Borgnakke Sonntag

Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 1

8e

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CONTENT CHAPTER 1

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In-Text Concept Questions

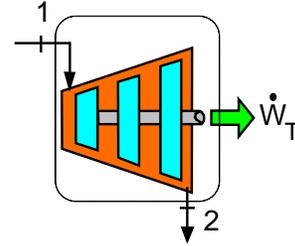Borgnakke and Sonntag

1.a

Make a control volume around the turbine in the steam power plant in Fig. 1.2 and list the flows of mass and energy that are there.

Solution:

We see hot high pressure steam flowing in at state 1 from the steam drum through a flow control (not shown). The steam leaves at a lower pressure to the condenser (heat exchanger) at state 2. A rotating shaft gives a rate of energy (power) to the electric generator set.
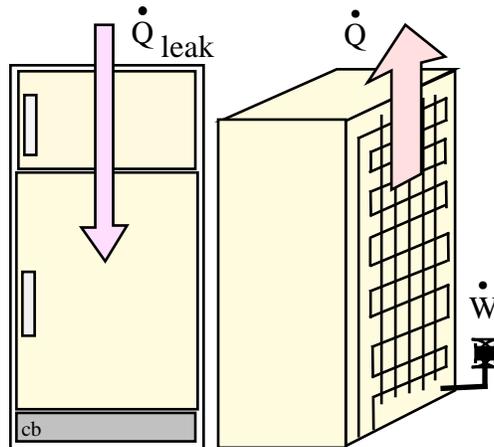
1.b

Take a control volume around your kitchen refrigerator and indicate where the components shown in Figure 1.3 are located and show all flows of energy transfers.

Solution:

The valve and the cold line, the evaporator, is inside close to the inside wall and usually a small blower distributes cold air from the freezer box to the refrigerator room.

$\dot{Q}_{leak}$

$\dot{Q}$

$\dot{W}$

cb

The black grille in the back or at the bottom is the condenser that gives heat to the room air.

The compressor sits at the bottom.

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1.c

Why do people float high in the water when swimming in the Dead Sea as compared with swimming in a fresh water lake?

As the dead sea is very salty its density is higher than fresh water density. The buoyancy effect gives a force up that equals the weight of the displaced water. Since density is higher the displaced volume is smaller for the same force.

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1.d

Density of liquid water is $\rho = 1008 - T/2$ [kg/m^3] with T in $^\circ$C. If the temperature increases, what happens to the density and specific volume?

Solution:

The density is seen to decrease as the temperature increases.

$$\Delta\rho = -\Delta T/2$$

Since the specific volume is the inverse of the density $v = 1/\rho$ it will increase.

1.e

A car tire gauge indicates 195 kPa; what is the air pressure inside?

The pressure you read on the gauge is a gauge pressure, ΔP, so the absolute pressure is found as
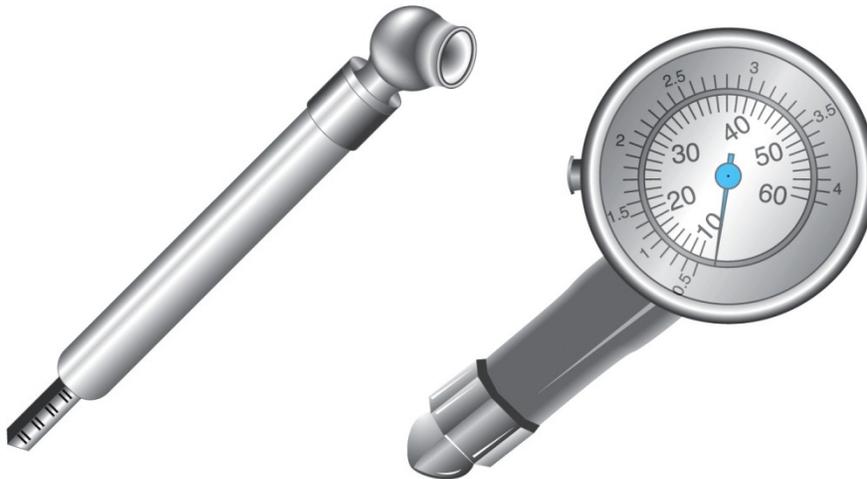
$$P = P_o + \Delta P = 101 + 195 = 296 \text{ kPa}$$

Figure 1.21
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1.f

Can I always neglect ΔP in the fluid above location A in figure 1.13? What does that depend on?
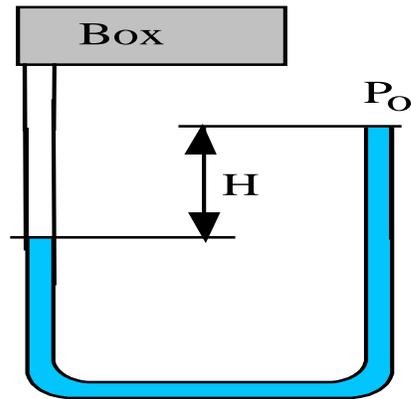
If the fluid density above A is low relative to the manometer fluid then you neglect the pressure variation above position A, say the fluid is a gas like air and the manometer fluid is like liquid water. However, if the fluid above A has a density of the same order of magnitude as the manometer fluid then the pressure variation with elevation is as large as in the manometer fluid and it must be accounted for.

1.g

A U tube manometer has the left branch connected to a box with a pressure of 110 kPa and the right branch open. Which side has a higher column of fluid?

Solution:

Since the left branch fluid surface feels 110 kPa and the right branch surface is at 100 kPa you must go further down to match the 110 kPa. The right branch has a higher column of fluid.
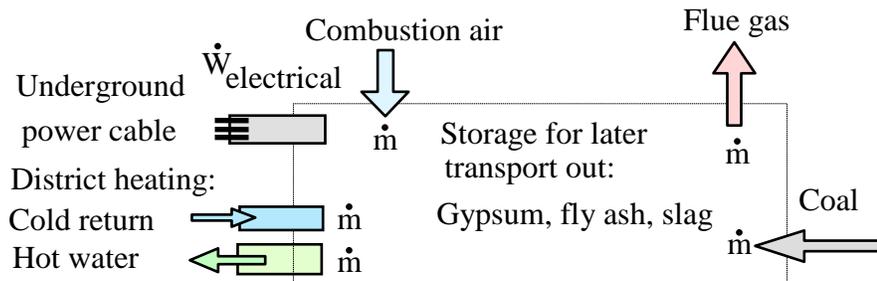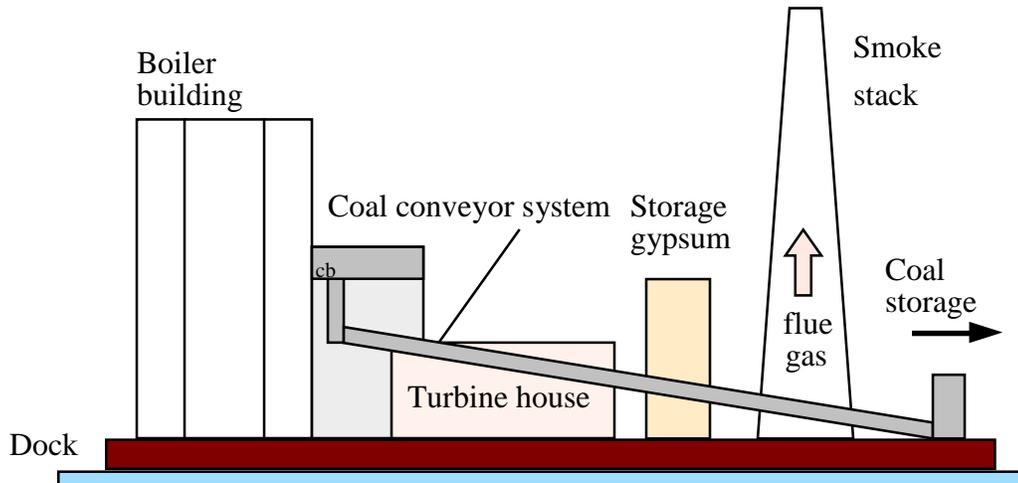
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Concept Problems

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1.1

Make a control volume around the whole power plant in Fig. 1.1 and with the help of Fig. 1.2 list what flows of mass and energy are in or out and any storage of energy. Make sure you know what is inside and what is outside your chosen C.V.
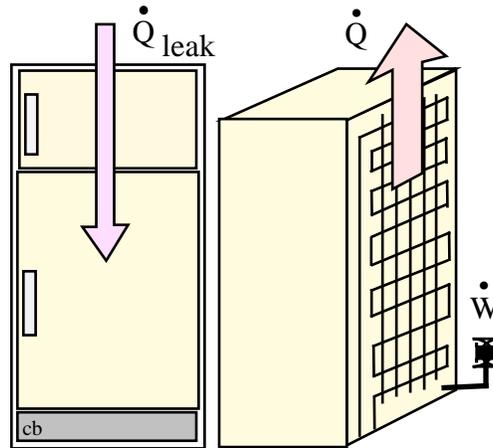
Solution:

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1.2

Make a control volume around the refrigerator in Fig. 1.3. Identify the mass flow of external air and show where you have significant heat transfer and where storage changes.

The valve and the cold line, the evaporator, is inside close to the inside wall and usually a small blower distributes cold air from the freezer box to the refrigerator room.

$\dot{Q}_{leak}$ $\dot{Q}$ $\dot{W}$

cb

The black grille in the back or at the bottom is the condenser that gives heat to the room air.

The compressor sits at the bottom.

The storage changes inside the box which is outside of the refrigeration cycle components of Fig. 1.3, when you put some warmer mass inside the refrigerator it is being cooled by the evaporator and the heat is leaving in the condenser.

The condenser warms outside air so the air flow over the condenser line carries away some energy. If natural convection is not enough to do this a small fan is used to blow air in over the condenser (forced convection). Likewise the air being cooled by the evaporator is redistributed inside the refrigerator by a small fan and some ducts.

Since the room is warmer than the inside of the refrigerator heat is transferred into the cold space through the sides and the seal around the door. Also when the door is opened warm air is pulled in and cold air comes out from the refrigerator giving a net energy transfer similar to a heat transfer.

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1.3

Separate the list P, F, V, v, ρ, T, a, m, L, t, and **V** into intensive, extensive, and non-properties.

Solution:

Intensive properties are independent upon mass: P, v, ρ, T
Extensive properties scales with mass: V, m
Non-properties: F, a, L, t, **V**

Comment: You could claim that acceleration a and velocity **V** are physical properties for the dynamic motion of the mass, but not thermal properties.

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1.4

A tray of liquid water is placed in a freezer where it cools from 20$^{\text{o}}$C to -5$^{\text{o}}$C. Show the energy flow(s) and storage and explain what changes.

Inside the freezer box, the walls are very cold as they are the outside of the evaporator, or the air is cooled and a small fan moves the air around to redistribute the cold air to all the items stored in the freezer box. The fluid in the evaporator absorbs the energy and the fluid flows over to the compressor on its way around the cycle, see Fig. 1.3. As the water is cooled it eventually reaches the freezing point and ice starts to form. After a significant amount of energy is removed from the water it is turned completely into ice (at 0$^{\text{o}}$C) and then cooled a little more to -5$^{\text{o}}$C. The water has a negative energy storage and the energy is moved by the refrigerant fluid out of the evaporator into the compressor and then finally out of the condenser into the outside room air.

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1.5

The overall density of fibers, rock wool insulation, foams and cotton is fairly low. Why is that?

Solution:

All these materials consist of some solid substance and mainly air or other gas. The volume of fibers (clothes) and rockwool that is solid substance is low relative to the total volume that includes air. The overall density is

$$\rho = \frac{m}{V} = \frac{m_{solid} + m_{air}}{V_{solid} + V_{air}}$$

where most of the mass is the solid and most of the volume is air. If you talk about the density of the solid only, it is high.
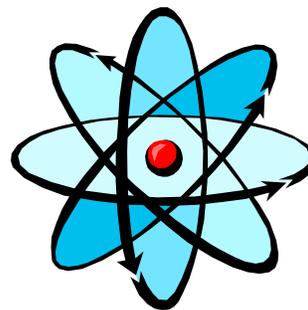
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1.6

Is density a unique measure of mass distribution in a volume? Does it vary? If so, on what kind of scale (distance)?

Solution:

Density is an average of mass per unit volume and we sense if it is not evenly distributed by holding a mass that is more heavy in one side than the other. Through the volume of the same substance (say air in a room) density varies only little from one location to another on scales of meter, cm or mm. If the volume you look at has different substances (air and the furniture in the room) then it can change abruptly as you look at a small volume of air next to a volume of hardwood.

Finally if we look at very small scales on the order of the size of atoms the density can vary infinitely, since the mass (electrons, neutrons and positrons) occupy very little volume relative to all the empty space between them.

1.7

Water in nature exists in different phases such as solid, liquid and vapor (gas). Indicate the relative magnitude of density and specific volume for the three phases.

Solution:

Values are indicated in Figure 1.8 as density for common substances. More accurate values are found in Tables A.3, A.4 and A.5

Water as solid (ice) has density of around 900 kg/m^3
Water as liquid has density of around 1000 kg/m^3
Water as vapor has density of around 1 kg/m^3 (sensitive to P and T)

Ice cube   Liquid drops falling   Cloud*

* Steam (water vapor) cannot be seen, what you see are tiny drops suspended in air from which we infer that there was some water vapor before it condensed.

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1.8

What is the approximate mass of 1 L of gasoline? Of helium in a balloon at T_o, P_o?

Solution:

Gasoline is a liquid slightly lighter than liquid water so its density is smaller than 1000 kg/m^3. 1 L is 0.001 m^3 which is a common volume used for food items. A more accurate density is listed in Table A.3 as 750 kg/m^3 so the mass becomes

$$m = \rho \, V = 750 \text{ kg/m}^3 \times 0.001 \text{ m}^3 = 0.75 \text{ kg}$$

The helium is a gas highly sensitive to P and T, so its density is listed at the standard conditions (100 kPa, 25C) in Table A.5 as $\rho = 0.1615$ kg/m^3,

$$m = \rho \, V = 0.1615 \text{ kg/m}^3 \times 0.001 \text{ m}^3 = 1.615 \times 10^{-4} \text{ kg}$$

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1.9

Can you carry 1 m^3 of liquid water?

Solution:

The density of liquid water is about 1000 kg/m^3 from Figure 1.7, see also Table A.3. Therefore the mass in one cubic meter is

$$m = \rho V = 1000 \text{ kg/m}^3 \times 1 \text{ m}^3 = 1000 \text{ kg}$$

and we can not carry that in the standard gravitational field.

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1.10

A heavy refrigerator has four height-adjustable feet. What feature of the feet will ensure that they do not make dents in the floor?

Answer:
The area that is in contact with the floor supports the total mass in the gravitational field.

$$F = PA = mg$$

so for a given mass the smaller the area is the larger the pressure becomes.

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1.11

A swimming pool has an evenly distributed pressure at the bottom. Consider a stiff steel plate lying on the ground. Is the pressure below it just as evenly distributed?
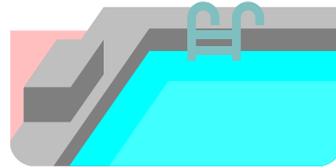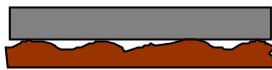
Solution:
The pressure is force per unit area from page 13:
$$P = F/A = mg/A$$

The steel plate can be reasonable plane and flat, but it is stiff and rigid. However, the ground is usually uneven so the contact between the plate and the ground is made over an area much smaller than the actual plate area. Thus the local pressure at the contact locations is much larger than the average indicated above.

The pressure at the bottom of the swimming pool is very even due to the ability of the fluid (water) to have full contact with the bottom by deforming itself. This is the main difference between a fluid behavior and a solid behavior.
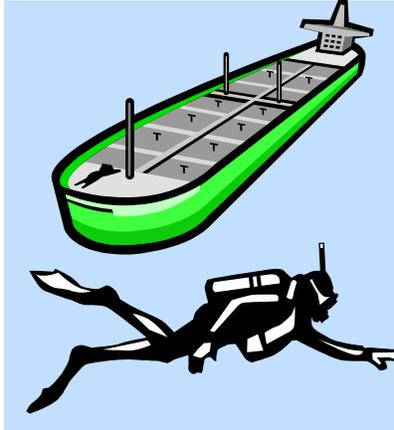
Steel plate
Ground

1.12

What physically determines the variation of the atmospheric pressure with elevation?

The total mass of the column of air over a unit area and the gravitation gives the force which per unit area is pressure. This is an integral of the density times gravitation over elevation as in Eq.1.4.

To perform the integral the density and gravitation as a function of height (elevation) should be known. Later we will learn that air density is a function of temperature and pressure (and compositions if it varies). Standard curve fits are known that describes this variation and you can find tables with the information about a standard atmosphere. See problems 1.28, 1.64, and 1.95 for some examples.

1.13

Two divers swim at 20 m depth. One of them swims right in under a supertanker; the other stays away from the tanker. Who feels a greater pressure?

Solution:

Each one feels the local pressure which is the static pressure only a function of depth.

$$P_{ocean} = P_0 + \Delta P = P_0 + \rho g H$$

So they feel exactly the same pressure.

1.14

A manometer with water shows a ΔP of $P_o/20$; what is the column height difference?

Solution:

$$\Delta P = P_o/20 = \rho Hg$$

$$H = P_o/(20 \; \rho \; g) = \frac{101.3 \times 1000 \text{ Pa}}{20 \times 997 \text{ kg/m}^3 \times 9.80665 \text{ m/s}^2}$$

$$= \textbf{0.502 m}$$

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1.15

    Does the pressure have to be uniform for equilibrium to exist?

    No.   It depends on what causes a pressure difference. Think about the pressure increasing as you move down into the ocean, the water at different levels are in equilibrium. However if the pressure is different at nearby locations at same elevation in the water or in air that difference induces a motion of the fluid from the higher towards the lower pressure. The motion will persist as long as the pressure difference exist.

1.16

A water skier does not sink too far down in the water if the speed is high enough. What makes that situation different from our static pressure calculations?

The water pressure right under the ski is not a static pressure but a static plus dynamic pressure that pushes the water away from the ski. The faster you go, the smaller amount of water is displaced, but at a higher velocity.
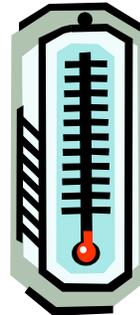
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1.17

What is the lowest temperature in degrees Celsuis? In degrees Kelvin?

Solution:

The lowest temperature is absolute zero which is
at zero degrees Kelvin at which point the
temperature in Celsius is negative

$$T_K = 0 \text{ K} = -273.15 \,^{\circ}C$$

1.18

Convert the formula for water density in In-text Concept Question "d" to be for T in degrees Kelvin.

