*. Therefore, in the absence of any friction, the process of raising a weight by a rotating shaft (or a flywheel) does not produce any entropy. Any process that does not produce a net entropy is reversible, and thus the process just described can be reversed by lowering the weight. Therefore, energy is not degraded during this process, and no potential to do work is lost.

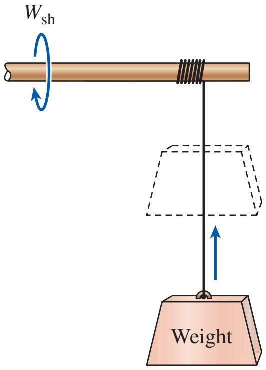


FIGURE 8–23
In the absence of friction, raising a weight by a rotating shaft does not create any disorder (entropy), and thus energy is not degraded during this process.

Instead of raising a weight, let us operate the paddle wheel in a container filled with a gas, as shown in Fig. 8–24. The paddle-wheel work in this case is converted to the internal energy of the gas, as evidenced by a rise in gas temperature, creating a higher level of molecular disorder in the container. This process is quite different from raising a weight since the organized paddle-wheel energy is now converted to a highly disorganized form of energy, which cannot be converted back to the paddle wheel as rotational kinetic energy. Only a portion of this energy can be converted to work by partially reorganizing it through the use of a heat engine. Therefore, energy is degraded during this process, the ability to do work is reduced, molecular disorder is produced, and associated with all this is an increase in entropy.

The *quantity* of energy is always preserved during an actual process (the first law), but the *quality* is bound to decrease (the second law). This decrease in quality is always accompanied by an increase in entropy. As an example, consider the transfer of 10 kJ of energy as heat from a hot medium to a cold one. At the end of the process, we still have the 10 kJ of energy, but at a lower temperature and thus at a lower quality.

Heat is, in essence, a form of *disorganized energy,* and some disorganization (entropy) flows with heat (Fig. 8–25). As a result, the entropy and the level of molecular disorder or randomness of the hot body decreases with the entropy, and the level of molecular disorder of the cold body increases. The second law requires that the increase in entropy of the cold body be greater than the decrease in entropy of the hot body, and thus the net entropy of the combined system (the cold body and the hot body) increases. That is, the combined system is at a state of greater disorder at the final state. Thus we can conclude that processes can occur only in the direction of increased overall entropy or molecular disorder. That is, the entire universe is getting more and more chaotic every day.

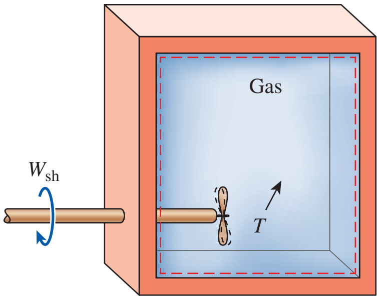


FIGURE 8–24
The paddle-wheel work done on a gas increases the level of disorder (entropy) of the gas, and thus energy is degraded during this process.

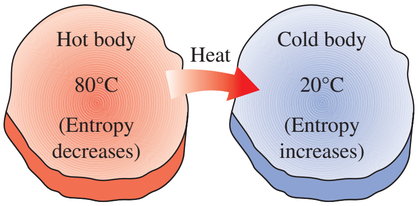


FIGURE 8–25
During a heat transfer process, the net entropy increases. (The increase in the entropy of the cold body more than offsets the decrease in the entropy of the hot body.)

Entropy and Entropy Generation in Daily Life

The concept of entropy can also be applied to other areas. Entropy can be viewed as a measure of disorder or disorganization in a system. Likewise, entropy generation can be viewed as a measure of disorder or disorganization generated during a process. The concept of entropy is not used in daily life nearly as extensively as the concept of energy, even though entropy is readily applicable to various aspects of daily life. The extension of the entropy concept to nontechnical fields is not a novel idea. It has been the topic of several articles, and even some books. Next, we present several ordinary events and show their relevance to the concept of entropy and entropy generation.

Efficient people lead low-entropy (highly organized) lives. They have a place for everything (minimum uncertainty), and it takes minimum energy for them to locate something. Inefficient people, on the other hand, are disorganized and lead high-entropy lives. It takes them minutes (if not hours) to find something they need, and they are likely to create a bigger disorder as they are searching since they will probably conduct the search in a disorganized manner. People leading high-entropy lifestyles are always on the run and never seem to catch up.

Consider two identical buildings, each containing one million books. In the first building, the books are *piled* on top of each other, whereas in the second building they are *highly organized, shelved, and indexed* for easy reference. There is no doubt about which building a student will prefer to go to for checking out a certain book. Yet, some may argue from the first-law point of view that these two buildings are equivalent since the mass and knowledge content of the two buildings are identical, despite the high level of disorganization (entropy) in the first building. This example illustrates that any realistic comparisons should involve the second-law point of view.

Two *textbooks* that seem to be identical because both cover basically the same topics and present the same information may actually be *very* different depending on *how* they cover the topics. After all, two seemingly identical cars are not so identical if one goes only half as many miles as the other one on the same amount of fuel. Likewise, two seemingly identical books are not so identical if it takes twice as long to learn a topic from one of them as it does from the other. Thus, comparisons made on the basis of the first law only may be highly misleading.

Having a disorganized (high-entropy) *army* is like having no army at all. It is no coincidence that the command centers of any armed forces are among the primary targets during a war. One army that consists of 10 divisions is 10 times more powerful than 10 armies each consisting of a single division. Likewise, one country that consists of 10 states is more powerful than 10 countries, each consisting of a single state. The *United States* would not be such a powerful country if there were 50 independent countries in its place instead of a single country with 50 states. The old cliché "divide and conquer" can be rephrased as "increase the entropy and conquer."

We know that mechanical friction is always accompanied by entropy generation, and thus reduced performance. We can generalize this to daily life: *friction in the workplace* with fellow workers is bound to generate entropy and thus adversely affect performance (Fig. 8–26). It results in reduced productivity.

We also know that *unrestrained expansion* (or explosion) and uncontrolled electron exchange (chemical reactions) generate entropy and are highly irreversible. Likewise, unrestrained opening of the mouth to scatter angry words is highly irreversible since this generates entropy, and it can cause considerable damage. A person who gets up in anger is bound to sit down at a loss. Hopefully, someday we will be able to come up with some procedures to quantify entropy generated during nontechnical activities, and maybe even pinpoint its primary sources and magnitude.

FIGURE 8–26
As in mechanical systems, friction in the workplace is bound to generate entropy and reduce performance.
Purestock/SuperStock

8–7 ▪ THE *T ds* RELATIONS

Recall that the quantity $(\delta Q/T)_{\text{int rev}}$ corresponds to a differential change in the property *entropy*. The entropy change for a process, then, can be evaluated by integrating $\delta Q/T$ along some imaginary internally reversible path between the actual end states. For isothermal internally reversible processes, this integration is straightforward. But when the temperature varies during the process, we have to have a relation between δQ and T to perform this integration. Finding such relations is what we intend to do in this section.

The differential form of the conservation of energy equation for a closed stationary system (a fixed mass) containing a simple compressible substance can be expressed for an internally reversible process as

$$\delta Q_{\text{int rev}} - \delta W_{\text{int rev,out}} = dU \tag{8–21}$$

But

$$\delta Q_{\text{int rev}} = T\,dS$$
$$\delta W_{\text{int rev,out}} = P\,dV$$

Thus,

$$T\,dS = dU + P\,dV \qquad \text{(kJ)} \tag{8–22}$$

or

$$T\,ds = du + P\,dv \qquad \text{(kJ/kg)} \tag{8–23}$$

This equation is known as the first *T ds,* or *Gibbs equation.* Notice that the only type of work interaction a simple compressible system may involve as it undergoes an internally reversible process is the boundary work.

The second *T ds* equation is obtained by eliminating *du* from Eq. 8–23 by using the definition of enthalpy ($h = u + Pv$):

$$\begin{matrix} h = u + Pv & \longrightarrow & dh = du + P\,dv + v\,dP \\ \text{(Eq. 8-23)} & \longrightarrow & T\,ds = du + P\,dv \end{matrix}\Bigg\}\; T\,ds = dh - v\,dP \tag{8–24}$$

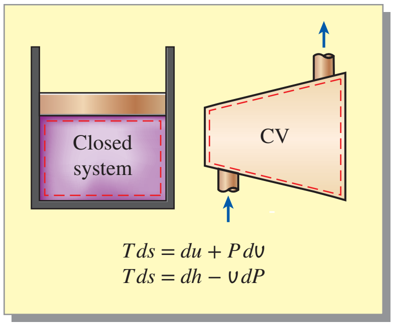


FIGURE 8–27
The *T ds* relations are valid for both reversible and irreversible processes and for both closed and open systems.

Equations 8–23 and 8–24 are extremely valuable since they relate entropy changes of a system to the changes in other properties. Unlike Eq. 8–4, they are property relations and therefore are independent of the type of the processes.

These *T ds* relations are developed with an internally reversible process in mind since the entropy change between two states must be evaluated along a reversible path. However, the results obtained are valid for both reversible and irreversible processes since entropy is a property and the change in a property between two states is independent of the type of process the system undergoes. Equations 8–23 and 8–24 are relations between the properties of a unit mass of a simple compressible system as it undergoes a change of state, and they are applicable whether the change occurs in a closed or an open system (Fig. 8–27).

Explicit relations for differential changes in entropy are obtained by solving for *ds* in Eqs. 8–23 and 8–24:

$$ds = \frac{du}{T} + \frac{P\,dv}{T} \tag{8–25}$$

and

$$ds = \frac{dh}{T} - \frac{v\,dP}{T} \tag{8–26}$$

The entropy change during a process can be determined by integrating either of these equations between the initial and the final states. To perform these integrations, however, we must know the relationship between *du* or *dh* and the temperature (such as $du = c_v\,dT$ and $dh = c_p\,dT$ for ideal gases) as well as the equation of state for the substance (such as the ideal-gas equation of state $Pv = RT$). For substances for which such relations exist, the integration of Eq. 8–25 or 8–26 is straightforward. For other substances, we have to rely on tabulated data.

The *T ds* relations for nonsimple systems, that is, systems that involve more than one mode of quasi-equilibrium work, can be obtained in a similar manner by including all the relevant quasi-equilibrium work modes.

8–8 ▪ ENTROPY CHANGE OF LIQUIDS AND SOLIDS

Recall that liquids and solids can be approximated as *incompressible substances* since their specific volumes remain nearly constant during a process. Thus, $dv \cong 0$ for liquids and solids, and Eq. 8–25 for this case reduces to

$$ds = \frac{du}{T} = \frac{c\,dT}{T} \tag{8–27}$$

since $c_p = c_v = c$ and $du = c\,dT$ for incompressible substances. Then the entropy change during a process is determined by integration to be

$$\textit{Liquids, solids:} \qquad s_2 - s_1 = \int_1^2 c(T)\frac{dT}{T} \cong c_{\text{avg}} \ln\frac{T_2}{T_1} \qquad \text{(kJ/kg·K)} \tag{8–28}$$

where c_{avg} is the *average* specific heat of the substance over the given temperature interval. Note that the entropy change of a truly incompressible substance depends on temperature only and is independent of pressure.

Equation 8–28 can be used to determine the entropy changes of solids and liquids with reasonable accuracy. However, for liquids that expand considerably with temperature, it may be necessary to consider the effects of volume change in calculations. This is especially the case when the temperature change is large.

A relation for isentropic processes of liquids and solids is obtained by setting the entropy change relation above equal to zero. It gives

$$\textit{Isentropic:} \qquad s_2 - s_1 = c_{\text{avg}} \ln\frac{T_2}{T_1} = 0 \quad \longrightarrow \quad T_2 = T_1 \tag{8–29}$$

That is, the temperature of a truly incompressible substance remains constant during an isentropic process. Therefore, the isentropic process of an incompressible substance is also isothermal. This behavior is closely approximated by liquids and solids.

EXAMPLE 8–7 Effect of Density of a Liquid on Entropy

Liquid methane is commonly used in various cryogenic applications. The critical temperature of methane is 191 K (or –82°C), and thus methane must be maintained below 191 K to keep it in liquid phase. The properties of liquid methane at various temperatures and pressures are given in Table 8–1. Determine the entropy change of liquid methane as it undergoes a process from 110 K and 1 MPa to 120 K and 5 MPa (*a*) using tabulated properties and (*b*) approximating liquid methane as an incompressible substance. What is the error involved in the latter case?

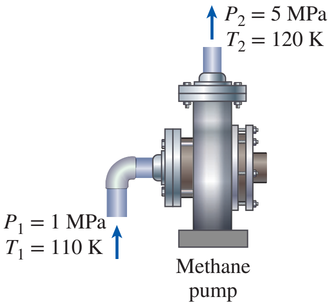


FIGURE 8–28
Schematic for Example 8–7.

SOLUTION Liquid methane undergoes a process between two specified states. The entropy change of methane is to be determined by using actual data and by assuming methane to be incompressible.

Analysis (*a*) We consider a unit mass of liquid methane (Fig. 8–28). The properties of the methane at the initial and final states are

TABLE 8–1

Properties of liquid methane

Temp., T, K	Pressure, P, MPa	Density, ρ, kg/m^3	Enthalpy, h, kJ/kg	Entropy, s, kJ/kg·K	Specific heat, c_p, kJ/kg·K
110	0.5	425.3	208.3	4.878	3.476
	1.0	425.8	209.0	4.875	3.471
	2.0	426.6	210.5	4.867	3.460
	5.0	429.1	215.0	4.844	3.432
120	0.5	410.4	243.4	5.185	3.551
	1.0	411.0	244.1	5.180	3.543
	2.0	412.0	245.4	5.171	3.528
	5.0	415.2	249.6	5.145	3.486

$$\textit{State 1:} \qquad \left.\begin{aligned} P_1 &= 1 \text{ MPa} \\ T_1 &= 110 \text{ K} \end{aligned}\right\} \quad \begin{aligned} s_1 &= 4.875 \text{ kJ/kg·K} \\ c_{p1} &= 3.471 \text{ kJ/kg·K} \end{aligned}$$

$$\textit{State 2:} \qquad \left.\begin{aligned} P_2 &= 5 \text{ MPa} \\ T_2 &= 120 \text{ K} \end{aligned}\right\} \quad \begin{aligned} s_2 &= 5.145 \text{ kJ/kg·K} \\ c_{p2} &= 3.486 \text{ kJ/kg·K} \end{aligned}$$

Therefore,

$$\Delta s = s_2 - s_1 = 5.145 - 4.875 = \mathbf{0.270\ kJ/kg·K}$$

(*b*) Approximating liquid methane as an incompressible substance, its entropy change is determined to be

$$\Delta s = c_{\text{avg}} \ln\frac{T_2}{T_1} = (3.4785 \text{ kJ/kg·K}) \ln\frac{120 \text{ K}}{110 \text{ K}} = \mathbf{0.303\ kJ/kg·K}$$

since

$$c_{\text{avg}} = \frac{c_{p1} + c_{p2}}{2} = \frac{3.471 + 3.486}{2} = 3.4785 \text{ kJ/kg·K}$$

Therefore, the error involved in approximating liquid methane as an incompressible substance is

$$\text{Error} = \frac{|\Delta s_{\text{actual}} - \Delta s_{\text{ideal}}|}{\Delta s_{\text{actual}}} = \frac{|0.270 - 0.303|}{0.270} = \mathbf{0.122} \text{ (or 12.2\%)}$$

Discussion This result is not surprising since the density of liquid methane changes during this process from 425.8 to 415.2 kg/m^3 (about 3 percent), which makes us question the validity of the incompressible substance assumption. Still, this assumption enables us to obtain reasonably accurate results with less effort, which proves to be very convenient in the absence of compressed liquid data.

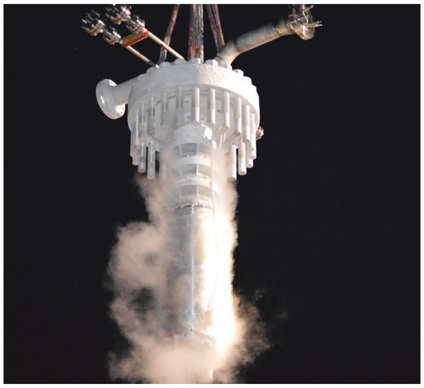


FIGURE 8–29
Liquefied natural gas (LNG) turbine after being removed from an LNG tank.
Courtesy of EBARA International Corporation, Cryodynamics Division, Sparks, Nevada

EXAMPLE 8–8 Economics of Replacing a Valve with a Turbine

A cryogenic manufacturing facility handles liquid methane at 115 K and 5 MPa at a rate of 0.280 m^3/s. A process requires dropping the pressure of liquid methane to 1 MPa, which is done by throttling the liquid methane by passing it through a flow resistance such as a valve. A recently hired engineer proposes to replace the throttling valve with a turbine in order to produce power while dropping the pressure to 1 MPa. Using data from Table 8–1, determine the maximum amount of power that can be produced by such a turbine. Also, determine how much this turbine will save the facility from electricity usage costs per year if the turbine operates continuously (8760 h/yr) and the facility pays $0.075/kWh for electricity.

SOLUTION Liquid methane is expanded in a turbine to a specified pressure at a specified rate. The maximum power that this turbine can produce and the amount of money it can save per year are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point, and thus $\Delta m_{\text{CV}} = 0$, $\Delta E_{\text{CV}} = 0$, and $\Delta S_{\text{CV}} = 0$. **2** The turbine is adiabatic, and thus there is no heat transfer. **3** The process is reversible. **4** Kinetic and potential energies are negligible.

Analysis We take the *turbine* as the system (Fig. 8–29). This is a *control volume* since mass crosses the system boundary during the process. We note that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$.

The preceding assumptions are reasonable since a turbine is normally well insulated and it must involve no irreversibilities for best performance and thus *maximum* power production. Therefore, the process through the turbine must be *reversible adiabatic* or *isentropic*. Then, $s_2 = s_1$ and

State 1:
$$\left.\begin{aligned} P_1 &= 5 \text{ MPa} \\ T_1 &= 115 \text{ K} \end{aligned}\right\} \begin{aligned} h_1 &= 232.3 \text{ kJ/kg} \\ s_1 &= 4.9945 \text{ kJ/kg·K} \\ \rho_1 &= 422.15 \text{ kg/m}^3 \end{aligned}$$

State 2:
$$\left.\begin{aligned} P_2 &= 1 \text{ MPa} \\ s_2 &= s_1 \end{aligned}\right\} h_2 = 222.8 \text{ kJ/kg}$$

Also, the mass flow rate of liquid methane is

$$\dot{m} = \rho_1 \dot{V}_1 = (422.15 \text{ kg/m}^3)(0.280 \text{ m}^3/\text{s}) = 118.2 \text{ kg/s}$$

Then the power output of the turbine is determined from the rate form of the energy balance to be

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}}\nearrow^{0\text{ (steady)}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}h_1 = \dot{W}_{\text{out}} + \dot{m}h_2 \qquad (\text{since } \dot{Q} = 0, \text{ke} \cong \text{pe} \cong 0)$$

$$\begin{aligned}\dot{W}_{\text{out}} &= \dot{m}(h_1 - h_2)\\ &= (118.2\text{ kg/s})(232.3 - 222.8)\text{ kJ/kg}\\ &= \mathbf{1123\ kW}\end{aligned}$$

For continuous operation (365 × 24 = 8760 h), the amount of power produced per year is

$$\begin{aligned}\text{Annual power production} &= \dot{W}_{\text{out}} \times \Delta t = (1123\text{ kW})(8760\text{ h/yr})\\ &= 0.9837 \times 10^7\text{ kWh/yr}\end{aligned}$$

At \$0.075/kWh, the amount of money this turbine can save the facility is

$$\begin{aligned}\text{Annual money savings} &= (\text{Annual power production})(\text{Unit cost of power})\\ &= (0.9837 \times 10^7\text{ kWh/yr})(\$0.075/\text{kWh})\\ &= \mathbf{\$737{,}800/yr}\end{aligned}$$

That is, this turbine can save the facility \$737,800 a year by simply taking advantage of the potential that is currently being wasted by a throttling valve, and the engineer who made this observation should be rewarded.

Discussion This example shows the importance of the property entropy since it enabled us to quantify the work potential that is being wasted. In practice, the turbine will not be isentropic, and thus the power produced will be less. The analysis above gave us the upper limit. An actual turbine-generator assembly can utilize about 80 percent of the potential and produce more than 900 kW of power while saving the facility more than \$600,000 a year.

It can also be shown that the temperature of methane drops to 113.9 K (a drop of 1.1 K) during the isentropic expansion process in the turbine instead of remaining constant at 115 K as would be the case if methane were assumed to be an incompressible substance. The temperature of methane would rise to 116.6 K (a rise of 1.6 K) during the throttling process.

8–9 ▪ THE ENTROPY CHANGE OF IDEAL GASES

An expression for the entropy change of an ideal gas can be obtained from Eq. 8–25 or 8–26 by employing the property relations for ideal gases (Fig. 8–30). By substituting $du = c_v\,dT$ and $P = RT/v$ into Eq. 8–25, the differential entropy change of an ideal gas becomes

$$ds = c_v\frac{dT}{T} + R\frac{dv}{v} \tag{8–30}$$

The entropy change for a process is obtained by integrating this relation between the end states:

$$s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R\ln\frac{v_2}{v_1} \tag{8–31}$$

FIGURE 8–30
A broadcast from channel IG.
Tony Cardoza/Getty Images

A second relation for the entropy change of an ideal gas is obtained in a similar manner by substituting $dh = c_p\,dT$ and $\upsilon = RT/P$ into Eq. 8–26 and integrating. The result is

$$s_2 - s_1 = \int_1^2 c_p(T)\frac{dT}{T} - R\ln\frac{P_2}{P_1} \tag{8–32}$$

The specific heats of ideal gases, with the exception of monatomic gases, depend on temperature, and the integrals in Eqs. 8–31 and 8–32 cannot be performed unless the dependence of c_υ and c_p on temperature is known. Even when the $c_\upsilon(T)$ and $c_p(T)$ functions are available, performing long integrations every time entropy change is calculated is not practical. Then two reasonable choices are left: either perform these integrations by simply assuming constant specific heats or evaluate those integrals once and tabulate the results. Both approaches are presented next.

Constant Specific Heats (Approximate Analysis)

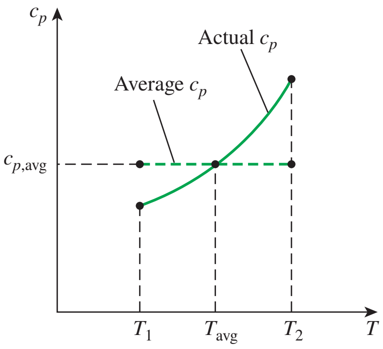


FIGURE 8–31
Under the constant-specific-heat assumption, the specific heat is assumed to be constant at some average value.

Assuming constant specific heats for ideal gases is a common approximation, and we used this assumption before on several occasions. It usually simplifies the analysis greatly, and the price we pay for this convenience is some loss in accuracy. The magnitude of the error introduced by this assumption depends on the situation at hand. For example, for monatomic ideal gases such as helium, the specific heats are independent of temperature, and therefore the constant-specific-heat assumption introduces no error. For ideal gases whose specific heats vary almost linearly in the temperature range of interest, the possible error is minimized by using specific heat values evaluated at the average temperature (Fig. 8–31). The results obtained in this way usually are sufficiently accurate if the temperature range is not greater than a few hundred degrees.

The entropy-change relations for ideal gases under the constant-specific-heat assumption are easily obtained by replacing $c_\upsilon(T)$ and $c_p(T)$ in Eqs. 8–31 and 8–32 with $c_{\upsilon,\text{avg}}$ and $c_{p,\text{avg}}$, respectively, and performing the integrations. We obtain

$$s_2 - s_1 = c_{\upsilon,\text{avg}}\ln\frac{T_2}{T_1} + R\ln\frac{\upsilon_2}{\upsilon_1} \qquad \text{(kJ/kg·K)} \tag{8–33}$$

and

$$s_2 - s_1 = c_{p,\text{avg}}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \qquad \text{(kJ/kg·K)} \tag{8–34}$$

Entropy changes can also be expressed on a unit-mole basis by multiplying these relations by molar mass:

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{\upsilon,\text{avg}}\ln\frac{T_2}{T_1} + R_u\ln\frac{\upsilon_2}{\upsilon_1} \qquad \text{(kJ/kmol·K)} \tag{8–35}$$

and

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{p,\text{avg}}\ln\frac{T_2}{T_1} - R_u\ln\frac{P_2}{P_1} \qquad \text{(kJ/kmol·K)} \tag{8–36}$$

Variable Specific Heats (Exact Analysis)

When the temperature change during a process is large and the specific heats of the ideal gas vary nonlinearly within the temperature range, the assumption of constant specific heats may lead to considerable errors in entropy-change calculations. For those cases, the variation of specific heats with temperature should be properly accounted for by utilizing accurate relations for the specific heats as a function of temperature. The entropy change during a process is then determined by substituting these $c_v(T)$ or $c_p(T)$ relations into Eq. 8–31 or 8–32 and performing the integrations.

Instead of performing these laborious integrals each time we have a new process, it is convenient to perform these integrals once and tabulate the results. For this purpose, we choose absolute zero as the reference temperature and define a function $s°$ as

$$s° = \int_0^T c_p(T)\frac{dT}{T} \tag{8–37}$$

Obviously, $s°$ is a function of temperature alone, and its value is zero at absolute zero temperature. The values of $s°$ are calculated at various temperatures, and the results are tabulated in the appendix as a function of temperature for air. Given this definition, the integral in Eq. 8–32 becomes

$$\int_1^2 c_p(T)\frac{dT}{T} = s_2^\circ - s_1^\circ \tag{8–38}$$

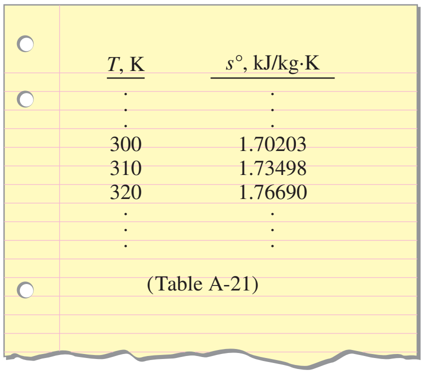
where s_2° is the value of $s°$ at T_2 and s_1° is the value at T_1. Thus,

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R\ln\frac{P_2}{P_1} \qquad \text{(kJ/kg·K)} \tag{8–39}$$

It can also be expressed on a unit-mole basis as

$$\bar{s}_2 - \bar{s}_1 = \bar{s}_2^\circ - \bar{s}_1^\circ - R_u\ln\frac{P_2}{P_1} \qquad \text{(kJ/kmol·K)} \tag{8–40}$$

Note that unlike internal energy and enthalpy, the entropy of an ideal gas varies with specific volume or pressure as well as the temperature. Therefore, entropy cannot be tabulated as a function of temperature alone. The $s°$ values in the tables account for the temperature dependence of entropy (Fig. 8–32). The variation of entropy with pressure is accounted for by the last term in Eq. 8–39. Another relation for entropy change can be developed based on Eq. 8–31, but this would require the definition of another function and tabulation of its values, which is not practical.



T, K	$s°$, kJ/kg·K
.	.
.	.
.	.
300	1.70203
310	1.73498
320	1.76690
.	.
.	.
.	.

(Table A-21)

FIGURE 8–32
The entropy of an ideal gas depends on both T and P. The function $s°$ represents only the temperature-dependent part of entropy.

EXAMPLE 8–9 Entropy Change of an Ideal Gas

Air is compressed from an initial state of 100 kPa and 17°C to a final state of 600 kPa and 57°C. Determine the entropy change of air during this compression process by using (*a*) property values from the air table and (*b*) average specific heats.

SOLUTION Air is compressed between two specified states. The entropy change of air is to be determined by using tabulated property values and also by using average specific heats.

Assumptions Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. Therefore, entropy change relations developed under the ideal-gas assumption are applicable.

Analysis A sketch of the system and the *T-s* diagram for the process are given in Fig. 8–33. We note that both the initial and the final states of air are completely specified.

(*a*) The properties of air are given in the air table (Table A–21). Reading $s°$ values at given temperatures and substituting, we find

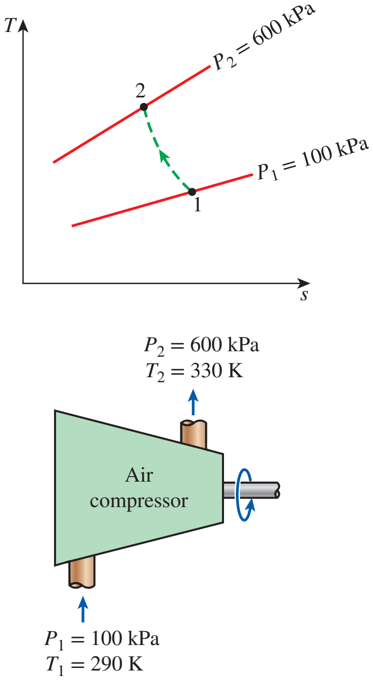


FIGURE 8–33
Schematic and *T-s* diagram for Example 8–9.

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R\ln\frac{P_2}{P_1}$$

$$= [(1.79783 - 1.66802)\ \text{kJ/kg·K}] - (0.287\ \text{kJ/kg·K})\ln\frac{600\ \text{kPa}}{100\ \text{kPa}}$$

$$= \mathbf{-0.3844\ kJ/kg·K}$$

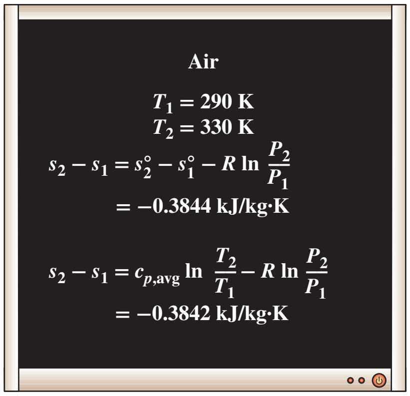
(*b*) The entropy change of air during this process can also be determined approximately from Eq. 8–34 by using a c_p value at the average temperature of 37°C (Table A–2*b*) and treating it as a constant:

$$s_2 - s_1 = c_{p,\text{avg}}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$= (1.006\ \text{kJ/kg·K})\ln\frac{330\ \text{K}}{290\ \text{K}} - (0.287\ \text{kJ/kg·K})\ln\frac{600\ \text{kPa}}{100\ \text{kPa}}$$

$$= \mathbf{-0.3842\ kJ/kg·K}$$

Discussion The two results above are almost identical since the change in temperature during this process is relatively small (Fig. 8–34). When the temperature change is large, however, they may differ significantly. For those cases, Eq. 8–39 should be used instead of Eq. 8–34 since it accounts for the variation of specific heats with temperature.



Air

$T_1 = 290$ K
$T_2 = 330$ K

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R\ln\frac{P_2}{P_1}$$
$$= -0.3844\ \text{kJ/kg·K}$$

$$s_2 - s_1 = c_{p,\text{avg}}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$
$$= -0.3842\ \text{kJ/kg·K}$$

FIGURE 8–34
For small temperature differences, the exact and approximate relations for entropy changes of ideal gases give almost identical results.

Isentropic Processes of Ideal Gases

Several relations for the isentropic processes of ideal gases can be obtained by setting the entropy-change relations developed previously equal to zero. Again, this is done first for the case of constant specific heats and then for the case of variable specific heats.

Constant Specific Heats (Approximate Analysis)

When the constant-specific-heat assumption is valid, the isentropic relations for ideal gases are obtained by setting Eqs. 8–33 and 8–34 equal to zero. From Eq. 8–33,

$$\ln\frac{T_2}{T_1} = -\frac{R}{c_\upsilon}\ln\frac{\upsilon_2}{\upsilon_1}$$

which can be rearranged as

$$\ln\frac{T_2}{T_1} = \ln\left(\frac{\upsilon_1}{\upsilon_2}\right)^{R/c_\upsilon} \qquad \textbf{(8–41)}$$

or

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{\upsilon_1}{\upsilon_2}\right)^{k-1} \qquad \text{(ideal gas)} \qquad \textbf{(8–42)}$$

since $R = c_p - c_\upsilon$, $k = c_p/c_\upsilon$, and thus $R/c_\upsilon = k - 1$.

Equation 8–42 is the *first isentropic relation* for ideal gases under the constant-specific-heat assumption. The *second isentropic relation* is obtained in a similar manner from Eq. 8–34 with the following result:

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \qquad \text{(ideal gas)} \qquad \textbf{(8–43)}$$

The *third isentropic relation* is obtained by substituting Eq. 8–43 into Eq. 8–42 and simplifying:

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \left(\frac{v_1}{v_2}\right)^k \qquad \text{(ideal gas)} \tag{8–44}$$

Equations 8–42 through 8–44 can also be expressed in a compact form as

$$Tv^{k-1} = \text{constant} \tag{8–45}$$

$$TP^{(1-k)/k} = \text{constant} \tag{8–46}$$

$$Pv^k = \text{constant} \tag{8–47}$$

The specific heat ratio k, in general, varies with temperature, and thus an average k value for the given temperature range should be used.

Note that the preceding ideal-gas isentropic relations, as the name implies, are strictly valid for isentropic processes only when the constant-specific-heat assumption is appropriate (Fig. 8–35).

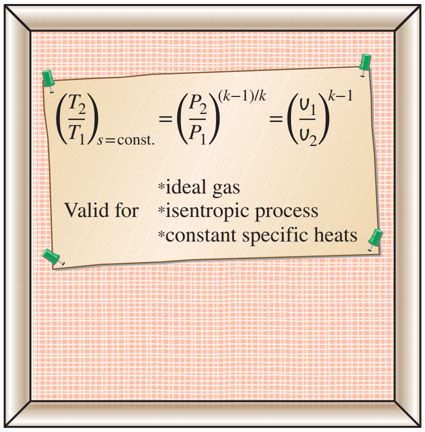


FIGURE 8–35
The isentropic relations of ideal gases are valid for the isentropic processes of ideal gases only.

Variable Specific Heats (Exact Analysis)

When the constant-specific-heat assumption is not appropriate, the isentropic relations developed previously yield results that are not quite accurate. For such cases, we should use an isentropic relation obtained from Eq. 8–39 that accounts for the variation of specific heats with temperature. Setting this equation equal to zero gives

$$0 = s_2^\circ - s_1^\circ - R\ln\frac{P_2}{P_1}$$

or

$$s_2^\circ = s_1^\circ + R\ln\frac{P_2}{P_1} \tag{8–48}$$

where s_2° is the s° value at the end of the isentropic process.

Relative Pressure and Relative Specific Volume

Equation 8–48 provides an accurate way of evaluating property changes of ideal gases during isentropic processes since it accounts for the variation of specific heats with temperature. However, it involves tedious iterations when the volume ratio is given instead of the pressure ratio. This is quite an inconvenience in optimization studies, which usually require many repetitive calculations. To remedy this deficiency, we define two new dimensionless quantities associated with isentropic processes.

The definition of the first is based on Eq. 8–48, which can be rearranged as

$$\frac{P_2}{P_1} = \exp\frac{s_2^\circ - s_1^\circ}{R}$$

or

$$\frac{P_2}{P_1} = \frac{\exp(s_2^\circ/R)}{\exp(s_1^\circ/R)}$$

The quantity $C_1\exp(s^\circ/R)$ is defined as the **relative pressure** P_r where C_1 is a constant. With this definition, the last relation becomes

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \frac{P_{r2}}{P_{r1}} \tag{8–49}$$

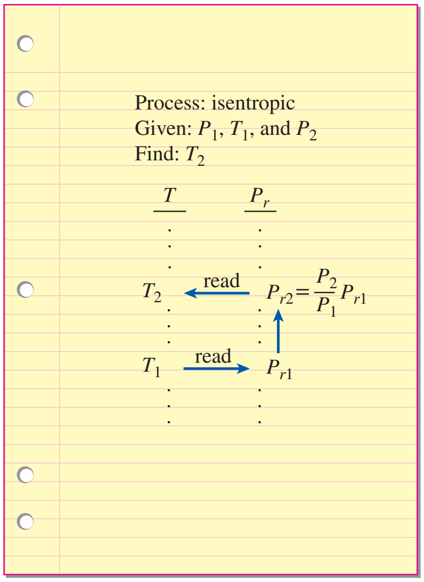


FIGURE 8–36
The use of P_r data for calculating the final temperature during an isentropic process.

Note that the relative pressure P_r is a *dimensionless* quantity that is a function of temperature only since $s°$ depends on temperature alone. Therefore, values of P_r can be tabulated against temperature. This is done for air in Table A–21. The use of P_r data is illustrated in Fig. 8–36.

Sometimes specific volume ratios are given instead of pressure ratios. This is particularly the case when automotive engines are analyzed. In such cases, one needs to work with volume ratios. Therefore, we define another quantity related to specific volume ratios for isentropic processes. This is done by utilizing the ideal-gas relation and Eq. 8–49:

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \rightarrow \frac{v_2}{v_1} = \frac{T_2 P_1}{T_1 P_2} = \frac{T_2 P_{r1}}{T_1 P_{r2}} = \frac{T_2/P_{r2}}{T_1/P_{r1}}$$

The quantity $C_2 T/P_r$ is a function of temperature only and is defined as **relative specific volume** v_r where C_2 is a constant. Thus,

$$\left(\frac{v_2}{v_1}\right)_{s = \text{const.}} = \frac{v_{r2}}{v_{r1}} \qquad \textbf{(8–50)}$$

Equations 8–49 and 8–50 are strictly valid for isentropic processes of ideal gases only. They account for the variation of specific heats with temperature and therefore give more accurate results than Eqs. 8–42 through 8–47. The values of P_r and v_r are listed for air in Table A–21.

EXAMPLE 8–10 Isentropic Compression of Air in a Car Engine

Air is compressed in a car engine from 22°C and 95 kPa in a reversible and adiabatic manner. If the compression ratio V_1/V_2 of this engine is 8, determine the final temperature of the air.

SOLUTION Air is compressed in a car engine isentropically. For a given compression ratio, the final air temperature is to be determined.

Assumptions At specified conditions, air can be treated as an ideal gas. Therefore, the isentropic relations for ideal gases are applicable.

Analysis A sketch of the system and the *T-s* diagram for the process are given in Fig. 8–37.

This process is easily recognized as being isentropic since it is both reversible and adiabatic. The final temperature for this isentropic process can be determined from Eq. 8–50 with the help of relative specific volume data (Table A–21), as illustrated in Fig. 8–38.

For closed systems: $$\frac{V_2}{V_1} = \frac{v_2}{v_1}$$

At $T_1 = 295$ K: $$v_{r1} = 647.9$$

From Eq. 8-50: $$v_{r2} = v_{r1}\left(\frac{v_2}{v_1}\right) = (647.9)\left(\frac{1}{8}\right) = 80.99 \rightarrow T_2 = \mathbf{662.7\ K}$$

Therefore, the temperature of air will increase by 367.7°C during this process.

ALTERNATIVE SOLUTION The final temperature could also be determined from Eq. 8–42 by assuming constant specific heats for air:

$$\left(\frac{T_2}{T_1}\right)_{s = \text{const.}} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

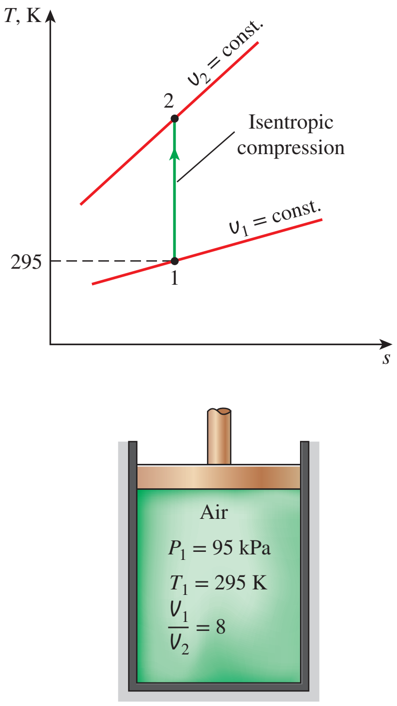


FIGURE 8–37
Schematic and *T-s* diagram for Example 8–10.

The specific heat ratio k also varies with temperature, and we need to use the value of k corresponding to the average temperature. However, the final temperature is not given, and so we cannot determine the average temperature in advance. For such cases, calculations can be started with a k value at the initial or the anticipated average temperature. This value could be refined later, if necessary, and the calculations can be repeated. We know that the temperature of the air will rise considerably during this adiabatic compression process, so we *guess* the average temperature to be about 450 K. The k value at this anticipated average temperature is determined from Table A–2*b* to be 1.391. Then, the final temperature of air becomes

$$T_2 = (295\text{ K})(8)^{1.391-1} = 665.2\text{ K}$$

This gives an average temperature value of 480.1 K, which is sufficiently close to the assumed value of 450 K. Therefore, it is not necessary to repeat the calculations by using the k value at this average temperature.

The result obtained by assuming constant specific heats for this case is in error by about 0.4 percent, which is rather small. This is not surprising since the temperature change of air is relatively small (only a few hundred degrees) and the specific heats of air vary almost linearly with temperature in this temperature range.

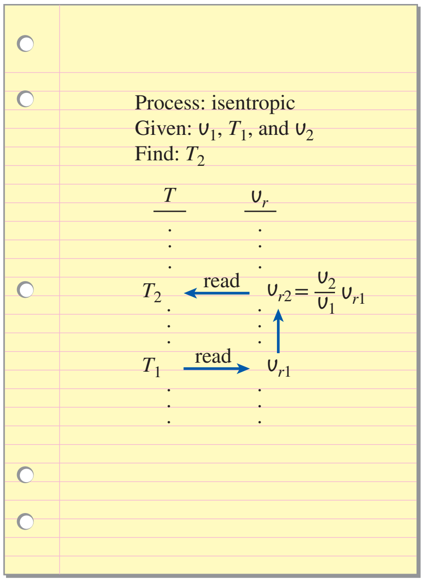


FIGURE 8–38

The use of υ_r data for calculating the final temperature during an isentropic process (Example 8–10).

EXAMPLE 8–11 Isentropic Expansion of an Ideal Gas

Air enters an isentropic turbine at 150 psia and 900°F through a 0.5-ft^2 inlet section with a velocity of 500 ft/s (Fig. 8–39). It leaves at 15 psia with a velocity of 100 ft/s. Calculate the air temperature at the turbine exit and the power produced, in hp, by this turbine.

SOLUTION Air is expanded in an isentropic turbine. The exit temperature of the air and the power produced are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** The process is isentropic (i.e., reversible-adiabatic). **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at an anticipated average temperature of 600°F are $c_p = 0.250$ Btu/lbm·R and $k = 1.377$ (Table A–2E*b*). The gas constant of air is $R = 0.3704$ psia·ft^3/lbm·R (Table A–1E).

Analysis There is only one inlet and one exit, and thus $\dot m_1 = \dot m_2 = \dot m$. We take the turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot E_{\text{in}} - \dot E_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\cancelto{0\text{ (steady)}}{\Delta \dot E_{\text{system}}/dt}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot E_{\text{in}} = \dot E_{\text{out}}$$

$$\dot m\left(h_1 + \frac{V_1^2}{2}\right) = \dot m\left(h_2 + \frac{V_2^2}{2}\right) + \dot W_{\text{out}}$$

$$\dot W_{\text{out}} = \dot m\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right)$$

$$= \dot m\left[c_p(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2}\right]$$

The exit temperature of the air for this isentropic process is

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (900 + 460\text{ R})\left(\frac{15\text{ psia}}{150\text{ psia}}\right)^{0.377/1.377} = \mathbf{724\ R}$$

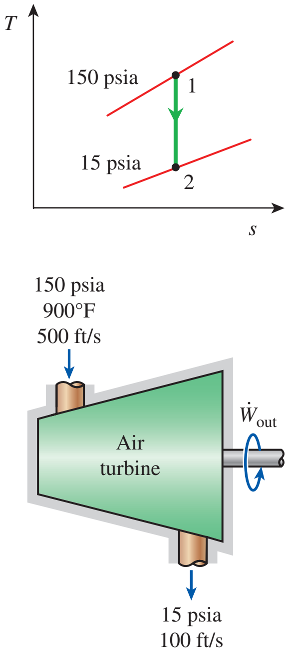


FIGURE 8–39

Schematic for Example 8–11.

The specific volume of air at the inlet and the mass flow rate are

$$\upsilon_1 = \frac{RT_1}{P_1} = \frac{(0.3704\ \text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(900 + 460\ \text{R})}{150\ \text{psia}} = 3.358\ \text{ft}^3/\text{lbm}$$

$$\dot{m} = \frac{A_1V_1}{\upsilon_1} = \frac{(0.5\ \text{ft}^2)(500\ \text{ft/s})}{3.358\ \text{ft}^3/\text{lbm}} = 74.45\ \text{lbm/s}$$

Substituting into the energy balance equation gives

$$\begin{aligned}\dot{W}_{\text{out}} &= \dot{m}\left[c_p(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2}\right]\\ &= (74.45\ \text{lbm/s})\Bigg[(0.250\ \text{Btu/lbm}\cdot\text{R})(1360 - 724)\text{R}\\ &\qquad + \frac{(500\ \text{ft/s})^2 - (100\ \text{ft/s})^2}{2}\left(\frac{1\ \text{Btu/lbm}}{25{,}037\ \text{ft}^2/\text{s}^2}\right)\Bigg]\\ &= 12{,}194\ \text{Btu/s}\left(\frac{1\ \text{hp}}{0.7068\ \text{Btu/s}}\right) = \mathbf{17{,}250\ hp}\end{aligned}$$

Discussion An actual adiabatic turbine will produce less power due to irreversibilities. Also, the exit temperature of the air will be higher in the actual turbine corresponding to a smaller enthalpy change between the inlet and the exit.

8–10 ▪ REVERSIBLE STEADY-FLOW WORK

The work done during a process depends on the path followed as well as on the properties at the end states. Recall that reversible (quasi-equilibrium) moving boundary work associated with closed systems is expressed in terms of the fluid properties as

$$W_b = \int_1^2 P\,dV$$

We mentioned that the quasi-equilibrium work interactions lead to the maximum work output for work-producing devices and the minimum work input for work-consuming devices.

It would also be very insightful to express the work associated with steady-flow devices in terms of fluid properties.

Taking the positive direction of work to be from the system (work output), the energy balance for a steady-flow device undergoing an internally reversible process can be expressed in differential form as

$$\delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + d\text{ke} + d\text{pe}$$

But

$$\left.\begin{array}{ll}\delta q_{\text{rev}} = T ds & \text{(Eq. 8–16)}\\ T ds = dh - \upsilon\,dP & \text{(Eq. 8–24)}\end{array}\right\}\ \delta q_{\text{rev}} = dh - \upsilon\,dP$$

Substituting this into the relation above and canceling *dh* yield

$$-\delta w_{\text{rev}} = \upsilon\,dP + d\text{ke} + d\text{pe}$$

Integrating, we find

$$w_{\text{rev}} = -\int_1^2 \upsilon\,dP - \Delta\text{ke} - \Delta\text{pe} \qquad \text{(kJ/kg)} \tag{8–51}$$

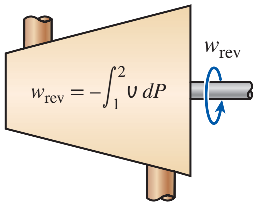
When the changes in kinetic and potential energies are negligible, this equation reduces to

$$w_{\text{rev}} = -\int_1^2 v\,dP \qquad (\text{kJ/kg}) \tag{8–52}$$

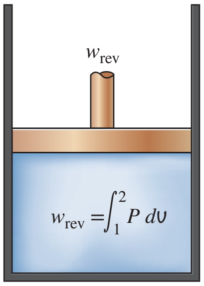
Equations 8–51 and 8–52 are relations for the *reversible work output* associated with an internally reversible process in a steady-flow device. They will give a negative result when work is done on the system. To avoid the negative sign, Eq. 8–51 can be written for work input to steady-flow devices such as compressors and pumps as

$$w_{\text{rev,in}} = \int_1^2 v\,dP + \Delta\text{ke} + \Delta\text{pe} \tag{8–53}$$

The resemblance between the $v\,dP$ in these relations and $P\,dv$ is striking. They should not be confused with each other, however, since $P\,dv$ is associated with reversible boundary work in closed systems (Fig. 8–40).



(*a*) Steady-flow system



(*b*) Closed system

FIGURE 8–40
Reversible work relations for steady-flow and closed systems.

Obviously, one needs to know v as a function of P for the given process to perform the integration. When the working fluid is *incompressible,* the specific volume v remains constant during the process and can be taken out of the integration. Then Eq. 8–51 simplifies to

$$w_{\text{rev}} = -v(P_2 - P_1) - \Delta\text{ke} - \Delta\text{pe} \qquad (\text{kJ/kg}) \tag{8–54}$$

For the steady flow of a liquid through a device that involves no work interactions (such as a nozzle or a pipe section), the work term is zero, and the preceding equation can be expressed as

$$v(P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0 \tag{8–55}$$

which is known as the **Bernoulli equation** in fluid mechanics. It is developed for an internally reversible process and thus is applicable to incompressible fluids that involve no irreversibilities such as friction or shock waves. This equation can be modified, however, to incorporate these effects.

Equation 8–52 has far-reaching implications in engineering for devices that produce or consume work steadily such as turbines, compressors, and pumps. It is obvious from this equation that the reversible steady-flow work is closely associated with the specific volume of the fluid flowing through the device. *The larger the specific volume, the larger the reversible work produced or consumed by the steady-flow device* (Fig. 8–41). This conclusion is equally valid for actual steady-flow devices. Therefore, every effort should be made to keep the specific volume of a fluid as small as possible during a compression process to minimize the work input and as large as possible during an expansion process to maximize the work output.

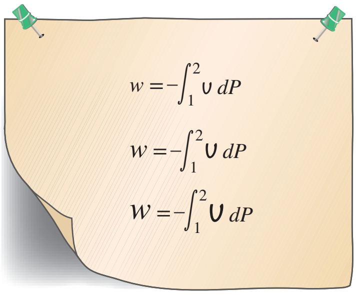
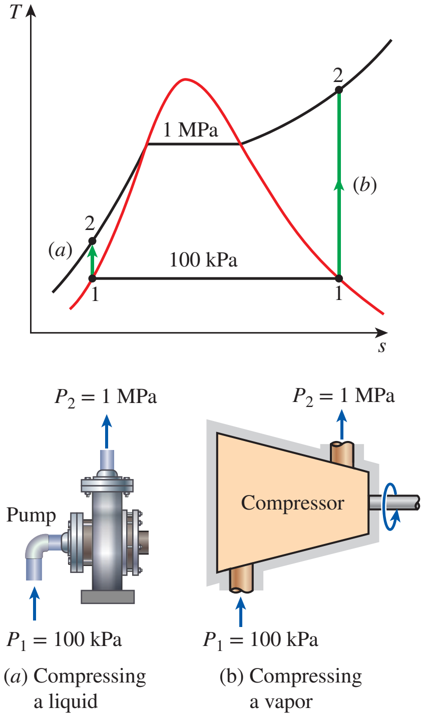


FIGURE 8–41
The larger the specific volume, the greater the work produced (or consumed) by a steady-flow device.

In steam or gas power plants, the pressure rise in the pump or compressor is equal to the pressure drop in the turbine if we disregard the pressure losses in various other components. In steam power plants, the pump handles liquid, which has a very small specific volume, and the turbine handles vapor, whose specific volume is many times larger. Therefore, the work output of the turbine is much larger than the work input to the pump. This is one of the reasons for the widespread use of steam power plants in electric power generation.

If we were to compress the steam exiting the turbine back to the turbine inlet pressure before cooling it first in the condenser in order to "save" the heat rejected, we would have to supply all the work produced by the turbine back to the compressor. In reality, the required work input would be even greater than the work output of the turbine because of the irreversibilities present in both processes.

In gas power plants, the working fluid (typically air) is compressed in the gas phase, and a considerable portion of the work output of the turbine is consumed by the compressor. As a result, a gas power plant delivers less net work per unit mass of the working fluid.



(*a*) Compressing a liquid

(b) Compressing a vapor

FIGURE 8–42

Schematic and *T-s* diagram for Example 8–12.

EXAMPLE 8–12 Compressing a Substance in the Liquid Versus Gas Phases

Determine the compressor work input required to compress steam isentropically from 100 kPa to 1 MPa, assuming that the steam exists as (*a*) saturated liquid and (*b*) saturated vapor at the inlet state.

SOLUTION Steam is to be compressed from a given pressure to a specified pressure isentropically. The work input is to be determined for the cases of steam being a saturated liquid and saturated vapor at the inlet.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible. **3** The process is given to be isentropic.

Analysis We take first the pump and then the compressor as the *system.* Both are *control volumes* since mass crosses the boundary. Sketches of the pump and the turbine together with the *T-s* diagram are given in Fig. 8–42.

(*a*) In this case, steam is a saturated liquid initially, and its specific volume is

$$v_1 = v_{f\,@\,100\text{ kPa}} = 0.001043\ \text{m}^3/\text{kg} \qquad \text{(Table A–5)}$$

which remains essentially constant during the process. Thus,

$$\begin{aligned} w_{\text{rev}} &= \int_1^2 v\,dP \cong v_1(P_2 - P_1) \\ &= (0.001043\ \text{m}^3/\text{kg})[(1000 - 100)\ \text{kPa}]\left(\frac{1\ \text{kJ}}{1\ \text{kPa·m}^3}\right) \\ &= \mathbf{0.94\ kJ/kg} \end{aligned}$$

(*b*) This time, steam is a saturated vapor initially and remains a vapor during the entire compression process. Since the specific volume of a gas changes considerably during a compression process, we need to know how v varies with P to perform the integration in Eq. 8–53. This relation, in general, is not readily available. But for an isentropic process, it is easily obtained from the second $T\,ds$ relation by setting $ds = 0$:

$$\left.\begin{aligned} T\,ds &= dh - v\,dP \quad \text{(Eq. 8–24)} \\ ds &= 0 \quad \text{(isentropic process)} \end{aligned}\right\}\ v\,dP = dh$$

Thus,

$$w_{\text{rev,in}} = \int_1^2 v\,dP = \int_1^2 dh = h_2 - h_1$$

This result could also be obtained from the energy balance relation for an isentropic steady-flow process. Next we determine the enthalpies:

$$\textit{State 1:}\quad \left.\begin{aligned} P_1 &= 100\ \text{kPa} \\ &\text{(sat. vapor)} \end{aligned}\right\}\ \begin{aligned} h_1 &= 2675.0\ \text{kJ/kg} \\ s_1 &= 7.3589\ \text{kJ/kg·K} \end{aligned} \qquad \text{(Table A–5)}$$

$$\textit{State 2:}\quad \left.\begin{aligned} P_2 &= 1\ \text{MPa} \\ s_2 &= s_1 \end{aligned}\right\}\ h_2 = 3194.5\ \text{kJ/kg} \qquad \text{(Table A–6)}$$

Thus,

$$w_{\text{rev,in}} = (3194.5 - 2675.0)\ \text{kJ/kg} = \mathbf{519.5\ kJ/kg}$$

Discussion Note that compressing steam in the vapor form would require over 500 times more work than compressing it in the liquid form between the same pressure limits.

Proof that Steady-Flow Devices Deliver the Most and Consume the Least Work When the Process Is Reversible

We have shown in Chap. 7 that cyclic devices (heat engines, refrigerators, and heat pumps) deliver the most work and consume the least when reversible processes are used. Now we demonstrate that this is also the case for individual devices such as turbines and compressors in steady operation.

Consider two steady-flow devices, one reversible and the other irreversible, operating between the same inlet and exit states. Again taking heat transfer to the system and work done by the system to be positive quantities, the energy balance for each of these devices can be expressed in the differential form as

Actual: $$\delta q_{\text{act}} - \delta w_{\text{act}} = dh + d\text{ke} + d\text{pe}$$

Reversible: $$\delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + d\text{ke} + d\text{pe}$$

The right-hand sides of these two equations are identical since both devices are operating between the same end states. Thus,

$$\delta q_{\text{act}} - \delta w_{\text{act}} = \delta q_{\text{rev}} - \delta w_{\text{rev}}$$

or

$$\delta w_{\text{rev}} - \delta w_{\text{act}} = \delta q_{\text{rev}} - \delta q_{\text{act}}$$

However,

$$\delta q_{\text{rev}} = T\,ds$$

Substituting this relation into the preceding equation and dividing each term by T, we obtain

$$\frac{\delta w_{\text{rev}} - \delta w_{\text{act}}}{T} = ds - \frac{\delta q_{\text{act}}}{T} \geq 0$$

since

$$ds \geq \frac{\delta q_{\text{act}}}{T}$$

Also, T is the absolute temperature, which is always positive. Thus,

$$\delta w_{\text{rev}} \geq \delta w_{\text{act}}$$

or

$$w_{\text{rev}} \geq w_{\text{act}}$$

Therefore, work-producing devices such as turbines (w is positive) deliver more work, and work-consuming devices such as pumps and compressors (w is negative) require less work when they operate reversibly (Fig. 8–43).

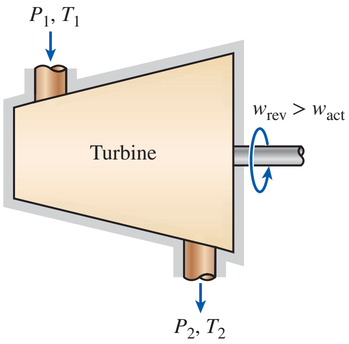


FIGURE 8–43
A reversible turbine delivers more work than an irreversible one if both operate between the same end states.

8–11 ▪ ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

We have said repeatedly that irreversibilities accompany all actual processes and that their effect is always to downgrade the performance of devices. In engineering analysis, it would be very useful to have some parameters that would enable us to quantify the degree of degradation of energy in these devices. In Chap. 7 we did this for cyclic devices, such as heat engines and refrigerators, by comparing the actual cycles to the idealized ones, such as the Carnot cycle. A cycle that was composed entirely of reversible processes served as the *model cycle* to which the actual cycles could be compared. This idealized model cycle enabled us to determine the theoretical limits of performance for cyclic devices under specified conditions and to examine how the performance of actual devices suffered as a result of irreversibilities.

Now we extend the analysis to discrete engineering devices working under steady-flow conditions, such as turbines, compressors, and nozzles, and we examine the degree of degradation of energy in these devices as a result of irreversibilities. However, first we need to define an ideal process that serves as a model for the actual processes.

Although some heat transfer between these devices and the surrounding medium is unavoidable, many steady-flow devices are intended to operate under adiabatic conditions. Therefore, the model process for these devices should be an adiabatic one. Furthermore, an ideal process should involve no irreversibilities since the effect of irreversibilities is always to downgrade the performance of engineering devices. Thus, the ideal process that can serve as a suitable model for adiabatic steady-flow devices is the *isentropic* process (Fig. 8–44).

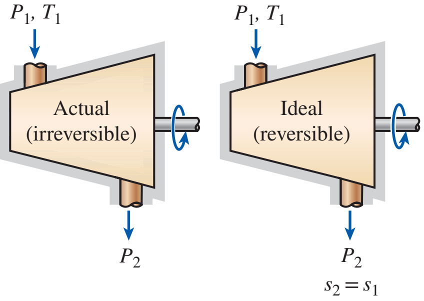


FIGURE 8–44
The isentropic process involves no irreversibilities and serves as the ideal process for adiabatic devices.

The more closely the actual process approximates the idealized isentropic process, the better the device performs. Thus, it would be desirable to have a parameter that expresses quantitatively how efficiently an actual device approximates an idealized one. This parameter is the **isentropic** or **adiabatic efficiency**, which is a measure of the deviation of actual processes from the corresponding idealized ones.

Isentropic efficiencies are defined differently for different devices since each device is set up to perform different tasks. Next, we define the isentropic efficiencies of turbines, compressors, and nozzles by comparing the actual performance of these devices to their performance under isentropic conditions for the same inlet state and exit pressure.

Isentropic Efficiency of Turbines

For a turbine under steady operation, the inlet state of the working fluid and the exhaust pressure are fixed. Therefore, the ideal process for an adiabatic turbine is an isentropic process between the inlet state and the exhaust pressure. The desired output of a turbine is the work produced, and the **isentropic efficiency of a turbine** is defined as *the ratio of the actual work output of the turbine to the work output that would be achieved if the process between the inlet state and the exit pressure were isentropic:*

$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s} \tag{8–56}$$

Usually the changes in kinetic and potential energies associated with a fluid stream flowing through a turbine are small relative to the change in enthalpy and can be neglected. Then, the work output of an adiabatic turbine simply becomes the change in enthalpy, and Eq. 8–56 becomes

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} \tag{8–57}$$

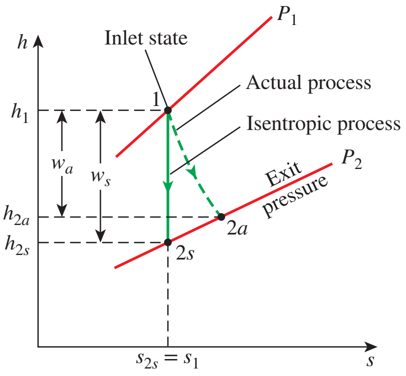


FIGURE 8–45
The *h*-*s* diagram for the actual and isentropic processes of an adiabatic turbine.

where h_{2a} and h_{2s} are the enthalpy values at the exit state for actual and isentropic processes, respectively (Fig. 8–45).

The value of η_T greatly depends on the design of the individual components that make up the turbine. Well-designed, large turbines have isentropic efficiencies above 90 percent. For small turbines, however, isentropic efficiency may drop below 70 percent. The value of the isentropic efficiency of a turbine is determined by measuring the actual work output of the turbine and by calculating the isentropic work output for the measured inlet conditions and the exit pressure. This value can then be used conveniently in the design of power plants.

EXAMPLE 8–13 Isentropic Efficiency of a Steam Turbine

Steam enters an adiabatic turbine steadily at 3 MPa and 400°C and leaves at 50 kPa and 100°C. If the power output of the turbine is 2 MW, determine (*a*) the isentropic efficiency of the turbine and (*b*) the mass flow rate of the steam flowing through the turbine.

SOLUTION Steam flows steadily in a turbine between inlet and exit states. For a specified power output, the isentropic efficiency and the mass flow rate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The changes in kinetic and potential energies are negligible.

Analysis A sketch of the system and the *T*-*s* diagram of the process are given in Fig. 8–46.

(*a*) The enthalpies at various states are

State 1:
$$\left.\begin{array}{l} P_1 = 3 \text{ MPa} \\ T_1 = 400°\text{C} \end{array}\right\} \begin{array}{l} h_1 = 3231.7 \text{ kJ/kg} \\ s_1 = 6.9235 \text{ kJ/kg·K} \end{array} \qquad \text{(Table A–6)}$$

State 2a:
$$\left.\begin{array}{l} P_{2a} = 50 \text{ kPa} \\ T_{2a} = 100°\text{C} \end{array}\right\} h_{2a} = 2682.4 \text{ kJ/kg} \qquad \text{(Table A–6)}$$

The exit enthalpy of the steam for the isentropic process h_{2s} is determined from the requirement that the entropy of the steam remain constant ($s_{2s} = s_1$):

State 2s:
$$\begin{array}{l} P_{2s} = 50 \text{ kPa} \\ (s_{2s} = s_1) \end{array} \longrightarrow \begin{array}{l} s_f = 1.0912 \text{ kJ/kg·K} \\ s_g = 7.5931 \text{ kJ/kg·K} \end{array} \qquad \text{(Table A–5)}$$

Obviously, at the end of the isentropic process steam exists as a saturated mixture since $s_f < s_{2s} < s_g$. Thus, we need to find the quality at state 2*s* first:

$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{6.9235 - 1.0912}{6.5019} = 0.897$$

and

$$h_{2s} = h_f + x_{2s} h_{fg} = 340.54 + 0.897(2304.7) = 2407.9 \text{ kJ/kg}$$

By substituting these enthalpy values into Eq. 8–57, the isentropic efficiency of this turbine is determined to be

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{3231.7 - 2682.4}{3231.7 - 2407.9} = \mathbf{0.667} \text{ (or 66.7\%)}$$

(*b*) The mass flow rate of steam through this turbine is determined from the energy balance for steady-flow systems:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$
$$\dot{m}h_1 = \dot{W}_{a,\text{out}} + \dot{m}h_{2a}$$
$$\dot{W}_{a,\text{out}} = \dot{m}(h_1 - h_{2a})$$
$$2 \text{ MW}\left(\frac{1000 \text{ kJ/s}}{1 \text{ MW}}\right) = \dot{m}(3231.7 - 2682.4) \text{ kJ/kg}$$
$$\dot{m} = \mathbf{3.64 \text{ kg/s}}$$

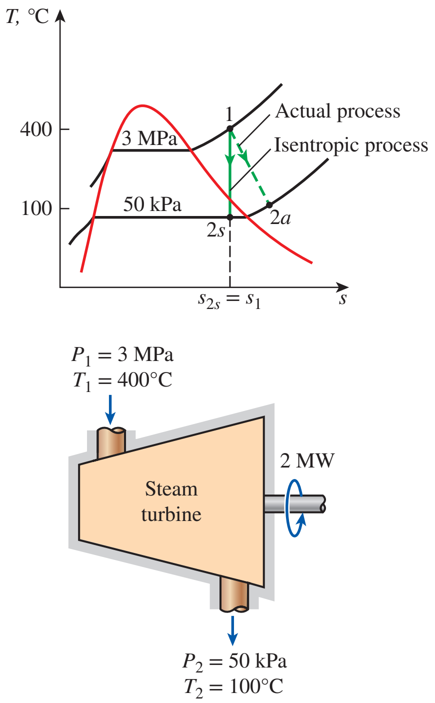


FIGURE 8–46
Schematic and *T*-*s* diagram for Example 8–13.

Isentropic Efficiencies of Compressors and Pumps

The **isentropic efficiency of a compressor** is defined as *the ratio of the work input required to raise the pressure of a gas to a specified value in an isentropic manner to the actual work input:*

$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a} \qquad \textbf{(8–58)}$$

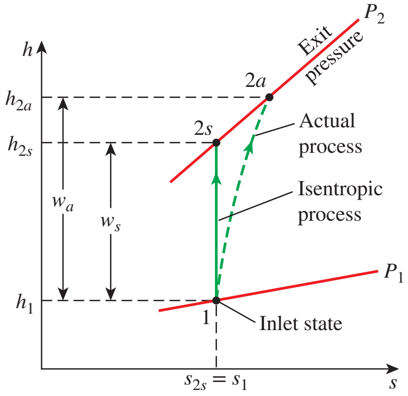


FIGURE 8–47
The *h-s* diagram of the actual and isentropic processes of an adiabatic compressor.

Notice that the isentropic compressor efficiency is defined with the *isentropic work input in the numerator* instead of in the denominator. This is because w_s is a smaller quantity than w_a, and this definition prevents η_C from becoming greater than 100 percent, which would falsely imply that the actual compressors performed better than the isentropic ones. Also notice that the inlet conditions and the exit pressure of the gas are the same for both the actual and the isentropic compressor.

When the changes in kinetic and potential energies of the gas being compressed are negligible, the work input to an adiabatic compressor becomes equal to the change in enthalpy, and Eq. 8–58 for this case becomes

$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \tag{8–59}$$

where h_{2a} and h_{2s} are the enthalpy values at the exit state for actual and isentropic compression processes, respectively, as illustrated in Fig. 8–47. Again, the value of η_C greatly depends on the design of the compressor. Well-designed compressors have isentropic efficiencies that range from 80 to 90 percent.

When the changes in potential and kinetic energies of a liquid are negligible, the isentropic efficiency of a pump is defined similarly as

$$\eta_P = \frac{w_s}{w_a} = \frac{\upsilon(P_2 - P_1)}{h_{2a} - h_1} \tag{8–60}$$

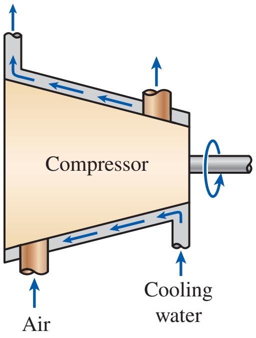


FIGURE 8–48
Compressors are sometimes intentionally cooled to minimize the work input.

When no attempt is made to cool the gas as it is compressed, the actual compression process is nearly adiabatic, and the reversible adiabatic (i.e., isentropic) process serves well as the ideal process. However, sometimes *compressors are cooled intentionally* by utilizing fins or a water jacket placed around the casing to reduce the work input requirements (Fig. 8–48). In this case, the isentropic process is not suitable as the model process since the device is no longer adiabatic and the isentropic compressor efficiency defined above is meaningless. A realistic model process for compressors that are intentionally cooled during the compression process is the *reversible isothermal process*. Then we can conveniently define an **isothermal efficiency** for such cases by comparing the actual process to a reversible isothermal one:

$$\eta_C = \frac{w_t}{w_a} \tag{8–61}$$

where w_t and w_a are the required work inputs to the compressor for the reversible isothermal and actual cases, respectively.

EXAMPLE 8–14 Effect of Efficiency on Compressor Power Input

Air is compressed by an adiabatic compressor from 100 kPa and 12°C to a pressure of 800 kPa at a steady rate of 0.2 kg/s. If the isentropic efficiency of the compressor is 80 percent, determine (*a*) the exit temperature of air and (*b*) the required power input to the compressor.

SOLUTION Air is compressed to a specified pressure at a specified rate. For a given isentropic efficiency, the exit temperature and the power input are to be determined.
Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The changes in kinetic and potential energies are negligible.
Analysis A sketch of the system and the *T-s* diagram of the process are given in Fig. 8–49.
(*a*) We know only one property (pressure) at the exit state, and we need to know one more to fix the state and thus determine the exit temperature. The property that can be

determined with minimal effort in this case is h_{2a} since the isentropic efficiency of the compressor is given. At the compressor inlet,

$$T_1 = 285 \text{ K} \rightarrow h_1 = 285.14 \text{ kJ/kg} \quad \text{(Table A–21)}$$
$$P_{r1} = 1.1584$$

The enthalpy of the air at the end of the isentropic compression process is determined by using one of the isentropic relations of ideal gases,

$$P_{r2} = P_{r1}\left(\frac{P_2}{P_1}\right) = 1.1584\left(\frac{800 \text{ kPa}}{100 \text{ kPa}}\right) = 9.2672$$

and

$$P_{r2} = 9.2672 \rightarrow h_{2s} = 517.05 \text{ kJ/kg}$$

Substituting the known quantities into the isentropic efficiency relation, we have

$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \rightarrow 0.80 = \frac{(517.05 - 285.14) \text{ kJ/kg}}{(h_{2a} - 285.14) \text{ kJ/kg}}$$

Thus,

$$h_{2a} = 575.03 \text{ kJ/kg} \rightarrow T_{2a} = \mathbf{569.5 \text{ K}}$$

(*b*) The required power input to the compressor is determined from the energy balance for steady-flow devices,

$$\begin{aligned} \dot{E}_{\text{in}} &= \dot{E}_{\text{out}} \\ \dot{m}h_1 + \dot{W}_{a,\text{in}} &= \dot{m}h_{2a} \\ \dot{W}_{a,\text{in}} &= \dot{m}(h_{2a} - h_1) \\ &= (0.2 \text{ kg/s})[(575.03 - 285.14) \text{ kJ/kg}] \\ &= \mathbf{58.0 \text{ kW}} \end{aligned}$$

Discussion Notice that in determining the power input to the compressor, we used h_{2a} instead of h_{2s} since h_{2a} is the actual enthalpy of the air as it exits the compressor. The quantity h_{2s} is a hypothetical enthalpy value that the air would have if the process were isentropic.

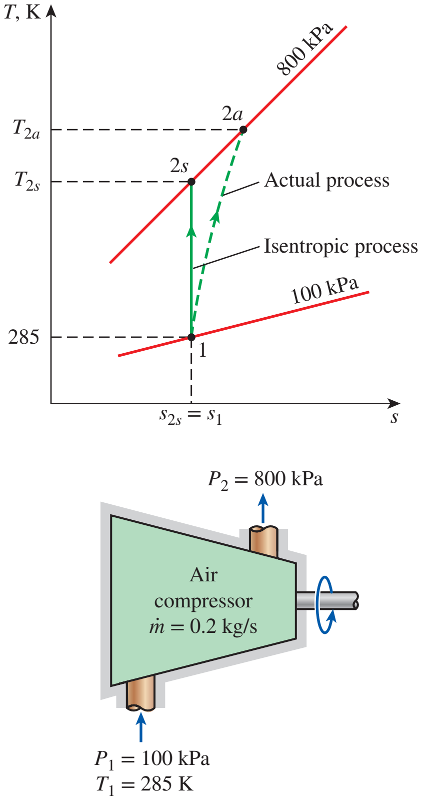


FIGURE 8–49
Schematic and *T-s* diagram for Example 8–14.

Isentropic Efficiency of Nozzles

Nozzles are essentially adiabatic devices and are used to accelerate a fluid. Therefore, the isentropic process serves as a suitable model for nozzles. The **isentropic efficiency of a nozzle** is defined as *the ratio of the actual kinetic energy of the fluid at the nozzle exit to the kinetic energy value at the exit of an isentropic nozzle for the same inlet state and exit pressure*. That is,

$$\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2} \qquad \text{(8–62)}$$

Note that the exit pressure is the same for both the actual and isentropic processes, but the exit state is different.

Nozzles involve no work interactions, and the fluid experiences little or no change in its potential energy as it flows through the device. If, in addition, the inlet velocity of the fluid is small relative to the exit velocity, the energy balance for this steady-flow device reduces to

$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

Then the isentropic efficiency of the nozzle can be expressed in terms of enthalpies as

$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} \tag{8–63}$$

where h_{2a} and h_{2s} are the enthalpy values at the nozzle exit for the actual and isentropic processes, respectively (Fig. 8–50). Isentropic efficiencies of nozzles are typically above 90 percent, and nozzle efficiencies above 95 percent are not uncommon.

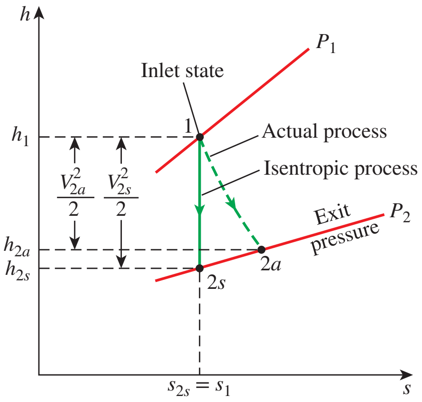


FIGURE 8–50
The *h-s* diagram of the actual and isentropic processes of an adiabatic nozzle.

EXAMPLE 8–15 Effect of Efficiency on Nozzle Exit Velocity

Air at 200 kPa and 950 K enters an adiabatic nozzle at low velocity and is discharged at a pressure of 110 kPa. If the isentropic efficiency of the nozzle is 92 percent, determine (*a*) the maximum possible exit velocity, (*b*) the exit temperature, and (*c*) the actual exit velocity of the air. Assume constant specific heats for air.

SOLUTION The acceleration of air in a nozzle is considered. For specified exit pressure and isentropic efficiency, the maximum and actual exit velocities and the exit temperature are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The inlet kinetic energy is negligible.

Analysis A sketch of the system and the *T-s* diagram of the process are given in Fig. 8–51.

The temperature of air will drop during this acceleration process because some of its internal energy is converted to kinetic energy. This problem can be solved accurately by using property data from the air table. But we will assume constant specific heats (thus sacrifice some accuracy) to demonstrate their use. Let us guess the average temperature of the air to be about 850 K. Then, the average values of c_p and k at this anticipated average temperature are determined from Table A–2*b* to be $c_p = 1.11$ kJ/kg·K and $k = 1.349$.

(*a*) The exit velocity of the air will be a maximum when the process in the nozzle involves no irreversibilities. The exit velocity in this case is determined from the steady-flow energy equation. However, first we need to determine the exit temperature. For the isentropic process of an ideal gas we have:

$$\frac{T_{2s}}{T_1} = \left(\frac{P_{2s}}{P_1}\right)^{(k-1)/k}$$

or

$$T_{2s} = T_1\left(\frac{P_{2s}}{P_1}\right)^{(k-1)/k} = (950\text{ K})\left(\frac{110\text{ kPa}}{200\text{ kPa}}\right)^{0.349/1.349} = 814\text{ K}$$

This gives an average temperature of 882 K, which is somewhat higher than the assumed average temperature (850 K). This result could be refined by reevaluating the k value at 882 K and repeating the calculations, but it is not warranted since the two average temperatures are sufficiently close (doing so would change the temperature by only 0.6 K, which is not significant).

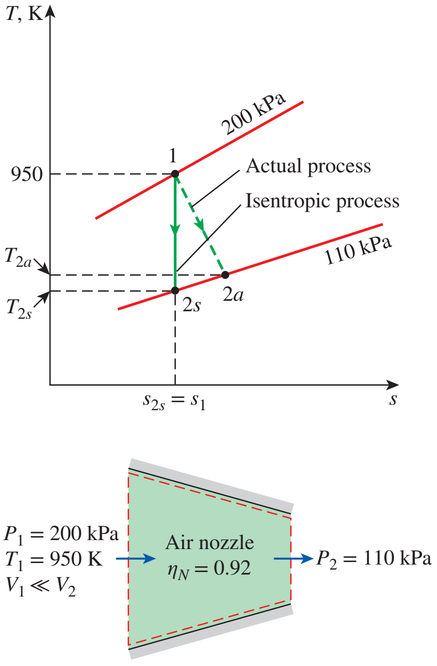


FIGURE 8–51
Schematic and *T-s* diagram for Example 8–15.

Now we can determine the isentropic exit velocity of the air from the energy balance for this isentropic steady-flow process:

$$e_{\text{in}} = e_{\text{out}}$$

$$h_1 + \frac{V_1^2}{2} = h_{2s} + \frac{V_{2s}^2}{2}$$

or

$$V_{2s} = \sqrt{2(h_1 - h_{2s})} = \sqrt{2c_{p,\text{avg}}(T_1 - T_{2s})}$$

$$= \sqrt{2(1.11\text{ kJ/kg·K})[(950 - 814)\text{K}]\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}$$

$$= \mathbf{549\text{ m/s}}$$

(*b*) The actual exit temperature of the air is higher than the isentropic exit temperature evaluated above and is determined from

$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{c_{p,\text{avg}}(T_1 - T_{2a})}{c_{p,\text{avg}}(T_1 - T_{2s})}$$

or

$$0.92 = \frac{950 - T_{2a}}{950 - 814} \rightarrow T_{2a} = \mathbf{825\text{ K}}$$

That is, the temperature is 11 K higher at the exit of the actual nozzle as a result of irreversibilities such as friction. It represents a loss since this rise in temperature comes at the expense of kinetic energy (Fig. 8–52).

(*c*) The actual exit velocity of air can be determined from the definition of isentropic efficiency of a nozzle,

$$\eta_N = \frac{V_{2a}^2}{V_{2s}^2} \rightarrow V_{2a} = \sqrt{\eta_N V_{2s}^2} = \sqrt{0.92(549\text{ m/s})^2} = \mathbf{527\text{ m/s}}$$

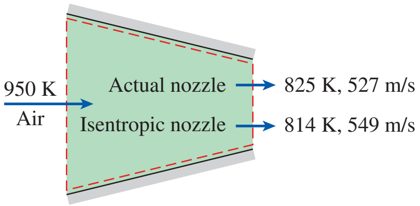


FIGURE 8–52
A substance leaves actual nozzles at a higher temperature (thus a lower velocity) as a result of friction.

8–12 ▪ ENTROPY BALANCE

The property *entropy* is a measure of molecular disorder or randomness of a system, and the second law of thermodynamics states that entropy can be created but it cannot be destroyed. Therefore, the entropy change of a system during a process is greater than the entropy transfer by an amount equal to the entropy generated during the process within the system, and the *increase of entropy principle* for any system is expressed as (Fig. 8–53)

$$\begin{pmatrix}\text{Total}\\\text{entropy}\\\text{entering}\end{pmatrix} - \begin{pmatrix}\text{Total}\\\text{entropy}\\\text{leaving}\end{pmatrix} + \begin{pmatrix}\text{Total}\\\text{entropy}\\\text{generated}\end{pmatrix} = \begin{pmatrix}\text{Change in the}\\\text{total entropy}\\\text{of the system}\end{pmatrix}$$

or

$$S_{\text{in}} - S_{\text{out}} + S_{\text{gen}} = \Delta S_{\text{system}} \qquad \textbf{(8–64)}$$

which is a verbal statement of Eq. 8–9. This relation is often referred to as the **entropy balance** and is applicable to any system undergoing any process. The entropy balance relation above can be stated as: *the entropy change of a system during a process is equal to the net entropy transfer through the system boundary plus the entropy generated within the system.* Next, we discuss the various terms in that relation.

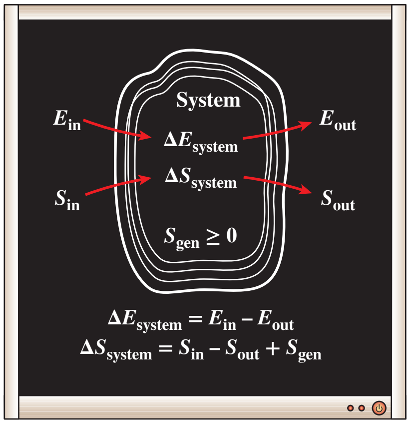


FIGURE 8–53
Energy and entropy balances for a system.

Entropy Change of a System, ΔS_{system}

Despite the reputation of entropy as being vague and abstract and the intimidation associated with it, entropy balance is actually easier to deal with than energy balance since, unlike energy, entropy does not exist in various forms. Therefore, the determination of entropy change of a system during a process involves evaluating the entropy of the system at the beginning and at the end of the process and taking their difference. That is,

Entropy change = Entropy at final state − Entropy at initial state

or

$$\Delta S_{\text{system}} = S_{\text{final}} - S_{\text{intial}} = S_2 - S_1 \tag{8–65}$$

Note that entropy is a property, and the value of a property does not change unless the state of the system changes. Therefore, the entropy change of a system is zero if the state of the system does not change during the process. For example, the entropy change of steady-flow devices such as nozzles, compressors, turbines, pumps, and heat exchangers is zero during steady operation.

When the properties of the system are not uniform, the entropy of the system can be determined by integration from

$$S_{\text{system}} = \int s\,\delta m = \int_V s\rho\,dV \tag{8–66}$$

where V is the volume of the system and ρ is density.

Mechanisms of Entropy Transfer, S_{in} and S_{out}

Entropy can be transferred to or from a system by two mechanisms: *heat transfer* and *mass flow* (in contrast, energy is transferred by work also). Entropy transfer is recognized at the system boundary as it crosses the boundary, and it represents the entropy gained or lost by a system during a process. The only form of entropy interaction associated with a fixed mass or closed system is *heat transfer,* and thus the entropy transfer for an adiabatic closed system is zero.

1 Heat Transfer

Heat is, in essence, a form of disorganized energy, and some disorganization (entropy) will flow with heat. Heat transfer to a system increases the entropy of that system and thus the level of molecular disorder or randomness, and heat transfer from a system decreases it. In fact, heat rejection is the only way the entropy of a fixed mass can be decreased. The ratio of the heat transfer Q at a location to the absolute temperature T at that location is called the *entropy flow* or *entropy transfer* and is expressed as (Fig. 8–54)

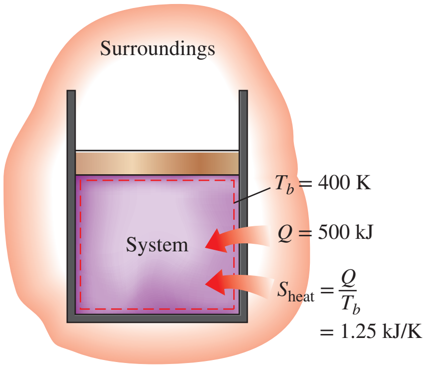


FIGURE 8–54
Heat transfer is always accompanied by entropy transfer in the amount of Q/T, where T is the boundary temperature.

Entropy transfer by heat transfer:
$$S_{\text{heat}} = \frac{Q}{T} \quad (T = \text{constant}) \tag{8–67}$$

The quantity Q/T represents the entropy transfer accompanied by heat transfer, and the direction of entropy transfer is the same as the direction of heat transfer since thermodynamic temperature T is always a positive quantity.

When the temperature T is not constant, the entropy transfer during a process 1-2 can be determined by integration (or by summation if appropriate) as

$$S_{\text{heat}} = \int_1^2 \frac{\delta Q}{T} \cong \sum \frac{Q_k}{T_k} \tag{8–68}$$

where Q_k is the heat transfer through the boundary at temperature T_k at location k.

When two systems are in contact, the entropy transfer from the warmer system is equal to the entropy transfer into the cooler one at the point of contact. That is, no entropy can be created or destroyed at the boundary since the boundary has no thickness and occupies no volume.

Note that **work** is entropy-free, and no entropy is transferred by work. Energy is transferred by both heat and work, whereas entropy is transferred only by heat. That is,

Entropy transfer by work: $$S_{\text{work}} = 0 \qquad \textbf{(8–69)}$$

The first law of thermodynamics makes no distinction between heat transfer and work; it considers them as *equals*. The distinction between heat transfer and work is brought out by the second law: *an energy interaction that is accompanied by entropy transfer is heat transfer, and an energy interaction that is not accompanied by entropy transfer is work*. That is, no entropy is exchanged during a work interaction between a system and its surroundings. Thus, only *energy* is exchanged during work interaction, whereas both *energy* and *entropy* are exchanged during heat transfer (Fig. 8–55).

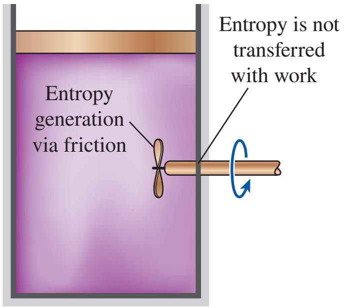


FIGURE 8–55
No entropy accompanies work as it crosses the system boundary. But entropy may be generated within the system as work is dissipated into a less useful form of energy.

2 Mass Flow

Mass contains entropy as well as energy, and the entropy and energy contents of a system are proportional to the mass. (When the mass of a system is doubled, so are the entropy and energy contents of the system.) Both entropy and energy are carried into or out of a system by streams of matter, and the rates of entropy and energy transport into or out of a system are proportional to the mass flow rate. Closed systems do not involve any mass flow and thus any entropy transfer by mass. When a mass in the amount of m enters or leaves a system, entropy in the amount of ms, where s is the specific entropy (entropy per unit mass entering or leaving), accompanies it (Fig. 8–56). That is,

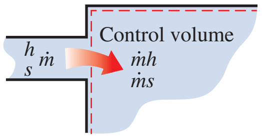


FIGURE 8–56
Mass contains entropy as well as energy, and thus mass flow into or out of a system is always accompanied by energy and entropy transfer.

Entropy transfer by mass flow: $$S_{\text{mass}} = ms \qquad \textbf{(8–70)}$$

Therefore, the entropy of a system increases by ms when mass in the amount of m enters and decreases by the same amount when the same amount of mass at the same state leaves the system. When the properties of the mass change during the process, the entropy transfer by mass flow can be determined by integration from

$$\dot{S}_{\text{mass}} = \int_{A_c} s\rho V_n \, dA_c \quad \text{and} \quad S_{\text{mass}} = \int s\,\delta m = \int_{\Delta t} \dot{S}_{\text{mass}}\, dt \qquad \textbf{(8–71)}$$

where A_c is the cross-sectional area of the flow and V_n is the local velocity normal to dA_c.

Entropy Generation, S_{gen}

Irreversibilities such as friction, mixing, chemical reactions, heat transfer through a finite temperature difference, unrestrained expansion, nonquasi-equilibrium compression, or expansion always cause the entropy of a system to increase, and entropy generation is a measure of the entropy created by such effects during a process.

For a *reversible process* (a process that involves no irreversibilities), the entropy generation is zero and thus the *entropy change* of a system is equal to the *entropy transfer*. Therefore, the entropy balance relation in the reversible case becomes analogous to the energy balance relation, which states that the *energy change* of a system during a process is equal to the *energy transfer* during that process.

However, note that the energy change of a system equals the energy transfer for *any* process, but the entropy change of a system equals the entropy transfer only for a *reversible* process.

The entropy transfer by heat Q/T is zero for adiabatic systems, and the entropy transfer by mass ms is zero for systems that involve no mass flow across their boundary (i.e., closed systems).

Entropy balance for *any system* undergoing *any process* can be expressed more explicitly as

$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\substack{\text{Net entropy transfer}\\ \text{by heat and mass}}} + \underbrace{S_{\text{gen}}}_{\substack{\text{Entropy}\\ \text{generation}}} = \underbrace{\Delta S_{\text{system}}}_{\substack{\text{Change}\\ \text{in entropy}}} \qquad (\text{kJ/K}) \tag{8–72}$$

or, in the **rate form**, as

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\substack{\text{Rate of net entropy}\\ \text{transfer by heat}\\ \text{and mass}}} + \underbrace{\dot{S}_{\text{gen}}}_{\substack{\text{Rate of entropy}\\ \text{generation}}} = \underbrace{dS_{\text{system}}/dt}_{\substack{\text{Rate of change}\\ \text{in entropy}}} \qquad (\text{kW/K}) \tag{8–73}$$

where the rates of entropy transfer by heat transferred at a rate of $\dot{Q}$ and mass flowing at a rate of $\dot{m}$ are $\dot{S}_{\text{heat}} = \dot{Q}/T$ and $\dot{S}_{\text{mass}} = \dot{m}s$. The entropy balance can also be expressed on a **unit-mass basis** as

$$(s_{\text{in}} - s_{\text{out}}) + s_{\text{gen}} = \Delta s_{\text{system}} \qquad (\text{kJ/kg·K}) \tag{8–74}$$

where all the quantities are expressed per unit mass of the system. Note that for a *reversible process,* the entropy generation term S_{gen} drops out from all of the relations above.

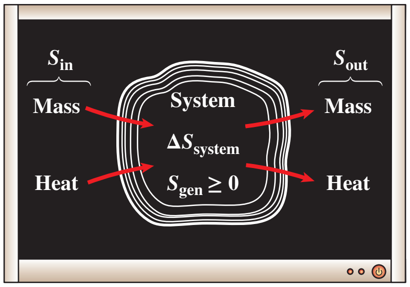


FIGURE 8–57
Mechanisms of entropy transfer for a general system.

The term S_{gen} represents the entropy generation *within the system boundary* only (Fig. 8–57), and not the entropy generation that may occur outside the system boundary during the process as a result of external irreversibilities. Therefore, a process for which $S_{\text{gen}} = 0$ is *internally reversible,* but not necessarily *totally* reversible. The *total* entropy generated during a process can be determined by applying the entropy balance to an *extended system* that includes the system itself and its immediate surroundings where external irreversibilities might be occurring (Fig. 8–58). Also, the entropy change in this case is equal to the sum of the entropy change of the system and the entropy change of the immediate surroundings. Note that under steady conditions, the state and thus the entropy of the immediate surroundings (let us call it the “buffer zone”) at any point does not change during the process, and the entropy change of the buffer zone is zero. The entropy change of the buffer zone, if any, is usually small relative to the entropy change of the system, and thus it is usually disregarded.

When evaluating the entropy transfer between an extended system and the surroundings, the boundary temperature of the extended system is simply taken to be the *environment temperature.*

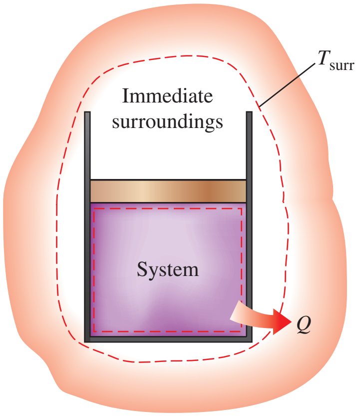


FIGURE 8–58
Entropy generation outside system boundaries can be accounted for by writing an entropy balance on an extended system that includes the system and its immediate surroundings.

Closed Systems

A closed system involves *no mass flow* across its boundaries, and its entropy change is simply the difference between the initial and final entropies of the system. The *entropy change* of a closed system is due to the *entropy transfer* accompanying heat transfer and the *entropy generation* within the system boundaries. Taking the positive direction of heat transfer to be *to* the system, the general entropy balance relation (Eq. 8–72) can be expressed for a closed system as

$$\textit{Closed system:} \quad \sum \frac{Q_k}{T_k} + S_{\text{gen}} = \Delta S_{\text{system}} = S_2 - S_1 \qquad (\text{kJ/K}) \tag{8–75}$$

The entropy balance relation above can be stated as:

> The entropy change of a closed system during a process is equal to the sum of the net entropy transferred through the system boundary by heat transfer and the entropy generated within the system boundaries.

For an *adiabatic process* ($Q = 0$), the entropy transfer term in the preceding relation drops out, and the entropy change of the closed system becomes equal to the entropy generation within the system boundaries. That is,

Adiabatic closed system:
$$S_{\text{gen}} = \Delta S_{\text{adiabatic system}} \qquad \textbf{(8–76)}$$

Noting that any closed system and its surroundings can be treated as an adiabatic system and the total entropy change of a system is equal to the sum of the entropy changes of its parts, the entropy balance for a closed system and its surroundings can be written as

System + Surroundings:
$$S_{\text{gen}} = \sum \Delta S = \Delta S_{\text{system}} + \Delta S_{\text{surroundings}} \qquad \textbf{(8–77)}$$

where $\Delta S_{\text{system}} = m(s_2 - s_1)$ and the entropy change of the surroundings can be determined from $\Delta S_{\text{surr}} = Q_{\text{surr}}/T_{\text{surr}}$ if its temperature is constant. At initial stages of studying entropy and entropy transfer, it is more instructive to start with the general form of the entropy balance (Eq. 8–72) and to simplify it for the problem under consideration. The specific relations above are convenient to use after a certain degree of intuitive understanding of the material is achieved.

Control Volumes

The entropy balance relations for control volumes differ from those for closed systems in that they involve one more mechanism of entropy exchange: *mass flow across the boundaries.* As mentioned earlier, mass possesses entropy as well as energy, and the amounts of these two extensive properties are proportional to the amount of mass (Fig. 8–59).

Taking the positive direction of heat transfer to be *to* the system, the general entropy balance relations (Eqs. 8–72 and 8–73) can be expressed for control volumes as

$$\sum \frac{Q_k}{T_k} + \sum m_i s_i - \sum m_e s_e + S_{\text{gen}} = (S_2 - S_1)_{\text{CV}} \qquad (\text{kJ/K}) \qquad \textbf{(8–78)}$$

or, in the rate form, as

$$\sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{\text{gen}} = dS_{\text{CV}}/dt \qquad (\text{kW/K}) \qquad \textbf{(8–79)}$$

This entropy balance relation can be stated as:

> The rate of entropy change within the control volume during a process is equal to the sum of the rate of entropy transfer through the control volume boundary by heat transfer, the net rate of entropy transfer into the control volume by mass flow, and the rate of entropy generation within the boundaries of the control volume as a result of irreversibilities.

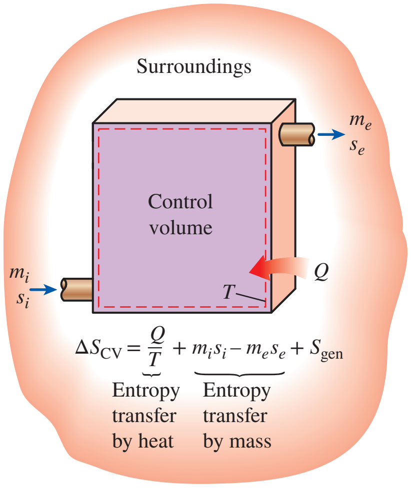


FIGURE 8–59
The entropy of a control volume changes as a result of mass flow as well as heat transfer.

Most control volumes encountered in practice such as turbines, compressors, nozzles, diffusers, heat exchangers, pipes, and ducts operate steadily, and thus they experience no change in their entropy. Therefore, the entropy balance relation for a general **steady-flow process** can be obtained from Eq. 8–79 by setting $dS_{\text{CV}}/dt = 0$ and rearranging to give

Steady-flow:
$$\dot{S}_{\text{gen}} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k} \qquad \textbf{(8–80)}$$

For *single-stream* (one inlet and one exit) steady-flow devices, the entropy balance relation simplifies to

Steady-flow, single-stream:
$$\dot S_{\text{gen}} = \dot m(s_e - s_i) - \sum \frac{\dot Q_k}{T_k} \tag{8–81}$$

For the case of an *adiabatic* single-stream device, the entropy balance relation further simplifies to

Steady-flow, single-stream, adiabatic:
$$\dot S_{\text{gen}} = \dot m(s_e - s_i) \tag{8–82}$$

which indicates that the specific entropy of the fluid must increase as it flows through an adiabatic device since $\dot S_{\text{gen}} \ge 0$ (Fig. 8–60). If the flow through the device is *reversible* and *adiabatic,* then the entropy remains constant, $s_e = s_i$, regardless of the changes in other properties.

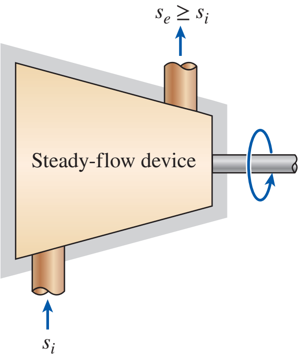


FIGURE 8–60
The entropy of a substance always increases (or remains constant in the case of a reversible process) as it flows through a single-stream, adiabatic, steady-flow device.

EXAMPLE 8–16 Entropy Generation in a Wall

Consider steady heat transfer through a 5-m × 7-m brick wall of a house of thickness 30 cm. On a day when the temperature of the outdoors is 0°C, the house is maintained at 27°C. The temperatures of the inner and outer surfaces of the brick wall are measured to be 20°C and 5°C, respectively, and the rate of heat transfer through the wall is 1035 W. Determine the rate of entropy generation in the wall and the rate of total entropy generation associated with this heat transfer process.

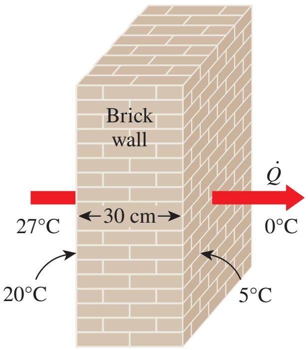


FIGURE 8–61
Schematic for Example 8–16.

SOLUTION Steady heat transfer through a wall is considered. For specified heat transfer rate, wall temperatures, and environment temperatures, the entropy generation rate within the wall and the total entropy generation rate are to be determined.

Assumptions **1** The process is steady, and thus the rate of heat transfer through the wall is constant. **2** Heat transfer through the wall is one-dimensional.

Analysis We first take the *wall* as the system (Fig. 8–61). This is a *closed system* since no mass crosses the system boundary during the process. We note that the entropy change of the wall is zero during this process since the state and thus the entropy of the wall do not change anywhere in the wall. Heat and entropy are entering from one side of the wall and leaving from the other side.

The rate form of the entropy balance for the wall simplifies to

$$\underbrace{\dot S_{\text{in}} - \dot S_{\text{out}}}_{\substack{\text{Rate of net entropy}\\ \text{transfer by heat}\\ \text{and mass}}} + \underbrace{\dot S_{\text{gen}}}_{\substack{\text{Rate of entropy}\\ \text{generation}}} = \underbrace{\cancelto{0\text{ (steady)}}{dS_{\text{system}}/dt}}_{\substack{\text{Rate of change}\\ \text{in entropy}}}$$

$$\left(\frac{\dot Q}{T}\right)_{\text{in}} - \left(\frac{\dot Q}{T}\right)_{\text{out}} + \dot S_{\text{gen}} = 0$$

$$\frac{1035\text{ W}}{293\text{ K}} - \frac{1035\text{ W}}{278\text{ K}} + \dot S_{\text{gen}} = 0$$

Therefore, the rate of entropy generation in the wall is

$$\dot S_{\text{gen}} = \mathbf{0.191\ W/K}$$

Note that entropy transfer by heat at any location is *Q*/*T* at that location, and the direction of entropy transfer is the same as the direction of heat transfer.

To determine the rate of total entropy generation during this heat transfer process, we extend the system to include the regions on both sides of the wall that experience a temperature change. Then, one side of the system boundary becomes room temperature while the other side becomes the temperature of the outdoors. The entropy balance for this *extended system* (system + immediate surroundings) is the same as that given above, except the two boundary temperatures are now 300 and 273 K instead of 293 and 278 K, respectively. Then the rate of total entropy generation becomes

$$\frac{1035 \text{ W}}{300 \text{ K}} - \frac{1035 \text{ W}}{273 \text{ K}} + \dot{S}_{\text{gen,total}} = 0 \quad \rightarrow \quad \dot{S}_{\text{gen,total}} = \mathbf{0.341 \text{ W/K}}$$

Discussion Note that the entropy change of this extended system is also zero since the state of air does not change at any point during the process. The difference between the two entropy generations is 0.150 W/K, and it represents the entropy generated in the air layers on both sides of the wall. The entropy generation in this case is entirely due to irreversible heat transfer through a finite temperature difference.

EXAMPLE 8–17 Entropy Generation During a Throttling Process

Steam at 7 MPa and 450°C is throttled in a valve to a pressure of 3 MPa during a steady-flow process. Determine the entropy generated during this process and check to see if the increase of entropy principle is satisfied.

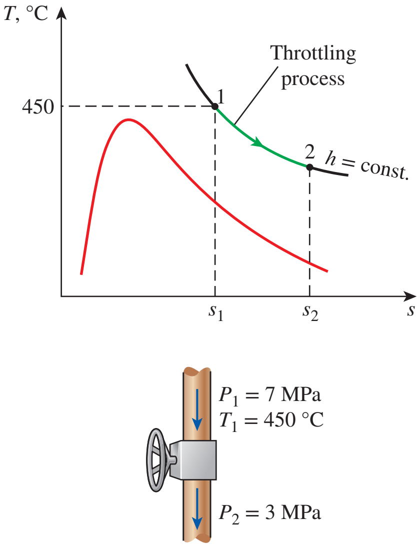


FIGURE 8–62
Schematic and *T*-*s* diagram for Example 8–17.

SOLUTION Steam is throttled to a specified pressure. The entropy generated during this process is to be determined, and the validity of the increase of entropy principle is to be verified.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\text{CV}} = 0$, $\Delta E_{\text{CV}} = 0$, and $\Delta S_{\text{CV}} = 0$. **2** Heat transfer to or from the valve is negligible. **3** The kinetic and potential energy changes are negligible, $\Delta\text{ke} = \Delta\text{pe} = 0$.

Analysis We take the throttling valve as the *system* (Fig. 8–62). This is a *control volume* since mass crosses the system boundary during the process. We note that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Also, the enthalpy of a fluid remains nearly constant during a throttling process, and thus $h_2 \cong h_1$.

The entropy of the steam at the inlet and the exit states is determined from the steam tables to be

$$\textit{State 1:} \qquad \left.\begin{array}{l} P_1 = 7 \text{ MPa} \\ T_1 = 450°\text{C} \end{array}\right\} \begin{array}{l} h_1 = 3288.3 \text{ kJ/kg} \\ s_1 = 6.6353 \text{ kJ/kg·K} \end{array}$$

$$\textit{State 2:} \qquad \left.\begin{array}{l} P_{2a} = 3 \text{ MPa} \\ h_2 = h_1 \end{array}\right\} s_2 = 7.0046 \text{ kJ/kg·K}$$

Then, the entropy generation per unit mass of the steam is determined from the entropy balance applied to the throttling valve,

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\substack{\text{Rate of net entropy} \\ \text{transfer by heat} \\ \text{and mass}}} + \underbrace{\dot{S}_{\text{gen}}}_{\substack{\text{Rate of entropy} \\ \text{generation}}} = \underbrace{dS_{\text{system}}/dt^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change} \\ \text{in entropy}}}$$

$$\dot{m}s_1 - \dot{m}s_2 + \dot{S}_{\text{gen}} = 0$$

$$\dot{S}_{\text{gen}} = \dot{m}(s_2 - s_1)$$

Dividing by mass flow rate and substituting gives

$$s_{\text{gen}} = s_2 - s_1 = 7.0046 - 6.6353 = \mathbf{0.3693 \text{ kJ/kg·K}}$$

This is the amount of entropy generated per unit mass of steam as it is throttled from the inlet state to the final pressure, and it is caused by unrestrained expansion. The increase of entropy principle is obviously satisfied during this process since the entropy generation is positive.

EXAMPLE 8–18 Entropy Generated when a Hot Block Is Dropped in a Lake

A 50-kg block of iron casting at 500 K is thrown into a large lake that is at a temperature of 285 K. The iron block eventually reaches thermal equilibrium with the lake water. Assuming an average specific heat of 0.45 kJ/kg·K for the iron, determine (*a*) the entropy change of the iron block, (*b*) the entropy change of the lake water, and (*c*) the entropy generated during this process.

SOLUTION A hot iron block is thrown into a lake and cools to the lake temperature. The entropy changes of the iron and of the lake as well as the entropy generated during this process are to be determined.

Assumptions **1** Both the water and the iron block are incompressible substances. **2** Constant specific heats can be used for the water and the iron. **3** The kinetic and potential energy changes of the iron are negligible, $\Delta KE = \Delta PE = 0$ and thus $\Delta E = \Delta U$.

Properties The specific heat of the iron is 0.45 kJ/kg·K (Table A–3).

Analysis We take the *iron casting* as the system (Fig. 8–63). This is a *closed system* since no mass crosses the system boundary during the process.

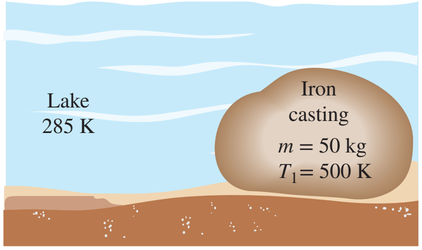


FIGURE 8–63
Schematic for Example 8–18.

To determine the entropy change for the iron block and for the lake, first we need to know the final equilibrium temperature. Given that the thermal energy capacity of the lake is very large relative to that of the iron block, the lake will absorb all the heat rejected by the iron block without experiencing any change in its temperature. Therefore, the iron block will cool to 285 K during this process while the lake temperature remains constant at 285 K.

(*a*) The entropy change of the iron block can be determined from

$$\Delta S_{\text{iron}} = m(s_2 - s_1) = mc_{\text{avg}} \ln \frac{T_2}{T_1}$$
$$= (50 \text{ kg})(0.45 \text{ kJ/kg·K}) \ln \frac{285 \text{ K}}{500 \text{ K}}$$
$$= \mathbf{-12.65 \text{ kJ/K}}$$

(*b*) The temperature of the lake water remains constant during this process at 285 K. Also, the amount of heat transfer from the iron block to the lake is determined from an energy balance on the iron block to be

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer by}\\ \text{heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$
$$-Q_{\text{out}} = \Delta U = mc_{\text{avg}}(T_2 - T_1)$$

or

$$Q_{\text{out}} = mc_{\text{avg}}(T_1 - T_2) = (50 \text{ kg})(0.45 \text{ kJ/kg·K})(500 - 285)\text{K} = 4838 \text{ kJ}$$

Then, the entropy change of the lake becomes

$$\Delta S_{\text{lake}} = \frac{Q_{\text{lake}}}{T_{\text{lake}}} = \frac{+4838 \text{ kJ}}{285 \text{ K}} = \mathbf{16.97 \text{ kJ/K}}$$

(*c*) The entropy generated during this process can be determined by applying an entropy balance on an *extended system* that includes the iron block and its immediate surroundings so that the boundary temperature of the extended system is at 285 K at all times:

$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\substack{\text{Net entropy transfer}\\ \text{by heat and mass}}} + \underbrace{S_{\text{gen}}}_{\substack{\text{Entropy}\\ \text{generation}}} = \underbrace{\Delta S_{\text{system}}}_{\substack{\text{Change}\\ \text{in entropy}}}$$
$$-\frac{Q_{\text{out}}}{T_b} + S_{\text{gen}} = \Delta S_{\text{system}}$$

or

$$S_{\text{gen}} = \frac{Q_{\text{out}}}{T_b} + \Delta S_{\text{system}} = \frac{4838\ \text{kJ}}{285\ \text{K}} - 12.65\ \text{kJ/K} = \mathbf{4.32\ kJ/K}$$

Discussion The entropy generated can also be determined by taking the iron block and the entire lake as the system, which is an isolated system, and applying an entropy balance. An isolated system involves no heat or entropy transfer, and thus the entropy generation in this case becomes equal to the total entropy change,

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{iron}} + \Delta S_{\text{lake}} = -12.65 + 16.97 = 4.32\ \text{kJ/K}$$

which is the same result obtained above.

EXAMPLE 8–19 Entropy Generation in a Heat Exchanger

Air in a large building is kept warm by heating it with steam in a heat exchanger (Fig. 8–64). Saturated water vapor enters this unit at 35°C at a rate of 10,000 kg/h and leaves as saturated liquid at 32°C. Air at 1-atm pressure enters the unit at 20°C and leaves at 30°C at about the same pressure. Determine the rate of entropy generation associated with this process.

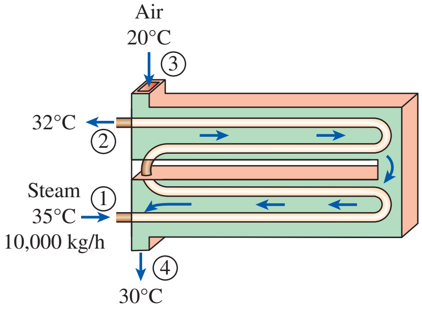


FIGURE 8–64
Schematic for Example 8–19.

SOLUTION Air is heated by steam in a heat exchanger. The rate of entropy generation associated with this process is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible, and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Air is an ideal gas with constant specific heats at room temperature. **5** The pressure of air remains constant.

Analysis The rate of entropy generation within the heat exchanger is determined by applying the rate form of the entropy balance on the entire heat exchanger:

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\substack{\text{Rate of net entropy}\\ \text{transfer by heat}\\ \text{and mass}}} + \underbrace{\dot{S}_{\text{gen}}}_{\substack{\text{Rate of entropy}\\ \text{generation}}} = \underbrace{dS_{\text{system}}/dt}_{\substack{\text{Rate of change}\\ \text{in entropy}}}\nearrow^{0\ \text{(steady)}}$$

$$\dot{m}_{\text{steam}}s_1 + \dot{m}_{\text{air}}s_3 - \dot{m}_{\text{steam}}s_2 - \dot{m}_{\text{air}}s_4 + \dot{S}_{\text{gen}} = 0$$

$$\dot{S}_{\text{gen}} = \dot{m}_{\text{steam}}(s_2 - s_1) + \dot{m}_{\text{air}}(s_4 - s_3)$$

The specific heat of air at room temperature is $c_p = 1.005$ kJ/kg·°C (Table A–2*a*). The properties of the steam at the inlet and exit states are

$$\left.\begin{array}{l} T_1 = 35°\text{C} \\ x_1 = 1 \end{array}\right\} \begin{array}{l} h_1 = 2564.6\ \text{kJ/kg} \\ s_1 = 8.3517\ \text{kJ/kg·K} \end{array} \qquad \text{(Table A–4)}$$

$$\left.\begin{array}{l} T_2 = 32°\text{C} \\ x_2 = 0 \end{array}\right\} \begin{array}{l} h_2 = 134.10\ \text{kJ/kg} \\ s_2 = 0.4641\ \text{kJ/kg·K} \end{array} \qquad \text{(Table A–4)}$$

From an energy balance the heat transferred from steam is equal to the heat transferred to the air. Then, the mass flow rate of air is determined to be

$$\dot{Q} = \dot{m}_{\text{steam}}(h_2 - h_1) = (10{,}000/3600\ \text{kg/s})(2564.6 - 134.10)\text{kJ/kg} = 6751\ \text{kW}$$

$$\dot{m}_{\text{air}} = \frac{\dot{Q}}{c_p(T_4 - T_3)} = \frac{6751\ \text{kW}}{(1.005\ \text{kJ/kg·°C})(30 - 20)°\text{C}} = 671.7\ \text{kg/s}$$

Substituting into the entropy balance relation, the rate of entropy generation becomes

$$\begin{aligned}\dot{S}_{\text{gen}} &= \dot{m}_{\text{steam}}(s_2 - s_1) + \dot{m}_{\text{air}}(s_4 - s_3)\\ &= \dot{m}_{\text{steam}}(s_2 - s_1) + \dot{m}_{\text{air}}c_p \ln\frac{T_4}{T_3}\\ &= (10{,}000/3600\text{ kg/s})(0.4641 - 8.3517)\text{kJ/kg·K}\\ &\quad + (671.1\text{ kg/s})(1.005\text{ kJ/kg·K})\ln\frac{303\text{ K}}{293\text{ K}}\\ &= \mathbf{0.745\ kW/K}\end{aligned}$$

Discussion Note that the pressure of air remains nearly constant as it flows through the heat exchanger, and thus the pressure term is not included in the entropy change expression for air.

SUMMARY

The second law of thermodynamics leads to the definition of a new property called *entropy,* which is a quantitative measure of microscopic disorder for a system. Any quantity whose cyclic integral is zero is a property, and entropy is defined as

$$dS = \left(\frac{dQ}{T}\right)_{\text{int rev}}$$

For the special case of an internally reversible, isothermal process, it gives

$$\Delta S = \frac{Q}{T_0}$$

The inequality part of the Clausius inequality combined with the definition of entropy yields an inequality known as the *increase of entropy principle,* expressed as

$$S_{\text{gen}} \geq 0$$

where S_{gen} is the *entropy generated* during the process. Entropy change is caused by heat transfer, mass flow, and irreversibilities. Heat transfer to a system increases the entropy, and heat transfer from a system decreases it. The effect of irreversibilities is always to increase the entropy.

The *entropy-change* and *isentropic relations* for a process can be summarized as follows:

1. *Pure substances:*

Any process: $\Delta s = s_2 - s_1$

Isentropic process: $s_2 = s_1$

2. *Incompressible substances:*

Any process: $s_2 - s_1 = c_{\text{avg}} \ln\frac{T_2}{T_1}$

Isentropic process: $T_2 = T_1$

3. *Ideal gases:*

a. Constant specific heats (approximate treatment):

Any process:

$$s_2 - s_1 = c_{v,\text{avg}} \ln\frac{T_2}{T_1} + R \ln\frac{v_2}{v_1}$$

$$s_2 - s_1 = c_{p,\text{avg}} \ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1}$$

Isentropic process:

$$\left(\frac{T_2}{T_1}\right)_{s\,=\,\text{const.}} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$\left(\frac{T_2}{T_1}\right)_{s\,=\,\text{const.}} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$\left(\frac{P_2}{P_1}\right)_{s\,=\,\text{const.}} = \left(\frac{v_1}{v_2}\right)^{k}$$

b. Variable specific heats (exact treatment):

Any process:

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln\frac{P_2}{P_1}$$

Isentropic process:

$$s_2^\circ = s_1^\circ + R \ln\frac{P_2}{P_1}$$

$$\left(\frac{P_2}{P_1}\right)_{s\,=\,\text{const.}} = \frac{P_{r2}}{P_{r1}}$$

$$\left(\frac{v_2}{v_1}\right)_{s\,=\,\text{const.}} = \frac{v_{r2}}{v_{r1}}$$

where P_r is the *relative pressure* and v_r is the *relative specific volume.* The function s° depends on temperature only.

The *steady-flow work* for a reversible process can be expressed in terms of the fluid properties as

$$w_{\text{rev}} = -\int_1^2 \upsilon\, dP - \Delta\text{ke} - \Delta\text{pe}$$

For incompressible substances (υ = constant) it simplifies to

$$w_{\text{rev}} = -\upsilon(P_2 - P_1) - \Delta\text{ke} - \Delta\text{pe}$$

The work done during a steady-flow process is proportional to the specific volume. Therefore, υ should be kept as small as possible during a compression process to minimize the work input and as large as possible during an expansion process to maximize the work output.

Most steady-flow devices operate under adiabatic conditions, and the ideal process for these devices is the isentropic process. The parameter that describes how efficiently a device approximates a corresponding isentropic device is called *isentropic* or *adiabatic efficiency.* It is expressed for turbines, compressors, and nozzles as follows:

$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s} \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

$$\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2} \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

In the preceding relations, $h_{2\text{a}}$ and $h_{2\text{s}}$ are the enthalpy values at the exit state for actual and isentropic *processes,* respectively.

The entropy balance for any system undergoing any process can be expressed in the general form as

$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\substack{\text{Net entropy transfer}\\ \text{by heat and mass}}} + \underbrace{S_{\text{gen}}}_{\substack{\text{Entropy}\\ \text{generation}}} = \underbrace{\Delta S_{\text{system}}}_{\substack{\text{Change}\\ \text{in entropy}}}$$

or, in the *rate form,* as

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\substack{\text{Rate of net entropy}\\ \text{transfer by heat}\\ \text{and mass}}} + \underbrace{\dot{S}_{\text{gen}}}_{\substack{\text{Rate of entropy}\\ \text{generation}}} = \underbrace{dS_{\text{system}}/dt}_{\substack{\text{Rate of change}\\ \text{in entropy}}}$$

For a general *steady-flow process* it simplifies to

$$\dot{S}_{\text{gen}} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$$

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PROBLEMS*

Entropy and the Increase of Entropy Principle

8–1C Does a cycle for which $\oint \delta Q > 0$ violate the Clausius inequality? Why?

8–2C Does the cyclic integral of heat have to be zero (i.e., does a system have to reject as much heat as it receives to complete a cycle)? Explain.

8–3C Is an isothermal process necessarily internally reversible? Explain your answer with an example.

8–4C Is the value of the integral $\int_1^2 \delta Q/T$ the same for all reversible processes between states 1 and 2? Why?

8–5C How do the values of the integral $\int_1^2 \delta Q/T$ compare for a reversible and an irreversible process between the same end states?

8–6C Is it possible to create entropy? Is it possible to destroy it?

8–7C The entropy of a hot baked potato decreases as it cools. Is this a violation of the increase of entropy principle? Explain.

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*Problems designated by a “C” are concept questions, and students are encouraged to answer them all. Problems designated by an “E” are in English units, and the SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

8–8C When a system is adiabatic, what can be said about the entropy change of the substance in the system?

8–9C Is it possible for the entropy change of a closed system to be zero during an irreversible process? Explain.

8–10C A piston–cylinder device contains helium gas. During a reversible, isothermal process, the entropy of the helium will (*never, sometimes, always*) increase.

8–11C A piston–cylinder device contains nitrogen gas. During a reversible, adiabatic process, the entropy of the nitrogen will (*never, sometimes, always*) increase.

8–12C A piston–cylinder device contains superheated steam. During an actual adiabatic process, the entropy of the steam will (*never, sometimes, always*) increase.

8–13C The entropy of steam will (*increase, decrease, remain the same*) as it flows through an actual adiabatic turbine.

8–14C During a heat transfer process, the entropy of a system (*always, sometimes, never*) increases.

8–15C Steam is accelerated as it flows through an actual adiabatic nozzle. The entropy of the steam at the nozzle exit will be (*greater than, equal to, less than*) the entropy at the nozzle inlet.

8–16 Heat is transferred at a rate of 2 kW from a hot reservoir at 800 K to a cold reservoir at 300 K. Calculate the rate at which the entropy of the two reservoirs changes and determine if the second law is satisfied. *Answer:* 0.00417 kW/K

8–17E A completely reversible air conditioner provides 36,000 Btu/h of cooling for a space maintained at 70°F while rejecting heat to the environmental air at 110°F. Calculate the rate at which the entropies of the two reservoirs change and verify that this air conditioner satisfies the increase of entropy principle.

8–18 Heat in the amount of 100 kJ is transferred directly from a hot reservoir at 1200 K to a cold reservoir at 600 K. Calculate the entropy change of the two reservoirs and determine if the increase of entropy principle is satisfied.

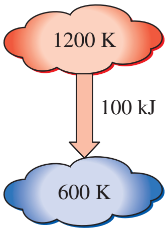


FIGURE P8–18

8–19 In Prob. 8–18, assume that the heat is transferred from the cold reservoir to the hot reservoir contrary to the Clausius statement of the second law. Prove that this violates the increase of entropy principle—as it must according to Clausius.

8–20 During the isothermal heat addition process of a Carnot cycle, 900 kJ of heat is added to the working fluid from a source at 400°C. Determine (*a*) the entropy change of the working fluid, (*b*) the entropy change of the source, and (*c*) the total entropy change for the process.

8–21 Reconsider Prob. 8–20. Using appropriate software, study the effects of the varying heat added to the working fluid and the source temperature on the entropy change of the working fluid, the entropy change of the source, and the total entropy change for the process. Let the source temperature vary from 100 to 1000°C. Plot the entropy changes of the source and of the working fluid against the source temperature for heat transfer amounts of 500 kJ, 900 kJ, and 1300 kJ, and discuss the results.

8–22E During the isothermal heat rejection process of a Carnot cycle, the working fluid experiences an entropy change of –0.7 Btu/R. If the temperature of the heat sink is 95°F, determine (*a*) the amount of heat transfer, (*b*) the entropy change of the sink, and (*c*) the total entropy change for this process. *Answers:* (*a*) 389 Btu, (*b*) 0.7 Btu/R, (*c*) 0

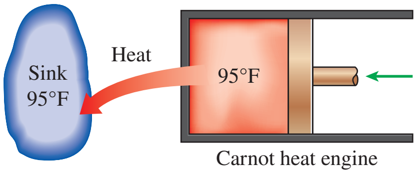


FIGURE P8–22E

8–23 Air is compressed by a 40-kW compressor from P_1 to P_2. The air temperature is maintained constant at 25°C during this process as a result of heat transfer to the surrounding medium at 20°C. Determine the rate of entropy change of the air. State the assumptions made in solving this problem. *Answer:* –0.134 kW/K

8–24 Refrigerant-134a enters the coils of the evaporator of a refrigeration system as a saturated liquid–vapor mixture at a pressure of 140 kPa. The refrigerant absorbs 180 kJ of heat from the cooled space, which is maintained at –10°C, and leaves as saturated vapor at the same pressure. Determine (*a*) the entropy change of the refrigerant, (*b*) the entropy change of the cooled space, and (*c*) the total entropy change for this process.

8–25 A rigid tank contains an ideal gas at 40°C that is being stirred by a paddle wheel. The paddle wheel does 200 kJ of work on the ideal gas. It is observed that the temperature of the ideal gas remains constant during this process as a result of heat transfer between the system and the surroundings at 30°C. Determine the entropy change of the ideal gas.

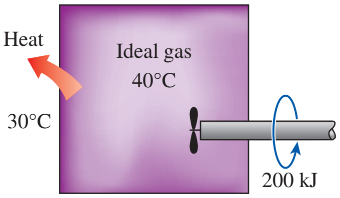


FIGURE P8–25

8–26 A rigid vessel is filled with a fluid from a source whose properties remain constant. How does the entropy of the surroundings change if the vessel is filled such that the specific entropy of the vessel contents remains constant?

8–27 A rigid vessel filled with a fluid is allowed to leak some fluid out through an opening. During this process, the specific entropy of the remaining fluid remains constant. How does the entropy of the environment change during this process?

Entropy Changes of Pure Substances

8–28C Is a process that is internally reversible and adiabatic necessarily isentropic? Explain.

8–29E One lbm of R-134a is expanded isentropically in a closed system from 100 psia and 100°F to 10 psia. Determine the total heat transfer and work production for this process.

8–30E Two lbm of water at 300 psia fill a weighted piston–cylinder device whose volume is 2.5 ft^3. The water is then heated at constant pressure until the temperature reaches 500°F. Determine the resulting change in the water's total entropy. *Answer:* 0.474 Btu/R

8–31 A well-insulated rigid tank contains 3 kg of a saturated liquid–vapor mixture of water at 200 kPa. Initially, three-quarters of the mass is in the liquid phase. An electric resistance heater placed in the tank is now turned on and kept on until all the liquid in the tank is vaporized. Determine the entropy change of the steam during this process. *Answer:* 11.1 kJ/K

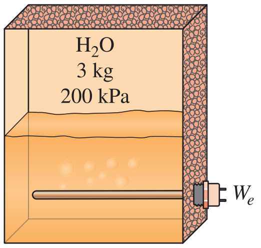


FIGURE P8–31

8–32 Using the relation $dS = (\delta Q/T)_{\text{int rev}}$ for the definition of entropy, calculate the change in the specific entropy of R-134a as it is heated at a constant pressure of 200 kPa from a saturated liquid to a saturated vapor. Use the R-134a tables to verify your answer.

8–33 The radiator of a steam heating system has a volume of 20 L and is filled with superheated water vapor at 200 kPa and 150°C. At this moment both the inlet and the exit valves to the radiator are closed. After a while the temperature of the steam drops to 40°C as a result of heat transfer to the room air. Determine the entropy change of the steam during this process. *Answer:* –0.132 kJ/K

8–34 A rigid tank is divided into two equal parts by a partition. One part of the tank contains 2.5 kg of compressed liquid water at 400 kPa and 60°C while the other part is evacuated. The partition is now removed, and the water expands to fill the entire tank. Determine the entropy change of water during this process, if the final pressure in the tank is 40 kPa. *Answer:* 0.492 kJ/K

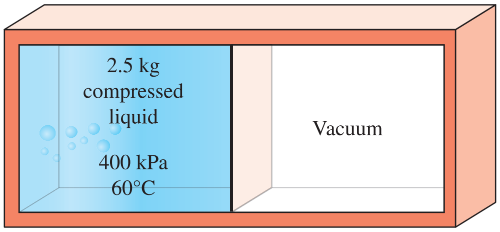


FIGURE P8–34

8–35 An insulated piston–cylinder device contains 0.05 m^3 of saturated refrigerant-134a vapor at 0.8-MPa pressure. The refrigerant is now allowed to expand in a reversible manner until the pressure drops to 0.4 MPa. Determine (*a*) the final temperature in the cylinder and (*b*) the work done by the refrigerant.

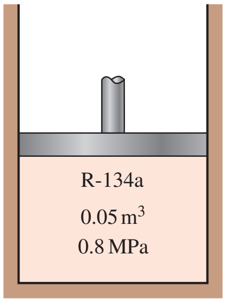


FIGURE P8–35

8–36 Reconsider Prob. 8–35. Using appropriate software, evaluate and plot the work done by the refrigerant as a function of final pressure as it varies from 0.8 to 0.4 MPa. Compare the work done for this process to one for which the temperature is constant over the same pressure range. Discuss your results.

8–37 An insulated piston–cylinder device contains 5 L of saturated liquid water at a constant pressure of 150 kPa. An electric resistance heater inside the cylinder is now turned on, and 1700 kJ of energy is transferred to the steam. Determine the entropy change of the water during this process. *Answer:* 4.42 kJ/K

8–38 One kg of R-134a initially at 600 kPa and 25°C undergoes a process during which the entropy is kept constant until the pressure drops to 100 kPa. Determine the final temperature of the R-134a and the final specific internal energy.

8–39 Refrigerant-134a is expanded isentropically from 600 kPa and 70°C at the inlet of a steady-flow turbine to 100 kPa at the outlet. The outlet area is 1 m^2, and the inlet area is 0.5 m^2. Calculate the inlet and outlet velocities when the mass flow rate is 0.75 kg/s. *Answers:* 0.0646 m/s, 0.171 m/s

8–40 Refrigerant-134a at 320 kPa and 40°C undergoes an isothermal process in a closed system until its quality is 45 percent. On a per-unit-mass basis, determine how much work and heat transfer are required. *Answers:* 40.6 kJ/kg, 130 kJ/kg

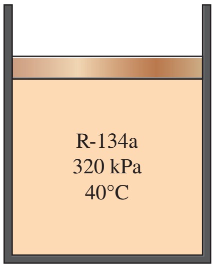


FIGURE P8–40

8–41 A rigid tank contains 5 kg of saturated vapor steam at 100°C. The steam is cooled to the ambient temperature of 25°C.

(*a*) Sketch the process with respect to the saturation lines on a *T*-υ diagram.
(*b*) Determine the entropy change of the steam, in kJ/K.
(*c*) For the steam and its surroundings, determine the total entropy change associated with this process, in kJ/K.

8–42 A 0.5-m^3 rigid tank contains refrigerant-134a initially at 200 kPa and 40 percent quality. Heat is transferred now to the refrigerant from a source at 35°C until the pressure rises to 400 kPa. Determine (*a*) the entropy change of the refrigerant, (*b*) the entropy change of the heat source, and (*c*) the total entropy change for this process.

8–43 Reconsider Prob. 8–42. Using appropriate software, investigate the effects of the source temperature and final pressure on the total entropy change for the process. Let the source temperature vary from 30 to 210°C, and let the final pressure vary from 250 to 500 kPa. Plot the total entropy change for the process as a function of the source temperature for final pressures of 250 kPa, 400 kPa, and 500 kPa, and discuss the results.

8–44 Steam enters a steady-flow adiabatic nozzle with a low inlet velocity as a saturated vapor at 6 MPa and expands to 1.2 MPa.

(*a*) Under the conditions that the exit velocity is to be the maximum possible value, sketch the *T*-*s* diagram with respect to the saturation lines for this process.
(*b*) Determine the maximum exit velocity of the steam, in m/s.

Answer: 764 m/s

8–45 Steam enters an adiabatic diffuser at 150 kPa and 120°C with a velocity of 550 m/s. Determine the minimum velocity that the steam can have at the outlet when the outlet pressure is 300 kPa.

8–46E R-134a vapor enters into a turbine at 250 psia and 175°F. The temperature of R-134a is reduced to 20°F in this turbine while its specific entropy remains constant. Determine the change in the enthalpy of R-134a as it passes through the turbine.

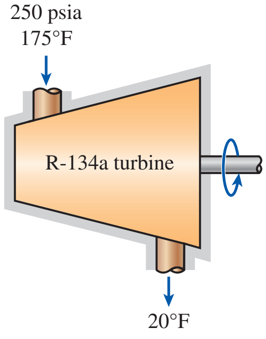


FIGURE P8–46E

8–47 Water vapor enters a compressor at 35 kPa and 160°C and leaves at 300 kPa with the same specific entropy as at the inlet. What are the temperature and the specific enthalpy of water at the compressor exit?

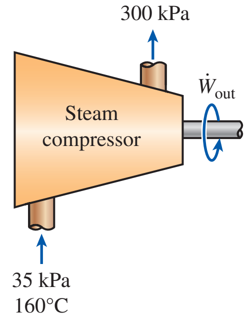


FIGURE P8–47

8–48 Refrigerant-134a enters an adiabatic compressor as saturated vapor at 160 kPa at a rate of 2 m^3/min and is compressed to a pressure of 900 kPa. Determine the minimum power that must be supplied to the compressor.

8–49E The compressor in a refrigerator compresses saturated R-134a vapor at 0°F to 200 psia. Calculate the work required by this compressor, in Btu/lbm, when the compression process is isentropic.

8–50 An isentropic steam turbine processes 2 kg/s of steam at 3 MPa, which is exhausted at 50 kPa and 100°C. Five percent of this flow is diverted for feedwater heating at 500 kPa. Determine the power produced by this turbine in kW. *Answer:* 2285 kW

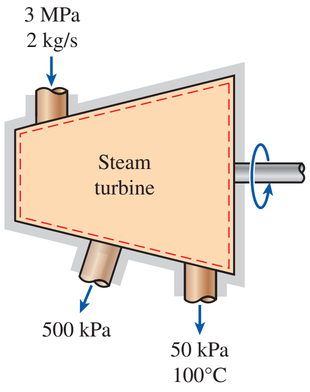


FIGURE P8–50

8–51 Water at 10°C and 81.4 percent quality is compressed isentropically in a closed system to 3 MPa. How much work does this process require in kJ/kg?

8–52 Two kg of saturated water vapor at 600 kPa are contained in a piston–cylinder device. The water expands adiabatically until the pressure is 100 kPa and is said to produce 700 kJ of work output.

(*a*) Determine the entropy change of the water, in kJ/kg·K.
(*b*) Is this process realistic? Using the *T*-*s* diagram for the process and the concepts of the second law, support your answer.

8–53 A piston–cylinder device contains 5 kg of steam at 100°C with a quality of 50 percent. This steam undergoes two processes as follows:

1-2 Heat is transferred to the steam in a reversible manner while the temperature is held constant until the steam exists as a saturated vapor.

2-3 The steam expands in an adiabatic, reversible process until the pressure is 15 kPa.

(*a*) Sketch these processes with respect to the saturation lines on a single *T-s* diagram.

(*b*) Determine the heat transferred to the steam in process 1-2, in kJ.

(*c*) Determine the work done by the steam in process 2-3, in kJ.

8–54 A rigid, 20-L steam cooker is arranged with a pressure relief valve set to release vapor and maintain the pressure once the pressure inside the cooker reaches 150 kPa. Initially, this cooker is filled with water at 175 kPa with a quality of 10 percent. Heat is now added until the quality inside the cooker is 40 percent. Determine the minimum entropy change of the thermal energy reservoir supplying this heat.

8–55 In Prob. 8–54, the water is stirred at the same time that it is being heated. Determine the minimum entropy change of the heat-supplying source if 100 kJ of work is done on the water as it is being heated.

8–56E A 0.55-ft^3 well-insulated rigid can initially contains refrigerant-134a at 90 psia and 30°F. Now a crack develops in the can, and the refrigerant starts to leak out slowly, Assuming the refrigerant remaining in the can has undergone a reversible, adiabatic process, determine the final mass in the can when the pressure drops to 20 psia.

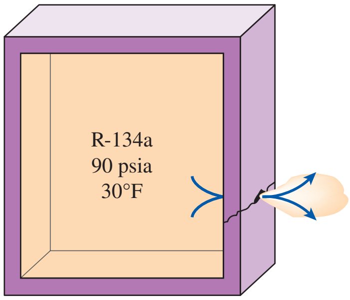


FIGURE P8–56E

8–57 Determine the total heat transfer for the reversible process 1-3 shown in Fig. P8–57.

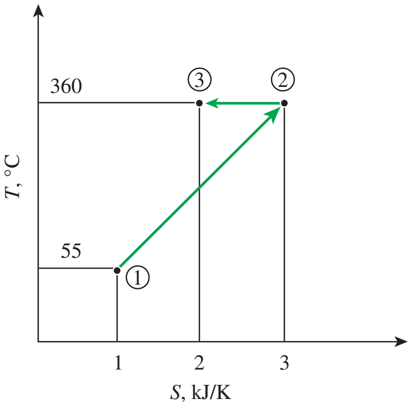


FIGURE P8–57

8–58 Determine the total heat transfer for the reversible process 1-2 shown in Fig. P8–58.

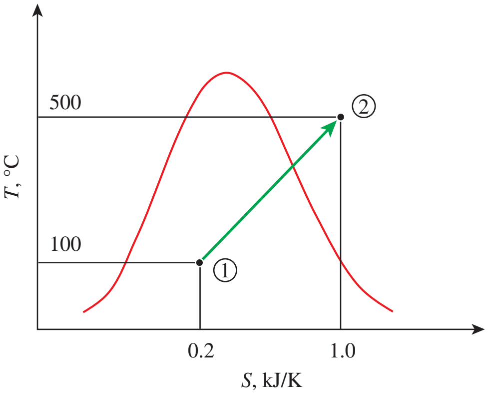


FIGURE P8–58

Entropy Change of Incompressible Substances

8–59C Consider two solid blocks, one hot and the other cold, brought into contact in an adiabatic container. After a while, thermal equilibrium is established in the container as a result of heat transfer. The first law requires that the amount of energy lost by the hot solid be equal to the amount of energy gained by the cold one. Does the second law require that the decrease in entropy of the hot solid be equal to the increase in entropy of the cold one?

8–60 An adiabatic pump is to be used to compress saturated liquid water at 10 kPa to a pressure to 15 MPa in a reversible manner. Determine the work input using (*a*) entropy data from the compressed liquid table, (*b*) inlet specific volume and pressure values, (*c*) average specific volume and pressure values. Also, determine the errors involved in parts *b* and *c*.

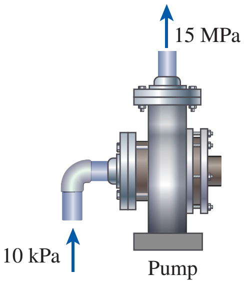


FIGURE P8–60

8–61 Ten grams of computer chips with a specific heat of 0.3 kJ/kg·K are initially at 20°C. These chips are cooled by placement in 5 grams of saturated liquid R-134a at –40°C. Presuming that the pressure remains constant while the chips are being cooled, determine the entropy change of (*a*) the chips, (*b*) the R-134a, and (*c*) the entire system. Is this process possible? Why?

8–62 A 25-kg iron block initially at 280°C is quenched in an insulated tank that contains 100 kg of water at 18°C. Assuming the water that vaporizes during the process condenses back in the tank, determine the total entropy change during this process.

8–63 A 30-kg aluminum block initially at 140°C is brought into contact with a 40-kg block of iron at 60°C in an insulated enclosure. Determine the final equilibrium temperature and the total entropy change for this process. *Answers:* 109°C, 0.251 kJ/K

8–64 Reconsider Prob. 8–63. Using appropriate software, study the effect of the mass of the iron block on the final equilibrium temperature and the total entropy change for the process. Let the mass of the iron vary from 10 to 100 kg. Plot the equilibrium temperature and the total entropy change as a function of iron mass, and discuss the results.

8–65 A 50-kg copper block initially at 140°C is dropped into an insulated tank that contains 90 L of water at 10°C. Determine the final equilibrium temperature and the total entropy change for this process.

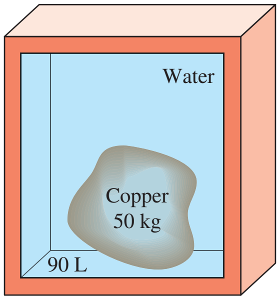


FIGURE P8–65

8–66 A 30-kg iron block and a 40-kg copper block, both initially at 80°C, are dropped into a large lake at 15°C. Thermal equilibrium is established after a while as a result of heat transfer between the blocks and the lake water. Determine the total entropy change for this process.

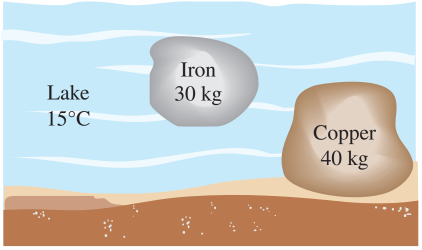


FIGURE P8–66

Entropy Change of Ideal Gases

8–67C What are P_r and v_r called? Is their use limited to isentropic processes? Explain.

8–68C Some properties of ideal gases such as internal energy and enthalpy vary with temperature only [that is, $u = u(T)$ and $h = h(T)$]. Is this also the case for entropy?

8–69C Can the entropy of an ideal gas change during an isothermal process?

8–70C An ideal gas undergoes a process between two specified temperatures, first at constant pressure and then at constant volume. For which case will the ideal gas experience a larger entropy change? Explain.

8–71 What is the difference between entropies of oxygen at 150 kPa and 39°C and oxygen at 150 kPa and 337°C on a per-unit-mass basis?

8–72E Air is expanded from 200 psia and 500°F to 100 psia and 50°F. Assuming constant specific heats, determine the change in the specific entropy of air. *Answer:* −0.106 Btu/lbm·R

8–73 Determine the final temperature when air is expanded isentropically from 1000 kPa and 477°C to 100 kPa in a piston–cylinder device.

8–74E Air is expanded isentropically from 100 psia and 500°F to 20 psia in a closed system. Determine its final temperature.

8–75 Which of the two gases—helium or nitrogen—has the higher final temperature as it is compressed isentropically from 100 kPa and 25°C to 1 MPa in a closed system?

8–76 Which of the two gases—neon or air—has the lower final temperature as it is expanded isentropically from 1000 kPa and 500°C to 100 kPa in a piston–cylinder device?

8–77 A 1.5-m^3 insulated rigid tank contains 2.7 kg of carbon dioxide at 100 kPa. Now paddle-wheel work is done on the system until the pressure in the tank rises to 150 kPa. Determine the entropy change of carbon dioxide during this process. Assume constant specific heats. *Answer:* 0.719 kJ/K

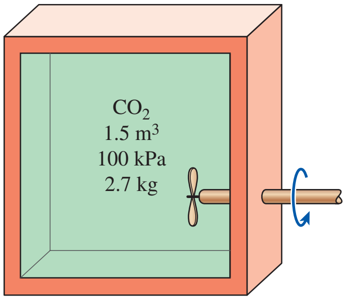


FIGURE P8–77

8–78 An insulated piston–cylinder device initially contains 300 L of air at 120 kPa and 17°C. Air is now heated for 15 min by a 200-W resistance heater placed inside the cylinder. The pressure of air is kept constant during this process. Determine the entropy change of air, assuming (*a*) constant specific heats and (*b*) variable specific heats.

8–79 A piston–cylinder device contains 0.75 kg of nitrogen gas at 140 kPa and 37°C. The gas is now compressed slowly in a polytropic process during which $PV^{1.3}$ = constant. The process ends when the volume is reduced by one-half. Determine the entropy change of nitrogen during this process. *Answer:* −0.0385 kJ/K

8–80 Reconsider Prob. 8–79. Using appropriate software, investigate the effect of varying the polytropic exponent from 1 to 1.4 on the entropy change of the nitrogen. Show the processes on a common P-v diagram.

8–81E A mass of 25 lbm of helium undergoes a process from an initial state of 50 ft^3/lbm and 60°F to a final state of 20 ft^3/lbm and 240°F. Determine the entropy change of helium during this process, assuming (*a*) the process is reversible and (*b*) the process is irreversible.

8–82 One kg of air at 200 kPa and 127°C is contained in a piston–cylinder device. Air is now allowed to expand in a reversible, isothermal process until its pressure is 100 kPa. Determine the amount of heat transferred to the air during this expansion.

8–83 An insulated rigid tank is divided into two equal parts by a partition. Initially, one part contains 12 kmol of an ideal gas at 330 kPa and 50°C, and the other side is evacuated. The partition is now removed, and the gas fills the entire tank. Determine the total entropy change during this process. *Answer:* 69.2 kJ/K

8–84 Air at 27°C and 100 kPa is contained in a piston–cylinder device. When the air is compressed adiabatically, a minimum work input of 1000 kJ will increase the pressure to 600 kPa. Assuming air has constant specific heats evaluated at 300 K, determine the mass of air in the device.

8–85 Air at 3.5 MPa and 500°C is expanded in an adiabatic gas turbine to 0.2 MPa. Calculate the maximum work that this turbine can produce in kJ/kg.

8–86 Air is compressed in a piston–cylinder device from 90 kPa and 20°C to 600 kPa in a reversible isothermal process. Determine (*a*) the entropy change of air and (*b*) the work done.

8–87 Helium gas is compressed from 90 kPa and 30°C to 450 kPa in a reversible, adiabatic process. Determine the final temperature and the work done, assuming the process takes place (*a*) in a piston–cylinder device and (*b*) in a steady-flow compressor.

8–88 Nitrogen at 120 kPa and 30°C is compressed to 600 kPa in an adiabatic compressor. Calculate the minimum work needed for this process in kJ/kg. *Answer:* 184 kJ/kg

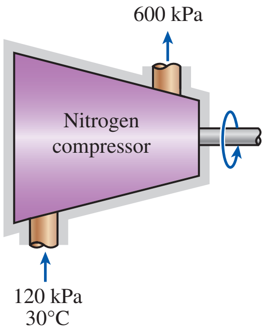


FIGURE P8–88

8–89 Five kg of air at 427°C and 600 kPa are contained in a piston–cylinder device. The air expands adiabatically until the pressure is 100 kPa and produces 600 kJ of work output. Assume air has constant specific heats evaluated at 300 K.

(*a*) Determine the entropy change of the air in kJ/kg·K.
(*b*) Since the process is adiabatic, is the process realistic? Using concepts of the second law, support your answer.

8–90 A container filled with 45 kg of liquid water at 95°C is placed in a 90-m^3 room that is initially at 12°C. Thermal equilibrium is established after a while as a result of heat transfer between the water and the air in the room. Using constant specific heats, determine (*a*) the final equilibrium temperature, (*b*) the amount of heat transfer between the water and the air in the room, and (*c*) the entropy generation. Assume the room is well sealed and heavily insulated.

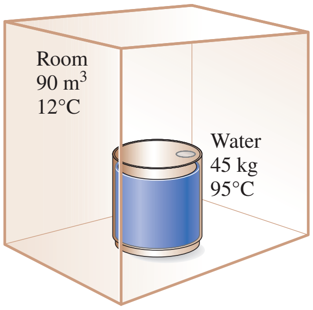


FIGURE P8–90

8–91 Oxygen at 300 kPa and 90°C flowing at an average velocity of 3 m/s is expanded in an adiabatic nozzle. What is the maximum velocity of the oxygen at the outlet of this nozzle when the outlet pressure is 120 kPa? *Answer:* 390 m/s

8–92 Air at 800 kPa and 400°C enters a steady-flow nozzle with a low velocity and leaves at 100 kPa. If the air undergoes an adiabatic expansion process through the nozzle, what is the maximum velocity of the air at the nozzle exit in m/s?

8–93E The well-insulated container shown in Fig. P8–93E is initially evacuated. The supply line contains air that is maintained at 150 psia and 140°F. The valve is opened until the pressure in the container is the same as the pressure in the supply line. Determine the minimum temperature in the container when the valve is closed.

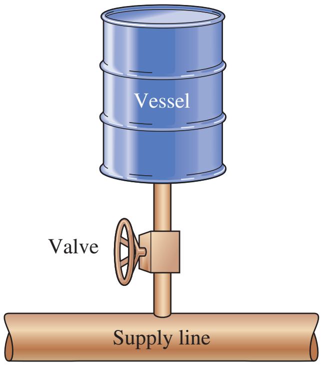


FIGURE P8–93E

8–94 An insulated rigid tank contains 4 kg of argon gas at 450 kPa and 30°C. A valve is now opened, and argon is allowed to escape until the pressure inside drops to 200 kPa. Assuming the argon remaining inside the tank has undergone a reversible, adiabatic process, determine the final mass in the tank. *Answer:* 2.46 kg

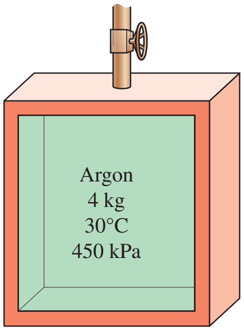


FIGURE P8–94

8–95 Reconsider Prob. 8–94. Using appropriate software, investigate the effect of the final pressure on the final mass in the tank as the pressure varies from 450 to 150 kPa, and plot the results.

Reversible Steady-Flow Work

8–96C In large compressors, the gas is often cooled while being compressed to reduce the power consumed by the compressor. Explain how cooling the gas during a compression process reduces the power consumption.

8–97C The turbines in steam power plants operate essentially under adiabatic conditions. A plant engineer suggests ending this practice. She proposes to run cooling water through the outer surface of the casing to cool the steam as it flows through the turbine. This way, she reasons, the entropy of the steam will decrease, the performance of the turbine will improve, and as a result the work output of the turbine will increase. How would you evaluate this proposal?

8–98C It is well known that the power consumed by a compressor can be reduced by cooling the gas during compression. Inspired by this, somebody proposes to cool the liquid as it flows through a pump, in order to reduce the power consumption of the pump. Would you support this proposal? Explain.

8–99 Calculate the work produced, in kJ/kg, for the reversible steady-flow process 1–3 shown in Fig. P8–99.

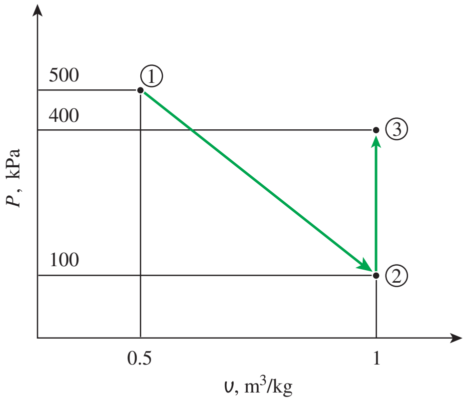


FIGURE P8–99

8–100E Calculate the work produced, in Btu/lbm, for the reversible steady-flow process 1–2 shown in Fig. P8–100E.

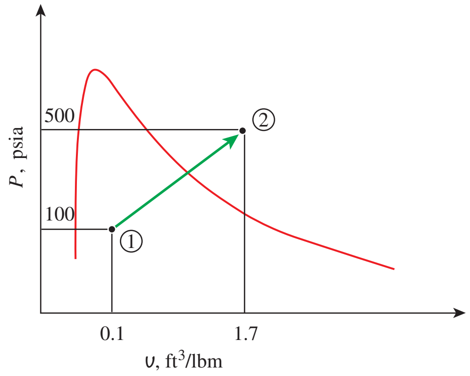


FIGURE P8–100E

8–101E Air is compressed isothermally from 13 psia and 55°F to 80 psia in a reversible steady-flow device. Calculate the work required, in Btu/lbm, for this compression. *Answer:* 64.2 Btu/lbm

8–102 Saturated water vapor at 150°C is compressed in a reversible steady-flow device to 1000 kPa while its specific volume remains constant. Determine the work required in kJ/kg.

8–103 Liquid water at 120 kPa enters a 7-kW pump where its pressure is raised to 5 MPa. If the elevation difference between the exit and the inlet levels is 10 m, determine the highest mass flow rate of liquid water this pump can handle. Neglect the kinetic energy change of water, and take the specific volume of water to be 0.001 m^3/kg.

8–104 Water enters the pump of a steam power plant as saturated liquid at 20 kPa at a rate of 45 kg/s and exits at 6 MPa. Neglecting the changes in kinetic and potential energies and assuming the process to be reversible, determine the power input to the pump.

8–105 Consider a steam power plant that operates between the pressure limits of 5 MPa and 10 kPa. Steam enters the pump as saturated liquid and leaves the turbine as saturated vapor. Determine the ratio of the work delivered by the turbine to the work consumed by the pump. Assume the entire cycle to be reversible and the heat losses from the pump and the turbine to be negligible.

8–106 Reconsider Prob. 8–105. Using appropriate software, investigate the effect of the quality of the steam at the turbine exit on the net work output. Vary the quality from 0.5 to 1.0, and plot the net work output as a function of this quality.

Isentropic Efficiencies of Steady-Flow Devices

8–107C Describe the ideal process for an (*a*) adiabatic turbine, (*b*) adiabatic compressor, and (*c*) adiabatic nozzle, and define the isentropic efficiency for each device.

8–108C Is the isentropic process a suitable model for compressors that are cooled intentionally? Explain.

8–109C On a T-s diagram, does the actual exit state (state 2) of an adiabatic turbine have to be on the right-hand side of the isentropic exit state (state 2s)? Why?

8–110 Argon gas enters an adiabatic turbine at 800°C and 1.5 MPa at a rate of 80 kg/min and exhausts at 200 kPa. If the power output of the turbine is 370 kW, determine the isentropic efficiency of the turbine.

8–111E Steam at 100 psia and 650°F is expanded adiabatically in a closed system to 10 psia. Determine the work produced, in Btu/lbm, and the final temperature of steam for an isentropic expansion efficiency of 80 percent. *Answers:* 132 Btu/lbm, 275°F

8–112E Combustion gases enter an adiabatic gas turbine at 1540°F and 120 psia and leave at 60 psia with a low velocity. Treating the combustion gases as air and assuming an isentropic efficiency of 82 percent, determine the work output of the turbine. *Answer:* 71.7 Btu/lbm

8–113 Steam at 4 MPa and 350°C is expanded in an adiabatic turbine to 120 kPa. What is the isentropic efficiency of this turbine if the steam is exhausted as a saturated vapor?

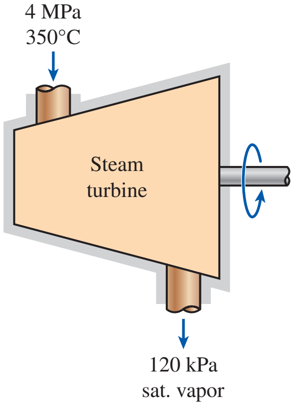


FIGURE P8–113

8–114 Steam at 3 MPa and 400°C is expanded to 30 kPa in an adiabatic turbine with an isentropic efficiency of 92 percent. Determine the power produced by this turbine, in kW, when the mass flow rate is 2 kg/s.

8–115 Repeat Prob. 8–114 for a turbine efficiency of 85 percent.

8–116 Refrigerant-134a enters an adiabatic compressor as saturated vapor at 100 kPa at a rate of 0.7 m^3/min and exits at 1-MPa pressure. If the isentropic efficiency of the compressor is 87 percent, determine (*a*) the temperature of the refrigerant at the exit of the compressor and (*b*) the power input, in kW. Also, show the process on a *T-s* diagram with respect to saturation lines.

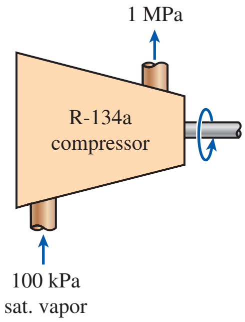


FIGURE P8–116

8–117 Reconsider Prob. 8–116. Using appropriate software, redo the problem by including the effects of the kinetic energy of the flow by assuming an inlet-to-exit area ratio of 1.5 for the compressor when the compressor exit pipe inside diameter is 2 cm.

8–118 The adiabatic compressor of a refrigeration system compresses saturated R-134a vapor at 0°C to 600 kPa and 50°C. What is the isentropic efficiency of this compressor?

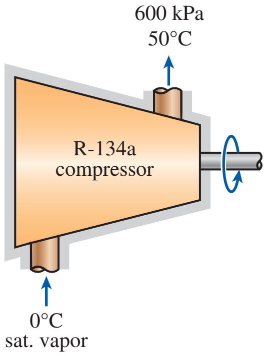


FIGURE P8–118

8–119 Air is compressed by an adiabatic compressor from 95 kPa and 27°C to 600 kPa and 277°C. Assuming variable specific heats and neglecting the changes in kinetic and potential energies, determine (*a*) the isentropic efficiency of the compressor and (*b*) the exit temperature of air if the process were reversible. *Answers:* (*a*) 81.9 percent, (*b*) 506 K

8–120E Argon gas enters an adiabatic compressor at 14 psia and 75°F with a velocity of 60 ft/s, and it exits at 200 psia and 240 ft/s. If the isentropic efficiency of the compressor is 87 percent, determine (*a*) the exit temperature of the argon and (*b*) the work input to the compressor.

8–121 An adiabatic steady-flow device compresses argon at 200 kPa and 27°C to 2 MPa. If the argon leaves this compressor at 550°C, what is the isentropic efficiency of the compressor?

8–122E Air enters an adiabatic nozzle at 45 psia and 940°F with low velocity and exits at 650 ft/s. If the isentropic efficiency of the nozzle is 85 percent, determine the exit temperature and pressure of the air.

8–123E Reconsider Prob. 8–122E. Using appropriate software, study the effect of varying the nozzle isentropic efficiency from 0.8 to 1.0 on both the exit temperature and pressure of the air, and plot the results.

8–124E An adiabatic diffuser at the inlet of a jet engine increases the pressure of the air that enters the diffuser at 11 psia and 30°F to 20 psia. What will the air velocity at the diffuser exit be if the diffuser isentropic efficiency, defined as the ratio of the actual kinetic energy change to the isentropic kinetic energy change, is 82 percent and the diffuser inlet velocity is 1200 ft/s? *Answer:* 735 ft/s

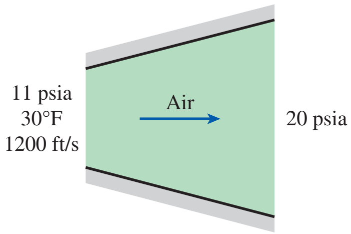


FIGURE P8–124E

8–125 Hot combustion gases enter the nozzle of a turbojet engine at 260 kPa, 747°C, and 80 m/s, and they exit at a pressure of 85 kPa. Assuming an isentropic efficiency of 92 percent and treating the combustion gases as air, determine (*a*) the exit velocity and (*b*) the exit temperature. *Answers:* (*a*) 728 m/s, (*b*) 786 K

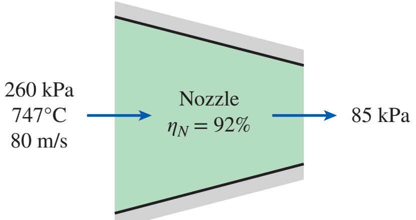


FIGURE P8–125

8–126 The exhaust nozzle of a jet engine expands air at 300 kPa and 180°C adiabatically to 100 kPa. Determine the air velocity at the exit when the inlet velocity is low and the nozzle isentropic efficiency is 93 percent.

Entropy Balance

8–127E An iron block of unknown mass at 185°F is dropped into an insulated tank that contains 0.8 ft^3 of water at 70°F. At the same time, a paddle wheel driven by a 200-W motor is activated to stir the water. Thermal equilibrium is established after 10 min with a final temperature of 75°F. Determine (*a*) the mass of the iron block and (*b*) the entropy generated during this process.

8–128E Refrigerant-134a is expanded adiabatically from 100 psia and 100°F to a pressure of 10 psia. Determine the entropy generation for this process, in Btu/lbm·R.

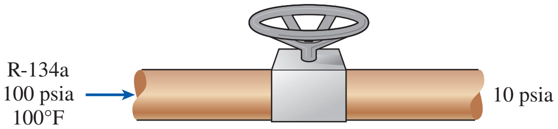


FIGURE P8–128E

8–129E A frictionless piston–cylinder device contains saturated liquid water at 40-psia pressure. Now 600 Btu of heat is transferred to water from a source at 1000°F, and part of the liquid vaporizes at constant pressure. Determine the total entropy generated during this process, in Btu/R.

8–130 Air enters a compressor steadily at the ambient conditions of 100 kPa and 22°C and leaves at 800 kPa. Heat is lost from the compressor in the amount of 120 kJ/kg, and the air experiences an entropy decrease of 0.40 kJ/kg·K. Using constant specific heats, determine (*a*) the exit temperature of the air, (*b*) the work input to the compressor, and (*c*) the entropy generation during this process.

8–131 Steam enters an adiabatic turbine steadily at 7 MPa, 500°C, and 45 m/s and leaves at 100 kPa and 75 m/s. If the power output of the turbine is 5 MW and the isentropic efficiency is 77 percent, determine (*a*) the mass flow rate of steam through the turbine, (*b*) the temperature at the turbine exit, and (*c*) the rate of entropy generation during this process.

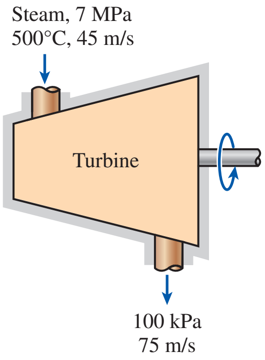


FIGURE P8–131

8–132 In an ice-making plant, water at 0°C is frozen at atmospheric pressure by evaporating saturated R-134a liquid at –16°C. The refrigerant leaves this evaporator as a saturated vapor, and the plant is sized to produce ice at 0°C at a rate of 5500 kg/h. Determine the rate of entropy generation in this plant. *Answer:* 0.115 kW/K

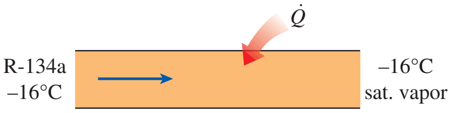


FIGURE P8–132

8–133 Oxygen enters an insulated 12-cm-diameter pipe with a velocity of 70 m/s. At the pipe entrance, the oxygen is at 240 kPa and 20°C, and at the exit it is at 200 kPa and 18°C. Calculate the rate at which entropy is generated in the pipe.

8–134E Water at 20 psia and 50°F enters a mixing chamber at a rate of 300 lbm/min where it is mixed steadily with steam entering at 20 psia and 240°F. The mixture leaves the chamber at 20 psia and 130°F, and heat is lost to the surrounding air at 70°F at a rate of 180 Btu/min. Neglecting the changes in kinetic and potential energies, determine the rate of entropy generation during this process.

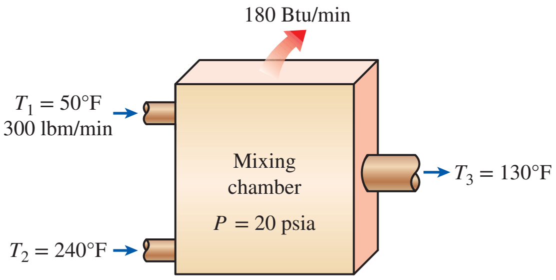


FIGURE P8–134E

8–135 Cold water (c_p = 4.18 kJ/kg·°C) leading to a shower enters a well-insulated, thin-walled, double-pipe, counterflow heat exchanger at 10°C at a rate of 0.95 kg/s and is heated to 70°C by hot water (c_p = 4.19 kJ/kg·°C) that enters at 85°C at a rate of 1.6 kg/s. Determine (*a*) the rate of heat transfer and (*b*) the rate of entropy generation in the heat exchanger.

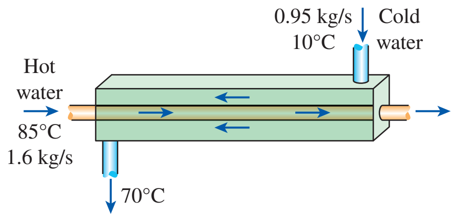


FIGURE P8–135

8–136 Air (c_p = 1.005 kJ/kg·°C) is to be preheated by hot exhaust gases in a crossflow heat exchanger before it enters the furnace. Air enters the heat exchanger at 95 kPa and 20°C at a rate of 1.6 m^3/s. The combustion gases (c_p = 1.10 kJ/kg·°C) enter at 180°C at a rate of 2.2 kg/s and leave at 95°C. Determine (*a*) the rate of heat transfer to the air, (*b*) the outlet temperature of the air, and (*c*) the rate of entropy generation.

8–137 In a dairy plant, milk at 4°C is pasteurized continuously at 72°C at a rate of 12 L/s for 24 hours a day and 365 days a year. The milk is heated to the pasteurizing temperature by hot water heated in a natural-gas-fired boiler that has an efficiency of 82 percent. The pasteurized milk is then cooled by cold water at 18°C before it is finally refrigerated back to 4°C. To save energy and money, the plant installs a regenerator that has an effectiveness of 82 percent. If the cost of natural gas is $1.30/therm (1 therm = 105,500 kJ), determine how much energy and money the regenerator will save this company per year and the annual reduction in entropy generation.

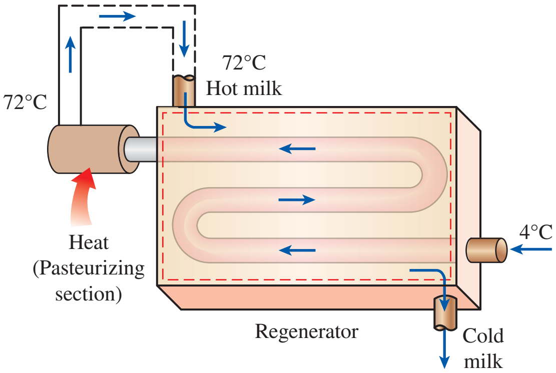


FIGURE P8–137

8–138 Steam is to be condensed in the condenser of a steam power plant at a temperature of 60°C with cooling water from a nearby lake, which enters the tubes of the condenser at 18°C at a rate of 75 kg/s and leaves at 27°C. Assuming the condenser to be perfectly insulated, determine (*a*) the rate of condensation of the steam and (*b*) the rate of entropy generation in the condenser. *Answers:* (*a*) 1.20 kg/s, (*b*) 1.06 kW/K

8–139 An ordinary egg can be approximated as a 5.5-cm-diameter sphere. The egg is initially at a uniform temperature of 8°C and is dropped into boiling water at 97°C. Taking the properties of the egg to be ρ = 1020 kg/m^3 and c_p = 3.32 kJ/kg·°C, determine (*a*) how much heat is transferred to the egg by the time the average temperature of the egg rises to 70°C and (*b*) the amount of entropy generation associated with this heat transfer process.

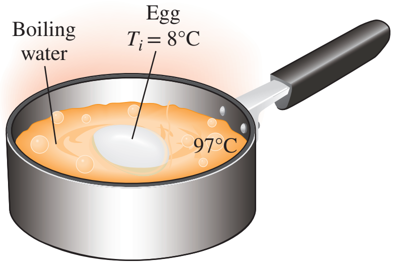


FIGURE P8–139

8–140 Chickens with an average mass of 2.2 kg and average specific heat of 3.54 kJ/kg·°C are to be cooled by chilled water that enters a continuous-flow-type immersion chiller at 0.5°C and leaves at 2.5°C. Chickens are dropped into the chiller at a uniform temperature of 15°C at a rate of 250 chickens per hour and are cooled to an average temperature of 3°C before they are taken out. The chiller gains heat from the surroundings at 25°C at a rate of 150 kJ/h. Determine (*a*) the rate of heat removal from the chickens, in kW, and (*b*) the rate of entropy generation during this chilling process.

8–141E In a production facility, 1.2-in-thick, 2-ft × 2-ft square brass plates (ρ = 532.5 lbm/ft^3 and c_p = 0.091 Btu/lbm·°F) that are initially at a uniform temperature of 75°F are heated by passing them through an oven at 1300°F at a rate of 450 per minute. If the plates remain in the oven until their average temperature rises to 1000°F, determine (*a*) the rate of heat transfer to the plates in the furnace and (*b*) the rate of entropy generation associated with this heat transfer process.

8–142 Long cylindrical steel rods (ρ = 7833 kg/m^3 and c_p = 0.465 kJ/kg·°C) of 10-cm diameter are heat treated by drawing them at a velocity of 3 m/min through a 7-m-long oven maintained at 900°C. If the rods enter the oven at 30°C and leave at 700°C, determine (*a*) the rate of heat transfer to the rods in the oven and (*b*) the rate of entropy generation associated with this heat transfer process.

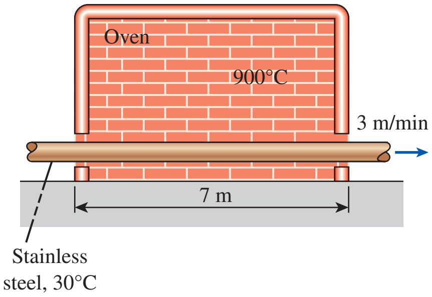


FIGURE P8–142

8–143 Stainless-steel ball bearings ($\rho = 8085 \text{ kg/m}^3$ and $c_p = 0.480 \text{ kJ/kg·°C}$) having a diameter of 1.8 cm are to be quenched in water at a rate of 1100 per minute. The balls leave the oven at a uniform temperature of 900°C and are exposed to air at 20°C for a while before they are dropped into the water. If the temperature of the balls drops to 850°C prior to quenching, determine (*a*) the rate of heat transfer from the balls to the air and (*b*) the rate of entropy generation due to heat loss from the balls to the air.

8–144 The inner and outer surfaces of a 4-m × 10-m brick wall of thickness 20 cm are maintained at temperatures of 16°C and 4°C, respectively. If the rate of heat transfer through the wall is 1800 W, determine the rate of entropy generation within the wall.

8–145E Steam enters a diffuser at 20 psia and 240°F with a velocity of 900 ft/s and exits as saturated vapor at 240°F and 100 ft/s. The exit area of the diffuser is 1 ft^2. Determine (*a*) the mass flow rate of the steam and (*b*) the rate of entropy generation during this process. Assume an ambient temperature of 77°F.

8–146 Steam enters an adiabatic nozzle at 2 MPa and 350°C with a velocity of 55 m/s and exits at 0.8 MPa and 390 m/s. If the nozzle has an inlet area of 7.5 cm^2, determine (*a*) the exit temperature and (*b*) the rate of entropy generation for this process. *Answers:* (*a*) 303°C, (*b*) 0.0854 kW/K

8–147 Steam expands in a turbine steadily at a rate of 40,000 kg/h, entering at 8 MPa and 500°C and leaving at 40 kPa as saturated vapor. If the power generated by the turbine is 8.2 MW, determine the rate of entropy generation for this process. Assume the surrounding medium is at 25°C. *Answer:* 11.4 kW/K

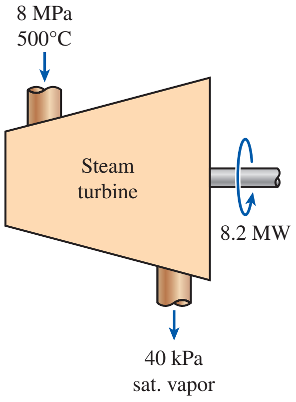


FIGURE P8–147

8–148 Liquid water at 200 kPa and 15°C is heated in a chamber by mixing it with superheated steam at 200 kPa and 150°C. Liquid water enters the mixing chamber at a rate of 4.3 kg/s, and the chamber is estimated to lose heat to the surrounding air at 20°C at a rate of 1200 kJ/min. If the mixture leaves the mixing chamber at 200 kPa and 80°C, determine (*a*) the mass flow rate of the superheated steam and (*b*) the rate of entropy generation during this mixing process. *Answers:* (*a*) 0.481 kg/s, (*b*) 0.746 kW/K

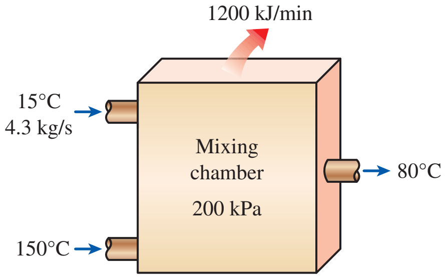


FIGURE P8–148

8–149 A 0.18-m^3 rigid tank is filled with saturated liquid water at 120°C. A valve at the bottom of the tank is now opened, and one-half of the total mass is withdrawn from the tank in liquid form. Heat is transferred to the water from a source at 230°C so that the temperature in the tank remains constant. Determine (*a*) the amount of heat transfer and (*b*) the total entropy generation for this process.

8–150 A rigid tank contains 7.5 kg of saturated water mixture at 400 kPa. A valve at the bottom of the tank is now opened, and liquid is withdrawn from the tank. Heat is transferred to the steam such that the pressure inside the tank remains constant. The valve is closed when no liquid is left in the tank. If it is estimated that a total of 5 kJ of heat is transferred to the tank, determine (*a*) the quality of steam in the tank at the initial state, (*b*) the amount of mass that has escaped, and (*c*) the entropy generation during this process if heat is supplied to the tank from a source at 500°C.

Review Problems

8–151 A proposed heat pump design creates a heating effect of 25 kW while using 5 kW of electrical power. The thermal energy reservoirs are at 300 K and 260 K. Is this possible according to the increase of entropy principle?

8–152 A refrigerator with a coefficient of performance of 4 transfers heat from a cold region at –20°C to a hot region at 30°C. Calculate the total entropy change of the regions when 1 kJ of heat is transferred from the cold region. Is the second law satisfied? Will this refrigerator still satisfy the second law if its coefficient of performance is 6?

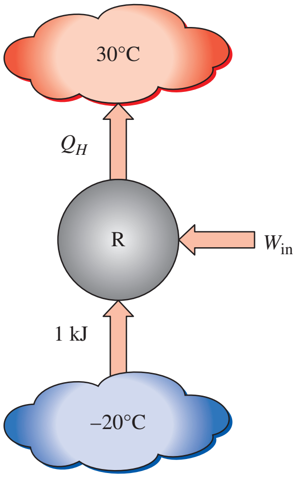


FIGURE P8–152

8–153 What is the minimum internal energy that steam can achieve as it is expanded adiabatically in a closed system from 1500 kPa and 320°C to 100 kPa?

8–154E Is it possible to expand water at 30 psia and 70 percent quality to 10 psia in a closed system undergoing an isothermal, reversible process while exchanging heat with an energy reservoir at 300°F?

8–155 What is the maximum volume that 3 kg of oxygen at 950 kPa and 373°C can be adiabatically expanded to in a piston–cylinder device if the final pressure is to be 100 kPa? *Answer:* 2.66 m^3

8–156E A 100-lbm block of a solid material whose specific heat is 0.5 Btu/lbm·R is at 80°F. It is heated with 10 lbm of saturated water vapor that has a constant pressure of 20 psia. Determine the final temperature of the block and water, and the entropy change of (*a*) the block, (*b*) the water, and (*c*) the entire system. Is this process possible? Why?

8–157 A piston–cylinder device contains air that undergoes a reversible thermodynamic cycle. Initially, air is at 400 kPa and 300 K with a volume of 0.3 m^3. Air is first expanded isothermally to 150 kPa, then compressed adiabatically to the initial pressure, and finally compressed at the constant pressure to the initial state. Accounting for the variation of specific heats with temperature, determine the work and heat transfer for each process.

8–158E A piston–cylinder device initially contains 15 ft^3 of helium gas at 25 psia and 70°F. Helium is now compressed in a polytropic process (PV^n = constant) to 70 psia and 300°F. Determine (*a*) the entropy change of helium, (*b*) the entropy change of the surroundings, and (*c*) whether this process is reversible, irreversible, or impossible. Assume the surroundings are at 70°F. *Answers:* (*a*) –0.016 Btu/R, (*b*) 0.019 Btu/R, (*c*) irreversible

8–159 A piston–cylinder device contains steam that undergoes a reversible thermodynamic cycle. Initially the steam is at 400 kPa and 350°C with a volume of 0.5 m^3. The steam is first expanded isothermally to 150 kPa, then compressed adiabatically to the initial pressure, and finally compressed at the constant pressure to the initial state. Determine the net work and heat transfer for the cycle after you calculate the work and heat interaction for each process.

8–160 One hundred kg of saturated steam at 100 kPa is to be adiabatically compressed in a closed system to 1000 kPa. How much work is required if the isentropic compression efficiency is 90 percent? *Answer:* 44,160 kJ

8–161E Ten lbm of R-134a is expanded without any heat transfer in a closed system from 120 psia and 100°F to 20 psia. If the isentropic expansion efficiency is 95 percent, what is the final volume of this refrigerant?

8–162 Refrigerant-134a at 700 kPa and 40°C is expanded adiabatically in a closed system to 60 kPa. Determine the work produced, in kJ/kg, and final enthalpy for an isentropic expansion efficiency of 80 percent. *Answer:* 37.9 kJ/kg, 238.4 kJ/kg

8–163 A 0.8-m^3 rigid tank contains carbon dioxide (CO_2) gas at 250 K and 100 kPa. A 500-W electric resistance heater placed in the tank is now turned on and kept on for 40 min, after which the pressure of CO_2 is measured to be 175 kPa. Assuming the surroundings to be at 300 K and using constant specific heats, determine (*a*) the final temperature of CO_2, (*b*) the net amount of heat transfer from the tank, and (*c*) the entropy generation during this process.

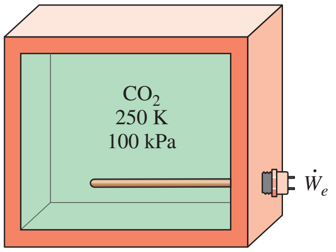


FIGURE P8–163

8–164 Air enters the evaporator section of a window air conditioner at 100 kPa and 27°C with a volume flow rate of 6 m^3/min. The refrigerant-134a at 120 kPa with a quality of 0.3 enters the evaporator at a rate of 2 kg/min and leaves as saturated vapor at the same pressure. Determine the exit temperature of the air and the rate of entropy generation for this process, assuming (*a*) the outer surfaces of the air conditioner are insulated and (*b*) heat is transferred to the evaporator of the air conditioner from the surrounding medium at 32°C at a rate of 30 kJ/min. *Answers:* (*a*) –15.9°C, 0.00196 kW/K, (*b*) –11.6°C, 0.00225 kW/K

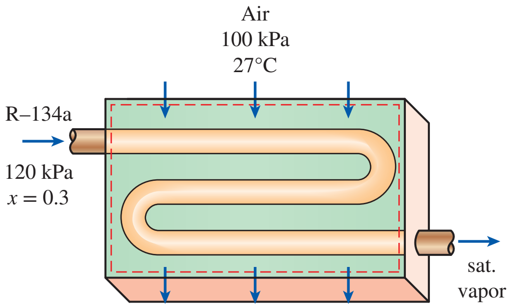


FIGURE P8–164

8–165 Air at 500 kPa and 400 K enters an adiabatic nozzle at a velocity of 30 m/s and leaves at 300 kPa and 350 K. Using variable specific heats, determine (*a*) the isentropic efficiency, (*b*) the exit velocity, and (*c*) the entropy generation.

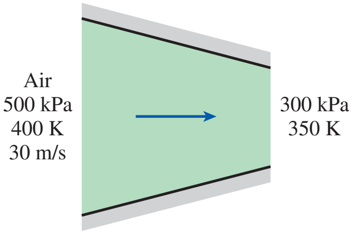


FIGURE P8–165

8–166E Helium gas enters a nozzle whose isentropic efficiency is 94 percent with a low velocity, and it exits at 14 psia, 180°F, and 1000 ft/s. Determine the pressure and temperature at the nozzle inlet.

8–167 An inventor claims to have invented an adiabatic steady-flow device with a single inlet-outlet that produces 230 kW when expanding 1 kg/s of air from 1200 kPa and 300°C to 100 kPa. Is this claim valid?

8–168 An adiabatic capillary tube is used in some refrigeration systems to drop the pressure of the refrigerant from the condenser level to the evaporator level. R-134a enters the capillary tube as a saturated liquid at 70°C and leaves at –20°C. Determine the rate of entropy generation in the capillary tube for a mass flow rate of 0.2 kg/s. *Answer:* 0.0166 kW/K

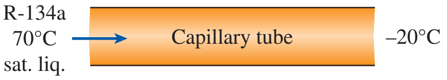


FIGURE P8–168

8–169 Helium gas is throttled steadily from 400 kPa and 60°C. Heat is lost from the helium in the amount of 1.75 kJ/kg to the surroundings at 25°C and 100 kPa. If the entropy of the helium increases by 0.34 kJ/kg·K in the valve, determine (*a*) the exit temperature and pressure and (*b*) the entropy generation during this process. *Answers:* (*a*) 59.7°C, 339 kPa, (*b*) 0.346 kJ/kg·K

8–170 Determine the work input and entropy generation during the compression of steam from 100 kPa to 1 MPa in (*a*) an adiabatic pump and (*b*) an adiabatic compressor if the inlet state is saturated liquid in the pump and saturated vapor in the compressor and the isentropic efficiency is 85 percent for both devices.

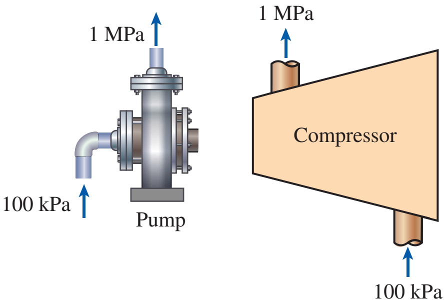


FIGURE P8–170

8–171 Carbon dioxide is compressed in a reversible, isothermal process from 100 kPa and 20°C to 400 kPa using a steady-flow device with one inlet and one outlet. Determine the work required and the heat transfer, both in kJ/kg, for this compression.

8–172 Reconsider Prob. 8–171. Determine the change in the work and heat transfer when the compression process is isentropic rather than isothermal.

8–173 The compressor of a refrigerator compresses saturated R-134a vapor at –10°C to 800 kPa. How much work, in kJ/kg, does this process require when the process is isentropic?

8–174 Three kg of helium gas at 100 kPa and 27°C are adiabatically compressed to 900 kPa. If the isentropic compression efficiency is 80 percent, determine the required work input and the final temperature of helium.

8–175 Steam at 6000 kPa and 500°C enters a steady-flow turbine. The steam expands in the turbine while doing work until the pressure is 1000 kPa. When the pressure is 1000 kPa, 10 percent of the steam is removed from the turbine for other uses. The remaining 90 percent of the steam continues to expand through the turbine while doing work and leaves the turbine at 10 kPa. The entire expansion process by the steam through the turbine is reversible and adiabatic.

(*a*) Sketch the process on a *T-s* diagram with respect to the saturation lines. Be sure to label the data states and the lines of constant pressure.

(*b*) If the turbine has an isentropic efficiency of 85 percent, what is the work done by the steam as it flows through the turbine per unit mass of steam flowing into the turbine, in kJ/kg?

8–176 Refrigerant-134a at 140 kPa and –10°C is compressed by an adiabatic 1.3-kW compressor to an exit state of 700 kPa and 60°C. Neglecting the changes in kinetic and potential energies, determine (*a*) the isentropic efficiency of the compressor, (*b*) the volume flow rate of the refrigerant at the compressor inlet in L/min, and (*c*) the maximum volume flow rate at the inlet conditions that this adiabatic 1.3-kW compressor can handle without violating the second law.

8–177 Refrigerant-134a enters a compressor as a saturated vapor at 160 kPa at a rate of 0.03 m^3/s and leaves at 800 kPa. The power input to the compressor is 10 kW. If the surroundings at 20°C experience an entropy increase of 0.008 kW/K, determine (*a*) the rate of heat loss from the compressor, (*b*) the exit temperature of the refrigerant, and (*c*) the rate of entropy generation.

8–178 Air is expanded in an adiabatic turbine of 90 percent isentropic efficiency from an inlet state of 2800 kPa and 400°C to an outlet pressure of 100 kPa. Calculate the outlet temperature of air, the work produced by this turbine, and the entropy generation. *Answers:* 303 K, 375 kJ/kg, 0.148 kJ/kg·K

8–179 A steam turbine is equipped to bleed 6 percent of the inlet steam for feedwater heating. It is operated with 4 MPa and 350°C steam at the inlet, a bleed pressure of 800 kPa, and an exhaust pressure of 30 kPa. Calculate the work produced by this turbine when the isentropic efficiency between the inlet and bleed point is 97 percent and the isentropic efficiency between the bleed point and exhaust is 95 percent. What is the overall isentropic efficiency of the turbine? *Hint:* Treat this turbine as two separate turbines, with one operating between the inlet and bleed conditions and the other operating between the bleed and exhaust conditions.

8–180 Two rigid tanks are connected by a valve. Tank A is insulated and contains 0.3 m^3 of steam at 400 kPa and 60 percent quality. Tank B is uninsulated and contains 2 kg of steam at 200 kPa and 250°C. The valve is now opened, and steam flows from tank A to tank B until the pressure in tank A drops to 200 kPa. During this process, 300 kJ of heat is transferred from tank B to the surroundings at 17°C. Assuming the steam remaining inside tank A to have undergone a reversible adiabatic process, determine (*a*) the final temperature in each tank and (*b*) the entropy generated during this process. *Answers:* (*a*) 120.2°C, 116.1°C, (*b*) 0.498 kJ/K

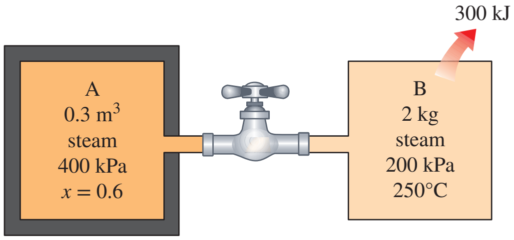


FIGURE P8–180

8–181 A 1200-W electric resistance heating element whose diameter is 0.5 cm is immersed in 40 kg of water initially at 20°C. Assuming the water container is well insulated, determine how long it will take for this heater to raise the water temperature to 50°C. Also, determine the entropy generated during this process in kJ/K.

8–182E A 15-ft^3 steel container that has a mass of 75 lbm when empty is filled with liquid water. Initially, both the steel tank and the water are at 120°F. Now heat is transferred, and the entire system cools to the surrounding air temperature of 70°F. Determine the total entropy generated during this process.

8–183 In order to cool 1 ton of water at 20°C in an insulated tank, a person pours 140 kg of ice at –5°C into the water. Determine (*a*) the final equilibrium temperature in the tank and (*b*) the entropy generation during this process. The melting temperature and the heat of fusion of ice at atmospheric pressure are 0°C and 333.7 kJ/kg.

8–184 One ton of liquid water at 80°C is brought into a well-insulated and well-sealed 4-m × 5-m × 7-m room initially at 22°C and 100 kPa. Assuming constant specific heats for both air and water at room temperature, determine (*a*) the final equilibrium temperature in the room and (*b*) the total entropy change during this process, in kJ/K.

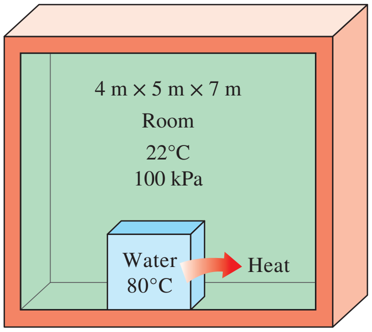


FIGURE P8–184

8–185 A well-insulated 4-m × 4-m × 5-m room initially at 10°C is heated by the radiator of a steam heating system. The radiator has a volume of 15 L and is filled with superheated vapor at 200 kPa and 200°C. At this moment both the inlet and the exit valves to the radiator are closed. A 120-W fan is used to distribute the air in the room. The pressure of the steam is observed to drop to 100 kPa after 30 min as a result of heat transfer to the room. Assuming constant specific heats for air at room temperature, determine (*a*) the average temperature of air in 30 min, (*b*) the entropy change of the steam, (*c*) the entropy change of the air in the room, and (*d*) the entropy generated during this process, in kJ/K. Assume the air pressure in the room remains constant at 100 kPa at all times.

8–186 A passive solar house that is losing heat to the outdoors at 3°C at an average rate of 50,000 kJ/h is maintained at 22°C at all times during a winter night for 10 h. The house is heated by 50 glass containers, each containing 20 L of water that is heated to 80°C during the day by absorbing solar energy. A thermostat-controlled 15-kW backup electric resistance heater turns on whenever necessary to keep the house at 22°C. Determine how long the electric heating system is on during this night and the amount of entropy generated during the night.

8–187 An insulated piston–cylinder device initially contains 0.02 m^3 of saturated liquid–vapor mixture of water with a quality of 0.1 at 100°C. Now some ice at –18°C is dropped into the cylinder. If the cylinder contains saturated liquid at 100°C when thermal equilibrium is established, determine (*a*) the amount of ice added and (*b*) the entropy generation during this process. The melting temperature and the heat of fusion of ice at atmospheric pressure are 0°C and 333.7 kJ/kg.

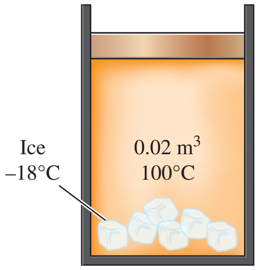


FIGURE P8–187

Design and Essay Problems

8–188 Compressors powered by natural gas engines are increasing in popularity. Several major manufacturing facilities have already replaced the electric motors that drive their compressors with gas-driven engines in order to reduce their energy bills since the cost of natural gas is much lower than the cost of electricity. Consider a facility that has a 130-kW compressor that runs 4400 h/yr at an average load factor of 0.6. Making reasonable assumptions and using unit costs for natural gas and electricity at your location, determine the potential cost savings per year by switching to gas-driven engines.

8–189 It is well known that the temperature of a gas rises while it is compressed as a result of the energy input in the form of compression work. At high compression ratios, the air temperature may rise above the autoignition temperature of some hydrocarbons, including some lubricating oil. Therefore, the presence of some lubricating oil vapor in high-pressure air raises the possibility of an explosion, creating a fire hazard. The concentration of the oil within the compressor is usually too low to create a real danger. However, the oil that collects on the inner walls of exhaust piping of the compressor may cause an explosion. Such explosions have largely been eliminated by using the proper lubricating oils, carefully designing the equipment, intercooling between compressor stages, and keeping the system clean.

A compressor is to be designed for an industrial application in Los Angeles. If the compressor exit temperature is not to exceed 250°C for safety reasons, determine the maximum allowable compression ratio that is safe for all possible weather conditions for that area.

8–190 Obtain the following information about a power plant that is closest to your town: the net power output; the type and amount of fuel; the power consumed by the pumps, fans, and other auxiliary equipment; stack gas losses; temperatures at several locations; and the rate of heat rejection at the condenser. Using these and other relevant data, determine the rate of entropy generation in that power plant.

8–191 You are designing a closed-system, isentropic-expansion process using an ideal gas that operates between the pressure limits of P_1 and P_2. The gases under consideration are hydrogen, nitrogen, air, helium, argon, and carbon dioxide. Which of these gases will produce the greatest amount of work? Which will require the least amount of work in a compression process?

CHAPTER 9

POWER AND REFRIGERATION CYCLES

Two important areas of application for thermodynamics are power generation and refrigeration. Both power generation and refrigeration are usually accomplished by systems that operate on a thermodynamic cycle. Thermodynamic cycles can be divided into two general categories: *power cycles* and *refrigeration cycles*.

The devices or systems used to produce a net power output are often called *engines*, and the thermodynamic cycles they operate on are called *power cycles*. The devices or systems used to produce refrigeration are called *refrigerators, air conditioners,* or *heat pumps*, and the cycles they operate on are called *refrigeration cycles*.

Thermodynamic cycles can also be categorized as *gas cycles* or *vapor cycles*, depending on the phase of the working fluid—the substance that circulates through the cyclic device. In gas cycles, the working fluid remains in the gaseous phase throughout the entire cycle, whereas in vapor cycles the working fluid exists in the vapor phase during one part of the cycle and in the liquid phase during another part.

Thermodynamic cycles can be categorized yet another way: *closed* and *open cycles*. In closed cycles, the working fluid is returned to the initial state at the end of the cycle and is recirculated. In open cycles, the working fluid is renewed at the end of each cycle instead of being recirculated. In automobile engines, for example, the combustion gases are exhausted and replaced by fresh air–fuel mixture at the end of each cycle. The engine operates on a mechanical cycle, but the working fluid in this type of device does not go through a complete thermodynamic cycle.

Heat engines are categorized as *internal combustion* or *external combustion engines*, depending on how the heat is supplied to the working fluid. In external combustion engines (such as steam power plants), energy is supplied to the working fluid from an external source such as a furnace, a geothermal well, a nuclear reactor, or even the sun. In internal combustion engines (such as automobile engines), this is done by burning the fuel within the system boundary. In this chapter, various gas power cycles are analyzed under some simplifying assumptions.

Steam is the most common working fluid used in vapor power cycles because of its many desirable characteristics, such as low cost, availability, and high enthalpy of vaporization. Other working fluids used include sodium, potassium, and mercury for high-temperature applications and some organic fluids such as benzene and the freons for low-temperature applications.

Steam power plants are commonly referred to as *coal plants, nuclear plants*, or *natural gas plants*, depending on the type of fuel used to supply heat to the steam. But the steam goes through the same basic cycle in all of them. Therefore, all can be analyzed in the same manner.

The most frequently used refrigeration cycle is the *vapor-compression refrigeration cycle* in which the refrigerant is vaporized and condensed alternately and is compressed in the vapor phase.

OBJECTIVES

The objectives of this chapter are to:

- Evaluate the performance of gas power cycles.
- Develop simplifying assumptions applicable to gas power cycles.
- Review the operation of reciprocating engines.
- Solve problems based on the Otto and Diesel cycles.
- Solve problems based on the Brayton cycle and the Brayton cycle with regeneration.
- Analyze vapor power cycles in which the working fluid is alternately vaporized and condensed.
- Investigate ways to modify the basic Rankine vapor power cycle to increase the cycle thermal efficiency.
- Analyze the reheat vapor power cycles.
- Analyze the ideal vapor-compression refrigeration cycle.
- Discuss the operation of refrigeration and heat pump systems

9–1 ■ BASIC CONSIDERATIONS IN THE ANALYSIS OF POWER CYCLES

Most power-producing devices operate on cycles, and the study of power cycles is an exciting and important part of thermodynamics. The cycles encountered in actual devices are difficult to analyze because of the presence of complicating effects, such as friction, and the absence of sufficient time for establishment of the equilibrium conditions during the cycle. To make an analytical study of a cycle feasible, we have to keep the complexities at a manageable level and utilize some idealizations (Fig. 9–1). When the actual cycle is stripped of all the internal irreversibilities and complexities, we end up with a cycle that resembles the actual cycle closely but is made up totally of internally reversible processes. Such a cycle is called an **ideal cycle** (Fig. 9–2).

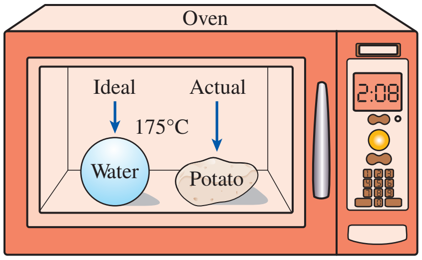


FIGURE 9–1
Modeling is a powerful engineering tool that provides great insight and simplicity at the expense of some loss in accuracy.

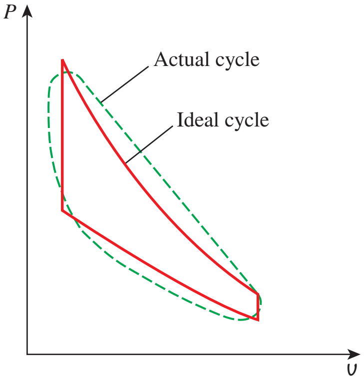


FIGURE 9–2
The analysis of many complex processes can be reduced to a manageable level by utilizing some idealizations.

A simple idealized model enables engineers to study the effects of the major parameters that dominate the cycle without getting bogged down in the details. The cycles discussed in this chapter are somewhat idealized, but they still retain the general characteristics of the actual cycles they represent. The conclusions reached from the analysis of ideal cycles are also applicable to actual cycles. The thermal efficiency of the Otto cycle, the ideal cycle for spark-ignition automobile engines, for example, increases with the compression ratio. This is also the case for actual automobile engines. The numerical values obtained from the analysis of an ideal cycle, however, are not necessarily representative of the actual cycles, and care should be exercised in their interpretation. The simplified analysis presented in this chapter for various power cycles of practical interest may also serve as the starting point for a more in-depth study.

Heat engines are designed for the purpose of converting thermal energy to work, and their performance is expressed in terms of the **thermal efficiency** η_{th}, which is the ratio of the net work produced by the engine to the total heat input:

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_{\text{in}}} \quad \text{or} \quad \eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} \tag{9–1}$$

Recall that heat engines that operate on a totally reversible cycle, such as the Carnot cycle, have the highest thermal efficiency of all heat engines operating between the same temperature levels. That is, nobody can develop a cycle more efficient than the *Carnot cycle.* Then the following question arises naturally: If the Carnot cycle is the best possible cycle, why do we not use it as the model cycle for all the heat engines instead of bothering with several so-called *ideal* cycles? The answer to this question is hardware-related. Most cycles encountered in practice differ significantly from the Carnot cycle, which makes it unsuitable as a realistic model. Each ideal cycle discussed in this chapter is related to a specific work-producing device and is an *idealized* version of the actual cycle.

The ideal cycles are internally reversible, but, unlike the Carnot cycle, they are not necessarily externally reversible. That is, they may involve irreversibilities external to the system such as heat transfer through a finite temperature difference. Therefore, the thermal efficiency of an ideal cycle, in general, is less than that of a totally reversible cycle operating between the same temperature limits. However, it is still considerably higher than the thermal efficiency of an actual cycle because of the idealizations utilized (Fig. 9–3).

The idealizations and simplifications commonly employed in the analysis of power cycles can be summarized as follows:

1. The cycle does not involve any *friction.* Therefore, the working fluid does not experience any pressure drop as it flows in pipes or devices such as heat exchangers.

FIGURE 9–3
An automotive engine with the combustion chamber exposed.
Idealink Photography/Alamy

2. All expansion and compression processes take place in a *quasi-equilibrium* manner.
3. The pipes connecting the various components of a system are well insulated, and *heat transfer* through them is negligible.

Neglecting the changes in *kinetic* and *potential energies* of the working fluid is another commonly utilized simplification in the analysis of power cycles. This is a reasonable assumption since in devices that involve shaft work, such as turbines, compressors, and pumps, the kinetic and potential energy terms are usually very small relative to the other terms in the energy equation. Fluid velocities encountered in devices such as condensers, boilers, and mixing chambers are typically low, and the fluid streams experience little change in their velocities, again making kinetic energy changes negligible. The only devices where the changes in kinetic energy are significant are the nozzles and diffusers, which are specifically designed to create large changes in velocity.

In the preceding chapters, *property diagrams* such as the P-υ and T-s diagrams have served as valuable aids in the analysis of thermodynamic processes. On both the P-υ and T-s diagrams, the area enclosed by the process curves of a cycle represents the net work produced during the cycle (Fig. 9–4), which is also equivalent to the net heat transfer for that cycle. The T-s diagram is particularly useful as a visual aid in the analysis of ideal power cycles. An ideal power cycle does not involve any internal irreversibilities, and so the only effect that can change the entropy of the working fluid during a process is heat transfer.

On a T-s diagram, a *heat-addition* process proceeds in the direction of increasing entropy, a *heat-rejection* process proceeds in the direction of decreasing entropy, and an *isentropic* (internally reversible, adiabatic) process proceeds at constant entropy. The area under the process curve on a T-s diagram represents the heat transfer for that process. The area under the heat addition process on a T-s diagram is a geometric measure of the total heat supplied during the cycle q_{in}, and the area under the heat rejection process is a measure of the total heat rejected q_{out}. The difference between these two (the area enclosed by the cyclic curve) is the net heat transfer, which is also the net work produced during the cycle. Therefore, on a T-s diagram, the ratio of the area enclosed by the cyclic curve to the area under the heat-addition process curve represents the thermal efficiency of the cycle. *Any modification that increases the ratio of these two areas will also increase the thermal efficiency of the cycle.*

Although the working fluid in an ideal power cycle operates on a closed loop, the type of individual process that comprises the cycle depends on the individual devices used to execute the cycle. In the Rankine cycle, which is the ideal cycle

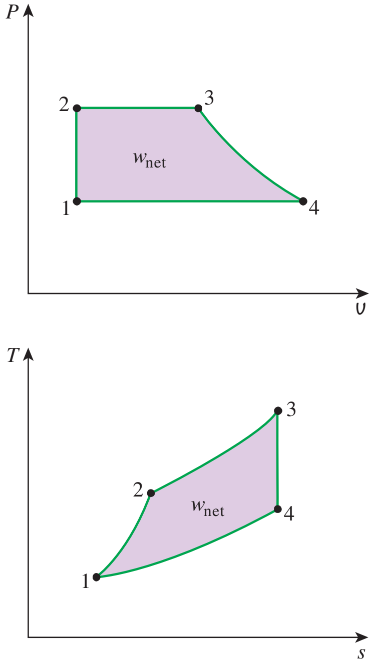


FIGURE 9–4
On both P-υ and T-s diagrams, the area enclosed by the process curve represents the net work of the cycle.

for steam power plants, the working fluid flows through a series of steady-flow devices such as the turbine and condenser, whereas in the Otto cycle, which is the ideal cycle for the spark-ignition automobile engine, the working fluid is alternately expanded and compressed in a piston–cylinder device. Therefore, equations pertaining to steady-flow systems should be used in the analysis of the Rankine cycle, and equations pertaining to closed systems should be used in the analysis of the Otto cycle.

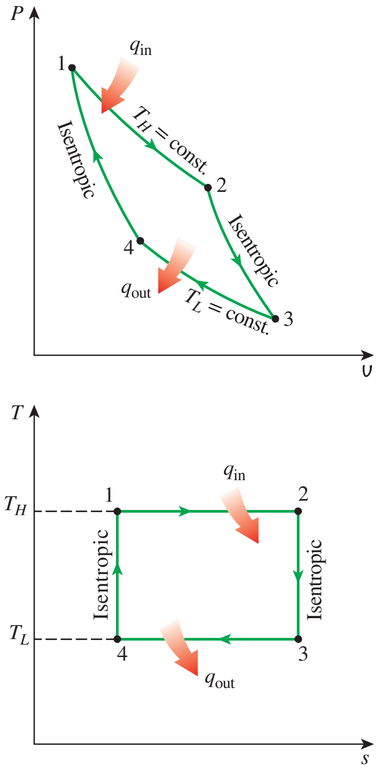


FIGURE 9–5
P-υ and *T*-*s* diagrams of a Carnot cycle.

9–2 ▪ THE CARNOT CYCLE AND ITS VALUE IN ENGINEERING

The Carnot cycle is composed of four totally reversible processes: isothermal heat addition, isentropic expansion, isothermal heat rejection, and isentropic compression. The *P*-υ and *T*-*s* diagrams of a Carnot cycle are replotted in Fig. 9–5. The Carnot cycle can be executed in a closed system (a piston–cylinder device) or a steady-flow system (utilizing two turbines and two compressors, as shown in Fig. 9–6), and either a gas or a vapor can be utilized as the working fluid. The Carnot cycle is the most efficient cycle that can be executed between a heat source at temperature T_H and a sink at temperature T_L, and its thermal efficiency is expressed as

$$\eta_{\text{th,Carnot}} = 1 - \frac{T_L}{T_H} \tag{9–2}$$

Reversible isothermal heat transfer is very difficult to achieve in reality because it would require very large heat exchangers and it would take a very long time (a power cycle in a typical engine is completed in a fraction of a second). Therefore, it is not practical to build an engine that would operate on a cycle that closely approximates the Carnot cycle.

The real value of the Carnot cycle comes from its being a standard against which the actual or the ideal cycles can be compared. The thermal efficiency of the Carnot cycle is a function of the sink and source temperatures only, and the thermal efficiency relation for the Carnot cycle (Eq. 9–2) conveys an important message that is equally applicable to both ideal and actual cycles: *Thermal efficiency increases with an increase in the average temperature at which heat is supplied to the system or with a decrease in the average temperature at which heat is rejected from the system.*

The source and sink temperatures that can be used in practice are not without limits, however. The highest temperature in the cycle is limited by the maximum temperature that the components of the heat engine, such as the piston or the turbine blades, can withstand. The lowest temperature is limited by the temperature of the cooling medium utilized in the cycle such as a lake, a river, or the atmospheric air.

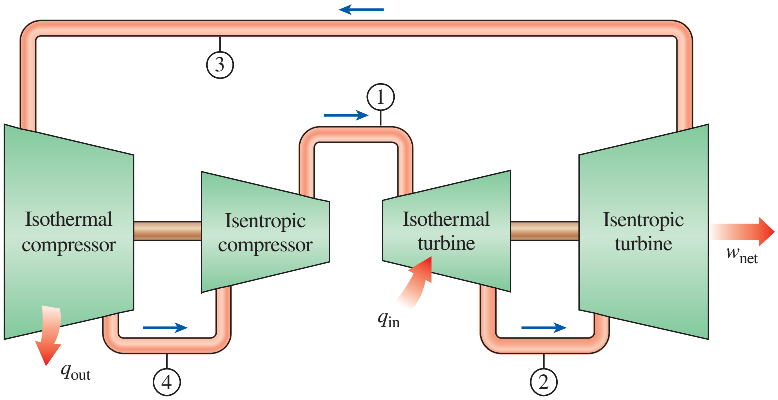


FIGURE 9–6
A steady-flow Carnot engine.

EXAMPLE 9–1 Derivation of the Efficiency of the Carnot Cycle

Show that the thermal efficiency of a Carnot cycle operating between the temperature limits of T_H and T_L is solely a function of these two temperatures and is given by Eq. 9–2.

SOLUTION It is to be shown that the efficiency of a Carnot cycle depends on the source and sink temperatures alone.

Analysis The *T-s* diagram of a Carnot cycle is redrawn in Fig. 9–7. All four processes that comprise the Carnot cycle are reversible, and thus the area under each process curve represents the heat transfer for that process. Heat is transferred to the system during process 1-2 and rejected during process 3-4. Therefore, the amount of heat input and heat output for the cycle can be expressed as

$$q_{\text{in}} = T_H(s_2 - s_1) \quad \text{and} \quad q_{\text{out}} = T_L(s_3 - s_4) = T_L(s_2 - s_1)$$

since processes 2-3 and 4-1 are isentropic, and thus $s_2 = s_3$ and $s_4 = s_1$. Substituting these into Eq. 9–1, we see that the thermal efficiency of a Carnot cycle is

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{T_L(s_2 - s_1)}{T_H(s_2 - s_1)} = 1 - \frac{T_L}{T_H}$$

Discussion Notice that the thermal efficiency of a Carnot cycle is independent of the type of the working fluid used (an ideal gas, steam, etc.) or whether the cycle is executed in a closed or steady-flow system.

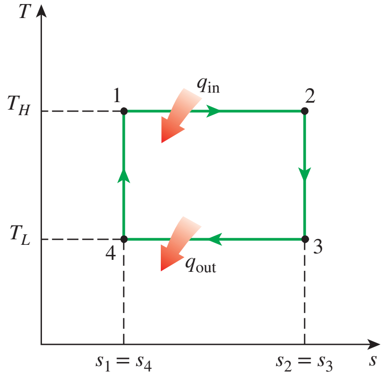


FIGURE 9–7
T-s diagram for Example 9–1.

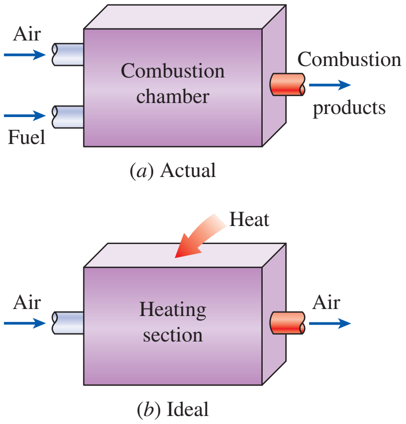
9–3 ▪ AIR-STANDARD ASSUMPTIONS

In gas power cycles, the working fluid remains a gas throughout the entire cycle. Spark-ignition engines, diesel engines, and conventional gas turbines are familiar examples of devices that operate on gas cycles. In all these engines, energy is provided by burning a fuel within the system boundaries. That is, they are *internal combustion engines.* Because of this combustion process, the composition of the working fluid changes from air and fuel to combustion products during the course of the cycle. However, considering that air is predominantly nitrogen that undergoes hardly any chemical reactions in the combustion chamber, the working fluid closely resembles air at all times.

Even though internal combustion engines operate on a mechanical cycle (the piston returns to its starting position at the end of each revolution), the working fluid does not undergo a complete thermodynamic cycle. It is thrown out of the engine at some point in the cycle (as exhaust gases) instead of being returned to the initial state. Working on an open cycle is the characteristic of all internal combustion engines.

The actual gas power cycles are rather complex. To reduce the analysis to a manageable level, we utilize the following approximations, commonly known as the **air-standard assumptions**:

1. The working fluid is air, which continuously circulates in a closed loop and always behaves as an ideal gas.
2. All the processes that make up the cycle are internally reversible.
3. The combustion process is replaced by a heat-addition process from an external source (Fig. 9–8).
4. The exhaust process is replaced by a heat-rejection process that restores the working fluid to its initial state.



(*a*) Actual

(*b*) Ideal

FIGURE 9–8
The combustion process is replaced by a heat-addition process in ideal cycles.

Another assumption that is often used to simplify the analysis even more is that air has constant specific heats whose values are determined at *room temperature* (25°C, or 77°F). When this assumption is used, the air-standard assumptions are called the **cold-air-standard assumptions**. A cycle for which the air-standard assumptions are applicable is frequently referred to as an **air-standard cycle**.

The air-standard assumptions previously stated provide considerable simplification in the analysis without significantly deviating from the actual cycles. This simplified model enables us to study qualitatively the influence of major parameters on the performance of the actual engines.

EXAMPLE 9–2 An Air-Standard Cycle

An air-standard cycle is executed in a closed system and is composed of the following four processes:

1-2 Isentropic compression from 100 kPa and 27°C to 1 MPa
2-3 P = *constant* heat addition in amount of 2800 kJ/kg
3-4 υ = *constant* heat rejection to 100 kPa
4-1 P = *constant* heat rejection to initial state

(*a*) Show the cycle on P-υ and T-s diagrams.
(*b*) Calculate the maximum temperature in the cycle.
(*c*) Determine the thermal efficiency.

Assume constant specific heats at room temperature.

SOLUTION The four processes of an air-standard cycle are described. The cycle is to be shown on P-υ and T-s diagrams, and the maximum temperature in the cycle and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K, c_υ = 0.718 kJ/kg·K, and k = 1.4 (Table A–2*a*).

Analysis (*a*) The cycle is shown on P-υ and T-s diagrams in Fig. 9–9.

(*b*) From the ideal gas isentropic relations and energy balance,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{ K})\left(\frac{1000\text{ kPa}}{100\text{ kPa}}\right)^{0.4/1.4} = 579.2\text{ K}$$

$$q_{\text{in}} = h_3 - h_2 = c_p(T_3 - T_1)$$

$$2800\text{ kJ/kg} = (1.005\text{ kJ/kg·K})(T_3 - 579.2) \longrightarrow T_{\text{max}} = T_3 = \mathbf{3360\ K}$$

(*c*) The temperature at state 4 is determined from the ideal gas relation for a fixed mass,

$$\frac{P_3\upsilon_3}{T_3} = \frac{P_4\upsilon_4}{T_4} \longrightarrow T_4 = \frac{P_4}{P_3}T_3 = \frac{100\text{ kPa}}{1000\text{ kPa}}(3360\text{ K}) = 336\text{ K}$$

The total amount of heat rejected from the cycle is

$$\begin{aligned} q_{\text{out}} &= q_{34,\text{out}} + q_{41,\text{out}} = (u_3 - u_4) + (h_4 - h_1) \\ &= c_\upsilon(T_3 - T_4) + c_p(T_4 - T_1) \\ &= (0.718\text{ kJ/kg·K})(3360 - 336)\text{K} + (1.005\text{ kJ/kg·K})(336 - 300)\text{K} \\ &= 2212\text{ kJ/kg} \end{aligned}$$

Then, the thermal efficiency is determined from its definition to be

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{2212\text{ kJ/kg}}{2800\text{ kJ/kg}} = 0.210 \text{ or } \mathbf{21.0\%}$$

Discussion The assumption of constant specific heats at room temperature is not realistic in this case since the temperature changes involved are too large.

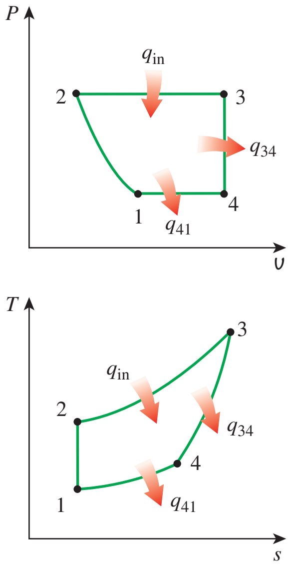


FIGURE 9–9
Schematic for Example 9–2.

9–4 ▪ AN OVERVIEW OF RECIPROCATING ENGINES

Despite its simplicity, the reciprocating engine (basically a piston–cylinder device) is one of the rare inventions that has proved to be very versatile and to have a wide range of applications. It is the powerhouse of the vast majority of automobiles, trucks, light aircraft, ships, and electric power generators, as well as many other devices.

The basic components of a reciprocating engine are shown in Fig. 9–10. The piston reciprocates in the cylinder between two fixed positions called the **top dead center** (TDC)—the position of the piston when it forms the smallest volume in the cylinder—and the **bottom dead center** (BDC)—the position of the piston when it forms the largest volume in the cylinder. The distance between the TDC and the BDC is the largest distance that the piston can travel in one direction, and it is called the **stroke** of the engine. The diameter of the piston is called the **bore**. The air or air–fuel mixture is drawn into the cylinder through the **intake valve**, and the combustion products are expelled from the cylinder through the **exhaust valve**.

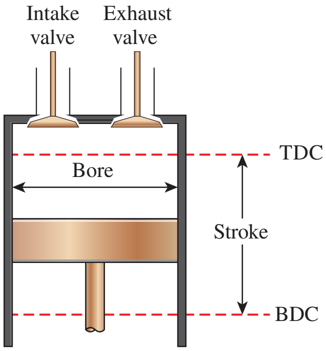
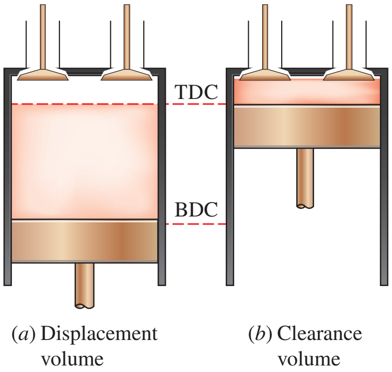


FIGURE 9–10
Nomenclature for reciprocating engines.

The minimum volume formed in the cylinder when the piston is at TDC is called the **clearance volume** (Fig. 9–11). The volume displaced by the piston as it moves between TDC and BDC is called the **displacement volume**. The ratio of the maximum volume formed in the cylinder to the minimum (clearance) volume is called the **compression ratio** r of the engine:

$$r = \frac{V_{\max}}{V_{\min}} = \frac{V_{\mathrm{BDC}}}{V_{\mathrm{TDC}}} \tag{9–3}$$



(a) Displacement volume
(b) Clearance volume

FIGURE 9–11
Displacement and clearance volumes of a reciprocating engine.

Notice that the compression ratio is a *volume ratio* and should not be confused with the pressure ratio.

Another term often used in conjunction with reciprocating engines is the **mean effective pressure** (MEP). It is a fictitious pressure that, if it acted on the piston during the entire power stroke, would produce the same amount of net work as that produced during the actual cycle (Fig. 9–12). That is,

$$W_{\mathrm{net}} = \mathrm{MEP} \times \text{Piston area} \times \text{Stroke} = \mathrm{MEP} \times \text{Displacement volume}$$

or

$$\mathrm{MEP} = \frac{W_{\mathrm{net}}}{V_{\max} - V_{\min}} = \frac{w_{\mathrm{net}}}{v_{\max} - v_{\min}} \quad (\mathrm{kPa}) \tag{9–4}$$

The mean effective pressure can be used as a parameter to compare the performances of reciprocating engines of equal size. The engine with a larger value of MEP delivers more net work per cycle and thus performs better.

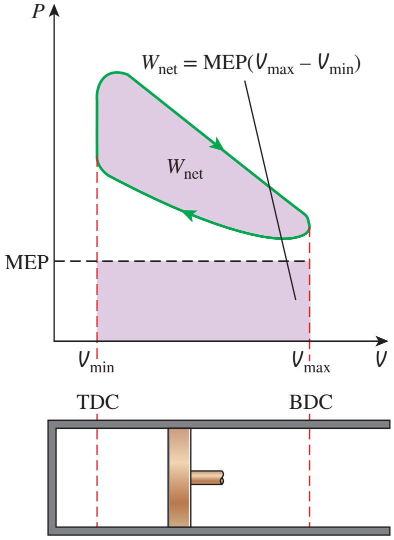
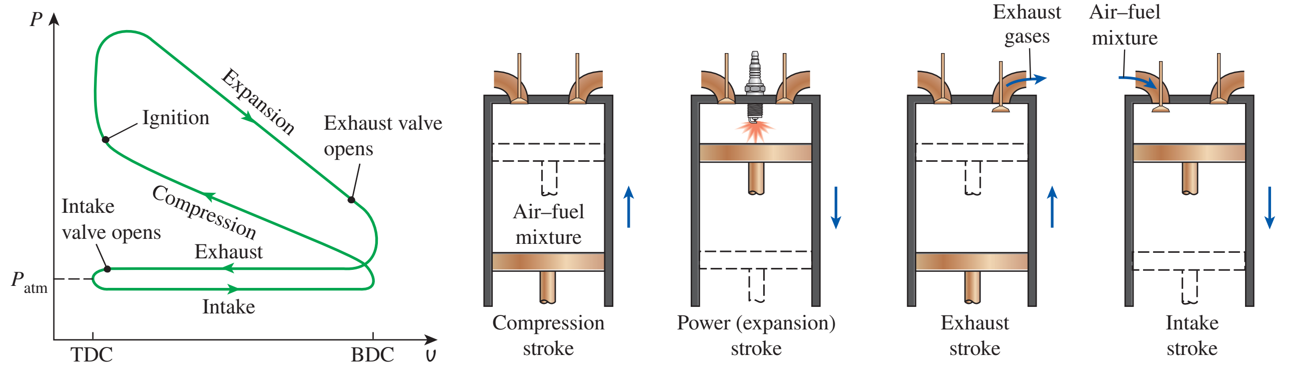


FIGURE 9–12
The net work output of a cycle is equivalent to the product of the mean effective pressure and the displacement volume.

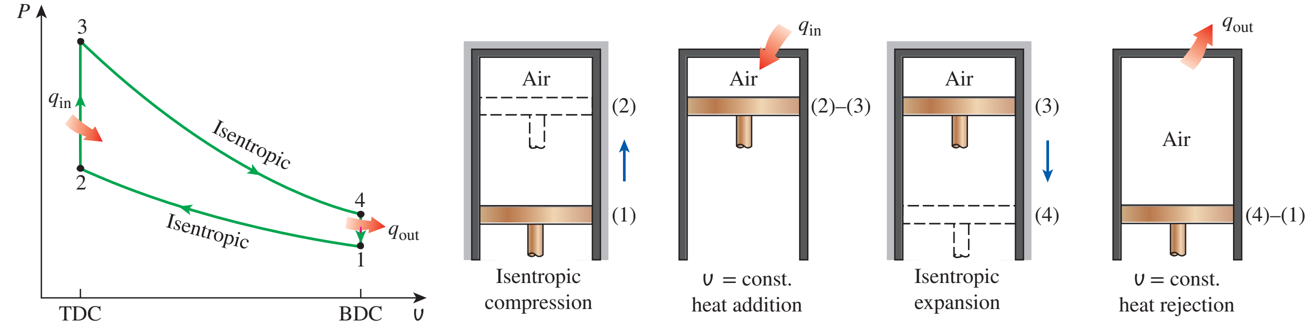
Reciprocating engines are classified as **spark-ignition (SI) engines** or **compression-ignition (CI) engines**, depending on how the combustion process in the cylinder is initiated. In SI engines, the combustion of the air–fuel mixture is initiated by a spark plug. In CI engines, the air–fuel mixture is self-ignited as a result of compressing the mixture above its self-ignition temperature. In the next two sections, we discuss the *Otto* and *Diesel cycles,* which are the ideal cycles for the SI and CI reciprocating engines, respectively.

9–5 ▪ OTTO CYCLE: THE IDEAL CYCLE FOR SPARK-IGNITION ENGINES

The Otto cycle is the ideal cycle for spark-ignition reciprocating engines. It is named after Nikolaus A. Otto, who built a successful four-stroke engine in 1876 in Germany using the cycle proposed by Frenchman Beau de Rochas in 1862.



(*a*) Actual four-stroke spark-ignition engine



(*b*) Ideal Otto cycle

FIGURE 9–13

Actual and ideal cycles in spark-ignition engines and their P-υ diagrams.

In most spark-ignition engines, the piston executes four complete strokes (two mechanical cycles) within the cylinder, and the crankshaft completes two revolutions for each thermodynamic cycle. These engines are called **four-stroke internal combustion engines**. A schematic of each stroke as well as a P-υ diagram for an actual four-stroke spark-ignition engine is given in Fig. 9–13*a*.

Initially, both the intake and the exhaust valves are closed, and the piston is at its lowest position (BDC). During the *compression stroke,* the piston moves upward, compressing the air–fuel mixture. Shortly before the piston reaches its highest position (TDC), the spark plug fires and the mixture ignites, increasing the pressure and temperature of the system. The high-pressure gases force the piston down, which in turn forces the crankshaft to rotate, producing a useful work output during the *expansion* or *power stroke.* Toward the end of expansion stroke, the exhaust valve opens and the combustion gases that are above the atmospheric pressure rush out of the cylinder through the open exhaust valve. This process is called **exhaust blowdown**, and most combustion gases leave the cylinder by the time the piston reaches BDC. The cylinder is still filled by the exhaust gases at a lower pressure at BDC. Now the piston moves upward one more time, purging the exhaust gases through the exhaust valve (the *exhaust stroke*), and down a second time, drawing in fresh air–fuel mixture through the intake valve (the *intake stroke*). Notice that the pressure in the cylinder is slightly above the atmospheric value during the exhaust stroke and slightly below during the intake stroke.

In **two-stroke engines**, all four functions described above are executed in just two strokes: the power stroke and the compression stroke. In these engines, the crankcase is sealed, and the outward motion of the piston is used to slightly pressurize the air–fuel mixture in the crankcase, as shown in Fig. 9–14. Also, the intake and exhaust valves are replaced by openings in the lower portion of the

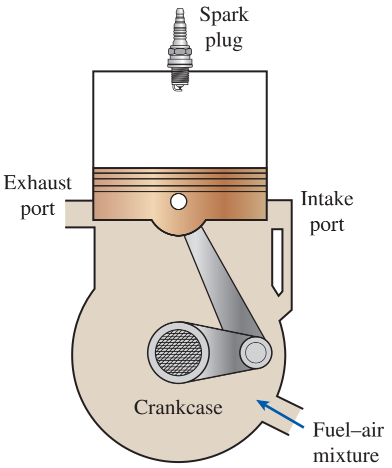


FIGURE 9–14

Schematic of a two-stroke reciprocating engine.

cylinder wall. During the latter part of the power stroke, the piston uncovers first the exhaust port, allowing the exhaust gases to be partially expelled, and then the intake port, allowing the fresh air–fuel mixture to rush in and drive most of the remaining exhaust gases out of the cylinder. This mixture is then compressed as the piston moves upward during the compression stroke and is subsequently ignited by a spark plug.

The two-stroke engines are generally less efficient than their four-stroke counterparts because of the incomplete expulsion of the exhaust gases and the partial expulsion of the fresh air–fuel mixture with the exhaust gases. However, they are relatively simple and inexpensive, and they have high power-to-weight and power-to-volume ratios, which make them suitable for applications requiring small size and weight such as for motorcycles, chain saws, and lawn mowers (Fig. 9–15).

Advances in several technologies—such as direct fuel injection, stratified charge combustion, and electronic controls—brought about a renewed interest in two-stroke engines that can offer high performance and fuel economy while satisfying stringent emission requirements. For a given weight and displacement, a well-designed two-stroke engine can provide significantly more power than its four-stroke counterpart because two-stroke engines produce power on every engine revolution instead of every other one. In the new two-stroke engines, the highly atomized fuel spray that is injected into the combustion chamber toward the end of the compression stroke burns much more completely. The fuel is sprayed after the exhaust valve is closed, which prevents unburned fuel from being ejected into the atmosphere. With stratified combustion, the flame that is initiated by igniting a small amount of the rich fuel–air mixture near the spark plug propagates through the combustion chamber filled with a much leaner mixture, and this results in much cleaner combustion. Also, the advances in electronics have made it possible to ensure the optimum operation under varying engine load and speed conditions. Major car companies have research programs underway on two-stroke engines, which are expected to make a comeback in the future.

The thermodynamic analysis of the actual four-stroke or two-stroke cycles described is not a simple task. However, the analysis can be simplified significantly if the air-standard assumptions are utilized. The resulting cycle, which closely resembles the actual operating conditions, is the ideal **Otto cycle**. It consists of four internally reversible processes:

1-2 Isentropic compression
2-3 Constant-volume heat addition
3-4 Isentropic expansion
4-1 Constant-volume heat rejection

The execution of the Otto cycle in a piston–cylinder device together with a P-υ diagram is illustrated in Fig. 9–13*b*. The T-s diagram of the Otto cycle is given in Fig. 9–16.

The ideal Otto cycle shown in Fig. 9–13*b* has one shortcoming. This ideal cycle consists of two strokes equivalent to one mechanical cycle or one crankshaft rotation. The actual engine operation shown in Fig. 9–13*a*, on the other hand, involves four strokes equivalent to two mechanical cycles or two crankshaft rotations. This can be corrected by including intake and exhaust strokes in the ideal Otto cycle, as shown in Fig. 9–17. In this modified cycle, air–fuel mixture (approximated as air due to air-standard assumptions) enters the cylinder through the open intake valve at atmospheric pressure P_0 during process 0-1 as the piston moves from TDC to BDC. The intake valve is closed at state 1 and air is compressed isentropically to state 2. Heat is transferred at constant volume (process 2-3); it is expanded isentropically to state 4; and heat is rejected at constant volume (process 4-1). Exhaust gases (again approximated as air) are expelled through the open exhaust valve (process 1-0) as the pressure remains constant at P_0.

FIGURE 9–15
Two-stroke engines are commonly used in motorcycles and lawn mowers.
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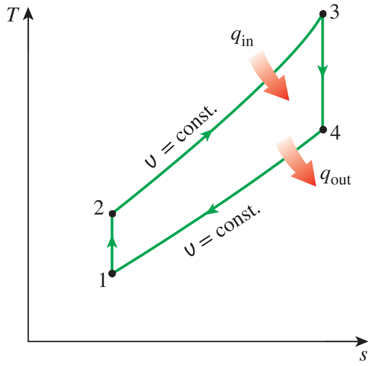


FIGURE 9–16
T-s diagram of the ideal Otto cycle.

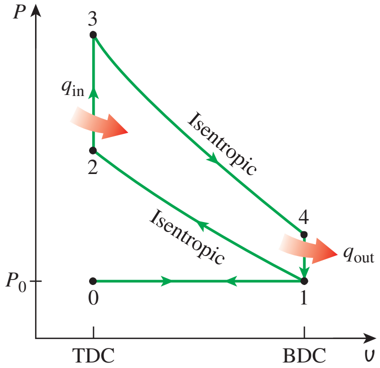


FIGURE 9–17
P-υ diagram of the ideal Otto cycle that includes intake and exhaust strokes.

The modified Otto cycle shown in Fig. 9–17 is executed in an open system during the intake and exhaust processes and in a closed system during the remaining four processes. We should point out that the constant-volume heat addition process (2-3) in the ideal Otto cycle replaces the combustion process of the actual engine operation while the constant-volume heat rejection process (4-1) replaces the exhaust blowdown.

The work interactions during the constant-pressure intake (0-1) and constant-pressure exhaust (1-0) processes can be expressed as

$$w_{\text{out,0-1}} = P_0(v_1 - v_0)$$
$$w_{\text{in,1-0}} = P_0(v_1 - v_0)$$

These two processes cancel each other as the work output during the intake is equal to work input during the exhaust. Then, the cycle reduces to the one in Fig. 9–13*b*. Therefore, inclusion of the intake and exhaust processes has no effect on the net work output from the cycle. However, when calculating power output from the cycle during an ideal Otto cycle analysis, we must consider the fact that the ideal Otto cycle has four strokes just like an actual four-stroke spark-ignition engine. This is illustrated in the last part of Example 9–3.

The Otto cycle is executed in a closed system, and disregarding the changes in kinetic and potential energies, the energy balance for any of the processes is expressed, on a unit-mass basis, as

$$(q_{\text{in}} - q_{\text{out}}) + (w_{\text{in}} - w_{\text{out}}) = \Delta u \qquad \text{(kJ/kg)} \tag{9–5}$$

No work is involved during the two heat transfer processes since both take place at constant volume. Therefore, heat transfer to and from the working fluid can be expressed as

$$q_{\text{in}} = u_3 - u_2 = c_v(T_3 - T_2) \tag{9–6a}$$

and

$$q_{\text{out}} = u_4 - u_1 = c_v(T_4 - T_1) \tag{9–6b}$$

Then the thermal efficiency of the ideal Otto cycle under the cold air standard assumptions becomes

$$\eta_{\text{th,Otto}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

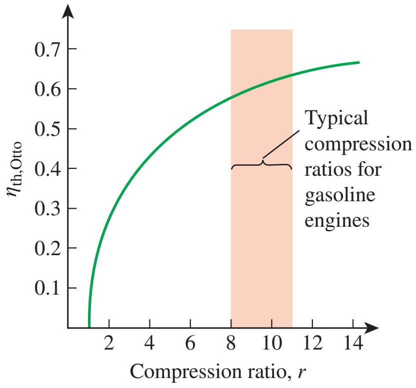


FIGURE 9–18
Thermal efficiency of the ideal Otto cycle as a function of compression ratio ($k = 1.4$).

Processes 1-2 and 3-4 are isentropic, and $v_2 = v_3$ and $v_4 = v_1$. Thus,

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1} = \left(\frac{v_3}{v_4}\right)^{k-1} = \frac{T_4}{T_3} \tag{9–7}$$

Substituting these equations into the thermal efficiency relation and simplifying give

$$\eta_{\text{th,Otto}} = 1 - \frac{1}{r^{k-1}} \tag{9–8}$$

where

$$r = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{V_1}{V_2} = \frac{v_1}{v_2} \tag{9–9}$$

is the compression ratio and k is the specific heat ratio c_p/c_v.

Equation 9–8 shows that under the cold-air-standard assumptions, the thermal efficiency of an ideal Otto cycle depends on the compression ratio of the engine and the specific heat ratio of the working fluid. The thermal efficiency of the ideal Otto cycle increases with both the compression ratio and the specific heat ratio. This is also true for actual spark-ignition internal combustion engines. A plot of thermal efficiency versus the compression ratio is given in Fig. 9–18 for $k = 1.4$, which is the specific heat ratio value of air at room temperature. For a given compression ratio, the thermal efficiency of an actual spark-ignition engine is less than that of an ideal Otto cycle because of the irreversibilities, such as friction, and other factors such as incomplete combustion.

We can observe from Fig. 9–18 that the thermal efficiency curve is rather steep at low compression ratios but flattens out starting with a compression ratio value of about 8. Therefore, the increase in thermal efficiency with the compression ratio is not as pronounced at high compression ratios. Also, when high compression ratios are used, the temperature in some regions of the air–fuel mixture rises above the autoignition temperature of the fuel (the temperature at which the fuel ignites without the help of a spark) during the combustion process, causing an early and rapid burn of the fuel at some point or points ahead of the flame front, followed by almost instantaneous inflammation of the end gas. This premature ignition of the fuel, called **autoignition**, produces an audible noise, which is called **engine knock**. Autoignition in spark-ignition engines cannot be tolerated because it hurts performance and can cause engine damage. The requirement that autoignition not be allowed places an upper limit on the compression ratios that can be used in spark-ignition internal combustion engines.

Improvement of the thermal efficiency of gasoline engines by utilizing higher compression ratios (up to about 12) without facing the autoignition problem has been made possible by using gasoline blends that have good antiknock characteristics, such as gasoline mixed with tetraethyl lead. Tetraethyl lead had been added to gasoline since the 1920s because it is an inexpensive method of raising the *octane rating,* which is a measure of the engine knock resistance of a fuel. Leaded gasoline, however, has a very undesirable side effect: it forms compounds during the combustion process that are hazardous to health and pollute the environment. In an effort to combat air pollution, the government adopted a policy in the mid-1970s that resulted in the eventual phase-out of leaded gasoline. Unable to use lead, the refiners developed other techniques to improve the antiknock characteristics of gasoline. Most cars made since 1975 have been designed to use unleaded gasoline, and the compression ratios had to be lowered to avoid engine knock. The ready availability of high-octane fuels made it possible to raise the compression ratios again in recent years. Also, owing to the improvements in other areas (reduction in overall automobile weight, improved aerodynamic design, using variable compression ratios by the use of a multilink system, hybrid engines that recover power lost during braking, individually controlled intake and exhaust valves, etc.), today's cars have better fuel economy and consequently get more miles per gallon of fuel. This is an example of how engineering decisions involve compromises, and efficiency is only one of the considerations in final design.

The second parameter affecting the thermal efficiency of an ideal Otto cycle is the specific heat ratio k. For a given compression ratio, an ideal Otto cycle using a monatomic gas (such as argon or helium, $k = 1.667$) as the working fluid will have the highest thermal efficiency. The specific heat ratio k, and thus the thermal efficiency of the ideal Otto cycle, decreases as the molecules of the working fluid get larger (Fig. 9–19). At room temperature it is 1.4 for air, 1.3 for carbon dioxide, and 1.2 for ethane. The working fluid in actual engines contains larger molecules such as carbon dioxide, and the specific heat ratio decreases with temperature, which is one of the reasons that the actual cycles have lower thermal efficiencies than the ideal Otto cycle. The thermal efficiencies of actual spark-ignition engines range from about 25 to 30 percent.

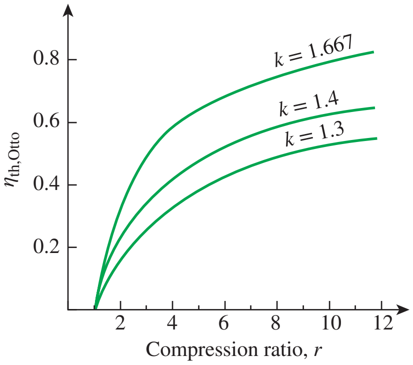


FIGURE 9–19
The thermal efficiency of the Otto cycle increases with the specific heat ratio k of the working fluid.

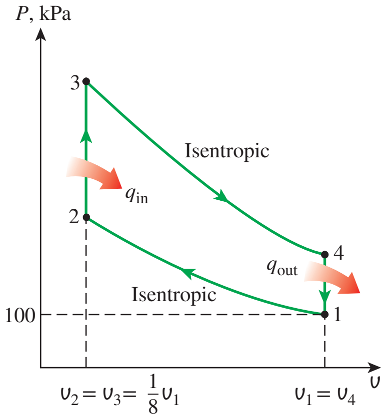


FIGURE 9–20

P-v diagram for the Otto cycle discussed in Example 9–3.

EXAMPLE 9–3 The Ideal Otto Cycle

An ideal Otto cycle has a compression ratio of 8. At the beginning of the compression process, air is at 100 kPa and 17°C, and 800 kJ/kg of heat is transferred to air during the constant-volume heat-addition process. Accounting for the variation of specific heats of air with temperature, determine (*a*) the maximum temperature and pressure that occur during the cycle, (*b*) the net work output, (*c*) the thermal efficiency, and (*d*) the mean effective pressure for the cycle.

(*e*) Also, determine the power output from the cycle, in kW, for an engine speed of 4000 rpm (rev/min). Assume that this cycle is operated on an engine that has four cylinders with a total displacement volume of 1.6 L.

SOLUTION An ideal Otto cycle is considered. The maximum temperature and pressure, the net work output, the thermal efficiency, the mean effective pressure, and the power output for a given engine speed are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** The variation of specific heats with temperature is to be accounted for.

Analysis The *P-v* diagram of the ideal Otto cycle described is shown in Fig. 9–20. We note that the air contained in the cylinder forms a closed system.

(*a*) The maximum temperature and pressure in an Otto cycle occur at the end of the constant-volume heat-addition process (state 3). But first we need to determine the temperature and pressure of air at the end of the isentropic compression process (state 2), using data from Table A–21:

$$T_1 = 290 \text{ K} \rightarrow u_1 = 206.91 \text{ kJ/kg}$$
$$v_{r1} = 676.1$$

Process 1-2 (isentropic compression of an ideal gas):

$$\frac{v_{r2}}{v_{r1}} = \frac{v_2}{v_1} = \frac{1}{r} \rightarrow v_{r2} = \frac{v_{r1}}{r} = \frac{676.1}{8} = 84.51 \rightarrow T_2 = 652.4 \text{ K}$$
$$u_2 = 475.11 \text{ kJ/kg}$$

$$\frac{P_2v_2}{T_2} = \frac{P_1v_1}{T_1} \rightarrow P_2 = P_1\left(\frac{T_2}{T_1}\right)\left(\frac{v_1}{v_2}\right)$$
$$= (100 \text{ kPa})\left(\frac{652.4 \text{ K}}{290 \text{ K}}\right)(8) = 1799.7 \text{ kPa}$$

Process 2-3 (constant-volume heat addition):

$$q_{in} = u_3 - u_2$$
$$800 \text{ kJ/kg} = u_3 - 475.11 \text{ kJ/kg}$$
$$u_3 = 1275.11 \text{ kJ/kg} \rightarrow T_3 = \mathbf{1575.1 \text{ K}}$$
$$v_{r3} = 6.108$$

$$\frac{P_3v_3}{T_3} = \frac{P_2v_2}{T_2} \rightarrow P_3 = P_2\left(\frac{T_3}{T_2}\right)\left(\frac{v_2}{v_3}\right)$$
$$= (1.7997 \text{ MPa})\left(\frac{1575.1 \text{ K}}{652.4 \text{ K}}\right)(1) = \mathbf{4.345 \text{ MPa}}$$

(*b*) The net work output for the cycle is determined either by finding the boundary ($P\,dV$) work involved in each process by integration and adding them or by finding the net heat transfer that is equivalent to the net work done during the cycle. We take the latter approach. However, first we need to find the internal energy of the air at state 4:

Process 3-4 (isentropic expansion of an ideal gas):

$$\frac{v_{r4}}{v_{r3}} = \frac{v_4}{v_3} = r \rightarrow v_{r4} = rv_{r3} = (8)(6.108) = 48.864 \rightarrow T_4 = 795.6 \text{ K}$$
$$u_4 = 588.74 \text{ kJ/kg}$$

Process 4-1 (constant-volume heat rejection):

$$-q_{\text{out}} = u_1 - u_4 \rightarrow q_{\text{out}} = u_4 - u_1$$
$$q_{\text{out}} = 588.74 - 206.91 = 381.83 \text{ kJ/kg}$$

Thus,

$$w_{\text{net}} = q_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 800 - 381.83 = \mathbf{418.17\ kJ/kg}$$

(*c*) The thermal efficiency of the cycle is determined from its definition:

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{418.17 \text{ kJ/kg}}{800 \text{ kJ/kg}} = 0.523 \quad \text{or} \quad \mathbf{52.3\%}$$

Under the cold-air-standard assumptions (constant specific heat values at room temperature), the thermal efficiency would be (Eq. 9–8)

$$\eta_{\text{th,Otto}} = 1 - \frac{1}{r^{k-1}} = 1 - r^{1-k} = 1 - (8)^{1-1.4} = 0.565 \quad \text{or} \quad 56.5\%$$

which is considerably different from the value obtained above. Therefore, care should be exercised in utilizing the cold-air-standard assumptions.

(*d*) The mean effective pressure is determined from its definition, Eq. 9–4:

$$\text{MEP} = \frac{w_{\text{net}}}{v_1 - v_2} = \frac{w_{\text{net}}}{v_1 - v_1/r} = \frac{w_{\text{net}}}{v_1(1 - 1/r)}$$

where

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa·m}^3\text{/kg·K})(290 \text{ K})}{100 \text{ kPa}} = 0.8323 \text{ m}^3\text{/kg}$$

Thus,

$$\text{MEP} = \frac{418.17 \text{ kJ/kg}}{(0.8323 \text{ m}^3\text{/kg})\left(1 - \frac{1}{8}\right)}\left(\frac{1 \text{ kPa·m}^3}{1 \text{ kJ}}\right) = \mathbf{574\ kPa}$$

(*e*) The total air mass taken by all four cylinders when they are charged is

$$m = \frac{V_d}{v_1} = \frac{0.0016 \text{ m}^3}{0.8323 \text{ m}^3\text{/kg}} = 0.001922 \text{ kg}$$

The net work produced by the cycle is

$$W_{\text{net}} = mw_{\text{net}} = (0.001922 \text{ kg})(418.17 \text{ kJ/kg}) = 0.8037 \text{ kJ}$$

That is, the net work produced per thermodynamic cycle is 0.8037 kJ/cycle. Noting that there are two revolutions per thermodynamic cycle (n_{rev} = 2 rev/cycle) in a four-stroke engine (or in the ideal Otto cycle including intake and exhaust strokes), the power produced by the engine is determined from

$$\dot{W}_{\text{net}} = \frac{W_{\text{net}}\dot{n}}{n_{\text{rev}}} = \frac{(0.8037 \text{ kJ/cycle})(4000 \text{ rev/min})}{2 \text{ rev/cycle}}\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \mathbf{26.8\ kW}$$

Discussion If we analyzed a two-stroke engine operating on an ideal Otto cycle with the same values, the power output would be calculated as

$$\dot{W}_{\text{net}} = \frac{W_{\text{net}}\dot{n}}{n_{\text{rev}}} = \frac{(0.8037 \text{ kJ/cycle})(4000 \text{ rev/min})}{1 \text{ rev/cycle}}\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 53.6 \text{ kW}$$

Note that there is one revolution in one thermodynamic cycle in two-stroke engines.

9–6 ▪ DIESEL CYCLE: THE IDEAL CYCLE FOR COMPRESSION-IGNITION ENGINES

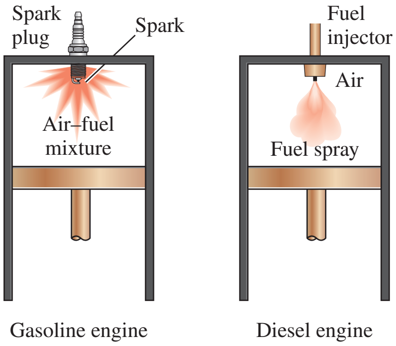
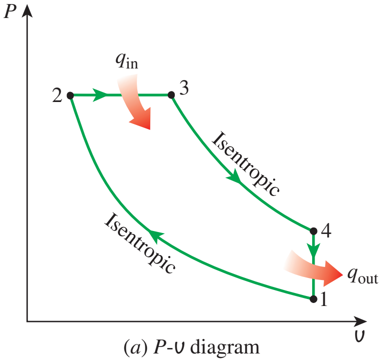


FIGURE 9–21
In diesel engines, the spark plug is replaced by a fuel injector, and only air is compressed during the compression process.

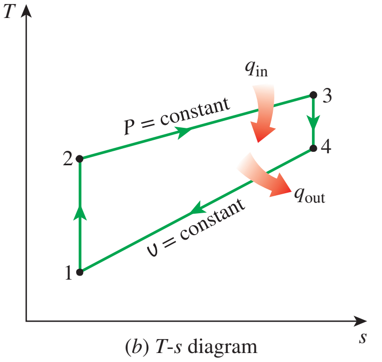
The Diesel cycle is the ideal cycle for CI reciprocating engines. The CI engine, first proposed by Rudolph Diesel in the 1890s, is very similar to the SI engine discussed in the last section, differing mainly in the method of initiating combustion. In spark-ignition engines (also known as *gasoline engines*), the air–fuel mixture is compressed to a temperature that is below the autoignition temperature of the fuel, and the combustion process is initiated by firing a spark plug. In CI engines (also known as *diesel engines*), the air is compressed to a temperature that is above the autoignition temperature of the fuel, and combustion starts on contact as the fuel is injected into this hot air. Therefore, the spark plug is replaced by a fuel injector in diesel engines (Fig. 9–21).

In gasoline engines, a mixture of air and fuel is compressed during the compression stroke, and the compression ratios are limited by the onset of autoignition or engine knock. In diesel engines, only air is compressed during the compression stroke, eliminating the possibility of autoignition. Therefore, diesel engines can be designed to operate at much higher compression ratios, typically between 12 and 24. Not having to deal with the problem of autoignition has another benefit: many of the stringent requirements placed on the gasoline can now be removed, and fuels that are less refined (thus less expensive) can be used in diesel engines.

The fuel injection process in diesel engines starts when the piston approaches TDC and continues during the first part of the power stroke. Therefore, the combustion process in these engines takes place over a longer interval. Because of this longer duration, the combustion process in the ideal Diesel cycle is approximated as a constant-pressure heat-addition process. In fact, this is the only process where the Otto and the Diesel cycles differ. The remaining three processes are the same for both ideal cycles. That is, process 1-2 is isentropic compression, 2-3 is constant-pressure heat addition, 3-4 is isentropic expansion, and 4-1 is constant-volume heat rejection. The similarity between the two cycles is also apparent from the P-v and T-s diagrams of the Diesel cycle, shown in Fig. 9–22.



(*a*) P-v diagram



(*b*) T-s diagram

FIGURE 9–22
T-s and P-v diagrams for the ideal Diesel cycle.

Noting that the Diesel cycle is executed in a piston–cylinder device, which forms a closed system, the amount of heat transferred to the working fluid at constant pressure and rejected from it at constant volume can be expressed as

$$\begin{aligned} q_{\text{in}} - w_{b,\text{out}} = u_3 - u_2 \rightarrow q_{\text{in}} &= P_2(v_3 - v_2) + (u_3 - u_2) \\ &= h_3 - h_2 = c_p(T_3 - T_2) \end{aligned} \tag{9–10a}$$

and

$$-q_{\text{out}} = u_1 - u_4 \rightarrow q_{\text{out}} = u_4 - u_1 = c_v(T_4 - T_1) \tag{9–10b}$$

Then the thermal efficiency of the ideal Diesel cycle under the cold-air-standard assumptions becomes

$$\eta_{\text{th,Diesel}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{kT_2(T_3/T_2 - 1)}$$

We now define a new quantity, the **cutoff ratio** r_c, as the ratio of the cylinder volumes after and before the combustion process:

$$r_c = \frac{V_3}{V_2} = \frac{v_3}{v_2} \tag{9–11}$$

Utilizing this definition and the isentropic ideal-gas relations for processes 1-2 and 3-4, we see that the thermal efficiency relation reduces to

$$\eta_{\text{th,Diesel}} = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right] \tag{9–12}$$

where r is the compression ratio defined by Eq. 9–9. Looking at Eq. 9–12 carefully, one would notice that under the cold-air-standard assumptions, the efficiency of a Diesel cycle differs from the efficiency of an Otto cycle by the quantity in the brackets. This quantity is always greater than 1. Therefore,

$$\eta_{\text{th,Otto}} > \eta_{\text{th,Diesel}} \tag{9–13}$$

when both cycles operate on the same compression ratio. Also, as the cutoff ratio decreases, the efficiency of the Diesel cycle increases (Fig. 9–23). For the limiting case of $r_c = 1$, the quantity in the brackets becomes unity (can you prove it?), and the efficiencies of the Otto and Diesel cycles become identical. Remember, though, that diesel engines operate at much higher compression ratios and thus are usually more efficient than the spark-ignition (gasoline) engines. The diesel engines also burn the fuel more completely since they usually operate at lower revolutions per minute and the air–fuel mass ratio is much higher than in spark-ignition engines. Thermal efficiencies of diesel automotive engines range from about 35 to 40 percent. Some very large, low-speed CI engines have thermal efficiencies over 50 percent.

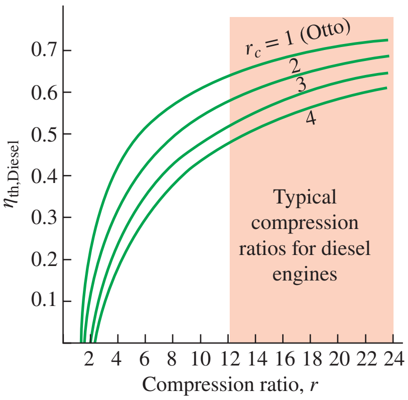


FIGURE 9–23
Thermal efficiency of the ideal Diesel cycle as a function of compression and cutoff ratios ($k = 1.4$).

The higher efficiency and lower fuel costs of diesel engines make them attractive in applications requiring relatively large amounts of power, such as in locomotive engines, emergency power generation units, large ships, and heavy trucks. As an example of how large a diesel engine can be, a 12-cylinder diesel engine built in 1964 by the Fiat Corporation of Italy had a normal power output of 25,200 hp (18.8 MW) at 122 rpm, a cylinder bore of 90 cm, and a stroke of 91 cm.

In modern high-speed compression-ignition engines, fuel is injected into the combustion chamber much sooner than in the early diesel engines. Fuel starts to ignite late in the compression stroke, and consequently part of the combustion occurs almost at constant volume. Fuel injection continues until the piston reaches the top dead center, and combustion of the fuel keeps the pressure high well into the expansion stroke. Thus, the entire combustion process can be better modeled as the combination of constant-volume and constant-pressure processes. The ideal cycle based on this concept is called the **dual cycle**, and the P-υ diagram for it is given in Fig. 9–24. The relative amounts of heat transferred during each process can be adjusted to approximate the actual cycle more closely. Note that both the Otto and the Diesel cycles can be obtained as special cases of the dual cycle. Dual cycle is a more realistic model than Diesel cycle for representing modern, high-speed compression-ignition engines.

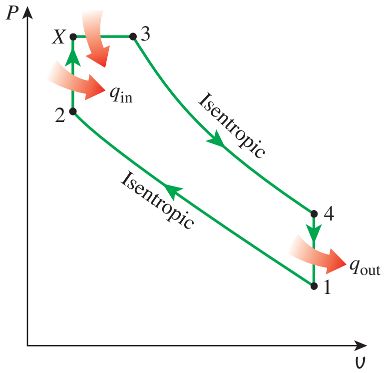


FIGURE 9–24
P-υ diagram of an ideal dual cycle.

EXAMPLE 9–4 The Ideal Diesel Cycle

An ideal Diesel cycle with air as the working fluid has a compression ratio of 18 and a cutoff ratio of 2. At the beginning of the compression process, the working fluid is at 14.7 psia, 80°F, and 117 in^3. Utilizing the cold-air-standard assumptions, determine (*a*) the temperature and pressure of air at the end of each process, (*b*) the net work output and the thermal efficiency, and (*c*) the mean effective pressure.

SOLUTION An ideal Diesel cycle is considered. The temperature and pressure at the end of each process, the net work output, the thermal efficiency, and the mean effective pressure are to be determined.

Assumptions **1** The cold-air-standard assumptions are applicable, and thus air can be assumed to have constant specific heats at room temperature. **2** Kinetic and potential energy changes are negligible.

Properties The gas constant of air is $R = 0.3704$ psia·ft^3/lbm·R, and its other properties at room temperature are $c_p = 0.240$ Btu/lbm·R, $c_\upsilon = 0.171$ Btu/lbm·R, and $k = 1.4$ (Table A–2E*a*).

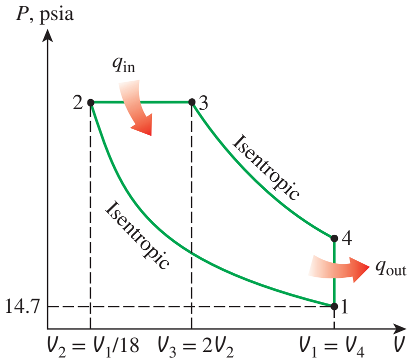


FIGURE 9–25
P-V diagram for the ideal Diesel cycle discussed in Example 9–4.

Analysis The *P-V* diagram of the ideal Diesel cycle described is shown in Fig. 9–25. We note that the air contained in the cylinder forms a closed system.

(*a*) The temperature and pressure values at the end of each process can be determined by utilizing the ideal-gas isentropic relations for processes 1-2 and 3-4. But first we determine the volumes at the end of each process from the definitions of the compression ratio and the cutoff ratio:

$$V_2 = \frac{V_1}{r} = \frac{117\text{ in}^3}{18} = 6.5\text{ in}^3$$
$$V_3 = r_c V_2 = (2)(6.5\text{ in}^3) = 13\text{ in}^3$$
$$V_4 = V_1 = 117\text{ in}^3$$

Process 1-2 (isentropic compression of an ideal gas, constant specific heats):

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = (540\text{ R})(18)^{1.4-1} = \mathbf{1716\text{ R}}$$
$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{k} = (14.7\text{ psia})(18)^{1.4} = \mathbf{841\text{ psia}}$$

Process 2-3 (constant-pressure heat addition to an ideal gas):

$$P_3 = P_2 = \mathbf{841\text{ psia}}$$
$$\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3} \rightarrow T_3 = T_2\left(\frac{V_3}{V_2}\right) = (1716\text{ R})(2) = \mathbf{3432\text{ R}}$$

Process 3-4 (isentropic expansion of an ideal gas, constant specific heats):

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{k-1} = (3432\text{ R})\left(\frac{13\text{ in}^3}{117\text{ in}^3}\right)^{1.4-1} = \mathbf{1425\text{ R}}$$
$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{k} = (841\text{ psia})\left(\frac{13\text{ in}^3}{117\text{ in}^3}\right)^{1.4} = \mathbf{38.8\text{ psia}}$$

(*b*) The net work for a cycle is equivalent to the net heat transfer. But first we find the mass of air:

$$m = \frac{P_1V_1}{RT_1} = \frac{(14.7\text{ psia})(117\text{ in}^3)}{(0.3704\text{ psia·ft}^3/\text{lbm·R})(540\text{ R})}\left(\frac{1\text{ ft}^3}{1728\text{ in}^3}\right) = 0.00498\text{ lbm}$$

Process 2-3 is a constant-pressure heat-addition process for which the boundary work and Δu terms can be combined into Δh. Thus,

$$\begin{aligned} Q_{in} &= m(h_3 - h_2) = mc_p(T_3 - T_2) \\ &= (0.00498\text{ lbm})(0.240\text{ Btu/lbm·R})[(3432 - 1716)\text{ R}] \\ &= 2.051\text{ Btu} \end{aligned}$$

Process 4-1 is a constant-volume heat-rejection process (it involves no work interactions), and the amount of heat rejected is

$$\begin{aligned} Q_{out} &= m(u_4 - u_1) = mc_v(T_4 - T_1) \\ &= (0.00498\text{ lbm})(0.171\text{ Btu/lbm·R})[(1425 - 540)\text{ R}] \\ &= 0.754\text{ Btu} \end{aligned}$$

Thus,

$$W_{net} = Q_{in} - Q_{out} = 2.051 - 0.754 = \mathbf{1.297\text{ Btu}}$$

Then the thermal efficiency becomes

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{1.297\text{ Btu}}{2.051\text{ Btu}} = 0.632 \quad\text{or}\quad \mathbf{63.2\%}$$

The thermal efficiency of this Diesel cycle under the cold-air-standard assumptions could also be determined from Eq. 9–12.

(*c*) The mean effective pressure is determined from its definition, Eq. 9–4:

$$\text{MEP} = \frac{W_{\text{net}}}{V_{\max} - V_{\min}} = \frac{W_{\text{net}}}{V_1 - V_2} = \frac{1.297\text{ Btu}}{(117 - 6.5)\text{ in}^3}\left(\frac{778.17\text{ lbf}\cdot\text{ft}}{1\text{ Btu}}\right)\left(\frac{12\text{ in}}{1\text{ ft}}\right)$$
$$= \mathbf{110\ psia}$$

Discussion Note that a constant pressure of 110 psia during the power stroke would produce the same net work output as the entire Diesel cycle.

9–7 ▪ BRAYTON CYCLE: THE IDEAL CYCLE FOR GAS-TURBINE ENGINES

The Brayton cycle was first proposed by George Brayton for use in the reciprocating oil-burning engine that he developed around 1870. Today, it is used for gas turbines only where both the compression and expansion processes take place in rotating machinery. Gas turbines usually operate on an *open cycle,* as shown in Fig. 9–26. Fresh air at ambient conditions is drawn into the compressor, where its temperature and pressure are raised. The high-pressure air proceeds into the combustion chamber, where the fuel is burned at constant pressure. The resulting high-temperature gases then enter the turbine, where they expand to the atmospheric pressure while producing power. The exhaust gases leaving the turbine are thrown out (not recirculated), causing the cycle to be classified as an open cycle.

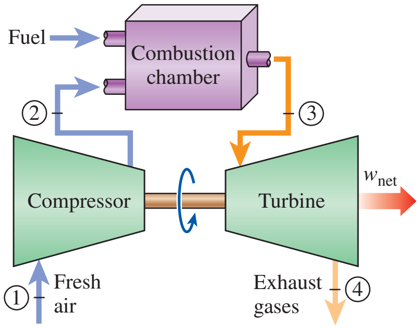


FIGURE 9–26
An open-cycle gas-turbine engine.

The open gas-turbine cycle just described can be modeled as a *closed cycle,* as shown in Fig. 9–27, by using the air-standard assumptions. Here the compression and expansion processes remain the same, but the combustion process is replaced by a constant-pressure heat-addition process from an external source, and the exhaust process is replaced by a constant-pressure heat-rejection process to the ambient air. The ideal cycle that the working fluid undergoes in this closed loop is the **Brayton cycle**, which is made up of four internally reversible processes:

1-2 Isentropic compression (in a compressor)
2-3 Constant-pressure heat addition
3-4 Isentropic expansion (in a turbine)
4-1 Constant-pressure heat rejection

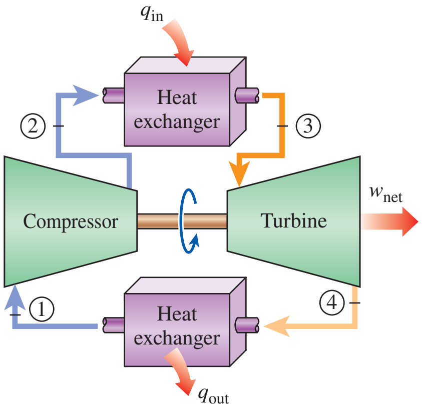


FIGURE 9–27
A closed-cycle gas-turbine engine.

The *T-s* and *P-v* diagrams of an ideal Brayton cycle are shown in Fig. 9–28. Notice that all four processes of the Brayton cycle are executed in steady-flow devices; thus, they should be analyzed as steady-flow processes. When the changes in kinetic and potential energies are neglected, the energy balance for a steady-flow process can be expressed, on a unit-mass basis, as

$$(q_{\text{in}} - q_{\text{out}}) + (w_{\text{in}} - w_{\text{out}}) = h_{\text{exit}} - h_{\text{inlet}} \tag{9–14}$$

Therefore, heat transfers to and from the working fluid are

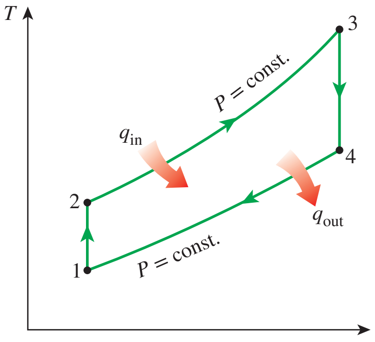
$$q_{\text{in}} = h_3 - h_2 = c_p(T_3 - T_2) \tag{9–15a}$$

and

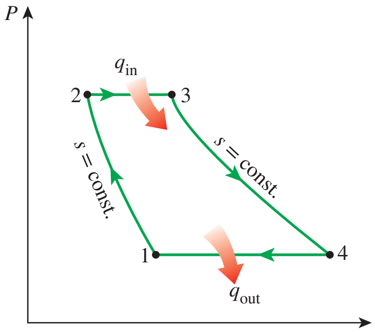
$$q_{\text{out}} = h_4 - h_1 = c_p(T_4 - T_1) \tag{9–15b}$$

Then the thermal efficiency of the ideal Brayton cycle under the cold-air-standard assumptions becomes

$$\eta_{\text{th,Brayton}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$



(*a*) *T-s* diagram



(*b*) *P*-υ diagram

FIGURE 9–28

T-s and *P*-υ diagrams for the ideal Brayton cycle.

Processes 1-2 and 3-4 are isentropic, and $P_2 = P_3$ and $P_4 = P_1$. Thus,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k} = \frac{T_3}{T_4}$$

Substituting these equations into the thermal efficiency relation and simplifying give

$$\eta_{\text{th,Brayton}} = 1 - \frac{1}{r_p^{(k-1)/k}} \tag{9–16}$$

where

$$r_p = \frac{P_2}{P_1} \tag{9–17}$$

is the **pressure ratio** and *k* is the specific heat ratio. Equation 9–16 shows that under the cold-air-standard assumptions, the thermal efficiency of an ideal Brayton cycle depends on the pressure ratio of the gas turbine and the specific heat ratio of the working fluid. The thermal efficiency increases with both of these parameters, which is also the case for actual gas turbines. A plot of thermal efficiency versus the pressure ratio is given in Fig. 9–29 for $k = 1.4$, which is the specific-heat-ratio value of air at room temperature.

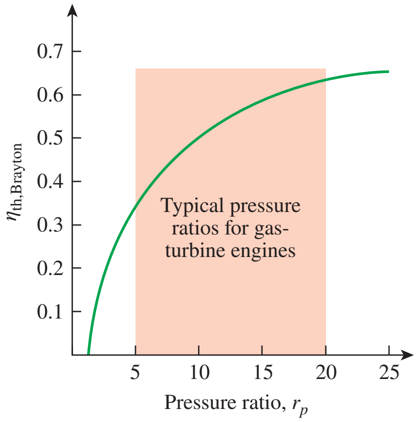


FIGURE 9–29

Thermal efficiency of the ideal Brayton cycle as a function of the pressure ratio.

The highest temperature in the cycle occurs at the end of the combustion process (state 3), and it is limited by the maximum temperature that the turbine blades can withstand. This also limits the pressure ratios that can be used in the cycle. For a fixed turbine inlet temperature T_3, the net work output per cycle increases with the pressure ratio, reaches a maximum, and then starts to decrease, as shown in Fig. 9–30. Therefore, there should be a compromise between the pressure ratio (thus the thermal efficiency) and the net work output. With less work output per cycle, a larger mass flow rate (thus a larger system) is needed to maintain the same power output, which may not be economical. In most common designs, the pressure ratio of gas turbines ranges from about 11 to 16.

The air in gas turbines performs two important functions: It supplies the necessary oxidant for the combustion of the fuel, and it serves as a coolant to keep the temperature of various components within safe limits. The second function is accomplished by drawing in more air than is needed for the complete combustion of the fuel. In gas turbines, an air–fuel mass ratio of 50 or above is not uncommon. Therefore, in a cycle analysis, treating the combustion gases as air does not cause any appreciable error. Also, the mass flow rate through the turbine is greater than that through the compressor, the difference being equal to the mass flow rate of the fuel. Thus, assuming a constant mass flow rate throughout the cycle yields conservative results for open-loop gas-turbine engines.

The two major application areas of gas-turbine engines are *aircraft propulsion* and *electric power generation.* When it is used for aircraft propulsion, the gas turbine produces just enough power to drive the compressor and a small generator to power the auxiliary equipment. The high-velocity exhaust gases are responsible for producing the necessary thrust to propel the aircraft. Gas turbines are also used as stationary power plants to generate electricity as stand-alone units or in conjunction with steam power plants on the high-temperature side. In these plants, the exhaust gases of the gas turbine serve as the heat source for the steam. The gas-turbine cycle can also be executed as a closed cycle for use in nuclear power plants. This time the working fluid is not limited to air, and a gas with more desirable characteristics (such as helium) can be used.

The majority of the Western world's naval fleets already use gas-turbine engines for propulsion and electric power generation. The General Electric LM2500 gas turbines used to power ships have a simple-cycle thermal efficiency of 37 percent. The General Electric WR-21 gas turbines equipped with intercooling and regeneration have a thermal efficiency of 43 percent and produce 21.6 MW (29,040 hp). The regeneration also reduces the exhaust temperature from 600°C (1100°F) to 350°C (650°F). Air is compressed to 3 atm before it enters the intercooler. Compared to steam-turbine and diesel-propulsion systems, the gas turbine offers greater power for a given size and weight, high reliability, long life, and more convenient operation. The engine start-up time has been reduced from 4 h required for a typical steam-propulsion system to less than 2 min for a gas turbine. Many modern marine propulsion systems use gas turbines together with diesel engines because of the high fuel consumption of simple-cycle gas-turbine engines. In combined diesel and gas-turbine systems, diesel is used to provide for efficient low-power and cruise operation, and gas turbine is used when high speeds are needed.

In gas-turbine power plants, the ratio of the compressor work to the turbine work, called the **back work ratio**, is very high (Fig. 9–31). Usually more than one-half of the turbine work output is used to drive the compressor. The situation is even worse when the isentropic efficiencies of the compressor and the turbine are low. This is quite in contrast to steam power plants, where the back work ratio is only a few percent. This is not surprising, however, since a liquid is compressed in steam power plants instead of a gas, and the steady-flow work is proportional to the specific volume of the working fluid.

A power plant with a high back work ratio requires a larger turbine to provide the additional power requirements of the compressor. Therefore, the turbines used in gas-turbine power plants are larger than those used in steam power plants of the same net power output.

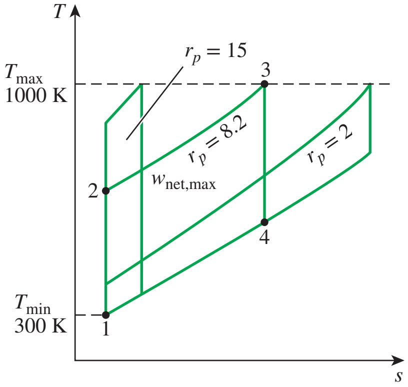


FIGURE 9–30

For fixed values of T_{min} and T_{max}, the net work of the Brayton cycle first increases with the pressure ratio, then reaches a maximum at $r_p = (T_{max}/T_{min})^{k/[2(k-1)]}$, and finally decreases.

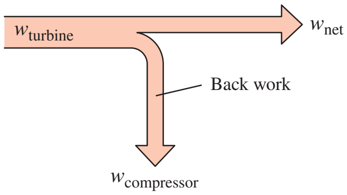


FIGURE 9–31

The fraction of the turbine work used to drive the compressor is called the back work ratio.

Development of Gas Turbines

The gas turbine has experienced phenomenal progress and growth since its first successful development in the 1930s. The early gas turbines built in the 1940s and even 1950s had simple-cycle efficiencies of about 17 percent because of the low compressor and turbine efficiencies and low turbine inlet temperatures due to metallurgical limitations of those times. Therefore, gas turbines found only limited use despite their versatility and their ability to burn a variety of fuels. The efforts to improve the cycle efficiency concentrated in three areas:

1. Increasing the turbine inlet (or firing) temperatures This has been the primary approach taken to improve gas-turbine efficiency. The turbine inlet temperatures have increased steadily from about 540°C (1000°F) in the 1940s to 1425°C (2600°F) and even higher today. These increases were made possible by the development of new materials and innovative cooling techniques for the critical components such as coating the turbine blades with ceramic layers and cooling the blades with the discharge air from the compressor. Maintaining high turbine inlet temperatures with an air-cooling technique requires the combustion temperature to be higher to compensate for the cooling effect of the cooling air. However, higher combustion temperatures increase the production of nitrogen oxides (NO_x), which are responsible for the formation of ozone at ground level and smog. Using steam as the coolant allowed an increase in the turbine inlet temperatures by 200°F without an increase in the combustion temperature. Steam is also a much more effective heat transfer medium than air.

2. Increasing the efficiencies of turbomachinery components The performance of early turbines suffered greatly from the inefficiencies of turbines and compressors. However, the advent of computers and advanced techniques for computer-aided design made it possible to design these components aerodynamically with minimal losses. The increased efficiencies of the turbines and compressors resulted in a significant increase in the cycle efficiency.

3. Adding modifications to the basic cycle The simple-cycle efficiencies of early gas turbines were practically doubled by incorporating intercooling, regeneration (or recuperation), and reheating, discussed in the next two sections. These improvements, of course, come at the expense of increased initial and operation costs, and they cannot be justified unless the decrease in fuel costs offsets the increase in other costs. Relatively low fuel prices, the general desire in the industry to minimize installation costs, and the tremendous increase in the simple-cycle efficiency to about 40 percent left little desire to make these modifications.

The first gas turbine for an electric utility was installed in 1949 in Oklahoma as part of a combined-cycle power plant. It was built by General Electric and produced 3.5 MW of power. Gas turbines installed until the mid-1970s suffered from low efficiency and poor reliability. In the past, base-load electric power generation was dominated by large coal and nuclear power plants. However, there has been an historic shift toward natural gas–fired gas turbines because of their higher efficiencies, lower capital costs, shorter installation times, and better emission characteristics, supported by an abundance of natural gas supplies, and more and more electric utilities are using gas turbines for base-load power production as well as for peaking. The construction costs for gas-turbine power plants are roughly half those of comparable conventional fossil-fuel steam power plants, which were the primary base-load power plants until the early 1980s. More than half of all power plants to be installed in the foreseeable future are forecast to be gas-turbine or combined gas–steam turbine types.

A gas turbine manufactured by General Electric in the early 1990s had a pressure ratio of 13.5 and generated 135.7 MW of net power at a thermal efficiency of 33 percent in simple-cycle operation. A more recent gas turbine manufactured by General Electric uses a turbine inlet temperature of 1425°C (2600°F) and produces up to 282 MW while achieving a thermal efficiency of 39.5 percent in the simple-cycle mode. A 1.3-ton small-scale gas turbine labeled OP-16, built by the Dutch firm Opra Optimal Radial Turbine, can run on gas or liquid fuel and can replace a 16-ton diesel engine. It has a pressure ratio of 6.5 and produces up to 2 MW of power. Its efficiency is 26 percent in the simple-cycle operation, which rises to 37 percent when equipped with a regenerator. The most recent simple-cycle-gas turbine power plants have a thermal efficiency of up to 44 percent, with a single unit producing over 500 MW of net power.

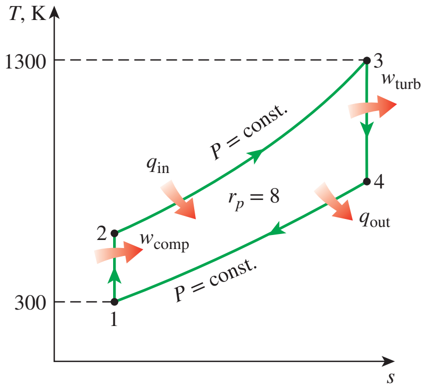


FIGURE 9–32
T-s diagram for the Brayton cycle discussed in Example 9–5.

EXAMPLE 9–5 The Simple Ideal Brayton Cycle

A gas-turbine power plant operating on an ideal Brayton cycle has a pressure ratio of 8. The gas temperature is 300 K at the compressor inlet and 1300 K at the turbine inlet. Using the air-standard assumptions, determine (*a*) the gas temperature at the exits of the compressor and the turbine, (*b*) the back work ratio, and (*c*) the thermal efficiency.

SOLUTION A power plant operating on the ideal Brayton cycle is considered. The compressor and turbine exit temperatures, back work ratio, and thermal efficiency are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** The variation of specific heats with temperature is to be considered.

Analysis The *T-s* diagram of the ideal Brayton cycle described is shown in Fig. 9–32. We note that the components involved in the Brayton cycle are steady-flow devices.

(*a*) The air temperatures at the compressor and turbine exits are determined from isentropic relations:

Process 1–2 (isentropic compression of an ideal gas):

$$T_1 = 300\text{ K} \rightarrow h_1 = 300.19\text{ kJ/kg}$$
$$P_{r1} = 1.386$$
$$P_{r2} = \frac{P_2}{P_1} P_{r1} = (8)(1.386) = 11.09 \rightarrow T_2 = \mathbf{540\ K} \qquad \text{(at compressor exit)}$$
$$h_2 = 544.35\text{ kJ/kg}$$

Process 3–4 (isentropic expansion of an ideal gas):

$$T_3 = 1300\text{ K} \rightarrow h_3 = 1395.97\text{ kJ/kg}$$
$$P_{r3} = 330.9$$
$$P_{r4} = \frac{P_4}{P_3} P_{r3} = \left(\frac{1}{8}\right)(330.9) = 41.36 \rightarrow T_4 = \mathbf{770\ K} \qquad \text{(at turbine exit)}$$
$$h_4 = 789.37\text{ kJ/kg}$$

(*b*) To find the back work ratio, we need to find the work input to the compressor and the work output of the turbine:

$$w_{\text{comp,in}} = h_2 - h_1 = 544.35 - 300.19 = 244.16\text{ kJ/kg}$$
$$w_{\text{turb,out}} = h_3 - h_4 = 1395.97 - 789.37 = 606.60\text{ kJ/kg}$$

Thus,

$$r_{\text{bw}} = \frac{w_{\text{comp,in}}}{w_{\text{turb,out}}} = \frac{244.16\text{ kJ/kg}}{606.60\text{ kJ/kg}} = \mathbf{0.403}$$

That is, 40.3 percent of the turbine work output is used just to drive the compressor.

(*c*) The thermal efficiency of the cycle is the ratio of the net power output to the total heat input:

$$q_{\text{in}} = h_3 - h_2 = 1395.97 - 544.35 = 851.62\text{ kJ/kg}$$
$$w_{\text{net}} = w_{\text{out}} - w_{\text{in}} = 606.60 - 244.16 = 362.4\text{ kJ/kg}$$

Thus,

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{362.4\text{ kJ/kg}}{851.62\text{ kJ/kg}} = 0.426 \quad \text{or} \quad \mathbf{42.6\%}$$

The thermal efficiency could also be determined from

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}}$$

where

$$q_{\text{out}} = h_4 - h_1 = 789.37 - 300.19 = 489.2\text{ kJ/kg}$$

Discussion Under the cold-air-standard assumptions (constant specific heat values at room temperature), the thermal efficiency would be, from Eq. 9–16,

$$\eta_{\text{th,Brayton}} = 1 - \frac{1}{r_p^{(k-1)/k}} = 1 - \frac{1}{8^{(1.4-1)/1.4}} = 0.448 \quad \text{or} \quad 44.8\%$$

which is sufficiently close to the value obtained by accounting for the variation of specific heats with temperature.

Deviation of Actual Gas-Turbine Cycles from Idealized Ones

The actual gas-turbine cycle differs from the ideal Brayton cycle on several accounts. For one thing, some pressure drop during the heat-addition and heat-rejection processes is inevitable. More importantly, the actual work input to the compressor is more, and the actual work output from the turbine is less because of irreversibilities. The deviation of actual compressor and turbine behavior from the idealized isentropic behavior can be accurately accounted for by utilizing the isentropic efficiencies of the turbine and compressor as

$$\eta_C = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \tag{9–18}$$

and

$$\eta_T = \frac{w_a}{w_s} \cong \frac{h_3 - h_{4a}}{h_3 - h_{4s}} \tag{9–19}$$

where states 2*a* and 4*a* are the actual exit states of the compressor and the turbine, respectively, and 2*s* and 4*s* are the corresponding states for the isentropic case, as illustrated in Fig. 9–33. The effect of the turbine and compressor efficiencies on the thermal efficiency of gas-turbine engines is illustrated in Example 9–6.

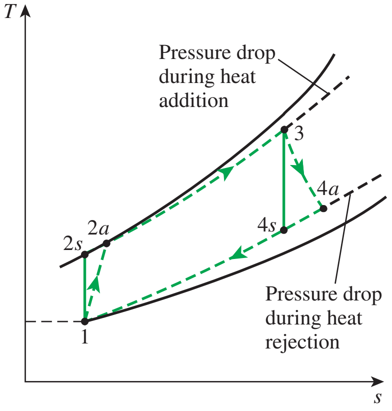


FIGURE 9–33

The deviation of an actual gas-turbine cycle from the ideal Brayton cycle as a result of irreversibilities.

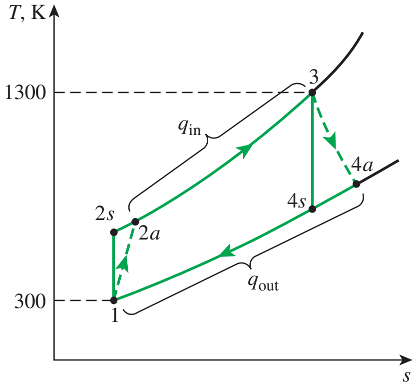


FIGURE 9–34

T-s diagram of the gas-turbine cycle discussed in Example 9–6.

EXAMPLE 9–6 An Actual Gas-Turbine Cycle

Assuming a compressor efficiency of 80 percent and a turbine efficiency of 85 percent, determine (*a*) the back work ratio, (*b*) the thermal efficiency, and (*c*) the turbine exit temperature of the gas-turbine cycle discussed in Example 9–5.

SOLUTION The Brayton cycle discussed in Example 9–5 is reconsidered. For specified turbine and compressor efficiencies, the back work ratio, the thermal efficiency, and the turbine exit temperature are to be determined.

Analysis (*a*) The *T-s* diagram of the cycle is shown in Fig. 9–34. The actual compressor work and turbine work are determined by using the definitions of compressor and turbine efficiencies, Eqs. 9–18 and 9–19:

Compressor: $$w_{\text{comp,in}} = \frac{w_s}{\eta_C} = \frac{244.16 \text{ kJ/kg}}{0.80} = 305.20 \text{ kJ/kg}$$

Turbine: $$w_{\text{turb,out}} = \eta_T w_s = (0.85)(606.60 \text{ kJ/kg}) = 515.61 \text{ kJ/kg}$$

Thus,

$$r_{\text{bw}} = \frac{w_{\text{comp,in}}}{w_{\text{turb,out}}} = \frac{305.20 \text{ kJ/kg}}{515.61 \text{ kJ/kg}} = \mathbf{0.592}$$

That is, the compressor is now consuming 59.2 percent of the work produced by the turbine (up from 40.3 percent). This increase is due to the irreversibilities that occur within the compressor and the turbine.

(*b*) In this case, air leaves the compressor at a higher temperature and enthalpy, which are determined to be

$$\begin{aligned} w_{\text{comp,in}} = h_{2a} - h_1 \rightarrow h_{2a} &= h_1 + w_{\text{comp,in}} \\ &= 300.19 + 305.20 \\ &= 605.39 \text{ kJ/kg} \qquad (\text{and } T_{2a} = 598 \text{ K}) \end{aligned}$$

Thus,

$$q_{\text{in}} = h_3 - h_{2a} = 1395.97 - 605.39 = 790.58 \text{ kJ/kg}$$
$$w_{\text{net}} = w_{\text{out}} - w_{\text{in}} = 515.61 - 305.20 = 210.41 \text{ kJ/kg}$$

and

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{210.41 \text{ kJ/kg}}{790.58 \text{ kJ/kg}} = 0.266 \quad \text{or} \quad \mathbf{26.6\%}$$

That is, the irreversibilities occurring within the turbine and compressor caused the thermal efficiency of the gas-turbine cycle to drop from 42.6 to 26.6 percent. This example shows how sensitive the performance of a gas-turbine power plant is to the efficiencies of the compressor and the turbine. In fact, gas-turbine efficiencies did not reach competitive values until significant improvements were made in the design of gas turbines and compressors.

(*c*) The air temperature at the turbine exit is determined from an energy balance on the turbine:

$$\begin{aligned} w_{\text{turb,out}} = h_3 - h_{4a} \rightarrow h_{4a} &= h_3 - w_{\text{turb,out}} \\ &= 1395.97 - 515.61 \\ &= 880.36 \text{ kJ/kg} \end{aligned}$$

Then, from Table A–21,

$$T_{4a} = \mathbf{853 \text{ K}}$$

Discussion The temperature at turbine exit is considerably higher than that at the compressor exit (T_{2a} = 598 K), which suggests the use of regeneration to reduce fuel cost.

9–8 ▪ THE BRAYTON CYCLE WITH REGENERATION

In gas-turbine engines, the temperature of the exhaust gas leaving the turbine is often considerably higher than the temperature of the air leaving the compressor. Therefore, the high-pressure air leaving the compressor can be heated by transferring heat to it from the hot exhaust gases in a counterflow heat exchanger, which is also known as a *regenerator* or a *recuperator.* A sketch of the gas-turbine engine utilizing a regenerator and the *T-s* diagram of the new cycle are shown in Figs. 9–35 and 9–36, respectively.

The thermal efficiency of the Brayton cycle increases as a result of regeneration since the portion of energy of the exhaust gases that is normally rejected to the surroundings is now used to preheat the air entering the combustion chamber. This, in turn, decreases the heat input (thus fuel) requirements for the same net work output. Note, however, that the use of a regenerator is recommended only when the turbine exhaust temperature is higher than the compressor exit temperature.

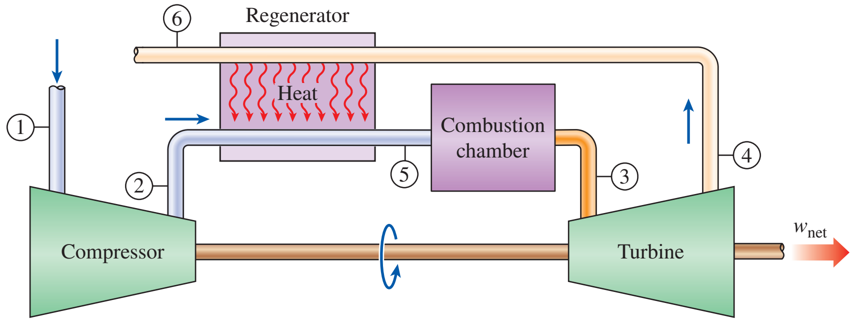


FIGURE 9–35
A gas-turbine engine with regenerator.

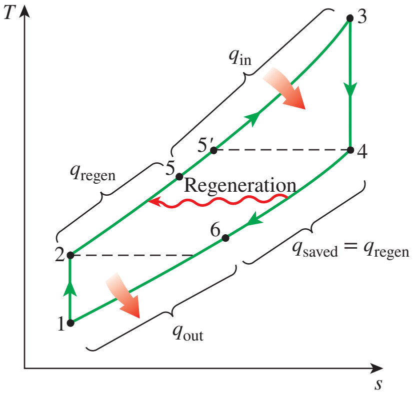


FIGURE 9–36
T-s diagram of a Brayton cycle with regeneration.

Otherwise, heat will flow in the reverse direction (*to* the exhaust gases), decreasing the efficiency. This situation is encountered in gas-turbine engines operating at very high pressure ratios.

The highest temperature occurring within the regenerator is T_4, the temperature of the exhaust gases leaving the turbine and entering the regenerator. Under no conditions can the air be preheated in the regenerator to a temperature above this value. Air normally leaves the regenerator at a lower temperature, T_5. In the limiting (ideal) case, the air exits the regenerator at the inlet temperature of the exhaust gases T_4. Assuming the regenerator to be well insulated and any changes in kinetic and potential energies to be negligible, the actual and maximum heat transfers from the exhaust gases to the air can be expressed as

$$q_{\text{regen,act}} = h_5 - h_2 \qquad \text{(9–20)}$$

and

$$q_{\text{regen,max}} = h_{5'} - h_2 = h_4 - h_2 \qquad \text{(9–21)}$$

The extent to which a regenerator approaches an ideal regenerator is called the **effectiveness** ϵ and is defined as

$$\epsilon = \frac{q_{\text{regen,act}}}{q_{\text{regen,max}}} = \frac{h_5 - h_2}{h_4 - h_2} \qquad \text{(9–22)}$$

When the cold-air-standard assumptions are utilized, it reduces to

$$\epsilon \cong \frac{T_5 - T_2}{T_4 - T_2} \qquad \text{(9–23)}$$

A regenerator with a higher effectiveness obviously saves a greater amount of fuel since it preheats the air to a higher temperature prior to combustion. However, achieving a higher effectiveness requires the use of a larger regenerator, which carries a higher price tag and causes a larger pressure drop. Therefore, the use of a regenerator with a very high effectiveness cannot be justified economically unless the savings from the fuel costs exceed the additional expenses involved. The effectiveness of most regenerators used in practice is below 0.85.

Under the cold-air-standard assumptions, the thermal efficiency of an ideal Brayton cycle with regeneration is

$$\eta_{\text{th,regen}} = 1 - \left(\frac{T_1}{T_3}\right)(r_p)^{(k-1)/k} \qquad \text{(9–24)}$$

Therefore, the thermal efficiency of an ideal Brayton cycle with regeneration depends on the ratio of the minimum to maximum temperatures as well as the pressure ratio. The thermal efficiency is plotted in Fig. 9–37 for various pressure ratios and minimum-to-maximum temperature ratios. This figure shows that regeneration is most effective at lower pressure ratios and low minimum-to-maximum temperature ratios.

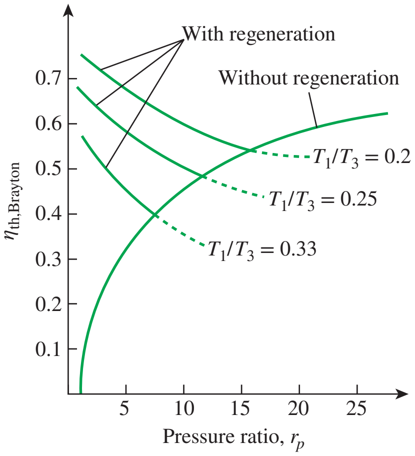


FIGURE 9–37
Thermal efficiency of the ideal Brayton cycle with and without regeneration.

EXAMPLE 9–7 Actual Gas-Turbine Cycle with Regeneration

Determine the thermal efficiency of the gas turbine described in Example 9–6 if a regenerator having an effectiveness of 80 percent is installed.

SOLUTION The gas turbine discussed in Example 9–6 is equipped with a regenerator. For a specified effectiveness, the thermal efficiency is to be determined.

Analysis The *T-s* diagram of the cycle is shown in Fig. 9–38. We first determine the enthalpy of the air at the exit of the regenerator, using the definition of effectiveness:

$$\epsilon = \frac{h_5 - h_{2a}}{h_{4a} - h_{2a}}$$

$$0.80 = \frac{(h_5 - 605.39)\ \text{kJ/kg}}{(880.36 - 605.39)\ \text{kJ/kg}} \rightarrow h_5 = 825.37\ \text{kJ/kg}$$

Thus,

$$q_{\text{in}} = h_3 - h_5 = (1395.97 - 825.37)\ \text{kJ/kg} = 570.60\ \text{kJ/kg}$$

This represents a savings of 220.0 kJ/kg from the heat input requirements. The addition of a regenerator (assumed to be frictionless) does not affect the net work output. Thus,

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{210.41\ \text{kJ/kg}}{570.60\ \text{kJ/kg}} = 0.369 \quad \text{or} \quad \mathbf{36.9\%}$$

Discussion Note that the thermal efficiency of the gas turbine has gone up from 26.6 to 36.9 percent as a result of installing a regenerator that helps to recover some of the thermal energy of the exhaust gases.

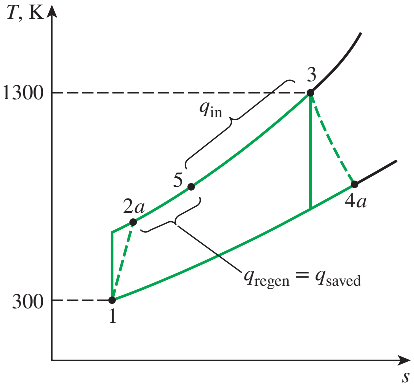


FIGURE 9–38
T-s diagram of the regenerative Brayton cycle described in Example 9–7.

9–9 ▪ THE CARNOT VAPOR CYCLE

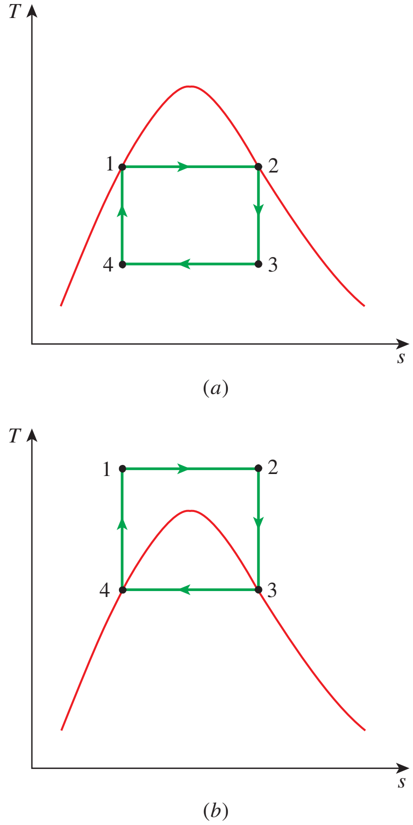
We have mentioned repeatedly that the Carnot cycle is the most efficient cycle operating between two specified temperature limits. Thus it is natural to look at the Carnot cycle first as a prospective ideal cycle for vapor power plants. If we could, we would certainly adopt it as the ideal cycle. As will be explained, however, the Carnot cycle is not a suitable model for power cycles. Throughout the discussions, we assume *steam* to be the working fluid since it is the working fluid predominantly used in vapor power cycles.

Consider a steady-flow *Carnot cycle* executed within the saturation dome of a pure substance, as shown in Fig. 9–39*a*. The fluid is heated reversibly and isothermally in a boiler (process 1-2), expanded isentropically in a turbine (process 2-3), condensed reversibly and isothermally in a condenser (process 3-4), and compressed isentropically by a compressor to the initial state (process 4-1).

Several impracticalities are associated with this cycle:

1. Isothermal heat transfer to or from a two-phase system is not difficult to achieve in practice since maintaining a constant pressure in the device automatically fixes the temperature at the saturation value. Therefore, processes 1-2 and 3-4 can be approached closely in actual boilers and condensers. Limiting the heat transfer processes to two-phase systems, however, severely limits the maximum temperature that can be used in the cycle (it has to remain under the critical-point value, which is 374°C for water). Limiting the maximum temperature in the cycle also limits the thermal efficiency. Any attempt to raise the maximum temperature in the cycle involves heat transfer to the working fluid in a single phase, which is not easy to accomplish isothermally.

2. The isentropic expansion process (process 2-3) can be approximated closely by a well-designed turbine. However, the quality of the steam decreases during this process, as shown on the *T-s* diagram in Fig. 9–39*a*. Thus the turbine has to handle steam with low quality, that is, steam with a high moisture content. The impingement of liquid droplets on the turbine blades causes erosion and is a major source of wear. Thus steam with qualities less than about 90 percent cannot be tolerated in the operation of power plants. This problem could be eliminated by using a working fluid with a very steep saturated vapor line.



(*a*)

(*b*)

FIGURE 9–39
T-s diagram of two Carnot vapor cycles.

3. The isentropic compression process (process 4-1) involves the compression of a liquid–vapor mixture to a saturated liquid. There are two difficulties associated with this process. First, it is not easy to control the condensation process so precisely as to end up with the desired quality at state 4. Second, it is not practical to design a compressor that handles two phases.

Some of these problems could be eliminated by executing the Carnot cycle in a different way, as shown in Fig. 9–39*b*. This cycle, however, presents other problems such as isentropic compression to extremely high pressures and isothermal heat transfer at variable pressures. Thus we conclude that the Carnot cycle cannot be approximated in actual devices and is not a realistic model for vapor power cycles.

9–10 ▪ RANKINE CYCLE: THE IDEAL CYCLE FOR VAPOR POWER CYCLES

Many of the impracticalities associated with the Carnot cycle can be eliminated by superheating the steam in the boiler and condensing it completely in the condenser, as shown schematically on a *T-s* diagram in Fig. 9–40. The cycle that results is the **Rankine cycle**, which is the ideal cycle for vapor power plants. The ideal Rankine cycle does not involve any internal irreversibilities and consists of the following four processes:

1-2 Isentropic compression in a pump
2-3 Constant-pressure heat addition in a boiler
3-4 Isentropic expansion in a turbine
4-1 Constant-pressure heat rejection in a condenser

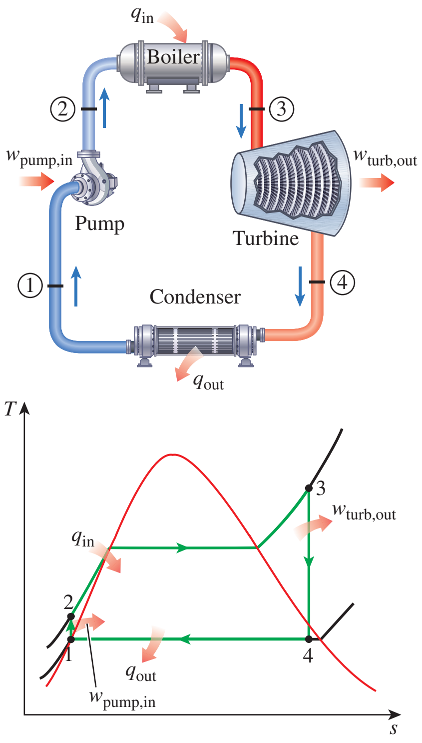


FIGURE 9–40
The simple ideal Rankine cycle.

Water enters the *pump* at state 1 as saturated liquid and is compressed isentropically to the operating pressure of the boiler. The water temperature increases somewhat during this isentropic compression process due to a slight decrease in the specific volume of water. The vertical distance between states 1 and 2 on the *T-s* diagram is greatly exaggerated for clarity. (If water were truly incompressible, would there be a temperature change at all during this process?)

Water enters the *boiler* as a compressed liquid at state 2 and leaves as a superheated vapor at state 3. The boiler is basically a large heat exchanger where the heat originating from combustion gases, nuclear reactors, or other sources is transferred to the water essentially at constant pressure. The boiler, together with the section where the steam is superheated (the super-heater), is often called the *steam generator.*

The superheated vapor at state 3 enters the *turbine,* where it expands isentropically and produces work by rotating the shaft connected to an electric generator. The pressure and the temperature of steam drop during this process to the values at state 4, where steam enters the *condenser.* At this state, steam is usually a saturated liquid–vapor mixture with a high quality. Steam is condensed at constant pressure in the condenser, which is basically a large heat exchanger, by rejecting heat to a cooling medium such as a lake, a river, or the atmosphere. Steam leaves the condenser as saturated liquid and enters the pump, completing the cycle. In areas where water is precious, the power plants are cooled by air instead of water. This method of cooling, which is also used in car engines, is called *dry cooling.* Several power plants in the world, including some in the United States, use dry cooling to conserve water.

Remembering that the area under the process curve on a *T-s* diagram represents the heat transfer for internally reversible processes, we see that the area under process curve 2-3 represents the heat transferred to the water in the boiler and the area under the process curve 4-1 represents the heat rejected in the condenser. The difference between these two (the area enclosed by the cycle curve) is the net work produced during the cycle.

Energy Analysis of the Ideal Rankine Cycle

All four components associated with the Rankine cycle (the pump, boiler, turbine, and condenser) are steady-flow devices, and thus all four processes that make up the Rankine cycle can be analyzed as steady-flow processes. The kinetic and potential energy changes of the steam are usually small relative to the work and heat transfer terms and are therefore usually neglected. Then the *steady-flow energy equation* per unit mass of steam reduces to

$$(q_{\text{in}} - q_{\text{out}}) + (w_{\text{in}} - w_{\text{out}}) = h_e - h_i \qquad \text{(kJ/kg)} \tag{9–25}$$

The boiler and the condenser do not involve any work, and the pump and the turbine are assumed to be isentropic. Then the conservation of energy relation for each device can be expressed as follows:

Pump ($q = 0$):
$$w_{\text{pump,in}} = h_2 - h_1 \tag{9–26}$$

or,

$$w_{\text{pump,in}} = \upsilon(P_2 - P_1) \tag{9–27}$$

where

$$h_1 = h_{f\,@\,P_1} \quad \text{and} \quad \upsilon \cong \upsilon_1 = \upsilon_{f\,@\,P_1} \tag{9–28}$$

Boiler ($w = 0$):
$$q_{\text{in}} = h_3 - h_2 \tag{9–29}$$

Turbine ($q = 0$):
$$w_{\text{turb,out}} = h_3 - h_4 \tag{9–30}$$

Condenser ($w = 0$):
$$q_{\text{out}} = h_4 - h_1 \tag{9–31}$$

The *thermal efficiency* of the Rankine cycle is determined from

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} \tag{9–32}$$

where

$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = w_{\text{turb,out}} - w_{\text{pump,in}}$$

The conversion efficiency of power plants in the United States is often expressed in terms of **heat rate**, which is the amount of heat supplied, in Btu's, to generate 1 kWh of electricity. The smaller the heat rate, the greater the efficiency. Considering that 1 kWh = 3412 Btu and disregarding the losses associated with the conversion of shaft power to electric power, the relation between the heat rate and the thermal efficiency can be expressed as

$$\eta_{\text{th}} = \frac{3412\ \text{(Btu/kWh)}}{\text{Heat rate (Btu/kWh)}} \tag{9–33}$$

For example, a heat rate of 11,363 Btu/kWh is equivalent to 30 percent efficiency.

The thermal efficiency can also be interpreted as the ratio of the area enclosed by the cycle on a *T-s* diagram to the area under the heat-addition process. The use of these relations is illustrated in the following example.

EXAMPLE 9–8 The Simple Ideal Rankine Cycle

Consider a steam power plant operating on the simple ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350°C and is condensed in the condenser at a pressure of 75 kPa. Determine the thermal efficiency of this cycle.

SOLUTION A steam power plant operating on the simple ideal Rankine cycle is considered. The thermal efficiency of the cycle is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis The schematic of the power plant and the *T-s* diagram of the cycle are shown in Fig. 9–41. We note that the power plant operates on the ideal Rankine cycle. Therefore, the pump and the turbine are isentropic, there are no pressure drops in the boiler and condenser, and steam leaves the condenser and enters the pump as saturated liquid at the condenser pressure.

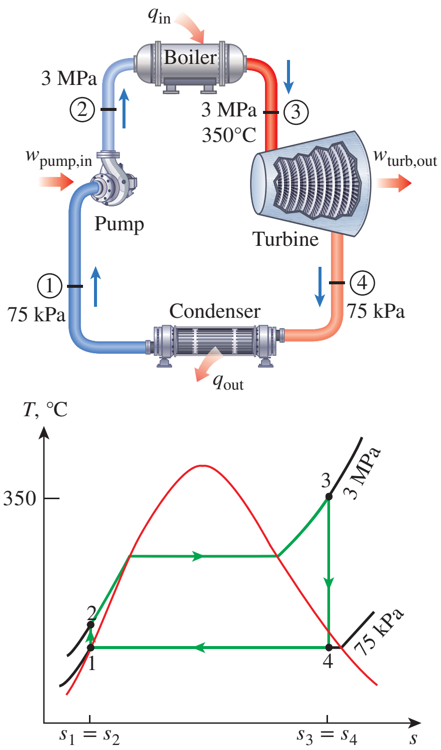


FIGURE 9–41
Schematic and *T-s* diagram for Example 9–8.

First we determine the enthalpies at various points in the cycle, using data from steam tables (Tables A–4, A–5, and A–6):

State 1:

$$\left.\begin{array}{l} P_1 = 75\text{ kPa} \\ \text{Sat. liquid} \end{array}\right\} \begin{array}{l} h_1 = h_{f\,@\,75\text{ kPa}} = 384.44\text{ kJ/kg} \\ v_1 = v_{f\,@\,75\text{ kPa}} = 0.001037\text{ m}^3\text{/kg} \end{array}$$

State 2: $P_2 = 3$ MPa

$$s_2 = s_1$$

$$w_{\text{pump,in}} = v_1(P_2 - P_1) = (0.001037\text{ m}^3\text{/kg})[(3000 - 75)\text{ kPa}]\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right)$$

$$= 3.03\text{ kJ/kg}$$

$$h_2 = h_1 + w_{\text{pump,in}} = (384.44 + 3.03)\text{ kJ/kg} = 387.47\text{ kJ/kg}$$

State 3:

$$\left.\begin{array}{l} P_3 = 3\text{ MPa} \\ T_3 = 350°\text{C} \end{array}\right\} \begin{array}{l} h_3 = 3116.1\text{ kJ/kg} \\ s_3 = 6.7450\text{ kJ/kg}\cdot\text{K} \end{array}$$

State 4: $P_4 = 75$ kPa (sat. mixture)

$$s_4 = s_3$$

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.7450 - 1.2132}{6.2426} = 0.8861$$

$$h_4 = h_f + x_4 h_{fg} = 384.44 + 0.8861(2278.0) = 2403.0\text{ kJ/kg}$$

Thus,

$$q_{\text{in}} = h_3 - h_2 = (3116.1 - 387.47)\text{ kJ/kg} = 2728.6\text{ kJ/kg}$$
$$q_{\text{out}} = h_4 - h_1 = (2403.0 - 384.44)\text{ kJ/kg} = 2018.6\text{ kJ/kg}$$

and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{2018.6\text{ kJ/kg}}{2728.6\text{ kJ/kg}} = 0.260\text{ or }\mathbf{26.0\%}$$

The thermal efficiency could also be determined from

$$w_{\text{turb,out}} = h_3 - h_4 = (3116.1 - 2403.0)\text{ kJ/kg} = 713.1\text{ kJ/kg}$$
$$w_{\text{net}} = w_{\text{turb,out}} - w_{\text{pump,in}} = (713.1 - 3.03)\text{ kJ/kg} = 710.1\text{ kJ/kg}$$

or

$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = (2728.6 - 2018.6)\text{ kJ/kg} = 710.0\text{ kJ/kg}$$

and

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{710.0\text{ kJ/kg}}{2728.6\text{ kJ/kg}} = 0.260\text{ or }\mathbf{26.0\%}$$

That is, this power plant converts 26 percent of the heat it receives in the boiler to net work. An actual power plant operating between the same temperature and pressure limits will have a lower efficiency because of the irreversibilities such as friction.

Discussion Notice that the back work ratio ($r_{bw} = w_{in}/w_{out}$) of this power plant is 0.004, and thus only 0.4 percent of the turbine work output is required to operate the pump. Having such low back work ratios is characteristic of vapor power cycles. This is in contrast to the gas power cycles, which typically involve very high back work ratios (about 40 to 80 percent).

It is also interesting to note the thermal efficiency of a Carnot cycle operating between the same temperature limits

$$\eta_{\text{th,Carnot}} = 1 - \frac{T_{\min}}{T_{\max}} = 1 - \frac{(91.76 + 273)\text{ K}}{(350 + 273)\text{ K}} = 0.415$$

Here $T_{\min}$ is taken as the saturation temperature of water at 75 kPa. The difference between the two efficiencies is due to the large external irreversibility in the Rankine cycle caused by the large temperature difference between steam and the heat source.

9–11 ▪ DEVIATION OF ACTUAL VAPOR POWER CYCLES FROM IDEALIZED ONES

The actual vapor power cycle differs from the ideal Rankine cycle, as illustrated in Fig. 9–42*a,* as a result of irreversibilities in various components. Fluid friction and heat loss to the surroundings are the two common sources of irreversibilities.

Fluid friction causes pressure drops in the boiler, the condenser, and the piping between various components. As a result, steam leaves the boiler at a somewhat lower pressure. Also, the pressure at the turbine inlet is somewhat lower than that at the boiler exit due to the pressure drop in the connecting pipes. The pressure drop in the condenser is usually very small. To compensate for these pressure drops, the water must be pumped to a sufficiently higher pressure than the ideal cycle calls for. This requires a larger pump and larger work input to the pump.

The other major source of irreversibility is the *heat loss* from the steam to the surroundings as the steam flows through various components. To maintain the same level of net work output, more heat needs to be transferred to the steam in the boiler to compensate for these undesired heat losses. As a result, cycle efficiency decreases.

Of particular importance are the irreversibilities occurring within the pump and the turbine. A pump requires a greater work input, and a turbine produces a smaller work output as a result of irreversibilities. Under ideal conditions, the flow through these devices is isentropic. The deviation of actual pumps and turbines from the isentropic ones can be accounted for by utilizing *isentropic efficiencies,* defined as

$$\eta_P = \frac{w_s}{w_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1} \tag{9–34}$$

and

$$\eta_T = \frac{w_a}{w_s} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}} \tag{9–35}$$

where states 2*a* and 4*a* are the actual exit states of the pump and the turbine, respectively, and 2*s* and 4*s* are the corresponding states for the isentropic case (Fig. 9-42*b*).

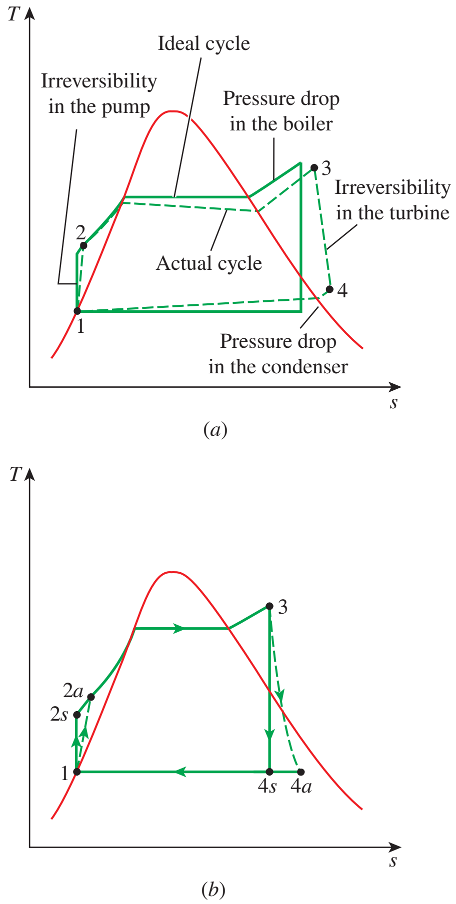


FIGURE 9–42
(*a*) Deviation of actual vapor power cycle from the ideal Rankine cycle. (*b*) The effect of pump and turbine irreversibilities on the ideal Rankine cycle.

Other factors also need to be considered in the analysis of actual vapor power cycles. In actual condensers, for example, the liquid is usually subcooled to prevent the onset of *cavitation,* the rapid vaporization and condensation of the fluid at the low-pressure side of the pump impeller, which may damage it. Additional losses occur at the bearings between the moving parts as a result of friction. Steam that leaks out during the cycle and air that leaks into the condenser represent two other sources of loss. Finally, the power consumed by the auxiliary equipment such as fans that supply air to the furnace should also be considered in evaluating the overall performance of power plants.

The effect of irreversibilities on the thermal efficiency of a steam power cycle is illustrated next with an example.

EXAMPLE 9–9 An Actual Steam Power Cycle

A steam power plant operates on the cycle shown in Fig. 9–43. If the isentropic efficiency of the turbine is 87 percent and the isentropic efficiency of the pump is 85 percent, determine (*a*) the thermal efficiency of the cycle and (*b*) the net power output of the plant for a mass flow rate of 15 kg/s.

SOLUTION A steam power cycle with specified turbine and pump efficiencies is considered. The thermal efficiency and the net power output are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis The schematic of the power plant and the *T-s* diagram of the cycle are shown in Fig. 9–43. The temperatures and pressures of steam at various points are also indicated on the figure. We note that the power plant involves steady-flow components and operates on the Rankine cycle, but the imperfections at various components are accounted for.

(*a*) The thermal efficiency of a cycle is the ratio of the net work output to the heat input, and it is determined as follows:

Pump work input:

$$
\begin{aligned}
w_{\text{pump,in}} &= \frac{w_{s,\text{pump,in}}}{\eta_P} = \frac{\text{v}_1(P_2 - P_1)}{\eta_P} \\
&= \frac{(0.001009\ \text{m}^3/\text{kg})[(16{,}000 - 9)\ \text{kPa}]}{0.85}\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right) \\
&= 19.0\ \text{kJ/kg}
\end{aligned}
$$

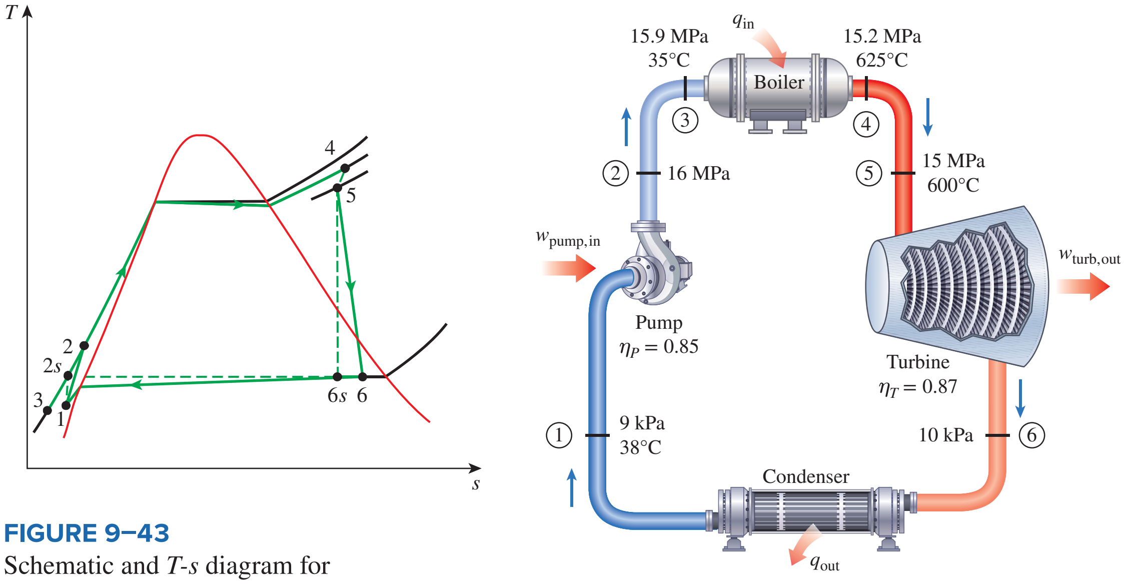


FIGURE 9–43
Schematic and *T-s* diagram for Example 9–9.

Turbine work output:

$$\begin{aligned} w_{\text{turb,out}} &= \eta_T w_{s,\text{turb,out}} \\ &= \eta_T(h_5 - h_{6s}) = 0.87(3583.1 - 2115.3)\ \text{kJ/kg} \\ &= 1277.0\ \text{kJ/kg} \end{aligned}$$

Boiler heat input: $q_{\text{in}} = h_4 - h_3 = (3647.6 - 160.1)\ \text{kJ/kg} = 3487.5\ \text{kJ/kg}$

Thus,

$$w_{\text{net}} = w_{\text{turb,out}} - w_{\text{pump,in}} = (1277.0 - 19.0)\ \text{kJ/kg} = 1258.0\ \text{kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1258.0\ \text{kJ/kg}}{3487.5\ \text{kJ/kg}} = 0.361 \text{ or } \mathbf{36.1\%}$$

(*b*) The power produced by this power plant is

$$\dot{W}_{\text{net}} = \dot{m}\, w_{\text{net}} = (15\ \text{kg/s})(1258.0\ \text{kJ/kg}) = \mathbf{18.9\ MW}$$

Discussion Without the irreversibilities, the thermal efficiency of this cycle would be 43.0 percent (see Example 9–10*c*).

9–12 ▪ HOW CAN WE INCREASE THE EFFICIENCY OF THE RANKINE CYCLE?

Steam power plants are responsible for the production of most electric power in the world, and even small increases in thermal efficiency can mean large savings from the fuel requirements. Therefore, every effort is made to improve the efficiency of the cycle on which steam power plants operate.

The basic idea behind all the modifications to increase the thermal efficiency of a power cycle is the same: *Increase the average temperature at which heat is transferred to the working fluid in the boiler, or decrease the average temperature at which heat is rejected from the working fluid in the condenser.* That is, the average fluid temperature should be as high as possible during heat addition and as low as possible during heat rejection. Next we discuss three ways of accomplishing this for the simple ideal Rankine cycle.

Lowering the Condenser Pressure (*Lowers $T_{\text{low,avg}}$*)

Steam exists as a saturated mixture in the condenser at the saturation temperature corresponding to the pressure inside the condenser. Therefore, lowering the operating pressure of the condenser automatically lowers the temperature of the steam, and thus the temperature at which heat is rejected.

The effect of lowering the condenser pressure on the Rankine cycle efficiency is illustrated on a *T-s* diagram in Fig. 9–44. For comparison purposes, the turbine inlet state is maintained the same. The colored area on this diagram represents the increase in net work output as a result of lowering the condenser pressure from P_4 to $P_{4'}$. The heat input requirements also increase (represented by the area under curve 2′–2), but this increase is very small. Thus the overall effect of lowering the condenser pressure is an increase in the thermal efficiency of the cycle.

To take advantage of the increased efficiencies at low pressures, the condensers of steam power plants usually operate well below the atmospheric pressure.

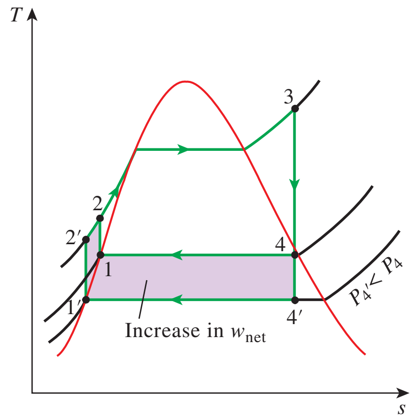


FIGURE 9–44
The effect of lowering the condenser pressure on the ideal Rankine cycle.

This does not present a major problem since the vapor power cycles operate in a closed loop. However, there is a lower limit on the condenser pressure that can be used. It cannot be lower than the saturation pressure corresponding to the temperature of the cooling medium. Consider, for example, a condenser that is to be cooled by a nearby river at 15°C. Allowing a temperature difference of 10°C for effective heat transfer, the steam temperature in the condenser must be above 25°C; thus, the condenser pressure must be above 3.2 kPa, which is the saturation pressure at 25°C.

Lowering the condenser pressure is not without any side effects, however. For one thing, it creates the possibility of air leakage into the condenser. More importantly, it increases the moisture content of the steam at the final stages of the turbine, as can be seen from Fig. 9–44. The presence of large quantities of moisture is highly undesirable in turbines because it decreases the turbine efficiency and erodes the turbine blades. Fortunately, this problem can be corrected, as discussed next.

Superheating the Steam to High Temperatures (*Increases* $T_{\text{high,avg}}$)

The average temperature at which heat is transferred to steam can be increased without increasing the boiler pressure by superheating the steam to high temperatures. The effect of superheating on the performance of vapor power cycles is illustrated on a *T*-*s* diagram in Fig. 9–45. The colored area on this diagram represents the increase in the net work. The total area under the process curve 3-3′ represents the increase in the heat input. Thus both the net work and heat input increase as a result of superheating the steam to a higher temperature. The overall effect is an increase in thermal efficiency, however, since the average temperature at which heat is added increases.

Superheating the steam to higher temperatures has another very desirable effect: It decreases the moisture content of the steam at the turbine exit, as can be seen from the *T*-*s* diagram (the quality at state 4′ is higher than that at state 4).

The temperature to which steam can be superheated is limited, however, by metallurgical considerations. At present the highest steam temperature allowed at the turbine inlet is about 620°C (1150°F). Any increase in this value depends on improving the present materials or finding new ones that can withstand higher temperatures. Ceramics are very promising in this regard.

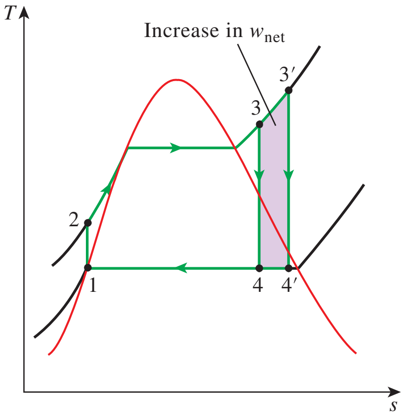


FIGURE 9–45
The effect of superheating the steam to higher temperatures on the ideal Rankine cycle.

Increasing the Boiler Pressure (*Increases* $T_{\text{high,avg}}$)

Another way of increasing the average temperature during the heat-addition process is to increase the operating pressure of the boiler, which automatically raises the temperature at which boiling takes place. This, in turn, raises the average temperature at which heat is transferred to the steam and thus raises the thermal efficiency of the cycle.

The effect of increasing the boiler pressure on the performance of vapor power cycles is illustrated on a *T*-*s* diagram in Fig. 9–46. Notice that for a fixed turbine inlet temperature, the cycle shifts to the left and the moisture content of steam at the turbine exit increases. This undesirable side effect can be corrected, however, by reheating the steam, as discussed in the next section.

Operating pressures of boilers have gradually increased over the years from about 2.7 MPa (400 psia) in 1922 to over 30 MPa (4500 psia) today, generating enough steam to produce a net power output of 1000 MW or more in a large power plant. Today many modern steam power plants operate at supercritical pressures ($P > 22.06$ MPa) and have thermal efficiencies of about 40 percent for fossil-fuel plants and 34 percent for nuclear plants. There are over 150 supercritical-pressure steam power plants in operation in the United States.

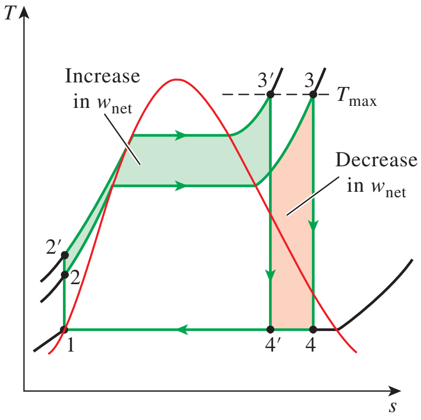


FIGURE 9–46
The effect of increasing the boiler pressure on the ideal Rankine cycle.

The lower efficiencies of nuclear power plants are due to the lower maximum temperatures used in those plants for safety reasons. The *T-s* diagram of a supercritical Rankine cycle is shown in Fig. 9–47.

The effects of lowering the condenser pressure, superheating to a higher temperature, and increasing the boiler pressure on the thermal efficiency of the Rankine cycle are illustrated next with an example.

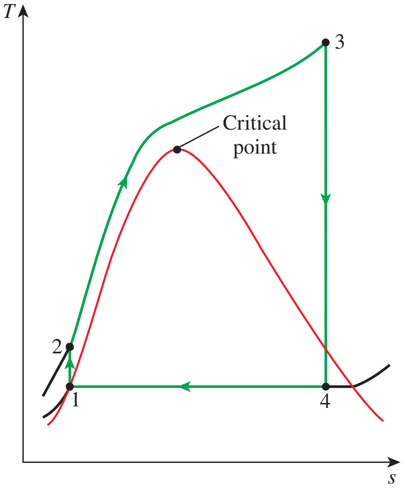


FIGURE 9–47
A supercritical Rankine cycle.

EXAMPLE 9–10 Effect of Boiler Pressure and Temperature on Efficiency

Consider a steam power plant operating on the ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350°C and is condensed in the condenser at a pressure of 10 kPa. Determine (*a*) the thermal efficiency of this power plant, (*b*) the thermal efficiency if steam is superheated to 600°C instead of 350°C, and (*c*) the thermal efficiency if the boiler pressure is raised to 15 MPa while the turbine inlet temperature is maintained at 600°C.

SOLUTION A steam power plant operating on the ideal Rankine cycle is considered. The effects of superheating the steam to a higher temperature and raising the boiler pressure on thermal efficiency are to be investigated.

Analysis The *T-s* diagrams of the cycle for all three cases are given in Fig. 9–48.

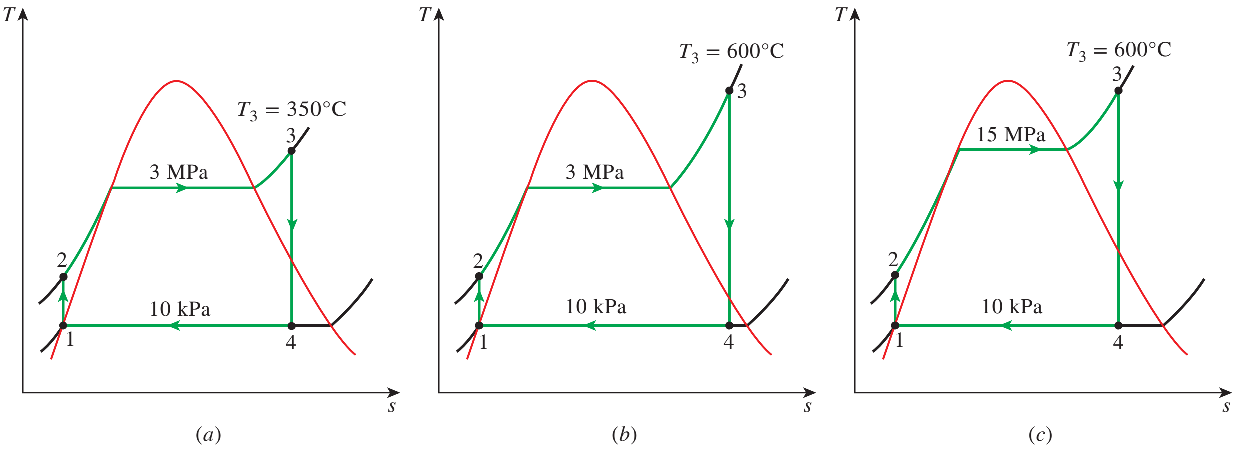
(*a*) This is the steam power plant discussed in Example 9–8, except that the condenser pressure is lowered to 10 kPa. The thermal efficiency is determined in a similar manner:

State 1:
$$\left.\begin{array}{l} P_1 = 10\ \text{kPa} \\ \text{Sat. liquid} \end{array}\right\} \begin{array}{l} h_1 = h_{f\,@\,10\ \text{kPa}} = 191.81\ \text{kJ/kg} \\ v_1 = v_{f\,@\,10\ \text{kPa}} = 0.00101\ \text{m}^3/\text{kg} \end{array}$$

State 2:
$$P_2 = 3\ \text{MPa}$$
$$s_2 = s_1$$

$$w_{\text{pump,in}} = v_1(P_2 - P_1) = (0.00101\ \text{m}^3/\text{kg})[(3000 - 10)\ \text{kPa}]\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right)$$
$$= 3.02\ \text{kJ/kg}$$
$$h_2 = h_1 + w_{\text{pump,in}} = (191.81 + 3.02)\ \text{kJ/kg} = 194.83\ \text{kJ/kg}$$

State 3:
$$\left.\begin{array}{l} P_3 = 3\ \text{MPa} \\ T_3 = 350^\circ\text{C} \end{array}\right\} \begin{array}{l} h_3 = 3116.1\ \text{kJ/kg} \\ s_3 = 6.7450\ \text{kJ/kg}\cdot\text{K} \end{array}$$



(*a*) (*b*) (*c*)

FIGURE 9–48
T-s diagrams of the three cycles discussed in Example 9–10.

State 4: $P_4 = 10 \text{ kPa}$ (sat. mixture)

$$s_4 = s_3$$

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.7450 - 0.6492}{7.4996} = 0.8128$$

Thus,

$$\begin{aligned} h_4 &= h_f + x_4 h_{fg} = 191.81 + 0.8128(2392.1) = 2136.1 \text{ kJ/kg} \\ q_{\text{in}} &= h_3 - h_2 = (3116.1 - 194.83) \text{ kJ/kg} = 2921.3 \text{ kJ/kg} \\ q_{\text{out}} &= h_4 - h_1 = (2136.1 - 191.81) \text{ kJ/kg} = 1944.3 \text{ kJ/kg} \end{aligned}$$

and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{1944.3 \text{ kJ/kg}}{2921.3 \text{ kJ/kg}} = 0.334 \text{ or } \mathbf{33.4\%}$$

Therefore, the thermal efficiency increases from 26.0 to 33.4 percent as a result of lowering the condenser pressure from 75 to 10 kPa. At the same time, however, the quality of the steam decreases from 88.6 to 81.3 percent (in other words, the moisture content increases from 11.4 to 18.7 percent).

(*b*) States 1 and 2 remain the same in this case, and the enthalpies at state 3 (3 MPa and 600°C) and state 4 (10 kPa and $s_4 = s_3$) are determined to be

$$\begin{aligned} h_3 &= 3682.8 \text{ kJ/kg} \\ h_4 &= 2380.3 \text{ kJ/kg} \quad (x_4 = 0.915) \end{aligned}$$

Thus,

$$\begin{aligned} q_{\text{in}} &= h_3 - h_2 = 3682.8 - 194.83 = 3488.0 \text{ kJ/kg} \\ q_{\text{out}} &= h_4 - h_1 = 2380.3 - 191.81 = 2188.5 \text{ kJ/kg} \end{aligned}$$

and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{2188.5 \text{ kJ/kg}}{3488.0 \text{ kJ/kg}} = 0.373 \text{ or } \mathbf{37.3\%}$$

Therefore, the thermal efficiency increases from 33.4 to 37.3 percent as a result of superheating the steam from 350 to 600°C. At the same time, the quality of the steam increases from 81.3 to 91.5 percent (in other words, the moisture content decreases from 18.7 to 8.5 percent).

(*c*) State 1 remains the same in this case, but the other states change. The enthalpies at state 2 (15 MPa and $s_2 = s_1$), state 3 (15 MPa and 600°C), and state 4 (10 kPa and $s_4 = s_3$) are determined in a similar manner to be

$$\begin{aligned} h_2 &= 206.95 \text{ kJ/kg} \\ h_3 &= 3583.1 \text{ kJ/kg} \\ h_4 &= 2115.3 \text{ kJ/kg} \quad (x_4 = 0.804) \end{aligned}$$

Thus,

$$\begin{aligned} q_{\text{in}} &= h_3 - h_2 = 3583.1 - 206.95 = 3376.2 \text{ kJ/kg} \\ q_{\text{out}} &= h_4 - h_1 = 2115.3 - 191.81 = 1923.5 \text{ kJ/kg} \end{aligned}$$

and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{1923.5 \text{ kJ/kg}}{3376.2 \text{ kJ/kg}} = 0.430 \text{ or } \mathbf{43.0\%}$$

Discussion The thermal efficiency increases from 37.3 to 43.0 percent as a result of raising the boiler pressure from 3 to 15 MPa while maintaining the turbine inlet temperature at 600°C. At the same time, however, the quality of the steam decreases from 91.5 to 80.4 percent (in other words, the moisture content increases from 8.5 to 19.6 percent).

9–13 ▪ THE IDEAL REHEAT RANKINE CYCLE

We noted in Sec. 9–12 that increasing the boiler pressure increases the thermal efficiency of the Rankine cycle, but it also increases the moisture content of the steam to unacceptable levels. Then it is natural to ask the following question:

> *How can we take advantage of the increased efficiencies at higher boiler pressures without facing the problem of excessive moisture at the final stages of the turbine?*

Two possibilities come to mind:

1. Superheat the steam to very high temperatures before it enters the turbine. This would be the desirable solution since the average temperature at which heat is added would also increase, thus increasing the cycle efficiency. This is not a viable solution, however, since it requires raising the steam temperature to metallurgically unsafe levels.

2. Expand the steam in the turbine in two stages, and reheat it in between. In other words, modify the simple ideal Rankine cycle with a **reheat** process. Reheating is a practical solution to the excessive moisture problem in turbines, and it is commonly used in modern steam power plants.

The *T-s* diagram of the ideal reheat Rankine cycle and the schematic of the power plant operating on this cycle are shown in Fig. 9–49. The ideal reheat Rankine cycle differs from the simple ideal Rankine cycle in that the expansion

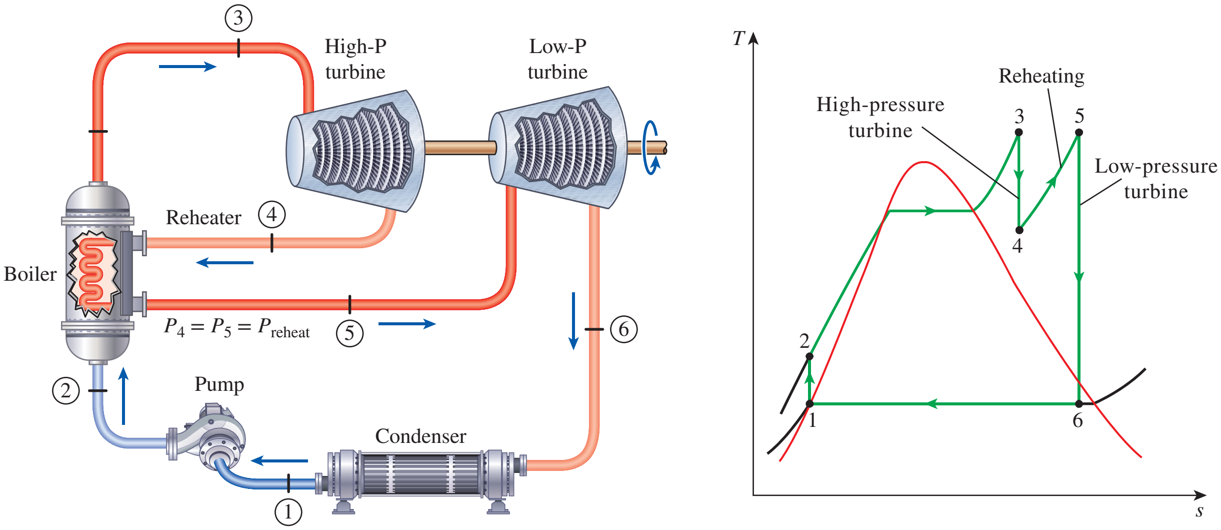


FIGURE 9–49
The ideal reheat Rankine cycle.

process takes place in two stages. In the first stage (the high-pressure turbine), steam is expanded isentropically to an intermediate pressure and sent back to the boiler, where it is reheated at constant pressure, usually to the inlet temperature of the first turbine stage. Steam then expands isentropically in the second stage (low-pressure turbine) to the condenser pressure. Thus the total heat input and the total turbine work output for a reheat cycle become

$$q_{\text{in}} = q_{\text{primary}} + q_{\text{reheat}} = (h_3 - h_2) + (h_5 - h_4) \quad \textbf{(9–36)}$$

and

$$w_{\text{turb,out}} = w_{\text{turb,I}} + w_{\text{turb,II}} = (h_3 - h_4) + (h_5 - h_6) \quad \textbf{(9–37)}$$

The incorporation of the single reheat in a modern power plant improves the cycle efficiency by 4 to 5 percent by increasing the average temperature at which heat is transferred to the steam.

The average temperature during the reheat process can be increased by increasing the number of expansion and reheat stages. As the number of stages is increased, the expansion and reheat processes approach an isothermal process at the maximum temperature, as shown in Fig. 9–50. The use of more than two reheat stages, however, is not practical. The theoretical improvement in efficiency from the second reheat is about half of that which results from a single reheat. If the turbine inlet pressure is not high enough, double reheat would result in superheated exhaust. This is undesirable as it would cause the average temperature for heat rejection to increase and thus the cycle efficiency to decrease. Therefore, double reheat is used only on supercritical-pressure ($P > 22.06$ MPa) power plants. A third reheat stage would increase the cycle efficiency by about half of the improvement attained by the second reheat. This gain is too small to justify the added cost and complexity.

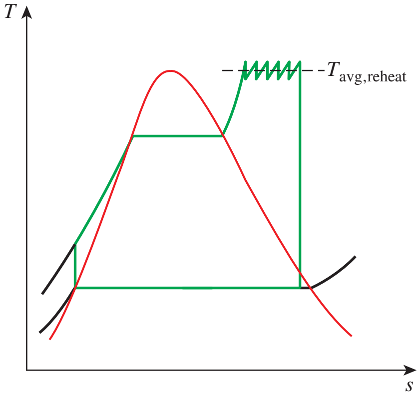


FIGURE 9–50
The average temperature at which heat is transferred during reheating increases as the number of reheat stages is increased.

The reheat cycle was introduced in the mid-1920s, but it was abandoned in the 1930s because of the operational difficulties. The steady increase in boiler pressures over the years made it necessary to reintroduce single reheat in the late 1940s and double reheat in the early 1950s.

The reheat temperatures are very close or equal to the turbine inlet temperature. The optimum reheat pressure is about one-fourth of the maximum cycle pressure. For example, the optimum reheat pressure for a cycle with a boiler pressure of 12 MPa is about 3 MPa.

Remember that the sole purpose of the reheat cycle is to reduce the moisture content of the steam at the final stages of the expansion process. If we had materials that could withstand sufficiently high temperatures, there would be no need for the reheat cycle.

EXAMPLE 9–11 The Ideal Reheat Rankine Cycle

Consider a steam power plant that operates on the ideal reheat Rankine cycle. The plant maintains the inlet of the high-pressure turbine at 600 psia and 600°F, the inlet of the low-pressure turbine at 200 psia and 600°F, and the condenser at 10 psia. The net power produced by this plant is 5000 kW. Determine the rate of heat addition and rejection and the thermal efficiency of the cycle.

Is there any advantage to operating the reheat section of the boiler at 100 psia rather than 200 psia while maintaining the same low-pressure turbine inlet temperature?

SOLUTION An ideal reheat steam Rankine cycle produces 5000 kW of power. The rates of heat addition and rejection and the thermal efficiency of the cycle are to be determined. Also, the effect of changing reheat pressure is to be investigated.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

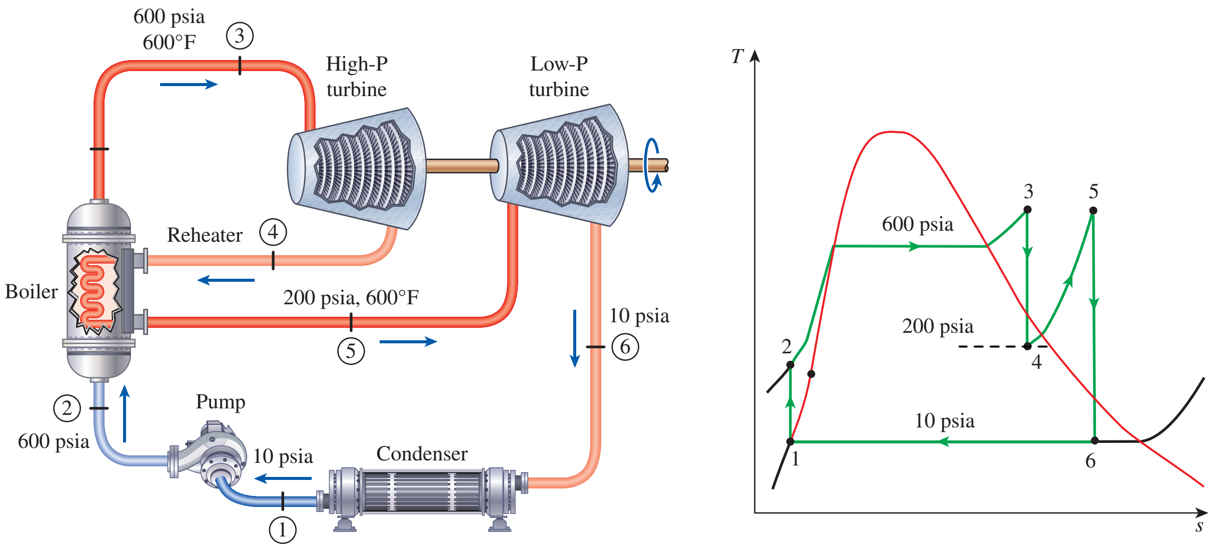


FIGURE 9–51
Schematic and *T*-*s* diagram for Example 9–11.

Analysis The schematic of the power plant and the *T*-*s* diagram of the cycle are shown in Fig. 9–51. The power plant operates on the ideal reheat Rankine cycle. Therefore, the pump and the turbines are isentropic, there are no pressure drops in the boiler and condenser, and steam leaves the condenser and enters the pump as saturated liquid at the condenser pressure. From the steam tables (Tables A-4E, A-5E, and A-6E),

$$h_1 = h_{f\,@\,10\text{ psia}} = 161.25\text{ Btu/lbm}$$
$$v_1 = v_{f\,@\,10\text{ psia}} = 0.01659\text{ ft}^3\text{/lbm}$$

$$\begin{aligned} w_{\text{pump,in}} &= v_1(P_2 - P_1) \\ &= (0.01659\text{ ft}^3\text{/lbm})[(600 - 10)\text{ psia}]\left(\frac{1\text{ Btu}}{5.404\text{ psia}\cdot\text{ft}^3}\right) \\ &= 1.81\text{ Btu/lbm} \end{aligned}$$

$$h_2 = h_1 + w_{\text{pump,in}} = 161.25 + 1.81 = 163.06\text{ Btu/lbm}$$

$$\left.\begin{aligned} P_3 &= 600\text{ psia} \\ T_3 &= 600°\text{F} \end{aligned}\right\} \begin{aligned} h_3 &= 1289.9\text{ Btu/lbm} \\ s_3 &= 1.5325\text{ Btu/lbm}\cdot\text{R} \end{aligned}$$

$$\left.\begin{aligned} P_4 &= 200\text{ psia} \\ s_4 &= s_3 \end{aligned}\right\} \begin{aligned} x_4 &= \frac{s_4 - s_f}{s_{fg}} = \frac{1.5325 - 0.54379}{1.00219} = 0.9865 \\ h_4 &= h_f + x_4 h_{fg} = 355.46 + (0.9865)(843.33) = 1187.5\text{ Btu/lbm} \end{aligned}$$

$$\left.\begin{aligned} P_5 &= 200\text{ psia} \\ T_5 &= 600°\text{F} \end{aligned}\right\} \begin{aligned} h_5 &= 1322.3\text{ Btu/lbm} \\ s_5 &= 1.6771\text{ Btu/lbm}\cdot\text{R} \end{aligned}$$

$$\left.\begin{aligned} P_6 &= 10\text{ psia} \\ s_6 &= s_5 \end{aligned}\right\} \begin{aligned} x_6 &= \frac{s_6 - s_f}{s_{fg}} = \frac{1.6771 - 0.28362}{1.50391} = 0.9266 \\ h_6 &= h_f + x_6 h_{fg} = 161.25 + (0.9266)(981.82) = 1071.0\text{ Btu/lbm} \end{aligned}$$

Thus,

$$q_{\text{in}} = (h_3 - h_2) + (h_5 - h_4) = 1289.9 - 163.06 + 1322.3 - 1187.5 = 1261.7 \text{ Btu/lbm}$$

$$q_{\text{out}} = h_6 - h_1 = 1071.0 - 161.25 = 909.7 \text{ Btu/lbm}$$

$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 1261.7 - 909.8 = 352.0 \text{ Btu/lbm}$$

The mass flow rate of steam in the cycle is determined from

$$\dot{W}_{\text{net}} = \dot{m}w_{\text{net}} \rightarrow \dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{5000 \text{ kJ/s}}{352.0 \text{ Btu/lbm}}\left(\frac{0.94782 \text{ Btu}}{1 \text{ kJ}}\right) = 13.47 \text{ lbm/s}$$

The rates of heat addition and rejection are

$$\dot{Q}_{\text{in}} = \dot{m}q_{\text{in}} = (13.47 \text{ lbm/s})(1261.7 \text{ Btu/lbm}) = \mathbf{16{,}995 \text{ Btu/s}}$$

$$\dot{Q}_{\text{out}} = \dot{m}q_{\text{out}} = (13.47 \text{ lbm/s})(909.7 \text{ Btu/lbm}) = \mathbf{12{,}250 \text{ Btu/s}}$$

and the thermal efficiency of the cycle is

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{5000 \text{ kJ/s}}{16{,}995 \text{ Btu/s}}\left(\frac{0.94782 \text{ Btu}}{1 \text{ kJ}}\right) = 0.279 \text{ or } \mathbf{27.9\%}$$

If we repeat the analysis for a reheat pressure of 100 psia at the same reheat temperature, we obtain a thermal efficiency of 27.3 percent. Thus, operating the reheater at 100 psia causes a slight decrease in the thermal efficiency.

Discussion Now we try to address this question: At what reheat pressure will the thermal efficiency be maximum? We repeat the analysis at various reheat pressures using appropriate software. The results are plotted in Fig. 9–52. The thermal efficiency reaches a maximum value of 28.1 percent at an optimum reheat pressure of about 325 psia.

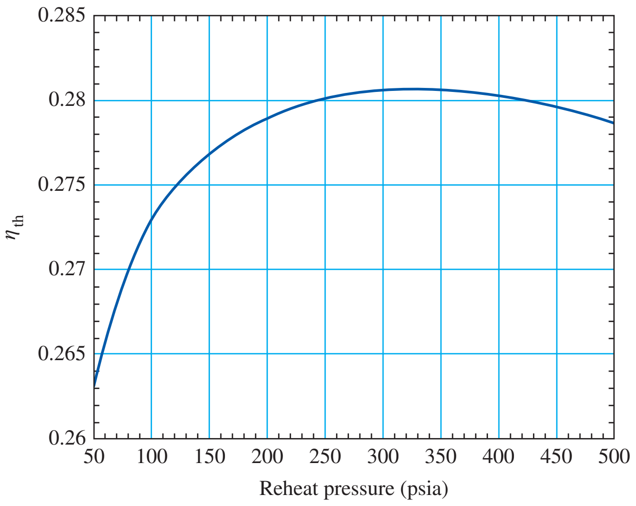


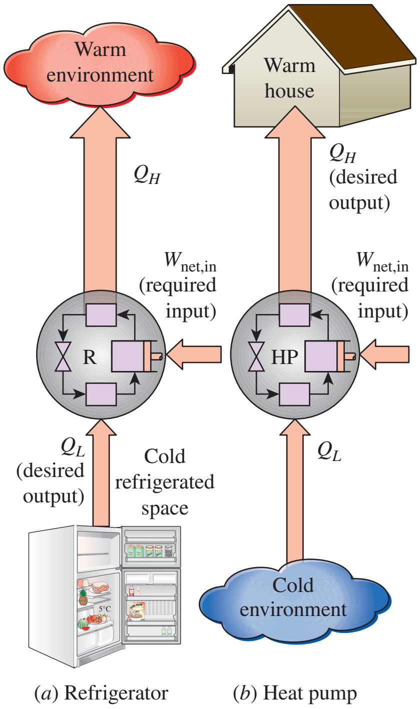
FIGURE 9–52
There is an optimum reheat pressure in the reheat Rankine cycle for which the thermal efficiency is maximum. The values refer to Example 9-11.

9–14 ▪ REFRIGERATORS AND HEAT PUMPS

We all know from experience that heat flows in the direction of decreasing temperature, that is, from high-temperature regions to low-temperature ones. This heat-transfer process occurs in nature without requiring any devices. The reverse process, however, cannot occur by itself. The transfer of heat from a low-temperature region to a high-temperature one requires special devices called **refrigerators**.

Refrigerators are cyclic devices, and the working fluids used in the refrigeration cycles are called **refrigerants**. A refrigerator is shown schematically in Fig. 9–53*a*. Here Q_L is the magnitude of the heat removed from the refrigerated space at temperature T_L, Q_H is the magnitude of the heat rejected to the warm space at temperature T_H, and $W_{\text{net,in}}$ is the net work input to the refrigerator. As discussed in Chap. 7, Q_L and Q_H represent magnitudes and thus are positive quantities.

Another device that transfers heat from a low-temperature medium to a high-temperature one is the **heat pump**. Refrigerators and heat pumps are essentially the same devices; they differ in their objectives only. The objective of a refrigerator is to maintain the refrigerated space at a low temperature by removing heat from it. Discharging this heat to a higher-temperature medium is merely a necessary part of the operation, not the purpose. The objective of a heat pump, however, is to maintain a heated space at a high temperature. This is accomplished by absorbing heat from a low-temperature source, such as well water or cold outside air in winter, and supplying this heat to a warmer medium such as a house (Fig. 9–53*b*).



(*a*) Refrigerator (*b*) Heat pump

FIGURE 9–53
The objective of a refrigerator is to remove heat (Q_L) from the cold medium; the objective of a heat pump is to supply heat (Q_H) to a warm medium.

The performance of refrigerators and heat pumps is expressed in terms of the **coefficient of performance** (COP), defined as

$$\text{COP}_\text{R} = \frac{\text{Desired output}}{\text{Required input}} = \frac{\text{Cooling effect}}{\text{Work input}} = \frac{Q_L}{W_{\text{net,in}}} \tag{9–38}$$

$$\text{COP}_\text{HP} = \frac{\text{Desired output}}{\text{Required input}} = \frac{\text{Heating effect}}{\text{Work input}} = \frac{Q_H}{W_{\text{net,in}}} \tag{9–39}$$

These relations can also be expressed in the rate form by replacing the quantities Q_L, Q_H, and $W_{\text{net,in}}$ with $\dot{Q}_L$, $\dot{Q}_H$, and $\dot{W}_{\text{net,in}}$, respectively. Notice that both COP_R and COP_HP can be greater than 1. A comparison of Eqs. 9–38 and 9–39 reveals that

$$\text{COP}_\text{HP} = \text{COP}_\text{R} + 1 \tag{9–40}$$

for fixed values of Q_L and Q_H. This relation implies that $\text{COP}_\text{HP} > 1$ since COP_R is a positive quantity. That is, a heat pump functions, at worst, as a resistance heater, supplying as much energy to the house as it consumes. In reality, however, part of Q_H is lost to the outside air through piping and other devices, and COP_HP may drop below unity when the outside air temperature is too low. When this happens, the system normally switches to the fuel (natural gas, propane, oil, etc.) or resistance-heating mode.

The *cooling capacity* of a refrigeration system—that is, the rate of heat removal from the refrigerated space—is often expressed in terms of **tons of refrigeration**. The capacity of a refrigeration system that can freeze 1 ton (2000 lbm) of liquid water at 0°C (32°F) into ice at 0°C in 24 h is said to be 1 ton. One ton of refrigeration is equivalent to 211 kJ/min or 200 Btu/min. The cooling load of a typical 200-m^2 residence is in the 3-ton (10-kW) range.

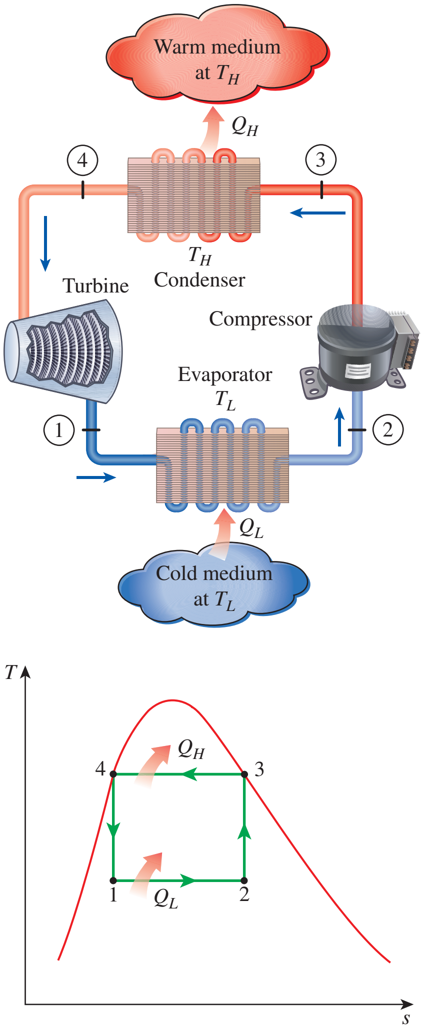


FIGURE 9–54
Schematic of a Carnot refrigerator and *T-s* diagram of the reversed Carnot cycle.

9–15 ▪ THE REVERSED CARNOT CYCLE

Recall from Chap. 7 that the Carnot cycle is a totally reversible cycle that consists of two reversible isothermal and two isentropic processes. It has the maximum thermal efficiency for given temperature limits, and it serves as a standard against which actual power cycles can be compared.

Since it is a reversible cycle, all four processes that comprise the Carnot cycle can be reversed. Reversing the cycle does also reverse the directions of any heat and work interactions. The result is a cycle that operates in the counterclockwise direction on a *T-s* diagram, which is called the **reversed Carnot cycle**. A refrigerator or heat pump that operates on the reversed Carnot cycle is called a **Carnot refrigerator** or a **Carnot heat pump**.

Consider a reversed Carnot cycle executed within the saturation dome of a refrigerant, as shown in Fig. 9–54. The refrigerant absorbs heat isothermally from a low-temperature source at T_L in the amount of Q_L (process 1-2), is compressed isentropically to state 3 (temperature rises to T_H), rejects heat isothermally to a high-temperature sink at T_H in the amount of Q_H (process 3-4), and expands isentropically to state 1 (temperature drops to T_L). The refrigerant changes from a saturated vapor state to a saturated liquid state in the condenser during process 3-4.

The coefficients of performance of Carnot refrigerators and heat pumps are expressed in terms of temperatures as

$$\text{COP}_{\text{R,Carnot}} = \frac{1}{T_H/T_L - 1} \tag{9–41}$$

and

$$\text{COP}_{\text{HP,Carnot}} = \frac{1}{1 - T_L/T_H} \tag{9–42}$$

Notice that both COPs increase as the difference between the two temperatures decreases, that is, as T_L rises or T_H falls.

The reversed Carnot cycle is the *most efficient* refrigeration cycle operating between two specified temperature levels. Therefore, it is natural to look at it first as a prospective ideal cycle for refrigerators and heat pumps. If we could, we certainly would adapt it as the ideal cycle. As explained below, however, the reversed Carnot cycle is not a suitable model for refrigeration cycles.

The two isothermal heat transfer processes are not difficult to achieve in practice since maintaining a constant pressure automatically fixes the temperature of a two-phase mixture at the saturation value. Therefore, processes 1-2 and 3-4 can be approached closely in actual evaporators and condensers. However, processes 2-3 and 4-1 cannot be approximated closely in practice. This is because process 2-3 involves the compression of a liquid–vapor mixture, which requires a compressor that will handle two phases, and process 4-1 involves the expansion of high-moisture-content refrigerant in a turbine.

It seems as if these problems could be eliminated by executing the reversed Carnot cycle outside the saturation region. But in this case we have difficulty maintaining isothermal conditions during the heat-absorption and heat-rejection processes. Therefore, we conclude that the reversed Carnot cycle cannot be approximated in actual devices and is not a realistic model for refrigeration cycles. However, the reversed Carnot cycle can serve as a standard against which actual refrigeration cycles are compared.

9–16 ▪ THE IDEAL VAPOR-COMPRESSION REFRIGERATION CYCLE

Many of the impracticalities associated with the reversed Carnot cycle can be eliminated by vaporizing the refrigerant completely before it is compressed and by replacing the turbine with a throttling device, such as an expansion valve or capillary tube. The cycle that results is called the **ideal vapor-compression refrigeration cycle**, and it is shown schematically and on a *T-s* diagram in Fig. 9–55. The vapor-compression refrigeration cycle is the most widely used cycle for refrigerators, air-conditioning systems, and heat pumps. It consists of four processes:

1-2 Isentropic compression in a compressor
2-3 Constant-pressure heat rejection in a condenser
3-4 Throttling in an expansion valve
4-1 Constant-pressure heat absorption in an evaporator

In an ideal vapor-compression refrigeration cycle, the refrigerant enters the compressor at state 1 as saturated vapor and is compressed isentropically to the condenser pressure. The temperature of the refrigerant increases during this isentropic compression process to well above the temperature of the surrounding medium. The refrigerant then enters the condenser as superheated vapor at state 2 and leaves as saturated liquid at state 3 as a result of heat rejection to the surroundings. The temperature of the refrigerant at this state is still above the temperature of the surroundings.

The saturated liquid refrigerant at state 3 is throttled to the evaporator pressure by passing it through an expansion valve or capillary tube. The temperature of the refrigerant drops below the temperature of the refrigerated space during this process. The refrigerant enters the evaporator at state 4 as a low-quality saturated mixture, and it completely evaporates by absorbing heat from the refrigerated space. The refrigerant leaves the evaporator as saturated vapor and reenters the compressor, completing the cycle.

In a household refrigerator, the tubes in the freezer compartment where heat is absorbed by the refrigerant serves as the evaporator. The coils behind the refrigerator, where heat is dissipated to the kitchen air, serve as the condenser (Fig. 9–56).

Remember that the area under the process curve on a *T-s* diagram represents the heat transfer for internally reversible processes. The area under the process curve 4-1 represents the heat absorbed by the refrigerant in the evaporator, and the area under the process curve 2-3 represents the heat rejected in the condenser. A rule of thumb is that the *COP improves by 2 to 4 percent for each °C the evaporating temperature is raised or the condensing temperature is lowered.*

Another diagram often used in the analysis of vapor-compression refrigeration cycles is the *P-h* diagram, as shown in Fig. 9–57. On this diagram, three of the four processes appear as straight lines, and the heat transfer in the condenser and the evaporator is proportional to the lengths of the corresponding process curves.

Notice that unlike the ideal cycles discussed before, the ideal vapor-compression refrigeration cycle is not an internally reversible cycle since it involves an irreversible (throttling) process. This process is maintained in the cycle to make it a more realistic model for the actual vapor-compression refrigeration cycle. If the throttling device were replaced by an isentropic turbine, the refrigerant would enter the evaporator at state 4′ instead of state 4. As a result, the refrigeration capacity would increase (by the area under process curve 4′–4 in Fig. 9–55) and the net work input would decrease (by the amount of work output of the turbine).

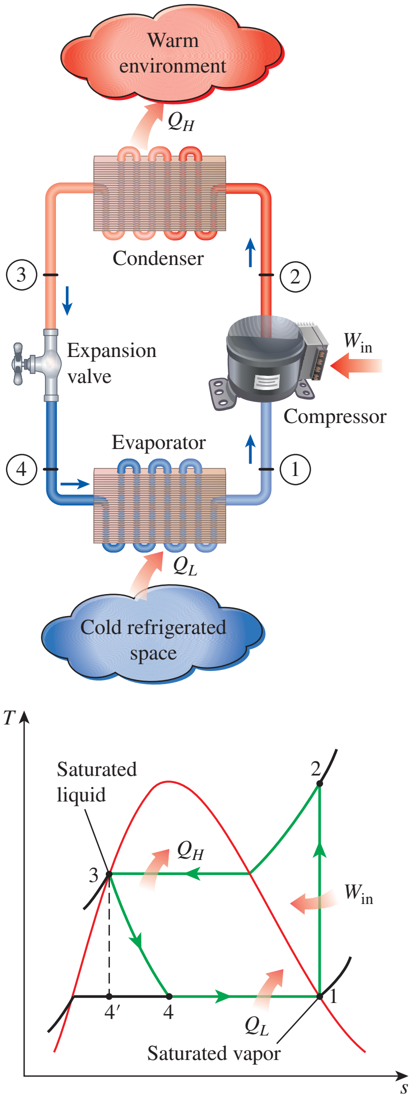


FIGURE 9–55
Schematic and *T-s* diagram for the ideal vapor-compression refrigeration cycle.