Solution:

$$\rho = 1008 - T_C/2 \quad [\text{kg/m}^3]$$

We need to express degrees Celsius in degrees Kelvin
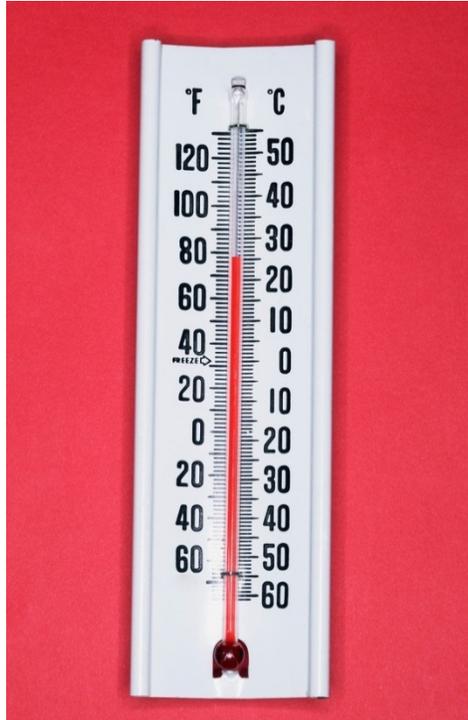
$$T_C = T_K - 273.15$$

and substitute into formula

$$\rho = 1008 - T_C/2 = 1008 - (T_K - 273.15)/2 = 1144.6 - T_K/2$$

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1.19

A thermometer that indicates the temperature with a liquid column has a bulb with a larger volume of liquid, why is that?

The expansion of the liquid volume with temperature is rather small so by having a larger volume expand with all the volume increase showing in the very small diameter column of fluid greatly increases the signal that can be read.

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1.20

What is the main difference between the macroscopic kinetic energy in a motion like the blowing of wind versus the microscopic kinetic energy of individual molecules? Which one can you sense with your hand?
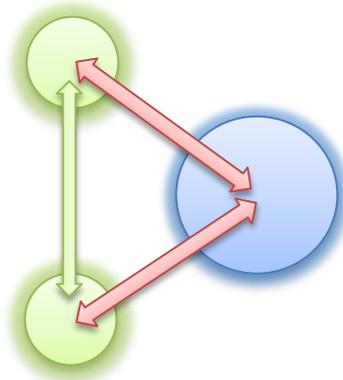
Answer:

The microscopic kinetic energy of individual molecules is too small for us to sense however when the combined action of billions (actually more like in the order of 1 E19) are added we get to the macroscopic magnitude we can sense. The wind velocity is the magnitude and direction of the averaged velocity over many molecules which we sense. The individual molecules are moving in a random motion (with zero average) on top of this mean (or average) motion. A characteristic velocity of this random motion is the speed of sound, around 340 m/s for atmospheric air and it changes with temperature.

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1.21

How can you illustrate the binding energy between the three atoms in water as they sit in a tri-atomic water molecule. Hint: imagine what must happen to create three separate atoms.

Answer:

If you want to separate the atoms you must pull them apart. Since they are bound together with strong forces (like non-linear springs) you apply a force over a distance which is work (energy in transfer) to the system and you could end up with two hydrogen atoms and one oxygen atom far apart so they no longer have strong forces between them. If you do not do anything else the atoms will sooner or later recombine and release all the energy you put in and the energy will come out as radiation or given to other molecules by collision interactions.

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Properties, Units, and Force

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1.22

An apple "weighs" 60 g and has a volume of 75 cm^3 in a refrigerator at 8°C. What is the apple density? List three intensive and two extensive properties of the apple.

Solution:

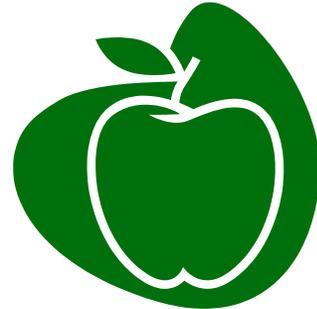$$\rho = \frac{m}{V} = \frac{0.06}{0.000\ 075}\ \frac{kg}{m^3} = 800\ \frac{kg}{m^3}$$

Intensive

$$\rho = 800\ \frac{kg}{m^3}\ ; \qquad v = \frac{1}{\rho} = 0.001\ 25\ \frac{m^3}{kg}$$

$$T = 8°C; \qquad P = 101\ kPa$$

Extensive

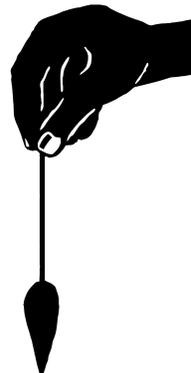m = 60 g = 0.06 kg

V = 75 cm^3 = 0.075 L = 0.000 075 m^3

1.23

One kilopond (1 kp) is the weight of 1 kg in the standard gravitational field. How many Newtons (N) is that?

$F = ma = mg$

$1 \text{ kp} = 1 \text{ kg} \times 9.807 \text{ m/s}^2 = \textbf{9.807 N}$

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1.24

A stainless steel storage tank contains 5 kg of oxygen gas and 7 kg of nitrogen gas. How many kmoles are in the tank?

Table A.2: $M_{O2} = 31.999$; $M_{N2} = 28.013$

$$n_{O2} = m_{O2} / M_{O2} = \frac{5}{31.999} = 0.15625 \text{ kmol}$$

$$n_{O2} = m_{N2} / M_{N2} = \frac{7}{28.013} = 0.24988 \text{ kmol}$$

$$n_{tot} = n_{O2} + n_{N2} = 0.15625 + 0.24988 = \textbf{0.40613 kmol}$$

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1.25

A steel cylinder of mass 4 kg contains 4 L of liquid water at 25°C at 100 kPa. Find the total mass and volume of the system. List two extensive and three intensive properties of the water

Solution:

Density of steel in Table A.3: $\quad \rho = 7820 \text{ kg/m}^3$

Volume of steel: $\qquad\qquad\qquad V = m/\rho = \dfrac{4 \text{ kg}}{7820 \text{ kg/m}^3} = 0.000\ 512 \text{ m}^3$

Density of water in Table A.4: $\quad \rho = 997 \text{ kg/m}^3$

Mass of water: $\qquad\qquad m = \rho V = 997 \text{ kg/m}^3 \times 0.004 \text{ m}^3 = 3.988 \text{ kg}$

Total mass: $\qquad\qquad\qquad m = m_{steel} + m_{water} = 4 + 3.988 = \textbf{7.988 kg}$

Total volume: $\qquad\qquad\quad V = V_{steel} + V_{water} = 0.000\ 512 + 0.004$

$$= \textbf{0.004 512 } \textbf{m}^3 = \textbf{4.51 L}$$

Extensive properties: m, V
Intensive properties: ρ (or $v = 1/\rho$), T, P

1.26

The "standard" acceleration (at sea level and 45° latitude) due to gravity is 9.80665 m/s^2. What is the force needed to hold a mass of 2 kg at rest in this gravitational field? How much mass can a force of 1 N support?
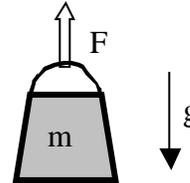
Solution:

$ma = 0 = \sum F = F - mg$

$F = mg = 2 \text{ kg} \times 9.80665 \text{ m/s}^2 =$ **19.613 N**

$F = mg \qquad\qquad =>$

$m = \dfrac{F}{g} = \dfrac{1 \text{ N}}{9.80665 \text{ m/s}^2} =$ **0.102 kg**
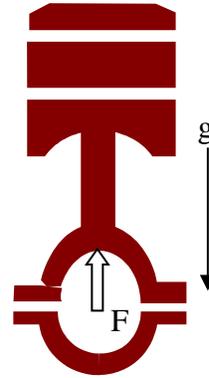
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1.27

An aluminum piston of 2.5 kg is in the standard gravitational field where a force of 25 N is applied vertically up. Find the acceleration of the piston.

Solution:

$$F_{up} = ma = F - mg$$

$$a = \frac{F - mg}{m} = \frac{F}{m} - g$$

$$= \frac{25 \text{ N}}{2.5 \text{ kg}} - 9.807 \text{ m/s}^2$$

$$= \mathbf{0.193 \text{ ms}^{-2}}$$

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1.28

When you move up from the surface of the earth the gravitation is reduced as $g = 9.807 - 3.32 \times 10^{-6} z$, with z as the elevation in meters. How many percent is the weight of an airplane reduced when it cruises at 11 000 m?

Solution:

$$g_O = 9.807 \text{ ms}^{-2}$$

$$g_H = 9.807 - 3.32 \times 10^{-6} \times 11\ 000 = 9.7705 \text{ ms}^{-2}$$

$$W_O = m\ g_O \quad ; \quad W_H = m\ g_H$$

$$W_H/W_O = g_H/g_O = \frac{9.7705}{9.807} = 0.9963$$

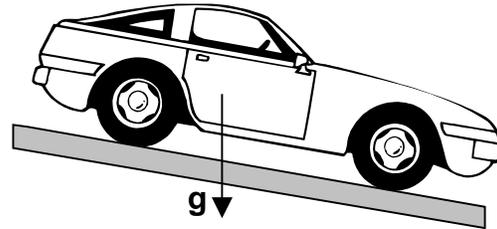Reduction $= 1 - 0.9963 = 0.0037$ or **0.37%**

i.e. we can neglect that for most applications.

1.29

A car rolls down a hill with a slope so the gravitational "pull" in the direction of motion is one tenth of the standard gravitational force (see Problem 1.26). If the car has a mass of 2500 kg find the acceleration.

Solution:

$ma = \sum F = mg / 10$

$a = mg / 10m = g/10$

$= 9.80665 \ (m/s^2) / 10$

$= \mathbf{0.981 \ m/s^2}$

This acceleration does not depend on the mass of the car.

1.30

A van is driven at 60 km/h and is brought to a full stop with constant deceleration in 5 seconds. If the total car and driver mass is 2075 kg find the necessary force.

Solution:

Acceleration is the time rate of change of velocity.

$$a = \frac{dV}{dt} = \frac{60 \times 1000}{3600 \times 5} = 3.333 \text{ m/s}^2$$

$ma = \sum F$;

$F_{net} = ma = 2075 \text{ kg} \times 3.333 \text{ m/s}^2 = \textbf{6916 N}$

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1.31

A 1500-kg car moving at 20 km/h is accelerated at a constant rate of 4 m/s^2 up to a speed of 75 km/h. What are the force and total time required?

Solution:

$$a = \frac{d\mathbf{V}}{dt} = \frac{\Delta \mathbf{V}}{\Delta t} \quad =>$$

$$\Delta t = \frac{\Delta \mathbf{V}}{a} = \frac{(75 - 20) \text{ km/h} \times 1000 \text{ m/km}}{3600 \text{ s/h} \times 4 \text{ m/s}^2} = \mathbf{3.82 \text{ sec}}$$
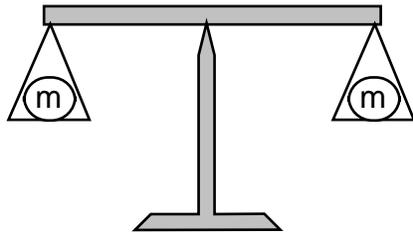
$$F = ma = 1500 \text{ kg} \times 4 \text{ m/s}^2 = \mathbf{6000 \text{ N}}$$
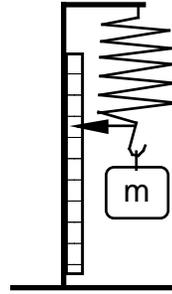
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1.32

On the moon the gravitational acceleration is approximately one-sixth that on the surface of the earth. A 5-kg mass is "weighed" with a beam balance on the surface on the moon. What is the expected reading? If this mass is weighed with a spring scale that reads correctly for standard gravity on earth (see Problem 1.26), what is the reading?

Solution:

Moon gravitation is: $g = g_{earth}/6$

Beam Balance Reading is **5 kg**   Spring Balance Reading is in kg units

This is mass comparison   Force comparison   length $\propto F \propto g$

$$\text{Reading will be } \frac{5}{6} \textbf{ kg}$$
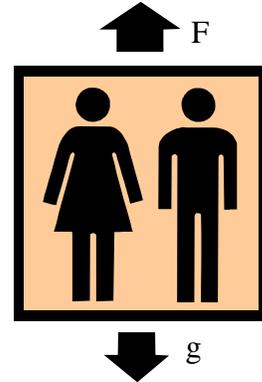
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1.33

The elevator in a hotel has a mass of 750 kg, and it carries six people with a total mass of 450 kg. How much force should the cable pull up with to have an acceleration of 1 m/s^2 in the upwards direction?

Solution:

The total mass moves upwards with an acceleration plus the gravitations acts with a force pointing down.

ma = Σ F = F – mg

F = ma + mg = m(a + g)
= (750 + 450) kg × (1 + 9.81) m/s^2
= **12 972 N**

1.34

One of the people in the previous problem weighs 80 kg standing still. How much weight does this person feel when the elevator starts moving?

Solution:

The equation of motion is
$$ma = \sum F = F - mg$$
so the force from the floor becomes
$$F = ma + mg = m(a + g)$$
$$= 80 \text{ kg} \times (1 + 9.81) \text{ m/s}^2$$
$$= 864.8 \text{ N}$$
$$= x \text{ kg} \times 9.81 \text{ m/s}^2$$

Solve for x

$$x = 864.8 \text{ N/ } 9.81 \text{ m/s}^2 = \textbf{88.15 kg}$$

The person then feels like having a mass of 88 kg instead of 80 kg. The weight is really force so to compare to standard mass we should use kp. So in this example the person is experiencing a force of 88 kp instead of the normal 80 kp.

1.35

A bottle of 12 kg steel has 1.75 kmole of liquid propane. It accelerates horizontal with 3 m/s^2, what is the needed force?
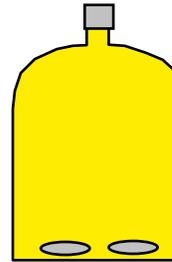
Solution:

The molecular weight for propane is M = 44.094 from Table A.2. The force must accelerate both the container mass and the propane mass.

$$m = m_{steel} + m_{propane} = 12 + (1.75 \times 44.094) = 90.645 \text{ kg}$$

$$ma = \sum F \quad \Rightarrow$$
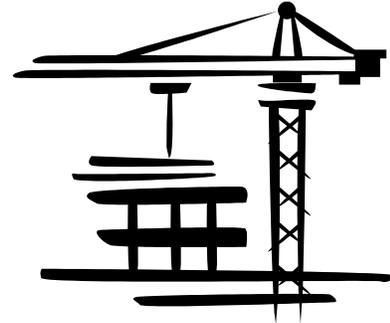$$F = ma = 90.645 \text{ kg} \times 3 \text{ m/s}^2 = \textbf{271.9 N}$$

1.36

Some steel beams with a total mass of 700 kg are raised by a crane with an acceleration of 2 m/s^2 relative to the ground at a location where the local gravitational acceleration is 9.5 m/s^2. Find the required force.

Solution:

$F = ma = F_{up} - mg$

$F_{up} = ma + mg = 700 \text{ kg} \ (\ 2 + 9.5 \ ) \ \text{m/s}^2$

$\qquad = \textbf{80 500 N}$

Borgnakke and Sonntag

Specific Volume

1.37

A 1 m^3 container is filled with 400 kg of granite stone, 200 kg dry sand and 0.2 m^3 of liquid 25°C water. Use properties from tables A.3 and A.4. Find the average specific volume and density of the masses when you exclude air mass and volume.

Solution:

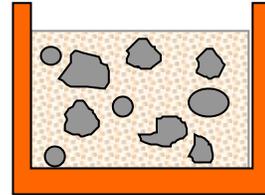Specific volume and density are ratios of total mass and total volume.

$$m_{liq} = V_{liq}/v_{liq} = V_{liq}\ \rho_{liq} = 0.2\ m^3 \times 997\ kg/m^3 = 199.4\ kg$$

$$m_{TOT} = m_{stone} + m_{sand} + m_{liq} = 400 + 200 + 199.4 = 799.4\ kg$$

$$V_{stone} = mv = m/\rho = 400\ kg/\ 2750\ kg/m^3 = 0.1455\ m^3$$

$$V_{sand} = mv = m/\rho = 200/\ 1500 = 0.1333\ m^3$$

$$V_{TOT} = V_{stone} + V_{sand} + V_{liq}$$
$$= 0.1455 + 0.1333 + 0.2 = 0.4788\ m^3$$

$$v = V_{TOT}\ /\ m_{TOT} = 0.4788/799.4 = \textbf{0.000599 m}^\textbf{3}\textbf{/kg}$$

$$\rho = 1/v = m_{TOT}/V_{TOT} = 799.4/0.4788 = \textbf{1669.6 kg/m}^\textbf{3}$$

Borgnakke and Sonntag

1.38

A power plant that separates carbon-dioxide from the exhaust gases compresses it to a density of 110 kg/m^3 and stores it in an un-minable coal seam with a porous volume of 100 000 m^3. Find the mass they can store.

Solution:

$$m = \rho \, V = 110 \text{ kg/m}^3 \times 100\,000 \text{ m}^3 = 11 \times 10^{6} \text{ kg}$$

Comment:
Just to put this in perspective a power plant that generates 2000 MW by burning coal would make about 20 million tons of carbon-dioxide a year. That is 2000 times the above mass so it is nearly impossible to store all the carbon-dioxide being produced.
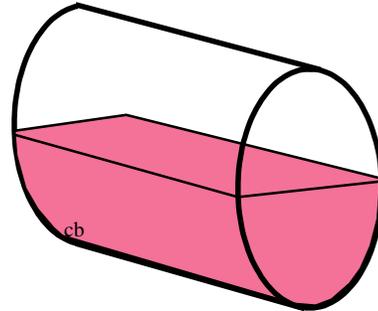
Borgnakke and Sonntag

1.39

A 15-kg steel gas tank holds 300 L of liquid gasoline, having a density of 800 kg/m^3. If the system is decelerated with 2g what is the needed force?

Solution:

$m = m_{tank} + m_{gasoline}$

$= 15 \text{ kg} + 0.3 \text{ m}^3 \times 800 \text{ kg/m}^3$

$= 255 \text{ kg}$

$F = ma = 255 \text{ kg} \times 2 \times 9.81 \text{ m/s}^2$

$= \textbf{5003 N}$

Borgnakke and Sonntag

1.40

A 5 m^3 container is filled with 900 kg of granite (density 2400 kg/m^3) and the rest of the volume is air with density 1.15 kg/m^3. Find the mass of air and the overall (average) specific volume.

Solution:

$$m_{air} = \rho\, V = \rho_{air}\, (\, V_{tot} - \frac{m_{granite}}{\rho}\,)$$

$$= 1.15\, [\, 5 - \frac{900}{2400}\,] = 1.15\, \times 4.625 = \textbf{5.32 kg}$$

$$v = \frac{V}{m} = \frac{5}{900 + 5.32} = \textbf{0.005 52 m}^\textbf{3}\textbf{/kg}$$

Comment: Because the air and the granite are not mixed or evenly distributed in the container the overall specific volume or density does not have much meaning.