Replacing the expansion valve with a turbine is not practical, however, since the added benefits cannot justify the added cost and complexity.

All four components associated with the vapor-compression refrigeration cycle are steady-flow devices, and thus all four processes that make up the cycle can be analyzed as steady-flow processes. The kinetic and potential energy changes of the refrigerant are usually small relative to the work and heat transfer terms, and therefore they can be neglected. Then the steady-flow energy equation on a unit-mass basis reduces to

$$(q_{\text{in}} - q_{\text{out}}) + (w_{\text{in}} - w_{\text{out}}) = h_e - h_i \tag{9–43}$$

The condenser and the evaporator do not involve any work, and the compressor can be approximated as adiabatic. Then the COPs of refrigerators and heat pumps operating on the vapor-compression refrigeration cycle can be expressed as

$$\text{COP}_\text{R} = \frac{q_L}{w_{\text{net,in}}} = \frac{h_1 - h_4}{h_2 - h_1} \tag{9–44}$$

and

$$\text{COP}_\text{HP} = \frac{q_H}{w_{\text{net,in}}} = \frac{h_2 - h_3}{h_2 - h_1} \tag{9–45}$$

where $h_1 = h_{g\,@\,P_1}$ and $h_3 = h_{f\,@\,P_3}$ for the ideal case.

Vapor-compression refrigeration dates back to 1834 when the Englishman Jacob Perkins received a patent for a closed-cycle ice machine using ether or other volatile fluids as refrigerants. A working model of this machine was built, but it was never produced commercially. In 1850, Alexander Twining began to design and build vapor-compression ice machines using ethyl ether, which is a commercially used refrigerant in vapor-compression systems. Initially, vapor-compression refrigeration systems were large and were mainly used for ice making, brewing, and cold storage. They lacked automatic controls and were steam-engine driven. In the 1890s, electric motor–driven smaller machines equipped with automatic controls started to replace the older units, and refrigeration systems began to appear in butcher shops and households. By 1930, the continued improvements made it possible to have vapor-compression refrigeration systems that were relatively efficient, reliable, small, and inexpensive.

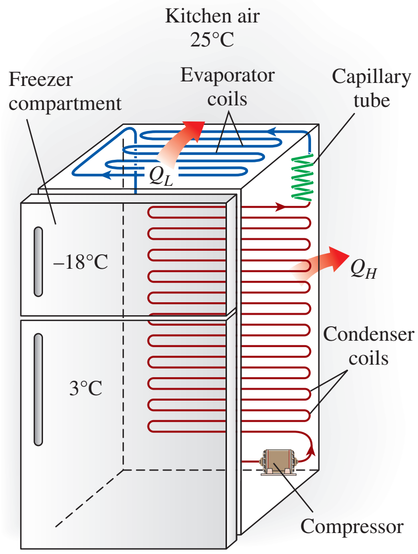


FIGURE 9–56
An ordinary household refrigerator.

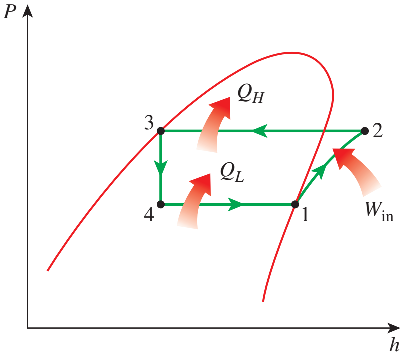


FIGURE 9–57
The *P-h* diagram of an ideal vapor-compression refrigeration cycle.

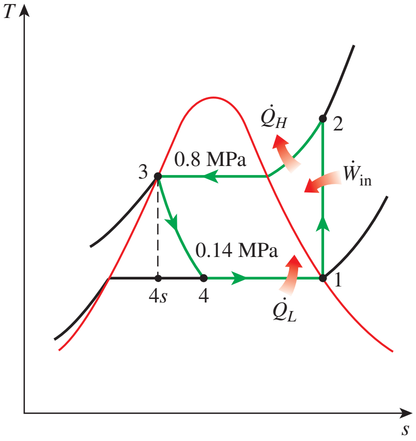


FIGURE 9–58
T-s diagram of the ideal vapor-compression refrigeration cycle described in Example 9–12.

EXAMPLE 9–12 The Ideal Vapor-Compression Refrigeration Cycle

A refrigerator uses refrigerant-134a as the working fluid and operates on an ideal vapor-compression refrigeration cycle between 0.14 and 0.8 MPa. If the mass flow rate of the refrigerant is 0.05 kg/s, determine (*a*) the rate of heat removal from the refrigerated space and the power input to the compressor, (*b*) the rate of heat rejection to the environment, and (*c*) the COP of the refrigerator.

SOLUTION A refrigerator operates on an ideal vapor-compression refrigeration cycle between two specified pressure limits. The rate of refrigeration, the power input, the rate of heat rejection, and the COP are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis The *T-s* diagram of the refrigeration cycle is shown in Fig. 9–58. We note that this is an ideal vapor-compression refrigeration cycle, and thus the compressor is isentropic and the refrigerant leaves the condenser as a saturated liquid and enters the compressor as saturated vapor. From the refrigerant-134a tables, the enthalpies of the refrigerant at all four states are determined as follows:

$$P_1 = 0.14\text{ MPa} \longrightarrow h_1 = h_{g\text{ @ 0.14 MPa}} = 239.19\text{ kJ/kg}$$

$$s_1 = s_{g\text{ @ 0.14 MPa}} = 0.94467\text{ kJ/kg·K}$$

$$\left.\begin{array}{l} P_2 = 0.8\text{ MPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 275.40\text{ kJ/kg}$$

$$P_3 = 0.8\text{ MPa} \longrightarrow h_3 = h_{f\text{ @ 0.8 MPa}} = 95.48\text{ kJ/kg}$$

$$h_4 \cong h_3\text{ (throttling)} \longrightarrow h_4 = 95.48\text{ kJ/kg}$$

(*a*) The rate of heat removal from the refrigerated space and the power input to the compressor are determined from their definitions:

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.05\text{ kg/s})[(239.19 - 95.48)\text{ kJ/kg}] = \mathbf{7.19\ kW}$$

and

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1) = (0.05\text{ kg/s})[(275.40 - 239.19)\text{ kJ/kg}] = \mathbf{1.81\ kW}$$

(*b*) The rate of heat rejection from the refrigerant to the environment is

$$\dot{Q}_H = \dot{m}(h_2 - h_3) = (0.05\text{ kg/s})[(275.40 - 95.48)\text{ kJ/kg}] = \mathbf{9.00\ kW}$$

It could also be determined from

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{\text{in}} = 7.19 + 1.81 = 9.00\text{ kW}$$

(*c*) The coefficient of performance of the refrigerator is

$$\text{COP}_\text{R} = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = \frac{7.19\text{ kW}}{1.81\text{ kW}} = \mathbf{3.97}$$

That is, this refrigerator removes about 4 units of thermal energy from the refrigerated space for each unit of electric energy it consumes.

Discussion It would be interesting to see what happens if the throttling valve were replaced by an isentropic turbine. The enthalpy at state 4*s* (the turbine exit with $P_{4s} = 0.14$ MPa, and $s_{4s} = s_3 = 0.35408$ kJ/kg·K) is 88.95 kJ/kg, and the turbine would produce 0.33 kW of power. This would decrease the power input to the refrigerator from 1.81 to 1.48 kW and increase the rate of heat removal from the refrigerated space from 7.19 to 7.51 kW. As a result, the COP of the refrigerator would increase from 3.97 to 5.07, an increase of 28 percent.

9–17 ▪ ACTUAL VAPOR-COMPRESSION REFRIGERATION CYCLE

An actual vapor-compression refrigeration cycle differs from the ideal one in several ways, owing mostly to the irreversibilities that occur in various components. Two common sources of irreversibilities are fluid friction (causes pressure drops) and heat transfer to or from the surroundings. The *T-s* diagram of an actual vapor-compression refrigeration cycle is shown in Fig. 9–59.

In the ideal cycle, the refrigerant leaves the evaporator and enters the compressor as *saturated vapor.* In practice, however, it may not be possible to control the state of the refrigerant so precisely. Instead, it is easier to design the system so that the refrigerant is slightly superheated at the compressor inlet. This slight overdesign ensures that the refrigerant is completely vaporized when it enters the compressor. Also, the line connecting the evaporator to the compressor is usually very long; thus, the pressure drop caused by fluid friction and heat transfer from the surroundings to the refrigerant can be very significant. The result of superheating, heat gain in the connecting line, and pressure drops in the evaporator and the

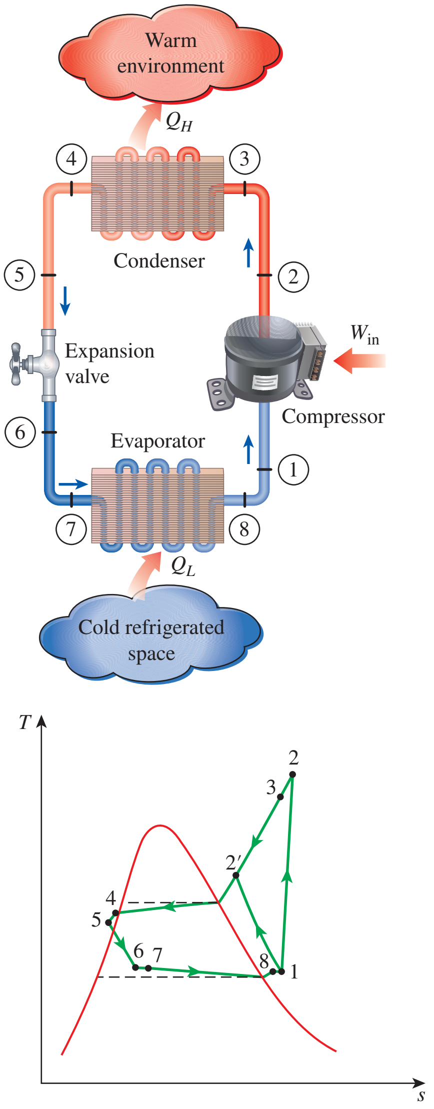


FIGURE 9–59
Schematic and *T-s* diagram for the actual vapor-compression refrigeration cycle.

connecting line is an increase in the specific volume, thus an increase in the power input requirements to the compressor since steady-flow work is proportional to the specific volume.

The *compression process* in the ideal cycle is internally reversible and adiabatic, and thus isentropic. The actual compression process, however, involves frictional effects, which increase the entropy, and heat transfer, which may increase or decrease the entropy, depending on the direction. Therefore, the entropy of the refrigerant may increase (process 1-2) or decrease (process 1-2′) during an actual compression process, depending on which effects dominate. The compression process 1-2′ may be even more desirable than the isentropic compression process since the specific volume of the refrigerant and thus the work input requirement are smaller in this case. Therefore, the refrigerant should be cooled during the compression process whenever it is practical and economical to do so.

In the ideal case, the refrigerant is assumed to leave the condenser as *saturated liquid* at the compressor exit pressure. In reality, however, it is unavoidable to have some pressure drop in the condenser as well as in the lines connecting the condenser to the compressor and to the throttling valve. Also, it is not easy to execute the condensation process with such precision that the refrigerant is a saturated liquid at the end, and it is undesirable to route the refrigerant to the throttling valve before the refrigerant is completely condensed. Therefore, the refrigerant is subcooled somewhat before it enters the throttling valve. We do not mind this at all, however, since the refrigerant in this case enters the evaporator with a lower enthalpy and thus can absorb more heat from the refrigerated space. The throttling valve and the evaporator are usually located very close to each other, so the pressure drop in the connecting line is small.

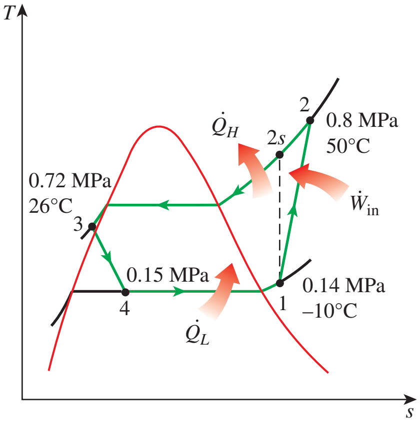


FIGURE 9–60

T-s diagram for Example 9–13.

EXAMPLE 9–13 The Actual Vapor-Compression Refrigeration Cycle

Refrigerant-134a enters the compressor of a refrigerator as superheated vapor at 0.14 MPa and −10°C at a rate of 0.05 kg/s and leaves at 0.8 MPa and 50°C. The refrigerant is cooled in the condenser to 26°C and 0.72 MPa and is throttled to 0.15 MPa. Disregarding any heat transfer and pressure drops in the connecting lines between the components, determine (*a*) the rate of heat removal from the refrigerated space and the power input to the compressor, (*b*) the isentropic efficiency of the compressor, and (*c*) the coefficient of performance of the refrigerator.

SOLUTION A refrigerator operating on a vapor-compression cycle is considered. The rate of refrigeration, the power input, the compressor efficiency, and the COP are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis The *T-s* diagram of the refrigeration cycle is shown in Fig. 9–60. We note that the refrigerant leaves the condenser as a compressed liquid and enters the compressor as superheated vapor. The enthalpies of the refrigerant at various states are determined from the refrigerant tables to be

$$\left.\begin{aligned} P_1 &= 0.14 \text{ MPa} \\ T_1 &= -10^\circ\text{C} \end{aligned}\right\} \quad h_1 = 246.37 \text{ kJ/kg}$$

$$\left.\begin{aligned} P_2 &= 0.8 \text{ MPa} \\ T_2 &= 50^\circ\text{C} \end{aligned}\right\} \quad h_2 = 286.71 \text{ kJ/kg}$$

$$\left.\begin{aligned} P_3 &= 0.72 \text{ MPa} \\ T_3 &= 26^\circ\text{C} \end{aligned}\right\} \quad h_3 \cong h_{f@\,26^\circ\text{C}} = 87.83 \text{ kJ/kg}$$

$$h_4 \cong h_3 \text{ (throttling)} \quad \rightarrow \quad h_4 = 87.83 \text{ kJ/kg}$$

(*a*) The rate of heat removal from the refrigerated space and the power input to the compressor are determined from their definitions:

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.05\ \text{kg/s})[(246.37 - 87.83)\ \text{kJ/kg}] = \mathbf{7.93\ kW}$$

and

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1) = (0.05\ \text{kg/s})[(286.71 - 246.37)\ \text{kJ/kg}] = \mathbf{2.02\ kW}$$

(*b*) The isentropic efficiency of the compressor is determined from

$$\eta_C \cong \frac{h_{2s} - h_1}{h_2 - h_1}$$

where the enthalpy at state 2*s* ($P_{2s} = 0.8$ MPa and $s_{2s} = s_1 = 0.9724$ kJ/kg·K) is 284.20 kJ/kg. Thus,

$$\eta_C = \frac{284.20 - 246.37}{286.71 - 246.37} = 0.938 \quad \text{or} \quad \mathbf{93.8\%}$$

(*c*) The coefficient of performance of the refrigerator is

$$\text{COP}_\text{R} = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = \frac{7.93\ \text{kW}}{2.02\ \text{kW}} = \mathbf{3.93}$$

Discussion This problem is identical to the one worked out in Example 9–12, except that the refrigerant is slightly superheated at the compressor inlet and subcooled at the condenser exit. Also, the compressor is not isentropic. As a result, the heat removal rate from the refrigerated space increases (by 10.3 percent), but the power input to the compressor increases even more (by 11.6 percent). Consequently, the COP of the refrigerator decreases from 3.97 to 3.93.

9–18 ▪ HEAT PUMP SYSTEMS

Heat pumps are generally more expensive to purchase and install than other heating systems, but they save money in the long run in some areas because they lower the heating bills. Despite their relatively higher initial costs, the popularity of heat pumps is increasing. About one-third of all single-family homes built in the United States in recent years are heated by heat pumps.

The most common energy source for heat pumps is atmospheric air (air-to-air systems), although water and soil are also used. The major problem with air-source systems is *frosting,* which occurs in humid climates when the temperature falls below 2 to 5°C. The frost accumulation on the evaporator coils is highly undesirable since it seriously disrupts heat transfer. The coils can be defrosted, however, by reversing the heat pump cycle (running it as an air conditioner). This results in a reduction in the efficiency of the system. Water-source systems usually use well water from depths of up to 80 m in the temperature range of 5 to 18°C, and they do not have a frosting problem. They typically have higher COPs but are more complex and require easy access to a large body of water such as underground water. Ground-source systems are also rather involved since they require long tubing to be placed deep in the ground where the soil temperature is relatively constant. The COP of heat pumps usually ranges between 1.5 and 4, depending on the particular system used and the temperature of the source. A new class of recently developed heat pumps that use variable-speed electric motor drives are at least twice as energy efficient as their predecessors.

Both the capacity and the efficiency of a heat pump fall significantly at low temperatures. Therefore, most air-source heat pumps require a supplementary heating system such as electric resistance heaters or an oil or gas furnace. Since water and soil temperatures do not fluctuate much, supplementary heating may not be required for water-source or ground-source systems. However, the heat pump system must be large enough to meet the maximum heating load.

Heat pumps and air conditioners have the same mechanical components. Therefore, it is not economical to have two separate systems to meet the heating and cooling requirements of a building. One system can be used as a heat pump in winter and an air conditioner in summer. This is accomplished by adding a reversing valve to the cycle, as shown in Fig. 9–61. As a result of this modification, the condenser of the heat pump (located indoors) functions as the evaporator of the air conditioner in summer. Also, the evaporator of the heat pump (located outdoors) serves as the condenser of the air conditioner. This feature increases the competitiveness of the heat pump. Such dual-purpose units are commonly used in motels.

Heat pumps are most competitive in areas that have a large cooling load during the cooling season and a relatively small heating load during the heating season, such as in the southern parts of the United States. In these areas, the heat pump can meet the entire cooling and heating needs of residential or commercial buildings. The heat pump is least competitive in areas where the heating load is very large and the cooling load is small, such as in the northern parts of the United States.

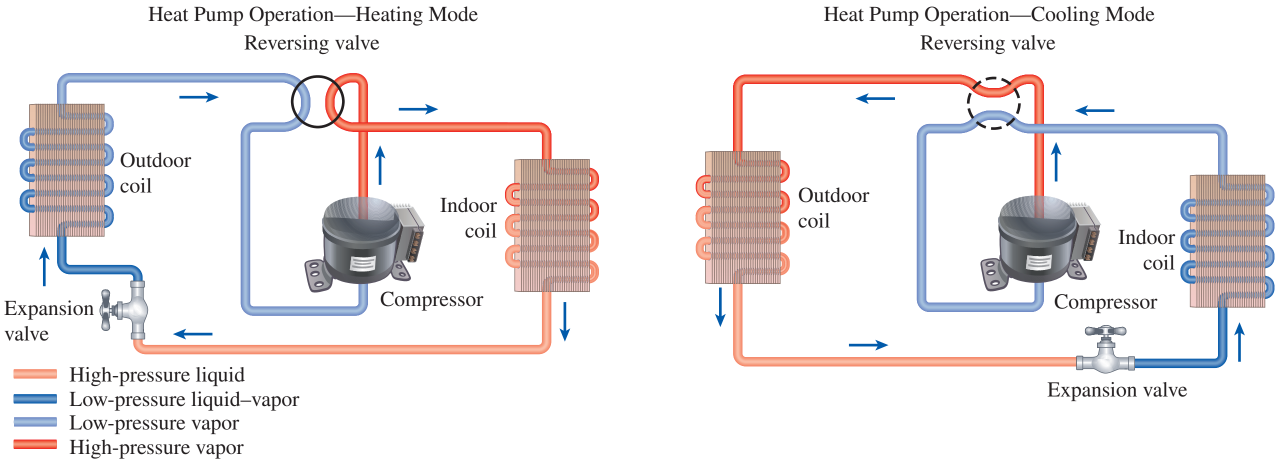


FIGURE 9–61

A heat pump can be used to heat a house in winter and to cool it in summer.

SUMMARY

The most efficient cycle operating between a heat source at temperature T_H and a sink at temperature T_L is the Carnot cycle, and its thermal efficiency is given by

$$\eta_{\text{th, Carnot}} = 1 - \frac{T_L}{T_H}$$

The actual gas cycles are rather complex. The approximations used to simplify the analysis are known as the *air-standard assumptions.* Under these assumptions, all the processes are assumed to be internally reversible; the working fluid is assumed to be air, which behaves as an ideal gas; and the combustion and exhaust processes are replaced by heat-addition and heat-rejection processes, respectively. The air-standard assumptions are called *cold-air-standard assumptions* if, in addition, air is assumed to have constant specific heats at room temperature.

In reciprocating engines, the *compression ratio r* and the *mean effective pressure* MEP are defined as

$$r = \frac{V_{\max}}{V_{\min}} = \frac{V_{\text{BDC}}}{V_{\text{TDC}}}$$

$$\text{MEP} = \frac{w_{\text{net}}}{v_{\max} - v_{\min}}$$

The *Otto cycle* is the ideal cycle for spark-ignition reciprocating engines, and it consists of four internally reversible processes: isentropic compression, constant-volume heat addition, isentropic expansion, and constant-volume heat rejection. Under cold-air-standard assumptions, the thermal efficiency of the ideal Otto cycle is

$$\eta_{\text{th, Otto}} = 1 - \frac{1}{r^{k-1}}$$

where r is the compression ratio and k is the specific heat ratio c_p/c_υ.

The *Diesel cycle* is the ideal cycle for compression-ignition reciprocating engines. It is very similar to the Otto cycle, except that the constant-volume heat-addition process is replaced by a constant-pressure heat-addition process. Its thermal efficiency under cold-air-standard assumptions is

$$\eta_{\text{th, Diesel}} = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right]$$

where r_c is the *cutoff ratio,* defined as the ratio of the cylinder volumes after and before the combustion process.

The ideal cycle for modern gas-turbine engines is the *Brayton cycle,* which is made up of four internally reversible processes: isentropic compression, constant-pressure heat addition, isentropic expansion, and constant-pressure heat rejection. Under cold-air-standard assumptions, its thermal efficiency is

$$\eta_{\text{th, Brayton}} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

where $r_p = P_{\text{max}}/P_{\text{min}}$ is the pressure ratio and k is the specific heat ratio. The thermal efficiency of the simple Brayton cycle increases with the pressure ratio.

The deviation of the actual compressor and the turbine from the idealized isentropic ones can be accurately accounted for by utilizing their isentropic efficiencies, defined as

$$\eta_C = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

and

$$\eta_T = \frac{w_a}{w_s} \cong \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

where states 1 and 3 are the inlet states, 2a and 4a are the actual exit states, and 2s and 4s are the isentropic exit states.

In gas-turbine engines, the temperature of the exhaust gas leaving the turbine is often considerably higher than the temperature of the air leaving the compressor. Therefore, the high-pressure air leaving the compressor can be heated by transferring heat to it from the hot exhaust gases in a counterflow heat exchanger, which is also known as a *regenerator.* The extent to which a regenerator approaches an ideal regenerator is called the *effectiveness* ϵ and is defined as

$$\epsilon = \frac{q_{\text{regen,act}}}{q_{\text{regen,max}}}$$

Under cold-air-standard assumptions, the thermal efficiency of an ideal Brayton cycle with regeneration becomes

$$\eta_{\text{th, regen}} = 1 - \left(\frac{T_1}{T_3}\right)\left(r_p\right)^{(k-1)/k}$$

where T_1 and T_3 are the minimum and maximum temperatures, respectively, in the cycle.

The *Carnot cycle* is not a suitable model for vapor power cycles because it cannot be approximated in practice. The model cycle for vapor power cycles is the *Rankine cycle,* which is composed of four internally reversible processes: constant-pressure heat addition in a boiler, isentropic expansion in a turbine, constant-pressure heat rejection in a condenser, and isentropic compression in a pump. Steam leaves the condenser as a saturated liquid at the condenser pressure.

The thermal efficiency of the Rankine cycle can be increased by increasing the average temperature at which heat is transferred to the working fluid and/or by decreasing the average temperature at which heat is rejected to the cooling medium. The average temperature during heat rejection can be decreased by lowering the turbine exit pressure. Consequently, the condenser pressure of most vapor power plants is well below the atmospheric pressure. The average temperature during heat addition can be increased by raising the boiler pressure or by superheating the fluid to high temperatures. There is a limit to the degree of superheating, however, since the fluid temperature is not allowed to exceed a metallurgically safe value.

Superheating has the added advantage of decreasing the moisture content of the steam at the turbine exit. Lowering the exhaust pressure or raising the boiler pressure, however, increases the moisture content. To take advantage of the improved efficiencies at higher boiler pressures and lower condenser pressures, steam is usually *reheated* after expanding partially in the high-pressure turbine. This is done by extracting the steam after partial expansion in the high-pressure turbine, sending it back to the boiler where it is reheated at constant pressure, and returning it to the low-pressure turbine for complete expansion to the condenser pressure. The average temperature during the reheat process, and thus the thermal efficiency of the cycle, can be increased by increasing the number of expansion and reheat stages. As the number of stages is increased, the expansion and reheat processes approach an isothermal process at maximum temperature. Reheating also decreases the moisture content at the turbine exit.

The transfer of heat from lower-temperature regions to higher-temperature ones is called *refrigeration.* Devices that produce refrigeration are called *refrigerators,* and the cycles on which they operate are called *refrigeration cycles*. The working fluids used in refrigerators are called *refrigerants*. Refrigerators used for the purpose of heating a space by transferring heat from a cooler medium are called *heat pumps*.

The performance of refrigerators and heat pumps is expressed in terms of *coefficient of performance* (COP), defined as

$$\text{COP}_\text{R} = \frac{\text{Desired output}}{\text{Required output}} = \frac{\text{Cooling effect}}{\text{Work input}} = \frac{Q_L}{W_{\text{net, in}}}$$

$$\text{COP}_\text{HP} = \frac{\text{Desired output}}{\text{Required output}} = \frac{\text{Heating effect}}{\text{Work input}} = \frac{Q_H}{W_{\text{net, in}}}$$

The standard of comparison for refrigeration cycles is the *reversed Carnot cycle*. A refrigerator or heat pump that operates on the reversed Carnot cycle is called a *Carnot refrigerator* or a *Carnot heat pump*, and their COPs are

$$\mathrm{COP}_{\mathrm{R,\ Carnot}} = \frac{1}{T_H/T_L - 1}$$

$$\mathrm{COP}_{\mathrm{HP,\ Carnot}} = \frac{1}{1 - T_L/T_H}$$

The most widely used refrigeration cycle is the *vapor-compression refrigeration cycle*. In an ideal vapor-compression refrigeration cycle, the refrigerant enters the compressor as a saturated vapor and is cooled to the saturated liquid state in the condenser. It is then throttled to the evaporator pressure and vaporizes as it absorbs heat from the refrigerated space.

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PROBLEMS*

Actual and Ideal Cycles, Carnot Cycle, Air-Standard Assumptions, Reciprocating Engines

9–1C What are the air-standard assumptions?

9–2C What is the difference between air-standard assumptions and the cold-air-standard assumptions?

9–3C Why is the Carnot cycle not suitable as an ideal cycle for all power-producing cyclic devices?

9–4C How does the thermal efficiency of an ideal cycle, in general, compare to that of a Carnot cycle operating between the same temperature limits?

9–5C How are the combustion and exhaust processes modeled under the air-standard assumptions?

9–6C What does the area enclosed by the cycle represent on a P-v diagram? How about on a T-s diagram?

9–7C Define the compression ratio for reciprocating engines.

9–8C Can the mean effective pressure of an automobile engine in operation be less than the atmospheric pressure?

9–9C What is the difference between spark-ignition and compression-ignition engines?

9–10C Define the following terms related to reciprocating engines: stroke, bore, top dead center, and clearance volume.

9–11C What is the difference between the clearance volume and the displacement volume of reciprocating engines?

9–12 Can any ideal gas power cycle have a thermal efficiency greater than 55 percent when using thermal energy reservoirs at 627°C and 17°C?

9–13 An air-standard cycle is executed within a closed piston–cylinder system and consists of three processes as follows:

1-2 V = *constant* heat addition from 100 kPa and 27°C to 850 kPa

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*Problems designated by a “C” are concept questions, and students are encouraged to answer them all. Problems designated by an “E” are in English units, and SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

2-3 Isothermal expansion until $\upsilon_3 = 7\upsilon_2$

3-1 P = *constant* heat rejection to the initial state

Assume air has constant properties with c_υ = 0.718 kJ/kg·K, c_p = 1.005 kJ/kg·K, R = 0.287 kJ/kg·K, and k = 1.4.

(*a*) Sketch the P-υ and T-s diagrams for the cycle.
(*b*) Determine the ratio of the compression work to the expansion work (the back work ratio).
(*c*) Determine the cycle thermal efficiency.
Answers: (*b*) 0.453, (*c*) 25.6 percent

9–14 An air-standard cycle with variable specific heats is executed in a closed system with 0.003 kg of air and consists of the following three processes:

1-2 υ = *constant* heat addition from 95 kPa and 17°C to 380 kPa

2-3 Isentropic expansion to 95 kPa

3-1 P = *constant* heat rejection to initial state

(*a*) Show the cycle on P-υ and T-s diagrams.
(*b*) Calculate the net work per cycle, in kJ.
(*c*) Determine the thermal efficiency.

9–15 Repeat Prob. 9–14 using constant specific heats at room temperature.

9–16E An air-standard cycle with variable specific heats is executed in a closed system and is composed of the following four processes:

1-2 υ = *constant* heat addition from 14.7 psia and 80°F in the amount of 300 Btu/lbm

2-3 P = *constant* heat addition to 3200 R

3-4 Isentropic expansion to 14.7 psia

4-1 P = *constant* heat rejection to initial state

(*a*) Show the cycle on P-υ and T-s diagrams.
(*b*) Calculate the total heat input per unit mass.
(*c*) Determine the thermal efficiency.
Answers: (*b*) 612 Btu/lbm, (*c*) 24.2 percent

9–17E Repeat Prob. 9–16E using constant specific heats at room temperature.

9–18 An air-standard Carnot cycle is executed in a closed system between the temperature limits of 350 and 1200 K. The pressures before and after the isothermal compression are 150 and 300 kPa, respectively. If the net work output per cycle is 0.5 kJ, determine (*a*) the maximum pressure in the cycle, (*b*) the heat transfer to air, and (*c*) the mass of air. Assume variable specific heats for air. *Answers:* (*a*) 30.0 MPa, (*b*) 0.706 kJ, (*c*) 0.00296 kg

9–19 Repeat Prob. 9–18 using helium as the working fluid.

9–20E The thermal energy reservoirs of an ideal gas Carnot cycle are at 1240°F and 40°F, and the device executing this cycle rejects 100 Btu of heat each time the cycle is executed. Determine the total heat supplied to and the total work produced by this cycle each time it is executed.

9–21 Consider a Carnot cycle executed in a closed system with 0.6 kg of air. The temperature limits of the cycle are 300 and 1100 K, and the minimum and maximum pressures that occur during the cycle are 20 and 3000 kPa. Assuming constant specific heats, determine the net work output per cycle.

Otto Cycle

9–22C What four processes make up the ideal Otto cycle?

9–23C Are the processes that make up the Otto cycle analyzed as closed-system or steady-flow processes? Why?

9–24C How do the efficiencies of the ideal Otto cycle and the Carnot cycle compare for the same temperature limits? Explain.

9–25C How does the thermal efficiency of an ideal Otto cycle change with the compression ratio of the engine and the specific heat ratio of the working fluid?

9–26C Why are high compression ratios not used in spark-ignition engines?

9–27C An ideal Otto cycle with a specified compression ratio is executed using (*a*) air, (*b*) argon, and (*c*) ethane as the working fluid. For which case will the thermal efficiency be the highest? Why?

9–28C How is the rpm (revolutions per minute) of an actual four-stroke gasoline engine related to the number of thermodynamic cycles? What would your answer be for a two-stroke engine?

9–29C What is the difference between fuel-injected gasoline engines and diesel engines?

9–30E Determine the mean effective pressure of an ideal Otto cycle that uses air as the working fluid; its state at the beginning of the compression is 14 psia and 60°F; its temperature at the end of the combustion is 1500°F; and its compression ratio is 9. Use constant specific heats at room temperature.

9–31E Reconsider Prob. 9–30E. Determine the rate of heat addition and rejection for this ideal Otto cycle when it produces 140 hp.

9–32 An ideal Otto cycle has a compression ratio of 8. At the beginning of the compression process, air is at 95 kPa and 27°C, and 750 kJ/kg of heat is transferred to air during the constant-volume heat-addition process. Taking into account the variation of specific heats with temperature, determine (*a*) the pressure and temperature at the end of the heat-addition process, (*b*) the net work output, (*c*) the thermal efficiency, and (*d*) the mean effective pressure for the cycle. *Answers:* (*a*) 3898 kPa, 1539 K, (*b*) 392 kJ/kg, (*c*) 52.3 percent, (*d*) 495 kPa

9–33 Reconsider Prob. 9–32. Using appropriate software, study the effect of varying the compression ratio from 5 to 10. Plot the net work output and thermal efficiency as a function of the compression ratio. Plot the T-s and P-υ diagrams for the cycle when the compression ratio is 8.

9–34 Repeat Prob. 9–32 using constant specific heats at room temperature.

9–35E A spark-ignition engine has a compression ratio of 10, an isentropic compression efficiency of 85 percent, and an isentropic expansion efficiency of 95 percent. At the beginning of the compression, the air in the cylinder is at 13 psia and 60°F. The maximum gas temperature is found to be 2300°F by measurement. Determine the heat supplied per unit mass, the thermal efficiency, and the mean effective pressure of this engine when modeled with the Otto cycle. Use constant specific heats at room temperature. *Answers:* 225 Btu/lbm, 49.7 percent, 45.3 psia

9–36 An ideal Otto cycle has a compression ratio of 7. At the beginning of the compression process, $P_1 = 90$ kPa, $T_1 = 27°\text{C}$, and $V_1 = 0.004\ \text{m}^3$. The maximum cycle temperature is 1127°C. For each repetition of the cycle, calculate the heat rejection and the net work production. Also calculate the thermal efficiency and mean effective pressure for this cycle. Use constant specific heats at room temperature. *Answers:* 1.03 kJ, 1.21 kJ, 54.1 percent, 354 kPa

9–37 A six-cylinder, 4-L spark-ignition engine operating on the ideal Otto cycle takes in air at 90 kPa and 20°C. The minimum enclosed volume is 15 percent of the maximum enclosed volume. When operated at 2500 rpm, this engine produces 90 hp. Determine the rate of heat addition to this engine. Use constant specific heats at room temperature.

9–38E An ideal Otto cycle with air as the working fluid has a compression ratio of 8. The minimum and maximum temperatures in the cycle are 540 and 2400 R. Accounting for the variation of specific heats with temperature, determine (*a*) the amount of heat transferred to the air during the heat-addition process, (*b*) the thermal efficiency, and (*c*) the thermal efficiency of a Carnot cycle operating between the same temperature limits.

9–39E Repeat Prob. 9–38E using argon as the working fluid.

9–40 When we double the compression ratio of an ideal Otto cycle, what happens to the maximum gas temperature and pressure when the state of the air at the beginning of the compression and the amount of heat addition remain the same? Use constant specific heats at room temperature.

Diesel Cycle

9–41C How does a diesel engine differ from a gasoline engine?

9–42C How does the ideal Diesel cycle differ from the ideal Otto cycle?

9–43C What is the cutoff ratio? How does it affect the thermal efficiency of a Diesel cycle?

9–44C For a specified compression ratio, is a diesel or gasoline engine more efficient?

9–45C Do diesel or gasoline engines operate at higher compression ratios? Why?

9–46 An ideal Diesel cycle has a compression ratio of 18 and a cutoff ratio of 1.5. Determine the maximum air temperature and the rate of heat addition to this cycle when it produces 200 hp of power; the cycle is repeated 1200 times per minute; and the state of the air at the beginning of the compression is 95 kPa and 17°C. Use constant specific heats at room temperature.

9–47 Rework Prob. 9–46 when the isentropic compression efficiency is 90 percent and the isentropic expansion efficiency is 95 percent.

9–48 An air-standard Diesel cycle has a compression ratio of 16 and a cutoff ratio of 2. At the beginning of the compression process, air is at 95 kPa and 27°C. Accounting for the variation of specific heats with temperature, determine (*a*) the temperature after the heat-addition process, (*b*) the thermal efficiency, and (*c*) the mean effective pressure. *Answers:* (*a*) 1725 K, (*b*) 56.3 percent, (*c*) 675.9 kPa

9–49 Repeat Prob. 9–48 using constant specific heats at room temperature.

9–50E An air-standard Diesel cycle has a compression ratio of 18.2. Air is at 120°F and 14.7 psia at the beginning of the compression process and at 3200 R at the end of the heat-addition process. Accounting for the variation of specific heats with temperature, determine (*a*) the cutoff ratio, (*b*) the heat rejection per unit mass, and (*c*) the thermal efficiency.

9–51E Repeat Prob. 9–50E using constant specific heats at room temperature.

9–52 An ideal Diesel cycle has a maximum cycle temperature of 2000°C. The state of the air at the beginning of the compression is $P_1 = 95$ kPa and $T_1 = 15°\text{C}$. This cycle is executed in a four-stroke, eight-cylinder engine with a cylinder bore of 10 cm and a piston stroke of 12 cm. The minimum volume enclosed in the cylinder is 5 percent of the maximum cylinder volume. Determine the power produced by this engine when it is operated at 1600 rpm. Use constant specific heats at room temperature. *Answer:* 96.5 kW

9–53 An ideal diesel engine has a compression ratio of 20 and uses air as the working fluid. The state of air at the beginning of the compression process is 95 kPa and 20°C. If the maximum temperature in the cycle is not to exceed 2200 K, determine (*a*) the thermal efficiency and (*b*) the mean effective pressure. Assume constant specific heats for air at room temperature. *Answers:* (*a*) 63.5 percent, (*b*) 933 kPa

9–54 Repeat Prob. 9–53, but replace the isentropic expansion process with a polytropic expansion process with the polytropic exponent $n = 1.35$. Use variable specific heats.

9–55 Reconsider Prob. 9–54. Using appropriate software, study the effect of varying the compression ratio from 14 to 24. Plot the net work output, mean effective pressure, and thermal efficiency as a function of the compression ratio. Plot the T-s and P-v diagrams for the cycle when the compression ratio is 20.

9–56 A four-cylinder, two-stroke 2.4-L diesel engine that operates on an ideal Diesel cycle has a compression ratio of 22 and a cutoff ratio of 1.8. Air is at 70°C and 97 kPa at the beginning of the compression process. Using the cold-air-standard assumptions, determine how much power the engine will deliver at 4250 rpm.

9–57 Repeat Prob. 9–56 using nitrogen as the working fluid.

9–58E An air-standard dual cycle has a compression ratio of 20 and a cutoff ratio of 1.3. The pressure ratio during the constant-volume heat addition process is 1.2. Determine the thermal efficiency, amount of heat added, and the maximum gas pressure and temperature when this cycle is operated at 14 psia and 70°F at the beginning of the compression. Use constant specific heats at room temperature.

9–59E Repeat Prob. 9–58E if the compression ratio is reduced to 12.

Ideal and Actual Gas-Turbine (Brayton) Cycles

9–60C What four processes make up the simple ideal Brayton cycle?

9–61C For fixed maximum and minimum temperatures, what is the effect of the pressure ratio on (*a*) the thermal efficiency and (*b*) the net work output of a simple ideal Brayton cycle?

9–62C What is the back work ratio? What are typical back work ratio values for gas-turbine engines?

9–63C Why are the back work ratios relatively high in gas-turbine engines?

9–64C How do the inefficiencies of the turbine and the compressor affect (*a*) the back work ratio and (*b*) the thermal efficiency of a gas-turbine engine?

9–65E A simple ideal Brayton cycle with air as the working fluid has a pressure ratio of 10. The air enters the compressor at 520 R and the turbine at 2000 R. Accounting for the variation of specific heats with temperature, determine (*a*) the air temperature at the compressor exit, (*b*) the back work ratio, and (*c*) the thermal efficiency.

9–66 A stationary gas-turbine power plant operates on a simple ideal Brayton cycle with air as the working fluid. The air enters the compressor at 95 kPa and 290 K and the turbine at 760 kPa and 1100 K. Heat is transferred to air at a rate of 35,000 kJ/s. Determine the power delivered by this plant (*a*) assuming constant specific heats at room temperature and (*b*) accounting for the variation of specific heats with temperature.

9–67 A gas-turbine power plant operates on the simple Brayton cycle with air as the working fluid and delivers 32 MW of power. The minimum and maximum temperatures in the cycle are 310 and 900 K, and the pressure of air at the compressor exit is 8 times the value at the compressor inlet. Assuming an isentropic efficiency of 80 percent for the compressor and 86 percent for the turbine, determine the mass flow rate of air through the cycle. Account for the variation of specific heats with temperature.

9–68 Repeat Prob. 9–67 using constant specific heats at room temperature.

9–69 A simple ideal Brayton cycle operates with air with minimum and maximum temperatures of 27°C and 727°C. It is designed so that the maximum cycle pressure is 2000 kPa and the minimum cycle pressure is 100 kPa. Determine the net work produced per unit mass of air each time this cycle is executed and the cycle's thermal efficiency. Use constant specific heats at room temperature.

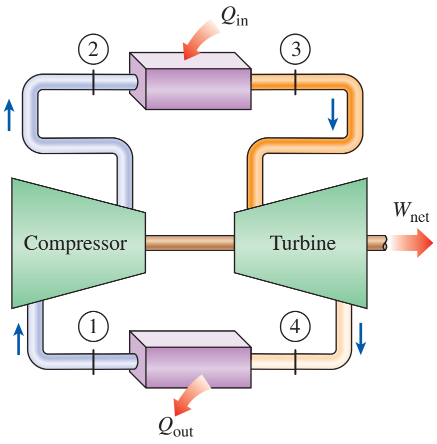


FIGURE P9-69

9–70 Repeat Prob. 9–69 when the isentropic efficiency of the turbine is 90 percent.

9–71 Repeat Prob. 9–69 when the isentropic efficiency of the turbine is 90 percent and that of the compressor is 80 percent.

9–72 Repeat Prob. 9–69 when the isentropic efficiencies of the turbine and compressor are 90 percent and 80 percent, respectively, and there is a 50-kPa pressure drop across the combustion chamber. *Answers:* 7.3 kJ, 3.8 percent

9–73 Air is used as the working fluid in a simple ideal Brayton cycle that has a pressure ratio of 12, a compressor inlet temperature of 300 K, and a turbine inlet temperature of 1000 K. Determine the required mass flow rate of air for a net power output of 70 MW, assuming both the compressor and the turbine have an isentropic efficiency of (*a*) 100 percent and (*b*) 85 percent. Assume constant specific heats at room temperature. *Answers:* (*a*) 352 kg/s, (*b*) 1037 kg/s

9–74 An aircraft engine operates on a simple ideal Brayton cycle with a pressure ratio of 10. Heat is added to the cycle at a rate of 500 kW; air passes through the engine at a rate of 1 kg/s; and the air at the beginning of the compression is at 70 kPa and 0°C. Determine the power produced by this engine and its thermal efficiency. Use constant specific heats at room temperature.

9–75 Repeat Prob. 9–74 for a pressure ratio of 15.

9–76 A gas-turbine power plant operates on the simple Brayton cycle between the pressure limits of 100 and 1600 kPa. The working fluid is air, which enters the compressor at 40°C at a rate of 850 m^3/min and leaves the turbine at 650°C. Assuming a compressor isentropic efficiency of 85 percent and a turbine isentropic efficiency of 88 percent, determine (*a*) the net power output, (*b*) the back work ratio, and (*c*) the thermal efficiency. Use constant specific heats with $c_v = 0.821$ kJ/kg·K, $c_p = 1.108$ kJ/kg·K, and $k = 1.35$. *Answers:* (*a*) 6488 kW, (*b*) 0.511, (*c*) 37.8 percent

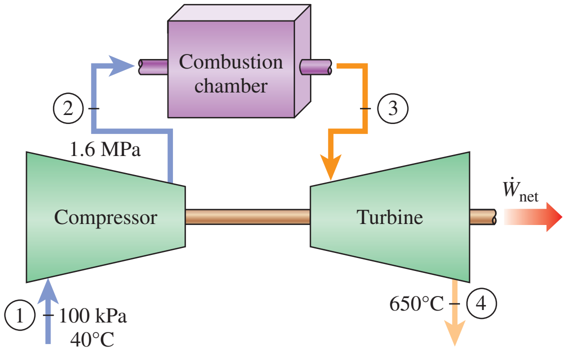


FIGURE P9–76

9–77E A simple ideal Brayton cycle uses argon as the working fluid. At the beginning of the compression, $P_1 = 15$ psia and $T_1 = 80$°F; the maximum cycle temperature is 1200°F; and the pressure in the combustion chamber is 150 psia. The argon enters the compressor through a 3 ft^2 opening with a velocity of 200 ft/s. Determine the rate of heat addition to this engine, the power produced, and the cycle's thermal efficiency.

9–78 A gas-turbine power plant operates on a modified Brayton cycle shown in the figure with an overall pressure ratio of 8. Air enters the compressor at 0°C and 100 kPa. The maximum cycle temperature is 1500 K. The compressor and the turbines are isentropic. The high-pressure turbine develops just enough power to run the compressor. Assume constant properties for air at 300 K with $c_v = 0.718$ kJ/kg·K, $c_p = 1.005$ kJ/kg·K, $R = 0.287$ kJ/kg·K, $k = 1.4$.

(*a*) Sketch the *T-s* diagram for the cycle. Label the data states.
(*b*) Determine the temperature and pressure at state 4, the exit of the high-pressure turbine.
(*c*) If the net power output is 200 MW, determine the mass flow rate of the air into the compressor in kg/s.

Answers: (*b*) 1279 K, 457 kPa, (*c*) 442 kg/s

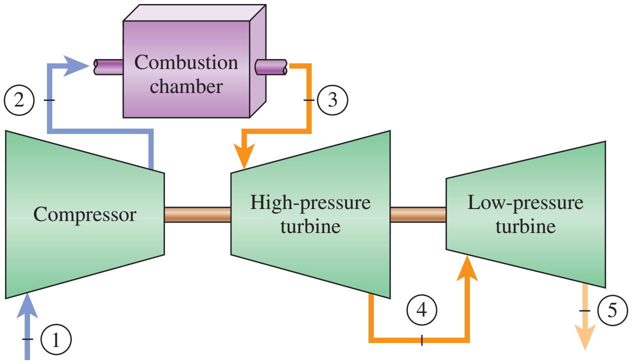


FIGURE P9–78

Brayton Cycle with Regeneration

9–79C How does regeneration affect the efficiency of a Brayton cycle, and how does it accomplish it?

9–80C Define the effectiveness of a regenerator used in gas-turbine cycles.

9–81C Somebody claims that at very high pressure ratios, the use of regeneration actually decreases the thermal efficiency of a gas-turbine engine. Is there any truth in this claim? Explain.

9–82C In an ideal regenerator, is the air leaving the compressor heated to the temperature at (*a*) the turbine inlet, (*b*) the turbine exit, (*c*) slightly above the turbine exit?

9–83C In 1903, Aegidius Elling of Norway designed and built an 11-hp gas turbine that used steam injection between the combustion chamber and the turbine to cool the combustion gases to a safe temperature for the materials available at the time. Currently there are several gas-turbine power plants that use steam injection to augment power and improve thermal efficiency. For example, the thermal efficiency of the General Electric LM5000 gas turbine is reported to increase from 35.8 percent in simple-cycle operation to 43 percent when steam injection is used. Explain why steam injection increases the power output and the efficiency of gas turbines. Also, explain how you would obtain the steam.

9–84 Develop an expression for the thermal efficiency of an ideal Brayton cycle with an ideal regenerator of effectiveness 100 percent. Use constant specific heats at room temperature.

9–85 A gas turbine for an automobile is designed with a regenerator. Air enters the compressor of this engine at 100 kPa and 30°C. The compressor pressure ratio is 8; the maximum cycle temperature is 800°C; and the cold airstream leaves the regenerator 10°C cooler than the hot airstream at the inlet of the regenerator. Assuming both the compressor and the turbine to be isentropic, determine the rates of heat addition and rejection for this cycle when it produces 115 kW. Use constant specific heats at room temperature. *Answers:* 240 kW, 125 kW

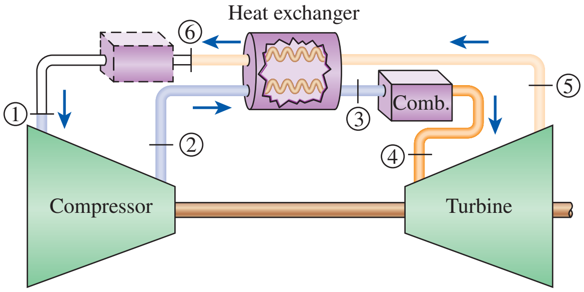


FIGURE P9–85

9–86 Rework Prob. 9–85 when the compressor isentropic efficiency is 87 percent and the turbine isentropic efficiency is 90 percent.

9–87 A gas-turbine engine operates on the ideal Brayton cycle with regeneration, as shown in Fig. P9–85. Now the regenerator is rearranged so that the airstreams of states 2 and 5 enter at one end of the regenerator and streams 3 and 6 exit at the other end (i.e., parallel flow arrangement of a heat exchanger). Consider such a system when air enters the compressor at 100 kPa and 20°C; the compressor pressure ratio is 7; the maximum cycle temperature is 727°C; and the difference between the hot and cold airstream temperatures is 6°C at the end of the regenerator where the cold stream leaves the regenerator. Is the cycle arrangement shown in the figure more or less efficient than this arrangement? Assume both the compressor and the turbine are isentropic, and use constant specific heats at room temperature.

9–88 A Brayton cycle with regeneration using air as the working fluid has a pressure ratio of 7. The minimum and maximum temperatures in the cycle are 310 and 1150 K. Assuming an isentropic efficiency of 75 percent for the compressor and 82 percent for the turbine and an effectiveness of 65 percent for the regenerator, determine (*a*) the air temperature at the turbine exit, (*b*) the net work output, and (*c*) the thermal efficiency. Use variable specific heats. *Answers:* (*a*) 783 K, (*b*) 108 kJ/kg, (*c*) 22.5 percent

9–89 A stationary gas-turbine power plant operates on an ideal regenerative Brayton cycle (ϵ = 100 percent) with air as the working fluid. Air enters the compressor at 95 kPa and 290 K and the turbine at 880 kPa and 1100 K. Heat is transferred to air from an external source at a rate of 30,000 kJ/s. Determine the power delivered by this plant (*a*) assuming constant specific heats for air at room temperature and (*b*) accounting for the variation of specific heats with temperature.

9–90 Air enters the compressor of a regenerative gas-turbine engine at 310 K and 100 kPa, where it is compressed to 900 kPa and 650 K. The regenerator has an effectiveness of 80 percent, and the air enters the turbine at 1400 K. For a turbine efficiency of 90 percent, determine (*a*) the amount of heat transfer in the regenerator and (*b*) the thermal efficiency. Assume variable specific heats for air. *Answers:* (*a*) 193 kJ/kg, (*b*) 40.0 percent

9–91 Repeat Prob. 9–90 using constant specific heats at room temperature.

9–92 Repeat Prob. 9–90 for a regenerator effectiveness of 70 percent.

Carnot Vapor Cycle

9–93C Why is the Carnot cycle not a realistic model for steam power plants?

9–94C Why is excessive moisture in steam undesirable in steam turbines? What is the highest moisture content allowed?

9–95 A steady-flow Carnot cycle uses water as the working fluid. Water changes from saturated liquid to saturated vapor as heat is transferred to it from a source at 250°C. Heat rejection takes place at a pressure of 20 kPa. Show the cycle on a *T-s* diagram relative to the saturation lines, and determine (*a*) the thermal efficiency, (*b*) the amount of heat rejected, and (*c*) the net work output.

9–96 Repeat Prob. 9–95 for a heat rejection pressure of 10 kPa.

9–97 Consider a steady-flow Carnot cycle with water as the working fluid. The maximum and minimum temperatures in the cycle are 350 and 60°C, respectively. The quality of water is 0.891 at the beginning of the heat-rejection process and 0.1 at the end. Show the cycle on a *T-s* diagram relative to the saturation lines, and determine (*a*) the thermal efficiency, (*b*) the pressure at the turbine inlet, and (*c*) the net work output. *Answers:* (*a*) 46.5 percent, (*b*) 1.40 MPa, (*c*) 1623 kJ/kg

9–98E Water enters the boiler of a steady-flow Carnot engine as a saturated liquid at 300 psia and leaves with a quality of 0.95. Steam leaves the turbine at a pressure of 20 psia. Show the cycle on a *T-s* diagram relative to the saturation lines, and determine (*a*) the thermal efficiency, (*b*) the quality at the end of the isothermal heat-rejection process, and (*c*) the net work output. *Answers:* (*a*) 21.6 percent, (*b*) 0.181, (*c*) 166 Btu/lbm

The Simple Rankine Cycle

9–99C What four processes make up the simple ideal Rankine cycle?

9–100C Consider a simple ideal Rankine cycle with fixed turbine inlet conditions. What is the effect of lowering the condenser pressure on

Pump work input:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Turbine work output:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Heat supplied:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Heat rejected:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Cycle efficiency:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Moisture content at turbine exit:	(*a*) increases, (*b*) decreases, (*c*) remains the same

9–101C Consider a simple ideal Rankine cycle with fixed turbine inlet temperature and condenser pressure. What is the effect of increasing the boiler pressure on

Pump work input:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Turbine work output:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Heat supplied:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Heat rejected:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Cycle efficiency:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Moisture content at turbine exit:	(*a*) increases, (*b*) decreases, (*c*) remains the same

9–102C Consider a simple ideal Rankine cycle with fixed boiler and condenser pressures. What is the effect of superheating the steam to a higher temperature on

Pump work input:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Turbine work output:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Heat supplied:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Heat rejected:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Cycle efficiency:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Moisture content at turbine exit:	(*a*) increases, (*b*) decreases, (*c*) remains the same

9–103C How do actual vapor power cycles differ from idealized ones?

9–104C Compare the pressures at the inlet and the exit of the boiler for (*a*) actual and (*b*) ideal cycles.

9–105C The entropy of steam increases in actual steam turbines as a result of irreversibilities. In an effort to control entropy increase, it is proposed to cool the steam in the turbine by running cooling water around the turbine casing. It is argued that this will reduce the entropy and the enthalpy of the steam at the turbine exit and thus increase the work output. How would you evaluate this proposal?

9–106C Is it possible to maintain a pressure of 10 kPa in a condenser that is being cooled by river water entering at 20°C?

9–107 A simple ideal Rankine cycle with water as the working fluid operates between the pressure limits of 3 MPa in the boiler and 30 kPa in the condenser. If the quality at the exit of the turbine cannot be less than 85 percent, what is the maximum thermal efficiency this cycle can have? *Answer:* 29.7 percent

9–108 A simple ideal Rankine cycle with water as the working fluid operates between the pressure limits of 4 MPa in the boiler and 20 kPa in the condenser and a turbine inlet temperature of

700°C. The boiler is sized to provide a steam flow of 50 kg/s. Determine the power produced by the turbine and consumed by the pump.

9–109 A simple ideal Rankine cycle which uses water as the working fluid operates its condenser at 40°C and its boiler at 250°C. Calculate the work produced by the turbine, the heat supplied in the boiler, and the thermal efficiency of this cycle when the steam enters the turbine without any superheating.

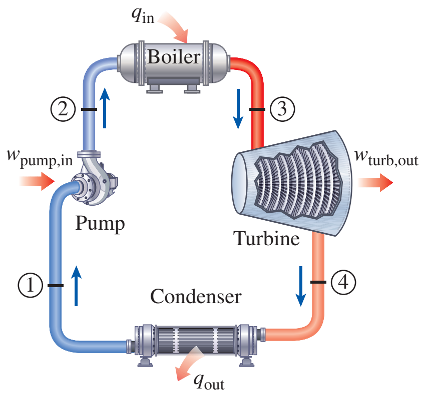


FIGURE P9–109

9–110 Consider a solar-pond power plant that operates on a simple ideal Rankine cycle with refrigerant-134a as the working fluid. The refrigerant enters the turbine as a saturated vapor at 1.4 MPa and leaves at 0.7 MPa. The mass flow rate of the refrigerant is 3 kg/s. Show the cycle on a *T-s* diagram with respect to saturation lines, and determine (*a*) the thermal efficiency of the cycle and (*b*) the power output of this plant.

9–111 Consider a 210-MW steam power plant that operates on a simple ideal Rankine cycle. Steam enters the turbine at 10 MPa and 500°C and is cooled in the condenser at a pressure of 10 kPa. Show the cycle on a *T-s* diagram with respect to saturation lines, and determine (*a*) the quality of the steam at the turbine exit, (*b*) the thermal efficiency of the cycle, and (*c*) the mass flow rate of the steam. *Answers:* (*a*) 0.793, (*b*) 40.2 percent, (*c*) 165 kg/s

9–112 Repeat Prob. 9–111 assuming an isentropic efficiency of 85 percent for both the turbine and the pump. *Answers:* (*a*) 0.874, (*b*) 34.1 percent, (*c*) 194 kg/s

9–113 A simple ideal Rankine cycle with water as the working fluid operates between the pressure limits of 15 MPa in the boiler and 100 kPa in the condenser. Saturated steam enters the turbine. Determine the work produced by the turbine, the heat transferred in the boiler, and thermal efficiency of the cycle. *Answers:* 699 kJ/kg, 2178 kJ/kg, 31.4 percent

9–114 Reconsider Prob. 9–113. Irreversibilities in the turbine cause the steam quality at the outlet of the turbine to be 70 percent. Determine the isentropic efficiency of the turbine and the thermal efficiency of the cycle. *Answers:* 87.7 percent, 27.4 percent

9–115E A steam Rankine cycle operates between the pressure limits of 1500 psia in the boiler and 2 psia in the condenser. The turbine inlet temperature is 800°F. The turbine isentropic efficiency is 90 percent, the pump losses are negligible, and the cycle is sized to produce 2500 kW of power. Calculate the mass flow rate through the boiler, the power produced by the turbine, the rate of heat supply in the boiler, and the thermal efficiency.

9–116E Reconsider Prob. 9–115E. How much error is caused in the thermal efficiency if the power required by the pump were completely neglected?

9–117 A simple Rankine cycle uses water as the working fluid. The boiler operates at 6000 kPa and the condenser at 50 kPa. At the entrance to the turbine, the temperature is 450°C. The isentropic efficiency of the turbine is 94 percent, pressure and pump losses are negligible, and the water leaving the condenser is subcooled by 6.3°C. The boiler is sized for a mass flow rate of 20 kg/s. Determine the rate at which heat is added in the boiler, the power required to operate the pumps, the net power produced by the cycle, and the thermal efficiency. *Answers:* 59,660 kW, 122 kW, 18,050 kW, 30.3 percent

9–118 Reconsider Prob. 9–117. Using appropriate software, determine how much the thermal efficiency of the cycle would change if there were a 50 kPa pressure drop across the boiler.

9–119 The net work output and the thermal efficiency for the Carnot and the simple ideal Rankine cycles with steam as the working fluid are to be calculated and compared. Steam enters the turbine in both cases at 5 MPa as a saturated vapor, and the condenser pressure is 50 kPa. In the Rankine cycle, the condenser exit state is saturated liquid and in the Carnot cycle, the boiler inlet state is saturated liquid. Draw the *T-s* diagrams for both cycles.

9–120 A binary geothermal power plant uses geothermal water at 160°C as the heat source. The plant operates on the simple Rankine cycle with isobutane as the working fluid. Heat is transferred to the cycle by a heat exchanger in which geothermal liquid water enters at 160°C at a rate of 555.9 kg/s and leaves at 90°C. Isobutane enters the turbine at 3.25 MPa and 147°C and leaves at 79.5°C and 410 kPa. Isobutane is condensed in an air-cooled condenser and pumped to the heat exchanger pressure. Assuming the pump to have an isentropic efficiency of 90 percent, determine (*a*) the isentropic efficiency of the turbine, (*b*) the net power output of the plant, and (*c*) the thermal efficiency of the plant.

The properties of isobutane are $h_1 = 273.01$ kJ/kg, $v_1 = 0.001842$ m^3/kg, $h_3 = 761.54$ kJ/kg, $h_4 = 689.74$ kJ/kg, $h_{4s} = 670.40$ kJ/kg. Take the specific heat of geothermal water to be $c_p = 4.258$ kJ/kg·°C.

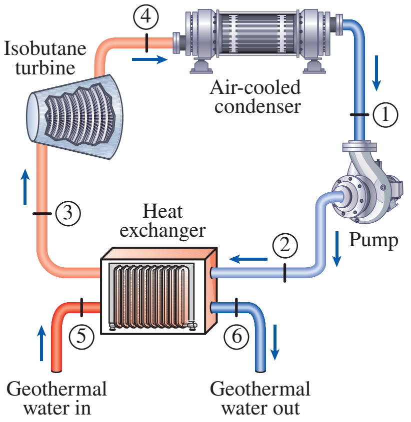


FIGURE P9–120

The Reheat Rankine Cycle

9–121C Show the ideal Rankine cycle with three stages of reheating on a *T-s* diagram. Assume the turbine inlet temperature is the same for all stages. How does the cycle efficiency vary with the number of reheat stages?

9–122C Is there an optimal pressure for reheating the steam of a Rankine cycle? Explain.

9–123C How do the following quantities change when a simple ideal Rankine cycle is modified with reheating? Assume the mass flow rate is maintained the same.

Pump work input:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Turbine work output:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Heat supplied:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Heat rejected:	(*a*) increases, (*b*) decreases, (*c*) remains the same
Moisture content at turbine exit:	(*a*) increases, (*b*) decreases, (*c*) remains the same

9–124C Consider a simple ideal Rankine cycle and an ideal Rankine cycle with three reheat stages. Both cycles operate between the same pressure limits. The maximum temperature is 700°C in the simple cycle and 450°C in the reheat cycle. Which cycle do you think will have a higher thermal efficiency?

9–125 Consider a steam power plant that operates on the ideal reheat Rankine cycle. The plant maintains the boiler at 17.5 MPa, the reheater at 2 MPa, and the condenser at 50 kPa. The temperature is 550°C at the entrance of the high-pressure turbine, and 300°C at the entrance of the low-pressure turbine. Determine the thermal efficiency of this system.

9–126 Reconsider Prob. 9–125. How much does the thermal efficiency of the cycle change when the temperature at the entrance to the low-pressure turbine is increased to 550°C?

9–127 An ideal reheat Rankine cycle with water as the working fluid operates the boiler at 15,000 kPa, the reheater at 2000 kPa, and the condenser at 100 kPa. The temperature is 450°C at the entrance of the high-pressure and low-pressure turbines. The mass flow rate through the cycle is 1.74 kg/s. Determine the power used by pumps, the power produced by the cycle, the rate of heat transfer in the reheater, and the thermal efficiency of this system.

9–128E Steam enters the high-pressure turbine of a steam power plant that operates on the ideal reheat Rankine cycle at 800 psia and 900°F and leaves as saturated vapor. Steam is then reheated to 800°F before it expands to a pressure of 1 psia. Heat is transferred to the steam in the boiler at a rate of 6×10^4 Btu/s. Steam is cooled in the condenser by the cooling water from a nearby river, which enters the condenser at 45°F. Show the cycle on a *T-s* diagram with respect to saturation lines, and determine (*a*) the pressure at which reheating takes place, (*b*) the net power output and thermal efficiency, and (*c*) the minimum mass flow rate of the cooling water required.

9–129 An ideal reheat Rankine cycle with water as the working fluid operates the inlet of the high-pressure turbine at 8000 kPa and 450°C, the inlet of the low-pressure turbine at 500 kPa and 500°C, and the condenser at 10 kPa. Determine the mass flow rate through the boiler needed for this system to produce a net 5000 kW of power and the thermal efficiency of the cycle.

9–130 A steam power plant operates on an ideal reheat Rankine cycle between the pressure limits of 15 MPa and 10 kPa. The mass flow rate of steam through the cycle is 12 kg/s. Steam enters both stages of the turbine at 500°C. If the moisture content of the steam at the exit of the low-pressure turbine is not to exceed 5 percent, determine (*a*) the pressure at which reheating takes place, (*b*) the total rate of heat input in the boiler, and (*c*) the thermal efficiency of the cycle. Also, show the cycle on a *T-s* diagram with respect to saturation lines.

9–131 Consider a steam power plant that operates on a reheat Rankine cycle and has a net power output of 80 MW. Steam enters the high-pressure turbine at 10 MPa and 500°C and the low-pressure turbine at 1 MPa and 500°C. Steam leaves the condenser as a saturated liquid at a pressure of 10 kPa. The isentropic efficiency of the turbine is 80 percent, and that of the pump is 95 percent. Show the cycle on a *T-s* diagram with respect to saturation lines, and determine (*a*) the quality (or temperature, if superheated) of the steam at the turbine exit, (*b*) the thermal efficiency of the cycle, and (*c*) the mass flow rate of the steam. *Answers:* (*a*) 88.1°C, (*b*) 34.1 percent, (*c*) 62.7 kg/s

9–132 Repeat Prob. 9–131 assuming both the pump and the turbine are isentropic. *Answers:* (*a*) 0.949, (*b*) 41.3 percent, (*c*) 50.0 kg/s

The Reversed Carnot Cycle

9–133C Why do we study the reversed Carnot cycle even though it is not a realistic model for refrigeration cycles?

9–134C Why is the reversed Carnot cycle executed within the saturation dome not a realistic model for refrigeration cycles?

9–135 A steady-flow Carnot refrigeration cycle uses refrigerant-134a as the working fluid. The refrigerant changes from saturated vapor to saturated liquid at 60°C in the condenser as it rejects heat. The evaporator pressure is 180 kPa. Show the cycle on a *T-s* diagram relative to saturation lines, and determine (*a*) the coefficient of performance, (*b*) the amount of heat absorbed from the refrigerated space, and (*c*) the net work input. *Answers:* (*a*) 3.58, (*b*) 109 kJ/kg, (*c*) 30.4 kJ/kg

9–136E Refrigerant-134a enters the condenser of a steady-flow Carnot refrigerator as a saturated vapor at 90 psia, and it leaves with a quality of 0.05. The heat absorption from the refrigerated space takes place at a pressure of 30 psia. Show the cycle on a *T-s* diagram relative to saturation lines, and determine (*a*) the coefficient of performance, (*b*) the quality at the beginning of the heat-absorption process, and (*c*) the net work input.

Ideal and Actual Vapor-Compression Refrigeration Cycles

9–137C Does the ideal vapor-compression refrigeration cycle involve any internal irreversibilities?

9–138C Why is the throttling valve not replaced by an isentropic turbine in the ideal vapor-compression refrigeration cycle?

9–139C In a refrigeration system, would you recommend condensing the refrigerant-134a at a pressure of 0.7 or 1.0 MPa if heat is to be rejected to a cooling medium at 15°C? Why?

9–140C Does the area enclosed by the cycle on a *T-s* diagram represent the net work input for the reversed Carnot cycle? How about for the ideal vapor-compression refrigeration cycle?

9–141C Consider two vapor-compression refrigeration cycles. The refrigerant enters the throttling valve as a saturated liquid at 30°C in one cycle and as subcooled liquid at 30°C in the other one. The evaporator pressure for both cycles is the same. Which cycle do you think will have a higher COP?

9–142C It is proposed to use water instead of refrigerant-134a as the working fluid in air-conditioning applications where the minimum temperature never falls below the freezing point. Would you support this proposal? Explain.

9–143C The COP of vapor-compression refrigeration cycles improves when the refrigerant is subcooled before it enters the throttling valve. Can the refrigerant be subcooled indefinitely to maximize this effect, or is there a lower limit? Explain.

9–144 A 10-kW cooling load is to be served by operating an ideal vapor-compression refrigeration cycle with its evaporator at 400 kPa and its condenser at 800 kPa. Calculate the refrigerant mass flow rate and the compressor power requirement when refrigerant-134a is used.

9–145E An ice-making machine operates on the ideal vapor-compression cycle, using refrigerant-134a. The refrigerant enters the compressor as saturated vapor at 20 psia and leaves the condenser as saturated liquid at 80 psia. Water enters the ice machine at 55°F and leaves as ice at 25°F. For an ice production rate of 15 lbm/h, determine the power input to the ice machine (169 Btu of heat needs to be removed from each 1 bm of water at 55°F to turn it into ice at 25°F).

9–146 An air conditioner using refrigerant-134a as the working fluid and operating on the ideal vapor-compression refrigeration cycle is to maintain a space at 22°C while operating its condenser at 1000 kPa. Determine the COP of the system when a temperature difference of 2°C is allowed for the transfer of heat in the evaporator.

9–147 An ideal vapor-compression refrigeration cycle using refrigerant-134a as the working fluid is used to cool a brine solution to −5°C. This solution is pumped to various buildings for the purpose of air conditioning. The refrigerant evaporates at −10°C with a total mass flow rate of 7 kg/s, and condenses at 600 kPa. Determine the COP of the cycle and the total cooling load.

9–148E A refrigerator operates on the ideal vapor-compression refrigeration cycle and uses refrigerant-134a as the working fluid. The condenser operates at 300 psia and the evaporator at 20°F. If an adiabatic, reversible expansion device were available and used to expand the liquid leaving the condenser, how much would the COP improve by using this device instead of the throttle device? *Answer:* 16.5 percent

9–149 A refrigerator uses refrigerant-134a as the working fluid and operates on the ideal vapor-compression refrigeration cycle except for the compression process. The refrigerant enters the evaporator at 120 kPa with a quality of 34 percent and leaves the compressor at 70°C. If the compressor consumes 450 W of power, determine (*a*) the mass flow rate of the refrigerant, (*b*) the condenser pressure, and (*c*) the COP of the refrigerator. *Answers:* (*a*) 0.00644 kg/s, (*b*) 800 kPa, (c) 2.03

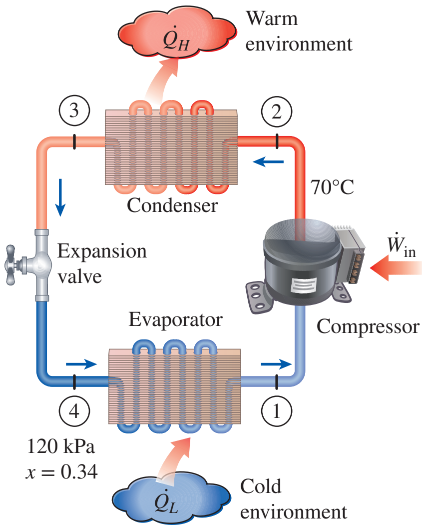


FIGURE P9–149

9–150 An ideal vapor-compression refrigeration cycle that uses refrigerant-134a as its working fluid maintains a condenser at 800 kPa and the evaporator at −20°C. Determine this system's COP and the amount of power required to service a 150 kW cooling load. *Answers:* 3.83, 39.2 kW

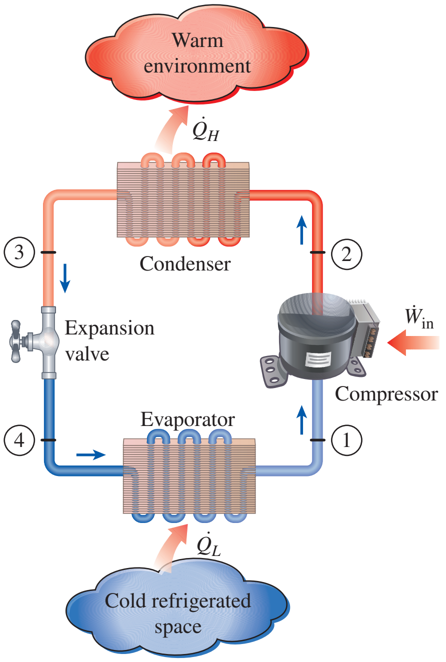


FIGURE P9–150

9–151E A refrigerator uses refrigerant-134a as its working fluid and operates on the ideal vapor-compression refrigeration cycle. The refrigerant evaporates at 5°F and condenses at 180 psia. This unit serves a 45,000 Btu/h cooling load. Determine the mass flow rate of the refrigerant and the power that this unit will require.

9–152E Repeat Prob. 9–151E using appropriate software if ammonia is used in place of refrigerant-134a.

9–153 A refrigerator uses refrigerant-134a as the working fluid and operates on the vapor-compression refrigeration cycle. The evaporator and condenser pressures are 200 kPa and 1400 kPa, respectively. The isentropic efficiency of the compressor is 88 percent. The refrigerant enters the compressor at a rate of 0.025 kg/s superheated by 10.1°C and leaves the condenser subcooled by 4.4°C. Determine (*a*) the rate of cooling provided by the evaporator, the power input, and the COP. Determine (*b*) the same parameters if the cycle operated on the ideal vapor-compression refrigeration cycle between the same pressure limits.

9–154 A commercial refrigerator with refrigerant-134a as the working fluid is used to keep the refrigerated space at −30°C by rejecting its waste heat to cooling water that enters the condenser at 18°C at a rate of 0.25 kg/s and leaves at 26°C. The refrigerant enters the condenser at 1.2 MPa and 65°C and leaves at 42°C. The inlet state of the compressor is 60 kPa and −34°C and the compressor is estimated to gain a net heat of 450 W from the surroundings. Determine (*a*) the quality of the refrigerant at the evaporator inlet, (*b*) the refrigeration load, (*c*) the COP of the refrigerator, and (*d*) the theoretical maximum refrigeration load for the same power input to the compressor.

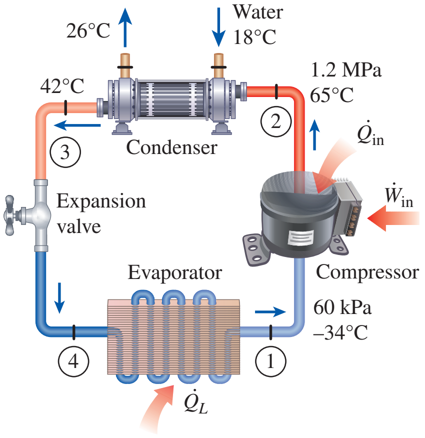


FIGURE P9–154

9–155 The manufacturer of an air conditioner claims a seasonal energy efficiency ratio (SEER) of 16 (Btu/h)/W for one of its units. This unit operates on the normal vapor-compression refrigeration cycle and uses refrigerant-22 as the working fluid. This SEER is for the operating conditions when the evaporator saturation temperature is −5°C and the condenser saturation temperature is 45°C. Selected data for refrigerant-22 are provided in the following table.

T, °C	P_{sat}, kPa	h_f, kJ/kg	h_g, kJ/kg	s_g, kJ/kg·K
–5	421.2	38.76	248.1	0.9344
45	1728	101	261.9	0.8682

(*a*) Sketch the hardware and the *T*-*s* diagram for this air conditioner.
(*b*) Determine the heat absorbed by the refrigerant in the evaporator per unit mass of refrigerant-22, in kJ/kg.
(*c*) Determine the work input to the compressor and the heat rejected in the condenser per unit mass of refrigerant-22, in kJ/kg.

9–156 An actual refrigerator operates on the vapor-compression refrigeration cycle with refrigerant-22 as the working fluid. The refrigerant evaporates at −15°C and condenses at 40°C. The isentropic efficiency of the compressor is 83 percent. The refrigerant is superheated by 5°C at the compressor inlet and subcooled by 5°C at the exit of the condenser. Determine (*a*) the heat removed from the cooled space and the work input, in kJ/kg and the COP of the cycle. Determine (*b*) the same parameters if the cycle operated on the ideal vapor-compression refrigeration cycle between the same evaporating and condensing temperatures.

The properties of R-22 in the case of actual operation are: h_1 = 402.49 kJ/kg, h_2 = 454.00 kJ/kg, h_3 = 243.19 kJ/kg. The properties of R-22 in the case of ideal operation are: h_1 = 399.04 kJ/kg, h_2 = 440.71 kJ/kg, h_3 = 249.80 kJ/kg. Note: state 1: compressor inlet, state 2: compressor exit, state 3: condenser exit, state 4: evaporator inlet.

Heat Pump Systems

9–157C Do you think a heat pump system will be more cost-effective in New York or in Miami? Why?

9–158C What is a water-source heat pump? How does the COP of a water-source heat pump system compare to that of an air-source system?

9–159 A heat pump operates on the ideal vapor-compression refrigeration cycle and uses refrigerant-134a as the working fluid. The condenser operates at 1000 kPa and the evaporator at 200 kPa. Determine this system's COP and the rate of heat supplied to the evaporator when the compressor consumes 6 kW.

9–160 Refrigerant-134a enters the condenser of a residential heat pump at 800 kPa and 50°C at a rate of 0.022 kg/s and leaves at 750 kPa subcooled by 3°C. The refrigerant enters the compressor at 200 kPa superheated by 4°C. Determine (*a*) the isentropic efficiency of the compressor, (*b*) the rate of heat supplied to the heated room, and (*c*) the COP of the heat pump. Also, determine (*d*) the COP and the rate of heat supplied to the heated room if this heat pump operated on the ideal vapor-compression cycle between the pressure limits of 200 and 800 kPa.

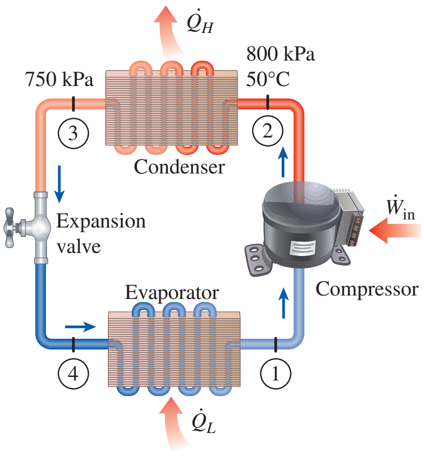


FIGURE P9–160

9–161E A heat pump that operates on the ideal vapor-compression cycle with refrigerant-134a is used to heat a house and maintain it at 75°F by using underground water at 50°F as the heat source. The house is losing heat at a rate of 80,000 Btu/h. The evaporator and condenser pressures are 50 and 120 psia, respectively. Determine the power input to the heat pump and the electric power saved by using a heat pump instead of a resistance heater. *Answers:* 3.28 hp, 28.2 hp

9–162 A heat pump using refrigerant-134a heats a house by using underground water at 8°C as the heat source. The house is losing heat at a rate of 60,000 kJ/h. The refrigerant enters the compressor at 280 kPa and 0°C, and it leaves at 1 MPa and 60°C. The refrigerant exits the condenser at 30°C. Determine (*a*) the power input to the heat pump, (*b*) the rate of heat absorption from the water, and (*c*) the increase in electric power input if an electric resistance heater is used instead of a heat pump. *Answers:* (*a*) 3.55 kW, (*b*) 13.12 kW, (*c*) 13.12 kW

9–163 Reconsider Prob. 9–162. Using appropriate software, investigate the effect of varying the compressor isentropic efficiency over the range 60 to 100 percent. Plot the power input to the compressor and the electric power saved by using a heat pump rather than electric resistance heating as functions of compressor efficiency, and discuss the results.

Review Problems

9–164 An air-standard cycle with variable specific heats is executed in a closed system with 0.003 kg of air, and it consists of the following three processes:

1-2 Isentropic compression from 100 kPa and 27°C to 700 kPa

2-3 $P =$ *constant* heat addition to initial specific volume

3-1 $\upsilon =$ *constant* heat rejection to initial state

(*a*) Show the cycle on P-υ and T-s diagrams.
(*b*) Calculate the maximum temperature in the cycle.
(*c*) Determine the thermal efficiency.

Answers: (*b*) 2100 K, (*c*) 15.8 percent

9–165 Repeat Prob. 9–164 using constant specific heats at room temperature.

9–166 A Carnot cycle is executed in a closed system and uses 0.0025 kg of air as the working fluid. The cycle efficiency is 60 percent, and the lowest temperature in the cycle is 300 K. The pressure at the beginning of the isentropic expansion is 700 kPa, and at the end of the isentropic compression it is 1 MPa. Determine the net work output per cycle.

9–167 An Otto cycle with a compression ratio of 8 begins its compression at 94 kPa and 10°C. The maximum cycle temperature is 900°C. Utilizing air-standard assumptions, determine the thermal efficiency of this cycle using (*a*) constant specific heats at room temperature and (*b*) variable specific heats. *Answers:* (*a*) 56.5 percent, (*b*) 53.7 percent

9–168 A Diesel cycle has a compression ratio of 22 and begins its compression at 85 kPa and 15°C. The maximum cycle temperature is 1200°C. Utilizing air-standard assumptions, determine the thermal efficiency of this cycle using (*a*) constant specific heats at room temperature and (*b*) variable specific heats.

9–169 Consider an engine operating on the ideal Diesel cycle with air as the working fluid. The volume of the cylinder is 1200 cm^3 at the beginning of the compression process, 75 cm^3 at the end, and 150 cm^3 after the heat-addition process. Air is at 17°C and 100 kPa at the beginning of the compression process. Determine (*a*) the pressure at the beginning of the heat-rejection process, (*b*) the net work per cycle in kJ, and (*c*) the mean effective pressure.

9–170 Repeat Prob. 9–169 using argon as the working fluid.

9–171 A four-cylinder, four-stroke, spark-ignition engine operates on the ideal Otto cycle with a compression ratio of 11 and a total displacement volume of 1.8 L. The air is at 90 kPa and 50°C at the beginning of the compression process. The heat input is 0.5 kJ per cycle per cylinder. Determine (*a*) the maximum temperature and pressure that occur during the cycle, (*b*) the net work per cycle per cylinder and the thermal efficiency of the cycle, (*c*) the mean effective pressure, and (*d*) the power output for an engine speed of 3000 rpm. Use constant specific heats with $c_\upsilon = 0.821$ kJ/kg·K, $c_p = 1.108$ kJ/kg·K, and $k = 1.35$.

9–172 A typical hydrocarbon fuel produces 43,000 kJ/kg of heat when used in a spark-ignition engine. Determine the compression ratio required for an ideal Otto cycle to use 0.039 g of fuel to produce 1 kJ of work. Use constant specific heats at room temperature. *Answer:* 9.66

9–173E An ideal dual cycle has a compression ratio of 14 and uses air as the working fluid. At the beginning of the compression process, air is at 14.7 psia and 120°F, and it occupies a volume of 98 in^3. During the heat-addition process, 0.6 Btu of heat is transferred to air at constant volume and 1.1 Btu at constant pressure. Using constant specific heats evaluated at room temperature, determine the thermal efficiency of the cycle.

9–174 Consider a simple ideal Brayton cycle operating between the temperature limits of 300 and 1250 K. Using constant specific heats at room temperature, determine the pressure ratio for which the compressor and the turbine exit temperatures of air are equal.

9–175 Helium is used as the working fluid in a Brayton cycle with regeneration. The pressure ratio of the cycle is 8, the compressor inlet temperature is 300 K, and the turbine inlet temperature is 1800 K. The effectiveness of the regenerator is 75 percent. Determine the thermal efficiency and the required mass flow rate of helium for a net power output of 60 MW, assuming both the compressor and the turbine have an isentropic efficiency of (*a*) 100 percent and (*b*) 80 percent.

9–176 Feedwater at 4000 kPa is heated at a rate of 6 kg/s from 200°C to 245°C in a closed feedwater heater of a regenerative Rankine cycle. Bleed steam enters this unit at 3000 kPa with a quality of 90 percent and leaves as a saturated liquid. Calculate the rate at which bleed steam is required.

9–177 Steam enters the turbine of a steam power plant that operates on a simple ideal Rankine cycle at a pressure of 6 MPa, and it leaves as a saturated vapor at 7.5 kPa. Heat is transferred to the steam in the boiler at a rate of 40,000 kJ/s. Steam is cooled in the condenser by the cooling water from a nearby river, which enters the condenser at 15°C. Show the cycle on a T-s diagram with respect to saturation lines, and determine (*a*) the turbine inlet temperature, (*b*) the net power output and thermal efficiency, and (*c*) the minimum mass flow rate of the cooling water required.

9–178 Consider a steam power plant operating on the ideal Rankine cycle with reheat between the pressure limits of 30 MPa and 10 kPa with a maximum cycle temperature of 700°C and a moisture content of 5 percent at the turbine exit. For a reheat temperature of 700°C, determine the reheat pressures of the cycle for the cases of (*a*) single and (*b*) double reheat.

9–179 A steam power plant operates on an ideal Rankine cycle with two stages of reheat and has a net power output of 75 MW. Steam enters all three stages of the turbine at 550°C. The maximum pressure in the cycle is 10 MPa, and the minimum pressure is 30 kPa. Steam is reheated at 4 MPa the first time and at 2 MPa the second time. Show the cycle on a *T-s* diagram with respect to saturation lines, and determine (*a*) the thermal efficiency of the cycle and (*b*) the mass flow rate of the steam. *Answers:* (*a*) 40.5 percent, (*b*) 48.5 kg/s

9–180 Rooms with floor areas of up to 15 m^2 are cooled adequately by window air conditioners whose cooling capacity is 5000 Btu/h. Assuming the COP of the air conditioner to be 3.5, determine the rate of heat gain of the room, in Btu/h, when the air conditioner is running continuously to maintain a constant room temperature.

9–181 Consider a steady-flow Carnot refrigeration cycle that uses refrigerant-134a as the working fluid. The maximum and minimum temperatures in the cycle are 30 and −20°C, respectively. The quality of the refrigerant is 0.15 at the beginning of the heat absorption process and 0.80 at the end. Show the cycle on a *T-s* diagram relative to saturation lines, and determine (*a*) the coefficient of performance, (*b*) the condenser and evaporator pressures, and (*c*) the net work input.

9–182 Consider an ice-producing plant that operates on the ideal vapor-compression refrigeration cycle and uses refrigerant-134a as the working fluid. The refrigeration cycle operating conditions require an evaporator pressure of 140 kPa and the condenser pressure of 1200 kPa. Cooling water flows through the water jacket surrounding the condenser and is supplied at the rate of 200 kg/s. The cooling water has a 10°C temperature rise as it flows through the water jacket. To produce ice, potable water is supplied to the chiller section of the refrigeration cycle. For each kg of ice produced, 333 kJ of energy must be removed from the potable water supply.

(*a*) Sketch the hardware for all three working fluids of this refrigerant–ice making system and the *T-s* diagram for the refrigeration cycle.
(*b*) Determine the mass flow rate of the refrigerant, in kg/s.
(*c*) Determine the mass flow rate of the potable water supply, in kg/s.

9–183 A heat pump that operates on the ideal vapor-compression cycle with refrigerant-134a is used to heat a house. The mass flow rate of the refrigerant is 0.25 kg/s. The condenser and evaporator pressures are 1400 and 320 kPa, respectively. Show the cycle on a *T-s* diagram with respect to saturation lines, and determine (*a*) the rate of heat supply to the house, (*b*) the volume flow rate of the refrigerant at the compressor inlet, and (*c*) the COP of this heat pump.

9–184 A large refrigeration plant is to be maintained at −15°C, and it requires refrigeration at a rate of 100 kW. The condenser of the plant is to be cooled by liquid water, which experiences a temperature rise of 8°C as it flows over the coils of the condenser. Assuming the plant operates on the ideal vapor-compression cycle using refrigerant-134a between the pressure limits of 120 and 700 kPa, determine (*a*) the mass flow rate of the refrigerant, (*b*) the power input to the compressor, and (*c*) the mass flow rate of the cooling water.

9–185 Reconsider Prob. 9–184. Using appropriate software, investigate the effect of evaporator pressure on the COP and the power input. Let the evaporator pressure vary from 120 to 380 kPa. Plot the COP and the power input as functions of evaporator pressure, and discuss the results.

Design and Essay Problems

9–186 The amount of fuel introduced into a spark-ignition engine is used in part to control the power produced by the engine. Gasoline produces approximately 42,000 kJ/kg when burned with air in a spark-ignition engine. Develop a schedule for gasoline consumption and maximum cycle temperature versus power production for an Otto cycle with a compression ratio of 8.

9–187 The weight of a diesel engine is directly proportional to the compression ratio ($W = kr$) because extra metal must be used to strengthen the engine for the higher pressures. Examine the net specific work produced by a diesel engine per unit of weight as the pressure ratio is varied and the specific heat input remains fixed. Do this for several heat inputs and proportionality constants k. Are there any optimal combinations of k and specific heat inputs?

9–188 In response to concerns about the environment, some major car manufacturers are currently marketing electric cars. Write an essay on the advantages and disadvantages of electric cars, and discuss when it is advisable to purchase an electric car instead of a traditional internal combustion car.

9–189 Intense research is underway to develop adiabatic engines that require no cooling of the engine block. Such engines are based on ceramic materials because of the ability of such materials to withstand high temperatures. Write an essay on the current status of adiabatic engine development. Also determine the highest possible efficiencies with these engines, and compare them to the highest possible efficiencies of current engines.

9–190 Exhaust gases from the turbine of a simple Brayton cycle are quite hot and may be used for other thermal purposes. One proposed use is generating saturated steam at 110°C from water at 30°C in a boiler. This steam will be distributed to several buildings on a college campus for space heating. A Brayton cycle with a pressure ratio of 6 is to be used for this purpose. Plot the power produced, the flow rate of produced steam, and the maximum cycle temperature as functions of the rate at which heat is added to the cycle. The temperature at the turbine inlet is not to exceed 2000°C.

9–191 Design a steam power cycle that can achieve a cycle thermal efficiency of at least 40 percent under the conditions that all turbines have isentropic efficiencies of 85 percent and all pumps have isentropic efficiencies of 60 percent. Prepare an engineering report describing your design. Your design report must include, but is not limited to, the following:

(*a*) Discussion of various cycles attempted to meet the goal as well as the positive and negative aspects of your design.
(*b*) System figures and *T-s* diagrams with labeled states and temperature, pressure, enthalpy, and entropy information for your design.
(*c*) Sample calculations.

9–192 Several geothermal power plants are in operation in the United States. The heat source of a geothermal plant is hot geothermal water, which is "free energy." An 8-MW geothermal power plant is being considered at a location where geothermal water at 160°C is available. Geothermal water is to serve as the heat source for a closed Rankine power cycle with refrigerant-134a as the working fluid. Specify suitable temperatures and pressures for the cycle, and determine the thermal efficiency of the cycle. Justify your selections.

9–193 Contact your power company and obtain information on the thermodynamic aspects of their most recently built power plant. If it is a conventional power plant, find out why it is preferred over a highly efficient combined power plant.

9–194 A company owns a refrigeration system whose refrigeration capacity is 200 tons (1 ton of refrigeration = 211 kJ/min), and you are to design a forced-air cooling system for fruits whose diameters do not exceed 7 cm under the following conditions: The fruits are to be cooled from 28°C to an average temperature of 8°C. The air temperature is to remain above −2°C and below 10°C at all times, and the velocity of air approaching the fruits must remain under 2 m/s. The cooling section can be as wide as 3.5 m and as high as 2 m.

Assuming reasonable values for the average fruit density, specific heat, and porosity (the fraction of air volume in a box), recommend reasonable values for (*a*) the air velocity approaching the cooling section, (*b*) the product-cooling capacity of the system, in kg·fruit/h, and (*c*) the volume flow rate of air.

PART
2

FLUID MECHANICS

CHAPTER

10

INTRODUCTION AND PROPERTIES OF FLUIDS

In the second part of the text we present the fundamentals of fluid mechanics. In this introductory chapter, we introduce the basic concepts commonly used in the analysis of fluid flow to avoid any misunderstandings. We start with a discussion of the no-slip condition, which is responsible for the development of boundary layers adjacent to the solid surfaces, and the no-temperature-jump condition. We continue with a discussion of the numerous ways of classification of fluid flow, such as *viscous versus inviscid regions of flow, internal versus external flow, compressible versus incompressible flow, laminar versus turbulent flow, natural versus forced flow*, and *steady versus unsteady flow*. Then we discuss the properties *vapor pressure* and *viscosity*, which play a dominant role in most aspects of fluid flow. Finally, we present the property *surface tension*, and determine the *capillary rise* from static equilibrium conditions.

OBJECTIVES

The objectives of this chapter are to:

- Understand the basic concepts of fluid mechanics.
- Recognize the various types of fluid flow problems encountered in practice.
- Understand the vapor pressure and its role in the occurrence of cavitation.
- Have a working knowledge of viscosity and the consequences of the frictional effects it causes in fluid flow.
- Calculate the capillary rises and drops due to the surface tension effects.

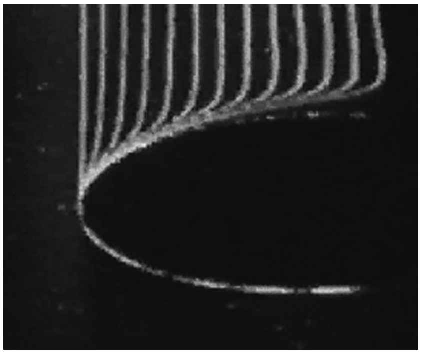


FIGURE 10–1
The development of a velocity profile due to the no-slip condition as a fluid flows over a blunt nose.

"Hunter Rouse: Laminar and Turbulent Flow Film." Copyright IIHR-Hydroscience & Engineering, The University of Iowa. Used by permission.

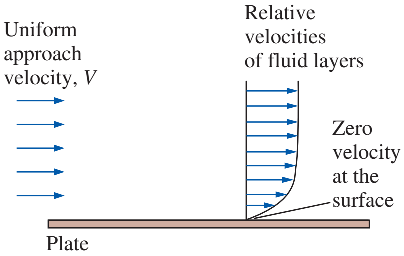


FIGURE 10–2
A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition.

10–1 ▪ THE NO-SLIP CONDITION

Fluid flow is often confined by solid surfaces, and it is important to understand how the presence of solid surfaces affects fluid flow. We know that water in a river cannot flow through large rocks, and must go around them. That is, the water velocity normal to the rock surface must be zero, and water approaching the surface normally comes to a complete stop at the surface. What is not as obvious is that water approaching the rock at any angle also comes to a complete stop at the rock surface, and thus the tangential velocity of water at the surface is also zero.

Consider the flow of a fluid in a stationary pipe or over a solid surface that is nonporous (i.e., impermeable to the fluid). All experimental observations indicate that a fluid in motion comes to a complete stop at the surface and assumes a zero velocity relative to the surface. That is, a fluid in direct contact with a solid "sticks" to the surface, and there is no slip. This is known as the **no-slip condition**. The fluid property responsible for the no-slip condition and the development of the boundary layer is *viscosity* and is discussed in Sec. 10–4.

The photograph in Fig. 10–1 clearly shows the evolution of a velocity gradient as a result of the fluid sticking to the surface of a blunt nose. The layer that sticks to the surface slows the adjacent fluid layer because of viscous forces between the fluid layers, which slows the next layer, and so on. A consequence of the no-slip condition is that all velocity profiles must have zero values with respect to the surface at the points of contact between a fluid and a solid surface (Fig. 10–2). Therefore, the no-slip condition is responsible for the development of the velocity profile. The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is called the **boundary layer**. Another consequence of the no-slip condition is the *surface drag,* or *skin friction drag,* which is the force a fluid exerts on a surface in the flow direction.

When a fluid is forced to flow over a curved surface, such as the back side of a cylinder, the boundary layer may no longer remain attached to the surface and separates from the surface—a process called **flow separation** (Fig. 10–3). We emphasize that the no-slip condition applies *everywhere* along the surface, even downstream of the separation point.

A phenomenon similar to the no-slip condition occurs in heat transfer. When two bodies at different temperatures are brought into contact, heat transfer occurs such that both bodies assume the same temperature at the points of contact. Therefore, a fluid and a solid surface have the same temperature at the points of contact. This is known as the **no-temperature-jump condition**.

10–2 ▪ CLASSIFICATION OF FLUID FLOWS

Earlier we defined *fluid mechanics* as the science that deals with the behavior of fluids at rest or in motion, and the interaction of fluids with solids or other fluids at the boundaries. There is a wide variety of fluid flow problems encountered

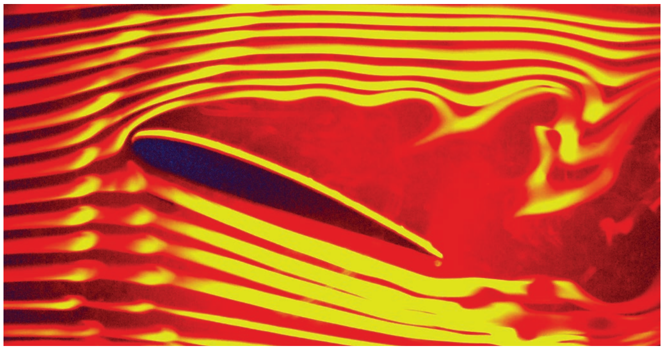


FIGURE 10–3
Flow separation during flow over a curved surface.

Gary S. Settles & Jason Listak/Science Source in practice, and it is usually convenient to classify them on the basis of some common characteristics to make it feasible to study them in groups. There are many ways to classify fluid flow problems, and here we present some general categories.

Viscous Versus Inviscid Regions of Flow

When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer. This internal resistance to flow is quantified by the fluid property *viscosity,* which is a measure of internal stickiness of the fluid. Viscosity is caused by cohesive forces between the molecules in liquids and by molecular collisions in gases. There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree. Flows in which the frictional effects are significant are called **viscous flows**. However, in many flows of practical interest, there are *regions* (typically regions not close to solid surfaces) where viscous forces are negligibly small compared to inertial or pressure forces. Neglecting the viscous terms in such **inviscid flow regions** greatly simplifies the analysis without much loss in accuracy.

The development of viscous and inviscid regions of flow as a result of inserting a flat plate parallel into a fluid stream of uniform velocity is shown in Fig. 10–4. The fluid sticks to the plate on both sides because of the no-slip condition, and the thin boundary layer in which the viscous effects are significant near the plate surface is the *viscous flow region*. The region of flow on both sides away from the plate and largely unaffected by the presence of the plate is the *inviscid flow region*.

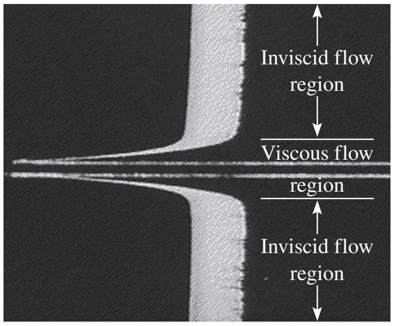


FIGURE 10–4
The flow of an originally uniform fluid stream over a flat plate, and the regions of viscous flow (next to the plate on both sides) and inviscid flow (away from the plate).
Fundamentals of Boundary Layers, National Committee for Fluid Mechanics Films, ©Education Development Center.

Internal Versus External Flow

A fluid flow is classified as being internal or external, depending on whether the fluid flows in a confined space or over a surface. The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is **external flow**. The flow in a pipe or duct is **internal flow** if the fluid is bounded by solid surfaces. Water flow in a pipe, for example, is internal flow, and airflow over a ball or over an exposed pipe during a windy day is external flow (Fig. 10–5). The flow of liquids in a duct is called *open-channel flow* if the duct is only partially filled with the liquid and there is a free surface. The flows of water in rivers and irrigation ditches are examples of such flows.

Internal flows are dominated by the influence of viscosity throughout the flow field. In external flows the viscous effects are limited to boundary layers near solid surfaces and to wake regions downstream of bodies.

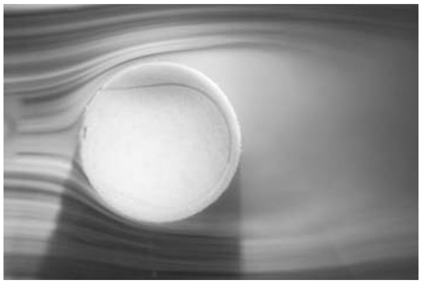


FIGURE 10–5
External flow over a tennis ball, and the turbulent wake region behind.
Courtesy NASA and Cislunar Aerospace, Inc.

Compressible Versus Incompressible Flow

A flow is classified as being *compressible* or *incompressible,* depending on the level of variation of density during flow. Incompressibility is an approximation, in which the flow is said to be **incompressible** if the density remains nearly constant throughout. Therefore, the volume of every portion of fluid remains unchanged over the course of its motion when the flow is approximated as incompressible.

The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible. Therefore, liquids are usually referred to as *incompressible substances.* A pressure of 210 atm, for example, causes the density of liquid water at 1 atm to change by just 1 percent. Gases, on the other hand, are highly compressible. A pressure change of just 0.01 atm, for example, causes a change of 1 percent in the density of atmospheric air.

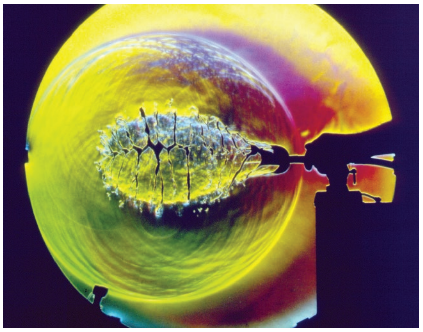


FIGURE 10–6
Schlieren image of the spherical shock wave produced by a bursting balloon at the Penn State Gas Dynamics Lab. Several secondary shocks are seen in the air surrounding the balloon.
Photo by G. S. Settles, Penn State University. Used by permission.

When analyzing rockets, spacecraft, and other systems that involve high-speed gas flows (Fig. 10–6), the flow speed is often expressed in terms of the dimensionless **Mach number** defined as

$$\text{Ma} = \frac{V}{c} = \frac{\text{Speed of flow}}{\text{Speed of sound}}$$

where c is the **speed of sound** whose value is 346 m/s in air at room temperature at sea level. A flow is called **sonic** when Ma = 1, **subsonic** when Ma < 1, **supersonic** when Ma > 1, and **hypersonic** when $\text{Ma} \gg 1$.

Liquid flows are incompressible to a high level of accuracy, but the level of variation of density in gas flows and the consequent level of approximation made when modeling gas flows as incompressible depends on the Mach number. Gas flows can often be approximated as incompressible if the density changes are under about 5 percent, which is usually the case when Ma < 0.3. Therefore, the compressibility effects of air at room temperature can be neglected at speeds under about 100 m/s. Compressibility effects should never be neglected for supersonic flows, however, since compressible flow phenomena like shock waves occur (Fig. 10–6).

Small density changes of liquids corresponding to large pressure changes can still have important consequences. The irritating "water hammer" in a water pipe, for example, is caused by the vibrations of the pipe generated by the reflection of pressure waves following the sudden closing of the valves.

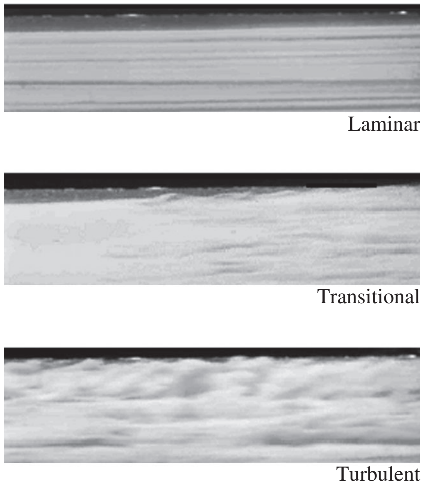


FIGURE 10–7
Laminar, transitional, and turbulent flows over a flat plate.
Courtesy ONERA, photograph by Werle

Laminar Versus Turbulent Flow

Some flows are smooth and orderly while others are rather chaotic. The highly ordered fluid motion characterized by smooth layers of fluid is called **laminar**. The word *laminar* comes from the movement of adjacent fluid particles together in "laminae." The flow of high-viscosity fluids such as oils at low velocities is typically laminar. The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations is called **turbulent** (Fig. 10–7). The flow of low-viscosity fluids such as air at high velocities is typically turbulent. A flow that alternates between being laminar and turbulent is called **transitional**. The experiments conducted by Osborne Reynolds in the 1880s resulted in the establishment of the dimensionless **Reynolds number**, **Re**, as the key parameter for the determination of the flow regime in pipes.

Natural (or Unforced) Versus Forced Flow

A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated. In **forced flow**, a fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In **natural flows**, fluid motion is due to natural means such as the buoyancy effect, which manifests itself as the rise of warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid (Fig. 10–8). In solar hot-water systems, for example, the thermosiphoning effect is commonly used to replace pumps by placing the water tank sufficiently above the solar collectors.

Steady Versus Unsteady Flow

The terms *steady* and *uniform* are used frequently in engineering, and thus it is important to have a clear understanding of their meanings. The term **steady** implies *no change of properties, velocity, temperature, etc., at a point with time.* The opposite of steady is **unsteady**. The term **uniform** implies *no change with location* over a specified region. These meanings are consistent with their everyday use (steady girlfriend, uniform distribution, etc.).

The terms *unsteady* and *transient* are often used interchangeably, but these terms are not synonyms. In fluid mechanics, *unsteady* is the most general term that applies to any flow that is not steady, but **transient** is typically used for developing flows. When a rocket engine is fired up, for example, there are transient effects (the pressure builds up inside the rocket engine, the flow accelerates, etc.) until the engine settles down and operates steadily. The term **periodic** refers to the kind of unsteady flow in which the flow oscillates about a steady mean.

Many devices such as turbines, compressors, boilers, condensers, and heat exchangers operate for long periods of time under the same conditions, and they are classified as *steady-flow devices.* (Note that the flow field near the rotating blades of a turbomachine is of course unsteady, but we consider the overall flow field rather than the details at some localities when we classify devices.) During steady flow, the fluid properties can change from point to point within a device, but at any fixed point they remain constant. Therefore, the volume, the mass, and the total energy content of a steady-flow device or flow section remain constant in steady operation. A simple analogy is shown in Fig. 10–9.

Steady-flow conditions can be closely approximated by devices that are intended for continuous operation such as turbines, pumps, boilers, condensers, and heat exchangers of power plants or refrigeration systems. Some cyclic devices, such as reciprocating engines or compressors, do not satisfy the steady-flow conditions since the flow at the inlets and the exits is pulsating and not steady. However, the fluid properties vary with time in a periodic manner, and the flow through these devices can still be analyzed as a steady-flow process by using time-averaged values for the properties.

Some fascinating visualizations of fluid flow are provided in the book *An Album of Fluid Motion* by Milton Van Dyke (1982). A nice illustration of an unsteady-flow field is shown in Fig. 10–10, taken from Van Dyke's book. Figure 10–10*a* is an instantaneous snapshot from a high-speed motion picture; it reveals large, alternating, swirling, turbulent eddies that are shed into the periodically oscillating wake from the blunt base of the object. The unsteady wake produces waves that move upstream alternately over the top and bottom surfaces of the airfoil in an unsteady fashion. Figure 10–10*b* shows the *same* flow field,

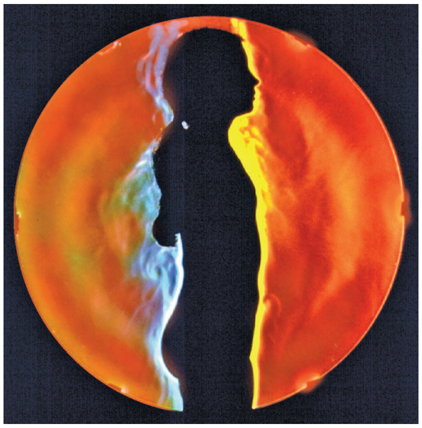
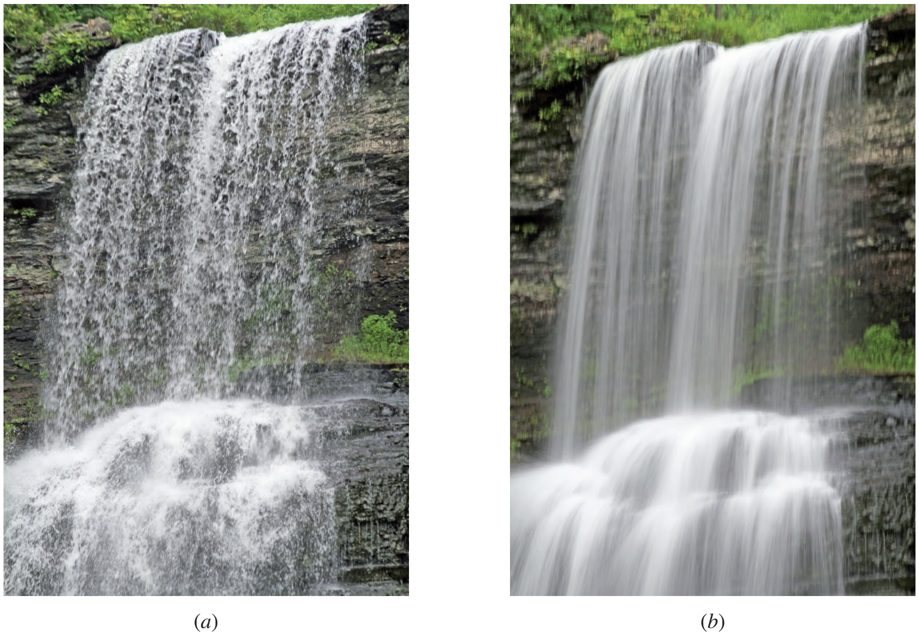


FIGURE 10–8

In this schlieren image of a girl in a swimming suit, the rise of lighter, warmer air adjacent to her body indicates that humans and warm-blooded animals are surrounded by thermal plumes of rising warm air.

G. S. Settles, Gas Dynamics Lab, Penn State University. Used by permission.

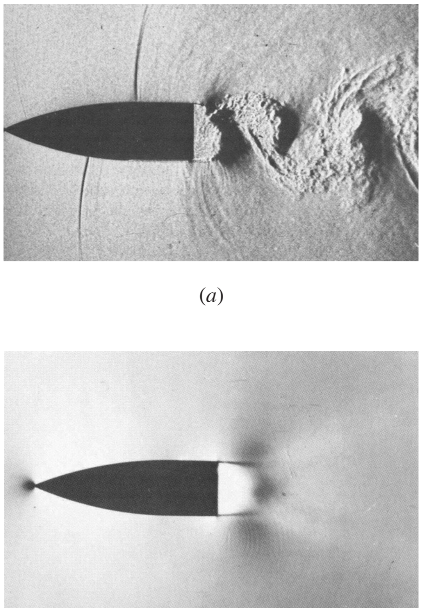


(*a*) (*b*)

FIGURE 10–9

Comparison of (*a*) instantaneous snapshot of an unsteady flow, and (*b*) long exposure picture of the same flow.

Photo by Eric G. Patterson. Used by permission


(*a*)

(*b*)

FIGURE 10–10
Oscillating wake of a blunt-based airfoil at Mach number 0.6. Photo (*a*) is an instantaneous image, while photo (*b*) is a long-exposure (time-averaged) image.

From Dyment, A., Flodrops, J. P. & Gryson, P. 1982. In Flow Visualization II, *W. Merzkirch, ed., 331–336. Washington: Hemisphere. Used by permission of Arthur Dyment.*

but the film is exposed for a longer time so that the image is time averaged over 12 cycles. The resulting time-averaged flow field appears "steady" since the details of the unsteady oscillations have been lost in the long exposure.

One of the most important jobs of an engineer is to determine whether it is sufficient to study only the time-averaged "steady" flow features of a problem, or whether a more detailed study of the unsteady features is required. If the engineer were interested only in the overall properties of the flow field (such as the time-averaged drag coefficient, the mean velocity, and pressure fields), a time-averaged description like that of Fig. 10–10*b*, time-averaged experimental measurements, or an analytical or numerical calculation of the time-averaged flow field would be sufficient. However, if the engineer were interested in details about the unsteady-flow field, such as flow-induced vibrations, unsteady pressure fluctuations, or the sound waves emitted from the turbulent eddies or the shock waves, a time-averaged description of the flow field would be insufficient.

Most of the analytical and computational examples provided in this textbook deal with steady or time-averaged flows, although we occasionally point out some relevant unsteady-flow features as well when appropriate.

One-, Two-, and Three-Dimensional Flows

A flow field is best characterized by its velocity distribution, and thus a flow is said to be one-, two-, or three-dimensional if the flow velocity varies in one, two, or three primary dimensions, respectively. A typical fluid flow involves a three-dimensional geometry, and the velocity may vary in all three dimensions, rendering the flow three-dimensional [$\vec{V}(x, y, z)$ in rectangular or $\vec{V}(r, \theta, z)$ in cylindrical coordinates]. However, the variation of velocity in certain directions can be small relative to the variation in other directions and can be ignored with negligible error. In such cases, the flow can be modeled conveniently as being one- or two-dimensional, which is easier to analyze.

Consider steady flow of a fluid entering from a large tank into a circular pipe. The fluid velocity everywhere on the pipe surface is zero because of the no-slip condition, and the flow is two-dimensional in the entrance region of the pipe since the velocity changes in both the r- and z-directions, but not in the θ-direction. The velocity profile develops fully and remains unchanged after some distance from the inlet (about 10 pipe diameters in turbulent flow, and typically farther than that in laminar pipe flow, as in Fig. 10–11), and the flow in this region is said to be *fully developed*. The fully developed flow in a circular pipe is *one-dimensional* since the velocity varies in the radial r-direction but not in the angular θ- or axial z-directions, as shown in Fig. 10–11. That is, the velocity profile is the same at any axial z-location, and it is symmetric about the axis of the pipe.

Note that the dimensionality of the flow also depends on the choice of coordinate system and its orientation. The pipe flow discussed, for example, is one-dimensional in cylindrical coordinates, but two-dimensional in Cartesian

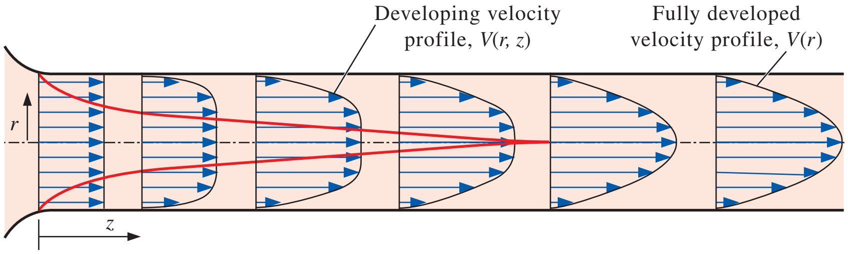


FIGURE 10–11
The development of the velocity profile in a circular pipe. $V = V(r, z)$ and thus the flow is two-dimensional in the entrance region, and becomes one-dimensional downstream when the velocity profile fully develops and remains unchanged in the flow direction, $V = V(r)$.

coordinates—illustrating the importance of choosing the most appropriate coordinate system. Also note that even in this simple flow, the velocity cannot be uniform across the cross section of the pipe because of the no-slip condition. However, at a well-rounded entrance to the pipe, the velocity profile may be approximated as being nearly uniform across the pipe, since the velocity is nearly constant at all radii except very close to the pipe wall.

A flow may be approximated as *two-dimensional* when the aspect ratio is large and the flow does not change appreciably along the longer dimension. For example, the flow of air over a car antenna can be considered two-dimensional except near its ends since the antenna's length is much greater than its diameter, and the airflow hitting the antenna is fairly uniform (Fig. 10–12).

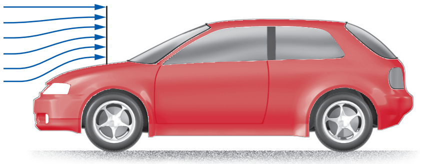


FIGURE 10–12
Flow over a car antenna is approximately two-dimensional except near the top and bottom of the antenna.

EXAMPLE 10–1 Axisymmetric Flow over a Bullet

Consider a bullet piercing through calm air during a short time interval in which the bullet's speed is nearly constant. Determine if the time-averaged airflow over the bullet during its flight is one-, two-, or three-dimensional (Fig. 10–13).

SOLUTION It is to be determined whether airflow over a bullet is one-, two-, or three-dimensional.

Assumptions There are no significant winds and the bullet is not spinning.

Analysis The bullet possesses an axis of symmetry and is therefore an axisymmetric body. The airflow upstream of the bullet is parallel to this axis, and we expect the time-averaged airflow to be rotationally symmetric about the axis—such flows are said to be axisymmetric. The velocity in this case varies with axial distance z and radial distance r, but not with angle θ. Therefore, the time-averaged airflow over the bullet is **two-dimensional**.

Discussion While the time-averaged airflow is axisymmetric, the *instantaneous* airflow is not, as illustrated in Fig. 10–10. In Cartesian coordinates, the flow would be three-dimensional. Finally, many bullets also spin.

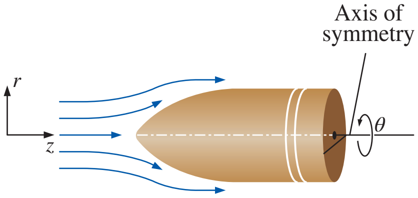


FIGURE 10–13
Axisymmetric flow over a bullet.

Uniform Versus Nonuniform Flow

Uniform flow implies that all fluid properties, such as velocity, pressure, temperature, etc., do not vary with position. A wind tunnel test section, for example, is designed such that the airflow is as uniform as possible. Even then, however, the flow does not remain uniform as we approach the wind tunnel walls, due to the no-slip condition and the presence of a boundary layer, as mentioned previously. The flow just downstream of a well-rounded pipe entrance (Fig. 10–11) is nearly uniform, again except for a very thin boundary layer near the wall. In engineering practice, it is common to approximate the flow in ducts and pipes and at inlets and outlets as uniform, even when it is not, for simplicity in calculations. For example, the fully developed pipe flow velocity profile of Fig. 10–11 is certainly not uniform, but for calculation purposes we sometimes approximate it as the uniform profile at the far left of the pipe, which has the same average velocity. Although this makes the calculations easier, it also introduces some errors that require correction factors; these are discussed in Chaps. 12 and 13 for kinetic energy and momentum, respectively.

10–3 ▪ VAPOR PRESSURE AND CAVITATION

It is well-established that temperature and pressure are dependent properties for pure substances during phase-change processes, and there is one-to-one correspondence between temperature and pressure. At a given pressure, the temperature at which a pure substance changes phase is called the **saturation temperature** T_{sat}. Likewise, at a given temperature, the pressure at which a pure

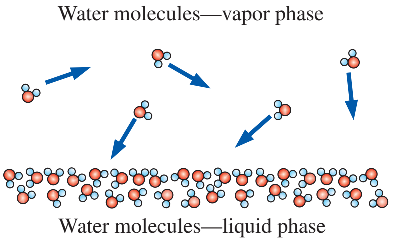


FIGURE 10–14
The vapor pressure (saturation pressure) of a pure substance (e.g., water) is the pressure exerted by its vapor molecules when the system is in phase equilibrium with its liquid molecules at a given temperature.

TABLE 10–1

Saturation (or vapor) pressure of water at various temperatures

Temperature T, °C	Saturation pressure P_{sat}, kPa
−10	0.260
−5	0.403
0	0.611
5	0.872
10	1.23
15	1.71
20	2.34
25	3.17
30	4.25
40	7.38
50	12.35
100	101.3 (1 atm)
150	475.8
200	1554
250	3973
300	8581

substance changes phase is called the **saturation pressure** P_{sat}. At an absolute pressure of 1 standard atmosphere (1 atm or 101.325 kPa), for example, the saturation temperature of water is 100°C. Conversely, at a temperature of 100°C, the saturation pressure of water is 1 atm.

The **vapor pressure** P_v of a pure substance is defined as *the pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature* (Fig. 10–14). P_v is a property of the pure substance, and turns out to be identical to the saturation pressure P_{sat} of the liquid ($P_v = P_{sat}$). We must be careful not to confuse vapor pressure with *partial pressure*. **Partial pressure** is defined as *the pressure of a gas or vapor in a mixture with other gases*. For example, atmospheric air is a mixture of dry air and water vapor, and atmospheric pressure is the sum of the partial pressure of dry air and the partial pressure of water vapor. The partial pressure of water vapor constitutes a small fraction (usually under 3 percent) of the atmospheric pressure since air is mostly nitrogen and oxygen. The partial pressure of a vapor must be less than or equal to the vapor pressure if there is no liquid present. However, when both vapor and liquid are present and the system is in phase equilibrium, the partial pressure of the vapor must equal the vapor pressure, and the system is said to be *saturated*. The rate of evaporation from open water bodies such as lakes is controlled by the difference between the vapor pressure and the partial pressure. For example, the vapor pressure of water at 20°C is 2.34 kPa. Therefore, a bucket of water at 20°C left in a room with dry air at 1 atm will continue evaporating until one of two things happens: the water evaporates away (there is not enough water to establish phase equilibrium in the room), or the evaporation stops when the partial pressure of the water vapor in the room rises to 2.34 kPa at which point phase equilibrium is established.

For phase-change processes between the liquid and vapor phases of a pure substance, the saturation pressure and the vapor pressure are equivalent since the vapor is pure. Note that the pressure value would be the same whether it is measured in the vapor or liquid phase (provided that it is measured at a location close to the liquid–vapor interface to avoid any hydrostatic effects). Vapor pressure increases with temperature. Thus, a substance at higher pressure boils at higher temperature. For example, water boils at 134°C in a pressure cooker operating at 3 atm absolute pressure, but it boils at 93°C in an ordinary pan at a 2000-m elevation, where the atmospheric pressure is 0.8 atm. The saturation (or vapor) pressures are given in Appendices 1 and 2 for various substances. An abridged table for water is given in Table 10–1 for easy reference.

The reason for our interest in vapor pressure is the possibility of the liquid pressure in liquid-flow systems dropping below the vapor pressure at some locations, and the resulting unplanned vaporization. For example, water at 10°C may vaporize and form bubbles at locations (such as the tip regions of impellers or suction sides of pumps) where the pressure drops below 1.23 kPa. The vapor bubbles (called **cavitation bubbles** since they form "cavities" in the liquid) collapse as they are swept away from the low-pressure regions, generating highly destructive, extremely high-pressure waves. This phenomenon, which is a common cause for drop in performance and even the erosion of impeller blades, is called **cavitation**, and it is an important consideration in the design of hydraulic turbines and pumps.

Cavitation must be avoided (or at least minimized) in most flow systems since it reduces performance, generates annoying vibrations and noise, and causes damage to equipment. We note that some flow systems use cavitation to their *advantage,* e.g., high-speed "supercavitating" torpedoes. The pressure spikes resulting from the large number of bubbles collapsing near a solid surface over a long period of time may cause erosion, surface pitting, fatigue failure, and the eventual destruction of the components or machinery (Fig. 10–15). The presence of cavitation in a flow system can be sensed by its characteristic tumbling sound.

EXAMPLE 10–2 Danger of Cavitation in a Propeller

The analysis of a propeller that operates in water at 20°C shows that the pressure at the tips of the propeller drops to 2 kPa at high speeds. Determine if there is a danger of cavitation for this propeller.

SOLUTION The minimum pressure in a propeller is given. It is to be determined if there is a danger of cavitation.

Properties The vapor pressure of water at 20°C is 2.34 kPa (Table 10–1).

Analysis To avoid cavitation, the pressure everywhere in the flow should remain above the vapor (or saturation) pressure at the given temperature, which is

$$P_v = P_{\text{sat@20°C}} = \mathbf{2.34\ kPa}$$

The pressure at the tip of the propeller is 2 kPa, which is less than the vapor pressure. Therefore, **there is a danger of cavitation for this propeller**.

Discussion Note that the vapor pressure increases with increasing temperature, and thus there is a greater danger of cavitation at higher fluid temperatures.

FIGURE 10–15
Cavitation damage on a 16-mm by 23-mm aluminum sample tested at 60 m/s for 2.5 hours. The sample was located at the cavity collapse region downstream of a cavity generator specifically designed to produce high damage potential.

Photo by David Stinebring, ARL/ Pennsylvania State University. Used by permission.

10–4 ▪ VISCOSITY

When two solid bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to motion. To move a table on the floor, for example, we have to apply a force to the table in the horizontal direction large enough to overcome the friction force. The magnitude of the force needed to move the table depends on the *friction coefficient* between the table legs and the floor.

The situation is similar when a fluid moves relative to a solid or when two fluids move relative to each other. We move with relative ease in air, but not so in water. Moving in oil would be even more difficult, as can be observed by the slower downward motion of a glass ball dropped in a tube filled with oil. It appears that there is a property that represents the internal resistance of a fluid to motion or the “fluidity,” and that property is the **viscosity**. The force a flowing fluid exerts on a body in the flow direction is called the **drag force**, and the magnitude of this force depends, in part, on viscosity (Fig. 10–16).

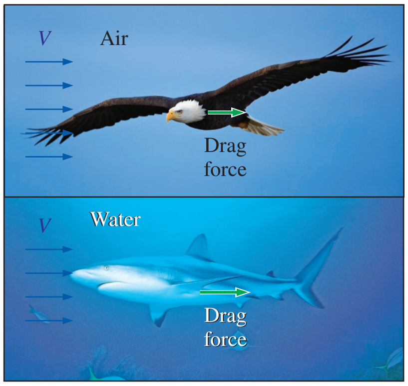


FIGURE 10–16
A fluid moving relative to a body exerts a drag force on the body, partly because of friction caused by viscosity.

Top: Adam Jones
Bottom: Digital Vision/Getty Images

To obtain a relation for viscosity, consider a fluid layer between two very large parallel plates (or equivalently, two parallel plates immersed in a large body of a fluid) separated by a distance ℓ (Fig. 10–17). Now a constant parallel force F is applied to the upper plate while the lower plate is held fixed. After the initial transients, it is observed that the upper plate moves continuously under the influence of this force at a constant speed V. The fluid in contact with the upper plate sticks to the plate surface and moves with it at the same speed, and the shear stress τ acting on this fluid layer is

$$\tau = \frac{F}{A} \tag{10–1}$$

where A is the contact area between the plate and the fluid. Note that the fluid layer deforms continuously under the influence of shear stress.

The fluid in contact with the lower plate assumes the velocity of that plate, which is zero (because of the no-slip condition). In steady laminar flow, the fluid velocity between the plates varies linearly between 0 and V, and thus the *velocity profile* and the *velocity gradient* are

$$u(y) = \frac{y}{\ell}V \quad \text{and} \quad \frac{du}{dy} = \frac{V}{\ell} \tag{10–2}$$

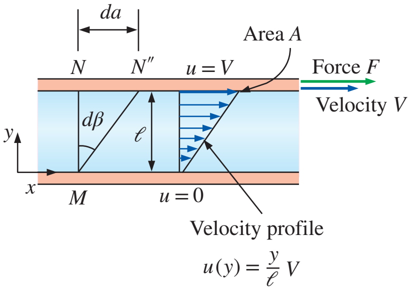


FIGURE 10–17
The behavior of a fluid in laminar flow between two parallel plates when the upper plate moves with a constant velocity.

where y is the vertical distance from the lower plate.

During a differential time interval dt, the sides of fluid particles along a vertical line MN rotate through a differential angle $d\beta$ while the upper plate moves a differential distance $da = V\,dt$. The angular displacement or deformation (or shear strain) can be expressed as

$$d\beta \approx \tan d\beta = \frac{da}{\ell} = \frac{V\,dt}{\ell} = \frac{du}{dy}dt \tag{10–3}$$

Rearranging, the rate of deformation under the influence of shear stress τ becomes

$$\frac{d\beta}{dt} = \frac{du}{dy} \tag{10–4}$$

Thus we conclude that the rate of deformation of a fluid element is equivalent to the velocity gradient du/dy. Further, it can be verified experimentally that for most fluids the rate of deformation (and thus the velocity gradient) is directly proportional to the shear stress τ,

$$\tau \propto \frac{d\beta}{dt} \quad \text{or} \quad \tau \propto \frac{du}{dy} \tag{10–5}$$

Fluids for which the rate of deformation is linearly proportional to the shear stress are called **Newtonian fluids** after Sir Isaac Newton, who expressed it first in 1687. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids. Blood and liquid plastics are examples of non-Newtonian fluids.

In one-dimensional shear flow of Newtonian fluids, shear stress can be expressed by the linear relationship

Shear stress:
$$\tau = \mu \frac{du}{dy} \qquad (\text{N/m}^2) \tag{10–6}$$

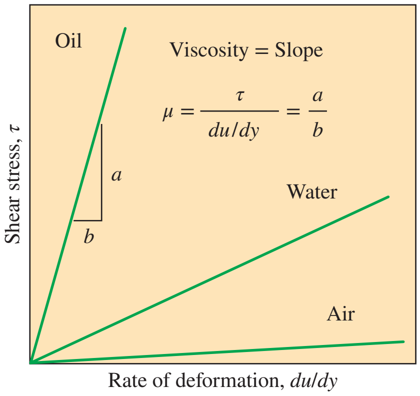


FIGURE 10–18
The rate of deformation (velocity gradient) of a Newtonian fluid is proportional to shear stress, and the constant of proportionality is the viscosity.

where the constant of proportionality μ is called the **coefficient of viscosity** or the **dynamic** (or **absolute**) **viscosity** of the fluid, whose unit is kg/m·s, or equivalently, N·s/m^2 (or Pa·s where Pa is the pressure unit pascal). A common viscosity unit is **poise**, which is equivalent to 0.1 Pa·s (or *centipoise,* which is one-hundredth of a poise). The viscosity of water at 20°C is 1.002 centipoise, and thus the unit centipoise serves as a useful reference. A plot of shear stress versus the rate of deformation (velocity gradient) for a Newtonian fluid is a straight line whose slope is the viscosity of the fluid, as shown in Fig. 10–18. Note that viscosity is independent of the rate of deformation for Newtonian fluids. Since the rate of deformation is proportional to the strain rate, Fig. 10–18 reveals that viscosity is actually a coefficient in a stress–strain relationship.

The **shear force** acting on a Newtonian fluid layer (or, by Newton's third law, the force acting on the plate) is

Shear force:
$$F = \tau A = \mu A \frac{du}{dy} \qquad (\text{N}) \tag{10–7}$$

where again A is the contact area between the plate and the fluid. Then the force F required to move the upper plate in Fig. 10–17 at a constant speed of V while the lower plate remains stationary is

$$F = \mu A \frac{V}{\ell} \qquad (\text{N}) \tag{10–8}$$

This relation can alternately be used to calculate μ when the force F is measured. Therefore, the experimental setup just described can be used to measure the viscosity of fluids. Note that under identical conditions, the force F would be very different for different fluids.

For non-Newtonian fluids, the relationship between shear stress and rate of deformation is not linear, as shown in Fig. 10–19. The slope of the curve on the τ versus du/dy chart is referred to as the *apparent viscosity* of the fluid. Fluids for which the apparent viscosity increases with the rate of deformation (such as solutions with suspended starch or sand) are referred to as *dilatant* or *shear thickening fluids,* and those that exhibit the opposite behavior (the fluid becoming less viscous as it is sheared harder, such as some paints, polymer solutions, and fluids with suspended particles) are referred to as *pseudoplastic* or *shear thinning fluids*. Some materials such as toothpaste can resist a finite shear stress and thus behave as a solid, but deform continuously when the shear stress exceeds the yield stress and behave as a fluid. Such materials are referred to as Bingham plastics after Eugene C. Bingham (1878–1945), who did pioneering work on fluid viscosity for the U.S. National Bureau of Standards in the early twentieth century.

In fluid mechanics and heat transfer, the ratio of dynamic viscosity to density appears frequently. For convenience, this ratio is given the name **kinematic viscosity** ν and is expressed as $\nu = \mu/\rho$. Two common units of kinematic viscosity are m^2/s and **stoke** (1 stoke = 1 cm^2/s = 0.0001 m^2/s).

In general, the viscosity of a fluid depends on both temperature and pressure, although the dependence on pressure is rather weak. For *liquids*, both the dynamic and kinematic viscosities are practically independent of pressure, and any small variation with pressure is usually disregarded, except at extremely high pressures. For *gases,* this is also the case for dynamic viscosity (at low to moderate pressures), but not for kinematic viscosity since the density of a gas is proportional to its pressure (Fig. 10–20).

The viscosity of a fluid is a measure of its resistance to the rate of deformation. Viscosity is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other.

The viscosity of a fluid is directly related to the pumping power needed to transport a fluid in a pipe or to move a body (such as a car in air or a submarine in the sea) through a fluid. Viscosity is caused by the cohesive forces between the molecules in liquids and by the molecular collisions in gases, and it varies greatly with temperature. The viscosity of liquids decreases with temperature, whereas the viscosity of gases increases with temperature (Fig. 10–21). This is because in a liquid the molecules possess more energy at higher temperatures, and they can oppose the large cohesive intermolecular forces more strongly. As a result, the energized liquid molecules can move more freely.

In a gas, on the other hand, the intermolecular forces are negligible, and the gas molecules at high temperatures move randomly at higher velocities. This results in more molecular collisions per unit volume per unit time and therefore in greater resistance to flow. The kinetic theory of gases predicts the viscosity of gases to be proportional to the square root of temperature. That is, $\mu_{\text{gas}} \propto \sqrt{T}$. This prediction is confirmed by practical observations, but deviations for different gases need to be accounted for by incorporating some correction factors. The viscosity of gases is expressed as a function of temperature by the Sutherland correlation (from The U.S. Standard Atmosphere) as

Gases:
$$\mu = \frac{aT^{1/2}}{1 + b/T} \tag{10–9}$$

where T is absolute temperature and a and b are experimentally determined constants. Note that measuring viscosity at two different temperatures is sufficient to determine these constants. For air at atmospheric conditions, the values of

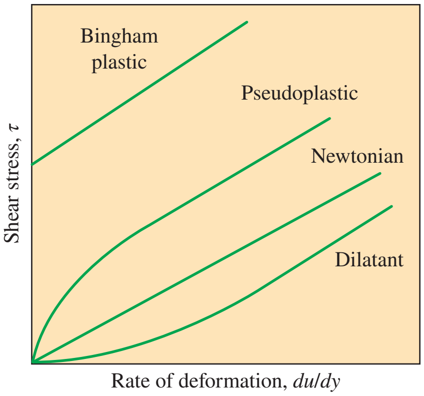


FIGURE 10–19
Variation of shear stress with the rate of deformation for Newtonian and non-Newtonian fluids (the slope of a curve at a point is the apparent viscosity of the fluid at that point).

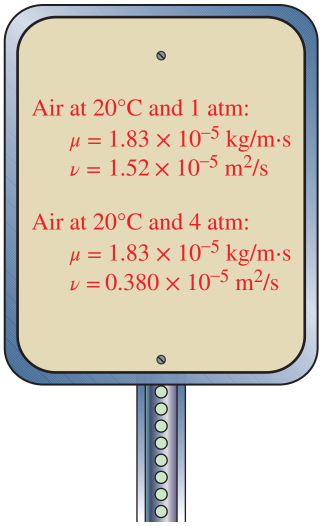


FIGURE 10–20
Dynamic viscosity, in general, does not depend on pressure, but kinematic viscosity does.

these constants are $a = 1.458 \times 10^{-6}$ kg/(m·s·K$^{1/2}$) and $b = 110.4$ K. The viscosity of gases is independent of pressure at low to moderate pressures (from a few percent of 1 atm to several atm). But viscosity increases at high pressures due to the increase in density.

For liquids, the viscosity is approximated as

$$\textit{Liquids:} \qquad \mu = a10^{b/(T-c)} \tag{10–10}$$

where again T is absolute temperature and a, b, and c are experimentally determined constants. For water, using the values $a = 2.414 \times 10^{-5}$ N·s/m^2, $b = 247.8$ K, and $c = 140$ K results in less than 2.5 percent error in viscosity in the temperature range of 0°C to 370°C (Touloukian et al., 1975).

The viscosities of some fluids at room temperature are listed in Table 10–2. They are plotted against temperature in Fig. 10–22. Note that the viscosities of different fluids differ by several orders of magnitude. Also note that it is more difficult to move an object in a higher-viscosity fluid such as engine oil than it is in a lower-viscosity fluid such as water. Liquids, in general, are much more viscous than gases.

Consider a fluid layer of thickness ℓ within a small gap between two concentric cylinders, such as the thin layer of oil in a journal bearing. The gap between the cylinders can be modeled as two parallel flat plates separated by the fluid.

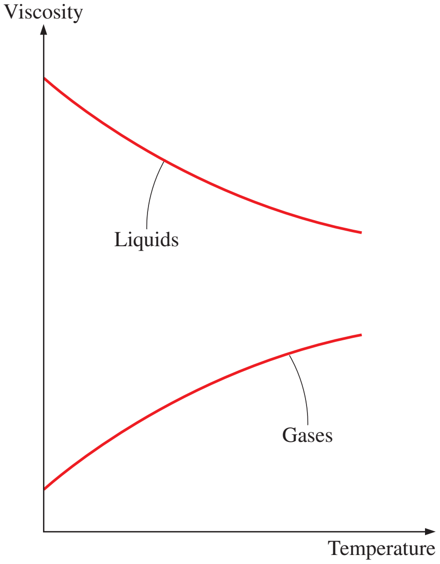


FIGURE 10–21

The viscosity of liquids decreases and the viscosity of gases increases with temperature.

TABLE 10–2

Dynamic viscosity of some fluids at 1 atm and 20°C (unless otherwise stated)

Fluid	Dynamic viscosity μ, kg/m·s
Glycerin:	
−20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.00040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088

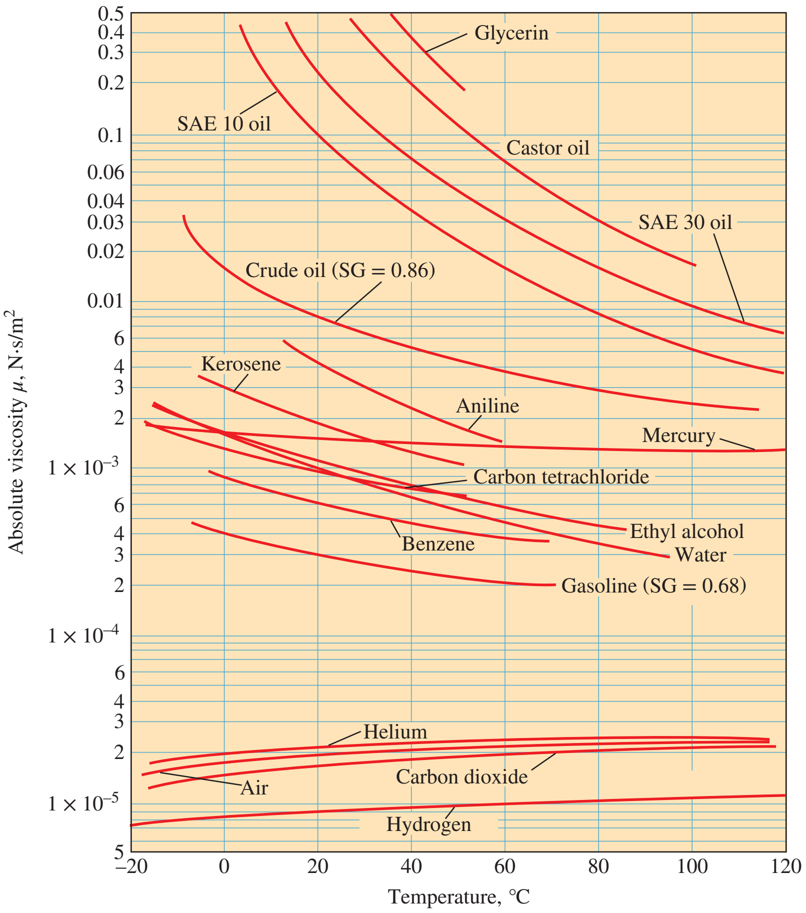


FIGURE 10–22

The variation of dynamic (absolute) viscosity of common fluids with temperature at 1 atm (1 N·s/m^2 = 1 kg/m·s = 0.020886 lbf·s/ft^2).

F. M. White, Fluid Mechanics *7e. Copyright ©2011 The McGraw-Hill Companies, Inc.*

Noting that torque is T = FR (force times the moment arm, which is the radius R of the inner cylinder in this case), the tangential velocity is $V = \omega R$ (angular velocity times the radius), and taking the wetted surface area of the inner cylinder to be $A = 2\pi RL$ by disregarding the shear stress acting on the two ends of the inner cylinder, torque can be expressed as

$$\mathrm{T} = FR = \mu \frac{2\pi R^3 \omega L}{\ell} = \mu \frac{4\pi^2 R^3 \dot{n} L}{\ell} \tag{10–11}$$

where L is the length of the cylinder and $\dot{n}$ is the number of revolutions per unit time, which is usually expressed in rpm (revolutions per minute). Note that the angular distance traveled during one rotation is 2π rad, and thus the relation between the angular velocity in rad/min and the rpm is $\omega = 2\pi\dot{n}$. Equation 10–11 can be used to calculate the viscosity of a fluid by measuring torque at a specified angular velocity. Therefore, two concentric cylinders can be used as a *viscometer,* a device that measures viscosity.

EXAMPLE 10–3 Determining the Viscosity of a Fluid

The viscosity of a fluid is to be measured by a viscometer constructed of two 40-cm-long concentric cylinders (Fig. 10–23). The outer diameter of the inner cylinder is 12 cm, and the gap between the two cylinders is 0.15 cm. The inner cylinder is rotated at 300 rpm, and the torque is measured to be 1.8 N·m. Determine the viscosity of the fluid.

SOLUTION The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

Assumptions **1** The inner cylinder is completely submerged in the fluid. **2** The viscous effects on the two ends of the inner cylinder are negligible.

Analysis The velocity profile is linear only when the curvature effects are negligible, and the profile can be approximated as being linear in this case since $\ell/R = 0.025 \ll 1$. Solving Eq. 10–11 for viscosity and substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{\mathrm{T}\ell}{4\pi^2 R^3 \dot{n} L} = \frac{(1.8\ \mathrm{N{\cdot}m})(0.0015\ \mathrm{m})}{4\pi^2 (0.06\ \mathrm{m})^3 \left(300\ \frac{1}{\mathrm{min}}\right)\left(\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right)(0.4\ \mathrm{m})} = \mathbf{0.158\ N{\cdot}s/m^2}$$

Discussion Viscosity is a strong function of temperature, and a viscosity value without a corresponding temperature has little use. Therefore, the temperature of the fluid should have also been measured during this experiment and reported with this calculation.

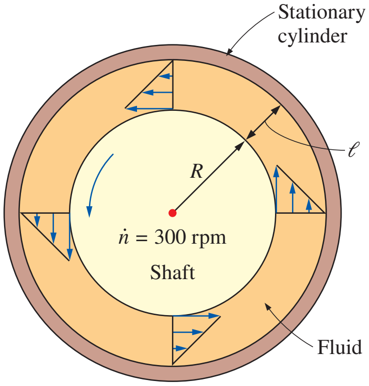


FIGURE 10–23
Schematic for Example 10–3 (not to scale).

10–5 ▪ SURFACE TENSION AND CAPILLARY EFFECT

It is often observed that a drop of blood forms a hump on a horizontal glass; a drop of mercury forms a near-perfect sphere and can be rolled just like a steel ball over a smooth surface; water droplets from rain or dew hang from branches or leaves of trees; a liquid fuel injected into an engine forms a mist of spherical droplets; water dripping from a leaky faucet falls as nearly spherical droplets; a soap bubble released into the air forms a nearly spherical shape; and water beads up into small drops on flower petals (Fig. 10–24*a*).

In these and other observations, liquid droplets behave like small balloons filled with the liquid, and the surface of the liquid acts like a stretched elastic

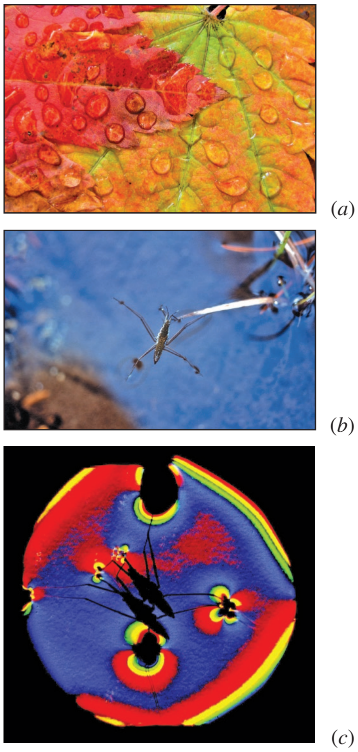


FIGURE 10–24
Some consequences of surface tension: (*a*) drops of water beading up on a leaf, (*b*) a water strider sitting on top of the surface of water, and (*c*) a color schlieren image of the water strider revealing how the water surface dips down where its feet contact the water (it looks like two insects but the second one is just a shadow).

(a) Don Paulson Photography/Purestock/SuperStock
(b) NPS Photo by Rosalie LaRue
(c) Photo courtesy of G. S. Settles, Gas Dynamics Lab, Penn State University, used by permission.

membrane under tension. The pulling force that causes this tension acts parallel to the surface and is due to the attractive forces between the molecules of the liquid. The magnitude of this force per unit length is called **surface tension** or *coefficient of surface tension* σ_s and is usually expressed in the unit N/m (or lbf/ft in English units). This effect is also called *surface energy* (per unit area) and is expressed in the equivalent unit of N·m/m^2 or J/m^2. In this case, σ_s represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount.

To visualize how surface tension arises, we present a microscopic view in Fig. 10–25 by considering two liquid molecules, one at the surface and one deep within the liquid body. The attractive forces applied on the interior molecule by the surrounding molecules balance each other because of symmetry. But the attractive forces acting on the surface molecule are not symmetric, and the attractive forces applied by the gas molecules above are usually very small. Therefore, there is a net attractive force acting on the molecule at the surface of the liquid, which tends to pull the molecules on the surface toward the interior of the liquid. This force is balanced by the repulsive forces from the molecules below the surface that are trying to be compressed. The result is that the liquid minimizes its surface area. This is the reason for the tendency of liquid droplets to attain a spherical shape, which has the minimum surface area for a given volume.

You also may have observed, with amusement, that some insects can land on water or even walk on water (Fig. 10–24*b*) and that small steel needles can float on water. These phenomena are made possible by surface tension, which balances the weights of these objects.

To understand the surface tension effect better, consider a liquid film (such as the film of a soap bubble) suspended on a U-shaped wire frame with a movable side (Fig. 10–26). Normally, the liquid film tends to pull the movable wire inward in order to minimize its surface area. A force F needs to be applied on the movable wire in the opposite direction to balance this pulling effect. Both sides of the thin film are surfaces exposed to air, and thus the length along which the surface tension acts in this case is $2b$. Then a force balance on the movable wire gives $F = 2b\sigma_s$, and thus the surface tension can be expressed as

$$\sigma_s = \frac{F}{2b} \tag{10–12}$$

Note that for $b = 0.5$ m, the measured force F (in N) is simply the surface tension in N/m. An apparatus of this kind with sufficient precision can be used to measure the surface tension of various liquids.

In the U-shaped wire frame apparatus, the movable wire is pulled to stretch the film and increase its surface area. When the movable wire is pulled a distance Δx, the surface area increases by $\Delta A = 2b\ \Delta x$, and the work W done during this stretching process is

$$W = \text{Force} \times \text{Distance} = F\ \Delta x = 2b\sigma_s\ \Delta x = \sigma_s\ \Delta A$$

where we have assumed that the force remains constant over the small distance. This result can also be interpreted as *the surface energy of the film is increased by an amount* $\sigma_s\ \Delta A$ *during this stretching process,* which is consistent with the alternative interpretation of σ_s as surface energy per unit area. This is similar to a rubber band having more potential (elastic) energy after it is stretched further. In the case of liquid film, the work is used to move liquid molecules from the interior parts to the surface against the attraction forces of other molecules. Therefore, surface tension also can be defined as *the work done per unit increase in the surface area of the liquid.*

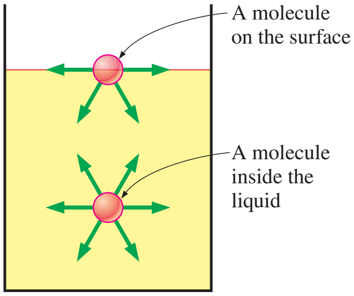


FIGURE 10–25
Attractive forces acting on a liquid molecule at the surface and deep inside the liquid.

The surface tension varies greatly from substance to substance and with temperature for a given substance, as shown in Table 10–3. At 20°C, for example, the surface tension is 0.073 N/m for water and 0.440 N/m for mercury surrounded by atmospheric air. The surface tension of mercury is large enough that mercury droplets form nearly spherical balls that can be rolled like a solid ball on a smooth surface. The surface tension of a liquid, in general, decreases with temperature and becomes zero at the critical point (and thus there is no distinct liquid–vapor interface at temperatures above the critical point). The effect of pressure on surface tension is usually negligible.

The surface tension of a substance can be changed considerably by *impurities*. Therefore, certain chemicals, called *surfactants,* can be added to a liquid to decrease its surface tension. For example, soaps and detergents lower the surface tension of water and enable it to penetrate the small openings between fibers for more effective washing. But this also means that devices whose operation depends on surface tension (such as heat pipes) can be destroyed by the presence of impurities due to poor workmanship.

We speak of surface tension for liquids only at liquid–liquid or liquid–gas interfaces. Therefore, it is imperative that the adjacent liquid or gas be specified when specifying surface tension. Surface tension determines the size of the liquid droplets that form, and so a droplet that keeps growing by the addition of more mass breaks down when the surface tension can no longer hold it together. This is like a balloon that bursts while being inflated when the pressure inside rises above the strength of the balloon material.

A curved interface indicates a pressure difference (or "pressure jump") across the interface with pressure being higher on the concave side. Consider, for example, a droplet of liquid in air, an air (or other gas) bubble in water, or a soap bubble in air. The excess pressure ΔP above atmospheric pressure can be determined by considering a free-body diagram of half the droplet or bubble (Fig. 10–27). Noting that surface tension acts along the circumference and the pressure acts on the area, horizontal force balances for the droplet or air bubble and the soap bubble give

Droplet or air bubble:
$$(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{droplet}} \rightarrow \Delta P_{\text{droplet}} = P_i - P_o = \frac{2\sigma_s}{R} \qquad \textbf{(10–13)}$$

Soap bubble:
$$2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{bubble}} \rightarrow \Delta P_{\text{bubble}} = P_i - P_o = \frac{4\sigma_s}{R} \qquad \textbf{(10–14)}$$

where P_i and P_o are the pressures inside and outside the droplet or bubble, respectively. When the droplet or bubble is in the atmosphere, P_o is simply atmospheric pressure. The extra factor of 2 in the force balance for the soap

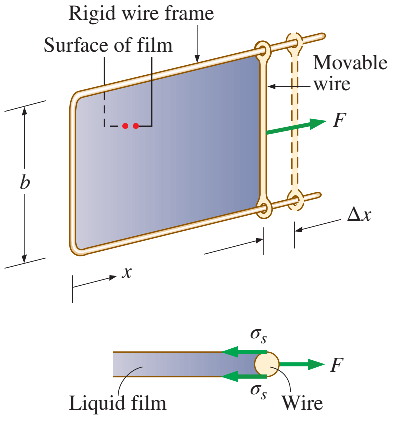


FIGURE 10–26

Stretching a liquid film with a U-shaped wire, and the forces acting on the movable wire of length b.

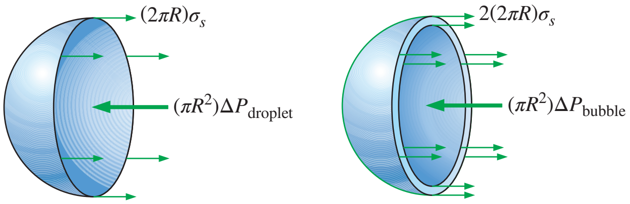
TABLE 10–3

Surface tension of some fluids in air at 1 atm and 20°C (unless otherwise stated)

Fluid	Surface tension σ_s, N/m*
†Water:	
0°C	0.076
20°C	0.073
100°C	0.059
300°C	0.014
Glycerin	0.063
SAE 30 oil	0.035
Mercury	0.440
Ethyl alcohol	0.023
Blood, 37°C	0.058
Gasoline	0.022
Ammonia	0.021
Soap solution	0.025
Kerosene	0.028

* Multiply by 0.06852 to convert to lbf/ft.

† See Appendices for more precise data for water.



(*a*) Half of a droplet or air bubble (*b*) Half of a soap bubble

FIGURE 10–27

The free-body diagram of half of a droplet or air bubble and half of a soap bubble.

bubble is due to the existence of a soap film with *two* surfaces (inner and outer surfaces) and thus two circumferences in the cross section.

The excess pressure in a droplet of liquid in a gas (or a bubble of gas in a liquid) can also be determined by considering a differential increase in the radius of the droplet due to the addition of a differential amount of mass and interpreting the surface tension as the increase in the surface energy per unit area. Then the increase in the surface energy of the droplet during this differential expansion process becomes

$$\delta W_{\text{surface}} = \sigma_s\, dA = \sigma_s\, d(4\pi R^2) = 8\pi R \sigma_s\, dR$$

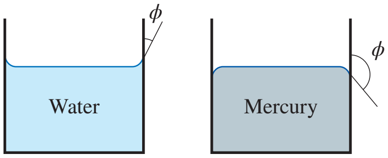
The expansion work done during this differential process is determined by multiplying the force by distance to obtain

$$\delta W_{\text{expansion}} = \text{Force} \times \text{Distance} = F\, dR = (\Delta PA)\, dR = 4\pi R^2\ \Delta P\, dR$$

Equating the two expressions above gives $\Delta P_{\text{droplet}} = 2\sigma_s/R$, which is the same relation obtained before and given in Eq. 10–13. Note that the excess pressure in a droplet or bubble is inversely proportional to the radius.

Capillary Effect

Another interesting consequence of surface tension is the **capillary effect**, which is the rise or fall of a liquid in a small-diameter tube inserted into the liquid. Such narrow tubes or confined flow channels are called **capillaries**. The rise of kerosene through a cotton wick inserted into the reservoir of a kerosene lamp is due to this effect. The capillary effect is also partially responsible for the rise of water to the top of tall trees. The curved free surface of a liquid in a capillary tube is called the **meniscus**.



(*a*) Wetting fluid

(*b*) Nonwetting fluid

FIGURE 10–28
The contact angle for wetting and nonwetting fluids.

It is commonly observed that water in a glass container curves up slightly at the edges where it touches the glass surface; but the opposite occurs for mercury: it curves down at the edges (Fig. 10–28). This effect is usually expressed by saying that water *wets* the glass (by sticking to it) while mercury does not. The strength of the capillary effect is quantified by the **contact** (or *wetting*) **angle** ϕ, defined as *the angle that the tangent to the liquid surface makes with the solid surface at the point of contact*. The surface tension force acts along this tangent line toward the solid surface. A liquid is said to wet the surface when $\phi < 90°$ and not to wet the surface when $\phi > 90°$. In atmospheric air, the contact angle of water (and most other organic liquids) with glass is nearly zero, $\phi \approx 0°$ (Fig. 10–29). Therefore, the surface tension force acts upward on water in a glass tube along the circumference, tending to pull the water up. As a result, water rises in the tube until the weight of the liquid in the tube above the liquid level of the reservoir balances the surface tension force. The contact angle is 130° for mercury–glass and 26° for kerosene–glass in air. Note that the contact angle, in general, is different in different environments (such as another gas or liquid in place of air).

FIGURE 10–29
The meniscus of colored water in two small-diameter glass tubes. Note that the edge of the meniscus meets the wall of the capillary tube at a very small contact angle.
Matt Meadows/McGraw-Hill Education

The phenomenon of the capillary effect can be explained microscopically by considering *cohesive forces* (the forces between like molecules, such as water and water) and *adhesive forces* (the forces between unlike molecules, such as water and glass). The liquid molecules at the solid–liquid interface are subjected to both cohesive forces by other liquid molecules and adhesive forces by the molecules of the solid. The relative magnitudes of these forces determine whether a liquid wets a solid surface or not. Obviously, the water molecules are more strongly attracted to the glass molecules than they are to other water molecules, and thus water tends to rise along the glass surface. The opposite occurs for mercury, which causes the liquid surface near the glass wall to be suppressed (Fig. 10–30).

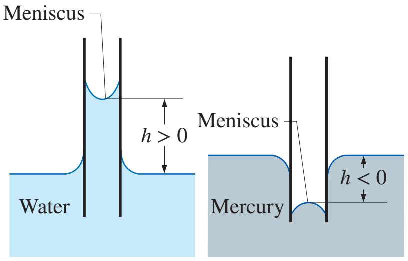


FIGURE 10–30
The capillary rise of water and the capillary fall of mercury in a small-diameter glass tube.

The magnitude of the capillary rise in a circular tube can be determined from a force balance on the cylindrical liquid column of height h in the tube (Fig. 10–31). The bottom of the liquid column is at the same level as the free surface of the reservoir, and thus the pressure there must be atmospheric pressure. This balances the atmospheric pressure acting at the top surface of the liquid column, and thus these two effects cancel each other. The weight of the liquid column is approximately

$$W = mg = \rho V g = \rho g(\pi R^2 h)$$

Equating the vertical component of the surface tension force to the weight gives

$$W = F_{\text{surface}} \rightarrow \rho g(\pi R^2 h) = 2\pi R \sigma_s \cos \phi$$

Solving for h gives the capillary rise to be

Capillary rise: $$h = \frac{2\sigma_s}{\rho g R} \cos \phi \qquad (R = \text{Constant}) \tag{10–15}$$

This relation is also valid for nonwetting liquids (such as mercury in glass) and gives the capillary drop. In this case $\phi > 90°$ and thus $\cos \phi < 0$, which makes h negative. Therefore, a negative value of capillary rise corresponds to a capillary drop (Fig. 10–30).

Note that the capillary rise is inversely proportional to the radius of the tube. Therefore, the thinner the tube is, the greater the rise (or fall) of the liquid in the tube. In practice, the capillary effect for water is usually negligible in tubes whose diameter is greater than 1 cm. When pressure measurements are made using manometers and barometers, it is important to use sufficiently large tubes to minimize the capillary effect. The capillary rise is also inversely proportional to the density of the liquid, as expected. Therefore, in general, lighter liquids experience greater capillary rises. Finally, it should be kept in mind that Eq. 10–15 is derived for constant-diameter tubes and should not be used for tubes of variable cross section.

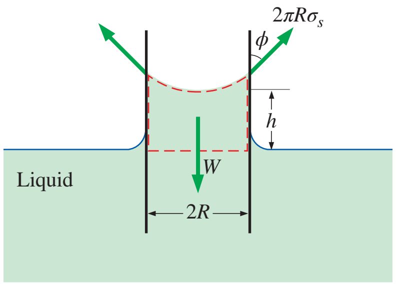


FIGURE 10–31
The forces acting on a liquid column that has risen in a tube due to the capillary effect.

EXAMPLE 10–4 The Capillary Rise of Water in a Tube

A 0.6-mm-diameter glass tube is inserted into water at 20°C in a cup. Determine the capillary rise of water in the tube (Fig. 10–32).

SOLUTION The rise of water in a slender tube as a result of the capillary effect is to be determined.

Assumptions **1** There are no impurities in the water and no contamination on the surfaces of the glass tube. **2** The experiment is conducted in atmospheric air.

Properties The surface tension of water at 20°C is 0.073 N/m (Table 10–3). The contact angle of water with glass is approximately 0° (from preceding text). We take the density of liquid water to be 1000 kg/m^3.

Analysis The capillary rise is determined directly from Eq. 10–15 by substituting the given values, yielding

$$\begin{aligned} h &= \frac{2\sigma_s}{\rho g R} \cos \phi = \frac{2(0.073 \text{ N/m})}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \times 10^{-3} \text{ m})}(\cos 0°)\left(\frac{1 \text{ kg·m/s}^2}{1 \text{ N}}\right) \\ &= 0.050 \text{ m} = \mathbf{5.0 \text{ cm}} \end{aligned}$$

Therefore, water rises in the tube 5 cm above the liquid level in the cup.

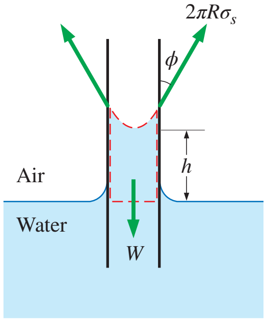


FIGURE 10–32
Schematic for Example 10–4.

Discussion Note that if the tube diameter were 1 cm, the capillary rise would be 0.3 mm, which is hardly noticeable to the eye. Actually, the capillary rise in a large-diameter tube occurs only at the rim. The center does not rise at all. Therefore, the capillary effect can be ignored for large-diameter tubes.

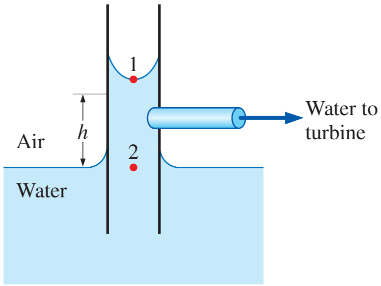


FIGURE 10–33
Schematic for Example 10–5.

EXAMPLE 10–5 Using Capillary Rise to Generate Power in a Hydraulic Turbine

Reconsider Example 10–4. Realizing that water rises by 5 cm under the influence of surface tension without requiring any energy input from an external source, a person conceives the idea that power can be generated by drilling a hole in the tube just below the water level and feeding the water spilling out of the tube into a turbine (Fig. 10–33). The person takes this idea even further by suggesting that a series of tube banks can be used for this purpose and cascading can be incorporated to achieve practically feasible flow rates and elevation differences. Determine if this idea has any merit.

SOLUTION Water that rises in tubes under the influence of the capillary effect is to be used to generate power by feeding it into a turbine. The validity of this suggestion is to be evaluated.

Analysis The proposed system may appear like a stroke of genius, since the commonly used hydroelectric power plants generate electric power by simply capturing the potential energy of elevated water, and the capillary rise provides the mechanism to raise the water to any desired height without requiring any energy input.

When viewed from a thermodynamic point of view, the proposed system immediately can be labeled as a perpetual motion machine (PMM) since it continuously generates electric power without requiring any energy input. That is, the proposed system creates energy, which is a clear violation of the first law of thermodynamics or the conservation of energy principle, and it does not warrant any further consideration. But the fundamental principle of conservation of energy did not stop many from dreaming about being the first to prove nature wrong, and to come up with a trick to permanently solve the world's energy problems. Therefore, the impossibility of the proposed system should be demonstrated.

As you may recall from your physics courses (also to be discussed in the next chapter), the pressure in a static fluid varies in the vertical direction only and increases with increasing depth linearly. Then the pressure difference across the 5-cm-high water column in the tube becomes

$$\begin{aligned}\Delta P_{\text{water column in tube}} &= P_2 - P_1 = \rho_{\text{water}} g h \\ &= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.05\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) \\ &= 0.49\ \text{kN/m}^2\ (\approx 0.005\ \text{atm})\end{aligned}$$

That is, the pressure at the top of the water column in the tube is 0.005 atm *less* than the pressure at the bottom. Noting that the pressure at the bottom of the water column is atmospheric pressure (since it is at the same horizontal line as the water surface in the cup), the pressure anywhere in the tube is below atmospheric pressure with the difference reaching 0.005 atm at the top. Therefore, if a hole were drilled at some elevation in the tube, the top of the meniscus would fall until its elevation was the same as that of the hole.

Discussion The water column in the tube is motionless, and thus, there cannot be any unbalanced force acting on it (zero net force). The force due to the pressure difference across the meniscus between the atmospheric air and the water at the top of water column is balanced by the surface tension. If this surface tension force were to disappear, the water in the tube would drop down under the influence of atmospheric pressure to the level of the free surface in the tube.

SUMMARY

A fluid in direct contact with a solid surface sticks to the surface and there is no slip. This is known as the *no-slip condition,* and it is due to the viscosity of the fluid. At a given temperature, the pressure at which a pure substance changes phase is called the *saturation pressure*. For phase-change processes between the liquid and vapor phases of a pure substance, the saturation pressure is commonly called the *vapor pressure* P_v.

At a given temperature, the pressure at which a pure substance changes phase is called the *saturation pressure*. For phase-change processes between the liquid and vapor phases of a pure substance, the saturation pressure is commonly called the *vapor pressure* P_v. Vapor bubbles that form in the low-pressure regions in a liquid (a phenomenon called *cavitation*) collapse as they are swept away from the low-pressure regions, generating highly destructive, extremely high-pressure waves.

The *viscosity* of a fluid is a measure of its resistance to deformation. The tangential force per unit area is called *shear stress* and is expressed for simple shear flow between plates (one-dimensional flow) as

$$\tau = \mu \frac{du}{dy}$$

where μ is the coefficient of viscosity or the *dynamic* (or *absolute*) *viscosity* of the fluid, u is the velocity component in the flow direction, and y is the direction normal to the flow direction. Fluids that obey this linear relationship are called *Newtonian fluids*. The ratio of dynamic viscosity to density is called the *kinematic viscosity* ν.

The pulling effect on the liquid molecules at an interface caused by the attractive forces of molecules per unit length is called *surface tension* s_s. The excess pressure ΔP inside a spherical droplet or soap bubble, respectively, is given by

$$\Delta P_{\text{droplet}} = P_i - P_o = \frac{2\sigma_s}{R} \quad \text{and} \quad \Delta P_{\text{soap bubble}} = P_i - P_o = \frac{4\sigma_s}{R}$$

where P_i and P_o are the pressures inside and outside the droplet or soap bubble. The rise or fall of a liquid in a small-diameter tube inserted into the liquid due to surface tension is called the *capillary effect*. The capillary rise or drop is given by

$$h = \frac{2\sigma_s}{\rho g R}\cos\phi$$

where ϕ is the *contact angle*. The capillary rise is inversely proportional to the radius of the tube; for water, it is negligible for tubes whose diameter is larger than about 1 cm.

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PROBLEMS*

No-Slip Condition and Classification of Fluid Flows

10–1C Define internal, external, and open-channel flows.

10–2C Define incompressible flow and incompressible fluid. Must the flow of a compressible fluid necessarily be treated as compressible?

10–3C Consider the flow of air over the wings of an aircraft. Is this flow internal or external? How about the flow of gases through a jet engine?

10–4C What is forced flow? How does it differ from natural flow? Is flow caused by winds forced or natural flow?

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*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and SI users can ignore them. Problems with the icon 💻 are comprehensive in nature and are intended to be solved with appropriate software.

10–5C What is the no-slip condition? What causes it?

10–6C What is a boundary layer? What causes a boundary layer to develop?

Vapor Pressure and Cavitation

10–7C What is vapor pressure? How is it related to saturation pressure?

10–8C Does water boil at higher temperatures at higher pressures? Explain.

10–9C If the pressure of a substance is increased during a boiling process, will the temperature also increase or will it remain constant? Why?

10–10C What is cavitation? What causes it?

10–11E The pressure on the suction side of pumps is typically low, and the surfaces on that side of the pump are susceptible to cavitation, especially at high fluid temperatures. If the minimum pressure on the suction side of a water pump is 0.70 psia absolute, determine the maximum water temperature to avoid the danger of cavitation.

10–12 A pump is used to transport water to a higher reservoir. If the water temperature is 20°C, determine the lowest pressure that can exist in the pump without cavitation.

10–13 In a piping system, the water temperature remains under 30°C. Determine the minimum pressure allowed in the system to avoid cavitation.

Viscosity

10–14C What is a Newtonian fluid? Is water a Newtonian fluid?

10–15C What is viscosity? What is the cause of it in liquids and in gases? Do liquids or gases have higher dynamic viscosities?

10–16C How does the kinematic viscosity of (*a*) liquids and (*b*) gases vary with temperature?

10–17C Consider two identical small glass balls dropped into two identical containers, one filled with water and the other with oil. Which ball will reach the bottom of the container first? Why?

10–18E The viscosity of a fluid is to be measured by a viscometer constructed of two 5-ft-long concentric cylinders. The inner diameter of the outer cylinder is 6 in, and the gap between the two cylinders is 0.035 in. The outer cylinder is rotated at 250 rpm, and the torque is measured to be 1.2 lbf·ft. Determine the viscosity of the fluid. *Answer:* 0.000272 lbf·s/ft²

10–19 The dynamic viscosity of carbon dioxide at 50°C and 200°C are 1.612×10^{-5} Pa·s and 2.276×10^{-5} Pa·s, respectively. Determine the constants *a* and *b* of the Sutherland correlation for carbon dioxide at atmospheric pressure. Then predict the viscosity of carbon dioxide at 100°C and compare your result against the value given in Table A–23.

10–20 Consider the flow of a fluid with viscosity μ through a circular pipe. The velocity profile in the pipe is given as $u(r) = u_{\max}(1 - r^n/R^n)$, where $u_{\max}$ is the maximum flow velocity, which occurs at the centerline; r is the radial distance from the centerline; and $u(r)$ is the flow velocity at any position r. Develop a relation for the drag force exerted on the pipe wall by the fluid in the flow direction per unit length of the pipe.

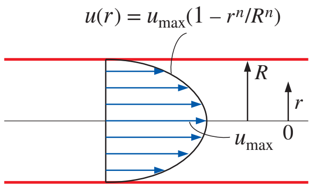


FIGURE P10–20

10–21 The viscosity of a fluid is to be measured by a viscometer constructed of two 75-cm-long concentric cylinders. The outer diameter of the inner cylinder is 15 cm, and the gap between the two cylinders is 1 mm. The inner cylinder is rotated at 300 rpm, and the torque is measured to be 0.8 N·m. Determine the viscosity of the fluid.

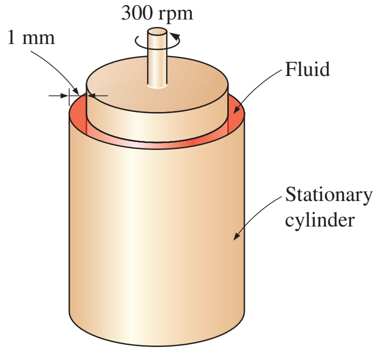


FIGURE P10–21

10–22 A thin 30-cm × 30-cm flat plate is pulled at 3 m/s horizontally through a 3.6-mm-thick oil layer sandwiched between two plates, one stationary and the other moving at a constant velocity of 0.3 m/s, as shown in Fig. P10–22. The dynamic viscosity of the oil is 0.027 Pa·s. Assuming the velocity in each oil layer to vary linearly, (*a*) plot the velocity profile and find the location where the oil velocity is zero and (*b*) determine the force that needs to be applied on the plate to maintain this motion.

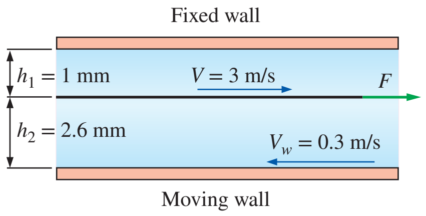


FIGURE P10–22

10–23 A rotating viscometer consists of two concentric cylinders—an inner cylinder of radius R_i rotating at angular velocity (rotation rate) ω_i, and a stationary outer cylinder of inside radius R_o. In the tiny gap between the two cylinders is the fluid of viscosity μ. The length of the cylinders (into the page in

Fig. P10–23) is *L*. *L* is large such that end effects are negligible (we can treat this as a two-dimensional problem). Torque (T) is required to rotate the inner cylinder at constant speed. (*a*) Showing all of your work and algebra, generate an approximate expression for T as a function of the other variables. (*b*) Explain why your solution is only an *approximation*. In particular, do you expect the velocity profile in the gap to remain linear as the gap becomes larger and larger (i.e., if the outer radius R_o were to increase, all else staying the same)?

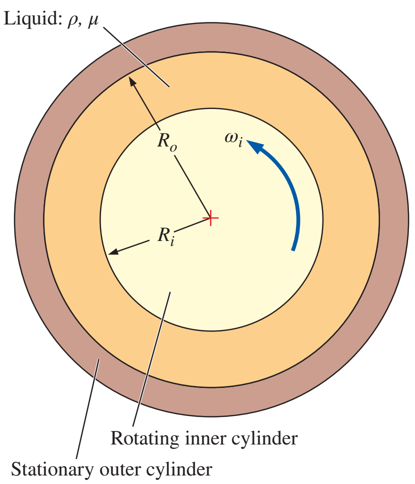


FIGURE P10–23

10–24 The clutch system shown in Fig. P10–24 is used to transmit torque through a 2-mm-thick oil film with $\mu = 0.38$ N·s/m^2 between two identical 30-cm-diameter disks. When the driving shaft rotates at a speed of 1200 rpm, the driven shaft is observed to rotate at 1125 rpm. Assuming a linear velocity profile for the oil film, determine the transmitted torque.

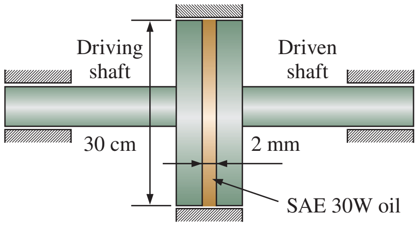


FIGURE P10–24

10–25 Reconsider Prob. 10–24. Using appropriate software, investigate the effect of oil film thickness on the torque transmitted. Let the film thickness vary from 0.1 mm to 10 mm. Plot your results, and state your conclusions.

10–26 A 50-cm × 30-cm × 20-cm block weighing 150 N is to be moved at a constant velocity of 1.10 m/s on an inclined surface with a friction coefficient of 0.27. (*a*) Determine the force *F* that needs to be applied in the horizontal direction. (*b*) If a 0.40-mm-thick oil film with a dynamic viscosity of 0.012 Pa·s is applied between the block and inclined surface, determine the percent reduction in the required force.

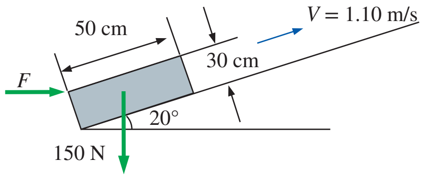


FIGURE P10–26

10–27 For flow over a plate, the variation of velocity with vertical distance *y* from the plate is given as $u(y) = ay - by^2$ where *a* and *b* are constants. Obtain a relation for the wall shear stress in terms of *a*, *b*, and *μ*.

10–28 In regions far from the entrance, fluid flow through a circular pipe is one dimensional, and the velocity profile for laminar flow is given by $u(r) = u_{\max}(1 - r^2/R^2)$, where *R* is the radius of the pipe, *r* is the radial distance from the center of the pipe, and $u_{\max}$ is the maximum flow velocity, which occurs at the center. Obtain (*a*) a relation for the drag force applied by the fluid on a section of the pipe of length *L* and (*b*) the value of the drag force for water flow at 20°C with $R = 0.08$ m, $L = 30$ m, $u_{\max} = 3$ m/s, and $\mu = 0.0010$ kg/m·s.

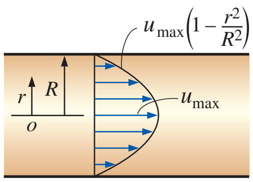


FIGURE P10–28

10–29 Repeat Prob. 10–28 for $u_{\max} = 6$ m/s. *Answer:* (*b*) 2.26 N

10–30 A frustum-shaped body is rotating at a constant angular speed of 200 rad/s in a container filled with SAE 10W oil at 20°C ($\mu = 0.100$ Pa·s), as shown in Fig. P10–30. If the thickness of the oil film on all sides is 1.2 mm, determine the power required to maintain this motion. Also determine the reduction in the required power input when the oil temperature rises to 80°C ($\mu = 0.0078$ Pa·s).

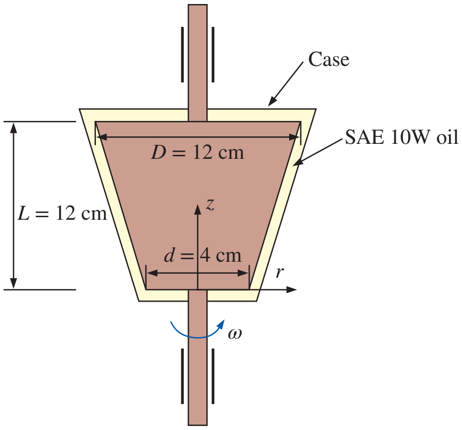


FIGURE P10–30

10–31 A rotating viscometer consists of two concentric cylinders—a stationary inner cylinder of radius R_i and an outer cylinder of inside radius R_o rotating at angular velocity (rotation rate) ω_o. In the tiny gap between the two cylinders is the fluid whose viscosity (μ) is to be measured. The length of the cylinders (into the page in Fig. P10–31) is L. L is large such that end effects are negligible (we can treat this as a two-dimensional problem). Torque (T) is required to rotate the inner cylinder at constant speed. Showing all your work and algebra, generate an approximate expression of T as a function of the other variables.

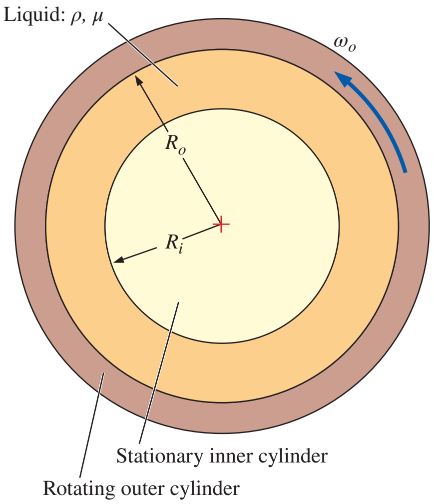


FIGURE P10–31

Surface Tension and Capillary Effect

10–32C What is surface tension? What is its cause? Why is the surface tension also called surface energy?

10–33C What is the capillary effect? What is its cause? How is it affected by the contact angle?

10–34C A small-diameter tube is inserted into a liquid whose contact angle is 110°. Will the level of liquid in the tube be higher or lower than the level of the rest of the liquid? Explain.

10–35C Consider a soap bubble. Is the pressure inside the bubble higher or lower than the pressure outside?

10–36C Is the capillary rise greater in small- or large-diameter tubes?

10–37 Determine the gage pressure inside a soap bubble of diameter (*a*) 0.2 cm and (*b*) 5 cm at 20°C.

10–38E A 2.4-in-diameter soap bubble is to be enlarged by blowing air into it. Taking the surface tension of soap solution to be 0.0027 lbf/ft, determine the work input required to inflate the bubble to a diameter of 2.7 in.

10–39 A 1.6-mm-diameter tube is inserted into an unknown liquid whose density is 960 kg/m^3, and it is observed that the liquid rises 5 mm in the tube, making a contact angle of 15°. Determine the surface tension of the liquid.

10–40 Consider a 0.15-mm-diameter air bubble in a liquid. Determine the pressure difference between the inside and outside of the air bubble if the surface tension at the air–liquid interface is (*a*) 0.080 N/m and (*b*) 0.12 N/m.

10–41 The surface tension of a liquid is to be measured using a liquid film suspended on a U-shaped wire frame with an 8-cm-long movable side. If the force needed to move the wire is 0.030 N, determine the surface tension of this liquid in air.

10–42 A capillary tube of 1.2 mm diameter is immersed vertically in water exposed to the atmosphere. Determine how high water will rise in the tube. Take the contact angle at the inner wall of the tube to be 6° and the surface tension to be 1.00 N/m. *Answer:* 0.338 m

10–43E A 0.018-in-diameter glass tube is inserted into mercury, which makes a contact angle of 140° with glass. Determine the capillary drop of mercury in the tube at 68°F. *Answer:* 0.874 in

10–44 A capillary tube is immersed vertically in a water container. Knowing that water starts to evaporate when the pressure drops below 2 kPa, determine the maximum capillary rise and tube diameter for this maximum-rise case. Take the contact angle at the inner wall of the tube to be 6° and the surface tension to be 1.00 N/m.

10–45 Contrary to what you might expect, a solid steel ball can float on water due to the surface tension effect. Determine the maximum diameter of a steel ball that would float on water at 10°C. What would your answer be for an aluminum ball? Take the densities of steel and aluminum balls to be 7800 kg/m^3 and 2700 kg/m^3, respectively.

10–46 Nutrients dissolved in water are carried to upper parts of plants by tiny tubes partly because of the capillary effect. Determine how high the water solution will rise in a tree in a 0.0026-mm-diameter tube as a result of the capillary effect. Treat the solution as water at 20°C with a contact angle of 15°. *Answer:* 11.1 m

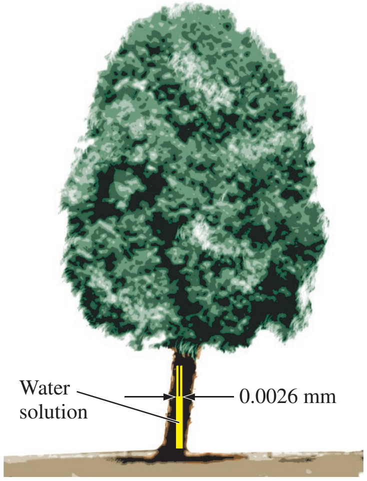


FIGURE P10–46

Review Problems

10–47 Consider a 55-cm-long journal bearing that is lubricated with oil whose viscosity is 0.1 kg/m·s at 20°C at the beginning of operation and 0.008 kg/m·s at the anticipated steady operating temperature of 80°C. The diameter of the shaft is 8 cm, and the average gap between the shaft and the journal is 0.08 cm. Determine the torque needed to overcome the bearing friction initially and during steady operation when the shaft is rotated at 1500 rpm.

10–48 The diameter of one arm of a U-tube is 5 mm while the other arm is large. If the U-tube contains some water, and both surfaces are exposed to atmospheric pressure, determine the difference between the water levels in the two arms.

10–49E The analysis of a propeller that operates in water at 60°F shows that the pressure at the tips of the propeller drops to 0.1 psia at high speeds. Determine if there is a danger of cavitation for this propeller.

10–50 The variation of the dynamic viscosity of water with absolute temperature is given as

T, K	μ, Pa·s
273.15	1.787×10^{-3}
278.15	1.519×10^{-3}
283.15	1.307×10^{-3}
293.15	1.002×10^{-3}
303.15	7.975×10^{-4}
313.15	6.529×10^{-4}
333.15	4.665×10^{-4}
353.15	3.547×10^{-4}
373.15	2.828×10^{-4}

Using these tabulated data, develop a relation for viscosity in the form of $\mu = \mu(T) = A + BT + CT^2 + DT^3 + ET^4$. Using the relation developed, predict the dynamic viscosity of water at 50°C at which the reported value is 5.468×10^{-4} Pa·s. Compare your result with the results of Andrade's equation, which is given in the form of $\mu = D{\cdot}e^{B/T}$, where D and B are constants whose values are to be determined using the viscosity data given.

10–51 Derive a relation for the capillary rise of a liquid between two large parallel plates a distance t apart inserted into the liquid vertically. Take the contact angle to be f.

10–52 A 10-cm-diameter cylindrical shaft rotates inside a 50-cm-long, 10.3-cm-diameter bearing. The space between the shaft and the bearing is completely filled with oil whose viscosity at anticipated operating temperature is 0.300 N·s/m^2. Determine the power required to overcome friction when the shaft rotates at a speed of (*a*) 600 rpm and (*b*) 1200 rpm.

10–53 Some rocks or bricks contain small air pockets in them and have a spongy structure. Assuming the air spaces form columns of an average diameter of 0.006 mm, determine how high water can rise in such a material. Take the surface tension of the air–water interface in that material to be 0.085 N/m.

10–54 A fluid between two very long parallel plates is heated in a way that its viscosity decreases linearly from 0.90 Pa·s at the lower plate to 0.50 Pa·s at the upper plate. The spacing between the two plates is 0.4 mm. The upper plate moves steadily at a velocity of 10 m/s, in a direction parallel to both plates. The pressure is constant everywhere, the fluid is Newtonian, and it is assumed to be incompressible. Neglect gravitational effects. (*a*) Obtain the fluid velocity u as a function of y, $u(y)$, where y is the vertical axis perpendicular to the plates. Plot the velocity profile across the gap between the plates. (*b*) Calculate the value of the shear stress. Show the direction of the shear stress on the moving plate and on the top surface of the fluid element adjacent to the moving plate.

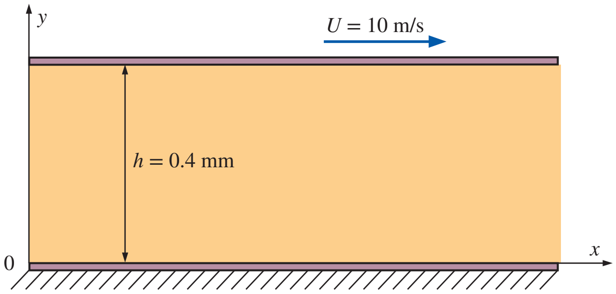


FIGURE P10–54

Design and Essay Problems

10–55 Design an experiment to measure the viscosity of liquids using a vertical funnel with a cylindrical reservoir of height h and a narrow flow section of diameter D and length L. Making appropriate assumptions, obtain a relation for viscosity in terms of easily measurable quantities such as density and volume flow rate.

10–56 Write the general relationship between shear stress τ and rate of deformation du/dy for non-Newtonian fluids. Also, write a report on how to measure the viscosity of non-Newtonian fluids.

10–57 Write an essay on the rise of the fluid to the tops of trees by capillary and other effects.

10–58 Write an essay on the oils used in car engines in different seasons and their viscosities.

10–59 Even though steel is about 7 to 8 times denser than water, a steel paper clip or razor blade can be made to float on water! Explain and discuss. Predict what would happen if you mix some soap with the water.

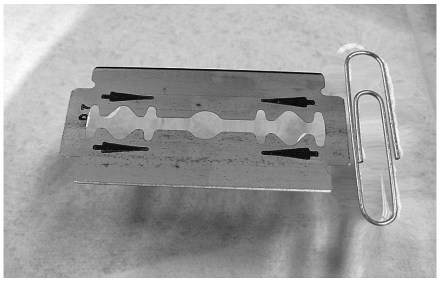


FIGURE P10–59

John M. Cimbala

CHAPTER

11

FLUID STATICS

This chapter deals with forces applied by fluids at rest. The fluid property responsible for those forces is *pressure,* which is a normal force exerted by a fluid per unit area. We start this chapter with a discussion of the *hydrostatic forces* applied on submerged bodies with plane or curved surfaces. We then consider the *buoyant force* applied by fluids on submerged or floating bodies and discuss the *stability* of such bodies. This chapter makes extensive use of force balances for bodies in static equilibrium, and it would be helpful if the relevant topics from statics are first reviewed.

OBJECTIVES

The objectives of this chapter are to:

- Calculate the forces and moments exerted by a fluid at rest on plane or curved submerged surfaces.
- Analyze the stability of floating and submerged bodies.

FIGURE 11–1
Hoover Dam.
Comstock/Getty Images

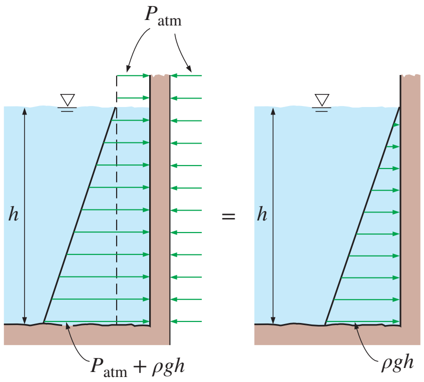
11–1 ▪ INTRODUCTION TO FLUID STATICS

Fluid statics deals with problems associated with fluids at rest. The fluid can be either gaseous or liquid. Fluid statics is generally referred to as *hydrostatics* when the fluid is a liquid and as *aerostatics* when the fluid is a gas. In fluid statics, there is no relative motion between adjacent fluid layers, and thus there are no shear (tangential) stresses in the fluid trying to deform it. The only stress we deal with in fluid statics is the *normal stress,* which is the pressure, and the variation of pressure is due only to the weight of the fluid. Therefore, the topic of fluid statics has significance only in gravity fields, and the force relations developed naturally involve the gravitational acceleration g. The force exerted on a surface by a fluid at rest is normal to the surface at the point of contact since there is no relative motion between the fluid and the solid surface, and thus there are no shear forces acting parallel to the surface.

Fluid statics is used to determine the forces acting on floating or submerged bodies and the forces developed by devices like hydraulic presses and car jacks. The design of many engineering systems such as water dams and liquid storage tanks requires the determination of the forces acting on their surfaces using fluid statics. The complete description of the resultant hydrostatic force acting on a submerged surface requires the determination of the magnitude, the direction, and the line of action of the force. In the following two sections, we consider the forces acting on both plane and curved surfaces of submerged bodies due to pressure.

11–2 ▪ HYDROSTATIC FORCES ON SUBMERGED PLANE SURFACES

A plate (such as a gate valve in a dam, the wall of a liquid storage tank, or the hull of a ship at rest) is subjected to fluid pressure distributed over its surface when exposed to a liquid (Fig. 11–1). On a *plane* surface, the hydrostatic forces form a system of parallel forces, and we often need to determine the *magnitude* of the force and its *point of application,* which is called the **center of pressure**. In most cases, the other side of the plate is open to the atmosphere (such as the dry side of a gate), and thus atmospheric pressure acts on both sides of the plate, yielding a zero resultant. In such cases, it is convenient to subtract atmospheric pressure and work with the gage pressure only (Fig. 11–2). For example, $P_{\text{gage}} = \rho gh$ at the bottom of the lake.



(*a*) P_{atm} considered

(*b*) P_{atm} subtracted

FIGURE 11–2
When analyzing hydrostatic forces on submerged surfaces, the atmospheric pressure can be subtracted for simplicity when it acts on both sides of the structure.

Consider the top surface of a flat plate of arbitrary shape completely submerged in a liquid, as shown in Fig. 11–3 together with its normal view. The plane of this surface (normal to the page) intersects the horizontal free surface at angle θ, and we take the line of intersection to be the x-axis (out of the page). The absolute pressure above the liquid is P_0, which is the local atmospheric pressure P_{atm} if the liquid is open to the atmosphere (but P_0 may be different than P_{atm} if the space above the liquid is evacuated or pressurized). Then the absolute pressure at any point on the plate is

$$P = P_0 + \rho gh = P_0 + \rho gy \sin \theta \qquad \textbf{(11–1)}$$

where h is the vertical distance of the point from the free surface and y is the distance of the point from the x-axis (from point O in Fig. 11–3). The resultant hydrostatic force F_R acting on the surface is determined by integrating the force $P\,dA$ acting on a differential area dA over the entire surface area,

$$F_R = \int_A P\,dA = \int_A (P_0 + \rho gy \sin \theta)\,dA = P_0 A + \rho g \sin \theta \int_A y\,dA \qquad \textbf{(11–2)}$$

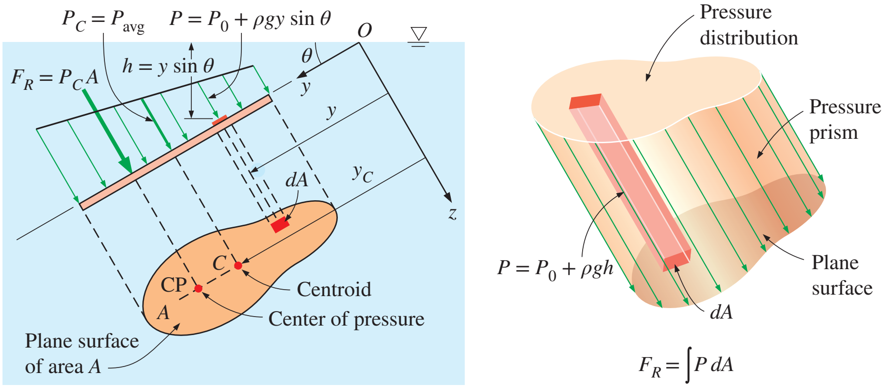


FIGURE 11–3
Hydrostatic force on an inclined plane surface completely submerged in a liquid.

But the *first moment of area* $\int_A y\, dA$ is related to the *y*-coordinate of the centroid (or center) of the surface by

$$y_C = \frac{1}{A}\int_A y\, dA \tag{11–3}$$

Substituting,

$$F_R = (P_0 + \rho g y_C \sin\theta)A = (P_0 + \rho g h_C)A = P_C A = P_{avg} A \tag{11–4}$$

where $P_C = P_0 + \rho g h_C$ is the pressure at the centroid of the surface, which is equivalent to the *average* pressure P_{avg} on the surface, and $h_C = y_C \sin\theta$ is the *vertical distance* of the centroid from the free surface of the liquid (Fig. 11–4). Thus we conclude that:

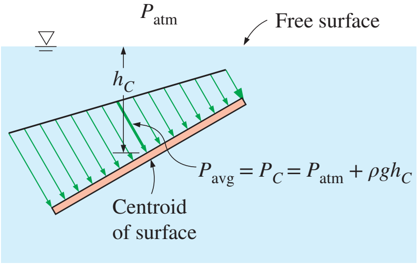


FIGURE 11–4
The pressure at the centroid of a plane surface is equivalent to the *average* pressure on the surface.

> The magnitude of the resultant force acting on a plane surface of a completely submerged plate in a homogeneous (constant density) fluid is equal to the product of the pressure P_C at the centroid of the surface and the area *A* of the surface (Fig. 11–5).

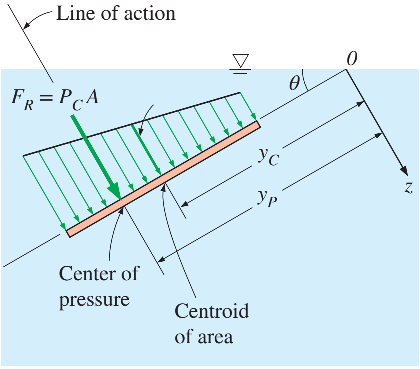


FIGURE 11–5
The resultant force acting on a plane surface is equal to the product of the pressure at the centroid of the surface and the surface area, and its line of action passes through the center of pressure.

The pressure P_0 is usually atmospheric pressure, which can be ignored in most force calculations since it acts on both sides of the plate. When this is not the case, a practical way of accounting for the contribution of P_0 to the resultant force is simply to add an equivalent depth $h_{equiv} = P_0/\rho g$ to h_C; that is, to assume the presence of an additional liquid layer of thickness h_{equiv} on top of the liquid with absolute vacuum above.

Next we need to determine the line of action of the resultant force F_R. Two parallel force systems are equivalent if they have the same magnitude and the same moment about any point. The line of action of the resultant hydrostatic force, in general, does not pass through the centroid of the surface—it lies underneath where the pressure is higher. The point of intersection of the line of action of the resultant force and the surface is the **center of pressure**. The vertical location of the line of action is determined by equating the moment of the resultant force to the moment of the distributed pressure force about the *x*-axis:

$$y_P F_R = \int_A yP\, dA = \int_A y(P_0 + \rho g y \sin\theta)\, dA = P_0 \int_A y\, dA + \rho g \sin\theta \int_A y\, dA$$

or

$$y_P F_R = P_0 y_C A + \rho g \sin\theta\, I_{xx,\,O} \tag{11–5}$$

where y_P is the distance of the center of pressure from the x-axis (point O in Fig. 11–5) and $I_{xx,\,O} = \int_A y^2$ is the *second moment of area* (also called the *area moment of inertia*) about the x-axis. The second moments of area are widely available for common shapes in engineering handbooks, but they are usually given about the axes passing through the centroid of the area. Fortunately, the second moments of area about two parallel axes are related to each other by the *parallel axis theorem,* which in this case is expressed as

$$I_{xx,\,O} = I_{xx,\,C} + y_C^2 A \qquad \textbf{(11–6)}$$

where $I_{xx,\,C}$ is the second moment of area about the x-axis passing through the centroid of the area and y_C (the y-coordinate of the centroid) is the distance between the two parallel axes. Substituting the F_R relation from Eq. 11–4 and the $I_{xx,\,O}$ relation from Eq. 11–6 into Eq. 11–5 and solving for y_P yields

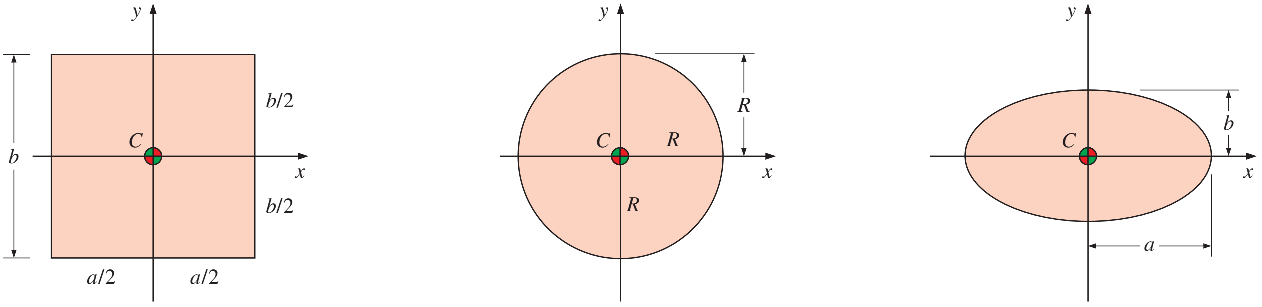
$$y_p = y_C + \frac{I_{xx,\,C}}{[y_C + P_0/(\rho g \sin\theta)]A} \qquad \textbf{(11–7a)}$$

For $P_0 = 0$, which is usually the case when the atmospheric pressure is ignored, it simplifies to

$$y_p = y_C + \frac{I_{xx,\,C}}{y_C A} \qquad \textbf{(11–7b)}$$

Knowing y_P, the vertical distance of the center of pressure from the free surface is determined from $h_P = y_P \sin\theta$.

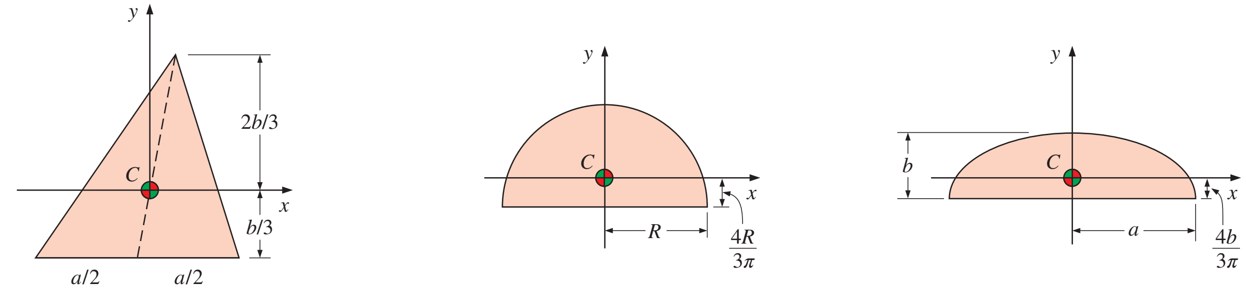
The $I_{xx,\,C}$ values for some common areas are given in Fig. 11–6. For areas that possess symmetry about the y-axis, the center of pressure lies on the y-axis



$A = ab,\ I_{xx,\,C} = ab^3/12$
(*a*) Rectangle

$A = \pi R^2,\ I_{xx,\,C} = \pi R^4/4$
(*b*) Circle

$A = \pi ab,\ I_{xx,\,C} = \pi ab^3/4$
(*c*) Ellipse



$A = ab/2,\ I_{xx,\,C} = ab^3/36$
(*d*) Triangle

$A = \pi R^2/2,\ I_{xx,\,C} = 0.109757R^4$
(*e*) Semicircle

$A = \pi ab/2,\ I_{xx,\,C} = 0.109757ab^3$
(*f*) Semiellipse

FIGURE 11–6
The centroid and the centroidal moments of inertia for some common geometries.

directly below the centroid. The location of the center of pressure in such cases is simply the point on the surface of the vertical plane of symmetry at a distance h_P from the free surface.

Pressure acts normal to the surface, and the hydrostatic forces acting on a flat plate of any shape form a volume whose base is the plate area and whose length is the linearly varying pressure, as shown in Fig. 11–7. This virtual **pressure prism** has an interesting physical interpretation: its *volume* is equal to the *magnitude* of the resultant hydrostatic force acting on the plate since $F_R = \int P\, dA$, and the line of action of this force passes through the *centroid* of this homogeneous prism. The projection of the centroid on the plate is the *pressure center*. Therefore, with the concept of pressure prism, the problem of describing the resultant hydrostatic force on a plane surface reduces to finding the volume and the two coordinates of the centroid of this pressure prism.

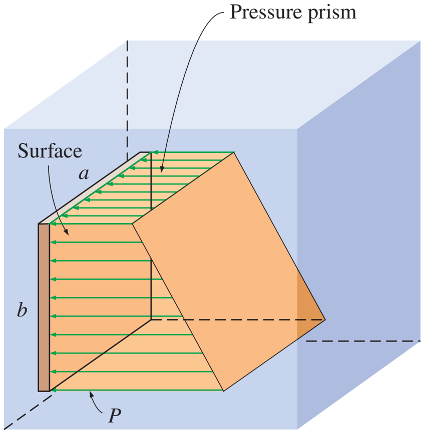


FIGURE 11–7
The hydrostatic forces acting on a plane surface form a pressure prism whose base (left face) is the surface and whose length is the pressure.

Special Case: Submerged Rectangular Plate

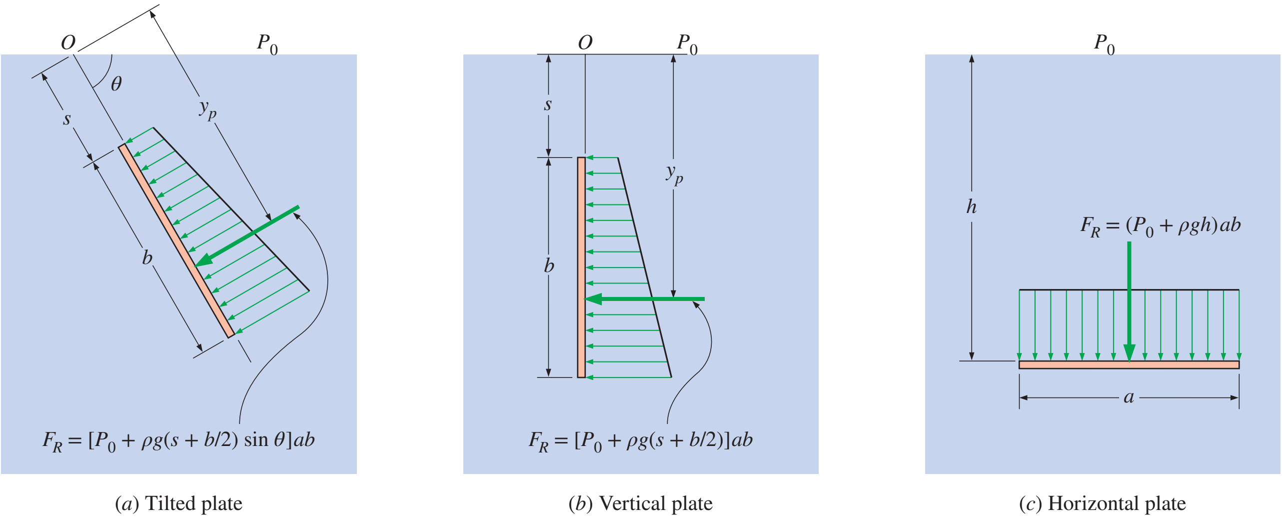
Consider a completely submerged rectangular flat plate of height b and width a tilted at an angle θ from the horizontal and whose top edge is horizontal and is at a distance s from the free surface along the plane of the plate, as shown in Fig. 11–8*a*. The resultant hydrostatic force on the upper surface is equal to the average pressure, which is the pressure at the midpoint of the surface, times the surface area A. That is,

Tilted rectangular plate:
$$F_R = P_C A = [P_0 + \rho g(s + b/2)\sin\theta]ab \tag{11–8}$$

The force acts at a vertical distance of $h_P = y_P \sin\theta$ from the free surface directly beneath the centroid of the plate where, from Eq. 11–7a,

$$\begin{aligned} y_P &= s + \frac{b}{2} + \frac{ab^3/12}{[s + b/2 + P_0/(\rho g \sin\theta)]ab} \\ &= s + \frac{b}{2} + \frac{b^2}{12[s + b/2 + P_0/(\rho g \sin\theta)]} \end{aligned} \tag{11–9}$$

When the upper edge of the plate is at the free surface and thus $s = 0$, Eq. 11–8 reduces to



(*a*) Tilted plate (*b*) Vertical plate (*c*) Horizontal plate

FIGURE 11–8
Hydrostatic force acting on the top surface of a submerged rectangular plate for tilted, vertical, and horizontal cases.

Tilted rectangular plate ($s = 0$): $$F_R = [P_0 + \rho g(b \sin \theta)/2]ab \qquad \textbf{(11–10)}$$

For a completely submerged *vertical* plate ($\theta = 90°$) whose top edge is horizontal, the hydrostatic force can be obtained by setting $\sin \theta = 1$ (Fig. 11–8*b*)

Vertical rectangular plate: $$F_R = [P_0 + \rho g(s + b/2)]ab \qquad \textbf{(11–11)}$$

Vertical rectangular plate ($s = 0$): $$F_R = (P_0 + \rho gb/2)ab \qquad \textbf{(11–12)}$$

When the effect of P_0 is ignored since it acts on both sides of the plate, the hydrostatic force on a vertical rectangular surface of height b whose top edge is horizontal and at the free surface is $F_R = \rho gab^2/2$ acting at a distance of $2b/3$ from the free surface directly beneath the centroid of the plate.

The pressure distribution on a submerged *horizontal* surface is uniform, and its magnitude is $P = P_0 + \rho gh$, where h is the distance of the surface from the free surface. Therefore, the hydrostatic force acting on a horizontal rectangular surface is

Horizontal rectangular plate: $$F_R = (P_0 + \rho gh)ab \qquad \textbf{(11–13)}$$

and it acts through the midpoint of the plate (Fig. 11–8*c*).

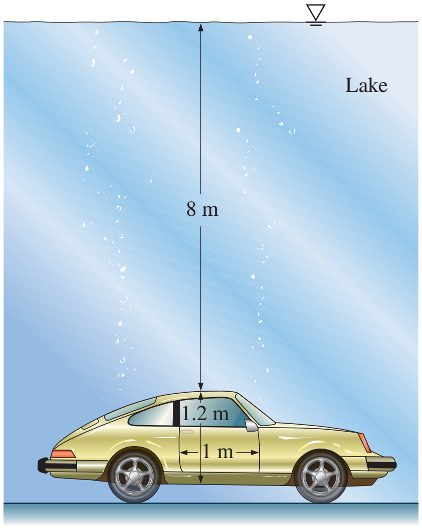


FIGURE 11–9
Schematic for Example 11–1.

EXAMPLE 11–1 Hydrostatic Force Acting on the Door of a Submerged Car

A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels (Fig. 11–9). The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of the pressure center, and discuss if the driver can open the door.

SOLUTION A car is submerged in water. The hydrostatic force on the door is to be determined, and the likelihood of the driver opening the door is to be assessed.

Assumptions **1** The bottom surface of the lake is horizontal. **2** The passenger cabin is well-sealed so that no water leaks inside. **3** The door can be approximated as a vertical rectangular plate. **4** The pressure in the passenger cabin remains at atmospheric value since there is no water leaking in, and thus no compression of the air inside. Therefore, atmospheric pressure cancels out in the calculations since it acts on both sides of the door. **5** The weight of the car is larger than the buoyant force acting on it.

Properties We take the density of lake water to be 1000 kg/m³ throughout.

Analysis The average (gage) pressure on the door is the pressure value at the centroid (midpoint) of the door and is determined to be

$$\begin{aligned} P_{\text{avg}} &= P_C = \rho g h_C = \rho g(s + b/2) \\ &= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(8 + 1.2/2 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg·m/s}^2}\right) \\ &= \mathbf{84.4 \text{ kN/m}^2} \end{aligned}$$

Then the resultant hydrostatic force on the door becomes

$$F_R = P_{\text{avg}} A = (84.4 \text{ kN/m}^2)(1 \text{ m} \times 1.2 \text{ m}) = \mathbf{101.3 \text{ kN}}$$

The pressure center is directly under the midpoint of the door, and its distance from the surface of the lake is determined from Eq. 11–9 by setting $P_0 = 0$, yielding

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 8 + \frac{1.2}{2} + \frac{1.2^2}{12(8 + 1.2/2)} = \mathbf{8.61 \text{ m}}$$

Discussion A strong person can lift 100 kg, which is a weight of 981 N or about 1 kN. Also, the person can apply the force at a point farthest from the hinges (1 m farther) for maximum effect and generate a moment of 1 kN·m. The resultant hydrostatic force acts under the midpoint of the door, and thus a distance of 0.5 m from the hinges. This generates a moment of 50.6 kN·m, which is about 50 times the moment the driver can possibly generate. Therefore, it is impossible for the driver to open the door of the car. The driver's best bet is to let some water in (by rolling the window down a little, for example) and to keep his or her head close to the ceiling. The driver should be able to open the door shortly before the car is filled with water since at that point the pressures on both sides of the door are nearly the same and opening the door in water is almost as easy as opening it in air.

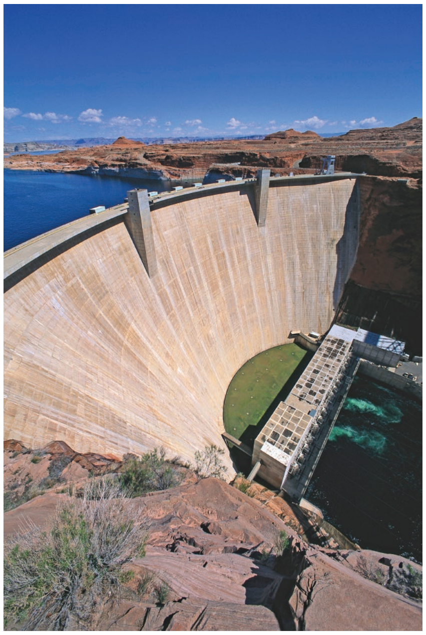


FIGURE 11–10
In many structures of practical application, the submerged surfaces are not flat, but curved as here at Glen Canyon Dam in Utah and Arizona.
Royalty-Free/CORBIS

11–3 ▪ HYDROSTATIC FORCES ON SUBMERGED CURVED SURFACES

In many practical applications, submerged surfaces are not flat (Fig. 11–10). For a submerged curved surface, the determination of the resultant hydrostatic force is more involved since it typically requires integration of the pressure forces that change direction along the curved surface. The concept of the pressure prism in this case is not much help either because of the complicated shapes involved.

The easiest way to determine the resultant hydrostatic force F_R acting on a two-dimensional curved surface is to determine the horizontal and vertical components F_H and F_V separately. This is done by considering the free-body diagram of the liquid block enclosed by the curved surface and the two plane surfaces (one horizontal and one vertical) passing through the two ends of the curved surface, as shown in Fig. 11–11. Note that the vertical surface of the liquid block considered is simply the projection of the curved surface on a *vertical plane,* and the horizontal surface is the projection of the curved surface on a *horizontal plane.* The resultant force acting on the curved solid surface is then equal and opposite to the force acting on the curved liquid surface (Newton's third law).

The force acting on the imaginary horizontal or vertical plane surface and its line of action can be determined as discussed in Section 11–2. The weight of

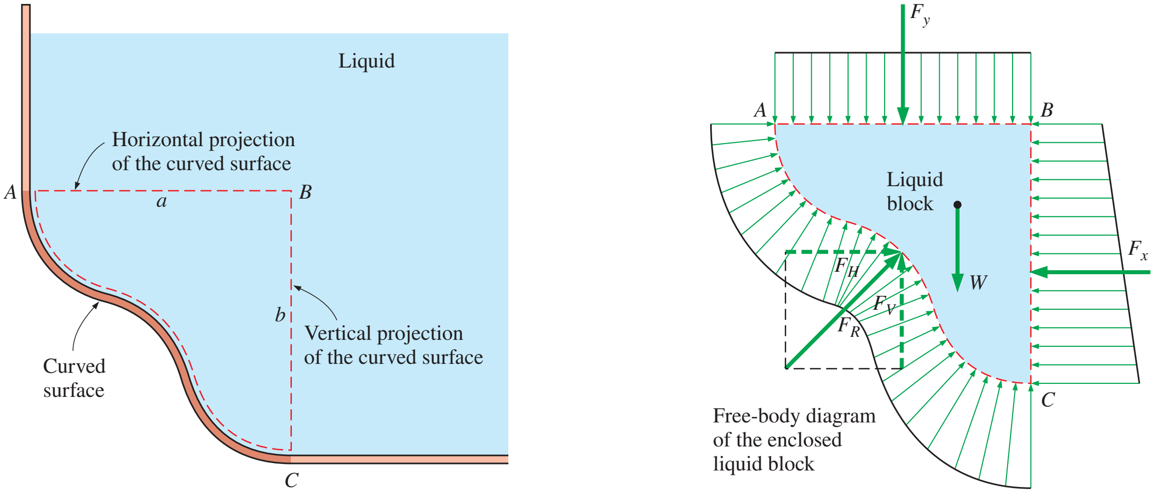


FIGURE 11–11
Determination of the hydrostatic force acting on a submerged curved surface.

the enclosed liquid block of volume V is simply $W = \rho g V$, and it acts downward through the centroid of this volume. Noting that the fluid block is in static equilibrium, the force balances in the horizontal and vertical directions give

Horizontal force component on curved surface:
$$F_H = F_x \tag{11–14}$$

Vertical force component on curved surface:
$$F_V = F_y \pm W \tag{11–15}$$

where the summation $F_y \pm W$ is a vector addition (i.e., add magnitudes if both act in the same direction and subtract if they act in opposite directions). Thus, we conclude that

1. The horizontal component of the hydrostatic force acting on a curved surface is equal (in both magnitude and the line of action) to the hydrostatic force acting on the vertical projection of the curved surface.
2. The vertical component of the hydrostatic force acting on a curved surface is equal to the hydrostatic force acting on the horizontal projection of the curved surface, plus (minus, if acting in the opposite direction) the weight of the fluid block.

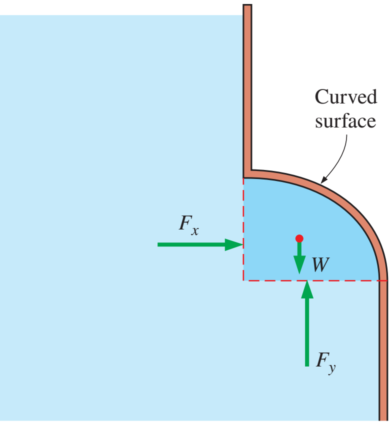


FIGURE 11–12
When a curved surface is above the liquid, the weight of the liquid and the vertical component of the hydrostatic force act in the opposite directions.

The magnitude of the resultant hydrostatic force acting on the curved surface is $F_R = \sqrt{F_H^2 + F_V^2}$, and the tangent of the angle it makes with the horizontal is $\tan \alpha = F_V/F_H$. The exact location of the line of action of the resultant force (e.g., its distance from one of the end points of the curved surface) can be determined by taking a moment about an appropriate point. These discussions are valid for all curved surfaces regardless of whether they are above or below the liquid. Note that in the case of a *curved surface above a liquid,* the weight of the liquid is *subtracted* from the vertical component of the hydrostatic force since they act in opposite directions (Fig. 11–12).

When the curved surface is a *circular arc* (full circle or any part of it), the resultant hydrostatic force acting on the surface always passes through the center of the circle. This is because the pressure forces are normal to the surface, and all lines normal to the surface of a circle pass through the center of the circle. Thus, the pressure forces form a concurrent force system at the center, which can be reduced to a single equivalent force at that point (Fig. 11–13).

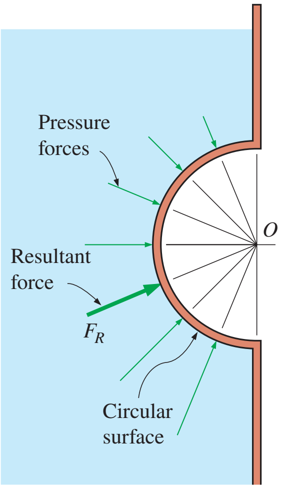


FIGURE 11–13
The hydrostatic force acting on a circular surface always passes through the center of the circle since the pressure forces are normal to the surface and they all pass through the center.

Finally, the hydrostatic force acting on a plane or curved surface submerged in a **multilayered fluid** of different densities can be determined by considering different parts of surfaces in different fluids as different surfaces, finding the force on each part, and then adding them using vector addition. For a plane surface, it can be expressed as (Fig. 11–14)

Plane surface in a multilayered fluid:
$$F_R = \sum F_{R,\,i} = \sum P_{C,\,i} A_i \tag{11–16}$$

where $P_{C,\,i} = P_0 + \rho_i g h_{C,\,i}$ is the pressure at the centroid of the portion of the surface in fluid i and A_i is the area of the plate in that fluid. The line of action of this equivalent force can be determined from the requirement that the moment of the equivalent force about any point is equal to the sum of the moments of the individual forces about the same point.

EXAMPLE 11–2 A Gravity-Controlled Cylindrical Gate

A long, solid cylinder of radius 0.8 m hinged at point A is used as an automatic gate, as shown in Fig. 11–15. When the water level reaches 5 m, the gate opens by turning about the hinge at point A. Determine (*a*) the hydrostatic force acting on the cylinder and its line of action when the gate opens and (*b*) the weight of the cylinder per m length of the cylinder.

SOLUTION The height of a water reservoir is controlled by a cylindrical gate hinged to the reservoir. The hydrostatic force on the cylinder and the weight of the cylinder per m length are to be determined.

Assumptions **1** Friction at the hinge is negligible. **2** Atmospheric pressure acts on both sides of the gate, and thus it cancels out.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis (*a*) We consider the free-body diagram of the liquid block enclosed by the circular surface of the cylinder and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as

Horizontal force on vertical surface:

$$\begin{aligned} F_H = F_x = P_{\text{avg}}A &= \rho g h_C A = \rho g(s + R/2)A \\ &= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4.2 + 0.8/2\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg·m/s}^2}\right) \\ &= 36.1\ \text{kN} \end{aligned}$$

Vertical force on horizontal surface (upward):

$$\begin{aligned} F_y = P_{\text{avg}}A &= \rho g h_C A = \rho g h_{\text{bottom}} A \\ &= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg·m/s}^2}\right) \\ &= 39.2\ \text{kN} \end{aligned}$$

Weight (downward) of fluid block for one m width into the page:

$$\begin{aligned} W = mg = \rho g V &= \rho g(R^2 - \pi R^2/4)(1\ \text{m}) \\ &= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.8\ \text{m})^2(1 - \pi/4)(1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg·m/s}^2}\right) \\ &= 1.3\ \text{kN} \end{aligned}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 39.2 - 1.3 = 37.9\ \text{kN}$$

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{36.1^2 + 37.9^2} = \mathbf{52.3\ kN}$$

$$\tan\theta = F_V/F_H = 37.9/36.1 = 1.05 \rightarrow \theta = 46.4°$$

Therefore, the magnitude of the hydrostatic force acting on the cylinder is 52.3 kN per m length of the cylinder, and its line of action passes through the center of the cylinder making an angle 46.4° with the horizontal.

(*b*) When the water level is 5 m high, the gate is about to open and thus the reaction force at the bottom of the cylinder is zero. Then the forces other than those at the hinge acting on the cylinder are its weight, acting through the center, and the hydrostatic force exerted by water. Taking a moment about point *A* at the location of the hinge and equating it to zero gives

$$F_R R \sin\theta - W_{\text{cyl}} R = 0 \ \rightarrow\ W_{\text{cyl}} = F_R \sin\theta = (52.3\ \text{kN}) \sin 46.4° = \mathbf{37.9\ kN}$$

Discussion The weight of the cylinder per m length is determined to be 37.9 kN. It can be shown that this corresponds to a mass of 3863 kg per m length and to a density of 1921 kg/m^3 for the material of the cylinder.

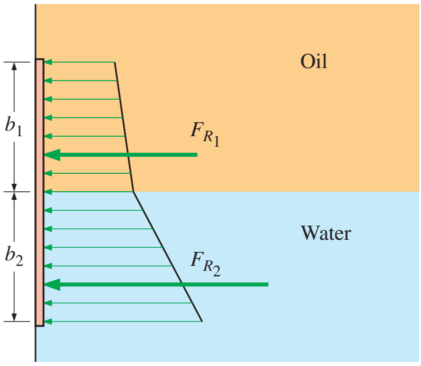


FIGURE 11–14
The hydrostatic force on a surface submerged in a multilayered fluid can be determined by considering parts of the surface in different fluids as different surfaces.

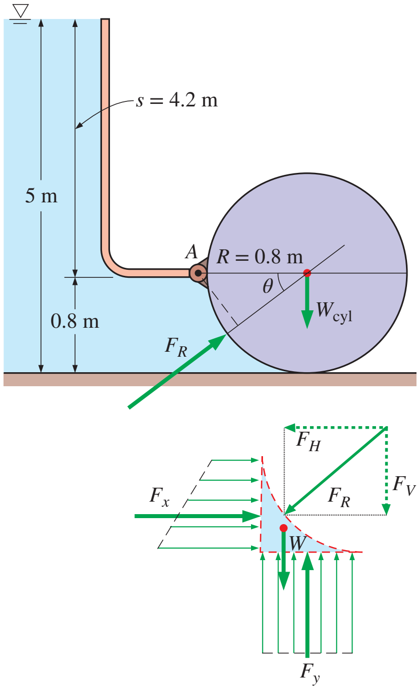


FIGURE 11–15
Schematic for Example 11–2 and the free-body diagram of the liquid underneath the cylinder.

11–4 ▪ BUOYANCY AND STABILITY

It is a common experience that an object feels lighter and weighs less in a liquid than it does in air. This can be demonstrated easily by weighing a heavy object in water with a waterproof spring scale. Also, objects made of wood or other light materials float on water. These and other observations suggest that a fluid exerts an upward force on a body immersed in it. This force that tends to lift the body is called the **buoyant force** and is denoted by F_B.

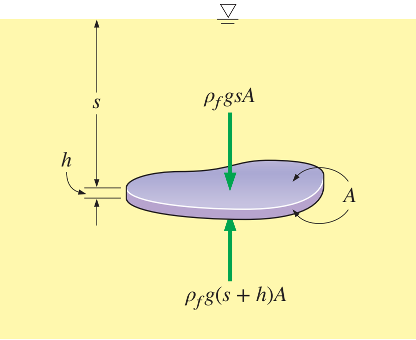


FIGURE 11–16
A flat plate of uniform thickness h submerged in a liquid parallel to the free surface.

The buoyant force is caused by the increase of pressure with depth in a fluid. Consider, for example, a flat plate of thickness h submerged in a liquid of density ρ_f parallel to the free surface, as shown in Fig. 11–16. The area of the top (and also bottom) surface of the plate is A, and its distance to the free surface is s. The gage pressures at the top and bottom surfaces of the plate are $\rho_f gs$ and $\rho_f g(s+h)$, respectively. Then the hydrostatic force $F_{\text{top}} = \rho_f gsA$ acts downward on the top surface, and the larger force $F_{\text{bottom}} = \rho_f g(s+h)A$ acts upward on the bottom surface of the plate. The difference between these two forces is a net upward force, which is the *buoyant force,*

$$F_B = F_{\text{bottom}} - F_{\text{top}} = \rho_f g(s+h)A - \rho_f gsA = \rho_f ghA = \rho_f gV \tag{11–17}$$

where $V = hA$ is the volume of the plate. But the relation $\rho_f gV$ is simply the weight of the liquid whose volume is equal to the volume of the plate. Thus, we conclude that *the buoyant force acting on the plate is equal to the weight of the liquid displaced by the plate.* For a fluid with constant density, the buoyant force is independent of the distance of the body from the free surface. It is also independent of the density of the solid body.

The relation in Eq. 11–17 is developed for a simple geometry, but it is valid for any body regardless of its shape. This can be shown mathematically by a force balance, or simply by this argument: Consider an arbitrarily shaped solid body submerged in a fluid at rest and compare it to a body of fluid of the same shape indicated by dashed lines at the same vertical location (Fig. 11–17). The buoyant forces acting on these two bodies are the same since the pressure distributions, which depend only on elevation, are the same at the boundaries of both. The imaginary fluid body is in static equilibrium, and thus the net force and net moment acting on it are zero. Therefore, the upward buoyant force must be equal to the weight of the imaginary fluid body whose volume is equal to the volume of the solid body. Further, the weight and the buoyant force must have the same line of action to have a zero moment. This is known as **Archimedes' principle**, after the Greek mathematician Archimedes (287–212 BC), and is expressed as

> The buoyant force acting on a body of uniform density immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.

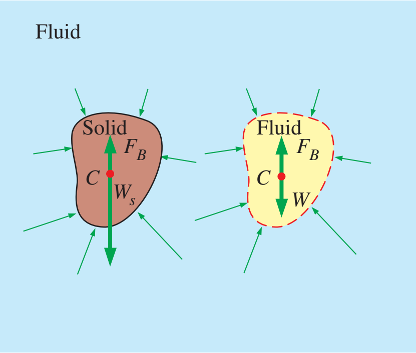


FIGURE 11–17
The buoyant forces acting on a solid body submerged in a fluid and on a fluid body of the same shape at the same depth are identical. The buoyant force F_B acts upward through the centroid C of the displaced volume and is equal in magnitude to the weight W of the displaced fluid, but is opposite in direction. For a solid of uniform density, its weight W_s also acts through the centroid, but its magnitude is not necessarily equal to that of the fluid it displaces. (Here $W_s > W$ and thus $W_s > F_B$; this solid body would sink.)

For *floating* bodies, the weight of the entire body must be equal to the buoyant force, which is the weight of the fluid whose volume is equal to the volume of the *submerged portion* of the floating body. That is,

$$F_B = W \rightarrow \rho_f gV_{\text{sub}} = \rho_{\text{avg, body}}\, gV_{\text{total}} \rightarrow \frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{\rho_{\text{avg, body}}}{\rho_f} \tag{11–18}$$

Therefore, the submerged volume fraction of a floating body is equal to the ratio of the average density of the body to the density of the fluid. Note that when the density ratio is equal to or greater than one, the floating body becomes completely submerged.

It follows from these discussions that a body immersed in a fluid (1) remains at rest at any location in the fluid where its average density is equal to the density of the fluid, (2) sinks to the bottom when its average density is greater than the density of the fluid, and (3) rises to the surface of the fluid and floats when the average density of the body is less than the density of the fluid (Fig. 11–18).

For a body floating on the surface of a liquid, the total weight of the body must obviously be *less* than that of the liquid it displaces. It turns out that a portion of the body volume is submerged (volume $V_{\text{submerged}}$), while the remaining portion is positioned above the surface of the liquid. Since the system is stationary, the two vertical forces W and F_B must still balance,

$$W = F_B = \rho_f g V_{\text{submerged}} \quad \rightarrow \quad V_{\text{submerged}} = W/\rho_f g$$

For a body of known weight W, we see that as the liquid density ρ_f increases, a smaller percentage of the body volume is submerged since ρ_f is in the denominator (see Fig. 11–19).

The buoyant force is proportional to the density of the fluid, and thus we might think that the buoyant force exerted by gases such as air is negligible. This is certainly the case in general, but there are significant exceptions. For example, the volume of a person is about 0.1 m^3, and taking the density of air to be 1.2 kg/m^3, the buoyant force exerted by air on the person is

$$F_B = \rho_f g V = (1.2\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m}^3) \cong 1.2\ \text{N}$$

The weight of an 80-kg person is $80 \times 9.81 = 788$ N. Therefore, ignoring the buoyancy in this case results in an error in weight of just 0.15 percent, which is negligible. But the buoyancy effects in gases dominate some important natural phenomena such as the rise of warm air in a cooler environment and thus the onset of natural convection currents, the rise of hot-air or helium balloons, and air movements in the atmosphere. A helium balloon, for example, rises as a result of the buoyancy effect until it reaches an altitude where the density of air (which decreases with altitude) equals the density of helium in the balloon—assuming the balloon does not burst by then, and ignoring the weight of the balloon's skin. Hot-air balloons (Fig. 11–20) work by similar principles.

Archimedes' principle is also used in geology by considering the continents to be floating on a sea of magma.

EXAMPLE 11–3 Measuring Specific Gravity with a Hydrometer

If you have a seawater aquarium, you have probably used a small cylindrical glass tube with a lead weight at its bottom to measure the salinity of the water by simply watching how deep the tube sinks. Such a device that floats in a vertical position and is used to measure the specific gravity of a liquid is called a *hydrometer* (Fig. 11–21). The top part of the hydrometer extends above the liquid surface, and the divisions on it allow one to read the specific gravity directly. The hydrometer is calibrated such that in pure water it reads exactly 1.0 at the air–water interface. (*a*) Obtain a relation for the specific gravity of a liquid as a function of distance Δz from the mark corresponding to pure water and (*b*) determine the mass of lead that must be poured into a 1-cm-diameter, 20-cm-long hydrometer if it is to float halfway (the 10-cm mark) in pure water.

SOLUTION The specific gravity of a liquid is to be measured by a hydrometer. A relation between specific gravity and the vertical distance from the reference level is to be obtained, and the amount of lead that needs to be added into the tube for a certain hydrometer is to be determined.

Assumptions **1** The weight of the glass tube is negligible relative to the weight of the lead added. **2** The curvature of the tube bottom is disregarded.

Properties We take the density of pure water to be 1000 kg/m^3.

Analysis (*a*) Noting that the hydrometer is in static equilibrium, the buoyant force F_B exerted by the liquid must always be equal to the weight W of the hydrometer. In pure water (subscript w), we let the vertical distance between

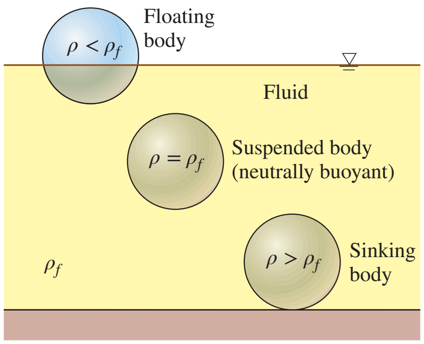


FIGURE 11–18
A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its average density relative to the density of the fluid.

FIGURE 11–19
The density of the water in the Dead Sea is about 24 percent higher than that of pure water. Therefore, people float much more easily (with more of their bodies above the water) in the Dead Sea than in fresh water or in normal seawater.

Photo by Andrew Cimbala

FIGURE 11–20
The altitude of a hot-air balloon is controlled by the temperature difference between the air inside and outside the balloon, since warm air is less dense than cold air. When the balloon is neither rising nor falling, the upward buoyant force exactly balances the downward weight.

PhotoLink/Getty Images

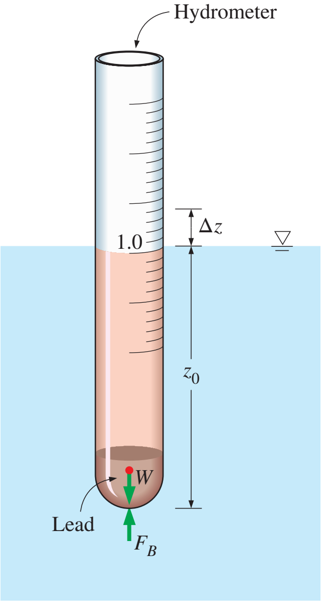


FIGURE 11–21
Schematic for Example 11–3.

the bottom of the hydrometer and the free surface of water be z_0. Setting $F_{B,w} = W$ in this case gives

$$W_{\text{hydro}} = F_{B,w} = \rho_w g V_{\text{sub}} = \rho_w g A z_0 \qquad \textbf{(1)}$$

where A is the cross-sectional area of the tube, and ρ_w is the density of pure water.

In a fluid lighter than water ($\rho_f < \rho_w$), the hydrometer will sink deeper, and the liquid level will be a distance of Δz above z_0. Again setting $F_B = W$ gives

$$W_{\text{hydro}} = F_{B,f} = \rho_f g V_{\text{sub}} = \rho_f g A(z_0 + \Delta z) \qquad \textbf{(2)}$$

This relation is also valid for fluids heavier than water by taking Δz to be a negative quantity. Setting Eqs. (1) and (2) here equal to each other since the weight of the hydrometer is constant and rearranging gives

$$\rho_w g A z_0 = \rho_f g A(z_0 + \Delta z) \quad \rightarrow \quad \text{SG}_f = \frac{\rho_f}{\rho_w} = \frac{z_0}{z_0 + \Delta z}$$

which is the relation between the specific gravity of the fluid and Δz. Note that z_0 is constant for a given hydrometer and Δz is negative for fluids heavier than pure water.

(*b*) Disregarding the weight of the glass tube, the amount of lead that needs to be added to the tube is determined from the requirement that the weight of the lead be equal to the buoyant force. When the hydrometer is floating with half of it submerged in water, the buoyant force acting on it is

$$F_B = \rho_w g V_{\text{sub}}$$

Equating F_B to the weight of lead gives

$$W = mg = \rho_w g V_{\text{sub}}$$

Solving for m and substituting, the mass of lead is determined to be

$$m = \rho_w V_{\text{sub}} = \rho_w(\pi R^2 h_{\text{sub}}) = (1000\ \text{kg/m}^3)[\pi(0.005\ \text{m})^2(0.1\ \text{m})] = \mathbf{0.00785\ kg}$$

Discussion Note that if the hydrometer were required to sink only 5 cm in water, the required mass of lead would be one-half of this amount. Also, the assumption that the weight of the glass tube is negligible is questionable since the mass of lead is only 7.85 g.

EXAMPLE 11–4 Height of Ice Block Below the Water Surface

Consider a large cubic ice block floating in seawater. The specific gravities of ice and seawater are 0.92 and 1.025, respectively. If a 25-cm-high portion of the ice block extends above the surface of the water, determine the height of the ice block below the surface.

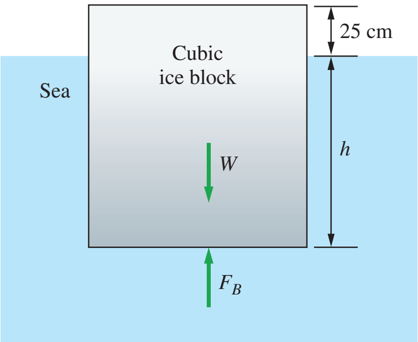


FIGURE 11–22
Schematic for Example 11–4.

SOLUTION The height of the portion of a cubic ice block that extends above the water surface is measured. The height of the ice block below the surface is to be determined.

Assumptions **1** The buoyancy force in air is negligible. **2** The top surface of the ice block is parallel to the surface of the sea.

Properties The specific gravities of ice and seawater are given to be 0.92 and 1.025, respectively, and thus the corresponding densities are 920 kg/m^3 and 1025 kg/m^3.

Analysis The weight of a body floating in a fluid is equal to the buoyant force acting on it (a consequence of vertical force balance from static equilibrium), as shown in Fig. 11–22. Therefore,

$$W = F_B \quad \rightarrow \quad \rho_{\text{body}} g V_{\text{total}} = \rho_{\text{fluid}} g V_{\text{submerged}}$$

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}}$$

The cross-sectional area of a cube is constant, and thus the "volume ratio" can be replaced by "height ratio." Then,

$$\frac{h_{\text{submerged}}}{h_{\text{total}}}=\frac{\rho_{\text{body}}}{\rho_{\text{fluid}}} \quad\rightarrow\quad \frac{h}{h+0.25}=\frac{\rho_{\text{ice}}}{\rho_{\text{water}}} \quad\rightarrow\quad \frac{h}{h+0.25\text{ m}}=\frac{920\text{ kg/m}^3}{1025\text{ kg/m}^3}$$

where h is the height of the ice block below the surface. Solving for h gives

$$h=\frac{(920\text{ kg/m}^3)(0.25\text{ m})}{(1025-920)\text{ kg/m}^3}=\mathbf{2.19\text{ m}}$$

Discussion Note that 0.92/1.025 = 0.898, so approximately 90 percent of the volume of an ice block remains under water. For symmetrical ice blocks this also represents the fraction of height that remains under water. This also applies to icebergs; the vast majority of an iceberg is submerged.

FIGURE 11–23
For floating bodies such as ships, stability is an important consideration for safety.
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Stability of Immersed and Floating Bodies

An important application of the buoyancy concept is the assessment of the stability of immersed and floating bodies with no external attachments. This topic is of great importance in the design of ships and submarines (Fig. 11–23). Here we provide some general qualitative discussions on vertical and rotational stability.

We use the classic "ball on the floor" analogy to explain the fundamental concepts of stability and instability. Shown in Fig. 11–24 are three balls at rest on the floor. Case (*a*) is **stable** since any small disturbance (someone moves the ball to the right or left) generates a restoring force (due to gravity) that returns it to its initial position. Case (*b*) is **neutrally stable** because if someone moves the ball to the right or left, it will stay put at its new location. It has no tendency to move back to its original location, nor does it continue to move away. Case (*c*) is a situation in which the ball may be at rest at the moment, but any disturbance, even an infinitesimal one, causes the ball to roll off the hill—it does not return to its original position; rather it *diverges* from it. This situation is **unstable**. What about a case where the ball is on an *inclined* floor? It is not appropriate to discuss stability for this case since the ball is not in a state of equilibrium. In other words, it cannot be at rest and would roll down the hill even without any disturbance.

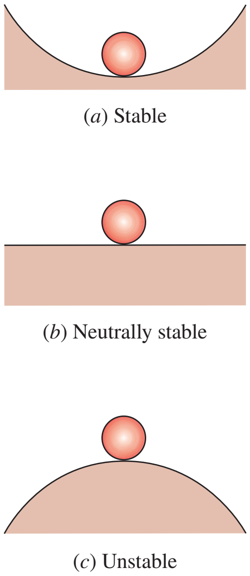

FIGURE 11–24
Stability is easily understood by analyzing a ball on the floor.

For an immersed or floating body in static equilibrium, the weight and the buoyant force acting on the body balance each other, and such bodies are inherently stable in the *vertical direction.* If an immersed neutrally buoyant body is raised or lowered to a different depth in an incompressible fluid, the body will remain in equilibrium at that location. If a floating body is raised or lowered somewhat by a vertical force, the body will return to its original position as soon as the external effect is removed. Therefore, a floating body possesses vertical stability, while an immersed neutrally buoyant body is neutrally stable since it does not return to its original position after a disturbance.

The *rotational stability* of an *immersed body* depends on the relative locations of the *center of gravity* G of the body and the *center of buoyancy* B, which is the centroid of the displaced volume. An immersed body is *stable* if the body is bottom-heavy and thus point G is directly below point B (Fig. 11–25*a*). A rotational disturbance of the body in such cases produces a *restoring moment* to return the body to its original stable position. Thus, a stable design for a submarine calls for the engines and the cabins for the crew to be located in the lower half in order to shift the weight to the bottom as much as possible. Hot-air or helium balloons (which can be viewed as being immersed in air) are also stable since the heavy cage that carries the load is at the bottom. An immersed body whose center of gravity G is directly above point B is *unstable,* and any disturbance will cause this body to turn upside down (Fig 11–25*c*). A body for which G and B coincide is *neutrally stable* (Fig 11–25*b*). This is the case for bodies whose density is constant throughout. For such bodies, there is no tendency to overturn or right themselves.

What about a case where the center of gravity is not vertically aligned with the center of buoyancy, as in Fig. 11–26? It is not appropriate to discuss stability for this case since the body is not in a state of equilibrium. In other words, it cannot be at rest and would rotate toward its stable state even without any disturbance. The restoring moment in the case shown in Fig. 11–26 is counterclockwise and causes the body to rotate counterclockwise so as to align point G vertically with point B. Note that there may be some oscillation, but eventually the body settles down at its stable equilibrium state [case (*a*) of Fig. 11–25]. The initial stability of the body of Fig. 11–26 is analogous to that of the ball on an inclined floor. Can you predict what would happen if the weight in the body of Fig. 11–26 were on the opposite side of the body?

The rotational stability criteria are similar for *floating bodies.* Again, if the floating body is bottom-heavy and thus the center of gravity G is directly below the center of buoyancy $B,$ the body is always stable. But unlike immersed bodies, a floating body may still be stable when G is directly above B (Fig. 11–27). This is because the centroid of the displaced volume shifts to the side to a point B' during a rotational disturbance while the center of gravity G of the body remains unchanged. If point B' is sufficiently far, these two forces create a restoring moment and return the body to the original position. A measure of stability for floating bodies is the **metacentric height** $GM,$ which is the distance between the center of gravity G and the metacenter M—the intersection point of the lines of action of the buoyant force through the body before and after rotation. The metacenter may be considered to be a fixed point for most hull shapes for small rolling angles up to about 20°. Typical values of metacentric height are 0.3–0.7 m for cruise ships, 0.9–1.5 m for sailboats, 0.6–0.9 m for cargo ships, and 0.75–1.3 m for warships. A floating body is stable if point M is above point $G,$ and thus GM is positive, and unstable if point M is below point $G,$ and thus GM is negative. In the latter case, the weight and the buoyant force acting on the tilted body generate an overturning

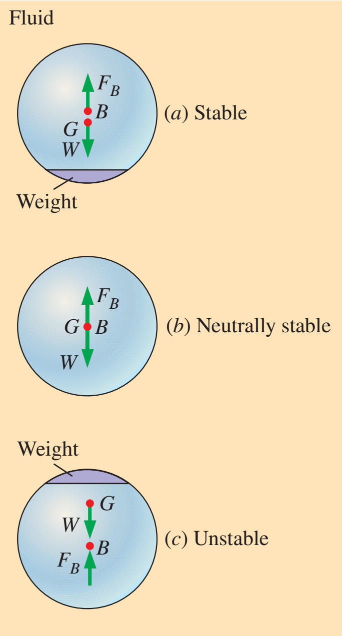

FIGURE 11–25
An immersed neutrally buoyant body is (*a*) stable if the center of gravity G is directly below the center of buoyancy B of the body, (*b*) neutrally stable if G and B are coincident, and (*c*) unstable if G is directly above B.

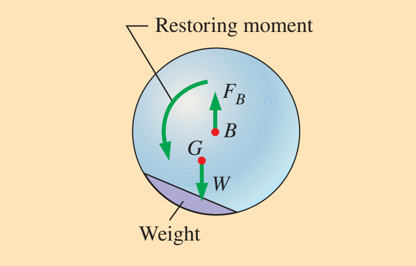

FIGURE 11–26
When the center of gravity G of an immersed neutrally buoyant body is not vertically aligned with the center of buoyancy B of the body, it is not in an equilibrium state and would rotate to its stable state, even without any disturbance.

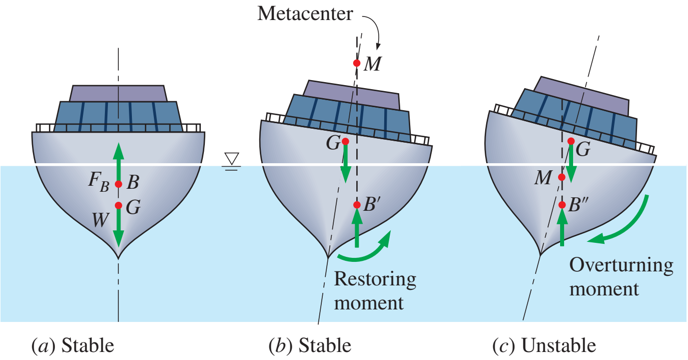

FIGURE 11–27
A floating body is *stable* if the body is (*a*) bottom-heavy and thus the center of gravity G is below the centroid B of the body, or (*b*) if the metacenter M is above point G. However, the body is (*c*) *unstable* if point M is below point G.

moment instead of a restoring moment, causing the body to capsize. The length of the metacentric height *GM* above *G* is a measure of the stability: the larger it is, the more stable is the floating body.

As already discussed, a boat can tilt to some maximum angle without capsizing, but beyond that angle it overturns (and sinks). We make a final analogy between the stability of floating objects and the stability of a ball rolling along the floor. Namely, imagine the ball in a trough between two hills (Fig. 11–28). The ball returns to its stable equilibrium position after being perturbed—up to a limit. If the perturbation amplitude is too great, the ball rolls down the opposite side of the hill and does not return to its equilibrium position. This situation is described as stable up to some limiting level of disturbance, but unstable beyond.

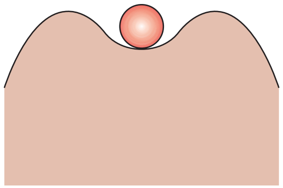

FIGURE 11–28

A ball in a trough between two hills is stable for small disturbances, but unstable for large disturbances.

SUMMARY

Fluid statics deals with problems associated with fluids at rest, and it is called *hydrostatics* when the fluid is a liquid. The magnitude of the resultant force acting on a plane surface of a completely submerged plate in a homogeneous fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface and is expressed as

$$F_R = (P_0 + \rho g h_C)A = P_C A = P_{\text{avg}} A$$

where $h_C = y_C \sin\theta$ is the *vertical distance* of the centroid from the free surface of the liquid. The pressure P_0 is usually atmospheric pressure, which cancels out in most cases since it acts on both sides of the plate. The point of intersection of the line of action of the resultant force and the surface is the *center of pressure*. The vertical location of the line of action of the resultant force is given by

$$y_P = y_C + \frac{I_{xx,\,C}}{[y_C + P_0/(\rho g \sin\theta)]A}$$

where $I_{xx,\,C}$ is the second moment of area about the x-axis passing through the centroid of the area.

A fluid exerts an upward force on a body immersed in it. This force is called the *buoyant force* and is expressed as

$$F_B = \rho_f g V$$

where V is the volume of the body. This is known as *Archimedes' principle* and is expressed as: the buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body; it acts upward through the centroid of the displaced volume. In a fluid with constant density, the buoyant force is independent of the distance of the body from the free surface. For *floating* bodies, the submerged volume fraction of the body is equal to the ratio of the average density of the body to the density of the fluid.

REFERENCES AND SUGGESTED READING

1. F. P. Beer, E. R. Johnston, Jr., E. R. Eisenberg, and G. H. Staab. *Vector Mechanics for Engineers, Statics,* 10th ed. New York: McGraw-Hill, 2012.
2. D. C. Giancoli. *Physics,* 6th ed. Upper Saddle River, NJ: Prentice Hall, 2012.

PROBLEMS[*]

Fluid Statics: Hydrostatic Forces on Plane and Curved Surfaces

11–1C Define the resultant hydrostatic force acting on a submerged surface, and the center of pressure.

11–2C You may have noticed that dams are much thicker at the bottom. Explain why dams are built that way.

11–3C Someone claims that she can determine the magnitude of the hydrostatic force acting on a plane surface submerged in water regardless of its shape and orientation if she knows the vertical distance of the centroid of the surface from the free surface and the area of the surface. Is this a valid claim? Explain.

11–4C A submerged horizontal flat plate is suspended in water by a string attached at the centroid of its upper surface. Now the plate is rotated 45° about an axis that passes through its centroid.

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[*] Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and SI users can ignore them. Problems with the icon 💻 are comprehensive in nature and are intended to be solved with appropriate software.

Discuss the change in the hydrostatic force acting on the top surface of this plate as a result of this rotation. Assume the plate remains submerged at all times.

11–5C Consider a submerged curved surface. Explain how you would determine the horizontal component of the hydrostatic force acting on this surface.

11–6C Consider a submerged curved surface. Explain how you would determine the vertical component of the hydrostatic force acting on this surface.

11–7C Consider a circular surface subjected to hydrostatic forces by a constant density liquid. If the magnitudes of the horizontal and vertical components of the resultant hydrostatic force are determined, explain how you would find the line of action of this force.

11–8E Consider a 200-ft-high, 1200-ft-wide dam filled to capacity. Determine (*a*) the hydrostatic force on the dam and (*b*) the force per unit area of the dam near the top and near the bottom.

11–9 A cylindrical tank is fully filled with water (Fig. P11–9). In order to increase the flow from the tank, an additional pressure is applied to the water surface by a compressor. For $P_0 = 0$, $P_0 = 5$ bar, and $P_0 = 10$ bar, calculate the hydrostatic force on the surface *A* exerted by water.

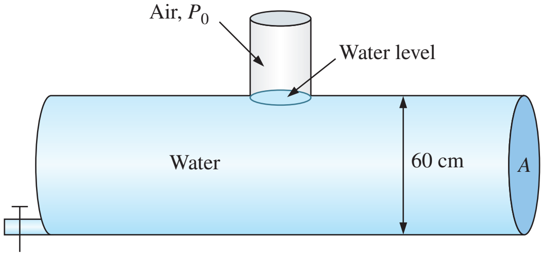

FIGURE P11–9

11–10 Consider an 8-m-long, 8-m-wide, and 2-m-high aboveground swimming pool that is filled with water to the rim. (*a*) Determine the hydrostatic force on each wall and the distance of the line of action of this force from the ground. (*b*) If the height of the walls of the pool is doubled and the pool is filled, will the hydrostatic force on each wall double or quadruple? Why? *Answer:* (*a*) 157 kN

11–11 Consider a heavy car submerged in water in a lake with a flat bottom. The driver's side door of the car is 1.1 m high and 0.9 m wide, and the top edge of the door is 10 m below the water surface. Determine the net force acting on the door (normal to its surface) and the location of the pressure center if (*a*) the car is well-sealed and it contains air at atmospheric pressure and (*b*) the car is filled with water.

11–12 A room in the lower level of a cruise ship has a 40-cm-diameter circular window. If the midpoint of the window is 2 m below the water surface, determine the hydrostatic force acting on the window, and the pressure center. Take the specific gravity of seawater to be 1.025. *Answers:* 2527 N, 2.005 m

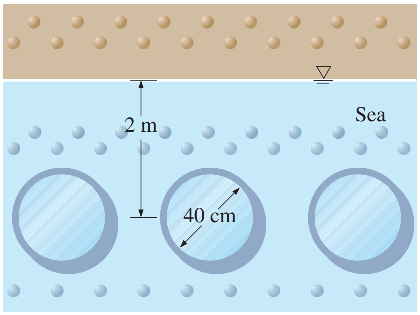

FIGURE P11–12

11–13 The water side of the wall of a 70-m-long dam is a quarter circle with a radius of 7 m. Determine the hydrostatic force on the dam and its line of action when the dam is filled to the rim.

11–14 A water trough of semicircular cross section of radius 0.6 m consists of two symmetric parts hinged to each other at the bottom, as shown in Fig. P11–14. The two parts are held together by a cable and turnbuckle placed every 3 m along the length of the trough. Calculate the tension in each cable when the trough is filled to the rim.

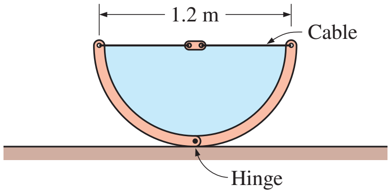

FIGURE P11–14

11–15 Determine the resultant force acting on the 0.7-m-high and 0.7-m-wide triangular gate shown in Fig. P11–15 and its line of action.

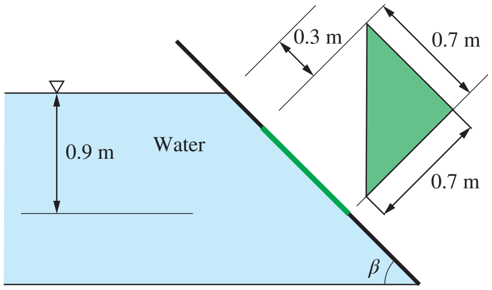

FIGURE P11–15

11–16 A 6-m-high, 5-m-wide rectangular plate blocks the end of a 5-m-deep freshwater channel, as shown in Fig. P11–16. The plate is hinged about a horizontal axis along its upper edge through a point *A* and is restrained from opening by a fixed ridge at point *B*. Determine the force exerted on the plate by the ridge.

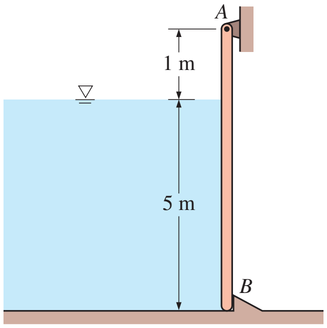

FIGURE P11–16

11–17 Reconsider Prob. 11–16. Using appropriate software, investigate the effect of water depth on the force exerted on the plate by the ridge. Let the water depth vary from 0 to 5 m in increments of 0.5 m. Tabulate and plot your results.

11–18E The flow of water from a reservoir is controlled by a 5-ft-wide L-shaped gate hinged at point *A*, as shown in Fig. P11–18E. If it is desired that the gate open when the water height is 12 ft, determine the mass of the required weight *W*. *Answer:* 30,900 lbm

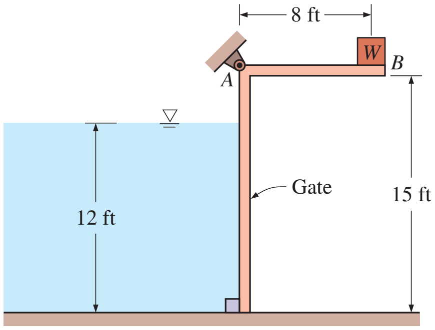

FIGURE P11–18E

11–19E Repeat Prob. 11–18E for a water height of 6 ft.

11–20 For a gate width of 2 m into the paper (Fig. P11–20), determine the force required to hold the gate *ABC* at its location. *Answer:* 17.8 kN

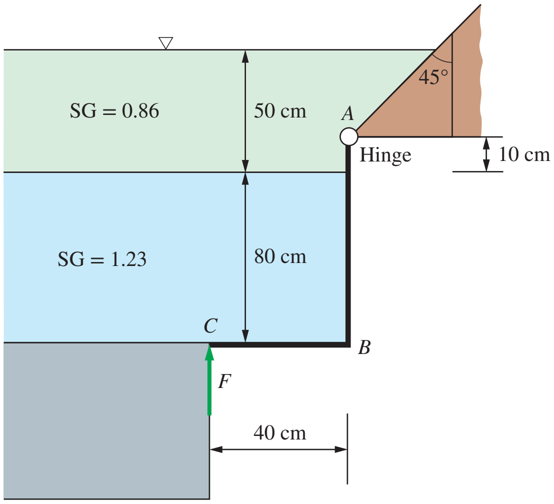

FIGURE P11–20

11–21E A long, solid cylinder of radius 2 ft hinged at point *A* is used as an automatic gate, as shown in Fig. P11–21E. When the water level reaches 12 ft, the cylindrical gate opens by turning about the hinge at point *A*. Determine (*a*) the hydrostatic force acting on the cylinder and its line of action when the gate opens and (*b*) the weight of the cylinder per ft length of the cylinder.

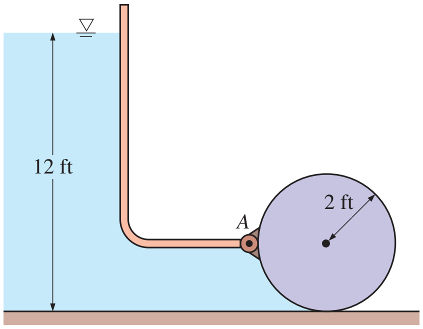

FIGURE P11–21E

11–22 The two sides of a V-shaped water trough are hinged to each other at the bottom where they meet, as shown in Fig. P11–22, making an angle of 45° with the ground from both sides. Each side is 0.75 m wide, and the two parts are held together by a cable and turnbuckle placed every 6 m along the length of the trough. Calculate the tension in each cable when the trough is filled to the rim. *Answer:* 5510 N

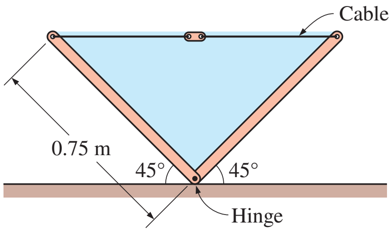

FIGURE P11–22

11–23 Repeat Prob. 11–22 for the case of a partially filled trough with a water height of 0.35 m directly above the hinge.

11–24 A 4-m-long quarter-circular gate of radius 3 m and of negligible weight is hinged about its upper edge *A*, as shown in Fig. P11–24. The gate controls the flow of water over the ledge at *B*, where the gate is pressed by a spring. Determine the minimum spring force required to keep the gate closed when the water level rises to *A* at the upper edge of the gate.

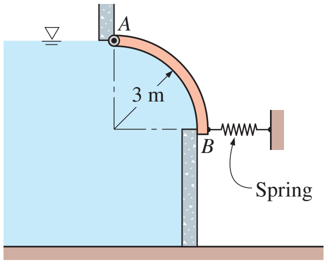

FIGURE P11–24

11–25 Repeat Prob. 11–24 for a radius of 2 m for the gate. *Answer:* 78.5 kN

Buoyancy

11–26C What is buoyant force? What causes it? What is the magnitude of the buoyant force acting on a submerged body whose volume is V? What are the direction and the line of action of the buoyant force?

11–27C Discuss the stability of (*a*) a submerged and (*b*) a floating body whose center of gravity is above the center of buoyancy.

11–28C Consider two 5-cm-diameter spherical balls—one made of aluminum, the other of iron—submerged in water. Will the buoyant forces acting on these two balls be the same or different? Explain.

11–29C Consider a 3-kg copper cube and a 3-kg copper ball submerged in a liquid. Will the buoyant forces acting on these two bodies be the same or different? Explain.

11–30C Consider two identical spherical balls submerged in water at different depths. Will the buoyant forces acting on these two balls be the same or different? Explain.

11–31 A 200-kg granite rock ($\rho = 2700$ kg/m^3) is dropped into a lake. A man dives in and tries to lift the rock. Determine how much force the man needs to apply to lift it from the bottom of the lake. Do you think he can do it?

11–32 The hull of a boat has a volume of 180 m^3, and the total mass of the boat when empty is 8560 kg. Determine how much load this boat can carry without sinking (*a*) in a lake and (*b*) in seawater with a specific gravity of 1.03.

11–33 The density of a liquid is to be determined by an old 1-cm-diameter cylindrical hydrometer whose division marks are completely wiped out. The hydrometer is first dropped in water, and the water level is marked. The hydrometer is then dropped into the other liquid, and it is observed that the mark for water has risen 0.3 cm above the liquid–air interface (Fig. P11–33). If the height of the original water mark is 12.3 cm, determine the density of the liquid.

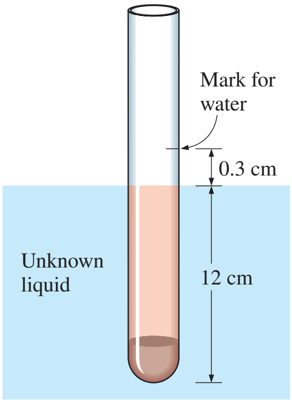

FIGURE P11–33

11–34 It is said that Archimedes discovered his principle during a bath while thinking about how he could determine if King Hiero's crown was actually made of pure gold. While in the bathtub, he conceived the idea that he could determine the average density of an irregularly shaped object by weighing it in air and also in water. If the crown weighed 3.55 kgf (= 34.8 N) in air and 3.25 kgf (= 31.9 N) in water, determine if the crown is made of pure gold. The density of gold is 19,300 kg/m^3. Discuss how you can solve this problem without weighing the crown in water but by using an ordinary bucket with no calibration for volume. You may weigh anything in air.

11–35 It is estimated that 90 percent of an iceberg's volume is below the surface, while only 10 percent is visible above the surface. For seawater with a density of 1025 kg/m^3, estimate the density of the iceberg.

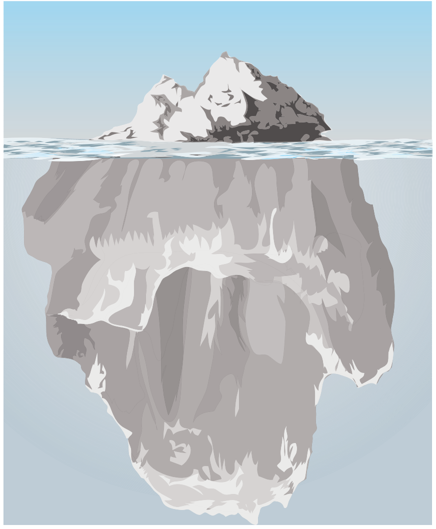

FIGURE P11–35

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11–36 One of the common procedures in fitness programs is to determine the fat-to-muscle ratio of the body. This is based on the principle that the muscle tissue is denser than the fat tissue, and, thus, the higher the average density of the body, the higher is the fraction of muscle tissue. The average density of the body can be determined by weighing the person in air and also while submerged in water in a tank. Treating all tissues and bones (other than fat) as muscle with an equivalent density of ρ_{muscle}, obtain a relation for the volume fraction of body fat x_{fat}. *Answer:* $x_{fat} = (\rho_{muscle} - \rho_{avg})/(\rho_{muscle} - \rho_{fat})$.

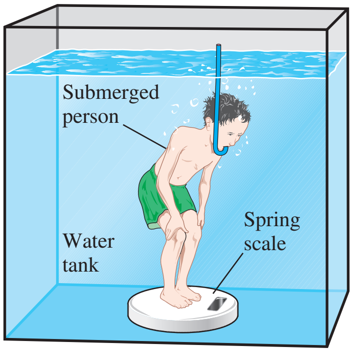


FIGURE P11–36

11–37 A cone floats in the glycerin (SG = 1.26), as shown in the figure. Find the mass of the cone.

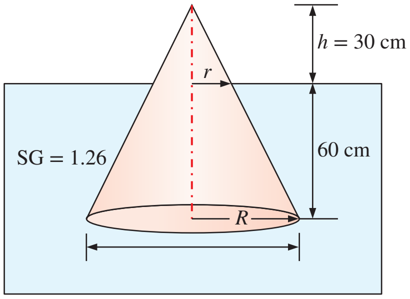


FIGURE P11–37

11–38 The weight of a body is usually measured by disregarding buoyancy force applied by the air. Consider a 20-cm-diameter spherical body of density 7800 kg/m^3. What is the percentage error associated with neglecting air buoyancy?

Review Problems

11–39 An air-conditioning system requires a 34-m-long section of 12-cm-diameter ductwork to be laid underwater. Determine the upward force the water will exert on the duct. Take the densities of air and water to be 1.3 kg/m^3 and 1000 kg/m^3, respectively.

11–40 The 0.5-m-radius semicircular gate shown in the figure is hinged through the top edge AB. Find the required force to be applied at the center of gravity to keep the gate closed. *Answer:* 11.3 kN

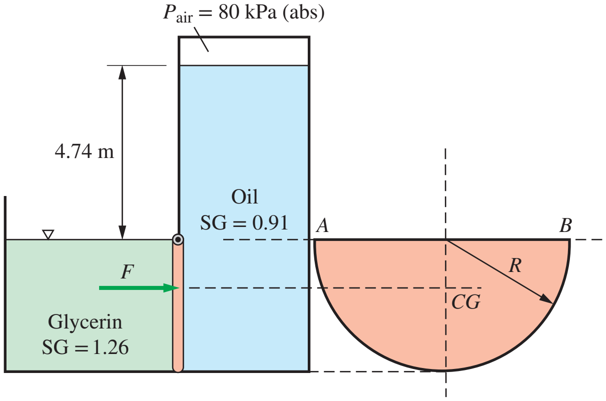


FIGURE P11–40

11–41 An elastic air balloon having a diameter of 30 cm is attached to the base of a container partially filled with water at +4°C, as shown in Fig. P11–41. If the pressure of the air above the water is gradually increased from 100 kPa to 1.6 MPa, will the force on the cable change? If so, what is the percent change in the force? Assume the pressure on the free surface and the diameter of the balloon are related by $P = CD^n$, where C is a constant and $n = -2$. The weight of the balloon and the air in it is negligible. *Answer:* 98.4 percent

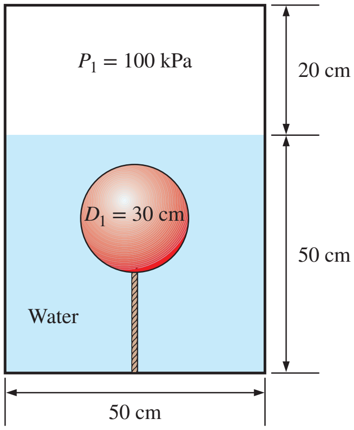


FIGURE P11–41

11–42 Reconsider Prob. 11–41. Using appropriate software, investigate the effect of air pressure above water on the cable force. Let this pressure vary from 0.5 MPa to 15 MPa. Plot the cable force versus the air pressure.

11–43 A 3-m-high, 5-m-wide rectangular gate is hinged at the top edge at A and is restrained by a fixed ridge at B. Determine the hydrostatic force exerted on the gate by the 5-m-high water and the location of the pressure center.

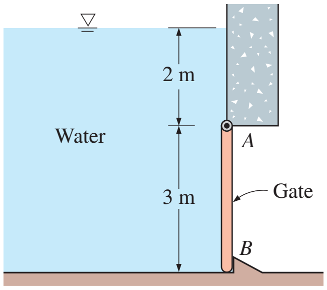


FIGURE P11–43

11–44 Repeat Prob. 11–43 for a total water height of 2 m.

11–45E A semicircular 40-ft-diameter tunnel is to be built under a 150-ft-deep, 800-ft-long lake, as shown in Fig. P11–45E. Determine the total hydrostatic force acting on the roof of the tunnel.

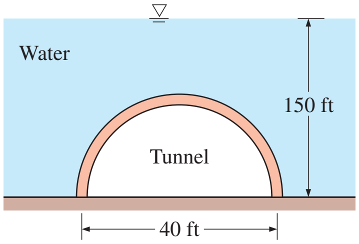


FIGURE P11–45E

11–46 A 30-ton, 4-m-diameter hemispherical dome on a level surface is filled with water, as shown in Fig. P11–46. Someone claims that he can lift this dome by making use of Pascal's law by attaching a long tube to the top and filling it with water. Determine the required height of water in the tube to lift the dome. Disregard the weight of the tube and the water in it. *Answer:* 1.72 m

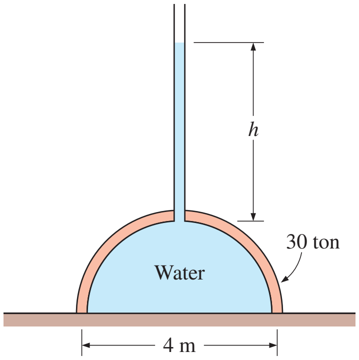


FIGURE P11–46

11–47 The water in a 25-m-deep reservoir is kept inside by a 90-m-wide wall whose cross section is an equilateral triangle, as shown in Fig. P11–47. Determine (*a*) the total force (hydrostatic + atmospheric) acting on the inner surface of the wall and its line of action and (*b*) the magnitude of the horizontal component of this force. Take $P_{atm} = 100$ kPa.

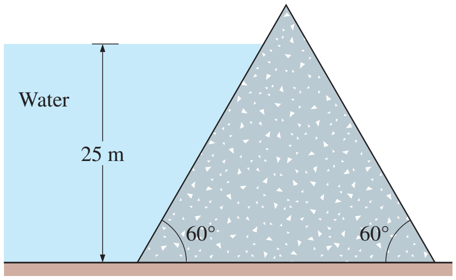


FIGURE P11–47

11–48 A raft is made using a number of logs 25 cm in diameter and 2 m long, as shown in the figure. It is desired that a maximum 90 percent volume of each log will be submerged when carrying two boys weighing 400 N each. Determine the minimum number of logs that must be used. The specific gravity of the wood is 0.75.

FIGURE P11–48

11–49 The 280-kg, 6-m-wide rectangular gate shown in Fig. P11–49 is hinged at *B* and leans against the floor at *A*, making an angle of 45° with the horizontal. The gate is to be opened from its lower edge by applying a normal force at its center. Determine the minimum force *F* required to open the water gate. *Answer:* 626 kN

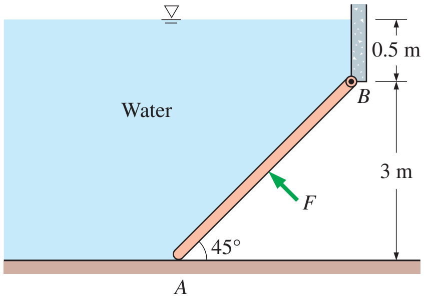


FIGURE P11–49

11–50 Repeat Prob. 11–49 for a water height of 0.8 m above the hinge at *B*.

11–51 Determine the vertical force applied by water on the container.

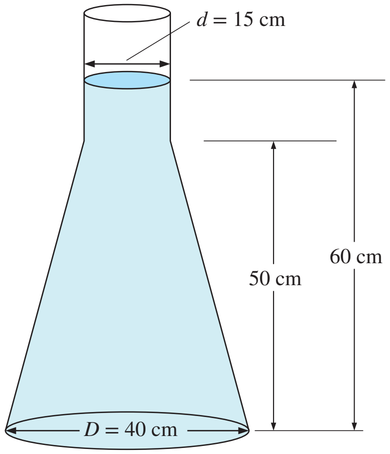


FIGURE P11–51

Design and Essay Problems

11–52 Shoes are to be designed to enable people of up to 80 kg to walk on fresh water or seawater. The shoes are to be made of blown plastic in the shape of a sphere, a (American) football, or a loaf of French bread. Determine the equivalent diameter of each shoe and comment on the proposed shapes from the stability point of view. What is your assessment of the marketability of these shoes?

11–53 The volume of a rock is to be determined without using any volume measurement devices. Explain how you would do this with a waterproof spring scale.

CHAPTER

12

BERNOULLI AND ENERGY EQUATIONS

This chapter deals with two equations commonly used in fluid mechanics: Bernoulli and energy equations. The *Bernoulli equation* is concerned with the conservation of kinetic, potential, and flow energies of a fluid stream and their conversion to each other in regions of flow where net viscous forces are negligible and where other restrictive conditions apply. The *energy equation* is a statement of the conservation of energy principle. In fluid mechanics, it is convenient to separate *mechanical energy* from *thermal energy* and to consider the conversion of mechanical energy to thermal energy as a result of frictional effects as *mechanical energy loss*. Then the energy equation becomes the *mechanical energy balance*.

In this chapter we derive the Bernoulli equation by applying Newton's second law to a fluid element along a streamline and demonstrate its use in a variety of applications. We continue with the development of the energy equation in a form suitable for use in fluid mechanics and introduce the concept of *head loss*. Finally, we apply the energy equation to various engineering systems.

OBJECTIVES

The objectives of this chapter are to:

- Understand the use and limitations of the Bernoulli equation, and apply it to solve a variety of fluid flow problems.
- Work with the energy equation expressed in terms of heads, and use it to determine turbine power output and pumping power requirements.

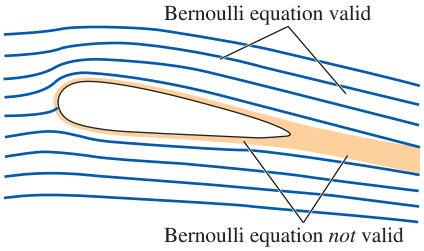


FIGURE 12–1
The *Bernoulli equation* is an *approximate* equation that is valid only in *inviscid regions of flow* where net viscous forces are negligibly small compared to inertial, gravitational, or pressure forces. Such regions occur outside of *boundary layers* and *wakes*.

12–1 ▪ THE BERNOULLI EQUATION

The Bernoulli equation is *an approximate relation between pressure, velocity, and elevation,* and is valid in *regions of steady, incompressible flow where net frictional forces are negligible* (Fig. 12–1). Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics. In this section, we derive the Bernoulli equation by applying the *conservation of linear momentum principle,* and we demonstrate both its usefulness and its limitations.

The key approximation in the derivation of the Bernoulli equation is that *viscous effects are negligibly small compared to inertial, gravitational, and pressure effects*. Since all fluids have viscosity (there is no such thing as an "inviscid fluid"), this approximation cannot be valid for an entire flow field of practical interest. In other words, we cannot apply the Bernoulli equation *everywhere* in a flow, no matter how small the fluid's viscosity. However, it turns out that the approximation *is* reasonable in certain *regions* of many practical flows. We refer to such regions as *inviscid regions of flow,* and we stress that they are *not* regions where the fluid itself is inviscid or frictionless, but rather they are regions where net viscous or frictional forces are negligibly small compared to other forces acting on fluid particles.

Care must be exercised when applying the Bernoulli equation since it is an approximation that applies only to inviscid regions of flow. In general, frictional effects are always important very close to solid walls (*boundary layers*) and directly downstream of bodies (*wakes*). Thus, the Bernoulli approximation is typically useful in flow regions outside of boundary layers and wakes, where the fluid motion is governed by the combined effects of pressure and gravity forces.

Acceleration of a Fluid Particle

The motion of a particle and the path it follows are described by the *velocity vector* as a function of time and space coordinates and the initial position of the particle. When the flow is *steady* (no change with time at a specified location), all particles that pass through the same point follow the same path (which is the *streamline*), and the velocity vectors remain tangent to the path at every point.

Often it is convenient to describe the motion of a particle in terms of its distance s along a streamline together with the radius of curvature along the streamline. The speed of the particle is related to the distance by $V = ds/dt$, which may vary along the streamline. In two-dimensional flow, the acceleration can be decomposed into two components: *streamwise acceleration* a_s along the streamline and *normal acceleration* a_n in the direction normal to the streamline, which is given as $a_n = V^2/R$. Note that streamwise acceleration is due to a change in speed along a streamline, and normal acceleration is due to a change in direction. For particles that move along a *straight path,* $a_n = 0$ since the radius of curvature is infinity and thus there is no change in direction. The Bernoulli equation results from a force balance along a streamline.

One may be tempted to think that acceleration is zero in steady flow since acceleration is the rate of change of velocity with time, and in steady flow there is no change with time. Well, a garden hose nozzle tells us that this understanding is not correct. Even in steady flow and thus constant mass flow rate, water accelerates through the nozzle (Fig. 12–2). *Steady* simply means *no change with time at a specified location,* but the value of a quantity may change from one location to another. In the case of a nozzle, the velocity of water remains constant at a specified point, but it changes from the inlet to the exit (water accelerates along the nozzle).

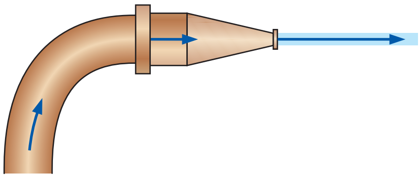


FIGURE 12–2
During steady flow, a fluid may not accelerate in time at a fixed point, but it may accelerate in space.

Mathematically, this can be expressed as follows: We take the velocity V of a fluid particle to be a function of s and t. Taking the total differential of $V(s, t)$ and dividing both sides by dt yield

$$dV = \frac{\partial V}{\partial s}ds + \frac{\partial V}{\partial t}dt \quad \text{and} \quad \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t} \tag{12–1}$$

In steady flow $\partial V/\partial t = 0$ and thus $V = V(s)$, and the acceleration in the s-direction becomes

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s}V = V\frac{dV}{ds} \tag{12–2}$$

where $V = ds/dt$ if we are following a fluid particle as it moves along a streamline. Therefore, acceleration in steady flow is due to the change of velocity with position.

Derivation of the Bernoulli Equation

Consider the motion of a fluid particle in a flow field in steady flow. Applying Newton's second law (which is referred to as the *linear momentum equation* in fluid mechanics) in the s-direction to a particle moving along a streamline gives

$$\sum F_s = ma_s \tag{12–3}$$

In regions of flow where net frictional forces are negligible, there is no pump or turbine, and there is no heat transfer along the streamline, the significant forces acting in the s-direction are the pressure (acting on both sides) and the component of the weight of the particle in the s-direction (Fig. 12–3). Therefore, Eq. 12–3 becomes

$$P\,dA - (P + dP)\,dA - W\sin\theta = mV\frac{dV}{ds} \tag{12–4}$$

where θ is the angle between the normal of the streamline and the vertical z-axis at that point, $m = \rho\mathcal{V} = \rho\,dA\,ds$ is the mass, $W = mg = \rho g\,dA\,ds$ is the weight of the fluid particle, and $\sin\theta = dz/ds$. Substituting,

$$-dP\,dA - \rho g\,dA\,ds\frac{dz}{ds} = \rho\,dA\,ds\,V\frac{dV}{ds} \tag{12–5}$$

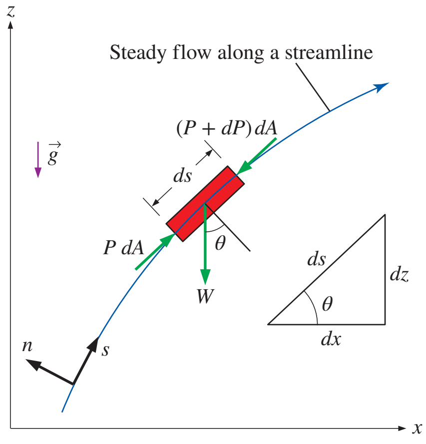


FIGURE 12–3
The forces acting on a fluid particle along a streamline.

Canceling dA from each term and simplifying,

$$-dP - \rho g\,dz = \rho V\,dV \tag{12–6}$$

Noting that $V\,dV = \frac{1}{2}d(V^2)$ and dividing each term by ρ gives

$$\frac{dP}{\rho} + \frac{1}{2}d(V^2) + g\,dz = 0 \tag{12–7}$$

Integrating,

Steady flow: $$\int\frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)} \tag{12–8}$$

since the last two terms are exact differentials. In the case of incompressible flow, the first term also becomes an exact differential, and integration gives

Steady, incompressible flow: $$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)} \tag{12–9}$$

This is the famous **Bernoulli equation** (Fig. 12–4), which is commonly used in fluid mechanics for steady, incompressible flow along a streamline in inviscid regions of flow. The Bernoulli equation was first stated in words by the Swiss mathematician Daniel Bernoulli (1700–1782) in a text written in 1738 when he was working in St. Petersburg, Russia. It was later derived in equation form by his associate Leonhard Euler (1707–1783) in 1755.

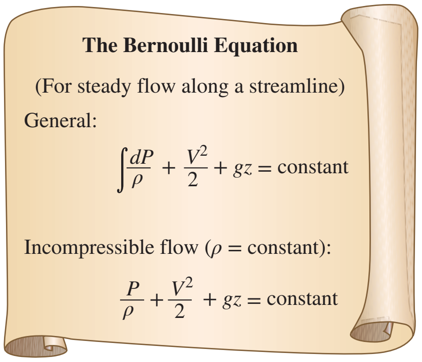


FIGURE 12–4
The incompressible Bernoulli equation is derived assuming incompressible flow, and thus it should not be used for flows with significant compressibility effects.

The value of the constant in Eq. 12–9 can be evaluated at any point on the streamline where the pressure, density, velocity, and elevation are known. The Bernoulli equation can also be written between any two points on the same streamline as

Steady, incompressible flow:
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 \qquad \textbf{(12–10)}$$

We recognize $V^2/2$ as *kinetic energy,* gz as *potential energy,* and P/ρ as *flow energy,* all per unit mass. Therefore, the Bernoulli equation can be viewed as an expression of *mechanical energy balance* and can be stated as follows (Fig. 12–5):

> The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when compressibility and frictional effects are negligible.

The kinetic, potential, and flow energies are the mechanical forms of energy, and the Bernoulli equation can be viewed as the "conservation of mechanical energy principle." This is equivalent to the general conservation of energy principle for systems that do not involve any conversion of mechanical energy and thermal energy to each other, and thus the mechanical energy and thermal energy are conserved separately. The Bernoulli equation states that during steady, incompressible flow with negligible friction, the various forms of mechanical energy are converted to each other, but their sum remains constant. In other words, there is no dissipation of mechanical energy during such flows since there is no friction that converts mechanical energy to sensible thermal (internal) energy.

Recall that energy is transferred to a system as work when a force is applied to the system through a distance. In the light of Newton's second law of motion, the Bernoulli equation can also be viewed as: *The work done by the pressure and gravity forces on the fluid particle is equal to the increase in the kinetic energy of the particle.*

The Bernoulli equation is obtained from Newton's second law for a fluid particle moving along a streamline. It can also be obtained from the *first law of thermodynamics* applied to a steady-flow system, as shown in Section 12–2.

Despite the highly restrictive approximations used in its derivation, the Bernoulli equation is commonly used in practice since a variety of practical fluid flow problems can be analyzed to reasonable accuracy with it. This is because many flows of practical engineering interest are steady (or at least steady in the mean), compressibility effects are relatively small, and net frictional forces are negligible in some regions of interest in the flow.

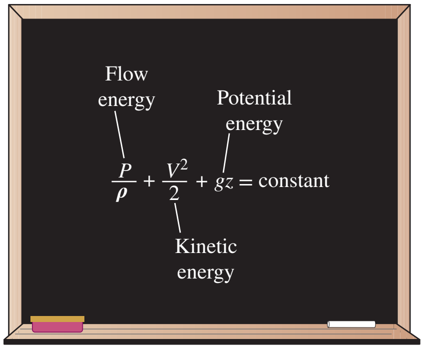


FIGURE 12–5
The Bernoulli equation states that the sum of the kinetic, potential, and flow energies (all per unit mass) of a fluid particle is constant along a streamline during steady flow.

Force Balance Across Streamlines

It is left as an exercise to show that a force balance in the direction n normal to the streamline yields the following relation applicable *across* the streamlines for steady, incompressible flow:

$$\frac{P}{\rho} + \int \frac{V^2}{R} dn + gz = \text{constant} \qquad \text{(across streamlines)} \qquad \textbf{(12–11)}$$

where R is the local radius of curvature of the streamline. For flow along curved streamlines (Fig. 12–6*a*), the pressure *decreases* towards the center of curvature, and fluid particles experience a corresponding centripetal force and centripetal acceleration due to this pressure gradient.

For flow along a straight line, $R \to \infty$ and Eq. 12–11 reduces to $P/\rho + gz =$ constant or $P = -\rho gz +$ constant, which is an expression for the variation of hydrostatic pressure with vertical distance for a stationary fluid body. Therefore, the variation of pressure with elevation in steady, incompressible flow along a straight line in an inviscid region of flow is the same as that in the stationary fluid (Fig. 12–6*b*).

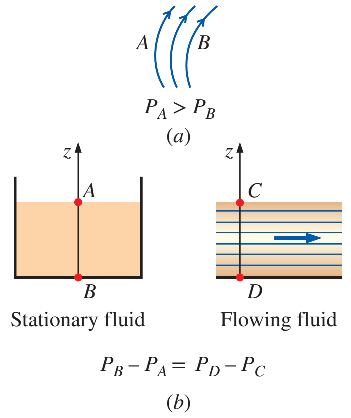


FIGURE 12–6
Pressure decreases towards the center of curvature when streamlines are curved (*a*), but the variation of pressure with elevation in steady, incompressible flow along a straight line (*b*) is the same as that in stationary fluid.

Unsteady, Compressible Flow

Similarly, using both terms in the acceleration expression (Eq. 12–11), it can be shown that the Bernoulli equation for *unsteady, compressible flow* is

Unsteady, compressible flow:
$$\int \frac{dP}{\rho} + \int \frac{\partial V}{\partial t} ds + \frac{V^2}{2} + gz = \text{constant} \qquad \textbf{(12–12)}$$

Static, Dynamic, and Stagnation Pressures

The Bernoulli equation states that the sum of the flow, kinetic, and potential energies of a fluid particle along a streamline is constant. Therefore, the kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. This phenomenon can be made more visible by multiplying the Bernoulli equation by the density ρ,

$$P + \rho\frac{V^2}{2} + \rho gz = \text{constant (along a streamline)} \qquad \textbf{(12–13)}$$

Each term in this equation has pressure units, and thus each term represents some kind of pressure:

- P is the **static pressure** (it does not incorporate any dynamic effects); it represents the actual thermodynamic pressure of the fluid. This is the same as the pressure used in thermodynamics and property tables.
- $\rho V^2/2$ is the **dynamic pressure**; it represents the pressure rise when the fluid in motion is brought to a stop isentropically.
- ρgz is the **hydrostatic pressure** term, which is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, i.e., fluid weight on pressure. (Be careful of the sign—unlike hydrostatic pressure ρgh which *increases* with fluid depth h, the hydrostatic pressure term ρgz *decreases* with fluid depth.)

The sum of the static, dynamic, and hydrostatic pressures is called the **total pressure**. Therefore, the Bernoulli equation states that *the total pressure along a streamline is constant*.

The sum of the static and dynamic pressures is called the **stagnation pressure**, and it is expressed as

$$P_{\text{stag}} = P + \rho\frac{V^2}{2} \qquad \text{(kPa)} \qquad \textbf{(12–14)}$$

The stagnation pressure represents the pressure at a point where the fluid is brought to a complete stop isentropically. The static, dynamic, and stagnation pressure heads are shown in Fig. 12–7, where head is an equivalent column height of the fluid. When static and stagnation pressures are measured at a specified location, the fluid velocity at that location is calculated from

$$V = \sqrt{\frac{2(P_{\text{stag}} - P)}{\rho}} \qquad \textbf{(12–15)}$$

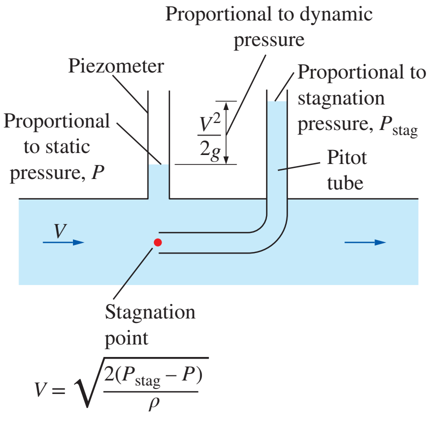


FIGURE 12–7
The static, dynamic, and stagnation pressure heads measured using piezometer tubes.

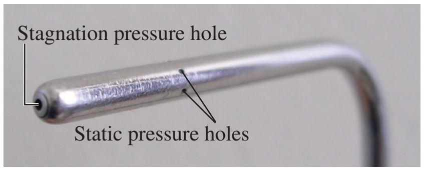


FIGURE 12–8
Close-up of a Pitot-static probe, showing the stagnation pressure hole and two of the five static circumferential pressure holes.
Photo by Po-Ya Abel Chuang. Used by permission

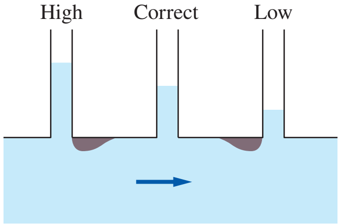


FIGURE 12–9
Careless drilling of the static pressure tap may result in an erroneous reading of the static pressure head.

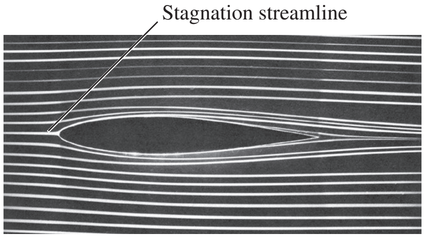


FIGURE 12–10
Streaklines produced by colored fluid introduced upstream of an airfoil; since the flow is steady, the streaklines are the same as streamlines and pathlines. The stagnation streamline is marked.
Courtesy ONERA. Photograph by Werlé.

Equation 12–15 is useful in the measurement of flow velocity when a combination of a static pressure tap and a Pitot tube is used, as illustrated in Fig. 12–7. A **static pressure tap** is simply a small hole drilled into a wall such that the plane of the hole is parallel to the flow direction. It measures the static pressure. A **Pitot tube** is a small tube with its open end aligned *into* the flow so as to sense the full impact pressure of the flowing fluid. It measures the stagnation pressure. In situations in which the static and stagnation pressure of a flowing *liquid* are greater than atmospheric pressure, a vertical transparent tube called a **piezometer tube** (or simply a **piezometer**) can be attached to the pressure tap and to the Pitot tube, as sketched in Fig. 12–7. The liquid rises in the piezometer tube to a column height (*head*) that is proportional to the pressure being measured. If the pressures to be measured are below atmospheric, or if measuring pressures in *gases,* piezometer tubes do not work. However, the static pressure tap and Pitot tube can still be used, but they must be connected to some other kind of pressure measurement device such as a U-tube manometer or a pressure transducer (Chap. 2). Sometimes it is convenient to integrate static pressure holes on a Pitot probe. The result is a **Pitot-static probe** (also called a **Pitot-Darcy probe**), as shown in Fig. 12–8. A Pitot-static probe connected to a pressure transducer or a manometer measures the dynamic pressure (and thus infers the fluid velocity) directly.

When the static pressure is measured by drilling a hole in the tube wall, care must be exercised to ensure that the opening of the hole is flush with the wall surface, with no extrusions before or after the hole (Fig. 12–9). Otherwise the reading would incorporate some dynamic effects, and thus it would be in error.

When a stationary body is immersed in a flowing stream, the fluid is brought to a stop at the nose of the body (the **stagnation point**). The flow streamline that extends from far upstream to the stagnation point is called the **stagnation streamline** (Fig. 12–10). For a two-dimensional flow in the *xy*-plane, the stagnation point is actually a *line* parallel to the *z*-axis, and the stagnation streamline is actually a *surface* that separates fluid that flows *over* the body from fluid that flows *under* the body. In an incompressible flow, the fluid decelerates nearly isentropically from its free-stream velocity to zero at the stagnation point, and the pressure at the stagnation point is thus the stagnation pressure.

Limitations on the Use of the Bernoulli Equation

The Bernoulli equation (Eq. 12–9) is one of the most frequently used and *misused* equations in fluid mechanics. Its versatility, simplicity, and ease of use make it a very valuable tool for use in analysis, but the same attributes also make it very tempting to misuse. Therefore, it is important to understand the restrictions on its applicability and observe the limitations on its use, as explained here:

1. **Steady flow** The first limitation on the Bernoulli equation is that it is applicable to *steady flow*. Therefore, it should not be used during the transient start-up and shut-down periods, or during periods of change in the flow conditions. Note that there is an unsteady form of the Bernoulli equation (Eq. 12–12), discussion of which is beyond the scope of the present text (see Panton, 2005).
2. **Negligible viscous effects** Every flow involves some friction, no matter how small, and *frictional effects* may or may not be negligible. The situation is complicated even more by the amount of error that can be tolerated. In general, frictional effects are negligible for short flow sections with large cross sections, especially at low flow velocities. Frictional effects are

usually significant in long and narrow flow passages, in the wake region downstream of an object, and in *diverging flow sections* such as diffusers because of the increased possibility of the fluid separating from the walls in such geometries. Frictional effects are also significant near solid surfaces, and thus the Bernoulli equation is usually applicable along a streamline in the core region of the flow, but not along a streamline close to the surface (Fig. 12–11).

A component that disturbs the streamlined structure of flow and thus causes considerable mixing and backflow such as a sharp entrance of a tube or a partially closed valve in a flow section can make the Bernoulli equation inapplicable.

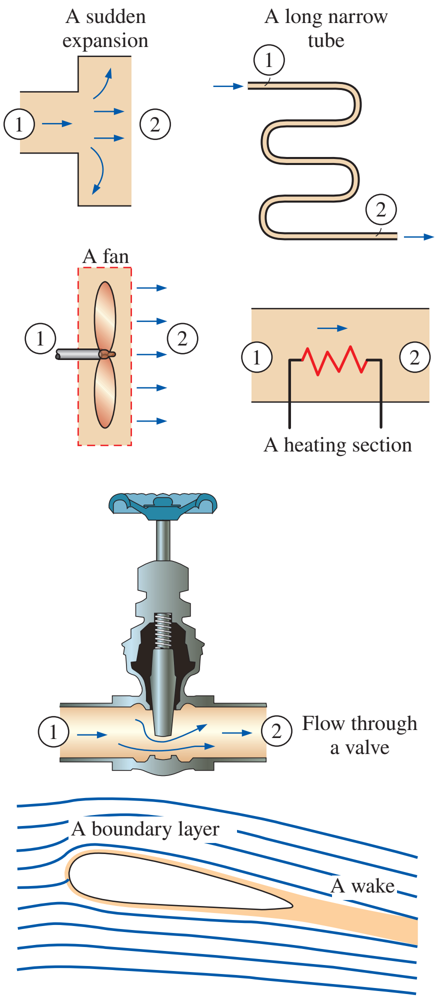
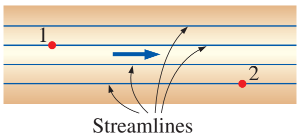


FIGURE 12–11
Frictional effects, heat transfer, and components that disturb the streamlined structure of flow make the Bernoulli equation invalid. It should *not* be used in any of the flows shown here.

3. **No shaft work** The Bernoulli equation was derived from a force balance on a particle moving along a streamline. Therefore, the Bernoulli equation is not applicable in a flow section that involves a pump, turbine, fan, or any other machine or impeller since such devices disrupt the streamlines and carry out energy interactions with the fluid particles. When the flow section considered involves any of these devices, the energy equation should be used instead to account for the shaft work input or output. However, the Bernoulli equation can still be applied to a flow section prior to or past a machine (assuming, of course, that the other restrictions on its use are satisfied). In such cases, the Bernoulli constant changes from upstream to downstream of the device.
4. **Incompressible flow** One of the approximations used in the derivation of the Bernoulli equation is that ρ = constant and thus the flow is incompressible. This condition is satisfied by liquids and also by gases at Mach numbers less than about 0.3 since compressibility effects and thus density variations of gases are negligible at such relatively low velocities. Note that there is a compressible form of the Bernoulli equation (Eqs. 12–8 and 12–12).
5. **Negligible heat transfer** The density of a gas is inversely proportional to temperature, and thus the Bernoulli equation should not be used for flow sections that involve significant temperature change such as heating or cooling sections.
6. **Flow along a streamline** Strictly speaking, the Bernoulli equation $P/\rho + V^2/2 + gz = C$ is applicable along a streamline, and the value of the constant C is generally different for different streamlines. However, when a region of the flow is *irrotational* and there is no *vorticity* in the flow field, the value of the constant C remains the same for all streamlines, and the Bernoulli equation becomes applicable *across* streamlines as well (Fig. 12–12). Therefore, we do not need to be concerned about the streamlines when the flow is irrotational, and we can apply the Bernoulli equation between any two points in the irrotational region of the flow.



$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

FIGURE 12–12
When the flow is irrotational, the Bernoulli equation becomes applicable between any two points along the flow (not just on the same streamline).

We derived the Bernoulli equation by considering two-dimensional flow in the *xz*-plane for simplicity, but the equation is valid for general three-dimensional flow as well, as long as it is applied along the same streamline. We should always keep in mind the approximations used in the derivation of the Bernoulli equation and make sure that they are valid before applying it.

Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)

It is often convenient to represent the level of mechanical energy graphically using *heights* to facilitate visualization of the various terms of the Bernoulli equation. This is done by dividing each term of the Bernoulli equation by g to give

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant} \qquad \text{(along a streamline)} \tag{12–16}$$

Each term in this equation has the dimension of length and represents some kind of "head" of a flowing fluid as follows:

- $P/\rho g$ is the **pressure head**; it represents the height of a fluid column that produces the static pressure P.
- $V^2/2g$ is the **velocity head**; it represents the elevation needed for a fluid to reach the velocity V during frictionless free fall.
- z is the **elevation head**; it represents the potential energy of the fluid.

Also, H is the **total head** for the flow. Therefore, the Bernoulli equation is expressed in terms of heads as: *The sum of the pressure, velocity, and elevation heads along a streamline is constant during steady flow when compressibility and frictional effects are negligible* (Fig. 12–13).

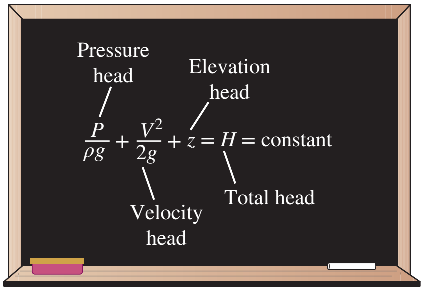


FIGURE 12–13
An alternative form of the Bernoulli equation is expressed in terms of heads as: *The sum of the pressure, velocity, and elevation heads is constant along a streamline.*

If a piezometer (which measures static pressure) is tapped into a pressurized pipe, as shown in Fig. 12–14, the liquid will rise to a height of $P/\rho g$ above the pipe center. The *hydraulic grade line* (HGL) is obtained by doing this at several locations along the pipe and drawing a curve through the liquid levels in the piezometers. The vertical distance above the pipe center is a measure of pressure within the pipe. Similarly, if a Pitot tube (measures static + dynamic pressure) is tapped into a pipe, the liquid will rise to a height of $P/\rho g + V^2/2g$ above the pipe center, or a distance of $V^2/2g$ above the HGL. The *energy grade line* (EGL) is obtained by doing this at several locations along the pipe and drawing a curve through the liquid levels in the Pitot tubes.

Noting that the fluid also has elevation head z (unless the reference level is taken to be the centerline of the pipe), the HGL and EGL are defined as follows: The line that represents the sum of the static pressure and the elevation heads, $P/\rho g + z$, is called the **hydraulic grade line**. The line that represents the total head of the fluid, $P/\rho g + V^2/2g + z$, is called the **energy grade line**. The difference between the heights of EGL and HGL is equal to the dynamic head, $V^2/2g$. We note the following about the HGL and EGL:

- For *stationary bodies* such as reservoirs or lakes, the EGL and HGL coincide with the free surface of the liquid. The elevation of the free surface z in such cases represents both the EGL and the HGL since the velocity is zero and the static (gage) pressure is zero.

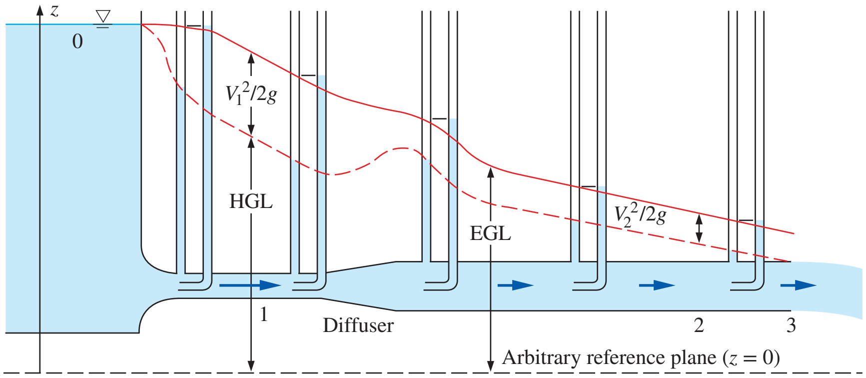


FIGURE 12–14
The *hydraulic grade line* (HGL) and the *energy grade line* (EGL) for free discharge from a reservoir through a horizontal pipe with a diffuser.

- The EGL is always a distance $V^2/2g$ above the HGL. These two curves approach each other as the velocity decreases, and they diverge as the velocity increases. The height of the HGL decreases as the velocity increases, and vice versa.
- In an *idealized Bernoulli-type flow,* EGL is horizontal and its height remains constant. This would also be the case for HGL when the flow velocity is constant (Fig. 12–15).
- For *open-channel flow,* the HGL coincides with the free surface of the liquid, and the EGL is a distance $V^2/2g$ above the free surface.
- At a *pipe exit,* the pressure head is zero (atmospheric pressure), and thus the HGL coincides with the pipe outlet (location 3 on Fig. 12–14).
- The *mechanical energy loss* due to frictional effects (conversion to thermal energy) causes the EGL and HGL to slope downward in the direction of flow. The slope is a measure of the head loss in the pipe (discussed in detail in Chap. 14). A component that generates significant frictional effects such as a valve causes a sudden drop in both EGL and HGL at that location.
- A *steep jump* occurs in EGL and HGL whenever mechanical energy is added to the fluid (by a pump, for example). Likewise, a *steep drop* occurs in EGL and HGL whenever mechanical energy is removed from the fluid (by a turbine, for example), as shown in Fig. 12–16.
- The gage pressure of a fluid is zero at locations where the HGL *intersects* the fluid. The pressure in a flow section that lies above the HGL is negative, and the pressure in a section that lies below the HGL is positive (Fig. 12–17). Therefore, an accurate drawing of a piping system overlaid with the HGL can be used to determine the regions where the gage pressure in the pipe is negative (below atmospheric pressure).

The last remark enables us to avoid situations in which the pressure drops below the vapor pressure of the liquid (which may cause *cavitation,* as discussed in Chap. 10). Proper consideration is necessary in the placement of a liquid pump to ensure that the suction side pressure does not fall too low, especially at elevated temperatures where vapor pressure is higher than it is at low temperatures.

Now we examine Fig. 12–14 more closely. At point 0 (at the liquid surface), EGL and HGL are even with the liquid surface since there is no flow there. HGL decreases rapidly as the liquid accelerates into the pipe; however, EGL decreases very slowly through the well-rounded pipe inlet. EGL declines continually along the flow direction due to friction and other irreversible losses in the flow. EGL cannot increase in the flow direction unless energy is supplied to the fluid. HGL can rise or fall in the flow direction, but can never exceed EGL. HGL rises in the diffuser section as the velocity decreases, and the static pressure recovers somewhat; the total pressure does *not* recover, however, and EGL decreases through the diffuser. The difference between EGL and HGL is $V_1^2/2g$ at point 1, and $V_2^2/2g$ at point 2. Since $V_1 > V_2$, the difference between the two grade lines is larger at point 1 than at point 2. The downward slope of both grade lines is larger for the smaller diameter section of pipe since the frictional head loss is greater. Finally, HGL decays to the liquid surface at the outlet since the pressure there is atmospheric. However, EGL is still higher than HGL by the amount $V_2^2/2g$ since $V_3 = V_2$ at the outlet.

Applications of the Bernoulli Equation

So far, we have discussed the fundamental aspects of the Bernoulli equation. Now, we demonstrate its use in a wide range of applications through examples.

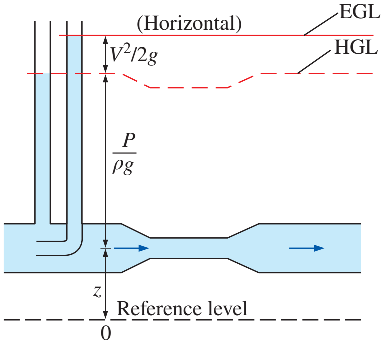


FIGURE 12–15
In an idealized Bernoulli-type flow, EGL is horizontal and its height remains constant. But this is not the case for HGL when the flow velocity varies along the flow.

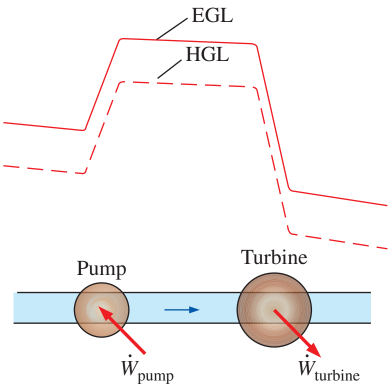


FIGURE 12–16
A *steep jump* occurs in EGL and HGL whenever mechanical energy is added to the fluid by a pump, and a *steep drop* occurs whenever mechanical energy is removed from the fluid by a turbine.

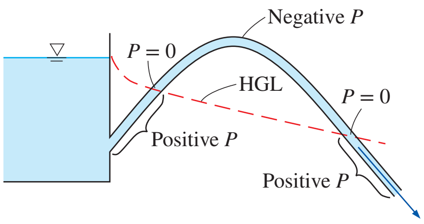


FIGURE 12–17
The gage pressure of a fluid is zero at locations where the HGL *intersects* the fluid, and the gage pressure is negative (vacuum) in a flow section that lies above the HGL.

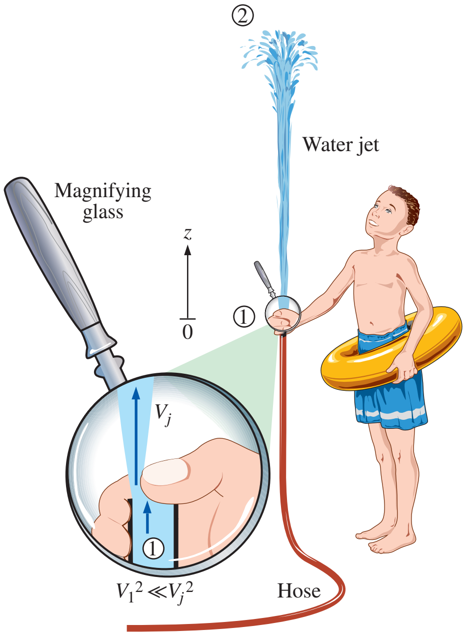


FIGURE 12–18
Schematic for Example 12–1. Inset shows a magnified view of the hose outlet region.

EXAMPLE 12–1 Spraying Water into the Air

Water is flowing from a garden hose (Fig. 12–18). A child places his thumb to cover most of the hose outlet, causing a thin jet of high-speed water to emerge. The pressure in the hose just upstream of his thumb is 400 kPa. If the hose is held upward, what is the maximum height that the jet could achieve?

SOLUTION Water from a hose attached to the water main is sprayed into the air. The maximum height the water jet can rise is to be determined.

Assumptions **1** The flow exiting into the air is steady, incompressible, and irrotational (so the Bernoulli equation is applicable). **2** The surface tension effects are negligible. **3** The friction between the water and air is negligible. **4** The irreversibilities that occur at the outlet of the hose due to abrupt contraction are not taken into account.

Properties We take the density of water to be 1000 kg/m^3.

Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, or wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is negligibly small compared to that of the jet ($V_1^2 \ll V_j^2$, see magnified portion of Fig. 12–18), and we take the elevation just below the hose outlet as the reference level ($z_1 = 0$). At the top of the water trajectory $V_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli equation along a streamline from 1 to 2 simplifies to

$$\frac{P_1}{\rho g} + \overset{\text{ignore}}{\cancel{\frac{V_1^2}{2g}}} + \overset{0}{\cancel{z_1}} = \frac{P_2}{\rho g} + \overset{0}{\cancel{\frac{V_2^2}{2g}}} + z_2 \quad \rightarrow \quad \frac{P_1}{\rho g} = \frac{P_{\text{atm}}}{\rho g} + z_2$$

Solving for z_2 and substituting,

$$z_2 = \frac{P_1 - P_{\text{atm}}}{\rho g} = \frac{P_{1,\text{gage}}}{\rho g} = \frac{400 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right)\left(\frac{1 \text{ kg·m/s}^2}{1 \text{ N}}\right)$$
$$= \mathbf{40.8\ m}$$

Therefore, the water jet can rise as high as 40.8 m into the sky in this case.

Discussion The result obtained by the Bernoulli equation represents the upper limit and should be interpreted accordingly. It tells us that the water cannot possibly rise more than 40.8 m, and, in all likelihood, the rise will be much less than 40.8 m due to irreversible losses that we neglected.

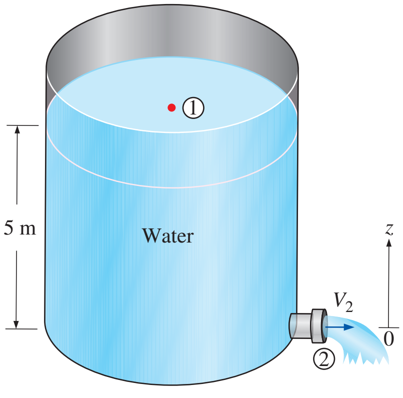


FIGURE 12–19
Schematic for Example 12–2.

EXAMPLE 12–2 Water Discharge from a Large Tank

A large tank open to the atmosphere is filled with water to a height of 5 m from the outlet tap (Fig. 12–19). A tap near the bottom of the tank is now opened, and water flows out from the smooth and rounded outlet. Determine the maximum water velocity at the outlet.

SOLUTION A tap near the bottom of a tank is opened. The maximum exit velocity of water from the tank is to be determined.

Assumptions **1** The flow is incompressible and irrotational (except very close to the walls). **2** The water drains slowly enough that the flow can be approximated as steady (actually quasi-steady when the tank begins to drain). **3** Irreversible losses in the tap region are neglected.

Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, or wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. We take point 1 to be at the free surface of water so that $P_1 = P_{\text{atm}}$ (open to the atmosphere), V_1 is negligibly small compared to V_2 (the tank diameter is very large relative to the outlet diameter), $z_1 = 5$ m, and $z_2 = 0$ (we take the reference level at the center of the outlet). Also, $P_2 = P_{\text{atm}}$ (water discharges into the atmosphere). For flow along a streamline from 1 to 2, the Bernoulli equation simplifies to

$$\cancel{\frac{P_1}{\rho g}} + \overset{\text{ignore}}{\cancel{\frac{V_1^2}{2g}}} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \overset{0}{\cancel{z_2}} \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Solving for V_2 and substituting,

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81\ \text{m/s}^2)(5\ \text{m})} = \mathbf{9.9\ m/s}$$

The relation $V = \sqrt{2gz}$ is called the **Torricelli equation**.

Therefore, the water leaves the tank with an initial maximum velocity of 9.9 m/s. This is the same velocity that would manifest if a solid were dropped a distance of 5 m in the absence of air friction drag. (What would the velocity be if the tap were at the bottom of the tank instead of on the side?)

Discussion If the orifice were sharp-edged instead of rounded, then the flow would be disturbed, and the average exit velocity would be less than 9.9 m/s. Care must be exercised when attempting to apply the Bernoulli equation to situations where abrupt expansions or contractions occur since the friction and flow disturbance in such cases may not be negligible. From conservation of mass, $(V_1/V_2)^2 = (D_2/D_1)^4$. So, for example, if $D_2/D_1 = 0.1$, then $(V_1/V_2)^2 = 0.0001$, and our approximation that $V_1^2 \ll V_2^2$ is justified.

EXAMPLE 12–3 Siphoning Out Gasoline from a Fuel Tank

During a trip to the beach ($P_{\text{atm}} = 1$ atm $= 101.3$ kPa), a car runs out of gasoline, and it becomes necessary to siphon gas out of the car of a Good Samaritan (Fig. 12–20). The siphon is a small-diameter hose, and to start the siphon it is necessary to insert one siphon end in the full gas tank, fill the hose with gasoline via suction, and then place the other end in a gas can below the level of the gas tank. The difference in pressure between point 1 (at the free surface of the gasoline in the tank) and point 2 (at the outlet of the tube) causes the liquid to flow from the higher to the lower elevation. Point 2 is located 0.75 m below point 1 in this case, and point 3 is located 2 m above point 1. The siphon diameter is 5 mm, and frictional losses in the siphon are to be disregarded. Determine (*a*) the minimum time to withdraw 4 L of gasoline from the tank to the can and (*b*) the pressure at point 3. The density of gasoline is 750 kg/m^3.

SOLUTION Gasoline is to be siphoned from a tank. The minimum time it takes to withdraw 4 L of gasoline and the pressure at the highest point in the system are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Even though the Bernoulli equation is not valid through the pipe because of frictional losses, we employ the Bernoulli equation anyway in order to obtain a *best-case estimate*. **3** The change in the gasoline surface level inside the tank is negligible compared to elevations z_1 and z_2 during the siphoning period.

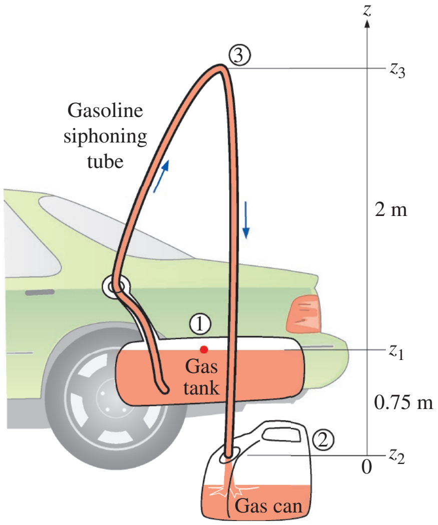


FIGURE 12–20
Schematic for Example 12–3.

Properties The density of gasoline is given to be 750 kg/m^3.

Analysis (*a*) We take point 1 to be at the free surface of gasoline in the tank so that $P_1 = P_{\text{atm}}$ (open to the atmosphere), $V_1 \cong 0$ (the tank is large relative to the tube diameter), and $z_2 = 0$ (point 2 is taken as the reference level). Also, $P_2 = P_{\text{atm}}$ (gasoline discharges into the atmosphere). Then the Bernoulli equation simplifies to

$$\cancel{\frac{P_1}{\rho g}} + \cancelto{\approx 0}{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \cancelto{0}{z_2} \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Solving for V_2 and substituting,

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81\ \text{m/s}^2)(0.75\ \text{m})} = 3.84\ \text{m/s}$$

The cross-sectional area of the tube and the flow rate of gasoline are

$$A = \pi D^2/4 = \pi(5 \times 10^{-3}\ \text{m})^2/4 = 1.96 \times 10^{-5}\ \text{m}^2$$

$$\dot{V} = V_2 A = (3.84\ \text{m/s})(1.96 \times 10^{-5}\ \text{m}^2) = 7.53 \times 10^{-5}\ \text{m}^3/\text{s} = 0.0753\ \text{L/s}$$

Then the time needed to siphon 4 L of gasoline becomes

$$\Delta t = \frac{V}{\dot{V}} = \frac{4\ \text{L}}{0.0753\ \text{L/s}} = \mathbf{53.1\ s}$$

(*b*) The pressure at point 3 is determined by writing the Bernoulli equation along a streamline between points 3 and 2. Noting that $V_2 = V_3$ (conservation of mass), $z_2 = 0$, and $P_2 = P_{\text{atm}}$,

$$\frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancelto{0}{z_2} = \frac{P_3}{\rho g} + \cancel{\frac{V_3^2}{2g}} + z_3 \quad \rightarrow \quad \frac{P_{\text{atm}}}{\rho g} = \frac{P_3}{\rho g} + z_3$$

Solving for P_3 and substituting,

$$\begin{aligned} P_3 &= P_{\text{atm}} - \rho g z_3 \\ &= 101.3\ \text{kPa} - (750\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.75\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= \mathbf{81.1\ kPa} \end{aligned}$$

Discussion The siphoning time is determined by neglecting frictional effects, and thus this is the *minimum time* required. In reality, the time will be longer than 53.1 s because of friction between the gasoline and the tube surface, along with other irreversible losses, as discussed in Chap. 14. Also, the pressure at point 3 is below the atmospheric pressure. If the elevation difference between points 1 and 3 is too high, the pressure at point 3 may drop below the vapor pressure of gasoline at the gasoline temperature, and some gasoline may evaporate (cavitate). The vapor then may form a pocket at the top and halt the flow of gasoline.

EXAMPLE 12–4 Velocity Measurement by a Pitot Tube

A piezometer and a Pitot tube are tapped into a horizontal water pipe, as shown in Fig. 12–21, to measure static and stagnation (static + dynamic) pressures. For the indicated water column heights, determine the velocity at the center of the pipe.

SOLUTION The static and stagnation pressures in a horizontal pipe are measured. The velocity at the center of the pipe is to be determined.

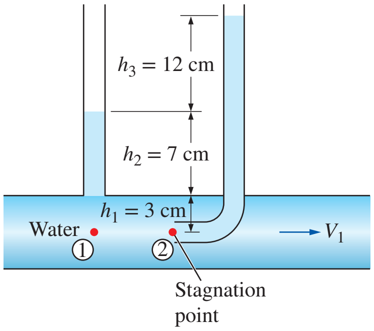


FIGURE 12–21
Schematic for Example 12–4.

Assumptions **1** The flow is steady and incompressible. **2** Points 1 and 2 are close enough together that the irreversible energy loss between these two points is negligible, and thus we can use the Bernoulli equation.

Analysis We take points 1 and 2 along the streamline at the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the tip of the Pitot tube. This is a steady flow with straight and parallel streamlines, and the gage pressures at points 1 and 2 can be expressed as

$$P_1 = \rho g(h_1 + h_2)$$
$$P_2 = \rho g(h_1 + h_2 + h_3)$$

Noting that $z_1 = z_2$, and point 2 is a stagnation point and thus $V_2 = 0$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{\cancelto{0}{V_2^2}}{2g} + \cancel{z_2} \quad \rightarrow \quad \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

Substituting the P_1 and P_2 expressions gives

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} = \frac{\rho g(h_1 + h_2 + h_3) - \rho g(h_1 + h_2)}{\rho g} = h_3$$

Solving for V_1 and substituting,

$$V_1 = \sqrt{2gh_3} = \sqrt{2(9.81\ \text{m/s}^2)(0.12\ \text{m})} = \mathbf{1.53\ m/s}$$

Discussion Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the Pitot tube compared to that in the piezometer tube.

12–2 ▪ ENERGY ANALYSIS OF STEADY FLOWS

For steady flows, the time rate of change of the energy content of the control volume is zero, and the energy equation can be expressed as

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) \qquad \textbf{(12–17)}$$

It states that *during steady flow the net rate of energy transfer to a control volume by heat and work transfers is equal to the difference between the rates of outgoing and incoming energy flows by mass flow*.

Many practical problems involve just one inlet and one outlet (Fig. 12–22). The mass flow rate for such **single-stream devices** is the same at the inlet and outlet, and Eq. 12–17 reduces to

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right) \qquad \textbf{(12–18)}$$

where subscripts 1 and 2 refer to the inlet and outlet, respectively. The steady-flow energy equation on a unit-mass basis is obtained by dividing Eq. 12–18 by the mass flow rate $\dot{m}$,

$$q_{\text{net in}} + w_{\text{shaft, net in}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \qquad \textbf{(12–19)}$$

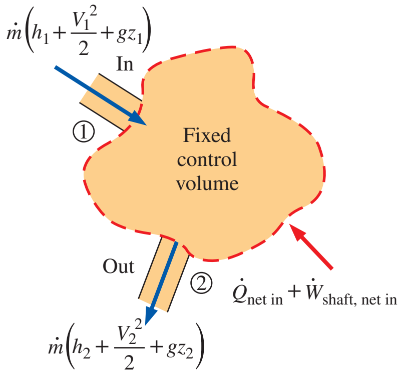


FIGURE 12–22
A control volume with only one inlet and one outlet and energy interactions.

where $q_{\text{net in}} = \dot{Q}_{\text{net in}}/\dot{m}$ is the net heat transfer to the fluid per unit mass and $w_{\text{shaft, net in}} = \dot{W}_{\text{shaft, net in}}/\dot{m}$ is the net shaft work input to the fluid per unit mass. Using the definition of enthalpy $h = u + P/\rho$ and rearranging, the steady-flow energy equation can also be expressed as

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + (u_2 - u_1 - q_{\text{net in}}) \tag{12–20}$$

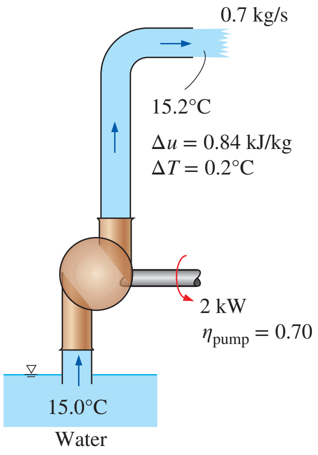


FIGURE 12–23
The lost mechanical energy in a fluid flow system results in an increase in the internal energy of the fluid and thus in a rise of fluid temperature.

where u is the *internal energy*, P/ρ is the *flow energy*, $V^2/2$ is the *kinetic energy*, and gz is the *potential energy* of the fluid, all per unit mass. These relations are valid for both compressible and incompressible flows.

The left side of Eq. 12–20 represents the mechanical energy input, while the first three terms on the right side represent the mechanical energy output. If the flow is ideal with no irreversibilities such as friction, the total mechanical energy must be conserved, and the term in parentheses $(u_2 - u_1 - q_{\text{net in}})$ must equal zero. That is,

Ideal flow (no mechanical energy loss): $$q_{\text{net in}} = u_2 - u_1 \tag{12–21}$$

Any increase in $u_2 - u_1$ above $q_{\text{net in}}$ is due to the irreversible conversion of mechanical energy to thermal energy, and thus $u_2 - u_1 - q_{\text{net in}}$ represents the mechanical energy loss per unit mass (Fig. 12–23). That is,

Real flow (with mechanical energy loss): $$e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}} \tag{12–22}$$

For single-phase fluids (a gas or a liquid), $u_2 - u_1 = c_\upsilon(T_2 - T_1)$ where c_υ is the constant-volume specific heat.

The steady-flow energy equation on a unit-mass basis can be written conveniently as a **mechanical energy** balance,

$$e_{\text{mech, in}} = e_{\text{mech, out}} + e_{\text{mech, loss}} \tag{12–23}$$

or

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + e_{\text{mech, loss}} \tag{12–24}$$

Noting that $w_{\text{shaft, net in}} = w_{\text{pump}} - w_{\text{turbine}}$, the mechanical energy balance can be written more explicitly as

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}} + e_{\text{mech, loss}} \tag{12–25}$$

where w_{pump} is the mechanical work input (due to the presence of a pump, fan, compressor, etc.) and w_{turbine} is the mechanical work output (due to a turbine). When the flow is incompressible, either absolute or gage pressure can be used for P since P_{atm}/ρ would appear on both sides and would cancel out.

Multiplying Eq. 12–25 by the mass flow rate $\dot{m}$ gives

$$\dot{m}\left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}} \tag{12–26}$$

FIGURE 12–24
A typical power plant has numerous pipes, elbows, valves, pumps, and turbines, all of which have irreversible losses.
Brand X Pictures/PunchStock

where $\dot{W}_{\text{pump}}$ is the shaft power input through the pump's shaft, $\dot{W}_{\text{turbine}}$ is the shaft power output through the turbine's shaft, and $\dot{E}_{\text{mech, loss}}$ is the *total* mechanical power loss, which consists of pump and turbine losses as well as the frictional losses in the piping network. That is,

$$\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, pump}} + \dot{E}_{\text{mech loss, turbine}} + \dot{E}_{\text{mech loss, piping}}$$

By convention, irreversible pump and turbine losses are treated separately from irreversible losses due to other components of the piping system (Fig. 12–24). Thus, the energy equation is expressed in its most common form in terms of *heads* by dividing each term in Eq. 12–26 by $\dot{m}g$. The result is

$$\frac{P_1}{\rho_1 g}+\frac{V_1^2}{2g}+z_1+h_{\text{pump},\,u}=\frac{P_2}{\rho_2 g}+\frac{V_2^2}{2g}+z_2+h_{\text{turbine},\,e}+h_L \qquad \textbf{(12–27)}$$

where

- $h_{\text{pump},\,u}=\dfrac{w_{\text{pump},\,u}}{g}+\dfrac{\dot{W}_{\text{pump},\,u}}{\dot{m}g}=\dfrac{\eta_{\text{pump}}\,\dot{W}_{\text{pump}}}{\dot{m}g}$ is the *useful head delivered to the fluid by the pump*. Because of irreversible losses in the pump, $h_{\text{pump},\,u}$ is *less* than $\dot{W}_{\text{pump}}/\dot{m}g$ by the factor η_{pump}.
- $h_{\text{turbine},\,e}=\dfrac{w_{\text{turbine},\,e}}{g}+\dfrac{\dot{W}_{\text{turbine},\,e}}{\dot{m}g}=\dfrac{\dot{W}_{\text{turbine}}}{\eta_{\text{pump}}\dot{m}g}$ is the *extracted head removed from the fluid by the turbine*. Because of irreversible losses in the turbine, $h_{\text{turbine},\,e}$ is *greater* than $\dot{W}_{\text{turbine}}/\dot{m}g$ by the factor η_{turbine}.
- $h_L=\dfrac{e_{\text{mech loss, piping}}}{g}=\dfrac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g}$ is the *irreversible head loss* between 1 and 2 due to all components of the piping system other than the pump or turbine.

Note that the head loss h_L represents the frictional losses associated with fluid flow in piping, and it does not include the losses that occur within the pump or turbine due to the inefficiencies of these devices—these losses are taken into account by η_{pump} and η_{turbine}. Equation 12–27 is illustrated schematically in Fig. 12–25.

The *pump head* is zero if the piping system does not involve a pump, a fan, or a compressor, and the *turbine head* is zero if the system does not involve a turbine.

Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction

When piping losses are negligible, there is negligible dissipation of mechanical energy into thermal energy, and thus $h_L = e_{\text{mech loss, piping}}/g \cong 0$. Also,

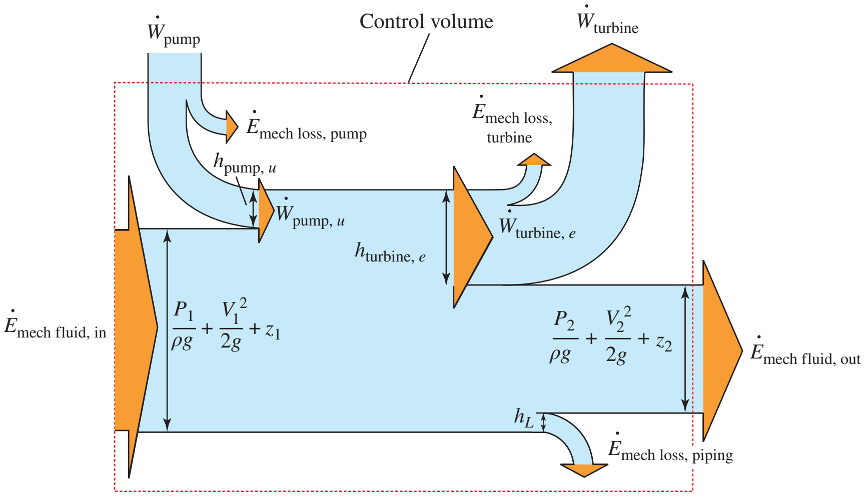


FIGURE 12–25
Mechanical energy flow chart for a fluid flow system that involves a pump and a turbine. Vertical dimensions show each energy term expressed as an equivalent column height of fluid, i.e., *head,* corresponding to each term of Eq. 12–27.

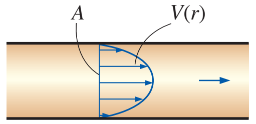
$h_{\text{pump},\,u} = h_{\text{turbine},\,e} = 0$ when there are no mechanical work devices such as fans, pumps, or turbines. Then Eq. 12–27 reduces to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \text{or} \quad \frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant} \tag{12–28}$$

which is the Bernoulli equation derived earlier using Newton's second law of motion. Thus, the Bernoulli equation can be thought of as a degenerate form of the energy equation.

Kinetic Energy Correction Factor, α

The average flow velocity V_{avg} was defined such that the relation $\rho V_{\text{avg}} A$ gives the actual mass flow rate. Therefore, there is no such thing as a correction factor for mass flow rate. However, as Gaspard Coriolis (1792–1843) showed, the kinetic energy of a fluid stream obtained from $V^2/2$ is not the same as the actual kinetic energy of the fluid stream since the square of a sum is not equal to the sum of the squares of its components (Fig. 12–26). This error can be corrected by replacing the kinetic energy terms $V^2/2$ in the energy equation with $\alpha V_{\text{avg}}^2/2$, where α is the **kinetic energy correction factor**. By using equations for the variation of velocity with the radial distance, it can be shown that the correction factor is 2.0 for fully developed laminar pipe flow, and it ranges between 1.04 and 1.11 for fully developed turbulent flow in a round pipe.



$$\dot{m} = \rho V_{\text{avg}} A, \qquad \rho = \text{constant}$$

$$\dot{\text{KE}}_{\text{act}} = \int \text{ke}\,\delta\dot{m} = \int_A \frac{1}{2}[V(r)]^2\,[\rho V(r)\,dA]$$

$$= \frac{1}{2}\rho \int_A [V(r)]^3\,dA$$

$$\dot{\text{KE}}_{\text{avg}} = \frac{1}{2}\dot{m}V_{\text{avg}}^2 = \frac{1}{2}\rho A V_{\text{avg}}^3$$

$$\alpha = \frac{\dot{\text{KE}}_{\text{act}}}{\dot{\text{KE}}_{\text{avg}}} = \frac{1}{A}\int_A \left(\frac{V(r)}{V_{\text{avg}}}\right)^3 dA$$

FIGURE 12–26
The determination of the *kinetic energy correction factor* using the actual velocity distribution $V(r)$ and the average velocity V_{avg} at a cross section.

The kinetic energy correction factors are often ignored (i.e., α is set equal to 1) in an elementary analysis since (1) most flows encountered in practice are turbulent, for which the correction factor is near unity, and (2) the kinetic energy terms are often small relative to the other terms in the energy equation, and multiplying them by a factor less than 2.0 does not make much difference. When the velocity and thus the kinetic energy are high, the flow turns turbulent, and a unity correction factor is more appropriate. However, you should keep in mind that you may encounter some situations for which these factors *are* significant, especially when the flow is laminar. Therefore, we recommend that you always include the kinetic energy correction factor when analyzing fluid flow problems. When the kinetic energy correction factors are included, the energy equations for *steady incompressible flow* (Eqs. 12–26 and 12–27) become

$$\dot{m}\left(\frac{P_1}{\rho} + \alpha_1\frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2\frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}} \tag{12–29}$$

$$\boxed{\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{\text{pump},\,u} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{\text{turbine},\,e} + h_L} \tag{12–30}$$

If the flow at an inlet or outlet is fully developed turbulent pipe flow, we recommend using $\alpha = 1.05$ as a reasonable estimate of the correction factor. This leads to a more conservative estimate of head loss, and it does not take much additional effort to include α in the equations.

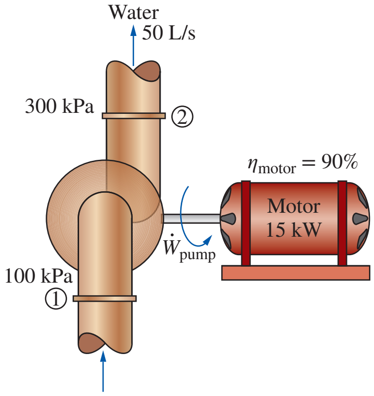


FIGURE 12–27
Schematic for Example 12–5.

EXAMPLE 12–5 Pumping Power and Frictional Heating in a Pump

The pump of a water distribution system is powered by a 15-kW electric motor whose efficiency is 90 percent (Fig. 12–27). The water flow rate through the pump is 50 L/s. The diameters of the inlet and outlet pipes are the same, and the elevation difference across the pump is negligible. If the absolute pressures at the inlet and outlet of the pump are measured to be 100 kPa and 300 kPa, respectively, determine (*a*) the mechanical efficiency of the pump and (*b*) the temperature rise of water as it flows through the pump due to mechanical inefficiencies.

SOLUTION The pressures across a pump are measured. The mechanical efficiency of the pump and the temperature rise of water are to be determined.
Assumptions **1** The flow is steady and incompressible. **2** The pump is driven by an external motor so that the heat generated by the motor is dissipated to the atmosphere. **3** The elevation difference between the inlet and outlet of the pump is negligible, $z_1 \cong z_2$. **4** The inlet and outlet diameters are the same and thus the average inlet and outlet velocities are equal, $V_1 = V_2$. **5** The kinetic energy correction factors are equal, $\alpha_1 = \alpha_2$.
Properties We take the density of water to be 1 kg/L = 1000 kg/m^3 and its specific heat to be 4.18 kJ/kg·°C.
Analysis (*a*) The mass flow rate of water through the pump is

$$\dot{m} = \rho \dot{V} = (1\text{ kg/L})(50\text{ L/s}) = 50\text{ kg/s}$$

The motor draws 15 kW of power and is 90 percent efficient. Thus the mechanical (shaft) power it delivers to the pump is

$$\dot{W}_{\text{pump, shaft}} = \eta_{\text{motor}} \dot{W}_{\text{electric}} = (0.90)(15\text{ kW}) = 13.5\text{ kW}$$

To determine the mechanical efficiency of the pump, we need to know the increase in the mechanical energy of the fluid as it flows through the pump, which is

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2\right) - \dot{m}\left(\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1\right)$$

Simplifying it for this case and substituting the given values,

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{m}\left(\frac{P_2 - P_1}{\rho}\right) = (50\text{ kg/s})\left(\frac{(300 - 100)\text{ kPa}}{1000\text{ kg/m}^3}\right)\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right) = 10.0\text{ kW}$$

Then the mechanical efficiency of the pump becomes

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump, }u}}{\dot{W}_{\text{pump, shaft}}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{pump, shaft}}} = \frac{10.0\text{ kW}}{13.5\text{ kW}} = \mathbf{0.741} \quad \text{or} \quad \mathbf{74.1\%}$$

(*b*) Of the 13.5-kW mechanical power supplied by the pump, only 10.0 kW is imparted to the fluid as mechanical energy. The remaining 3.5 kW is converted to thermal energy due to frictional effects, and this "lost" mechanical energy manifests itself as a heating effect in the fluid,

$$\dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump, shaft}} - \Delta \dot{E}_{\text{mech, fluid}} = 13.5 - 10.0 = 3.5\text{ kW}$$

The temperature rise of water due to this mechanical inefficiency is determined from the thermal energy balance, $\dot{E}_{\text{mech, loss}} = \dot{m}(u_2 - u_1) = \dot{m}c\Delta T$. Solving for ΔT,

$$\Delta T = \frac{\dot{E}_{\text{mech, loss}}}{\dot{m}c} = \frac{3.5\text{ kW}}{(50\text{ kg/s})(4.18\text{ kJ/kg}\cdot{}^\circ\text{C})} = \mathbf{0.017^\circ C}$$

Therefore, the water experiences a temperature rise of 0.017°C which is very small, due to mechanical inefficiency, as it flows through the pump.
Discussion In an actual application, the temperature rise of water would probably be less since part of the heat generated would be transferred to the casing of the pump and from the casing to the surrounding air. If the pump and motor were completely submerged in water, then the 1.5 kW dissipated due to motor inefficiency would also be transferred to the surrounding water as heat.

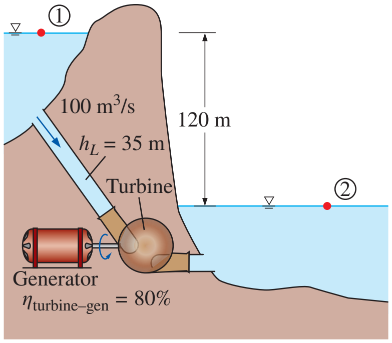


FIGURE 12–28
Schematic for Example 12–6.

EXAMPLE 12–6 Hydroelectric Power Generation from a Dam

In a hydroelectric power plant, 100 m³/s of water flows from an elevation of 120 m to a turbine, where electric power is generated (Fig. 12–28). The total irreversible head loss in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 m. If the overall efficiency of the turbine–generator is 80 percent, estimate the electric power output.

SOLUTION The available head, flow rate, head loss, and efficiency of a hydroelectric turbine are given. The electric power output is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Water levels at the reservoir and the discharge site remain constant.

Properties We take the density of water to be 1000 kg/m³.

Analysis The mass flow rate of water through the turbine is

$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(100\ \text{m}^3/\text{s}) = 10^5\ \text{kg/s}$$

We take point 2 as the reference level, and thus $z_2 = 0$. Also, both points 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$), and the flow velocities are negligible at both points ($V_1 = V_2 = 0$). Then the energy equation for steady, incompressible flow reduces to

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1\cancel{\frac{V_1^2}{2g}} + z_1 + \overset{0}{\cancel{h_{\text{pump},\,u}}} = \cancel{\frac{P_2}{\rho g}} + \alpha_2\cancel{\frac{V_2^2}{2g}} + \overset{0}{\cancel{z_2}} + h_{\text{turbine},\,e} + h_L$$

or

$$h_{\text{turbine},\,e} = z_1 - h_L$$

Substituting, the extracted turbine head and the corresponding turbine power are

$$h_{\text{turbine},\,e} = z_1 - h_L = 120 - 35 = 85\ \text{m}$$

$$\dot{W}_{\text{turbine},\,e} = \dot{m}h_{\text{turbine},\,e} = (10^5\ \text{kg/s})(9.81\ \text{m/s}^2)(85\ \text{m})\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 83{,}400\ \text{kW}$$

Therefore, a perfect turbine–generator would generate 83,400 kW of electricity from this resource. The electric power generated by the actual unit is

$$\dot{W}_{\text{electric}} = \eta_{\text{turbine–gen}}\dot{W}_{\text{turbine},\,e} = (0.80)(83.4\ \text{MW}) = \mathbf{66.7\ MW}$$

Discussion Note that the power generation would increase by almost 1 MW for each percentage point improvement in the efficiency of the turbine–generator unit. You will learn how to determine h_L in Chap. 14.

EXAMPLE 12–7 Fan Selection for Air Cooling of a Computer

A fan is to be selected to cool a computer case whose dimensions are 12 cm × 40 cm × 40 cm (Fig. 12–29). Half of the volume in the case is expected to be filled with components and the other half to be airspace. A 5-cm-diameter hole is available at the back of the case for the installation of the fan that is to replace the air in the void spaces of the case once every second. Small low-power fan–motor combined units are available in the market, and their efficiency is estimated to be 30 percent. Determine (*a*) the wattage of the fan–motor unit to be purchased and (*b*) the pressure difference across the fan. Take the air density to be 1.20 kg/m³.

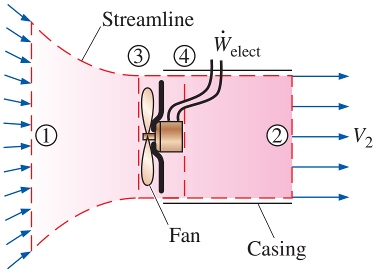


FIGURE 12–29
Schematic for Example 12–7.

SOLUTION A fan is to cool a computer case by completely replacing the air inside once every second. The power of the fan and the pressure difference across it are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Losses other than those due to the inefficiency of the fan–motor unit are negligible. **3** The flow at the outlet is fairly uniform except near the center (due to the wake of the fan motor), and the kinetic energy correction factor at the outlet is 1.10.

Properties The density of air is given to be 1.20 kg/m^3.

Analysis (*a*) Noting that half of the volume of the case is occupied by the components, the air volume in the computer case is

$$\Delta V_{\text{air}} = (\text{Void fraction})(\text{Total case volume})$$
$$= 0.5(12\text{ cm} \times 40\text{ cm} \times 40\text{ cm}) = 9600\text{ cm}^3$$

Therefore, the volume and mass flow rates of air through the case are

$$\dot V = \frac{\Delta V_{\text{air}}}{\Delta t} = \frac{9600\text{ cm}^3}{1\text{ s}} = 9600\text{ cm}^3/\text{s} = 9.6 \times 10^{-3}\text{ m}^3/\text{s}$$

$$\dot m = \rho \dot V = (1.20\text{ kg/m}^3)(9.6 \times 10^{-3}\text{ m}^3/\text{s}) = 0.0115\text{ kg/s}$$

The cross-sectional area of the opening in the case and the average air velocity through the outlet are

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.05\text{ m})^2}{4} = 1.96 \times 10^{-3}\text{ m}^2$$

$$V = \frac{\dot V}{A} = \frac{9.6 \times 10^{-3}\text{ m}^3/\text{s}}{1.96 \times 10^{-3}\text{ m}^2} = 4.90\text{ m/s}$$

We draw the control volume around the fan such that both the inlet and the outlet are at atmospheric pressure ($P_1 = P_2 = P_{\text{atm}}$), as shown in Fig. 12–29, where the inlet section 1 is large and far from the fan so that the flow velocity at the inlet section is negligible ($V_1 \cong 0$). Noting that $z_1 = z_2$ and frictional losses in the flow are disregarded, the mechanical losses consist of fan losses only, and the energy equation (Eq. 12–29) simplifies to

$$\dot m\left(\frac{\cancel{P_1}}{\rho} + \alpha_1 \frac{\overset{0}{\cancel{V_1^2}}}{2} + \cancel{gz_1}\right) + \dot W_{\text{fan}} = \dot m\left(\frac{\cancel{P_2}}{\rho} + \alpha_2 \frac{V_2^2}{2} + \cancel{gz_2}\right) + \overset{0}{\cancel{\dot W_{\text{turbine}}}} + \dot E_{\text{mech loss, fan}}$$

Solving for $\dot W_{\text{fan}} - \dot E_{\text{mech loss, fan}} = \dot W_{\text{fan, }u}$ and substituting,

$$\dot W_{\text{fan, }u} = \dot m \alpha_2 \frac{V_2^2}{2} = (0.0115\text{ kg/s})(1.10)\frac{(4.90\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 0.152\text{ W}$$

Then the required electric power input to the fan is determined to be

$$\dot W_{\text{elect}} = \frac{\dot W_{\text{fan, }u}}{\eta_{\text{fan–motor}}} = \frac{0.152\text{ W}}{0.3} = \mathbf{0.506\text{ W}}$$

Therefore, a fan–motor rated at about a half watt is adequate for this job (Fig. 12–30).

(*b*) To determine the pressure difference across the fan unit, we take points 3 and 4 to be on the two sides of the fan on a horizontal line. This time $z_3 = z_4$ again and $V_3 = V_4$ since the fan is a narrow cross section, and the energy equation reduces to

$$\dot m \frac{P_3}{\rho} + \dot W_{\text{fan}} = \dot m \frac{P_4}{\rho} + \dot E_{\text{mech loss, fan}} \quad \rightarrow \quad \dot W_{\text{fan, }u} = \dot m \frac{P_4 - P_3}{\rho}$$

Solving for $P_4 - P_3$ and substituting,

$$P_4 - P_3 = \frac{\rho \dot W_{\text{fan, }u}}{\dot m} = \frac{(1.2\text{ kg/m}^3)(0.152\text{ W})}{0.0115\text{ kg/s}}\left(\frac{1\text{ Pa}\cdot\text{ m}^3}{1\text{ Ws}}\right) = \mathbf{15.8\text{ Pa}}$$

Therefore, the pressure rise across the fan is 15.8 Pa.

Discussion The efficiency of the fan–motor unit is given to be 30 percent, which means 30 percent of the electric power $\dot W_{\text{electric}}$ consumed by the unit is converted to useful mechanical energy while the rest (70 percent) is "lost" and converted to thermal energy. Also, a more powerful fan is required in an actual system to overcome frictional losses inside the computer case. Note that if we had ignored the kinetic energy correction factor at the outlet, the required electrical power and pressure rise would have been 10 percent lower in this case (0.460 W and 14.4 Pa, respectively).

FIGURE 12–30

The cooling fans used in computers and computer power supplies are typically small and consume only a few watts of electrical power.

PhotoDisc/Getty Images

SUMMARY

This chapter deals with the Bernoulli and energy equations and their applications.

The *Bernoulli equation* is a relation between pressure, velocity, and elevation in steady, incompressible flow and is expressed along a streamline and in regions where net viscous forces are negligible as

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

It can also be expressed between any two points on a streamline as

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

The Bernoulli equation is an expression of mechanical energy balance and can be stated as: *The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when the compressibility and frictional effects are negligible.* Multiplying the Bernoulli equation by density gives

$$P + \rho\frac{V^2}{2} + \rho gz = \text{constant}$$

where P is the *static pressure,* which represents the actual pressure of the fluid; $\rho V^2/2$ is the *dynamic pressure,* which represents the pressure rise when the fluid in motion is brought to a stop isentropically; and ρgz is the *hydrostatic pressure,* which accounts for the effects of fluid weight on pressure. The sum of the static, dynamic, and hydrostatic pressures is called the *total pressure*. The Bernoulli equation states that *the total pressure along a streamline is constant*. The sum of the static and dynamic pressures is called the *stagnation pressure*, which represents the pressure at a point where the fluid is brought to a complete stop in an isentropic manner. The Bernoulli equation can also be represented in terms of "heads" by dividing each term by g,

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant}$$

where $P/\rho g$ is the *pressure head,* which represents the height of a fluid column that produces the static pressure P; $V^2/2g$ is the *velocity head,* which represents the elevation needed for a fluid to reach the velocity V during frictionless free fall; and z is the *elevation head,* which represents the potential energy of the fluid. Also, H is the *total head* for the flow. The curve that represents the sum of the static pressure and the elevation heads, $P/\rho g + z$, is called the *hydraulic grade line* (HGL), and the curve that represents the total head of the fluid, $P/\rho g + V^2/2g + z$, is called the *energy grade line* (EGL).

The *energy equation* for steady, incompressible flow is

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{\text{pump, }u} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{\text{turbine, }e} + h_L$$

where

$$h_{\text{pump, }u} = \frac{w_{\text{pump, }u}}{g} + \frac{\dot{W}_{\text{pump, }u}}{\dot{m}g} = \frac{\eta_{\text{pump}}\ \dot{W}_{\text{pump}}}{\dot{m}g}$$

$$h_{\text{turbine, }e} = \frac{w_{\text{turbine, }e}}{g} + \frac{\dot{W}_{\text{turbine, }e}}{\dot{m}g} = \frac{\dot{W}_{\text{turbine}}}{\eta_{\text{turbine}}\dot{m}g}$$

$$h_L = \frac{e_{\text{mech loss, piping}}}{g} = \frac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g}$$

$$e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}}$$

The Bernoulli and energy equations are two of the most fundamental relations in fluid mechanics.

REFERENCES AND SUGGESTED READING

1. R. C. Dorf, ed. in chief. *The Engineering Handbook,* 2nd ed. Boca Raton, FL: CRC Press, 2004.
2. R. L. Panton. *Incompressible Flow*, 3rd ed. New York: Wiley, 2005.
3. M. Van Dyke. *An Album of Fluid Motion*. Stanford, CA: The Parabolic Press, 1982.

PROBLEMS*

Bernoulli Equation

12–1C Express the Bernoulli equation in three different ways using (*a*) energies, (*b*) pressures, and (*c*) heads.

12–2C What are the three major assumptions used in the derivation of the Bernoulli equation?

12–3C Define static, dynamic, and hydrostatic pressure. Under what conditions is their sum constant for a flow stream?

12–4C What is streamwise acceleration? How does it differ from normal acceleration? Can a fluid particle accelerate in steady flow?

12–5C What is stagnation pressure? Explain how it can be measured.

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* Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

12–6C Define pressure head, velocity head, and elevation head for a fluid stream and express them for a fluid stream whose pressure is P, velocity is V, and elevation is z.

12–7C How is the location of the hydraulic grade line determined for open-channel flow? How is it determined at the outlet of a pipe discharging to the atmosphere?

12–8C In a certain application, a siphon must go over a high wall. Can water or oil with a specific gravity of 0.8 go over a higher wall? Why?

12–9C What is the hydraulic grade line? How does it differ from the energy grade line? Under what conditions do both lines coincide with the free surface of a liquid?

12–10C A glass manometer with oil as the working fluid is connected to an air duct as shown in Fig. P12–10C. Will the oil levels in the manometer be as in Fig. P12–10C*a* or *b*? Explain. What would your response be if the flow direction is reversed?

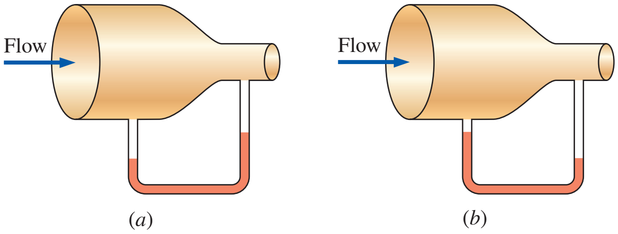


FIGURE P12–10C

12–11C The velocity of a fluid flowing in a pipe is to be measured by two different Pitot-type mercury manometers shown in Fig. P12–11C. Would you expect both manometers to predict the same velocity for flowing water? If not, which would be more accurate? Explain. What would your response be if air were flowing in the pipe instead of water?

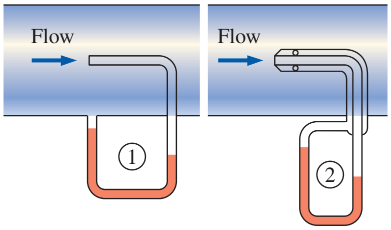


FIGURE P12–11C

12–12C The water level of a tank on a building roof is 20 m above the ground. A hose leads from the tank bottom to the ground. The end of the hose has a nozzle, which is pointed straight up. What is the maximum height to which the water could rise? What factors would reduce this height?

12–13C Explain how and why a siphon works. Someone proposes siphoning cold water over a 7-m-high wall. Is this feasible? Explain.

12–14C A student siphons water over an 8.5-m-high wall at sea level. She then climbs to the summit of Mount Shasta (elevation 4390 m, P_{atm} = 58.5 kPa) and attempts the same experiment. Comment on her prospects for success.

12–15 In a hydroelectric power plant, water enters the turbine nozzles at 800 kPa absolute with a low velocity. If the nozzle outlets are exposed to atmospheric pressure of 100 kPa, determine the maximum velocity to which water can be accelerated by the nozzles before striking the turbine blades.

12–16 A Pitot-static probe is used to measure the speed of an aircraft flying at 3000 m. If the differential pressure reading is 3 kPa, determine the speed of the aircraft.

12–17 The air velocity in the duct of a heating system is to be measured by a Pitot-static probe inserted into the duct parallel to the flow. If the differential height between the water columns connected to the two outlets of the probe is 3.2 cm, determine (*a*) the flow velocity and (*b*) the pressure rise at the tip of the probe. The air temperature and pressure in the duct are 45°C and 98 kPa, respectively.

12–18 A piezometer and a Pitot tube are tapped into a 4-cm-diameter horizontal water pipe, and the heights of the water columns are measured to be 26 cm in the piezometer and 35 cm in the Pitot tube (both measured from the top surface of the pipe). Determine the velocity at the center of the pipe.

12–19 The diameter of a cylindrical water tank is D_o and its height is H. The tank is filled with water, which is open to the atmosphere. An orifice of diameter D with a smooth entrance (i.e., negligible losses) is open at the bottom. Develop a relation for the time required for the tank (*a*) to empty halfway and (*b*) to empty completely.

12–20E A siphon pumps water from a large reservoir to a lower tank that is initially empty. The tank also has a rounded orifice 20 ft below the reservoir surface where the water leaves the tank. Both the siphon and the orifice diameters are 2 in. Ignoring frictional losses, determine to what height the water will rise in the tank at equilibrium.

12–21E Water flows through a horizontal pipe at a rate of 2.4 gal/s. The pipe consists of two sections of diameters 4 in and 2 in with a smooth reducing section. The pressure difference between the two pipe sections is measured by a mercury manometer. Neglecting frictional effects, determine the differential height of mercury between the two pipe sections. *Answer:* 3.0 in

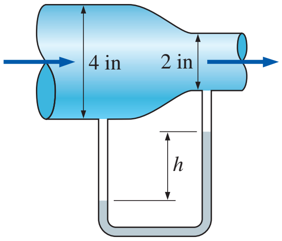


FIGURE P12–21E

12–22 An airplane is flying at an altitude of 10,500 m. Determine the gage pressure at the stagnation point on the nose of the plane if the speed of the plane is 450 km/h. How would you solve this problem if the speed were 1050 km/h? Explain.

12–23 While traveling on a dirt road, the bottom of a car hits a sharp rock and a small hole develops at the bottom of its gas tank. If the height of the gasoline in the tank is 30 cm, determine the initial velocity of the gasoline at the hole. Discuss how the velocity will change with time and how the flow will be affected if the lid of the tank is closed tightly. *Answer:* 2.43 m/s

12–24 The water in an 8-m-diameter, 3-m-high above-ground swimming pool is to be emptied by unplugging a 3-cm-diameter, 25-m-long horizontal pipe attached to the bottom of the pool. Determine the maximum discharge rate of water through the pipe. Also, explain why the actual flow rate will be less.

12–25 Reconsider Prob. 12–24. Determine how long it will take to empty the swimming pool completely. *Answer:* 15.4 h

12–26 Reconsider Prob. 12–25. Using appropriate software, investigate the effect of the discharge pipe diameter on the time required to empty the pool completely. Let the diameter vary from 1 to 10 cm in increments of 1 cm. Tabulate and plot the results.

12–27 Air at 105 kPa and 37°C flows upward through a 6-cm-diameter inclined duct at a rate of 65 L/s. The duct diameter is then reduced to 4 cm through a reducer. The pressure change across the reducer is measured by a water manometer. The elevation difference between the two points on the pipe where the two arms of the manometer are attached is 0.20 m. Determine the differential height between the fluid levels of the two arms of the manometer.

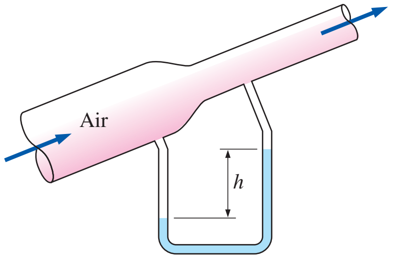


FIGURE P12–27

12–28 Water at 20°C is siphoned from a reservoir as shown in Fig. P12–28. For $d = 8$ cm and $D = 16$ cm, determine (*a*) the minimum flow rate that can be achieved without cavitation occurring in the piping system and (*b*) the maximum elevation of the highest point of the piping system to avoid cavitation. (*c*) Also, discuss the ways of increasing the maximum elevation of the highest point of the piping system to avoid cavitation.

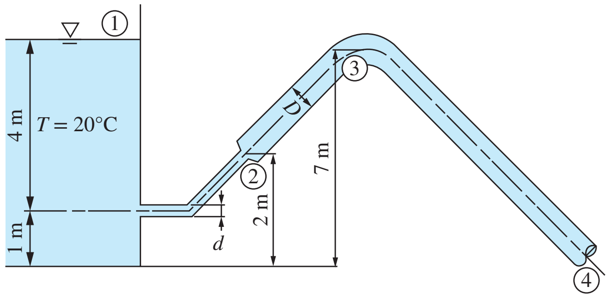


FIGURE P12–28

12–29 The water pressure in the mains of a city at a particular location is 270 kPa gage. Determine if this main can serve water to neighborhoods that are 25 m above this location.