Borgnakke and Sonntag

1.41

A tank has two rooms separated by a membrane. Room A has 1 kg air and volume 0.5 m^3, room B has 0.75 m^3 air with density 0.8 kg/m^3. The membrane is broken and the air comes to a uniform state. Find the final density of the air.
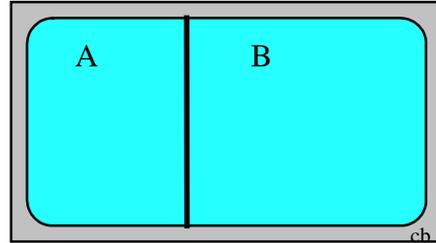
Solution:

Density is mass per unit volume

$m = m_A + m_B = m_A + \rho_B V_B = 1 + 0.8 \times 0.75 = 1.6$ kg

$V = V_A + V_B = 0.5 + 0.75 = 1.25$ m^3

$\rho = \dfrac{m}{V} = \dfrac{1.6}{1.25} \dfrac{kg}{m^3} =$ **1.28 kg/m^3**

Borgnakke and Sonntag

1.42

One kilogram of diatomic oxygen (O_2 molecular weight 32) is contained in a 500-L tank. Find the specific volume on both a mass and mole basis (v and $\bar{v}$).

Solution:

From the definition of the specific volume

$$v = \frac{V}{m} = \frac{0.5}{1} \frac{m^3}{kg} = \mathbf{0.5\ m^3/kg}$$

$$\bar{v} = \frac{V}{n} = \frac{V}{m/M} = M\ v = 32\ kg/kmol \times 0.5\ m^3/kg = \mathbf{16\ m^3/kmol}$$
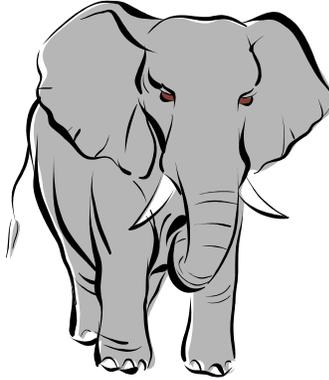
Borgnakke and Sonntag

Pressure

Borgnakke and Sonntag

1.43

A 5000-kg elephant has a cross sectional area of 0.02 m^2 on each foot. Assuming an even distribution, what is the pressure under its feet?

Force balance: ma = 0 = PA – mg

$$P = mg/A = 5000 \text{ kg} \times 9.81 \text{ m/s}^2 /(4 \times 0.02 \text{ m}^2)$$
$$= 613\ 125 \text{ Pa} = \textbf{613 kPa}$$

1.44

A valve in a cylinder has a cross sectional area of 11 cm^2 with a pressure of 735 kPa inside the cylinder and 99 kPa outside. How large a force is needed to open the valve?

$$F_{net} = P_{in}A - P_{out}A$$

$$= (735 - 99) \text{ kPa} \times 11 \text{ cm}^2$$

$$= 6996 \text{ kPa cm}^2$$

$$= 6996 \times \frac{kN}{m^2} \times 10^{-4} \text{ m}^2$$

$$= \textbf{700 N}$$

P_{cyl}

cb

1.45

The hydraulic lift in an auto-repair shop has a cylinder diameter of 0.2 m. To what pressure should the hydraulic fluid be pumped to lift 40 kg of piston/arms and 700 kg of a car?

Solution:
Force acting on the mass by the gravitational field

$$F{\downarrow} = ma = mg = 740 \times 9.80665 = 7256.9 \text{ N} = 7.257 \text{ kN}$$

Force balance: $\qquad F{\uparrow} = (P - P_0)\,A = F{\downarrow} \qquad => \quad P = P_0 + F{\downarrow} / A$

$$A = \pi\,D^2\,(1/4) = 0.031416 \text{ m}^2$$

$$P = 101 \text{ kPa} + \frac{7.257 \text{ kN}}{0.031416 \text{ m}^2} = \textbf{332 kPa}$$

1.46

A hydraulic lift has a maximum fluid pressure of 500 kPa. What should the piston-cylinder diameter be so it can lift a mass of 850 kg?

Solution:

With the piston at rest the static force balance is

$$F\uparrow = P\,A = F\downarrow = mg$$
$$A = \pi\,r^2 = \pi\,D^2/4$$
$$PA = P\,\pi\,D^2/4 = mg \quad\Rightarrow\quad D^2 = \frac{4mg}{P\,\pi}$$

$$D = 2\sqrt{\frac{mg}{P\pi}} = 2\sqrt{\frac{850\ \text{kg} \times 9.807\ \text{m/s}^2}{500\ \text{kPa} \times \pi \times 1000\ (\text{Pa/kPa})}} = \mathbf{0.146\ m}$$

1.47

A laboratory room keeps a vacuum of 0.1 kPa. What net force does that put on the door of size 2 m by 1 m?

Solution:

The net force on the door is the difference between the forces on the two sides as the pressure times the area

$$F = P_{outside}\ A - P_{inside}\ A = \Delta P\ A = 0.1\ kPa \times 2\ m \times 1\ m = \textbf{200 N}$$

Remember that kPa is kN/m^2.

$P_{abs} = P_O - \Delta P$

$\Delta P = 0.1\ kPa$

1.48

A vertical hydraulic cylinder has a 125-mm diameter piston with hydraulic fluid inside the cylinder and an ambient pressure of 1 bar. Assuming standard gravity, find the piston mass that will create a pressure inside of 1500 kPa.

Solution:

Force balance:

$$F\uparrow = PA = F\downarrow = P_0A + m_pg;$$

$$P_0 = 1 \text{ bar} = 100 \text{ kPa}$$

$$A = (\pi/4)\,D^2 = (\pi/4) \times 0.125^2 = 0.01227 \text{ m}^2$$

$$m_p = (P - P_0)\,\frac{A}{g} = (\,1500 - 100\,)\text{ kPa} \times 1000 \text{ Pa/kPa} \times \frac{0.01227}{9.80665}\frac{\text{m}^2}{\text{m/s}^2}$$

$$= \mathbf{1752 \text{ kg}}$$

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1.49

A 75-kg human footprint is 0.05 m^2 when the human is wearing boots. Suppose you want to walk on snow that can at most support an extra 3 kPa; what should the total snowshoe area be?

Force balance: $ma = 0 = PA - mg$

$$A = \frac{mg}{P} = \frac{75 \text{ kg} \times 9.81 \text{ m/s}^2}{3 \text{ kPa}} = 0.245 \text{ m}^2$$

1.50

A piston/cylinder with cross sectional area of 0.01 m^2 has a piston mass of 100 kg resting on the stops, as shown in Fig. P1.50. With an outside atmospheric pressure of 100 kPa, what should the water pressure be to lift the piston?

Solution:

The force acting down on the piston comes from gravitation and the outside atmospheric pressure acting over the top surface.

Force balance: $\quad$ $F\uparrow = F\downarrow = PA = m_p g + P_0 A$

Now solve for P (divide by 1000 to convert to kPa for 2nd term)

$$P = P_0 + \frac{m_p g}{A} = 100 \text{ kPa} + \frac{100 \times 9.80665}{0.01 \times 1000} \text{ kPa}$$

$$= 100 \text{ kPa} + 98.07 \text{ kPa} = \textbf{198 kPa}$$

cb

Water

1.51

A large exhaust fan in a laboratory room keeps the pressure inside at 10 cm water vacuum relative to the hallway. What is the net force on the door measuring 1.9 m by 1.1 m?

Solution:

The net force on the door is the difference between the forces on the two sides as the pressure times the area

$$F = P_{outside} \, A - P_{inside} \, A = \Delta P \times A$$
$$= 10 \text{ cm H}_2\text{O} \times 1.9 \text{ m} \times 1.1 \text{ m}$$
$$= 0.10 \times 9.80638 \text{ kPa} \times 2.09 \text{ m}^2$$
$$= \textbf{2049 N}$$

Table A.1: 1 m H_2O is 9.80638 kPa and kPa is kN/m^2.

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1.52

A tornado rips off a 100 m^2 roof with a mass of 1000 kg. What is the minimum vacuum pressure needed to do that if we neglect the anchoring forces?

Solution:

The net force on the roof is the difference between the forces on the two sides as the pressure times the area

$$F = P_{inside}\, A - P_{outside}A = \Delta P\, A$$

That force must overcome the gravitation mg, so the balance is

$$\Delta P\, A = mg$$

$$\Delta P = mg/A = (1000 \text{ kg} \times 9.807 \text{ m/s}^2)/100 \text{ m}^2 = \textbf{98 Pa = 0.098 kPa}$$

Remember that kPa is kN/m^2.

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1.53

A 5-kg cannon-ball acts as a piston in a cylinder with a diameter of 0.15 m. As the gun-powder is burned a pressure of 7 MPa is created in the gas behind the ball. What is the acceleration of the ball if the cylinder (cannon) is pointing horizontally?

Solution:

The cannon ball has 101 kPa on the side facing the atmosphere.

$$ma = F = P_1 \times A - P_0 \times A = (P_1 - P_0) \times A$$

$$= (7000 - 101) \text{ kPa} \times \pi (0.15^2 / 4) \text{ m}^2 = 121.9 \text{ kN}$$

$$a = \frac{F}{m} = \frac{121.9 \text{ kN}}{5 \text{ kg}} = \mathbf{24\ 380\ m/s^2}$$

1.54

Repeat the previous problem for a cylinder (cannon) pointing 40 degrees up relative to the horizontal direction.

Solution:

$$ma = F = (P_1 - P_0) A - mg \sin 40^0$$

$$ma = (7000 - 101) \text{ kPa} \times \pi \times (0.15^2 / 4) \text{ m}^2 - 5 \times 9.807 \times 0.6428 \text{ N}$$
$$= 121.9 \text{ kN} - 31.52 \text{ N} = 121.87 \text{ kN}$$

$$a = \frac{F}{m} = \frac{121.87 \text{ kN}}{5 \text{ kg}} = \textbf{24 374 m/s}^2$$

1.55

A 2.5 m tall steel cylinder has a cross sectional area of 1.5 m^2. At the bottom with a height of 0.5 m is liquid water on top of which is a 1 m high layer of gasoline. This is shown in Fig. P1.55. The gasoline surface is exposed to atmospheric air at 101 kPa. What is the highest pressure in the water?

Solution:

The pressure in the fluid goes up with the depth as

$$P = P_{top} + \Delta P = P_{top} + \rho g h$$

and since we have two fluid layers we get

$$P = P_{top} + [(\rho h)_{gasoline} + (\rho h)_{water}] \, g$$

The densities from Table A.4 are:

$$\rho_{gasoline} = 750 \text{ kg/m}^3; \quad \rho_{water} = 997 \text{ kg/m}^3$$

$$P = 101 \text{ kPa} + [750 \times 1 + 997 \times 0.5] \text{ kg/m}^2 \times \frac{9.807}{1000} \text{ (m/s}^2) \text{ (kPa/Pa)}$$

$$= \textbf{113.2 kPa}$$

1.56

An underwater buoy is anchored at the seabed with a cable, and it contains a total mass of 250 kg. What should the volume be so that the cable holds it down with a force of 1000 N?

Solution:

We need to do a force balance on the system at rest and the combined pressure over the buoy surface is the buoyancy (lift) equal to the "weight" of the displaced water volume

$$ma = 0 = m_{H2O}g - mg - F$$
$$= \rho_{H2O}Vg - mg - F$$
$$V = (mg + F)/\rho_{H2O}g = (m + F/g)/\rho_{H2O}$$
$$= (250 \text{ kg} + 1000 \text{ N}/9.81 \text{ m/s}^2) / 997 \text{ kg/m}^3$$
$$= \mathbf{0.353 \text{ m}^3}$$

1.57

At the beach, atmospheric pressure is 1025 mbar. You dive 15 m down in the ocean and you later climb a hill up to 250 m elevation. Assume the density of water is about 1000 kg/m^3 and the density of air is 1.18 kg/m^3. What pressure do you feel at each place?

Solution:

$\Delta P = \rho gh$,

Units from A.1:   1 mbar = 100 Pa   (1 bar = 100 kPa).

$$P_{ocean} = P_0 + \Delta P = 1025 \times 100 \text{ Pa} + 1000 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 15 \text{ m}$$

$$= 2.4965 \times 10^5 \text{ Pa} = \textbf{250 kPa}$$

$$P_{hill} = P_0 - \Delta P = 1025 \times 100 \text{ Pa} - 1.18 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 250 \text{ m}$$

$$= 0.99606 \times 10^5 \text{ Pa} = \textbf{99.61 kPa}$$

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1.58

What is the pressure at the bottom of a 5 m tall column of fluid with atmospheric pressure 101 kPa on the top surface if the fluid is

a) water at 20°C b) glycerine 25°C or c) gasoline 25°C

Solution:

Table A.4: $\rho_{H2O} = 997 \text{ kg/m}^3$; $\rho_{Glyc} = 1260 \text{ kg/m}^3$; $\rho_{gasoline} = 750 \text{ kg/m}^3$

$$\Delta P = \rho gh \qquad\qquad P = P_{top} + \Delta P$$
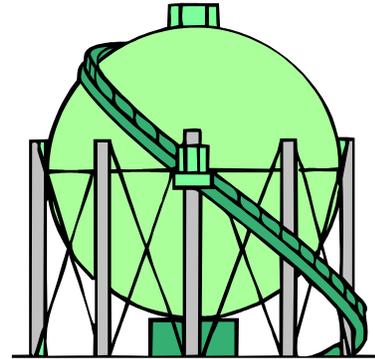
a) $\Delta P = \rho gh = 997 \times 9.807 \times 5 = 48\ 888 \text{ Pa}$
    $P = 101 + 48.99 = \textbf{149.9 kPa}$

b) $\Delta P = \rho gh = 1260 \times 9.807 \times 5 = 61\ 784 \text{ Pa}$
    $P = 101 + 61.8 = \textbf{162.8 kPa}$

c) $\Delta P = \rho gh = 750 \times 9.807 \times 5 = 36\ 776 \text{ Pa}$
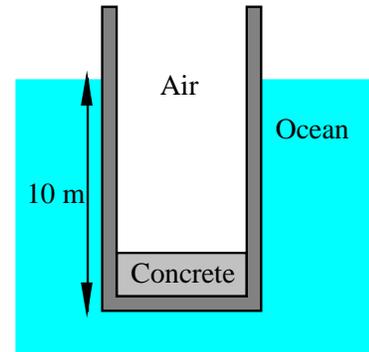 $P = 101 + 36.8 = \textbf{137.8 kPa}$

1.59

A steel tank of cross sectional area 3 m^2 and 16 m tall weighs 10 000 kg and it is open at the top, as shown in Fig. P1.59. We want to float it in the ocean so it sticks 10 m straight down by pouring concrete into the bottom of it. How much concrete should I put in?

Solution:

The force up on the tank is from the water pressure at the bottom times its area. The force down is the gravitation times mass and the atmospheric pressure.

$$F\uparrow = PA = (\rho_{ocean}gh + P_0)A$$

$$F\downarrow = (m_{tank} + m_{concrete})g + P_0A$$

The force balance becomes

$$F\uparrow = F\downarrow = (\rho_{ocean}gh + P_0)A = (m_{tank} + m_{concrete})g + P_0A$$

Solve for the mass of concrete

$$m_{concrete} = (\rho_{ocean}hA - m_{tank}) = 997 \times 10 \times 3 - 10\ 000 = \textbf{19 910 kg}$$

Notice: The first term is the mass of the displaced ocean water. The force up is the weight (mg) of this mass called buoyancy which balances with gravitation and the force from P$_0$ cancel.

1.60

A piston, $m_p = 5$ kg, is fitted in a cylinder, $A = 15$ cm^2, that contains a gas. The setup is in a centrifuge that creates an acceleration of 25 m/s^2 in the direction of piston motion towards the gas. Assuming standard atmospheric pressure outside the cylinder, find the gas pressure.

Solution:

Force balance: $\quad F\uparrow = F\downarrow = P_0 A + m_p g = PA$

$P = P_0 + \dfrac{m_p g}{A}$

$= 101.325 + \dfrac{5 \times 25}{1000 \times 0.0015} \dfrac{\text{kPa kg m/s}^2}{\text{Pa m}^2}$

$= \mathbf{184.7 \text{ kPa}}$

1.61

Liquid water with density ρ is filled on top of a thin piston in a cylinder with cross-sectional area A and total height H, as shown in Fig. P1.61. Air is let in under the piston so it pushes up, spilling the water over the edge. Derive the formula for the air pressure as a function of piston elevation from the bottom, h.

Solution:

Force balance

P0

Piston: $F\!\uparrow = F\!\downarrow$

$PA = P_0A + m_{H2O}g$

$P = P_0 + m_{H2O}g/A$

$\mathbf{P = P_0 + (H - h)\rho g}$

H

h

P

P_0

h, V   air

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Manometers and Barometers

Borgnakke and Sonntag

1.62

A probe is lowered 16 m into a lake. Find the absolute pressure there?

Solution:

The pressure difference for a column is from Eq.1.2 and the density of water is from Table A.4.

$$\Delta P = \rho g H$$
$$= 997 \ \text{kg/m}^3 \times 9.81 \ \text{m/s}^2 \times 16 \ \text{m}$$
$$= 156 \ 489 \ \text{Pa} = 156.489 \ \text{kPa}$$
$$P_{ocean} = P_0 + \Delta P$$
$$= 101.325 + 156.489$$
$$= \mathbf{257.8 \ kPa}$$

1.63

The density of atmospheric air is about 1.15 kg/m^3, which we assume is constant. How large an absolute pressure will a pilot see when flying 2000 m above ground level where the pressure is 101 kPa.

Solution:

Assume g and ρ are constant then the pressure difference to carry a column of height 2000 m is from Fig.2.10

$$\Delta P = \rho g h = 1.15 \text{ kg/m}^3 \times 9.807 \text{ ms}^{-2} \times 2000 \text{ m}$$
$$= 22\ 556 \text{ Pa} = 22.6 \text{ kPa}$$

The pressure on top of the column of air is then

$$P = P_0 - \Delta P = 101 - 22.6 = \textbf{78.4 kPa}$$

1.64

The standard pressure in the atmosphere with elevation (H) above sea level can be correlated as $P = P_0 (1 - H/L)^{5.26}$ with $L = 44\ 300$ m. With the local sea level pressure P_0 at 101 kPa, what is the pressure at 10 000 m elevation?

$$P = P_0 (1 - H/L)^{5.26}$$
$$= 101 \text{ kPa } (1 - 10\ 000/44\ 300)^{5.26}$$
$$= \mathbf{26.3 \text{ kPa}}$$

1.65

A barometer to measure absolute pressure shows a mercury column height of 725 mm. The temperature is such that the density of the mercury is 13 550 kg/m^3. Find the ambient pressure.

Solution:

Hg : L = 725 mm = 0.725 m; ρ = 13 550 kg/m^3

The external pressure P balances the column of height L so from Fig. 1.14

$P = \rho\, L\, g$ = 13 550 kg/m^3 × 9.80665 m/s^2 × 0.725 m × 10^{-3} kPa/Pa

     = **96.34 kPa**

This is a more common type that does not involve mercury as an older wall mounted unit.

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1.66

A differential pressure gauge mounted on a vessel shows 1.25 MPa and a local barometer gives atmospheric pressure as 0.96 bar. Find the absolute pressure inside the vessel.

Solution:

Convert all pressures to units of kPa.

$$P_{gauge} = 1.25 \text{ MPa} = 1250 \text{ kPa};$$

$$P_0 = 0.96 \text{ bar} = 96 \text{ kPa}$$

$$P = P_{gauge} + P_0 = 1250 + 96 = \textbf{1346 kPa}$$

1.67

A manometer shows a pressure difference of 1 m of liquid mercury. Find ΔP in kPa.

Solution:

Hg : L = 1 m; ρ = 13 580 kg/m^3 from Table A.4 (or read Fig 1.8)

The pressure difference ΔP balances the column of height L so from Eq.1.2

$\Delta P = \rho\ g\ L$ = 13 580 kg/m^3 × 9.80665 m/s^2 × 1.0 m × 10^{-3} kPa/Pa

     = **133.2 kPa**

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1.68

Blue manometer fluid of density 925 kg/m^3 shows a column height difference of 3 cm vacuum with one end attached to a pipe and the other open to $P_0 = 101$ kPa.

What is the absolute pressure in the pipe?

Solution:

Since the manometer shows a vacuum we have

$$P_{PIPE} = P_0 - \Delta P$$

$$\Delta P = \rho gh = 925 \text{ kg/m}^3 \times 9.807 \text{ m/s}^2 \times 0.03 \text{ m}$$

$$= 272.1 \text{ Pa} = 0.272 \text{ kPa}$$

$$P_{PIPE} = 101 - 0.272 = \textbf{100.73 kPa}$$

Pipe

P_O

1.69

What pressure difference does a 10 m column of atmospheric air show?

Solution:

The pressure difference for a column is from Eq.1.2

$$\Delta P = \rho g H$$

So we need density of air from Fig. 2.8 or Table A.5, $\rho = 1.2$ kg/m^3

$$\Delta P = 1.2 \text{ kg/m}^3 \times 9.81 \text{ ms}^{-2} \times 10 \text{ m} = 117.7 \text{ Pa} = \textbf{0.12 kPa}$$

1.70

A barometer measures 760 mmHg at street level and 735 mmHg on top of a building. How tall is the building if we assume air density of 1.15 kg/m^3?

Solution:

$\Delta P = \rho g H$

$$H = \Delta P/\rho g = \frac{760 - 735}{1.15 \times 9.807} \frac{\text{mmHg}}{\text{kg/m}^2\text{s}^2} \frac{133.32 \text{ Pa}}{\text{mmHg}} = \textbf{295 m}$$

1.71

The pressure gauge on an air tank shows 75 kPa when the diver is 10 m down in the ocean. At what depth will the gauge pressure be zero? What does that mean?

Ocean H_2O pressure at 10 m depth is

$$P_{water} = P_0 + \rho Lg = 101.3 + \frac{997 \times 10 \times 9.80665}{1000} = 199 \text{ kPa}$$

Air Pressure (absolute) in tank

$P_{tank} = 199 + 75 = 274$ kPa

Tank Pressure (gauge) reads zero at H_2O local pressure

$$274 = 101.3 + \frac{997 \times 9.80665}{1000} L$$

L = **17.66 m**

At this depth you will have to suck the air in, it can no longer push itself through a valve.

Borgnakke and Sonntag

1.72

An exploration submarine should be able to go 1200 m down in the ocean. If the ocean density is 1020 kg/m^3 what is the maximum pressure on the submarine hull?

Solution:

Assume we have atmospheric pressure inside the submarine then the pressure difference to the outside water is

$$\Delta P = \rho Lg = (1020 \text{ kg/m}^3 \times 1200 \text{ m} \times 9.807 \text{ m/s}^2) / (1000 \text{ Pa/kPa})$$
$$= 12\ 007 \text{ kPa} \approx \textbf{12 MPa}$$

1.73

A submarine maintains 101 kPa inside it and it dives 240 m down in the ocean having an average density of 1030 kg/m^3. What is the pressure difference between the inside and the outside of the submarine hull?

Solution:

Assume the atmosphere over the ocean is at 101 kPa, then ΔP is from the 240 m column water.

$\Delta P = \rho Lg$

$= (1030 \text{ kg/m}^3 \times\ 240 \text{ m} \times\ 9.807 \text{ m/s}^2) / 1000 = \textbf{2424 kPa}$

Borgnakke and Sonntag

1.74

Assume we use a pressure gauge to measure the air pressure at street level and at the roof of a tall building. If the pressure difference can be determined with an accuracy of 1 mbar (0.001 bar) what uncertainty in the height estimate does that corresponds to?

Solution:

$\rho_{air} = 1.169 \ \text{kg/m}^3$ from Table A.5

$\Delta P = 0.001 \ \text{bar} = 100 \ \text{Pa}$

$L = \dfrac{\Delta P}{\rho g} = \dfrac{100}{1.169 \times 9.807} = \textbf{8.72 m}$

As you can see that is not really accurate enough for many purposes.

Barometer (mb)

990
1040 995
1035 1000
1030 1005
1025 1010
1020 1015

1020

1021 ↑   1017 ↓

1.75

The absolute pressure in a tank is 115 kPa and the local ambient absolute pressure is 97 kPa. If a U-tube with mercury, density 13550 kg/m^3, is attached to the tank to measure the gage pressure, what column height difference would it show?

Solution:

$\Delta P = P_{tank} - P_0 = \rho g\ H$

$H = (P_{tank} - P_0)/\rho g = [(115 - 97) \times 1000]$ Pa $/ (13550$ kg/m$^3 \times 9.81$ m/s$^2)$

= 0.135 m = 13.5 cm

1.76

An absolute pressure gauge attached to a steel cylinder shows 135 kPa. We want to attach a manometer using liquid water a day that P_{atm} = 101 kPa. How high a fluid level difference must we plan for?

Solution:

Since the manometer shows a pressure difference we have

$$\Delta P = P_{CYL} - P_{atm} = \rho \, L \, g$$

$$L = \Delta P / \rho g = \frac{(135 - 101) \text{ kPa}}{997 \text{ kg m}^{-3} \times 10 \times 9.807 \text{ m/s}^2} \frac{1000 \text{ Pa}}{\text{kPa}}$$

$$= \textbf{3.467 m}$$

Borgnakke and Sonntag

1.77

A U-tube manometer filled with water, density 1000 kg/m^3, shows a height difference of 25 cm. What is the gauge pressure? If the right branch is tilted to make an angle of 30° with the horizontal, as shown in Fig. P1.77, what should the length of the column in the tilted tube be relative to the U-tube?

Solution:

Same height in the two sides in the direction of g.

$\Delta P = F/A = mg/A = V\rho g/A = h\rho g$

$= 0.25 \text{ m} \times 1000 \text{ kg/m}^3 \times 9.807 \text{m/s}^2$

$= 2452.5 \text{ Pa}$

$= \textbf{2.45 kPa}$

$h = H \times \sin 30°$

$\Rightarrow H = h/\sin 30° = 2h = \textbf{50 cm}$

Borgnakke and Sonntag

1.78

A pipe flowing light oil has a manometer attached as shown in Fig. P1.78. What is the absolute pressure in the pipe flow?

Solution:

Table A.3: $\rho_{oil} = 910$ kg/m^3; $\rho_{water} = 997$ kg/m^3

$$P_{BOT} = P_0 + \rho_{water}\, g\, H_{tot} = P_0 + 997 \text{ kg/m}^3 \times 9.807 \text{ m/s}^2 \times 0.8 \text{ m}$$
$$= P_o + 7822 \text{ Pa}$$

$$P_{PIPE} = P_{BOT} - \rho_{water}\, g\, H_1 - \rho_{oil}\, g\, H_2$$
$$= P_{BOT} - 997 \times 9.807 \times 0.1 - 910 \times 9.807 \times 0.2$$
$$= P_{BOT} - 977.7 \text{ Pa} - 1784.9 \text{ Pa}$$

$$P_{PIPE} = P_o + (7822 - 977.7 - 1784.9) \text{ Pa}$$
$$= P_o + 5059.4 \text{ Pa} = 101.325 + 5.06 = \mathbf{106.4 \text{ kPa}}$$

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1.79

The difference in height between the columns of a manometer is 200 mm with a fluid of density 900 kg/m^3. What is the pressure difference? What is the height difference if the same pressure difference is measured using mercury, density 13600 kg/ m^3, as manometer fluid?

Solution:

$$\Delta P = \rho_1 gh_1 = 900 \text{ kg/m}^3 \times 9.807 \text{ m/s}^2 \times 0.2 \text{ m} = 1765.26 \text{ Pa} = \textbf{1.77 kPa}$$

$$h_{Hg} = \Delta P/ (\rho_{hg} \text{ g}) = (\rho_1 \text{ gh}_1) / (\rho_{hg} \text{ g}) = \frac{900}{13600} \times 0.2 \text{ m}$$

$$= \textbf{0.0132 m = 13.2 mm}$$

Borgnakke and Sonntag

1.80

Two cylinders are filled with liquid water, $\rho = 1000$ kg/m^3, and connected by a line with a closed valve, as shown in Fig. P1.80. A has 100 kg and B has 500 kg of water, their cross-sectional areas are $A_A = 0.1$ m^2 and $A_B = 0.25$ m^2 and the height h is 1 m. Find the pressure on each side of the valve. The valve is opened and water flows to an equilibrium. Find the final pressure at the valve location.

Solution:

$$V_A = v_{H_2O}m_A = m_A/\rho = 0.1 = A_A h_A \quad \Rightarrow \quad h_A = 1 \text{ m}$$

$$V_B = v_{H_2O}m_B = m_B/\rho = 0.5 = A_B h_B \quad \Rightarrow \quad h_B = 2 \text{ m}$$

$$P_{VB} = P_0 + \rho g(h_B + H) = 101325 + 1000 \times 9.81 \times 3 = 130\ 755 \text{ Pa}$$

$$P_{VA} = P_0 + \rho g h_A = 101325 + 1000 \times 9.81 \times 1 = 111\ 135 \text{ Pa}$$

Equilibrium: same height over valve in both

$$V_{tot} = V_A + V_B = h_2 A_A + (h_2 - H)A_B \Rightarrow h_2 = \frac{h_A A_A + (h_B + H)A_B}{A_A + A_B} = 2.43 \text{ m}$$

$$P_{V2} = P_0 + \rho g h_2 = 101.325 + (1000 \times 9.81 \times 2.43)/1000 = \textbf{125.2 kPa}$$

1.81

Two piston/cylinder arrangements, A and B, have their gas chambers connected by a pipe. Cross-sectional areas are $A_A = 75$ cm^2 and $A_B = 25$ cm^2 with the piston mass in A being $m_A = 25$ kg. Outside pressure is 100 kPa and standard gravitation. Find the mass m_B so that none of the pistons have to rest on the bottom.

Solution:

Force balance for both pistons: $\quad F\uparrow = F\downarrow$

A: $\quad m_{PA}g + P_0A_A = PA_A$

B: $\quad m_{PB}g + P_0A_B = PA_B$

Same P in A and B gives no flow between them.

$$\frac{m_{PA}g}{A_A} + P_0 = \frac{m_{PB}g}{A_B} + P_0$$

$$\Rightarrow m_{PB} = m_{PA} \, A_A / A_B = 25 \times 25/75 = \mathbf{8.33 \ kg}$$

Borgnakke and Sonntag

1.82

Two hydraulic piston/cylinders are of same size and setup as in Problem 1.81, but with negligible piston masses. A single point force of 250 N presses down on piston A. Find the needed extra force on piston B so that none of the pistons have to move.

Solution:

$A_A = 75 \text{ cm}^2$;

$A_B = 25 \text{ cm}^2$

No motion in connecting pipe: $P_A = P_B$

Forces on pistons balance

$$P_A = P_0 + F_A / A_A = P_B = P_0 + F_B / A_B$$

$$F_B = F_A \times \frac{A_B}{A_A} = 250 \text{ N} \times \frac{25}{75} = \textbf{83.33 N}$$

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1.83

A piece of experimental apparatus is located where $g = 9.5$ m/s^2 and the temperature is 5°C. An air flow inside the apparatus is determined by measuring the pressure drop across an orifice with a mercury manometer (see Problem 1.91 for density) showing a height difference of 200 mm. What is the pressure drop in kPa?

Solution:

$$\Delta P = \rho g h ; \qquad \rho_{Hg} = 13600 \text{ kg/m}^3$$

$$\Delta P = 13\ 600 \text{ kg/m}^3 \times 9.5 \text{ m/s}^2 \times 0.2 \text{ m} = 25840 \text{ Pa} = \textbf{25.84 kPa}$$

Air

g

Borgnakke and Sonntag

Energy and Temperature

1.84

An escalator brings four people, whose total mass is 300 kg, 25 m up in a building. Explain what happens with respect to energy transfer and stored energy.

The four people (300 kg) have their potential energy raised, which is how the energy is stored. The energy is supplied as electrical power to the motor that pulls the escalator with a cable.

1.85

A car moves at 75 km/h; its mass, including people, is 3200 kg. How much kinetic energy does the car have?

$$KE = \tfrac{1}{2}\, m\, \mathbf{V}^2 = \tfrac{1}{2} \times\ 3200\ \text{kg} \times \left(\frac{75 \times 1000}{3600}\right)^2\ \text{m}^2/\text{s}^2$$
$$= 694\ 444\ \text{J} = \mathbf{694\ kJ}$$

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1.86

A 52-kg package is lifted up to the top shelf in a storage bin that is 4 m above the ground floor. How much increase in potential energy does the package get?

The potential energy is from Eq.1.5

$$pe = gz$$

so for a certain mass we get

$$PE = mgH = 52 \text{ kg} \times 9.81 \text{ m/s}^2 \times 4 \text{ m} = 2040 \text{ J} = \textbf{2.04 kJ}$$

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1.87

A car of mass 1775 kg travels with a velocity of 100 km/h. Find the kinetic energy. How high should it be lifted in the standard gravitational field to have a potential energy that equals the kinetic energy?

Solution:

Standard kinetic energy of the mass is

$$KE = \tfrac{1}{2}\, m\, \mathbf{V}^2 = \tfrac{1}{2} \times 1775 \text{ kg} \times \left(\frac{100 \times 1000}{3600}\right)^2 \text{ m}^2/\text{s}^2$$

$$= \tfrac{1}{2} \times 1775 \times 27.778 \text{ Nm} = 684\ 800 \text{ J}$$

$$= \mathbf{684.8\ kJ}$$

Standard potential energy is

$$POT = mgh$$

$$h = \tfrac{1}{2}\, m\, \mathbf{V}^2 / mg = \frac{684\ 800 \text{ Nm}}{1775 \text{ kg} \times 9.807 \text{ m/s}^2} = \mathbf{39.3\ m}$$

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1.88

An oxygen molecule with mass m = M m_o = 32 × 1.66 × 10^{-27} kg moves with a velocity of 240 m/s. What is the kinetic energy of the molecule? What temperature does that corresponds to if it has to equal (3/2) *kT* where k is Boltzmans constant and *T* is the absolute temperature in Kelvin.

$$KE = \tfrac{1}{2}\, m\, \mathbf{V}^2 = \tfrac{1}{2} \times 32 \times 1.66 \times 10^{-27}\ \text{kg} \times 240^2\ \text{m}^2/\text{s}^2$$
$$= \mathbf{1.53\ E{-}21\ J}$$

So if the KE equals 1.5 kT then we get

$$KE = 1.5\ kT = 1.5 \times 1.38065 \times 10^{-23}\ \text{J/K} \times T$$

$$T = (2/3)\ KE/k = (2/3) \times 1.53\ E{-}21\ /\ 1.38065 \times 10^{-23} = \mathbf{73.9\ K}$$

1.89

What is a temperature of –5°C in degrees Kelvin?

Solution:

The offset from Celsius to Kelvin is 273.15 K,
so we get

$$T_K = T_C + 273.15 = -5 + 273.15$$
$$= \mathbf{268.15\ K}$$

1.90

The human comfort zone is between 18 and 24$^{\text{o}}$C, what is the range in Kelvin? What is the maximum relative change from the low to the high temperature?

Solution:

$$T_K = T_C + 273.15$$

The range in K becomes from 291.15 to 297.15 K.
The relative change is 6 degrees up from 291.15 K which is

$$\text{Relative change \%} = \frac{6}{291.15} \times 100 = \textbf{2.06\%}$$

1.91

The density of mercury changes approximately linearly with temperature as

$$\rho_{Hg} = 13595 - 2.5\,T \;\; kg/m^3 \quad T \;\; \text{in Celsius}$$

so the same pressure difference will result in a manometer reading that is influenced by temperature. If a pressure difference of 100 kPa is measured in the summer at 35°C and in the winter at −15°C, what is the difference in column height between the two measurements?

Solution:

The manometer reading h relates to the pressure difference as

$$\Delta P = \rho\, L\, g \quad \Rightarrow \quad L = \frac{\Delta P}{\rho g}$$

The manometer fluid density from the given formula gives

$$\rho_{su} = 13595 - 2.5 \times 35 = 13507.5 \; kg/m^3$$

$$\rho_{w} = 13595 - 2.5 \times (-15) = 13632.5 \; kg/m^3$$

The two different heights that we will measure become

$$L_{su} = \frac{100 \times 10^3}{13507.5 \times 9.807} \frac{kPa\,(Pa/kPa)}{(kg/m^3)\,m/s^2} = 0.7549 \; m$$

$$L_{w} = \frac{100 \times 10^3}{13632.5 \times 9.807} \frac{kPa\,(Pa/kPa)}{(kg/m^3)\,m/s^2} = 0.7480 \; m$$

$$\Delta L = L_{su} - L_{w} = \textbf{0.0069 m = 6.9 mm}$$

1.92

A mercury thermometer measures temperature by measuring the volume expansion of a fixed mass of liquid Hg due to a change in the density, see problem 1.91. Find the relative change (%) in volume for a change in temperature from 10°C to 20°C.

Solution:

From 10°C to 20°C

At 10°C : ρ_{Hg} = 13595 – 2.5 × 10 = 13570 kg/m^3

At 20°C : ρ_{Hg} = 13595 – 2.5 × 20 = 13545 kg/m^3

The volume from the mass and density is: $V = m/\rho$

$$\text{Relative Change} = \frac{V_{20} - V_{10}}{V_{10}} = \frac{(m/\rho_{20}) - (m/\rho_{10})}{m/\rho_{10}}$$

$$= \frac{\rho_{10}}{\rho_{20}} - 1 = \frac{13570}{13545} - 1 = \mathbf{0.0018\ (0.18\%)}$$

1.93

Density of liquid water is $\rho = 1008 - T/2$ [kg/m^3] with T in ^{o}C. If the temperature increases 10°C how much deeper does a 1 m layer of water become?

Solution:

The density change for a change in temperature of 10°C becomes

$$\Delta\rho = -\Delta T/2 = -5 \text{ kg/m}^3$$

from an ambient density of
$$\rho = 1008 - T/2 = 1008 - 25/2 = 995.5 \text{ kg/m}^3$$

Assume the area is the same and the mass is the same $\quad m = \rho V = \rho AH,$ then we have

$$\Delta m = 0 = V\Delta\rho + \rho\Delta V \quad \Rightarrow \quad \Delta V = - V\Delta\rho/\rho$$

and the change in the height is

$$\Delta H = \frac{\Delta V}{A} = \frac{H\Delta V}{V} = \frac{-H\Delta\rho}{\rho} = \frac{-1 \times (-5)}{995.5} = \textbf{0.005 m}$$

barely measurable.