12–30 A pressurized tank of water has a 10-cm-diameter orifice at the bottom, where water discharges to the atmosphere. The water level is 2.5 m above the outlet. The tank air pressure above the water level is 250 kPa (absolute) while the atmospheric pressure is 100 kPa. Neglecting frictional effects, determine the initial discharge rate of water from the tank. *Answer:* 0.147 m^3/s

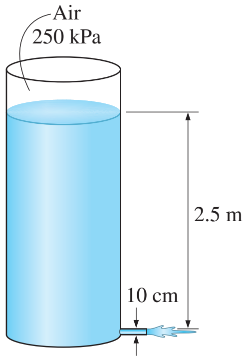


FIGURE P12–30

12–31 Reconsider Prob. 12–30. Using appropriate software, investigate the effect of water height in the tank on the discharge velocity. Let the water height vary from 0 to 5 m in increments of 0.5 m. Tabulate and plot the results.

12–32E Air is flowing through a venturi meter whose diameter is 2.6 in at the entrance part (location 1) and 1.8 in at the throat (location 2). The gage pressure is measured to be 12.2 psia at the entrance and 11.8 psia at the throat. Neglecting frictional effects, show that the volume flow rate can be expressed as

$$\dot{V} = A_2\sqrt{\frac{2(P_1 - P_2)}{\rho(1 - A_2^2/A_1^2)}}$$

and determine the flow rate of air. Take the air density to be 0.075 lbm/ft^3.

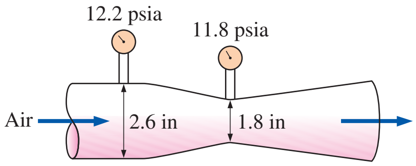


FIGURE P12–32E

12–33 The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank cover is airtight, and the air pressure above the water surface is 2 atm gage. The system is at sea level. Determine the maximum height to which the water stream could rise. *Answer:* 40.7 m

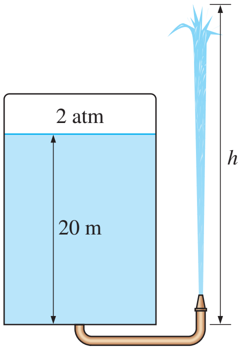


FIGURE P12–33

12–34E The air velocity in a duct is measured by a Pitot-static probe connected to a differential pressure gage. If the air is at 13.4 psia absolute and 70°F and the reading of the differential pressure gage is 0.15 psi, determine the air velocity. *Answer:* 143 ft/s

12–35 In cold climates, water pipes may freeze and burst if proper precautions are not taken. In such an occurrence, the exposed part of a pipe on the ground ruptures, and water shoots up to 55 m. Estimate the gage pressure of water in the pipe. State your assumptions and discuss if the actual pressure is more or less than the value you predicted.

12–36 Water enters a tank of diameter D_T steadily at a mass flow rate of $\dot{m}_{\text{in}}$. An orifice at the bottom with diameter D_o allows water to escape. The orifice has a rounded entrance, so the frictional losses are negligible. If the tank is initially empty, (*a*) determine the maximum height that the water will reach in the tank and (*b*) obtain a relation for water height *z* as a function of time.

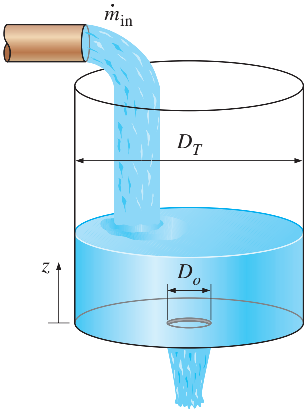


FIGURE P12–36

Energy Equation

12–37C What is irreversible head loss? How is it related to the mechanical energy loss?

12–38C What is useful pump head? How is it related to the power input to the pump?

12–39C Consider the steady adiabatic flow of an incompressible fluid. Can the temperature of the fluid decrease during flow? Explain.

12–40C Consider the steady adiabatic flow of an incompressible fluid. If the temperature of the fluid remains constant during flow, is it accurate to say that the frictional effects are negligible?

12–41C What is the kinetic energy correction factor? Is it significant?

12–42C The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The water stream from the nozzle is observed to rise 25 m above the ground. Explain what may cause the water from the hose to rise above the tank level.

12–43C A 3-m-high tank filled with water has a discharge valve near the bottom and another near the top. (*a*) If these two valves are opened, will there be any difference between the discharge velocities of the two water streams? (*b*) If a hose whose discharge end is left open on the ground is first connected to the lower valve and then to the higher valve, will there be any difference between the discharge rates of water for the two cases? Disregard any frictional effects.

12–44C A person is filling a knee-high bucket with water using a garden hose and holding it such that water discharges from the hose at the level of his waist. Someone suggests that the bucket will fill faster if the hose is lowered such that water discharges from the hose at knee level. Do you agree with this suggestion? Explain. Disregard any frictional effects.

12–45 Water is being pumped from a large lake to a reservoir 25 m above at a rate of 25 L/s by a 10-kW (shaft) pump. If the irreversible head loss of the piping system is 5 m, determine the mechanical efficiency of the pump. *Answer:* 73.6 percent

12–46 Reconsider Prob. 12–45. Using appropriate software, investigate the effect of irreversible head loss on the mechanical efficiency of the pump. Let the head loss vary from 0 to 15 m in increments of 1 m. Plot the results, and discuss them.

12–47 A 15-hp (shaft) pump is used to raise water to a 45-m higher elevation. If the mechanical efficiency of the pump is 82 percent, determine the maximum volume flow rate of the water.

12–48 Water flows at a rate of 0.040 m^3/s in a horizontal pipe whose diameter is reduced from 15 cm to 8 cm by a reducer. If the pressure at the centerline is measured to be 480 kPa and 440 kPa before and after the reducer, respectively, determine the irreversible head loss in the reducer. Take the kinetic energy correction factors to be 1.05. *Answer:* 0.963 m

12–49 The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank is at sea level, and the water surface is open to the atmosphere. In the line leading from the tank to the nozzle is a pump, which increases the pressure of water. If the water jet rises to a height of 27 m from the ground, determine the minimum pressure rise supplied by the pump to the water line.

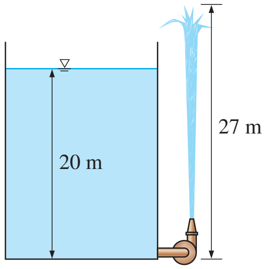


FIGURE P12–49

12–50 A hydraulic turbine has 50 m of head available at a flow rate of 1.30 m^3/s, and its overall turbine–generator efficiency is 78 percent. Determine the electric power output of this turbine.

12–51E In a hydroelectric power plant, water flows from an elevation of 400 ft to a turbine, where electric power is generated. For an overall turbine–generator efficiency of 85 percent, determine the minimum flow rate required to generate 100 kW of electricity. *Answer:* 217 lbm/s

12–52E Reconsider Prob. 12–51E. Determine the flow rate of water if the irreversible head loss of the piping system between the free surfaces of the inlet and the outlet is 36 ft.

12–53 A fan is to be selected to ventilate a bathroom whose dimensions are 2 m × 3 m × 3 m. The air velocity is not to exceed 7 m/s to minimize vibration and noise. The combined efficiency of the fan–motor unit to be used can be taken to be 50 percent. If the fan is to replace the entire volume of air in 15 min, determine (*a*) the wattage of the fan–motor unit to be purchased, (*b*) the diameter of the fan casing, and (*c*) the pressure difference across the fan. Take the air density to be 1.25 kg/m^3 and disregard the effect of the kinetic energy correction factors.

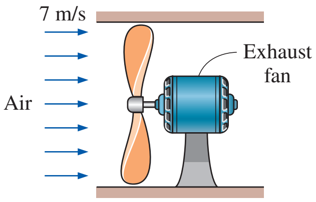


FIGURE P12–53

12–54 Water flows at a rate of 20 L/s through a horizontal pipe whose diameter is constant at 3 cm. The pressure drop across a valve in the pipe is measured to be 2 kPa, as shown in Fig. P12–54. Determine the irreversible head loss of the valve and the useful pumping power needed to overcome the resulting pressure drop. *Answers:* 0.204 m, 40 W

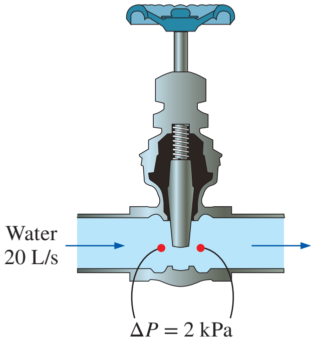


FIGURE P12–54

12–55E The water level in a tank is 34 ft above the ground. A hose is connected to the bottom of the tank at the ground level, and the nozzle at the end of the hose is pointed straight up. The tank cover is airtight, but the pressure over the water surface is unknown. Determine the minimum tank air pressure (gage) that will cause a water stream from the nozzle to rise 72 ft from the ground.

12–56 A large tank is initially filled with water 4 m above the center of a sharp-edged 10-cm-diameter orifice. The tank water surface is open to the atmosphere, and the orifice drains to the atmosphere. If the total irreversible head loss in the system is 0.2 m, determine the initial discharge velocity of water from the tank. Take the kinetic energy correction factor at the orifice to be 1.2.

12–57 Water enters a hydraulic turbine through a 30-cm-diameter pipe at a rate of 0.6 m^3/s and exits through a 25-cm-diameter pipe. The pressure drop in the turbine is measured by a mercury manometer to be 1.2 m. For a combined turbine–generator efficiency of 83 percent, determine the net electric power output. Disregard the effect of the kinetic energy correction factors.

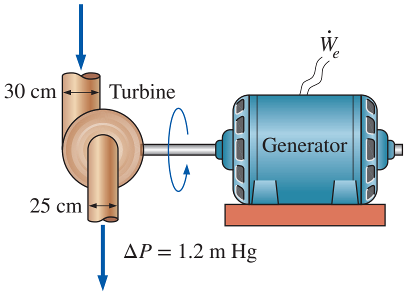


FIGURE P12–57

12–58E A 78-percent-efficient 12-hp pump is pumping water from a lake to a nearby pool at a rate of 1.2 ft^3/s through a constant-diameter pipe. The free surface of the pool is 32 ft above that of the lake. Determine the irreversible head loss of the piping system, in ft, and the mechanical power used to overcome it.

12–59 Water is pumped from a lower reservoir to a higher reservoir by a pump that provides 23 kW of useful mechanical power to the water. The free surface of the upper reservoir is 57 m higher than the surface of the lower reservoir. If the flow rate of water is measured to be 0.03 m^3/s, determine the irreversible head loss of the system and the lost mechanical power during this process.

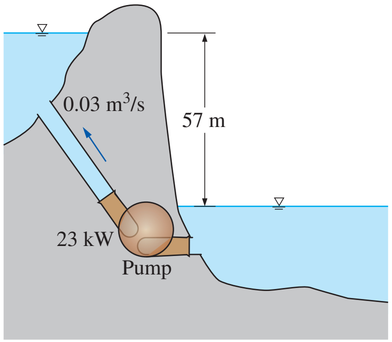


FIGURE P12–59

12–60 Water in a partially filled large tank is to be supplied to the rooftop, which is 8 m above the water level in the tank, through a 2.5-cm-internal-diameter pipe by maintaining a constant air pressure of 300 kPa (gage) in the tank. If the head loss in the piping is 2 m of water, determine the discharge rate of the supply of water to the rooftop.

12–61 Underground water is to be pumped by a 78-percent-efficient 5-kW submerged pump to a pool whose free surface is 30 m above the underground water level. The diameter of the pipe is 7 cm on the intake side and 5 cm on the discharge side. Determine (*a*) the maximum flow rate of water and (*b*) the pressure difference across the pump. Assume the elevation difference between the pump inlet and the outlet and the effect of the kinetic energy correction factors to be negligible.

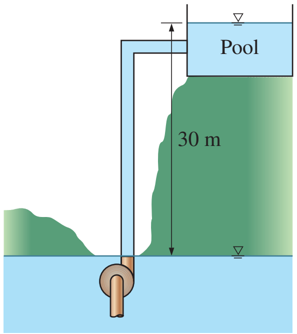


FIGURE P12–61

12–62 Reconsider Prob. 12–61. Determine the flow rate of water and the pressure difference across the pump if the irreversible head loss of the piping system is 4 m.

12–63 The velocity profile for turbulent flow in a circular pipe is approximated as $u(r) = u_{\max}(1 - r/R)^{1/n}$, where $n = 9$. Determine the kinetic energy correction factor for this flow. *Answer:* 1.04

12–64 The demand for electric power is usually much higher during the day than it is at night, and utility companies often sell power at night at much lower prices to encourage consumers to use the available power generation capacity and to avoid building new, expensive power plants that will be used only a short time during peak periods. Utilities are also willing to purchase power produced during the day from private parties at a high price.

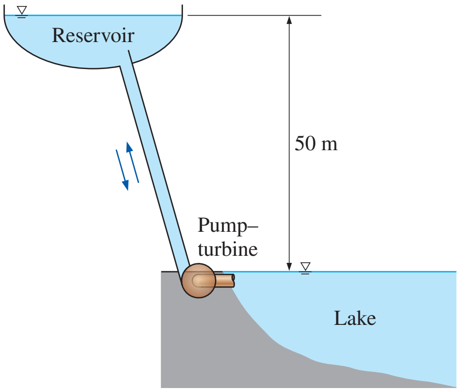


FIGURE P12–64

Suppose a utility company is selling electric power for \$0.06/kWh at night and is willing to pay \$0.13/kWh for power produced during the day. To take advantage of this opportunity, an entrepreneur is considering building a large reservoir 50 m above the lake level, pumping water from the lake to the reservoir at night using cheap power, and letting the water flow from the reservoir back to the lake during the day, producing power as the pump–motor operates as a turbine–generator during reverse flow. Preliminary analysis shows that a water flow rate of 2 m^3/s can be used in either direction, and the irreversible head loss of the piping system is 4 m. The combined pump–motor and turbine–generator efficiencies are expected to be 75 percent each. Assuming the system operates for 10 h each in the pump and turbine modes during a typical day, determine the potential revenue this pump–turbine system can generate per year.

12–65 A fireboat is to fight fires at coastal areas by drawing seawater with a density of 1030 kg/m^3 through a 10-cm-diameter pipe at a rate of 0.04 m^3/s and discharging it through a hose nozzle with an exit diameter of 5 cm. The total irreversible head loss of the system is 3 m, and the position of the nozzle is 3 m above sea level. For a pump efficiency of 70 percent, determine the required shaft power input to the pump and the water discharge velocity. *Answers:* 15.7 kW, 20.4 m/s

FIGURE P12–65

Review Problems

12–66E The water level in a tank is 70 ft above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank is at sea level, and the water surface is open to the atmosphere. In the line leading from the tank to the nozzle is a pump, which increases the water pressure by 15 psia. Determine the maximum height to which the water stream could rise.

12–67 A pressurized 2-m-diameter tank of water has a 10-cm-diameter orifice at the bottom, where water discharges to the atmosphere. The water level initially is 3 m above the outlet. The tank air pressure above the water level is maintained at 450 kPa absolute and the atmospheric pressure is 100 kPa. Neglecting frictional effects, determine (*a*) how long it will take for half of the water in the tank to be discharged and (*b*) the water level in the tank after 10 s.

12–68 A very large tank contains air at 102 kPa at a location where the atmospheric air is at 100 kPa and 20°C. Now a 2-cm-diameter tap is opened. Determine the maximum flow rate of air through the hole. What would your response be if air is discharged through a 2-m-long, 4-cm-diameter tube with a 2-cm-diameter nozzle? Would you solve the problem the same way if the pressure in the storage tank were 300 kPa?

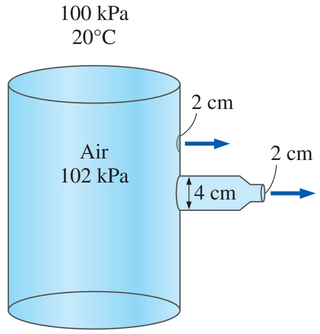


FIGURE P12–68

12–69 Water is flowing through a Venturi meter whose diameter is 7 cm at the entrance part and 4 cm at the throat. The pressure is measured to be 380 kPa at the entrance and 200 kPa at the throat. Neglecting frictional effects, determine the flow rate of water. *Answer:* 0.0252 m^3/s

12–70 Water flows at a rate of 0.011 m^3/s in a horizontal pipe whose diameter increases from 6 to 11 cm by an enlargement section. If the head loss across the enlargement section is 0.65 m and the kinetic energy correction factor at both the inlet and the outlet is 1.05, determine the pressure change.

12–71 Air flows through a pipe at a rate of 120 L/s. The pipe consists of two sections of diameters 22 cm and 10 cm with a smooth reducing section that connects them. The pressure difference between the two pipe sections is measured by a water manometer. Neglecting frictional effects, determine the differential height of water between the two pipe sections. Take the air density to be 1.20 kg/m^3. *Answer:* 1.37 cm

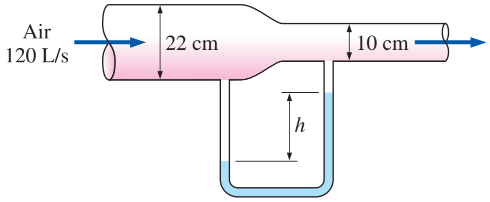


FIGURE P12–71

12–72 A 3-m-high large tank is initially filled with water. The tank water surface is open to the atmosphere, and a sharp-edged 10-cm-diameter orifice at the bottom drains to the atmosphere through a horizontal 80-m-long pipe. If the total irreversible head loss of the system is determined to be 1.5 m, determine the initial velocity of the water from the tank. Disregard the effect of the kinetic energy correction factors. *Answer:* 5.42 m/s

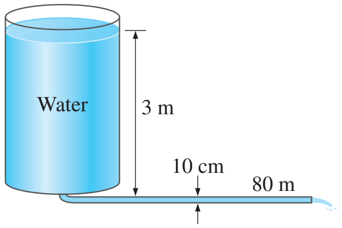


FIGURE P12–72

12–73 Reconsider Prob. 12–72. Using appropriate software, investigate the effect of the tank height on the initial discharge velocity of water from the completely filled tank. Let the tank height vary from 2 to 15 m in increments of 1 m, and assume the irreversible head loss remains constant. Tabulate and plot the results.

12–74 Reconsider Prob. 12–72. In order to drain the tank faster, a pump is installed near the tank exit. Determine the pump head input necessary to establish an average water velocity of 6.5 m/s when the tank is full.

12–75 An oil pump is drawing 18 kW of electric power while pumping oil with $\rho = 860\ kg/m^3$ at a rate of 0.1 m^3/s. The inlet and outlet diameters of the pipe are 8 cm and 12 cm, respectively. If the pressure rise of oil in the pump is measured to be 250 kPa and the motor efficiency is 95 percent, determine the mechanical efficiency of the pump. Take the kinetic energy correction factor to be 1.05.

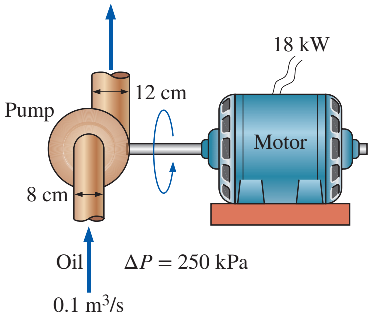


FIGURE P12–75

12–76 A wind tunnel draws atmospheric air at 20°C and 101.3 kPa with a large fan located near the exit of the tunnel. If the air velocity in the tunnel is 80 m/s, determine the pressure in the tunnel.

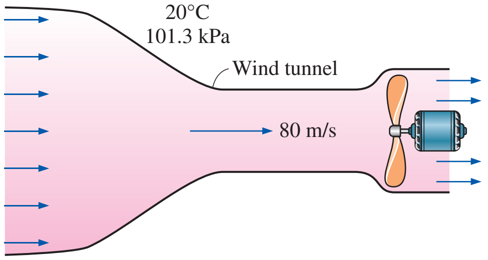


FIGURE P12–76

12–77 A pump-storage plant uses a turbine to generate hydropower during the day, letting water from the upper reservoir flow through the turbine into the lower reservoir as sketched. The plant then pumps water back up to the upper reservoir during the night. In a typical pump-storage plant, the same turbomachine is used as both the pump and the turbine, and is called a ***pump–turbine***. The plant is profitable because the demand for electric power is much higher during the day than it is at night, and utility companies sell power at much lower prices at night to encourage customers to use the available power generation capacity and to avoid building new expensive power plants that would be used only a short time during peak periods. Utilities are also willing to purchase power produced during the day from private parties at a high price. Suppose a utility company is selling electric power for \$0.030/kWh at night and is willing to pay \$0.120/kWh for power produced during the day. A pump-storage plant has a gross head of 90.0 m and can handle a volume flow rate of 4.6 m^3/s in either direction. The irreversible head losses in the system are estimated as 5.0 m in either direction at this flow rate. The efficiency of the combined pump–motor is 88 percent, and that of the combined turbine–generator is 92 percent. The plant runs in pump mode for 10 h each night and runs in turbine mode for 10 h each day. It operates 340 days in a year. How much net revenue (\$) does this pump-storage plant generate in one year?

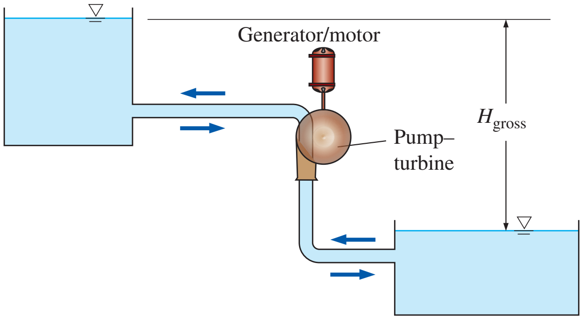


FIGURE P12–77

Design and Essay Problems

12–78 Using a handheld bicycle pump to generate an air jet, a soda can as the water reservoir, and a straw as the tube, design and build an atomizer. Study the effects of various parameters such as the tube length, the diameter of the exit hole, and the pumping speed on performance.

12–79 Using a flexible drinking straw and a ruler, explain how you would measure the water flow velocity in a river.

12–80 The power generated by a wind turbine is proportional to the cube of the wind velocity. Inspired by the acceleration of a fluid in a nozzle, someone proposes to install a reducer casing to capture the wind energy from a larger area and accelerate it before the wind strikes the turbine blades, as shown in Fig. P12–80. Evaluate if the proposed modification should be given a consideration in the design of new wind turbines.

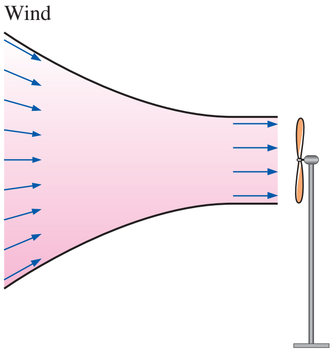


FIGURE P12–80

CHAPTER

13

MOMENTUM ANALYSIS OF FLOW SYSTEMS

When dealing with engineering problems, we wish to obtain fast and accurate solutions at minimal cost. Most engineering problems, including those associated with fluid flow, can be analyzed using one of three basic approaches: differential, experimental, and control volume. In *differential approaches,* the problem is formulated accurately using differential quantities, but the solution of the resulting differential equations is difficult, usually requiring the use of numerical methods with extensive computer codes. *Experimental approaches* complemented with dimensional analysis are highly accurate, but they are typically time consuming and expensive. The *finite control volume approach* described in this chapter is remarkably fast and simple and usually gives answers that are sufficiently accurate for most engineering purposes. Therefore, despite the approximations involved, the basic finite control volume analysis performed with paper and pencil has always been an indispensable tool for engineers.

In Chap. 12, the control volume energy analysis of fluid flow systems was presented. In this chapter, we present the finite control volume momentum analysis of fluid flow problems. First we give an overview of Newton's laws and the conservation relations for linear momentum. Then using the Reynolds transport theorem, we develop the linear momentum equation for control volumes and use it to determine the forces associated with fluid flow.

OBJECTIVES

The objectives of this chapter are to:

- Identify the various kinds of forces acting on a control volume.
- Use control volume analysis to determine the forces associated with fluid flow.

13–1 ▪ NEWTON'S LAWS

Newton's laws are relations between motions of bodies and the forces acting on them. Newton's first law states that *a body at rest remains at rest, and a body in motion remains in motion at the same velocity in a straight path when the net force acting on it is zero.* Therefore, a body tends to preserve its state of inertia. Newton's second law states that *the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass.* Newton's third law states that *when a body exerts a force on a second body, the second body exerts an equal and opposite force on the first.* Therefore, the direction of an exposed reaction force depends on the body taken as the system.

For a rigid body of mass m, Newton's second law is expressed as

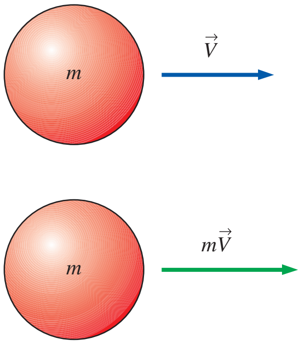


FIGURE 13–1
Linear momentum is the product of mass and velocity, and its direction is the direction of velocity.

Newton's second law:

$$\vec F = m\vec a = m\frac{d\vec V}{dt} = \frac{d(m\vec V)}{dt} \tag{13–1}$$

where $\vec F$ is the net force acting on the body and $\vec a$ is the acceleration of the body under the influence of $\vec F$.

The product of the mass and the velocity of a body is called the *linear momentum* or just the *momentum* of the body. The momentum of a rigid body of mass m moving with velocity $\vec V$ is $m\vec V$ (Fig. 13–1). Then Newton's second law expressed in Eq. 13–1 can also be stated as *the rate of change of the momentum of a body is equal to the net force acting on the body* (Fig. 13–2). This statement is more in line with Newton's original statement of the second law, and it is more appropriate for use in fluid mechanics when studying the forces generated as a result of velocity changes of fluid streams. Therefore, in fluid mechanics, Newton's second law is usually referred to as the *linear momentum equation.*

The momentum of a system remains constant only when the net force acting on it is zero, and thus the momentum of such a system is conserved. This is known as the *conservation of momentum principle.* This principle has proven to be a very useful tool when analyzing collisions such as those between balls; between balls and rackets, bats, or clubs; and between atoms or subatomic particles. It is also used to analyze explosions such as those that occur in rockets, missiles, and guns. In fluid mechanics, however, the net force acting on a system is typically *not* zero, and we prefer to work with the linear momentum equation rather than the conservation of momentum principle.

Note that force, acceleration, velocity, and momentum are vector quantities, and as such they have direction as well as magnitude. Also, momentum is a constant multiple of velocity, and thus the direction of momentum is the direction of velocity as shown in Fig. 13–1. Any vector equation can be written in scalar form for a specified direction using magnitudes, e.g., $F_x = ma_x = d(mV_x)/dt$ in the x-direction.

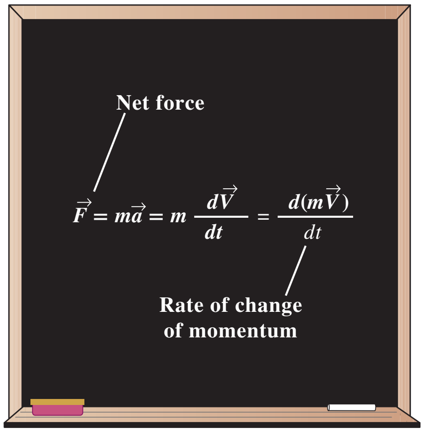


FIGURE 13–2
Newton's second law is also expressed as *the rate of change of the momentum of a body is equal to the net force acting on it.*

The counterpart of Newton's second law for rotating rigid bodies is expressed as $\vec M = I\vec\alpha$, where $\vec M$ is the net moment or torque applied on the body, I is the moment of inertia of the body about the axis of rotation, and $\vec\alpha$ is the angular acceleration. It can also be expressed in terms of the rate of change of angular momentum $d\vec H/dt$ as

Angular momentum equation:

$$\vec M = I\vec\alpha = I\frac{d\vec\omega}{dt} = \frac{d(I\vec\omega)}{dt} = \frac{d\vec H}{dt} \tag{13–2}$$

where $\vec\omega$ is the angular velocity. For a rigid body rotating about a fixed x-axis, the angular momentum equation is written in scalar form as

Angular momentum about x-axis:

$$M_x = I_x\frac{d\omega_x}{dt} = \frac{dH_x}{dt} \tag{13–3}$$

The angular momentum equation can be stated as *the rate of change of the angular momentum of a body is equal to the net torque acting on it* (Fig. 13–3).

The total angular momentum of a rotating body remains constant when the net torque acting on it is zero, and thus the angular momentum of such systems is conserved. This is known as the *conservation of angular momentum principle* and is expressed as $I\omega$ = constant. Many interesting phenomena such as ice skaters spinning faster when they bring their arms close to their bodies and divers rotating faster when they curl after the jump can be explained easily with the help of the conservation of angular momentum principle (in both cases, the moment of inertia I is decreased and thus the angular velocity ω is increased as the outer parts of the body are brought closer to the axis of rotation).

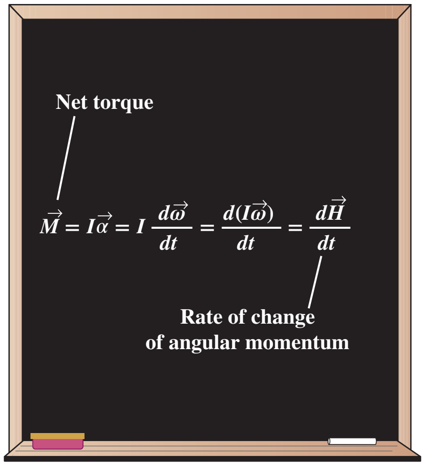


FIGURE 13–3
The rate of change of the angular momentum of a body is equal to the net torque acting on it.

13–2 ▪ CHOOSING A CONTROL VOLUME

We now briefly discuss how to *wisely* select a control volume. A control volume can be selected as any arbitrary region in space through which fluid flows, and its bounding control surface can be fixed, moving, and even deforming during flow. The application of a basic conservation law is a systematic procedure for bookkeeping or accounting of the quantity under consideration, and thus it is extremely important that the boundaries of the control volume are well defined during an analysis. Also, the flow rate of any quantity into or out of a control volume depends on the flow velocity *relative to the control surface,* and thus it is essential to know if the control volume remains at rest during flow or if it moves.

Many flow systems involve stationary hardware firmly fixed to a stationary surface, and such systems are best analyzed using *fixed* control volumes. When determining the reaction force acting on a tripod holding the nozzle of a hose, for example, a natural choice for the control volume is one that passes perpendicularly through the nozzle exit flow and through the bottom of the tripod legs (Fig. 13–4*a*). This is a fixed control volume, and the water velocity relative to a fixed point on the ground is the same as the water velocity relative to the nozzle exit plane.

When analyzing flow systems that are moving or deforming, it is usually more convenient to allow the control volume to *move* or *deform*. When determining the thrust developed by the jet engine of an airplane cruising at constant velocity, for example, a wise choice of control volume is one that encloses the airplane and cuts through the nozzle exit plane (Fig. 13–4*b*). The control volume in this case moves with velocity $\vec{V}_{\text{CV}}$, which is identical to the cruising velocity of the airplane relative to a fixed point on earth. When determining the flow rate of exhaust gases leaving the nozzle, the proper velocity to use is the velocity of the exhaust gases relative to the nozzle exit plane, that is, the *relative velocity* $\vec{V}_r$. Since the entire control volume moves at velocity $\vec{V}_{\text{CV}}$, the relative velocity becomes $\vec{V}_r = \vec{V} - \vec{V}_{\text{CV}}$, where $\vec{V}$ is the *absolute velocity* of the exhaust gases, i.e., the velocity relative to a fixed point on earth. Note that $\vec{V}_r$ is the fluid velocity expressed relative to a coordinate system moving *with* the control volume. Also, this is a vector equation, and velocities in opposite directions have opposite signs. For example, if the airplane is cruising at 500 km/h to the left, and the velocity of the exhaust gases is 800 km/h to the right relative to the ground, the velocity of the exhaust gases relative to the nozzle exit is

$$\vec{V}_r = \vec{V} - \vec{V}_{\text{CV}} = 800\vec{i} - (-500\vec{i}) = 1300\vec{i} \text{ km/h}$$

That is, the exhaust gases leave the nozzle at 1300 km/h to the right relative to the nozzle exit (in the direction opposite to that of the airplane); this is the velocity that should be used when evaluating the outflow of exhaust gases through the control surface (Fig. 13–4*b*). Note that the exhaust gases would

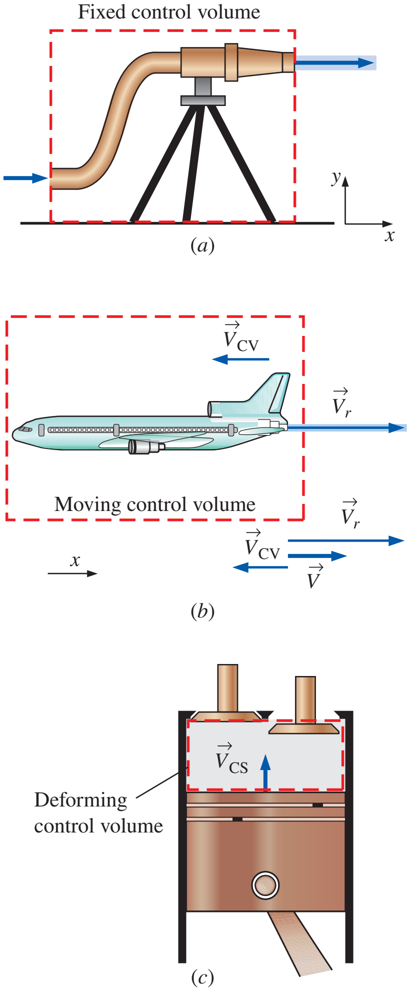


FIGURE 13–4
Examples of (*a*) fixed, (*b*) moving, and (*c*) deforming control volumes.

appear motionless to an observer on the ground if the relative velocity were equal in magnitude to the airplane velocity.

When analyzing the purging of exhaust gases from a reciprocating internal combustion engine, a wise choice for the control volume is one that comprises the space between the top of the piston and the cylinder head (Fig. 13–4*c*). This is a *deforming* control volume, since part of the control surface moves relative to other parts. The relative velocity for an inlet or outlet on the deforming part of a control surface (there are no such inlets or outlets in Fig. 13–4*c*) is then given by $\vec{V}_r = \vec{V} - \vec{V}_{\text{CS}}$ where $\vec{V}$ is the absolute fluid velocity and $\vec{V}_{\text{CS}}$ is the control surface velocity, both relative to a fixed point outside the control volume. Note that $\vec{V}_{\text{CS}} = \vec{V}_{\text{CV}}$ for moving but nondeforming control volumes, and $\vec{V}_{\text{CS}} = \vec{V}_{\text{CV}} = 0$ for fixed ones.

13–3 ▪ FORCES ACTING ON A CONTROL VOLUME

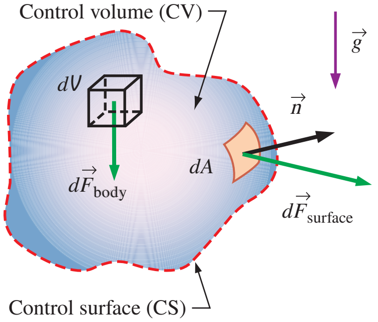


FIGURE 13–5
The total force acting on a control volume is composed of body forces and surface forces; body force is shown on a differential volume element, and surface force is shown on a differential surface element.

The forces acting on a control volume consist of **body forces** that act throughout the entire body of the control volume (such as gravity, electric, and magnetic forces) and **surface forces** that act on the control surface (such as pressure and viscous forces and reaction forces at points of contact). Only external forces are considered in the analysis. Internal forces (such as the pressure force between a fluid and the inner surfaces of the flow section) are not considered in a control volume analysis unless they are exposed by passing the control surface through that area.

In control volume analysis, the sum of all forces acting on the control volume at a particular instant in time is represented by $\Sigma\vec{F}$ and is expressed as

Total force acting on control volume:
$$\sum \vec{F} = \sum \vec{F}_{\text{body}} + \sum \vec{F}_{\text{surface}} \qquad \textbf{(13–4)}$$

Body forces act on each volumetric portion of the control volume. The body force acting on a differential element of fluid of volume $d\mathcal{V}$ within the control volume is shown in Fig. 13–5, and we must perform a volume integral to account for the net body force on the entire control volume. *Surface forces* act on each portion of the control surface. A differential surface element of area dA and unit outward normal $\vec{n}$ on the control surface is shown in Fig. 13–5, along with the surface force acting on it. We must perform an area integral to obtain the net surface force acting on the entire control surface. As sketched, the surface force may act in a direction independent of that of the outward normal vector.

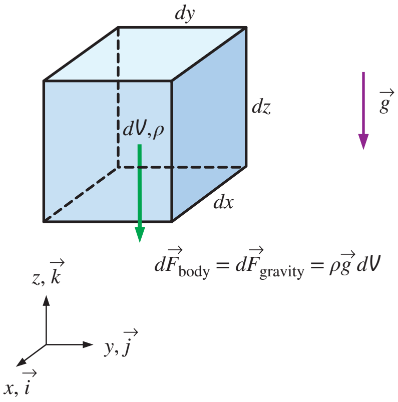


FIGURE 13–6
The gravitational force acting on a differential volume element of fluid is equal to its weight; the axes are oriented so that the gravity vector acts *downward* in the negative z-direction.

The most common body force is that of **gravity**, which exerts a downward force on every differential element of the control volume. While other body forces, such as electric and magnetic forces, may be important in some analyses, we consider only gravitational forces here.

The differential body force $d\vec{F}_{\text{body}} + d\vec{F}_{\text{gravity}}$ acting on the small fluid element shown in Fig. 13–6 is simply its weight,

Gravitational force acting on a fluid element:
$$d\vec{F}_{\text{gravity}} = \rho\vec{g}\,d\mathcal{V} \qquad \textbf{(13–5)}$$

where ρ is the average density of the element and $\vec{g}$ is the gravitational vector. In Cartesian coordinates we adopt the convention that $\vec{g}$ acts in the negative z-direction, as in Fig. 13–6, so that

Gravitational vector in Cartesian coordinates:
$$\vec{g} = -g\vec{k} \qquad \textbf{(13–6)}$$

Note that the coordinate axes in Fig. 13–6 are oriented so that the gravity vector acts *downward* in the $-z$-direction. On earth at sea level, the gravitational constant g is equal to 9.807 m/s^2. Since gravity is the only body force being considered, integration of Eq. 13–5 yields

Total body force acting on control volume:
$$\sum \vec{F}_{\text{body}} = \int_{\text{CV}} \rho \vec{g}\, dV = m_{\text{CV}}\vec{g} \tag{13–7}$$

Surface forces are not as simple to analyze since they consist of both *normal* and *tangential* components. **Normal stresses** are composed of pressure (which always act inwardly normal) and viscous stresses. **Shear stresses** are composed entirely of viscous stresses.

A careful selection of the control volume enables us to write the total force acting on the control volume, $\Sigma\vec{F}$, as the sum of more readily available quantities like weight, pressure, and reaction forces. We recommend the following for control volume analysis:

Total force:
$$\underbrace{\sum \vec{F}}_{\text{total force}} = \underbrace{\sum \vec{F}_{\text{gravity}}}_{\text{body force}} + \underbrace{\sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}}}_{\text{surface force}} \tag{13–8}$$

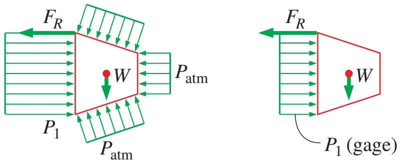
The first term on the right-hand side of Eq. 13–8 is the body force *weight,* since gravity is the only body force we are considering. The other three terms combine to form the net surface force; they are pressure forces, viscous forces, and “other” forces acting on the control surface. $\Sigma\vec{F}_{\text{other}}$ is composed of reaction forces required to turn the flow; forces at bolts, cables, struts, or walls through which the control surface cuts; etc.

All these surface forces arise as the control volume is isolated from its surroundings for analysis, and the effect of any detached object is accounted for by a force at that location. This is similar to drawing a free-body diagram in your statics and dynamics classes. We should choose the control volume such that forces that are not of interest remain internal, and thus they do not complicate the analysis. A well-chosen control volume exposes only the forces that are to be determined (such as reaction forces) and a minimum number of other forces.

A common simplification in the application of Newton’s laws of motion is to subtract the *atmospheric pressure* and work with gage pressures. This is because atmospheric pressure acts in all directions, and its effect cancels out in every direction (Fig. 13–7). This means we can also ignore the pressure forces at outlet sections where the fluid is discharged at subsonic velocities to the atmosphere since the discharge pressures in such cases are very near atmospheric pressure.

As an example of how to wisely choose a control volume, consider control volume analysis of water flowing steadily through a faucet with a partially closed gate valve spigot (Fig. 13–8). We wish to calculate the net force on the flange to ensure that the flange bolts are strong enough. There are many possible choices for the control volume. Some engineers restrict their control volumes to the fluid itself, as indicated by CV A (the purple control volume) in Fig. 13–8. With this control volume, there are pressure forces that vary along the control surface, there are viscous forces along the pipe wall and at locations inside the valve, and there is a body force, namely, the weight of the water in the control volume. Fortunately, to calculate the net force on the flange, we do *not* need to integrate the pressure and viscous stresses all along the control surface. Instead, we can lump the unknown pressure and viscous forces together into one reaction force, representing the net force of the walls on the water. This force, plus the weight of the faucet and the water, is equal to the net force on the flange. (We must be very careful with our signs, of course.)

When choosing a control volume, you are not limited to the fluid alone. Often it is more convenient to slice the control surface *through* solid objects such as walls, struts, or bolts as illustrated by CV B (the red control volume) in Fig. 13–8. A control volume may even surround an entire object, like the one



With atmospheric pressure considered

With atmospheric pressure cancelled out

FIGURE 13–7
Atmospheric pressure acts in all directions, and thus it can be ignored when performing force balances since its effect cancels out in every direction.

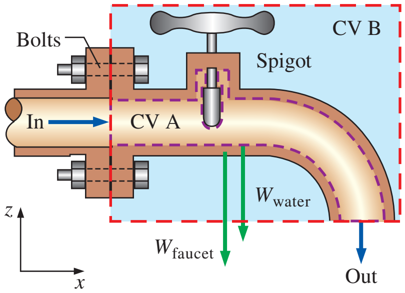


FIGURE 13–8
Cross section through a faucet assembly, illustrating the importance of choosing a control volume wisely; CV B is much easier to work with than CV A.

shown here. Control volume B is a wise choice because we are not concerned with any details of the flow or even the geometry inside the control volume. For the case of CV B, we assign a net reaction force acting at the portions of the control surface that slice through the flange bolts. Then, the only other things we need to know are the gage pressure of the water at the flange (the inlet to the control volume) and the weights of the water and the faucet assembly. The pressure everywhere else along the control surface is atmospheric (zero gage pressure) and cancels out.

13–4 ▪ THE REYNOLDS TRANSPORT THEOREM

In thermodynamics and solid mechanics we often work with a *system* (also called a *closed system*), defined as a *quantity of matter of fixed identity*. In fluid dynamics, it is more common to work with a *control volume* (also called an *open system*), defined as a *region in space chosen for study*. The size and shape of a system may change during a process, but no mass crosses its boundaries. A control volume, on the other hand, allows mass to flow in or out across its boundaries, which are called the **control surface**. A control volume may also move and deform during a process, but many real-world applications involve fixed, nondeformable control volumes.

Figure 13–9 illustrates both a system and a control volume for the case of deodorant being sprayed from a spray can. When analyzing the spraying process, a natural choice for our analysis is either the moving, deforming fluid (a system) or the volume bounded by the inner surfaces of the can (a control volume). These two choices are identical before the deodorant is sprayed. When some contents of the can are discharged, the system approach considers the discharged mass as part of the system and tracks it (a difficult job indeed); thus the mass of the system remains constant. Conceptually, this is equivalent to attaching a flat balloon to the nozzle of the can and letting the spray inflate the balloon. The inner surface of the balloon now becomes part of the boundary of the system. The control volume approach, however, is not concerned at all with the deodorant that has escaped the can (other than its properties at the exit), and thus the mass of the control volume decreases during this process while its volume remains constant. Therefore, the system approach treats the spraying process as an expansion of the system's volume, whereas the control volume approach considers it as a fluid discharge through the control surface of the fixed control volume.

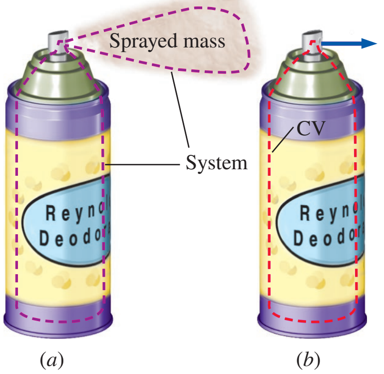


FIGURE 13–9
Two methods of analyzing the spraying of deodorant from a spray can: (*a*) We follow the fluid as it moves and deforms. This is the *system approach*—no mass crosses the boundary, and the total mass of the system remains fixed. (*b*) We consider a fixed interior volume of the can. This is the *control volume approach*—mass crosses the boundary.

Most principles of fluid mechanics are adopted from solid mechanics, where the physical laws dealing with the time rates of change of extensive properties are expressed for systems. In fluid mechanics, it is usually more convenient to work with control volumes, and thus there is a need to relate the changes in a control volume to the changes in a system. The relationship between the time rates of change of an extensive property for a system and for a control volume is expressed by the **Reynolds transport theorem (RTT)**, which provides the link between the system and control volume approaches (Fig. 13–10). RTT is named after the English engineer Osborne Reynolds (1842–1912), who did much to advance its application in fluid mechanics.

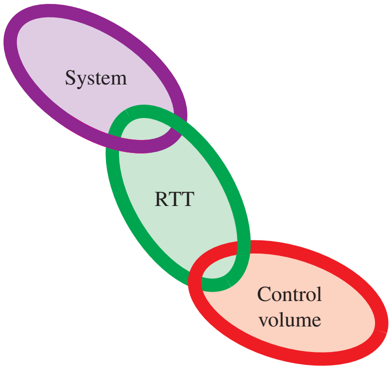


FIGURE 13–10
The *Reynolds transport theorem* (RTT) provides a link between the system approach and the control volume approach.

The general form of the Reynolds transport theorem can be derived by considering a system with an arbitrary shape and arbitrary interactions, but the derivation is rather involved. To help you grasp the fundamental meaning of the theorem, we derive it first in a straightforward manner using a simple geometry and then generalize the results.

Consider flow from left to right through a diverging (expanding) portion of a flow field as sketched in Fig. 13–11. The upper and lower bounds of the fluid under consideration are *streamlines* of the flow, and we assume uniform flow through any cross section between these two streamlines. We choose the control volume to be fixed between sections (1) and (2) of the flow field. Both (1) and (2) are normal to the direction of flow. At some initial time *t*, the system coincides with the control volume, and thus the system and control volume are identical (the greenish-shaded region in Fig. 13–11). During time interval Δt, the system moves in the flow direction at uniform speeds V_1 at section (1) and V_2 at section (2). The system at this later time is indicated by the hatched region. The region uncovered by the system during this motion is designated as section I (part of the CV), and the new region covered by the system is designated as section II (not part of the CV). Therefore, at time $t + \Delta t$, the system consists of the same fluid, but it occupies the region CV − I + II. The control volume is fixed in space, and thus it remains as the shaded region marked CV at all times.

Let *B* represent any **extensive property** (such as mass, energy, or momentum), and let $b = B/m$ represent the corresponding **intensive property**. Noting that extensive properties are additive, the extensive property *B* of the system at times *t* and $t + \Delta t$ is expressed as

$$B_{\text{sys},\,t} = B_{\text{CV},\,t} \quad \text{(the system and CV coincide at time } t\text{)}$$
$$B_{\text{sys},\,t+\Delta t} = B_{\text{CV},\,t+\Delta t} - B_{\text{I},\,t+\Delta t} + B_{\text{II},\,t+\Delta t}$$

Subtracting the first equation from the second one and dividing by Δt gives

$$\frac{B_{\text{sys},\,t+\Delta t} - B_{\text{sys},\,t}}{\Delta t} = \frac{B_{\text{CV},\,t+\Delta t} - B_{\text{CV},\,t}}{\Delta t} - \frac{B_{\text{I},\,t+\Delta t}}{\Delta t} + \frac{B_{\text{II},\,t+\Delta t}}{\Delta t}$$

Taking the limit as $\Delta t \to 0$, and using the definition of derivative, we get

$$\frac{dB_{\text{sys}}}{dt} = \frac{dB_{\text{CV}}}{dt} - \dot{B}_{\text{in}} + \dot{B}_{\text{out}} \tag{13–9}$$

or

$$\frac{dB_{\text{sys}}}{dt} = \frac{dB_{\text{CV}}}{dt} - b_1\rho_1 V_1 A_1 + b_2\rho_2 V_2 A_2$$

since

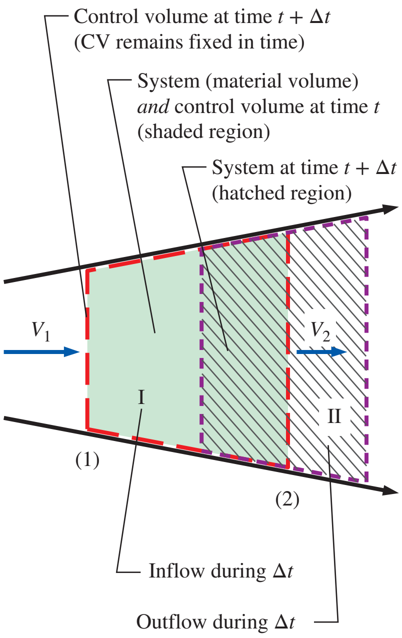
$$B_{\text{I},\,t+\Delta t} = b_1 m_{\text{I},\,t+\Delta t} = b_1\rho_1 \mathcal{V}_{\text{I},\,t+\Delta t} = b_1\rho_1 V_1\,\Delta t\, A_1$$
$$B_{\text{II},\,t+\Delta t} = b_2 m_{\text{II},\,t+\Delta t} = b_2\rho_2 \mathcal{V}_{\text{II},\,t+\Delta t} = b_2\rho_2 V_2\,\Delta t\, A_2$$

and

$$\dot{B}_{\text{in}} = \dot{B}_{\text{I}} = \lim_{\Delta t\to 0}\frac{B_{\text{I},\,t+\Delta t}}{\Delta t} = \lim_{\Delta t\to 0}\frac{b_1\rho_1 V_1\Delta t\, A_1}{\Delta t} = b_1\rho_1 V_1 A_1$$
$$\dot{B}_{\text{out}} = \dot{B}_{\text{II}} = \lim_{\Delta t\to 0}\frac{B_{\text{II},\,t+\Delta t}}{\Delta t} = \lim_{\Delta t\to 0}\frac{b_2\rho_2 V_2\Delta t\, A_2}{\Delta t} = b_2\rho_2 V_2 A_2$$

where A_1 and A_2 are the cross-sectional areas at locations 1 and 2. Equation 13–9 states that *the time rate of change of the property B of the system is equal to the time rate of change of B of the control volume plus the net flux of B out of the control volume by mass crossing the control surface*. This is the desired relation since it relates the change of a property of a system to the change of that property for a control volume. Note that Eq. 13–9 applies at any instant in time, where it is assumed that the system and the control volume occupy the same space at that particular instant in time.

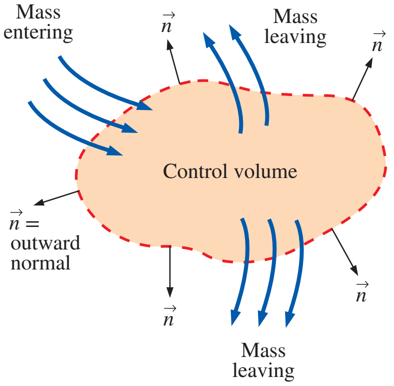
The influx $\dot{B}_{\text{in}}$ and outflux $\dot{B}_{\text{out}}$ of the property *B* in this case are easy to determine since there is only one inlet and one outlet, and the velocities are approximately normal to the surfaces at sections (1) and (2). In general, however, we may have several inlet and outlet ports, and the velocity may not be normal to the control surface at the point of entry. Also, the velocity may not be uniform. To generalize the process, we consider a differential surface area *dA* on the control surface and denote its **unit outer normal** by $\vec{n}$. The flow rate of property *b* through *dA* is $\rho b\vec{V}\cdot\vec{n}\, dA$ since the dot product $\vec{V}\cdot\vec{n}$ gives the normal component of the velocity.



At time *t*: Sys = CV
At time $t + \Delta t$: Sys = CV − I + II

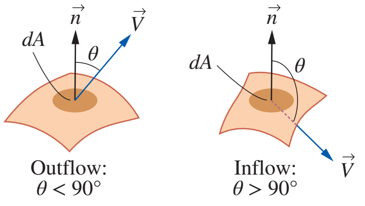
FIGURE 13–11

A moving *system* (hatched region) and a fixed *control volume* (shaded region) in a diverging portion of a flow field at times *t* and $t + \Delta t$. The upper and lower bounds are streamlines of the flow.



$$\dot{B}_{\text{net}} = \dot{B}_{\text{out}} - \dot{B}_{\text{in}} = \int_{\text{CS}} \rho b \vec{V} \cdot \vec{n}\, dA$$

FIGURE 13–12
The integral of $\rho b \vec{V}\cdot\vec{n}\, dA$ over the control surface gives the net amount of the property B flowing out of the control volume (into the control volume if it is negative) per unit time.



$\vec{V}\cdot\vec{n} = |\vec{V}|\,|\vec{n}| \cos\theta = V\cos\theta$
If $\theta < 90°$, then $\cos\theta > 0$ (outflow).
If $\theta > 90°$, then $\cos\theta < 0$ (inflow).
If $\theta = 90°$, then $\cos\theta = 0$ (no flow).

FIGURE 13–13
Outflow and inflow of mass across the differential area of a control surface.

Then the net rate of outflow through the entire control surface is determined by integration to be (Fig. 13–12)

$$\dot{B}_{\text{net}} = \dot{B}_{\text{out}} - \dot{B}_{\text{in}} = \int_{\text{CS}} \rho b \vec{V}\cdot\vec{n}\, dA \qquad \text{(inflow if negative)} \tag{13–10}$$

An important aspect of this relation is that it automatically subtracts the inflow from the outflow, as explained next. The dot product of the velocity vector at a point on the control surface and the outer normal at that point is $\vec{V}\cdot\vec{n} = |\vec{V}|\,|\vec{n}|\cos\theta = |\vec{V}|\cos\theta$, where θ is the angle between the velocity vector and the outer normal, as shown in Fig. 13–13. For $\theta < 90°$, $\cos\theta > 0$ and thus $\vec{V}\cdot\vec{n} > 0$ for outflow of mass from the control volume, and for $\theta > 90°$, $\cos\theta < 0$ and thus $\vec{V}\cdot\vec{n} < 0$ for inflow of mass into the control volume. Therefore, the differential quantity $\rho b \vec{V}\cdot\vec{n}\, dA$ is positive for mass flowing out of the control volume, and negative for mass flowing into the control volume, and its integral over the entire control surface gives the rate of net outflow of the property B by mass.

The properties within the control volume may vary with position, in general. In such a case, the total amount of property B within the control volume must be determined by integration:

$$B_{\text{CV}} = \int_{\text{CV}} \rho b\, d\mathcal{V} \tag{13–11}$$

The term dB_{CV}/dt in Eq. 13–9 is thus equal to $\dfrac{d}{dt}\displaystyle\int_{\text{CV}} \rho b\, d\mathcal{V}$ and represents the time rate of change of the property B content of the control volume. A positive value for dB_{CV}/dt indicates an increase in the B content, and a negative value indicates a decrease. Substituting Eqs. 13–10 and 13–11 into Eq. 13–9 yields the Reynolds transport theorem, also known as the *system-to-control-volume transformation* for a fixed control volume:

RTT, fixed CV:

$$\frac{dB_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} \rho b\, d\mathcal{V} + \int_{\text{CS}} \rho b \vec{V}\cdot\vec{n}\, dA \tag{13–12}$$

Since the control volume is not moving or deforming with time, the time derivative on the right-hand side can be moved inside the integral, since the domain of integration does not change with time. (In other words, it is irrelevant whether we differentiate or integrate first.) But the time derivative in that case must be expressed as a *partial* derivative ($\partial/\partial t$) since density and the quantity b may depend not only on time, but also on the position within the control volume. Thus, an alternate form of the Reynolds transport theorem for a fixed control volume is

Alternate RTT, fixed CV:

$$\frac{dB_{\text{sys}}}{dt} = \int_{\text{CV}} \frac{\partial}{\partial t}(\rho b)\, d\mathcal{V} + \int_{\text{CS}} \rho b \vec{V}\cdot\vec{n}\, dA \tag{13–13}$$

It turns out that Eq. 13–13 is also valid for the most general case of a moving and/or deforming control volume, provided that velocity vector $\vec{V}$ is an *absolute* velocity (as viewed from a fixed reference frame).

Next we consider yet *another* alternative form of the RTT. Equation 13–12 was derived for a *fixed* control volume. However, many practical systems such as turbine and propeller blades involve nonfixed control volumes. Fortunately, Eq. 13–12 is also valid for *moving* and/or *deforming* control volumes provided that the absolute fluid velocity $\vec{V}$ in the last term is replaced by the **relative velocity** $\vec{V}_r$,

Relative velocity:

$$\vec{V}_r = \vec{V} - \vec{V}_{\text{CS}} \tag{13–14}$$

where $\vec{V}_{CS}$ is the local velocity of the control surface (Fig. 13–14). The most general form of the Reynolds transport theorem is thus

RTT, nonfixed CV:
$$\frac{dB_{\text{sys}}}{dt} = \frac{d}{dt}\int_{CV} \rho b\, dV + \int_{CS} \rho b \vec{V}_r \cdot \vec{n}\, dA \qquad \textbf{(13–15)}$$

Note that for a control volume that moves and/or deforms with time, the time derivative is applied *after* integration in Eq. 13–15. As a simple example of a moving control volume, consider a toy car moving at a constant absolute velocity $\vec{V}_{\text{car}}$ = 10 km/h to the right. A high-speed jet of water (absolute velocity = $\vec{V}_{\text{jet}}$ = 25 km/h to the right) strikes the back of the car and propels it (Fig. 13–15). If we draw a control volume around the car, the relative velocity is $\vec{V}_r = 25 - 10 =$ 15 km/h to the right. This represents the velocity at which an observer moving with the control volume (moving with the car) would observe the fluid crossing the control surface. In other words, $\vec{V}_r$ is the fluid velocity expressed relative to a coordinate system moving *with* the control volume.

Finally, by application of the Leibniz theorem, it can be shown that the Reynolds transport theorem for a general moving and/or deforming control volume (Eq. 13–15) is equivalent to the form given by Eq. 13–13, which is repeated here:

Alternate RTT, nonfixed CV:
$$\frac{dB_{\text{sys}}}{dt} = \int_{CV} \frac{\partial}{\partial t}(\rho b)\, dV + \int_{CS} \rho b \vec{V} \cdot \vec{n}\, dA \qquad \textbf{(13–16)}$$

In contrast to Eq. 13–15, the velocity vector $\vec{V}$ in Eq. 13–16 must be taken as the *absolute* velocity (as viewed from a fixed reference frame) in order to apply to a nonfixed control volume.

During steady flow, the amount of the property B within the control volume remains constant in time, and thus the time derivative in Eq. 13–15 becomes zero. Then the Reynolds transport theorem reduces to

RTT, steady flow:
$$\frac{dB_{\text{sys}}}{dt} = \int_{CS} \rho b\, \vec{V}_r \cdot \vec{n}\, dA \qquad \textbf{(13–17)}$$

Note that unlike the control volume, the property B content of the system may still change with time during a steady process. But in this case the change must be equal to the net property transported by mass across the control surface (an advective rather than an unsteady effect).

In most practical engineering applications of the RTT, fluid crosses the boundary of the control volume at a finite number of well-defined inlets and outlets (Fig. 13–16). In such cases, it is convenient to cut the control surface directly across each inlet and outlet and replace the surface integral in Eq. 13–15 with approximate algebraic expressions at each inlet and outlet based on the *average* values of fluid properties crossing the boundary. We define ρ_{avg}, b_{avg}, and $V_{r,\,\text{avg}}$ as the average values of ρ, b, and V_r, respectively, across an inlet or outlet of cross-sectional area A, e.g., $b_{\text{avg}} = \frac{1}{A}\int_A b\, dA$. The surface integrals in the RTT (Eq. 13–15), when applied over an inlet or outlet of cross-sectional area A, are then *approximated* by pulling property b out of the surface integral and replacing it with its average. This yields

$$\int_A \rho b\, \vec{V}_r \cdot \vec{n}\, dA \cong b_{\text{avg}} \int_A \rho \vec{V}_r \cdot \vec{n}\, dA = b_{\text{avg}}\, \dot{m}_r$$

where $\dot{m}_r$ is the mass flow rate through the inlet or outlet relative to the (moving) control surface. The approximation in this equation is exact when property b is uniform over cross-sectional area A. Equation 13–15 thus becomes

$$\frac{dB_{\text{sys}}}{dt} = \frac{d}{dt}\int_{CV} \rho b\, dV + \sum_{\text{out}} \underbrace{\dot{m}_r b_{\text{avg}}}_{\text{for each outlet}} - \sum_{\text{in}} \underbrace{\dot{m}_r b_{\text{avg}}}_{\text{for each inlet}} \qquad \textbf{(13–18)}$$

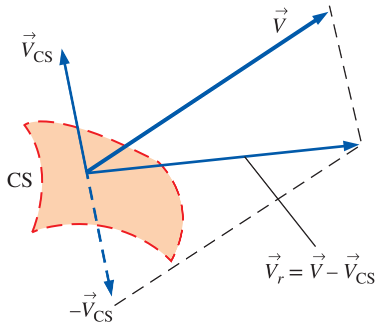
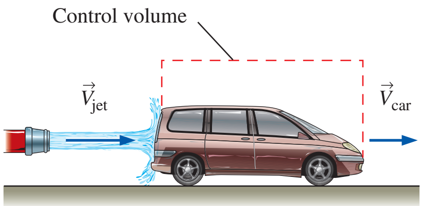


FIGURE 13–14
Relative velocity crossing a control surface is found by vector addition of the absolute velocity of the fluid and the negative of the local velocity of the control surface.

Absolute reference frame:



Relative reference frame:

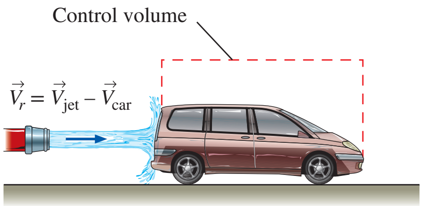


FIGURE 13–15
Reynolds transport theorem applied to a control volume moving at constant velocity.

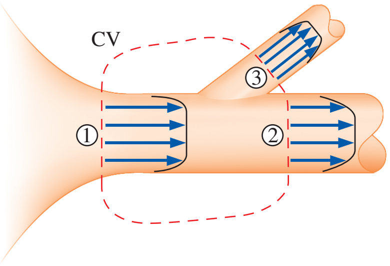


FIGURE 13–16
An example control volume in which there is one well-defined inlet (1) and two well-defined outlets (2 and 3). In such cases, the control surface integral in the RTT can be more conveniently written in terms of the average values of fluid properties crossing each inlet and outlet.

In some applications, we may wish to rewrite Eq. 13–18 in terms of volume (rather than mass) flow rate. In such cases, we make a further approximation that $\dot m_r \approx \rho_{\text{avg}}\dot V_r = \rho_{\text{avg}} V_{r,\,\text{avg}} A$. This approximation is exact when fluid density ρ is uniform over A. Equation 13–18 then reduces to

Approximate RTT for well-defined inlets and outlets:

$$\frac{dB_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} \rho b\, dV + \sum_{\text{out}} \underbrace{\rho_{\text{avg}} b_{\text{avg}} V_{r,\,\text{avg}} A}_{\text{for each outlet}} - \sum_{\text{in}} \underbrace{\rho_{\text{avg}} b_{\text{avg}} V_{r,\,\text{avg}} A}_{\text{for each inlet}} \tag{13–19}$$

Note that these approximations simplify the analysis greatly but may not always be accurate, especially in cases where the velocity distribution across the inlet or outlet is not very uniform (e.g., pipe flows; Fig. 13–16). In particular, the control surface integral of Eq. 13–16 becomes *nonlinear* when property b contains a velocity term (e.g., when applying RTT to the linear momentum equation, $b = \vec V$), and the approximation of Eq. 13–19 leads to errors. Fortunately we can eliminate the errors by including *correction factors* in Eq. 13–19.

Equations 13–18 and 13–19 apply to fixed *or* moving control volumes, but as discussed previously, the *relative velocity* must be used for the case of a nonfixed control volume. In Eq. 13–18 for example, the mass flow rate $\dot m_r$ is relative to the (moving) control surface, hence the r subscript.

An Application: Conservation of Mass

The general conservation of mass relation for a control volume can also be derived using the Reynolds transport theorem by taking the property B to be the mass, m. Then we have $b = 1$ since dividing the mass by mass to get the property per unit mass gives unity. Also, the mass of a system is constant, and thus its time derivative is zero. That is, $dm_{\text{sys}}/dt = 0$. Then the Reynolds transport equation in this case reduces to (Fig. 13–17)

$$\begin{array}{ccccc} \dfrac{dB_{sys}}{dt} & = & \dfrac{d}{dt}\displaystyle\int_{CV}\rho b\,dV & + & \displaystyle\int_{CS}\rho b(\vec V\cdot\vec n)\,dA \\ \Big\downarrow B=m & & \Big\downarrow b=1 & & \Big\downarrow b=1 \\ \dfrac{dm_{sys}}{dt} & = & \dfrac{d}{dt}\displaystyle\int_{CV}\rho\,dV & + & \displaystyle\int_{CS}\rho(\vec V\cdot\vec n)\,dA \end{array}$$

FIGURE 13–17
The conservation of mass equation is obtained by replacing B in the Reynolds transport theorem by mass m, and b by 1 (m per unit mass $= m/m = 1$).

General conservation of mass:

$$\frac{d}{dt}\int_{\text{CV}} \rho\, dV + \int_{\text{CS}} \rho(\vec V\cdot\vec n)\, dA = 0 \tag{13–20}$$

It states that *the time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero*.

This demonstrates that the Reynolds transport theorem is a very powerful tool, and we can use it with confidence.

Splitting the surface integral in Eq. 13–20 into two parts—one for the outgoing flow streams (positive) and one for the incoming streams (negative)—the general conservation of mass relation can also be expressed as

$$\frac{d}{dt}\int_{\text{CV}} \rho\, dV + \sum_{\text{out}}\int_A \rho V_n\, dA - \sum_{\text{in}}\int_A \rho V_n\, dA = 0 \tag{13–21}$$

Using the definition of mass flow rate, it can also be expressed as

$$\frac{d}{dt}\int_{\text{CV}} \rho\, dV = \sum_{\text{in}} \dot m - \sum_{\text{out}} \dot m \qquad \text{or} \qquad \frac{dm_{\text{CV}}}{dt} = \sum_{\text{in}} \dot m - \sum_{\text{out}} \dot m \tag{13–22}$$

where the summation signs are used to emphasize that *all* the inlets and outlets are to be considered.

13–5 ▪ THE LINEAR MOMENTUM EQUATION

Newton's second law for a system of mass m subjected to net force $\Sigma\vec F$ is expressed as

$$\sum \vec F = m\vec a = m\frac{d\vec V}{dt} = \frac{d}{dt}(m\vec V) \tag{13–23}$$

where $m\vec{V}$ is the **linear momentum** of the system. Noting that both the density and velocity may change from point to point within the system, Newton's second law can be expressed more generally as

$$\sum \vec{F} = \frac{d}{dt}\int_{\text{sys}} \rho \vec{V}\, dV \qquad \textbf{(13–24)}$$

where $\rho \vec{V}\, dV$ is the momentum of a differential element dV, which has mass $\delta m = \rho\, dV$. Therefore, Newton's second law can be stated as *the sum of all external forces acting on a system is equal to the time rate of change of linear momentum of the system.* This statement is valid for a coordinate system that is at rest or moves with a constant velocity, called an *inertial coordinate system* or *inertial reference frame.* Accelerating systems such as aircraft during takeoff are best analyzed using noninertial (or accelerating) coordinate systems fixed to the aircraft. Note that Eq. 13–24 is a vector relation, and thus the quantities $\vec{F}$ and $\vec{V}$ have direction as well as magnitude.

Equation 13–24 is for a given mass of a solid or fluid and is of limited use in fluid mechanics since most flow systems are analyzed using control volumes. The *Reynolds transport theorem* developed in Section 13–4 provides the necessary tools to shift from the system formulation to the control volume formulation. Setting $b = \vec{V}$ and thus $B = m\vec{V}$, the Reynolds transport theorem is expressed for linear momentum as (Fig. 13–18)

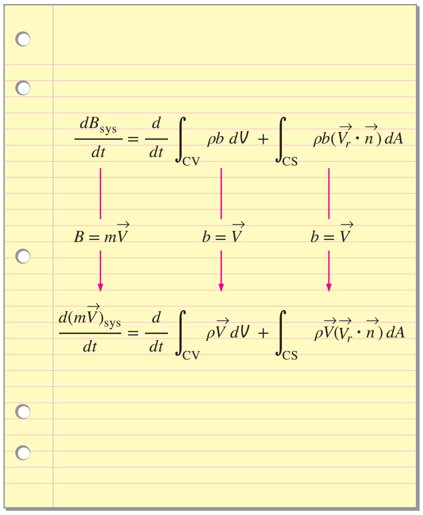


FIGURE 13–18
The linear momentum equation is obtained by replacing B in the Reynolds transport theorem with the momentum $m\vec{V}$ and b with the momentum per unit mass $\vec{V}$.

$$\frac{d(m\vec{V})_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} \rho \vec{V}\, dV + \int_{\text{CS}} \rho \vec{V}(\vec{V}_r \cdot \vec{n})\, dA \qquad \textbf{(13–25)}$$

The left-hand side of this equation is, from Eq. 13–23, equal to $\Sigma\vec{F}$. Substituting, the general form of the linear momentum equation that applies to fixed, moving, or deforming control volumes is

General:
$$\sum \vec{F} = \frac{d}{dt}\int_{\text{CV}} \rho \vec{V}\, dV + \int_{\text{CS}} \rho \vec{V}(\vec{V}_r \cdot \vec{n})\, dA \qquad \textbf{(13–26)}$$

which is stated in words as

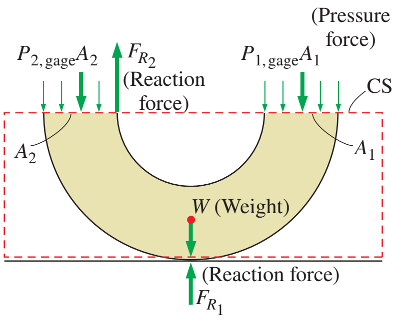
$$\left(\begin{array}{c}\text{The sum of all}\\ \text{external forces}\\ \text{acting on a CV}\end{array}\right) = \left(\begin{array}{c}\text{The time rate of change}\\ \text{of the linear momentum}\\ \text{of the contents of the CV}\end{array}\right) + \left(\begin{array}{c}\text{The net flow rate of}\\ \text{linear momentum out of the}\\ \text{control surface by mass flow}\end{array}\right)$$

Here $\vec{V}_r = \vec{V} - \vec{V}_{\text{CS}}$ is the fluid velocity relative to the control surface (for use in mass flow rate calculations at all locations where the fluid crosses the control surface), and $\vec{V}$ is the fluid velocity as viewed from an inertial reference frame. The product $\rho(\vec{V}_r \cdot \vec{n})\, dA$ represents the mass flow rate through area element dA into or out of the control volume.

For a fixed control volume (no motion or deformation of the control volume), $\vec{V}_r = \vec{V}$, and the linear momentum equation becomes

Fixed CV:
$$\sum \vec{F} = \frac{d}{dt}\int_{\text{CV}} \rho \vec{V}\, dV + \int_{\text{CS}} \rho \vec{V}(\vec{V} \cdot \vec{n})\, dA \qquad \textbf{(13–27)}$$

Note that the momentum equation is a *vector equation,* and thus each term should be treated as a vector. Also, the components of this equation can be resolved along orthogonal coordinates (such as *x, y,* and *z* in the Cartesian coordinate system) for convenience. The sum of forces $\Sigma\vec{F}$ in most cases consists of weights, pressure forces, and reaction forces (Fig. 13–19). The momentum equation is commonly used to calculate the forces (usually on support systems or connectors) induced by the flow.



An 180° elbow supported by the ground

FIGURE 13–19
In most flow systems, the sum of forces $\Sigma\vec{F}$ consists of weights, pressure forces, and reaction forces. Gage pressures are used here since atmospheric pressure cancels out on all sides of the control surface.

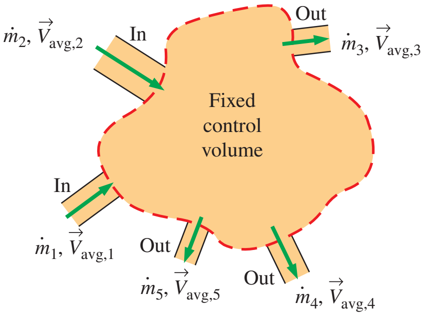


FIGURE 13–20
In a typical engineering problem, the control volume may contain multiple inlets and outlets; at each inlet or outlet we define the mass flow rate $\dot m$ and the average velocity $\vec V_{avg}$.

Special Cases

Most momentum problems considered in this text are steady. During *steady flow,* the amount of momentum within the control volume remains constant, and thus the time rate of change of linear momentum of the contents of the control volume (the second term of Eq. 13–26) is zero. Thus,

Steady flow:
$$\sum \vec F = \int_{CS} \rho \vec V (\vec V_r \cdot \vec n)\, dA \qquad \textbf{(13–28)}$$

For a case in which a nondeforming control volume moves at constant velocity (an inertial reference frame), the *first* $\vec V$ in Eq. 13–28 may *also* be taken relative to the moving control surface.

While Eq. 13–27 is exact for fixed control volumes, it is not always convenient when solving practical engineering problems because of the integrals. Instead, as we did for conservation of mass, we would like to rewrite Eq. 13–27 in terms of average velocities and mass flow rates through inlets and outlets. In other words, our desire is to rewrite the equation in *algebraic* rather than *integral* form. In many practical applications, fluid crosses the boundaries of the control volume at one or more inlets and one or more outlets, and carries with it some momentum into or out of the control volume. For simplicity, we always draw our control surface such that it slices normal to the inflow or outflow velocity at each such inlet or outlet (Fig. 13–20).

The mass flow rate $\dot m$ into or out of the control volume across an inlet or outlet at which ρ is nearly constant is

Mass flow rate across an inlet or outlet:
$$\dot m = \int_{A_c} \rho(\vec V \cdot \vec n)\, dA_c = \rho V_{avg} A_c \qquad \textbf{(13–29)}$$

Comparing Eq. 13–29 to Eq. 13–27, we notice an extra velocity in the control surface integral of Eq. 13–27. If $\vec V$ were uniform ($\vec V = \vec V_{avg}$) across the inlet or outlet, we could simply take it outside the integral. Then we could write the rate of inflow or outflow of momentum through the inlet or outlet in simple algebraic form,

Momentum flow rate across a uniform inlet or outlet:

$$\int_{A_c} \rho \vec V (\vec V \cdot \vec n)\, dA_c = \rho V_{avg} A_c \vec V_{avg} = \dot m \vec V_{avg} \qquad \textbf{(13–30)}$$

The uniform flow approximation is reasonable at some inlets and outlets, e.g., the well-rounded entrance to a pipe, the flow at the entrance to a wind tunnel test section, and a slice through a water jet moving at nearly uniform speed through air (Fig. 13–21). At each such inlet or outlet, Eq. 13–30 can be applied directly.

Momentum-Flux Correction Factor, β

Unfortunately, the velocity across most inlets and outlets of practical engineering interest is *not* uniform. Nevertheless, it turns out that we can still convert the control surface integral of Eq. 13–27 into algebraic form, but a dimensionless correction factor β, called the **momentum-flux correction factor**, is required, as first shown by the French scientist Joseph Boussinesq (1842–1929). The algebraic form of Eq. 13–27 for a fixed control volume is then written as

$$\sum \vec F = \frac{d}{dt}\int_{CV} \rho \vec V\, dV + \sum_{out} \beta \dot m \vec V_{avg} - \sum_{in} \beta \dot m \vec V_{avg} \qquad \textbf{(13–31)}$$

where a unique value of momentum-flux correction factor is applied to each inlet and outlet in the control surface. Note that $\beta = 1$ *for the case of uniform flow* over an inlet or outlet, as in Fig. 13–21. For the general case, we define β such that the integral form of the momentum flux into or out of the control surface at an inlet or outlet of cross-sectional area A_c can be expressed in terms of mass flow rate $\dot{m}$ through the inlet or outlet and average velocity $\vec{V}_{\text{avg}}$ through the inlet or outlet,

Momentum flux across an inlet or outlet:
$$\int_{A_c} \rho\vec{V}(\vec{V}\cdot\vec{n})\, dA_c = \beta\dot{m}\vec{V}_{\text{avg}} \tag{13–32}$$

For the case in which density is uniform over the inlet or outlet and $\vec{V}$ is in the same direction as $\vec{V}_{\text{avg}}$ over the inlet or outlet, we solve Eq. 13–32 for β,

$$\beta = \frac{\int_{A_c} \rho V(\vec{V}\cdot\vec{n})\, dA_c}{\dot{m}V_{\text{avg}}} = \frac{\int_{A_c} \rho V(\vec{V}\cdot\vec{n})\, dA_c}{\rho V_{\text{avg}} A_c V_{\text{avg}}} \tag{13–33}$$

where we have substituted $\rho V_{\text{avg}} A_c$ for $\dot{m}$ in the denominator. The densities cancel, and since V_{avg} is constant, it can be brought inside the integral. Furthermore, if the control surface slices normal to the inlet or outlet area, $(\vec{V}\cdot\vec{n})\, dA_c = V\, dA_c$. Then, Eq. 13–33 simplifies to

Momentum-flux correction factor:
$$\beta = \frac{1}{A_c}\int_{A_c}\left(\frac{V}{V_{\text{avg}}}\right)^2 dA_c \tag{13–34}$$

It may be shown that β is always greater than or equal to unity.

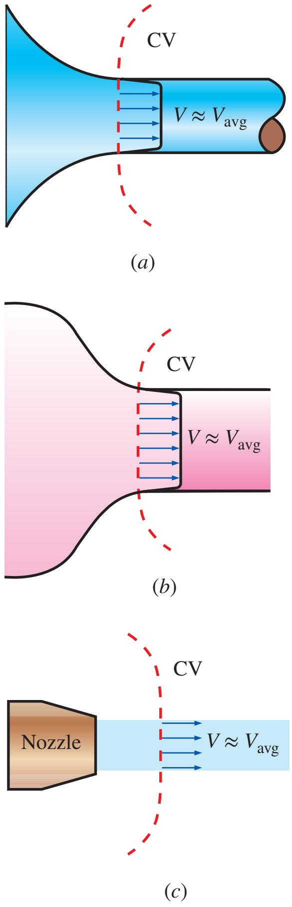


FIGURE 13–21
Examples of inlets or outlets in which the uniform flow approximation is reasonable: (*a*) the well-rounded entrance to a pipe, (*b*) the entrance to a wind tunnel test section, and (*c*) a slice through a free water jet in air.

EXAMPLE 13–1 Momentum-Flux Correction Factor for Laminar Pipe Flow

Consider laminar flow through a very long straight section of round pipe. It is shown in Chap. 14 that the velocity profile through a cross-sectional area of the pipe is parabolic (Fig. 13–22), with the axial velocity component given by

$$V = 2V_{\text{avg}}\left(1 - \frac{r^2}{R^2}\right) \tag{1}$$

where R is the radius of the inner wall of the pipe and V_{avg} is the average velocity. Calculate the momentum-flux correction factor through a cross section of the pipe for the case in which the pipe flow represents an outlet of the control volume, as sketched in Fig. 13–22.

SOLUTION For a given velocity distribution we are to calculate the momentum-flux correction factor.

Assumptions **1** The flow is incompressible and steady. **2** The control volume slices through the pipe normal to the pipe axis, as sketched in Fig. 13–22.

Analysis We substitute the given velocity profile for V in Eq. 13–34 and integrate, noting that $dA_c = 2\pi r\, dr$,

$$\beta = \frac{1}{A_c}\int_{A_c}\left(\frac{V}{V_{\text{avg}}}\right)^2 dA_c = \frac{4}{\pi R^2}\int_0^R \left(1 - \frac{r^2}{R^2}\right)^2 2\pi r\, dr \tag{2}$$

Defining a new integration variable $y = 1 - r^2/R^2$ and thus $dy = -2r\, dr/R^2$ (also, $y = 1$ at $r = 0$, and $y = 0$ at $r = R$) and performing the integration, the momentum-flux correction factor for fully developed laminar flow becomes

Laminar flow:
$$\beta = -4\int_1^0 y^2\, dy = -4\left[\frac{y^3}{3}\right]_1^0 = \frac{4}{3} \tag{3}$$

Discussion We have calculated β for an outlet, but the same result would have been obtained if we had considered the cross section of the pipe as an inlet to the control volume.

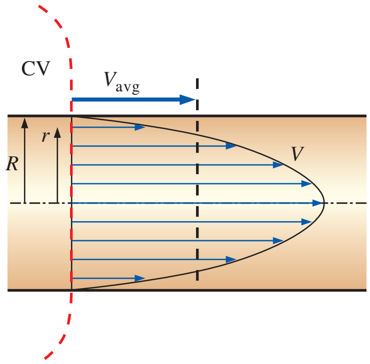


FIGURE 13–22
Velocity profile over a cross section of a pipe in which the flow is fully developed and laminar.

From Example 13–1 we see that β is not very close to unity for fully developed laminar pipe flow, and ignoring β could potentially lead to significant error. If we were to perform the same kind of integration as in Example 13–1 but for fully developed *turbulent* rather than laminar pipe flow, we would find that β ranges from about 1.01 to 1.04. Since these values are so close to unity, many practicing engineers completely disregard the momentum-flux correction factor. While the neglect of β in turbulent flow calculations may have an insignificant effect on the final results, it is wise to keep it in our equations. Doing so not only improves the accuracy of our calculations, but reminds us to include the momentum-flux correction factor when solving laminar flow control volume problems.

For turbulent flow β may have an insignificant effect at inlets and outlets, but for laminar flow β may be important and should not be neglected. It is wise to include β in all momentum control volume problems.

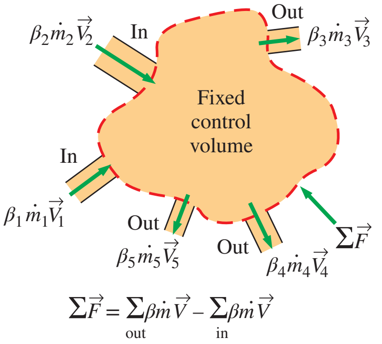


FIGURE 13–23
The net force acting on the control volume during steady flow is equal to the difference between the outgoing and the incoming momentum fluxes.

Steady Flow

If the flow is also *steady,* the time derivative term in Eq. 13–31 vanishes, and we are left with

Steady linear momentum equation:
$$\sum\vec{F} = \sum_{\text{out}}\beta\dot{m}\vec{V} - \sum_{\text{in}}\beta\dot{m}\vec{V} \tag{13–35}$$

where we have dropped the subscript "avg" from average velocity. Equation 13–35 states that *the net force acting on the control volume during steady flow is equal to the difference between the rates of outgoing and incoming momentum flows.* This statement is illustrated in Fig. 13–23. It can also be expressed for any direction, since Eq. 13–35 is a vector equation.

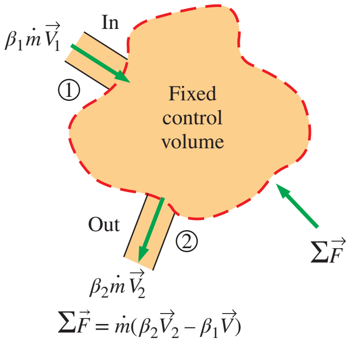


FIGURE 13–24
A control volume with only one inlet and one outlet.

Steady Flow with One Inlet and One Outlet

Many practical engineering problems involve just one inlet and one outlet (Fig. 13–24). The mass flow rate for such **single-stream systems** remains constant, and Eq. 13–35 reduces to

One inlet and one outlet:
$$\sum\vec{F} = \dot{m}(\beta_2\vec{V}_2 - \beta_1\vec{V}_1) \tag{13–36}$$

where we have adopted the usual convention that subscript 1 implies the inlet and subscript 2 the outlet, and $\vec{V}_1$ and $\vec{V}_2$ denote the *average* velocities across the inlet and outlet, respectively.

We emphasize again that all the preceding relations are *vector* equations, and thus all the additions and subtractions are *vector* additions and subtractions. Recall that subtracting a vector is equivalent to adding it after reversing its direction (Fig. 13–25). When writing the momentum equation for a specified coordinate direction (such as the x-axis), we use the projections of the vectors on that axis. For example, Eq. 13–36 is written along the x-coordinate as

Along x-coordinate:
$$\sum F_x = \dot{m}(\beta_2 V_{2,x} - \beta_1 V_{1,x}) \tag{13–37}$$

where ΣF_x is the vector sum of the x-components of the forces, and $V_{2,x}$ and $V_{1,x}$ are the x-components of the outlet and inlet velocities of the fluid stream, respectively. The force or velocity components in the positive x-direction are positive quantities, and those in the negative x-direction are negative quantities. Also, it is good practice to take the direction of unknown forces in the positive directions (unless the problem is very straightforward). A negative value obtained for an unknown force indicates that the assumed direction is wrong and should be reversed.

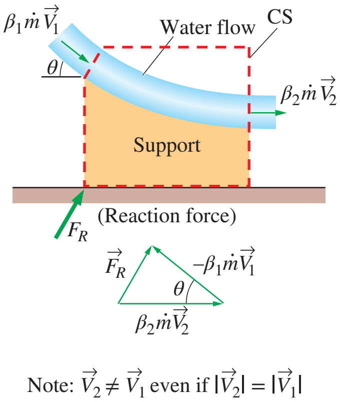


FIGURE 13–25
The determination by vector addition of the reaction force on the support caused by a change of direction of water.

Flow with No External Forces

An interesting situation arises when there are no external forces (such as weight, pressure, and reaction forces) acting on the body in the direction of motion—a common situation for space vehicles and satellites. For a control volume with multiple inlets and outlets, Eq. 13–31 reduces in this case to

No external forces:
$$0 = \frac{d(m\vec{V})_{\text{CV}}}{dt} + \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \textbf{(13–38)}$$

This is an expression of the conservation of momentum principle, which is stated in words as *in the absence of external forces, the rate of change of the momentum of a control volume is equal to the difference between the rates of incoming and outgoing momentum flow rates.*

When the mass m of the control volume remains nearly constant, the first term of Eq. 13–38 becomes simply mass times acceleration, since

$$\frac{d(m\vec{V})_{\text{CV}}}{dt} = m_{\text{CV}} \frac{d\vec{V}_{\text{CV}}}{dt} = (m\vec{a})_{\text{CV}} = m_{\text{CV}}\vec{a}$$

Therefore, the control volume in this case can be treated as a solid body (a fixed-mass system) with a net thrusting force (or just **thrust**) of

Thrust:
$$\vec{F}_{\text{thrust}} = m_{\text{body}}\vec{a} = \sum_{\text{in}} \beta \dot{m} \vec{V} - \sum_{\text{out}} \beta \dot{m} \vec{V} \quad \textbf{(13–39)}$$

acting on the body. In Eq 13–39, fluid velocities are relative to an inertial reference frame—that is, a coordinate system that is fixed in space or is moving uniformly at constant velocity on a straight path. When analyzing the motion of bodies moving at constant velocity on a straight path, it is convenient to choose an inertial reference frame that moves with the body at the same velocity on the same path. In this case the velocities of fluid streams relative to the inertial reference frame are identical to the velocities relative to the moving body, which are much easier to apply. This approach, while not strictly valid for noninertial reference frames, can also be used to calculate the *initial* acceleration of a space vehicle when its rocket is fired (Fig. 13–26).

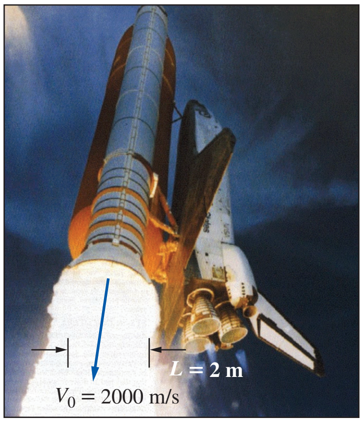


FIGURE 13–26
The thrust needed to lift the space shuttle is generated by the rocket engines as a result of momentum change of the fuel as it is accelerated from about zero to an exit speed of about 2000 m/s after combustion.
NASA

Recall that thrust is a mechanical force typically generated through the reaction of an accelerating fluid. In the jet engine of an aircraft, for example, hot exhaust gases are accelerated by the action of expansion and outflow of gases through the back of the engine, and a thrusting force is produced by a reaction in the opposite direction. The generation of thrust is based on Newton's third law of motion, which states that *for every action at a point there is an equal and opposite reaction.* In the case of a jet engine, if the engine exerts a force on exhaust gases, then the exhaust gases exert an equal force on the engine in the opposite direction. That is, the pushing force exerted on the departing gases by the engine is equal to the thrusting force the departing gases exert on the remaining mass of the aircraft in the opposite direction $\vec{F}_{\text{thrust}} = -\vec{F}_{\text{push}}$. On the free-body diagram of an aircraft, the effect of outgoing exhaust gases is accounted for by the insertion of a force in the opposite direction of motion of the exhaust gases.

EXAMPLE 13–2 **The Force to Hold a Deflector Elbow in Place**

A reducing elbow is used to deflect water flow at a rate of 14 kg/s in a horizontal pipe upward 30° while accelerating it (Fig. 13–27). The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 113 cm^2 at the inlet and 7 cm^2 at the outlet. The elevation difference between the centers of the outlet and the inlet is 30 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (*a*) the gage pressure at the center of the inlet of the elbow and (*b*) the anchoring force needed to hold the elbow in place.

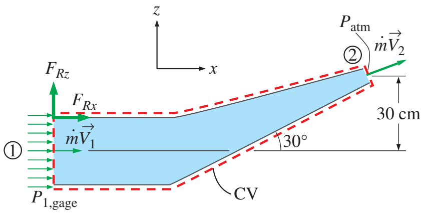


FIGURE 13–27
Schematic for Example 13–2.

SOLUTION A reducing elbow deflects water upward and discharges it to the atmosphere. The pressure at the inlet of the elbow and the force needed to hold the elbow in place are to be determined.

Assumptions **1** The flow is steady, and the frictional effects are negligible. **2** The weight of the elbow and the water in it is negligible. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **4** The flow is turbulent and fully developed at both the inlet and outlet of the control volume, and we take the momentum-flux correction factor to be $\beta = 1.03$ (as a conservative estimate) at both the inlet and the outlet.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the elbow as the control volume and designate the inlet by 1 and the outlet by 2. We also take the *x*- and *z*-coordinates as shown. The continuity equation for this one-inlet, one-outlet, steady-flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 14$ kg/s. Noting that $\dot{m} = \rho A V$, the inlet and outlet velocities of water are

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{14\ \text{kg/s}}{(1000\ \text{kg/m}^3)(0.0113\ \text{m}^2)} = 1.24\ \text{m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{14\ \text{kg/s}}{(1000\ \text{kg/m}^3)(7\times 10^{-4}\ \text{m}^2)} = 20.0\ \text{m/s}$$

We use the Bernoulli equation (Chap. 12) as a first approximation to calculate the pressure. In Chap. 14 we will learn how to account for frictional losses along the walls. Taking the center of the inlet cross section as the reference level ($z_1 = 0$) and noting that $P_2 = P_{\text{atm}}$, the Bernoulli equation for a streamline going through the center of the elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$P_1 - P_2 = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1\right)$$

$$P_1 - P_{\text{atm}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2) \times \left(\frac{(20\ \text{m/s})^2 - (1.24\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0.3 - 0\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$P_{1,\,\text{gage}} = 202.2\ \text{kN/m}^2 = \mathbf{202.0\ kPa} \quad \text{(gage)}$$

(*b*) The momentum equation for steady flow is

$$\sum \vec{F} = \sum_{\text{out}} \beta\dot{m}\vec{V} - \sum_{\text{in}} \beta\dot{m}\vec{V}$$

We let the *x*- and *z*-components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive direction. We also use gage pressure since the atmospheric pressure acts on the entire control surface. Then the momentum equations along the *x*- and *z*-axes become

$$F_{Rx} + P_{1,\,\text{gage}}A_1 = \beta\dot{m}V_2\cos\theta - \beta\dot{m}V_1$$
$$F_{Rz} = \beta\dot{m}V_2\sin\theta$$

where we have set $\beta = \beta_1 = \beta_2$. Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$\begin{aligned} F_{Rx} &= \beta\dot{m}(V_2\cos\theta - V_1) - P_{1,\,\text{gage}}A_1 \\ &= 1.03(14\ \text{kg/s})[(20\cos 30^\circ - 1.24)\ \text{m/s}]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) \\ &\quad -(202{,}200\ \text{N/m}^2)(0.0113\ \text{m}^2) \\ &= 232 - 2285 = \mathbf{-2053\ N} \end{aligned}$$

$$F_{Rz} = \beta\dot{m}V_2\sin\theta = (1.03)(14\ \text{kg/s})(20\sin 30^\circ\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{144\ N}$$

The negative result for F_{Rx} indicates that the assumed direction is wrong, and it should be reversed. Therefore, F_{Rx} acts in the negative *x*-direction.

Discussion There is a nonzero pressure distribution along the inside walls of the elbow, but since the control volume is outside the elbow, these pressures do not appear in our analysis. The weight of the elbow and the water in it could be added to the vertical force for better accuracy. The actual value of $P_{1,\text{ gage}}$ will be higher than that calculated here because of frictional and other irreversible losses in the elbow.

EXAMPLE 13–3 The Force to Hold a Reversing Elbow in Place

The deflector elbow in Example 13–2 is replaced by a reversing elbow such that the fluid makes a 180° U-turn before it is discharged, as shown in Fig. 13–28. The elevation difference between the centers of the inlet and the exit sections is still 0.3 m. Determine the anchoring force needed to hold the elbow in place.

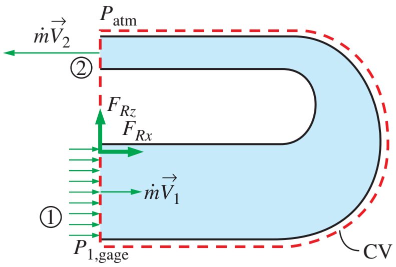


FIGURE 13–28
Schematic for Example 13–3.

SOLUTION The inlet and the outlet velocities and the pressure at the inlet of the elbow remain the same, but the vertical component of the anchoring force at the connection of the elbow to the pipe is zero in this case ($F_{Rz} = 0$) since there is no other force or momentum flux in the vertical direction (we are neglecting the weight of the elbow and the water). The horizontal component of the anchoring force is determined from the momentum equation written in the x-direction. Noting that the outlet velocity is negative since it is in the negative x-direction, we have

$$F_{Rx} + P_{1,\text{ gage}}A_1 = \beta\dot{m}(-V_2) - \beta\dot{m}V_1 = -\beta\dot{m}(V_2 + V_1)$$

Solving for F_{Rx} and substituting the known values,

$$\begin{aligned} F_{Rx} &= -\beta\dot{m}(V_2 + V_1) - P_{1,\text{ gage}}A_1 \\ &= -(1.03)(14\text{ kg/s})[(20 + 1.24)\text{ m/s}]\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) - (202{,}200\text{ N/m}^2)(0.0113\text{ m}^2) \\ &= -306 - 2285 = \mathbf{-2591\text{ N}} \end{aligned}$$

Therefore, the horizontal force on the flange is 2591 N acting in the negative x-direction (the elbow is trying to separate from the pipe). This force is equivalent to the weight of about 260 kg mass, and thus the connectors (such as bolts) used must be strong enough to withstand this force.

Discussion The reaction force in the x-direction is larger than that of Example 13–2 since the walls turn the water over a much greater angle. If the reversing elbow is replaced by a straight nozzle (like one used by firefighters) such that water is discharged in the positive x-direction, the momentum equation in the x-direction becomes

$$F_{Rx} + P_{1,\text{ gage}}A_1 = \beta\dot{m}V_2 - \beta\dot{m}V_1 \quad \rightarrow \quad F_{Rx} = \beta\dot{m}(V_2 - V_1) - P_{1,\text{ gage}}A_1$$

since both V_1 and V_2 are in the positive x-direction. This shows the importance of using the correct sign (positive if in the positive direction and negative if in the opposite direction) for velocities and forces.

EXAMPLE 13–4 Water Jet Striking a Moving Cart

Water accelerated by a nozzle to 35 m/s strikes the vertical back surface of a cart moving horizontally at a constant velocity of 10 m/s in the flow direction (Fig. 13–29). The mass flow rate of water through the stationary nozzle is 30 kg/s. After the strike, the water stream splatters off in all directions in the plane of the back surface. (*a*) Determine the force that needs to be applied by the brakes of the cart to prevent it from accelerating. (*b*) If this force were used to generate power instead of wasting it on the brakes, determine the maximum amount of power that could ideally be generated. (*c*) If the mass of the cart is 400 kg and the brakes fail, determine the acceleration of the cart when the water first strikes it. Assume the mass of water that wets the back surface is negligible.

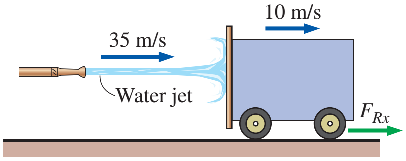


FIGURE 13–29
Schematic for Example 13–4.

SOLUTION Water accelerated by a nozzle strikes the back surface of a cart moving horizontally at a constant velocity. The braking force, the power wasted by the brakes, and the acceleration of the cart if the brakes fail are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The water splatters off the sides of the plate in all directions in the plane of the back surface. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on all surfaces. **4** Friction during motion is negligible. **5** The motions of the water jet and the cart are horizontal. **6** Jet flow is nearly uniform, and thus the effect of the momentum-flux correction factor is negligible, $\beta \cong 1$.

Analysis We take the cart as the control volume, and the direction of flow as the positive direction of the x-axis. The relative velocity between the cart and the jet is

$$V_r = V_{\text{jet}} - V_{\text{cart}} = 35 - 10 = 25 \text{ m/s}$$

Therefore, we can view the cart as being stationary and the jet moving with a velocity of 25 m/s. Noting that water leaves the nozzle at 20 m/s and the corresponding mass flow rate relative to the nozzle exit is 30 kg/s, the mass flow rate of water striking the cart corresponding to a water jet velocity of 25 m/s relative to the cart is

$$\dot{m}_r = \frac{V_r}{V_{\text{jet}}}\dot{m}_{\text{jet}} = \frac{25 \text{ m/s}}{35 \text{ m/s}}(30 \text{ kg/s}) = 21.43 \text{ kg/s}$$

The momentum equation for steady flow in the x (flow)-direction reduces in this case to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad F_{Rx} = -\dot{m}_i V_i \quad \rightarrow \quad F_{\text{brake}} = -\dot{m}_r V_r$$

We note that the brake force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x-direction. Substituting the given values,

$$F_{\text{brake}} = -\dot{m}_r V_r = -(21.43 \text{ kg/s})(+25 \text{ m/s})\left(\frac{1 \text{ N}}{1 \text{ kg·m/s}^2}\right) = -535.8 \text{ N} \cong \mathbf{-536\ N}$$

FIGURE 13–30
The downwash of a helicopter is similar to the jet discussed in Example 13–4. The jet impinges on the surface of the water in this case, causing circular waves as seen here.
Purestock/Superstock

The negative sign indicates that the braking force acts in the opposite direction to motion, as expected. Just as the water jet here imparts a force to the cart, the air jet from a helicopter (downwash) imparts a force on the surface of the water (Fig. 13–30). Noting that work is force times distance and the distance traveled by the cart per unit time is the cart velocity, the power wasted by the brakes is

$$\dot{W} = F_{\text{brake}} V_{\text{cart}} = (535.8 \text{ N})(10 \text{ m/s})\left(\frac{1 \text{ W}}{1 \text{ N·m/s}}\right) = 5358 \text{ W} \cong \mathbf{5.36\ kW}$$

Note that the power wasted is equivalent to the maximum power that can be generated as the cart velocity is maintained constant.

(c) When the brakes fail, the braking force will propel the cart forward, and the acceleration will be

$$a = \frac{F}{m_{\text{cart}}} = \frac{535.8 \text{ N}}{400 \text{ kg}}\left(\frac{1 \text{ kg·m/s}^2}{1 \text{ N}}\right) = \mathbf{1.34\ m/s^2}$$

Discussion This is the acceleration at the moment the brakes fail. The acceleration will decrease as the relative velocity between the water jet and the cart (and thus the force) decreases.

EXAMPLE 13–5 Power Generation and Wind Loading of a Wind Turbine

A wind generator with a 30-ft-diameter blade span has a cut-in wind speed (minimum speed for power generation) of 7 mph, at which velocity the turbine generates 0.4 kW of electric power (Fig. 13–31). Determine (a) the efficiency of the wind turbine–generator unit and (b) the horizontal force exerted by the wind on the supporting mast of the wind

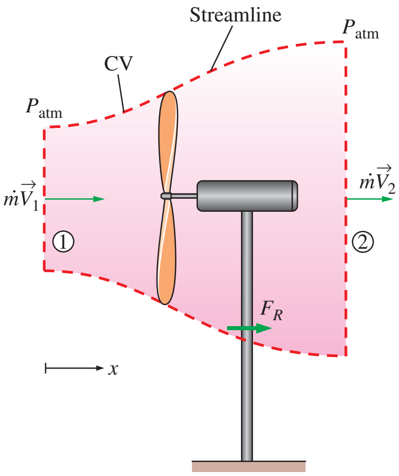


FIGURE 13–31
Schematic for Example 13–5.

turbine. What is the effect of doubling the wind velocity to 14 mph on power generation and the force exerted? Assume the efficiency remains the same, and take the density of air to be 0.076 lbm/ft^3.

SOLUTION The power generation and loading of a wind turbine are to be analyzed. The efficiency and the force exerted on the mast are to be determined, and the effects of doubling the wind velocity are to be investigated.

Assumptions **1** The wind flow is steady and incompressible. **2** The efficiency of the turbine–generator is independent of wind speed. **3** The frictional effects are negligible, and thus none of the incoming kinetic energy is converted to thermal energy. **4** The average velocity of air through the wind turbine is the same as the wind velocity (actually, it is considerably less). **5** The wind flow is nearly uniform upstream and downstream of the wind turbine, and thus the momentum-flux correction factor is $\beta = \beta_1 = \beta_2 \cong 1$.

Properties The density of air is given to be 0.076 lbm/ft^3.

Analysis Kinetic energy is a mechanical form of energy, and thus it can be converted to work entirely. Therefore, the power potential of the wind is proportional to its kinetic energy, which is $V^2/2$ per unit mass, and thus the maximum power is $\dot{m}V^2/2$ for a given mass flow rate:

$$V_1 = (7\text{ mph})\left(\frac{1.4667\text{ ft/s}}{1\text{ mph}}\right) = 10.27\text{ ft/s}$$

$$\dot{m} = \rho_1 V_1 A_1 = \rho_1 V_1 \frac{\pi D^2}{4} = (0.076\text{ lbm/ft}^3)(10.27\text{ ft/s})\frac{\pi(30\text{ ft})^2}{4} = 551.7\text{ lbm/s}$$

$$\begin{aligned}\dot{W}_{\text{max}} &= \dot{m}\text{ke}_1 = \dot{m}\frac{V_1^2}{2}\\ &= (551.7\text{ lbm/s})\frac{(10.27\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.3\text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right)\\ &= 1.225\text{ kW}\end{aligned}$$

Therefore, the available power to the wind turbine is 1.225 kW at the wind velocity of 7 mph. Then the turbine–generator efficiency becomes

$$\eta_{\text{wind turbine}} = \frac{\dot{W}_{\text{act}}}{\dot{W}_{\text{max}}} = \frac{0.4\text{ kW}}{1.225\text{ kW}} = \mathbf{0.327} \qquad (\text{or } \mathbf{32.7\%})$$

(*b*) The frictional effects are assumed to be negligible, and thus the portion of incoming kinetic energy not converted to electric power leaves the wind turbine as outgoing kinetic energy. Noting that the mass flow rate remains constant, the exit velocity is determined to be

$$\dot{m}\text{ke}_2 = \dot{m}\text{ke}_1(1 - \eta_{\text{wind turbine}}) \rightarrow \dot{m}\,\frac{V_2^2}{2} = \dot{m}\,\frac{V_1^2}{2}(1 - \eta_{\text{wind turbine}}) \qquad \textbf{(1)}$$

or

$$V_2 = V_1\sqrt{1 - \eta_{\text{wind turbine}}} = (10.27\text{ ft/s})\sqrt{1 - 0.327} = 8.43\text{ ft/s}$$

To determine the force on the mast (Fig. 13–32), we draw a control volume around the wind turbine such that the wind is normal to the control surface at the inlet and the outlet and the entire control surface is at atmospheric pressure (Fig. 13–30). The momentum equation for steady flow is given as

$$\sum \vec{F} = \sum_{\text{out}} \beta\dot{m}\vec{V} - \sum_{\text{in}} \beta\dot{m}\vec{V} \qquad \textbf{(2)}$$

Writing Eq. 2 along the x-direction and noting that $\beta = 1$, $V_{1,\,x} = V_1$, and $V_{2,\,x} = V_2$ give

$$F_R = \dot{m}V_2 - \dot{m}V_1 = \dot{m}(V_2 - V_1) \qquad \textbf{(3)}$$

FIGURE 13–32
Forces and moments on the supporting mast of a modern wind turbine can be substantial, and they increase in proportion to the square of the wind velocity; thus the mast is typically quite large and strong.
Ingram Publishing/SuperStock

Substituting the known values into Eq. 3 gives

$$F_R = \dot{m}(V_2 - V_1) = (551.7 \text{ lbm/s})(8.43 - 10.27 \text{ ft/s})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= -31.5 \text{ lbf}$$

The negative sign indicates that the reaction force acts in the negative x-direction, as expected. Then the force exerted by the wind on the mast becomes $F_{\text{mast}} = -F_R =$ **31.5 lbf**.

The power generated is proportional to V^3 since the mass flow rate is proportional to V and the kinetic energy to V^2. Therefore, doubling the wind velocity to 14 mph will increase the power generation by a factor of $2^3 = 8$ to $0.4 \times 8 = 3.2$ kW. The force exerted by the wind on the support mast is proportional to V^2. Therefore, doubling the wind velocity to 14 mph will increase the wind force by a factor of $2^2 = 4$ to $31.5 \times 4 = 126$ lbf.

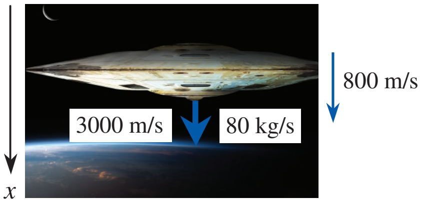


FIGURE 13–33
Schematic for Example 13–6.
Marc Ward/Stocktrek Images/Getty Images

EXAMPLE 13–6 Deceleration of a Spacecraft

A spacecraft with a mass of 12,000 kg is dropping vertically toward a planet at a constant speed of 800 m/s (Fig. 13–33). To slow down the spacecraft, a solid-fuel rocket at the bottom is fired, and combustion gases leave the rocket at a constant rate of 80 kg/s and at a velocity of 3000 m/s relative to the spacecraft in the direction of motion of the spacecraft for a period of 5 s. Disregarding the small change in the mass of the spacecraft, determine (*a*) the deceleration of the spacecraft during this period, (*b*) the change of velocity of the spacecraft, and (*c*) the thrust exerted on the spacecraft.

SOLUTION The rocket of a spacecraft is fired in the direction of motion. The deceleration, the velocity change, and the thrust are to be determined.

Assumptions **1** The flow of combustion gases is steady and one-dimensional during the firing period, but the flight of the spacecraft is unsteady. **2** There are no external forces acting on the spacecraft, and the effect of pressure force at the nozzle outlet is negligible. **3** The mass of discharged fuel is negligible relative to the mass of the spacecraft, and thus, the spacecraft may be treated as a solid body with a constant mass. **4** The nozzle is well designed such that the effect of the momentum-flux correction factor is negligible, and thus, $\beta \cong 1$.

Analysis (*a*) For convenience, we choose an inertial reference frame that moves with the spacecraft at the same initial velocity. Then the velocities of the fluid stream relative to an inertial reference frame become simply the velocities relative to the spacecraft. We take the direction of motion of the spacecraft as the positive direction along the x-axis. There are no external forces acting on the spacecraft, and its mass is essentially constant. Therefore, the spacecraft can be treated as a solid body with constant mass, and the momentum equation in this case is, from Eq. 13–39,

$$\vec{F}_{\text{thrust}} = m_{\text{spacecraft}}\,\vec{a}_{\text{spacecraft}} = \sum_{\text{in}} \beta\dot{m}\vec{V} - \sum_{\text{out}} \beta\dot{m}\vec{V}$$

where the fluid stream velocities relative to the inertial reference frame in this case are identical to the velocities relative to the spacecraft. Noting that the motion is on a straight line and the discharged gases move in the positive x-direction, we write the momentum equation using magnitudes as

$$m_{\text{spacecraft}}a_{\text{spacecraft}} = m_{\text{spacecraft}}\frac{dV_{\text{spacecraft}}}{dt} = -\dot{m}_{\text{gas}}V_{\text{gas}}$$

Noting that gases leave in the positive x-direction and substituting, the acceleration of the spacecraft during the first 5 seconds is determined to be

$$a_{\text{spacecraft}} = \frac{dV_{\text{spacecraft}}}{dt} = -\frac{\dot{m}_{\text{gas}}}{m_{\text{spacecraft}}}V_{\text{gas}} = -\frac{80 \text{ kg/s}}{12{,}000 \text{ kg}}(+3000 \text{ m/s}) = \mathbf{-20 \text{ m/s}^2}$$

The negative value confirms that the spacecraft is decelerating in the positive x-direction at a rate of 20 m/s^2.

(b) Knowing the deceleration, which is constant, the velocity change of the spacecraft during the first 5 seconds is determined from the definition of acceleration to be

$$dV_{\text{spacecraft}} = a_{\text{spacecraft}}dt \rightarrow \Delta V_{\text{spacecraft}} = a_{\text{spacecraft}}\Delta t = (-20\text{ m/s}^2)(5\text{ s})$$
$$= \mathbf{-100\ m/s}$$

(c) The thrusting force exerted on the space aircraft is, from Eq. 13–39,

$$F_{\text{thrust}} = 0 - \dot{m}_{\text{gas}}V_{\text{gas}} = 0 - (80\text{ kg/s})(+3000\text{ m/s})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{-240\ kN}$$

The negative sign indicates that the thrusting force due to firing of the rocket acts on the aircraft in the negative x-direction.

Discussion Note that if this fired rocket were attached somewhere on a test stand, it would exert a force of 240 kN (equivalent to the weight of about 24 tons of mass) on its support in the opposite direction of the discharged gases.

SUMMARY

This chapter deals mainly with the conservation of momentum for finite control volumes. The forces acting on the control volume consist of *body forces* that act throughout the entire body of the control volume (such as gravity, electric, and magnetic forces) and *surface forces* that act on the control surface (such as the pressure forces and reaction forces at points of contact). The sum of all forces acting on the control volume at a particular instant in time is represented by $\Sigma\vec{F}$ and is expressed as

$$\underbrace{\sum\vec{F}}_{\text{total force}} = \underbrace{\sum\vec{F}_{\text{gravity}}}_{\text{body force}} + \underbrace{\sum\vec{F}_{\text{pressure}} + \sum\vec{F}_{\text{viscous}} + \sum\vec{F}_{\text{other}}}_{\text{surface forces}}$$

Newton's second law can be stated as *the sum of all external forces acting on a system is equal to the time rate of change of linear momentum of the system.* Setting $b = \vec{V}$ and thus $B = m\vec{V}$ in the Reynolds transport theorem and utilizing Newton's second law gives the *linear momentum equation* for a control volume as

$$\sum\vec{F} = \frac{d}{dt}\int_{\text{CV}}\rho\vec{V}\,d\mathcal{V} + \int_{\text{CS}}\rho\vec{V}(\vec{V}_r\cdot\vec{n})\,dA$$

which reduces to the following special cases:

Steady flow: $$\sum\vec{F} = \int_{\text{CS}}\rho\vec{V}(\vec{V}_r\cdot\vec{n})\,dA$$

Unsteady flow (*algebraic form*):

$$\sum\vec{F} = \frac{d}{dt}\int_{\text{CV}}\rho\vec{V}\,d\mathcal{V} + \sum_{\text{out}}\beta\dot{m}\vec{V} - \sum_{\text{in}}\beta\dot{m}\vec{V}$$

Steady flow (*algebraic form*): $$\sum\vec{F} = \sum_{\text{out}}\beta\dot{m}\vec{V} - \sum_{\text{in}}\beta\dot{m}\vec{V}$$

No external forces: $$0 = \frac{d(m\vec{V})_{\text{CV}}}{dt} + \sum_{\text{out}}\beta\dot{m}\vec{V} - \sum_{\text{in}}\beta\dot{m}\vec{V}$$

where β is the momentum-flux correction factor. A control volume whose mass m remains constant can be treated as a solid body (a fixed-mass system) with a *net thrusting force* (also called simply the *thrust*) of

$$\vec{F}_{\text{thrust}} = m_{\text{CV}}\vec{a} = \sum_{\text{in}}\beta\dot{m}\vec{V} - \sum_{\text{out}}\beta\dot{m}\vec{V}$$

acting on the body.

REFERENCES AND SUGGESTED READING

1. Kundu, P. K., Cohen, I. M., and Dowling, D. R., *Fluid Mechanics,* ed. 5. San Diego, CA: Academic Press, 2011.
2. Terry Wright, *Fluid Machinery: Performance, Analysis, and Design,* Boca Raton, FL: CRC Press, 1999.

PROBLEMS*

Newton's Laws and Conservation of Momentum

13–1C Express Newton's second law of motion for rotating bodies. What can you say about the angular velocity and angular momentum of a rotating nonrigid body of constant mass if the net torque acting on it is zero?

13–2C Express Newton's first, second, and third laws.

13–3C Is momentum a vector? If so, in what direction does it point?

13–4C Express the conservation of momentum principle. What can you say about the momentum of a body if the net force acting on it is zero?

Linear Momentum Equation

13–5C How do surface forces arise in the momentum analysis of a control volume? How can we minimize the number of surface forces exposed during analysis?

13–6C Explain the importance of the Reynolds transport theorem in fluid mechanics, and describe how the linear momentum equation is obtained from it.

13–7C What is the importance of the momentum-flux correction factor in the momentum analysis of flow systems? For which type(s) of flow is it significant and must it be considered in analysis: laminar flow, turbulent flow, or jet flow?

13–8C Write the momentum equation for steady one-dimensional flow for the case of no external forces, and explain the physical significance of its terms.

13–9C In the application of the momentum equation, explain why we can usually disregard the atmospheric pressure and work with gage pressures only.

13–10C Two firefighters are fighting a fire with identical water hoses and nozzles, except that one is holding the hose straight so that the water leaves the nozzle in the same direction it comes, while the other holds it backward so that the water makes a U-turn before being discharged. Which firefighter will experience a greater reaction force?

13–11C A rocket in space (no friction or resistance to motion) can expel gases relative to itself at some high velocity *V*. Is *V* the upper limit to the rocket's ultimate velocity?

13–12C Describe in terms of momentum and airflow how a helicopter is able to hover.

FIGURE P13–12C

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13–13C Does it take more, equal, or less power for a helicopter to hover at the top of a high mountain than it does at sea level? Explain.

13–14C In a given location, would a helicopter require more energy in summer or winter to achieve a specified performance? Explain.

13–15C Describe body forces and surface forces, and explain how the net force acting on a control volume is determined. Is fluid weight a body force or surface force? How about pressure?

13–16C A constant-velocity horizontal water jet from a stationary nozzle impinges normally on a vertical flat plate that rides on a nearly frictionless track. As the water jet hits the plate, it begins to move due to the water force. Will the acceleration of the plate remain constant or change? Explain.

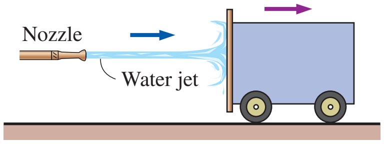

FIGURE P13–16C

13–17C A horizontal water jet of constant velocity *V* from a stationary nozzle impinges normally on a vertical flat plate that rides on a nearly frictionless track. As the water jet hits the plate, it begins to move due to the water force. What is the highest velocity the plate can attain? Explain.

13–18C A horizontal water jet from a nozzle of constant exit cross section impinges normally on a stationary vertical flat plate. A certain force *F* is required to hold the plate against the water stream. If the water velocity is doubled, will the necessary holding force also be doubled? Explain.

13–19 A 2.5-cm-diameter horizontal water jet with a speed of $V_j = 40$ m/s relative to the ground is deflected by a 60° stationary cone whose base diameter is 25 cm. Water velocity along the cone varies linearly from zero at the cone surface to the incoming jet speed of 40 m/s at the free surface. Disregarding the effect of gravity and the shear forces, determine the horizontal force *F* needed to hold the cone stationary.

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* Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and SI users can ignore them. Problems with the icon 💻 are comprehensive in nature and are intended to be solved with appropriate software.

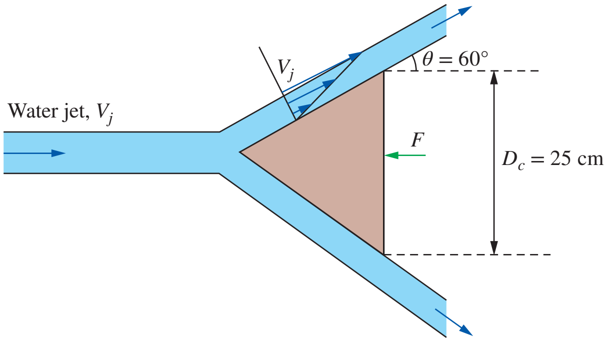

FIGURE P13–19

13–20 A 90° elbow in a horizontal pipe is used to direct water flow upward at a rate of 40 kg/s. The diameter of the entire elbow is 10 cm. The elbow discharges water into the atmosphere, and thus the pressure at the exit is the local atmospheric pressure. The elevation difference between the centers of the exit and the inlet of the elbow is 50 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (*a*) the gage pressure at the center of the inlet of the elbow and (*b*) the anchoring force needed to hold the elbow in place. Take the momentum-flux correction factor to be 1.03 at both the inlet and the outlet.

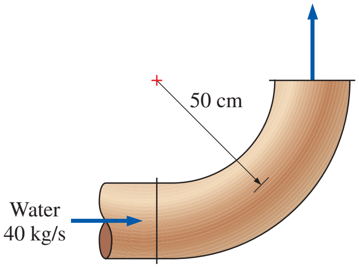

FIGURE P13–20

13–21 Repeat Prob. 13–20 for the case of another (identical) elbow attached to the existing elbow so that the fluid makes a U-turn. *Answers:* (*a*) 9.81 kPa, (*b*) 497 N

13–22E A horizontal water jet impinges against a vertical flat plate at 30 ft/s and splashes off the sides in the vertical plane. If a horizontal force of 500 lbf is required to hold the plate against the water stream, determine the volume flow rate of the water.

13–23 Water enters a 7-cm-diameter pipe steadily with a uniform velocity of 2 m/s and exits with the turbulent flow velocity distribution given by $u = u_{\max}(1 - r/R)^{1/7}$. If the pressure drop along the pipe is 10 kPa, determine the drag force exerted on the pipe by water flow.

13–24 A reducing elbow in a horizontal pipe is used to deflect water flow by an angle $\theta = 45°$ from the flow direction while accelerating it. The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 150 cm^2 at the inlet and 25 cm^2 at the exit. The elevation difference between the centers of the exit and the inlet is 40 cm. The mass of the elbow and the water in it is 50 kg. Determine the anchoring force needed to hold the elbow in place. Take the momentum-flux correction factor to be 1.03 at both the inlet and outlet.

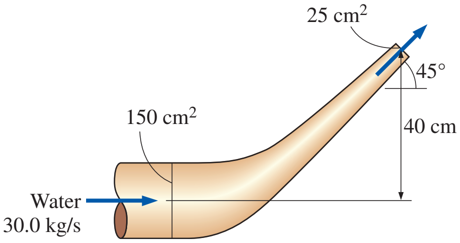

FIGURE P13–24

13–25 Repeat Prob. 13–24 for the case of $\theta = 125°$.

13–26E A 100-ft^3/s water jet is moving in the positive *x*-direction at 18 ft/s. The stream hits a stationary splitter, such that half of the flow is diverted upward at 45° and the other half is directed downward, and both streams have a final average speed of 18 ft/s. Disregarding gravitational effects, determine the *x*- and *z*-components of the force required to hold the splitter in place against the water force.

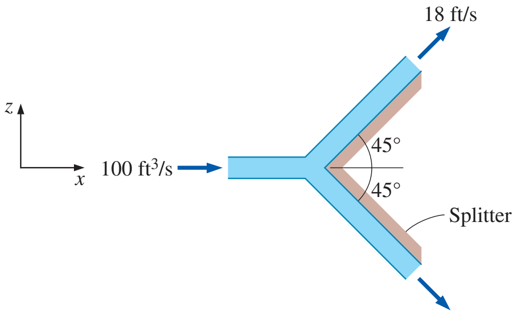

FIGURE P13–26E

13–27E Reconsider Prob. 13–26E. Using appropriate software, investigate the effect of the splitter angle on the force exerted on the splitter in the incoming flow direction. Let the half splitter angle vary from 0° to 180° in increments of 10°. Tabulate and plot your results, and draw some conclusions.

13–28 Commercially available large wind turbines have blade span diameters larger than 100 m and generate over 3 MW of electric power at peak design conditions. Consider a wind turbine with a 75-m blade span subjected to 25-km/h steady winds. If the combined turbine–generator efficiency of the wind turbine is 32 percent, determine (*a*) the power generated by the turbine and (*b*) the horizontal force exerted by the wind on the supporting mast of the turbine. Take the density of air to be 1.25 kg/m^3, and disregard frictional effects on the mast.

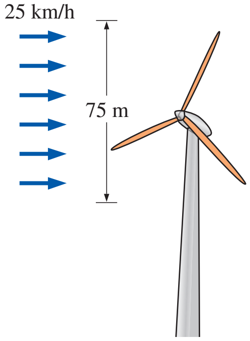

FIGURE P13–28

13–29E A fan with 24-in-diameter blades moves 2000 cfm (cubic feet per minute) of air at 70°F at sea level. Determine (*a*) the force required to hold the fan and (*b*) the minimum power input required for the fan. Choose a control volume sufficiently large to contain the fan, with the inlet sufficiently far upstream so that the gage pressure at the inlet is nearly zero. Assume air approaches the fan through a large area with negligible velocity and air exits the fan with a uniform velocity at atmospheric pressure through an imaginary cylinder whose diameter is the fan blade diameter. *Answers:* (*a*) 0.820 lbf, (*b*) 5.91 W

13–30E A 3-in-diameter horizontal jet of water, with velocity 140 ft/s, strikes a bent plate, which deflects the water by 135° from its original direction. How much force is required to hold the plate against the water stream, and what is its direction? Disregard frictional and gravitational effects.

13–31 Firefighters are holding a nozzle at the end of a hose while trying to extinguish a fire. If the nozzle exit diameter is 8 cm and the water flow rate is 12 m^3/min, determine (*a*) the average water exit velocity and (*b*) the horizontal resistance force required of the firefighters to hold the nozzle. *Answers:* (*a*) 39.8 m/s, (*b*) 7958 N

FIGURE P13–31

13–32 A 5-cm-diameter horizontal jet of water with a velocity of 30 m/s relative to the ground strikes a flat plate that is moving in the same direction as the jet at a velocity of 20 m/s. The water splatters in all directions in the plane of the plate. How much force does the water stream exert on the plate?

13–33 Reconsider Prob. 13–32. Using appropriate software, investigate the effect of the plate velocity on the force exerted on the plate. Let the plate velocity vary from 0 to 30 m/s, in increments of 3 m/s. Tabulate and plot your results.

13–34E A 3-in-diameter horizontal water jet having a velocity of 90 ft/s strikes a curved plate, which deflects the water 180° at the same speed. Ignoring the frictional effects, determine the force required to hold the plate against the water stream.

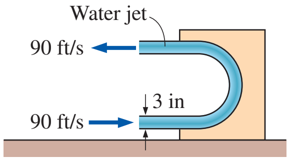

FIGURE P13–34E

13–35 An unloaded helicopter of mass 12,000 kg hovers at sea level while it is being loaded. In the unloaded hover mode, the blades rotate at 550 rpm. The horizontal blades above the helicopter cause a 18-m-diameter air mass to move downward at an average velocity proportional to the overhead blade rotational velocity (rpm). A load of 14,000 kg is loaded onto the helicopter, and the helicopter slowly rises. Determine (*a*) the volumetric airflow rate downdraft that the helicopter generates during unloaded hover and the required power input and (*b*) the rpm of the helicopter blades to hover with the 14,000-kg load and the required power input. Take the density of atmospheric air to be 1.18 kg/m^3. Assume air approaches the blades from the top through a large area with negligible velocity and air is forced by the blades to move down with a uniform velocity through an imaginary cylinder whose base is the blade span area.

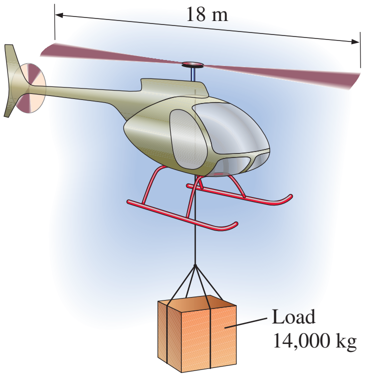

FIGURE P13–35

13–36 Reconsider the helicopter in Prob. 13–35, except that it is hovering on top of a 2200-m-high mountain where the air density is 0.987 kg/m^3. Noting that the unloaded helicopter blades must rotate at 550 rpm to hover at sea level, determine the blade rotational velocity to hover at the higher altitude. Also determine the percent increase in the required power input to hover at 2200-m altitude relative to that at sea level. *Answers:* 601 rpm, 9.3 percent

13–37 Water is flowing through a 10-cm-diameter water pipe at a rate of 0.1 m^3/s. Now a diffuser with an outlet diameter of 20 cm is bolted to the pipe in order to slow down water, as shown in Fig. P13–37. Disregarding frictional effects, determine the force exerted on the bolts due to the water flow.

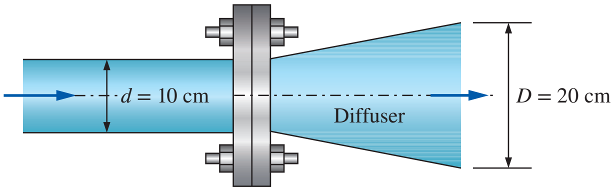

FIGURE P13–37

13–38 Water flowing in a horizontal 25-cm-diameter pipe at 8 m/s and 300 kPa gage enters a 90° bend reducing section, which connects to a 15-cm-diameter vertical pipe. The inlet of the bend is 50 cm above the exit. Neglecting any frictional and gravitational effects, determine the net resultant force exerted on the reducer by the water. Take the momentum-flux correction factor to be 1.04.

13–39 A horizontal 4-cm-diameter water jet with a velocity of 18 m/s impinges normally upon a vertical plate of mass

750 kg. The plate rides on a nearly frictionless track and is initially stationary. When the jet strikes the plate, the plate begins to move in the direction of the jet. The water always splatters in the plane of the retreating plate. Determine (*a*) the acceleration of the plate when the jet first strikes it (time = 0), (*b*) the time it takes for the plate to reach a velocity of 9 m/s, and (*c*) the plate velocity 20 s after the jet first strikes the plate. For simplicity, assume the velocity of the jet is increased as the cart moves such that the impulse force exerted by the water jet on the plate remains constant.

13–40 Water enters a centrifugal pump axially at atmospheric pressure at a rate of 0.09 m^3/s and at a velocity of 5 m/s and leaves in the normal direction along the pump casing, as shown in Fig. P13–40. Determine the force acting on the shaft (which is also the force acting on the bearing of the shaft) in the axial direction.

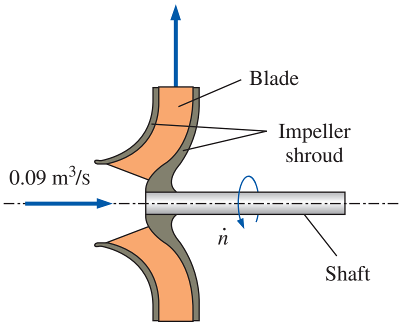

FIGURE P13–40

13–41 An incompressible fluid of density ρ and viscosity μ flows through a curved duct that turns the flow 180°. The duct cross-sectional area remains constant. The average velocity, momentum-flux correction factor, and gage pressure are known at the inlet (1) and outlet (2), as in Fig. P13–41. (*a*) Write an expression for the horizontal force F_x of the fluid on the walls of the duct in terms of the given variables. (*b*) Verify your expression by plugging in the following values: $\rho = 998.2\ \text{kg/m}^3$, $\mu = 1.003 \times 10^{-3}$ kg/m·s, $A_1 = A_2 = 0.025\ \text{m}^2$, $\beta_1 = 1.01$, $\beta_2 = 1.03$, $V_1 = 10$ m/s, $P_{1,\text{gage}} = 78.47$ kPa, and $P_{2,\text{gage}} = 65.23$ kPa. *Answer:* (*b*) $F_x = 8680$ N to the right

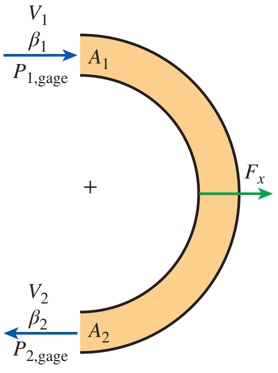

FIGURE P13–41

13–42 Consider the curved duct of Prob. 13–41, except allow the cross-sectional area to vary along the duct ($A_1 \neq A_2$). (*a*) Write an expression for the horizontal force F_x of the fluid on the walls of the duct in terms of the given variables. (*b*) Verify your expression by plugging in the following values: $\rho = 998.2\ \text{kg/m}^3$, $A_1 = 0.025\ \text{m}^2$, $A_2 = 0.015\ \text{m}^2$, $\beta_1 = 1.02$, $\beta_2 = 1.04$, $V_1 = 20$ m/s, $P_{1,\text{gage}} = 88.34$ kPa, and $P_{2,\text{gage}} = 67.48$ kPa. *Answer:* (*b*) $F_x = 30{,}700$ N to the right

13–43 As a follow-up to Prob. 13–41, it turns out that for a large enough area ratio A_2/A_1, the inlet pressure is actually *smaller* than the outlet pressure! Explain how this can be true in light of the fact that there is friction and there are other irreversibilities due to turbulence, and pressure must be lost along the axis of the duct to overcome these irreversibilities.

13–44 Water of density $\rho = 998.2\ \text{kg/m}^3$ flows through a fireman's nozzle—a converging section of pipe that accelerates the flow. The inlet diameter is $d_1 = 0.100$ m, and the outlet diameter is $d_2 = 0.050$ m. The average velocity, momentum-flux correction factor, and gage pressure are known at the inlet (1) and outlet (2), as in Fig. P13–44. (*a*) Write an expression for the horizontal force F_x of the fluid on the walls of the nozzle in terms of the given variables. (*b*) Verify your expression by plugging in the following values: $\beta_1 = 1.03$, $\beta_2 = 1.02$, $V_1 = 3$ m/s, $P_{1,\text{gage}} = 137{,}000$ Pa, and $P_{2,\text{gage}} = 0$ Pa. *Answer:* (*b*) $F_x = 861$ N to the right

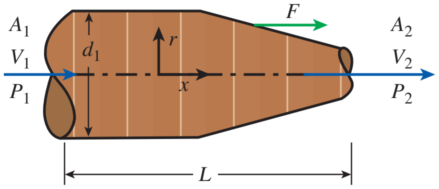

FIGURE P13–44

13–45 A sluice gate, which controls flow rate in a channel by simply raising or lowering a vertical plate, is commonly used in irrigation systems. A force is exerted on the gate due to the difference between the water heights y_1 and y_2 and the flow velocities V_1 and V_2 upstream and downstream from the gate, respectively. Take the width of the sluice gate (into the page) to be w. Wall shear stresses along the channel walls may be ignored, and for simplicity, we assume steady, uniform flow at locations 1 and 2. Develop a relationship for the force F_R acting on the sluice gate as a function of depths y_1 and y_2, mass flow rate $\dot{m}$, gravitational constant g, gate width w, and water density ρ.

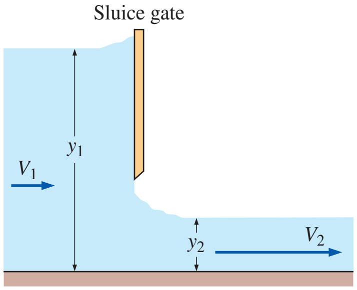

FIGURE P13–45

Review Problems

13–46 An 8-cm-diameter horizontal water jet having a velocity of 35 m/s strikes a vertical stationary flat plate. The water splatters in all directions in the plane of the plate. How much force is required to hold the plate against the water stream? *Answer:* 6110 N

13–47 Water flowing steadily at a rate of 0.16 m^3/s is deflected downward by an angled elbow as shown in Fig. P13–47. For $D = 30$ cm, $d = 10$ cm, and $h = 50$ cm, determine the force acting on the flanges of the elbow and the angle its line of action makes with the horizontal. Take the internal volume of the elbow to be 0.03 m^3 and disregard the weight of the elbow material and the frictional effects.

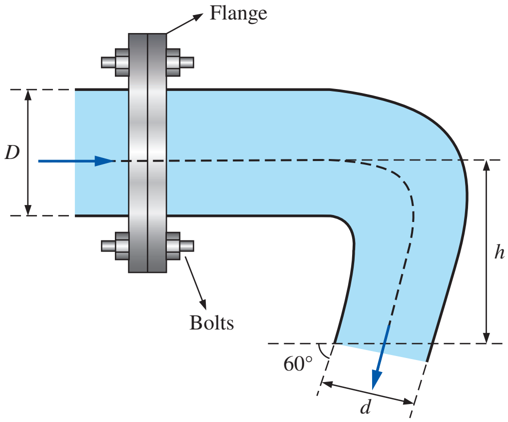

FIGURE P13–47

13–48 Repeat Prob. 13–47 by taking into consideration the weight of the elbow whose mass is 5 kg.

13–49 A 16-cm-diameter horizontal water jet with a speed of $V_j = 20$ m/s relative to the ground is deflected by a 40° cone moving to the left at $V_c = 10$ m/s. Determine the external force, F, needed to maintain the motion of the cone. Disregard the gravity and surface shear effects, and assume the cross-sectional area of water jet that is normal to the direction of motion remains constant throughout the flow. *Answer:* 4230 N

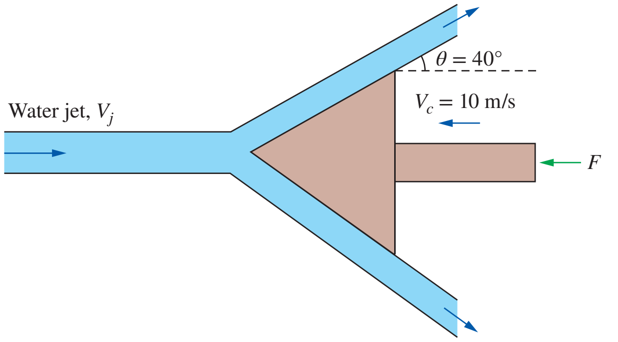

FIGURE P13–49

13–50 A tripod holding a nozzle, which directs a 5-cm-diameter stream of water from a hose, is shown in Fig. P13–50. The nozzle mass is 10 kg when filled with water. The tripod is rated to provide 1800 N of holding force. A firefighter was standing 60 cm behind the nozzle and was hit by the nozzle when the tripod suddenly failed and released the nozzle. You have been hired as an accident reconstructionist and, after testing the tripod, have determined that as water flow rate increased, it did collapse at 1800 N. In your final report you must state the water velocity and the flow rate consistent with the failure and the nozzle velocity when it hit the firefighter. For simplicity, ignore pressure and momentum effects in the upstream portion of the hose. *Answers:* 30.3 m/s, 0.0595 m^3/s, 14.7 m/s

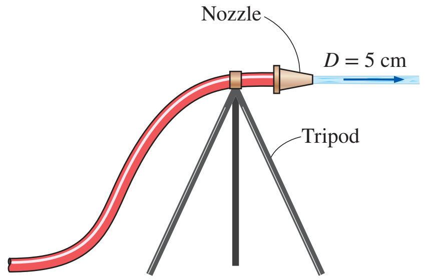

FIGURE P13–50

13–51 Consider an airplane with a jet engine attached to the tail section that expels combustion gases at a rate of 18 kg/s with a velocity of $V = 300$ m/s relative to the plane. During landing, a thrust reverser (which serves as a brake for the aircraft and facilitates landing on a short runway) is lowered in the path of the exhaust jet, which deflects the exhaust from rearward to 120°. Determine (*a*) the thrust (forward force) that the engine produces prior to the insertion of the thrust reverser and (*b*) the braking force produced after the thrust reverser is deployed.

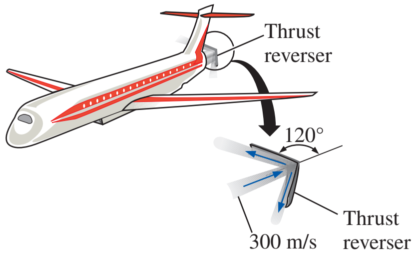

FIGURE P13–51

13–52 Reconsider Prob. 13–51. Using appropriate software, investigate the effect of thrust reverser angle on the braking force exerted on the airplane. Let the reverser angle vary from 0° (no reversing) to 180° (full reversing) in increments of 10°. Tabulate and plot your results and draw conclusions.

13–53E A spacecraft cruising in space at a constant velocity of 2000 ft/s has a mass of 25,000 lbm. To slow down the spacecraft, a solid fuel rocket is fired, and the combustion gases leave the rocket at a constant rate of 150 lbm/s at a velocity of 5000 ft/s in the same direction as the spacecraft for a period of 5 s. Assuming the mass of the spacecraft remains constant, determine (*a*) the deceleration of the spacecraft during this 5-s period, (*b*) the change of velocity of the spacecraft during this time period, and (*c*) the thrust exerted on the spacecraft.

13–54 A 60-kg ice skater is standing on ice with ice skates (negligible friction). She is holding a flexible hose (essentially weightless) that directs a 2-cm-diameter stream of water horizontally parallel to her skates. The water velocity at the hose outlet is 10 m/s relative to the skater. If she is initially standing still, determine (*a*) the velocity of the skater and the distance she travels in 5 s and (*b*) how long it will take to move 5 m and the velocity at that moment. *Answers:* (*a*) 2.62 m/s, 6.54 m, (*b*) 4.4 s, 2.3 m/s

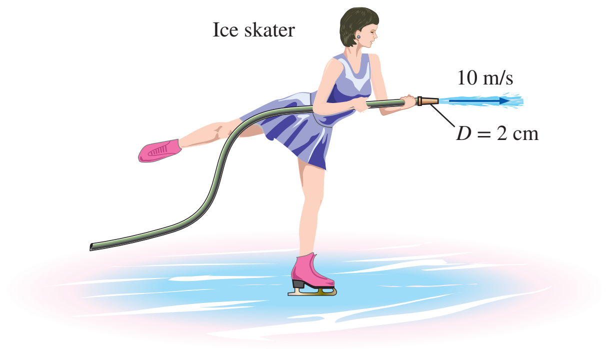

FIGURE P13–54

13–55 A 5-cm-diameter horizontal jet of water, with velocity 30 m/s, strikes the tip of a horizontal cone, which deflects the water by 60° from its original direction. How much force is required to hold the cone against the water stream?

13–56 Water is flowing into and discharging from a pipe U-section as shown in Fig. P13–56. At flange (1), the total absolute pressure is 200 kPa, and 55 kg/s flows into the pipe. At flange (2), the total pressure is 150 kPa. At location (3), 15 kg/s of water discharges to the atmosphere, which is at 100 kPa. Determine the total *x*- and *z*-forces at the two flanges connecting the pipe. Discuss the significance of gravity force for this problem. Take the momentum-flux correction factor to be 1.03 throughout the pipes.

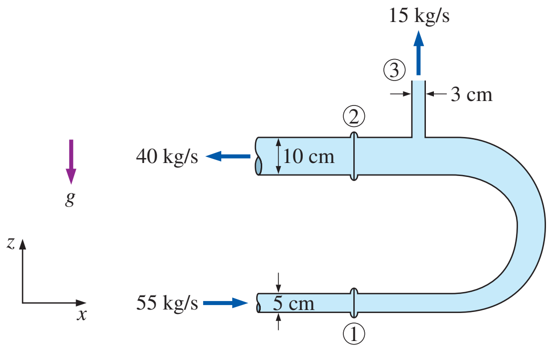

FIGURE P13–56

13–57 Indiana Jones needs to ascend a 10-m-high building. There is a large hose filled with pressurized water hanging down from the building top. He builds a square platform and mounts four 4-cm-diameter nozzles pointing down at each corner. By connecting hose branches, a water jet with 15-m/s velocity can be produced from each nozzle. Jones, the platform, and the nozzles have a combined mass of 150 kg. Determine (*a*) the minimum water jet velocity needed to raise the system, (*b*) how long it takes for the system to rise 10 m when the water jet velocity is 18 m/s and the velocity of the platform at that moment, and (*c*) how much higher the momentum will raise Jones if he shuts off the water at the moment the platform reaches 10 m above the ground. How much time does he have to jump from the platform to the roof? *Answers:* (*a*) 17.1 m/s, (*b*) 4.37 s, 4.57 m/s, (*c*) 1.07 m, 0.933 s

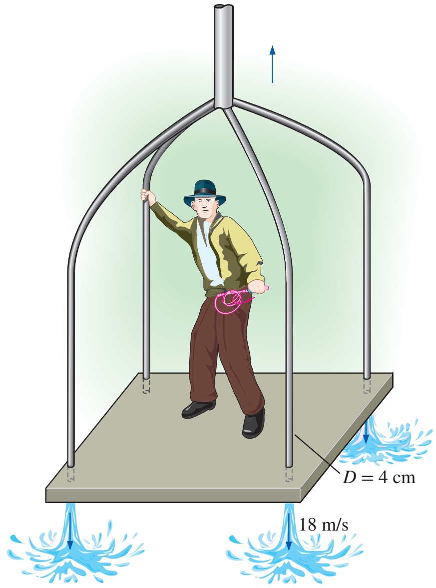

FIGURE P13–57

13–58E An engineering student considers using a fan as a levitation demonstration. She plans to face the box-enclosed fan so the air blast is directed facedown through a 2-ft-diameter blade span area. The system weighs 3 lbf, and the student will secure the system from rotating. By increasing the power to the fan, she plans to increase the blade rpm and air exit velocity until the exhaust provides sufficient upward force to cause the box fan to hover in the air. Determine (*a*) the air exit velocity to produce 3 lbf, (*b*) the volumetric flow rate needed, and (*c*) the minimum mechanical power that must be supplied to the airstream. Take the air density to be 0.078 lbm/ft^3.

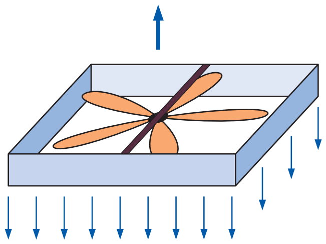

FIGURE P13–58E

13–59 A 7-cm-diameter vertical water jet is injected upwards by a nozzle at a speed of 15 m/s. Determine the maximum weight of a flat plate that can be supported by this water jet at a height of 2 m from the nozzle.

13–60 Repeat Prob. 13–59 for a height of 8 m from the nozzle.

13–61 Consider steady developing laminar flow of water in a constant-diameter horizontal discharge pipe attached to a tank. The fluid enters the pipe with nearly uniform velocity V and pressure P_1. The velocity profile becomes parabolic after a certain distance with a momentum correction factor of 2 while the pressure drops to P_2. Obtain a relation for the horizontal force acting on the bolts that hold the pipe attached to the tank.

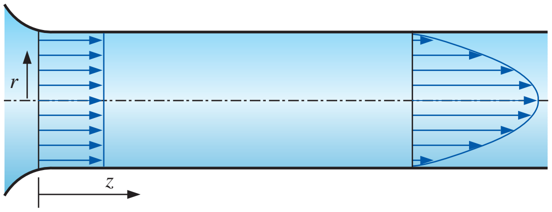

FIGURE P13–61

13–62 A soldier jumps from a plane and opens his parachute when his velocity reaches the terminal velocity V_T. The parachute slows him down to his landing velocity of V_F. After the parachute is deployed, the air resistance is proportional to the velocity squared (i.e., $F = kV^2$). The soldier, his parachute, and his gear have a total mass of m. Show that $k = mg/V_F^2$ and develop a relation for the soldier's velocity after he opens the parachute at time $t = 0$.

Answer: $V = V_F \dfrac{V_T + V_F + (V_T - V_F)e^{-2gt/V_F}}{V_T + V_F + (V_T - V_F)e^{-2gt/V_F}}$

FIGURE P13–62

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13–63 Nearly frictionless vertical guide rails maintain a plate of mass m_p in a horizontal position, such that it can slide freely in the vertical direction. A nozzle directs a water stream of area A against the plate's underside. The water jet splatters in the plate plane, applying an upward force against the plate. The water flow rate $\dot{m}$ (kg/s) can be controlled. Assume that distances are short, so the velocity of the rising jet can be considered constant with height. (*a*) Determine the minimum mass flow rate $\dot{m}_{\min}$ necessary to just levitate the plate, and obtain a relation for the steady-state velocity of the upward moving plate for $\dot{m} > \dot{m}_{\min}$. (*b*) At time $t = 0$, the plate is at rest, and the water jet with $\dot{m} > \dot{m}_{\min}$ is suddenly turned on. Apply a force balance to the plate and obtain the integral that relates velocity to time (do not solve).

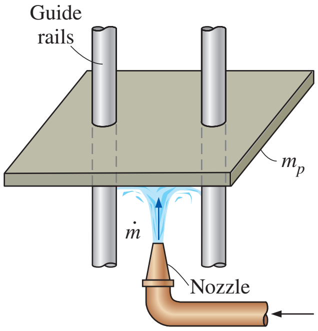


FIGURE P13–63

13–64 Water shoots out of a large tank sitting on a cart with frictionless wheels. The water jet velocity is $V_j = 7.00$ m/s, its cross-sectional area is $A_j = 20.0\ \text{mm}^2$, and the momentum-flux correction factor of the jet is 1.04. The water is deflected 135° as shown ($\theta = 45°$), and all of the water flows back into the tank. The density of the water is $1000\ \text{kg/m}^3$. Calculate the horizontal force F (in units of N) required to hold the cart in place.

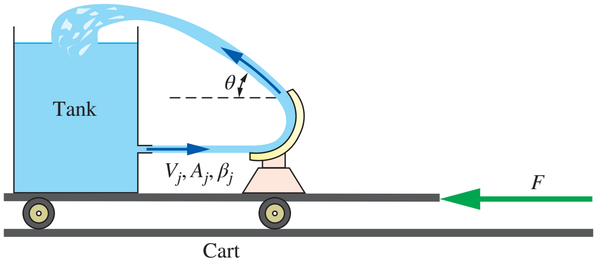


FIGURE P13–64

Design and Essay Problem

13–65 Visit a fire station and obtain information about flow rates through hoses and discharge diameters. Using this information, calculate the impulse force to which the firefighters are subjected when holding a fire hose.

CHAPTER

14

INTERNAL FLOW

Fluid flow is classified as *external* or *internal,* depending on whether the fluid is forced to flow over a surface or in a conduit. Internal and external flows exhibit very different characteristics. In this chapter we consider *internal flow* where the conduit is completely filled with the fluid, and the flow is driven primarily by a pressure difference. This should not be confused with *open-channel flow,* where the conduit is partially filled by the fluid and thus the flow is partially bounded by solid surfaces, as in an irrigation ditch, and the flow is driven by gravity alone.

We start this chapter with a general physical description of internal flow through pipes and ducts, including the *entrance region* and the *fully developed* region. We continue with a discussion of the dimensionless *Reynolds number* and its physical significance. We then introduce the *pressure drop* correlations associated with pipe flow for both laminar and turbulent flows. Finally, we discuss minor losses and determine the pressure drop and pumping power requirements for real-world piping systems.

OBJECTIVES

The objectives of this chapter are to:

- Have a deeper understanding of laminar and turbulent flow in pipes and the analysis of fully developed flow.
- Calculate the major and minor losses associated with pipe flow in piping networks and determine the pumping power requirements.

14–1 ▪ INTRODUCTION

Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications and fluid distribution networks. The fluid in such applications is usually forced to flow by a fan or pump through a flow section. We pay particular attention to *friction,* which is directly related to the *pressure drop* and *head loss* during flow through pipes and ducts. The pressure drop is then used to determine the pumping power requirement. A typical piping system involves pipes of different diameters connected to each other by various fittings or elbows to route the fluid, valves to control the flow rate, and pumps to pressurize the fluid.

The terms *pipe, duct,* and *conduit* are usually used interchangeably for flow sections. In general, flow sections of circular cross section are referred to as *pipes* (especially when the fluid is a liquid), and flow sections of noncircular cross section as *ducts* (especially when the fluid is a gas). Small-diameter pipes are usually referred to as *tubes.* Given this uncertainty, we will use more descriptive phrases (such as *a circular pipe* or *a rectangular duct*) whenever necessary to avoid any misunderstandings.

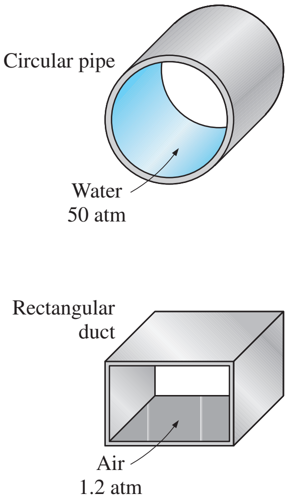


FIGURE 14–1
Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot.

You have probably noticed that most fluids, especially liquids, are transported in *circular pipes.* This is because pipes with a circular cross section can withstand large pressure differences between the inside and the outside without undergoing significant distortion. *Noncircular pipes* are usually used in applications such as the heating and cooling systems of buildings where the pressure difference is relatively small, the manufacturing and installation costs are lower, and the available space is limited for ductwork (Fig. 14–1).

Although the theory of fluid flow is reasonably well understood, theoretical solutions are obtained only for a few simple cases such as fully developed laminar flow in a circular pipe. Therefore, we must rely on experimental results and empirical relations for most fluid flow problems rather than closed-form analytical solutions. Noting that the experimental results are obtained under carefully controlled laboratory conditions and that no two systems are exactly alike, we must not be so naive as to view the results obtained as "exact." An error of 10 percent (or more) in friction factors calculated using the relations in this chapter is the "norm" rather than the "exception."

The fluid velocity in a pipe changes from *zero* at the wall because of the no-slip condition to a maximum at the pipe center. In fluid flow, it is convenient to work with an *average* velocity V_{avg}, which remains constant in incompressible flow when the cross-sectional area of the pipe is constant (Fig. 14–2). The average velocity in heating and cooling applications may change somewhat because of changes in density with temperature. But, in practice, we evaluate the fluid properties at some average temperature and treat them as constants. The convenience of working with constant properties usually more than justifies the slight loss in accuracy.

Also, the friction between the fluid particles in a pipe does cause a slight rise in fluid temperature as a result of the mechanical energy being converted to sensible thermal energy. But this temperature rise due to *frictional heating* is usually too small to warrant any consideration in calculations and thus is disregarded. For example, in the absence of any heat transfer, no noticeable difference can be detected between the inlet and outlet temperatures of water flowing in a pipe. The primary consequence of friction in fluid flow is pressure drop, and thus any significant temperature change in the fluid is due to heat transfer.

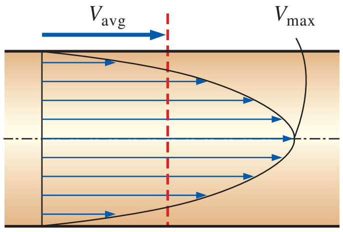


FIGURE 14–2
Average velocity V_{avg} is defined as the average speed through a cross section. For fully developed laminar pipe flow, V_{avg} is half of the maximum velocity.

The value of the average velocity V_{avg} at some streamwise cross section is determined from the requirement that the *conservation of mass* principle be satisfied (Fig. 14–2). That is,

$$\dot{m} = \rho V_{avg} A_c = \int_{A_c} \rho u(r)\, dA_c \tag{14–1}$$

where $\dot{m}$ is the mass flow rate, ρ is the density, A_c is the cross-sectional area, and $u(r)$ is the velocity profile. Then the average velocity for incompressible flow in a circular pipe of radius R is expressed as

$$V_{\text{avg}} = \frac{\int_{A_c} \rho u(r)\, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2}\int_0^R u(r) r\, dr \tag{14–2}$$

Therefore, when we know the flow rate or the velocity profile, the average velocity can be determined easily.

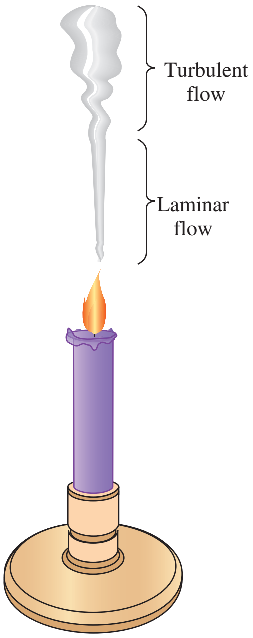


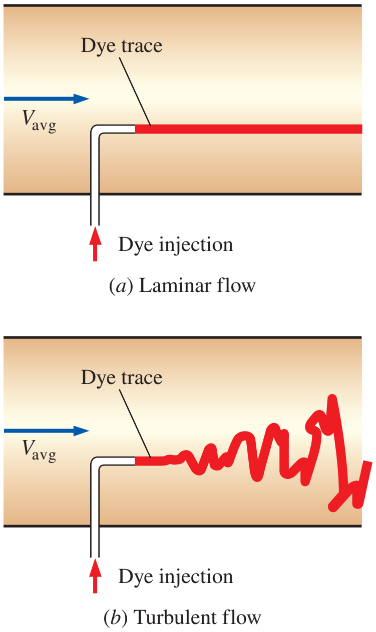
FIGURE 14–3
Laminar and turbulent flow regimes of a candle smoke plume.

14–2 ▪ LAMINAR AND TURBULENT FLOWS

If you have been around smokers, you probably noticed that the cigarette smoke rises in a smooth plume for the first few centimeters and then starts fluctuating randomly in all directions as it continues its rise. Other plumes behave similarly (Fig. 14–3). Likewise, a careful inspection of flow in a pipe reveals that the fluid flow is streamlined at low velocities but turns chaotic as the velocity is increased above a critical value, as shown in Fig. 14–4. The flow regime in the first case is said to be **laminar**, characterized by *smooth streamlines* and *highly ordered motion,* and **turbulent** in the second case, where it is characterized by *velocity fluctuations* and *highly disordered motion.* The **transition** from laminar to turbulent flow does not occur suddenly; rather, it occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent. Most flows encountered in practice are turbulent. Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.

We can verify the existence of these laminar, transitional, and turbulent flow regimes by injecting some dye streaks into the flow in a glass pipe, as the British engineer Osborne Reynolds (1842–1912) did over a century ago. We observe that the dye streak forms a *straight and smooth line* at low velocities when the flow is laminar (we may see some blurring because of molecular diffusion), has *bursts of fluctuations* in the transitional regime, and *zigzags rapidly and disorderly* when the flow becomes fully turbulent. These zigzags and the dispersion of the dye are indicative of the fluctuations in the main flow and the rapid mixing of fluid particles from adjacent layers.

The *intense mixing* of the fluid in turbulent flow as a result of rapid fluctuations enhances momentum transfer between fluid particles, which increases the friction force on the pipe wall and thus the required pumping power. The friction factor reaches a maximum when the flow becomes fully turbulent.



(*a*) Laminar flow
(*b*) Turbulent flow

FIGURE 14–4
The behavior of colored fluid injected into the flow in (*a*) laminar and (*b*) turbulent flow in a pipe.

Reynolds Number

The transition from laminar to turbulent flow depends on the *geometry, surface roughness, flow velocity, surface temperature,* and *type of fluid,* among other things. After exhaustive experiments in the 1880s, Osborne Reynolds discovered that the flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* in the fluid (Fig. 14–5). This ratio is called the **Reynolds number** and is expressed for internal flow in a circular pipe as

$$\text{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{\text{avg}} D}{\nu} = \frac{\rho V_{\text{avg}} D}{\mu} \tag{14–3}$$

where V_{avg} = average flow velocity (m/s), D = characteristic length of the geometry (diameter in this case, in m), and $\nu = \mu/\rho$ = kinematic viscosity of the fluid (m^2/s). Note that the Reynolds number is a *dimensionless* quantity. Also, kinematic viscosity has units m^2/s, and can be viewed as *viscous diffusivity* or *diffusivity for momentum.*

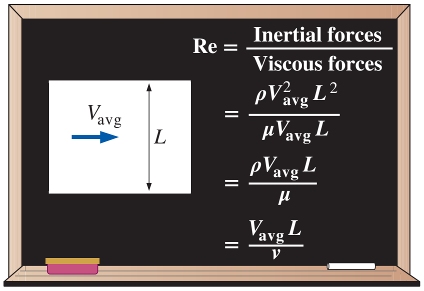


FIGURE 14–5
The Reynolds number can be viewed as the ratio of inertial forces to viscous forces acting on a fluid element.

At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid. At *small* or *moderate* Reynolds numbers, however, the viscous forces are large enough to suppress these fluctuations and to keep the fluid "in line." Thus the flow is *turbulent* in the first case and *laminar* in the second.

The Reynolds number at which the flow becomes turbulent is called the **critical Reynolds number**, Re_{cr}. The value of the critical Reynolds number is different for different geometries and flow conditions. For internal flow in a circular pipe, the generally accepted value of the critical Reynolds number is $\text{Re}_{cr} = 2300$.

For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter** D_h defined as (Fig. 14–6)

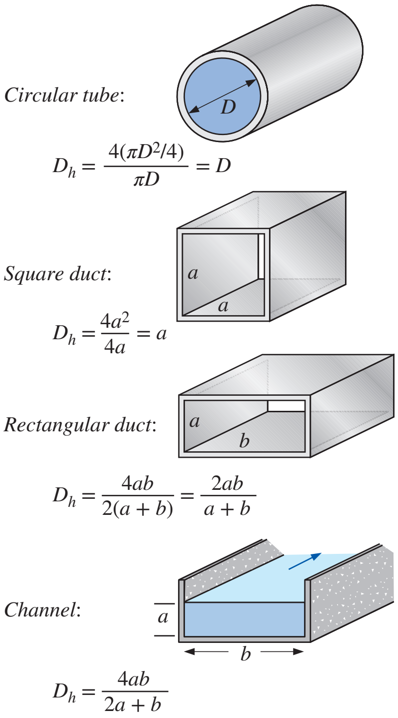


FIGURE 14–6
The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes. When there is a free surface, such as in open-channel flow, the wetted perimeter includes only the walls in contact with the fluid.

Hydraulic diameter:
$$D_h = \frac{4A_c}{p} \tag{14–4}$$

where A_c is the cross-sectional area of the pipe and p is its wetted perimeter. The hydraulic diameter is defined such that it reduces to ordinary diameter D for circular pipes,

Circular pipes:
$$D_h = \frac{4A_c}{p} = \frac{4(\pi D^2/4)}{\pi D} = D$$

It certainly is desirable to have precise values of Reynolds numbers for laminar, transitional, and turbulent flows, but this is not the case in practice. It turns out that the transition from laminar to turbulent flow also depends on the degree of disturbance of the flow by *surface roughness, pipe vibrations,* and *fluctuations in the upstream flow.* Under most practical conditions, the flow in a circular pipe is laminar for $\text{Re} \lesssim 2300$, turbulent for $\text{Re} \gtrsim 4000$, and transitional in between. That is,

$$\begin{aligned} \text{Re} &\lesssim 2300 && \text{laminar flow} \\ 2300 \lesssim \text{Re} &\lesssim 4000 && \text{transitional flow} \\ \text{Re} &\gtrsim 4000 && \text{turbulent flow} \end{aligned}$$

In transitional flow, the flow switches between laminar and turbulent in a disorderly fashion (Fig. 14–7). It should be kept in mind that laminar flow can be maintained at much higher Reynolds numbers in very smooth pipes by avoiding flow disturbances and pipe vibrations. In such carefully controlled laboratory experiments, laminar flow has been maintained at Reynolds numbers of up to 100,000.

14–3 ▪ THE ENTRANCE REGION

Consider a fluid entering a circular pipe at a uniform velocity. Because of the no-slip condition, the fluid particles in the layer in contact with the wall of the pipe come to a complete stop. This layer also causes the fluid particles in the adjacent layers to slow down gradually as a result of friction. To make up for this velocity reduction, the velocity of the fluid at the midsection of the pipe has to increase to keep the mass flow rate through the pipe constant. As a result, a velocity gradient develops along the pipe.

The region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the **velocity boundary layer** or just the **boundary layer**. The hypothetical boundary surface divides the flow in a pipe into two regions: the **boundary layer region**, in which the viscous effects and the velocity changes are significant, and the **irrotational (core) flow region**, in which the frictional effects are negligible and the velocity remains essentially constant in the radial direction.

The thickness of this boundary layer increases in the flow direction until the boundary layer reaches the pipe center and thus fills the entire pipe, as shown in Fig. 14–8, and the velocity becomes fully developed a little farther downstream. The region from the pipe inlet to the point at which the velocity profile is fully developed is called the **hydrodynamic entrance region**, and the length of this region is called the **hydrodynamic entry length** L_h. Flow in the entrance region is called *hydrodynamically developing flow* since this is the region where the velocity profile develops. The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged is called the **hydrodynamically fully developed region**. The flow is said to be **fully developed** when the normalized temperature profile remains unchanged as well. Hydrodynamically fully developed flow is equivalent to fully developed flow when the fluid in the pipe is not heated or cooled since the fluid temperature in this case remains essentially constant throughout. The velocity profile in the fully developed region is *parabolic* in laminar flow and much *flatter* (or *fuller*) in turbulent flow due to eddy motion and more vigorous mixing in the radial direction. The time-averaged velocity profile remains unchanged when the flow is fully developed, and thus

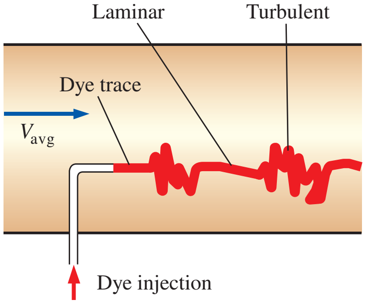


FIGURE 14–7
In the transitional flow region of 2300 ≤ Re ≤ 4000, the flow switches between laminar and turbulent somewhat randomly.

Hydrodynamically fully developed:
$$\frac{\partial u(r,x)}{\partial x} = 0 \quad \rightarrow \quad u = u(r) \tag{14–5}$$

The shear stress at the pipe wall τ_w is related to the slope of the velocity profile at the surface. Noting that the velocity profile remains unchanged in the hydrodynamically fully developed region, the wall shear stress also remains constant in that region (Fig. 14–9).

Consider fluid flow in the hydrodynamic entrance region of a pipe. The wall shear stress is the *highest* at the pipe inlet where the thickness of the boundary layer is smallest, and decreases gradually to the fully developed value, as shown in Fig. 14–10. Therefore, the pressure drop is *higher* in the entrance regions of a pipe, and the effect of the entrance region is always to *increase* the average friction factor for the entire pipe. This increase may be significant for short pipes but is negligible for long ones.

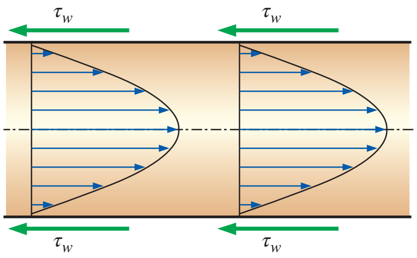


FIGURE 14–9
In the fully developed flow region of a pipe, the velocity profile does not change downstream, and thus the wall shear stress remains constant as well.

Entry Lengths

The hydrodynamic entry length is usually taken to be the distance from the pipe entrance to where the wall shear stress (and thus the friction factor) reaches within about 2 percent of the fully developed value. In *laminar flow,* the nondimensional hydrodynamic entry length is given approximately as [see Kays, Crawford, and Weigand (2004) and Bhatti and Shah (1987)]

$$\frac{L_{h,\text{ laminar}}}{D} \cong 0.05\text{Re} \tag{14–6}$$

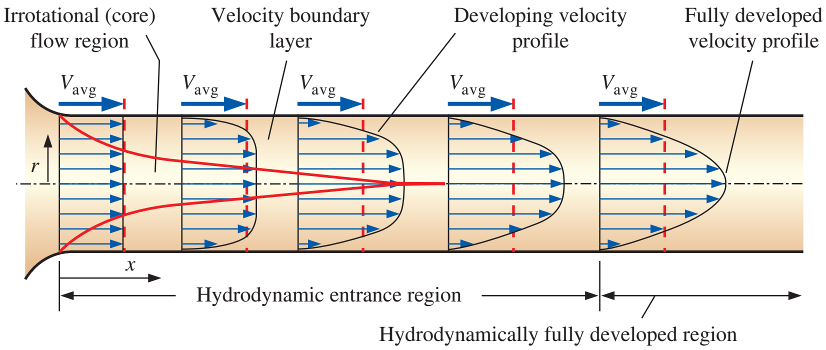


FIGURE 14–8
The development of the velocity boundary layer in a pipe. (The developed average velocity profile is parabolic in laminar flow, as shown, but much flatter or fuller in turbulent flow.)

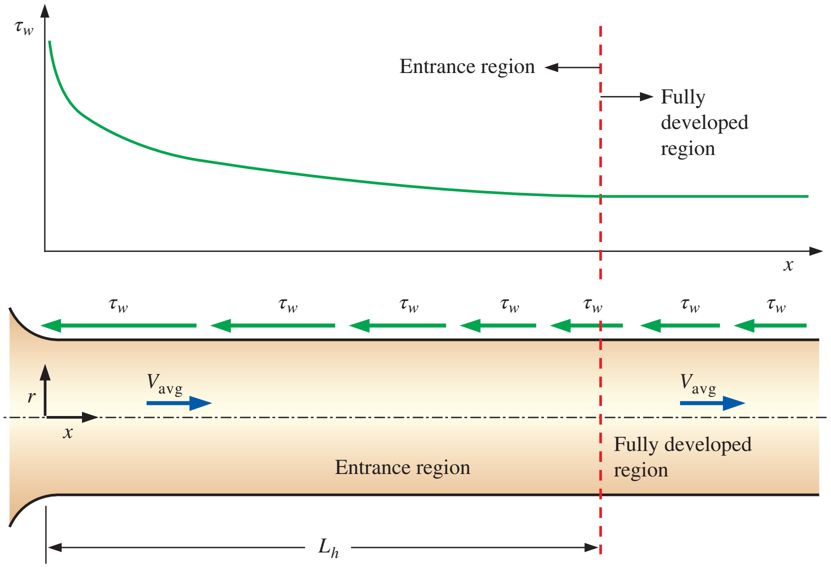


FIGURE 14–10
The variation of wall shear stress in the flow direction for flow in a pipe from the entrance region into the fully developed region.

For Re = 20, the hydrodynamic entry length is about the size of the diameter but increases linearly with velocity. In the limiting laminar case of Re = 2300, the hydrodynamic entry length is 115*D*.

In *turbulent flow,* the intense mixing during random fluctuations usually overshadows the effects of molecular diffusion. The nondimensional hydrodynamic entry length for turbulent flow is approximated as [see Bhatti and Shah (1987) and Zhi-qing (1982)]

$$\frac{L_{h,\text{ turbulent}}}{D} = 1.359\,\text{Re}^{1/4} \tag{14–7}$$

Compared to high Reynolds number laminar flow, the entry length is much shorter in turbulent flow, as expected, and its dependence on the Reynolds number is weaker. In many pipe flows of practical engineering interest, the entrance effects become insignificant beyond a pipe length of about 10 diameters, and the nondimensional hydrodynamic entry length is approximated as

$$\frac{L_{h,\text{ turbulent}}}{D} \approx 10 \tag{14–8}$$

Precise correlations for calculating the frictional head losses in entrance regions are available in the literature. However, the pipes used in practice are usually several times the length of the entrance region, and thus the flow through the pipes is often assumed to be fully developed for the entire length of the pipe. This simplistic approach gives *reasonable* results for long pipes but sometimes poor results for short ones since it underpredicts the wall shear stress and thus the friction factor.

14–4 ▪ LAMINAR FLOW IN PIPES

We mentioned in Section 14–2 that flow in pipes is laminar for Re $\lesssim$ 2300, and that the flow is fully developed if the pipe is sufficiently long (relative to the entry length) so that the entrance effects are negligible. In this section, we consider the steady, laminar, incompressible flow of fluid with constant properties in the fully developed region of a straight circular pipe. We obtain the momentum equation by applying a momentum balance to a differential volume element, and we obtain the velocity profile by solving it. Then we use it to obtain a relation for the friction factor. An important aspect of the analysis here is that it is one of the few available for viscous flow.

In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline, and the velocity profile $u(r)$ remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component in the direction normal to the pipe axis is everywhere zero. There is no acceleration since the flow is steady and fully developed.

Now consider a ring-shaped differential volume element of radius r, thickness dr, and length dx oriented coaxially with the pipe, as shown in Fig. 14–11. The volume element involves only pressure and viscous effects, and thus the pressure and shear forces must balance each other. The pressure force acting on a submerged plane surface is the product of the pressure at the centroid of the surface and the surface area. A force balance on the volume element in the flow direction gives

$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} = 0 \qquad \textbf{(14–9)}$$

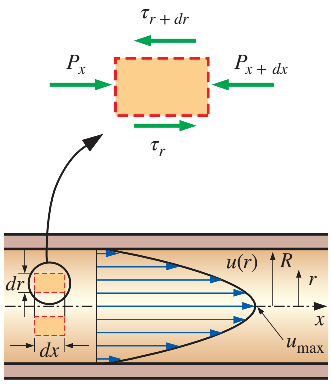


FIGURE 14–11
Free-body diagram of a ring-shaped differential fluid element of radius r, thickness dr, and length dx oriented coaxially with a horizontal pipe in fully developed laminar flow. (The size of the fluid element is greatly exaggerated for clarity.)

which indicates that in fully developed flow in a horizontal pipe, the viscous and pressure forces balance each other. Dividing by $2\pi\, dr\, dx$ and rearranging,

$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dx} - (r\tau)_r}{dr} = 0 \qquad \textbf{(14–10)}$$

But we recognize that the two numerators in Eq. 14–10 are dP and $d(r\tau)$, respectively. Thus,

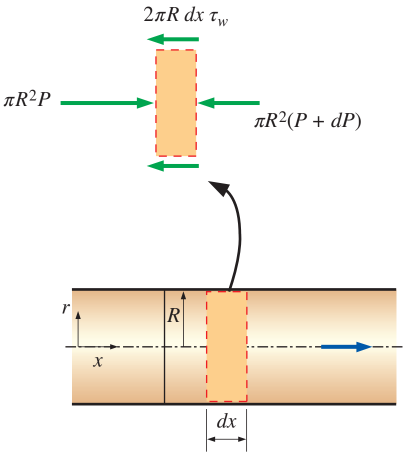
$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0 \qquad \textbf{(14–11)}$$

Substituting $\tau = -\mu(du/dr)$, dividing by r, and taking $\mu =$ constant gives the desired equation,

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} \qquad \textbf{(14–12)}$$

The quantity du/dr is negative in pipe flow, and the negative sign is included to obtain positive values for τ. (Or, $du/dr = -du/dy$ if we define $y = R - r$.) The left side of Eq. 14–12 is a function of r, and the right side is a function of x. The equality must hold for any value of r and x, and an equality of the form $f(r) = g(x)$ can be satisfied only if both $f(r)$ and $g(x)$ are equal to the same constant. Thus we conclude that $dP/dx =$ constant. This is verified by writing a force balance on a volume element of radius R and thickness dx (a slice of the pipe as in Fig. 14–12), which gives

$$\frac{dP}{dx} = -\frac{2\tau_w}{R} \qquad \textbf{(14–13)}$$



Force balance:

$$\pi R^2 P - \pi R^2(P + dP) - 2\pi R\, dx\, \tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

FIGURE 14–12
Free-body diagram of a fluid disk element of radius R and length dx in fully developed laminar flow in a horizontal pipe.

Here τ_w is constant since the viscosity and the velocity profile are constants in the fully developed region. Therefore, $dP/dx =$ constant.

Equation 14–12 is solved by rearranging and integrating it twice to give

$$u(r) = \frac{r^2}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2 \qquad \textbf{(14–14)}$$

The velocity profile $u(r)$ is obtained by applying the boundary conditions $\partial u/\partial r = 0$ at $r = 0$ (because of symmetry about the centerline) and $u = 0$ at $r = R$ (the no-slip condition at the pipe wall),

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) \qquad \textbf{(14–15)}$$

Therefore, the velocity profile in fully developed laminar flow in a pipe is *parabolic* with a maximum at the centerline and a minimum (zero) at the pipe wall. Also, the axial velocity u is positive for any r, and thus the axial pressure gradient dP/dx must be negative (i.e., pressure must decrease in the flow direction because of viscous effects—it takes pressure to push the fluid through the pipe).

The average velocity is determined from its definition by substituting Eq. 14–15 into Eq. 14–2, and performing the integration, yielding

$$V_{\text{avg}} = \frac{2}{R^2}\int_0^R u(r)r\,dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right)r\,dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right) \qquad \textbf{(14–16)}$$

Combining the last two equations, the velocity profile is rewritten as

$$u(r) = 2V_{\text{avg}}\left(1-\frac{r^2}{R^2}\right) \qquad \textbf{(14–17)}$$

This is a convenient form for the velocity profile since V_{avg} can be determined easily from the flow rate information.

The maximum velocity occurs at the centerline and is determined from Eq. 14–17 by substituting $r = 0$,

$$u_{\max} = 2V_{\text{avg}} \qquad \textbf{(14–18)}$$

Therefore, *the average velocity in fully developed laminar pipe flow is one-half of the maximum velocity*.

Pressure Drop and Head Loss

A quantity of interest in the analysis of pipe flow is the *pressure drop* ΔP since it is directly related to the power requirements of the fan or pump to maintain flow. We note that dP/dx = constant, and integrating from $x = x_1$ where the pressure is P_1 to $x = x_1 + L$ where the pressure is P_2 gives

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} \qquad \textbf{(14–19)}$$

Substituting Eq. 14–19 into the V_{avg} expression in Eq. 14–16, the pressure drop is expressed as

Laminar flow:
$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{\text{avg}}}{R^2} = \frac{32\mu L V_{\text{avg}}}{D^2} \qquad \textbf{(14–20)}$$

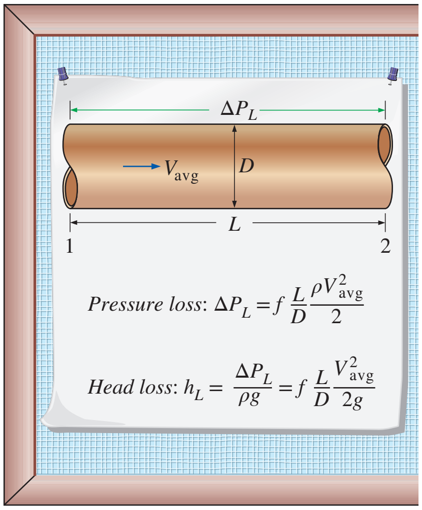


FIGURE 14–13
The relation for pressure loss (and head loss) is one of the most general relations in fluid mechanics, and it is valid for laminar or turbulent flows, circular or noncircular pipes, and pipes with smooth or rough surfaces.

The symbol Δ is typically used to indicate the difference between the final and initial values, like $\Delta y = y_2 - y_1$. But in fluid flow, ΔP is used to designate pressure drop, and thus it is $P_1 - P_2$. A pressure drop due to viscous effects represents an irreversible pressure loss, and it is sometimes called **pressure loss** ΔP_L to emphasize that it is a *loss* (just like the head loss h_L, which as we shall see is proportional to ΔP_L).

Note from Eq. 14–20 that the pressure drop is proportional to the viscosity μ of the fluid, and ΔP would be zero if there were no friction. Therefore, the drop of pressure from P_1 to P_2 in this case is due entirely to viscous effects, and Eq. 14–20 represents the pressure loss ΔP_L when a fluid of viscosity μ flows through a pipe of constant diameter D and length L at average velocity V_{avg}.

In practice, it is convenient to express the pressure loss for all types of fully developed internal flows (laminar or turbulent flows, circular or noncircular pipes, smooth or rough surfaces, horizontal or inclined pipes) as (Fig. 14–13)

Pressure loss:
$$\Delta P_L = f\,\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} \qquad \textbf{(14–21)}$$

where $\rho V_{\text{avg}}^2/2$ is the *dynamic pressure* and f is the **Darcy friction factor**,

$$f = \frac{8\tau_w}{\rho V_{\text{avg}}^2} \tag{14–22}$$

It is also called the **Darcy–Weisbach friction factor**, named after the Frenchman Henry Darcy (1803–1858) and the German Julius Weisbach (1806–1871), the two engineers who provided the greatest contribution to its development. It should not be confused with the *friction coefficient* C_f [also called the *Fanning friction factor,* named after the American engineer John Fanning (1837–1911)], which is defined as $C_f = 2\tau_w/(\rho V_{\text{avg}}^2) = f/4$.

Setting Eqs. 14–20 and 14–21 equal to each other and solving for f gives the friction factor for fully developed laminar flow in a circular pipe,

Circular pipe, laminar:
$$f = \frac{64\mu}{\rho D V_{\text{avg}}} = \frac{64}{\text{Re}} \tag{14–23}$$

This equation shows that *in laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface (assuming, of course, that the roughness is not extreme)*.

In the analysis of piping systems, pressure losses are commonly expressed in terms of the *equivalent fluid column height,* called the **head loss** h_L. Noting from fluid statics that $\Delta P = \rho g h$ and thus a pressure difference of ΔP corresponds to a fluid height of $h = \Delta P/\rho g$, the *pipe head loss* is obtained by dividing ΔP_L by ρg to give

Head loss:
$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{\text{avg}}^2}{2g} \tag{14–24}$$

The head loss h_L represents *the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe*. The head loss is caused by viscosity, and it is directly related to the wall shear stress. Equations 14–21 and 14–24 are valid for both laminar and turbulent flows in both circular and noncircular pipes, but Eq. 14–23 is valid only for fully developed laminar flow in circular pipes.

Once the pressure loss (or head loss) is known, the required pumping power *to overcome the pressure loss* is determined from

$$\dot{W}_{\text{pump}, L} = \dot{V}\,\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L \tag{14–25}$$

where $\dot{V}$ is the volume flow rate and $\dot{m}$ is the mass flow rate.

The average velocity for laminar flow in a horizontal pipe is, from Eq. 14–20,

Horizontal pipe:
$$V_{\text{avg}} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P D^2}{32\mu L} \tag{14–26}$$

Then the volume flow rate for laminar flow through a horizontal pipe of diameter D and length L becomes

$$\dot{V} = V_{\text{avg}} A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P \pi D^4}{128\mu L} \tag{14–27}$$

This equation is known as **Poiseuille's law**, and this flow is called *Hagen–Poiseuille flow* in honor of the works of G. Hagen (1797–1884) and J. Poiseuille (1799–1869) on the subject. Note from Eq. 14–27 that *for a specified flow rate, the pressure drops and thus the required pumping power is proportional to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the radius (or diameter) of the pipe*. Therefore, the pumping power requirement for a laminar-flow piping system can be reduced by a factor of 16 by doubling the pipe diameter (Fig. 14–14). Of course the benefits of the reduction in the energy costs must be weighed against the increased cost of construction due to using a larger-diameter pipe.

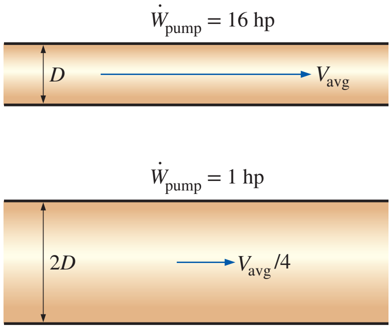


FIGURE 14–14
The pumping power requirement for a laminar-flow piping system can be reduced by a factor of 16 by doubling the pipe diameter.

The pressure drop ΔP equals the pressure loss ΔP_L in the case of a horizontal pipe, but this is not the case for inclined pipes or pipes with variable cross-sectional area. This can be demonstrated by writing the energy equation for steady, incompressible one-dimensional flow in terms of heads as (see Chap. 12)

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, } u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, } e} + h_L \tag{14–28}$$

where $h_{\text{pump, } u}$ is the useful pump head delivered to the fluid, $h_{\text{turbine, } e}$ is the turbine head extracted from the fluid, h_L is the irreversible head loss between sections 1 and 2, V_1 and V_2 are the average velocities at sections 1 and 2, respectively, and α_1 and α_2 are the *kinetic energy correction factors* at sections 1 and 2 (it can be shown that $\alpha = 2$ for fully developed laminar flow and about 1.05 for fully developed turbulent flow). Equation 14–28 can be rearranged as

$$P_1 - P_2 = \rho(\alpha_2 V_2^2 - \alpha_1 V_1^2)/2 + \rho g[(z_2 - z_1) + h_{\text{turbine, } e} - h_{\text{pump, } u} + h_L] \tag{14–29}$$

Therefore, the pressure drop $\Delta P = P_1 - P_2$ and pressure loss $\Delta P_L = \rho g h_L$ for a given flow section are equivalent if (1) the flow section is horizontal so that there are no hydrostatic or gravity effects ($z_1 = z_2$), (2) the flow section does not involve any work devices such as a pump or a turbine since they change the fluid pressure ($h_{\text{pump, } u} = h_{\text{turbine, } e} = 0$), (3) the cross-sectional area of the flow section is constant and thus the average flow velocity is constant ($V_1 = V_2$), and (4) the velocity profiles at sections 1 and 2 are the same shape ($\alpha_1 = \alpha_2$).

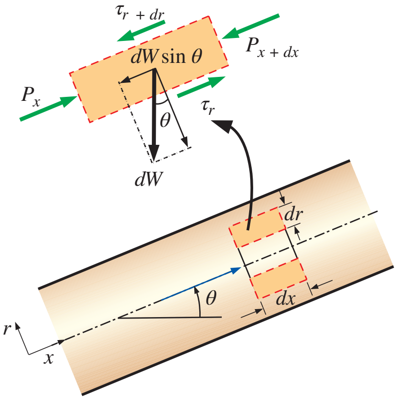


FIGURE 14–15
Free-body diagram of a ring-shaped differential fluid element of radius r, thickness dr, and length dx oriented coaxially with an inclined pipe in fully developed laminar flow.

Effect of Gravity on Velocity and Flow Rate in Laminar Flow

Gravity has no effect on flow in horizontal pipes, but it has a significant effect on both the velocity and the flow rate in uphill or downhill pipes. Relations for inclined pipes can be obtained in a similar manner from a force balance in the direction of flow. The only additional force in this case is the component of the fluid weight in the flow direction, whose magnitude is

$$dW_x = dW \sin\theta = \rho g\, dV_{\text{element}} \sin\theta = \rho g(2\pi r\, dr\, dx) \sin\theta \tag{14–30}$$

where θ is the angle between the horizontal and the flow direction (Fig. 14–15). The force balance in Eq. 14–9 now becomes

$$\begin{aligned}(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dr\, \tau)_r \\ - (2\pi r\, dx\, \tau)_{r+dr} - \rho g(2\pi r\, dr\, dx)\sin\theta = 0\end{aligned} \tag{14–31}$$

which results in the differential equation

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} + \rho g \sin\theta \tag{14–32}$$

Following the same solution procedure as previously, the velocity profile is

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g \sin\theta\right)\left(1 - \frac{r^2}{R^2}\right) \tag{14–33}$$

From Eq. 14–33, the *average velocity* and the *volume flow rate* relations for laminar flow through inclined pipes are, respectively,

$$V_{\text{avg}} = \frac{(\Delta P - \rho g L \sin\theta)D^2}{32\mu L} \quad \text{and} \quad \dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L} \tag{14–34}$$

which are identical to the corresponding relations for horizontal pipes, except that ΔP is replaced by $\Delta P - \rho g L \sin\theta$. Therefore, the results already obtained for horizontal pipes can also be used for inclined pipes provided that ΔP is replaced by $\Delta P - \rho g L \sin\theta$ (Fig. 14–16). Note that $\theta > 0$ and thus $\sin\theta > 0$ for uphill flow, and $\theta < 0$ and thus $\sin\theta < 0$ for downhill flow.

In inclined pipes, the combined effect of pressure difference and gravity drives the flow. Gravity helps downhill flow but opposes uphill flow. Therefore, much greater pressure differences need to be applied to maintain a specified flow rate in uphill flow, although this becomes important only for liquids because the density of gases is generally low. In the special case of *no flow* ($\dot{V} = 0$), Eq. 14–34 yields $\Delta P = \rho g L \sin\theta$, which is what we would obtain from fluid statics (Chap. 11).

Laminar Flow in Circular Pipes

(Fully developed flow with no pump or turbine in the flow section, and $\Delta P = P_1 - P_2$)

Horizontal pipe: $\dot{V} = \dfrac{\Delta P\,\pi D^4}{128\mu L}$

Inclined pipe: $\dot{V} = \dfrac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$

Uphill flow: $\theta > 0$ and $\sin\theta > 0$

Downhill flow: $\theta < 0$ and $\sin\theta < 0$

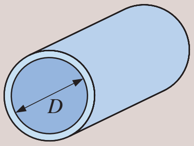
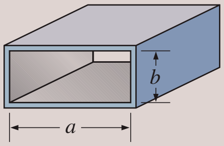
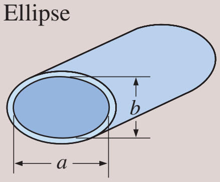
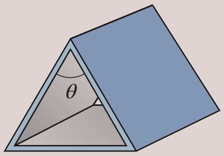
FIGURE 14–16
The relations developed for fully developed laminar flow through horizontal pipes can also be used for inclined pipes by replacing ΔP with $\Delta P - \rho g L \sin\theta$.

Laminar Flow in Noncircular Pipes

The friction factor f relations are given in Table 14–1 for *fully developed laminar flow* in pipes of various cross sections. The Reynolds number for flow in these pipes is based on the hydraulic diameter $D_h = 4A_c/p$, where A_c is the cross-sectional area of the pipe and p is its wetted perimeter.

TABLE 14–1

Friction factor for fully developed *laminar flow* in pipes of various cross sections ($D_h = 4A_c/p$ and $\text{Re} = V_{\text{avg}} D_h/\nu$)

Tube geometry	*a/b* or θ°	Friction factor f
Circle  	—	64.00/Re
Rectangle  	*a/b*	
	1	56.92/Re
	2	62.20/Re
	3	68.36/Re
	4	72.92/Re
	6	78.80/Re
	8	82.32/Re
	∞	96.00/Re
Ellipse  	*a/b*	
	1	64.00/Re
	2	67.28/Re
	4	72.96/Re
	8	76.60/Re
	16	78.16/Re
Isosceles triangle  	θ	
	10°	50.80/Re
	30°	52.28/Re
	60°	53.32/Re
	90°	52.60/Re
	120°	50.96/Re

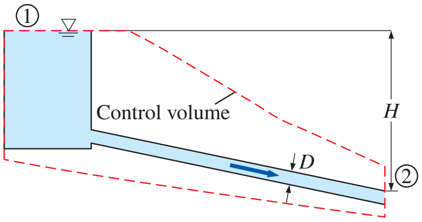


FIGURE 14–17
Schematic for Example 14–1.

EXAMPLE 14–1 Laminar Draining from a Pool

At the end of the summer, a swimming pool is being drained through a very long, small-diameter hose (Fig. 14–17). The hose is smooth, of inner diameter $D = 6.0$ cm, and of length $L = 65$ m. The initial height difference from the pool surface to the outlet of the hose is $H = 2.20$ m. Calculate the volume flow rate in liters per minute (Lpm) at the start of draining.

SOLUTION Water is drained from a pool. The volume flow rate is to be determined.
Assumptions **1** The flow is incompressible and *quasi-steady* (we approximate it as steady at the start of the draining since the pool volume is large and drains slowly). **2** Entrance effects are negligible since the hose is so long; the flow is fully developed. **3** Any other losses such as elbows in the hose are negligible.
Properties The properties of water are $\rho = 998\ \text{kg/m}^3$ and $\mu = 0.001002$ kg/m·s.
Analysis The first step in a problem like this is to wisely choose a control volume. The one shown in the sketch cuts just below the surface of the water in the pool (inlet, 1) and slices through the hose discharge (outlet, 2). We assume that the flow is laminar, but we will need to verify this at the end. For fully developed laminar pipe flow at the outlet of the hose, $\alpha_2 = 2$. Letting V be the average velocity through the hose, the appropriate equations are:

$$\text{Re} = \frac{\rho VD}{\mu} \qquad f = \frac{64}{\text{Re}} \qquad h_L = f\frac{L}{D}\frac{V^2}{2g} \tag{1}$$

and the head form of the energy equation from Chap. 12,

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{\text{pump, }u} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{\text{turbine, }e} + h_L \tag{2}$$

In this problem, there are no pumps or turbines, so those terms are zero. Both P_1 and P_2 are equal to atmospheric pressure, so the pressure terms cancel. Water speed V_1 at the control volume inlet (the pool surface) is negligibly small since the pool is draining slowly. At the outlet, $V_2 = V$. Thus, the energy equation reduces to

$$\alpha_2\frac{V^2}{2g} = z_1 - z_2 - h_L = H - h_L \tag{3}$$

At this point, we can enter this set of simultaneous equations into an equation solver to obtain the solution. Or, if we are calculating by hand, we must perform algebra on Eqs. 1 and 3 to get one equation and one unknown (V),

$$V^2 + \frac{64\mu L}{\rho D^2\alpha_2}V - \frac{2gH}{\alpha_2} = 0 \tag{4}$$

Since Eq. (4) is in standard form for solution by the quadratic rule, and all the constants in the equation are known, we easily solve for $V = 0.36969$ m/s. Finally, we calculate the volume flow rate,

$$\dot{V} = VA_c = V\frac{\pi D^2}{4} = (0.36969\ \text{m/s})\frac{\pi(0.0060\ \text{m})^2}{4}\left(\frac{1000\ \text{L}}{\text{m}^3}\right)\left(\frac{60\ \text{s}}{\text{min}}\right) = 0.6272\frac{\text{L}}{\text{min}}$$

which we round to our final answer: $\dot{V} \approx$ **0.627 Lpm**

We check the Reynolds number,

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(998\ \text{kg/m}^3)(0.36969\ \text{m/s})(0.0060\ \text{m})}{0.001002\ \text{kg/m·s}} = 2209$$

Since Re < 2300, we verify that the flow is laminar, although it is close to transitioning to turbulent flow.

Discussion It is important to verify assumptions. It would be wise and useful practice for students to verify the algebra leading to Eq. 4, and to solve it using the quadratic rule, including all units. The hose in the sketch is inclined at some angle. However, the angle was never used in the solution because the only thing that matters is height H, and the result would be the same regardless of whether the pool is shallow with a large inclination of the hose or the pool is deep with a small or even zero inclination of the hose.

EXAMPLE 14–2 Pumping Power Requirement for Oil Flow in a Pipe

Consider the flow of oil with $\rho = 894\ \text{kg/m}^3$ and $\mu = 2.33\ \text{kg/m·s}$ in a 28-cm-diameter pipeline at an average velocity of 0.5 m/s. A 330-m-long section of the pipeline passes through the icy waters of a lake (Fig. 14–18). Disregarding the entrance effects, determine the pumping power required to overcome the pressure losses and to maintain the flow of oil in the pipe.

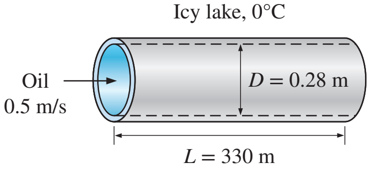


FIGURE 14–18
Schematic for Example 14–2. Drawing not to scale.

SOLUTION Oil flows through a pipeline that passes through icy waters of a lake. The pumping power needed to overcome pressure losses is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The flow section considered is away from the entrance, and thus the flow is fully developed. **3** The roughness effects are negligible, and thus the inner surfaces are considered to be smooth, $\varepsilon \approx 0$.

Properties The properties of oil are given to be $\rho = 894\ \text{kg/m}^3$ and $\mu = 2.33\ \text{kg/m·s}$.

Analysis The volume flow rate and the Reynolds number in this case are

$$\dot{V} = VA_c = V\frac{\pi D^2}{4} = (0.5\ \text{m/s})\frac{\pi(0.28\ \text{m})^2}{4} = 0.03079\ \text{m}^3/\text{s}$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(894\ \text{kg/m}^3)(0.5\ \text{m/s})(0.28\ \text{m})}{2.33\ \text{kg/m·s}} = 53.72$$

which is less than 2300. Therefore, the flow is laminar, and the friction factor is

$$f = \frac{64}{\text{Re}} = \frac{64}{53.72} = 1.191$$

Then the pressure drop in the pipe and the required pumping power become

$$\begin{aligned}\Delta P = \Delta P_L &= f\frac{L}{D}\frac{\rho V^2}{2}\\ &= 1.191\frac{330\ \text{m}}{0.28\ \text{m}}\frac{(894\ \text{kg/m}^3)(0.5\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg·m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)\\ &= 156.9\ \text{kPa}\end{aligned}$$

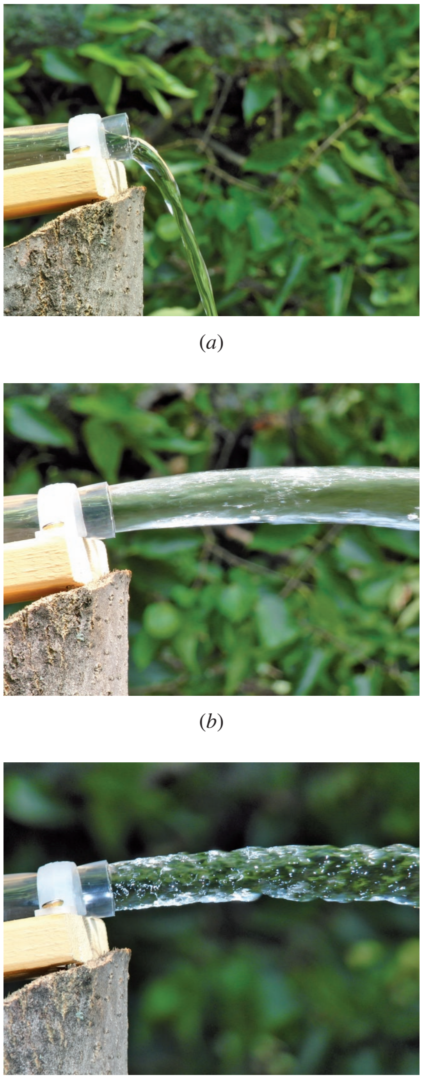
$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.03079\ \text{m}^3/\text{s})(156.9\ \text{kPa})\left(\frac{1\ \text{kW}}{1\ \text{kPa·m}^3/\text{s}}\right) = \mathbf{4.83\ kW}$$

Discussion The power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

14–5 ▪ TURBULENT FLOW IN PIPES

Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress. However, turbulent flow is a complex mechanism dominated by fluctuations, and despite tremendous amounts of work done in this area by researchers, turbulent flow still is not fully understood. Therefore, we must rely on experiments and the empirical or semi-empirical correlations developed for various situations.

Turbulent flow is characterized by disorderly and rapid fluctuations of swirling regions of fluid, called **eddies**, throughout the flow (Fig. 14–19). These fluctuations provide an additional mechanism for momentum and energy transfer. In laminar flow, fluid particles flow in an orderly manner along pathlines, and momentum and energy are transferred across streamlines by molecular diffusion. In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer. As a result, turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients (Fig. 14–20).



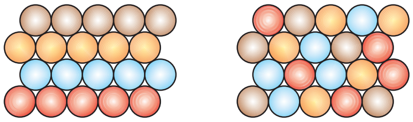
(*a*)

(*b*)

(*c*)

FIGURE 14–19
Water exiting a tube: (*a*) laminar flow at low flow rate, (*b*) turbulent flow at high flow rate, and (*c*) same as (*b*) but with a short shutter exposure to capture individual eddies.

Photo by Alex Wouden



(*a*) Before turbulence

(*b*) After turbulence

FIGURE 14–20
The intense mixing in turbulent flow brings fluid particles at different momentums into close contact and thus enhances momentum transfer.

Turbulent Velocity Profile

Unlike laminar flow, the expressions for the velocity profile in a turbulent flow are based on both analysis and measurements, and thus they are semi-empirical in nature with constants determined from experimental data. Consider fully developed turbulent flow in a pipe, and let u denote the time-averaged velocity in the axial direction (and thus drop the overbar from $\overline{u}$ for simplicity).

Typical velocity profiles for fully developed laminar and turbulent flows are given in Fig. 14–21. Note that the velocity profile is parabolic in laminar flow but is much fuller in turbulent flow, with a sharp drop near the pipe wall. Turbulent flow along a wall can be considered to consist of four regions, characterized by the distance from the wall (Fig. 14–21). The very thin layer next to the wall where viscous effects are dominant is the **viscous** (or **laminar** or **linear** or **wall**) sublayer. The velocity profile in this layer is very nearly *linear,* and the flow is streamlined. Next to the viscous sublayer is the **buffer layer**, in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects. Above the buffer layer is the **overlap** (or **transition**) **layer**, also called the **inertial sublayer**, in which the turbulent effects are much more significant, but still not dominant. Above that is the **outer** (or **turbulent**) **layer** in the remaining part of the flow in which turbulent effects dominate over molecular diffusion (viscous) effects.

Flow characteristics are quite different in different regions, and thus it is difficult to come up with an analytic relation for the velocity profile for the entire flow as we did for laminar flow. The best approach in the turbulent case turns out to be to identify the key variables and functional forms using dimensional analysis, and then to use experimental data to determine the numerical values of any constants. Several equations have been generated in this manner to approximate the velocity profile shape in fully developed turbulent pipe flow. Discussion of these equations is beyond the scope of this text.

The Moody Chart and Its Associated Equations

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the **relative roughness** ε/D, which is the ratio of the mean height of roughness of the pipe to the pipe diameter. The functional form of this dependence cannot be obtained from a theoretical analysis, and all available results are obtained from painstaking experiments using artificially roughened surfaces (usually by gluing sand grains of a known size on the inner surfaces of the pipes). Most such experiments were conducted by Prandtl's student J. Nikuradse in 1933, followed by the works of others. The friction factor was calculated from measurements of the flow rate and the pressure drop.

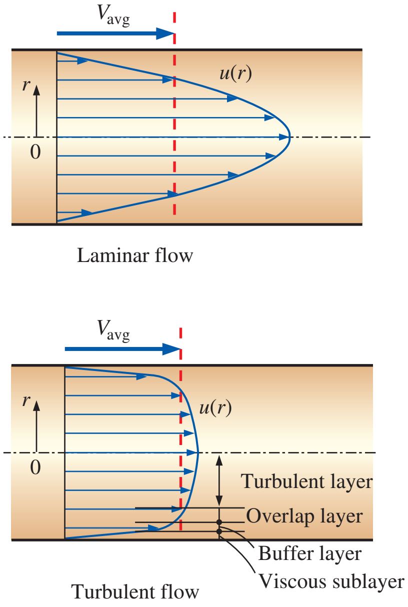
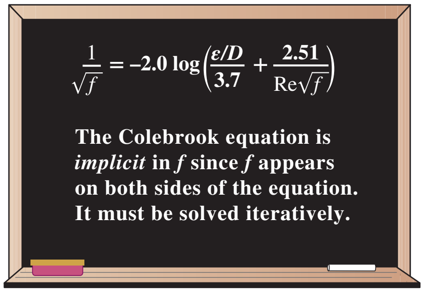


FIGURE 14–21
The velocity profile in fully developed pipe flow is parabolic in laminar flow, but much fuller in turbulent flow. Note that $u(r)$ in the turbulent case is the *time-averaged* velocity component in the axial direction (the overbar on u has been dropped for simplicity).

The experimental results are presented in tabular, graphical, and functional forms obtained by curve-fitting experimental data. In 1939, Cyril F. Colebrook (1910–1997) combined the available data for transition and turbulent flow in smooth as well as rough pipes into the following implicit relation (Fig. 14–22) known as the **Colebrook equation**:

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \quad \text{(turbulent flow)} \qquad \textbf{(14–35)}$$



$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

The Colebrook equation is *implicit* in f since f appears on both sides of the equation. It must be solved iteratively.

FIGURE 14–22
The Colebrook equation.

We note that the logarithm in Eq. 14–35 is a base 10 rather than a natural logarithm. In 1942, the American engineer Hunter Rouse (1906–1996) verified Colebrook's equation and produced a graphical plot of f as a function of Re and the product $\mathrm{Re}\sqrt{f}$. He also presented the laminar flow relation and a table of commercial pipe roughness. Two years later, Lewis F. Moody (1880–1953) redrew Rouse's diagram into the form commonly used today. The now famous **Moody chart** is given in the appendix as Fig. A–27. It presents the Darcy friction factor for pipe flow as a function of Reynolds number and ε/D over a wide range. It is probably one of the most widely accepted and used charts in engineering. Although it is developed for circular pipes, it can also be used for noncircular pipes by replacing the diameter with the hydraulic diameter.

Commercially available pipes differ from those used in the experiments because the roughness of pipes in the market is not uniform, and it is difficult to give a precise description of it. Equivalent roughness values for some commercial pipes are given in Table 14–2 as well as on the Moody chart. But it should be kept in mind that these values are for new pipes, and the relative roughness of pipes may increase with use as a result of corrosion, scale buildup, and precipitation. As a result, the friction factor may increase by a factor of 5 to 10. Actual operating conditions must be considered in the design of piping systems. Also, the Moody chart and its equivalent Colebrook equation involve several uncertainties (the roughness size, experimental error, curve fitting of data, etc.), and thus the results obtained should not be treated as "exact." They are usually considered to be accurate to ±15 percent over the entire range in the figure.

We make the following observations from the Moody chart:

- For laminar flow, the friction factor decreases with increasing Reynolds number, and it is independent of surface roughness.
- The friction factor is a minimum for a smooth pipe (but still not zero because of the no-slip condition) and increases with roughness (Fig. 14–23). The Colebrook equation in this case ($\varepsilon = 0$) reduces to the **Prandtl equation** expressed as $1/\sqrt{f} = 2.0\log(\mathrm{Re}\sqrt{f}) - 0.8$.
- The transition region from the laminar to turbulent regime ($2300 < \mathrm{Re} < 4000$) is indicated by the shaded area in the Moody chart (Figs. 14–24 and A–27). The flow in this region may be laminar or turbulent, depending on flow disturbances, or it may alternate between laminar and turbulent, and thus the friction factor may also alternate between the values for laminar and turbulent flow. The data in this range are the least reliable. At small relative roughnesses, the friction factor increases in the transition region and approaches the value for smooth pipes.

TABLE 14–2

Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

* The uncertainty in these values can be as much as ±60 percent.

Relative roughness, ε/D	Friction factor, f
0.0*	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

* Smooth surface. All values are for Re = 10^6 and are calculated from the Colebrook equation.

FIGURE 14–23
The friction factor is minimum for a smooth pipe and increases with roughness.

- At very large Reynolds numbers (to the right of the dashed line on the Moody chart) the friction factor curves corresponding to specified relative roughness curves are nearly horizontal, and thus the friction factors are independent of the Reynolds number (Fig. 14–24). The flow in that region is called *fully rough turbulent flow* or just *fully rough flow* because the thickness of the viscous sublayer decreases with increasing Reynolds number, and it becomes so thin that it is negligibly small compared to the surface roughness height. The viscous effects in this case are produced in the main flow primarily by the protruding roughness elements, and the contribution of the viscous sublayer is negligible. The Colebrook equation in the *fully rough* zone (Re → ∞) reduces to the **von Kármán equation** expressed as $1/\sqrt{f} = -2.0\log[(\varepsilon/D)/3.7]$, which is explicit in f. Some authors call this zone *completely* (or *fully*) *turbulent flow,* but this is misleading since the flow to the left of the dashed blue line in Fig. 14–24 is also fully turbulent.

In calculations, we should make sure that we use the actual internal diameter of the pipe, which may be different than the nominal diameter. For example, the internal diameter of a steel pipe whose nominal diameter is 1 in is 1.049 in (Table 14–3).

Types of Fluid Flow Problems

In the design and analysis of piping systems that involve the use of the Moody chart (or the Colebrook equation), we usually encounter three types of problems (the fluid and the roughness of the pipe are assumed to be specified in all cases) (Fig. 14–25):

1. Determining the **pressure drop** (or head loss) when the pipe length and diameter are given for a specified flow rate (or velocity)
2. Determining the **flow rate** when the pipe length and diameter are given for a specified pressure drop (or head loss)
3. Determining the **pipe diameter** when the pipe length and flow rate are given for a specified pressure drop (or head loss)

Problems of the *first type* are straightforward and can be solved directly by using the Moody chart. Problems of the *second* and *third types* are commonly encountered in engineering design (for example, in the selection of pipe diameter that minimizes the sum of the construction and pumping costs), but the use

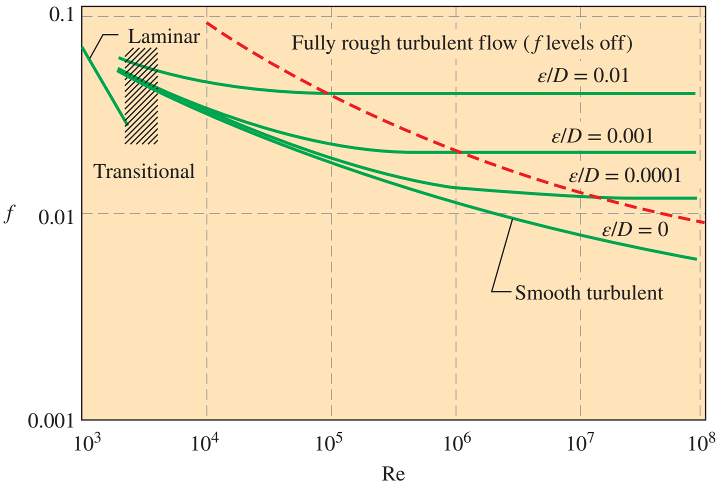


FIGURE 14–24
At very large Reynolds numbers, the friction factor curves on the Moody chart are nearly horizontal, and thus the friction factors are independent of the Reynolds number. See Fig. A–27 for a full-page, more detailed Moody chart.

of the Moody chart with such problems requires an iterative approach—an equation solver is recommended.

In problems of the *second type,* the diameter is given, but the flow rate is unknown. A good guess for the friction factor in that case is obtained from the completely turbulent flow region for the given roughness. This is true for large Reynolds numbers, which often occur in practice. Once the flow rate is obtained, the friction factor is corrected using the Moody chart or the Colebrook equation, and the process is repeated until the solution converges. (Typically only a few iterations are required for convergence to three or four digits of precision.)

In problems of the *third type,* the diameter is not known, and thus the Reynolds number and the relative roughness cannot be calculated. Therefore, we start calculations by assuming a pipe diameter. The pressure drop calculated for the assumed diameter is then compared to the specified pressure drop, and calculations are repeated with another pipe diameter in an iterative fashion until convergence.

To avoid tedious iterations in head loss, flow rate, and diameter calculations, Swamee and Jain (1976) proposed the following explicit relations that are accurate to within 2 percent of the Moody chart:

$$h_L = 1.07\frac{\dot{V}^2 L}{gD^5}\left\{\ln\left[\frac{\varepsilon}{3.7D}+4.62\left(\frac{\nu D}{\dot{V}}\right)^{0.9}\right]\right\}^{-2} \qquad \begin{array}{c} 10^{-6} < \varepsilon/D < 10^{-2} \\ 3000 < \text{Re} < 3\times 10^{8} \end{array} \tag{14–36}$$

$$\dot{V} = -0.965\left(\frac{gD^5 h_L}{L}\right)^{0.5}\ln\left[\frac{\varepsilon}{3.7D}+\left(\frac{3.17\nu^2 L}{gD^3 h_L}\right)^{0.5}\right] \qquad \text{Re} > 2000 \tag{14–37}$$

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75}+\nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} \qquad \begin{array}{c} 10^{-6} < \varepsilon/D < 10^{-2} \\ 5000 < \text{Re} < 3\times 10^{8} \end{array} \tag{14–38}$$

Note that all quantities are dimensional, and the units simplify to the desired unit (for example, to m or ft in the last relation) when consistent units are used. Noting that the Moody chart is accurate to within 15 percent of experimental data, we should have no reservation in using these approximate relations in the design of piping systems.

The Colebrook equation is implicit in f, and thus the determination of the friction factor requires iteration. An approximate *explicit* relation for f was given by S. E. Haaland in 1983 as

$$\frac{1}{\sqrt{f}} \cong -1.8\log\left[\frac{6.9}{\text{Re}}+\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right] \tag{14–39}$$

The results obtained from this relation are within 2 percent of those obtained from the Colebrook equation. If more accurate results are desired, Eq. 14–39 can be used as a good *first guess* in a Newton iteration when using a programmable calculator or a spreadsheet to solve for f with Eq. 14–35.

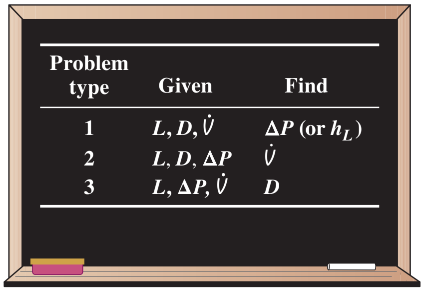
We routinely use the Colebrook equation to calculate the friction factor f for fully developed turbulent pipe flow. Indeed, the Moody chart is *created* using the Colebrook equation. However, in addition to being implicit, the Colebrook equation is valid only for *turbulent* pipe flow (when the flow is laminar, $f = 64/\text{Re}$). Thus, we need to verify that the Reynolds number is in the turbulent range. An equation was generated by Churchill (1977) that is not only explicit, but is also useful for *any* Re and *any* roughness, even for laminar flow, and even in the fuzzy transitional region between laminar and turbulent flow. The **Churchill equation** is

$$f = 8\left[\left(\frac{8}{\text{Re}}\right)^{12}+(A+B)^{-1.5}\right]^{\frac{1}{12}} \tag{14–40}$$

TABLE 14–3

Standard sizes for Schedule 40 steel pipes

Nominal size, in	Actual inside diameter, in
$\frac{1}{8}$	0.269
$\frac{1}{4}$	0.364
$\frac{3}{8}$	0.493
$\frac{1}{2}$	0.622
$\frac{3}{4}$	0.824
1	1.049
$1\frac{1}{2}$	1.610
2	2.067
$2\frac{1}{2}$	2.469
3	3.068
5	5.047
10	10.02



Problem type	Given	Find
1	$L, D, \dot{V}$	ΔP (or h_L)
2	$L, D, \Delta P$	$\dot{V}$
3	$L, \Delta P, \dot{V}$	D

FIGURE 14–25
The three types of problems encountered in pipe flow.

where

$$A = \left\{ -2.457 \ln\left[\left(\frac{7}{\text{Re}}\right)^{0.9} + 0.27\frac{\varepsilon}{D}\right]\right\}^{16} \qquad \text{and} \qquad B = \left(\frac{37{,}530}{\text{Re}}\right)^{16}$$

The difference between the Colebrook and Churchill equations is less than 1 percent. Because it is explicit and valid over the entire range of Reynolds numbers and roughnesses, it is recommended that the Churchill equation be used for determination of friction factor *f*.

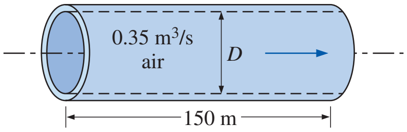


FIGURE 14–26
Schematic for Example 14–3.

EXAMPLE 14–3 Determining the Diameter of an Air Duct

Heated air at 1 atm and 35°C is to be transported in a 150-m-long circular plastic duct at a rate of 0.35 m³/s (Fig. 14–26). If the head loss in the pipe is not to exceed 20 m, determine the minimum diameter of the duct.

SOLUTION The flow rate and the head loss in an air duct are given. The diameter of the duct is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The duct involves no components such as bends, valves, and connectors. **4** Air is an ideal gas. **5** The duct is smooth since it is made of plastic. **6** The flow is turbulent (to be verified).

Properties The density, dynamic viscosity, and kinematic viscosity of air at 35°C are $\rho = 1.145 \text{ kg/m}^3$, $\mu = 1.895 \times 10^{-5}$ kg/m·s, and $\nu = 1.655 \times 10^{-5} \text{ m}^2/\text{s}$.

Analysis This is a problem of the third type since it involves the determination of diameter for specified flow rate and head loss. We can solve this problem using three different approaches: (1) an iterative approach by assuming a pipe diameter, calculating the head loss, comparing the result to the specified head loss, and repeating calculations until the calculated head loss matches the specified value; (2) writing all the relevant equations (leaving the diameter as an unknown) and solving them simultaneously using an equation solver; and (3) using the third Swamee–Jain formula. We will demonstrate the use of the last two approaches.

The average velocity, the Reynolds number, the friction factor, and the head loss relations are expressed as (*D* is in m, *V* is in m/s, and Re and *f* are dimensionless)

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.35 \text{ m}^3/\text{s}}{\pi D^2/4}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{VD}{1.655 \times 10^{-5} \text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) = -2.0 \log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} \quad \rightarrow \quad 20 \text{ m} = f\frac{150 \text{ m}}{D}\frac{V^2}{2(9.81 \text{ m/s}^2)}$$

The roughness is approximately zero for a plastic pipe (Table 14–2). Therefore, this is a set of four equations and four unknowns, and solving them with an equation solver gives

$$D = \mathbf{0.267 \text{ m}}, \qquad f = 0.0180, \qquad V = 6.24 \text{ m/s}, \qquad \text{and} \qquad \text{Re} = 100{,}800$$

Therefore, the diameter of the duct should be more than 26.7 cm if the head loss is not to exceed 20 m. Note that Re > 4000, and thus the turbulent flow assumption is verified.

The diameter can also be determined directly from the third Swamee–Jain formula to be

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04}$$

$$= 0.66\left[0 + (1.655\times10^{-5}\text{ m}^2/\text{s})(0.35\text{ m}^3/\text{s})^{9.4}\left(\frac{150\text{ m}}{(9.81\text{ m/s}^2)(20\text{ m})}\right)^{5.2}\right]^{0.04}$$

$$= \mathbf{0.271\text{ m}}$$

Discussion Note that the difference between the two results is less than 2 percent. Therefore, the simple Swamee–Jain relation can be used with confidence. Finally, the first (iterative) approach requires an initial guess for D. If we use the Swamee–Jain result as our initial guess, the diameter converges to $D = 0.267$ m in short order.

EXAMPLE 14–4 Determining the Pressure Drop in a Water Pipe

Water at 60°F (ρ = 62.36 lbm/ft^3 and $\mu = 7.536 \times 10^{-4}$ lbm/ft·s) is flowing steadily in a 2-in-diameter horizontal pipe made of stainless steel at a rate of 0.2 ft^3/s (Fig. 14–27). Determine the pressure drop, the head loss, and the required pumping power input for flow over a 200-ft-long section of the pipe.

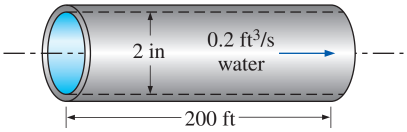


FIGURE 14–27
Schematic for Example 14–4.

SOLUTION The flow rate through a specified water pipe is given. The pressure drop, the head loss, and the pumping power requirements are to be determined.
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as a pump or a turbine.
Properties The density and dynamic viscosity of water are given to be ρ = 62.36 lbm/ft^3 and $\mu = 7.536 \times 10^{-4}$ lbm/ft·s, respectively.
Analysis We recognize this as a problem of the first type, since flow rate, pipe length, and pipe diameter are known. First we calculate the average velocity and the Reynolds number to determine the flow regime:

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.2\text{ ft}^3/\text{s}}{\pi(2/12\text{ ft})^2/4} = 9.17\text{ ft/s}$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(62.36\text{ lbm/ft}^3)(9.17\text{ ft/s})(2/12\text{ ft})}{7.536\times10^{-4}\text{ lbm/ft·s}} = 126{,}400$$

Since Re is greater than 4000, the flow is turbulent. The relative roughness of the pipe is estimated using Table 14–3

$$\varepsilon/D = \frac{0.000007\text{ ft}}{2/12\text{ ft}} = 0.000042$$

The friction factor corresponding to this relative roughness and Reynolds number is determined from the Moody chart. To avoid any reading error, we determine f from the Colebrook equation on which the Moody chart is based:

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.000042}{3.7} + \frac{2.51}{126{,}400\sqrt{f}}\right)$$

Using an equation solver or an iterative scheme, the friction factor is determined to be $f = 0.0174$. Then the pressure drop (which is equivalent to pressure loss in this case), head loss, and the required power input become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.0174\frac{200\text{ ft}}{2/12\text{ ft}}\frac{(62.36\text{ lbm/ft}^3)(9.17\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)$$

$$= \mathbf{1700\ lbf/ft^2 = 11.8\ psi}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = 0.0174\frac{200\text{ ft}}{2/12\text{ ft}}\frac{(9.17\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = \mathbf{27.3\ ft}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.2\text{ ft}^3\text{/s})(1700\text{ lbf/ft}^2)\left(\frac{1\text{ W}}{0.737\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{461\ W}$$

Therefore, power input in the amount of 461 W is needed to overcome the frictional losses in the pipe.

Discussion It is common practice to write our final answers to three significant digits, even though we know that the results are accurate to at most two significant digits because of inherent inaccuracies in the Colebrook equation, as discussed previously. The friction factor could also be determined easily from the explicit Haaland relation (Eq. 14–39). It would give $f = 0.0172$, which is sufficiently close to 0.0174. The Churchill equation (Eq. 14–40) gives $f = 0.0173$, which is also very close to 0.0174, but does not require iteration. Also, the friction factor corresponding to $\varepsilon = 0$ in this case is 0.0171, which indicates that this stainless-steel pipe can be approximated as smooth with minimal error.

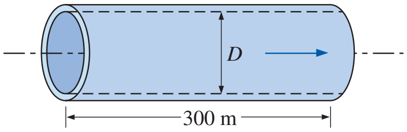


FIGURE 14–28
Schematic for Example 14–5.

EXAMPLE 14–5 Determining the Flow Rate of Air in a Duct

Reconsider Example 14–3. Now the duct length is doubled while its diameter is maintained constant (Fig. 14–28). If the total head loss is to remain constant, determine the drop in the flow rate through the duct.

SOLUTION The diameter and the head loss in an air duct are given. The drop in the flow rate is to be determined.

Analysis This is a problem of the second type since it involves the determination of the flow rate for a specified pipe diameter and head loss. The solution involves an iterative approach since the flow rate (and thus the flow velocity) is not known.

The average velocity, Reynolds number, friction factor, and the head loss relations are expressed as (D is in m, V is in m/s, and Re and f are dimensionless)

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad\rightarrow\quad V = \frac{\dot{V}}{\pi(0.267\text{ m})^2/4}$$

$$\text{Re} = \frac{VD}{\nu} \quad\rightarrow\quad \text{Re} = \frac{V(0.267\text{ m})}{1.655\times10^{-5}\text{ m}^2\text{/s}}$$

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad\rightarrow\quad \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} \quad\rightarrow\quad 20\text{ m} = f\frac{300\text{ m}}{0.267\text{ m}}\frac{V^2}{2(9.81\text{ m/s}^2)}$$

This is a set of four equations in four unknowns and solving them with an equation solver gives

$$\dot{V} = 0.24\text{ m}^3\text{/s},\qquad f = 0.0195,\qquad V = 4.23\text{ m/s},\qquad \text{and}\qquad \text{Re} = 68{,}300$$

Then the drop in the flow rate becomes

$$\dot{V}_{\text{drop}} = \dot{V}_{\text{old}} - \dot{V}_{\text{new}} = 0.35 - 0.24 = \mathbf{0.11\ m^3/s} \qquad \left(\text{a drop of 31 percent}\right)$$

Therefore, for a specified head loss (or available head or fan pumping power), the flow rate drops by about 31 percent from 0.35 to 0.24 m^3/s when the duct length doubles.

Alternative Solution If a computer is not available (as in an exam situation), another option is to set up a *manual iteration loop*. We have found that the best convergence is usually realized by first guessing the friction factor f, and then solving for the velocity V. The equation for V as a function of f is

Average velocity through the pipe: $$V = \sqrt{\frac{2gh_L}{fL/D}}$$

Once V is calculated, the Reynolds number can be calculated, from which a *corrected* friction factor is obtained from the Moody chart or the Colebrook equation. We repeat the calculations with the corrected value of f until convergence. We guess $f = 0.04$ for illustration:

Iteration	f (guess)	V, m/s	Re	Corrected f
1	0.04	2.955	4.724×10^4	0.0212
2	0.0212	4.059	6.489×10^4	0.01973
3	0.01973	4.207	6.727×10^4	0.01957
4	0.01957	4.224	6.754×10^4	0.01956
5	0.01956	4.225	6.756×10^4	0.01956

Notice that the iteration has converged to three digits in only three iterations and to four digits in only four iterations. The final results are identical to those obtained with the equation solver, yet do not require a computer.

Discussion The new flow rate can also be determined directly from the second Swamee–Jain formula to be

$$\begin{aligned}
\dot{V} &= -0.965\left(\frac{gD^5h_L}{L}\right)^{0.5}\ln\left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17\nu^2L}{gD^3h_L}\right)^{0.5}\right] \\
&= -0.965\left(\frac{(9.81\ \text{m/s}^2)(0.267\ \text{m})^5(20\ \text{m})}{300\ \text{m}}\right)^{0.5} \\
&\quad \times \ln\left[0 + \left(\frac{3.17\,(1.655 \times 10^{-5}\ \text{m}^2\text{/s})^2(300\ \text{m})}{(9.81\ \text{m/s}^2)(0.267\ \text{m})^3(20\ \text{m})}\right)^{0.5}\right] \\
&= 0.24\ \text{m}^3\text{/s}
\end{aligned}$$

Note that the result from the Swamee–Jain relation is the same (to two significant digits) as that obtained with the Colebrook equation using the equation solver or using our manual iteration technique. Therefore, the simple Swamee–Jain relation can be used with confidence.

EXAMPLE 14–6 Turbulent Draining from a Pool

Reconsider the draining pool example, Example 14–1. The flow rate was very low, so the pool owner uses a larger diameter hose (Fig. 14–29). The new hose has inner diameter D = 2.00 cm and average roughness height ε = 0.0020 cm. All other parameters remain the same as the previous problem, including the length of the hose. Calculate the volume flow rate in liters per minute (Lpm) at the start of draining.

SOLUTION Water is drained from a pool. The volume flow rate is to be determined.

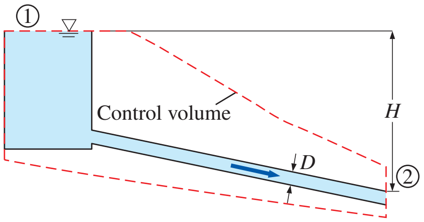


FIGURE 14–29
Schematic for Example 14–6.

Assumptions 1 The flow is incompressible and *quasi-steady*. 2 Entrance effects are negligible since the hose is so long; the flow is fully developed. 3 Any other losses such as elbows in the hose are negligible.

Properties The properties of water are $\rho = 998$ kg/m^3 and $\mu = 0.001002$ kg/m·s.

Analysis We choose the same (wise) control volume as in Example 14–1. The equations and analysis are in fact identical to the previous problem except that friction factor f is no longer 64/Re since we suspect that the flow will be turbulent since the hose diameter is larger. We must use instead either the Colebrook or Churchill equation to obtain f. Following the previous example, the energy equation reduces to

$$\alpha_2 \frac{V^2}{2g} = H - h_L \qquad \textbf{(1)}$$

For fully developed turbulent pipe flow at the outlet of the hose, $\alpha_2 = 1.05$. For turbulent flow, we cannot obtain a simple quadratic equation for average velocity V. Instead, we must solve Eq. (1) simultaneously with either the Colebrook or Churchill equation and the other equations defining Re and h_L from Example 14–1. This can be done "by hand" in similar fashion to Example 14–5 or with an equation solver. We choose the latter and calculate $V = 0.6536$ m/s using the Churchill equation. Finally, we calculate the volume flow rate,

$$\dot{V} = VA_c = V\frac{\pi D^2}{4} = (0.6536 \text{ m/s})\frac{\pi(0.020 \text{ m})^2}{4}\left(\frac{1000 \text{ L}}{\text{m}^3}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 12.32\frac{\text{L}}{\text{min}}$$

which we round to our final answer: $\dot{V} \approx$ **12.3 Lpm**

We check the Reynolds number,

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(998 \text{ kg/m}^3)(0.6536 \text{ m/s})(0.020 \text{ m})}{0.001002 \text{ kg/m·s}} = 13{,}020$$

Since Re > 4000, we verify that the flow is indeed turbulent.

Discussion If we use the Colebrook equation, the volume flow rate is 12.38 Lpm, only about 0.5 percent different from the Churchill result. Compared to Example 14–1, the flow rate is significantly higher, and the pool would drain much faster with the larger diameter hose, as expected.

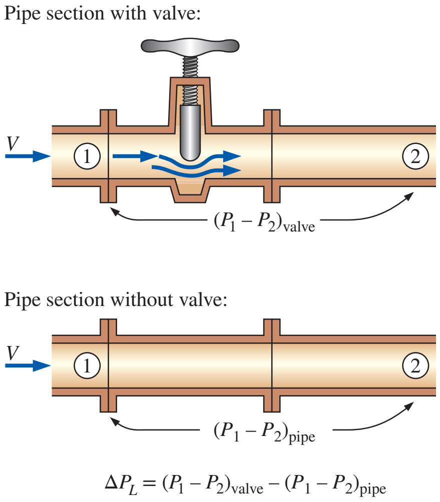


FIGURE 14–30
For a constant-diameter section of a pipe with a minor loss component, the loss coefficient of the component (such as the gate valve shown) is determined by measuring the additional pressure loss it causes and dividing it by the dynamic pressure in the pipe.

14–6 ▪ MINOR LOSSES

The fluid in a typical piping system passes through various fittings, valves, bends, elbows, tees, inlets, exits, expansions, and contractions in addition to the straight sections of piping. These components interrupt the smooth flow of the fluid and cause additional losses because of the flow separation and mixing they induce. In a typical system with long pipes, these losses are minor compared to the head loss in the straight sections (the **major losses**) and are called **minor losses**. Although this is generally true, in some cases the minor losses may be greater than the major losses. This is the case, for example, in systems with several turns and valves in a short distance. The head loss introduced by a completely open valve, for example, may be negligible. But a partially closed valve may cause the largest head loss in the system, as evidenced by the drop in the flow rate. Flow through valves and fittings is very complex, and a theoretical analysis is generally not plausible. Therefore, minor losses are determined experimentally, usually by the manufacturers of the components.

Minor losses are usually expressed in terms of the **loss coefficient** K_L (also called the **resistance coefficient**), defined as (Fig. 14–30)

Loss coefficient:
$$K_L = \frac{h_L}{V^2/(2g)} \qquad \textbf{(14–41)}$$

where h_L is the *additional* irreversible head loss in the piping system caused by insertion of the component, and is defined as $h_L = \Delta P_L/\rho g$. For example, imagine replacing the valve in Fig. 14–30 with a section of constant diameter pipe from location 1 to location 2. ΔP_L is defined as the pressure drop from 1 to 2 for the case *with* the valve, $(P_1 - P_2)_{\text{valve}}$, *minus* the pressure drop that would occur in the imaginary straight pipe section from 1 to 2 *without* the valve, $(P_1 - P_2)_{\text{pipe}}$ at the same flow rate. While the majority of the irreversible head loss occurs locally near the valve, some of it occurs downstream of the valve due to induced swirling turbulent eddies that are produced in the valve and continue downstream. These eddies "waste" mechanical energy because they are ultimately dissipated into heat while the flow in the downstream section of pipe eventually returns to fully developed conditions. When measuring minor losses in some minor loss components, such as *elbows,* for example, location 2 must be considerably far downstream (tens of pipe diameters) in order to fully account for the additional irreversible losses due to these decaying eddies.

When the pipe diameter downstream of the component *changes,* determination of the minor loss is even more complicated. In all cases, however, it is based on the *additional* irreversible loss of mechanical energy that would otherwise not exist if the minor loss component were not there. For simplicity, you may think of the minor loss as occurring *locally* across the minor loss component, but keep in mind that the component influences the flow for several pipe diameters downstream. By the way, this is the reason why most flow meter manufacturers recommend installing their flow meter at least 10 to 20 pipe diameters downstream of any elbows or valves—this allows the swirling turbulent eddies generated by the elbow or valve to largely disappear and the velocity profile to become fully developed before entering the flow meter. (Most flow meters are calibrated with a fully developed velocity profile at the flow meter inlet and yield the best accuracy when such conditions also exist in the actual application.)

When the inlet diameter equals the outlet diameter, the loss coefficient of a component can also be determined by measuring the pressure loss across the component and dividing it by the dynamic pressure, $K_L = \Delta P_L/(\frac{1}{2}\rho V^2)$. When the loss coefficient for a component is available, the head loss for that component is determined from

Minor loss:

$$h_L = K_L \frac{V^2}{2g} \tag{14–42}$$

The loss coefficient, in general, depends on the geometry of the component and the Reynolds number, just like the friction factor. However, it is usually assumed to be independent of the Reynolds number. This is a reasonable approximation since most flows in practice have large Reynolds numbers and the loss coefficients (including the friction factor) tend to be independent of the Reynolds number at large Reynolds numbers.

Minor losses are also expressed in terms of the **equivalent length** L_{equiv}, defined as (Fig. 14–31)

Equivalent length:

$$h_L = K_L \frac{V^2}{2g} = f \frac{L_{\text{equiv}}}{D} \frac{V^2}{2g} \quad \rightarrow \quad L_{\text{equiv}} = \frac{D}{f} K_L \tag{14–43}$$

where f is the friction factor and D is the diameter of the pipe that contains the component. The head loss caused by the component is equivalent to the head loss caused by a section of the pipe whose length is L_{equiv}. Therefore, the contribution of a component to the head loss is accounted for by simply adding L_{equiv} to the total pipe length.

Both approaches are used in practice, but the use of loss coefficients is more common. Therefore, we also use that approach in this book. Once all the loss coefficients are available, the total head loss in a piping system is determined from

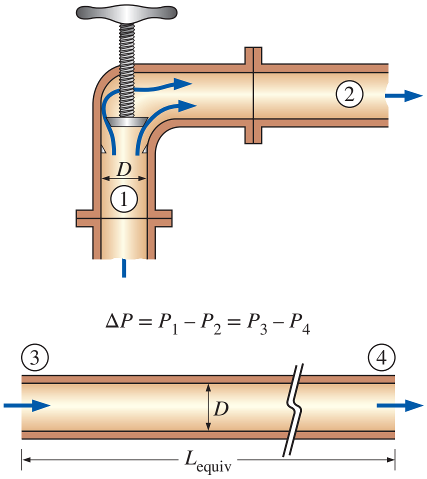


FIGURE 14–31
The head loss caused by a component (such as the angle valve shown) is equivalent to the head loss caused by a section of the pipe whose length is the equivalent length.

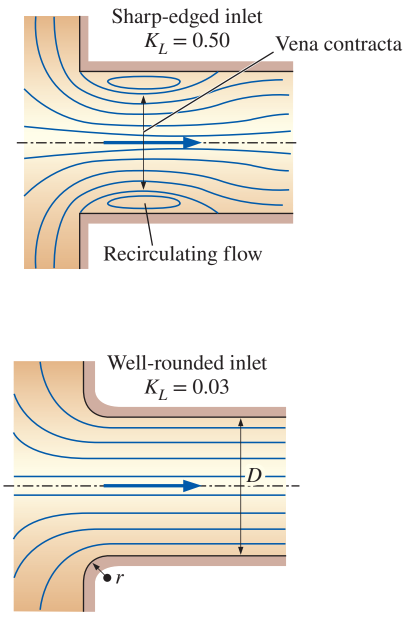


FIGURE 14–32
The head loss at the inlet of a pipe is almost negligible for well-rounded inlets ($K_L = 0.03$ for $r/D > 0.2$) but increases to about 0.50 for sharp-edged inlets.

Total head loss (general):
$$h_{L,\text{ total}} = h_{L,\text{ major}} + h_{L,\text{ minor}}$$
$$= \sum_i f_i \frac{L_i}{D_i}\frac{V_i^2}{2g} + \sum_j K_{L,j}\frac{V_j^2}{2g} \tag{14–44}$$

where i represents each pipe section with constant diameter and j represents each component that causes a minor loss. If the entire piping system being analyzed has a constant diameter, Eq. 14–44 reduces to

Total head loss (D = constant):
$$h_{L,\text{ total}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} \tag{14–45}$$

where V is the average flow velocity through the entire system (note that $V =$ constant since $D =$ constant).

Representative loss coefficients K_L are given in Table 14–4 for inlets, exits, bends, sudden and gradual area changes, and valves. There is considerable uncertainty in these values since the loss coefficients, in general, vary with the pipe diameter, the surface roughness, the Reynolds number, and the details of the design. The loss coefficients of two seemingly identical valves by two different manufacturers, for example, can differ by a factor of 2 or more. Therefore, the particular manufacturer's data should be consulted in the final design of piping systems rather than relying on the representative values in handbooks.

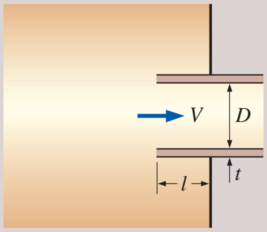
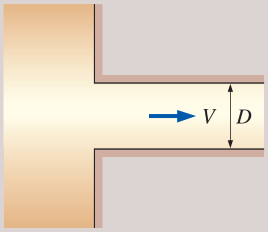
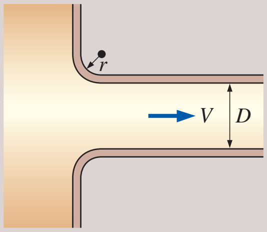
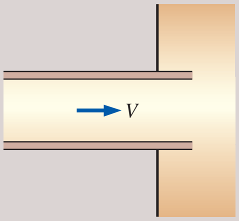
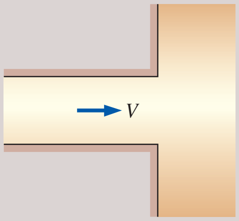
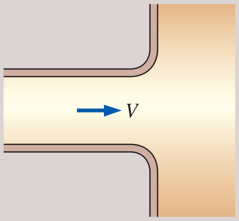
The head loss at the inlet of a pipe is a strong function of geometry. It is almost negligible for well-rounded inlets ($K_L = 0.03$ for $r/D > 0.2$) but increases to about 0.50 for sharp-edged inlets (Fig. 14–32). That is, a sharp-edged inlet causes half of the velocity head to be lost as the fluid enters the pipe. This is because the fluid cannot make sharp 90° turns easily, especially at high velocities. As a result, the flow separates at the corners, and the flow is constricted into the **vena contracta** region formed in the midsection of the pipe (Fig. 14–33). Therefore, a sharp-edged inlet acts like a flow constriction. The velocity increases in the vena contracta region (and the pressure decreases) because of the reduced effective flow area and then decreases as the flow fills the entire cross section of the pipe. There would be negligible loss if the pressure were increased in accordance with Bernoulli's equation (the velocity head would simply be converted into pressure head). However, this deceleration process is far from ideal, and the viscous dissipation caused by intense mixing and the turbulent eddies converts part of the kinetic energy into frictional heating, as evidenced by a slight rise in fluid temperature. The end result is a drop in velocity without much pressure recovery, and the inlet loss is a measure of this irreversible pressure drop.

Even slight rounding of the edges can result in significant reduction of K_L, as shown in Fig. 14–34. The loss coefficient rises sharply (to about $K_L = 0.8$) when the pipe protrudes into the reservoir since some fluid near the edge in this case is forced to make a 180° turn.

The loss coefficient for a submerged pipe exit is often listed in handbooks as $K_L = 1$. More precisely, however, K_L is equal to the kinetic energy correction factor α at the exit of the pipe. Although α is indeed close to 1 for fully developed *turbulent* pipe flow, it is equal to 2 for fully developed *laminar* pipe flow. To avoid possible errors when analyzing laminar pipe flow, then, it is best to always set $K_L = \alpha$ at a submerged pipe exit. At any such exit, whether laminar or turbulent, the fluid leaving the pipe loses *all* of its kinetic energy as it mixes with the reservoir fluid and eventually comes to rest through the irreversible action of viscosity. This is true regardless of the shape of the exit (Table 14–4 and Fig. 14–35). Therefore, there is no advantage to rounding off the sharp edges of pipe exits.

TABLE 14–4

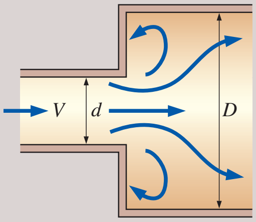
Loss coefficients K_L of various pipe components for turbulent flow (for use in the relation $h_L = K_L V^2/(2g)$, where V is the average velocity in the pipe that contains the component)*

Pipe Inlet *Reentrant:* $K_L = 0.80$ ($t \ll D$ and $l \approx 0.1D$)  	*Sharp-edged:* $K_L = 0.50$  	*Well-rounded* ($r/D > 0.2$): $K_L = 0.03$ *Slightly rounded* ($r/D = 0.1$): $K_L = 0.12$ (see Fig. 14–34) 
Pipe Exit *Reentrant:* $K_L = \alpha$  	*Sharp-edged:* $K_L = \alpha$  	*Rounded:* $K_L = \alpha$ 

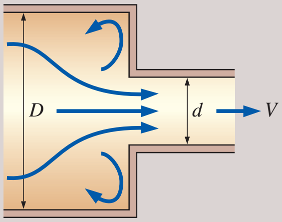
Note: The kinetic energy correction factor is $\alpha = 2$ for fully developed laminar flow, and $\alpha \approx 1.05$ for fully developed turbulent flow.

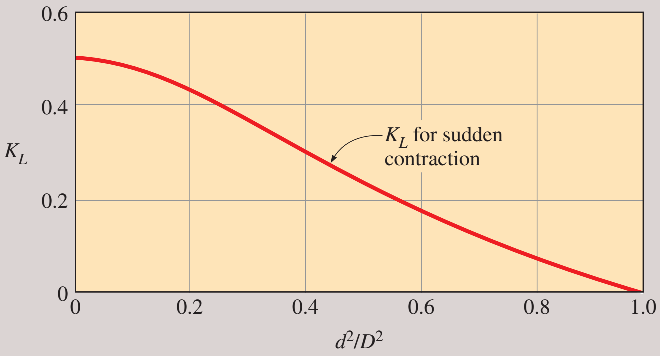
Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Sudden expansion: $K_L = \alpha\left(1 - \dfrac{d^2}{D^2}\right)^2$



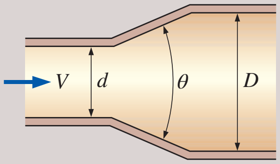
Sudden contraction: See chart.



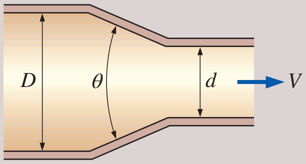


Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Expansion (for $\theta = 20°$):
$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$

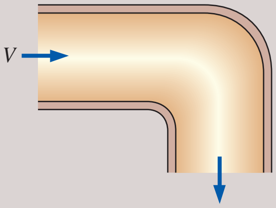
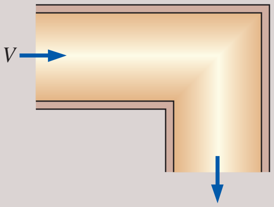
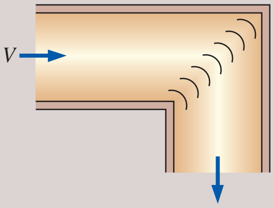
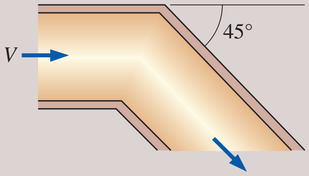
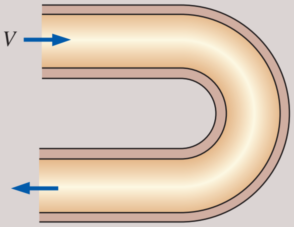
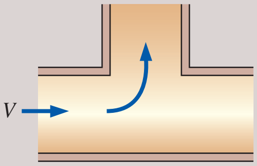
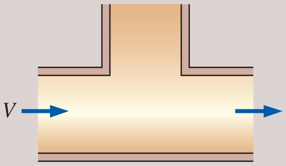
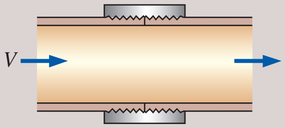


Contraction:
$K_L = 0.02$ for $\theta = 30°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$



(*continues*)

TABLE 14–4 (Continued)

Bends and Branches 90° *smooth bend:* Flanged: $K_L = 0.3$ Threaded: $K_L = 0.9$  	90° *miter bend* (without vanes): $K_L = 1.1$  	90° *miter bend* (with vanes): $K_L = 0.2$  	45° *threaded elbow:* $K_L = 0.4$ 
180° *return bend:* Flanged: $K_L = 0.2$ Threaded: $K_L = 1.5$  	*Tee* (branch flow): Flanged: $K_L = 1.0$ Threaded: $K_L = 2.0$  	*Tee* (line flow): Flanged: $K_L = 0.2$ Threaded: $K_L = 0.9$  	*Threaded union:* $K_L = 0.08$ 
Valves *Globe valve,* fully open: $K_L = 10$ *Angle valve,* fully open: $K_L = 5$ *Ball valve,* fully open: $K_L = 0.05$ *Swing check valve:* $K_L = 2$	*Gate valve,* fully open: $K_L = 0.2$ $\frac{1}{4}$ closed: $K_L = 0.3$ $\frac{1}{2}$ closed: $K_L = 2.1$ $\frac{3}{4}$ closed: $K_L = 17$		

*These are representative values for loss coefficients. Actual values strongly depend on the design and manufacture of the components and may differ from the given values considerably (especially for valves). Actual manufacturer's data should be used in the final design.

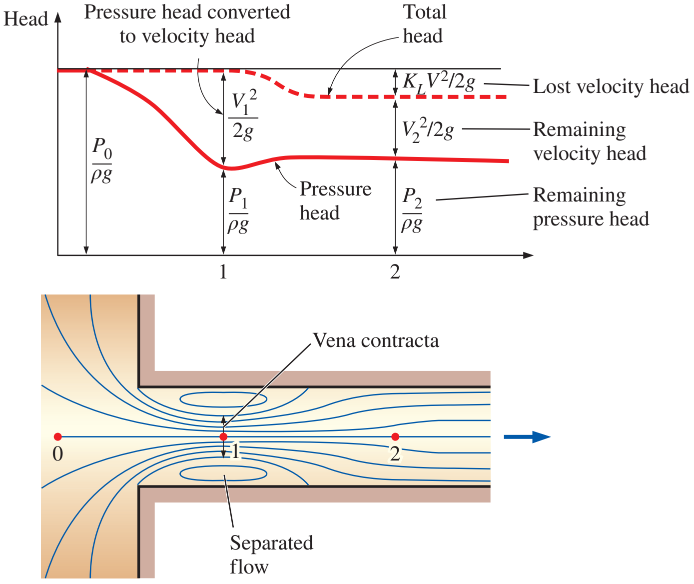


FIGURE 14–33
Graphical representation of flow contraction and the associated head loss at a sharp-edged pipe inlet.

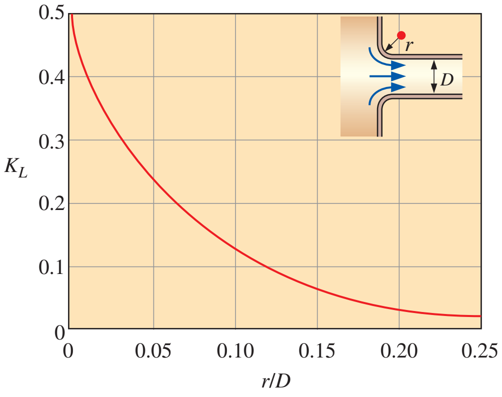


FIGURE 14–34
The effect of rounding of a pipe inlet on the loss coefficient.
Data from ASHRAE Handbook of Fundamentals.

Piping systems often involve *sudden* or *gradual* expansion or contraction sections to accommodate changes in flow rates or properties such as density and velocity. The losses are usually much greater in the case of *sudden* expansion and contraction (or wide-angle expansion) because of flow separation. By combining the equations of mass, momentum, and energy balance, the loss coefficient for the case of a **sudden expansion** is approximated as

$$K_L = \alpha\left(1 - \frac{A_{\text{small}}}{A_{\text{large}}}\right)^2 \qquad \text{(sudden expansion)} \qquad \textbf{(14–46)}$$

where A_{small} and A_{large} are the cross-sectional areas of the small and large pipes, respectively. Note that $K_L = 0$ when there is no area change ($A_{\text{small}} = A_{\text{large}}$) and $K_L = \alpha$ when a pipe discharges into a reservoir ($A_{\text{large}} \gg A_{\text{small}}$). No such relation exists for a sudden contraction, and the K_L values in that case must be read from a chart or table (e.g., Table 14–4). The losses due to expansions and contractions can be reduced significantly by installing conical gradual area changers (nozzles and diffusers) between the small and large pipes. The K_L values for representative cases of gradual expansion and contraction are given in Table 14–4. Note that in head loss calculations, the velocity in the *small pipe* is to be used as the reference velocity in Eq. 14–42. Losses during expansion are usually much higher than the losses during contraction because of flow separation.

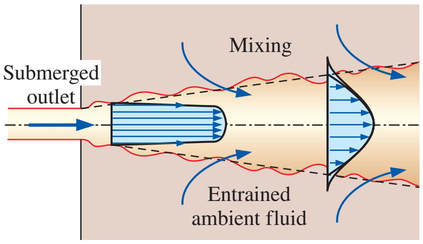


FIGURE 14–35
All the kinetic energy of the flow is "lost" (turned into thermal energy) through friction as the jet decelerates and mixes with ambient fluid downstream of a submerged outlet.

Piping systems also involve changes in direction without a change in diameter, and such flow sections are called *bends* or *elbows*. The losses in these devices are due to flow separation (just like a car being thrown off the road when it enters a turn too fast) on the inner side and the swirling secondary flows that result. The losses during changes of direction can be minimized by making the turn "easy" on the fluid by using circular arcs (like 90° elbows) instead of sharp turns (like miter bends) (Fig. 14–36). But the use of sharp turns (and the resulting penalty in loss coefficient) may be necessary when the turning space is limited. In such cases, the losses can be minimized by utilizing properly placed guide vanes to help the flow turn in an orderly manner without being thrown off the course. The loss coefficients for some elbows and miter bends as well as tees are given in Table 14–4. These coefficients do not include the frictional losses along the pipe bend. Such losses should be calculated as in straight pipes (using the length of the centerline as the pipe length) and added to other losses.

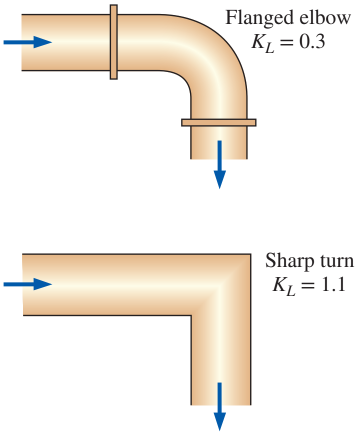


FIGURE 14–36
The losses during changes of direction can be minimized by making the turn "easy" on the fluid by using circular arcs instead of sharp turns.

Valves are commonly used in piping systems to control flow rates by simply altering the head loss until the desired flow rate is achieved. For valves it is desirable to have a very low loss coefficient when they are fully open, such as with a *ball valve,* so that they cause minimal head loss during full-load operation

(Fig. 14–37*b*). Several different valve designs, each with its own advantages and disadvantages, are in common use today. The *gate valve* slides up and down like a gate, the *globe valve* (Fig. 14–37*a*) closes a hole placed in the valve, the *angle valve* is a globe valve with a 90° turn, and the *check valve* allows the fluid to flow only in one direction like a diode in an electric circuit. Table 14–4 lists the representative loss coefficients of the popular designs. Note that the loss coefficient increases drastically as a valve is closed. Also, the deviation in the loss coefficients for different manufacturers is greatest for valves because of their complex geometries.

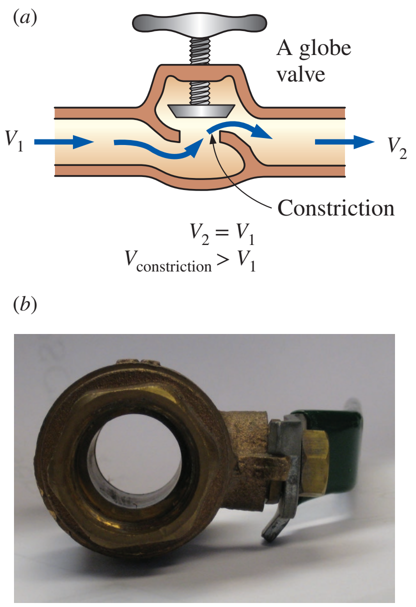


FIGURE 14–37
(*a*) The large head loss in a partially closed globe valve is due to irreversible deceleration, flow separation, and mixing of high-velocity fluid coming from the narrow valve passage. (*b*) The head loss through a fully-open ball valve, on the other hand, is quite small.
John M. Cimbala

EXAMPLE 14–7 Head Loss and Pressure Rise During Gradual Expansion

A 6-cm-diameter horizontal water pipe expands gradually to a 9-cm-diameter pipe (Fig. 14–38). The walls of the expansion section are angled 10° from the axis. The average velocity and pressure of water before the expansion section are 7 m/s and 150 kPa, respectively. Determine the head loss in the expansion section and the pressure in the larger-diameter pipe.

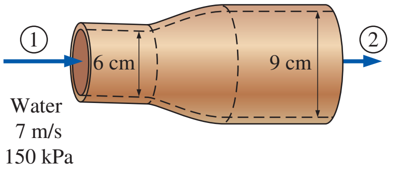


FIGURE 14–38
Schematic for Example 14–7.

SOLUTION A horizontal water pipe expands gradually into a larger-diameter pipe. The head loss and pressure after the expansion are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The flow at sections 1 and 2 is fully developed and turbulent with $\alpha_1 = \alpha_2 \cong 1.06$.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$. The loss coefficient for a gradual expansion of total included angle $\theta = 20°$ and diameter ratio $d/D = 6/9$ is $K_L = 0.133$ (by interpolation using Table 14–4).

Analysis Noting that the density of water remains constant, the downstream velocity of water is determined from conservation of mass to be

$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho V_1 A_1 = \rho V_2 A_2 \quad \rightarrow \quad V_2 = \frac{A_1}{A_2} V_1 = \frac{D_1^2}{D_2^2} V_1$$

$$V_2 = \frac{(0.06\ \text{m})^2}{(0.09\ \text{m})^2}(7\ \text{m/s}) = 3.11\ \text{m/s}$$

Then the irreversible head loss in the expansion section becomes

$$h_L = K_L \frac{V_1^2}{2g} = (0.133)\frac{(7\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = \mathbf{0.333\ m}$$

Noting that $z_1 = z_2$ and there are no pumps or turbines involved, the energy equation for the expansion section is expressed in terms of heads as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + \cancel{z_1} + \cancelto{0}{h_{\text{pump}, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + \cancel{z_2} + \cancelto{0}{h_{\text{turbine, e}}} + h_L$$

or

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + h_L$$

Solving for P_2 and substituting,

$$P_2 = P_1 + \rho\left\{\frac{\alpha_1 V_1^2 - \alpha_2 V_2^2}{2} - g h_L\right\} = 150\ \text{kPa} + (1000\ \text{kg/m}^3)$$

$$\times \left\{\frac{1.06\,(7\ \text{m/s})^2 - 1.06\,(3.11\ \text{m/s})^2}{2} - (9.81\ \text{m/s}^2)(0.333\ \text{m})\right\}$$

$$\times \left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)$$

$$= \mathbf{168\ kPa}$$

Therefore, despite the head (and pressure) loss, the pressure *increases* from 150 to 168 kPa after the expansion. This is due to the conversion of dynamic pressure to static pressure when the average flow velocity is decreased in the larger pipe.

Discussion It is common knowledge that higher pressure upstream is necessary to cause flow, and it may come as a surprise to you that the downstream pressure has *increased* after the expansion, despite the loss. This is because the flow is driven by the sum of the three heads that comprise the total head (namely, pressure head, velocity head, and elevation head). During flow expansion, the higher-velocity head upstream is converted to pressure head downstream, and this increase outweighs the nonrecoverable head loss. Also, you may be tempted to solve this problem using the Bernoulli equation. Such a solution would ignore the head loss (and the associated pressure loss) and result in an incorrect higher pressure for the fluid downstream.

14–7 ▪ PIPING NETWORKS AND PUMP SELECTION

Series and Parallel Pipes

Most piping systems encountered in practice such as the water distribution systems in cities or commercial or residential establishments involve numerous parallel and series connections as well as several sources (supply of fluid into the system) and loads (discharges of fluid from the system) (Fig. 14–39). A piping project may involve the design of a new system or the expansion of an existing system. The engineering objective in such projects is to design a piping system that will reliably deliver the specified flow rates at specified pressures at minimum total (initial plus operating and maintenance) cost. Once the layout of the system is prepared, the determination of the pipe diameters and the pressures throughout the system, while remaining within the budget constraints, typically requires solving the system repeatedly until the optimal solution is reached. Computer modeling and analysis of such systems make this tedious task a simple chore.

FIGURE 14–39

A piping network in an industrial facility.

Vithun Khamsong/Getty Images

Piping systems typically involve several pipes connected to each other in series and/or in parallel, as shown in Figs. 14–40 and 14–41. When the pipes are connected **in series**, the flow rate through the entire system remains constant regardless of the diameters of the individual pipes in the system. This is a natural consequence of the conservation of mass principle for steady incompressible flow. The total head loss in this case is equal to the sum of the head losses in individual pipes in the system, including the minor losses. The expansion or contraction losses at connections are considered to belong to the smaller-diameter pipe since the expansion and contraction loss coefficients are defined on the basis of the average velocity in the smaller-diameter pipe.

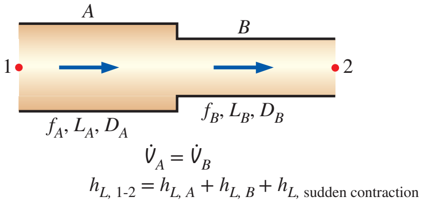


FIGURE 14–40

For pipes *in series,* the flow rate is the same in each pipe, and the total head loss is the sum of the head losses in the individual pipes.

For a pipe that branches out into two (or more) **parallel pipes** and then rejoins at a junction downstream, the total flow rate is the sum of the flow rates in the individual pipes. The pressure drop (or head loss) in each individual pipe connected in parallel must be the same since $\Delta P = P_A - P_B$ and the junction pressures P_A and P_B are the same for all the individual pipes. For a system of two parallel pipes 1 and 2 between junctions A and B with negligible minor losses, this is expressed as

$$h_{L,1} = h_{L,2} \qquad \rightarrow \qquad f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g}$$

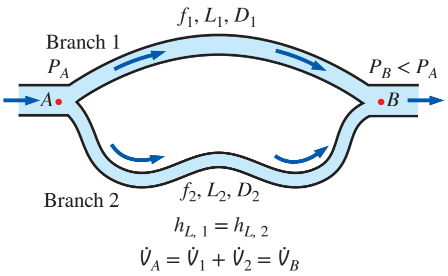


FIGURE 14–41

For pipes *in parallel,* the head loss is the same in each pipe, and the total flow rate is the sum of the flow rates in individual pipes.

Then the ratio of the average velocities and the flow rates in the two parallel pipes become

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2} \qquad \text{and} \qquad \frac{\dot{V}_1}{\dot{V}_2} = \frac{A_{c,1}V_1}{A_{c,2}V_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2}$$

Therefore, the relative flow rates in parallel pipes are established from the requirement that the head loss in each pipe be the same. This result can be extended to any number of pipes connected in parallel. The result is also valid for pipes for which the minor losses are significant if the equivalent lengths for components that contribute to minor losses are added to the pipe length. Note that the flow rate in one of the parallel branches is proportional to its diameter to the power 5/2 and is inversely proportional to the square root of its length and friction factor.

The analysis of piping networks, no matter how complex they are, is based on two simple principles:

1. *Conservation of mass throughout the system must be satisfied.* This is done by requiring the total flow into a junction to be equal to the total flow out of the junction for all junctions in the system. Also, the flow rate must remain constant in pipes connected in series regardless of the changes in diameters.
2. *Pressure drop (and thus head loss) between two junctions must be the same for all paths between the two junctions.* This is because pressure is a point function and it cannot have two values at a specified point. In practice this rule is used by requiring that the algebraic sum of head losses in a loop (for all loops) be equal to zero. (A head loss is taken to be positive for flow in the clockwise direction and negative for flow in the counterclockwise direction.)

Therefore, the analysis of piping networks is very similar to the analysis of electric circuits (Kirchhoff's laws), with flow rate corresponding to electric current and pressure corresponding to electric potential. However, the situation is much more complex here since, unlike the electrical resistance, the "flow resistance" is a highly nonlinear function. Therefore, the analysis of piping networks requires the simultaneous solution of a system of nonlinear equations, which requires software such as EES, Excel, Mathcad, Matlab, etc., or commercially available software designed specifically for such applications.

Piping Systems with Pumps and Turbines

When a piping system involves a pump and/or turbine, the steady-flow energy equation on a unit-mass basis is expressed as (see Section 12–2)

$$\frac{P_1}{\rho} + \alpha_1\frac{V_1^2}{2} + gz_1 + w_{\text{pump, }u} = \frac{P_2}{\rho} + \alpha_2\frac{V_2^2}{2} + gz_2 + w_{\text{turbine, }e} + gh_L \tag{14–47}$$

or in terms of heads as

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{\text{pump, }u} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{\text{turbine, }e} + h_L \tag{14–48}$$

where $h_{\text{pump, }u} = w_{\text{pump, }u}/g$ is the useful pump head delivered to the fluid, $h_{\text{turbine, }e} = w_{\text{turbine, }e}/g$ is the turbine head extracted from the fluid, α is the kinetic energy correction factor whose value is about 1.05 for most (turbulent) flows encountered in practice, and h_L is the total head loss in the piping (including the minor losses if they are significant) between points 1 and 2. The pump head is zero if the piping system does not involve a pump or a fan, the turbine head is zero if the system does not involve a turbine, and both are zero if the system does not involve any mechanical work-producing or work-consuming devices.

Many practical piping systems involve a pump to move a fluid from one reservoir to another. Taking points 1 and 2 to be at the *free surfaces* of the reservoirs (Fig. 14–42), the energy equation is solved for the required useful pump head, yielding

$$h_{\text{pump}, u} = (z_2 - z_1) + h_L \tag{14–49}$$

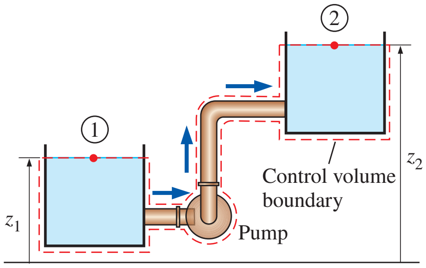
since the velocities at free surfaces are negligible for large reservoirs and the pressures are at atmospheric pressure. Therefore, the useful pump head is equal to the elevation difference between the two reservoirs plus the head loss. If the head loss is negligible compared to $z_2 - z_1$, the useful pump head is equal to the elevation difference between the two reservoirs. In the case of $z_1 > z_2$ (the first reservoir being at a higher elevation than the second one) with no pump, the flow is driven by gravity at a flow rate that causes a head loss equal to the elevation difference. A similar argument can be given for the turbine head for a hydroelectric power plant by replacing $h_{\text{pump}, u}$ in Eq. 14–49 by $-h_{\text{turbine}, e}$.

Once the useful pump head is known, the *mechanical power that needs to be delivered by the pump to the fluid* and the *electric power consumed by the motor of the pump* for a specified flow rate are determined from

$$\dot{W}_{\text{pump, shaft}} = \frac{\rho \dot{V} g h_{\text{pump}, u}}{\eta_{\text{pump}}} \quad \text{and} \quad \dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump}, u}}{\eta_{\text{pump–motor}}} \tag{14–50}$$

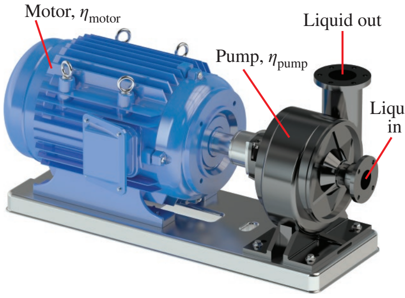
where $\eta_{\text{pump–motor}}$ is the *efficiency of the pump–motor combination,* which is the product of the pump and the motor efficiencies (Fig. 14–43). The pump–motor efficiency is defined as the ratio of the net mechanical energy delivered to the fluid by the pump to the electric energy consumed by the motor of the pump, and it typically ranges between 50 and 85 percent.

The head loss of a piping system increases (usually quadratically) with the flow rate. A plot of required useful pump head $h_{\text{pump}, u}$ as a function of flow rate is called the **system** (or **demand**) **curve**. The head produced by a pump is not a constant either. Both the pump head and the pump efficiency vary with the flow rate, and pump manufacturers supply this variation in tabular or graphical form, as shown in Fig. 14–44. These experimentally determined $h_{\text{pump}, u}$ and $\eta_{\text{pump}, u}$ versus $\dot{V}$ curves are called **characteristic** (or **supply** or **performance**) **curves**. Note that the flow rate of a pump increases as the required head decreases. The intersection point of the pump head curve with the vertical axis typically represents the *maximum head*



$$h_{\text{pump}, u} = (z_2 - z_1) + h_L$$
$$\dot{W}_{\text{pump}, u} = \rho \dot{V} g h_{\text{pump}, u}$$

FIGURE 14–42
When a pump moves a fluid from one reservoir to another, the useful pump head requirement is equal to the elevation difference between the two reservoirs plus the head loss.



$$\eta_{\text{pump–motor}} = \eta_{\text{pump}} \eta_{\text{motor}}$$

FIGURE 14–43
The efficiency of the pump–motor combination is the product of the pump and the motor efficiencies.
Yunus Cengel

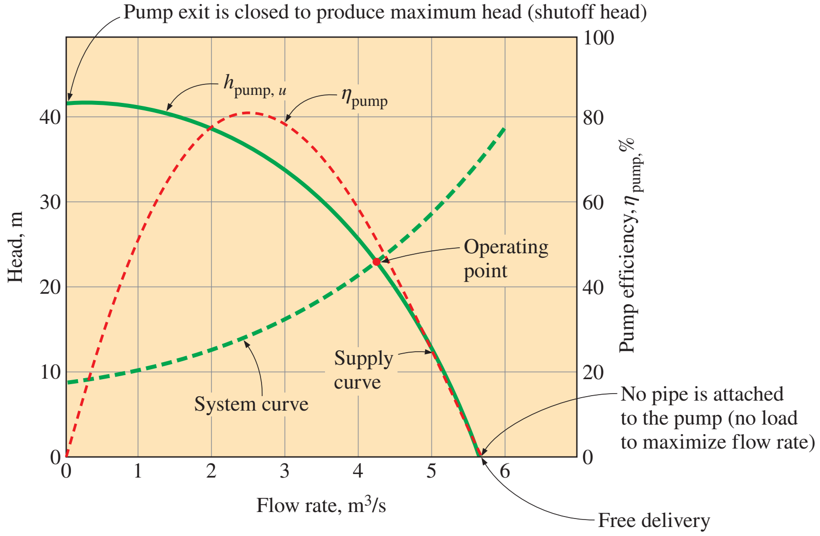


FIGURE 14–44
Characteristic pump curves for centrifugal pumps, the system curve for a piping system, and the operating point.

(called the **shutoff head**) the pump can provide, while the intersection point with the horizontal axis indicates the *maximum flow rate* (called the **free delivery**) that the pump can supply.

The *efficiency* of a pump is highest at a certain combination of head and flow rate. Therefore, a pump that can supply the required head and flow rate is not necessarily a good choice for a piping system unless the efficiency of the pump at those conditions is sufficiently high. The pump installed in a piping system will operate at the point where the *system curve* and the *characteristic curve* intersect. This point of intersection is called the **operating point**, as shown in Fig. 14–41. The useful head produced by the pump at this point matches the head requirements of the system at that flow rate. Also, the efficiency of the pump during operation is the value corresponding to that flow rate.

EXAMPLE 14–8 Pumping Water Through Two Parallel Pipes

Water at 20°C is to be pumped from a reservoir (z_A = 5 m) to another reservoir at a higher elevation (z_B = 13 m) through two 36-m-long pipes connected in parallel, as shown in Fig. 14–45. The pipes are made of commercial steel, and the diameters of the two pipes are 4 and 8 cm. Water is to be pumped by a 70 percent efficient motor–pump combination that draws 8 kW of electric power during operation. The minor losses and the head loss in pipes that connect the parallel pipes to the two reservoirs are considered to be negligible. Determine the total flow rate between the reservoirs and the flow rate through each of the parallel pipes.

SOLUTION The pumping power input to a piping system with two parallel pipes is given. The flow rates are to be determined.

Assumptions **1** The flow is steady (since the reservoirs are large) and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The elevations of the reservoirs remain constant. **4** The minor losses and the head loss in pipes other than the parallel pipes are negligible. **5** Flows through both pipes are turbulent (to be verified).

Properties The density and dynamic viscosity of water at 20°C are $\rho = 998$ kg/m^3 and $\mu = 1.002 \times 10^{-3}$ kg/m·s. The roughness of commercial steel pipe is $\varepsilon = 0.000045$ m (Table 14–3).

Analysis This problem cannot be solved directly since the velocities (or flow rates) in the pipes are not known. Therefore, we would normally use a trial-and-error approach here. However, equation solvers are widely available, and thus, we simply set up the equations to be solved by an equation solver. The useful head supplied by the pump to the fluid is determined from

$$\dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump},\,u}}{\eta_{\text{pump–motor}}} \quad \rightarrow \quad 8000 \text{ W} = \frac{(998 \text{ kg/m}^3)\dot{V}(9.81 \text{ m/s}^2)h_{\text{pump},\,u}}{0.70} \tag{1}$$

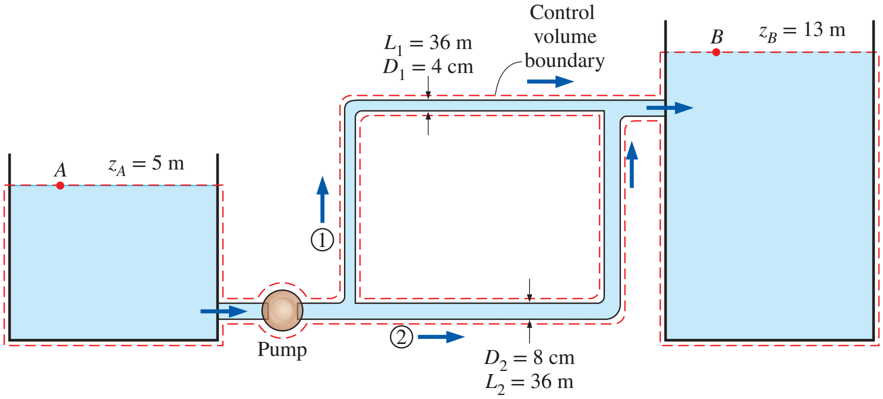


FIGURE 14–45
The piping system discussed in Example 14–8.

We choose points A and B at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_A = P_B = P_{\text{atm}}$) and that the fluid velocities at both points are nearly zero ($V_A \approx V_B \approx 0$) since the reservoirs are large, the energy equation for a control volume between these two points simplifies to

$$\cancel{\frac{P_A}{\rho g}} + \alpha_A \cancelto{0}{\frac{V_A^2}{2g}} + z_A + h_{\text{pump},\,u} = \cancel{\frac{P_B}{\rho g}} + \alpha_B \cancelto{0}{\frac{V_B^2}{2g}} + z_B + h_L$$

or

$$h_{\text{pump},\,u} = (z_B - z_A) + h_L$$

or

$$h_{\text{pump},\,u} = (13\text{ m} - 5\text{ m}) + h_L \qquad \textbf{(2)}$$

where

$$h_L = h_{L,1} = h_{L,2} \qquad \textbf{(3)(4)}$$

We designate the 4-cm-diameter pipe by 1 and the 8-cm-diameter pipe by 2. Equations for the average velocity, the Reynolds number, the friction factor, and the head loss in each pipe are

$$V_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \quad \rightarrow \quad V_1 = \frac{\dot{V}_1}{\pi(0.04\text{ m})^2/4} \qquad \textbf{(5)}$$

$$V_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \quad \rightarrow \quad V_2 = \frac{\dot{V}_2}{\pi(0.08\text{ m})^2/4} \qquad \textbf{(6)}$$

$$\text{Re}_1 = \frac{\rho V_1 D_1}{\mu} \quad \rightarrow \quad \text{Re}_1 = \frac{(998\text{ kg/m}^3)V_1(0.04\text{ m})}{1.002 \times 10^{-3}\text{ kg/m·s}} \qquad \textbf{(7)}$$

$$\text{Re}_2 = \frac{\rho V_2 D_2}{\mu} \quad \rightarrow \quad \text{Re}_2 = \frac{(998\text{ kg/m}^3)V_2(0.08\text{ m})}{1.002 \times 10^{-3}\text{ kg/m·s}} \qquad \textbf{(8)}$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$
$$\rightarrow \quad \frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{0.000045}{3.7 \times 0.04} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \qquad \textbf{(9)}$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right)$$
$$\rightarrow \quad \frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{0.000045}{3.7 \times 0.08} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \qquad \textbf{(10)}$$

$$h_{L,1} = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} \quad \rightarrow \quad h_{L,1} = f_1 \frac{36\text{ m}}{0.04\text{ m}}\frac{V_1^2}{2(9.81\text{ m/s}^2)} \qquad \textbf{(11)}$$

$$h_{L,2} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} \quad \rightarrow \quad h_{L,2} = f_2 \frac{36\text{ m}}{0.08\text{ m}}\frac{V_2^2}{2(9.81\text{ m/s}^2)} \qquad \textbf{(12)}$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \qquad \textbf{(13)}$$

This is a system of 13 equations in 13 unknowns, and their simultaneous solution by an equation solver gives

$$\dot{V} = \mathbf{0.0300\ m^3/s}, \qquad \dot{V}_1 = \mathbf{0.00415\ m^3/s}, \qquad \dot{V}_2 = \mathbf{0.0259\ m^3/s}$$

$$V_1 = 3.30\text{ m/s}, \quad V_2 = 5.15\text{ m/s}, \quad h_L = h_{L,1} = h_{L,2} = 11.1\text{ m}, \quad h_{\text{pump}} = 19.1\text{ m}$$

$$\text{Re}_1 = 131{,}600, \quad \text{Re}_2 = 410{,}000, \quad f_1 = 0.0221, \quad f_2 = 0.0182$$

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

Discussion The two parallel pipes have the same length and roughness, but the diameter of the first pipe is half the diameter of the second one. Yet only 14 percent of the water flows through the first pipe. This shows the strong dependence of the flow rate on diameter. Also, it can be shown that if the free surfaces of the two reservoirs were at the same elevation (and thus $z_A = z_B$), the flow rate would increase by 20 percent from 0.0300 to 0.0361 m³/s. Alternately, if the reservoirs were as given but the irreversible head losses were negligible, the flow rate would become 0.0715 m³/s (an increase of 138 percent).

EXAMPLE 14–9 Gravity-Driven Water Flow in a Pipe

Water at 10°C flows from a large reservoir to a smaller one through a 5-cm-diameter cast iron piping system, as shown in Fig. 14–46. Determine the elevation z_1 for a flow rate of 6 L/s.

SOLUTION The flow rate through a piping system connecting two reservoirs is given. The elevation of the source is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The elevations of the reservoirs remain constant. **3** There are no pumps or turbines in the line.

Properties The density and dynamic viscosity of water at 10°C are $\rho = 999.7\text{ kg/m}^3$ and $\mu = 1.307 \times 10^{-3}$ kg/m·s. The roughness of cast iron pipe is $\varepsilon = 0.00026$ m (Table 14–3).

Analysis The piping system involves 89 m of piping, a sharp-edged entrance ($K_L = 0.5$), two standard flanged elbows ($K_L = 0.3$ each), a fully open gate valve ($K_L = 0.2$), and a submerged exit ($K_L = 1.06$). We choose points 1 and 2 at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{\text{atm}}$) and that the fluid velocities at both points are nearly zero ($V_1 \approx V_2 \approx 0$), the energy equation for a control volume between these two points simplifies to

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancelto{0}{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancelto{0}{\frac{V_2^2}{2g}} + z_2 + h_L \quad \rightarrow \quad z_1 = z_2 + h_L$$

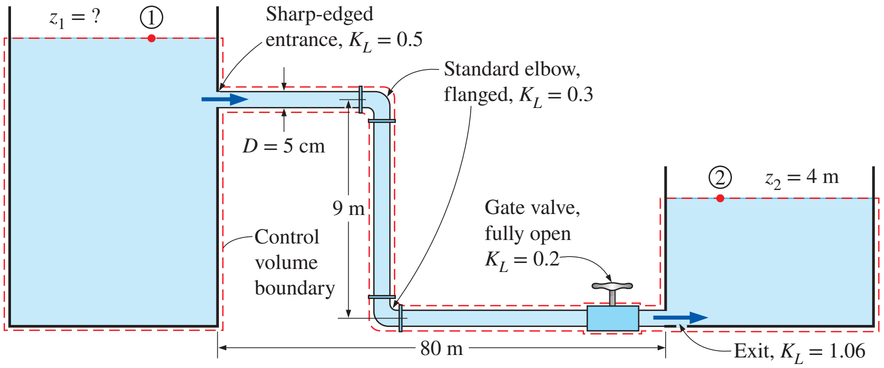


FIGURE 14–46
The piping system discussed in Example 14–9.

where

$$h_L = h_{L,\text{ total}} = h_{L,\text{ major}} + h_{L,\text{ minor}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.006\text{ m}^3/\text{s}}{\pi(0.05\text{ m})^2/4} = 3.06\text{ m/s}$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(999.7\text{ kg/m}^3)(3.06\text{ m/s})(0.05\text{ m})}{1.307\times10^{-3}\text{ kg/m·s}} = 117{,}000$$

The flow is turbulent since Re > 4000. Noting that $\varepsilon/D = 0.00026/0.05 = 0.0052$, the friction factor is determined from the Colebrook equation (or the Moody chart),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad\rightarrow\quad \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.0052}{3.7} + \frac{2.51}{117{,}000\sqrt{f}}\right)$$

It gives $f = 0.0315$. The sum of the loss coefficients is

$$\begin{aligned}\sum K_L &= K_{L,\text{ entrane}} + 2K_{L,\text{ elbow}} + K_{L,\text{ valve}} + K_{L,\text{ exit}}\\ &= 0.5 + 2\times0.3 + 0.2 + 1.06 = 2.36\end{aligned}$$

Then the total head loss and the elevation of the source become

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} = \left(0.0315\frac{89\text{ m}}{0.05\text{ m}} + 2.36\right)\frac{(3.06\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 27.9\text{ m}$$

$$z_1 = z_2 + h_L = 4 + 27.9 = \mathbf{31.9\text{ m}}$$

Therefore, the free surface of the first reservoir must be 31.9 m above the ground level to ensure water flow between the two reservoirs at the specified rate.

Discussion Note that $fL/D = 56.1$ in this case, which is about 24 times the total minor loss coefficient. Therefore, ignoring the sources of minor losses in this case would result in about 4 percent error. It can be shown that at the same flow rate, the total head loss would be 35.9 m (instead of 27.9 m) if the valve were three-fourths closed, and it would drop to 24.8 m if the pipe between the two reservoirs were straight at the ground level (thus eliminating the elbows and the vertical section of the pipe). The head loss could be reduced further (from 24.8 to 24.6 m) by rounding the entrance. The head loss can be reduced significantly (from 27.9 to 16.0 m) by replacing the cast iron pipes with smooth pipes such as those made of plastic.

SUMMARY

In *internal flow,* a pipe is completely filled with a fluid. *Laminar flow* is characterized by smooth streamlines and highly ordered motion, and *turbulent flow* is characterized by unsteady disorderly velocity fluctuations and highly disordered motion. The *Reynolds number* is defined as

$$\text{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{\text{avg}}D}{\nu} = \frac{\rho V_{\text{avg}}D}{\mu}$$

Under most practical conditions, the flow in a pipe is laminar at Re < 2300, turbulent at Re > 4000, and transitional in between.

The region of the flow in which the effects of the viscous shearing forces are felt is called the *velocity boundary layer.* The region from the pipe inlet to the point at which the flow becomes fully developed is called the *hydrodynamic entrance region,* and the length of this region is called the *hydrodynamic entry length* L_h. It is given by

$$\frac{L_{h,\text{ laminar}}}{D} \cong 0.05\text{ Re} \quad\text{and}\quad \frac{L_{h,\text{ turbulent}}}{D} \cong 10$$

The friction coefficient in the fully developed flow region is constant. The *maximum* and *average* velocities in fully developed laminar flow in a circular pipe are

$$u_{\max} = 2V_{\text{avg}} \quad\text{and}\quad V_{\text{avg}} = \frac{\Delta PD^2}{32\mu L}$$

The *volume flow rate* and the *pressure drop* for laminar flow in a horizontal pipe are

$$\dot{V} = V_{\text{avg}} A_c = \frac{\Delta P \pi D^4}{128\mu L} \quad \text{and} \quad \Delta P = \frac{32\mu L V_{\text{avg}}}{D^2}$$

The *pressure loss* and *head loss* for all types of internal flows (laminar or turbulent, in circular or noncircular pipes, smooth or rough surfaces) are expressed as

$$\Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} \quad \text{and} \quad h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g}$$

where $\rho V^2/2$ is the *dynamic pressure* and the dimensionless quantity f is the *friction factor*. For fully developed laminar flow in a round pipe, the friction factor is $f = 64/\text{Re}$.

For noncircular pipes, the diameter in the previous relations is replaced by the *hydraulic diameter* defined as $D_h = 4A_c/p$, where A_c is the cross-sectional area of the pipe and p is its wetted perimeter.

In fully developed turbulent flow, the friction factor depends on the Reynolds number and the *relative roughness* ε/D. The friction factor in turbulent flow is given by the *Colebrook equation,* expressed as

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

The plot of this formula is known as the *Moody chart*. The design and analysis of piping systems involve the determination of the head loss, flow rate, or the pipe diameter. Tedious iterations in these calculations can be avoided by the approximate Swamee–Jain formulas expressed as

$$h_L = 1.07\frac{\dot{V}^2 L}{gD^5}\left\{\ln\left[\frac{\varepsilon}{3.7D} + 4.62\left(\frac{\nu D}{\dot{V}}\right)^{0.9}\right]\right\}^{-2}$$

$$10^{-6} < \varepsilon/D < 10^{-2}$$
$$3000 < \text{Re} < 3 \times 10^8$$

$$\dot{V} = -0.965\left(\frac{gD^5 h_L}{L}\right)^{0.5} \ln\left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17\nu^2 L}{gD^3 h_L}\right)^{0.5}\right]$$

$$\text{Re} > 2000$$

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04}$$

$$10^{-6} < \varepsilon/D < 10^{-2}$$
$$5000 < \text{Re} < 3 \times 10^8$$

The losses that occur in piping components such as fittings, valves, bends, elbows, tees, inlets, exits, expansions, and contractions are called *minor losses*. The minor losses are usually expressed in terms of the *loss coefficient* K_L. The head loss for a component is determined from

$$h_L = K_L\frac{V^2}{2g}$$

When all the loss coefficients are available, the total head loss in a piping system is

$$h_{L,\text{ total}} = h_{L,\text{ major}} + h_{L,\text{ minor}} = \sum_i f_i \frac{L_i}{D_i}\frac{V_i^2}{2g} + \sum_j K_{L,j}\frac{V_j^2}{2g}$$

If the entire piping system is of constant diameter, the total head loss reduces to

$$h_{L,\text{ total}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

The analysis of a piping system is based on two simple principles: (1) The conservation of mass throughout the system must be satisfied and (2) the pressure drop between two points must be the same for all paths between the two points. When the pipes are connected *in series,* the flow rate through the entire system remains constant regardless of the diameters of the individual pipes. For a pipe that branches out into two (or more) *parallel pipes* and then rejoins at a junction downstream, the total flow rate is the sum of the flow rates in the individual pipes but the head loss in each branch is the same.

When a piping system involves a pump and/or turbine, the steady-flow energy equation is expressed as

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{\text{pump},\, u} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{\text{turbine},\, e} + h_L$$

When the useful pump head $h_{\text{pump},\, u}$ is known, the mechanical power that needs to be supplied by the pump to the fluid and the electric power consumed by the motor of the pump for a specified flow rate are

$$\dot{W}_{\text{pump, shaft}} = \frac{\rho\dot{V}g\,h_{\text{pump},\, u}}{\eta_{\text{pump}}} \quad \text{and} \quad \dot{W}_{\text{elect}} = \frac{\rho\dot{V}g\,h_{\text{pump},\, u}}{\eta_{\text{pump–motor}}}$$

where $\eta_{\text{pump–motor}}$ is the *efficiency of the pump–motor combination,* which is the product of the pump and the motor efficiencies.

The plot of the head loss versus the flow rate $\dot{V}$ is called the *system curve*. The head produced by a pump is not a constant, and the curves of $h_{\text{pump},\, u}$ and η_{pump} versus $\dot{V}$ are called the *characteristic curves*. A pump installed in a piping system operates at the *operating point,* which is the point of intersection of the system curve and the characteristic curve.

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PROBLEMS[*]

Laminar and Turbulent Flow

14–1C How is the hydrodynamic entry length defined for flow in a pipe? Is the entry length longer in laminar or turbulent flow?

14–2C Why are liquids usually transported in circular pipes?

14–3C What is the physical significance of the Reynolds number? How is it defined for (*a*) flow in a circular pipe of inner diameter D and (*b*) flow in a rectangular duct of cross section $a \times b$?

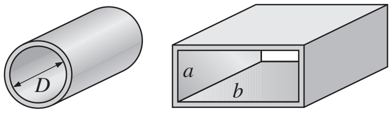

FIGURE P14–3C

14–4C Consider a person walking first in air and then in water at the same speed. For which motion will the Reynolds number be higher?

14–5C Show that the Reynolds number for flow in a circular pipe of diameter D can be expressed as $\text{Re} = 4\dot{m}/(\pi D\mu)$.

14–6C Which fluid at room temperature requires a larger pump to flow at a specified velocity in a given pipe: water or engine oil? Why?

14–7C Consider the flow of air and water in pipes of the same diameter, at the same temperature, and at the same mean velocity. Which flow is more likely to be turbulent? Why?

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[*] Problems designated by a “C” are concept questions, and students are encouraged to answer them all. Problems designated by an “E” are in English units, and SI users can ignore them. Problems with the icon 💻 are comprehensive in nature and are intended to be solved with appropriate software.

14–8C Consider laminar flow in a circular pipe. Is the wall shear stress τ_w higher near the inlet of the pipe or near the exit? Why? What would your response be if the flow were turbulent?

14–9C How does surface roughness affect the pressure drop in a pipe if the flow is turbulent? What would your response be if the flow were laminar?

14–10C What is hydraulic diameter? How is it defined? What is it equal to for a circular pipe of diameter D?

14–11E Shown here is a cool picture of water being released at 300,000 gallons per second in the spring of 2008. This was part of a revitalization effort for the ecosystem of the Grand Canyon and the Colorado River. Estimate the Reynolds number of the pipe flow. Is it laminar or turbulent? (*Hint*: For a length scale, approximate the height of the man in the blue shirt directly above the pipe to be 6 ft.)

FIGURE P14–11E

Courtesy U.S. Bureau of Reclamation, Public Affairs PN Region

Fully Developed Flow in Pipes

14–12C In the fully developed region of flow in a circular pipe, does the velocity profile change in the flow direction?

14–13C Someone claims that the volume flow rate in a circular pipe with laminar flow can be determined by measuring the velocity at the centerline in the fully developed region, multiplying it by the cross-sectional area, and dividing the result by 2. Do you agree? Explain.

14–14C Someone claims that the average velocity in a circular pipe in fully developed laminar flow can be determined by simply measuring the velocity at $R/2$ (midway between the wall surface and the centerline). Do you agree? Explain.

14–15C Someone claims that the shear stress at the center of a circular pipe during fully developed laminar flow is zero. Do you agree with this claim? Explain.

14–16C Someone claims that in fully developed turbulent flow in a pipe, the shear stress is a maximum at the pipe wall. Do you agree with this claim? Explain.

14–17C How does the wall shear stress τ_w vary along the flow direction in the fully developed region in (*a*) laminar flow and (*b*) turbulent flow?

14–18C How is the friction factor for flow in a pipe related to the pressure loss? How is the pressure loss related to the pumping power requirement for a given mass flow rate?

14–19C Discuss whether fully developed pipe flow is one-, two-, or three-dimensional.

14–20C Consider fully developed flow in a circular pipe with negligible entrance effects. If the length of the pipe is doubled, the head loss will (*a*) double, (*b*) more than double, (*c*) less than double, (*d*) reduce by half, or (*e*) remain constant.

14–21C Consider fully developed laminar flow in a circular pipe. If the diameter of the pipe is reduced by half while the flow rate and the pipe length are held constant, the head loss will (*a*) double, (*b*) triple, (*c*) quadruple, (*d*) increase by a factor of 8, or (*e*) increase by a factor of 16.

14–22C Explain why the friction factor is independent of the Reynolds number at very large Reynolds numbers.

14–23C Consider laminar flow of air in a circular pipe with perfectly smooth surfaces. Do you think the friction factor for this flow is zero? Explain.

14–24C Consider fully developed laminar flow in a circular pipe. If the viscosity of the fluid is reduced by half by heating while the flow rate is held constant, how does the head loss change?

14–25C How is head loss related to pressure loss? For a given fluid, explain how you would convert head loss to pressure loss.

14–26C What is turbulent viscosity? What causes it?

14–27C What is the physical mechanism that causes the friction factor to be higher in turbulent flow?

14–28 The velocity profile for the fully developed laminar flow of a Newtonian fluid between two large parallel plates is given by

$$u(y) = \frac{3\,u_0}{2}\left[1 - \left(\frac{y}{h}\right)^2\right]$$

where $2h$ is the distance between the two plates, u_0 is the velocity at the center plane, and y is the vertical coordinate from the center plane. For a plate width of b, obtain a relation for the flow rate through the plates.

14–29 Water at 15°C (ρ = 999.1 kg/m^3 and $\mu = 1.138 \times 10^{-3}$ kg/m·s) is flowing steadily in a 30-m-long and 6-cm-diameter horizontal pipe made of stainless steel at a rate of 10 L/s. Determine (*a*) the pressure drop, (*b*) the head loss, and (*c*) the pumping power requirement to overcome this pressure drop.

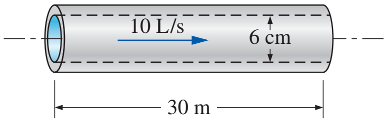


FIGURE P14–29

14–30E Water at 70°F passes through 0.75-in-internal-diameter copper tubes at a rate of 0.5 lbm/s. Determine the pumping power per ft of pipe length required to maintain this flow at the specified rate.

14–31E Heated air at 1 atm and 100°F is to be transported in a 400-ft-long circular plastic duct at a rate of 12 ft^3/s. If the head loss in the pipe is not to exceed 50 ft, determine the minimum diameter of the duct.

14–32 In fully developed laminar flow in a circular pipe, the velocity at $R/2$ (midway between the wall surface and the centerline) is measured to be 11 m/s. Determine the velocity at the center of the pipe. *Answer:* 14.7 m/s

14–33 The velocity profile in fully developed laminar flow in a circular pipe of inner radius $R = 2$ cm, in m/s, is given by $u(r) = 4(1 - r^2/R^2)$. Determine the average and maximum velocities in the pipe and the volume flow rate.

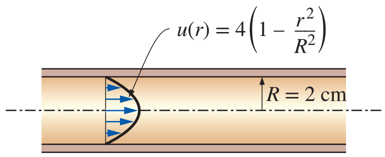


FIGURE P14–33

14–34 Repeat Prob. 14–33 for a pipe of inner radius 7 cm.

14–35 Water at 10°C ($\rho = 999.7\ \text{kg/m}^3$ and $\mu = 1.307 \times 10^{-3}$ kg/m·s) is flowing steadily in a 0.12-cm-diameter, 15-m-long pipe at an average velocity of 0.9 m/s. Determine (*a*) the pressure drop, (*b*) the head loss, and (*c*) the pumping power requirement to overcome this pressure drop. *Answers:* (*a*) 392 kPa, (*b*) 40.0 m, (*c*) 0.399 W

14–36 Consider laminar flow of a fluid through a square channel with smooth surfaces. Now the average velocity of the fluid is doubled. Determine the change in the head loss of the fluid. Assume the flow regime remains unchanged.

14–37 Repeat Prob. 14–36 for turbulent flow in smooth pipes for which the friction factor is given as $f = 0.184\text{Re}^{-0.2}$. What would your answer be for fully turbulent flow in a rough pipe?

14–38 Air enters a 10-m-long section of a rectangular duct of cross section 15 cm × 20 cm made of commercial steel at 1 atm and 35°C at an average velocity of 5 m/s. Disregarding the entrance effects, determine the fan power needed to overcome the pressure losses in this section of the duct. *Answer:* 2.55 W

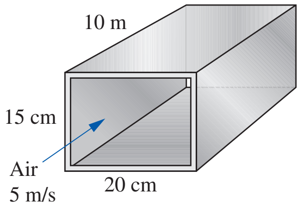


FIGURE P14–38

14–39 Consider an air solar collector that is 1 m wide and 4 m long and has a constant spacing of 3 cm between the glass cover and the collector plate. Air flows at an average temperature of 45°C at a rate of 0.12 m^3/s through the 1-m-wide edge of the collector along the 4-m-long passageway. Disregarding the entrance and roughness effects and the 90° bend, determine the pressure drop in the collector. *Answer:* 17.5 Pa

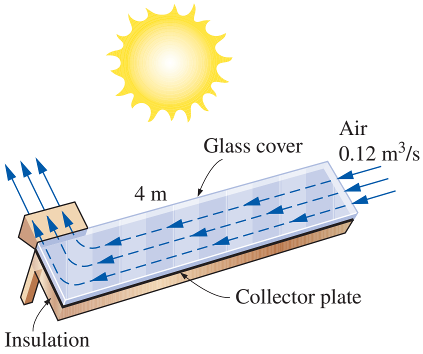


FIGURE P14–39

14–40 Glycerin at 40°C with $\rho = 1252\ \text{kg/m}^3$ and $\mu = 0.27$ kg/m·s is flowing through a 3-cm-diameter, 25-m-long pipe that discharges into the atmosphere at 100 kPa. The flow rate through the pipe is 0.075 L/s. (*a*) Determine the absolute pressure 25 m before the pipe exit. (*b*) At what angle θ must the pipe be inclined downward from the horizontal for the pressure in the entire pipe to be atmospheric pressure and the flow rate to be maintained the same?

14–41E Air at 1 atm and 60°F is flowing through a 1 ft × 1 ft square duct made of commercial steel at a rate of 1600 cfm. Determine the pressure drop and head loss per ft of the duct.

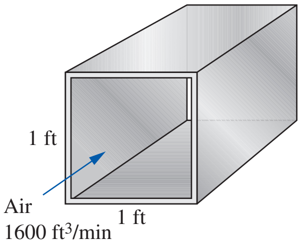


FIGURE P14–41E

14–42 Oil with a density of 850 kg/m^3 and kinematic viscosity of 0.00062 m^2/s is being discharged by a 8-mm-diameter, 40-m-long horizontal pipe from a storage tank open to the atmosphere. The height of the liquid level above the center of the pipe is 4 m. Disregarding the minor losses, determine the flow rate of oil through the pipe.

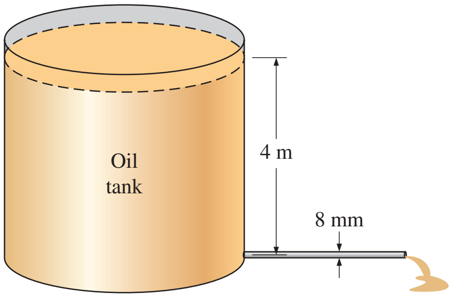


FIGURE P14–42

14–43 In an air heating system, heated air at 40°C and 105 kPa absolute is distributed through a 0.2 m × 0.3 m rectangular duct made of commercial steel at a rate of 0.5 m^3/s. Determine the pressure drop and head loss through a 40-m-long section of the duct. *Answers:* 124 Pa, 10.8 m

14–44 Glycerin at 40°C with ρ = 1252 kg/m^3 and μ = 0.27 kg/m·s is flowing through a 6-cm-diameter horizontal smooth pipe with an average velocity of 3.5 m/s. Determine the pressure drop per 10 m of the pipe.

14–45 Reconsider Prob. 14–44. Using appropriate software, investigate the effect of the pipe diameter on the pressure drop for the same constant flow rate. Let the pipe diameter vary from 1 to 10 cm in increments of 1 cm. Tabulate and plot the results, and draw conclusions.

14–46 Liquid ammonia at −20°C is flowing through a 20-m-long section of a 5-mm-diameter copper tube at a rate of 0.09 kg/s. Determine the pressure drop, the head loss, and the pumping power required to overcome the frictional losses in the tube. *Answers:* 1240 kPa, 189 m, 0.167 kW

14–47 Consider the fully developed flow of glycerin at 40°C through a 70-m-long, 4-cm-diameter, horizontal, circular pipe. If the flow velocity at the centerline is measured to be 6 m/s, determine the velocity profile and the pressure difference across this 70-m-long section of the pipe, and the useful pumping power required to maintain this flow.

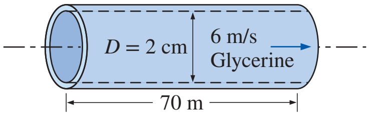


FIGURE P14–47

Minor Losses

14–48C What is minor loss in pipe flow? How is the minor loss coefficient K_L defined?

14–49C Define equivalent length for minor loss in pipe flow. How is it related to the minor loss coefficient?

14–50C The effect of rounding of a pipe exit on the loss coefficient is (*a*) negligible, (*b*) somewhat significant, or (*c*) very significant.

14–51C Which has a greater minor loss coefficient during pipe flow: gradual expansion or gradual contraction? Why?

14–52C A piping system involves sharp turns, and thus large minor head losses. One way of reducing the head loss is to replace the sharp turns with circular elbows. What is another way?

14–53C During a retrofitting project of a fluid flow system to reduce the pumping power, it is proposed to install vanes into the miter elbows or to replace the sharp turns in 90° miter elbows by smooth curved bends. Which approach will result in a greater reduction in pumping power requirements?

14–54 A horizontal pipe has an abrupt expansion from D_1 = 5 cm to D_2 = 10 cm. The water velocity in the smaller section is 8 m/s and the flow is turbulent. The pressure in the smaller section is P_1 = 410 kPa. Taking the kinetic energy correction factor to be 1.06 at both the inlet and the outlet, determine the downstream pressure P_2, and estimate the error that would have occurred if Bernoulli's equation had been used. *Answers:* 423 kPa, 17.3 kPa

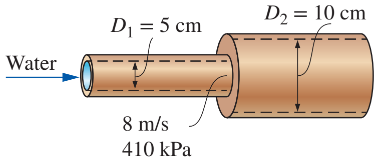
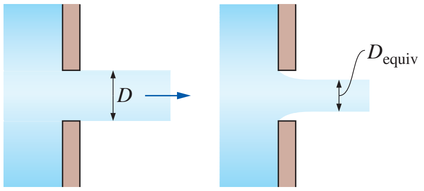


FIGURE P14–54

14–55 Consider flow from a water reservoir through a circular hole of diameter *D* at the side wall at a vertical distance *H* from the free surface. The flow rate through an actual hole with a sharp-edged entrance (K_L = 0.5) is considerably less than the flow rate calculated assuming "frictionless" flow and thus zero loss for the hole. Disregarding the effect of the kinetic energy correction factor, obtain a relation for the "equivalent diameter" of the sharp-edged hole for use in frictionless flow relations.



Frictionless flow

Actual flow

FIGURE P14–55

14–56 Repeat Prob. 14–55 for a slightly rounded entrance (K_L = 0.12).

14–57 Water is to be withdrawn from an 8-m-high water reservoir by drilling a 2.2-cm-diameter hole at the bottom surface. Disregarding the effect of the kinetic energy correction factor, determine the flow rate of water through the hole if (*a*) the entrance of the hole is well-rounded and (*b*) the entrance is sharp-edged.

Piping Systems and Pump Selection

14–58C A piping system equipped with a pump is operating steadily. Explain how the operating point (the flow rate and the head loss) is established.

14–59C Water is pumped from a large lower reservoir to a higher reservoir. Someone claims that if the head loss is negligible, the required pump head is equal to the elevation difference between the free surfaces of the two reservoirs. Do you agree?

14–60C For a piping system, define the system curve, the characteristic curve, and the operating point on a head versus flow rate chart.

14–61C A person filling a bucket with water using a garden hose suddenly remembers that attaching a nozzle to the hose increases the discharge velocity of water and wonders if this increased velocity would decrease the filling time of the bucket. What would happen to the filling time if a nozzle were attached to the hose: increase it, decrease it, or have no effect? Why?

14–62C Consider two identical 2-m-high open tanks filled with water on top of a 1-m-high table. The discharge valve of one of the tanks is connected to a hose whose other end is left open on the ground while the other tank does not have a hose connected to its discharge valve. Now the discharge valves of both tanks are opened. Disregarding any frictional loses in the hose, which tank do you think empties completely first? Why?

14–63C A piping system involves two pipes of different diameters (but of identical length, material, and roughness) connected in series. How would you compare the (*a*) flow rates and (*b*) pressure drops in these two pipes?

14–64C A piping system involves two pipes of different diameters (but of identical length, material, and roughness) connected in parallel. How would you compare the (*a*) flow rates and (*b*) pressure drops in these two pipes?

14–65 Water at 15°C is drained from a large reservoir using two horizontal plastic pipes connected in series. The first pipe is 13 m long and has a 10-cm diameter, while the second pipe is 35 m long and has a 5-cm diameter. The water level in the reservoir is 18 m above the centerline of the pipe. The pipe entrance is sharp-edged, and the contraction between the two pipes is sudden. Neglecting the effect of the kinetic energy correction factor, determine the discharge rate of water from the reservoir.

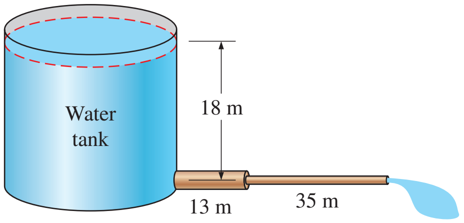


FIGURE P14–65

14–66 A semi-spherical tank of radius R is completely filled with water. Now a hole of cross-sectional area A_h and discharge coefficient C_d at the bottom of the tank is fully opened and water starts to flow out. Develop an expression for the time needed to empty the tank completely.

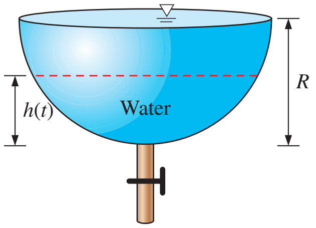


FIGURE P14–66

14–67 The water needs of a small farm are to be met by pumping water from a well that can supply water continuously at a rate of 5 L/s. The water level in the well is 20 m below the ground level, and water is to be pumped to a large tank on a hill, which is 58 m above the ground level of the well, using 6-cm internal diameter plastic pipes. The required length of piping is measured to be 510 m, and the total minor loss coefficient due to the use of elbows, vanes, etc. is estimated to be 12. Taking the efficiency of the pump to be 75 percent, determine the rated power of the pump that needs to be purchased, in kW. The density and viscosity of water at anticipated operation conditions are taken to be 1000 kg/m^3 and 0.00131 kg/m·s, respectively. Is it wise to purchase a suitable pump that meets the total power requirements, or is it necessary to also pay particular attention to the large elevation head in this case? Explain. *Answer:* 6.89 kW

14–68E Water at 70°F flows by gravity from a large reservoir at a high elevation to a smaller one through a 60-ft-long, 2-in-diameter cast iron piping system that includes four standard flanged elbows, a well-rounded entrance, a sharp-edged exit, and a fully open gate valve. Taking the free surface of the lower reservoir as the reference level, determine the elevation z_1 of the higher reservoir for a flow rate of 10 ft^3/min. *Answer:* 12.6 ft

14–69 A 2.4-m-diameter tank is initially filled with water 4 m above the center of a sharp-edged 10-cm-diameter orifice. The tank water surface is open to the atmosphere, and the orifice drains to the atmosphere. Neglecting the effect of the kinetic energy correction factor, calculate (*a*) the initial velocity from the tank and (*b*) the time required to empty the tank. Does the loss coefficient of the orifice cause a significant increase in the draining time of the tank?

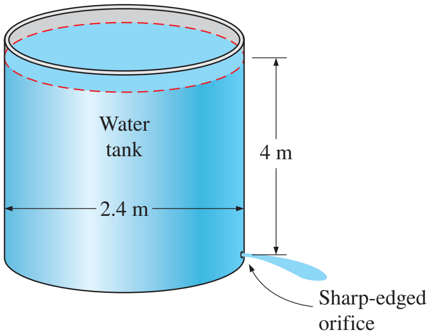


FIGURE P14–69

14–70 A 3-m-diameter tank is initially filled with water 2 m above the center of a sharp-edged 10-cm-diameter orifice. The tank water surface is open to the atmosphere, and the orifice drains to the atmosphere through a 100-m-long pipe. The friction coefficient of the pipe is taken to be 0.015 and the effect of the kinetic energy correction factor can be neglected. Determine (*a*) the initial velocity from the tank and (*b*) the time required to empty the tank.

14–71 Reconsider Prob. 14–70. In order to drain the tank faster, a pump is installed near the tank exit as in Fig. P14–71. Determine how much pump power input is necessary to establish an average water velocity of 4 m/s when the tank is full at $z = 2$ m. Also, assuming the discharge velocity to remain constant, estimate the time required to drain the tank.

Someone suggests that it makes no difference whether the pump is located at the beginning or at the end of the pipe, and that the performance will be the same in either case, but another person argues that placing the pump near the end of the pipe may cause cavitation. The water temperature is 30°C, so the water vapor pressure is $P_v = 4.246$ kPa $= 0.43$ m H_2O, and the system is located at sea level. Investigate if there is the possibility of cavitation and if we should be concerned about the location of the pump.

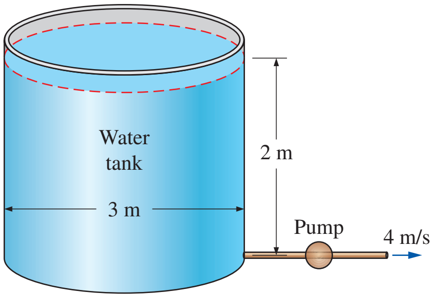


FIGURE P14–71

14–72 Gasoline ($\rho = 680$ kg/m^3 and $\nu = 4.29 \times 10^{-7}$ m^2/s) is transported at a rate of 240 L/s for a distance of 2 km. The surface roughness of the piping is 0.03 mm. If the head loss due to pipe friction is not to exceed 10 m, determine the minimum diameter of the pipe.

14–73 A 4-m-high cylindrical tank having a cross-sectional area of $A_T = 1.5$ m^2 is filled with equal volumes of water and oil whose specific gravity is SG = 0.75. Now a 1-cm-diameter hole at the bottom of the tank is opened, and water starts to flow out. If the discharge coefficient of the hole is $C_d = 0.85$, determine how long it will take for the water in the tank, which is open to the atmosphere, to empty completely.

14–74E A farmer is to pump water at 70°F from a river to a water storage tank nearby using a 125-ft-long, 5-in-diameter plastic pipe with three flanged 90° smooth bends. The water velocity near the river surface is 6 ft/s, and the pipe inlet is placed in the river normal to the flow direction of water to take advantage of the dynamic pressure. The elevation difference between the river and the free surface of the tank is 12 ft. For a flow rate of 1.5 ft^3/s and an overall pump efficiency of 70 percent, determine the required electric power input to the pump.

14–75E Reconsider Prob. 14–74E. Using appropriate software, investigate the effect of the pipe diameter on the required electric power input to the pump. Let the pipe diameter vary from 1 to 10 in, in increments of 1 in. Tabulate and plot the results, and draw conclusions.

14–76 A water tank filled with solar-heated water at 40°C is to be used for showers in a field using gravity-driven flow. The system includes 35 m of 1.5-cm-diameter galvanized iron piping with four miter bends (90°) without vanes and a wide-open globe valve. If water is to flow at a rate of 1.2 L/s through the shower head, determine how high the water level in the tank must be from the exit level of the shower. Disregard the losses at the entrance and at the shower head, and neglect the effect of the kinetic energy correction factor.

14–77 Two water reservoirs *A* and *B* are connected to each other through a 40-m-long, 2-cm-diameter cast iron pipe with a sharp-edged entrance. The pipe also involves a swing check valve and a fully open gate valve. The water level in both reservoirs is the same, but reservoir *A* is pressurized by compressed air while reservoir *B* is open to the atmosphere at 95 kPa. If the initial flow rate through the pipe is 1.5 L/s, determine the absolute air pressure on top of reservoir *A*. Take the water temperature to be 10°C. *Answer:* 1100 kPa

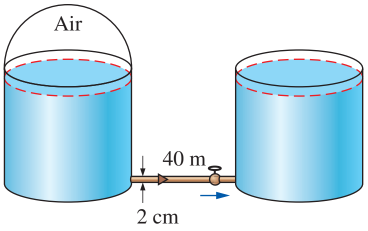


FIGURE P14–77

14–78 A vented tanker is to be filled with fuel oil with $\rho = 920$ kg/m^3 and $\mu = 0.045$ kg/m·s from an underground reservoir using a 25-m-long, 4-cm-diameter plastic hose with a slightly rounded entrance and two 90° smooth bends. The elevation difference between the oil level in the reservoir and the top of the tanker where the hose is discharged is 5 m. The capacity of the tanker is 18 m^3 and the filling time is 30 min. Taking the kinetic energy correction factor at the hose discharge to be 1.05 and assuming an overall pump efficiency of 82 percent, determine the required power input to the pump.

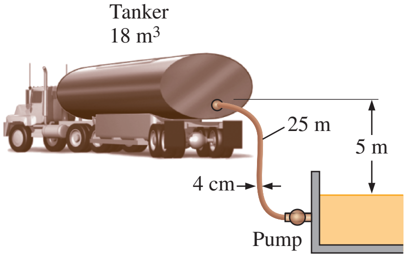


FIGURE P14–78

14–79 Water at 15°C is to be pumped from a reservoir ($z_A = 2$ m) to another reservoir at a higher elevation ($z_B = 9$ m) through two 25-m-long plastic pipes connected in parallel. The diameters of the two pipes are 3 cm and 5 cm. Water is to be pumped by a 68 percent efficient motor–pump unit that draws 8 kW of electric power during operation. The minor losses and the head loss in the pipes that connect the parallel pipes to the two reservoirs are considered to be negligible. Determine the total flow rate between the reservoirs and the flow rates through each of the parallel pipes.

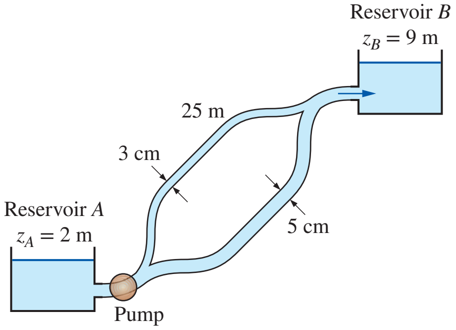


FIGURE P14–79

14–80 A certain part of cast iron piping of a water distribution system involves a parallel section. Both parallel pipes have a diameter of 30 cm, and the flow is fully turbulent. One of the branches (pipe *A*) is 1500 m long while the other branch (pipe *B*) is 2500 m long. If the flow rate through pipe *A* is 0.4 m^3/s, determine the flow rate through pipe *B*. Disregard minor losses and assume the water temperature to be 15°C. Show that the flow is fully rough, and thus the friction factor is independent of Reynolds number. *Answer:* 0.310 m^3/s

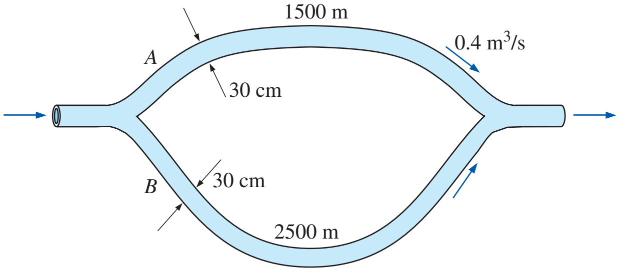


FIGURE P14–80

14–81 Repeat Prob. 14–80 assuming pipe *A* has a halfway-closed gate valve ($K_L = 2.1$) while pipe *B* has a fully open globe valve ($K_L = 10$), and the other minor losses are negligible.

14–82 A geothermal district heating system involves the transport of geothermal water at 110°C from a geothermal well to a city at about the same elevation for a distance of 12 km at a rate of 1.5 m^3/s in 60-cm-diameter stainless-steel pipes. The fluid pressures at the wellhead and the arrival point in the city are to be the same. The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. (*a*) Assuming the pump–motor efficiency to be 80 percent, determine the electric power consumption of the system for pumping. Would you recommend the use of a single large pump or several smaller pumps of the same total pumping power scattered along the pipeline? Explain. (*b*) Determine the daily cost of power consumption of the system if the unit cost of electricity is \$0.06/kWh. (*c*) The temperature of geothermal water is estimated to drop 0.5°C during this long flow. Determine if the frictional heating during flow can make up for this drop in temperature.

14–83 Repeat Prob. 14–82 for cast iron pipes of the same diameter.

14–84 Water is transported by gravity through a 10-cm-diameter 550-m-long plastic pipe with an elevation gradient of 0.01 (i.e., an elevation drop of 1 m per 100 m of pipe length). Taking $\rho = 1000$ kg/m^3 and $\nu = 1 \times 10^{-6}$ m^2/s for water, determine the flow rate of water through the pipe. If the pipe were horizontal, what would the power requirements be to maintain the same flow rate?

14–85 Water to a residential area is transported at a rate of 1.5 m^3/s via 70-cm-internal-diameter concrete pipes with a surface roughness of 3 mm and a total length of 1500 m. In order to reduce pumping power requirements, it is proposed to line the interior surfaces of the concrete pipe with 2-cm-thick petroleum-based lining that has a surface roughness thickness of 0.04 mm. There is a concern that the reduction of pipe diameter to 66 cm and the increase in average velocity may offset any gains. Taking $\rho = 1000$ kg/m^3 and $\nu = 1 \times 10^{-6}$ m^2/s for water, determine the percent increase or decrease in the pumping power requirements due to pipe frictional losses as a result of lining the concrete pipes.

14–86 Two pipes of identical length and material are connected in parallel. The diameter of pipe *A* is twice the diameter of pipe *B*. Assuming the friction factor to be the same in both cases and disregarding minor losses, determine the ratio of the flow rates in the two pipes.

Review Problems

14–87 In a laminar flow through a circular tube of radius of *R*, the velocity and temperature profiles at a cross section are given by $u = u_0(1 - r^2/R^2)$ and $T(r) = A + Br^2 - Cr^4$ where *A*, *B*, and *C* are positive constants. Obtain a relation for the bulk fluid temperature at that cross section.

14–88 In a geothermal district heating system, 10,000 kg/s of hot water must be delivered a distance of 10 km in a horizontal pipe. The minor losses are negligible, and the only significant energy loss arises from pipe friction. The friction factor is taken to be 0.015. Specifying a larger-diameter pipe would reduce water velocity, velocity head, pipe friction, and thus power consumption. But a larger pipe would also cost more money initially to purchase and install. Otherwise stated, there is an optimum pipe diameter that will minimize the sum of pipe cost and future electric power cost.

Assume the system will run 24 h/day, every day, for 30 years. During this time the cost of electricity remains constant at \$0.06/kWh. Assume system performance stays constant over the decades (this may not be true, especially if highly mineralized water is passed through the pipeline—scale may form). The pump has an overall efficiency of 80 percent. The cost to purchase, install, and insulate a 10-km pipe depends on the diameter *D* and is given by Cost = $\$10^6 D^2$, where *D* is in m. Assuming zero inflation and interest rate for simplicity and zero salvage value and zero maintenance cost, determine the optimum pipe diameter.

14–89 The compressed air requirements of a manufacturing facility are met by a 120-hp compressor that draws in air from the outside through a 9-m-long, 22-cm-diameter duct made of thin galvanized iron sheets. The compressor takes in air at a rate of 0.27 m^3/s at the outdoor conditions of 15°C and 95 kPa. Disregarding any minor losses, determine the useful power used by the compressor to overcome the frictional losses in this duct. *Answer:* 6.74 W

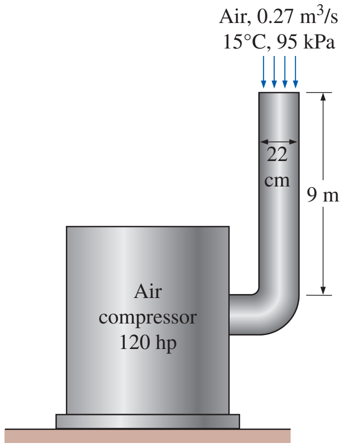


FIGURE P14–89

14–90 A house built on a riverside is to be cooled in summer by utilizing the cool water of the river. A 20-m-long section of a circular stainless-steel duct of 20 cm diameter passes through the water. Air flows through the underwater section of the duct at 4 m/s at an average temperature of 15°C. For an overall fan efficiency of 62 percent, determine the fan power needed to overcome the flow resistance in this section of the duct.

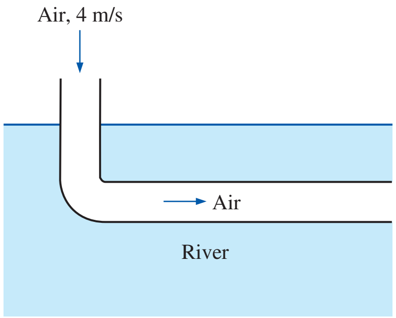


FIGURE P14–90

14–91 The velocity profile in fully developed laminar flow in a circular pipe, in m/s, is given by $u(r) = 6(1 - 100r^2)$, where r is the radial distance from the centerline of the pipe in m. Determine (*a*) the radius of the pipe, (*b*) the average velocity through the pipe, and (*c*) the maximum velocity in the pipe.

14–92 Two pipes of identical diameter and material are connected in parallel. The length of pipe *A* is five times the length of pipe *B*. Assuming the flow is fully turbulent in both pipes and thus the friction factor is independent of the Reynolds number and disregarding minor losses, determine the ratio of the flow rates in the two pipes. *Answer:* 0.447

14–93 Repeat Prob. 14–92 except let the length of pipe *A* be three times that of pipe *B*. Compare this result to that of Prob. 14–92. Does the difference agree with your intuition? Explain.

14–94 Shell-and-tube heat exchangers with hundreds of tubes housed in a shell are commonly used in practice for heat transfer between two fluids. Such a heat exchanger used in an active solar hot-water system transfers heat from a water-antifreeze solution flowing through the shell and the solar collector to fresh water flowing through the tubes at an average temperature of 60°C at a rate of 15 L/s. The heat exchanger contains 80 brass tubes 1 cm in inner diameter and 1.5 m in length. Disregarding inlet, exit, and header losses, determine the pressure drop across a single tube and the pumping power required by the tube-side fluid of the heat exchanger.

After operating a long time, 1-mm-thick scale builds up on the inner surfaces with an equivalent roughness of 0.4 mm. For the same pumping power input, determine the percent reduction in the flow rate of water through the tubes.

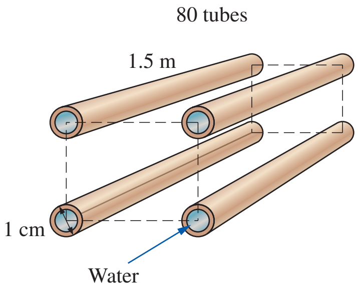


FIGURE P14–94

14–95 Water at 15°C is to be discharged from a reservoir at a rate of 18 L/s using two horizontal cast iron pipes connected in series and a pump between them. The first pipe is 20 m long and has a 6 cm diameter, while the second pipe is 35 m long and has a 3 cm diameter. The water level in the reservoir is 30 m above the centerline of the pipe. The pipe entrance is sharp-edged, and losses associated with the connection of the pump are negligible. Neglecting the effect of the kinetic energy correction factor, determine the required pumping head and the minimum pumping power to maintain the indicated flow rate.

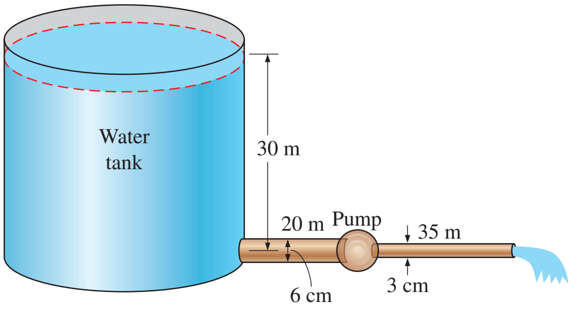


FIGURE P14–95

14–96 Reconsider Prob. 14–95. Using appropriate software, investigate the effect of the second pipe diameter on the required pumping head to maintain the indicated flow rate. Let the diameter vary from 1 to 10 cm in increments of 1 cm. Tabulate and plot the results.

14–97 A pipeline that transports oil at 40°C at a rate of 3 m^3/s branches out into two parallel pipes made of commercial steel that reconnect downstream. Pipe *A* is 500 m long and has a diameter of 30 cm, while pipe *B* is 800 m long and has a diameter of 45 cm. The minor losses are considered to be negligible. Determine the flow rate through each of the parallel pipes.

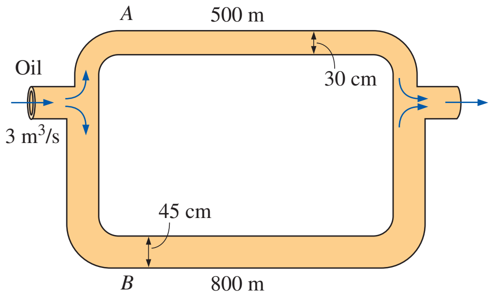


FIGURE P14–97

14–98 Repeat Prob. 14–97 for hot-water flow of a district heating system at 100°C.

14–99 Water is to be withdrawn from a 7-m-high water reservoir by drilling a well-rounded 5-cm-diameter hole with negligible loss near the bottom and attaching a horizontal 90° bend of negligible length. Taking the kinetic energy correction factor to be 1.05, determine the flow rate of water through the bend if (*a*) the bend is a flanged smooth bend and (*b*) the bend is a miter bend without vanes. *Answers:* (*a*) 19.8 L/s, (*b*) 15.7 L/s

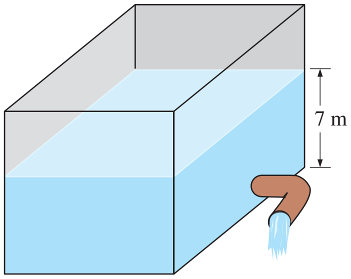


FIGURE P14–99

14–100 The compressed air requirements of a textile factory are met by a large compressor that draws in 0.6 m^3/s air at atmospheric conditions of 20°C and 1 bar (100 kPa) and consumes 300 kW electric power when operating. Air is compressed to a gage pressure of 8 bar (absolute pressure of 900 kPa), and compressed air is transported to the production area through a 15-cm-internal-diameter, 83-m-long, galvanized steel pipe with a surface roughness of 0.15 mm. The average temperature of compressed air in the pipe is 60°C. The compressed air line has 8 elbows with a loss coefficient of 0.6 each. If the compressor efficiency is 85 percent, determine the pressure drop and the power wasted in the transportation line. *Answers:* 1.40 kPa, 0.125 kW

14–101 Reconsider Prob. 14–100. In order to reduce the head losses in the piping and thus the power wasted, someone suggests doubling the diameter of the 83-m-long compressed air pipes. Calculate the reduction in wasted power, and determine if this is a worthwhile idea. Considering the cost of replacement, does this proposal make sense to you?

14–102E A water fountain is to be installed at a remote location by attaching a cast iron pipe directly to a water main through which water is flowing at 70°F and 60 psig. The entrance to the pipe is sharp-edged, and the 70-ft-long piping system involves three 90° miter bends without vanes, a fully open gate valve, and an angle valve with a loss coefficient of 5 when fully open. If the system is to provide water at a rate of 15 gal/min and the elevation difference between the pipe and the fountain is negligible, determine the minimum diameter of the piping system. *Answer:* 0.713 in

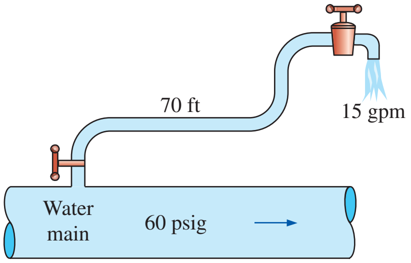


FIGURE P14–102E

14–103E Repeat Prob. 14–102E for plastic (smooth) pipes.

14–104 In a hydroelectric power plant, water at 20°C is supplied to the turbine at a rate of 0.55 m^3/s through a 200-m-long, 0.35-m-diameter cast iron pipe. The elevation difference between the free surface of the reservoir and the turbine discharge is 140 m, and the combined turbine–generator efficiency is 85 percent. Disregarding the minor losses because of the large length-to-diameter ratio, determine the electric power output of this plant.

14–105 In Prob. 14–104, the pipe diameter is tripled in order to reduce the pipe losses. Determine the percent increase in the net power output as a result of this modification.

14–106E The drinking water needs of an office are met by large water bottles. One end of a 0.35-in-diameter, 6-ft-long plastic hose is inserted into the bottle placed on a high stand, while the other end with an on/off valve is maintained 3 ft below the bottom of the bottle. If the water level in the bottle is 1 ft when it is full, determine how long it would take to fill an 8-oz glass (= 0.00835 ft^3) (*a*) when the bottle is first opened and (*b*) when the bottle is almost empty. Take the total minor loss coefficient, including the on/off valve, to be 2.8 when it is fully open. Assume the water temperature to be the same as the room temperature of 70°F. *Answers:* (*a*) 2.4 s, (*b*) 2.8 s

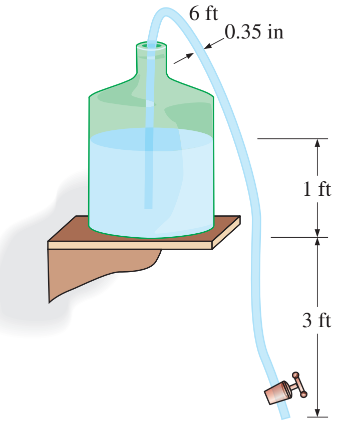


FIGURE P14–106E

14–107E Reconsider Prob. 14–106E. Using appropriate software, investigate the effect of the hose diameter on the time required to fill a glass when the bottle is full. Let the diameter vary from 0.2 to 2 in, in increments of 0.2 in. Tabulate and plot the results.

14–108E Reconsider Prob. 14–106E. The office worker who set up the siphoning system purchased a 12-ft-long reel of the plastic tube and wanted to use the whole thing to avoid cutting it in pieces, thinking that it is the elevation difference that makes siphoning work, and the length of the tube is not important. So he used the entire 12-ft-long tube. Assuming the turns or constrictions in the tube are not significant (being very optimistic) and the same elevation is maintained, determine the time it takes to fill a glass of water for both cases (bottle nearly full and bottle nearly empty).

14–109 The water at 20°C in a 10-m-diameter, 2-m-high aboveground swimming pool is to be emptied by unplugging a 5-cm-diameter, 25-m-long horizontal plastic pipe attached to the bottom of the pool. Determine the initial rate of discharge of water through the pipe and the time (hours) it would take to empty the swimming pool completely assuming the entrance to the pipe is well-rounded with negligible loss. Take the friction factor of the pipe to be 0.022. Using the initial discharge velocity, check if this is a reasonable value for the friction factor. *Answers:* 3.55 L/s, 24.6 h

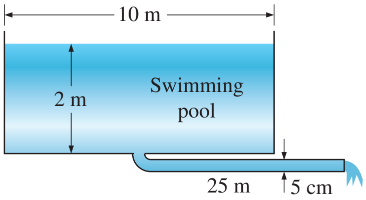


FIGURE P14–109

14–110 Reconsider Prob. 14–109. Using appropriate software, investigate the effect of the discharge pipe diameter on the time required to empty the pool completely. Let the diameter vary from 1 to 10 cm, in increments of 1 cm. Tabulate and plot the results.

14–111 Repeat Prob. 14–109 for a sharp-edged entrance to the pipe with $K_L = 0.5$. Is this "minor loss" truly "minor" or not?

Design and Essay Problems

14–112 Discuss equations used to calculate unknown flow rates or diameters for each pipe section in piping systems such as pipe networks and branching pipes, and how their solution is carried out. Define the analogy between electric current in electric circuits and fluid flow in pipe networks.

14–113 A pump is to be selected for a waterfall in a garden. The water collects in a pond at the bottom, and the elevation difference between the free surface of the pond and the location where the water is discharged is 3 m. The flow rate of water is to be at least 8 L/s. Select an appropriate motor–pump unit for this job, and identify three manufacturers with product model numbers and prices. Make a selection and explain why you selected that particular product. Also estimate the cost of annual power consumption of this unit assuming continuous operation.

14–114 During a camping trip, you notice that water is discharged from a high reservoir to a stream in the valley through a 30-cm-diameter plastic pipe. The elevation difference between the free surface of the reservoir and the stream is 70 m. You conceive the idea of generating power from this water. Design a power plant that will produce the most power from this resource. Also, investigate the effect of power generation on the discharge rate of water. What discharge rate maximizes the power production?

CHAPTER 15

EXTERNAL FLOW: DRAG AND LIFT

In this chapter we consider *external flow*—flow over bodies that are immersed in a fluid, with emphasis on the resulting lift and drag forces. In external flow, the viscous effects are confined to a portion of the flow field such as the boundary layers and wakes, which are surrounded by an outer flow region that involves small velocity and temperature gradients.

When a fluid moves over a solid body, it exerts pressure forces normal to the surface and shear forces parallel to the surface of the body. We are usually interested in the *resultant* of the pressure and shear forces acting on the body rather than the details of the distributions of these forces along the entire surface of the body. The component of the resultant pressure and shear forces that acts in the flow direction is called the *drag force* (or just *drag*), and the component that acts normal to the flow direction is called the *lift force* (or just *lift*).

We start this chapter with a discussion of drag and lift, and we explore the concepts of pressure drag, friction drag, and flow separation. We continue with the drag coefficients of various two- and three-dimensional geometries encountered in practice and determine the drag force using experimentally determined drag coefficients. We then examine the development of the velocity boundary layer during parallel flow over a flat surface and develop relations for the skin friction and drag coefficients for flow over flat plates, cylinders, and spheres. Finally, we discuss the lift developed by airfoils and the factors that affect the lift characteristics of bodies.

OBJECTIVES

The objectives of this chapter are to:

- Have an intuitive understanding of the various physical phenomena associated with external flow such as drag, friction and pressure drag, drag reduction, and lift.
- Calculate the drag force associated with flow over common geometries.
- Understand the effects of flow regime on the drag coefficients associated with flow over cylinders and spheres.
- Understand the fundamentals of flow over airfoils, and calculate the drag and lift forces acting on airfoils.

15–1 ▪ INTRODUCTION

Fluid flow over solid bodies frequently occurs in practice, and it is responsible for numerous physical phenomena such as the *drag force* acting on automobiles, power lines, trees, and underwater pipelines; the *lift* developed by bird or airplane wings; *upward draft* of rain, snow, hail, and dust particles in high winds; the transportation of red blood cells by blood flow; the entrainment and disbursement of liquid droplets by sprays; the vibration and noise generated by bodies moving in a fluid; and the power generated by wind turbines (Fig. 15–1). Therefore, developing a good understanding of external flow is important in the design of many engineering systems such as aircraft, automobiles, buildings, ships, submarines, and all kinds of turbines. Late-model cars, for example, have been designed with particular emphasis on aerodynamics. This has resulted in significant reductions in fuel consumption and noise, and considerable improvement in handling.

Sometimes a fluid moves over a stationary body (such as the wind blowing over a building), and other times a body moves through a quiescent fluid (such as a car moving through air). These two seemingly different processes are equivalent to each other; what matters is the relative motion between the fluid and the body. Such motions are conveniently analyzed by fixing the coordinate system on the body and are referred to as **flow over bodies** or **external flow**. The aerodynamic aspects of different airplane wing designs, for example, are studied conveniently in a lab by placing the wings in a wind tunnel and blowing air over them with large fans. Also, a flow can be classified as being steady or unsteady, depending on the reference frame selected. Flow around an airplane, for example, is always unsteady with respect to the ground, but it is steady with respect to a frame of reference moving with the airplane at cruise conditions.

The flow fields and geometries for most external flow problems are too complicated to be solved analytically, and thus we have to rely on correlations based on experimental data. The availability of high-speed computers has made it possible to conduct a series of "numerical experiments" quickly by solving the governing equations numerically, and to resort to expensive and time-consuming testing and experimentation only in the final stages of design. Such testing is done in wind tunnels. H. F. Phillips (1845–1912) built the first wind tunnel in 1894 and measured lift and drag. In this chapter we mostly rely on relations developed experimentally.

The velocity of the fluid approaching a body is called the **free-stream velocity** and is denoted by V. It is also denoted by u_∞ or U_∞ when the flow is aligned with the x-axis since u is used to denote the x-component of velocity. The fluid velocity ranges from zero at the body surface (the no-slip condition) to the free-stream value away from the body surface, and the subscript "infinity" serves as a reminder that this is the value at a distance where the presence of the body is not felt. The free-stream velocity may vary with location and time (e.g., the wind blowing past a building). But in design and analysis, the free-stream velocity is usually assumed to be *uniform* and *steady* for convenience, and this is what we do in this chapter.

The shape of a body has a profound influence on the flow over the body and the velocity field. The flow over a body is said to be **two-dimensional** when the body is very long and of constant cross section and the flow is normal to the body. The wind blowing over a long pipe perpendicular to its axis is an example of two-dimensional flow. Note that the velocity component in the axial direction is zero in this case, and thus the velocity is two-dimensional.

(*a*) (b) (*c*)

(*d*) (*e*)

(*f*) (*g*)

FIGURE 15–1

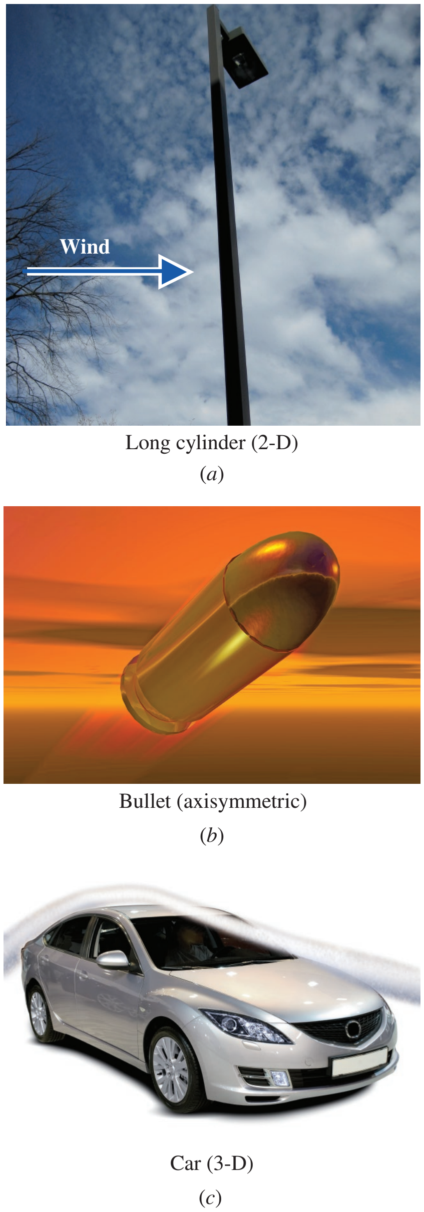
Flow over bodies is commonly encountered in practice.

(*a*) *Royalty-Free/CORBIS*; (*b*) *John Foxx/Getty Images*; (*c*) *IT Stock/age fotostock*; (*d*) *Royalty-Free/CORBIS*; (*e*) *StockTrek/SuperStock*; (*f*) *Corbis*; (*g*) *ROY H PHOTOGRAPHY/Getty Images*

The two-dimensional idealization is appropriate when the body is sufficiently long so that the end effects are negligible and the approach flow is uniform. Another simplification occurs when the body possesses rotational symmetry about an axis in the flow direction. The flow in this case is also two-dimensional and is said to be **axisymmetric**. A bullet piercing through air is an example of axisymmetric flow. The velocity in this case varies with the axial distance x and the radial distance r. Flow over a body that cannot be modeled as two-dimensional or axisymmetric, such as flow over a car, is **three-dimensional** (Fig. 15–2).

Flow over bodies can also be classified as **incompressible flows** (e.g., flows over automobiles, submarines, and buildings) and **compressible flows** (e.g., flows over high-speed aircraft, rockets, and missiles). Compressibility effects are negligible at low velocities (flows with $\text{Ma} \lesssim 0.3$), and such flows can be treated as incompressible with little loss in accuracy. Compressible flow and flows that involve partially immersed bodies with a free surface (such as a ship cruising in water) are beyond the scope of this introductory text.

Bodies subjected to fluid flow are classified as being streamlined or bluff, depending on their overall shape. A body is said to be **streamlined** if a conscious effort is made to align its shape with the anticipated streamlines in the flow. Streamlined bodies such as race cars and airplanes appear to be contoured and sleek. Otherwise, a body (such as a building) tends to block the flow and is said to be **bluff** or blunt. Usually it is much easier to force a streamlined body through a fluid, and thus streamlining has been of great importance in the design of vehicles and airplanes (Fig. 15–3).



Long cylinder (2-D)
(*a*)

Bullet (axisymmetric)
(*b*)

Car (3-D)
(*c*)

FIGURE 15–2
Two-dimensional, axisymmetric, and three-dimensional flows.
(a) John M. Cimbala; (b) Royalty-Free/CORBIS (c) Hannu Liivaar/Alamy

15–2 ▪ DRAG AND LIFT

It is a common experience that a body meets some resistance when it is forced to move through a fluid, especially a liquid. As you may have noticed, it is very difficult to walk in water because of the much greater resistance it offers to motion compared to air. Also, you may have seen high winds knocking down trees, power lines, and even trailers and felt the strong “push” the wind exerts on your body (Fig. 15–4). You experience the same feeling when you extend your arm out of the window of a moving car. A fluid may exert forces and moments on a body in and about various directions. The force a flowing fluid exerts on a body in the flow direction is called **drag**. The drag force can be measured directly by simply attaching the body subjected to fluid flow to a calibrated spring and measuring the displacement in the flow direction (just like measuring weight with a spring scale). More sophisticated drag-measuring devices, called drag balances, use flexible beams fitted with strain gages to measure the drag electronically.

Drag is usually an undesirable effect, like friction, and we do our best to minimize it. Reduction of drag is closely associated with the reduction of fuel consumption in automobiles, submarines, and aircraft; improved safety and durability of structures subjected to high winds; and reduction of noise and vibration. But in some cases drag produces a beneficial effect and we try to maximize it. Friction, for example, is a “life saver” in the brakes of automobiles. Likewise, it is the drag that makes it possible for people to parachute, for pollens to fly to distant locations, and for us all to enjoy the waves of the oceans and the relaxing movements of the leaves of trees.

A stationary fluid exerts only normal pressure forces on the surface of a body immersed in it. A moving fluid, however, also exerts tangential shear forces on the surface because of the no-slip condition caused by viscous effects. Both of these forces, in general, have components in the direction of flow, and thus the drag force is due to the combined effects of pressure and wall shear forces in the flow direction. The components of the pressure and wall shear forces in the direction *normal* to the flow tend to move the body in that direction, and their sum is called **lift**.

For two-dimensional flows, the resultant of the pressure and shear forces can be split into two components: one in the direction of flow, which is the drag force, and another in the direction normal to flow, which is the lift, as shown in Fig. 15–5. For three-dimensional flows, there is also a side force component in the direction normal to the page that tends to move the body in that direction.

The fluid forces may also generate moments and cause the body to rotate. The moment about the flow direction is called the *rolling moment,* the moment about the lift direction is called the *yawing moment,* and the moment about the side force direction is called the *pitching moment.* For bodies that possess symmetry about the lift–drag plane such as cars, airplanes, and ships, the time-averaged side force, yawing moment, and rolling moment are zero when the wind and wave forces are aligned with the body. What remain for such bodies are the drag and lift forces and the pitching moment. For axisymmetric bodies aligned with the flow, such as a bullet, the only time-averaged force exerted by the fluid on the body is the drag force.

The pressure and shear forces acting on a differential area dA on the surface are $P\,dA$ and $\tau_w\,dA$, respectively. The differential drag force and the lift force acting on dA in two-dimensional flow are (Fig. 15–5)

$$dF_D = -P\,dA\cos\theta + \tau_w\,dA\sin\theta \tag{15–1}$$

and

$$dF_L = -P\,dA\sin\theta - \tau_w\,dA\cos\theta \tag{15–2}$$

where θ is the angle that the outer normal of dA makes with the positive flow direction. The total drag and lift forces acting on the body are determined by integrating Eqs. 15–1 and 15–2 over the entire surface of the body,

Drag force: $$F_D = \int_A dF_D = \int_A (-P\cos\theta + \tau_w\sin\theta)\,dA \tag{15–3}$$

and

Lift force: $$F_L = \int_A dF_L = -\int_A (P\sin\theta + \tau_w\cos\theta)\,dA \tag{15–4}$$

These are the equations used to predict the net drag and lift forces on bodies when the flow is simulated on a computer. However, when we perform experimental analyses, Eqs. 15–3 and 15–4 are not practical since the detailed distributions of pressure and shear forces are difficult to obtain by measurements. Fortunately, this information is often not needed. Usually all we need to know is the resultant drag force and lift acting on the entire body, which can be measured directly and easily in a wind tunnel.

Equations 15–1 and 15–2 show that both the skin friction (wall shear) and pressure, in general, contribute to the drag and the lift. In the special case of a thin *flat plate* aligned parallel to the flow direction, the drag force depends on the wall shear only and is independent of pressure since $\theta = 90°$. When the flat plate is placed normal to the flow direction, however, the drag force depends on the pressure only and is independent of wall shear since the shear stress in this case acts in the direction normal to flow and $\theta = 0°$ (Fig. 15–6). If the flat plate is tilted at an angle relative to the flow direction, then the drag force depends on both the pressure and the shear stress.

The wings of airplanes are shaped and positioned specifically to generate lift with minimal drag. This is done by maintaining an angle of attack during cruising, as shown in Fig. 15–7. Both lift and drag are strong functions of the angle of attack, as we discuss later in this chapter. The pressure difference between the

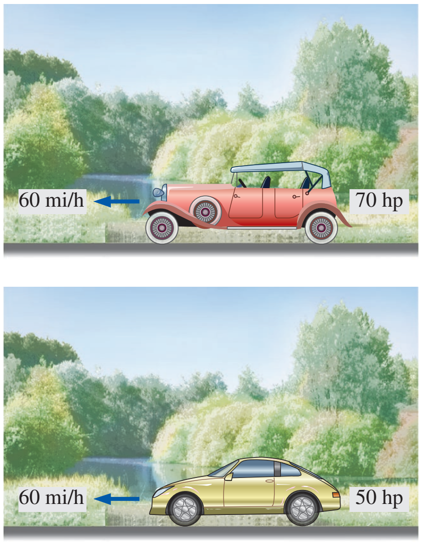


FIGURE 15–3
It is much easier to force a streamlined body than a blunt body through a fluid.

FIGURE 15–4
High winds knock down trees, power lines, and even people as a result of the drag force.

top and bottom surfaces of the wing generates an upward force that tends to lift the wing and thus the airplane to which it is connected. For slender bodies such as wings, the shear force acts nearly parallel to the flow direction, and thus its contribution to the lift is small. The drag force for such slender bodies is mostly due to shear forces (the skin friction).

The drag and lift forces depend on the density ρ of the fluid, the upstream velocity V, and the size, shape, and orientation of the body, among other things, and it is not practical to list these forces for a variety of situations. Instead, it is more convenient to work with appropriate dimensionless numbers that represent the drag and lift characteristics of the body. These numbers are the **drag coefficient** C_D, and the **lift coefficient** C_L, and they are defined as

Drag coefficient:
$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} \tag{15–5}$$

Lift coefficient:
$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} \tag{15–6}$$

where A is ordinarily the **frontal area** (the area projected on a plane normal to the direction of flow) of the body. In other words, A is the area seen by a person looking at the body from the direction of the approaching fluid. The frontal area of a cylinder of diameter D and length L, for example, is $A = LD$. In lift and drag calculations of some thin bodies, such as airfoils, A is taken to be the **planform area**, which is the area seen by a person looking at the body from above in a direction normal to the body. The drag and lift coefficients are primarily functions of the shape of the body, but in some cases they also depend on the Reynolds number and the surface roughness. The term $\frac{1}{2}\rho V^2$ in Eqs. 15–5 and 15–6 is the **dynamic pressure**.

The local drag and lift coefficients vary along the surface as a result of the changes in the velocity boundary layer in the flow direction. We are usually interested in the drag and lift forces for the *entire* surface, which can be determined using the *average* drag and lift coefficients. Therefore, we present correlations for both local (identified with the subscript x) and average drag and lift coefficients. When relations for local drag and lift coefficients for a surface of length L are available, the *average* drag and lift coefficients for the entire surface are determined by integration from

$$C_D = \frac{1}{L}\int_0^L C_{D,x}\,dx \tag{15–7}$$

and

$$C_L = \frac{1}{L}\int_0^L C_{L,x}\,dx \tag{15–8}$$

The forces acting on a falling body are usually the drag force, the buoyant force, and the weight of the body. When a body is dropped into the atmosphere or a lake, it first accelerates under the influence of its weight. The motion of the body is resisted by the drag force, which acts in the direction opposite to motion. As the velocity of the body increases, so does the drag force. This continues until all the forces balance each other and the net force acting on the body (and thus its acceleration) is zero. Then the velocity of the body remains constant during the rest of its fall if the properties of the fluid in the path of the body remain essentially constant. This is the maximum velocity a falling body can attain and is called the **terminal velocity** (Fig. 15–8).

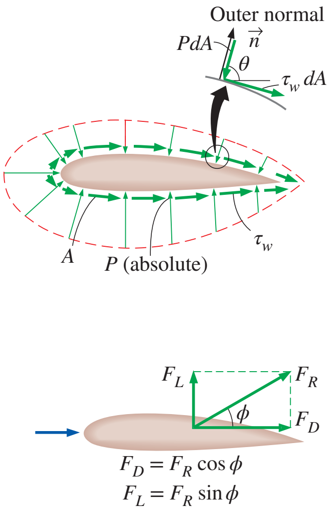


FIGURE 15–5
The pressure and viscous forces acting on a two-dimensional body and the resultant lift and drag forces.

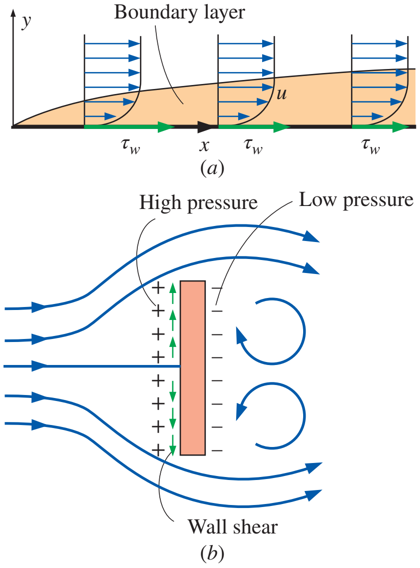


FIGURE 15–6
(*a*) Drag force acting on a flat plate parallel to the flow depends on wall shear only. (*b*) Drag force acting on a flat plate normal to the flow depends on the pressure only and is independent of the wall shear, which acts normal to the free-stream flow.

EXAMPLE 15–1 **Measuring the Drag Coefficient of a Car**

The drag coefficient of a car at the design conditions of 1 atm, 70°F, and 60 mi/h is to be determined experimentally in a large wind tunnel in a full-scale test (Fig. 15–9). The frontal area of the car is 22.26 ft^2. If the force acting on the car in the flow direction is measured to be 68 lbf, determine the drag coefficient of this car.

SOLUTION The drag force acting on a car is measured in a wind tunnel. The drag coefficient of the car at test conditions is to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The cross section of the tunnel is large enough to simulate free flow over the car. **3** The bottom of the tunnel is also moving at the speed of air to approximate actual driving conditions or this effect is negligible.

Properties The density of air at 1 atm and 70°F is $\rho = 0.07489$ lbm/ft^3.

Analysis The drag force acting on a body and the drag coefficient are given by

$$F_D = C_D A \frac{\rho V^2}{2} \quad \text{and} \quad C_D = \frac{2F_D}{\rho A V^2}$$

where A is the frontal area. Substituting and noting that 1 mi/h = 1.467 ft/s, the drag coefficient of the car is determined to be

$$C_D = \frac{2 \times (68 \text{ lbf})}{(0.07489 \text{ lbm/ft}^3)(22.26 \text{ ft}^2)(60 \times 1.467 \text{ ft/s})^2}\left(\frac{32.2 \text{ lbm·ft/s}^2}{1 \text{ lbf}}\right) = \mathbf{0.34}$$

Discussion Note that the drag coefficient depends on the design conditions, and its value may be different at different conditions such as the Reynolds number. Therefore, the published drag coefficients of different vehicles can be compared meaningfully only if they are determined under dynamically similar conditions or if Reynolds number independence is demonstrated. This shows the importance of developing standard testing procedures.

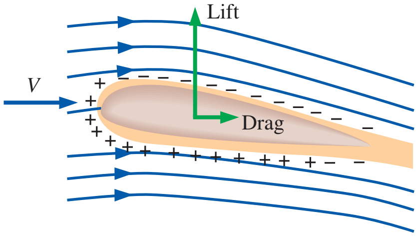


FIGURE 15–7
Airplane wings are shaped and positioned to generate sufficient lift during flight while keeping drag at a minimum. Pressures above and below atmospheric pressure are indicated by plus and minus signs, respectively.

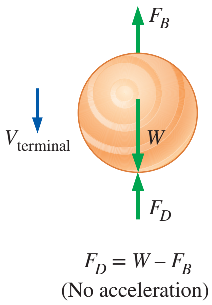


FIGURE 15–8
During a free fall, a body reaches its *terminal velocity* when the drag force equals the weight of the body minus the buoyant force.

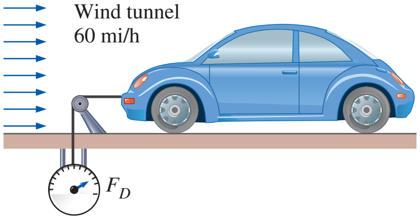


FIGURE 15–9
Schematic for Example 15–1.

15–3 ▪ FRICTION AND PRESSURE DRAG

As mentioned in Section 15–2, the drag force is the net force exerted by a fluid on a body in the direction of flow due to the combined effects of wall shear and pressure forces. It is often instructive to separate the two effects and study them separately.

The part of drag that is due directly to wall shear stress τ_w is called the **skin friction drag** (or just *friction drag* $F_{D,\text{ friction}}$) since it is caused by frictional effects, and the part that is due directly to pressure P is called the **pressure drag** (also called the *form drag* because of its strong dependence on the form or shape of the body). The friction and pressure drag coefficients are defined as

$$C_{D,\text{friction}} = \frac{F_{D,\text{friction}}}{\frac{1}{2}\rho V^2 A} \quad \text{and} \quad C_{D,\text{pressure}} = \frac{F_{D,\text{pressure}}}{\frac{1}{2}\rho V^2 A} \tag{15–9}$$

When the friction and pressure drag coefficients (based on the same area A) or forces are available, the total drag coefficient or drag force is determined by simply adding them,

$$C_D = C_{D,\text{friction}} + C_{D,\text{pressure}} \quad \text{and} \quad F_D = F_{D,\text{friction}} + F_{D,\text{pressure}} \tag{15–10}$$

The *friction drag* is the component of the wall shear force in the direction of flow, and thus it depends on the orientation of the body as well as the magnitude of the wall shear stress τ_w. The friction drag is *zero* for a flat surface normal to the flow and *maximum* for a flat surface parallel to the flow since the friction drag in this case equals the total shear force on the surface. Therefore, for parallel flow over a flat surface, the drag coefficient is equal to the *friction drag coefficient,* or simply the *friction coefficient*. Friction drag is a strong function of viscosity and increases with increasing viscosity.

The Reynolds number is inversely proportional to the viscosity of the fluid. Therefore, the contribution of friction drag to total drag for blunt bodies is less at higher Reynolds numbers and may be negligible at very high Reynolds numbers. The drag in such cases is mostly due to pressure drag. At low Reynolds numbers, most drag is due to friction drag. This is especially the case for highly streamlined bodies such as airfoils. The friction drag is also proportional to the surface area. Therefore, bodies with a larger surface area experience a larger friction drag. Large commercial airplanes, for example, reduce their total surface area and thus their drag by retracting their wing extensions when they reach cruising altitudes to save fuel. The friction drag coefficient is independent of *surface roughness* in laminar flow, but is a strong function of surface roughness in turbulent flow due to surface roughness elements protruding further into the boundary layer. The *friction drag coefficient* is analogous to the *friction factor* in pipe flow discussed in Chap. 14, and its value depends on the flow regime.

The pressure drag is proportional to the frontal area and to the *difference* between the pressures acting on the front and back of the immersed body. Therefore, the pressure drag is usually dominant for blunt bodies, small for streamlined bodies such as airfoils, and zero for thin flat plates parallel to the flow (Fig. 15–10). The pressure drag becomes most significant when the velocity of the fluid is too high for the fluid to be able to follow the curvature of the body, and thus the fluid *separates* from the body at some point and creates a very low-pressure region in the back. The pressure drag in this case is due to the large pressure difference between the front and back sides of the body.

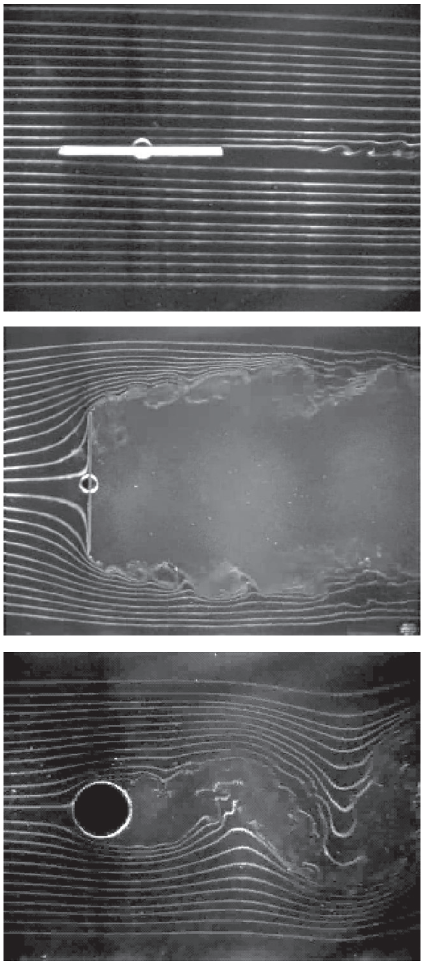


FIGURE 15–10

Drag is due entirely to *friction drag* for a flat plate parallel to the flow; it is due entirely to pressure drag for a flat plate normal to the flow; and it is due to *both* (but mostly *pressure drag*) for a cylinder normal to the flow. The total drag coefficient C_D is lowest for a parallel flat plate, highest for a vertical flat plate, and in between (but close to that of a vertical flat plate) for a cylinder.

From G. M. Homsy et al., "Multi-Media Fluid Mechanics," Cambridge Univ. Press (2001). Image ©Stanford University (2000). Reprinted by permission.

Reducing Drag by Streamlining

The first thought that comes to mind to reduce drag is to streamline a body in order to reduce flow separation and thus to reduce pressure drag. Even car salespeople are quick to point out the low drag coefficients of their cars, owing to streamlining. But streamlining has opposite effects on pressure and friction drag forces. It decreases pressure drag by delaying boundary layer separation and thus reducing the pressure difference between the front and back of the body and increases the friction drag by increasing the surface area. The end result depends on which effect dominates. Therefore, any optimization study to reduce the drag of a body must consider both effects and must attempt to minimize the *sum* of the two, as shown in Fig. 15–11. The minimum total drag occurs at $D/L = 0.25$ for the case shown in Fig. 15–11. For the case of a circular cylinder with the same thickness as the streamlined shape of Fig. 15–11, the drag coefficient would be about five times as much. Therefore, it is possible to reduce the drag of a cylindrical component to nearly one-fifth by the use of proper fairings.

The effect of streamlining on the drag coefficient is described best by considering long elliptical cylinders with different aspect (or length-to-thickness) ratios L/D, where L is the length in the flow direction and D is the thickness, as shown in Fig. 15–12. Note that the drag coefficient decreases drastically as the ellipse becomes slimmer. For the special case of $L/D = 1$

(a circular cylinder), the drag coefficient is $C_D \cong 1$ at this Reynolds number. As the aspect ratio is decreased and the cylinder resembles a flat plate, the drag coefficient increases to 1.9, the value for a flat plate normal to flow. Note that the curve becomes nearly flat for aspect ratios greater than about 4. Therefore, for a given diameter D, elliptical shapes with an aspect ratio of about $L/D \cong 4$ usually offer a good compromise between the total drag coefficient and length L. The reduction in the drag coefficient at high aspect ratios is primarily due to the boundary layer staying attached to the surface longer and the resulting pressure recovery. The pressure drag on an elliptical cylinder with an aspect ratio of 4 or greater is negligible (less than 2 percent of total drag at this Reynolds number).

As the aspect ratio of an elliptical cylinder is increased by flattening it (i.e., decreasing D while holding L constant), the drag coefficient starts increasing and tends to infinity as $L/D \rightarrow \infty$ (i.e., as the ellipse resembles a flat plate parallel to flow). This is due to the frontal area, which appears in the denominator in the definition of C_D, approaching zero. It does not mean that the drag force increases drastically (actually, the drag force decreases) as the body becomes flat. This shows that the frontal area is inappropriate for use in the drag force relations for slim bodies such as thin airfoils and flat plates. In such cases, the drag coefficient is defined on the basis of the *planform area,* which is simply the surface area of one side (top or bottom) of a flat plate parallel to the flow. This is quite appropriate since for slim bodies the drag is almost entirely due to friction drag, which is proportional to the surface area.

Streamlining has the added benefit of *reducing vibration and noise*. Streamlining should be considered only for bluff bodies that are subjected to high-velocity fluid flow (and thus high Reynolds numbers) for which flow separation is a real possibility. It is not necessary for bodies that typically involve low Reynolds number flows (e.g., creeping flows in which Re < 1) since the drag in those cases is almost entirely due to friction drag, and streamlining would only increase the surface area and thus the total drag. Therefore, careless streamlining may actually increase drag instead of decreasing it.

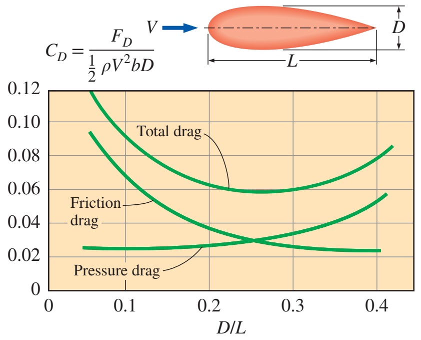


FIGURE 15–11
The variation of friction, pressure, and total drag coefficients of a two-dimensional streamlined strut with thickness-to-chord length ratio for Re = 4 × 10^4. Note that C_D for airfoils and other thin bodies is based on *planform* area *bL* rather than frontal area *bD*, where *b* is the width of the 2-D body into the page.
Data from Abbott and von Doenhoff (1959).

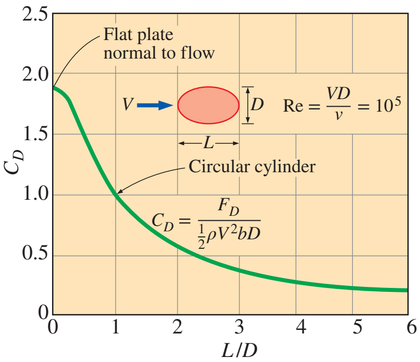


FIGURE 15–12
The variation of the drag coefficient of a long elliptical cylinder with aspect ratio. Here C_D is based on the frontal area *bD* where *b* is the width of the 2-D body into the page.
Data from Blevins (1984).

Flow Separation

When driving on country roads, it is a common safety measure to slow down at sharp turns in order to avoid being thrown off the road. Many drivers have learned the hard way that a car refuses to comply when forced to turn curves at excessive speeds. We can view this phenomenon as "the separation of cars" from roads. This phenomenon is also observed when fast vehicles jump off hills. At low velocities, the wheels of the vehicle always remain in contact with the road surface. But at high velocities, the vehicle is too fast to follow the curvature of the road and takes off at the hill, losing contact with the road.

A fluid acts much the same way when forced to flow over a curved surface at high velocities. A fluid follows the front portion of the curved surface with no problem, but it has difficulty remaining attached to the surface on the back side. At sufficiently high velocities, the fluid stream detaches itself from the surface of the body. This is called **flow separation** (Fig. 15–13). Flow can separate from a surface even if it is fully submerged in a liquid or immersed in a gas (Fig. 15–14). The location of the separation point depends on several factors such as the Reynolds number, the surface roughness, and the level of fluctuations in the free stream, and it is usually difficult to predict exactly where separation will occur unless there are sharp corners or abrupt changes in the shape of the solid surface.

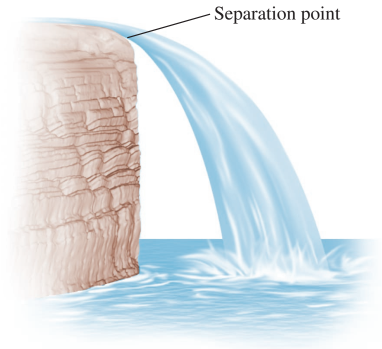


FIGURE 15–13
Flow separation in a waterfall.

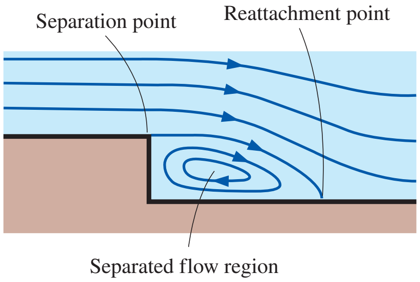


FIGURE 15–14

Flow separation over a backward-facing step along a wall.

FIGURE 15–15

Flow separation and the wake region for flow over a tennis ball.

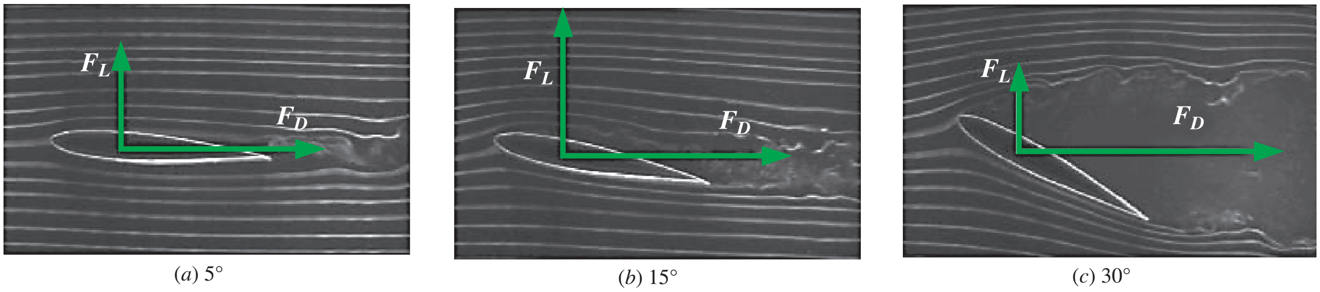
Courtesy NASA and the Cislunar Aerospace, Inc.

When a fluid separates from a body, it forms a separated region between the body and the fluid stream. This low-pressure region behind the body where recirculating and backflows occur is called the **separated region**. The larger the separated region, the larger the pressure drag. The effects of flow separation are felt far downstream in the form of reduced velocity (relative to the upstream velocity). The region of flow trailing the body where the effects of the body on velocity are felt is called the **wake** (Fig. 15–15). The separated region comes to an end when the two flow streams reattach. Therefore, the separated region is an enclosed volume, whereas the wake keeps growing behind the body until the fluid in the wake region regains its velocity and the velocity profile becomes nearly flat again. Viscous and rotational effects are the most significant in the boundary layer, the separated region, and the wake.

The occurrence of separation is not limited to bluff bodies. Complete separation over the entire back surface may also occur on a streamlined body such as an airplane wing at a sufficiently large **angle of attack** (larger than about 15° for most airfoils), which is the angle the incoming fluid stream makes with the **chord** (the line that connects the nose and the trailing edge) of the wing. Flow separation on the top surface of a wing reduces lift drastically and may cause the airplane to **stall**. Stalling has been blamed for many airplane accidents and loss of efficiencies in turbomachinery (Fig. 15–16).

Note that drag and lift are strongly dependent on the shape of the body, and any effect that causes the shape to change has a profound effect on the drag and lift. For example, snow accumulation and ice formation on airplane wings may change the shape of the wings sufficiently to cause significant loss of lift. This phenomenon has caused many airplanes to lose altitude and crash and many others to abort takeoff. Therefore, it has become a routine safety measure to check for ice or snow buildup on critical components of airplanes before takeoff in bad weather. This is especially important for airplanes that have waited a long time on the runway before takeoff because of heavy traffic.

An important consequence of flow separation is the formation and shedding of circulating fluid structures, called **vortices**, in the wake region. The periodic generation of these vortices downstream is referred to as **vortex shedding**. This phenomenon usually occurs during normal flow over long cylinders or spheres for Re $\gtrsim$ 90. The vibrations generated by vortices near the body may cause the body to resonate to dangerous levels if the frequency of the vortices is close to the natural frequency of the body—a situation that must be avoided in the design of equipment that is subjected to high-velocity fluid flow such as the wings of airplanes and suspended bridges subjected to steady high winds.



(*a*) 5° (*b*) 15° (*c*) 30°

FIGURE 15–16

At large angles of attack (usually larger than 15°), flow may separate completely from the top surface of an airfoil, reducing lift drastically and causing the airfoil to stall.

From G. M. Homsy et al., “Multi-Media Fluid Mechanics,” Cambridge Univ. Press (2001). Image ©Stanford University (2000). Reprinted by permission.

15–4 ▪ DRAG COEFFICIENTS OF COMMON GEOMETRIES

The concept of drag has important consequences in daily life, and the drag behavior of various natural and human-made bodies is characterized by their drag coefficients measured under typical operating conditions. Although drag is caused by two different effects (friction and pressure), it is usually difficult to determine them separately. Besides, in most cases, we are interested in the *total* drag rather than the individual drag components, and thus usually the *total* drag coefficient is reported. The determination of drag coefficients has been the topic of numerous studies (mostly experimental), and there is a huge amount of drag coefficient data in the literature for just about any geometry of practical interest.

The drag coefficient, in general, depends on the *Reynolds number,* especially for Reynolds numbers below about 10^4. At higher Reynolds numbers, the drag coefficients for most geometries remain essentially constant (Fig. 15–17). This is due to the flow at high Reynolds numbers becoming fully turbulent. However, this is not the case for rounded bodies such as circular cylinders and spheres, as we discuss later in this section. The reported drag coefficients are usually applicable only to flows at high Reynolds numbers.

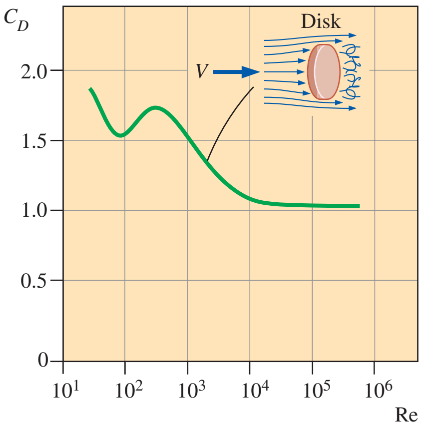


FIGURE 15–17
The drag coefficients for most geometries (but not all) remain essentially constant at Reynolds numbers above about 10^4.

The drag coefficient exhibits different behavior in the low (creeping), moderate (laminar), and high (turbulent) regions of the Reynolds number. The inertia effects are negligible in low Reynolds number flows ($\text{Re} \lesssim 1$), called *creeping flows,* and the fluid wraps around the body smoothly. The drag coefficient in this case is inversely proportional to the Reynolds number, and for a sphere it is determined to be

$$\textit{Sphere:} \qquad C_D = \frac{24}{\text{Re}} \qquad (\text{Re} \lesssim 1) \tag{15–11}$$

Then the drag force acting on a spherical object at low Reynolds numbers becomes

$$F_D = C_D A \frac{\rho V^2}{2} = \frac{24}{\text{Re}} A \frac{\rho V^2}{2} = \frac{24}{\rho VD/\mu} \frac{\pi D^2}{4} \frac{\rho V^2}{2} = 3\pi\mu VD \tag{15–12}$$

which is known as **Stokes' law**, after British mathematician and physicist G. G. Stokes (1819–1903). This relation shows that at very low Reynolds numbers, the drag force acting on spherical objects is proportional to the diameter, the velocity, and the viscosity of the fluid. This relation is often applicable to dust particles in the air and suspended solid particles in water.

The drag coefficients for low Reynolds number flows past some other geometries are given in Fig. 15–18. Note that at low Reynolds numbers, the shape of the body does not have a major influence on the drag coefficient.

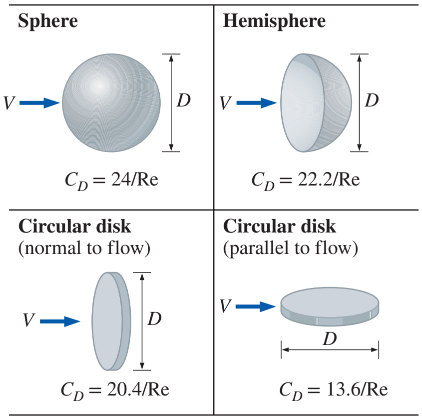


FIGURE 15–18
Drag coefficients C_D at low Reynolds numbers ($\text{Re} \lesssim 1$ where $\text{Re} = VD/\nu$ and $A = \pi D^2/4$).

The drag coefficients for various two- and three-dimensional bodies are given in Tables 15–1 and 15–2 for large Reynolds numbers. We make several observations from these tables about the drag coefficient at high Reynolds numbers. First of all, the *orientation* of the body relative to the direction of flow has a major influence on the drag coefficient. For example, the drag coefficient for flow over a hemisphere is 0.4 when the spherical side faces the flow, but it increases threefold to 1.2 when the flat side faces the flow (Fig. 15–19).

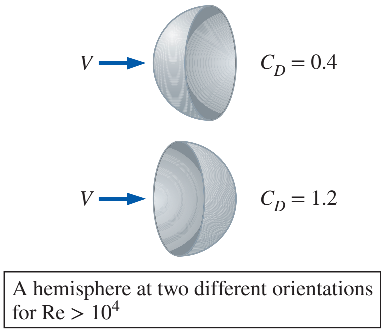


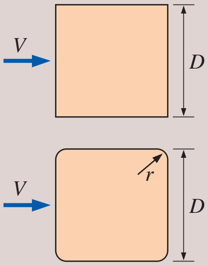
FIGURE 15–19
The drag coefficient of a body may change drastically by changing the body's orientation (and thus shape) relative to the direction of flow.

For blunt bodies with sharp corners, such as flow over a rectangular block or a flat plate normal to the flow, separation occurs at the edges of the front and back surfaces, with no significant change in the character of flow. Therefore, the drag coefficient of such bodies is nearly independent of the Reynolds number. Note that the drag coefficient of a long rectangular rod is reduced almost by half from 2.2 to 1.2 by rounding the corners.

TABLE 15–1

Drag coefficients C_D of various two-dimensional bodies for Re > 10^4 based on the frontal area $A = bD$, where b is the length in direction normal to the page (for use in the drag force relation $F_D = C_D A \rho V^2/2$ where V is the upstream velocity)

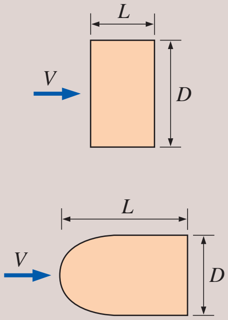
Square rod



Sharp corners: $C_D = 2.2$

Round corners ($r/D = 0.2$): $C_D = 1.2$

Rectangular rod



Sharp corners:

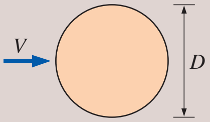
L/D	C_D
0.0*	1.9
0.1	1.9
0.5	2.5
1.0	2.2
2.0	1.7
3.0	1.3

* Corresponds to thin plate

Round front edge:

L/D	C_D
0.5	1.2
1.0	0.9
2.0	0.7
4.0	0.7

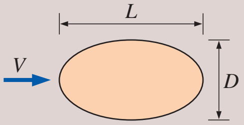
Circular rod (cylinder)



Laminar: $C_D = 1.2$

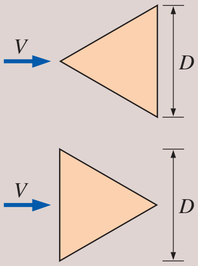
Turbulent: $C_D = 0.3$

Elliptical rod



	C_D	
L/D	Laminar	Turbulent
2	0.60	0.20
4	0.35	0.15
8	0.25	0.10

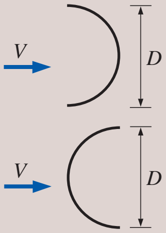
Equilateral triangular rod



$C_D = 1.5$

$C_D = 2.0$

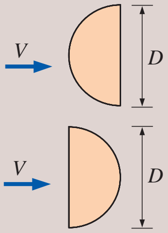
Semicircular shell



$C_D = 2.3$

$C_D = 1.2$

Semicircular rod



$C_D = 1.2$

$C_D = 1.7$

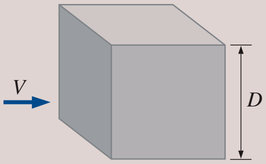
Biological Systems and Drag

The concept of drag also has important consequences for biological systems. For example, the bodies of *fish,* especially the ones that swim fast for long distances (such as dolphins), are highly streamlined to minimize drag (the drag coefficient of dolphins based on the wetted skin area is about 0.0035, comparable to the value for a flat plate in turbulent flow). So it is no surprise that we build submarines that mimic large fish. Tropical fish with fascinating beauty and elegance, on the other hand, swim short distances only. Obviously grace, not high speed and drag, was the primary consideration in their design. Birds teach us a lesson on drag

TABLE 15–2

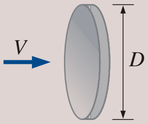
Representative drag coefficients C_D for various three-dimensional bodies based on the frontal area for $Re > 10^4$ unless stated otherwise (for use in the drag force relation $F_D = C_D A \rho V^2/2$ where V is the upstream velocity)

Cube, $A = D^2$



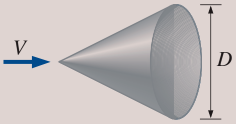
$C_D = 1.05$

Thin circular disk, $A = \pi D^2/4$



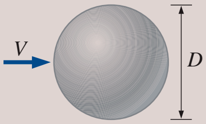
$C_D = 1.1$

Cone (for $\theta = 30°$), $A = \pi D^2/4$



$C_D = 0.5$

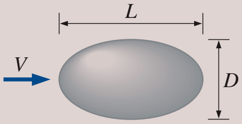
Sphere, $A = \pi D^2/4$



Laminar:
$Re \lesssim 2 \times 10^5$
$C_D = 0.5$
Turbulent:
$Re \gtrsim 2 \times 10^6$
$C_D = 0.2$

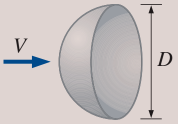
See Fig. 15–36 for C_D vs. Re for smooth and rough spheres.

Ellipsoid, $A = \pi D^2/4$

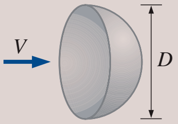


	C_D	
L/D	Laminar $Re \lesssim 2 \times 10^5$	Turbulent $Re \gtrsim 2 \times 10^6$
0.75	0.5	0.2
1	0.5	0.2
2	0.3	0.1
4	0.3	0.1
8	0.2	0.1

Hemisphere, $A = \pi D^2/4$

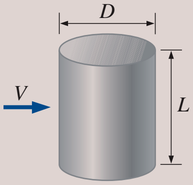


$C_D = 0.4$



$C_D = 1.2$

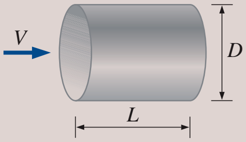
Finite cylinder, vertical, $A = LD$



L/D	C_D
1	0.6
2	0.7
5	0.8
10	0.9
40	1.0
∞	1.2

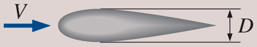
Values are for laminar flow ($Re \lesssim 2 \times 10^5$)

Finite cylinder, horizontal, $A = \pi D^2/4$



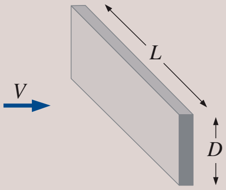
L/D	C_D
0.5	1.1
1	0.9
2	0.9
4	0.9
8	1.0

Streamlined body, $A = \pi D^2/4$



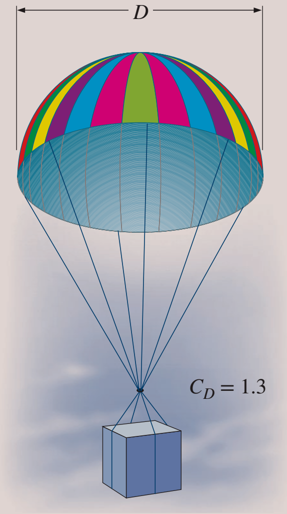
$C_D = 0.04$

Rectangular plate, $A = LD$



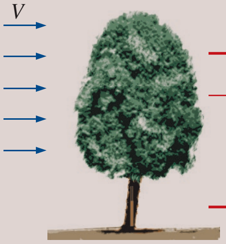
$C_D = 1.10 + 0.02\,(L/D + D/L)$
for $1/30 < (L/D) < 30$

Parachute, $A = \pi D^2/4$



$C_D = 1.3$

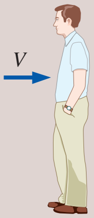
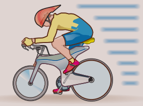
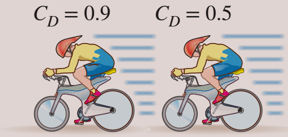
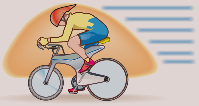
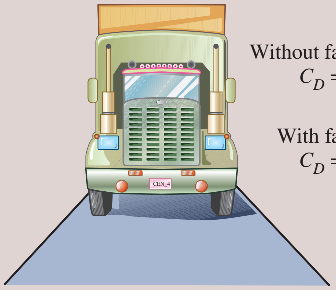
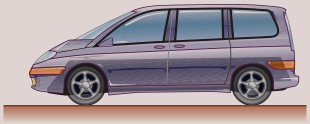
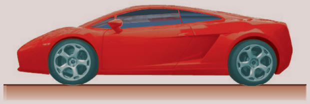
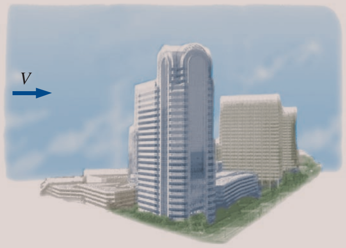
Tree, A = frontal area



V, m/s	C_D
10	0.4–1.2
20	0.3–1.0
30	0.2–0.7

(*continues*)

TABLE 15–2 (Continued)

Person (average)  Standing: $C_D A = 9\ \text{ft}^2 = 0.84\ \text{m}^2$ Sitting: $C_D A = 6\ \text{ft}^2 = 0.56\ \text{m}^2$	Bikes  Upright: $A = 5.5\ \text{ft}^2 = 0.51\ \text{m}^2$ $C_D = 1.1$  Racing: $A = 3.9\ \text{ft}^2 = 0.36\ \text{m}^2$ $C_D = 0.9$	 Drafting: $A = 3.9\ \text{ft}^2 = 0.36\ \text{m}^2$ $C_D = 0.50$  With fairing: $A = 5.0\ \text{ft}^2 = 0.46\ \text{m}^2$ $C_D = 0.12$
Semitrailer, A = frontal area  Without fairing: $C_D = 0.96$ With fairing: $C_D = 0.76$	Automotive, A = frontal area  Minivan: $C_D = 0.4$  Passenger car or sports car: $C_D = 0.3$	High-rise buildings, A = frontal area $C_D \approx 1.0$ to 1.4 

reduction by extending their beaks forward and folding their feet backward during flight (Fig. 15–20). Airplanes, which look somewhat like large birds, retract their wheels after takeoff in order to reduce drag and thus fuel consumption.

The flexible structure of plants enables them to reduce drag in high winds by changing their shapes. Large flat leaves, for example, curl into a low-drag conical shape at high wind speeds, while tree branches cluster to reduce drag. Flexible trunks bend under the influence of the wind to reduce drag, and the bending moment is lowered by reducing frontal area.

If you watch the Olympic Games, you have probably observed many instances of conscious effort by the competitors to reduce drag. Some examples: During 100-m running, the runners hold their fingers together and straight and move their hands parallel to the direction of motion to reduce the drag on their hands. Swimmers with long hair cover their heads with a tight and smooth cover to reduce head drag. They also wear well-fitting one-piece swimming suits. Horse and bicycle riders lean forward as much as they can to reduce drag (by reducing both the drag coefficient and frontal area). Speed skiers do the same thing.

Drag Coefficients of Vehicles

The term *drag coefficient* is commonly used in various areas of daily life. Car manufacturers try to attract consumers by pointing out the *low drag coefficients* of their cars (Fig. 15–21). The drag coefficients of vehicles range from about 1.0 for large semitrailers to 0.4 for minivans and 0.3 for passenger cars. In general, the more blunt the vehicle, the higher the drag coefficient. Installing a fairing reduces the drag coefficient of tractor-trailer rigs by about 20 percent by making the frontal surface more streamlined. As a rule of thumb, the percentage of fuel savings due to reduced drag is about half the percentage of drag reduction at highway speeds.

When the effect of the road on air motion is disregarded, the ideal shape of a *vehicle* is the basic *teardrop,* with a drag coefficient of about 0.1 for the turbulent flow case. But this shape needs to be modified to accommodate several necessary external components such as wheels, mirrors, axles, and door handles. Also, the vehicle must be high enough for comfort, and there must be a minimum clearance from the road. Further, a vehicle cannot be too long to fit in garages and parking spaces. Controlling the material and manufacturing costs requires minimizing or eliminating any "dead" volume that cannot be utilized. The result is a shape that resembles more a box than a teardrop, and this was the shape of early cars with a drag coefficient of about 0.8 in the 1920s. This wasn't a problem in those days since the velocities were low, fuel was cheap, and drag was not a major design consideration.

The average drag coefficients of cars dropped to about 0.70 in the 1940s, to 0.55 in the 1970s, to 0.45 in the 1980s, and to 0.30 in the 1990s as a result of improved manufacturing techniques for metal forming and paying more attention to the shape of the car and streamlining (Fig. 15–22). The drag coefficient for well-built racing cars is about 0.2, but this is achieved after making the comfort of drivers a secondary consideration. Noting that the theoretical lower limit of C_D is about 0.1 and the value for racing cars is 0.2, it appears that there is only a little room for further improvement in the drag coefficient of passenger cars from the current value of about 0.3. The drag coefficient of a Mazda 3, for example, is 0.29. For trucks and buses, the drag coefficient can be reduced further by optimizing the front and rear contours (by rounding, for example) to the extent it is practical while keeping the overall length of the vehicle the same.

When traveling as a group, a sneaky way of reducing drag is **drafting**, a phenomenon well known to bicycle riders and car racers. It involves approaching a moving body from behind and being *drafted* into the low-pressure region in the rear of the body. The drag coefficient of a racing bicyclist, for example, is reduced from 0.9 to 0.5 (Table 15–2) by drafting, as also shown in Fig. 15–23.

We also can help reduce the overall drag of a vehicle and thus fuel consumption by being more conscientious drivers. For example, drag force is proportional to the square of velocity. Therefore, driving over the speed limit on the highways not only increases the chances of getting speeding tickets or getting into an accident, but it also increases the amount of fuel consumption per mile. Therefore, driving at moderate speeds is safe and economical. Also, anything that extends from the car, even an arm, increases the drag coefficient. Driving with the windows rolled down also increases the drag and fuel consumption. At highway speeds, a driver can often save fuel in hot weather by running the air conditioner instead of driving with the windows rolled down. For many low-drag automobiles, the turbulence and additional drag generated by open windows consume more fuel than does the air conditioner, but this is not the case for high-drag vehicles.

Superposition

The shapes of many bodies encountered in practice are not simple. But such bodies can be treated conveniently in drag force calculations by considering them to be composed of two or more simple bodies. A satellite dish mounted on a roof with a cylindrical bar, for example, can be considered to be a combination of a hemispherical body and a cylinder. Then the drag coefficient of the body can be determined approximately by using **superposition**. Such a

FIGURE 15–20
Birds teach us a lesson on drag reduction by extending their beaks forward and folding their feet backward during flight.
Adam Jones

FIGURE 15–21
This sleek-looking Toyota Prius has a drag coefficient of 0.26—one of the lowest for a passenger car.
©Hannu Liivaar/Alamy Stock Photo

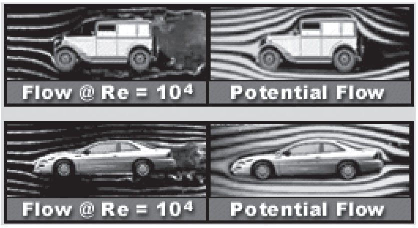


FIGURE 15–22
Streamlines around an aerodynamically designed modern car closely resemble the streamlines around the car in the ideal potential flow (assumes negligible friction), except near the rear end, resulting in a low drag coefficient.
Left: From G. M. Homsy et al., "Multi-Media Fluid Mechanics," Cambridge Univ. Press (2001). Image ©Stanford University (2000). Reprinted by permission.
Right: From G. M. Homsy et al., "Multi-Media Fluid Mechanics," Cambridge Univ. Press (2001). Image ©Stanford University (2000) and Sigurdur D. Thoroddsen. Reprinted by permission.

simplistic approach does not account for the effects of components on each other, and thus the results obtained should be interpreted accordingly.

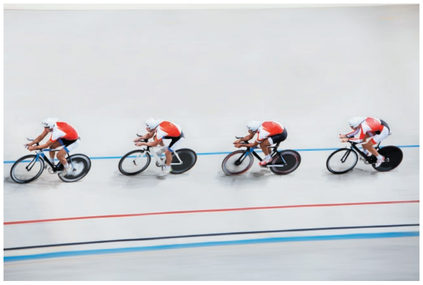


FIGURE 15–23
The drag coefficients of bodies following other moving bodies closely is reduced considerably due to drafting (i.e., entering into the low-pressure region created by the body in front).
Clerkenwell/Getty Images

FIGURE 15–24
Schematic for Example 15–2.

EXAMPLE 15–2 Effect of Frontal Area on Fuel Efficiency of a Car

Two common methods of improving the fuel efficiency of a vehicle are to reduce the drag coefficient and the frontal area of the vehicle. Consider a car (Fig. 15–24) whose width (W) and height (H) are 1.85 m and 1.70 m, respectively, with a drag coefficient of 0.30. Determine the amount of fuel and money saved per year as a result of reducing the car height to 1.55 m while keeping its width the same. Assume the car is driven 18,000 km a year at an average speed of 95 km/h. Take the density and price of gasoline to be 0.74 kg/L and \$0.95/L, respectively. Also take the density of air to be 1.20 kg/m^3, the heating value of gasoline to be 44,000 kJ/kg, and the overall efficiency of the car's drive train to be 30 percent.

SOLUTION The frontal area of a car is reduced by redesigning it. The resulting fuel and money savings per year are to be determined.

Assumptions **1** The car is driven 18,000 km a year at an average speed of 95 km/h. **2** The effect of reduction of the frontal area on the drag coefficient is negligible.

Properties The densities of air and gasoline are given to be 1.20 kg/m^3 and 0.74 kg/L, respectively. The heating value of gasoline is given to be 44,000 kJ/kg.

Analysis The drag force acting on a body is

$$F_D = C_D A \frac{\rho V^2}{2}$$

where A is the frontal area of the body. The drag force acting on the car before redesigning is

$$\begin{aligned} F_D &= 0.3(1.85 \times 1.70\ \text{m}^2)\frac{(1.20\ \text{kg/m}^3)(95\ \text{km/h})^2}{2}\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)^2\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) \\ &= 393\ \text{N} \end{aligned}$$

Noting that work is force times distance, the amount of work done to overcome this drag force and the required energy input for a distance of 18,000 km are

$$\begin{aligned} W_{\text{drag}} &= F_D L = (394\ \text{N})(18{,}000\ \text{km/yr})\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{kJ}}{1000\ \text{N}\cdot\text{m}}\right) \\ &= 7.092 \times 10^6\ \text{kJ/yr} \end{aligned}$$

$$E_{\text{in}} = \frac{W_{\text{drag}}}{\eta_{\text{car}}} = \frac{7.092 \times 10^6\ \text{kJ/yr}}{0.30} = 2.364 \times 10^7\ \text{kJ/yr}$$

The amount and the cost of the fuel that supplies this much energy are

$$\begin{aligned} \text{Amount of fuel} &= \frac{m_{\text{fuel}}}{\rho_{\text{fuel}}} = \frac{E_{\text{in}}/\text{HV}}{\rho_{\text{fuel}}} = \frac{(2.364 \times 10^7\ \text{kJ/yr})/(44{,}000\ \text{kJ/kg})}{0.74\ \text{kg/L}} \\ &= 726\ \text{L/yr} \end{aligned}$$

$$\text{Cost} = (\text{Amount of fuel})(\text{Unit cost}) = (726\ \text{L/yr})(\$0.95/\text{L}) = \$690/\text{yr}$$

That is, the car uses about 730 liters of gasoline at a total cost of about \$690 per year to overcome the drag.

The drag force and the work done to overcome it are directly proportional to the frontal area. Then the percent reduction in the fuel consumption due to reducing the frontal area is equal to the percent reduction in the frontal area:

$$\text{Reduction ratio} = \frac{A - A_{\text{new}}}{A} = \frac{H - H_{\text{new}}}{H} = \frac{1.70 - 1.55}{1.70} = 0.0882$$

$$\text{Amount reduction} = (\text{Reduction ratio})(\text{Amount})$$

$$\text{Fuel reduction} = 0.0882(726\ \text{L/yr}) = \mathbf{64\ L/yr}$$

$$\text{Cost reduction} = (\text{Reduction ratio})(\text{Cost}) = 0.0882(\$690/\text{yr}) = \mathbf{\$61/yr}$$

Therefore, reducing the car's height reduces the fuel consumption due to drag by nearly 9 percent.

Discussion Answers are given to 2 significant digits. This example demonstrates that significant reductions in drag and fuel consumption can be achieved by reducing the frontal area of a vehicle as well as its drag coefficient.

Example 15–2 is indicative of the tremendous amount of effort put into redesigning various parts of cars such as the window moldings, the door handles, the windshield, and the front and rear ends in order to reduce aerodynamic drag. For a car moving on a level road at constant speed, the power developed by the engine is used to overcome rolling resistance, friction between moving components, aerodynamic drag, and driving the auxiliary equipment. The aerodynamic drag is negligible at low speeds, but it becomes significant at speeds above about 30 mi/h. Reduction of the frontal area of the cars (to the dislike of tall drivers) has also contributed greatly to the reduction of drag and fuel consumption.

15–5 ▪ PARALLEL FLOW OVER FLAT PLATES

Consider the flow of a fluid over a *flat plate,* as shown in Fig. 15–25. Surfaces that are slightly contoured (such as turbine blades) also can be approximated as flat plates with reasonable accuracy. The x-coordinate is measured along the plate surface from the *leading edge* of the plate in the direction of the flow, and y is measured from the surface in the normal direction. The fluid approaches the plate in the x-direction with a uniform velocity V, which is equivalent to the velocity over the plate away from the surface.

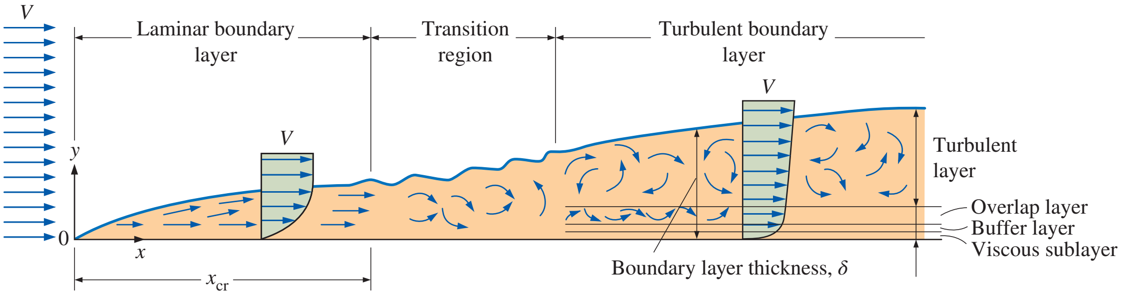


FIGURE 15–25
The development of the boundary layer for flow over a flat plate, and the different flow regimes. Not to scale.

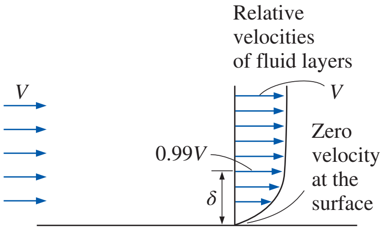
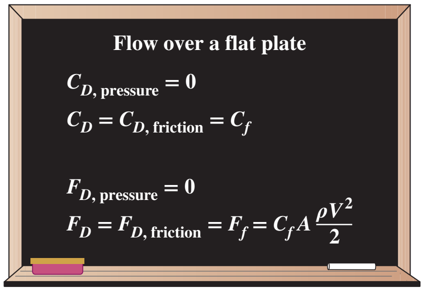


FIGURE 15–26
The development of a boundary layer on a surface is due to the no-slip condition and friction.



Flow over a flat plate

$C_{D,\text{ pressure}} = 0$

$C_D = C_{D,\text{ friction}} = C_f$

$F_{D,\text{ pressure}} = 0$

$F_D = F_{D,\text{ friction}} = F_f = C_f A \dfrac{\rho V^2}{2}$

FIGURE 15–27
For parallel flow over a flat plate, the pressure drag is zero, and thus the drag coefficient is equal to the friction coefficient and the drag force is equal to the friction force.

For the sake of discussion, we consider the fluid to consist of adjacent layers piled on top of each other. The velocity of the particles in the first fluid layer adjacent to the plate is zero because of the no-slip condition. This motionless layer slows down the particles of the neighboring fluid layer as a result of friction between the particles of these two adjoining fluid layers at different velocities. This fluid layer then slows down the molecules of the next layer, and so on. Thus, the presence of the plate is felt up to some normal distance δ from the plate beyond which the free-stream velocity remains virtually unchanged. As a result, the x-component of the fluid velocity, u, varies from 0 at $y = 0$ to nearly V (typically $0.99V$) at $y = \delta$ (Fig. 15–26).

The region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the **velocity boundary layer**. The *boundary layer thickness* δ is typically defined as the distance y from the surface at which $u = 0.99V$.

The hypothetical line of $u = 0.99V$ divides the flow over a plate into two regions: the **boundary layer region**, in which the viscous effects and the velocity changes are significant, and the **irrotational flow region**, in which the frictional effects are negligible and the velocity remains essentially constant.

For parallel flow over a flat plate, the pressure drag is zero, and thus the drag coefficient is equal to the *friction drag coefficient,* or simply the *friction coefficient* (Fig. 15–27). That is,

Flat plate: $$C_D = C_{D,\text{ friction}} = C_f \tag{15–13}$$

Once the average friction coefficient C_f is available, the drag (or friction) force over the surface is determined from

Friction force on a flat plate: $$F_D = F_f = \tfrac{1}{2} C_f A \rho V^2 \tag{15–14}$$

where A is the surface area of the plate exposed to fluid flow. When both sides of a thin plate are subjected to flow, A becomes the total area of the top and bottom surfaces. Note that both the average friction coefficient C_f and the local friction coefficient $C_{f,\,x}$, in general, vary with location along the surface.

Typical average velocity profiles in laminar and turbulent flow are sketched in Fig. 15–25. Note that the velocity profile in turbulent flow is much fuller than that in laminar flow, with a sharp drop near the surface. The turbulent boundary layer can be considered to consist of four regions, characterized by the distance from the wall. The very thin layer next to the wall where viscous effects are dominant is the **viscous sublayer**. The velocity profile in this layer is very nearly *linear*, and the flow is nearly parallel. Next to the viscous sublayer is the **buffer layer**, in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects. Above the buffer layer is the **overlap layer**, in which the turbulent effects are much more significant, but still not dominant. Above that is the **turbulent** (or **outer**) **layer** in which turbulent effects dominate over viscous effects. Note that the turbulent boundary layer profile on a flat plate closely resembles the boundary layer profile in fully developed turbulent pipe flow.

The transition from laminar to turbulent flow depends on the *surface geometry, surface roughness, upstream velocity, surface temperature,* and the *type of fluid,* among other things, and is best characterized by the Reynolds number. The Reynolds number at a distance x from the leading edge of a flat plate is expressed as

$$\text{Re}_x = \frac{\rho V x}{\mu} = \frac{Vx}{\nu} \tag{15–15}$$

where V is the upstream velocity and x is the characteristic length of the geometry, which, for a flat plate, is the length of the plate in the flow direction. Note that unlike pipe flow, the Reynolds number varies for a flat plate along the flow, reaching $\text{Re}_L = VL/\nu$ at the end of the plate. For any point on a flat plate, the characteristic length is the distance x of the point from the leading edge in the flow direction.

For flow over a smooth flat plate, transition from laminar to turbulent begins at about $\text{Re} \cong 1 \times 10^5$ but does not become fully turbulent before the Reynolds number reaches much higher values, typically around 3×10^6. In engineering analysis, a generally accepted value for the critical Reynolds number is

$$\text{Re}_{x,\text{cr}} = \frac{\rho V x_{\text{cr}}}{\mu} = 5 \times 10^5$$

The actual value of the engineering critical Reynolds number for a flat plate may vary somewhat from about 10^5 to 3×10^6 depending on the surface roughness, the turbulence level, and the variation of pressure along the surface.

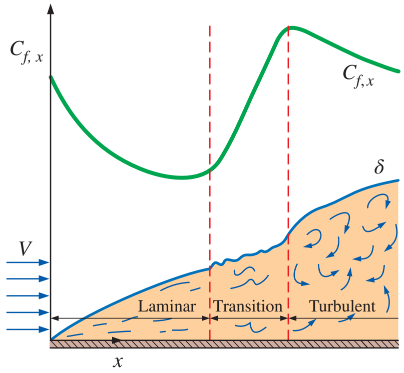


FIGURE 15–28
The variation of the local friction coefficient for flow over a flat plate. Note that the vertical scale of the boundary layer is greatly exaggerated in this sketch.

Friction Coefficient

The friction coefficient for laminar flow over a flat plate can be determined theoretically by solving the conservation of mass and linear momentum equations numerically. For turbulent flow, however, it must be determined experimentally and expressed by empirical correlations.

The local friction coefficient *varies* along the surface of the flat plate as a result of the changes in the velocity boundary layer in the flow direction. We are usually interested in the drag force on the *entire* surface, which can be determined using the *average* friction coefficient. But sometimes we are also interested in the drag force at a certain location, and in such cases, we need to know the *local* value of the friction coefficient. With this in mind, we present correlations for both local (identified with the subscript x) and average friction coefficients over a flat plate for *laminar, turbulent,* and *combined laminar and turbulent* flow conditions. Once the local values are available, the *average* friction coefficient for the entire plate is determined by integration as

$$C_f = \frac{1}{L}\int_0^L C_{f,x}\,dx \tag{15–16}$$

Based on analysis, the boundary layer thickness and the local friction coefficient at location x for laminar flow over a flat plate are determined to be

Laminar: $$\delta = \frac{4.91x}{\text{Re}_x^{1/2}} \quad \text{and} \quad C_{f,x} = \frac{0.664}{\text{Re}_x^{1/2}} \qquad \text{Re}_x \lesssim 5 \times 10^5 \tag{15–17}$$

The corresponding relations for turbulent flow are

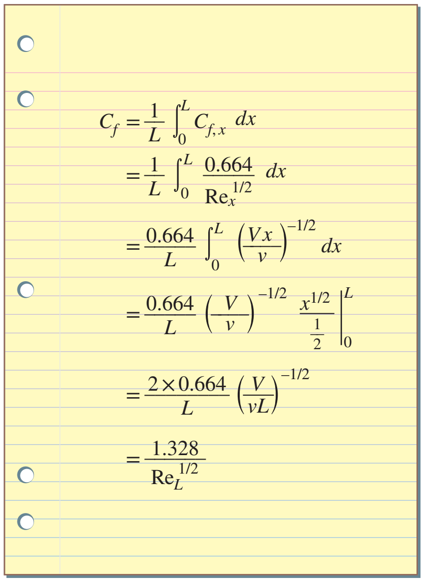
Turbulent: $$\delta = \frac{0.38x}{\text{Re}_x^{1/5}} \quad \text{and} \quad C_{f,x} = \frac{0.059}{\text{Re}_x^{1/5}} \qquad 5 \times 10^5 \lesssim \text{Re}_x \lesssim 10^7 \tag{15–18}$$

where x is the distance from the leading edge of the plate and $\text{Re}_x = Vx/\nu$ is the Reynolds number at location x. Note that $C_{f,x}$ is proportional to $1/\text{Re}_x^{1/2}$ and thus to $x^{-1/2}$ for laminar flow, and it is proportional to $x^{-1/5}$ for turbulent flow. In either case, $C_{f,x}$ is infinite at the leading edge ($x = 0$), and therefore Eqs. 15–17 and 15–18 are not valid close to the leading edge. The variation of the boundary layer thickness δ and the friction coefficient $C_{f,x}$ along a flat plate is sketched in Fig. 15–28. The local friction coefficients are higher in turbulent flow than they are in laminar flow because of the intense mixing that occurs in the turbulent boundary layer. Note that $C_{f,x}$ reaches its highest values when the flow becomes fully turbulent and then decreases by a factor of $x^{-1/5}$ in the flow direction, as shown in the figure.

The *average* friction coefficient over the entire plate is determined by substituting Eqs. 15–17 and 15–18 into Eq. 15–16 and performing the integrations (Fig. 15–29). We get

Laminar: $$C_f = \frac{1.33}{\text{Re}_L^{1/2}} \qquad \text{Re}_L \lesssim 5 \times 10^5 \tag{15–19}$$

Turbulent: $$C_f = \frac{0.074}{\text{Re}_L^{1/5}} \qquad 5 \times 10^5 \lesssim \text{Re}_L \lesssim 10^7 \tag{15–20}$$



$$\begin{aligned}
C_f &= \frac{1}{L}\int_0^L C_{f,x}\,dx \\
&= \frac{1}{L}\int_0^L \frac{0.664}{\text{Re}_x^{1/2}}\,dx \\
&= \frac{0.664}{L}\int_0^L \left(\frac{Vx}{\nu}\right)^{-1/2} dx \\
&= \frac{0.664}{L}\left(\frac{V}{\nu}\right)^{-1/2} \frac{x^{1/2}}{\frac{1}{2}}\Bigg|_0^L \\
&= \frac{2 \times 0.664}{L}\left(\frac{V}{\nu L}\right)^{-1/2} \\
&= \frac{1.328}{\text{Re}_L^{1/2}}
\end{aligned}$$

FIGURE 15–29
The average friction coefficient over a surface is determined by integrating the local friction coefficient over the entire surface. The values shown here are for a laminar flat plate boundary layer.

Relative Roughness, ε/L	Friction Coefficient, C_f
0.0*	0.0029
1×10^{-5}	0.0032
1×10^{-4}	0.0049
1×10^{-3}	0.0084

* Smooth surface for Re = 10^7. Others calculated from Eq. 15-23 for fully rough flow.

FIGURE 15–30
For turbulent flow, surface roughness may cause the friction coefficient to increase severalfold.

The first of these relations gives the average friction coefficient for the entire plate when the flow is *laminar* over the *entire* plate. The second relation gives the average friction coefficient for the entire plate only when the flow is *turbulent* over the *entire* plate, or when the laminar flow region of the plate is negligibly small relative to the turbulent flow region (that is, $x_{cr} \ll L$ where the length of the plate x_{cr} over which the flow is laminar is determined from $Re_{cr} = 5 \times 10^5 = Vx_{cr}/\nu$).

In some cases, a flat plate is sufficiently long for the flow to become turbulent, but not long enough to disregard the laminar flow region. In such cases, the *average* friction coefficient over the entire plate is determined by performing the integration in Eq. 15–16 over two parts: the laminar region $0 \le x \le x_{cr}$ and the turbulent region $x_{cr} < x \le L$ as

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,\,x,\,\text{laminar}}\,dx + \int_{x_{cr}}^{L} C_{f,\,x,\,\text{turbulent}}\,dx\right) \qquad \textbf{(15–21)}$$

Note that we included the transition region with the turbulent region. Again taking the critical Reynolds number to be $Re_{cr} = 5 \times 10^5$ and performing these integrations after substituting the indicated expressions, the *average* friction coefficient over the *entire* plate is determined to be

$$C_f = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} \qquad 5 \times 10^5 \lesssim Re_L \lesssim 10^7 \qquad \textbf{(15–22)}$$

The constants in this relation would be different for different critical Reynolds numbers. Also, the surfaces are assumed to be *smooth* and the free stream to be of very low turbulence intensity. For laminar flow, the friction coefficient depends on only the Reynolds number, and the surface roughness has no effect. For turbulent flow, however, surface roughness causes the friction coefficient to increase severalfold, to the point that in the fully rough turbulent regime the friction coefficient is a function of surface roughness alone and is independent of the Reynolds number (Fig. 15–30). This is analogous to flow in pipes.

A curve fit of experimental data for the average friction coefficient in this regime is given by Schlichting (1979) as

Fully rough turbulent regime:
$$C_f = \left(1.89 - 1.62\log\frac{\varepsilon}{L}\right)^{-2.5} \qquad \textbf{(15–23)}$$

where ε is the surface roughness and L is the length of the plate in the flow direction. In the absence of a better one, this relation can be used for turbulent flow on rough surfaces for $Re > 10^6$, especially when $\varepsilon/L > 10^{-4}$.

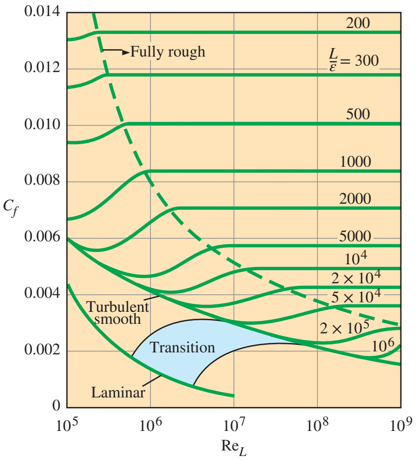


FIGURE 15–31
Friction coefficient for parallel flow over smooth and rough flat plates.
Data from White (2010).

Friction coefficients C_f for parallel flow over smooth and rough flat plates are plotted in Fig. 15–31 for both laminar and turbulent flows. Note that C_f increases severalfold with roughness in turbulent flow. Also note that C_f is independent of the Reynolds number in the fully rough region. This chart is the flat-plate analog of the Moody chart for pipe flows.

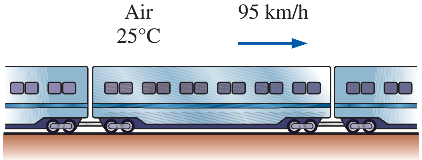


FIGURE 15–32
Schematic for Example 15–3.

EXAMPLE 15–3 Drag Force on a Train Surface

The top surface of the passenger car of a train moving at a velocity of 95 km/h is 2.1 m wide and 8 m long (Fig. 15–32). If the outdoor air is at 1 atm and 25°C, determine the drag force acting on the top surface of the car. Ignore any upstream boundary layer from the car(s) in front of this one; in other words, let the boundary layer start at the front edge of the top of the car.

SOLUTION A train is cruising at a specified velocity. The drag force acting on the top surface of a passenger car of the train is to be determined.

Assumptions **1** The airflow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5 \times 10^5$. **3** Air is an ideal gas. **4** The top surface of the train is smooth (in reality it can be rough). **5** The air is calm (no significant winds).

Properties The density and kinematic viscosity of air at 1 atm and 25°C are $\rho = 1.184\ \text{kg/m}^3$ and $\nu = 1.562 \times 10^{-5}\ \text{m}^2/\text{s}$.

Analysis The Reynolds number is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{[(95/3.6)\ \text{m/s}](8\ \text{m})}{1.562 \times 10^{-5}\ \text{m}^2/\text{s}} = 1.352 \times 10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient is determined to be

$$C_f = \frac{0.074}{\text{Re}_L^{1/5}} - \frac{1742}{\text{Re}_L} = \frac{0.074}{(1.352 \times 10^7)^{1/5}} - \frac{1742}{1.352 \times 10^7} = 0.002645$$

Noting that the pressure drag is zero, and thus $C_D = C_f$ for a flat plate, the drag force acting on the surface becomes

$$F_D = C_f A \frac{\rho V^2}{2} = (0.002645)[(8 \times 2.1)\ \text{m}^2]\,\frac{(1.184\ \text{kg/m}^3)[(95/3.6)\ \text{m/s}]^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= \mathbf{18.3\ N}$$

Discussion Note that we can solve this problem using the turbulent flow relation (instead of the combined laminar–turbulent flow relation) without much loss in accuracy since the Reynolds number is much greater than the critical value. Also, the actual drag force will probably be greater because of surface roughness effects. In a real train, however, the upstream boundary layers would significantly affect the drag on the car in question, perhaps even *decreasing* the drag on this car.

15–6 ▪ FLOW OVER CYLINDERS AND SPHERES

Flow over cylinders and spheres is frequently encountered in practice. For example, the tubes in a shell-and-tube heat exchanger involve both *internal flow* through the tubes and *external flow* over the tubes, and both flows must be considered in the analysis of the heat exchanger. Also, many sports such as soccer, tennis, and golf involve flow over spherical balls.

The characteristic length for a circular cylinder or sphere is taken to be the *external diameter D*. Thus, the Reynolds number is defined as $\text{Re} = VD/\nu$ where V is the uniform velocity of the fluid as it approaches the cylinder or sphere. The critical Reynolds number for flow across a circular cylinder or sphere is about $\text{Re}_{\text{cr}} \cong 2 \times 10^5$. That is, the boundary layer remains laminar for about $\text{Re} \lesssim 2 \times 10^5$, is transitional for $2 \times 10^5 \lesssim \text{Re} \lesssim 2 \times 10^6$, and becomes fully turbulent for $\text{Re} \gtrsim 2 \times 10^6$.

Crossflow over a cylinder exhibits complex flow patterns, as shown in Fig. 15–33. The fluid approaching the cylinder branches out and encircles the cylinder, forming a boundary layer that wraps around the cylinder. The fluid particles on the midplane strike the cylinder at the stagnation point, bringing the fluid to a complete stop and thus raising the pressure at that point. The pressure decreases in the flow direction while the fluid velocity increases.

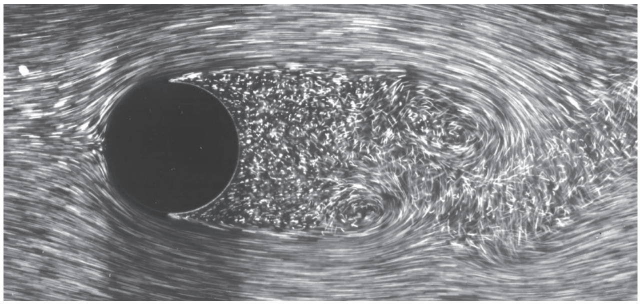


FIGURE 15–33
Laminar boundary layer separation with a turbulent wake; flow over a circular cylinder at Re = 2000.
Courtesy of ONERA. Photo by Werlé

At very low upstream velocities ($\text{Re} \lesssim 1$), the fluid completely wraps around the cylinder and the two arms of the fluid meet on the rear side of the cylinder in an orderly manner. Thus, the fluid follows the curvature of the cylinder. At higher velocities, the fluid still hugs the cylinder on the frontal side, but it is too fast to remain attached to the surface as it approaches the top (or bottom) of the cylinder. As a result, the boundary layer detaches from the surface, forming a separation region behind the cylinder. Flow in the wake region is characterized by periodic vortex formation and pressures much lower than the stagnation point pressure.

The nature of the flow across a cylinder or sphere strongly affects the total drag coefficient C_D. Both the *friction drag* and the *pressure drag* can be significant. The high pressure in the vicinity of the stagnation point and the low pressure on the opposite side in the wake produce a net force on the body in the direction of flow. The drag force is primarily due to friction drag at low Reynolds numbers ($\text{Re} \lesssim 10$) and to pressure drag at high Reynolds numbers ($\text{Re} \gtrsim 5000$). Both effects are significant at intermediate Reynolds numbers.

The average drag coefficients C_D for crossflow over a smooth single circular cylinder and a sphere are given in Fig. 15–34. The curves exhibit different behaviors in different ranges of Reynolds numbers:

- For $\text{Re} \lesssim 1$, we have creeping flow, and the drag coefficient decreases with increasing Reynolds number. For a sphere, it is $C_D = 24/\text{Re}$. There is no flow separation in this regime.
- At about $\text{Re} \cong 10$, separation starts occurring on the rear of the body with vortex shedding starting at about $\text{Re} \cong 90$. The region of separation increases with increasing Reynolds number up to about $\text{Re} \cong 10^3$. At this point, the drag is mostly (about 95 percent) due to pressure drag. The drag coefficient continues to decrease with increasing Reynolds number in this range of $10 \lesssim \text{Re} \lesssim 10^3$. (A decrease in the drag coefficient does not necessarily indicate a decrease in drag. The drag force is proportional to the square of the velocity, and the increase in velocity at higher Reynolds numbers usually more than offsets the decrease in the drag coefficient.)

  Some useful empirical relations for drag coefficient over a sphere in this range of Reynolds numbers are:

$$C_D = \frac{24}{\text{Re}}(1 + 0.0916\ \text{Re}) \quad \text{for } 0.1 < \text{Re} < 5$$

$$C_D = \frac{24}{\text{Re}}(1 + 0.158\ \text{Re}^{2/3}) \quad \text{for } 5 < \text{Re} < 1000$$

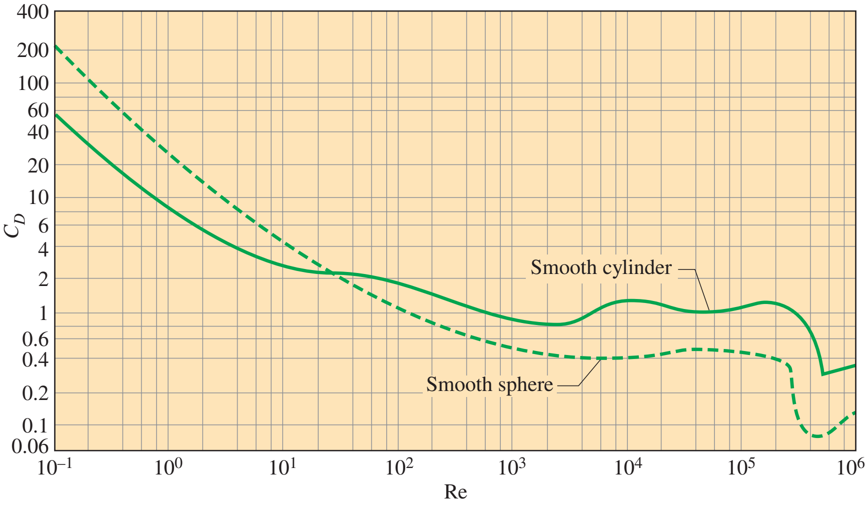
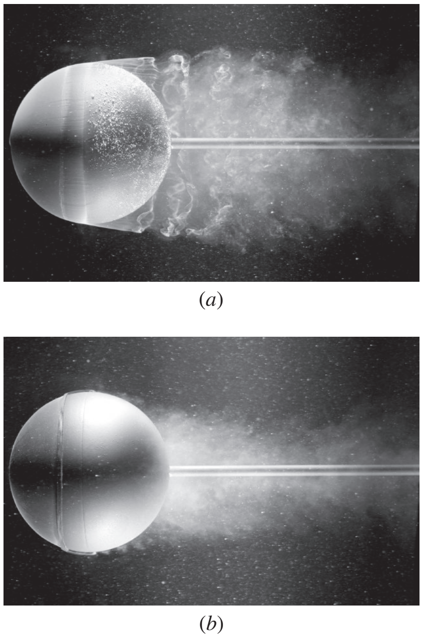


FIGURE 15–34
Average drag coefficient for crossflow over a smooth circular cylinder and a smooth sphere.
Data from H. Schlichting.

- In the moderate range of $10^3 \lesssim \text{Re} \lesssim 10^5$, the drag coefficient remains relatively constant. This behavior is characteristic of bluff bodies. The flow in the boundary layer is laminar in this range, but the flow in the separated region past the cylinder or sphere is highly turbulent with a wide turbulent wake.
- There is a sudden drop in the drag coefficient somewhere in the range of $10^5 \lesssim \text{Re} \lesssim 10^6$ (usually, at about 2×10^5). This large reduction in C_D is due to the flow in the boundary layer becoming *turbulent,* which moves the separation point further on the rear of the body, reducing the size of the wake and thus the magnitude of the pressure drag. This is in contrast to streamlined bodies, which experience an increase in the drag coefficient (mostly due to friction drag) when the boundary layer becomes turbulent.
- There is a "transitional" regime for $2 \times 10^5 \lesssim \text{Re} \lesssim 2 \times 10^6$, in which C_D dips to a minimum value and then slowly rises to its final turbulent value.

Flow separation occurs at about $\theta \cong 80°$ (measured from the front stagnation point of a cylinder) when the boundary layer is *laminar* and at about $\theta \cong 140°$ when it is *turbulent* (Fig. 15–35). The delay of separation in turbulent flow is caused by the rapid fluctuations of the fluid in the transverse direction, which enables the turbulent boundary layer to travel farther along the surface before separation occurs, resulting in a narrower wake and a smaller pressure drag. Keep in mind that turbulent flow has a fuller velocity profile as compared to the laminar case, and thus it requires a stronger adverse pressure gradient to overcome the additional momentum close to the wall. In the range of Reynolds numbers where the flow changes from laminar to turbulent, even the drag force F_D decreases as the velocity (and thus the Reynolds number) increases. This results in a sudden decrease in drag of a flying body (sometimes called the *drag crisis*) and instabilities in flight.


(*a*)

(*b*)

FIGURE 15–35
Flow visualization of flow over (*a*) a smooth sphere at Re = 15,000, and (*b*) a sphere at Re = 30,000 with a trip wire. The delay of boundary layer separation is clearly seen by comparing the two photographs.
Courtesy of ONERA. Photo by Werlé

Effect of Surface Roughness

We mentioned earlier that *surface roughness,* in general, increases the drag coefficient in turbulent flow. This is especially the case for streamlined bodies. For blunt bodies such as a circular cylinder or sphere, however, an increase in the surface roughness may actually *decrease* the drag coefficient, as shown in Fig. 15–36 for a sphere. This is done by tripping the boundary layer into turbulence at a lower Reynolds number, and thus delaying flow separation, causing the fluid to close in behind the body, narrowing the wake, and reducing pressure drag considerably. This results in a

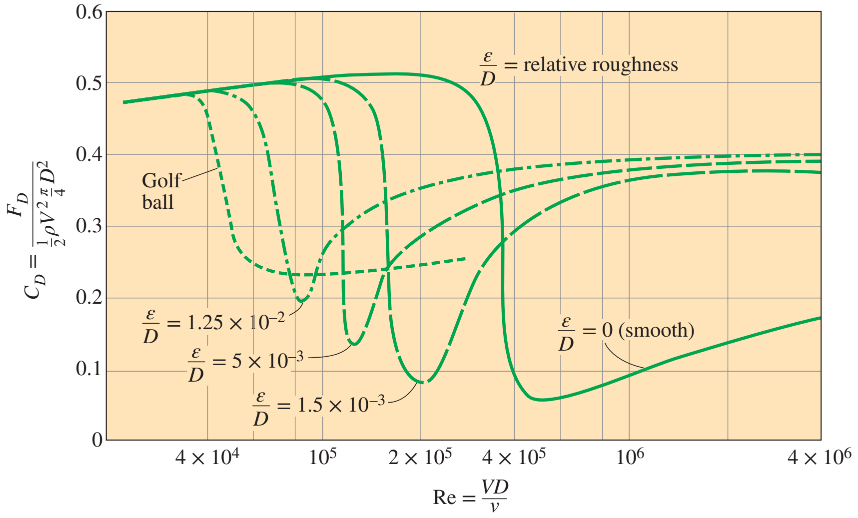


FIGURE 15–36
The effect of surface roughness on the drag coefficient of a sphere.
Data from Blevins (1984).

much smaller drag coefficient and thus drag force for a rough-surfaced cylinder or sphere in a certain range of Reynolds number compared to a smooth one of identical size at the same velocity. At Re = 2×10^5, for example, $C_D \cong 0.1$ for a rough sphere with $\varepsilon/D = 0.0015$, whereas $C_D \cong 0.5$ for a smooth one. Therefore, the drag coefficient in this case is reduced by a factor of 5 by simply roughening the surface. Note, however, that at Re = 10^6, $C_D \cong 0.4$ for a very rough sphere while $C_D \cong 0.1$ for the smooth one. Obviously, roughening the sphere in this case increases the drag by a factor of 4 (Fig. 15–37).

	C_D	
Re	Smooth Surface	Rough Surface, $\varepsilon/D = 0.0015$
2×10^5	0.5	0.1
10^6	0.1	0.4

FIGURE 15–37
Surface roughness may increase or decrease the drag coefficient of a spherical object, depending on the value of the Reynolds number.

The preceding discussion shows that roughening the surface can be used to great advantage in reducing drag, but it can also backfire on us if we are not careful—specifically, if we do not operate in the right range of the Reynolds number. With this consideration, golf balls are intentionally roughened to induce *turbulence* at a lower Reynolds number to take advantage of the sharp *drop* in the drag coefficient at the onset of turbulence in the boundary layer (the typical velocity range of golf balls is 15 to 150 m/s, and the Reynolds number is less than 4×10^5). The critical Reynolds number of dimpled golf balls is about 4×10^4. The occurrence of turbulent flow at this Reynolds number reduces the drag coefficient of a golf ball by about half, as shown in Fig. 15–36. For a given hit, this means a longer distance for the ball. Experienced golfers also give the ball a spin during the hit, which helps the rough ball develop a lift and thus travel higher and farther. A similar argument can be given for a tennis ball. For a table tennis ball, however, the speeds are slower and the ball is smaller—it never reaches the turbulent range. Therefore, the surfaces of table tennis balls are smooth.

Once the drag coefficient is available, the drag force acting on a body in crossflow is determined from Eq. 15–5 where A is the *frontal area* ($A = LD$ for a cylinder of length L and $A = \pi D^2/4$ for a sphere). It should be kept in mind that free-stream turbulence and disturbances by other bodies in the flow (such as flow over tube bundles) may affect the drag coefficient significantly.

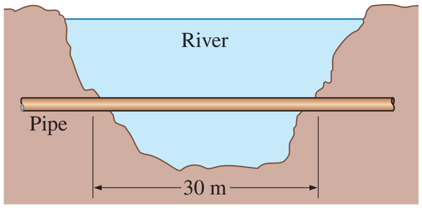


FIGURE 15–38
Schematic for Example 15–4.

EXAMPLE 15–4 Drag Force Acting on a Pipe in a River

A 2.2-cm-outer-diameter pipe is to span a river at a 30-m-wide section while being completely immersed in water (Fig. 15–38). The average flow velocity of water is 4 m/s, and the water temperature is 15°C. Determine the drag force exerted on the pipe by the river.

SOLUTION A pipe is submerged in a river. The drag force that acts on the pipe is to be determined.

Assumptions **1** The outer surface of the pipe is smooth, so Fig. 15–34 can be used to determine the drag coefficient. **2** Water flow in the river is steady. **3** The direction of water flow is normal to the pipe. **4** Turbulence in river flow is not considered.

Properties The density and dynamic viscosity of water at 15°C are $\rho = 999.1\ \text{kg/m}^3$ and $\mu = 1.138 \times 10^{-3}\ \text{kg/m·s}$.

Analysis Noting that $D = 0.022$ m, the Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{\rho VD}{\mu} = \frac{(999.1\ \text{kg/m}^3)(4\ \text{m/s})(0.022\ \text{m})}{1.138 \times 10^{-3}\ \text{kg/m·s}} = 7.73 \times 10^4$$

The drag coefficient corresponding to this value is, from Fig. 15–34, $C_D = 1.0$. Also, the frontal area for flow past a cylinder is $A = LD$. Then the drag force acting on the pipe becomes

$$F_D = C_D A \frac{\rho V^2}{2} = 1.0(30 \times 0.022\ \text{m}^2)\frac{(999.1\ \text{kg/m}^3)(4\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg·m/s}^2}\right)$$

$$= 5275\ \text{N} \cong \mathbf{5300\ N}$$

Discussion Note that this force is equivalent to the weight of a mass over 500 kg. Therefore, the drag force the river exerts on the pipe is equivalent to hanging a total of over 500 kg in mass on the pipe supported at its ends 30 m apart. The necessary precautions should be taken if the pipe cannot support this force. If the river were to flow at a faster speed or if turbulent fluctuations in the river were more significant, the drag force would be even larger. *Unsteady* forces on the pipe might then be significant.

15–7 ▪ LIFT

Lift was defined earlier as the component of the net force (due to viscous and pressure forces) that is perpendicular to the flow direction, and the lift coefficient was expressed in Eq. 15–6 as

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A}$$

where A in this case is normally the *planform area,* which is the area that would be seen by a person looking at the body from above in a direction normal to the body, and V is the upstream velocity of the fluid (or, equivalently, the velocity of a flying body in a quiescent fluid). For an airfoil of width (or span) b and chord length c (the length between the leading and trailing edges), the planform area is $A = bc$. The distance between the two ends of a wing or airfoil is called the **wingspan** or just the **span**. For an aircraft, the wingspan is taken to be the total distance between the tips of the two wings, which includes the width of the fuselage between the wings (Fig. 15–39). The average lift per unit planform area F_L/A is called the **wing loading**, which is simply the ratio of the weight of the aircraft to the planform area of the wings (since lift equals weight when flying at constant altitude).

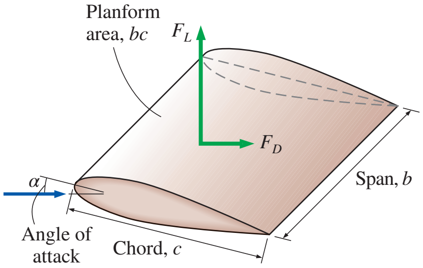


FIGURE 15–39
Definition of various terms associated with an airfoil.

Airplane flight is based on lift, and thus developing a better understanding of lift as well as improving the lift characteristics of bodies have been the focus of numerous studies. Our emphasis in this section is on devices such as *airfoils* that are specifically designed to generate lift while keeping the drag at a minimum. But it should be kept in mind that some devices such as *spoilers* and *inverted airfoils* on racing cars are designed for the opposite purpose of avoiding lift or even generating negative lift to improve traction and control (some early race cars actually "took off" at high speeds as a result of the lift produced, which alerted the engineers to come up with ways to reduce lift in their designs).

For devices that are intended to generate lift such as airfoils, the contribution of *viscous effects* to lift is usually negligible since the bodies are streamlined, and wall shear is parallel to the surfaces of such devices and thus nearly normal to the direction of lift (Fig. 15–40). Therefore, lift in practice can be approximated as due entirely to the pressure distribution on the surfaces of the body, and thus the shape of the body has the primary influence on lift. Then the primary consideration in the design of airfoils is minimizing the average pressure at the upper surface while maximizing it at the lower surface. The Bernoulli equation can be used as a guide in identifying the high- and low-pressure regions: *Pressure is low at locations where the flow velocity is high, and pressure is high at locations where the flow velocity is low.* Also, at moderate angles of attack, lift is practically independent of the surface roughness since roughness affects the wall shear, not the pressure. The contribution of shear to lift is significant only for very small (lightweight) bodies that fly at low velocities (and thus low Reynolds numbers).

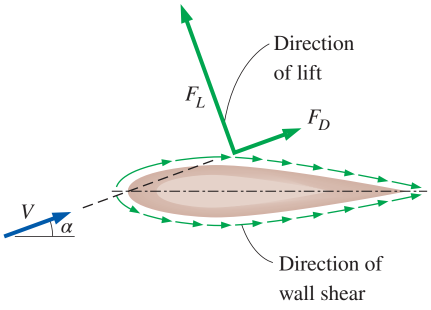
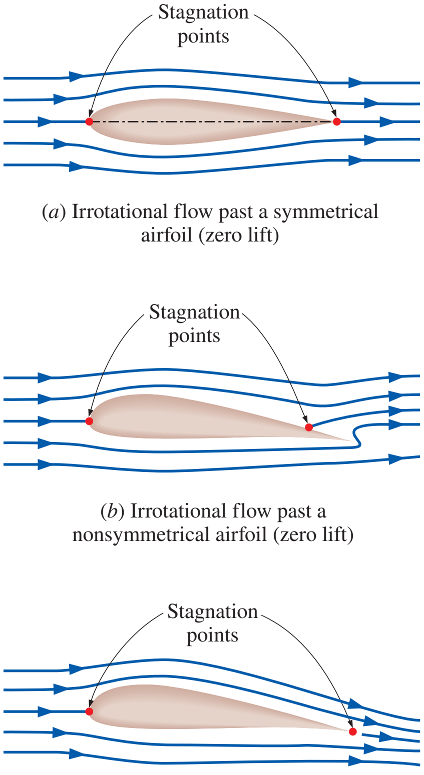


FIGURE 15–40
For airfoils, the contribution of viscous effects to lift is usually negligible since wall shear is parallel to the surfaces and thus nearly normal to the direction of lift.

Noting that the contribution of viscous effects to lift is negligible, we should be able to determine the lift acting on an airfoil by simply integrating the



(*a*) Irrotational flow past a symmetrical airfoil (zero lift)

(*b*) Irrotational flow past a nonsymmetrical airfoil (zero lift)

(*c*) Actual flow past a nonsymmetrical airfoil (positive lift)

FIGURE 15–41
Irrotational and actual flow past symmetrical and nonsymmetrical two-dimensional airfoils.

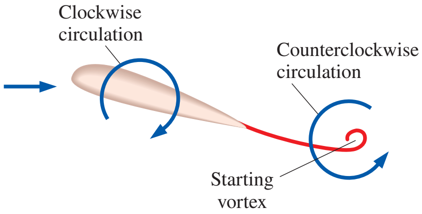


FIGURE 15–42
Shortly after a sudden increase in angle of attack, a counterclockwise starting vortex is shed from the airfoil, while clockwise circulation appears around the airfoil, causing lift to be generated.

pressure distribution around the airfoil. The pressure changes in the flow direction along the surface, but it remains essentially constant through the boundary layer in a direction normal to the surface. Therefore, it seems reasonable to ignore the very thin boundary layer on the airfoil and calculate the pressure distribution around the airfoil from the relatively simple potential flow theory (zero vorticity, irrotational flow) for which net viscous forces are zero for flow past an airfoil.

The flow fields obtained from such calculations are sketched in Fig. 15–41 for both symmetrical and nonsymmetrical airfoils by ignoring the thin boundary layer. At zero angle of attack, the lift produced by the symmetrical airfoil is zero, as expected because of symmetry, and the stagnation points are at the leading and trailing edges. For the nonsymmetrical airfoil, which is at a small angle of attack, the front stagnation point has moved down below the leading edge, and the rear stagnation point has moved up to the upper surface close to the trailing edge. To our surprise, the lift produced is calculated again to be zero—a clear contradiction of experimental observations and measurements. Obviously, the theory needs to be modified to bring it in line with the observed phenomenon.

The source of inconsistency is the rear stagnation point being on the upper surface instead of on the trailing edge. This requires the lower-side fluid to make a near U-turn and flow around the sharp trailing edge toward the stagnation point while remaining attached to the surface, which is a physical impossibility since the observed phenomenon is the separation of flow at sharp turns (imagine a car attempting to make this turn at high speed). Therefore, the lower-side fluid separates smoothly off the trailing edge, and the upper-side fluid responds by pushing the rear stagnation point downstream. In fact, the stagnation point at the upper surface moves all the way to the trailing edge. This way the two flow streams from the top and the bottom sides of the airfoil meet at the trailing edge, yielding a smooth flow downstream parallel to the sharp trailing edge. Lift is generated because the flow velocity at the top surface is higher, and thus the pressure on that surface is lower due to the Bernoulli effect.

The potential flow theory and the observed phenomenon can be reconciled as follows: Flow starts out as predicted by theory, with no lift, but the lower fluid stream separates at the trailing edge when the velocity reaches a certain value. This forces the separated upper fluid stream to close in at the trailing edge, initiating clockwise circulation around the airfoil. This clockwise circulation increases the velocity of the upper stream while decreasing that of the lower stream, causing lift. A **starting vortex** of opposite sign (counterclockwise circulation) is then shed downstream (Fig. 15–42), and smooth streamlined flow is established over the airfoil. When the potential flow theory is modified by the addition of an appropriate amount of circulation to move the stagnation point down to the trailing edge, excellent agreement is obtained between theory and experiment for both the flow field and the lift.

It is desirable for airfoils to generate the most lift while producing the least drag. Therefore, a measure of performance for airfoils is the **lift-to-drag ratio**, which is equivalent to the ratio of the lift-to-drag coefficients C_L/C_D. This information is provided either by plotting C_L versus C_D for different values of the angle of attack (a lift–drag polar) or by plotting the ratio C_L/C_D versus the angle of attack. The latter is done for a particular airfoil design in Fig. 15–43. Note that the C_L/C_D ratio increases with the angle of attack until the airfoil stalls, and the value of the lift-to-drag ratio can be on the order of 100 for a two-dimensional airfoil.

One obvious way to change the lift and drag characteristics of an airfoil is to change the angle of attack. On an airplane, for example, the entire plane is pitched up to increase lift, since the wings are fixed relative to the fuselage. Another approach is to change the shape of the airfoil by the use of movable *leading edge* and *trailing edge flaps,* as is commonly done in modern large aircraft (Fig. 15–44). The flaps are used to alter the shape of the wings during takeoff and landing to maximize lift and to enable the aircraft to land or take off at low speeds. The increase in drag during this takeoff and landing is not much of a concern because of the relatively short time periods involved. Once at cruising altitude, the flaps are retracted, and the wing is returned to its "normal" shape with minimal drag coefficient and adequate lift coefficient to minimize fuel consumption while cruising at a constant altitude. Note that even a small lift coefficient can generate a large lift force during normal operation because of the large cruising velocities of aircraft and the proportionality of lift to the square of flow velocity.

The effects of flaps on the lift and drag coefficients are shown in Fig. 15–45 for an airfoil. Note that the maximum lift coefficient increases from about 1.5 for the airfoil with no flaps to 3.5 for the double-slotted flap case. But also note that the maximum drag coefficient increases from about 0.06 for the airfoil with no flaps to about 0.3 for the double-slotted flap case. This is a fivefold increase in the drag coefficient, and the engines must work much harder to provide the necessary thrust to overcome this drag. The angle of attack of the flaps can be increased to maximize the lift coefficient. Also, the flaps extend the chord length and thus enlarge the wing area A. The Boeing 727 uses a triple-slotted flap at the trailing edge and a slot at the leading edge.

The minimum flight velocity is determined from the requirement that the total weight W of the aircraft be equal to lift and $C_L = C_{L,\,\max}$. That is,

$$W = F_L = \tfrac{1}{2} C_{L,\max}\, \rho V_{\min}^2 A \qquad \rightarrow \qquad V_{\min} = \sqrt{\frac{2W}{\rho C_{L,\max} A}} \tag{15–24}$$

For a given weight, the landing or takeoff speed can be minimized by maximizing the product of the lift coefficient and the wing area, $C_{L,\,\max}A$. One way of doing that is to use flaps, as already discussed. Another way is to control the boundary layer, which can be accomplished simply by leaving flow sections (slots) between the flaps, as shown in Fig. 15–46. Slots are used to prevent the separation of the boundary layer from the upper surface of the wings and the flaps. This is done by allowing air to move from the high-pressure region under the wing into the low-pressure region at the top surface. Note that the lift coefficient reaches its maximum value $C_L = C_{L,\,\max}$, and thus the flight velocity reaches its minimum, at stall conditions, which is a region of unstable operation and must be avoided. The Federal Aviation Administration (FAA) does not allow operation below 1.2 times the stall speed for safety.

Another thing we notice from this equation is that the minimum velocity for takeoff or landing is inversely proportional to the square root of density. Noting that air density decreases with altitude (by about 15 percent at 1500 m), longer runways are required at airports at higher altitudes such as Denver to accommodate higher minimum takeoff and landing velocities. The situation becomes even more critical on hot summer days since the density of air is inversely proportional to temperature.

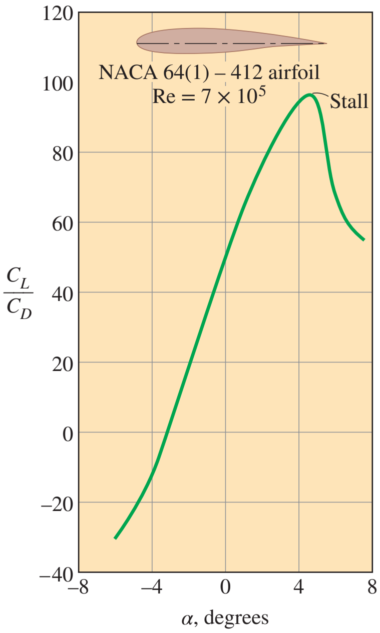


FIGURE 15–43
The variation of the lift-to-drag ratio with angle of attack for a two-dimensional airfoil.
Data from Abbott, von Doenhoff, and Stivers (1945).


(*a*) Flaps extended (landing)
(*b*) Flaps retracted (cruising)

FIGURE 15–44
The lift and drag characteristics of an airfoil during takeoff and landing are changed by changing the shape of the airfoil by the use of movable flaps.
Yunus Çengel

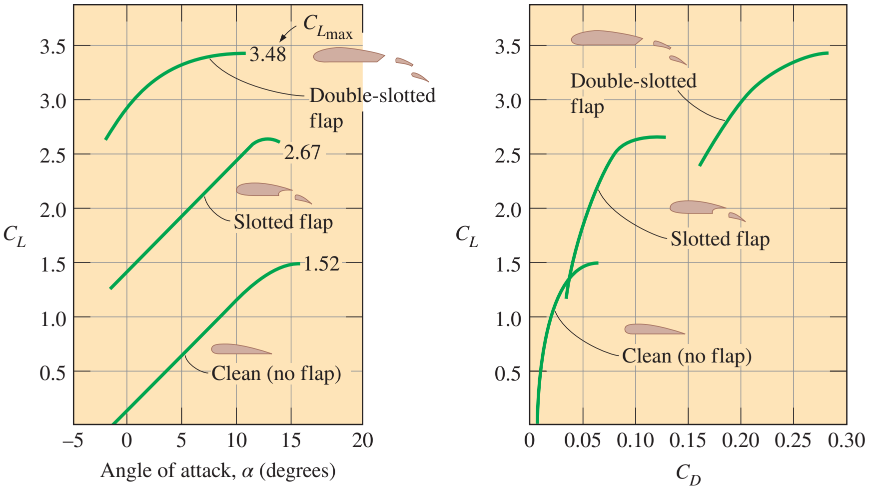


FIGURE 15–45
Effect of flaps on the lift and drag coefficients of an airfoil.
Data from Abbott and von Doenhoff, for NACA 23012 (1959).

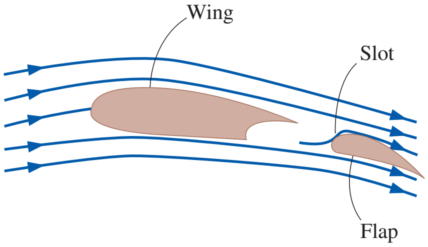


FIGURE 15–46
A flapped airfoil with a slot to prevent the separation of the boundary layer from the upper surface and to increase the lift coefficient.

The development of efficient (low-drag) airfoils was the subject of intense experimental investigations in the 1930s. These airfoils were standardized by the National Advisory Committee for Aeronautics (NACA, which is now NASA), and extensive lists of data on lift coefficients were reported. The variation of the lift coefficient C_L with angle of attack for two 2-D (infinite span) airfoils (NACA 0012 and NACA 2412) is given in Fig. 15–47. We make the following observations from this figure:

- The lift coefficient increases almost linearly with angle of attack α, reaches a maximum at about $\alpha = 16°$, and then starts to decrease sharply. This decrease of lift with further increase in the angle of attack is called *stall,* and it is caused by flow separation and the formation of a wide wake region over the top surface of the airfoil. Stall is highly undesirable since it also increases drag.
- At zero angle of attack ($\alpha = 0°$), the lift coefficient is zero for symmetrical airfoils but nonzero for nonsymmetrical ones with greater curvature at the top surface. Therefore, planes with symmetrical wing sections must fly with their wings at higher angles of attack in order to produce the same lift.
- The lift coefficient is increased by severalfold by adjusting the angle of attack (from 0.25 at $\alpha = 0°$ for the nonsymmetrical airfoil to 1.25 at $\alpha = 10°$).

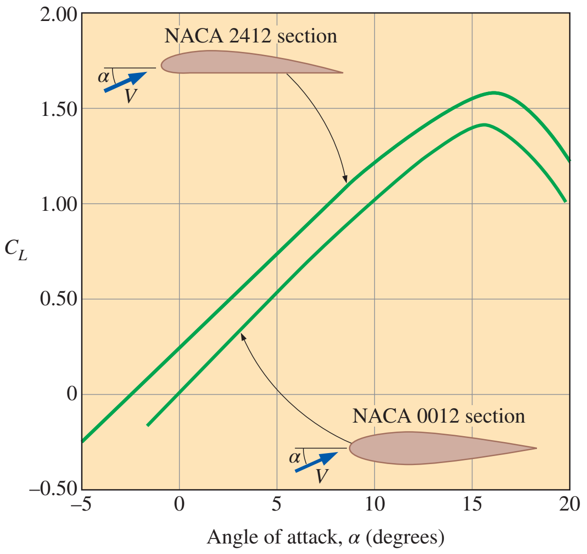


FIGURE 15–47
The variation of the lift coefficient with angle of attack for a symmetrical and a nonsymmetrical airfoil.
Data from Abbott (1945, 1959).

- The drag coefficient also increases with angle of attack, often exponentially (Fig. 15–48). Therefore, large angles of attack should be used sparingly for short periods of time for fuel efficiency.

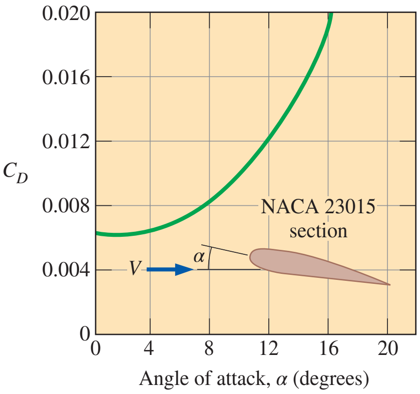


FIGURE 15–48
The variation of the drag coefficient of an airfoil with angle of attack.
Data from Abbott and von Doenhoff (1959).

Finite-Span Wings and Induced Drag

For airplane wings and other airfoils of finite span, the end effects at the tips become important because of the fluid leakage between the lower and upper surfaces. The pressure difference between the lower surface (high-pressure region) and the upper surface (low-pressure region) drives the fluid at the tips upward while the fluid is swept toward the back because of the relative motion between the fluid and the wing. This results in a swirling motion that spirals along the flow, called the **tip vortex**, at the tips of both wings. Vortices are also formed along the airfoil between the tips of the wings. These distributed vortices collect toward the edges after being shed from the trailing edges of the wings and combine with the tip vortices to form two streaks of powerful **trailing vortices** along the tips of the wings (Fig. 15–49). Trailing vortices generated by large aircraft persist for a long time for long distances (over 10 km) before they gradually disappear due to viscous dissipation. Such vortices and the accompanying downdraft are strong enough to cause a small aircraft to lose control and flip over if it flies through the wake of a larger aircraft. Therefore, following a large aircraft closely (within 10 km) poses a real danger for smaller aircraft. This issue is the controlling factor that governs the spacing of aircraft at takeoff, which limits the flight capacity at airports. In nature, this effect is used to advantage by birds that migrate in V-formation by utilizing the updraft generated by the bird in front. It has been determined that the birds in a typical flock can fly to their destination in V-formation with one-third less energy. Military jets also occasionally fly in V-formation for the same reason (Fig. 15–50).

Tip vortices that interact with the free stream impose forces on the wing tips in all directions, including the flow direction. The component of the force in the flow direction adds to drag and is called **induced drag**. The total drag of a wing is then the sum of the induced drag (3-D effects) and the drag of the airfoil section (2-D effects).

The ratio of the square of the average span of an airfoil to the planform area is called the **aspect ratio**. For an airfoil with a rectangular planform of chord c and span b, it is expressed as


(*a*) (*b*) (*c*)

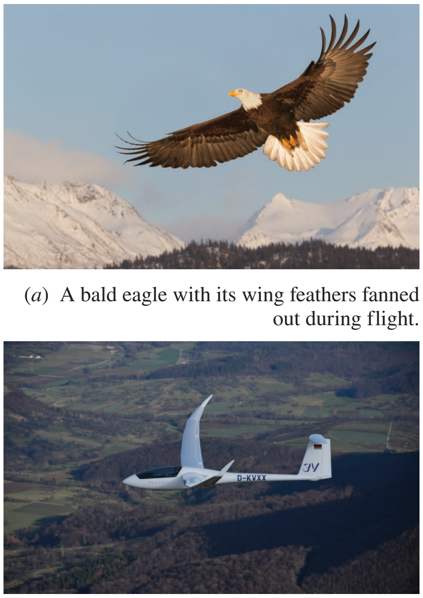
FIGURE 15–49
Trailing vortices visualized in various ways: (*a*) Smoke streaklines in a wind tunnel show vortex cores leaving the trailing edge of a rectangular wing; (*b*) Four contrails initially formed by condensation of water vapor in the low-pressure region behind the jet engines eventually merge into the two counter-rotating trailing vortices that persist very far downstream; (*c*) A crop duster flies through smoky air which swirls around in one of the tip vortices from the aircraft's wing.
(a) From Head, Malcolm R. 1982 in Flow Visualization II, W. Merzkirch. Ed., 399–403, Washington: Hemisphere; (b) Geostock/Getty Images; (c) NASA Langley Research Center

$$\mathrm{AR}=\frac{b^2}{A}=\frac{b^2}{bc}=\frac{b}{c} \tag{15–25}$$

Therefore, the aspect ratio is a measure of how (relatively) narrow an airfoil is in the flow direction. The lift coefficient of wings, in general, increases while the drag coefficient decreases with increasing aspect ratio. This is because a long narrow wing (large aspect ratio) has a shorter tip length and thus smaller tip losses and smaller induced drag than a short and wide wing of the same planform area. Therefore, bodies with large aspect ratios fly more efficiently, but they are less maneuverable because of their larger moment of inertia (owing to the greater distance from the center). Bodies with smaller aspect ratios maneuver better since the wings are closer to the central part. So it is no surprise that *fighter planes* (and fighter birds like falcons) have short and wide wings while *large commercial planes* (and soaring birds like albatrosses) have long and narrow wings.

The end effects can be minimized by attaching **endplates** or **winglets** at the tips of the wings perpendicular to the top surface. The endplates function by blocking some of the leakage around the wing tips, which results in a considerable reduction in the strength of the tip vortices and the induced drag. Wing tip feathers on birds fan out for the same purpose (Fig. 15–51).

FIGURE 15–50
(*a*) Geese flying in their characteristic V-formation to save energy.
(*b*) Military jets imitating nature.
(a) Corbis
(b) Charles Smith/Corbis



(*a*) A bald eagle with its wing feathers fanned out during flight.

(*b*) Winglets are used on this sailplane to reduce induced drag.

FIGURE 15–51
Induced drag is reduced by (*a*) wing tip feathers on bird wings and (*b*) endplates or other disruptions on airplane wings.
(a) Ken Canning/E+/Getty Images; (b) Courtesy Jacques Noel, Schempp-Hirth. Used by permission

EXAMPLE 15–5 Lift and Drag of a Commercial Airplane

A commercial airplane has a total mass of 70,000 kg and a wing planform area of 150 m^2 (Fig. 15–52). The plane has a cruising speed of 558 km/h and a cruising altitude of 12,000 m, where the air density is 0.312 kg/m^3. The plane has double-slotted flaps for use during takeoff and landing, but it cruises with all flaps retracted. Assuming the lift and the drag characteristics of the wings can be approximated by NACA 23012 (Fig. 15–45), determine (*a*) the minimum safe speed for takeoff and landing with and without extending the flaps, (*b*) the angle of attack to cruise steadily at the cruising altitude, and (*c*) the power that needs to be supplied to provide enough thrust to overcome wing drag.

SOLUTION The cruising conditions of a passenger plane and its wing characteristics are given. The minimum safe landing and takeoff speeds, the angle of attack during cruising, and the power required are to be determined.

Assumptions **1** The drag and lift produced by parts of the plane other than the wings, such as the fuselage, are not considered. **2** The wings are assumed to be two-dimensional airfoil sections, and the tip effects of the wings are not considered. **3** The lift and the drag characteristics of the wings are approximated by NACA 23012 so that Fig. 15–45 is applicable. **4** The average density of air on the ground is 1.20 kg/m^3.

Properties The density of air is 1.20 kg/m^3 on the ground and 0.312 kg/m^3 at cruising altitude. The maximum lift coefficient $C_{L,\,\max}$ of the wing is 3.48 and 1.52 with and without flaps, respectively (Fig. 15–45).

Analysis (*a*) The weight and cruising speed of the airplane are

$$W=mg=(70{,}000\text{ kg})(9.81\text{ m/s}^2)\left(\frac{1\text{ N}}{1\text{ kg·m/s}^2}\right)=686{,}700\text{ N}$$

$$V=(558\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)=155\text{ m/s}$$

The minimum velocities corresponding to the stall conditions without and with flaps, respectively, are obtained from Eq. 15–24,

$$V_{\min 1}=\sqrt{\frac{2W}{\rho C_{L,\max 1}A}}=\sqrt{\frac{2(686{,}700\text{ N})}{(1.2\text{ kg/m}^3)(1.52)(150\text{ m}^2)}\left(\frac{1\text{ kg·m/s}^2}{1\text{ N}}\right)}=70.9\text{ m/s}$$

$$V_{\min 2}=\sqrt{\frac{2W}{\rho C_{L,\max 2}A}}=\sqrt{\frac{2(686{,}700\text{ N})}{(1.2\text{ kg/m}^3)(3.48)(150\text{ m}^2)}\left(\frac{1\text{ kg·m/s}^2}{1\text{ N}}\right)}=46.8\text{ m/s}$$

Then the "safe" minimum velocities to avoid the stall region are obtained by multiplying the values above by 1.2:

$$\textit{Without flaps:}\quad V_{\text{min 1, safe}} = 1.2\,V_{\text{min 1}} = 1.2(70.9\text{ m/s}) = 85.1\text{ m/s} = \mathbf{306\ km/h}$$

$$\textit{with flaps:}\quad V_{\text{min 2, safe}} = 1.2\,V_{\text{min 2}} = 1.2(46.8\text{ m/s}) = 56.2\text{ m/s} = \mathbf{202\ km/h}$$

since 1 m/s = 3.6 km/h. Note that the use of flaps allows the plane to take off and land at considerably lower velocities, and thus on a shorter runway.

(*b*) When an aircraft is cruising steadily at a constant altitude, the lift must be equal to the weight of the aircraft, $F_L = W$. Then the lift coefficient is

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} = \frac{686{,}700\text{ N}}{\frac{1}{2}(0.312\text{ kg/m}^3)(155\text{ m/s})^2(150\text{ m}^2)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = 1.22$$

For the case with no flaps, the angle of attack corresponding to this value of C_L is determined from Fig. 15–45 to be $\alpha \cong \mathbf{10°}$.

(*c*) When the aircraft is cruising steadily at a constant altitude, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force. The drag coefficient corresponding to the cruising lift coefficient of 1.22 is determined from Fig. 15–45 to be $C_D \cong 0.03$ for the case with no flaps. Then the drag force acting on the wings becomes

$$\begin{aligned} F_D = C_D A\frac{\rho V^2}{2} &= (0.03)(150\text{ m}^2)\frac{(0.312\text{ kg/m}^3)(155\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) \\ &= 16.9\text{ kN} \end{aligned}$$

Noting that power is force times velocity (distance per unit time), the power required to overcome this drag is equal to the thrust times the cruising velocity:

$$\begin{aligned} \text{Power} = \text{Thrust} \times \text{Velocity} = F_D V &= (16.9\text{ kN})(155\text{ m/s})\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) \\ &= \mathbf{2620\ kW} \end{aligned}$$

Therefore, the engines must supply 2620 kW of power to overcome the drag on the wings during cruising. For a propulsion efficiency of 30 percent (i.e., 30 percent of the energy of the fuel is utilized to propel the aircraft), the plane requires energy input at a rate of 8730 kJ/s.

Discussion The power determined is the power to overcome the drag that acts on the wings only and does not include the drag that acts on the remaining parts of the aircraft (the fuselage, the tail, etc.). Therefore, the total power required during cruising will be much greater. Also, it does not consider induced drag, which can be dominant during takeoff when the angle of attack is high (Fig. 15–45 is for a 2-D airfoil and does not include 3-D effects).

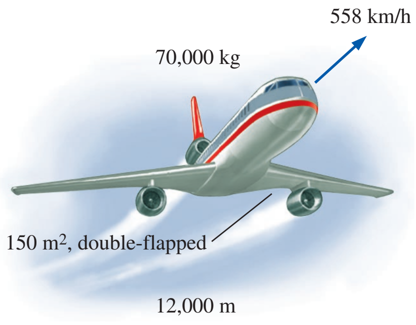


FIGURE 15–52
Schematic for Example 15–5.

No discussion on lift and drag would be complete without mentioning the contributions of Wilbur (1867–1912) and Orville (1871–1948) Wright. The Wright Brothers are truly the most impressive engineering team of all time. Self-taught, they were well informed of the contemporary theory and practice in aeronautics. They both corresponded with other leaders in the field and published in technical journals. While they cannot be credited with developing the concepts of lift and drag, they used them to achieve the first powered, manned, heavier-than-air, controlled flight (Fig. 15–53). They succeeded, while so many before them failed, because they evaluated and designed parts separately. Before the Wrights, experimenters were building and testing whole airplanes. While intuitively appealing, the approach did not allow the determination of how to make the craft better. When a flight lasts only a moment, you can only

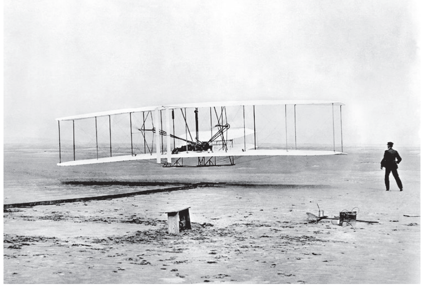


FIGURE 15–53
The Wright Brothers take flight at Kitty Hawk.
Library of Congress Prints & Photographs Division [LC-DIG-ppprs-00626].

guess at the weakness in the design. Thus, a new craft did not necessarily perform any better than its predecessor. Testing was simply one belly flop followed by another. The Wrights changed all that. They studied each part using scale and full-size models in wind tunnels and in the field. Well before the first powered flyer was assembled, they knew the area required for their best wing shape to support a plane carrying a man and the engine horsepower required to provide adequate thrust with their improved impeller. The Wright Brothers not only showed the world how to fly, they showed engineers how to use the equations presented here to design even better aircraft.

SUMMARY

In this chapter, we study the flow of fluids over immersed bodies with emphasis on the resulting lift and drag forces. A fluid may exert forces and moments on a body in and about various directions. The force a flowing fluid exerts on a body in the flow direction is called *drag,* while that in the direction normal to the flow is called *lift.* The part of drag that is due directly to wall shear stress τ_w is called the *skin friction drag* since it is caused by frictional effects, and the part that is due directly to pressure P is called the *pressure drag* or *form drag* because of its strong dependence on the form or shape of the body.

The *drag coefficient* C_D and the *lift coefficient* C_L are dimensionless numbers that represent the drag and the lift characteristics of a body and are defined as

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} \quad \text{and} \quad C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A}$$

where A is usually the *frontal area* (the area projected on a plane normal to the direction of flow) of the body. For plates and airfoils, A is taken to be the *planform area,* which is the area that would be seen by a person looking at the body from directly above. The drag coefficient, in general, depends on the *Reynolds number,* especially for Reynolds numbers below 10^4. At higher Reynolds numbers, the drag coefficients for many geometries remain essentially constant.

A body is said to be *streamlined* if a conscious effort is made to align its shape with the anticipated streamlines in the flow in order to reduce drag. Otherwise, a body (such as a building) tends to block the flow and is said to be *bluff.* At sufficiently high velocities, the fluid stream detaches itself from the surface of the body. This is called *flow separation.* When a fluid stream separates from the body, it forms a *separated region* between the body and the fluid stream. Separation may also occur on a streamlined body such as an airplane wing at a sufficiently large *angle of attack,* which is the angle the incoming fluid stream makes with the *chord* (the line that connects the nose and the end) of the body. Flow separation on the top surface of a wing reduces lift drastically and may cause the airplane to *stall.*

The region of flow above a surface in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the *velocity boundary layer* or just the *boundary layer.* The *thickness* of the boundary layer, δ, is defined as the distance from the surface at which the velocity is $0.99V$. The hypothetical line of velocity $0.99V$ divides the flow over a plate into two regions: the *boundary layer region,* in which the viscous effects and the velocity changes are significant, and the *irrotational outer flow region,* in which the frictional effects are negligible and the velocity remains essentially constant.

For external flow, the Reynolds number is expressed as

$$\text{Re}_L = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$$

where V is the upstream velocity and L is the characteristic length of the geometry, which is the length of the plate in the flow direction for a flat plate and the diameter D for a cylinder or sphere. The *average* friction coefficients over an entire flat plate are

Laminar flow: $$C_f = \frac{1.33}{\text{Re}_L^{1/2}} \qquad \text{Re}_L \lesssim 5 \times 10^5$$

Turbulent flow: $$C_f = \frac{0.074}{\text{Re}_L^{1/5}} \qquad 5 \times 10^5 \lesssim \text{Re}_L \lesssim 10^7$$

If the flow is approximated as laminar up to the engineering critical number of $\text{Re}_{\text{cr}} = 5 \times 10^5$, and then turbulent beyond, the average friction coefficient over the entire flat plate becomes

$$C_f = \frac{0.074}{\text{Re}_L^{1/5}} - \frac{1742}{\text{Re}_L} \qquad 5 \times 10^5 \lesssim \text{Re}_L \lesssim 10^7$$

A curve fit of experimental data for the average friction coefficient in the fully rough turbulent regime is

Rough surface: $$C_f = \left(1.89 - 1.62 \log \frac{\varepsilon}{L}\right)^{-2.5}$$

where ε is the surface roughness and L is the length of the plate in the flow direction. In the absence of a better one, this relation can be used for turbulent flow on rough surfaces for $\text{Re} > 10^6$, especially when $\varepsilon/L > 10^{-4}$.

Surface roughness, in general, increases the drag coefficient in turbulent flow. For bluff bodies such as a circular cylinder or sphere, however, an increase in the surface roughness may *decrease* the drag coefficient. This is done by tripping the flow into turbulence at a lower Reynolds number, and thus causing the fluid to close in behind the body, narrowing the wake and reducing pressure drag considerably.

It is desirable for airfoils to generate the most lift while producing the least drag. Therefore, a measure of performance for airfoils is the *lift-to-drag ratio,* C_L/C_D.

The minimum safe flight velocity of an aircraft is determined from

$$V_{\min} = \sqrt{\frac{2W}{\rho C_{L,\,\max} A}}$$

For a given weight, the landing or takeoff speed can be minimized by maximizing the product of the lift coefficient and the wing area, $C_{L,\,\max}A$.

For airplane wings and other airfoils of finite span, the pressure difference between the lower and the upper surfaces drives the fluid at the tips upward. This results in swirling eddies, called *tip vortices.* Tip vortices that interact with the free stream impose forces on the wing tips in all directions, including the flow direction. The component of the force in the flow direction adds to drag and is called *induced drag*. The total drag of a wing is then the sum of the induced drag (3-D effects) and the drag of the airfoil section (2-D effects).

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PROBLEMS*

Drag, Lift, and Drag Coefficients

15–1C What is drag? What causes it? Why do we usually try to minimize it?

15–2C What is lift? What causes it? Does wall shear contribute to the lift?

15–3C Which bicyclist is more likely to go faster: one who keeps his head and his body in the most upright position or one who leans down and brings his body closer to his knees? Why?

15–4C Define the frontal area of a body subjected to external flow. When is it appropriate to use the frontal area in drag and lift calculations?

15–5C Define the planform area of a body subjected to external flow. When is it appropriate to use the planform area in drag and lift calculations?

15–6C What is the difference between the upstream velocity and the free-stream velocity? For what types of flow are these two velocities equal to each other?

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* Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and SI users can ignore them. Problems with the icon 💻 are comprehensive in nature and are intended to be solved with appropriate software.

15–7C What is the difference between streamlined and bluff bodies? Is a tennis ball a streamlined or bluff body?

15–8C Name some applications in which a large drag is desired.

15–9C During flow over a given body, the drag force, the upstream velocity, and the fluid density are measured. Explain how you would determine the drag coefficient. What area would you use in the calculations?

15–10C What is terminal velocity? How is it determined?

15–11C What is the difference between skin friction drag and pressure drag? Which is usually more significant for slender bodies such as airfoils?

15–12C What is the effect of surface roughness on the friction drag coefficient in laminar and turbulent flows?

15–13C What is the effect of streamlining on (*a*) friction drag and (*b*) pressure drag? Does the total drag acting on a body necessarily decrease as a result of streamlining? Explain.

15–14C What is flow separation? What causes it? What is the effect of flow separation on the drag coefficient?

15–15C What is drafting? How does it affect the drag coefficient of the drafted body?

15–16C Consider laminar flow over a flat plate. How does the local friction coefficient change with position?

15–17C In general, how does the drag coefficient vary with the Reynolds number at (*a*) low and moderate Reynolds numbers and (*b*) at high Reynolds numbers ($\text{Re} > 10^4$)?

15–18 A car is moving at a constant velocity of 110 km/h. Determine the upstream velocity to be used in fluid flow analysis if (*a*) the air is calm, (*b*) wind is blowing against the direction of motion of the car at 30 km/h, and (*c*) wind is blowing in the same direction of motion of the car at 30 km/h.

15–19 The resultant of the pressure and wall shear forces acting on a body is measured to be 430 N, making 30° with the direction of flow. Determine the drag and the lift forces acting on the body.

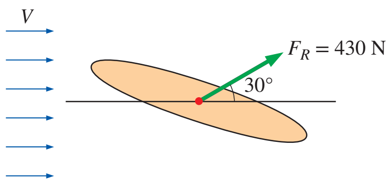


FIGURE P15–19

15–20 During a high Reynolds number experiment, the total drag force acting on a spherical body of diameter $D = 12$ cm subjected to airflow at 1 atm and 5°C is measured to be 5.2 N. The pressure drag acting on the body is calculated by integrating the pressure distribution (measured by the use of pressure sensors throughout the surface) to be 4.9 N. Determine the friction drag coefficient of the sphere. *Answer:* 0.0115

15–21 The drag coefficient of a car at the design conditions of 1 atm, 25°C, and 90 km/h is to be determined experimentally in a large wind tunnel in a full-scale test. The height and width of the car are 1.25 m and 1.65 m, respectively. If the horizontal force acting on the car is measured to be 220 N, determine the total drag coefficient of this car. *Answer:* 0.29

15–22E To reduce the drag coefficient and thus to improve the fuel efficiency, the frontal area of a car is to be reduced. Determine the amount of fuel and money saved per year as a result of reducing the frontal area from 20 to 13 ft^2. Assume the car is driven 12,000 mi a year at an average speed of 55 mi/h. Take the density and price of gasoline to be 50 lbm/ft^3 and \$3.10/gal, respectively; the density of air to be 0.075 lbm/ft^3, the heating value of gasoline to be 20,000 Btu/lbm; and the overall efficiency of the drive train to be 30 percent.

15–23E Reconsider Prob. 15–22E. Using appropriate software, investigate the effect of frontal area on the annual fuel consumption of the car. Let the frontal area vary from 10 to 30 ft^2 in increments of 2 ft^2. Tabulate and plot the results.

15–24 A circular sign has a diameter of 50 cm and is subjected to normal winds up to 150 km/h at 10°C and 100 kPa. Determine the drag force acting on the sign. Also determine the bending moment at the bottom of its pole whose height from the ground to the bottom of the sign is 1.5 m. Disregard the drag on the pole.

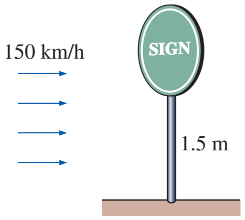


FIGURE P15–24

15–25 Suzy likes to drive with a silly sun ball on her car antenna. The frontal area of the ball is $A = 2.08 \times 10^{-3}\ \text{m}^2$. As gas prices rise, her husband is concerned that she is wasting fuel because of the additional drag on the ball. He runs a quick test in the wind tunnel at his university and measures the drag coefficient to be $C_D = 0.87$ at nearly all air speeds. Estimate how many liters of fuel she wastes per year by having this ball on her antenna. Use the following additional information: She drives about 15,000 km per year at an average speed of 20.8 m/s. The overall car efficiency is 0.312, $\rho_{\text{fuel}} = 0.802$ kg/L, and the heating value of the fuel is 44,020 kJ/kg. Use standard air properties. Is the amount of wasted fuel significant?

FIGURE P15–25

Suzanne Cimbala

15–26 Advertisement signs are commonly carried by taxicabs for additional income, but they also increase the fuel cost. Consider a sign that consists of a 0.30-m-high, 0.9-m-wide, and 0.9-m-long rectangular block mounted on top of a taxicab such that the sign has a frontal area of 0.3 m by 0.9 m from all four sides. Determine the increase in the annual fuel cost of this taxicab due to this sign. Assume the taxicab is driven 60,000 km a year at an average speed of 50 km/h and the overall efficiency of the drive train is 28 percent. Take the density, unit price, and heating value of gasoline to be 0.72 kg/L, \$1.10/L, and 42,000 kJ/kg, respectively, and the density of air to be 1.25 kg/m^3.

FIGURE P15–26

15–27E At highway speeds, about half of the power generated by the car's engine is used to overcome aerodynamic drag, and thus the fuel consumption is nearly proportional to the drag force on a level road. Determine the percentage increase in fuel consumption of a car per unit time when a person who normally drives at 55 mi/h now starts driving at 70 mi/h.

15–28 A submarine can be treated as an ellipsoid with a diameter of 5 m and a length of 25 m. Determine the power required for this submarine to cruise horizontally and steadily at 40 km/h in seawater whose density is 1025 kg/m^3. Also determine the power required to tow this submarine in air whose density is 1.30 kg/m^3. Assume the flow is turbulent in both cases.

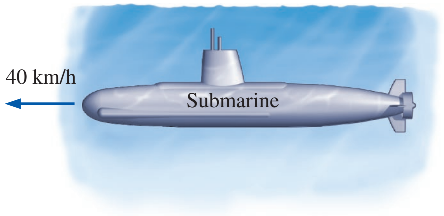


FIGURE P15–28

15–29 A 70-kg bicyclist is riding her 15-kg bicycle downhill on a road with a slope of 8° without pedaling or braking. The bicyclist has a frontal area of 0.45 m^2 and a drag coefficient of 1.1 in the upright position, and a frontal area of 0.4 m^2 and a drag coefficient of 0.9 in the racing position. Disregarding the rolling resistance and friction at the bearings, determine the terminal velocity of the bicyclist for both positions. Take the air density to be 1.25 kg/m^3. *Answers:* 70 km/h, 82 km/h

15–30 A wind turbine with two or four hollow hemispherical cups connected to a pivot is commonly used to measure wind speed. Consider a wind turbine with four 8-cm-diameter cups with a center-to-center distance of 40 cm, as shown in Fig. P15–30. The pivot is stuck as a result of some malfunction, and the cups stop rotating. For a wind speed of 15 m/s and air density of 1.25 kg/m^3, determine the maximum torque this turbine applies on the pivot.

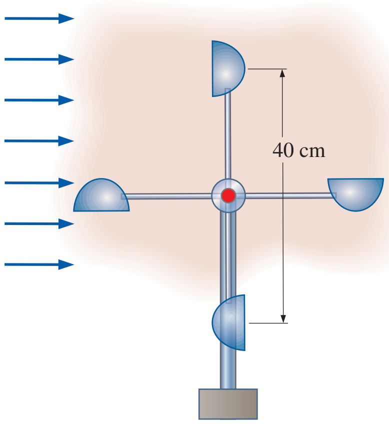


FIGURE P15–30

15–31 Reconsider Prob. 15–30. Using appropriate software, investigate the effect of wind speed on the torque applied on the pivot. Let the wind speed vary from 0 to 50 m/s in increments of 5 m/s. Tabulate and plot the results.

15–32 During steady motion of a vehicle on a level road, the power delivered to the wheels is used to overcome aerodynamic drag and rolling resistance (the product of the rolling resistance coefficient and the weight of the vehicle), assuming the friction at the bearings of the wheels is negligible. Consider a car that has a total mass of 950 kg, a drag coefficient of 0.32, a frontal area of 1.8 m^2, and a rolling resistance coefficient of 0.04. The maximum power the engine can deliver to the wheels is 80 kW. Determine (*a*) the speed at which the rolling resistance is equal to the aerodynamic drag force and (*b*) the maximum speed of this car. Take the air density to be 1.20 kg/m^3.

15–33 Reconsider Prob. 15–32. Using appropriate software, investigate the effect of car speed on the required power to overcome (*a*) rolling resistance, (*b*) the aerodynamic drag, and (*c*) their combined effect. Let the car speed vary from 0 to 150 km/h in increments of 15 km/h. Tabulate and plot the results.

15–34E Bill gets a job delivering pizzas. The pizza company makes him mount a sign on the roof of his car. The frontal area of the sign is $A = 0.612$ ft^2, and he estimates the drag coefficient to be $C_D = 0.94$ at nearly all air speeds. Estimate how much additional money it costs Bill per year in fuel to drive with the sign on his roof compared to without the sign. Use the following additional information: He drives about 10,000 miles per year at an average speed of 45 mph. The overall car efficiency is 0.332, $\rho_{\text{fuel}} = 50.2$ lbm/ft^3, and the heating value of the fuel is 1.53×10^7 ft·lbf/lbm. The fuel costs \$3.50 per gallon. Use standard air properties. Be careful with unit conversions.

15–35 A 0.80-m-diameter, 1.2-m-high garbage can is found in the morning tipped over due to high winds during the night. Assuming the average density of the garbage inside to be 150 kg/m^3 and taking the air density to be 1.25 kg/m^3, estimate the wind velocity during the night when the can was tipped over. Take the drag coefficient of the can to be 0.7. *Answer:* 135 km/h

15–36 An 8-mm-diameter plastic sphere whose density is 1150 kg/m^3 is dropped into water at 20°C. Determine the terminal velocity of the sphere in water.

15–37 A 7-m-diameter hot air balloon that has a total mass of 350 kg is standing still in air on a windless day. The balloon is suddenly subjected to 40 km/h winds. Determine the initial acceleration of the balloon in the horizontal direction.

15–38 During major windstorms, high vehicles such as RVs and semis may be thrown off the road and boxcars off their tracks, especially when they are empty and in open areas. Consider a 6000-kg semi that is 10 m long, 2.5 m high, and 2 m wide. The distance between the bottom of the truck and the road is 0.8 m. Now the truck is exposed to winds from its side surface. Determine the wind velocity that will tip the truck over to its side. Take the air density to be 1.1 kg/m^3, and assume the weight to be uniformly distributed.

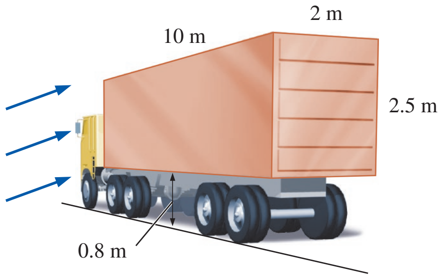


FIGURE P15–38

Flow over Flat Plates

15–39C What does the friction coefficient represent in flow over a flat plate? How is it related to the drag force acting on the plate?

15–40C What fluid property is responsible for the development of the velocity boundary layer? What is the effect of the velocity on the thickness of the boundary layer?

15–41C How is the average friction coefficient determined in flow over a flat plate?

15–42E Light oil at 75°F flows over a 17-ft-long flat plate with a free-stream velocity of 5 ft/s. Determine the total drag force per unit width of the plate.

15–43 The local atmospheric pressure in Denver, Colorado, (elevation 1610 m) is 83.4 kPa. Air at this pressure and at 25°C flows with a velocity of 9 m/s over a 2.5-m × 5-m flat plate. Determine the drag force acting on the top surface of the plate if the air flows parallel to the (*a*) 5-m-long side and (*b*) the 2.5-m-long side.

15–44E Air at 70°F flows over a 10-ft-long flat plate at 25 ft/s. Determine the local friction coefficient at intervals of 1 ft, and plot the results against the distance from the leading edge.

15–45 Consider laminar flow of a fluid over a flat plate. Now the free-stream velocity of the fluid is tripled. Determine the change in the drag force on the plate. Assume the flow remains laminar. *Answer:* A 5.20-fold increase

15–46E Consider a refrigeration truck traveling at 70 mi/h at a location where the air is at 1 atm and 80°F. The refrigerated compartment of the truck can be considered to be a 9-ft-wide, 8-ft-high, and 20-ft-long rectangular box. Assuming the airflow over the entire outer surface to be turbulent and attached (no flow separation), determine the drag force acting on the top and side surfaces and the power required to overcome this drag.

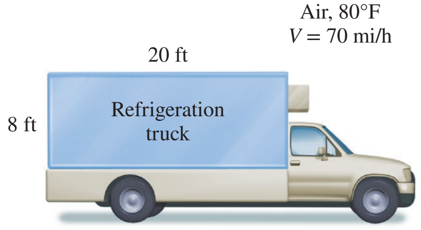


FIGURE P15–46E

15–47E Reconsider Prob. 15–46E. Using appropriate software, investigate the effect of truck speed on the total drag force acting on the top and side surfaces and the power required to overcome it. Let the truck speed vary from 0 to 100 mi/h in increments of 10 mi/h. Tabulate and plot the results.

15–48 Air at 25°C and 1 atm is flowing over a long flat plate with a velocity of 8 m/s. Determine the distance from the leading edge of the plate where the flow becomes turbulent, and the thickness of the boundary layer at that location.

15–49 Repeat Prob. 15–48 for water.

15–50 The weight of a thin flat plate 50 cm × 50 cm in size is balanced by a counterweight that has a mass of 2 kg, as shown in Fig. P15–50. Now a fan is turned on, and air at 1 atm and 25°C flows downward over both surfaces of the plate (front and back in the sketch) with a free-stream velocity of 10 m/s. Determine the mass of the counterweight that needs to be added in order to balance the plate in this case.

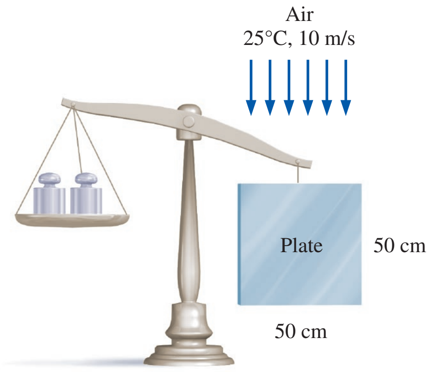


FIGURE P15–50

15–51 The forming section of a plastics plant puts out a continuous sheet of plastic that is 1.2 m wide and 2 mm thick at a rate of 18 m/min. The sheet is subjected to airflow at a velocity of 4 m/s on both top and bottom surfaces normal to the direction of motion of the sheet. The width of the air cooling section is such that a fixed point on the plastic sheet passes through that section in 2 s. Using properties of air at 1 atm and 60°C, determine the drag force the air exerts on the plastic sheet in the direction of airflow.

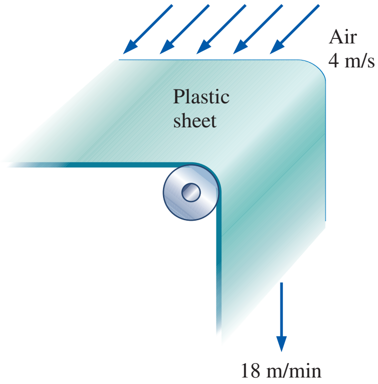


FIGURE P15–51

Flow Across Cylinders and Spheres

15–52C In flow over bluff bodies such as a cylinder, how does the pressure drag differ from the friction drag?

15–53C Why is flow separation in flow over cylinders delayed when the boundary layer is turbulent?

15–54C In flow over cylinders, why does the drag coefficient suddenly drop when the boundary layer becomes turbulent? Isn't turbulence supposed to increase the drag coefficient instead of decreasing it?

15–55 A 5-mm-diameter electrical transmission line is exposed to windy air. Determine the drag force exerted on a 160-m-long section of the wire during a windy day when the air is at 1 atm and 15°C and the wind is blowing across the transmission line at 50 km/h.

15–56 A long 5-cm-diameter steam pipe passes through some area open to the wind. Determine the drag force acting on the pipe per unit of its length when the air is at 1 atm and 10°C and the wind is blowing across the pipe at a speed of 50 km/h.

15–57 Consider 0.8-cm-diameter hail that is falling freely in atmospheric air at 1 atm and 5°C. Determine the terminal velocity of the hail. Take the density of hail to be 910 kg/m^3.

15–58E A 1.2-in-outer-diameter pipe is to span a river at a 115-ft-wide section while being completely immersed in water. The average flow velocity of the water is 8 ft/s, and its temperature is 70°F. Determine the drag force exerted on the pipe by the river. *Answer:* 748 lbf

15–59 A 2-m-long, 0.2-m-diameter cylindrical pine log (density = 513 kg/m^3) is suspended by a crane in the horizontal position. The log is subjected to normal winds of 40 km/h at 5°C and 88 kPa. Disregarding the weight of the cable and its drag, determine the angle θ the cable will make with the horizontal and the tension on the cable.

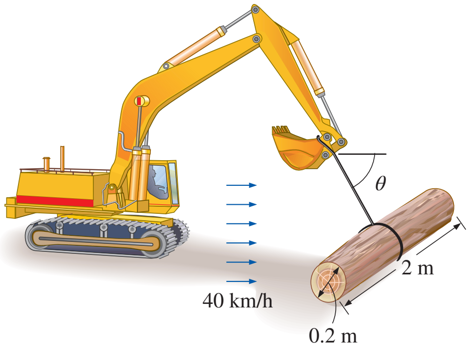


FIGURE P15–59

15–60 A 0.12-mm-diameter dust particle whose density is 2.1 g/cm^3 is observed to be suspended in the air at 1 atm and 20°C at a fixed point. Estimate the updraft velocity of air motion at that location. Assume Stokes' law to be applicable. Is this a valid assumption? *Answer:* 0.90 m/s

15–61E A person extends his uncovered arms into the windy air outside at 1 atm and 60°F and 25 mi/h in order to feel nature closely. Treating the arm as a 2-ft-long and 4-in-diameter cylinder, determine the combined drag force on both arms. *Answer:* 2.12 lbf

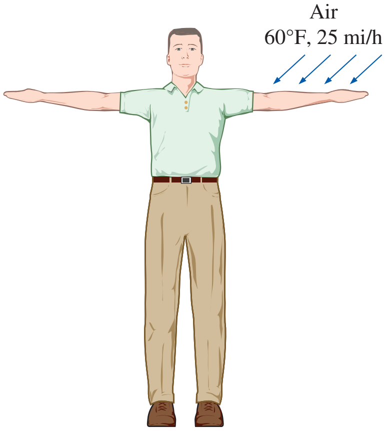


FIGURE P15–61E

15–62 Dust particles of diameter 0.06 mm and density 1.6 g/cm^3 are unsettled during high winds and rise to a height of 200 m by the time things calm down. Estimate how long it takes for the dust particles to fall back to the ground in still air at 1 atm and 30°C, and their velocity. Disregard the initial transient period during which the dust particles accelerate to their terminal velocity, and assume Stokes' law to be applicable.

Lift

15–63C What is stall? What causes an airfoil to stall? Why are commercial aircraft not allowed to fly at conditions near stall?

15–64C Air is flowing past a symmetrical airfoil at zero angle of attack. Is the (*a*) lift and (*b*) drag acting on the airfoil zero or nonzero?

15–65C Both the lift and the drag of an airfoil increase with an increase in the angle of attack. In general, which increases at a higher rate, the lift or the drag?

15–66C Why are flaps used at the leading and trailing edges of the wings of large aircraft during takeoff and landing? Can an aircraft take off or land without them?

15–67C Why is the contribution of viscous effects to lift usually negligible for airfoils?

15–68C Air is flowing past a symmetrical airfoil at an angle of attack of 5°. Is the (*a*) lift and (*b*) drag acting on the airfoil zero or nonzero?

15–69C Air is flowing past a spherical ball. Is the lift exerted on the ball zero or nonzero? Answer the same question if the ball is spinning.

15–70C What is the effect of wing tip vortices (the air circulation from the lower part of the wings to the upper part) on the drag and the lift?

15–71C What is induced drag on wings? Can induced drag be minimized by using long and narrow wings or short and wide wings?

15–72C Explain why endplates or winglets are added to some airplane wings.

15–73C How do flaps affect the lift and the drag of wings?

15–74E A 2.4-in-diameter smooth ball rotating at 500 rpm is dropped in a water stream at 60°F flowing at 4 ft/s. Determine the lift and the drag force acting on the ball when it is first dropped in the water.

15–75 Consider an aircraft that takes off at 260 km/h when it is fully loaded. If the weight of the aircraft is increased by 10 percent as a result of overloading, determine the speed at which the overloaded aircraft will take off. *Answer:* 273 km/h

15–76 Consider an airplane whose takeoff speed is 220 km/h and that takes 15 s to take off at sea level. For an airport at an elevation of 1600 m (such as Denver), determine (*a*) the takeoff speed, (*b*) the takeoff time, and (*c*) the additional runway length required for this airplane. Assume constant acceleration for both cases.

FIGURE P15–76

15–77 A jumbo jet airplane has a mass of about 400,000 kg when fully loaded with over 400 passengers and takes off at a speed of 250 km/h. Determine the takeoff speed when the airplane has 150 empty seats. Assume each passenger with luggage is 140 kg and the wing and flap settings are maintained the same. *Answer:* 243 km/h

15–78 Reconsider Prob. 15–77. Using appropriate software, investigate the effect of passenger count on the takeoff speed of the aircraft. Let the number of passengers vary from 0 to 500 in increments of 50. Tabulate and plot the results.

15–79 A small aircraft has a wing area of 40 m^2, a lift coefficient of 0.45 at takeoff settings, and a total mass of 4000 kg. Determine (*a*) the takeoff speed of this aircraft at sea level at standard atmospheric conditions, (*b*) the wing loading, and (*c*) the required power to maintain a constant cruising speed of 360 km/h for a cruising drag coefficient of 0.035.

15–80 Consider a light plane that has a total weight of 11,000 N and a wing area of 39 m^2 and whose wings resemble the NACA 23012 airfoil with no flaps. Using data from Fig. 15–45, determine the takeoff speed at an angle of attack of 5° at sea level. Also determine the stall speed. *Answers:* 99.7 km/h, 62.7 km/h

15–81 A small airplane has a total mass of 1800 kg and a wing area of 42 m^2. Determine the lift and drag coefficients of this airplane while cruising at an altitude of 4000 m at a constant speed of 280 km/h and generating 190 kW of power.

15–82 An airplane has a mass of 48,000 kg, a wing area of 300 m^2, a maximum lift coefficient of 3.2, and a cruising drag coefficient of 0.03 at an altitude of 12,000 m. Determine (*a*) the takeoff speed at sea level, assuming it is 20 percent over the stall speed, and (*b*) the thrust that the engines must deliver for a cruising speed of 900 km/h.

15–83E An airplane is consuming fuel at a rate of 7 gal/min when cruising at a constant altitude of 10,000 ft at constant speed. Assuming the drag coefficient and the engine efficiency remain the same, determine the rate of fuel consumption at an altitude of 30,000 ft at the same speed.

Review Problems

15–84 A 1.2-m-external-diameter spherical tank is located outdoors at 1 atm and 25°C and is subjected to winds at 48 km/h. Determine the drag force exerted on it by the wind. *Answer:* 16.7 N

15–85E A 7-ft-diameter spherical tank completely submerged in fresh water is being towed by a ship at 8 ft/s. Assuming turbulent flow, determine the required towing power.

15–86 A 2-m-high, 4-m-wide rectangular advertisement panel is attached to a 4-m-wide, 0.15-m-high rectangular concrete block (density = 2300 kg/m^3) by two 5-cm-diameter, 4-m-high (exposed part) poles, as shown in Fig. P15–86. If the sign is to withstand 150 km/h winds from any direction, determine (*a*) the maximum drag force on the panel, (*b*) the drag force acting on the poles, and (*c*) the minimum length *L* of the concrete block for the panel to resist the winds. Take the density of air to be 1.30 kg/m^3.

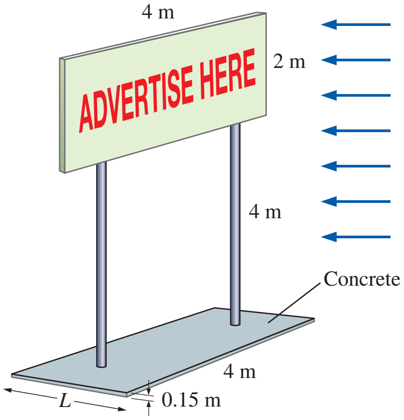


FIGURE P15–86

15–87 A plastic boat whose bottom surface can be approximated as a 1.5-m-wide, 2-m-long flat surface is to move through water at 15°C at speeds up to 45 km/h. Determine the friction drag exerted on the boat by the water and the power needed to overcome it.

FIGURE P15–87

15–88 Reconsider Prob. 15–87. Using appropriate software, investigate the effect of boat speed on the drag force acting on the bottom surface of the boat, and the power needed to overcome it. Let the boat speed vary from 0 to 100 km/h in increments of 10 km/h. Tabulate and plot the results.

15–89 Stokes' law can be used to determine the viscosity of a fluid by dropping a spherical object in it and measuring the terminal velocity of the object in that fluid. This can be done by plotting the distance traveled against time and observing when the curve becomes linear. During such an experiment a 3-mm-diameter glass ball (ρ = 2500 kg/m^3) is dropped into a fluid whose density is 875 kg/m^3, and the terminal velocity is measured to be 0.15 m/s. Disregarding the wall effects, determine the viscosity of the fluid.

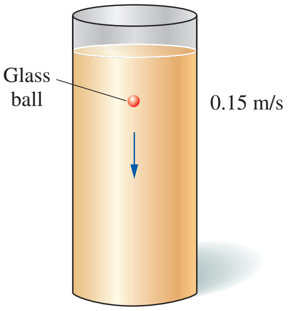


FIGURE P15–89

15–90E The passenger compartment of a minivan traveling at 50 mi/h in ambient air at 1 atm and 80°F is modeled as a 4.5-ft-high, 6-ft-wide, and 11-ft-long rectangular box. The airflow over the exterior surfaces is assumed to be turbulent because of the intense vibrations involved. Determine the drag force acting on the top and the two side surfaces of the van and the power required to overcome it.

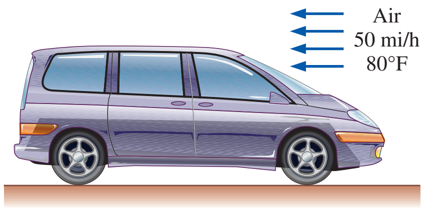


FIGURE P15–90E

15–91E A commercial airplane has a total mass of 150,000 lbm and a wing planform area of 1700 ft^2. The plane has a cruising speed of 625 mi/h and a cruising altitude of 38,000 ft where the air density is 0.0208 lbm/ft^3. The plane has double-slotted flaps for use during takeoff and landing, but it cruises with all flaps retracted. Assuming the lift and drag characteristics of the wings can be approximated by NACA 23012, determine (*a*) the minimum safe speed for takeoff and landing with and without extending the flaps, (*b*) the angle of attack to cruise steadily at the cruising altitude, and (*c*) the power that needs to be supplied to provide enough thrust to overcome drag. Take the air density on the ground to be 0.075 lbm/ft^3.

15–92 An automotive engine can be approximated as a 0.4-m-high, 0.60-m-wide, and 0.7-m-long rectangular block. The ambient air is at 1 atm and 15°C. Determine the drag force acting on the bottom surface of the engine block as the car travels at a velocity of 120 km/h. Assume the flow to be turbulent over the entire surface because of the constant agitation of the engine block. *Answer:* 1.22 N

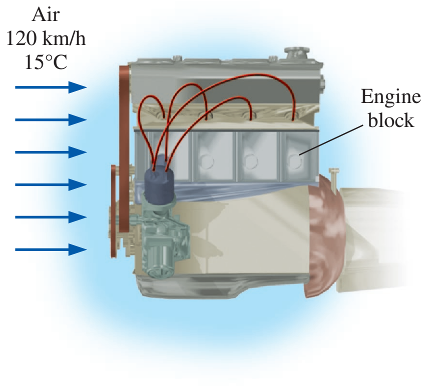


FIGURE P15–92

15–93 A paratrooper and his 7-m-diameter parachute weigh 1200 N. Taking the average air density to be 1.2 kg/m^3, determine the terminal velocity of the paratrooper. *Answer:* 6.3 m/s

FIGURE P15–93

15–94 It is proposed to meet the water needs of a recreational vehicle (RV) by installing a 3-m-long, 0.5-m-diameter cylindrical tank on top of the vehicle. Determine the additional power needed at a speed of 80 km/h when the tank is installed such that its circular surfaces face (*a*) the front and back (as sketched) and (*b*) the sides of the RV. Assume atmospheric conditions are 87 kPa and 20°C. *Answers:* (*a*) 1.05 kW, (*b*) 6.77 kW

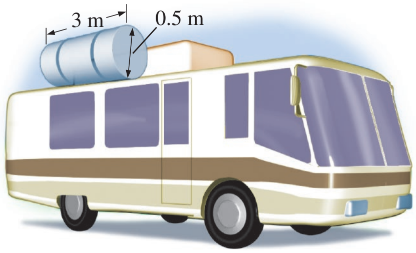


FIGURE P15–94

15–95 Consider a blimp that can be approximated as a 3-m diameter, 8-m long ellipsoid and is connected to the ground. On a windless day, the rope tension due to the net buoyancy effect is measured to be 120 N. Determine the rope tension when there are 50 km/h winds blowing along the blimp (parallel to the blimp axis).

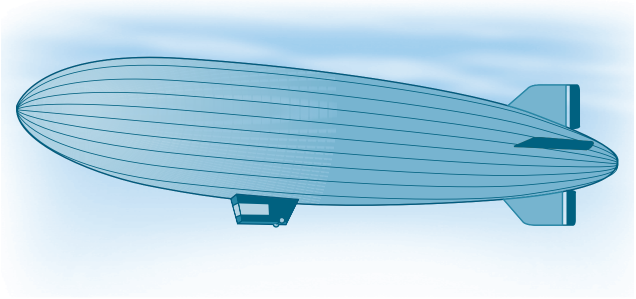


FIGURE P15–95

15–96 A 17,000-kg tractor-trailer rig has a frontal area of 9.2 m^2, a drag coefficient of 0.96, a rolling resistance coefficient of 0.05 (multiplying the weight of a vehicle by the rolling resistance coefficient gives the rolling resistance), a bearing friction resistance of 350 N, and a maximum speed of 110 km/h on a level road during steady cruising in calm weather with an air density of 1.25 kg/m^3. Now a fairing is installed on the front of the rig to suppress separation and to streamline the flow to the top surface, and the drag coefficient is reduced to 0.76. Determine the maximum speed of the rig with the fairing. *Answer:* 133 km/h

15–97 During an experiment, three aluminum balls ($\rho_s = 2600$ kg/m^3) having diameters 2, 4, and 10 mm, respectively, are dropped into a tank filled with glycerin at 22°C ($\rho_f = 1274$ kg/m^3 and $\mu = 1$ kg/m·s). The terminal settling velocities of the balls are measured to be 3.2, 12.8, and 60.4 mm/s, respectively. Compare these values with the velocities predicted by Stokes' law for drag force $F_D = 3\pi\mu DV$, which is valid for very low Reynolds numbers (Re $\ll$ 1). Determine the error involved for each case and assess the accuracy of Stokes' law.

15–98 Repeat Prob. 15–97 by considering the more general form of Stokes' law expressed as $F_D = 3\pi\mu DV + (9\pi/16)\rho V^2 D^2$ where ρ is the fluid density.

15–99 Engine oil at 40°C is flowing over a long flat plate with a velocity of 6 m/s. Determine the distance x_{cr} from the leading edge of the plate where the flow becomes turbulent, and calculate and plot the thickness of the boundary layer over a length of $2x_{cr}$.

15–100 The cylindrical chimney of a factory has an external diameter of 1.1 m and is 20 m high. Determine the bending moment at the base of the chimney when winds at 110 km/h are blowing across it. Take the atmospheric conditions to be 20°C and 1 atm.

Design and Essay Problems

15–101 Write a report on the flaps used at the leading and trailing edges of the wings of large commercial aircraft. Discuss how the flaps affect the drag and lift coefficients during takeoff and landing.

15–102 Large commercial airplanes cruise at high altitudes (up to about 40,000 ft) to save fuel. Discuss how flying at high altitudes reduces drag and saves fuel. Also discuss why small planes fly at relatively low altitudes.

15–103 Many drivers turn off their air conditioners and roll down the car windows in hopes of saving fuel. But it is claimed that this apparent "free cooling" actually increases the fuel consumption of some cars. Investigate this matter and write a report on which practice saves gasoline under what conditions.

PART

3

HEAT TRANSFER

CHAPTER

16

MECHANISMS OF HEAT TRANSFER

The science of thermodynamics deals with the *amount* of heat transfer as a system undergoes a process from one equilibrium state to another, and it makes no reference to *how long* the process will take. But in engineering, we are often interested in the *rate* of heat transfer, which is the topic of the science of *heat transfer*.

We start this chapter with an overview of the three basic mechanisms of heat transfer, which are conduction, convection, and radiation, and we discuss thermal conductivity. *Conduction* is the transfer of energy from the more energetic particles of a substance to the adjacent, less energetic ones as a result of interactions between the particles. *Convection* is the mode of heat transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion. *Radiation* is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules. We close this chapter with a discussion of simultaneous heat transfer.

OBJECTIVES

The objectives of this chapter are to:

- Understand the basic mechanisms of heat transfer, which are conduction, convection, and radiation, and Fourier's law of heat conduction, Newton's law of cooling, and the Stefan–Boltzmann law of radiation.
- Identify the mechanisms of heat transfer that occur simultaneously in practice.
- Develop an awareness of the cost associated with heat losses.
- Solve various heat transfer problems encountered in practice.

16–1 ▪ INTRODUCTION

In Chap. 3, we defined *heat* as the form of energy that can be transferred from one system to another as a result of temperature difference. A thermodynamic analysis is concerned with the *amount* of heat transfer as a system undergoes a process from one equilibrium state to another. The science that deals with the determination of the *rates* of such energy transfers is the science of *heat transfer.* The transfer of energy as heat is always from the higher-temperature medium to the lower-temperature one, and heat transfer stops when the two media reach the same temperature.

Heat can be transferred in three different modes: *conduction, convection,* and *radiation.* All modes of heat transfer require the existence of a temperature difference, and all modes are from the high-temperature medium to a lower-temperature one. Next we give a brief description of each mode. A detailed study of these modes is given in later chapters of this text.

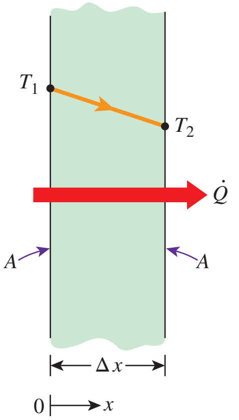


FIGURE 16–1
Heat conduction through a large plane wall of thickness Δx and area A.

16–2 ▪ CONDUCTION

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Conduction can take place in solids, liquids, or gases. In gases and liquids, conduction is due to the *collisions* and *diffusion* of the molecules during their random motion. In solids, it is due to the combination of *vibrations* of the molecules in a lattice and the energy transport by *free electrons.* A cold canned drink in a warm room, for example, eventually warms up to the room temperature as a result of heat transfer from the room to the drink through the aluminum can by conduction.

The *rate* of heat conduction through a medium depends on the *geometry* of the medium, its *thickness,* and the *material* of the medium, as well as the *temperature difference* across the medium. We know that wrapping a hot water tank with glass wool (an insulating material) reduces the rate of heat loss from the tank. The thicker the insulation, the smaller the heat loss. We also know that a hot water tank loses heat at a higher rate when the temperature of the room housing the tank is lowered. Further, the larger the tank, the larger the surface area and thus the rate of heat loss.

Consider steady heat conduction through a large plane wall of thickness $\Delta x = L$ and area A, as shown in Fig. 16–1. The temperature difference across the wall is $\Delta T = T_2 - T_1$. Experiments have shown that the rate of heat transfer $\dot{Q}$ through the wall is *doubled* when the temperature difference ΔT across the wall or the area A normal to the direction of heat transfer is doubled, but is *halved* when the wall thickness L is doubled. Thus we conclude that *the rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area but is inversely proportional to the thickness of the layer.* That is,

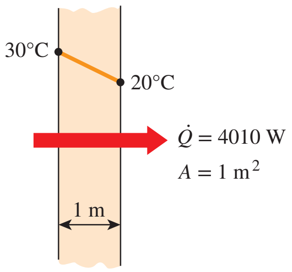
$$\text{Rate of heat conduction} \propto \frac{\text{(Area)(Temperature difference)}}{\text{Thickness}}$$

or,

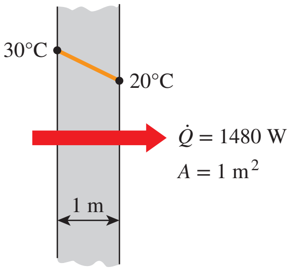
$$\dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{\Delta T}{\Delta x} \quad \text{(W)} \qquad \textbf{(16–1)}$$

where the constant of proportionality k is the **thermal conductivity** of the material, which is a *measure of the ability of a material to conduct heat* (Fig. 16–2). In the limiting case of $\Delta x \to 0$, Eq. 16–1 reduces to the differential form

$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx} \quad \text{(W)} \qquad \textbf{(16–2)}$$



(*a*) Copper ($k = 401$ W/m·K)



(*b*) Silicon ($k = 148$ W/m·K)

FIGURE 16–2
The rate of heat conduction through a solid is directly proportional to its thermal conductivity.

which is called **Fourier's law of heat conduction** after J. Fourier, who expressed it first in his heat transfer text in 1822. Here dT/dx is the **temperature gradient**, which is the slope of the temperature curve on a T-x diagram (the rate of change of T with x), at location x. Equation 16–2 indicates that the rate of heat conduction in a given direction is proportional to the temperature gradient in that direction. Heat is conducted in the direction of decreasing temperature, and the temperature gradient becomes negative when temperature decreases with increasing x. The *negative sign* in Eq. 16–2 ensures that heat transfer in the positive x-direction is a positive quantity.

The heat transfer area A is always *normal* to the direction of heat transfer. For heat loss through a 5-m-long, 3-m-high, and 25-cm-thick wall, for example, the heat transfer area is $A = 15\ \text{m}^2$. Note that the thickness of the wall has no effect on A (Fig. 16–3).

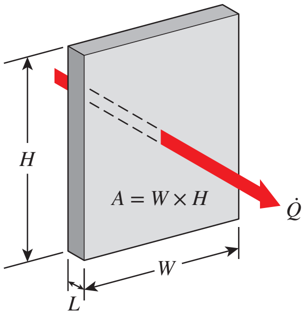


FIGURE 16–3
In heat conduction analysis, A represents the area *normal* to the direction of heat transfer.

EXAMPLE 16–1 The Cost of Heat Loss Through a Roof

The roof of an electrically heated home is 6 m long, 8 m wide, and 0.25 m thick, and it is made of a flat layer of concrete whose thermal conductivity is $k = 0.8$ W/m·K (Fig. 16–4). The temperatures of the inner and the outer surfaces of the roof one night are measured to be 15°C and 4°C, respectively, for a period of 10 h. Determine (*a*) the rate of heat loss through the roof that night and (*b*) the cost of that heat loss to the homeowner if the cost of electricity is \$0.08/kWh.

SOLUTION The inner and outer surfaces of the flat concrete roof of an electrically heated home are maintained at specified temperatures during a night. The heat loss through the roof and its cost that night are to be determined.

Assumptions **1** Steady operating conditions exist during the entire night since the surface temperatures of the roof remain constant at the specified values. **2** Constant properties can be used for the roof.

Properties The thermal conductivity of the roof is given to be $k = 0.8$ W/m·K.

Analysis (*a*) Noting that heat transfer through the roof is by conduction and the area of the roof is $A = 6\ \text{m} \times 8\ \text{m} = 48\ \text{m}^2$, the steady rate of heat transfer through the roof is

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.8\ \text{W/m·K})(48\ \text{m}^2)\frac{(15-4)^\circ\text{C}}{0.25\ \text{m}} = \mathbf{1690\ W} = \mathbf{1.69\ kW}$$

(*b*) The amount of heat lost through the roof during a 10-h period and its cost are

$$Q = \dot{Q}\,\Delta t = (1.69\ \text{kW})(10\ \text{h}) = 16.9\ \text{kWh}$$
$$\text{Cost} = (\text{Amount of energy})(\text{Unit cost of energy})$$
$$= (16.9\ \text{kWh})(\$0.08/\text{kWh}) = \mathbf{\$1.35}$$

Discussion The cost to the homeowner of the heat loss through the roof that night was \$1.35. The total heating bill of the house will be much larger since the heat losses through the walls are not considered in these calculations.

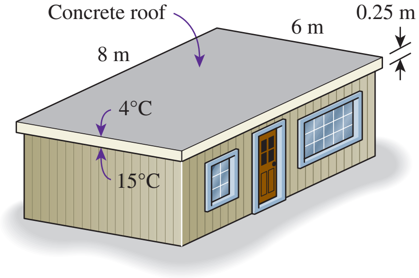


FIGURE 16–4
Schematic for Example 16–1.

Thermal Conductivity

We have seen that different materials store heat differently, and we have defined the property specific heat c_p as a measure of a material's ability to store thermal energy. For example, $c_p = 4.18$ kJ/kg·°C for water and $c_p = 0.45$ kJ/kg·°C for iron at room temperature, which indicates that water can store almost 10 times the energy that iron can per unit mass. Likewise, the thermal conductivity k is a measure of a material's ability to conduct heat. For example, $k = 0.607$ W/m·K for water and $k = 80.2$ W/m·K for iron at room temperature, which indicates that iron conducts heat more than 100 times faster than water can. Thus we say that water is

TABLE 16–1

The thermal conductivities of some materials at room temperature

Material	k, W/m·K*
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.607
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

*Multiply by 0.5778 to convert to Btu/h·ft·°F.

a poor heat conductor relative to iron, although water is an excellent medium for storing thermal energy.

Equation 16–1 for the rate of conduction heat transfer under steady conditions can also be viewed as the defining equation for thermal conductivity. Thus the **thermal conductivity** of a material can be defined as *the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.* The thermal conductivity of a material is a measure of the ability of the material to conduct heat. A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or *insulator.* The thermal conductivities of some common materials at room temperature are given in Table 16–1. The thermal conductivity of pure copper at room temperature is $k = 401$ W/m·K, which indicates that a 1-m-thick copper wall will conduct heat at a rate of 401 W per m^2 area per K temperature difference across the wall. Note that materials such as copper and silver that are good electric conductors are also good heat conductors and have high values of thermal conductivity. Materials such as rubber, wood, and Styrofoam are poor conductors of heat and have low conductivity values.

A layer of material of known thickness and area can be heated from one side by an electric resistance heater of known output. If the outer surfaces of the heater are well insulated, all the heat generated by the resistance heater will be transferred through the material whose conductivity is to be determined. Then measuring the two surface temperatures of the material when steady heat transfer is reached and substituting them into Eq. 16–1 together with other known quantities give the thermal conductivity (Fig. 16–5).

The thermal conductivities of materials vary over a wide range, as shown in Fig. 16–6. The thermal conductivities of gases such as air vary by a factor of 10^4 from those of pure metals such as copper. Note that pure crystals and metals have the highest thermal conductivities, and gases and insulating materials have the lowest.

Temperature is a measure of the kinetic energies of the particles such as the molecules or atoms of a substance. In a liquid or gas, the kinetic energy of the molecules is due to their random translational motion as well as their vibrational and rotational motions. When two molecules possessing different kinetic energies collide, part of the kinetic energy of the more energetic (higher-temperature) molecule is transferred to the less energetic (lower-temperature) molecule, much the same as when two elastic balls of the same mass at different velocities collide and part of the kinetic energy of the faster ball is transferred to the slower one. The higher the temperature, the faster the molecules move, and the higher the number of such collisions, the better the heat transfer.

The *kinetic theory* of gases predicts, and experiments confirm, that the thermal conductivity of gases is proportional to the *square root of the thermodynamic temperature T* and inversely proportional to the *square root of the molar mass M.* Therefore, for a particular gas (fixed M), the thermal conductivity increases with increasing temperature, and at a fixed temperature the thermal conductivity decreases with increasing M. For example, at a fixed temperature of 1000 K, the thermal conductivity of helium ($M = 4$) is 0.343 W/m·K, and that of air ($M = 29$) is 0.0667 W/m·K, which is much lower than that of helium.

The thermal conductivities of *gases* at 1 atm pressure are listed in Table A-23. However, they can also be used at pressures other than 1 atm since the thermal conductivity of gases is *independent of pressure* in a wide range of pressures encountered in practice.

The mechanism of heat conduction in a *liquid* is complicated by the fact that the molecules are more closely spaced, and they exert a stronger intermolecular force field. The thermal conductivities of liquids usually lie between those of solids and gases. The thermal conductivity of a substance is normally highest in the solid phase and lowest in the gas phase. The thermal conductivity of liquids

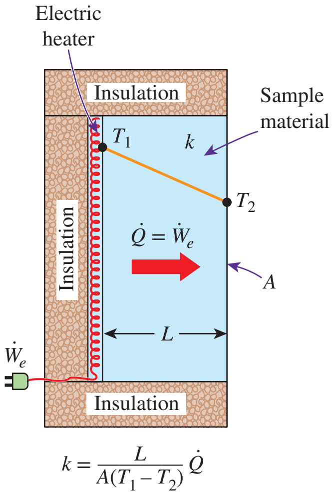


FIGURE 16–5

A simple experimental setup to determine the thermal conductivity of a material.

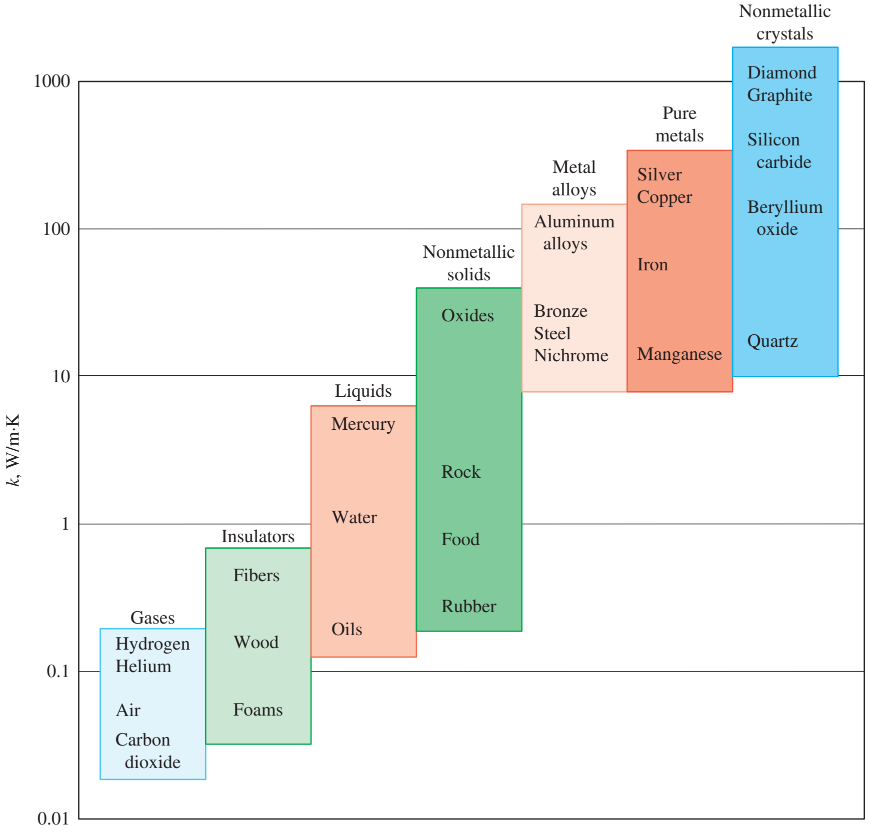


FIGURE 16–6
The range of thermal conductivity of various materials at room temperature.

is generally insensitive to pressure except near the thermodynamic critical point. Unlike gases, the thermal conductivities of most liquids decrease with increasing temperature, with water being a notable exception. Like gases, the conductivity of liquids decreases with increasing molar mass. Liquid metals such as mercury and sodium have high thermal conductivities and are very suitable for use in applications where a high heat transfer rate to a liquid is desired, as in nuclear power plants.

In *solids,* heat conduction is due to two effects: (1) the *lattice vibrational waves* induced by the vibrational motions of the molecules positioned at relatively fixed positions in a periodic manner called a lattice and (2) the energy transported via the *free flow of electrons* in the solid (Fig. 16–7). The thermal conductivity of a solid is obtained by adding the lattice and electronic components. The relatively high thermal conductivities of pure metals are primarily due to the electronic component. The lattice component of thermal conductivity strongly depends on the way the molecules are arranged. For example, diamond, which is a highly ordered crystalline solid, has the highest known thermal conductivity at room temperature.

Unlike metals, which are good electrical and heat conductors, *crystalline solids* such as diamond and semiconductors such as silicon are good heat conductors but poor electrical conductors. As a result, such materials find widespread use in the electronics industry. Despite their higher price, diamond heat sinks are used in the cooling of sensitive electronic components because of the excellent thermal conductivity of diamond. Silicon oils and gaskets are commonly used in the packaging of electronic components because they provide both good thermal contact and good electrical insulation.

Pure metals have high thermal conductivities, and one would think that *metal alloys* should also have high conductivities. One would expect an alloy made of two metals of thermal conductivities k_1 and k_2 to have a conductivity k between

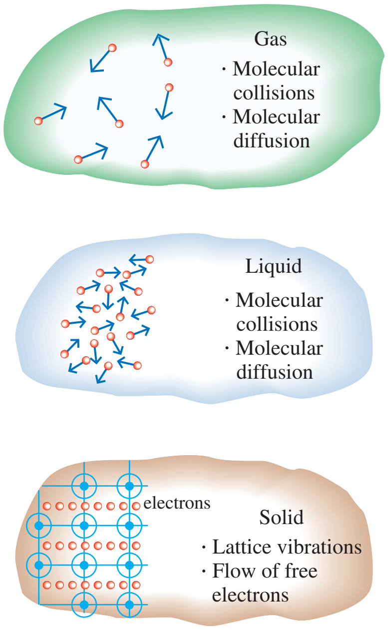


FIGURE 16–7
The mechanisms of heat conduction in different phases of a substance.

k_1 and k_2. But this turns out not to be the case. The thermal conductivity of an alloy of two metals is usually much lower than that of either metal, as shown in Table 16–2. Even small amounts in a pure metal of "foreign" molecules that are good conductors themselves seriously disrupt the transfer of heat in that metal. For example, the thermal conductivity of steel containing just 1 percent of chrome is 62 W/m·K, while the thermal conductivities of iron and chromium are 83 and 95 W/m·K, respectively.

The thermal conductivities of materials vary with temperature (Table 16–3). The variation of thermal conductivity over certain temperature ranges is negligible for some materials but significant for others, as shown in Fig. 16–8. The thermal conductivities of certain solids exhibit dramatic increases at temperatures near absolute zero, when these solids become *superconductors*. For example, the conductivity of copper reaches a maximum value of about 20,000 W/m·K at 20 K, which is about 50 times the conductivity at room temperature. The thermal conductivities and other thermal properties of various materials are given in Tables A–15 to A–25.

As shown in Table 16–1 and Fig. 16–8, most liquids have relatively low thermal conductivities when compared to the thermal conductivity of solids. The thermal conductivity of commonly used heat transfer fluids (such as water or oil) can be considerably increased by adding small solid particles to the fluid. The name **nanofluid** refers to the suspension of nanometer-sized particles (with typical length scales of 1 to 100 nm), called nanoparticles, in conventional fluids. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene glycol, and oil. Nanofluids have been shown to enhance the thermal conductivity and convective heat transfer performance of the working (base) liquids.

Values of thermal conductivity given in Tables A–15 to A–25 are appropriate when the physical dimensions of the material under consideration are relatively

TABLE 16–2

The thermal conductivity of an alloy is usually much lower than the thermal conductivity of either metal of which it is composed

Pure metal or alloy	k, W/m·K, at 300 K
Copper	401
Nickel	91
Constantan (55% Cu, 45% Ni)	23
Copper	401
Aluminum	237
Commercial bronze (90% Cu, 10% Al)	52

TABLE 16–3

Thermal conductivities of materials vary with temperature

	k, W/m·K	
T, K	Copper	Aluminum
100	482	302
200	413	237
300	401	237
400	393	240
600	379	231
800	366	218

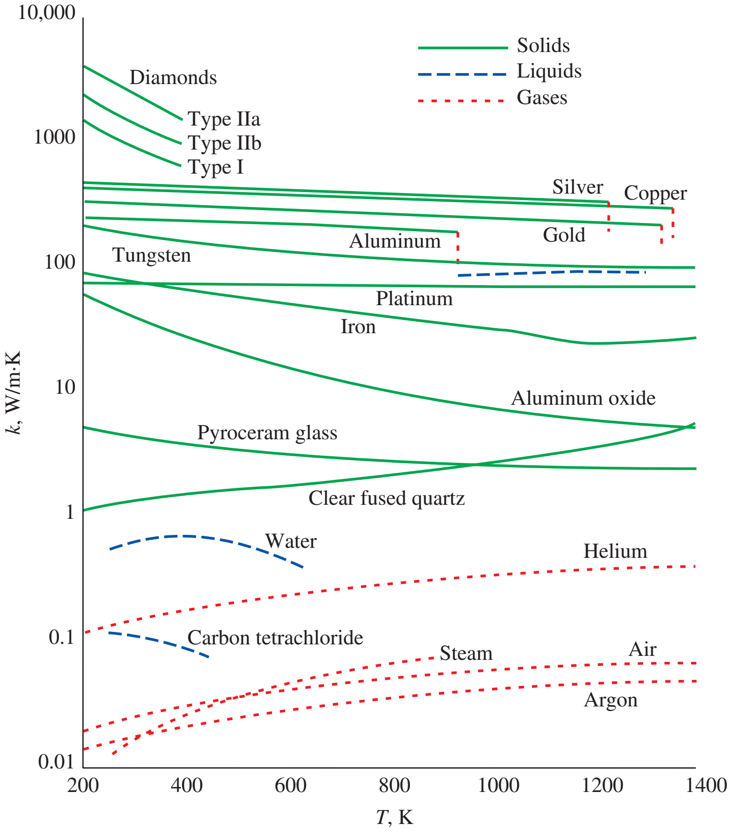


FIGURE 16–8
The variation of the thermal conductivity of various solids, liquids, and gases with temperature.

large. In some of the emerging areas of technology, such as microelectronics, the physical dimensions are typically in micro- or nanometers. For these applications, the small physical dimensions most likely will influence the value of thermal conductivity in the solid and liquid states. In these situations, as the physical dimensions decrease, the average net distance traveled by the energy carriers typically decreases, and this reduces the value of thermal conductivity.

The temperature dependence of thermal conductivity causes considerable complexity in conduction analysis. Therefore, it is common practice to evaluate the thermal conductivity k at the *average temperature* and treat it as a *constant* in calculations.

In heat transfer analysis, a material is normally assumed to be *isotropic*—that is, to have uniform properties in all directions. This assumption is realistic for most materials, except those that exhibit different structural characteristics in different directions, such as laminated composite materials and wood. The thermal conductivity of wood across the grain, for example, is different than that parallel to the grain.

Thermal Diffusivity

The product ρc_p, which is often encountered in heat transfer analysis, is called the **heat capacity** of a material. Both the specific heat c_p and the heat capacity ρc_p represent the heat storage capability of a material. But c_p expresses it *per unit mass,* whereas ρc_p expresses it *per unit volume,* as can be noticed from their units J/kg·K and J/m^3·K, respectively.

Another material property that appears in the transient heat conduction analysis is the **thermal diffusivity**, which represents how fast heat diffuses through a material and is defined as

$$\alpha = \frac{\text{Heat conduction}}{\text{Heat storage}} = \frac{k}{\rho c_p} \quad (\text{m}^2/\text{s}) \tag{16–3}$$

Note that the thermal conductivity k represents how well a material conducts heat, and the heat capacity ρc_p represents how much energy a material stores per unit volume. Therefore, the thermal diffusivity of a material can be viewed as the ratio of the *heat conducted* through the material to the *heat stored* per unit volume. A material that has a high thermal conductivity or a low heat capacity will obviously have a large thermal diffusivity. The larger the thermal diffusivity, the faster the propagation of heat into the medium. A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat is conducted further.

The thermal diffusivities of some common materials at 20°C are given in Table 16–4. Note that the thermal diffusivity ranges from $\alpha = 0.14 \times 10^{-6}$ m^2/s for water to 149×10^{-6} m^2/s for silver, which is a difference of more than a thousand times. Also note that the thermal diffusivities of beef and water are the same. This is not surprising, since meat as well as fresh vegetables and fruits are mostly water, and thus they possess the thermal properties of water.

TABLE 16–4

The thermal diffusivities of some materials at room temperature

Material	α, m^2/s*
Silver	149×10^{-6}
Gold	127×10^{-6}
Copper	113×10^{-6}
Aluminum	97.5×10^{-6}
Iron	22.8×10^{-6}
Mercury (*l*)	4.7×10^{-6}
Marble	1.2×10^{-6}
Ice	1.2×10^{-6}
Concrete	0.75×10^{-6}
Brick	0.52×10^{-6}
Heavy soil (dry)	0.52×10^{-6}
Glass	0.34×10^{-6}
Glass wool	0.23×10^{-6}
Water (*l*)	0.14×10^{-6}
Beef	0.14×10^{-6}
Wood (oak)	0.13×10^{-6}

*Multiply by 10.76 to convert to ft^2/s.

EXAMPLE 16–2 Measuring the Thermal Conductivity of a Material

A common way of measuring the thermal conductivity of a material is to sandwich an electric thermofoil heater between two identical samples of the material, as shown in Fig. 16–9. The thickness of the resistance heater, including its cover, which is made of thin silicon rubber, is usually less than 0.5 mm. A circulating fluid such as tap water keeps the exposed ends of the samples at constant temperature. The lateral surfaces of the samples are well insulated to ensure that heat transfer through the samples is one-dimensional. Two

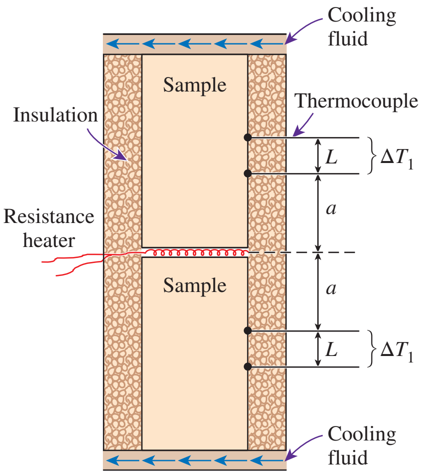


FIGURE 16–9

Apparatus to measure the thermal conductivity of a material using two identical samples and a thin resistance heater (Example 16–2).

thermocouples are embedded into each sample some distance L apart, and a differential thermometer reads the temperature drop ΔT across this distance along each sample. When steady operating conditions are reached, the total rate of heat transfer through both samples becomes equal to the electric power drawn by the heater.

In a certain experiment, cylindrical samples of diameter 5 cm and length 10 cm are used. The two thermocouples in each sample are placed 3 cm apart. After initial transients, the electric heater is observed to draw 0.4 A at 110 V, and both differential thermometers read a temperature difference of 15°C. Determine the thermal conductivity of the sample.

SOLUTION The thermal conductivity of a material is to be determined by ensuring one-dimensional heat conduction and by measuring temperatures when steady operating conditions are reached.

Assumptions **1** Steady operating conditions exist since the temperature readings do not change with time. **2** Heat losses through the lateral surfaces of the apparatus are negligible since those surfaces are well insulated, and thus all heat generated by the heater is conducted through the samples. **3** The apparatus possesses thermal symmetry.

Analysis The electrical power consumed by the resistance heater and converted to heat is

$$\dot{W}_e = \mathbf{V}I = (110\text{ V})(0.4\text{ A}) = 44\text{ W}$$

The rate of heat flow through each sample is

$$\dot{Q} = \frac{1}{2}\dot{W}_e = \frac{1}{2} \times (44\text{ W}) = 22\text{ W}$$

since only half of the heat generated flows through each sample because of symmetry. Reading the same temperature difference across the same distance in each sample also confirms that the apparatus possesses thermal symmetry. The heat transfer area is the area normal to the direction of heat transfer, which is the cross-sectional area of the cylinder in this case:

$$A = \frac{1}{4}\pi D^2 = \frac{1}{4}\pi(0.05\text{ m})^2 = 0.001963\text{ m}^2$$

Noting that the temperature drops by 15°C within 3 cm in the direction of heat flow, the thermal conductivity of the sample is determined to be

$$\dot{Q} = kA\frac{\Delta T}{L} \rightarrow k = \frac{\dot{Q}L}{A\,\Delta T} = \frac{(22\text{ W})(0.03\text{ m})}{(0.001963\text{ m}^2)(15°\text{C})} = \mathbf{22.4\ W/m{\cdot}K}$$

Discussion Perhaps you are wondering if we really need to use two samples in the apparatus, since the measurements on the second sample do not give any additional information. It seems like we can replace the second sample with insulation. Indeed, we do not need the second sample; however, it enables us to verify the temperature measurements on the first sample and provides thermal symmetry, which reduces experimental error.

EXAMPLE 16–3 Conversion Between SI and English Units

An engineer who is working on the heat transfer analysis of a brick building in English units needs the thermal conductivity of brick. But the only value he can find from his handbooks is 0.72 W/m·°C, which is in SI units. To make matters worse, the engineer does not have a direct conversion factor between the two unit systems for thermal conductivity. Can you help him out?

SOLUTION The situation this engineer is facing is not unique, and most engineers often find themselves in a similar position. When converting units, one must be careful to avoid some common pitfalls that lead to costly mistakes. Although unit conversion is a simple process, it requires care and careful reasoning.

The conversion factors for W and m are straightforward and are given in conversion tables to be

$$1\ \text{W} = 3.41214\ \text{Btu/h}$$
$$1\ \text{m} = 3.2808\ \text{ft}$$

But the conversion of °C into °F is not so simple, and it can be a source of error if one is not careful. Perhaps the first thought that comes to mind is to replace °C with (°F – 32)/1.8 since $T(°\text{C}) = [T(°\text{F}) - 32]/1.8$. But this will be wrong since the °C in the unit W/m·°C represents per *°C change in temperature.* Noting that 1°C change in temperature corresponds to 1.8°F, so the proper conversion factor is

$$1°\text{C} = 1.8°\text{F}$$

Substituting, we get

$$1\ \text{W/m}\cdot°\text{C} = \frac{3.41214\ \text{Btu/h}}{(3.2808\ \text{ft})(1.8°\text{F})} = 0.5778\ \text{Btu/h}\cdot\text{ft}\cdot°\text{F}$$

which is the desired conversion factor. Therefore, the thermal conductivity of the brick in English units is

$$\begin{aligned} k_{\text{brick}} &= 0.72\ \text{W/m}\cdot°\text{C} \\ &= 0.72 \times (0.5778\ \text{Btu/h}\cdot\text{ft}\cdot°\text{F}) \\ &= \mathbf{0.42\ Btu/h\cdot ft\cdot °F} \end{aligned}$$

Discussion Note that the thermal conductivity value of a material in English units is about half that in SI units (Fig. 16–10). Also note that we rounded the result to two significant digits (the same number in the original value) since expressing the result in more significant digits (such as 0.4160 instead of 0.42) would falsely imply a more accurate value than the original one.

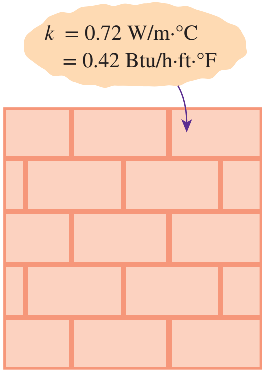


FIGURE 16–10
The thermal conductivity value in English units is obtained by multiplying the value in SI units by 0.5778.

16–3 ▪ CONVECTION

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of *conduction* and *fluid motion.* The faster the fluid motion, the greater the convection heat transfer. In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction. The presence of bulk motion of the fluid enhances the heat transfer between the solid surface and the fluid, but it also complicates the determination of heat transfer rates.

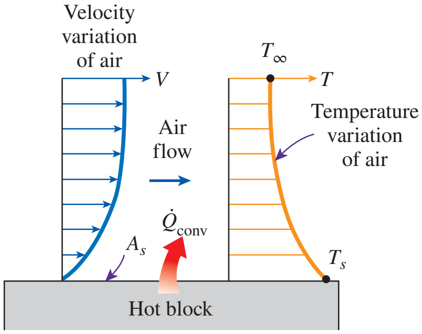


FIGURE 16–11
Heat transfer from a hot surface to air by convection.

Consider the cooling of a hot block by blowing cool air over its top surface (Fig. 16–11). Heat is first transferred to the air layer adjacent to the block by conduction. This heat is then carried away from the surface by convection, that is, by the combined effects of conduction within the air that is due to the random motion of air molecules and the bulk or macroscopic motion of the air that removes the heated air near the surface and replaces it with the cooler air.

Convection is called **forced convection** if the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind. In contrast, convection is called **natural** (or **free**) **convection** if the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid (Fig. 16–12). For example, in the absence of a fan, heat transfer from the surface of the hot block in Fig. 16–11 is by natural convection since any motion in the air in this case is due to the rise of the warmer (and thus lighter) air near the surface and the fall of the cooler (and thus heavier) air to fill its place. Heat transfer between the block and the surrounding air is by conduction if the temperature difference between the air and the block is not large enough to overcome the resistance of air to movement and thus to initiate natural convection currents.

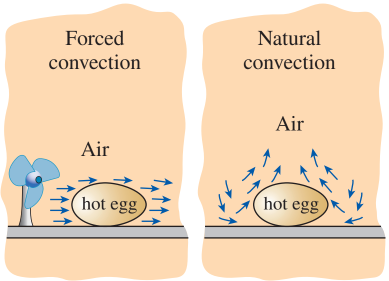


FIGURE 16–12
The cooling of a boiled egg by forced and natural convection.

TABLE 16–5

Typical values of convection heat transfer coefficient

Type of convection	h, W/m^2·K*
Free convection of gases	2–25
Free convection of liquids	10–1000
Forced convection of gases	25–250
Forced convection of liquids	50–20,000
Boiling and condensation	2500–100,000

*Multiply by 0.176 to convert to Btu/h·ft^2·°F.

Heat transfer processes that involve *change of phase* of a fluid are also considered to be convection because of the fluid motion induced during the process, such as the rise of the vapor bubbles during boiling or the fall of the liquid droplets during condensation.

Despite the complexity of convection, the rate of *convection heat transfer* is observed to be proportional to the temperature difference, and it is conveniently expressed by **Newton's law of cooling** as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \quad \text{(W)} \tag{16–4}$$

where h is the *convection heat transfer coefficient* in W/m^2·K or Btu/h·ft^2·°F, A_s is the surface area through which convection heat transfer takes place, T_s is the surface temperature, and T_∞ is the temperature of the fluid sufficiently far from the surface. Note that at the surface, the fluid temperature equals the surface temperature of the solid.

The convection heat transfer coefficient h is not a property of the fluid. It is an experimentally determined parameter whose value depends on all the variables influencing convection such as the surface geometry, the nature of fluid motion, the properties of the fluid, and the bulk fluid velocity. Typical values of h are given in Table 16–5.

Some people do not consider convection to be a fundamental mechanism of heat transfer since it is essentially heat conduction in the presence of fluid motion. But we still need to give this combined phenomenon a name, unless we are willing to keep referring to it as "conduction with fluid motion." Thus, it is practical to recognize convection as a separate heat transfer mechanism despite the valid arguments to the contrary.

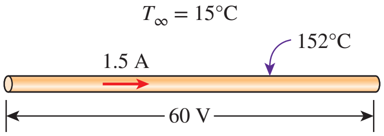


FIGURE 16–13
Schematic for Example 16–4.

EXAMPLE 16–4 Measuring Convection Heat Transfer Coefficient

A 2-m-long, 0.3-cm-diameter electrical wire extends across a room at 15°C, as shown in Fig. 16–13. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 152°C in steady operation. Also, the voltage drop and electric current through the wire are measured to be 60 V and 1.5 A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.

SOLUTION The convection heat transfer coefficient for heat transfer from an electrically heated wire to air is to be determined by measuring temperatures when steady operating conditions are reached and the electric power consumed.

Assumptions **1** Steady operating conditions exist since the temperature readings do not change with time. **2** Radiation heat transfer is negligible.

Analysis When steady operating conditions are reached, the rate of heat loss from the wire equals the rate of heat generation in the wire as a result of resistance heating. That is,

$$\dot{Q} = \dot{E}_{\text{generated}} = \mathbf{V}I = (60\text{ V})(1.5\text{ A}) = 90\text{ W}$$

The surface area of the wire is

$$A_s = \pi DL = \pi(0.003\text{ m})(2\text{ m}) = 0.01885\text{ m}^2$$

Newton's law of cooling for convection heat transfer is expressed as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

Disregarding any heat transfer by radiation and thus assuming all the heat loss from the wire occurs by convection, the convection heat transfer coefficient is determined to be

$$h = \frac{\dot{Q}_{\text{conv}}}{A_s(T_s - T_\infty)} = \frac{90\text{ W}}{(0.01885\text{ m}^2)(152 - 15)°\text{C}} = \mathbf{34.9\ W/m^2{\cdot}K}$$

Discussion Note that the simple setup described above can be used to determine the average heat transfer coefficients from a variety of surfaces in air. Also, heat transfer by radiation can be eliminated by keeping the surrounding surfaces at the temperature of the wire.

16–4 ▪ RADIATION

Radiation is the energy emitted by matter in the form of *electromagnetic waves* (or *photons*) as a result of the changes in the electronic configurations of the atoms or molecules. Unlike conduction and convection, the transfer of heat by radiation does not require the presence of an *intervening medium.* In fact, heat transfer by radiation is fastest (it occurs at the speed of light), and it suffers no attenuation in a vacuum. This is how the energy of the sun reaches the earth.

In heat transfer studies we are interested in *thermal radiation,* which is the form of radiation emitted by bodies because of their temperature. It differs from other forms of electromagnetic radiation such as X-rays, gamma rays, microwaves, radio waves, and television waves that are not related to temperature. All bodies at a temperature above absolute zero emit thermal radiation.

Radiation is a *volumetric phenomenon,* and all solids, liquids, and gases emit, absorb, or transmit radiation to varying degrees. However, radiation is usually considered to be a *surface phenomenon* for solids that are opaque to thermal radiation such as metals, wood, and rocks since the radiation emitted by the interior regions of such material can never reach the surface, and the radiation incident on such bodies is usually absorbed within a few microns from the surface.

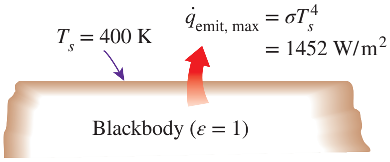


FIGURE 16–14
Blackbody radiation represents the *maximum amount of radiation that can be emitted from a surface at a specified temperature.*

The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature T_s (in K or R) is given by the **Stefan–Boltzmann law** as

$$\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4 \quad \text{(W)} \tag{16–5}$$

where $\sigma = 5.670 \times 10^{-8}$ W/m$^2{\cdot}$K^4 or 0.1714×10^{-8} Btu/h·ft$^2{\cdot}$R^4 is the *Stefan–Boltzmann constant.* The idealized surface that emits radiation at this maximum rate is called a **blackbody**, and the radiation emitted by a blackbody is called **blackbody radiation** (Fig. 16–14). The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature and is expressed as

$$\dot{Q}_{\text{emit}} = \varepsilon \sigma A_s T_s^4 \quad \text{(W)} \tag{16–6}$$

where ε is the **emissivity** of the surface. The property emissivity, whose value is in the range $0 \le \varepsilon \le 1$, is a measure of how closely a surface approximates a blackbody for which $\varepsilon = 1$. The emissivities of some surfaces are given in Table 16–6.

Another important radiation property of a surface is its **absorptivity** α, which is the fraction of the radiation energy incident on a surface that is absorbed by the surface. Like emissivity, its value is in the range $0 \le \alpha \le 1$. A blackbody absorbs all radiation incident on it. That is, a blackbody is a perfect absorber ($\alpha = 1$) as well as a perfect emitter.

In general, both ε and α of a surface depend on the temperature and the wavelength of the radiation. **Kirchhoff's law** of radiation states that the emissivity and

TABLE 16–6
Emissivities of some materials at 300 K

Material	Emissivity
Aluminum foil	0.07
Anodized aluminum	0.82
Polished copper	0.03
Polished gold	0.03
Polished silver	0.02
Polished stainless steel	0.17
Black paint	0.98
White paint	0.90
White paper	0.92–0.97
Asphalt pavement	0.85–0.93
Red brick	0.93–0.96
Human skin	0.95
Wood	0.82–0.92
Soil	0.93–0.96
Water	0.96
Vegetation	0.92–0.96

the absorptivity of a surface at a given temperature and wavelength are equal. In many practical applications, the surface temperature and the temperature of the source of incident radiation are of the same order of magnitude, and the average absorptivity of a surface is taken to be equal to its average emissivity. The rate at which a surface absorbs radiation is determined from (Fig. 16–15)

$$\dot{Q}_{\text{absorbed}} = \alpha \dot{Q}_{\text{incident}} \quad \text{(W)} \tag{16–7}$$

where $\dot{Q}_{\text{incident}}$ is the rate at which radiation is incident on the surface and α is the absorptivity of the surface. For opaque (nontransparent) surfaces, the portion of incident radiation not absorbed by the surface is reflected back.

The difference between the rates of radiation emitted by the surface and the radiation absorbed is the *net* radiation heat transfer. If the rate of radiation absorption is greater than the rate of radiation emission, the surface is said to be *gaining* energy by radiation. Otherwise, the surface is said to be *losing* energy by radiation. In general, the determination of the net rate of heat transfer by radiation between two surfaces is a complicated matter since it depends on the properties of the surfaces, their orientation relative to each other, and the interaction of the medium between the surfaces with radiation.

When a surface of emissivity ε and surface area A_s at a *thermodynamic temperature* T_s is *completely enclosed* by a much larger (or black) surface at thermodynamic temperature T_{surr} separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by (Fig. 16–16)

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4) \quad \text{(W)} \tag{16–8}$$

In this special case, the emissivity and the surface area of the surrounding surface do not have any effect on the net radiation heat transfer.

Radiation heat transfer to or from a surface surrounded by a gas such as air occurs *parallel* to conduction (or convection, if there is bulk gas motion) between the surface and the gas. Thus the total heat transfer is determined by *adding* the contributions of both heat transfer mechanisms. For simplicity and convenience, this is often done by defining a **combined heat transfer coefficient** h_{combined} that includes the effects of both convection and radiation. Then the *total* heat transfer rate to or from a surface by convection and radiation is expressed as

$$\begin{aligned} \dot{Q}_{\text{total}} &= \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = h_{\text{conv}} A_s(T_s - T_{\text{surr}}) + \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4) \\ \dot{Q}_{\text{total}} &= h_{\text{combined}} A_s(T_s - T_{\text{surr}}) \quad \text{(W)} \\ h_{\text{combined}} &= h_{\text{conv}} + h_{\text{rad}} = h_{\text{conv}} + \varepsilon\sigma(T_s + T_{\text{surr}})(T_s^2 + T_{\text{surr}}^2) \end{aligned} \tag{16–9}$$

Note that the combined heat transfer coefficient is essentially a convection heat transfer coefficient modified to include the effects of radiation.

Radiation is usually significant relative to conduction or natural convection, but negligible relative to forced convection. Thus radiation in forced convection applications is usually disregarded, especially when the surfaces involved have low emissivities and low to moderate temperatures.

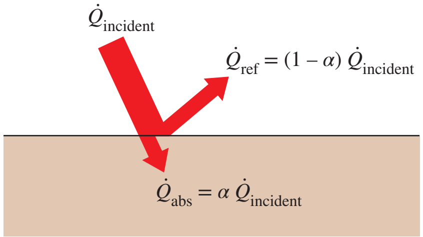


FIGURE 16–15
The absorption of radiation incident on an opaque surface of absorptivity α.

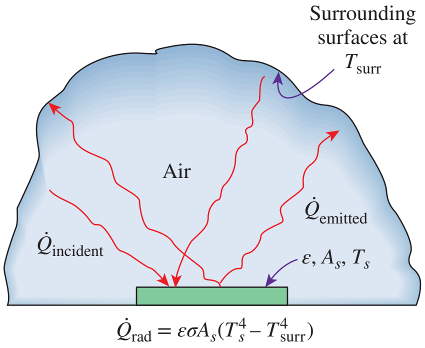


FIGURE 16–16
Radiation heat transfer between a surface and the surfaces surrounding it.

EXAMPLE 16–5 Radiation Effect on Thermal Comfort

It is a common experience to feel chilly in winter and warm in summer in our homes even when the thermostat setting is kept the same. This is due to the so-called radiation effect resulting from radiation heat exchange between our bodies and the surrounding surfaces of the walls and the ceiling.

Consider a person standing in a room maintained at 22°C at all times. The inner surfaces of the walls, floors, and the ceiling of the house are at an average temperature of 10°C in winter and 25°C in summer. Determine the rate of radiation heat transfer between this person and the surrounding surfaces if the exposed surface area and the average outer surface temperature of the person are 1.4 m^2 and 30°C, respectively (Fig. 16–17).

SOLUTION The rates of radiation heat transfer between a person and the surrounding surfaces at specified temperatures are to be determined in summer and winter.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by convection is not considered. **3** The person is completely surrounded by the interior surfaces of the room. **4** The surrounding surfaces are at a uniform temperature.

Properties The emissivity of a person is $\varepsilon = 0.95$ (Table 16–6).

Analysis The net rates of radiation heat transfer from the body to the surrounding walls, ceiling, and floor in winter and summer are

$$\begin{aligned}\dot{Q}_{\text{rad, winter}} &= \varepsilon\sigma A_s(T_s^4 - T_{\text{surr, winter}}^4)\\ &= (0.95)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.4\ \text{m}^2)\\ &\quad\times[(30+273)^4-(10+273)^4]\ \text{K}^4\\ &= \mathbf{152\ W}\end{aligned}$$

and

$$\begin{aligned}\dot{Q}_{\text{rad, summer}} &= \varepsilon\sigma A_s(T_s^4 - T_{\text{surr, summer}}^4)\\ &= (0.95)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.4\ \text{m}^2)\\ &\quad\times[(30+273)^4-(25+273)^4]\ \text{K}^4\\ &= \mathbf{40.9\ W}\end{aligned}$$

Discussion Note that we must use *thermodynamic (i.e., absolute) temperatures* in radiation calculations. Also note that the rate of heat loss from the person by radiation is almost four times as large in winter as it is in summer, which explains the chill we feel in winter even if the thermostat setting is kept the same.

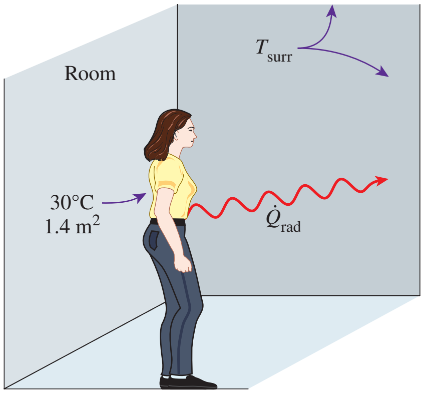


FIGURE 16–17
Schematic for Example 16–5.

16–5 ▪ SIMULTANEOUS HEAT TRANSFER MECHANISMS

We mentioned that there are three mechanisms of heat transfer, but not all three can exist simultaneously in a medium. For example, heat transfer is only by conduction in *opaque solids,* but by conduction and radiation in *semitransparent solids.* Thus, a solid may involve conduction and radiation but not convection. However, a solid may involve heat transfer by convection and/or radiation on its surfaces exposed to a fluid or other surfaces. For example, the outer surfaces of a cold piece of rock will warm up in a warmer environment as a result of heat gain by convection (from the air) and radiation (from the sun or the warmer surrounding surfaces). But the inner parts of the rock will warm up as this heat is transferred to the inner region of the rock by conduction.

Heat transfer is by conduction and possibly by radiation in a *still fluid* (no bulk fluid motion) and by convection and radiation in a *flowing fluid.* In the absence of radiation, heat transfer through a fluid is either by conduction or convection, depending on the presence of any bulk fluid motion. Convection can be viewed as combined conduction and fluid motion, and conduction in a fluid can be viewed as a special case of convection in the absence of any fluid motion (Fig. 16–18).

Thus, when we deal with heat transfer through a *fluid,* we have either *conduction* or *convection,* but not both. Also, gases are practically transparent to radiation, except that some gases are known to absorb radiation strongly at certain

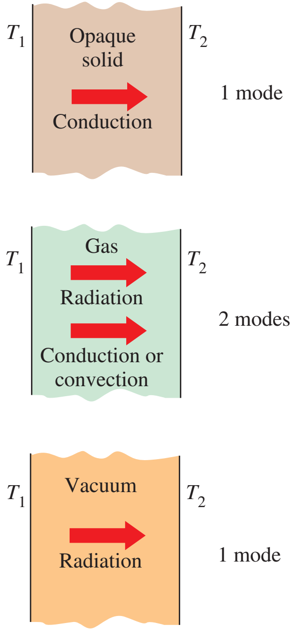


FIGURE 16–18
Although there are three mechanisms of heat transfer, a medium may involve only two of them simultaneously.

wavelengths. Ozone, for example, strongly absorbs ultraviolet radiation. But in most cases, a gas between two solid surfaces does not interfere with radiation, and it acts effectively as a vacuum. Liquids, on the other hand, are usually strong absorbers of radiation.

Finally, heat transfer through a *vacuum* is by radiation only since conduction or convection requires the presence of a material medium.

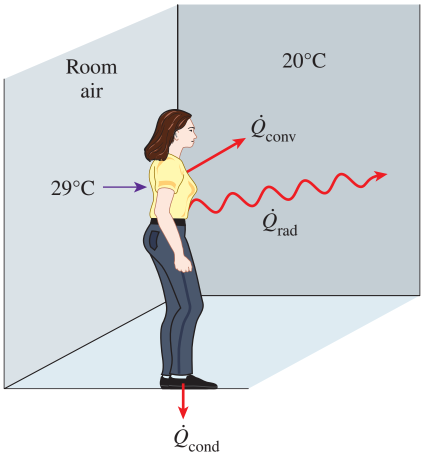


FIGURE 16–19
Heat transfer from the person described in Example 16–6.

EXAMPLE 16–6 Heat Loss from a Person

Consider a person standing in a breezy room at 20°C. Determine the total rate of heat transfer from this person if the exposed surface area and the average outer surface temperature of the person are 1.6 m^2 and 29°C, respectively, and the convection heat transfer coefficient is 6 $W/m^2 \cdot K$ (Fig. 16–19).

SOLUTION The total rate of heat transfer from a person by both convection and radiation to the surrounding air and surfaces at specified temperatures is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The person is completely surrounded by the interior surfaces of the room. **3** The surrounding surfaces are at the same temperature as the air in the room. **4** Heat conduction to the floor through the feet is negligible.

Properties The emissivity of a person is $\varepsilon = 0.95$ (Table 16–6).

Analysis The heat transfer between the person and the air in the room is by convection (instead of conduction) since it is conceivable that the air in the vicinity of the skin or clothing warms up and rises as a result of heat transfer from the body, initiating natural convection currents. It appears that the experimentally determined value for the rate of convection heat transfer in this case is 6 W per unit surface area (m^2) per unit temperature difference (in K or °C) between the person and the air away from the person. Thus, the rate of convection heat transfer from the person to the air in the room is

$$\begin{aligned}\dot{Q}_{\text{conv}} &= hA_s(T_s - T_\infty)\\ &= (6\ \text{W/m}^2\cdot\text{K})(1.6\ \text{m}^2)(29 - 20)^\circ\text{C}\\ &= 86.4\ \text{W}\end{aligned}$$

The person also loses heat by radiation to the surrounding wall surfaces. We take the temperature of the surfaces of the walls, ceiling, and floor to be equal to the air temperature in this case for simplicity, but we recognize that this does not need to be the case. These surfaces may be at a higher or lower temperature than the average temperature of the room air, depending on the outdoor conditions and the structure of the walls. Considering that air does not intervene with radiation and the person is completely enclosed by the surrounding surfaces, the net rate of radiation heat transfer from the person to the surrounding walls, ceiling, and floor is

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4)\\ &= (0.95)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.6\,\text{m}^2)\\ &\quad\times[(29+273)^4 - (20+273)^4]\text{K}^4\\ &= 81.7\ \text{W}\end{aligned}$$

Note that we must use *thermodynamic* temperatures in radiation calculations. Also note that we used the emissivity value for the skin and clothing at room temperature since the emissivity is not expected to change significantly at a slightly higher temperature.

Then the rate of total heat transfer from the body is determined by adding these two quantities:

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = (86.4 + 81.7)\ \text{W} \cong \mathbf{168\ W}$$

Discussion The heat transfer would be much higher if the person were not dressed since the exposed surface temperature would be higher. Thus, an important function of the clothes is to serve as a barrier against heat transfer.

In these calculations, heat transfer through the feet to the floor by conduction, which is usually very small, is neglected. Heat transfer from the skin by perspiration, which is the dominant mode of heat transfer in hot environments, is not considered here.

Also, the units W/m^2·°C and W/m^2·K for heat transfer coefficient are equivalent and can be interchanged.

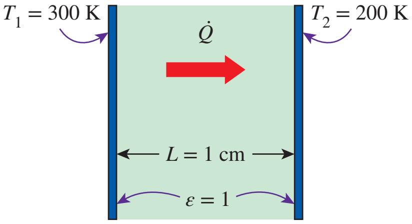


FIGURE 16–20
Schematic for Example 16–7.

EXAMPLE 16–7 Heat Transfer Between Two Isothermal Plates

Consider steady heat transfer between two large parallel plates at constant temperatures of $T_1 = 300$ K and $T_2 = 200$ K that are $L = 1$ cm apart, as shown in Fig. 16–20. Assuming the surfaces to be black (emissivity $\varepsilon = 1$), determine the rate of heat transfer between the plates per unit surface area, assuming the gap between the plates is (*a*) filled with atmospheric air, (*b*) evacuated, (*c*) filled with urethane insulation, and (*d*) filled with superinsulation that has an apparent thermal conductivity of 0.00002 W/m·K.

SOLUTION The total rate of heat transfer between two large parallel plates at specified temperatures is to be determined for four different cases.

Assumptions **1** Steady operating conditions exist. **2** There are no natural convection currents in the air between the plates. **3** The surfaces are black, and thus $\varepsilon = 1$.

Properties The thermal conductivity at the average temperature of 250 K is $k = 0.0219$ W/m·K for air (Table A–22), 0.026 W/m·K for urethane insulation, and 0.00002 W/m·K for the superinsulation.

Analysis (*a*) The rates of conduction and radiation heat transfer between the plates through the air layer are

$$\dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.0219\ \text{W/m·K})(1\ \text{m}^2)\frac{(300 - 200)\text{K}}{0.01\ \text{m}} = 219\ \text{W}$$

and

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A(T_1^4 - T_2^4)\\ &= (1)(5.67 \times 10^{-8}\ \text{W/m}^2\text{·K}^4)(1\ \text{m}^2)\left[(300\ \text{K})^4 - (200\ \text{K})^4\right] = 369\ \text{W}\end{aligned}$$

Therefore,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} + \dot{Q}_{\text{rad}} = 219 + 369 = \mathbf{588\ W}$$

The heat transfer rate in reality will be higher because of the natural convection currents that are likely to occur in the airspace between the plates.

(*b*) When the airspace between the plates is evacuated, there will be no conduction or convection, and the only heat transfer between the plates will be by radiation. Therefore,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{rad}} = \mathbf{369\ W}$$

(*c*) An opaque solid material placed between two plates blocks direct radiation heat transfer between the plates. Also, the thermal conductivity of an insulating material accounts for the radiation heat transfer that may be occurring through the voids in the insulating material. The rate of heat transfer through the urethane insulation is

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.026\ \text{W/m·K})(1\ \text{m}^2)\frac{(300 - 200)\ \text{K}}{0.01\ \text{m}} = \mathbf{260\ W}$$

Note that heat transfer through the urethane material is less than the heat transfer through the air determined in (*a*), although the thermal conductivity of the insulation is higher than that of air. This is because the insulation blocks the radiation, whereas air transmits it.

(*d*) The layers of the superinsulation prevent any direct radiation heat transfer between the plates. However, radiation heat transfer between the sheets of superinsulation does

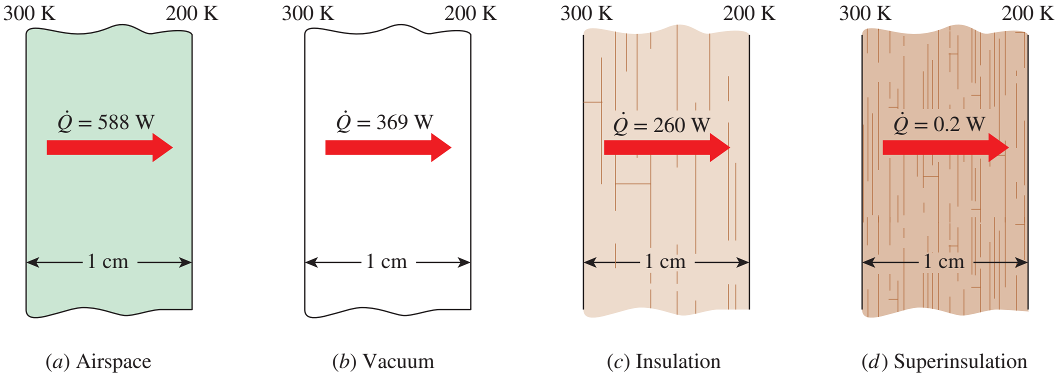


FIGURE 16–21
Different ways of reducing heat transfer between two isothermal plates, and their effectiveness.

occur, and the apparent thermal conductivity of the superinsulation accounts for this effect. Therefore,

$$\dot{Q}_{\text{total}} = kA\frac{T_1 - T_2}{L} = (0.00002\text{ W/m·K})(1\text{ m}^2)\frac{(300 - 200)\text{ K}}{0.01\text{ m}} = \mathbf{0.2\ W}$$

which is $\frac{1}{1845}$ of the heat transfer through the vacuum. The results of this example are summarized in Fig. 16–21 to put them into perspective.

Discussion This example demonstrates the effectiveness of superinsulations and explains why they are the insulation of choice in critical applications despite their high cost.

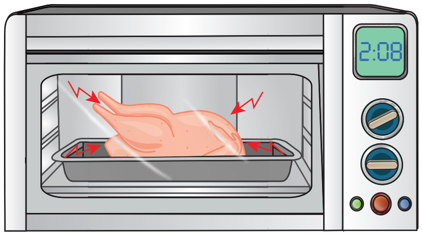


FIGURE 16–22
A chicken being cooked in a microwave oven (Example 16–8).

EXAMPLE 16–8 Heat Transfer in Conventional and Microwave Ovens

The fast and efficient cooking of microwave ovens has made them one of the essential appliances in modern kitchens (Fig. 16–22). Discuss the heat transfer mechanisms associated with the cooking of a chicken in microwave and conventional ovens, and explain why cooking in a microwave oven is more efficient.

SOLUTION Food is cooked in a microwave oven by absorbing the electromagnetic radiation energy generated by the microwave tube, called the magnetron. The radiation emitted by the magnetron is not thermal radiation since its emission is not due to the temperature of the magnetron; rather, it is due to the conversion of electrical energy into electromagnetic radiation at a specified wavelength. The wavelength of the microwave radiation is such that it is *reflected* by metal surfaces; *transmitted* by cookware made of glass, ceramic, or plastic; and *absorbed* and converted to internal energy by food (especially water, sugar, and fat) molecules.

In a microwave oven, the *radiation* that strikes the chicken is absorbed by the skin of the chicken and the outer parts. As a result, the temperature of the chicken at and near the skin rises. Heat is then *conducted* toward the inner parts of the chicken from its outer parts. Of course, some of the heat absorbed by the outer surface of the chicken is lost to the air in the oven by *convection.*

In a conventional oven, the air in the oven is first heated to the desired temperature by the electric or gas heating element. This preheating may take several minutes. The heat is then transferred from the air to the skin of the chicken by *natural convection* in older ovens or by *forced convection* in the newer convection ovens that utilize a fan. The air motion in convection ovens increases the convection heat transfer coefficient and thus decreases the cooking time. Heat is then *conducted* toward the inner parts of the chicken from its outer parts as in microwave ovens.

Microwave ovens replace the slow convection heat transfer process in conventional ovens with the instantaneous radiation heat transfer. As a result, microwave ovens transfer energy to the food at full capacity the moment they are turned on, and thus they cook faster while consuming less energy.

EXAMPLE 16–9 Heating of a Plate by Solar Energy

A thin metal plate is insulated on the back and exposed to solar radiation at the front surface (Fig. 16–23). The exposed surface of the plate has an absorptivity of 0.6 for solar radiation. If solar radiation is incident on the plate at a rate of 700 W/m^2 and the surrounding air temperature is 25°C, determine the surface temperature of the plate when the heat loss by convection and radiation equals the solar energy absorbed by the plate. Assume the combined convection and radiation heat transfer coefficient to be 50 W/m^2·K.

SOLUTION The back side of the thin metal plate is insulated, and the front side is exposed to solar radiation. The surface temperature of the plate is to be determined when it stabilizes.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the insulated side of the plate is negligible. **3** The heat transfer coefficient remains constant.

Properties The solar absorptivity of the plate is given to be $\alpha = 0.6$.

Analysis The absorptivity of the plate is 0.6, and thus 60 percent of the solar radiation incident on the plate is absorbed continuously. As a result, the temperature of the plate rises, and the temperature difference between the plate and the surroundings increases. This increasing temperature difference causes the rate of heat loss from the plate to the surroundings to increase. At some point, the rate of heat loss from the plate equals the rate of solar energy absorbed, and the temperature of the plate no longer changes. The temperature of the plate when steady operation is established is determined from

$$\dot{E}_{\text{gained}} = \dot{E}_{\text{lost}} \quad \text{or} \quad \alpha A_s \dot{q}_{\text{incident, solar}} = h_{\text{combined}} A_s (T_s - T_\infty)$$

Solving for T_s and substituting, the plate surface temperature is determined to be

$$T_s = T_\infty + \alpha \frac{\dot{q}_{\text{incident, solar}}}{h_{\text{combined}}} = 25°\text{C} + \frac{0.6 \times (700\ \text{W/m}^2)}{50\ \text{W/m}^2\cdot\text{K}} = \mathbf{33.4°C}$$

Discussion Note that the heat losses prevent the plate temperature from rising above 33.4°C. Also, the combined heat transfer coefficient accounts for the effects of both convection and radiation, and thus it is very convenient to use in heat transfer calculations when its value is known with reasonable accuracy.

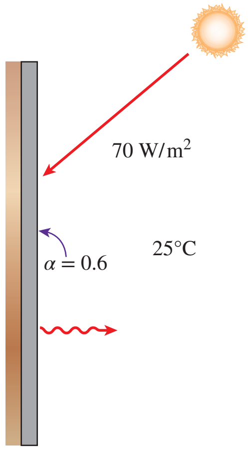


FIGURE 16–23
Schematic for Example 16–9.

SUMMARY

Heat can be transferred in three different modes: conduction, convection, and radiation. *Conduction* is the transfer of heat from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles and is expressed by *Fourier's law of heat conduction* as

$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx}$$

where k is the *thermal conductivity* of the material in W/m·K or Btu/h·ft·R, A is the *area* normal to the direction of heat transfer, and dT/dx is the *temperature gradient.* The magnitude of the rate of heat conduction across a plane layer of thickness L is given by

$$\dot{Q}_{\text{cond}} = kA\frac{\Delta T}{L}$$

where ΔT is the temperature difference across the layer.

Convection is the mode of heat transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion. The rate of convection heat transfer is expressed by *Newton's law of cooling* as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

where h is the *convection heat transfer coefficient* in W/m^2·K or Btu/h·ft^2·R, A_s is the *surface area* through which convection heat transfer takes place, T_s is the *surface temperature,* and T_∞ is the *temperature of the fluid* sufficiently far from the surface.

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules. The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature T_s is given by the *Stefan–Boltzmann law* as $\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4$, where $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4 or 0.1714×10^{-8} Btu/h·ft^2·R^4 is the *Stefan–Boltzmann constant.*

When a surface of emissivity ε and surface area A_s at a temperature T_s is completely enclosed by a much larger (or black) surface at a temperature T_{surr} separated by a gas (such as air) that does not block radiation, the net rate of radiation heat transfer between these two surfaces is given by

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s\,(T_s^4 - T_{\text{surr}}^4)$$

In this case, the emissivity (ε) and the surface area of the surrounding surface do not have any effect on the net radiation heat transfer.

The rate at which a surface absorbs radiation is determined from $\dot{Q}_{\text{absorbed}} = \alpha\dot{Q}_{\text{incident}}$ where $\dot{Q}_{\text{incident}}$ is the rate at which radiation is incident on the surface and α is the absorptivity of the surface.

REFERENCES AND SUGGESTED READING

1. Y. A. Çengel and A. J. Ghajar. *Heat and Mass Transfer: Fundamentals and Applications*, 6th ed. New York: McGraw-Hill, 2020.
2. Y. A. Çengel, M. A. Boles, and M. Kanoğlu. *Thermodynamics: An Engineering Approach.* 9th ed. New York: McGraw-Hill, 2019.
3. R. J. Ribando. *Heat Transfer Tools.* New York: McGraw-Hill, 2002.

PROBLEMS*

Heat Transfer Mechanisms

16–1C Define thermal conductivity, and explain its significance in heat transfer.

16–2C Which is a better heat conductor, diamond or silver?

16–3C How do the thermal conductivity of gases and liquids vary with temperature?

16–4C Why is the thermal conductivity of superinsulation orders of magnitude lower than the thermal conductivity of ordinary insulation?

16–5C Why do we characterize the heat conduction ability of insulators in terms of their apparent thermal conductivity instead of their ordinary thermal conductivity?

16–6C What are the mechanisms of heat transfer? How are they distinguished from each other?

16–7C Write down the expressions for the physical laws that govern each mode of heat transfer, and identify the variables involved in each relation.

16–8C How does heat conduction differ from convection?

16–9C Does any of the energy of the sun reach the earth by conduction or convection?

16–10C How does forced convection differ from natural convection?

16–11C What is the physical mechanism of heat conduction in a solid, a liquid, and a gas?

16–12C Consider heat transfer through a windowless wall of a house on a winter day. Discuss the parameters that affect the rate of heat conduction through the wall.

16–13C Consider heat loss through two walls of a house on a winter night. The walls are identical except that one of them has a tightly fitted glass window. Through which wall will the house lose more heat? Explain.

16–14C Consider two houses that are identical except that the walls are built using bricks in one house and wood in the other. If the walls of the brick house are twice as thick, which house do you think will be more energy efficient?

16–15C Consider two walls of a house that are identical except that one is made of 10-cm-thick wood while the other is made of 25-cm-thick brick. Through which wall will the house lose more heat in winter?

16–16C Define emissivity and absorptivity. What is Kirchhoff's law of radiation?

16–17C What is a blackbody? How do real bodies differ from blackbodies?

16–18 A wood slab with a thickness of 0.05 m is subjected to a heat flux of 40 W/m^2. The left and right surface temperatures of the wood slab are kept at constant temperatures of 40°C and 20°C, respectively. What is the thermal conductivity of the wood slab?

16–19 The inner and outer surfaces of a 4-m × 7-m brick wall of thickness 30 cm and thermal conductivity 0.69 W/m·K are maintained at temperatures of 20°C and 5°C, respectively. Determine the rate of heat transfer through the wall in W.

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*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

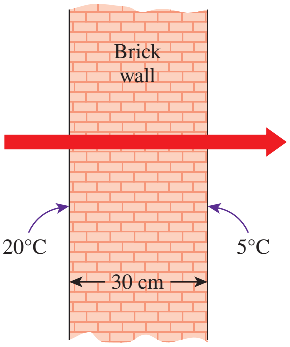


FIGURE P16–19

16–20E The inner and outer glasses of a 4-ft × 4-ft double-pane window are at 60°F and 48°F, respectively. If the 0.25-in space between the two glasses is filled with still air, determine the rate of heat transfer through the window. *Answer:* 131 Btu/h

16–21 The inner and outer surfaces of a 0.5-cm thick 2-m × 2-m window glass in winter are 10°C and 3°C, respectively. If the thermal conductivity of the glass is 0.78 W/m·K, determine the amount of heat loss through the glass over a period of 5 h. What would your answer be if the glass were 1 cm thick? *Answers:* 78.6 MJ, 39.3 MJ

16–22 Reconsider Prob. 16–21. Using appropriate software, plot the amount of heat loss through the glass as a function of the window glass thickness in the range of 0.1 cm to 1.0 cm. Discuss the results.

16–23 An aluminum pan whose thermal conductivity is 237 W/m·K has a flat bottom with diameter 15 cm and thickness 0.4 cm. Heat is transferred steadily to boiling water in the pan through its bottom at a rate of 800 W. If the inner surface of the bottom of the pan is at 105°C, determine the temperature of the outer surface of the bottom of the pan.

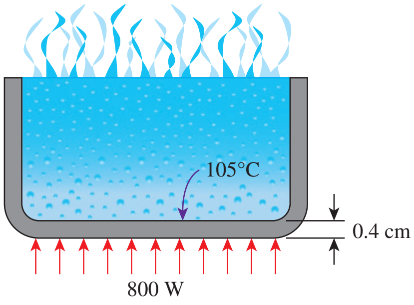


FIGURE P16–23

16–24E The north wall of an electrically heated home is 20 ft long, 10 ft high, and 1 ft thick and is made of brick whose thermal conductivity is $k = 0.42$ Btu/h·ft·°F. On a certain winter night, the temperatures of the inner and the outer surfaces of the wall are measured to be at about 62°F and 25°F, respectively, for a period of 8 h. Determine (*a*) the rate of heat loss through the wall that night and (*b*) the cost of that heat loss to the homeowner if the cost of electricity is \$0.07/kWh.

16–25 In a certain experiment, cylindrical samples of diameter 4 cm and length 7 cm are used (see Fig. 16–9). The two thermocouples in each sample are placed 3 cm apart. After initial transients, the electric heater is observed to draw 0.6 A at 110 V, and both differential thermometers read a temperature difference of 10°C. Determine the thermal conductivity of the sample. *Answer:* 78.8 W/m·K

16–26 One way of measuring the thermal conductivity of a material is to sandwich an electric thermofoil heater between two identical rectangular samples of the material and to heavily insulate the four outer edges, as shown in Fig. P16–26. Thermocouples attached to the inner and outer surfaces of the samples record the temperatures.

During an experiment, two 0.5-cm-thick samples 10 cm × 10 cm in size are used. When steady operation is reached, the heater is observed to draw 25 W of electric power, and the temperature of each sample is observed to drop from 82°C at the inner surface to 74°C at the outer surface. Determine the thermal conductivity of the material at the average temperature.

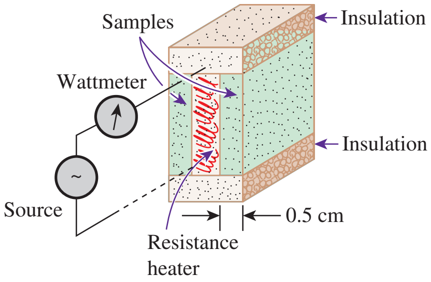


FIGURE P16–26

16–27 A concrete wall with a surface area of 20 m^2 and a thickness of 0.30 m separates conditioned room air from ambient air. The temperature of the inner surface of the wall (T_1) is maintained at 25°C. (*a*) Determine the heat loss $\dot{Q}$(W) through the concrete wall for three thermal conductivity values of 0.75, 1, and 1.25 W/m·K and outer wall surface temperatures of $T_2 = -15, -10, -5, 0, 5, 10, 15, 20, 25, 30,$ and 38°C (a total of 11 data points for each thermal conductivity value). Tabulate the results for all three cases in one table. Also provide a computer-generated graph [Heat loss, $\dot{Q}$(W) vs. Outside wall temperature, T_2(°C)] for the display of your results. The results for all three cases should be plotted on the same graph. (*b*) Discuss your results for the three cases.

16–28 A hollow spherical iron container with outer diameter 20 cm and thickness 0.2 cm is filled with iced water at 0°C. If the outer surface temperature is 5°C, determine the approximate rate of heat gain by the iced water in kW and the rate at which ice melts in the container. The heat of fusion of water is 333.7 kJ/kg. Treat the spherical shell as a plain wall, and use the outer area.

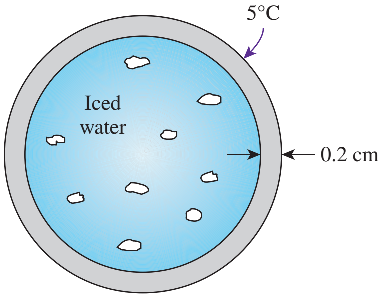


FIGURE P16–28

16–29 Reconsider Prob. 16–28. Using appropriate software, plot the rate at which ice melts as a function of the container thickness in the range of 0.1 cm to 1.0 cm. Discuss the results.

16–30E An engineer who is working on the heat transfer analysis of a house in English units needs the convection heat transfer coefficient on the outer surface of the house. But the only value he can find from his handbooks is 14 W/m^2·K, which is in SI units. The engineer does not have a direct conversion factor between the two unit systems for the convection heat transfer coefficient. Using the conversion factors between W and Btu/h, m and ft, and °C and °F, express the given convection heat transfer coefficient in Btu/h·ft^2·°F. *Answer:* 2.47 Btu/h·ft^2·°F

16–31 Air at 20°C with a convection heat transfer coefficient of 20 W/m^2·K blows over a pond. The surface temperature of the pond is at 40°C. Determine the heat flux between the surface of the pond and the air.

16–32 In a power plant, pipes transporting superheated vapor are very common. Superheated vapor is flowing at a rate of 0.3 kg/s inside a pipe 5 cm in diameter and 10 m in length. The pipe is located in a power plant at 20°C and has a uniform surface temperature of 100°C. If the temperature drop between the inlet and exit of the pipe is 30°C, and the specific heat of the vapor is 2190 J/kg·K, determine the heat transfer coefficient as a result of convection between the pipe surface and the surroundings.

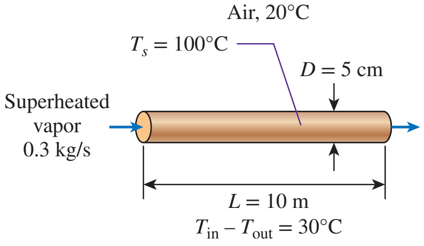


FIGURE P16–32

16–33 An electric current of 5 A passing through a resistor has a measured voltage of 6 V across the resistor. The resistor is cylindrical with a diameter of 2.5 cm and length of 15 cm. The resistor has a uniform temperature of 90°C, and the room air temperature is 20°C. Assuming that heat transfer by radiation is negligible, determine the heat transfer coefficient by convection.

16–34 A transistor with a height of 0.4 cm and a diameter of 0.6 cm is mounted on a circuit board. The transistor is cooled by air flowing over it with an average heat transfer coefficient of 30 W/m^2·K. If the air temperature is 55°C and the transistor case temperature is not to exceed 70°C, determine the amount of power this transistor can dissipate safely. Disregard any heat transfer from the transistor base.

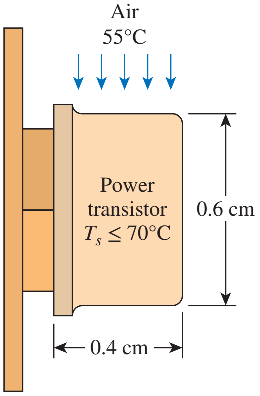


FIGURE P16–34

16–35E A 200-ft-long section of a steam pipe whose outer diameter is 4 in passes through an open space at 50°F. The average temperature of the outer surface of the pipe is measured to be 280°F, and the average heat transfer coefficient on that surface is determined to be 6 Btu/h·ft^2·°F. Determine (*a*) the rate of heat loss from the steam pipe and (*b*) the annual cost of this energy loss if steam is generated in a natural gas furnace having an efficiency of 86 percent and the price of natural gas is $1.10/therm (1 therm = 100,000 Btu). *Answers:* (*a*) 289,027 Btu/h, (*b*) $32,384/yr

16–36 The boiling temperature of nitrogen at atmospheric pressure at sea level (1 atm) is −196°C. Therefore, nitrogen is commonly used in low-temperature scientific studies since the temperature of liquid nitrogen in a tank open to the atmosphere remains constant at −196°C until the liquid nitrogen in the tank is depleted. Any heat transfer to the tank results in the evaporation of some liquid nitrogen, which has a heat of vaporization of 198 kJ/kg and a density of 810 kg/m^3 at 1 atm.

Consider a 4-m-diameter spherical tank initially filled with liquid nitrogen at 1 atm and −196°C. The tank is exposed to 20°C ambient air with a heat transfer coefficient of 25 W/m^2·K. The temperature of the thin-shelled spherical tank is observed to be almost the same as the temperature of the nitrogen inside. Disregarding any radiation heat exchange, determine the rate of evaporation of the liquid nitrogen in the tank as a result of the heat transfer from the ambient air.

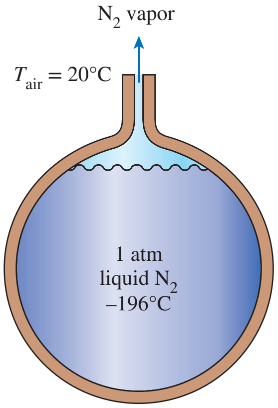


FIGURE P16–36

16–37 A 1.4-m-long, 0.2-cm-diameter electrical wire extends across a room that is maintained at 20°C. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 240°C in steady operation. Also, the voltage drop and electric current through the wire are measured to be 110 V and 3 A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room. *Answer:* 170.5 W/m²·K

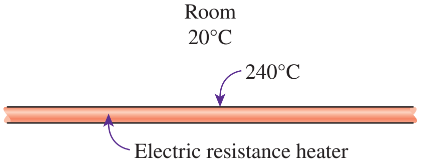


FIGURE P16–37

16–38 Reconsider Prob. 16–37. Using appropriate software, plot the convection heat transfer coefficient as a function of the wire surface temperature in the range of 100°C to 300°C. Discuss the results.

16–39E Using the conversion factors between W and Btu/h, m and ft, and K and R, express the Stefan–Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴ in the English unit Btu/h·ft²·R⁴.

16–40 The outer surface of a spacecraft in space has an emissivity of 0.8 and a solar absorptivity of 0.3. If solar radiation is incident on the spacecraft at a rate of 950 W/m², determine the surface temperature of the spacecraft when the radiation emitted equals the solar energy absorbed.

16–41 Consider a person whose exposed surface area is 1.7 m², emissivity is 0.5, and surface temperature is 32°C. Determine the rate of heat loss from that person by radiation in a large room having walls at a temperature of (*a*) 300 K and (*b*) 280 K. *Answers:* (*a*) 26.7 W, (*b*) 121 W

16–42 Consider a sealed 20-cm-high electronic box whose base dimensions are 40 cm × 40 cm placed in a vacuum chamber. The emissivity of the outer surface of the box is 0.95. If the electronic components in the box dissipate a total of 100 W of power and the outer surface temperature of the box is not to exceed 55°C, determine the temperature at which the surrounding surfaces must be kept if this box is to be cooled by radiation alone. Assume the heat transfer from the bottom surface of the box to the stand to be negligible.

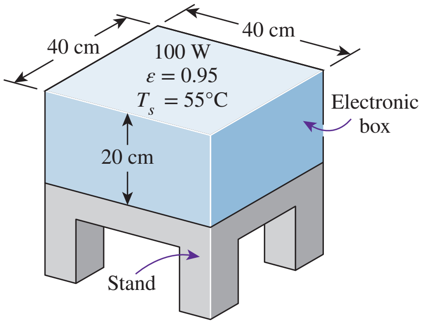


FIGURE P16–42

16–43 Two surfaces, one highly polished and the other heavily oxidized, are found to be emitting the same amount of energy per unit area. The highly polished surface has an emissivity of 0.1 at 1070°C, while the emissivity of the heavily oxidized surface is 0.78. Determine the temperature of the heavily oxidized surface.

16–44 A spherical interplanetary probe with a diameter of 2 m is sent out into the solar system. The probe surface is made of material having an emissivity of 0.9 and an absorptivity of 0.1. Signals from the sensors monitoring the probe surface temperatures are indicating an average value of −40°C for a space temperature of 0 K. If the electronics inside the probe are generating heat at a rate of 100 W/m³, determine the incident radiation rate on the probe surface.

16–45 An electronic package in the shape of a sphere with an outer diameter of 100 mm is placed in a large laboratory room. The surface emissivity of the package can assume three different values (0.2, 0.25, and 0.3). The walls of the room are maintained at a constant temperature of 77 K. The electronics in this package can only operate in the surface temperature range of $40°C \le T_s \le 85°C$. Determine the range of power dissipation ($\dot{W}$) for the electronic package over this temperature range for the three surface emissivity values (ε). Plot the results in terms of $\dot{W}$(W) vs. T_s(°C) for the three different values of emissivity over a surface temperature range of 40 to 85°C with temperature increments of 5°C (total of 10 data points for each ε value). Provide a computer-generated graph for the display of your results, and tabulate the data used for the graph. Comment on the results obtained.

Simultaneous Heat Transfer Mechanisms

16–46C Can all three modes of heat transfer occur simultaneously (in parallel) in a medium?

16–47C Can a medium involve (*a*) conduction and convection, (*b*) conduction and radiation, or (*c*) convection and radiation simultaneously? Give examples for the "yes" answers.

16–48C The deep human body temperature of a healthy person remains constant at 37°C while the temperature and the humidity of the environment change with time. Discuss the heat transfer mechanisms between the human body and the environment in both summer and winter, and explain how a person can keep cooler in summer and warmer in winter.

16–49C We often turn the fan on in summer to help us cool. Explain how a fan makes us feel cooler in the summer. Also explain why some people use ceiling fans also in winter.

16–50 Consider a 20-cm-thick granite wall with a thermal conductivity of 2.79 W/m·K. The temperature of the left surface is held constant at 50°C, whereas the right face is exposed to a flow of 22°C air with a convection heat transfer coefficient of 15 W/m²·K. Neglecting heat transfer by radiation, find the right wall surface temperature and the heat flux through the wall.

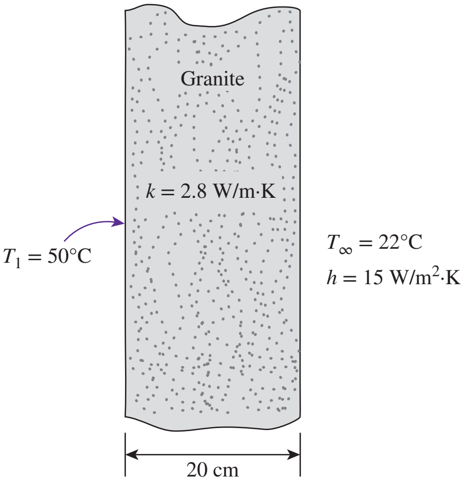


FIGURE P16–50

16–51 A solid plate, with a thickness of 15 cm and a thermal conductivity of 80 W/m·K, is being cooled at the upper surface by air. The air temperature is 10°C, while the temperatures at the upper and lower surfaces of the plate are 50 and 60°C, respectively. Determine the convection heat transfer coefficient of air at the upper surface, and discuss whether the value is reasonable or not for forced convection of air.

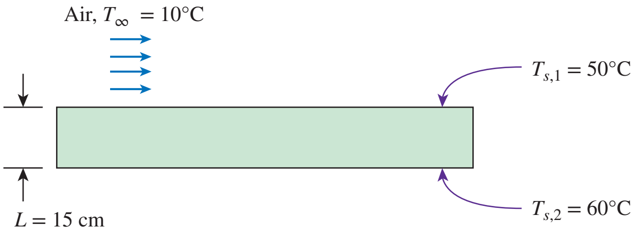


FIGURE P16–51

16–52 Air at 20°C with a convection heat transfer coefficient of 25 W/m²·K blows over a horizontal steel hot plate ($k = 43$ W/m·K). The surface area of the plate is 0.38 m² with a thickness of 2 cm. The plate surface is maintained at a constant temperature of $T_s = 250°C$, and the plate loses 300 W from its surface by radiation. Calculate the inside plate temperature (T_i).

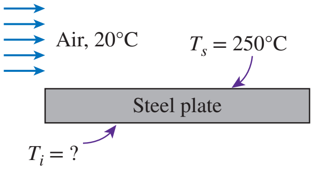


FIGURE P16–52

16–53 An electronic package with a surface area of 1 m² placed in an orbiting space station is exposed to space. The electronics in this package dissipate all 1 kW of its power to the space through its exposed surface. The exposed surface has an emissivity of 1.0 and an absorptivity of 0.25. Determine the steady-state exposed surface temperature of the electronic package (*a*) if the surface is exposed to a solar flux of 750 W/m², and (*b*) if the surface is not exposed to the sun.

16–54 Consider steady heat transfer between two large parallel plates at constant temperatures of $T_1 = 290$ K and $T_2 = 150$ K that are $L = 2$ cm apart. Assuming the surfaces to be black (emissivity $\varepsilon = 1$), determine the rate of heat transfer between the plates per unit surface area assuming the gap between the plates is (*a*) filled with atmospheric air, (*b*) evacuated, (*c*) filled with fiberglass insulation ($k = 0.036$ W/m·K), and (*d*) filled with superinsulation having an apparent thermal conductivity of 0.00015 W/m·K.

16–55 Consider a person standing in a room at 23°C. Determine the total rate of heat transfer from this person if the exposed surface area and the skin temperature of the person are 1.7 m² and 32°C, respectively, and the convection heat transfer coefficient is 5 W/m²·K. Take the emissivity of the skin and the clothes to be 0.9, and assume the temperature of the inner surfaces of the room to be the same as the air temperature.

16–56E A 2-in-diameter spherical ball whose surface is maintained at a temperature of 170°F is suspended in the middle of a room at 70°F. If the convection heat transfer coefficient is 15 Btu/h·ft²·°F and the emissivity of the surface is 0.8, determine the total rate of heat transfer from the ball.

16–57 A 1000-W iron is left on an ironing board with its base exposed to the air at 20°C. The convection heat transfer

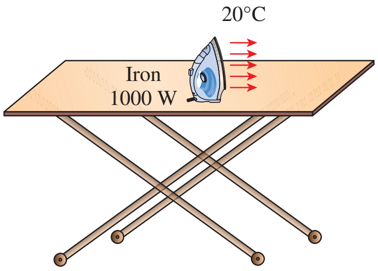


FIGURE P16–57

coefficient between the base surface and the surrounding air is 35 W/m²·K. If the base has an emissivity of 0.6 and a surface area of 0.02 m², determine the temperature of the base of the iron. *Answer:* 674°C

16–58 A 3-m-internal-diameter spherical tank made of 1-cm-thick stainless steel is used to store iced water at 0°C. The tank is located outdoors at 25°C. Assuming the entire steel tank to be at 0°C and thus the thermal resistance of the tank to be negligible, determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period. The heat of fusion of water at atmospheric pressure is $h_{if} = 333.7$ kJ/kg. The emissivity of the outer surface of the tank is 0.75, and the convection heat transfer coefficient on the outer surface can be taken to be 30 W/m²·K. Assume the average surrounding surface temperature for radiation exchange to be 15°C. *Answers:* (*a*) 23.1 kW, (*b*) 5980 kg

16–59 In the metal processing industry, heat treatment of metals is commonly done using electrically heated draw batch furnaces. Consider a furnace that is situated in a room with surrounding air temperature of 30°C and an average convection heat transfer coefficient of 12 W/m²·K. The furnace front is made of a steel plate with thickness of 20 mm and a thermal conductivity of 25 W/m·K. The outer furnace front surface has an emissivity of 0.23, and the inside surface is subjected to a heat flux of 8 kW/m². Determine the outside surface temperature of the furnace front.

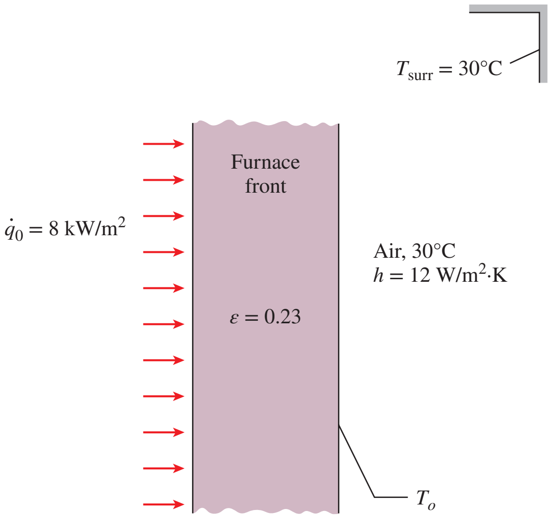


FIGURE P16–59

16–60 Solar radiation is incident on a 5-m² solar absorber plate surface at a rate of 800 W/m². Ninety-three percent of the solar radiation is absorbed by the absorber plate, while the remaining 7 percent is reflected away. The solar absorber plate has a surface temperature of 40°C with an emissivity of 0.9 that experiences radiation exchange with the surrounding temperature of −5°C. In addition, convective heat transfer occurs between the absorber plate surface and the ambient air of 20°C with a convection heat transfer coefficient of 7 W/m²·K. Determine the efficiency of the solar absorber, which is defined as the ratio of the usable heat collected by the absorber to the incident solar radiation on the absorber.

16–61 A flat-plate solar collector is used to heat water by having water flow through tubes attached at the back of the thin solar absorber plate. The absorber plate has a surface area of 2 m² with emissivity and absorptivity of 0.9. The surface temperature of the absorber is 35°C, and solar radiation is incident on the absorber at 500 W/m² with a surrounding temperature of 0°C. The convection heat transfer coefficient at the absorber surface is 5 W/m²·K, while the ambient temperature is 25°C. Net heat absorbed by the solar collector heats the water from an inlet temperature (T_{in}) to an outlet temperature (T_{out}). If the water flow rate is 5 g/s with a specific heat of 4.2 kJ/kg·K, determine the temperature rise of the water.

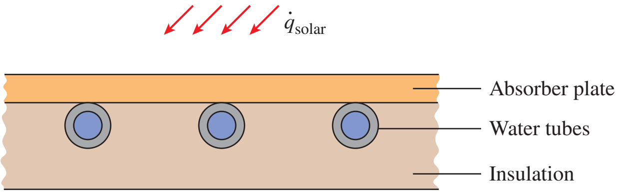


FIGURE P16–61

16–62E Consider a flat-plate solar collector placed horizontally on the flat roof of a house. The collector is 5 ft wide and 15 ft long, and the average temperature of the exposed surface of the collector is 100°F. The emissivity of the exposed surface of the collector is 0.9. Determine the rate of heat loss from the collector by convection and radiation during a calm day when the ambient air temperature is 70°F and the effective sky temperature for radiation exchange is 50°F. Take the convection heat transfer coefficient on the exposed surface to be 2.5 Btu/h·ft²·°F.

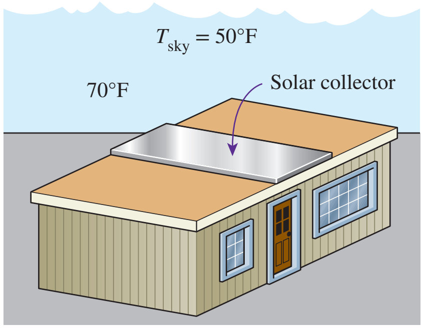


FIGURE P16–62E

16–63 An AISI 304 stainless steel sheet is going through an annealing process inside an electrically heated oven. The ambient air inside the oven is 600°C, while the surrounding surfaces of the oven are at a uniform temperature of 750°C. If the emissivity of the stainless steel sheet is 0.40 and the average convection heat transfer coefficient is 10 W/m²·K, determine the temperature of the stainless steel sheet.

16–64 The roof of a house consists of a 15-cm-thick concrete slab ($k = 2$ W/m·K) that is 15 m wide and 20 m long. The emissivity of the outer surface of the roof is 0.9, and the convection heat transfer coefficient on that surface is estimated to be 15 W/m^2·K. The inner surface of the roof is maintained at 15°C. On a clear winter night, the ambient air is reported to be at 10°C, while the night sky temperature for radiation heat transfer is 255 K. Considering both radiation and convection heat transfer, determine the outer surface temperature and the rate of heat transfer through the roof.

If the house is heated by a furnace burning natural gas with an efficiency of 85 percent, and the unit cost of natural gas is $0.60/therm (1 therm = 105,500 kJ of energy content), determine the money lost through the roof that night during a 14-h period.

16–65 The inner and outer surfaces of a 25-cm-thick wall in summer are at 27°C and 44°C, respectively. The outer surface of the wall exchanges heat by radiation with surrounding surfaces at 40°C and by convection with ambient air also at 40°C with a convection heat transfer coefficient of 8 W/m^2·K. Solar radiation is incident on the surface at a rate of 150 W/m^2. If both the emissivity and the solar absorptivity of the outer surface are 0.8, determine the effective thermal conductivity of the wall.

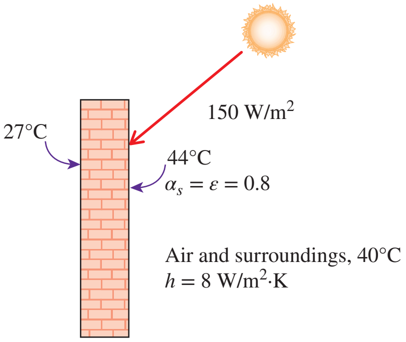


FIGURE P16–65

Review Problems

16–66 A cylindrical resistor element on a circuit board dissipates 0.8 W of power. The resistor is 2 cm long and has a diameter of 0.4 cm. Assuming heat to be transferred uniformly from all surfaces, determine (*a*) the amount of heat this resistor dissipates during a 24-h period, (*b*) the heat flux, and (*c*) the fraction of heat dissipated from the top and bottom surfaces.

16–67 The heat generated in the circuitry on the surface of a silicon chip ($k = 130$ W/m·K) is conducted to the ceramic substrate to which it is attached. The chip is 6 mm × 6 mm in size and 0.5 mm thick and dissipates 3 W of power. Disregarding any heat transfer through the 0.5-mm-high side surfaces, determine the temperature difference between the front and back surfaces of the chip in steady operation.

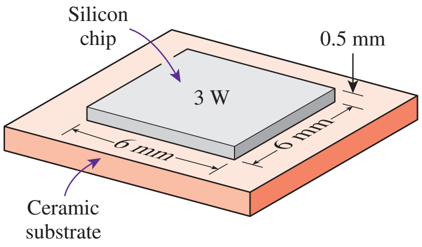


FIGURE P16–67

16–68 A 0.3-cm-thick, 12-cm-high, and 18-cm-long circuit board houses 80 closely spaced logic chips on one side, each dissipating 0.06 W. The board is impregnated with copper fillings and has an effective thermal conductivity of 16 W/m·K. All the heat generated in the chips is conducted across the circuit board and is dissipated from the back side of the board to the ambient air. Determine the temperature difference between the two sides of the circuit board. *Answer:* 0.042°C

16–69 A 40-cm-long, 800-W electric resistance heating element with diameter 0.5 cm and surface temperature 120°C is immersed in 75 kg of water initially at 20°C. Determine how long it will take for this heater to raise the water temperature to 80°C. Also, determine the convection heat transfer coefficients at the beginning and at the end of the heating process.

16–70 It is well known that wind makes the cold air feel much colder as a result of the *wind-chill effect* that is due to an increase in the convection heat transfer coefficient with increasing air velocity. The wind-chill effect is usually expressed in terms of the *wind-chill temperature* (WCT), which is the apparent temperature felt by exposed skin. For an outdoor air temperature of 0°C, for example, the wind-chill temperature is −5°C with 20 km/h winds and −9°C with 60 km/h winds. That is, a person exposed to 0°C windy air at 20 km/h will feel as cold as a person exposed to −5°C calm air (air motion under 5 km/h).

For heat transfer purposes, a standing man can be modeled as a 30-cm-diameter, 170-cm-long vertical cylinder with both the top and bottom surfaces insulated and with the side surface at an average temperature of 34°C. For a convection heat transfer coefficient of 15 W/m^2·K, determine the rate of heat loss from this man by convection in still air at 20°C. What will your answer be if the convection heat transfer coefficient is increased to 30 W/m^2·K as a result of winds? What is the wind-chill temperature in this case? *Answers:* 336 W, 672 W, 6°C

16–71 An engine block with a surface area measured to be 0.95 m^2 generates a power output of 50 kW with a net engine efficiency of 35 percent. The engine block operates inside a compartment at 157°C, and the average convection heat transfer coefficient is 50 W/m^2·K. If convection is the only heat transfer mechanism occurring, determine the engine block surface temperature. *Answer:* 841°C

16–72 Consider an electrical wire submerged in liquid water at atmospheric conditions. The wire has a diameter of 1 mm and a length of 15 cm. The current through the wire is increased until the water reaches a temperature of 100°C. For this situation (boiling water) use an average value of the upper and lower values of the convection heat transfer coefficients given in Table 16–5. The wire surface temperature is to be maintained at 115°C. How much electric power is required for this case?

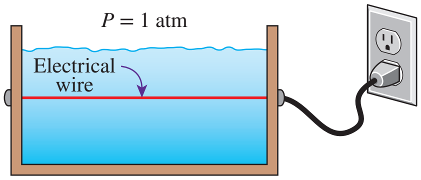


FIGURE P16–72

16–73 A cylindrical fuel rod 2 cm in diameter is encased in a concentric tube and cooled by water. The fuel generates heat uniformly at a rate of 150 MW/m^3. The convection heat transfer coefficient on the fuel rod is 5000 W/m^2·K, and the average temperature of the cooling water, sufficiently far from the fuel rod, is 70°C. Determine the surface temperature of the fuel rod, and discuss whether the value of the given convection heat transfer coefficient on the fuel rod is reasonable.

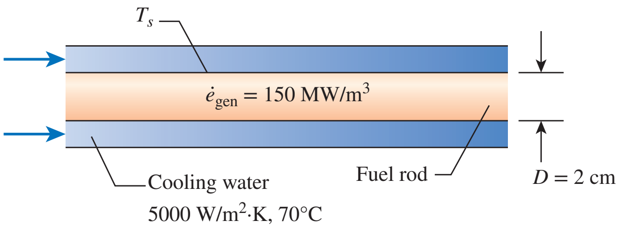


FIGURE P16–73

16–74 Consider a person standing in a room maintained at 20°C at all times. The inner surfaces of the walls, floors, and ceiling of the house are observed to be at an average temperature of 12°C in winter and 23°C in summer. Determine the rates of radiation heat transfer between this person and the surrounding surfaces in both summer and winter if the exposed surface area, emissivity, and the average outer surface temperature of the person are 1.6 m^2, 0.95, and 32°C, respectively.

16–75 Consider a 3-m × 3-m × 3-m cubical furnace whose top and side surfaces closely approximate black surfaces at a temperature of 1200 K. The base surface has an emissivity of $\varepsilon = 0.7$ and is maintained at 800 K. Determine the net rate of radiation heat transfer to the base surface from the top and side surfaces. *Answer:* 594 kW

16–76 A soldering iron has a cylindrical tip of 2.5 mm in diameter and 20 mm in length. With age and usage, the tip has oxidized and has an emissivity of 0.80. Assuming that the average convection heat transfer coefficient over the soldering iron tip is 25 W/m^2·K and the surrounding air temperature is 20°C, determine the power required to maintain the tip at 400°C.

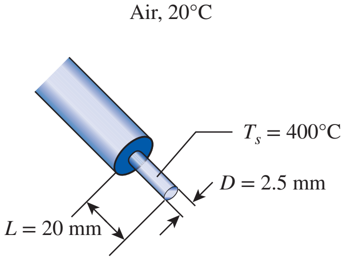


FIGURE P16–76

16–77 A thin metal plate is insulated on the back and exposed to solar radiation on the front surface. The exposed surface of the plate has an absorptivity of 0.7 for solar radiation. If solar radiation is incident on the plate at a rate of 550 W/m^2 and the surrounding air temperature is 10°C, determine the surface temperature of the plate when the heat loss by convection equals the solar energy absorbed by the plate. Take the convection heat transfer coefficient to be 25 W/m^2·K, and disregard any heat loss by radiation.

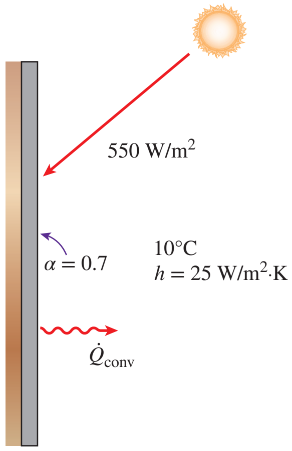


FIGURE P16–77

16–78 Consider a flat-plate solar collector placed on the roof of a house. The temperatures at the inner and outer surfaces of the glass cover are measured to be 28°C and 25°C, respectively. The glass cover has a surface area of 2.5 m^2, a thickness of 0.6 cm, and a thermal conductivity of 0.7 W/m·K. Heat is lost from the outer surface of the cover by convection and radiation with a convection heat transfer coefficient of 10 W/m^2·K and an ambient temperature of 15°C. Determine the fraction of heat lost from the glass cover by radiation.

16–79 An electric heater with the total surface area of 0.25 m^2 and emissivity 0.75 is in a room where the air has a temperature of 20°C and the walls are at 10°C. When the heater consumes 500 W of electric power, its surface has a steady temperature of 120°C. Determine the temperature of the heater surface when it consumes 700 W. Solve the problem (*a*) assuming negligible radiation and (*b*) taking radiation into consideration. Based on your results, comment on the assumption made in part (*a*).

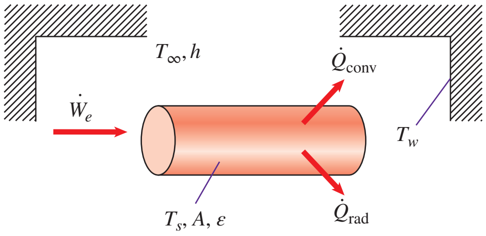


FIGURE P16–79

Design and Essay Problems

16–80 Write an essay on how microwave ovens work, and explain how they cook much faster than conventional ovens. Discuss whether conventional electric or microwave ovens consume more electricity for the same task.

16–81 Using information from the utility bills for the coldest month last year, estimate the average rate of heat loss from your house for that month. In your analysis, consider the contribution of the internal heat sources such as people, lights, and appliances. Identify the primary sources of heat loss from your house, and propose ways of improving the energy efficiency of your house.

16–82 Conduct this experiment to determine the combined heat transfer coefficient between an incandescent lightbulb and the surrounding air and surfaces using a 60-W lightbulb. You will need a thermometer, which can be purchased in a hardware store, and metal glue. You will also need a piece of string and a ruler to calculate the surface area of the lightbulb. First, measure the air temperature in the room, and then glue the tip of the thermocouple wire of the thermometer to the glass of the lightbulb. Turn the light on and wait until the temperature reading stabilizes. The temperature reading will give the surface temperature of the lightbulb. Assuming 10 percent of the rated power of the bulb is converted to light and is transmitted by the glass, calculate the heat transfer coefficient from Newton's law of cooling.

CHAPTER

17

STEADY HEAT CONDUCTION

In heat transfer analysis, we are often interested in the rate of heat transfer through a medium under steady conditions and surface temperatures. Such problems can be solved easily without involving any differential equations by the introduction of the *thermal resistance concept* in a manner analogous to electrical circuit problems. In this case, the thermal resistance corresponds to electrical resistance, temperature difference corresponds to voltage, and the heat transfer rate corresponds to electric current.

We start this chapter with *one-dimensional steady heat conduction* in a plane wall, a cylinder, and a sphere, and we develop relations for *thermal resistances* in these geometries. We also develop thermal resistance relations for convection and radiation conditions at the boundaries. We apply this concept to heat conduction problems in *multilayer* plane walls, cylinders, and spheres and generalize it to systems that involve heat transfer in two or three dimensions. We also discuss the *thermal contact resistance* and the *overall heat transfer coefficient* and develop relations for the critical radius of insulation for a cylinder and a sphere. Finally, we discuss steady heat transfer from *finned surfaces.*

OBJECTIVES

The objectives of this chapter are to:

- Understand the concept of thermal resistance and its limitations, and develop thermal resistance networks for practical heat conduction problems.
- Solve steady conduction problems that involve multilayer rectangular, cylindrical, or spherical geometries.
- Develop an intuitive understanding of thermal contact resistance and circumstances under which it may be significant.
- Identify applications in which insulation may actually increase heat transfer.
- Analyze finned surfaces, and assess how efficiently and effectively fins enhance heat transfer.

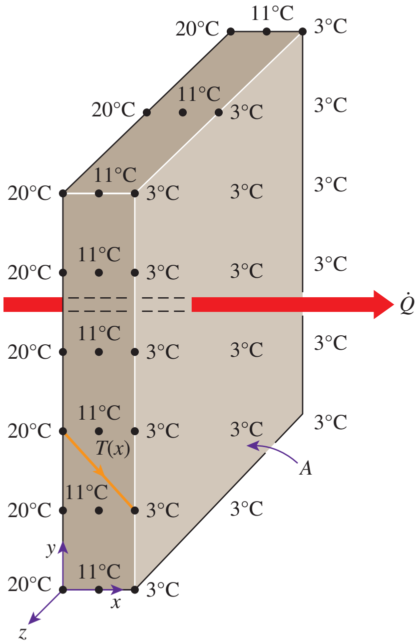


FIGURE 17–1
Heat transfer through a wall is one-dimensional when the temperature of the wall varies in one direction only.

17–1 ▪ STEADY HEAT CONDUCTION IN PLANE WALLS

Consider steady heat conduction through the walls of a house during a winter day. We know that heat is continuously lost to the outdoors through the wall. We intuitively feel that heat transfer through the wall is in the *normal direction* to the wall surface, and no significant heat transfer takes place in the wall in other directions (Fig. 17–1).

Recall that heat transfer in a certain direction is driven by the *temperature gradient* in that direction. There is no heat transfer in a direction in which there is no change in temperature. Temperature measurements at several locations on the inner or outer wall surface will confirm that a wall surface is nearly *isothermal.* That is, the temperatures at the top and bottom of a wall surface as well as at the right and left ends are almost the same. Therefore, there is no heat transfer through the wall from the top to the bottom, or from left to right, but there is considerable temperature difference between the inner and the outer surfaces of the wall, and thus significant heat transfer in the direction from the inner surface to the outer one.

The small thickness of the wall causes the temperature gradient in that direction to be large. Further, if the air temperatures inside and outside the house remain constant, then heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional.* The temperature of the wall in this case depends on one direction only (say the x-direction) and can be expressed as $T(x)$.

Noting that heat transfer is the only energy interaction involved in this case and there is no heat generation, the *energy balance* for the wall can be expressed as

$$\begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{pmatrix}$$

or

$$\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \frac{dE_{\text{wall}}}{dt} \qquad \textbf{(17–1)}$$

But $dE_{\text{wall}}/dt = 0$ for *steady* operation, since there is no change in the temperature of the wall with time at any point. Therefore, the rate of heat transfer into the wall must be equal to the rate of heat transfer out of it. In other words, *the rate of heat transfer through the wall must be constant,* $\dot{Q}_{\text{cond, wall}} = \text{constant}$.

Consider a plane wall of thickness L and average thermal conductivity k. The two surfaces of the wall are maintained at constant temperatures of T_1 and T_2. For one-dimensional steady heat conduction through the wall, we have $T(x)$. Then Fourier's law of heat conduction for the wall can be expressed as

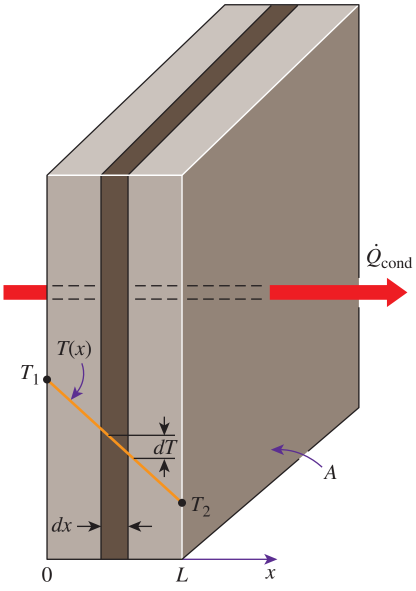


FIGURE 17–2
Under steady conditions, the temperature distribution in a plane wall is a straight line.

$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx} \quad \text{(W)} \qquad \textbf{(17–2)}$$

where the rate of conduction heat transfer $\dot{Q}_{\text{cond, wall}}$ and the wall area A are constant. Thus $dT/dx = \text{constant}$, which means that *the temperature through the wall varies linearly with* x. That is, the temperature distribution in the wall under steady conditions is a *straight line* (Fig. 17–2).

Separating the variables in the preceding equation and integrating from $x = 0$, where $T(0) = T_1$, to $x = L$, where $T(L) = T_2$, we get

$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

Performing the integrations and rearranging gives

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L} \quad \text{(W)} \tag{17–3}$$

which is identical to Eq. 16–1. Again, *the rate of heat conduction through a plane wall is proportional to the average thermal conductivity, the wall area, and the temperature difference, but is inversely proportional to the wall thickness.* Also, once the rate of heat conduction is available, the temperature $T(x)$ at any location x can be determined by replacing T_2 in Eq. 17–3 with T and L with x.

Thermal Resistance Concept

Equation 17–3 for heat conduction through a plane wall can be rearranged as

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \quad \text{(W)} \tag{17–4}$$

where

$$R_{\text{wall}} = \frac{L}{kA} \quad \text{(K/W)} \tag{17–5}$$

is the *thermal resistance* of the wall against heat conduction or simply the **conduction resistance** of the wall. Note that the thermal resistance of a medium depends on the *geometry* and the *thermal properties* of the medium. Note that thermal resistance can also be expressed as $R_{\text{wall}} = \Delta T/\dot{Q}_{\text{cond, wall}}$, which is the ratio of the driving potential ΔT to the corresponding transfer rate $\dot{Q}_{\text{cond, wall}}$.

This equation for heat transfer is analogous to the relation for *electric current flow I,* expressed as

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e} \tag{17–6}$$

where $R_e = L/\sigma_e A$ is the *electrical resistance* and $\mathbf{V}_1 - \mathbf{V}_2$ is the *voltage difference* across the resistance (σ_e is the electrical conductivity). Thus, the *rate of heat transfer* through a layer corresponds to the *electric current,* the *thermal resistance* corresponds to *electrical resistance,* and the *temperature difference* corresponds to *voltage difference* across the layer (Fig. 17–3).

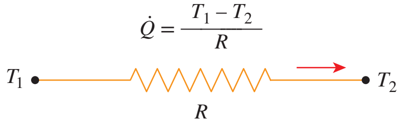
Consider convection heat transfer from a solid surface of area A_s and temperature T_s to a fluid whose temperature sufficiently far from the surface is T_∞, with a convection heat transfer coefficient h. Newton's law of cooling for convection heat transfer rate $\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$ can be rearranged as

$$\dot{Q}_{\text{conv}} = \frac{T_s - T_\infty}{R_{\text{conv}}} \quad \text{(W)} \tag{17–7}$$

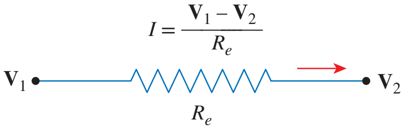
where

$$R_{\text{conv}} = \frac{1}{hA_s} \quad \text{(K/W)} \tag{17–8}$$

is the *thermal resistance* of the surface against heat convection, or simply the **convection resistance** of the surface (Fig. 17–4). Note that when the convection heat transfer coefficient is very large ($h \to \infty$), the convection resistance becomes *zero* and $T_s \approx T_\infty$. That is, the surface offers *no resistance to convection,* and thus it does not slow down the heat transfer process. This situation is approached in practice at surfaces where boiling and condensation occur. Also note that the surface does not have to be a plane surface. Equation 17–8 for convection resistance is valid for surfaces of any shape, provided that the assumption of $h =$ constant and uniform is reasonable.



(*a*) Heat flow



(*b*) Electric current flow

FIGURE 17–3
Analogy between thermal and electrical resistance concepts.

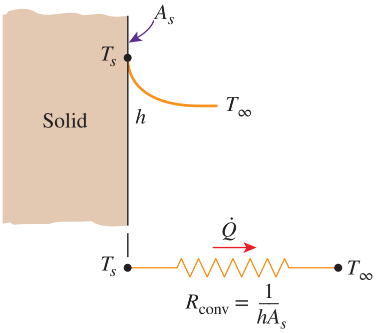


FIGURE 17–4
Schematic for convection resistance at a surface.

When the wall is surrounded by a gas, the *radiation effects,* which we have ignored so far, can be significant and may need to be considered. The rate of radiation heat transfer between a surface of emissivity ε and area A_s at temperature T_s and the surrounding surfaces at some average temperature T_{surr} can be expressed as

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4) = h_{\text{rad}} A_s(T_s - T_{\text{surr}}) = \frac{T_s - T_{\text{surr}}}{R_{\text{rad}}} \quad \text{(W)} \qquad \textbf{(17–9)}$$

where

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_s} \quad \text{(K/W)} \qquad \textbf{(17–10)}$$

is the *thermal resistance* of a surface against radiation, or the **radiation resistance**, and

$$h_{\text{rad}} = \frac{\dot{Q}_{\text{rad}}}{A_s(T_s - T_{\text{surr}})} = \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}}) \quad \text{(W/m}^2\text{·K)} \qquad \textbf{(17–11)}$$

is the **radiation heat transfer coefficient**. Note that both T_s and T_{surr} *must* be in K in the evaluation of h_{rad}. The definition of the radiation heat transfer coefficient enables us to express radiation conveniently in a manner analogous to convection in terms of a temperature difference. But h_{rad} depends strongly on temperature, while h_{conv} usually does not.

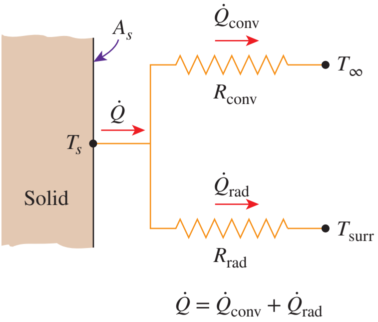


FIGURE 17–5
Schematic for convection and radiation resistances at a surface.

A surface exposed to the surrounding air involves convection and radiation simultaneously, and the total heat transfer at the surface is determined by adding (or subtracting, if in the opposite direction) the radiation and convection components. The convection and radiation resistances are parallel to each other, as shown in Fig. 17–5, and they may cause some complication in the thermal resistance network. When $T_{\text{surr}} \approx T_\infty$, the radiation effect can properly be accounted for by replacing h in the convection resistance relation (Eq. 17–8) with

$$h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}} \quad \text{(W/m}^2\text{·K)} \qquad \textbf{(17–12)}$$

where h_{combined} is the **combined heat transfer coefficient** discussed in Chap. 16. This way all complications associated with radiation are avoided.

Thermal Resistance Network

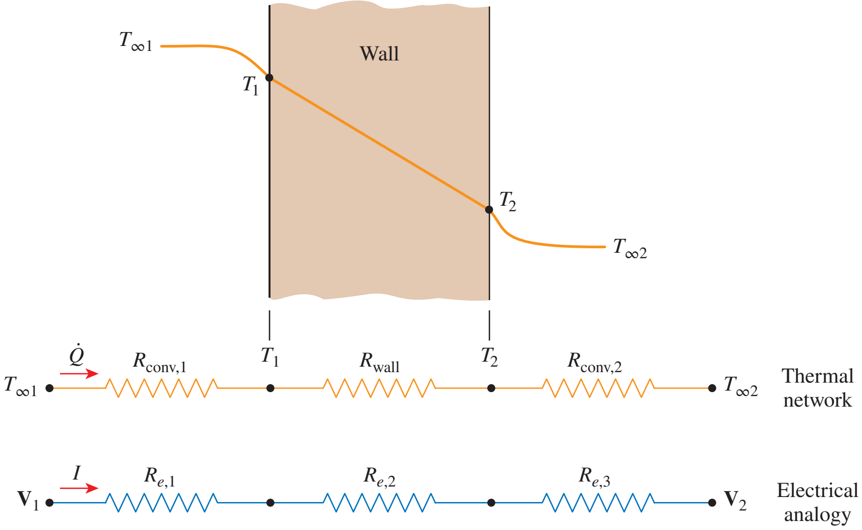
Now consider steady one-dimensional heat transfer through a plane wall of thickness L, area A, and thermal conductivity k that is exposed to convection on both sides to fluids at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ with heat transfer coefficients h_1 and h_2, respectively, as shown in Fig. 17–6. Assuming $T_{\infty 2} < T_{\infty 1}$, the variation of temperature will be as shown in the figure. Note that the temperature varies linearly in the wall and asymptotically approaches $T_{\infty 1}$ and $T_{\infty 2}$ in the fluids as we move away from the wall.

Under steady conditions we have

$$\left(\begin{array}{c}\text{Rate of}\\ \textit{heat convection}\\ \text{into the wall}\end{array}\right) = \left(\begin{array}{c}\text{Rate of}\\ \textit{heat conduction}\\ \text{through the wall}\end{array}\right) = \left(\begin{array}{c}\text{Rate of}\\ \textit{heat convection}\\ \text{from the wall}\end{array}\right)$$

or

$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA\frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2}) \qquad \textbf{(17–13)}$$



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{conv},1} + R_{\text{wall}} + R_{\text{conv},2}}$$

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_{e,1} + R_{e,2} + R_{e,3}}$$

FIGURE 17–6
The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

which can be rearranged as

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}} \qquad \textbf{(17–14)}$$

Once the rate of heat transfer is calculated, Eq. 17–14 can also be used to determine the intermediate temperatures T_1 or T_2. Adding the numerators and denominators yields (Fig. 17–7)

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \quad \text{(W)} \qquad \textbf{(17–15)}$$

where

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \quad \text{(K/W)} \qquad \textbf{(17–16)}$$

Note that the heat transfer area A is constant for a plane wall, and the rate of heat transfer through a wall separating two media is equal to the overall temperature difference $(T_{\infty 1} - T_{\infty 2})$ divided by the total thermal resistance between the media. Also note that the thermal resistances are in *series,* and the equivalent thermal resistance is determined by simply *adding* the individual resistances, just like the electrical resistances connected in series. Thus, the electrical analogy still applies. We summarize this as *the rate of steady heat transfer between two surfaces is equal to the temperature difference divided by the total thermal resistance between those two surfaces.*

Another observation that can be made from Eq. 17–15 is that the ratio of the temperature drop to the thermal resistance across any layer is constant, and thus the temperature drop across any layer is proportional to the thermal resistance of the layer. The larger the resistance, the larger the temperature drop. In fact, the equation $\dot{Q} = \Delta T/R$ can be rearranged as

$$\Delta T = \dot{Q}R \quad (^\circ\text{C}) \qquad \textbf{(17–17)}$$

If

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \cdots = \frac{a_n}{b_n} = c$$

then

$$\frac{a_1 + a_2 + \cdots + a_n}{b_1 + b_2 + \cdots + b_n} = c$$

For example,

$$\frac{1}{4} = \frac{2}{8} = \frac{5}{20} = 0.25$$

and

$$\frac{1 + 2 + 5}{4 + 8 + 20} = 0.25$$

FIGURE 17–7
A useful mathematical identity.

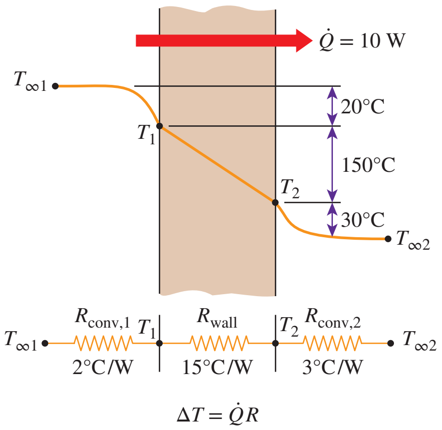


FIGURE 17–8
The temperature drop across a layer is proportional to its thermal resistance.

which indicates that the *temperature drop* across any layer is equal to the *rate of heat transfer* times the *thermal resistance* across that layer (Fig. 17–8). You may recall that this is also true for voltage drop across an electrical resistance when the electric current is constant.

It is sometimes convenient to express heat transfer through a medium in an manner analogous to Newton's law of cooling as

$$\dot{Q} = UA\,\Delta T \quad \text{(W)} \tag{17–18}$$

where U is the **overall heat transfer coefficient** with the unit W/m^2·K. The overall heat transfer coefficient is usually used in heat transfer calculations associated with heat exchangers. It is also used in heat transfer calculations through windows, commonly referred to as U-factor. A comparison of Eqs. 17–15 and 17–18 reveals that

$$UA = \frac{1}{R_{\text{total}}} \quad \text{(W/K)} \tag{17–19}$$

Therefore, for a unit area, the overall heat transfer coefficient is equal to the inverse of the total thermal resistance.

Note that we do not need to know the surface temperatures of the wall in order to evaluate the rate of steady heat transfer through it. All we need to know is the convection heat transfer coefficients and the fluid temperatures on both sides of the wall. The *surface temperature* of the wall can be determined as described above using the thermal resistance concept, but by taking the surface at which the temperature is to be determined as one of the terminal surfaces. For example, once $\dot{Q}$ is evaluated, the surface temperature T_1 can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_{\infty 1} - T_1}{1/h_1 A} \tag{17–20}$$

Multilayer Plane Walls

In practice, we often encounter plane walls that consist of several layers of different materials. The thermal resistance concept can still be used to determine the rate of steady heat transfer through such *composite* walls. As you may have already guessed, this is done by simply noting that the conduction resistance of each wall is L/kA connected in series, and using the electrical analogy—that is, by dividing the *temperature difference* between two surfaces at known temperatures by the *total thermal resistance* between them.

Consider a plane wall that consists of two layers (such as a brick wall with a layer of insulation). The rate of steady heat transfer through this two-layer composite wall can be expressed as (Fig. 17–9)

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \tag{17–21}$$

where R_{total} is the *total thermal resistance,* expressed as

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{wall, 1}} + R_{\text{wall, 2}} + R_{\text{conv, 2}} \\ &= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A} \end{aligned} \tag{17–22}$$

The subscripts 1 and 2 in the R_{wall} relations indicate the first and the second layers, respectively. We could also obtain this result by following the approach

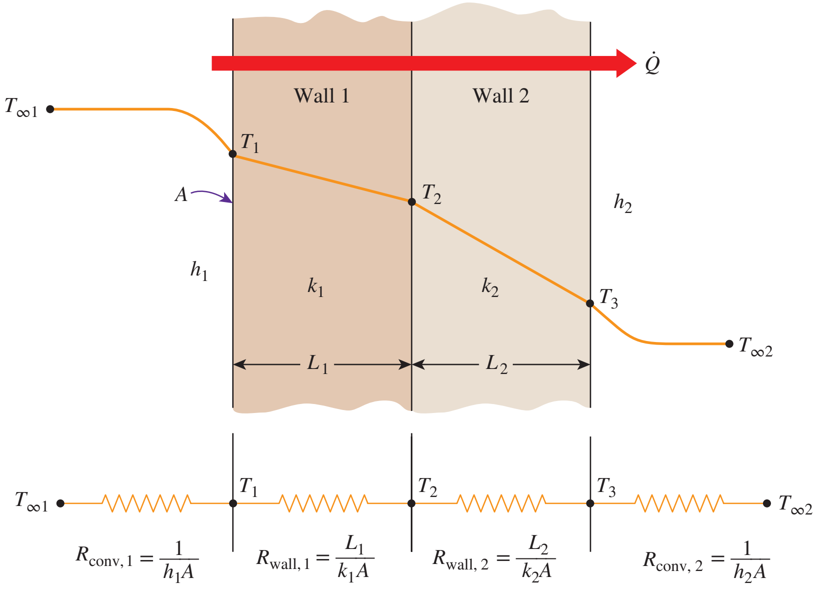


FIGURE 17–9
The thermal resistance network for heat transfer through a two-layer plane wall subjected to convection on both sides.

already used for the single-layer case by noting that the rate of steady heat transfer $\dot{Q}$ through a multilayer medium is constant, and thus it must be the same through each layer. Note from the thermal resistance network that the resistances are *in series,* and thus the *total thermal resistance* is simply the *arithmetic sum* of the individual thermal resistances in the path of heat transfer.

This result for the *two-layer* case is analogous to the *single-layer* case, except that an *additional resistance* is added for the *additional layer.* This result can be extended to plane walls that consist of *three* or *more layers* by adding an *additional resistance* for each *additional layer.*

Once $\dot{Q}$ is *known,* an unknown surface temperature T_j at any surface or interface j can be determined from

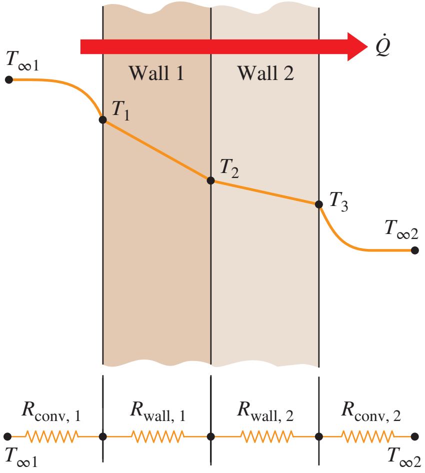
$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total},\, i-j}} \qquad \textbf{(17–23)}$$

where T_i is a *known* temperature at location i and $R_{\text{total},\, i-j}$ is the total thermal resistance between locations i and j. For example, when the fluid temperatures $T_{\infty 1}$ and $T_{\infty 2}$ for the two-layer case shown in Fig. 17–9 are available and $\dot{Q}$ is calculated from Eq. 17–21, the interface temperature T_2 between the two walls can be determined from (Fig. 17–10)

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_{\text{wall},1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A}} \qquad \textbf{(17–24)}$$

The temperature drop across a layer is easily determined from Eq. 17–17 by multiplying $\dot{Q}$ by the thermal resistance of that layer.

The thermal resistance concept is widely used in practice because it is intuitively easy to understand and it has proven to be a powerful tool in the solution of a wide range of heat transfer problems. But its use is limited to systems through which the rate of heat transfer $\dot{Q}$ remains *constant;* that is, to systems involving *steady* heat transfer with *no heat generation* (such as resistance heating or chemical reactions) within the medium.



To find T_1: $\dot{Q} = \dfrac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$

To find T_2: $\dot{Q} = \dfrac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_{\text{wall},1}}$

To find T_3: $\dot{Q} = \dfrac{T_3 - T_{\infty 2}}{R_{\text{conv},2}}$

FIGURE 17–10
The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and $\dot{Q}$ is calculated.

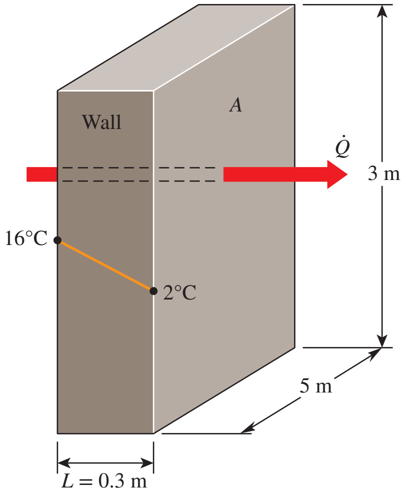


FIGURE 17–11
Schematic for Example 17–1.

EXAMPLE 17–1 Heat Loss Through a Wall

Consider a 3-m-high, 5-m-wide, and 0.3-m-thick wall whose thermal conductivity is $k = 0.9$ W/m·K (Fig. 17–11). On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 16°C and 2°C, respectively. Determine the rate of heat loss through the wall on that day.

SOLUTION The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.

Assumptions **1** Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.9$ W/m·K.

Analysis Noting that heat transfer through the wall is by conduction and the area of the wall is $A = 3\text{ m} \times 5\text{ m} = 15\text{ m}^2$, the steady rate of heat transfer through the wall can be determined from Eq. 17–3 to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.9\text{ W/m·°C})(15\text{ m}^2)\frac{(16 - 2)\text{°C}}{0.3\text{ m}} = \mathbf{630\ W}$$

We could also determine the steady rate of heat transfer through the wall by making use of the thermal resistance concept from

$$\dot{Q} = \frac{\Delta T_{\text{wall}}}{R_{\text{wall}}}$$

where

$$R_{\text{wall}} = \frac{L}{kA} = \frac{0.3\text{ m}}{(0.9\text{ W/m·°C})(15\text{ m}^2)} = 0.02222\text{°C/W}$$

Substituting, we get

$$\dot{Q} = \frac{(16 - 2)\text{°C}}{0.02222\text{°C/W}} = 630\text{ W}$$

Discussion This is the same result obtained earlier. Note that heat conduction through a plane wall with specified surface temperatures can be determined directly and easily without utilizing the thermal resistance concept. However, the thermal resistance concept serves as a valuable tool in more complex heat transfer problems, as you will see in the following examples. Also, the units W/m·°C and W/m·K for thermal conductivity are equivalent, and thus interchangeable. This is also the case for °C and K for temperature differences.

EXAMPLE 17–2 Heat Loss Through a Single-Pane Window

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of $k = 0.78$ W/m·K. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10\text{ W/m}^2\text{·K}$ and $h_2 = 40\text{ W/m}^2\text{·K}$, which includes the effects of radiation.

SOLUTION Heat loss through a window glass is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

Assumptions **1** Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer through the wall is one-dimensional since any significant temperature gradients exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.78$ W/m·K.

Analysis This problem involves conduction through the glass window and convection at its surfaces, and it can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in Fig. 17–12. Noting that the area of the window is $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.08333°\text{C/W}$$

$$R_{\text{glass}} = \frac{L}{kA} = \frac{0.008 \text{ m}}{(0.78 \text{ W/m}\cdot\text{K})(1.2 \text{ m}^2)} = 0.00855°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.02083°\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{glass}} + R_{\text{conv, 2}} = 0.08333 + 0.00855 + 0.02083 \\ &= 0.1127°\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]°\text{C}}{0.1127°\text{C/W}} = \mathbf{266\ W}$$

Knowing the rate of heat transfer, the inner surface temperature of the window glass can be determined from

$$\begin{aligned} \dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} \quad \rightarrow \quad T_1 &= T_{\infty 1} - \dot{Q} R_{\text{conv, 1}} \\ &= 20°\text{C} - (266 \text{ W})(0.08333°\text{C/W}) \\ &= \mathbf{-2.2°C} \end{aligned}$$

Discussion Note that the inner surface temperature of the window glass is −2.2°C even though the temperature of the air in the room is maintained at 20°C. Such low surface temperatures are highly undesirable since they cause the formation of fog or even frost on the inner surfaces of the glass when the humidity in the room is high.

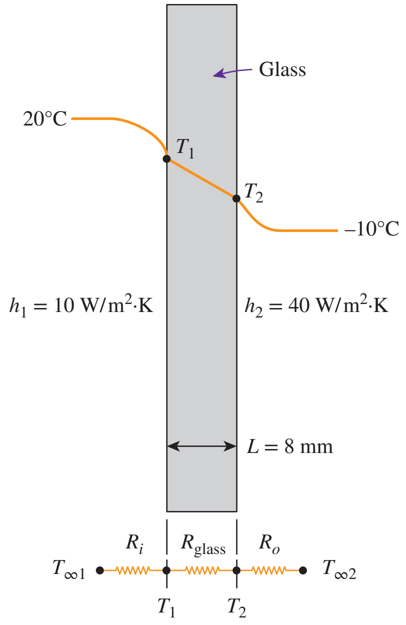


FIGURE 17–12
Schematic for Example 17–2.

EXAMPLE 17–3 Heat Loss Through Double-Pane Windows

Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass ($k = 0.78$ W/m·K) separated by a 10-mm-wide stagnant airspace ($k = 0.026$ W/m·K). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m²·K and $h_2 = 40$ W/m²·K, which includes the effects of radiation.

SOLUTION A double-pane window is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

Analysis This example problem is identical to the previous one except that the single 8-mm-thick window glass is replaced by two 4-mm-thick glasses that enclose a 10-mm-wide stagnant airspace. Therefore, the thermal resistance network of this problem involves two additional conduction resistances corresponding to the two additional layers, as shown in Fig. 17–13. Noting that the area of the window is

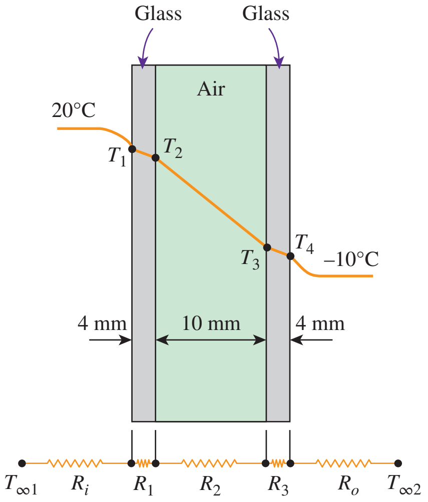


FIGURE 17–13
Schematic for Example 17–3.

again $A = 0.8\ \text{m} \times 1.5\ \text{m} = 1.2\ \text{m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2\cdot\text{K})(1.2\ \text{m}^2)} = 0.08333°\text{C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.004\ \text{m}}{(0.78\ \text{W/m}\cdot\text{K})(1.2\ \text{m}^2)} = 0.00427°\text{C/W}$$

$$R_2 = R_{\text{air}} = \frac{L_2}{k_2 A} = \frac{0.01\ \text{m}}{(0.026\ \text{W/m}\cdot\text{K})(1.2\ \text{m}^2)} = 0.3205°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40\ \text{W/m}^2\cdot\text{K})(1.2\ \text{m}^2)} = 0.02083°\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{glass, 1}} + R_{\text{air}} + R_{\text{glass, 2}} + R_{\text{conv, 2}} \\ &= 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083 \\ &= 0.4332°\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]°\text{C}}{0.4332°\text{C/W}} = \mathbf{69.2\ W}$$

which is about one-fourth of the result obtained in Example 17–2. This explains the popularity of the double- and even triple-pane windows in cold climates. The drastic reduction in the heat transfer rate in this case is due to the large thermal resistance of the air layer between the glasses.

The inner surface temperature of the window in this case will be

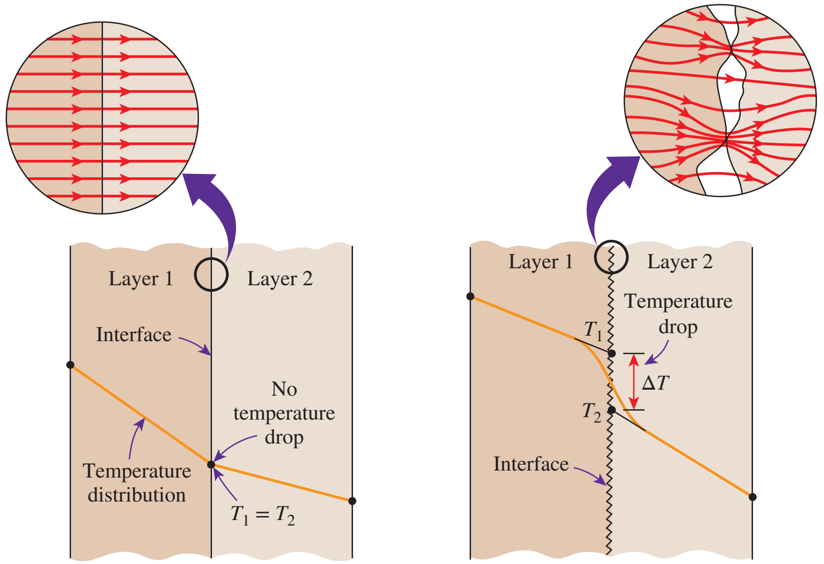
$$T_1 = T_{\infty 1} - \dot{Q}R_{\text{conv, 1}} = 20°\text{C} - (69.2\ \text{W})(0.08333°\text{C/W}) = \mathbf{14.2°C}$$

which is considerably higher than the −2.2°C obtained in Example 17–2. Therefore, a double-pane window will rarely get fogged. A double-pane window will also reduce heat gain in summer and thus reduce air-conditioning costs.

17–2 ▪ THERMAL CONTACT RESISTANCE

In the analysis of heat conduction through multilayer solids, we assumed “perfect contact” at the interface of two layers, and thus no temperature drop at the interface. This would be the case when the surfaces are perfectly smooth and they produce a perfect contact at each point. In reality, however, even flat surfaces that appear smooth to the eye turn out to be rather rough when examined under a microscope, as shown in Fig. 17–14, with numerous peaks and valleys. That is, a surface is *microscopically rough* no matter how smooth it appears to be.

When two such surfaces are pressed against each other, the peaks form good material contact but the valleys form voids filled with air in most cases. As a result, an interface contains numerous *air gaps* of varying sizes that act as *insulation* because of the low thermal conductivity of air. Thus, an interface offers some resistance to heat transfer, and this resistance for a unit interface area is called the **thermal contact resistance**, R_c. The value of R_c is determined experimentally using a setup like the one shown in Fig. 17–15, and as expected, there is considerable scatter of data because of the difficulty in characterizing the surfaces.



(*a*) Ideal (perfect) thermal contact

(*b*) Actual (imperfect) thermal contact

FIGURE 17–14

Temperature distribution and heat flow lines along two solid plates pressed against each other for the case of perfect and imperfect contact.

Consider heat transfer through two metal rods of cross-sectional area A that are pressed against each other. Heat transfer through the interface of these two rods is the sum of the heat transfers through the *solid contact spots* (solid-to-solid conduction) and the *gaps* (conduction and/or radiation across the gaps) in the noncontact areas (which is a major contributor to heat transfer) and can be expressed as

$$\dot{Q} = \dot{Q}_{\text{contact}} + \dot{Q}_{\text{gap}} \tag{17–25}$$

It can also be expressed in a manner analogous to Newton's law of cooling as

$$\dot{Q} = h_c A \, \Delta T_{\text{interface}} \tag{17–26}$$

where A is the apparent interface area (which is the same as the cross-sectional area of the rods) and $\Delta T_{\text{interface}}$ is the effective temperature difference at the interface. The quantity h_c, which corresponds to the convection heat transfer coefficient, is called the **thermal contact conductance** and is expressed as

$$h_c = \frac{\dot{Q}/A}{\Delta T_{\text{interface}}} \qquad (\text{W/m}^2\cdot\text{K}) \tag{17–27}$$

It is related to thermal contact resistance by

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{\text{interface}}}{\dot{Q}/A} \qquad (\text{m}^2\cdot\text{K/W}) \tag{17–28}$$

That is, thermal contact resistance is the inverse of thermal contact conductance. Usually, thermal contact conductance is reported in the literature, but the concept of thermal contact resistance serves as a better vehicle for explaining the effect of interface on heat transfer. Note that R_c represents thermal contact resistance for a *unit area.* The thermal resistance for the entire interface is obtained by dividing R_c by the apparent interface area A.

The thermal contact resistance can be determined from Eq. 17–28 by measuring the temperature drop at the interface and dividing it by the heat flux under steady conditions. The value of thermal contact resistance depends on the *surface roughness* and the *material properties* as well as the *temperature* and *pressure* at

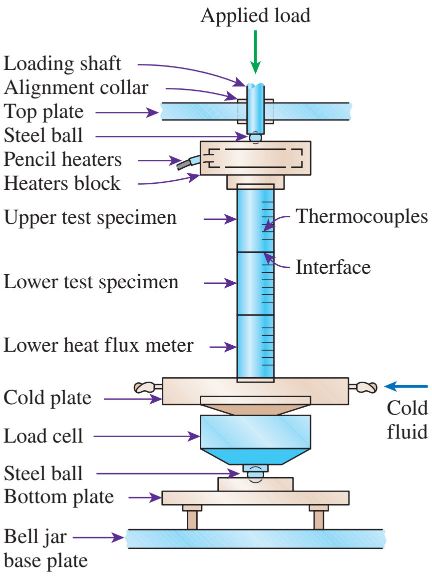


FIGURE 17–15

A typical experimental setup for the determination of thermal contact resistance.

Source: Song et al., 1993.

the interface and the *type of fluid* trapped at the interface. The situation becomes more complex when plates are fastened by bolts, screws, or rivets since the interface pressure in this case is nonuniform. The thermal contact resistance in that case also depends on the plate thickness, the bolt radius, and the size of the contact zone. Thermal contact resistance is observed to *decrease* with *decreasing surface roughness* and *increasing interface pressure,* as expected. Most experimentally determined values of the thermal contact resistance fall between 0.000005 and 0.0005 $m^2 \cdot K/W$ (the corresponding range of thermal contact conductance is 2000 to 200,000 $W/m^2 \cdot K$).

TABLE 17–1

Thermal contact conductance for aluminum plates with different fluids at the interface for a surface roughness of 10 μm and interface pressure of 1 atm

Fluid at the interface	Contact conductance, h_c, $W/m^2 \cdot K$
Air	3640
Helium	9520
Hydrogen	13,900
Silicone oil	19,000
Glycerin	37,700

Source: E. Fried. "Thermal Conduction Contribution to Heat Transfer at Contacts." Thermal Conductivity, *vol. 2, ed. R. P. Tye. London: Academic Press, 1969.*

When we analyze heat transfer in a medium consisting of two or more layers, the first thing we need to know is whether the thermal contact resistance is *significant* or not. We can answer this question by comparing the magnitudes of the thermal resistances of the layers with typical values of thermal contact resistance. For example, the thermal resistance of a 1-cm-thick layer of an insulating material for a unit surface area is

$$R_{c,\text{ insulation}} = \frac{L}{k} = \frac{0.01\text{ m}}{0.04\text{ W/m}\cdot\text{K}} = 0.25\text{ m}^2\cdot\text{K/W}$$

whereas for a 1-cm-thick layer of copper, it is

$$R_{c,\text{ copper}} = \frac{L}{k} = \frac{0.01\text{ m}}{386\text{ W/m}\cdot\text{K}} = 0.000026\text{ m}^2\cdot\text{K/W}$$

Comparing the values preceding with typical values of thermal contact resistance, we conclude that thermal contact resistance is significant and can even dominate the heat transfer for good heat conductors such as metals, but it can be disregarded for poor heat conductors such as insulations. This is not surprising since insulating materials consist mostly of airspace just like the interface itself.

The thermal contact resistance can be minimized by applying a thermally conducting liquid called a *thermal grease* such as silicon oil on the surfaces before they are pressed against each other. This is commonly done when attaching electronic components such as power transistors to heat sinks. The thermal contact resistance can also be reduced by replacing the air at the interface with a *better conducting gas* such as helium or hydrogen, as shown in Table 17–1.

Another way to minimize the contact resistance is to insert a *soft metallic foil* such as tin, silver, copper, nickel, or aluminum between the two surfaces. Experimental studies show that the thermal contact resistance can be reduced by a factor of up to 7 by a metallic foil at the interface. For maximum effectiveness, the foils must be very thin. The effect of metallic coatings on thermal contact conductance is shown in Fig. 17–16 for various metal surfaces.

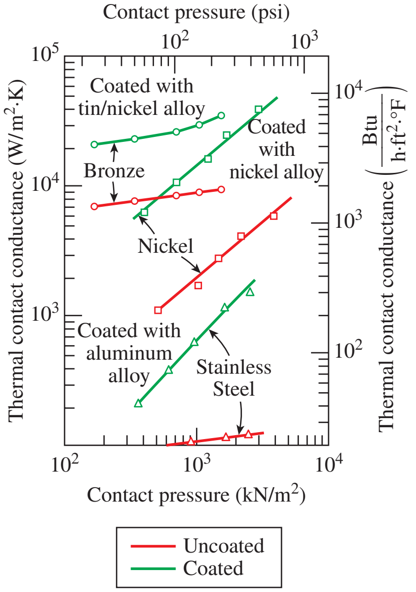


FIGURE 17–16

Effect of metallic coatings on thermal contact conductance.

Source: Peterson, 1987.

There is considerable uncertainty in the contact conductance data reported in the literature, and care should be exercised when using them. In Table 17–2, some experimental results are given for the contact conductance between similar and dissimilar metal surfaces for use in preliminary design calculations. Note that the *thermal contact conductance* is *highest* (and thus the contact resistance is lowest) for *soft metals* with *smooth surfaces* at *high pressure.*

EXAMPLE 17–4 Equivalent Thickness for Contact Resistance

The thermal contact conductance at the interface of two 1-cm-thick copper plates is measured to be 18,000 $W/m^2 \cdot K$. Determine the thickness of the copper plate whose thermal resistance is equal to the thermal resistance of the interface between the plates (Fig. 17–17).

SOLUTION The thickness of the copper plate whose thermal resistance is equal to the thermal contact resistance is to be determined.

TABLE 17–2

Thermal contact conductance of some metal surfaces in air (from various sources)

Material	Surface condition	Roughness, μm	Temperature, °C	Pressure, MPa	h_c,* W/m^2·K
Identical metal pairs					
416 Stainless steel	Ground	2.54	90–200	0.17–2.5	3,800
304 Stainless steel	Ground	1.14	20	4–7	1,900
Aluminum	Ground	2.54	150	1.2–2.5	11,400
Copper	Ground	1.27	20	1.2–20	143,000
Copper	Milled	3.81	20	1–5	55,500
Copper (vacuum)	Milled	0.25	30	0.17–7	11,400
Dissimilar metal pairs					
Stainless steel–				10	2,900
Aluminum		20–30	20	20	3,600
Stainless steel–				10	16,400
Aluminum		1.0–2.0	20	20	20,800
Steel Ct-30–				10	50,000
Aluminum	Ground	1.4–2.0	20	15–35	59,000
Steel Ct-30–				10	4,800
Aluminum	Milled	4.5–7.2	20	30	8,300
				5	42,000
Aluminum-Copper	Ground	1.17–1.4	20	15	56,000
				10	12,000
Aluminum-Copper	Milled	4.4–4.5	20	20–35	12,000

*Divide the given values by 5.678 to convert to Btu/h·ft^2·°F.

Properties The thermal conductivity of copper at room temperature is $k = 401$ W/m·K (Table A–24).

Analysis Noting that thermal contact resistance is the inverse of thermal contact conductance, the thermal contact resistance is

$$R_c = \frac{1}{h_c} = \frac{1}{18{,}000 \text{ W/m}^2\cdot\text{K}} = 5.556 \times 10^{-5} \text{ m}^2\cdot\text{K/W}$$

For a unit surface area, the thermal resistance of a flat plate is defined as

$$R = \frac{L}{k}$$

where L is the thickness of the plate and k is the thermal conductivity. Setting $R = R_c$, the equivalent thickness is determined from the relation above to be

$$L = kR_c = (401 \text{ W/m·K})(5.556 \times 10^{-5} \text{ m}^2\cdot\text{K/W}) = 0.0223 \text{ m} = \mathbf{2.23 \text{ cm}}$$

Discussion Note that the interface between the two plates offers as much resistance to heat transfer as a 2.23-cm-thick copper plate. It is interesting that the thermal contact resistance in this case is greater than the sum of the thermal resistances of both plates.

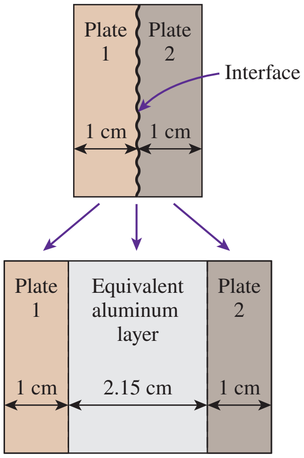


FIGURE 17–17
Schematic for Example 17–4.

EXAMPLE 17–5 Contact Resistance of Transistors

Four identical power transistors with aluminum casing are attached on one side of a 1-cm-thick 20-cm × 20-cm square copper plate ($k = 386$ W/m·K) by screws that exert an average pressure of 6 MPa (Fig. 17–18). The base area of each transistor is

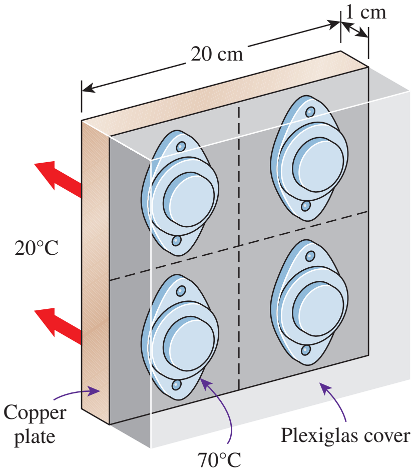


FIGURE 17–18
Schematic for Example 17–5.

8 cm^2, and each transistor is placed at the center of a 10-cm × 10-cm quarter section of the plate. The interface roughness is estimated to be about 1.5 μm. All transistors are covered by a thick Plexiglas layer, which is a poor conductor of heat, and thus all the heat generated at the junction of the transistor must be dissipated to the ambient at 20°C through the back surface of the copper plate. The combined convection/radiation heat transfer coefficient at the back surface can be taken to be 25 $\text{W/m}^2\cdot\text{K}$. If the case temperature of the transistor is not to exceed 70°C, determine the maximum power each transistor can dissipate safely and the temperature jump at the case/plate interface.

SOLUTION Four identical power transistors are attached on a copper plate. For a maximum case temperature of 70°C, the maximum power dissipation and the temperature jump at the interface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer can be approximated as being one-dimensional, although it is recognized that heat conduction in some parts of the plate will be two-dimensional since the plate area is much larger than the base area of the transistor. But the large thermal conductivity of copper will minimize this effect. **3** All the heat generated at the junction is dissipated through the back surface of the plate since the transistors are covered by a thick Plexiglas layer. **4** Thermal conductivities are constant.

Properties The thermal conductivity of copper is given to be $k = 386$ W/m·K. The contact conductance is obtained from Table 17–2 to be $h_c = 42{,}000\ \text{W/m}^2\cdot\text{K}$, which corresponds to a copper/aluminum interface for the case of 1.17 – 1.4 μm roughness and 5 MPa pressure, which is sufficiently close to what we have.

Analysis The contact area between the case and the plate is given to be 8 cm^2, and the plate area for each transistor is 100 cm^2. The thermal resistance network of this problem consists of three resistances in series (interface, plate, and convection), which are determined to be

$$R_{\text{interface}} = \frac{1}{h_c A_c} = \frac{1}{(42{,}000\ \text{W/m}^2\cdot\text{K})(8\times10^{-4}\ \text{m}^2)} = 0.030^\circ\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.01\ \text{m}}{(386\ \text{W/m}\cdot\text{K})(0.01\ \text{m}^2)} = 0.0026^\circ\text{C/W}$$

$$R_{\text{conv}} = \frac{1}{h_o A} = \frac{1}{(25\ \text{W/m}^2\cdot\text{K})(0.01\ \text{m}^2)} = 4.0^\circ\text{C/W}$$

The total thermal resistance is then

$$R_{\text{total}} = R_{\text{interface}} + R_{\text{plate}} + R_{\text{ambient}} = 0.030 + 0.0026 + 4.0 = 4.0326^\circ\text{C/W}$$

Note that the thermal resistance of a copper plate is very small and can be ignored altogether. Then the rate of heat transfer is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{(70-20)^\circ\text{C}}{4.0326^\circ\text{C/W}} = \mathbf{12.4\ W}$$

Therefore, the power transistor should not be operated at power levels greater than 12.4 W if the case temperature is not to exceed 70°C.

The temperature jump at the interface is determined from

$$\Delta T_{\text{interface}} = \dot{Q} R_{\text{interface}} = (12.4\ \text{W})(0.030^\circ\text{C/W}) = \mathbf{0.37^\circ C}$$

which is not very large. Therefore, even if we eliminate the thermal contact resistance at the interface completely, we lower the operating temperature of the transistor in this case by less than 0.4°C.

17–3 ▪ GENERALIZED THERMAL RESISTANCE NETWORKS

The *thermal resistance* concept or the *electrical analogy* can also be used to solve steady heat transfer problems that involve parallel layers or combined series-parallel arrangements. Although such problems are often two- or even three-dimensional, approximate solutions can be obtained by assuming one-dimensional heat transfer and using the thermal resistance network.

Consider the composite wall shown in Fig. 17–19, which consists of two parallel layers. The thermal resistance network, which consists of two parallel resistances, can be represented as shown in the figure. Noting that the total heat transfer is the sum of the heat transfers through each layer, we have

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{17–29}$$

Utilizing the electrical analogy, we get

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{total}}} \tag{17–30}$$

where

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \rightarrow \quad R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2} \tag{17–31}$$

since the resistances are in parallel.

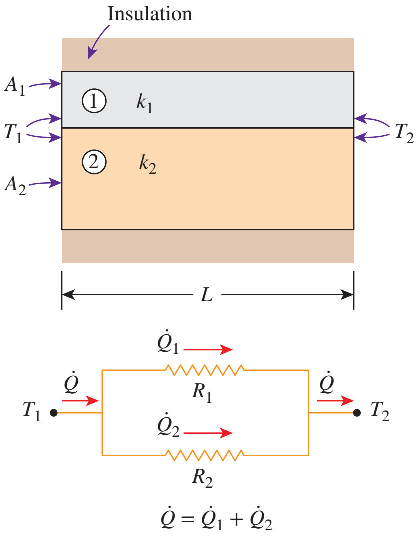


FIGURE 17–19
Thermal resistance network for two parallel layers.

Now consider the combined series-parallel arrangement shown in Fig. 17–20. The total rate of heat transfer through this composite system can again be expressed as

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \tag{17–32}$$

where

$$R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}} \tag{17–33}$$

and

$$R_1 = \frac{L_1}{k_1 A_1} \quad R_2 = \frac{L_2}{k_2 A_2} \quad R_3 = \frac{L_3}{k_3 A_3} \quad R_{\text{conv}} = \frac{1}{hA_3} \tag{17–34}$$

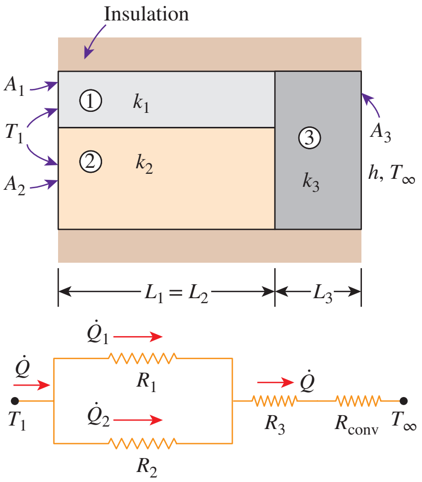


FIGURE 17–20
Thermal resistance network for combined series-parallel arrangement.

Once the individual thermal resistances are evaluated, the total resistance and the total rate of heat transfer can easily be determined from the relations above.

The result obtained is somewhat approximate, since the surfaces of the third layer are probably not isothermal, and heat transfer between the first two layers is likely to occur.

Two assumptions commonly used in solving complex multidimensional heat transfer problems by treating them as one-dimensional (say, in the x-direction) using the thermal resistance network are (1) any plane wall normal to the x-axis is *isothermal* (i.e., assumes the temperature varies in the x-direction only) and (2) any plane parallel to the x-axis is *adiabatic* (i.e., assumes heat transfer occurs in the x-direction only). These two assumptions result in different resistance networks, and thus different (but usually close) values for the total thermal resistance and thus heat transfer. The actual result lies between these two values. In geometries in which heat transfer occurs predominantly in one direction, either approach gives satisfactory results.

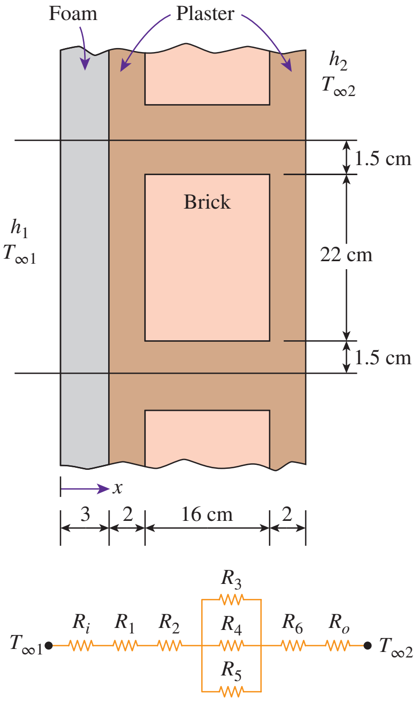


FIGURE 17–21
Schematic for Example 17–6.

EXAMPLE 17–6 Heat Loss Through a Composite Wall

A 3-m-high and 5-m-wide wall consists of long 16-cm × 22-cm cross section horizontal bricks ($k = 0.72$ W/m·K) separated by 3-cm-thick plaster layers ($k = 0.22$ W/m·K). There are also 2-cm-thick plaster layers on each side of the brick and a 3-cm-thick rigid foam ($k = 0.026$ W/m·K) on the inner side of the wall, as shown in Fig. 17–21. The indoor and the outdoor temperatures are 20°C and −10°C, respectively, and the convection heat transfer coefficients on the inner and the outer sides are $h_1 = 10$ W/m²·K and $h_2 = 25$ W/m²·K, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

SOLUTION The composition of a composite wall is given. The rate of heat transfer through the wall is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer can be approximated as being one-dimensional since it is predominantly in the *x*-direction. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivities are given to be $k = 0.72$ W/m·K for bricks, $k = 0.22$ W/m·K for plaster layers, and $k = 0.026$ W/m·K for the rigid foam.

Analysis There is a pattern in the construction of this wall that repeats itself every 25-cm distance in the vertical direction. There is no variation in the horizontal direction. Therefore, we consider a 1-m-deep and 0.25-m-high portion of the wall, since it is representative of the entire wall.

Assuming any cross section of the wall normal to the *x*-direction to be *isothermal,* the thermal resistance network for the representative section of the wall becomes as shown in Fig. 17–21. The individual resistances are evaluated as:

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2\cdot\text{K})(0.25 \times 1\ \text{m}^2)} = 0.40^\circ\text{C/W}$$

$$R_1 = R_{\text{foam}} = \frac{L}{kA} = \frac{0.03\ \text{m}}{(0.026\ \text{W/m}\cdot\text{K})(0.25 \times 1\ \text{m}^2)} = 4.62^\circ\text{C/W}$$

$$R_2 = R_6 = R_{\text{plaster, side}} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(0.22\ \text{W/m}\cdot\text{K})(0.25 \times 1\ \text{m}^2)}$$
$$= 0.36^\circ\text{C/W}$$

$$R_3 = R_5 = R_{\text{plaster, center}} = \frac{L}{kA} = \frac{0.16\ \text{m}}{(0.22\ \text{W/m}\cdot\text{K})(0.015 \times 1\ \text{m}^2)}$$
$$= 48.48^\circ\text{C/W}$$

$$R_4 = R_{\text{brick}} = \frac{L}{kA} = \frac{0.16\ \text{m}}{(0.72\ \text{W/m}\cdot\text{K})(0.22 \times 1\ \text{m}^2)} = 1.01^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(25\ \text{W/m}^2\cdot\text{K})(0.25 \times 1\ \text{m}^2)} = 0.16^\circ\text{C/W}$$

The three resistances R_3, R_4, and R_5 in the middle are parallel, and their equivalent resistance is determined from

$$\frac{1}{R_{\text{mid}}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03\ \text{W/}^\circ\text{C}$$

which gives

$$R_{\text{mid}} = 0.97^\circ\text{C/W}$$

Now all the resistances are in series, and the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_i + R_1 + R_2 + R_{\text{mid}} + R_6 + R_o \\ &= 0.40 + 4.62 + 0.36 + 0.97 + 0.36 + 0.16 \\ &= 6.87^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the wall becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{6.87^\circ\text{C/W}} = 4.37\ \text{W} \qquad (\text{per } 0.25\ \text{m}^2 \text{ surface area})$$

or 4.37/0.25 = 17.5 W per m^2 area. The total area of the wall is $A = 3\ \text{m} \times 5\ \text{m} = 15\ \text{m}^2$. Then the rate of heat transfer through the entire wall becomes

$$\dot{Q}_{\text{total}} = (1.75\ \text{W/m}^2)(15\ \text{m}^2) = \mathbf{263\ W}$$

Of course, this result is approximate, since we assumed the temperature within the wall to vary in one direction only and ignored any temperature change (and thus heat transfer) in the other two directions.

Discussion In the preceding solution, we assumed the temperature at any cross section of the wall normal to the x-direction was *isothermal.* We could also solve this problem by going to the other extreme and assuming the surfaces parallel to the x-direction are *adiabatic.* The thermal resistance network in this case will be as shown in Fig. 17–22. By following the approach outlined above, the total thermal resistance in this case is determined to be $R_{\text{total}} = 6.97^\circ\text{C/W}$, which is very close to the value 6.87°C/W obtained before. Thus, either approach gives roughly the same result in this case. This example demonstrates that either approach can be used in practice to obtain satisfactory results.

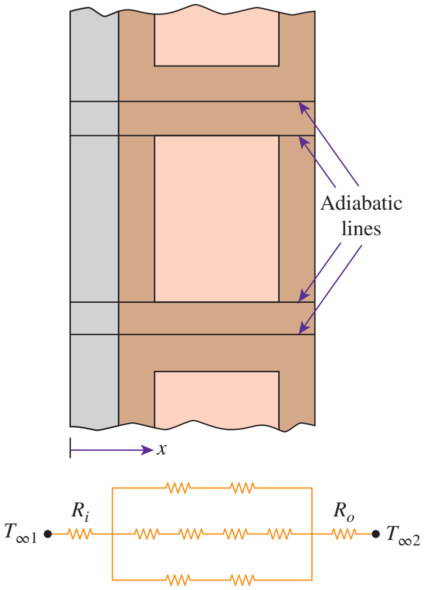


FIGURE 17–22

Alternative thermal resistance network for Example 17–6 for the case of surfaces parallel to the primary direction of heat transfer being adiabatic.

17–4 ▪ HEAT CONDUCTION IN CYLINDERS AND SPHERES

Consider steady heat conduction through a hot-water pipe. Heat is continuously lost to the outdoors through the wall of the pipe, and we intuitively feel that heat transfer through the pipe is in the normal direction to the pipe surface and that no significant heat transfer takes place in the pipe in other directions (Fig. 17–23). The wall of the pipe, whose thickness is rather small, separates two fluids at different temperatures, and thus the temperature gradient in the radial direction is relatively large. Further, if the fluid temperatures inside and outside the pipe remain constant, then heat transfer through the pipe is *steady.* Thus heat transfer through the pipe can be modeled as *steady* and *one-dimensional.* The temperature of the pipe in this case depends on one direction only (the radial r-direction) and can be expressed as $T = T(r)$. The temperature is independent of the azimuthal angle or the axial distance. This situation is approximated in practice in long cylindrical pipes and spherical containers.

In *steady* operation, there is no change in the temperature of the pipe with time at any point. Therefore, the rate of heat transfer into the pipe must be equal to the rate of heat transfer out of it. In other words, heat transfer through the pipe must be constant, $\dot{Q}_{\text{cond, cyl}} = \text{constant}$.

Consider a long cylindrical layer (such as a circular pipe) of inner radius r_1, outer radius r_2, length L, and average thermal conductivity k (Fig. 17–24). The two surfaces of the cylindrical layer are maintained at constant temperatures T_1 and T_2. There is no heat generation in the layer, and the thermal conductivity is constant. For one-dimensional heat conduction through the cylindrical layer, we have $T(r)$. Then Fourier's law of heat conduction for heat transfer through the cylindrical layer can be expressed as

$$\dot{Q}_{\text{cond, cyl}} = -kA\frac{dT}{dr} \qquad (\text{W}) \tag{17–35}$$

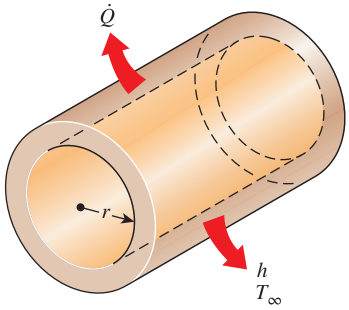


FIGURE 17–23

Heat is lost from a hot-water pipe to the air outside in the radial direction, and thus heat transfer from a long pipe is one-dimensional.

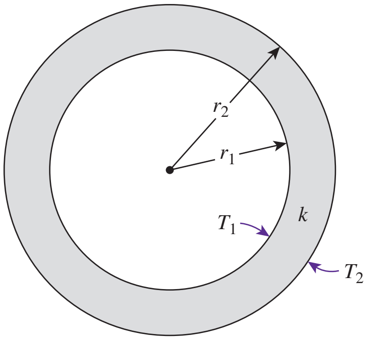


FIGURE 17–24
A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures T_1 and T_2.

where $A = 2\pi rL$ is the heat transfer area at location r. Note that A depends on r, and thus it *varies* in the direction of heat transfer. Separating the variables in Eq. 17–35 and integrating from $r = r_1$, where $T(r_1) = T_1$, to $r = r_2$, where $T(r_2) = T_2$, gives

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A}\,dr = -\int_{T=T_1}^{T_2} k\,dT \tag{17–36}$$

Substituting $A = 2\pi rL$ and performing the integrations give

$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad \text{(W)} \tag{17–37}$$

since $\dot{Q}_{\text{cond, cyl}}$ = constant. This equation can be rearranged as

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \qquad \text{(W)} \tag{17–38}$$

where

$$R_{\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times \text{Length} \times \text{Thermal conductivity}} \tag{17–39}$$

is the *thermal resistance* of the cylindrical layer against heat conduction, or simply the *conduction resistance* of the cylinder layer.

We can repeat the analysis for a *spherical layer* by taking $A = 4\pi r^2$ and performing the integrations in Eq. 17–36. The result can be expressed as

$$\dot{Q}_{\text{cond, sph}} = \frac{T_1 - T_2}{R_{\text{sph}}} \tag{17–40}$$

where

$$R_{\text{sph}} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})} \tag{17–41}$$

is the *thermal resistance* of the spherical layer against heat conduction, or simply the *conduction resistance* of the spherical layer.

Now consider steady one-dimensional heat transfer through a cylindrical or spherical layer that is exposed to convection on both sides to fluids at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ with heat transfer coefficients h_1 and h_2, respectively, as shown in Fig. 17–25. The thermal resistance network in this case consists of one conduction and two convection resistances in series, just like the one for the plane wall, and the rate of heat transfer under steady conditions can be expressed as

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \tag{17–42}$$

where

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{cyl}} + R_{\text{conv, 2}} \\ &= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi Lk} + \frac{1}{(2\pi r_2 L)h_2} \end{aligned} \tag{17–43}$$

for a *cylindrical* layer, and

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{sph}} + R_{\text{conv, 2}} \\ &= \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2} \end{aligned} \tag{17–44}$$

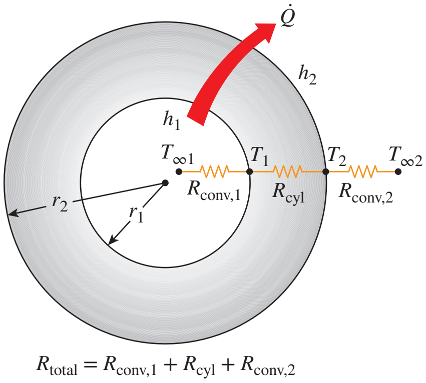


FIGURE 17–25
The thermal resistance network for a cylindrical (or spherical) shell subjected to convection from both the inner and the outer sides.

for a *spherical* layer. Note that A in the convection resistance relation $R_{\text{conv}} = 1/hA$ is the *surface area at which convection occurs.* It is equal to $A = 2\pi rL$ for a cylindrical surface and $A = 4\pi r^2$ for a spherical surface of radius r. Also note that the thermal resistances are in series, and thus the total thermal resistance is determined by simply adding the individual resistances, just like the electrical resistances connected in series.

Multilayered Cylinders and Spheres

Steady heat transfer through multilayered cylindrical or spherical shells can be handled just like multilayered plane walls discussed earlier by simply adding an *additional resistance* in series for each *additional layer.* For example, the steady heat transfer rate through the three-layered composite cylinder of length L shown in Fig. 17–26 with convection on both sides can be expressed as

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \tag{17–45}$$

where R_{total} is the *total thermal resistance,* expressed as

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{cyl, 1}} + R_{\text{cyl, 2}} + R_{\text{cyl, 3}} + R_{\text{conv, 2}} \\ &= \frac{1}{h_1A_1} + \frac{\ln(r_2/r_1)}{2\pi Lk_1} + \frac{\ln(r_3/r_2)}{2\pi Lk_2} + \frac{\ln(r_4/r_3)}{2\pi Lk_3} + \frac{1}{h_2A_4} \end{aligned} \tag{17–46}$$

where $A_1 = 2\pi r_1 L$ and $A_4 = 2\pi r_4 L$. Equation 17–46 can also be used for a three-layered spherical shell by replacing the thermal resistances of cylindrical layers with the corresponding spherical ones. Again, note from the thermal resistance network that the resistances are in series, and thus the total thermal resistance is simply the *arithmetic sum* of the individual thermal resistances in the path of heat flow.

Once $\dot{Q}$ is known, we can determine any intermediate temperature T_j by applying the relation $\dot{Q} = (T_i - T_j)/R_{\text{total, } i-j}$ across any layer or layers such that T_i is

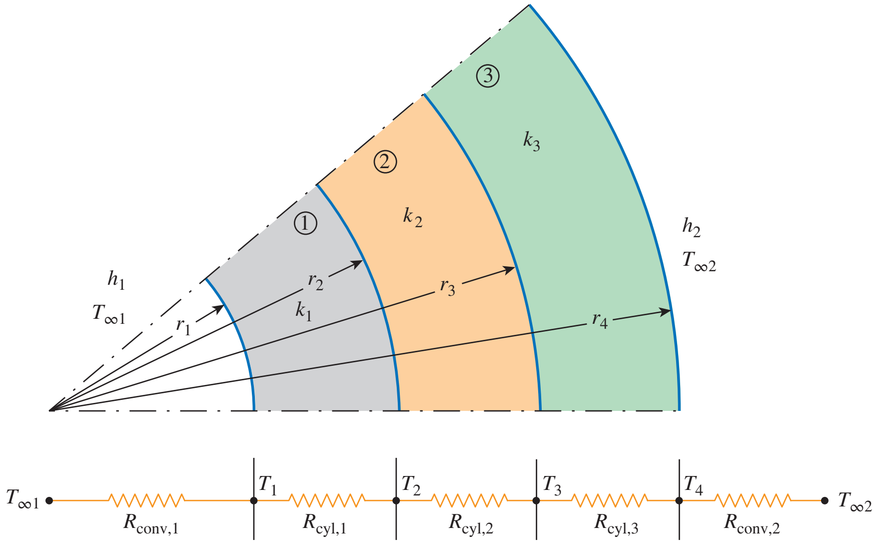
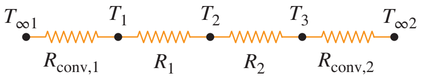


FIGURE 17–26
The thermal resistance network for heat transfer through a three-layered composite cylinder subjected to convection on both sides.



$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$$

$$= \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1}$$

$$= \frac{T_1 - T_3}{R_1 + R_2}$$

$$= \frac{T_2 - T_3}{R_2}$$

$$= \frac{T_2 - T_{\infty 2}}{R_2 + R_{\text{conv},2}}$$

$$= \cdots$$

FIGURE 17–27

The ratio $\Delta T/R$ across any layer is equal to $\dot{Q}$, which remains constant in one-dimensional steady conduction.

a *known* temperature at location i and $R_{\text{total},\, i-j}$ is the total thermal resistance between locations i and j (Fig. 17–27). For example, once $\dot{Q}$ has been calculated, the interface temperature T_2 between the first and second cylindrical layers can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv, 1}} + R_{\text{cyl, 1}}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1(2\pi r_1 L)} + \dfrac{\ln(r_2/r_1)}{2\pi L k_1}} \qquad \textbf{(17–47)}$$

We could also calculate T_2 from

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{\text{conv, 2}}} = \frac{T_2 - T_{\infty 2}}{\dfrac{\ln(r_3/r_2)}{2\pi L k_2} + \dfrac{\ln(r_4/r_3)}{2\pi L k_3} + \dfrac{1}{h_o(2\pi r_4 L)}} \qquad \textbf{(17–48)}$$

Although both relations give the same result, we prefer the first one since it involves fewer terms and thus less work.

The thermal resistance concept can also be used for *other geometries,* provided that the proper conduction resistances and the proper surface areas in convection resistances are used.

EXAMPLE 17–7 Heat Transfer to a Spherical Container

A 3-m internal diameter spherical tank made of 2-cm-thick stainless steel ($k = 15$ W/m·K) is used to store iced water at $T_{\infty 1} = 0°\text{C}$. The tank is located in a room whose temperature is $T_{\infty 2} = 22°\text{C}$. The walls of the room are also at 22°C. The outer surface of the tank is black, and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are $h_1 = 80$ W/m²·K and $h_2 = 10$ W/m²·K, respectively. Determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period.

SOLUTION A spherical container filled with iced water is subjected to convection and radiation heat transfer at its outer surface. The rate of heat transfer and the amount of ice that melts per day are to be determined.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal conductivity is constant.

Properties The thermal conductivity of steel is given to be $k = 15$ W/m·K. The heat of fusion of water at atmospheric pressure is $h_{if} = 333.7$ kJ/kg. The outer surface of the tank is black, and thus its emissivity is $\varepsilon = 1$.

Analysis (*a*) The thermal resistance network for this problem is given in Fig. 17–28. Noting that the inner diameter of the tank is $D_1 = 3$ m and the outer diameter is $D_2 = 3.04$ m, the inner and the outer surface areas of the tank are

$$A_1 = \pi D_1^2 = \pi(3\text{ m})^2 = 28.2\text{ m}^2$$

$$A_2 = \pi D_2^2 = \pi(3.04\text{ m})^2 = 29.0\text{ m}^2$$

Also, the radiation heat transfer coefficient is given by

$$h_{\text{rad}} = \varepsilon\sigma(T_2^2 + T_{\infty 2}^2)(T_2 + T_{\infty 2})$$

But we do not know the outer surface temperature T_2 of the tank, and thus we cannot calculate h_{rad}. Therefore, we need to assume a T_2 value now and check the accuracy of this assumption later. We will repeat the calculations if necessary using a revised value for T_2.

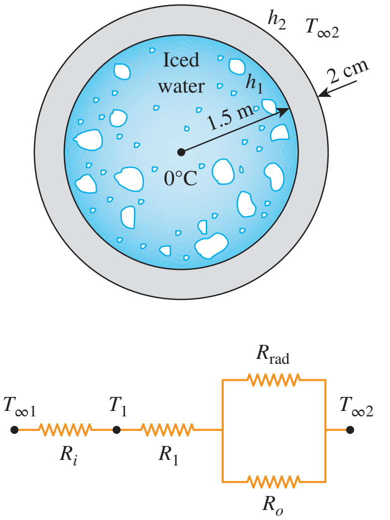


FIGURE 17–28

Schematic for Example 17–7.

We note that T_2 must be between 0°C and 22°C, but it must be closer to 0°C, since the heat transfer coefficient inside the tank is much larger. Taking $T_2 = 5°\text{C} = 278$ K, the radiation heat transfer coefficient is determined to be

$$\begin{aligned} h_{\text{rad}} &= (1)(5.67 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4)\,[(295 \text{ K})^2 + (278 \text{ K})^2][(295 + 278) \text{ K}] \\ &= 5.34 \text{ W/m}^2\cdot\text{K} = 5.34 \text{ W/m}^2\cdot°\text{C} \end{aligned}$$

Then the individual thermal resistances become

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A_1} = \frac{1}{(80 \text{ W/m}^2\cdot\text{K})(28.3 \text{ m}^2)} = 0.000442°\text{C/W}$$

$$\begin{aligned} R_1 = R_{\text{sphere}} &= \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52 - 1.50) \text{ m}}{4\pi(15 \text{ W/m}\cdot\text{K})(1.52 \text{ m})(1.50 \text{ m})} \\ &= 0.000047°\text{C/W} \end{aligned}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A_2} = \frac{1}{(10 \text{ W/m}^2\cdot\text{K})(29.0 \text{ m}^2)} = 0.00345°\text{C/W}$$

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_2} = \frac{1}{(5.34 \text{ W/m}^2\cdot\text{K})(29.0 \text{ m}^2)} = 0.00646°\text{C/W}$$

The two parallel resistances R_o and R_{rad} can be replaced by an equivalent resistance R_{equiv} determined from

$$\frac{1}{R_{\text{equiv}}} = \frac{1}{R_o} + \frac{1}{R_{\text{rad}}} = \frac{1}{0.00345} + \frac{1}{0.00646} = 444.7 \text{ W/°C}$$

which gives

$$R_{\text{equiv}} = 0.00225°\text{C/W}$$

Now all the resistances are in series, and the total resistance is

$$R_{\text{total}} = R_i + R_1 + R_{\text{equiv}} = 0.000442 + 0.000047 + 0.00225 = 0.00274°\text{C/W}$$

Then the steady rate of heat transfer to the iced water becomes

$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{\text{total}}} = \frac{(22 - 0)°\text{C}}{0.00274°\text{C/W}} = \mathbf{8029\ W} \qquad (\text{or } \dot{Q} = 8.029 \text{ kJ/s})$$

To check the validity of our original assumption, we now determine the outer surface temperature from

$$\begin{aligned} \dot{Q} = \frac{T_{\infty 2} - T_2}{R_{\text{equiv}}} \quad \longrightarrow \quad T_2 &= T_{\infty 2} - \dot{Q} R_{\text{equiv}} \\ &= 22°\text{C} - (8029 \text{ W})(0.00225°\text{C/W}) = 4°\text{C} \end{aligned}$$

which is sufficiently close to the 5°C assumed in the determination of the radiation heat transfer coefficient. Therefore, there is no need to repeat the calculations using 4°C for T_2.

(*b*) The total amount of heat transfer during a 24-h period is

$$Q = \dot{Q}\,\Delta t = (8.029 \text{ kJ/s})(24 \times 3600 \text{ s}) = 693{,}700 \text{ kJ}$$

Noting that it takes 333.7 kJ of energy to melt 1 kg of ice at 0°C, the amount of ice that will melt during a 24-h period is

$$m_{\text{ice}} = \frac{Q}{h_{if}} = \frac{693{,}700 \text{ kJ}}{333.7 \text{ kJ/kg}} = \mathbf{2079\ kg}$$

Therefore, about 2 metric tons of ice will melt in the tank every day.

Discussion An easier way to deal with combined convection and radiation at a surface when the surrounding medium and surfaces are at the same temperature is to add the radiation and convection heat transfer coefficients and to treat the result as the convection heat transfer coefficient. That is, to take $h = 10 + 5.34 = 15.34$ W/m²·K in this case. This way, we can ignore radiation since its contribution is accounted for in the convection heat transfer coefficient. The convection resistance of the outer surface in this case would be

$$R_{\text{combined}} = \frac{1}{h_{\text{combined}} A_2} = \frac{1}{(15.34 \text{ W/m}^2\cdot\text{K})(29.0 \text{ m}^2)} = 0.00225^\circ\text{C/W}$$

which is identical to the value obtained for equivalent resistance for the parallel convection and the radiation resistances.

EXAMPLE 17–8 Heat Loss Through an Insulated Steam Pipe

Steam at $T_{\infty 1} = 320^\circ$C flows in a cast iron pipe ($k = 80$ W/m·K) whose inner and outer diameters are $D_1 = 5$ cm and $D_2 = 5.5$ cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation with $k = 0.05$ W/m·K. Heat is lost to the surroundings at $T_{\infty 2} = 5^\circ$C by natural convection and radiation, with a combined heat transfer coefficient of $h_2 = 18$ W/m²·K. Taking the heat transfer coefficient inside the pipe to be $h_1 = 60$ W/m²·K, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

SOLUTION A steam pipe covered with glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be $k = 80$ W/m·K for cast iron and $k = 0.05$ W/m·K for glass wool insulation.

Analysis The thermal resistance network for this problem involves four resistances in series and is given in Fig. 17–29. Taking $L = 1$ m, the areas of the surfaces exposed to convection are determined to be

$$A_1 = 2\pi r_1 L = 2\pi(0.025 \text{ m})(1 \text{ m}) = 0.157 \text{ m}^2$$
$$A_3 = 2\pi r_3 L = 2\pi(0.0575 \text{ m})(1 \text{ m}) = 0.361 \text{ m}^2$$

Then the individual thermal resistances become

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A_1} = \frac{1}{(60 \text{ W/m}^2\cdot\text{K})(0.157 \text{ m}^2)} = 0.106^\circ\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80 \text{ W/m}\cdot\text{K})(1 \text{ m})} = 0.0002^\circ\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05 \text{ W/m}\cdot\text{K})(1 \text{ m})} = 2.35^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A_3} = \frac{1}{(18 \text{ W/m}^2\cdot\text{K})(0.361 \text{ m}^2)} = 0.154^\circ\text{C/W}$$

Noting that all resistances are in series, the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61^\circ\text{C/W}$$

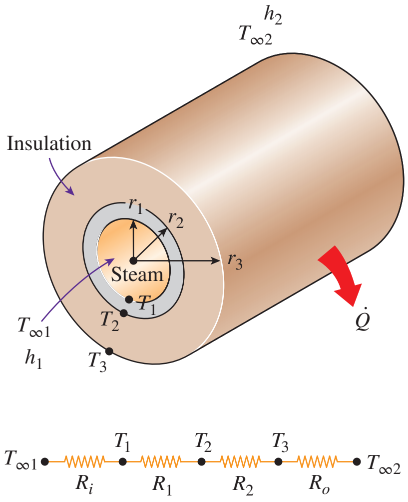


FIGURE 17–29
Schematic for Example 17–8.

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(320 - 5)^\circ\text{C}}{2.61^\circ\text{C/W}} = \mathbf{121\ W} \qquad \text{(per m pipe length)}$$

The heat loss for a given pipe length can be determined by multiplying the preceding quantity by the pipe length *L*.

The temperature drops across the pipe and the insulation are determined from Eq. 17–17 to be

$$\Delta T_{\text{pipe}} = \dot{Q} R_{\text{pipe}} = (121\ \text{W})(0.0002^\circ\text{C/W}) = \mathbf{0.02^\circ C}$$

$$\Delta T_{\text{insulation}} = \dot{Q} R_{\text{insulation}} = (121\ \text{W})(2.35^\circ\text{C/W}) = \mathbf{284^\circ C}$$

That is, the temperatures between the inner and the outer surfaces of the pipe differ by 0.02°C, whereas the temperatures between the inner and the outer surfaces of the insulation differ by 284°C.

Discussion Note that the thermal resistance of the pipe is too small relative to the other resistances and can be neglected without causing any significant error. Also note that the temperature drop across the pipe is practically zero, and thus the pipe can be assumed to be isothermal. The resistance to heat flow in insulated pipes is primarily due to insulation.

17–5 ▪ CRITICAL RADIUS OF INSULATION

We know that adding more insulation to a wall or to the attic always decreases heat transfer. The thicker the insulation, the lower the heat transfer rate. This is expected, since the heat transfer area *A* is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance.

Adding insulation to a cylindrical pipe or a spherical shell, however, is a different matter. The additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection. The heat transfer from the pipe may increase or decrease, depending on which effect dominates.

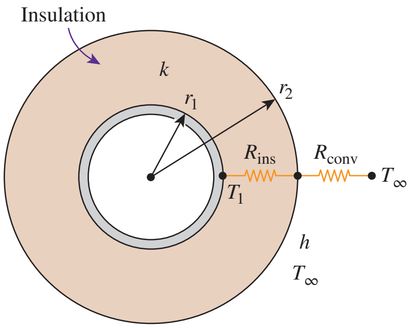


FIGURE 17–30
An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.

Consider a cylindrical pipe of outer radius r_1 whose outer surface temperature T_1 is kept constant (Fig. 17–30). The pipe is now insulated with a material whose thermal conductivity is *k* and whose outer radius is r_2. Heat is lost from the pipe to the surrounding medium at temperature T_∞, with a convection heat transfer coefficient *h*. The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as (Fig. 17–31)

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{h(2\pi r_2 L)}} \tag{17–49}$$

The variation of $\dot{Q}$ with the outer radius of the insulation r_2 is plotted in Fig. 17–31. The value of r_2 at which $\dot{Q}$ reaches a maximum is determined from the requirement that $d\dot{Q}/dr_2 = 0$ (zero slope). Performing the differentiation and solving for r_2 yields the **critical radius of insulation** for a cylindrical body:

$$r_{\text{cr, cylinder}} = \frac{k}{h} \qquad \text{(m)} \tag{17–50}$$

Note that the critical radius of insulation depends on the thermal conductivity of the insulation *k* and the external convection heat transfer coefficient *h*. The rate of heat transfer from the cylinder increases with the addition of insulation for $r_2 < r_{\text{cr}}$, reaches a maximum when $r_2 = r_{\text{cr}}$, and starts to decrease for $r_2 > r_{\text{cr}}$. Thus, insulating the pipe may actually increase the rate of heat transfer from the pipe instead of decreasing it when $r_2 < r_{\text{cr}}$.

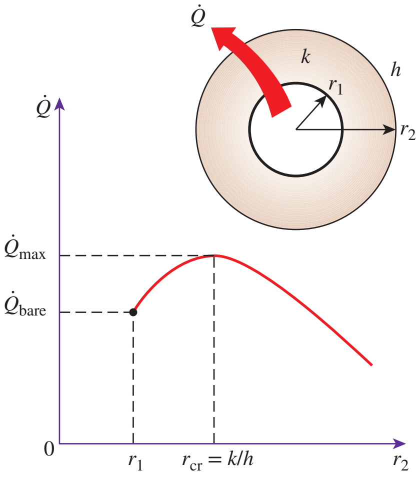


FIGURE 17–31
The variation of heat transfer rate with the outer radius of the insulation r_2 when $r_1 < r_{\text{cr}}$.

The important question to answer at this point is whether we need to be concerned about the critical radius of insulation when insulating hot-water pipes or even hot-water tanks. Should we always check and make sure that the outer radius of insulation sufficiently exceeds the critical radius before we install any insulation? Probably not, as explained here.

The value of the critical radius r_{cr} is largest when k is large and h is small. Noting that the lowest value of h encountered in practice is about 5 W/m^2·K for the case of natural convection of gases, and that the thermal conductivity of common insulating materials is about 0.05 W/m·K, the largest value of the critical radius we are likely to encounter is

$$r_{\text{cr, max}} = \frac{k_{\text{max, insulation}}}{h_{\text{min}}} \approx \frac{0.05 \text{ W/m·K}}{5 \text{ W/m}^2\text{·K}} = 0.01 \text{ m} = 1 \text{ cm}$$

This value would be even smaller when the radiation effects are considered. The critical radius would be much less in forced convection, often less than 1 mm, because of much larger h values associated with forced convection. Therefore, we can insulate hot-water or steam pipes freely without worrying about the possibility of increasing the heat transfer by insulating the pipes.

The radius of electric wires may be smaller than the critical radius. Therefore, the plastic electrical insulation may actually *enhance* the heat transfer from electric wires and thus keep their steady operating temperatures at lower and thus safer levels.

The discussions above can be repeated for a sphere, and it can be shown in a similar manner that the critical radius of insulation for a spherical shell is

$$r_{\text{cr, sphere}} = \frac{2k}{h} \tag{17–51}$$

where k is the thermal conductivity of the insulation and h is the convection heat transfer coefficient on the outer surface.

EXAMPLE 17–9 Heat Loss from an Insulated Electric Wire

A 3-mm-diameter and 5-m-long electric wire is tightly wrapped with a 2-mm-thick plastic cover whose thermal conductivity is $k = 0.15$ W/m·K. Electrical measurements indicate that a current of 10 A passes through the wire, and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30°\text{C}$ with a heat transfer coefficient of $h = 12$ W/m^2·K, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.

SOLUTION An electric wire is tightly wrapped with a plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient incorporates the radiation effects, if any.

Properties The thermal conductivity of plastic is given to be $k = 0.15$ W/m·K.

Analysis Heat is generated in the wire, and its temperature rises as a result of resistance heating. We assume heat is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined to be

$$\dot{Q} = \dot{W}_e = \mathbf{V}I = (8 \text{ V})(10 \text{ A}) = 80 \text{ W}$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in Fig. 17–32. The values of these two resistances are

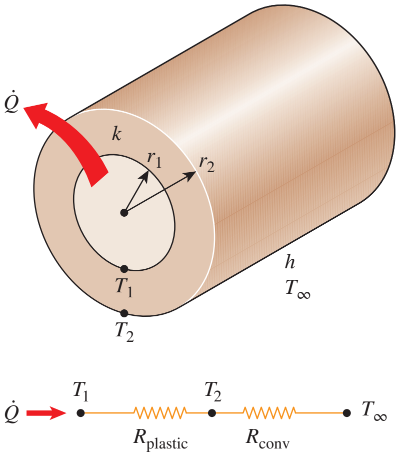


FIGURE 17–32
Schematic for Example 17–9.

$$A_2 = (2\pi r_2)\,L = 2\pi(0.0035\text{ m})(5\text{ m}) = 0.110\text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_2} = \frac{1}{(12\text{ W/m}^2\cdot\text{K})(0.110\text{ m}^2)} = 0.76^\circ\text{C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15\text{ W/m}\cdot\text{K})(5\text{ m})} = 0.18^\circ\text{C/W}$$

and therefore

$$R_{\text{total}} = R_{\text{plastic}} + R_{\text{conv}} = 0.76 + 0.18 = 0.94^\circ\text{C/W}$$

Then the interface temperature can be determined from

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \quad\longrightarrow\quad \begin{aligned} T_1 &= T_\infty + \dot{Q}R_{\text{total}} \\ &= 30^\circ\text{C} + (80\text{ W})(0.94^\circ\text{C/W}) = \mathbf{105^\circ C} \end{aligned}$$

Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation.

To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover. It is determined from Eq. 17–50 to be

$$r_{\text{cr}} = \frac{k}{h} = \frac{0.15\text{ W/m}\cdot\text{K}}{12\text{ W/m}^2\cdot\text{K}} = 0.0125\text{ m} = 12.5\text{ mm}$$

which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will *enhance* heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer $\dot{Q}$ will *increase* when the interface temperature T_1 is held constant, or T_1 will *decrease* when $\dot{Q}$ is held constant, which is the case here.

Discussion It can be shown by repeating the calculations for a 4-mm-thick plastic cover that the interface temperature drops to 90.6°C when the thickness of the plastic cover is doubled. It can also be shown in a similar manner that the interface reaches a minimum temperature of 83°C when the outer radius of the plastic cover equals the critical radius.

17–6 ▪ HEAT TRANSFER FROM FINNED SURFACES

The rate of heat transfer from a surface at a temperature T_s to the surrounding medium at T_∞ is given by Newton's law of cooling as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

where A_s is the heat transfer surface area and h is the convection heat transfer coefficient. When the temperatures T_s and T_∞ are fixed by design considerations, as is often the case, there are *two ways* to increase the rate of heat transfer: to increase the *convection heat transfer coefficient* h or to increase the *surface area* A_s. Increasing h may require the installation of a pump or fan, or replacing the existing one with a larger one, but this approach may or may not be practical. Besides, it may not be adequate. The alternative is to increase the surface area by attaching to the surface *extended surfaces* called *fins* made of highly conductive materials such as aluminum. Finned surfaces are manufactured by extruding, welding, or wrapping a thin metal sheet on a surface. Fins enhance heat transfer from a surface by exposing a larger surface area to convection and radiation.

An interesting application of fins from about 150 million years ago, the Jurassic era, is shown in Fig. 17–33. The dinosaur stegosaurus lived during this era, and

FIGURE 17–33
Presumed cooling fins on dinosaur stegosaurus.
Christian Darkin/Alamy

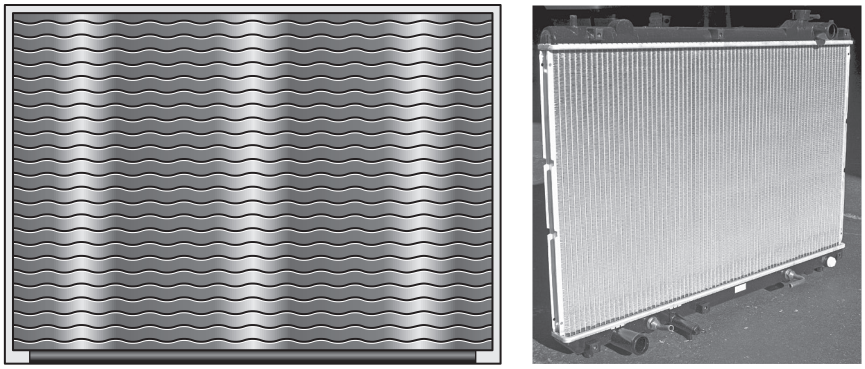


FIGURE 17–34
The thin plate fins of a car radiator greatly increase the rate of heat transfer to the air.
Left: Yunus Cengel
right: McGraw-Hill Education/Christopher Kerrigan

it had two rows of big (and bizarre) bony plates down its back. For a long time, scientists thought that the plates were some kind of armor to protect the vegetarian from predators. We now know that a lot of blood flowed through the plates, and they may have acted like a car radiator. The heart pumped blood through the plates, and the plates acted like cooling fins to cool the blood down.

Finned surfaces are commonly used in practice to enhance heat transfer, and they often increase the rate of heat transfer from a surface severalfold. The car radiator shown in Fig. 17–34 is an example of a finned surface. The closely packed thin metal sheets attached to the hot-water tubes increase the surface area for convection and thus the rate of convection heat transfer from the tubes to the air many times. There are a variety of innovative fin designs available in the market, and they seem to be limited only by imagination (Fig. 17–35).

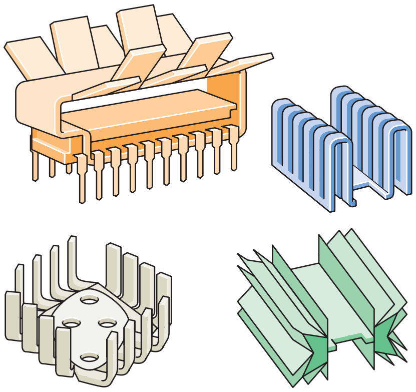


FIGURE 17–35
Some innovative fin designs.

In the analysis of fins, we consider *steady* operation with *no heat generation* in the fin, and we assume the thermal conductivity k of the material to remain constant. We also assume the convection heat transfer coefficient h to be *constant* and *uniform* over the entire surface of the fin for convenience in the analysis. We recognize that the convection heat transfer coefficient h, in general, varies along the fin as well as along its circumference, and its value at a point is a strong function of the *fluid motion* at that point. The value of h is usually much lower at the *fin base* than it is at the *fin tip* because the fluid is surrounded by solid surfaces near the base, which seriously disrupt its motion to the point of "suffocating" it, while the fluid near the fin tip has little contact with a solid surface and thus encounters little resistance to flow. Therefore, adding too many fins on a surface may actually decrease the overall heat transfer when the decrease in h offsets any gain resulting from the increase in the surface area.

Fin Equation

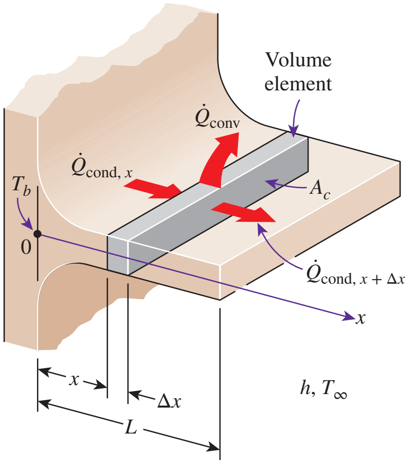


FIGURE 17–36
Volume element of a fin at location x having a length of Δx, cross-sectional area of A_c, and perimeter of p.

Consider a volume element of a fin at location x having a length of Δx, a cross-sectional area of A_c, and a perimeter of p, as shown in Fig. 17–36. Under steady conditions, the energy balance on this volume element can be expressed as

$$\left(\begin{array}{c}\text{Rate of } \textit{heat} \\ \textit{conduction into} \\ \text{the element at } x\end{array}\right) = \left(\begin{array}{c}\text{Rate of } \textit{heat} \\ \textit{conduction from} \text{ the} \\ \text{element at } x + \Delta x\end{array}\right) + \left(\begin{array}{c}\text{Rate of } \textit{heat} \\ \textit{convection from} \\ \text{the element}\end{array}\right)$$

or

$$\dot{Q}_{\text{cond},\,x} = \dot{Q}_{\text{cond},\,x+\Delta x} + \dot{Q}_{\text{conv}}$$

where

$$\dot{Q}_{\text{conv}} = h(p\,\Delta x)(T - T_\infty)$$

Substituting and dividing by Δx, we obtain

$$\frac{\dot{Q}_{\text{cond}, x+\Delta x} - \dot{Q}_{\text{cond}, x}}{\Delta x} + hp(T - T_\infty) = 0 \tag{17–52}$$

Taking the limit as $\Delta x \rightarrow 0$ gives

$$\frac{d\dot{Q}_{\text{cond}}}{dx} + hp(T - T_\infty) = 0 \tag{17–53}$$

From Fourier's law of heat conduction we have

$$\dot{Q}_{\text{cond}} = -kA_c \frac{dT}{dx} \tag{17–54}$$

where A_c is the cross-sectional area of the fin at location x. Substitution of this relation into Eq. 17–53 gives the differential equation governing heat transfer in fins,

$$\frac{d}{dx}\left(kA_c \frac{dT}{dx}\right) - hp(T - T_\infty) = 0 \tag{17–55}$$

In general, the cross-sectional area A_c and the perimeter p of a fin vary with x, which makes this differential equation difficult to solve. In the special case of *constant cross section* and *constant thermal conductivity,* the differential equation Eq. 17–55 reduces to

$$\frac{d^2T}{dx^2} - \frac{hp}{kA_c}(T - T_\infty) = 0 \quad \text{or} \quad \frac{d^2\theta}{dx^2} - m^2\theta = 0 \tag{17–56}$$

where

$$m^2 = \frac{hp}{kA_c} \tag{17–57}$$

and $\theta = T - T_\infty$ is the *temperature excess.* At the fin base we have $\theta_b = T_b - T_\infty$.

Equation 17–56 is a linear, homogeneous, second-order differential equation with constant coefficients. A fundamental theory of differential equations states that such an equation has two linearly independent solution functions, and its general solution is the linear combination of those two solution functions. A careful examination of the differential equation reveals that subtracting a constant multiple of the solution function θ from its second derivative yields zero. Thus we conclude that the function θ and its second derivative must be *constant multiples* of each other. The only functions whose derivatives are constant multiples of the functions themselves are the *exponential functions* (or a linear combination of exponential functions such as sine and cosine hyperbolic functions). Therefore, the solution functions of the preceding differential equation are the exponential functions e^{-mx} or e^{mx} or constant multiples of them. This can be verified by direct substitution. For example, the second derivative of e^{-mx} is m^2e^{-mx}, and its substitution into Eq. 17–56 yields zero. Therefore, the general solution of the differential equation Eq. 17–56 is

$$\theta(x) = C_1e^{mx} + C_2e^{-mx} \tag{17–58}$$

where C_1 and C_2 are arbitrary constants whose values are to be determined from the boundary conditions at the base and at the tip of the fin. Note that we need only two conditions to determine C_1 and C_2 uniquely.

The temperature of the plate to which the fins are attached is normally known in advance. Therefore, at the fin base we have a *specified temperature* boundary condition, expressed as

Boundary condition at fin base:

$$\theta(0) = \theta_b = T_b - T_\infty \tag{17–59}$$

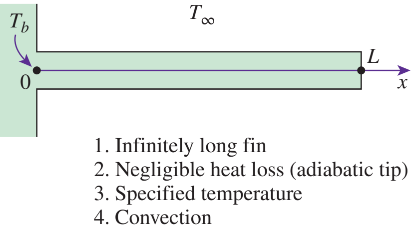


FIGURE 17–37
Boundary conditions at the fin base and the fin tip.

At the fin tip we have several possibilities, including infinitely long fins, negligible heat loss (idealized as an adiabatic tip), specified temperature, and convection (Fig. 17–37). Next, we consider each case separately.

1 Infinitely Long Fin ($T_{\text{fin tip}} = T_\infty$)

For a sufficiently long fin of *uniform* cross section (A_c = constant), the temperature of the fin at the fin tip approaches the environment temperature T_∞ and thus θ approaches zero. That is,

Boundary condition at fin tip: $$\theta(L) = T(L) - T_\infty = 0 \quad \text{as} \quad L \to \infty$$

This condition is satisfied by the function e^{-mx}, but not by the other prospective solution function e^{mx} since it tends to infinity as x gets larger. Therefore, the general solution in this case will consist of a constant multiple of e^{-mx}. The value of the constant multiple is determined from the requirement that at the fin base where $x = 0$, the value of θ is θ_b. Noting that $e^{-mx} = e^0 = 1$, the proper value of the constant is θ_b, and the solution function we are looking for is $\theta(x) = \theta_b e^{-mx}$. This function satisfies the differential equation as well as the requirements that the solution reduce to θ_b at the fin base and approach zero at the fin tip for large x. Noting that $\theta = T - T_\infty$ and $m = \sqrt{hp/kA_c}$, the variation of temperature along the fin in this case can be expressed as

Very long fin: $$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-x\sqrt{hp/kA_c}} \qquad \textbf{(17–60)}$$

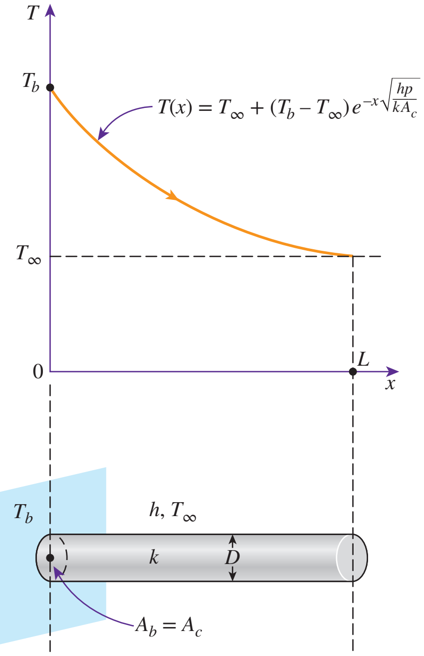
Note that the temperature along the fin in this case decreases *exponentially* from T_b to T_∞, as shown in Fig. 17–38. The steady rate of *heat transfer* from the entire fin can be determined from Fourier's law of heat conduction

Very long fin: $$\dot{Q}_{\text{long fin}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty) \qquad \textbf{(17–61)}$$

where p is the perimeter, A_c is the cross-sectional area of the fin, and x is the distance from the fin base. Alternatively, the rate of heat transfer from the fin could also be determined by considering heat transfer from a differential volume element of the fin and integrating it over the entire surface of the fin:

$$\dot{Q}_{\text{fin}} = \int_{A_{\text{fin}}} h[T(x) - T_\infty]\, dA_{\text{fin}} = \int_{A_{\text{fin}}} h\theta(x)\, dA_{\text{fin}} \qquad \textbf{(17–62)}$$

The two approaches described are equivalent and give the same result since, under steady conditions, the heat transfer from the exposed surfaces of the fin is equal to the heat transfer to the fin at the base (Fig. 17–39).



($p = \pi D$, $A_c = \pi D^2/4$ for a cylindrical fin)

FIGURE 17–38
A long circular fin of uniform cross section and the variation of temperature along it.

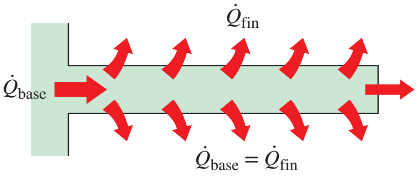


FIGURE 17–39
Under steady conditions, heat transfer from the exposed surfaces of the fin is equal to heat conduction to the fin at the base.

2 Negligible Heat Loss from the Fin Tip (Adiabatic Fin Tip, $\dot{Q}_{\text{fin tip}} = 0$)

Fins are not likely to be so long that their temperature approaches the surrounding temperature at the tip. A more realistic situation is for heat transfer from the fin tip to be negligible since the heat transfer from the fin is proportional to its surface area, and the surface area of the fin tip is usually a negligible fraction of the total fin area. Then the fin tip can be assumed to be adiabatic, and the condition at the fin tip can be expressed as

Boundary condition at fin tip: $$\left.\frac{d\theta}{dx}\right|_{x=L} = 0 \qquad \textbf{(17–63)}$$

The condition at the fin base remains the same as expressed in Eq. 17–59. The application of the boundary conditions given by Eqs. 17–59 and 17–63 on the general solution (Eq. 17–58) requires that $\theta(0) = \theta_b = C_1 + C_2$ and $mC_1e^{mL} - mC_2e^{-mL} = 0$, respectively. Solving these two equations simultaneously for C_1 and C_2 yields $C_1 = \theta_b/(1 + e^{2mL})$ and $C_2 = \theta_b/(1 + e^{-2mL})$. Substituting the relations for C_1 and C_2 into Eq. 17–58 and using the definition of the hyperbolic cosine function $\cosh x = (e^x + e^{-x})/2$ gives the desired relation for the temperature distribution:

Adiabatic fin tip:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL} \tag{17–64}$$

The rate of heat transfer from the fin can be determined again from Fourier's law of heat conduction:

Adiabatic fin tip:

$$\dot{Q}_{\text{adiabatic tip}} = -kA_c \frac{dT}{dx}\bigg|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty) \tanh mL \tag{17–65}$$

where the equation for the hyperbolic tangent function is

$$\tanh x = \sinh x/\cosh x = (e^x - e^{-x})/(e^x + e^{-x}).$$

Note that the heat transfer relations for the very long fin and the fin with negligible heat loss at the tip differ by the factor tanh *mL*, which approaches 1 as *L* becomes very large.

3 Specified Temperature ($T_{\text{fin, tip}} = T_L$)

In this case the temperature at the end of the fin (the fin tip) is fixed at a specified temperature T_L. This case could be considered a generalization of the case of *Infinitely Long Fin* where the fin tip temperature was fixed at T_∞. The condition at the fin tip for this case is

Boundary condition at fin tip:

$$\theta(L) = \theta_L = T_L - T_\infty \tag{17–66}$$

The fin base boundary condition remains the same as given in Eq. 17–59. Applying the boundary conditions given by Eqs. 17–59 and 17–66 on the general solution (Eq. 17–58) gives, after some lengthy algebra and using the definition of the hyperbolic sine function, $\sinh x = (e^x - e^{-x})/2$, the desired temperature distribution:

Specified fin tip temperature:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{[(T_L - T_\infty)/(T_b - T_\infty)] \sinh mx + \sinh m(L - x)}{\sinh mL} \tag{17–67}$$

Using Fourier's law of heat conduction, the *rate of heat transfer* from the fin is

Specified fin tip temperature:

$$\begin{aligned}\dot{Q}_{\text{specified temp.}} &= -kA_c \frac{dT}{dx}\bigg|_{x=0} \\ &= \sqrt{hpkA_c}(T_b - T_\infty)\, \frac{\cosh mL - [(T_L - T_\infty)/(T_b - T_\infty)]}{\sinh mL}\end{aligned} \tag{17–68}$$

Note that Eqs. 17–67 and 17–68 reduce to Eqs. 17–60 and 17–61 for the case of *infinitely long fin* ($L \to \infty$).

4 Convection from Fin Tip

The fin tips, in practice, are exposed to the surroundings, and thus the proper boundary condition for the fin tip is convection that may also include the effects of radiation. Consider the case of convection only at the tip. The condition at the fin tip can be obtained from an energy balance at the fin tip ($\dot{Q}_{\text{cond}} = \dot{Q}_{\text{conv}}$). That is,

Boundary condition at fin tip:

$$-kA_c \frac{dT}{dx}\bigg|_{x=L} = hA_c[T(L) - T_\infty] \tag{17–69}$$

The boundary condition at the fin base is Eq. 17–59, which is the same as the three previous cases. Substituting the two boundary conditions given by Eqs. 17–59 and 17–69 into the general solution (Eq. 17–58), it may be shown after some lengthy manipulation that the temperature distribution is

Convection from fin tip:
$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh m(L-x)+(h/mk)\sinh m(L-x)}{\cosh mL+(h/mk)\sinh mL} \tag{17–70}$$

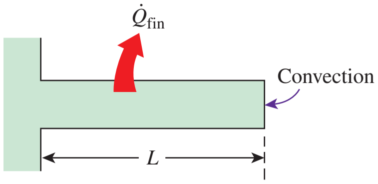
The *rate of heat transfer* from the fin can be found by substituting the temperature gradient at the base of the fin, obtained from Eq. 17–70, into Fourier's law of heat conduction. The result is

Convection from fin tip:

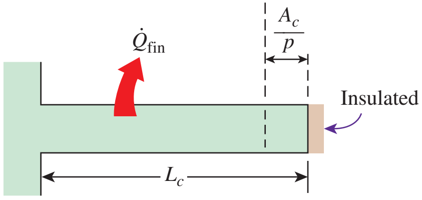
$$\begin{aligned}\dot{Q}_{\text{convection}} &= -kA_c\left.\frac{dT}{dx}\right|_{x=0}\\ &= \sqrt{hpkA_c}\,(T_b-T_\infty)\,\frac{\sinh mL+(h/mk)\cosh mL}{\cosh mL+(h/mk)\sinh mL}\end{aligned} \tag{17–71}$$

The solution to the general fin equation for the case of *convection from fin tip* is rather complex. An approximate, yet practical and accurate, way of accounting for the loss from the fin tip is to replace the *fin length L* in the relation for the *insulated tip* case with a **corrected fin length** defined as (Fig. 17–40)

Corrected fin length:
$$L_c = L+\frac{A_c}{p} \tag{17–72}$$



(*a*) Actual fin with convection at the tip



(*b*) Equivalent fin with insulated tip

FIGURE 17–40
Corrected fin length L_c is defined such that heat transfer from a fin of length L_c with insulated tip is equal to heat transfer from the actual fin of length L with convection at the fin tip.

where A_c is the cross-sectional area and p is the perimeter of the fin at the tip. Multiplying the preceding relation by the perimeter gives $A_{\text{corrected}} = A_{\text{fin(lateral)}} + A_{\text{tip}}$, which indicates that the fin area determined using the corrected length is equivalent to the sum of the lateral fin area plus the fin tip area.

The corrected length approximation gives very good results when the variation of temperature near the fin tip is small (which is the case when $mL \geq 1$) and the heat transfer coefficient at the fin tip is about the same as that at the lateral surface of the fin. Therefore, *fins subjected to convection at their tips can be treated as fins with insulated tips by replacing the actual fin length with the corrected length in Eqs. 17–64 and 17–65.*

Using the proper relations for A_c and p, the corrected lengths for rectangular and cylindrical fins are easily determined to be

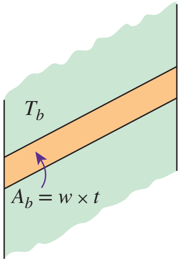
$$L_{c,\text{ rectangular fin}} = L+\frac{t}{2} \quad \text{and} \quad L_{c,\text{ cylindrical fin}} = L+\frac{D}{4}$$

where t is the thickness of the rectangular fins and D is the diameter of the cylindrical fins.