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1.94

Using the freezing and boiling point temperatures for water in both Celsius and Fahrenheit scales, develop a conversion formula between the scales. Find the conversion formula between Kelvin and Rankine temperature scales.

Solution:

$$T_{Freezing} = 0 \, ^{O}C = 32 \, F; \qquad T_{Boiling} = 100 \, ^{O}C = 212 \, F$$

$$\Delta T = 100 \, ^{O}C = 180 \, F \implies T_{o_C} = (T_F - 32)/1.8 \quad or \quad T_F = 1.8 \, T_{o_C} + 32$$

For the absolute K & R scales both are zero at absolute zero.

$$T_R = 1.8 \times T_K$$

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1.95

The atmosphere becomes colder at higher elevation. As an average the standard atmospheric absolute temperature can be expressed as $T_{atm} = 288 - 6.5 \times 10^{-3} z$, where z is the elevation in meters. How cold is it outside an airplane cruising at 12 000 m expressed in Kelvin and in Celsius?

Solution:

For an elevation of z = 12 000 m we get

$$T_{atm} = 288 - 6.5 \times 10^{-3} z = \textbf{210 K}$$

To express that in degrees Celsius we get
$$T_C = T - 273.15 = \textbf{--63.15}^{\textbf{o}}\textbf{C}$$

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Review Problems

1.96

Repeat problem 1.83 if the flow inside the apparatus is liquid water, $\rho \cong 1000$ kg/m^3, instead of air. Find the pressure difference between the two holes flush with the bottom of the channel. You cannot neglect the two unequal water columns.

Solution: Balance forces in the manometer:

$$(H - h_2) - (H - h_1) = \Delta h_{Hg} = h_1 - h_2$$

$$P_1 A + \rho_{H2O} h_1 gA + \rho_{Hg}(H - h_1)gA$$
$$= P_2 A + \rho_{H2O} h_2 gA + \rho_{Hg}(H - h_2)gA$$

$$\Rightarrow P_1 - P_2 = \rho_{H2O}(h_2 - h_1)g + \rho_{Hg}(h_1 - h_2)g$$

$$P_1 - P_2 = \rho_{Hg}\Delta h_{Hg}g - \rho_{H2O}\Delta h_{Hg}g$$

$$= (13\ 600 \times 0.2 \times 9.5 - 1000 \times 0.2 \times 9.5)\ \text{kg/m}^3 \times \text{m} \times \text{m/s}^2$$
$$= (25\ 840 - 1900)\ \text{Pa} = 23\ 940\ \text{Pa} = \textbf{23.94 kPa}$$

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1.97

A dam retains a lake 6 m deep. To construct a gate in the dam we need to know the net horizontal force on a 5 m wide and 6 m tall port section that then replaces a 5 m section of the dam. Find the net horizontal force from the water on one side and air on the other side of the port.

Solution:

$$P_{bot} = P_0 + \Delta P$$

$$\Delta P = \rho g h = 997 \text{ kg/m}^3 \times 9.807 \text{ m/s}^2 \times 6 \text{ m} = 58\ 665 \text{ Pa} = 58.66 \text{ kPa}$$

Neglect ΔP in air

$$F_{net} = F_{right} - F_{left} = P_{avg} A - P_0 A$$

$$P_{avg} = P_0 + 0.5\ \Delta P \quad \text{Since a linear pressure variation with depth.}$$

$$F_{net} = (P_0 + 0.5\ \Delta P)A - P_0 A = 0.5\ \Delta P\ A = 0.5 \times 58.66 \times 5 \times 6 = \textbf{880 kN}$$

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1.98

In the city water tower, water is pumped up to a level 25 m above ground in a pressurized tank with air at 125 kPa over the water surface. This is illustrated in Fig. P1.98. Assuming the water density is 1000 kg/m^3 and standard gravity, find the pressure required to pump more water in at ground level.

Solution:

$\Delta P = \rho\ L\ g$

$\quad = 1000 \text{ kg/m}^3 \times 25 \text{ m} \times 9.807 \text{ m/s}^2$

$\quad = 245\ 175 \text{ Pa} = 245.2 \text{ kPa}$

$P_{bottom} = P_{top} + \Delta P$

$\quad\quad = 125 + 245.2$

$\quad\quad = \textbf{370 kPa}$

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1.99

The main waterline into a tall building has a pressure of 600 kPa at 5 m elevation below ground level. How much extra pressure does a pump need to add to ensure a water line pressure of 200 kPa at the top floor 150 m above ground?

Solution:

The pump exit pressure must balance the top pressure plus the column ΔP. The pump inlet pressure provides part of the absolute pressure.

$P_{after\ pump} = P_{top} + \Delta P$

$\Delta P = \rho g h = 997\ kg/m^3 \times 9.807\ m/s^2 \times (150 + 5)\ m$

$\qquad = 1\ 515\ 525\ Pa = 1516\ kPa$

$P_{after\ pump} = 200 + 1516 = 1716\ kPa$

$\Delta P_{pump} = 1716 - 600 = \textbf{1116 kPa}$

1.100

Two cylinders are connected by a piston as shown in Fig. P1.100. Cylinder A is used as a hydraulic lift and pumped up to 500 kPa. The piston mass is 25 kg and there is standard gravity. What is the gas pressure in cylinder B?
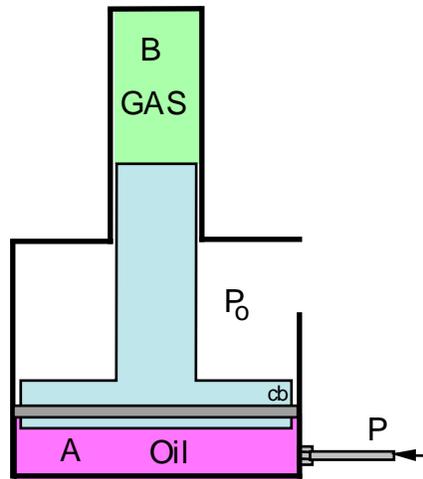
Solution:

Force balance for the piston: $P_BA_B + m_pg + P_0(A_A - A_B) = P_AA_A$

$A_A = (\pi/4)0.1^2 = 0.00785 \text{ m}^2$; $A_B = (\pi/4)0.025^2 = 0.000\ 491 \text{ m}^2$

$P_BA_B = P_AA_A - m_pg - P_0(A_A - A_B) = 500 \times 0.00785 - (25 \times 9.807/1000)$

$- 100\ (0.00785 - 0.000\ 491) = 2.944 \text{ kN}$

$P_B = 2.944/0.000\ 491 = 5996 \text{ kPa} = \textbf{6.0 MPa}$

1.101

A 5-kg piston in a cylinder with diameter of 100 mm is loaded with a linear spring and the outside atmospheric pressure of 100 kPa as shown in Fig. P1.101. The spring exerts no force on the piston when it is at the bottom of the cylinder and for the state shown, the pressure is 400 kPa with volume 0.4 L. The valve is opened to let some air in, causing the piston to rise 2 cm. Find the new pressure.

Solution:

A linear spring has a force linear proportional to displacement. $F = k \, x$, so the equilibrium pressure then varies linearly with volume: $P = a + bV$, with an intersect a and a slope $b = dP/dV$. Look at the balancing pressure at zero volume ($V \rightarrow 0$) when there is no spring force $F = PA = P_oA + m_pg$ and the initial state. These two points determine the straight line shown in the P-V diagram.

Piston area $= A_p = (\pi/4) \times 0.1^2 = 0.00785 \text{ m}^2$

$$a = P_0 + \frac{m_pg}{A_p} = 100 \text{ kPa} + \frac{5 \times 9.80665}{0.00785} \text{ Pa}$$

$$= 106.2 \text{ kPa} \quad \text{intersect for zero volume.}$$

$$V_2 = 0.4 + 0.00785 \times 20 = 0.557 \text{ L}$$

$$P_2 = P_1 + \frac{dP}{dV} \Delta V$$

$$= 400 + \frac{(400\text{-}106.2)}{0.4 \text{ - } 0} (0.557 \text{ - } 0.4)$$

$$= \mathbf{515.3 \text{ kPa}}$$

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 1
English Units

8e

UPDATED JULY 2013

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CHAPTER 1

Borgnakke and Sonntag

Concept Problems

1.102E

A mass of 2 lbm has acceleration of 5 ft/s^2, what is the needed force in lbf?

Solution:

Newtons 2nd law: $\quad$ F = ma

$F = ma = 2 \text{ lbm} \times 5 \text{ ft/s}^2 = 10 \text{ lbm ft/s}^2$

$\quad = \dfrac{10}{32.174} \text{ lbf} = \textbf{0.31 lbf}$

1.103E

How much mass is in 1 gallon of gasoline? If helium in a balloon at atmospheric *P* and *T*?

Solution:

A volume of 1 gal equals 231 in^3, see Table A.1. From Table F.3 the density is 46.8 lbm/ft^3, so we get

$$m = \rho V = 46.8 \text{ lbm/ft}^3 \times 1 \times (231/12^3) \text{ ft}^3 = \textbf{6.256 lbm}$$

A more accurate value from Table F.3 is $\rho = 848$ lbm/ft^3.

For the helium we see Table F.4 that density is 10.08×10^{-3} lbm/ft^3 so we get

$$m = \rho V = 10.08 \times 10^{-3} \text{ lbm/ft}^3 \times 1 \times (231/12^3) \text{ ft}^3 = \textbf{0.00135 lbm}$$

1.104E

Can you easily carry a one gallon bar of solid gold?

Solution:

The density of solid gold is about 1205 lbm/ft^3 from Table F.2, we could also have read Figure 1.7 and converted the units.

$$V = 1 \text{ gal} = 231 \text{ in}^3 = 231 \times 12^{-3} \text{ ft}^3 = 0.13368 \text{ ft}^3$$

Therefore the mass in one gallon is

$$m = \rho V = 1205 \text{ lbm/ft}^3 \times 0.13368 \text{ ft}^3$$
$$= 161 \text{ lbm}$$

and some people can just about carry that in the standard gravitational field.

1.105E

What is the temperature of –5F in degrees Rankine?

Solution:

The offset from Fahrenheit to Rankine is
459.67 R, so we get

$$T_R = T_F + 459.67 = -5 + 459.67$$
$$= \textbf{454.7 R}$$

1.106E

What is the smallest temperature in degrees Fahrenheit you can have? Rankine?

Solution:

The lowest temperature is absolute zero which is at zero degrees Rankine at which point the temperature in Fahrenheit is negative

$$T_R = 0 \text{ R} = -459.67 \text{ F}$$

1.107E

What is the relative magnitude of degree Rankine to degree Kelvin

Look in Table A.1 p. 757:

$$1 \text{ K} = 1 \text{ }^{o}\text{C} = 1.8 \text{ R} = 1.8 \text{ F}$$

$$1 \text{ R} = \frac{5}{9} \text{ K} = 0.5556 \text{ K}$$

1.108E

Chemical reaction rates genrally double for a 10 K increase in temperature. How large an increase is that in Fahrenheit?

From the Conversion Table A.1: $1 \text{ K} = 1 \,^{\text{o}}\text{C} = 1.8 \text{ R} = 1.8 \text{ F}$

So the 10 K increase becomes

$$10 \text{ K} = 18 \text{ F}$$

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Properties and Units

1.109E

An apple weighs 0.2 lbm and has a volume of 6 in^3 in a refrigerator at 38 F. What is the apple density? List three intensive and two extensive properties for the apple.

Solution:

$$\rho = \frac{m}{V} = \frac{0.2}{6} \frac{\text{lbm}}{\text{in}^3} = 0.0333 \frac{\text{lbm}}{\text{in}^3} = 57.6 \frac{\text{lbm}}{\text{ft}^3}$$

Intensive

$$\rho = 57.6 \frac{\text{lbm}}{\text{ft}^3} ; \qquad v = \frac{1}{\rho} = 0.0174 \frac{\text{ft}^3}{\text{lbm}}$$

$$T = 38 \text{ F}; \qquad P = 14.696 \text{ lbf/in}^2$$

Extensive

$$m = 0.2 \text{ lbm}$$

$$V = 6 \text{ in}^3 = 0.026 \text{ gal} = 0.00347 \text{ ft}^3$$

1.110E

A steel piston of 10 lbm is in the standard gravitational field where a force of 10 lbf is applied vertically up. Find the acceleration of the piston.

Solution:

$$F_{up} = ma = F - mg$$

$$a = \frac{F - mg}{m} = \frac{F}{m} - g$$

$$= \frac{10 \text{ lbf}}{10 \text{ lbm}} - 32.174 \text{ ft/s}^2$$

$$= (1 \times 32.174 - 32.174) \text{ ft/s}^2$$

$$= \mathbf{0 \text{ fts}^{-2}}$$

The mass does not move, it is held stationary.

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Force, Energy, Density

1.111E

A 2500-lbm car moving at 25 mi/h is accelerated at a constant rate of 15 ft/s^2 up to a speed of 50 mi/h. What are the force and total time required?

Solution:

$$a = \frac{d\mathbf{V}}{dt} = \frac{\Delta\mathbf{V}}{\Delta t} \quad \Rightarrow \quad \Delta t = \frac{\Delta\mathbf{V}}{a}$$

$$\Delta t = \frac{(50 - 25) \text{ mi/h} \times 1609.34 \text{ m/mi} \times 3.28084 \text{ ft/m}}{3600 \text{ s/h} \times 15 \text{ ft/s}^2} = \textbf{2.44 sec}$$

F = ma = 2500 lbm × 15 ft/s^2 / (32.174 lbm ft /lbf-s^2)

= **1165 lbf**

1.112E

An escalator brings four people of total 600 lbm and a 1000 lbm cage up with an acceleration of 3 ft/s^2 what is the needed force in the cable?

Solution:

The total mass moves upwards with an acceleration plus the gravitations acts with a force pointing down.

$ma = \sum F = F - mg$

$F = ma + mg = m(a + g)$
  $= (1000 + 600) \text{ lbm} \times (3 + 32.174) \text{ ft/s}^2$
  $= \textbf{56 278 lbm ft/s}^2 = \textbf{56 278 lbf}$

1.113E

One pound-mass of diatomic oxygen (O_2 molecular weight 32) is contained in a 100-gal tank. Find the specific volume on both a mass and mole basis (v and $\bar{v}$).

Solution:

$$V = 100 \times 231 \text{ in}^3 = (23\ 100/12^3)\ \text{ft}^3 = 13.37 \text{ ft}^3$$

conversion seen in Table A.1

This is based on the definition of the specific volume

$$v = V/m = 13.37 \text{ ft}^3/1 \text{ lbm} = \mathbf{13.37\ ft^3/lbm}$$

$$\bar{v} = V/n = \frac{V}{m/M} = Mv = 32 \times 13.37 = \mathbf{427.8\ ft^3/lbmol}$$

1.114E

A 30-lbm steel gas tank holds 10 ft^3 of liquid gasoline, having a density of 50 lbm/ft^3. What force is needed to accelerate this combined system at a rate of 15 ft/s^2?

Solution:

$$m = m_{tank} + m_{gasoline}$$
$$= 30 \text{ lbm} + 10 \text{ ft}^3 \times 50 \text{ lbm/ft}^3$$
$$= 530 \text{ lbm}$$

$$F = ma = (530 \text{ lbm} \times 15 \text{ ft/s}^2) / (32.174 \text{ lbm ft/s}^2\text{- lbf}) = \textbf{247.1 lbf}$$

1.115E

A powerplant that separates carbon-dioxide from the exhaust gases compresses it to a density of 8 lbm/ft^3 and stores it in an un-minable coal seam with a porous volume of 3 500 000 ft^3. Find the mass they can store.

Solution:

$$m = \rho \, V = 8 \text{ lbm/ft}^3 \times 3\ 500\ 000 \text{ ft}^3 = 2.8 \times 10^{\,7} \text{ lbm}$$

Just to put this in perspective a power plant that generates 2000 MW by burning coal would make about 20 million tons of carbon-dioxide a year. That is 2000 times the above mass so it is nearly impossible to store all the carbon-dioxide being produced.

1.116E

A laboratory room keeps a vacuum of 4 in. of water due to the exhaust fan. What is the net force on a door of size 6 ft by 3 ft?

Solution:

The net force on the door is the difference between the forces on the two sides as the pressure times the area

$$F = P_{outside} A - P_{inside} A = \Delta P \times A$$
$$= 4 \text{ in } H_2O \times 6 \text{ ft} \times 3 \text{ ft}$$
$$= 4 \times 0.036126 \text{ lbf/in}^2 \times 18 \text{ ft}^2 \times 144 \text{ in}^2/\text{ft}^2$$
$$= \textbf{374.6 lbf}$$

Table A.1: 1 in. H_2O is 0.036 126 lbf/in^2, a unit also often listed as psi.

1.117E

A 150-lbm human total footprint is 0.5 ft when the person is wearing boots. If snow can support an extra 1 psi, what should the total snow shoe area be?

Force balance: $ma = 0 = PA - mg$

$$A = \frac{mg}{P} = \frac{150 \text{ lbm} \times 32.174 \text{ ft/s}^2}{1 \text{ lbf/in}^2} = \frac{150 \text{ lbm} \times 32.174 \text{ ft/s}^2}{32.174 \text{ lbm-ft/(s}^2\text{in}^2)}$$

$$= \mathbf{150 \text{ in}^2} = 1.04 \text{ ft}^2$$

1.118E

A tornado rips off a 1000 ft^2 roof with a mass of 2000 lbm. What is the minimum vacuum pressure needed to do that if we neglect the anchoring forces?

Solution:

The net force on the roof is the difference between the forces on the two sides as the pressure times the area

$$F = P_{inside} \, A - P_{outside} A = \Delta P \, A$$

That force must overcome the gravitation mg, so the balance is

$$\Delta P \, A = mg$$

$$\Delta P = mg/A = (2000 \text{ lbm} \times 32.174 \text{ ft/s}^2 \,)/1000 \text{ ft}^2$$
$$= 2000 \,/(1000 \times 144) \text{ psi} = \textbf{0.0139 psi}$$

Remember that psi = lbf/in^2.

1.119E

A manometer shows a pressure difference of 3.5 in of liquid mercury. Find ΔP in psi.

Solution:

Hg : L = 3.5 in; ρ = 848 lbm/ft^3 from Table F.3

Pressure: 1 psi = 1 lbf/ in^2

The pressure difference ΔP balances the column of height L so from Eq.2.2

$\Delta P = \rho\ g\ L\ = 848$ lbm/ft$^3 \times 32.174$ ft/s$^2 \times (3.5/12)$ ft

$= 247.3$ lbf/ft$^2 = (247.3 / 144)$ lbf/in^2

$= \textbf{1.72 psi}$

1.120E

A 7 ft m tall steel cylinder has a cross sectional area of 15 ft^2. At the bottom with a height of 2 ft m is liquid water on top of which is a 4 ft high layer of gasoline. The gasoline surface is exposed to atmospheric air at 14.7 psia. What is the highest pressure in the water?