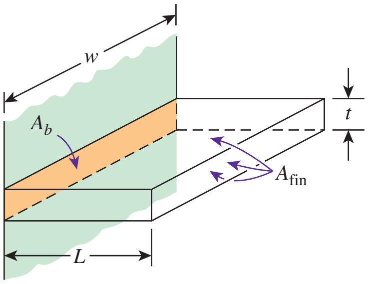
Fin Efficiency

Consider the surface of a *plane wall* at temperature T_b exposed to a medium at temperature T_∞. Heat is lost from the surface to the surrounding medium by convection with a heat transfer coefficient of h. Disregarding radiation or accounting for its contribution in the convection coefficient h, heat transfer from a surface area A_s is expressed as $\dot{Q} = hA_s(T_s - T_\infty)$.

Now let us consider a fin of constant cross-sectional area $A_c = A_b$ and length L that is attached to the surface with perfect contact (Fig. 17–41). This time heat is transferred from the surface to the fin *by conduction* and from the fin to the surrounding medium *by convection* with the same heat transfer coefficient h. The temperature of the fin is T_b at the fin base and gradually decreases toward the fin tip. Convection from the fin surface causes the temperature at any cross section to drop somewhat from the midsection toward the outer surfaces. However, the cross-sectional area of the fins is usually very small, and thus the temperature



(*a*) Surface without fins



(*b*) Surface with a fin

$$\begin{aligned}A_{\text{fin}} &= 2\times w\times L + w\times t\\ &\cong 2\times w\times L\end{aligned}$$

FIGURE 17–41
Fins enhance heat transfer from a surface by enhancing surface area.

at any cross section can be considered to be uniform. Also, the fin tip can be assumed for convenience and simplicity to be adiabatic by using the corrected length for the fin instead of the actual length.

In the limiting case of *zero thermal resistance* or *infinite thermal conductivity* ($k \to \infty$), the temperature of the fin is uniform at the base value of T_b. The heat transfer from the fin is *maximum* in this case and can be expressed as

$$\dot{Q}_{\text{fin, max}} = hA_{\text{fin}}(T_b - T_\infty) \tag{17–73}$$

In reality, however, the temperature of the fin drops along the fin, and thus the heat transfer from the fin is less because of the decreasing temperature difference $T(x) - T_\infty$ toward the fin tip, as shown in Fig. 17–42. To account for the effect of this decrease in temperature on heat transfer, we define **fin efficiency** as

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\begin{array}{c}\text{Ideal heat transfer rate from the fin}\\ \text{if the entire fin were at base temperature}\end{array}} \tag{17–74}$$

or

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) \tag{17–75}$$

where A_{fin} is the total surface area of the fin. This relation enables us to determine the heat transfer from a fin when its efficiency is known. For the cases of constant cross section of *very long fins* and *fins with adiabatic tips,* the fin efficiency can be expressed as

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}(T_b - T_\infty)}{hA_{\text{fin}}(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{mL} \tag{17–76}$$

and

$$\eta_{\text{adiabatic tip}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}(T_b - T_\infty)\tanh mL}{hA_{\text{fin}}(T_b - T_\infty)} = \frac{\tanh mL}{mL} \tag{17–77}$$

since $A_{\text{fin}} = pL$ for fins with constant cross section. Equation 17–77 can also be used for fins subjected to convection provided that the fin length L is replaced by the corrected length L_c.

Table 17–3 provides fin efficiency relations for fins with uniform and nonuniform cross section. For fins with nonuniform profile, Eq. 17–56 is no longer valid, and the general form of the differential equation governing heat transfer in fins of arbitrary shape, Eq. 17–55, must be used. For these cases the solution is no longer in the form of simple exponential or hyperbolic functions. The mathematical functions I and K that appear in some of these relations are the *modified Bessel functions,* and their values are given in Table 17–4. Efficiencies are plotted in Fig. 17–43 for fins on a *plain surface* and in Fig. 17–44 for *circular fins* of constant thickness. For most fins of constant thickness encountered in practice, the fin thickness t is too small relative to the fin length $L,$ and thus the fin tip area is negligible.

Note that fins with triangular and parabolic profiles contain less material and are more efficient than the ones with rectangular profiles, and thus are more suitable for applications requiring minimum weight such as space applications.

An important consideration in the design of finned surfaces is the selection of the proper *fin length L.* Normally the *longer* the fin, the *larger* the heat transfer area and thus the *higher* the rate of heat transfer from the fin. But also the larger the fin, the bigger the mass, the higher the price, and the larger the fluid friction. Therefore, increasing the length of the fin beyond a certain value cannot be

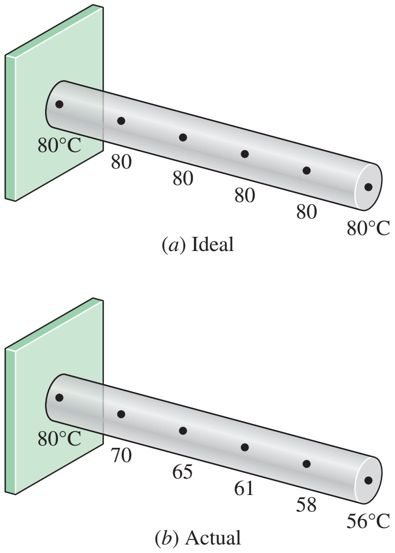


FIGURE 17–42
Ideal and actual temperature distribution along a fin.

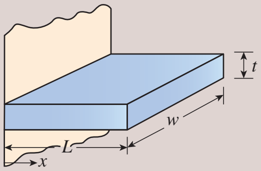
TABLE 17–3

Efficiency and surface areas of common fin configurations

Straight rectangular fins

$m = \sqrt{2h/kt}$
$L_c = L + t/2$
$A_{\text{fin}} = 2wL_c$

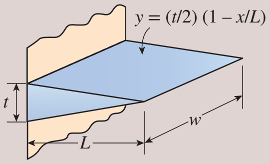
$$\eta_{\text{fin}} = \frac{\tanh mL_c}{mL_c}$$



Straight triangular fins

$m = \sqrt{2h/kt}$
$A_{\text{fin}} = 2w\sqrt{L^2 + (t/2)^2}$

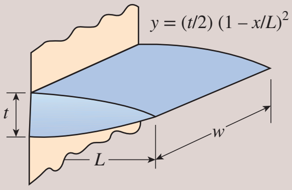
$$\eta_{\text{fin}} = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$



Straight parabolic fins

$m = \sqrt{2h/kt}$
$A_{\text{fin}} = wL[C_1 + (L/t)\ln(t/L + C_1)]$
$C_1 = \sqrt{1 + (t/L)^2}$

$$\eta_{\text{fin}} = \frac{2}{1 + \sqrt{(2mL)^2 + 1}}$$

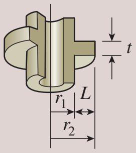


Circular fins of rectangular profile

$m = \sqrt{2h/kt}$
$r_{2c} = r_2 + t/2$
$A_{\text{fin}} = 2\pi(r_{2c}^2 - r_1^2)$

$$\eta_{\text{fin}} = C_2\frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$$

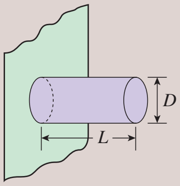
$$C_2 = \frac{2r_1/m}{r_{2c}^2 - r_1^2}$$



Pin fins of rectangular profile

$m = \sqrt{4h/kD}$
$L_c = L + D/4$
$A_{\text{fin}} = \pi D L_c$

$$\eta_{\text{fin}} = \frac{\tanh mL_c}{mL_c}$$

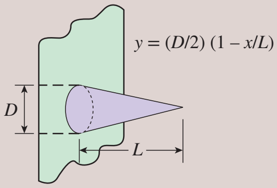


Pin fins of triangular profile

$m = \sqrt{4h/kD}$
$A_{\text{fin}} = \dfrac{\pi D}{2}\sqrt{L^2 + (D/2)^2}$

$$\eta_{\text{fin}} = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$$

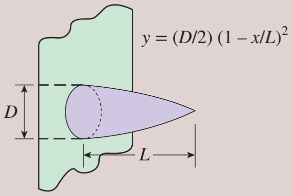
$I_2(x) = I_0(x) - (2/x)\,I_1(x)$ where $x = 2mL$



Pin fins of parabolic profile

$m = \sqrt{4h/kD}$
$A_{\text{fin}} = \dfrac{\pi L^3}{8D}\left[C_3C_4 - \dfrac{L}{2D}\ln[(2DC_4/L) + C_3]\right]$
$C_3 = 1 + 2(D/L)^2$
$C_4 = \sqrt{1 + (D/L)^2}$

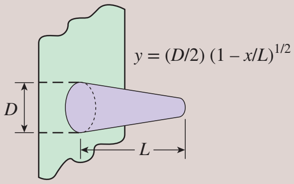
$$\eta_{\text{fin}} = \frac{2}{1 + \sqrt{(2mL/3)^2 + 1}}$$



Pin fins of parabolic profile (blunt tip)

$m = \sqrt{4h/kD}$
$A_{\text{fin}} = \dfrac{\pi D^4}{96L^2}\{[16(L/D)^2 + 1]^{3/2} - 1\}$

$$\eta_{\text{fin}} = \frac{3}{2mL}\frac{I_1(4mL/3)}{I_0(4mL/3)}$$



justified unless the added benefits outweigh the added cost. Also, the fin efficiency decreases with increasing fin length because of the decrease in fin temperature with length. Fin lengths that cause the fin efficiency to drop below 60 percent usually cannot be justified economically and should be avoided. The efficiency of most fins used in practice is above 90 percent.

Fin Effectiveness

Fins are used to *enhance* heat transfer, and the use of fins on a surface cannot be recommended unless the enhancement in heat transfer justifies the added cost and complexity associated with the fins. In fact, there is no assurance that adding fins on a surface will *enhance* heat transfer. The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case. The performance of fins is expressed in terms of the *fin effectiveness* ε_{fin} defined as (Fig. 17–45)

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of } \textit{base area } A_b}{\text{Heat transfer rate from the surface of } \textit{area } A_b} \tag{17–78}$$

Hence, A_b is the cross-sectional area of the fin at the base and $\dot{Q}_{\text{no fin}}$ represents the rate of heat transfer from this area if no fins are attached to the surface. An effectiveness of $\varepsilon_{\text{fin}} = 1$ indicates that the addition of fins to the surface does not affect heat transfer at all. That is, heat conducted to the fin through the base area A_b is equal to the heat transferred from the same area A_b to the surrounding medium. An effectiveness of $\varepsilon_{\text{fin}} < 1$ indicates that the fin actually acts as *insulation,* slowing down the heat transfer from the surface. This situation can occur when fins made of low thermal conductivity materials are used. An effectiveness of $\varepsilon_{\text{fin}} > 1$ indicates that fins are *enhancing* heat transfer from the surface, as they should. However, the use of fins cannot be justified unless ε_{fin} is sufficiently larger than 1. Finned surfaces are designed on the basis of *maximizing* effectiveness for a specified cost or *minimizing* cost for a desired effectiveness.

Note that both the fin efficiency and fin effectiveness are related to the performance of the fin, but they are different quantities. However, they are related to each other by

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \frac{A_{\text{fin}}}{A_b}\eta_{\text{fin}} \tag{17–79}$$

Therefore, the fin effectiveness can be determined easily when the fin efficiency is known, or vice versa.

The rate of heat transfer from a sufficiently *long* fin of *uniform* cross section under steady conditions is given by Eq. 17–61. Substituting this relation into Eq. 17–78, the effectiveness of such a long fin is determined to be

$$\varepsilon_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\sqrt{hpkA_c}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}} \tag{17–80}$$

since $A_c = A_b$ in this case. We can draw several important conclusions from the fin effectiveness relation above for consideration in the design and selection of the fins:

- The *thermal conductivity k* of the fin material should be as high as possible. Thus it is no coincidence that fins are made from metals, with copper, aluminum, and iron being the most common ones. Perhaps the most widely used fins are made of aluminum because of its low cost and weight and its resistance to corrosion.

TABLE 17–4

Modified Bessel functions of the first and second kinds*

x	$e^{-x}I_0(x)$	$e^{-x}I_1(x)$	$e^{-x}K_0(x)$	$e^{-x}K_1(x)$
0.0	1.0000	0.0000	∞	∞
0.2	0.8269	0.0823	2.1408	5.8334
0.4	0.6974	0.1368	1.6627	3.2587
0.6	0.5993	0.1722	1.4167	2.3739
0.8	0.5241	0.1945	1.2582	1.9179
1.0	0.4658	0.2079	1.1445	1.6362
1.2	0.4198	0.2153	1.0575	1.4429
1.4	0.3831	0.2185	0.9881	1.3011
1.6	0.3533	0.2190	0.9309	1.1919
1.8	0.3289	0.2177	0.8828	1.1048
2.0	0.3085	0.2153	0.8416	1.0335
2.2	0.2913	0.2121	0.8057	0.9738
2.4	0.2766	0.2085	0.7740	0.9229
2.6	0.2639	0.2047	0.7459	0.8790
2.8	0.2528	0.2007	0.7206	0.8405
3.0	0.2430	0.1968	0.6978	0.8066
3.2	0.2343	0.1930	0.6770	0.7763
3.4	0.2264	0.1892	0.6580	0.7491
3.6	0.2193	0.1856	0.6405	0.7245
3.8	0.2129	0.1821	0.6243	0.7021
4.0	0.2070	0.1788	0.6093	0.6816
4.2	0.2016	0.1755	0.5953	0.6627
4.4	0.1966	0.1725	0.5823	0.6454
4.6	0.1919	0.1695	0.5701	0.6292
4.8	0.1876	0.1667	0.5586	0.6143
5.0	0.1835	0.1640	0.5478	0.6003
5.2	0.1797	0.1614	0.5376	0.5872
5.4	0.1762	0.1589	0.5280	0.5749
5.6	0.1728	0.1565	0.5188	0.5634
5.8	0.1697	0.1542	0.5101	0.5525
6.0	0.1667	0.1521	0.5019	0.5422
6.5	0.1598	0.1469	0.4828	0.5187
7.0	0.1537	0.1423	0.4658	0.4981
7.5	0.1483	0.1380	0.4505	0.4797
8.0	0.1434	0.1341	0.4366	0.4631
8.5	0.1390	0.1305	0.4239	0.4482
9.0	0.1350	0.1272	0.4123	0.4346
9.5	0.1313	0.1241	0.4016	0.4222
10.0	0.1278	0.1213	0.3916	0.4108

*Evaluated from EES using the mathematical functions Bessel_I(x) and Bessel_K(x)

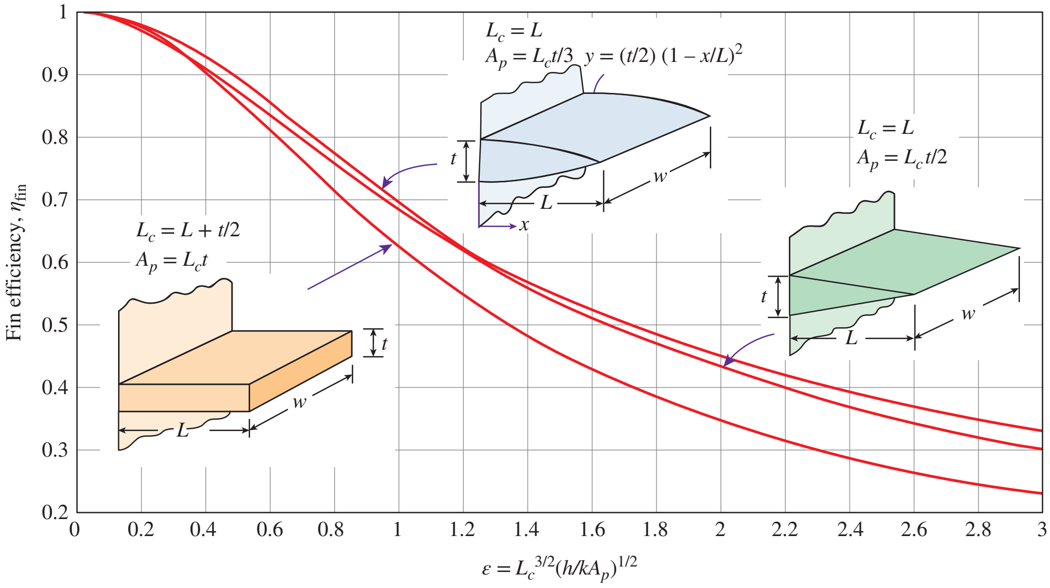


FIGURE 17–43

Efficiency of straight fins of rectangular, triangular, and parabolic profiles.

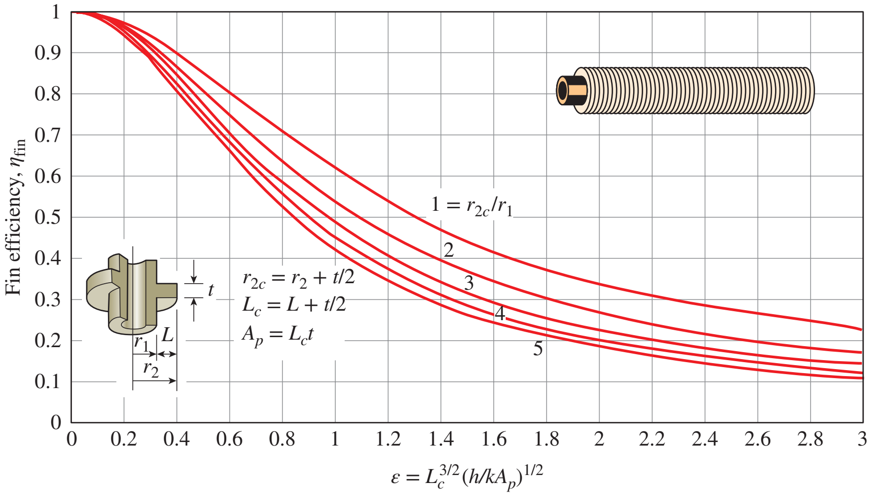


FIGURE 17–44

Efficiency of annular fins of constant thickness *t*.

- The ratio of the *perimeter* to the *cross-sectional area* of the fin p/A_c should be as high as possible. This criterion is satisfied by *thin* plate fins and *slender* pin fins.
- The use of fins is *most effective* in applications involving a *low convection heat transfer coefficient.* Thus, the use of fins is more easily justified when the medium is a *gas* instead of a liquid and the heat transfer is by *natural convection* instead of by forced convection. Therefore, it is no coincidence that in liquid-to-gas heat exchangers such as the car radiator, fins are placed on the *gas* side.

When determining the rate of heat transfer from a finned surface, we must consider the *unfinned portion* of the surface as well as the *fins.* Therefore, the rate of heat transfer for a surface containing *n* fins can be expressed as

$$\begin{aligned}\dot{Q}_{\text{total, fin}} &= \dot{Q}_{\text{unfin}} + \dot{Q}_{\text{fin}}\\ &= hA_{\text{unfin}}(T_b - T_\infty) + \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)\\ &= h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)\end{aligned} \tag{17–81}$$

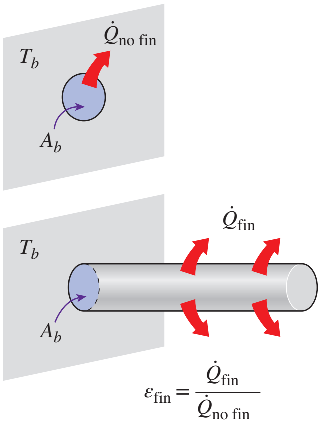


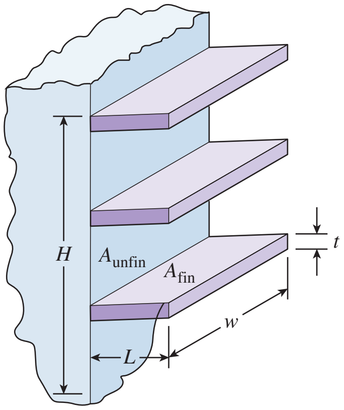
FIGURE 17–45

The effectiveness of a fin.

We can also define an **overall effectiveness** for a finned surface as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins,

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}(T_b - T_\infty)} = \frac{A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}}}{A_{\text{no fin}}} \tag{17–82}$$

where $A_{\text{no fin}}$ is the area of the surface when there are no fins, A_{fin} is the total surface area of all the fins on the surface, and A_{unfin} is the area of the unfinned portion of the surface (Fig. 17–46). Note that the overall fin effectiveness depends on the fin density (number of fins per unit length) as well as the effectiveness of the individual fins. The overall effectiveness is a better measure of the performance of a finned surface than the effectiveness of the individual fins.



$A_{\text{no fin}} = w \times H$
$A_{\text{unfin}} = w \times H - 3 \times (t \times w)$
$A_{\text{fin}} = 2 \times L \times w + t \times w$
$\cong 2 \times L \times w$ (one fin)

FIGURE 17–46
Various surface areas associated with a rectangular surface with three fins.

Proper Length of a Fin

An important step in the design of a fin is the determination of the appropriate length of the fin once the fin material and the fin cross section are specified. You may be tempted to think that the longer the fin, the larger the surface area and thus the higher the rate of heat transfer. Therefore, for maximum heat transfer, the fin should be infinitely long. However, the temperature drops along the fin exponentially and reaches the environment temperature at some length. The part of the fin beyond this length does not contribute to heat transfer since it is at the temperature of the environment, as shown in Fig. 17–47. Therefore, designing such an "extra long" fin is out of the question since it results in material waste, excessive weight, and increased size and thus increased cost with no benefit in return (in fact, such a long fin will hurt performance since it will suppress fluid motion and thus reduce the convection heat transfer coefficient). Fins that are so long that the temperature approaches the environment temperature cannot be recommended either since the little increase in heat transfer at the tip region cannot justify the disproportionate increase in the weight and cost.

To get a sense of the proper length of a fin, we compare heat transfer from a fin of finite length to heat transfer from an infinitely long fin under the same conditions. The ratio of these two heat transfers is

Heat transfer ratio:

$$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL}{\sqrt{hpkA_c}\,(T_b - T_\infty)} = \tanh mL \tag{17–83}$$

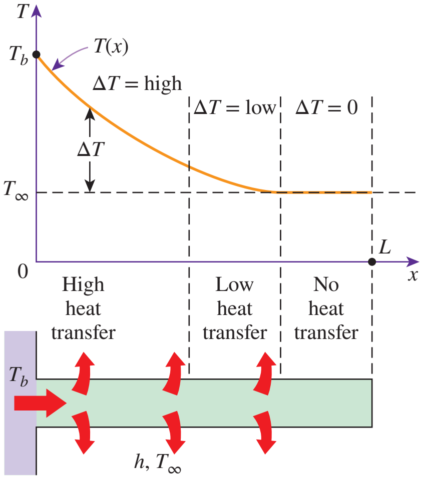


FIGURE 17–47
Because of the gradual temperature drop along the fin, the region near the fin tip makes little or no contribution to heat transfer.

Using a hand calculator, the values of tanh mL are evaluated for some values of mL, and the results are given in Table 17–5. We observe from the table that heat transfer from a fin increases with mL almost linearly at first, but the curve reaches a plateau later and reaches a value for the infinitely long fin at about $mL = 5$. Therefore, a fin whose length is $L = 5/m$ can be considered to be an infinitely long fin. We also observe that reducing the fin length by half in that case (from $mL = 5$ to $mL = 2.5$) causes a drop of just 1 percent in heat transfer. We certainly would not hesitate sacrificing 1 percent in heat transfer performance in return for 50 percent reduction in the size and possibly the cost of the fin. In practice, a fin length that corresponds to about $mL = 1$ will transfer 76.2 percent of the heat that can be transferred by an infinitely long fin, and thus it should offer a good compromise between heat transfer performance and the fin size.

A common approximation used in the analysis of fins is to assume the fin temperature to vary in one direction only (along the fin length) and the temperature variation along other directions is negligible. Perhaps you are wondering if this one-dimensional approximation is a reasonable one. This is certainly the case for fins made of thin metal sheets such as the fins on a car radiator, but we wouldn't be so sure for fins made of thick materials. Studies have shown that the error involved in one-dimensional fin analysis is negligible (less than about 1 percent) when

$$\frac{h\delta}{k} < 0.2$$

where δ is the characteristic thickness of the fin, which is taken to be the plate thickness t for rectangular fins and the diameter D for cylindrical ones.

Specially designed finned surfaces called *heat sinks,* which are commonly used in the cooling of electronic equipment, involve one-of-a-kind complex geometries, as shown in Table 17–6. The heat transfer performance of heat sinks is usually expressed in terms of their *thermal resistances* R in °C/W, which is defined as

$$\dot{Q}_{\text{fin}} = \frac{T_b - T_\infty}{R} = hA_{\text{fin}}\eta_{\text{fin}}(T_b - T_\infty) \qquad \textbf{(17–84)}$$

A small value of thermal resistance indicates a small temperature drop across the heat sink, and thus a high fin efficiency.

TABLE 17–5

The variation of heat transfer from a fin relative to that from an infinitely long fin

mL	$\dfrac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \tanh mL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

EXAMPLE 17–10 Maximum Power Dissipation of a Transistor

Power transistors that are commonly used in electronic devices consume large amounts of electric power. The failure rate of electronic components increases almost exponentially with operating temperature. As a rule of thumb, the failure rate of electronic components is halved for each 10°C reduction in the junction operating temperature. Therefore, the operating temperature of electronic components is kept below a safe level to minimize the risk of failure.

The sensitive electronic circuitry of a power transistor at the junction is protected by its case, which is a rigid metal enclosure. Heat transfer characteristics of a power transistor are usually specified by the manufacturer in terms of the case-to-ambient thermal resistance, which accounts for both the natural convection and radiation heat transfers.

The case-to-ambient thermal resistance of a power transistor that has a maximum power rating of 10 W is given to be 20°C/W. If the case temperature of the transistor is not to exceed 85°C, determine the power at which this transistor can be operated safely in an environment at 25°C.

SOLUTION The maximum power rating of a transistor whose case temperature is not to exceed 85°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 85°C.

Properties The case-to-ambient thermal resistance is given to be 20°C/W.

Analysis The power transistor and the thermal resistance network associated with it are shown in Fig. 17–48. We notice from the thermal resistance network that there is a single resistance of 20°C/W between the case at $T_c = 85$°C and the ambient at $T_\infty = 22$°C, and thus the rate of heat transfer is

$$\dot{Q} = \left(\frac{\Delta T}{R}\right)_{\text{case-ambient}} = \frac{T_c - T_\infty}{R_{\text{case-ambient}}} = \frac{(85 - 25)^\circ\text{C}}{20^\circ\text{C/W}} = \mathbf{3\ W}$$

Therefore, this power transistor should not be operated at power levels above 3 W if its case temperature is not to exceed 85°C.

Discussion This transistor can be used at higher power levels by attaching it to a heat sink (which lowers the thermal resistance by increasing the heat transfer surface area, as discussed in the next example) or by using a fan (which lowers the thermal resistance by increasing the convection heat transfer coefficient).

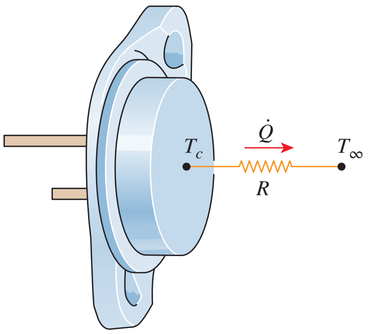
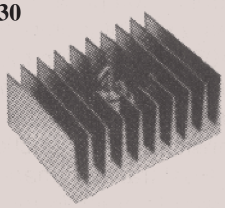
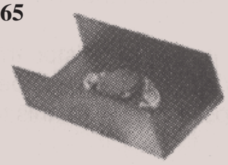
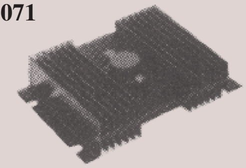
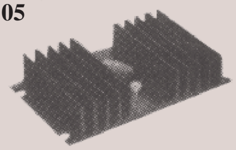
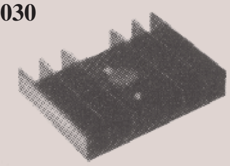


FIGURE 17–48
Schematic for Example 17–10.

TABLE 17–6

Combined natural convection and radiation thermal resistance of various heat sinks used in the cooling of electronic devices between the heat sink and the surroundings. All fins are made of aluminum 6063T-5, are black anodized, and are 76 mm (3 in) long. (Source: Vemaline Products, Inc.)

HS 5030		$R = 0.9$°C/W (vertical) $R = 1.2$°C/W (horizontal) Dimensions: 76 mm × 105 mm × 44 mm Surface area: 677 cm^2
HS 6065		$R = 5$°C/W Dimensions: 76 mm × 38 mm × 24 mm Surface area: 387 cm^2
HS 6071		$R = 1.4$°C/W (vertical) $R = 1.8$°C/W (horizontal) Dimensions: 76 mm × 92 mm × 26 mm Surface area: 968 cm^2
HS 6105		$R = 1.8$°C/W (vertical) $R = 2.1$°C/W (horizontal) Dimensions: 76 mm × 127 mm × 91 mm Surface area: 677 cm^2
HS 6115		$R = 1.1$°C/W (vertical) $R = 1.3$°C/W (horizontal) Dimensions: 76 mm × 102 mm × 25 mm Surface area: 929 cm^2
HS 7030		$R = 2.9$°C/W (vertical) $R = 3.1$°C/W (horizontal) Dimensions: 76 mm × 97 mm × 19 mm Surface area: 290 cm^2

EXAMPLE 17–11 Selecting a Heat Sink for a Transistor

A 25-W power transistor is to be cooled by attaching it to one of the commercially available heat sinks shown in Table 17–6. Select a heat sink that will allow the case temperature of the transistor not to exceed 55°C in the ambient air at 18°C.

SOLUTION A commercially available heat sink from Table 17–6 is to be selected to keep the case temperature of a transistor below 55°C.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 55°C. **3** The contact resistance between the transistor and the heat sink is negligible.

Analysis The rate of heat transfer from a 25-W transistor at full power is $\dot{Q} = 25$ W. The thermal resistance between the transistor attached to the heat sink and the ambient air for the specified temperature difference is determined to be

$$\dot{Q} = \frac{\Delta T}{R} \longrightarrow R = \frac{\Delta T}{\dot{Q}} = \frac{(55 - 18)°\text{C}}{25 \text{ W}} = 1.5°\text{C/W}$$

Therefore, the thermal resistance of the heat sink should be below 1.5°C/W. An examination of Table 17–6 reveals that the HS 5030 in both horizontal and vertical positions, HS 6071 in vertical position, and HS 6115 in both horizontal and vertical positions can be selected.

EXAMPLE 17–12 Heat Transfer from Fins of Variable Cross Section

Aluminum pin fins of parabolic profile with blunt tips are attached on a plane wall with surface temperature of 200°C (Fig. 17–49). Each fin has a length of 20 mm and a base diameter of 5 mm. The fins are exposed to an ambient air condition of 25°C, and the convection heat transfer coefficient is 50 W/m²·K. If the thermal conductivity of the fins is 240 W/m·K, determine the efficiency, heat transfer rate, and effectiveness of each fin.

SOLUTION The efficiency, heat transfer rate, and effectiveness of a pin fin of parabolic profile with a blunt tip are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fin is given as 240 W/m·K.

Analysis From Table 17–3, for pin fins of parabolic profile (blunt tip), we have

$$mL = \sqrt{\frac{4h}{kD}}L = \sqrt{\frac{4(50 \text{ W/m}^2\cdot\text{K})}{(240 \text{ W/m}\cdot\text{K})(0.005 \text{ m})}}(0.020 \text{ m}) = 0.2582$$

$$A_{\text{fin}} = \frac{\pi D^4}{96L^2}\left\{\left[16\left(\frac{L}{D}\right)^2 + 1\right]^{3/2} - 1\right\} = \frac{\pi(0.005 \text{ m})^4}{96(0.020 \text{ m})^2}\left\{\left[16\left(\frac{0.020 \text{ m}}{0.005 \text{ m}}\right)^2 + 1\right]^{3/2} - 1\right\}$$

$$= 2.106 \times 10^{-4} \text{ m}^2$$

$$\eta_{\text{fin}} = \frac{3}{2mL}\frac{I_1(4mL/3)}{I_0(4mL/3)} = \frac{3}{2(0.2582)}\frac{I_1[4(0.2582)/3]}{I_0[4(0.2582)/3]} = 5.8095\frac{I_1[0.3443]}{I_0[0.3443]}$$

The values of the Bessel functions corresponding to $x = 0.3443$ are determined from Table 17–4 to be $I_0 = 1.0350$ and $I_1 = 0.1716$. Substituting, the fin efficiency is determined to be

$$\eta_{\text{fin}} = 5.8095\frac{0.1716}{1.0350} = \mathbf{0.9632}$$

The heat transfer rate for a single fin is

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} h A_{\text{fin}}(T_b - T_\infty)$$
$$= (0.9632)(50 \text{ W/m}^2\cdot\text{K})(2.106 \times 10^{-4} \text{ m}^2)(200 - 25)°\text{C} = \mathbf{1.77 \text{ W}}$$

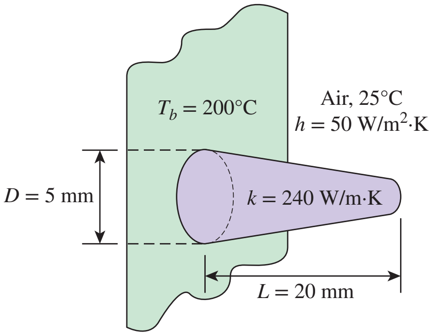


FIGURE 17–49
Schematic for Example 17–12.

The fin effectiveness is

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\dot{Q}_{\text{fin}}}{h(\pi D^2/4)(T_b - T_\infty)}$$

$$= \frac{1.77 \text{ W}}{(50 \text{ W/m}^2\cdot\text{K})\,[\pi(0.005 \text{ m})^2/4](200 - 25)°\text{C}}$$

$$= \mathbf{10.3}$$

That is, over a tenfold increase in heat transfer is achieved by using a pin fin in this case.

SUMMARY

One-dimensional heat transfer through a simple or composite body exposed to convection from both sides to media at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ can be expressed as

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

where R_{total} is the total thermal resistance between the two media. For a plane wall exposed to convection on both sides, the total resistance is expressed as

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

This relation can be extended to plane walls that consist of two or more layers by adding an additional resistance for each additional layer. The elementary thermal resistance relations can be expressed as follows:

Conduction resistance (plane wall): $R_{\text{wall}} = \dfrac{L}{kA}$

Conduction resistance (cylinder): $R_{\text{cyl}} = \dfrac{\ln(r_2/r_1)}{2\pi Lk}$

Conduction resistance (sphere): $R_{\text{sph}} = \dfrac{r_2 - r_1}{4\pi r_1 r_2 k}$

Convection resistance: $R_{\text{conv}} = \dfrac{1}{hA}$

Interface resistance: $R_{\text{interface}} = \dfrac{1}{h_c A} = \dfrac{R_c}{A}$

Radiation resistance: $R_{\text{rad}} = \dfrac{1}{h_{\text{rad}} A}$

where h_c is the thermal contact conductance, R_c is the thermal contact resistance, and the radiation heat transfer coefficient is defined as

$$h_{\text{rad}} = \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}})$$

Once the rate of heat transfer is available, the *temperature drop* across any layer can be determined from

$$\Delta T = \dot{Q}R$$

The thermal resistance concept can also be used to solve steady heat transfer problems involving parallel layers or combined series-parallel arrangements.

Adding insulation to a cylindrical pipe or a spherical shell increases the rate of heat transfer if the outer radius of the insulation is less than the *critical radius of insulation,* defined as

$$r_{\text{cr, cylinder}} = \frac{k_{\text{ins}}}{h}$$

$$r_{\text{cr, sphere}} = \frac{2k_{\text{ins}}}{h}$$

The effectiveness of an insulation is often given in terms of its *R-value,* the thermal resistance of the material for a unit surface area, expressed as

$$R\text{-value} = \frac{L}{k} \qquad \text{(flat insulation)}$$

where L is the thickness and k is the thermal conductivity of the material.

Finned surfaces are commonly used in practice to enhance heat transfer. Fins enhance heat transfer from a surface by exposing a larger surface area to convection. The temperature distribution along the fin is given by

Very long fin: $\dfrac{T(x) - T_\infty}{T_b - T_\infty} = e^{-x\sqrt{hp/kA_c}}$

Adiabatic fin tip: $\dfrac{T(x) - T_\infty}{T_b - T_\infty} = \dfrac{\cosh m(L - x)}{\cosh mL}$

Specified temperature at fin tip:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{[(T_L - T_\infty)/(T_b - T_\infty)]\sinh mx + \sinh m(L - x)}{\sinh mL}$$

Convection from fin tip:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x) + (h/mk)\sinh m(L - x)}{\cosh mL + (h/mk)\sinh mL}$$

where $m = \sqrt{hp/kA_c}$, p is the perimeter, and A_c is the cross-sectional area of the fin. The rates of heat transfer for these cases are given to be

Very long fin:

$$\dot{Q}_{\text{long fin}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}(T_b - T_\infty)$$

Adiabatic fin tip:

$$\dot{Q}_{\text{adiabatic tip}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}(T_b - T_\infty)\tanh mL$$

Specified temperature at fin tip:

$$\dot{Q}_{\text{specified temp.}} = \sqrt{hpkA_c}(T_b - T_\infty)\frac{\cosh mL - [(T_L - T_\infty)/(T_b - T_\infty)]}{\sinh mL}$$

Convection from the fin tip:

$$\dot{Q}_{\text{convection}} = \sqrt{hpkA_c}(T_b - T_\infty)\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

Fins exposed to convection at their tips can be treated as fins with adiabatic tips by using the corrected length $L_c = L + A_c/p$ instead of the actual fin length.

The temperature of a fin drops along the fin, and thus the heat transfer from the fin is less because of the decreasing temperature difference toward the fin tip. To account for the effect of this decrease in temperature on heat transfer, we define *fin efficiency* as

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$

When the fin efficiency is available, the rate of heat transfer from a fin can be determined from

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)$$

The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case and is expressed in terms of the *fin effectiveness* ε_{fin}, defined as

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of } \textit{base area } A_b}{\text{Heat transfer rate from the surface of } \textit{area } A_b}$$

Here, A_b is the cross-sectional area of the fin at the base, and $\dot{Q}_{\text{no fin}}$ represents the rate of heat transfer from this area if no fins are attached to the surface. The *overall effectiveness* for a finned surface is defined as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins,

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}}A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}(T_b - T_\infty)}$$

Fin efficiency and fin effectiveness are related to each other by

$$\varepsilon_{\text{fin}} = \frac{A_{\text{fin}}}{A_b}\eta_{\text{fin}}$$

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PROBLEMS*

Steady Heat Conduction in Plane Walls

17–1C Consider heat conduction through a wall of thickness L and area A. Under what conditions will the temperature distributions in the wall be a straight line?

17–2C Consider heat conduction through a plane wall. Does the energy content of the wall change during steady heat conduction? How about during transient conduction? Explain.

17–3C What does the thermal resistance of a medium represent?

17–4C Can we define the convection resistance for a unit surface area as the inverse of the convection heat transfer coefficient?

17–5C Consider steady heat transfer through the wall of a room in winter. The convection heat transfer coefficient at the outer surface of the wall is three times that of the inner surface as a result of the

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*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and SI users can ignore them. Problems with the icon are comprehensive in nature and are intended to be solved with appropriate software.

winds. On which surface of the wall do you think the temperature will be closer to the surrounding air temperature? Explain.

17–6C How is the combined heat transfer coefficient defined? What convenience does it offer in heat transfer calculations?

17–7C Why are the convection and the radiation resistances at a surface in parallel instead of being in series?

17–8C Consider steady one-dimensional heat transfer through a plane wall exposed to convection from both sides to environments at known temperatures $T_{\infty 1}$ and $T_{\infty 2}$ with known heat transfer coefficients h_1 and h_2. Once the rate of heat transfer $\dot{Q}$ has been evaluated, explain how you would determine the temperature of each surface.

17–9C Someone comments that a microwave oven can be viewed as a conventional oven with zero convection resistance at the surface of the food. Is this an accurate statement?

17–10C Consider two cold canned drinks, one wrapped in a blanket and the other placed on a table in the same room. Which drink will warm up faster?

17–11C Consider a surface of area A at which the convection and radiation heat transfer coefficients are h_{conv} and h_{rad}, respectively. Explain how you would determine (*a*) the single equivalent heat transfer coefficient, and (*b*) the equivalent thermal resistance. Assume the medium and the surrounding surfaces are at the same temperature.

17–12C Consider steady one-dimensional heat transfer through a multilayer medium. If the rate of heat transfer $\dot{Q}$ is known, explain how you would determine the temperature drop across each layer.

17–13C Consider a window glass consisting of two 4-mm-thick glass sheets pressed tightly against each other. Compare the heat transfer rate through this window with that of one consisting of a single 8-mm-thick glass sheet under identical conditions.

17–14 Consider a 3-m-high, 6-m-wide, and 0.25-m-thick brick wall whose thermal conductivity is $k = 0.8$ W/m·K. On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 14°C and 5°C, respectively. Determine the rate of heat loss through the wall on that day.

17–15 Consider a person standing in a room at 20°C with an exposed surface area of 1.7 m^2. The deep body temperature of the human body is 37°C, and the thermal conductivity of the human tissue near the skin is about 0.3 W/m·K. The body is losing heat at a rate of 150 W by natural convection and radiation to the surroundings. Taking the body temperature 0.5 cm beneath the skin to be 37°C, determine the skin temperature of the person. *Answer: 35.5°C*

17–16E Consider an electrically heated brick house ($k = 0.40$ Btu/h·ft·°F) whose walls are 9 ft high and 1 ft thick. Two of the walls of the house are 50 ft long and the others are 35 ft long. The house is maintained at 70°F at all times while the temperature of the outdoors varies. On a certain day, the temperature of the inner surface of the walls is measured to be at 55°F while the average temperature of the outer surface is observed to remain at 45°F during the day for 10 h and at 35°F at night for 14 h. Determine the amount of heat lost from the house that day. Also determine the cost of that heat loss to the homeowner for an electricity price of $0.09/kWh.

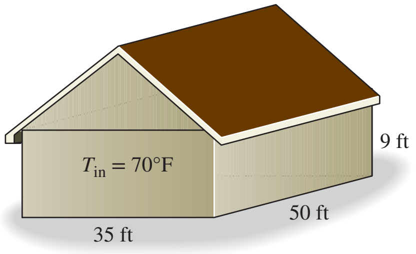


FIGURE P17–16E

17–17 A 12-cm × 18-cm circuit board houses on its surface 100 closely spaced logic chips, each dissipating 0.06 W in an environment at 40°C. The heat transfer from the back surface of the board is negligible. If the heat transfer coefficient on the surface of the board is 10 W/m^2·K, determine (*a*) the heat flux on the surface of the circuit board, in W/m^2; (*b*) the surface temperature of the chips; and (*c*) the thermal resistance between the surface of the circuit board and the cooling medium, in °C/W.

17–18 Water is boiling in a 25-cm-diameter aluminum pan ($k = 237$ W/m·K) at 95°C. Heat is transferred steadily to the boiling water in the pan through its 0.5-cm-thick flat bottom at a rate of 800 W. If the inner surface temperature of the bottom of the pan is 108°C, determine (*a*) the boiling heat transfer coefficient on the inner surface of the pan and (*b*) the outer surface temperature of the bottom of the pan.

17–19 A cylindrical resistor element on a circuit board dissipates 0.15 W of power in an environment at 35°C. The resistor is 1.2 cm long and has a diameter of 0.3 cm. Assuming heat to be transferred uniformly from all surfaces, determine (*a*) the amount of heat this resistor dissipates during a 24-h period; (*b*) the heat flux on the surface of the resistor, in W/m^2; and (*c*) the surface temperature of the resistor for a combined convection and radiation heat transfer coefficient of 9 W/m^2·K.

17–20 Consider a power transistor that dissipates 0.15 W of power in an environment at 30°C. The transistor is 0.4 cm long and has a diameter of 0.5 cm. Assuming heat to be transferred uniformly from all surfaces, determine (*a*) the amount of heat this resistor dissipates during a 24-h period, in kWh; (*b*) the heat flux on the surface of the transistor, in W/m^2; and (*c*) the surface temperature of the resistor for a combined convection and radiation heat transfer coefficient of 18 W/m^2·K.

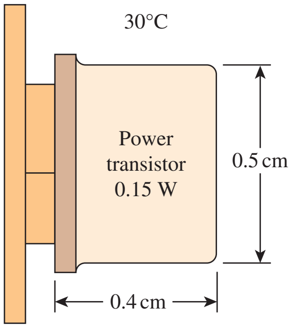


FIGURE P17–20

17–21 Consider a 1.5-m-high and 2.4-m-wide glass window whose thickness is 6 mm and thermal conductivity is $k = 0.78$ W/m·K. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 24°C while the temperature of the outdoors is −5°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2·K and $h_2 = 25$ W/m^2·K, and disregard any heat transfer by radiation.

17–22 Consider a 1.5-m-high and 2.4-m-wide double-pane window consisting of two 3-mm-thick layers of glass ($k = 0.78$ W/m·K) separated by a 12-mm-wide stagnant airspace ($k = 0.026$ W/m·K). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 21°C while the temperature of the outdoors is −5°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2·K and $h_2 = 25$ W/m^2·K, and disregard any heat transfer by radiation. *Answers:* 154 W, 16.7°C

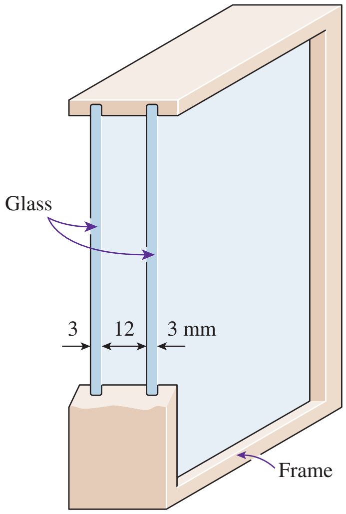


FIGURE P17–22

17–23 Repeat Prob. 17–22, assuming the space between the two glass layers is evacuated.

17–24 Reconsider Prob. 17–22. Using appropriate software, plot the rate of heat transfer through the window as a function of the width of airspace in the range of 2 mm to 20 mm, assuming pure conduction through the air. Discuss the results.

17–25E A wall is constructed of two layers of 0.6-in-thick sheetrock ($k = 0.10$ Btu/h·ft·°F), which is a plasterboard made of two layers of heavy paper separated by a layer of gypsum, placed 7 in apart. The space between the sheetrocks is filled with fiberglass insulation ($k = 0.020$ Btu/h·ft·°F). Determine (*a*) the thermal resistance of the wall and (*b*) its *R*-value of insulation in English units.

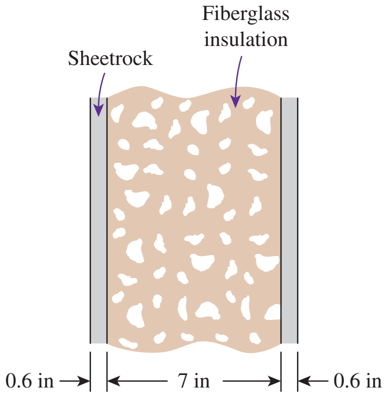


FIGURE P17–25E

17–26 To defog the rear window of an automobile, a very thin transparent heating element is attached to the inner surface of the window. A uniform heat flux of 1300 W/m^2 is provided to the heating element for defogging a rear window with thickness of 5 mm. The interior temperature of the automobile is 22°C, and the convection heat transfer coefficient is 15 W/m^2·K. The outside ambient temperature is −5°C, and the convection heat transfer coefficient is 100 W/m^2·K. If the thermal conductivity of the window is 1.2 W/m·K, determine the inner surface temperature of the window.

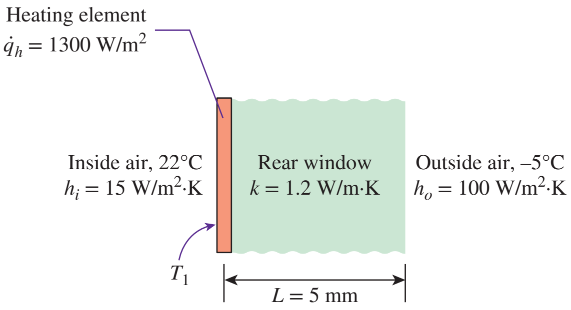


FIGURE P17–26

17–27 A transparent film is to be bonded onto the top surface of a solid plate inside a heated chamber. For the bond to cure properly, a temperature of 70°C is to be maintained at the bond, between the film and the solid plate. The transparent film has a thickness of 1 mm and thermal conductivity of 0.05 W/m·K, while the solid plate is 13 mm thick and has a thermal conductivity of 1.2 W/m·K. Inside the heated chamber, the convection heat transfer coefficient is 70 W/m^2·K. If the bottom surface of the solid plate is maintained at 52°C, determine the temperature inside the heated chamber and the surface temperature of the transparent film. Assume thermal contact resistance is negligible.

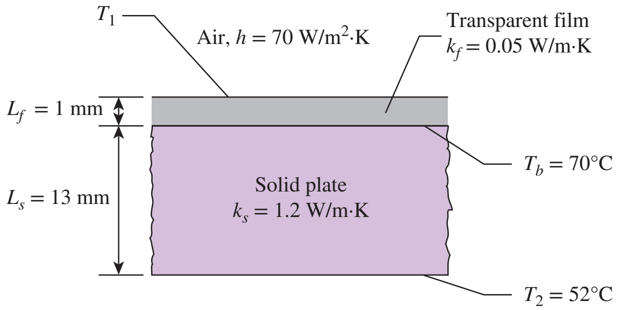


FIGURE P17–27

17–28 To defrost ice accumulated on the outer surface of an automobile windshield, warm air is blown over the inner surface of the windshield. Consider an automobile windshield with thickness of 5 mm and thermal conductivity of 1.4 W/m·K. The outside ambient temperature is −10°C and the convection heat transfer coefficient is 200 W/m²·K, while the ambient temperature inside the automobile is 25°C. Determine the value of the convection heat transfer coefficient for the warm air blowing over the inner surface of the windshield necessary to cause the accumulated ice to begin melting.

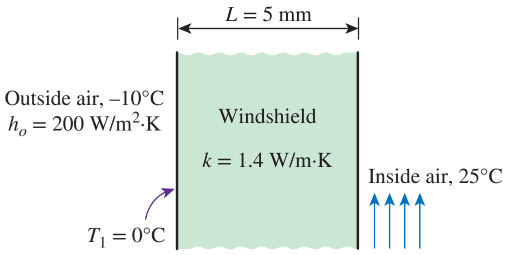


FIGURE P17–28

17–29 The roof of a house consists of a 15-cm-thick concrete slab ($k = 2$ W/m·K) that is 15 m wide and 20 m long. The convection heat transfer coefficients on the inner and outer surfaces of the roof are 5 and 12 W/m²·K, respectively. On a clear winter night, the ambient air is reported to be at 10°C, while the night sky temperature is 100 K. The house and the interior surfaces of the wall are maintained at a constant temperature of 20°C. The emissivity of both surfaces of the concrete roof is 0.9. Considering both radiation and convection heat transfers, determine the rate of heat transfer through the roof and the inner surface temperature of the roof.

If the house is heated by a furnace burning natural gas with an efficiency of 80 percent, and the price of natural gas is \$1.20/therm (1 therm = 105,500 kJ of energy content), determine the money lost through the roof that night during a 14-h period.

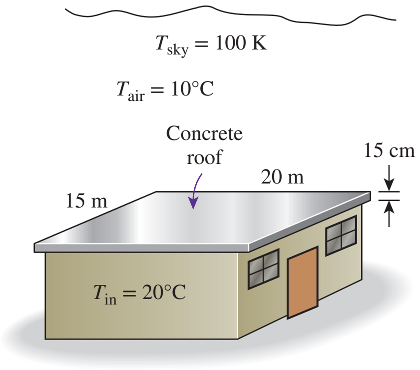


FIGURE P17–29

17–30 A 2-m × 1.5-m section of wall of an industrial furnace burning natural gas is not insulated, and the temperature at the outer surface of this section is measured to be 110°C. The temperature of the furnace room is 32°C, and the combined convection and radiation heat transfer coefficient at the surface of the outer furnace is 10 W/m²·K. It is proposed to insulate this section of the furnace wall with glass wool insulation ($k = 0.038$ W/m·K) in order to reduce the heat loss by 90 percent. Assuming the outer surface temperature of the metal section remains at about 110°C, determine the thickness of the insulation that needs to be used.

The furnace operates continuously and has an efficiency of 78 percent. The price of the natural gas is \$1.10/therm (1 therm = 105,500 kJ of energy content). If the installation of the insulation will cost \$250 for materials and labor, determine how long it will take for the insulation to pay for itself from the energy it saves.

17–31 The wall of a refrigerator is constructed of fiberglass insulation ($k = 0.035$ W/m·K) sandwiched between two layers of 1-mm-thick sheet metal ($k = 15.1$ W/m·K). The refrigerated space is maintained at 2°C, and the average heat transfer coefficients at the inner and outer surfaces of the wall are 4 W/m²·K and 9 W/m²·K, respectively. The kitchen temperature averages 24°C. It is observed that condensation occurs on the outer surfaces of the refrigerator when the temperature of the outer surface drops to 20°C. Determine the minimum thickness of fiberglass insulation that needs to be used in the wall in order to avoid condensation on the outer surfaces.

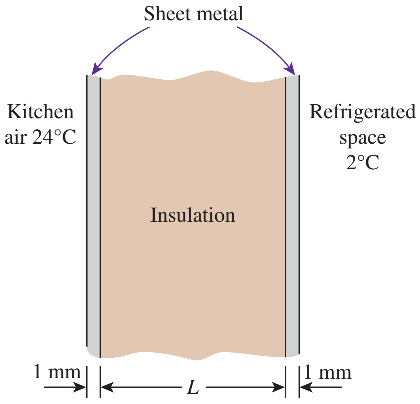


FIGURE P17–31

17–32 Reconsider Prob. 17–31. Using appropriate software, investigate the effects of the thermal conductivities of the insulation material and the sheet metal on the thickness of the insulation. Let the thermal conductivity vary from 0.02 W/m·K to 0.08 W/m·K for insulation and 10 W/m·K to 400 W/m·K for sheet metal. Plot the thickness of the insulation as the functions of the thermal conductivities of the insulation and the sheet metal, and discuss the results.

17–33 Heat is to be conducted along a circuit board that has a copper layer on one side. The circuit board is 15 cm long and 15 cm wide, and the thicknesses of the copper and epoxy layers are 0.1 mm and 1.2 mm, respectively. Disregarding heat transfer from side surfaces, determine the percentages of heat conduction along the copper ($k = 386$ W/m·K) and epoxy ($k = 0.26$ W/m·K) layers. Also determine the effective thermal conductivity of the board. *Answers:* 0.8 percent, 99.2 percent, 29.9 W/m·K

17–34E A 0.05-in-thick copper plate ($k = 223$ Btu/h·ft·°F) is sandwiched between two 0.15-in-thick epoxy boards ($k = 0.15$ Btu/h·ft·°F) that are 7 in × 9 in in size. Determine the effective thermal conductivity of the board along its 9-in-long side. What fraction of the heat conducted along that side is conducted through copper?

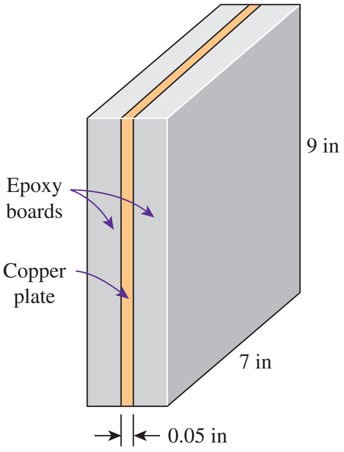


FIGURE P17–34E

17–35 Consider a house that has a 10-m × 20-m base and a 4-m-high wall. All four walls of the house have an *R*-value of 2.31 m^2·°C/W. The two 10-m × 4-m walls have no windows. The third wall has five windows made of 0.5-cm-thick glass ($k = 0.78$ W/m·K), 1.2 m × 1.8 m in size. The fourth wall has the same size and number of windows, but they are double-paned with a 1.5-cm-thick stagnant airspace ($k = 0.026$ W/m·K) enclosed between two 0.5-cm-thick glass layers. The thermostat in the house is set at 24°C, and the average temperature outside at that location is 8°C during the seven-month-long heating season. Disregarding any direct radiation gain or loss through the windows and taking the heat transfer coefficients at the inner and outer surfaces of the house to be 7 and 18 W/m^2·K, respectively, determine the average rate of heat transfer through each wall.

If the house is electrically heated and the price of electricity is \$0.08/kWh, determine the amount of money this household will save per heating season by converting the single-pane windows to double-pane windows.

17–36E Consider a house whose walls are 12 ft high and 40 ft long. Two of the walls of the house have no windows, while each of the other two walls has four windows made of 0.25-in-thick glass ($k = 0.45$ /Btu·h·ft·°F), 3 ft × 5 ft in size. The walls are certified to have an *R*-value of 19 (i.e., an *L*/*k* value of 19 h·ft^2·°F/Btu). Disregarding any direct radiation gain or loss through the windows and taking the heat transfer coefficients at the inner and outer surfaces of the house to be 2 and 4 Btu/h·ft^2·°F, respectively, determine the ratio of the heat transfer through the walls with and without windows.

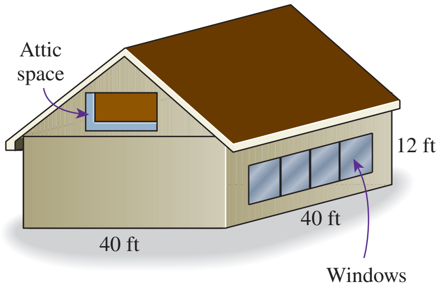


FIGURE P17–36E

Thermal Contact Resistance

17–37C What is thermal contact resistance? How is it related to thermal contact conductance?

17–38C Will the thermal contact resistance be greater for smooth or rough plain surfaces?

17–39C Explain how the thermal contact resistance can be minimized.

17–40C A wall consists of two layers of insulation pressed against each other. Do we need to be concerned about the thermal contact resistance at the interface in a heat transfer analysis or can we just ignore it?

17–41C A plate consists of two thin metal layers pressed against each other. Do we need to be concerned about the thermal contact resistance at the interface in a heat transfer analysis or can we just ignore it?

17–42C Consider two surfaces pressed against each other. Now the air at the interface is evacuated. Will the thermal contact resistance at the interface increase or decrease as a result?

17–43 The thermal contact conductance at the interface of two 1-cm-thick aluminum plates is measured to be 11,000 W/m^2·K. Determine the thickness of the aluminum plate whose thermal resistance is equal to the thermal resistance of the interface between the plates.

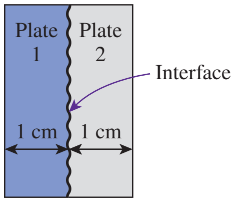


FIGURE P17–43

17–44 Two 5-cm-diameter, 15-cm-long aluminum bars ($k = 176$ W/m·K) with ground surfaces are pressed against each other with a pressure of 20 atm. The bars are enclosed in an insulation sleeve and, thus, heat transfer from the lateral surfaces is negligible. If the top and bottom surfaces of the two-bar system are maintained at temperatures of 150°C and 20°C, respectively, determine (*a*) the rate of heat transfer along the cylinders under steady conditions and (*b*) the temperature drop at the interface. *Answers:* (*a*) 142.4 W, (*b*) 6.4°C

17–45 A 1-mm-thick copper plate ($k = 401$ W/m·K) is sandwiched between two 7-mm-thick epoxy boards ($k = 0.26$ W/m·K) that are 15 cm × 20 cm in size. If the thermal contact conductance on both sides of the copper plate is estimated to be 6000 W/m·K, determine the error involved in the total thermal resistance of the plate if the thermal contact conductances are ignored.

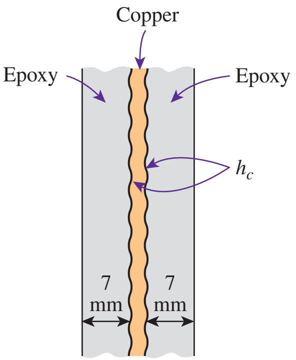


FIGURE P17–45

17–46 Two identical aluminum plates with thickness of 30 cm are pressed against each other at an average pressure of 1 atm. The interface, sandwiched between the two plates, is filled with glycerin. On the left outer surface, it is subjected to a uniform heat flux of 7800 W/m^2 at a constant temperature of 50°C. On the right outer surface, the temperature is maintained constant at 30°C. Determine the thermal contact conductance of the glycerin at the interface if the thermal conductivity of the aluminum plates is 237 W/m·K. Discuss whether the value of the thermal contact conductance is reasonable or not.

17–47 A two-layer wall is made of two metal plates, with surface roughness of about 25 μm, pressed together at an average pressure of 10 MPa. The first layer is a stainless steel plate with a thickness of 5 mm and a thermal conductivity of 14 W/m·K. The second layer is an aluminum plate with a thickness of 15 mm and a thermal conductivity of 237 W/m·K. On the stainless steel side of the wall, the surface is subjected to a heat flux of 800 W/m^2. On the aluminum side of the wall, the surface experiences convection heat transfer at an ambient temperature of 20°C, where the convection coefficient is 12 W/m^2·K. Determine the surface temperature of the stainless steel plate.

17–48 An aluminum plate 25 mm thick ($k = 235$ W/m·K) is attached to a copper plate with thickness of 10 mm. The copper plate is heated electrically to dissipate a uniform heat flux of 5300 W/m^2. The upper surface of the aluminum plate is exposed to convection heat transfer in a condition such that the convection heat transfer coefficient is 67 W/m^2·K and the surrounding room temperature is 20°C. Other surfaces of the two attached plates are insulated such that heat only dissipates through the upper surface of the aluminum plate. If the surface of the copper plate that is attached to the aluminum plate has a temperature of 100°C, determine the thermal contact conductance of the aluminum/copper interface.

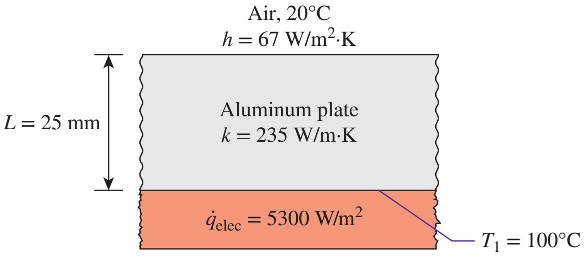


FIGURE P17–48

17–49 An aluminum plate and a stainless steel plate are pressed against each other at an average pressure of 20 MPa. Both plates have a surface roughness of 2 μm. Determine the impact on the temperature drop at the interface if the surface roughness of the plates is increased tenfold.

17–50 A thin electronic component with a surface area of 950 cm^2 is cooled by having a heat sink attached on its top surface. The thermal contact conductance of the interface between the electronic component and the heat sink is 2000 W/m^2·K. According to the manufacturer, the heat sink has combined convection and radiation thermal resistance of 0.3 K/W. If the electronic component dissipates 45 W of heat through the heat sink in a surrounding temperature of 30°C, determine the temperature of the electronic component. Does the contact resistance at the interface of the electronic component and the heat sink play a significant role in the heat dissipation?

17–51 Consider an engine cover that is made with two layers of metal plates. The inner layer is stainless steel ($k_1 = 14$ W/m·K) with a thickness of 10 mm, and the outer layer is aluminum ($k_2 = 237$ W/m·K) with a thickness of 5 mm. Both metal plates have a surface roughness of about 23 μm. The aluminum plate is attached on the stainless steel plate by screws that exert an average pressure of 20 MPa at the interface. The inside stainless steel surface of the cover is exposed to heat from the engine with a convection heat transfer coefficient of 10 W/m^2·K at an ambient temperature of 150°C. The outside aluminum surface is exposed to a convection heat transfer coefficient of 25 W/m^2·K at an ambient temperature of 40°C. Determine the heat flux through the engine cover.

17–52 Inconel® refers to a class of nickel-chromium-based superalloys that are used in high-temperature applications, such as gas turbine blades. For further improvement in the performance of gas turbine engines, the outer blade surface is coated with ceramic-based thermal barrier coating (TBC). A flat Inconel® plate with a thickness of 12 mm is coated with a layer of TBC with a thickness of 300 μm on its surface. At the interface between the Inconel® and the TBC, the thermal contact conductance is 3500 W/m^2·K. The thermal conductivities of the Inconel® and the TBC are 25 W/m·K and 1.5 W/m·K, respectively. The plate is in a surrounding of hot combustion gases at 1500°C, and the convection heat transfer coefficient is 750 W/m^2·K. Determine the temperature at the mid-plane of the Inconel® plate if the outer surface temperature is 1200°C.

Generalized Thermal Resistance Networks

17–53C What are the two approaches used in the development of the thermal resistance network for two-dimensional problems?

17–54C The thermal resistance networks can also be used approximately for multidimensional problems. For what kind of multidimensional problems will the thermal resistance approach give adequate results?

17–55C When plotting the thermal resistance network associated with a heat transfer problem, explain when two resistances are in series and when they are in parallel.

17–56 A 10-cm-thick wall is to be constructed with 2.5-m-long wood studs ($k = 0.11$ W/m·K) that have a cross section of 10 cm × 10 cm. At some point the builder ran out of those studs and started using pairs of 2.5-m-long wood studs that have a cross section of 5 cm × 10 cm nailed to each other instead. The manganese steel nails ($k = 50$ W/m·K) are 10 cm long and have a diameter of 0.4 cm. A total of 50 nails are used to connect the two studs, which are mounted to the wall such that the nails cross the wall. The temperature difference between the inner and outer surfaces of the wall is 8°C. Assuming the thermal contact resistance between the two layers to be negligible, determine the rate of heat transfer (*a*) through a solid stud and (*b*) through a stud pair of equal length and width nailed to each other. (*c*) Also determine the effective conductivity of the nailed stud pair.

17–57E Consider a 10-in × 12-in epoxy glass laminate ($k = 0.10$ Btu/h·ft·°F) whose thickness is 0.05 in. In order to reduce the thermal resistance across its thickness, cylindrical copper fillings ($k = 223$ Btu/h·ft·°F) of 0.02 in diameter are to be planted throughout the board, with a center-to-center distance of 0.06 in. Determine the new value of the thermal resistance of the epoxy board for heat conduction across its thickness as a result of this modification. *Answer:* 0.000256 h·°F/Btu

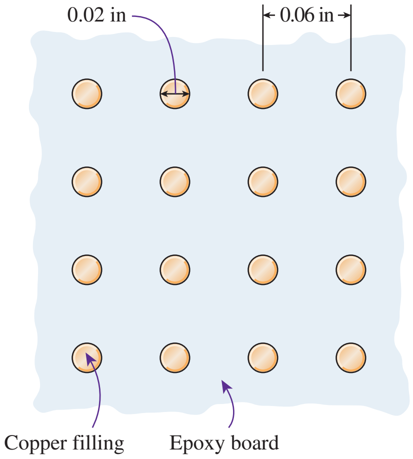


FIGURE P17–57E

17–58 Clothing made of several thin layers of fabric with trapped air in between, often called ski clothing, is commonly used in cold climates because it is light, fashionable, and a very effective thermal insulator. So it is no surprise that such clothing has largely replaced thick and heavy old-fashioned coats.

Consider a jacket made of five layers of 0.15-mm-thick synthetic fabric ($k = 0.13$ W/m·K) with 1.5-mm-thick airspace ($k = 0.026$ W/m·K) between the layers. Assuming the inner surface temperature of the jacket to be 25°C and the surface area to be 1.25 m^2, determine the rate of heat loss through the jacket when the temperature of the outdoors is 0°C and the heat transfer coefficient at the outer surface is 25 W/m^2·K.

What would your response be if the jacket is made of a single layer of 0.75-mm-thick synthetic fabric? What should be the thickness of a wool fabric ($k = 0.035$ W/m·K) if the person is to achieve the same level of thermal comfort wearing a thick wool coat instead of a five-layer ski jacket?

17–59 A typical section of a building wall is shown in Fig. P17–59. This section extends in and out of the page and is repeated in the vertical direction. The wall support members are made of steel ($k = 50$ W/m·K). The support members are 8 cm(t_{23}) × 0.5 cm(L_B). The remainder of the inner wall space is filled with insulation ($k = 0.03$ W/m·K) and measures 8 cm (t_{23}) × 60 cm (L_B). The inner wall is made of gypsum board ($k = 0.5$ W/m·K) that is 1 cm thick (t_{12}) and the outer wall is made of brick ($k = 1.0$ W/m·K) that is 10 cm thick (t_{34}). What is the average heat flux through this wall when $T_1 = 20$°C and $T_4 = 35$°C?

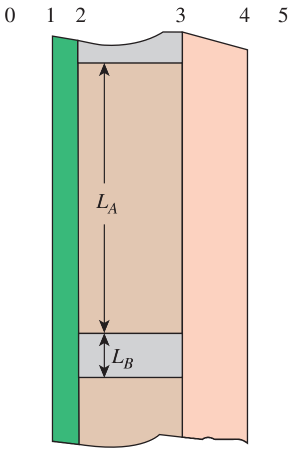


FIGURE P17–59

17–60 A 4-m-high and 6-m-wide wall consists of a long 15-cm × 25-cm cross section of horizontal bricks ($k = 0.72$ W/m·K) separated by 3-cm-thick plaster layers ($k = 0.22$ W/m·K). There are also 2-cm-thick plaster layers on each side of the wall, and a 2-cm-thick rigid foam ($k = 0.026$ W/m·K) on the inner side of the wall. The indoor and the outdoor temperatures are 22°C and −4°C, and the convection heat transfer coefficients on the inner and the outer sides are $h_1 = 10$ W/m^2·K and $h_2 = 20$ W/m^2·K, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

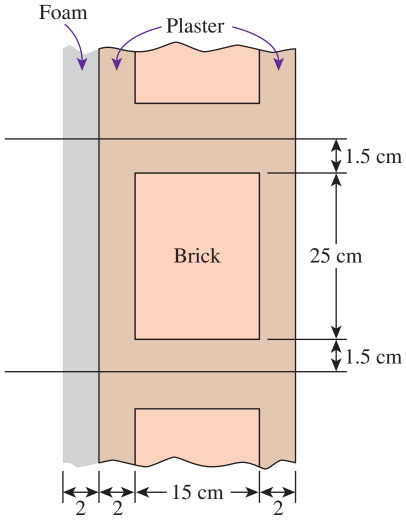


FIGURE P17–60

17–61 Reconsider Prob. 17–60. Using appropriate software, plot the rate of heat transfer through the wall as a function of the thickness of the rigid foam in the range of 1 cm to 10 cm. Discuss the results.

17–62 A 12-m-long and 5-m-high wall is constructed of two layers of 1-cm-thick sheetrock ($k = 0.17$ W/m·K) spaced 16 cm by wood studs ($k = 0.11$ W/m·K) whose cross section is 16 cm × 5 cm. The studs are placed vertically 60 cm apart, and the space between them is filled with fiberglass insulation ($k = 0.034$ W/m·K). The house is maintained at 20°C and the ambient temperature outside is −9°C. Taking the heat transfer coefficients at the inner and outer surfaces of the house to be 8.3 and 34 W/m^2·K, respectively, determine (*a*) the thermal resistance of the wall considering a representative section of it and (*b*) the rate of heat transfer through the wall.

17–63E A 10-in-thick, 30-ft-long, and 10-ft-high wall is to be constructed using 9-in-long solid bricks ($k = 0.40$ Btu/h·ft·°F) of cross section 7 in × 7 in, or identical-size bricks with nine square air holes ($k = 0.015$ Btu/h·ft·°F) that are 9 in long and have a cross section of 1.5 in × 1.5 in. There is a 0.5-in-thick plaster layer ($k = 0.10$ Btu/h·ft·°F) between two adjacent bricks on all four sides and on both sides of the wall. The house is maintained at 80°F and the ambient temperature outside is 35°F. Taking the heat transfer coefficients at the inner and outer surfaces of the wall to be 1.5 and 6 Btu/h·ft^2·°F, respectively, determine the rate of heat transfer through the wall constructed of (*a*) solid bricks and (*b*) bricks with air holes.

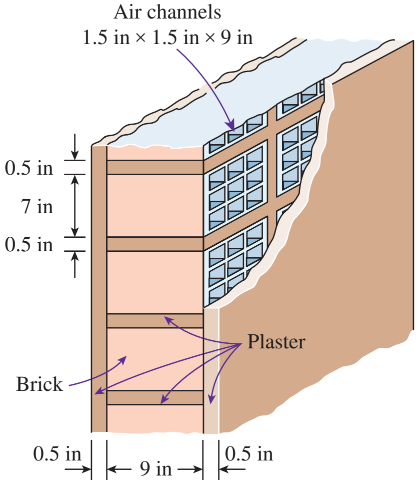


FIGURE P17–63E

17–64 Consider a 5-m-high, 8-m-long, and 0.22-m-thick wall whose representative cross section is as given in the figure. The thermal conductivities of various materials used, in W/m·K, are $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, and $k_E = 35$. The left and right surfaces of the wall are maintained at uniform temperatures of 300°C and 100°C, respectively. Assuming heat transfer through the wall to be one-dimensional, determine (*a*) the rate of heat transfer through the wall; (*b*) the temperature at the point where the sections *B*, *D*, and *E* meet; and (*c*) the temperature drop across the section *F*. Disregard any contact resistances at the interfaces.

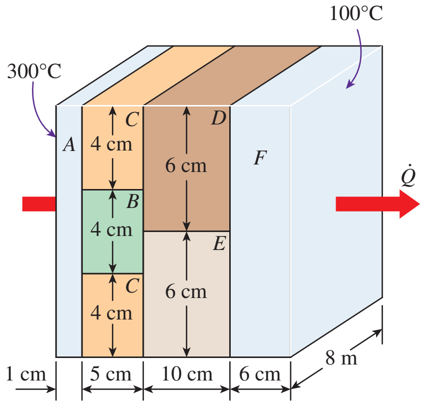


FIGURE P17–64

17–65 In an experiment to measure convection heat transfer coefficients, a very thin metal foil of very low emissivity (e.g., highly polished copper) is attached on the surface of a slab of material with very low thermal conductivity. The other surface of the metal foil is exposed to convection heat transfer by flowing fluid over the foil surface. This setup diminishes heat conduction through the slab and radiation on the metal foil surface, while heat convection plays the prominent role. The slab to which the metal foil is attached has a thickness of 25 mm and a thermal conductivity of 0.023 W/m·K. In a condition where the surrounding room temperature is 20°C, the metal foil is heated electrically with a uniform heat flux of 5000 W/m^2. If the bottom surface of the slab is 20°C and the metal foil has an emissivity of 0.02, determine (*a*) the convection heat transfer coefficient if air is flowing over the metal foil and the surface temperature of the foil is 150°C; and (*b*) the convection heat transfer coefficient if water is flowing over the metal foil and the surface temperature of the foil is 30°C.

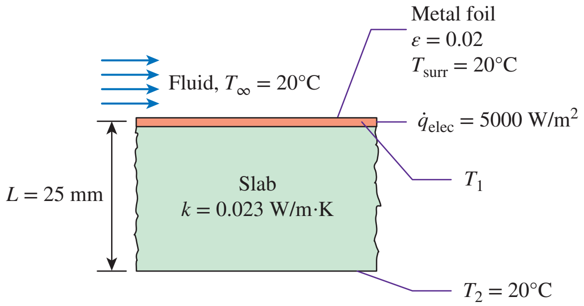


FIGURE P17–65

Heat Conduction in Cylinders and Spheres

17–66C What is an infinitely long cylinder? When is it proper to treat an actual cylinder as being infinitely long, and when is it not?

17–67C Can the thermal resistance concept be used for a solid cylinder or sphere in steady operation? Explain.

17–68C Consider a short cylinder whose top and bottom surfaces are insulated. The cylinder is initially at a uniform temperature T_i and is subjected to convection from its side surface to a medium at temperature T_∞, with a heat transfer coefficient of h. Is the heat transfer in this short cylinder one- or two-dimensional? Explain.

17–69 Steam at 280°C flows in a stainless steel pipe ($k = 15$ W/m·K) whose inner and outer diameters are 5 cm and 5.5 cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation ($k = 0.038$ W/m·K). Heat is lost to the surroundings at 5°C by natural convection and radiation, with a combined natural convection and radiation heat transfer coefficient of 22 W/m^2·K. Taking the heat transfer coefficient inside the pipe to be 80 W/m^2·K, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

17–70 Reconsider Prob. 17–69. Using appropriate software, investigate the effect of the thickness of the insulation on the rate of heat loss from the steam and the temperature drop across the insulation layer. Let the insulation thickness vary from 1 cm to 10 cm. Plot the rate of heat loss and the temperature drop as a function of insulation thickness, and discuss the results.

17–71 A 50-m-long section of a steam pipe whose outer diameter is 10 cm passes through an open space at 15°C. The average temperature of the outer surface of the pipe is measured to be 150°C. If the combined heat transfer coefficient on the outer surface of the pipe is 20 W/m^2·K, determine (*a*) the rate of heat loss from the steam pipe; (*b*) the annual cost of this energy lost if steam is generated in a natural gas furnace that has an efficiency of 75 percent and the price of natural gas is \$0.52/therm (1 therm = 105,500 kJ); and (*c*) the thickness of fiberglass insulation ($k = 0.035$ W/m·K) needed to save 90 percent of the heat lost. Assume the pipe temperature remains constant at 150°C.

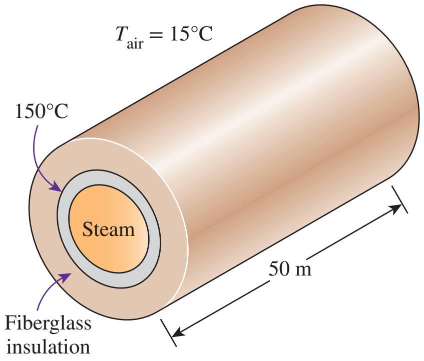


FIGURE P17–71

17–72 Superheated steam at an average temperature 200°C is transported through a steel pipe ($k = 50$ W/m·K, $D_o = 8.0$ cm, $D_i = 6.0$ cm, and $L = 20.0$ m). The pipe is insulated with a 4-cm-thick layer of gypsum plaster ($k = 0.5$ W/m·K). The insulated pipe is placed horizontally inside a warehouse where the average air temperature is 10°C. The steam and the air heat transfer coefficients are estimated to be 800 and 200 W/m^2·K, respectively. Calculate (*a*) the daily rate of heat transfer from the superheated steam, and (*b*) the temperature on the outside surface of the gypsum plaster insulation.

17–73E Steam exiting the turbine of a steam power plant at 100°F is to be condensed in a large condenser by cooling water flowing through copper pipes ($k = 223$ Btu/h·ft·°F) of inner diameter 0.4 in and outer diameter 0.6 in at an average temperature of 70°F. The heat of vaporization of water at 100°F is 1037 Btu/lbm. The heat transfer coefficients are 2400 Btu/h·ft^2·°F on the steam side and 35 Btu/h·ft^2·°F on the water side. Determine the length of the tube required to condense steam at a rate of 250 lbm/h. *Answer:* 2380 ft

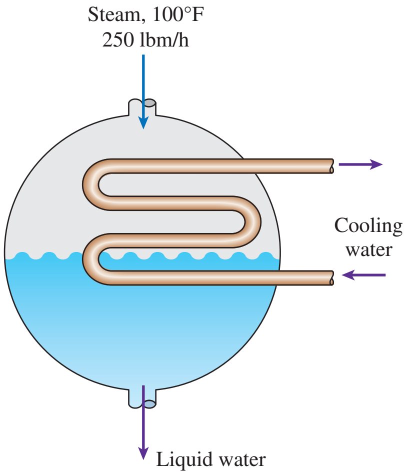


FIGURE P17–73E

17–74E Repeat Prob. 17–73E, assuming that a 0.01-in-thick layer of mineral deposit ($k = 0.5$ Btu/h·ft·°F) has formed on the inner surface of the pipe.

17–75E Reconsider Prob. 17–73E. Using appropriate software, investigate the effects of the thermal conductivity of the pipe material and the outer diameter of the pipe on the length of the tube required. Let the thermal conductivity vary from 10 Btu/h·ft·°F to 400 Btu/h·ft·°F and the outer diameter from 0.5 in to 1.0 in. Plot the length of the tube as functions of pipe conductivity and the outer pipe diameter, and discuss the results.

17–76 A 2.2-mm-diameter and 14-m-long electric wire is tightly wrapped with a 1-mm-thick plastic cover whose thermal conductivity is $k = 0.15$ W/m·K. Electrical measurements indicate that a current of 13 A passes through the wire, and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30$°C with a heat transfer coefficient of $h = 24$ W/m^2·K, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine if doubling the thickness of the plastic cover will increase or decrease this interface temperature.

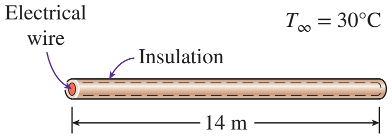


FIGURE P17–76

17–77 Consider a 1.5-m-high electric hot-water heater that has a diameter of 40 cm and maintains the hot water at 60°C. The tank is located in a small room whose average temperature is 27°C, and the heat transfer coefficients on the inner and outer surfaces of the heater are 50 and 12 W/m^2·K, respectively. The tank is placed in another 46-cm-diameter sheet metal tank of negligible thickness, and the space between the two tanks is filled with foam insulation ($k = 0.03$ W/m·K). The thermal resistances of the water tank and the outer thin sheet metal shell are very small and can be neglected. The price of electricity is \$0.08/kWh, and the homeowner pays \$280 a year for water heating. Determine the fraction of the hot-water energy cost of this household that is due to the heat loss from the tank.

Hot-water tank insulation kits consisting of 3-cm-thick fiberglass insulation ($k = 0.035$ W/m·K) large enough to wrap the entire tank are available in the market for about \$30. If such an insulation is installed on this water tank by the homeowner himself, how long will it take for this additional insulation to pay for itself? *Answers:* 15.2 percent, 21 months

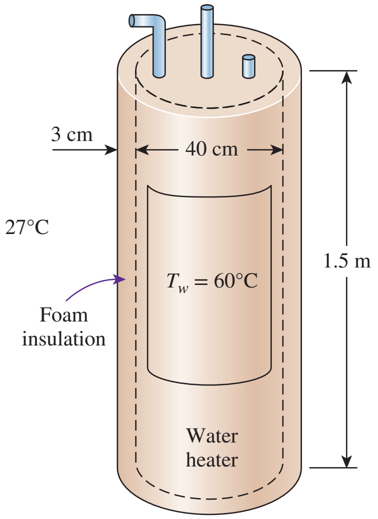


FIGURE P17–77

17–78 Chilled water enters a thin-shelled 4-cm-diameter, 200-m-long pipe at 7°C at a rate of 0.98 kg/s and leaves at 8°C. The pipe is exposed to ambient air at 30°C with a heat transfer coefficient of 9 W/m^2·K. If the pipe is to be insulated with glass wool insulation ($k = 0.05$ W/m·K) in order to decrease the temperature rise of the water to 0.25°C, determine the required thickness of the insulation.

17–79E Steam at 450°F is flowing through a steel pipe ($k = 8.7$ Btu/h·ft·°F) whose inner and outer diameters are 3.5 in and 4.0 in, respectively, in an environment at 55°F. The pipe is insulated with 2-in-thick fiberglass insulation ($k = 0.020$ Btu/h·ft·°F). If the heat transfer coefficients on the inside and the outside of the pipe are 30 and 5 Btu/h·ft^2·°F, respectively, determine the rate of heat loss from the steam per foot length of the pipe. What is the error involved in neglecting the thermal resistance of the steel pipe in calculations?

17–80 Hot water at an average temperature of 90°C is flowing through a 15-m section of a cast iron pipe ($k = 52$ W/m·K) whose inner and outer diameters are 4 cm and 4.6 cm, respectively. The outer surface of the pipe, whose emissivity is 0.7, is exposed to the cold air at 10°C in the basement, with a heat transfer coefficient of 15 W/m^2·K. The heat transfer coefficient at the inner surface of the pipe is 120 W/m^2·K. Taking the walls of the basement to be at 10°C also, determine the rate of heat loss from the hot water. Also, determine the average velocity of the water in the pipe if the temperature of the water drops by 3°C as it passes through the basement.

17–81 In a pharmaceutical plant, a copper pipe ($k_c = 400$ W/m·K) with inner diameter of 20 mm and wall thickness of 2.5 mm is used for carrying liquid oxygen to a storage tank. The liquid oxygen flowing in the pipe has an average temperature of −200°C and a convection heat transfer coefficient of 120 W/m^2·K. The condition surrounding the pipe has an ambient air temperature of 20°C and a combined heat transfer coefficient of 20 W/m^2·K. If the dew point is 10°C, determine the thickness of the insulation ($k_i = 0.05$ W/m·K) around the copper pipe to avoid condensation on the outer surface. Assume thermal contact resistance is negligible.

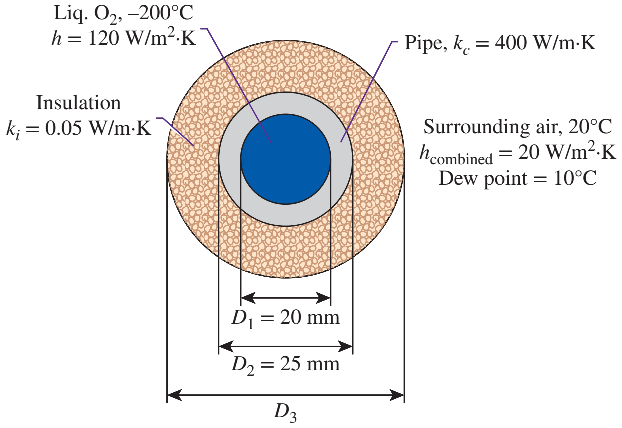


FIGURE P17–81

17–82 Liquid hydrogen is flowing through an insulated pipe ($k = 23$ W/m·K, $D_i = 3$ cm, $D_o = 4$ cm, and $L = 20$ m). The pipe is situated in a chemical plant, where the average air temperature is 40°C. The convection heat transfer coefficients of the liquid hydrogen and the ambient air are 200 W/m^2·K and 50 W/m^2·K, respectively. If the outer surface temperature of the insulated pipe is 5°C, determine the thickness of the pipe insulation ($k = 0.6$ W/m·K) needed to keep the liquid hydrogen flowing at an average temperature of −300°C.

17–83 An 8-m-internal-diameter spherical tank made of 1.5-cm-thick stainless steel ($k = 15$ W/m·K) is used to store iced water at 0°C. The tank is located in a room whose temperature is 25°C. The walls of the room are also at 25°C. The outer surface of the tank is black (emissivity $\varepsilon = 1$), and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are 80 W/m^2·K and 10 W/m^2·K, respectively. Determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period. The heat of fusion of water at atmospheric pressure is $h_{if} = 333.7$ kJ/kg.

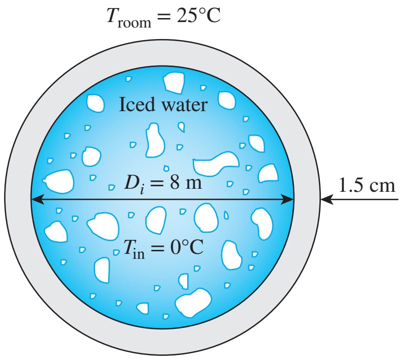


FIGURE P17–83

17–84 The boiling temperature of nitrogen at atmospheric pressure at sea level (1 atm pressure) is −196°C. Therefore, nitrogen is commonly used in low-temperature scientific studies since the temperature of liquid nitrogen in a tank open to the atmosphere will remain constant at −196°C until it is depleted. Any heat transfer to the tank will result in the evaporation of some liquid nitrogen, which has a heat of vaporization of 198 kJ/kg and a density of 810 kg/m^3 at 1 atm.

Consider a 3-m-diameter spherical tank that is initially filled with liquid nitrogen at 1 atm and −196°C. The tank is exposed to ambient air at 15°C, with a combined convection and radiation heat transfer coefficient of 35 W/m^2·K. The temperature of the thin-shelled spherical tank is observed to be almost the same as the temperature of the nitrogen inside. Determine the rate of evaporation of the liquid nitrogen in the tank as a result of the heat transfer from the ambient air if the tank is (*a*) not insulated, (*b*) insulated with 5-cm-thick fiberglass insulation ($k = 0.035$ W/m·K), and (*c*) insulated with 2-cm-thick superinsulation which has an effective thermal conductivity of 0.00005 W/m·K.

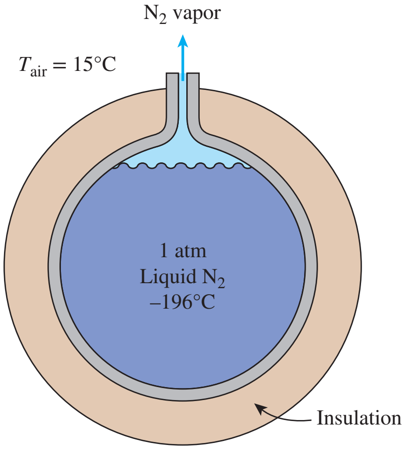


FIGURE P17–84

17–85 Repeat Prob. 17–84 for liquid oxygen, which has a boiling temperature of −183°C, a heat of vaporization of 213 kJ/kg, and a density of 1140 kg/m^3 at 1 atm pressure.

Critical Radius of Insulation

17–86C What is the critical radius of insulation? How is it defined for a cylindrical layer?

17–87C Consider an insulated pipe exposed to the atmosphere. Will the critical radius of insulation be greater on calm days or on windy days? Why?

17–88C A pipe is insulated to reduce the heat loss from it. However, measurements indicate that the rate of heat loss has increased instead of decreasing. Can the measurements be right?

17–89C Consider a pipe at a constant temperature whose radius is greater than the critical radius of insulation. Someone claims that the rate of heat loss from the pipe has increased when some insulation is added to the pipe. Is this claim valid?

17–90C A pipe is insulated such that the outer radius of the insulation is less than the critical radius. Now the insulation is taken off. Will the rate of heat transfer from the pipe increase or decrease for the same pipe surface temperature?

17–91E A 0.083-in-diameter electrical wire at 90°F is covered by 0.02-in-thick plastic insulation ($k = 0.075$ Btu/h·ft·°F). The wire is exposed to a medium at 50°F, with a combined convection and radiation heat transfer coefficient of 2.5 Btu/h·ft^2·°F. Determine if the plastic insulation on the wire will increase or decrease heat transfer from the wire. *Answer:* It helps

17–92E Repeat Prob. 17–91E, assuming a thermal contact resistance of 0.01 h·ft^2·°F/Btu at the interface of the wire and the insulation.

17–93 A 4-mm-diameter spherical ball at 50°C is covered by 1-mm-thick plastic insulation ($k = 0.13$ W/m·K). The ball is exposed to a medium at 15°C, with a combined convection and radiation heat transfer coefficient of 20 W/m^2·K. Determine if the plastic insulation on the ball will help or hurt heat transfer from the ball.

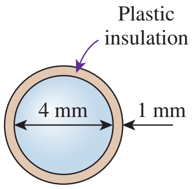


FIGURE P17–93

17–94 Reconsider Prob. 17–93. Using appropriate software, plot the rate of heat transfer from the ball as a function of the plastic insulation thickness in the range of 0.5 mm to 20 mm. Discuss the results.

Heat Transfer from Finned Surfaces

17–95C Hot air is to be cooled as it is forced to flow through the tubes exposed to atmospheric air. Fins are to be added in order to enhance heat transfer. Would you recommend attaching the fins inside or outside the tubes? Why? When would you recommend attaching fins both inside and outside the tubes?

17–96C What is the reason for the widespread use of fins on surfaces?

17–97C What is the difference between the fin effectiveness and the fin efficiency?

17–98C The fins attached to a surface are determined to have an effectiveness of 0.9. Do you think the rate of heat transfer from the surface has increased or decreased as a result of the addition of these fins?

17–99C Explain how the fins enhance heat transfer from a surface. Also, explain how the addition of fins may actually decrease heat transfer from a surface.

17–100C How does the overall effectiveness of a finned surface differ from the effectiveness of a single fin?

17–101C Hot water is to be cooled as it flows through the tubes exposed to atmospheric air. Fins are to be attached in order to enhance heat transfer. Would you recommend attaching the fins inside or outside the tubes? Why?

17–102C Consider two finned surfaces that are identical except that the fins on the first surface are formed by casting or extrusion, whereas they are attached to the second surface afterwards by welding or tight fitting. For which case do you think the fins will provide greater enhancement in heat transfer? Explain.

17–103C The heat transfer surface area of a fin is equal to the sum of all surfaces of the fin exposed to the surrounding medium, including the surface area of the fin tip. Under what conditions can we neglect heat transfer from the fin tip?

17–104C Does the (*a*) efficiency and (*b*) effectiveness of a fin increase or decrease as the fin length is increased?

17–105C Two pin fins are identical, except that the diameter of one of them is twice the diameter of the other. For which fin is the (*a*) fin effectiveness and (*b*) fin efficiency higher? Explain.

17–106C Two plate fins of constant rectangular cross section are identical, except that the thickness of one of them is twice the thickness of the other. For which fin is the (*a*) fin effectiveness and (*b*) fin efficiency higher? Explain.

17–107C Two finned surfaces are identical, except that the convection heat transfer coefficient of one of them is twice that of the other. For which finned surface is the (*a*) fin effectiveness and (*b*) fin efficiency higher? Explain.

17–108 Obtain a relation for the fin efficiency for a fin of constant cross-sectional area A_c, perimeter p, length L, and thermal conductivity k exposed to convection to a medium at T_∞ with a heat transfer coefficient h. Assume the fins are sufficiently long so that the temperature of the fin at the tip is nearly T_∞. Take the temperature of the fin at the base to be T_b and neglect heat transfer from the fin tips. Simplify the relation for (*a*) a circular fin of diameter D and (*b*) rectangular fins of thickness t.

17–109 A 4-mm-diameter and 10-cm-long aluminum fin ($k = 237$ W/m·K) is attached to a surface. If the heat transfer coefficient is 12 W/m²·K, determine the percent error in the rate of heat transfer from the fin when the infinitely long fin assumption is used instead of the adiabatic fin tip assumption.

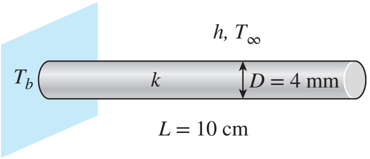


FIGURE P17–109

17–110 Consider a very long rectangular fin attached to a flat surface such that the temperature at the end of the fin is essentially that of the surrounding air, i.e., 20°C. Its width is 5.0 cm; thickness is 1.0 mm; thermal conductivity is 200 W/m·K; and base temperature is 40°C. The heat transfer coefficient is 20 W/m²·K. Estimate the fin temperature at a distance of 5.0 cm from the base and the rate of heat loss from the entire fin.

17–111 Consider a very long, slender rod. One end of the rod is attached to a base surface maintained at T_b, while the surface of the rod is exposed to an air temperature of 400°C. Thermocouples imbedded in the rod at locations 25 and 120 mm from the base surface register temperatures of 325°C and 375°C, respectively. (*a*) Calculate the rod base temperature (°C). (*b*) Determine the rod length (mm) for the case where the ratio of the heat transfer from a finite length fin to the heat transfer from a very long fin under the same conditions is 99 percent.

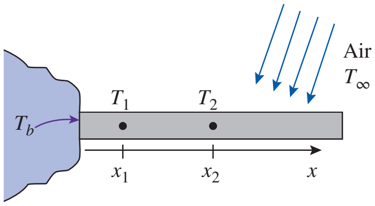


FIGURE P17–111

17–112 Two very long, slender rods of the same diameter and length are given. One rod (Rod 1) is made of aluminum and has a thermal conductivity $k_1 = 200$ W/m·K, but the thermal conductivity of Rod 2, k_2, is not known. To determine the thermal conductivity of Rod 2, both rods are thermally attached at one end to a metal surface which is maintained at a constant temperature T_b. Both rods are losing heat by convection, with a convection heat transfer coefficient h into the ambient air at T_∞. The surface temperature of each rod is measured at various distances from the hot base surface. The measurements reveal that the temperature of the aluminum rod (Rod 1) at $x_1 = 40$ cm from the base is the same as that of the rod of unknown thermal conductivity (Rod 2) at $x_2 = 20$ cm from the base. Determine the thermal conductivity k_2 of the second rod (W/m·K).

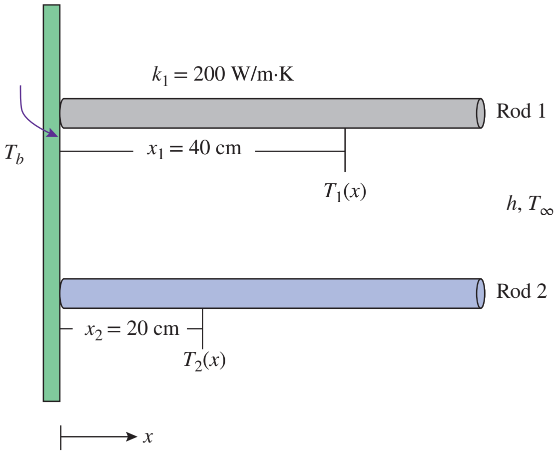


FIGURE P17–112

17–113 A turbine blade made of a metal alloy ($k = 17$ W/m·K) has a length of 5.3 cm, a perimeter of 11 cm, and a cross-sectional area of 5.13 cm². The turbine blade is exposed to hot gas from the combustion chamber at 973°C with a convection heat transfer coefficient of 538 W/m²·K. The base of the turbine blade maintains a constant temperature of 450°C, and the tip is adiabatic. Determine the heat transfer rate to the turbine blade and the temperature at the tip.

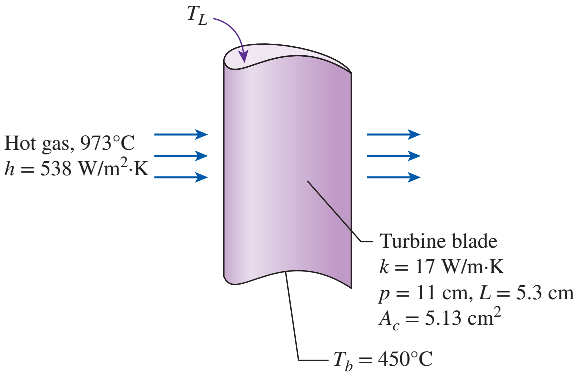


FIGURE P17–113

17–114E Consider a stainless steel spoon ($k = 8.7$ Btu/h·ft·°F) partially immersed in boiling water at 200°F in a kitchen at 75°F. The handle of the spoon has a cross section of 0.08 in × 0.5 in and extends 7 in in the air from the free surface of the water. If the heat transfer coefficient at the exposed surfaces of the spoon handle is 3 Btu/h·ft^2·°F, determine the temperature difference across the exposed surface of the spoon handle. State your assumptions. *Answer:* 124.6°F

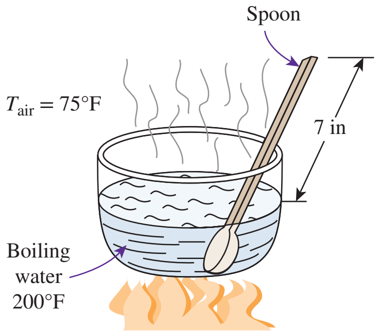


FIGURE P17–114E

17–115E Reconsider Prob. 17–114E. Using appropriate software, investigate the effects of the thermal conductivity of the spoon material and the length of its extension in the air on the temperature difference across the exposed surface of the spoon handle. Let the thermal conductivity vary from 5 Btu/h·ft^2·°F to 225 Btu/h·ft^2·°F and the length from 5 in to 12 in. Plot the temperature difference as the function of thermal conductivity and length, and discuss the results.

17–116 A DC motor delivers mechanical power to a rotating stainless steel shaft ($k = 15.1$ W/m·K) with a length of 25 cm and a diameter of 25 mm. In a surrounding with ambient air temperature of 20°C and convection heat transfer coefficient of 25 W/m^2·K, the surface area of the motor housing that is exposed to the ambient air is 0.075 m^2. The motor uses 300 W of electrical power and delivers 55 percent of it as mechanical power to rotate the stainless steel shaft. If the tip of the stainless steel shaft has a temperature of 22°C, determine the surface temperature of the motor housing. Assume the base temperature of the shaft is equal to the surface temperature of the motor housing.

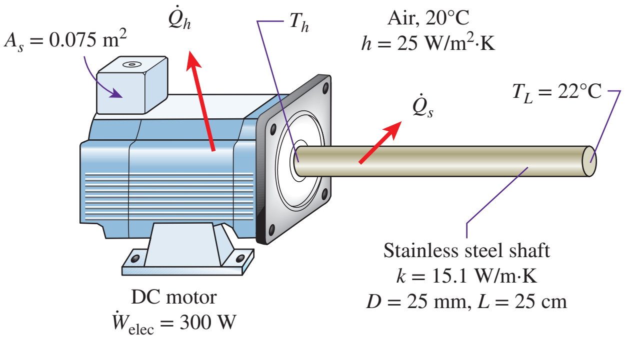


FIGURE P17–116

17–117 A plane wall with surface temperature of 350°C is attached with straight rectangular fins ($k = 235$ W/m·K). The fins are exposed to an ambient air condition of 25°C, and the convection heat transfer coefficient is 154 W/m^2·K. Each fin has a length of 50 mm, a base 5 mm thick, and a width of 100 mm. Determine the efficiency, heat transfer rate, and effectiveness of each fin, using (*a*) Table 17–3 and (*b*) Figure 17–43.

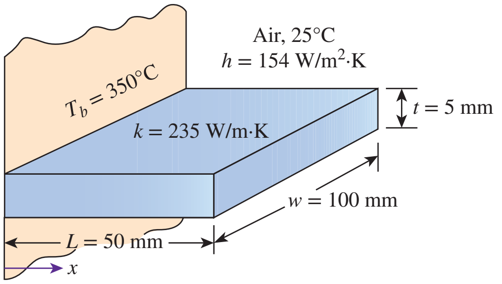


FIGURE P17–117

17–118 Pipes with inner and outer diameters of 50 mm and 60 mm, respectively, are used for transporting superheated vapor in a manufacturing plant. The pipes with thermal conductivity of 16 W/m·K are connected together by flanges with combined thickness of 20 mm and outer diameter of 90 mm. Air condition surrounding the pipes has a temperature of 25°C and a convection heat transfer coefficient of 10 W/m^2·K. If the inner surface temperature of the pipe is maintained at a constant temperature of 150°C, determine the temperature at the base of the flange and the rate of heat loss through the flange.

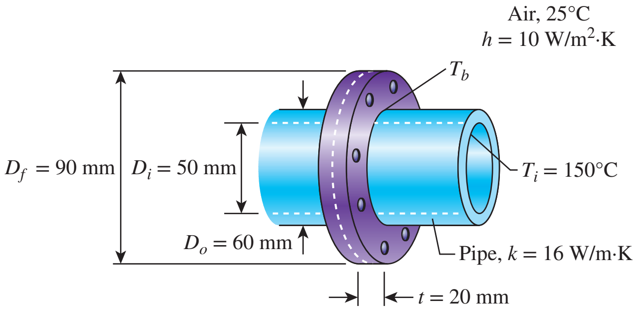


FIGURE P17–118

17–119 Steam in a heating system flows through tubes whose outer diameter is 5 cm and whose walls are maintained at a temperature of 130°C. Circular aluminum alloy 2024-T6 fins ($k = 186$ W/m·K) of outer diameter 6 cm and constant thickness 1 mm are attached to the tube. The space between the fins is 3 mm, and thus there are 250 fins per meter length of the tube. Heat is transferred to the surrounding air at $T_\infty = 25°C$, with a heat transfer coefficient of 40 W/m^2·K. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins. *Answer:* 1788 W

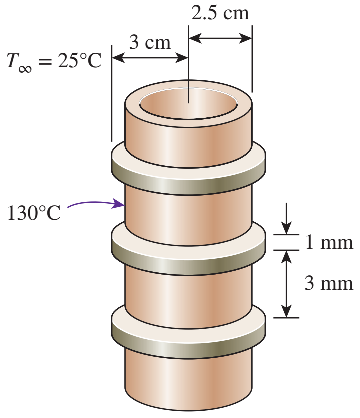


FIGURE P17–119

17–120 A 0.4-cm-thick, 12-cm-high, and 18-cm-long circuit board houses 80 closely spaced logic chips on one side, each dissipating 0.04 W. The board is impregnated with copper fillings and has an effective thermal conductivity of 30 W/m·K. All the heat generated in the chips is conducted across the circuit board and is dissipated from the back side of the board to a medium at 40°C, with a heat transfer coefficient of 52 W/m^2·K. (*a*) Determine the temperatures on the two sides of the circuit board. (*b*) Now a 0.2-cm-thick, 12-cm-high, and 18-cm-long aluminum plate ($k = 237$ W/m·K) with 864 2-cm-long aluminum pin fins of diameter 0.25 cm is attached to the back side of the circuit board with a 0.02-cm-thick epoxy adhesive ($k = 1.8$ W/m·K). Determine the new temperatures on the two sides of the circuit board.

17–121 A hot surface at 100°C is to be cooled by attaching 3-cm-long, 0.25-cm-diameter aluminum pin fins ($k = 237$ W/m·K) to it, with a center-to-center distance of 0.6 cm. The temperature of the surrounding medium is 30°C, and the heat transfer coefficient on the surfaces is 35 W/m^2·K. Determine the rate of heat transfer from the surface for a 1-m × 1-m section of the plate. Also determine the overall effectiveness of the fins.

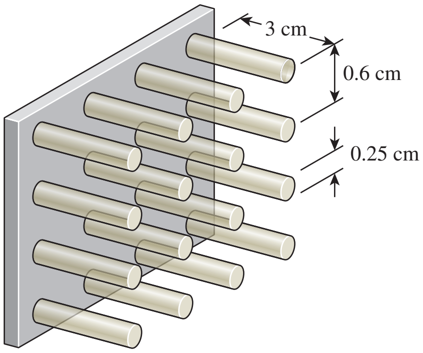


FIGURE P17–121

17–122 Reconsider Prob. 17–121. Using appropriate software, investigate the effect of the center-to-center distance of the fins on the rate of heat transfer from the surface and the overall effectiveness of the fins. Let the center-to-center distance vary from 0.4 cm to 2.0 cm. Plot the rate of heat transfer and the overall effectiveness as a function of the center-to-center distance, and discuss the results.

17–123 A 40-W power transistor is to be cooled by attaching it to one of the commercially available heat sinks shown in Table 17–6. Select a heat sink that will allow the case temperature of the transistor not to exceed 90°C in the ambient air at 20°C.

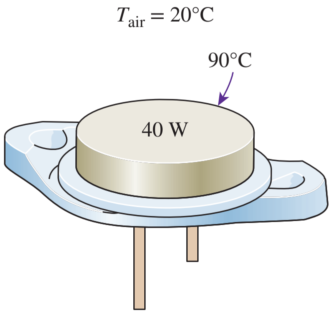


FIGURE P17–123

Review Problems

17–124 Consider two identical people each generating 60 W of metabolic heat steadily while doing sedentary work and dissipating it by convection and perspiration. The first person is wearing clothes made of 1-mm-thick leather ($k = 0.159$ W/m·K) that covers half of the body, while the second one is wearing clothes made of 1-mm-thick synthetic fabric ($k = 0.13$ W/m·K) that covers the body completely. The ambient air is at 30°C, the heat transfer coefficient at the outer surface is 15 W/m^2·K, and the inner surface temperature of the clothes can be taken to be 32°C. Treating the body of each person as a 25-cm-diameter, 1.7-m-long cylinder, determine the fractions of heat lost from each person by perspiration.

17–125 Cold conditioned air at 12°C is flowing inside a 1.5-cm-thick square aluminum ($k = 237$ W/m·K) duct of inner cross section 22 cm × 22 cm at a mass flow rate of 0.8 kg/s. The duct is exposed to air at 33°C with a combined convection-radiation heat transfer coefficient of 13 W/m^2·K. The convection heat transfer coefficient at the inner surface is 75 W/m^2·K. If the air temperature in the duct should not increase by more than 1°C, determine the maximum length of the duct.

17–126 Hot water is flowing at an average velocity of 1.5 m/s through a cast iron pipe ($k = 52$ W/m·K) whose inner and outer diameters are 3 cm and 3.5 cm, respectively. The pipe passes through a 15-m-long section of a basement whose temperature is 15°C. If the temperature of the water drops from 70°C to 67°C as it passes through the basement and the heat transfer coefficient on the inner surface of the pipe is 400 W/m^2·K, determine the combined convection and radiation heat transfer coefficient at the outer surface of the pipe. *Answer:* 272.5 W/m^2·K

17–127 The plumbing system of a house involves a 0.5-m section of a plastic pipe ($k = 0.16$ W/m·K) of inner diameter 2 cm and outer diameter 2.4 cm exposed to the ambient air. During a cold and windy night, the ambient air temperature remains at about −5°C for 14 h. The combined convection and radiation heat transfer coefficient on the outer surface of the pipe is estimated to be 40 W/m^2·K, and the heat of fusion of water is 333.7 kJ/kg. Assuming the pipe to contain stationary water initially at 0°C, determine if the water in that section of the pipe will completely freeze that night.

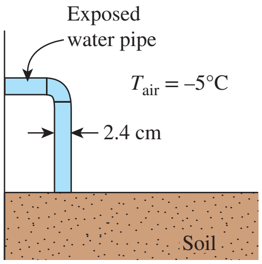


FIGURE P17–127

17–128 Steam at 260°C is flowing inside a steel pipe ($k = 61$ W/m·K) whose inner and outer diameters are 10 cm and 12 cm, respectively, in an environment at 20°C. The heat transfer coefficients inside and outside the pipe are 120 W/m^2·K and 14 W/m^2·K, respectively. Determine (*a*) the thickness of the insulation ($k = 0.038$ W/m·K) needed to reduce the heat loss by

95 percent and (*b*) the thickness of the insulation needed to reduce the exposed surface temperature of insulated pipe to 40°C for safety reasons.

17–129 A spherical vessel, 3.0 m in diameter (and negligible wall thickness), is used for storing a fluid at a temperature of 0°C. The vessel is covered with a 5.0-cm-thick layer of an insulation ($k = 0.20$ W/m·K). The surrounding air is at 22°C. The inside and outside heat transfer coefficients are 40 and 10 W/m^2·K, respectively. Calculate (*a*) all thermal resistances, in K/W, (*b*) the steady rate of heat transfer, and (*c*) the temperature difference across the insulation layer.

17–130 One wall of a refrigerated warehouse is 10.0 m high and 5.0 m wide. The wall is made of three layers: 1.0-cm-thick aluminum ($k = 200$ W/m·K), 8.0-cm-thick fiberglass ($k = 0.038$ W/m·K), and 3.0-cm-thick gypsum board ($k = 0.48$ W/m·K). The warehouse inside and outside temperatures are -10°C and 20°C, respectively, and the average value of both inside and outside heat transfer coefficients is 40 W/m^2·K.

- (*a*) Calculate the rate of heat transfer across the warehouse wall in steady operation.
- (*b*) Suppose that 400 metal bolts ($k = 43$ W/m·K), each 2.0 cm in diameter and 12.0 cm long, are used to fasten (i.e., hold together) the three wall layers. Calculate the rate of heat transfer for the "bolted" wall.
- (*c*) What is the percent change in the rate of heat transfer across the wall due to metal bolts?

17–131 A 4-m-high and 6-m-long wall is constructed of two large 0.8-cm-thick steel plates ($k = 15$ W/m·K) separated by 1-cm-thick and 22-cm-wide steel bars placed 99 cm apart. The remaining space between the steel plates is filled with fiberglass insulation ($k = 0.035$ W/m·K). If the temperature difference between the inner and the outer surfaces of the walls is 22°C, determine the rate of heat transfer through the wall. Can we ignore the steel bars between the plates in heat transfer analysis since they occupy only 1 percent of the heat transfer surface area?

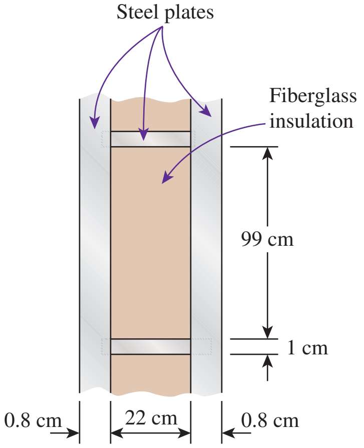


FIGURE P17–131

17–132 A typical section of a building wall is shown in Fig. P17–132. This section extends in and out of the page and is repeated in the vertical direction. The wall support members are made of steel ($k = 50$ W/m·K). The support members are 8 cm (t_{23}) × 0.5 cm (L_B). The remainder of the inner wall space is filled with insulation ($k = 0.03$ W/m·K) and measures 8 cm (t_{23}) × 60 cm (L_B). The inner wall is made of gypsum board ($k = 0.5$ W/m·K) that is 1 cm thick (t_{12}), and the outer wall is made of brick ($k = 1.0$ W/m·K) that is 10 cm thick (t_{34}). What is the temperature on the interior brick surface, 3, when $T_1 = 20$°C and $T_4 = 35$°C?

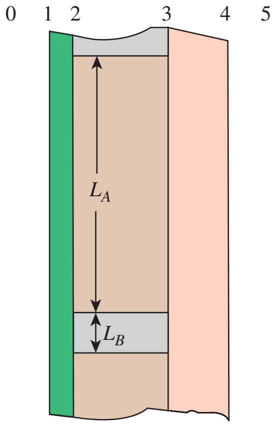


FIGURE P17–132

17–133 A 12-cm-long bar with a square cross section, as shown in Fig. P17–133, consists of a 1-cm-thick copper layer ($k = 380$ W/m·K) and a 1-cm-thick epoxy composite layer ($k = 0.4$ W/m·K). Calculate the rate of heat transfer under a thermal driving force of 50°C when the direction of steady one-dimensional heat transfer is (*a*) from front to back (i.e., along its length), (*b*) from left to right, and (*c*) from top to bottom.

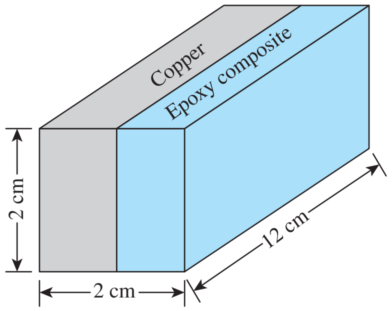


FIGURE P17–133

17–134 Circular fins of uniform cross section, with diameter of 10 mm and length of 50 mm, are attached to a wall with surface temperature of 350°C. The fins are made of material with thermal conductivity of 240 W/m·K, they are exposed to an ambient air condition of 25°C, and the convection heat transfer coefficient is 250 W/m^2·K. Determine the heat transfer rate and plot the temperature variation of a single fin for the following boundary conditions:

- (*a*) Infinitely long fin
- (*b*) Adiabatic fin tip
- (*c*) Fin with tip temperature of 250°C
- (*d*) Convection from the fin tip

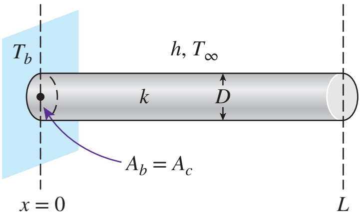


FIGURE P17–134

17–135 A total of 10 rectangular aluminum fins ($k = 203$ W/m·K) are placed on the outside flat surface of an electronic device. Each fin is 100 mm wide, 20 mm high, and 4 mm thick. The fins are located parallel to each other at a center-to-center distance of 8 mm. The temperature at the outside surface of the electronic device is 72°C. The air is at 20°C, and the heat transfer coefficient is 80 W/m^2·K. Determine (*a*) the rate of heat loss from the electronic device to the surrounding air and (*b*) the fin effectiveness.

17–136 A plane wall with surface temperature of 300°C is attached with straight aluminum triangular fins ($k = 236$ W/m·K). The fins are exposed to an ambient air condition of 25°C, and the convection heat transfer coefficient is 25 W/m^2·K. Each fin has a length of 55 mm, a base 4 mm thick, and a width of 110 mm. Using Table 17–3, determine the efficiency, heat transfer rate, and effectiveness of each fin.

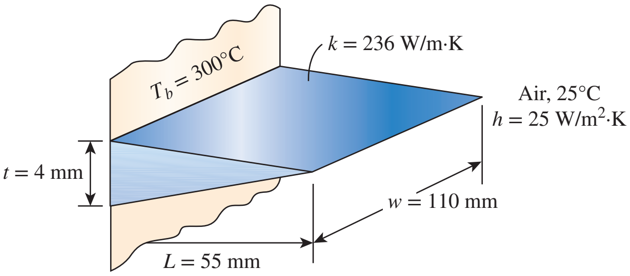


FIGURE P17–136

17–137 A plane wall surface at 200°C is to be cooled with aluminum pin fins of parabolic profile with blunt tips. Each fin has a length of 25 mm and a base diameter of 4 mm. The fins are exposed to ambient air at 25°C, and the heat transfer coefficient is 45 W/m^2·K. If the thermal conductivity of the fins is 230 W/m·K, determine the heat transfer rate from a single fin and the increase in the rate of heat transfer per m^2 surface area as a result of attaching fins. Assume there are 100 fins per m^2 surface area.

17–138 Steam in a heating system flows through tubes whose outer diameter is 3 cm and whose walls are maintained at a temperature of 120°C. Circular aluminum alloy fins ($k = 180$ W/m·K) of outer diameter 6 cm and constant thickness $t = 2$ mm are attached to the tube, as shown in Fig. P17–138. The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at 25°C, with a combined heat transfer coefficient of 60 W/m^2·K. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.

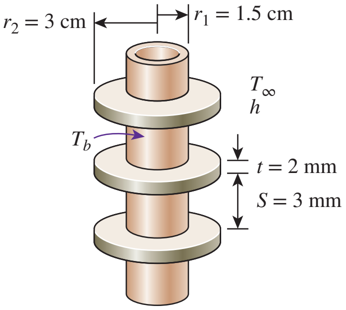


FIGURE P17–138

17–139 A 0.2-cm-thick, 10-cm-high, and 15-cm-long circuit board houses electronic components on one side that dissipate a total of 15 W of heat uniformly. The board is impregnated with conducting metal fillings and has an effective thermal conductivity of 12 W/m·K. All the heat generated in the components is conducted across the circuit board and is dissipated from the back side of the board to a medium at 37°C, with a heat transfer coefficient of 45 W/m^2·K. (*a*) Determine the surface temperatures on the two sides of the circuit board. (*b*) Now a 0.1-cm-thick, 10-cm-high, and 15-cm-long aluminum plate ($k = 237$ W/m·K) with 20 0.2-cm-thick, 2-cm-long, and 15-cm-wide aluminum fins of rectangular profile are attached to the back side of the circuit board with a 0.03-cm-thick epoxy adhesive ($k = 1.8$ W/m·K). Determine the new temperatures on the two sides of the circuit board.

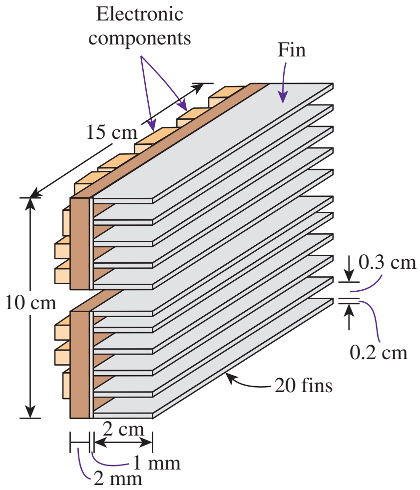


FIGURE P17–139

Design and Essay Problems

17–140 The temperature in deep space is close to absolute zero, which presents thermal challenges for the astronauts who do space walks. Propose a design for the clothing of the astronauts that will be most suitable for the thermal environment in space. Defend the selections in your design.

17–141 In the design of electronic components, it is desirable to attach the electronic circuitry to a substrate material that is a very good thermal conductor but also a very effective electrical insulator. If high cost is not a major concern, what material would you propose for the substrate?

17–142 Find out about the wall construction of the cabins of large commercial airplanes, the range of ambient conditions under which they operate, typical heat transfer coefficients on the inner and outer surfaces of the wall, and the heat generation rates inside. Determine the size of the heating and air-conditioning system that will be able to maintain the cabin at 20°C at all times for an airplane capable of carrying 400 people.

17–143 Repeat Prob. 17–142 for a submarine with a crew of 60 people.

17–144 A house with 200-m^2 floor space is to be heated with geothermal water flowing through pipes laid in the ground under the floor. The walls of the house are 4 m high, and there are 10 single-paned windows in the house that are 1.2 m wide and 1.8 m high. The house has R-19 (in $\text{h·ft}^2\text{·°F/Btu}$) insulation in the walls and R-30 in the ceiling. The floor temperature is not to exceed 40°C. Hot geothermal water is available at 90°C, and the inner and outer diameters of the pipes to be used are 2.4 cm and 3.0 cm. Design such a heating system for this house in your area.

17–145 Using a timer (or watch) and a thermometer, conduct this experiment to determine the rate of heat gain of your refrigerator. First, make sure that the door of the refrigerator is not opened for at least a few hours to make sure that steady operating conditions are established. Start the timer when the refrigerator stops running, and measure the time Δt_1 it stays off before it kicks in. Then measure the time Δt_2 it stays on. Noting that the heat removed during Δt_2 is equal to the heat gain of the refrigerator during $\Delta t_1 + \Delta t_2$ and using the power consumed by the refrigerator when it is running, determine the average rate of heat gain for your refrigerator, in watts. Take the COP (coefficient of performance) of your refrigerator to be 1.3 if it is not available.

Now, clean the condenser coils of the refrigerator and remove any obstacles in the way of airflow through the coils. Then determine the improvement in the COP of the refrigerator.

CHAPTER

18

TRANSIENT HEAT CONDUCTION

The temperature of a body, in general, varies with time as well as position. In rectangular coordinates, this variation is expressed as $T(x, y, z, t)$, where (x, y, z) indicate variation in the x-, y-, and z-directions, and t indicates variation with time. In Chap. 17, we considered heat conduction under *steady* conditions, for which the temperature of a body at any point does not change with time. This certainly simplified the analysis, especially when the temperature varied in one direction only, and we were able to obtain analytical solutions. In this chapter, we consider the variation of temperature with *time* as well as *position* in one- and multidimensional systems.

We start this chapter with the analysis of *lumped systems* in which the temperature of a body varies with time but remains uniform throughout at any time. Then we consider the variation of temperature with time as well as position for one-dimensional heat conduction problems such as those associated with a large plane wall, a long cylinder, a sphere, and a semi-infinite medium using *analytical solutions.* Finally, we consider transient heat conduction in multidimensional systems by utilizing the *product solution.*

OBJECTIVES

The objectives of this chapter are to:

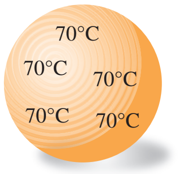
- Assess when the spatial variation of temperature is negligible and temperature varies nearly uniformly with time, making the simplified lumped system analysis applicable.
- Obtain analytical solutions for transient one-dimensional conduction problems in rectangular, cylindrical, and spherical geometries using the method of separation of variables, and understand why a one-term solution is usually a reasonable approximation.
- Solve the transient conduction problem in large media using the similarity variable, and predict the variation of temperature with time and distance from the exposed surface.
- Construct solutions for multidimensional transient conduction problems using the product solution approach.

18–1 ▪ LUMPED SYSTEM ANALYSIS

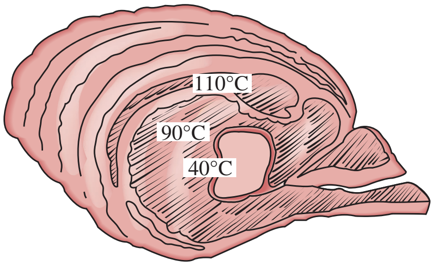
In heat transfer analysis, some bodies are observed to behave like a "lump" whose interior temperature remains essentially uniform at any time during a heat transfer process. The temperature of such bodies can be taken to be a function of time only, $T(t)$. Heat transfer analysis that utilizes this idealization is known as **lumped system analysis**, which provides great simplification in certain classes of heat transfer problems without much sacrifice in accuracy.

Consider a small hot copper ball coming out of an oven (Fig. 18–1). Measurements indicate that the temperature of the copper ball changes with time, but it does not change much with position at any given time. Thus the temperature of the ball remains nearly uniform at all times, and we can talk about the temperature of the ball with no reference to a specific location.

Now let us go to the other extreme and consider a large roast in an oven. If you have done any roasting, you must have noticed that the temperature distribution within the roast is not even close to being uniform. You can easily verify this by taking the roast out before it is completely done and cutting it in half. You will see that the outer parts of the roast are well done while the center part is barely warm. Thus, lumped system analysis is not applicable in this case. Before presenting a criterion about the applicability of lumped system analysis, we develop the formulation associated with it.



(*a*) Copper ball



(*b*) Roast beef

FIGURE 18–1
A small copper ball can be modeled as a lumped system, but a roast beef cannot.

Consider a body of arbitrary shape of mass m, volume V, surface area A_s, density ρ, and specific heat c_p initially at a uniform temperature T_i (Fig. 18–2). At time $t = 0$, the body is placed into a medium at temperature T_∞, and heat transfer takes place between the body and its environment, with a heat transfer coefficient h. For the sake of discussion, we assume that $T_\infty > T_i$, but the analysis is equally valid for the opposite case. We assume lumped system analysis to be applicable, so the temperature remains uniform within the body at all times and changes with time only, $T = T(t)$.

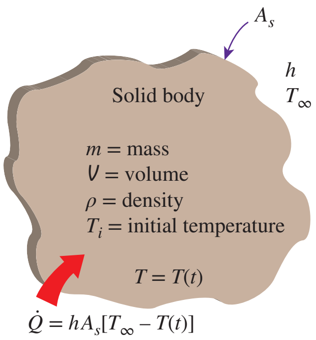


FIGURE 18–2
The geometry and parameters involved in lumped system analysis.

During a differential time interval dt, the temperature of the body rises by a differential amount dT. An energy balance of the solid for the time interval dt can be expressed as

$$\left(\begin{array}{c}\text{Heat transfer into the body}\\ \text{during } dt\end{array}\right) = \left(\begin{array}{c}\text{The increase in}\\ \text{the energy of the}\\ \text{body during } dt\end{array}\right)$$

or

$$hA_s(T_\infty - T)\,dt = mc_p\,dT \tag{18–1}$$

Noting that $m = \rho V$ and $dT = d(T - T_\infty)$ since $T_\infty = \text{constant}$, Eq. 18–1 can be rearranged as

$$\frac{d(T - T_\infty)}{T - T_\infty} = \frac{hA_s}{\rho V c_p}\,dt \tag{18–2}$$

Integrating from $t = 0$, at which $T = T_i$, to any time t, at which $T = T(t)$, gives

$$\ln\frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p}t \tag{18–3}$$

Taking the exponential of both sides and rearranging, we obtain

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \tag{18–4}$$

where

$$b = \frac{hA_s}{\rho V c_p} \qquad (1/\text{s}) \tag{18–5}$$

is a positive quantity whose dimension is $(\text{time})^{-1}$. The reciprocal of b has time units (usually seconds) and is called the **time constant**. Equation 18–4 is plotted in Fig. 18–3 for different values of b. There are two observations that can be made from this figure and the relation above:

1. Equation 18–4 enables us to determine the temperature $T(t)$ of a body at time t, or alternatively, the time t required for the temperature to reach a specified value $T(t)$.
2. The temperature of a body approaches the ambient temperature T_∞ exponentially. The temperature of the body changes rapidly at the beginning, but rather slowly later on. A large value of b indicates that the body approaches the environment temperature in a short time. The larger the value of the exponent b, the higher the rate of decay in temperature. Note that b is proportional to the surface area but inversely proportional to the mass and the specific heat of the body. This is not surprising since it takes longer to heat or cool a larger mass, especially when it has a large specific heat.

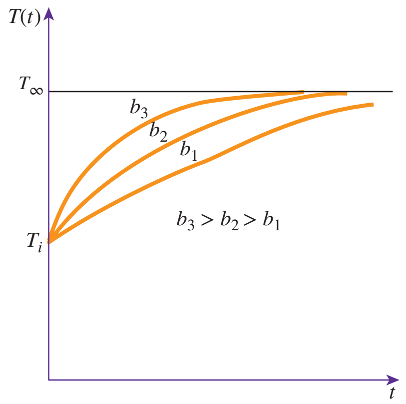


FIGURE 18–3
The temperature of a lumped system approaches the environment temperature as time gets larger.

Once the temperature $T(t)$ at time t is available from Eq. 18–4, the *rate* of convection heat transfer between the body and its environment at that time can be determined from Newton's law of cooling as

$$\dot{Q}(t) = hA_s[T_\infty - T(t)] \qquad \text{(W)} \tag{18–6}$$

The *total amount* of heat transfer between the body and the surrounding medium over the time interval $t = 0$ to t is simply the change in the energy content of the body:

$$Q = mc_p[T(t) - T_i] \qquad \text{(kJ)} \tag{18–7}$$

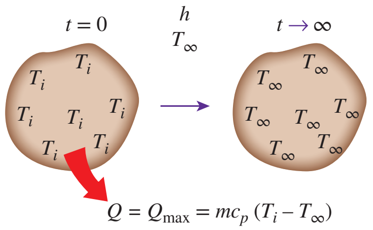


FIGURE 18–4
Heat transfer to or from a body reaches its maximum value when the body reaches the environment temperature.

The amount of heat transfer reaches its upper limit when the body reaches the surrounding temperature T_∞. Therefore, the *maximum* heat transfer between the body and its surroundings is (Fig. 18–4)

$$Q_{\max} = mc_p(T_\infty - T_i) \qquad \text{(kJ)} \tag{18–8}$$

We could also obtain this equation by substituting the $T(t)$ relation from Eq. 18–4 into the $\dot{Q}(t)$ relation in Eq. 18–6 and integrating it from $t = 0$ to $t \to \infty$.

Criteria for Lumped System Analysis

Lumped system analysis certainly provides great convenience in heat transfer analysis, and naturally we would like to know when it is appropriate to use it. The first step in establishing a criterion for the applicability of lumped system analysis is to define a **characteristic length** as

$$L_c = \frac{V}{A_s}$$

and a dimensionless **Biot number** Bi as

$$\text{Bi} = \frac{hL_c}{k} \tag{18–9}$$

The characteristic length L_c to be used in the Biot number for simple geometries in which heat transfer is one-dimensional, such as a large plane wall of thickness $2L$, a long cylinder of radius r_o, and a sphere of radius r_o, becomes L (half thickness), $r_o/2$, and $r_o/3$, respectively. Equation 18–9 can also be expressed as (Fig. 18–5)

$$\text{Bi} = \frac{h}{k/L_c}\,\frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

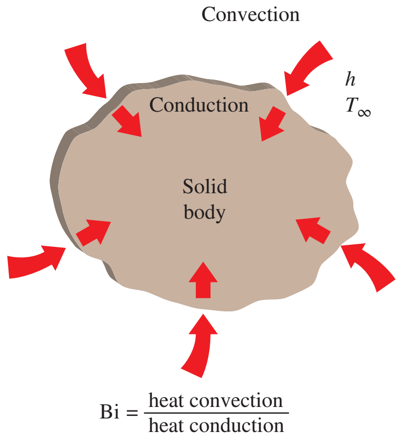


FIGURE 18–5
The Biot number can be viewed as the ratio of the convection at the surface to conduction within the body.

or

$$\mathrm{Bi} = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

When a solid body is being heated by the hotter fluid surrounding it (such as a potato being baked in an oven), heat is first *convected* to the body and subsequently *conducted* within the body. The Biot number is the *ratio* of the internal resistance of a body to *heat conduction* to its external resistance to *heat convection.* Therefore, a small Biot number represents small resistance to heat conduction, and thus small temperature gradients within the body.

Lumped system analysis assumes a *uniform* temperature distribution throughout the body, which is the case only when the thermal resistance of the body to heat conduction (the *conduction resistance*) is zero. Thus, lumped system analysis is *exact* when $\mathrm{Bi} = 0$ and *approximate* when $\mathrm{Bi} > 0$. Of course, the smaller the Bi number, the more accurate the lumped system analysis. Then the question we must answer is, how much accuracy are we willing to sacrifice for the convenience of the lumped system analysis?

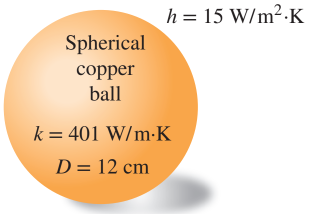
Before answering this question, we should mention that a 15 percent uncertainty in the convection heat transfer coefficient h in most cases is considered normal and expected. Assuming h to be *constant* and *uniform* is also an approximation of questionable validity, especially for irregular geometries. Therefore, in the absence of sufficient experimental data for the specific geometry under consideration, we cannot claim our results to be better than ± 15 percent, even when $\mathrm{Bi} = 0$. This being the case, introducing another source of uncertainty into the problem will not have much effect on the overall uncertainty, provided that it is minor. It is generally accepted that lumped system analysis is *applicable* if

$$\mathrm{Bi} \leq 0.1$$

When this criterion is satisfied, the temperatures within the body relative to the surroundings (i.e., $T - T_\infty$) remain within 5 percent of each other even for well-rounded geometries such as a spherical ball. Thus, when $\mathrm{Bi} < 0.1$, the variation of temperature with location within the body is slight and can reasonably be approximated as being uniform.

The first step in the application of lumped system analysis is the calculation of the *Biot number* and the assessment of the applicability of this approach. One may still wish to use lumped system analysis even when the criterion $\mathrm{Bi} < 0.1$ is not satisfied if high accuracy is not a major concern.

Note that the Biot number is the ratio of the *convection* at the surface to *conduction* within the body, and this number should be as small as possible for lumped system analysis to be applicable. Therefore, *small bodies* with *high thermal conductivity* are good candidates for lumped system analysis, especially when they are in a medium that is a poor conductor of heat (such as air or another gas) and motionless. Thus, the hot small copper ball placed in quiescent air, discussed earlier, is most likely to satisfy the criterion for lumped system analysis (Fig. 18–6).



$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D = 0.02 \text{ m}$$

$$\mathrm{Bi} = \frac{hL_c}{k} = \frac{15 \times 0.02}{401} = 0.00075 < 1$$

FIGURE 18–6
Small bodies with high thermal conductivities and low convection coefficients are most likely to satisfy the criterion for lumped system analysis.

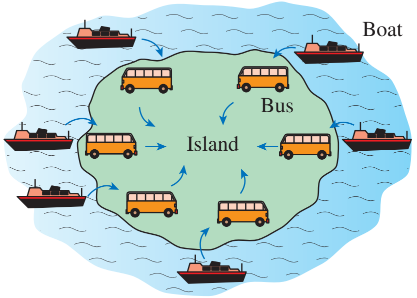


FIGURE 18–7
Analogy between heat transfer to a solid and passenger traffic to an island.

Some Remarks on Heat Transfer in Lumped Systems

To understand the heat transfer mechanism during the heating or cooling of a solid by the fluid surrounding it, and the criterion for lumped system analysis, consider this analogy (Fig. 18–7). People from the mainland are to go *by boat* to an island whose entire shore is a harbor, and from the harbor to their destinations on the island *by bus.* The overcrowding of people at the harbor depends on the boat traffic to the island and the ground transportation system on the island. If there is an excellent ground transportation system with plenty of buses, there will be no overcrowding at the harbor, especially when the boat traffic is light. But when the opposite is true, there will be a huge overcrowding at the harbor, creating a large difference between the populations at the harbor and inland. The chance of overcrowding is much lower in a small island with plenty of fast buses.

In heat transfer, a poor ground transportation system corresponds to poor heat conduction in a body, and overcrowding at the harbor to the accumulation of thermal energy and the subsequent rise in temperature near the surface of the body relative to its inner parts. Lumped system analysis is obviously not applicable when there is overcrowding at the surface. Of course, we have disregarded radiation in this analogy and thus the air traffic to the island. Like passengers at the harbor, heat changes *vehicles* at the surface from *convection* to *conduction.* Noting that a surface has zero thickness and thus cannot store any energy, heat reaching the surface of a body by convection must continue its journey within the body by conduction.

Consider heat transfer from a hot body to its cooler surroundings. Heat is transferred from the body to the surrounding fluid as a result of a temperature difference. But this energy comes from the region near the surface, and thus the temperature of the body near the surface will drop. This creates a *temperature gradient* between the inner and outer regions of the body and initiates heat transfer by conduction from the interior of the body toward the outer surface.

When the convection heat transfer coefficient h and thus the rate of convection from the body are high, the temperature of the body near the surface drops quickly (Fig. 18–8). This creates a larger temperature difference between the inner and outer regions unless the body is able to transfer heat from the inner to the outer regions just as fast. Thus, the magnitude of the maximum temperature difference within the body depends strongly on the ability of a body to conduct heat toward its surface relative to the ability of the surrounding medium to convect heat away from the surface. The Biot number is a measure of the relative magnitudes of these two competing effects.

Recall that heat conduction in a specified direction n per unit surface area is expressed as $\dot{q} = -k\,\partial T/\partial n$, where $\partial T/\partial n$ is the temperature gradient and k is the thermal conductivity of the solid. Thus, the temperature distribution in the body will be *uniform* only when its thermal conductivity is *infinite,* and no such material is known to exist. Therefore, temperature gradients and thus temperature differences must exist within the body, no matter how small, in order for heat conduction to take place. Of course, the temperature gradient and the thermal conductivity are inversely proportional for a given heat flux. Therefore, the larger the thermal conductivity, the smaller the temperature gradient.

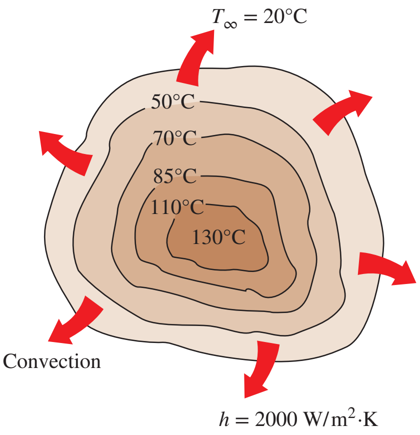


FIGURE 18–8
When the convection coefficient h is high and k is low, large temperature differences occur between the inner and outer regions of a large solid.

EXAMPLE 18–1 Temperature Measurement by Thermocouples

The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1-mm-diameter sphere, as shown in Fig. 18–9. The properties of the junction are $k = 35$ W/m·K, $\rho = 8500$ kg/m^3, and $c_p = 320$ J/kg·K, and the convection heat transfer coefficient between the junction and the gas is $h = 210$ W/m^2·K. Determine how long it will take for the thermocouple to read 99 percent of the initial temperature difference.

SOLUTION The temperature of a gas stream is to be measured by a thermocouple. The time it takes to register 99 percent of the initial ΔT is to be determined.

Assumptions **1** The junction is spherical with a diameter of $D = 0.001$ m. **2** The thermal properties of the junction and the heat transfer coefficient are constant. **3** Radiation effects are negligible.

Properties The properties of the junction are given in the problem statement.

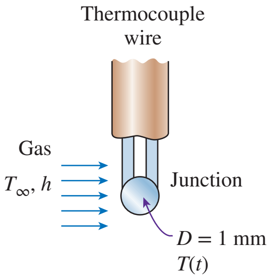


FIGURE 18–9
Schematic for Example 18–1.

Analysis The characteristic length of the junction is

$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D = \frac{1}{6}(0.001 \text{ m}) = 1.67 \times 10^{-4} \text{ m}$$

Then the Biot number becomes

$$\text{Bi} = \frac{hL_c}{k} = \frac{(210 \text{ W/m}^2\cdot\text{K})(1.67 \times 10^{-4} \text{ m})}{35 \text{ W/m}\cdot\text{K}} = 0.001 < 0.1$$

Therefore, lumped system analysis is applicable, and the error involved in this approximation is negligible.

In order to read 99 percent of the initial temperature difference $T_i - T_\infty$ between the junction and the gas, we must have

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = 0.01$$

For example, when $T_i = 0°\text{C}$ and $T_\infty = 100°\text{C}$, a thermocouple is considered to have read 99 percent of this applied temperature difference when its reading indicates $T(t) = 99°\text{C}$.

The value of the exponent b is

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{210 \text{ W/m}^2\cdot\text{K}}{(8500 \text{ kg/m}^3)\,(320 \text{ J/kg}\cdot\text{K})\,(1.67 \times 10^{-4} \text{ m})} = 0.462 \text{ s}^{-1}$$

We now substitute these values into Eq. 18–4 and obtain

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \quad \longrightarrow \quad 0.01 = e^{-(0.462 \text{ s}^{-1})t}$$

which yields

$$t = \mathbf{10 \text{ s}}$$

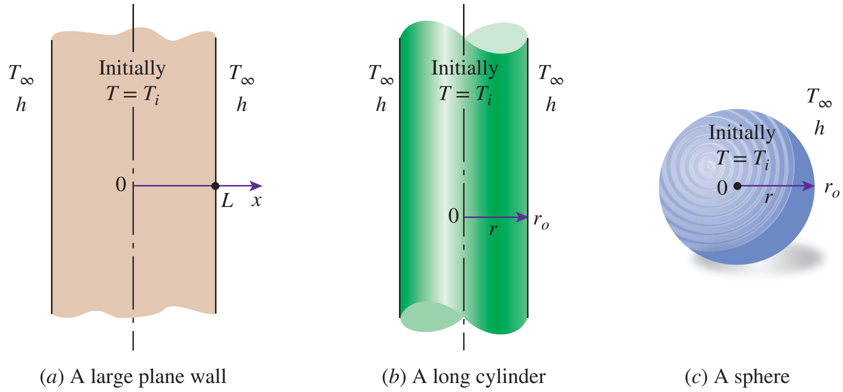
Therefore, we must wait at least 10 s for the temperature of the thermocouple junction to approach within 99 percent of the initial junction-gas temperature difference.

Discussion Note that conduction through the wires and radiation exchange with the surrounding surfaces affect the result, and they should be considered in a more refined analysis.

18–2 ▪ TRANSIENT HEAT CONDUCTION IN LARGE PLANE WALLS, LONG CYLINDERS, AND SPHERES WITH SPATIAL EFFECTS

In Sec. 18–1, we considered bodies in which the variation of temperature within the body is negligible; that is, bodies that remain nearly *isothermal* during a process. Relatively small bodies of highly conductive materials approximate this behavior. In general, however, the temperature within a body changes from point to point as well as with time. In this section, we consider the variation of temperature with *time* and *position* in one-dimensional problems such as those associated with a large plane wall, a long cylinder, and a sphere.

Consider a plane wall of thickness $2L$, a long cylinder of radius r_o, and a sphere of radius r_o initially at a *uniform temperature* T_i, as shown in Fig. 18–10. At time $t = 0$, each geometry is placed in a large medium that is at a constant temperature T_∞ and kept in that medium for $t > 0$. Heat transfer takes place between these bodies and their environments by convection with a *uniform* and *constant* heat



(*a*) A large plane wall (*b*) A long cylinder (*c*) A sphere

FIGURE 18–10
Schematic of the simple geometries in which heat transfer is one-dimensional.

transfer coefficient *h*. Note that all three cases possess geometric and thermal symmetry: the plane wall is symmetric about its *center plane* ($x = 0$), the cylinder is symmetric about its *centerline* ($r = 0$), and the sphere is symmetric about its *center point* ($r = 0$). We neglect *radiation* heat transfer between these bodies and their surrounding surfaces, or we incorporate the radiation effect into the convection heat transfer coefficient *h*.

The variation of the temperature profile with *time* in the plane wall is illustrated in Fig. 18–11. When the wall is first exposed to the surrounding medium at $T_\infty < T_i$ at $t = 0$, the entire wall is at its initial temperature T_i. But the wall temperature at and near the surfaces starts to drop as a result of heat transfer from the wall to the surrounding medium. This creates a *temperature gradient* in the wall and initiates heat conduction from the inner parts of the wall toward its outer surfaces. Note that the temperature at the center of the wall remains at T_i until $t = t_2$, and that the temperature profile within the wall remains symmetric at all times about the center plane. The temperature profile gets flatter and flatter as time passes as a result of heat transfer, and it eventually becomes uniform at $T = T_\infty$. That is, the wall reaches *thermal equilibrium* with its surroundings. At that point, heat transfer stops since there is no longer a temperature difference. Similar discussions can be given for the long cylinder or sphere.

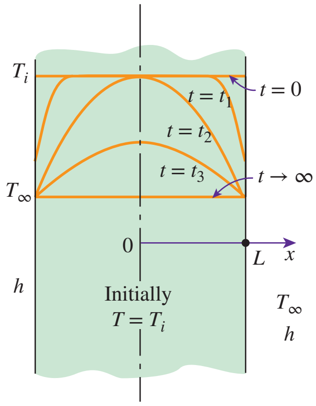


FIGURE 18–11
Transient temperature profiles in a plane wall exposed to convection from its surfaces for $T_i > T_\infty$.

Nondimensionalized One-Dimensional Transient Conduction Problem

The formulation of heat conduction problems for the determination of the one-dimensional transient temperature distribution in a plane wall, a cylinder, or a sphere results in a partial differential equation whose solution typically involves infinite series and transcendental equations, which are inconvenient to use. But the analytical solution provides valuable insight into the physical problem, and thus it is important to go through the steps involved. Next we demonstrate the solution procedure for the case of a plane wall.

Consider a plane wall of thickness $2L$ initially at a uniform temperature of T_i, as shown in Fig. 18–10*a*. At time $t = 0$, the wall is immersed in a fluid at temperature T_∞ and is subjected to convection heat transfer from both sides with a convection coefficient of *h*. The height and the width of the wall are large relative to its thickness, and thus heat conduction in the wall can be approximated to be one-dimensional. Also, there is thermal symmetry about the midplane passing through $x = 0$, and thus the temperature distribution must be symmetrical about the midplane. Therefore, the value of temperature at any $-x$ value in $-L \le x \le 0$ at any time t must be equal to the value at $+x$ in $0 \le x \le L$ at the same time. This means we can formulate and solve the heat conduction problem in the positive half domain $0 \le x \le L$, and then apply the solution to the other half.

Under the conditions of constant thermophysical properties, no heat generation, thermal symmetry about the midplane, uniform initial temperature, and constant convection coefficient, the one-dimensional transient heat conduction problem in the half-domain $0 \le x \le L$ of the plane wall can be expressed as (see Chap. 2 of Çengel and Ghajar, 2020)

Differential equation: $$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{18–10a}$$

Boundary conditions: $$\frac{\partial T(0, t)}{\partial x} = 0 \quad \text{and} \quad -k\frac{\partial T(L, t)}{\partial x} = h[T(L, t) - T_\infty] \tag{18–10b}$$

Initial condition: $$T(x, 0) = T_i \tag{18–10c}$$

where the property $\alpha = k/\rho c_p$ is the thermal diffusivity of the material.

We now attempt to nondimensionalize the problem by defining a dimensionless space variable $X = x/L$ and dimensionless temperature $\theta(x, t) = [T(x, t) - T_\infty]/[T_i - T_\infty]$. These are convenient choices since both X and θ vary between 0 and 1. However, there is no clear guidance for the proper form of the dimensionless time variable and the h/k ratio, so we will let the analysis indicate them. We note that

$$\frac{\partial \theta}{\partial X} = \frac{\partial \theta}{\partial (x/L)} = \frac{L}{T_i - T_\infty}\frac{\partial T}{\partial x}, \quad \frac{\partial^2 \theta}{\partial X^2} = \frac{L^2}{T_i - T_\infty}\frac{\partial^2 T}{\partial x^2} \qquad \text{and} \qquad \frac{\partial \theta}{\partial t} = \frac{1}{T_i - T_\infty}\frac{\partial T}{\partial t}$$

Substituting into Eqs. 18–10*a* and 18–10*b* and rearranging give

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{L^2}{\alpha}\frac{\partial \theta}{\partial t} \quad \text{and} \quad \frac{\partial \theta(1, t)}{\partial X} = \frac{-hL}{k}\theta(1, t) \tag{18–11}$$

Therefore, the proper form of the dimensionless time is $\tau = \alpha t/L^2$, which is called the **Fourier number** Fo (named after Jean Baptiste Joseph Fourier), and we recognize $\text{Bi} = hL/k$ as the Biot number defined in Sec. 18–1. Then the formulation of the one-dimensional transient heat conduction problem in a plane wall can be expressed in nondimensional form as

Dimensionless differential equation: $$\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau} \tag{18–12a}$$

Dimensionless BC's: $$\frac{\partial \theta(0, \tau)}{\partial X} = 0 \quad \text{and} \quad \frac{\partial \theta(1, \tau)}{\partial X} = -\text{Bi}\,\theta(1, \tau) \tag{18–12b}$$

Dimensionless initial condition: $$\theta(X, 0) = 1 \tag{18–12c}$$

where

$\theta(X, \tau) = \dfrac{T(x, t) - T_\infty}{T_i - T_\infty}$ *Dimensionless temperature*

$X = \dfrac{x}{L}$ *Dimensionless distance from the center*

$\text{Bi} = \dfrac{hL}{k}$ *Dimensionless heat transfer coefficient* (Biot number)

$\tau = \dfrac{\alpha t}{L^2} = \text{Fo}$ *Dimensionless time* (Fourier number)

The heat conduction equation in cylindrical or spherical coordinates can be nondimensionalized in a similar way. Note that nondimensionalization reduces the number of independent variables and parameters from 8 to 3—from $x, L, t, k, \alpha, h, T_i$, and T_∞ to X, Bi, and Fo (Fig. 18–12). That is,

$$\theta = f(X, \text{Bi}, \text{Fo}) \tag{18–13}$$

(*a*) Original heat conduction problem:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, \; T(x, 0) = T_i$$

$$\frac{\partial T(0, t)}{\partial x} = 0, \; -k\frac{\partial T(L, t)}{\partial x} = h[T(L, t) - T_\infty]$$

$$T = F(x, L, t, k, \alpha, h, T_i, T_\infty)$$

(*b*) Nondimensionalized problem:

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau}, \; \theta(X, 0) = 1$$

$$\frac{\partial \theta(0, \tau)}{\partial X} = 0, \; \frac{\partial \theta(1, \tau)}{\partial X} = -\text{Bi}\,\theta(1, \tau)$$

$$\theta = f(X, \text{Bi}, \tau)$$

FIGURE 18–12
Nondimensionalization reduces the number of independent variables in one-dimensional transient conduction problems from 8 to 3, offering great convenience in the presentation of results.

This makes it very practical to conduct parametric studies and avoid results in graphical form. Equation 18–13 is the generalized version of Eq. 18–4 for the lumped system analysis (no space variables). This can be shown by using the definitions of θ, α, L_c, Bi, and Fo in Eq. 18–4. The final result is

$$\theta = \frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} = e^{-\frac{hA_s t}{\rho V c_p}} = e^{-\text{BiFo}}$$

or $\theta = f(\text{Fo, Bi})$ which is the special case of Eq. 18–13 with no space variable.

The nondimensionalized partial differential equation given in Eqs. 18–4 through 18–12, together with its boundary and initial conditions, can be solved using several analytical and numerical techniques, including the Laplace or other transform methods, the method of separation of variables, the finite difference method, and the finite-element method. Using the method of separation of variables (see Chap. 4 of Çengel and Ghajar, 2020, for mathematical details), solutions for all three geometries are obtained in terms of infinite series and are summarized in Table 18–1. The solution for the plane wall is also applicable for a plane wall of thickness L whose left surface at $x = 0$ is insulated and whose right surface at $x = L$ is subjected to convection since this is precisely the mathematical problem we solved.

The analytical solutions of transient conduction problems typically involve infinite series, and thus the evaluation of an infinite number of terms to determine the temperature at a specified location and time. This may look intimidating at first, but there is no need to worry. As demonstrated in Fig. 18–13, the terms in the summation decline rapidly as n and thus λ_n increase because of the exponential decay function $e^{-\lambda_n^2 \tau}$. This is especially the case when the dimensionless time τ is large. Therefore, the evaluation of the first few terms of the infinite series (in this case just the first term) is usually adequate to determine the dimensionless temperature θ.

$$\theta_n = A_n e^{-\lambda_n^2 \tau} \cos(\lambda_n X)$$

$$A_n = \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)}$$

$$\lambda_n \tan \lambda_n = \text{Bi}$$

For Bi = 5, $X = 1$, and $\tau = 0.2$:

n	λ_n	A_n	θ_n
1	1.3138	1.2402	0.22321
2	4.0336	–0.3442	0.00835
3	6.9096	0.1588	0.00001
4	9.8928	–0.876	0.00000

FIGURE 18–13
The terms in the series solution of transient conduction problems decline rapidly as n and thus λ_n increase because of the exponential decay function with the exponent $-\lambda_n \tau$.

Approximate Analytical Solutions

The analytical solution we obtained for one-dimensional transient heat conduction in a plane wall involves infinite series and implicit equations, which are difficult to evaluate. Therefore, we have an incentive to simplify the analytical solutions and to present the solutions in *tabular* form using simple relations.

The dimensionless quantities defined above for a plane wall can also be used for a *cylinder* or *sphere* by replacing the space variable x with r and the half-thickness L with the outer radius r_o. Note that the characteristic length in the definition

TABLE 18–1

Summary of the solutions for one-dimensional transient conduction in a plane wall of thickness $2L$, a cylinder of radius r_o, and a sphere of radius r_o subjected to convection from all surfaces.*

Geometry	Solution	λ_n's are the roots of
Plane wall	$\theta = \sum_{n=1}^{\infty} \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \cos(\lambda_n x/L)$	$\lambda_n \tan \lambda_n = \text{Bi}$
Cylinder	$\theta = \sum_{n=1}^{\infty} \frac{2}{\lambda_n} \frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 \tau} J_0(\lambda_n r/r_o)$	$\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = \text{Bi}$
Sphere	$\theta = \sum_{n=1}^{\infty} \frac{4(\sin \lambda_n - \lambda_n \cos \lambda_n)}{2\lambda_n - \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \frac{\sin(\lambda_n r/r_o)}{\lambda_n r/r_o}$	$1 - \lambda_n \cot \lambda_n = \text{Bi}$

*Here $\theta = (T - T_\infty)/(T_i - T_\infty)$ is the dimensionless temperature, $\text{Bi} = hL/k$ or hr_o/k is the Biot number, $\text{Fo} = \tau = \alpha t/L^2$ or $\alpha t/r_o^2$ is the Fourier number, and J_0 and J_1 are the Bessel functions of the first kind whose values are given in Table 18–3. Note that the characteristic length used for each geometry in the equations for the Biot and Fourier numbers is different for the exact (analytical) solution than the one used for the lumped system analysis.

of the Biot number is taken to be the *half-thickness* L for the plane wall and the *radius* r_o for the long cylinder and sphere instead of V/A used in lumped system analysis.

We mentioned earlier that the terms in the series solutions in Table 18–1 converge rapidly with increasing time, and for $\tau > 0.2$, keeping the first term and neglecting all the remaining terms in the series results in an error under 2 percent. We are usually interested in the solution for times with $\tau > 0.2$, and thus it is very convenient to express the solution using this **one-term approximation**, given as

$$\textit{Plane wall:}\quad \theta_{\text{wall}} = \frac{T(x, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L) \qquad \tau > 0.2 \tag{18–14}$$

$$\textit{Cylinder:}\quad \theta_{\text{cyl}} = \frac{T(r, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o) \qquad \tau > 0.2 \tag{18–15}$$

$$\textit{Sphere:}\quad \theta_{\text{sph}} = \frac{T(r, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o} \qquad \tau > 0.2 \tag{18–16}$$

where the constants A_1 and λ_1 are functions of the Bi number only, and their values are listed in Table 18–2 against the Bi number for all three geometries. The function J_0 is the zeroth-order Bessel function of the first kind, whose value can be determined from Table 18–3. Noting that $\cos(0) = J_0(0) = 1$ and the limit of $(\sin x)/x$ is also 1, these relations simplify to the next ones at the center of a plane wall, cylinder, or sphere:

$$\textit{Center of plane wall } (x = 0)\textit{:}\qquad \theta_{0,\text{ wall}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \tag{18–17}$$

$$\textit{Center of cylinder } (r = 0)\textit{:}\qquad \theta_{0,\text{ cyl}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \tag{18–18}$$

$$\textit{Center of sphere } (r = 0)\textit{:}\qquad \theta_{0,\text{ sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \tag{18–19}$$

Comparing the two sets of equations above, we notice that the dimensionless temperatures anywhere in a plane wall, cylinder, or sphere are related to the center temperature by

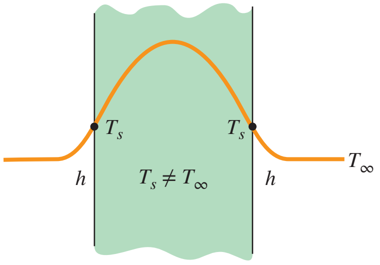
$$\frac{\theta_{\text{wall}}}{\theta_{0,\text{ wall}}} = \cos\left(\frac{\lambda_1 x}{L}\right), \quad \frac{\theta_{\text{cyl}}}{\theta_{0,\text{ cyl}}} = J_0\left(\frac{\lambda_1 r}{r_o}\right), \quad \text{and} \quad \frac{\theta_{\text{sph}}}{\theta_{0,\text{ sph}}} = \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o} \tag{18–20}$$

which shows that time dependence of dimensionless temperature within a given geometry is the same throughout. That is, if the dimensionless center temperature θ_0 drops by 20 percent at a specified time, so does the dimensionless temperature θ_0 anywhere else in the medium at the same time.

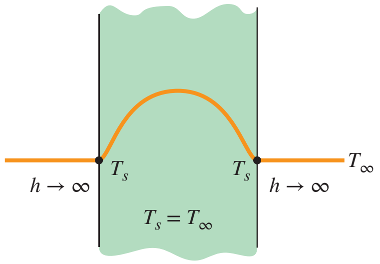
Once the Bi number is known, these relations can be used to determine the temperature anywhere in the medium. The determination of the constants A_1 and λ_1 from Table 18–2 usually requires interpolation.

Graphical representation of the one-term approximation solutions for a large plane wall, long cylinder, and sphere were presented by M. P. Heisler in 1947 and are called transient temperature charts or **Heisler charts**. They were supplemented in 1961 with transient heat transfer charts by H. Gröber. The charts are sometimes difficult to read, and they are subject to reading errors. Therefore, the one-term approximate expressions should be preferred to the charts and will be used in this chapter.

Note that the case $\text{Bi} = \infty$ in Table 18–2 corresponds to $h \to \infty$, which corresponds to the case of *specified surface temperature* T_∞. That is, the case in which the surfaces of the body are suddenly brought to the temperature T_∞ at $t = 0$ and kept at T_∞ at all times can be handled by setting h to infinity (Fig. 18–14).



(*a*) Finite convection coefficient



(*b*) Infinite convection coefficient

FIGURE 18–14
The specified surface temperature corresponds to the case of convection to an environment at T_∞ with a convection coefficient h that is *infinite*.

TABLE 18–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/k for a plane wall of thickness $2L$, and Bi = hr_o/k for a cylinder or sphere of radius r_o)

	Plane wall		Cylinder		Sphere	
Bi	λ_1(rad)	A_1	λ_1(rad)	A_1	λ_1(rad)	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 18–3

The zeroth- and first-order Bessel functions of the first kind

η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	0.4708
2.8	−0.1850	0.4097
3.0	−0.2601	0.3391
3.2	−0.3202	0.2613

The temperature of the body changes from the initial temperature T_i to the temperature of the surroundings T_∞ at the end of the transient heat conduction process. Thus, the *maximum* amount of heat that a body can gain (or lose if $T_i > T_\infty$) is simply the *change* in the *energy content* of the body. That is,

$$Q_{\max} = mc_p(T_\infty - T_i) = \rho V c_p(T_\infty - T_i) \quad \text{(kJ)} \tag{18–21}$$

where m is the mass, V is the volume, ρ is the density, and c_p is the specific heat of the body. Thus, $Q_{\max}$ represents the amount of heat transfer for $t \to \infty$. The amount of heat transfer Q at a finite time t is obviously less than this maximum, and it can be expressed as the sum of the internal energy changes throughout the entire geometry,

$$Q = \int_V \rho c_p [T(x, t) - T_i] dV \tag{18–22}$$

where $T(x, t)$ is the temperature distribution in the medium at time t. Assuming constant properties, the ratio of $Q/Q_{\max}$ becomes

$$\frac{Q}{Q_{\max}} = \frac{\int_V \rho c_p [T(x, t) - T_i] dV}{\rho c_p (T_\infty - T_i) V} = \frac{1}{V}\int_V (1 - \theta) dV \tag{18–23}$$

Using the appropriate nondimensional temperature relations based on the one-term approximation for the plane wall, cylinder, and sphere, and performing the indicated integrations, we obtain the following relations for the fraction of heat transfer in those geometries:

Plane wall:
$$\left(\frac{Q}{Q_{\max}}\right)_{\text{wall}} = 1 - \theta_{0,\,\text{wall}}\frac{\sin\lambda_1}{\lambda_1} \tag{18–24}$$

Cylinder:
$$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\,\text{cyl}}\frac{J_1(\lambda_1)}{\lambda_1} \tag{18–25}$$

Sphere:
$$\left(\frac{Q}{Q_{\max}}\right)_{\text{sph}} = 1 - 3\theta_{0,\,\text{sph}}\frac{\sin\lambda_1 - \lambda_1\cos\lambda_1}{\lambda_1^3} \tag{18–26}$$

Note that once the *fraction* of heat transfer $Q/Q_{\max}$ based on the one-term approximation relations given above has been determined for the given t, the actual amount of heat transfer by that time can be evaluated by multiplying this fraction by $Q_{\max}$ given by Eq. 18–21. A *negative* sign for $Q_{\max}$ indicates that the body is *rejecting* heat.

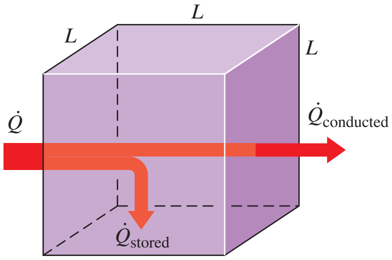
The use of the one-term approximate solutions already discussed is limited to the conditions specified at the beginning of this section: the body is initially at a *uniform* temperature, the temperature of the medium surrounding the body and the convection heat transfer coefficient are *constant* and *uniform,* and there is no *heat generation* in the body.

We discussed the physical significance of the *Biot number* earlier and indicated that it is a measure of the relative magnitudes of the two heat transfer mechanisms: *convection* at the surface and *conduction* through the solid. A *small* value of Bi indicates that the inner resistance of the body to heat conduction is *small* relative to the resistance to convection between the surface and the fluid. As a result, the temperature distribution within the solid becomes fairly uniform, and lumped system analysis becomes applicable. Recall that when $\text{Bi} < 0.1$, the error in assuming the temperature within the body to be *uniform* is negligible.

To understand the physical significance of the *Fourier number* τ (or Fo), we express it as (Fig 18.15)

$$\tau = \frac{\alpha t}{L^2} = \frac{kL^2(1/L)\ \Delta T}{\rho c_p L^3/t\ \ \Delta T} = \frac{\begin{array}{c}\text{The rate at which heat is } \textit{conducted}\\ \text{across a body of thickness } L \text{ and}\\ \text{normal area } L^2 \text{ (and thus volume } L^3\text{)}\end{array}}{\begin{array}{c}\text{The rate at which heat is stored}\\ \text{in a body of volume } L^3\end{array}} \tag{18–27}$$

Therefore, the Fourier number is a measure of *heat conducted* through a body relative to *heat stored.* Thus, a large value of the Fourier number indicates faster propagation of heat through a body.



Fourier number: $\tau = \dfrac{\alpha t}{L^2} = \dfrac{\dot{Q}_{\text{conducted}}}{\dot{Q}_{\text{stored}}}$

FIGURE 18–15
Fourier number at time t can be viewed as the ratio of the rate of heat conducted to the rate of heat stored at that time.

Perhaps you are wondering about what constitutes an infinitely large plate or an infinitely long cylinder. After all, nothing in this world is infinite. A plate whose thickness is small relative to the other dimensions can be modeled as an infinitely large plate, except very near the outer edges. But the edge effects on large bodies are usually negligible, and thus a large plane wall such as the wall of a house can be modeled as an infinitely large wall for heat transfer purposes. Similarly, a long cylinder whose diameter is small relative to its length can be analyzed as an infinitely long cylinder. The use of the one-term approximate solutions is illustrated in Examples 18–2, 18–3, and 18–4.

EXAMPLE 18–2 Heating of Brass Plates in an Oven

In a production facility, large brass plates of 4-cm thickness that are initially at a uniform temperature of 20°C are heated by passing them through an oven that is maintained at 500°C (Fig. 18–16). The plates remain in the oven for 7 min. Taking the combined convection and radiation heat transfer coefficient to be $h = 120$ W/m^2·K, determine the surface temperature of the plates when they come out of the oven.

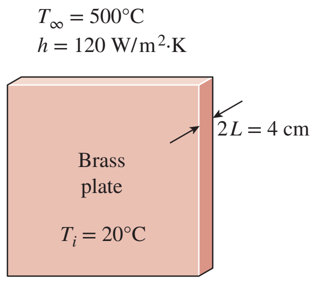


FIGURE 18–16
Schematic for Example 18–2.

SOLUTION Large brass plates are heated in an oven. The surface temperature of the plates leaving the oven is to be determined.

Assumptions **1** Heat conduction in the plate is one-dimensional since the plate is large relative to its thickness and there is thermal symmetry about the center plane. **2** The thermal properties of the plate and the heat transfer coefficient are constant. **3** The Fourier number is $\tau > 0.2$, so the one-term approximate solutions are applicable.

Properties The properties of brass at room temperature are $k = 110$ W/m·K, $\rho = 8530$ kg/m^3, $c_p = 380$ J/kg·K, and $\alpha = 33.9 \times 10^{-6}$ m^2/s (Table A–24). More accurate results are obtained by using properties at average temperature.

Analysis The temperature at a specified location at a given time can be determined from the one-term solutions. Noting that the half-thickness of the plate is $L = 0.02$ m, the Biot number for this problem is

$$\text{Bi} = \frac{hL}{k} = \frac{(120\ \text{W/m}^2\cdot\text{K})\ (0.02\ \text{m})}{52\ \text{W/m}\cdot\text{K}} = 0.022$$

The coefficients λ_1 and A_1 for a plane wall corresponding to this Bi are determined from Table 18–2 to be

$$\lambda_1 = 0.1468, \quad A_1 = 1.0036$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(33.9 \times 10^{-6}\ \text{m}^2/\text{s})\ (7 \times 60\ \text{s})}{(0.02\ \text{m})^2} = 35.6$$

which is greater than 0.2, and thus the one-term approximate solution is applicable.

The surface ($x = L$) temperature of the plates is determined by substituting these and other values into Eq. 18–14 and solving for $T(L, t)$

$$\frac{T(x, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L) \rightarrow \frac{T(L, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1)$$

$$T(L, t) = (T_i - T_\infty)\, A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1) + T_\infty$$

and thus

$$T(L, t) = (20 - 500)\ (1.0036) e^{-(0.1468)^2 (35.6)} \cos(0.1468) - 500 = 279°\text{C}$$

Therefore, the surface temperature of the plates will be 279°C when they leave the oven.

Discussion We notice that the Biot number in this case is Bi = 0.022, which is much less than 0.1. Therefore, we expect the lumped system analysis to be applicable, and the lumped system analysis in this case may yield just as accurate results with less effort.

The heat transfer surface area of the plate is $2A$, where A is the face area of the plate (the plate transfers heat through both of its surfaces), and the volume of the plate is $V = (2L)A$, where L is the half-thickness of the plate. The exponent b used in the lumped system analysis is

$$b = \frac{hA_s}{\rho c_p V} = \frac{h(2A)}{\rho c_p (2LA)} = \frac{h}{\rho c_p L}$$

$$= \frac{120\ \text{W/m}^2\cdot\text{K}}{(8530\ \text{kg/m}^3)\ (380\ \text{J/kg}\cdot\text{K})\ (0.02\ \text{m})} = 0.00185\ \text{s}^{-1}$$

Then the temperature of the plate at $t = 7$ min $= 420$ s is determined from

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T(t) - 500}{20 - 500} = e^{-(0.00185\ \text{s}^{-1})(420\ \text{s})}$$

It yields

$$T(t) = 279°\text{C}$$

which is identical to the result obtained above using the one-term approximate solution. Therefore, we can use lumped system analysis with confidence when the Biot number is sufficiently small.

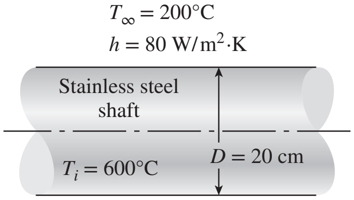


FIGURE 18–17
Schematic for Example 18–3.

EXAMPLE 18–3 Cooling of a Long Stainless Steel Cylindrical Shaft

A long 20-cm-diameter cylindrical shaft made of stainless steel 304 comes out of an oven at a uniform temperature of 600°C (Fig. 18–17). The shaft is then allowed to cool slowly in an environment chamber at 200°C with an average heat transfer coefficient of $h = 80$ W/m²·K. Determine the temperature at the center of the shaft 45 min after the start of the cooling process. Also, determine the heat transfer per unit length of the shaft during this time period.

SOLUTION A long cylindrical shaft is allowed to cool slowly. The center temperature and the heat transfer per unit length are to be determined.

Assumptions **1** Heat conduction in the shaft is one-dimensional since it is long and it has thermal symmetry about the centerline. **2** The thermal properties of the shaft and the heat transfer coefficient are constant. **3** The Fourier number is $\tau > 0.2$, so the one-term approximate solutions are applicable.

Properties The properties of stainless steel 304 at room temperature are $k = 14.9$ W/m·K, $\rho = 7900$ kg/m³, $c_p = 477$ J/kg·K, and $\alpha = 3.95 \times 10^{-6}$ m²/s (Table A–24). More accurate results can be obtained by using properties at average temperature.

Analysis The temperature within the shaft may vary with the radial distance r as well as with time, and the temperature at a specified location at a given time can be determined from the one-term approximate solutions. Noting that the radius of the shaft is $r_o = 0.1$ m, the Biot and Fourier numbers for this problem are

$$\text{Bi} = \frac{hr_o}{k} = \frac{(80\ \text{W/m}^2\cdot\text{K})\ (0.1\ \text{m})}{14.9\ \text{W/m}\cdot\text{K}} = 0.537$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(3.95 \times 10^{-5}\ \text{m}^2/\text{s})\ (45 \times 60\ \text{s})}{(0.1\ \text{m})^2} = 1.07$$

The coefficients λ_1 and A_1 for a cylinder corresponding to this Bi are determined from Table 18–2 to be

$$\lambda_1 = 0.970, \quad A_1 = 1.122$$

Substituting these values into Eq. 18–18 gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = 1.122 e^{-(0.970)^2(1.07)} = 0.41$$

and thus the temperature at the center of the shaft is

$$T_0 = T_\infty + 0.41(T_i - T_\infty) = 200 + 0.41(600 - 200) = \mathbf{364°C}$$

To determine the actual heat transfer, we first need to calculate the maximum heat that can be transferred from the cylinder, which is the sensible energy of the cylinder relative to its environment. Taking $L = 1$ m,

$$m = \rho V = \rho\pi r_o^2 L = (7900\ \text{kg/m}^3)\pi(0.1\ \text{m})^2(1\ \text{m}) = 248.2\ \text{kg}$$

$$\begin{aligned} Q_{\max} &= mc_p(T_i - T_\infty) = (248.2\ \text{kg})(0.477\ \text{kJ/kg}\cdot\text{K})(600 - 200)°\text{C} \\ &= 47{,}350\ \text{kJ} \end{aligned}$$

Then the fractional heat transfer ($Q/Q_{\max}$) for the cylinder is determined from Eq. 18–25 to be

$$\frac{Q}{Q_{\max}} = 1 - 2\theta_0 \frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2 \times 0.41 \frac{0.430}{0.970} = 0.636$$

where the value of $J_1(\lambda_1)$ for $\lambda_1 = 0.970$ is determined from Table 18–3 to be 0.430.

The total heat transfer from the shaft during the first 45 min of the cooling is

$$Q = 0.636 Q_{\max} = 0.636 \times (47{,}350 \text{ kJ}) = \mathbf{30{,}120 \text{ kJ}}$$

Discussion During the 45 minutes of the cooling period, the center temperature drops by 236°C, and the heat transferred from the shaft to the environment during this cooling process is about 64 percent of the maximum heat that can be transferred from the shaft.

EXAMPLE 18–4 Boiling Eggs

An ordinary egg can be approximated as a 5-cm-diameter sphere (Fig. 18–18). The egg is initially at a uniform temperature of 5°C and is dropped into boiling water at 95°C. Taking the convection heat transfer coefficient to be $h = 1200$ W/m^2·K, determine how long it will take for the center of the egg to reach 70°C.

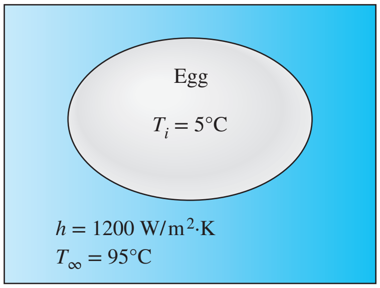


FIGURE 18–18
Schematic for Example 18–4.

SOLUTION An egg is cooked in boiling water. The cooking time of the egg is to be determined.

Assumptions **1** The egg is spherical with a radius of $r_o = 2.5$ cm. **2** Heat conduction in the egg is one-dimensional because of thermal symmetry about the midpoint. **3** The thermal properties of the egg and the heat transfer coefficient are constant. **4** The Fourier number is $\tau > 0.2$, so the one-term approximate solutions are applicable.

Properties The water content of eggs is about 74 percent, and thus the thermal conductivity and diffusivity of eggs can be approximated by those of water at the average temperature of $(5 + 70)/2 = 37.5°C$; $k = 0.627$ W/m·K and $\alpha = k/\rho c_p = 0.151 \times 10^{-6}$ m^2/s (Table A–15).

Analysis Egg white begins to thicken at 63°C and turns solid at 65°C. The yolk begins to thicken at 65°C and sets at 70°C. The whole egg sets at temperatures above 70°C. Therefore, the egg in this case will qualify as hard boiled. The temperature within the egg varies with radial distance as well as time, and the temperature at a specified location at a given time can be determined from the one-term approximate solution. The Biot number for this problem is

$$\text{Bi} = \frac{hr_o}{k} = \frac{(1200 \text{ W/m}^2\cdot\text{K})(0.025 \text{ m})}{0.627 \text{ W/m}\cdot\text{K}} = 47.8$$

which is much greater than 0.1, and thus the lumped system analysis is not applicable. The coefficients λ_1 and A_1 for a sphere corresponding to this Bi are, from Table 18–2,

$$\lambda_1 = 3.0754, \quad A_1 = 1.9958$$

Substituting these and other values into Eq. 18–19 and solving for τ gives

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70 - 95}{5 - 95} = 1.9958 e^{-(3.0754)^2 \tau} \longrightarrow \tau = 0.209$$

which is greater than 0.2, and thus the one-term approximate solution is applicable with an error of less than 2 percent. Then the cooking time is determined from the definition of the Fourier number to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.209)(0.025 \text{ m})^2}{0.151 \times 10^{-6} \text{ m}^2/\text{s}} = 865 \text{ s} \approx \mathbf{14.4 \text{ min}}$$

Therefore, it will take about 15 min for the center of the egg to be heated from 5°C to 70°C.

Discussion Note that the Biot number in lumped system analysis was defined differently as $\text{Bi} = hL_c/k = h(r_o/3)/k$. However, either definition can be used in determining the applicability of the lumped system analysis unless $\text{Bi} \approx 0.1$.

Also note that the cooking time depends on many parameters, such as the size of the egg, its temperature before cooking, the boiling temperature of water (and thus altitude), and the heat transfer coefficient (and thus the level of bubble motion during boiling). Therefore, there is a considerable amount of science or a good amount of experience behind boiling eggs to the correct amount of doneness.

18–3 ▪ TRANSIENT HEAT CONDUCTION IN SEMI-INFINITE SOLIDS

A semi-infinite solid is an idealized body that has a *single plane surface* and extends to infinity in all directions, as shown in Fig. 18–19. This idealized body is used to indicate that the temperature change in the part of the body in which we are interested (the region close to the surface) is due to the thermal conditions on a single surface. The earth, for example, can be considered to be a semi-infinite medium in determining the variation of temperature near its surface. Also, a thick wall can be modeled as a semi-infinite medium if all we are interested in is the variation of temperature in the region near one of the surfaces, and the other surface is too far away to have any impact on the region of interest during the time of observation. The temperature in the core region of the wall remains unchanged in this case.

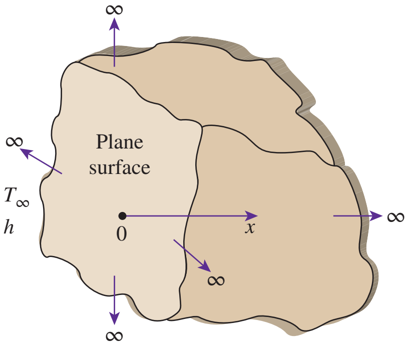


FIGURE 18–19
Schematic of a semi-infinite body.

For short periods of time, most bodies can be modeled as semi-infinite solids since heat does not have sufficient time to penetrate deep into the body, and the thickness of the body does not enter into the heat transfer analysis. A steel piece of any shape, for example, can be treated as a semi-infinite solid when it is quenched rapidly to harden its surface. A body whose surface is heated by a laser pulse can be treated the same way.

Consider a semi-infinite solid with constant thermophysical properties, no internal heat generation, uniform thermal conditions on its exposed surface, and initially a uniform temperature of T_i throughout. Heat transfer in this case occurs only in the direction normal to the surface (the x direction), and thus it is one-dimensional. Differential equations are independent of the boundary or initial conditions, and thus Eq. 18–10*a* for one-dimensional transient conduction in Cartesian coordinates applies. The depth of the solid is large ($x \to \infty$) compared to the depth that heat can penetrate, and these phenomena can be expressed mathematically as a boundary condition as $T(x \to \infty, t) = T_i$.

Heat conduction in a semi-infinite solid is governed by the thermal conditions imposed on the exposed surface, and thus the solution depends strongly on the boundary condition at $x = 0$. Below we present a detailed analytical solution for the case of constant temperature T_s on the surface and give the results for other more complicated boundary conditions. When the surface temperature is changed to T_s at $t = 0$ and held constant at that value at all times, the formulation of the problem can be expressed as

Differential equation:
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{18–28a}$$

Boundary conditions:
$$T(0,\ t) = T_s \quad \text{and} \quad T(x \to \infty, t) = T_i \tag{18–28b}$$

Initial condition:
$$T(x, 0) = T_i \tag{18–28c}$$

The separation of variables technique does not work in this case since the medium is infinite. But another clever approach that converts the partial differential equation into an ordinary differential equation by combining the two independent variables x

and t into a single variable η, called the **similarity variable**, works well. For transient conduction in a semi-infinite medium, it is defined as

Similarity variable:
$$\eta = \frac{x}{\sqrt{4\alpha t}} \tag{18–29}$$

Assuming $T = T(\eta)$ (to be verified) and using the chain rule, all derivatives in the heat conduction equation can be transformed into the new variable, as shown in Fig. 18–20. Noting that $\eta = 0$ at $x = 0$ and $\eta \to \infty$ as $x \to \infty$ (and also at $t = 0$) and substituting into Eqs. 18–28 give, after simplification,

$$\frac{d^2T}{d\eta^2} = -2\eta\frac{dT}{d\eta} \tag{18–30a}$$

$$T(0) = T_s \quad \text{and} \quad T(\eta \to \infty) = T_i \tag{18–30b}$$

Note that the second boundary condition and the initial condition result in the same boundary condition. Both the transformed equation and the boundary conditions depend on η only and are independent of x and t. Therefore, transformation is successful, and η is indeed a similarity variable.

To solve the second-order ordinary differential equation in Eqs. 18–30, we define a new variable w as $w = dT/d\eta$. This reduces Eq. 18–30a into a first-order differential equation that can be solved by separating variables,

$$\frac{dw}{d\eta} = -2\eta w \quad \rightarrow \quad \frac{dw}{w} = -2\eta d\eta \quad \rightarrow \quad \ln w = -\eta^2 + C_0 \rightarrow w = C_1 e^{-\eta^2}$$

where $C_1 = \ln C_0$. Back substituting $w = dT/d\eta$ and integrating again,

$$T = C_1\int_0^{\eta} e^{-u^2}du + C_2 \tag{18–31}$$

where u is a dummy integration variable. The boundary condition at $\eta = 0$ gives $C_2 = T_s$, and the one for $\eta \to \infty$ gives

$$T_i = C_1\int_0^{\infty} e^{-u^2}\,du + C_2 = C_1\frac{\sqrt{\pi}}{2} + T_s \rightarrow C_1 = \frac{2(T_i - T_s)}{\sqrt{\pi}} \tag{18–32}$$

Substituting the C_1 and C_2 expressions into Eq. 18–31 and rearranging, the variation of temperature becomes

$$\frac{T - T_s}{T_i - T_s} = \frac{2}{\sqrt{\pi}}\int_0^{\eta} e^{-u^2}du = \text{erf}(\eta) = 1 - \text{erfc}(\eta) \tag{18–33}$$

where the mathematical functions

$$\text{erf}(\eta) = \frac{2}{\sqrt{\pi}}\int_0^{\eta} e^{-u^2}du \quad \text{and} \quad \text{erfc}(\eta) = 1 - \text{erf}(\eta) = 1 - \frac{2}{\sqrt{\pi}}\int_0^{\eta} e^{-u^2}du \tag{18–34}$$

are called the **error function** and the **complementary error function**, respectively, of argument η (Fig. 18–21). Despite its simple appearance, the integral in the definition of the error function cannot be performed analytically. Therefore, the function erfc(η) is evaluated numerically for different values of η, and the results are listed in Table 18–4.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{and} \quad \eta = \frac{x}{\sqrt{4\alpha t}}$$

$$\frac{\partial T}{\partial t} = \frac{dT}{d\eta}\frac{\partial \eta}{\partial t} = \frac{-x}{2t\sqrt{4\alpha t}}\frac{dT}{d\eta}$$

$$\frac{\partial T}{\partial x} = \frac{dT}{d\eta}\frac{\partial \eta}{\partial x} = \frac{1}{\sqrt{4\alpha t}}\frac{dT}{d\eta}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{d}{d\eta}\left(\frac{\partial T}{\partial x}\right)\frac{\partial \eta}{\partial x} = \frac{1}{4\alpha t}\frac{d^2T}{d\eta^2}$$

FIGURE 18–20
Transformation of variables in the derivatives of the heat conduction equation by the use of the chain rule.

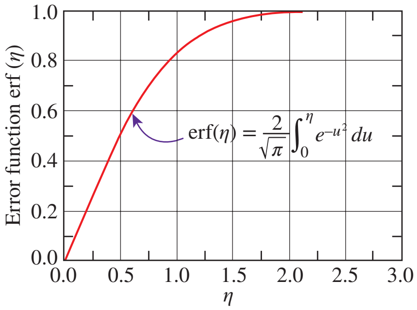


FIGURE 18–21
Error function is a standard mathematical function, just like the sinus and tangent functions, whose value varies between 0 and 1.

TABLE 18–4

The complementary error function

η	erfc (η)	η	erfc (η)	η	erfc (η)	η	erfc (η)	η	erfc (η)	η	erfc (η)
0.00	1.00000	0.38	0.5910	0.76	0.2825	1.14	0.1069	1.52	0.03159	1.90	0.00721
0.02	0.9774	0.40	0.5716	0.78	0.2700	1.16	0.10090	1.54	0.02941	1.92	0.00662
0.04	0.9549	0.42	0.5525	0.80	0.2579	1.18	0.09516	1.56	0.02737	1.94	0.00608
0.06	0.9324	0.44	0.5338	0.82	0.2462	1.20	0.08969	1.58	0.02545	1.96	0.00557
0.08	0.9099	0.46	0.5153	0.84	0.2349	1.22	0.08447	1.60	0.02365	1.98	0.00511
0.10	0.8875	0.48	0.4973	0.86	0.2239	1.24	0.07950	1.62	0.02196	2.00	0.00468
0.12	0.8652	0.50	0.4795	0.88	0.2133	1.26	0.07476	1.64	0.02038	2.10	0.00298
0.14	0.8431	0.52	0.4621	0.90	0.2031	1.28	0.07027	1.66	0.01890	2.20	0.00186
0.16	0.8210	0.54	0.4451	0.92	0.1932	1.30	0.06599	1.68	0.01751	2.30	0.00114
0.18	0.7991	0.56	0.4284	0.94	0.1837	1.32	0.06194	1.70	0.01612	2.40	0.00069
0.20	0.7773	0.58	0.4121	0.96	0.1746	1.34	0.05809	1.72	0.01500	2.50	0.00041
0.22	0.7557	0.60	0.3961	0.98	0.1658	1.36	0.05444	1.74	0.01387	2.60	0.00024
0.24	0.7343	0.62	0.3806	1.00	0.1573	1.38	0.05098	1.76	0.01281	2.70	0.00013
0.26	0.7131	0.64	0.3654	1.02	0.1492	1.40	0.04772	1.78	0.01183	2.80	7.5E-05
0.28	0.6921	0.66	0.3506	1.04	0.1413	1.42	0.04462	1.80	0.01091	2.90	4.1E-05
0.30	0.6714	0.68	0.3362	1.06	0.1339	1.44	0.04170	1.82	0.01006	3.00	2.2E-05
0.32	0.6509	0.70	0.3222	1.08	0.1267	1.46	0.03895	1.84	0.00926	3.20	6.0E-06
0.34	0.6306	0.72	0.3086	1.10	0.1198	1.48	0.03635	1.86	0.00853	3.40	1.5E-06
0.36	0.6107	0.74	0.2953	1.12	0.1132	1.50	0.03390	1.88	0.00784	3.60	3.6E-07

Knowing the temperature distribution, the heat flux at the surface can be determined from Fourier's law to be

$$\dot{q}_s = -k\frac{\partial T}{\partial x}\bigg|_{x=0} = -k\frac{dT}{d\eta}\frac{\partial \eta}{\partial x}\bigg|_{\eta=0} = -kC_1 e^{-\eta^2}\frac{1}{\sqrt{4\alpha t}}\bigg|_{\eta=0} = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}} \qquad \textbf{(18–35)}$$

The solutions in Eqs. 18–33 and 18–35 correspond to the case when the temperature of the exposed surface of the medium is suddenly raised (or lowered) to T_s at $t = 0$ and is maintained at that value at all times. The specified surface temperature case is closely approximated in practice when condensation or boiling takes place on the surface. Using a similar approach or the Laplace transform technique, analytical solutions can be obtained for other boundary conditions on the surface, with the following results.

Case 1: Specified Surface Temperature, T_s = constant (Fig. 18–22).

$$\frac{T(x, t) - T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \quad \text{and} \quad \dot{q}_s(t) = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}} \qquad \textbf{(18–36)}$$

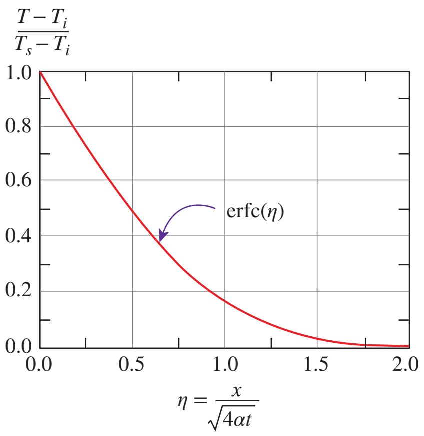


FIGURE 18–22
Dimensionless temperature distribution for transient conduction in a semi-infinite solid whose surface is maintained at a constant temperature T_s.

Case 2: Specified Surface Heat Flux, $\dot{q}_s$ = constant.

$$T(x, t) - T_i = \frac{\dot{q}_s}{k}\left[\sqrt{\frac{4\alpha t}{\pi}}\exp\left(-\frac{x^2}{4\alpha t}\right) - x\,\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right] \qquad \textbf{(18–37)}$$

Case 3: Convection on the Surface, $\dot{q}_s(t) = h[T_\infty - T(0, t)]$.

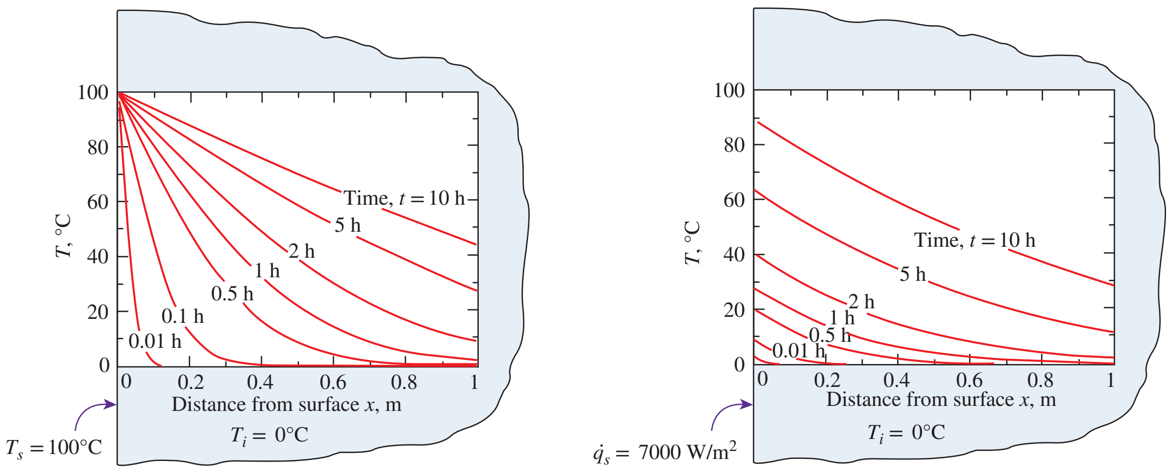
$$\frac{T(x, t) - T_i}{T_\infty - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right)\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right) \qquad \textbf{(18–38)}$$

Case 4: Energy Pulse at Surface, e_s = constant.
Energy in the amount of e_s per unit surface area (in J/m^2) is supplied to the semi-infinite body instantaneously at time $t = 0$ (by a laser pulse, for example), and all energy is assumed to enter the body, with no heat loss from the surface.

$$T(x, t) - T_i = \frac{e_s}{k\sqrt{\pi t/\alpha}} \exp\left(-\frac{x^2}{4\alpha t}\right) \tag{18–39}$$

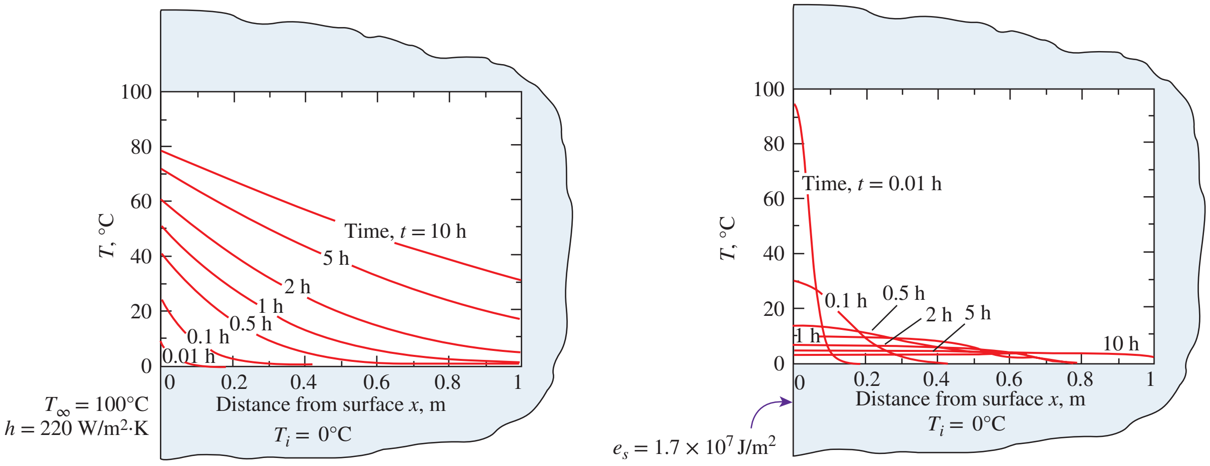
Note that Cases 1 and 3 are closely related. In Case 1, the surface $x = 0$ is brought to a temperature T_s at time $t = 0$ and kept at that value at all times. In Case 3, the surface is exposed to convection by a fluid at a constant temperature T_∞ with a heat transfer coefficient h.

The solutions for all four cases are plotted in Fig. 18–23 for a representative case using a large cast iron block initially at 0°C throughout. In Case 1, the surface temperature remains constant at the specified value of T_s, and temperature



(a) Specified surface temperature, T_s = constant

(b) Specified surface heat flux, $\dot{q}_s$ = constant



(c) Convection at the surface

(d) Energy pulse at the surface, e_s = constant

FIGURE 18–23
Variations of temperature with position and time in a large cast iron block ($\alpha = 2.31 \times 10^{-5}$ m^2/s, $k = 80.2$ W/m·K) initially at 0°C under different thermal conditions on the surface.

increases gradually within the medium as heat penetrates deeper into the solid. Note that during initial periods only a thin slice near the surface is affected by heat transfer. Also, the temperature gradient at the surface and thus the rate of heat transfer into the solid decreases with time. In Case 2, heat is continually supplied to the solid, and thus the temperature within the solid, including the surface, increases with time. This is also the case with convection (Case 3), except that the surrounding fluid temperature T_∞ is the highest temperature that the solid body can rise to. In Case 4, the surface is subjected to an instant burst of heat supply at time $t = 0$, such as heating by a laser pulse, and then the surface is covered with insulation. The result is an instant rise in surface temperature, followed by a temperature drop as heat is conducted deeper into the solid. Note that the temperature profile is always normal to the surface at all times. (Why?)

The variation of temperature with position and time in a semi-infinite solid subjected to convection heat transfer is plotted in Fig. 18–24 for the nondimensionalized temperature against the dimensionless similarity variable $\eta = x/\sqrt{4\alpha t}$ for various values of the parameter $h\sqrt{\alpha t}/k$. Although the graphical solution given in Fig. 18–24 is simply a plot of the exact analytical solution, it is subject to reading errors, and thus is of limited accuracy compared to the analytical solution. Also, the values on the vertical axis of Fig. 18–24 correspond to $x = 0$ and thus represent the surface temperature. The curve $h\sqrt{\alpha t}/k = \infty$ corresponds to $h \to \infty$, which corresponds to the case of *specified temperature* T_∞ at the surface at $x = 0$. That is, the case in which the surface of the semi-infinite body is suddenly brought to temperature T_∞ at $t = 0$ and kept at T_∞ at all times can be handled by setting h to infinity. For a *finite* heat transfer coefficient h, the surface temperature approaches the fluid temperature T_∞ as the time t approaches infinity.

Contact of Two Semi-Infinite Solids

When two large bodies A and B, initially at uniform temperatures $T_{A,i}$ and $T_{B,i}$ are brought into contact, they instantly achieve temperature equality at the contact surface (temperature equality is achieved over the entire surface if the contact resistance is negligible). If the two bodies are of the same material with constant properties, thermal symmetry requires the contact surface temperature to be the arithmetic average, $T_s = (T_{A,i} + T_{B,i})/2$, and to remain constant at that value at all times.

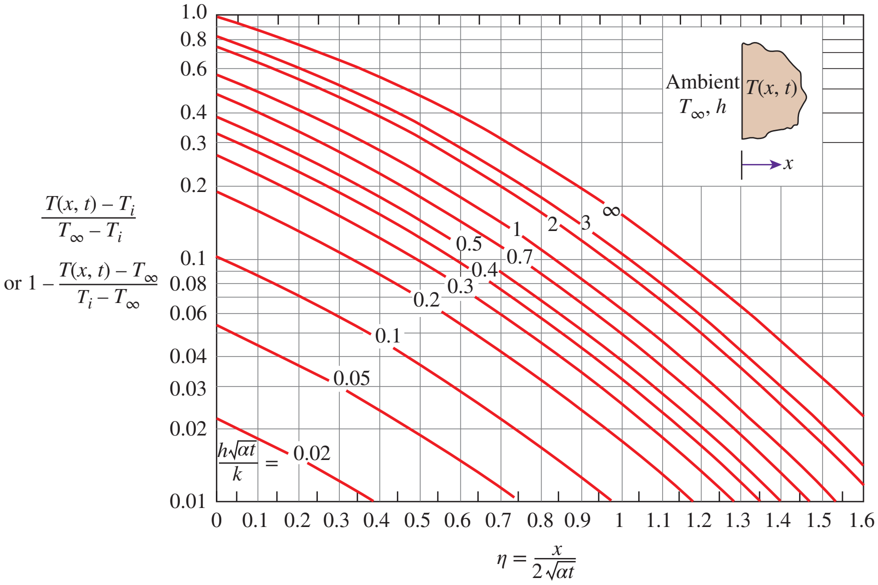


FIGURE 18–24

Variation of temperature with position and time in a semi-infinite solid initially at temperature T_i subjected to convection to an environment at T_∞ with a convection heat transfer coefficient of h.

If the bodies are of different materials, they still achieve a temperature equality, but the surface temperature T_s in this case will be different than the arithmetic average. Noting that both bodies can be treated as semi-infinite solids with the same specified surface temperature, the energy balance on the contact surface gives, from Eq. 18–36,

$$\dot{q}_{s,A} = \dot{q}_{s,B} \rightarrow -\frac{k_A(T_s - T_{A,i})}{\sqrt{\pi\alpha_A t}} = \frac{k_B(T_s - T_{B,i})}{\sqrt{\pi\alpha_B t}} \rightarrow \frac{T_{A,i} - T_s}{T_s - T_{B,i}} = \sqrt{\frac{(k\rho c_p)_B}{(k\rho c_p)_A}}$$

Then T_s is determined to be (Fig. 18–25)

$$T_s = \frac{\sqrt{(k\rho c_p)_A}\,T_{A,i} + \sqrt{(k\rho c_p)_B}\,T_{B,i}}{\sqrt{(k\rho c_p)_A} + \sqrt{(k\rho c_p)_B}} \tag{18–40}$$

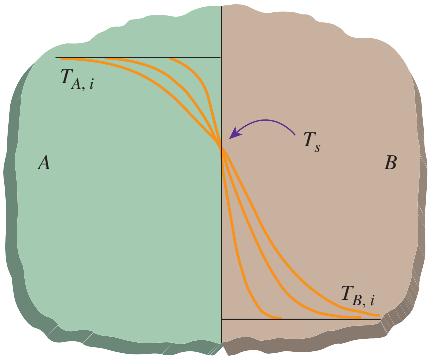


FIGURE 18–25
Contact of two semi-infinite solids of different initial temperatures.

Therefore, the interface temperature of two bodies brought into contact is dominated by the body with the larger $k\rho c_p$. This also explains why a metal at room temperature feels colder than wood at the same temperature. At room temperature, the $\sqrt{k\rho c_p}$ value is 24 kJ/m²·K for aluminum, 0.38 kJ/m²·K for wood, and 1.1 kJ/m²·K for human flesh. Using Eq. 18–40, it can be shown that when a person with a skin temperature of 35°C touches an aluminum block and then a wood block both at 15°C, the contact surface temperature will be 15.9°C in the case of aluminum and 30°C in the case of wood.

EXAMPLE 18–5 Minimum Burial Depth of Water Pipes to Avoid Freezing

In areas where the air temperature remains below 0°C for prolonged periods of time, the freezing of water in underground pipes is a major concern. Fortunately, the soil remains relatively warm during those periods, and it takes weeks for the subfreezing temperatures to reach the water mains in the ground. Thus, the soil effectively serves as insulation to protect the water from subfreezing temperatures in winter.

The ground at a particular location is covered with snowpack at −10°C for a continuous period of three months, and the average soil properties at that location are $k = 0.4$ W/m·K and $\alpha = 0.15 \times 10^{-6}$ m²/s (Fig. 18–26). Assuming an initial uniform temperature of 15°C for the ground, determine the minimum burial depth to prevent the water pipes from freezing.

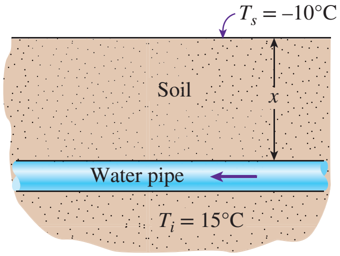


FIGURE 18–26
Schematic for Example 18–5.

SOLUTION The water pipes are buried in the ground to prevent freezing. The minimum burial depth at a particular location is to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium. **2** The thermal properties of the soil are constant.

Properties The properties of the soil are as given in the problem statement.

Analysis The temperature of the soil surrounding the pipes will be 0°C after three months in the case of minimum burial depth. Therefore, from Fig. 18–24, we have

$$\left.\begin{aligned} &\frac{h\sqrt{\alpha t}}{k} = \infty \quad (\text{since } h \rightarrow \infty) \\ &\frac{T(x,t) - T_i}{T_\infty - T_i} = \frac{0 - 15}{-10 - 15} = 0.6 \end{aligned}\right\} \eta = \frac{x}{2\sqrt{\alpha t}} = 0.36$$

We note that

$$t = (90 \text{ days})(24 \text{ h/day})(3600 \text{ s/h}) = 7.78 \times 10^6 \text{ s}$$

and thus

$$x = 2\eta\sqrt{\alpha t} = 2 \times 0.36\sqrt{(0.15 \times 10^{-6}\ \text{m}^2/\text{s})(7.78 \times 10^6\ \text{s})} = \mathbf{0.78\ m}$$

Therefore, the water pipes must be buried to a depth of at least 78 cm to avoid freezing under the specified harsh winter conditions.

ALTERNATIVE SOLUTION The solution of this problem could also be determined from Eq. 18–36:

$$\frac{T(x, t) - T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \longrightarrow \frac{0 - 15}{-10 - 15} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) = 0.60$$

The argument that corresponds to this value of the complementary error function is determined from Table 18–4 to be $\eta = 0.37$. Therefore,

$$x = 2\eta\sqrt{\alpha t} = 2 \times 0.37\sqrt{(0.15 \times 10^{-6}\ \text{m}^2/\text{s})(7.78 \times 10^6\ \text{s})} = \mathbf{0.80\ m}$$

Again, the slight difference is due to the reading error of the chart.

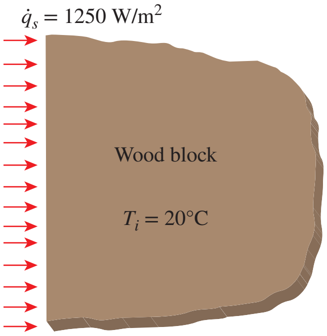


FIGURE 18–27
Schematic for Example 18–6.

EXAMPLE 18–6 Surface Temperature Rise of Heated Blocks

A thick, black-painted wood block at 20°C is subjected to constant solar heat flux of 1250 W/m^2 (Fig. 18–27). Determine the exposed surface temperature of the block after 20 minutes. What would your answer be if the block were made of aluminum?

SOLUTION A wood block is subjected to solar heat flux. The surface temperature of the block is to be determined and compared to the value for an aluminum block.
Assumptions **1** All incident solar radiation is absorbed by the block. **2** Heat loss from the block is disregarded (and thus the result obtained is the maximum temperature). **3** The block is sufficiently thick to be treated as a semi-infinite solid, and the properties of the block are constant.
Properties Thermal conductivity and diffusivity values at room temperature are $k = 0.159$ W/m·K and $\alpha = k/\rho c_p = 1.75 \times 10^{-7}$ m^2/s for hardwoods and $k = 237$ W/m·K and $\alpha = 9.71 \times 10^{-5}$ m^2/s for pure aluminum.
Analysis This is a transient conduction problem in a semi-infinite medium subjected to constant surface heat flux, and the surface temperature can be expressed from Eq. 18–37 as

$$T_s = T(0, t) = T_i + \frac{\dot{q}_s}{k}\sqrt{\frac{4\alpha t}{\pi}}$$

Substituting the given values, the surface temperatures for both the wood and aluminum blocks are determined to be

$$T_{s,\text{ wood}} = 20°\text{C} + \frac{1250\ \text{W/m}^2}{0.159\ \text{W/m·K}}\sqrt{\frac{4(1.75 \times 10^{-7}\ \text{m}^2/\text{s})(20 \times 60\ \text{s})}{\pi}} = \mathbf{149°C}$$

$$T_{s,\text{ Al}} = 20°\text{C} + \frac{1250\ \text{W/m}^2}{237\ \text{W/m·K}}\sqrt{\frac{4(9.71 \times 10^{-5}\ \text{m}^2/\text{s})(20 \times 60\ \text{s})}{\pi}} = \mathbf{22.0°C}$$

Note that thermal energy supplied to the wood accumulates near the surface because of the low conductivity and diffusivity of wood, causing the surface temperature to rise to high values. Metals, on the other hand, conduct the heat they receive to inner parts of the block because of their high conductivity and diffusivity, resulting in minimal temperature rise at the surface. In reality, both temperatures will be lower because of heat losses.

Discussion The temperature profiles for both wood and aluminum blocks at $t = 20$ min are evaluated and plotted in Fig. 18–28 using appropriate software. At a depth of $x = 0.044$ m the temperature in both blocks is 21.8°C. At a depth of 0.08 m, the temperatures become 20.0°C for wood and 21.6°C for aluminum block, which confirms that heat penetrates faster and further in metals than in nonmetals.

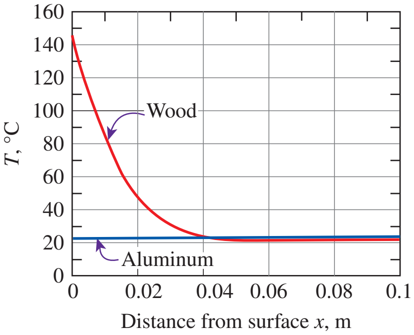


FIGURE 18–28
Variation of temperature within the wood and aluminum blocks at $t = 20$ min.

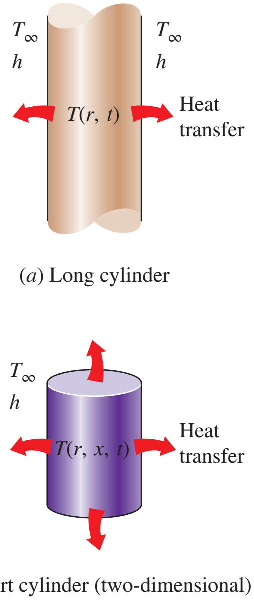
18–4 ▪ TRANSIENT HEAT CONDUCTION IN MULTIDIMENSIONAL SYSTEMS

The analytical solutions presented earlier can be used to determine the temperature distribution and heat transfer in *one-dimensional* heat conduction problems associated with a large plane wall, a long cylinder, a sphere, and a semi-infinite medium. Using a superposition approach called the **product solution**, these solutions can also be used to construct solutions for the *two-dimensional* transient heat conduction problems encountered in geometries such as a short cylinder, a long rectangular bar, or a semi-infinite cylinder or plate, and even *three-dimensional* problems associated with geometries such as a rectangular prism or a semi-infinite rectangular bar, provided that *all* surfaces of the solid are subjected to convection to the *same* fluid at temperature T_∞, with the *same* heat transfer coefficient *h*, and the body involves no heat generation (Fig. 18–29). The solution in such multidimensional geometries can be expressed as the *product* of the solutions for the one-dimensional geometries whose intersection is the multidimensional geometry.

Consider a *short cylinder* of height *a* and radius r_o initially at a uniform temperature T_i. There is no heat generation in the cylinder. At time $t = 0$, the cylinder is subjected to convection from all surfaces to a medium at temperature T_∞ with a heat transfer coefficient *h*. The temperature within the cylinder will change with *x* as well as *r* and time *t* since heat transfer occurs from the top and bottom of the cylinder as well as its side surfaces. That is, $T = T(r, x, t)$, and thus this is a two-dimensional transient heat conduction problem. When the properties are assumed to be constant, it can be shown that the solution to this two-dimensional problem can be expressed as

$$\left(\frac{T(r, x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{short}\\\text{cylinder}}} = \left(\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{plane}\\\text{wall}}} \left(\frac{T(r, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{infinite}\\\text{cylinder}}} \tag{18–41}$$

That is, the solution for the two-dimensional short cylinder of height *a* and radius r_o is equal to the *product* of the nondimensionalized solutions for the one-dimensional plane wall of thickness *a* and the long cylinder of radius r_o, which are the two geometries whose intersection is the short cylinder, as shown in Fig. 18–30. We generalize this as follows: *the solution for a multidimensional geometry is the product of the solutions of the one-dimensional geometries whose intersection is the multidimensional body.*



(*a*) Long cylinder

(*b*) Short cylinder (two-dimensional)

FIGURE 18–29
The temperature in a short cylinder exposed to convection from all surfaces varies in both the radial and axial directions, and thus heat is transferred in both directions.

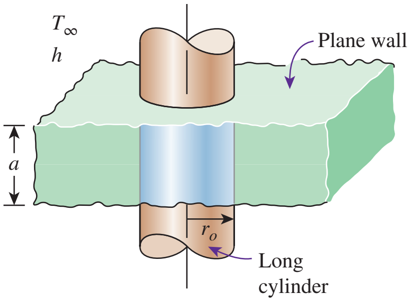


FIGURE 18–30
A short cylinder of radius r_o and height a is the *intersection* of a long cylinder of radius r_o and a plane wall of thickness a.

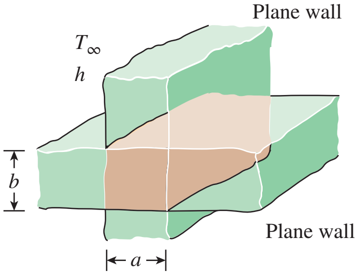


FIGURE 18–31
A long, solid bar of rectangular profile $a \times b$ is the *intersection* of two plane walls of thicknesses a and b.

For convenience, the one-dimensional solutions are denoted by

$$\begin{aligned}\theta_{\text{wall}}(x, t) &= \left(\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{plane}\\ \text{wall}}}\\ \theta_{\text{cyl}}(r, t) &= \left(\frac{T(r, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{infinite}\\ \text{cylinder}}}\\ \theta_{\text{semi-inf}}(x, t) &= \left(\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{semi-infinite}\\ \text{solid}}}\end{aligned} \tag{18–42}$$

For example, the solution for a long, solid bar whose cross section is an $a \times b$ rectangle is the intersection of the two infinite plane walls of thicknesses a and b, as shown in Fig. 18–31, and thus the transient temperature distribution for this rectangular bar can be expressed as

$$\left(\frac{T(x, y, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{rectangular}\\ \text{bar}}} = \theta_{\text{wall}}(x, t)\,\theta_{\text{wall}}(y, t) \tag{18–43}$$

The proper forms of the product solutions for some other geometries are given in Table 18–5. It is important to note that the x-coordinate is measured from the *surface* in a semi-infinite solid and from the *midplane* in a plane wall. The radial distance r is always measured from the centerline.

Note that the solution of a *two-dimensional* problem involves the product of *two* one-dimensional solutions, whereas the solution of a *three-dimensional* problem involves the product of *three* one-dimensional solutions.

A modified form of the product solution can also be used to determine the total transient heat transfer to or from a multidimensional geometry by using the one-dimensional values, as shown by L. S. Langston in 1982. The transient heat transfer for a two-dimensional geometry formed by the intersection of two one-dimensional geometries 1 and 2 is

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 2D}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right] \tag{18–44}$$

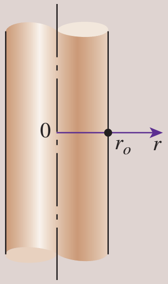
Transient heat transfer for a three-dimensional body formed by the intersection of three one-dimensional bodies 1, 2, and 3 is given by

$$\begin{aligned}\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 3D}} &= \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]\\ &\quad + \left(\frac{Q}{Q_{\max}}\right)_3 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]\left[1 - \left(\frac{Q}{Q_{\max}}\right)_2\right]\end{aligned} \tag{18–45}$$

The use of the product solution in transient two- and three-dimensional heat conduction problems is illustrated in the following examples.

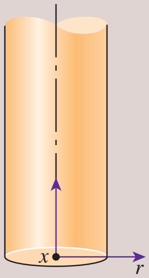
TABLE 18–5

Multidimensional solutions expressed as products of one-dimensional solutions for bodies that are initially at a uniform temperature T_i and exposed to convection from all surfaces to a medium at T_∞



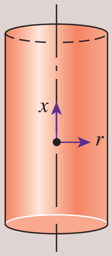
$\theta(r, t) = \theta_{\text{cyl}}(r, t)$

Infinite cylinder



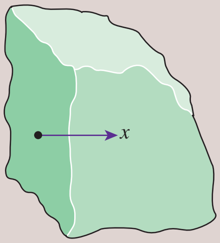
$\theta(x, r, t) = \theta_{\text{cyl}}(r, t)\,\theta_{\text{semi-inf}}(x, t)$

Semi-infinite cylinder



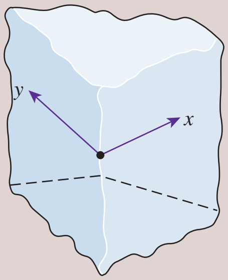
$\theta(x, r, t) = \theta_{\text{cyl}}(r, t)\,\theta_{\text{wall}}(x, t)$

Short cylinder



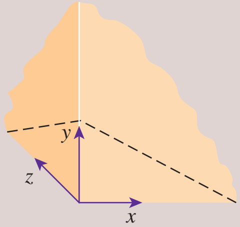
$\theta(x, t) = \theta_{\text{semi-inf}}(x, t)$

Semi-infinite medium



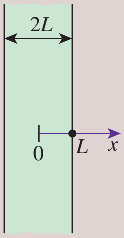
$\theta(x, y, t) = \theta_{\text{semi-inf}}(x, t)\,\theta_{\text{semi-inf}}(y, t)$

Quarter-infinite medium



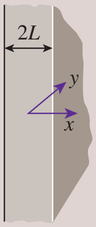
$\theta(x, y, z, t) =$
$\theta_{\text{semi-inf}}(x, t)\,\theta_{\text{semi-inf}}(y, t)\,\theta_{\text{semi-inf}}(z, t)$

Corner region of a large medium



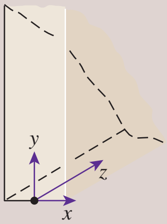
$\theta(x, t) = \theta_{\text{wall}}(x, t)$

Infinite plate (or plane wall)



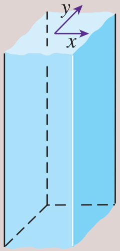
$\theta(x, y, t) = \theta_{\text{wall}}(x, t)\,\theta_{\text{semi-inf}}(y, t)$

Semi-infinite plate



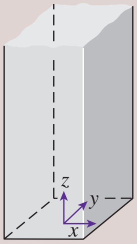
$\theta(x, y, z, t) =$
$\theta_{\text{wall}}(x, t)\,\theta_{\text{semi-inf}}(y, t)\,\theta_{\text{semi-inf}}(z, t)$

Quarter-infinite plate



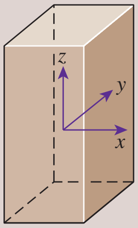
$\theta(x, y, t) = \theta_{\text{wall}}(x, t)\,\theta_{\text{wall}}(y, t)$

Infinite rectangular bar



$\theta(x, y, z, t) =$
$\theta_{\text{wall}}(x, t)\,\theta_{\text{wall}}(y, t)\,\theta_{\text{semi-inf}}(z, t)$

Semi-infinite rectangular bar



$\theta(x, y, z, t) =$
$\theta_{\text{wall}}(x, t)\,\theta_{\text{wall}}(y, t)\,\theta_{\text{wall}}(z, t)$

Rectangular parallelepiped

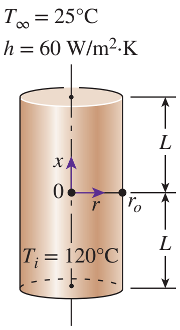


FIGURE 18–32
Schematic for Example 18–7.

EXAMPLE 18–7 Cooling of a Short Brass Cylinder

A short brass cylinder of diameter $D = 10$ cm and height $H = 12$ cm is initially at a uniform temperature $T_i = 120°C$. The cylinder is now placed in atmospheric air at 25°C, where heat transfer takes place by convection, with a heat transfer coefficient of $h = 60\ W/m^2·K$. Calculate the temperature at (*a*) the center of the cylinder and (*b*) the center of the top surface of the cylinder 15 min after the start of the cooling.

SOLUTION A short cylinder is allowed to cool in atmospheric air. The temperatures at the centers of the cylinder and the top surface are to be determined.

Assumptions **1** Heat conduction in the short cylinder is two-dimensional, and thus the temperature varies in both the axial x- and the radial r-directions. **2** The thermal properties of the cylinder and the heat transfer coefficient are constant. **3** The Fourier number is $\tau > 0.2$, so the one-term approximate solutions are applicable.

Properties The properties of brass at room temperature are $k = 110$ W/m·K and $\alpha = 33.9 \times 10^{-6}\ m^2/s$ (Table A–24). More accurate results can be obtained by using properties at average temperature.

Analysis (*a*) This short cylinder can physically be formed by the intersection of a long cylinder of radius $r_o = 5$ cm and a plane wall of thickness $2L = 12$ cm, as shown in Fig. 18–32. The dimensionless temperature at the center of the plane wall is determined from Eq. 18–17.

First we find the Biot and Fourier numbers for the plane wall

$$\text{Bi} = \frac{hL}{k} = \frac{(60\ \text{W/m}^2\text{·K})\ (0.06\ \text{m})}{110\ \text{W/m·K}} = 0.0327$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.39 \times 10^{-5}\ \text{m}^2\text{/s})\ (15 \times 60\ \text{s})}{(0.06\ \text{m})^2} = 8.48$$

The coefficients λ_1 and A_1 for a plane wall corresponding to Bi = 0.0327 are determined from Table 18–2 to be $\lambda_1 = 0.1776$ and $A_1 = 1.0054$.

Substituting these and other values into Eq. 18–17 gives

$$\theta_{0,\ \text{wall}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = 1.0054\ e^{-(0.1776)^2(8.48)} = 0.77$$

Similarly, the temperature at the center of the cylinder is determined from the one-term approximate solutions Eq. 18–18. First we find the Biot and Fourier numbers for the cylinder

$$\text{Bi} = \frac{hr_o}{k} = \frac{(60\ \text{W/m}^2\text{·K})\ (0.05\ \text{m})}{110\ \text{W/m·K}} = 0.0273$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.39 \times 10^{-5}\ \text{m}^2\text{/s})(15 \times 60\ \text{s})}{(0.05\ \text{m})^2} = 12.2$$

The coefficients λ_1 and A_1 for a cylinder corresponding to Bi = 0.0273 are determined from Table 18–2 to be $\lambda_1 = 0.2294$ and $A_1 = 1.0068$.

Substituting these and other values into Eq. 18–18 gives

$$\theta_{0,\ \text{cyl}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = 1.0068 e^{-(0.2294)^2(12.2)} = 0.53$$

Therefore,

$$\left(\frac{T(0, 0, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{0,\ \text{wall}}(0, t) \times \theta_{0,\ \text{cyl}}(0, t) = 0.77 \times 0.53 = 0.41$$

and

$$T(0, 0, t) = T_\infty + 0.41(T_i - T_\infty) = 25 + 0.41(120 - 25) = \mathbf{64°C}$$

This is the temperature at the center of the short cylinder, which is also the center of both the long cylinder and the plate.

(b) The center of the top surface of the cylinder is still at the center of the long cylinder ($r = 0$), but at the outer surface of the plane wall ($x = L$). Therefore, we first need to find the surface temperature of the wall using the one-term approximate solutions Eq. 18–14. Noting that $x = L = 0.06$ m, the values of the Biot number, Fourier number, and the constants λ_1 and A_1 remain the same as was determined in part (a) for the plane wall. Substituting these and other values into Eq. 18–14 gives

$$\theta_{\text{wall}} = \frac{T(x, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L)$$

$$\theta_{\text{wall}} = \frac{T(x, L) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1) = 1.0054 e^{-(0.1776)^2(8.48)} \cos(0.1776) = 0.757$$

Therefore,

$$\left(\frac{T(L, 0, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{\text{wall}}(L, t)\, \theta_{0,\text{ cyl}}(0, t) = 0.757 \times 0.53 = 0.40$$

and

$$T(L, 0, t) = T_\infty + 0.40(T_i - T_\infty) = 25 + 0.40(120 - 25) = \mathbf{63°C}$$

which is the temperature at the center of the top surface of the cylinder.

EXAMPLE 18–8 Heat Transfer from a Short Cylinder

Determine the total heat transfer from the short brass cylinder ($\rho = 8530$ kg/m^3, $c_p = 0.380$ kJ/kg·K) discussed in Example 18–7.

SOLUTION We first determine the maximum heat that can be transferred from the cylinder, which is the sensible energy content of the cylinder relative to its environment:

$$m = \rho V = \rho \pi r_o^2 H = (8530 \text{ kg/m}^3)\pi(0.05 \text{ m})^2(0.12 \text{ m}) = 8.04 \text{ kg}$$
$$Q_{\max} = mc_p(T_i - T_\infty) = (8.04 \text{ kg})(0.380 \text{ kJ/kg·K})(120 - 25)°\text{C} = 290.2 \text{ kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries. For the plane wall, it is determined from the one-term approximate solutions Eq. 18–24 to be

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{wall}} = 1 - \theta_{0,\text{ wall}} \frac{\sin(\lambda_1)}{\lambda_1} = 1 - (0.77)\frac{\sin(0.1776)}{0.1776} = 0.23$$

Similarly, for the cylinder using the one-term approximate relation Eq. 18–25, we have

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\text{ cyl}} \frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.53)\frac{0.11385}{0.2294} = 0.47$$

since $J_1(0.2294) = 0.11385$ from Table 18–3. The heat transfer ratio for the short cylinder is, from Eq. 18–44,

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{short cyl}} = \left(\frac{Q}{Q_{\max}}\right)_{\text{plane wall}} + \left(\frac{Q}{Q_{\max}}\right)_{\text{long cyl}} \left[1 - \left(\frac{Q}{Q_{\max}}\right)_{\text{plane wall}}\right]$$
$$= 0.23 + 0.47(1 - 0.23) = 0.592$$

Therefore, the total heat transfer from the cylinder during the first 15 min of cooling is

$$Q = 0.592\, Q_{\max} = 0.592 \times (290.2 \text{ kJ}) = \mathbf{172 \text{ kJ}}$$

SUMMARY

In this chapter, we considered the variation of temperature with time as well as position in one- or multidimensional systems. We first considered the *lumped systems* in which the temperature varies with time but remains uniform throughout the system at any time. The temperature of a lumped body of arbitrary shape of mass m, volume V, surface area A_s, density ρ, and specific heat c_p initially at a uniform temperature T_i that is exposed to convection at time $t = 0$ in a medium at temperature T_∞ with a heat transfer coefficient h is expressed as

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

where

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

is a positive quantity whose dimension is (time)$^{-1}$. This relation can be used to determine the temperature $T(t)$ of a body at time t or, alternatively, the time t required for the temperature to reach a specified value $T(t)$. Once the temperature $T(t)$ at time t is available, the *rate* of convection heat transfer between the body and its environment at that time can be determined from Newton's law of cooling as

$$\dot{Q}(t) = hA_s[T(t) - T_\infty]$$

The *total amount* of heat transfer between the body and the surrounding medium over the time interval $t = 0$ to t is simply the change in the energy content of the body,

$$Q = mc_p[T(t) - T_i]$$

The *maximum* heat transfer between the body and its surroundings is

$$Q_{\text{max}} = mc_p(T_\infty - T_i)$$

The error involved in lumped system analysis is negligible when

$$\text{Bi} = \frac{hL_c}{k} < 0.1$$

where Bi is the *Biot number* and $L_c = V/A_s$ is the *characteristic length.*

When the lumped system analysis is not applicable, the variation of temperature with position as well as time can be determined using the *one-term approximation solutions* for a large plane wall, a long cylinder, and a sphere, respectively. These solutions are applicable for one-dimensional heat transfer in those geometries. Therefore, their use is limited to situations in which the body is initially at a uniform temperature, all surfaces are subjected to the same thermal conditions, and the body does not involve any heat generation. These solutions can also be used to determine the total heat transfer from the body up to a specified time t.

Using the *one-term approximation,* the solutions of one-dimensional transient heat conduction problems are expressed analytically as

Plane wall: $$\theta_{\text{wall}} = \frac{T(x, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 x/L)$$

Cylinder: $$\theta_{\text{cyl}} = \frac{T(r, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r/r_o)$$

Sphere: $$\theta_{\text{sph}} = \frac{T(r, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}$$

where the constants A_1 and λ_1 are functions of the Bi number only, and their values are listed in Table 18–2 against the Bi number for all three geometries. The error involved in one-term solutions is less than 2 percent when $\tau > 0.2$.

Using the one-term solutions, the fractional heat transfers in different geometries are expressed as

Plane wall: $$\left(\frac{Q}{Q_{\text{max}}}\right)_{\text{wall}} = 1 - \theta_{0,\text{ wall}} \frac{\sin \lambda_1}{\lambda_1}$$

Cylinder: $$\left(\frac{Q}{Q_{\text{max}}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\text{ cyl}} \frac{J_1(\lambda_1)}{\lambda_1}$$

Sphere: $$\left(\frac{Q}{Q_{\text{max}}}\right)_{\text{sph}} = 1 - 3\theta_{0,\text{ sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

The solutions of transient heat conduction in a semi-infinite solid with constant properties under various boundary conditions at the surface are given as follows:

Specified Surface Temperature, $T_s =$ constant:

$$\frac{T(x, t) - T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \quad \text{and} \quad \dot{q}_s(t) = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}$$

Specified Surface Heat Flux, $\dot{q}_s =$ constant:

$$T(x, t) - T_i = \frac{\dot{q}_s}{k}\left[\sqrt{\frac{4\alpha t}{\pi}} \exp\left(-\frac{x^2}{4\alpha t}\right) - x\,\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$$

Convection on the Surface, $\dot{q}_s(t) = h[T_\infty - T(0, t)]$:

$$\frac{T(x, t) - T_i}{T_\infty - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right) \times \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)$$

Energy Pulse at Surface, $e_s =$ constant:

$$T(x, t) - T_i = \frac{e_s}{k\sqrt{\pi t/\alpha}} \exp\left(-\frac{x^2}{4\alpha t}\right)$$

where erfc (η) is the *complementary error function* of argument η.

Using a superposition principle called the *product solution,* these one-term approximate solutions can also be used to construct solutions for the *two-dimensional* transient heat conduction problems encountered in geometries such as a short cylinder, a long rectangular bar, or a semi-infinite cylinder or plate, and even *three-dimensional* problems associated with geometries such as a rectangular prism or a semi-infinite rectangular bar, provided that all surfaces of the solid are subjected to convection to the same fluid at temperature T_∞, with the same convection heat transfer coefficient h, and the body involves no heat generation. The solution in such multidimensional geometries can be expressed as the product of the solutions for the one-dimensional geometries whose intersection is the multidimensional geometry.

The total heat transfer to or from a multidimensional geometry can also be determined by using the one-dimensional values. The transient heat transfer for a two-dimensional geometry formed by the intersection of two one-dimensional geometries 1 and 2 is

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 2D}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]$$

Transient heat transfer for a three-dimensional body formed by the intersection of three one-dimensional bodies 1, 2, and 3 is given by

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 3D}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2 = \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right] + \left(\frac{Q}{Q_{\max}}\right)_3 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]\left[1 - \left(\frac{Q}{Q_{\max}}\right)_2\right]$$

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PROBLEMS*

Lumped System Analysis

18–1C What is the physical significance of the Biot number? Is the Biot number more likely to be larger for highly conducting solids or poorly conducting ones?

18–2C What is lumped system analysis? When is it applicable?

18–3C In what medium is the lumped system analysis more likely to be applicable: in water or in air? Why?

18–4C For which solid is the lumped system analysis more likely to be applicable: an actual apple or a golden apple of the same size? Why?

18–5C For which kinds of bodies made of the same material is the lumped system analysis more likely to be applicable: slender ones or well-rounded ones of the same volume? Why?

18–6C Consider heat transfer between two identical hot solid bodies and the air surrounding them. The first solid is being cooled by a fan while the second one is allowed to cool naturally. For which solid is the lumped system analysis more likely to be applicable? Why?

18–7C Consider heat transfer between two identical hot solid bodies and their environments. The first solid is dropped in a large container filled with water, while the second one is allowed to cool naturally in the air. For which solid is the lumped system analysis more likely to be applicable? Why?

18–8C Consider a potato being baked in an oven that is maintained at a constant temperature. The temperature of the potato is observed to rise by 5°C during the first minute. Will the temperature rise during the second minute be less than, equal to, or more than 5°C? Why?

18–9C Consider two identical 4-kg pieces of roast beef. The first piece is baked as a whole, while the second is baked after being cut into two equal pieces in the same oven. Will there be any difference between the cooking times of the whole and cut roasts? Why?

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*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

18–10C Consider a sphere and a cylinder of equal volume made of copper. Both the sphere and the cylinder are initially at the same temperature and are exposed to convection in the same environment. Which do you think will cool faster, the cylinder or the sphere? Why?

18–11 Obtain relations for the characteristic lengths of a large plane wall of thickness $2L$, a very long cylinder of radius r_o, and a sphere of radius r_o.

18–12 Obtain a relation for the time required for a lumped system to reach the average temperature $\frac{1}{2}(T_i + T_\infty)$, where T_i is the initial temperature and T_∞ is the temperature of the environment.

18–13 A brick of $203 \times 102 \times 57$ mm in dimension is being burned in a kiln to 1100°C and then allowed to cool in a room with ambient air temperature of 30°C and convection heat transfer coefficient of 5 W/m²·K. If the brick has properties of $\rho = 1920$ kg/m³, $c_p = 790$ J/kg·K, and $k = 0.90$ W/m·K, determine the time required to cool the brick to a temperature difference of 5°C from the ambient air temperature.

18–14 Consider an 800-W iron whose base plate is made of 0.5-cm-thick aluminum alloy 2024–T6 ($\rho = 2770$ kg/m³, $c_p = 875$ J/kg·K, $\alpha = 7.3 \times 10^{-5}$ m²/s). The base plate has a surface area of 0.03 m². Initially, the iron is in thermal equilibrium with the ambient air at 22°C. Taking the heat transfer coefficient at the surface of the base plate to be 12 W/m²·K and assuming 85 percent of the heat generated in the resistance wires is transferred to the plate, determine how long it will take for the plate temperature to reach 140°C. Is it realistic to assume the plate temperature to be uniform at all times?

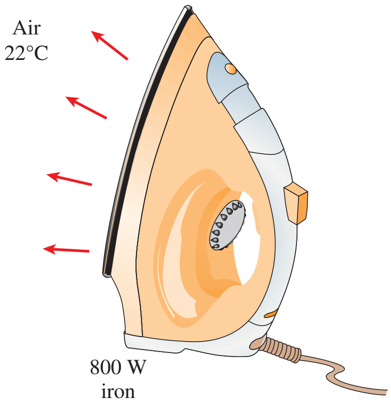


FIGURE P18–14

18–15 Reconsider Prob. 18–14. Using appropriate software, investigate the effects of the heat transfer coefficient and the final plate temperature on the time it will take for the plate to reach this temperature. Let the heat transfer coefficient vary from 5 W/m²·K to 25 W/m²·K and the temperature from 30°C to 200°C. Plot the time as functions of the heat transfer coefficient and the temperature, and discuss the results.

18–16 Metal plates ($k = 180$ W/m·K, $\rho = 2800$ kg/m³, and $c_p = 880$ J/kg·K) with a thickness of 1 cm are being heated in an oven for 2 min. Air in the oven is maintained at 800°C with a convection heat transfer coefficient of 200 W/m²·K. If the initial temperature of the plates is 20°C, determine the temperature of the plates when they are removed from the oven.

18–17 A 5-mm-thick stainless steel strip ($k = 21$ W/m·K, $\rho = 8000$ kg/m³, and $c_p = 570$ J/kg·K) is being heat treated as it moves through a furnace at a speed of 1 cm/s. The air temperature in the furnace is maintained at 900°C with a convection heat transfer coefficient of 80 W/m²·K. If the furnace length is 3 m and the stainless steel strip enters it at 20°C, determine the temperature of the strip as it exits the furnace.

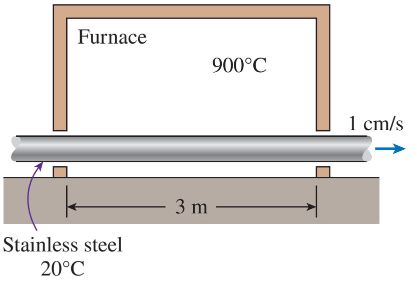


FIGURE P18–17

18–18 A batch of 2-cm-thick stainless steel plates ($k = 21$ W/m·K, $\rho = 8000$ kg/m³, and $c_p = 570$ J/kg·K) are conveyed through a furnace to be heat treated. The plates enter the furnace at 18°C, and they travel a distance of 3 m inside the furnace. The air temperature in the furnace is maintained at 950°C with a convection heat transfer coefficient of 150 W/m²·K. Using appropriate software, determine how the velocity of the plates affects the temperature of the plates at the end of the heat treatment. Let the velocity of the plates vary from 5 to 60 mm/s, and plot the temperature of the plates at the furnace exit as a function of the velocity.

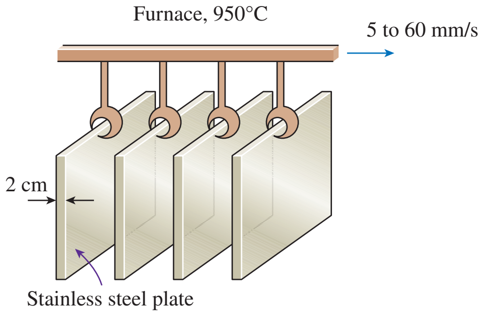


FIGURE P18–18

18–19 A long copper rod of diameter 2.0 cm is initially at a uniform temperature of 100°C. It is now exposed to an airstream at 20°C with a heat transfer coefficient of 200 W/m²·K. How long would it take for the copper rod to cool to an average temperature of 25°C?

18–20 Springs in automobile suspension systems are made of steel rods heated and wound into coils while ductile. Consider steel rods ($\rho = 7832$ kg/m³, $c_p = 434$ J/kg·K, and $k = 63.9$ W/m·K) with diameter of 2.5 cm and length of 1.27 m. The steel rods are heated in an oven with a uniform convection heat transfer coefficient of 20 W/m²·K. The steel rods were heated from an initial temperature of 20°C to the desired temperature of 450°C before being wound into coils. Determine the ambient temperature in the oven if the steel rods were to be heated to the desired temperature within 10 min.

18–21 Steel rods ($\rho = 7832$ kg/m^3, $c_p = 434$ J/kg·K, and $k = 63.9$ W/m·K) are heated in a furnace to 850°C and then quenched in a water bath at 50°C for 40 s as part of a hardening process. The convection heat transfer coefficient is 650 W/m^2·K. If the steel rods have diameter of 40 mm and length of 2 m, determine their average temperature when they are taken out of the water bath.

18–22 To warm up some milk for a baby, a mother pours milk into a thin-walled cylindrical container whose diameter is 6 cm. The height of the milk in the container is 7 cm. She then places the container into a large pan filled with hot water at 70°C. The milk is stirred constantly so that its temperature is uniform at all times. If the heat transfer coefficient between the water and the container is 120 W/m^2·K, determine how long it will take for the milk to warm up from 3°C to 38°C. Assume the entire surface area of the cylindrical container (including the top and bottom) is in thermal contact with the hot water. Take the properties of the milk to be the same as those of water. Can the milk in this case be treated as a lumped system? Why? *Answer:* 4.50 min

18–23 The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1.2-mm-diameter sphere. The properties of the junction are $k = 35$ W/m·K, $\rho = 8500$ kg/m^3, and $c_p = 320$ J/kg·K, and the heat transfer coefficient between the junction and the gas is $h = 110$ W/m^2·K. Determine how long it will take for the thermocouple to read 99 percent of the initial temperature difference. *Answer:* 22.8 s

18–24 In an experiment, the temperature of a hot gas stream is to be measured by a thermocouple with a spherical junction. Due to the nature of this experiment, the response time of the thermocouple to register 99 percent of the initial temperature difference must be within 5 s. The properties of the thermocouple junction are $k = 35$ W/m·K, $\rho = 8500$ kg/m^3, and $c_p = 320$ J/kg·K. If the heat transfer coefficient between the thermocouple junction and the gas is 250 W/m^2·K, determine the diameter of the junction.

18–25 A thermocouple with a spherical junction diameter of 0.5 mm is used for measuring the temperature of hot airflow in a circular duct. The convection heat transfer coefficient of the airflow can be related with the diameter (D) of the spherical junction and the average airflow velocity (V) as $h = 2.2(V/D)^{0.5}$, where D, h, and V are in m, W/m^2·K, and m/s, respectively. The properties of the thermocouple junction are $k = 35$ W/m·K, $\rho = 8500$ kg/m^3, and $c_p = 320$ J/kg·K. Determine the minimum airflow velocity that the thermocouple can use, if the maximum response time of the thermocouple to register 99 percent of the initial temperature difference is 5 s.

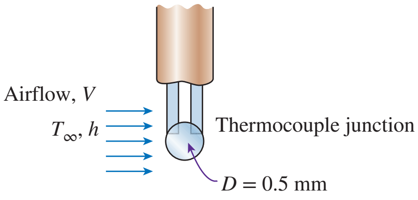


FIGURE P18–25

18–26 Pulverized coal particles are used in oxy-fuel combustion power plants for electricity generation. Consider a situation where coal particles are suspended in hot air flowing through a heated tube, where the convection heat transfer coefficient is 100 W/m^2·K. If the average surface area and volume of the coal particles are 3.1 mm^2 and 0.5 mm^3, respectively, determine how much time it would take to heat the coal particles to two-thirds of the initial temperature difference. The properties of coal are $k = 0.26$ W/m·K, $\rho = 1350$ kg/m^3, and $c_p = 1260$ J/kg·K.

18–27 Oxy-fuel combustion power plants use pulverized coal particles as fuel to burn in a pure oxygen environment to generate electricity. Before entering the furnace, pulverized spherical coal particles with an average diameter of 300 μm are transported at 2 m/s through a 3-m-long heated tube while suspended in hot air. The air temperature in the tube is 900°C, and the average convection heat transfer coefficient is 250 W/m^2·K. Determine the temperature of the coal particles at the exit of the heated tube if the initial temperature of the particles is 20°C. The properties of coal are $k = 0.26$ W/m·K, $\rho = 1350$ kg/m^3, and $c_p = 1260$ J/kg·K.

18–28 Plasma spraying is a process used for coating a material surface with a protective layer to prevent the material from degradation. In a plasma spraying process, the protective layer in powder form is injected into a plasma jet. The powder is then heated to molten droplets and propelled onto the material surface. Once deposited on the material surface, the molten droplets solidify and form a layer of protective coating. Consider a plasma spraying process using alumina ($k = 30$ W/m·K, $\rho = 3970$ kg/m^3, and $c_p = 800$ J/kg·K) powder that is injected into a plasma jet at $T_\infty = 15{,}000$°C and $h = 10{,}000$ W/m^2·K. The alumina powder is made of spherical particles with an average diameter of 60 μm and a melting point at 2300°C. Determine the amount of time it would take for the particles, with an initial temperature of 20°C, to reach their melting point from the moment they are injected into the plasma jet.

18–29 Consider a spherical shell satellite with outer diameter of 4 m and shell thickness of 10 mm that is reentering the atmosphere. The shell satellite is made of stainless steel with properties of $\rho = 8238$ kg/m^3, $c_p = 468$ J/kg·K, and $k = 13.4$ W/m·K. During the reentry, the effective atmosphere temperature surrounding the satellite is 1250°C with a convection heat transfer coefficient of 130 W/m^2·K. If the initial temperature of the shell is 10°C, determine the shell temperature after 5 min of reentry. Assume heat transfer occurs only on the satellite shell.

18–30E In a manufacturing facility, 2-in-diameter brass balls ($k = 64.1$ Btu/h·ft·°F, $\rho = 532$ lbm/ft^3, and $c_p = 0.092$ Btu/lbm·°F) initially at 250°F are quenched in a water bath at 120°F for 2 min at a rate of 120 balls per minute. If the convection heat transfer coefficient is 42 Btu/h·ft^2·°F, determine (*a*) the temperature of the balls after quenching and (*b*) the rate at which heat needs to be removed from the water in order to keep its temperature constant at 120°F.

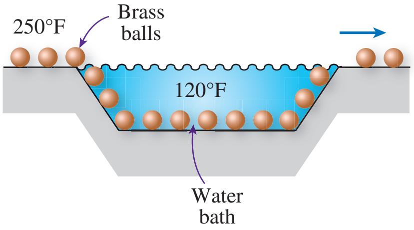


FIGURE P18–30E

18–31 Consider a sphere of diameter 5 cm, a cube of side length 5 cm, and a rectangular prism of dimension 4 cm × 5 cm × 6 cm, all initially at 0°C and all made of silver ($k = 429$ W/m·K, $\rho = 10{,}500$ kg/m^3, $c_p = 0.235$ kJ/kg·K). Now all three of these geometries are exposed to ambient air at 33°C on all of their surfaces with a heat transfer coefficient of 12 W/m^2·K. Determine how long it will take for the temperature of each geometry to rise to 25°C.

18–32 An electronic device dissipating 18 W has a mass of 20 g, a specific heat of 850 J/kg·K, and a surface area of 4 cm^2. The device is lightly used, and it is on for 5 min and then off for several hours, during which it cools to the ambient temperature of 25°C. Taking the heat transfer coefficient to be 12 W/m^2·K, determine the temperature of the device at the end of the 5-min operating period. What would your answer be if the device were attached to an aluminum heat sink having a mass of 200 g and a surface area of 80 cm^2? Assume the device and the heat sink to be nearly isothermal.

Transient Heat Conduction in Large Plane Walls, Long Cylinders, and Spheres with Spatial Effects

18–33C An egg is to be cooked to a certain level of doneness by being dropped into boiling water. Can the cooking time be shortened by turning up the heat and bringing water to a more rapid boil?

18–34C What is an infinitely long cylinder? When is it proper to treat an actual cylinder as being infinitely long, and when is it not? For example, is it proper to use this model when finding the temperatures near the bottom or top surfaces of a cylinder? Explain.

18–35C What is the physical significance of the Fourier number? Will the Fourier number for a specified heat transfer problem double when the time is doubled?

18–36C Can the one-term approximate solutions for a plane wall exposed to convection on both sides be used for a plane wall with one side exposed to convection while the other side is insulated? Explain.

18–37C How can we use the one-term approximate solutions when the surface temperature of the geometry is specified instead of the temperature of the surrounding medium and the convection heat transfer coefficient?

18–38C The Biot number during a heat transfer process between a sphere and its surroundings is determined to be 0.02. Would you use lumped system analysis or the one-term approximate solutions when determining the midpoint temperature of the sphere? Why?

18–39C A body at an initial temperature of T_i is brought into a medium at a constant temperature of T_∞. How can you determine the maximum possible amount of heat transfer between the body and the surrounding medium?

18–40 In a meat processing plant, 2-cm-thick steaks ($k = 0.45$ W/m·K and $\alpha = 0.91 \times 10^{-7}$ m^2/s) that are initially at 25°C are to be cooled by passing them through a refrigeration room at −11°C. The heat transfer coefficient on both sides of the steaks is 9 W/m^2·K. If both surfaces of the steaks are to be cooled to 2°C, determine how long the steaks should be kept in the refrigeration room. Solve this problem using the analytical one-term approximation method.

18–41 A 10-cm-thick aluminum plate ($\alpha = 97.1 \times 10^{-6}$ m^2/s) is being heated in liquid with temperature of 500°C. The aluminum plate has a uniform initial temperature of 25°C. If the surface temperature of the aluminum plate is approximately the liquid temperature, determine the temperature at the center plane of the aluminum plate after 15 s of heating. Solve this problem using the analytical one-term approximation method.

18–42 Layers of 23-cm-thick meat slabs ($k = 0.47$ W/m·K and $\alpha = 0.13 \times 10^{-6}$ m^2/s) initially at a uniform temperature of 7°C are to be frozen by refrigerated air at −30°C flowing at a velocity of 1.4 m/s. The average heat transfer coefficient between the meat and the air is 20 W/m^2·K. Assuming the size of the meat slabs to be large relative to their thickness, determine how long it will take for the center temperature of the slabs to drop to −18°C. Also, determine the surface temperature of the meat slabs at that time.

18–43 A heated 6-mm-thick Pyroceram plate ($\rho = 2600$ kg/m^3, $c_p = 808$ J/kg·K, $k = 3.98$ W/m·K, and $\alpha = 1.89 \times 10^{-6}$ m^2/s) is being cooled in a room with air temperature of 25°C and convection heat transfer coefficient of 13.3 W/m^2·K. The heated Pyroceram plate had an initial temperature of 500°C, and it is allowed to cool for 286 s. If the mass of the Pyroceram plate is 10 kg, determine the heat transfer from the Pyroceram plate during the cooling process using the analytical one-term approximation method.

18–44 After a long, hard week on the books, you and your friend are ready to relax and enjoy the weekend. You take a steak 50 mm thick from the freezer. (*a*) How long (in hours) do you have to let the good times roll before the steak has thawed? Assume that the steak is initially at −8°C, that it thaws when the temperature at the center of the steak reaches 4°C, and that the room temperature is 22°C with a convection heat transfer coefficient of 10 W/m^2·K. Neglect the heat of fusion associated with the melting phase change. Treat the steak as a one-dimensional plane wall having the following properties: $\rho = 1000$ kg/m^3, $c_p = 4472$ J/kg·K, and $k = 0.625$ W/m·K. (*b*) How much energy per unit area (in J/m^2) has been removed from the steak during this period of thawing? (*c*) Show whether or not the thawing of this steak can be analyzed by neglecting the internal thermal resistance of the steak.

18–45 A long cylindrical wood log ($k = 0.17$ W/m·K and $\alpha = 1.28 \times 10^{-7}$ m^2/s) is 10 cm in diameter and is initially at a uniform temperature of 25°C. It is exposed to hot gases at 525°C in a fireplace with a heat transfer coefficient of 13.6 W/m^2·K on the surface. If the ignition temperature of the wood is 375°C, determine how long it will be before the log ignites. Solve this problem using the analytical one-term approximation method.

18–46E Long cylindrical AISI stainless steel rods ($k = 7.74$ Btu/h·ft·°F and $\alpha = 0.135$ ft^2/h) of 4-in diameter are heat

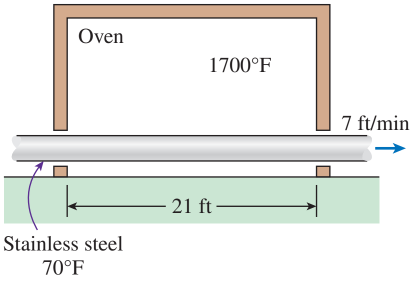


FIGURE P18–46E

treated by drawing them at a velocity of 7 ft/min through a 21-ft-long oven maintained at 1700°F. The heat transfer coefficient in the oven is 20 Btu/h·ft^2·°F. If the rods enter the oven at 70°F, determine their centerline temperature when they leave. Solve this problem using the analytical one-term approximation method.

18–47 A long iron rod ($\rho = 7870$ kg/m^3, $c_p = 447$ J/kg·K, $k = 80.2$ W/m·K, and $\alpha = 23.1 \times 10^{-6}$ m^2/s) with diameter of 25 mm is initially heated to a uniform temperature of 700°C. The iron rod is then quenched in a large water bath that is maintained at constant temperature of 50°C and with a convection heat transfer coefficient of 128 W/m^2·K. Determine the time required for the iron rod surface temperature to cool to 200°C. Solve this problem using the analytical one-term approximation method.

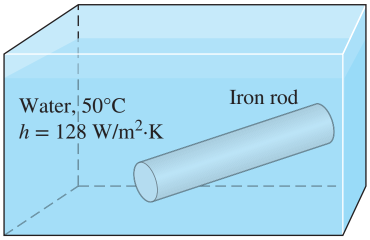


FIGURE P18–47

18–48 A 2-cm-diameter plastic rod has a thermocouple inserted to measure temperature at the center of the rod. The plastic rod ($\rho = 1190$ kg/m^3, $c_p = 1465$ J/kg·K, and $k = 0.19$ W/m·K) was initially heated to a uniform temperature of 70°C and allowed to be cooled in ambient air at 25°C. After 1388 s of cooling, the thermocouple measured the temperature at the center of the rod to be 30°C. Determine the convection heat transfer coefficient for this process. Solve this problem using the analytical one-term approximation method.

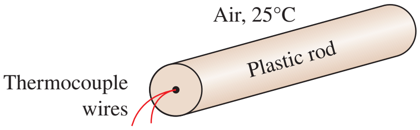


FIGURE P18–48

18–49 A 65-kg beef carcass ($k = 0.47$ W/m·K and $\alpha = 0.13 \times 10^{-6}$ m^2/s) initially at a uniform temperature of 37°C is to be cooled by refrigerated air at -10°C flowing at a velocity of 1.2 m/s. The average heat transfer coefficient between the carcass and the air is 22 W/m^2·K. Treating the carcass as a cylinder of diameter 24 cm and height 1.4 m and disregarding heat transfer from the base and top surfaces, determine how long it will take for the center temperature of the carcass to drop to 4°C. Also, determine if any part of the carcass will freeze during this process. Solve this problem using the analytical one-term approximation method. *Answer:* 12.2 h

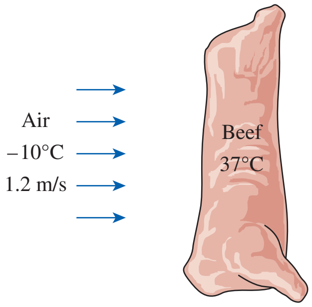


FIGURE P18–49

18–50 A 30-cm-diameter, 4-m-high cylindrical column of a house made of concrete ($k = 0.79$ W/m·K, $\alpha = 5.94 \times 10^{-7}$ m^2/s, $\rho = 1600$ kg/m^3, and $c_p = 0.84$ kJ/kg·K) cooled to 14°C during a cold night is heated again during the day by being exposed to ambient air at an average temperature of 28°C with an average heat transfer coefficient of 14 W/m^2·K. Using the analytical one-term approximation method, determine (*a*) how long it will take for the column surface temperature to rise to 27°C, (*b*) the amount of heat transfer until the center temperature reaches 28°C, and (*c*) the amount of heat transfer until the surface temperature reaches 27°C.

18–51 A long 35-cm-diameter cylindrical shaft made of stainless steel 304 ($k = 14.9$ W/m·K, $\rho = 7900$ kg/m^3, $c_p = 477$ J/kg·K, and $\alpha = 3.95 \times 10^{-6}$ m^2/s) comes out of an oven at a uniform temperature of 500°C. The shaft is then allowed to cool slowly in a chamber at 150°C with an average convection heat transfer coefficient of $h = 60$ W/m^2·K. Determine the temperature at the center of the shaft 20 min after the start of the cooling process. Also, determine the heat transfer per unit length of the shaft during this time period. Solve this problem using the analytical one-term approximation method. *Answers:* 486°C, 22,270 kJ

18–52 Reconsider Prob. 18–51. Using appropriate software, investigate the effect of the cooling time on the final center temperature of the shaft and the amount of heat transfer. Let the time vary from 5 min to 60 min. Plot the center temperature and the heat transfer as a function of the time, and discuss the results.

18–53 Steel rods, 2 m in length and 60 mm in diameter, are being drawn through an oven that maintains a temperature of 800°C and convection heat transfer coefficient of 128 W/m^2·K. The steel rods ($\rho = 7832$ kg/m^3, $c_p = 434$ J/kg·K, $k = 63.9$ W/m·K, and $\alpha = 18.8 \times 10^{-6}$ m^2/s) were initially at a uniform temperature of 30°C. Using the analytical one-term approximation method, determine the amount of heat transferred to the steel rods after 133 s of heating.

18–54 For heat transfer purposes, an egg can be considered to be a 5.5-cm-diameter sphere having the properties of water. An egg that is initially at 4.3°C is dropped into boiling water at 100°C. The heat transfer coefficient at the surface of the egg is estimated to be 800 W/m^2·K. If the egg is considered cooked when its center temperature reaches 71°C, determine how long the egg should be kept in the boiling water. Solve this problem using the analytical one-term approximation method.

18–55 Citrus fruits are very susceptible to cold weather, and extended exposure to subfreezing temperatures can destroy them. Consider an 8-cm-diameter orange that is initially at 15°C. A cold front moves in one night, and the ambient temperature suddenly drops to -6°C, with a heat transfer coefficient of 15 W/m^2·K. Using the properties of water for the orange and assuming the ambient conditions remain constant for 4 h before the cold front moves out, determine if any part of the orange will freeze that night. Solve this problem using the analytical one-term approximation method.

18–56 Chickens with an average mass of 1.7 kg ($k = 0.45$ W/m·K and $\alpha = 0.13 \times 10^{-6}$ m^2/s) initially at a uniform temperature of 15°C are to be chilled in agitated brine at −7°C. The average heat transfer coefficient between the chicken and the brine is determined experimentally to be 440 W/m^2·K. Taking the average density of the chicken to be 0.95 g/cm^3 and treating the chicken as a spherical lump, determine the center and the surface temperatures of the chicken in 2 h and 45 min. Also, determine if any part of the chicken will freeze during this process. Solve this problem using the analytical one-term approximation method.

18–57 Hailstones are formed in high-altitude clouds at 253 K. Consider a hailstone with diameter of 20 mm that is falling through air at 15°C with convection heat transfer coefficient of 163 W/m^2·K. Assuming the hailstone can be modeled as a sphere and has properties of ice at 253 K, determine how long it takes to reach melting point at the surface of the falling hailstone. Solve this problem using the analytical one-term approximation method. The properties of ice at 253 K are ρ = 922 kg/m^3, c_p = 1945 J/kg·K, and k = 2.03 W/m·K.

18–58 An ordinary egg can be approximated as a 5.5-cm-diameter sphere whose properties are roughly $k = 0.6$ W/m·K and $\alpha = 0.14 \times 10^{-6}$ m^2/s. The egg is initially at a uniform temperature of 4°C and is dropped into boiling water at 97°C. Taking the convection heat transfer coefficient to be $h = 1400$ W/m^2·K, determine how long it will take for the center of the egg to reach 70°C. Solve this problem using the analytical one-term approximation method.

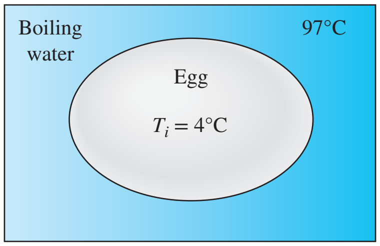


FIGURE P18–58

18–59 Reconsider Prob. 18–58. Using appropriate software, investigate the effect of the final center temperature of the egg on the time it will take for the center to reach this temperature. Let the temperature vary from 50°C to 95°C. Plot the time versus the temperature, and discuss the results.

18–60E Oranges of 2.5-in diameter ($k = 0.26$ Btu/h·ft·°F and $\alpha = 1.4 \times 10^{-6}$ ft^2/s) initially at a uniform temperature of 78°F are to be cooled by refrigerated air at 25°F flowing at a velocity of 1 ft/s. The average heat transfer coefficient between the oranges and the air is experimentally determined to be 4.6 Btu/h·ft^2·°F. Determine how long it will take for the center temperature of the oranges to drop to 40°F. Also, determine if any part of the oranges will freeze during this process. Solve this problem using the analytical one-term approximation method.

18–61 White potatoes ($k = 0.50$ W/m·K and $\alpha = 0.13 \times 10^{-6}$ m^2/s) that are initially at a uniform temperature of 20°C and have an average diameter of 6 cm are to be cooled by refrigerated air at 2°C flowing at a velocity of 4 m/s. The average heat transfer coefficient between the potatoes and the air is experimentally determined to be 19 W/m^2·K. Determine how long it will take for the center temperature of the potatoes to drop to 6°C. Also, determine if any part of the potatoes will experience chilling injury during this process. Solve this problem using the analytical one-term approximation method.

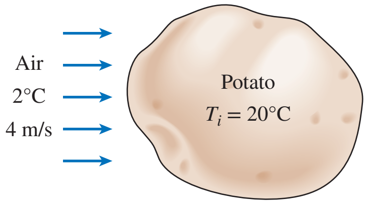


FIGURE P18–61

18–62 An experiment is to be conducted to determine the heat transfer coefficient on the surfaces of tomatoes that are placed in cold water at 7°C. The tomatoes ($k = 0.59$ W/m·K, $\alpha = 0.141 \times 10^{-6}$ m^2/s, $\rho = 999$ kg/m^3, $c_p = 3.99$ kJ/kg·K) with an initial uniform temperature of 30°C are spherical with a diameter of 8 cm. After 2 h, the temperatures at the center and the surface of the tomatoes are measured to be 10.0°C and 7.1°C, respectively. Using the analytical one-term approximation method, determine the heat transfer coefficient and the amount of heat transfer during this period if there are eight such tomatoes in water.

18–63 A 9-cm-diameter potato ($\rho = 1100$ kg/m^3, $c_p = 3900$ J/kg·K, $k = 0.6$ W/m·K, and $\alpha = 1.4 \times 10^{-7}$ m^2/s) that is initially at a uniform temperature of 25°C is baked in an oven at 170°C until a temperature sensor inserted into the center of the potato indicates a reading of 70°C. The potato is then taken out of the oven and wrapped in thick towels so that almost no heat is lost from the baked potato. Assuming the heat transfer coefficient in the oven to be 40 W/m^2·K, determine (*a*) how long the potato is baked in the oven and (*b*) the final equilibrium temperature of the potato after it is wrapped. Solve this problem using the analytical one-term approximation method.

18–64 In *Betty Crocker's Cookbook,* it is stated that it takes 2 h 45 min to roast a 3.2-kg rib initially at 4.5°C to "rare" in an oven maintained at 163°C. It is recommended that a meat thermometer be used to monitor the cooking, and the rib is considered rare when the thermometer inserted into the center of the thickest part of the meat registers 60°C. The rib can be treated as a homogeneous spherical object with the properties $\rho = 1200$ kg/m^3, $c_p = 4.1$ kJ/kg·K, $k = 0.45$ W/m·K, and $\alpha = 0.91 \times 10^{-7}$ m^2/s. Determine (*a*) the heat transfer coefficient at the surface of the rib; (*b*) the temperature of the outer surface of the rib when it is done; and (*c*) the amount of heat transferred to the rib. (*d*) Using the values obtained, predict how long it will take to roast this rib to "medium" level, which occurs when the innermost temperature of the rib reaches 71°C. Compare your result to the listed value of 3 h 20 min.

If the roast rib is to be set on the counter for about 15 min before it is sliced, it is recommended that the rib be taken out of the oven when the thermometer registers about 4°C below the indicated value because the rib will continue cooking even after it is taken out of the oven. Do you agree with this recommendation? Solve this problem using the analytical one-term approximation method. *Answers:* (*a*) 156.9 W/m^2·K, (*b*) 159.5°C, (*c*) 1629 kJ, (*d*) 3.0 h

18–65 Repeat Prob. 18–64 for a roast rib that is to be "well-done" instead of "rare." A rib is considered to be well-done when its center temperature reaches 77°C, and the roasting in this case takes about 4 h 15 min.

Transient Heat Conduction in Semi-Infinite Solids

18–66C Under what conditions can a plane wall be treated as a semi-infinite medium?

18–67C What is a semi-infinite medium? Give examples of solid bodies that can be treated as semi-infinite media for heat transfer purposes.

18–68C Consider a hot semi-infinite solid at an initial temperature of T_i that is exposed to convection to a cooler medium at a constant temperature of T_∞, with a heat transfer coefficient of h. Explain how you can determine the total amount of heat transfer from the solid up to a specified time t_o.

18–69E The walls of a furnace are made of 1.5-ft-thick concrete ($k = 0.64$ Btu/h·ft·°F and $\alpha = 0.023$ ft^2/h). Initially, the furnace and the surrounding air are in thermal equilibrium at 70°F. The furnace is then fired, and the inner surfaces of the furnace are subjected to hot gases at 1800°F with a very large heat transfer coefficient. Determine how long it will take for the temperature of the outer surface of the furnace walls to rise to 70.1°F. *Answer:* 3.0 h

18–70 Consider a curing kiln whose walls are made of 30-cm-thick concrete with a thermal diffusivity of $\alpha = 0.23 \times 10^{-5}$ m^2/s. Initially, the kiln and its walls are in equilibrium with the surroundings at 6°C. Then all the doors are closed and the kiln is heated by steam so that the temperature of the inner surface of the walls is raised to 42°C, and the temperature is maintained at that level for 2.5 h. The curing kiln is then opened and exposed to the atmospheric air after the steam flow is turned off. If the outer surfaces of the walls of the kiln were insulated, would it save any energy that day during the period the kiln was used for curing for 2.5 h only, or would it make no difference? Base your answer on calculations.

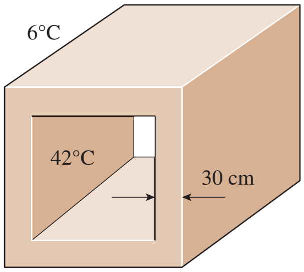


FIGURE P18–70

18–71 In areas where the air temperature remains below 0°C for prolonged periods of time, the freezing of water in underground pipes is a major concern. Fortunately, the soil remains relatively warm during those periods, and it takes weeks for the subfreezing temperatures to reach the water mains in the ground. Thus, the soil effectively serves as an insulation to protect the water from the freezing atmospheric temperatures in winter.

The ground at a particular location is covered with snowpack at −8°C for a continuous period of 60 days, and the average soil properties at that location are $k = 0.35$ W/m·K and $\alpha = 0.15 \times 10^{-6}$ m^2/s. Assuming an initial uniform temperature of 8°C for the ground, determine the minimum burial depth to prevent the water pipes from freezing.

18–72 A large cast iron container ($k = 52$ W/m·K and $\alpha = 1.70 \times 10^{-5}$ m^2/s) with 4-cm-thick walls is initially at a uniform temperature of 0°C and is filled with ice at 0°C. Now the outer surfaces of the container are exposed to hot water at 55°C with a very large heat transfer coefficient. Determine how long it will be before the ice inside the container starts melting. Also, taking the heat transfer coefficient on the inner surface of the container to be 250 W/m^2·K, determine the rate of heat transfer to the ice through a 1.2-m-wide and 2-m-high section of the wall when steady operating conditions are reached. Assume the ice starts melting when its inner surface temperature rises to 0.1°C.

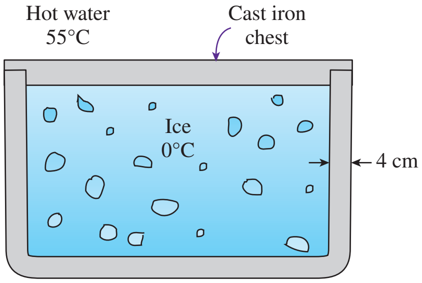


FIGURE P18–72

18–73 A highway made of asphalt is initially at a uniform temperature of 55°C. Suddenly the highway surface temperature is reduced to 25°C by rain. Determine the temperature at the depth of 3 cm from the highway surface and the heat flux transferred from the highway after 60 min. Assume the highway surface temperature is maintained at 25°C. The properties of asphalt are ρ = 2115 kg/m^3, c_p = 920 J/kg·K, and k = 0.062 W/m·K. *Answers:* 53.6°C, 98 W/m^2

18–74 A thick aluminum block initially at 20°C is subjected to constant heat flux of 4000 W/m^2 by an electric resistance heater whose top surface is insulated. Determine how much the surface temperature of the block will rise after 30 min.

18–75 Refractory bricks are used as linings for furnaces, and they generally have low thermal conductivity to minimize heat loss through the furnace walls. Consider a thick furnace wall lined with refractory bricks ($k = 1.0$ W/m·K and $\alpha = 5.08 \times 10^{-7}$ m^2/s), where initially the wall has a uniform temperature of 15°C. If the wall surface is subjected to uniform heat flux of 20 kW/m^2, determine the temperature at the depth of 10 cm from the surface after an hour of heating time.

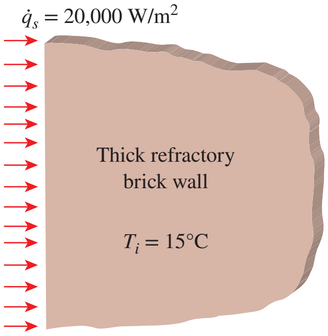


FIGURE P18–75

18–76 Thick slabs of stainless steel ($k = 14.9$ W/m·K and $\alpha = 3.95 \times 10^{-6}$ m^2/s) and copper ($k = 401$ W/m·K and $\alpha = 117 \times 10^{-6}$ m^2/s) are subjected to uniform heat flux of 8 kW/m^2 at the surface. The two slabs have a uniform initial temperature of 20°C. Determine the temperatures of both slabs, at 1 cm from the surface, after 60 s of exposure to the heat flux.

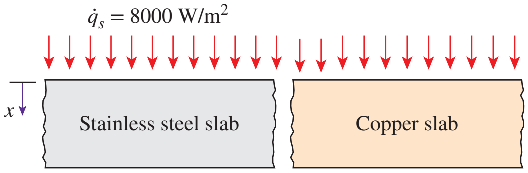


FIGURE P18–76

18–77 A thick wood slab ($k = 0.17$ W/m·K and $\alpha = 1.28 \times 10^{-7}$ m^2/s) that is initially at a uniform temperature of 25°C is exposed to hot gases at 550°C for 5 min. The heat transfer coefficient between the gases and the wood slab is 35 W/m^2·K. If the ignition temperature of the wood is 450°C, determine if the wood will ignite.

18–78 The soil temperature in the upper layers of the earth varies with variations in the atmospheric conditions. Before a cold front moves in, the earth at a location is initially at a uniform temperature of 15°C. Then the area is subjected to a temperature of −8°C and high winds that result in a convection heat transfer coefficient of 40 W/m^2·K on the earth's surface for 10 h. Taking the properties of the soil at that location to be $k = 0.9$ W/m·K and $\alpha = 1.6 \times 10^{-5}$ m^2/s, determine the soil temperature at distances 0, 10, 20, and 50 cm from the earth's surface at the end of this 10-h period.

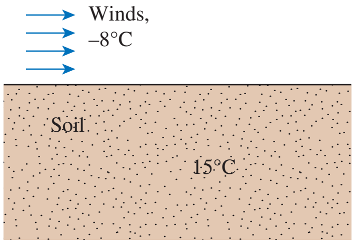


FIGURE P18–78

18–79 Reconsider Prob. 18–78. Using appropriate software, plot the soil temperature as a function of the distance from the earth's surface as the distance varies from 0 m to 1 m, and discuss the results.

18–80 We often cut a watermelon in half and put it into the freezer to cool it quickly. But usually we forget to check on it and end up having a watermelon with a frozen layer on top. To avoid this potential problem, a person wants to set a timer so that it will go off when the temperature of the exposed surface of the watermelon drops to 3°C. Consider a 25-cm-diameter spherical watermelon that is cut into two equal parts and put into a freezer at −12°C. Initially, the entire watermelon is at a uniform temperature of 25°C, and the heat transfer coefficient on the surfaces is 22 W/m^2·K. Assuming the watermelon to have the properties of water, determine how long it will take for the center of the exposed cut surfaces of the watermelon to drop to 3°C.

18–81 Thick slabs of stainless steel ($k = 14.9$ W/m·K and $\alpha = 3.95 \times 10^{-6}$ m^2/s) and copper ($k = 401$ W/m·K and $\alpha = 117 \times 10^{-6}$ m^2/s) are placed under an array of laser diodes, which supply an energy pulse of 5×10^7 J/m^2 instantaneously at $t = 0$ to both materials. The two slabs have a uniform initial temperature of 20°C. Determine the temperatures of both slabs at 5 cm from the surface and 60 s after receiving an energy pulse from the laser diodes.

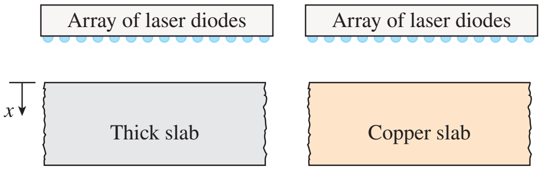


FIGURE P18–81

18–82 A barefooted person whose feet are at 32°C steps on a large aluminum block at 20°C. Treating both the feet and the aluminum block as semi-infinite solids, determine the contact surface temperature. What would your answer be if the person stepped on a wood block instead? At room temperature, the $\sqrt{k\rho c_p}$ value is 24 kJ/m^2·°C for aluminum, 0.38 kJ/m^2·°C for wood, and 1.1 kJ/m^2·°C for human flesh.

Transient Heat Conduction in Multidimensional Systems

18–83C What is the product solution method? How is it used to determine the transient temperature distribution in a two-dimensional system?

18–84C How is the product solution used to determine the variation of temperature with time and position in three-dimensional systems?

18–85C A short cylinder initially at a uniform temperature T_i is subjected to convection from all of its surfaces to a medium at temperature T_∞. Explain how you can determine the temperature of the midpoint of the cylinder at a specified time t.

18–86C Consider a short cylinder whose top and bottom surfaces are insulated. The cylinder is initially at a uniform temperature T_i and is subjected to convection from its side surface to a medium at temperature T_∞ with a heat transfer coefficient of h. Is the heat transfer in this short cylinder one- or two-dimensional? Explain.

18–87 Consider a cubic block whose sides are 5 cm long and a cylindrical block whose height and diameter are also 5 cm. Both blocks are initially at 20°C and are made of granite

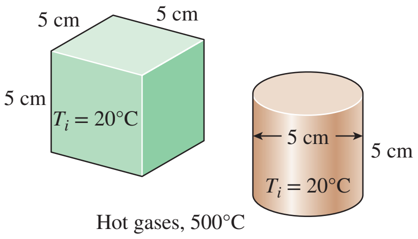


FIGURE P18–87

($k = 2.5$ W/m·K and $\alpha = 1.15 \times 10^{-6}$ m^2/s). Now both blocks are exposed to hot gases at 500°C in a furnace on all of their surfaces with a heat transfer coefficient of 40 W/m^2·K. Determine the center temperature of each geometry after 10, 20, and 60 min. Solve this problem using the analytical one-term approximation method.

18–88 Repeat Prob. 18–87 with the heat transfer coefficient at the top and the bottom surfaces of each block being doubled to 80 W/m^2·K.

18–89E A hot dog can be considered to be a cylinder 5 in long and 0.8 in in diameter whose properties are $\rho = 61.2$ lbm/ft^3, $c_p = 0.93$ Btu/lbm·°F, $k = 0.44$ Btu/h·ft·°F, and $\alpha = 0.0077$ ft^2/h. A hot dog initially at 40°F is dropped into boiling water at 212°F. If the heat transfer coefficient at the surface of the hot dog is estimated to be 120 Btu/h·ft^2·°F, determine the center temperature of the hot dog after 5, 10, and 15 min by treating the hot dog as (*a*) a finite cylinder and (*b*) an infinitely long cylinder. Solve this problem using the analytical one-term approximation method.

18–90 A 2-cm-high cylindrical ice block ($k = 2.22$ W/m·K and $\alpha = 0.124 \times 10^{-7}$ m^2/s) is placed on a table on its base of diameter 2 cm in a room at 24°C. The heat transfer coefficient on the exposed surfaces of the ice block is 13 W/m^2·K, and heat transfer from the base of the ice block to the table is negligible. If the ice block is not to start melting at any point for at least 3 h, determine what the initial temperature of the ice block should be. Solve this problem using the analytical one-term approximation method.

18–91 A short brass cylinder ($\rho = 8530$ kg/m^3, $c_p = 0.389$ kJ/kg·K, $k = 110$ W/m·K, and $\alpha = 3.39 \times 10^{-5}$ m^2/s) of diameter 4 cm and height 20 cm is initially at a uniform temperature of 150°C. The cylinder is now placed in atmospheric air at 20°C, where heat transfer takes place by convection with a heat transfer coefficient of 40 W/m^2·K. Calculate (*a*) the center temperature of the cylinder; (*b*) the center temperature of the top surface of the cylinder; and (*c*) the total heat transfer from the cylinder 15 min after the start of the cooling. Solve this problem using the analytical one-term approximation method.

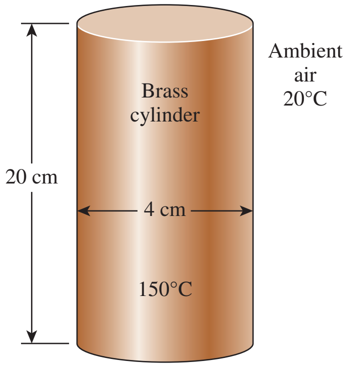


FIGURE P18–91

18–92 Reconsider Prob. 18–91. Using appropriate software, investigate the effect of the cooling time on the center temperature of the cylinder, the center temperature of the top surface of the cylinder, and the total heat transfer. Let the time vary from 5 min to 60 min. Plot the center temperature of the cylinder, the center temperature of the top surface, and the total heat transfer as a function of the time, and discuss the results.

18–93 A semi-infinite aluminum cylinder ($k = 237$ W/m·K, $\alpha = 9.71 \times 10^{-5}$ m^2/s) of diameter $D = 15$ cm is initially at a uniform temperature of $T_i = 115$°C. The cylinder is now placed in water at 10°C, where heat transfer takes place by convection with a heat transfer coefficient of $h = 140$ W/m^2·K. Determine the temperature at the center of the cylinder 5 cm from the end surface 8 min after the start of cooling. Solve this problem using the analytical one-term approximation method.

Review Problems

18–94 Large steel plates 1.0 cm in thickness are quenched from 600°C to 100°C by submerging them in an oil reservoir held at 30°C. The average heat transfer coefficient for both faces of steel plates is 400 W/m^2·K. Average steel properties are $k = 45$ W/m·K, $\rho = 7800$ kg/m^3, and $c_p = 470$ J/kg·K. Calculate the quench time for steel plates.

18–95 A long roll of 2-m-wide and 0.5-cm-thick 1-Mn manganese steel plate coming off a furnace at 820°C is to be quenched in an oil bath ($c_p = 2.0$ kJ/kg·K) at 45°C. The metal sheet is moving at a steady velocity of 20 m/min, and the oil bath is 9 m long. Taking the convection heat transfer coefficient on both sides of the plate to be 860 W/m^2·K, determine the temperature of the sheet metal when it leaves the oil bath. Also, determine the required rate of heat removal from the oil to keep its temperature constant at 45°C.

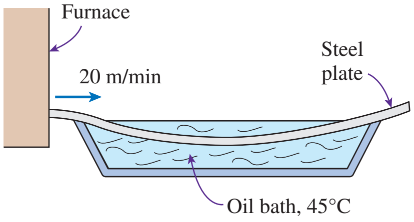


FIGURE P18–95

18–96 Aluminum wires 4 mm in diameter are produced by extrusion. The wires leave the extruder at an average temperature of 350°C and at a linear rate of 10 m/min. Before leaving the extrusion room, the wires are cooled to an average temperature of 50°C by transferring heat to the surrounding air at 25°C with a heat transfer coefficient of 50 W/m^2·K. Calculate the necessary length of the wire cooling section in the extrusion room.

18–97E During a picnic on a hot summer day, the only available drinks were those at the ambient temperature of 90°F. In an effort to cool a 12-fluid-oz drink in a can, which is 5 in high and has a diameter of 2.5 in, a person grabs the can and starts shaking it in the iced water of the chest at 32°F. The temperature of the drink can be assumed to be uniform at all times, and the heat transfer coefficient between the iced water and the aluminum can is 30 Btu/h·ft^2·°F . Using the properties of water for the drink, estimate how long it will take for the canned drink to cool to 40°F. Solve this problem using lumped system analysis. Is the lumped system analysis applicable to this problem? Why?

18–98 In an annealing process, a 50-mm-thick stainless steel plate ($\rho = 8238$ kg/m^3, $c_p = 468$ J/kg·K, $k = 13.4$ W/m·K, and $\alpha = 3.48 \times 10^{-6}$ m^2/s) was reheated in a furnace from an initial uniform temperature of 230°C. The ambient temperature inside the furnace is uniform at 1000°C, and the convection heat transfer coefficient is 215 W/m^2·K. If the entire stainless steel plate is to be heated to at least 600°C, determine the time that the plate should be heated in the furnace using the analytical one-term approximation method.

18–99 During a fire, the trunks of some dry oak trees ($k = 0.17$ W/m·K and $\alpha = 1.28 \times 10^{-7}$ m^2/s) that are initially at a uniform temperature of 30°C are exposed to hot gases at 600°C for 4 h, with a heat transfer coefficient of 65 W/m^2·K on the surface. The ignition temperature of the trees is 410°C. Treating the trunks of the trees as long cylindrical rods of diameter 20 cm, determine if these dry trees will ignite as the fire sweeps through them. Solve this problem using the analytical one-term approximation method.

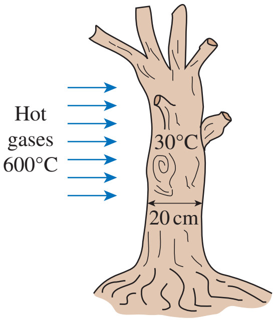


FIGURE P18–99

18–100E In *Betty Crocker's Cookbook,* it is stated that it takes 5 h to roast a 14-lb stuffed turkey initially at 40°F in an oven maintained at 325°F. It is recommended that a meat thermometer be used to monitor the cooking, and the turkey is considered done when the thermometer inserted deep into the thickest part of the breast or thigh without touching the bone registers 185°F. The turkey can be treated as a homogeneous spherical object with the properties $\rho = 75$ lbm/ft^3, $c_p = 0.98$ Btu/lbm·°F, $k = 0.26$ Btu/h·ft·°F, and $\alpha = 0.0035$ ft^2/h. Assuming the tip of the thermometer is at one-third radial distance from the center of the turkey, determine (*a*) the average heat transfer coefficient at the surface of the turkey, (*b*) the temperature of the skin of the turkey when it is done, and (*c*) the total amount of heat transferred to the turkey in the oven. Will the reading of the thermometer be more or less than 185°F 5 min after the turkey is taken out of the oven?

18–101 A watermelon initially at 35°C is to be cooled by dropping it into a lake at 15°C. After 4 h and 40 min of cooling, the center temperature of the watermelon is measured to be 20°C. Treating the watermelon as a 20-cm-diameter sphere and using the properties $k = 0.618$ W/m·K, $\alpha = 0.15 \times 10^{-6}$ m^2/s, $\rho = 995$ kg/m^3, and $c_p = 4.18$ kJ/kg·K, determine the average heat transfer coefficient and the surface temperature of the watermelon at the end of the cooling period. Solve this problem using the analytical one-term approximation method.

18–102 Spherical glass beads coming out of a kiln are allowed to cool in a room temperature of 30°C. A glass bead with a diameter of 10 mm and an initial temperature of 400°C is allowed to cool for 3 min. If the convection heat transfer coefficient is 28 W/m^2·K, determine the temperature at the center of the glass bead using the analytical one-term approximation method. The glass bead has properties of $\rho = 2800$ kg/m^3, $c_p = 750$ J/kg·K, and $k = 0.7$ W/m·K.

18–103 Water mains must be placed at sufficient depth below the earth's surface to avoid freezing during extended periods of subfreezing temperatures. Determine the minimum depth at which the water main must be placed at a location where the soil is initially at 15°C and the earth's surface temperature under the worst conditions is expected to remain at −10°C for 75 days. Take the properties of soil at that location to be $k = 0.7$ W/m·K and $\alpha = 1.4 \times 10^{-5}$ m^2/s. *Answer:* 7.05 m

18–104 A 40-cm-thick brick wall ($k = 0.72$ W/m·K, and $\alpha = 1.6 \times 10^{-6}$ m^2/s) is heated to an average temperature of 18°C by the heating system and the solar radiation incident on it during the day. During the night, the outer surface of the wall is exposed to cold air at −3°C with an average heat transfer coefficient of 20 W/m^2·K. Determine the wall temperatures at distances 15, 30, and 40 cm from the outer surface for 2 h.

18–105 In a volcano eruption, lava at 1200°C is found flowing on the ground. The ground was initially at 15°C, and the lava flow has a convection heat transfer coefficient of 3500 W/m^2·K. Determine the ground surface (*a*) temperature and (*b*) heat flux after 2 s of lava flow. The properties of the ground (dry soil) are $\rho = 1500$ kg/m^3, $c_p = 1900$ J/kg·K, and $k = 1.0$ W/m·K.

18–106 A large iron slab ($\rho = 7870$ kg/m^3, $c_p = 447$ J/kg·K, and $k = 80.2$ W/m·K) was initially heated to a uniform temperature of 150°C and then placed on a concrete floor ($\rho = 1600$ kg/m^3, $c_p = 840$ J/kg·K, and $k = 0.79$ W/m·K). The concrete floor was initially at a uniform temperature of 30°C. Determine (*a*) the surface temperature between the iron slab and concrete floor and (*b*) the temperature of the concrete floor at the depth of 25 mm, if the surface temperature remains constant after 15 min.

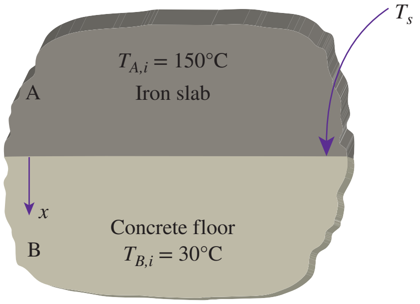


FIGURE P18–106

18–107 A hot dog can be considered to be a 12-cm-long cylinder whose diameter is 2 cm and whose properties are $\rho = 980$ kg/m^3, $c_p = 3.9$ kJ/kg·K, $k = 0.76$ W/m·K, and $\alpha = 2 \times 10^{-7}$ m^2/s. A hot dog initially at 5°C is dropped into boiling water at 100°C. The heat transfer coefficient at the surface of the hot dog is estimated to be 600 W/m^2·K. If the hot dog is considered cooked when its center temperature reaches 80°C, determine how long it will take to cook it in the boiling water. Solve this problem using the analytical one-term approximation method.

18–108 Consider the engine block of a car made of cast iron ($k = 52$ W/m·K and $\alpha = 1.7 \times 10^{-5}$ m^2/s). The engine can be considered to be a rectangular block whose sides are 80 cm, 40 cm, and 40 cm. The engine is at a temperature of 150°C when it is turned off. The engine is then exposed to atmospheric air at 17°C with a heat transfer coefficient of 6 W/m^2·K. Determine (*a*) the center temperature of the top surface whose sides are 80 cm and 40 cm and (*b*) the corner temperature after 45 min of cooling. Solve this problem using the analytical one-term approximation method.

Design and Essay Problems

18–109 Conduct the following experiment at home to determine the combined convection and radiation heat transfer coefficient at the surface of an apple exposed to the room air. You will need two thermometers and a clock.

First, weigh the apple and measure its diameter. You can measure its volume by placing it in a large measuring cup halfway filled with water, and measuring the change in volume when it is completely immersed in the water. Refrigerate the apple overnight so that it is at a uniform temperature in the morning, and measure the air temperature in the kitchen. Then take the apple out and stick one of the thermometers to its middle and the other just under the skin. Record both temperatures every 5 min for an hour. Using these two temperatures, calculate the heat transfer coefficient for each interval and take their average. The result is the combined convection and radiation heat transfer coefficient for this heat transfer process. Using your experimental data, also calculate the thermal conductivity and thermal diffusivity of the apple and compare them to the values given above.

18–110 Repeat Prob. 18–109 using a banana instead of an apple. The thermal properties of bananas are practically the same as those of apples.

18–111 Conduct the following experiment to determine the time constant for a can of soda and then predict the temperature of the soda at different times. Leave the soda in the refrigerator overnight. Measure the air temperature in the kitchen and the temperature of the soda while it is still in the refrigerator by taping the sensor of the thermometer to the outer surface of the can. Then take the soda out and measure its temperature again in 5 min. Using these values, calculate the exponent *b*. Using this *b*-value, predict the temperatures of the soda in 10, 15, 20, 30, and 60 min and compare the results with the actual temperature measurements. Do you think the lumped system analysis is valid in this case?

CHAPTER

19

FORCED CONVECTION

So far, we have considered *conduction*, which is the mechanism of heat transfer through a solid or a quiescent fluid. We now consider *convection*, which is the mechanism of heat transfer through a fluid in the presence of bulk fluid motion.

Convection is classified as *natural* (or *free*) *convection* or *forced convection*, depending on how the fluid motion is initiated. In forced convection, the fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In natural convection, any fluid motion is caused by natural means such as the buoyancy effect, which manifests itself as the rise of warmer fluid and the fall of cooler fluid. Convection is also classified as *external* or *internal*, depending on whether the fluid is forced to flow over a surface or in a pipe.

We start this chapter with a general physical description of the convection mechanism and thermal boundary layer. We continue with the discussion of the dimensionless *Prandtl* and *Nusselt numbers* and their physical significance. We then present empirical relations for the heat transfer coefficients for flow over various geometries such as a flat plate, cylinder, and sphere, for both laminar and turbulent flow conditions. Finally, we discuss the characteristics of flow inside tubes and present the heat transfer correlations associated with it. The relevant concepts from Chaps. 14 and 15 should be reviewed before this chapter is studied.

OBJECTIVES

The objectives of this chapter are to:

- Understand the physical mechanism of convection and its classification.
- Visualize the development of a thermal boundary layer during flow over surfaces.
- Gain a working knowledge of the dimensionless Prandtl and Nusselt numbers.
- Develop an understanding of the mechanism of heat transfer in turbulent flow.
- Evaluate the heat transfer associated with flow over a flat plate for both laminar and turbulent flow and flow over cylinders and spheres.
- Have a visual understanding of different flow regions in internal flow, and calculate hydrodynamic and thermal entry lengths.
- Analyze heating and cooling of a fluid flowing in a tube under constant surface temperature and constant surface heat flux conditions, and work with the logarithmic mean temperature difference.
- Determine the Nusselt number in fully developed turbulent flow using empirical relations, and calculate the heat transfer rate.

19–1 ▪ PHYSICAL MECHANISM OF CONVECTION

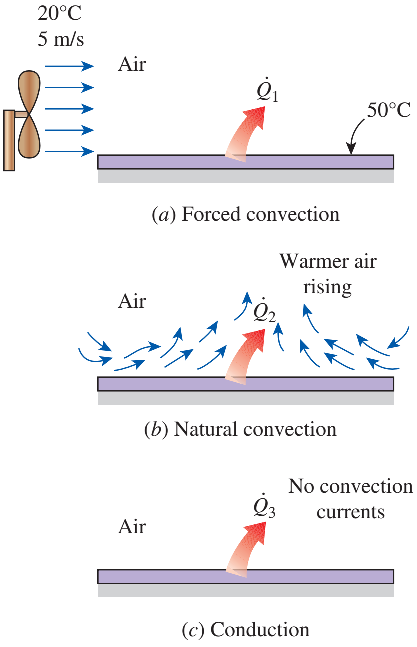


FIGURE 19–1
Heat transfer from a hot surface to the surrounding fluid by convection and conduction.

We mentioned in Chap. 16 that there are three basic mechanisms of heat transfer: conduction, convection, and radiation. Conduction and convection are similar in that both mechanisms require the presence of a material medium. But they differ in that convection requires the presence of fluid motion.

Heat transfer through a solid is always by conduction, since the molecules of a solid remain at relatively fixed positions. Heat transfer through a liquid or gas, however, can be by conduction or convection, depending on the presence of any bulk fluid motion. Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it. Therefore, conduction in a fluid can be viewed as the limiting case of convection, corresponding to the case of quiescent fluid (Fig. 19–1).

Convection heat transfer is complicated by the fact that it involves fluid motion as well as heat conduction. The fluid motion enhances heat transfer, since it brings warmer and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in a fluid. Therefore, the rate of heat transfer through a fluid is much higher by convection than it is by conduction. In fact, the higher the fluid velocity, the higher the rate of heat transfer.

To clarify this point further, consider steady heat transfer through a fluid contained between two parallel plates maintained at different temperatures, as shown in Fig. 19–2. The temperatures of the fluid and the plate are the same at the points of contact because of the continuity of temperature. Assuming no fluid motion, the energy of the hotter fluid molecules near the hot plate is transferred to the adjacent cooler fluid molecules. This energy is then transferred to the next layer of the cooler fluid molecules. This energy is then transferred to the next layer of the cooler fluid, and so on, until it is finally transferred to the other plate. This is what happens during conduction through a fluid. Now let us use a syringe to draw some fluid near the hot plate and inject it next to the cold plate repeatedly. You can imagine that this will speed up the heat transfer process considerably, since some energy is carried to the other side as a result of fluid motion.

Consider the cooling of a hot block with a fan blowing air over its top surface. We know that heat is transferred from the hot block to the surrounding cooler air, and the block eventually cools. We also know that the block cools faster if the fan is switched to a higher speed. Replacing air with water enhances the convection heat transfer even more.

Experience shows that convection heat transfer strongly depends on the fluid properties *dynamic viscosity* μ, *thermal conductivity* k, *density* ρ, and *specific heat* c_p, as well as the *fluid velocity* V. It also depends on the *geometry* and the *roughness* of the solid surface, in addition to the *type of fluid flow* (such as being streamlined or turbulent). Thus, we expect the convection heat transfer relations to be rather complex because of the dependence of convection on so many variables. This is not surprising, since convection is the most complex mechanism of heat transfer.

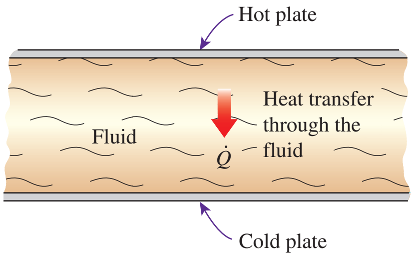


FIGURE 19–2
Heat transfer through a fluid sandwiched between two parallel plates.

Despite the complexity of convection, the rate of convection heat transfer is observed to be proportional to the temperature difference and is conveniently expressed by **Newton's law of cooling** as

$$\dot{q}_{\text{conv}} = h(T_s - T_\infty) \qquad (\text{W/m}^2) \tag{19–1}$$

or

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \qquad (\text{W}) \tag{19–2}$$

where

h = convection heat transfer coefficient, W/m^2·K
A_s = heat transfer surface area, m^2
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C

Judging from its units, the **convection heat transfer coefficient** h can be defined as *the rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference.*

You should not be deceived by the simple appearance of this relation, because the convection heat transfer coefficient h depends on several of the mentioned variables and thus is difficult to determine.

Fluid flow is often confined by solid surfaces, and it is important to understand how the presence of solid surfaces affects fluid flow. Consider the flow of a fluid in a stationary pipe or over a solid surface that is nonporous (i.e., impermeable to the fluid). All experimental observations indicate that a fluid in motion comes to a complete stop at the surface and assumes a zero velocity relative to the surface. That is, a fluid in direct contact with a solid "sticks" to the surface due to viscous effects, and there is no slip. This is known as the **no-slip condition**.

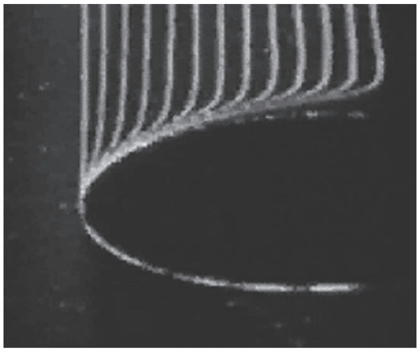


FIGURE 19–3

The development of a velocity profile due to the no-slip condition as a fluid flows over a blunt nose.

"Hunter Rouse: Laminar and Turbulent Flow Film." Copyright IIHR-Hydroscience & Engineering, The University of Iowa. Used by permission.

Fig. 19–3, obtained from a video clip, clearly shows the evolution of a velocity gradient as a result of the fluid sticking to the surface of a blunt nose. The layer that sticks to the surface slows the adjacent fluid layer because of viscous forces between the fluid layers, which slows the next layer, and so on. Therefore, the no-slip condition is responsible for the development of the velocity profile. The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is called the **boundary layer**. The fluid property responsible for the no-slip condition and the development of the boundary layer is *viscosity*.

A fluid layer adjacent to a moving surface has the same velocity as the surface. A consequence of the no-slip condition is that all velocity profiles must have zero values with respect to the surface at the points of contact between a fluid and a solid surface (Fig. 19–4). Another consequence of the no-slip condition is the *surface drag,* which is the force a fluid exerts on a surface in the flow direction.

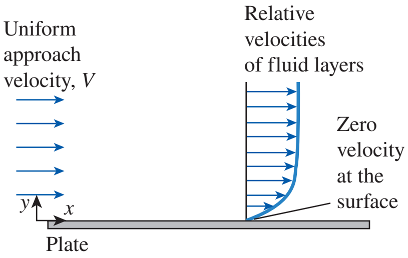


FIGURE 19–4

A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition.

An implication of the no-slip condition is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by *pure conduction,* since the fluid layer is motionless, and it can be expressed as

$$\dot{q}_{\text{conv}} = \dot{q}_{\text{cond}} = -k_{\text{fluid}} \left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad (\text{W/m}^2) \tag{19–3}$$

where T represents the temperature distribution in the fluid and $(\partial T/\partial y)_{y=0}$ is the *temperature gradient* at the surface. Heat is then *convected away* from the surface as a result of fluid motion. Note that convection heat transfer from a solid surface to a fluid is merely the conduction heat transfer from the solid surface to the fluid layer adjacent to the surface. Therefore, we can equate Eqs. 19–1 and 19–3 for the heat flux to obtain

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty} \qquad (\text{W/m}^2\text{·K}) \tag{19–4}$$

for the determination of the *convection heat transfer coefficient* when the temperature distribution within the fluid is known.

The convection heat transfer coefficient, in general, varies along the flow (or x-) direction. The *average* or *mean* convection heat transfer coefficient for a surface in such cases is determined by properly averaging the *local* convection heat transfer coefficients over the entire surface.

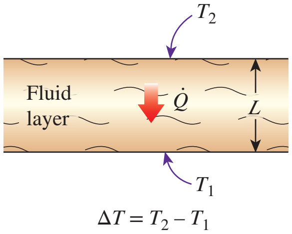


FIGURE 19–5
Heat transfer through a fluid layer of thickness L and temperature difference ΔT.

Nusselt Number

In convection studies, it is common practice to nondimensionalize the governing equations and combine the variables, which group together into *dimensionless numbers* in order to reduce the number of total variables. It is also common practice to nondimensionalize the heat transfer coefficient h with the Nusselt number, defined as

$$\mathrm{Nu} = \frac{hL_c}{k} \tag{19–5}$$

where k is the thermal conductivity of the fluid and L_c is the *characteristic length.* The Nusselt number is named after Wilhelm Nusselt, who made significant contributions to convective heat transfer in the first half of the 20th century, and it is viewed as the *dimensionless convection heat transfer coefficient.*

To understand the physical significance of the Nusselt number, consider a fluid layer of thickness L and temperature difference $\Delta T = T_2 - T_1$, as shown in Fig. 19–5. Heat transfer through the fluid layer is by *convection* when the fluid involves some motion and by *conduction* when the fluid layer is motionless. Heat flux (the rate of heat transfer per unit surface area) in either case is

$$\dot{q}_{\text{conv}} = h\,\Delta T \tag{19–6}$$

and

$$\dot{q}_{\text{cond}} = k\frac{\Delta T}{L} \tag{19–7}$$

Taking their ratio gives

$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\,\Delta T}{k\,\Delta T/L} = \frac{hL}{k} = \mathrm{Nu} \tag{19–8}$$

which is the Nusselt number. Therefore, the Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer. The larger the Nusselt number, the more effective the convection. A Nusselt number of Nu = 1 for a fluid layer represents heat transfer across the layer by pure conduction.

We use forced convection in daily life more often than you might think (Fig. 19–6). We resort to forced convection whenever we want to increase the rate of heat transfer from a hot object. For example, we turn on the fan on hot summer days to help our bodies to cool more effectively. The higher the fan speed, the better we feel. We *stir* our soup and *blow* on a hot slice of pizza to make them cool faster. The air on windy winter days feels much colder than it actually is. The simplest solution to heating problems in electronics packaging is to use a large enough fan.

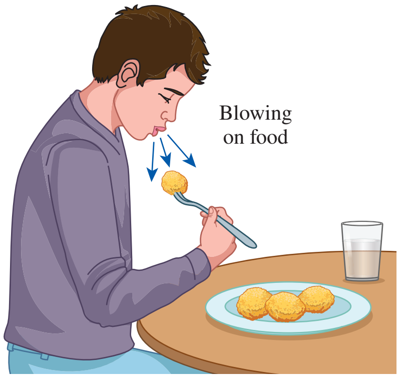


FIGURE 19–6
We resort to forced convection whenever we need to increase the rate of heat transfer.

19–2 ▪ THERMAL BOUNDARY LAYER

We have seen that a velocity boundary layer develops when a fluid flows over a surface as a result of the fluid layer adjacent to the surface assuming the surface velocity (i.e., zero velocity relative to the surface). Also, we defined the velocity boundary layer as the region in which the fluid velocity varies from zero to 0.99V. Likewise, a *thermal boundary layer* develops when a fluid at a specified temperature flows over a surface that is at a different temperature, as shown in Fig. 19–7.

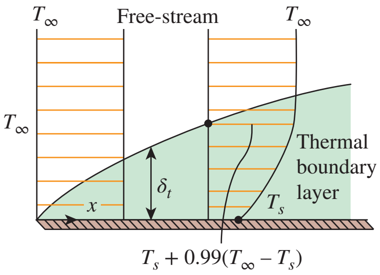


FIGURE 19–7
Thermal boundary layer on a flat plate (the fluid is hotter than the plate surface).

Consider the flow of a fluid at a uniform temperature of T_∞ over an isothermal flat plate at temperature T_s. The fluid particles in the layer adjacent to the surface reach thermal equilibrium with the plate and assume the surface temperature T_s. These fluid particles then exchange energy with the particles in the adjoining fluid layer, and so on. As a result, a temperature profile develops in the flow field that ranges from T_s at the surface to T_∞ sufficiently far from the surface. The flow region over the surface in which the temperature variation in the direction normal to the surface is significant is the **thermal boundary layer**. The *thickness* of the thermal boundary layer δ_t at any location along the surface is defined as *the distance from the surface at which the temperature difference* $T - T_s$ *equals* $0.99(T_\infty - T_s)$. Note that for the special case of $T_s = 0$, we have $T = 0.99T_\infty$ at the outer edge of the thermal boundary layer, which is analogous to $u = 0.99V$ for the velocity boundary layer.

The thickness of the thermal boundary layer increases in the flow direction, since the effects of heat transfer are felt at greater distances from the surface further downstream.

The convection heat transfer rate anywhere along the surface is directly related to the temperature gradient at that location. Therefore, the shape of the temperature profile in the thermal boundary layer dictates the convection heat transfer between a solid surface and the fluid flowing over it. In flow over a heated (or cooled) surface, both velocity and thermal boundary layers develop simultaneously. Noting that the fluid velocity has a strong influence on the temperature profile, the development of the velocity boundary layer relative to the thermal boundary layer will have a strong effect on the convection heat transfer.

Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the *dimensionless* parameter **Prandtl number**, defined as

$$\text{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k} \tag{19–9}$$

It is named after Ludwig Prandtl, who introduced the concept of boundary layer in 1904 and made significant contributions to boundary layer theory. The Prandtl numbers of fluids range from less than 0.01 for liquid metals to more than 100,000 for heavy oils (Table 19–1). Note that the Prandtl number is on the order of 10 for water.

The Prandtl numbers of gases are about 1, which indicates that both momentum and heat dissipate through the fluid at about the same rate. Heat diffuses very quickly in liquid metals ($\text{Pr} \ll 1$) and very slowly in oils ($\text{Pr} \gg 1$) relative to momentum. Consequently the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.

TABLE 19–1

Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

19–3 ▪ PARALLEL FLOW OVER FLAT PLATES

Consider the parallel flow of a fluid over a flat plate of length L in the flow direction, as shown in Fig. 19–8. The x-coordinate is measured along the plate surface from the leading edge in the direction of the flow. The fluid approaches the plate in the x-direction with a uniform velocity V and temperature T_∞. The flow in the velocity boundary layers starts out as laminar, but if the plate is sufficiently long, the flow becomes turbulent at a distance x_{cr} from the leading edge where the Reynolds number reaches its critical value for transition.

The transition from laminar to turbulent flow depends on the *surface geometry, surface roughness, upstream velocity, surface temperature,* and the *type of fluid,* among other things, and it is best characterized by the Reynolds number. The Reynolds number at a distance x from the leading edge of a flat plate is expressed as

$$\text{Re}_x = \frac{\rho V x}{\mu} = \frac{Vx}{\nu} \tag{19–10}$$

Note that the value of the Reynolds number varies for a flat plate along the flow, reaching $\text{Re}_L = VL/\nu$ at the end of the plate.

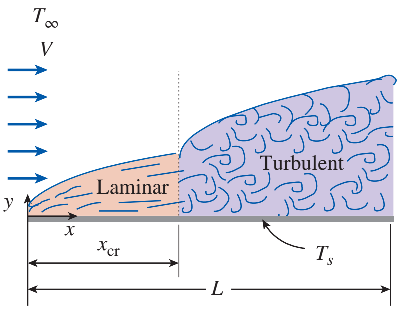


FIGURE 19–8
Laminar and turbulent regions of the boundary layer during flow over a flat plate.

For flow over a flat plate, the transition from laminar to turbulent begins at about $\mathrm{Re} \cong 1 \times 10^5$, but it does not become fully turbulent before the Reynolds number reaches much higher values, typically around 3×10^6. In engineering analysis, a generally accepted value for the critical Reynolds number is

$$\mathrm{Re}_{\mathrm{cr}} = \frac{\rho V x_{\mathrm{cr}}}{\mu} = 5 \times 10^5 \qquad \textbf{(19–11)}$$

The actual value of the engineering critical Reynolds number for a flat plate may vary somewhat from 10^5 to 3×10^6, depending on the surface roughness, the turbulence level, and the variation of pressure along the surface.

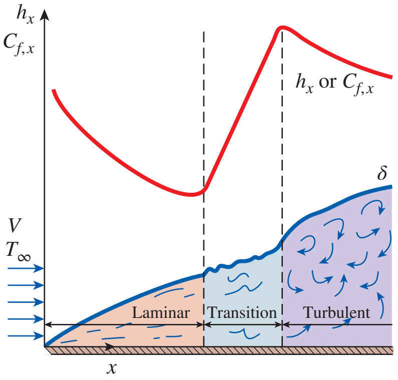


FIGURE 19–9
The variation of the local friction and heat transfer coefficients for flow over a flat plate.

The local Nusselt number at a location x for laminar flow over a flat plate may be obtained by solving the differential energy equation to be

$$\textit{Laminar:} \quad \mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.6, \quad \mathrm{Re}_x < 5 \times 10^5 \qquad \textbf{(19–12)}$$

From experiments, the corresponding relation for turbulent flow is

$$\textit{Turbulent:} \quad \mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \end{array} \qquad \textbf{(19–13)}$$

Note that h_x is proportional to $\mathrm{Re}_x^{0.5}$ and thus to $x^{-0.5}$ for laminar flow. Therefore, h_x is *infinite* at the leading edge ($x = 0$) and decreases by a factor of $x^{-0.5}$ in the flow direction. The variation of the boundary layer thickness δ and the friction and heat transfer coefficients along an isothermal flat plate are shown in Fig. 19–9. The local friction and heat transfer coefficients are higher in turbulent flow than they are in laminar flow. Also, h_x reaches its highest values when the flow becomes fully turbulent, and then decreases by a factor of $x^{-0.2}$ in the flow direction, as shown in the figure.

The *average* Nusselt number over the entire plate is determined by using the relations for the local convection coefficient above and integrating them over the entire surface. We get

$$\textit{Laminar:} \quad \mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Re}_L < 5 \times 10^5, \quad \mathrm{Pr} > 0.6 \qquad \textbf{(19–14)}$$

$$\textit{Turbulent:} \quad \mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \end{array} \qquad \textbf{(19–15)}$$

The first relation gives the average heat transfer coefficient for the entire plate when the flow is *laminar* over the *entire* plate. Note that the average Nusselt number over the entire plate in the case of laminar flow is twice the value of the local Nusselt number at the end of the plate, $\mathrm{Nu} = 2\mathrm{Nu}_{x=L}$ or $h = 2h_{x=L}$. This is only true for laminar flow and does not carry over to the turbulent flow. The second relation gives the average heat transfer coefficient for the entire plate only when the flow is *turbulent* over the *entire* plate, or when the laminar flow region of the plate is too small relative to the turbulent flow region.

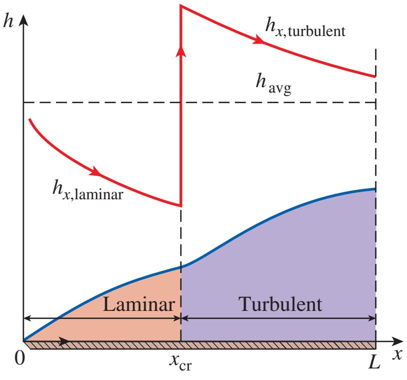


FIGURE 19–10
Graphical representation of the average heat transfer coefficient for a flat plate with combined laminar and turbulent flow.

In some cases, a flat plate is sufficiently long for the flow to become turbulent, but not long enough to disregard the laminar flow region. In such cases, the *average* heat transfer coefficient over the entire plate is determined by performing the integration over two parts as

$$h = \frac{1}{L}\left(\int_0^{x_{\mathrm{cr}}} h_{x,\,\mathrm{laminar}}\,dx + \int_{x_{\mathrm{cr}}}^{L} h_{x,\,\mathrm{turbulent}}\,dx\right) \qquad \textbf{(19–16)}$$

Again taking the critical Reynolds number to be $\mathrm{Re}_{\mathrm{cr}} = 5 \times 10^5$ and performing the integrations in Eq. 19–16 after substituting the indicated expressions, the *average* Nusselt number over the *entire* plate is determined to be (Fig. 19–10)

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \end{array} \qquad \textbf{(19–17)}$$

For a completely turbulent boundary layer ($\mathrm{Re}_{\mathrm{cr}} = 0$) or a very short x_{cr} ($L \gg x_{\mathrm{cr}}$ or $\mathrm{Re}_L \gg \mathrm{Re}_{\mathrm{cr}}$), Eq. 19–17 simplifies to the equation for turbulent flow, Eq. 19–15. The constants in Eq. 19–17 will be different for different critical Reynolds numbers.

Liquid metals such as mercury have high thermal conductivities, and they are commonly used in applications that require high heat transfer rates. Liquid metals have very small Prandtl numbers, and Eq. 19–12 does not apply. However, for this case the thermal boundary layer develops much faster than the velocity boundary layer. Then we can assume the velocity in the thermal boundary layer to be constant at the free stream value and solve the energy equation. It gives

$$\mathrm{Nu}_x = 0.565(\mathrm{Re}_x\,\mathrm{Pr})^{1/2} = 0.565\,\mathrm{Pe}_x^{1/2} \qquad \mathrm{Pr} \le 0.05,\ \mathrm{Pe}_x \ge 100 \tag{19–18}$$

where $\mathrm{Pe}_x = \mathrm{Re}_x\mathrm{Pr}$ is the dimensionless **Peclet number**.

It is desirable to have a single correlation that applies to *all fluids*, including liquid metals. By curve-fitting existing data, Churchill and Ozoe (1973) proposed the following relation for laminar flow over an isothermal flat plate which is applicable for *all Prandtl numbers* and is claimed to be accurate to ±1 percent,

$$\mathrm{Nu}_x = \frac{h_x x}{k} = \frac{0.3387\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}}{[1 + (0.0468/\mathrm{Pr})^{2/3}]^{1/4}} \qquad \mathrm{Re}_x\,\mathrm{Pr} \ge 100 \tag{19–19}$$

with $\mathrm{Nu} = 2\mathrm{Nu}_{x=L}$. For the *uniform heat flux* case, 0.3387 is changed to 0.4637 and 0.0468 is changed to 0.02052. Properties are still evaluated at the film temperature.

These relations have been obtained for the case of *isothermal* surfaces, but they could also be used approximately for the case of nonisothermal surfaces by assuming the surface temperature to be constant at some average value. Also, the surfaces are assumed to be *smooth* and the free stream to be *turbulent free*. The effect of variable properties can be accounted for by evaluating all properties at the film temperature.

Flat Plate with Unheated Starting Length

So far we have limited our consideration to situations for which the entire plate is heated from the leading edge. But many practical applications involve surfaces with an unheated starting section of length ξ, shown in Fig. 19–11, and thus there is no heat transfer for $0 < x < \xi$. In such cases, the velocity boundary layer starts to develop at the leading edge ($x = 0$), but the thermal boundary layer starts to develop where heating starts ($x = \xi$).

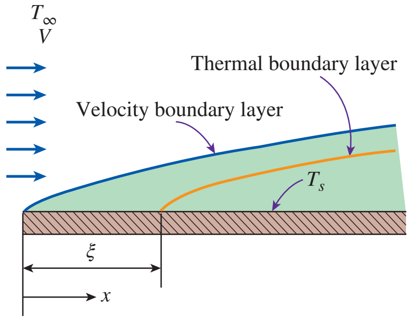


FIGURE 19–11
Flow over a flat plate with an unheated starting length.

Consider a flat plate whose heated section is maintained at a constant temperature ($T = T_s$ constant for $x > \xi$). Using integral solution methods, the local Nusselt numbers for both laminar and turbulent flows are determined to be

Laminar:
$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for }\xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \frac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}} \tag{19–20}$$

Turbulent:
$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for }\xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \frac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}} \tag{19–21}$$

for $x > \xi$. Note that for $\xi = 0$, these Nu_x relations reduce to $\mathrm{Nu}_{x(\text{for }\xi=0)}$, which is the Nusselt number relation for a flat plate without an unheated starting length. Therefore, the terms in brackets in the denominator serve as correction factors for plates with unheated starting lengths.

The determination of the average Nusselt number for the heated section of a plate requires the integration of the local Nusselt number relations above, which cannot be done analytically. Therefore, integrations must be done numerically.

The results of numerical integrations have been correlated for the average Nusselt number (Ameel, 1997) as

Laminar: $$\text{Nu} = \text{Nu}_{(\text{for } \xi=0)} \frac{L}{L-\xi}[1-(\xi/L)^{3/4}]^{2/3} \qquad \textbf{(19–22)}$$

Turbulent: $$\text{Nu} = \text{Nu}_{(\text{for } \xi=0)} \frac{L}{L-\xi}[1-(\xi/L)^{9/10}]^{8/9} \qquad \textbf{(19–23)}$$

The quantity $\text{Nu}_{(\text{for } \xi=0)}$ is the average Nusselt number for a plate of length L when heating starts at the leading edge of the plate. For laminar flow, it can be obtained from Eq. 19–14; for turbulent flow, it is obtained from Eq. 19–15 (assuming turbulent flow over the entire surface). Note that Nu is equal to hL/k, where h is averaged over the heated portion of the plate only, which has the length $(L - \xi)$. To determine the total rate of heat transfer from the plate, the averaged value of h over the heated portion of the plate must be multiplied by the area of the heated section.

Uniform Heat Flux

When a flat plate is subjected to *uniform heat flux* instead of uniform temperature, the local Nusselt number is given by

Laminar: $$\text{Nu}_x = 0.453\,\text{Re}_x^{0.5}\,\text{Pr}^{1/3} \qquad \text{Pr} > 0.6, \qquad \text{Re}_x < 5\times10^5 \qquad \textbf{(19–24)}$$

Turbulent: $$\text{Nu}_x = 0.0308\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3} \qquad 0.6 \le \text{Pr} \le 60, \qquad 5\times10^5 \le \text{Re}_x \le 10^7 \qquad \textbf{(19–25)}$$

These relations give values that are 36 percent higher for laminar flow and 4 percent higher for turbulent flow relative to the isothermal plate case. When the plate involves an unheated starting length, the relations developed for the uniform surface temperature case can still be used provided that Eqs. 19–24 and 19–25 are used for $\text{Nu}_{x(\text{for } \xi=0)}$ in Eqs. 19–20 and 19–21, respectively. To obtain the *average* Nusselt number for a flat plate subjected to *uniform heat flux*, it is sufficiently accurate to use the average Nusselt number equations developed for the case of isothermal surfaces for laminar flow (Eq. 19–14) and turbulent flow (Eq. 19–15).

When heat flux $\dot{q}_s$ is prescribed, the rate of heat transfer to or from the plate and the surface temperature at a distance x are determined from

$$\dot{Q} = \dot{q}_s A_s \qquad \textbf{(19–26)}$$

and

$$\dot{q}_s = h_x[T_s(x) - T_\infty)] \rightarrow T_s(x) = T_\infty + \frac{\dot{q}_s}{h_x} \qquad \textbf{(19–27)}$$

where A_s is the heat transfer surface area.

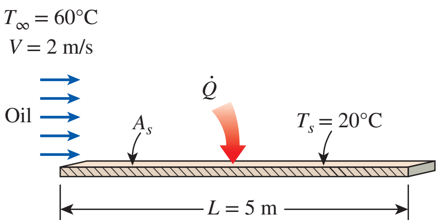


FIGURE 19–12
Schematic for Example 19–1.

EXAMPLE 19–1 Flow of Hot Oil Over a Flat Plate

Engine oil at 60°C flows over the upper surface of a 5-m-long flat plate whose temperature is 20°C with a velocity of 2 m/s (Fig. 19–12). Determine the total drag force and the rate of heat transfer per unit width of the entire plate.

SOLUTION Engine oil flows over a flat plate. The total drag force and the rate of heat transfer per unit width of the plate are to be determined.
Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\text{Re}_{\text{cr}} = 5 \times 10^5$.

Properties The properties of engine oil at the film temperature of $T_f = (T_s + T_\infty)/2 = (20 + 60)/2 = 40°\text{C}$ are (Table A–19)

$$\rho = 876\ \text{kg/m}^3 \qquad \text{Pr} = 2962$$
$$k = 0.1444\ \text{W/m·K} \qquad \nu = 2.485 \times 10^{-4}\ \text{m}^2\text{/s}$$

Analysis Noting that $L = 5$ m, the Reynolds number at the end of the plate is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(2\ \text{m/s})(5\ \text{m})}{2.485 \times 10^{-4}\ \text{m}^2\text{/s}} = 4.024 \times 10^4$$

which is less than the critical Reynolds number. Thus we have *laminar flow* over the entire plate, and the average friction coefficient is

$$C_f = 1.33\ \text{Re}_L^{-0.5} = 1.33 \times (4.024 \times 10^4)^{-0.5} = 0.00663$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for parallel flow over a flat plate, the drag force acting on the plate per unit width becomes

$$F_D = C_f A \frac{\rho V^2}{2} = 0.00663(5 \times 1\ \text{m}^2)\frac{(876\ \text{kg/m}^3)(2\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg·m/s}^2}\right) = \mathbf{58.1\ N}$$

The total drag force acting on the entire plate can be determined by multiplying the value obtained above by the width of the plate.

This force per unit width corresponds to the weight of a mass of about 6 kg. Therefore, a person who applies an equal and opposite force to the plate to keep it from moving will feel like he or she is using as much force as is necessary to hold a 6-kg mass from dropping.

Similarly, the Nusselt number is determined using the laminar flow relations for a flat plate,

$$\text{Nu} = \frac{hL}{k} = 0.664\ \text{Re}_L^{0.5}\ \text{Pr}^{1/3} = 0.664 \times (4.024 \times 10^4)^{0.5} \times 2962^{1/3} = 1913$$

Then,

$$h = \frac{k}{L}\text{Nu} = \frac{0.144\ \text{W/m·K}}{5\ \text{m}}(1913) = 55.25\ \text{W/m}^2\text{·K}$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (55.25\ \text{W/m}^2\text{·K})(5 \times 1\ \text{m}^2)(60 - 20)°\text{C} = \mathbf{11{,}050\ W}$$

Discussion Note that heat transfer is always from the higher-temperature medium to the lower-temperature one. In this case, it is from the oil to the plate. The heat transfer rate is per meter width of the plate. The heat transfer for the entire plate can be obtained by multiplying the value obtained by the actual width of the plate.

EXAMPLE 19–2 Cooling of Plastic Sheets by Forced Air

The forming section of a plastics plant puts out a continuous sheet of plastic that is 4 ft wide and 0.04 in thick at a velocity of 30 ft/min. The temperature of the plastic sheet is 200°F when it is exposed to the surrounding air, and a 2-ft-long section of the plastic sheet is subjected to airflow at 80°F at a velocity of 10 ft/s on both sides along its surfaces normal to the direction of motion of the sheet, as shown in Fig. 19–13. Determine (*a*) the rate of heat transfer from the plastic sheet to air by forced convection and radiation and (*b*) the temperature of the plastic sheet at the end of the cooling section. Take the density, specific heat, and emissivity of the plastic sheet to be $\rho = 75\ \text{lbm/ft}^3$, $c_p = 0.4\ \text{Btu/lbm·°F}$, and $\varepsilon = 0.9$.

SOLUTION Plastic sheets are cooled as they leave the forming section of a plastics plant. The rate of heat loss from the plastic sheet by convection and radiation and the exit temperature of the plastic sheet are to be determined.

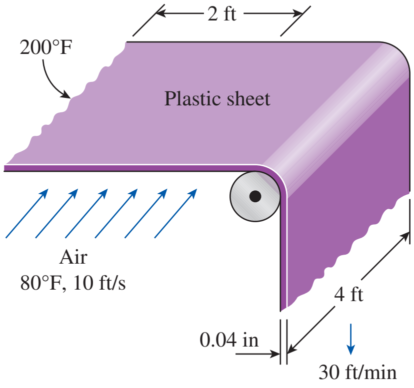


FIGURE 19–13
Schematic for Example 19–2.

Assumptions 1 Steady operating conditions exist. 2 The critical Reynolds number is $\text{Re}_{cr} = 5 \times 10^5$. 3 Air is an ideal gas. 4 The local atmospheric pressure is 1 atm. 5 The surrounding surfaces are at the temperature of the room air.

Properties The properties of the plastic sheet are given in the problem statement. The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (200 + 80)/2 = 140°\text{F}$ and 1 atm pressure are (Table A–22E)

$$k = 0.01623 \text{ Btu/h·ft·°F} \qquad \text{Pr} = 0.7202$$
$$\nu = 0.204 \times 10^{-3} \text{ ft}^2\text{/s}$$

Analysis (*a*) We expect the temperature of the plastic sheet to drop somewhat as it flows through the 2-ft-long cooling section, but at this point we do not know the magnitude of that drop. Therefore, we assume the plastic sheet to be isothermal at 200°F to get started. We will repeat the calculations if necessary to account for the temperature drop of the plastic sheet.

Noting that $L = 4$ ft, the Reynolds number at the end of the airflow across the plastic sheet is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(10 \text{ ft/s})(4 \text{ ft})}{0.204 \times 10^{-3} \text{ ft}^2\text{/s}} = 1.961 \times 10^5$$

which is less than the critical Reynolds number. Thus, we have *laminar flow* over the entire sheet, and the Nusselt number is determined from the laminar flow relations for a flat plate to be

$$\text{Nu} = \frac{hL}{k} = 0.664 \text{ Re}_L^{0.5} \text{ Pr}^{1/3} = 0.664 \times (1.961 \times 10^5)^{0.5} \times (0.7202)^{1/3} = 263.6$$

Then,

$$h = \frac{k}{L}\text{Nu} = \frac{0.01623 \text{ Btu/h·ft·°F}}{4 \text{ ft}}(263.6) = 1.07 \text{ Btu/h·ft}^2\text{·°F}$$
$$A_s = (2 \text{ ft})(4 \text{ ft})(2 \text{ sides}) = 16 \text{ ft}^2$$

and

$$\begin{aligned}\dot{Q}_{\text{conv}} &= hA_s(T_s - T_\infty)\\ &= (1.07 \text{ Btu/h·ft}^2\text{·°F})(16 \text{ ft}^2)(200 - 80)\text{°F}\\ &= 2054 \text{ Btu/h}\\ \dot{Q}_{\text{rad}} &= \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4)\\ &= (0.9)(0.1714 \times 10^{-8} \text{ Btu/h·ft}^2\text{·R}^4)(16 \text{ ft}^2)[(660 \text{ R})^4 - (540 \text{ R})^4]\\ &= 2585 \text{ Btu/h}\end{aligned}$$

Therefore, the rate of cooling of the plastic sheet by combined convection and radiation is

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 2054 + 2585 = \mathbf{4639 \text{ Btu/h}}$$

(*b*) To find the temperature of the plastic sheet at the end of the cooling section, we need to know the mass of the plastic rolling out per unit time (or the mass flow rate), which is determined from

$$\dot{m} = \rho A_c V_{\text{plastic}} = (75 \text{ lbm/ft}^3)\left(\frac{4 \times 0.04}{12}\text{ft}^2\right)\left(\frac{30}{60}\text{ft/s}\right) = 0.5 \text{ lbm/s}$$

Then, an energy balance on the cooled section of the plastic sheet yields

$$\dot{Q} = \dot{m}c_p(T_2 - T_1) \rightarrow T_2 = T_1 + \frac{\dot{Q}}{\dot{m}c_p}$$

Noting that $\dot{Q}$ is a negative quantity (heat loss) for the plastic sheet and substituting, the temperature of the plastic sheet as it leaves the cooling section is determined to be

$$T_2 = 200\text{°F} + \frac{-4639 \text{ Btu/h}}{(0.5 \text{ lbm/s})(0.4 \text{ Btu/lbm·°F})}\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = \mathbf{193.6°F}$$

Discussion The average temperature of the plastic sheet drops by about 6.4°F as it passes through the cooling section. The calculations now can be repeated by taking the average temperature of the plastic sheet to be 196.8°F instead of 200°F for better accuracy, but the change in the results will be insignificant because of the small change in temperature.

19–4 ▪ FLOW ACROSS CYLINDERS AND SPHERES

Flows across cylinders and spheres, in general, involve *flow separation,* which is difficult to handle analytically. Therefore, such flows must be studied experimentally or numerically. Indeed, flow across cylinders and spheres has been studied experimentally by many investigators, and several empirical correlations have been developed for the heat transfer coefficient.

The complicated flow pattern across a cylinder greatly influences heat transfer. The variation of the local Nusselt number Nu_θ around the periphery of a cylinder subjected to crossflow of air is given in Fig. 19–14. Note that, for all cases, the value of Nu_θ starts out relatively high at the stagnation point ($\theta = 0°$) but decreases with increasing θ as a result of the thickening of the laminar boundary layer. On the two curves at the bottom corresponding to Re = 70,800 and 101,300, Nu_θ reaches a minimum at $\theta \approx 80°$, which is the separation point in laminar flow. Then Nu_θ increases with increasing θ as a result of the intense mixing in the separated flow region (the wake). The curves at the top corresponding to Re = 140,000 to 219,000 differ from the first two curves in that they have *two* minima for Nu_θ. The sharp increase in Nu_θ at about $\theta \approx 90°$ is due to the transition from laminar to turbulent flow. The later decrease in Nu_θ is again due to the thickening of the boundary layer. Nu_θ reaches its second minimum at about $\theta \approx 140°$, which is the flow separation point in turbulent flow, and increases with θ as a result of the intense mixing in the turbulent wake region.

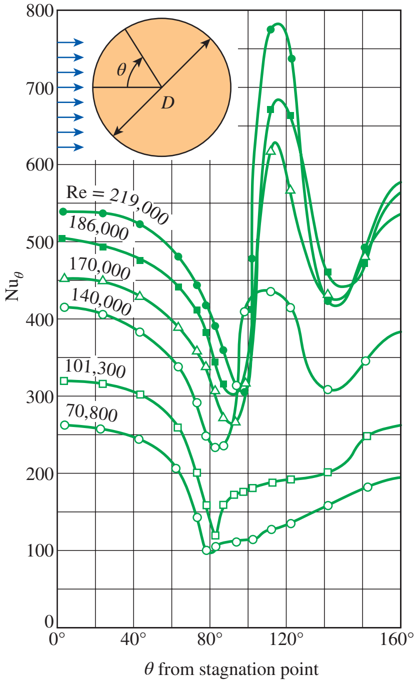


FIGURE 19–14

Variation of the local heat transfer coefficient along the circumference of a circular cylinder in crossflow of air.

Source: Giedt, 1949.

The preceding discussions on the local heat transfer coefficients are insightful; however, they are of limited value in heat transfer calculations since the calculation of heat transfer requires the *average* heat transfer coefficient over the entire surface. Of the several such relations available in the literature for the average Nusselt number for crossflow over a cylinder, we present the one proposed by Churchill and Bernstein (1977):

$$\text{Nu}_{\text{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{1/2}\,\text{Pr}^{1/3}}{[1 + (0.4/\text{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5} \qquad \textbf{(19–28)}$$

This relation is quite comprehensive in that it correlates available data well for RePr > 0.2. The fluid properties are evaluated at the *film temperature* $T_f = \frac{1}{2}(T_\infty + T_s)$, which is the average of the free-stream and surface temperatures.

For flow over a *sphere,* Whitaker (1972) recommends the following comprehensive correlation:

$$\text{Nu}_{\text{sph}} = \frac{hD}{k} = 2 + [0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \qquad \textbf{(19–29)}$$

which is valid for $3.5 \le \text{Re} \le 8 \times 10^4$, $0.7 \le \text{Pr} \le 380$, and $1.0 \le (\mu_\infty/\mu_s) \le 3.2$. The fluid properties in this case are evaluated at the free-stream temperature T_∞, except for μ_s, which is evaluated at the surface temperature T_s. Although the two relations above are considered to be quite accurate, the results obtained from them can be off by as much as 30 percent.

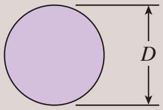
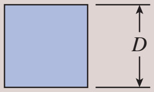
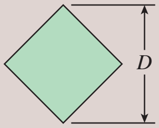
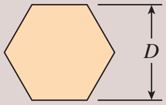
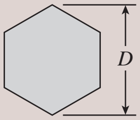
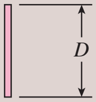
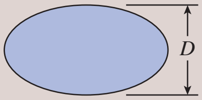
The average Nusselt number for flow across cylinders can be expressed compactly as

$$\text{Nu}_{\text{cyl}} = \frac{hD}{k} = C\,\text{Re}^m\,\text{Pr}^n \qquad \textbf{(19–30)}$$

where $n = \frac{1}{3}$ and the experimentally determined constants C and m are given in Table 19–2 for circular as well as various noncircular cylinders. The characteristic length D for use in the calculation of the Reynolds and the Nusselt numbers for different geometries is as indicated on each figure in Table 19–2. All fluid properties are evaluated at the film temperature. Note that the values presented in Table 19–2 for noncircular geometries have been updated based on the recommendations of Sparrow et al. (2004). Correlations for square and chamfered cylinders for all possible directions of incoming fluid are available in Tong et al. (2016).

TABLE 19–2

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in crossflow

Cross section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$\mathrm{Nu} = 0.989\,\mathrm{Re}^{0.330}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.911\,\mathrm{Re}^{0.385}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.683\,\mathrm{Re}^{0.466}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.193\,\mathrm{Re}^{0.618}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.027\,\mathrm{Re}^{0.805}\,\mathrm{Pr}^{1/3}$
Square 	Gas	3900–79,000 32,000–1,600,000	$\mathrm{Nu} = 0.094\,\mathrm{Re}^{0.675}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.0249\,\mathrm{Re}^{0.811}\,\mathrm{Pr}^{1/3}$
Square (tilted 45°) 	Gas	5600–111,000 46,000–2,300,000	$\mathrm{Nu} = 0.258\,\mathrm{Re}^{0.588}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.0260\,\mathrm{Re}^{0.839}\,\mathrm{Pr}^{1/3}$
Hexagon 	Gas	4500–90,700	$\mathrm{Nu} = 0.148\,\mathrm{Re}^{0.638}\,\mathrm{Pr}^{1/3}$
Hexagon (tilted 45°) 	Gas	5200–20,400 20,400–105,000	$\mathrm{Nu} = 0.162\,\mathrm{Re}^{0.638}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.039\,\mathrm{Re}^{0.782}\,\mathrm{Pr}^{1/3}$
Vertical plate 	Gas	6300–23,600	$\mathrm{Nu} = 0.257\,\mathrm{Re}^{0.731}\,\mathrm{Pr}^{1/3}$
Ellipse 	Gas	1400–8200	$\mathrm{Nu} = 0.197\,\mathrm{Re}^{0.612}\,\mathrm{Pr}^{1/3}$

Source: Zukauskas, 1972, Jakob, 1949, Sparrow et al., 2004, and Tong et al., 2016.

The relations for cylinders presented here are for *single* cylinders or cylinders oriented such that the flow over them is not affected by the presence of others. Also, they are applicable to *smooth* surfaces. *Surface roughness* and the *free-stream turbulence* may affect the drag and heat transfer coefficients significantly. Equation 19–30 provides a simpler alternative to Eq. 19–28 for flow over cylinders. However, Eq. 19–28 is more accurate, and thus it should be preferred in calculations whenever possible.

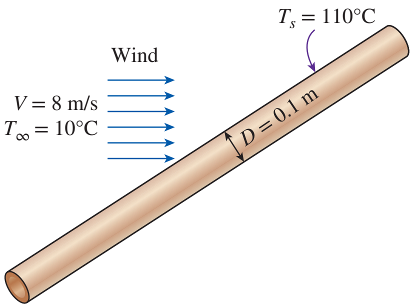


FIGURE 19–15
Schematic for Example 19–3.

EXAMPLE 19–3 Heat Loss from a Steam Pipe in Windy Air

A long 10-cm-diameter steam pipe whose external surface temperature is 110°C passes through some open area that is not protected against the winds (Fig. 19–15). Determine the rate of heat loss from the pipe per unit of its length when the air is at 1 atm pressure and 10°C and the wind is blowing across the pipe at a velocity of 8 m/s.

SOLUTION A steam pipe is exposed to windy air. The rate of heat loss from the steam is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas.

Properties The properties of air at the average film temperature of $T_f = (T_s + T_\infty)/2 = (110 + 10)/2 = 60°\text{C}$ and 1 atm pressure are (Table A–22)

$$k = 0.02808 \text{ W/m·K} \qquad \text{Pr} = 0.7202 \qquad \nu = 1.896 \times 10^{-5} \text{ m}^2\text{/s}$$

Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(8 \text{ m/s})(0.1 \text{ m})}{1.896 \times 10^{-5} \text{ m}^2\text{/s}} = 4.219 \times 10^4$$

The Nusselt number can be determined from

$$\begin{aligned}\text{Nu} = \frac{hD}{k} &= 0.3 + \frac{0.62 \text{ Re}^{1/2} \text{ Pr}^{1/3}}{[1 + (0.4/\text{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5} \\ &= 0.3 + \frac{0.62(4.219 \times 10^4)^{1/2}(0.7202)^{1/3}}{[1 + (0.4/0.7202)^{2/3}]^{1/4}}\left[1 + \left(\frac{4.219 \times 10^4}{282{,}000}\right)^{5/8}\right]^{4/5} \\ &= 124\end{aligned}$$

and

$$h = \frac{k}{D}\text{Nu} = \frac{0.02808 \text{ W/m·K}}{0.1 \text{ m}}(124) = 34.8 \text{ W/m}^2\text{·K}$$

Then, the rate of heat transfer from the pipe per unit of its length becomes

$$A_s = pL = \pi DL = \pi(0.1 \text{ m})(1 \text{ m}) = 0.314 \text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (34.8 \text{ W/m}^2\text{·K})(0.314 \text{ m}^2)(110 - 10)°\text{C} = \mathbf{1093 \text{ W}}$$

The rate of heat loss from the entire pipe can be obtained by multiplying the value above by the length of the pipe in m.

Discussion The simpler Nusselt number relation in Table 19–2 in this case would give Nu = 128, which is 3 percent higher than the value obtained above using Eq. 19–28.

EXAMPLE 19–4 Cooling of a Steel Ball by Forced Air

A 25-cm-diameter stainless steel ball ($\rho = 8055$ kg/m^3, $c_p = 480$ J/kg·K) is removed from the oven at a uniform temperature of 300°C (Fig. 19–16). The ball is then subjected to the flow of air at 1 atm pressure and 25°C with a velocity of 3 m/s. The surface temperature of the ball eventually drops to 200°C. Determine the average convection heat transfer coefficient during this cooling process, and estimate how long the process will take.

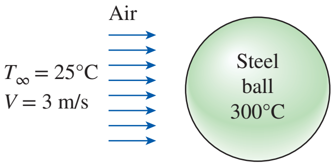


FIGURE 19–16
Schematic for Example 19–4.

SOLUTION A hot stainless steel ball is cooled by forced air. The average convection heat transfer coefficient and the cooling time are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas. **4** The outer surface temperature of the ball is uniform at all times. **5** The surface temperature of the ball during cooling is changing. Therefore, the convection heat transfer coefficient between the ball and the air will also change. To avoid this complexity, we take the surface temperature of the ball to be constant at the average temperature of (300 + 200)/2 = 250°C in the evaluation of the heat transfer coefficient and use the value obtained for the entire cooling process.

Properties The dynamic viscosity of air at the average surface temperature is $\mu_s = \mu_{@\,250°\text{C}} = 2.76 \times 10^{-5}$ kg/m·s. The properties of air at the free-stream temperature of 25°C and 1 atm are (Table A–22)

$$\begin{aligned} k &= 0.02551 \text{ W/m·K} & \nu &= 1.562 \times 10^{-5} \text{ m}^2\text{/s} \\ \mu &= 1.849 \times 10^{-5} \text{ kg/m·s} & \text{Pr} &= 0.7296 \end{aligned}$$

Analysis The Reynolds number is determined from

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(3\ \mathrm{m/s})\,(0.25\ \mathrm{m})}{1.562 \times 10^{-5}\ \mathrm{m^2/s}} = 4.802 \times 10^4$$

The Nusselt number is

$$\begin{aligned}\mathrm{Nu} = \frac{hD}{k} &= 2 + [0.4\ \mathrm{Re}^{1/2} + 0.06\ \mathrm{Re}^{2/3}]\ \mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \\ &= 2 + [0.4\,(4.802 \times 10^4)^{1/2} + 0.06\,(4.802 \times 10^4)^{2/3}](0.7296)^{0.4} \\ &\quad \times \left(\frac{1.849 \times 10^{-5}}{2.76 \times 10^{-5}}\right)^{1/4} \\ &= 135\end{aligned}$$

Then the average convection heat transfer coefficient becomes

$$h = \frac{k}{D}\mathrm{Nu} = \frac{0.02551\ \mathrm{W/m{\cdot}K}}{0.25\ \mathrm{m}}(135) = \mathbf{13.8\ W/m^2{\cdot}K}$$

In order to estimate the time of cooling of the ball from 300°C to 200°C, we determine the *average* rate of heat transfer from Newton's law of cooling by using the *average* surface temperature. That is,

$$A_s = \pi D^2 = \pi(0.25\ \mathrm{m})^2 = 0.1963\ \mathrm{m^2}$$

$$\dot{Q}_{\mathrm{avg}} = hA_s(T_{s,\,\mathrm{avg}} - T_\infty) = (13.8\ \mathrm{W/m^2{\cdot}K})(0.1963\ \mathrm{m^2})(250 - 25)^\circ\mathrm{C} = 610\ \mathrm{W}$$

Next we determine the *total* heat transferred from the ball, which is simply the change in the energy of the ball as it cools from 300°C to 200°C:

$$m = \rho V = \rho\tfrac{1}{6}\pi D^3 = (8055\ \mathrm{kg/m^3})\,\tfrac{1}{6}\pi(0.25\ \mathrm{m})^3 = 65.9\ \mathrm{kg}$$

$$Q_{\mathrm{total}} = mc_p(T_2 - T_1) = (65.9\ \mathrm{kg})(480\ \mathrm{J/kg{\cdot}K})(300 - 200)^\circ\mathrm{C} = 3{,}163{,}000\ \mathrm{J}$$

In this calculation, we assumed that the entire ball is at 200°C, which is not necessarily true. The inner region of the ball will probably be at a higher temperature than its surface. With this assumption, the time of cooling is determined to be

$$\Delta t \approx \frac{Q}{\dot{Q}_{\mathrm{avg}}} = \frac{3{,}163{,}000\ \mathrm{J}}{610\ \mathrm{J/s}} = 5185\ \mathrm{s} = \mathbf{1\ h\ 26\ min}$$

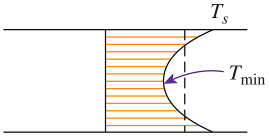
Discussion The time of cooling could also be determined more accurately using the relations introduced in Chap. 18. But the simplifying assumptions we made above can be justified if all we need is a ballpark value. It will be naive to expect the time of cooling to be exactly 1 h 26 min, but, using our engineering judgment, it is realistic to expect the time of cooling to be somewhere between one and two hours.

19–5 ▪ GENERAL CONSIDERATIONS FOR PIPE FLOW

Liquid or gas flow through pipes or ducts is commonly used in practice in heating and cooling applications. The fluid in such applications is forced to flow by a fan or pump through a conduit that is sufficiently long to accomplish the desired heat transfer.

When a fluid is heated or cooled as it flows through a tube, the temperature of the fluid at any cross section changes from T_s at the surface of the wall to some maximum (or minimum in the case of heating) at the tube center. In fluid flow, it is convenient to work with an **average** or **mean temperature** T_m, which remains constant at a cross section. Unlike the mean velocity, the mean temperature T_m *changes* in the flow direction whenever the fluid is heated or cooled.

The value of the mean temperature T_m is determined from the requirement that the *conservation of energy* principle be satisfied. That is, the energy transported by the fluid through a cross section in actual flow must be equal to the energy that would be transported through the same cross section if the fluid were at a constant temperature T_m. This can be expressed mathematically as (Fig. 19–17)



(*a*) Actual

(*b*) Idealized

FIGURE 19–17
Actual and idealized temperature profiles for flow in a tube (the rate at which energy is transported with the fluid is the same for both cases).

$$\dot{E}_{\text{fluid}} = \dot{m}c_pT_m = \int_{\dot{m}} c_pT(r)\delta\dot{m} = \int_{A_c} \rho c_pT(r)u(r)\,dA_c \tag{19–31}$$

where c_p is the specific heat of the fluid. Note that the product $\dot{m}c_pT_m$ at any cross section along the tube represents the *energy flow* with the fluid at that cross section. Then, the mean temperature of a fluid with constant density and specific heat flowing in a circular pipe of radius R can be expressed as

$$T_m = \frac{\int_{\dot{m}} c_pT(r)\delta\dot{m}}{\dot{m}c_p} = \frac{\int_0^R c_pT(r)\rho u(r)2\pi r\,dr}{\rho V_{\text{avg}}(\pi R^2)c_p} = \frac{2}{V_{\text{avg}}R^2}\int_0^R T(r)u(r)r\,dr \tag{19–32}$$

Note that the mean temperature T_m of a fluid changes during heating or cooling. Also, the fluid properties in internal flow are usually evaluated at the *bulk mean fluid temperature,* which is the arithmetic average of the mean temperatures at the inlet and the exit. That is, $T_b = (T_i + T_e)/2$.

The general aspects of flow in pipes were considered in Chap. 14. The Reynolds number for flow through a pipe of inside diameter D was defined as

$$\text{Re} = \frac{V_{\text{avg}}D}{\nu} = \frac{\rho V_{\text{avg}}D}{\mu} = \frac{\rho D}{\mu}\left(\frac{\dot{m}}{\rho\pi D^2/4}\right) = \frac{4\dot{m}}{\mu\pi D} \tag{19–33}$$

where V_{avg} is the average flow velocity, D is the diameter of the tube, and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid.

It certainly is desirable to have precise values of Reynolds numbers for laminar, transitional, and turbulent flows, but this is not the case in practice. This is because the transition from laminar to turbulent flow also depends on the degree of disturbance of the flow by *surface roughness, pipe vibrations,* and the *fluctuations in the flow.* Under most practical conditions, the flow in a tube is laminar for $\text{Re} < 2300$, fully turbulent for $\text{Re} > 10{,}000$, and transitional in between. But it should be kept in mind that in many cases the flow becomes fully turbulent for $\text{Re} > 4000$. When designing piping networks and determining pumping power, a conservative approach is taken, and flows with $\text{Re} > 4000$ are assumed to be turbulent.

Thermal Entrance Region

The development of the velocity boundary layer was discussed in Chap. 14 (Fig. 19–18).

Now consider a fluid at a uniform temperature entering a circular tube whose surface is maintained at a different temperature. This time, the fluid particles in the layer in contact with the surface of the tube assume the surface temperature. This initiates convection heat transfer in the tube and the development of a *thermal boundary layer* along the tube. The thickness of this boundary layer also increases

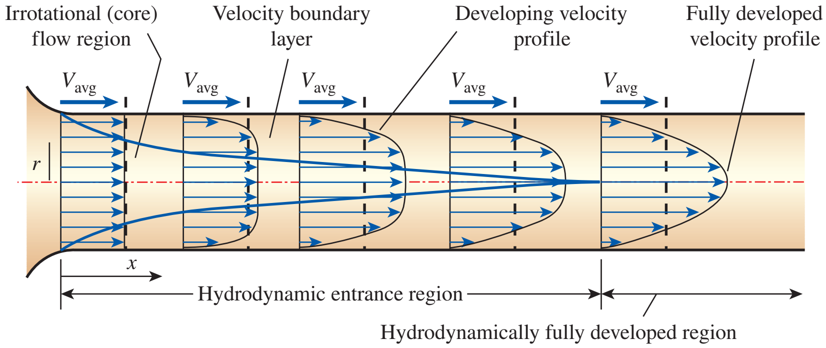


FIGURE 19–18
The development of the velocity boundary layer in a pipe. (The developed average velocity profile is parabolic in laminar flow, as shown, but much flatter or fuller in turbulent flow.)

in the flow direction until the boundary layer reaches the tube center and thus fills the entire tube, as shown in Fig. 19–19.

The region of flow over which the thermal boundary layer develops and reaches the tube center is called the **thermal entrance region**, and the length of this region is called the **thermal entry length** L_t. Flow in the thermal entrance region is called *thermally developing flow* since this is the region where the temperature profile develops. The region beyond the thermal entrance region in which the dimensionless temperature profile expressed as $(T_s - T)/(T_s - T_m)$ remains unchanged is called the **thermally fully developed region**. The region in which the flow is both hydrodynamically and thermally developed and thus both the velocity and dimensionless temperature profiles remain unchanged is called *fully developed flow.* That is,

Hydrodynamically fully developed:
$$\frac{\partial u(r, x)}{\partial x} = 0 \longrightarrow u = u(r) \tag{19–34}$$

Thermally fully developed:
$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)}\right] = 0 \tag{19–35}$$

The shear stress at the tube wall τ_w is related to the slope of the velocity profile at the surface. Noting that the velocity profile remains unchanged in the hydrodynamically fully developed region, the wall shear stress also remains constant in that region. A similar argument can be given for the heat transfer coefficient in the thermally fully developed region.

In a thermally fully developed region, the derivative of $(T_s - T)/(T_s - T_m)$ with respect to x is zero by definition, and thus $(T_s - T)/(T_s - T_m)$ is independent of x. Then, the derivative of $(T_s - T)/(T_s - T_m)$ with respect to r must also be independent of x. That is,

$$\left.\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right)\right|_{r=R} = \frac{-(\partial T/\partial r)|_{r=R}}{T_s - T_m} \neq f(x) \tag{19–36}$$

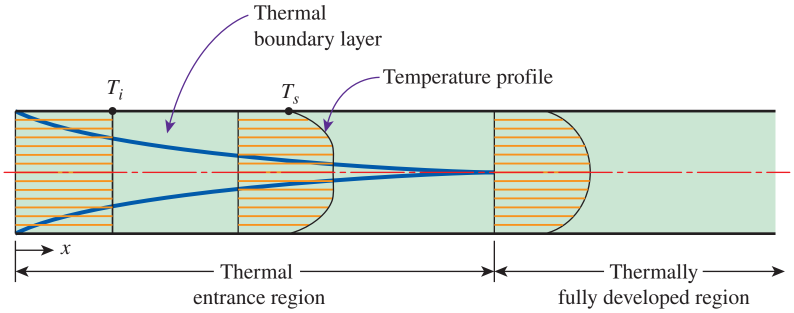


FIGURE 19–19
The development of the thermal boundary layer in a tube. (The fluid in the tube is being cooled.)

Surface heat flux can be expressed as

$$\dot{q}_s = h_x(T_s - T_m) = k\frac{\partial T}{\partial r}\bigg|_{r=R} \longrightarrow h_x = \frac{k(\partial T/\partial r)|_{r=R}}{T_s - T_m} \tag{19–37}$$

which, from Eq. 19–36, is independent of x. Thus, we conclude that *in the thermally fully developed region of a tube, the local convection coefficient is constant* (does not vary with x). Therefore, both *the local friction factor* f_x (*which is related to the local wall shear stress*) and the local convection coefficient h_x remain constant in the hydrodynamically and thermally fully developed regions, respectively, as shown in Fig. 19–20 for $\text{Pr} > 1$.

Note that the *temperature profile* in the thermally fully developed region may vary with x in the flow direction. That is, unlike the velocity profile, the temperature profile can be different at different cross sections of the tube in the developed region, and it usually is. However, the dimensionless temperature profile defined previously remains unchanged in the thermally developed region when the temperature or heat flux at the tube surface remains constant.

During laminar flow in a tube, the magnitude of the dimensionless Prandtl number Pr is a measure of the relative growth of the velocity and thermal boundary layers. For fluids with $\text{Pr} \approx 1$, such as gases, the two boundary layers essentially coincide with each other. For fluids with $\text{Pr} \gg 1$, such as oils, the velocity boundary layer outgrows the thermal boundary layer. As a result, the hydrodynamic entry length is smaller than the thermal entry length. The opposite is true for fluids with $\text{Pr} \ll 1$ such as liquid metals.

Consider a fluid that is being heated (or cooled) in a tube as it flows through it. The wall shear stress and the heat transfer coefficient are *highest* at the tube inlet where the thickness of the boundary layers is smallest, and they decrease gradually to the fully developed values, as shown in Fig. 19–20. Therefore, the pressure drop and heat flux are *higher* in the entrance regions of a tube, and the effect of the entrance region is always to *increase* the average friction factor and heat transfer coefficient for the entire tube. This enhancement can be significant for short tubes but negligible for long ones.

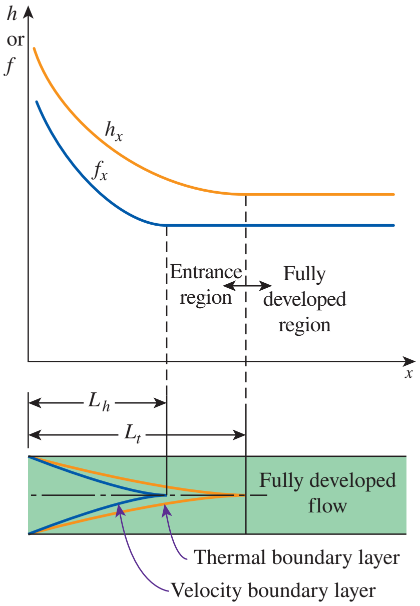


FIGURE 19–20
Variation of the friction factor and the convection heat transfer coefficient in the flow direction for flow in a tube ($\text{Pr} > 1$).

Entry Lengths

The hydrodynamic entry length is usually taken to be the distance from the tube entrance where the wall shear stress (and thus the friction factor) reaches within about 2 percent of the fully developed value. In *laminar flow,* the hydrodynamic and thermal entry lengths are given approximately as [see Kays and Crawford (1993) and Shah and Bhatti (1987)]

$$L_{h,\,\text{laminar}} \approx 0.05\ \text{Re}\ D \tag{19–38}$$

$$L_{t,\,\text{laminar}} \approx 0.05\ \text{Re}\ \text{Pr}\ D = \text{Pr}\ L_{h,\,\text{laminar}} \tag{19–39}$$

For $\text{Re} = 20$, the hydrodynamic entry length is about the size of the diameter, but it increases linearly with velocity. In the limiting case of $\text{Re} = 2300$, the hydrodynamic entry length is $115D$. When the flow is simultaneously hydrodynamically and thermally developing, a longer thermal entrance length is required than for hydrodynamically fully developed flow. Meyer and Everts (2018) suggest replacing the coefficient of 0.05 used in Eq. 19–39 with 0.12 for simultaneously hydrodynamically and thermally developing forced convection laminar flow.

In *turbulent flow,* the intense mixing during random fluctuations usually overshadows the effects of molecular diffusion, and therefore the hydrodynamic and thermal entry lengths are about the same size and are independent of the Prandtl number.

The entry length is much shorter in turbulent flow, as expected, and its dependence on the Reynolds number is weaker. In many tube flows of practical interest, the entrance effects become insignificant beyond a tube length of 10 diameters, and the hydrodynamic and thermal entry lengths are approximately taken to be

$$L_{h,\text{ turbulent}} \approx L_{t,\text{ turbulent}} \approx 10D \tag{19–40}$$

The variation of local Nusselt number along a tube in turbulent flow for both constant surface temperature and constant surface heat flux is given in Fig. 19–21 for the range of Reynolds numbers encountered in heat transfer equipment. We make these important observations from this figure:

- The Nusselt numbers and thus the convection heat transfer coefficients are much higher in the entrance region.
- The Nusselt number reaches a constant value at a distance of less than 10 diameters, and thus the flow can be assumed to be fully developed for $x > 10D$.
- The Nusselt numbers for the constant surface temperature and constant surface heat flux conditions are identical in the fully developed regions and nearly identical in the entrance regions. Therefore, Nusselt number is insensitive to the type of thermal boundary condition, and the turbulent flow correlations can be used for either type of boundary condition.

Precise correlations for the friction and heat transfer coefficients for the entrance regions are available in the literature. However, the tubes used in practice in forced convection are usually several times the length of either entrance region, and thus the flow through the tubes is often assumed to be fully developed for the entire length of the tube. This simplistic approach gives *reasonable* results for the rate of heat transfer for long tubes and *conservative* results for short ones.

It should be noted that the preceding observations are only valid for turbulent flow. In laminar flow, Nusselt number values are much lower than turbulent flow values, the distance for the Nusselt number to reach a constant value is typically much longer, and the flow is sensitive to the thermal boundary conditions imposed on the flow.

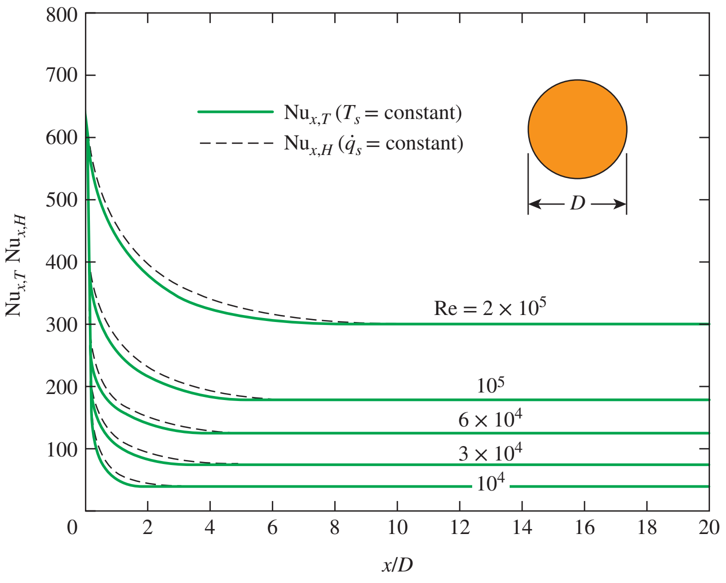


FIGURE 19–21
Variation of local Nusselt number along a tube in turbulent flow for both constant surface temperature and constant surface heat flux

Source: Deissler, 1953.

19–6 ▪ GENERAL THERMAL ANALYSIS

In the absence of any work interactions (such as electric resistance heating), the conservation of energy equation for the steady flow of a fluid in a tube can be expressed as (Fig. 19–22)

$$\dot{Q} = \dot{m}c_p(T_e - T_i) \quad \text{(W)} \tag{19–41}$$

where T_i and T_e are the mean fluid temperatures at the inlet and exit of the tube, respectively, and $\dot{Q}$ is the rate of heat transfer to or from the fluid. Note that the temperature of a fluid flowing in a tube remains constant in the absence of any energy interactions through the wall of the tube.

The thermal conditions at the surface can usually be approximated with reasonable accuracy to be *constant surface temperature* (T_s = constant) or *constant surface heat flux* ($\dot{q}_s$ = constant). For example, the constant surface temperature condition is realized when a phase change process such as boiling or condensation occurs at the outer surface of a tube. The constant surface heat flux condition is realized when the tube is subjected to radiation or electric resistance heating uniformly from all directions.

Surface heat flux is expressed as

$$\dot{q}_s = h_x(T_s - T_m) \qquad \text{(W/m}^2\text{)} \tag{19–42}$$

where h_x is the *local* heat transfer coefficient and T_s and T_m are the surface and the mean fluid temperatures at that location. Note that the mean fluid temperature T_m of a fluid flowing in a tube must change during heating or cooling. Therefore, when $h_x = h$ = constant, the surface temperature T_s must change when $\dot{q}_s$ = constant, and the surface heat flux $\dot{q}_s$ must change when T_s = constant. Thus we may have either T_s = constant or $\dot{q}_s$ = constant at the surface of a tube, but not both. Next we consider convection heat transfer for these two common cases.

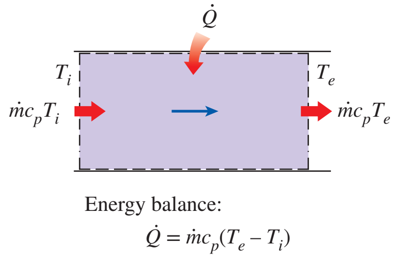


FIGURE 19–22
The heat transfer to a fluid flowing in a tube is equal to the increase in the energy of the fluid.

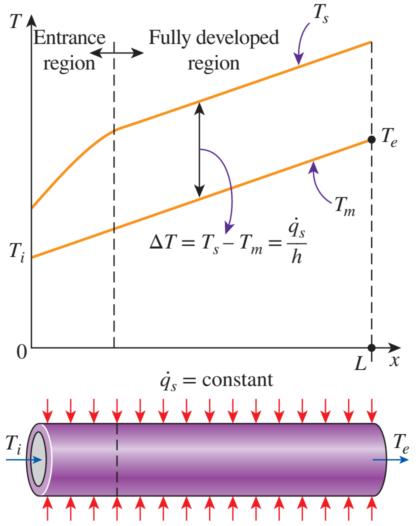


FIGURE 19–23
Variation of the *tube surface* and the *mean fluid* temperatures along the tube for the case of constant surface heat flux.

Constant Surface Heat Flux ($\dot{q}_s$ = constant)

In the case of $\dot{q}_s$ = constant, the rate of heat transfer can also be expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m}c_p(T_e - T_i) \qquad \text{(W)} \tag{19–43}$$

Then the mean fluid temperature at the tube exit becomes

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m}c_p} \tag{19–44}$$

Note that the mean fluid temperature increases *linearly* in the flow direction in the case of constant surface heat flux, since the surface area increases linearly in the flow direction (A_s is equal to the perimeter, which is constant, times the tube length).

The surface temperature in the case of constant surface heat flux $\dot{q}_s$ can be determined from

$$\dot{q}_s = h(T_s - T_m) \quad \longrightarrow \quad T_s = T_m + \frac{\dot{q}_s}{h} \tag{19–45}$$

In the fully developed region, the surface temperature T_s will also increase linearly in the flow direction since h is constant, and thus $T_s - T_m$ = constant (Fig. 19–23). Of course this is true when the fluid properties remain constant during flow.

The slope of the mean fluid temperature T_m on a T-x diagram can be determined by applying the steady-flow energy balance to a tube slice of thickness dx shown in Fig. 19–24. It gives

$$\dot{m}c_p\,dT_m = \dot{q}_s(p\,dx) \quad \longrightarrow \quad \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}c_p} = \text{constant} \tag{19–46}$$

where p is the perimeter of the tube.

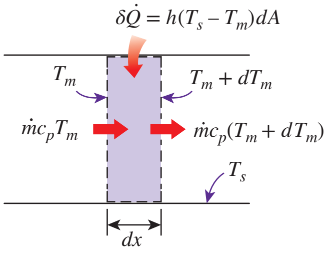


FIGURE 19–24
Energy interactions for a differential control volume in a tube.

Noting that both $\dot{q}_s$ and h are constants, the differentiation of Eq. 19–45 with respect to x gives

$$\frac{dT_m}{dx} = \frac{dT_s}{dx} \tag{19–47}$$

Also, the requirement that the dimensionless temperature profile remain unchanged in the fully developed region gives

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0 \quad \longrightarrow \quad \frac{1}{T_s - T_m}\left(\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x}\right) = 0 \quad \longrightarrow \quad \frac{\partial T}{\partial x} = \frac{dT_s}{dx} \tag{19–48}$$

since $T_s - T_m$ = constant. Combining Eqs. 19–46, 19–47, and 19–48 gives

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}c_p} = \text{constant} \tag{19–49}$$

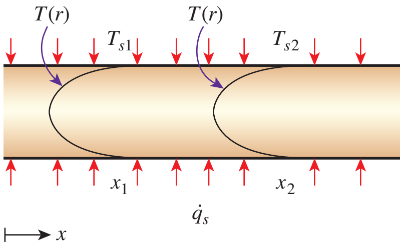


FIGURE 19–25
The shape of the temperature profile remains unchanged in the fully developed region of a tube subjected to constant surface heat flux.

Then we conclude that *in fully developed flow in a tube subjected to constant surface heat flux, the temperature gradient is independent of x, and thus the shape of the temperature profile does not change along the tube* (Fig. 19–25).

Integrating Eq. 19–49 from $x = 0$ (tube inlet where $T_m = T_i$) we obtain an expression for the variation of mean temperature along the tube

$$T_m = T_i + \frac{\dot{q}_s p}{\dot{m}c_p}x \tag{19–50}$$

We evaluate this equation at $x = L$ (tube exit where $T_m = T_e$). Then when we recognize that $A_s = pL$, Eq. 19–43 is obtained. From Eq. 19–50 we can conclude again that the mean temperature varies linearly with x along the tube for the case of constant heat flux.

For a circular tube, $p = 2\pi R$ and $\dot{m} = \rho V_{\text{avg}} A_c = \rho V_{\text{avg}}(\pi R^2)$. Then Eq. 19–49 becomes

Circular tube:
$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho V_{\text{avg}} c_p R} = \text{constant} \tag{19–51}$$

where V_{avg} is the mean velocity of the fluid.

Constant Surface Temperature (T_s = constant)

From Newton's law of cooling, the rate of heat transfer to or from a fluid flowing in a tube can be expressed as

$$\dot{Q} = hA_s\Delta T_{\text{avg}} = hA_s(T_s - T_m)_{\text{avg}} \qquad \text{(W)} \tag{19–52}$$

where h is the average convection heat transfer coefficient, A_s is the heat transfer surface area (it is equal to πDL for a circular pipe of length L), and ΔT_{avg} is some appropriate *average* temperature difference between the fluid and the surface. Next we discuss two suitable ways of expressing ΔT_{avg}.

In the constant surface temperature (T_s = constant) case, ΔT_{avg} can be expressed *approximately* by the **arithmetic mean temperature difference** ΔT_{am} as

$$\begin{aligned}\Delta T_{\text{avg}} \approx \Delta T_{\text{am}} &= \frac{\Delta T_i + \Delta T_e}{2} = \frac{(T_s - T_i) + (T_s - T_e)}{2} = T_s - \frac{T_i + T_e}{2} \\ &= T_s - T_b\end{aligned} \tag{19–53}$$

where $T_b = (T_i + T_e)/2$ is the *bulk mean fluid temperature,* which is the *arithmetic average* of the mean fluid temperatures at the inlet and the exit of the tube.

Note that the *arithmetic mean temperature difference* ΔT_{am} is simply the *average* of the *temperature differences* between the surface and the fluid at the inlet and the exit of the tube. Inherent in this definition is the assumption that the mean fluid temperature varies linearly along the tube, which is hardly ever the case when T_s = constant. This simple approximation often gives acceptable results, but not always. Therefore, we need a better way to evaluate ΔT_{avg}.

Consider the heating of a fluid in a tube of constant cross section whose inner surface is maintained at a constant temperature of T_s. We know that the mean temperature of the fluid T_m increases in the flow direction as a result of heat transfer. The energy balance on a differential control volume shown in Fig. 19–24 gives

$$\dot{m}c_p\, dT_m = h(T_s - T_m)dA_s \tag{19–54}$$

That is, the increase in the energy of the fluid (represented by an increase in its mean temperature by dT_m) is equal to the heat transferred to the fluid from the tube surface by convection. Noting that the differential surface area is $dA_s = p\,dx$, where p is the perimeter of the tube, and that $dT_m = -d(T_s - T_m)$, since T_s is constant, the relation above can be rearranged as

$$\frac{d(T_s - T_m)}{T_s - T_m} = -\frac{hp}{\dot{m}c_p}dx \tag{19–55}$$

Integrating from $x = 0$ (tube inlet where $T_m = T_i$) to $x = L$ (tube exit where $T_m = T_e$) gives

$$\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}c_p} \tag{19–56}$$

where $A_s = pL$ is the surface area of the tube and h is the constant *average* convection heat transfer coefficient. Taking the exponential of both sides and solving for T_e gives the following relation, which is very useful for the determination of the *mean fluid temperature at the tube exit:*

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}c_p) \tag{19–57}$$

This relation can also be used to determine the mean fluid temperature $T_m(x)$ at any x by replacing $A_s = pL$ with px.

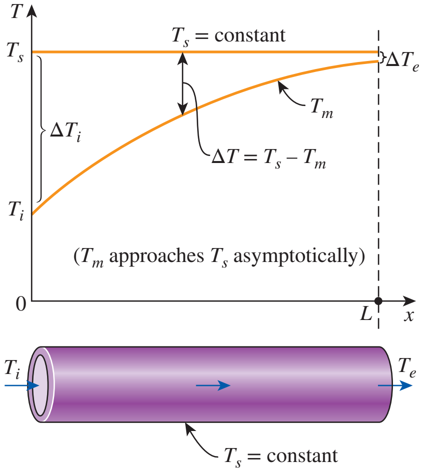
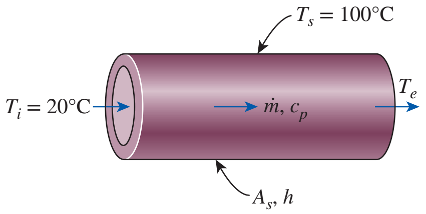


FIGURE 19–26

The variation of the *mean fluid temperature* along the tube for the case of constant temperature.

Note that the temperature difference between the fluid and the surface *decays exponentially* in the flow direction, and the rate of decay depends on the magnitude of the exponent $hA_s/\dot{m}c_p$, as shown in Fig. 19–26. This dimensionless parameter is called the *number of transfer units,* denoted by NTU, and is a measure of the effectiveness of the heat transfer systems. For NTU > 5, the exit temperature of the fluid becomes almost equal to the surface temperature, $T_e \approx T_s$ (Fig. 19–27). Noting that the fluid temperature can approach the surface temperature but cannot cross it, an NTU of about 5 indicates that the limit is reached for heat transfer, and the heat transfer does not increase no matter how much we extend the length of the tube. A small value of NTU, on the other hand, indicates more opportunities for heat transfer, and the heat transfer continues to increase as the tube length is increased. A large NTU and thus a large heat transfer surface area (which means a large tube) may be desirable from a heat transfer point of view, but it may be unacceptable from an economic point of view. The selection of heat transfer equipment usually reflects a compromise between heat transfer performance and cost.



NTU = $hA_s/\dot{m}c_p$	T_e, °C
0.01	20.8
0.05	23.9
0.10	27.6
0.50	51.5
1.00	70.6
5.00	99.5
10.00	100.0

FIGURE 19–27

An NTU greater than 5 indicates that the fluid flowing in a tube will reach the surface temperature at the exit regardless of the inlet temperature.

Solving Eq. 19–56 for $\dot{m}c_p$ gives

$$\dot{m}c_p = -\frac{hA_s}{\ln[(T_s - T_e)/(T_s - T_i)]} \tag{19–58}$$

Substituting this into Eq. 19–41, we obtain

$$\dot{Q} = hA_s\Delta T_{\text{lm}} \tag{19–59}$$

where

$$\Delta T_{\text{lm}} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} \tag{19–60}$$

is the **log mean temperature difference**. Note that $\Delta T_i = T_s - T_i$ and $\Delta T_e = T_s - T_e$ are the temperature differences between the surface and the fluid at the inlet and the exit of the tube, respectively. This ΔT_{lm} relation appears to be prone to misuse, but it is practically fail-safe, since using T_i in place of T_e and vice versa in the numerator and/or the denominator will, at most, affect the sign, not the magnitude. Also, it can be used for both heating ($T_s > T_i$ and T_e) and cooling ($T_s < T_i$ and T_e) of a fluid in a tube.

The log mean temperature difference ΔT_{lm} is obtained by tracing the actual temperature profile of the fluid along the tube and is an *exact* representation of the *average temperature difference* between the fluid and the surface. It truly reflects the exponential decay of the local temperature difference. When ΔT_e differs from ΔT_i by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent. But the error increases to undesirable levels when ΔT_e differs from ΔT_i by greater amounts. Therefore, we should always use the log mean temperature difference when determining the convection heat transfer in a tube whose surface is maintained at a constant temperature T_s.

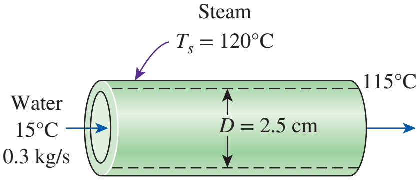


FIGURE 19–28
Schematic for Example 19–5.

EXAMPLE 19–5 Heating of Water in a Tube by Steam

Water enters a 2.5-cm-internal-diameter thin copper tube of a heat exchanger at 15°C at a rate of 0.3 kg/s, and is heated by steam condensing outside at 120°C. If the average heat transfer coefficient is 800 W/m²·K, determine the length of the tube required in order to heat the water to 115°C (Fig. 19–28).

SOLUTION Water is heated by steam in a circular tube. The tube length required to heat the water to a specified temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Fluid properties are constant. **3** The convection heat transfer coefficient is constant. **4** The conduction resistance of copper tube is negligible, so the inner surface temperature of the tube is equal to the condensation temperature of steam.

Properties The specific heat of water at the bulk mean temperature of (15 + 115)/2 = 65°C is 4187 J/kg·K. The heat of condensation of steam at 120°C is 2203 kJ/kg (Table A–15).

Analysis Knowing the inlet and exit temperatures of water, the rate of heat transfer is determined to be

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.3 \text{ kg/s})(4.187 \text{ kJ/kg·K})(115°\text{C} - 15°\text{C}) = 125.6 \text{ kW}$$

The log mean temperature difference is

$$\Delta T_e = T_s - T_e = 120°\text{C} - 115°\text{C} = 5°\text{C}$$

$$\Delta T_i = T_s - T_i = 120°\text{C} - 15°\text{C} = 105°\text{C}$$

$$\Delta T_{\text{lm}} = \frac{\Delta T_e - T_i}{\ln(\Delta T_e/\Delta T_i)} = \frac{5 - 105}{\ln(5/105)} = 32.85°\text{C}$$

The heat transfer surface area is

$$\dot{Q} = hA_s\Delta T_{\text{lm}} \longrightarrow A_s = \frac{\dot{Q}}{h\Delta T_{\text{lm}}} = \frac{125.6 \text{ kW}}{(0.8 \text{ kW/m}^2\text{·K})(32.85°\text{C})} = 4.78 \text{ m}^2$$

Then the required tube length becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{4.78 \text{ m}^2}{\pi(0.025 \text{ m})} = \mathbf{61 \text{ m}}$$

Discussion The bulk mean temperature of water during this heating process is 65°C, and thus the *arithmetic* mean temperature difference is $\Delta T_{\text{am}} = 120 - 65 = 55°\text{C}$. Using ΔT_{am} instead of ΔT_{lm} would give $L = 36$ m, which is grossly in error. This shows the importance of using the log mean temperature in calculations.

19–7 ▪ LAMINAR FLOW IN TUBES

Consider steady laminar flow of a fluid in a circular tube of radius R. The fluid properties ρ, k, and c_p are constant, and the work done by viscous forces is negligible. The fluid flows along the x-axis with velocity u. The flow is fully developed so that u is independent of x and thus $u = u(r)$. Noting that energy is transferred by mass in the x-direction and by conduction in the r-direction (heat conduction in the x-direction is assumed to be negligible), the steady-flow energy balance for a cylindrical shell element of thickness dr and length dx can be expressed as (Fig. 19–29)

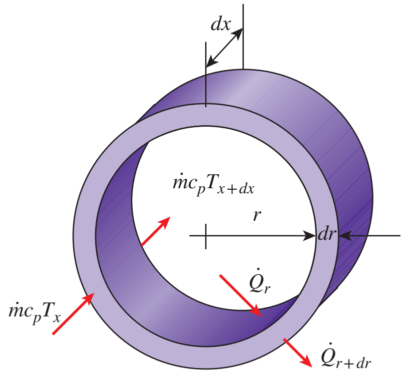


FIGURE 19–29
The differential volume element used in the derivation of energy balance relation.

$$\dot{m}c_pT_x - \dot{m}c_pT_{x+dx} + \dot{Q}_r - \dot{Q}_{r+dr} = 0 \tag{19–61}$$

where $\dot{m} = \rho u A_c = \rho u(2\pi r\,dr)$. Substituting and dividing by $2\pi r\,dr\,dx$ gives, after rearranging,

$$\rho c_p u \frac{T_{x+dx} - T_x}{dx} = -\frac{1}{2\pi r\,dx}\frac{\dot{Q}_{r+dr} - \dot{Q}_r}{dr} \tag{19–62}$$

or

$$u\frac{\partial T}{\partial x} = -\frac{1}{2\rho c_p \pi r\,dx}\frac{\partial \dot{Q}}{\partial r} \tag{19–63}$$

But from Fourier's law of heat conduction in the radial direction,

$$\frac{\partial \dot{Q}}{\partial r} = \frac{\partial}{\partial r}\left(-2\pi r k\,dx\frac{\partial T}{\partial r}\right) = -2\pi k\,dx\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{19–64}$$

Substituting and using $\alpha = k/\rho c_p$ gives

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{19–65}$$

which states that *the rate of net energy transfer to the control volume by mass flow is equal to the net rate of heat conduction in the radial direction.*

Constant Surface Heat Flux

For fully developed flow in a circular tube subjected to constant surface heat flux, we have, from Eq. 19–51,

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho V_{\text{avg}} c_p R} = \text{constant} \tag{19–66}$$

If heat conduction in the x-direction were considered in the derivation of Eq. 19–65, it would give an additional term $\alpha\partial^2 T/\partial x^2$, which would be equal to zero since $\partial T/\partial x =$ constant and thus $T = T(r)$. Therefore, the assumption that there is no axial heat conduction is satisfied exactly in this case.

Substituting Eq. 19–66 and the relation for velocity profile from Chap. 14 into Eq. 19–65 gives

$$\frac{4\dot{q}_s}{kR}\left(1 - \frac{r^2}{R^2}\right) = \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) \tag{19–67}$$

which is a second-order ordinary differential equation. Its general solution is obtained by separating the variables and integrating twice to be

$$T = \frac{\dot{q}_s}{kR}\left(r^2 - \frac{r^4}{4R^2}\right) + C_1 \ln r + C_2 \tag{19–68}$$

The desired solution to the problem is obtained by applying the boundary conditions $\partial T/\partial r = 0$ at $r = 0$ (because of symmetry) and $T = T_s$ at $r = R$. We get

$$T = T_s - \frac{\dot{q}_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right) \tag{19–69}$$

The mean temperature T_m is determined by substituting the velocity and temperature profile relations into Eq. 19–32 and performing the integration. It gives

$$T_m = T_s - \frac{11}{24}\frac{\dot{q}_s R}{k} \tag{19–70}$$

Combining this relation with $\dot{q}_s = h(T_s - T_m)$ gives

$$h = \frac{24}{11}\frac{k}{R} = \frac{48}{11}\frac{k}{D} = 4.36\frac{k}{D} \tag{19–71}$$

or

Circular tube, laminar ($\dot{q}_s$ = constant): $$\text{Nu} = \frac{hD}{k} = 4.36 \tag{19–72}$$

Therefore, for fully developed laminar flow in a circular tube subjected to constant surface heat flux, the Nusselt number is a constant. There is no dependence on the Reynolds or the Prandtl numbers.

Constant Surface Temperature

A similar analysis can be performed for fully developed laminar flow in a circular tube for the case of constant surface temperature T_s. The solution procedure in this case is more complex than the constant surface heat flux case, but the resulting Nusselt number relation obtained is equally simple. This problem has been solved iteratively as reported by Kays and Crawford (1993) and more recently by Shokouhmand and Hooman (2003) using an exact analytical procedure. The resulting Nusselt number relation obtained is (Fig. 19–30):

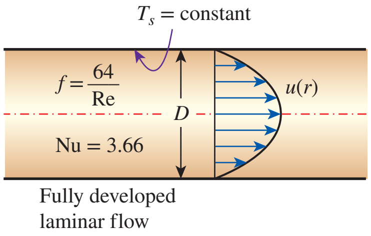


FIGURE 19–30
In laminar flow in a tube with constant surface temperature, both the *friction factor* and the *heat transfer coefficient* remain constant in the fully developed region.

Circular tube, laminar (T_s = constant): $$\text{Nu} = \frac{hD}{k} = 3.66 \tag{19–73}$$

Comparison of Eqs. 19–72 and 19–73 shows that the Nusselt number for the case of constant surface heat flux is 16 percent higher than the case of constant surface temperature for the fully developed laminar pipe flow. This shows that laminar flow is sensitive to the applied surface thermal boundary condition and for applications requiring higher rates of heat transfer, whenever possible; the constant surface heat flux boundary condition should be used. This is contrary to the results shown in Fig. 19–21 for the turbulent flow, which showed no sensitivity to the different surface thermal boundary conditions in the fully developed region.

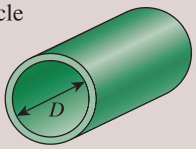
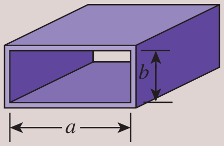
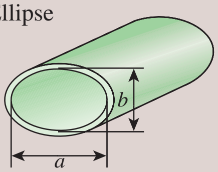
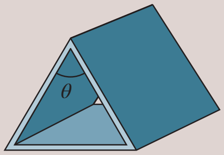
The thermal conductivity k for use in the Nu relations above should be evaluated at the bulk mean fluid temperature, which is the arithmetic average of the mean fluid temperatures at the inlet and the exit of the tube. For laminar flow, the effect of *surface roughness* on the friction factor and the heat transfer coefficient is negligible.

Laminar Flow in Noncircular Tubes

The friction factor f and the Nusselt number relations are given in Table 19–3 for *fully developed laminar flow* in tubes of various cross sections. The Reynolds and Nusselt numbers for flow in these tubes are based on the hydraulic diameter $D_h = 4A_c/p$, where A_c is the cross-sectional area of the tube and p is its perimeter. Once the Nusselt number is available, the convection heat transfer coefficient is determined from $h = k\,\text{Nu}/D_h$.

TABLE 19–3

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $\text{Re} = V_{\text{avg}}D_h/\nu$, and $\text{Nu} = hD_h/k$)

		Nusselt number		
Tube geometry	*a/b* or θ	T_s = Const.	$\dot{q}_s$ = Const.	Friction factor *f*
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	*a/b*			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	*a/b*			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Isosceles triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

Developing Laminar Flow in the Entrance Region

For a circular tube of length L subjected to constant surface temperature, the average Nusselt number for the *thermal entrance region* can be determined from (Edwards et al., 1979)

Entry region, laminar:
$$\text{Nu} = 3.66 + \frac{0.065(D/L)\,\text{Re Pr}}{1 + 0.04[(D/L)\,\text{Re Pr}]^{2/3}} \qquad \textbf{(19–74)}$$

Note that the average Nusselt number is larger at the entrance region, as expected, and it approaches asymptotically to the fully developed value of 3.66 as $L \to \infty$. This relation assumes that the flow is hydrodynamically developed when the fluid enters the heating section, but it can also be used for flow developing hydrodynamically when $\text{Pr} \geq 5$.

When the difference between the surface and the fluid temperatures is large, it may be necessary to account for the variation of viscosity with temperature.

The average Nusselt number for hydrodynamically and thermally developing laminar flow in a circular tube in that case can be determined from (Sieder and Tate, 1936)

Entry region, laminar:
$$\text{Nu} = 1.86\left(\frac{\text{Re Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} \qquad \textbf{(19–75)}$$

The preceding equation is recommended for $0.60 \leq \text{Pr} \leq 5$ and $0.0044 \leq (\mu_b/\mu_s) \leq 9.75$. All properties appearing in Eqs. 19–74 and 19–75 should be evaluated at the bulk mean fluid temperature, $T_b = (T_i + T_e)/2$, except for μ_s, which is evaluated at the surface temperature.

The average Nusselt number for the thermal entrance region of flow between *isothermal parallel plates* of length L is expressed as (Edwards et al., 1979)

Entry region, laminar:
$$\text{Nu} = 7.54 + \frac{0.03(D_h/L)\text{Re Pr}}{1 + 0.016[(D_h/L)\text{Re Pr}]^{2/3}} \tag{19–76}$$

where D_h is the hydraulic diameter, which is twice the spacing of the plates. This relation can be used for $\text{Re} \leq 2800$.

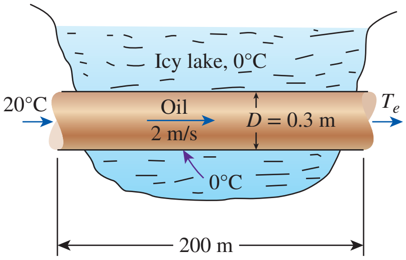


FIGURE 19–31
Schematic for Example 19–6.

EXAMPLE 19–6 Flow of Oil in a Pipeline Through a Lake

Consider the flow of oil at 20°C in a 30-cm-diameter pipeline at an average velocity of 2 m/s (Fig. 19–31). A 200-m-long section of the horizontal pipeline passes through icy waters of a lake at 0°C. Measurements indicate that the surface temperature of the pipe is very nearly 0°C. Disregarding the thermal resistance of the pipe material, determine (*a*) the temperature of the oil when the pipe leaves the lake, (*b*) the rate of heat transfer from the oil, and (*c*) the pumping power required to overcome the pressure losses and to maintain the flow of the oil in the pipe.

SOLUTION Oil flows in a pipeline that passes through icy waters of a lake at 0°C. The exit temperature of the oil, the rate of heat loss, and the pumping power needed to overcome pressure losses are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is very nearly 0°C. **3** The thermal resistance of the pipe is negligible. **4** The inner surfaces of the pipeline are smooth. **5** The flow is hydrodynamically developed when the pipeline reaches the lake.

Properties We do not know the exit temperature of the oil, and thus we cannot determine the bulk mean temperature, which is the temperature at which the properties of oil are to be evaluated. The mean temperature of the oil at the inlet is 20°C, and we expect this temperature to drop somewhat as a result of heat loss to the icy waters of the lake. We evaluate the properties of the oil at the inlet temperature, but we will repeat the calculations, if necessary, using properties at the evaluated bulk mean temperature. At 20°C we read (Table A–19)

$$\rho = 888.1 \text{ kg/m}^3 \qquad \nu = 9.429 \times 10^{-4} \text{ m}^2\text{/s}$$
$$k = 0.145 \text{ W/m·K} \qquad c_p = 1880 \text{ J/kg·K} \qquad \text{Pr} = 10{,}863$$

Analysis (*a*) The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}}D}{\nu} = \frac{(2 \text{ m/s})(0.3 \text{ m})}{9.429 \times 10^{-4} \text{ m}^2\text{/s}} = 636$$

which is less than the critical Reynolds number of 2300. Therefore, the flow is laminar, and the thermal entry length in this case is roughly

$$L_t \approx 0.05 \text{ Re Pr } D = 0.05 \times 636 \times 10{,}863 \times (0.3 \text{ m}) \approx 103{,}600 \text{ m}$$

which is much greater than the total length of the pipe. This is typical of fluids with high Prandtl numbers. Therefore, we assume thermally developing flow and determine the Nusselt number from

$$\text{Nu} = \frac{hD}{k} = 3.66 + \frac{0.065(D/L) \text{ Re Pr}}{1 + 0.04 [(D/L) \text{ Re Pr}]^{2/3}}$$
$$= 3.66 + \frac{0.065(0.3/200) \times 636 \times 10{,}863}{1 + 0.04 [(0.3/200) \times 636 \times 10{,}863]^{2/3}}$$
$$= 37.3$$

Note that this Nusselt number is considerably higher than the fully developed value of 3.66. Then,

$$h = \frac{k}{D}\text{Nu} = \frac{0.145 \text{ W/m·K}}{0.3 \text{ m}}(37.3) = 18.0 \text{ W/m}^2\text{·K}$$

Also,

$$A_s = \pi DL = \pi(0.3 \text{ m})(200 \text{ m}) = 188.5 \text{ m}^2$$
$$\dot{m} = \rho A_c V_{\text{avg}} = (888.1 \text{ kg/m}^3)[\tfrac{1}{4}\pi(0.3 \text{ m})^2](2 \text{ m/s}) = 125.6 \text{ kg/s}$$

Next we determine the exit temperature of the oil,

$$\begin{aligned} T_e &= T_s - (T_s - T_i)\exp(-hA_s/\dot{m}c_p) \\ &= 0°\text{C} - [(0 - 20)°\text{C}]\exp\left[-\frac{(18.0 \text{ W/m}^2\text{·K})(188.5 \text{ m}^2)}{(125.6 \text{ kg/s})(1881 \text{ J/kg·K})}\right] \\ &= \mathbf{19.71°C} \end{aligned}$$

Thus, the mean temperature of the oil drops by a mere 0.29°C as it crosses the lake. This makes the bulk mean oil temperature 19.86°C, which is practically identical to the inlet temperature of 20°C. Therefore, we do not need to reevaluate the properties.

(*b*) The log mean temperature difference and the rate of heat loss from the oil are

$$\Delta T_{\text{lm}} = \frac{T_i - T_e}{\ln\dfrac{T_s - T_e}{T_s - T_i}} = \frac{20 - 19.71}{\ln\dfrac{0 - 19.71}{0 - 20}} = -19.85°\text{C}$$

$$\dot{Q} = hA_s\,\Delta T_{\text{lm}} = (18.0 \text{ W/m}^2\text{·K})(188.5 \text{ m}^2)(-19.85°\text{C}) = \mathbf{-6.75 \times 10^4\ W}$$

Therefore, the oil will lose heat at a rate of 67.5 kW as it flows through the pipe in the icy waters of the lake. Note that ΔT_{lm} is identical to the arithmetic mean temperature in this case, since $\Delta T_i \approx \Delta T_e$.

(*c*) The laminar flow of oil is hydrodynamically developed. Therefore, the friction factor can be determined from

$$f = \frac{64}{\text{Re}} = \frac{64}{636} = 0.1006$$

Then the pressure drop in the pipe and the required pumping power become

$$\Delta P = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.1006\frac{200 \text{ m}}{0.3 \text{ m}}\frac{(888.1 \text{ kg/m}^3)(2 \text{ m/s})^2}{2} = 1.19 \times 10^5 \text{ N/m}^2$$

$$\dot{W}_{\text{pump}} = \frac{\dot{m}\,\Delta P}{\rho} = \frac{(125.6 \text{ kg/s})(1.19 \times 10^5 \text{ N/m}^2)}{888.1 \text{ kg/m}^3} = \mathbf{16.8\ kW}$$

Discussion We need a 16.8-kW pump just to overcome the friction in the pipe as the oil flows in the 200-m-long pipe through the lake.

19–8 ▪ TURBULENT FLOW IN TUBES

We mentioned earlier that flow in smooth tubes is usually fully turbulent for Re > 10,000. Turbulent flow is commonly used in practice because of the higher heat transfer coefficients associated with it. Most correlations for the friction and heat transfer coefficients in turbulent flow are based on experimental studies because of the difficulty in dealing with turbulent flow theoretically.

For *smooth* tubes, the friction factor in turbulent flow can be determined from the explicit *first Petukhov equation* (Petukhov, 1970), given as

$$\textit{Smooth tubes:} \quad f = (0.790 \ln \text{Re} - 1.64)^{-2} \qquad 3000 < \text{Re} < 5 \times 10^6 \qquad \textbf{(19–77)}$$

The Nusselt number in turbulent flow is related to the friction factor through the *Chilton–Colburn analogy,* expressed as

$$\mathrm{Nu} = 0.125\, f\, \mathrm{Re}\mathrm{Pr}^{1/3} \tag{19–78}$$

Once the friction factor is available, this equation can be used conveniently to evaluate the Nusselt number for both smooth and rough tubes.

For fully developed turbulent flow in *smooth tubes,* a simple relation for the Nusselt number can be obtained by substituting the simple power law relation $f = 0.184\ \mathrm{Re}^{-0.2}$ for the friction factor into Eq. 19–78. It gives

$$\mathrm{Nu} = 0.023\ \mathrm{Re}^{0.8}\ \mathrm{Pr}^{1/3} \qquad \begin{pmatrix} 0.7 \le \mathrm{Pr} \le 160 \\ \mathrm{Re} > 10{,}000 \end{pmatrix} \tag{19–79}$$

which is known as the *Colburn equation.* The accuracy of this equation can be improved by modifying it as

$$\mathrm{Nu} = 0.023\ \mathrm{Re}^{0.8}\,\mathrm{Pr}^{n} \tag{19–80}$$

where $n = 0.4$ for *heating* and 0.3 for *cooling* of the fluid flowing through the tube. This equation is known as the *Dittus–Boelter equation* (Dittus and Boelter, 1930), and it is preferred over the Colburn equation.

The preceding equations can be used when the temperature difference between the fluid and wall surface is not large by evaluating all fluid properties at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$. When the variation in properties is large due to a large temperature difference, the following equation due to Sieder and Tate (1936) can be used:

$$\mathrm{Nu} = 0.027\ \mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} \qquad \begin{pmatrix} 0.7 \le \mathrm{Pr} \le 16{,}700 \\ \mathrm{Re} \ge 10{,}000 \end{pmatrix} \tag{19–81}$$

Here all properties are evaluated at T_b except μ_s, which is evaluated at T_s.

The preceding Nusselt number relations are fairly simple, but they may give errors as large as 25 percent. This error can be reduced considerably to less than 10 percent by using more complex but accurate relations, such as the *second Petukhov equation,* expressed as

$$\mathrm{Nu} = \frac{(f/8)\mathrm{Re}\ \mathrm{Pr}}{1.07 + 12.7(f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 10^4 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix} \tag{19–82}$$

The accuracy of this relation at lower Reynolds numbers is improved by modifying it as (Gnielinski, 1976)

$$\mathrm{Nu} = \frac{(f/8)(\mathrm{Re} - 1000)\mathrm{Pr}}{1 + 12.7(f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 3 \times 10^3 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix} \tag{19–83}$$

where the friction factor f can be determined from an appropriate relation such as the first Petukhov equation. Gnielinski's equation should be preferred in calculations. Again properties should be evaluated at the bulk mean fluid temperature.

The preceding relations are not very sensitive to the *thermal conditions* at the tube surfaces, and they can be used for both T_s = constant and $\dot{q}_s$ = constant cases. Despite their simplicity, the correlations already presented give sufficiently accurate results for most engineering purposes. They can also be used to obtain rough estimates of the friction factor and the heat transfer coefficients in the transition region.

The relations given so far do not apply to liquid metals because of their very low Prandtl numbers. For liquid metals ($0.004 < \mathrm{Pr} < 0.01$), the following relations are recommended by Sleicher and Rouse (1975) for $10^4 < \mathrm{Re} < 10^6$:

$$\textit{Liquid metals, } T_s = \text{constant:} \qquad \mathrm{Nu} = 4.8 + 0.0156\ \mathrm{Re}^{0.85}\ \mathrm{Pr}_s^{0.93} \tag{19–84}$$

$$\text{Liquid metals, } \dot{q}_s = \text{constant:} \quad \text{Nu} = 6.3 + 0.0167\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93} \tag{19–85}$$

where the subscript s indicates that the Prandtl number is to be evaluated at the surface temperature.

In turbulent flow, wall roughness increases the heat transfer coefficient h by a factor of 2 or more (Dipprey and Sabersky, 1963). The convection heat transfer coefficient for rough tubes can be calculated approximately from the Nusselt number relations such as Eq. 19–83 by using the friction factor determined from the Moody chart or the Colebrook equation. However, this approach is not very accurate since there is no further increase in h with f for $f > 4f_{\text{smooth}}$ (Norris, 1970), and correlations developed specifically for rough tubes should be used when more accuracy is desired.

Developing Turbulent Flow in the Entrance Region

The entry lengths for turbulent flow are typically short, often just 10 tube diameters long, and thus the Nusselt number determined for fully developed turbulent flow can be used approximately for the entire tube. This simple approach gives reasonable results for pressure drop and heat transfer for long tubes and conservative results for short ones. Correlations for the friction and heat transfer coefficients for the entrance regions are available in the literature for better accuracy.

Turbulent Flow in Noncircular Tubes

The velocity and temperature profiles in turbulent flow are nearly straight lines in the core region, and any significant velocity and temperature gradients occur in the viscous sublayer (Fig. 19–32). Despite the small thickness of the viscous sublayer (usually much less than 1 percent of the pipe diameter), the characteristics of the flow in this layer are very important since they set the stage for flow in the rest of the pipe. Therefore, pressure drop and heat transfer characteristics of turbulent flow in tubes are dominated by the very thin viscous sublayer next to the wall surface, and the shape of the core region is not of much significance. Consequently, the turbulent flow relations given above for circular tubes can also be used for noncircular tubes with reasonable accuracy by replacing the diameter D in the evaluation of the Reynolds number by the hydraulic diameter $D_h = 4A_c/p$.

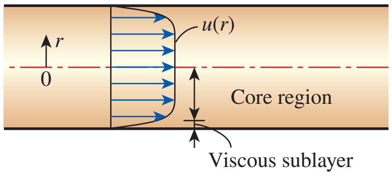


FIGURE 19–32
In turbulent flow, the velocity profile is nearly a straight line in the core region, and any significant velocity gradients occur in the viscous sublayer.

Flow Through Tube Annulus

Some simple heat transfer devices consist of two concentric tubes and are properly called *double-tube heat exchangers* (Fig. 19–33). In such devices, one fluid flows through the tube while the other flows through the annular space. The governing differential equations for both flows are identical. Therefore, steady laminar flow through an annulus can be studied analytically by using suitable boundary conditions.

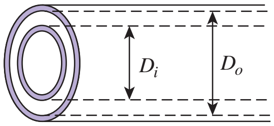


FIGURE 19–33
A double-tube heat exchanger that consists of two concentric tubes.

Consider a concentric annulus of inner diameter D_i and outer diameter D_o. The hydraulic diameter of the annulus is

$$D_h = \frac{4A_c}{p} = \frac{4\pi(D_o^2 - D_i^2)/4}{\pi(D_o + D_i)} = D_o - D_i$$

Annular flow is associated with two Nusselt numbers—Nu_i on the inner tube surface and Nu_o on the outer tube surface—since it may involve heat transfer on both surfaces. The Nusselt numbers for fully developed laminar flow with one surface isothermal and the other adiabatic are given in Table 19–4. When Nusselt numbers are known, the convection heat transfer coefficients for the inner and the outer surfaces are determined from

$$\text{Nu}_i = \frac{h_i D_h}{k} \quad \text{and} \quad \text{Nu}_o = \frac{h_o D_h}{k} \tag{19–86}$$

TABLE 19–4

Nusselt number for fully developed laminar flow in an annulus with one surface isothermal and the other adiabatic

D_i/D_o	Nu_i	Nu_o
0	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

Source: Kays and Perkins, 1972

For fully developed turbulent flow, the inner and outer convection heat transfer coefficients are approximately equal to each other, and the tube annulus can be treated as a noncircular duct with a hydraulic diameter of $D_h = D_o - D_i$. The Nusselt number in this case can be determined from a suitable turbulent flow relation such as the Gnielinski equation (Eq. 19–83). To improve the accuracy of Nusselt numbers obtained from these relations for annular flow, Petukhov and Roizen (1964) recommend multiplying them by the following correction factors when one of the tube walls is adiabatic and heat transfer is through the other wall:

$$F_i = 0.86\left(\frac{D_i}{D_o}\right)^{-0.16} \quad \text{(outer wall adiabatic)} \tag{19–87}$$

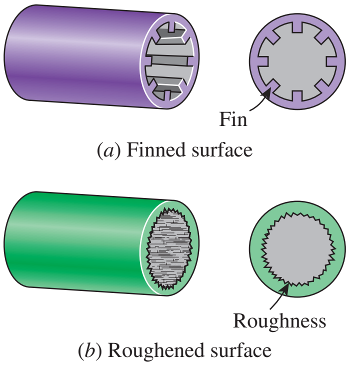
$$F_o = 1 - 0.14\left(\frac{D_i}{D_o}\right)^{0.6} \quad \text{(inner wall adiabatic)} \tag{19–88}$$

Gnielinski (2015), on the basis of a large amount of recent experimental data from the literature, improved his earlier heat transfer correlations for turbulent annular flow (Gnielinski, 2009) and developed a single comprehensive correlation for the calculation of the heat transfer coefficient for turbulent flow in tubes and concentric annular ducts. Only different correlations for the friction factor are required, and the appropriate hydraulic diameter must be used. Specific additional factors are needed that take into consideration the effect of the diameter ratio of the annulus and the different boundary conditions for heating and cooling. Examples include heat transfer at the inner wall with the outer wall insulated and heat transfer at the outer wall with the inner wall insulated. No experimental data were found for heat transfer from both walls to the annular flow.

Heat Transfer Enhancement

Tubes with rough surfaces have much higher heat transfer coefficients than tubes with smooth surfaces. Therefore, tube surfaces are often intentionally *roughened, corrugated,* or *finned* in order to *enhance* the convection heat transfer coefficient and thus the convection heat transfer rate (Fig. 19–34). Heat transfer in turbulent flow in a tube has been increased by as much as 400 percent by roughening the surface. Roughening the surface, of course, also increases the friction factor and thus the power requirement for the pump or the fan.

The convection heat transfer coefficient can also be increased by inducing pulsating flow with pulse generators, by inducing swirl by inserting a twisted tape into the tube, or by inducing secondary flows by coiling the tube.



(*a*) Finned surface

(*b*) Roughened surface

FIGURE 19–34
Tube surfaces are often *roughened, corrugated,* or *finned* in order to *enhance* convection heat transfer.

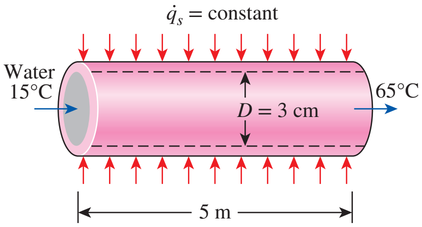


FIGURE 19–35
Schematic for Example 19–7.

EXAMPLE 19–7 Heating of Water by Resistance Heaters with Turbulent Flow in a Tube

Water is to be heated from 15°C to 65°C as it flows through a 3-cm-internal-diameter 5-m-long tube (Fig. 19–35). The tube is equipped with an electric resistance heater that provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so in steady operation, all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 10 L/min, determine the power rating of the resistance heater. Also estimate the inner surface temperature of the tube at the exit.

SOLUTION Water is to be heated in a tube equipped with an electric resistance heater on its surface. The power rating of the heater and the inner surface temperature at the exit are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface heat flux is uniform. **3** The inner surfaces of the tube are smooth.

Properties The properties of water at the bulk mean temperature of $T_b = (T_i + T_e)/2 = (15 + 65)/2 = 40°\text{C}$ are (Table A–15)

$$\rho = 992.1 \text{ kg/m}^3 \qquad c_p = 4179 \text{ J/kg·K}$$

$$k = 0.631 \text{ W/m·K} \qquad \text{Pr} = 4.32$$

$$\nu = \mu/\rho = 0.658 \times 10^{-6} \text{ m}^2\text{/s}$$

Analysis The cross-sectional and heat transfer surface areas are

$$A_c = \tfrac{1}{4}\pi D^2 = \tfrac{1}{4}\pi(0.03 \text{ m})^2 = 7.069 \times 10^{-4} \text{ m}^2$$

$$A_s = \pi DL = \pi(0.03 \text{ m})(5 \text{ m}) = 0.471 \text{ m}^2$$

The volume flow rate of water is given as $\dot{V} = 10 \text{ L/min} = 0.01 \text{ m}^3\text{/min}$. Then the mass flow rate becomes

$$\dot{m} = \rho\dot{V} = (992.1 \text{ kg/m}^3)(0.01 \text{ m}^3\text{/min}) = 9.921 \text{ kg/min} = 0.1654 \text{ kg/s}$$

To heat the water at this mass flow rate from 15°C to 65°C, heat must be supplied to the water at a rate of

$$\begin{aligned}\dot{Q} &= \dot{m}c_p(T_e - T_i)\\ &= (0.1654 \text{ kg/s})(4.179 \text{ kJ/kg·K})(65 - 15)°\text{C}\\ &= 34.6 \text{ kJ/s} = 34.6 \text{ kW}\end{aligned}$$

All of this energy must come from the resistance heater. Therefore, the power rating of the heater must be **34.6 kW**.

The surface temperature T_s of the tube at any location can be determined from

$$\dot{q}_s = h(T_s - T_m) \rightarrow T_s = T_m + \frac{\dot{q}_s}{h}$$

where h is the heat transfer coefficient and T_m is the mean temperature of the fluid at that location. The surface heat flux is constant in this case, and its value can be determined from

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = \frac{34.6 \text{ kW}}{0.471 \text{ m}^2} = 73.46 \text{ kW/m}^2$$

To determine the heat transfer coefficient, we first need to find the mean velocity of water and the Reynolds number:

$$V_{\text{avg}} = \frac{\dot{V}}{A_c} = \frac{0.010 \text{ m}^3\text{/min}}{7.069 \times 10^{-4} \text{ m}^2} = 14.15 \text{ m/min} = 0.236 \text{ m/s}$$

$$\text{Re} = \frac{V_{\text{avg}} D}{\nu} = \frac{(0.236 \text{ m/s})(0.03 \text{ m})}{0.658 \times 10^{-6} \text{ m}^2\text{/s}} = 10{,}760$$

which is greater than 10,000. Therefore, the flow is turbulent, and the entry length is roughly

$$L_h \approx L_t \approx 10D = 10 \times 0.03 = 0.3 \text{ m}$$

which is much shorter than the total length of the tube. Therefore, we can assume fully developed turbulent flow in the entire tube and determine the Nusselt number from

$$\text{Nu} = \frac{hD}{k} = 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.4} = 0.023\,(10{,}760)^{0.8}\,(4.32)^{0.4} = 69.4$$

Then,

$$h = \frac{k}{D}\text{Nu} = \frac{0.631 \text{ W/m·K}}{0.03 \text{ m}}(69.4) = 1460 \text{ W/m}^2\text{·K}$$

and the inner surface temperature of the tube at the exit becomes

$$T_s = T_m + \frac{\dot{q}_s}{h} = 65^\circ\text{C} + \frac{73{,}460\ \text{W/m}^2}{1460\ \text{W/m}^2\cdot\text{K}} = \mathbf{115^\circ C}$$

Discussion Note that the inner surface temperature of the tube will be 50°C higher than the mean water temperature at the tube exit. This temperature difference of 50°C between the water and the surface will remain constant throughout the fully developed flow region.

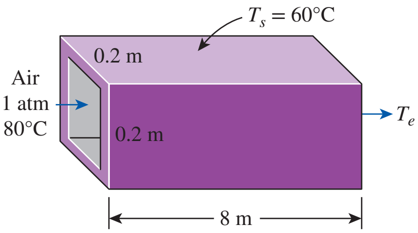


FIGURE 19–36
Schematic for Example 19–8.

EXAMPLE 19–8 Heat Loss from the Ducts of a Heating System

Hot air at atmospheric pressure and 80°C enters an 8-m-long uninsulated square duct of cross section 0.2 m × 0.2 m that passes through the attic of a house at a rate of 0.15 m^3/s (Fig. 19–36). The duct is observed to be nearly isothermal at 60°C. Determine the exit temperature of the air and the rate of heat loss from the duct to the attic space.

SOLUTION Heat loss from uninsulated square ducts of a heating system in the attic is considered. The exit temperature and the rate of heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Air is an ideal gas.

Properties We do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk mean temperature of air, which is the temperature at which the properties are to be determined. The temperature of air at the inlet is 80°C, and we expect this temperature to drop somewhat as a result of heat loss through the duct whose surface is at 60°C. At 80°C and 1 atm, we read (Table A–22)

$$\rho = 0.9994\ \text{kg/m}^3 \qquad c_p = 1008\ \text{J/kg}\cdot\text{K}$$
$$k = 0.02953\ \text{W/m}\cdot\text{K} \quad \text{Pr} = 0.7154$$
$$\nu = 2.097 \times 10^{-5}\ \text{m}^2/\text{s}$$

Analysis The characteristic length (which is the hydraulic diameter), the mean velocity, and the Reynolds number in this case are

$$D_h = \frac{4A_c}{p} = \frac{4a^2}{4a} = a = 0.2\ \text{m}$$

$$V_{\text{avg}} = \frac{\dot{V}}{A_c} = \frac{0.15\ \text{m}^3/\text{s}}{(0.2\ \text{m})^2} = 3.75\ \text{m/s}$$

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(3.75\ \text{m/s})(0.2\ \text{m})}{2.097 \times 10^{-5}\ \text{m}^2/\text{s}} = 35{,}765$$

which is greater than 10,000. Therefore, the flow is turbulent, and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10 \times 0.2\ \text{m} = 2\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct and determine the Nusselt number from

$$\text{Nu} = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\ \text{Pr}^{0.3} = 0.023\,(35{,}765)^{0.8}\,(0.7154)^{0.3} = 91.4$$

Then,

$$h = \frac{k}{D_h}\text{Nu} = \frac{0.02953\ \text{W/m}\cdot\text{K}}{0.2\ \text{m}}(91.4) = 13.5\ \text{W/m}^2\cdot\text{K}$$

$$A_s = 4aL = 4 \times (0.2\ \text{m})(8\ \text{m}) = 6.4\ \text{m}^2$$

$$\dot{m} = \rho\dot{V} = (0.9994\ \text{kg/m}^3)(0.15\ \text{m}^3/\text{s}) = 0.150\ \text{kg/s}$$

Next, we determine the exit temperature of air from

$$\begin{aligned} T_e &= T_s - (T_s - T_i)\exp(-hA_s/\dot{m}c_p) \\ &= 60°\text{C} - [(60 - 80°\text{C})]\exp\left[-\frac{(13.5\ \text{W/m}^2\cdot\text{K})(6.4\ \text{m}^2)}{(0.150\ \text{kg/s})(1008\ \text{J/kg}\cdot\text{K})}\right] \\ &= \mathbf{71.3°C} \end{aligned}$$

Then the log mean temperature difference and the rate of heat loss from the air become

$$\Delta T_{\text{lm}} = \frac{T_i - T_e}{\ln \dfrac{T_s - T_e}{T_s - T_i}} = \frac{80 - 71.3}{\ln \dfrac{60 - 71.3}{60 - 80}} = -15.2°\text{C}$$

$$\dot{Q} = hA_s\,\Delta T_{\text{lm}} = (13.5\ \text{W/m}^2\cdot\text{K})(6.4\ \text{m}^2)(-15.2°\text{C}) = \mathbf{-1313\ W}$$

Therefore, air will lose heat at a rate of 1313 W as it flows through the duct in the attic.

Discussion The average fluid temperature is (80 + 71.3)/2 = 75.7°C, which is sufficiently close to 80°C, at which we evaluated the properties of air. Therefore, it is not necessary to reevaluate the properties at this temperature and to repeat the calculations.

SUMMARY

Convection is the mode of heat transfer that involves conduction as well as bulk fluid motion. The rate of convection heat transfer in external flow is expressed by *Newton's law of cooling* as

$$\dot{Q} = hA_s(T_s - T_\infty)$$

where T_s is the surface temperature and T_∞ is the free-stream temperature. The heat transfer coefficient h is usually expressed in the dimensionless form as the *Nusselt number* as $\text{Nu} = hL_c/k$ where L_c is the *characteristic length*. The characteristic length for noncircular tubes is the *hydraulic diameter* D_h defined as $D_h = 4A_c/p$ where A_c is the cross-sectional area of the tube and p is its perimeter. The value of the critical Reynolds number is about 5×10^5 for flow over a flat plate, 2×10^5 for flow over cylinders and spheres, and 2300 for flow inside tubes.

The average Nusselt number relations for flow over a flat plate are

Laminar: $$\text{Nu} = \frac{hL}{k} = 0.664\ \text{Re}_L^{0.5}\ \text{Pr}^{1/3} \qquad \text{Re}_L < 5 \times 10^5$$

Turbulent:

$$\text{Nu} = \frac{hL}{k} = 0.037\ \text{Re}_L^{0.8}\ \text{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \text{Pr} \le 60 \\ 5 \times 10^5 \le \text{Re}_L \le 10^7 \end{array}$$

Combined:

$$\text{Nu} = \frac{hL}{k} = (0.037\ \text{Re}_L^{0.8} - 871)\ \text{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \text{Pr} \le 60 \\ 5 \times 10^5 \le \text{Re}_L \le 10^7 \end{array}$$

For isothermal surfaces with an unheated starting section of length ξ, the local Nusselt number and the average convection coefficient relations are

Laminar: $$\text{Nu}_x = \frac{\text{Nu}_{x\,(\text{for } \xi = 0)}}{[1 - (\xi/x)^{3/4}]^{1/3}} = \frac{0.332\ \text{Re}_x^{0.5}\ \text{Pr}^{1/3}}{[1 - (\xi/x)^{3/4}]^{1/3}}$$

Turbulent: $$\text{Nu}_x = \frac{\text{Nu}_{x\,(\text{for } \xi = 0)}}{[1 - (\xi/x)^{9/10}]^{1/9}} = \frac{0.0296\ \text{Re}_x^{0.8}\ \text{Pr}^{1/3}}{[1 - (\xi/x)^{9/10}]^{1/9}}$$

Laminar: $$h = \frac{2[1 - (\xi/x)^{3/4}]}{1 - \xi/L} h_{x=L}$$

Turbulent: $$h = \frac{5[1 - (\xi/x)^{9/10}]}{1 - \xi/L} h_{x=L}$$

These relations are for the case of *isothermal* surfaces. When a flat plate is subjected to *uniform heat flux,* the local Nusselt number is given by

Laminar: $$\text{Nu}_x = 0.453\ \text{Re}_x^{0.5}\ \text{Pr}^{1/3}$$

Turbulent: $$\text{Nu}_x = 0.0308\ \text{Re}_x^{0.8}\ \text{Pr}^{1/3}$$

The average Nusselt numbers for crossflow over a *cylinder* and *sphere* are

$$\text{Nu}_{\text{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\ \text{Re}^{1/2}\ \text{Pr}^{1/3}}{[1 + (0.4/\text{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

which is valid for $\text{Re Pr} > 0.2$, and

$$\text{Nu}_{\text{sph}} = \frac{hD}{k} = 2 + [0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

which is valid for $3.5 \le \text{Re} \le 80{,}000$ and $0.7 \le \text{Pr} \le 380$. The fluid properties are evaluated at the film temperature $T_f = (T_\infty + T_s)/2$ in the case of a cylinder, and at the free-stream temperature T_∞ (except for μ_s, which is evaluated at the surface temperature T_s) in the case of a sphere.

The Reynolds number for internal flow and the hydraulic diameter are defined as

$$\text{Re} = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{V_{\text{avg}} D}{\nu} \qquad \text{and} \qquad D_h = \frac{4A_c}{p}$$

The flow in a tube is laminar for $\text{Re} < 2300$, turbulent for about $\text{Re} > 10{,}000$, and transitional in between.

The length of the region from the tube inlet to the point at which the boundary layer merges at the centerline is the *hydrodynamic entry length* L_h. The region beyond the entrance region in which the velocity profile is fully developed is the *hydrodynamically fully developed region*. The length of the region of flow over which the thermal boundary layer develops and reaches the tube center is the *thermal entry length* L_t. The region in which the flow is both hydrodynamically and thermally developed is the *fully developed flow region*. The entry lengths are given by

$$L_{h,\,\text{laminar}} \approx 0.05\,\text{Re}\,D$$

$$L_{t,\,\text{laminar}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = \text{Pr}\,L_{h,\,\text{laminar}}$$

$$L_{h,\,\text{turbulent}} \approx L_{t,\,\text{turbulent}} = 10D$$

For $\dot{q}_s = \text{constant}$, the rate of heat transfer is expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m}c_p(T_e - T_i)$$

For $T_s = \text{constant}$, we have

$$\dot{Q} = hA_s\Delta T_{\text{ln}} = \dot{m}c_p(T_e - T_i)$$

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}c_p)$$

$$\Delta T_{\text{ln}} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

For fully developed laminar flow in a circular pipe, we have

Circular tube, laminar ($\dot{q}_s = \text{constant}$): $\quad \text{Nu} = \dfrac{hD}{k} = 4.36$

Circular tube, laminar ($T_s = \text{constant}$): $\quad \text{Nu} = \dfrac{hD}{k} = 3.66$

For *developing laminar flow* in the entrance region with constant surface temperature, we have

Circular tube: $\quad \text{Nu} = 3.66 + \dfrac{0.065(D/L)\text{Re Pr}}{1 + 0.04[(D/L)\text{Re Pr}]^{2/3}}$

Circular tube: $\quad \text{Nu} = 1.86\left(\dfrac{\text{Re Pr}\,D}{L}\right)^{1/3}\left(\dfrac{\mu_b}{\mu_s}\right)^{0.14}$

Parallel plates: $\quad \text{Nu} = 7.54 + \dfrac{0.03(D_h/L)\text{Re Pr}}{1 + 0.016[(D_h/L)\text{Re Pr}]^{2/3}}$

For *fully developed turbulent flow with smooth surfaces,* we have

$$f = (0.790\ln\text{Re} - 1.64)^{-2} \qquad 10^4 < \text{Re} < 10^6$$

$$\text{Nu} = 0.125f\ \text{Re}\,\text{Pr}^{1/3}$$

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{1/3} \quad \begin{pmatrix} 0.7 \le \text{Pr} \le 160 \\ \text{Re} > 10{,}000 \end{pmatrix}$$

$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^n$ with $n = 0.4$ for *heating* and 0.3 for *cooling* of fluid

$$\text{Nu} = \frac{(f/8)(\text{Re} - 1000)\text{Pr}}{1 + 12.7(f/8)^{0.5}(\text{Pr}^{2/3} - 1)} \quad \begin{pmatrix} 0.5 \le \text{Pr} \le 2000 \\ 3 \times 10^3 < \text{Re} < 5 \times 10^6 \end{pmatrix}$$

The fluid properties are evaluated at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$. For liquid metal flow in the range of $10^4 < \text{Re} < 10^6$ we have

$T_s = \text{constant}$: $\quad \text{Nu} = 4.8 + 0.0156\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93}$

$\dot{q}_s = \text{constant}$: $\quad \text{Nu} = 6.3 + 0.0167\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93}$

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PROBLEMS*

19–1C What is forced convection? How does it differ from natural convection? Is convection that is caused by winds forced or natural convection?

19–2C What is external forced convection? How does it differ from internal forced convection? Can a heat transfer system involve both internal and external convection at the same time? Give an example.

19–3C In which mode of heat transfer is the convection heat transfer coefficient usually higher, natural convection or forced convection? Why?

19–4C Consider a hot baked potato. Will the potato cool faster or slower when we blow the warm air coming from our lungs on it instead of letting it cool naturally in the cooler air in the room? Explain.

19–5C What is the physical significance of the Nusselt number? How is it defined?

19–6C When is heat transfer through a fluid conduction and when is it convection? For what case is the rate of heat transfer higher? How does the convection heat transfer coefficient differ from the thermal conductivity of a fluid?

19–7 An average man has a body surface area of 1.8 m^2 and a skin temperature of 33°C. The convection heat transfer coefficient for a clothed person walking in still air is expressed as $h = 8.6V^{0.53}$ for $0.5 < V < 2$ m/s, where V is the walking velocity in m/s. Assuming the average surface temperature of the clothed person to be 30°C, determine the rate of heat loss from an average man walking in still air at 7°C by convection at a walking velocity of (*a*) 0.5 m/s, (*b*) 1.0 m/s, (*c*) 1.5 m/s, and (*d*) 2.0 m/s.

19–8 The convection heat transfer coefficient for a clothed person standing in moving air is expressed as $h = 14.8V^{0.69}$ for $0.15 < V < 1.5$ m/s, where V is the air velocity. For a person with a body surface area of 1.7 m^2 and an average surface temperature of 29°C, determine the rate of heat loss from the person in windy air at 10°C by convection for air velocities of (*a*) 0.5 m/s, (*b*) 1.0 m/s, and (*c*) 1.5 m/s.

19–9 During air cooling of potatoes, the heat transfer coefficient for combined convection, radiation, and evaporation is determined experimentally to be as shown:

Air velocity, m/s	Heat transfer coefficient, W/m²·K
0.66	14.0
1.00	19.1
1.36	20.2
1.73	24.4

Consider an 8-cm-diameter potato initially at 20°C. Potatoes are cooled by refrigerated air at 5°C at a velocity of 1 m/s. Determine the initial rate of heat transfer from a potato and the initial value of the temperature gradient in the potato at the surface.

19–10 The upper surface of a 50-cm-thick solid plate ($k = 237$ W/m·K) is being cooled by water with temperature of 20°C. The upper and lower surfaces of the solid plate are maintained at constant temperatures of 60°C and 120°C, respectively. Determine the water convection heat transfer coefficient and the water temperature gradient at the upper plate surface.

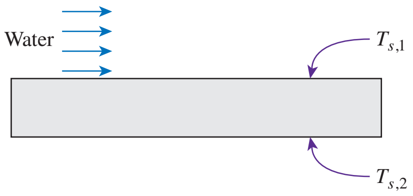


FIGURE P19–10

19–11 Consider airflow over a plate surface maintained at a temperature of 220°C. The temperature profile of the airflow is given as

$$T(y) = T_\infty - (T_\infty - T_s)\exp\left(-\frac{V}{\alpha_{\text{fluid}}}y\right)$$

The airflow at 1 atm has a free stream velocity and temperature of 0.08 m/s and 20°C, respectively. Determine the heat flux on the plate surface and the convection heat transfer coefficient of the airflow.

19–12 During air cooling of oranges, grapefruit, and tangelos, the heat transfer coefficient for combined convection, radiation, and evaporation for air velocities of $0.11 < V < 0.33$ m/s is determined experimentally and is expressed as $h = 5.05\,k_{\text{air}}\,\text{Re}^{1/3}/D$, where the diameter D is the characteristic length. Oranges are cooled by refrigerated air at 3°C and 1 atm at a velocity of 0.3 m/s. Determine (*a*) the initial rate of heat transfer from a 7-cm-diameter orange initially at 15°C with a thermal conductivity of 0.7 W/m·K, (*b*) the value of the initial temperature gradient inside the orange at the surface, and (*c*) the value of the Nusselt number.

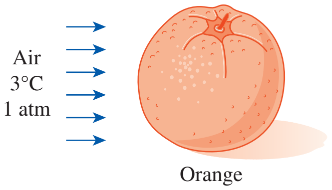


FIGURE P19–12

Flow Over Flat Plates

19–13 In an experiment, the local heat transfer over a flat plate was correlated in the form of the local Nusselt number as expressed by the following correlation

$$\text{Nu}_x = 0.035\text{Re}_x^{0.8}\,\text{Pr}^{1/3}$$

Determine the ratio of the average convection heat transfer coefficient (h) over the entire plate length to the local convection heat transfer coefficient (h_x) at $x = L$.

19–14 Water at 43.3°C flows over a large plate at a velocity of 30.0 cm/s. The plate is 1.0 m long (in the flow direction), and its surface is maintained at a uniform temperature of 10.0°C. Calculate the steady rate of heat transfer per unit width of the plate.

19–15 Hot carbon dioxide exhaust gas at 1 atm is being cooled by flat plates. The gas at 220°C flows in parallel over the upper and lower surfaces of a 1.5-m-long flat plate at a velocity of 3 m/s. If the flat plate surface temperature is maintained at 80°C, determine (*a*) the local convection heat transfer coefficient at 1 m from the leading edge, (*b*) the average convection heat transfer coefficient over the entire plate, and (*c*) the total heat flux transfer to the plate.

19–16 Hot engine oil at 150°C is flowing in parallel over a flat plate at a velocity of 2 m/s. Surface temperature of the 0.5-m-long flat plate is constant at 50°C. Determine (*a*) the local convection heat transfer coefficient at 0.2 m from the leading edge and the average convection heat transfer coefficient, and (*b*) repeat part (*a*) using the Churchill and Ozoe (1973) relation.

19–17 Parallel plates form a solar collector that covers a roof, as shown in the figure. The plates are maintained at 15°C, while ambient air at 10°C flows over the roof with $V = 2$ m/s. Determine the rate of convective heat loss from (*a*) the first plate and (*b*) the third plate.

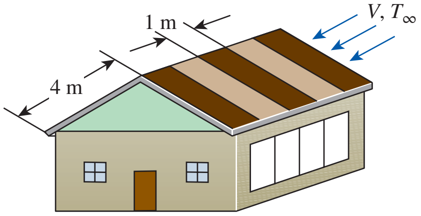


FIGURE P19–17

19–18 Hydrogen gas at 1 atm is flowing in parallel over the upper and lower surfaces of a 3-m-long flat plate at a velocity of 2.5 m/s. The gas temperature is 120°C, and the surface temperature of the plate is maintained at 30°C. Using appropriate software, investigate the local convection heat transfer coefficient and the local total convection heat flux along the plate. By varying the location along the plate for $0.2 \leq x \leq 3$ m, plot the local convection heat transfer coefficient and the local total convection heat flux as functions of x. Assume flow is laminar, but make sure to verify this assumption.

19–19 An array of power transistors, dissipating 5 W of power each, are to be cooled by mounting them on a 25-cm × 25-cm-square aluminum plate and blowing air at 35°C over the plate with a fan at a velocity of 4 m/s. The average temperature of the plate is not to exceed 65°C. Assuming the heat transfer from the back side of the plate to be negligible and disregarding radiation, determine the number of transistors that can be placed on this plate.

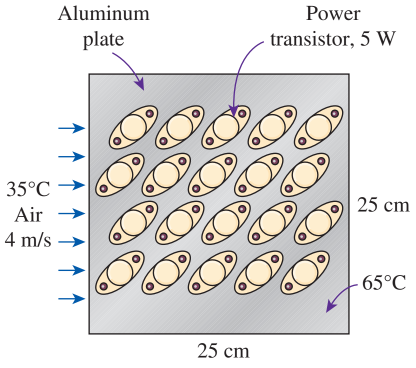


FIGURE P19–19

19–20 Repeat Prob. 19–19 for a location at an elevation of 1610 m where the atmospheric pressure is 83.4 kPa. *Answer:* 5

19–21E Consider a refrigeration truck traveling at 70 mph at a location where the air temperature is 80°F. The refrigerated compartment of the truck can be considered to be a 9-ft-wide, 7-ft-high, and 20-ft-long rectangular box. The refrigeration system of the truck can provide 3 tons of refrigeration (i.e., it can remove heat at a rate of 600 Btu/min). The outer surface of the truck is coated with a low-emissivity material, and thus radiation heat transfer is very small. Determine the average temperature of the outer surface of the refrigeration compartment of the truck if the refrigeration system is observed to be operating at half the capacity. Assume the airflow over the entire outer surface to be turbulent and the heat transfer coefficient at the front and rear surfaces to be equal to that on side surfaces. For air properties evaluations, assume a film temperature of 80°F. Is this a good assumption?

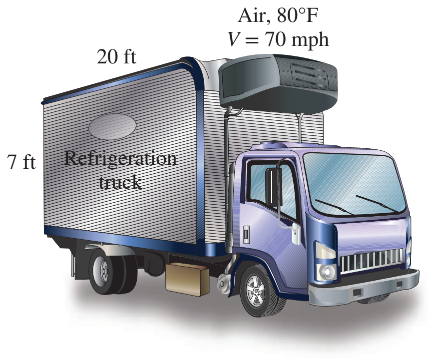


FIGURE P19–21E

19–22 Consider a hot automotive engine, which can be approximated as a 0.5-m-high, 0.40-m-wide, and 0.8-m-long rectangular block. The bottom surface of the block is at a temperature of 100°C and has an emissivity of 0.95. The ambient air is at 20°C, and the road surface is at 25°C. Determine the rate of heat transfer from the bottom surface of the engine block by convection and radiation as the car travels at a velocity of 80 km/h. Assume the flow to be turbulent over the entire surface because of the constant agitation of the engine block.

19–23 A 5-m-long strip of sheet metal is being transported on a conveyor at a velocity of 5 m/s, while the coating on the upper surface is being cured by infrared lamps. The coating on the upper surface of the metal strip has an absorptivity of 0.6 and an emissivity of 0.7, while the surrounding ambient air temperature is 25°C. Radiation heat transfer occurs only on the upper surface, while convection heat transfer occurs on both upper and lower

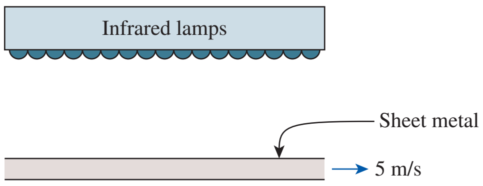


FIGURE P19–23

surfaces of the sheet metal. If the infrared lamps supply a constant heat flux of 5000 W/m^2, determine the surface temperature of the sheet metal. Evaluate the properties of air at 80°C.

19–24 Reconsider Prob. 19–23. Using appropriate software, evaluate the effect of the sheet metal velocity on its surface temperature. By varying the sheet metal velocity from 3 to 30 m/s, plot the surface temperature of the sheet metal as a function of velocity. If the surface temperature of the sheet metal should be maintained above 100°C, determine the minimum velocity that is necessary to satisfy this condition. Evaluate the properties of air at 80°C.

19–25 The local atmospheric pressure in Denver, Colorado (elevation 1610 m), is 83.4 kPa. Air at this pressure and at 30°C flows with a velocity of 6 m/s over a 2.5-m × 8-m flat plate whose temperature is 120°C. Determine the rate of heat transfer from the plate if the air flows parallel to (*a*) the 8-m-long side and (*b*) the 2.5-m side.

19–26E Warm air is blown over the inner surface of an automobile windshield to defrost ice accumulated on the outer surface of the windshield. Consider an automobile windshield ($k_w = 0.8$ Btu/h·ft·R) with an overall height of 20 in and thickness of 0.2 in. The outside air (1 atm) ambient temperature is 8°F, and the average airflow velocity over the outer windshield surface is 50 mph, while the ambient temperature inside the automobile is 77°F. Determine the value of the convection heat transfer coefficient for the warm air blowing over the inner surface of the windshield that is needed to cause the accumulated ice to begin melting. Assume the windshield surface can be treated as a flat-plate surface.

19–27 The top surface of the passenger car of a train moving at a velocity of 95 km/h is 2.8 m wide and 8 m long. The top surface is absorbing solar radiation at a rate of 380 W/m^2, and the temperature of the ambient air is 30°C. Assuming the roof of the car to be perfectly insulated and the radiation heat exchange with the surroundings to be small relative to convection, determine the equilibrium temperature of the top surface of the car. *Answer:* 37.5°C

FIGURE P19–27

19–28 Reconsider Prob. 19–27. Using appropriate software, investigate the effects of the train velocity and the rate of absorption of solar radiation on the equilibrium temperature of the top surface of the car. Let the train velocity vary from 10 km/h to 120 km/h and the rate of solar absorption from 100 W/m^2 to 500 W/m^2. Plot the equilibrium temperature as functions of train velocity and solar radiation absorption rate, and discuss the results.

19–29 Solar radiation is incident on the glass cover of a solar collector at a rate of 700 W/m^2. The glass transmits 88 percent of the incident radiation and has an emissivity of 0.90. The entire hot water needs of a family in summer can be met by two collectors 1.2 m high and 1 m wide. The two collectors are attached to each other on one side so that they appear like a single collector 1.2-m × 2-m in size. The temperature of the glass cover is measured to be 35°C on a day when the surrounding air temperature is 25°C and the wind is blowing at 30 km/h. The effective sky temperature for radiation exchange between the glass cover and the open sky is −40°C. Water enters the tubes attached to the absorber plate at a rate of 1 kg/min. Assuming the back surface of the absorber plate to be heavily insulated and the only heat loss to occur through the glass cover, determine (*a*) the total rate of heat loss from the collector, (*b*) the collector efficiency, which is the ratio of the amount of heat transferred to the water from the solar energy incident on the collector, and (*c*) the temperature rise of water as it flows through the collector.

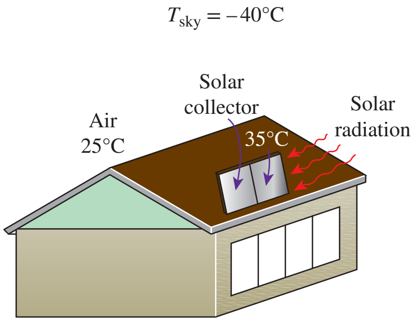


FIGURE P19–29

19–30 Liquid mercury at 250°C is flowing in parallel over a flat plate at a velocity of 0.3 m/s. The surface temperature of the 0.1-m-long flat plate is constant at 50°C. Determine (*a*) the local convection heat transfer coefficient at 5 cm from the leading edge and (*b*) the average convection heat transfer coefficient over the entire plate.

19–31 Water vapor at 250°C is flowing with a velocity of 5 m/s in parallel over a 2-m-long flat plate where there is an unheated starting length of 0.5 m. The heated section of the flat plate is maintained at a constant temperature of 50°C. Determine (*a*) the local convection heat transfer coefficient at the trailing edge, (*b*) the average convection heat transfer coefficient for the heated section, and (*c*) the rate of heat transfer per unit width for the heated section.

19–32 A 15-mm × 15-mm silicon chip is mounted such that the edges are flush in a substrate. The chip dissipates 1.4 W of power uniformly, while air at 20°C (1 atm) with a velocity of 25 m/s is used to cool the upper surface of the chip. If the substrate provides an unheated starting length of 15 mm, determine the surface temperature at the trailing edge of the chip. Evaluate the air properties at 50°C.

19–33 Air is flowing in parallel over the upper surface of a flat plate with a length of 4 m. The first half of the plate length, from the leading edge, has a constant surface temperature of 50°C. The second half of the plate length is subjected to a uniform heat flux of 86 W/m^2. The air has a free-stream velocity and temperature of 2 m/s and 10°C, respectively. Determine the local convection heat transfer coefficients at 1 m and 3 m from the leading edge. As a first approximation, assume the boundary layer over the second portion of the plate with uniform heat flux has not been affected by the first half of the plate with constant surface temperature. Evaluate the air properties at a film temperature of 30°C. Is the film temperature $T_f = 30°\text{C}$ applicable at $x = 3$ m?

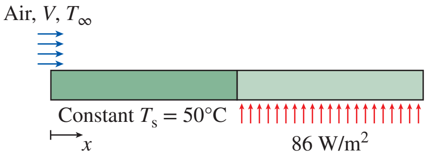


FIGURE P19–33

19–34 Reconsider Prob. 19–33. Using appropriate software, evaluate the local convection heat transfer coefficient, the local surface temperature, and the local film temperature along the plate. By varying the location along the plate for $0.2 \le x \le 4$ m, plot the local convection heat transfer coefficient, the local surface temperature, and the local film temperature as functions of x.

19–35 A 15-cm × 15-cm circuit board dissipating 20 W of power uniformly is cooled by air, which approaches the circuit board at 20°C with a velocity of 6 m/s. Disregarding any heat transfer from the back surface of the board, determine the surface temperature of the electronic components (*a*) at the leading edge and (*b*) at the end of the board. Assume the flow to be turbulent since the electronic components are expected to act as turbulators. For air properties evaluations, assume a film temperature of 35°C. Is this a good assumption?

Flow Across Cylinders and Spheres

19–36C Consider laminar flow of air across a hot circular cylinder. At what point on the cylinder will the heat transfer be highest? What would your answer be if the flow were turbulent?

19–37 A long 12-cm-diameter steam pipe whose external surface temperature is 90°C passes through some open area that is not protected against the winds. Determine the rate of heat loss from the pipe per unit of its length when the air is at 1 atm pressure and 7°C and the wind is blowing across the pipe at a velocity of 65 km/h.

19–38 A heated long cylindrical rod is placed in a crossflow of air at 20°C (1 atm) with velocity of 10 m/s. The rod has a diameter of 5 mm, and its surface has an emissivity of 0.95. If the surrounding temperature is 20°C and the heat flux dissipated from the rod is 16,000 W/m^2, determine the surface temperature of the rod. Evaluate the air properties at 70°C.

19–39E A person extends his uncovered arms into the windy air outside at 54°F and 30 mph in order to feel nature closely (Fig. P19–39E). Initially, the skin temperature of the arm is 84°F. Treating the arm as a 2-ft-long and 3-in-diameter cylinder, determine the rate of heat loss from the arm.

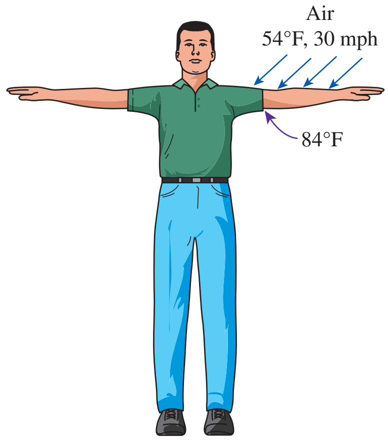


FIGURE P19–39E

19–40E Reconsider Prob. 19–39E. Using appropriate software, investigate the effects of air temperature and wind velocity on the rate of heat loss from the arm. Let the air temperature vary from 20°F to 80°F and the wind velocity from 10 mph to 40 mph. Plot the rate of heat loss as a function of air temperature and of wind velocity, and discuss the results.

19–41 In a geothermal power plant, the used geothermal water at 80°C enters a 15-cm-diameter and 400-m-long uninsulated pipe at a rate of 8.5 kg/s and leaves at 70°C before being reinjected back into the ground. Windy air at 15°C flows normal to the pipe. Disregarding radiation, determine the average wind velocity in km/h.

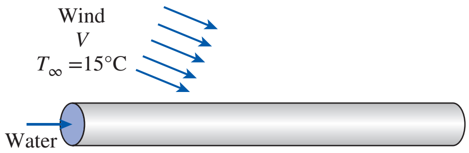


FIGURE P19–41

19–42 A 5-mm-diameter electrical transmission line carries an electric current of 50 A and has a resistance of 0.002 ohm per meter length. Determine the surface temperature of the wire during a windy day when the air temperature is 10°C and the wind is blowing across the transmission line at 50 km/h.

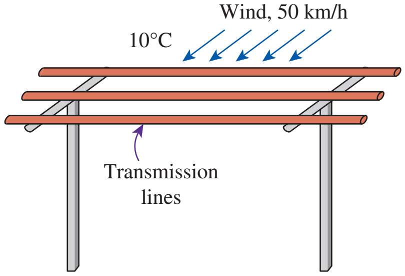


FIGURE P19–42

19–43 Reconsider Prob. 19–42. Using appropriate software, investigate the effect of the wind velocity on the surface temperature of the wire. Let the wind velocity vary from 10 km/h to 80 km/h. Plot the surface temperature as a function of wind velocity, and discuss the results.

19–44 A long aluminum wire of diameter 3 mm is extruded at a temperature of 280°C. The wire is subjected to cross airflow at 20°C at a velocity of 6 m/s. Determine the rate of heat transfer from the wire to the air per meter length when it is first exposed to the air.

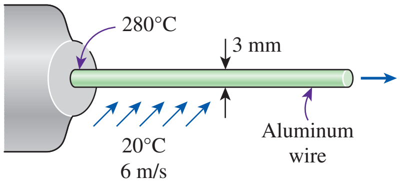


FIGURE P19–44

19–45E Consider a person who is trying to keep cool on a hot summer day by turning a fan on and exposing his entire body to airflow. The air temperature is 85°F, and the fan is blowing air at a velocity of 6 ft/s. If the person is doing light work and generating sensible heat at a rate of 300 Btu/h, determine the average temperature of the outer surface (skin or clothing) of the person. The average human body can be treated as a 1-ft-diameter cylinder with an exposed surface area of 18 ft^2. Disregard any heat transfer by radiation. What would your answer be if the air velocity were doubled? Evaluate the air properties at 100°F. *Answers:* 95.1°F, 91.6°F

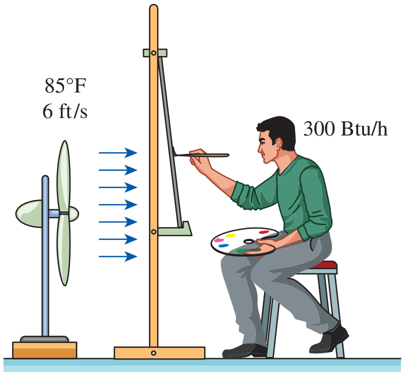


FIGURE P19–45E

19–46E A 12-ft-long, 1.5-kW electrical resistance wire is made of 0.1-in-diameter stainless steel ($k = 8.7$ Btu/h·ft·°F). The resistance wire operates in an environment at 85°F. Determine the surface temperature of the wire if it is cooled by a fan blowing air at a velocity of 20 ft/s. For evaluations of the air properties, the film temperature has to be found iteratively. As an initial guess, assume the film temperature to be 200°F.

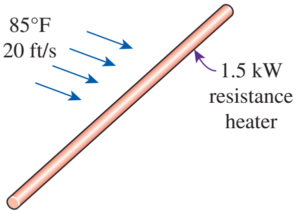


FIGURE P19–46E

19–47 A 0.4-W cylindrical electronic component with diameter 0.3 cm and length 1.8 cm and mounted on a circuit board is cooled by air flowing across it at a velocity of 240 m/min. If the air temperature is 35°C, determine the surface temperature of the component. For air properties evaluations, assume a film temperature of 50°C. Is this a good assumption?

19–48 Consider a 50-cm-diameter and 95-cm-long hot water tank. The tank is placed on the roof of a house. The water inside the tank is heated to 80°C by a flat-plate solar collector during the day. The tank is then exposed to windy air at 18°C with an average velocity of 40 km/h during the night. Estimate the temperature of the tank after a 45-min period. Assume the tank surface to be at the same temperature as the water inside and the heat transfer coefficient on the top and bottom surfaces to be the same as that on the side surface. Evaluate the air properties at 50°C.

19–49 Reconsider Prob. 19–48. Using appropriate software, plot the temperature of the tank as a function of the cooling time as the time varies from 30 min to 5 h, and discuss the results.

19–50 A 10-cm-diameter, 30-cm-high cylindrical bottle contains cold water at 3°C. The bottle is placed in windy air at 27°C. The water temperature is measured to be 11°C after 45 min of cooling. Disregarding radiation effects and heat transfer from the top and bottom surfaces, estimate the average wind velocity.

19–51 A stainless steel ball ($\rho = 8055$ kg/m^3, $c_p = 480$ J/kg·K) of diameter $D = 15$ cm is removed from the oven at a uniform temperature of 125°C. The ball is then subjected to the flow of air at 1 atm pressure and 30°C with a velocity of 6 m/s. The surface temperature of the ball eventually drops to 75°C. Determine the average convection heat transfer coefficient during this cooling process, and estimate how long this process has taken.

19–52 Reconsider Prob. 19–51. Using appropriate software, investigate the effect of air velocity on the average convection heat transfer coefficient and the cooling time. Let the air velocity vary from 1 m/s to 10 m/s. Plot the heat transfer coefficient and the cooling time as a function of air velocity, and discuss the results.

19–53 An average person generates heat at a rate of 84 W while resting. Assuming one-quarter of this heat is lost from the head and disregarding radiation, determine the average surface temperature of the head when it is not covered and is subjected to winds at 10°C and 12 km/h. The head can be approximated as a 30-cm-diameter sphere. Assume a surface temperature of 15°C for evaluation of μ_s. Is this a good assumption? *Answer:* 14.9°C

19–54 An incandescent lightbulb is an inexpensive but highly inefficient device that converts electrical energy into light. It converts about 10 percent of the electrical energy it consumes into light while converting the remaining 90 percent into heat. (A fluorescent lightbulb will give the same amount of light while consuming only one-fourth of the electrical energy, and it will last 10 times longer than an incandescent lightbulb.) The glass bulb of the lamp heats up very quickly as a result of absorbing all that heat and dissipating it to the surroundings by convection and radiation.

Consider a 10-cm-diameter, 100-W lightbulb cooled by a fan that blows air at 30°C to the bulb at a velocity of 2 m/s. The surrounding surfaces are also at 30°C, and the emissivity of the glass is 0.9. Assuming 10 percent of the energy passes through the glass bulb as light with negligible absorption and the rest of the energy is absorbed and dissipated by the bulb itself, determine the equilibrium temperature of the glass bulb. Assume a surface temperature of 100°C for evaluation of μ_s. Is this a good assumption?

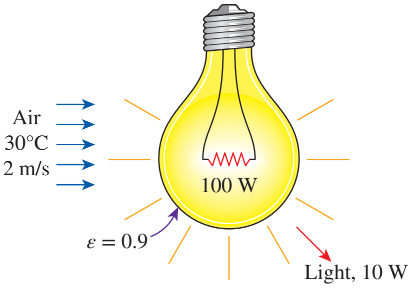


FIGURE P19–54

19–55 A 0.4-m-diameter spherical tank of negligible thickness contains iced water at 0°C. Air at 25°C flows over the tank with a velocity of 3 m/s. Determine the rate of heat transfer to the tank and the rate at which ice melts. The heat of fusion of water at 0°C is 333.7 kJ/kg.

Flow in Tubes

19–56C How is the thermal entry length defined for flow in a tube? In what region is the flow in a tube fully developed?

19–57C Consider laminar forced convection in a circular tube. Will the heat flux be higher near the inlet of the tube or near the exit? Why?

19–58C Consider turbulent forced convection in a circular tube. Will the heat flux be higher near the inlet of the tube or near the exit? Why?

19–59C What does the logarithmic mean temperature difference represent for flow in a tube whose surface temperature is constant? Why do we use the logarithmic mean temperature instead of the arithmetic mean temperature?

19–60C Consider fluid flow in a tube whose surface temperature remains constant. What is the appropriate temperature difference for use in Newton's law of cooling with an average heat transfer coefficient?

19–61C What is the physical significance of the number of transfer units $\text{NTU} = hA_s/\dot{m}c_p$? What do small and large NTU values tell us about a heat transfer system?

19–62 Air enters an 18-cm-diameter, 12-m-long underwater duct at 50°C and 1 atm at a mean velocity of 7 m/s and is cooled by the water outside. If the average heat transfer coefficient is 65 $\text{W/m}^2\cdot\text{K}$ and the tube temperature is nearly equal to the water temperature of 10°C, determine the exit temperature of air and the rate of heat transfer. Evaluate air properties at a bulk mean temperature of 30°C. Is this a good assumption?

19–63 Combustion gases passing through a 5-cm-internal-diameter circular tube are used to vaporize wastewater at atmospheric pressure. Hot gases enter the tube at 115 kPa and 250°C at a mean velocity of 5 m/s and leave at 150°C. If the average heat transfer coefficient is 120 $\text{W/m}^2\cdot\text{K}$ and the inner surface temperature of the tube is 110°C, determine (*a*) the tube length and (*b*) the rate of evaporation of water. Use air properties for the combustion gases.

19–64 Repeat Prob. 19–63 for a heat transfer coefficient of 40 $\text{W/m}^2\cdot\text{K}$.

19–65 Cooling water available at 10°C is used to condense steam at 30°C in the condenser of a power plant at a rate of 0.15 kg/s by circulating the cooling water through a bank of 5-m-long, 1.2-cm-internal-diameter thin copper tubes. Water enters the tubes at a mean velocity of 4 m/s and leaves at a temperature of 24°C. The tubes are nearly isothermal at 30°C. Determine the average heat transfer coefficient between the water and the tubes, and determine the number of tubes needed to achieve the indicated heat transfer rate in the condenser.

19–66 Repeat Prob. 19–65 for steam condensing at a rate of 0.60 kg/s.

19–67 Reconsider Prob. 19–65. Using appropriate software, investigate the effect of the cooling water average (mean) velocity on the number of tubes needed to achieve the indicated heat transfer rate in the condenser. By varying the cooling water average velocity from 0.3 to 6 m/s, plot the number of tubes as a function of the water's average velocity.

19–68 Inside a condenser, there is a bank of seven copper tubes with cooling water flowing in them. Steam condenses at a rate of 0.6 kg/s on the outer surfaces of the tubes, which are at a constant temperature of 68°C. Each copper tube is 5 m long and has an inner diameter of 25 mm. Cooling water enters each tube at 5°C and exits at 60°C. Determine the average heat transfer coefficient of the cooling water flowing inside each tube and the cooling water mean velocity needed to achieve the indicated heat transfer rate in the condenser.

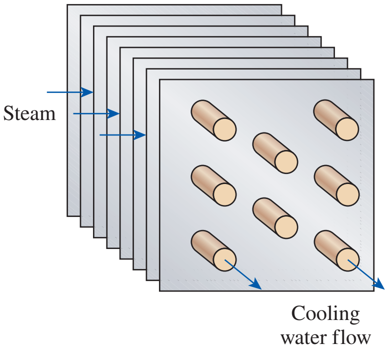


FIGURE P19–68

19–69 Reconsider Prob. 19–68. Using appropriate software, evaluate the effect of the cooling water mean velocity on the rate of steam condensation in the condenser. By varying the cooling water mean velocity for $0 < V_{\text{avg}} \leq 2$ m/s, plot the steam condensation rate as a function of the water mean velocity.

19–70 In a gas-fired boiler, water is being boiled at 120°C by hot air flowing through a 5-m-long, 5-cm-diameter tube submerged in water. Hot air enters the tube at 1 atm and 300°C at a mean velocity of 7 m/s and leaves at 150°C. If the surface temperature of the tube is 120°C, determine the average convection heat transfer coefficient of the air and the rate of water evaporation, in kg/h.

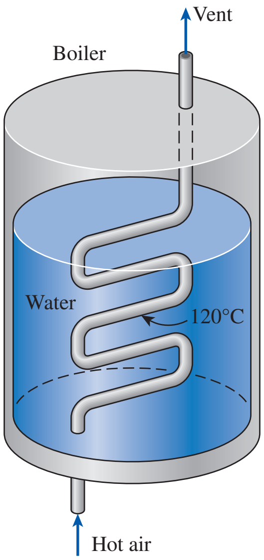


FIGURE P19–70

19–71 Reconsider Prob. 19–70. Using appropriate software, evaluate the effect of the tube length on the average convection heat transfer coefficient of air. By varying the tube length from 3 to 18 m, plot the average convection heat transfer coefficient as a function of the tube length.

Laminar and Turbulent Flow in Tubes

19–72 Consider a 25-mm-diameter and 15-m-long smooth tube that is used for heating fluids. The wall is heated electrically to provide a constant surface heat flux along the entire tube. Fluids enter the tube at 50°C and exit at 150°C. If the mass flow rate is maintained at 0.01 kg/s, determine the convection heat transfer coefficients at the tube outlet for water, engine oil, and liquid mercury.

19–73 Consider a 25-mm-diameter and 15-m-long smooth tube that is maintained at a constant surface temperature. Fluids enter the tube at 50°C with a mass flow rate of 0.01 kg/s. Determine the tube surface temperatures necessary to heat water, engine oil, and liquid mercury to the desired outlet temperature of 150°C.

19–74 In a thermal system, water enters a 25-mm-diameter and 23-m-long circular tube with a mass flow rate of 0.1 kg/s at 25°C. The heat transfer from the tube surface to the water can be expressed in terms of heat flux as $\dot{q}_s(x) = ax$. The coefficient a is 400 W/m^3, and the axial distance from the tube inlet is x measured in meters. Determine (*a*) an expression for the mean temperature $T_m(x)$ of the water, (*b*) the outlet mean temperature of the water, and (*c*) the value of a uniform heat flux $\dot{q}_s$ on the tube surface that would result in the same outlet mean temperature calculated in part (*b*). Evaluate water properties at 35°C.

19–75 Consider a 10-m-long smooth rectangular tube, with $a = 50$ mm and $b = 25$ mm, that is maintained at a constant surface temperature. Liquid water enters the tube at 20°C with a mass flow rate of 0.01 kg/s. Determine the tube surface temperature necessary to heat the water to the desired outlet temperature of 80°C.

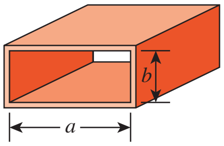


FIGURE P19–75

19–76 A computer cooled by a fan contains eight printed circuit boards (PCBs), each dissipating 12 W of power. The height of the PCBs is 12 cm and the length is 15 cm. The clearance between the tips of the components on the PCB and the back surface of the adjacent PCB is 0.3 cm. The cooling air is supplied by a 10-W fan mounted at the inlet. If the temperature rise of air as it flows through the case of the computer is not to exceed 10°C, determine (*a*) the flow rate of the air that the fan needs to deliver, (*b*) the fraction of the temperature rise of air that is due to the heat generated by the fan and its motor, and (*c*) the highest allowable inlet air temperature if the surface temperature of the components is not to exceed 70°C anywhere in the system. As a first approximation, assume flow is fully developed in the channel. Evaluate properties of air at a bulk mean temperature of 25°C. Is this a good assumption?

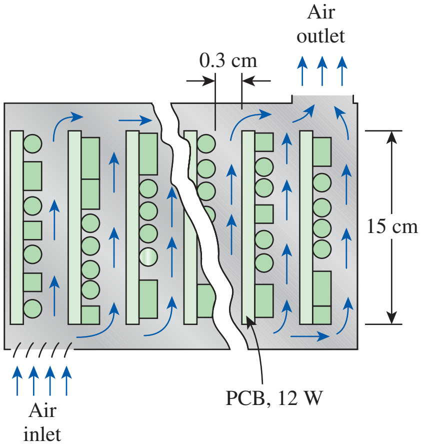


FIGURE P19–76

19–77 In a manufacturing plant that produces cosmetic products, glycerin is being heated by flowing through a 25-mm-diameter and 10-m-long tube. With a mass flow rate of 0.5 kg/s, the flow of glycerin enters the tube at 25°C. The tube surface is maintained at a constant surface temperature of 140°C. Determine the outlet mean temperature and the total rate of heat transfer for the tube. Evaluate the properties for glycerin at 30°C.

19–78 Liquid glycerin is flowing through a 25-mm-diameter and 10-m-long tube. The liquid glycerin enters the tube at 20°C with a mass flow rate of 0.5 kg/s. If the outlet mean temperature is 40°C and the tube surface temperature is constant, determine the surface temperature of the tube.

19–79 Air at 20°C (1 atm) enters into a 5-mm-diameter and 10-cm-long circular tube at an average velocity of 5 m/s. The tube wall is maintained at a constant surface temperature of 160°C. Determine the convection heat transfer coefficient and the outlet mean temperature. Evaluate the air properties at 50°C.

19–80 Glycerin is being heated by flowing between two parallel 1-m-wide and 10-m-long plates with 12.5-mm spacing. The glycerin enters the parallel plates with a temperature of 25°C and a mass flow rate of 0.7 kg/s. The plates have a constant surface temperature of 40°C. Determine the outlet mean temperature of the glycerin and the total rate of heat transfer. Evaluate the properties for glycerin at 30°C. Is this a good assumption?

19–81 Reconsider Prob. 19–80. Using appropriate software, evaluate the effect of the glycerin mass flow rate on the surface temperature of the parallel plates and the total rate of heat transfer necessary to keep the outlet mean temperature of the glycerin at 35°C. By varying the mass flow rate from 0.05 to 6 kg/s, plot the surface temperature of the parallel plates and the total rate of heat transfer as a function of the mass flow rate.

19–82E Air flows in an isothermal tube under fully developed conditions. The inlet temperature is 60°F and the tube surface temperature is 120°F. The tube is 10 ft long, and the inner diameter is 2 in. The air mass flow rate is 18.2 lbm/h. Calculate the exit temperature of the air and the total rate of heat transfer from the tube wall to the air. Evaluate the air properties at a temperature 80°F. Is this a good assumption?

19–83 Water flows through a circular pipe. The pipe's inner diameter is 2 cm, and its length is 6 m. The average velocity of the water is 15 cm/s, and its inlet temperature is 20°C. Assuming that the heating is uniform and the flow is fully developed, what is the heat flux required to raise the water temperature by 1°C by the time it reaches the end of the pipe? What is the surface temperature of the pipe at the exit? Evaluate the properties of water at 20°C.

19–84 Air flows in a pipe under fully developed conditions with an average velocity of 1.25 m/s and a temperature of 20°C. The pipe's inner diameter is 4 cm, and its length is 4 m. The first half of the pipe is kept at a constant wall temperature of 100°C. The second half of the pipe is subjected to a constant heat flux of 200 W. Determine (*a*) the air temperature at the 2 m length, (*b*) the air temperature at the exit, (*c*) the total heat transfer to the air, and (*d*) the wall temperature at the exit of the tube. Evaluate the properties of air at 80°C.

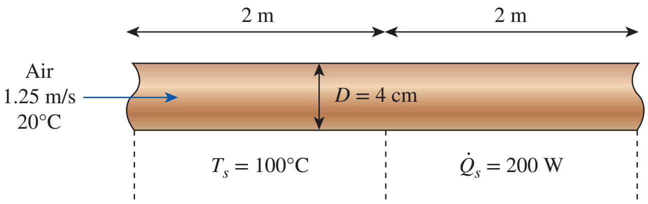


FIGURE P19–84

19–85E Air flows with an average velocity of 3 ft/s in a duct that is 1 ft wide and 1 in high and 40 ft long. The aspect ratio of the duct is very small (0.083), and the duct can be treated as a parallel-plate channel. The duct passes through a heated oven, and the temperature of the duct is approximately constant and equal to 100°F. The air enters the duct at 50°F. For fully developed flow of air in the duct, determine (*a*) the temperature of the air as it leaves the duct, (*b*) the rate of heat transfer from the duct to the air, and (c) the heat flux at the duct exit. Evaluate the properties of air at 75°F. Is this a good assumption?

19–86 Determine the convection heat transfer coefficient for the flow of (*a*) air and (*b*) water at a velocity of 5 m/s in an 8-cm-diameter and 10-m-long tube when the tube is subjected to uniform heat flux from all surfaces. Use fluid properties at 25°C.

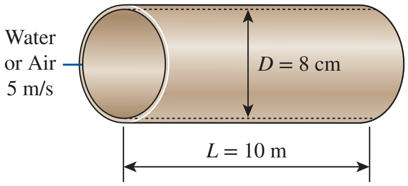


FIGURE P19–86

19–87 Water is to be heated from 10°C to 80°C as it flows through a 2-cm-internal-diameter, 13-m-long tube. The tube is equipped with an electric resistance heater, which provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 5 L/min, determine the power rating of the resistance heater. Also, estimate the inner surface temperature of the pipe at the exit.

19–88 Consider a fluid with a Prandtl number of 7 flowing through a smooth circular tube. Using the Colburn, Petukhov, and Gnielinski equations, determine the Nusselt numbers for Reynolds numbers at 3500, 10^4, and 5×10^5. Compare and discuss the results.

19–89E The hot water needs of a household are to be met by heating water at 55°F to 180°F with a parabolic solar collector at a rate of 5 lbm/s. Water flows through a 1.25-in-diameter thin aluminum tube whose outer surface is anodized black in order to maximize its solar absorption ability. The centerline of the tube coincides with the focal line of the collector, and a glass sleeve is placed outside the tube to minimize the heat losses. If solar energy is transferred to water at a net rate of 350 Btu/h per ft length of the tube, determine the required length of the parabolic collector to meet the hot water requirements of this house. Also, determine the surface temperature of the tube at the exit.

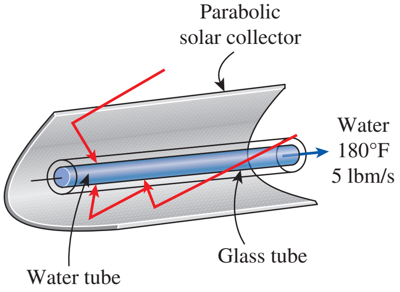


FIGURE P19–89E

19–90 Air (1 atm) enters into a 5-cm-diameter circular tube at 20°C with an average velocity of 5 m/s. The tube wall is maintained at a constant surface temperature of 160°C, and the outlet mean temperature is 80°C. Estimate the length of the tube. Is the flow fully developed?

19–91 An 8-m-long, uninsulated square duct of cross section 0.2 m × 0.2 m and relative roughness 10^{-3} passes through the attic space of a house. Hot air enters the duct at 1 atm and 80°C at a volume flow rate of 0.15 m^3/s. The duct surface is nearly isothermal at 60°C. Determine the rate of heat loss from the duct to the attic space and the pressure difference between the inlet and outlet sections of the duct. Evaluate air properties at a bulk mean temperature of 80°C. Is this a good assumption?

19–92 Hot air at atmospheric pressure and 75°C enters a 10-m-long uninsulated square duct of cross section 0.15 m × 0.15 m that passes through the attic of a house at a rate of 0.2 m^3/s. The duct is observed to be nearly isothermal at 70°C. Determine the exit temperature of the air and the rate of heat loss from the duct to the airspace in the attic. Evaluate air properties at a bulk mean temperature of 75°C. Is this a good assumption?

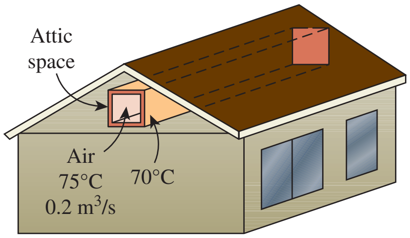


FIGURE P19–92

19–93 Reconsider Prob. 19–92. Using appropriate software, investigate the effect of the volume flow rate of air on the exit temperature of air and the rate of heat loss. Let the flow rate vary from 0.05 m^3/s to 0.15 m^3/s. Plot the exit temperature and the rate of heat loss as a function of flow rate, and discuss the results.

19–94 A concentric annulus tube has inner and outer diameters of 25 mm and 100 mm, respectively. Liquid water flows at a mass flow rate of 0.05 kg/s through the annulus with the inlet and outlet mean temperatures of 20°C and 80°C, respectively. The inner tube wall is maintained with a constant surface temperature of 120°C, while the outer tube surface is insulated. Determine the length of the concentric annulus tube. Assume flow is fully developed.

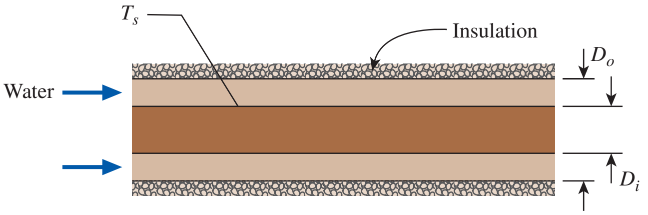


FIGURE P19–94

Review Problems

19–95 Four power transistors, each dissipating 10 W, are mounted on a thin vertical aluminum plate ($k = 237$ W/m·K) 22 cm × 22 cm in size. The heat generated by the transistors is to be dissipated by both surfaces of the plate to the surrounding air at 20°C, which is blown over the plate by a fan at a velocity of 5 m/s. The entire plate can be assumed to be nearly isothermal, and the exposed surface area of the transistor can be taken to be equal to its base area. Determine the temperature of the aluminum plate. Evaluate the air properties at a film temperature of 40°C and 1 atm.

19–96 Oil at 60°C flows at a velocity of 20 cm/s over a 5.0-m-long and 1.0-m-wide flat plate maintained at a constant temperature of 20°C. Determine the rate of heat transfer from the oil to the plate if the average oil properties are $\rho = 880$ kg/m³, $\mu = 0.005$ kg/m·s, $k = 0.15$ W/m·K, and $c_p = 2.0$ kJ/kg·K.

19–97E The passenger compartment of a minivan traveling at 70 mph can be modeled as a 3.2-ft-high, 6-ft-wide, and 12-ft-long rectangular box whose walls have an insulating value of *R*-3 (i.e., a wall thickness-to-thermal conductivity ratio of 3 h·ft²·°F/Btu). The interior of a minivan is maintained at an average temperature of 70°F during a trip at night while the outside air temperature is 90°F.

The average heat transfer coefficient on the interior surfaces of the van is 1.2 Btu/h·ft²·°F. The airflow over the exterior surfaces can be assumed to be turbulent because of the intense vibrations involved, and the heat transfer coefficient on the front and back surfaces can be taken to be equal to that on the top surface. Disregarding any heat gain or loss by radiation, determine the rate of heat transfer from the ambient air to the van. Assume the airflow to be entirely turbulent because of the intense vibrations involved. Use a film temperature of 80°F for evaluations of air properties at 1 atm.

FIGURE P19–97E

19–98 Consider a house that is maintained at a constant temperature of 22°C. One of the walls of the house has three single-pane glass windows that are 1.5 m high and 1.8 m long. The glass ($k = 0.78$ W/m·K) is 0.5 cm thick, and the heat transfer coefficient on the inner surface of the glass is 8 W/m²·K. Now winds at 35 km/h start to blow parallel to the surface of this wall. If the air temperature outside is −2°C, determine the rate of heat loss through the windows of this wall. Assume radiation heat transfer to be negligible. Evaluate the air properties at a film temperature of 5°C and 1 atm.

19–99 An automotive engine can be approximated as a 0.4-m-high, 0.60-m-wide, and 0.7-m-long rectangular block. The bottom surface of the block is at a temperature of 75°C and has an emissivity of 0.92. The ambient air is at 5°C, and the road surface is at 10°C. Determine the rate of heat transfer from the bottom surface of the engine block by convection and radiation as the car travels at a velocity of 60 km/h. Assume the flow to be turbulent over the entire surface because of the constant agitation of the engine block. How will the heat transfer be affected when a 2-mm-thick layer of gunk ($k = 3$ W/m·K) has formed on the bottom surface as a result of the dirt and oil collected on that surface over time? Assume the metal temperature under the gunk is still 75°C.

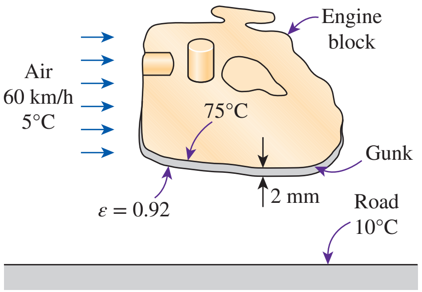


FIGURE P19–99

19–100 To defrost ice accumulated on the outer surface of an automobile windshield, warm air is blown over the inner surface of the windshield. Consider an automobile windshield ($k_w = 1.4$ W/m·K) with an overall height of 0.5 m and thickness of 5 mm. The outside air (1 atm) ambient temperature is −20°C, and the average airflow velocity over the outer windshield surface is 80 km/h, while the ambient temperature inside the automobile is 25°C. Determine the value of the convection heat transfer coefficient for the warm air blowing over the inner surface of the windshield that is needed to cause the accumulated ice to begin melting. Assume the windshield surface can be treated as a flat plate surface.

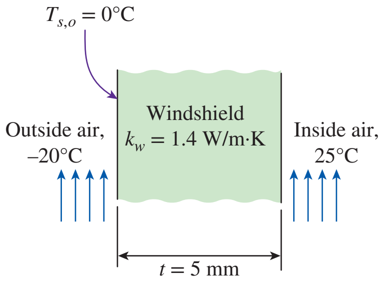


FIGURE P19–100

19–101 The local atmospheric pressure in Denver, Colorado (elevation 1610 m), is 83.4 kPa. Air at this pressure and 20°C flows with a velocity of 8 m/s over a 1.5-m × 6-m flat plate whose temperature is 140°C. Determine the rate of heat transfer from the plate if the air flows parallel to (*a*) the 6-m-long side and (*b*) the 1.5-m side.

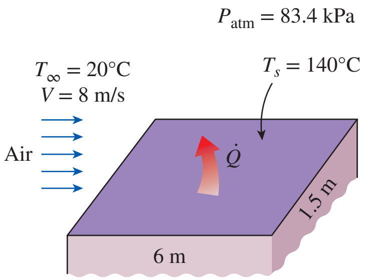


FIGURE P19–101

19–102 A 20-mm × 20-mm silicon chip is mounted such that the edges are flush in a substrate. The substrate provides an unheated starting length of 20 mm that acts as a turbulator. A turbulator is a device that trips the velocity boundary layer to turbulence. Hence the flow is turbulent over the chip. Airflow at 25°C (1 atm) with a velocity of 25 m/s is used to cool the upper surface of the chip. If the maximum surface temperature of the chip cannot exceed 75°C, determine the maximum allowable power dissipation on the chip surface.

19–103 An airstream at 1 atm flows, with a velocity of 15 m/s, in parallel over a 3-m-long flat plate where there is an unheated starting length of 1 m. The airstream has a temperature of 20°C, and the heated section of the flat plate is maintained at a constant temperature of 80°C. Determine (*a*) the local convection heat transfer coefficient at the trailing edge and (*b*) the average convection heat transfer coefficient for the heated section.

19–104 In the effort to increase the removal of heat from a hot surface at 120°C, a cylindrical pin fin ($k_f = 237$ W/m·K) with a diameter of 5 mm is attached to the hot surface. Air at 20°C (1 atm) is flowing across the pin fin with a velocity of 10 m/s. Determine the maximum possible rate of heat transfer from the pin fin. Evaluate the air properties at 70°C.

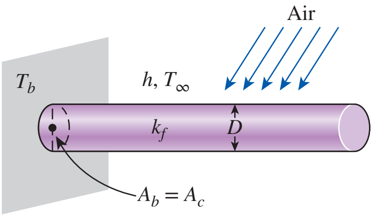


FIGURE P19–104

19–105 Consider a person who is trying to keep cool on a hot summer day by turning a fan on and exposing his body to airflow. The air temperature is 32°C, and the fan is blowing air at a velocity of 5 m/s. The surrounding surfaces are at 40°C, and the emissivity of the person can be taken to be 0.9. If the person is doing light work and generating sensible heat at a rate of 90 W, determine the average temperature of the outer surface (skin or clothing) of the person. The average human body can be treated as a 30-cm-diameter cylinder with an exposed surface area of 1.7 m^2. Evaluate the air properties at a film temperature of 35°C and 1 atm. *Answer:* 36.2°C

19–106E A transistor with a height of 0.32 in and a diameter of 0.22 in is mounted on a circuit board. The transistor is cooled by air flowing over it at a velocity of 600 ft/min. If the air temperature is 120°F and the transistor case temperature is not to exceed 180°F, determine the amount of power this transistor can dissipate safely.

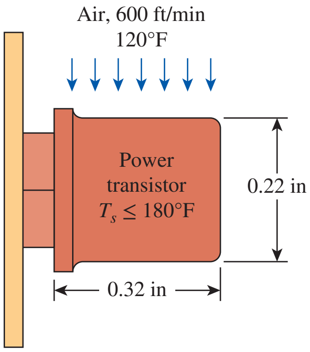


FIGURE P19–106E

19–107 Steam at 250°C flows in a stainless steel pipe ($k = 15$ W/m·K) whose inner and outer diameters are 4 cm and 4.6 cm, respectively. The pipe is covered with 3.5-cm-thick glass wool insulation ($k = 0.038$ W/m·K) whose outer surface has an emissivity of 0.3. Heat is lost to the surrounding air and surfaces at 3°C by convection and radiation. Taking the heat transfer coefficient inside the pipe to be 80 W/m^2·K, determine the rate of heat loss from the steam per unit length of the pipe when air is flowing across the pipe at 4 m/s. Evaluate the air properties at a film temperature of 10°C and 1 atm.

19–108 A 0.55-m-internal-diameter spherical tank made of 1-cm-thick stainless steel ($k = 15$ W/m·K) is used to store iced water at 0°C. The tank is located outdoors at 30°C and is subjected to winds at 8 km/h. Assuming the entire steel tank to be at 0°C and thus its thermal resistance to be negligible, determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period. The heat of fusion of water at atmospheric pressure is $h_{if} = 333.7$ kJ/kg. Disregard any heat transfer by radiation.

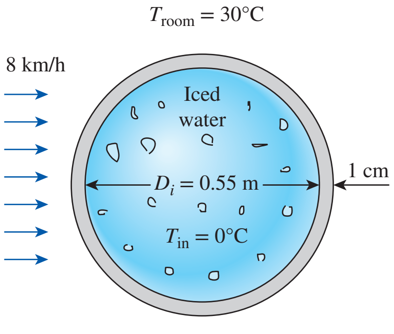


FIGURE P19–108

19–109 Repeat Prob. 19–108, assuming the inner surface of the tank to be at 0°C but by taking the thermal resistance of the tank and heat transfer by radiation into consideration. Assume the average surrounding surface temperature for radiation exchange to be 25°C and the outer surface of the tank to have an emissivity of 0.75. *Answers:* (*a*) 379 W, (*b*) 98.1 kg

19–110 A fluid ($\rho = 1000$ kg/m^3, $\mu = 1.4 \times 10^{-3}$ kg/m·s, $c_p = 4.2$ kJ/kg·K, and $k = 0.58$ W/m·K) flows with an average velocity of 0.3 m/s through a 14-m-long tube with inside diameter of 0.01 m. Heat is uniformly added to the entire tube at the rate of 1500 W/m^2. Determine (*a*) the value of convection heat transfer coefficient at the exit, (*b*) the value of $T_s - T_m$, and (*c*) the value of $T_e - T_i$.

19–111 Crude oil at 22°C enters a 20-cm-diameter pipe with an average velocity of 32 cm/s. The average pipe wall temperature is 2°C. Crude oil properties are as given below. Calculate the rate of heat transfer and the pipe length if the crude oil outlet temperature is 20°C.

T °C	ρ kg/m^3	k W/m·K	μ mPa·s	c_p kJ/kg·K
2.0	900	0.145	60.0	1.80
22.0	890	0.145	20.0	1.90

19–112 Air (1 atm) enters a 5-mm-diameter circular tube at an average velocity of 5 m/s. The tube wall is maintained at a constant surface temperature. Determine the convection heat transfer coefficient for (*a*) a 10-cm-long tube and (*b*) a 50-cm-long tube. Evaluate the air properties at 50°C.

19–113 To cool a storehouse in the summer without using a conventional air-conditioning system, the owner decided to hire an engineer to design an alternative system that would make use of the water in the nearby lake. The engineer decided to flow air through a thin, smooth, 10-cm-diameter copper tube that is submerged in the lake. The water in the lake is typically at a constant temperature of 15°C and a convection heat transfer coefficient of 1000 W/m^2·K. If air (1 atm) enters the copper tube at a mean temperature of 30°C with an average velocity of 2.5 m/s, determine the necessary copper tube length so that the outlet mean temperature of the air is 20°C.

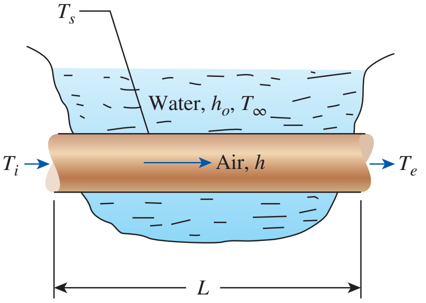


FIGURE P19–113

19–114 Liquid mercury is flowing at 0.6 kg/s through a 5-cm-diameter tube with inlet and outlet mean temperatures of 100°C and 200°C, respectively. The tube surface temperature is kept constant at 250°C. Determine the tube length using (*a*) the appropriate Nusselt number relation for liquid metals and (*b*) the Dittus–Boelter equation. (*c*) Compare the results of (*a*) and (*b*).

19–115 In the effort to find the best way to cool a smooth, thin-walled copper tube, an engineer decided to flow air either through the tube or across the outer tube surface. The tube has a diameter of 5 cm, and the surface temperature is held constant. Determine (*a*) the convection heat transfer coefficient when air is flowing through its inside at 25 m/s with a bulk mean temperature of 50°C and (*b*) the convection heat transfer coefficient when air is flowing across its outer surface at 25 m/s with a film temperature of 50°C.

19–116 Water is heated at a rate of 10 kg/s from a temperature of 15°C to 35°C by passing it through five identical tubes, each 5.0 cm in diameter, whose surface temperature is 60.0°C. Estimate (*a*) the steady rate of heat transfer and (*b*) the length of tubes needed to accomplish this task.

19–117 Repeat Prob. 19–116 for a flow rate of 20 kg/s.

19–118 Water at 1500 kg/h and 10°C enters a 10-mm-diameter smooth tube whose wall temperature is maintained at 49°C. Calculate (*a*) the tube length necessary to heat the water to 40°C, and (*b*) the water outlet temperature if the tube length is doubled. Assume average water properties to be the same as in (*a*).

19–119E The exhaust gases of an automotive engine leave the combustion chamber and enter an 8-ft-long and 3.5-in-diameter thin-walled steel exhaust pipe at 800°F and 15.5 psia at a rate of 0.05 lbm/s. The surrounding ambient air is at a temperature of 80°F, and the heat transfer coefficient on the outer surface of the exhaust pipe is 3 Btu/h·ft^2·°F. Assuming the exhaust gases to have the properties of air, determine (*a*) the velocity of the exhaust gases at the inlet of the exhaust pipe and (*b*) the temperature at which the exhaust gases will leave the pipe and enter the air.

19–120E Air is flowing through a smooth, thin-walled, 4-in-diameter copper tube that is submerged in water. The water maintains a constant temperature of 60°F and a convection heat transfer coefficient of 176 Btu/h·ft^2·R. If air (1 atm) enters the copper tube at a mean temperature of 90°F with an average velocity of 8 ft/s, determine the necessary copper tube length so that the outlet mean temperature of the air is 70°F.

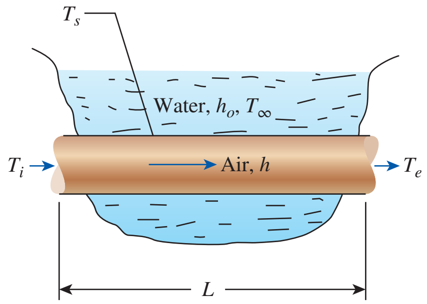


FIGURE P19–120E

19–121 Hot water at 90°C enters a 15-m section of a cast iron pipe (k = 52 W/m·K) whose inner and outer diameters are 4 and 4.6 cm, respectively, at an average velocity of 1.2 m/s. The outer surface of the pipe, whose emissivity is 0.7, is exposed to the cold air at 10°C in a basement, with a convection heat transfer coefficient of 12 W/m^2·K. Taking the walls of the basement to be at 10°C also, determine (*a*) the rate of heat loss from the water and (*b*) the temperature at which the water leaves the basement.

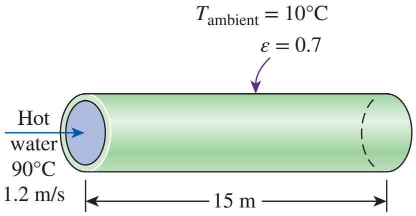


FIGURE P19–121

19–122 Repeat Prob. 19–121 for a pipe made of copper (k = 386 W/m·K) instead of cast iron.

19–123 Hot exhaust gases leaving a stationary diesel engine at 450°C enter a 15-cm-diameter pipe at an average velocity of 7.2 m/s. The surface temperature of the pipe is 180°C. Determine the pipe length if the exhaust gases are to leave the pipe at 250°C after transferring heat to water in a heat recovery unit. Use the properties of air for exhaust gases.

19–124 Geothermal steam at 165°C condenses in the shell side of a heat exchanger over the tubes through which water flows. Water enters the 4-cm-diameter, 14-m-long tubes at 20°C at a rate of 0.8 kg/s. Determine the exit temperature of water and the rate of condensation of geothermal steam.

19–125 Cold air at 5°C enters a 12-cm-diameter, 20-m-long isothermal pipe at a velocity of 2.5 m/s and leaves at 19°C. Estimate the surface temperature of the pipe.

19–126 100 kg/s of crude oil is heated from 20°C to 40°C through the tube side of a multitube heat exchanger. The crude oil flow is divided evenly among all 100 tubes in the tube bundle. The ID of each tube is 10 mm, and the inside tube-wall temperature is maintained at 100°C. Average properties of the crude oil are $\rho = 950$ kg/m^3, $c_p = 1.9$ kJ/kg·K, $k =$ 0.25 W/m·K, $\mu = 12$ mPa·s, and $\mu_w = 4$ mPa·s. Estimate the rate of heat transfer and the tube length.

Design and Essay problems

19–127 Conduct this experiment to determine the heat loss coefficient of your house or apartment in W/°C or Btu/h·°F. First make sure that the conditions in the house are steady and the house is at the set temperature of the thermostat. Use an outdoor thermometer to monitor outdoor temperature. One evening, using a watch or timer, determine how long the heater was on during a 3-h period and the average outdoor temperature during that period. Then using the heat output rating of your heater, determine the amount of heat supplied. Also, estimate the amount of heat generation in the house during that period by noting the number of people, the total wattage of lights that were on, and the heat generated by the appliances and equipment. Using that information, calculate the average rate of heat loss from the house and the heat loss coefficient.

19–128 Electronic boxes such as computers are commonly cooled by a fan. Write an essay on forced air cooling of electronic boxes and on the selection of the fan for electronic devices.

19–129 Design a heat exchanger to pasteurize milk with steam in a dairy plant. Milk is to flow through a bank of 1.2-cm-internal-diameter tubes while steam condenses outside the tubes at 1 atm. Milk is to enter the tubes at 4°C, and it is to be heated to 72°C at a rate of 15 L/s. Making reasonable assumptions, you are to specify the tube length and the number of tubes and the pump for the heat exchanger.

19–130 A desktop computer is to be cooled by a fan. The electronic components of the computer consume 80 W of power under full-load conditions. The computer is to operate in environments at temperatures up to 50°C and at elevations up to 3000 m where the atmospheric pressure is 70.12 kPa. The exit temperature of air is not to exceed 60°C to meet reliability requirements. Also, the average velocity of air is not to exceed 120 m/min at the exit of the computer case where the fan is installed; this is to keep the noise level down. Specify the flow rate of the fan that needs to be installed and the diameter of the casing of the fan.

CHAPTER

20

NATURAL CONVECTION

In Chap. 19, we considered heat transfer by *forced convection,* where a fluid was *forced* to move over a surface or in a tube by external means such as a pump or a fan. In this chapter, we consider *natural convection,* alternatively called free convection, where any fluid motion occurs by natural means such as buoyancy. The fluid motion in forced convection is quite noticeable, since a fan or a pump can transfer enough momentum to the fluid to move it in a certain direction. The fluid motion in natural convection, however, is often not noticeable because of the low velocities involved.

The convection heat transfer coefficient is a strong function of *velocity:* the higher the velocity, the higher the convection heat transfer coefficient. The fluid velocities associated with natural convection are low, typically less than 1 m/s. Therefore, the heat transfer coefficients encountered in natural convection are usually much lower than those encountered in forced convection. Yet several types of heat transfer equipment are designed to operate under natural convection conditions instead of forced convection because natural convection does not require the use of a fluid mover.

We start this chapter with a discussion of the physical mechanism of *natural convection* and the *Grashof number.* We then present the correlations to evaluate heat transfer by natural convection for various geometries, including enclosures.

OBJECTIVES

The objectives of this chapter are to:

- Understand the physical mechanism of natural convection.
- Derive the governing equations of natural convection, and obtain the dimensionless Grashof number by nondimensionalizing them.
- Evaluate the Nusselt number for natural convection associated with vertical, horizontal, and inclined plates as well as cylinders and spheres.
- Analyze natural convection inside enclosures such as double-pane windows.

20–1 ▪ PHYSICAL MECHANISM OF NATURAL CONVECTION

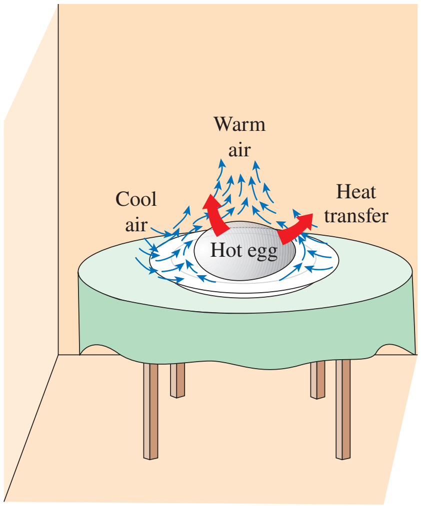


FIGURE 20–1
The cooling of a boiled egg in a cooler environment by natural convection.

Many familiar heat transfer applications involve natural convection as the primary mechanism of heat transfer. Some examples are cooling of electronic equipment such as power transistors, TVs, and DVDs; heat transfer from electric baseboard heaters or steam radiators; heat transfer from the refrigeration coils and power transmission lines; and heat transfer from the bodies of animals and human beings. Natural convection in gases is usually accompanied by radiation of comparable magnitude except for low-emissivity surfaces.

We know that a hot boiled egg (or a hot baked potato) on a plate eventually cools to the surrounding air temperature (Fig. 20–1). The egg is cooled by transferring heat by convection to the air and by radiation to the surrounding surfaces. Disregarding heat transfer by radiation, the physical mechanism of cooling a hot egg (or any hot object) in a cooler environment can be explained as follows:

As soon as the hot egg is exposed to cooler air, the temperature of the outer surface of the eggshell drops somewhat, and the temperature of the air adjacent to the shell rises as a result of heat conduction from the shell to the air. Consequently, the egg is surrounded by a thin layer of warmer air, and heat is then transferred from this warmer layer to the outer layers of air. The cooling process in this case is rather slow since the egg would always be blanketed by warm air, and it has no direct contact with the cooler air farther away. We may not notice any air motion in the vicinity of the egg, but careful measurements would indicate otherwise.

The temperature of the air adjacent to the egg is higher and thus its density is lower, since at constant pressure the density of a gas is inversely proportional to its temperature. Thus, we have a situation in which some low-density or "light" gas is surrounded by a high-density or "heavy" gas, and the natural laws dictate that the *light gas rise.* This is no different than the oil in a vinegar-and-oil salad dressing rising to the top (since $\rho_{\text{oil}} < \rho_{\text{vinegar}}$). This phenomenon is characterized incorrectly by the phrase "heat rises," which is understood to mean *heated air rises.* The space vacated by the warmer air in the vicinity of the egg is replaced by the cooler air nearby, and the presence of cooler air in the vicinity of the egg speeds up the cooling process. The rise of warmer air and the flow of cooler air into its place continues until the egg is cooled to the temperature of the surrounding air. The motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a **natural convection current**, and the heat transfer that is enhanced as a result of this natural convection current is called **natural convection heat transfer**. Note that in the absence of natural convection currents, heat transfer from the egg to the air surrounding it would be by conduction only, and the rate of heat transfer from the egg would be much lower.

Natural convection is just as effective in the heating of cold surfaces in a warmer environment as it is in the cooling of hot surfaces in a cooler environment, as shown in Fig. 20–2. Note that the direction of fluid motion is reversed in this case.

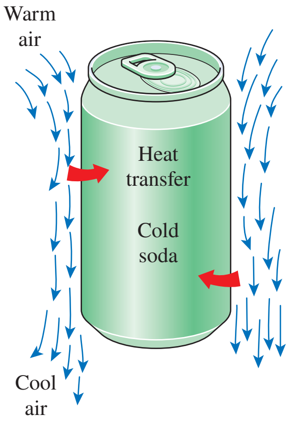


FIGURE 20–2
The warming up of a cold drink in a warmer environment by natural convection.

In a gravitational field, there is a net force that pushes upward a light fluid placed in a heavier fluid. The upward force exerted by a fluid on a body completely or partially immersed in it is called the **buoyancy force**. The magnitude of the buoyancy force is equal to the weight of the *fluid displaced* by the body. That is,

$$F_{\text{buoyancy}} = \rho_{\text{fluid}}\, g V_{\text{body}} \tag{20–1}$$

where ρ_{fluid} is the average density of the *fluid* (not the body), g is the gravitational acceleration, and V_{body} is the volume of the portion of the body immersed in the fluid (for bodies completely immersed in the fluid, it is the total volume of the body). In the absence of other forces, the net vertical force acting on a body is the difference between the weight of the body and the buoyancy force. That is,

$$\begin{aligned} F_{\text{net}} &= W - F_{\text{buoyancy}} \\ &= \rho_{\text{body}}\, g V_{\text{body}} - \rho_{\text{fluid}}\, g V_{\text{body}} \\ &= (\rho_{\text{body}} - \rho_{\text{fluid}})\, g V_{\text{body}} \end{aligned} \tag{20–2}$$

Note that this force is proportional to the difference in the densities of the fluid and the body immersed in it. Thus, a body immersed in a fluid will experience a "weight loss" in an amount equal to the weight of the fluid it displaces. This is known as *Archimedes' principle.*

To have a better understanding of the buoyancy effect, consider an egg dropped into water. If the average density of the egg is greater than the density of water (a sign of freshness), the egg settles at the bottom of the container. Otherwise, it rises to the top. When the density of the egg equals the density of water, the egg settles somewhere in the water while remaining completely immersed, acting like a "weightless object" in space. This occurs when the upward buoyancy force acting on the egg equals the weight of the egg, which acts downward.

The *buoyancy effect* has far-reaching implications in life. For one thing, without buoyancy, heat transfer between a hot (or cold) surface and the fluid surrounding it would be by *conduction* instead of by *natural convection.* The natural convection currents encountered in the oceans, lakes, and the atmosphere owe their existence to buoyancy. Also, light boats as well as heavy warships made of steel float on water because of buoyancy (Fig. 20–3). Ships are designed on the basis of the principle that the entire weight of a ship and its contents is equal to the weight of the water that the submerged volume of the ship can contain. The "chimney effect" that induces the upward flow of hot combustion gases through a chimney is also due to the buoyancy effect, and the upward force acting on the gases in the chimney is proportional to the difference between the densities of the hot gases in the chimney and the cooler air outside. Note that there is *no noticeable gravity* in space, and thus there can be no natural convection heat transfer in a spacecraft, even if the spacecraft is filled with atmospheric air.

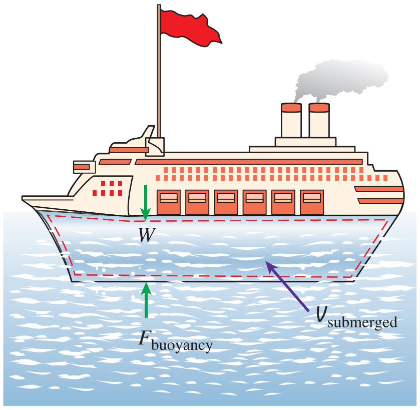


FIGURE 20–3
It is the buoyancy force that keeps ships afloat in water ($W = F_{\text{buoyancy}}$ for floating objects).

In heat transfer studies, the primary variable is *temperature,* and it is desirable to express the net buoyancy force (Eq. 20–2) in terms of temperature differences. But this requires expressing the density difference in terms of a temperature difference, which requires a knowledge of a property that represents the *variation of the density of a fluid with temperature at constant pressure. The property that provides that information is the* **volume expansion coefficient** β, defined as (Fig. 20–4)

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad \text{(1/K)} \tag{20–3}$$

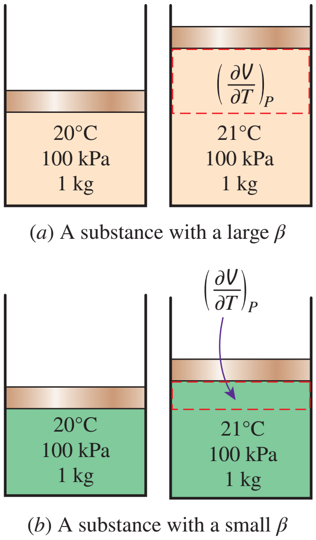
In natural convection studies, the condition of the fluid sufficiently far from the hot or cold surface is indicated by the subscript "infinity" to serve as a reminder that this is the value at a distance where the presence of the surface is not felt. In such cases, the volume expansion coefficient can be expressed approximately by replacing differential quantities with differences as

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \qquad \text{(at constant } P\text{)} \tag{20–4}$$

or

$$\rho_\infty - \rho = \rho\beta(T - T_\infty) \qquad \text{(at constant } P\text{)} \tag{20–5}$$

where ρ_∞ is the density and T_∞ is the temperature of the quiescent fluid away from the surface.



(*a*) A substance with a large β

(*b*) A substance with a small β

FIGURE 20–4
The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure.

We can show easily that the volume expansion coefficient β of an *ideal gas* ($P = \rho RT$) at a temperature T is equivalent to the inverse of the temperature:

$$\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\text{K}) \tag{20–6}$$

where T is the *thermodynamic* temperature. Note that a large value of β for a fluid means a large change in density with temperature, and that the product $\beta\Delta T$ represents the fraction of volume change of a fluid that corresponds to a temperature change ΔT at constant pressure. Also note that the buoyancy force is proportional to the *density difference,* which is proportional to the *temperature difference* at constant pressure. Therefore, the larger the temperature difference between the fluid adjacent to a hot (or cold) surface and the fluid away from it, the *larger* the buoyancy force and the *stronger* the natural convection currents, and thus the *higher* the heat transfer rate.

The magnitude of the natural convection heat transfer between a surface and a fluid is directly related to the *flow rate* of the fluid. The higher the flow rate, the higher the heat transfer rate. In fact, it is the very high flow rates that increase the heat transfer coefficient by orders of magnitude when forced convection is used. In natural convection, no blowers are used, and therefore the flow rate cannot be controlled externally. The flow rate in this case is established by the dynamic balance of *buoyancy* and *friction.*

As we have discussed, the buoyancy force is caused by the density difference between the heated (or cooled) fluid adjacent to the surface and the fluid surrounding it, and it is proportional to this density difference and the volume occupied by the warmer fluid. It is also well known that whenever two bodies in contact (solid–solid, solid–fluid, or fluid–fluid) move relative to each other, a *friction force* develops at the contact surface in the direction opposite to that of the motion. This opposing force slows down the fluid and thus reduces the flow rate of the fluid. Under steady conditions, the airflow rate driven by buoyancy is established at the point where these two effects *balance* each other. The friction force increases as more and more solid surfaces are introduced, seriously disrupting the fluid flow and heat transfer. For that reason, heat sinks with closely spaced fins are not suitable for natural convection cooling.

Most heat transfer correlations in natural convection are based on experimental measurements. The instrument often used in natural convection experiments is the *Mach–Zehnder interferometer,* which gives a plot of isotherms in the fluid in the vicinity of a surface. The operation principle of interferometers is based on the fact that at low pressure, the lines of constant temperature for a gas correspond to the lines of constant density, and that the index of refraction of a gas is a function of its density. Therefore, the degree of refraction of light at some point in a gas is a measure of the temperature gradient at that point. An interferometer produces a map of interference fringes, which can be interpreted as lines of *constant temperature* as shown in Fig. 20–5. The smooth and parallel lines in (a) indicate that the flow is *laminar,* whereas the eddies and irregularities in (b) indicate that the flow is *turbulent.* Note that the lines are closest near the surface, indicating a *higher temperature gradient.*

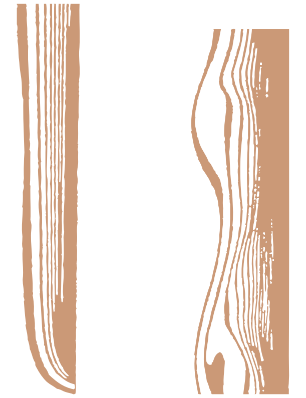

(*a*) Laminar flow (*b*) Turbulent flow

FIGURE 20–5
Isotherms in natural convection over a hot plate in air.

20–2 ▪ EQUATION OF MOTION AND THE GRASHOF NUMBER

In this section we derive the equation of motion that governs the natural convection flow in laminar boundary layer. The conservation of mass and energy equations for forced convection are also applicable for natural convection, but the momentum equation needs to be modified to incorporate buoyancy.

Consider a vertical hot flat plate immersed in a quiescent fluid body. We assume the natural convection flow to be steady, laminar, and two-dimensional, and the fluid to be Newtonian with constant properties, including density, with one exception: the density difference $\rho - \rho_\infty$ is to be considered since it is this density difference between the inside and the outside of the boundary layer that gives rise to buoyancy force and sustains flow. (This is known as the *Boussinesq approximation.*) We take the upward direction along the plate to be x, and the direction normal to the surface to be y, as shown in Fig. 20–6. Therefore, gravity acts in the $-x$-direction. Noting that the flow is steady and two-dimensional, the x- and y-components of velocity within the boundary layer are $u = u(x, y)$ and $v = v(x, y)$, respectively.

The velocity and temperature profiles for natural convection over a vertical hot plate are also shown in Fig. 20–6. Note that as in forced convection, the thickness of the boundary layer increases in the flow direction. Unlike forced convection, however, the fluid velocity is *zero* at the outer edge of the velocity boundary layer as well as at the surface of the plate. This is expected since the fluid beyond the boundary layer is motionless. Thus, the fluid velocity increases with distance from the surface, reaches a maximum, and gradually decreases to zero at a distance sufficiently far from the surface. At the surface, the fluid temperature is equal to the plate temperature, and gradually decreases to the temperature of the surrounding fluid at a distance sufficiently far from the surface, as shown in the figure. In the case of *cold surfaces,* the shape of the velocity and temperature profiles remains the same, but their direction is reversed.

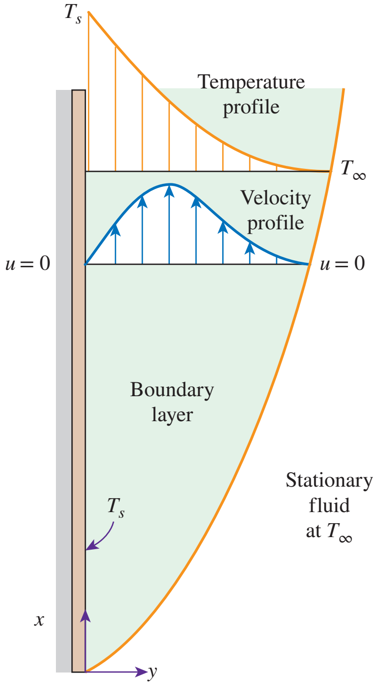


FIGURE 20–6
Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature T_s inserted in a fluid at temperature T_∞.

Consider a differential volume element of height dx, length dy, and unit depth in the z-direction (normal to the paper) for analysis. The forces acting on this volume element are shown in Fig. 20–7. Newton's second law of motion for this volume element can be expressed as

$$\delta m \cdot a_x = F_x \qquad \textbf{(20–7)}$$

where $\delta m = \rho(dx \cdot dy \cdot 1)$ is the mass of the fluid within the volume element. The acceleration in the x-direction is obtained by taking the total differential of $u(x, y)$, which is $du = (\partial u/\partial x)dx + (\partial u/\partial y)dy$, and dividing it by dt. We get

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \qquad \textbf{(20–8)}$$

The forces acting on the differential volume element in the vertical direction are the pressure forces acting on the top and bottom surfaces, the shear stresses acting on the side surfaces (the normal stresses acting on the top and bottom surfaces are small and are disregarded), and the force of gravity acting on the entire volume element. Then the net surface force acting in the x-direction becomes

$$\begin{aligned} F_x &= \left(\frac{\partial \tau}{\partial y}dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x}dx\right)(dy \cdot 1) - \rho g(dx \cdot dy \cdot 1) \\ &= \left(\mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g\right)(dx \cdot dy \cdot 1) \end{aligned} \qquad \textbf{(20–9)}$$

since $\tau = \mu(\partial u/\partial y)$. Substituting Eqs. 20–8 and 20–9 into Eq. 20–7 and dividing by $\rho \cdot dx \cdot dy \cdot 1$ gives Newton's second law of motion in the x-direction (the x-momentum equation) as

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g \qquad \textbf{(20–10)}$$

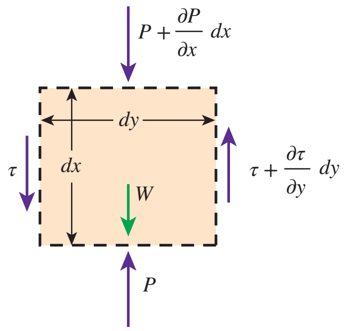


FIGURE 20–7
Forces acting on a differential volume element in the natural convection boundary layer over a vertical flat plate.

The x-momentum equation in the quiescent fluid outside the boundary layer can be obtained from the relation above as a special case by setting $u = 0$. It gives

$$\frac{\partial P_\infty}{\partial x} = -\rho_\infty g \qquad \textbf{(20–11)}$$

which is simply the relation for the variation of hydrostatic pressure in a quiescent fluid with height, as expected. Also, noting that $v \ll u$ in the boundary layer and thus $\partial v/\partial x \approx \partial v/\partial y \approx 0$, and that there are no body forces (including gravity) in the y-direction, the force balance in that direction gives $\partial P/\partial y = 0$. That is, the variation of pressure in the direction normal to the surface is negligible, and for a given x the pressure in the boundary layer is equal to the pressure in the quiescent fluid. Therefore, $P = P(x) = P_\infty(x)$ and $\partial P/\partial x = \partial P_\infty/\partial x = -\rho_\infty g$. Substituting into Eq. 20–10,

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g \tag{20–12}$$

The last term represents the net upward force per unit volume of the fluid (the difference between the buoyant force and the fluid weight). This is the force that initiates and sustains convection currents.

From Eq. 20–5, we have $\rho_\infty - \rho = \rho\beta(T - T_\infty)$. Substituting it into the last equation and dividing both sides by ρ gives the desired form of the x-momentum equation,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \tag{20–13}$$

This is the equation that governs the fluid motion in the boundary layer due to the effect of buoyancy. Note that the momentum equation involves the temperature, and thus the momentum and energy equations must be solved simultaneously.

The complete set of conservation equations, continuity, momentum, and energy that govern natural convection flow over vertical isothermal plates are:

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

Energy: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

with the following boundary conditions (see Fig. 20–6):

At $y = 0$: $u(x, 0) = 0,\ v(x, 0) = 0,$ $T(x, 0) = T_s$
At $y \to \infty$: $u(x, \infty) \to 0,$ $T(x, \infty) \to T_\infty$
At $x = 0$: $u(0, y) = 0,$ $T(0, y) = T_\infty$

The Grashof Number

The governing equations of natural convection and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by suitable constant quantities: all lengths by a characteristic length L_c, all velocities by an arbitrary reference velocity V (which, from the definition of Reynolds number, is taken to be $V = \text{Re}_L\,\nu/L_c$), and temperature by a suitable temperature difference (which is taken to be $T_s - T_\infty$) as

$$x^* = \frac{x}{L_c},\quad y^* = \frac{y}{L_c},\quad u^* = \frac{u}{V},\quad v^* = \frac{v}{V},\quad \text{and}\quad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

where asterisks are used to denote nondimensional variables. The nondimensionalized forms of the continuity and energy equations are still applicable here.

However, the nondimensionalized momentum equation due to buoyancy effects will be different. Nondimensionalizing the momentum equation (Eq. 20–13) with the preceding parameters and simplifying gives

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\right]\frac{T^*}{\mathrm{Re}_L^2} + \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} \tag{20–14}$$

The dimensionless parameter in the brackets represents the natural convection effects and is called the **Grashof number** Gr_L,

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \tag{20–15}$$

where

g = gravitational acceleration, m/s^2
β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C
L_c = characteristic length of the geometry, m
ν = kinematic viscosity of the fluid, m^2/s

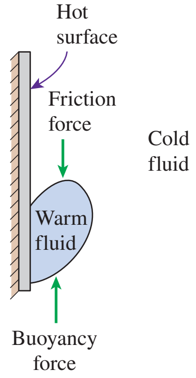
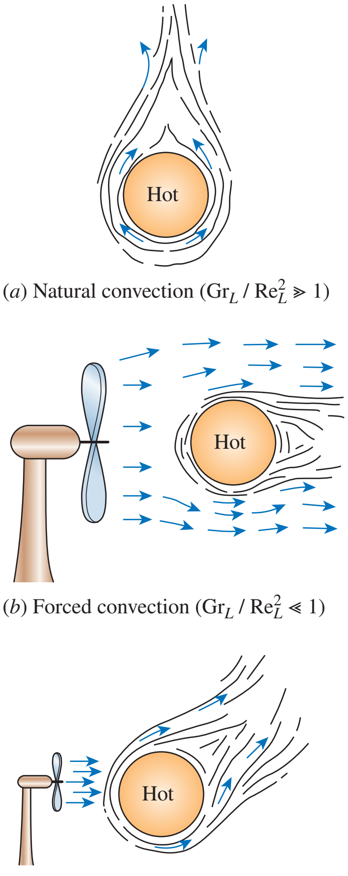


FIGURE 20–8
The Grashof number Gr is a measure of the relative magnitudes of the *buoyancy force* and the opposing *viscous force* acting on the fluid.

We mentioned in the preceding chapters that the flow regime in forced convection is governed by the dimensionless *Reynolds number,* which represents the ratio of inertial forces to viscous forces acting on the fluid. The flow regime in natural convection is governed by the dimensionless *Grashof number,* which represents the ratio of the *buoyancy force* to the *viscous force* acting on the fluid (Fig. 20–8).

The role played by the Reynolds number in forced convection is played by the Grashof number in natural convection. As such, the Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection. For vertical plates, for example, the critical Grashof number is observed to be about 10^9. Therefore, the flow regime on a vertical plate becomes *turbulent* at Grashof numbers greater than 10^9.

When a surface is subjected to external flow, the problem involves both natural and forced convection. The relative importance of each mode of heat transfer is determined by the value of the coefficient $\mathrm{Gr}_L/\mathrm{Re}_L^2$, which appears in Eq. 20–14 (Fig. 20–9). If $\mathrm{Gr}_L/\mathrm{Re}_L^2 \gg 1$, inertia forces are negligible, and natural convection effects dominate. Conversely, if $\mathrm{Gr}_L/\mathrm{Re}_L^2 \ll 1$, buoyancy forces are negligible, and forced convection must be considered. For the case that $\mathrm{Gr}_L/\mathrm{Re}_L^2 \approx 1$, both inertia and buoyancy forces are equally present, and both natural and forced convection effects must be considered. In this case, the flow is referred to as **mixed convection**.



(*a*) Natural convection ($\mathrm{Gr}_L / \mathrm{Re}_L^2 \gg 1$)

(*b*) Forced convection ($\mathrm{Gr}_L / \mathrm{Re}_L^2 \ll 1$)

(*c*) Mixed convection ($\mathrm{Gr}_L / \mathrm{Re}_L^2 \approx 1$)

FIGURE 20–9
The relative importance of convection heat transfer regimes for flow near a hot sphere.

20–3 ▪ NATURAL CONVECTION OVER SURFACES

Natural convection heat transfer on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermophysical properties of the fluid involved.

Although we understand the mechanism of natural convection well, the complexities of fluid motion make it very difficult to obtain simple analytical relations for heat transfer by solving the governing equations of motion and energy. Some analytical solutions exist for natural convection, but such solutions lack generality since they are obtained for simple geometries under some simplifying

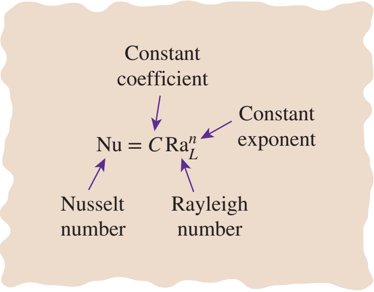


FIGURE 20–10
Natural convection heat transfer correlations are usually expressed in terms of the Rayleigh number raised to a constant *n* multiplied by another constant *C,* both of which are determined experimentally.

assumptions. Therefore, with the exception of some simple cases, heat transfer relations in natural convection are based on experimental studies. Of the many such correlations of varying complexity and claimed accuracy available in the literature for any given geometry, we present here the ones that are best known and most widely used.

The simple empirical correlations for the average *Nusselt number* Nu in natural convection are of the form (Fig. 20–10)

$$\text{Nu} = \frac{hL_c}{k} = C(\text{Gr}_L\,\text{Pr})^n = C\,\text{Ra}_L^n \qquad \textbf{(20–16)}$$

where Ra_L is the **Rayleigh number**, which is the product of the Grashof number, which describes the relationship between buoyancy and viscosity within the fluid, and the Prandtl number, which describes the relationship between momentum diffusivity and thermal diffusivity. Hence the Rayleigh number itself may also be viewed as the ratio of buoyancy forces and (the products of) thermal and momentum diffusivities.

$$\text{Ra}_L = \text{Gr}_L\,\text{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\,\text{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu\alpha} \qquad \textbf{(20–17)}$$

The values of the constants *C* and *n* depend on the *geometry* of the surface and the *flow regime,* which is characterized by the range of the Rayleigh number. The value of *n* is usually $\frac{1}{4}$ for laminar flow and $\frac{1}{3}$ for turbulent flow. The value of the constant *C* is normally less than 1.

Simple relations for the average Nusselt number for various geometries are given in Table 20–1, together with sketches of the geometries. Also given in this table are the characteristic lengths of the geometries and the ranges of Rayleigh number in which the relation is applicable. All fluid properties are to be evaluated at the film temperature $T_f = \frac{1}{2}(T_s + T_\infty)$.

When the average Nusselt number and thus the average convection coefficient is known, the rate of heat transfer by natural convection from a solid surface at a uniform temperature T_s to the surrounding fluid is expressed by Newton's law of cooling as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \qquad (\text{W}) \qquad \textbf{(20–18)}$$

where A_s is the heat transfer surface area and *h* is the average heat transfer coefficient on the surface.

Vertical Plates (T_s = constant)

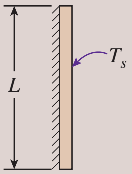
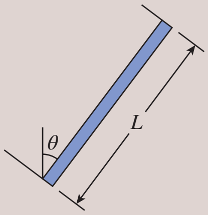
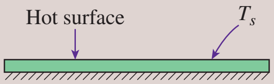
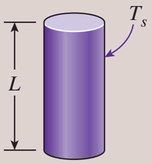
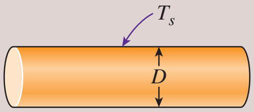
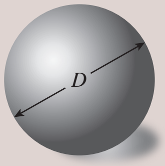
For a vertical flat plate, the characteristic length is the plate height *L.* In Table 20–1 we give three relations for the average Nusselt number for an isothermal vertical plate. The first two relations are very simple. Despite its complexity, we suggest using the third one (Eq. 20–21) recommended by Churchill and Chu (1975b) since it is applicable over the entire range of the Rayleigh number. This relation is most accurate in the range of $10^{-1} < \text{Ra}_L < 10^9$.

Vertical Plates ($\dot{q}_s$ = constant)

In the case of constant surface heat flux, the rate of heat transfer is known (it is simply $\dot{Q} = \dot{q}_s A_s$), but the surface temperature T_s is not. In fact, T_s increases with height along the plate. It turns out that the Nusselt number relations for the constant surface temperature and constant surface heat flux cases are nearly identical (Churchill and Chu, 1975b). Therefore, the relations for isothermal plates can also be used for plates subjected to constant heat flux, provided that the plate midpoint temperature $T_{L/2}$ is used for T_s in the evaluation of the film temperature,

TABLE 20–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu	
Vertical plate  	L	10^4–10^9 10^9–10^{13} Entire range	$\text{Nu} = 0.59\ \text{Ra}_L^{1/4}$ $\text{Nu} = 0.1\ \text{Ra}_L^{1/3}$ $\text{Nu} = \left\{0.825 + \dfrac{0.387\ \text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)	**(20–19)** **(20–20)** **(20–21)**
Inclined plate  	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g with $g\cos\theta$ for $0 < \theta < 60°$	
Horizontal plate (surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate)  	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	(a) $\text{Nu} = 0.54\,\text{Ra}_L^{1/4}$ (a) $\text{Nu} = 0.15\,\text{Ra}_L^{1/3}$ (b) $\text{Nu} = 0.27\,\text{Ra}_L^{1/4}$	**(20–22)** **(20–23)** **(20–24)**
Vertical cylinder  	L		A vertical cylinder can be treated as a vertical plate when $D \geq \dfrac{35L}{\text{Gr}_L^{1/4}}$	
Horizontal cylinder  	D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{0.6 + \dfrac{0.387\,\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$	**(20–25)**
Sphere  	D	$\text{Ra}_D \leq 10^{11}$ $(\text{Pr} \geq 0.7)$	$\text{Nu} = 2 + \dfrac{0.589\ \text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$	**(20–26)**

the Rayleigh number, and the Nusselt number. Noting that $h = \dot{q}_s/(T_{L/2} - T_\infty)$, the average Nusselt number in this case can be expressed as

$$\text{Nu} = \frac{hL}{k} = \frac{\dot{q}_s L}{k(T_{L/2} - T_\infty)} \tag{20–27}$$

The midpoint temperature $T_{L/2}$ is determined by iteration so that the Nusselt numbers determined from Eqs. 20–21 and 20–27 match.

Vertical Cylinders

An outer surface of a vertical cylinder can be treated as a vertical plate when the diameter of the cylinder is large enough that the curvature effects are negligible. This condition is satisfied if

$$D \geq \frac{35L}{\text{Gr}_L^{1/4}} \tag{20–28}$$

When this criterion is met, the relations for vertical plates can also be used for vertical cylinders. Nusselt number relations for slender cylinders that do not meet this criterion are available in the literature (e.g., Cebeci, 1974).

Inclined Plates

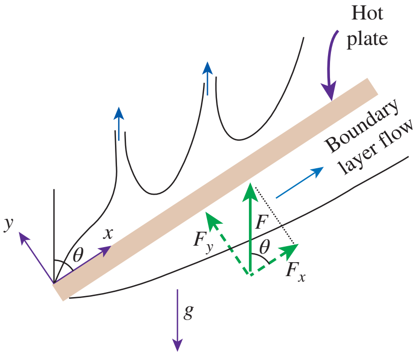


FIGURE 20–11
Natural convection flows on the upper and lower surfaces of an inclined hot plate.

Consider an inclined hot plate that makes an angle θ from the vertical, as shown in Fig. 20–11, in a cooler environment. The net force $F = g(\rho_\infty - \rho)$ (the difference between the buoyancy and gravity) acting on a unit volume of the fluid in the boundary layer is always in the vertical direction. In the case of an inclined plate, this force can be resolved into two components: $F_x = F\cos\theta$ parallel to the plate that drives the flow along the plate, and $F_y = F\sin\theta$ normal to the plate. Noting that the force that drives the motion is reduced, we expect the convection currents to be weaker and the rate of heat transfer to be lower relative to the vertical plate case.

The experiments confirm what we suspect for the lower surface of a hot plate, but the opposite is observed on the upper surface. The reason for this curious behavior for the upper surface is that the force component F_y initiates upward motion in addition to the parallel motion along the plate, and thus the boundary layer breaks up and forms plumes, as shown in the figure. As a result, the thickness of the boundary layer and thus the resistance to heat transfer decreases, and the rate of heat transfer increases relative to the vertical orientation.

In the case of a cold plate in a warmer environment, the opposite occurs as expected: The boundary layer on the upper surface remains intact with weaker boundary layer flow and thus lower rate of heat transfer, and the boundary layer on the lower surface breaks apart (the colder fluid falls down) and thus enhances heat transfer.

When the boundary layer remains intact (the lower surface of a hot plate or the upper surface of a cold plate), the Nusselt number can be determined from the vertical plate relations provided that g in the Rayleigh number relation is replaced by $g\cos\theta$ for $\theta < 60°$. Nusselt number relations for the other two surfaces (the upper surface of a hot plate or the lower surface of a cold plate) are available in the literature (e.g., Fujii and Imura, 1972).

Horizontal Plates

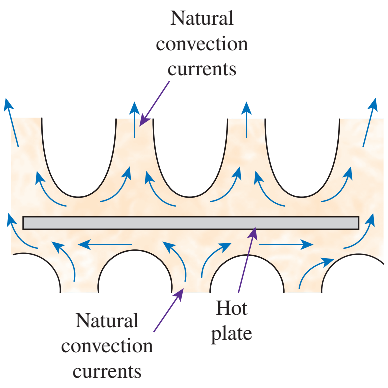


FIGURE 20–12
Natural convection flows on the upper and lower surfaces of a horizontal hot plate.

The rate of heat transfer to or from a horizontal surface depends on whether the surface is facing upward or downward. For a hot surface in a cooler environment, the net force acts upward, forcing the heated fluid to rise. If the hot surface is facing upward, the heated fluid rises freely, inducing strong natural convection currents and thus effective heat transfer, as shown in Fig. 20–12. But if the hot surface is facing downward, the plate blocks the heated fluid that tends to rise (except near the edges), impeding heat transfer. The opposite is true for a cold plate in a warmer environment since the net force (weight minus buoyancy force) in this case acts downward, and the cooled fluid near the plate tends to descend.

The average Nusselt number for horizontal surfaces can be determined from the simple power-law relations given in Table 20–1. Depending on the Ra range, use either Eq. 20–22 or Eq. 20–23 for a horizontal plate with the hot surface facing up or Eq. 20–24 for a horizontal plate with the hot surface facing down. The characteristic length to be used in these equations for horizontal surfaces is calculated from

$$L_c = \frac{A_s}{p} \tag{20–29}$$

where A_s is the surface area and p is the perimeter. Note that $L_c = a/4$ for a horizontal square surface of length a and $D/4$ for a horizontal circular surface of diameter D.

Horizontal Cylinders and Spheres

The boundary layer over a hot horizontal cylinder starts to develop at the bottom, increasing in thickness along the circumference and forming a rising plume at the top, as shown in Fig. 20–13. Therefore, the local Nusselt number is highest at the bottom and lowest at the top of the cylinder when the boundary layer flow remains laminar. The opposite is true in the case of a cold horizontal cylinder in a warmer medium; the boundary layer in this case starts to develop at the top of the cylinder and ends with a descending plume at the bottom.

The average Nusselt number over the entire surface can be determined from Eq. 20–25 (Churchill and Chu, 1975a) for an isothermal horizontal cylinder and from Eq. 20–26 for an isothermal sphere (Churchill, 1983), both given in Table 20–1.

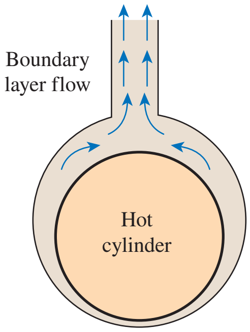


FIGURE 20–13
Natural convection flow over a horizontal hot cylinder.

EXAMPLE 20–1 Heat Loss from Hot-Water Pipes

A 6-m-long section of an 8-cm-diameter horizontal hot-water pipe shown in Fig. 20–14 passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection.

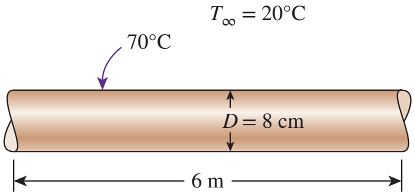


FIGURE 20–14
Schematic for Example 20–1.

SOLUTION A horizontal hot-water pipe passes through a large room. The rate of heat loss from the pipe by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (70 + 20)/2 = 45°C$ and 1 atm are (Table A–22)

$$k = 0.02699 \text{ W/m·K} \qquad \text{Pr} = 0.7241$$
$$\nu = 1.750 \times 10^{-5} \text{ m}^2\text{/s} \qquad \beta = \frac{1}{T_f} = \frac{1}{318 \text{ K}}$$

Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.08$ m. Then the Rayleigh number becomes

$$\text{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81 \text{ m/s}^2)[1/(318 \text{ K})](70 - 20 \text{ K})(0.08 \text{ m})^3}{(1.750 \times 10^{-5} \text{ m}^2\text{/s})^2}(0.7241) = 1.867 \times 10^6$$

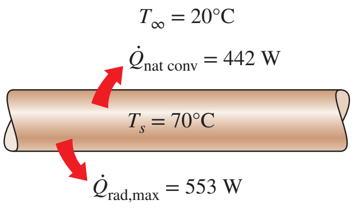


FIGURE 20–15
Radiation heat transfer is usually comparable to natural convection in magnitude and should be considered in heat transfer analysis.

The natural convection Nusselt number in this case can be determined from Eq. 20–25 to be

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.867 \times 10^6)^{1/6}}{[1 + (0.559/0.7241)^{9/16}]^{8/27}}\right\}^2$$
$$= 17.39$$

Then,

$$h = \frac{k}{D}\text{Nu} = \frac{0.02699 \text{ W/m·K}}{0.08 \text{ m}}(17.39) = 5.867 \text{ W/m}^2\text{·K}$$
$$A_s = \pi DL = \pi(0.08 \text{ m})(6 \text{ m}) = 1.508 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.867 \text{ W/m}^2\text{·K})(1.508 \text{ m}^2)(70 - 20)°\text{C} = \mathbf{442 \text{ W}}$$

Therefore, the pipe loses heat to the air in the room at a rate of 442 W by natural convection.

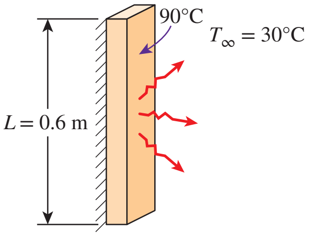
Discussion The pipe loses heat to the surroundings by radiation as well as by natural convection. Assuming the outer surface of the pipe to be black (emissivity $\varepsilon = 1$) and the inner surfaces of the walls of the room to be at room temperature, the radiation heat transfer is determined to be (Fig. 20–15)

$$\dot{Q}_{\text{rad}} = \varepsilon A_s \sigma(T_s^4 - T_{\text{surr}}^4)$$
$$= (1)(1.508 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2\text{·K}^4)[(70 + 273 \text{ K})^4 - (20 + 273 \text{ K})^4]$$
$$= 553 \text{ W}$$

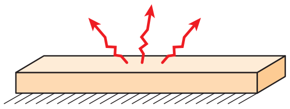
which is larger than natural convection. The emissivity of a real surface is less than 1, and thus the radiation heat transfer for a real surface will be less. But radiation will still be significant for most systems cooled by natural convection. Therefore, a radiation analysis should normally accompany a natural convection analysis unless the emissivity of the surface is low.

EXAMPLE 20–2 Cooling of a Plate in Different Orientations

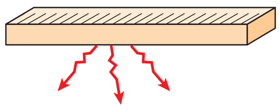
Consider a 0.6-m × 0.6-m thin square plate in a room at 30°C. One side of the plate is maintained at a temperature of 90°C, while the other side is insulated, as shown in Fig. 20–16. Determine the rate of heat transfer from the plate by natural convection if the plate is (*a*) vertical, (*b*) horizontal with hot surface facing up, and (*c*) horizontal with hot surface facing down.



(*a*) Vertical



(*b*) Hot surface facing up



(*c*) Hot surface facing down

FIGURE 20–16
Schematic for Example 20–2.

SOLUTION A hot plate with an insulated back is considered. The rate of heat loss by natural convection is to be determined for different orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (90 + 30)/2 = 60°\text{C}$ and 1 atm are (Table A–22)

$$k = 0.02808 \text{ W/m·K} \qquad \text{Pr} = 0.7202$$
$$\nu = 1.896 \times 10^{-5} \text{ m}^2\text{/s} \qquad \beta = \frac{1}{T_f} = \frac{1}{333 \text{ K}}$$

Analysis (*a*) *Vertical.* The characteristic length in this case is the height of the plate, which is $L = 0.6$ m. The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81 \text{ m/s}^2)[1/(333 \text{ K})](90 - 30 \text{ K})(0.6 \text{ m})^3}{(1.896 \times 10^{-5} \text{ m}^2\text{/s})^2}(0.7202) = 7.649 \times 10^8$$

Then the natural convection Nusselt number can be determined from Eq. 20–21 to be

$$\begin{aligned}\text{Nu} &= \left\{0.825 + \frac{0.387\,\text{Ra}_L^{1/6}}{[1+(0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2 \\ &= \left\{0.825 + \frac{0.387(7.649\times 10^8)^{1/6}}{[1+(0.492/0.7202)^{9/16}]^{8/27}}\right\}^2 = 113.3\end{aligned}$$

Note that the simpler relation Eq. 20–19 would give $\text{Nu} = 0.59\,\text{Ra}_L^{1/4} = 98.12$, which is 13 percent lower. Then,

$$\begin{aligned}h &= \frac{k}{L}\text{Nu} = \frac{0.02808\text{ W/m·K}}{0.6\text{ m}}(113.3) = 5.302\text{ W/m}^2\text{·K} \\ A_s &= L^2 = (0.6\text{ m})^2 = 0.36\text{ m}^2\end{aligned}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.302\text{ W/m}^2\text{·K})(0.36\text{ m}^2)(90-30)^\circ\text{C} = \mathbf{115\ W}$$

(*b*) *Horizontal with hot surface facing up.* The characteristic length and the Rayleigh number in this case are

$$\begin{aligned}L_c &= \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6\text{ m}}{4} = 0.15\text{ m} \\ \text{Ra}_L &= \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} \\ &= \frac{(9.81\text{ m/s}^2)[1/(333\text{ K})](90-30\text{ K})(0.15\text{ m})^3}{(1.896\times 10^{-5}\text{ m}^2\text{/s})^2}(0.7202) = 1.195\times 10^7\end{aligned}$$

The natural convection Nusselt number can be determined from Eq. 20–22 to be

$$\text{Nu} = 0.54\,\text{Ra}_L^{1/4} = 0.54(1.195\times 10^7)^{1/4} = 31.75$$

Then,

$$h = \frac{k}{L_c}\text{Nu} = \frac{0.02808\text{ W/m·K}}{0.15\text{ m}}(31.75) = 5.944\text{ W/m}^2\text{·K}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.944\text{ W/m}^2\text{·K})(0.36\text{ m}^2)(90-30)^\circ\text{C} = \mathbf{128\ W}$$

Note that for the calculated Ra value in part (*b*), we could have used Eq. 20–23 given in Table 20–1. The results in this case would have been overpredicted by 8 percent, which is well within the accuracy of the relations given in the table.

(*c*) *Horizontal with hot surface facing down.* The characteristic length and the Rayleigh number in this case are the same as those determined in (*b*). But the natural convection Nusselt number is to be determined from Eq. 20–24,

$$\text{Nu} = 0.27\,\text{Ra}_L^{1/4} = 0.27(1.195\times 10^7)^{1/4} = 15.87$$

Then,

$$h = \frac{k}{L_c}\text{Nu} = \frac{0.02808\text{ W/m·K}}{0.15\text{ m}}(15.87) = 2.971\text{ W/m}^2\text{·K}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (2.971\text{ W/m}^2\text{·K})(0.36\text{ m}^2)(90-30)^\circ\text{C} = \mathbf{64.2\ W}$$

Note that the natural convection heat transfer is the lowest in the case of the hot surface facing down. This is not surprising, since the hot air is "trapped" under the plate in this case and cannot get away from the plate easily. As a result, the cooler air in the vicinity of the plate will have difficulty reaching the plate, which results in a reduced rate of heat transfer.

Discussion The plate will lose heat to the surroundings by radiation as well as by natural convection. Assuming the surface of the plate to be black (emissivity $\varepsilon = 1$) and the inner surfaces of the walls of the room to be at room temperature, the radiation heat transfer in this case is determined to be

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4)\\ &= (1)(0.36\text{ m}^2)(5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(90 + 273\text{ K})^4 - (30 + 273\text{ K})^4]\\ &= 182\text{ W}\end{aligned}$$

which is larger than that for natural convection heat transfer for each case. Therefore, radiation can be significant and needs to be considered in surfaces cooled by natural convection.

20–4 ▪ NATURAL CONVECTION INSIDE ENCLOSURES

A considerable portion of heat loss from a typical residence occurs through the windows. We certainly would insulate the windows, if we could, in order to conserve energy. The problem is finding an insulating material that is transparent. An examination of the thermal conductivities of the insulating materials reveals that *air* is a *better insulator* than most common insulating materials. Besides, it is transparent. Therefore, it makes sense to insulate the windows with a layer of air. Of course, we need to use another sheet of glass to trap the air. The result is an *enclosure,* which is known as a *double-pane window* in this case. Other examples of enclosures include wall cavities, solar collectors, and cryogenic chambers involving concentric cylinders or spheres.

Enclosures are often encountered in practice, and heat transfer through them is of practical interest. Heat transfer in enclosed spaces is complicated by the fact that the fluid in the enclosure, in general, does not remain stationary. In a vertical enclosure, the fluid adjacent to the hotter surface rises and the fluid adjacent to the cooler one falls, setting off a rotationary motion within the enclosure that enhances heat transfer through the enclosure. Typical flow patterns in vertical and horizontal rectangular enclosures are shown in Figs. 20–17 and 20–18.

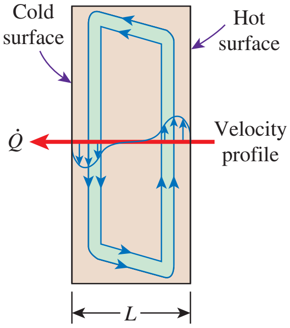
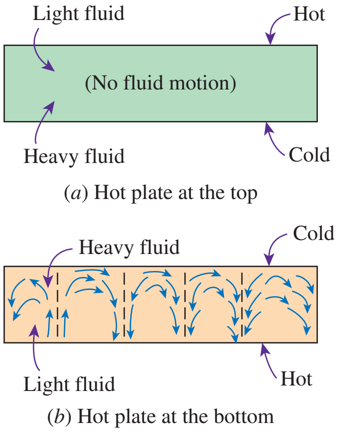


FIGURE 20–17
Convective currents in a vertical rectangular enclosure.



(*a*) Hot plate at the top

(*b*) Hot plate at the bottom

FIGURE 20–18
Convective currents in a horizontal enclosure with (*a*) hot plate at the top and (*b*) hot plate at the bottom.

The characteristics of heat transfer through a horizontal enclosure depend on whether the hotter plate is at the top or at the bottom, as shown in Fig. 20–18. When the hotter plate is at the *top,* no convection currents develop in the enclosure, since the lighter fluid is always on top of the heavier fluid. Heat transfer in this case is by *pure conduction,* and we have Nu = 1. When the hotter plate is at the *bottom,* the heavier fluid will be on top of the lighter fluid, and there will be a tendency for the lighter fluid to topple the heavier fluid and rise to the top, where it comes in contact with the cooler plate and cools down. Until that happens, however, heat transfer is still by *pure conduction* and Nu = 1. When $\text{Ra}_L > 1708$, the buoyant force overcomes the fluid resistance and initiates natural convection currents, which are observed to be in the form of hexagonal cells called *Bénard cells.* For $\text{Ra}_L > 3 \times 10^5$, the cells break down, and the fluid motion becomes turbulent.

The Rayleigh number for an enclosure is determined from

$$\text{Ra}_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\text{Pr} \tag{20–30}$$

where the characteristic length L_c is the distance between the hot and cold surfaces, and T_1 and T_2 are the temperatures of the hot and cold surfaces, respectively. All fluid properties are to be evaluated at the average fluid temperature $T_{\text{avg}} = (T_1 + T_2)/2$.

Effective Thermal Conductivity

When the Nusselt number is known, the rate of heat transfer through the enclosure can be determined from

$$\dot{Q} = hA_s(T_1 - T_2) = k\text{Nu}A_s\frac{T_1 - T_2}{L_c} \qquad \textbf{(20–31)}$$

since $h = k\text{Nu}/L$. The rate of steady heat conduction across a layer of thickness L_c, area A_s, and thermal conductivity k is expressed as

$$\dot{Q}_{\text{cond}} = kA_s\frac{T_1 - T_2}{L_c} \qquad \textbf{(20–32)}$$

where T_1 and T_2 are the temperatures on the two sides of the layer. A comparison of this relation with Eq. 20–31 reveals that the convection heat transfer in an enclosure is analogous to heat conduction across the fluid layer in the enclosure provided that the thermal conductivity k is replaced by kNu. That is, *the fluid in an enclosure behaves like a fluid whose thermal conductivity is* kNu *as a result of convection currents.* Therefore, the quantity kNu is called the **effective thermal conductivity** of the enclosure. That is,

$$k_{\text{eff}} = k\text{Nu} \qquad \textbf{(20–33)}$$

Note that for the special case of Nu = 1, the effective thermal conductivity of the enclosure becomes equal to the conductivity of the fluid. This is expected since this case corresponds to pure conduction (Fig. 20–19).

Natural convection heat transfer in enclosed spaces has been the subject of many experimental and numerical studies, and numerous correlations for the Nusselt number exist. Simple power-law-type relations in the form of $\text{Nu} = C\text{Ra}_L^n$, where C and n are constants, are sufficiently accurate, but they are usually applicable to a narrow range of Prandtl and Rayleigh numbers and aspect ratios. The relations that are more comprehensive are naturally more complex. Next we present some widely used relations for various types of enclosures.

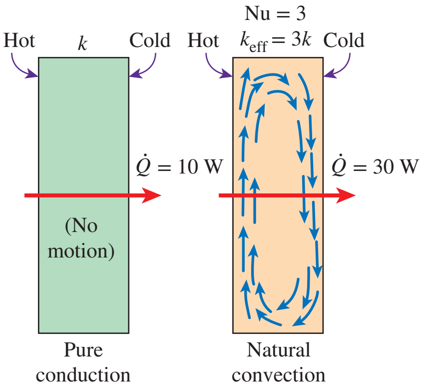


FIGURE 20–19
A Nusselt number of 3 for an enclosure indicates that heat transfer through the enclosure by *natural convection* is three times that by *pure conduction.*

Horizontal Rectangular Enclosures

We need no Nusselt number relations for the case of the hotter plate being at the top, since there are no convection currents in this case and heat transfer is downward by conduction (Nu = 1). When the hotter plate is at the bottom, however, significant convection currents set in for $\text{Ra}_L > 1708$, and the rate of heat transfer increases (Fig. 20–20).

For horizontal enclosures that contain air, Jakob (1949) recommends the following simple correlations

$$\text{Nu} = 0.195\ \text{Ra}_L^{1/4} \qquad 10^4 < \text{Ra}_L < 4 \times 10^5 \qquad \textbf{(20–34)}$$

$$\text{Nu} = 0.068\ \text{Ra}_L^{1/3} \qquad 4 \times 10^5 < \text{Ra}_L < 10^7 \qquad \textbf{(20–35)}$$

These relations can also be used for other gases with $0.5 < \text{Pr} < 2$. Using water, silicone oil, and mercury in their experiments, Globe and Dropkin (1959) obtained this correlation for horizontal enclosures heated from below,

$$\text{Nu} = 0.069\ \text{Ra}_L^{1/3}\ \text{Pr}^{0.074} \qquad 3 \times 10^5 < \text{Ra}_L < 7 \times 10^9 \qquad \textbf{(20–36)}$$

Based on experiments with air, Hollands et al. (1976) recommend this correlation for horizontal enclosures,

$$\text{Nu} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}_L}\right]^{+} + \left[\frac{\text{Ra}_L^{1/3}}{18} - 1\right]^{+} \qquad \text{Ra}_L < 10^8 \qquad \textbf{(20–37)}$$

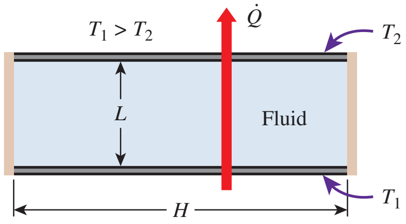


FIGURE 20–20
A horizontal rectangular enclosure with isothermal surfaces.

The notation $[\ ]^+$ indicates that if the quantity in the bracket is negative, it should be set equal to zero. This relation also correlates data well for liquids with moderate Prandtl numbers for $\mathrm{Ra}_L < 10^5$, and thus it can also be used for water.

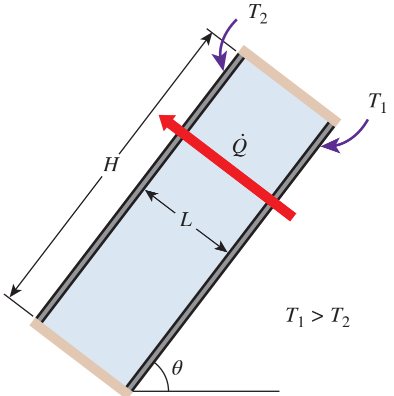


FIGURE 20–21
An inclined rectangular enclosure with isothermal surfaces.

TABLE 20–2

Critical angles for inclined rectangular enclosures

Aspect ratio, H/L	Critical angle, θ_{cr}
1	25°
3	53°
6	60°
12	67°
>12	70°

Inclined Rectangular Enclosures

Airspaces between two inclined parallel plates are commonly encountered in flat-plate solar collectors (between the glass cover and the absorber plate) and the double-pane skylights on inclined roofs. Heat transfer through an inclined enclosure depends on the **aspect ratio** H/L as well as the tilt angle θ from the horizontal (Fig. 20–21).

For large aspect ratios ($H/L \geq 12$), this equation (Hollands et al., 1976) correlates experimental data extremely well for tilt angles up to 70°,

$$\mathrm{Nu} = 1 + 1.44\left[1 - \frac{1708}{\mathrm{Ra}_L \cos\theta}\right]^+ \left(1 - \frac{1708(\sin 1.8\theta)^{1.6}}{\mathrm{Ra}_L \cos\theta}\right) + \left[\frac{(\mathrm{Ra}_L \cos\theta)^{1/3}}{18} - 1\right]^+ \tag{20–38}$$

for $\mathrm{Ra}_L < 10^5$, $0 < \theta < 70°$, and $H/L \geq 12$. Again, any quantity in $[\ ]^+$ should be set equal to zero if it is negative. This is to ensure that $\mathrm{Nu} = 1$ for $\mathrm{Ra}_L \cos\theta < 1708$. Note that this relation reduces to Eq. 20–37 for horizontal enclosures for $\theta = 0°$, as expected.

For enclosures with smaller aspect ratios ($H/L < 12$), the next correlation can be used, provided that the tilt angle is less than the critical value θ_{cr} listed in Table 20–2 (Catton, 1978)

$$\mathrm{Nu} = \mathrm{Nu}_{\theta=0°}\left(\frac{\mathrm{Nu}_{\theta=90°}}{\mathrm{Nu}_{\theta=0°}}\right)^{\theta/\theta_{cr}} (\sin\theta_{cr})^{\theta/(4\theta_{cr})} \qquad 0° < \theta < \theta_{cr} \tag{20–39}$$

For tilt angles greater than the critical value ($\theta_{cr} < \theta < 90°$), the Nusselt number can be obtained by multiplying the Nusselt number for a vertical enclosure by $(\sin\theta)^{1/4}$ (Ayyaswamy and Catton, 1973),

$$\mathrm{Nu} = \mathrm{Nu}_{\theta=90°}(\sin\theta)^{1/4} \qquad \theta_{cr} < \theta < 90°\text{, any } H/L \tag{20–40}$$

For enclosures tilted more than 90°, the recommended relation is [Arnold et al. (1975)]

$$\mathrm{Nu} = 1 + (\mathrm{Nu}_{\theta=90°} - 1)\sin\theta \qquad 90° < \theta < 180°\text{, any } H/L \tag{20–41}$$

More recent but more complex correlations are also available in the literature (e.g., ElSherbiny et al., 1982).

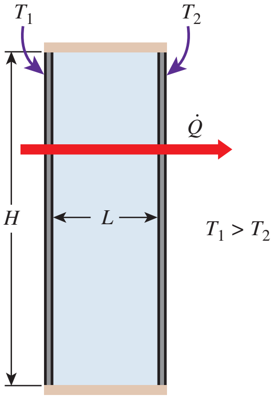


FIGURE 20–22
A vertical rectangular enclosure with isothermal surfaces.

Vertical Rectangular Enclosures

For vertical enclosures (Fig. 20–22), Catton (1978) recommends these two correlations due to Berkovsky and Polevikov (1977),

$$\mathrm{Nu} = 0.18\left(\frac{\mathrm{Pr}}{0.2 + \mathrm{Pr}}\mathrm{Ra}_L\right)^{0.29} \qquad \begin{array}{l} 1 < H/L < 2 \\ \text{any Prandtl number} \\ \mathrm{Ra}_L\mathrm{Pr}/(0.2 + \mathrm{Pr}) > 10^3 \end{array} \tag{20–42}$$

$$\mathrm{Nu} = 0.22\left(\frac{\mathrm{Pr}}{0.2 + \mathrm{Pr}}\mathrm{Ra}_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} \qquad \begin{array}{l} 2 < H/L < 10 \\ \text{any Prandtl number} \\ \mathrm{Ra}_L < 10^{10} \end{array} \tag{20–43}$$

For vertical enclosures with larger aspect ratios, the following correlations can be used (MacGregor and Emery, 1969)

$$\text{Nu} = 0.42\,\text{Ra}_L^{1/4}\,\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} \qquad \begin{aligned} 10 &< H/L < 40 \\ 1 &< \text{Pr} < 2\times10^4 \\ 10^4 &< \text{Ra}_L < 10^7 \end{aligned} \tag{20–44}$$

$$\text{Nu} = 0.046\,\text{Ra}_L^{1/3} \qquad \begin{aligned} 1 &< H/L < 40 \\ 1 &< \text{Pr} < 20 \\ 10^6 &< \text{Ra}_L < 10^9 \end{aligned} \tag{20–45}$$

Again, all fluid properties are to be evaluated at the average temperature $(T_1 + T_2)/2$.

Concentric Cylinders

Consider two long concentric horizontal cylinders maintained at uniform but different temperatures of T_i and T_o, as shown in Fig. 20–23. The diameters of the inner and outer cylinders are D_i and D_o, respectively, and the characteristic length is the spacing between the cylinders, $L_c = (D_o - D_i)/2$. The rate of heat transfer through the annular space between the cylinders by natural convection per unit length is expressed as

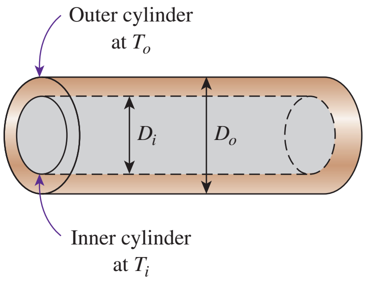


FIGURE 20–23
Two concentric horizontal isothermal cylinders.

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) \qquad (\text{W/m}) \tag{20–46}$$

The recommended relation for effective thermal conductivity is (Raithby and Hollands, 1975)

$$\frac{k_{\text{eff}}}{k} = 0.386\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra}_L)^{1/4} \tag{20–47}$$

where the geometric factor for concentric cylinders F_{cyl} is

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} \tag{20–48}$$

The k_{eff} relation in Eq. 20–47 is applicable for $0.70 \le \text{Pr} \le 6000$ and $10^2 \le F_{\text{cyl}}\text{Ra}_L \le 10^7$. For $F_{\text{cyl}}\text{Ra}_L < 100$, natural convection currents are negligible, and thus $k_{\text{eff}} = k$. Note that k_{eff} cannot be less than k, and thus we should set $k_{\text{eff}} = k$ if $k_{\text{eff}}/k < 1$. The fluid properties are evaluated at the average temperature of $(T_i + T_o)/2$.

Concentric Spheres

For concentric isothermal spheres, the rate of heat transfer through the gap between the spheres by natural convection is expressed as (Fig. 20–24)

$$\dot{Q} = k_{\text{eff}}\frac{\pi D_i D_o}{L_c}(T_i - T_o) \qquad (\text{W}) \tag{20–49}$$

where $L_c = (D_o - D_i)/2$ is the characteristic length. The recommended relation for effective thermal conductivity is (Raithby and Hollands, 1975)

$$\frac{k_{\text{eff}}}{k} = 0.74\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{sph}}\text{Ra}_L)^{1/4} \tag{20–50}$$

where the geometric factor for concentric spheres F_{sph} is

$$F_{\text{sph}} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5} \tag{20–51}$$

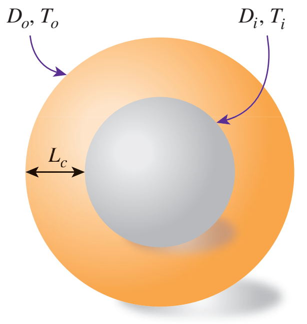


FIGURE 20–24
Two concentric isothermal spheres.

The k_{eff} relation in Eq. 20–50 is applicable for $0.70 \leq \text{Pr} \leq 4200$ and $10^2 \leq F_{\text{sph}}\text{Ra}_L \leq 10^4$. If $k_{\text{eff}}/k < 1$, we should set $k_{\text{eff}} = k$.

Combined Natural Convection and Radiation

Gases are nearly transparent to radiation, and thus heat transfer through a gas layer is by simultaneous convection (or conduction, if the gas is quiescent) and radiation. Natural convection heat transfer coefficients are typically very low compared to those for forced convection. Therefore, radiation is usually disregarded in forced convection problems, but it must be considered in natural convection problems that involve a gas. This is especially the case for surfaces with high emissivities. For example, about half of the heat transfer through the airspace of a double-pane window is by radiation. The total rate of heat transfer is determined by adding the convection and radiation components,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} \tag{20–52}$$

Radiation heat transfer from a surface at temperature T_s surrounded by surfaces at a temperature T_{surr} (both in K) is determined from

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4) \qquad \text{(W)} \tag{20–53}$$

where ε is the emissivity of the surface, A_s is the surface area, and $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴ is the Stefan–Boltzmann constant.

When the end effects are negligible, radiation heat transfer between two large parallel plates at temperatures T_1 and T_2 is expressed as (see Chap. 21 for details)

$$\dot{Q}_{\text{rad}} = \frac{\sigma A_s(T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \varepsilon_{\text{effective}}\,\sigma A_s(T_1^4 - T_2^4) \qquad \text{(W)} \tag{20–54}$$

where ε_1 and ε_2 are the emissivities of the plates, and $\varepsilon_{\text{effective}}$ is the *effective emissivity* defined as

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} \tag{20–55}$$

The emissivity of an ordinary glass surface, for example, is 0.84. Therefore, the effective emissivity of two parallel glass surfaces facing each other is 0.72. Radiation heat transfer between concentric cylinders and spheres is discussed in Chap. 21.

Note that in some cases the temperature of the surrounding medium may be below the surface temperature ($T_\infty < T_s$), while the temperature of the surrounding surfaces is above the surface temperature ($T_{\text{surr}} > T_s$). In such cases, convection and radiation heat transfers are subtracted from each other instead of being added since they are in opposite directions. Also, for a metal surface, the radiation effect can be reduced to negligible levels by polishing the surface and thus lowering the surface emissivity to a value near zero.

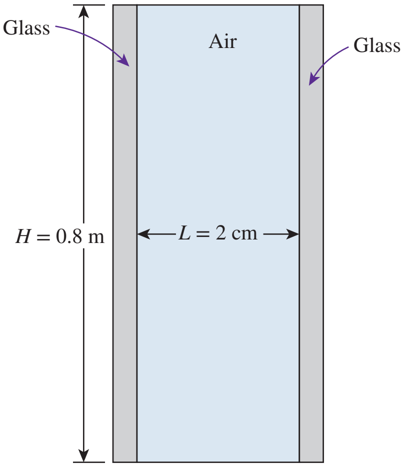


FIGURE 20–25
Schematic for Example 20–3.

EXAMPLE 20–3 Heat Loss Through a Double-Pane Window

The vertical 0.8-m-high, 2-m-wide double-pane window shown in Fig. 20–25 consists of two sheets of glass separated by a 2-cm air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 12°C and 2°C, determine the rate of heat transfer through the window.

SOLUTION Two glasses of a double-pane window are maintained at specified temperatures. The rate of heat transfer through the window is to be determined.
Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Radiation heat transfer is not considered.

Properties The properties of air at the average temperature of $T_{\text{avg}} = (T_1 + T_2)/2 = (12 + 2)/2 = 7°\text{C}$ and 1 atm pressure are (Table A–22)

$$k = 0.02416 \text{ W/m·K} \qquad \text{Pr} = 0.7344$$

$$\nu = 1.400 \times 10^{-5} \text{ m}^2\text{/s} \qquad \beta = \frac{1}{T_{\text{avg}}} = \frac{1}{280 \text{ K}}$$

Analysis We have a rectangular enclosure filled with air. The characteristic length in this case is the distance between the two glasses, $L_c = L = 0.02$ m. Then the Rayleigh number becomes

$$\text{Ra}_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\text{Pr}$$

$$= \frac{(9.81 \text{ m/s}^2)[1/(280 \text{ K})](12 - 2 \text{ K})(0.02 \text{ m})^3}{(1.400 \times 10^{-5} \text{ m}^2\text{/s})^2}(0.7344) = 1.050 \times 10^4$$

The aspect ratio of the geometry is $H/L = 0.8/0.02 = 40$. Then the Nusselt number in this case can be determined from Eq. 20–44 to be

$$\text{Nu} = 0.42 \text{ Ra}_L^{1/4} \text{ Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3}$$

$$= 0.42(1.050 \times 10^4)^{1/4}(0.7344)^{0.012}\left(\frac{0.8}{0.02}\right)^{-0.3} = 1.40$$

Then,

$$A_s = H \times W = (0.8 \text{ m})(2 \text{ m}) = 1.6 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_1 - T_2) = k\text{Nu}A_s\frac{T_1 - T_2}{L}$$

$$= (0.02416 \text{ W/m·K})(1.40)(1.6 \text{ m}^2)\frac{(12 - 2)°\text{C}}{0.02 \text{ m}} = \mathbf{27.1 \text{ W}}$$

Therefore, heat is lost through the window at a rate of 27.1 W.

Discussion Recall that a Nusselt number of $\text{Nu} = 1$ for an enclosure corresponds to pure conduction heat transfer through the enclosure. The air in the enclosure in this case remains still, and no natural convection currents occur in the enclosure. The Nusselt number in our case is 1.40, which indicates that heat transfer through the enclosure is 1.40 times that by pure conduction. The increase in heat transfer is due to the natural convection currents that develop in the enclosure.

EXAMPLE 20–4 Heating Water in a Tube by Solar Energy

A solar collector consists of a horizontal aluminum tube having an outer diameter of 2 in enclosed in a concentric thin glass tube of 4-in diameter (Fig. 20–26). Water is heated as it flows through the tube, and the annular space between the aluminum and the glass tubes is filled with air at 1 atm pressure. The pump circulating the water fails during a clear day, and the water temperature in the tube starts rising. The aluminum tube absorbs solar radiation at a rate of 30 Btu/h per foot length, and the temperature of the ambient air outside is 70°F. Disregarding any heat loss by radiation, determine the temperature of the aluminum tube when steady operation is established (i.e., when the rate of heat loss from the tube equals the amount of solar energy gained by the tube).

SOLUTION The circulating pump of a solar collector that consists of a horizontal tube and its glass cover fails. The equilibrium temperature of the tube is to be determined.

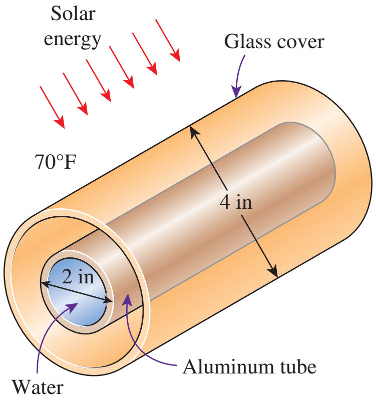


FIGURE 20–26
Schematic for Example 20–4.

Assumptions **1** Steady operating conditions exist. **2** The tube and its cover are isothermal. **3** Air is an ideal gas. **4** Heat loss by radiation is negligible.

Properties The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and we cannot evaluate the average temperatures. Therefore, we assume the glass temperature to be 110°F, and we use properties at an anticipated average temperature of (70 + 110)/2 = 90°F (Table A–22E),

$$k = 0.01505\ \text{Btu/h·ft·°F} \qquad \text{Pr} = 0.7275$$

$$\nu = 1.753 \times 10^{-4}\ \text{ft}^2\text{/s} \qquad \beta = \frac{1}{T_{\text{avg}}} = \frac{1}{550\ \text{R}}$$

Analysis We have a horizontal cylindrical enclosure filled with air at 1 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{\text{tube-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q}_{\text{solar gain}} = 30\ \text{Btu/h} \qquad \text{(per foot of tube)}$$

The heat transfer surface area of the glass cover is

$$A_o = A_{\text{glass}} = (\pi D_o L) = \pi(4/12\ \text{ft})(1\ \text{ft}) = 1.047\ \text{ft}^2 \qquad \text{(per foot of tube)}$$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, it is clear that the solution will require a trial-and-error approach. Assuming the glass cover temperature to be 110°F, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$\begin{aligned}\text{Ra}_{D_o} &= \frac{g\beta(T_s - T_\infty)D_o^3}{\nu^2}\,\text{Pr}\\ &= \frac{(32.2\ \text{ft/s}^2)[1/(550\ \text{R})](110 - 70\ \text{R})(4/12\ \text{ft})^3}{(1.753 \times 10^{-4}\ \text{ft}^2\text{/s})^2}(0.7275) = 2.054 \times 10^6\end{aligned}$$

$$\begin{aligned}\text{Nu} &= \left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.054 \times 10^6)^{1/6}}{[1 + (0.559/0.7275)^{9/16}]^{8/27}}\right\}^2\\ &= 17.89\end{aligned}$$

$$\begin{aligned}h_o &= \frac{k}{D_o}\text{Nu} = \frac{0.01505\ \text{Btu/h·ft·°F}}{4/12\ \text{ft}}(17.89) = 0.8077\ \text{Btu/h·ft}^2\text{·°F}\\ \dot{Q}_o &= h_o A_o(T_o - T_\infty) = (0.8077\ \text{Btu/h·ft}^2\text{·°F})(1.047\ \text{ft}^2)(110 - 70)\text{°F}\\ &= 33.8\ \text{Btu/h}\end{aligned}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 110°F for the glass cover is high. Repeating the calculations with lower temperatures, the glass cover temperature corresponding to 30 Btu/h is determined to be 106°F.

The temperature of the aluminum tube is determined in a similar manner using the natural convection relations for two horizontal concentric cylinders. The characteristic length in this case is the distance between the two cylinders, which is

$$L_c = (D_o - D_i)/2 = (4 - 2)/2 = 1\ \text{in} = 1/12\ \text{ft}$$

We start the calculations by assuming the tube temperature to be 200°F, and thus an average temperature of (106 + 200)/2 = 153°F = 613 R. Using air properties at this temperature gives

$$\mathrm{Ra}_L = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\mathrm{Pr}$$

$$= \frac{(32.2\ \mathrm{ft/s^2})[1/(613\ \mathrm{R})](200 - 106\ \mathrm{R})(1/12\ \mathrm{ft})^3}{(2.117 \times 10^{-4}\ \mathrm{ft^2/s})^2}(0.7184) = 4.580 \times 10^4$$

The effective thermal conductivity is

$$F_{\mathrm{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5}$$

$$= \frac{[\ln(4/2)]^4}{(1/12\ \mathrm{ft})^3[(2/12\ \mathrm{ft})^{-3/5} + (4/12\ \mathrm{ft})^{-3/5}]^5} = 0.1466$$

$$k_{\mathrm{eff}} = 0.386k\left(\frac{\mathrm{Pr}}{0.861 + \mathrm{Pr}}\right)^{1/4}(F_{\mathrm{cyl}}\mathrm{Ra}_L)^{1/4}$$

$$= 0.386(0.01653\ \mathrm{Btu/h{\cdot}ft{\cdot}^\circ F})\left(\frac{0.7184}{0.861 + 0.7184}\right)^{1/4}$$

$$\times (0.1466 \times 4.580 \times 10^4)^{1/4}$$

$$= 0.04743\ \mathrm{Btu/h{\cdot}ft{\cdot}^\circ F}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q} = \frac{2\pi k_{\mathrm{eff}}}{\ln(D_o/D_i)}(T_i - T_o)$$

$$= \frac{2\pi(0.04743\ \mathrm{Btu/h{\cdot}ft{\cdot}^\circ F})}{\ln(4/2)}(200 - 106)^\circ\mathrm{F} = 40.4\ \mathrm{Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 200°F for the tube is high. By trying other values, the tube temperature corresponding to 30 Btu/h is determined to be **180°F**. Therefore, the tube will reach an equilibrium temperature of 180°F when the pump fails.

Discussion Note that we have not considered heat loss by radiation in the calculations, and thus the tube temperature determined is probably too high.

SUMMARY

In this chapter, we have considered natural convection heat transfer where any fluid motion occurs by natural means such as buoyancy. The volume expansion coefficient of a substance represents the variation of the density of that substance with temperature at constant pressure, and for an ideal gas, it is expressed as $\beta = 1/T$, where T is the absolute temperature in K or R.

The flow regime in natural convection is governed by a dimensionless number called the *Grashof number,* which represents the ratio of the buoyancy force to the viscous force acting on the fluid and is expressed as

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

where L_c is the *characteristic length,* which is the height L for a vertical plate and the diameter D for a horizontal cylinder. The correlations for the Nusselt number $\mathrm{Nu} = hL_c/k$ in natural convection are expressed in terms of the *Rayleigh number* defined as

$$\mathrm{Ra}_L = \mathrm{Gr}_L\,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\mathrm{Pr}$$

Nusselt number relations for various surfaces are given in Table 20–1. All fluid properties are evaluated at the film temperature of $T_f = \frac{1}{2}(T_s + T_\infty)$. The outer surface of a vertical cylinder can be treated as a vertical plate when the curvature effects are negligible. The characteristic length for a horizontal surface is $L_c = A_s/p$, where A_s is the surface area and p is the perimeter.

In a *horizontal rectangular enclosure* with the hotter plate at the top, heat transfer is by pure conduction and Nu = 1. When the hotter plate is at the bottom, the Nusselt number is

$$\text{Nu} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}_L}\right]^+ + \left[\frac{\text{Ra}_L^{1/3}}{18} - 1\right]^+ \qquad \text{Ra}_L < 10^8$$

The notation [ ]$^+$ indicates that if the quantity in the bracket is negative, it should be set equal to zero. For *vertical rectangular enclosures,* the Nusselt number can be determined from

$$\text{Nu} = 0.18\left(\frac{\text{Pr}}{0.2 + \text{Pr}}\text{Ra}_L\right)^{0.29} \qquad \begin{array}{c} 1 < H/L < 2 \\ \text{any Prandtl number} \\ \text{Ra}_L\text{Pr}/(0.2 + \text{Pr}) > 10^3 \end{array}$$

$$\text{Nu} = 0.22\left(\frac{\text{Pr}}{0.2 + \text{Pr}}\text{Ra}_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} \qquad \begin{array}{c} 2 < H/L < 10 \\ \text{any Prandtl number} \\ \text{Ra}_L < 10^{10} \end{array}$$

For aspect ratios greater than 10, Eqs. 20–44 and 20–45 should be used. For inclined enclosures, Eqs. 20–38 through 20–41 should be used.

For *concentric horizontal cylinders,* the rate of heat transfer through the annular space between the cylinders by natural convection per unit length is

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o)$$

where

$$\frac{k_{\text{eff}}}{k} = 0.386\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra}_L)^{1/4}$$

and

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5}$$

For a *spherical enclosure,* the rate of heat transfer through the space between the spheres by natural convection is expressed as

$$\dot{Q} = k_{\text{eff}}\frac{\pi D_i D_o}{L_c}(T_i - T_o)$$

where

$$\frac{k_{\text{eff}}}{k} = 0.74\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{sph}}\text{Ra}_L)^{1/4}$$

$$L_c = (D_o - D_i)/2$$

$$F_{\text{sph}} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5}$$

The quantity kNu is called the *effective thermal conductivity* of the enclosure, since a fluid in an enclosure behaves like a quiescent fluid whose thermal conductivity is kNu as a result of convection currents. The fluid properties are evaluated at the average temperature of $(T_i + T_o)/2$.

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PROBLEMS *

Physical Mechanism of Natural Convection

20–1C What is natural convection? How does it differ from forced convection? What force causes natural convection currents?

20–2C In which mode of heat transfer is the convection heat transfer coefficient usually higher, natural convection or forced convection? Why?

20–3C Consider a hot boiled egg in a spacecraft that is filled with air at atmospheric pressure and temperature at all times. Will the egg cool faster or slower when the spacecraft is in space instead of on the ground? Explain.

20–4C When will the hull of a ship sink deeper in the water: when the ship is sailing in fresh water or in seawater? Why?

20–5C What is buoyancy force? Compare the relative magnitudes of the buoyancy force acting on a body immersed in these media: (*a*) air, (*b*) water, (*c*) mercury, and (*d*) an evacuated chamber.

20–6C Consider two fluids, one with a large coefficient of volume expansion and the other with a small one. In what fluid will a hot surface initiate stronger natural convection currents? Why? Assume the viscosity of the fluids to be the same.

20–7C Physically, what does the Grashof number represent? How does the Grashof number differ from the Reynolds number?

20–8 Show that the volume expansion coefficient of an ideal gas is $\beta = 1/T$, where T is the absolute temperature.

20–9 The density of liquid water can be correlated as $\rho(T) = 1000 - 0.0736T - 0.00355\,T^2$ where ρ and T are in kg/m^3 and °C, respectively. Determine the volume expansion coefficient at 70°C. Compare the result with the value tabulated in Table A–15.

20–10 A 10-cm × 10-cm plate has a constant surface temperature of 150°C. Determine the Grashof number if the chip is placed in the following fluids: air (1 atm, 30°C), liquid water (30°C), engine oil (10°C). Discuss how the Grashof number affects the natural convection flow.

20–11E A long 2-in-diameter rod with surface temperature of 200°F is submerged in a bath of fluid. Determine the Grashof and Rayleigh numbers if the fluid is (*a*) liquid water at 40°F, (*b*) liquid ammonia at 40°F, (*c*) engine oil at 50°F, and (*d*) air at 40°F (1 atm).

Natural Convection over Surfaces

20–12C Will a hot horizontal plate whose back side is insulated cool faster or slower when its hot surface is facing down instead of up?

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*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

20–13C Consider laminar natural convection from a vertical hot plate. Will the heat flux be higher at the top or at the bottom of the plate? Why?

20–14C Under what conditions can the outer surface of a vertical cylinder be treated as a vertical plate in natural convection calculations?

20–15 A 0.2-m-long and 25-mm-thick vertical plate ($k = 15$ W/m·K) separates the hot water from the cold water. The plate surface exposed to the hot water has a temperature of 100°C, and the temperature of the cold water is 7°C. Determine the temperature of the plate surface exposed to the cold water ($T_{s,c}$). *Hint:* The $T_{s,c}$ has to be found iteratively. Start the iteration process with an initial guess of 53.5°C for the $T_{s,c}$.

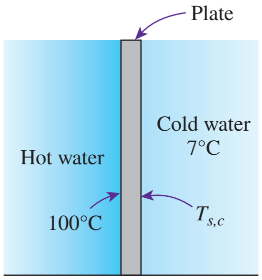


FIGURE P20–15

20–16 Reconsider Prob. 20–15. Using appropriate software, investigate the effect of the plate thermal conductivity on the surface temperature exposed to the cold water. By varying the plate thermal conductivity from 3 to 200 W/m·K, plot the plate surface temperature on the cold water side as a function of the plate thermal conductivity.

20–17 A 0.2-m-long and 25-mm-thick vertical plate ($k = 1.5$ W/m·K) separates the hot water from the cold air at 2°C. The plate surface exposed to the hot water has a temperature of 100°C, and the surface exposed to the cold air has an emissivity of 0.73. Determine the temperature of the plate surface exposed to the cold air ($T_{s,c}$). *Hint:* The $T_{s,c}$ has to be found iteratively. Start the iteration process with an initial guess of 51°C for the $T_{s,c}$.

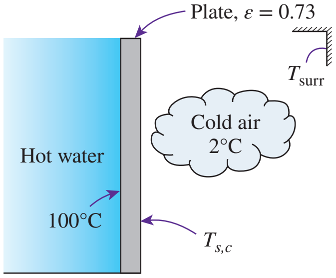


FIGURE P20–17

20–18 Reconsider Prob. 20–17. Using appropriate software, evaluate the effect of the plate thickness on the surface temperature exposed to the cold air. By varying the plate thickness from 0.01 to 0.1 m, plot the plate surface temperature on the cold air side as a function of the plate thickness.

20–19 A 0.5-m-long thin vertical plate is subjected to uniform heat flux on one side, while the other side is exposed to cool air at 5°C. The plate surface has an emissivity of 0.73, and its midpoint temperature is 55°C. Determine the heat flux on the plate surface.

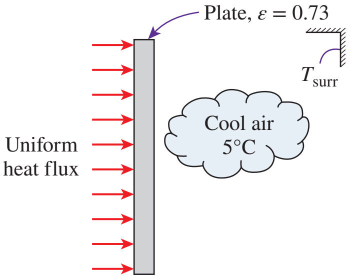
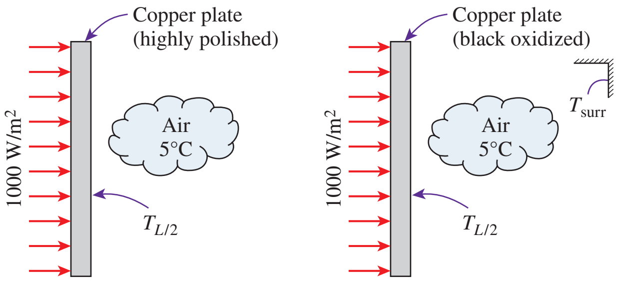


FIGURE P20–19

20–20 A 0.5-m-long thin vertical copper plate is subjected to a uniform heat flux of 1000 W/m^2 on one side, while the other side is exposed to air at 5°C. Determine the plate midpoint temperature for (*a*) a highly polished surface and (*b*) a black oxidized surface. *Hint:* The plate midpoint temperature ($T_{L/2}$) has to be found iteratively. Begin the calculations by using a film temperature of 30°C.



(*a*) (*b*)

FIGURE P20–20

20–21 Reconsider Prob. 20–20. Using appropriate software, evaluate the effect of the uniform surface heat flux on the plate midpoint temperature for (*a*) the highly polished surface and (*b*) the black oxidized surface. By varying the surface heat flux from 500 to 1500 W/m^2, plot the plate midpoint temperature as a function of the surface heat flux.

20–22 Consider a 1.2-m-high and 2-m-wide glass window with a thickness of 6 mm, thermal conductivity $k = 0.78$ W/m·K, and emissivity $\varepsilon = 0.9$. The room and the walls that face the window are maintained at 25°C, and the average temperature of the inner surface of the window is measured to be 5°C. If the temperature of the outdoors is −5°C, determine (*a*) the convection heat transfer coefficient on the inner surface of the window, (*b*) the rate of total heat transfer through the window, and (*c*) the combined natural convection and radiation heat transfer coefficient on the outer surface of the window. Is it reasonable to neglect the thermal resistance of the glass in this case?

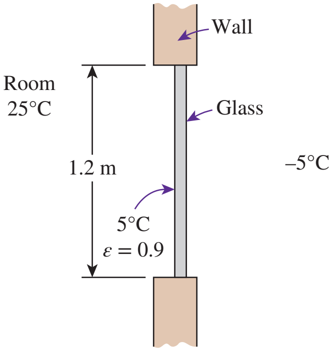


FIGURE P20–22

20–23E Consider a 2-ft × 2-ft thin square plate in a room at 75°F. One side of the plate is maintained at a temperature of 130°F, while the other side is insulated. Determine the rate of heat transfer from the plate by natural convection if the plate is (*a*) vertical, (*b*) horizontal with hot surface facing up, and (*c*) horizontal with hot surface facing down.

20–24E Reconsider Prob. 20–23E. Using appropriate software, plot the rate of natural convection heat transfer for different orientations of the plate as a function of the plate temperature as the temperature varies from 80°F to 180°F, and discuss the results.

20–25 The side surfaces of a 3-m-high cubic industrial furnace burning natural gas are not insulated, and the temperature at the outer surface of this section is measured to be 110°C. The temperature of the furnace room, including its surfaces, is 30°C, and the emissivity of the outer surface of the furnace is 0.7. It is proposed that this section of the furnace wall be insulated with glass wool insulation ($k = 0.038$ W/m·K) wrapped by a reflective sheet ($\varepsilon = 0.2$) in order to reduce the heat loss by 90 percent. Assuming the outer surface temperature of the metal section still remains at about 110°C, determine the thickness of the insulation that needs to be used.

The furnace operates continuously throughout the year and has an efficiency of 78 percent. The price of the natural gas is \$1.10/therm (1 therm = 105,500 kJ of energy content). If the installation of the insulation will cost \$550 for materials and labor, determine how long it will take for the insulation to pay for itself from the energy it saves.

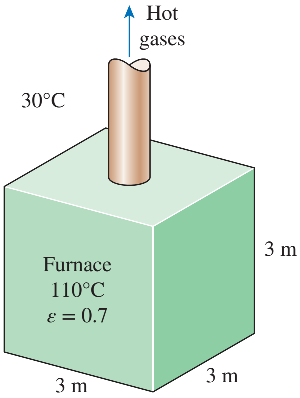


FIGURE P20–25

20–26 Consider a vertical plate with length *L*, placed in quiescent air. If the film temperature is 20°C and the average Nusselt number in natural convection is of the form $\text{Nu} = C\text{Ra}_L^n$, show that the average heat transfer coefficient can be expressed as

$$h = 1.51(\Delta T/L)^{1/4} \qquad 10^4 < \text{Ra}_L < 10^9$$
$$h = 1.19\,\Delta T^{1/3} \qquad 10^{10} < \text{Ra}_L < 10^{13}$$

20–27 A 50-cm × 50-cm circuit board that contains 121 square chips on one side is to be cooled by combined natural convection and radiation by mounting it on a vertical surface in a room at 25°C. Each chip dissipates 0.18 W of power, and the emissivity of the chip surfaces is 0.7. Assuming the heat transfer from the back side of the circuit board to be negligible, and the temperature of the surrounding surfaces to be the same as the air temperature of the room, determine the surface temperature of the chips. Evaluate air properties at a film temperature of 30°C and 1 atm pressure. Is this a good assumption? *Answer:* 36.2°C

20–28 Repeat Prob. 20–27 assuming the circuit board to be positioned horizontally with (*a*) chips facing up and (*b*) chips facing down.

20–29 Consider a 15-cm × 20-cm printed circuit board (PCB) that has electronic components on one side. The board is placed in a room at 20°C. The heat loss from the back surface of the board is negligible. If the circuit board is dissipating 8 W of power in steady operation, determine the average temperature of the hot surface of the board, assuming the board is (*a*) vertical, (*b*) horizontal with hot surface facing up, and (*c*) horizontal with hot surface facing down. Take the emissivity of the surface of the board to be 0.8 and assume the surrounding surfaces to be at the same temperature as the air in the room. Evaluate air properties at a film temperature of 32.5°C and 1 atm pressure. Is this a good assumption? *Answers:* (*a*) 46.6°C, (*b*) 42.6°C, (*c*) 50.7°C

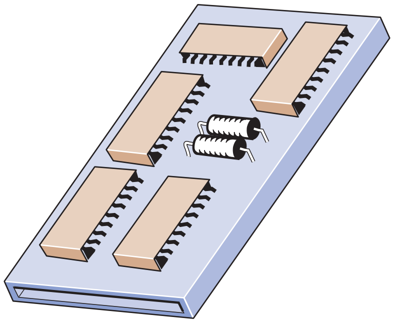


FIGURE P20–29

20–30 Reconsider Prob. 20–29. Using appropriate software, investigate the effects of the room temperature and the emissivity of the board on the temperature of the hot surface of the board for different orientations of the board. Let the room temperature vary from 5°C to 35°C and the emissivity from 0.1 to 1.0. Plot the hot surface temperature for different orientations of the board as the functions of the room temperature and the emissivity, and discuss the results.

20–31 Consider a thin 24-cm-long and 20-cm-wide horizontal plate suspended in air at 20°C. The plate is equipped with electric resistance heating elements with a rating of 20 W. Now the heater is turned on and the plate temperature rises. Determine the temperature of the plate when steady operating conditions are reached. The plate has an emissivity of 0.90, and the surrounding surfaces are at 17°C. As an initial guess, assume a surface temperature of 50°C. Is this a good assumption?

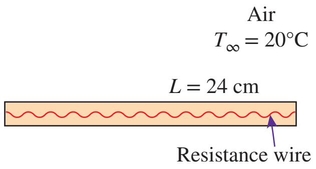


FIGURE P20–31

20–32 A manufacturer makes absorber plates that are 1.2 m × 0.8 m in size for use in solar collectors. The back side of the plate is heavily insulated, while its front surface is coated with black chrome, which has an absorptivity of 0.87 for solar radiation and an emissivity of 0.09. Consider such a plate placed horizontally outdoors in calm air at 25°C. Solar radiation is incident on the plate at a rate of 600 W/m^2. Taking the effective sky temperature to be 10°C, determine the equilibrium temperature of the absorber plate. What would your answer be if the absorber plate is made of ordinary aluminum plate that has a solar absorptivity of 0.28 and an emissivity of 0.07? Evaluate air properties at a film temperature of 70°C and 1 atm pressure. Is this a good assumption?

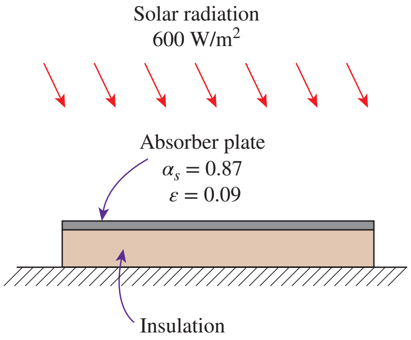


FIGURE P20–32

20–33 Repeat Prob. 20–32 for an aluminum plate painted flat black (solar absorptivity 0.98 and emissivity 0.98) and also for a plate painted white (solar absorptivity 0.26 and emissivity 0.90). Evaluate air properties at a film temperature of 45°C and 1 atm pressure. Is this a good assumption?

20–34 A circular grill of diameter 0.25 m has an emissivity of 0.8. If the surface temperature is maintained at 150°C, determine the required electrical power when the room air and surroundings are at 30°C.

20–35 A can of engine oil with a length of 150 mm and a diameter of 100 mm is placed vertically in the trunk of a car. On a hot summer day, the temperature in the trunk is 43°C. If the surface temperature of the can is 17°C, determine heat transfer rate from the can surface. Neglect the heat transfer from the ends of the can.

20–36 Flue gases from an incinerator are released to atmosphere using a stack that is 0.6 m in diameter and 10.0 m high. The outer surface of the stack is at 40°C and the surrounding air is at 10°C. Determine the rate of heat transfer from the stack, assuming (*a*) there is no wind and (*b*) the stack is exposed to 20 km/h winds.

20–37 In a plant that manufactures canned aerosol paints, the cans are temperature-tested in water baths at 60°C before they are shipped to ensure that they withstand temperatures up to 55°C during transportation and shelving (as shown in Fig. P20–37). The cans, moving on a conveyor, enter the open hot water bath, which is 0.5 m deep, 1 m wide, and 3.5 m long, and they move slowly in the hot water toward the other end. Some of the cans fail the test and explode in the water bath. The water container is made of sheet metal, and the entire container is at about the same temperature as the hot water. The emissivity of the outer surface of the container is 0.7. If the temperature of the surrounding air and surfaces is 20°C, determine the rate of heat loss from the four side surfaces of the container (disregard the top surface, which is open).

The water is heated electrically by resistance heaters, and the cost of electricity is $0.085/kWh. If the plant operates 24 h a day 365 days a year and thus 8760 h a year, determine the annual cost of the heat losses from the container for this facility.

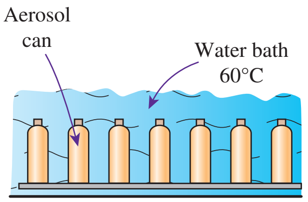


FIGURE P20–37

20–38 Reconsider Prob. 20–37. In order to reduce the heating cost of the hot water, it is proposed to insulate the side and bottom surfaces of the container with 5-cm-thick fiberglass insulation ($k = 0.035$ W/m·K) and to wrap the insulation with aluminum foil ($\varepsilon = 0.1$) in order to minimize the heat loss by radiation. An estimate is obtained from a local insulation contractor, who proposes to do the insulation job for $350, including materials and labor. Would you support this proposal? How long will it take for the insulation to pay for itself from the energy it saves? Evaluate air properties at a film temperature of 23°C and 1 atm pressure. Is this a good assumption?

20–39 A room is to be heated by a coal-burning stove, which is a cylindrical cavity with an outer diameter of 50 cm and a height of 120 cm. The rate of heat loss from the room is estimated to be 1.5 kW when the air temperature in the room is maintained constant at 24°C. The emissivity of the stove surface is 0.85, and the average temperature of the surrounding wall surfaces is 14°C. Determine the surface temperature of the stove. Neglect the heat transfer from the bottom surface and take the heat transfer coefficient at the top surface to be the same as that on the side surface.

The heating value of the coal is 30,000 kJ/kg, and the combustion efficiency is 65 percent. Determine the amount of coal burned in a day if the stove operates 14 h a day. Evaluate air properties at a film temperature of 77°C and 1 atm pressure. Is this a good assumption?

20–40 Consider a cylinder with a length of 15 cm and a diameter of 10 cm. The cylinder has a surface temperature of 43°C, while the room air temperature is 17°C. Determine whether placing the cylinder horizontally or vertically would achieve a higher heat transfer rate.

20–41 An aluminum soda can 150 mm in length and 60 mm in diameter is placed horizontally inside a refrigerator compartment that maintains a temperature of 4°C. If the surface temperature of the can is 36°C, estimate the heat transfer rate from the can. Neglect the heat transfer from the ends of the can.

20–42 Thermal energy generated by the electrical resistance of a 5-mm-diameter and 4-m-long bare cable is dissipated to the surrounding air at 20°C. The voltage drop and the electric current across the cable in steady operation are measured to be 60 V and 1.5 A, respectively. Disregarding radiation, estimate the surface temperature of the cable. Evaluate air properties at a film temperature of 60°C and 1 atm pressure. Is this a good assumption?

20–43 A 300-W cylindrical resistance heater is 0.75 m long and 0.5 cm in diameter. The resistance wire is placed horizontally in a fluid at 20°C. Determine the outer surface temperature of the resistance wire in steady operation if the fluid is (*a*) air and (*b*) water. Ignore any heat transfer by radiation. Use properties at 500°C for air and 40°C for water.

20–44 A 12-m-long section of a 5-cm-diameter horizontal hot-water pipe passes through a large room whose temperature is 27°C. If the temperature and the emissivity of the outer surface of the pipe are 73°C and 0.8, respectively, determine the rate of heat loss from the pipe by (*a*) natural convection and (*b*) radiation.

20–45 Thick fluids such as asphalt and waxes and the pipes in which they flow are often heated in order to reduce the viscosity of the fluids and thus to reduce the pumping costs. Consider the flow of such a fluid through a 100-m-long pipe of outer diameter 30 cm in calm, ambient air at 0°C. The pipe is heated electrically, and a thermostat keeps the outer surface temperature of the pipe constant at 25°C. The emissivity of the outer surface of the pipe is 0.8, and the effective sky temperature is −30°C. Determine the power rating of the electric resistance heater, in kW, that needs to be used. Also, determine the cost of electricity associated with heating the pipe during a 15-h period under the above conditions if the price of electricity is \$0.09/kWh. *Answers:* 29.1 kW, \$39.30

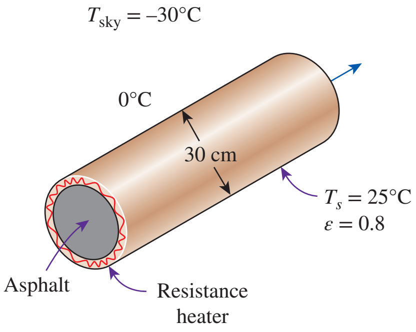


FIGURE P20–45

20–46 Reconsider Prob. 20–45. To reduce the cost of heating the pipe, it is proposed to insulate it with enough fiberglass insulation ($k = 0.035$ W/m·K) wrapped in aluminum foil ($\varepsilon = 0.1$) to cut down the heat losses by 85 percent. Assuming the pipe temperature must remain constant at 25°C, determine the thickness of the insulation that needs to be used. How much money will the insulation save during this 15-h period? Evaluate air properties at a film temperature of 5°C and 1 atm pressure. Is this a good assumption? *Answers:* 1.3 cm, \$33.40

20–47 A 3-mm-diameter and 12-m-long electric wire is tightly wrapped with a 1.5-mm-thick plastic cover whose thermal conductivity and emissivity are $k = 0.20$ W/m·K and $\varepsilon = 0.9$. Electrical measurements indicate that a current of 10 A passes through the wire, and there is a voltage drop of 7 V along the wire. If the insulated wire is exposed to calm atmospheric air at $T_\infty = 30°C$, determine the temperature at the interface of the wire and the plastic cover in steady operation. Take the surrounding surfaces to be at about the same temperature as the air. Evaluate air properties at a film temperature of 40°C and 1 atm pressure. Is this a good assumption?

20–48 During a visit to a plastic sheeting plant, it was observed that a 45-m-long section of a 2-in nominal (6.03-cm-outer-diameter) steam pipe extended from one end of the plant to the other with no insulation on it. The temperature measurements at several locations revealed that the average temperature of the exposed surfaces of the steam pipe was 170°C, while the temperature of the surrounding air was 20°C. The outer surface of the pipe appeared to be oxidized, and its emissivity can be taken to be 0.7. Taking the temperature of the surrounding surfaces to be 20°C also, determine the rate of heat loss from the steam pipe.

Steam is generated in a gas furnace that has an efficiency of 84 percent, and the plant pays \$1.10 per therm (1 therm = 105,500 kJ) of natural gas. The plant operates 24 h a day, 365 days a year, and thus 8760 h a year. Determine the annual cost of the heat losses from the steam pipe for this facility.

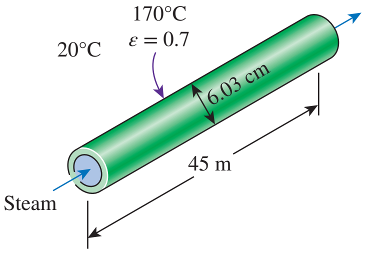


FIGURE P20–48

20–49 Reconsider Prob. 20–48. Using appropriate software, investigate the effect of the surface temperature of the steam pipe on the rate of heat loss from the pipe and the annual cost of this heat loss. Let the surface temperature vary from 100°C to 200°C. Plot the rate of heat loss and the annual cost as a function of the surface temperature, and discuss the results.

20–50 A 1.5-m-diameter, 4-m-long cylindrical propane tank is initially filled with liquid propane, whose density is 581 kg/m^3. The tank is exposed to the ambient air at 25°C in calm weather. The outer surface of the tank is polished so that the radiation heat transfer is negligible. Now a crack develops at the top of the tank, and the pressure inside drops to 1 atm while the temperature drops to −42°C, which is the boiling temperature of propane at 1 atm. The heat of vaporization of propane at

1 atm is 425 kJ/kg. The propane is slowly vaporized as a result of the heat transfer from the ambient air into the tank, and the propane vapor escapes the tank at −42°C through the crack. Assuming the propane tank to be at about the same temperature as the propane inside at all times, determine how long it will take for the tank to empty if it is not insulated.

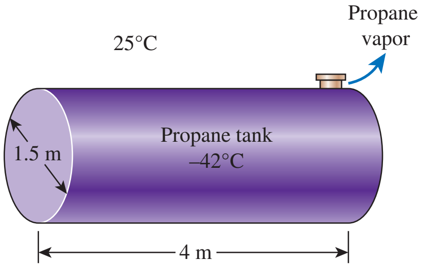


FIGURE P20–50

20–51 Hot water is being transported in a horizontal pipe ($k = 0.15$ W/m·K, $D_i = 2.5$ cm, $D_o = 4.5$ cm), where the pipe inner surface temperature is at 100°C. A length of 2 m of the pipe's outer surface is exposed to cool air at 12°C. Assuming that the properties of air can be evaluated at 40°C and 1 atm pressure, determine the outer surface temperature of the pipe. Is 40°C an appropriate film temperature for evaluation of the air properties?

20–52 Hot engine oil is being transported in a horizontal pipe ($k = 15$ W/m·K, $D_i = 5$ cm) with a wall thickness of 5 mm. The pipe is covered with a 5-mm-thick layer of insulation ($k = 0.15$ W/m·K). A length of 2 m of the outer surface is exposed to cool air at 10°C. If the pipe inner surface temperature is at 90°C, determine the outer surface temperature. *Hint:* The pipe outer surface temperature has to be found iteratively. Begin the calculations by using a film temperature of 50°C.

20–53 Reconsider Prob. 20–52. Using appropriate software, evaluate the effect of the insulation layer thickness on the outer surface temperature. By varying the insulation layer thickness from 5 to 15 mm, plot the outer surface temperature as a function of the thickness of the insulation layer.

20–54 A hot fluid ($k_{\text{fluid}} = 0.72$ W/m·K) is flowing as a laminar fully developed flow inside a pipe with an inner diameter of 35 mm and a wall thickness of 5 mm. The pipe is 10 m long, and the outer surface is exposed to air at 10°C. The average temperature difference between the hot fluid and the pipe inner surface is $\Delta T_{\text{avg}} = 10$°C, and the inner and outer surface temperatures are constant. Determine the outer surface temperature of the pipe. Evaluate the air properties at 50°C. Is this a good assumption?

20–55 A hot liquid ($c_p = 1000$ J/kg·K) flows at a flow rate of 0.05 kg/s inside a copper pipe with an inner diameter of 45 mm and a wall thickness of 5 mm. At the pipe exit, the liquid temperature decreases by 10°C from its temperature at the inlet. The outer surface of the 5-m-long copper pipe is black oxidized, which subjects the outer surface to radiation heat transfer. The air temperature surrounding the pipe is 10°C. Assuming that the properties of air can be evaluated at 35°C and 1 atm pressure, determine the outer surface temperature of the pipe. Is 35°C an appropriate film temperature for evaluation of the air properties?

20–56E An average person generates heat at a rate of 240 Btu/h while resting in a room at 70°F. Assuming one-quarter of this heat is lost from the head and taking the emissivity of the skin to be 0.9, determine the average surface temperature of the head when it is not covered. The head can be approximated as a 12-in-diameter sphere, and the interior surfaces of the room can be assumed to be at the room temperature.

20–57 An incandescent lightbulb is an inexpensive but highly inefficient device that converts electrical energy into light. It converts about 5 percent of the electrical energy it consumes into light while converting the remaining 95 percent into heat. The glass bulb of the lamp heats up very quickly as a result of absorbing all that heat and dissipating it to the surroundings by convection and radiation. Consider an 8-cm-diameter, 60-W lightbulb in a room at 25°C. The emissivity of the glass is 0.9. Assuming that 5 percent of the energy passes through the glass bulb as light with negligible absorption and the rest of the energy is absorbed and dissipated by the bulb itself by natural convection and radiation, determine the equilibrium temperature of the glass bulb. Assume the interior surfaces of the room to be at room temperature. *Answer:* 175°C

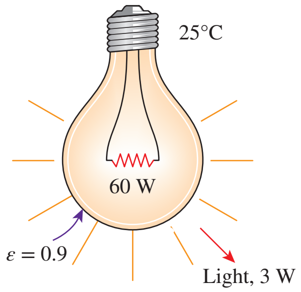


FIGURE P20–57

20–58 The water in a 40-L tank is to be heated from 15°C to 45°C by a 6-cm-diameter spherical heater whose surface temperature is maintained at 85°C. Determine how long the heater should be kept on.

20–59 A 2-m-diameter, thin-walled stainless steel spherical tank is filled with chemicals undergoing a reaction. The reaction releases heat through the tank, where the tank outer surface temperature is 50°C and is exposed to air at 20°C. The stainless steel surface of the tank has an emissivity of 0.35. Determine the rate of heat release from the chemical reaction inside the tank. Is the given emissivity value reasonable for a stainless steel surface? What is the contribution of natural convection to the overall rate of heat loss at the tank surface?

20–60 A hot liquid is poured into a spherical tank with an inner diameter of 3 m and a wall thickness of 3 cm. The tank wall is made of a material with a thermal conductivity of 0.15 W/m·K. The hot liquid in the tank causes the inner surface temperature to be 100°C, while the tank outer surface is exposed to air at 20°C and has an emissivity of 0.35. Determine the outer surface temperature of the tank. Assume that the properties of air can be evaluated at 40°C and 1 atm pressure. Is this a good assumption?

Natural Convection Inside Enclosures

20–61C The upper and lower compartments of a well-insulated container are separated by two parallel sheets of glass with an airspace between them. One of the compartments is to be filled with a hot fluid and the other with a cold fluid. If it is desired that heat transfer between the two compartments be minimal, would you recommend putting the hot fluid into the upper or the lower compartment of the container? Why?

20–62C Someone claims that the airspace in a double-pane window enhances the heat transfer from a house because of the natural convection currents that occur in the airspace and recommends that the double-pane window be replaced by a single sheet of glass whose thickness is equal to the sum of the thicknesses of the two glasses of the double-pane window to save energy. Do you agree with this claim?

20–63C Consider a double-pane window consisting of two glass sheets separated by a 1-cm-wide airspace. Someone suggests inserting a thin vinyl sheet between the two glass sheets to form two 0.5-cm-wide compartments in the window in order to reduce natural convection heat transfer through the window. From a heat transfer point of view, would you be in favor of this idea to reduce heat losses through the window?

20–64C What does the effective conductivity of an enclosure represent? How is the ratio of the effective conductivity to thermal conductivity related to the Nusselt number?

20–65 Show that the thermal resistance of a rectangular enclosure can be expressed as $R = L_c/(Ak\text{Nu})$, where k is the thermal conductivity of the fluid in the enclosure.

20–66 Consider a 3-m-high rectangular enclosure consisting of two surfaces separated by a 0.1-m air gap at 1 atm. If the surface temperatures across the air gap are 30°C and −10°C, determine the ratio of the heat transfer rate for the horizontal orientation (with hotter surface at the bottom) to that for vertical orientation.

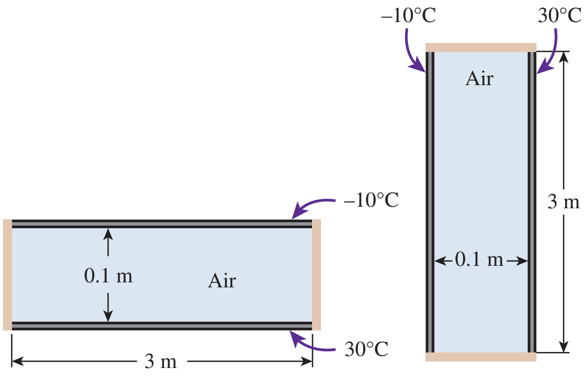


FIGURE P20–66

20–67 Flat-plate solar collectors are often tilted up toward the sun in order to intercept a greater amount of direct solar radiation. The tilt angle from the horizontal also affects the rate of heat loss from the collector. Consider a 1.5-m-high and 3-m-wide solar collector that is tilted at an angle θ from the horizontal. The back side of the absorber is heavily insulated. The absorber plate and the glass cover, which are spaced 2.5 cm from each other, are maintained at temperatures of 80°C and 40°C, respectively. Determine the rate of heat loss from the absorber plate by natural convection for $\theta = 0°$, 30°, and 90°.

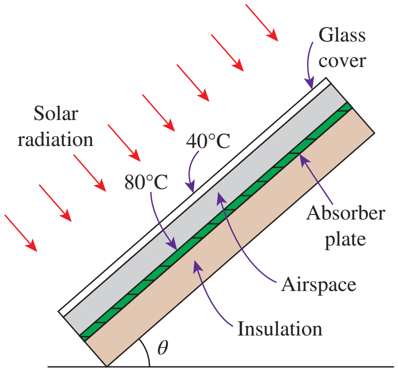


FIGURE P20–67

20–68 A vertical 1.5-m-high and 3.0-m-wide enclosure consists of two surfaces separated by a 0.4-m air gap at atmospheric pressure. If the surface temperatures across the air gap are measured to be 280 K and 336 K and the surface emissivities to be 0.15 and 0.90, determine the fraction of heat transferred through the enclosure by radiation. *Answer:* 0.30

20–69 A vertical 1.5-m-high, 2.8-m-wide double-pane window consists of two layers of glass separated by a 2.0-cm air gap at atmospheric pressure. The room temperature is 26°C while the inner glass temperature is 18°C. Disregarding radiation heat transfer, determine the temperature of the outer glass layer and the rate of heat loss through the window by natural convection.

20–70E A vertical 4-ft-high and 6-ft-wide double-pane window consists of two sheets of glass separated by a 1-in air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 65°F and 40°F, determine the rate of heat transfer through the window by (*a*) natural convection and (*b*) radiation. Also, determine the *R*-value of insulation of this window such that multiplying the inverse of the *R*-value by the surface area and the temperature difference gives the total rate of heat transfer through the window. The effective emissivity for use in radiation calculations between two large parallel glass plates can be taken to be 0.82.

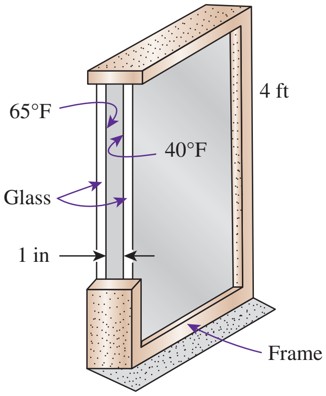


FIGURE P20–70E

20–71E Reconsider Prob. 20–70E. Using appropriate software, investigate the effect of the air gap thickness on the rates of heat transfer by natural convection and radiation and the *R*-value of insulation. Let the air gap thickness vary from 0.2 in to 2.0 in. Plot the rates of heat transfer by natural convection and radiation and the *R*-value of insulation as a function of the air gap thickness, and discuss the results.

20–72 A simple solar collector is built by placing a 5-cm-diameter clear plastic tube around a garden hose whose outer diameter is 1.6 cm. The hose is painted black to maximize solar absorption, and some plastic rings are used to keep the spacing between the hose and the clear plastic cover constant. During a clear day, the temperature of the hose is measured to be 65°C, while the ambient air temperature is 26°C. Determine the rate of heat loss from the water in the hose per meter of its length by natural convection. Also, discuss how the performance of this solar collector can be improved. Evaluate air properties at an average temperature of 50°C and 1 atm pressure. Is this a good assumption? *Answer:* 8.2 W

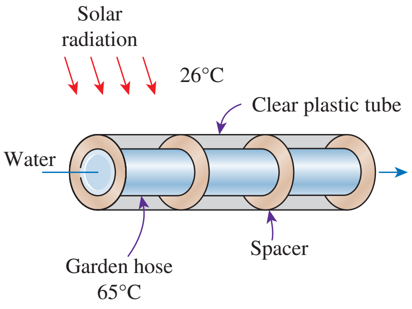


FIGURE P20–72

20–73 Reconsider Prob. 20–72. Using appropriate software, plot the rate of heat loss from the water by natural convection as a function of the ambient air temperature as the temperature varies from 4°C to 40°C, and discuss the results.

20–74 Consider two concentric horizontal cylinders of diameters 55 cm and 65 cm and length 125 cm. The surfaces of the inner and outer cylinders are maintained at 54°C and 106°C, respectively. Determine the rate of heat transfer between the cylinders by natural convection if the annular space is filled with (*a*) water and (*b*) air.

20–75 Two concentric spheres with diameters of 5 cm and 10 cm have their surface temperatures maintained at 100°C and 200°C, respectively. The enclosure between the two concentric spherical surfaces is filled with nitrogen gas at 1 atm. Determine the rate of heat transfer through the enclosure.

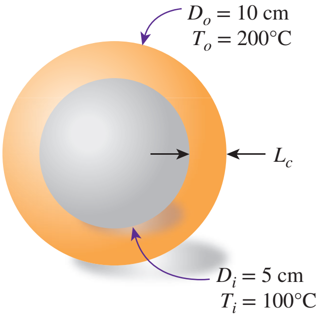


FIGURE P20–75

20–76 Two concentric spheres of diameters 15 cm and 25 cm are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are T_1 = 350 K and T_2 = 275 K, respectively. Determine the rate of heat transfer from the inner sphere to the outer sphere by natural convection.

20–77 Reconsider Prob. 20–76. Using appropriate software, plot the rate of natural convection heat transfer as a function of the hot surface temperature of the sphere as the temperature varies from 250 K to 450 K, and discuss the results.

20–78 Two concentric spheres of diameters D_i = 20 cm and D_o = 30 cm are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are T_i = 320 K and T_o = 280 K, respectively. Determine the rate of heat transfer from the inner sphere to the outer sphere by natural convection.

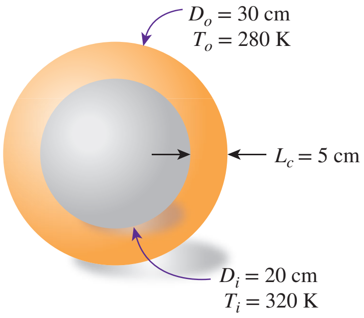


FIGURE P20–78

Review Problems

20–79 An electric resistance space heater is designed such that it resembles a rectangular box 50 cm high, 80 cm long, and 15 cm wide filled with 45 kg of oil. The heater is to be placed against a wall, and thus heat transfer from its back surface is negligible. The surface temperature of the heater is not to exceed 75°C in a room at 25°C for safety considerations. Disregarding heat transfer from the bottom and top surfaces of the heater in anticipation that the top surface will be used as a shelf, determine the power rating of the heater in W. Take the emissivity of the outer surface of the heater to be 0.8 and the average temperature of the ceiling and wall surfaces to be the same as the room air temperature.

Also, determine how long it will take for the heater to reach steady operation when it is first turned on (i.e., for the oil temperature to rise from 25°C to 75°C). State your assumptions in the calculations.

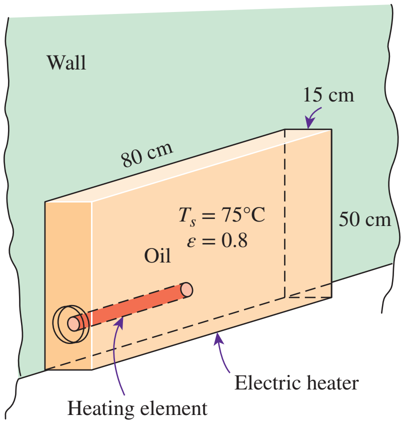


FIGURE P20–79

20–80 During a plant visit, it was observed that a 1.5-m-high and 1-m-wide section of the vertical front section of a natural gas furnace wall was too hot to touch. The temperature measurements on the surface revealed that the average temperature of the exposed hot surface was 110°C, while the temperature of the surrounding air was 25°C. The surface appeared to be oxidized, and its emissivity can be taken to be 0.7. Taking the temperature of the surrounding surfaces to be 25°C also, determine the rate of heat loss from this furnace.

The furnace has an efficiency of 79 percent, and the plant pays \$1.20 per therm of natural gas. If the plant operates 10 h a day, 310 days a year, and thus 3100 h a year, determine the annual cost of the heat loss from this vertical hot surface on the front section of the furnace wall.

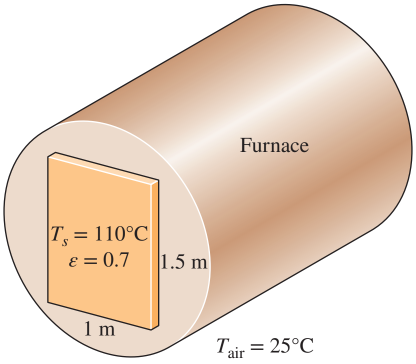


FIGURE P20–80

20–81 A plate (0.5 m × 0.5 m) is inclined at an angle of 30°. The top surface of the plate is well insulated. Estimate the rate of heat loss from the plate when the bottom surface is maintained at 60°C and the surrounding atmospheric quiescent air is at 0°C.

20–82 A group of 25 power transistors, dissipating 1.5 W each, are to be cooled by attaching them to a black-anodized square aluminum plate and mounting the plate on the wall of a room at 30°C. The emissivity of the transistor and the plate surfaces is 0.9. Assuming the heat transfer from the back side of the plate to be negligible and the temperature of the surrounding surfaces to be the same as the air temperature of the room, determine the size of the plate if the average surface temperature of the plate is not to exceed 50°C. *Answer:* 43 cm × 43 cm

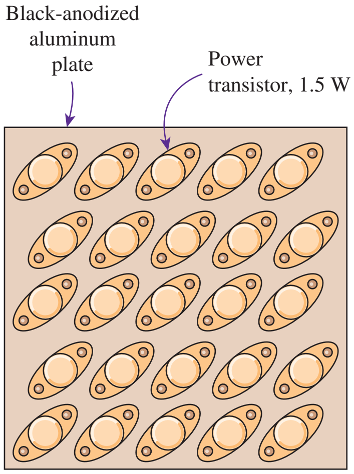


FIGURE P20–82

20–83 Repeat Prob. 20–82 assuming the plate to be positioned horizontally with (*a*) transistors facing up and (*b*) transistors facing down.

20–84 Consider an $L \times L$ horizontal plate that is placed in quiescent air with the hot surface facing up. If the film temperature is 20°C and the average Nusselt number in natural convection is of the form $\text{Nu} = C\text{Ra}_L^n$, show that the average heat transfer coefficient can be expressed as

$$h = 1.95\,(\Delta T/L)^{1/4} \qquad 10^4 < \text{Ra}_L < 10^7$$
$$h = 1.79\Delta T^{1/3} \qquad 10^7 < \text{Ra}_L < 10^{11}$$

20–85 Consider a flat-plate solar collector placed horizontally on the flat roof of a house. The collector is 1.5 m wide and 4.5 m long, and the average temperature of the exposed surface of the collector is 42°C. Determine the rate of heat loss from the collector by natural convection during a calm day when the ambient air temperature is 8°C. Also, determine the heat loss by radiation by taking the emissivity of the collector surface to be 0.85 and the effective sky temperature to be −15°C. *Answers:* 1314 W, 1762 W

20–86 An electronic box that consumes 200 W of power is cooled by a fan blowing air into the box enclosure. The dimensions of the electronic box are 15 cm × 50 cm × 50 cm, and all surfaces of the box are exposed to the ambient environment except the base surface. Temperature measurements indicate that the box is at an average temperature of 32°C when the ambient temperature and the temperature of the surrounding walls are 25°C. If the emissivity of the outer surface of the box is 0.75, determine the fraction of the heat lost from the outer surfaces of the electronic box.

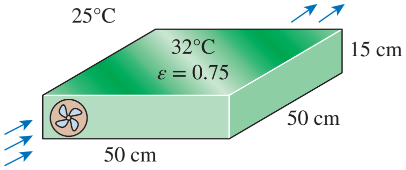


FIGURE P20–86

20–87E The components of an electronic system dissipating 150 W are located in a 5-ft-long horizontal duct whose cross section is 6 in × 6 in. The components in the duct are cooled by forced air, which enters at 85°F at a rate of 22 cfm and leaves at 100°F. The surfaces of the sheet metal duct are not painted, and thus radiation heat transfer from the outer surfaces is negligible. If the ambient air temperature is 80°F, determine (*a*) the heat transfer from the outer surfaces of the duct to the ambient air by natural convection and (*b*) the average temperature of the duct. Evaluate air properties at a film temperature of 100°F and 1 atm pressure. Is this a good assumption?

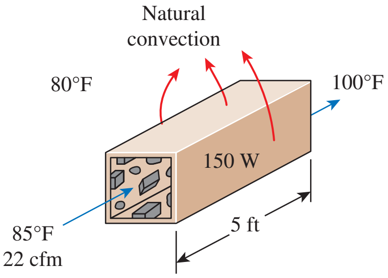


FIGURE P20–87E

20–88E Repeat Prob. 20–87E assuming the fan fails and thus all heat generated inside the duct must be rejected to the ambient air by natural convection through the outer surfaces of the duct. Evaluate air properties at a film temperature of 120°F and 1 atm pressure. Is this a good assumption?

20–89E Repeat Prob. 20–87E for a circular horizontal duct of diameter 4 in. Evaluate air properties at a film temperature of 115°F and 1 atm pressure. Is this a good assumption?

20–90 Exhaust gases from a manufacturing plant are being discharged through a 10-m-tall exhaust stack with outer diameter of 1 m. The exhaust gases are discharged at a rate of 0.125 kg/s, while temperature drop between inlet and exit of the exhaust stack is 30°C, and the constant pressure-specific heat of the exhaust gases is 1600 J/kg·K. On a particular calm day, the surrounding quiescent air temperature is 33°C. Solar radiation is incident on the exhaust stack outer surface at a rate of 500 W/m^2, and both the emissivity and solar absorptivity of the outer surface are 0.9. Determine the exhaust stack outer surface temperature. Assume the film temperature is 60°C.

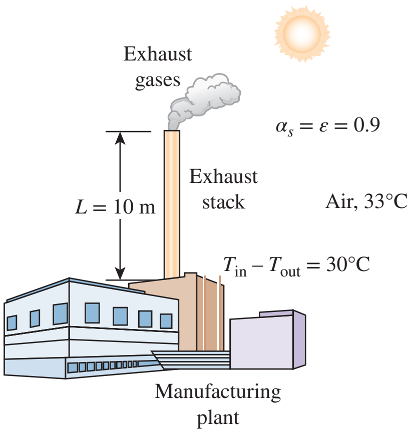


FIGURE P20–90

20–91 A 40-cm-diameter, 110-cm-high cylindrical hot-water tank is located in the bathroom of a house maintained at 20°C. The surface temperature of the tank is measured to be 44°C and its emissivity is 0.4. Taking the surrounding surface temperature to be also 20°C, determine the rate of heat loss from all surfaces of the tank by natural convection and radiation.

20–92 A 12-cm-diameter and 15-m-long cylinder with a surface temperature of 10°C is placed horizontally in air at 40°C. Calculate the steady rate of heat transfer for the cases of (*a*) free-stream air velocity of 10 m/s due to normal winds and (*b*) no winds and thus a free-stream velocity of zero.

20–93E Hot water is flowing at an average velocity of 4 ft/s through a cast iron pipe ($k = 30$ Btu/h·ft·°F) whose inner and outer diameters are 1.0 in and 1.2 in, respectively. The pipe passes through a 50-ft-long section of a basement whose temperature is 60°F. The emissivity of the outer surface of the pipe is 0.5, and the walls of the basement are also at about 60°F. If the inlet temperature of the water is 150°F and the heat transfer coefficient on the inner surface of the pipe is 30 Btu/h·ft^2·°F, determine the temperature drop of water as it passes through the basement. Evaluate air properties at a film temperature of 105°C and 1 atm pressure. Is this a good assumption?

20–94E Consider an industrial furnace that resembles a 13-ft-long horizontal cylindrical enclosure 8 ft in diameter whose end surfaces are well insulated. The furnace burns natural gas at a rate of 48 therms/h. The combustion efficiency of the furnace is 82 percent (i.e., 18 percent of the chemical energy of the fuel is lost through the flue gases as a result of incomplete combustion and the flue gases leaving the furnace at high temperature). If the heat loss from the outer surfaces of the furnace by natural convection and radiation is not to exceed 1 percent of the heat generated inside, determine the highest allowable surface temperature of the furnace. Assume the air and wall surface temperature of the room to be 75°F, and take the emissivity of the outer surface of the furnace to be 0.85. If the cost of natural gas is \$1.15/therm and the furnace operates 2800 h per year, determine the annual cost of this heat loss to the plant. Evaluate properties of air at a film temperature of 107.5°F and 1 atm pressure. Is this a good assumption?

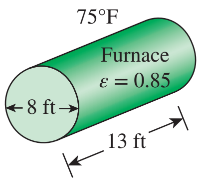


FIGURE P20–94E

20–95 A 6-m-internal-diameter spherical tank made of 1.5-cm-thick stainless steel ($k = 15$ W/m·K) is used to store iced water at 0°C in a room at 20°C. The walls of the room are also at 20°C. The outer surface of the tank is black (emissivity $\varepsilon = 1$), and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. Assuming the entire steel tank to be at 0°C and thus the thermal resistance of the tank to be negligible, determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period. The heat of fusion of water is 333.7 kJ/kg. *Answers:* (*a*) 15.4 kW, (*b*) 3988 kg

20–96 A spherical vessel with a 30.0-cm outside diameter is used as a reactor for a slow endothermic reaction. The vessel is completely submerged in a large water-filled tank, held at a constant temperature of 30°C. The outside surface temperature of the vessel is 20°C. Calculate the rate of heat transfer in steady operation for the following cases: (*a*) the water in the tank is still, (*b*) the water in the tank is still (as in a part *a*), however, the buoyancy force caused by the difference in water density is assumed to be negligible, and (*c*) the water in the tank is circulated at an average velocity of 20 cm/s.

20–97 Consider a 1.2-m-high and 2-m-wide double-pane window consisting of two 3-mm-thick layers of glass ($k = 0.78$ W/m·K) separated by a 2.5-cm-wide airspace. Determine the steady rate of heat transfer through this window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is 0°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2·K and $h_2 = 25$ W/m^2·K, and disregard any heat transfer by radiation. Evaluate air properties at a film temperature of 10°C and 1 atm pressure. Is this a good assumption?

20–98 A solar collector consists of a horizontal copper tube of outer diameter 5 cm enclosed in a concentric thin glass tube of 9 cm diameter. Water is heated as it flows through the tube, and the annular space between the copper and glass tube is filled with air at 1 atm pressure. During a clear day, the temperatures of the tube surface and the glass cover are measured to be 60°C and 32°C, respectively. Determine the rate of heat loss from the collector by natural convection per meter length of the tube. *Answer:* 17.4 W

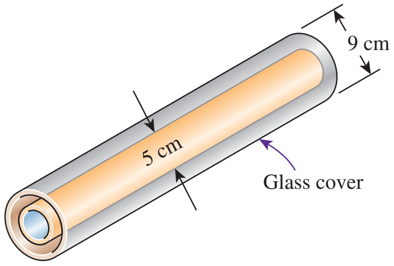


FIGURE P20–98

20–99 A solar collector consists of a horizontal aluminum tube of outer diameter 5 cm enclosed in a concentric thin glass tube of 7 cm diameter. Water is heated as it flows through the aluminum tube, and the annular space between the aluminum and glass tubes is filled with air at 1 atm pressure. The pump circulating the water fails during a clear day, and the water temperature in the tube starts rising. The aluminum tube absorbs solar radiation at a rate of 20 W per meter length, and the temperature of the ambient air outside is 30°C. Approximating the surfaces of the tube and the glass cover as being black (emissivity $\varepsilon = 1$) in radiation calculations and taking the effective sky temperature to be 20°C, determine the temperature of the aluminum tube when equilibrium is established (i.e., when the net heat loss from the tube by convection and radiation equals the amount of solar energy absorbed by the tube). For evaluation of air properties at 1 atm pressure, assume 33°C for the surface temperature of the glass cover and 45°C for the aluminum tube temperature. Are these good assumptions?

20–100 Two concentric spheres with diameters of 5 cm and 10 cm are having the surface temperatures maintained at 200°C and 100°C, respectively (see Fig. P20–100). The enclosure between the two concentric spherical surfaces is filled with nitrogen gas at 1 atm. If both concentric sphere surfaces are black ($\varepsilon = 1$), determine the rate of heat transfer on the inner surface.

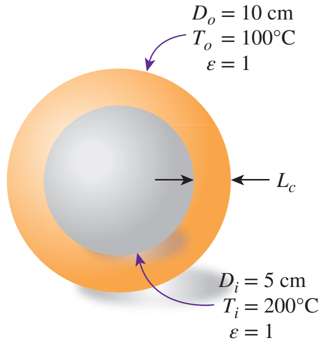


FIGURE P20–100

Design and Essay Problems

20–101 Contact a manufacturer of aluminum heat sinks and obtain its product catalog for cooling electronic components by natural convection and radiation. Write an essay on how to select a suitable heat sink for an electronic component when its maximum power dissipation and maximum allowable surface temperature are specified.

20–102 The top surfaces of practically all flat-plate solar collectors are covered with glass in order to reduce the heat losses from the absorber plate underneath. Although the glass cover reflects or absorbs about 15 percent of the incident solar radiation, it saves much more from the potential heat losses from the absorber plate, and thus it is considered to be an essential part of a well-designed solar collector. Inspired by the energy efficiency of double-pane windows, someone proposes to use double glazing on solar collectors instead of a single sheet of glass. Investigate if this is a good idea for the town in which you live. Use local weather data, and base your conclusion on heat transfer analysis and economic considerations.

CHAPTER

21

RADIATION HEAT TRANSFER

So far, we have considered the conduction and convection modes of heat transfer, which are related to the nature of the media involved and the presence of fluid motion, among other things. We now turn our attention to the third mechanism of heat transfer: *radiation,* which is characteristically different from the other two.

We start this chapter with a discussion of *electromagnetic waves* and the *electromagnetic spectrum,* with particular emphasis on *thermal radiation.* Then we introduce the idealized *blackbody, blackbody radiation,* and *blackbody radiation function,* together with the *Stefan–Boltzmann law, Planck's law,* and *Wien's displacement law.* Various radiation fluxes such as *emissive power, irradiation,* and *radiosity* are expressed. This is followed by a discussion of radiative properties of materials such as *emissivity, absorptivity, reflectivity,* and *transmissivity* and their dependence on wavelength and temperature. The *greenhouse effect* is presented as an example of the consequences of the wavelength dependence of radiation properties.

We continue with a discussion of view factors and the rules associated with them. View factor expressions and charts for some common configurations are given, and the crossed-strings method is presented. Finally, we discuss radiation heat transfer, first between black surfaces and then between nonblack surfaces using the radiation network approach.

OBJECTIVES

The objectives of this chapter are to:

- Classify electromagnetic radiation, and identify thermal radiation.
- Understand the idealized blackbody, and calculate the total and spectral blackbody emissive power.
- Calculate the fraction of radiation emitted in a specified wavelength band using the blackbody radiation functions.
- Develop a clear understanding of the properties emissivity, absorptivity, reflectivity, and transmissivity on a spectral, directional, and total basis.
- Apply Kirchhoff's law to determine the absorptivity of a surface when its emissivity is known.
- Define view factor, and understand its importance in radiation heat transfer calculations.
- Calculate radiation heat transfer between black surfaces.
- Obtain relations for the net rate of radiation heat transfer between the surfaces of a two-zone enclosure, including two large parallel plates, two long concentric cylinders, and two concentric spheres.
- Understand radiation heat transfer in three-surface enclosures.

21–1 ▪ INTRODUCTION

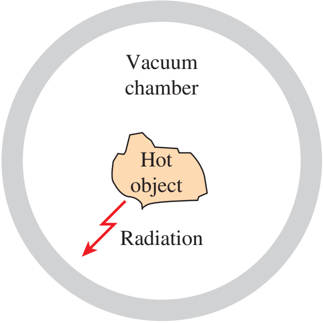


FIGURE 21–1
A hot object in a vacuum chamber loses heat by radiation only.

Consider a hot object that is suspended in an evacuated chamber whose walls are at room temperature (Fig. 21–1). The hot object will eventually cool down and reach thermal equilibrium with its surroundings. That is, it will lose heat until its temperature reaches the temperature of the walls of the chamber. Heat transfer between the object and the chamber could not have taken place by conduction or convection, because these two mechanisms cannot occur in a vacuum. Therefore, heat transfer must have occurred through another mechanism that involves the emission of the internal energy of the object. This mechanism is *radiation.*

Radiation differs from the other two heat transfer mechanisms in that it does not require the presence of a material medium to take place. In fact, energy transfer by radiation is fastest (at the speed of light), and it suffers no attenuation in a *vacuum.* Also, radiation transfer occurs in solids as well as liquids and gases. In most practical applications, all three modes of heat transfer occur concurrently at varying degrees. But heat transfer through an evacuated space can occur only by radiation. For example, the energy of the sun reaches the earth by radiation.

You will recall that heat transfer by conduction or convection takes place in the direction of decreasing temperature; that is, from a high-temperature medium to a lower-temperature one. It is interesting that radiation heat transfer can occur between two bodies separated by a medium colder than both bodies (Fig. 21–2). For example, solar radiation reaches the surface of the earth after passing through cold air layers at high altitudes. Also, the radiation-absorbing surfaces inside a greenhouse reach high temperatures even when its plastic or glass cover remains relatively cool.

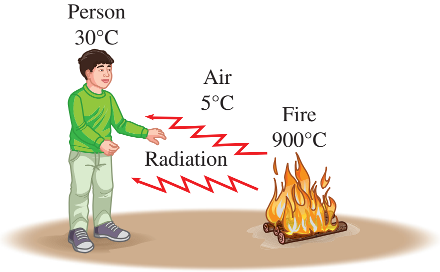


FIGURE 21–2
Unlike conduction and convection, heat transfer by radiation can occur between two bodies, even when they are separated by a medium colder than both.

The theoretical foundation of radiation was established in 1864 by physicist James Clerk Maxwell, who postulated that accelerated charges or changing electric currents give rise to electric and magnetic fields. These rapidly moving fields are called **electromagnetic waves** or **electromagnetic radiation**, and they represent the energy emitted by matter as a result of the changes in the electronic configurations of the atoms or molecules. In 1887, Heinrich Hertz experimentally demonstrated the existence of such waves. Electromagnetic waves transport energy just like other waves, and all electromagnetic waves travel at the *speed of light* in a vacuum, which is $c_0 = 2.9979 \times 10^8$ m/s. Electromagnetic waves are characterized by their *frequency* ν or *wavelength* λ. These two properties in a medium are related by

$$\lambda = \frac{c}{\nu} \tag{21–1}$$

where c is the speed of propagation of a wave in that medium. The speed of propagation in a medium is related to the speed of light in a vacuum by $c = c_0/n$, where n is the *index of refraction* of that medium. The refractive index is essentially unity for air and most gases, about 1.5 for glass, and 1.33 for water. The commonly used unit of wavelength is the *micrometer* (μm) or micron, where 1 μm = 10^{-6} m. Unlike the wavelength and the speed of propagation, the frequency of an electromagnetic wave depends only on the source and is independent of the medium through which the wave travels. The *frequency* (the number of oscillations per second) of an electromagnetic wave can range from less than a million Hz to a septillion Hz or higher, depending on the source. Note from Eq. 21–1 that the wavelength and the frequency of electromagnetic radiation are inversely proportional.

It has proven useful to view electromagnetic radiation as the propagation of a collection of discrete packets of energy called **photons** or **quanta**, as proposed by Max Planck in 1900 in conjunction with his *quantum theory.* In this view, each photon of frequency ν is considered to have an energy of

$$e = h\nu = \frac{hc}{\lambda} \tag{21–2}$$

where $h = 6.626069 \times 10^{-34}$ J·s is *Planck's constant.* Note from the second part of Eq. 21–2 that the energy of a photon is inversely proportional to its wavelength. Therefore, shorter-wavelength radiation possesses larger photon energies. It is no wonder that we try to avoid very-short-wavelength radiation such as gamma rays and X-rays since they are highly destructive.

21–2 ▪ THERMAL RADIATION

Although all electromagnetic waves have the same general features, waves of different wavelength differ significantly in their behavior. The electromagnetic radiation encountered in practice covers a wide range of wavelengths, varying from less than 10^{-10} μm for cosmic rays to more than 10^{10} μm for electrical power waves. The **electromagnetic spectrum** also includes gamma rays, X-rays, ultraviolet radiation, visible light, infrared radiation, thermal radiation, microwaves, and radio waves, as shown in Fig. 21–3.

Different types of electromagnetic radiation are produced through various mechanisms. For example, *gamma rays* are produced by nuclear reactions, *X-rays* by the bombardment of metals with high-energy electrons, *microwaves* by special types of electron tubes such as klystrons and magnetrons, and *radio waves* by the excitation of some crystals or by the flow of alternating current through electric conductors.

The short-wavelength gamma rays and X-rays are primarily of concern to nuclear engineers, while the long-wavelength microwaves and radio waves are of concern to electrical engineers. The type of electromagnetic radiation that is pertinent to heat transfer is the **thermal radiation** emitted as a result of energy transitions of molecules, atoms, and electrons of a substance. Temperature is a measure of the strength of these activities at the microscopic level, and the rate of thermal radiation emission increases with increasing temperature. Thermal radiation is continuously emitted by all matter whose temperature is above absolute zero. That is, everything around us, such as walls, furniture, and our friends, constantly emits (and absorbs) radiation (Fig. 21–4). Thermal radiation is also defined as the portion of the electromagnetic spectrum that extends from about 0.1 to 100 μm, since the radiation emitted by bodies due to their temperature falls almost entirely into this wavelength range. Thus, thermal radiation includes the entire visible and infrared (IR) radiation bands as well as a portion of the ultraviolet (UV) radiation band.

What we call **light** is simply the *visible* portion of the electromagnetic spectrum that lies between 0.40 and 0.76 μm. Light is characteristically no different than other electromagnetic radiation, except that it happens to trigger the sensation of seeing in the human eye. Light, or the visible spectrum, consists of narrow bands of color from violet (0.40–0.44 μm) to red (0.63–0.76 μm), as shown in Table 21–1.

A body that emits some radiation in the visible range is called a light source. The sun is obviously our primary light source. The electromagnetic radiation emitted by the sun is known as **solar radiation**, and nearly all of it falls into the wavelength band 0.3–3 μm. Almost *half* of solar radiation is light (i.e., it falls into the visible range), with the remaining portion being ultraviolet and infrared.

The radiation emitted by bodies at room temperature falls into the **infrared** region of the spectrum, which extends from 0.76 to 100 μm. Bodies start emitting noticeable visible radiation at temperatures above 800 K. The tungsten filament of a lightbulb must be heated to temperatures above 2000 K before it can emit any significant amount of radiation in the visible range.

The **ultraviolet** radiation includes the low-wavelength end of the thermal radiation spectrum and lies between the wavelengths 0.01 and 0.40 μm. Ultraviolet rays are to be avoided since they can kill microorganisms and cause serious damage to humans and other living beings. About 12 percent of solar radiation is in the ultraviolet range, and it would be devastating if it were to reach the surface of the earth.

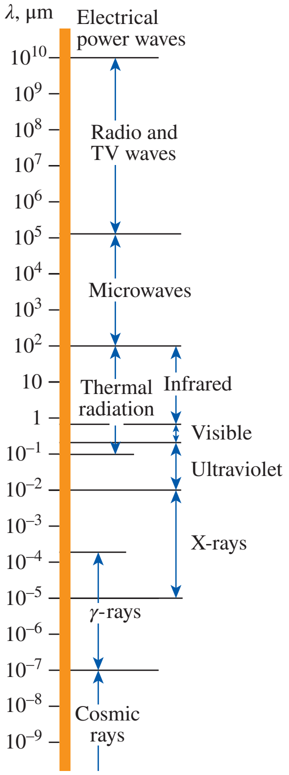


FIGURE 21–3

The electromagnetic wave spectrum.

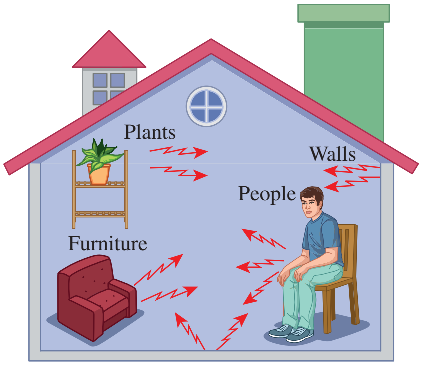


FIGURE 21–4

Everything around us constantly emits thermal radiation.

TABLE 21–1

The wavelength ranges of different colors

Color	Wavelength band
Violet	0.40–0.44 μm
Blue	0.44–0.49 μm
Green	0.49–0.54 μm
Yellow	0.54–0.60 μm
Orange	0.60–0.67 μm
Red	0.63–0.76 μm

Fortunately, the ozone (O_3) layer in the atmosphere acts as a protective blanket and absorbs most of this ultraviolet radiation. The ultraviolet rays that remain in sunlight are still sufficient to cause serious sunburns to sun worshippers, and prolonged exposure to direct sunlight is the leading cause of skin cancer, which can be lethal. Recent discoveries of "holes" in the ozone layer have prompted the international community to ban the use of ozone-destroying chemicals such as the refrigerant Freon-12 in order to save the earth. Ultraviolet radiation is also produced artificially in fluorescent lamps for use in medicine as a bacteria killer and in tanning parlors as an artificial tanner.

Microwave ovens utilize electromagnetic radiation in the **microwave** region of the spectrum generated by microwave tubes called *magnetrons.* Microwaves in the range of 10^2–10^5 μm are very suitable for use in cooking since they are *reflected* by metals, *transmitted* by glass and plastics, and *absorbed* by food (especially water) molecules. Thus, the electric energy converted to radiation in a microwave oven eventually becomes part of the internal energy of the food. The fast and efficient cooking of microwave ovens has made them one of the essential appliances in modern kitchens (Fig. 21–5).

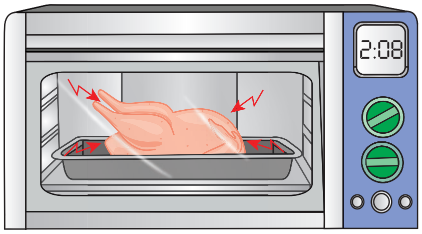


FIGURE 21–5
Food is heated or cooked in a microwave oven by absorbing the electromagnetic radiation energy generated by the magnetron of the oven.

Radars and cordless telephones also use electromagnetic radiation in the microwave region. The wavelength of the electromagnetic waves used in radio and TV broadcasting usually ranges between 1 and 1000 m in the **radio wave** region of the spectrum.

In heat transfer studies, we are interested in the energy emitted by bodies because of their temperature only. Therefore, we limit our consideration to *thermal radiation,* which we simply call *radiation.* The relations developed in this chapter are restricted to thermal radiation only and may not be applicable to other forms of electromagnetic radiation.

The electrons, atoms, and molecules of all solids, liquids, and gases above absolute zero temperature are constantly in motion, and thus radiation is constantly emitted, as well as being absorbed or transmitted throughout the entire volume of matter. That is, radiation is a **volumetric phenomenon**. However, for opaque (nontransparent) solids such as metals, wood, and rocks, radiation is considered to be a **surface phenomenon**, since the radiation emitted by the interior regions can never reach the surface, and the radiation incident on such bodies is usually absorbed within a few microns from the surface (Fig. 21–6). Note that the radiation characteristics of surfaces can be changed completely by applying thin layers of coatings on them.

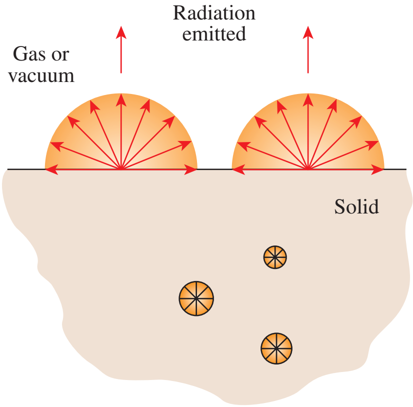


FIGURE 21–6
Radiation in opaque solids is considered a surface phenomenon since the radiation emitted only by the molecules at the surface can escape the solid.

21–3 ▪ BLACKBODY RADIATION

A body at a thermodynamic (or absolute) temperature above zero emits radiation in all directions over a wide range of wavelengths. The amount of radiation energy emitted from a surface at a given wavelength depends on the material of the body and the condition of its surface as well as the surface temperature. Therefore, different bodies may emit different amounts of radiation per unit surface area, even when they are at the same temperature. Thus, it is natural to be curious about the *maximum* amount of radiation that can be emitted by a surface at a given temperature. Satisfying this curiosity requires the definition of an idealized body, called a *blackbody,* to serve as a standard against which the radiative properties of real surfaces may be compared.

A **blackbody** is defined as *a perfect emitter and absorber of radiation.* At a specified temperature and wavelength, no surface can emit more energy than a blackbody. A blackbody absorbs *all* incident radiation, regardless of wavelength and direction. Also, a blackbody emits radiation energy uniformly in all directions per unit area normal to the direction of emission (Fig. 21–7). That is, a blackbody is a *diffuse* emitter. The term *diffuse* means "independent of direction."

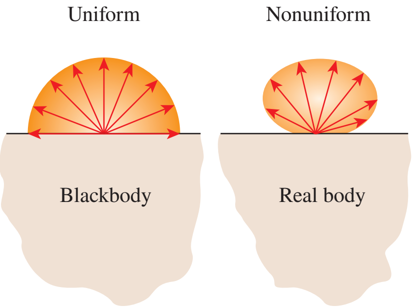


FIGURE 21–7
A blackbody is said to be a *diffuse* emitter since it emits radiation energy uniformly in all directions.

The radiation energy emitted by a blackbody per unit time and per unit surface area was determined experimentally by Joseph Stefan in 1879 and expressed as

$$E_b(T) = \sigma T^4 \qquad (\text{W/m}^2) \tag{21–3}$$

where $\sigma = 5.670 \times 10^{-8}$ W/m²·K⁴ is the *Stefan–Boltzmann constant* and T is the absolute temperature of the surface in K. This relation was theoretically verified in 1884 by Ludwig Boltzmann. Equation 21–3 is known as the **Stefan–Boltzmann law**, and E_b is called the **blackbody emissive power**. Note that the emission of thermal radiation is proportional to the *fourth power* of the absolute temperature.

Although a blackbody would appear *black* to the eye, a distinction should be made between the idealized blackbody and an ordinary black surface. Any surface that absorbs light (the visible portion of radiation) would appear black to the eye, and a surface that reflects it completely would appear white. Considering that visible radiation occupies a very narrow band of the spectrum from 0.4 to 0.76 μm, we cannot make any judgments about the blackness of a surface on the basis of visual observations. For example, snow and white paint reflect light and thus appear white. But they are essentially black for infrared radiation since they strongly absorb long-wavelength radiation. Surfaces coated with lampblack paint approach idealized blackbody behavior.

Another type of body that closely resembles a blackbody is a *large cavity with a small opening,* as shown in Fig. 21–8. Radiation coming in through the opening of area A undergoes multiple reflections, and thus it has several chances to be absorbed by the interior surfaces of the cavity before any part of it can possibly escape. Also, if the surface of the cavity is isothermal at temperature T, the radiation emitted by the interior surfaces streams through the opening after undergoing multiple reflections, and thus it has a diffuse nature. Therefore, the cavity acts as a perfect absorber and perfect emitter, and the opening will resemble a blackbody of surface area A at temperature T, regardless of the actual radiative properties of the cavity.

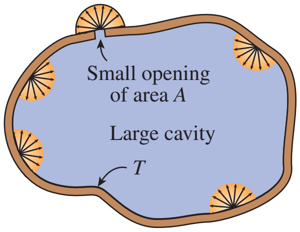


FIGURE 21–8

A large isothermal cavity at temperature T with a small opening of area A closely resembles a blackbody of surface area A at the same temperature.

The Stefan–Boltzmann law in Eq. 21–3 gives the *total* blackbody emissive power E_b, which is the sum of the radiation emitted over all wavelengths. Sometimes we need to know the **spectral blackbody emissive power**, which is *the amount of radiation energy emitted by a blackbody at a thermodynamic temperature T per unit time, per unit surface area, and per unit wavelength about the wavelength λ.* For example, we are more interested in the amount of radiation an incandescent lightbulb emits in the visible wavelength spectrum than we are in the total amount emitted.

The relation for the spectral blackbody emissive power $E_{b\lambda}$ was developed by Max Planck in 1901 in conjunction with his famous quantum theory. This relation is known as **Planck's law** and is expressed as

$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]} \qquad (\text{W/m}^2\cdot\mu\text{m}) \tag{21–4}$$

where

$$C_1 = 2\pi h c_0^2 = 3.74177 \times 10^8 \text{ W}\cdot\mu\text{m}^4/\text{m}^2$$

$$C_2 = hc_0/k = 1.43878 \times 10^4 \ \mu\text{m}\cdot\text{K}$$

Also, T is the absolute temperature of the surface, λ is the wavelength of the radiation emitted, and $k = 1.38065 \times 10^{-23}$ J/K is *Boltzmann's constant.* This relation is valid for a surface in a *vacuum* or a *gas.* For other media, it needs to be modified by replacing C_1 with C_1/n^2, where n is the index of refraction of the medium. Note that the term *spectral* indicates dependence on wavelength.

The variation of the spectral blackbody emissive power with wavelength is plotted in Fig. 21–9 for selected temperatures. Several observations can be made from this figure:

1. The emitted radiation is a continuous function of *wavelength.* At any specified temperature, it increases with wavelength, reaches a peak, and then decreases with increasing wavelength.
2. At any wavelength, the amount of emitted radiation *increases* with increasing temperature.
3. As temperature increases, the curves shift to the left to the shorter-wavelength region. Consequently, a larger fraction of the radiation is emitted at *shorter wavelengths* at higher temperatures.
4. The radiation emitted by the *sun,* which is considered to be a blackbody at 5780 K (or roughly at 5800 K), reaches its peak in the visible region of the spectrum. Therefore, the sun is in tune with our eyes. On the other hand, surfaces at $T \leq 800$ K emit almost entirely in the infrared region and thus are not visible to the eye unless they reflect light coming from other sources.

As the temperature increases, the peak of the curve in Fig. 21–9 shifts toward shorter wavelengths. The wavelength at which the peak occurs for a specified temperature is given by **Wien's displacement law** as

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K} \qquad \textbf{(21–5)}$$

This relation was originally developed by Willy Wien in 1894 using classical thermodynamics, but it can also be obtained by differentiating Eq. 21–4

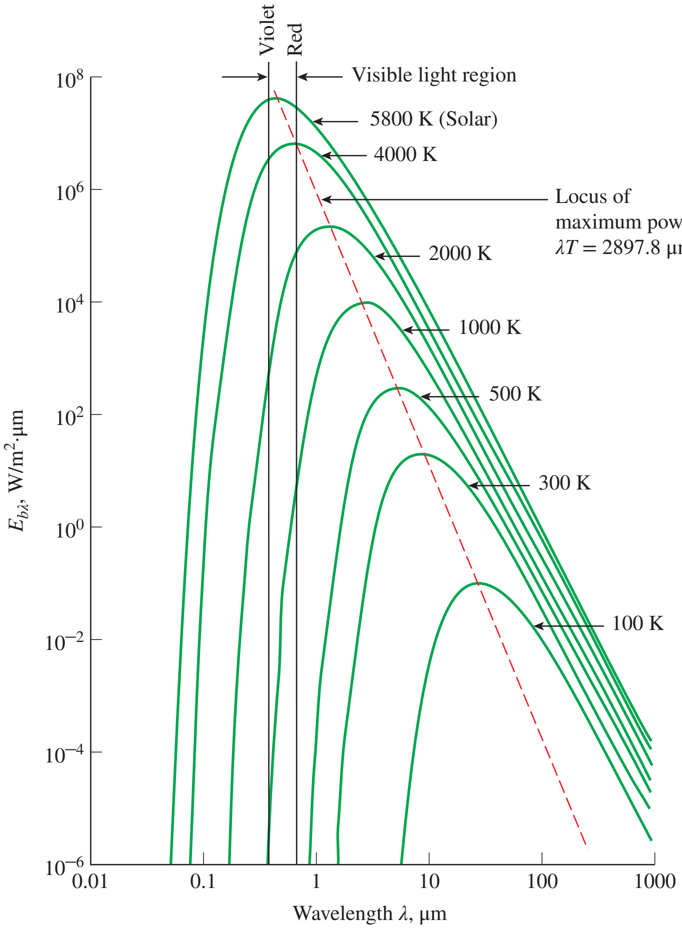


FIGURE 21–9
The variation of the blackbody emissive power with wavelength for several temperatures.

with respect to λ while holding T constant and setting the result equal to zero. A plot of Wien's displacement law, which is the locus of the peaks of the radiation emission curves, is also given in Fig. 21–9.

The peak of the solar radiation, for example, occurs at $\lambda = 2897.8/5780 = 0.50\ \mu\text{m}$, which is near the middle of the visible range. The peak of the radiation emitted by a surface at room temperature ($T = 298$ K) occurs at 9.72 μm, which is well into the infrared region of the spectrum.

An electrical resistance heater starts radiating heat soon after it is plugged in, and we can feel the emitted radiation energy by holding our hands against the heater. But this radiation is entirely in the infrared region and thus cannot be sensed by our eyes. The heater will appear dull red when its temperature reaches about 1000 K, since it starts emitting a detectable amount (about 1 $\text{W/m}^2\cdot\mu\text{m}$) of visible red radiation at that temperature. As the temperature rises even more, the heater appears bright red and is said to be *red hot.* When the temperature reaches about 1500 K, the heater emits enough radiation in the entire visible range of the spectrum to appear almost *white* to the eye, and it is called *white hot.*

Although it cannot be sensed directly by the human eye, infrared radiation can be detected by infrared cameras, which transmit the information to microprocessors to display visual images of objects at night. *Rattlesnakes* can sense the infrared radiation or the "body heat" coming off warm-blooded animals, and thus they can see at night without using any instruments. Similarly, honeybees are sensitive to ultraviolet radiation. A surface that reflects all of the light appears *white,* while a surface that absorbs all of the light incident on it appears black. (Then how do we see a black surface?)

It should be clear from this discussion that the color of an object is not due to emission, which is primarily in the infrared region, unless the surface temperature of the object exceeds about 1000 K. Instead, the color of a surface depends on the absorption and reflection characteristics of the surface and is due to selective absorption and reflection of the incident visible radiation coming from a light source such as the sun or an incandescent lightbulb. A piece of clothing containing a pigment that reflects red while absorbing the remaining parts of the incident light appears "red" to the eye (Fig. 21–10). Leaves appear "green" because their cells contain the pigment chlorophyll, which strongly reflects green while absorbing other colors.

It is left as an exercise to show that integration of the *spectral* blackbody emissive power $E_{b\lambda}$ over the entire wavelength spectrum gives the *total* blackbody emissive power E_b:

$$E_b(T) = \int_0^\infty E_{b\lambda}(\lambda, T)\, d\lambda = \sigma T^4 \quad (\text{W/m}^2) \tag{21–6}$$

Thus, we obtained the Stefan–Boltzmann law (Eq. 21–3) by integrating Planck's law (Eq. 21–4) over all wavelengths. Note that on an $E_{b\lambda}$–λ chart, $E_{b\lambda}$ corresponds to any value on the curve, whereas E_b corresponds to the area under the entire curve for a specified temperature (Fig. 21–11). Also, the term *total* means "integrated over all wavelengths."

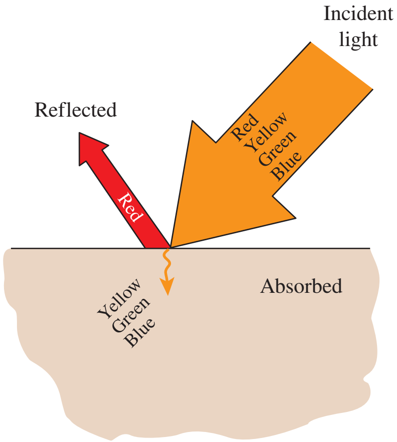


FIGURE 21–10
A surface that reflects red while absorbing the remaining parts of the incident light appears red to the eye.

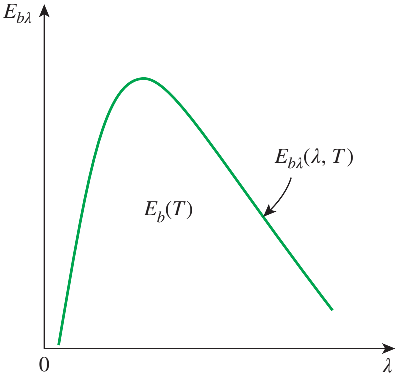


FIGURE 21–11
On an $E_{b\lambda}$–λ chart, the area under a curve for a given temperature represents the total radiation energy emitted by a blackbody at that temperature.

EXAMPLE 21–1 Radiation Emission from a Black Ball

Consider a 20-cm-diameter spherical ball at 800 K suspended in air as shown in Fig. 21–12. Assuming the ball closely approximates a blackbody, determine (*a*) the total blackbody emissive power, (*b*) the total amount of radiation emitted by the ball in 5 min, (*c*) the spectral blackbody emissive power at a wavelength of 3 μm, and (*d*) the wavelength for the maximum spectral blackbody emissive power at the ball surface temperature of 800 K.

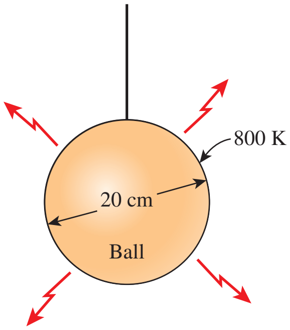


FIGURE 21–12
The spherical ball considered in Example 21–1.

SOLUTION An isothermal sphere is suspended in air. The total blackbody emissive power, the total radiation emitted in 5 min, the spectral blackbody emissive power at 3 μm, and the wavelength for the maximum spectral blackbody emissive power at the ball surface temperature of 800 K are to be determined.

Assumptions **1** The ball behaves as a blackbody. **2** Uniform surface temperature.

Analysis (*a*) The total blackbody emissive power is determined from the Stefan–Boltzmann law to be

$$E_b = \sigma T^4 = (5.67 \times 10^{-8}\ \text{W/m}^2{\cdot}\text{K}^4)(800\ \text{K})^4 = \mathbf{23.2\ kW/m^2}$$

That is, the ball emits 23.2 kJ of energy in the form of electromagnetic radiation per second per m^2 of the surface area of the ball.

(*b*) The total amount of radiation energy emitted from the entire ball in 5 min is determined by multiplying the blackbody emissive power obtained above by the total surface area of the ball and the given time interval:

$$A_s = \pi D^2 = \pi(0.2\ \text{m})^2 = 0.1257\ \text{m}^2$$

$$\Delta t = (5\ \text{min})\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = 300\ \text{s}$$

$$Q_{\text{rad}} = E_b A_s \Delta t = (23.2\ \text{kW/m}^2)(0.1257\ \text{m}^2)(300\ \text{s})\left(\frac{1\ \text{kJ}}{1\ \text{kW}{\cdot}\text{s}}\right)$$

$$= \mathbf{875\ kJ}$$

That is, the ball loses 875 kJ of its internal energy in the form of electromagnetic waves to the surroundings in 5 min, which is enough energy to heat 20 kg of water from 0°C to 100°C. Note that the surface temperature of the ball cannot remain constant at 800 K unless there is an equal amount of energy flow to the surface from the surroundings or from the interior regions of the ball through some mechanisms such as chemical or nuclear reactions.

(*c*) The spectral blackbody emissive power at a wavelength of 3 μm is determined from Planck's distribution law to be

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\dfrac{C_2}{\lambda T}\right) - 1\right]} = \frac{3.74177 \times 10^8\ \text{W}{\cdot}\mu\text{m}^4/\text{m}^2}{(3\ \mu\text{m})^5\left[\exp\left(\dfrac{1.43878 \times 10^4\ \mu\text{m}{\cdot}\text{K}}{(3\ \mu\text{m})(800\ \text{K})}\right) - 1\right]}$$

$$= \mathbf{3846\ W/m^2{\cdot}\boldsymbol{\mu}m}$$

(*d*) The wavelength for the maximum spectral blackbody emissive power at the ball surface temperature of 800 K is determined from Wien's displacement law to be

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}{\cdot}\text{K}$$

So,

$$\lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}{\cdot}\text{K}}{800\ \text{K}} = \mathbf{3.62\ \boldsymbol{\mu}m}$$

Discussion The maximum spectral blackbody emmisive power at the $\lambda_{\text{max power}} = 3.62\ \mu\text{m}$ is 4229 W/m^2·μm. This is about 10 percent higher than the value calculated in part (*c*) for a wavelength of 3 μm.

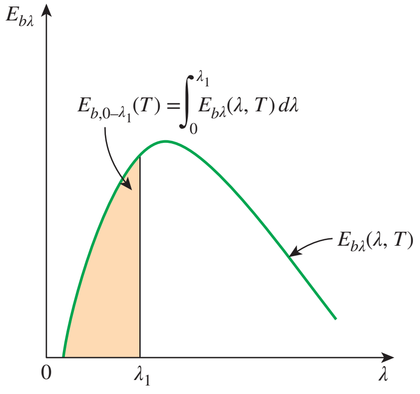


FIGURE 21–13
On an $E_{b\lambda}$–λ chart, the area under the curve to the left of the $\lambda = \lambda_1$ line represents the radiation energy emitted by a blackbody in the wavelength range 0–λ_1 for the given temperature.

The Stefan–Boltzmann law $E_b(T) = \sigma T^4$ gives the *total* radiation emitted by a blackbody at all wavelengths from $\lambda = 0$ to $\lambda = \infty$. But we are often interested in the amount of radiation emitted over *some wavelength band.* For example, an incandescent lightbulb is judged on the basis of the radiation it emits in the visible range rather than the radiation it emits at all wavelengths.

The radiation energy emitted by a blackbody per unit area over a wavelength band from $\lambda = 0$ to λ is determined from (Fig. 21–13)

$$E_{b,\,0-\lambda}(T) = \int_0^{\lambda} E_{b\lambda}(\lambda, T)\,d\lambda \quad (\text{W/m}^2) \qquad \textbf{(21–7)}$$

It looks like we can determine $E_{b,\,0-\lambda}$ by substituting the $E_{b\lambda}$ relation from Eq. 21–4 and performing this integration. But it turns out that this integration does not have a simple closed-form solution, and performing a numerical integration each time we need a value of $E_{b,\,0-\lambda}$ is not practical. Therefore, we define a dimensionless quantity f_λ called the **blackbody radiation function** as

$$f_\lambda(T) = \frac{\displaystyle\int_0^\lambda E_{b\lambda}(\lambda, T)\,d\lambda}{\sigma T^4} \qquad \textbf{(21–8)}$$

The function f_λ represents *the fraction of radiation emitted from a blackbody at temperature T in the wavelength band from* $\lambda = 0$ *to* λ. In reference to Fig. 21–13, f_λ can be interpreted as the ratio of the shaded region (radiation energy emitted by the blackbody in the wavelength range $0 - \lambda_1$) over the total area under the curve (total radiation energy emitted by the blackbody in the wavelength range $0 - \infty$). The values of f_λ are listed in Table 21–2 as a function of λT, where λ is in μm and T is in K.

The fraction of radiation energy emitted by a blackbody at temperature T over a finite wavelength band from $\lambda = \lambda_1$ to $\lambda = \lambda_2$ is determined from (Fig. 21–14)

$$f_{\lambda_1-\lambda_2}(T) = \frac{\displaystyle\int_0^{\lambda_2} E_{b\lambda}(\lambda, T)\,d\lambda - \int_0^{\lambda_1} E_{b\lambda}(\lambda, T)\,d\lambda}{\sigma T^4} = f_{\lambda_2}(T) - f_{\lambda_1}(T) \qquad \textbf{(21–9)}$$

where $f_{\lambda_1}(T)$ and $f_{\lambda_2}(T)$ are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, respectively.

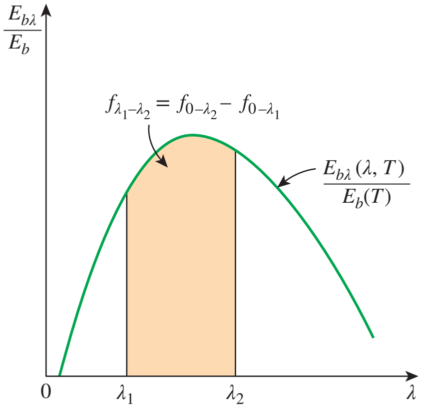


FIGURE 21–14
Graphical representation of the fraction of radiation emitted in the wavelength band from λ_1 to λ_2.

TABLE 21–2

Blackbody radiation functions f_λ

λT, μm·K	f_λ	λT, μm·K	f_λ
200	0.000000	6200	0.754140
400	0.000000	6400	0.769234
600	0.000000	6600	0.783199
800	0.000016	6800	0.796129
1000	0.000321	7000	0.808109
1200	0.002134	7200	0.819217
1400	0.007790	7400	0.829527
1600	0.019718	7600	0.839102
1800	0.039341	7800	0.848005
2000	0.066728	8000	0.856288
2200	0.100888	8500	0.874608
2400	0.140256	9000	0.890029
2600	0.183120	9500	0.903085
2800	0.227897	10,000	0.914199
3000	0.273232	10,500	0.923710
3200	0.318102	11,000	0.931890
3400	0.361735	11,500	0.939959
3600	0.403607	12,000	0.945098
3800	0.443382	13,000	0.955139
4000	0.480877	14,000	0.962898
4200	0.516014	15,000	0.969981
4400	0.548796	16,000	0.973814
4600	0.579280	18,000	0.980860
4800	0.607559	20,000	0.985602
5000	0.633747	25,000	0.992215
5200	0.658970	30,000	0.995340
5400	0.680360	40,000	0.997967
5600	0.701046	50,000	0.998953
5800	0.720158	75,000	0.999713
6000	0.737818	100,000	0.999905

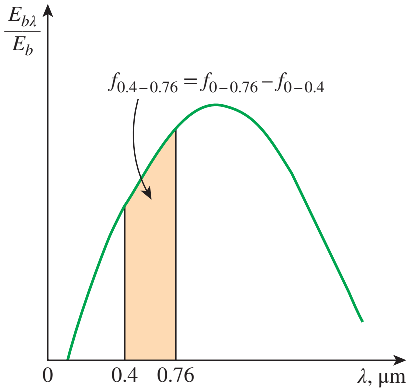


FIGURE 21–15
Graphical representation of the fraction of radiation emitted in the visible range in Example 21–2.

EXAMPLE 21–2 Light Emitted by the Sun and by a Lightbulb

Charge-coupled device (CCD) image sensors, which are common in modern digital cameras, respond differently to light sources with different spectral distributions. Daylight and incandescent light sources may be approximated as blackbodies at the effective surface temperatures of 5800 K and 2800 K, respectively. Determine the fraction of radiation emitted within the visible spectrum wavelengths, from 0.40 μm (violet) to 0.76 μm (red), for each of the lighting sources.

SOLUTION For specified blackbody temperatures, the fraction of visible radiation emitted by the sun and the filament of an incandescent lightbulb are to be determined.

Assumptions The sun and the incandescent light filament behave as blackbodies.

Analysis The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40$ μm to $\lambda_2 = 0.76$ μm. For the sun at $T = 5800$ K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 21–2 to be (Fig. 21–15)

$$\lambda_1 T = (0.40\ \mu\text{m})(5800\ \text{K}) = 2320\ \mu\text{m}\cdot\text{K} \rightarrow f_{\lambda_1,\ \text{daylight}} = 0.124509$$
$$\lambda_2 T = (0.76\ \mu\text{m})(5800\ \text{K}) = 4408\ \mu\text{m}\cdot\text{K} \rightarrow f_{\lambda_2,\ \text{daylight}} = 0.550015$$

Then the fraction of visible radiation emitted by the sun becomes

$$f_{\lambda_1-\lambda_2,\ \text{daylight}} = 0.550015 - 0.124509 = \mathbf{0.426} \text{ or } \mathbf{42.6\ percent}$$

For an incandescent lightbulb at $T = 2800$ K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 21–2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(2800\ \text{K}) = 1120\ \mu\text{m}\cdot\text{K} \rightarrow f_{\lambda_1,\ \text{incandescent}} = 0.001409$$
$$\lambda_2 T = (0.76\ \mu\text{m})(5800\ \text{K}) = 2128\ \mu\text{m}\cdot\text{K} \rightarrow f_{\lambda_2,\ \text{incandescent}} = 0.088590$$

Then the fraction of radiation the lightbulb emits in the visible range becomes

$$f_{\lambda_1-\lambda_2,\ \text{incandescent}} = 0.088590 - 0.001409 = \mathbf{0.087} \text{ or } \mathbf{8.7\ percent}$$

Discussion Note that almost half of the radiation emitted by the sun is in the visible range, and thus the sun is a very efficient light source. But less than 10 percent of the radiation emitted by the incandescent lightbulb is in the form of visible light, and thus incandescent lightbulbs are inefficient as light sources. Consequently, they are being replaced by the highly efficient fluorescent and LED light sources.

21–4 ▪ RADIATIVE PROPERTIES

Most materials encountered in practice, such as metals, wood, and bricks, are *opaque* to thermal radiation, and radiation is considered to be a *surface phenomenon* for such materials. That is, thermal radiation is emitted or absorbed within the first few microns of the surface, and thus we speak of radiative properties of *surfaces* for opaque materials.

Some other materials, such as glass and water, allow visible radiation to penetrate to considerable depths before any significant absorption takes place. Radiation through such *semitransparent* materials obviously cannot be considered to be a surface phenomenon since the entire volume of the material interacts with radiation. On the other hand, both glass and water are practically opaque to infrared radiation. Therefore, materials can exhibit different behavior at different wavelengths, and the dependence on wavelength is an important consideration in the study of radiative properties such as emissivity, absorptivity, reflectivity, and transmissivity of materials.

In the preceding section, we defined a *blackbody* as a perfect emitter and absorber of radiation and said that no body can emit more radiation than a blackbody at the same temperature. Therefore, a blackbody can serve as a convenient *reference* in describing the emission and absorption characteristics of real surfaces.

Emissivity

The **emissivity** of a surface represents *the ratio of the radiation emitted by the surface at a given temperature to the radiation emitted by a blackbody at the same temperature.* The emissivity of a surface is denoted by ε, and it varies between zero and one, $0 \le \varepsilon \le 1$. Emissivity is a measure of how closely a real surface approximates a blackbody, for which $\varepsilon = 1$.

The emissivity of a real surface is not a constant. Rather, it varies with the *temperature* of the surface as well as the *wavelength* and the *direction* of the emitted radiation. Therefore, different emissivities can be defined for a surface, depending on the effects considered. For example, the emissivity of a surface at a specified wavelength is called *spectral emissivity* and is denoted by ε_λ. Likewise, the emissivity in a specified direction is called *directional emissivity*, denoted by ε_θ, where the angle is between the direction of radiation and the normal of the surface. The most elemental emissivity of a surface at a given temperature is the *spectral directional emissivity* $\varepsilon_{\lambda,\theta}$, which is the emissivity at a specified wavelength in a specified direction. Note that blackbody radiation is independent of direction.

In practice, it is usually more convenient to work with radiation properties averaged over all directions, called *hemispherical properties.* Noting that the integral of the rate of radiation energy emitted at a specified wavelength per unit surface area over the entire hemisphere is *spectral emissive power,* the **spectral hemispherical emissivity** can be expressed as

$$\varepsilon_\lambda(\lambda, T) = \frac{E_\lambda(\lambda, T)}{E_{b\lambda}(\lambda, T)} \tag{21–10}$$

Note that the emissivity of a surface at a given wavelength can be different at different temperatures since the spectral distribution of emitted radiation (and thus the amount of radiation emitted at a given wavelength) changes with temperature.

The **total hemispherical emissivity** is defined in terms of the radiation energy emitted over all wavelengths in all directions as

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} \tag{21–11}$$

Therefore, the total hemispherical emissivity (or simply the "average emissivity") of a surface at a given temperature represents the ratio of the total radiation energy emitted by the surface to the radiation emitted by a blackbody of the same surface area at the same temperature.

Noting that $E = \int_0^\infty E_\lambda \, d\lambda$ and $E_\lambda(\lambda, T) = \varepsilon_\lambda(\lambda, T)\, E_{b\lambda}(\lambda, T)$, the total hemispherical emissivity can also be expressed as

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda, T) E_{b\lambda}(\lambda, T)\, d\lambda}{\sigma T^4} \tag{21–12}$$

since $E_b(T) = \sigma T^4$. To perform this integration, we need to know the variation of spectral emissivity with wavelength at the specified temperature. The integrand is usually a complicated function, and the integration has to be performed numerically. However, the integration can be performed quite easily by dividing the spectrum into a sufficient number of *wavelength bands* and assuming the emissivity to remain constant over each band; that is, by expressing the function $\varepsilon_\lambda(\lambda, T)$ as

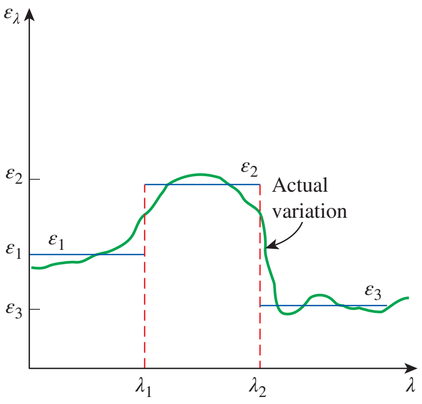


FIGURE 21–16
Approximating the actual variation of emissivity with wavelength by a step function.

a step function. This simplification offers great convenience for little sacrifice of accuracy, since it allows us to transform the integration into a summation in terms of blackbody emission functions.

As an example, consider the emissivity function plotted in Fig. 21–16. It seems like this function can be approximated reasonably well by a step function of the form

$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = \text{constant}, & 0 \le \lambda < \lambda_1 \\ \varepsilon_2 = \text{constant}, & \lambda_1 \le \lambda < \lambda_2 \\ \varepsilon_3 = \text{constant}, & \lambda_2 \le \lambda < \infty \end{cases} \tag{21–13}$$

Then the average emissivity can be determined from Eq. 21–12 by breaking the integral into three parts and utilizing the definition of the blackbody radiation function as

$$\begin{aligned} \varepsilon(T) &= \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda}\, d\lambda}{E_b} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b\lambda}\, d\lambda}{E_b} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda}\, d\lambda}{E_b} \\ &= \varepsilon_1 f_{0-\lambda_1}(T) + \varepsilon_2 f_{\lambda_1-\lambda_2}(T) + \varepsilon_3 f_{\lambda_2-\infty}(T) \end{aligned} \tag{21–14}$$

Radiation is a complex phenomenon as it is, and the consideration of the wavelength and direction dependence of properties, assuming sufficient data exist, makes it even more complicated. Therefore, the *diffuse* and *gray* approximations are often used in radiation calculations. A surface is said to be *diffuse* if its properties are *independent of direction,* and *gray* if its properties are *independent of wavelength.* Therefore, the emissivity of a gray, diffuse surface is simply the total hemispherical emissivity of that surface because of independence of direction and wavelength (Fig. 21–17).

Real surface:
$\varepsilon_\theta \neq$ constant
$\varepsilon_\lambda \neq$ constant

Diffuse surface:
$\varepsilon_\theta =$ constant

Gray surface:
$\varepsilon_\lambda =$ constant

Diffuse, gray surface:
$\varepsilon = \varepsilon_\lambda = \varepsilon_\theta =$ constant

FIGURE 21–17
The effect of diffuse and gray approximations on the emissivity of a surface.

A few comments about the validity of the diffuse approximation are in order. Although real surfaces do not emit radiation in a perfectly diffuse manner as a blackbody does, they often come close. The variation of emissivity with direction for both electrical conductors and nonconductors is given in Fig. 21–18. Here θ is the angle measured from the normal of the surface, and thus $\theta = 0$ for radiation emitted in a direction normal to the surface. Note that ε_θ remains nearly constant for about $\theta < 40°$ for conductors such as metals and for $\theta < 70°$ for nonconductors such as plastics. Therefore, the directional emissivity of a surface in the normal direction is representative of the hemispherical emissivity of the surface. In radiation analysis, it is common practice to assume the surfaces to be diffuse emitters with an emissivity equal to the value in the normal ($\theta = 0$) direction.

The effect of the gray approximation on emissivity and emissive power of a real surface is illustrated in Fig. 21–19. Note that the radiation emission from a real surface, in general, differs from the Planck distribution, and the emission curve may have several peaks and valleys. A gray surface should emit as much radiation as the real surface it represents at the same temperature. Therefore, the areas under the emission curves of the real and gray surfaces must be equal.

The emissivities of common materials are listed in Table A-26, and the variation of emissivity with wavelength and temperature is illustrated in Fig. 21–20. Typical ranges of emissivity of various materials are given in Fig. 21–21. Note that metals generally have low emissivities, as low as 0.02 for polished surfaces, and nonmetals such as ceramics and organic materials have high ones. The emissivity of metals increases with temperature. Also, oxidation causes significant increases in the emissivity of metals. Heavily oxidized metals can have emissivities comparable to those of nonmetals.

Care should be exercised in the use and interpretation of radiation property data reported in the literature, since the properties strongly depend on the

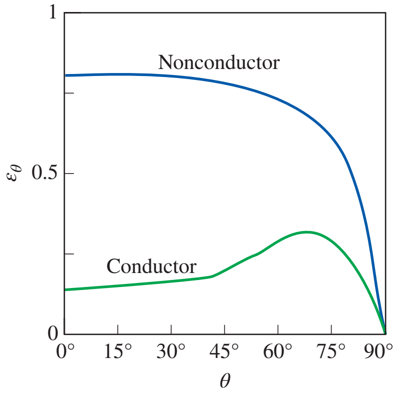
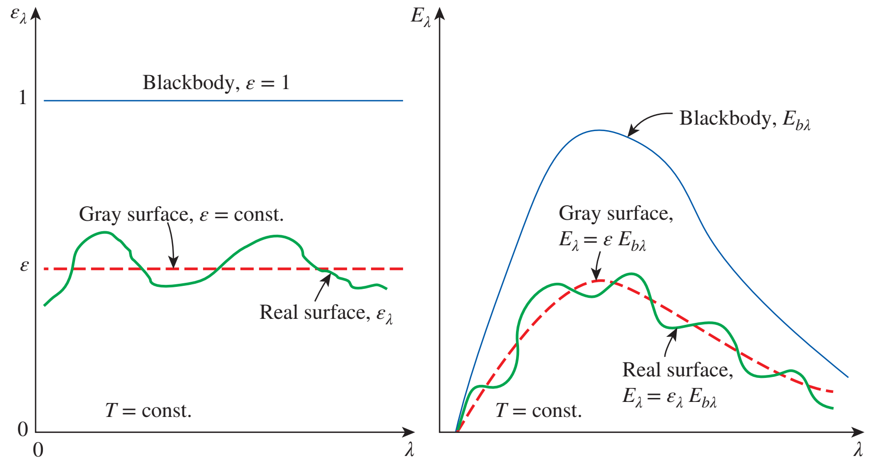
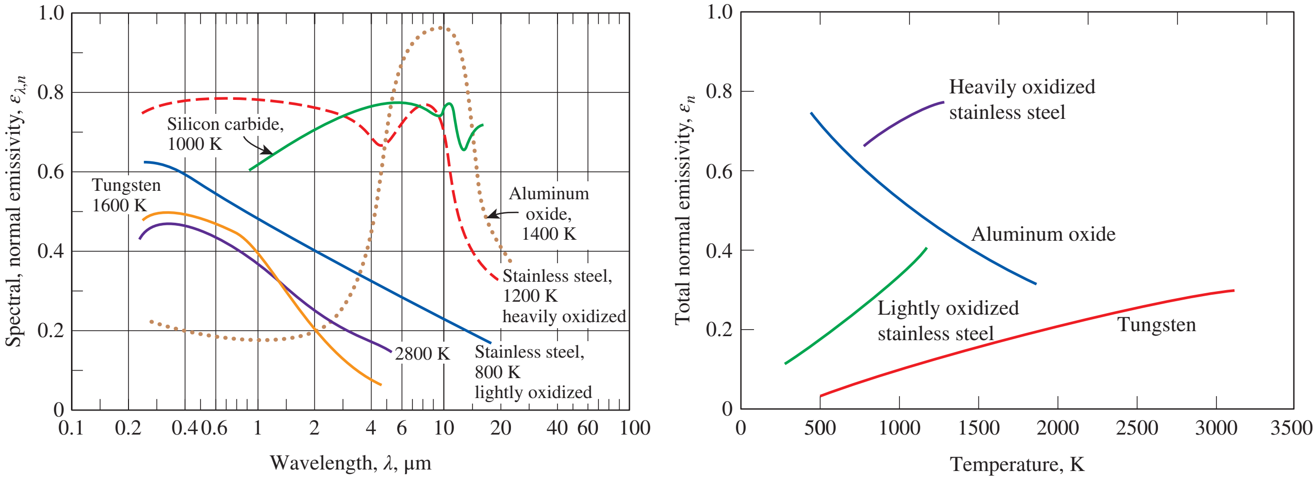


FIGURE 21–18
Typical variations of emissivity with direction for electrical conductors and nonconductors.



(*a*) (*b*)

FIGURE 21–19
Comparison of the emissivity (*a*) and emissive power (*b*) of a real surface with those of a gray surface and a blackbody at the same temperature.



(*a*) (*b*)

FIGURE 21–20
The variation of normal emissivity with (*a*) wavelength and (*b*) temperature for various materials.

surface conditions such as oxidation, roughness, type of finish, and cleanliness. Consequently, there is considerable discrepancy and uncertainty in the reported values. This uncertainty is largely due to the difficulty in characterizing and describing the surface conditions precisely.

EXAMPLE 21–3 Emissivity of a Surface and Emissive Power

The spectral emissivity function of an opaque surface at 800 K is approximated as (Fig. 21–22)

$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = 0.3, & 0 \le \lambda < 3\ \mu\text{m} \\ \varepsilon_2 = 0.8, & 3\ \mu\text{m} \le \lambda < 7\ \mu\text{m} \\ \varepsilon_3 = 0.1, & 7\ \mu\text{m} \le \lambda < \infty \end{cases}$$

Determine the average emissivity of the surface and its emissive power.

SOLUTION The variation of emissivity of a surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

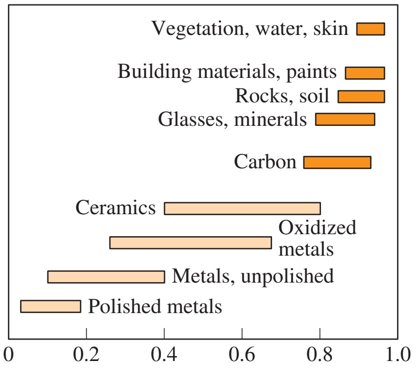


FIGURE 21–21
Typical ranges of emissivity for various materials.

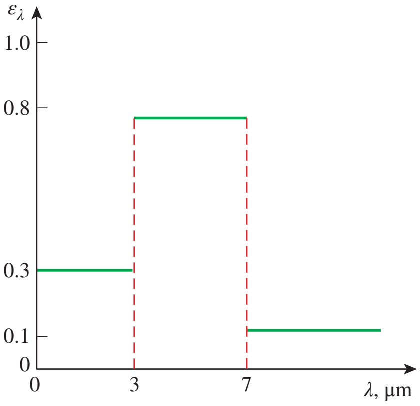


FIGURE 21–22
The spectral emissivity of the surface considered in Example 21–3.

Analysis The variation of the emissivity with wavelength is given as a step function. Therefore, the average emissivity of the surface can be determined from Eq. 21–12 by breaking the integral into three parts,

$$\begin{aligned}\varepsilon(T) &= \frac{\varepsilon_1\int_0^{\lambda_1} E_{b\lambda}\,d\lambda}{\sigma T^4} + \frac{\varepsilon_2\int_{\lambda_1}^{\lambda_2} E_{b\lambda}\,d\lambda}{\sigma T^4} + \frac{\varepsilon_3\int_{\lambda_2}^{\infty} E_{b\lambda}\,d\lambda}{\sigma T^4}\\ &= \varepsilon_1 f_{0-\lambda_1}(T) + \varepsilon_2 f_{\lambda_1-\lambda_2}(T) + \varepsilon_3 f_{\lambda_2-\infty}(T)\\ &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2(f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3(1 - f_{\lambda_2})\end{aligned}$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions and are determined from Table 21–2 to be

$$\lambda_1 T = (3\ \mu\text{m})(800\ \text{K}) = 2400\ \mu\text{m}\cdot\text{K} \to f_{\lambda_1} = 0.140256$$
$$\lambda_2 T = (7\ \mu\text{m})(800\ \text{K}) = 5600\ \mu\text{m}\cdot\text{K} \to f_{\lambda_2} = 0.701046$$

Note that $f_{0-\lambda_1} = f_{\lambda_1} - f_0 = f_{\lambda_1}$ since $f_0 = 0$, and $f_{\lambda_2-\infty} = f_\infty - f_{\lambda_2} = 1 - f_{\lambda_2}$ since $f_\infty = 1$. Substituting,

$$\begin{aligned}\varepsilon &= 0.3 \times 0.140256 + 0.8(0.701046 - 0.140256) + 0.1(1 - 0.701046)\\ &= \mathbf{0.521}\end{aligned}$$

That is, the surface will emit as much radiation energy at 800 K as a gray surface having a constant emissivity of $\varepsilon = 0.521$. The emissive power of the surface is

$$E = \varepsilon\sigma T^4 = 0.521(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(800\ \text{K})^4 = \mathbf{12{,}100\ W/m^2}$$

Discussion Note that the surface emits 12.1 kJ of radiation energy per second per m^2 area of the surface.

Absorptivity, Reflectivity, and Transmissivity

Everything around us constantly emits radiation, and the emissivity represents the emission characteristics of those bodies. This means that every body, including our own, is constantly bombarded by radiation coming from all directions over a range of wavelengths. Recall that radiation flux *incident on a surface* is called **irradiation** and is denoted by G.

When radiation strikes a surface, part of it is absorbed, part of it is reflected, and the remaining part, if any, is transmitted, as illustrated in Fig. 21–23. *The fraction of irradiation absorbed by the surface* is called the **absorptivity** α, *the fraction reflected by the surface* is called the **reflectivity** ρ, and *the fraction transmitted* is called the **transmissivity** τ. That is,

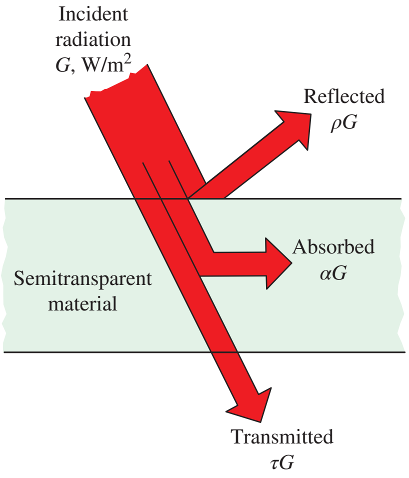


FIGURE 21–23
The absorption, reflection, and transmission of incident radiation by a semitransparent material.

Absorptivity: $$\alpha = \frac{\text{Absorbed radiation}}{\text{Incident radiation}} = \frac{G_{\text{abs}}}{G} \qquad 0 \le \alpha \le 1 \tag{21–15}$$

Reflectivity: $$\rho = \frac{\text{Reflected radiation}}{\text{Incident radiation}} = \frac{G_{\text{ref}}}{G} \qquad 0 \le \rho \le 1 \tag{21–16}$$

Transmissivity: $$\tau = \frac{\text{Transmitted radiation}}{\text{Incident radiation}} = \frac{G_{\text{tr}}}{G} \qquad 0 \le \tau \le 1 \tag{21–17}$$

where G is the radiation flux incident on the surface, and G_{abs}, G_{ref}, and G_{tr} are the absorbed, reflected, and transmitted portions of it, respectively. The first law of thermodynamics requires that the sum of the absorbed, reflected, and transmitted radiation be equal to the incident radiation. That is,

$$G_{\text{abs}} + G_{\text{ref}} + G_{\text{tr}} = G$$

Dividing each term of this relation by G yields

$$\alpha + \rho + \tau = 1 \tag{21–18}$$

For idealized blackbodies which are perfect absorbers, $\rho = 0$ and $\tau = 0$, and Eq. 21–18 reduces to $\alpha = 1$. For opaque surfaces such as most solids and liquids, $\tau = 0$, and thus

$$\alpha + \rho = 1 \tag{21–19}$$

For most gases, the reflectance is absent, $\rho = 0$, and Eq. 21–18 reduces in this case to

$$\alpha + \tau = 1 \tag{21–20}$$

The preceding two equations are important property relations since the knowledge of one property (either α or ρ in Eq. 21–19 and α or τ in Eq. 21–20) implies the knowledge of the other property.

These definitions are for *total hemispherical* properties, since G represents the radiation flux incident on the surface from all directions over the hemispherical space and over all wavelengths. Thus, α, ρ, and τ are the *average* properties of a medium for all directions and all wavelengths. However, like emissivity, these properties can also be defined for a specific wavelength and/or direction. For example,

$$\alpha_\lambda(\lambda) = \frac{G_{\lambda,\,\text{abs}}(\lambda)}{G_\lambda(\lambda)} \quad \text{and} \quad \rho_\lambda(\lambda) = \frac{G_{\lambda,\,\text{ref}}(\lambda)}{G_\lambda(\lambda)} \tag{21–21}$$

where G_λ is the spectral irradiation (in W/m^2·μm) incident on the surface, and $G_{\lambda,\,\text{abs}}$ and $G_{\lambda,\,\text{ref}}$ are the reflected and absorbed portions of it, respectively.

Similar quantities can be defined for the transmissivity of semitransparent materials. For example, the **spectral hemispherical transmissivity** of a medium can be expressed as

$$\tau_\lambda(\lambda) = \frac{G_{\lambda,\,\text{tr}}(\lambda)}{G_\lambda(\lambda)} \tag{21–22}$$

The average absorptivity, reflectivity, and transmissivity of a surface can also be defined in terms of their spectral counterparts as

$$\alpha = \frac{\displaystyle\int_0^\infty \alpha_\lambda G_\lambda \, d\lambda}{\displaystyle\int_0^\infty G_\lambda \, d\lambda} \qquad \rho = \frac{\displaystyle\int_0^\infty \rho_\lambda G_\lambda \, d\lambda}{\displaystyle\int_0^\infty G_\lambda \, d\lambda} \qquad \tau = \frac{\displaystyle\int_0^\infty \tau_\lambda G_\lambda \, d\lambda}{\displaystyle\int_0^\infty G_\lambda \, d\lambda} \tag{21–23}$$

The reflectivity differs somewhat from the other properties in that it is *bidirectional* in nature. That is, the value of the reflectivity of a surface depends not only on the direction of the incident radiation but also on the direction of reflection. Therefore, the reflected rays of a radiation beam incident on a real surface in a specified direction form an irregular shape, as shown in Fig. 21–24. Such detailed reflectivity data do not exist for most surfaces, and even if they did, they would be of little value in radiation calculations since this would usually add more complication to the analysis.

In practice, for simplicity, surfaces are assumed to reflect in a perfectly *specular* or *diffuse* manner. In **specular** (or *mirrorlike*) **reflection**, *the angle of reflection equals the angle of incidence of the radiation beam.* In **diffuse reflection**, *radiation is reflected equally in all directions,* as shown in Fig. 21–24. Reflection from smooth and polished surfaces approximates specular reflection, whereas reflection from rough surfaces approximates diffuse reflection. In radiation analysis, smoothness is defined relative to wavelength. A surface is said to be *smooth* if the height of the surface roughness is much smaller than the wavelength of the incident radiation.

Unlike emissivity, the absorptivity of a material is practically independent of surface temperature. However, the absorptivity depends strongly on the temperature of the source from which the incident radiation is originating. This is also evident from Fig. 21–25, which shows the absorptivities of various materials at room temperature as functions of the temperature of the radiation source. For example, the absorptivity of the concrete roof of a house is about 0.6 for solar radiation (source temperature: 5780 K) and 0.9 for radiation originating from the surrounding trees and buildings (source temperature: 300 K), as illustrated in Fig. 21–26.

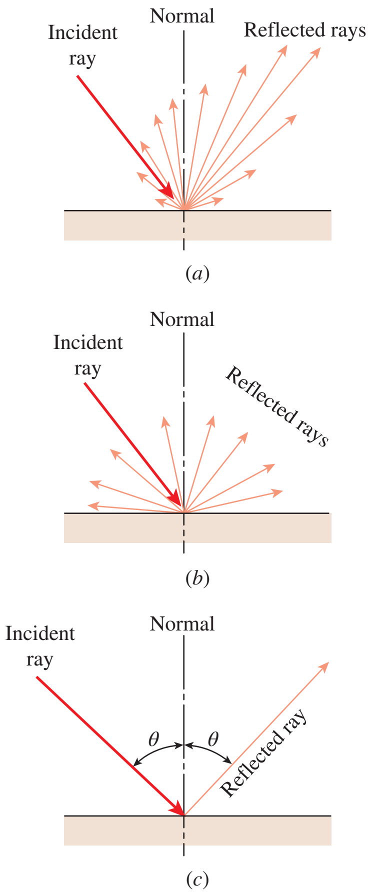


FIGURE 21–24
Different types of reflection from a surface: (*a*) actual or irregular, (*b*) diffuse, and (*c*) specular or mirrorlike.

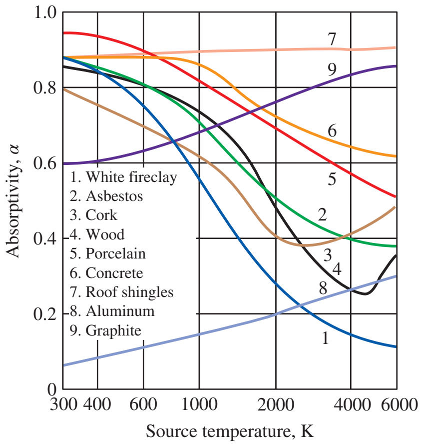


FIGURE 21–25
Variation of absorptivity with the temperature of the source of irradiation for various common materials at room temperature.

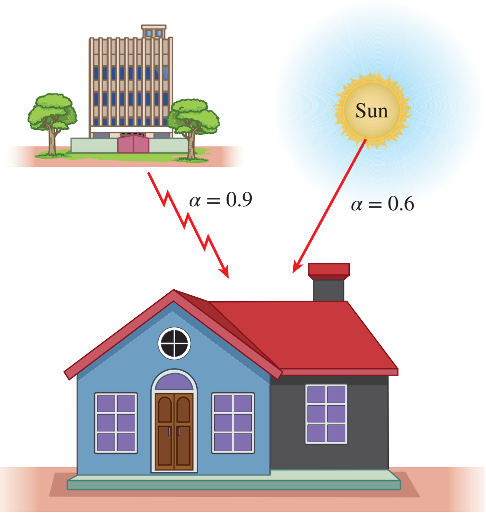


FIGURE 21–26
The absorptivity of a material may be quite different for radiation originating from sources at different temperatures.

Notice that the absorptivity of aluminum increases with the source temperature, a characteristic for metals, and the absorptivity of electric nonconductors, in general, decreases with temperature. This decrease is most pronounced for surfaces that appear white to the eye. For example, the absorptivity of a white painted surface is low for solar radiation, but it is rather high for infrared radiation.

Kirchhoff's Law

Consider a small body of surface area A_s, emissivity ε, and absorptivity α at temperature T contained in a large isothermal enclosure at the same temperature, as shown in Fig. 21–27. Recall that a large isothermal enclosure forms a blackbody cavity regardless of the radiative properties of the enclosure surface, and the body in the enclosure is too small to interfere with the blackbody nature of the cavity. Therefore, the radiation incident on any part of the surface of the small body is equal to the radiation emitted by a blackbody at temperature T. That is, $G = E_b(T) = \sigma T^4$, and the radiation absorbed by the small body per unit of its surface area is

$$G_{\text{abs}} = \alpha G = \alpha\sigma T^4$$

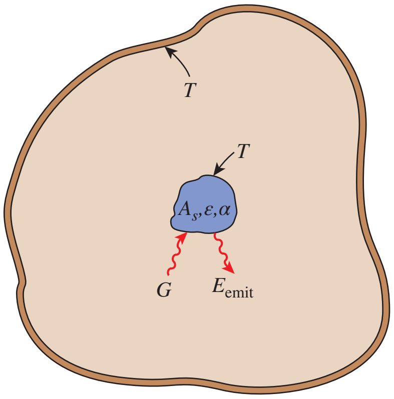


FIGURE 21–27
The small body contained in a large isothermal enclosure used in the development of Kirchhoff's law.

The radiation emitted by the small body is

$$E_{\text{emit}} = \varepsilon\sigma T^4$$

Considering that the small body is in thermal equilibrium with the enclosure, the net rate of heat transfer to the body must be zero. Therefore, the radiation emitted by the body must be equal to the radiation absorbed by it:

$$A_s \varepsilon\sigma T^4 = A_s \alpha\sigma T^4$$

Thus, we conclude that for any surface in the enclosure,

$$\varepsilon(T) = \alpha(T) \tag{21–24}$$

That is, *the total hemispherical emissivity of a surface at temperature T is equal to its total hemispherical absorptivity for radiation coming from a blackbody at the same temperature.* This relation, which greatly simplifies the radiation analysis, was first developed by Gustav Kirchhoff in 1860 and is now called **Kirchhoff's law**. Note that this relation is derived under the condition that the surface temperature is equal to the temperature of the source of irradiation, and the reader is cautioned against using it when considerable difference (more than a few hundred degrees) exists between the surface temperature and the temperature of the source of irradiation.

For any surface in the enclosure, the preceding derivation can also be repeated for radiation at a specified wavelength to obtain the *spectral* form of Kirchhoff's law:

$$\varepsilon_\lambda(T) = \alpha_\lambda(T) \tag{21–25}$$

This relation is valid when the irradiation or the emitted radiation is independent of direction. The form of Kirchhoff's law that involves no restrictions is the *spectral directional* form expressed as $\varepsilon_{\lambda,\theta}(T) = \alpha_{\lambda,\theta}(T)$. That is, the emissivity of a surface at a specified wavelength, direction, and temperature is always equal to its absorptivity at the same wavelength, direction, and temperature.

It is very tempting to use Kirchhoff's law in radiation analysis since the relation $\varepsilon = \alpha$ together with $\rho = 1 - \alpha$ enables us to determine all three properties of an opaque surface from a knowledge of only *one* property. Although Eq. 21–24 gives acceptable results in most cases, in practice, care should be exercised when there is considerable difference between the surface temperature and the temperature of the source of incident radiation.

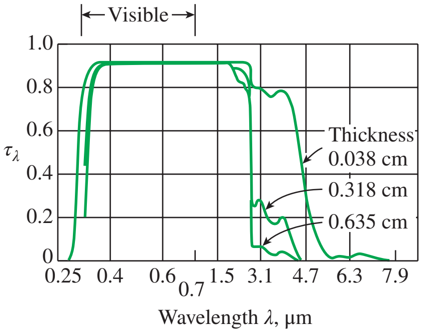


FIGURE 21–28

The spectral transmissivity of low-iron glass at room temperature for different thicknesses.

The Greenhouse Effect

You have probably noticed that when you leave your car under direct sunlight on a sunny day, the interior of the car gets much warmer than the air outside, and you may have wondered why the car acts like a *heat trap.* The answer lies in the spectral transmissivity curve of the *glass,* which resembles an inverted U, as shown in Fig. 21–28. We observe from this figure that glass at thicknesses encountered in practice transmits over 90 percent of radiation in the visible range and is practically opaque (nontransparent) to radiation in the longer-wavelength infrared regions of the electromagnetic spectrum (roughly $\lambda > 3$ μm). Therefore, glass has a transparent window in the wavelength range $0.3\ \mu\text{m} < \lambda < 3\ \mu\text{m}$ in which over 90 percent of solar radiation is emitted. On the other hand, the radiation emitted by surfaces at room temperature falls entirely in the infrared region. Consequently, glass allows the solar radiation to enter but does not allow the infrared radiation from the interior surfaces to escape. This causes a rise in the interior temperature as a result of the energy buildup in the car. This heating effect, which is due to the nongray characteristic of glass (or clear plastics), is known as the **greenhouse effect**, since it is used extensively in greenhouses (Fig. 21–29).

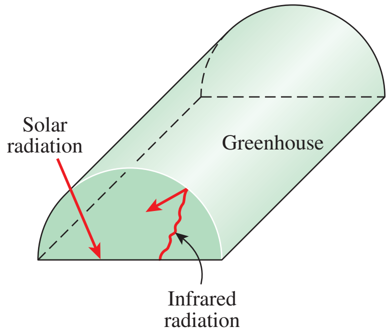


FIGURE 21–29

A greenhouse traps energy by allowing the solar radiation to come in but not allowing the infrared radiation to go out.

The greenhouse effect is also experienced on a larger scale on earth. The surface of the earth, which warms up during the day as a result of the absorption of solar energy, cools down at night by radiating its energy into deep space as infrared radiation. The combustion gases such as CO_2 and water vapor in the atmosphere transmit the bulk of the solar radiation but absorb the infrared radiation emitted by the surface of the earth. Thus, there is concern that the energy trapped on earth will eventually cause global warming and thus drastic changes in weather patterns.

In *humid* places such as coastal areas, there is not a large change between the daytime and nighttime temperatures because the humidity acts as a barrier in the path of the infrared radiation coming from the earth, and thus it slows down the cooling process at night. In areas with clear skies such as deserts, there is a large swing between the daytime and nighttime temperatures because of the absence of such barriers for infrared radiation.

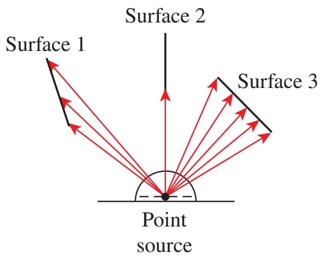


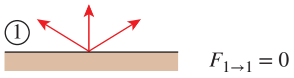
FIGURE 21–30

Radiation heat exchange between surfaces depends on the *orientation* of the surfaces relative to each other, and this dependence on orientation is accounted for by the *view factor.*

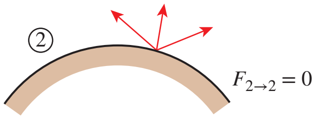
21–5 ▪ THE VIEW FACTOR

Radiation heat transfer between surfaces depends on the *orientation* of the surfaces relative to each other as well as their radiation properties and temperatures, as illustrated in Fig. 21–30. For example, a camper can make the most use of a campfire on a cold night by standing as close to the fire as possible and by blocking as much of the radiation coming from the fire as possible by facing the fire instead of turning sideways to it. Likewise, one can maximize the amount of solar radiation incident on oneself while sunbathing by lying down instead of standing.

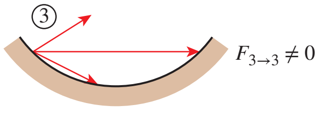
To account for the effects of orientation on radiation heat transfer between two surfaces, we define a new parameter called the *view factor,* which is a purely geometric quantity and is independent of the surface properties and temperature. It is also called the *shape factor, configuration factor,* and *angle factor.* The view factor based on the assumption that the surfaces are diffuse emitters and diffuse reflectors is called the *diffuse view factor,* and the view factor based on the assumption that the surfaces are diffuse emitters but specular reflectors is called the *specular view factor.* In this book, we consider radiation exchange between diffuse surfaces only, and thus the term *view factor* simply means *diffuse view factor.*



(*a*) Plane surface



(*b*) Convex surface



(*c*) Concave surface

FIGURE 21–31
The view factor from a surface to itself is *zero* for *plane* or *convex* surfaces and *nonzero* for *concave* surfaces.

The view factor from a surface i to a surface j is denoted by $F_{i \to j}$ or just F_{ij}, and is defined as

$$F_{ij} = \text{the fraction of the radiation leaving surface } i \text{ that strikes surface } j \text{ directly}$$

The notation $F_{i \to j}$ is instructive for beginners, since it emphasizes that the view factor is for radiation that travels from surface i to surface j. However, this notation becomes rather awkward when it has to be used many times in a problem. In such cases, it is convenient to replace it with its shorthand version F_{ij}.

The view factor F_{12} represents the fraction of radiation leaving surface 1 that strikes surface 2 directly, and F_{21} represents the fraction of radiation leaving surface 2 that strikes surface 1 directly. Note that the radiation that strikes a surface does not need to be absorbed by that surface. Also, radiation that strikes a surface after being reflected by other surfaces is not considered in the evaluation of view factors.

For the special case of $j = i$, we have

$$F_{i \to i} = \text{the fraction of radiation leaving surface } i \text{ that strikes itself directly}$$

Noting that in the absence of strong electromagnetic fields radiation beams travel in straight paths, the view factor from a surface to itself is zero unless the surface "sees" itself. Therefore, $F_{i \to i} = 0$ for *plane* or *convex* surfaces and $F_{i \to i} \neq 0$ for concave surfaces, as illustrated in Fig. 21–31.

The value of the view factor ranges between *zero* and *one.* The limiting case $F_{i \to i} = 0$ indicates that the two surfaces do not have a direct view of each other, and thus radiation leaving surface i cannot strike surface j directly. The other limiting case $F_{i \to j} = 1$ indicates that surface j completely surrounds surface $i,$ so all radiation leaving surface i is intercepted by surface $j.$ For example, in a geometry consisting of two concentric spheres, all radiation leaving the surface of the smaller sphere (surface 1) strikes the larger sphere (surface 2), and thus $F_{1 \to 2} = 1$, as illustrated in Fig. 21–32.

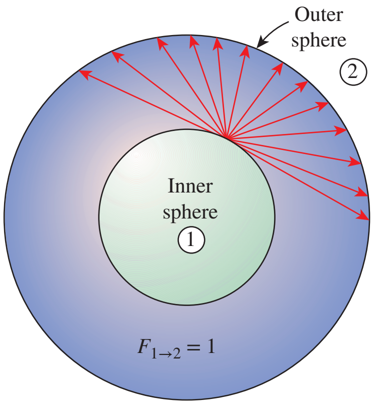


FIGURE 21–32
In a geometry that consists of two concentric spheres, the view factor $F_{1 \to 2} = 1$ since all radiation leaving the surface of the smaller sphere is intercepted by the larger sphere.

The view factor has proven to be very useful in radiation analysis because it allows us to express the *fraction of radiation* leaving a surface that strikes another surface in terms of the orientation of these two surfaces relative to each other. The underlying assumption in this process is that the radiation a surface receives from a source is directly proportional to the angle the surface subtends when viewed from the source. This would be the case only if the radiation coming off the source is *uniform* in all directions throughout its surface and the medium between the surfaces does not *absorb, emit,* or *scatter* radiation. That is, it is the case when the surfaces are *isothermal* and *diffuse* emitters and reflectors and the surfaces are separated by a *nonparticipating* medium such as a vacuum or air.

The view factor $F_{1 \to 2}$ between two surfaces A_1 and A_2 can be determined in a systematic manner first by expressing the view factor between two differential areas dA_1 and dA_2 in terms of the spatial variables and then by performing the necessary integrations. However, this approach is not practical, since even for simple geometries the resulting integrations are usually very complex and difficult to perform.

View factors for hundreds of common geometries are evaluated and the results given in analytical, graphical, and tabular form in several publications. Early tables and charts for simple configurations were given by Hamilton and Morgan (1952). Fairly extensive tabulations were given in the books by Siegel and Howell (2002) and Modest (2003). Siegel and Howell (2002) also give an exhaustive listing of sources for more involved view factors. The most complete tabulation is given in a catalog by Howell (1982). Some of the view factor expressions that appear in heat transfer books and catalogs are very difficult to use. Thyageswaran (2017) has derived an exact, closed-form and easy-to–use analytical expression to simplify view factor calculations for radiation exchange between a pair of directly opposed, unequal rectangles in parallel planes. View factors for selected geometries are given in Tables 21–3 and 21–4 in *analytical* form and in Figs. 21–33 to 21–36 in *graphical* form. The view factors in Table 21–3 are for three-dimensional geometries. The view factors in Table 21–4, on the other hand, are for geometries that are *infinitely long* in the direction perpendicular to the plane of the paper and are therefore two-dimensional.

TABLE 21–3

View factor expressions for some common geometries of finite size (3-D)

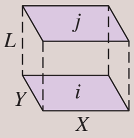
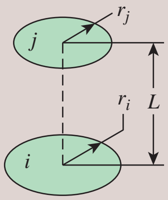
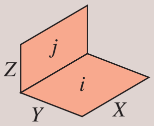
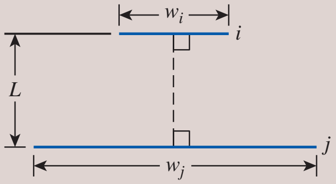
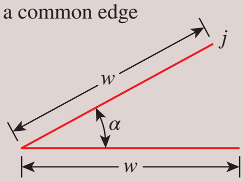
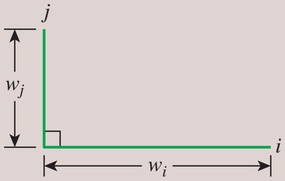
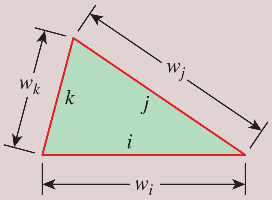
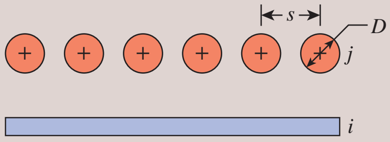
Geometry	Relation
Aligned parallel rectangles  	$\bar{X} = X/L$ and $\bar{Y} = Y/L$ $F_{i \to j} = \dfrac{2}{\pi \bar{X}\bar{Y}} \left\{ \ln \left[\dfrac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2} \right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2} \tan^{-1} \dfrac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} \right.$ $\left. + \bar{Y}(1+\bar{X}^2)^{1/2} \tan^{-1} \dfrac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X} \tan^{-1} \bar{X} - \bar{Y} \tan^{-1} \bar{Y} \right\}$
Coaxial parallel disks  	$R_i = r_i/L$ and $R_j = r_j/L$ $S = 1 + \dfrac{1+R_j^2}{R_i^2}$ $F_{i \to j} = \dfrac{1}{2} \left\{ S - \left[S^2 - 4\left(\dfrac{r_j}{r_i}\right)^2 \right]^{1/2} \right\}$ For $r_i = r_j = r$ and $R = r/L$: $F_{i \to j} = F_{j \to i} = 1 + \dfrac{1 - \sqrt{4R^2+1}}{2R^2}$
Perpendicular rectangles with a common edge  	$H = Z/X$ and $W = Y/X$ $F_{i \to j} = \dfrac{1}{\pi W} \left(W \tan^{-1} \dfrac{1}{W} + H \tan^{-1} \dfrac{1}{H} - (H^2+W^2)^{1/2} \tan^{-1} \dfrac{1}{(H^2+W^2)^{1/2}} \right.$ $+ \dfrac{1}{4} \ln \left\{ \dfrac{(1+W^2)(1+H^2)}{1+W^2+H^2} \left[\dfrac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)} \right]^{W^2} \right.$ $\left. \left. \times \left[\dfrac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)} \right]^{H^2} \right\} \right)$

TABLE 21–4

View factor expressions for some infinitely long (2-D) geometries

Geometry	Relation
Parallel plates with midlines connected by perpendicular line 	$W_i = w_i/L$ and $W_j = w_j/L$ $F_{i\rightarrow j} = \dfrac{[(W_i + W_j)^2 + 4]^{1/2} - [(W_j - W_i)^2 + 4]^{1/2}}{2W_i}$
Inclined plates of equal width and with a common edge 	$F_{i\rightarrow j} = 1 - \sin \frac{1}{2}\alpha$
Perpendicular plates with a common edge 	$F_{i\rightarrow j} = \frac{1}{2}\left\{1 + \frac{w_j}{w_i} - \left[1 + \left(\frac{w_j}{w_i}\right)^2\right]^{1/2}\right\}$
Three-sided enclosure 	$F_{i\rightarrow j} = \dfrac{w_i + w_j - w_k}{2w_i}$
Infinite plane and row of cylinders 	$F_{i\rightarrow j} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \frac{D}{s}\tan^{-1}\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}$

21–6 ▪ VIEW FACTOR RELATIONS

Radiation analysis on an enclosure consisting of N surfaces requires the evaluation of N^2 view factors, and this evaluation process is probably the most time-consuming part of a radiation analysis. However, it is neither practical nor necessary to evaluate all of the view factors directly. Once a sufficient number of view factors are available, the rest of them can be determined by utilizing some fundamental relations for view factors, as discussed next.

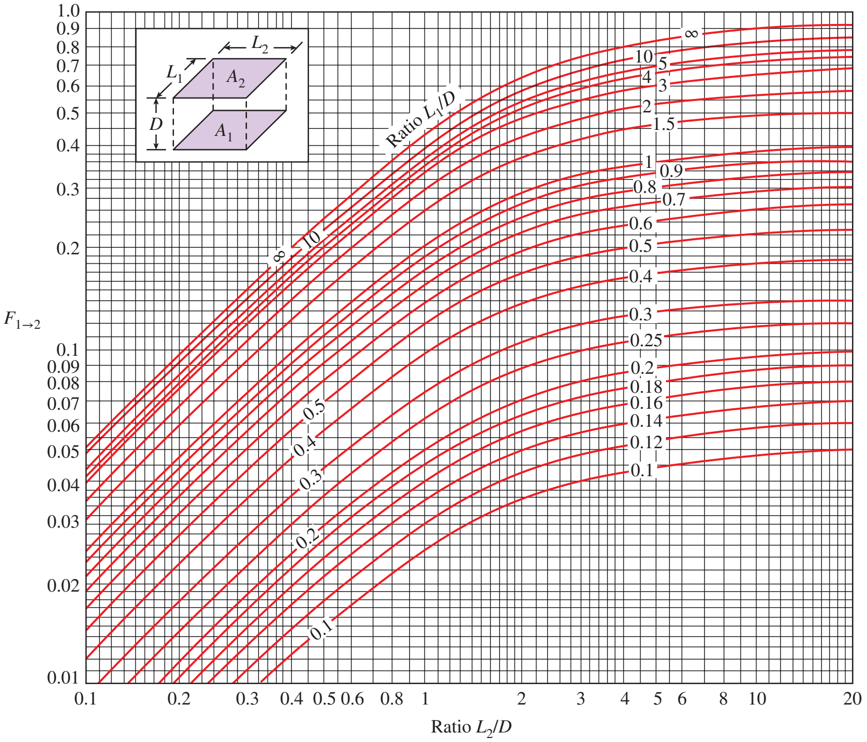


FIGURE 21–33
View factor between two aligned parallel rectangles of equal size.

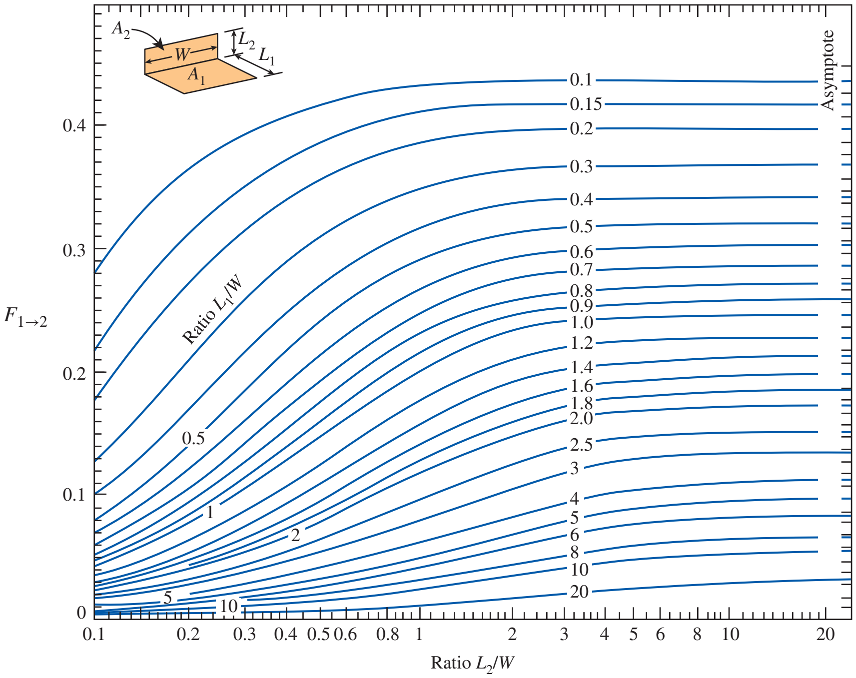


FIGURE 21–34
View factor between two perpendicular rectangles with a common edge.

1 The Reciprocity Relation

The view factors $F_{i \to j}$ and $F_{j \to i}$ are *not* equal to each other unless the areas of the two surfaces are. That is,

$$F_{j \to i} = F_{i \to j} \quad \text{when} \quad A_i = A_j$$
$$F_{j \to i} \neq F_{i \to j} \quad \text{when} \quad A_i \neq A_j$$

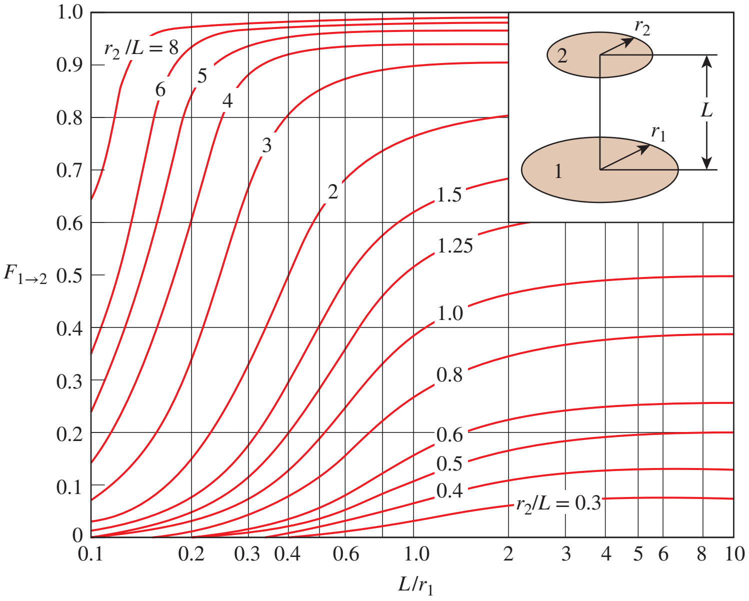
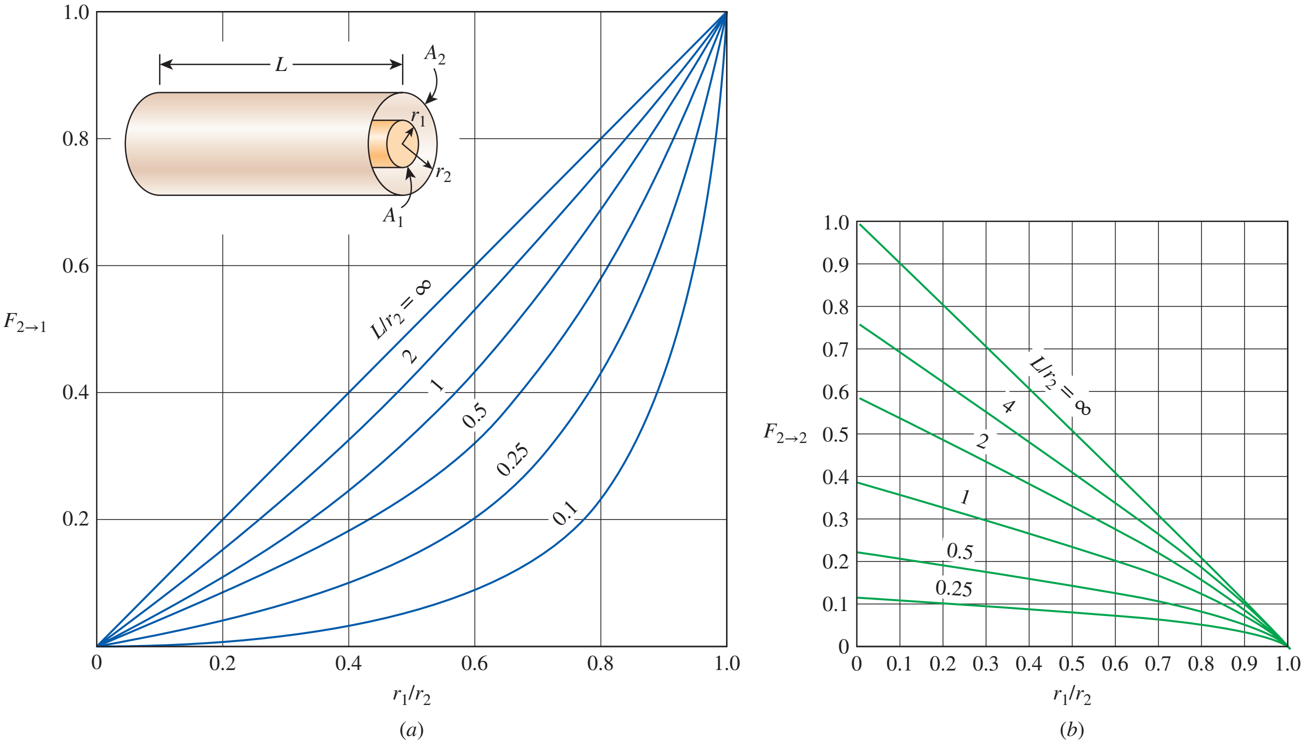


FIGURE 21–35

View factor between two coaxial parallel disks.



(*a*) (*b*)

FIGURE 21–36

View factors for two concentric cylinders of finite length: (*a*) outer cylinder to inner cylinder; (*b*) outer cylinder to itself.

We have shown earlier that the pair of view factors $F_{i\to j}$ and $F_{j\to i}$ are related to each other by

$$A_iF_{i\to j} = A_jF_{j\to i} \tag{21–26}$$

This relation is referred to as the **reciprocity relation** or the **reciprocity rule**, and it enables us to determine the counterpart of a view factor from a knowledge of the view factor itself and the areas of the two surfaces. When determining the pair of view factors $F_{i \to j}$ and $F_{j \to i}$, it makes sense to evaluate first the easier one directly and then the more difficult one by applying the reciprocity relation.

2 The Summation Rule

The radiation analysis of a surface normally requires the consideration of the radiation coming in or going out in all directions. Therefore, most radiation problems encountered in practice involve enclosed spaces. When formulating a radiation problem, we usually form an *enclosure* consisting of the surfaces interacting radiatively. Even openings are treated as imaginary surfaces with radiation properties equivalent to those of the opening.

The conservation of energy principle requires that all radiation leaving any surface i of an enclosure be intercepted by the surfaces of the enclosure. Therefore, *the sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself, must equal unity.* This is known as the **summation rule** for an enclosure and is expressed as (Fig. 21–37)

$$\sum_{j=1}^{N} F_{i \to j} = 1 \tag{21–27}$$

where N is the number of surfaces of the enclosure. For example, applying the summation rule to surface 1 of a three-surface enclosure yields

$$\sum_{j=1}^{3} F_{1 \to j} = F_{1 \to 1} + F_{1 \to 2} + F_{1 \to 3} = 1$$

The summation rule can be applied to each surface of an enclosure by varying i from 1 to N. Therefore, the summation rule applied to each of the N surfaces of an enclosure gives N relations for the determination of the view factors. Also, the reciprocity rule gives $\frac{1}{2}N(N-1)$ additional relations. Then the total number of view factors that need to be evaluated directly for an N-surface enclosure becomes

$$N^2 - [N + \tfrac{1}{2}N(N-1)] = \tfrac{1}{2}N(N-1)$$

For example, for a six-surface enclosure, we need to determine only $\frac{1}{2} \times 6\ (6-1) = 15$ of the $6^2 = 36$ view factors directly. The remaining 21 view factors can be determined from the 21 equations that are obtained by applying the reciprocity and the summation rules.

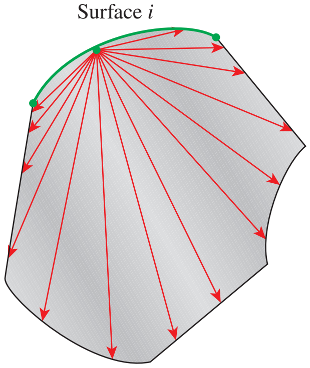


FIGURE 21–37
Radiation leaving any surface i of an enclosure must be intercepted completely by the surfaces of the enclosure. Therefore, the sum of the view factors from surface i to each one of the surfaces of the enclosure must be unity.

EXAMPLE 21–4 View Factors Associated with Two Concentric Spheres

Determine the view factors associated with an enclosure formed by two concentric spheres, shown in Fig. 21–38.

SOLUTION The view factors associated with two concentric spheres are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis The outer surface of the smaller sphere (surface 1) and the inner surface of the larger sphere (surface 2) form a two-surface enclosure. Therefore, $N = 2$, and this enclosure involves $N^2 = 2^2 = 4$ view factors, which are F_{11}, F_{12}, F_{21}, and F_{22}. In this two-surface enclosure, we need to determine only

$$\tfrac{1}{2}N(N-1) = \tfrac{1}{2} \times 2(2-1) = 1$$

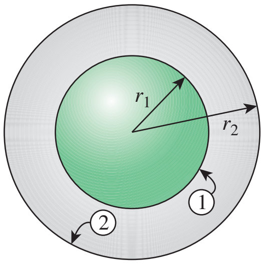


FIGURE 21–38
The geometry considered in Example 21–4.

view factor directly. The remaining three view factors can be determined by the application of the summation and reciprocity rules. But it turns out that we can determine not only one but *two* view factors directly in this case by a simple *inspection:*

$F_{11} = \mathbf{0}$ since no radiation leaving surface 1 strikes itself

$F_{12} = \mathbf{1}$ since all radiation leaving surface 1 strikes surface 2

Actually it would be sufficient to determine only one of these view factors by inspection, since we could always determine the other one from the summation rule applied to surface 1 as $F_{11} + F_{12} = 1$.

The view factor F_{21} is determined by applying the reciprocity relation to surfaces 1 and 2:

$$A_1 F_{12} = A_2 F_{21}$$

which yields

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{4\pi r_1^2}{4\pi r_2^2} \times 1 = \left(\frac{r_1}{r_2}\right)^2$$

Finally, the view factor F_{22} is determined by applying the summation rule to surface 2:

$$F_{21} + F_{22} = 1$$

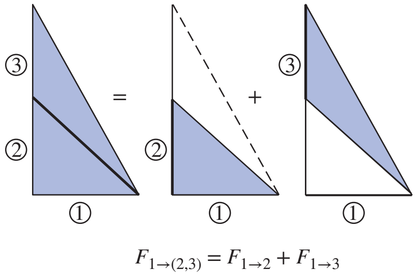
and thus

$$F_{22} = 1 - F_{21} = 1 - \left(\frac{r_1}{r_2}\right)^2$$

Discussion Note that when the outer sphere is much larger than the inner sphere ($r_2 \gg r_1$), F_{22} approaches 1. This is expected, since the fraction of radiation leaving the outer sphere that is intercepted by the inner sphere will be negligible in that case. Also note that the two spheres considered above do not need to be concentric. However, the radiation analysis will be most accurate for the case of concentric spheres, since the radiation is most likely to be uniform on the surfaces in that case.

3 The Superposition Rule

Sometimes the view factor associated with a given geometry is not available in standard tables and charts. In such cases, it is desirable to express the given geometry as the sum or difference of some geometries with known view factors, and then to apply the **superposition rule**, which can be expressed as *the view factor from a surface i to a surface j is equal to the sum of the view factors from surface i to the parts of surface j.* Note that the reverse of this is not true. That is, the view factor from a surface *j* to a surface *i* is *not* equal to the sum of the view factors from the parts of surface *j* to surface *i.*



$F_{1\to(2,3)} = F_{1\to 2} + F_{1\to 3}$

FIGURE 21–39
The view factor from a surface to a composite surface is equal to the sum of the view factors from the surface to the parts of the composite surface.

Consider the geometry in Fig. 21–39, which is infinitely long in the direction perpendicular to the plane of the paper. The radiation that leaves surface 1 and strikes the combined surfaces 2 and 3 is equal to the sum of the radiation that strikes surfaces 2 and 3. Therefore, the view factor from surface 1 to the combined surfaces of 2 and 3 is

$$F_{1\to(2,3)} = F_{1\to 2} + F_{1\to 3} \tag{21–28}$$

Suppose we need to find the view factor $F_{1\to 3}$. A quick check of the view factor expressions and charts in this section reveals that such a view factor cannot be evaluated directly. However, the view factor $F_{1\to 3}$ can be determined from Eq. 21–28 after determining both $F_{1\to 2}$ and $F_{1\to(2,3)}$ from the chart in Table 21–4. Therefore, it may be possible to determine some difficult view factors with relative ease by expressing one or both of the areas as the sum or differences of areas and then applying the superposition rule.

To obtain a relation for the view factor $F_{(2,\,3) \to 1}$, we multiply Eq. 21–28 by A_1,

$$A_1 F_{1 \to (2,\,3)} = A_1 F_{1 \to 2} + A_1 F_{1 \to 3}$$

and apply the reciprocity relation to each term to get

$$(A_2 + A_3) F_{(2,\,3) \to 1} = A_2 F_{2 \to 1} + A_3 F_{3 \to 1}$$

or

$$F_{(2,\,3) \to 1} = \frac{A_2 F_{2 \to 1} + A_3 F_{3 \to 1}}{A_2 + A_3} \tag{21–29}$$

Areas that are expressed as the sum of more than two parts can be handled in a similar manner.