Solution:

The pressure in the fluid goes up with the depth as

$$P = P_{top} + \Delta P = P_{top} + \rho g h$$

and since we have two fluid layers we get

$$P = P_{top} + [(\rho h)_{gasoline} + (\rho h)_{water}]g$$

The densities from Table F.3 are:

$$\rho_{gasoline} = 46.8 \text{ lbm/ft}^3; \quad \rho_{water} = 62.2 \text{ lbm/ft}^3$$

$$P = 14.7 + [46.8 \times 4 + 62.2 \times 2]\frac{32.174}{144 \times 32.174} = \textbf{16.86 lbf/in}^2$$

1.121E

A U-tube manometer filled with water, density 62.3 lbm/ft^3, shows a height difference of 10 in. What is the gauge pressure? If the right branch is tilted to make an angle of 30° with the horizontal, as shown in Fig. P1.77, what should the length of the column in the tilted tube be relative to the U-tube?

Solution:

$\Delta P = F/A = mg/A = h\rho g$

$$= \frac{(10/12) \times 62.3 \times 32.174}{32.174 \times 144}$$

$= P_{gauge} = \textbf{0.36 lbf/in}^2$

$h = H \times \sin 30°$

$\Rightarrow H = h/\sin 30° = 2h = 20 \text{ in} = \textbf{0.833 ft}$

1.122E

A piston/cylinder with cross-sectional area of 0.1 ft^2 has a piston mass of 200 lbm resting on the stops, as shown in Fig. P1.50. With an outside atmospheric pressure of 1 atm, what should the water pressure be to lift the piston?

Solution:

The force acting down on the piston comes from gravitation and the outside atmospheric pressure acting over the top surface.

Force balance: $F\uparrow = F\downarrow = PA = m_pg + P_0A$

Now solve for P (multiply by 144 to convert from ft^2 to in^2)

$$P = P_0 + \frac{m_pg}{A} = 14.696 + \frac{200 \times 32.174}{0.1 \times 144 \times 32.174}$$

$$= 14.696 \text{ psia} + 13.88 \text{ psia} = \textbf{28.58 lbf/in}^2$$

cb

Water

1.123E

The main waterline into a tall building has a pressure of 90 psia at 16 ft elevation below ground level. How much extra pressure does a pump need to add to ensure a waterline pressure of 30 psia at the top floor 450 ft above ground?

Solution:

The pump exit pressure must balance the top pressure plus the column ΔP. The pump inlet pressure provides part of the absolute pressure.

$P_{\text{after pump}} = P_{\text{top}} + \Delta P$

$\Delta P = \rho gh = 62.2 \text{ lbm/ft}^3 \times 32.174 \text{ ft/s}^2 \times (450 + 16) \text{ ft} \times \dfrac{1 \text{ lbf s}^2}{32.174 \text{ lbm ft}}$

$= 28\,985 \text{ lbf/ft}^2 = 201.3 \text{ lbf/in}^2$

$P_{\text{after pump}} = 30 + 201.3 = 231.3 \text{ psia}$

$\Delta P_{\text{pump}} = 231.3 - 90 = \textbf{141.3 psi}$

1.124E

A piston, m_p = 10 lbm, is fitted in a cylinder, A = 2.5 in.^{2}, that contains a gas. The setup is in a centrifuge that creates an acceleration of 75 ft/s^2. Assuming standard atmospheric pressure outside the cylinder, find the gas pressure.

Solution:

Force balance: $F\uparrow = F\downarrow = P_0A + m_pg = PA$

$$P = P_0 + \frac{m_pg}{A}$$

$$= 14.696 + \frac{10 \times 75}{2.5 \times 32.174} \ \frac{\text{lbm ft/s}^2}{\text{in}^2} \ \frac{\text{lbf-s}^2}{\text{lbm-ft}}$$

$$= 14.696 + 9.324 = \textbf{24.02 lbf/in}^2$$

P$_o$

g

gas

Temperature

1.125E

The human comfort zone is between 18 and 24°C. What is that range in Fahrenheit?

$$T = 18°C = 32 + 1.8 \times 18 = 64.4 \text{ F}$$
$$T = 24°C = 32 + 1.8 \times 24 = 75.2 \text{ F}$$

So the range is like **64 to 75 F**.

1.126E

The atmosphere becomes colder at higher elevation. As an average the standard atmospheric absolute temperature can be expressed as $T_{atm} = 518 - 3.84 \times 10^{-3} z$, where z is the elevation in feet. How cold is it outside an airplane cruising at 32 000 ft expressed in Rankine and in Fahrenheit?

Solution:

For an elevation of z = 32 000 ft we get

$$T_{atm} = 518 - 3.84 \times 10^{-3} z = \mathbf{395.1\ R}$$

To express that in degrees Fahrenheit we get
$$T_F = T - 459.67 = \mathbf{-64.55\ F}$$

1.127E

The density of mercury changes approximately linearly with temperature as

$\rho_{Hg} = 851.5 - 0.086\ T$ lbm/ft^3 T in degrees Fahrenheit

so the same pressure difference will result in a manometer reading that is influenced by temperature. If a pressure difference of 14.7 lbf/in.^{2} is measured in the summer at 95 F and in the winter at 5 F, what is the difference in column height between the two measurements?

Solution:

$$\Delta P = \rho g h \quad \Rightarrow \quad h = \Delta P / \rho g$$

$$\rho_{su} = 843.33\ \text{lbm/ft}^3; \qquad \rho_w = 851.07\ \text{lbm/ft}^3$$

$$h_{su} = \frac{14.7 \times 144 \times 32.174}{843.33 \times 32.174} = 2.51\ \text{ft} = 30.12\ \text{in}$$

$$h_w = \frac{14.7 \times 144 \times 32.174}{851.07 \times 32.174} = 2.487\ \text{ft} = 29.84\ \text{in}$$

$$\Delta h = h_{su} - h_w = 0.023\ \text{ft} = \textbf{0.28 in}$$

Borgnakke Sonntag

Fundamentals of
Thermodynamics

SOLUTION MANUAL
CHAPTER 2

8e

Updated June 2013

CONTENT CHAPTER 2

In-Text Concept Questions

2.a

If the pressure is smaller than the smallest P_{sat} at a given T, what is the phase?

Refer to the phase diagrams in Figures 2.4
and 2.5. For a lower P you are below the
vaporization curve (or the sublimation curve)
and that is the superheated vapor region. You
have the gas phase.

2.b

An external water tap has the valve activated by a long spindle so the closing
mechanism is located well inside the wall. Why is that?

Solution:
By having the spindle inside the wall the coldest location with water when the valve
is closed is kept at a temperature above the freezing point. If the valve spindle was
outside there would be some amount of water that could freeze while it is trapped
inside the pipe section potentially rupturing the pipe.

2.c

What is the lowest temperature (approximately) at which water can be liquid?

Look at the phase diagram in Fig. 2.4. At the border between ice I, ice III and the liquid region is a triple point which is the lowest T where you can have liquid. From the figure it is estimated to be about 255 K i.e. at -18$^{\text{o}}$C.

T $\approx$ 255 K $\approx$ - 18°C

2.d

Some tools should be cleaned in water at a least 150°C. How high a P is needed?

Solution:

If I need liquid water at 150°C I must have a pressure that is at least the saturation pressure for this temperature.

Table B.1.1: 150°C, P_{sat} = 475.9 kPa.

2.e

Water at 200 kPa has a quality of 50%. Is the volume fraction $V_g/V_{tot} < 50\%$ or $> 50\%$?

This is a two-phase state at a given pressure and without looking in the table we know that v_f is much smaller than v_g.

From the definition of quality we get the masses from total mass, m, as

$$m_f = (1 - x)\, m, \qquad m_g = x\, m$$

The volumes are

$$V_f = m_f\, v_f = (1 - x)\, m\, v_f, \qquad V_g = m_g\, v_g = x\, m\, v_g$$

So when half the mass is liquid and the other half is vapor the liquid volume is much smaller that the vapor volume. The vapor volume is thus much more than 50% of the total volume. Only right at the critical point is $v_f = v_g$ for all other states $v_g > v_f$ and the difference is larger for smaller pressures.

2.f

Why are most of the compressed liquid or solid regions not included in the printed tables?

For the compressed liquid and the solid phases the specific volume and thus density is nearly constant. These surfaces are very steep nearly constant v and there is then no reason to fill up a table with the same value of v for different P and T.

2.g

Why is it not typical to find tables for Ar, He, Ne or air like an Appendix B table?

The temperature at which these substances are close to the two-phase region is very low. For technical applications with temperatures around atmospheric or higher they are ideal gases. Look in Table A.2 and we can see the critical temperatures as

Ar : 150.8 K   He: 5.19 K   Ne: 44.4 K

It requires a special refrigerator in a laboratory to bring a substance down to these cryogenic temperatures.

2.h

What is the percent change in volume as liquid water freezes? Mention some effects the volume change can have in nature and in our households.

The density of water in the different phases can be found in Tables A.3 and A.4 and in Table B.1.

From Table B.1.1 $v_f = 0.00100 \text{ m}^3/\text{kg}$

From Table B.1.5 $v_i = 0.0010908 \text{ m}^3/\text{kg}$

Percent change: $100\,\dfrac{v_i - v_f}{v_f} = 100 \times \dfrac{0.0010908 - 0.001}{0.001} = 9.1\ \%\ \text{increase}$

Liquid water that seeps into cracks or other confined spaces and then freezes will expand and widen the cracks. This is what destroys any porous material exposed to the weather on buildings, roads and mountains. It can burst water pipes and crack engine blocks (that is why you put anti-freeze in it).

2.i

How accurate is it to assume that methane is an ideal gas at room conditions?

From Table A.2: $T_c = 190.4$ K, $P_c = 4.60$ MPa

So at room conditions we have much higher $T > T_c$ and $P << P_c$ so this is the ideal gas region. To confirm look in Table B.7.2

$$100 \text{ kPa, } 300 \text{ K, }   v = 1.55215 \text{ m}^3/\text{kg}$$

Find the compressibility factor (R from Table A.5) as

$$Z = Pv/RT = \frac{100 \text{ kPa} \times 1.55215 \text{ m}^3/\text{kg}}{0.5183 \text{ kJ/kg-K} \times 300 \text{ K}} = 0.99823$$

so Z is 1 with an accuracy of 0.2% better than most measurements can be done.

2.j

I want to determine a state of some substance, and I know that P = 200 kPa; is it helpful to write PV = mRT to find the second property?

NO. You need a second property.

Notice that two properties are needed to determine a state. The EOS can give you a third property if you know two, like (P,T) gives v just as you would get by entering a table with a set (P,T). This EOS substitutes for a table when it is applicable.

2.k

A bottle at 298 K should have liquid propane; how high a pressure is needed? (use Fig. D.1)

To have a liquid the pressure must be higher than or equal to the saturation pressure. There is no printed propane table so we use the compressibility chart and Table A.2

Propane Table A.2: $T_c = 369.8$ K, $P_c = 4.25$ MPa

The reduced temperature is:

$$T_r = \frac{T}{T_c} = \frac{298}{369.8} = 0.806,$$

for which we find in Fig. D.1: $P_{r\ sat} = 0.25$

$$P = P_{r\ sat}\ P_c = 0.25 \times 4.25 \text{ MPa} = \textbf{1.06 MPa}$$

2.1

A bottle at 298 K should have liquid propane; how high a pressure is needed? (use the software)

To have a liquid the pressure must be higher than or equal to the saturation pressure. There is no printed propane table but the software has propane included

Start CATT3, select cryogenic substances, propane
              select calculator,   select case 4 (T, x) = (25°C, 0)

⇨   P = **0.9518 MPa**

Concept Problems

2.1

Are the pressures in the tables absolute or gauge pressures?

Solution:

The behavior of a pure substance depends on the absolute pressure, so P in the tables is absolute.

2.2

What is the minimum pressure for which I can have liquid carbon dioxide?

Look at the phase diagram in Fig. 2.5. The minimum P in the liquid phase is at the triple point. From Table 2.2 this is at **520 kPa** (a similar value around 4-500 kPa is seen in Fig. 2.5).

The 100 kPa is below the triple point.

2.3

When you skate on ice, a thin liquid film forms under the skate. How can that be?

The ice is at some temperature below the freezing temperature for the atmospheric pressure of 100 kPa = 0.1 MPa and thus to the left of the fusion line in the solid ice I region of Fig. 2.4. As the skate comes over the ice the pressure is increased dramatically right under the blade so it brings the state straight up in the diagram crossing the fusion line and brings it into a liquid state at same temperature.

The very thin liquid film under the skate changes the friction to be viscous rather than a solid to solid contact friction. Friction is thus significantly reduced.

Comment: The latest research has shown that the pressure may not be enough to generate the liquid, but that such a liquid layer always exist on an ice surface, maybe only a few molecules thick (dependent upon temperature). At really low T say -40$^{\text{o}}$C no such liquid layer exists which is why your finger can stick to such a surface.

2.4

At a higher elevation like in mountains the pressure is lower, what effect does that have for cooking food?

A lower pressure means that water will boil at a lower temperature, see the vaporization line in Fig. 2.4, or in Table B.1.2 showing the saturated temperature as a function of the pressure. You therefore must increase the cooking time a little.

2.5

Water at room temperature and room pressure has $v \approx 1 \times 10^n$ m^3/kg what is n?

See Table B.1.1 or B.1.2 to determine it is in the liquid phase
(you should know this already).

Table A.4 or from B1.1 at 20^oC: n = –3   (v = 0.00100 m^3/kg)

2.6

Can a vapor exist below the triple point temperature?

Look at the phase diagrams in Figs 2.4 and 2.5. Below the triple point the
sublimation curve has very small pressures, but not zero. So for pressures below the
saturation pressure the substance is a vapor. If the phase diagram is plotted in linear
coordinates the small vapor region is nearly not visible.

2.7

In Example 2.1 b is there any mass at the indicated specific volume? Explain.

This state is a two-phase mixture of liquid and vapor. There is no mass at the
indicated state, the v value is an average for all the mass, so there is some mass at the
saturated vapor state (fraction is the quality x) and the remainder of the mass is
saturated liquid (fraction 1-x).

2.8

Sketch two constant-pressure curves (500 kPa and 30 000 kPa) in a T-v diagram and indicate on the curves where in the water tables you see the properties.

The 30 MPa line in Table B.1.4 starts at 0°C and table ends at 380°C, the line is continued in Table B.1.3 starting at 375°C and table ends at 1300°C.

The 500 kPa line in Table B.1.4 starts at 0.01°C and table ends at the saturated liquid state (151.86°C). The line is continued in Table B.1.3 starting at the saturated vapor state (151.86°C) continuing up to 1300°C.

2.9

If I have 1 L of R-410A at 1 MPa, 20°C how much mass is that?

R-410A Tables B.4:

B.4.1 P_{sat} = 1444.2 kPa at 20°C so superheated vapor.

B.4.2 v = 0.02838 m^3/kg   under subheading 1000 kPa

$$m = \frac{V}{v} = \frac{0.001 \text{ m}^3}{0.02838 \text{ m}^3/\text{kg}} = \textbf{0.0352 kg} = \textbf{35.2 g}$$

The P-v (log-log) diagram from CATT3, P in MPa and v in m^3/kg.

2.10

Locate the state of ammonia at 200 kPa, -10°C. Indicate in both the P-v and the T-v
diagrams the location of the nearest states listed in the printed Table B.2

2.11

Why are most of the compressed liquid or solid regions not included in the printed tables?

For the compressed liquid and the solid phases the specific volume and thus density is nearly constant. These surfaces are very steep nearly constant v and there is then no reason to fill up a table with the same value of v for different P and T.

2.12

How does a constant v-process look like for an ideal gas in a P-T diagram?

For an ideal gas: $Pv = RT$ so then $P = (R/v)\,T$

Constant v is a straight line with slope (R/v) in the P-T diagram

2.13

If $v = RT/P$ for an ideal gas what is the similar equation for a liquid?

The equation for a liquid is: $v = \text{Constant} = v_o$

If you include that v increases a little with T then: $v = v_o + C\,(T - T_o)$
where C is a small constant with units $m^3/kg\text{-}K$.

2.14

To solve for v given (P, T) in Eq.2.14 , what is the mathematical problem?

To solve for v in Eq. 2.14 multiply with the de-numerators so we get the form

$$P\, f_1(v) = RT\, f_2(v) - a\,(v - b)$$

This is a 3rd order polynomial (a cubic) function in v. The problem then is to find the roots or zero points in this cubic equation. The mathematical subject to study is to find zero points of functions (or roots). Typically you will do it by some iteration technique. Successive substitutions, bi-section, Newton-Raphson are some of the methods you should learn.

2.15

As the pressure of a gas becomes larger, Z becomes larger than 1. What does that imply?

$$Pv = Z\,RT$$

So for a given P, the specific volume v is then larger than predicted by the ideal gas law. The molecules are pressed so close together that they have repulsive forces between them (the electron clouds are getting closer). The ideal gas law assumes the atoms (molecules) are point masses with no interactions between them and thus has a limit of zero specific volume as P goes to infinity. Real molecules occupy some volume and the outer shell has a number of electrons with negative charges which can interact with one another if they are close enough.

Phase Diagrams, Triple and Critical Points

2.16

Carbon dioxide at 280 K can be in different phases. Indicate the pressure range you have for each of the three phases (vapor, liquid and solid).

Look at the P-T phase diagram in Fig. 2.5 at 280 K:

$P < 4000$ kPa   vapor
4000 kPa $< P < 400$ MPa   liquid
400 MPa $< P$ solid

2.17

Modern extraction techniques can be based on dissolving material in supercritical fluids such as carbon dioxide. How high are pressure and density of carbon dioxide when the pressure and temperature are around the critical point? Repeat for ethyl alcohol.

Solution:

CO_2 :

Table A.2: $P_c =$ **7.38 MPa**, $T_c = 304$ K, $v_c = 0.00212$ m^3/kg

$$\rho_c = 1/v_c = 1/0.00212 = \textbf{472 kg/m}^3$$

C_2H_5OH:

Table A.2: $P_c =$ **6.14 MPa**, $T_c = 514$ K, $v_c = 0.00363$ m^3/kg

$$\rho_c = 1/v_c = 1/0.00363 = \textbf{275 kg/m}^3$$

2.18

The ice cap on the North Pole could be 1000 m thick with a density of 920 kg/m^3. Find the pressure at the bottom and the corresponding melting temperature.

Solution:

$$\rho_{ICE} = 920 \text{ kg/m}^3$$

$$\Delta P = \rho g H = 920 \text{ kg/m}^3 \times 9.80665 \text{ m/s}^2 \times 1000 \text{ m} = 9\ 022\ 118 \text{ Pa}$$

$$P = P_O + \Delta P = 101.325 + 9022 = \textbf{9123 kPa}$$

See figure 3.7 liquid solid interphase => $\textbf{T}_{\textbf{LS}} = \textbf{--1°C}$

2.19

Find the lowest temperature at which it is possible to have water in the liquid phase. At what pressure must the liquid exist?

Solution:

There is no liquid at lower temperatures than on the fusion line, see Fig. 3.7, saturated ice III to liquid phase boundary is at

$T \approx 263K \approx - 10°C$ **and**
$P \approx 210$ **MPa**

2.20

Water at 27°C can exist in different phases dependent upon the pressure. Give the approximate pressure range in kPa for water being in each one of the three phases vapor, liquid or solid.