EXAMPLE 21–5 Fraction of Radiation Leaving Through an Opening

Determine the fraction of the radiation leaving the base of the cylindrical enclosure shown in Fig. 21–40 that escapes through a coaxial ring opening at its top surface. The radius and the length of the enclosure are $r_1 = 10$ cm and $L = 10$ cm, while the inner and outer radii of the ring are $r_2 = 5$ cm and $r_3 = 8$ cm, respectively.

SOLUTION The fraction of radiation leaving the base of a cylindrical enclosure through a coaxial ring opening at its top surface is to be determined.

Assumptions The base surface is a diffuse emitter and reflector.

Analysis We are asked to determine the fraction of the radiation leaving the base of the enclosure that escapes through an opening at the top surface. Actually, what we are asked to determine is simply the view factor $F_{1 \to \text{ring}}$ from the base of the enclosure to the ring-shaped surface at the top.

We do not have an analytical expression or chart for view factors between a circular area and a coaxial ring, and so we cannot determine $F_{1 \to \text{ring}}$ directly. However, we do have a chart for view factors between two coaxial parallel disks, and we can always express a ring in terms of disks.

Let the base surface of radius $r_1 = 10$ cm be surface 1, the circular area of $r_2 = 5$ cm at the top be surface 2, and the circular area of $r_3 = 8$ cm be surface 3. Using the superposition rule, the view factor from surface 1 to surface 3 can be expressed as

$$F_{1 \to 3} = F_{1 \to 2} + F_{1 \to \text{ring}}$$

since surface 3 is the sum of surface 2 and the ring area. The view factors $F_{1 \to 2}$ and $F_{1 \to 3}$ are determined from the chart in Fig. 21–35.

$$\frac{L}{r_1} = \frac{10\text{ cm}}{10\text{ cm}} = 1 \quad \text{and} \quad \frac{r_2}{L} = \frac{5\text{ cm}}{10\text{ cm}} = 0.5 \xrightarrow{\text{(Fig. 21–35)}} = 0.11$$

$$\frac{L}{r_1} = \frac{10\text{ cm}}{10\text{ cm}} = 1 \quad \text{and} \quad \frac{r_3}{L} = \frac{8\text{ cm}}{10\text{ cm}} = 0.8 \xrightarrow{\text{(Fig. 21–35)}} = 0.28$$

Therefore,

$$F_{1 \to \text{ring}} = F_{1 \to 3} - F_{1 \to 2} = 0.28 - 0.11 = \mathbf{0.17}$$

which is the desired result. Note that $F_{1 \to 2}$ and $F_{1 \to 3}$ represent the fractions of radiation leaving the base that strike the circular surfaces 2 and 3, respectively, and their difference gives the fraction that strikes the ring area.

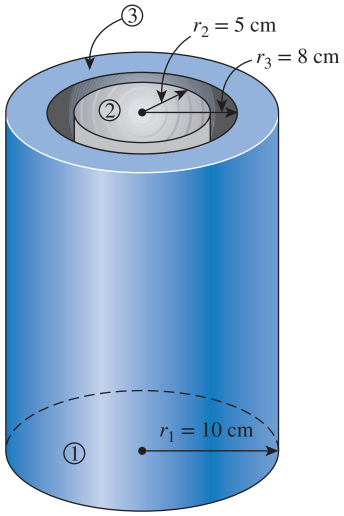
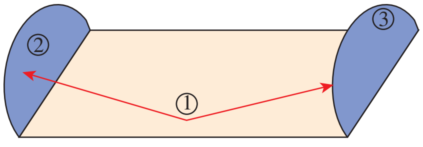


FIGURE 21–40
The cylindrical enclosure considered in Example 21–5.

4 The Symmetry Rule

The determination of the view factors in a problem can be simplified further if the geometry involved possesses some sort of symmetry. Therefore, it is good practice to check for the presence of any *symmetry* in a problem before attempting to determine the view factors directly. The presence of symmetry can be determined *by inspection,* keeping the definition of the view factor in mind. Identical surfaces that are oriented in an identical manner with respect to another surface will intercept identical amounts of radiation leaving that surface. Therefore, the **symmetry rule** can be expressed as *two (or more) surfaces that possess symmetry about a third surface will have identical view factors from that surface* (Fig. 21–41).

The symmetry rule can also be expressed as *if the surfaces j and k are symmetric about the surface i then* $F_{i\to j} = F_{i\to k}$. Using the reciprocity rule, we can show that the relation $F_{j\to i} = F_{k\to i}$ is also true in this case.



$F_{1\to 2} = F_{1\to 3}$
(Also, $F_{2\to 1} = F_{3\to 1}$)

FIGURE 21–41
Two surfaces that are symmetric about a third surface will have the same view factor from the third surface.

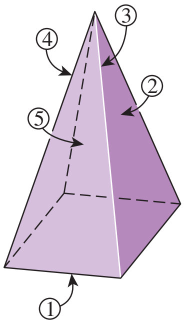


FIGURE 21–42
The pyramid considered in Example 21–6.

EXAMPLE 21–6 View Factors Associated with a Tetragon

Determine the view factors from the base of the pyramid shown in Fig. 21–42 to each of its four side surfaces. The base of the pyramid is a square, and its side surfaces are isosceles triangles.

SOLUTION The view factors from the base of a pyramid to each of its four side surfaces for the case of a square base are to be determined.
Assumptions The surfaces are diffuse emitters and reflectors.
Analysis The base of the pyramid (surface 1) and its four side surfaces (surfaces 2, 3, 4, and 5) form a five-surface enclosure. The first thing we notice about this enclosure is its symmetry. The four side surfaces are symmetric about the base surface. Then, from the *symmetry rule,* we have

$$F_{12} = F_{13} = F_{14} = F_{15}$$

Also, the *summation rule* applied to surface 1 yields

$$\sum_{j=1}^{5} F_{1j} = F_{11} + F_{12} + F_{13} + F_{14} + F_{15} = 1$$

However, $F_{11} = 0$, since the base is a *flat* surface. Then the two relations above yield

$$F_{12} = F_{13} = F_{14} = F_{15} = \mathbf{0.25}$$

Discussion Note that each of the four side surfaces of the pyramid receive one-fourth of all radiation leaving the base surface, as expected. Also note that the presence of symmetry greatly simplified the determination of the view factors.

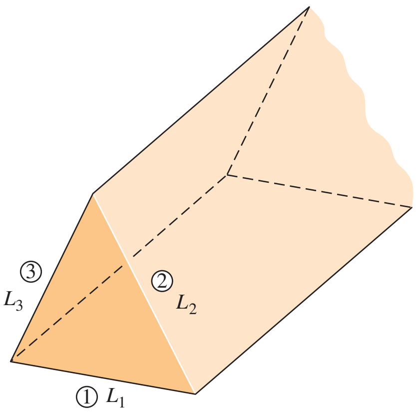


FIGURE 21–43
The infinitely long triangular duct considered in Example 21–7.

EXAMPLE 21–7 View Factors Associated with a Triangular Duct

Determine the view factor from any one side to any other side of the infinitely long triangular duct whose cross section is given in Fig. 21–43.

SOLUTION The view factors associated with an infinitely long triangular duct are to be determined.
Assumptions The surfaces are diffuse emitters and reflectors.
Analysis The widths of the sides of the triangular cross section of the duct are L_1, L_2, and L_3, and the surface areas corresponding to them are A_1, A_2, and A_3, respectively. Since the duct is infinitely long, the fraction of radiation leaving any surface that escapes through the

ends of the duct is negligible. Therefore, the infinitely long duct can be considered to be a three-surface enclosure, $N = 3$.

This enclosure involves $N^2 = 3^2 = 9$ view factors, and we need to determine

$$\tfrac{1}{2}N(N-1) = \tfrac{1}{2} \times 3(3-1) = 3$$

of these view factors directly. Fortunately, we can determine all three of them by inspection to be

$$F_{11} = F_{22} = F_{33} = 0$$

since all three surfaces are flat. The remaining six view factors can be determined by the application of the summation and reciprocity rules.

Applying the summation rule to each of the three surfaces gives

$$\begin{aligned} F_{11} + F_{12} + F_{13} &= 1 \\ F_{21} + F_{22} + F_{23} &= 1 \\ F_{31} + F_{32} + F_{33} &= 1 \end{aligned}$$

Noting that $F_{11} = F_{22} = F_{33} = 0$ and multiplying the first equation by A_1, the second by A_2, and the third by A_3 gives

$$\begin{aligned} A_1F_{12} + A_1F_{13} &= A_1 \\ A_2F_{21} + A_2F_{23} &= A_2 \\ A_3F_{31} + A_3F_{32} &= A_3 \end{aligned}$$

Finally, applying the three reciprocity relations $A_1F_{12} = A_2F_{21}$, $A_1F_{13} = A_3F_{31}$, and $A_2F_{23} = A_3F_{32}$ gives

$$\begin{aligned} A_1F_{12} + A_1F_{13} &= A_1 \\ A_1F_{12} + A_2F_{23} &= A_2 \\ A_1F_{13} + A_2F_{23} &= A_3 \end{aligned}$$

This is a set of three algebraic equations with three unknowns, which can be solved to obtain

$$\begin{aligned} F_{12} &= \frac{A_1 + A_2 - A_3}{2A_1} = \frac{L_1 + L_2 - L_3}{2L_1} \\ F_{13} &= \frac{A_1 + A_3 - A_2}{2A_1} = \frac{L_1 + L_3 - L_2}{2L_1} \qquad \textbf{(21–30)} \\ F_{23} &= \frac{A_2 + A_3 - A_1}{2A_2} = \frac{L_2 + L_3 - L_1}{2L_2} \end{aligned}$$

Discussion Note that we have replaced the areas of the side surfaces with their corresponding widths for simplicity, since $A = Ls$ and the length s can be factored out and canceled. We can generalize this result as *the view factor from a surface of a very long triangular duct to another surface is equal to the sum of the widths of these two surfaces minus the width of the third surface, divided by twice the width of the first surface.*

View Factors Between Infinitely Long Surfaces: The Crossed-Strings Method

Many problems encountered in practice involve geometries of constant cross section such as channels and ducts that are *very long* in one direction relative to the other directions. Such geometries can conveniently be considered to be *two-dimensional,* since any radiation interaction through their end surfaces is negligible. These geometries can subsequently be modeled as being *infinitely long,* and the view factor between their surfaces can be determined by the amazingly simple *crossed-strings method* developed by H. C. Hottel in the 1950s. The surfaces of the geometry do not need to be flat; they can be convex, concave, or any irregular shape.

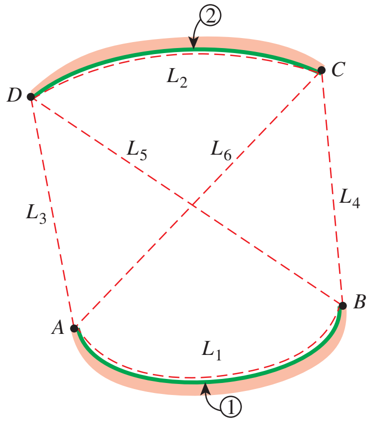


FIGURE 21–44
Determination of the view factor $F_{1\rightarrow 2}$ by the application of the crossed-strings method.

To demonstrate this method, consider the geometry shown in Fig. 21–44, and let us try to find the view factor $F_{1\to 2}$ between surfaces 1 and 2. The first thing we do is identify the end points of the surfaces (the points A, B, C, and D) and connect them to each other with tightly stretched strings, which are indicated by dashed lines. Hottel has shown that the view factor $F_{1\to 2}$ can be expressed in terms of the lengths of these stretched strings, which are straight lines, as

$$F_{1\to 2} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1} \tag{21–31}$$

Note that $L_5 + L_6$ is the sum of the lengths of the *crossed strings,* and $L_3 + L_4$ is the sum of the lengths of the *uncrossed strings* attached to the end points. Therefore, Hottel's crossed-strings method can be expressed verbally as

$$F_{i\to j} = \frac{\sum(\text{Crossed strings}) - \sum(\text{Uncrossed strings})}{2 \times (\text{Length of surface } i)} \tag{21–32}$$

The crossed-strings method is applicable even when the two surfaces considered share a common edge, as in a triangle. In such cases, the common edge can be treated as an imaginary string of zero length. The method can also be applied to surfaces that are partially blocked by other surfaces by allowing the strings to bend around the blocking surfaces.

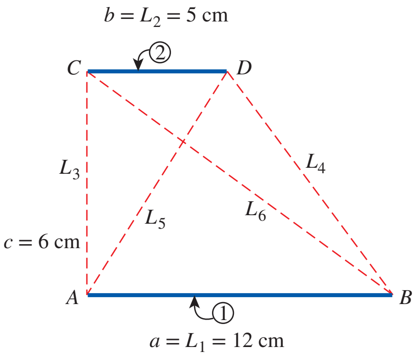


FIGURE 21–45
The two infinitely long parallel plates considered in Example 21–8.

EXAMPLE 21–8 The Crossed-Strings Method for View Factors

Two infinitely long parallel plates of widths $a = 12$ cm and $b = 5$ cm are located a distance $c = 6$ cm apart, as shown in Fig. 21–45. (*a*) Determine the view factor $F_{1\to 2}$ from surface 1 to surface 2 by using the crossed-strings method. (*b*) Derive the crossed-strings formula by forming triangles on the given geometry and using Eq. 21–30 for view factors between the sides of triangles.

SOLUTION The view factors between two infinitely long parallel plates are to be determined using the crossed-strings method, and the formula for the view factor is to be derived.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis (*a*) First we label the end points of both surfaces and draw straight dashed lines between the end points, as shown in Fig. 21–45. Then we identify the crossed and uncrossed strings and apply the crossed-strings method (Eq. 21–32) to determine the view factor $F_{1\to 2}$:

$$F_{1\to 2} = \frac{\sum(\text{Crossed strings}) - \sum(\text{Uncrossed strings})}{2 \times (\text{Length of surface 1})} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$

where

$$L_1 = a = 12 \text{ cm} \qquad L_4 = \sqrt{7^2 + 6^2} = 9.22 \text{ cm}$$
$$L_2 = b = 5 \text{ cm} \qquad L_5 = \sqrt{5^2 + 6^2} = 7.81 \text{ cm}$$
$$L_3 = c = 6 \text{ cm} \qquad L_6 = \sqrt{12^2 + 6^2} = 13.42 \text{ cm}$$

Substituting,

$$F_{1\to 2} = \frac{[(7.81 + 13.42) - (6 + 9.22)] \text{ cm}}{2 \times 12 \text{ cm}} = \mathbf{0.250}$$

(*b*) The geometry is infinitely long in the direction perpendicular to the plane of the paper, and thus the two plates (surfaces 1 and 2) and the two openings (imaginary surfaces 3 and 4) form a four-surface enclosure. Then applying the summation rule to surface 1 yields

$$F_{11} + F_{12} + F_{13} + F_{14} = 1$$

But $F_{11} = 0$ since it is a flat surface. Therefore,

$$F_{12} = 1 - F_{13} - F_{14}$$

where the view factors F_{13} and F_{14} can be determined by considering the triangles *ABC* and *ABD,* respectively, and applying Eq. 21–30 for view factors between the sides of triangles. We obtain

$$F_{13} = \frac{L_1 + L_3 - L_6}{2L_1}, \quad F_{14} = \frac{L_1 + L_4 - L_5}{2L_1}$$

Substituting,

$$\begin{aligned} F_{12} &= 1 - \frac{L_1 + L_3 - L_6}{2L_1} - \frac{L_1 + L_4 - L_5}{2L_1} \\ &= \boldsymbol{\frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}} \end{aligned}$$

which is the desired result. This is also a miniproof of the crossed-strings method for the case of two infinitely long plain parallel surfaces.

21–7 ▪ RADIATION HEAT TRANSFER: BLACK SURFACES

So far, we have considered the nature of radiation, the radiation properties of materials, and the view factors, and we are now in a position to consider the rate of heat transfer between surfaces by radiation. The analysis of radiation exchange between surfaces, in general, is complicated because of reflection: a radiation beam leaving a surface may be reflected several times, with partial reflection occurring at each surface, before it is completely absorbed. The analysis is simplified greatly when the surfaces involved can be approximated as blackbodies because of the absence of reflection. In this section, we consider radiation exchange between *black surfaces* only; we extend the analysis to reflecting surfaces in the next section.

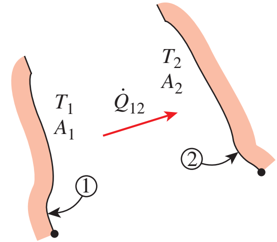


FIGURE 21–46
Two general black surfaces maintained at uniform temperatures T_1 and T_2.

Consider two black surfaces of arbitrary shape maintained at uniform temperatures T_1 and T_2, as shown in Fig. 21–46. Radiation leaves a black surface at a rate of $E_b = \sigma T^4$ per unit surface area, the view factor $F_{1\to 2}$ represents the fraction of radiation leaving surface 1 that strikes surface 2, and $F_{2\to 1}$ represents the fraction of radiation leaving surface 2 that strikes surface 1. The *net* rate of radiation heat transfer from surface 1 to surface 2 can be expressed as

$$\begin{aligned} \dot{Q}_{1\to 2} &= \begin{pmatrix} \text{Radiation leaving} \\ \text{the entire surface 1} \\ \text{that strikes surface 2} \end{pmatrix} - \begin{pmatrix} \text{Radiation leaving} \\ \text{the entire surface 2} \\ \text{that strikes surface 1} \end{pmatrix} \\ &= A_1 E_{b1} F_{1\to 2} - A_2 E_{b2} F_{2\to 1} \qquad \text{(W)} \end{aligned} \tag{21–33}$$

Applying the reciprocity relation $A_1 F_{1\to 2} = A_2 F_{2\to 1}$ yields

$$\dot{Q}_{1\to 2} = A_1 F_{1\to 2}\,\sigma(T_1^4 - T_2^4) \qquad \text{(W)} \tag{21–34}$$

which is the desired relation. Equation 21–34 provides the *net* rate at which radiation *leaves* surface 1 as a result of its interaction with surface 2, which is equal to the *net* rate at which surface 2 *gains* radiation due to its interaction with surface 1. A negative value for $\dot{Q}_{1\to 2}$ indicates that *net* radiation heat transfer is from surface 2 to surface 1.

Now consider an *enclosure* consisting of *N black* surfaces maintained at specified temperatures. The *net* radiation heat transfer *from* any surface *i* of this enclosure is determined by adding up the net radiation heat transfers from surface i to each of the surfaces of the enclosure:

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i \to j} = \sum_{j=1}^{N} A_i F_{i \to j} \sigma (T_i^4 - T_j^4) \qquad \text{(W)} \tag{21–35}$$

Again a negative value for $\dot{Q}$ indicates that *net* radiation heat transfer is *to* surface i (i.e., surface i *gains* radiation energy instead of losing). Also, the *net* heat transfer from a surface to itself is zero, regardless of the shape of the surface.

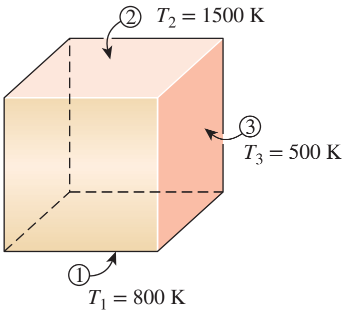


FIGURE 21–47
The cubical furnace of black surfaces considered in Example 21–9.

EXAMPLE 21–9 Radiation Heat Transfer in a Black Furnace

Consider the 5-m × 5-m × 5-m cubical furnace shown in Fig. 21–47, whose surfaces closely approximate black surfaces. The base, top, and side surfaces of the furnace are maintained at uniform temperatures of 800 K, 1500 K, and 500 K, respectively. Determine (*a*) the net rate of radiation heat transfer between the base and the side surfaces, (*b*) the net rate of radiation heat transfer between the base and the top surface, and (*c*) the net radiation heat transfer from the base surface.

SOLUTION The surfaces of a cubical furnace are black and are maintained at uniform temperatures. The *net* rate of radiation heat transfer between the base and side surfaces, between the base and the top surface, and from the base surface are to be determined.

Assumptions The surfaces are black and isothermal.

Analysis (*a*) The geometry involves six surfaces, and thus we may be tempted at first to treat the furnace as a six-surface enclosure. However, the four side surfaces possess the same properties, and thus we can treat them as a single side surface in radiation analysis. We consider the base surface to be surface 1, the top surface to be surface 2, and the side surfaces to be surface 3. Then the problem reduces to determining $\dot{Q}_{1 \to 3}$, $\dot{Q}_{1 \to 2}$, and $\dot{Q}_1$.

The *net* rate of radiation heat transfer $\dot{Q}_{1 \to 3}$ from surface 1 to surface 3 can be determined from Eq. 21–34, since both surfaces involved are black, by replacing the subscript 2 with 3:

$$\dot{Q}_{1 \to 3} = A_1 F_{1 \to 3} \sigma (T_1^4 - T_3^4)$$

But first we need to evaluate the view factor $F_{1 \to 3}$. After checking the view factor charts and tables, we realize that we cannot determine this view factor directly. However, we can determine the view factor $F_{1 \to 2}$ from Fig. 21–33 to be $F_{1 \to 2} = 0.2$, and we know that $F_{1 \to 1} = 0$ since surface 1 is a plane. Then applying the summation rule to surface 1 yields

$$F_{1 \to 1} + F_{1 \to 2} + F_{1 \to 3} = 1$$

or

$$F_{1 \to 3} = 1 - F_{1 \to 1} - F_{1 \to 2} = 1 - 0 - 0.2 = 0.8$$

Substituting,

$$\begin{aligned} \dot{Q}_{1 \to 3} &= (25 \text{ m}^2)(0.8)(5.67 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(800 \text{ K})^4 - (500 \text{ K})^4] \\ &= \mathbf{394 \text{ kW}} \end{aligned}$$

(*b*) The net rate of radiation heat transfer $\dot{Q}_{1 \to 2}$ from surface 1 to surface 2 is determined in a similar manner from Eq. 21–34 to be

$$\begin{aligned} \dot{Q}_{1 \to 2} &= A_1 F_{1 \to 2} \sigma (T_1^4 - T_2^4) \\ &= (25 \text{ m}^2)(0.2)(5.67 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(800 \text{ K})^4 - (1500 \text{ K})^4] \\ &= \mathbf{-1319 \text{ kW}} \end{aligned}$$

The negative sign indicates that *net* radiation heat transfer is from surface 2 to surface 1.

(*c*) The *net* radiation heat transfer from the base surface $\dot{Q}_1$ is determined from Eq. 21–35 by replacing the subscript *i* with 1 and taking $N = 3$:

$$\begin{aligned}\dot{Q}_1 &= \sum_{j=1}^{3} \dot{Q}_{1\to j} = \dot{Q}_{1\to 1} + \dot{Q}_{1\to 2} + \dot{Q}_{1\to 3}\\ &= 0 + (-1319\ \text{kW}) + (394\ \text{kW})\\ &= \mathbf{-925\ kW}\end{aligned}$$

Again the negative sign indicates that *net* radiation heat transfer is *to* surface 1. That is, the base of the furnace is gaining *net* radiation at a rate of 925 kW.

21–8 ▪ RADIATION HEAT TRANSFER: DIFFUSE, GRAY SURFACES

The analysis of radiation transfer in enclosures consisting of black surfaces is relatively easy, as we have seen, but most enclosures encountered in practice involve nonblack surfaces, which allow multiple reflections to occur. Radiation analysis of such enclosures becomes very complicated unless some simplifying assumptions are made.

To make a simple radiation analysis possible, it is common to assume the surfaces of an enclosure to be *opaque, diffuse,* and *gray*. That is, the surfaces are nontransparent, they are diffuse emitters and diffuse reflectors, and their radiation properties are independent of wavelength. Also, each surface of the enclosure is *isothermal,* and both the incoming and outgoing radiation are *uniform* over each surface. But first we review the concept of radiosity.

Radiosity

Surfaces emit radiation as well as reflect it, and thus the radiation leaving a surface consists of emitted and reflected parts. The calculation of radiation heat transfer between surfaces involves the *total* radiation energy streaming away from a surface, with no regard for its origin. The *total radiation energy leaving a surface per unit time and per unit area* is the **radiosity** and is denoted by *J* (Fig. 21–48).

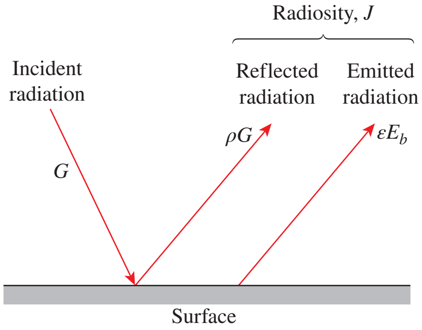


FIGURE 21–48
Radiosity represents the sum of the radiation energy emitted and reflected by a surface.

For a surface *i* that is *gray* and *opaque* ($\varepsilon_i = \alpha_i$ and $\alpha_i + \rho_i = 1$), the radiosity can be expressed as

$$\begin{aligned}J_i &= \begin{pmatrix}\text{Radiation emitted}\\ \text{by surface } i\end{pmatrix} + \begin{pmatrix}\text{Radiation reflected}\\ \text{by surface } i\end{pmatrix}\\ &= \varepsilon_i E_{bi} + \rho_i G_i\\ &= \varepsilon_i E_{bi} + (1-\varepsilon_i)G_i \qquad (\text{W/m}^2)\end{aligned} \tag{21–36}$$

where $E_{bi} = \sigma T_i^4$ is the blackbody emissive power of surface *i* and G_i is irradiation (i.e., the radiation energy incident on surface *i* per unit time per unit area).

For a surface that can be approximated as a *blackbody* ($\varepsilon_i = 1$), the radiosity relation reduces to

$$J_i = E_{bi} = \sigma T_i^4 \qquad \text{(blackbody)} \tag{21–37}$$

That is, *the radiosity of a blackbody is equal to its emissive power.* This is expected, since a blackbody does not reflect any radiation, and thus radiation coming from a blackbody is due to emission only.

Net Radiation Heat Transfer to or from a Surface

During a radiation interaction, a surface *loses* energy by emitting radiation and *gains* energy by absorbing radiation emitted by other surfaces. A surface experiences a net gain or a net loss of energy, depending on which quantity is larger. The *net* rate of radiation heat transfer from a surface i of surface area A_i is denoted by $\dot{Q}_i$ and is expressed as

$$\dot{Q}_i = \begin{pmatrix}\text{Radiation leaving}\\ \text{entire surface } i\end{pmatrix} - \begin{pmatrix}\text{Radiation incident}\\ \text{on entire surface } i\end{pmatrix}$$

$$= A_i(J_i - G_i) \quad \text{(W)} \tag{21–38}$$

Solving for G_i from Eq. 21–36 and substituting into Eq. 21–38 yields

$$\dot{Q}_i = A_i\left(J_i - \frac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i}\right) = \frac{A_i \varepsilon_i}{1 - \varepsilon_i}(E_{bi} - J_i) \qquad \text{(W)} \tag{21–39}$$

In an electrical analogy to Ohm's law, this equation can be rearranged as

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i} \qquad \text{(W)} \tag{21–40}$$

where

$$R_i = \frac{1 - \varepsilon_i}{A_i \varepsilon_i} \tag{21–41}$$

is the **surface resistance** to radiation. The quantity $E_{bi} - J_i$ corresponds to a *potential difference* and the net rate of radiation heat transfer corresponds to *current* in the electrical analogy, as illustrated in Fig. 21–49.

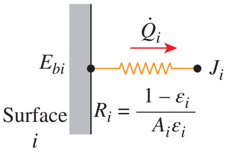


FIGURE 21–49
Electrical analogy of surface resistance to radiation.

The direction of the net radiation heat transfer depends on the relative magnitudes of J_i (the radiosity) and E_{bi} (the emissive power of a blackbody at the temperature of the surface). It is *from* the surface if $E_{bi} > J_i$ and *to* the surface if $J_i > E_{bi}$. A negative value for $\dot{Q}_i$ indicates that heat transfer is *to* the surface. All of this radiation energy gained must be removed from the other side of the surface through some mechanism if the surface temperature is to remain constant.

The surface resistance to radiation for a *blackbody* is *zero* since $\varepsilon_i = 1$ and $J_i = E_{bi}$. The net rate of radiation heat transfer in this case is determined directly from Eq. 21–38.

Some surfaces encountered in numerous practical heat transfer applications are modeled as being *adiabatic* since their back sides are well insulated and the net heat transfer through them is zero. When the convection effects on the front (heat transfer) side of such a surface is negligible and steady-state conditions are reached, the surface must lose as much radiation energy as it gains, and thus $\dot{Q}_i = 0$. In such cases, the surface is said to *reradiate* all the radiation energy it receives, and such a surface is called a **reradiating surface**. Setting $\dot{Q}_i = 0$ in Eq. 21–40 yields

$$J_i = E_{bi} = \sigma T_i^4 \qquad \text{(W/m}^2\text{)} \tag{21–42}$$

Therefore, the *temperature* of a reradiating surface under steady conditions can easily be determined from Eq. 21–42 once its radiosity is known. Note that the temperature of a reradiating surface is *independent of its emissivity*. In radiation analysis, the surface resistance of a reradiating surface is disregarded since there is no net heat transfer through it. (This is like the fact that there is no need to consider a resistance in an electrical network if no current is flowing through it.)

Net Radiation Heat Transfer Between Any Two Surfaces

Consider two diffuse, gray, and opaque surfaces of arbitrary shape maintained at uniform temperatures, as shown in Fig. 21–50. Recognizing that the radiosity J represents the rate of radiation leaving a surface per unit surface area and that the view factor $F_{i \to j}$ represents the fraction of radiation leaving surface i that strikes surface *j*, the *net* rate of radiation heat transfer from surface *i* to surface *j* can be expressed as

$$\dot{Q}_{i \to j} = \left(\begin{array}{c}\text{Radiation leaving}\\ \text{the entire surface } i\\ \text{that strikes surface } j\end{array}\right) - \left(\begin{array}{c}\text{Radiation leaving}\\ \text{the entire surface } j\\ \text{that strikes surface } i\end{array}\right)$$

$$= A_i J_i F_{i \to j} - A_j J_j F_{j \to i} \qquad \text{(W)} \qquad \textbf{(21–43)}$$

Applying the reciprocity relation $A_i F_{i \to j} = A_j F_{j \to i}$ yields

$$\dot{Q}_{i \to j} = A_i F_{i \to j}(J_i - J_j) \qquad \text{(W)} \qquad \textbf{(21–44)}$$

Again in analogy to Ohm's law, this equation can be rearranged as

$$\dot{Q}_{i \to j} = \frac{J_i - J_j}{R_{i \to j}} \qquad \text{(W)} \qquad \textbf{(21–45)}$$

where

$$R_{i \to j} = \frac{1}{A_i F_{i \to j}} \qquad \textbf{(21–46)}$$

is the **space resistance** to radiation. Again the quantity $J_i - J_j$ corresponds to a *potential difference,* and the net rate of heat transfer between two surfaces corresponds to *current* in the electrical analogy, as illustrated in Fig. 21–50.

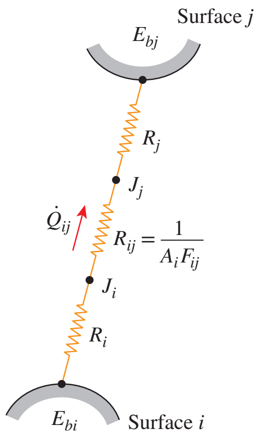


FIGURE 21–50
Electrical analogy of space resistance to radiation.

The direction of the net radiation heat transfer between two surfaces depends on the relative magnitudes of J_i and J_j. A positive value for $\dot{Q}_{i \to j}$ indicates that net heat transfer is *from* surface *i* *to* surface *j*. A negative value indicates the opposite.

In an *N*-surface enclosure, the conservation of energy principle requires that the net heat transfer from surface *i* be equal to the sum of the net heat transfers from surface *i* to each of the *N* surfaces of the enclosure. That is,

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i \to j} = \sum_{j=1}^{N} A_i F_{i \to j}(J_i - J_j) = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{i \to j}} \qquad \text{(W)} \qquad \textbf{(21–47)}$$

The network representation of net radiation heat transfer from surface *i* to the remaining surfaces of an *N*-surface enclosure is given in Fig. 21–51. Note that $\dot{Q}_{i \to i}$ (the net rate of heat transfer from a surface to itself) is zero regardless of the shape of the surface. Combining Eqs. 21–40 and 21–47 gives

$$\frac{E_{bi} - J_i}{R_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{i \to j}} \qquad \text{(W)} \qquad \textbf{(21–48)}$$

which has the electrical analogy interpretation that *the net radiation flow from a surface through its surface resistance is equal to the sum of the radiation flows from that surface to all other surfaces through the corresponding space resistances.*

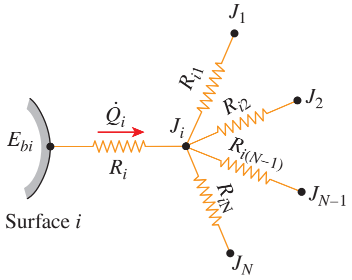


FIGURE 21–51
Network representation of net radiation heat transfer from surface *i* to the remaining surfaces of an *N*-surface enclosure.

Methods of Solving Radiation Problems

In the radiation analysis of an enclosure, either the temperature or the net rate of heat transfer must be given for each of the surfaces to obtain a unique solution for the unknown surface temperatures and heat transfer rates. There are two methods commonly used to solve radiation problems. In the first method, Eqs. 21–47 (for surfaces with specified heat transfer rates) and 21–48 (for surfaces with specified temperatures) are simplified and rearranged as

Surfaces with specified net heat transfer rate $\dot{Q}$

$$\dot{Q}_i = A_i \sum_{j=1}^{N} F_{i \to j}(J_i - J_j) \qquad \textbf{(21–49)}$$

Surfaces with specified temperature T_i

$$\sigma T_i^4 = J_i + \frac{1 - \varepsilon_i}{\varepsilon_i} \sum_{j=1}^{N} F_{i \to j}(J_i - J_j) \qquad \textbf{(21–50)}$$

Note that $\dot{Q}_i = 0$ for insulated (or reradiating) surfaces, and $\sigma T_i^4 = J_i$ for black surfaces since $\varepsilon_i = 1$ in that case. Also, the term corresponding to $j = i$ drops out from either relation since $J_i - J_j = J_i - J_i = 0$ in that case.

The equations above give N linear algebraic equations for the determination of the N unknown radiosities for an N-surface enclosure. Once the radiosities J_1, $J_2, \ldots, J_N$ are available, the unknown heat transfer rates can be determined from Eq. 21–49, while the unknown surface temperatures can be determined from Eq. 21–50. The temperatures of insulated or reradiating surfaces can be determined from $\sigma T_i^4 = J_i$. A positive value for $\dot{Q}_i$ indicates net radiation heat transfer *from* surface i to other surfaces in the enclosure while a negative value indicates net radiation heat transfer *to* the surface.

The systematic approach described above for solving radiation heat transfer problems is very suitable for use with today's popular equation solvers such as EES, Mathcad, and Matlab, especially when there are a large number of surfaces, and it is known as the **direct method** (formerly the *matrix method,* since it resulted in matrices and the solution required a knowledge of linear algebra). The second method, which is described next, is called the **network method**, and it is based on the electrical network analogy.

The network method was first introduced by A. K. Oppenheim in the 1950s and found widespread acceptance because of its simplicity and emphasis on the physics of the problem. The application of the method is straightforward: Draw a surface resistance associated with each surface of an enclosure and connect them with space resistances. Then solve the radiation problem by treating it as an electrical network problem where the radiation heat transfer replaces the current and radiosity replaces the potential.

The network method is not practical for enclosures with more than three or four surfaces, however, because of the increased complexity of the network. Next we apply the method to solve radiation problems in two- and three-surface enclosures.

Radiation Heat Transfer in Two-Surface Enclosures

Consider an enclosure consisting of two opaque surfaces at specified temperatures T_1 and T_2, as shown in Fig. 21–52, and we try to determine the net rate of radiation heat transfer between the two surfaces with the network method. Surfaces 1 and 2 have emissivities ε_1 and ε_2 and surface areas A_1 and A_2 and are maintained at uniform temperatures T_1 and T_2, respectively. There are only two surfaces in the enclosure, and thus we can write

$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$

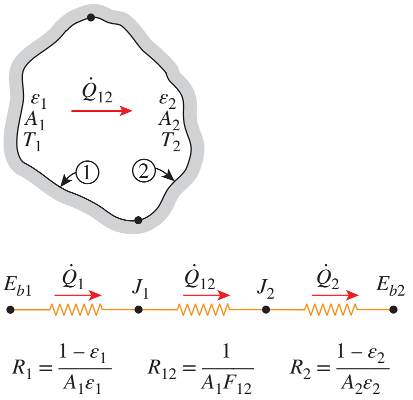


FIGURE 21–52
Schematic of a two-surface enclosure and the radiation network associated with it.

That is, the net rate of radiation heat transfer from surface 1 to surface 2 must equal the net rate of radiation heat transfer *from* surface 1 and the net rate of radiation heat transfer *to* surface 2.

The radiation network of this two-surface enclosure consists of two surface resistances and one space resistance, as shown in Fig. 21–52. In an electrical network, the electric current flowing through these resistances connected in series would be determined by dividing the potential difference between points A and B by the total resistance between the same two points. The net rate of radiation transfer is determined in the same manner and is expressed as

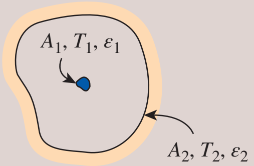
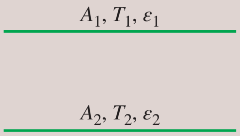
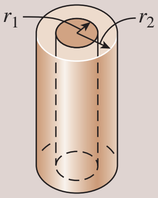
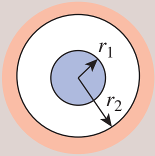
$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2} = \dot{Q}_1 = -\dot{Q}_2$$

or

$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} \qquad \text{(W)} \tag{21–51}$$

TABLE 21–5

Radiation heat transfer relations for some familiar two-surface arrangements

Small object in a large cavity  	$\frac{A_1}{A_2} \approx 0$ $F_{12} = 1$	$\dot{Q}_{12} = A_1\sigma\varepsilon_1(T_1^4 - T_2^4)$	**(21–52)**
Infinitely large parallel plates  	$A_1 = A_2 = A$ $F_{12} = 1$	$\dot{Q}_{12} = \dfrac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$	**(21–53)**
Infinitely long concentric cylinders  	$\frac{A_1}{A_2} = \frac{r_1}{r_2}$ $F_{12} = 1$	$\dot{Q}_{12} = \dfrac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$	**(21–54)**
Concentric spheres  	$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$ $F_{12} = 1$	$\dot{Q}_{12} = \dfrac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2}$	**(21–55)**

This important result is applicable to any two gray, diffuse, and opaque surfaces that form an enclosure. The view factor F_{12} depends on the geometry and must be determined first. Simplified forms of Eq. 21–51 for some familiar arrangements that form a two-surface enclosure are given in Table 21–5. Note that $F_{12} = 1$ for all of these special cases.

EXAMPLE 21–10 Radiation Heat Transfer Between Parallel Plates

Two very large parallel plates are maintained at uniform temperatures $T_1 = 800$ K and $T_2 = 500$ K and have emissivities $\varepsilon_1 = 0.2$ and $\varepsilon_2 = 0.7$, respectively, as shown in Fig. 21–53. Determine the *net* rate of radiation heat transfer between the two surfaces per unit surface area of the plates.

SOLUTION Two large parallel plates are maintained at uniform temperatures. The net rate of radiation heat transfer between the plates is to be determined.

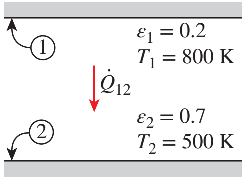


FIGURE 21–53
The two parallel plates considered in Example 21–10.

Assumptions Both surfaces are opaque, diffuse, and gray.

Analysis The net rate of radiation heat transfer between the two plates per unit area is readily determined from Eq. 21–53 to be

$$\dot{q}_{12} = \frac{\dot{Q}_{12}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} = \frac{(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(800\ \text{K})^4 - (500\ \text{K})^4]}{\dfrac{1}{0.2} + \dfrac{1}{0.7} - 1}$$

$$= \mathbf{3625\ W/m^2}$$

Discussion Note that heat at a *net* rate of 3625 W is transferred from plate 1 to plate 2 by radiation per unit surface area of either plate.

Radiation Heat Transfer in Three-Surface Enclosures

We now consider an enclosure consisting of three opaque, diffuse, and gray surfaces, as shown in Fig. 21–54. Surfaces 1, 2, and 3 have surface areas A_1,A_2, and A_3; emissivities ε_1, ε_2, and ε_3; and uniform temperatures T_1, T_2, and T_3, respectively. The radiation network of this geometry is constructed by following the standard procedure: draw a surface resistance associated with each of the three surfaces and connect these surface resistances with space resistances, as shown in the figure. Relations for the surface and space resistances are given by Eqs. 21–41 and 21–46. The three end point potentials E_{b1}, E_{b2}, and E_{b3} are considered known, since the surface temperatures are specified. Then all we need to find are the radiosities J_1, J_2, and J_3. The three equations for the determination of these three unknowns are obtained from the requirement that *the algebraic sum of the currents (net radiation heat transfer) at each node must equal zero.* That is,

$$\begin{aligned}
&\frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0 \\
&\frac{J_1 - J_2}{R_{12}} + \frac{E_{b2} - J_2}{R_2} + \frac{J_3 - J_2}{R_{23}} = 0 \\
&\frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} + \frac{E_{b3} - J_3}{R_3} = 0
\end{aligned} \tag{21–56}$$

Once the radiosities J_1, J_2, and J_3 are available, the net rate of radiation heat transfers at each surface can be determined from Eq. 21–47.

The set of equations above simplifies further if one or more surfaces are "special" in some way. For example, $J_i = E_{bi} = \sigma T_i^4$ for a *black* or *reradiating* surface. Also, $\dot{Q}_i = 0$ for a reradiating surface. Finally, when the *net* rate of radiation heat transfer $\dot{Q}_i$ is specified at surface i instead of the temperature, the term $(E_{bi} - J_i)/R_i$ should be replaced by the specified $\dot{Q}_i$.

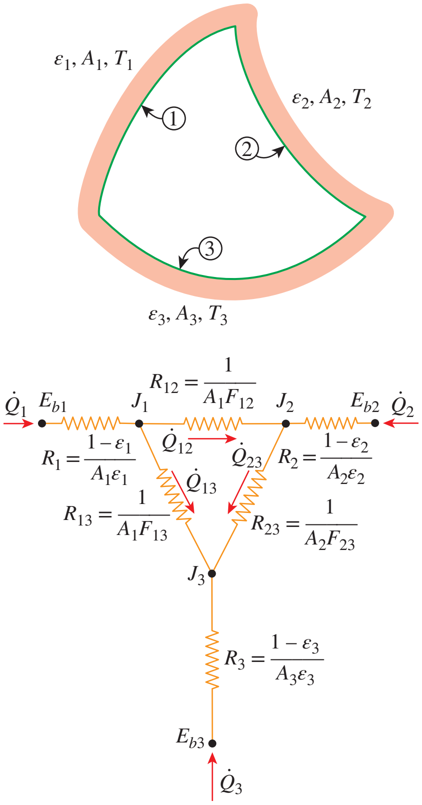


FIGURE 21–54
Schematic of a three-surface enclosure and the radiation network associated with it.

EXAMPLE 21–11 Radiation Heat Transfer in a Cylindrical Furnace

Consider a cylindrical furnace with $r_o = H = 1$ m, as shown in Fig. 21–55. The top (surface 1) and the base (surface 2) of the furnace have emissivities $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.4$, respectively, and are maintained at uniform temperatures $T_1 = 700$ K and $T_2 = 500$ K. The side surface closely approximates a blackbody and is maintained at a temperature of $T_3 = 400$ K. Determine the *net* rate of radiation heat transfer at each surface during steady operation, and explain how these surfaces can be maintained at specified temperatures.

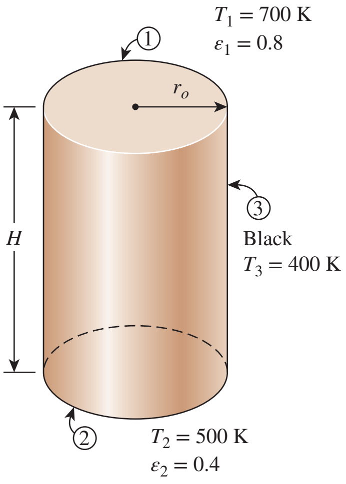


FIGURE 21–55
The cylindrical furnace considered in Example 21–11.

SOLUTION The surfaces of a cylindrical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer at each surface during steady operation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Analysis We will solve this problem systematically using the direct method to demonstrate its use. The cylindrical furnace can be considered to be a three-surface enclosure with surface areas of

$$A_1 = A_2 = \pi r_0^2 = \pi(1\text{ m})^2 = 3.14\text{ m}^2$$

$$A_3 = 2\pi r_o H = 2\pi(1\text{ m})(1\text{ m}) = 6.28\text{ m}^2$$

The view factor from the base to the top surface is, from Fig. 21–35, $F_{12} = 0.38$. Then the view factor from the base to the side surface is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \rightarrow F_{13} = 1 - F_{11} - F_{12} = 1 - 0 - 0.38 = 0.62$$

since the base surface is flat and thus $F_{11} = 0$. Noting that the top and bottom surfaces are symmetric about the side surface, $F_{21} = F_{12} = 0.38$ and $F_{23} = F_{13} = 0.62$. The view factor F_{31} is determined from the reciprocity relation,

$$A_1F_{13} = A_3F_{31} \rightarrow F_{31} = F_{13}(A_1/A_3) = (0.62)(0.314/0.628) = 0.31$$

Also, $F_{32} = F_{31} = 0.31$ because of symmetry. Now that all the view factors are available, we apply Eq. 21–50 to each surface to determine the radiosities:

$$\textit{Top surface } (i = 1): \quad \sigma T_1^4 = J_1 + \frac{1 - \varepsilon_1}{\varepsilon_1}[F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)]$$

$$\textit{Bottom surface } (i = 2): \quad \sigma T_2^4 = J_2 + \frac{1 - \varepsilon_2}{\varepsilon_2}[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)]$$

$$\textit{Side surface } (i = 3): \quad \sigma T_3^4 = J_3 + 0 \text{ (since surface 3 is black and thus } \varepsilon_3 = 1)$$

Substituting the known quantities,

$$(5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)(700\text{ K})^4 = J_1 + \frac{1 - 0.8}{0.8}[0.38(J_1 - J_2) + 0.62(J_1 - J_3)]$$

$$(5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)(500\text{ K})^4 = J_2 + \frac{1 - 0.4}{0.4}[0.38(J_2 - J_1) + 0.62(J_2 - J_3)]$$

$$(5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)(400\text{ K})^4 = J_3$$

Solving these equations for J_1, J_2, and J_3 gives

$$J_1 = 11{,}418\text{ W/m}^2, \quad J_2 = 4562\text{ W/m}^2, \quad \text{and} \quad J_3 = 1452\text{ W/m}^2$$

Then the net rates of radiation heat transfer at the three surfaces are determined from Eq. 21–49 to be

$$\begin{aligned}\dot{Q}_1 &= A_1[F_{1\rightarrow 2}(J_1 - J_2) + F_{1\rightarrow 3}(J_1 - J_3)] \\ &= (3.14\text{ m}^2)[0.38(11{,}418 - 4562) + 0.62(11{,}418 - 1452)]\text{ W/m}^2 \\ &= \mathbf{27.6\ kW}\end{aligned}$$

$$\begin{aligned}\dot{Q}_2 &= A_2[F_{2\rightarrow 1}(J_2 - J_1) + F_{2\rightarrow 3}(J_2 - J_3)] \\ &= (3.14\text{ m}^2)[0.38(4562 - 11{,}418) + 0.62(4562 - 1452)]\text{ W/m}^2 \\ &= \mathbf{-2.13\ kW}\end{aligned}$$

$$\begin{aligned}\dot{Q}_3 &= A_3[F_{3\rightarrow 1}(J_3 - J_1) + F_{3\rightarrow 2}(J_3 - J_2)] \\ &= (6.28\text{ m}^2)[0.31(1452 - 11{,}418) + 0.31(1452 - 4562)]\text{ W/m}^2 \\ &= \mathbf{-25.5\ kW}\end{aligned}$$

Note that the direction of *net* radiation heat transfer is *from* the top surface *to* the base and side surfaces, and the algebraic sum of these three quantities must be equal to zero. That is,

$$\dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 = 27.6 + (-2.13) + (-25.5) \cong 0$$

Discussion To maintain the surfaces at the specified temperatures, we must supply heat to the top surface continuously at a rate of 27.6 kW while removing 2.13 kW from the base and 25.5 kW from the side surfaces.

The direct method presented here is straightforward, and it does not require the evaluation of radiation resistances. Also, it can be applied to enclosures with any number of surfaces in the same manner.

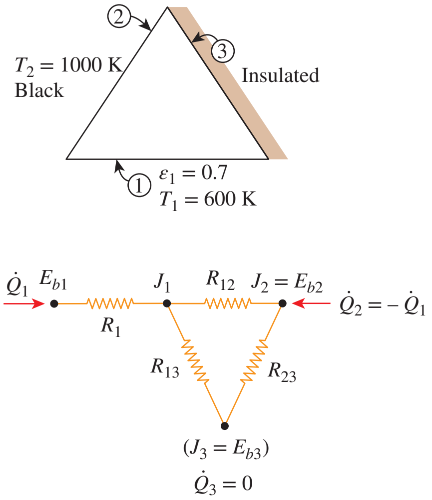


FIGURE 21–56
The triangular furnace considered in Example 21–12.

EXAMPLE 21–12 Radiation Heat Transfer in a Triangular Furnace

A furnace is shaped like a long equilateral triangular duct, as shown in Fig. 21–56. The width of each side is 1 m. The base surface has an emissivity of 0.7 and is maintained at a uniform temperature of 600 K. The heated left-side surface closely approximates a blackbody at 1000 K. The right-side surface is well insulated. Determine the rate at which heat must be supplied to the heated side externally per unit length of the duct in order to maintain these operating conditions.

SOLUTION Two of the surfaces of a long equilateral triangular furnace are maintained at uniform temperatures while the third surface is insulated. The external rate of heat transfer to the heated side per unit length of the duct during steady operation is to be determined.
Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.
Analysis The furnace can be considered to be a three-surface enclosure with a radiation network as shown in the figure, since the duct is very long and thus the end effects are negligible. We observe that the view factor from any surface to any other surface in the enclosure is 0.5 because of symmetry. Surface 3 is a reradiating surface since the net rate of heat transfer at that surface is zero. Then we must have $\dot{Q}_1 = -\dot{Q}_2$, since all heat lost by surface 1 must be gained by surface 2. The radiation network in this case is a simple series–parallel connection, and we can determine $\dot{Q}_1$ directly from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{R_1 + \left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1}} = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \left(A_1F_{12} + \dfrac{1}{1/A_1F_{13} + 1/A_2F_{23}}\right)^{-1}}$$

where

$$A_1 = A_2 = A_3 = wL = 1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2 \quad \text{(per unit length of the duct)}$$

$$F_{12} = F_{13} = F_{23} = 0.5 \qquad \text{(symmetry)}$$

$$E_{b1} = \sigma T_1^4 = (5.67 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4)(600 \text{ K})^4 = 7348 \text{ W/m}^2$$

$$E_{b2} = \sigma T_2^4 = (5.67 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4)(1000 \text{ K})^4 = 56{,}700 \text{ W/m}^2$$

Substituting,

$$\dot{Q}_1 = \frac{(56{,}700 - 7348) \text{ W/m}^2}{\dfrac{1-0.7}{0.7 \times 1 \text{ m}^2} + \left[(0.5 \times 1 \text{ m}^2) + \dfrac{1}{1/(0.5 \times 1 \text{ m}^2) + 1/(0.5 \times 1 \text{ m}^2)}\right]^{-1}}$$

$$= \mathbf{28.0 \text{ kW}}$$

Therefore, heat at a rate of 28 kW must be supplied to the heated surface per unit length of the duct to maintain steady operation in the furnace.

SUMMARY

Radiation propagates in the form of electromagnetic waves. The *frequency* ν and *wavelength* λ of electromagnetic waves in a medium are related by $\lambda = c/\nu$, where c is the speed of propagation in that medium. All matter continuously emits *thermal radiation* as a result of vibrational and rotational motions of molecules, atoms, and electrons of a substance.

A *blackbody* is defined as a perfect emitter and absorber of radiation. At a specified temperature and wavelength, no surface can emit more energy than a blackbody. A blackbody absorbs *all* incident radiation, regardless of wavelength and direction. The radiation energy emitted by a blackbody per unit time and per unit surface area is called the *blackbody emissive power* E_b and is expressed by the *Stefan–Boltzmann law* as

$$E_b(T) = \sigma T^4$$

where $\sigma = 5.670 \times 10^{-8}$ W/m²·K⁴ is the *Stefan–Boltzmann constant* and T is the absolute temperature of the surface in K. At any specified temperature, the spectral blackbody emissive power $E_{b\lambda}$ increases with wavelength, reaches a peak, and then decreases with increasing wavelength. The wavelength at which the peak occurs for a specified temperature is given by *Wien's displacement law* as

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K}$$

The *blackbody radiation function* f_λ represents the fraction of radiation emitted by a blackbody at temperature T in the wavelength band from $\lambda = 0$ to λ. The fraction of radiation energy emitted by a blackbody at temperature T over a finite wavelength band from $\lambda = \lambda_1$ to $\lambda = \lambda_2$ is determined from

$$f_{\lambda_1-\lambda_2}(T) = f_{\lambda_2}(T) - f_{\lambda_1}(T)$$

where $f_{\lambda_1}(T)$ and $f_{\lambda_2}(T)$ are the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, respectively.

The *emissivity* of a surface represents the ratio of the radiation emitted by the surface at a given temperature to the radiation emitted by a blackbody at the same temperature. Different emissivities are defined as

Spectral hemispherical emissivity:

$$\varepsilon_\lambda(\lambda, T) = \frac{E_\lambda(\lambda, T)}{E_{b\lambda}(\lambda, T)}$$

Total hemispherical emissivity:

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda, T) E_{b\lambda}(\lambda, T)\, d\lambda}{\sigma T^4}$$

Emissivity can also be expressed as a step function by dividing the spectrum into a sufficient number of *wavelength bands* of constant emissivity as, for example,

$$\varepsilon(T) = \varepsilon_1 f_{0-\lambda_1}(T) + \varepsilon_2 f_{\lambda_1-\lambda_2}(T) + \varepsilon_3 f_{\lambda_2-\infty}(T)$$

The *total hemispherical emissivity* ε of a surface is the average emissivity over all directions and wavelengths.

When radiation strikes a surface, part of it is absorbed, part of it is reflected, and the remaining part, if any, is transmitted. The fraction of incident radiation (irradiation G) absorbed by the surface is called the *absorptivity*, the fraction reflected by the surface is called the *reflectivity*, and the fraction transmitted is called the *transmissivity*. Various absorptivities, reflectivities, and transmissivities for a medium are expressed as

$$\alpha_\lambda(\lambda) = \frac{G_{\lambda,\text{abs}}(\lambda)}{G_\lambda(\lambda)}, \quad \rho_\lambda(\lambda) = \frac{G_{\lambda,\text{ref}}(\lambda)}{G_\lambda(\lambda)}, \quad \text{and} \quad \tau_\lambda(\lambda) = \frac{G_{\lambda,\text{tr}}(\lambda)}{G_\lambda(\lambda)}$$

$$\alpha = \frac{G_\text{abs}}{G}, \quad \rho = \frac{G_\text{ref}}{G}, \quad \text{and} \quad \tau = \frac{G_\text{tr}}{G}$$

The consideration of wavelength and direction dependence of properties makes radiation calculations very complicated. Therefore, the *gray* and *diffuse* approximations are commonly utilized in radiation calculations. A surface is said to be *diffuse* if its properties are independent of direction and *gray* if its properties are independent of wavelength.

The sum of the absorbed, reflected, and transmitted fractions of radiation energy must be equal to unity:

$$\alpha + \rho + \tau = 1$$

For *opaque* surfaces, $\tau = 0$, and thus

$$\alpha + \rho = 1$$

Surfaces are usually assumed to reflect in a perfectly *specular* or *diffuse* manner for simplicity. In *specular* (or *mirrorlike*) *reflection*, the angle of reflection equals the angle of incidence of the radiation beam. In *diffuse reflection,* radiation is reflected equally in all directions. Reflection from smooth and polished surfaces approximates specular reflection, whereas reflection from rough surfaces approximates diffuse reflection. *Kirchhoff's law* of radiation is expressed as

$$\varepsilon_{\lambda,\theta}(T) = \alpha_{\lambda,\theta}(T), \quad \varepsilon_\lambda(T) = \alpha_\lambda(T), \quad \text{and} \quad \varepsilon(T) = \alpha(T)$$

Radiation heat transfer between surfaces depends on the orientation of the surfaces relative to each other. In a radiation analysis, this effect is accounted for by the geometric parameter *view factor*. The *view factor* from a surface i to a surface j is denoted by $F_{i\rightarrow j}$ or F_{ij}, and is defined as the fraction of the radiation leaving surface i that strikes surface j directly. The view factor $F_{i\rightarrow i}$ represents the fraction of the radiation leaving a surface i that strikes itself directly: $F_{i\rightarrow i} = 0$ for *plane* or *convex surfaces* and $F_{i\rightarrow i} \neq 0$ for *concave surfaces*.

For view factors, the *reciprocity rule* is expressed as

$$A_i F_{i\rightarrow j} = A_j F_{j\rightarrow i}$$

The sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself, must equal unity. This is known as the *summation rule* for an enclosure. The *superposition rule* is expressed as the view factor from a surface i to a surface j is equal to the sum of the view factors from surface i to the parts of surface j. The symmetry rule is expressed as if the surfaces j and k are symmetric about the surface i then $F_{i\to j} = F_{i\to k}$.

The rate of net radiation heat transfer between two *black* surfaces is determined from

$$\dot{Q}_{1\to 2} = A_1 F_{1\to 2}\sigma(T_1^4 - T_2^4)$$

The *net* radiation heat transfer from any surface i of a *black* enclosure is determined by adding up the net radiation heat transfers from surface i to each of the surfaces of the enclosure:

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i\to j} = \sum_{j=1}^{N} A_i F_{i\to j}\sigma(T_i^4 - T_j^4)$$

The total radiation energy leaving a surface per unit time and per unit area is called the *radiosity* and is denoted by J. The *net* rate of radiation heat transfer from a surface i of surface area A_i is expressed as

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i} \quad \text{where} \quad R_i = \frac{1-\varepsilon_i}{A_i\varepsilon_i}$$

is the *surface resistance* to radiation. The *net* rate of radiation heat transfer from surface i to surface j can be expressed as

$$\dot{Q}_{i\to j} = \frac{J_i - J_j}{R_{i\to j}} \quad \text{where} \quad R_{i\to j} = \frac{1}{A_i F_{i\to j}}$$

is the *space resistance* to radiation. The *network method* is applied to radiation enclosure problems by drawing a surface resistance associated with each surface of an enclosure and connecting them with space resistances. Then the problem is solved by treating it as an electrical network problem where the radiation heat transfer replaces the current and the radiosity replaces the potential. The *direct method* is based on the following two equations:

Surfaces with specified net heat transfer rate $\dot{Q}_i$
$$\dot{Q}_i = A_i \sum_{j=1}^{N} F_{i\to j}(J_i - J_j)$$

Surfaces with specified temperature T_i
$$\sigma T_i^4 = J_i + \frac{1-\varepsilon_i}{\varepsilon_i}\sum_{j=1}^{N} F_{i\to j}(J_i - J_j)$$

The first and the second groups of equations give N linear algebraic equations for the determination of the N unknown radiosities for an N-surface enclosure. Once the radiosities $J_1, J_2, \ldots, J_N$ are available, the unknown surface temperatures and heat transfer rates can be determined from the equations just shown.

The net rate of radiation transfer between any two gray, diffuse, opaque surfaces that form an enclosure is given by

$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

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PROBLEMS*

Electromagnetic and Thermal Radiation

21–1C What is an electromagnetic wave? How does it differ from a sound wave?

21–2C By what properties is an electromagnetic wave characterized? How are these properties related to each other?

21–3C What is thermal radiation? How does it differ from the other forms of electromagnetic radiation?

21–4C How does microwave cooking differ from conventional cooking?

21–5C What is visible light? How does it differ from the other forms of electromagnetic radiation?

21–6C What is the cause of color? Why do some objects appear blue to the eye while others appear red? Is the color of a surface at room temperature related to the radiation it emits?

21–7C Why do skiers get sunburned so easily?

21–8C How do ultraviolet and infrared radiation differ? Do you think your body emits any radiation in the ultraviolet range? Explain.

21–9 Electricity is generated and transmitted in power lines at a frequency of 50 Hz (1 Hz = 1 cycle per second). Determine the wavelength of the electromagnetic waves generated by the passage of electricity in power lines.

21–10 The speed of light in a vacuum is given to be 3.0×10^8 m/s. Determine the speed of light in air ($n = 1$), in water ($n = 1.33$), and in glass ($n = 1.5$).

21–11 A radio station is broadcasting radio waves at a wavelength of 150 m. Determine the frequency of these waves.

21–12 An electromagnetic wave with a wavelength of 0.5 μm is being propagated in different media: air, water, and glass. The refractive index of air, water, and glass are 1, 1.33, and 1.5, respectively. Determine the photon energy, in eV, of the electromagnetic wave in each medium ($1 \text{ eV} = 1.6022 \times 10^{-19}$ J).

21–13 The electromagnetic spectrum that lies between 0.40 and 0.76 μm is what we call visible light. Within this spectrum, the color violet has the shortest wavelength while the color red has the longest wavelength. Determine which of these colors, violet ($\lambda = 0.40$ μm) or red ($\lambda = 0.76$ μm), propagates more photon energy.

Blackbody Radiation

21–14C What is a blackbody? Does a blackbody actually exist?

21–15C Define the total and spectral blackbody emissive powers. How are they related to each other? How do they differ?

21–16C Why did we define the blackbody radiation function? What does it represent? For what is it used?

21–17 Consider a surface at a uniform temperature of 1000 K. Determine the maximum rate of thermal radiation that can be emitted by this surface, in W/m^2.

21–18 A flame from a match may be approximated as a blackbody at the effective surface temperature of 1700 K, while moonlight may be approximated as a blackbody at the effective surface temperature of 4000 K, respectively. Determine the peak spectral blackbody emissive power for both lighting sources (match flame and moonlight).

21–19 At a wavelength of 0.7 μm, the blackbody emissive power is equal to 10^8 W/m^3. Determine (*a*) the temperature of the blackbody and (*b*) the total emissive power at this temperature.

21–20 💻 The sun can be treated as a blackbody at 5780 K. Using appropriate software, calculate and plot the spectral blackbody emissive power $E_{b\lambda}$ of the sun versus wavelength in the range of 0.01 μm to 1000 μm. Discuss the results.

21–21 A small body is placed inside a spherical stainless steel chamber with a diameter of 3 m. The chamber is evacuated and has a constant surface temperature of 500 K. Determine the radiation incident on the surface of the small body inside the chamber if the interior surface of the chamber is (*a*) coated black and (*b*) well polished.

21–22 Consider a black spherical ball, with a diameter of 25 cm, that is suspended in air. Determine the surface temperature of the ball that should be maintained in order to heat 10 kg of air from 20 to 30°C in 5 minutes.

21–23 A thin vertical copper plate is subjected to a uniform heat flux of 1000 W/m^2 on one side, while the other side is exposed to ambient surroundings at 5°C. The surface of the plate is oxidized black and can be treated as a blackbody. The heat transfer coefficient due to natural convection on the plate surface is 5.3 W/m^2·K. Determine the surface temperature of the plate. Discuss the contribution of the net radiation heat transfer on the total heat loss from the plate.

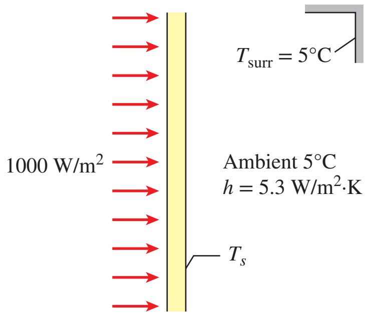

FIGURE P21–23

21–24 A circular ceramic plate that can be modeled as a blackbody is being heated by an electrical heater. The plate is 30 cm in diameter and is situated in a surrounding ambient temperature of 15°C where the natural convection heat transfer coefficient is 12 W/m^2·K. If the efficiency of the electrical heater to transfer heat to the plate is 80 percent, determine the electric power that the heater needs to keep the surface temperature of the plate at 200°C.

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*Problems designated by a “C” are concept questions, and students are encouraged to answer them all. Problems designated by an “E” are in English units, and SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

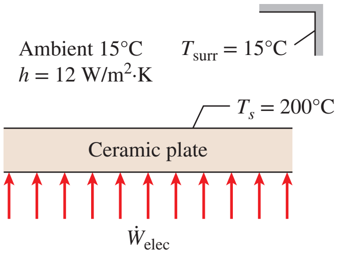


FIGURE P21–24

21–25 We wish an incandescent lightbulb to emit at least 15 percent of its energy at wavelengths shorter than 0.8 μm. Determine the minimum temperature to which the filament of the lightbulb must be heated.

21–26 The temperature of the filament of an incandescent lightbulb is 2500 K. Assuming the filament to be a blackbody, determine the fraction of the radiant energy emitted by the filament that falls in the visible range. Also, determine the wavelength at which the emission of radiation from the filament peaks.

21–27 The temperature of the filament of an incandescent lightbulb is 2800 K. Treating the filament as a blackbody, determine the fraction of the radiant energy emitted by the filament that falls in the visible range. Also, determine the wavelength at which the emission of radiation from the filament peaks.

21–28 Reconsider Prob. 21–27. Using appropriate software, investigate the effect of temperature on the fraction of radiation emitted in the visible range. Let the surface temperature vary from 1000 K to 4000 K, and plot the fraction of radiation emitted in the visible range versus the surface temperature.

21–29 It is desired that the radiation energy emitted by a light source reach a maximum in the blue range ($\lambda = 0.47$ μm). Determine the temperature of this light source and the fraction of radiation it emits in the visible range ($\lambda = 0.40 - 0.76$ μm).

21–30E The sun can be treated as a blackbody at an effective surface temperature of 10,400 R. Determine the rate at which infrared radiation energy ($\lambda = 0.76 - 100$ μm) is emitted by the sun, in Btu/h·ft^2.

21–31 A 3-mm-thick glass window transmits 90 percent of the radiation between $\lambda = 0.3$ and 3.0 μm and is essentially opaque for radiation at other wavelengths. Determine the rate of radiation transmitted through a 3-m × 3-m glass window from blackbody sources at (*a*) 5800 K and (*b*) 1000 K.

21–32 Consider the sun, which is considered to be a blackbody with a surface temperature of roughly 5800 K. Determine the percentage of solar energy (*a*) in the visible range, (*b*) at wavelengths shorter than the visible range, and (*c*) at wavelengths longer than the visible range.

Radiation Properties

21–33C What is a gray body? How does it differ from a blackbody? What is a diffuse gray surface?

21–34C Define the properties emissivity and absorptivity. When are these two properties equal to each other?

21–35C Define the properties reflectivity and transmissivity, and discus the different forms of reflection.

21–36C What is the greenhouse effect? Why is it a matter of great concern among atmospheric scientists?

21–37 A furnace that has a 40-cm × 40-cm glass window can be considered to be a blackbody at 1200 K. If the transmissivity of the glass is 0.7 for radiation at wavelengths less than 3 μm and zero for radiation at wavelengths greater than 3 μm, determine the fraction and the rate of radiation coming from the furnace and transmitted through the window.

21–38 The spectral emissivity function of an opaque surface at 1000 K is approximated as

$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = 0.4, & 0 \le \lambda < 3\ \mu\text{m} \\ \varepsilon_2 = 0.7, & 3\ \mu\text{m} \le \lambda < 6\ \mu\text{m} \\ \varepsilon_3 = 0.3, & 6\ \mu\text{m} \le \lambda < \infty \end{cases}$$

Determine the average emissivity of the surface and the rate of radiation emission from the surface, in W/m^2.

21–39 The emissivity of a tungsten filament can be approximated to be 0.5 for radiation at wavelengths less than 1 μm and 0.15 for radiation at greater than 1 μm. Determine the average emissivity of the filament at (*a*) 1500 K and (*b*) 2500 K. Also, determine the absorptivity and reflectivity of the filament at both temperatures.

21–40 The variations of the spectral emissivity of two surfaces are as given in Fig. P21–40. Determine the average emissivity of each surface at $T = 3000$ K. Also, determine the average absorptivity and reflectivity of each surface for radiation coming from a source at 3000 K. Which surface is more suitable to serve as a solar absorber?

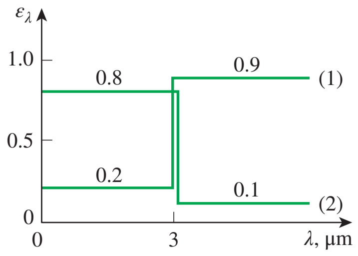


FIGURE P21–40

21–41 The emissivity of a surface coated with aluminum oxide can be approximated to be 0.15 for radiation at wavelengths less than 5 μm and 0.9 for radiation at wavelengths greater than 5 μm. Determine the average emissivity of this surface at (*a*) 5800 K and (*b*) 300 K. What can you say about the absorptivity of this surface for radiation coming from sources at 5800 K and 300 K? *Answers:* (*a*) 0.154, (*b*) 0.89

21–42 The variation of the spectral absorptivity of a surface is as given in Fig. P21–42. Determine the average absorptivity and reflectivity of the surface for radiation that originates from a source at $T = 2500$ K. Also, determine the average emissivity of this surface at 3000 K.

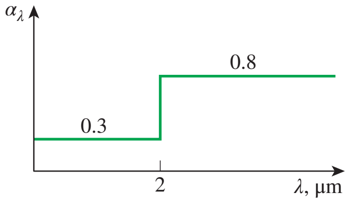


FIGURE P21–42

21–43 The reflectivity of aluminum coated with lead sulfate is 0.35 for radiation at wavelengths less than 3 μm and 0.95 for radiation greater than 3 μm. Determine the average reflectivity of this surface for solar radiation ($T \approx 5800$ K) and radiation coming from surfaces at room temperature ($T \approx 300$ K). Also, determine the emissivity and absorptivity of this surface at both temperatures. Do you think this material is suitable for use in solar collectors?

21–44 An opaque horizontal plate is well insulated on the edges and the lower surface. The irradiation on the plate is 3000 W/m^2, of which 500 W/m^2 is reflected. The plate has a uniform temperature of 700 K and has an emissive power of 5000 W/m^2. Determine the total emissivity and absorptivity of the plate.

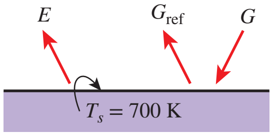


FIGURE P21–44

21–45 Irradiation on a semitransparent medium is at a rate of 520 W/m^2. If 160 W/m^2 of the irradiation is reflected from the medium and 130 W/m^2 is transmitted through the medium, determine the medium's absorptivity, reflectivity, transmissivity, and emissivity.

21–46 Consider an opaque horizontal plate that is well insulated on the edges and the lower surface. The plate is uniformly irradiated from above while air at $T_\infty = 300$ K flows over the surface, providing a uniform convection heat transfer coefficient of 40 W/m^2·K. Under steady-state conditions, the surface has a radiosity of 4000 W/m^2, and the plate temperature is maintained uniformly at 350 K. If the total absorptivity of the plate is 0.40, determine (*a*) the irradiation on the plate, (*b*) the total reflectivity of the plate, (*c*) the emissive power of the plate, and (*d*) the total emissivity of the plate.

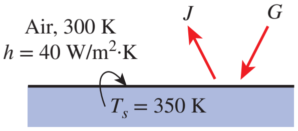


FIGURE P21–46

21–47 Consider an opaque plate that is well insulated on the edges and heated at the bottom with an electric heater. The plate has an emissivity of 0.67 and is situated in an ambient surrounding temperature of 7°C where the natural convection heat transfer coefficient is 7 W/m^2·K. To maintain a surface temperature of 80°C, the electric heater supplies 1000 W/m^2 of uniform heat flux to the plate. Determine the radiosity of the plate under these conditions.

21–48 A horizontal plate is experiencing uniform irradiation on both upper and lower surfaces. The ambient air temperature surrounding the plate is 290 K with a convection heat transfer coefficient of 30 W/m^2·K. Both upper and lower surfaces of the plate have a radiosity of 4000 W/m^2, and the plate temperature is maintained uniformly at 390 K. If the plate is not opaque and has an absorptivity of 0.527, determine the irradiation and emissivity of the plate.

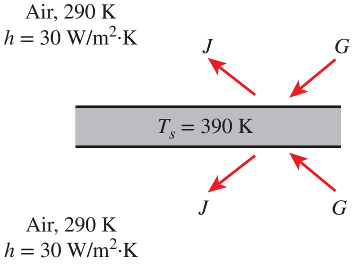


FIGURE P21–48

The View Factor

21–49C What does the view factor represent? When is the view factor from a surface to itself not zero?

21–50C How can you determine the view factor F_{12} when the view factor F_{21} and the surface areas are available?

21–51C What are the summation rule and the superposition rule for view factors?

21–52C What is the crossed-strings method? For what kinds of geometries is the crossed-strings method applicable?

21–53 Consider two coaxial parallel circular disks of equal diameter $D = 1$ m spaced apart by 1 m, and two aligned parallel square plates (1 m × 1 m) also spaced apart by 1 m. Determine the view factors F_{12} between the circular disks and the square plates. Which of the two geometries has the higher view factor value?

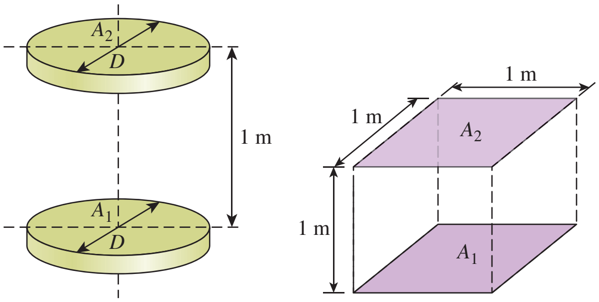


FIGURE P21–53

21–54 Consider two coaxial parallel circular disks of equal diameter D that are spaced apart by a distance L. If the view factor is $F_{12} = 0.1$, without altering the diameter of the disks, determine a solution that would increase the view factor F_{12} by a factor of 5.

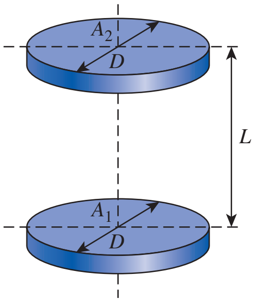


FIGURE P21–54

21–55 Cylindrical heaters are spaced equally at 5 cm apart in a row, and the heaters are positioned between two large parallel plates. If the diameter of the cylinders is 35 mm, determine the view factors between the plate and the row of cylinders, F_{12} and F_{32}.

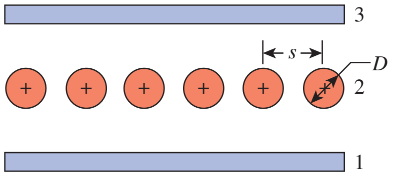


FIGURE P21–55

21–56 Consider an enclosure consisting of 13 surfaces. How many view factors does this geometry involve? How many of these view factors can be determined by the application of the reciprocity and the summation rules?

21–57 Consider a hemispherical furnace with a flat circular base of diameter D. Determine the view factor from the dome of this furnace to its base. *Answer:* 0.5

21–58 Consider a conical enclosure of height h and base diameter D. Determine the view factor from the conical side surface to a hole of diameter d located at the center of the base.

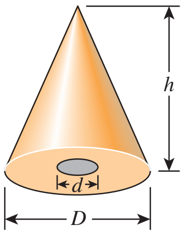
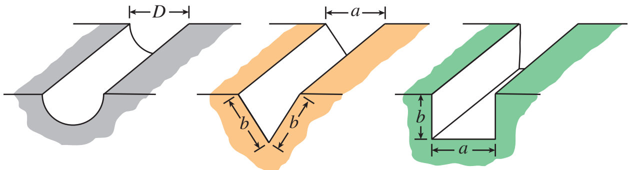


FIGURE P21–58

21–59 Determine the four view factors associated with an enclosure formed by two very long concentric cylinders of radii r_1 and r_2. Neglect the end effects.

21–60 Determine the view factors from the very long grooves shown in Fig. P21–60 to the surroundings without using any view factor tables or charts. Neglect end effects.



(*a*) Semicylindrical (*b*) Triangular groove (*c*) Rectangular groove

FIGURE P21–60

21–61 Consider a cylindrical enclosure with A_1, A_2, and A_3 representing the internal base, top, and side surfaces, respectively. Using the length-to-diameter ratio, $K = L/D$, determine (*a*) the expression for the view factor between the base and the side surface F_{13} in terms of K and (*b*) the value of the view factor F_{13} for $L = D$. *Answers:* (*a*) $F_{13} = 2K\sqrt{(K^2 + 1)} - 2K^2$, (*b*) 0.828

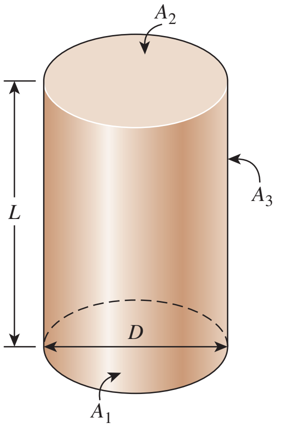


FIGURE P21–61

21–62 Determine the view factors from the base of a cube to each of the other five surfaces.

21–63 Determine the view factors F_{13} and F_{23} between the rectangular surfaces shown in Fig. P21–63.

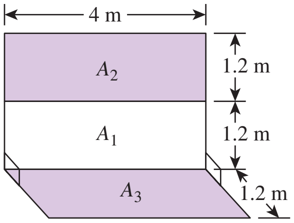


FIGURE P21–63

21–64 Determine the view factor F_{12} between the rectangular surfaces shown in Fig. P21–64.

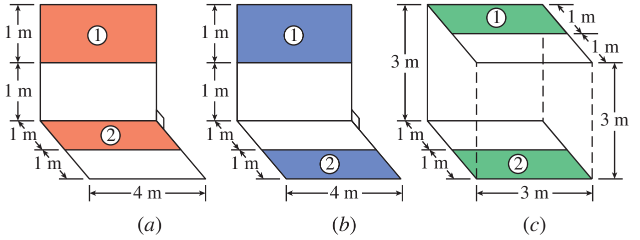


FIGURE P21–64

21–65 Consider a cylindrical enclosure whose height is twice the diameter of its base. Determine the view factor from the side surface of this cylindrical enclosure to its base surface.

21–66 For the internal surfaces of the right circular cylinder shown in Fig. P21–66, determine F_{13} and F_{33}.

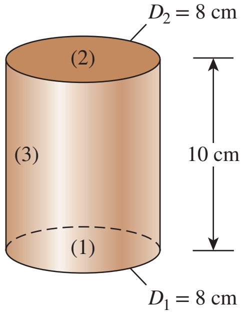


FIGURE P21–66

21–67 Two infinitely long parallel plates of width w are located at w distance apart, as shown in Fig. P21-67. Using the Hottel's crossed-strings method, determine the view factor F_{12}.

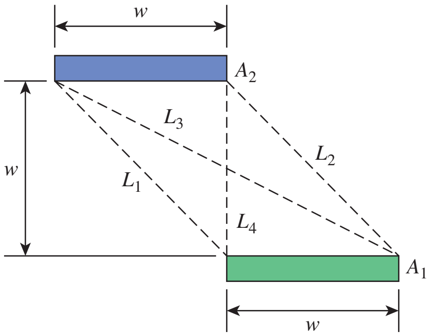


FIGURE P21–67

21–68 Two infinitely long parallel cylinders of diameter D are located a distance s apart from each other. Determine the view factor F_{12} between these two cylinders.

Radiation Heat Transfer Between Surfaces

21–69C Why is the radiation analysis of enclosures that consist of black surfaces relatively easy? How is the rate of radiation heat transfer between two surfaces expressed in this case?

21–70C How does radiosity for a surface differ from the emitted energy? For what kinds of surfaces are these two quantities identical?

21–71C What are the radiation surface and space resistances? How are they expressed? For what kinds of surfaces is the radiation surface resistance zero?

21–72C What is a reradiating surface? What simplifications does a reradiating surface offer in the radiation analysis?

21–73C What are the two methods used in radiation analysis? How do these two methods differ?

21–74 Consider a person whose exposed surface area is 1.9 m^2, emissivity is 0.85, and surface temperature is 30°C. Determine the rate of heat loss from that person by radiation in a large room whose walls are at a temperature of (*a*) 295 K and (*b*) 260 K.

21–75 Consider two black coaxial parallel circular disks of equal diameter D that are spaced apart by a distance L. The top and bottom disks have uniform temperatures of 500°C and 520°C, respectively. Determine the radiation heat transfer coefficient h_{rad} between the disks if they are spaced apart by $L = D$.

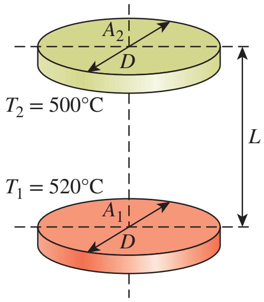


FIGURE P21–75

21–76 Reconsider Prob. 21–75. Using appropriate software, evaluate the effect of the distance L between the black coaxial parallel disks ($D = 1$ m) on the radiation heat transfer coefficient. By varying the distance L between the disks from 0.05 to 3 m, plot the radiation heat transfer coefficient h_{rad} as a function of the distance L between the disks.

21–77E Two black parallel rectangles with dimensions 3 ft × 5 ft are spaced apart by a distance of 1 ft. The two parallel rectangles are experiencing radiation heat transfer as black surfaces, where the top rectangle receives a total of 180,000 Btu/h radiation heat transfer rate from the bottom rectangle. If the top rectangle has a uniform temperature of 60°F, determine the temperature of the bottom rectangle.

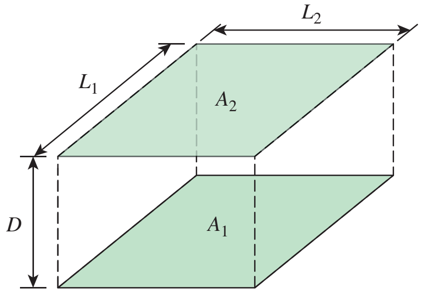


FIGURE P21–77E

21–78E The room shown in Fig. P21–78E is 20 ft by 20 ft wide and 9 ft high. The floor is at 100°F, the walls are at 60°F, and the ceiling is at 40°F. All surfaces are assumed to be black. Calculate the net radiation heat transfer (*a*) from floor to walls and (*b*) from floor to ceiling.

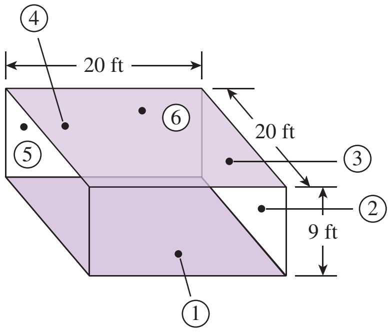


FIGURE P21–78E

21–79 A dryer is shaped like a long semicylindrical duct of diameter 1.5 m. The base of the dryer is occupied with water-soaked materials to be dried. The base is maintained at a temperature of 370 K, while the dome of the dryer is maintained at 1000 K. If both surfaces behave as blackbodies, determine the drying rate per unit length experienced by the wet materials.

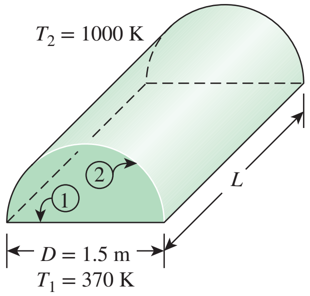


FIGURE P21–79

21–80 A furnace is of cylindrical shape with $R = H = 3$ m. The base, top, and side surfaces of the furnace are all black and are maintained at uniform temperatures of 500, 700, and 1400 K, respectively. Determine the net rate of radiation heat transfer to or from the top surface during steady operation.

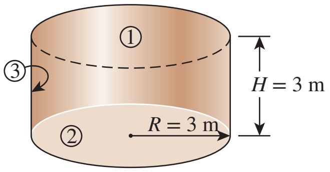


FIGURE P21–80

21–81 Two parallel disks of diameter $D = 0.6$ m separated by $L = 0.4$ m are located directly on top of each other. Both disks are black and are maintained at a temperature of 450 K. The back sides of the disks are insulated, and the environment that the disks are in can be considered to be blackbodies at 300 K. Determine the net rate of radiation heat transfer from the disks to the environment. *Answer:* 781 W

21–82 Consider a hemispherical furnace of diameter $D = 5$ m with a flat base, as shown in Fig. P21–82. The dome of the furnace is black, and the base has an emissivity of 0.7. The base and the dome of the furnace are maintained at uniform temperatures of 400 and 1000 K, respectively. Determine the net rate of radiation heat transfer from the dome to the base surface during steady operation. *Answer:* 759 kW

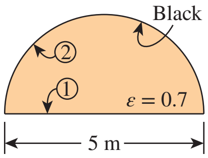


FIGURE P21–82

21–83 Two parallel black disks are positioned coaxially at a distance of 0.25 m apart in surroundings with a constant temperature of 300 K. The lower disk is 0.2 m in diameter and the upper disk is 0.4 m in diameter. If the lower disk is heated electrically at 100 W to maintain a uniform temperature of 500 K, determine the temperature of the upper disk.

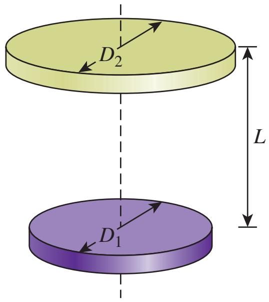


FIGURE P21–83

21–84E Two parallel disks of diameter $D = 3$ ft separated by $L = 2$ ft are located directly on top of each other. The disks are separated by a radiation shield whose emissivity is 0.15. Both disks are black and are maintained at temperatures of 1350 R and 650 R, respectively. The environment that the disks are in can be considered to be a blackbody at 540 R. Determine the net rate of radiation heat transfer through the shield under steady conditions.

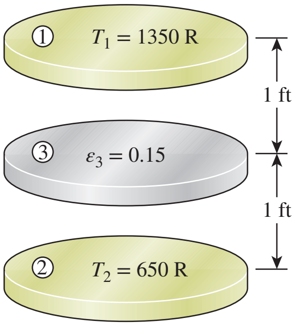


FIGURE P21–84E

21–85 Two infinitely long parallel plates of width w are located at w distance apart, as shown in Fig. P21-85. The two plates behave as black surfaces, where surface A_1 has a temperature of 700 K and surface A_2 has a temperature of 300 K. Determine the radiation heat flux between the two surfaces.

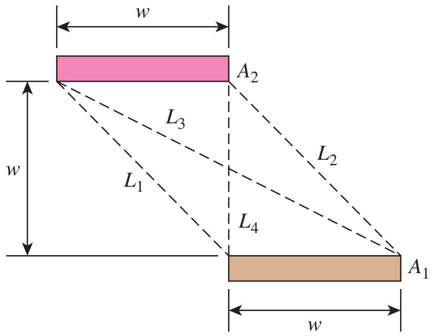


FIGURE P21–85

21–86 Two long, parallel 20-cm-diameter cylinders are located 30 cm apart from each other. Both cylinders are black and are maintained at temperatures 425 K and 275 K. The surroundings can be treated as a blackbody at 300 K. For a 1-m-long section of the cylinders, determine the rates of radiation heat transfer between the cylinders and between the hot cylinder and the surroundings.

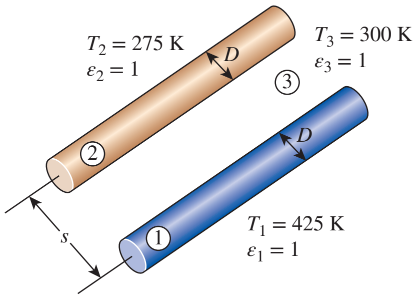


FIGURE P21–86

21–87 Consider two rectangular surfaces perpendicular to each other with a common edge that is 1.6 m long. The horizontal surface is 0.8 m wide, and the vertical surface is 1.2 m high. The horizontal surface has an emissivity of 0.75 and is maintained at 450 K. The vertical surface is black and is maintained at 700 K. The back sides of the surfaces are insulated. The surrounding surfaces are at 290 K and can be considered to have an emissivity of 0.85. Determine the net rate of radiation heat transfer between the two surfaces and between the horizontal surface and the surroundings.

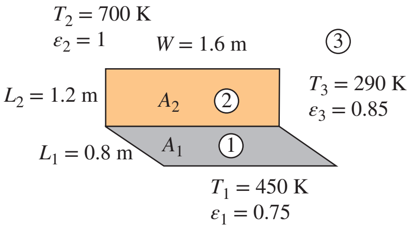


FIGURE P21–87

21–88 A furnace is shaped like a long equilateral-triangular duct where the width of each side is 2 m. Heat is supplied from the base surface, whose emissivity is $\varepsilon_1 = 0.8$, at a rate of 800 W/m^2 while the side surfaces, whose emissivities are 0.4, are maintained at 600 K. Neglecting the end effects, determine the temperature of the base surface. Can you treat this geometry as a two-surface enclosure?

21–89 Reconsider Prob. 21–88. Using appropriate software, investigate the effects of the rate of the heat transfer at the base surface and the temperature of the side surfaces on the temperature of the base surface. Let the rate of heat transfer vary from 500 W/m^2 to 1000 W/m^2 and the temperature from 300 K to 700 K. Plot the temperature of the base surface as functions of the rate of heat transfer and the temperature of the side surfaces, and discuss the results.

21–90 Consider a 30-cm-diameter hemispherical enclosure. The dome is maintained at 600 K, and heat is supplied from the dome at a rate of 65 W while the base surface with an emissivity of 0.55 is maintained at 400 K. Determine the emissivity of the dome.

21–91E A furnace is shaped like a long semicylindrical duct of diameter $D = 15$ ft. The base and the dome of the furnace have emissivities of 0.5 and 0.9 and are maintained at uniform temperatures of 550 and 1800 R, respectively. Determine the net rate of radiation heat transfer from the dome to the base surface per unit length during steady operation.

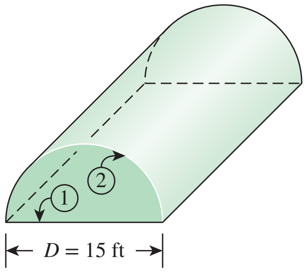


FIGURE P21–91E

21–92 Air is flowing between two infinitely large parallel plates. The upper plate is at 500 K and has an emissivity of 0.7, while the lower plate is a black surface with temperature at 330 K. If the air temperature is 290 K, determine the convection heat transfer coefficient associated with the air.

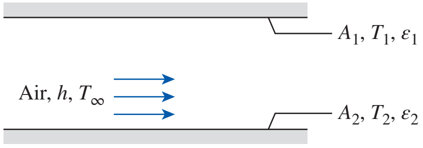


FIGURE P21–92

21–93 Two very long concentric cylinders of diameters $D_1 = 0.35$ m and $D_2 = 0.5$ m are maintained at uniform temperatures of $T_1 = 950$ K and $T_2 = 500$ K and have emissivities $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.55$, respectively. Determine the net rate of radiation heat transfer between the two cylinders per unit length of the cylinders.

21–94 This experiment is conducted to determine the emissivity of a certain material. A long cylindrical rod of diameter $D_1 = 0.01$ m is coated with this new material and is placed in an evacuated long cylindrical enclosure of diameter $D_2 = 0.1$ m and emissivity $\varepsilon_2 = 0.95$, which is cooled externally and maintained at a temperature of 200 K at all times. The rod is heated by passing electric current through it. When steady operating conditions are reached, it is observed that the rod is dissipating electric power at a rate of 12 W per unit of its length, and its surface temperature is 600 K. Based on these measurements, determine the emissivity of the coating on the rod.

21–95 Liquid nitrogen is stored in a spherical tank of 1 m diameter where the tank surface is maintained uniformly at 80 K. The spherical tank is enclosed by a 1.6-m-diameter concentric sphere with uniform surface temperature of 273 K. Both spherical surfaces have an emissivity of 0.01, and the gap between the inner sphere and outer sphere is vacuumed. Determine the rate of vaporization for the liquid nitrogen.

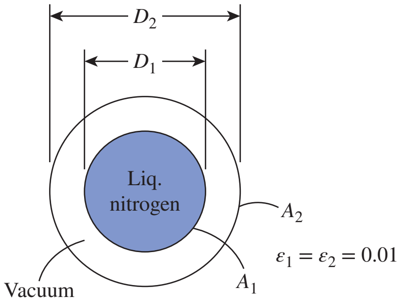


FIGURE P21–95

21–96 Two concentric spheres of diameters $D_1 = 0.3$ m and $D_2 = 0.6$ m are maintained at uniform temperatures $T_1 = 800$ K and $T_2 = 500$ K and have emissivities $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.7$, respectively. Determine the net rate of radiation heat transfer between the two spheres. Also, determine the convection heat transfer coefficient at the outer surface if both the surrounding medium and the surrounding surfaces are at 30°C. Assume the emissivity of the outer surface is 0.35.

21–97 A spherical tank of diameter $D = 2$ m that is filled with liquid nitrogen at 100 K is kept in an evacuated cubic enclosure whose sides are 3 m long. The emissivities of the spherical tank and the enclosure are $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.8$, respectively. If the temperature of the cubic enclosure is measured to be 240 K, determine the net rate of radiation heat transfer to the liquid nitrogen. *Answer:* 228 W

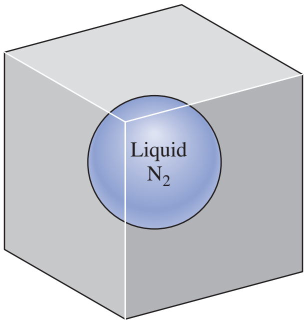


FIGURE P21–97

21–98 Repeat Prob. 21–97 by replacing the cubic enclosure with a spherical enclosure whose diameter is 3 m.

21–99 Reconsider Prob. 21–97. Using appropriate software, investigate the effects of the side length and the emissivity of the cubic enclosure, and the emissivity of the spherical tank on the net rate of radiation heat transfer. Let the side length vary from 2 m to 12 m and both emissivities from 0.1 to 0.9. Plot the net rate of radiation heat transfer as functions of side length and emissivities, and discuss the results.

21–100E A 9-ft-high room with a base area of 12 ft × 12 ft is to be heated by electric resistance heaters placed on the ceiling, which is maintained at a uniform temperature of 90°F at all times. The floor of the room is at 65°F and has an emissivity of 0.8. The side surfaces are well insulated. Treating the ceiling as a blackbody, determine the rate of heat loss from the room through the floor.

21–101 Consider a 4-m × 4-m × 4-m cubical furnace whose floor and ceiling are black and whose side surfaces are reradiating. The floor and the ceiling of the furnace are maintained at temperatures of 550 K and 1100 K, respectively. Determine the net rate of radiation heat transfer between the floor and the ceiling of the furnace.

21–102 Consider a circular grill whose diameter is 0.3 m. The bottom of the grill is covered with hot coal bricks at 950 K, while the wire mesh on top of the grill is covered with steaks initially at 5°C. The distance between the coal bricks and the steaks is 0.20 m. Treating both the steaks and the coal bricks as blackbodies, determine the initial rate of radiation heat transfer from the coal bricks to the steaks. Also, determine the initial rate of radiation heat transfer to the steaks if the side opening of the grill is covered by aluminum foil, which can be approximated as a reradiating surface. *Answers:* 928 W, 2085 W

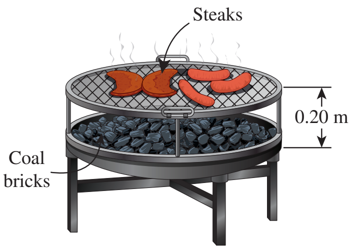


FIGURE P21–102

21–103E Consider a 10-ft × 10-ft × 10-ft cubical furnace whose top and side surfaces closely approximate black surfaces and whose base surface has an emissivity $\varepsilon = 0.4$. The base, top, and side surfaces of the furnace are maintained at uniform temperatures of 800 R, 1600 R, and 2400 R, respectively. Determine the net rate of radiation heat transfer between (*a*) the base and the side surfaces and (*b*) the base and the top surfaces. Also, determine the net rate of radiation heat transfer to the base surface.

21–104E Reconsider Prob. 21–103E. Using appropriate software, investigate the effect of base surface emissivity on the net rates of radiation heat transfer between the base and the side surfaces, between the base and top surfaces, and to the base surface. Let the emissivity vary from 0.1 to 0.9. Plot the rates of heat transfer as a function of emissivity, and discuss the results.

Review Problems

21–105 A 1-m-diameter spherical cavity is maintained at a uniform temperature of 600 K. Now a 5-mm-diameter hole is drilled. Determine the maximum rate of radiation energy streaming through the hole. What would your answer be if the diameter of the cavity were 3 m?

21–106 Daylight and incandescent light may be approximated as blackbodies at the effective surface temperatures of 5800 K and 2800 K, respectively. Determine the wavelength at maximum emission of radiation for each of the lighting sources.

21–107 The human skin is "selective" when it comes to the absorption of the solar radiation that strikes it perpendicularly. The skin absorbs only 50 percent of the incident radiation with wavelengths between $\lambda_1 = 0.517$ μm and $\lambda_2 = 1.552$ μm. The radiation with wavelengths shorter than λ_1 and longer than λ_2 is fully absorbed. The solar surface may be modeled as a blackbody with effective surface temperature of 5800 K. Calculate the fraction of the incident solar radiation that is absorbed by the human skin.

21–108 The spectral emissivity of an opaque surface at 1200 K is approximated as

$$\begin{aligned} &\varepsilon_1 = 0 && \text{for} && \lambda < 2\ \mu\text{m} \\ &\varepsilon_2 = 0.85 && \text{for} && 2 \le \lambda \le 6\ \mu\text{m} \\ &\varepsilon_3 = 0 && \text{for} && \lambda > 6\ \mu\text{m} \end{aligned}$$

Determine the total emissivity and the emissive flux of the surface.

21–109 The spectral absorptivity of an opaque surface is as shown in Fig. P21-109. Determine the absorptivity of the surface for radiation emitted by a source at (*a*) 1000 K and (*b*) 3000 K.

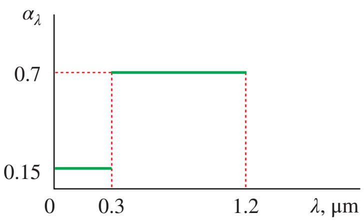


FIGURE P21–109

21–110 The surface in Prob. 21–109 receives solar radiation at a rate of 470 W/m^2. Determine the solar absorptivity of the surface and the rate of absorption of solar radiation.

21–111 Solar radiation is incident on the front surface of a thin plate with direct and diffuse components of 300 and 250 W/m^2, respectively. The direct radiation makes a 30° angle with the normal of the surface. The plate surfaces have a solar absorptivity of 0.63 and an emissivity of 0.93. The air temperature is 5°C and the convection heat transfer coefficient is 20 W/m^2·K. The effective sky temperature for the front surface is −33°C while the surrounding surfaces are at 5°C for the back surface. Determine the equilibrium temperature of the plate.

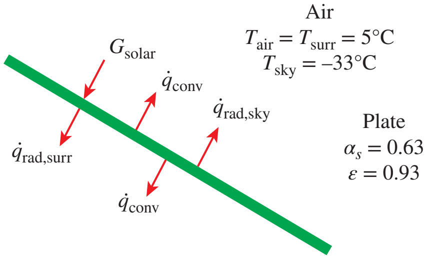


FIGURE P21–111

21–112 A horizontal opaque flat plate is well insulated on the edges and the lower surface. The top surface has an area of 5 m^2, and it experiences uniform irradiation at a rate of 5000 W. The plate absorbs 4000 W of the irradiation, and the surface is losing heat at a rate of 500 W by convection. If the plate maintains a uniform temperature of 350 K, determine the absorptivity, reflectivity, and emissivity of the plate.

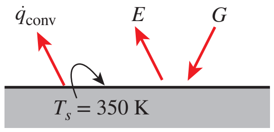


FIGURE P21–112

21–113 Consider an enclosure consisting of eight surfaces. How many view factors does this geometry involve? How many of these view factors can be determined by the application of the reciprocity and the summation rules?

21–114 Consider a cylindrical enclosure with A_1, A_2, and A_3 representing the internal base, top, and side surfaces, respectively. Using the length-to-diameter ratio, $K = L/D$, determine (*a*) the expression for the view factor from the side surface to itself F_{33} in terms of K and (*b*) the value of the view factor F_{33} for $L = D$.

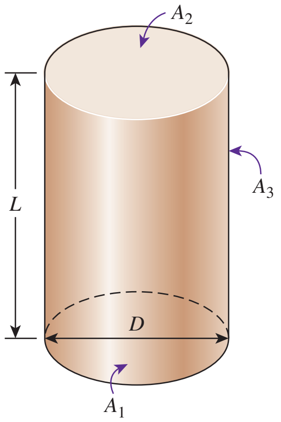


FIGURE P21–114

21–115 Two parallel black disks are positioned coaxially with a distance of 0.25 m apart. The lower disk is 0.2 m in diameter and the upper disk is 0.4 m in diameter. If the lower disk is heated electrically at 20 W to maintain a uniform temperature of 500 K, determine the temperature of the upper disk.

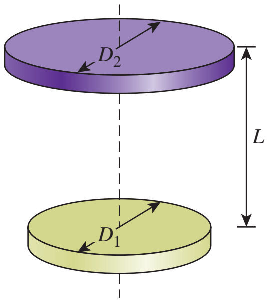


FIGURE P21–115

21–116 Two parallel concentric disks, 20 cm and 40 cm in diameter, are separated by a distance of 10 cm. The smaller disk ($\varepsilon = 0.80$) is at a temperature of 300°C. The larger disk ($\varepsilon = 0.60$) is at a temperature of 800°C.

(*a*) Calculate the radiation view factors.
(*b*) Determine the rate of radiation heat exchange between the two disks.
(*c*) Suppose that the space between the two disks is completely surrounded by a reflective surface. Estimate the rate of radiation heat exchange between the two disks.

21–117 A dryer is shaped like a long semicylindrical duct of diameter 1.5 m. The base of the dryer is occupied by water-soaked materials to be dried, and it is maintained at a temperature of 370 K and emissivity of 0.5. The dome of the dryer is maintained at 1000 K with emissivity of 0.8. Determine the drying rate per unit length experienced by the wet materials.

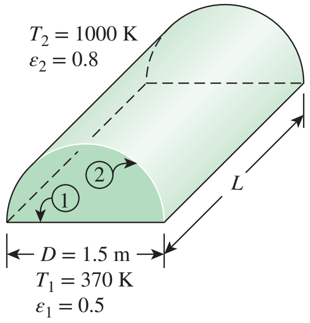


FIGURE P21–117

21–118 A large number of long tubes, each of diameter D, are placed parallel to each other and at a center-to-center distance of s. Since all of the tubes are geometrically similar and at the same temperature, these could be treated collectively as one surface (A_j) for radiation heat transfer calculations. As shown in Fig. P21–118, the tube bank (A_j) is placed opposite a large flat wall (A_i) such that the tube bank is parallel to the wall.

(*a*) Calculate the view factors F_{ij} and F_{ji} for $s = 3.0$ cm and $D = 1.5$ cm.

(*b*) Calculate the net rate of radiation heat transfer between the wall and the tube bank per unit area of the wall when $T_i = 900°\text{C}$, $T_j = 60°\text{C}$, $\varepsilon_i = 0.8$, and $\varepsilon_j = 0.9$.

(*c*) A fluid flows through the tubes at an average temperature of 40°C, resulting in a heat transfer coefficient of 2.0 kW/m²·K. Assuming $T_i = 900°\text{C}$, $\varepsilon_i = 0.8$ and $\varepsilon_j = 0.9$ (as above) and neglecting the tube wall thickness and convection from the outer surface, calculate the temperature of the tube surface in steady operation.

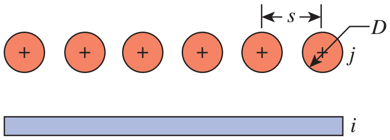


FIGURE P21–118

21–119 A vertical 2-m-high and 5-m-wide double-pane window consists of two sheets of glass separated by a 3-cm-thick air gap. In order to reduce heat transfer through the window, the air space between the two glasses is partially evacuated to 0.3 atm pressure. The emissivities of the glass surfaces are 0.9. Taking the glass surface temperatures across the air gap to be 15°C and 5°C, determine the rate of heat transfer through the window by natural convection and radiation.

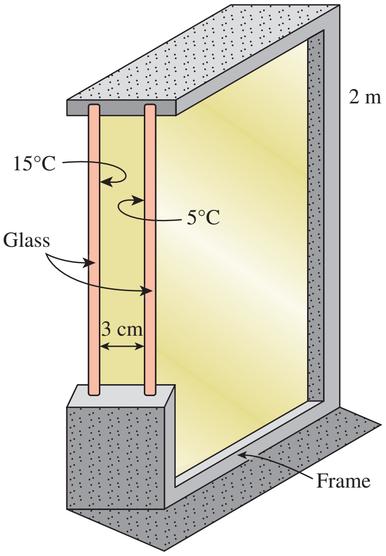


FIGURE P21–119

21–120 A 2-m-internal-diameter double-walled spherical tank is used to store iced water at 0°C. Each wall is 0.5 cm thick, and the 1.5-cm-thick airspace between the two walls of the tank is evacuated in order to minimize heat transfer. The surfaces surrounding the evacuated space are polished so that each surface has an emissivity of 0.15. The temperature of the outer wall of the tank is measured to be 20°C. Assuming the inner wall of the steel tank to be at 0°C, determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period.

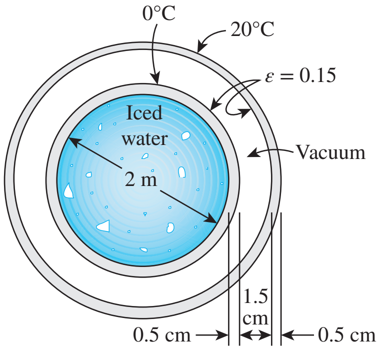


FIGURE P21–120

21–121 Two concentric spheres of diameters $D_1 = 15$ cm and $D_2 = 25$ cm are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are $T_1 = 350$ K and $T_2 = 275$ K, respectively, and their emissivities are 0.75. Determine the rate of heat transfer from the inner sphere to the outer sphere by (*a*) natural convection and (*b*) radiation.

21–122 Consider a cubical furnace with a side length of 3 m. The top surface is maintained at 700 K. The base surface has an emissivity of 0.90 and is maintained at 950 K. The side surface is black and is maintained at 450 K. Heat is supplied from the base surface at a rate of 340 kW. Determine the emissivity of the top surface and the net rates of heat transfer between the top and the bottom surfaces and between the bottom and side surfaces.

21–123 A furnace is of cylindrical shape with a diameter of 1.2 m and a length of 1.2 m. The top surface has an emissivity of 0.60 and is maintained at 450 K. The bottom surface has an emissivity of 0.50 and is maintained at 800 K. The side surface has an emissivity of 0.40. Heat is supplied from the base surface at a net rate of 1400 W. Determine the temperature of the side surface and the net rates of heat transfer between the top and the bottom surfaces and between the bottom and side surfaces.

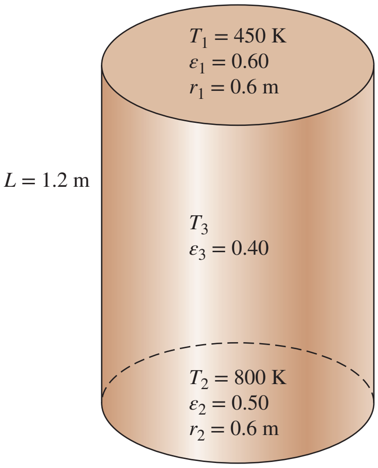


FIGURE P21–123

Design and Essay Problems

21–124 Write an essay on the radiation properties of selective surfaces used on the absorber plates of solar collectors. Find out about the various kinds of such surfaces, and discuss the performance and cost of each type. Recommend a selective surface that optimizes cost and performance.

21–125 According to an Atomic Energy Commission report, a hydrogen bomb can be approximated as a large fireball at a temperature of 7200 K. You are to assess the impact if such a bomb exploded 5 km above a city. Assume the diameter of the fireball to be 1 km and the blast to last 15 s. Investigate the level of radiation energy people, plants, and houses will be exposed to, and how adversely they will be affected by the blast.

21–126 Radiation shields are commonly used in the design of superinsulations for use in space and cryogenics applications. Write an essay on superinsulations and how they are used in different applications.

21–127 Thermal comfort in a house is strongly affected by the so-called radiation effect, which is due to radiation heat transfer between the person and the surrounding surfaces. A person feels much colder in the morning, for example, because of the lower surface temperature of the walls at that time, although the thermostat setting of the house is fixed. Write an essay on the radiation effect, how it affects human comfort, and how it is accounted for in heating and air-conditioning applications.

CHAPTER

22

HEAT EXCHANGERS

Heat exchangers are devices that facilitate the *exchange of heat* between two fluids that are at different temperatures while keeping them from mixing with each other. Heat exchangers are commonly used in practice in a wide range of applications, from heating and air-conditioning systems in a household to chemical processing and power production in large plants. Heat exchangers differ from mixing chambers in that they do not allow the two fluids involved to mix.

Heat transfer in a heat exchanger usually involves *convection* in each fluid and *conduction* through the wall separating the two fluids. In the analysis of heat exchangers, it is convenient to work with an *overall heat transfer coefficient U* that accounts for the contribution of all these effects on heat transfer. The rate of heat transfer between the two fluids at a location in a heat exchanger depends on the magnitude of the temperature difference at that location, which varies along the heat exchanger.

Heat exchangers are manufactured in a variety of types, and thus we start this chapter with the *classification* of heat exchangers. We then discuss the determination of the overall heat transfer coefficient in heat exchangers and the *log mean temperature difference* (LMTD) for some configurations. We then introduce the *correction factor F* to account for the deviation of the mean temperature difference from the LMTD in complex configurations. Finally, we discuss the effectiveness–NTU method, which enables us to analyze heat exchangers when the outlet temperatures of the fluids are not known.

OBJECTIVES

The objectives of this chapter are to:

- Recognize numerous types of heat exchangers, and classify them.
- Develop an awareness of fouling on surfaces, and determine the overall heat transfer coefficient for a heat exchanger.
- Perform a general energy analysis on heat exchangers.
- Obtain a relation for the logarithmic mean temperature difference for use in the LMTD method, and modify it for different types of heat exchangers using the correction factor.
- Develop relations for effectiveness, and analyze heat exchangers when outlet temperatures are not known using the effectiveness-NTU method.

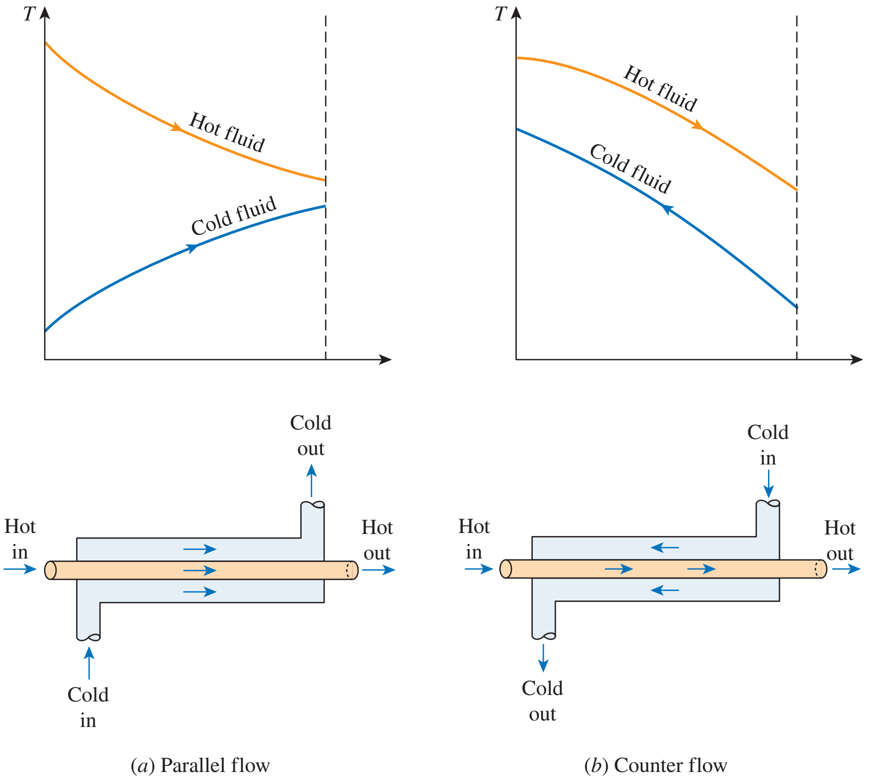
22–1 ▪ TYPES OF HEAT EXCHANGERS

Different heat transfer applications require different types of hardware and different configurations of heat transfer equipment. The attempt to match the heat transfer hardware to the heat transfer requirements within the specified constraints has resulted in numerous types of innovative heat exchanger designs.

The simplest type of heat exchanger consists of two concentric pipes of different diameters, as shown in Fig. 22–1, called the **double-pipe** heat exchanger. One fluid in a double-pipe heat exchanger flows through the smaller pipe, while the other fluid flows through the annular space between the two pipes. Two types of flow arrangement are possible in a double-pipe heat exchanger: in **parallel flow**, both the hot and cold fluids enter the heat exchanger at the same end and move in the *same* direction. In **counterflow**, on the other hand, the hot and cold fluids enter the heat exchanger at opposite ends and flow in *opposite* directions.

Another type of heat exchanger, which is specifically designed to realize a large heat transfer surface area per unit volume, is the **compact** heat exchanger. The ratio of the heat transfer surface area of a heat exchanger to its volume is called the *area density* β. A heat exchanger with $\beta > 700\ \text{m}^2/\text{m}^3$ (or $200\ \text{ft}^2/\text{ft}^3$) is classified as being compact. Examples of compact heat exchangers are car radiators ($\beta \approx 1000\ \text{m}^2/\text{m}^3$), glass-ceramic gas turbine heat exchangers ($\beta \approx 6000\ \text{m}^2/\text{m}^3$), the regenerator of a Stirling engine ($\beta \approx 15{,}000\ \text{m}^2/\text{m}^3$), and the human lung ($\beta \approx 20{,}000\ \text{m}^2/\text{m}^3$). The flow passages in these compact heat exchangers are usually small, and the flow can be considered to be laminar. Compact heat exchangers enable us to achieve high heat transfer rates between two fluids in a small volume, and they are commonly used in applications with strict limitations on the weight and volume of heat exchangers (Fig. 22–2).

The large surface area in compact heat exchangers is obtained by attaching closely spaced *thin plate* or *corrugated fins* to the walls separating the two fluids. Compact heat exchangers are commonly used in gas-to-gas and gas-to-liquid



(*a*) Parallel flow (*b*) Counter flow

FIGURE 22–1
Different flow regimes and associated temperature profiles in a double-pipe heat exchanger.

(or liquid-to-gas) heat exchangers to counteract the low heat transfer coefficient associated with gas flow with increased surface area. In a car radiator, which is a water-to-air compact heat exchanger, for example, it is no surprise that fins are attached to the air side of the tube surface.

Another example of compact heat exchangers widely used today in industrial applications such as chemical processing, fuel processing, waste heat recovery, and refrigeration is the **printed circuit heat exchanger** referred to as PCHE (Fig. 22–3). PCHE derives its name from the manufacturing process in which the flat metal plates that form the core of this heat exchanger are chemically etched to a depth of 1 to 3 mm. The chemically etched plates are then stacked together and joined by diffusion bonding. There may be different etching patterns on the plates to make the heat exchanger as a parallel, counter, or crossflow heat exchanger. Furthermore, Li et al. (2011) reported that unlike conventional heat exchangers with straight flow channels, the flow channels in PCHE can be made in zigzag, S-shape, or aero foil shape in order to induce flow turbulence and hence enhance the heat transfer coefficient. PCHEs have high surface density, typically greater than 2500 m^2/m^3. The PCHEs are usually made from stainless steel, titanium, copper, nickel, and nickel alloys and can withstand operating pressures up to 500 bar. The major advantages of printed circuit heat exchangers are their wide operating temperature range, which is from −250°C to 900°C, very high heat transfer coefficients, and their size, which is about four to six times smaller and lighter than conventional heat exchangers. One of the major disadvantages of using PCHEs is very high pressure drop and that they require very clean fluid to be passed through them, otherwise blockages can occur easily in the fine channel spacing (0.5 to 2 mm).

In compact heat exchangers, the two fluids usually move *perpendicular* to each other, and such flow configuration is called **crossflow**. The crossflow is further classified as *unmixed* or *mixed flow,* depending on the flow configuration, as shown in Fig. 22–4. In (*a*), the crossflow is said to be *unmixed* since the plate fins force the fluid to flow through a particular interfin spacing and prevent it from moving in the transverse direction (i.e., parallel to the tubes). The crossflow in (*b*) is said to be *mixed* since the fluid now is free to move in the transverse direction. Both fluids are unmixed in a car radiator. The presence of mixing in the fluid can have a significant effect on the heat transfer characteristics of the heat exchanger.

Perhaps the most common type of heat exchanger in industrial applications is the **shell-and-tube** heat exchanger, shown in Fig. 22–5. Shell-and-tube heat exchangers contain a large number of tubes (sometimes several hundred) packed in a shell with their axes parallel to that of the shell. Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the tubes through the shell. *Baffles* are commonly placed in the shell to force the shell-side fluid to flow across the shell to enhance heat transfer and to maintain uniform spacing between the tubes. Despite their widespread use, shell-and-tube heat exchangers are not

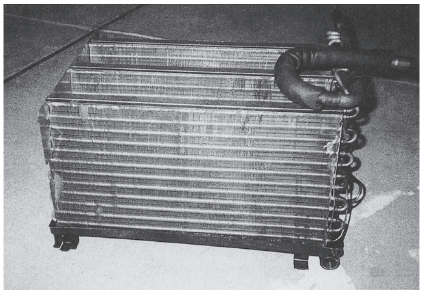


FIGURE 22–2
A gas-to-liquid compact heat exchanger for a residential air-conditioning system.
Yunus Çengel

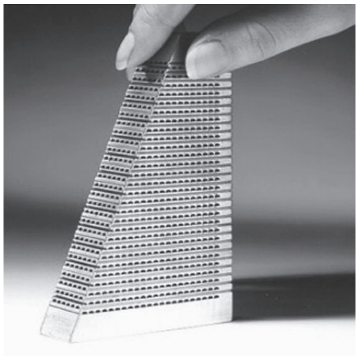
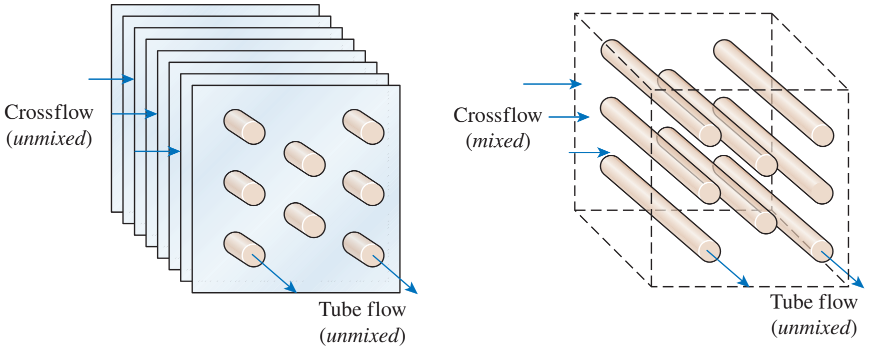


FIGURE 22–3
Photograph of a core section from a printed circuit heat exchanger (Heatric, a Meggitt Company, Dorset, UK).
Courtesy of Heatric, a Meggitt Company, Dorset, UK



(*a*) Both fluids unmixed

(*b*) One fluid mixed, one fluid unmixed

FIGURE 22–4
Different flow configurations in crossflow heat exchangers.

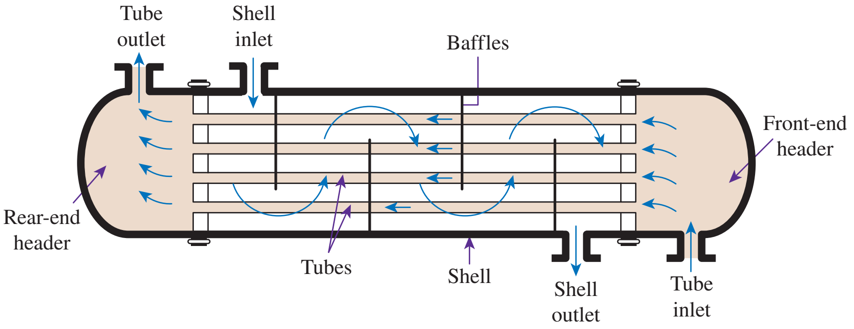


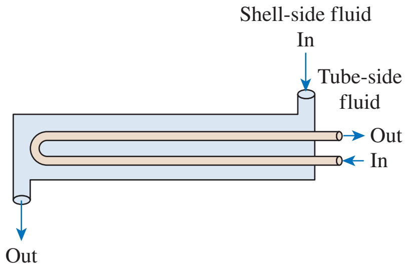
FIGURE 22–5
The schematic of a shell-and-tube heat exchanger (one shell pass and one tube pass).

suitable for use in automotive and aircraft applications because of their relatively large size and weight. Note that the tubes in a shell-and-tube heat exchanger open to some large flow areas called *headers* at both ends of the shell, where the tube-side fluid accumulates before entering the tubes and after leaving them.

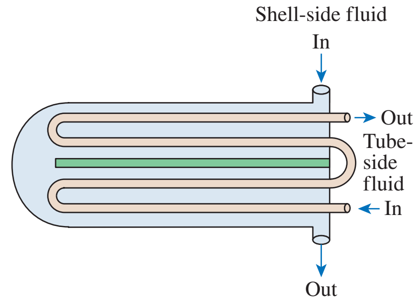
Shell-and-tube heat exchangers are further classified according to the number of shell and tube passes involved. Heat exchangers in which all the tubes make one U-turn in the shell, for example, are called *one-shell-pass and two-tube-passes* heat exchangers. Likewise, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a *two-shell-passes and four-tube-passes* heat exchanger (Fig. 22–6).

An innovative type of heat exchanger that has found widespread use is the **plate-and-frame** (or just plate) heat exchanger, which consists of a series of plates with corrugated flat flow passages (Fig. 22–7). The hot and cold fluids flow in alternate passages, and thus each cold fluid stream is surrounded by two hot fluid streams, resulting in very effective heat transfer. Also, plate heat exchangers can grow with increasing demand for heat transfer by simply mounting more plates. They are well suited for liquid-to-liquid heat exchange applications, provided that the hot and cold fluid streams are at about the same pressure.

Another type of heat exchanger that involves the alternate passage of the hot and cold fluid streams through the same flow area is the **regenerative** heat exchanger.

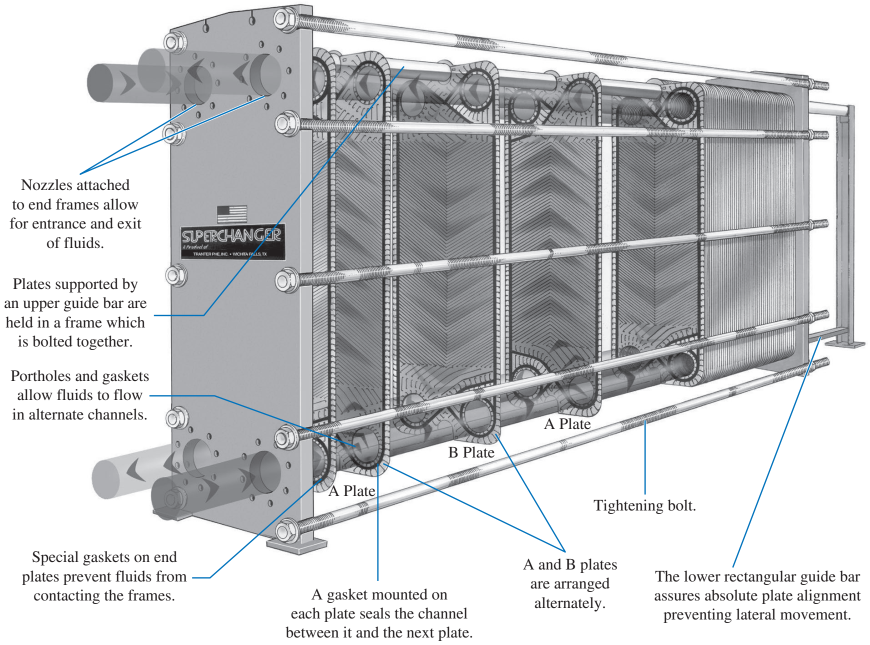


(*a*) One shell pass and two tube passes



(*b*) Two shell passes and four tube passes

FIGURE 22–6
Multipass flow arrangements in shell-and-tube heat exchangers.



Nozzles attached to end frames allow for entrance and exit of fluids.

Plates supported by an upper guide bar are held in a frame which is bolted together.

Portholes and gaskets allow fluids to flow in alternate channels.

Special gaskets on end plates prevent fluids from contacting the frames.

A gasket mounted on each plate seals the channel between it and the next plate.

A and B plates are arranged alternately.

The lower rectangular guide bar assures absolute plate alignment preventing lateral movement.

FIGURE 22–7
A plate-and-frame liquid-to-liquid heat exchanger.

The *static*-type regenerative heat exchanger is basically a porous mass that has a large heat storage capacity, such as a ceramic wire mesh. Hot and cold fluids flow through this porous mass alternately. Heat is transferred from the hot fluid to the matrix of the regenerator during the flow of the hot fluid, and from the matrix to the cold fluid during the flow of the cold fluid. Thus, the matrix serves as a temporary heat storage medium.

The *dynamic*-type regenerator involves a rotating drum and continuous flow of the hot and cold fluid through different portions of the drum so that any portion of the drum passes periodically through the hot stream, storing heat, and then through the cold stream, rejecting this stored heat. Again the drum serves as the medium to transport the heat from the hot to the cold fluid stream.

Heat exchangers are often given specific names to reflect the specific application for which they are used. For example, a *condenser* is a heat exchanger in which one of the fluids is cooled and condenses as it flows through the heat exchanger. A *boiler* is another heat exchanger in which one of the fluids absorbs heat and vaporizes. A *space radiator* is a heat exchanger that transfers heat from the hot fluid to the surrounding space by radiation.

22–2 ▪ THE OVERALL HEAT TRANSFER COEFFICIENT

A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by *convection,* through the wall by *conduction,* and from the wall to the cold fluid again by *convection.* Any radiation effects are usually included in the convection heat transfer coefficients.

The thermal resistance network associated with this heat transfer process involves two convection resistances and one conduction resistance, as shown in Fig. 22–8. The analysis is very similar to the material presented in Chap. 17. Here the subscripts *i* and *o* represent the inner and outer surfaces of the inner tube. For a double-pipe heat exchanger, the *thermal resistance* of the tube wall is

$$R_{\text{wall}} = \frac{\ln(D_o/D_i)}{2\pi kL} \tag{22–1}$$

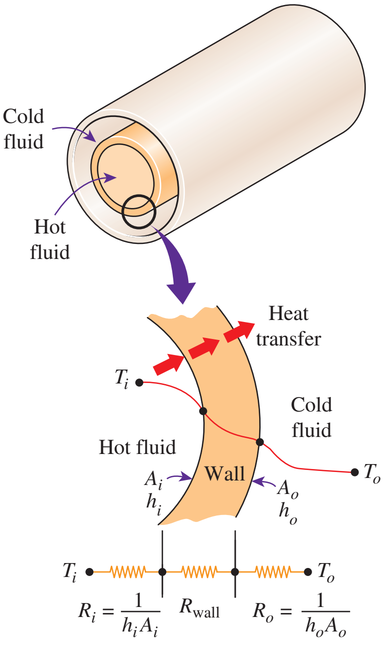


FIGURE 22–8
Thermal resistance network associated with heat transfer in a double-pipe heat exchanger.

where k is the thermal conductivity of the wall material and L is the length of the tube. Then the *total thermal resistance* becomes

$$R = R_{\text{total}} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_iA_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{1}{h_oA_o} \tag{22–2}$$

Here A_i is the area of the inner surface of the wall that separates the two fluids, and A_o is the area of the outer surface of the wall. In other words, A_i and A_o are surface areas of the separating wall wetted by the inner and the outer fluids, respectively. When one fluid flows inside a circular tube and the other outside of it, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$ (Fig. 22–9).

In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold one into a single resistance R, and to express the rate of heat transfer between the two fluids as

$$\dot{Q} = \frac{\Delta T}{R} = UA_s\Delta T = U_iA_i\Delta T = U_oA_o\Delta T \tag{22–3}$$

where A_s is the surface area and U is the **overall heat transfer coefficient**, whose unit is W/m²·K, which is identical to the unit of the ordinary convection coefficient h. Canceling ΔT, Eq. 22–3 reduces to

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + R_{\text{wall}} + \frac{1}{h_oA_o} \tag{22–4}$$

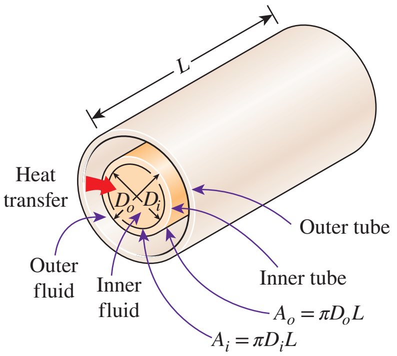


FIGURE 22–9
The two heat transfer surface areas associated with a double-pipe heat exchanger (for thin tubes, $D_i \approx D_o$ and thus $A_i \approx A_o$).

Perhaps you are wondering why we have two overall heat transfer coefficients U_i and U_o for a heat exchanger. The reason is that every heat exchanger has two heat transfer surface areas A_i and A_o, which, in general, are not equal to each other.

Note that $U_iA_i = U_oA_o$, but $U_i \neq U_o$ unless $A_i = A_o$. Therefore, the overall heat transfer coefficient U of a heat exchanger is meaningless unless the area on which it is based is specified. This is especially the case when one side of the tube wall is finned and the other side is not, since the surface area of the finned side is several times that of the unfinned side.

When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, as is usually the case, the thermal resistance of the tube is negligible ($R_{\text{wall}} \approx 0$) and the inner and outer surfaces of the tube are almost identical ($A_i \approx A_o \approx A_s$). Then Eq. 22–4 for the overall heat transfer coefficient simplifies to

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o} \tag{22–5}$$

where $U \approx U_i \approx U_o$. The individual convection heat transfer coefficients inside and outside the tube, h_i and h_o, are determined using the convection relations discussed in Chap. 19 for internal and external forced convection flows.

The overall heat transfer coefficient U in Eq. 22–5 is dominated by the *smaller* convection coefficient, since the inverse of a large number is small. When one of the convection coefficients is *much smaller* than the other (say, $h_i \ll h_o$), we have $1/h_i \gg 1/h_o$, and thus $U \approx h_i$. Therefore, the smaller heat transfer coefficient creates a *bottleneck* on the path of heat transfer and seriously impedes heat transfer. This situation arises frequently when one of the fluids is a gas and the other is a liquid. In such cases, fins are commonly used on the gas side to enhance the product UA and thus the heat transfer on that side.

Representative values of the overall heat transfer coefficient U are given in Table 22–1. Note that the overall heat transfer coefficient ranges from about

TABLE 22–1

Representative values of the overall heat transfer coefficients in heat exchangers

Type of heat exchanger	U, W/m^2·K*
Water-to-water	850–1700
Water-to-oil	100–350
Water-to-gasoline or kerosene	300–1000
Water-to-brine	600–1200
Feedwater heaters	1000–8500
Steam-to-light fuel oil	200–400
Steam-to-heavy fuel oil	50–200
Steam condenser	1000–6000
Freon condenser (water cooled)	300–1000
Ammonia condenser (water cooled)	800–1400
Alcohol condensers (water cooled)	250–700
Gas-to-gas	10–40
Gas-to-brine	10–250
Oil-to-oil	50–400
Organic vapors-to-water	700–1000
Organic solvents-to-organic solvents	100–300
Water-to-air in finned tubes (water in tubes)	30–60† 400–850‡
Steam-to-air in finned tubes (steam in tubes)	30–300† 400–4000‡

*Multiply the listed values by 0.176 to convert them to Btu/h·ft^2·°F.

†Based on air-side surface area.

‡Based on water- or steam-side surface area.

10 W/m^2·K for gas-to-gas heat exchangers to about 10,000 W/m^2·K for heat exchangers that involve phase changes. This is not surprising, since gases have very low thermal conductivities, and phase-change processes involve very high heat transfer coefficients.

When the tube is *finned* on one side to enhance heat transfer, the total heat transfer surface area on the finned side becomes

$$A_s = A_{\text{total}} = A_{\text{fin}} + A_{\text{unfinned}} \tag{22–6}$$

where A_{fin} is the surface area of the fins and A_{unfinned} is the area of the unfinned portion of the tube surface. For short fins of high thermal conductivity, we can use this total area in the convection resistance relation $R_{\text{conv}} = 1/hA_s$ since the fins in this case will be very nearly isothermal. Otherwise, we should determine the effective surface area from

$$A_s = A_{\text{unfinned}} + \eta_{\text{fin}} A_{\text{fin}} \tag{22–7}$$

where η_{fin} is the fin efficiency, which can be obtained from the relationships discussed in Chap. 17. This way, the temperature drop along the fins is accounted for. Note that $\eta_{\text{fin}} = 1$ for isothermal fins, and thus Eq. 22–7 reduces to Eq. 22–6 in that case. Note that for the finned surface, the relevant area (A_i or A_o) in Eq. 22–4 should be calculated from Eq. 22–7.

Fouling Factor

The performance of heat exchangers usually deteriorates with time as a result of accumulation of *deposits* on heat transfer surfaces. The layer of deposits represents *additional resistance* to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease. The net effect of these accumulations on heat transfer is represented by a **fouling factor** R_f, which is a measure of the *thermal resistance* introduced by fouling.

The most common type of fouling is the *precipitation* of solid deposits in a fluid on the heat transfer surfaces. You can observe this type of fouling even in your house. If you check the inner surfaces of your teapot after prolonged use, you will probably notice a layer of calcium-based deposits on the surfaces at which boiling occurs. This is especially the case in areas where the water is hard. The scales of such deposits come off by scratching, and the surfaces can be cleaned of such deposits by chemical treatment. Now imagine those mineral deposits forming on the inner surfaces of fine tubes in a heat exchanger (Fig. 22–10) and the detrimental effect it may have on the flow passage area and the heat transfer. To avoid this potential problem, water in power and process plants is extensively treated, and its solid contents are removed before it is allowed to circulate through the system. The solid ash particles in the flue gases accumulating on the surfaces of air preheaters create similar problems.

Another form of fouling, which is common in the chemical process industry, is *corrosion* and other *chemical fouling*. In this case, the surfaces are fouled by the accumulation of the products of chemical reactions on the surfaces. This form of fouling can be avoided by coating metal pipes with glass or using plastic pipes instead of metal ones. Heat exchangers may also be fouled by the growth of algae in warm fluids. This type of fouling is called *biological fouling* and can be prevented by chemical treatment.

In applications where it is likely to occur, fouling should be considered in the design and selection of heat exchangers. In such applications, it may be necessary to select a larger and thus more expensive heat exchanger to ensure that it meets the design heat transfer requirements even after fouling occurs. The periodic

FIGURE 22–10
Precipitation fouling of ash particles on superheater tubes.
Babcock & Wilcox Enterprises. Reprinted by permission.

TABLE 22–2

Representative fouling factors (thermal resistance due to fouling for a unit surface area)

Fluid	R_f, m^2·K/W
Distilled water, seawater, river water, boiler feedwater:	
Below 50°C	0.0001
Above 50°C	0.0002
Fuel oil	0.0009
Transformer, lubricating or hydraulic oil	0.0002
Quenching oil	0.0007
Vegetable oil	0.0005
Steam (oil-free)	0.0001
Steam (with oil traces)	0.0002
Organic solvent vapors, natural gas	0.0002
Engine exhaust and fuel gases	0.0018
Refrigerants (liquid)	0.0002
Refrigerants (vapor)	0.0004
Ethylene and methylene glycol (antifreeze) and amine solutions	0.00035
Alcohol vapors	0.0001
Air	0.0004

Source: Tubular Exchange Manufacturers Association.

cleaning of heat exchangers and the resulting downtime are additional penalties associated with fouling.

The fouling factor is obviously zero for a new heat exchanger and increases with time as the solid deposits build up on the heat exchanger surface. The fouling factor depends on the *operating temperature* and the *velocity* of the fluids, as well as the length of service. Fouling increases with *increasing temperature* and *decreasing velocity.*

The overall heat transfer coefficient relation Eq. 22–4 or 22–5 for an unfinned double-pipe heat exchanger is valid for clean surfaces and needs to be modified to account for the effects of fouling on both the inner and the outer surfaces of the tube. For an unfinned double-pipe heat exchanger, it can be expressed as

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o} \tag{22–8}$$

where $R_{f,i}$ and $R_{f,o}$ are the fouling factors at those surfaces.

Representative values of fouling factors are given in Table 22–2. More comprehensive tables of fouling factors are available in handbooks. As you would expect, considerable uncertainty exists in these values, and they should be used as a guide in the selection and evaluation of heat exchangers to account for the effects of anticipated fouling on heat transfer. Note that most fouling factors in the table are of the order of 10^{-4} m^2·K/W, which is equivalent to the thermal resistance of a 0.2-mm-thick limestone layer ($k = 2.9$ W/m·K) per unit surface area. Therefore, in the absence of specific data, we can assume the surfaces to be coated with 0.2 mm of limestone as a starting point to account for the effects of fouling.

EXAMPLE 22–1 Overall Heat Transfer Coefficient of a Heat Exchanger

Hot oil is to be cooled in a double-tube counterflow heat exchanger. The copper inner tubes have a diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (the shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg/s, and the oil flows through the shell at a rate of 0.8 kg/s. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this heat exchanger.

SOLUTION Hot oil is cooled by water in a double-tube counterflow heat exchanger. The overall heat transfer coefficient is to be determined.

Assumptions **1** The thermal resistance of the inner tube is negligible since the tube material is highly conductive and its thickness is negligible. **2** Both the oil and water flow are fully developed. **3** Properties of the oil and water are constant.

Properties The properties of water at 45°C are (Table A–15)

$$\rho = 990.1 \text{ kg/m}^3 \qquad \Pr = 3.91$$
$$k = 0.637 \text{ W/m·K} \qquad \nu = \mu/\rho = 0.602 \times 10^{-6} \text{ m}^2\text{/s}$$

The properties of oil at 80°C are (Table A–19)

$$\rho = 852 \text{ kg/m}^3 \qquad \Pr = 499.3$$
$$k = 0.138 \text{ W/m·K} \qquad \nu = 3.794 \times 10^{-5} \text{ m}^2\text{/s}$$

Analysis The schematic of the heat exchanger is given in Fig. 22–11. The overall heat transfer coefficient U can be determined from Eq. 22–5:

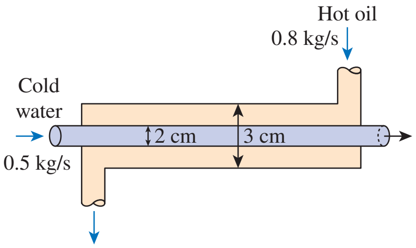


FIGURE 22–11
Schematic for Example 22–1.

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where h_i and h_o are the convection heat transfer coefficients inside and outside the tube, respectively, which are to be determined using the forced convection relations.

The hydraulic diameter for a circular tube is the diameter of the tube itself, $D_h = D = 0.02$ m. The average velocity of water in the tube and the Reynolds number are

$$V = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\frac{1}{4}\pi D^2)} = \frac{0.5 \text{ kg/s}}{(990.1 \text{ kg/m}^3)[\frac{1}{4}\pi(0.02 \text{ m})^2]} = 1.61 \text{ m/s}$$

and

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.61 \text{ m/s})(0.02 \text{ m})}{0.602 \times 10^{-6} \text{ m}^2\text{/s}} = 53{,}490$$

which is greater than 10,000. Therefore, the flow of water is turbulent. Assuming the flow to be fully developed, the Nusselt number can be determined from

$$\text{Nu} = \frac{hD}{k} = 0.023 \text{ Re}^{0.8}\text{Pr}^{0.4} = 0.023(53{,}490)^{0.8}(3.91)^{0.4} = 240.6$$

Then,

$$h = \frac{k}{D}\text{Nu} = \frac{0.637 \text{ W/m·K}}{0.02 \text{ m}}(240.6) = 7663 \text{ W/m}^2\text{·K}$$

Now we repeat the analysis for oil.

The hydraulic diameter for the annular space is

$$D_h = D_o - D_i = 0.03 - 0.02 = 0.01 \text{ m}$$

The average velocity and the Reynolds number in this case are

$$V = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho[\frac{1}{4}\pi(D_o^2 - D_i^2)]} = \frac{0.8 \text{ kg/s}}{(852 \text{ kg/m}^3)[\frac{1}{4}\pi(0.03^2 - 0.02^2)] \text{ m}^2} = 2.39 \text{ m/s}$$

and

$$\text{Re} = \frac{VD_h}{\nu} = \frac{(2.39 \text{ m/s})(0.01 \text{ m})}{3.794 \times 10^{-5} \text{ m}^2\text{/s}} = 630$$

which is less than 2300. Therefore, the flow of oil is laminar. Assuming fully developed flow, the Nusselt number on the tube side of the annular space Nu_i corresponding to $D_i/D_o = 0.02/0.03 = 0.667$ can be determined from Table 22–3 by interpolation to be

$$\text{Nu} = 5.45$$

and

$$h_o = \frac{k}{D_h}\text{Nu} = \frac{0.138 \text{ W/m·K}}{0.01 \text{ m}}(5.45) = 75.2 \text{ W/m}^2\text{·K}$$

Then the overall heat transfer coefficient for this heat exchanger becomes

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{7663 \text{ W/m}^2\text{·K}} + \frac{1}{75.2 \text{ W/m}^2\text{·K}}} = \mathbf{74.5 \text{ W/m}^2\text{·K}}$$

TABLE 22–3

Nusselt number for fully developed laminar flow in a circular annulus with one surface insulated and the other isothermal

D_i/D_o	Nu_i	Nu_o
0.00	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

Kays and Perkins, 1972

Discussion Note that $U \approx h_o$ in this case, since $h_i \gg h_o$. This confirms our earlier statement that the overall heat transfer coefficient in a heat exchanger is dominated by the smaller heat transfer coefficient when the difference between the two values is large.

To improve the overall heat transfer coefficient and thus the heat transfer in this heat exchanger, we must use some enhancement techniques on the oil side, such as a finned surface.

EXAMPLE 22–2 Effect of Fouling on the Overall Heat Transfer Coefficient

A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel ($k = 15.1$ W/m·K) inner tube of inner diameter $D_i = 1.5$ cm and outer diameter $D_o = 1.9$ cm and an outer shell of inner diameter 3.2 cm. The convection heat transfer coefficient is given to be $h_i = 800$ W/m^2·K on the inner surface of the tube and $h_o = 1200$ W/m^2·K on the outer surface. For a fouling factor of $R_{f,i} = 0.0004$ m^2·K/W on the tube side and $R_{f,o} = 0.0001$ m^2·K/W on the shell side, determine (*a*) the thermal resistance of the heat exchanger per unit length and (*b*) the overall heat transfer coefficients U_i and U_o based on the inner and outer surface areas of the tube, respectively.

SOLUTION The heat transfer coefficients and the fouling factors on the tube and shell sides of a heat exchanger are given. The thermal resistance and the overall heat transfer coefficients based on the inner and outer areas are to be determined.

Assumptions The heat transfer coefficients and the fouling factors are constant and uniform.

Analysis (*a*) The schematic of the heat exchanger is given in Fig. 22–12. The thermal resistance for an unfinned shell-and-tube heat exchanger with fouling on both heat transfer surfaces is given by Eq. 22–8 as

$$R = \frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

where

$$A_i = \pi D_iL = \pi(0.015 \text{ m})(1 \text{ m}) = 0.0471 \text{ m}^2$$
$$A_o = \pi D_oL = \pi(0.019 \text{ m})(1 \text{ m}) = 0.0597 \text{ m}^2$$

Substituting, the total thermal resistance is determined to be

$$\begin{aligned} R &= \frac{1}{(800 \text{ W/m}^2\text{·K})(0.0471 \text{ m}^2)} + \frac{0.0004 \text{ m}^2\text{·K/W}}{0.0471 \text{ m}^2} \\ &\quad + \frac{\ln(0.019/0.015)}{2\pi(15.1 \text{ W/m·K})(1 \text{ m})} \\ &\quad + \frac{0.0001 \text{ m}^2\text{·K/W}}{0.0597 \text{ m}^2} + \frac{1}{(1200 \text{ W/m}^2\text{·K})(0.0597 \text{ m}^2)} \\ &= (0.02654 + 0.00849 + 0.0025 + 0.00168 + 0.01396)\text{K/W} \\ &= \mathbf{0.0532°C/W} \end{aligned}$$

Note that about 19 percent of the total thermal resistance in this case is due to fouling and about 5 percent of it is due to the steel tube separating the two fluids. The rest (76 percent) is due to the convection resistances.

(*b*) Knowing the total thermal resistance and the heat transfer surface areas, the overall heat transfer coefficients based on the inner and outer surfaces of the tube are

$$U_i = \frac{1}{RA_i} = \frac{1}{(0.0532 \text{ K/W})(0.0471 \text{ m}^2)} = \mathbf{399 \text{ W/m}^2\text{·K}}$$

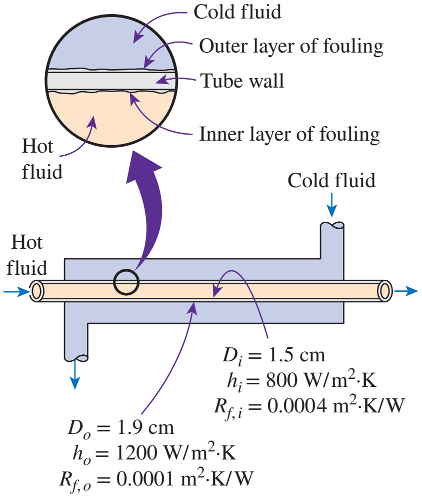


FIGURE 22–12
Schematic for Example 22–2.

and

$$U_o = \frac{1}{RA_o} = \frac{1}{(0.0532\ \text{K/W})(0.0597\ \text{m}^2)} = \mathbf{315\ W/m^2{\cdot}K}$$

Discussion Note that the two overall heat transfer coefficients differ significantly (by 27 percent) in this case because of the considerable difference between the heat transfer surface areas on the inner and the outer sides of the tube. For tubes of negligible thickness, the difference between the two overall heat transfer coefficients would be negligible.

22–3 ▪ ANALYSIS OF HEAT EXCHANGERS

Heat exchangers are commonly used in practice, and an engineer often finds himself or herself in a position to *select a heat exchanger* that will achieve a *specified temperature change* in a fluid stream of known mass flow rate, or to *predict the outlet temperatures* of the hot and cold fluid streams in a *specified heat exchanger.*

In upcoming sections, we discuss the two methods used in the analysis of heat exchangers. Of these, the *log mean temperature difference* (or LMTD) method is best suited for the first task and the *effectiveness*–NTU (the number of transfer units) method for the second task. But first we present some general considerations.

Heat exchangers usually operate for long periods of time with no change in their operating conditions. Therefore, they can be modeled as *steady-flow* devices. As such, the mass flow rate of each fluid remains constant, and the fluid properties such as temperature and velocity at any inlet or outlet remain the same. Also, the fluid streams experience little or no change in their velocities and elevations, and thus the kinetic and potential energy changes are negligible. The specific heat of a fluid, in general, changes with temperature. But in a specified temperature range, it can be treated as a constant at some average value with little loss in accuracy. Axial heat conduction along the tube is usually insignificant and can be considered negligible. Finally, the outer surface of the heat exchanger is assumed to be *perfectly insulated,* so that there is no heat loss to the surrounding medium, and any heat transfer occurs between the two fluids only.

The idealizations stated above are closely approximated in practice, and they greatly simplify the analysis of a heat exchanger with little sacrifice in accuracy. Therefore, they are commonly used. Under these assumptions, the *first law of thermodynamics* requires that the rate of heat transfer from the hot fluid be equal to the rate of heat transfer to the cold one. That is,

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,\,\text{out}} - T_{c,\,\text{in}}) \tag{22–9}$$

and

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,\,\text{in}} - T_{h,\,\text{out}}) \tag{22–10}$$

where the subscripts *c* and *h* stand for *cold* and *hot* fluids, respectively, and

$\dot{m}_c, \dot{m}_h$ = mass flow rates
c_{pc}, c_{ph} = specific heats
$T_{c,\,\text{out}}, T_{h,\,\text{out}}$ = outlet temperatures
$T_{c,\,\text{in}}, T_{h,\,\text{in}}$ = inlet temperatures

Note that the heat transfer rate $\dot{Q}$ is taken to be a positive quantity, and its direction is understood to be from the hot fluid to the cold one in accordance with the second law of thermodynamics.

In heat exchanger analysis, it is often convenient to combine the product of the mass flow rate and the specific heat of a fluid into a single quantity. This quantity is called the **heat capacity rate** and is defined for the hot and cold fluid streams as

$$C_h = \dot{m}_h c_{ph} \qquad \text{and} \qquad C_c = \dot{m}_c c_{pc} \tag{22–11}$$

The heat capacity rate of a fluid stream represents the rate of heat transfer needed to change the temperature of the fluid stream by 1°C as it flows through a heat exchanger. Note that in a heat exchanger, the fluid with a *large* heat capacity rate experiences a *small* temperature change, and the fluid with a *small* heat capacity rate experiences a *large* temperature change. Therefore, *doubling* the mass flow rate of a fluid while leaving everything else unchanged will *halve* the temperature change of that fluid.

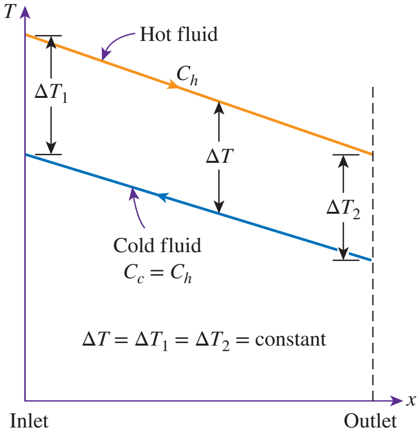


FIGURE 22–13
Two fluid streams that have the same capacity rates experience the same temperature change in a well-insulated heat exchanger.

Given the preceding definition of the heat capacity rate, Eqs. 22–9 and 22–10 can also be expressed as

$$\dot{Q} = C_c(T_{c,\,\text{out}} - T_{c,\,\text{in}}) \tag{22–12}$$

and

$$\dot{Q} = C_h(T_{h,\,\text{in}} - T_{h,\,\text{out}}) \tag{22–13}$$

That is, the heat transfer rate in a heat exchanger is equal to the heat capacity rate of either fluid multiplied by the temperature change of that fluid. Note that *the only time the temperature rise of a cold fluid is equal to the temperature drop of the hot fluid is when the heat capacity rates of the two fluids are equal to each other* (Fig. 22–13).

Two special types of heat exchangers commonly used in practice are *condensers* and *boilers*. One of the fluids in a condenser or a boiler undergoes a phase-change process, and the rate of heat transfer is expressed as

$$\dot{Q} = \dot{m}h_{fg} \tag{22–14}$$

where $\dot{m}$ is the rate of evaporation or condensation of the fluid and h_{fg} is the enthalpy of vaporization of the fluid at the specified temperature or pressure.

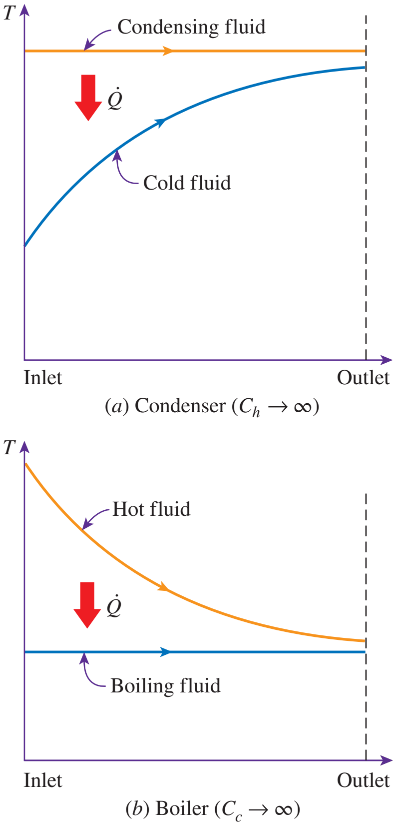
An ordinary fluid absorbs or releases a large amount of heat essentially at constant temperature during a phase-change process, as shown in Fig. 22–14. The heat capacity rate of a fluid during a phase-change process must approach infinity since the temperature change is practically zero. That is, $C = \dot{m}c_p \to \infty$ when $\Delta T \to 0$, so that the heat transfer rate $\dot{Q} = \dot{m}c_p\,\Delta T$ is a finite quantity. Therefore, in heat exchanger analysis, a condensing or boiling fluid is conveniently modeled as a fluid whose heat capacity rate is *infinity*.

The rate of heat transfer in a heat exchanger can also be expressed in an analogous manner to Newton's law of cooling as

$$\dot{Q} = UA_s\,\Delta T_m \tag{22–15}$$

where U is the overall heat transfer coefficient, A_s is the heat transfer surface area, and ΔT_m is an appropriate mean temperature difference between the two fluids. Here the surface area A_s can be determined precisely using the dimensions of the heat exchanger. However, the overall heat transfer coefficient U and the temperature difference ΔT between the hot and cold fluids, in general, may vary along the heat exchanger.

The average value of the overall heat transfer coefficient can be determined as described in Sec. 22–2 by using the average convection coefficients for each fluid. It turns out that the appropriate form of the average temperature difference between the two fluids is *logarithmic* in nature, and its determination is presented in Sec. 22–4. It should be noted that the mean temperature difference ΔT_m is dependent on the heat exchanger flow arrangement and its type of construction.



(*a*) Condenser ($C_h \to \infty$)

(*b*) Boiler ($C_c \to \infty$)

FIGURE 22–14
Variation of fluid temperatures in a heat exchanger when one of the fluids condenses or boils.

22–4 ▪ THE LOG MEAN TEMPERATURE DIFFERENCE METHOD

Earlier, we mentioned that the temperature difference between the hot and cold fluids varies along the heat exchanger, and it is convenient to have a *mean temperature difference* ΔT_m for use in the relation $\dot{Q} = UA_s \Delta T_m$.

In order to develop a relation for the equivalent average temperature difference between the two fluids, consider the *parallel-flow double-pipe* heat exchanger shown in Fig. 22–15. Note that the temperature difference ΔT between the hot and cold fluids is large at the inlet of the heat exchanger but decreases exponentially toward the outlet. As you would expect, the temperature of the hot fluid decreases and the temperature of the cold fluid increases along the heat exchanger, but the temperature of the cold fluid can never exceed that of the hot fluid no matter how long the heat exchanger is.

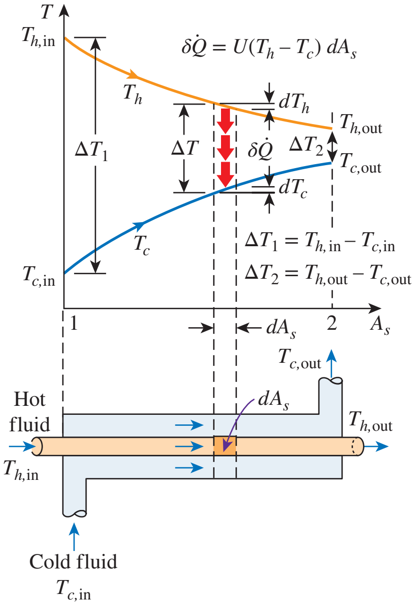


FIGURE 22–15
Variation of the fluid temperatures in a parallel-flow double-pipe heat exchanger.

Assuming the outer surface of the heat exchanger to be well insulated so that any heat transfer occurs between the two fluids, and disregarding any changes in kinetic and potential energy, an energy balance on each fluid in a differential section of the heat exchanger can be expressed as

$$\delta\dot{Q} = -\dot{m}_h c_{ph}\, dT_h \tag{22–16}$$

and

$$\delta\dot{Q} = \dot{m}_c c_{pc}\, dT_c \tag{22–17}$$

That is, the rate of heat loss from the hot fluid at any section of a heat exchanger is equal to the rate of heat gain by the cold fluid in that section. The temperature change of the hot fluid is a *negative* quantity, and so a *negative sign* is added to Eq. 22–16 to make the heat transfer rate $\dot{Q}$ a positive quantity. Solving the equations above for dT_h and dT_c gives

$$dT_h = -\frac{\delta\dot{Q}}{\dot{m}_h c_{ph}} \tag{22–18}$$

and

$$dT_c = \frac{\delta\dot{Q}}{\dot{m}_c c_{pc}} \tag{22–19}$$

Taking their difference, we get

$$dT_h - dT_c = d(T_h - T_c) = -\delta\dot{Q}\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right) \tag{22–20}$$

The rate of heat transfer in the differential section of the heat exchanger can also be expressed as

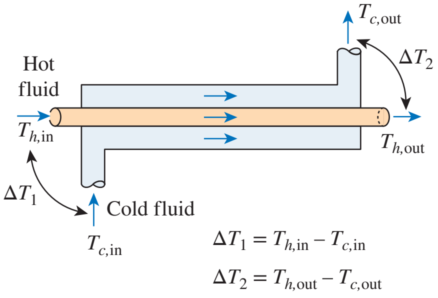
$$\delta\dot{Q} = U(T_h - T_c)\, dA_s \tag{22–21}$$

Substituting this equation into Eq. 22–20 and rearranging give

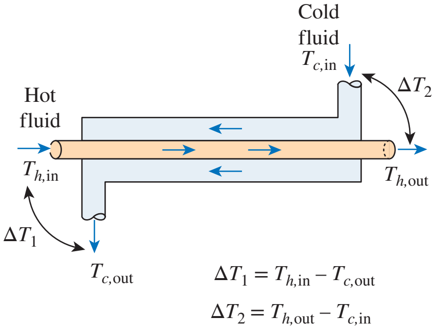
$$\frac{d(T_h - T_c)}{T_h - T_c} = -U\, dA_s\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right) \tag{22–22}$$

Integrating from the inlet of the heat exchanger to its outlet, we obtain

$$\ln\frac{T_{h,\text{out}} - T_{c,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}} = -UA_s\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{cp}}\right) \tag{22–23}$$



(*a*) Parallel-flow heat exchangers



(*b*) Counterflow heat exchangers

FIGURE 22–16

The ΔT_1 and ΔT_2 expressions in parallel-flow and counterflow heat exchangers.

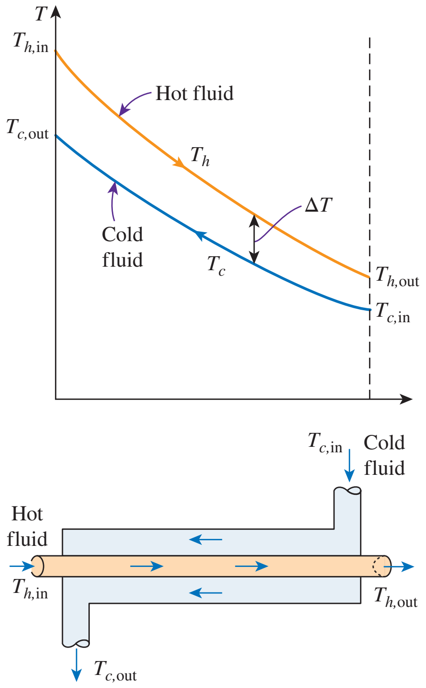


FIGURE 22–17

The variation of the fluid temperatures in a counterflow double-pipe heat exchanger.

Finally, solving Eqs. 22–9 and 22–10 for $\dot{m}_c c_{pc}$ and $\dot{m}_h c_{ph}$ and substituting into Eq. 22–23 give, after some rearrangement,

$$\dot{Q} = UA_s \Delta T_{\text{lm}} \tag{22–24}$$

where

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \tag{22–25}$$

is the **log mean temperature difference**, which is the suitable form of the average temperature difference for use in the analysis of heat exchangers. Here ΔT_1 and ΔT_2 represent the temperature difference between the two fluids at the two ends (inlet and outlet) of the heat exchanger. It makes no difference which end of the heat exchanger is designated as the inlet or the outlet (Fig. 22–16). It should be noted that Eqs. 22–24 and 22–25 are good for any heat exchanger provided that the end point temperature differences are defined properly. For multipass and crossflow heat exchangers, as will be shown later, the log mean temperature difference should be corrected through a correction factor.

The temperature difference between the two fluids decreases from ΔT_1 at the inlet to ΔT_2 at the outlet. Thus, it is tempting to use the arithmetic mean temperature $\Delta T_{\text{am}} = \frac{1}{2}(\Delta T_1 + \Delta T_2)$ as the average temperature difference. The logarithmic mean temperature difference ΔT_{lm} is obtained by tracing the actual temperature profile of the fluids along the heat exchanger and is an *exact* representation of the *average temperature difference* between the hot and cold fluids. It truly reflects the exponential decay of the local temperature difference.

Note that ΔT_{lm} is always less than ΔT_{am}. Therefore, using ΔT_{am} in calculations instead of ΔT_{lm} will overestimate the rate of heat transfer in a heat exchanger between the two fluids. When ΔT_1 differs from ΔT_2 by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent. But the error increases to undesirable levels when ΔT_1 differs from ΔT_2 by greater amounts. Therefore, we should always use the *logarithmic mean temperature difference* when determining the rate of heat transfer in a heat exchanger.

Counterflow Heat Exchangers

The variation of temperatures of hot and cold fluids in a counterflow heat exchanger is given in Fig. 22–17. Note that the hot and cold fluids enter the heat exchanger from opposite ends, and the outlet temperature of the *cold fluid* in this case may exceed the outlet temperature of the *hot fluid.* In the limiting case, the cold fluid will be heated to the inlet temperature of the hot fluid. However, the outlet temperature of the cold fluid can *never* exceed the inlet temperature of the hot fluid, since this would be a violation of the second law of thermodynamics.

The relation already given for the log mean temperature difference is developed using a parallel-flow heat exchanger, but it can shown that the analysis is also applicable to counterflow heat exchangers. But this time, ΔT_1 and ΔT_2 are expressed as shown in Fig. 22–16.

For specified inlet and outlet temperatures, the log mean temperature difference for a counterflow heat exchanger is always greater than that for a parallel-flow heat exchanger. That is, $\Delta T_{\text{lm, CF}} > \Delta T_{\text{lm, PF}}$, and thus a smaller surface area (and thus a smaller heat exchanger) is needed to achieve a specified heat transfer rate in a counterflow heat exchanger, assuming the same value of the overall heat transfer coefficient. Therefore, it is common practice to use counterflow arrangements in heat exchangers.

In a counterflow heat exchanger, the temperature difference between the hot and the cold fluids remains constant along the heat exchanger when the *heat capacity rates* of the two fluids are *equal* (that is, $\Delta T = \text{constant}$ when $C_h = C_c$ or $\dot{m}_h c_{ph} = \dot{m}_c c_{pc}$). Then we have $\Delta T_1 = \Delta T_2$, and the log mean temperature difference relation gives $\Delta T_{\text{lm}} = \frac{0}{0}$, which is indeterminate. It can be shown by the application of l'Hôpital's rule that in this case we have $\Delta T_{\text{lm}} = \Delta T_1 = \Delta T_2$, as expected.

A *condenser* or a *boiler* can be considered to be either a parallel- or counterflow heat exchanger since both approaches give the same result.

Multipass and Crossflow Heat Exchangers: Use of a Correction Factor

The log mean temperature difference ΔT_{lm} relation developed earlier is limited to parallel-flow and counterflow heat exchangers only. Similar relations are also developed for *crossflow* and *multipass shell-and-tube* heat exchangers, but the resulting expressions are too complicated because of the complex flow conditions.

In such cases, it is convenient to relate the equivalent temperature difference to the log mean temperature difference relation for the counterflow case as

$$\Delta T_{\text{lm}} = F\,\Delta T_{\text{lm, CF}} \tag{22–26}$$

where F is the **correction factor**, which depends on the *geometry* of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams. The $\Delta T_{\text{lm, CF}}$ is the log mean temperature difference for the case of a *counterflow* heat exchanger with the same inlet and outlet temperatures and is determined from Eq. 22–25 by taking $\Delta T_1 = T_{h,\text{ in}} - T_{c,\text{ out}}$ and $\Delta T_2 = T_{h,\text{ out}} - T_{c,\text{ in}}$ (Fig. 22–18).

The correction factor is less than unity for a crossflow and multipass shell-and-tube heat exchanger. That is, $F \leq 1$. The limiting value of $F = 1$ corresponds to the counterflow heat exchanger. Thus, the correction factor F for a heat exchanger is *a measure of deviation of the* ΔT_{lm} *from the corresponding values for the counterflow case.*

The correction factor F for common crossflow and shell-and-tube heat exchanger configurations is given in Fig. 22–19 versus two temperature ratios P and R defined as

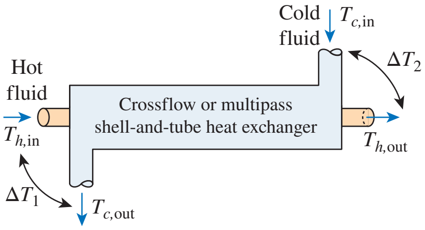
$$P = \frac{t_2 - t_1}{T_1 - t_1} \tag{22–27}$$

and

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}c_p)_{\text{tube side}}}{(\dot{m}c_p)_{\text{shell side}}} \tag{22–28}$$

where the subscripts 1 and 2 represent the *inlet* and *outlet,* respectively. Note that for a shell-and-tube heat exchanger, T and t represent the *shell-* and *tube-side* temperatures, respectively, as shown in the correction factor charts. It makes no difference whether the hot or the cold fluid flows through the shell or the tube. The determination of the correction factor F requires the availability of the inlet and the outlet temperatures for both the cold and hot fluids.

Note that the value of P ranges from 0 to 1. The value of R, on the other hand, ranges from 0 to infinity, with $R = 0$ corresponding to the phase change (condensation or boiling) on the shell side and $R \to \infty$ to phase change on the tube side. The correction factor is $F = 1$ for both of these limiting cases. Therefore, the correction factor for a condenser or boiler is $F = 1$, regardless of the configuration of the heat exchanger.



Heat transfer rate:

$$\dot{Q} = UA_s F\,\Delta T_{\text{lm,CF}}$$

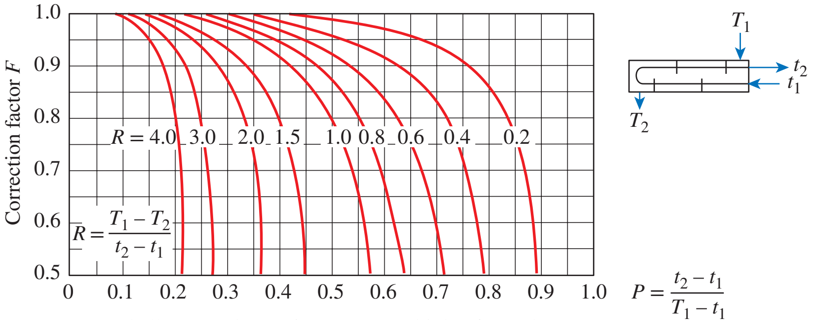
where $\Delta T_{\text{lm,CF}} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$

$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}}$

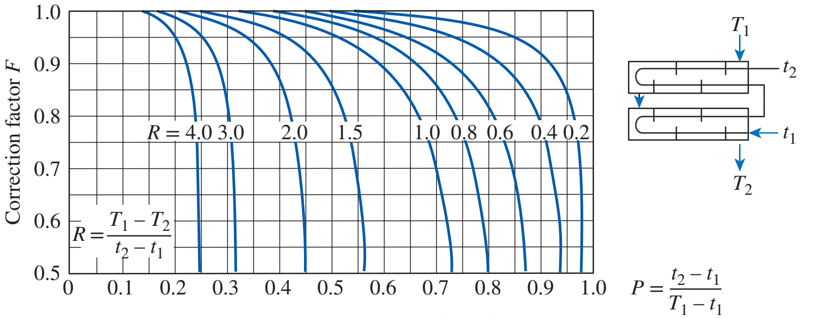
$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}}$

and $F = \ldots$ (Fig. 22–19)

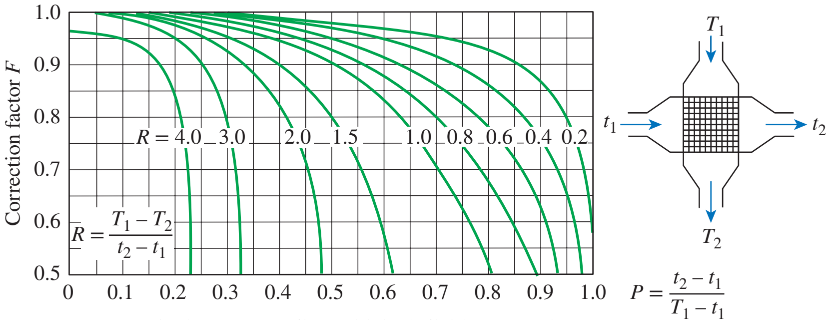
FIGURE 22–18
The determination of the heat transfer rate for crossflow and multipass shell-and-tube heat exchangers using the correction factor.



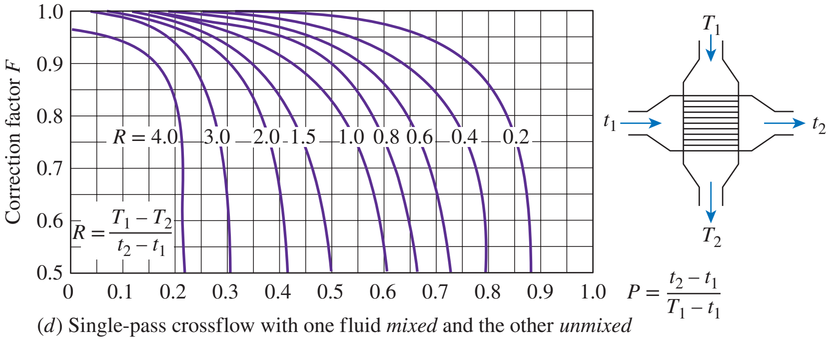
(a) One shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes



(b) Two shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes



(c) Single-pass crossflow with both fluids *unmixed*



(d) Single-pass crossflow with one fluid *mixed* and the other *unmixed*

FIGURE 22–19

Correction factor F charts for common shell-and-tube and crossflow heat exchangers.

Source: Bowman, Mueller, and Nagle, 1940.

EXAMPLE 22–3 The Condensation of Steam in a Condenser

Steam in the condenser of a power plant is to be condensed at a temperature of 30°C with cooling water from a nearby lake, which enters the tubes of the condenser at 14°C and leaves at 22°C. The surface area of the tubes is 45 m^2, and the overall heat transfer coefficient is 2100 $W/m^2 \cdot K$. Determine the mass flow rate of the cooling water needed and the rate of condensation of the steam in the condenser.

SOLUTION Steam is condensed by cooling water in the condenser of a power plant. The mass flow rate of the cooling water and the rate of condensation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The heat of vaporization of water at 30°C is $h_{fg} = 2431$ kJ/kg and the specific heat of cold water at the average temperature of 18°C is $c_p = 4184$ J/kg·K (Table A–15).

Analysis The schematic of the condenser is given in Fig. 22–20. The condenser can be treated as a counterflow heat exchanger since the temperature of one of the fluids (the steam) remains constant.

The temperature difference between the steam and the cooling water at the two ends of the condenser is

$$\Delta T_1 = T_{h,\,\text{in}} - T_{c,\,\text{out}} = (30 - 22)^\circ\text{C} = 8^\circ\text{C}$$
$$\Delta T_2 = T_{h,\,\text{out}} - T_{c,\,\text{in}} = (30 - 14)^\circ\text{C} = 16^\circ\text{C}$$

That is, the temperature difference between the two fluids varies from 8°C at one end to 16°C at the other. The proper average temperature difference between the two fluids is the *log mean temperature difference* (not the arithmetic), which is determined from

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{8 - 16}{\ln(8/16)} = 11.5^\circ\text{C}$$

This is a little less than the arithmetic mean temperature difference of $\frac{1}{2}(8 + 16) = 12^\circ\text{C}$. Then the heat transfer rate in the condenser is determined from

$$\dot{Q} = UA_s\,\Delta T_{\text{lm}} = (2100\ \text{W/m}^2\cdot\text{K})(45\ \text{m}^2)(11.5^\circ\text{C}) = 1.087 \times 10^6\ \text{W} = 1087\ \text{kW}$$

Therefore, steam will lose heat at a rate of 1087 kW as it flows through the condenser, and the cooling water will gain practically all of it, since the condenser is well insulated.

The mass flow rate of the cooling water and the rate of the condensation of the steam are determined from $\dot{Q} = [\dot{m}c_p(T_{\text{out}} - T_{\text{in}})]_{\text{cooling water}} = (\dot{m}h_{fg})_{\text{steam}}$ to be

$$\dot{m}_{\text{cooling water}} = \frac{\dot{Q}}{c_p(T_{\text{out}} - T_{\text{in}})} = \frac{1087\ \text{kJ/s}}{(4.184\ \text{kJ/kg}\cdot\text{K})(22 - 14)^\circ\text{C}} = \mathbf{32.5\ kg/s}$$

and

$$\dot{m}_{\text{steam}} = \frac{\dot{Q}}{h_{fg}} = \frac{1087\ \text{kJ/s}}{2431\ \text{kJ/kg}} = \mathbf{0.45\ kg/s}$$

Therefore, we need to circulate about 72 kg of cooling water for each 1 kg of steam condensing to remove the heat released during the condensation process.

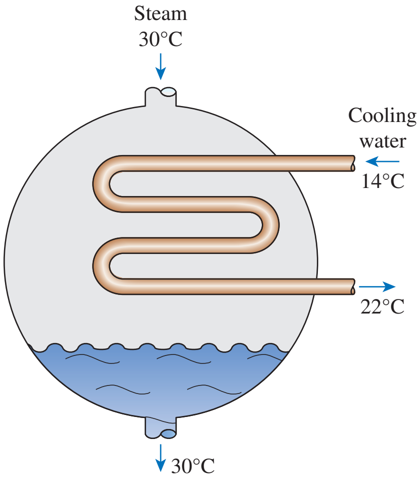


FIGURE 22–20
Schematic for Example 22–3.

EXAMPLE 22–4 Heating Water in a Counterflow Heat Exchanger

A counterflow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m²·K, determine the length of the heat exchanger required to achieve the desired heating.

SOLUTION Water is heated in a counterflow double-pipe heat exchanger by geothermal water. The required length of the heat exchanger is to be determined.

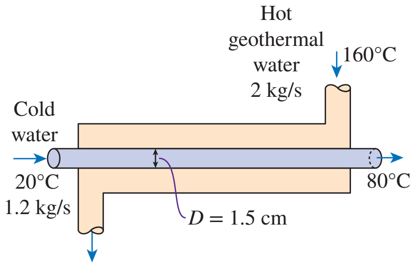


FIGURE 22–21
Schematic for Example 22–4.

Assumptions 1 Steady operating conditions exist. 2 The heat exchanger is well insulated so that heat loss to the surroundings is negligible. 3 Changes in the kinetic and potential energies of fluid streams are negligible. 4 There is no fouling. 5 Fluid properties are constant.

Properties We take the specific heats of water and geothermal fluid to be 4.18 and 4.31 kJ/kg·K, respectively.

Analysis The schematic of the heat exchanger is given in Fig. 22–21. The rate of heat transfer in the heat exchanger can be determined from

$$\dot{Q} = [\dot{m}c_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (1.2\text{ kg/s})(4.18\text{ kJ/kg·K})(80 - 20)°\text{C} = 301\text{ kW}$$

Noting that all of this heat is supplied by the geothermal water, the outlet temperature of the geothermal water is determined to be

$$\dot{Q} = [\dot{m}c_p(T_{\text{in}} - T_{\text{out}})]_{\text{geothermal}}$$

$$T_{\text{out}} = T_{\text{in}} - \frac{\dot{Q}}{\dot{m}c_p}$$

$$= 160°\text{C} - \frac{301\text{ kW}}{(2\text{ kg/s})(4.31\text{ kJ/kg·K})}$$

$$= 125°\text{C}$$

Knowing the inlet and outlet temperatures of both fluids, the logarithmic mean temperature difference for this counterflow heat exchanger becomes

$$\Delta T_1 = T_{h,\text{ in}} - T_{c,\text{ out}} = (160 - 80)°\text{C} = 80°\text{C}$$
$$\Delta T_2 = T_{h,\text{ out}} - T_{c,\text{ in}} = (125 - 20)°\text{C} = 105°\text{C}$$

and

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{80 - 105}{\ln(80/105)} = 91.9°\text{C}$$

Then the surface area of the heat exchanger is determined to be

$$\dot{Q} = UA_s\Delta T_{\text{lm}} \longrightarrow A_s = \frac{\dot{Q}}{U\Delta T_{\text{lm}}} = \frac{301{,}000\text{ W}}{(640\text{ W/m}^2\text{·K})(91.9°\text{C})} = 5.12\text{ m}^2$$

To provide this much heat transfer surface area, the length of the tube must be

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{5.12\text{ m}^2}{\pi(0.015\text{ m})} = \mathbf{109\text{ m}}$$

Discussion The inner tube of this counterflow heat exchanger (and thus the heat exchanger itself) needs to be over 100 m long to achieve the desired heat transfer, which is impractical. In cases like this, we need to use a plate heat exchanger or a multipass shell-and-tube heat exchanger with multiple passes of tube bundles.

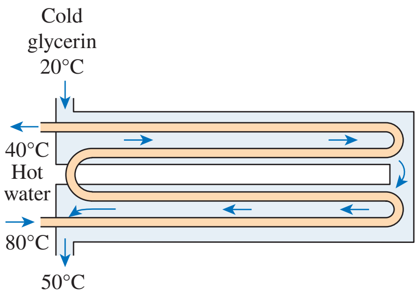


FIGURE 22–22
Schematic for Example 22–5.

EXAMPLE 22–5 Heating of Glycerin in a Multipass Heat Exchanger

A two-shell-passes and four-tube-passes heat exchanger is used to heat glycerin from 20°C to 50°C with hot water, which enters the thin-walled 2-cm-diameter tubes at 80°C and leaves at 40°C (Fig. 22–22). The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 W/m^2·K on the glycerin (shell) side and 160 W/m^2·K on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (*a*) before any fouling and (*b*) after fouling with a fouling factor of 0.0006 m^2·K/W occurs on the outer surfaces of the tubes.

SOLUTION Glycerin is heated in a two-shell-passes and four-tube-passes heat exchanger by hot water. The rate of heat transfer for the cases of fouling and no fouling are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Heat transfer coefficients and fouling factors are constant and uniform. **5** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Analysis The tubes are said to be thin-walled, and thus it is reasonable to assume the inner and outer surface areas of the tubes to be equal. Then the heat transfer surface area becomes

$$A_s = \pi DL = \pi(0.02\text{ m})(60\text{ m}) = 3.77\text{ m}^2$$

The rate of heat transfer in this heat exchanger can be determined from

$$\dot{Q} = UA_sF\,\Delta T_{\text{lm, CF}}$$

where F is the correction factor and $\Delta T_{\text{lm, CF}}$ is the log mean temperature difference for the counterflow arrangement. These two quantities are determined from

$$\begin{aligned}\Delta T_1 &= T_{h,\text{ in}} - T_{c,\text{ out}} = (80 - 50)^\circ\text{C} = 30^\circ\text{C}\\ \Delta T_2 &= T_{h,\text{ out}} - T_{c,\text{ in}} = (40 - 20)^\circ\text{C} = 20^\circ\text{C}\\ \Delta T_{\text{lm, CF}} &= \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{30 - 20}{\ln(30/20)} = 24.7^\circ\text{C}\end{aligned}$$

and

$$\left.\begin{aligned}P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{40 - 80}{20 - 80} = 0.67\\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 50}{40 - 80} = 0.75\end{aligned}\right\} F = 0.91 \qquad \text{(Fig. 22–19}b\text{)}$$

(*a*) In the case of no fouling, the overall heat transfer coefficient U is

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{160\text{ W/m}^2\cdot\text{K}} + \dfrac{1}{25\text{ W/m}^2\cdot\text{K}}} = 21.6\text{ W/m}^2\cdot\text{K}$$

Then the rate of heat transfer becomes

$$\dot{Q} = UA_sF\,\Delta T_{\text{lm, CF}} = (21.6\text{ W/m}^2\cdot\text{K})(3.77\text{ m}^2)(0.91)(24.7^\circ\text{C}) = \mathbf{1830\text{ W}}$$

(*b*) When there is fouling on one of the surfaces, we have

$$\begin{aligned}U &= \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o} + R_f} = \frac{1}{\dfrac{1}{160\text{ W/m}^2\cdot\text{K}} + \dfrac{1}{25\text{ W/m}^2\cdot\text{K}} + 0.0006\text{ m}^2\cdot\text{K/W}}\\ &= 21.3\text{ W/m}^2\cdot\text{K}\end{aligned}$$

and

$$\dot{Q} = UA_sF\,\Delta T_{\text{lm, CF}} = (21.3\text{ W/m}^2\cdot\text{K})(3.77\text{ m}^2)(0.91)(24.7^\circ\text{C}) = \mathbf{1805\text{ W}}$$

Discussion Note that the rate of heat transfer decreases as a result of fouling, as expected. The decrease is not dramatic, however, because of the relatively low convection heat transfer coefficients involved.

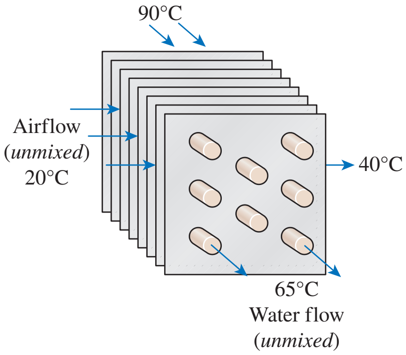


FIGURE 22–23
Schematic for Example 22–6.

EXAMPLE 22–6 Cooling of Water in an Automotive Radiator

A test is conducted to determine the overall heat transfer coefficient in an automotive radiator that is a compact crossflow water-to-air heat exchanger with both fluids (air and water) unmixed (Fig. 22–23). The radiator has 40 tubes of internal diameter 0.5 cm and length 65 cm in a closely spaced plate-finned matrix. Hot water enters the tubes at 90°C at a rate of 0.6 kg/s and leaves at 65°C. Air flows across the radiator through the interfin spaces and is heated from 20°C to 40°C. Determine the overall heat transfer coefficient U_i of this radiator based on the inner surface area of the tubes.

SOLUTION During an experiment involving an automotive radiator, the inlet and exit temperatures of water and air and the mass flow rate of water are measured. The overall heat transfer coefficient based on the inner surface area is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Changes in the kinetic and potential energies of fluid streams are negligible. **3** Fluid properties are constant.

Properties The specific heat of water at the average temperature of (90 + 65)/2 = 77.5°C is 4.195 kJ/kg·K (Table A–15).

Analysis The rate of heat transfer in this radiator from the hot water to the air is determined from an energy balance on water flow,

$$\begin{aligned}\dot{Q} &= [\dot{m}c_p(T_{\text{in}} - T_{\text{out}})]_{\text{water}} = (0.6\text{ kg/s})(4.195\text{ kJ/kg·K})(90 - 65)\text{°C} \\ &= 62.93\text{ kW}\end{aligned}$$

The tube-side heat transfer area is the total surface area of the tubes and is determined from (where n is the number of tubes)

$$A_i = n\pi D_i L = (40)\pi(0.005\text{ m})(0.65\text{ m}) = 0.408\text{ m}^2$$

Knowing the rate of heat transfer and the surface area, the overall heat transfer coefficient can be determined from

$$\dot{Q} = U_i A_i F\,\Delta T_{\text{lm, CF}} \longrightarrow U_i = \frac{\dot{Q}}{A_i F\,\Delta T_{\text{lm, CF}}}$$

where F is the correction factor and $\Delta T_{\text{lm, CF}}$ is the log mean temperature difference for the counterflow arrangement. These two quantities are found to be

$$\begin{aligned}\Delta T_1 &= T_{h,\text{ in}} - T_{c,\text{ out}} = (90 - 40)\text{°C} = 50\text{°C} \\ \Delta T_2 &= T_{h,\text{ out}} - T_{c,\text{ in}} = (65 - 20)\text{°C} = 45\text{°C} \\ \Delta T_{\text{lm, CF}} &= \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{50 - 45}{\ln(50/45)} = 47.5\text{°C}\end{aligned}$$

and

$$\left.\begin{aligned}P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{65 - 90}{20 - 90} = 0.36 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 40}{65 - 90} = 0.80\end{aligned}\right\} F = 0.97 \qquad \text{(Fig. 22–19}c\text{)}$$

Substituting, the overall heat transfer coefficient U_i is determined to be

$$U_i = \frac{\dot{Q}}{A_i F\,\Delta T_{\text{lm, CF}}} = \frac{62{,}930\text{ W}}{(0.408\text{ m}^2)(0.97)(47.5\text{°C})} = \mathbf{3347\ W/m^2{\cdot}K}$$

Discussion Note that the overall heat transfer coefficient on the air side will be much lower because of the large surface area involved on that side.

22–5 ▪ THE EFFECTIVENESS–NTU METHOD

The log mean temperature difference (LMTD) method discussed in Sec. 22–4 is easy to use in heat exchanger analysis when the inlet and the outlet temperatures of the hot and cold fluids are known or can be determined from an energy balance. Once ΔT_{lm}, the mass flow rates, and the overall heat transfer coefficient are available, the heat transfer surface area of the heat exchanger can be determined from

$$\dot{Q} = UA_s \Delta T_{\text{lm}}$$

Therefore, the LMTD method is very suitable for determining the *size* of a heat exchanger to realize prescribed outlet temperatures when the mass flow rates and the inlet and outlet temperatures of the hot and cold fluids are specified.

With the LMTD method, the task is to select a heat exchanger that will meet the prescribed heat transfer requirements. The procedure to be followed in the selection process is:

1. Select the type of heat exchanger suitable for the application.
2. Determine any unknown inlet or outlet temperature and the heat transfer rate using an energy balance.
3. Calculate the log mean temperature difference ΔT_{lm} and the correction factor F, if necessary.
4. Obtain (select or calculate) the value of the overall heat transfer coefficient U.
5. Calculate the heat transfer surface area A_s.

The task is completed by selecting a heat exchanger that has a heat transfer surface area equal to or larger than A_s.

A second kind of problem encountered in heat exchanger analysis is the determination of the *heat transfer rate* and the *outlet temperatures* of the hot and cold fluids for prescribed fluid mass flow rates and inlet temperatures when the *type* and *size* of the heat exchanger are specified. The heat transfer surface area of the heat exchanger in this case is known, but the *outlet temperatures* are not. Here the task is to determine the heat transfer performance of a specified heat exchanger or to determine if a heat exchanger available in storage will do the job.

The LMTD method could still be used for this alternative problem, but the procedure would require tedious iterations, and thus it is not practical. In an attempt to eliminate the iterations from the solution of such problems, Kays and London came up with a method in 1955 called the **effectiveness–NTU method**, which greatly simplified heat exchanger analysis.

This method is based on a dimensionless parameter called the **heat transfer effectiveness** ε, defined as

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}} \tag{22–29}$$

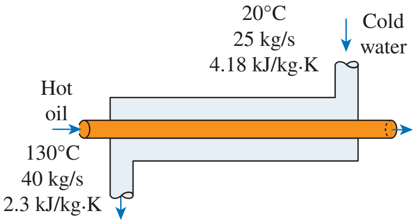
The *actual* heat transfer rate in a heat exchanger can be determined from an energy balance on the hot or cold fluids and can be expressed as

$$\dot{Q} = C_c(T_{c,\text{ out}} - T_{c,\text{ in}}) = C_h(T_{h,\text{ in}} - T_{h,\text{ out}}) \tag{22–30}$$

where $C_c = \dot{m}_c c_{pc}$ and $C_h = \dot{m}_h c_{ph}$ are the heat capacity rates of the cold and hot fluids, respectively.

To determine the maximum possible heat transfer rate in a heat exchanger, we first recognize that the *maximum temperature difference* in a heat exchanger is the difference between the *inlet* temperatures of the hot and cold fluids. That is,

$$\Delta T_{\text{max}} = T_{h,\text{ in}} - T_{c,\text{ in}} \tag{22–31}$$



$$C_c = \dot{m}_c c_{pc} = 104.5 \text{ kW/K}$$
$$C_h = \dot{m}_h c_{ph} = 92 \text{ kW/K}$$
$$C_{\text{min}} = 92 \text{ kW/K}$$
$$\Delta T_{\text{max}} = T_{h,\text{in}} - T_{c,\text{in}} = 110°\text{C}$$
$$\dot{Q}_{\text{max}} = C_{\text{min}}\,\Delta T_{\text{max}} = 10{,}120 \text{ kW}$$

FIGURE 22–24
The determination of the maximum rate of heat transfer in a heat exchanger.

The heat transfer in a heat exchanger will reach its maximum value when (1) the cold fluid is heated to the inlet temperature of the hot fluid or (2) the hot fluid is cooled to the inlet temperature of the cold fluid. These two limiting conditions will not be reached simultaneously unless the heat capacity rates of the hot and cold fluids are identical (i.e., $C_c = C_h$). When $C_c \not\approx C_h$, which is usually the case, the fluid with the *smaller* heat capacity rate will experience a larger temperature change, and thus it will be the first to experience the maximum temperature, at which point the heat transfer will come to a halt. Therefore, the maximum possible heat transfer rate in a heat exchanger is (Fig. 22–24)

$$\dot{Q}_{\text{max}} = C_{\text{min}}(T_{h,\text{ in}} - T_{c,\text{ in}}) \tag{22–32}$$

where C_{min} is the smaller of C_h and C_c. This is further clarified by Example 22–7.

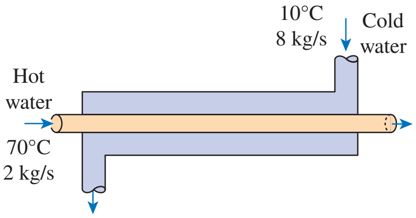


FIGURE 22–25
Schematic for Example 22–7.

EXAMPLE 22–7 Upper Limit for Heat Transfer in a Heat Exchanger

Cold water enters a counterflow heat exchanger at 10°C at a rate of 8 kg/s, where it is heated by a hot-water stream that enters the heat exchanger at 70°C at a rate of 2 kg/s. Assuming the specific heat of water to remain constant at $c_p = 4.18$ kJ/kg·K, determine the maximum heat transfer rate and the outlet temperatures of the cold- and the hot-water streams for this limiting case.

SOLUTION Cold- and hot-water streams enter a heat exchanger at specified temperatures and flow rates. The maximum rate of heat transfer in the heat exchanger and the outlet temperatures are to be determined.
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.
Properties The specific heat of water is given to be $c_p = 4.18$ kJ/kg·K.
Analysis A schematic of the heat exchanger is given in Fig. 22–25. The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (2 \text{ kg/s})(4.18 \text{ kJ/kg·K}) = 8.36 \text{ kW/K}$$

and

$$C_c = \dot{m}_c c_{pc} = (8 \text{ kg/s})(4.18 \text{ kJ/kg·K}) = 33.4 \text{ kW/K}$$

Therefore,

$$C_{\text{min}} = C_h = 8.36 \text{ kW/K}$$

which is the smaller of the two heat capacity rates. Then the maximum heat transfer rate is determined from Eq. 22–32 to be

$$\begin{aligned}\dot{Q}_{\text{max}} &= C_{\text{min}}(T_{h,\text{ in}} - T_{c,\text{ in}})\\ &= (8.36 \text{ kW/K})(70 - 10)°\text{C}\\ &= \mathbf{502 \text{ kW}}\end{aligned}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 502 kW. This value would be approached in a counterflow heat exchanger with a very large heat transfer surface area.

The maximum temperature difference in this heat exchanger is $\Delta T_{\text{max}} = T_{h,\text{ in}} - T_{c,\text{ in}} = (70 - 10)°\text{C} = 60°\text{C}$. Therefore, the hot water cannot be cooled by more than 60°C (to 10°C) in this heat exchanger, and the cold water cannot be heated by more than 60°C (to 70°C), no matter what we do. The outlet temperatures of the cold and the hot streams in this limiting case are determined to be

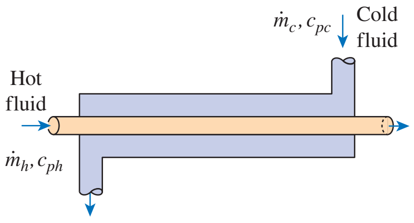
$$\dot{Q} = C_c(T_{c,\text{ out}} - T_{c,\text{ in}}) \longrightarrow T_{c,\text{ out}} = T_{c,\text{ in}} + \frac{\dot{Q}}{C_c} = 10^\circ\text{C} + \frac{502\text{ kW}}{33.4\text{ kW/K}} = \mathbf{25^\circ C}$$

$$\dot{Q} = C_h(T_{h,\text{ in}} - T_{h,\text{ out}}) \longrightarrow T_{h,\text{ out}} = T_{h,\text{ in}} - \frac{\dot{Q}}{C_h} = 70^\circ\text{C} - \frac{502\text{ kW}}{8.38\text{ kW/K}} = \mathbf{10^\circ C}$$

Discussion Note that the hot water is cooled to the limit of 10°C (the inlet temperature of the cold-water stream), but the cold water is heated to 25°C only when maximum heat transfer occurs in the heat exchanger. This is not surprising, since the mass flow rate of the hot water is only one-fourth that of the cold water, and, as a result, the temperature of the cold water increases by 0.25°C for each 1°C drop in the temperature of the hot water.

You may be tempted to think that the cold water should be heated to 70°C in the limiting case of maximum heat transfer. But this will require the temperature of the hot water to drop to −170°C (below 10°C), which is impossible. Therefore, heat transfer in a heat exchanger reaches its maximum value when the fluid with the smaller heat capacity rate (or the smaller mass flow rate when both fluids have the same specific heat value) experiences the maximum temperature change. This example explains why we use $C_{\min}$ in the evaluation of $\dot{Q}_{\max}$ instead of $C_{\max}$.

We can show that the hot water will leave at the inlet temperature of the cold water and vice versa in the limiting case of maximum heat transfer when the mass flow rates of the hot- and cold-water streams are identical (Fig. 22–26). We can also show that the outlet temperature of the cold water will reach the 70°C limit when the mass flow rate of the hot water is greater than that of the cold water.



$$\dot{Q} = \dot{m}_h c_{ph} \Delta T_h$$
$$= \dot{m}_c c_{pc} \Delta T_c$$

If $\quad \dot{m}_c c_{pc} = \dot{m}_h c_{ph}$

then $\quad \Delta T_h = \Delta T_c$

FIGURE 22–26

The temperature rise of the cold fluid in a heat exchanger will be equal to the temperature drop of the hot fluid when the heat capacity rates of the hot and cold fluids are identical.

The determination of $\dot{Q}_{\max}$ requires the availability of the *inlet temperature* of the hot and cold fluids and their *mass flow rates,* which are usually specified. Then, once the effectiveness of the heat exchanger is known, the actual heat transfer rate $\dot{Q}$ can be determined from

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}}) \tag{22–33}$$

where

if $C_c = C_{\min}$: $$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c(T_{c,\text{ out}} - T_{c,\text{ in}})}{C_c(T_{h,\text{ in}} - T_{c,\text{ in}})} = \frac{T_{c,\text{ out}} - T_{c,\text{ in}}}{T_{h,\text{ in}} - T_{c,\text{ in}}}$$

if $C_h = C_{\min}$: $$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,\text{ in}} - T_{h,\text{ out}})}{C_h(T_{h,\text{ in}} - T_{c,\text{ in}})} = \frac{T_{h,\text{ in}} - T_{h,\text{ out}}}{T_{h,\text{ in}} - T_{c,\text{ in}}}$$

Therefore, the effectiveness of a heat exchanger enables us to determine the heat transfer rate without knowing the *outlet temperatures* of the fluids.

The effectiveness of a heat exchanger depends on the *geometry* of the heat exchanger as well as the *flow arrangement.* Therefore, different types of heat exchangers have different effectiveness relations. Next we illustrate the development of the effectiveness ε relation for the double-pipe *parallel-flow* heat exchanger.

Equation 22–23 developed in Sec. 22–4 for a parallel-flow heat exchanger can be rearranged as

$$\ln \frac{T_{h,\text{ out}} - T_{c,\text{ out}}}{T_{h,\text{ in}} - T_{c,\text{ in}}} = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right) \tag{22–34}$$

Also, solving Eq. 22–30 for $T_{h,\,\text{out}}$ gives

$$T_{h,\,\text{out}} = T_{h,\,\text{in}} - \frac{C_c}{C_h}(T_{c,\,\text{out}} - T_{c,\,\text{in}}) \tag{22–35}$$

Substituting this relation into Eq. 22–34 after adding and subtracting $T_{c,\,\text{in}}$ gives

$$\ln \frac{T_{h,\,\text{in}} - T_{c,\,\text{in}} + T_{c,\,\text{in}} - T_{c,\,\text{out}} - \dfrac{C_c}{C_h}(T_{c,\,\text{out}} - T_{c,\,\text{in}})}{T_{h,\,\text{in}} - T_{c,\,\text{in}}} = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

which simplifies to

$$\ln\left[1 - \left(1 + \frac{C_c}{C_h}\right)\frac{T_{c,\,\text{out}} - T_{c,\,\text{in}}}{T_{h,\,\text{in}} - T_{c,\,\text{in}}}\right] = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right) \tag{22–36}$$

We now manipulate the definition of effectiveness to obtain

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} = \frac{C_c(T_{c,\,\text{out}} - T_{c,\,\text{in}})}{C_{\text{min}}(T_{h,\,\text{in}} - T_{c,\,\text{in}})} \longrightarrow \frac{T_{c,\,\text{out}} - T_{c,\,\text{in}}}{T_{h,\,\text{in}} - T_{c,\,\text{in}}} = \varepsilon\frac{C_{\text{min}}}{C_c}$$

Substituting this result into Eq. 22–36 and solving for ε gives the following relation for the effectiveness of a *parallel-flow* heat exchanger:

$$\varepsilon_{\text{parallel flow}} = \frac{1 - \exp\left[-\dfrac{UA_s}{C_c}\left(1 + \dfrac{C_c}{C_h}\right)\right]}{\left(1 + \dfrac{C_c}{C_h}\right)\dfrac{C_{\text{min}}}{C_c}} \tag{22–37}$$

Taking either C_c or C_h to be C_{min} (both approaches give the same result), the preceding relation can be expressed more conveniently as

$$\varepsilon_{\text{parallel flow}} = \frac{1 - \exp\left[-\dfrac{UA_s}{C_{\text{min}}}\left(1 + \dfrac{C_{\text{min}}}{C_{\text{max}}}\right)\right]}{1 + \dfrac{C_{\text{min}}}{C_{\text{max}}}} \tag{22–38}$$

Again C_{min} is the *smaller* heat capacity rate and C_{max} is the larger one, and it makes no difference whether C_{min} belongs to the hot or cold fluid.

Effectiveness relations of the heat exchangers typically involve the *dimensionless* group UA_s/C_{min}. This quantity is called the **number of transfer units NTU** and is expressed as

$$\text{NTU} = \frac{UA_s}{C_{\text{min}}} = \frac{UA_s}{(\dot{m}c_p)_{\text{min}}} \tag{22–39}$$

where U is the overall heat transfer coefficient and A_s is the heat transfer surface area of the heat exchanger. Note that NTU is proportional to A_s. Therefore, for specified values of U and C_{min}, the value of NTU *is a measure of the heat transfer surface area* A_s. Thus, the larger the NTU, the larger the heat exchanger.

In heat exchanger analysis, it is also convenient to define another dimensionless quantity called the **capacity ratio** c as

$$c = \frac{C_{\text{min}}}{C_{\text{max}}} \tag{22–40}$$

TABLE 22–4

Effectiveness relations for heat exchangers: NTU $= UA_s/C_{\min}$ and $c = C_{\min}/C_{\max} = (\dot{m}c_p)_{\min}/(\dot{m}c_p)_{\max}$

Heat exchanger type	Effectiveness relation
1 *Double pipe:* Parallel-flow	$\varepsilon = \dfrac{1 - \exp\,[-\text{NTU}(1+c)]}{1+c}$
Counterflow	$\varepsilon = \dfrac{1 - \exp\,[-\text{NTU}(1-c)]}{1 - c\exp\,[-\text{NTU}(1-c)]}$ (for $c < 1$) $\varepsilon = \dfrac{\text{NTU}}{1+\text{NTU}}$ (for $c = 1$)
2 *Shell-and-tube:* One shell pass 2, 4, . . . tube passes	$\varepsilon_1 = 2\left\{1 + c + \sqrt{1+c^2}\,\dfrac{1 + \exp\left[-\text{NTU}_1\sqrt{1+c^2}\right]}{1 - \exp\left[-\text{NTU}_1\sqrt{1+c^2}\right]}\right\}^{-1}$
n shell passes $2n$, $4n$, . . . tube passes	$\varepsilon = \left[\left(\dfrac{1-\varepsilon_1 c}{1-\varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1-\varepsilon_1 c}{1-\varepsilon_1}\right)^n - c\right]^{-1}$
3 Crossflow (*single-pass*) Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{c}\left[\exp(-c\text{NTU}^{0.78}) - 1\right]\right\}$
$C_{\max}$ mixed, $C_{\min}$ unmixed	$\varepsilon = \dfrac{1}{c}(1 - \exp\{-c[1 - \exp(-\text{NTU})]\})$
$C_{\min}$ mixed, $C_{\max}$ unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{c}[1 - \exp(-c\,\text{NTU})]\right\}$
4 All heat exchangers with $c = 0$	$\varepsilon = 1 - \exp(-\text{NTU})$

Source: W. M. Kays and A. L. London. Compact Heat Exchangers, 3/e. McGraw-Hill, 1984.

It can be shown that the effectiveness of a heat exchanger is a function of the number of transfer units NTU and the capacity ratio c. That is,

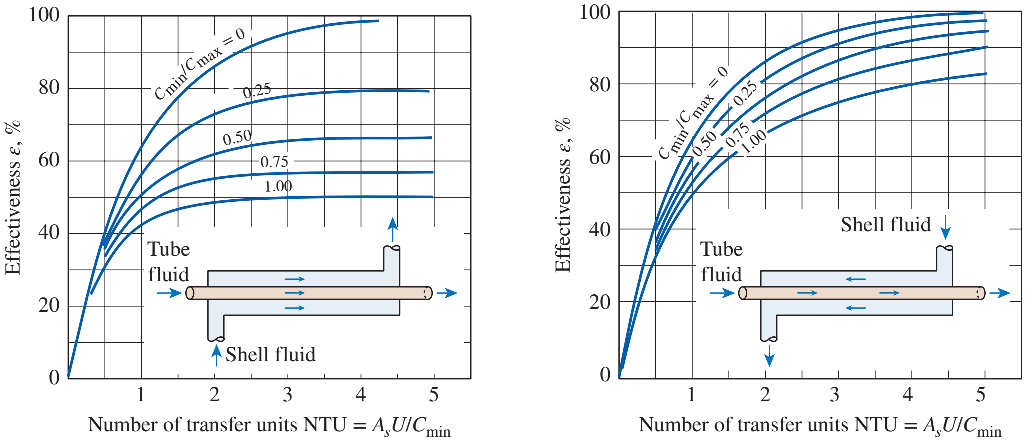
$$\varepsilon = \text{function}\ (UA_s/C_{\min},\ C_{\min}/C_{\max}) = \text{function}\ (\text{NTU},\ c)$$

Effectiveness relations have been developed for a large number of heat exchangers, and the results are given in Table 22–4. The effectivenesses of some common types of heat exchangers are also plotted in Fig. 22–27.

More extensive effectiveness charts and relations are available in the literature. The dashed lines in Fig. 22–27*f* are for the case of $C_{\min}$ unmixed and $C_{\max}$ mixed, and the solid lines are for the opposite case. The analytic relations for the effectiveness give more accurate results than the charts, since reading errors in charts are unavoidable, and the relations are very suitable for computerized analysis of heat exchangers.

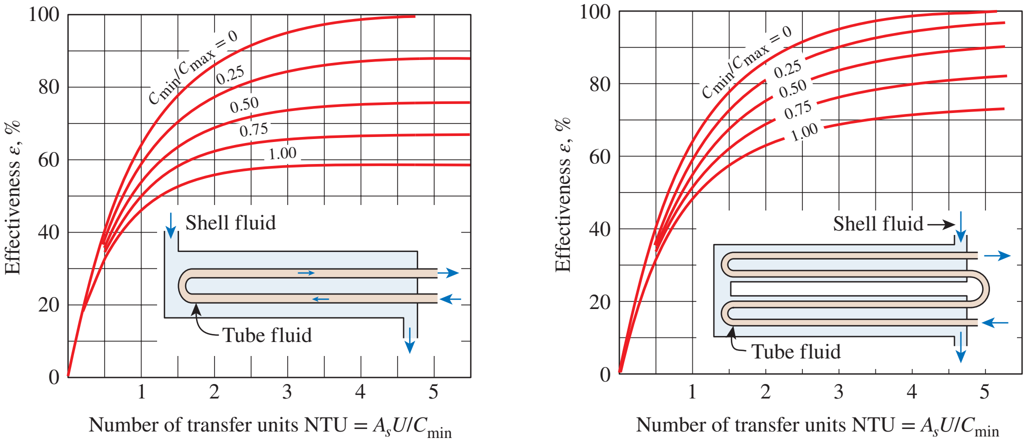
We make these observations from the effectiveness relations and charts already given:

1. The value of the effectiveness ranges from 0 to 1. It increases rapidly with NTU for small values (up to about NTU = 1.5) but rather slowly for larger values. Therefore, the use of a heat exchanger with a large NTU (usually



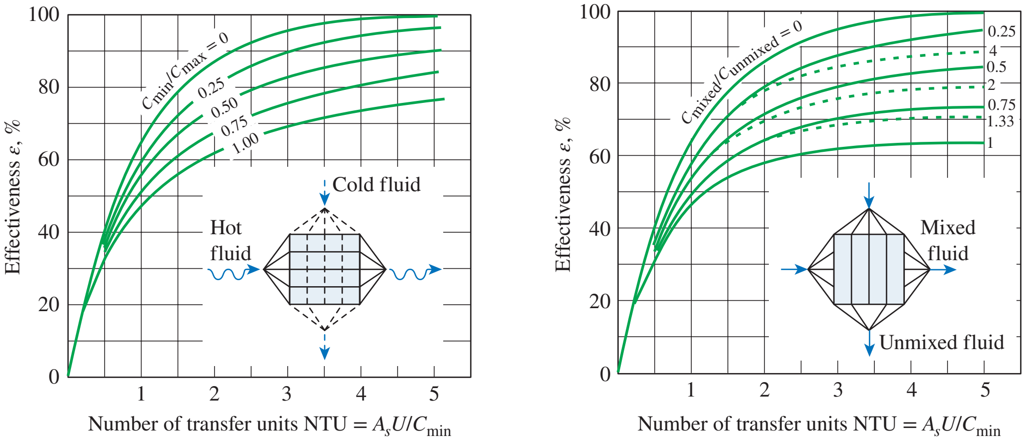
(*a*) Parallel-flow

(*b*) Counterflow



(*c*) One shell pass and 2, 4, 6, … tube passes

(*d*) Two shell passes and 4, 8, 12, … tube passes



(*e*) Crossflow with both fluids unmixed

(*f*) Crossflow with one fluid mixed and the other unmixed

FIGURE 22–27

Effectiveness for heat exchangers.

Source: W. M. Kays and A. L. London. Compact Heat Exchangers, 3/e. McGraw-Hill, 1984.

larger than 3) and thus a large size cannot be justified economically, since a large increase in NTU in this case corresponds to a small increase in effectiveness. Thus, a heat exchanger with a very high effectiveness may be desirable from a heat transfer point of view but undesirable from an economical point of view.

2. For a given NTU and capacity ratio $c = C_{\min}/C_{\max}$, the *counterflow* heat exchanger has the *highest* effectiveness, followed closely by the crossflow heat exchangers with both fluids unmixed. As you might expect, the lowest effectiveness values are encountered in parallel-flow heat exchangers (Fig. 22–28).
3. The effectiveness of a heat exchanger is independent of the capacity ratio c for NTU values of less than about 0.3.
4. The value of the capacity ratio c ranges between 0 and 1. For a given NTU, the effectiveness becomes a *maximum* for $c = 0$ and a *minimum* for $c = 1$. The case $c = C_{\min}/C_{\max} \to 0$ corresponds to $C_{\max} \to \infty$, which is realized during a phase-change process in a *condenser* or *boiler.* All effectiveness relations in this case reduce to

$$\varepsilon = \varepsilon_{\max} = 1 - \exp(-\text{NTU}) \tag{22–41}$$

regardless of the type of heat exchanger (Fig. 22–29). Note that the temperature of the condensing or boiling fluid remains constant in this case. The effectiveness is the *lowest* in the other limiting case of $c = C_{\min}/C_{\max} = 1$, which is realized when the heat capacity rates of the two fluids are equal. In industrial applications such as hot water cooling or chilled water heating pertaining to the heating, ventilating, and air conditioning (HVAC) industry, the hot or cold fluid is channeled from one location to another with the pipe immersed in ambient environment. Typical examples of this flow situation are the fluid-carrying heat exchanger pipe exposed to ambient air or submerged in large liquid media such as a lake. In such cases, since the effective change in reference temperature of the ambient environment is virtually zero and the relative mass of the ambient environment is infinitely large, the maximum heat capacity rate $C_{\max}$ is a very large number. Thus the capacity ratio $c = C_{\min}/C_{\max} \to 0$, and hence Eq. 22–41 is applicable in cases when the heat exchanger is in contact with the ambient environment.

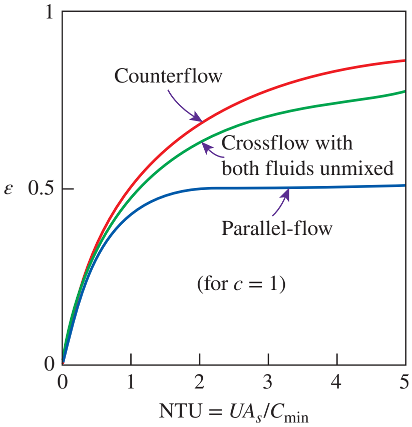


FIGURE 22–28

For a specified NTU and capacity ratio c, the counterflow heat exchanger has the highest effectiveness and the parallel-flow the lowest.

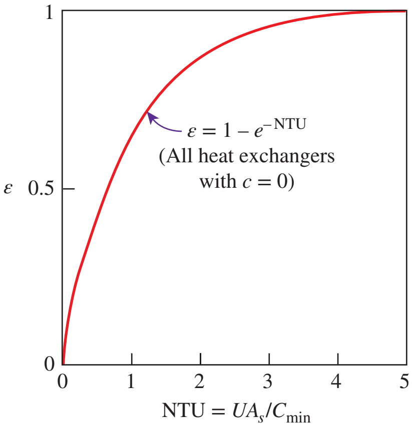


FIGURE 22–29

The effectiveness relation reduces to $\varepsilon = \varepsilon_{\max} = 1 - \exp(-\text{NTU})$ for all heat exchangers when the capacity ratio $c = 0$.

Once the quantities $c = C_{\min}/C_{\max}$ and $\text{NTU} = UA_s/C_{\min}$ have been evaluated, the effectiveness ε can be determined from either the charts or the effectiveness relation for the specified type of heat exchanger. Then the rate of heat transfer $\dot{Q}$ and the outlet temperatures $T_{h,\text{ out}}$ and $T_{c,\text{ out}}$ can be determined from Eqs. 22–33 and 22–30, respectively. Note that the analysis of heat exchangers with unknown outlet temperatures is a straightforward matter with the effectiveness–NTU method but requires rather tedious iterations with the LMTD method.

We mentioned earlier that when all the inlet and outlet temperatures are specified, the *size* of the heat exchanger can easily be determined using the LMTD method. Alternatively, it can also be determined from the effectiveness–NTU method by first evaluating the effectiveness ε from its definition (Eq. 22–29) and then the NTU from the appropriate NTU relation in Table 22–5.

Note that the relations in Table 22–5 are equivalent to those in Table 22–4. Both sets of relations are given for convenience. The relations in Table 22–4 give the effectiveness directly when NTU is known, and the relations in Table 22–5 give the NTU directly when the effectiveness ε is known.

TABLE 22–5

NTU relations for heat exchangers: $\text{NTU} = UA_s/C_{\min}$ and $c = C_{\min}/C_{\max} = (\dot{m}c_p)_{\min}/(\dot{m}c_p)_{\max}$

Heat exchanger type		NTU relation
1	*Double-pipe:*	
	Parallel-flow	$\text{NTU} = -\dfrac{\ln[1 - \varepsilon(1 + c)]}{1 + c}$
	Counterflow	$\text{NTU} = \dfrac{1}{c - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon c - 1}\right)$ (for $c < 1$) $\text{NTU} = \dfrac{\varepsilon}{1 - \varepsilon}$ (for $c = 1$)
2	*Shell and tube:* One shell pass 2, 4, . . . tube passes	$\text{NTU}_1 = -\dfrac{1}{\sqrt{1 + c^2}}\ln\left(\dfrac{2/\varepsilon_1 - 1 - c - \sqrt{1 + c^2}}{2/\varepsilon_1 - 1 - c + \sqrt{1 + c^2}}\right)$
	n shell passes 2*n*, 4*n*, . . . tube passes	$\text{NTU} = n(\text{NTU})_1$ To find effectiveness of the heat exchanger with one shell pass use, $\varepsilon_1 = \dfrac{F - 1}{F - c}$ where $F = \left(\dfrac{\varepsilon c - 1}{\varepsilon - 1}\right)^{1/n}$
3	*Crossflow (single-pass):*	
	$C_{\max}$ mixed, $C_{\min}$ unmixed	$\text{NTU} = -\ln\left[1 + \dfrac{\ln(1 - \varepsilon c)}{c}\right]$
	$C_{\min}$ mixed, $C_{\max}$ unmixed	$\text{NTU} = -\dfrac{\ln[c\ln(1 - \varepsilon) + 1]}{c}$
4	All heat exchangers with $c = 0$	$\text{NTU} = -\ln(1 - \varepsilon)$

Source: W. M. Kays and A. L. London. Compact Heat Exchangers, 3/e. McGraw-Hill, 1984.

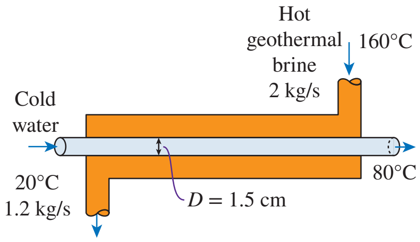


FIGURE 22–30
Schematic for Example 22–8.

EXAMPLE 22–8 The Effectiveness-NTU Method

A counterflow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s (Fig. 22–30). The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. The overall heat transfer coefficient of the heat exchanger is 640 W/m^2·K. Using the effectiveness-NTU method, determine the length of the heat exchanger required to achieve the desired heating.

SOLUTION Water is heated in a counterflow double-pipe heat exchanger by geothermal water. Using the ε-NTU method, the required length of the heat exchanger is to be determined.

Analysis In the effectiveness-NTU method, we first determine the heat capacity rates of the hot and cold fluids and identify the smaller one.

$$C_h = \dot{m}_h c_{ph} = (2\text{ kg/s})(4.31\text{ kJ/kg·K}) = 8.62\text{ kW/K}$$
$$C_c = \dot{m}_c c_{pc} = (1.2\text{ kg/s})(4.18\text{ kJ/kg·K}) = 5.02\text{ kW/K}$$

Therefore,

$$C_{\min} = C_c = 5.02\text{ kW/K}$$

and

$$c = C_{\min}/C_{\max} = 5.02/8.62 = 0.582$$

Then the maximum heat transfer rate is determined from Eq. 22–32 to be

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}})\\ &= (5.02\text{ kW/K})(160 - 20)^\circ\text{C}\\ &= 702.8\text{ kW}\end{aligned}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 702.8 kW. The actual rate of heat transfer is

$$\begin{aligned}\dot{Q} &= [\dot{m}c_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (1.2\text{ kg/s})(4.18\text{ kJ/kg·K})(80 - 20)^\circ\text{C}\\ &= 301.0\text{ kW}\end{aligned}$$

Thus, the effectiveness of the heat exchanger is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{301.0\text{ kW}}{702.8\text{ kW}} = 0.428$$

Knowing the effectiveness, the NTU of this counterflow heat exchanger can be determined from Fig. 22–27*b* or the appropriate relation from Table 22–5 for $c < 1$. We choose the latter approach for greater accuracy:

$$\text{NTU} = \frac{1}{c-1}\ln\left(\frac{\varepsilon - 1}{\varepsilon c - 1}\right) = \frac{1}{0.582 - 1}\ln\left(\frac{0.428 - 1}{0.428 \times 0.582 - 1}\right) = 0.651$$

Then the heat transfer surface area becomes

$$\text{NTU} = \frac{UA_s}{C_{\min}} \longrightarrow A_s = \frac{\text{NTU}\,C_{\min}}{U} = \frac{(0.651)(5020\text{ W/K})}{640\text{ W/m}^2\text{·K}} = 5.11\text{ m}^2$$

To provide this much heat transfer surface area, the length of the tube must be

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{5.11\text{ m}^2}{\pi(0.015\text{ m})} = \mathbf{108\text{ m}}$$

Discussion This problem was solved in Example 22–4 using the LMTD method. Note that we obtained practically the same result in a systematic and straightforward manner using the effectiveness–NTU method.

EXAMPLE 22–9 Cooling Hot Oil with Water in a Multipass Heat Exchanger

Hot oil is to be cooled by water in a one-shell-pass and eight-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is 310 W/m^2·K. Water flows through the tubes at a rate of 0.2 kg/s and the oil through the shell at a rate of 0.3 kg/s. The water and the oil enter at temperatures of 20°C and 150°C, respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.

SOLUTION Hot oil is to be cooled by water in a heat exchanger. The mass flow rates and the inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined.

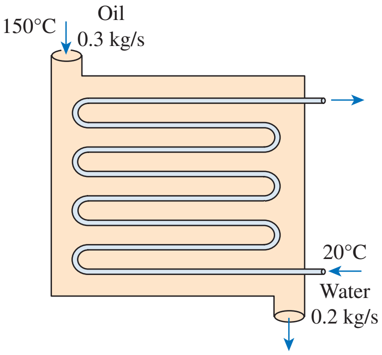


FIGURE 22–31
Schematic for Example 22–9.

Assumptions 1 Steady operating conditions exist. 2 The heat exchanger is well insulated so that heat loss to the surroundings is negligible. 3 The thickness of the tube is negligible since it is thin-walled. 4 Changes in the kinetic and potential energies of fluid streams are negligible. 5 The overall heat transfer coefficient is constant and uniform.

Properties We take the specific heats of water and oil to be 4.18 and 2.13 kJ/kg·°C, respectively.

Analysis The schematic of the heat exchanger is given in Fig. 22–31. The outlet temperatures are not specified, and they cannot be determined from an energy balance. The use of the LMTD method in this case will involve tedious iterations, and thus the ε–NTU method is indicated. The first step in the ε–NTU method is to determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h c_{ph} = (0.3 \text{ kg/s})(2.13 \text{ kJ/kg·°C}) = 0.639 \text{ kW/K}$$
$$C_c = \dot{m}_c c_{pc} = (0.2 \text{ kg/s})(4.18 \text{ kJ/kg·°C}) = 0.836 \text{ kW/K}$$

Therefore,

$$C_{\min} = C_h = 0.639 \text{ kW/K} \qquad \text{and} \qquad c = \frac{C_{\min}}{C_{\max}} = \frac{0.639}{0.836} = 0.764$$

Then the maximum heat transfer rate is determined from Eq. 22–32 to be

$$\dot{Q}_{\max} = C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}}) = (0.639 \text{ kW/K})(150 - 20)\text{°C} = 83.1 \text{ kW}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 83.1 kW. The heat transfer surface area is

$$A_s = n(\pi DL) = 8\pi(0.014 \text{ m})(5 \text{ m}) = 1.76 \text{ m}^2$$

Then the NTU of this heat exchanger becomes

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(310 \text{ W/m}^2\text{·K})(1.76 \text{ m}^2)}{639 \text{ W/K}} = 0.854$$

The effectiveness of this heat exchanger corresponding to $c = 0.764$ and NTU = 0.854 is determined from Fig. 22–27*c* to be

$$\varepsilon = 0.47$$

We could also determine the effectiveness from the third relation in Table 22–4 more accurately but with more labor. Then the actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.47)(83.1 \text{ kW}) = \mathbf{39.1 \text{ kW}}$$

Finally, the outlet temperatures of the cold and the hot fluid streams are determined to be

$$\dot{Q} = C_c(T_{c,\text{ out}} - T_{c,\text{ in}})$$
$$T_{c,\text{ out}} = T_{c,\text{ in}} + \frac{\dot{Q}}{C_c}$$
$$= 20\text{°C} + \frac{39.1 \text{ kW}}{0.836 \text{ kW/K}} = \mathbf{66.8°C}$$
$$\dot{Q} = C_h(T_{h,\text{ in}} - T_{h,\text{ out}})$$
$$T_{h,\text{ out}} = T_{h,\text{ in}} - \frac{\dot{Q}}{C_h}$$
$$= 150\text{°C} - \frac{39.1 \text{ kW}}{0.639 \text{ kW/K}} = \mathbf{88.8°C}$$

Therefore, the temperature of the cooling water will rise from 20°C to 66.8°C as it cools the hot oil from 150°C to 88.8°C in this heat exchanger.

SUMMARY

Heat exchangers are devices that allow the exchange of heat between two fluids without allowing them to mix with each other. Heat exchangers are manufactured in a variety of types, the simplest being the *double-pipe* heat exchanger. In a *parallel-flow* type, both the hot and cold fluids enter the heat exchanger at the same end and move in the same direction, whereas in a *counterflow* type, the hot and cold fluids enter the heat exchanger at opposite ends and flow in opposite directions. In *compact* heat exchangers, the two fluids move perpendicular to each other, and such a flow configuration is called *crossflow.* Other common types of heat exchangers in industrial applications are the *plate* and the *shell-and-tube* heat exchangers.

Heat transfer in a heat exchanger usually involves convection in each fluid and conduction through the wall separating the two fluids. In the analysis of heat exchangers, it is convenient to work with an *overall heat transfer coefficient U* or a *total thermal resistance R,* expressed as

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + R_{\text{wall}} + \frac{1}{h_oA_o}$$

where the subscripts i and o stand for the inner and outer surfaces of the wall that separates the two fluids, respectively. When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, the relation simplifies to

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where $U \approx U_i \approx U_o$. The effects of fouling on both the inner and the outer surfaces of the tubes of a heat exchanger can be accounted for by

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R$$
$$= \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

where $A_i = \pi D_iL$ and $A_o = \pi D_oL$ are the areas of the inner and outer surfaces and $R_{f,i}$ and $R_{f,o}$ are the fouling factors at those surfaces.

In a well-insulated heat exchanger, the rate of heat transfer from the hot fluid is equal to the rate of heat transfer to the cold one. That is,

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,\text{out}} - T_{c,\text{in}}) = C_c(T_{c,\text{out}} - T_{c,\text{in}})$$

and

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,\text{in}} - T_{h,\text{out}}) = C_h(T_{h,\text{in}} - T_{h,\text{out}})$$

where the subscripts c and h stand for the cold and hot fluids, respectively, and the product of the mass flow rate and the specific heat of a fluid $\dot{m}c_p$ is called the *heat capacity rate.*

Of the two methods used in the analysis of heat exchangers, the *log mean temperature difference* (or LMTD) method is best suited for determining the size of a heat exchanger when all the inlet and the outlet temperatures are known. The *effectiveness–NTU* method is best suited to predict the outlet temperatures of the hot and cold fluid streams in a specified heat exchanger. In the LMTD method, the rate of heat transfer is determined from

$$\dot{Q} = UA_s\Delta T_{\text{lm}}$$

where

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

is the *log mean temperature difference,* which is the suitable form of the average temperature difference for use in the analysis of heat exchangers. Here ΔT_1 and ΔT_2 represent the temperature differences between the two fluids at the two ends (inlet and outlet) of the heat exchanger. For crossflow and multipass shell-and-tube heat exchangers, the logarithmic mean temperature difference is related to the counterflow one $\Delta T_{\text{lm, CF}}$ as

$$\Delta T_{\text{lm}} = F\,\Delta T_{\text{lm, CF}}$$

where F is the *correction factor,* which depends on the geometry of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams.

The *effectiveness* of a heat exchanger is defined as

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

where

$$\dot{Q}_{\max} = C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})$$

and $C_{\min}$ is the smaller of $C_h = \dot{m}_h c_{ph}$ and $C_c = \dot{m}_c c_{pc}$. The effectiveness of heat exchangers can be determined from effectiveness relations or charts.

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PROBLEMS*

Types of Heat Exchangers

22–1C Classify heat exchangers according to flow type, and explain the characteristics of each type.

22–2C When is a heat exchanger classified as being compact? Do you think a double-pipe heat exchanger can be classified as a compact heat exchanger?

22–3C What is a regenerative heat exchanger? How does a static type of regenerative heat exchanger differ from a dynamic type?

22–4C What is the role of the baffles in a shell-and-tube heat exchanger? How does the presence of baffles affect the heat transfer and the pumping power requirements? Explain.

22–5C How does a crossflow heat exchanger differ from a counterflow one? What is the difference between mixed and unmixed fluids in crossflow?

The Overall Heat Transfer Coefficient

22–6C What are the heat transfer mechanisms involved during heat transfer in a liquid-to-liquid heat exchanger from the hot to the cold fluid?

22–7C Under what conditions is the thermal resistance of the tube in a heat exchanger negligible?

22–8C Consider a double-pipe parallel-flow heat exchanger of length L. The inner and outer diameters of the inner tube are D_1 and D_2, respectively, and the inner diameter of the outer tube is D_3. Explain how you would determine the two heat transfer surface areas A_i and A_o. When is it reasonable to assume $A_i \approx A_o \approx A_s$?

22–9C How is the thermal resistance due to fouling in a heat exchanger accounted for? How do the fluid velocity and temperature affect fouling?

22–10C In a thin-walled double-pipe heat exchanger, when is the approximation $U = h_i$ a reasonable one? Here U is the overall heat transfer coefficient and h_i is the convection heat transfer coefficient inside the tube.

22–11C What are the common causes of fouling in a heat exchanger? How does fouling affect heat transfer and pressure drop?

22–12C Under what conditions can the overall heat transfer coefficient of a heat exchanger be determined from $U = (1/h_i + 1/h_o)^{-1}$?

22–13E The tube in a heat exchanger has a 2-in inner diameter and a 3-in outer diameter. The thermal conductivity of the tube material is 0.5 Btu/h·ft·°F, while the inner surface heat transfer coefficient is 50 Btu/h·ft^2·°F and the outer surface heat transfer coefficient is 10 Btu/h·ft^2·°F. Determine the overall heat transfer coefficients based on the outer and inner surfaces.

22–14 A long, thin-walled double-pipe heat exchanger with tube and shell diameters of 1.0 cm and 2.5 cm, respectively, is used to condense refrigerant-134a with water at 20°C. The refrigerant flows through the tube, with a convection heat transfer coefficient of $h_i = 4100$ W/m^2·K. Water flows through the shell at a rate of 0.3 kg/s. Determine the overall heat transfer coefficient of this heat exchanger.

22–15 Repeat Prob. 22–14 by assuming a 2-mm-thick layer of limestone ($k = 1.3$ W/m·K) forms on the outer surface of the inner tube.

22–16 💻 Reconsider Prob. 22–15. Using appropriate software, plot the overall heat transfer coefficient as a function of the limestone thickness as it varies from 1 mm to 3 mm, and discuss the results.

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*Problems designated by a “C” are concept questions, and students are encouraged to answer them all. Problems designated by an “E” are in English units, and SI users can ignore them. Problems with the 💻 icon are comprehensive in nature and are intended to be solved with appropriate software.

22–17E Water at an average temperature of 180°F and an average velocity of 4 ft/s flows through a thin-walled $\frac{3}{4}$-in-diameter tube. The water is cooled by air that flows across the tube with a velocity of 12 ft/s at an average temperature of 80°F. Determine the overall heat transfer coefficient.

22–18 A jacketed-agitated vessel, fitted with a turbine agitator, is used for heating a water stream from 10°C to 54°C. The average heat transfer coefficient for water at the vessel's inner wall can be estimated from $\text{Nu} = 0.76\,\text{Re}^{2/3}\,\text{Pr}^{1/3}$. Saturated steam at 100°C condenses in the jacket, for which the average heat transfer coefficient in kW/m^2·K is $h_o = 13.1(T_g - T_w)^{-0.25}$. The vessel dimensions are: $D_t = 0.6$ m, $H = 0.6$ m, and $D_a = 0.2$ m. The agitator speed is 60 rpm. Calculate the mass rate of water that can be heated in this agitated vessel steadily.

22–19E A counterflow heat exchanger is stated to have an overall heat transfer coefficient based on outside tube area of 50 Btu/h·ft^2·°F when operating at design and clean conditions. After a period of use, scale buildup in the heat exchanger gives a fouling factor of 0.002 h·ft^2·°F/Btu. Determine (*a*) the overall heat transfer coefficient of the heat exchanger and (*b*) the percentage change in the overall heat transfer coefficient due to the scale buildup.

22–20 Water at an average temperature of 110°C and an average velocity of 3.5 m/s flows through a 7-m-long stainless steel tube ($k = 14.2$ W/m·K) in a boiler. The inner and outer diameters of the tube are $D_i = 1.0$ cm and $D_o = 1.4$ cm, respectively. If the convection heat transfer coefficient at the outer surface of the tube where boiling is taking place is $h_o = 7200$ W/m^2·K, determine the overall heat transfer coefficient U_i of this boiler based on the inner surface area of the tube.

22–21 Repeat Prob. 22–20, assuming a fouling factor $R_{f,\,i} = 0.0005$ m^2·K/W on the inner surface of the tube.

22–22 Reconsider Prob. 22–21. Using appropriate software, plot the overall heat transfer coefficient based on the inner surface as a function of fouling factor as it varies from 0.0001 m^2·K/W to 0.0008 m^2·K/W, and discuss the results.

22–23 Hot engine oil with a heat capacity rate of 4440 W/K (product of mass flow rate and specific heat) and an inlet temperature of 150°C flows through a double-pipe heat exchanger. The double-pipe heat exchanger is constructed of a 1.5-m-long copper pipe ($k = 250$ W/m·K) with an inner tube of inside diameter 2 cm and outside tube diameter of 2.25 cm. The inner diameter of the outer tube of the double-pipe heat exchanger is 6 cm. Oil flowing at a rate of 2 kg/s through the inner tube exits the heat exchanger at a temperature of 50°C. The cold fluid, i.e., water, enters the heat exchanger at 20°C and exits at 70°C. Assuming the fouling factor on the oil side and water side to be 0.00015 m^2·K/W and 0.0001 m^2·K/W, respectively, determine the overall heat transfer coefficient on the inner and outer surfaces of the copper tube.

Analysis of Heat Exchangers

22–24C What are the common approximations made in the analysis of heat exchangers?

22–25C Under what conditions is the heat transfer relation

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,\,\text{out}} - T_{c,\,\text{in}}) = \dot{m}_h c_{ph}(T_{h,\,\text{in}} - T_{h,\,\text{out}})$$

valid for a heat exchanger?

22–26C Consider a condenser in which steam at a specified temperature is condensed by rejecting heat to the cooling water. If the heat transfer rate in the condenser and the temperature rise of the cooling water are known, explain how the rate of condensation of the steam and the mass flow rate of the cooling water can be determined. Also, explain how the total thermal resistance *R* of this condenser can be evaluated in this case.

22–27C What is the heat capacity rate? What can you say about the temperature changes of the hot and cold fluids in a heat exchanger if both fluids have the same capacity rate? What does a heat capacity of infinity for a fluid in a heat exchanger mean?

22–28C Under what conditions will the temperature rise of the cold fluid in a heat exchanger be equal to the temperature drop of the hot fluid?

The Log Mean Temperature Difference Method

22–29C In the heat transfer relation $\dot{Q} = UA_s\,\Delta T_{\text{lm}}$ for a heat exchanger, what is ΔT_{lm} called? How is it calculated for a parallel-flow and a counterflow heat exchanger?

22–30C How does the log mean temperature difference for a heat exchanger differ from the arithmetic mean temperature difference? For specified inlet and outlet temperatures, which one of these two quantities is larger?

22–31C The temperature difference between the hot and cold fluids in a heat exchanger is given to be ΔT_1 at one end and ΔT_2 at the other end. Can the logarithmic temperature difference ΔT_{lm} of this heat exchanger be greater than both ΔT_1 and ΔT_2? Explain.

22–32C Can the outlet temperature of the cold fluid in a heat exchanger be higher than the outlet temperature of the hot fluid in a parallel-flow heat exchanger? How about in a counterflow heat exchanger? Explain.

22–33C Explain how the LMTD method can be used to determine the heat transfer surface area of a multipass shell-and-tube heat exchanger when all the necessary information, including the outlet temperatures, is given.

22–34C In the heat transfer relation $\dot{Q} = UA_s F\,\Delta T_{\text{lm}}$ for a heat exchanger, what is the quantity *F* called? What does it represent? Can *F* be greater than 1?

22–35C For specified inlet and outlet temperatures, for what kind of heat exchanger will the ΔT_{lm} be greatest: double-pipe parallel-flow, double-pipe counterflow, crossflow, or multipass shell-and-tube heat exchanger?

22–36 A double-pipe parallel-flow heat exchanger is used to heat cold tap water with hot water. Hot water ($c_p = 4.25$ kJ/kg·K) enters the tube at 85°C at a rate of 1.4 kg/s and leaves at 50°C. The heat exchanger is not well insulated, and it is estimated that 3 percent of the heat given up by the hot fluid is lost from the heat exchanger. If the overall heat transfer coefficient and the surface area of the heat exchanger are 1150 W/m^2·K and 4 m^2, respectively, determine the rate of heat transfer to the cold water and the log mean temperature difference for this heat exchanger.

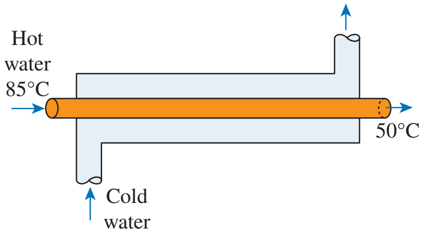


FIGURE P22–36

22–37 A stream of hydrocarbon ($c_p = 2.2$ kJ/kg·K) is cooled at a rate of 720 kg/h from 150°C to 40°C in the tube side of a double-pipe counterflow heat exchanger. Water ($c_p = 4.18$ kJ/kg·K) enters the heat exchanger at 10°C at a rate of 540 kg/h. The outside diameter of the inner tube is 2.5 cm, and its length is 6.0 m. Calculate the overall heat transfer coefficient.

22–38 A double-pipe parallel-flow heat exchanger is to heat water ($c_p = 4180$ J/kg·K) from 25°C to 60°C at a rate of 0.2 kg/s. The heating is to be accomplished by geothermal water ($c_p = 4310$ J/kg·K) available at 140°C at a mass flow rate of 0.3 kg/s. The inner tube is thin-walled and has a diameter of 0.8 cm. If the overall heat transfer coefficient of the heat exchanger is 550 W/m²·K, determine the length of the tube required to achieve the desired heating.

22–39 Reconsider Prob. 22–38. Using appropriate software, investigate the effects of temperature and mass flow rate of geothermal water on the length of the tube. Let the temperature vary from 100°C to 200°C, and the mass flow rate from 0.1 kg/s to 0.5 kg/s. Plot the length of the tube as functions of temperature and mass flow rate, and discuss the results.

22–40 Glycerin ($c_p = 2400$ J/kg·K) at 20°C and 0.5 kg/s is to be heated by ethylene glycol ($c_p = 2500$ J/kg·K) at 70°C in a thin-walled double-pipe parallel-flow heat exchanger. The temperature difference between the two fluids is 15°C at the outlet of the heat exchanger. If the overall heat transfer coefficient is 240 W/m²·K and the heat transfer surface area is 3.2 m², determine (*a*) the rate of heat transfer, (*b*) the outlet temperature of the glycerin, and (*c*) the mass flow rate of the ethylene glycol.

22–41 A heat exchanger contains 400 tubes with inner diameter of 23 mm and outer diameter of 25 mm. The length of each tube is 3.7 m. The corrected log mean temperature difference is 23°C, while the inner surface convection heat transfer coefficient is 3410 W/m²·K and the outer surface convection heat transfer coefficient is 6820 W/m²·K. If the thermal resistance of the tubes is negligible, determine the heat transfer rate.

22–42E A single-pass heat exchanger is to be designed to heat 100,000 lbm of water in an hour from 60°F to 100°F by condensation of water vapor at 230°F on the shell side. Each tube has an inner diameter of 1.2 in and a wall thickness of 0.12 in. The inner surface convection heat transfer coefficient is 480 Btu/h·ft²·°F and the outer surface convection heat transfer coefficient is 2000 Btu/h·ft²·°F. If the inlet velocity of water ($c_{pc} = 1$ Btu/lbm·°F and $\rho = 62.3$ lbm/ft³) on the tube side is 4 ft/s, determine the required number of tubes and length of tubes. Assume the thermal resistance of the tubes is negligible.

22–43 Consider the flow of engine oil ($c_p = 2048$ J/kg·K) through a thin-walled copper tube at a rate of 0.3 kg/s. The engine oil that enters the copper tube at an inlet temperature of 80°C is to be cooled by cold water ($c_p = 4180$ J/kg·K) flowing at a temperature of 20°C. We wish to have the exit temperature of the engine oil not exceed 40°C. The individual convective heat transfer coefficients on oil and water sides are 750 W/m²·K and 350 W/m²·K, respectively. If a thermocouple probe installed on the downstream side of the cooling water measures a temperature of 32°C, for a double-pipe parallel-flow heat exchanger determine (*a*) the mass flow rate of the cooling water, (*b*) the log mean temperature difference, and (*c*) the area of the heat exchanger.

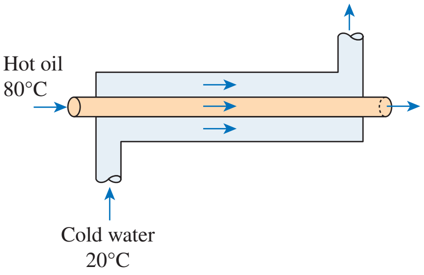


FIGURE P22–43

22–44 In a parallel-flow heat exchanger, hot fluid enters the heat exchanger at a temperature of 150°C and a mass flow rate of 3 kg/s. The cooling medium enters the heat exchanger at a temperature of 30°C with a mass flow rate of 0.5 kg/s and leaves at a temperature of 70°C. The specific heat capacities of the hot and cold fluids are 1150 J/kg·K and 4180 J/kg·K, respectively. The convection heat transfer coefficient on the inner and outer sides of the tube are 300 W/m²·K and 800 W/m²·K, respectively. For a fouling factor of 0.0003 m²·K/W on the tube side and 0.0001 m²·K/W on the shell side, determine (*a*) the overall heat transfer coefficient, (*b*) the exit temperature of the hot fluid, and (*c*) the surface area of the heat exchanger.

22–45 Ethylene glycol is heated from 25°C to 40°C at a rate of 2.5 kg/s in a horizontal copper tube ($k = 386$ W/m·K) with an inner diameter of 2.0 cm and an outer diameter of 2.5 cm. A saturated vapor ($T_g = 110$°C) condenses on the outside-tube surface with the heat transfer coefficient (in kW/m²·K) given by $9.2/(T_g - T_w)^{0.25}$, where T_w is the average outside-tube wall temperature. What tube length must be used? Take the properties of ethylene glycol to be $\rho = 1109$ kg/m³, $c_p = 2428$ J/kg·K, $k = 0.253$ W/m·K, $\mu = 0.01545$ kg/m·s, and Pr = 148.5.

22–46 A counterflow heat exchanger is stated to have an overall heat transfer coefficient of 284 W/m²·K when operating at design and clean conditions. Hot fluid enters the tube side at 93°C and exits at 71°C, while cold fluid enters the shell side at 27°C and exits at 38°C. After a period of use, built-up scale in the heat exchanger gives a fouling factor of 0.0004 m²·K/W. If the surface area is 93 m², determine (*a*) the rate of heat transfer in the heat exchanger and (*b*) the mass flow rates of both hot and cold fluids. Assume both hot and cold fluids have a specific heat of 4.2 kJ/kg·K.

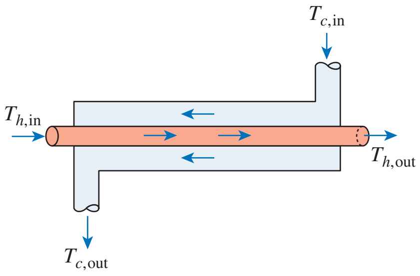


FIGURE P22–46

22–47 A thin-walled double-pipe counterflow heat exchanger is to be used to cool oil ($c_p = 2200$ J/kg·K) from 150°C to 50°C at a rate of 2.5 kg/s with water ($c_p = 4180$ J/kg·K) that enters at 22°C at a rate of 1.5 kg/s. The diameter of the tube is 2.5 cm, and its length is 6 m. Determine the overall heat transfer coefficient of this heat exchanger.

22–48 Reconsider Prob. 22–47. Using appropriate software, investigate the effects of oil exit temperature and water inlet temperature on the overall heat transfer coefficient of the heat exchanger. Let the oil exit temperature vary from 30°C to 70°C and the water inlet temperature from 5°C to 25°C. Plot the overall heat transfer coefficient as a function of the two temperatures, and discuss the results.

22–49 Cold water ($c_p = 4180$ J/kg·K) leading to a shower enters a thin-walled double-pipe counterflow heat exchanger at 15°C at a rate of 1.25 kg/s and is heated to 60°C by hot water ($c_p = 4190$ J/kg·K) that enters at 100°C at a rate of 4 kg/s. If the overall heat transfer coefficient is 880 W/m^2·K, determine the rate of heat transfer and the heat transfer surface area of the heat exchanger.

22–50E Steam is to be condensed on the shell side of a one-shell-pass and eight-tube-passes condenser, with 60 tubes in each pass at 90°F ($h_{fg} = 1043$ Btu/lbm). Cooling water ($c_p = 1.0$ Btu/lbm·°F) enters the tubes at 55°F and leaves at 70°F. The tubes are thin walled and have a diameter of 3/4 in and a length of 5 ft per pass. If the overall heat transfer coefficient is 600 Btu/h·ft^2·°F, determine (*a*) the rate of heat transfer, (*b*) the rate of condensation of steam, and (*c*) the mass flow rate of the cold water.

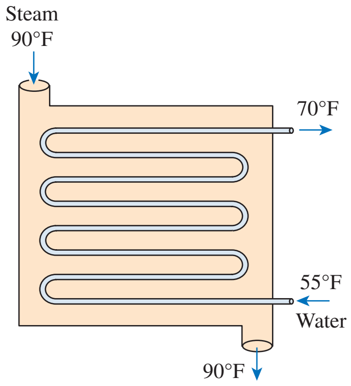


FIGURE P22–50E

22–51E Reconsider Prob. 22–50E. Using appropriate software, investigate the effect of the condensing steam temperature on the rate of heat transfer, the rate of condensation of steam, and the mass flow rate of the cold water. Let the steam temperature vary from 80°F to 120°F. Plot the rate of heat transfer, the condensation rate of steam, and the mass flow rate of the cold water as a function of steam temperature, and discuss the results.

22–52 Hot exhaust gases of a stationary diesel engine are to be used to generate steam in an evaporator. Exhaust gases ($c_p = 1051$ J/kg·K) enter the heat exchanger at 550°C at a rate of 0.25 kg/s while water enters as saturated liquid and evaporates at 200°C ($h_{fg} = 1941$ kJ/kg). The heat transfer surface area of the heat exchanger based on the water side is 0.5 m^2, and the overall heat transfer coefficient is 1780 W/m^2·K. Determine the rate of heat transfer, the exit temperature of the exhaust gases, and the rate of evaporation of the water.

22–53 Reconsider Prob. 22–52. Using appropriate software, investigate the effect of the exhaust gas inlet temperature on the rate of heat transfer, the exit temperature of exhaust gases, and the rate of evaporation of water. Let the temperature of exhaust gases vary from 300°C to 600°C. Plot the rate of heat transfer, the exit temperature of exhaust gases, and the rate of evaporation of the water as a function of the temperature of the exhaust gases, and discuss the results.

22–54 In a textile manufacturing plant, the waste dyeing water ($c_p = 4295$ J/kg·K) at 80°C is to be used to preheat fresh water ($c_p = 4180$ J/kg·K) at 10°C at the same flow rate in a double-pipe counterflow heat exchanger. The heat transfer surface area of the heat exchanger is 1.65 m^2, and the overall heat transfer coefficient is 625 W/m^2·K. If the rate of heat transfer in the heat exchanger is 35 kW, determine the outlet temperature and the mass flow rate of each fluid stream.

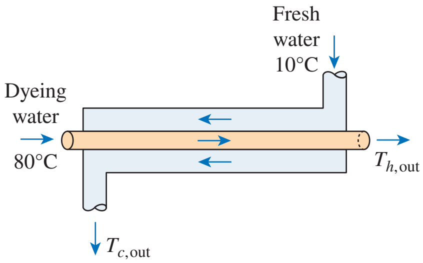


FIGURE P22–54

22–55 A performance test is being conducted on a double-pipe counterflow heat exchanger that carries engine oil and water at a flow rate of 2.5 kg/s and 1.75 kg/s, respectively. Since the heat exchanger has been in service for a long time, it is suspected that fouling might have developed inside the heat exchanger that could affect the overall heat transfer coefficient. The test to be carried out is such that, for a designed value of the overall heat transfer coefficient of 450 W/m^2·K and a surface area of 7.5 m^2, the oil must be heated from 25°C to 55°C by passing hot water at 100°C ($c_p = 4206$ J/kg·K) at the flow rates mentioned above. Determine if the fouling has affected the overall heat transfer coefficient. If yes, then what is the magnitude of the fouling resistance?

22–56 In an industrial facility, a counterflow double-pipe heat exchanger uses superheated steam at a temperature of 250°C to heat feedwater at 30°C. The superheated steam experiences a temperature drop of 70°C as it exits the heat exchanger. The water to be heated flows through the heat exchanger tube of negligible thickness at a constant rate of 3.47 kg/s. The convective heat transfer coefficient on the superheated steam and water side is 850 W/m^2·K and 1250 W/m^2·K, respectively. To account for the fouling due to chemical impurities that might be present in the feedwater, use an appropriate fouling factor for the water side. Determine (*a*) the heat exchanger area required to maintain the exit temperature of the water at a minimum of 70°C. (*b*) What would be the required heat exchanger area in the case of a parallel flow arrangement?

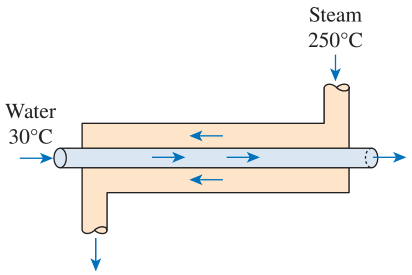


FIGURE P22–56

22–57 A test is conducted to determine the overall heat transfer coefficient in a shell-and-tube oil-to-water heat exchanger that has 24 tubes of internal diameter 1.2 cm and length 2 m in a single shell. Cold water ($c_p = 4180$ J/kg·K) enters the tubes at 20°C at a rate of 3 kg/s and leaves at 55°C. Oil ($c_p = 2150$ J/kg·K) flows through the shell and is cooled from 120°C to 45°C. Determine the overall heat transfer coefficient U_i of this heat exchanger based on the inner surface area of the tubes. *Answer:* 8.31 kW/m^2·K

22–58 A shell-and-tube heat exchanger is used for heating 14 kg/s of oil ($c_p = 2.0$ kJ/kg·K) from 20°C to 46°C. The heat exchanger has one shell pass and six tube passes. Water enters the shell side at 80°C and leaves at 60°C. The overall heat transfer coefficient is estimated to be 1000 W/m^2·K. Calculate the rate of heat transfer and the heat transfer area.

22–59E A one-shell-pass and eight-tube-passes heat exchanger is used to heat glycerin ($c_p = 0.60$ Btu/lbm·°F) from 80°F to 140°F by hot water ($c_p = 1.0$ Btu/lbm·°F) that enters the thin-walled 0.5-in-diameter tubes at 175°F and leaves at 120°F. The total length of the tubes in the heat exchanger is 400 ft. The convection heat transfer coefficient is 4 Btu/h·ft^2·°F on the glycerin (shell) side and 50 Btu/h·ft^2·°F on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (*a*) before any fouling occurs and (*b*) after fouling with a fouling factor of 0.002 h·ft^2·°F/Btu on the outer surfaces of the tubes.

22–60 A shell-and-tube heat exchanger with two shell passes and 12 tube passes is used to heat water ($c_p = 4180$ J/kg·K) with ethylene glycol ($c_p = 2680$ J/kg·K). Water enters the tubes at 22°C at a rate of 0.8 kg/s and leaves at 70°C. Ethylene glycol enters the shell at 110°C and leaves at 60°C. If the overall heat transfer coefficient based on the tube side is 280 W/m^2·K, determine the rate of heat transfer and the heat transfer surface area on the tube side.

22–61 Reconsider Prob. 22–60. Using appropriate software, investigate the effect of the mass flow rate of water on the rate of heat transfer and the tube-side surface area. Let the mass flow rate vary from 0.4 kg/s to 2.2 kg/s. Plot the rate of heat transfer and the surface area as a function of the mass flow rate, and discuss the results.

22–62 A shell-and-tube heat exchanger with two shell passes and 12 tube passes is used to heat water ($c_p = 4180$ J/kg·K) in the tubes from 20°C to 70°C at a rate of 4.5 kg/s. Heat is supplied by hot oil ($c_p = 2300$ J/kg·K) that enters the shell side at 170°C at a rate of 10 kg/s. For a tube-side overall heat transfer coefficient of 350 W/m^2·K, determine the heat transfer surface area on the tube side. *Answer:* 25.7 m^2

22–63 Repeat Prob. 22–62 for a mass flow rate of 3 kg/s for water.

22–64 A shell-and-tube heat exchanger with two shell passes and eight tube passes is used to heat ethyl alcohol ($c_p = 2670$ J/kg·K) in the tubes from 25°C to 70°C at a rate of 2.1 kg/s. The heating is to be done by water ($c_p = 4190$ J/kg·K) that enters the shell side at 95°C and leaves at 45°C. If the overall heat transfer coefficient is 950 W/m^2·K, determine the heat transfer surface area of the heat exchanger.

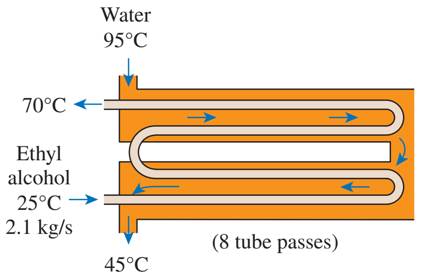


FIGURE P22–64

22–65E A single-pass crossflow heat exchanger is used to cool jacket water ($c_p = 1.0$ Btu/lbm·°F) of a diesel engine from 190°F to 140°F, using air ($c_p = 0.245$ Btu/lbm·°F) with an inlet temperature of 90°F. Both airflow and water flow are unmixed. If the water and air mass flow rates are 92,000 lbm/h and 400,000 lbm/h, respectively, determine the log mean temperature difference for this heat exchanger.

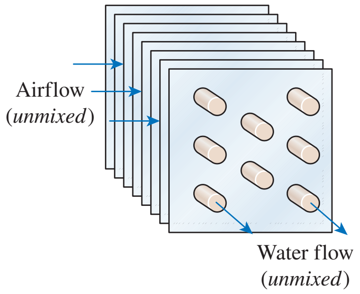


FIGURE P22–65E

The Effectiveness–NTU Method

22–66C What does the effectiveness of a heat exchanger represent? Can effectiveness be greater than 1? On what factors does the effectiveness of a heat exchanger depend?

22–67C For a specified fluid pair, inlet temperatures, and mass flow rates, what kind of heat exchanger will have the highest effectiveness: double-pipe parallel-flow, double-pipe counterflow, crossflow, or multipass shell-and-tube heat exchanger?

22–68C Explain how you can evaluate the outlet temperatures of the cold and hot fluids in a heat exchanger after its effectiveness is determined.

22–69C Can the temperature of the hot fluid drop below the inlet temperature of the cold fluid at any location in a heat exchanger? Explain.

22–70C Consider a heat exchanger in which both fluids have the same specific heats but different mass flow rates. Which fluid will experience a larger temperature change: the one with the lower or higher mass flow rate?

22–71C Under what conditions can a counterflow heat exchanger have an effectiveness of 1? What would your answer be for a parallel-flow heat exchanger?

22–72C Consider a double-pipe counterflow heat exchanger. In order to enhance heat transfer, the length of the heat exchanger is now doubled. Do you think its effectiveness will also double?

22–73C Consider a shell-and-tube water-to-water heat exchanger with identical mass flow rates for both the hot- and cold-water streams. Now the mass flow rate of the cold water is reduced by half. Will the effectiveness of this heat exchanger increase, decrease, or remain the same as a result of this modification? Explain. Assume the overall heat transfer coefficient and the inlet temperatures remain the same.

22–74C Consider two double-pipe counterflow heat exchangers that are identical except that one is twice as long as the other one. Which heat exchanger is more likely to have a higher effectiveness?

22–75C How is the NTU of a heat exchanger defined? What does it represent? Is a heat exchanger with a very large NTU (say, 10) necessarily a good one to buy?

22–76C Consider a heat exchanger that has an NTU of 4. Someone proposes to double the size of the heat exchanger and thus double the NTU to 8 in order to increase the effectiveness of the heat exchanger and thus save energy. Would you support this proposal?

22–77C Consider a heat exchanger that has an NTU of 0.1. Someone proposes to triple the size of the heat exchanger and thus triple the NTU to 0.3 in order to increase the effectiveness of the heat exchanger and thus save energy. Would you support this proposal?

22–78 The radiator in an automobile is a crossflow heat exchanger ($UA_s = 10$ kW/K) that uses air ($c_p = 1.00$ kJ/kg·K) to cool the engine-coolant fluid ($c_p = 4.00$ kJ/kg·K). The engine fan draws 22°C air through this radiator at a rate of 8 kg/s while the coolant pump circulates the engine coolant at a rate of 5 kg/s. The coolant enters this radiator at 80°C. Under these conditions, the effectiveness of the radiator is 0.4. Determine (*a*) the outlet temperature of the air and (*b*) the rate of heat transfer between the two fluids.

22–79 Consider an oil-to-oil double-pipe heat exchanger whose flow arrangement is not known. The temperature measurements indicate that the cold oil enters at 15°C and leaves at 55°C, while the hot oil enters at 80°C and leaves at 40°C. Do you think this is a parallel-flow or counterflow heat exchanger? Why? Assuming the mass flow rates of both fluids to be identical, determine the effectiveness of this heat exchanger.

22–80E Hot water enters a double-pipe counterflow water-to-oil heat exchanger at 190°F and leaves at 100°F. Oil enters at 70°F and leaves at 130°F. Determine which fluid has the smaller heat capacity rate, and calculate the effectiveness of this heat exchanger.

22–81 Glycerin ($c_p = 2400$ J/kg·K) at 20°C and 0.5 kg/s is to be heated by ethylene glycol ($c_p = 2500$ J/kg·K) at 60°C and the same mass flow rate in a thin-walled double-pipe parallel-flow heat exchanger. If the overall heat transfer coefficient is 380 W/m^2·K and the heat transfer surface area is 6.5 m^2, determine (*a*) the rate of heat transfer and (*b*) the outlet temperatures of the glycerin and the glycol.

22–82 Water ($c_p = 4180$ J/kg·K) is to be heated by solar-heated hot air ($c_p = 1010$ J/kg·K) in a double-pipe counterflow heat exchanger. Air enters the heat exchanger at 90°C at a rate of 0.3 kg/s, while water enters at 22°C at a rate of 0.1 kg/s. The overall heat transfer coefficient based on the inner side of the tube is given to be 80 W/m^2·K. The length of the tube is 12 m, and the internal diameter of the tube is 1.2 cm. Determine the outlet temperatures of the water and the air.

22–83 Reconsider Prob. 22–82. Using appropriate software, investigate the effects of the mass flow rate of water and the tube length on the outlet temperatures of water and air. Let the mass flow rate vary from 0.05 kg/s to 1.0 kg/s and the tube length from 5 m to 25 m. Plot the outlet temperatures of the water and the air as functions of the mass flow rate and the tube length, and discuss the results.

22–84 Cold water ($c_p = 4180$ J/kg·K) leading to a shower enters a thin-walled double-pipe counterflow heat exchanger at 15°C at a rate of 0.25 kg/s and is heated to 45°C by hot water ($c_p = 4190$ J/kg·K) that enters at 100°C at a rate of 3 kg/s. If the overall heat transfer coefficient is 950 W/m^2·K, determine the rate of heat transfer and the heat transfer surface area of the heat exchanger using the ε–NTU method. *Answers: 31.35 kW, 0.482 m^2*

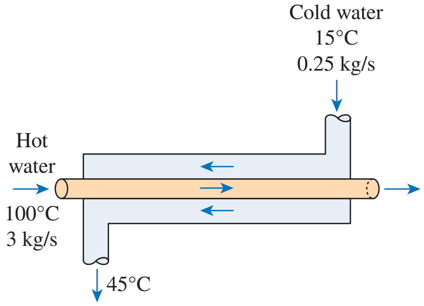


FIGURE P22–84

22–85 Reconsider Prob. 22–84. Using appropriate software, investigate the effects of the inlet temperature of hot water and the heat transfer coefficient on the rate of heat transfer and the surface area. Let the inlet temperature vary from 60°C to 120°C and the overall heat transfer coefficient from 750 W/m^2·K to 1250 W/m^2·K. Plot the rate of heat transfer and surface area as functions of the inlet temperature and the heat transfer coefficient, and discuss the results.

22–86E A thin-walled double-pipe counterflow heat exchanger is to be used to cool oil ($c_p = 0.525$ Btu/lbm·°F) from 300°F to 105°F at a rate of 5 lbm/s with water ($c_p = 1.0$ Btu/lbm·°F) that enters at 70°F at a rate of 3 lbm/s. The diameter of the tube is 5 in, and its length is 200 ft. Determine the overall heat transfer coefficient of this heat exchanger using (*a*) the LMTD method and (*b*) the ε–NTU method.

22–87E A one-shell and two-tube-type heat exchanger has an overall heat transfer coefficient of 300 Btu/h·ft^2·°F. The shell-side fluid has a heat capacity rate of 20,000 Btu/h·°F, while the tube-side fluid has a heat capacity rate of 40,000 Btu/h·°F. The inlet temperatures on the shell side and tube side are 200°F and 90°F, respectively. If the total heat transfer area is 100 ft^2, determine (*a*) the heat transfer effectiveness and (*b*) the actual heat transfer rate in the heat exchanger.

22–88 In a one-shell and two-tube heat exchanger, cold water with inlet temperature of 20°C is heated by hot water supplied at the inlet at 80°C. The cold and hot water flow rates are 5000 kg/h and 10,000 kg/h, respectively. If the shell-and-tube heat exchanger has a UA_s value of 11,600 W/K, determine the cold water and hot water outlet temperatures. Assume $c_{pc} = 4178$ J/kg·K and $c_{ph} = 4188$ J/kg·K.

22–89 Hot oil ($c_p = 2200$ J/kg·K) is to be cooled by water ($c_p = 4180$ J/kg·K) in a two-shell-passes and 12-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with a diameter of 1.8 cm. The length of each tube pass in the heat exchanger is 3 m, and the overall heat transfer coefficient is 340 W/m^2·K. Water flows through the tubes at a total rate of 0.1 kg/s, and the oil flows through the shell at a rate of 0.2 kg/s. The water and the oil enter at temperatures 18°C and 160°C, respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil. *Answers:* 36.2 kW, 104.6°C, 77.7°C

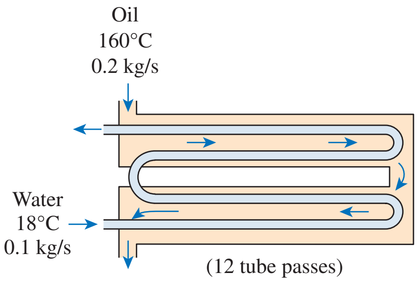


FIGURE P22–89

22–90E Oil is being cooled from 180°F to 120°F in a one-shell and two-tube heat exchanger with an overall heat transfer coefficient of 40 Btu/h·ft^2·°F. Water ($c_{pc} = 1.0$ Btu/lbm·°F) enters at 80°F and exits at 100°F with a mass flow rate of 20,000 lbm/h. Determine (*a*) the NTU value and (*b*) the surface area of the heat exchanger.

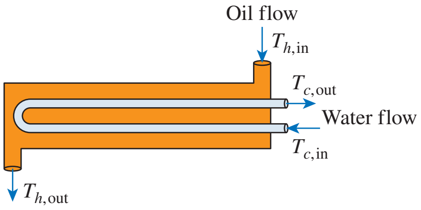


FIGURE P22–90E

22–91 A shell-and-tube heat exchanger with two shell passes and eight tube passes is used to heat ethyl alcohol ($c_p = 2670$ J/kg·K) in the tubes from 25°C to 70°C at a rate of 2.1 kg/s. The heating is to be done by water ($c_p = 4190$ J/kg·K) that enters the shell at 95°C and leaves at 60°C. If the overall heat transfer coefficient is 800 W/m^2·K, determine the heat transfer surface area of the heat exchanger using (*a*) the LMTD method and (*b*) the ε–NTU method. *Answer:* (*a*) 11.4 m^2

22–92 Cold water ($c_p = 4180$ J/kg·K) enters the tubes of a heat exchanger with two shell passes and 23 tube passes at 14°C at a rate of 3 kg/s, while hot oil ($c_p = 2200$ J/kg·K) enters the shell at 200°C at the same mass flow rate. The overall heat transfer coefficient based on the outer surface of the tube is 300 W/m^2·K, and the heat transfer surface area on that side is 20 m^2. Determine the rate of heat transfer using (*a*) the LMTD method and (*b*) the ε–NTU method.

22–93E In a one-shell and eight-tube-pass heat exchanger, the temperature of water flowing at rate of 50,000 lbm/h is raised from 70°F to 150°F. Hot air ($c_p = 0.25$ Btu/lbm·°F) that flows on the tube side enters the heat exchanger at 600°F and exits at 300°F. If the convection heat transfer coefficient on the outer surface of the tubes is 30 Btu/h·ft^2·°F, determine the surface area of the heat exchanger using both LMTD and ε–NTU methods. Account for the possible fouling resistance of 0.0015 and 0.001 h·ft^2·°F/Btu on the water and air sides, respectively.

22–94 Air ($c_p = 1005$ J/kg·K) enters a crossflow heat exchanger at 20°C at a rate of 3 kg/s, where it is heated by a hot water stream ($c_p = 4190$ J/kg·K) that enters the heat exchanger at 70°C at a rate of 1 kg/s. Determine the maximum heat transfer rate and the outlet temperatures of both fluids for that case.

22–95 A crossflow heat exchanger with both fluids unmixed has an overall heat transfer coefficient of 200 W/m^2·K and a heat transfer surface area of 400 m^2. The hot fluid has a heat capacity of 40,000 W/K, while the cold fluid has a heat capacity of 80,000 W/K. If the inlet temperatures of both hot and cold fluids are 80°C and 20°C, respectively, determine the exit temperature of the cold fluid.

22–96 A crossflow air-to-water heat exchanger with an effectiveness of 0.65 is used to heat water ($c_p = 4180$ J/kg·K) with hot air ($c_p = 1010$ J/kg·K). Water enters the heat exchanger at 20°C at a rate of 4 kg/s, while air enters at 100°C at a rate of 9 kg/s. If the overall heat transfer coefficient based on the water side is 260 W/m^2·K, determine the heat transfer surface area of the heat exchanger on the water side. Assume both fluids are unmixed. *Answer:* 52.4 m^2

22–97 Cold water ($c_p = 4.18$ kJ/kg·K) enters a crossflow heat exchanger at 14°C at a rate of 0.35 kg/s where it is heated by hot air ($c_p = 1.0$ kJ/kg·K) that enters the heat exchanger at 65°C at a rate of 0.8 kg/s and leaves at 25°C. Determine the maximum outlet temperature of the cold water and the effectiveness of this heat exchanger.

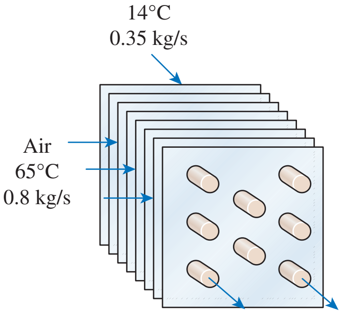


FIGURE P22–97

22–98 Oil in an engine is being cooled by air in a crossflow heat exchanger, where both fluids are unmixed. Oil ($c_{ph} = 2047$ J/kg·K) flowing with a flow rate of 0.026 kg/s enters the heat exchanger at 75°C, while air ($c_{pc} = 1007$J/kg·K) enters at 30°C with a flow rate of 0.21 kg/s. The overall heat transfer coefficient of the heat exchanger is 53 W/m²·K and the total surface area is 1 m². Determine (*a*) the heat transfer effectiveness and (*b*) the outlet temperature of the oil.

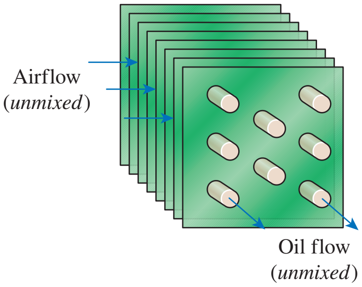


FIGURE P22–98

22–99 Consider a recuperative crossflow heat exchanger (both fluids unmixed) used in a gas turbine system that carries the exhaust gases at a flow rate of 7.5 kg/s and a temperature of 500°C. The air initially at 30°C and flowing at a rate of 15 kg/s is to be heated in the recuperator. The convective heat transfer coefficients on the exhaust gas and air sides are 750 W/m²·K and 300 W/m²·K, respectively. Due to long-term use of the gas turbine, the recuperative heat exchanger is subject to fouling on both gas and air sides that offers a resistance of 0.0004 m²·K/W each. Take the properties of exhaust gas to be the same as that of air ($c_p = 1069$ J/kg·K). If the exit temperature of the exhaust gas is 320°C, determine (*a*) if the air could be heated to a temperature of 150°C and (*b*) the area of the heat exchanger. (*c*) If the answer to part (*a*) is no, then determine what should be the air mass flow rate in order to attain the desired exit temperature of 150°C and (*d*) plot the variation of the exit air temperature over a range of 75°C to 300°C with the air mass flow rate, assuming all the other conditions remain the same.

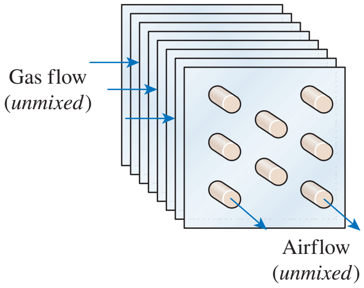


FIGURE P22–99

22–100 Consider the operation of a single-pass crossflow heat exchanger with water ($c_p = 4193$ J/kg·K) (mixed) and methanol ($c_p = 2577$ J/kg·K) (unmixed). Water entering and exiting the heat exchanger at 90°C and 60°C, respectively is used to heat the methanol, which is initially at 10°C. The overall heat transfer coefficient and the total heat transfer rate are estimated to be 650 W/m²·K and 250 kW, respectively. In order to achieve an effectiveness of at least 0.5, (*a*) what would be the mass flow rate of water and the heat exchanger surface area? (*b*) If the mass flow rate of water is changed within ±30 percent of that determined in part (*a*) with the same surface area (A_s), plot the variation of heat transfer rate, methanol exit temperature, overall heat transfer coefficient, and the effectiveness of the heat exchanger with respect to change in the water flow rate.

22–101 Water ($c_p = 4180$ J/kg·K) enters the 2.5-cm-internal-diameter tube of a double-pipe counterflow heat exchanger at 20°C at a rate of 2.2 kg/s. Water is heated by steam condensing at 120°C ($h_{fg} = 2203$ kJ/kg) in the shell. If the overall heat transfer coefficient of the heat exchanger is 700 W/m²·K, determine the length of the tube required in order to heat the water to 80°C using (*a*) the LMTD method and (*b*) the ε–NTU method.

22–102 Ethanol is vaporized at 78°C ($h_{fg} = 846$ kJ/kg) in a double-pipe parallel-flow heat exchanger at a rate of 0.04 kg/s by hot oil ($c_p = 2200$ J/kg·K) that enters at 115°C. If the heat transfer surface area and the overall heat transfer coefficients are 6.2 m² and 320 W/m²·K, respectively, determine the outlet temperature and the mass flow rate of oil using (*a*) the LMTD method and (*b*) the ε–NTU method.

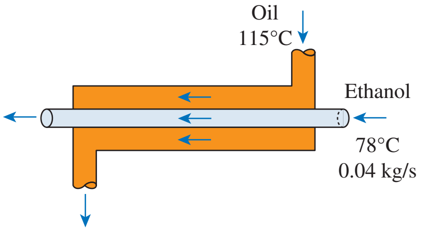


FIGURE P22–102

22–103 Saturated water vapor at 100°C condenses in a one-shell and two-tube heat exchanger with a surface area of 0.5 m² and an overall heat transfer coefficient of 2000 W/m²·K. Cold water ($c_{pc} = 4179$ J/kg·K) flowing at 0.5 kg/s enters the tube side at 15°C. Determine (*a*) the heat transfer effectiveness, (*b*) the outlet temperature of the cold water, and (*c*) the heat transfer rate for the heat exchanger.

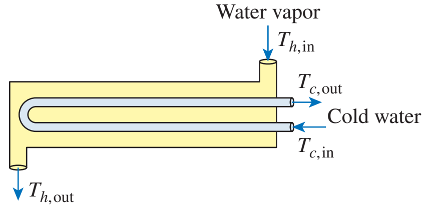


FIGURE P22–103

22–104 Steam is to be condensed on the shell side of a one-shell-pass and eight-tube-passes condenser, with 50 tubes in each pass, at 30°C (h_{fg} = 2431 kJ/kg). Cooling water (c_p = 4180 J/kg·K) enters the tubes at 18°C at a rate of 2200 kg/h. The tubes are thin-walled and have a diameter of 1.5 cm and a length of 2 m per pass. If the overall heat transfer coefficient is 3000 W/m^2·K, determine (*a*) the rate of heat transfer and (*b*) the rate of condensation of steam.

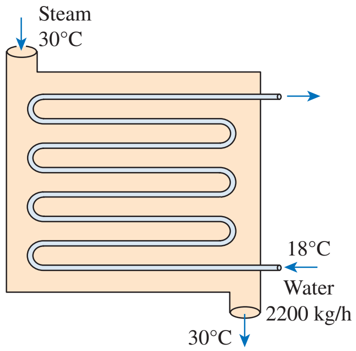


FIGURE P22–104

22–105 Reconsider Prob. 22–104. Using appropriate software, investigate the effects of the condensing steam temperature and the tube diameter on the rate of heat transfer and the rate of condensation of steam. Let the steam temperature vary from 20°C to 70°C and the tube diameter from 1.0 cm to 2.0 cm. Plot the rate of heat transfer and the rate of condensation as functions of steam temperature and tube diameter, and discuss the results.

22–106E Consider a closed-loop heat exchanger that carries exit water (c_p = 1 Btu/lbm·°F and ρ = 62.4 lbm/ft^3) of a condenser side initially at 100°F. The water flows through a 500-ft-long stainless steel pipe of 1 in inner diameter immersed in a large lake. The temperature of lake water surrounding the heat exchanger is 45°F. The overall heat transfer coefficient of the heat exchanger is estimated to be 250 Btu/h·ft^2·°F. What is the exit temperature of the water from the immersed heat exchanger if it flows through the pipe at an average velocity of 9 ft/s? Use the ε–NTU method for analysis.

22–107 Consider the flow of saturated steam at 270.1 kPa that flows through the shell side of a shell-and-tube heat exchanger while the water flows through four tubes of diameter 1.25 cm at a rate of 0.25 kg/s through each tube. The water enters the tubes of the heat exchanger at 20°C and exits at 60°C. Due to the heat exchange with the cold fluid, steam is condensed on the tube's external surface. The convection heat transfer coefficient on the steam side is 1500 W/m^2·K, while the fouling resistance for the steam and water may be taken as 0.00015 and 0.0001 m^2·K/W, respectively. Using the NTU method, determine (*a*) the effectiveness of the heat exchanger, (*b*) the length of the tube, and (*c*) the rate of steam condensation.

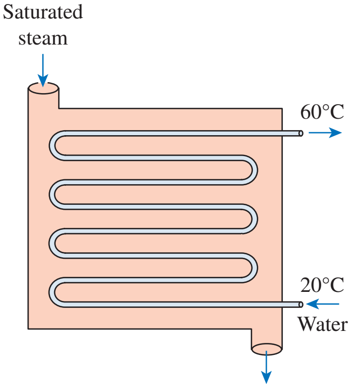


FIGURE P22–107

Review Problems

22–108 Consider a water-to-water double-pipe heat exchanger whose flow arrangement is not known. The temperature measurements indicate that the cold water enters at 20°C and leaves at 50°C, while the hot water enters at 80°C and leaves at 45°C. Do you think this is a parallel-flow or counterflow heat exchanger? Explain.

22–109 By taking the limit as $\Delta T_2 \to \Delta T_1$, show that when $\Delta T_1 = \Delta T_2$ for a heat exchanger, the ΔT_{lm} relation reduces to $\Delta T_{\text{lm}} = \Delta T_1 = \Delta T_2$.

22–110E Water (c_p = 1.0 Btu/lbm·°F) is to be heated by solar-heated hot air (c_p = 0.24 Btu/lbm·°F) in a double-pipe counter-flow heat exchanger. Air enters the heat exchanger at 190°F at a rate of 0.7 lbm/s and leaves at 135°F. Water enters at 70°F at a rate of 0.35 lbm/s. The overall heat transfer coefficient based on the inner side of the tube is given to be 20 Btu/h·ft^2·°F. Determine the length of the tube required for a tube internal diameter of 0.5 in.

22–111 A shell-and-tube heat exchanger with one shell pass and 14 tube passes is used to heat water in the tubes with geothermal steam condensing at 120°C (h_{fg} = 2203 kJ/kg) on the shell side. The tubes are thin-walled and have a diameter of 2.4 cm and a length of 3.2 m per pass. Water (c_p = 4180 J/kg·K) enters the tubes at 18°C at a rate of 6.2 kg/s. If the temperature difference between the two fluids at the exit is 46°C, determine (*a*) the rate of heat transfer, (*b*) the rate of condensation of steam, and (*c*) the overall heat transfer coefficient.

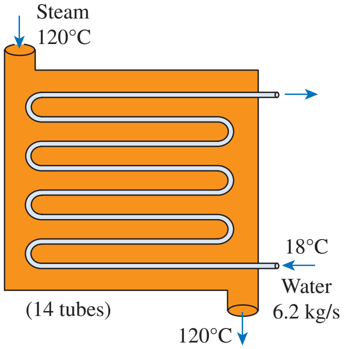


FIGURE P22–111

22–112 Hot water at 60°C is cooled to 36°C through the tube side of a one-shell-pass and two-tube-passes heat exchanger. The coolant is also a water stream, for which the inlet and outlet temperatures are 7°C and 31°C, respectively. The overall heat transfer coefficient and the heat transfer area are 950 W/m^2·K and 15 m^2, respectively. Calculate the mass flow rates of hot and cold water streams in steady operation.

22–113 Cold water ($c_p = 4180$ J/kg·K) enters the tubes of a heat exchanger with two shell passes and 20 tube passes at 15°C at a rate of 4 kg/s, while hot oil ($c_p = 2200$ J/kg·K) enters the shell at 130°C at the same mass flow rate and leaves at 60°C. If the overall heat transfer coefficient based on the outer surface of the tube is 220 W/m^2·K, determine (*a*) the rate of heat transfer and (*b*) the heat transfer surface area on the outer side of the tube.

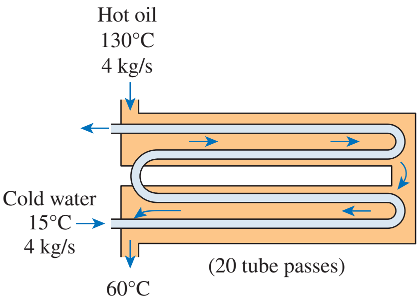


FIGURE P22–113

22–114 A shell-and-tube heat exchanger is used for cooling 47 kg/s of a process stream flowing through the tubes from 160°C to 100°C. This heat exchanger has a total of 100 identical tubes, each with an inside diameter of 2.5 cm and negligible wall thickness. The average properties of the process stream are: $\rho = 950$ kg/m^3, $k = 0.50$ W/m·K, $c_p = 3.5$ kJ/kg·K, and $\mu = 0.002$ kg/m·s. The coolant stream is water $c_p = 4.18$ kJ/kg·K at a flow rate of 66 kg/s and an inlet temperature of 10°C, which yields an average shell-side heat transfer coefficient of 4.0 kW/m^2·K. Calculate the tube length if the heat exchanger has (*a*) one shell pass and one tube pass and (*b*) one shell pass and four tube passes.

22–115 A two-shell-pass and four-tube-pass heat exchanger is used for heating a hydrocarbon stream ($c_p = 2.0$ kJ/kg·K) steadily from 20°C to 50°C. A water stream enters the shell side at 80°C and leaves at 40°C. There are 160 thin-walled tubes, each with a diameter of 2.0 cm and length of 1.5 m. The tube-side and shell-side heat transfer coefficients are 1.6 and 2.5 kW/m^2·K, respectively. (*a*) Calculate the rate of heat transfer and the mass rates of water and hydrocarbon streams. (*b*) With usage, the outlet hydrocarbon-stream temperature was found to decrease by 5°C due to the deposition of solids on the tube surface. Estimate the magnitude of the fouling factor.

22–116 A shell-and-tube heat exchanger with two shell passes and four tube passes is used for cooling oil ($c_p = 2.0$ kJ/kg·K) from 125°C to 55°C. The coolant is water, which enters the shell side at 25°C and leaves at 46°C. The overall heat transfer coefficient is 900 W/m^2·K. For an oil flow rate of 10 kg/s, calculate the cooling water flow rate and the heat transfer area.

22–117 Saturated water vapor at 100°C condenses in the shell side of a one-shell and two-tube heat exchanger with a surface area of 0.5 m^2 and an overall heat transfer coefficient of 2000 W/m^2·K. If cold water ($c_{pc} = 4179$ J/kg·K) flowing at 0.5 kg/s enters the tube side at 15°C, determine the outlet temperature of the cold water and the heat transfer rate for the heat exchanger.

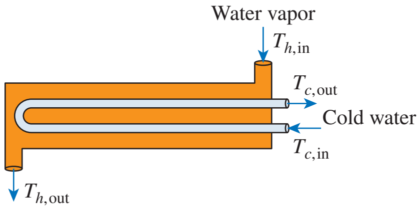


FIGURE P22–117

22–118 A single-pass crossflow heat exchanger uses hot air (mixed) to heat water (unmixed), flowing with a mass flow rate of 3 kg/s, from 30°C to 80°C. The hot air enters and exits the heat exchanger at 220°C and 100°C, respectively. If the overall heat transfer coefficient is 200 W/m^2·K, determine the required surface area.

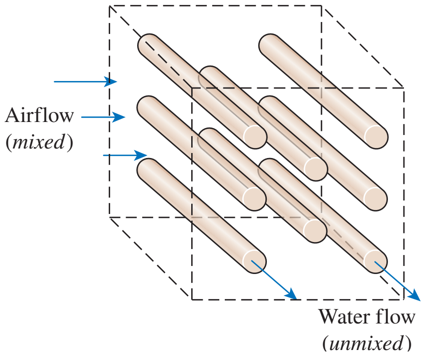


FIGURE P22–118

22–119 Oil in an engine is being cooled by air in a crossflow heat exchanger, where both fluids are unmixed. Oil ($c_{ph} = 2047$ J/kg·K) flowing with a flow rate of 0.026 kg/s enters the tube side at 75°C, while air ($c_{pc} = 1007$ J/kg·K) enters the shell side at 30°C with a flow rate of 0.21 kg/s. The overall heat transfer coefficient of the heat exchanger is 53 W/m^2·K, and the total surface area is 1 m^2. If the correction factor is $F = 0.96$, determine the outlet temperatures of the oil and air.

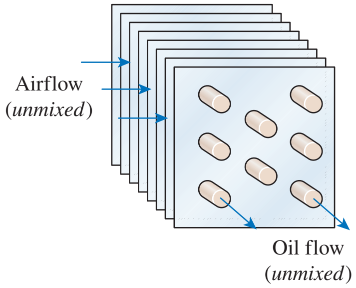


FIGURE P22–119

22–120 Consider a water-to-water counterflow heat exchanger with these specifications. Hot water enters at 90°C while cold water enters at 20°C. The exit temperature of the hot water is 15°C greater than that of the cold water, and the mass flow rate of the hot water is 50 percent greater than that of the cold water. The product of heat transfer surface area and the overall heat transfer coefficient is 2200 W/K. Taking the specific heat of both cold and hot water to be $c_p = 4180$ J/kg·K, determine (*a*) the outlet temperature of the cold water, (*b*) the effectiveness of the heat exchanger, (*c*) the mass flow rate of the cold water, and (*d*) the heat transfer rate.

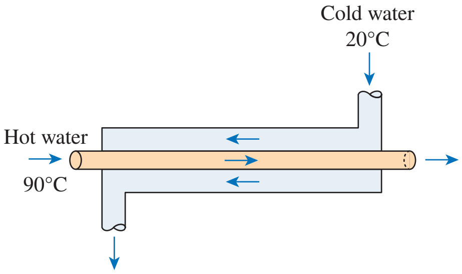


FIGURE P22–120

22–121 Geothermal water ($c_p = 4250$ J/kg·K) at 75°C is to be used to heat fresh water ($c_p = 4180$ J/kg·K) at 17°C at a rate of 1.2 kg/s in a double-pipe counterflow heat exchanger. The heat transfer surface area is 25 m^2, the overall heat transfer coefficient is 480 W/m^2·K, and the mass flow rate of geothermal water is larger than that of fresh water. If the effectiveness of the heat exchanger must be 0.823, determine the mass flow rate of geothermal water and the outlet temperatures of both fluids.

22–122 A crossflow heat exchanger with both fluids unmixed has an overall heat transfer coefficient of 200 W/m^2·K, and a heat transfer surface area of 400 m^2. The hot fluid has a heat capacity of 40,000 W/K, while the cold fluid has a heat capacity of 80,000 W/K. If the inlet temperatures of both hot and cold fluids are 80°C and 20°C, respectively, determine (*a*) the exit temperature of the hot fluid and (*b*) the rate of heat transfer in the heat exchanger.

22–123 In a chemical plant, a certain chemical is heated by hot water supplied by a natural gas furnace. The hot water ($c_p = 4180$ J/kg·K) is then discharged at 60°C at a rate of 8 kg/min. The plant operates 8 h a day, 5 days a week, 52 weeks a year. The furnace has an efficiency of 78 percent, and the cost of the natural gas is \$1.00 per therm (1 therm = 105,500 kJ). The average temperature of the cold water entering the furnace throughout the year is 14°C. In order to save energy, it is proposed to install a water-to-water heat exchanger to preheat the incoming cold water with the drained hot water. Assuming that the heat exchanger will recover 72 percent of the available heat in the hot water, determine the heat transfer rating of the heat exchanger that needs to be purchased, and suggest a suitable type. Also, determine the amount of money this heat exchanger will save the company per year from natural gas savings.

22–124 A single-pass crossflow heat exchanger with both fluids unmixed has water entering at 16°C and exiting at 33°C, while oil ($c_p = 1.93$ kJ/kg·K and $\rho = 870$ kg/m^3) flowing at 0.19 m^3/min enters at 38°C and exits at 29°C. If the surface area of the heat exchanger is 20 m^2, determine (*a*) the NTU value and (*b*) the value of the overall heat transfer coefficient.

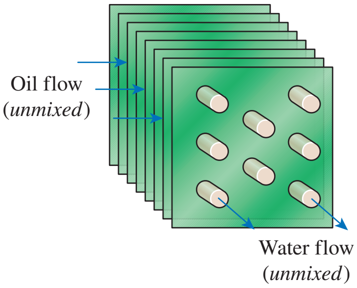


FIGURE P22–124

22–125 Air at 18°C ($c_p = 1006$ J/kg·K) is to be heated to 58°C by hot oil at 80°C ($c_p = 2150$ J/kg·K) in a crossflow heat exchanger with air mixed and oil unmixed. The product of the heat transfer surface area and the overall heat transfer coefficient is 750 W/K, and the mass flow rate of air is twice that of oil. Determine (*a*) the effectiveness of the heat exchanger, (*b*) the mass flow rate of air, and (*c*) the rate of heat transfer.

22–126 The mass flow rate, specific heat, and inlet temperature of the tube-side stream in a double-pipe, parallel-flow heat exchanger are 3200 kg/h, 2.0 kJ/kg·K, and 120°C, respectively. The mass flow rate, specific heat, and inlet temperature of the other stream are 2000 kg/h, 4.2 kJ/kg·K, and 20°C, respectively. The heat transfer area and overall heat transfer coefficient are 0.50 m^2 and 2.0 kW/m^2·K, respectively. Find the outlet temperatures of both streams in steady operation using (*a*) the LMTD method and (*b*) the effectiveness–NTU method.

Design and Essay Problems

22–127 Water flows through a shower head steadily at a rate of 8 kg/min. The water is heated in an electric water heater from 15°C to 45°C. In an attempt to conserve energy, it is proposed to pass the drained warm water at a temperature of 38°C through a heat exchanger to preheat the incoming cold water. Design a heat exchanger that is suitable for this task, and discuss the potential savings in energy and money for your area.

22–128 Design a hydrocooling unit that can cool fruits and vegetables from 30°C to 5°C at a rate of 20,000 kg/h under the following conditions:

The unit will be of flood type that will cool the products as they are conveyed into the channel filled with water. The products will be dropped into the channel filled with water at one end and picked up at the other end. The channel can be as wide as 3 m and as high as 90 cm. The water is to be circulated and cooled by the evaporator section of a refrigeration system. The refrigerant temperature inside the coils is to be −2°C, and the water temperature is not to drop below 1°C and not to exceed 6°C.

Assuming reasonable values for the average product density, specific heat, and porosity (the fraction of air volume in a box), recommend reasonable values for the quantities related to the thermal aspects of the hydrocooler, including (*a*) how long the fruits and vegetables need to remain in the channel, (*b*) the length of the channel, (*c*) the water velocity through the channel, (*d*) the velocity of the conveyor and thus the fruits and vegetables through the channel, (*e*) the refrigeration capacity of the refrigeration system, and (*f*) the type of heat exchanger for the evaporator and the surface area on the water side.

22–129 A company owns a refrigeration system whose refrigeration capacity is 200 tons (1 ton of refrigeration =211 kJ/min), and you are to design a forced-air cooling system for fruits whose diameters do not exceed 7 cm under the following conditions:

The fruits are to be cooled from 28°C to an average temperature of 8°C. The air temperature is to remain above −2°C and below 10°C at all times, and the velocity of air approaching the fruits must remain under 2 m/s. The cooling section can be as wide as 3.5 m and as high as 2 m.

Assuming reasonable values for the average fruit density, specific heat, and porosity (the fraction of air volume in a box), recommend reasonable values for the quantities related to the thermal aspects of the forced-air cooling, including (*a*) how long the fruits need to remain in the cooling section, (*b*) the length of the cooling section, (*c*) the air velocity approaching the cooling section, (*d*) the product cooling capacity of the system, in kg fruit/h, (*e*) the volume flow rate of the air, and (*f*) the type of heat exchanger for the evaporator and the surface area on the air side.

APPENDIX
1

PROPERTY TABLES AND CHARTS (SI UNITS)

TABLE A–1

Molar mass, gas constant, and critical-point properties

				Critical-point properties		
Substance	Formula	Molar mass, M kg/kmol	Gas constant, R kJ/kg·K*	Temperature, K	Pressure, MPa	Volume, m^3/kmol
Air	—	28.97	0.2870	132.5	3.77	0.0883
Ammonia	NH_3	17.03	0.4882	405.5	11.28	0.0724
Argon	Ar	39.948	0.2081	151	4.86	0.0749
Benzene	C_6H_6	78.115	0.1064	562	4.92	0.2603
Bromine	Br_2	159.808	0.0520	584	10.34	0.1355
n-Butane	C_4H_{10}	58.124	0.1430	425.2	3.80	0.2547
Carbon dioxide	CO_2	44.01	0.1889	304.2	7.39	0.0943
Carbon monoxide	CO	28.011	0.2968	133	3.50	0.0930
Carbon tetrachloride	CCl_4	153.82	0.05405	556.4	4.56	0.2759
Chlorine	Cl_2	70.906	0.1173	417	7.71	0.1242
Chloroform	$CHCl_3$	119.38	0.06964	536.6	5.47	0.2403
Dichlorodifluoromethane (R–12)	CCl_2F_2	120.91	0.06876	384.7	4.01	0.2179
Dichlorofluoromethane (R–21)	$CHCl_2F$	102.92	0.08078	451.7	5.17	0.1973
Ethane	C_2H_6	30.070	0.2765	305.5	4.48	0.1480
Ethyl alcohol	C_2H_5OH	46.07	0.1805	516	6.38	0.1673
Ethylene	C_2H_4	28.054	0.2964	282.4	5.12	0.1242
Helium	He	4.003	2.0769	5.3	0.23	0.0578
n-Hexane	C_6H_{14}	86.179	0.09647	507.9	3.03	0.3677
Hydrogen (normal)	H_2	2.016	4.1240	33.3	1.30	0.0649
Krypton	Kr	83.80	0.09921	209.4	5.50	0.0924
Methane	CH_4	16.043	0.5182	191.1	4.64	0.0993
Methyl alcohol	CH_3OH	32.042	0.2595	513.2	7.95	0.1180
Methyl chloride	CH_3Cl	50.488	0.1647	416.3	6.68	0.1430
Neon	Ne	20.183	0.4119	44.5	2.73	0.0417
Nitrogen	N_2	28.013	0.2968	126.2	3.39	0.0899
Nitrous oxide	N_2O	44.013	0.1889	309.7	7.27	0.0961
Oxygen	O_2	31.999	0.2598	154.8	5.08	0.0780
Propane	C_3H_8	44.097	0.1885	370	4.26	0.1998
Propylene	C_3H_6	42.081	0.1976	365	4.62	0.1810
Sulfur dioxide	SO_2	64.063	0.1298	430.7	7.88	0.1217
Tetrafluoroethane (R–134a)	CF_3CH_2F	102.03	0.08149	374.2	4.059	0.1993
Trichlorofluoromethane (R–11)	CCl_3F	137.37	0.06052	471.2	4.38	0.2478
Water	H_2O	18.015	0.4615	647.1	22.06	0.0560
Xenon	Xe	131.30	0.06332	289.8	5.88	0.1186

*The unit kJ/kg·K is equivalent to kPa·m^3/kg·K. The gas constant is calculated from $R = R_u/M$, where $R_u = 8.31447$ kJ/kmol·K and M is the molar mass.

Source of Data: K. A. Kobe and R. E. Lynn, Jr., *Chemical Review* 52 (1953), pp. 117–236; and ASHRAE, *Handbook of Fundamentals* (Atlanta, GA: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., 1993), pp. 16.4 and 36.1.

TABLE A–2

Ideal-gas specific heats of various common gases

(*a*) At 300 K

Gas	Formula	Gas constant, R kJ/kg·K	c_p kJ/kg·K	c_v kJ/kg·K	k
Air	—	0.2870	1.005	0.718	1.400
Argon	Ar	0.2081	0.5203	0.3122	1.667
Butane	C_4H_{10}	0.1433	1.7164	1.5734	1.091
Carbon dioxide	CO_2	0.1889	0.846	0.657	1.289
Carbon monoxide	CO	0.2968	1.040	0.744	1.400
Ethane	C_2H_6	0.2765	1.7662	1.4897	1.186
Ethylene	C_2H_4	0.2964	1.5482	1.2518	1.237
Helium	He	2.0769	5.1926	3.1156	1.667
Hydrogen	H_2	4.1240	14.307	10.183	1.405
Methane	CH_4	0.5182	2.2537	1.7354	1.299
Neon	Ne	0.4119	1.0299	0.6179	1.667
Nitrogen	N_2	0.2968	1.039	0.743	1.400
Octane	C_8H_{18}	0.0729	1.7113	1.6385	1.044
Oxygen	O_2	0.2598	0.918	0.658	1.395
Propane	C_3H_8	0.1885	1.6794	1.4909	1.126
Steam	H_2O	0.4615	1.8723	1.4108	1.327

Note: The unit kJ/kg·K is equivalent to kJ/kg·°C.

Source of Data: B. G. Kyle, *Chemical and Process Thermodynamics,* 3rd ed. (Upper Saddle River, NJ: Prentice Hall, 2000).

TABLE A–2

Ideal-gas specific heats of various common gases (*Continued*)

(*b*) At various temperatures

Temperature, K	c_p kJ/kg·K	c_v kJ/kg·K	k	c_p kJ/kg·K	c_v kJ/kg·K	k	c_p kJ/kg·K	c_v kJ/kg·K	k
		Air			*Carbon dioxide, CO_2*			*Carbon monoxide, CO*	
250	1.003	0.716	1.401	0.791	0.602	1.314	1.039	0.743	1.400
300	1.005	0.718	1.400	0.846	0.657	1.288	1.040	0.744	1.399
350	1.008	0.721	1.398	0.895	0.706	1.268	1.043	0.746	1.398
400	1.013	0.726	1.395	0.939	0.750	1.252	1.047	0.751	1.395
450	1.020	0.733	1.391	0.978	0.790	1.239	1.054	0.757	1.392
500	1.029	0.742	1.387	1.014	0.825	1.229	1.063	0.767	1.387
550	1.040	0.753	1.381	1.046	0.857	1.220	1.075	0.778	1.382
600	1.051	0.764	1.376	1.075	0.886	1.213	1.087	0.790	1.376
650	1.063	0.776	1.370	1.102	0.913	1.207	1.100	0.803	1.370
700	1.075	0.788	1.364	1.126	0.937	1.202	1.113	0.816	1.364
750	1.087	0.800	1.359	1.148	0.959	1.197	1.126	0.829	1.358
800	1.099	0.812	1.354	1.169	0.980	1.193	1.139	0.842	1.353
900	1.121	0.834	1.344	1.204	1.015	1.186	1.163	0.866	1.343
1000	1.142	0.855	1.336	1.234	1.045	1.181	1.185	0.888	1.335
		Hydrogen, H_2			*Nitrogen, N_2*			*Oxygen, O_2*	
250	14.051	9.927	1.416	1.039	0.742	1.400	0.913	0.653	1.398
300	14.307	10.183	1.405	1.039	0.743	1.400	0.918	0.658	1.395
350	14.427	10.302	1.400	1.041	0.744	1.399	0.928	0.668	1.389
400	14.476	10.352	1.398	1.044	0.747	1.397	0.941	0.681	1.382
450	14.501	10.377	1.398	1.049	0.752	1.395	0.956	0.696	1.373
500	14.513	10.389	1.397	1.056	0.759	1.391	0.972	0.712	1.365
550	14.530	10.405	1.396	1.065	0.768	1.387	0.988	0.728	1.358
600	14.546	10.422	1.396	1.075	0.778	1.382	1.003	0.743	1.350
650	14.571	10.447	1.395	1.086	0.789	1.376	1.017	0.758	1.343
700	14.604	10.480	1.394	1.098	0.801	1.371	1.031	0.771	1.337
750	14.645	10.521	1.392	1.110	0.813	1.365	1.043	0.783	1.332
800	14.695	10.570	1.390	1.121	0.825	1.360	1.054	0.794	1.327
900	14.822	10.698	1.385	1.145	0.849	1.349	1.074	0.814	1.319
1000	14.983	10.859	1.380	1.167	0.870	1.341	1.090	0.830	1.313

Source of Data: Kenneth Wark, *Thermodynamics,* 4th ed. (New York: McGraw-Hill, 1983), p. 783, Table A–4M. Originally published in *Tables of Thermal Properties of Gases,* NBS Circular 564, 1955.

TABLE A–2

Ideal-gas specific heats of various common gases (*Concluded*)

(*c*) As a function of temperature

$$\bar{c}_p = a + bT + cT^2 + dT^3$$

(T in K, c_p in kJ/kmol·K)

Substance	Formula	*a*	*b*	*c*	*d*	Temperature range, K	% error	
							Max.	Avg.
Nitrogen	N_2	28.90	-0.1571×10^{-2}	0.8081×10^{-5}	-2.873×10^{-9}	273–1800	0.59	0.34
Oxygen	O_2	25.48	1.520×10^{-2}	-0.7155×10^{-5}	1.312×10^{-9}	273–1800	1.19	0.28
Air	—	28.11	0.1967×10^{-2}	0.4802×10^{-5}	-1.966×10^{-9}	273–1800	0.72	0.33
Hydrogen	H_2	29.11	-0.1916×10^{-2}	0.4003×10^{-5}	-0.8704×10^{-9}	273–1800	1.01	0.26
Carbon monoxide	CO	28.16	0.1675×10^{-2}	0.5372×10^{-5}	-2.222×10^{-9}	273–1800	0.89	0.37
Carbon dioxide	CO_2	22.26	5.981×10^{-2}	-3.501×10^{-5}	7.469×10^{-9}	273–1800	0.67	0.22
Water vapor	H_2O	32.24	0.1923×10^{-2}	1.055×10^{-5}	-3.595×10^{-9}	273–1800	0.53	0.24
Nitric oxide	NO	29.34	-0.09395×10^{-2}	0.9747×10^{-5}	-4.187×10^{-9}	273–1500	0.97	0.36
Nitrous oxide	N_2O	24.11	5.8632×10^{-2}	-3.562×10^{-5}	10.58×10^{-9}	273–1500	0.59	0.26
Nitrogen dioxide	NO_2	22.9	5.715×10^{-2}	-3.52×10^{-5}	7.87×10^{-9}	273–1500	0.46	0.18
Ammonia	NH_3	27.568	2.5630×10^{-2}	0.99072×10^{-5}	-6.6909×10^{-9}	273–1500	0.91	0.36
Sulfur	S	27.21	2.218×10^{-2}	-1.628×10^{-5}	3.986×10^{-9}	273–1800	0.99	0.38
Sulfur dioxide	SO_2	25.78	5.795×10^{-2}	-3.812×10^{-5}	8.612×10^{-9}	273–1800	0.45	0.24
Sulfur trioxide	SO_3	16.40	14.58×10^{-2}	-11.20×10^{-5}	32.42×10^{-9}	273–1300	0.29	0.13
Acetylene	C_2H_2	21.8	9.2143×10^{-2}	-6.527×10^{-5}	18.21×10^{-9}	273–1500	1.46	0.59
Benzene	C_6H_6	−36.22	48.475×10^{-2}	-31.57×10^{-5}	77.62×10^{-9}	273–1500	0.34	0.20
Methanol	CH_4O	19.0	9.152×10^{-2}	-1.22×10^{-5}	-8.039×10^{-9}	273–1000	0.18	0.08
Ethanol	C_2H_6O	19.9	20.96×10^{-2}	-10.38×10^{-5}	20.05×10^{-9}	273–1500	0.40	0.22
Hydrogen chloride	HCl	30.33	-0.7620×10^{-2}	1.327×10^{-5}	-4.338×10^{-9}	273–1500	0.22	0.08
Methane	CH_4	19.89	5.024×10^{-2}	1.269×10^{-5}	-11.01×10^{-9}	273–1500	1.33	0.57
Ethane	C_2H_6	6.900	17.27×10^{-2}	-6.406×10^{-5}	7.285×10^{-9}	273–1500	0.83	0.28
Propane	C_3H_8	−4.04	30.48×10^{-2}	-15.72×10^{-5}	31.74×10^{-9}	273–1500	0.40	0.12
n-Butane	C_4H_{10}	3.96	37.15×10^{-2}	-18.34×10^{-5}	35.00×10^{-9}	273–1500	0.54	0.24
i-Butane	C_4H_{10}	−7.913	41.60×10^{-2}	-23.01×10^{-5}	49.91×10^{-9}	273–1500	0.25	0.13
n-Pentane	C_5H_{12}	6.774	45.43×10^{-2}	-22.46×10^{-5}	42.29×10^{-9}	273–1500	0.56	0.21
n-Hexane	C_6H_{14}	6.938	55.22×10^{-2}	-28.65×10^{-5}	57.69×10^{-9}	273–1500	0.72	0.20
Ethylene	C_2H_4	3.95	15.64×10^{-2}	-8.344×10^{-5}	17.67×10^{-9}	273–1500	0.54	0.13
Propylene	C_3H_6	3.15	23.83×10^{-2}	-12.18×10^{-5}	24.62×10^{-9}	273–1500	0.73	0.17

Source of Data: B. G. Kyle, *Chemical and Process Thermodynamics* (Englewood Cliffs, NJ: Prentice–Hall, 1984).

TABLE A–3

Properties of common liquids, solids, and foods

(*a*) Liquids

	Boiling data at 1 atm		*Freezing data*		*Liquid properties*		
Substance	Normal boiling point, °C	Latent heat of vaporization h_{fg}, kJ/kg	Freezing point, °C	Latent heat of fusion h_{if}, kJ/kg	Temperature, °C	Density ρ, kg/m^3	Specific heat c_p, kJ/kg·K
Ammonia	−33.3	1357	−77.7	322.4	−33.3	682	4.43
					−20	665	4.52
					0	639	4.60
					25	602	4.80
Argon	−185.9	161.6	−189.3	28	−185.6	1394	1.14
Benzene	80.2	394	5.5	126	20	879	1.72
Brine (20% sodium chloride by mass)	103.9	—	−17.4	—	20	1150	3.11
n-Butane	−0.5	385.2	−138.5	80.3	−0.5	601	2.31
Carbon dioxide	−78.4[*]	230.5 (at 0°C)	−56.6		0	298	0.59
Ethanol	78.2	838.3	−114.2	109	25	783	2.46
Ethyl alcohol	78.6	855	−156	108	20	789	2.84
Ethylene glycol	198.1	800.1	−10.8	181.1	20	1109	2.84
Glycerine	179.9	974	18.9	200.6	20	1261	2.32
Helium	−268.9	22.8	—	—	−268.9	146.2	22.8
Hydrogen	−252.8	445.7	−259.2	59.5	−252.8	70.7	10.0
Isobutane	−11.7	367.1	−160	105.7	−11.7	593.8	2.28
Kerosene	204–293	251	−24.9	—	20	820	2.00
Mercury	356.7	294.7	−38.9	11.4	25	13,560	0.139
Methane	−161.5	510.4	−182.2	58.4	−161.5	423	3.49
					−100	301	5.79
Methanol	64.5	1100	−97.7	99.2	25	787	2.55
Nitrogen	−195.8	198.6	−210	25.3	−195.8	809	2.06
					−160	596	2.97
Octane	124.8	306.3	−57.5	180.7	20	703	2.10
Oil (light)					25	910	1.80
Oxygen	−183	212.7	−218.8	13.7	−183	1141	1.71
Petroleum	—	230–384			20	640	2.0
Propane	−42.1	427.8	−187.7	80.0	−42.1	581	2.25
					0	529	2.53
					50	449	3.13
Refrigerant-134a	−26.1	217.0	−96.6	—	−50	1443	1.23
					−26.1	1374	1.27
					0	1295	1.34
					25	1207	1.43
Water	100	2257	0.0	333.7	0	1000	4.22
					25	997	4.18
					50	988	4.18
					75	975	4.19
					100	958	4.22

[*] Sublimation temperature. (At pressures below the triple-point pressure of 518 kPa, carbon dioxide exists as a solid or gas. Also, the freezing-point temperature of carbon dioxide is the triple-point temperature of −56.5°C.)

TABLE A–3

Properties of common liquids, solids, and foods (*Concluded*)

(*b*) Solids (values are for room temperature unless indicated otherwise)

Substance	Density, ρ kg/m^3	Specific heat, c_p kJ/kg·K	Substance	Density, ρ kg/m^3	Specific heat, c_p kJ/kg·K
Metals			Nonmetals		
Aluminum			Asphalt	2110	0.920
200 K		0.797	Brick, common	1922	0.79
250 K		0.859	Brick, fireclay (500°C)	2300	0.960
300 K	2,700	0.902	Concrete	2300	0.653
350 K		0.929	Clay	1000	0.920
400 K		0.949	Diamond	2420	0.616
450 K		0.973	Glass, window	2700	0.800
500 K		0.997	Glass, pyrex	2230	0.840
Bronze (76% Cu, 2% Zn, 2% Al)	8,280	0.400	Graphite	2500	0.711
			Granite	2700	1.017
Brass, yellow (65% Cu, 35% Zn)	8,310	0.400	Gypsum or plaster board	800	1.09
			Ice		
Copper			200 K		1.56
−173°C		0.254	220 K		1.71
−100°C		0.342	240 K		1.86
−50°C		0.367	260 K		2.01
0°C		0.381	273 K	921	2.11
27°C	8,900	0.386	Limestone	1650	0.909
100°C		0.393	Marble	2600	0.880
200°C		0.403	Plywood (Douglas Fir)	545	1.21
Iron	7,840	0.45	Rubber (soft)	1100	1.840
Lead	11,310	0.128	Rubber (hard)	1150	2.009
Magnesium	1,730	1.000	Sand	1520	0.800
Nickel	8,890	0.440	Stone	1500	0.800
Silver	10,470	0.235	Woods, hard (maple, oak, etc.)	721	1.26
Steel, mild	7,830	0.500	Woods, soft (fir, pine, etc.)	513	1.38
Tungsten	19,400	0.130			

(*c*) Foods

Food	Water content, % (mass)	Freezing point, °C	*Specific heat,* kJ/kg·K		Latent heat of fusion, kJ/kg	Food	Water content, % (mass)	Freezing point, °C	*Specific heat,* kJ/kg·K		Latent heat of fusion, kJ/kg
			Above freezing	Below freezing					Above freezing	Below freezing	
Apples	84	−1.1	3.65	1.90	281	Lettuce	95	−0.2	4.02	2.04	317
Bananas	75	−0.8	3.35	1.78	251	Milk, whole	88	−0.6	3.79	1.95	294
Beef round	67	—	3.08	1.68	224	Oranges	87	−0.8	3.75	1.94	291
Broccoli	90	−0.6	3.86	1.97	301	Potatoes	78	−0.6	3.45	1.82	261
Butter	16	—	—	1.04	53	Salmon fish	64	−2.2	2.98	1.65	214
Cheese, swiss	39	−10.0	2.15	1.33	130	Shrimp	83	−2.2	3.62	1.89	277
Cherries	80	−1.8	3.52	1.85	267	Spinach	93	−0.3	3.96	2.01	311
Chicken	74	−2.8	3.32	1.77	247	Strawberries	90	−0.8	3.86	1.97	301
Corn, sweet	74	−0.6	3.32	1.77	247	Tomatoes, ripe	94	−0.5	3.99	2.02	314
Eggs, whole	74	−0.6	3.32	1.77	247	Turkey	64	—	2.98	1.65	214
Ice cream	63	−5.6	2.95	1.63	210	Watermelon	93	−0.4	3.96	2.01	311

Source of Data: Values are obtained from various handbooks and other sources or are calculated. Water content and freezing-point data of foods are from *ASHRAE, Handbook of Fundamentals,* SI version (Atlanta, GA: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., 1993), Chapter 30, Table1. Freezing point is the temperature at which freezing starts for fruits and vegetables, and the average freezing temperature for other foods.

TABLE A–4

Saturated water—Temperature table

		Specific volume, m^3/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
Temp., T °C	Sat. press., P_{sat} kPa	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
0.01	0.6117	0.001000	206.00	0.000	2374.9	2374.9	0.001	2500.9	2500.9	0.0000	9.1556	9.1556
5	0.8725	0.001000	147.03	21.019	2360.8	2381.8	21.020	2489.1	2510.1	0.0763	8.9487	9.0249
10	1.2281	0.001000	106.32	42.020	2346.6	2388.7	42.022	2477.2	2519.2	0.1511	8.7488	8.8999
15	1.7057	0.001001	77.885	62.980	2332.5	2395.5	62.982	2465.4	2528.3	0.2245	8.5559	8.7803
20	2.3392	0.001002	57.762	83.913	2318.4	2402.3	83.915	2453.5	2537.4	0.2965	8.3696	8.6661
25	3.1698	0.001003	43.340	104.83	2304.3	2409.1	104.83	2441.7	2546.5	0.3672	8.1895	8.5567
30	4.2469	0.001004	32.879	125.73	2290.2	2415.9	125.74	2429.8	2555.6	0.4368	8.0152	8.4520
35	5.6291	0.001006	25.205	146.63	2276.0	2422.7	146.64	2417.9	2564.6	0.5051	7.8466	8.3517
40	7.3851	0.001008	19.515	167.53	2261.9	2429.4	167.53	2406.0	2573.5	0.5724	7.6832	8.2556
45	9.5953	0.001010	15.251	188.43	2247.7	2436.1	188.44	2394.0	2582.4	0.6386	7.5247	8.1633
50	12.352	0.001012	12.026	209.33	2233.4	2442.7	209.34	2382.0	2591.3	0.7038	7.3710	8.0748
55	15.763	0.001015	9.5639	230.24	2219.1	2449.3	230.26	2369.8	2600.1	0.7680	7.2218	7.9898
60	19.947	0.001017	7.6670	251.16	2204.7	2455.9	251.18	2357.7	2608.8	0.8313	7.0769	7.9082
65	25.043	0.001020	6.1935	272.09	2190.3	2462.4	272.12	2345.4	2617.5	0.8937	6.9360	7.8296
70	31.202	0.001023	5.0396	293.04	2175.8	2468.9	293.07	2333.0	2626.1	0.9551	6.7989	7.7540
75	38.597	0.001026	4.1291	313.99	2161.3	2475.3	314.03	2320.6	2634.6	1.0158	6.6655	7.6812
80	47.416	0.001029	3.4053	334.97	2146.6	2481.6	335.02	2308.0	2643.0	1.0756	6.5355	7.6111
85	57.868	0.001032	2.8261	355.96	2131.9	2487.8	356.02	2295.3	2651.4	1.1346	6.4089	7.5435
90	70.183	0.001036	2.3593	376.97	2117.0	2494.0	377.04	2282.5	2659.6	1.1929	6.2853	7.4782
95	84.609	0.001040	1.9808	398.00	2102.0	2500.1	398.09	2269.6	2667.6	1.2504	6.1647	7.4151
100	101.42	0.001043	1.6720	419.06	2087.0	2506.0	419.17	2256.4	2675.6	1.3072	6.0470	7.3542
105	120.90	0.001047	1.4186	440.15	2071.8	2511.9	440.28	2243.1	2683.4	1.3634	5.9319	7.2952
110	143.38	0.001052	1.2094	461.27	2056.4	2517.7	461.42	2229.7	2691.1	1.4188	5.8193	7.2382
115	169.18	0.001056	1.0360	482.42	2040.9	2523.3	482.59	2216.0	2698.6	1.4737	5.7092	7.1829
120	198.67	0.001060	0.89133	503.60	2025.3	2528.9	503.81	2202.1	2706.0	1.5279	5.6013	7.1292
125	232.23	0.001065	0.77012	524.83	2009.5	2534.3	525.07	2188.1	2713.1	1.5816	5.4956	7.0771
130	270.28	0.001070	0.66808	546.10	1993.4	2539.5	546.38	2173.7	2720.1	1.6346	5.3919	7.0265
135	313.22	0.001075	0.58179	567.41	1977.3	2544.7	567.75	2159.1	2726.9	1.6872	5.2901	6.9773
140	361.53	0.001080	0.50850	588.77	1960.9	2549.6	589.16	2144.3	2733.5	1.7392	5.1901	6.9294
145	415.68	0.001085	0.44600	610.19	1944.2	2554.4	610.64	2129.2	2739.8	1.7908	5.0919	6.8827
150	476.16	0.001091	0.39248	631.66	1927.4	2559.1	632.18	2113.8	2745.9	1.8418	4.9953	6.8371
155	543.49	0.001096	0.34648	653.19	1910.3	2563.5	653.79	2098.0	2751.8	1.8924	4.9002	6.7927
160	618.23	0.001102	0.30680	674.79	1893.0	2567.8	675.47	2082.0	2757.5	1.9426	4.8066	6.7492
165	700.93	0.001108	0.27244	696.46	1875.4	2571.9	697.24	2065.6	2762.8	1.9923	4.7143	6.7067
170	792.18	0.001114	0.24260	718.20	1857.5	2575.7	719.08	2048.8	2767.9	2.0417	4.6233	6.6650
175	892.60	0.001121	0.21659	740.02	1839.4	2579.4	741.02	2031.7	2772.7	2.0906	4.5335	6.6242
180	1002.8	0.001127	0.19384	761.92	1820.9	2582.8	763.05	2014.2	2777.2	2.1392	4.4448	6.5841
185	1123.5	0.001134	0.17390	783.91	1802.1	2586.0	785.19	1996.2	2781.4	2.1875	4.3572	6.5447
190	1255.2	0.001141	0.15636	806.00	1783.0	2589.0	807.43	1977.9	2785.3	2.2355	4.2705	6.5059
195	1398.8	0.001149	0.14089	828.18	1763.6	2591.7	829.78	1959.0	2788.8	2.2831	4.1847	6.4678
200	1554.9	0.001157	0.12721	850.46	1743.7	2594.2	852.26	1939.8	2792.0	2.3305	4.0997	6.4302

TABLE A–4

Saturated water—Temperature table *(Concluded)*

		Specific volume, m^3/kg		*Internal energy,* kJ/kg			*Enthalpy,* kJ/kg			*Entropy,* kJ/kg·K		
Temp., T °C	Sat. press., P_{sat} kPa	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
205	1724.3	0.001164	0.11508	872.86	1723.5	2596.4	874.87	1920.0	2794.8	2.3776	4.0154	6.3930
210	1907.7	0.001173	0.10429	895.38	1702.9	2598.3	897.61	1899.7	2797.3	2.4245	3.9318	6.3563
215	2105.9	0.001181	0.094680	918.02	1681.9	2599.9	920.50	1878.8	2799.3	2.4712	3.8489	6.3200
220	2319.6	0.001190	0.086094	940.79	1660.5	2601.3	943.55	1857.4	2801.0	2.5176	3.7664	6.2840
225	2549.7	0.001199	0.078405	963.70	1638.6	2602.3	966.76	1835.4	2802.2	2.5639	3.6844	6.2483
230	2797.1	0.001209	0.071505	986.76	1616.1	2602.9	990.14	1812.8	2802.9	2.6100	3.6028	6.2128
235	3062.6	0.001219	0.065300	1010.0	1593.2	2603.2	1013.7	1789.5	2803.2	2.6560	3.5216	6.1775
240	3347.0	0.001229	0.059707	1033.4	1569.8	2603.1	1037.5	1765.5	2803.0	2.7018	3.4405	6.1424
245	3651.2	0.001240	0.054656	1056.9	1545.7	2602.7	1061.5	1740.8	2802.2	2.7476	3.3596	6.1072
250	3976.2	0.001252	0.050085	1080.7	1521.1	2601.8	1085.7	1715.3	2801.0	2.7933	3.2788	6.0721
255	4322.9	0.001263	0.045941	1104.7	1495.8	2600.5	1110.1	1689.0	2799.1	2.8390	3.1979	6.0369
260	4692.3	0.001276	0.042175	1128.8	1469.9	2598.7	1134.8	1661.8	2796.6	2.8847	3.1169	6.0017
265	5085.3	0.001289	0.038748	1153.3	1443.2	2596.5	1159.8	1633.7	2793.5	2.9304	3.0358	5.9662
270	5503.0	0.001303	0.035622	1177.9	1415.7	2593.7	1185.1	1604.6	2789.7	2.9762	2.9542	5.9305
275	5946.4	0.001317	0.032767	1202.9	1387.4	2590.3	1210.7	1574.5	2785.2	3.0221	2.8723	5.8944
280	6416.6	0.001333	0.030153	1228.2	1358.2	2586.4	1236.7	1543.2	2779.9	3.0681	2.7898	5.8579
285	6914.6	0.001349	0.027756	1253.7	1328.1	2581.8	1263.1	1510.7	2773.7	3.1144	2.7066	5.8210
290	7441.8	0.001366	0.025554	1279.7	1296.9	2576.5	1289.8	1476.9	2766.7	3.1608	2.6225	5.7834
295	7999.0	0.001384	0.023528	1306.0	1264.5	2570.5	1317.1	1441.6	2758.7	3.2076	2.5374	5.7450
300	8587.9	0.001404	0.021659	1332.7	1230.9	2563.6	1344.8	1404.8	2749.6	3.2548	2.4511	5.7059
305	9209.4	0.001425	0.019932	1360.0	1195.9	2555.8	1373.1	1366.3	2739.4	3.3024	2.3633	5.6657
310	9865.0	0.001447	0.018333	1387.7	1159.3	2547.1	1402.0	1325.9	2727.9	3.3506	2.2737	5.6243
315	10,556	0.001472	0.016849	1416.1	1121.1	2537.2	1431.6	1283.4	2715.0	3.3994	2.1821	5.5816
320	11,284	0.001499	0.015470	1445.1	1080.9	2526.0	1462.0	1238.5	2700.6	3.4491	2.0881	5.5372
325	12,051	0.001528	0.014183	1475.0	1038.5	2513.4	1493.4	1191.0	2684.3	3.4998	1.9911	5.4908
330	12,858	0.001560	0.012979	1505.7	993.5	2499.2	1525.8	1140.3	2666.0	3.5516	1.8906	5.4422
335	13,707	0.001597	0.011848	1537.5	945.5	2483.0	1559.4	1086.0	2645.4	3.6050	1.7857	5.3907
340	14,601	0.001638	0.010783	1570.7	893.8	2464.5	1594.6	1027.4	2622.0	3.6602	1.6756	5.3358
345	15,541	0.001685	0.009772	1605.5	837.7	2443.2	1631.7	963.4	2595.1	3.7179	1.5585	5.2765
350	16,529	0.001741	0.008806	1642.4	775.9	2418.3	1671.2	892.7	2563.9	3.7788	1.4326	5.2114
355	17,570	0.001808	0.007872	1682.2	706.4	2388.6	1714.0	812.9	2526.9	3.8442	1.2942	5.1384
360	18,666	0.001895	0.006950	1726.2	625.7	2351.9	1761.5	720.1	2481.6	3.9165	1.1373	5.0537
365	19,822	0.002015	0.006009	1777.2	526.4	2303.6	1817.2	605.5	2422.7	4.0004	0.9489	4.9493
370	21,044	0.002217	0.004953	1844.5	385.6	2230.1	1891.2	443.1	2334.3	4.1119	0.6890	4.8009
373.95	22,064	0.003106	0.003106	2015.7	0	2015.7	2084.3	0	2084.3	4.4070	0	4.4070

Source of Data: Tables A–4 through A–8 are generated using the Engineering Equation Solver (EES) software developed by S. A. Klein and F. L. Alvarado. The routine used in calculations is the highly accurate Steam_IAPWS, which incorporates the 1995 Formulation for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use, issued by The International Association for the Properties of Water and Steam (IAPWS). This formulation replaces the 1984 formulation of Haar, Gallagher, and Kell (*NBS/NRC Steam Tables,* Hemisphere Publishing Co., 1984), which is also available in EES as the routine STEAM. The new formulation is based on the correlations of Saul and Wagner (*J. Phys. Chem. Ref. Data,* 16, 893, 1987) with modifications to adjust to the International Temperature Scale of 1990. The modifications are described by Wagner and Pruss (*J. Phys. Chem. Ref. Data,* 22, 783, 1993). The properties of ice are based on Hyland and Wexler, "Formulations for the Thermodynamic Properties of the Saturated Phases of H_2O from 173.15 K to 473.15 K," *ASHRAE Trans.*, Part 2A, Paper 2793, 1983.

TABLE A–5

Saturated water—Pressure table

		Specific volume, m^3/kg		*Internal energy,* kJ/kg			*Enthalpy,* kJ/kg			*Entropy,* kJ/kg·K		
Press., P kPa	Sat. temp., T_{sat} °C	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
1.0	6.97	0.001000	129.19	29.302	2355.2	2384.5	29.303	2484.4	2513.7	0.1059	8.8690	8.9749
1.5	13.02	0.001001	87.964	54.686	2338.1	2392.8	54.688	2470.1	2524.7	0.1956	8.6314	8.8270
2.0	17.50	0.001001	66.990	73.431	2325.5	2398.9	73.433	2459.5	2532.9	0.2606	8.4621	8.7227
2.5	21.08	0.001002	54.242	88.422	2315.4	2403.8	88.424	2451.0	2539.4	0.3118	8.3302	8.6421
3.0	24.08	0.001003	45.654	100.98	2306.9	2407.9	100.98	2443.9	2544.8	0.3543	8.2222	8.5765
4.0	28.96	0.001004	34.791	121.39	2293.1	2414.5	121.39	2432.3	2553.7	0.4224	8.0510	8.4734
5.0	32.87	0.001005	28.185	137.75	2282.1	2419.8	137.75	2423.0	2560.7	0.4762	7.9176	8.3938
7.5	40.29	0.001008	19.233	168.74	2261.1	2429.8	168.75	2405.3	2574.0	0.5763	7.6738	8.2501
10	45.81	0.001010	14.670	191.79	2245.4	2437.2	191.81	2392.1	2583.9	0.6492	7.4996	8.1488
15	53.97	0.001014	10.020	225.93	2222.1	2448.0	225.94	2372.3	2598.3	0.7549	7.2522	8.0071
20	60.06	0.001017	7.6481	251.40	2204.6	2456.0	251.42	2357.5	2608.9	0.8320	7.0752	7.9073
25	64.96	0.001020	6.2034	271.93	2190.4	2462.4	271.96	2345.5	2617.5	0.8932	6.9370	7.8302
30	69.09	0.001022	5.2287	289.24	2178.5	2467.7	289.27	2335.3	2624.6	0.9441	6.8234	7.7675
40	75.86	0.001026	3.9933	317.58	2158.8	2476.3	317.62	2318.4	2636.1	1.0261	6.6430	7.6691
50	81.32	0.001030	3.2403	340.49	2142.7	2483.2	340.54	2304.7	2645.2	1.0912	6.5019	7.5931
75	91.76	0.001037	2.2172	384.36	2111.8	2496.1	384.44	2278.0	2662.4	1.2132	6.2426	7.4558
100	99.61	0.001043	1.6941	417.40	2088.2	2505.6	417.51	2257.5	2675.0	1.3028	6.0562	7.3589
101.325	99.97	0.001043	1.6734	418.95	2087.0	2506.0	419.06	2256.5	2675.6	1.3069	6.0476	7.3545
125	105.97	0.001048	1.3750	444.23	2068.8	2513.0	444.36	2240.6	2684.9	1.3741	5.9100	7.2841
150	111.35	0.001053	1.1594	466.97	2052.3	2519.2	467.13	2226.0	2693.1	1.4337	5.7894	7.2231
175	116.04	0.001057	1.0037	486.82	2037.7	2524.5	487.01	2213.1	2700.2	1.4850	5.6865	7.1716
200	120.21	0.001061	0.88578	504.50	2024.6	2529.1	504.71	2201.6	2706.3	1.5302	5.5968	7.1270
225	123.97	0.001064	0.79329	520.47	2012.7	2533.2	520.71	2191.0	2711.7	1.5706	5.5171	7.0877
250	127.41	0.001067	0.71873	535.08	2001.8	2536.8	535.35	2181.2	2716.5	1.6072	5.4453	7.0525
275	130.58	0.001070	0.65732	548.57	1991.6	2540.1	548.86	2172.0	2720.9	1.6408	5.3800	7.0207
300	133.52	0.001073	0.60582	561.11	1982.1	2543.2	561.43	2163.5	2724.9	1.6717	5.3200	6.9917
325	136.27	0.001076	0.56199	572.84	1973.1	2545.9	573.19	2155.4	2728.6	1.7005	5.2645	6.9650
350	138.86	0.001079	0.52422	583.89	1964.6	2548.5	584.26	2147.7	2732.0	1.7274	5.2128	6.9402
375	141.30	0.001081	0.49133	594.32	1956.6	2550.9	594.73	2140.4	2735.1	1.7526	5.1645	6.9171
400	143.61	0.001084	0.46242	604.22	1948.9	2553.1	604.66	2133.4	2738.1	1.7765	5.1191	6.8955
450	147.90	0.001088	0.41392	622.65	1934.5	2557.1	623.14	2120.3	2743.4	1.8205	5.0356	6.8561
500	151.83	0.001093	0.37483	639.54	1921.2	2560.7	640.09	2108.0	2748.1	1.8604	4.9603	6.8207
550	155.46	0.001097	0.34261	655.16	1908.8	2563.9	655.77	2096.6	2752.4	1.8970	4.8916	6.7886
600	158.83	0.001101	0.31560	669.72	1897.1	2566.8	670.38	2085.8	2756.2	1.9308	4.8285	6.7593
650	161.98	0.001104	0.29260	683.37	1886.1	2569.4	684.08	2075.5	2759.6	1.9623	4.7699	6.7322
700	164.95	0.001108	0.27278	696.23	1875.6	2571.8	697.00	2065.8	2762.8	1.9918	4.7153	6.7071
750	167.75	0.001111	0.25552	708.40	1865.6	2574.0	709.24	2056.4	2765.7	2.0195	4.6642	6.6837

TABLE A–5

Saturated water—Pressure table *(Concluded)*

		Specific volume, m^3/kg		*Internal energy,* kJ/kg			*Enthalpy,* kJ/kg			*Entropy,* kJ/kg·K		
Press., P kPa	Sat. temp., T_{sat} °C	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
800	170.41	0.001115	0.24035	719.97	1856.1	2576.0	720.87	2047.5	2768.3	2.0457	4.6160	6.6616
850	172.94	0.001118	0.22690	731.00	1846.9	2577.9	731.95	2038.8	2770.8	2.0705	4.5705	6.6409
900	175.35	0.001121	0.21489	741.55	1838.1	2579.6	742.56	2030.5	2773.0	2.0941	4.5273	6.6213
950	177.66	0.001124	0.20411	751.67	1829.6	2581.3	752.74	2022.4	2775.2	2.1166	4.4862	6.6027
1000	179.88	0.001127	0.19436	761.39	1821.4	2582.8	762.51	2014.6	2777.1	2.1381	4.4470	6.5850
1100	184.06	0.001133	0.17745	779.78	1805.7	2585.5	781.03	1999.6	2780.7	2.1785	4.3735	6.5520
1200	187.96	0.001138	0.16326	796.96	1790.9	2587.8	798.33	1985.4	2783.8	2.2159	4.3058	6.5217
1300	191.60	0.001144	0.15119	813.10	1776.8	2589.9	814.59	1971.9	2786.5	2.2508	4.2428	6.4936
1400	195.04	0.001149	0.14078	828.35	1763.4	2591.8	829.96	1958.9	2788.9	2.2835	4.1840	6.4675
1500	198.29	0.001154	0.13171	842.82	1750.6	2593.4	844.55	1946.4	2791.0	2.3143	4.1287	6.4430
1750	205.72	0.001166	0.11344	876.12	1720.6	2596.7	878.16	1917.1	2795.2	2.3844	4.0033	6.3877
2000	212.38	0.001177	0.099587	906.12	1693.0	2599.1	908.47	1889.8	2798.3	2.4467	3.8923	6.3390
2250	218.41	0.001187	0.088717	933.54	1667.3	2600.9	936.21	1864.3	2800.5	2.5029	3.7926	6.2954
2500	223.95	0.001197	0.079952	958.87	1643.2	2602.1	961.87	1840.1	2801.9	2.5542	3.7016	6.2558
3000	233.85	0.001217	0.066667	1004.6	1598.5	2603.2	1008.3	1794.9	2803.2	2.6454	3.5402	6.1856
3500	242.56	0.001235	0.057061	1045.4	1557.6	2603.0	1049.7	1753.0	2802.7	2.7253	3.3991	6.1244
4000	250.35	0.001252	0.049779	1082.4	1519.3	2601.7	1087.4	1713.5	2800.8	2.7966	3.2731	6.0696
5000	263.94	0.001286	0.039448	1148.1	1448.9	2597.0	1154.5	1639.7	2794.2	2.9207	3.0530	5.9737
6000	275.59	0.001319	0.032449	1205.8	1384.1	2589.9	1213.8	1570.9	2784.6	3.0275	2.8627	5.8902
7000	285.83	0.001352	0.027378	1258.0	1323.0	2581.0	1267.5	1505.2	2772.6	3.1220	2.6927	5.8148
8000	295.01	0.001384	0.023525	1306.0	1264.5	2570.5	1317.1	1441.6	2758.7	3.2077	2.5373	5.7450
9000	303.35	0.001418	0.020489	1350.9	1207.6	2558.5	1363.7	1379.3	2742.9	3.2866	2.3925	5.6791
10,000	311.00	0.001452	0.018028	1393.3	1151.8	2545.2	1407.8	1317.6	2725.5	3.3603	2.2556	5.6159
11,000	318.08	0.001488	0.015988	1433.9	1096.6	2530.4	1450.2	1256.1	2706.3	3.4299	2.1245	5.5544
12,000	324.68	0.001526	0.014264	1473.0	1041.3	2514.3	1491.3	1194.1	2685.4	3.4964	1.9975	5.4939
13,000	330.85	0.001566	0.012781	1511.0	985.5	2496.6	1531.4	1131.3	2662.7	3.5606	1.8730	5.4336
14,000	336.67	0.001610	0.011487	1548.4	928.7	2477.1	1571.0	1067.0	2637.9	3.6232	1.7497	5.3728
15,000	342.16	0.001657	0.010341	1585.5	870.3	2455.7	1610.3	1000.5	2610.8	3.6848	1.6261	5.3108
16,000	347.36	0.001710	0.009312	1622.6	809.4	2432.0	1649.9	931.1	2581.0	3.7461	1.5005	5.2466
17,000	352.29	0.001770	0.008374	1660.2	745.1	2405.4	1690.3	857.4	2547.7	3.8082	1.3709	5.1791
18,000	356.99	0.001840	0.007504	1699.1	675.9	2375.0	1732.2	777.8	2510.0	3.8720	1.2343	5.1064
19,000	361.47	0.001926	0.006677	1740.3	598.9	2339.2	1776.8	689.2	2466.0	3.9396	1.0860	5.0256
20,000	365.75	0.002038	0.005862	1785.8	509.0	2294.8	1826.6	585.5	2412.1	4.0146	0.9164	4.9310
21,000	369.83	0.002207	0.004994	1841.6	391.9	2233.5	1888.0	450.4	2338.4	4.1071	0.7005	4.8076
22,000	373.71	0.002703	0.003644	1951.7	140.8	2092.4	2011.1	161.5	2172.6	4.2942	0.2496	4.5439
22,064	373.95	0.003106	0.003106	2015.7	0	2015.7	2084.3	0	2084.3	4.4070	0	4.4070

TABLE A–6

Superheated water

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
	P = 0.01 MPa (45.81°C)[*]				P = 0.05 MPa (81.32°C)				P = 0.10 MPa (99.61°C)			
Sat.[†]	14.670	2437.2	2583.9	8.1488	3.2403	2483.2	2645.2	7.5931	1.6941	2505.6	2675.0	7.3589
50	14.867	2443.3	2592.0	8.1741								
100	17.196	2515.5	2687.5	8.4489	3.4187	2511.5	2682.4	7.6953	1.6959	2506.2	2675.8	7.3611
150	19.513	2587.9	2783.0	8.6893	3.8897	2585.7	2780.2	7.9413	1.9367	2582.9	2776.6	7.6148
200	21.826	2661.4	2879.6	8.9049	4.3562	2660.0	2877.8	8.1592	2.1724	2658.2	2875.5	7.8356
250	24.136	2736.1	2977.5	9.1015	4.8206	2735.1	2976.2	8.3568	2.4062	2733.9	2974.5	8.0346
300	26.446	2812.3	3076.7	9.2827	5.2841	2811.6	3075.8	8.5387	2.6389	2810.7	3074.5	8.2172
400	31.063	2969.3	3280.0	9.6094	6.2094	2968.9	3279.3	8.8659	3.1027	2968.3	3278.6	8.5452
500	35.680	3132.9	3489.7	9.8998	7.1338	3132.6	3489.3	9.1566	3.5655	3132.2	3488.7	8.8362
600	40.296	3303.3	3706.3	10.1631	8.0577	3303.1	3706.0	9.4201	4.0279	3302.8	3705.6	9.0999
700	44.911	3480.8	3929.9	10.4056	8.9813	3480.6	3929.7	9.6626	4.4900	3480.4	3929.4	9.3424
800	49.527	3665.4	4160.6	10.6312	9.9047	3665.2	4160.4	9.8883	4.9519	3665.0	4160.2	9.5682
900	54.143	3856.9	4398.3	10.8429	10.8280	3856.8	4398.2	10.1000	5.4137	3856.7	4398.0	9.7800
1000	58.758	4055.3	4642.8	11.0429	11.7513	4055.2	4642.7	10.3000	5.8755	4055.0	4642.6	9.9800
1100	63.373	4260.0	4893.8	11.2326	12.6745	4259.9	4893.7	10.4897	6.3372	4259.8	4893.6	10.1698
1200	67.989	4470.9	5150.8	11.4132	13.5977	4470.8	5150.7	10.6704	6.7988	4470.7	5150.6	10.3504
1300	72.604	4687.4	5413.4	11.5857	14.5209	4687.3	5413.3	10.8429	7.2605	4687.2	5413.3	10.5229
	P = 0.20 MPa (120.21°C)				P = 0.30 MPa (133.52°C)				P = 0.40 MPa (143.61°C)			
Sat.	0.88578	2529.1	2706.3	7.1270	0.60582	2543.2	2724.9	6.9917	0.46242	2553.1	2738.1	6.8955
150	0.95986	2577.1	2769.1	7.2810	0.63402	2571.0	2761.2	7.0792	0.47088	2564.4	2752.8	6.9306
200	1.08049	2654.6	2870.7	7.5081	0.71643	2651.0	2865.9	7.3132	0.53434	2647.2	2860.9	7.1723
250	1.19890	2731.4	2971.2	7.7100	0.79645	2728.9	2967.9	7.5180	0.59520	2726.4	2964.5	7.3804
300	1.31623	2808.8	3072.1	7.8941	0.87535	2807.0	3069.6	7.7037	0.65489	2805.1	3067.1	7.5677
400	1.54934	2967.2	3277.0	8.2236	1.03155	2966.0	3275.5	8.0347	0.77265	2964.9	3273.9	7.9003
500	1.78142	3131.4	3487.7	8.5153	1.18672	3130.6	3486.6	8.3271	0.88936	3129.8	3485.5	8.1933
600	2.01302	3302.2	3704.8	8.7793	1.34139	3301.6	3704.0	8.5915	1.00558	3301.0	3703.3	8.4580
700	2.24434	3479.9	3928.8	9.0221	1.49580	3479.5	3928.2	8.8345	1.12152	3479.0	3927.6	8.7012
800	2.47550	3664.7	4159.8	9.2479	1.65004	3664.3	4159.3	9.0605	1.23730	3663.9	4158.9	8.9274
900	2.70656	3856.3	4397.7	9.4598	1.80417	3856.0	4397.3	9.2725	1.35298	3855.7	4396.9	9.1394
1000	2.93755	4054.8	4642.3	9.6599	1.95824	4054.5	4642.0	9.4726	1.46859	4054.3	4641.7	9.3396
1100	3.16848	4259.6	4893.3	9.8497	2.11226	4259.4	4893.1	9.6624	1.58414	4259.2	4892.9	9.5295
1200	3.39938	4470.5	5150.4	10.0304	2.26624	4470.3	5150.2	9.8431	1.69966	4470.2	5150.0	9.7102
1300	3.63026	4687.1	5413.1	10.2029	2.42019	4686.9	5413.0	10.0157	1.81516	4686.7	5412.8	9.8828
	P = 0.50 MPa (151.83°C)				P = 0.60 MPa (158.83°C)				P = 0.80 MPa (170.41°C)			
Sat.	0.37483	2560.7	2748.1	6.8207	0.31560	2566.8	2756.2	6.7593	0.24035	2576.0	2768.3	6.6616
200	0.42503	2643.3	2855.8	7.0610	0.35212	2639.4	2850.6	6.9683	0.26088	2631.1	2839.8	6.8177
250	0.47443	2723.8	2961.0	7.2725	0.39390	2721.2	2957.6	7.1833	0.29321	2715.9	2950.4	7.0402
300	0.52261	2803.3	3064.6	7.4614	0.43442	2801.4	3062.0	7.3740	0.32416	2797.5	3056.9	7.2345
350	0.57015	2883.0	3168.1	7.6346	0.47428	2881.6	3166.1	7.5481	0.35442	2878.6	3162.2	7.4107
400	0.61731	2963.7	3272.4	7.7956	0.51374	2962.5	3270.8	7.7097	0.38429	2960.2	3267.7	7.5735
500	0.71095	3129.0	3484.5	8.0893	0.59200	3128.2	3483.4	8.0041	0.44332	3126.6	3481.3	7.8692
600	0.80409	3300.4	3702.5	8.3544	0.66976	3299.8	3701.7	8.2695	0.50186	3298.7	3700.1	8.1354
700	0.89696	3478.6	3927.0	8.5978	0.74725	3478.1	3926.4	8.5132	0.56011	3477.2	3925.3	8.3794
800	0.98966	3663.6	4158.4	8.8240	0.82457	3663.2	4157.9	8.7395	0.61820	3662.5	4157.0	8.6061
900	1.08227	3855.4	4396.6	9.0362	0.90179	3855.1	4396.2	8.9518	0.67619	3854.5	4395.5	8.8185
1000	1.17480	4054.0	4641.4	9.2364	0.97893	4053.8	4641.1	9.1521	0.73411	4053.3	4640.5	9.0189
1100	1.26728	4259.0	4892.6	9.4263	1.05603	4258.8	4892.4	9.3420	0.79197	4258.3	4891.9	9.2090
1200	1.35972	4470.0	5149.8	9.6071	1.13309	4469.8	5149.6	9.5229	0.84980	4469.4	5149.3	9.3898
1300	1.45214	4686.6	5412.6	9.7797	1.21012	4686.4	5412.5	9.6955	0.90761	4686.1	5412.2	9.5625

[*] The temperature in parentheses is the saturation temperature at the specified pressure.

[†] Properties of saturated vapor at the specified pressure.

TABLE A–6

Superheated water *(Continued)*

T °C	υ m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/kg·K	υ m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/kg·K	υ m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/kg·K
	P = 1.00 MPa (179.88°C)				P = 1.20 MPa (187.96°C)				P = 1.40 MPa (195.04°C)			
Sat.	0.19437	2582.8	2777.1	6.5850	0.16326	2587.8	2783.8	6.5217	0.14078	2591.8	2788.9	6.4675
200	0.20602	2622.3	2828.3	6.6956	0.16934	2612.9	2816.1	6.5909	0.14303	2602.7	2803.0	6.4975
250	0.23275	2710.4	2943.1	6.9265	0.19241	2704.7	2935.6	6.8313	0.16356	2698.9	2927.9	6.7488
300	0.25799	2793.7	3051.6	7.1246	0.21386	2789.7	3046.3	7.0335	0.18233	2785.7	3040.9	6.9553
350	0.28250	2875.7	3158.2	7.3029	0.23455	2872.7	3154.2	7.2139	0.20029	2869.7	3150.1	7.1379
400	0.30661	2957.9	3264.5	7.4670	0.25482	2955.5	3261.3	7.3793	0.21782	2953.1	3258.1	7.3046
500	0.35411	3125.0	3479.1	7.7642	0.29464	3123.4	3477.0	7.6779	0.25216	3121.8	3474.8	7.6047
600	0.40111	3297.5	3698.6	8.0311	0.33395	3296.3	3697.0	7.9456	0.28597	3295.1	3695.5	7.8730
700	0.44783	3476.3	3924.1	8.2755	0.37297	3475.3	3922.9	8.1904	0.31951	3474.4	3921.7	8.1183
800	0.49438	3661.7	4156.1	8.5024	0.41184	3661.0	4155.2	8.4176	0.35288	3660.3	4154.3	8.3458
900	0.54083	3853.9	4394.8	8.7150	0.45059	3853.3	4394.0	8.6303	0.38614	3852.7	4393.3	8.5587
1000	0.58721	4052.7	4640.0	8.9155	0.48928	4052.2	4639.4	8.8310	0.41933	4051.7	4638.8	8.7595
1100	0.63354	4257.9	4891.4	9.1057	0.52792	4257.5	4891.0	9.0212	0.45247	4257.0	4890.5	8.9497
1200	0.67983	4469.0	5148.9	9.2866	0.56652	4468.7	5148.5	9.2022	0.48558	4468.3	5148.1	9.1308
1300	0.72610	4685.8	5411.9	9.4593	0.60509	4685.5	5411.6	9.3750	0.51866	4685.1	5411.3	9.3036
	P = 1.60 MPa (201.37°C)				P = 1.80 MPa (207.11°C)				P = 2.00 MPa (212.38°C)			
Sat.	0.12374	2594.8	2792.8	6.4200	0.11037	2597.3	2795.9	6.3775	0.09959	2599.1	2798.3	6.3390
225	0.13293	2645.1	2857.8	6.5537	0.11678	2637.0	2847.2	6.4825	0.10381	2628.5	2836.1	6.4160
250	0.14190	2692.9	2919.9	6.6753	0.12502	2686.7	2911.7	6.6088	0.11150	2680.3	2903.3	6.5475
300	0.15866	2781.6	3035.4	6.8864	0.14025	2777.4	3029.9	6.8246	0.12551	2773.2	3024.2	6.7684
350	0.17459	2866.6	3146.0	7.0713	0.15460	2863.6	3141.9	7.0120	0.13860	2860.5	3137.7	6.9583
400	0.19007	2950.8	3254.9	7.2394	0.16849	2948.3	3251.6	7.1814	0.15122	2945.9	3248.4	7.1292
500	0.22029	3120.1	3472.6	7.5410	0.19551	3118.5	3470.4	7.4845	0.17568	3116.9	3468.3	7.4337
600	0.24999	3293.9	3693.9	7.8101	0.22200	3292.7	3692.3	7.7543	0.19962	3291.5	3690.7	7.7043
700	0.27941	3473.5	3920.5	8.0558	0.24822	3472.6	3919.4	8.0005	0.22326	3471.7	3918.2	7.9509
800	0.30865	3659.5	4153.4	8.2834	0.27426	3658.8	4152.4	8.2284	0.24674	3658.0	4151.5	8.1791
900	0.33780	3852.1	4392.6	8.4965	0.30020	3851.5	4391.9	8.4417	0.27012	3850.9	4391.1	8.3925
1000	0.36687	4051.2	4638.2	8.6974	0.32606	4050.7	4637.6	8.6427	0.29342	4050.2	4637.1	8.5936
1100	0.39589	4256.6	4890.0	8.8878	0.35188	4256.2	4889.6	8.8331	0.31667	4255.7	4889.1	8.7842
1200	0.42488	4467.9	5147.7	9.0689	0.37766	4467.6	5147.3	9.0143	0.33989	4467.2	5147.0	8.9654
1300	0.45383	4684.8	5410.9	9.2418	0.40341	4684.5	5410.6	9.1872	0.36308	4684.2	5410.3	9.1384
	P = 2.50 MPa (223.95°C)				P = 3.00 MPa (233.85°C)				P = 3.50 MPa (242.56°C)			
Sat.	0.07995	2602.1	2801.9	6.2558	0.06667	2603.2	2803.2	6.1856	0.05706	2603.0	2802.7	6.1244
225	0.08026	2604.8	2805.5	6.2629								
250	0.08705	2663.3	2880.9	6.4107	0.07063	2644.7	2856.5	6.2893	0.05876	2624.0	2829.7	6.1764
300	0.09894	2762.2	3009.6	6.6459	0.08118	2750.8	2994.3	6.5412	0.06845	2738.8	2978.4	6.4484
350	0.10979	2852.5	3127.0	6.8424	0.09056	2844.4	3116.1	6.7450	0.07680	2836.0	3104.9	6.6601
400	0.12012	2939.8	3240.1	7.0170	0.09938	2933.6	3231.7	6.9235	0.08456	2927.2	3223.2	6.8428
450	0.13015	3026.2	3351.6	7.1768	0.10789	3021.2	3344.9	7.0856	0.09198	3016.1	3338.1	7.0074
500	0.13999	3112.8	3462.8	7.3254	0.11620	3108.6	3457.2	7.2359	0.09919	3104.5	3451.7	7.1593
600	0.15931	3288.5	3686.8	7.5979	0.13245	3285.5	3682.8	7.5103	0.11325	3282.5	3678.9	7.4357
700	0.17835	3469.3	3915.2	7.8455	0.14841	3467.0	3912.2	7.7590	0.12702	3464.7	3909.3	7.6855
800	0.19722	3656.2	4149.2	8.0744	0.16420	3654.3	4146.9	7.9885	0.14061	3652.5	4144.6	7.9156
900	0.21597	3849.4	4389.3	8.2882	0.17988	3847.9	4387.5	8.2028	0.15410	3846.4	4385.7	8.1304
1000	0.23466	4049.0	4635.6	8.4897	0.19549	4047.7	4634.2	8.4045	0.16751	4046.4	4632.7	8.3324
1100	0.25330	4254.7	4887.9	8.6804	0.21105	4253.6	4886.7	8.5955	0.18087	4252.5	4885.6	8.5236
1200	0.27190	4466.3	5146.0	8.8618	0.22658	4465.3	5145.1	8.7771	0.19420	4464.4	5144.1	8.7053
1300	0.29048	4683.4	5409.5	9.0349	0.24207	4682.6	5408.8	8.9502	0.20750	4681.8	5408.0	8.8786

TABLE A–6

Superheated water *(Continued)*

T °C	v m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/kg·K	v m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/kg·K	v m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/kg·K
	P = 4.0 MPa (250.35°C)				*P* = 4.5 MPa (257.44°C)				*P* = 5.0 MPa (263.94°C)			
Sat.	0.04978	2601.7	2800.8	6.0696	0.04406	2599.7	2798.0	6.0198	0.03945	2597.0	2794.2	5.9737
275	0.05461	2668.9	2887.3	6.2312	0.04733	2651.4	2864.4	6.1429	0.04144	2632.3	2839.5	6.0571
300	0.05887	2726.2	2961.7	6.3639	0.05138	2713.0	2944.2	6.2854	0.04535	2699.0	2925.7	6.2111
350	0.06647	2827.4	3093.3	6.5843	0.05842	2818.6	3081.5	6.5153	0.05197	2809.5	3069.3	6.4516
400	0.07343	2920.8	3214.5	6.7714	0.06477	2914.2	3205.7	6.7071	0.05784	2907.5	3196.7	6.6483
450	0.08004	3011.0	3331.2	6.9386	0.07076	3005.8	3324.2	6.8770	0.06332	3000.6	3317.2	6.8210
500	0.08644	3100.3	3446.0	7.0922	0.07652	3096.0	3440.4	7.0323	0.06858	3091.8	3434.7	6.9781
600	0.09886	3279.4	3674.9	7.3706	0.08766	3276.4	3670.9	7.3127	0.07870	3273.3	3666.9	7.2605
700	0.11098	3462.4	3906.3	7.6214	0.09850	3460.0	3903.3	7.5647	0.08852	3457.7	3900.3	7.5136
800	0.12292	3650.6	4142.3	7.8523	0.10916	3648.8	4140.0	7.7962	0.09816	3646.9	4137.7	7.7458
900	0.13476	3844.8	4383.9	8.0675	0.11972	3843.3	4382.1	8.0118	0.10769	3841.8	4380.2	7.9619
1000	0.14653	4045.1	4631.2	8.2698	0.13020	4043.9	4629.8	8.2144	0.11715	4042.6	4628.3	8.1648
1100	0.15824	4251.4	4884.4	8.4612	0.14064	4250.4	4883.2	8.4060	0.12655	4249.3	4882.1	8.3566
1200	0.16992	4463.5	5143.2	8.6430	0.15103	4462.6	5142.2	8.5880	0.13592	4461.6	5141.3	8.5388
1300	0.18157	4680.9	5407.2	8.8164	0.16140	4680.1	5406.5	8.7616	0.14527	4679.3	5405.7	8.7124
	P = 6.0 MPa (275.59°C)				*P* = 7.0 MPa (285.83°C)				*P* = 8.0 MPa (295.01°C)			
Sat.	0.03245	2589.9	2784.6	5.8902	0.027378	2581.0	2772.6	5.8148	0.023525	2570.5	2758.7	5.7450
300	0.03619	2668.4	2885.6	6.0703	0.029492	2633.5	2839.9	5.9337	0.024279	2592.3	2786.5	5.7937
350	0.04225	2790.4	3043.9	6.3357	0.035262	2770.1	3016.9	6.2305	0.029975	2748.3	2988.1	6.1321
400	0.04742	2893.7	3178.3	6.5432	0.039958	2879.5	3159.2	6.4502	0.034344	2864.6	3139.4	6.3658
450	0.05217	2989.9	3302.9	6.7219	0.044187	2979.0	3288.3	6.6353	0.038194	2967.8	3273.3	6.5579
500	0.05667	3083.1	3423.1	6.8826	0.048157	3074.3	3411.4	6.8000	0.041767	3065.4	3399.5	6.7266
550	0.06102	3175.2	3541.3	7.0308	0.051966	3167.9	3531.6	6.9507	0.045172	3160.5	3521.8	6.8800
600	0.06527	3267.2	3658.8	7.1693	0.055665	3261.0	3650.6	7.0910	0.048463	3254.7	3642.4	7.0221
700	0.07355	3453.0	3894.3	7.4247	0.062850	3448.3	3888.3	7.3487	0.054829	3443.6	3882.2	7.2822
800	0.08165	3643.2	4133.1	7.6582	0.069856	3639.5	4128.5	7.5836	0.061011	3635.7	4123.8	7.5185
900	0.08964	3838.8	4376.6	7.8751	0.076750	3835.7	4373.0	7.8014	0.067082	3832.7	4369.3	7.7372
1000	0.09756	4040.1	4625.4	8.0786	0.083571	4037.5	4622.5	8.0055	0.073079	4035.0	4619.6	7.9419
1100	0.10543	4247.1	4879.7	8.2709	0.090341	4245.0	4877.4	8.1982	0.079025	4242.8	4875.0	8.1350
1200	0.11326	4459.8	5139.4	8.4534	0.097075	4457.9	5137.4	8.3810	0.084934	4456.1	5135.5	8.3181
1300	0.12107	4677.7	5404.1	8.6273	0.103781	4676.1	5402.6	8.5551	0.090817	4674.5	5401.0	8.4925
	P = 9.0 MPa (303.35°C)				*P* = 10.0 MPa (311.00°C)				*P* = 12.5 MPa (327.81°C)			
Sat.	0.020489	2558.5	2742.9	5.6791	0.018028	2545.2	2725.5	5.6159	0.013496	2505.6	2674.3	5.4638
325	0.023284	2647.6	2857.1	5.8738	0.019877	2611.6	2810.3	5.7596				
350	0.025816	2725.0	2957.3	6.0380	0.022440	2699.6	2924.0	5.9460	0.016138	2624.9	2826.6	5.7130
400	0.029960	2849.2	3118.8	6.2876	0.026436	2833.1	3097.5	6.2141	0.020030	2789.6	3040.0	6.0433
450	0.033524	2956.3	3258.0	6.4872	0.029782	2944.5	3242.4	6.4219	0.023019	2913.7	3201.5	6.2749
500	0.036793	3056.3	3387.4	6.6603	0.032811	3047.0	3375.1	6.5995	0.025630	3023.2	3343.6	6.4651
550	0.039885	3153.0	3512.0	6.8164	0.035655	3145.4	3502.0	6.7585	0.028033	3126.1	3476.5	6.6317
600	0.042861	3248.4	3634.1	6.9605	0.038378	3242.0	3625.8	6.9045	0.030306	3225.8	3604.6	6.7828
650	0.045755	3343.4	3755.2	7.0954	0.041018	3338.0	3748.1	7.0408	0.032491	3324.1	3730.2	6.9227
700	0.048589	3438.8	3876.1	7.2229	0.043597	3434.0	3870.0	7.1693	0.034612	3422.0	3854.6	7.0540
800	0.054132	3632.0	4119.2	7.4606	0.048629	3628.2	4114.5	7.4085	0.038724	3618.8	4102.8	7.2967
900	0.059562	3829.6	4365.7	7.6802	0.053547	3826.5	4362.0	7.6290	0.042720	3818.9	4352.9	7.5195
1000	0.064919	4032.4	4616.7	7.8855	0.058391	4029.9	4613.8	7.8349	0.046641	4023.5	4606.5	7.7269
1100	0.070224	4240.7	4872.7	8.0791	0.063183	4238.5	4870.3	8.0289	0.050510	4233.1	4864.5	7.9220
1200	0.075492	4454.2	5133.6	8.2625	0.067938	4452.4	5131.7	8.2126	0.054342	4447.7	5127.0	8.1065
1300	0.080733	4672.9	5399.5	8.4371	0.072667	4671.3	5398.0	8.3874	0.058147	4667.3	5394.1	8.2819

TABLE A–6

Superheated water *(Concluded)*

T °C	υ m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/kg·K	υ m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/kg·K	υ m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/kg·K
	P = 15.0 MPa (342.16°C)				*P* = 17.5 MPa (354.67°C)				*P* = 20.0 MPa (365.75°C)			
Sat.	0.010341	2455.7	2610.8	5.3108	0.007932	2390.7	2529.5	5.1435	0.005862	2294.8	2412.1	4.9310
350	0.011481	2520.9	2693.1	5.4438								
400	0.015671	2740.6	2975.7	5.8819	0.012463	2684.3	2902.4	5.7211	0.009950	2617.9	2816.9	5.5526
450	0.018477	2880.8	3157.9	6.1434	0.015204	2845.4	3111.4	6.0212	0.012721	2807.3	3061.7	5.9043
500	0.020828	2998.4	3310.8	6.3480	0.017385	2972.4	3276.7	6.2424	0.014793	2945.3	3241.2	6.1446
550	0.022945	3106.2	3450.4	6.5230	0.019305	3085.8	3423.6	6.4266	0.016571	3064.7	3396.2	6.3390
600	0.024921	3209.3	3583.1	6.6796	0.021073	3192.5	3561.3	6.5890	0.018185	3175.3	3539.0	6.5075
650	0.026804	3310.1	3712.1	6.8233	0.022742	3295.8	3693.8	6.7366	0.019695	3281.4	3675.3	6.6593
700	0.028621	3409.8	3839.1	6.9573	0.024342	3397.5	3823.5	6.8735	0.021134	3385.1	3807.8	6.7991
800	0.032121	3609.3	4091.1	7.2037	0.027405	3599.7	4079.3	7.1237	0.023870	3590.1	4067.5	7.0531
900	0.035503	3811.2	4343.7	7.4288	0.030348	3803.5	4334.6	7.3511	0.026484	3795.7	4325.4	7.2829
1000	0.038808	4017.1	4599.2	7.6378	0.033215	4010.7	4592.0	7.5616	0.029020	4004.3	4584.7	7.4950
1100	0.042062	4227.7	4858.6	7.8339	0.036029	4222.3	4852.8	7.7588	0.031504	4216.9	4847.0	7.6933
1200	0.045279	4443.1	5122.3	8.0192	0.038806	4438.5	5117.6	7.9449	0.033952	4433.8	5112.9	7.8802
1300	0.048469	4663.3	5390.3	8.1952	0.041556	4659.2	5386.5	8.1215	0.036371	4655.2	5382.7	8.0574
	P = 25.0 MPa				*P* = 30.0 MPa				*P* = 35.0 MPa			
375	0.001978	1799.9	1849.4	4.0345	0.001792	1738.1	1791.9	3.9313	0.001701	1702.8	1762.4	3.8724
400	0.006005	2428.5	2578.7	5.1400	0.002798	2068.9	2152.8	4.4758	0.002105	1914.9	1988.6	4.2144
425	0.007886	2607.8	2805.0	5.4708	0.005299	2452.9	2611.8	5.1473	0.003434	2253.3	2373.5	4.7751
450	0.009176	2721.2	2950.6	5.6759	0.006737	2618.9	2821.0	5.4422	0.004957	2497.5	2671.0	5.1946
500	0.011143	2887.3	3165.9	5.9643	0.008691	2824.0	3084.8	5.7956	0.006933	2755.3	2997.9	5.6331
550	0.012736	3020.8	3339.2	6.1816	0.010175	2974.5	3279.7	6.0403	0.008348	2925.8	3218.0	5.9093
600	0.014140	3140.0	3493.5	6.3637	0.011445	3103.4	3446.8	6.2373	0.009523	3065.6	3399.0	6.1229
650	0.015430	3251.9	3637.7	6.5243	0.012590	3221.7	3599.4	6.4074	0.010565	3190.9	3560.7	6.3030
700	0.016643	3359.9	3776.0	6.6702	0.013654	3334.3	3743.9	6.5599	0.011523	3308.3	3711.6	6.4623
800	0.018922	3570.7	4043.8	6.9322	0.015628	3551.2	4020.0	6.8301	0.013278	3531.6	3996.3	6.7409
900	0.021075	3780.2	4307.1	7.1668	0.017473	3764.6	4288.8	7.0695	0.014904	3749.0	4270.6	6.9853
1000	0.023150	3991.5	4570.2	7.3821	0.019240	3978.6	4555.8	7.2880	0.016450	3965.8	4541.5	7.2069
1100	0.025172	4206.1	4835.4	7.5825	0.020954	4195.2	4823.9	7.4906	0.017942	4184.4	4812.4	7.4118
1200	0.027157	4424.6	5103.5	7.7710	0.022630	4415.3	5094.2	7.6807	0.019398	4406.1	5085.0	7.6034
1300	0.029115	4647.2	5375.1	7.9494	0.024279	4639.2	5367.6	7.8602	0.020827	4631.2	5360.2	7.7841
	P = 40.0 MPa				*P* = 50.0 MPa				*P* = 60.0 MPa			
375	0.001641	1677.0	1742.6	3.8290	0.001560	1638.6	1716.6	3.7642	0.001503	1609.7	1699.9	3.7149
400	0.001911	1855.0	1931.4	4.1145	0.001731	1787.8	1874.4	4.0029	0.001633	1745.2	1843.2	3.9317
425	0.002538	2097.5	2199.0	4.5044	0.002009	1960.3	2060.7	4.2746	0.001816	1892.9	2001.8	4.1630
450	0.003692	2364.2	2511.8	4.9449	0.002487	2160.3	2284.7	4.5896	0.002086	2055.1	2180.2	4.4140
500	0.005623	2681.6	2906.5	5.4744	0.003890	2528.1	2722.6	5.1762	0.002952	2393.2	2570.3	4.9356
550	0.006985	2875.1	3154.4	5.7857	0.005118	2769.5	3025.4	5.5563	0.003955	2664.6	2901.9	5.3517
600	0.008089	3026.8	3350.4	6.0170	0.006108	2947.1	3252.6	5.8245	0.004833	2866.8	3156.8	5.6527
650	0.009053	3159.5	3521.6	6.2078	0.006957	3095.6	3443.5	6.0373	0.005591	3031.3	3366.8	5.8867
700	0.009930	3282.0	3679.2	6.3740	0.007717	3228.7	3614.6	6.2179	0.006265	3175.4	3551.3	6.0814
800	0.011521	3511.8	3972.6	6.6613	0.009073	3472.2	3925.8	6.5225	0.007456	3432.6	3880.0	6.4033
900	0.012980	3733.3	4252.5	6.9107	0.010296	3702.0	4216.8	6.7819	0.008519	3670.9	4182.1	6.6725
1000	0.014360	3952.9	4527.3	7.1355	0.011441	3927.4	4499.4	7.0131	0.009504	3902.0	4472.2	6.9099
1100	0.015686	4173.7	4801.1	7.3425	0.012534	4152.2	4778.9	7.2244	0.010439	4130.9	4757.3	7.1255
1200	0.016976	4396.9	5075.9	7.5357	0.013590	4378.6	5058.1	7.4207	0.011339	4360.5	5040.8	7.3248
1300	0.018239	4623.3	5352.8	7.7175	0.014620	4607.5	5338.5	7.6048	0.012213	4591.8	5324.5	7.5111

TABLE A–7

Compressed liquid water

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
	P = 5 MPa (263.94°C)				P = 10 MPa (311.00°C)				P = 15 MPa (342.16°C)			
Sat.	0.0012862	1148.1	1154.5	2.9207	0.0014522	1393.3	1407.9	3.3603	0.0016572	1585.5	1610.3	3.6848
0	0.0009977	0.04	5.03	0.0001	0.0009952	0.12	10.07	0.0003	0.0009928	0.18	15.07	0.0004
20	0.0009996	83.61	88.61	0.2954	0.0009973	83.31	93.28	0.2943	0.0009951	83.01	97.93	0.2932
40	0.0010057	166.92	171.95	0.5705	0.0010035	166.33	176.37	0.5685	0.0010013	165.75	180.77	0.5666
60	0.0010149	250.29	255.36	0.8287	0.0010127	249.43	259.55	0.8260	0.0010105	248.58	263.74	0.8234
80	0.0010267	333.82	338.96	1.0723	0.0010244	332.69	342.94	1.0691	0.0010221	331.59	346.92	1.0659
100	0.0010410	417.65	422.85	1.3034	0.0010385	416.23	426.62	1.2996	0.0010361	414.85	430.39	1.2958
120	0.0010576	501.91	507.19	1.5236	0.0010549	500.18	510.73	1.5191	0.0010522	498.50	514.28	1.5148
140	0.0010769	586.80	592.18	1.7344	0.0010738	584.72	595.45	1.7293	0.0010708	582.69	598.75	1.7243
160	0.0010988	672.55	678.04	1.9374	0.0010954	670.06	681.01	1.9316	0.0010920	667.63	684.01	1.9259
180	0.0011240	759.47	765.09	2.1338	0.0011200	756.48	767.68	2.1271	0.0011160	753.58	770.32	2.1206
200	0.0011531	847.92	853.68	2.3251	0.0011482	844.32	855.80	2.3174	0.0011435	840.84	858.00	2.3100
220	0.0011868	938.39	944.32	2.5127	0.0011809	934.01	945.82	2.5037	0.0011752	929.81	947.43	2.4951
240	0.0012268	1031.6	1037.7	2.6983	0.0012192	1026.2	1038.3	2.6876	0.0012121	1021.0	1039.2	2.6774
260	0.0012755	1128.5	1134.9	2.8841	0.0012653	1121.6	1134.3	2.8710	0.0012560	1115.1	1134.0	2.8586
280					0.0013226	1221.8	1235.0	3.0565	0.0013096	1213.4	1233.0	3.0410
300					0.0013980	1329.4	1343.3	3.2488	0.0013783	1317.6	1338.3	3.2279
320									0.0014733	1431.9	1454.0	3.4263
340									0.0016311	1567.9	1592.4	3.6555
	P = 20 MPa (365.75°C)				P = 30 MPa				P = 50 MPa			
Sat.	0.0020378	1785.8	1826.6	4.0146								
0	0.0009904	0.23	20.03	0.0005	0.0009857	0.29	29.86	0.0003	0.0009767	0.29	49.13	−0.0010
20	0.0009929	82.71	102.57	0.2921	0.0009886	82.11	111.77	0.2897	0.0009805	80.93	129.95	0.2845
40	0.0009992	165.17	185.16	0.5646	0.0009951	164.05	193.90	0.5607	0.0009872	161.90	211.25	0.5528
60	0.0010084	247.75	267.92	0.8208	0.0010042	246.14	276.26	0.8156	0.0009962	243.08	292.88	0.8055
80	0.0010199	330.50	350.90	1.0627	0.0010155	328.40	358.86	1.0564	0.0010072	324.42	374.78	1.0442
100	0.0010337	413.50	434.17	1.2920	0.0010290	410.87	441.74	1.2847	0.0010201	405.94	456.94	1.2705
120	0.0010496	496.85	517.84	1.5105	0.0010445	493.66	525.00	1.5020	0.0010349	487.69	539.43	1.4859
140	0.0010679	580.71	602.07	1.7194	0.0010623	576.90	608.76	1.7098	0.0010517	569.77	622.36	1.6916
160	0.0010886	665.28	687.05	1.9203	0.0010823	660.74	693.21	1.9094	0.0010704	652.33	705.85	1.8889
180	0.0011122	750.78	773.02	2.1143	0.0011049	745.40	778.55	2.1020	0.0010914	735.49	790.06	2.0790
200	0.0011390	837.49	860.27	2.3027	0.0011304	831.11	865.02	2.2888	0.0011149	819.45	875.19	2.2628
220	0.0011697	925.77	949.16	2.4867	0.0011595	918.15	952.93	2.4707	0.0011412	904.39	961.45	2.4414
240	0.0012053	1016.1	1040.2	2.6676	0.0011927	1006.9	1042.7	2.6491	0.0011708	990.55	1049.1	2.6156
260	0.0012472	1109.0	1134.0	2.8469	0.0012314	1097.8	1134.7	2.8250	0.0012044	1078.2	1138.4	2.7864
280	0.0012978	1205.6	1231.5	3.0265	0.0012770	1191.5	1229.8	3.0001	0.0012430	1167.7	1229.9	2.9547
300	0.0013611	1307.2	1334.4	3.2091	0.0013322	1288.9	1328.9	3.1761	0.0012879	1259.6	1324.0	3.1218
320	0.0014450	1416.6	1445.5	3.3996	0.0014014	1391.7	1433.7	3.3558	0.0013409	1354.3	1421.4	3.2888
340	0.0015693	1540.2	1571.6	3.6086	0.0014932	1502.4	1547.1	3.5438	0.0014049	1452.9	1523.1	3.4575
360	0.0018248	1703.6	1740.1	3.8787	0.0016276	1626.8	1675.6	3.7499	0.0014848	1556.5	1630.7	3.6301
380					0.0018729	1782.0	1838.2	4.0026	0.0015884	1667.1	1746.5	3.8102

TABLE A–8

Saturated ice–water vapor

		Specific volume, m^3/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
Temp., T °C	Sat. press., P_{sat} kPa	Sat. ice, v_i	Sat. vapor, v_g	Sat. ice, u_i	Subl., u_{ig}	Sat. vapor, u_g	Sat. ice, h_i	Subl., h_{ig}	Sat. vapor, h_g	Sat. ice, s_i	Subl., s_{ig}	Sat. vapor, s_g
0.01	0.61169	0.001091	205.99	−333.40	2707.9	2374.5	−333.40	2833.9	2500.5	−1.2202	10.374	9.154
0	0.61115	0.001091	206.17	−333.43	2707.9	2374.5	−333.43	2833.9	2500.5	−1.2204	10.375	9.154
−2	0.51772	0.001091	241.62	−337.63	2709.4	2371.8	−337.63	2834.5	2496.8	−1.2358	10.453	9.218
−4	0.43748	0.001090	283.84	−341.80	2710.8	2369.0	−341.80	2835.0	2493.2	−1.2513	10.533	9.282
−6	0.36873	0.001090	334.27	−345.94	2712.2	2366.2	−345.93	2835.4	2489.5	−1.2667	10.613	9.347
−8	0.30998	0.001090	394.66	−350.04	2713.5	2363.5	−350.04	2835.8	2485.8	−1.2821	10.695	9.413
−10	0.25990	0.001089	467.17	−354.12	2714.8	2360.7	−354.12	2836.2	2482.1	−1.2976	10.778	9.480
−12	0.21732	0.001089	554.47	−358.17	2716.1	2357.9	−358.17	2836.6	2478.4	−1.3130	10.862	9.549
−14	0.18121	0.001088	659.88	−362.18	2717.3	2355.2	−362.18	2836.9	2474.7	−1.3284	10.947	9.618
−16	0.15068	0.001088	787.51	−366.17	2718.6	2352.4	−366.17	2837.2	2471.0	−1.3439	11.033	9.689
−18	0.12492	0.001088	942.51	−370.13	2719.7	2349.6	−370.13	2837.5	2467.3	−1.3593	11.121	9.761
−20	0.10326	0.001087	1131.3	−374.06	2720.9	2346.8	−374.06	2837.7	2463.6	−1.3748	11.209	9.835
−22	0.08510	0.001087	1362.0	−377.95	2722.0	2344.1	−377.95	2837.9	2459.9	−1.3903	11.300	9.909
−24	0.06991	0.001087	1644.7	−381.82	2723.1	2341.3	−381.82	2838.1	2456.2	−1.4057	11.391	9.985
−26	0.05725	0.001087	1992.2	−385.66	2724.2	2338.5	−385.66	2838.2	2452.5	−1.4212	11.484	10.063
−28	0.04673	0.001086	2421.0	−389.47	2725.2	2335.7	−389.47	2838.3	2448.8	−1.4367	11.578	10.141
−30	0.03802	0.001086	2951.7	−393.25	2726.2	2332.9	−393.25	2838.4	2445.1	−1.4521	11.673	10.221
−32	0.03082	0.001086	3610.9	−397.00	2727.2	2330.2	−397.00	2838.4	2441.4	−1.4676	11.770	10.303
−34	0.02490	0.001085	4432.4	−400.72	2728.1	2327.4	−400.72	2838.5	2437.7	−1.4831	11.869	10.386
−36	0.02004	0.001085	5460.1	−404.40	2729.0	2324.6	−404.40	2838.4	2434.0	−1.4986	11.969	10.470
−38	0.01608	0.001085	6750.5	−408.07	2729.9	2321.8	−408.07	2838.4	2430.3	−1.5141	12.071	10.557
−40	0.01285	0.001084	8376.7	−411.70	2730.7	2319.0	−411.70	2838.3	2426.6	−1.5296	12.174	10.644

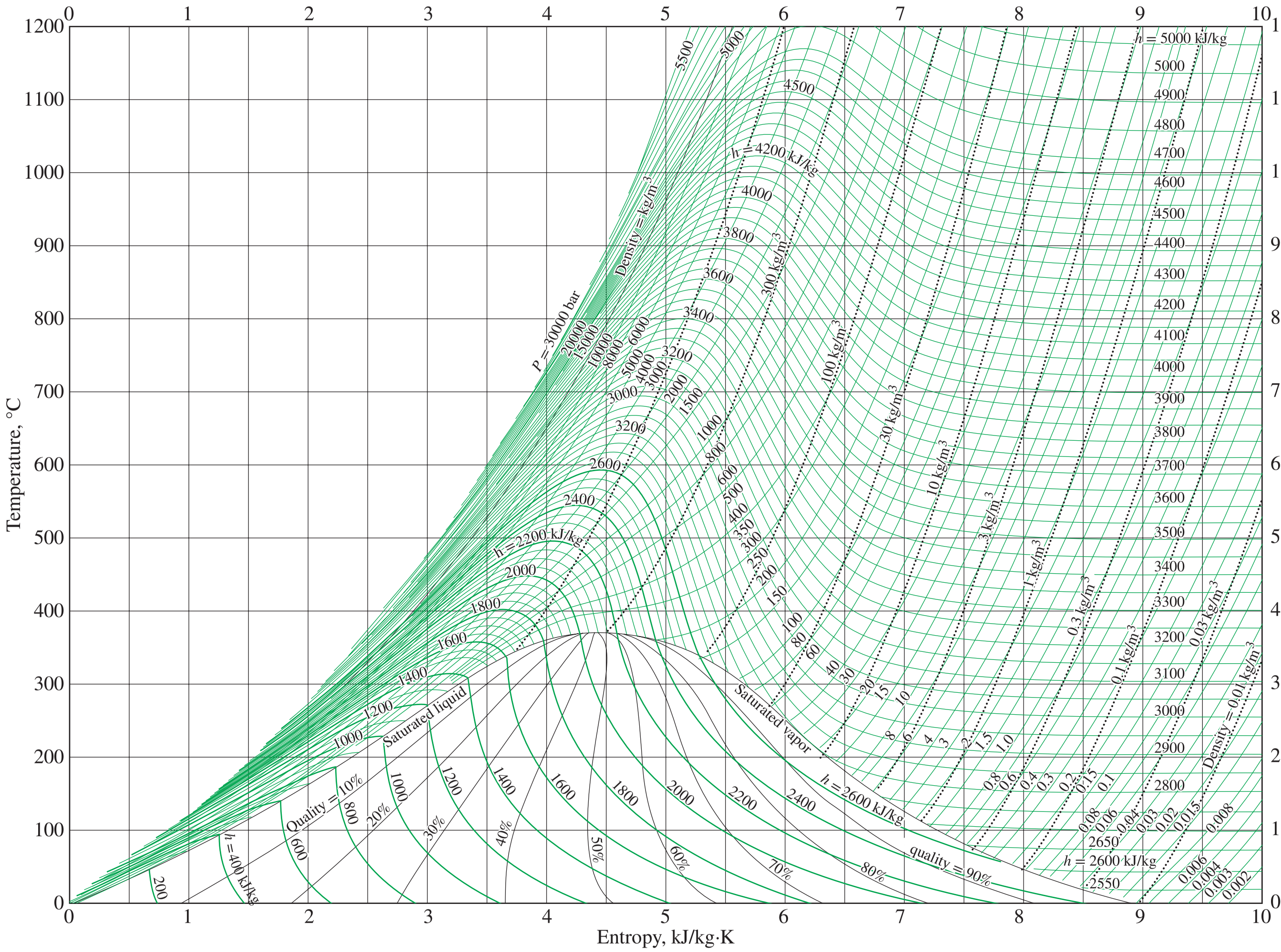


FIGURE A–9

T-s diagram for water.

Source of Data: From NBS/NRC Steam Tables/1 *by Lester Haar, John S. Gallagher, and George S. Kell. Routledge/Taylor & Francis Books, Inc.,* 1984.

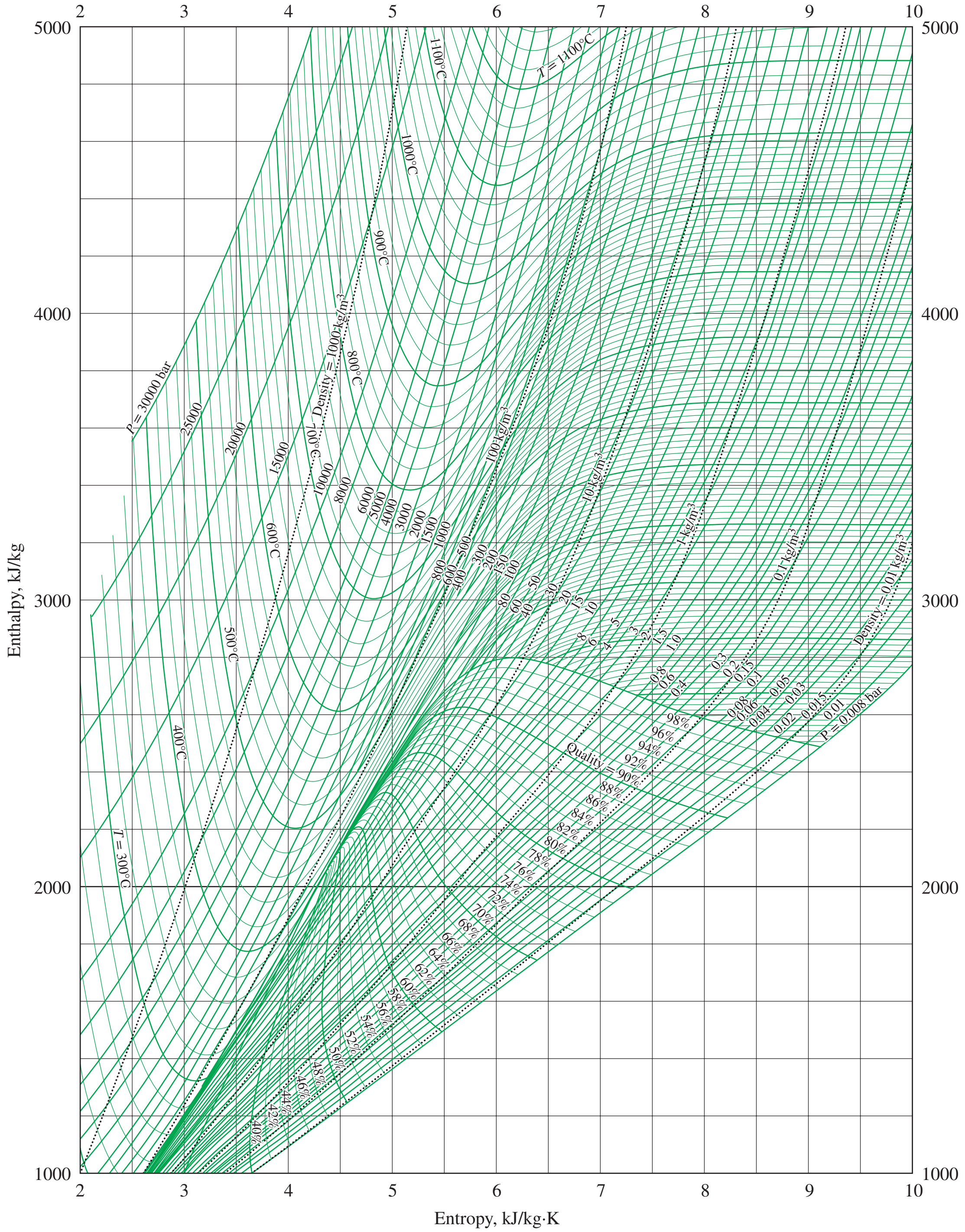


FIGURE A–10

Mollier diagram for water.

Source of Data: From NBS/NRC Steam Tables/1 *by Lester Haar, John S. Gallagher, and George S. Kell. Routledge/Taylor & Francis Books, Inc.,* 1984.

TABLE A–11

Saturated refrigerant-134a—Temperature table

		Specific volume, m^3/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
Temp., T °C	Sat. press., P_{sat} kPa	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
−40	51.25	0.0007053	0.36064	−0.036	207.42	207.38	0.00	225.86	225.86	0.00000	0.96869	0.96869
−38	56.86	0.0007082	0.32718	2.472	206.06	208.53	2.512	224.62	227.13	0.01071	0.95516	0.96588
−36	62.95	0.0007111	0.29740	4.987	204.69	209.68	5.032	223.37	228.40	0.02137	0.94182	0.96319
−34	69.56	0.0007141	0.27082	7.509	203.32	210.83	7.559	222.10	229.66	0.03196	0.92867	0.96063
−32	76.71	0.0007171	0.24706	10.04	201.94	211.97	10.09	220.83	230.93	0.04249	0.91569	0.95819
−30	84.43	0.0007201	0.22577	12.58	200.55	213.12	12.64	219.55	232.19	0.05297	0.90289	0.95586
−28	92.76	0.0007232	0.20666	15.12	199.15	214.27	15.19	218.25	233.44	0.06339	0.89024	0.95364
−26	101.73	0.0007264	0.18947	17.67	197.75	215.42	17.75	216.95	234.70	0.07376	0.87776	0.95152
−24	111.37	0.0007296	0.17398	20.23	196.34	216.57	20.31	215.63	235.94	0.08408	0.86542	0.94950
−22	121.72	0.0007328	0.15999	22.80	194.92	217.71	22.89	214.30	237.19	0.09435	0.85323	0.94758
−20	132.82	0.0007361	0.14735	25.37	193.49	218.86	25.47	212.96	238.43	0.10456	0.84119	0.94575
−18	144.69	0.0007394	0.13589	27.96	192.05	220.00	28.07	211.60	239.67	0.11473	0.82927	0.94401
−16	157.38	0.0007428	0.12550	30.55	190.60	221.15	30.67	210.23	240.90	0.12486	0.81749	0.94234
−14	170.93	0.0007463	0.11605	33.15	189.14	222.29	33.28	208.84	242.12	0.13493	0.80583	0.94076
−12	185.37	0.0007498	0.10744	35.76	187.66	223.42	35.90	207.44	243.34	0.14497	0.79429	0.93925
−10	200.74	0.0007533	0.099600	38.38	186.18	224.56	38.53	206.02	244.55	0.15496	0.78286	0.93782
−8	217.08	0.0007570	0.092438	41.01	184.69	225.69	41.17	204.59	245.76	0.16491	0.77154	0.93645
−6	234.44	0.0007607	0.085888	43.64	183.18	226.82	43.82	203.14	246.95	0.17482	0.76033	0.93514
−4	252.85	0.0007644	0.079889	46.29	181.66	227.94	46.48	201.66	248.14	0.18469	0.74921	0.93390
−2	272.36	0.0007683	0.074388	48.94	180.12	229.07	49.15	200.17	249.33	0.19452	0.73819	0.93271
0	293.01	0.0007722	0.069335	51.61	178.58	230.18	51.83	198.67	250.50	0.20432	0.72726	0.93158
2	314.84	0.0007761	0.064690	54.28	177.01	231.30	54.53	197.14	251.66	0.21408	0.71641	0.93050
4	337.90	0.0007802	0.060412	56.97	175.44	232.40	57.23	195.58	252.82	0.22381	0.70565	0.92946
6	362.23	0.0007843	0.056469	59.66	173.84	233.51	59.95	194.01	253.96	0.23351	0.69496	0.92847
8	387.88	0.0007886	0.052829	62.37	172.23	234.60	62.68	192.42	255.09	0.24318	0.68435	0.92752
10	414.89	0.0007929	0.049466	65.09	170.61	235.69	65.42	190.80	256.22	0.25282	0.67380	0.92661
12	443.31	0.0007973	0.046354	67.82	168.96	236.78	68.17	189.16	257.33	0.26243	0.66331	0.92574
14	473.19	0.0008018	0.043471	70.56	167.30	237.86	70.94	187.49	258.43	0.27201	0.65289	0.92490
16	504.58	0.0008064	0.040798	73.31	165.62	238.93	73.72	185.80	259.51	0.28157	0.64252	0.92409
18	537.52	0.0008112	0.038317	76.07	163.92	239.99	76.51	184.08	260.59	0.29111	0.63219	0.92330
20	572.07	0.0008160	0.036012	78.85	162.19	241.04	79.32	182.33	261.64	0.30062	0.62192	0.92254
22	608.27	0.0008209	0.033867	81.64	160.45	242.09	82.14	180.55	262.69	0.31012	0.61168	0.92180
24	646.18	0.0008260	0.031869	84.44	158.68	243.13	84.98	178.74	263.72	0.31959	0.60148	0.92107
26	685.84	0.0008312	0.030008	87.26	156.89	244.15	87.83	176.90	264.73	0.32905	0.59131	0.92036
28	727.31	0.0008366	0.028271	90.09	155.08	245.17	90.70	175.03	265.73	0.33849	0.58117	0.91967
30	770.64	0.0008421	0.026648	92.93	153.24	246.17	93.58	173.13	266.71	0.34792	0.57105	0.91897
32	815.89	0.0008477	0.025131	95.79	151.37	247.17	96.49	171.19	267.67	0.35734	0.56095	0.91829
34	863.11	0.0008535	0.023712	98.67	149.48	248.15	99.41	169.21	268.61	0.36675	0.55086	0.91760
36	912.35	0.0008595	0.022383	101.56	147.55	249.11	102.34	167.19	269.53	0.37615	0.54077	0.91692
38	963.68	0.0008657	0.021137	104.47	145.60	250.07	105.30	165.13	270.44	0.38554	0.53068	0.91622
40	1017.1	0.0008720	0.019968	107.39	143.61	251.00	108.28	163.03	271.31	0.39493	0.52059	0.91552
42	1072.8	0.0008786	0.018870	110.34	141.59	251.92	111.28	160.89	272.17	0.40432	0.51048	0.91480
44	1130.7	0.0008854	0.017837	113.30	139.53	252.83	114.30	158.70	273.00	0.41371	0.50036	0.91407

TABLE A–11

Saturated refrigerant-134a—Temperature table *(Concluded)*

Temp., T °C	Sat. press., P_{sat} kPa	*Specific volume,* m^3/kg		*Internal energy,* kJ/kg			*Enthalpy,* kJ/kg			*Entropy,* kJ/kg·K		
		Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
46	1191.0	0.0008924	0.016866	116.28	137.43	253.71	117.34	156.46	273.80	0.42311	0.49020	0.91331
48	1253.6	0.0008997	0.015951	119.28	135.30	254.58	120.41	154.17	274.57	0.43251	0.48001	0.91252
52	1386.2	0.0009151	0.014276	125.35	130.89	256.24	126.62	149.41	276.03	0.45136	0.45948	0.91084
56	1529.1	0.0009317	0.012782	131.52	126.29	257.81	132.94	144.41	277.35	0.47028	0.43870	0.90898
60	1682.8	0.0009498	0.011434	137.79	121.45	259.23	139.38	139.09	278.47	0.48930	0.41746	0.90676
65	1891.0	0.0009751	0.009959	145.80	115.06	260.86	147.64	132.05	279.69	0.51330	0.39048	0.90379
70	2118.2	0.0010037	0.008650	154.03	108.17	262.20	156.15	124.37	280.52	0.53763	0.36239	0.90002
75	2365.8	0.0010373	0.007486	162.55	100.62	263.17	165.01	115.87	280.88	0.56252	0.33279	0.89531
80	2635.3	0.0010774	0.006439	171.43	92.22	263.66	174.27	106.35	280.63	0.58812	0.30113	0.88925
85	2928.2	0.0011273	0.005484	180.81	82.64	263.45	184.11	95.39	279.51	0.61487	0.26632	0.88120
90	3246.9	0.0011938	0.004591	190.94	71.19	262.13	194.82	82.22	277.04	0.64354	0.22638	0.86991
95	3594.1	0.0012945	0.003713	202.49	56.25	258.73	207.14	64.94	272.08	0.67605	0.17638	0.85243
100	3975.1	0.0015269	0.002657	218.73	29.72	248.46	224.80	34.22	259.02	0.72224	0.09169	0.81393

Source of Data: Tables A–11 through A–13 are generated using the Engineering Equation Solver (EES) software developed by S. A. Klein and F. L. Alvarado. The routine used in calculations is the R134a, which is based on the fundamental equation of state developed by R. Tillner–Roth and H.D. Baehr, "An International Standard Formulation for the Thermodynamic Properties of 1,1,1,2-Tetrafluoroethane (HFC-134a) for temperatures from 170 K to 455 K and pressures up to 70 MPa," *J. Phys. Chem, Ref. Data,* Vol. 23, No. 5, 1994. The enthalpy and entropy values of saturated liquid are set to zero at −40°C (and −40°F).

TABLE A–12

Saturated refrigerant-134a—Pressure table

		Specific volume, m^3/kg		*Internal energy,* kJ/kg			*Enthalpy,* kJ/kg			*Entropy,* kJ/kg·K		
Press., P kPa	Sat. temp., T_{sat} °C	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
60	−36.95	0.0007097	0.31108	3.795	205.34	209.13	3.837	223.96	227.80	0.01633	0.94812	0.96445
70	−33.87	0.0007143	0.26921	7.672	203.23	210.90	7.722	222.02	229.74	0.03264	0.92783	0.96047
80	−31.13	0.0007184	0.23749	11.14	201.33	212.48	11.20	220.27	231.47	0.04707	0.91009	0.95716
90	−28.65	0.0007222	0.21261	14.30	199.60	213.90	14.36	218.67	233.04	0.06003	0.89431	0.95434
100	−26.37	0.0007258	0.19255	17.19	198.01	215.21	17.27	217.19	234.46	0.07182	0.88008	0.95191
120	−22.32	0.0007323	0.16216	22.38	195.15	217.53	22.47	214.52	236.99	0.09269	0.85520	0.94789
140	−18.77	0.0007381	0.14020	26.96	192.60	219.56	27.06	212.13	239.19	0.11080	0.83387	0.94467
160	−15.60	0.0007435	0.12355	31.06	190.31	221.37	31.18	209.96	241.14	0.12686	0.81517	0.94202
180	−12.73	0.0007485	0.11049	34.81	188.20	223.01	34.94	207.95	242.90	0.14131	0.79848	0.93979
200	−10.09	0.0007532	0.099951	38.26	186.25	224.51	38.41	206.09	244.50	0.15449	0.78339	0.93788
240	−5.38	0.0007618	0.083983	44.46	182.71	227.17	44.64	202.68	247.32	0.17786	0.75689	0.93475
280	−1.25	0.0007697	0.072434	49.95	179.54	229.49	50.16	199.61	249.77	0.19822	0.73406	0.93228
320	2.46	0.0007771	0.063681	54.90	176.65	231.55	55.14	196.78	251.93	0.21631	0.71395	0.93026
360	5.82	0.0007840	0.056809	59.42	173.99	233.41	59.70	194.15	253.86	0.23265	0.69591	0.92856
400	8.91	0.0007905	0.051266	63.61	171.49	235.10	63.92	191.68	255.61	0.24757	0.67954	0.92711
450	12.46	0.0007983	0.045677	68.44	168.58	237.03	68.80	188.78	257.58	0.26462	0.66093	0.92555
500	15.71	0.0008058	0.041168	72.92	165.86	238.77	73.32	186.04	259.36	0.28021	0.64399	0.92420
550	18.73	0.0008129	0.037452	77.09	163.29	240.38	77.54	183.44	260.98	0.29460	0.62842	0.92302
600	21.55	0.0008198	0.034335	81.01	160.84	241.86	81.50	180.95	262.46	0.30799	0.61398	0.92196
650	24.20	0.0008265	0.031680	84.72	158.51	243.23	85.26	178.56	263.82	0.32052	0.60048	0.92100
700	26.69	0.0008331	0.029392	88.24	156.27	244.51	88.82	176.26	265.08	0.33232	0.58780	0.92012
750	29.06	0.0008395	0.027398	91.59	154.11	245.70	92.22	174.03	266.25	0.34348	0.57582	0.91930
800	31.31	0.0008457	0.025645	94.80	152.02	246.82	95.48	171.86	267.34	0.35408	0.56445	0.91853
850	33.45	0.0008519	0.024091	97.88	150.00	247.88	98.61	169.75	268.36	0.36417	0.55362	0.91779
900	35.51	0.0008580	0.022703	100.84	148.03	248.88	101.62	167.69	269.31	0.37383	0.54326	0.91709
950	37.48	0.0008640	0.021456	103.70	146.11	249.82	104.52	165.68	270.20	0.38307	0.53333	0.91641
1000	39.37	0.0008700	0.020329	106.47	144.24	250.71	107.34	163.70	271.04	0.39196	0.52378	0.91574
1200	46.29	0.0008935	0.016728	116.72	137.12	253.84	117.79	156.12	273.92	0.42449	0.48870	0.91320
1400	52.40	0.0009167	0.014119	125.96	130.44	256.40	127.25	148.92	276.17	0.45325	0.45742	0.91067
1600	57.88	0.0009400	0.012134	134.45	124.05	258.50	135.96	141.96	277.92	0.47921	0.42881	0.90802
1800	62.87	0.0009639	0.010568	142.36	117.85	260.21	144.09	135.14	279.23	0.50304	0.40213	0.90517
2000	67.45	0.0009887	0.009297	149.81	111.75	261.56	151.78	128.36	280.15	0.52519	0.37684	0.90204
2500	77.54	0.0010567	0.006941	167.02	96.47	263.49	169.66	111.18	280.84	0.57542	0.31701	0.89243
3000	86.16	0.0011410	0.005272	183.09	80.17	263.26	186.51	92.57	279.08	0.62133	0.25759	0.87893

TABLE A–13

Superheated refrigerant-134a

T °C	υ m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	υ m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	υ m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
	$P = 0.06$ MPa ($T_{sat} = -36.95$°C)				$P = 0.10$ MPa ($T_{sat} = -26.37$°C)				$P = 0.14$ MPa ($T_{sat} = -18.77$°C)			
Sat.	0.31108	209.13	227.80	0.9645	0.19255	215.21	234.46	0.9519	0.14020	219.56	239.19	0.9447
–20	0.33608	220.62	240.78	1.0175	0.19841	219.68	239.52	0.9721				
–10	0.35048	227.57	248.60	1.0478	0.20743	226.77	247.51	1.0031	0.14605	225.93	246.37	0.9724
0	0.36476	234.67	256.56	1.0775	0.21630	233.97	255.60	1.0333	0.15263	233.25	254.61	1.0032
10	0.37893	241.94	264.68	1.1067	0.22506	241.32	263.82	1.0628	0.15908	240.68	262.95	1.0331
20	0.39302	249.37	272.95	1.1354	0.23373	248.81	272.18	1.0919	0.16544	248.24	271.40	1.0625
30	0.40705	256.97	281.39	1.1637	0.24233	256.46	280.69	1.1204	0.17172	255.95	279.99	1.0913
40	0.42102	264.73	289.99	1.1916	0.25088	264.27	289.36	1.1485	0.17794	263.80	288.72	1.1196
50	0.43495	272.66	298.75	1.2192	0.25937	272.24	298.17	1.1762	0.18412	271.81	297.59	1.1475
60	0.44883	280.75	307.68	1.2464	0.26783	280.36	307.15	1.2036	0.19025	279.97	306.61	1.1750
70	0.46269	289.01	316.77	1.2732	0.27626	288.65	316.28	1.2306	0.19635	288.29	315.78	1.2021
80	0.47651	297.43	326.02	1.2998	0.28465	297.10	325.57	1.2573	0.20242	296.77	325.11	1.2289
90	0.49032	306.02	335.43	1.3261	0.29303	305.71	335.01	1.2836	0.20847	305.40	334.59	1.2554
100	0.50410	314.76	345.01	1.3521	0.30138	314.48	344.61	1.3097	0.21449	314.19	344.22	1.2815
	$P = 0.18$ MPa ($T_{sat} = -12.73$°C)				$P = 0.20$ MPa ($T_{sat} = -10.09$°C)				$P = 0.24$ MPa ($T_{sat} = -5.38$°C)			
Sat.	0.11049	223.01	242.90	0.9398	0.09995	224.51	244.50	0.9379	0.08398	227.17	247.32	0.9348
–10	0.11189	225.04	245.18	0.9485	0.09991	224.57	244.56	0.9381				
0	0.11722	232.49	253.59	0.9799	0.10481	232.11	253.07	0.9699	0.08617	231.30	251.98	0.9520
10	0.12240	240.02	262.05	1.0103	0.10955	239.69	261.60	1.0005	0.09026	239.00	260.66	0.9832
20	0.12748	247.66	270.60	1.0400	0.11418	247.36	270.20	1.0304	0.09423	246.76	269.38	1.0134
30	0.13248	255.43	279.27	1.0691	0.11874	255.16	278.91	1.0596	0.09812	254.63	278.17	1.0429
40	0.13741	263.33	288.07	1.0976	0.12322	263.09	287.74	1.0882	0.10193	262.61	287.07	1.0718
50	0.14230	271.38	297.00	1.1257	0.12766	271.16	296.70	1.1164	0.10570	270.73	296.09	1.1002
60	0.14715	279.58	306.07	1.1533	0.13206	279.38	305.79	1.1441	0.10942	278.98	305.24	1.1281
70	0.15196	287.93	315.28	1.1806	0.13641	287.75	315.03	1.1714	0.11310	287.38	314.53	1.1555
80	0.15673	296.43	324.65	1.2075	0.14074	296.27	324.41	1.1984	0.11675	295.93	323.95	1.1826
90	0.16149	305.09	334.16	1.2340	0.14504	304.93	333.94	1.2250	0.12038	304.62	333.51	1.2093
100	0.16622	313.90	343.82	1.2603	0.14933	313.75	343.62	1.2513	0.12398	313.46	343.22	1.2356
	$P = 0.28$ MPa ($T_{sat} = -1.25$°C)				$P = 0.32$ MPa ($T_{sat} = 2.46$°C)				$P = 0.40$ MPa ($T_{sat} = 8.91$°C)			
Sat.	0.07243	229.49	249.77	0.9323	0.06368	231.55	251.93	0.9303	0.051266	235.10	255.61	0.9271
0	0.07282	230.46	250.85	0.9362								
10	0.07646	238.29	259.70	0.9681	0.06609	237.56	258.70	0.9545	0.051506	235.99	256.59	0.9306
20	0.07997	246.15	268.54	0.9987	0.06925	245.51	267.67	0.9856	0.054213	244.19	265.88	0.9628
30	0.08338	254.08	277.42	1.0285	0.07231	253.52	276.66	1.0158	0.056796	252.37	275.09	0.9937
40	0.08672	262.12	286.40	1.0577	0.07530	261.62	285.72	1.0452	0.059292	260.60	284.32	1.0237
50	0.09000	270.28	295.48	1.0862	0.07823	269.83	294.87	1.0739	0.061724	268.92	293.61	1.0529
60	0.09324	278.58	304.69	1.1143	0.08111	278.17	304.12	1.1022	0.064104	277.34	302.98	1.0814
70	0.09644	287.01	314.01	1.1419	0.08395	286.64	313.50	1.1299	0.066443	285.88	312.45	1.1095
80	0.09961	295.59	323.48	1.1690	0.08675	295.24	323.00	1.1572	0.068747	294.54	322.04	1.1370
90	0.10275	304.30	333.07	1.1958	0.08953	303.99	332.64	1.1841	0.071023	303.34	331.75	1.1641
100	0.10587	313.17	342.81	1.2223	0.09229	312.87	342.41	1.2106	0.073274	312.28	341.59	1.1908
110	0.10897	322.18	352.69	1.2484	0.09503	321.91	352.31	1.2368	0.075504	321.35	351.55	1.2172
120	0.11205	331.34	362.72	1.2742	0.09775	331.08	362.36	1.2627	0.077717	330.56	361.65	1.2432
130	0.11512	340.65	372.88	1.2998	0.10045	340.41	372.55	1.2883	0.079913	339.92	371.89	1.2689
140	0.11818	350.11	383.20	1.3251	0.10314	349.88	382.89	1.3136	0.082096	349.42	382.26	1.2943

TABLE A–13

Superheated refrigerant-134a *(Concluded)*

T °C	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
	$P = 0.50$ MPa ($T_{sat} = 15.71$°C)				$P = 0.60$ MPa ($T_{sat} = 21.55$°C)				$P = 0.70$ MPa ($T_{sat} = 26.69$°C)			
Sat.	0.041168	238.77	259.36	0.9242	0.034335	241.86	262.46	0.9220	0.029392	244.51	265.08	0.9201
20	0.042115	242.42	263.48	0.9384								
30	0.044338	250.86	273.03	0.9704	0.035984	249.24	270.83	0.9500	0.029966	247.49	268.47	0.9314
40	0.046456	259.27	282.50	1.0011	0.037865	257.88	280.60	0.9817	0.031696	256.41	278.59	0.9642
50	0.048499	267.73	291.98	1.0309	0.039659	266.50	290.30	1.0122	0.033322	265.22	288.54	0.9955
60	0.050485	276.27	301.51	1.0600	0.041389	275.17	300.00	1.0417	0.034875	274.03	298.44	1.0257
70	0.052427	284.91	311.12	1.0884	0.043069	283.91	309.75	1.0706	0.036373	282.88	308.34	1.0550
80	0.054331	293.65	320.82	1.1163	0.044710	292.74	319.57	1.0988	0.037829	291.81	318.29	1.0835
90	0.056205	302.52	330.63	1.1436	0.046318	301.69	329.48	1.1265	0.039250	300.84	328.31	1.1115
100	0.058053	311.52	340.55	1.1706	0.047900	310.75	339.49	1.1536	0.040642	309.96	338.41	1.1389
110	0.059880	320.65	350.59	1.1971	0.049458	319.93	349.61	1.1804	0.042010	319.21	348.61	1.1659
120	0.061687	329.91	360.75	1.2233	0.050997	329.24	359.84	1.2068	0.043358	328.57	358.92	1.1925
130	0.063479	339.31	371.05	1.2492	0.052519	338.69	370.20	1.2328	0.044688	338.06	369.34	1.2186
140	0.065256	348.85	381.47	1.2747	0.054027	348.26	380.68	1.2585	0.046004	347.67	379.88	1.2445
150	0.067021	358.52	392.04	1.3000	0.055522	357.98	391.29	1.2838	0.047306	357.42	390.54	1.2700
160	0.068775	368.34	402.73	1.3250	0.057006	367.83	402.03	1.3089	0.048597	367.31	401.32	1.2952
	$P = 0.80$ MPa ($T_{sat} = 31.31$°C)				$P = 0.90$ MPa ($T_{sat} = 35.51$°C)				$P = 1.00$ MPa ($T_{sat} = 39.37$°C)			
Sat.	0.025645	246.82	267.34	0.9185	0.022686	248.82	269.25	0.9169	0.020319	250.71	271.04	0.9157
40	0.027035	254.84	276.46	0.9481	0.023375	253.15	274.19	0.9328	0.020406	251.32	271.73	0.9180
50	0.028547	263.87	286.71	0.9803	0.024809	262.46	284.79	0.9661	0.021796	260.96	282.76	0.9526
60	0.029973	272.85	296.82	1.0111	0.026146	271.62	295.15	0.9977	0.023068	270.33	293.40	0.9851
70	0.031340	281.83	306.90	1.0409	0.027413	280.74	305.41	1.0280	0.024261	279.61	303.87	1.0160
80	0.032659	290.86	316.99	1.0699	0.028630	289.88	315.65	1.0574	0.025398	288.87	314.27	1.0459
90	0.033941	299.97	327.12	1.0982	0.029806	299.08	325.90	1.0861	0.026492	298.17	324.66	1.0749
100	0.035193	309.17	337.32	1.1259	0.030951	308.35	336.21	1.1141	0.027552	307.52	335.08	1.1032
110	0.036420	318.47	347.61	1.1531	0.032068	317.72	346.58	1.1415	0.028584	316.96	345.54	1.1309
120	0.037625	327.89	357.99	1.1798	0.033164	327.19	357.04	1.1684	0.029592	326.49	356.08	1.1580
130	0.038813	337.42	368.47	1.2062	0.034241	336.78	367.59	1.1949	0.030581	336.12	366.70	1.1847
140	0.039985	347.08	379.07	1.2321	0.035302	346.48	378.25	1.2211	0.031554	345.87	377.42	1.2110
150	0.041143	356.86	389.78	1.2577	0.036349	356.30	389.01	1.2468	0.032512	355.73	388.24	1.2369
160	0.042290	366.78	400.61	1.2830	0.037384	366.25	399.89	1.2722	0.033457	365.71	399.17	1.2624
170	0.043427	376.83	411.57	1.3081	0.038408	376.33	410.89	1.2973	0.034392	375.82	410.22	1.2876
180	0.044554	387.01	422.65	1.3328	0.039423	386.54	422.02	1.3221	0.035317	386.06	421.38	1.3125
	$P = 1.20$ MPa ($T_{sat} = 46.29$°C)				$P = 1.40$ MPa ($T_{sat} = 52.40$°C)				$P = 1.60$ MPa ($T_{sat} = 57.88$°C)			
Sat.	0.016728	253.84	273.92	0.9132	0.014119	256.40	276.17	0.9107	0.012134	258.50	277.92	0.9080
50	0.017201	257.64	278.28	0.9268								
60	0.018404	267.57	289.66	0.9615	0.015005	264.46	285.47	0.9389	0.012372	260.91	280.71	0.9164
70	0.019502	277.23	300.63	0.9939	0.016060	274.62	297.10	0.9733	0.013430	271.78	293.27	0.9536
80	0.020529	286.77	311.40	1.0249	0.017023	284.51	308.34	1.0056	0.014362	282.11	305.09	0.9875
90	0.021506	296.28	322.09	1.0547	0.017923	294.28	319.37	1.0364	0.015215	292.19	316.53	1.0195
100	0.022442	305.81	332.74	1.0836	0.018778	304.01	330.30	1.0661	0.016014	302.16	327.78	1.0501
110	0.023348	315.40	343.41	1.1119	0.019597	313.76	341.19	1.0949	0.016773	312.09	338.93	1.0795
120	0.024228	325.05	354.12	1.1395	0.020388	323.55	352.09	1.1230	0.017500	322.03	350.03	1.1081
130	0.025086	334.79	364.90	1.1665	0.021155	333.41	363.02	1.1504	0.018201	332.02	361.14	1.1360
140	0.025927	344.63	375.74	1.1931	0.021904	343.34	374.01	1.1773	0.018882	342.06	372.27	1.1633
150	0.026753	354.57	386.68	1.2192	0.022636	353.37	385.07	1.2038	0.019545	352.19	383.46	1.1901
160	0.027566	364.63	397.71	1.2450	0.023355	363.51	396.20	1.2298	0.020194	362.40	394.71	1.2164
170	0.028367	374.80	408.84	1.2704	0.024061	373.75	407.43	1.2554	0.020830	372.71	406.04	1.2422
180	0.029158	385.10	420.09	1.2955	0.024757	384.12	418.78	1.2808	0.021456	383.13	417.46	1.2677

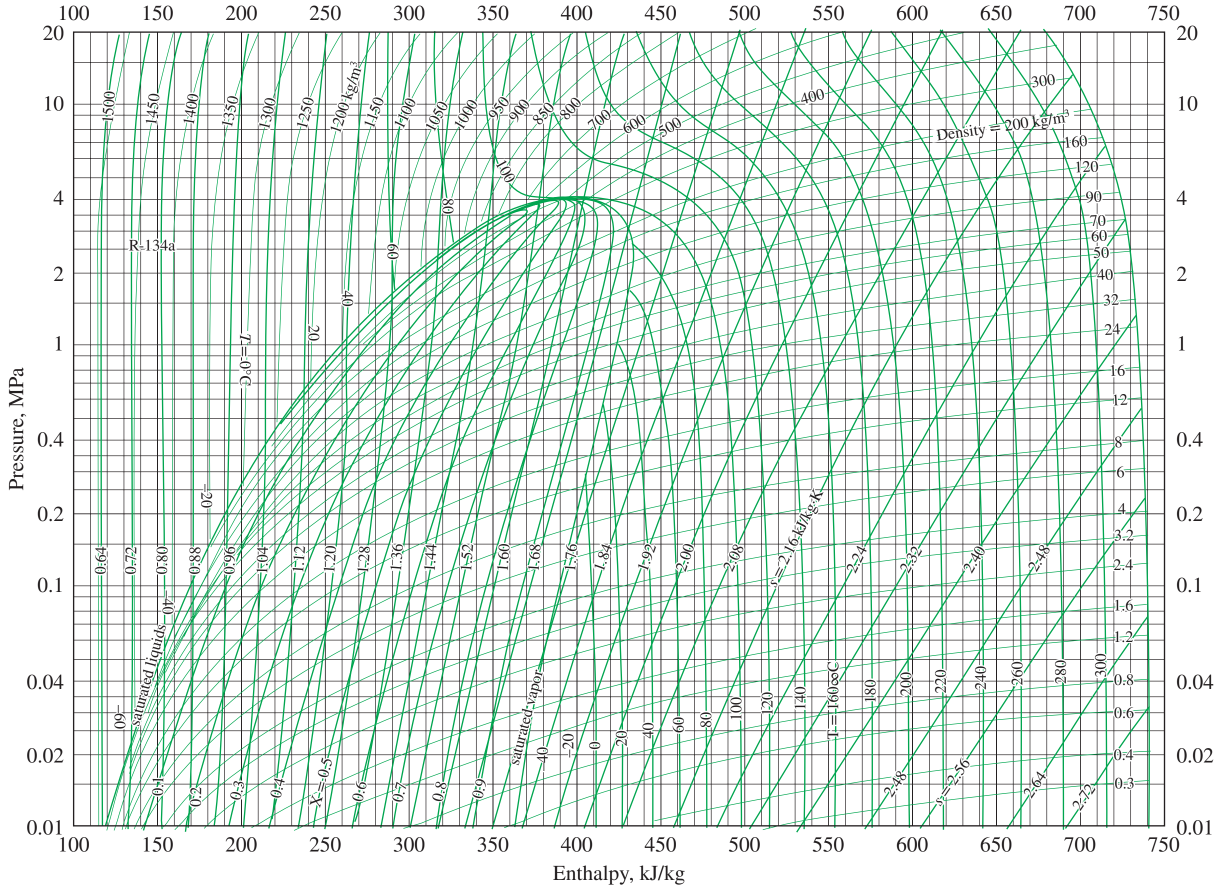


FIGURE A–14

P-h diagram for refrigerant-134a.

Note: The reference point used for the chart is different than that used in the R-134a tables. Therefore, problems should be solved using all property data either from the tables or from the chart, but not from both.

Source of Data: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc., Atlanta, GA.

TABLE A–15

Properties of saturated water

Temp. T, °C	Saturation pressure P_{sat}, kPa	Density ρ, kg/m^3		Enthalpy of vaporization h_{fg}, kJ/kg	Specific heat c_p, J/kg·K		Thermal conductivity k, W/m·K		Dynamic viscosity μ, kg/m·s		Prandtl number Pr		Volume expansion coefficient β, 1/K
		Liquid	Vapor		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid
0.01	0.6113	999.8	0.0048	2501	4217	1854	0.561	0.0171	1.792×10^{-3}	0.922×10^{-5}	13.5	1.00	-0.068×10^{-3}
5	0.8721	999.9	0.0068	2490	4205	1857	0.571	0.0173	1.519×10^{-3}	0.934×10^{-5}	11.2	1.00	0.015×10^{-3}
10	1.2276	999.7	0.0094	2478	4194	1862	0.580	0.0176	1.307×10^{-3}	0.946×10^{-5}	9.45	1.00	0.733×10^{-3}
15	1.7051	999.1	0.0128	2466	4185	1863	0.589	0.0179	1.138×10^{-3}	0.959×10^{-5}	8.09	1.00	0.138×10^{-3}
20	2.339	998.0	0.0173	2454	4182	1867	0.598	0.0182	1.002×10^{-3}	0.973×10^{-5}	7.01	1.00	0.195×10^{-3}
25	3.169	997.0	0.0231	2442	4180	1870	0.607	0.0186	0.891×10^{-3}	0.987×10^{-5}	6.14	1.00	0.247×10^{-3}
30	4.246	996.0	0.0304	2431	4178	1875	0.615	0.0189	0.798×10^{-3}	1.001×10^{-5}	5.42	1.00	0.294×10^{-3}
35	5.628	994.0	0.0397	2419	4178	1880	0.623	0.0192	0.720×10^{-3}	1.016×10^{-5}	4.83	1.00	0.337×10^{-3}
40	7.384	992.1	0.0512	2407	4179	1885	0.631	0.0196	0.653×10^{-3}	1.031×10^{-5}	4.32	1.00	0.377×10^{-3}
45	9.593	990.1	0.0655	2395	4180	1892	0.637	0.0200	0.596×10^{-3}	1.046×10^{-5}	3.91	1.00	0.415×10^{-3}
50	12.35	988.1	0.0831	2383	4181	1900	0.644	0.0204	0.547×10^{-3}	1.062×10^{-5}	3.55	1.00	0.451×10^{-3}
55	15.76	985.2	0.1045	2371	4183	1908	0.649	0.0208	0.504×10^{-3}	1.077×10^{-5}	3.25	1.00	0.484×10^{-3}
60	19.94	983.3	0.1304	2359	4185	1916	0.654	0.0212	0.467×10^{-3}	1.093×10^{-5}	2.99	1.00	0.517×10^{-3}
65	25.03	980.4	0.1614	2346	4187	1926	0.659	0.0216	0.433×10^{-3}	1.110×10^{-5}	2.75	1.00	0.548×10^{-3}
70	31.19	977.5	0.1983	2334	4190	1936	0.663	0.0221	0.404×10^{-3}	1.126×10^{-5}	2.55	1.00	0.578×10^{-3}
75	38.58	974.7	0.2421	2321	4193	1948	0.667	0.0225	0.378×10^{-3}	1.142×10^{-5}	2.38	1.00	0.607×10^{-3}
80	47.39	971.8	0.2935	2309	4197	1962	0.670	0.0230	0.355×10^{-3}	1.159×10^{-5}	2.22	1.00	0.653×10^{-3}
85	57.83	968.1	0.3536	2296	4201	1977	0.673	0.0235	0.333×10^{-3}	1.176×10^{-5}	2.08	1.00	0.670×10^{-3}
90	70.14	965.3	0.4235	2283	4206	1993	0.675	0.0240	0.315×10^{-3}	1.193×10^{-5}	1.96	1.00	0.702×10^{-3}
95	84.55	961.5	0.5045	2270	4212	2010	0.677	0.0246	0.297×10^{-3}	1.210×10^{-5}	1.85	1.00	0.716×10^{-3}
100	101.33	957.9	0.5978	2257	4217	2029	0.679	0.0251	0.282×10^{-3}	1.227×10^{-5}	1.75	1.00	0.750×10^{-3}
110	143.27	950.6	0.8263	2230	4229	2071	0.682	0.0262	0.255×10^{-3}	1.261×10^{-5}	1.58	1.00	0.798×10^{-3}
120	198.53	943.4	1.121	2203	4244	2120	0.683	0.0275	0.232×10^{-3}	1.296×10^{-5}	1.44	1.00	0.858×10^{-3}
130	270.1	934.6	1.496	2174	4263	2177	0.684	0.0288	0.213×10^{-3}	1.330×10^{-5}	1.33	1.01	0.913×10^{-3}
140	361.3	921.7	1.965	2145	4286	2244	0.683	0.0301	0.197×10^{-3}	1.365×10^{-5}	1.24	1.02	0.970×10^{-3}
150	475.8	916.6	2.546	2114	4311	2314	0.682	0.0316	0.183×10^{-3}	1.399×10^{-5}	1.16	1.02	1.025×10^{-3}
160	617.8	907.4	3.256	2083	4340	2420	0.680	0.0331	0.170×10^{-3}	1.434×10^{-5}	1.09	1.05	1.145×10^{-3}
170	791.7	897.7	4.119	2050	4370	2490	0.677	0.0347	0.160×10^{-3}	1.468×10^{-5}	1.03	1.05	1.178×10^{-3}
180	1,002.1	887.3	5.153	2015	4410	2590	0.673	0.0364	0.150×10^{-3}	1.502×10^{-5}	0.983	1.07	1.210×10^{-3}
190	1,254.4	876.4	6.388	1979	4460	2710	0.669	0.0382	0.142×10^{-3}	1.537×10^{-5}	0.947	1.09	1.280×10^{-3}
200	1,553.8	864.3	7.852	1941	4500	2840	0.663	0.0401	0.134×10^{-3}	1.571×10^{-5}	0.910	1.11	1.350×10^{-3}
220	2,318	840.3	11.60	1859	4610	3110	0.650	0.0442	0.122×10^{-3}	1.641×10^{-5}	0.865	1.15	1.520×10^{-3}
240	3,344	813.7	16.73	1767	4760	3520	0.632	0.0487	0.111×10^{-3}	1.712×10^{-5}	0.836	1.24	1.720×10^{-3}
260	4,688	783.7	23.69	1663	4970	4070	0.609	0.0540	0.102×10^{-3}	1.788×10^{-5}	0.832	1.35	2.000×10^{-3}
280	6,412	750.8	33.15	1544	5280	4835	0.581	0.0605	0.094×10^{-3}	1.870×10^{-5}	0.854	1.49	2.380×10^{-3}
300	8,581	713.8	46.15	1405	5750	5980	0.548	0.0695	0.086×10^{-3}	1.965×10^{-5}	0.902	1.69	2.950×10^{-3}
320	11,274	667.1	64.57	1239	6540	7900	0.509	0.0836	0.078×10^{-3}	2.084×10^{-5}	1.00	1.97	
340	14,586	610.5	92.62	1028	8240	11,870	0.469	0.110	0.070×10^{-3}	2.255×10^{-5}	1.23	2.43	
360	18,651	528.3	144.0	720	14,690	25,800	0.427	0.178	0.060×10^{-3}	2.571×10^{-5}	2.06	3.73	
374.14	22,090	317.0	317.0	0	—	—	—	—	0.043×10^{-3}	4.313×10^{-5}			

Note 1: Kinematic viscosity ν and thermal diffusivity α can be calculated from their definitions, $\nu = \mu/\rho$ and $\alpha = k/\rho c_p = \nu/\text{Pr}$. The temperatures 0.01°C, 100°C, and 374.14°C are the triple-, boiling-, and critical-point temperatures of water, respectively. The properties listed above (except the vapor density) can be used at any pressure with negligible error except at temperatures near the critical-point value.

Note 2: The unit kJ/kg·°C for specific heat is equivalent to kJ/kg·K, and the unit W/m·°C for thermal conductivity is equivalent to W/m·K.

Source: Viscosity and thermal conductivity data are from J. V. Sengers and J. T. R. Watson, *Journal of Physical and Chemical Reference Data* 15 (1986), pp. 1291–1322. Other data are obtained from various sources or calculated.

TABLE A–16

Properties of saturated refrigerant-134a

Temp. T, °C	Saturation pressure P, kPa	Density ρ, kg/m^3		Enthalpy of vaporization h_{fg}, kJ/kg	Specific heat c_p, J/kg·K		Thermal conductivity k, W/m·K		Dynamic viscosity μ, kg/m·s		Prandtl number Pr		Volume expansion coefficient β, 1/K	Surface tension, N/m
		Liquid	Vapor		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	
−40	51.2	1418	2.773	225.9	1254	748.6	0.1101	0.00811	4.878×10^{-4}	2.550×10^{-6}	5.558	0.235	0.00205	0.01760
−35	66.2	1403	3.524	222.7	1264	764.1	0.1084	0.00862	4.509×10^{-4}	3.003×10^{-6}	5.257	0.266	0.00209	0.01682
−30	84.4	1389	4.429	219.5	1273	780.2	0.1066	0.00913	4.178×10^{-4}	3.504×10^{-6}	4.992	0.299	0.00215	0.01604
−25	106.5	1374	5.509	216.3	1283	797.2	0.1047	0.00963	3.882×10^{-4}	4.054×10^{-6}	4.757	0.335	0.00220	0.01527
−20	132.8	1359	6.787	213.0	1294	814.9	0.1028	0.01013	3.614×10^{-4}	4.651×10^{-6}	4.548	0.374	0.00227	0.01451
−15	164.0	1343	8.288	209.5	1306	833.5	0.1009	0.01063	3.371×10^{-4}	5.295×10^{-6}	4.363	0.415	0.00233	0.01376
−10	200.7	1327	10.04	206.0	1318	853.1	0.0989	0.01112	3.150×10^{-4}	5.982×10^{-6}	4.198	0.459	0.00241	0.01302
−5	243.5	1311	12.07	202.4	1330	873.8	0.0968	0.01161	2.947×10^{-4}	6.709×10^{-6}	4.051	0.505	0.00249	0.01229
0	293.0	1295	14.42	198.7	1344	895.6	0.0947	0.01210	2.761×10^{-4}	7.471×10^{-6}	3.919	0.553	0.00258	0.01156
5	349.9	1278	17.12	194.8	1358	918.7	0.0925	0.01259	2.589×10^{-4}	8.264×10^{-6}	3.802	0.603	0.00269	0.01084
10	414.9	1261	20.22	190.8	1374	943.2	0.0903	0.01308	2.430×10^{-4}	9.081×10^{-6}	3.697	0.655	0.00280	0.01014
15	488.7	1244	23.75	186.6	1390	969.4	0.0880	0.01357	2.281×10^{-4}	9.915×10^{-6}	3.604	0.708	0.00293	0.00944
20	572.1	1226	27.77	182.3	1408	997.6	0.0856	0.01406	2.142×10^{-4}	1.075×10^{-5}	3.521	0.763	0.00307	0.00876
25	665.8	1207	32.34	177.8	1427	1028	0.0833	0.01456	2.012×10^{-4}	1.160×10^{-5}	3.448	0.819	0.00324	0.00808
30	770.6	1188	37.53	173.1	1448	1061	0.0808	0.01507	1.888×10^{-4}	1.244×10^{-5}	3.383	0.877	0.00342	0.00742
35	887.5	1168	43.41	168.2	1471	1098	0.0783	0.01558	1.772×10^{-4}	1.327×10^{-5}	3.328	0.935	0.00364	0.00677
40	1017.1	1147	50.08	163.0	1498	1138	0.0757	0.01610	1.660×10^{-4}	1.408×10^{-5}	3.285	0.995	0.00390	0.00613
45	1160.5	1125	57.66	157.6	1529	1184	0.0731	0.01664	1.554×10^{-4}	1.486×10^{-5}	3.253	1.058	0.00420	0.00550
50	1318.6	1102	66.27	151.8	1566	1237	0.0704	0.01720	1.453×10^{-4}	1.562×10^{-5}	3.231	1.123	0.00455	0.00489
55	1492.3	1078	76.11	145.7	1608	1298	0.0676	0.01777	1.355×10^{-4}	1.634×10^{-5}	3.223	1.193	0.00500	0.00429
60	1682.8	1053	87.38	139.1	1659	1372	0.0647	0.01838	1.260×10^{-4}	1.704×10^{-5}	3.229	1.272	0.00554	0.00372
65	1891.0	1026	100.4	132.1	1722	1462	0.0618	0.01902	1.167×10^{-4}	1.771×10^{-5}	3.255	1.362	0.00624	0.00315
70	2118.2	996.2	115.6	124.4	1801	1577	0.0587	0.01972	1.077×10^{-4}	1.839×10^{-5}	3.307	1.471	0.00716	0.00261
75	2365.8	964	133.6	115.9	1907	1731	0.0555	0.02048	9.891×10^{-5}	1.908×10^{-5}	3.400	1.612	0.00843	0.00209
80	2635.2	928.2	155.3	106.4	2056	1948	0.0521	0.02133	9.011×10^{-5}	1.982×10^{-5}	3.558	1.810	0.01031	0.00160
85	2928.2	887.1	182.3	95.4	2287	2281	0.0484	0.02233	8.124×10^{-5}	2.071×10^{-5}	3.837	2.116	0.01336	0.00114
90	3246.9	837.7	217.8	82.2	2701	2865	0.0444	0.02357	7.203×10^{-5}	2.187×10^{-5}	4.385	2.658	0.01911	0.00071
95	3594.1	772.5	269.3	64.9	3675	4144	0.0396	0.02544	6.190×10^{-5}	2.370×10^{-5}	5.746	3.862	0.03343	0.00033
100	3975.1	651.7	376.3	33.9	7959	8785	0.0322	0.02989	4.765×10^{-5}	2.833×10^{-5}	11.77	8.326	0.10047	0.00004

Note 1: Kinematic viscosity ν and thermal diffusivity α can be calculated from their definitions, $\nu = \mu/\rho$ and $\alpha = k/\rho c_p = \nu/\text{Pr}$. The properties listed here (except the vapor density) can be used at any pressures with negligible error except at temperatures near the critical-point value.

Note 2: The unit kJ/kg·°C for specific heat is equivalent to kJ/kg·K, and the unit W/m·°C for thermal conductivity is equivalent to W/m·K.

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Original sources: R. Tillner-Roth and H. D. Baehr, "An International Standard Formulation for the Thermodynamic Properties of 1,1,1,2-Tetrafluoroethane (HFC-134a) for Temperatures from 170 K to 455 K and Pressures up to 70 MPa," *J. Phys. Chem, Ref. Data,* Vol. 23, No. 5, 1994; M.J. Assael, N. K. Dalaouti, A. A. Griva, and J. H. Dymond, "Viscosity and Thermal Conductivity of Halogenated Methane and Ethane Refrigerants," *IJR,* Vol. 22, pp. 525–535, 1999; NIST REFPROP 6 program (M. O. McLinden, S. A. Klein, E. W. Lemmon, and A. P. Peskin, Physical and Chemical Properties Division, National Institute of Standards and Technology, Boulder, CO 80303, 1995).

TABLE A–17

Properties of saturated ammonia

Temp. T, °C	Saturation pressure P, kPa	Density ρ, kg/m^3		Enthalpy of vaporization h_{fg}, kJ/kg	Specific heat c_p, J/kg·K		Thermal conductivity k, W/m·K		Dynamic viscosity μ, kg/m·s		Prandtl number Pr		Volume expansion coefficient β, 1/K	Surface tension, N/m
		Liquid	Vapor		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	
−40	71.66	690.2	0.6435	1389	4414	2242	—	0.01792	2.926×10^{-4}	7.957×10^{-6}	—	0.9955	0.00176	0.03565
−30	119.4	677.8	1.037	1360	4465	2322	—	0.01898	2.630×10^{-4}	8.311×10^{-6}	—	1.017	0.00185	0.03341
−25	151.5	671.5	1.296	1345	4489	2369	0.5968	0.01957	2.492×10^{-4}	8.490×10^{-6}	1.875	1.028	0.00190	0.03229
−20	190.1	665.1	1.603	1329	4514	2420	0.5853	0.02015	2.361×10^{-4}	8.669×10^{-6}	1.821	1.041	0.00194	0.03118
−15	236.2	658.6	1.966	1313	4538	2476	0.5737	0.02075	2.236×10^{-4}	8.851×10^{-6}	1.769	1.056	0.00199	0.03007
−10	290.8	652.1	2.391	1297	4564	2536	0.5621	0.02138	2.117×10^{-4}	9.034×10^{-6}	1.718	1.072	0.00205	0.02896
−5	354.9	645.4	2.886	1280	4589	2601	0.5505	0.02203	2.003×10^{-4}	9.218×10^{-6}	1.670	1.089	0.00210	0.02786
0	429.6	638.6	3.458	1262	4617	2672	0.5390	0.02270	1.896×10^{-4}	9.405×10^{-6}	1.624	1.107	0.00216	0.02676
5	516	631.7	4.116	1244	4645	2749	0.5274	0.02341	1.794×10^{-4}	9.593×10^{-6}	1.580	1.126	0.00223	0.02566
10	615.3	624.6	4.870	1226	4676	2831	0.5158	0.02415	1.697×10^{-4}	9.784×10^{-6}	1.539	1.147	0.00230	0.02457
15	728.8	617.5	5.729	1206	4709	2920	0.5042	0.02492	1.606×10^{-4}	9.978×10^{-6}	1.500	1.169	0.00237	0.02348
20	857.8	610.2	6.705	1186	4745	3016	0.4927	0.02573	1.519×10^{-4}	1.017×10^{-5}	1.463	1.193	0.00245	0.02240
25	1003	602.8	7.809	1166	4784	3120	0.4811	0.02658	1.438×10^{-4}	1.037×10^{-5}	1.430	1.218	0.00254	0.02132
30	1167	595.2	9.055	1144	4828	3232	0.4695	0.02748	1.361×10^{-4}	1.057×10^{-5}	1.399	1.244	0.00264	0.02024
35	1351	587.4	10.46	1122	4877	3354	0.4579	0.02843	1.288×10^{-4}	1.078×10^{-5}	1.372	1.272	0.00275	0.01917
40	1555	579.4	12.03	1099	4932	3486	0.4464	0.02943	1.219×10^{-4}	1.099×10^{-5}	1.347	1.303	0.00287	0.01810
45	1782	571.3	13.8	1075	4993	3631	0.4348	0.03049	1.155×10^{-4}	1.121×10^{-5}	1.327	1.335	0.00301	0.01704
50	2033	562.9	15.78	1051	5063	3790	0.4232	0.03162	1.094×10^{-4}	1.143×10^{-5}	1.310	1.371	0.00316	0.01598
55	2310	554.2	18.00	1025	5143	3967	0.4116	0.03283	1.037×10^{-4}	1.166×10^{-5}	1.297	1.409	0.00334	0.01493
60	2614	545.2	20.48	997.4	5234	4163	0.4001	0.03412	9.846×10^{-5}	1.189×10^{-5}	1.288	1.452	0.00354	0.01389
65	2948	536.0	23.26	968.9	5340	4384	0.3885	0.03550	9.347×10^{-5}	1.213×10^{-5}	1.285	1.499	0.00377	0.01285
70	3312	526.3	26.39	939.0	5463	4634	0.3769	0.03700	8.879×10^{-5}	1.238×10^{-5}	1.287	1.551	0.00404	0.01181
75	3709	516.2	29.90	907.5	5608	4923	0.3653	0.03862	8.440×10^{-5}	1.264×10^{-5}	1.296	1.612	0.00436	0.01079
80	4141	505.7	33.87	874.1	5780	5260	0.3538	0.04038	8.030×10^{-5}	1.292×10^{-5}	1.312	1.683	0.00474	0.00977
85	4609	494.5	38.36	838.6	5988	5659	0.3422	0.04232	7.646×10^{-5}	1.322×10^{-5}	1.338	1.768	0.00521	0.00876
90	5116	482.8	43.48	800.6	6242	6142	0.3306	0.04447	7.284×10^{-5}	1.354×10^{-5}	1.375	1.871	0.00579	0.00776
95	5665	470.2	49.35	759.8	6561	6740	0.3190	0.04687	6.946×10^{-5}	1.389×10^{-5}	1.429	1.999	0.00652	0.00677
100	6257	456.6	56.15	715.5	6972	7503	0.3075	0.04958	6.628×10^{-5}	1.429×10^{-5}	1.503	2.163	0.00749	0.00579

Note 1: Kinematic viscosity ν and thermal diffusivity α can be calculated from their definitions, $\nu = \mu/\rho$ and $\alpha = k/\rho c_p = \nu/\text{Pr}$. The properties listed here (except the vapor density) can be used at any pressures with negligible error except at temperatures near the critical-point value.

Note 2: The unit kJ/kg·°C for specific heat is equivalent to kJ/kg·K, and the unit W/m·°C for thermal conductivity is equivalent to W/m·K.

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Original sources: Tillner-Roth, Harms-Watzenberg, and Baehr, "Eine neue Fundamentalgleichung fur Ammoniak," *DKV-Tagungsbericht* 20:167–181, 1993; Liley and Desai, "Thermophysical Properties of Refrigerants," *ASHRAE,* 1993, ISBN 1-1883413-10-9.

TABLE A–18

Properties of saturated propane

Temp. T, °C	Saturation pressure P, kPa	Density ρ, kg/m^3		Enthalpy of vaporization h_{fg}, kJ/kg	Specific heat c_p, J/kg·K		Thermal conductivity k, W/m·K		Dynamic viscosity μ, kg/m·s		Prandtl number Pr		Volume expansion coefficient β, 1/K	Surface tension, N/m
		Liquid	Vapor		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	
−120	0.4053	664.7	0.01408	498.3	2003	1115	0.1802	0.00589	6.136×10^{-4}	4.372×10^{-6}	6.820	0.827	0.00153	0.02630
−110	1.157	654.5	0.03776	489.3	2021	1148	0.1738	0.00645	5.054×10^{-4}	4.625×10^{-6}	5.878	0.822	0.00157	0.02486
−100	2.881	644.2	0.08872	480.4	2044	1183	0.1672	0.00705	4.252×10^{-4}	4.881×10^{-6}	5.195	0.819	0.00161	0.02344
−90	6.406	633.8	0.1870	471.5	2070	1221	0.1606	0.00769	3.635×10^{-4}	5.143×10^{-6}	4.686	0.817	0.00166	0.02202
−80	12.97	623.2	0.3602	462.4	2100	1263	0.1539	0.00836	3.149×10^{-4}	5.409×10^{-6}	4.297	0.817	0.00171	0.02062
−70	24.26	612.5	0.6439	453.1	2134	1308	0.1472	0.00908	2.755×10^{-4}	5.680×10^{-6}	3.994	0.818	0.00177	0.01923
−60	42.46	601.5	1.081	443.5	2173	1358	0.1407	0.00985	2.430×10^{-4}	5.956×10^{-6}	3.755	0.821	0.00184	0.01785
−50	70.24	590.3	1.724	433.6	2217	1412	0.1343	0.01067	2.158×10^{-4}	6.239×10^{-6}	3.563	0.825	0.00192	0.01649
−40	110.7	578.8	2.629	423.1	2258	1471	0.1281	0.01155	1.926×10^{-4}	6.529×10^{-6}	3.395	0.831	0.00201	0.01515
−30	167.3	567.0	3.864	412.1	2310	1535	0.1221	0.01250	1.726×10^{-4}	6.827×10^{-6}	3.266	0.839	0.00213	0.01382
−20	243.8	554.7	5.503	400.3	2368	1605	0.1163	0.01351	1.551×10^{-4}	7.136×10^{-6}	3.158	0.848	0.00226	0.01251
−10	344.4	542.0	7.635	387.8	2433	1682	0.1107	0.01459	1.397×10^{-4}	7.457×10^{-6}	3.069	0.860	0.00242	0.01122
0	473.3	528.7	10.36	374.2	2507	1768	0.1054	0.01576	1.259×10^{-4}	7.794×10^{-6}	2.996	0.875	0.00262	0.00996
5	549.8	521.8	11.99	367.0	2547	1814	0.1028	0.01637	1.195×10^{-4}	7.970×10^{-6}	2.964	0.883	0.00273	0.00934
10	635.1	514.7	13.81	359.5	2590	1864	0.1002	0.01701	1.135×10^{-4}	8.151×10^{-6}	2.935	0.893	0.00286	0.00872
15	729.8	507.5	15.85	351.7	2637	1917	0.0977	0.01767	1.077×10^{-4}	8.339×10^{-6}	2.909	0.905	0.00301	0.00811
20	834.4	500.0	18.13	343.4	2688	1974	0.0952	0.01836	1.022×10^{-4}	8.534×10^{-6}	2.886	0.918	0.00318	0.00751
25	949.7	492.2	20.68	334.8	2742	2036	0.0928	0.01908	9.702×10^{-5}	8.738×10^{-6}	2.866	0.933	0.00337	0.00691
30	1076	484.2	23.53	325.8	2802	2104	0.0904	0.01982	9.197×10^{-5}	8.952×10^{-6}	2.850	0.950	0.00358	0.00633
35	1215	475.8	26.72	316.2	2869	2179	0.0881	0.02061	8.710×10^{-5}	9.178×10^{-6}	2.837	0.971	0.00384	0.00575
40	1366	467.1	30.29	306.1	2943	2264	0.0857	0.02142	8.240×10^{-5}	9.417×10^{-6}	2.828	0.995	0.00413	0.00518
45	1530	458.0	34.29	295.3	3026	2361	0.0834	0.02228	7.785×10^{-5}	9.674×10^{-6}	2.824	1.025	0.00448	0.00463
50	1708	448.5	38.79	283.9	3122	2473	0.0811	0.02319	7.343×10^{-5}	9.950×10^{-5}	2.826	1.061	0.00491	0.00408
60	2110	427.5	49.66	258.4	3283	2769	0.0765	0.02517	6.487×10^{-5}	1.058×10^{-5}	2.784	1.164	0.00609	0.00303
70	2580	403.2	64.02	228.0	3595	3241	0.0717	0.02746	5.649×10^{-5}	1.138×10^{-5}	2.834	1.343	0.00811	0.00204
80	3127	373.0	84.28	189.7	4501	4173	0.0663	0.03029	4.790×10^{-5}	1.249×10^{-5}	3.251	1.722	0.01248	0.00114
90	3769	329.1	118.6	133.2	6977	7239	0.0595	0.03441	3.807×10^{-5}	1.448×10^{-5}	4.465	3.047	0.02847	0.00037

Note 1: Kinematic viscosity ν and thermal diffusivity α can be calculated from their definitions, $\nu = \mu/\rho$ and $\alpha = k/\rho c_\rho = \nu/\mathrm{Pr}$. The properties listed here (except the vapor density) can be used at any pressures with negligible error except at temperatures near the critical-point value.

Note 2: The unit kJ/kg·°C for specific heat is equivalent to kJ/kg·K, and the unit W/m·°C for thermal conductivity is equivalent to W/m·K.

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Original sources: Reiner Tillner-Roth, “Fundamental Equations of State,” Shaker, Verlag, Aachan, 1998; B. A. Younglove and J. F. Ely, “Thermophysical Properties of Fluids. II Methane, Ethane, Propane, Isobutane, and Normal Butane,” *J. Phys. Chem. Ref. Data,* Vol. 16, No. 4, 1987; G.R. Somayajulu, “A Generalized Equation for Surface Tension from the Triple-Point to the Critical-Point,” *International Journal of Thermophysics,* Vol. 9, No. 4, 1988.

TABLE A–19

Properties of liquids

Temp. T, °C	Density ρ, kg/m^3	Specific heat c_p, J/kg·K	Thermal conductivity k, W/m·K	Thermal diffusivity α, m^2/s	Dynamic viscosity μ, kg/m·s	Kinematic viscosity ν, m^2/s	Prandtl number Pr	Volume expansion coeff. β, 1/K
Methane [CH_4]								
−160	420.2	3492	0.1863	1.270×10^{-7}	1.133×10^{-4}	2.699×10^{-7}	2.126	0.00352
−150	405.0	3580	0.1703	1.174×10^{-7}	9.169×10^{-5}	2.264×10^{-7}	1.927	0.00391
−140	388.8	3700	0.1550	1.077×10^{-7}	7.551×10^{-5}	1.942×10^{-7}	1.803	0.00444
−130	371.1	3875	0.1402	9.749×10^{-8}	6.288×10^{-5}	1.694×10^{-7}	1.738	0.00520
−120	351.4	4146	0.1258	8.634×10^{-8}	5.257×10^{-5}	1.496×10^{-7}	1.732	0.00637
−110	328.8	4611	0.1115	7.356×10^{-8}	4.377×10^{-5}	1.331×10^{-7}	1.810	0.00841
−100	301.0	5578	0.0967	5.761×10^{-8}	3.577×10^{-5}	1.188×10^{-7}	2.063	0.01282
−90	261.7	8902	0.0797	3.423×10^{-8}	2.761×10^{-5}	1.055×10^{-7}	3.082	0.02922
Methanol [$CH_3(OH)$]								
20	788.4	2515	0.1987	1.002×10^{-7}	5.857×10^{-4}	7.429×10^{-7}	7.414	0.00118
30	779.1	2577	0.1980	9.862×10^{-8}	5.088×10^{-4}	6.531×10^{-7}	6.622	0.00120
40	769.6	2644	0.1972	9.690×10^{-8}	4.460×10^{-4}	5.795×10^{-7}	5.980	0.00123
50	760.1	2718	0.1965	9.509×10^{-8}	3.942×10^{-4}	5.185×10^{-7}	5.453	0.00127
60	750.4	2798	0.1957	9.320×10^{-8}	3.510×10^{-4}	4.677×10^{-7}	5.018	0.00132
70	740.4	2885	0.1950	9.128×10^{-8}	3.146×10^{-4}	4.250×10^{-7}	4.655	0.00137
Isobutane (R600a)								
−100	683.8	1881	0.1383	1.075×10^{-7}	9.305×10^{-4}	1.360×10^{-6}	12.65	0.00142
−75	659.3	1970	0.1357	1.044×10^{-7}	5.624×10^{-4}	8.531×10^{-7}	8.167	0.00150
−50	634.3	2069	0.1283	9.773×10^{-8}	3.769×10^{-4}	5.942×10^{-7}	6.079	0.00161
−25	608.2	2180	0.1181	8.906×10^{-8}	2.688×10^{-4}	4.420×10^{-7}	4.963	0.00177
0	580.6	2306	0.1068	7.974×10^{-8}	1.993×10^{-4}	3.432×10^{-7}	4.304	0.00199
25	550.7	2455	0.0956	7.069×10^{-8}	1.510×10^{-4}	2.743×10^{-7}	3.880	0.00232
50	517.3	2640	0.0851	6.233×10^{-8}	1.155×10^{-4}	2.233×10^{-7}	3.582	0.00286
75	478.5	2896	0.0757	5.460×10^{-8}	8.785×10^{-5}	1.836×10^{-7}	3.363	0.00385
100	429.6	3361	0.0669	4.634×10^{-8}	6.483×10^{-5}	1.509×10^{-7}	3.256	0.00628
Glycerin								
0	1276	2262	0.2820	9.773×10^{-8}	10.49	8.219×10^{-3}	84,101	
5	1273	2288	0.2835	9.732×10^{-8}	6.730	5.287×10^{-3}	54,327	
10	1270	2320	0.2846	9.662×10^{-8}	4.241	3.339×10^{-3}	34,561	
15	1267	2354	0.2856	9.576×10^{-8}	2.496	1.970×10^{-3}	20,570	
20	1264	2386	0.2860	9.484×10^{-8}	1.519	1.201×10^{-3}	12,671	
25	1261	2416	0.2860	9.388×10^{-8}	0.9934	7.878×10^{-4}	8,392	
30	1258	2447	0.2860	9.291×10^{-8}	0.6582	5.232×10^{-4}	5,631	
35	1255	2478	0.2860	9.195×10^{-8}	0.4347	3.464×10^{-4}	3,767	
40	1252	2513	0.2863	9.101×10^{-8}	0.3073	2.455×10^{-4}	2,697	
Engine oil (unused)								
0	899.0	1797	0.1469	9.097×10^{-8}	3.814	4.242×10^{-3}	46,636	0.00070
20	888.1	1881	0.1450	8.680×10^{-8}	0.8374	9.429×10^{-4}	10,863	0.00070
40	876.0	1964	0.1444	8.391×10^{-8}	0.2177	2.485×10^{-4}	2,962	0.00070
60	863.9	2048	0.1404	7.934×10^{-8}	0.07399	8.565×10^{-5}	1,080	0.00070
80	852.0	2132	0.1380	7.599×10^{-8}	0.03232	3.794×10^{-5}	499.3	0.00070
100	840.0	2220	0.1367	7.330×10^{-8}	0.01718	2.046×10^{-5}	279.1	0.00070
120	828.9	2308	0.1347	7.042×10^{-8}	0.01029	1.241×10^{-5}	176.3	0.00070
140	816.8	2395	0.1330	6.798×10^{-8}	0.006558	8.029×10^{-6}	118.1	0.00070
150	810.3	2441	0.1327	6.708×10^{-8}	0.005344	6.595×10^{-6}	98.31	0.00070

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Originally based on various sources.

TABLE A–20

Properties of liquid metals

Temp. T, °C	Density ρ, kg/m^3	Specific heat c_p, J/kg·K	Thermal conductivity k, W/m·K	Thermal diffusivity α, m^2/s	Dynamic viscosity μ, kg/m·s	Kinematic viscosity ν, m^2/s	Prandtl number Pr	Volume expansion coeff. β, 1/K
Mercury (Hg) melting point: −39°C								
0	13595	140.4	8.18200	4.287×10^{-6}	1.687×10^{-3}	1.241×10^{-7}	0.0289	1.810×10^{-4}
25	13534	139.4	8.51533	4.514×10^{-6}	1.534×10^{-3}	1.133×10^{-7}	0.0251	1.810×10^{-4}
50	13473	138.6	8.83632	4.734×10^{-6}	1.423×10^{-3}	1.056×10^{-7}	0.0223	1.810×10^{-4}
75	13412	137.8	9.15632	4.956×10^{-6}	1.316×10^{-3}	9.819×10^{-8}	0.0198	1.810×10^{-4}
100	13351	137.1	9.46706	5.170×10^{-6}	1.245×10^{-3}	9.326×10^{-8}	0.0180	1.810×10^{-4}
150	13231	136.1	10.07780	5.595×10^{-6}	1.126×10^{-3}	8.514×10^{-8}	0.0152	1.810×10^{-4}
200	13112	135.5	10.65465	5.996×10^{-6}	1.043×10^{-3}	7.959×10^{-8}	0.0133	1.815×10^{-4}
250	12993	135.3	11.18150	6.363×10^{-6}	9.820×10^{-4}	7.558×10^{-8}	0.0119	1.829×10^{-4}
300	12873	135.3	11.68150	6.705×10^{-6}	9.336×10^{-4}	7.252×10^{-8}	0.0108	1.854×10^{-4}
Bismuth (Bi) melting point: 271°C								
350	9969	146.0	16.28	1.118×10^{-5}	1.540×10^{-3}	1.545×10^{-7}	0.01381	
400	9908	148.2	16.10	1.096×10^{-5}	1.422×10^{-3}	1.436×10^{-7}	0.01310	
500	9785	152.8	15.74	1.052×10^{-5}	1.188×10^{-3}	1.215×10^{-7}	0.01154	
600	9663	157.3	15.60	1.026×10^{-5}	1.013×10^{-3}	1.048×10^{-7}	0.01022	
700	9540	161.8	15.60	1.010×10^{-5}	8.736×10^{-4}	9.157×10^{-8}	0.00906	
Lead (Pb) melting point: 327°C								
400	10506	158	15.97	9.623×10^{-6}	2.277×10^{-3}	2.167×10^{-7}	0.02252	
450	10449	156	15.74	9.649×10^{-6}	2.065×10^{-3}	1.976×10^{-7}	0.02048	
500	10390	155	15.54	9.651×10^{-6}	1.884×10^{-3}	1.814×10^{-7}	0.01879	
550	10329	155	15.39	9.610×10^{-6}	1.758×10^{-3}	1.702×10^{-7}	0.01771	
600	10267	155	15.23	9.568×10^{-6}	1.632×10^{-3}	1.589×10^{-7}	0.01661	
650	10206	155	15.07	9.526×10^{-6}	1.505×10^{-3}	1.475×10^{-7}	0.01549	
700	10145	155	14.91	9.483×10^{-6}	1.379×10^{-3}	1.360×10^{-7}	0.01434	
Sodium (Na) melting point: 98°C								
100	927.3	1378	85.84	6.718×10^{-5}	6.892×10^{-4}	7.432×10^{-7}	0.01106	
200	902.5	1349	80.84	6.639×10^{-5}	5.385×10^{-4}	5.967×10^{-7}	0.008987	
300	877.8	1320	75.84	6.544×10^{-5}	3.878×10^{-4}	4.418×10^{-7}	0.006751	
400	853.0	1296	71.20	6.437×10^{-5}	2.720×10^{-4}	3.188×10^{-7}	0.004953	
500	828.5	1284	67.41	6.335×10^{-5}	2.411×10^{-4}	2.909×10^{-7}	0.004593	
600	804.0	1272	63.63	6.220×10^{-5}	2.101×10^{-4}	2.614×10^{-7}	0.004202	
Potassium (K) melting point: 64°C								
200	795.2	790.8	43.99	6.995×10^{-5}	3.350×10^{-4}	4.213×10^{-7}	0.006023	
300	771.6	772.8	42.01	7.045×10^{-5}	2.667×10^{-4}	3.456×10^{-7}	0.004906	
400	748.0	754.8	40.03	7.090×10^{-5}	1.984×10^{-4}	2.652×10^{-7}	0.00374	
500	723.9	750.0	37.81	6.964×10^{-5}	1.668×10^{-4}	2.304×10^{-7}	0.003309	
600	699.6	750.0	35.50	6.765×10^{-5}	1.487×10^{-4}	2.126×10^{-7}	0.003143	
Sodium-potassium (%22Na-%78K) melting point: −11°C								
100	847.3	944.4	25.64	3.205×10^{-5}	5.707×10^{-4}	6.736×10^{-7}	0.02102	
200	823.2	922.5	26.27	3.459×10^{-5}	4.587×10^{-4}	5.572×10^{-7}	0.01611	
300	799.1	900.6	26.89	3.736×10^{-5}	3.467×10^{-4}	4.339×10^{-7}	0.01161	
400	775.0	879.0	27.50	4.037×10^{-5}	2.357×10^{-4}	3.041×10^{-7}	0.00753	
500	751.5	880.1	27.89	4.217×10^{-5}	2.108×10^{-4}	2.805×10^{-7}	0.00665	
600	728.0	881.2	28.28	4.408×10^{-5}	1.859×10^{-4}	2.553×10^{-7}	0.00579	

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Originally based on various sources.

TABLE A–21

Ideal-gas properties of air

T	h	P_r	u	v_r	s°
K	kJ/kg		kJ/kg		kJ/kg·K
200	199.97	0.3363	142.56	1707.0	1.29559
210	209.97	0.3987	149.69	1512.0	1.34444
220	219.97	0.4690	156.82	1346.0	1.39105
230	230.02	0.5477	164.00	1205.0	1.43557
240	240.02	0.6355	171.13	1084.0	1.47824
250	250.05	0.7329	178.28	979.0	1.51917
260	260.09	0.8405	185.45	887.8	1.55848
270	270.11	0.9590	192.60	808.0	1.59634
280	280.13	1.0889	199.75	738.0	1.63279
285	285.14	1.1584	203.33	706.1	1.65055
290	290.16	1.2311	206.91	676.1	1.66802
295	295.17	1.3068	210.49	647.9	1.68515
298	298.18	1.3543	212.64	631.9	1.69528
300	300.19	1.3860	214.07	621.2	1.70203
305	305.22	1.4686	217.67	596.0	1.71865
310	310.24	1.5546	221.25	572.3	1.73498
315	315.27	1.6442	224.85	549.8	1.75106
320	320.29	1.7375	228.42	528.6	1.76690
325	325.31	1.8345	232.02	508.4	1.78249
330	330.34	1.9352	235.61	489.4	1.79783
340	340.42	2.149	242.82	454.1	1.82790
350	350.49	2.379	250.02	422.2	1.85708
360	360.58	2.626	257.24	393.4	1.88543
370	370.67	2.892	264.46	367.2	1.91313
380	380.77	3.176	271.69	343.4	1.94001
390	390.88	3.481	278.93	321.5	1.96633
400	400.98	3.806	286.16	301.6	1.99194
410	411.12	4.153	293.43	283.3	2.01699
420	421.26	4.522	300.69	266.6	2.04142
430	431.43	4.915	307.99	251.1	2.06533
440	441.61	5.332	315.30	236.8	2.08870
450	451.80	5.775	322.62	223.6	2.11161
460	462.02	6.245	329.97	211.4	2.13407
470	472.24	6.742	337.32	200.1	2.15604
480	482.49	7.268	344.70	189.5	2.17760
490	492.74	7.824	352.08	179.7	2.19876
500	503.02	8.411	359.49	170.6	2.21952
510	513.32	9.031	366.92	162.1	2.23993
520	523.63	9.684	374.36	154.1	2.25997
530	533.98	10.37	381.84	146.7	2.27967
540	544.35	11.10	389.34	139.7	2.29906
550	554.74	11.86	396.86	133.1	2.31809
560	565.17	12.66	404.42	127.0	2.33685
570	575.59	13.50	411.97	121.2	2.35531
580	586.04	14.38	419.55	115.7	2.37348
590	596.52	15.31	427.15	110.6	2.39140
600	607.02	16.28	434.78	105.8	2.40902
610	617.53	17.30	442.42	101.2	2.42644
620	628.07	18.36	450.09	96.92	2.44356
630	638.63	19.84	457.78	92.84	2.46048
640	649.22	20.64	465.50	88.99	2.47716
650	659.84	21.86	473.25	85.34	2.49364
660	670.47	23.13	481.01	81.89	2.50985
670	681.14	24.46	488.81	78.61	2.52589
680	691.82	25.85	496.62	75.50	2.54175
690	702.52	27.29	504.45	72.56	2.55731
700	713.27	28.80	512.33	69.76	2.57277
710	724.04	30.38	520.23	67.07	2.58810
720	734.82	32.02	528.14	64.53	2.60319
730	745.62	33.72	536.07	62.13	2.61803
740	756.44	35.50	544.02	59.82	2.63280
750	767.29	37.35	551.99	57.63	2.64737
760	778.18	39.27	560.01	55.54	2.66176
780	800.03	43.35	576.12	51.64	2.69013
800	821.95	47.75	592.30	48.08	2.71787
820	843.98	52.59	608.59	44.84	2.74504
840	866.08	57.60	624.95	41.85	2.77170
860	888.27	63.09	641.40	39.12	2.79783
880	910.56	68.98	657.95	36.61	2.82344
900	932.93	75.29	674.58	34.31	2.84856
920	955.38	82.05	691.28	32.18	2.87324
940	977.92	89.28	708.08	30.22	2.89748
960	1000.55	97.00	725.02	28.40	2.92128
980	1023.25	105.2	741.98	26.73	2.94468
1000	1046.04	114.0	758.94	25.17	2.96770
1020	1068.89	123.4	776.10	23.72	2.99034
1040	1091.85	133.3	793.36	23.29	3.01260
1060	1114.86	143.9	810.62	21.14	3.03449
1080	1137.89	155.2	827.88	19.98	3.05608
1100	1161.07	167.1	845.33	18.896	3.07732
1120	1184.28	179.7	862.79	17.886	3.09825
1140	1207.57	193.1	880.35	16.946	3.11883
1160	1230.92	207.2	897.91	16.064	3.13916
1180	1254.34	222.2	915.57	15.241	3.15916
1200	1277.79	238.0	933.33	14.470	3.17888
1220	1301.31	254.7	951.09	13.747	3.19834
1240	1324.93	272.3	968.95	13.069	3.21751

TABLE A–21

Ideal-gas properties of air *(Concluded)*

T K	h kJ/kg	P_r	u kJ/kg	v_r	$s°$ kJ/kg·K
1260	1348.55	290.8	986.90	12.435	3.23638
1280	1372.24	310.4	1004.76	11.835	3.25510
1300	1395.97	330.9	1022.82	11.275	3.27345
1320	1419.76	352.5	1040.88	10.747	3.29160
1340	1443.60	375.3	1058.94	10.247	3.30959
1360	1467.49	399.1	1077.10	9.780	3.32724
1380	1491.44	424.2	1095.26	9.337	3.34474
1400	1515.42	450.5	1113.52	8.919	3.36200
1420	1539.44	478.0	1131.77	8.526	3.37901
1440	1563.51	506.9	1150.13	8.153	3.39586
1460	1587.63	537.1	1168.49	7.801	3.41247
1480	1611.79	568.8	1186.95	7.468	3.42892
1500	1635.97	601.9	1205.41	7.152	3.44516
1520	1660.23	636.5	1223.87	6.854	3.46120
1540	1684.51	672.8	1242.43	6.569	3.47712
1560	1708.82	710.5	1260.99	6.301	3.49276
1580	1733.17	750.0	1279.65	6.046	3.50829
1600	1757.57	791.2	1298.30	5.804	3.52364
1620	1782.00	834.1	1316.96	5.574	3.53879
1640	1806.46	878.9	1335.72	5.355	3.55381
1660	1830.96	925.6	1354.48	5.147	3.56867
1680	1855.50	974.2	1373.24	4.949	3.58335
1700	1880.1	1025	1392.7	4.761	3.5979
1750	1941.6	1161	1439.8	4.328	3.6336
1800	2003.3	1310	1487.2	3.994	3.6684
1850	2065.3	1475	1534.9	3.601	3.7023
1900	2127.4	1655	1582.6	3.295	3.7354
1950	2189.7	1852	1630.6	3.022	3.7677
2000	2252.1	2068	1678.7	2.776	3.7994
2050	2314.6	2303	1726.8	2.555	3.8303
2100	2377.7	2559	1775.3	2.356	3.8605
2150	2440.3	2837	1823.8	2.175	3.8901
2200	2503.2	3138	1872.4	2.012	3.9191
2250	2566.4	3464	1921.3	1.864	3.9474

Note: The properties P_r (relative pressure) and v_r (relative specific volume) are dimensionless quantities used in the analysis of isentropic processes and should not be confused with the properties pressure and specific volume.

Source of Data: Kenneth Wark, *Thermodynamics,* 4th ed. (New York: McGraw-Hill, 1983), pp. 785–86, Table A–5. Originally published in J. H. Keenan and J. Kaye, *Gas Tables* (New York: John Wiley & Sons, 1948).

TABLE A–22

Properties of air at 1 atm pressure

Temp. T, °C	Density ρ, kg/m^3	Specific heat c_p, J/kg·K	Thermal conductivity k, W/m·K	Thermal diffusivity α, m^2/s	Dynamic viscosity μ, kg/m·s	Kinematic viscosity ν, m^2/s	Prandtl number Pr
−150	2.866	983	0.01171	4.158×10^{-6}	8.636×10^{-6}	3.013×10^{-6}	0.7246
−100	2.038	966	0.01582	8.036×10^{-6}	1.189×10^{-5}	5.837×10^{-6}	0.7263
−50	1.582	999	0.01979	1.252×10^{-5}	1.474×10^{-5}	9.319×10^{-6}	0.7440
−40	1.514	1002	0.02057	1.356×10^{-5}	1.527×10^{-5}	1.008×10^{-5}	0.7436
−30	1.451	1004	0.02134	1.465×10^{-5}	1.579×10^{-5}	1.087×10^{-5}	0.7425
−20	1.394	1005	0.02211	1.578×10^{-5}	1.630×10^{-5}	1.169×10^{-5}	0.7408
−10	1.341	1006	0.02288	1.696×10^{-5}	1.680×10^{-5}	1.252×10^{-5}	0.7387
0	1.292	1006	0.02364	1.818×10^{-5}	1.729×10^{-5}	1.338×10^{-5}	0.7362
5	1.269	1006	0.02401	1.880×10^{-5}	1.754×10^{-5}	1.382×10^{-5}	0.7350
10	1.246	1006	0.02439	1.944×10^{-5}	1.778×10^{-5}	1.426×10^{-5}	0.7336
15	1.225	1007	0.02476	2.009×10^{-5}	1.802×10^{-5}	1.470×10^{-5}	0.7323
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177
80	0.9994	1008	0.02953	2.931×10^{-5}	2.096×10^{-5}	2.097×10^{-5}	0.7154
90	0.9718	1008	0.03024	3.086×10^{-5}	2.139×10^{-5}	2.201×10^{-5}	0.7132
100	0.9458	1009	0.03095	3.243×10^{-5}	2.181×10^{-5}	2.306×10^{-5}	0.7111
120	0.8977	1011	0.03235	3.565×10^{-5}	2.264×10^{-5}	2.522×10^{-5}	0.7073
140	0.8542	1013	0.03374	3.898×10^{-5}	2.345×10^{-5}	2.745×10^{-5}	0.7041
160	0.8148	1016	0.03511	4.241×10^{-5}	2.420×10^{-5}	2.975×10^{-5}	0.7014
180	0.7788	1019	0.03646	4.593×10^{-5}	2.504×10^{-5}	3.212×10^{-5}	0.6992
200	0.7459	1023	0.03779	4.954×10^{-5}	2.577×10^{-5}	3.455×10^{-5}	0.6974
250	0.6746	1033	0.04104	5.890×10^{-5}	2.760×10^{-5}	4.091×10^{-5}	0.6946
300	0.6158	1044	0.04418	6.871×10^{-5}	2.934×10^{-5}	4.765×10^{-5}	0.6935
350	0.5664	1056	0.04721	7.892×10^{-5}	3.101×10^{-5}	5.475×10^{-5}	0.6937
400	0.5243	1069	0.05015	8.951×10^{-5}	3.261×10^{-5}	6.219×10^{-5}	0.6948
450	0.4880	1081	0.05298	1.004×10^{-4}	3.415×10^{-5}	6.997×10^{-5}	0.6965
500	0.4565	1093	0.05572	1.117×10^{-4}	3.563×10^{-5}	7.806×10^{-5}	0.6986
600	0.4042	1115	0.06093	1.352×10^{-4}	3.846×10^{-5}	9.515×10^{-5}	0.7037
700	0.3627	1135	0.06581	1.598×10^{-4}	4.111×10^{-5}	1.133×10^{-4}	0.7092
800	0.3289	1153	0.07037	1.855×10^{-4}	4.362×10^{-5}	1.326×10^{-4}	0.7149
900	0.3008	1169	0.07465	2.122×10^{-4}	4.600×10^{-5}	1.529×10^{-4}	0.7206
1000	0.2772	1184	0.07868	2.398×10^{-4}	4.826×10^{-5}	1.741×10^{-4}	0.7260
1500	0.1990	1234	0.09599	3.908×10^{-4}	5.817×10^{-5}	2.922×10^{-4}	0.7478
2000	0.1553	1264	0.11113	5.664×10^{-4}	6.630×10^{-5}	4.270×10^{-4}	0.7539

Note: For ideal gases, the properties c_p, k, μ, and Pr are independent of pressure. The properties ρ, ν, and α at a pressure P (in atm) other than 1 atm are determined by multiplying the values of P at the given temperature by ρ and by dividing ν and α by P.

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Original sources: Keenan, Chao, Keyes, *Gas Tables*, Wiley, 1984; and *Thermophysical Properties of Matter*. Vol. 3: *Thermal Conductivity*, Y. S. Touloukian, P. E. Liley, S. C. Saxena, Vol. 11: *Viscosity*, Y. S. Touloukian, S. C. Saxena, and P. Hestermans, IFI/Plenun, NY, 1970, ISBN 0-306067020-8.

TABLE A–23

Properties of gases at 1 atm pressure

Temp. T, °C	Density ρ, kg/m^3	Specific heat c_p, J/kg·K	Thermal conductivity k, W/m·K	Thermal diffusivity α, m^2/s	Dynamic viscosity μ, kg/m·s	Kinematic viscosity ν, m^2/s	Prandtl number Pr
			Carbon dioxide, CO_2				
−50	2.4035	746	0.01051	5.860×10^{-6}	1.129×10^{-5}	4.699×10^{-6}	0.8019
0	1.9635	811	0.01456	9.141×10^{-6}	1.375×10^{-5}	7.003×10^{-6}	0.7661
50	1.6597	866.6	0.01858	1.291×10^{-5}	1.612×10^{-5}	9.714×10^{-6}	0.7520
100	1.4373	914.8	0.02257	1.716×10^{-5}	1.841×10^{-5}	1.281×10^{-5}	0.7464
150	1.2675	957.4	0.02652	2.186×10^{-5}	2.063×10^{-5}	1.627×10^{-5}	0.7445
200	1.1336	995.2	0.03044	2.698×10^{-5}	2.276×10^{-5}	2.008×10^{-5}	0.7442
300	0.9358	1060	0.03814	3.847×10^{-5}	2.682×10^{-5}	2.866×10^{-5}	0.7450
400	0.7968	1112	0.04565	5.151×10^{-5}	3.061×10^{-5}	3.842×10^{-5}	0.7458
500	0.6937	1156	0.05293	6.600×10^{-5}	3.416×10^{-5}	4.924×10^{-5}	0.7460
1000	0.4213	1292	0.08491	1.560×10^{-4}	4.898×10^{-5}	1.162×10^{-4}	0.7455
1500	0.3025	1356	0.10688	2.606×10^{-4}	6.106×10^{-5}	2.019×10^{-4}	0.7745
2000	0.2359	1387	0.11522	3.521×10^{-4}	7.322×10^{-5}	3.103×10^{-4}	0.8815
			Carbon monoxide, CO				
−50	1.5297	1081	0.01901	1.149×10^{-5}	1.378×10^{-5}	9.012×10^{-6}	0.7840
0	1.2497	1048	0.02278	1.739×10^{-5}	1.629×10^{-5}	1.303×10^{-5}	0.7499
50	1.0563	1039	0.02641	2.407×10^{-5}	1.863×10^{-5}	1.764×10^{-5}	0.7328
100	0.9148	1041	0.02992	3.142×10^{-5}	2.080×10^{-5}	2.274×10^{-5}	0.7239
150	0.8067	1049	0.03330	3.936×10^{-5}	2.283×10^{-5}	2.830×10^{-5}	0.7191
200	0.7214	1060	0.03656	4.782×10^{-5}	2.472×10^{-5}	3.426×10^{-5}	0.7164
300	0.5956	1085	0.04277	6.619×10^{-5}	2.812×10^{-5}	4.722×10^{-5}	0.7134
400	0.5071	1111	0.04860	8.628×10^{-5}	3.111×10^{-5}	6.136×10^{-5}	0.7111
500	0.4415	1135	0.05412	1.079×10^{-4}	3.379×10^{-5}	7.653×10^{-5}	0.7087
1000	0.2681	1226	0.07894	2.401×10^{-4}	4.557×10^{-5}	1.700×10^{-4}	0.7080
1500	0.1925	1279	0.10458	4.246×10^{-4}	6.321×10^{-5}	3.284×10^{-4}	0.7733
2000	0.1502	1309	0.13833	7.034×10^{-4}	9.826×10^{-5}	6.543×10^{-4}	0.9302
			Methane, CH_4				
−50	0.8761	2243	0.02367	1.204×10^{-5}	8.564×10^{-6}	9.774×10^{-6}	0.8116
0	0.7158	2217	0.03042	1.917×10^{-5}	1.028×10^{-5}	1.436×10^{-5}	0.7494
50	0.6050	2302	0.03766	2.704×10^{-5}	1.191×10^{-5}	1.969×10^{-5}	0.7282
100	0.5240	2443	0.04534	3.543×10^{-5}	1.345×10^{-5}	2.567×10^{-5}	0.7247
150	0.4620	2611	0.05344	4.431×10^{-5}	1.491×10^{-5}	3.227×10^{-5}	0.7284
200	0.4132	2791	0.06194	5.370×10^{-5}	1.630×10^{-5}	3.944×10^{-5}	0.7344
300	0.3411	3158	0.07996	7.422×10^{-5}	1.886×10^{-5}	5.529×10^{-5}	0.7450
400	0.2904	3510	0.09918	9.727×10^{-5}	2.119×10^{-5}	7.297×10^{-5}	0.7501
500	0.2529	3836	0.11933	1.230×10^{-4}	2.334×10^{-5}	9.228×10^{-5}	0.7502
1000	0.1536	5042	0.22562	2.914×10^{-4}	3.281×10^{-5}	2.136×10^{-4}	0.7331
1500	0.1103	5701	0.31857	5.068×10^{-4}	4.434×10^{-5}	4.022×10^{-4}	0.7936
2000	0.0860	6001	0.36750	7.120×10^{-4}	6.360×10^{-5}	7.395×10^{-4}	1.0386
			Hydrogen, H_2				
−50	0.11010	12635	0.1404	1.009×10^{-4}	7.293×10^{-6}	6.624×10^{-5}	0.6562
0	0.08995	13920	0.1652	1.319×10^{-4}	8.391×10^{-6}	9.329×10^{-5}	0.7071
50	0.07603	14349	0.1881	1.724×10^{-4}	9.427×10^{-6}	1.240×10^{-4}	0.7191
100	0.06584	14473	0.2095	2.199×10^{-4}	1.041×10^{-5}	1.582×10^{-4}	0.7196
150	0.05806	14492	0.2296	2.729×10^{-4}	1.136×10^{-5}	1.957×10^{-4}	0.7174
200	0.05193	14482	0.2486	3.306×10^{-4}	1.228×10^{-5}	2.365×10^{-4}	0.7155

TABLE A–23

Properties of gases at 1 atm pressure (*Concluded*)

Temp. T, °C	Density ρ, kg/m^3	Specific heat c_p, J/kg·K	Thermal conductivity k, W/m·K	Thermal diffusivity α, m^2/s	Dynamic viscosity μ, kg/m·s	Kinematic viscosity ν, m^2/s	Prandtl number Pr
300	0.04287	14481	0.2843	4.580×10^{-4}	1.403×10^{-5}	3.274×10^{-4}	0.7149
400	0.03650	14540	0.3180	5.992×10^{-4}	1.570×10^{-5}	4.302×10^{-4}	0.7179
500	0.03178	14653	0.3509	7.535×10^{-4}	1.730×10^{-5}	5.443×10^{-4}	0.7224
1000	0.01930	15577	0.5206	1.732×10^{-3}	2.455×10^{-5}	1.272×10^{-3}	0.7345
1500	0.01386	16553	0.6581	2.869×10^{-3}	3.099×10^{-5}	2.237×10^{-3}	0.7795
2000	0.01081	17400	0.5480	2.914×10^{-3}	3.690×10^{-5}	3.414×10^{-3}	1.1717
				Nitrogen, N_2			
−50	1.5299	957.3	0.02001	1.366×10^{-5}	1.390×10^{-5}	9.091×10^{-6}	0.6655
0	1.2498	1035	0.02384	1.843×10^{-5}	1.640×10^{-5}	1.312×10^{-5}	0.7121
50	1.0564	1042	0.02746	2.494×10^{-5}	1.874×10^{-5}	1.774×10^{-5}	0.7114
100	0.9149	1041	0.03090	3.244×10^{-5}	2.094×10^{-5}	2.289×10^{-5}	0.7056
150	0.8068	1043	0.03416	4.058×10^{-5}	2.300×10^{-5}	2.851×10^{-5}	0.7025
200	0.7215	1050	0.03727	4.921×10^{-5}	2.494×10^{-5}	3.457×10^{-5}	0.7025
300	0.5956	1070	0.04309	6.758×10^{-5}	2.849×10^{-5}	4.783×10^{-5}	0.7078
400	0.5072	1095	0.04848	8.727×10^{-5}	3.166×10^{-5}	6.242×10^{-5}	0.7153
500	0.4416	1120	0.05358	1.083×10^{-4}	3.451×10^{-5}	7.816×10^{-5}	0.7215
1000	0.2681	1213	0.07938	2.440×10^{-4}	4.594×10^{-5}	1.713×10^{-4}	0.7022
1500	0.1925	1266	0.11793	4.839×10^{-4}	5.562×10^{-5}	2.889×10^{-4}	0.5969
2000	0.1502	1297	0.18590	9.543×10^{-4}	6.426×10^{-5}	4.278×10^{-4}	0.4483
				Oxygen, O_2			
−50	1.7475	984.4	0.02067	1.201×10^{-5}	1.616×10^{-5}	9.246×10^{-6}	0.7694
0	1.4277	928.7	0.02472	1.865×10^{-5}	1.916×10^{-5}	1.342×10^{-5}	0.7198
50	1.2068	921.7	0.02867	2.577×10^{-5}	2.194×10^{-5}	1.818×10^{-5}	0.7053
100	1.0451	931.8	0.03254	3.342×10^{-5}	2.451×10^{-5}	2.346×10^{-5}	0.7019
150	0.9216	947.6	0.03637	4.164×10^{-5}	2.694×10^{-5}	2.923×10^{-5}	0.7019
200	0.8242	964.7	0.04014	5.048×10^{-5}	2.923×10^{-5}	3.546×10^{-5}	0.7025
300	0.6804	997.1	0.04751	7.003×10^{-5}	3.350×10^{-5}	4.923×10^{-5}	0.7030
400	0.5793	1025	0.05463	9.204×10^{-5}	3.744×10^{-5}	6.463×10^{-5}	0.7023
500	0.5044	1048	0.06148	1.163×10^{-4}	4.114×10^{-5}	8.156×10^{-5}	0.7010
1000	0.3063	1121	0.09198	2.678×10^{-4}	5.732×10^{-5}	1.871×10^{-4}	0.6986
1500	0.2199	1165	0.11901	4.643×10^{-4}	7.133×10^{-5}	3.243×10^{-4}	0.6985
2000	0.1716	1201	0.14705	7.139×10^{-4}	8.417×10^{-5}	4.907×10^{-4}	0.6873
				Water vapor, H_2O			
−50	0.9839	1892	0.01353	7.271×10^{-6}	7.187×10^{-6}	7.305×10^{-6}	1.0047
0	0.8038	1874	0.01673	1.110×10^{-5}	8.956×10^{-6}	1.114×10^{-5}	1.0033
50	0.6794	1874	0.02032	1.596×10^{-5}	1.078×10^{-5}	1.587×10^{-5}	0.9944
100	0.5884	1887	0.02429	2.187×10^{-5}	1.265×10^{-5}	2.150×10^{-5}	0.9830
150	0.5189	1908	0.02861	2.890×10^{-5}	1.456×10^{-5}	2.806×10^{-5}	0.9712
200	0.4640	1935	0.03326	3.705×10^{-5}	1.650×10^{-5}	3.556×10^{-5}	0.9599
300	0.3831	1997	0.04345	5.680×10^{-5}	2.045×10^{-5}	5.340×10^{-5}	0.9401
400	0.3262	2066	0.05467	8.114×10^{-5}	2.446×10^{-5}	7.498×10^{-5}	0.9240
500	0.2840	2137	0.06677	1.100×10^{-4}	2.847×10^{-5}	1.002×10^{-4}	0.9108
1000	0.1725	2471	0.13623	3.196×10^{-4}	4.762×10^{-5}	2.761×10^{-4}	0.8639
1500	0.1238	2736	0.21301	6.288×10^{-4}	6.411×10^{-5}	5.177×10^{-4}	0.8233
2000	0.0966	2928	0.29183	1.032×10^{-3}	7.808×10^{-5}	8.084×10^{-4}	0.7833

Note: For ideal gases, the properties c_p, k, μ, and Pr are independent of pressure. The properties ρ, ν, and α at a pressure P (in atm) other than 1 atm are determined by multiplying the values of P at the given temperature by ρ and by dividing ν and α by P.

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Originally based on various sources.

TABLE A–24

Properties of solid metals

Composition	Melting point, K	Properties at 300 K				Properties at various temperatures (K), k(W/m·K)/c_p(J/kg·K)					
		ρ kg/m^3	c_p J/kg·K	k W/m·K	$\alpha \times 10^6$ m^2/s	100	200	400	600	800	1000
Aluminum:											
Pure	933	2702	903	237	97.1	302	237	240	231	218	
						482	798	949	1033	1146	
Alloy 2024-T6 (4.5% Cu, 1.5% Mg, 0.6% Mn)	775	2770	875	177	73.0	65	163	186	186		
						473	787	925	1042		
Alloy 195, Cast (4.5% Cu)		2790	883	168	68.2			174	185		
Beryllium	1550	1850	1825	200	59.2	990	301	161	126	106	90.8
						203	1114	2191	2604	2823	3018
Bismuth	545	9780	122	7.86	6.59	16.5	9.69	7.04			
						112	120	127			
Boron	2573	2500	1107	27.0	9.76	190	55.5	16.8	10.6	9.60	9.85
						128	600	1463	1892	2160	2338
Cadmium	594	8650	231	96.8	48.4	203	99.3	94.7			
							198	222	242		
Chromium	2118	7160	449	93.7	29.1	159	111	90.9	80.7	71.3	65.4
						192	384	484	542	581	616
Cobalt	1769	8862	421	99.2	26.6	167	122	85.4	67.4	58.2	52.1
						236	379	450	503	550	628
Copper:											
Pure	1358	8933	385	401	117	482	413	393	379	366	352
						252	356	397	417	433	451
Commercial bronze (90% Cu, 10% Al)	1293	8800	420	52	14		42	52	59		
							785	160	545		
Phosphor gear bronze (89% Cu, 11% Sn)	1104	8780	355	54	17		41	65	74		
							—	—	—		
Cartridge brass (70% Cu, 30% Zn)	1188	8530	380	110	33.9	75	95	137	149		
							360	395	425		
Constantan (55% Cu, 45% Ni)	1493	8920	384	23	6.71	17	19				
						237	362				
Germanium	1211	5360	322	59.9	34.7	232	96.8	43.2	27.3	19.8	17.4
						190	290	337	348	357	375
Gold	1336	19,300	129	317	127	327	323	311	298	284	270
						109	124	131	135	140	145
Iridium	2720	22,500	130	147	50.3	172	153	144	138	132	126
						90	122	133	138	144	153
Iron:											
Pure	1810	7870	447	80.2	23.1	134	94.0	69.5	54.7	43.3	32.8
						216	384	490	574	680	975
Armco (99.75% pure)		7870	447	72.7	20.7	95.6	80.6	65.7	53.1	42.2	32.3
						215	384	490	574	680	975
Carbon steels:											
Plain carbon ($Mn \leq 1\%$ $Si \leq 0.1\%$)		7854	434	60.5	17.7			56.7	48.0	39.2	30.0
								487	559	685	1169
AISI 1010		7832	434	63.9	18.8			58.7	48.8	39.2	31.3
								487	559	685	1168
Carbon-silicon ($Mn \leq 1\%$ $0.1\% < Si \leq 0.6\%$)		7817	446	51.9	14.9			49.8	44.0	37.4	29.3
								501	582	699	971

TABLE A–24

Properties of solid metals (*Continued*)

Composition	Melting point, K	*Properties at 300 K*				*Properties at various temperatures* (K), k(W/m·K)/c_p(J/kg·K)					
		ρ kg/m^3	c_p J/kg·K	k W/m·K	$\alpha \times 10^6$ m^2/s	100	200	400	600	800	1000
Carbon–manganese–silicon		8131	434	41.0	11.6			42.2	39.7	35.0	27.6
(1% < Mn < 1.65%								487	559	685	1090
0.1% < Si < 0.6%)											
Chromium (low) steels:											
$\frac{1}{2}$Cr–$\frac{1}{4}$ Mo–Si (0.18% C,		7822	444	37.7	10.9			38.2	36.7	33.3	26.9
0.65% Cr, 0.23% Mo,											
0.6% Si)								492	575	688	969
1 Cr–$\frac{1}{2}$ Mo											
(0.16% C, 1% Cr,		7858	442	42.3	12.2			42.0	39.1	34.5	27.4
0.54% Mo,								492	575	688	969
0.39% Si)		7836	443	48.9	14.1			46.8	42.1	36.3	28.2
1 Cr–V											
(0.2% C, 1.02% Cr,											
0.15% V)								492	575	688	969
Stainless steels:											
AISI 302		8055	480	15.1	3.91			17.3	20.0	22.8	25.4
								512	559	585	606
AISI 304	1670	7900	477	14.9	3.95	9.2	12.6	16.6	19.8	22.6	25.4
						272	402	515	557	582	611
AISI 316		8238	468	13.4	3.48			15.2	18.3	21.3	24.2
								504	550	576	602
AISI 347		7978	480	14.2	3.71			15.8	18.9	21.9	24.7
								513	559	585	606
Lead	601	11,340	129	35.3	24.1	39.7	36.7	34.0	31.4		
								118	125	132	142
Magnesium	923	1740	1024	156	87.6	169	159	153	149	146	
							649	934	1074	1170	1267
Molybdenum	2894	10,240	251	138	53.7	179	143	134	126	118	112
						141	224	261	275	285	295
Nickel:											
Pure	1728	8900	444	90.7	23.0	164	107	80.2	65.6	67.6	71.8
						232	383	485	592	530	562
Nichrome	1672	8400	420	12	3.4			14	16	21	
(80% Ni, 20% Cr)								480	525	545	
Inconel X-750	1665	8510	439	11.7	3.1	8.7	10.3	13.5	17.0	20.5	24.0
(73% Ni, 15% Cr,											
6.7% Fe)						—	372	473	510	546	626
Niobium	2741	8570	265	53.7	23.6	55.2	52.6	55.2	58.2	61.3	64.4
						188	249	274	283	292	301
Palladium	1827	12,020	244	71.8	24.5	76.5	71.6	73.6	79.7	86.9	94.2
						168	227	251	261	271	281
Platinum: Pure	2045	21,450	133	71.6	25.1	77.5	72.6	71.8	73.2	75.6	78.7
						100	125	136	141	146	152
Alloy 60Pt–40Rh	1800	16,630	162	47	17.4			52	59	65	69
(60% Pt, 40% Rh)								—	—	—	—
Rhenium	3453	21,100	136	47.9	16.7	58.9	51.0	46.1	44.2	44.1	44.6
						97	127	139	145	151	156
Rhodium	2236	12,450	243	150	49.6	186	154	146	136	127	121
						147	220	253	274	293	311

TABLE A–24

Properties of solid metals (*Concluded*)

		Properties at 300 K				*Properties at various temperatures* (K), k(W/m·K)/c_p(J/kg·K)					
Composition	Melting point, K	ρ kg/m^3	c_p J/kg·K	k W/m·K	$\alpha \times 10^6$ m^2/s	100	200	400	600	800	1000
Silicon	1685	2330	712	148	89.2	884	264	98.9	61.9	42.4	31.2
						259	556	790	867	913	946
Silver	1235	10,500	235	429	174	444	430	425	412	396	379
						187	225	239	250	262	277
Tantalum	3269	16,600	140	57.5	24.7	59.2	57.5	57.8	58.6	59.4	60.2
						110	133	144	146	149	152
Thorium	2023	11,700	118	54.0	39.1	59.8	54.6	54.5	55.8	56.9	56.9
						99	112	124	134	145	156
Tin	505	7310	227	66.6	40.1	85.2	73.3	62.2			
						188	215	243			
Titanium	1953	4500	522	21.9	9.32	30.5	24.5	20.4	19.4	19.7	20.7
						300	465	551	591	633	675
Tungsten	3660	19,300	132	174	68.3	208	186	159	137	125	118
						87	122	137	142	146	148
Uranium	1406	19,070	116	27.6	12.5	21.7	25.1	29.6	34.0	38.8	43.9
						94	108	125	146	176	180
Vanadium	2192	6100	489	30.7	10.3	35.8	31.3	31.3	33.3	35.7	38.2
						258	430	515	540	563	597
Zinc	693	7140	389	116	41.8	117	118	111	103		
						297	367	402	436		
Zirconium	2125	6570	278	22.7	12.4	33.2	25.2	21.6	20.7	21.6	23.7
						205	264	300	332	342	362

Source: Frank P. Incropera and David P. DeWitt, *Fundamentals of Heat and Mass Transfer,* 3rd ed., 1990.

TABLE A–25

Properties of solid nonmetals

		Properties at 300 K				Properties at various temperatures (K), k(W/m·K)/c_p(J/kg·K)					
Composition	Melting point, K	ρ kg/m^3	c_p J/kg·K	k W/m·K	$\alpha \times 10^6$ m^2/s	100	200	400	600	800	1000
Aluminum oxide, sapphire	2323	3970	765	46	15.1	450	82	32.4	18.9	13.0	10.5
						—	—	940	1110	1180	1225
Aluminum oxide, polycrystalline	2323	3970	765	36.0	11.9	133	55	26.4	15.8	10.4	7.85
						—	—	940	1110	1180	1225
Beryllium oxide	2725	3000	1030	272	88.0			196	111	70	47
								1350	1690	1865	1975
Boron	2573	2500	1105	27.6	9.99	190	52.5	18.7	11.3	8.1	6.3
						—	—	1490	1880	2135	2350
Boron fiber epoxy (30% vol.) composite	590	2080									
k, ∥ to fibers				2.29		2.10	2.23	2.28			
k, ⊥ to fibers				0.59		0.37	0.49	0.60			
c_p			1122			364	757	1431			
Carbon, amorphous	1500	1950	—	1.60	—	0.67	1.18	1.89	21.9	2.37	2.53
						—	—	—	—	—	—
Diamond, type IIa insulator	—	3500	509	2300		10,000	4000	1540			
						21	194	853			
Graphite, pyrolytic	2273	2210									
k, ∥ to layers				1950		4970	3230	1390	892	667	534
k, ⊥ to layers				5.70		16.8	9.23	4.09	2.68	2.01	1.60
c_p			709			136	411	992	1406	1650	1793
Graphite fiber epoxy (25% vol.) composite	450	1400									
k, heat flow ∥ to fibers				11.1		5.7	8.7	13.0			
k, heat flow ⊥ to fibers			0.87		0.46	0.68	1.1				
c_p			935			337	642	1216			
Pyroceram, Corning 9606	1623	2600	808	3.98	1.89	5.25	4.78	3.64	3.28	3.08	2.96
						—	—	908	1038	1122	1197
Silicon carbide	3100	3160	675	490	230			—	—	—	87
								880	1050	1135	1195
Silicon dioxide, crystalline (quartz)	1883	2650									
k, ∥ to c-axis				10.4		39	16.4	7.6	5.0	4.2	
k, ⊥ to c-axis				6.21		20.8	9.5	4.70	3.4	3.1	
c_p			745			—	—	885	1075	1250	
Silicon dioxide, polycrystalline (fused silica)	1883	2220	745	1.38	0.834	0.69	1.14	1.51	1.75	2.17	2.87
						—	—	905	1040	1105	1155
Silicon nitride	2173	2400	691	16.0	9.65	—	—	13.9	11.3	9.88	8.76
						—	578	778	937	1063	1155
Sulfur	392	2070	708	0.206	0.141	0.165	0.185				
							403	606			
Thorium dioxide	3573	9110	235	13	6.1			10.2	6.6	4.7	3.68
								255	274	285	295
Titanium dioxide, polycrystalline	2133	4157	710	8.4	2.8			7.01	5.02	8.94	3.46
								805	880	910	930

TABLE A–26

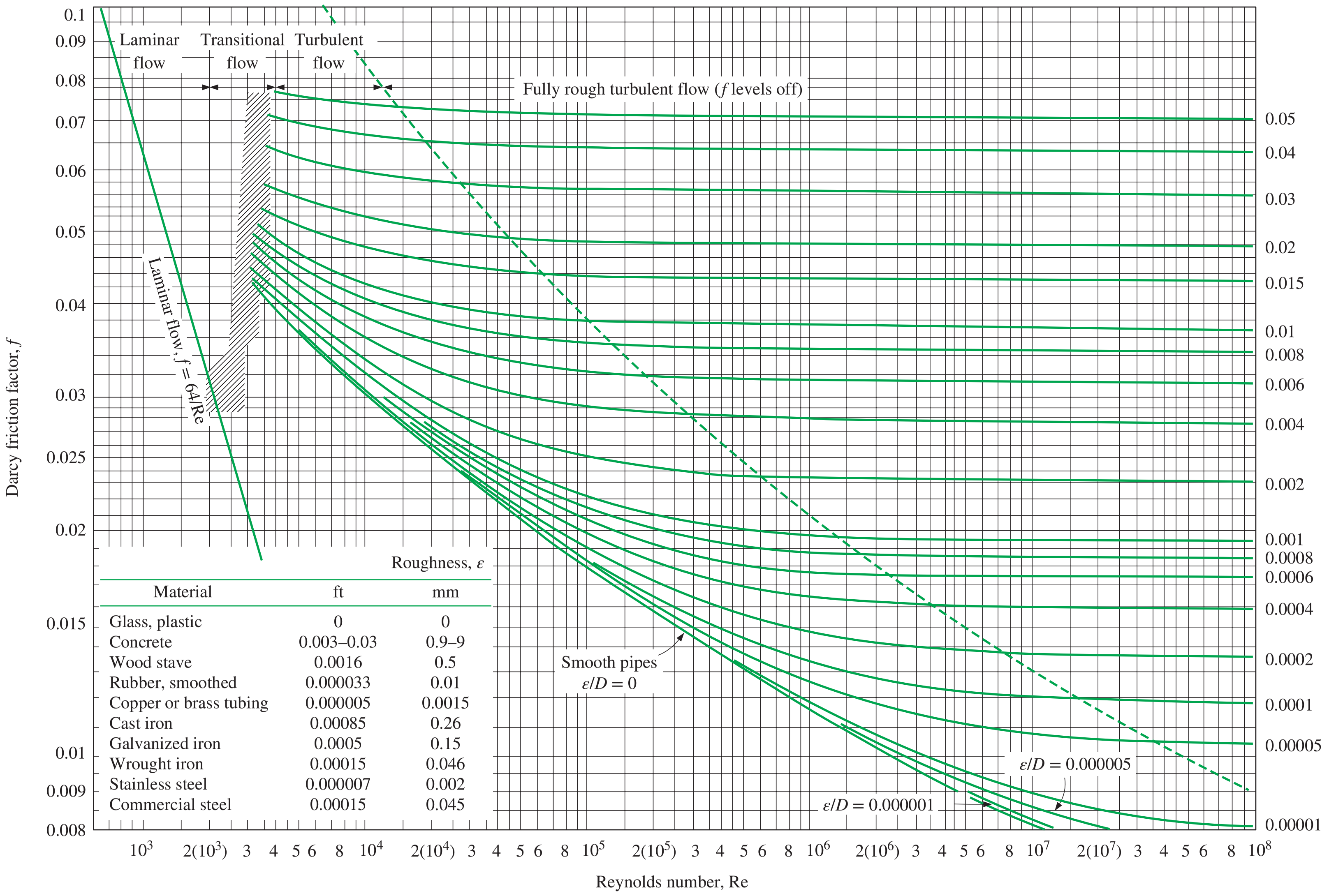
Emissivities of surfaces
(*a*) Metals

Material	Temperature, K	Emissivity, ε
Aluminum		
Polished	300–900	0.04–0.06
Commercial sheet	400	0.09
Heavily oxidized	400–800	0.20–0.33
Anodized	300	0.8
Bismuth, bright	350	0.34
Brass		
Highly polished	500–650	0.03–0.04
Polished	350	0.09
Dull plate	300–600	0.22
Oxidized	450–800	0.6
Chromium, polished	300–1400	0.08–0.40
Copper		
Highly polished	300	0.02
Polished	300–500	0.04–0.05
Commercial sheet	300	0.15
Oxidized	600–1000	0.5–0.8
Black oxidized	300	0.78
Gold		
Highly polished	300–1000	0.03–0.06
Bright foil	300	0.07
Iron		
Highly polished	300–500	0.05–0.07
Case iron	300	0.44
Wrought iron	300–500	0.28
Rusted	300	0.61
Oxidized	500–900	0.64–0.78
Lead		
Polished	300–500	0.06–0.08
Unoxidized, rough	300	0.43
Oxidized	300	0.63
Magnesium, polished	300–500	0.07–0.13
Mercury	300–400	0.09–0.12
Molybdenum		
Polished	300–2000	0.05–0.21
Oxidized	600–800	0.80–0.82
Nickel		
Polished	500–1200	0.07–0.17
Oxidized	450–1000	0.37–0.57
Platinum, polished	500–1500	0.06–0.18
Silver, polished	300–1000	0.02–0.07
Stainless steel		
Polished	300–1000	0.17–0.30
Lightly oxidized	600–1000	0.30–0.40
Highly oxidized	600–1000	0.70–0.80
Steel		
Polished sheet	300–500	0.08–0.14
Commercial sheet	500–1200	0.20–0.32
Heavily oxidized	300	0.81
Tin, polished	300	0.05
Tungsten		
Polished	300–2500	0.03–0.29
Filament	3500	0.39
Zinc		
Polished	300–800	0.02–0.05
Oxidized	300	0.25

TABLE A–26

Emissivities of surfaces
(*b*) Nonmetals (*Concluded*)

Material	Temperature, K	Emissivity, ε
Alumina	800–1400	0.65–0.45
Aluminum oxide	600–1500	0.69–0.41
Asbestos	300	0.96
Asphalt pavement	300	0.85–0.93
Brick		
Common	300	0.93–0.96
Fireclay	1200	0.75
Carbon filament	2000	0.53
Cloth	300	0.75–0.90
Concrete	300	0.88–0.94
Glass		
Window	300	0.90–0.95
Pyrex	300–1200	0.82–0.62
Pyroceram	300–1500	0.85–0.57
Ice	273	0.95–0.99
Magnesium oxide	400–800	0.69–0.55
Masonry	300	0.80
Paints		
Aluminum	300	0.40–0.50
Black, lacquer, shiny	300	0.88
Oils, all colors	300	0.92–0.96
Red primer	300	0.93
White acrylic	300	0.90
White enamel	300	0.90
Paper, white	300	0.90
Plaster, white	300	0.93
Porcelain, glazed	300	0.92
Quartz, rough, fused	300	0.93
Rubber		
Hard	300	0.93
Soft	300	0.86
Sand	300	0.90
Silicon carbide	600–1500	0.87–0.85
Skin, human	300	0.95
Snow	273	0.80–0.90
Soil, earth	300	0.93–0.96
Soot	300–500	0.95
Teflon	300–500	0.85–0.92
Water, deep	273–373	0.95–0.96
Wood		
Beech	300	0.94
Oak	300	0.90

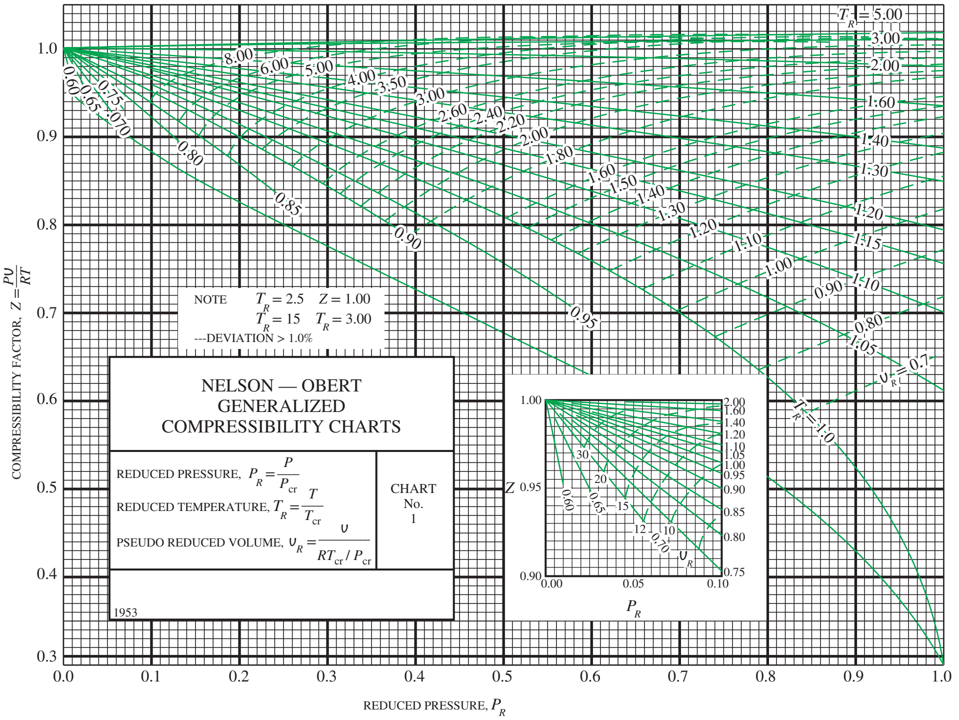


	Roughness, ε	
Material	ft	mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

FIGURE A–27

The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $\Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$.

(*a*) Low pressures, $0 < P_R < 1.0$



(*b*) Intermediate pressures, $0 < P_R < 7$

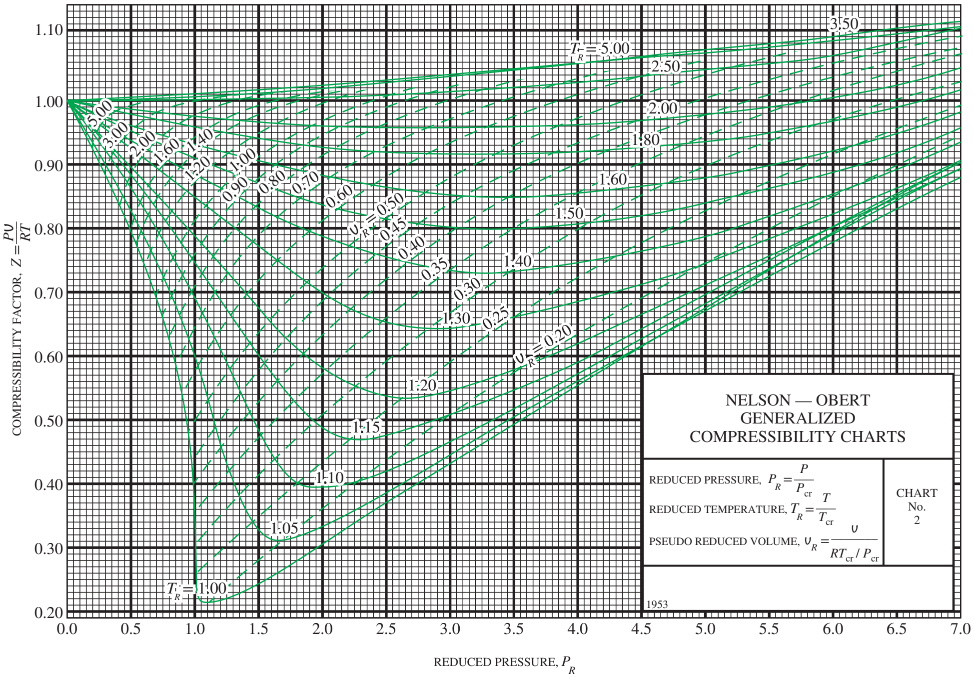


FIGURE A–28

Nelson–Obert generalized compressibility chart.

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APPENDIX

2

PROPERTY TABLES AND CHARTS (ENGLISH UNITS)

TABLE A–1E

Molar mass, gas constant, and critical-point properties

			Gas constant, R^*		*Critical-point properties*		
Substance	Formula	Molar mass, M lbm/lbmol	Btu/ lbm·R	psia·ft^3/ lbm·R	Temperature, R	Pressure, psia	Volume, ft^3/lbmol
Air	—	28.97	0.06855	0.3704	238.5	547	1.41
Ammonia	NH_3	17.03	0.1166	0.6301	729.8	1636	1.16
Argon	Ar	39.948	0.04971	0.2686	272	705	1.20
Benzene	C_6H_6	78.115	0.02542	0.1374	1012	714	4.17
Bromine	Br_2	159.808	0.01243	0.06714	1052	1500	2.17
n-Butane	C_4H_{10}	58.124	0.03417	0.1846	765.2	551	4.08
Carbon dioxide	CO_2	44.01	0.04513	0.2438	547.5	1071	1.51
Carbon monoxide	CO	28.011	0.07090	0.3831	240	507	1.49
Carbon tetrachloride	CCl_4	153.82	0.01291	0.06976	1001.5	661	4.42
Chlorine	Cl_2	70.906	0.02801	0.1517	751	1120	1.99
Chloroform	$CHCl_3$	119.38	0.01664	0.08988	965.8	794	3.85
Dichlorodifluoromethane (R-12)	CCl_2F_2	120.91	0.01643	0.08874	692.4	582	3.49
Dichlorofluoromethane (R-21)	$CHCl_2F$	102.92	0.01930	0.1043	813.0	749	3.16
Ethane	C_2H_6	30.020	0.06616	0.3574	549.8	708	2.37
Ethyl alcohol	C_2H_5OH	46.07	0.04311	0.2329	929.0	926	2.68
Ethylene	C_2H_4	28.054	0.07079	0.3825	508.3	742	1.99
Helium	He	4.003	0.4961	2.6809	9.5	33.2	0.926
n-Hexane	C_6H_{14}	86.178	0.02305	0.1245	914.2	439	5.89
Hydrogen (normal)	H_2	2.016	0.9851	5.3224	59.9	188.1	1.04
Krypton	Kr	83.80	0.02370	0.1280	376.9	798	1.48
Methane	CH_4	16.043	0.1238	0.6688	343.9	673	1.59
Methyl alcohol	CH_3OH	32.042	0.06198	0.3349	923.7	1154	1.89
Methyl chloride	CH_3Cl	50.488	0.03934	0.2125	749.3	968	2.29
Neon	Ne	20.183	0.09840	0.5316	80.1	395	0.668
Nitrogen	N_2	28.013	0.07090	0.3830	227.1	492	1.44
Nitrous oxide	N_2O	44.013	0.04512	0.2438	557.4	1054	1.54
Oxygen	O_2	31.999	0.06206	0.3353	278.6	736	1.25
Propane	C_3H_8	44.097	0.04504	0.2433	665.9	617	3.20
Propylene	C_3H_6	42.081	0.04719	0.2550	656.9	670	2.90
Sulfur dioxide	SO_2	64.063	0.03100	1.1675	775.2	1143	1.95
Tetrafluoroethane (R-134a)	CF_3CH_2F	102.03	0.01946	0.1052	673.6	588.7	3.19
Trichlorofluoromethane (R-11)	CCl_3F	137.37	0.01446	0.07811	848.1	635	3.97
Water	H_2O	18.015	0.1102	0.5956	1164.8	3200	0.90
Xenon	Xe	131.30	0.01513	0.08172	521.55	852	1.90

[*] Calculated from $R = R_u/M$, where R_u = 1.98588 Btu/lbmol·R = 10.7316 psia·ft^3/lbmol·R and M is the molar mass.

Source of Data: K. A. Kobe and R. E. Lynn, Jr., *Chemical Review* 52 (1953), pp. 117–236, and ASHRAE, *Handbook of Fundamentals* (Atlanta, GA: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc., 1993), pp. 16.4 and 36.1.

TABLE A–2E

Ideal-gas specific heats of various common gases
(*a*) At 80°F

Gas	Formula	Gas constant, R Btu/lbm·R	c_p Btu/lbm·R	c_v Btu/lbm·R	k
Air	—	0.06855	0.240	0.171	1.400
Argon	Ar	0.04971	0.1253	0.0756	1.667
Butane	C_4H_{10}	0.03424	0.415	0.381	1.09
Carbon dioxide	CO_2	0.04513	0.203	0.158	1.285
Carbon monoxide	CO	0.07090	0.249	0.178	1.399
Ethane	C_2H_6	0.06616	0.427	0.361	1.183
Ethylene	C_2H_4	0.07079	0.411	0.340	1.208
Helium	He	0.4961	1.25	0.753	1.667
Hydrogen	H_2	0.9851	3.43	2.44	1.404
Methane	CH_4	0.1238	0.532	0.403	1.32
Neon	Ne	0.09840	0.246	0.1477	1.667
Nitrogen	N_2	0.07090	0.248	0.177	1.400
Octane	C_8H_{18}	0.01742	0.409	0.392	1.044
Oxygen	O_2	0.06206	0.219	0.157	1.395
Propane	C_3H_8	0.04504	0.407	0.362	1.124
Steam	H_2O	0.1102	0.445	0.335	1.329

Source of Data: Gordon J. Van Wylen and Richard E. Sonntag, *Fundamentals of Classical Thermodynamics,* English/SI Version, 3rd ed. (New York: John Wiley & Sons, 1986), p. 687, Table A–8E.

TABLE A–2E

Ideal-gas specific heats of various common gases (*Continued*)
(*b*) At various temperatures

Temp., °F	c_p Btu/lbm·R	c_v Btu/lbm·R	k	c_p Btu/lbm·R	c_v Btu/lbm·R	k	c_p Btu/lbm·R	c_v Btu/lbm·R	k
	Air			*Carbon dioxide,* CO_2			*Carbon monoxide, CO*		
40	0.240	0.171	1.401	0.195	0.150	1.300	0.248	0.177	1.400
100	0.240	0.172	1.400	0.205	0.160	1.283	0.249	0.178	1.399
200	0.241	0.173	1.397	0.217	0.172	1.262	0.249	0.179	1.397
300	0.243	0.174	1.394	0.229	0.184	1.246	0.251	0.180	1.394
400	0.245	0.176	1.389	0.239	0.193	1.233	0.253	0.182	1.389
500	0.248	0.179	1.383	0.247	0.202	1.223	0.256	0.185	1.384
600	0.250	0.182	1.377	0.255	0.210	1.215	0.259	0.188	1.377
700	0.254	0.185	1.371	0.262	0.217	1.208	0.262	0.191	1.371
800	0.257	0.188	1.365	0.269	0.224	1.202	0.266	0.195	1.364
900	0.259	0.191	1.358	0.275	0.230	1.197	0.269	0.198	1.357
1000	0.263	0.195	1.353	0.280	0.235	1.192	0.273	0.202	1.351
1500	0.276	0.208	1.330	0.298	0.253	1.178	0.287	0.216	1.328
2000	0.286	0.217	1.312	0.312	0.267	1.169	0.297	0.226	1.314
	Hydrogen, H_2			*Nitrogen,* N_2			*Oxygen,* O_2		
40	3.397	2.412	1.409	0.248	0.177	1.400	0.219	0.156	1.397
100	3.426	2.441	1.404	0.248	0.178	1.399	0.220	0.158	1.394
200	3.451	2.466	1.399	0.249	0.178	1.398	0.223	0.161	1.387
300	3.461	2.476	1.398	0.250	0.179	1.396	0.226	0.164	1.378
400	3.466	2.480	1.397	0.251	0.180	1.393	0.230	0.168	1.368
500	3.469	2.484	1.397	0.254	0.183	1.388	0.235	0.173	1.360
600	3.473	2.488	1.396	0.256	0.185	1.383	0.239	0.177	1.352
700	3.477	2.492	1.395	0.260	0.189	1.377	0.242	0.181	1.344
800	3.494	2.509	1.393	0.262	0.191	1.371	0.246	0.184	1.337
900	3.502	2.519	1.392	0.265	0.194	1.364	0.249	0.187	1.331
1000	3.513	2.528	1.390	0.269	0.198	1.359	0.252	0.190	1.326
1500	3.618	2.633	1.374	0.283	0.212	1.334	0.263	0.201	1.309
2000	3.758	2.773	1.355	0.293	0.222	1.319	0.270	0.208	1.298

Note: The unit Btu/lbm·R is equivalent to Btu/lbm·°F.

Source of Data: Kenneth Wark, *Thermodynamics,* 4th ed. (New York: McGraw-Hill, 1983), p. 830, Table A–4. Originally published in *Tables of Properties of Gases,* NBS Circular 564, 1955.

TABLE A–2E

Ideal-gas specific heats of various common gases (*Concluded*)
(*c*) As a function of temperature

$$\bar{c}_p = a + bT + cT^2 + dT^3$$
(T in R, c_p in Btu/lbmol·R)

Substance	Formula	a	b	c	d	Temperature range, R	% error Max.	% error Avg.
Nitrogen	N_2	6.903	-0.02085×10^{-2}	0.05957×10^{-5}	-0.1176×10^{-9}	491–3240	0.59	0.34
Oxygen	O_2	6.085	0.2017×10^{-2}	-0.05275×10^{-5}	0.05372×10^{-9}	491–3240	1.19	0.28
Air	—	6.713	0.02609×10^{-2}	0.03540×10^{-5}	-0.08052×10^{-9}	491–3240	0.72	0.33
Hydrogen	H_2	6.952	-0.02542×10^{-2}	0.02952×10^{-5}	-0.03565×10^{-9}	491–3240	1.02	0.26
Carbon monoxide	CO	6.726	0.02222×10^{-2}	0.03960×10^{-5}	-0.09100×10^{-9}	491–3240	0.89	0.37
Carbon dioxide	CO_2	5.316	0.79361×10^{-2}	-0.2581×10^{-5}	0.3059×10^{-9}	491–3240	0.67	0.22
Water vapor	H_2O	7.700	0.02552×10^{-2}	0.07781×10^{-5}	-0.1472×10^{-9}	491–3240	0.53	0.24
Nitric oxide	NO	7.008	-0.01247×10^{-2}	0.07185×10^{-5}	-0.1715×10^{-9}	491–2700	0.97	0.36
Nitrous oxide	N_2O	5.758	0.7780×10^{-2}	-0.2596×10^{-5}	0.4331×10^{-9}	491–2700	0.59	0.26
Nitrogen dioxide	NO_2	5.48	0.7583×10^{-2}	-0.260×10^{-5}	0.322×10^{-9}	491–2700	0.46	0.18
Ammonia	NH_3	6.5846	0.34028×10^{-2}	0.073034×10^{-5}	-0.27402×10^{-9}	491–2700	0.91	0.36
Sulfur	S	6.499	0.2943×10^{-2}	-0.1200×10^{-5}	0.1632×10^{-9}	491–3240	0.99	0.38
Sulfur dioxide	SO_2	6.157	0.7689×10^{-2}	-0.2810×10^{-5}	0.3527×10^{-9}	491–3240	0.45	0.24
Sulfur trioxide	SO_3	3.918	1.935×10^{-2}	-0.8256×10^{-5}	1.328×10^{-9}	491–2340	0.29	0.13
Acetylene	C_2H_2	5.21	1.2227×10^{-2}	-0.4812×10^{-5}	0.7457×10^{-9}	491–2700	1.46	0.59
Benzene	C_6H_6	−8.650	6.4322×10^{-2}	-2.327×10^{-5}	3.179×10^{-9}	491–2700	0.34	0.20
Methanol	CH_4O	4.55	1.214×10^{-2}	-0.0898×10^{-5}	-0.329×10^{-9}	491–1800	0.18	0.08
Ethanol	C_2H_6O	4.75	2.781×10^{-2}	-0.7651×10^{-5}	0.821×10^{-9}	491–2700	0.40	0.22
Hydrogen chloride	HCl	7.244	-0.1011×10^{-2}	0.09783×10^{-5}	-0.1776×10^{-9}	491–2740	0.22	0.08
Methane	CH_4	4.750	0.6666×10^{-2}	0.09352×10^{-5}	-0.4510×10^{-9}	491–2740	1.33	0.57
Ethane	C_2H_6	1.648	2.291×10^{-2}	-0.4722×10^{-5}	0.2984×10^{-9}	491–2740	0.83	0.28
Propane	C_3H_8	−0.966	4.044×10^{-2}	-1.159×10^{-5}	1.300×10^{-9}	491–2740	0.40	0.12
n-Butane	C_4H_{10}	0.945	4.929×10^{-2}	-1.352×10^{-5}	1.433×10^{-9}	491–2740	0.54	0.24
i-Butane	C_4H_{10}	−1.890	5.520×10^{-2}	-1.696×10^{-5}	2.044×10^{-9}	491–2740	0.25	0.13
n-Pentane	C_5H_{12}	1.618	6.028×10^{-2}	-1.656×10^{-5}	1.732×10^{-9}	491–2740	0.56	0.21
n-Hexane	C_6H_{14}	1.657	7.328×10^{-2}	-2.112×10^{-5}	2.363×10^{-9}	491–2740	0.72	0.20
Ethylene	C_2H_4	0.944	2.075×10^{-2}	-0.6151×10^{-5}	0.7326×10^{-9}	491–2740	0.54	0.13
Propylene	C_3H_6	0.753	3.162×10^{-2}	-0.8981×10^{-5}	1.008×10^{-9}	491–2740	0.73	0.17

Source of Data: B. G. Kyle, *Chemical and Process Thermodynamics,* 3rd ed. (Upper Saddle River, NJ: Prentice Hall, 2000).

TABLE A–3E

Properties of common liquids, solids, and foods

(*a*) Liquids

	Boiling data at 1 atm		*Freezing data*		*Liquid properties*		
Substance	Normal boiling point, °F	Latent heat of vaporization, h_{fg} Btu/lbm	Freezing point, °F	Latent heat of fusion, h_{if} Btu/lbm	Temperature, °F	Density, ρ lbm/ft^3	Specific heat, c_p Btu/lbm·R
Ammonia	−27.9	24.54	−107.9	138.6	−27.9	42.6	1.06
					0	41.3	1.083
					40	39.5	1.103
					80	37.5	1.135
Argon	−302.6	69.5	−308.7	12.0	−302.6	87.0	0.272
Benzene	176.4	169.4	41.9	54.2	68	54.9	0.411
Brine (20% sodium chloride by mass)	219.0	—	0.7	—	68	71.8	0.743
n-Butane	31.1	165.6	−217.3	34.5	31.1	37.5	0.552
Carbon dioxide	−109.2[*]	99.6 (at 32°F)	−69.8	—	32	57.8	0.583
Ethanol	172.8	360.5	−173.6	46.9	77	48.9	0.588
Ethyl alcohol	173.5	368	−248.8	46.4	68	49.3	0.678
Ethylene glycol	388.6	344.0	12.6	77.9	68	69.2	0.678
Glycerine	355.8	419	66.0	86.3	68	78.7	0.554
Helium	−452.1	9.80	—	—	−452.1	9.13	5.45
Hydrogen	−423.0	191.7	−434.5	25.6	−423.0	4.41	2.39
Isobutane	10.9	157.8	−255.5	45.5	10.9	37.1	0.545
Kerosene	399–559	108	−12.8	—	68	51.2	0.478
Mercury	674.1	126.7	−38.0	4.90	77	847	0.033
Methane	−258.7	219.6	296.0	25.1	−258.7	26.4	0.834
					−160	20.0	1.074
Methanol	148.1	473	−143.9	42.7	77	49.1	0.609
Nitrogen	−320.4	85.4	−346.0	10.9	−320.4	50.5	0.492
					−260	38.2	0.643
Octane	256.6	131.7	−71.5	77.9	68	43.9	0.502
Oil (light)	—	—			77	56.8	0.430
Oxygen	−297.3	91.5	−361.8	5.9	−297.3	71.2	0.408
Petroleum	—	99–165			68	40.0	0.478
Propane	−43.7	184.0	−305.8	34.4	−43.7	36.3	0.538
					32	33.0	0.604
					100	29.4	0.673
Refrigerant-134a	−15.0	93.3	−141.9	—	−40	88.5	0.283
					−15	86.0	0.294
					32	80.9	0.318
					90	73.6	0.348
Water	212	970.1	32	143.5	32	62.4	1.01
					90	62.1	1.00
					150	61.2	1.00
					212	59.8	1.01

[*] Sublimation temperature. (At pressures below the triple-point pressure of 75.1 psia, carbon dioxide exists as a solid or gas. Also, the freezing-point temperature of carbon dioxide is the triple-point temperature of −69.8°F.)

TABLE A–3E

Properties of common liquids, solids, and foods (*Concluded*)

(*b*) Solids (values are for room temperature unless indicated otherwise)

Substance	Density, ρ lbm/ft^3	Specific heat, c_p Btu/lbm·R	Substance	Density, ρ lbm/ft^3	Specific heat, c_p Btu/lbm·R
Metals			Nonmetals		
Aluminum			Asphalt	132	0.220
−100°F		0.192	Brick, common	120	0.189
32°F		0.212	Brick, fireclay (500°C)	144	0.229
100°F	170	0.218	Concrete	144	0.156
200°F		0.224	Clay	62.4	0.220
300°F		0.229	Diamond	151	0.147
400°F		0.235	Glass, window	169	0.191
500°F		0.240	Glass, pyrex	139	0.200
Bronze (76% Cu, 2% Zn, 2% Al)	517	0.0955	Graphite	156	0.170
			Granite	169	0.243
			Gypsum or plaster board		
Brass, yellow (65% Cu, 35% Zn)	519	0.0955	Ice	50	0.260
Copper			−50°F		0.424
−60°F		0.0862	0°F		0.471
0°F		0.0893	20°F		0.491
100°F	555	0.0925	32°F	57.5	0.502
200°F		0.0938	Limestone	103	0.217
390°F		0.0963	Marble 162	0.210	
Iron	490	0.107	Plywood (Douglas fir)	34.0	
Lead	705	0.030	Rubber (hard)	68.7	
Magnesium	108	0.239	Rubber (soft)	71.8	
Nickel	555	0.105	Sand	94.9	
Silver	655	0.056	Stone	93.6	
Steel, mild	489	0.119	Woods, hard (maple, oak, etc.)	45.0	
Tungsten	1211	0.031	Woods, soft (fir, pine, etc.)	32.0	

(*c*) Foods

Food	Water content, % (mass)	Freezing point, °F	*Specific heat,* Btu/lbm·R		Latent heat of fusion, Btu/lbm	Food	Water content, % (mass)	Freezing point, °F	*Specific heat,* Btu/lbm·R		Latent heat of fusion, Btu/lbm
			Above freezing	Below freezing					Above freezing	Below freezing	
Apples	84	30	0.873	0.453	121	Lettuce	95	32	0.961	0.487	136
Bananas	75	31	0.801	0.426	108	Milk, whole	88	31	0.905	0.465	126
Beef round	67	—	0.737	0.402	96	Oranges	87	31	0.897	0.462	125
Broccoli	90	31	0.921	0.471	129	Potatoes	78	31	0.825	0.435	112
Butter	16	—	—	0.249	23	Salmon fish	64	28	0.713	0.393	92
Cheese, Swiss	39	14	0.513	0.318	56	Shrimp	83	28	0.865	0.450	119
Cherries	80	29	0.841	0.441	115	Spinach	93	31	0.945	0.481	134
Chicken	74	27	0.793	0.423	106	Strawberries	90	31	0.921	0.471	129
Corn, sweet	74	31	0.793	0.423	106	Tomatoes, ripe	94	31	0.953	0.484	135
Eggs, whole	74	31	0.793	0.423	106	Turkey	64	—	0.713	0.393	92
Ice cream	63	22	0.705	0.390	90	Watermelon	93	31	0.945	0.481	134

Source of Data: Values are obtained from various handbooks and other sources or are calculated. Water content and freezing-point data of foods are from ASHRAE, *Handbook of Fundamentals,* I-P version (Atlanta, GA: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc., 1993), Chap. 30, Table 1.
Freezing point is the temperature at which freezing starts for fruits and vegetables, and the average freezing temperature for other foods.

TABLE A–4E

Saturated water—Temperature table

		Specific volume, ft^3/lbm		*Internal energy,* Btu/lbm			*Enthalpy,* Btu/lbm			*Entropy,* Btu/lbm·R		
Temp., T °F	Sat. press., P_{sat} psia	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
32.018	0.08871	0.01602	3299.9	0.000	1021.0	1021.0	0.000	1075.2	1075.2	0.00000	2.18672	2.1867
35	0.09998	0.01602	2945.7	3.004	1019.0	1022.0	3.004	1073.5	1076.5	0.00609	2.17011	2.1762
40	0.12173	0.01602	2443.6	8.032	1015.6	1023.7	8.032	1070.7	1078.7	0.01620	2.14271	2.1589
45	0.14756	0.01602	2035.8	13.05	1012.2	1025.3	13.05	1067.8	1080.9	0.02620	2.11587	2.1421
50	0.17812	0.01602	1703.1	18.07	1008.9	1026.9	18.07	1065.0	1083.1	0.03609	2.08956	2.1256
55	0.21413	0.01603	1430.4	23.07	1005.5	1028.6	23.07	1062.2	1085.3	0.04586	2.06377	2.1096
60	0.25638	0.01604	1206.1	28.08	1002.1	1030.2	28.08	1059.4	1087.4	0.05554	2.03847	2.0940
65	0.30578	0.01604	1020.8	33.08	998.76	1031.8	33.08	1056.5	1089.6	0.06511	2.01366	2.0788
70	0.36334	0.01605	867.18	38.08	995.39	1033.5	38.08	1053.7	1091.8	0.07459	1.98931	2.0639
75	0.43016	0.01606	739.27	43.07	992.02	1035.1	43.07	1050.9	1093.9	0.08398	1.96541	2.0494
80	0.50745	0.01607	632.41	48.06	988.65	1036.7	48.07	1048.0	1096.1	0.09328	1.94196	2.0352
85	0.59659	0.01609	542.80	53.06	985.28	1038.3	53.06	1045.2	1098.3	0.10248	1.91892	2.0214
90	0.69904	0.01610	467.40	58.05	981.90	1040.0	58.05	1042.4	1100.4	0.11161	1.89630	2.0079
95	0.81643	0.01612	403.74	63.04	978.52	1041.6	63.04	1039.5	1102.6	0.12065	1.87408	1.9947
100	0.95052	0.01613	349.83	68.03	975.14	1043.2	68.03	1036.7	1104.7	0.12961	1.85225	1.9819
110	1.2767	0.01617	264.96	78.01	968.36	1046.4	78.02	1031.0	1109.0	0.14728	1.80970	1.9570
120	1.6951	0.01620	202.94	88.00	961.56	1049.6	88.00	1025.2	1113.2	0.16466	1.76856	1.9332
130	2.2260	0.01625	157.09	97.99	954.73	1052.7	97.99	1019.4	1117.4	0.18174	1.72877	1.9105
140	2.8931	0.01629	122.81	107.98	947.87	1055.9	107.99	1013.6	1121.6	0.19855	1.69024	1.8888
150	3.7234	0.01634	96.929	117.98	940.98	1059.0	117.99	1007.8	1125.7	0.21508	1.65291	1.8680
160	4.7474	0.01639	77.185	127.98	934.05	1062.0	128.00	1001.8	1129.8	0.23136	1.61670	1.8481
170	5.9999	0.01645	61.982	138.00	927.08	1065.1	138.02	995.88	1133.9	0.24739	1.58155	1.8289
180	7.5197	0.01651	50.172	148.02	920.06	1068.1	148.04	989.85	1137.9	0.26318	1.54741	1.8106
190	9.3497	0.01657	40.920	158.05	912.99	1071.0	158.08	983.76	1141.8	0.27874	1.51421	1.7930
200	11.538	0.01663	33.613	168.10	905.87	1074.0	168.13	977.60	1145.7	0.29409	1.48191	1.7760
210	14.136	0.01670	27.798	178.15	898.68	1076.8	178.20	971.35	1149.5	0.30922	1.45046	1.7597
212	14.709	0.01671	26.782	180.16	897.24	1077.4	180.21	970.09	1150.3	0.31222	1.44427	1.7565
220	17.201	0.01677	23.136	188.22	891.43	1079.6	188.28	965.02	1153.3	0.32414	1.41980	1.7439
230	20.795	0.01684	19.374	198.31	884.10	1082.4	198.37	958.59	1157.0	0.33887	1.38989	1.7288
240	24.985	0.01692	16.316	208.41	876.70	1085.1	208.49	952.06	1160.5	0.35342	1.36069	1.7141
250	29.844	0.01700	13.816	218.54	869.21	1087.7	218.63	945.41	1164.0	0.36779	1.33216	1.6999
260	35.447	0.01708	11.760	228.68	861.62	1090.3	228.79	938.65	1167.4	0.38198	1.30425	1.6862
270	41.877	0.01717	10.059	238.85	853.94	1092.8	238.98	931.76	1170.7	0.39601	1.27694	1.6730
280	49.222	0.01726	8.6439	249.04	846.16	1095.2	249.20	924.74	1173.9	0.40989	1.25018	1.6601
290	57.573	0.01735	7.4607	259.26	838.27	1097.5	259.45	917.57	1177.0	0.42361	1.22393	1.6475
300	67.028	0.01745	6.4663	269.51	830.25	1099.8	269.73	910.24	1180.0	0.43720	1.19818	1.6354
310	77.691	0.01755	5.6266	279.79	822.11	1101.9	280.05	902.75	1182.8	0.45065	1.17289	1.6235
320	89.667	0.01765	4.9144	290.11	813.84	1104.0	290.40	895.09	1185.5	0.46396	1.14802	1.6120
330	103.07	0.01776	4.3076	300.46	805.43	1105.9	300.80	887.25	1188.1	0.47716	1.12355	1.6007
340	118.02	0.01787	3.7885	310.85	796.87	1107.7	311.24	879.22	1190.5	0.49024	1.09945	1.5897
350	134.63	0.01799	3.3425	321.29	788.16	1109.4	321.73	870.98	1192.7	0.50321	1.07570	1.5789
360	153.03	0.01811	2.9580	331.76	779.28	1111.0	332.28	862.53	1194.8	0.51607	1.05227	1.5683
370	173.36	0.01823	2.6252	342.29	770.23	1112.5	342.88	853.86	1196.7	0.52884	1.02914	1.5580
380	195.74	0.01836	2.3361	352.87	761.00	1113.9	353.53	844.96	1198.5	0.54152	1.00628	1.5478
390	220.33	0.01850	2.0842	363.50	751.58	1115.1	364.25	835.81	1200.1	0.55411	0.98366	1.5378

TABLE A–4E

Saturated water—Temperature table *(Concluded)*

		Specific volume, ft^3/lbm		*Internal energy,* Btu/lbm			*Enthalpy,* Btu/lbm			*Entropy,* Btu/lbm·R		
Temp., T °F	Sat. press., P_{sat} psia	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
400	247.26	0.01864	1.8639	374.19	741.97	1116.2	375.04	826.39	1201.4	0.56663	0.96127	1.5279
410	276.69	0.01878	1.6706	384.94	732.14	1117.1	385.90	816.71	1202.6	0.57907	0.93908	1.5182
420	308.76	0.01894	1.5006	395.76	722.08	1117.8	396.84	806.74	1203.6	0.59145	0.91707	1.5085
430	343.64	0.01910	1.3505	406.65	711.80	1118.4	407.86	796.46	1204.3	0.60377	0.89522	1.4990
440	381.49	0.01926	1.2178	417.61	701.26	1118.9	418.97	785.87	1204.8	0.61603	0.87349	1.4895
450	422.47	0.01944	1.0999	428.66	690.47	1119.1	430.18	774.94	1205.1	0.62826	0.85187	1.4801
460	466.75	0.01962	0.99510	439.79	679.39	1119.2	441.48	763.65	1205.1	0.64044	0.83033	1.4708
470	514.52	0.01981	0.90158	451.01	668.02	1119.0	452.90	751.98	1204.9	0.65260	0.80885	1.4615
480	565.96	0.02001	0.81794	462.34	656.34	1118.7	464.43	739.91	1204.3	0.66474	0.78739	1.4521
490	621.24	0.02022	0.74296	473.77	644.32	1118.1	476.09	727.40	1203.5	0.67686	0.76594	1.4428
500	680.56	0.02044	0.67558	485.32	631.94	1117.3	487.89	714.44	1202.3	0.68899	0.74445	1.4334
510	744.11	0.02067	0.61489	496.99	619.17	1116.2	499.84	700.99	1200.8	0.70112	0.72290	1.4240
520	812.11	0.02092	0.56009	508.80	605.99	1114.8	511.94	687.01	1199.0	0.71327	0.70126	1.4145
530	884.74	0.02118	0.51051	520.76	592.35	1113.1	524.23	672.47	1196.7	0.72546	0.67947	1.4049
540	962.24	0.02146	0.46553	532.88	578.23	1111.1	536.70	657.31	1194.0	0.73770	0.65751	1.3952
550	1044.8	0.02176	0.42465	545.18	563.58	1108.8	549.39	641.47	1190.9	0.75000	0.63532	1.3853
560	1132.7	0.02207	0.38740	557.68	548.33	1106.0	562.31	624.91	1187.2	0.76238	0.61284	1.3752
570	1226.2	0.02242	0.35339	570.40	532.45	1102.8	575.49	607.55	1183.0	0.77486	0.59003	1.3649
580	1325.5	0.02279	0.32225	583.37	515.84	1099.2	588.95	589.29	1178.2	0.78748	0.56679	1.3543
590	1430.8	0.02319	0.29367	596.61	498.43	1095.0	602.75	570.04	1172.8	0.80026	0.54306	1.3433
600	1542.5	0.02362	0.26737	610.18	480.10	1090.3	616.92	549.67	1166.6	0.81323	0.51871	1.3319
610	1660.9	0.02411	0.24309	624.11	460.73	1084.8	631.52	528.03	1159.5	0.82645	0.49363	1.3201
620	1786.2	0.02464	0.22061	638.47	440.14	1078.6	646.62	504.92	1151.5	0.83998	0.46765	1.3076
630	1918.9	0.02524	0.19972	653.35	418.12	1071.5	662.32	480.07	1142.4	0.85389	0.44056	1.2944
640	2059.3	0.02593	0.18019	668.86	394.36	1063.2	678.74	453.14	1131.9	0.86828	0.41206	1.2803
650	2207.8	0.02673	0.16184	685.16	368.44	1053.6	696.08	423.65	1119.7	0.88332	0.38177	1.2651
660	2364.9	0.02767	0.14444	702.48	339.74	1042.2	714.59	390.84	1105.4	0.89922	0.34906	1.2483
670	2531.2	0.02884	0.12774	721.23	307.22	1028.5	734.74	353.54	1088.3	0.91636	0.31296	1.2293
680	2707.3	0.03035	0.11134	742.11	269.00	1011.1	757.32	309.57	1066.9	0.93541	0.27163	1.2070
690	2894.1	0.03255	0.09451	766.81	220.77	987.6	784.24	253.96	1038.2	0.95797	0.22089	1.1789
700	3093.0	0.03670	0.07482	801.75	146.50	948.3	822.76	168.32	991.1	0.99023	0.14514	1.1354
705.10	3200.1	0.04975	0.04975	866.61	0	866.6	896.07	0	896.1	1.05257	0	1.0526

Source of Data: Tables A–4E through A–8E are generated using the Engineering Equation Solver (EES) software developed by S. A. Klein and F. L. Alvarado. The routine used in calculations is the highly accurate Steam_IAPWS, which incorporates the 1995 Formulation for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use, issued by The International Association for the Properties of Water and Steam (IAPWS). This formulation replaces the 1984 formulation of Haar, Gallagher, and Kell *(NBS/NRC Steam Tables,* Hemisphere Publishing Co., 1984), which is also available in EES as the routine STEAM. The new formulation is based on the correlations of Saul and Wagner (*J. Phys. Chem. Ref. Data,* 16, 893, 1987) with modifications to adjust to the International Temperature Scale of 1990. The modifications are described by Wagner and Pruss (*J. Phys. Chem. Ref. Data,* 22, 783, 1993). The properties of ice are based on Hyland and Wexler, "Formulations for the Thermodynamic Properties of the Saturated Phases of H_2O from 173.15 K to 473.15 K," *ASHRAE Trans.*, Part 2A, Paper 2793, 1983.

TABLE A–5E

Saturated water—Pressure table

		Specific volume, ft^3/lbm		*Internal energy,* Btu/lbm			*Enthalpy,* Btu/lbm			*Entropy,* Btu/lbm·R		
Press., P psia	Sat. temp., T_{sat} °F	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
1	101.69	0.01614	333.49	69.72	973.99	1043.7	69.72	1035.7	1105.4	0.13262	1.84495	1.9776
2	126.02	0.01623	173.71	94.02	957.45	1051.5	94.02	1021.7	1115.8	0.17499	1.74444	1.9194
3	141.41	0.01630	118.70	109.39	946.90	1056.3	109.40	1012.8	1122.2	0.20090	1.68489	1.8858
4	152.91	0.01636	90.629	120.89	938.97	1059.9	120.90	1006.0	1126.9	0.21985	1.64225	1.8621
5	162.18	0.01641	73.525	130.17	932.53	1062.7	130.18	1000.5	1130.7	0.23488	1.60894	1.8438
6	170.00	0.01645	61.982	138.00	927.08	1065.1	138.02	995.88	1133.9	0.24739	1.58155	1.8289
8	182.81	0.01652	47.347	150.83	918.08	1068.9	150.86	988.15	1139.0	0.26757	1.53800	1.8056
10	193.16	0.01659	38.425	161.22	910.75	1072.0	161.25	981.82	1143.1	0.28362	1.50391	1.7875
14.696	211.95	0.01671	26.805	180.12	897.27	1077.4	180.16	970.12	1150.3	0.31215	1.44441	1.7566
15	212.99	0.01672	26.297	181.16	896.52	1077.7	181.21	969.47	1150.7	0.31370	1.44441	1.7549
20	227.92	0.01683	20.093	196.21	885.63	1081.8	196.27	959.93	1156.2	0.33582	1.39606	1.7319
25	240.03	0.01692	16.307	208.45	876.67	1085.1	208.52	952.03	1160.6	0.35347	1.36060	1.7141
30	250.30	0.01700	13.749	218.84	868.98	1087.8	218.93	945.21	1164.1	0.36821	1.33132	1.6995
35	259.25	0.01708	11.901	227.92	862.19	1090.1	228.03	939.16	1167.2	0.38093	1.30632	1.6872
40	267.22	0.01715	10.501	236.02	856.09	1092.1	236.14	933.69	1169.8	0.39213	1.28448	1.6766
45	274.41	0.01721	9.4028	243.34	850.52	1093.9	243.49	928.68	1172.2	0.40216	1.26506	1.6672
50	280.99	0.01727	8.5175	250.05	845.39	1095.4	250.21	924.03	1174.2	0.41125	1.24756	1.6588
55	287.05	0.01732	7.7882	256.25	840.61	1096.9	256.42	919.70	1176.1	0.41958	1.23162	1.6512
60	292.69	0.01738	7.1766	262.01	836.13	1098.1	262.20	915.61	1177.8	0.42728	1.21697	1.6442
65	297.95	0.01743	6.6560	267.41	831.90	1099.3	267.62	911.75	1179.4	0.43443	1.20341	1.6378
70	302.91	0.01748	6.2075	272.50	827.90	1100.4	272.72	908.08	1180.8	0.44112	1.19078	1.6319
75	307.59	0.01752	5.8167	277.31	824.09	1101.4	277.55	904.58	1182.1	0.44741	1.17895	1.6264
80	312.02	0.01757	5.4733	281.87	820.45	1102.3	282.13	901.22	1183.4	0.45335	1.16783	1.6212
85	316.24	0.01761	5.1689	286.22	816.97	1103.2	286.50	898.00	1184.5	0.45897	1.15732	1.6163
90	320.26	0.01765	4.8972	290.38	813.62	1104.0	290.67	894.89	1185.6	0.46431	1.14737	1.6117
95	324.11	0.01770	4.6532	294.36	810.40	1104.8	294.67	891.89	1186.6	0.46941	1.13791	1.6073
100	327.81	0.01774	4.4327	298.19	807.29	1105.5	298.51	888.99	1187.5	0.47427	1.12888	1.6032
110	334.77	0.01781	4.0410	305.41	801.37	1106.8	305.78	883.44	1189.2	0.48341	1.11201	1.5954
120	341.25	0.01789	3.7289	312.16	795.79	1107.9	312.55	878.20	1190.8	0.49187	1.09646	1.5883
130	347.32	0.01796	3.4557	318.48	790.51	1109.0	318.92	873.21	1192.1	0.49974	1.08204	1.5818
140	353.03	0.01802	3.2202	324.45	785.49	1109.9	324.92	868.45	1193.4	0.50711	1.06858	1.5757
150	358.42	0.01809	3.0150	330.11	780.69	1110.8	330.61	863.88	1194.5	0.51405	1.05595	1.5700
160	363.54	0.01815	2.8347	335.49	776.10	1111.6	336.02	859.49	1195.5	0.52061	1.04405	1.5647
170	368.41	0.01821	2.6749	340.62	771.68	1112.3	341.19	855.25	1196.4	0.52682	1.03279	1.5596
180	373.07	0.01827	2.5322	345.53	767.42	1113.0	346.14	851.16	1197.3	0.53274	1.02210	1.5548
190	377.52	0.01833	2.4040	350.24	763.31	1113.6	350.89	847.19	1198.1	0.53839	1.01191	1.5503
200	381.80	0.01839	2.2882	354.78	759.32	1114.1	355.46	843.33	1198.8	0.54379	1.00219	1.5460
250	400.97	0.01865	1.8440	375.23	741.02	1116.3	376.09	825.47	1201.6	0.56784	0.95912	1.5270
300	417.35	0.01890	1.5435	392.89	724.77	1117.7	393.94	809.41	1203.3	0.58818	0.92289	1.5111
350	431.74	0.01912	1.3263	408.55	709.98	1118.5	409.79	794.65	1204.4	0.60590	0.89143	1.4973
400	444.62	0.01934	1.1617	422.70	696.31	1119.0	424.13	780.87	1205.0	0.62168	0.86350	1.4852
450	456.31	0.01955	1.0324	435.67	683.52	1119.2	437.30	767.86	1205.2	0.63595	0.83828	1.4742
500	467.04	0.01975	0.92819	447.68	671.42	1119.1	449.51	755.48	1205.0	0.64900	0.81521	1.4642
550	476.97	0.01995	0.84228	458.90	659.91	1118.8	460.93	743.60	1204.5	0.66107	0.79388	1.4550
600	486.24	0.02014	0.77020	469.46	648.88	1118.3	471.70	732.15	1203.9	0.67231	0.77400	1.4463

TABLE A–5E

Saturated water—Pressure table *(Concluded)*

		Specific volume, ft^3/lbm		*Internal energy,* Btu/lbm			*Enthalpy,* Btu/lbm			*Entropy,* Btu/lbm·R		
Press., P psia	Sat. temp., T_{sat} °F	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
700	503.13	0.02051	0.65589	488.96	627.98	1116.9	491.62	710.29	1201.9	0.69279	0.73771	1.4305
800	518.27	0.02087	0.56920	506.74	608.30	1115.0	509.83	689.48	1199.3	0.71117	0.70502	1.4162
900	532.02	0.02124	0.50107	523.19	589.54	1112.7	526.73	669.46	1196.2	0.72793	0.67505	1.4030
1000	544.65	0.02159	0.44604	538.58	571.49	1110.1	542.57	650.03	1192.6	0.74341	0.64722	1.3906
1200	567.26	0.02232	0.36241	566.89	536.87	1103.8	571.85	612.39	1184.2	0.77143	0.59632	1.3677
1400	587.14	0.02307	0.30161	592.79	503.50	1096.3	598.76	575.66	1174.4	0.79658	0.54991	1.3465
1600	604.93	0.02386	0.25516	616.99	470.69	1087.7	624.06	539.18	1163.2	0.81972	0.50645	1.3262
1800	621.07	0.02470	0.21831	640.03	437.86	1077.9	648.26	502.35	1150.6	0.84144	0.46482	1.3063
2000	635.85	0.02563	0.18815	662.33	404.46	1066.8	671.82	464.60	1136.4	0.86224	0.42409	1.2863
2500	668.17	0.02860	0.13076	717.67	313.53	1031.2	730.90	360.79	1091.7	0.91311	0.31988	1.2330
3000	695.41	0.03433	0.08460	783.39	186.41	969.8	802.45	214.32	1016.8	0.97321	0.18554	1.1587
3200.1	705.10	0.04975	0.04975	866.61	0	866.6	896.07	0	896.1	1.05257	0	1.0526

TABLE A–6E

Superheated water

T °F	ʋ ft^3/lbm	*u* Btu/lbm	*h* Btu/lbm	*s* Btu/lbm·R	ʋ ft^3/lbm	*u* Btu/lbm	*h* Btu/lbm	*s* Btu/lbm·R	ʋ ft^3/lbm	*u* Btu/lbm	*h* Btu/lbm	*s* Btu/lbm·R
	P = 1.0 psia (101.69°F)[*]				*P* = 5.0 psia (162.18°F)				*P* = 10 psia (193.16°F)			
Sat.[†]	333.49	1043.7	1105.4	1.9776	73.525	1062.7	1130.7	1.8438	38.425	1072.0	1143.1	1.7875
200	392.53	1077.5	1150.1	2.0509	78.153	1076.2	1148.5	1.8716	38.849	1074.5	1146.4	1.7926
240	416.44	1091.2	1168.3	2.0777	83.009	1090.3	1167.1	1.8989	41.326	1089.1	1165.5	1.8207
280	440.33	1105.0	1186.5	2.1030	87.838	1104.3	1185.6	1.9246	43.774	1103.4	1184.4	1.8469
320	464.20	1118.9	1204.8	2.1271	92.650	1118.4	1204.1	1.9490	46.205	1117.6	1203.1	1.8716
360	488.07	1132.9	1223.3	2.1502	97.452	1132.5	1222.6	1.9722	48.624	1131.9	1221.8	1.8950
400	511.92	1147.1	1241.8	2.1722	102.25	1146.7	1241.3	1.9944	51.035	1146.2	1240.6	1.9174
440	535.77	1161.3	1260.4	2.1934	107.03	1160.9	1260.0	2.0156	53.441	1160.5	1259.4	1.9388
500	571.54	1182.8	1288.6	2.2237	114.21	1182.6	1288.2	2.0461	57.041	1182.2	1287.8	1.9693
600	631.14	1219.4	1336.2	2.2709	126.15	1219.2	1335.9	2.0933	63.029	1219.0	1335.6	2.0167
700	690.73	1256.8	1384.6	2.3146	138.09	1256.7	1384.4	2.1371	69.007	1256.5	1384.2	2.0605
800	750.31	1295.1	1433.9	2.3553	150.02	1294.9	1433.7	2.1778	74.980	1294.8	1433.5	2.1013
1000	869.47	1374.2	1535.1	2.4299	173.86	1374.2	1535.0	2.2524	86.913	1374.1	1534.9	2.1760
1200	988.62	1457.1	1640.0	2.4972	197.70	1457.0	1640.0	2.3198	98.840	1457.0	1639.9	2.2433
1400	1107.8	1543.7	1748.7	2.5590	221.54	1543.7	1748.7	2.3816	110.762	1543.6	1748.6	2.3052
	P = 15 psia (212.99°F)				*P* = 20 psia (227.92°F)				*P* = 40 psia (267.22°F)			
Sat.	26.297	1077.7	1150.7	1.7549	20.093	1081.8	1156.2	1.7319	10.501	1092.1	1169.8	1.6766
240	27.429	1087.8	1163.9	1.7742	20.478	1086.5	1162.3	1.7406				
280	29.085	1102.4	1183.2	1.8010	21.739	1101.4	1181.9	1.7679	10.713	1097.3	1176.6	1.6858
320	30.722	1116.9	1202.2	1.8260	22.980	1116.1	1201.2	1.7933	11.363	1112.9	1197.1	1.7128
360	32.348	1131.3	1221.1	1.8496	24.209	1130.7	1220.2	1.8171	11.999	1128.1	1216.9	1.7376
400	33.965	1145.7	1239.9	1.8721	25.429	1145.1	1239.3	1.8398	12.625	1143.1	1236.5	1.7610
440	35.576	1160.1	1258.8	1.8936	26.644	1159.7	1258.3	1.8614	13.244	1157.9	1256.0	1.7831
500	37.986	1181.9	1287.3	1.9243	28.458	1181.6	1286.9	1.8922	14.165	1180.2	1285.0	1.8143
600	41.988	1218.7	1335.3	1.9718	31.467	1218.5	1334.9	1.9398	15.686	1217.5	1333.6	1.8625
700	45.981	1256.3	1383.9	2.0156	34.467	1256.1	1383.7	1.9837	17.197	1255.3	1382.6	1.9067
800	49.967	1294.6	1433.3	2.0565	37.461	1294.5	1433.1	2.0247	18.702	1293.9	1432.3	1.9478
1000	57.930	1374.0	1534.8	2.1312	43.438	1373.8	1534.6	2.0994	21.700	1373.4	1534.1	2.0227
1200	65.885	1456.9	1639.8	2.1986	49.407	1456.8	1639.7	2.1668	24.691	1456.5	1639.3	2.0902
1400	73.836	1543.6	1748.5	2.2604	55.373	1543.5	1748.4	2.2287	27.678	1543.3	1748.1	2.1522
1600	81.784	1634.0	1861.0	2.3178	61.335	1633.9	1860.9	2.2861	30.662	1633.7	1860.7	2.2096
	P = 60 psia (292.69°F)				*P* = 80 psia (312.02°F)				*P* = 100 psia (327.81°F)			
Sat.	7.1766	1098.1	1177.8	1.6442	5.4733	1102.3	1183.4	1.6212	4.4327	1105.5	1187.5	1.6032
320	7.4863	1109.6	1192.7	1.6636	5.5440	1105.9	1187.9	1.6271				
360	7.9259	1125.5	1213.5	1.6897	5.8876	1122.7	1209.9	1.6545	4.6628	1119.8	1206.1	1.6263
400	8.3548	1140.9	1233.7	1.7138	6.2187	1138.7	1230.8	1.6794	4.9359	1136.4	1227.8	1.6521
440	8.7766	1156.1	1253.6	1.7364	6.5420	1154.3	1251.2	1.7026	5.2006	1152.4	1248.7	1.6759
500	9.4005	1178.8	1283.1	1.7682	7.0177	1177.3	1281.2	1.7350	5.5876	1175.9	1279.3	1.7088
600	10.4256	1216.5	1332.2	1.8168	7.7951	1215.4	1330.8	1.7841	6.2167	1214.4	1329.4	1.7586
700	11.4401	1254.5	1381.6	1.8613	8.5616	1253.8	1380.5	1.8289	6.8344	1253.0	1379.5	1.8037
800	12.4484	1293.3	1431.5	1.9026	9.3218	1292.6	1430.6	1.8704	7.4457	1292.0	1429.8	1.8453
1000	14.4543	1373.0	1533.5	1.9777	10.8313	1372.6	1532.9	1.9457	8.6575	1372.2	1532.4	1.9208
1200	16.4525	1456.2	1638.9	2.0454	12.3331	1455.9	1638.5	2.0135	9.8615	1455.6	1638.1	1.9887
1400	18.4464	1543.0	1747.8	2.1073	13.8306	1542.8	1747.5	2.0755	11.0612	1542.6	1747.2	2.0508
1600	20.438	1633.5	1860.5	2.1648	15.3257	1633.3	1860.2	2.1330	12.2584	1633.2	1860.0	2.1083
1800	22.428	1727.6	1976.6	2.2187	16.8192	1727.5	1976.5	2.1869	13.4541	1727.3	1976.3	2.1622
2000	24.417	1825.2	2096.3	2.2694	18.3117	1825.0	2096.1	2.2376	14.6487	1824.9	2096.0	2.2130

[*] The temperature in parentheses is the saturation temperature at the specified pressure.

[†] Properties of saturated vapor at the specified pressure.

TABLE A–6E

Superheated water *(Continued)*

T °F	v ft³/lbm	*u* Btu/lbm	*h* Btu/lbm	*s* Btu/lbm·R	v ft³/lbm	*u* Btu/lbm	*h* Btu/lbm	*s* Btu/lbm·R	v ft³/lbm	*u* Btu/lbm	*h* Btu/lbm	*s* Btu/lbm·R
	P = 120 psia (341.25°F)				*P* = 140 psia (353.03°F)				*P* = 160 psia (363.54°F)			
Sat.	3.7289	1107.9	1190.8	1.5883	3.2202	1109.9	1193.4	1.5757	2.8347	1111.6	1195.5	1.5647
360	3.8446	1116.7	1202.1	1.6023	3.2584	1113.4	1197.8	1.5811				
400	4.0799	1134.0	1224.6	1.6292	3.4676	1131.5	1221.4	1.6092	3.0076	1129.0	1218.0	1.5914
450	4.3613	1154.5	1251.4	1.6594	3.7147	1152.6	1248.9	1.6403	3.2293	1150.7	1246.3	1.6234
500	4.6340	1174.4	1277.3	1.6872	3.9525	1172.9	1275.3	1.6686	3.4412	1171.4	1273.2	1.6522
550	4.9010	1193.9	1302.8	1.7131	4.1845	1192.7	1301.1	1.6948	3.6469	1191.4	1299.4	1.6788
600	5.1642	1213.4	1328.0	1.7375	4.4124	1212.3	1326.6	1.7195	3.8484	1211.3	1325.2	1.7037
700	5.6829	1252.2	1378.4	1.7829	4.8604	1251.4	1377.3	1.7652	4.2434	1250.6	1376.3	1.7498
800	6.1950	1291.4	1429.0	1.8247	5.3017	1290.8	1428.1	1.8072	4.6316	1290.2	1427.3	1.7920
1000	7.2083	1371.7	1531.8	1.9005	6.1732	1371.3	1531.3	1.8832	5.3968	1370.9	1530.7	1.8682
1200	8.2137	1455.3	1637.7	1.9684	7.0367	1455.0	1637.3	1.9512	6.1540	1454.7	1636.9	1.9363
1400	9.2149	1542.3	1746.9	2.0305	7.8961	1542.1	1746.6	2.0134	6.9070	1541.8	1746.3	1.9986
1600	10.2135	1633.0	1859.8	2.0881	8.7529	1632.8	1859.5	2.0711	7.6574	1632.6	1859.3	2.0563
1800	11.2106	1727.2	1976.1	2.1420	9.6082	1727.0	1975.9	2.1250	8.4063	1726.9	1975.7	2.1102
2000	12.2067	1824.8	2095.8	2.1928	10.4624	1824.6	2095.7	2.1758	9.1542	1824.5	2095.5	2.1610
	P = 180 psia (373.07°F)				*P* = 200 psia (381.80°F)				*P* = 225 psia (391.80°F)			
Sat.	2.5322	1113.0	1197.3	1.5548	2.2882	1114.1	1198.8	1.5460	2.0423	1115.3	1200.3	1.5360
400	2.6490	1126.3	1214.5	1.5752	2.3615	1123.5	1210.9	1.5602	2.0728	1119.7	1206.0	1.5427
450	2.8514	1148.7	1243.7	1.6082	2.5488	1146.7	1241.0	1.5943	2.2457	1144.1	1237.6	1.5783
500	3.0433	1169.8	1271.2	1.6376	2.7247	1168.2	1269.0	1.6243	2.4059	1166.2	1266.3	1.6091
550	3.2286	1190.2	1297.7	1.6646	2.8939	1188.9	1296.0	1.6516	2.5590	1187.2	1293.8	1.6370
600	3.4097	1210.2	1323.8	1.6897	3.0586	1209.1	1322.3	1.6771	2.7075	1207.7	1320.5	1.6628
700	3.7635	1249.8	1375.2	1.7361	3.3796	1249.0	1374.1	1.7238	2.9956	1248.0	1372.7	1.7099
800	4.1104	1289.5	1426.5	1.7785	3.6934	1288.9	1425.6	1.7664	3.2765	1288.1	1424.5	1.7528
900	4.4531	1329.7	1478.0	1.8179	4.0031	1329.2	1477.3	1.8059	3.5530	1328.5	1476.5	1.7925
1000	4.7929	1370.5	1530.1	1.8549	4.3099	1370.1	1529.6	1.8430	3.8268	1369.5	1528.9	1.8296
1200	5.4674	1454.3	1636.5	1.9231	4.9182	1454.0	1636.1	1.9113	4.3689	1453.6	1635.6	1.8981
1400	6.1377	1541.6	1746.0	1.9855	5.5222	1541.4	1745.7	1.9737	4.9068	1541.1	1745.4	1.9606
1600	6.8054	1632.4	1859.1	2.0432	6.1238	1632.2	1858.8	2.0315	5.4422	1632.0	1858.6	2.0184
1800	7.4716	1726.7	1975.6	2.0971	6.7238	1726.5	1975.4	2.0855	5.9760	1726.4	1975.2	2.0724
2000	8.1367	1824.4	2095.4	2.1479	7.3227	1824.3	2095.3	2.1363	6.5087	1824.1	2095.1	2.1232
	P = 250 psia (400.97°F)				*P* = 275 psia (409.45°F)				*P* = 300 psia (417.35°F)			
Sat.	1.8440	1116.3	1201.6	1.5270	1.6806	1117.0	1202.6	1.5187	1.5435	1117.7	1203.3	1.5111
450	2.0027	1141.3	1234.0	1.5636	1.8034	1138.5	1230.3	1.5499	1.6369	1135.6	1226.4	1.5369
500	2.1506	1164.1	1263.6	1.5953	1.9415	1162.0	1260.8	1.5825	1.7670	1159.8	1257.9	1.5706
550	2.2910	1185.6	1291.5	1.6237	2.0715	1183.9	1289.3	1.6115	1.8885	1182.1	1287.0	1.6001
600	2.4264	1206.3	1318.6	1.6499	2.1964	1204.9	1316.7	1.6380	2.0046	1203.5	1314.8	1.6270
650	2.5586	1226.8	1345.1	1.6743	2.3179	1225.6	1343.5	1.6627	2.1172	1224.4	1341.9	1.6520
700	2.6883	1247.0	1371.4	1.6974	2.4369	1246.0	1370.0	1.6860	2.2273	1244.9	1368.6	1.6755
800	2.9429	1287.3	1423.5	1.7406	2.6699	1286.5	1422.4	1.7294	2.4424	1285.7	1421.3	1.7192
900	3.1930	1327.9	1475.6	1.7804	2.8984	1327.3	1474.8	1.7694	2.6529	1326.6	1473.9	1.7593
1000	3.4403	1369.0	1528.2	1.8177	3.1241	1368.5	1527.4	1.8068	2.8605	1367.9	1526.7	1.7968
1200	3.9295	1453.3	1635.0	1.8863	3.5700	1452.9	1634.5	1.8755	3.2704	1452.5	1634.0	1.8657
1400	4.4144	1540.8	1745.0	1.9488	4.0116	1540.5	1744.6	1.9381	3.6759	1540.2	1744.2	1.9284
1600	4.8969	1631.7	1858.3	2.0066	4.4507	1631.5	1858.0	1.9960	4.0789	1631.3	1857.7	1.9863
1800	5.3777	1726.2	1974.9	2.0607	4.8882	1726.0	1974.7	2.0501	4.4803	1725.8	1974.5	2.0404
2000	5.8575	1823.9	2094.9	2.1116	5.3247	1823.8	2094.7	2.1010	4.8807	1823.6	2094.6	2.0913

TABLE A–6E

Superheated water *(Continued)*

T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
°F	ft^3/lbm	Btu/lbm	Btu/lbm	Btu/lbm·R	ft^3/lbm	Btu/lbm	Btu/lbm	Btu/lbm·R	ft^3/lbm	Btu/lbm	Btu/lbm	Btu/lbm·R
	$P = 350$ psia (431.74°F)				$P = 400$ psia (444.62°F)				$P = 450$ psia (456.31°F)			
Sat.	1.3263	1118.5	1204.4	1.4973	1.1617	1119.0	1205.0	1.4852	1.0324	1119.2	1205.2	1.4742
450	1.3739	1129.3	1218.3	1.5128	1.1747	1122.5	1209.4	1.4901				
500	1.4921	1155.2	1251.9	1.5487	1.2851	1150.4	1245.6	1.5288	1.1233	1145.4	1238.9	1.5103
550	1.6004	1178.6	1282.2	1.5795	1.3840	1174.9	1277.3	1.5610	1.2152	1171.1	1272.3	1.5441
600	1.7030	1200.6	1310.9	1.6073	1.4765	1197.6	1306.9	1.5897	1.3001	1194.6	1302.8	1.5737
650	1.8018	1221.9	1338.6	1.6328	1.5650	1219.4	1335.3	1.6158	1.3807	1216.9	1331.9	1.6005
700	1.8979	1242.8	1365.8	1.6567	1.6507	1240.7	1362.9	1.6401	1.4584	1238.5	1360.0	1.6253
800	2.0848	1284.1	1419.1	1.7009	1.8166	1282.5	1417.0	1.6849	1.6080	1280.8	1414.7	1.6706
900	2.2671	1325.3	1472.2	1.7414	1.9777	1324.0	1470.4	1.7257	1.7526	1322.7	1468.6	1.7117
1000	2.4464	1366.9	1525.3	1.7791	2.1358	1365.8	1523.9	1.7636	1.8942	1364.7	1522.4	1.7499
1200	2.7996	1451.7	1633.0	1.8483	2.4465	1450.9	1632.0	1.8331	2.1718	1450.1	1631.0	1.8196
1400	3.1484	1539.6	1743.5	1.9111	2.7527	1539.0	1742.7	1.8960	2.4450	1538.4	1742.0	1.8827
1600	3.4947	1630.8	1857.1	1.9691	3.0565	1630.3	1856.5	1.9541	2.7157	1629.8	1856.0	1.9409
1800	3.8394	1725.4	1974.0	2.0233	3.3586	1725.0	1973.6	2.0084	2.9847	1724.6	1973.2	1.9952
2000	4.1830	1823.3	2094.2	2.0742	3.6597	1823.0	2093.9	2.0594	3.2527	1822.6	2093.5	2.0462
	$P = 500$ psia (467.04°F)				$P = 600$ psia (486.24°F)				$P = 700$ psia (503.13°F)			
Sat.	0.92815	1119.1	1205.0	1.4642	0.77020	1118.3	1203.9	1.4463	0.65589	1116.9	1201.9	1.4305
500	0.99304	1140.1	1231.9	1.4928	0.79526	1128.2	1216.5	1.4596				
550	1.07974	1167.1	1267.0	1.5284	0.87542	1158.7	1255.9	1.4996	0.72799	1149.5	1243.8	1.4730
600	1.15876	1191.4	1298.6	1.5590	0.94605	1184.9	1289.9	1.5325	0.79332	1177.9	1280.7	1.5087
650	1.23312	1214.3	1328.4	1.5865	1.01133	1209.0	1321.3	1.5614	0.85242	1203.4	1313.8	1.5393
700	1.30440	1236.4	1357.0	1.6117	1.07316	1231.9	1351.0	1.5877	0.90769	1227.2	1344.8	1.5666
800	1.44097	1279.2	1412.5	1.6576	1.19038	1275.8	1408.0	1.6348	1.01125	1272.4	1403.4	1.6150
900	1.57252	1321.4	1466.9	1.6992	1.30230	1318.7	1463.3	1.6771	1.10921	1316.0	1459.7	1.6581
1000	1.70094	1363.6	1521.0	1.7376	1.41097	1361.4	1518.1	1.7160	1.20381	1359.2	1515.2	1.6974
1100	1.82726	1406.2	1575.3	1.7735	1.51749	1404.4	1572.9	1.7522	1.29621	1402.5	1570.4	1.7341
1200	1.95211	1449.4	1630.0	1.8075	1.62252	1447.8	1627.9	1.7865	1.38709	1446.2	1625.9	1.7685
1400	2.1988	1537.8	1741.2	1.8708	1.82957	1536.6	1739.7	1.8501	1.56580	1535.4	1738.2	1.8324
1600	2.4430	1629.4	1855.4	1.9291	2.0340	1628.4	1854.2	1.9085	1.74192	1627.5	1853.1	1.8911
1800	2.6856	1724.2	1972.7	1.9834	2.2369	1723.4	1971.8	1.9630	1.91643	1722.7	1970.9	1.9457
2000	2.9271	1822.3	2093.1	2.0345	2.4387	1821.7	2092.4	2.0141	2.08987	1821.0	2091.7	1.9969
	$P = 800$ psia (518.27°F)				$P = 1000$ psia (544.65°F)				$P = 1250$ psia (572.45°F)			
Sat.	0.56920	1115.0	1199.3	1.4162	0.44604	1110.1	1192.6	1.3906	0.34549	1102.0	1181.9	1.3623
550	0.61586	1139.4	1230.5	1.4476	0.45375	1115.2	1199.2	1.3972				
600	0.67799	1170.5	1270.9	1.4866	0.51431	1154.1	1249.3	1.4457	0.37894	1129.5	1217.2	1.3961
650	0.73279	1197.6	1306.0	1.5191	0.56411	1185.1	1289.5	1.4827	0.42703	1167.5	1266.3	1.4414
700	0.78330	1222.4	1338.4	1.5476	0.60844	1212.4	1325.0	1.5140	0.46735	1198.7	1306.8	1.4771
750	0.83102	1246.0	1369.1	1.5735	0.64944	1237.6	1357.8	1.5418	0.50344	1226.4	1342.9	1.5076
800	0.87678	1268.9	1398.7	1.5975	0.68821	1261.7	1389.0	1.5670	0.53687	1252.2	1376.4	1.5347
900	0.96434	1313.3	1456.0	1.6413	0.76136	1307.7	1448.6	1.6126	0.59876	1300.5	1439.0	1.5826
1000	1.04841	1357.0	1512.2	1.6812	0.83078	1352.5	1506.2	1.6535	0.65656	1346.7	1498.6	1.6249
1100	1.13024	1400.7	1568.0	1.7181	0.89783	1396.9	1563.1	1.6911	0.71184	1392.2	1556.8	1.6635
1200	1.21051	1444.6	1623.8	1.7528	0.96327	1441.4	1619.7	1.7263	0.76545	1437.4	1614.5	1.6993
1400	1.36797	1534.2	1736.7	1.8170	1.09101	1531.8	1733.7	1.7911	0.86944	1528.7	1729.8	1.7649
1600	1.52283	1626.5	1851.9	1.8759	1.21610	1624.6	1849.6	1.8504	0.97072	1622.2	1846.7	1.8246
1800	1.67606	1721.9	1970.0	1.9306	1.33956	1720.3	1968.2	1.9053	1.07036	1718.4	1966.0	1.8799
2000	1.82823	1820.4	2091.0	1.9819	1.46194	1819.1	2089.6	1.9568	1.16892	1817.5	2087.9	1.9315

TABLE A–6E

Superheated water *(Concluded)*

T °F	*v* ft^3/lbm	*u* Btu/lbm	*h* Btu/lbm	*s* Btu/lbm·R	*v* ft^3/lbm	*u* Btu/lbm	*h* Btu/lbm	*s* Btu/lbm·R	*v* ft^3/lbm	*u* Btu/lbm	*h* Btu/lbm	*s* Btu/lbm·R
	P = 1500 psia (596.26°F)				*P* = 1750 psia (617.17°F)				*P* = 2000 psia (635.85°F)			
Sat.	0.27695	1092.1	1169.0	1.3362	0.22681	1080.5	1153.9	1.3112	0.18815	1066.8	1136.4	1.2863
600	0.28189	1097.2	1175.4	1.3423								
650	0.33310	1147.2	1239.7	1.4016	0.26292	1122.8	1207.9	1.3607	0.20586	1091.4	1167.6	1.3146
700	0.37198	1183.6	1286.9	1.4433	0.30252	1166.8	1264.7	1.4108	0.24894	1147.6	1239.8	1.3783
750	0.40535	1214.4	1326.9	1.4771	0.33455	1201.5	1309.8	1.4489	0.28074	1187.4	1291.3	1.4218
800	0.43550	1242.2	1363.1	1.5064	0.36266	1231.7	1349.1	1.4807	0.30763	1220.5	1334.3	1.4567
850	0.46356	1268.2	1396.9	1.5328	0.38835	1259.3	1385.1	1.5088	0.33169	1250.0	1372.8	1.4867
900	0.49015	1293.1	1429.2	1.5569	0.41238	1285.4	1419.0	1.5341	0.35390	1277.5	1408.5	1.5134
1000	0.54031	1340.9	1490.8	1.6007	0.45719	1334.9	1482.9	1.5796	0.39479	1328.7	1474.9	1.5606
1100	0.58781	1387.3	1550.5	1.6402	0.49917	1382.4	1544.1	1.6201	0.43266	1377.5	1537.6	1.6021
1200	0.63355	1433.3	1609.2	1.6767	0.53932	1429.2	1603.9	1.6572	0.46864	1425.1	1598.5	1.6400
1400	0.72172	1525.7	1726.0	1.7432	0.61621	1522.6	1722.1	1.7245	0.53708	1519.5	1718.3	1.7081
1600	0.80714	1619.8	1843.8	1.8033	0.69031	1617.4	1840.9	1.7852	0.60269	1615.0	1838.0	1.7693
1800	0.89090	1716.4	1963.7	1.8589	0.76273	1714.5	1961.5	1.8410	0.66660	1712.5	1959.2	1.8255
2000	0.97358	1815.9	2086.1	1.9108	0.83406	1814.2	2084.3	1.8931	0.72942	1812.6	2082.6	1.8778
	P = 2500 psia (668.17°F)				*P* = 3000 psia (695.41°F)				*P* = 3500 psia			
Sat.	0.13076	1031.2	1091.7	1.2330	0.08460	969.8	1016.8	1.1587				
650									0.02492	663.7	679.9	0.8632
700	0.16849	1098.4	1176.3	1.3072	0.09838	1005.3	1059.9	1.1960	0.03065	760.0	779.9	0.9511
750	0.20327	1154.9	1249.0	1.3686	0.14840	1114.1	1196.5	1.3118	0.10460	1057.6	1125.4	1.2434
800	0.22949	1195.9	1302.0	1.4116	0.17601	1167.5	1265.3	1.3676	0.13639	1134.3	1222.6	1.3224
850	0.25174	1230.1	1346.6	1.4463	0.19771	1208.2	1317.9	1.4086	0.15847	1183.8	1286.5	1.3721
900	0.27165	1260.7	1386.4	1.4761	0.21640	1242.8	1362.9	1.4423	0.17659	1223.4	1337.8	1.4106
950	0.29001	1289.1	1423.3	1.5028	0.23321	1273.9	1403.3	1.4716	0.19245	1257.8	1382.4	1.4428
1000	0.30726	1316.1	1458.2	1.5271	0.24876	1302.8	1440.9	1.4978	0.20687	1289.0	1423.0	1.4711
1100	0.33949	1367.3	1524.4	1.5710	0.27732	1356.8	1510.8	1.5441	0.23289	1346.1	1496.9	1.5201
1200	0.36966	1416.6	1587.6	1.6103	0.30367	1408.0	1576.6	1.5850	0.25654	1399.3	1565.4	1.5627
1400	0.42631	1513.3	1710.5	1.6802	0.35249	1507.0	1702.7	1.6567	0.29978	1500.7	1694.8	1.6364
1600	0.48004	1610.1	1832.2	1.7424	0.39830	1605.3	1826.4	1.7199	0.33994	1600.4	1820.5	1.7006
1800	0.53205	1708.6	1954.8	1.7991	0.44237	1704.7	1950.3	1.7773	0.37833	1700.8	1945.8	1.7586
2000	0.58295	1809.4	2079.1	1.8518	0.48532	1806.1	2075.6	1.8304	0.41561	1802.9	2072.1	1.8121
	P = 4000 psia				*P* = 5000 psia				*P* = 6000 psia			
650	0.02448	657.9	676.1	0.8577	0.02379	648.3	670.3	0.8485	0.02325	640.3	666.1	0.8408
700	0.02871	742.3	763.6	0.9347	0.02678	721.8	746.6	0.9156	0.02564	708.1	736.5	0.9028
750	0.06370	962.1	1009.2	1.1410	0.03373	821.8	853.0	1.0054	0.02981	788.7	821.8	0.9747
800	0.10520	1094.2	1172.1	1.2734	0.05937	986.9	1041.8	1.1581	0.03949	897.1	941.0	1.0711
850	0.12848	1156.7	1251.8	1.3355	0.08551	1092.4	1171.5	1.2593	0.05815	1018.6	1083.1	1.1819
900	0.14647	1202.5	1310.9	1.3799	0.10390	1155.9	1252.1	1.3198	0.07584	1103.5	1187.7	1.2603
950	0.16176	1240.7	1360.5	1.4157	0.11863	1203.9	1313.6	1.3643	0.09010	1163.7	1263.7	1.3153
1000	0.17538	1274.6	1404.4	1.4463	0.13128	1244.0	1365.5	1.4004	0.10208	1211.4	1324.7	1.3578
1100	0.19957	1335.1	1482.8	1.4983	0.15298	1312.2	1453.8	1.4590	0.12211	1288.4	1424.0	1.4237
1200	0.22121	1390.3	1554.1	1.5426	0.17185	1372.1	1531.1	1.5070	0.13911	1353.4	1507.8	1.4758
1300	0.24128	1443.0	1621.6	1.5821	0.18902	1427.8	1602.7	1.5490	0.15434	1412.5	1583.8	1.5203
1400	0.26028	1494.3	1687.0	1.6182	0.20508	1481.4	1671.1	1.5868	0.16841	1468.4	1655.4	1.5598
1600	0.29620	1595.5	1814.7	1.6835	0.23505	1585.6	1803.1	1.6542	0.19438	1575.7	1791.5	1.6294
1800	0.33033	1696.8	1941.4	1.7422	0.26320	1689.0	1932.5	1.7142	0.21853	1681.1	1923.7	1.6907
2000	0.36335	1799.7	2068.6	1.7961	0.29023	1793.2	2061.7	1.7689	0.24155	1786.7	2054.9	1.7463

TABLE A–7E

Compressed liquid water

T °F	υ ft^3/lbm	*u* Btu/lbm	*h* Btu/lbm	*s* Btu/lbm·R	υ ft^3/lbm	*u* Btu/lbm	*h* Btu/lbm	*s* Btu/lbm·R	υ ft^3/lbm	*u* Btu/lbm	*h* Btu/lbm	*s* Btu/lbm·R
	P = 500 psia (467.04°F)				P = 1000 psia (544.65°F)				P = 1500 psia (596.26°F)			
Sat.	0.019750	447.68	449.51	0.64900	0.021595	538.58	542.57	0.74341	0.023456	605.07	611.58	0.80836
32	0.015994	0.01	1.49	0.00001	0.015966	0.03	2.99	0.00005	0.015939	0.05	4.48	0.00008
50	0.015998	18.03	19.51	0.03601	0.015972	17.99	20.95	0.03593	0.015946	17.95	22.38	0.03584
100	0.016107	67.86	69.35	0.12930	0.016083	67.69	70.67	0.12899	0.016059	67.53	71.98	0.12869
150	0.016317	117.70	119.21	0.21462	0.016292	117.42	120.43	0.21416	0.016267	117.14	121.66	0.21369
200	0.016607	167.70	169.24	0.29349	0.016580	167.31	170.38	0.29289	0.016553	166.92	171.52	0.29229
250	0.016972	218.04	219.61	0.36708	0.016941	217.51	220.65	0.36634	0.016911	217.00	221.69	0.36560
300	0.017417	268.92	270.53	0.43641	0.017380	268.24	271.46	0.43551	0.017345	267.57	272.39	0.43463
350	0.017954	320.64	322.30	0.50240	0.017910	319.77	323.08	0.50132	0.017866	318.91	323.87	0.50025
400	0.018609	373.61	375.33	0.56595	0.018552	372.48	375.91	0.56463	0.018496	371.37	376.51	0.56333
450	0.019425	428.44	430.24	0.62802	0.019347	426.93	430.51	0.62635	0.019271	425.47	430.82	0.62472
500					0.020368	484.03	487.80	0.68764	0.020258	482.01	487.63	0.68550
550									0.021595	542.50	548.50	0.74731
	P = 2000 psia (635.85°F)				P = 3000 psia (695.41°F)				P = 5000 psia			
Sat.	0.025634	662.33	671.82	0.86224	0.034335	783.39	802.45	0.97321				
32	0.015912	0.07	5.96	0.00010	0.015859	0.10	8.90	0.00011	0.015756	0.13	14.71	0.00002
50	0.015921	17.91	23.80	0.03574	0.015870	17.83	26.64	0.03554	0.015773	17.65	32.25	0.03505
100	0.016035	67.36	73.30	0.12838	0.015988	67.04	75.91	0.12776	0.015897	66.41	81.12	0.12652
200	0.016527	166.54	172.66	0.29170	0.016475	165.79	174.94	0.29053	0.016375	164.36	179.51	0.28824
300	0.017310	266.92	273.33	0.43376	0.017242	265.65	275.22	0.43204	0.017112	263.24	279.07	0.42874
400	0.018442	370.30	377.12	0.56205	0.018338	368.22	378.41	0.55959	0.018145	364.35	381.14	0.55492
450	0.019199	424.06	431.16	0.62314	0.019062	421.36	431.94	0.62010	0.018812	416.40	433.80	0.61445
500	0.020154	480.08	487.54	0.68346	0.019960	476.45	487.53	0.67958	0.019620	469.94	488.10	0.67254
560	0.021739	552.21	560.26	0.75692	0.021405	546.59	558.47	0.75126	0.020862	537.08	556.38	0.74154
600	0.023317	605.77	614.40	0.80898	0.022759	597.42	610.06	0.80086	0.021943	584.42	604.72	0.78803
640					0.024765	654.52	668.27	0.85476	0.023358	634.95	656.56	0.83603
680					0.028821	728.63	744.64	0.92288	0.025366	690.67	714.14	0.88745
700									0.026777	721.78	746.56	0.91564

TABLE A–8E

Saturated ice–water vapor

		Specific volume, ft^3/lbm		*Internal energy,* Btu/lbm			*Enthalpy,* Btu/lbm			*Entropy,* Btu/lbm·R		
Temp., T °F	Sat. press., P_{sat} psia	Sat. ice, v_i	Sat. vapor, v_g	Sat. ice, u_i	Subl., u_{ig}	Sat. vapor, u_g	Sat. ice, h_i	Subl., h_{ig}	Sat. vapor, h_g	Sat. ice, s_i	Subl., s_{ig}	Sat. vapor, s_g
32.018	0.08871	0.01747	3299.6	−143.34	1164.2	1020.9	−143.34	1218.3	1075.0	−0.29146	2.4779	2.1864
32	0.08864	0.01747	3302.6	−143.35	1164.2	1020.9	−143.35	1218.4	1075.0	−0.29148	2.4779	2.1865
30	0.08086	0.01747	3605.8	−144.35	1164.6	1020.2	−144.35	1218.5	1074.2	−0.29353	2.4883	2.1948
25	0.06405	0.01746	4505.8	−146.85	1165.4	1018.6	−146.85	1218.8	1072.0	−0.29865	2.5146	2.2160
20	0.05049	0.01746	5657.6	−149.32	1166.2	1016.9	−149.32	1219.1	1069.8	−0.30377	2.5414	2.2376
15	0.03960	0.01745	7138.9	−151.76	1167.0	1015.2	−151.76	1219.3	1067.6	−0.30889	2.5687	2.2598
10	0.03089	0.01744	9054.0	−154.18	1167.8	1013.6	−154.18	1219.5	1065.4	−0.31401	2.5965	2.2825
5	0.02397	0.01743	11,543	−156.57	1168.5	1011.9	−156.57	1219.7	1063.1	−0.31913	2.6248	2.3057
0	0.01850	0.01743	14,797	−158.94	1169.2	1010.3	−158.94	1219.9	1060.9	−0.32426	2.6537	2.3295
−5	0.01420	0.01742	19,075	−161.28	1169.9	1008.6	−161.28	1220.0	1058.7	−0.32938	2.6832	2.3538
−10	0.01083	0.01741	24,731	−163.60	1170.6	1007.0	−163.60	1220.1	1056.5	−0.33451	2.7133	2.3788
−15	0.00821	0.01740	32,257	−165.90	1171.2	1005.3	−165.90	1220.2	1054.3	−0.33964	2.7440	2.4044
−20	0.00619	0.01740	42,335	−168.16	1171.8	1003.6	−168.16	1220.3	1052.1	−0.34478	2.7754	2.4306
−25	0.00463	0.01739	55,917	−170.41	1172.4	1002.0	−170.41	1220.3	1049.9	−0.34991	2.8074	2.4575
−30	0.00344	0.01738	74,345	−172.63	1173.0	1000.3	−172.63	1220.3	1047.7	−0.35505	2.8401	2.4850
−35	0.00254	0.01738	99,526	−174.83	1173.5	998.7	−174.83	1220.3	1045.5	−0.36019	2.8735	2.5133
−40	0.00186	0.01737	134,182	−177.00	1174.0	997.0	−177.00	1220.3	1043.3	−0.36534	2.9076	2.5423

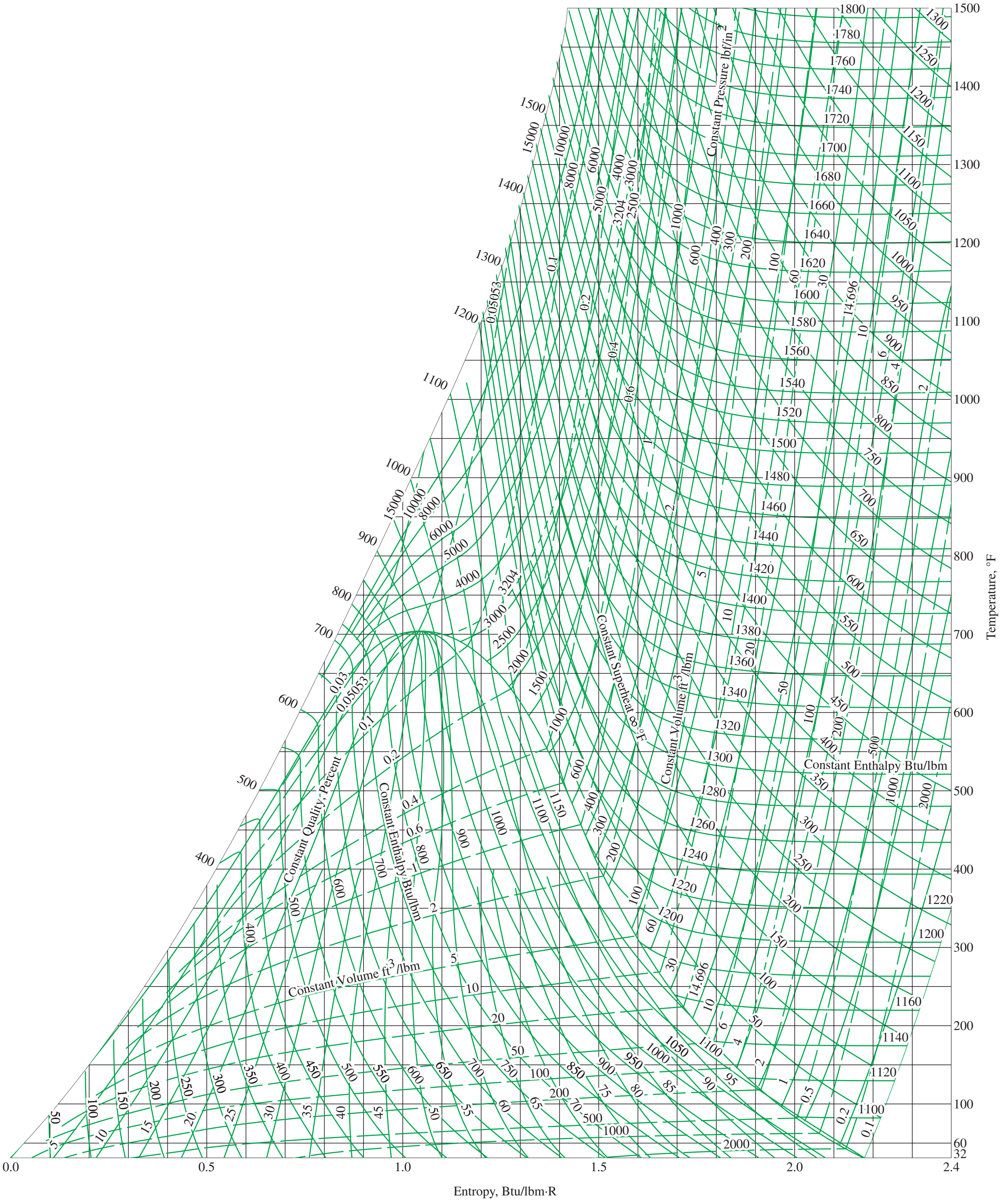

FIGURE A–9E

T-s diagram for water.

Source of Data: Joseph H. Keenan, Frederick G. Keyes, Philip G. Hill, and Joan G. Moore. Steam Tables (*New York: John Wiley & Sons, 1969*).

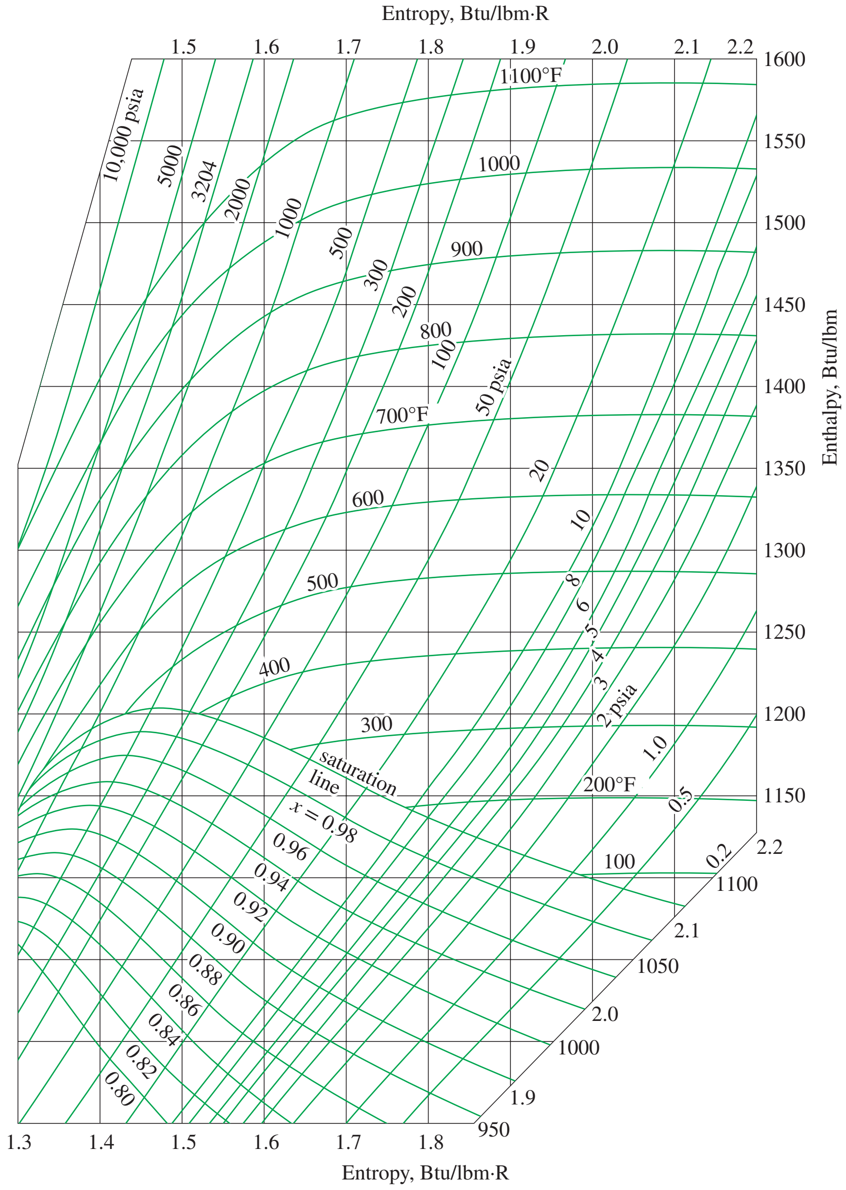

FIGURE A–10E

Mollier diagram for water.

Source of Data: Joseph H. Keenan, Frederick G. Keyes, Philip G. Hill, and Joan G. Moore. Steam Tables (*New York: John Wiley & Sons, 1969*).

TABLE A–11E

Saturated refrigerant-134a–Temperature table

		Specific volume, ft^3/lbm		*Internal energy,* Btu/lbm			*Enthalpy,* Btu/lbm			*Entropy,* Btu/lbm·R		
Temp., T °F	Sat. press., P_{sat} psia	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
−40	7.432	0.01130	5.7769	−0.016	89.174	89.16	0.000	97.104	97.10	0.00000	0.23136	0.23136
−35	8.581	0.01136	5.0489	1.483	88.360	89.84	1.501	96.360	97.86	0.00355	0.22689	0.23044
−30	9.869	0.01143	4.4286	2.987	87.542	90.53	3.008	95.608	98.62	0.00707	0.22250	0.22957
−25	11.306	0.01149	3.8980	4.497	86.717	91.21	4.522	94.849	99.37	0.01057	0.21819	0.22876
−20	12.906	0.01156	3.4424	6.014	85.887	91.90	6.041	94.080	100.12	0.01404	0.21396	0.22800
−15	14.680	0.01163	3.0495	7.536	85.050	92.59	7.568	93.303	100.87	0.01748	0.20981	0.22729
−10	16.642	0.01170	2.7097	9.065	84.206	93.27	9.102	92.515	101.62	0.02090	0.20572	0.22662
−5	18.806	0.01178	2.4146	10.601	83.355	93.96	10.642	91.717	102.36	0.02430	0.20171	0.22600
0	21.185	0.01185	2.1575	12.143	82.496	94.64	12.190	90.907	103.10	0.02767	0.19775	0.22542
5	23.793	0.01193	1.9328	13.693	81.628	95.32	13.745	90.085	103.83	0.03103	0.19385	0.22488
10	26.646	0.01200	1.7358	15.249	80.751	96.00	15.308	89.251	104.56	0.03436	0.19001	0.22437
15	29.759	0.01208	1.5625	16.813	79.865	96.68	16.879	88.403	105.28	0.03767	0.18623	0.22390
20	33.147	0.01216	1.4097	18.384	78.969	97.35	18.459	87.541	106.00	0.04097	0.18249	0.22345
25	36.826	0.01225	1.2746	19.963	78.062	98.03	20.047	86.665	106.71	0.04424	0.17880	0.22304
30	40.813	0.01233	1.1548	21.550	77.144	98.69	21.643	85.772	107.42	0.04750	0.17515	0.22265
35	45.124	0.01242	1.0482	23.145	76.214	99.36	23.249	84.863	108.11	0.05074	0.17154	0.22228
40	49.776	0.01251	0.95323	24.749	75.272	100.02	24.864	83.937	108.80	0.05397	0.16797	0.22194
45	54.787	0.01261	0.86837	26.361	74.317	100.68	26.489	82.993	109.48	0.05718	0.16443	0.22162
50	60.175	0.01270	0.79236	27.983	73.347	101.33	28.124	82.029	110.15	0.06038	0.16093	0.22131
55	65.957	0.01280	0.72414	29.614	72.363	101.98	29.770	81.046	110.82	0.06357	0.15746	0.22103
60	72.152	0.01290	0.66277	31.254	71.364	102.62	31.426	80.041	111.47	0.06674	0.15401	0.22075
65	78.780	0.01301	0.60744	32.904	70.348	103.25	33.094	79.014	112.11	0.06991	0.15058	0.22049
70	85.858	0.01311	0.55746	34.565	69.315	103.88	34.773	77.964	112.74	0.07306	0.14718	0.22024
75	93.408	0.01323	0.51222	36.237	68.264	104.50	36.465	76.889	113.35	0.07621	0.14379	0.22000
80	101.45	0.01334	0.47119	37.920	67.193	105.11	38.170	75.788	113.96	0.07934	0.14042	0.21976
85	110.00	0.01346	0.43391	39.614	66.102	105.72	39.888	74.660	114.55	0.08247	0.13706	0.21953
90	119.08	0.01359	0.39997	41.321	64.989	106.31	41.620	73.503	115.12	0.08560	0.13371	0.21931
95	128.72	0.01372	0.36902	43.041	63.852	106.89	43.367	72.315	115.68	0.08872	0.13036	0.21908
100	138.93	0.01386	0.34074	44.774	62.690	107.46	45.130	71.094	116.22	0.09183	0.12702	0.21885
105	149.73	0.01400	0.31486	46.521	61.501	108.02	46.909	69.838	116.75	0.09495	0.12367	0.21862
110	161.16	0.01415	0.29113	48.284	60.284	108.57	48.706	68.544	117.25	0.09806	0.12031	0.21838
115	173.23	0.01430	0.26933	50.063	59.035	109.10	50.521	67.210	117.73	0.10118	0.11694	0.21813
120	185.96	0.01446	0.24928	51.858	57.753	109.61	52.356	65.833	118.19	0.10430	0.11356	0.21786
130	213.53	0.01482	0.21373	55.505	55.075	110.58	56.091	62.935	119.03	0.11056	0.10672	0.21728
140	244.06	0.01522	0.18331	59.237	52.221	111.46	59.925	59.813	119.74	0.11686	0.09973	0.21660
150	277.79	0.01567	0.15707	63.070	49.151	112.22	63.875	56.419	120.29	0.12324	0.09253	0.21577
160	314.94	0.01619	0.13423	67.022	45.811	112.83	67.965	52.690	120.66	0.12971	0.08502	0.21473
170	355.80	0.01682	0.11413	71.139	42.101	113.24	72.246	48.509	120.75	0.13637	0.07703	0.21340
180	400.66	0.01759	0.09619	75.464	37.893	113.36	76.768	43.721	120.49	0.14327	0.06834	0.21161
190	449.90	0.01861	0.07982	80.093	32.929	113.02	81.642	38.025	119.67	0.15057	0.05852	0.20909
200	504.00	0.02010	0.06441	85.297	26.629	111.93	87.172	30.761	117.93	0.15872	0.04662	0.20534
210	563.76	0.02309	0.04722	91.993	16.498	108.49	94.402	19.015	113.42	0.16924	0.02839	0.19763

Source of Data: Tables A-11E through A-13E are generated using the Engineering Equation Solver (EES) software developed by S. A. Klein and F. L. Alvarado. The routine used in calculations is the R134a, which is based on the fundamental equation of state developed by R. Tillner-Roth and H. D. Baehr, "An International Standard Formulation for the Thermodynamic Properties of 1,1,1,2-Tetrafluoroethane (HFC-134a) for temperatures from 170 K to 455 K and pressures up to 70 MPa," *J. Phys. Chem, Ref. Data,* Vol. 23, No. 5, 1994. The enthalpy and entropy values of saturated liquid are set to zero at −40°C (and −40°F).

TABLE A–12E

Saturated refrigerant-134a–Pressure table

		Specific volume, ft^3/lbm		*Internal energy,* Btu/lbm			*Enthalpy,* Btu/lbm			*Entropy,* Btu/lbm·R		
Press., P psia	Sat. temp., T_{sat} °F	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
5	−53.09	0.01113	8.3740	−3.914	91.283	87.37	−3.903	99.021	95.12	−0.00944	0.24353	0.23409
10	−29.52	0.01143	4.3740	3.132	87.463	90.59	3.153	95.536	98.69	0.00741	0.22208	0.22949
15	−14.15	0.01164	2.9882	7.796	84.907	92.70	7.828	93.170	101.00	0.01806	0.20911	0.22717
20	−2.43	0.01181	2.2781	11.393	82.915	94.31	11.436	91.302	102.74	0.02603	0.19967	0.22570
25	7.17	0.01196	1.8442	14.367	81.249	95.62	14.422	89.725	104.15	0.03247	0.19218	0.22465
30	15.37	0.01209	1.5506	16.929	79.799	96.73	16.996	88.340	105.34	0.03792	0.18595	0.22386
35	22.57	0.01221	1.3382	19.195	78.504	97.70	19.274	87.093	106.37	0.04265	0.18058	0.22324
40	29.01	0.01232	1.1773	21.236	77.326	98.56	21.327	85.950	107.28	0.04686	0.17586	0.22272
45	34.86	0.01242	1.0510	23.101	76.240	99.34	23.205	84.889	108.09	0.05065	0.17164	0.22229
50	40.23	0.01252	0.94909	24.824	75.228	100.05	24.939	83.894	108.83	0.05412	0.16780	0.22192
55	45.20	0.01261	0.86509	26.428	74.277	100.70	26.556	82.954	109.51	0.05732	0.16429	0.22160
60	49.84	0.01270	0.79462	27.932	73.378	101.31	28.073	82.060	110.13	0.06028	0.16104	0.22132
65	54.20	0.01278	0.73462	29.351	72.523	101.87	29.505	81.205	110.71	0.06306	0.15801	0.22107
70	58.30	0.01287	0.68290	30.696	71.705	102.40	30.862	80.385	111.25	0.06567	0.15518	0.22084
75	62.19	0.01295	0.63784	31.975	70.921	102.90	32.155	79.594	111.75	0.06813	0.15251	0.22064
80	65.89	0.01303	0.59822	33.198	70.167	103.36	33.391	78.830	112.22	0.07047	0.14998	0.22045
85	69.41	0.01310	0.56309	34.369	69.438	103.81	34.575	78.089	112.66	0.07269	0.14758	0.22027
90	72.78	0.01318	0.53173	35.494	68.733	104.23	35.713	77.369	113.08	0.07481	0.14529	0.22011
95	76.02	0.01325	0.50356	36.577	68.048	104.63	36.810	76.668	113.48	0.07684	0.14311	0.21995
100	79.12	0.01332	0.47811	37.623	67.383	105.01	37.870	75.984	113.85	0.07879	0.14101	0.21981
110	85.00	0.01346	0.43390	39.614	66.102	105.72	39.888	74.660	114.55	0.08247	0.13706	0.21953
120	90.49	0.01360	0.39681	41.489	64.878	106.37	41.791	73.388	115.18	0.08590	0.13338	0.21928
130	95.64	0.01374	0.36523	43.263	63.704	106.97	43.594	72.159	115.75	0.08912	0.12993	0.21905
140	100.51	0.01387	0.33800	44.951	62.570	107.52	45.311	70.967	116.28	0.09215	0.12668	0.21883
150	105.12	0.01400	0.31426	46.563	61.473	108.04	46.952	69.807	116.76	0.09502	0.12359	0.21861
160	109.50	0.01413	0.29339	48.109	60.406	108.51	48.527	68.674	117.20	0.09776	0.12064	0.21840
170	113.69	0.01426	0.27487	49.595	59.366	108.96	50.043	67.564	117.61	0.10036	0.11783	0.21819
180	117.69	0.01439	0.25833	51.027	58.349	109.38	51.507	66.475	117.98	0.10286	0.11513	0.21799
190	121.53	0.01452	0.24346	52.412	57.353	109.76	52.922	65.402	118.32	0.10526	0.11252	0.21778
200	125.22	0.01464	0.23001	53.753	56.375	110.13	54.295	64.345	118.64	0.10757	0.11000	0.21757
220	132.21	0.01490	0.20662	56.321	54.462	110.78	56.927	62.267	119.19	0.11195	0.10519	0.21714
240	138.73	0.01516	0.18694	58.757	52.596	111.35	59.430	60.225	119.65	0.11606	0.10063	0.21669
260	144.85	0.01543	0.17012	61.082	50.763	111.84	61.824	58.205	120.03	0.11994	0.09627	0.21622
280	150.62	0.01570	0.15555	63.313	48.951	112.26	64.126	56.197	120.32	0.12364	0.09207	0.21571
300	156.09	0.01598	0.14279	65.460	47.154	112.61	66.347	54.195	120.54	0.12717	0.08800	0.21517
350	168.64	0.01672	0.11673	70.567	42.632	113.20	71.651	49.109	120.76	0.13545	0.07815	0.21360
400	179.86	0.01758	0.09643	75.401	37.957	113.36	76.702	43.794	120.50	0.14317	0.06847	0.21164
450	190.02	0.01860	0.07979	80.112	32.909	113.02	81.662	38.003	119.67	0.15060	0.05849	0.20909
500	199.29	0.01997	0.06533	84.900	27.096	112.00	86.748	31.292	118.04	0.15810	0.04748	0.20558

TABLE A–13E

Superheated refrigerant-134a

T °F	v ft^3/lbm	u Btu/lbm	h Btu/lbm	s Btu/lbm·R	v ft^3/lbm	u Btu/lbm	h Btu/lbm	s Btu/lbm·R	v ft^3/lbm	u Btu/lbm	h Btu/lbm	s Btu/lbm·R
	$P = 10$ psia ($T_{sat} = -29.52$°F)				$P = 15$ psia ($T_{sat} = -14.15$°F)				$P = 20$ psia ($T_{sat} = -2.43$°F)			
Sat.	4.3740	90.59	98.69	0.22949	2.9882	92.70	101.00	0.22717	2.2781	94.31	102.74	0.22570
−20	4.4856	92.14	100.44	0.23351								
0	4.7135	95.42	104.14	0.24175	3.1001	95.08	103.69	0.23312	2.2922	94.73	103.21	0.22673
20	4.9380	98.77	107.91	0.24978	3.2551	98.49	107.52	0.24129	2.4130	98.19	107.12	0.23506
40	5.1600	102.21	111.76	0.25763	3.4074	101.96	111.42	0.24924	2.5306	101.71	111.07	0.24313
60	5.3802	105.73	115.68	0.26533	3.5577	105.51	115.38	0.25702	2.6461	105.29	115.08	0.25099
80	5.5989	109.33	119.69	0.27290	3.7064	109.14	119.42	0.26465	2.7600	108.94	119.15	0.25868
100	5.8165	113.02	123.78	0.28035	3.8540	112.85	123.54	0.27214	2.8726	112.67	123.30	0.26623
120	6.0331	116.80	127.96	0.28768	4.0006	116.64	127.75	0.27952	2.9842	116.48	127.53	0.27364
140	6.2490	120.66	132.23	0.29492	4.1464	120.52	132.03	0.28678	3.0950	120.38	131.83	0.28094
160	6.4642	124.62	136.58	0.30205	4.2915	124.49	136.40	0.29395	3.2051	124.35	136.22	0.28814
180	6.6789	128.66	141.01	0.30910	4.4361	128.53	140.85	0.30102	3.3146	128.41	140.68	0.29523
200	6.8930	132.78	145.54	0.31606	4.5802	132.67	145.38	0.30800	3.4237	132.56	145.23	0.30223
220	7.1068	136.99	150.14	0.32293	4.7239	136.89	150.00	0.31489	3.5324	136.78	149.86	0.30914
	$P = 30$ psia ($T_{sat} = 15.37$°F)				$P = 40$ psia ($T_{sat} = 29.01$°F)				$P = 50$ psia ($T_{sat} = 40.23$°F)			
Sat.	1.5506	96.73	105.34	0.22386	1.1773	98.56	107.28	0.22272	0.9491	100.05	108.83	0.22192
20	1.5691	97.56	106.27	0.22583								
40	1.6528	101.18	110.35	0.23416	1.2126	100.61	109.59	0.22740				
60	1.7338	104.83	114.45	0.24220	1.2768	104.35	113.80	0.23567	1.0019	103.85	113.12	0.23033
80	1.8130	108.54	118.60	0.25003	1.3389	108.12	118.03	0.24365	1.0540	107.69	117.44	0.23849
100	1.8908	112.31	122.81	0.25769	1.3995	111.94	122.30	0.25142	1.1043	111.56	121.78	0.24639
120	1.9675	116.16	127.08	0.26519	1.4588	115.83	126.63	0.25902	1.1534	115.49	126.16	0.25408
140	2.0434	120.08	131.43	0.27256	1.5173	119.79	131.02	0.26646	1.2015	119.48	130.60	0.26160
160	2.1185	124.09	135.85	0.27981	1.5750	123.82	135.47	0.27377	1.2488	123.54	135.09	0.26898
180	2.1931	128.17	140.34	0.28695	1.6321	127.92	140.00	0.28096	1.2955	127.67	139.66	0.27622
200	2.2671	132.33	144.92	0.29399	1.6887	132.10	144.60	0.28805	1.3416	131.87	144.29	0.28335
220	2.3408	136.58	149.57	0.30094	1.7449	136.37	149.28	0.29503	1.3873	136.15	148.99	0.29037
240	2.4141	140.90	154.30	0.30780	1.8007	140.70	154.03	0.30192	1.4326	140.51	153.76	0.29730
260	2.4871	145.30	159.11	0.31458	1.8562	145.12	158.86	0.30873	1.4776	144.94	158.61	0.30413
280	2.5598	149.79	164.00	0.32128	1.9114	149.62	163.77	0.31545	1.5223	149.45	163.53	0.31087
	$P = 60$ psia ($T_{sat} = 49.84$°F)				$P = 70$ psia ($T_{sat} = 58.30$°F)				$P = 80$ psia ($T_{sat} = 65.89$°F)			
Sat.	0.7946	101.31	110.13	0.22132	0.6829	102.40	111.25	0.22084	0.5982	103.36	112.22	0.22045
60	0.8179	103.31	112.39	0.22572	0.6857	102.74	111.62	0.22157				
80	0.8636	107.24	116.82	0.23408	0.7271	106.77	116.18	0.23018	0.6243	106.27	115.51	0.22663
100	0.9072	111.17	121.24	0.24212	0.7662	110.77	120.69	0.23838	0.6601	110.35	120.12	0.23501
120	0.9495	115.14	125.69	0.24992	0.8037	114.79	125.20	0.24630	0.6941	114.43	124.70	0.24305
140	0.9908	119.17	130.17	0.25753	0.8401	118.86	129.74	0.25399	0.7270	118.53	129.29	0.25084
160	1.0312	123.26	134.71	0.26497	0.8756	122.98	134.32	0.26151	0.7589	122.69	133.92	0.25843
180	1.0709	127.42	139.31	0.27227	0.9105	127.16	138.95	0.26886	0.7900	126.89	138.59	0.26585
200	1.1101	131.64	143.97	0.27945	0.9447	131.40	143.64	0.27608	0.8206	131.17	143.31	0.27312
220	1.1489	135.94	148.69	0.28651	0.9785	135.72	148.40	0.28318	0.8507	135.50	148.09	0.28026
240	1.1872	140.31	153.49	0.29346	1.0118	140.11	153.22	0.29017	0.8803	139.91	152.94	0.28728
260	1.2252	144.76	158.36	0.30032	1.0449	144.57	158.10	0.29706	0.9096	144.38	157.85	0.29420
280	1.2629	149.28	163.30	0.30709	1.0776	149.10	163.06	0.30386	0.9386	148.93	162.82	0.30102
300	1.3004	153.88	168.31	0.31378	1.1101	153.71	168.09	0.31057	0.9674	153.55	167.87	0.30775
320	1.3377	158.55	173.40	0.32039	1.1424	158.40	173.20	0.31720	0.9959	158.25	172.99	0.31440

TABLE A–13E

Superheated refrigerant-134a *(Concluded)*

T °F	v ft^3/lbm	u Btu/lbm	h Btu/lbm	s Btu/lbm·R	v ft^3/lbm	u Btu/lbm	h Btu/lbm	s Btu/lbm·R	v ft^3/lbm	u Btu/lbm	h Btu/lbm	s Btu/lbm·R
	P = 90 psia (T_{sat} = 72.78°F)				P = 100 psia (T_{sat} = 79.12°F)				P = 120 psia (T_{sat} = 90.49°F)			
Sat.	0.53173	104.23	113.08	0.22011	0.47811	105.01	113.85	0.21981	0.39681	106.37	115.18	0.21928
80	0.54388	105.75	114.81	0.22332	0.47906	105.19	114.06	0.22018				
100	0.57729	109.91	119.53	0.23191	0.51076	109.46	118.91	0.22902	0.41013	108.49	117.59	0.22364
120	0.60874	114.05	124.19	0.24009	0.54022	113.66	123.66	0.23735	0.43692	112.85	122.55	0.23234
140	0.63885	118.20	128.84	0.24799	0.56821	117.86	128.38	0.24535	0.46190	117.16	127.42	0.24059
160	0.66796	122.39	133.51	0.25565	0.59513	122.09	133.10	0.25310	0.48563	121.47	132.25	0.24853
180	0.69629	126.63	138.22	0.26313	0.62122	126.36	137.85	0.26065	0.50844	125.80	137.09	0.25621
200	0.72399	130.92	142.98	0.27045	0.64667	130.68	142.64	0.26802	0.53054	130.18	141.96	0.26370
220	0.75119	135.28	147.79	0.27763	0.67158	135.05	147.48	0.27525	0.55206	134.60	146.86	0.27102
240	0.77796	139.70	152.66	0.28469	0.69605	139.50	152.38	0.28234	0.57312	139.08	151.80	0.27819
260	0.80437	144.19	157.59	0.29164	0.72016	144.00	157.33	0.28932	0.59379	143.62	156.80	0.28523
280	0.83048	148.75	162.58	0.29849	0.74396	148.58	162.34	0.29620	0.61413	148.22	161.86	0.29216
300	0.85633	153.39	167.65	0.30524	0.76749	153.22	167.42	0.30297	0.63420	152.89	166.97	0.29898
320	0.88195	158.09	172.78	0.31191	0.79079	157.94	172.57	0.30966	0.65402	157.62	172.15	0.30571
	P = 140 psia (T_{sat} = 100.51°F)				P = 160 psia (T_{sat} = 109.50°F)				P = 180 psia (T_{sat} = 117.69°F)			
Sat.	0.33800	107.52	116.28	0.21883	0.29339	108.51	117.20	0.21840	0.25833	109.38	117.98	0.21799
120	0.36243	111.97	121.36	0.22775	0.30578	111.01	120.07	0.22339	0.26083	109.95	118.64	0.21912
140	0.38551	116.42	126.40	0.23630	0.32774	115.63	125.33	0.23232	0.28231	114.78	124.18	0.22852
160	0.40711	120.82	131.37	0.24444	0.34790	120.14	130.44	0.24070	0.30154	119.43	129.47	0.23720
180	0.42766	125.23	136.31	0.25229	0.36686	124.63	135.49	0.24872	0.31936	124.01	134.65	0.24542
200	0.44743	129.66	141.25	0.25990	0.38494	129.13	140.52	0.25647	0.33619	128.58	139.77	0.25332
220	0.46657	134.13	146.22	0.26731	0.40234	133.65	145.56	0.26399	0.35228	133.16	144.89	0.26095
240	0.48522	138.65	151.22	0.27457	0.41921	138.21	150.62	0.27133	0.36779	137.76	150.01	0.26838
260	0.50345	143.22	156.26	0.28168	0.43564	142.82	155.72	0.27851	0.38284	142.41	155.16	0.27564
280	0.52134	147.85	161.36	0.28866	0.45171	147.48	160.86	0.28555	0.39751	147.11	160.35	0.28275
300	0.53895	152.55	166.51	0.29553	0.46748	152.21	166.05	0.29248	0.41186	151.86	165.58	0.28972
320	0.55630	157.31	171.72	0.30230	0.48299	156.99	171.29	0.29929	0.42594	156.67	170.85	0.29658
340	0.57345	162.14	176.99	0.30898	0.49828	161.84	176.59	0.30600	0.43980	161.53	176.18	0.30333
360	0.59041	167.03	182.33	0.31557	0.51338	166.75	181.95	0.31262	0.45347	166.47	181.57	0.30998
	P = 200 psia (T_{sat} = 125.22°F)				P = 300 psia (T_{sat} = 156.09°F)				P = 400 psia (T_{sat} = 179.86°F)			
Sat.	0.23001	110.13	118.64	0.21757	0.14279	112.61	120.54	0.21517	0.09643	113.36	120.50	0.21164
140	0.24541	113.86	122.94	0.22483								
160	0.26412	118.67	128.44	0.23386	0.14656	113.82	121.96	0.21747				
180	0.28115	123.36	133.77	0.24231	0.16355	119.53	128.61	0.22803	0.09658	113.42	120.56	0.21174
200	0.29704	128.01	139.00	0.25037	0.17776	124.79	134.66	0.23734	0.11440	120.53	128.99	0.22473
220	0.31212	132.65	144.20	0.25813	0.19044	129.86	140.43	0.24596	0.12746	126.45	135.88	0.23502
240	0.32658	137.31	149.39	0.26566	0.20211	134.83	146.05	0.25412	0.13853	131.96	142.21	0.24420
260	0.34054	141.99	154.60	0.27300	0.21306	139.77	151.60	0.26193	0.14844	137.27	148.26	0.25272
280	0.35410	146.73	159.83	0.28017	0.22347	144.71	157.11	0.26949	0.15756	142.48	154.15	0.26079
300	0.36733	151.50	165.10	0.28720	0.23346	149.66	162.62	0.27683	0.16611	147.65	159.95	0.26853
320	0.38029	156.34	170.41	0.29410	0.24310	154.63	168.13	0.28399	0.17423	152.81	165.71	0.27601
340	0.39300	161.23	175.77	0.30089	0.25246	159.65	173.66	0.29100	0.18201	157.97	171.45	0.28328
360	0.40552	166.18	181.19	0.30758	0.26159	164.71	179.23	0.29788	0.18951	163.16	177.19	0.29037

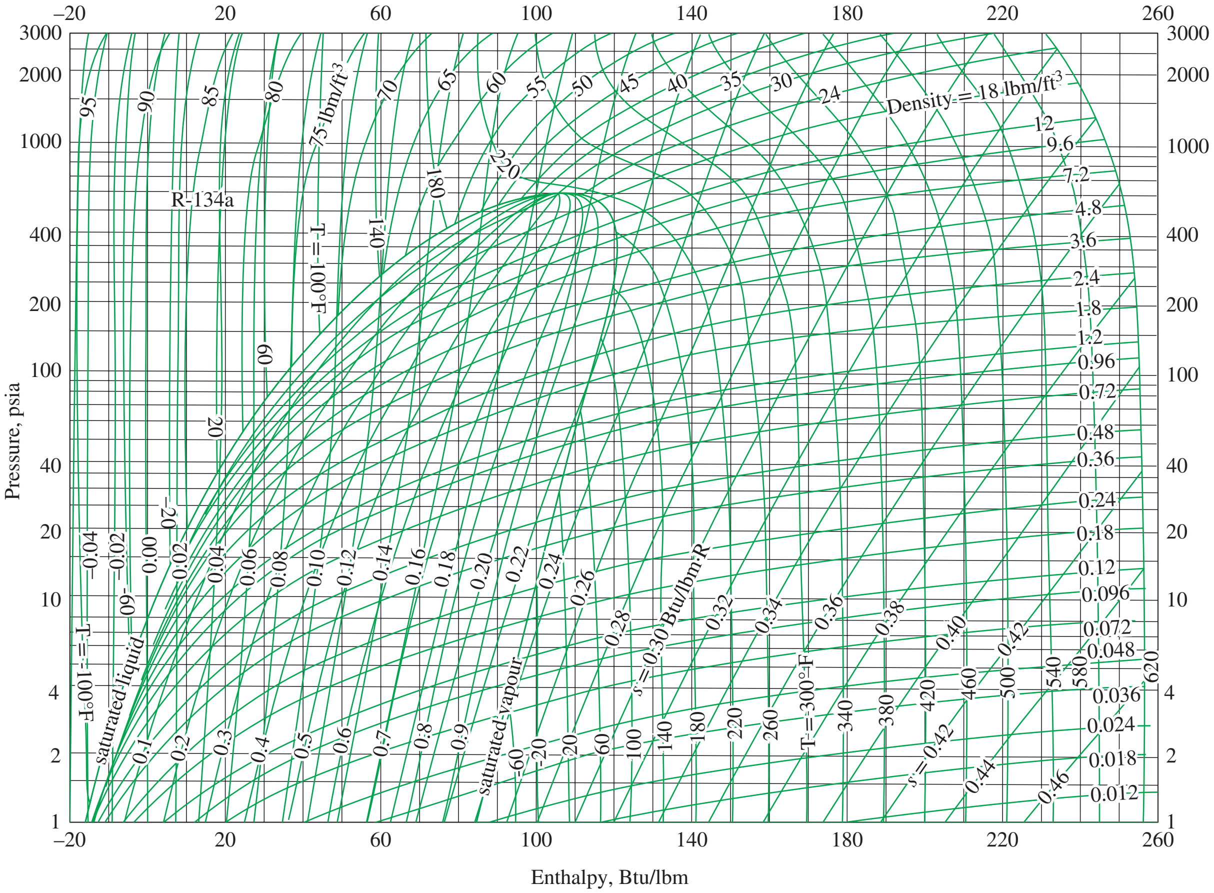

FIGURE A–14E

P-h diagram for refrigerant-134a.

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TABLE A–15E

Properties of saturated water

Temp. T, °F	Saturation pressure P_{sat}, psia	Density ρ, lbm/ft^3		Enthalpy of vaporization h_{fg}, Btu/lbm	Specific heat c_p, Btu/lbm·R		Thermal conductivity k, Btu/h·ft·R		Dynamic viscosity μ, lbm/ft·s		Prandtl number Pr		Volume expansion coefficient β, 1/R
		Liquid	Vapor		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid
32.02	0.0887	62.41	0.00030	1075	1.010	0.446	0.324	0.0099	1.204×10^{-3}	6.194×10^{-6}	13.5	1.00	-0.038×10^{-3}
40	0.1217	62.42	0.00034	1071	1.004	0.447	0.329	0.0100	1.308×10^{-3}	6.278×10^{-6}	11.4	1.01	0.003×10^{-3}
50	0.1780	62.41	0.00059	1065	1.000	0.448	0.335	0.0102	8.781×10^{-4}	6.361×10^{-6}	9.44	1.01	0.047×10^{-3}
60	0.2563	62.36	0.00083	1060	0.999	0.449	0.341	0.0104	7.536×10^{-4}	6.444×10^{-6}	7.95	1.00	0.080×10^{-3}
70	0.3632	62.30	0.00115	1054	0.999	0.450	0.347	0.0106	6.556×10^{-4}	6.556×10^{-6}	6.79	1.00	0.115×10^{-3}
80	0.5073	62.22	0.00158	1048	0.999	0.451	0.352	0.0108	5.764×10^{-4}	6.667×10^{-6}	5.89	1.00	0.145×10^{-3}
90	0.6988	62.12	0.00214	1043	0.999	0.453	0.358	0.0110	5.117×10^{-4}	6.778×10^{-6}	5.14	1.00	0.174×10^{-3}
100	0.9503	62.00	0.00286	1037	0.999	0.454	0.363	0.0112	4.578×10^{-4}	6.889×10^{-6}	4.54	1.01	0.200×10^{-3}
110	1.2763	61.86	0.00377	1031	0.999	0.456	0.367	0.0115	4.128×10^{-4}	7.000×10^{-6}	4.05	1.00	0.224×10^{-3}
120	1.6945	61.71	0.00493	1026	0.999	0.458	0.371	0.0117	3.744×10^{-4}	7.111×10^{-6}	3.63	1.00	0246×10^{-3}
130	2.225	61.55	0.00636	1020	0.999	0.460	0.375	0.0120	3.417×10^{-4}	7.222×10^{-6}	3.28	1.00	0.267×10^{-3}
140	2.892	61.38	0.00814	1014	0.999	0.463	0.378	0.0122	3.136×10^{-4}	7.333×10^{-6}	2.98	1.00	0.287×10^{-3}
150	3.722	61.19	0.0103	1008	1.000	0.465	0.381	0.0125	2.889×10^{-4}	7.472×10^{-6}	2.73	1.00	0.306×10^{-3}
160	4.745	60.99	0.0129	1002	1.000	0.468	0.384	0.0128	2.675×10^{-4}	7.583×10^{-6}	2.51	1.00	0.325×10^{-3}
170	5.996	60.79	0.0161	996	1.001	0.472	0.386	0.0131	2.483×10^{-4}	7.722×10^{-6}	2.90	1.00	0.346×10^{-3}
180	7.515	60.57	0.0199	990	1.002	0.475	0.388	0.0134	2.317×10^{-4}	7.833×10^{-6}	2.15	1.00	0.367×10^{-3}
190	9.343	60.35	0.0244	984	1.004	0.479	0.390	0.0137	2.169×10^{-4}	7.972×10^{-6}	2.01	1.00	0.382×10^{-3}
200	11.53	60.12	0.0297	978	1.005	0.483	0.391	0.0141	2.036×10^{-4}	8.083×10^{-6}	1.88	1.00	0.395×10^{-3}
210	14.125	59.87	0.0359	972	1.007	0.487	0.392	0.0144	1.917×10^{-4}	8.222×10^{-6}	1.77	1.00	0.412×10^{-3}
212	14.698	59.82	0.0373	970	1.007	0.488	0.392	0.0145	1.894×10^{-4}	8.250×10^{-6}	1.75	1.00	0.417×10^{-3}
220	17.19	59.62	0.0432	965	1.009	0.492	0.393	0.0148	1808×10^{-4}	8.333×10^{-6}	1.67	1.00	0.429×10^{-3}
230	20.78	59.36	0.0516	959	1.011	0.497	0.394	0.0152	1.711×10^{-4}	8.472×10^{-6}	1.58	1.00	0.443×10^{-3}
240	24.97	59.09	0.0612	952	1.013	0.503	0.394	0.0156	1.625×10^{-4}	8.611×10^{-6}	1.50	1.00	0.462×10^{-3}
250	29.82	58.82	0.0723	946	1.015	0.509	0.395	0.0160	1.544×10^{-4}	8.611×10^{-6}	1.43	1.00	0.480×10^{-3}
260	35.42	58.53	0.0850	939	1.018	0.516	0.395	0.0164	1.472×10^{-4}	8.861×10^{-6}	1.37	1.00	0.497×10^{-3}
270	41.85	58.24	0.0993	932	1.020	0.523	0.395	0.0168	1.406×10^{-4}	9.000×10^{-6}	1.31	1.01	0.514×10^{-3}
280	49.18	57.94	0.1156	926	1.023	0.530	0.395	0.0172	1.344×10^{-4}	9.111×10^{-6}	1.25	1.01	0.532×10^{-3}
290	57–53	57.63	0.3390	918	1.026	0.538	0.395	0.0177	1.289×10^{-4}	9.250×10^{-6}	1.21	1.01	0.549×10^{-3}
300	66.98	57.31	0.1545	910	1.029	0.547	0.394	0.0182	1.236×10^{-4}	9.389×10^{-6}	1.16	1.02	0.566×10^{-3}
320	89.60	56.65	0.2033	895	1.036	0.567	0.393	0.0191	1.144×10^{-4}	9.639×10^{-6}	1.09	1.03	0.636×10^{-3}
340	117.93	55.95	0.2637	880	1.044	0.590	0.391	0.0202	1.063×10^{-4}	9.889×10^{-6}	1.02	1.04	0.656×10^{-3}
360	152.92	56.22	0.3377	863	1.054	0.617	0.389	0.0213	9.972×10^{-5}	1.013×10^{-5}	0.973	1.06	0.681×10^{-3}
380	195.60	54.46	0.4275	845	1.065	0.647	0.385	0.0224	9.361×10^{-5}	1.041×10^{-5}	0.932	1.08	0.720×10^{-3}
400	241.1	53.65	0.5359	827	1.078	0.683	0.382	0.0237	8.833×10^{-5}	1.066×10^{-5}	0.893	1.11	0.771×10^{-3}
450	422.1	51.46	0.9082	775	1.121	0.799	0.370	0.0271	7.722×10^{-5}	1.130×10^{-5}	0.842	1.20	0.912×10^{-3}
500	680.0	48.95	1.479	715	1.188	0.972	0.352	0.0312	6.833×10^{-5}	1.200×10^{-5}	0.830	1.35	1.111×10^{-3}
550	1046.7	45.96	4.268	641	1.298	1.247	0.329	0.0368	6.083×10^{-5}	1.280×10^{-5}	0.864	1.56	1.445×10^{-3}
600	1541	42.32	3.736	550	1.509	1.759	0.299	0.0461	5.389×10^{-5}	1.380×10^{-5}	0.979	1.90	1.883×10^{-3}
650	2210	37.31	6.152	422	2.086	3.103	0.267	0.0677	4.639×10^{-5}	1.542×10^{-5}	1.30	2.54	
700	3090	27.28	13.44	168	13.80	25.90	0.254	0.1964	3.417×10^{-5}	2.044×10^{-5}	6.68	9.71	
705.44	3204	19.79	19.79	0	∞	∞	∞	∞	2.897×10^{-5}	2.897×10^{-5}			

Note 1: Kinematic viscosity ν and thermal diffusivity α can be calculated from their definitions, $\nu = \mu/\rho$ and $\alpha = k/\rho c_p = \nu/\text{Pr}$. The temperatures 32.02°F, 212°F, and 705.44°F are the triple-, boiling-, and critical-point temperatures of water, respectively. All properties listed above (except the vapor density) can be used at any pressures with negligible error except at temperatures near the critical-point value.

Note 2: The unit Btu/lbm·°F for specific heat is equivalent to Btu/lbm·R, and the unit Btu/h·ft·°F for thermal conductivity is equivalent to Btu/h·ft·R.

Source: Viscosity and thermal conductivity data are from J. V. Sengers and J. T. T. Watson, *Journal of Physical and Chemical Reference Data* 15 (1986), pp. 1291–1322. Other data are obtained from various sources or calculated.

TABLE A–16E

Properties of saturated refrigerant-134a

Temp. T, °F	Saturation pressure P_{sat}, psia	Density ρ, lbm/ft^3		Enthalpy of vaporization h_{fg}, Btu/lbm	Specific heat c_p, Btu/lbm·R		Thermal conductivity k, Btu/h·ft·R		Dynamic viscosity μ, lbm/ft·s		Prandtl number Pr		Volume expansion coefficient β, 1/R	Surface tension lbf/ft
		Liquid	Vapor		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	
−40	7.4	88.51	0.1731	97.1	0.2996	0.1788	0.0636	0.00466	3.278×10^{-4}	1.714×10^{-6}	5.558	0.237	0.00114	0.001206
−30	9.9	87.5	0.2258	95.6	0.3021	0.1829	0.0626	0.00497	3.004×10^{-4}	2.053×10^{-6}	5.226	0.272	0.00117	0.001146
−20	12.9	86.48	0.2905	94.1	0.3046	0.1872	0.0613	0.00529	2.762×10^{-4}	2.433×10^{-6}	4.937	0.310	0.00120	0.001087
−10	16.6	85.44	0.3691	92.5	0.3074	0.1918	0.0602	0.00559	2.546×10^{-4}	2.856×10^{-6}	4.684	0.352	0.00124	0.001029
0	21.2	84.38	0.4635	90.9	0.3103	0.1966	0.0589	0.00589	2.345×10^{-4}	3.314×10^{-6}	4.463	0.398	0.00128	0.000972
10	26.6	83.31	0.5761	89.3	0.3134	0.2017	0.0576	0.00619	2.181×10^{-4}	3.811×10^{-4}	4.269	0.447	0.00132	0.000915
20	33.1	82.2	0.7094	87.5	0.3167	0.2070	0.0563	0.00648	2.024×10^{-4}	4.342×10^{-6}	4.098	0.500	0.00132	0.000859
30	40.8	81.08	0.866	85.8	0.3203	0.2127	0.0550	0.00676	1.883×10^{-4}	4.906×10^{-6}	3.947	0.555	0.00142	0.000803
40	49.8	79.92	1.049	83.9	0.3240	0.2188	0.0536	0.00704	1.752×10^{-4}	5.494×10^{-6}	3.814	0.614	0.00149	0.000749
50	60.2	78.73	1.262	82.0	0.3281	0.2253	0.0522	0.00732	1.633×10^{-4}	6.103×10^{-6}	3.697	0.677	0.00156	0.000695
60	72.2	77.51	1.509	80.0	0.3325	0.2323	0.0507	0.00758	1.522×10^{-4}	6.725×10^{-6}	3.594	0.742	0.00163	0.000642
70	85.9	76.25	1.794	78.0	0.3372	0.2398	0.0492	0.00785	1.420×10^{-4}	7.356×10^{-6}	3.504	0.810	0.00173	0.000590
80	101.4	74.94	2.122	75.8	0.3424	0.2481	0.0476	0.00810	1.324×10^{-4}	7.986×10^{-6}	3.425	0.880	0.00183	0.000538
90	119.1	73.59	2.5	73.5	0.3481	0.2572	0.0460	0.00835	1.234×10^{-4}	8.611×10^{-6}	3.357	0.955	0.00195	0.000488
100	138.9	72.17	2.935	71.1	0.3548	0.2674	0.0444	0.00860	1.149×10^{-4}	9.222×10^{-6}	3.303	1.032	0.00210	0.000439
110	161.2	70.69	3.435	68.5	0.3627	0.2790	0.0427	0.00884	1.068×10^{-4}	9.814×10^{-6}	3.262	1.115	0.00227	0.000391
120	186.0	69.13	4.012	65.8	0.3719	0.2925	0.0410	0.00908	9.911×10^{-5}	1.038×10^{-5}	3.235	1.204	0.00248	0.000344
130	213.5	67.48	4.679	62.9	0.3829	0.3083	0.0392	0.00931	9.175×10^{-5}	1.092×10^{-5}	3.223	1.303	0.00275	0.000299
140	244.1	65.72	5.455	59.8	0.3963	0.3276	0.0374	0.00954	8.464×10^{-5}	1.144×10^{-5}	3.229	1.416	0.00308	0.000255
150	277.8	63.83	6.367	56.4	0.4131	0.3520	0.0355	0.00976	7.778×10^{-5}	1.195×10^{-5}	3.259	1.551	0.00351	0.000212
160	314.9	61.76	7.45	52.7	0.4352	0.3839	0.0335	0.00998	7.108×10^{-5}	1.245×10^{-5}	3.324	1.725	0.00411	0.000171
170	355.8	59.47	8.762	48.5	0.4659	0.4286	0.0314	0.01020	6.450×10^{-5}	1.298×10^{-5}	3.443	1.963	0.00498	0.000132
180	400.7	56.85	10.4	43.7	0.5123	0.4960	0.0292	0.01041	5.792×10^{-5}	1.366×10^{-5}	3.661	2.327	0.00637	0.000095
190	449.9	53.75	12.53	38.0	0.5929	0.6112	0.0267	0.01063	5.119×10^{-5}	1.431×10^{-5}	4.090	2.964	0.00891	0.000061
200	504.0	49.75	15.57	30.7	0.7717	0.8544	0.0239	0.01085	4.397×10^{-5}	1.544×10^{-5}	5.119	4.376	0.01490	0.000031
210	563.8	43.19	21.18	18.9	1.4786	1.6683	0.0199	0.01110	3.483×10^{-5}	1.787×10^{-5}	9.311	9.669	0.04021	0.000006

Note 1: Kinematic viscosity ν and thermal diffusivity α can be calculated from their definitions, $\nu = \mu/\rho$ and $\alpha = k/\rho c_\rho = \nu/\mathrm{Pr}$. The properties listed here (except the vapor density) can be used at any pressures with negligible error except at temperatures near the critical-point value.

Note 2: The unit Btu/lbm·°F for specific heat is equivalent to Btu/lbm·R, and the unit Btu/h·ft·°F for thermal conductivity is equivalent to Btu/h·ft·R.

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Original sources: R. Tilner-Roth and H. D. Baehr, "An International Standard Formulation for the Thermodynamic Properties of 1,1,1,2-Tetrafluorethane (HFC-134a) for Temperatures from 170 K to 455 K and Pressures up to 70 Mpa," *J. Phys. Chem. Ref. Data,* Vol. 23, No.5, 1994: M. J. Assael, N. K. Dalaouti, A. A. Griva, and J. H. Dymond, "Viscosity and Thermal Conductivity of Halogenated Methane and Ethane Refrigerants," *IJR,* Vol. 22, pp. 525–535, 1999: NIST REPROP 6 program (M. O. McLinden, S. A. Klein, E. W. Lemmon, and A. P. Peskin, Physicial and Chemical Properties Division, National Institute of Standards and Technology, Boulder, CO 80303. 1995).

TABLE A–17E

Properties of saturated ammonia

Temp. T, °F	Saturation pressure P_{sat}, psia	Density ρ, lbm/ft^3		Enthalpy of vaporization h_{fg}, Btu/lbm	Specific heat c_p, Btu/lbm·R		Thermal conductivity k, Btu/h·ft·R		Dynamic viscosity μ, lbm/ft·s		Prandtl number Pr		Volume expansion coefficient β, 1/R	Surface tension lbf/ft
		Liquid	Vapor		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	
−40	10.4	43.08	0.0402	597.0	1.0542	0.5354	–	0.01026	1.966×10^{-4}	5.342×10^{-6}	–	1.003	0.00098	0.002443
−30	13.9	42.66	0.0527	590.2	1.0610	0.5457	–	0.01057	1.853×10^{-4}	5.472×10^{-6}	–	1.017	0.00101	0.002357
−20	18.3	42.33	0.0681	583.2	1.0677	0.5571	0.3501	0.01089	1.746×10^{-4}	5.600×10^{-6}	1.917	1.031	0.00103	0.002272
−10	23.7	41.79	0.0869	575.9	1.0742	0.5698	0.3426	0.01121	1.645×10^{-4}	5.731×10^{-6}	1.856	1.048	0.00106	0.002187
0	30–4	41.34	0.1097	568.4	1.0807	0.5838	0.3352	0.01154	1.549×10^{-4}	5.861×10^{-6}	1.797	1.068	0.00109	0.002103
10	38.5	40.89	0.1370	560.7	1.0873	0.5992	0.3278	0.01187	1.458×10^{-4}	5.994×10^{-6}	1.740	1.089	0.00112	0.002018
20	48.2	40.43	0.1694	552.6	1.0941	0.6160	0.3203	0.01220	1.371×10^{-4}	6.125×10^{-6}	1.686	1.113	0.00116	0.001934
30	59.8	39.96	0.2075	544.4	1.1012	0.6344	0.3129	0.01254	1290×10^{-4}	6.256×10^{-6}	1.634	1.140	0.00119	0.001850
40	73.4	39.48	0.2521	535.8	1.1087	0.6544	0.3055	0.01288	1.213×10^{-4}	6.389×10^{-6}	1.585	1.168	0.00123	0.001767
50	89.2	38.99	0.3040	526.9	1.1168	0.6762	0.2980	0.01323	1.140×10^{-4}	6.522×10^{-6}	1.539	1.200	0.00128	0.001684
60	107.7	38.50	0.3641	517.7	1.1256	0.6999	0.2906	0.01358	1.072×10^{-4}	6.656×10^{-6}	1.495	1.234	0.00132	0.001601
70	128.9	37.99	0.4332	508.1	1.1353	0.7257	0.2832	0.01394	1.008×10^{-4}	6.786×10^{-6}	1.456	1.272	0.00137	0.001518
80	153.2	37.47	0.5124	498.2	1.1461	0.7539	0.2757	0.01431	9.486×10^{-5}	6.922×10^{-6}	1.419	1.313	0.00143	0.001436
90	180.8	36.94	0.6029	487.8	1.1582	0.7846	0.2683	0.01468	8.922×10^{-5}	7.056×10^{-6}	1.387	1.358	0.00149	0.001354
100	212.0	36.40	0.7060	477.0	1.1719	0.8183	0.2609	0.01505	8.397×10^{-5}	7.189×10^{-6}	1.358	1.407	0.00156	0.001273
110	247.2	35.83	0.8233	465.8	1.1875	0.8554	0.2535	0.01543	7.903×10^{-5}	7.325×10^{-6}	1.333	1.461	0.00164	0.001192
120	286.5	35.26	0.9564	454.1	1.2054	0.8965	0.2460	0.01582	7.444×10^{-5}	7.458×10^{-6}	1.313	1.522	0.00174	0.001111
130	330.4	34.66	1.1074	441.7	1.2261	0.9425	0.2386	0.01621	7.017×10^{-5}	7.594×10^{-6}	1.298	1.589	0.00184	0.001031
140	379.4	34.04	1.2786	428.8	1.2502	0.9943	0.2312	0.01661	6.617×10^{-5}	7.731×10^{-6}	1.288	1.666	0.00196	0.000951
150	433.2	33.39	1.4730	415.2	1.2785	1.0533	0.2237	0.01702	6.244×10^{-5}	7.867×10^{-4}	1.285	1.753	0.00211	0.000872
160	492.7	32.72	1.6940	400.8	1.3120	1.1214	0.2163	0.01744	5.900×10^{-5}	8.006×10^{-6}	1.288	1.853	0.00228	0.000794
170	558.2	32.01	1.9460	385.4	1.3523	1.2012	0.2089	0.01786	5.578×10^{-5}	8.142×10^{-6}	1.300	1.971	0.00249	0.000716
180	630.1	31.26	2.2346	369.1	1.4015	1.2965	0.2014	0.01829	5.278×10^{-5}	8.281×10^{-6}	1.322	2.113	0.00274	0.000638
190	708.5	30.47	2.5670	351.6	1.4624	1.4128	0.1940	0.01874	5.000×10^{-5}	8.419×10^{-6}	1.357	2.286	0.00306	0.000562
200	794.4	29.62	2.9527	332.7	1.5397	1.5586	0.1866	0.01919	4.742×10^{-5}	8.561×10^{-6}	1.409	2.503	0.00348	0.000486
210	887.9	28.70	3.4053	312.0	1.6411	1.7473	0.1791	0.01966	4.500×10^{-5}	8.703×10^{-6}	1.484	2.784	0.00403	0.000411
220	989.5	27.69	3.9440	289.2	1.7798	2.0022	0.1717	0.02015	4.275×10^{-5}	8.844×10^{-6}	1.595	3.164	0.00480	0.000338
230	1099.0	25.57	4.5987	263.5	1.9824	2.3659	0.1643	0.02065	4.064×10^{-5}	8.989×10^{-6}	1.765	3.707	0.00594	0.000265
240	1219.4	25.28	5.4197	234.0	2.3100	2.9264	0.1568	0.02119	3.864×10^{-5}	9.136×10^{-6}	2.049	4.542	0.00784	0.000194

Note 1: Kinematic viscosity ν and thermal diffusivity α can be calculated from their definitions, $\nu = \mu/\rho$ and $\alpha = k/\rho c_\rho = \nu/\text{Pr}$. The properties listed here (except the vapor density) can be used at any pressures with negligible error except at temperatures near the critical-point value.

Note 2: The unit Btu/lbm·°F for specific heat is equivalent to Btu/lbm·R, and the unit Btu/lbm·°F, for thermal conductivity is equivalent to Btu/h·ft·R.

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Orginal sources: Tillner-Roth, Harms-Watzenterg, and Baehr, "Eine neue Fundamentalgleichung fur Ammoniak," *DKV-Tagungsbericht* 20: 167–181, 1993; Liley and Desai, "Thermophysical Properties of Refrigerants," *ASHRAE,* 1993, ISBN 1-1883413-10-9.

TABLE A–18E

Properties of saturated propane

Temp. T, °F	Saturation pressure P_{sat}, psia	Density ρ, lbm/ft³		Enthalpy of vaporization h_{fg}, Btu/lbm	Specific heat c_p, Btu/lbm·R		Thermal conductivity k, Btu/h·ft·R		Dynamic viscosity μ, lbm/ft·s		Prandtl number Pr		Volume expansion coefficient β, 1/R	Surface tension lbf/ft
		Liquid	Vapor		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	
−200	0.0201	42.06	0.0003	217.7	0.4750	0.2595	0.1073	0.00313	5.012×10^{-4}	2.789×10^{-6}	7.991	0.833	0.00083	0.001890
−180	0.0752	41.36	0.0011	213.4	0.4793	0.2680	0.1033	0.00347	3.941×10^{-4}	2.975×10^{-6}	6.582	0.826	0.00086	0.001780
−160	0.2307	40.65	0.0032	209.1	0.4845	0.2769	0.0992	0.00384	3.199×10^{-4}	3.164×10^{-6}	5.626	0.821	0.00088	0.001671
−140	0.6037	39.93	0.0078	204.8	0.4907	0.2866	0.0949	0.00423	2.660×10^{-4}	3.358×10^{-6}	4.951	0.818	0.00091	0.001563
−120	1.389	39.20	0.0170	200.5	0.4982	0.2971	0.0906	0.00465	2.252×10^{-4}	3.556×10^{-6}	4.457	0.817	0.00094	0.001455
−100	2.878	38.46	0.0334	196.1	0.5069	0.3087	0.0863	0.00511	1.934×10^{-4}	3.756×10^{-6}	4.087	0.817	0.00097	0.001349
−90	4.006	38.08	0.0453	193.9	0.5117	0.3150	0.0842	0.00534	1.799×10^{-4}	3.858×10^{-6}	3.936	0.819	0.00099	0.001297
−80	5.467	37.70	0.0605	191.6	0.5169	0.3215	0.0821	0.00559	1.678×10^{-4}	3.961×10^{-6}	3.803	0.820	0.00101	0.001244
−70	7.327	37.32	0.0793	189.3	0.5224	0.3284	0.0800	0.00585	1.569×10^{-4}	4.067×10^{-6}	3.686	0.822	0.00104	0.001192
−60	9.657	36.93	0.1024	186.9	0.5283	0.3357	0.0780	0.00611	1.469×10^{-4}	4.172×10^{-6}	3.582	0.825	0.00106	0.001140
−50	12.54	36.54	0.1305	184.4	0.5345	0.3433	0.0760	0.00639	1.378×10^{-4}	4.278×10^{-6}	3.490	0.828	0.00109	0.001089
−40	16.05	36.13	0.1641	181.9	0.5392	0.3513	0.0740	0.00568	1.294×10^{-4}	4.386×10^{-6}	3.395	0.831	0.00112	0.001038
−30	20.29	35.73	0.2041	179.3	0.5460	0.3596	0.0721	0.00697	1.217×10^{-4}	4.497×10^{-6}	3.320	0.835	0.00115	0.000987
−20	25.34	35.31	0.2512	176.6	0.5531	0.3684	0.0702	0.00728	1.146×10^{-4}	4.611×10^{-6}	3.253	0.840	0.00119	0.000937
−10	31.3	34.89	0.3063	173.8	0.5607	0.3776	0.0683	0.00761	1.079×10^{-4}	4.725×10^{-6}	3.192	0.845	0.00123	0.000887
0	38.28	34.46	0.3703	170.9	0.5689	0.3874	0.0665	0.00794	1.018×10^{-4}	4.842×10^{-6}	3.137	0.850	0.00127	0.000838
10	46.38	34.02	0.4441	167.9	0.5775	0.3976	0.0647	0.00829	9.606×10^{-5}	4.961×10^{-6}	3.088	0.857	0.00132	0.000789
20	55.7	33.56	0.5289	164.8	0.5867	0.4084	0.0629	0.00865	9.067×10^{-5}	5.086×10^{-6}	3.043	0.864	0.00138	0.000740
30	66.35	33.10	0.6259	161.6	0.5966	0.4199	0.0512	0.00903	8.561×10^{-5}	5.211×10^{-6}	3.003	0.873	0.00144	0.000692
40	78.45	32.62	0.7365	158.1	0.6072	0.4321	0.0595	0.00942	8.081×10^{-5}	5.342×10^{-6}	2.967	0.882	0.00151	0.000644
50	92.12	32.13	0.8621	154.6	0.6187	0.4452	0.0579	0.00983	7.631×10^{-5}	5.478×10^{-6}	2.935	0.893	0.00159	0.000597
60	107.5	31.63	1.0046	150.8	0.6311	0.4593	0.0563	0.01025	7.200×10^{-5}	5.617×10^{-6}	2.906	0.906	0.00168	0.000551
70	124.6	31.11	1.1659	146.8	0.6447	0.4746	0.0547	0.01070	6.794×10^{-5}	5.764×10^{-6}	2.881	0.921	0.00179	0.000505
80	143.7	30.56	1.3484	142.7	0.6596	0.4915	0.0532	0.01116	6.406×10^{-5}	5.919×10^{-6}	2.860	0.938	0.00191	0.000460
90	164.8	30.00	1.5549	138.2	0.6762	0.5103	0.0517	0.01165	6.033×10^{-5}	6.081×10^{-6}	2.843	0.959	0.00205	0.000416
100	188.1	29.41	1.7887	133.6	0.6947	0.5315	0.0501	0.01217	5.675×10^{-5}	6.256×10^{-6}	2.831	0.984	0.00222	0.000372
120	241.8	28.13	2.3562	123.2	0.7403	0.5844	0.0472	0.01328	5.000×10^{-6}	6.644×10^{-6}	2.825	1.052	0.00267	0.000288
140	306.1	26.69	3.1003	111.1	0.7841	0.6613	0.0442	0.01454	4.358×10^{-5}	7.111×10^{-6}	2.784	1.164	0.00338	0.000208
160	382.4	24.98	4.1145	96.4	0.8696	0.7911	0.0411	0.01603	3.733×10^{-5}	7.719×10^{-6}	2.845	1.371	0.00459	0.000133
180	472.9	22.79	5.6265	77.1	1.1436	1.0813	0.0376	0.01793	3.083×10^{-5}	8.617×10^{-6}	3.380	1.870	0.00791	0.000065

Note 1: Kinematic viscosity ν and thermal diffusivity α can be calculated from their definitions, $\nu = \mu/\rho$ and $\alpha = k/\rho c_p = \nu/\text{Pr}$. The properties listed here (except the vapor density) can be used at any pressures with negligible error at temperatures near the critical-point value.

Note 2: The unit Btu/lbm·°F for specific heat is equivalent to Btu/lbm·°R, and the unit Btu/h·ft·°F for thermal conductivity is equivalent to Btu/h·ft·R.

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Original sources: Reiner Tillner-Roth, "Fundamental Equations of State," Shaker, Verlag, Aachan, 1998; B. A. Younglove and J. F. Ely. "Thermophysical Properties of Fluids. II Methane, Ethane, Propane, Isobutane, and Normal Butane," *J. Phys. Chem. Ref. Data,* Vol. 16, No. 4, 1987; G. R. Somayajulu, "A Generalized Equation for Surface Tension from the Triple-Point to the Critical-Point," *International Journal of Thermophysics,* Vol. 9, No. 4, 1988.

TABLE A–19E

Properties of liquids

Temp. T, °F	Density ρ, lbm/ft^3	Specific heat c_p, Btu/lbm·R	Thermal conductivity k, Btu/h·ft·R	Thermal diffusivity α, ft^2/s	Dynamic viscosity μ, lbm/ft·s	Kinematic viscosity ν, ft^2/s	Prandtl number Pr	Volume expansion coeff. β, 1/R
Methane (CH_4)								
−280	27.41	0.8152	0.1205	1.497×10^{-6}	1.057×10^{-4}	3.857×10^{-6}	2.575	0.00175
−260	26.43	0.8301	0.1097	1.389×10^{-6}	8.014×10^{-5}	3.032×10^{-6}	2.183	0.00192
−240	25.39	0.8523	0.0994	1.276×10^{-6}	6.303×10^{-5}	2.482×10^{-6}	1.945	0.00215
−220	24.27	0.8838	0.0896	1.159×10^{-6}	5.075×10^{-5}	2.091×10^{-6}	1.803	0.00247
−200	23.04	0.9314	0.0801	1.036×10^{-6}	4.142×10^{-5}	1.798×10^{-6}	1.734	0.00295
−180	21.64	1.010	0.0709	9.008×10^{-7}	3.394×10^{-5}	1.568×10^{-6}	1.741	0.00374
−160	19.99	1.158	0.0616	7.397×10^{-7}	2.758×10^{-5}	1.379×10^{-6}	1.865	0.00526
−140	17.84	1.542	0.0518	5.234×10^{-7}	2.168×10^{-5}	1.215×10^{-6}	2.322	0.00943
Methanol [$CH_3(OH)$]								
70	49.15	0.6024	0.1148	1.076×10^{-6}	3.872×10^{-4}	7.879×10^{-3}	7.317	0.000656
90	48.50	0.6189	0.1143	1.057×10^{-6}	3.317×10^{-4}	6.840×10^{-6}	6.468	0.000671
110	47.85	0.6373	0.1138	1.036×10^{-6}	2.872×10^{-4}	6.005×10^{-6}	5.793	0.000691
130	47.18	0.6576	0.1133	1.014×10^{-6}	2.513×10^{-4}	5.326×10^{-6}	5.250	0.000716
150	46.50	0.6796	0.1128	9.918×10^{-7}	2.218×10^{-4}	4.769×10^{-6}	4.808	0.000749
170	45.80	0.7035	0.1124	9.687×10^{-7}	1.973×10^{-4}	4.308×10^{-6}	4.447	0.000789
Isobutane (R600a)								
−150	42.75	0.4483	0.0799	1.157×10^{-6}	6.417×10^{-4}	1.500×10^{-5}	12.96	0.000785
−100	41.06	0.4721	0.0782	1.120×10^{-3}	3.669×10^{-4}	8.939×10^{-6}	7.977	0.000836
−50	39.31	0.4986	0.0731	1.036×10^{-6}	2.376×10^{-4}	6.043×10^{-6}	5.830	0.000908
0	37.48	0.5289	0.0664	9.299×10^{-7}	1.651×10^{-4}	4.406×10^{-6}	4.738	0.001012
50	35.52	0.5643	0.0591	8.187×10^{-7}	1.196×10^{-4}	3.368×10^{-6}	4.114	0.001169
100	33.35	0.6075	0.0521	7.139×10^{-7}	8.847×10^{-5}	2.653×10^{-3}	3.716	0.001421
150	30.84	0.6656	0.0457	6.188×10^{-7}	6.558×10^{-5}	2.127×10^{-6}	3.437	0.001883
200	27.73	0.7635	0.0400	5.249×10^{-7}	4.750×10^{-5}	1.713×10^{-6}	3.264	0.002970
Glycerin								
32	79.65	0.5402	0.163	1.052×10^{-6}	7.047	0.08847	84101	
40	79.49	0.5458	0.1637	1.048×10^{-3}	4.803	0.06042	57655	
50	79.28	0.5541	0.1645	1.040×10^{-6}	2.850	0.03594	34561	
60	79.07	0.5632	0.1651	1.029×10^{-6}	1.547	0.01956	18995	
70	78.86	0.5715	0.1652	1.018×10^{-6}	0.9422	0.01195	11730	
80	78.66	0.5794	0.1652	1.007×10^{-6}	0.5497	0.00699	6941	
90	78.45	0.5878	0.1652	9.955×10^{-7}	0.3756	0.004787	4809	
100	78.24	0.5964	0.1653	9.841×10^{-7}	0.2277	0.00291	2957	
Engine oil (unused)								
32	56.12	0.4291	0.0849	9.792×10^{-7}	2.563	4.566×10^{-2}	46636	0.000389
50	55.79	0.4395	0.08338	9.448×10^{-7}	1.210	2.169×10^{-2}	22963	0.000389
75	55.3	0.4531	0.08378	9.288×10^{-7}	0.4286	7.751×10^{-3}	8345	0.000389
100	54.77	0.4669	0.08367	9.089×10^{-7}	0.1630	2.977×10^{-3}	3275	0.000389
125	54.24	0.4809	0.08207	8.740×10^{-7}	7.617×10^{-2}	1.404×10^{-3}	1607	0.000389
150	53.73	0.4946	0.08046	8.411×10^{-7}	3.833×10^{-2}	7.135×10^{-4}	848.3	0.000389
200	52.68	0.5231	0.07936	7.999×10^{-7}	1.405×10^{-2}	2.668×10^{-4}	333.6	0.000389
250	51.71	0.5523	0.07776	7.563×10^{-7}	6.744×10^{-3}	1.304×10^{-4}	172.5	0.000389
300	50.63	0.5818	0.07673	7.236×10^{-7}	3.661×10^{-3}	7.232×10^{-5}	99.94	0.000389

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Originally based on various sources.

TABLE A–20E

Properties of liquid metals

Temp. T, °F	Density ρ, lbm/ft^3	Specific heat c_p, Btu/lbm·R	Thermal conductivity k, Btu/h·ft·R	Thermal diffusivity α, ft^2/s	Dynamic viscosity μ, lbm/ft·s	Kinematic viscosity ν, ft^2/s	Prandtl number Pr	Volume expansion coeff. β, 1/R
Mercury (Hg) melting point: −38°F								
32	848.7	0.03353	4.727	4.614×10^{-5}	1.133×10^{-3}	1.335×10^{-6}	0.02895	1.005×10^{-4}
50	847.2	0.03344	4.805	4.712×10^{-5}	1.092×10^{-3}	1.289×10^{-6}	0.02737	1.005×10^{-4}
100	842.9	0.03319	5.015	4.980×10^{-5}	9.919×10^{-4}	1.176×10^{-6}	0.02363	1.005×10^{-4}
150	838.7	0.03298	5.221	5.244×10^{-5}	9.122×10^{-4}	1.087×10^{-6}	0.02074	1.005×10^{-4}
200	834.5	0.03279	5.422	5.504×10^{-5}	8.492×10^{-4}	1.017×10^{-6}	0.01849	1.005×10^{-4}
300	826.2	0.03252	5.815	6.013×10^{-5}	7.583×10^{-4}	9.180×10^{-7}	0.01527	1.005×10^{-4}
400	817.9	0.03236	6.184	6.491×10^{-5}	6.972×10^{-4}	8.524×10^{-7}	0.01313	1.008×10^{-4}
500	809.6	0.03230	6.518	6.924×10^{-5}	6.525×10^{-4}	8.061×10^{-7}	0.01164	1.018×10^{-4}
600	801.3	0.03235	6.839	7.329×10^{-5}	6.186×10^{-4}	7.719×10^{-7}	0.01053	1.035×10^{-4}
Bismuth (Bi) melting point: 520°F								
700	620.7	0.03509	9.361	1.193×10^{-4}	1.001×10^{-3}	1.614×10^{-6}	0.01352	
800	616.5	0.03569	9.245	1.167×10^{-4}	9.142×10^{-4}	1.482×10^{-6}	0.01271	
900	612.2	0.0363	9.129	1.141×10^{-4}	8.267×10^{-4}	1.350×10^{-6}	0.01183	
1000	608.0	0.0369	9.014	1.116×10^{-4}	7.392×10^{-4}	1.215×10^{-6}	0.0109	
1100	603.7	0.0375	9.014	1.105×10^{-4}	6.872×10^{-4}	1.138×10^{-6}	0.01029	
Lead (Pb) melting point: 621°F								
700	658	0.03797	9.302	1.034×10^{-4}	1.612×10^{-3}	2.450×10^{-6}	0.02369	
800	654	0.03750	9.157	1.037×10^{-4}	1.453×10^{-3}	2.223×10^{-6}	0.02143	
900	650	0.03702	9.013	1.040×10^{-4}	1.296×10^{-3}	1.994×10^{-6}	0.01917	
1000	645.7	0.03702	8.912	1.035×10^{-4}	1.202×10^{-3}	1.862×10^{-6}	0.01798	
1100	641.5	0.03702	8.810	1.030×10^{-4}	1.108×10^{-3}	1.727×10^{-6}	0.01676	
1200	637.2	0.03702	8.709	1.025×10^{-4}	1.013×10^{-3}	1.590×10^{-6}	0.01551	
Sodium (Na) melting point: 280°F								
300	57.13	0.3258	48.19	7.192×10^{-4}	4.136×10^{-4}	7.239×10^{-6}	0.01007	
400	56.28	0.3219	46.58	7.142×10^{-4}	3.572×10^{-4}	6.350×10^{-6}	0.008891	
500	55.42	0.3181	44.98	7.087×10^{-4}	3.011×10^{-4}	5.433×10^{-6}	0.007667	
600	54.56	0.3143	43.37	7.026×10^{-4}	2.448×10^{-4}	4.488×10^{-6}	0.006387	
800	52.85	0.3089	40.55	6.901×10^{-4}	1.772×10^{-4}	3.354×10^{-6}	0.004860	
1000	51.14	0.3057	38.12	6.773×10^{-4}	1.541×10^{-4}	3.014×10^{-6}	0.004449	
Potassium (K) melting point: 147°F								
300	50.40	0.1911	26.00	7.500×10^{-4}	2.486×10^{-4}	4.933×10^{-6}	0.006577	
400	49.58	0.1887	25.37	7.532×10^{-4}	2.231×10^{-4}	4.500×10^{-6}	0.005975	
500	48.76	0.1863	24.73	7.562×10^{-4}	1.976×10^{-4}	4.052×10^{-6}	0.005359	
600	47.94	0.1839	24.09	7.591×10^{-4}	1.721×10^{-4}	3.589×10^{-6}	0.004728	
800	46.31	0.1791	22.82	7.643×10^{-4}	1.210×10^{-4}	2.614×10^{-6}	0.003420	
1000	44.62	0.1791	21.34	7.417×10^{-4}	1.075×10^{-4}	2.409×10^{-6}	0.003248	
Sodium-potassium (%22Na-%78K) melting point: 12°F								
200	52.99	0.2259	14.79	3.432×10^{-4}	3.886×10^{-4}	7.331×10^{-6}	0.02136	
300	52.16	0.2230	14.99	3.580×10^{-4}	3.467×10^{-4}	6.647×10^{-6}	0.01857	
400	51.32	0.2201	15.19	3.735×10^{-4}	3.050×10^{-4}	5.940×10^{-6}	0.0159	
600	49.65	0.2143	15.59	4.070×10^{-4}	2.213×10^{-4}	4.456×10^{-6}	0.01095	
800	47.99	0.2100	15.95	4.396×10^{-4}	1.539×10^{-4}	3.207×10^{-6}	0.007296	
1000	46.36	0.2103	16.20	4.615×10^{-4}	1.353×10^{-4}	2.919×10^{-6}	0.006324	

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Originally based on various sources.

TABLE A–21E

Ideal-gas properties of air

T R	*h* Btu/lbm	P_r	*u* Btu/lbm	v_r	*s*° Btu/lbm·R	*T* R	*h* Btu/lbm	P_r	*u* Btu/lbm	v_r	*s*° Btu/lbm·R
360	85.97	0.3363	61.29	396.6	0.50369	1600	395.74	71.13	286.06	8.263	0.87130
380	90.75	0.4061	64.70	346.6	0.51663	1650	409.13	80.89	296.03	7.556	0.87954
400	95.53	0.4858	68.11	305.0	0.52890	1700	422.59	90.95	306.06	6.924	0.88758
420	100.32	0.5760	71.52	270.1	0.54058	1750	436.12	101.98	316.16	6.357	0.89542
440	105.11	0.6776	74.93	240.6	0.55172	1800	449.71	114.0	326.32	5.847	0.90308
460	109.90	0.7913	78.36	215.33	0.56235	1850	463.37	127.2	336.55	5.388	0.91056
480	114.69	0.9182	81.77	193.65	0.57255	1900	477.09	141.5	346.85	4.974	0.91788
500	119.48	1.0590	85.20	174.90	0.58233	1950	490.88	157.1	357.20	4.598	0.92504
520	124.27	1.2147	88.62	158.58	0.59173	2000	504.71	174.0	367.61	4.258	0.93205
537	128.10	1.3593	91.53	146.34	0.59945	2050	518.71	192.3	378.08	3.949	0.93891
540	129.06	1.3860	92.04	144.32	0.60078	2100	532.55	212.1	388.60	3.667	0.94564
560	133.86	1.5742	95.47	131.78	0.60950	2150	546.54	223.5	399.17	3.410	0.95222
580	138.66	1.7800	98.90	120.70	0.61793	2200	560.59	256.6	409.78	3.176	0.95919
600	143.47	2.005	102.34	110.88	0.62607	2250	574.69	281.4	420.46	2.961	0.96501
620	148.28	2.249	105.78	102.12	0.63395	2300	588.82	308.1	431.16	2.765	0.97123
640	153.09	2.514	109.21	94.30	0.64159	2350	603.00	336.8	441.91	2.585	0.97732
660	157.92	2.801	112.67	87.27	0.64902	2400	617.22	367.6	452.70	2.419	0.98331
680	162.73	3.111	116.12	80.96	0.65621	2450	631.48	400.5	463.54	2.266	0.98919
700	167.56	3.446	119.58	75.25	0.66321	2500	645.78	435.7	474.40	2.125	0.99497
720	172.39	3.806	123.04	70.07	0.67002	2550	660.12	473.3	485.31	1.996	1.00064
740	177.23	4.193	126.51	65.38	0.67665	2600	674.49	513.5	496.26	1.876	1.00623
760	182.08	4.607	129.99	61.10	0.68312	2650	688.90	556.3	507.25	1.765	1.01172
780	186.94	5.051	133.47	57.20	0.68942	2700	703.35	601.9	518.26	1.662	1.01712
800	191.81	5.526	136.97	53.63	0.69558	2750	717.83	650.4	529.31	1.566	1.02244
820	196.69	6.033	140.47	50.35	0.70160	2800	732.33	702.0	540.40	1.478	1.02767
840	201.56	6.573	143.98	47.34	0.70747	2850	746.88	756.7	551.52	1.395	1.03282
860	206.46	7.149	147.50	44.57	0.71323	2900	761.45	814.8	562.66	1.318	1.03788
880	211.35	7.761	151.02	42.01	0.71886	2950	776.05	876.4	573.84	1.247	1.04288
900	216.26	8.411	154.57	39.64	0.72438	3000	790.68	941.4	585.04	1.180	1.04779
920	221.18	9.102	158.12	37.44	0.72979	3050	805.34	1011	596.28	1.118	1.05264
940	226.11	9.834	161.68	35.41	0.73509	3100	820.03	1083	607.53	1.060	1.05741
960	231.06	10.61	165.26	33.52	0.74030	3150	834.75	1161	618.82	1.006	1.06212
980	236.02	11.43	168.83	31.76	0.74540	3200	849.48	1242	630.12	0.955	1.06676
1000	240.98	12.30	172.43	30.12	0.75042	3250	864.24	1328	641.46	0.907	1.07134
1040	250.95	14.18	179.66	27.17	0.76019	3300	879.02	1418	652.81	0.8621	1.07585
1080	260.97	16.28	186.93	24.58	0.76964	3350	893.83	1513	664.20	0.8202	1.08031
1120	271.03	18.60	194.25	22.30	0.77880	3400	908.66	1613	675.60	0.7807	1.08470
1160	281.14	21.18	201.63	20.29	0.78767	3450	923.52	1719	687.04	0.7436	1.08904
1200	291.30	24.01	209.05	18.51	0.79628	3500	938.40	1829	698.48	0.7087	1.09332
1240	301.52	27.13	216.53	16.93	0.80466	3550	953.30	1946	709.95	0.6759	1.09755
1280	311.79	30.55	224.05	15.52	0.81280	3600	968.21	2068	721.44	0.6449	1.10172
1320	322.11	34.31	231.63	14.25	0.82075	3650	983.15	2196	732.95	0.6157	1.10584
1360	332.48	38.41	239.25	13.12	0.82848	3700	998.11	2330	744.48	0.5882	1.10991
1400	342.90	42.88	246.93	12.10	0.83604	3750	1013.1	2471	756.04	0.5621	1.11393
1440	353.37	47.75	254.66	11.17	0.84341	3800	1028.1	2618	767.60	0.5376	1.11791
1480	363.89	53.04	262.44	10.34	0.85062	3850	1043.1	2773	779.19	0.5143	1.12183
1520	374.47	58.78	270.26	9.578	0.85767	3900	1058.1	2934	790.80	0.4923	1.12571
1560	385.08	65.00	278.13	8.890	0.86456	3950	1073.2	3103	802.43	0.4715	1.12955

TABLE A–21E

Ideal-gas properties of air *(Concluded)*

T R	h Btu/lbm	P_r	u Btu/lbm	v_r	s° Btu/lbm·R
4000	1088.3	3280	814.06	0.4518	1.13334
4050	1103.4	3464	825.72	0.4331	1.13709
4100	1118.5	3656	837.40	0.4154	1.14079
4150	1133.6	3858	849.09	0.3985	1.14446
4200	1148.7	4067	860.81	0.3826	1.14809
4300	1179.0	4513	884.28	0.3529	1.15522
4400	1209.4	4997	907.81	0.3262	1.16221
4500	1239.9	5521	931.39	0.3019	1.16905
4600	1270.4	6089	955.04	0.2799	1.17575
4700	1300.9	6701	978.73	0.2598	1.18232
4800	1331.5	7362	1002.5	0.2415	1.18876
4900	1362.2	8073	1026.3	0.2248	1.19508
5000	1392.9	8837	1050.1	0.2096	1.20129
5100	1423.6	9658	1074.0	0.1956	1.20738
5200	1454.4	10,539	1098.0	0.1828	1.21336
5300	1485.3	11,481	1122.0	0.1710	1.2192

Note: The properties P_r (relative pressure) and v_r (relative specific volume) are dimensionless quantities used in the analysis of isentropic processes and should not be confused with the properties pressure and specific volume.

Source of Data: Kenneth Wark, *Thermodynamics,* 4th ed. (New York: McGraw-Hill, 1983), pp. 832–33, Table A–5. Originally published in J. H. Keenan and J. Kaye, *Gas Tables* (New York: John Wiley & Sons, 1948).

TABLE A–22E

Properties of air at 1 atm pressure

Temp. T, °F	Density ρ, lbm/ft^3	Specific heat c_p, Btu/lbm·R	Thermal conductivity k, Btu/h·ft·R	Thermal diffusivity α, ft^2/s	Dynamic viscosity μ, lbm/ft·s	Kinematic viscosity ν, ft^2/s	Prandtl number Pr
−300	0.24844	0.5072	0.00508	1.119×10^{-5}	4.039×10^{-6}	1.625×10^{-5}	1.4501
−200	0.15276	0.2247	0.00778	6.294×10^{-5}	6.772×10^{-6}	4.433×10^{-5}	0.7042
−100	0.11029	0.2360	0.01037	1.106×10^{-4}	9.042×10^{-6}	8.197×10^{-5}	0.7404
−50	0.09683	0.2389	0.01164	1.397×10^{-4}	1.006×10^{-5}	1.039×10^{-4}	0.7439
0	0.08630	0.2401	0.01288	1.726×10^{-4}	1.102×10^{-5}	1.278×10^{-4}	0.7403
10	0.08446	0.2402	0.01312	1.797×10^{-4}	1.121×10^{-5}	1.328×10^{-4}	0.7391
20	0.08270	0.2403	0.01336	1.868×10^{-4}	1.140×10^{-5}	1.379×10^{-4}	0.7378
30	0.08101	0.2403	0.01361	1.942×10^{-4}	1.158×10^{-5}	1.430×10^{-4}	0.7365
40	0.07939	0.2404	0.01385	2.016×10^{-4}	1.176×10^{-5}	1.482×10^{-4}	0.7350
50	0.07783	0.2404	0.01409	2.092×10^{-4}	1.194×10^{-5}	1.535×10^{-4}	0.7336
60	0.07633	0.2404	0.01433	2.169×10^{-4}	1.212×10^{-5}	1.588×10^{-4}	0.7321
70	0.07489	0.2404	0.01457	2.248×10^{-4}	1.230×10^{-5}	1.643×10^{-4}	0.7306
80	0.07350	0.2404	0.01481	2.328×10^{-4}	1.247×10^{-5}	1.697×10^{-4}	0.7290
90	0.07217	0.2404	0.01505	2.409×10^{-4}	1.265×10^{-5}	1.753×10^{-4}	0.7275
100	0.07088	0.2405	0.01529	2.491×10^{-4}	1.281×10^{-5}	1.809×10^{-4}	0.7260
110	0.06963	0.2405	0.01552	2.575×10^{-4}	1.299×10^{-5}	1.866×10^{-4}	0.7245
120	0.06843	0.2405	0.01576	2.660×10^{-4}	1.316×10^{-5}	1.923×10^{-4}	0.7230
130	0.06727	0.2405	0.01599	2.746×10^{-4}	1.332×10^{-5}	1.981×10^{-4}	0.7216
140	0.06615	0.2406	0.01623	2.833×10^{-4}	1.349×10^{-5}	2.040×10^{-4}	0.7202
150	0.06507	0.2406	0.01646	2.921×10^{-4}	1.365×10^{-5}	2.099×10^{-4}	0.7188
160	0.06402	0.2406	0.01669	3.010×10^{-4}	1.382×10^{-5}	2.159×10^{-4}	0.7174
170	0.06300	0.2407	0.01692	3.100×10^{-4}	1.398×10^{-5}	2.220×10^{-4}	0.7161
180	0.06201	0.2408	0.01715	3.191×10^{-4}	1.414×10^{-5}	2.281×10^{-4}	0.7148
190	0.06106	0.2408	0.01738	3.284×10^{-4}	1.430×10^{-5}	2.343×10^{-4}	0.7136
200	0.06013	0.2409	0.01761	3.377×10^{-4}	1.446×10^{-5}	2.406×10^{-4}	0.7124
250	0.05590	0.2415	0.01874	3.857×10^{-4}	1.524×10^{-5}	2.727×10^{-4}	0.7071
300	0.05222	0.2423	0.01985	4.358×10^{-4}	1.599×10^{-5}	3.063×10^{-4}	0.7028
350	0.04899	0.2433	0.02094	4.879×10^{-4}	1.672×10^{-5}	3.413×10^{-4}	0.6995
400	0.04614	0.2445	0.02200	5.419×10^{-4}	1.743×10^{-5}	3.777×10^{-4}	0.6971
450	0.04361	0.2458	0.02305	5.974×10^{-4}	1.812×10^{-5}	4.154×10^{-4}	0.6953
500	0.04134	0.2472	0.02408	6.546×10^{-4}	1.878×10^{-5}	4.544×10^{-4}	0.6942
600	0.03743	0.2503	0.02608	7.732×10^{-4}	2.007×10^{-5}	5.361×10^{-4}	0.6934
700	0.03421	0.2535	0.02800	8.970×10^{-4}	2.129×10^{-5}	6.225×10^{-4}	0.6940
800	0.03149	0.2568	0.02986	1.025×10^{-3}	2.247×10^{-5}	7.134×10^{-4}	0.6956
900	0.02917	0.2599	0.03164	1.158×10^{-3}	2.359×10^{-5}	8.087×10^{-4}	0.6978
1000	0.02718	0.2630	0.03336	1.296×10^{-3}	2.467×10^{-5}	9.080×10^{-4}	0.7004
1500	0.02024	0.2761	0.04106	2.041×10^{-3}	2.957×10^{-5}	1.460×10^{-3}	0.7158
2000	0.01613	0.2855	0.04752	2.867×10^{-3}	3.379×10^{-5}	2.095×10^{-3}	0.7308
2500	0.01340	0.2922	0.05309	3.765×10^{-3}	3.750×10^{-5}	2.798×10^{-3}	0.7432
3000	0.01147	0.2972	0.05811	4.737×10^{-3}	4.082×10^{-5}	3.560×10^{-3}	0.7516
3500	0.01002	0.3010	0.06293	5.797×10^{-3}	4.381×10^{-5}	4.373×10^{-3}	0.7543
4000	0.00889	0.3040	0.06789	6.975×10^{-3}	4.651×10^{-5}	5.229×10^{-3}	0.7497

Note: For ideal gases, the properties c_p, k, μ, and Pr are independent of pressure. The properties ρ, ν, and α at a pressure P (in atm) other than 1 atm are determined by multiplying the values of ρ at the given temperature by P and by dividing ν and α by P.

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Original sources: Keenan, Chao, Keyes, Gas Tables, Wiley, 1984; and *Thermophysical Properties of Matter,* Vol. 3: *Thermal Conductivity,* Y. S. Touloukian, P. E. Liley, S. C. Saxena, Vol. 11: *Viscosity,* Y. S. Touloukian, S. C. Saxena, and P. Hestermans, IFI/Plenun, NY, 1970, ISBN 0-306067020-8.

TABLE A–23E

Properties of gases at 1 atm pressure

Temp. T, °F	Density ρ, lbm/ft^3	Specific heat c_p, Btu/lbm·R	Thermal conductivity k, Btu/h·ft·R	Thermal diffusivity α, ft^2/s	Dynamic viscosity μ, lbm/ft·s	Kinematic viscosity ν, ft^2/s	Prandtl number Pr
Carbon dioxide, CO_2							
−50	0.14712	0.1797	0.00628	6.600×10^{-5}	7.739×10^{-6}	5.261×10^{-5}	0.7970
0	0.13111	0.1885	0.00758	8.522×10^{-5}	8.661×10^{-6}	6.606×10^{-5}	0.7751
50	0.11825	0.1965	0.00888	1.061×10^{-4}	9.564×10^{-6}	8.086×10^{-5}	0.7621
100	0.10769	0.2039	0.01017	1.286×10^{-4}	1.045×10^{-5}	9.703×10^{-5}	0.7543
200	0.09136	0.2171	0.01273	1.784×10^{-4}	1.217×10^{-5}	1.332×10^{-4}	0.7469
300	0.07934	0.2284	0.01528	2.341×10^{-4}	1.382×10^{-5}	1.743×10^{-4}	0.7445
500	0.06280	0.2473	0.02027	3.626×10^{-4}	1.696×10^{-5}	2.700×10^{-4}	0.7446
1000	0.04129	0.2796	0.03213	7.733×10^{-4}	2.381×10^{-5}	5.767×10^{-4}	0.7458
1500	0.03075	0.2995	0.04281	1.290×10^{-3}	2.956×10^{-5}	9.610×10^{-4}	0.7445
2000	0.02450	0.3124	0.05193	1.885×10^{-3}	3.451×10^{-5}	1.408×10^{-3}	0.7474
Carbon monoxide, CO							
−50	0.09363	0.2571	0.01118	1.290×10^{-4}	9.419×10^{-6}	1.005×10^{-4}	0.7798
0	0.08345	0.2523	0.01240	1.636×10^{-4}	1.036×10^{-5}	1.242×10^{-4}	0.7593
50	0.07526	0.2496	0.01359	2.009×10^{-4}	1.127×10^{-5}	1.498×10^{-4}	0.7454
100	0.06854	0.2484	0.01476	2.408×10^{-4}	1.214×10^{-5}	1.772×10^{-4}	0.7359
200	0.05815	0.2485	0.01702	3.273×10^{-4}	1.379×10^{-5}	2.372×10^{-4}	0.7247
300	0.05049	0.2505	0.01920	4.217×10^{-4}	1.531×10^{-5}	3.032×10^{-4}	0.7191
500	0.03997	0.2567	0.02331	6.311×10^{-4}	1.802×10^{-5}	4.508×10^{-4}	0.7143
1000	0.02628	0.2732	0.03243	1.254×10^{-3}	2.334×10^{-5}	8.881×10^{-4}	0.7078
1500	0.01957	0.2862	0.04049	2.008×10^{-3}	2.766×10^{-5}	1.413×10^{-3}	0.7038
2000	0.01559	0.2958	0.04822	2.903×10^{-3}	3.231×10^{-5}	2.072×10^{-3}	0.7136
Methane, CH_4							
−50	0.05363	0.5335	0.01401	1.360×10^{-4}	5.861×10^{-6}	1.092×10^{-4}	0.8033
0	0.04779	0.5277	0.01616	1.780×10^{-4}	6.506×10^{-6}	1.361×10^{-4}	0.7649
50	0.04311	0.5320	0.01839	2.228×10^{-4}	7.133×10^{-6}	1.655×10^{-4}	0.7428
100	0.03925	0.5433	0.02071	2.698×10^{-4}	7.742×10^{-6}	1.972×10^{-4}	0.7311
200	0.03330	0.5784	0.02559	3.690×10^{-4}	8.906×10^{-6}	2.674×10^{-4}	0.7245
300	0.02892	0.6226	0.03077	4.748×10^{-4}	1.000×10^{-5}	3.457×10^{-4}	0.7283
500	0.02289	0.7194	0.04195	7.075×10^{-4}	1.200×10^{-5}	5.244×10^{-4}	0.7412
1000	0.01505	0.9438	0.07346	1.436×10^{-3}	1.620×10^{-5}	1.076×10^{-3}	0.7491
1500	0.01121	1.1162	0.10766	2.390×10^{-3}	1.974×10^{-5}	1.760×10^{-3}	0.7366
2000	0.00893	1.2419	0.14151	3.544×10^{-3}	2.327×10^{-5}	2.605×10^{-3}	0.7353
Hydrogen, H_2							
−50	0.00674	3.0603	0.08246	1.110×10^{-3}	4.969×10^{-6}	7.373×10^{-4}	0.6638
0	0.00601	3.2508	0.09049	1.287×10^{-3}	5.381×10^{-6}	8.960×10^{-4}	0.6960
50	0.00542	3.3553	0.09818	1.500×10^{-3}	5.781×10^{-6}	1.067×10^{-3}	0.7112
100	0.00493	3.4118	0.10555	1.742×10^{-3}	6.167×10^{-6}	1.250×10^{-3}	0.7177
200	0.00419	3.4549	0.11946	2.295×10^{-3}	6.911×10^{-6}	1.652×10^{-3}	0.7197
300	0.00363	3.4613	0.13241	2.924×10^{-3}	7.622×10^{-6}	2.098×10^{-3}	0.7174
500	0.00288	3.4572	0.15620	4.363×10^{-3}	8.967×10^{-6}	3.117×10^{-3}	0.7146
1000	0.00189	3.5127	0.20989	8.776×10^{-3}	1.201×10^{-5}	6.354×10^{-3}	0.7241
1500	0.00141	3.6317	0.26381	1.432×10^{-2}	1.477×10^{-5}	1.048×10^{-2}	0.7323
2000	0.00112	3.7656	0.31923	2.098×10^{-2}	1.734×10^{-5}	1.544×10^{-2}	0.7362
Nitrogen, N_2							
−50	0.09364	0.2320	0.01176	1.504×10^{-4}	9.500×10^{-6}	1.014×10^{-4}	0.6746
0	0.08346	0.2441	0.01300	1.773×10^{-4}	1.043×10^{-5}	1.251×10^{-4}	0.7056
50	0.07527	0.2480	0.01420	2.113×10^{-4}	1.134×10^{-5}	1.507×10^{-4}	0.7133

TABLE A–23E

Properties of gases at 1 atm pressure (*Concluded*)

Temp. T, °F	Density ρ, lbm/ft^3	Specific heat c_p, Btu/lbm·R	Thermal conductivity k, Btu/h·ft·R	Thermal diffusivity α, ft^2/s	Dynamic viscosity μ, lbm/ft·s	Kinematic viscosity ν, ft^2/s	Prandtl number Pr
100	0.06854	0.2489	0.01537	2.502×10^{-4}	1.221×10^{-5}	1.783×10^{-4}	0.7126
200	0.05815	0.2487	0.01760	3.379×10^{-4}	1.388×10^{-5}	2.387×10^{-4}	0.7062
300	0.05050	0.2492	0.01970	4.349×10^{-4}	1.543×10^{-5}	3.055×10^{-4}	0.7025
500	0.03997	0.2535	0.02359	6.466×10^{-4}	1.823×10^{-5}	4.559×10^{-4}	0.7051
1000	0.02628	0.2697	0.03204	1.255×10^{-3}	2.387×10^{-5}	9.083×10^{-4}	0.7232
1500	0.01958	0.2831	0.04002	2.006×10^{-3}	2.829×10^{-5}	1.445×10^{-3}	0.7202
2000	0.01560	0.2927	0.04918	2.992×10^{-3}	3.212×10^{-5}	2.059×10^{-3}	0.6882
			Oxygen, O_2				
−50	0.10697	0.2331	0.01216	1.355×10^{-4}	1.104×10^{-5}	1.032×10^{-4}	0.7622
0	0.09533	0.2245	0.01346	1.747×10^{-4}	1.218×10^{-5}	1.277×10^{-4}	0.7312
50	0.08598	0.2209	0.01475	2.157×10^{-4}	1.326×10^{-5}	1.543×10^{-4}	0.7152
100	0.07830	0.2200	0.01601	2.582×10^{-4}	1.429×10^{-5}	1.826×10^{-4}	0.7072
200	0.06643	0.2221	0.01851	3.484×10^{-4}	1.625×10^{-5}	2.446×10^{-4}	0.7020
300	0.05768	0.2262	0.02096	4.463×10^{-4}	1.806×10^{-5}	3.132×10^{-4}	0.7018
500	0.04566	0.2352	0.02577	6.665×10^{-4}	2.139×10^{-5}	4.685×10^{-4}	0.7029
1000	0.03002	0.2520	0.03698	1.357×10^{-3}	2.855×10^{-5}	9.509×10^{-4}	0.7005
1500	0.02236	0.2626	0.04701	2.224×10^{-3}	3.474×10^{-5}	1.553×10^{-3}	0.6985
2000	0.01782	0.2701	0.05614	3.241×10^{-3}	4.035×10^{-5}	2.265×10^{-3}	0.6988
			Water vapor, H_2O				
−50	0.06022	0.4512	0.00797	8.153×10^{-5}	4.933×10^{-6}	8.192×10^{-5}	1.0050
0	0.05367	0.4484	0.00898	1.036×10^{-4}	5.592×10^{-6}	1.041×10^{-4}	1.0049
50	0.04841	0.4472	0.01006	1.291×10^{-4}	6.261×10^{-6}	1.293×10^{-4}	1.0018
100	0.04408	0.4473	0.01121	1.579×10^{-4}	6.942×10^{-6}	1.574×10^{-4}	0.9969
200	0.03740	0.4503	0.01372	2.263×10^{-4}	8.333×10^{-6}	2.228×10^{-4}	0.9845
300	0.03248	0.4557	0.01648	3.093×10^{-4}	9.756×10^{-6}	3.004×10^{-4}	0.9713
500	0.02571	0.4707	0.02267	5.204×10^{-4}	1.267×10^{-5}	4.931×10^{-4}	0.9475
1000	0.01690	0.5167	0.04134	1.314×10^{-3}	2.014×10^{-5}	1.191×10^{-3}	0.9063
1500	0.01259	0.5625	0.06315	2.477×10^{-3}	2.742×10^{-5}	2.178×10^{-3}	0.8793
2000	0.01003	0.6034	0.08681	3.984×10^{-3}	3.422×10^{-5}	3.411×10^{-3}	0.8563

Note: For ideal gases, the properties c_p, k, μ, and Pr are independent of pressure. The properties ρ, ν, and α at a pressure P (in atm) other than 1 atm are determined by multiplying the values of ρ at the given temperature by P and by dividing ν and α by P.

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Originally based on various sources.

TABLE A–24E

Properties of solid metals

		Properties at 540 R				*Properties at various temperatures* (R), k(Btu/h·ft·R)/c_p(Btu/lbm·R)					
Composition	Melting point, R	ρ lbm/ft^3	c_p(Btu/ lbm·R)	k(Btu/ h·ft·R)	$\alpha \times 10^6$ ft^2/s	180	360	720	1080	1440	1800
Aluminum	1679	168	0.216	137	1045	174.5	137	138.6	133.4	126	
Pure						0.115	0.191	0.226	0.246	0.273	
Alloy 2024-T6	1395	173	0.209	102.3	785.8	37.6	94.2	107.5	107.5		
(4.5% Cu, 1.5% Mg, 0.6% Mn)						0.113	0.188	0.22	0.249		
Alloy 195, cast (4.5% Cu)		174.2	0.211	97	734		100.5	106.9			
Beryllium	2790	115.5	0.436	115.6	637.2	572	174	93	72.8	61.3	52.5
						0.048	0.266	0.523	0.621	0.624	0.72
Bismuth	981	610.5	0.029	4.6	71	9.5	5.6	4.06			
						0.026	0.028	0.03			
Boron	4631	156	0.264	15.6	105	109.7	32.06	9.7	6.1	5.5	5.7
						0.03	0.143	0.349	0.451	0.515	0.558
Cadmium	1069	540	0.055	55.6	521	117.3	57.4	54.7			
						0.047	0.053	0.057			
Chromium	3812	447	0.107	54.1	313.2	91.9	64.1	52.5	46.6	41.2	37.8
						0.045	0.091	0.115	0.129	0.138	0.147
Cobalt	3184	553.2	0.101	57.3	286.3	96.5	70.5	49.3	39	33.6	80.1
						0.056	0.09	0.107	0.12	0.131	0.145
Copper	2445	559	0.092	231.7	1259.3	278.5	238.6	227.07	219	212	203.4
Pure						0.06	0.085	0.094	0.01	0.103	0.107
Commercial bronze	2328	550	0.1	30	150.7	24.3	30	34			
(90% Cu, 10% Al)						0.187	0.109	0.130			
Phosphor gear bronze	1987	548.1	0.084	31.2	183	23.7	37.6	42.8			
(89% Cu, 11% Sn)						—	—	—			
Cartridge brass	2139	532.5	0.09	63.6	364.9	43.3	54.9	79.2	86.0		
(70% Cu, 30% Zn)						0.09	0.09	0.101			
Constantan	2687	557	0.092	13.3	72.3	9.8	1.1				
(55% Cu, 45% Ni)						0.06	0.09				
Germanium	2180	334.6	0.08	34.6	373.5	134	56	25	15.7	11.4	10.05
						0.045	0.069	0.08	0.083	0.085	0.089
Gold	2405	1205	0.03	183.2	1367	189	186.6	179.7	172.2	164.09	156
						0.026	0.029	0.031	0.032	0.033	0.034
Iridium	4896	1404.6	0.031	85	541.4	99.4	88.4	83.2	79.7	76.3	72.8
						0.021	0.029	0.031	0.032	0.034	0.036
Iron:	3258	491.3	0.106	46.4	248.6	77.4	54.3	40.2	31.6	25.01	19
Pure						0.051	0.091	0.117	0.137	0.162	0.232
Armco		491.3	0.106	42	222.8	55.2	46.6	38	30.7	24.4	18.7
(99.75% pure)						0.051	0.091	0.117	0.137	0.162	0.233
Carbon steels		490.3	0.103	35	190.6			32.8	27.7	22.7	17.4
Plain carbon (Mn $\leq$ 1%,											
Si $\leq$ 0.1%)								0.116	0.113	0.163	0.279
AISI 1010		489	0.103	37	202.4			33.9	28.2	22.7	18
								0.116	0.133	0.163	0.278
Carbon-silicon		488	0.106	30	160.4			28.8	25.4	21.6	17
(Mn $\leq$ 1%,											
0.1% < Si $\leq$ 0.6%)								0.119	0.139	0.166	0.231
Carbon-manganese-silicon		508	0.104	23.7	125			24.4	23	20.2	16
(1% < Mn $\leq$ 1.65%,											
0.1% < Si $\leq$ 0.6%)								0.116	0.133	0.163	0.260
Chromium (low) steels:		488.3	0.106	21.8	117.4			22	21.2	19.3	15.6
$\frac{1}{2}$ Cr–$\frac{1}{4}$ Mo–Si								0.117	0.137	0.164	0.23
(0.18% C, 0.65% Cr,											
0.23% Mo, 0.6% Si)											
1 Cr –$\frac{1}{2}$Mo (0.16% C,		490.6	0.106	24.5	131.3			24.3	22.6	20	15.8
1% Cr, 0.54% Mo, 0.39% Si)								0.117	0.137	0.164	0.231
1 Cr–V		489.2	0.106	28.3	151.8			27.0	24.3	21	16.3
(0.2% C, 1.02% Cr, 0.15% V)								0.117	0.137	0.164	0.231

TABLE A–24E

Properties of solid metals (*Concluded*)

Composition	Melting point, R	Properties at 540 R				Properties at various temperatures (R), k(Btu/h·ft·R)/c_p(Btu/lbm·R)					
		ρ lbm/ft^3	c_p(Btu/ lbm·R)	k(Btu/ h·ft·R)	$\alpha \times 10^6$ ft^2/s	180	360	720	1080	1440	1800
Stainless steels:		503	0.114	8.7	42			10	11.6	13.2	14.7
AISI 302								0.122	0.133	0.140	0.144
AISI 304	3006	493.2	0.114	8.6	42.5	5.31	7.3	9.6	11.5	13	14.7
						0.064	0.096	0.123	0.133	0.139	0.145
AISI 316		514.3	0.111	7.8	37.5			8.8	10.6	12.3	14
						0.12	0.131	0.137	0.143		
AISI 347		498	0.114	8.2	40			9.1	1.1	12.7	14.3
								0.122	0.133	0.14	0.144
Lead	1082	708	0.03	20.4	259.4	23	21.2	19.7	18.1		
						0.028	0.029	0.031	0.034		
Magnesium	1661	109	0.245	90.2	943	87.9	91.9	88.4	86.0	84.4	
						0.155	0.223	0.256	0.279	0.302	
Molybdenum	5209	639.3	0.06	79.7	578	1034	82.6	77.4	72.8	68.2	64.7
						0.038	0.053	0.062	0.065	0.068	0.070
Nickel:	3110	555.6	0.106	52.4	247.6	94.8	61.8	46.3	37.9	39	41.4
Pure						0.055	0.091	0.115	0.141	0.126	0.134
Nichrome	3010	524.4	0.1	6.9	36.6			8.0	9.3	12.2	
(80% Ni, 20% Cr)							0.114	0.125	0.130		
Inconel X-750	2997	531.3	0.104	6.8	33.4	5	5.9	7.8	9.8	11.8	13.9
(73% Ni, 15% Cr,											
6.7% Fe)						—	0.088	0.112	0.121	0.13	0.149
Niobium	4934	535	0.063	31	254	31.9	30.4	32	33.6	35.4	32.2
						0.044	0.059	0.065	0.067	0.069	0.071
Palladium	3289	750.4	0.058	41.5	263.7	44.2	41.4	42.5	46	50	54.4
						0.04	0.054	0.059	0.062	0.064	0.067
Platinum:	3681	1339	0.031	41.4	270	44.7	42	41.5	42.3	43.7	45.5
Pure						0.024	0.03	0.032	0.034	0.035	0.036
Alloy 60Pt-40Rh	3240	1038.2	0.038	27.2	187.3			30	34	37.5	40
(60% Pt, 40% Rh)								—	—	—	
Rhenium	6215	1317.2	0.032	27.7	180	34	30	26.6	25.5	25.4	25.8
						0.023	0.03	0.033	0.034	0.036	0.037
Rhodium	4025	777.2	0.058	86.7	534	107.5	89	84.3	78.5	73.4	70
						0.035	0.052	0.06	0.065	0.069	0.074
Silicon	3033	145.5	0.17	85.5	960.2	510.8	152.5	57.2	35.8	24.4	18.0
						0.061	0.132	0.189	0.207	0.218	0.226
Silver	2223	656	0.056	248	1873	257	248.4	245.5	238	228.8	219
						0.044	0.053	0.057	0.059	0.062	0.066
Tantalum	5884	1036.3	0.033	33.2	266	34.2	33.2	33.4	34	34.3	34.8
						0.026	0.031	0.034	0.035	0.036	0.036
Thorium	3641	730.4	0.028	31.2	420.9	34.6	31.5	31.4	32.2	32.9	32.9
						0.024	0.027	0.029	0.032	0.035	0.037
Tin	909	456.3	0.054	38.5	431.6	49.2	42.4	35.9			
						0.044	0.051	0.058			
Titanium	3515	281	0.013	12.7	100.3	17.6	14.2	11.8	11.2	11.4	12
						0.071	0.111	0.131	0.141	0.151	0.161
Tungsten	6588	1204.9	0.031	100.5	735.2	120.2	107.5	92	79.2	72.2	68.2
						0.020	0.029	0.032	0.033	0.034	0.035
Uranium	2531	1190.5	0.027	16	134.5	12.5	14.5	17.1	19.6	22.4	25.4
						0.022	0.026	0.029	0.035	0.042	0.043
Vanadium	3946	381	0.117	17.7	110.9	20.7	18	18	19.3	20.6	22.0
						0.061	0.102	0.123	0.128	0.134	0.142
Zinc	1247	445.7	0.093	67	450	67.6	68.2	64.1	59.5		
						0.07	0.087	0.096	0.104		
Zirconium	3825	410.2	0.067	13.1	133.5	19.2	14.6	12.5	12	12.5	13.7
						0.049	0.063	0.072	0.77	0.082	0.087

Source: Tables A–24E and A–25E are obtained from the respective tables in SI units in Appendix 1 using proper conversion factors.

TABLE A–25E

Properties of solid nonmentals

		Properties at 540 R				*Properties at various temperatures* (R), k(Btu/h·ft·R)/c_p(Btu/lbm·R)					
Composition	Melting point, R	ρ lbm/ft^3	c_p(Btu/ lbm·R)	k(Btu/ h·ft·R)	$\alpha \times 10^6$ ft^2/s	180	360	720	1080	1440	1800
Aluminum oxide,											
sapphire	4181	247.8	0.182	26.6	162.5	260	47.4	18.7	11	7.5	6
Aluminum oxide						—	—	0.224	0.265	0.281	0.293
polycrystalline	4181	247.8	0.182	20.8	128	76.8	31.7	15.3	9.3	6	4.5
								0.244	0.265	0.281	0.293
Beryllium oxide	4905	187.3	0.246	157.2	947.3			113.2	64.2	40.4	27.2
								0.322	0.40	0.44	0.459
Boron	4631	156	0.264	16	107.5	109.8	30.3	10.8	6.5	4.6	3.6
								0.355	0.445	0.509	0.561
Boron fiber epoxy	1062	130									
(30% vol.) composite											
k, ∥ to fibers				1.3		1.2	1.3	1.31			
k, ⊥ to fibers				0.34		0.21	0.28	0.34			
c_p			0.268			0.086	0.18	0.34			
Carbon, amorphous	2700	121.7	—	0.92	—	0.38	0.68	1.09	1.26	1.36	1.46
Diamond,	—	219	0.121	1329	—	5778	2311.2	889.8			
type IIa							0.005	0.046	0.203		
insulator	4091	138									
Graphite, pyrolytic				1126.7		2871.6	1866.3	803.2	515.4	385.4	308.5
k, ∥ to layers				3.3		9.7	5.3	2.4	1.5	1.16	0.92
k, ⊥ to layers											
c_p			0.169			0.32	0.098	0.236	0.335	0.394	0.428
Graphite fiber epoxy	810	87.4									
(25% vol.) composite				6.4		3.3	5.0	7.5			
k, heat flow ∥ to fibers				0.5	5	0.4	0.63				
k, heat flow ⊥ to fibers			0.223			0.08	0.153	0.29			
c_p	2921	162.3	0.193	2.3	20.3	3.0	2.3	2.1	1.9	1.7	1.7
Pyroceram, Corning 9606	5580	197.3	0.161	283.1	2475.7			—	—	—	50.3
Silicon carbide											
								0.210	0.25	0.27	0.285
Silicon dioxide,											
crystalline (quartz)	3389	165.4									
k , ∥ to c-axis				6		22.5	9.5	4.4	2.9	2.4	
k , ⊥ to c-axis				3.6		12.0	5.9	2.7	2	1.8	
c_p			0.177					0.211	0.256	0.298	
Silicon dioxide,	3389	138.6	0.177	0.79	9	0.4	0.65	0.87	1.01	1.25	1.65
polycrystalline (fused silica)					—	—	—	0.216	0.248	0.264	0.276
Silicon nitride	3911	150	0.165	9.2	104	—	—	8.0	6.5	5.7	5.0
						—	0.138	0.185	0.223	0.253	0.275
Sulfur	706	130	0.169	0.1	1.51	0.095	0.1				
						0.962	0.144				
Thorium dioxide	6431	568.7	0.561	7.5	65.7			5.9	3.8	2.7	2.12
								0.609	0.654	0.680	0.704
Titanium dioxide,	3840	259.5	0.170	4.9	30.1			4.0	2.9	2.3	2
polycrystalline								0.192	0.210	0.217	0.222

Source: Tables A–24E and A–25E are obtained from the respective tables in SI units in Appendix 1 using proper conversion factors.

INDEX

NOMENCLATURE

a	Acceleration, m/s^2
A_s	Surface area, m^2
A_c	Cross-sectional area, m^2
Bi	Biot number
c	Speed of sound, m/s
c	Specific heat, kJ/kg·K
C_c, C_h	Heat capacity rate, W/°C
C_D	Drag coefficient
C_f	Fanning friction factor or friction coefficient
C_L	Lift coefficient
c_p	Constant-pressure specific heat, kJ/kg·K
c_υ	Constant-volume specific heat, kJ/kg·K
COP	Coefficient of performance
COP_{HP}	Coefficient of performance of a heat pump
COP_{R}	Coefficient of performance of a refrigerator
d, D	Diameter, m
D_h	Hydraulic diameter, m
e	Specific total energy, kJ/kg
erfc	Complementary error function
E	Total energy, kJ
E_b	Blackbody emissive flux, W/m^2
EER	Energy efficiency rating
f	Darcy friction factor
f_λ	Blackbody radiation function
F	Force, N
F_D	Drag force, N
F_{ij}, $F_{i \to j}$	View factor
F_L	Lift force, N
Fo	Fourier number
g	Gravitational acceleration, m/s^2
G	Incident radiation, W/m^2
Gr	Grashof number
h	Convection heat transfer coefficient, W/m^2·°C
h	Specific enthalpy, $u + P\upsilon$, kJ/kg
h_c	Thermal contact conductance, W/m^2·°C
h_{fg}	Enthalpy of vaporization, kJ/kg
h_{if}	Latent heat of fusion, kJ/kg
h_L	Head loss, m
H	Total enthalpy, $U + PV$, kJ
I	Electric current, A; Bessel function
J	Radiosity, W/m^2; Bessel function
k	Specific heat ratio, c_p/c_υ
k	Thermal conductivity, W/m·°C
k_{eff}	Effective thermal conductivity, W/m·°C
k_s	Spring constant
ke	Specific kinetic energy, $V^2/2$, kJ/kg
K_L	Minor loss coefficient
KE	Total kinetic energy, $mV^2/2$, kJ
L	Length; half thickness of a plane wall
L_c	Characteristic or corrected length
L_h	Hydrodynamic entry length
L_t	Thermal entry length
m	Mass, kg
$\dot{m}$	Mass flow rate, kg/s
M	Molar mass, kg/kmol
M	Moment of force, N·m
Ma	Mach number
n	Polytropic exponent
N	Number of moles, kmol

NTU	Number of transfer units
Nu	Nusselt number
p	Perimeter or wetted perimeter, m
pe	Specific potential energy, gz, kJ/kg
P	Pressure, kPa
P_{cr}	Critical pressure, kPa
P_r	Relative pressure
P_R	Reduced pressure
P_v	Vapor pressure, kPa
PE	Total potential energy, mgz, kJ
Pr	Prandtl number
q	Heat transfer per unit mass, kJ/kg
$\dot{q}$	Heat flux, W/m^2
Q	Total heat transfer, kJ
$\dot{Q}$	Heat transfer rate, kW
Q_H	Heat transfer with high-temperature body, kJ
Q_L	Heat transfer with low-temperature body, kJ
r_{cr}	Critical radius of insulation, m
R	Gas constant, kJ/kg·K
R	Radius, m; Thermal resistance, °C/W
R_c	Thermal contact resistance, m^2·°C/W
R_f	Fouling factor, m^2·°C/W
R_u	Universal gas constant, kJ/kmol·K
R-value	R-value of insulation
Ra	Rayleigh number
Re	Reynolds number
s	Specific entropy, kJ/kg·K
s_{gen}	Specific entropy generation, kJ/kg·K
S	Total entropy, kJ/K
S_{gen}	Total entropy generation, kJ/K
t	Time, s; Thickness, m
T	Temperature, °C or K
T	Torque, N·m
T_b	Bulk fluid temperature, °C
T_{cr}	Critical temperature, K
T_f	Film temperature, °C
T_H	Temperature of high-temperature body, K
T_i	Initial temperature, °C
T_∞	Free stream or surrounding medium temperature, °C
T_L	Temperature of low-temperature body, K
T_R	Reduced temperature
T_s	Surface temperature, °C or K
T_{sat}	Saturation temperature, °C
u	Specific internal energy, kJ/kg
u,v	x- and y-components of velocity, m/s
U	Total internal energy, kJ
U	Overall heat transfer coefficient, W/m^2·°C
υ	Specific volume, m^3/kg
υ_{cr}	Critical specific volume, m^3/kg
υ_r	Relative specific volume
$\mathcal{V}$	Total volume, m^3
$\dot{\mathcal{V}}$	Volume flow rate, m^3/s
V	Velocity, m/s
V_m	Mean velocity, m/s
V_∞	Free-stream velocity, m/s
w	Work per unit mass, kJ/kg
W	Total work, kJ
$\dot{W}$	Power, kW
W_{in}	Work input, kJ
W_{out}	Work output, kJ
x	Quality
z	Elevation, m
Z	Compressibility factor

Greek Letters

α	Absorptivity
α	Thermal diffusivity, m^2/s
α_s	Solar absorptivity
β	Volume expansivity, 1/K
δ	Velocity boundary layer thickness, m
δ_t	Thermal boundary layer thickness, m
ΔP	Pressure drop, Pa
ΔT_{lm}	Log mean temperature difference, °C
ε	Emissivity; heat exchanger or fin effectiveness
ε	Mean surface roughness, m
ζ	Vorticity, 1/s
η_{fin}	Fin efficiency
η_{th}	Thermal efficiency
θ	Dimensionless temperature
μ	Dynamic viscosity, kg/m·s or $N{\cdot}s/m^2$
ν	Kinematic viscosity $= \mu/\rho$, m^2/s; frequency, 1/s
ρ	Density, kg/m^3
σ	Stefan–Boltzmann constant
σ_n	Normal stress, N/m^2
σ_s	Surface tension, N/m
τ	Shear stress, N/m^2
τ	Transmissivity; Fourier number
τ_w	Wall shear stress, N/m^2

Subscripts

a	Air
abs	Absolute
act	Actual
atm	Atmospheric
b	Boundary; bulk fluid
cond	Conduction
conv	Convection
cr	Critical point
CV	Control volume
e	Exit conditions
f	Saturated liquid; film
fg	Difference in property between saturated liquid and saturated vapor
g	Saturated vapor
H	High temperature as in T_H and Q_H
i	Inlet, initial, or indoor conditions
L	Low temperature as in T_L and Q_L
o	Outlet or outdoor conditions
r	Relative
rad	Radiation
s	Surface
surr	Surrounding surfaces
sat	Saturated
sys	System
v	Water vapor
1	Initial or inlet state
2	Final or exit state
∞	Property of a far field; free-flow conditions

Superscripts

˙ (over dot)	Quantity per unit time
¯ (over bar)	Quantity per unit mole
° (circle)	Standard reference state

CONVERSION FACTORS

Dimension	Metric	Metric/English
Acceleration	1 m/s^2 = 100 cm/s^2	1 m/s^2 = 3.2808 ft/s^2 1 ft/s^2 = 0.3048* m/s^2
Area	1 m^2 = 10^4 cm^2 = 10^6 mm^2 = 10^{-6} km^2	1 m^2 = 1550 in^2 = 10.764 ft^2 1 ft^2 = 144 in^2 = 0.09290304* m^2
Density	1 g/cm^3 = 1 kg/L = 1000 kg/m^3	1 g/cm^3 = 62.428 lbm/ft^3 = 0.036127 lbm/in^3 1 lbm/in^3 = 1728 lbm/ft^3 1 kg/m^3 = 0.062428 lbm/ft^3
Energy, heat, work, and specific energy	1 kJ = 1000 J = 1000 N·m = 1 kPa·m^3 1 kJ/kg = 1000 m^2/s^2 1 kWh = 3600 kJ	1 kJ = 0.94782 Btu 1 Btu = 1.055056 kJ = 5.40395 psia·ft^3 = 778.169 lbf·ft 1 Btu/lbm = 25,037 ft^2/s^2 = 2.326* kJ/kg 1 kWh = 3412.14 Btu
Force	1 N = 1 kg·m/s^2 = 10^5 dyne 1 kgf = 9.80665 N	1 N = 0.22481 lbf 1 lbf = 32.174 lbm·ft/s^2 = 4.44822 N 1 lbf = 1 slug·ft/s^2
Length	1 m = 100 cm = 1000 mm = 10^6 μm 1 km = 1000 m	1 m = 39.370 in = 3.2808 ft = 1.0926 yd 1 ft = 12 in = 0.3048* m 1 mile = 5280 ft = 1.6093 km 1 in = 2.54* cm
Mass	1 kg = 1000 g 1 metric ton = 1000 kg	1 kg = 2.2046226 lbm 1 lbm = 0.45359237* kg 1 ounce = 28.3495 g 1 slug = 32.174 lbm = 14.5939 kg 1 short ton = 2000 lbm = 907.1847 kg
Power	1 W = 1 J/s 1 kW = 1000 W = 1 kJ/s 1 hp‡ = 745.7 W	1 kW = 3412.14 Btu/h = 1.341 hp = 737.56 lbf·ft/s 1 hp = 550 lbf·ft/s = 0.7068 Btu/s = 42.41 Btu/min = 2544.5 Btu/h = 0.74570 kW 1 Btu/h = 1.055056 kJ/h
Pressure or stress, and pressure expressed as a head	1 Pa = 1 N/m^2 1 kPa = 10^3 Pa = 10^{-3} MPa 1 atm = 101.325 kPa = 1.01325 bar = 760 mm Hg at 0°C = 1.03323 kgf/cm^2 1 mm Hg = 0.1333 kPa	1 Pa = 1.4504×10^{-4} psi = 0.020886 lbf/ft^2 1 psi = 144 lbf/ft^2 = 6.894757 kPa 1 atm = 14.696 psi = 29.92 inches Hg at 30°F 1 inch Hg = 13.60 inches H_2O = 3.387 kPa
Specific heat	1 kJ/kg·°C = 1 kJ/kg·K = 1 J/g·°C	1 Btu/lbm·°F = 4.1868 kJ/kg·°C 1 Btu/lbmol·R = 4.1868 kJ/kmol·K 1 kJ/kg·°C = 0.23885 Btu/lbm·°F = 0.23885 Btu/lbm·R
Specific volume	1 m^3/kg = 1000 L/kg = 1000 cm^3/g	1 m^3/kg = 16.02 ft^3/lbm 1 ft^3/lbm = 0.062428 m^3/kg
Temperature	$T(\text{K}) = T(°\text{C}) + 273.15$ $\Delta T(\text{K}) = \Delta T(°\text{C})$	$T(\text{R}) = T(°\text{F}) + 459.67 = 1.8T(\text{K})$ $T(°\text{F}) = 1.8\,T(°\text{C}) + 32$ $\Delta T(°\text{F}) = \Delta T(\text{R}) = 1.8\Delta T(\text{K})$
Velocity	1 m/s = 3.60 km/h	1 m/s = 3.2808 ft/s = 2.237 mi/h 1 mi/h = 1.46667 ft/s 1 mi/h = 1.6093 km/h
Viscosity, dynamic	1 kg/m·s = 1 N·s/m^2 = 1 Pa·s = 10 poise	1 kg/m·s = 2419.1 lbm/ft·h = 0.020886 lbf·s/ft^2 = 0.67197 lbm/ft·s

*Exact conversion factor between metric and English units.

‡Mechanical horsepower. The electrical horsepower is taken to be exactly 746 W.

Dimension	Metric	Metric/English
Viscosity, kinematic	$1\ \text{m}^2/\text{s} = 10^4\ \text{cm}^2/\text{s}$ $1\ \text{stoke} = 1\ \text{cm}^2/\text{s} = 10^{-4}\ \text{m}^2/\text{s}$	$1\ \text{m}^2/\text{s} = 10.764\ \text{ft}^2/\text{s} = 3.875 \times 10^4\ \text{ft}^2/\text{h}$ $1\ \text{m}^2/\text{s} = 10.764\ \text{ft}^2/\text{s}$
Volume	$1\ \text{m}^3 = 1000\ \text{L} = 10^6\ \text{cm}^3$ (cc)	$1\ \text{m}^3 = 6.1024 \times 10^4\ \text{in}^3 = 35.315\ \text{ft}^3$ $= 264.17$ gal (U.S.) 1 U.S. gallon $= 231\ \text{in}^3 = 3.7854$ L 1 fl ounce $= 29.5735\ \text{cm}^3 = 0.0295735$ L 1 U.S. gallon = 128 fl ounces
Volume flow rate	$1\ \text{m}^3/\text{s} = 60{,}000\ \text{L/min} = 10^6\ \text{cm}^3/\text{s}$	$1\ \text{m}^3/\text{s} = 15{,}850\ \text{gal/min} = 35.315\ \text{ft}^3/\text{s}$ $= 2118.9\ \text{ft}^3/\text{min}$ (CFM)

Some physical constants

Physical Constant	Metric	English
Standard acceleration of gravity	$g = 9.80665\ \text{m/s}^2$	$g = 32.174\ \text{ft/s}^2$
Standard atmospheric pressure	$P_{\text{atm}} = 1\ \text{atm} = 101.325\ \text{kPa}$ $= 1.01325$ bar $= 760$ mm Hg (0°C) $= 10.3323\ \text{m H}_2\text{O}$ (4°C)	$P_{\text{atm}} = 1\ \text{atm} = 14.696\ \text{psia}$ $= 2116.2\ \text{lbf/ft}^2$ $= 29.9213$ inches Hg (32°F) $= 406.78\ \text{inches H}_2\text{O}$ (39.2°F)
Universal gas constant	$R_u = 8.31447\ \text{kJ/kmol·K}$ $= 8.31447\ \text{kN·m/kmol·K}$	$R_u = 1.9859\ \text{Btu/lbmol·R}$ $= 1545.37\ \text{ft·lbf/lbmol·R}$

Commonly used properties

Property	Metric	English
Air at 20°C (68°F) and 1 atm		
Specific gas constant*	$R_{\text{air}} = 0.2870\ \text{kJ/kg·K}$ $= 287.0\ \text{m}^2/\text{s}^2\text{·K}$	$R_{\text{air}} = 0.06855\ \text{Btu/lbm·R}$ $= 53.34\ \text{ft·lbf/lbm·R}$ $= 1716\ \text{ft}^2/\text{s}^2\text{·R}$
Specific heat ratio	$k = c_p/c_v = 1.40$	$k = c_p/c_v = 1.40$
Specific heats	$c_p = 1.005\ \text{kJ/kg·K}$ $= 1005\ \text{m}^2/\text{s}^2\text{·K}$ $c_v = 0.718\ \text{kJ/kg·K}$ $= 718\ \text{m}^2/\text{s}^2\text{·K}$	$c_p = 0.240\ \text{Btu/lbm·R}$ $= 187\ \text{ft·lbf/lbm·R}$ $= 6009\ \text{ft}^2/\text{s}^2\text{·R}$ $c_v = 0.171\ \text{Btu/lbm·R}$ $= 133\ \text{ft·lbf/lbm·R}$ $= 4281\ \text{ft}^2/\text{s}^2\text{·R}$
Speed of sound	$c = 343.2\ \text{m/s} = 1236\ \text{km/h}$	$c = 1126\ \text{ft/s} = 767.7\ \text{mi/h}$
Density	$\rho = 1.204\ \text{kg/m}^3$	$\rho = 0.07518\ \text{lbm/ft}^3$
Viscosity	$\mu = 1.825 \times 10^{-5}\ \text{kg/m·s}$	$\mu = 1.227 \times 10^{-5}\ \text{lbm/ft·s}$
Kinematic viscosity	$\nu = 1.516 \times 10^{-5}\ \text{m}^2/\text{s}$	$\nu = 1.632 \times 10^{-4}\ \text{ft}^2/\text{s}$
Liquid water at 20°C (68°F) and 1 atm		
Specific heat ($c = c_p = c_v$)	$c = 4.182\ \text{kJ/kg·K}$ $= 4182\ \text{m}^2/\text{s}^2\text{·K}$	$c = 0.9989\ \text{Btu/lbm·R}$ $= 777.3\ \text{ft·lbf/lbm·R}$ $= 25{,}009\ \text{ft}^2/\text{s}^2\text{·R}$
Density	$\rho = 998.0\ \text{kg/m}^3$	$\rho = 62.30\ \text{lbm/ft}^3$
Viscosity	$\mu = 1.002 \times 10^{-3}\ \text{kg/m·s}$	$\mu = 6.733 \times 10^{-4}\ \text{lbm/ft·s}$
Kinematic viscosity	$\nu = 1.004 \times 10^{-6}\ \text{m}^2/\text{s}$	$\nu = 1.081 \times 10^{-5}\ \text{ft}^2/\text{s}$

*Independent of pressure or temperature