Solution:

The phases can be seen in Fig. 2.4, a sketch of which is shown to the right.

$T = 27 \; °C = 300 \; K$

From Fig. 2.4:

$\qquad P_{VL} \approx 4 \times 10^{-3} \; MPa = 4 \; kPa,$

$\qquad P_{LS} = 10^3 \; MPa$

$$0 < P < \qquad 4 \; \textbf{kPa} \qquad \textbf{VAPOR}$$
$$\textbf{0.004 MPa} < P < \textbf{1000 MPa} \qquad \textbf{LIQUID}$$
$$P > \textbf{1000 MPa} \qquad \textbf{SOLID (ICE)}$$

2.21

Dry ice is the name of solid carbon dioxide. How cold must it be at atmospheric (100 kPa) pressure? If it is heated at 100 kPa what eventually happens?

Solution:

The phase boundaries are shown in Figure 2.5

At 100 kPa the carbon dioxide is solid if   **T < 190 K**

It goes directly to a vapor state without becoming a liquid hence its name.

The 100 kPa is below
the triple point.

2.22

What is the lowest temperature in Kelvin for which you can see metal as a liquid if the metal is *a.* mercury   *b.* zinc

Solution:

Assume the two substances have a phase diagram similar to Fig. 2.5, then the triple point is the lowest T with liquid possible the data is from Table 2.1

$$T_a = -39^oC = \textbf{234 K}$$

$$T_b = 419^oC = \textbf{692 K}$$

2.23

A substance is at 2 MPa, 17°C in a rigid tank. Using only the critical properties can the phase of the mass be determined if the substance is oxygen, water or propane?

Solution:

Find state relative to critical point properties which are from Table A.2:

        a) Oxygen O_2 : 5.04 MPa   154.6 K

        b) Water H_2O : 22.12 MPa   647.3 K

        c) Propane C_3H_8 : 4.25 MPa   369.8 K

State is at 17 °C = 290 K and   2 MPa < P_c
for all cases:

O_2 : T $\gg T_c$ Superheated vapor P < Pc

H_2O : T $\ll T_c$;   P $\ll$ P_c
               you cannot say.

C_3H_8 : T < T_c ; P < P_c you cannot say

2.24

Give the phase for the following states.

Solution:

a. CO_2 $T = 40°C$ $P = 0.5$ MPa   Table A.2

$T > T_c$ =>   also $P << P_c$

superheated vapor   assume ideal gas Table A.5

b. Air $T = 20°C$ $P = 200$ kPa   Table A.2

superheated vapor   assume ideal gas Table A.5

c. NH_3 $T = 170°C$ $P = 600$ kPa   Table B.2.2 or A.2

$T > T_c$ =>   **superheated vapor**

General Tables

2.25

Give the phase for the following states.

Solution:

a. H_2O $T = 260°C$ $P = 5$ MPa   Table B.1.1 **or** B.1.2

B.1.1 For given T read: $P_{sat} = 4.689$ MPa

$P > P_{sat}$ => **compressed liquid**

B.1.2 For given P read: $T_{sat} = 264°C$

$T < T_{sat}$ => **compressed liquid**

b. H_2O $T = -2°C$ $P = 100$ kPa   Table B.1.1 $T < T_{triple\ point}$

Table B.1.5 at $-2°C$ read: $P_{sat} = 0.518$ kPa

since $P > P_{sat}$ =>   **compressed solid**

Note state b in P-v, see the
3-D figure, is up on the
solid face.

2.26

Determine the phase of the substance at the given state using Appendix B tables

a)	Water	100°C, 500 kPa
b)	Ammonia	-10°C, 150 kPa
c)	R-410A	0°C, 350 kPa

Solution:

a) From Table B.1.1 $P_{sat}(100°C) = 101.3$ kPa

500 kPa > P_{sat} then it is compressed liquid

OR from Table B.1.2 $T_{sat}(500$ kPa$) = 152°C$

$100°C < T_{sat}$ then it is subcooled liquid = compressed liquid

b) Ammonia NH_3 :

Table B.2.1: $P < P_{sat}(-10\ °C) = 291$ kPa

Superheated vapor

c) R-410A

Table B.4.1: $P < P_{sat}(0\ °C) = 799$ kPa

Superheated vapor.

The S-L fusion line goes slightly to the left for water. It tilts slightly to the right for most other substances.

2.27

Give the missing property of P-v-T and x for water at

      a.   P = 10 MPa, v = 0.003 m³/kg   b.   1 MPa, 190°C

      c.   200°C, 0.1 m³/kg   d.   10 kPa, 10°C

Solution:

For all states start search in table B.1.1 (if T given) or B.1.2 (if P given)

a. P = 10 MPa, v = 0.003 m³/kg   so look in B.1.2 at 10 MPa

$$T = 311°C; \ v_f = 0.001452 < v < v_g = 0.01803 \text{ m}^3/\text{kg, so L+V}$$

$$\Rightarrow \quad x = (v - v_f)/v_{fg} = (0.003 - 0.001452)/0.01657 = 0.093$$

b. 1 MPa, 190°C :   Only one of the two look-ups is needed

      B.1.1: $P < P_{sat}$ = 1254.4 kPa so it is superheated vapor

      B.1.2: $T > T_{sat}$ = 179.91°C so it is superheated vapor

      B.1.3: $v = 0.19444 + (0.20596 - 0.19444)\dfrac{190 - 179.91}{200 - 179.91} = 0.2002 \text{ m}^3/\text{kg}$

c. 200°C, 0.1 m³/kg: look in B.1.1: $P = P_{sat}$ = 1553.8 kPa

$$v_f = 0.001156 \text{ m}^3/\text{kg} \ < v < \ v_g = 0.12736 \text{ m}^3/\text{kg, so L+V}$$

$$\Rightarrow \quad x = (v - v_f)/v_{fg} = (0.1 - 0.001156)/0.1262 = 0.7832$$

d. 10 kPa, 10°C :   Only one of the two look-ups is needed

      From B.1.1: $P > P_g$ = 1.2276 kPa so compressed liquid

      From B.1.2: $T < T_{sat}$ = 45.8°C so compressed liquid

      From B.1.1: $v = v_f$ = 0.001 m³/kg (at given T, not given P)

States shown are
placed relative to the
two-phase region, not
to each other.

2.28

For water at 200 kPa with a quality of 10%, find the volume fraction of vapor.

This is a two-phase state at a given pressure:

Table B.1.2: $v_f = 0.001\ 061\ m^3/kg,\quad v_g = 0.88573\ m^3/kg$

From the definition of quality we get the masses from total mass, m, as

$$m_f = (1-x)\,m, \qquad m_g = x\,m$$

The volumes are

$$V_f = m_f\,v_f = (1-x)\,m\,v_f, \qquad V_g = m_g\,v_g = x\,m\,v_g$$

So the volume fraction of vapor is

$$\text{Fraction} = \frac{V_g}{V} = \frac{V_g}{V_g + V_f} = \frac{x\,m\,v_g}{x\,m\,v_g + (1-x)m\,v_f}$$

$$= \frac{0.1 \times 0.88573}{0.1 \times 0.88573 + 0.9 \times 0.001061} = \frac{0.088573}{0.0895279} = \mathbf{0.9893}$$

Notice that the liquid volume is only about 1% of the total. We could also have found the overall $v = v_f + x v_{fg}$ and then $V = m\,v$.

2.29

Determine whether refrigerant R-410A in each of the following states is a compressed liquid, a superheated vapor, or a mixture of saturated liquid and vapor.

Solution:

All cases are seen in Table B.4.1

a. $50°C$, $0.05 \text{ m}^3/\text{kg}$ From table B.4.1 at $50°C$ $v_g = 0.00707 \text{ m}^3/\text{kg}$

since $v > v_g$ we have **superheated vapor**

b. 1.0 MPa, 20°C   From table B.4.1 at $20°C$ $P_g = 909.9$ kPa

since $P < P_g$ we have **superheated vapor**

c. 0.1 MPa, $0.1 \text{ m}^3/\text{kg}$ From table B.4.1 at 0.1 MPa (use 101 kPa)

$v_f = 0.00074$ and $v_g = 0.2395 \text{ m}^3/\text{kg}$

as $v_f < v < v_g$ we have a **mixture of liquid & vapor**

d $-20°C$, 200 kPa   **superheated vapor**, $P < P_g = 400$ kPa at $-20°C$

2.30

Show the states in Problem 2.29 in a sketch of the P-v diagram.

States shown are placed relative to the two-phase region, not to each other.

2.31

How great is the change in liquid specific volume for water at 20°C as you move up from state i towards state j in Fig. 2.14, reaching 15 000 kPa?

State "i", here "a", is saturated liquid and up is then compressed liquid states

a	Table B.1.1:	$v_f = 0.001\ 002\ m^3/kg$	at	2.34 kPa
b	Table B.1.4:	$v_f = 0.001\ 002\ m^3/kg$	at	500 kPa
c	Table B.1.4:	$v_f = 0.001\ 001\ m^3/kg$	at	2000 kPa
d	Table B.1.4:	$v_f = 0.001\ 000\ m^3/kg$	at	5000 kPa
e	Table B.1.4:	$v_f = 0.000\ 995\ m^3/kg$	at	15 000 kPa
f	Table B.1.4:	$v_f = 0.000\ 980\ m^3/kg$	at	50 000 kPa

Notice how small the changes in v are for very large changes in P.

2.32

Fill out the following table for substance ammonia:
Solution:

	P [kPa]	T [^{O}C]	v [m^3/kg]	x
a)	**1003**	25	0.1185	**0.9238**
b)	**119.5**	-30	**0.4824**	0.5

a)   B.2.1 $v_f < v < v_g$ =>   two-phase mix   Look in B.2.1
   => $x = (v - v_f)/v_{fg} = (0.1185 – 0.001658)/0.12647 = 0.9238$

b)   B.2.1 $P = P_{sat} = 119.5$ kPa
     $v = v_f + x\ v_{fg} = 0.001476 + 0.5 \times 0.96192 = 0.4824$ m^3/kg

2.33

Place the two states a-b listed in Problem 2.32 as labeled dots in a sketch of the P-v and T-v diagrams.
Solution:

2.34

Give the missing property of P, T, v and x for R-410A at

a.   T = -20°C, P = 450 kPa

b.   P = 300 kPa, v = 0.092 m^3/kg

Solution:

a) B.4.1   P > P$_{sat}$ = 399.6 kPa $\Rightarrow$ **compressed liquid**

v ~ v$_f$ = 0.000803 m^3/kg

x = undefined

b) B.4.2   v > v$_g$ at 300 kPa $\Rightarrow$ **superheated vapor**

$$T = -20 + [0 - (-20)]\left(\frac{0.092 - 0.08916}{0.09845 - 0.08916}\right) = -13.89°C$$

x = undefined

2.35

Fill out the following table for substance water:
Solution:

	P [kPa]	T [^oC]	v [m^3/kg]	x
a)	500	20	**0.001002**	**Undefined**
b)	500	**151.86**	0.20	**0.532**
c)	1400	200	**0.14302**	**Undefined**
d)	**8581**	300	**0.01762**	0.8

a) Table B.1.1 $P > P_{sat}$ so it is compressed liquid => Table B.1.4

b) Table B.1.2 $v_f < v < v_g$ so two phase   L + V

$$x = \frac{v - v_f}{v_{fg}} = (0.2 - 0.001093) / 0.3738 = 0.532$$

$$T = T_{sat} = 151.86^oC$$

c) Only one of the two look-up is needed

Table B.1.1   200^oC $P < P_{sat} =$ => superheated vapor

Table B.1.2   1400 kPa $T > T_{sat} = 195^oC$

Table B.1.3   sub-table for 1400 kPa gives the state properties

d) Table B.1.1   since quality is given it is two-phase

$$v = v_f + x \times v_{fg} = 0.001404 + 0.8 \times 0.02027 = 0.01762 \text{ m}^3/\text{kg}$$

2.36

Place the four states a-d listed in Problem 2.35 as labeled dots in a sketch of the P-v and T-v diagrams.

Solution:

2.37

Determine the specific volume for R-410A at these states:

      a. -15^oC, 400 kPa

      b. 20^oC, 1500 kPa

      c. 20^oC, quality 25%

a)   Table B.4.1: $P < P_{sat} = 480.4$ kPa, so superheated vapor.

     B.4.2 $v = 0.06475 + (0.07227 - 0.06475)\dfrac{-15 - (-19.98)}{0 - (-19.98)} = 0.06662$ m^3/kg

b)   Table B.4.1: $P > P_{sat} = 1444$ kPa, so compressed liquid

           $v \approx v_f = 0.000923$ m^3/kg

c)   Table B.4.1: $v_f = 0.000923$ m^3/kg, $v_{fg} = 0.01666$ m^3/kg   so

        $v = v_f + x\, v_{fg} = 0.000923 + 0.25 \times 0.01666 = 0.00509$ m^3/kg

States shown are placed relative to the two-phase region, not to each other.

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2.38

Give the missing property of P, T, v and x for CH_4 at:

a.   T = 155 K, v = 0.04 m^3/kg

b.   T = 350 K, v = 0.25 m^3/kg

a) B.7.1 $v < v_g = 0.04892$ m^3/kg $\Rightarrow$ 2-phase

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.04 - 0.002877}{0.04605} = \mathbf{0.806}$$

$P = P_{sat} = \mathbf{1296\ kPa}$

b) B.7.1 $T > T_c$ and $v \gg v_c$ $\Rightarrow$ superheated vapor B.7.2
located between 600 & 800 kPa

$$P = 600 + 200 \frac{0.25 - 0.30067}{0.2251 - 0.30067} = \mathbf{734\ kPa}$$

2.39

Give the specific volume of carbon-dioxide at -20$^{\text{o}}$C for 2000 kPa and repeat for 1400 kPa.

Table B.3.1:   -20$^{\text{o}}$C P_{sat} = 1969 kPa,

at 2000 kPa state is compressed liquid: $v = v_f = 0.000969$ m^3/kg

at 1400 kPa state is superheated vapor: $v = 0.0296$ m^3/kg

The 2000 kPa is above and
the 1400 kPa is below
the vaporization line.

2.40

Calculate the following specific volumes

    a. CO_2 $10^{\circ}C$, 80% quality

    b.   Water   4 MPa, 90% quality

    c.   Nitrogen   120 K, 60% quality

Solution:

All states are two-phase with quality given. The overall specific volume is given by Eq.2.1 or 2.2

$$v = v_f + x\, v_{fg} = (1\text{-}x)v_f + x\, v_g$$

    a. CO_2 $10^{\circ}C$, 80% quality in Table B.3.1

         $v = 0.001161 + x \times 0.00624 = 0.006153 \text{ m}^3/\text{kg}$

    b.   Water   4 MPa, 90% quality in Table B.1.2

         $v = 0.001252(1\text{-}x) + x \times 0.04978 = 0.04493 \text{ m}^3/\text{kg}$

    c.   Nitrogen   120 K, 60% quality in Table B.6.1

         $v = 0.001915 + x \times 0.00608 = 0.005563 \text{ m}^3/\text{kg}$

2.41

Give the missing property of P, T, v, and x for:

a) R-410A at $T = 25^oC$, $v = 0.01 \text{ m}^3/\text{kg}$

b) R-410A at 400 kPa, $v = 0.075 \text{ m}^3/\text{kg}$

c) NH3 at 10^oC, $v = 0.1 \text{ m}^3/\text{kg}$

Solution:

a)   Table B.4.1 $v < v_g = 0.01514 \text{ m}^3/\text{kg}$ so state is two-phase L + V

$P = P_{sat} = \textbf{1653.6 kPa}$

$x = \dfrac{v - v_f}{v_{fg}} = (0.01 - 0.000944)/0.01420 = \textbf{0.6377}$

b)   Table B.4.1 $v > v_g$ so go to B.4.2 superheated vapor 400 kPa.

Make linear interpolation

$$T = 0 + (20 - 0)\frac{0.075 - 0.07227}{0.07916 - 0.07227} = 7.92, \quad x \text{ is undefined}$$

c)   Table B.2.1 $v_f < v < v_g = 0.01514 \text{ m}^3/\text{kg}$ so state is two-phase L + V

$P = P_{sat} = \textbf{615.2 kPa}$

$x = \dfrac{v - v_f}{v_{fg}} = (0.1 - 0.0016)/0.20381 = \textbf{0.4828}$

Each state is
positioned
relative to the
two-phase region
not to each other

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2.42

You want a pot of water to boil at 105°C. How heavy a lid should you put on the 15 cm diameter pot when $P_{atm} = 101$ kPa?

Solution:

Table B.1.1 at 105°C : $P_{sat} = 120.8$ kPa

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} 0.15^2 = 0.01767 \text{ m}^2$$

$$F_{net} = (P_{sat} - P_{atm}) A = (120.8 - 101) \text{ kPa} \times 0.01767 \text{ m}^2$$
$$= 0.3498 \text{ kN} = 350 \text{ N}$$

$$F_{net} = m_{lid} \, g$$

$$m_{lid} = F_{net}/g = \frac{350}{9.807} \frac{\text{N}}{\text{m/s}^2} = \textbf{35.7 kg}$$

Some lids are clamped on, the problem deals with one that stays on due to its weight.

2.43

Water at 400 kPa with a quality of 25% has its pressure raised 50 kPa in a constant volume process. What is the new quality and temperature?

Solution:

State 1 from Table B.1.2 at 400 kPa

$$v = v_f + x\, v_{fg} = 0.001084 + 0.25 \times 0.46138 = 0.11643 \text{ m}^3/\text{kg}$$

State 2 has same v at P = 450 kPa also from Table B.1.2

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.11643 - 0.001088}{0.41289} = \mathbf{0.279}$$

$$T = T_{sat} = \mathbf{147.93 \text{ °C}}$$

2.44

A sealed rigid vessel has volume of 1 m^3 and contains 2 kg of water at 100°C. The vessel is now heated. If a safety pressure valve is installed, at what pressure should the valve be set to have a maximum temperature of 200°C?

Solution:

Process: v = V/m = constant

State 1: v_1 = 1/2 = 0.5 m^3/kg

from Table B.1.1
it is 2-phase

State 2: 200°C, 0.5 m^3/kg
Table B.1.3 between 400
and 500 kPa so interpolate

$$P \cong 400 + \frac{0.5 - 0.53422}{0.42492 - 0.53422} \times (500 - 400) = \textbf{431.3 kPa}$$

2.45

Saturated water vapor at 200 kPa is in a constant pressure piston cylinder. At this state the piston is 0.1 m from the cylinder bottom. How much is this distance and the temperature if the water is cooled to occupy half the original volume?

Solution:

State 1: B 1.2 $v_1 = v_g$ (200 kPa) = 0.8857 m^3/kg, T_1 = 120.2°C

Process: P = constant = 200 kPa

State 2: P, $v_2 = v_1/2 = 0.44285$ m^3/kg

Table B.1.2 $v_2 < v_g$ so two phase $T_2 = T_{sat}$ = 120.2°C

Height is proportional to volume

$h_2 = h_1 \times v_2/v_1 = 0.1$ m $\times 0.5 =$ **0.05m**

2.46

Saturated liquid water at 60°C is put under pressure to decrease the volume by 1% keeping the temperature constant. To what pressure should it be compressed?

Solution:

State 1: $T = 60°C$, $x = 0.0$;   Table B.1.1: $v = 0.001017 \text{ m}^3/\text{kg}$

Process: $T = \text{constant} = 60°C$

State 2:   T, $v = 0.99 \times v_{f\,(60°C)} = 0.99 \times 0.001017 = 0.0010068 \text{ m}^3/\text{kg}$

Between 20 & 30 MPa in Table B.1.4, $P \cong$ **23.8 MPa**

2.47

Water at 400 kPa with a quality of 25% has its temperature raised 20°C in a constant pressure process. What is the new quality and specific volume?

Solution:

State 1 from Table B.1.2 at 400 kPa

$$v = v_f + x\, v_{fg} = 0.001084 + 0.25 \times 0.46138 = 0.126185 \text{ m}^3/\text{kg}$$

State 2 has same P from Table B.1.2 at 400 kPa

$$T_2 = T_{sat} + 20 = 143.63 + 20 = 163.63^O\text{C}$$

so state is superheated vapor look in B.1.3 and interpolate between 150 and 200 C in the 400 kPa superheated vapor table.

x = **undefined**

$$v_2 = 0.47084 + (0.53422 - 0.47084)\, \frac{163.63 \text{ - } 150}{200 \text{ - } 150} = \textbf{0.4881 m}^3\textbf{/kg}$$

2.48

In your refrigerator the working substance evaporates from liquid to vapor at -20 ^{O}C inside a pipe around the cold section. Outside (on the back or below) is a black grille inside which the working substance condenses from vapor to liquid at +45 ^{O}C. For each location find the pressure and the change in specific volume (v) if the substance is ammonia.

Solution:
The properties come from the saturated tables where each phase change takes place at constant pressure and constant temperature.

Substance	TABLE	T	P_{sat} , kPa	$\Delta v = v_{fg},\ m^3/kg$
Ammonia	B.2.1	45 ^{O}C	1782	0.0707
Ammonia	B.2.1	-20 ^{O}C	190	0.622

2.49

Repeat the previous problem with the substances
a) R-134a   b) R-410A

In your refrigerator, the working substance evaporates from liquid to vapor at -20 ^{O}C inside a pipe around the cold section. Outside (on the back or below) is a black grille inside which the working substance condenses from vapor to liquid at +45 ^{O}C. For each location find the pressure and the change in specific volume (v).

Solution:
The properties come from the saturated tables where each phase change takes place at constant pressure and constant temperature.

Substance	TABLE	T	P_{sat} , kPa	$\Delta v = v_{fg}$, m^3/kg
R-134a	B.5.1	45 ^{O}C	1160	0.0165
R-134a	B.5.1	-20 ^{O}C	134	0.146
R-410A	B.4.1	45 ^{O}C	2728	0.00723
R-410A	B.4.1	-20 ^{O}C	400	0.064

2.50

Repeat Problem 2.48 with CO_2, condenser at $+ 20^{\circ}C$ and evaporator at $-30^{\circ}C$.

Solution:

The properties come from the saturated tables where each phase change takes place at constant pressure and constant temperature.

Substance	TABLE	T	P_{sat} , kPa	$\Delta v = v_{fg}$, m^3/kg
CO_2	B.3.1	$20\ ^{O}C$	5729	0.00386
CO_2	B.3.1	$-30\ ^{O}C$	1428	0.026

2.51

A glass jar is filled with saturated water at 500 kPa, quality 25%, and a tight lid is put on. Now it is cooled to $-10°C$. What is the mass fraction of solid at this temperature?

Solution:

Constant volume and mass $\Rightarrow v_1 = v_2 = V/m$

From Table B.1.2: $v_1 = 0.001093 + 0.25 \times 0.3738 = 0.094543 \ m^3/kg$

From Table B.1.5: $v_2 = 0.0010891 + x_2 \times 446.756 = v_1 = 0.094543 \ m^3/kg$

$$\Rightarrow x_2 = 0.0002 \ \text{mass fraction vapor}$$

$x_{solid} = 1 - x_2 = 0.9998$ or   **99.98 %**

2.52

Two tanks are connected as shown in Fig. P2.52, both containing water. Tank A is at 200 kPa, $v = 0.5$ m^3/kg, $V_A = 1$ m^3 and tank B contains 3.5 kg at 0.5 MPa, 400°C. The valve is now opened and the two come to a uniform state. Find the final specific volume.

Solution:

Control volume: both tanks. Constant total volume and mass process.

State A1: (P, v) $m_A = V_A/v_A = 1/0.5 = 2$ kg

State B1: (P, T)   Table B.1.3 $v_B = 0.6173$ m^3/kg

$$\Rightarrow V_B = m_B v_B = 3.5 \text{ kg} \times 0.6173 \text{ m}^3\text{/kg} = 2.1606 \text{ m}^3$$

Final state: $m_{tot} = m_A + m_B = 5.5$ kg

$$V_{tot} = V_A + V_B = 3.1606 \text{ m}^3$$

$$v_2 = V_{tot}/m_{tot} = \textbf{0.5746 m}^3\textbf{/kg}$$

2.53

Saturated vapor R-134a at 60°C changes volume at constant temperature. Find the new pressure, and quality if saturated, if the volume doubles. Repeat the question for the case the volume is reduced to half the original volume.

Solution:

1: (T, x) B.4.1: $v_1 = v_g = 0.01146$ m^3/kg, $P_1 = P_{sat} = 1681.8$ kPa

2: $v_2 = 2v_1 = 0.02292$ m^3/kg superheated vapor
Interpolate between 1000 kPa and 1200 kPa
$$P_2 = 1000 + 200 \times \frac{0.02292 - 0.02311}{0.01844 - 0.02311} = \mathbf{1008 \ kPa}$$

3: $v_3 = v_1/2 = 0.00573$ m^3/kg $< v_g$: two phase
$$x_3 = \frac{v_3 - v_f}{v_{fg}} = \frac{0.00573 - 0.000951}{0.01051} = \mathbf{0.4547}$$
$$P_3 = P_{sat} = \mathbf{1681.8 \ kPa}$$

2.54

A steel tank contains 6 kg of propane (liquid + vapor) at 20°C with a volume of 0.015 m^3. The tank is now slowly heated. Will the liquid level inside eventually rise to the top or drop to the bottom of the tank? What if the initial mass is 1 kg instead of 6 kg?

Solution:

Constant volume and mass $v_2 = v_1 = \dfrac{V}{m} = \dfrac{0.015 \text{ m}^3}{6 \text{ kg}} = 0.0025 \text{ m}^3/\text{kg}$

A.2: $v_c = 0.00454 \text{ m}^3/\text{kg} > v_1$

eventually reaches sat. liquid.
   $\Rightarrow$ **level rises to top**

If m = 1 kg $\Rightarrow$ $v_1 = 0.015 \text{ m}^3/\text{kg} > v_c$

then it will reach saturated vapor.
   $\Rightarrow$ **level falls**

2.55

Saturated water vapor at 60°C has its pressure decreased to increase the volume by 10% keeping the temperature constant. To what pressure should it be expanded?

Solution:

Initial state: $v = 7.6707 \text{ m}^3/\text{kg}$ from table B.1.1

Final state: $v = 1.10 \times v_g = 1.1 \times 7.6707 = 8.4378 \text{ m}^3/\text{kg}$

Interpolate at 60°C between saturated (P = 19.94 kPa) and superheated vapor P = 10 kPa in Tables B.1.1 and B.1.3

$$P \cong 19.941 + (10 - 19.941)\,\frac{8.4378 - 7.6707}{15.3345 - 7.6707} = \textbf{18.9 kPa}$$

Comment: T,v $\Rightarrow$ P = 18 kPa (software) v is not linear in P, more like 1/P, so the linear interpolation in P is not very accurate.

2.56

Ammonia at 20^OC with a quality of 50% and total mass 2 kg is in a rigid tank with an outlet valve at the top. How much vapor (mass) can you take out through the valve assuming the temperature stays constant?

Solution:

The top has saturated vapor at 20^OC as long as there is a two phase inside. When there is no more liquid and vapor will be taken out pressure will drop for the remaining vapor so we can take it out until we reach P_0.

$$V = m_1 v_1 = 2 \text{ kg} \times 0.5 \ (0.001638 + 0.14922) \ \text{m}^3/\text{kg} = 0.15086 \ \text{m}^3$$

$$v_2 = 1.4153 \ \text{m}^3/\text{kg} \quad \text{(from Table B.2.2 at 100 kPa)}$$

$$m_2 = V/v_2 = 0.15086 \ \text{m}^3/ \ 1.4153 \ \text{m}^3/\text{kg} = 0.1066 \ \text{kg}$$

$$m = m_1 - m_2 = 2 - 0.1066 = \textbf{1.893 kg}$$

2.57

A sealed rigid vessel of 2 m^3 contains a saturated mixture of liquid and vapor R-134a at 10°C. If it is heated to 50°C, the liquid phase disappears. Find the pressure at 50°C and the initial mass of the liquid.

Solution:

Process: constant volume and constant mass.

State 2 is saturated vapor, from table B.5.1

$$P_2 = P_{sat}(50°C) = \textbf{1.318 MPa}$$

State 1: same specific volume as state 2

$v_1 = v_2 = 0.015124 \ m^3/kg$

$v_1 = 0.000794 + x_1 \times 0.048658$

$$\Rightarrow x_1 = 0.2945$$

$m = V/v_1 = 2 \ m^3/0.015124 \ m^3/kg = 132.24 \ kg;$

$m_{liq} = (1 - x_1) \ m = \textbf{93.295 kg}$

2.58

A storage tank holds methane at 120 K, with a quality of 25 %, and it warms up by 5°C per hour due to a failure in the refrigeration system. How long time will it take before the methane becomes single phase and what is the pressure then?

Solution: Use Table B.7.1

Assume rigid tank $v = \text{constant} = v_1$

$$v_1 = 0.002439 + 0.25 \times 0.30367$$

$$= 0.078366 \ \text{m}^3/\text{kg}$$

We then also see that

$$v_1 > v_c = 0.00615 \ \text{m}^3/\text{kg}$$

All single phase when

$$v = v_g \ \Rightarrow \ T \cong 145 \ \text{K}$$

$$\Delta t = \frac{\Delta T}{(5°\text{C/h})} \cong \frac{145 - 120}{5} = \textbf{5 hours;} \qquad P = P_{\text{sat}} = \textbf{824 kPa}$$

2.59

Ammonia at 10°C and mass 10 kg is in a piston cylinder with an initial volume of 1 m^3. The piston initially resting on the stops has a mass such that a pressure of 900 kPa will float it. Now the ammonia is slowly heated to 50°C. Find the final pressure and volume.

Solution:

C.V. Ammonia, constant mass.

Process: V = constant, unless P = P$_{float}$

State 1: T = 10 ^{o}C,

$$v_1 = \frac{V}{m} = \frac{1}{10} = 0.1 \ m^3/kg$$

From Table B.2.1 $v_f < v < v_g$

$$x_1 = \frac{v - v_f}{v_{fg}} = \frac{0.1 - 0.0016}{0.20381} = 0.4828$$

State 1a: P = 900 kPa, v = v$_1$ = 0.1 m^3/kg $< v_g$ at 900 kPa

This state is two-phase T$_{1a}$ = 21.52^{o}C

Since T$_2$ > T$_{1a}$ then v$_2$ > v$_{1a}$

State 2: 50°C and on line(s) means

P$_2$ = **900 kPa** which is superheated vapor.

Table B.2.2 : v$_2$ = (0.18465 + 0.14499)/2 = 0.16482 m^3/kg

V$_2$ = mv$_2$ = **1.6482 m^3**

2.60

A 400-m^3 storage tank is being constructed to hold LNG, liquified natural gas, which may be assumed to be essentially pure methane. If the tank is to contain 90% liquid and 10% vapor, by volume, at 100 kPa, what mass of LNG (kg) will the tank hold? What is the quality in the tank?

Solution:

CH$_4$ is in the section B tables.

From Table B.7.1: $v_f \cong 0.002366 \text{ m}^3/\text{kg}$,   (interpolated)

From Table B.7.2: $v_g \cong 0.55665 \text{ m}^3/\text{kg}$ (first entry 100 kPa)

$$m_{liq} = \frac{V_{liq}}{v_f} = \frac{0.9 \times 400}{0.002366} = 152\ 155.5 \text{ kg}; \quad m_{vap} = \frac{V_{vap}}{v_g} = \frac{0.1 \times 400}{0.55665} = 71.86 \text{ kg}$$

$$m_{tot} = \textbf{152 227 kg,} \quad x = m_{vap} / m_{tot} = \textbf{4.72} \times \textbf{10}^{\textbf{-4}}$$

2.61

A boiler feed pump delivers 0.05 m³/s of water at 240°C, 20 MPa. What is the mass flowrate (kg/s)? What would be the percent error if the properties of saturated liquid at 240°C were used in the calculation? What if the properties of saturated liquid at 20 MPa were used?

Solution:

State 1:   (T, P) compressed liquid seen in B.1.4: $v = 0.001205$ m³/kg

$$\dot{m} = \dot{V}/v = 0.05/0.001205 = \textbf{41.5 kg/s}$$

$$v_{f\,(240°C)} = 0.001229 \text{ m}^3/\text{kg} \Rightarrow \dot{m} = 40.68 \text{ kg/s} \quad \textbf{error 2\%}$$

$$v_{f\,(20\,MPa)} = 0.002036 \text{ m}^3/\text{kg} \Rightarrow \dot{m} = 24.56 \text{ kg/s} \quad \textbf{error 41\%}$$

The constant T line is nearly vertical for the liquid phase in the P-v diagram.
The state is at so high P, T that the saturated liquid line is not extremely steep.

2.62

A piston/cylinder arrangement is loaded with a linear spring and the outside atmosphere. It contains water at 5 MPa, 400°C with the volume being 0.1 m^3. If the piston is at the bottom, the spring exerts a force such that P_{lift} = 200 kPa. The system now cools until the pressure reaches 1200 kPa. Find the mass of water, the final state (T_2, v_2) and plot the *P–v* diagram for the process.

Solution:

1: Table B.1.3 $\Rightarrow$ v_1= 0.05781 m^3/kg

$\quad$ m = V/v_1 = 0.1/0.05781 = 1.73 kg

Straight line: $P = P_a + C \times v$

$$v_2 = v_1 \frac{P_2 - P_a}{P_1 - P_a} = \textbf{0.01204 m}^3\textbf{/kg}$$

$v_2 < v_g$(1200 kPa) so two-phase $T_2 = \textbf{188°C}$

$\Rightarrow$ x_2 = (v_2 - 0.001139)/0.1622 = 0.0672

2.63

A pressure cooker (closed tank) contains water at 100°C with the liquid volume being 1/20 of the vapor volume. It is heated until the pressure reaches 2.0 MPa. Find the final temperature. Has the final state more or less vapor than the initial state?

Solution:

State 1: $V_f = m_f v_f = V_g/20 = m_g v_g/20$;

Table B.1.1: $v_f = 0.001044$ m^3/kg, $v_g = 1.6729$ m^3/kg

$$x_1 = \frac{m_g}{m_g + m_f} = \frac{20\, m_f v_f / v_g}{m_f + 20\, m_f v_f / v_g} = \frac{20\, v_f}{20\, v_f + v_g} = \frac{0.02088}{0.02088 + 1.6729} = 0.01233$$

$v_1 = 0.001044 + 0.01233 \times 1.67185 = 0.02166$ m^3/kg

State 2: $v_2 = v_1 = 0.02166$ m^3/kg $< v_g$(2MPa) from B.1.2 so two-phase

At state 2: $v_2 = v_f + x_2\, v_{fg}$

$0.02166 = 0.001177 + x_2 \times 0.09845$

=> $x_2 = 0.208$

More vapor at final state

$T_2 = T_{sat}$(2MPa) = **212.4°C**

2.64

A pressure cooker has the lid screwed on tight. A small opening with A = 5 mm^2 is covered with a petcock that can be lifted to let steam escape. How much mass should the petcock have to allow boiling at 120$^\text{o}$C with an outside atmosphere at 101.3 kPa?

Table B.1.1.: P_{sat} = 198.5 kPa

$F = mg = \Delta P \times A$

$m = \Delta P \times A/g$

$$= \frac{(198.5\text{-}101.3)\times1000\times5\times10^{-6}}{9.807} \frac{\text{N}}{\text{ms}^{-2}}$$

$= 0.0496 \text{ kg} = \textbf{50 g}$

Ideal Gas Law

2.65

What is the relative (%) change in P if we double the absolute temperature of an ideal gas keeping mass and volume constant? What will it be if we double V having m, T constant.

Ideal gas law: $PV = mRT$

State 2: $P_2V = mRT_2 = mR2T_1 = 2P_1V$ $\Rightarrow$ $P_2 = 2P_1$
                 Relative change $= \Delta P/P_1 = P_1/P_1 = 1 = $ **100%**

State 3: $P_3V_3 = mRT_1 = P_1V_1$ $\Rightarrow$ $P_3 = P_1V_1/V_3 = P_1/2$
                 Relative change $= \Delta P/P_1 = -P_1/2P_1 = -0.5 = $ **-50%**

2.66

A 1-m^3 tank is filled with a gas at room temperature 20°C and pressure 200 kPa. How much mass is there if the gas is a) air, b) neon or c) propane ?

Solution:

Use Table A.2 to compare T and P to the critical T and P with

$\quad$ T = 20°C = 293.15 K ;   P = 200 kPa << P$_c$ for all

Air : T >> T$_{C,N2}$; T$_{C,O2}$ = 154.6 K   so ideal gas; R= 0.287 kJ/kg K

Neon: T >> T$_c$ = 44.4 K   so ideal gas; R = 0.41195 kJ/kg K

Propane: T < T$_c$ = 370 K, but P << P$_c$ = 4.25 MPa

$\qquad\qquad$ so gas R = 0.18855 kJ/kg K

All states are ideal gas states so the ideal gas law applies

$$PV = mRT$$

a) $\quad m = \dfrac{PV}{RT} = \dfrac{200 \text{ kPa} \times 1 \text{ m}^3}{0.287 \text{ kJ/kgK} \times 293.15 \text{ K}} = \textbf{2.377 kg}$

b) $\quad m = \dfrac{PV}{RT} = \dfrac{200 \text{ kPa} \times 1 \text{ m}^3}{0.41195 \text{ kJ/kgK} \times 293.15 \text{ K}} = \textbf{1.656 kg}$

c) $\quad m = \dfrac{PV}{RT} = \dfrac{200 \text{ kPa} \times 1 \text{ m}^3}{0.18855 \text{ kJ/kgK} \times 293.15 \text{ K}} = \textbf{3.618 kg}$

2.67

Calculate the ideal gas constant for argon and hydrogen based on table A.2 and verify the value with Table A.5

argon: $\quad R = \dfrac{\bar{R}}{M} = \dfrac{8.3145}{39.948} = 0.2081 \text{ kJ/kgK} \quad$ same as Table A.5

hydrogen: $\quad R = \dfrac{\bar{R}}{M} = \dfrac{8.3145}{2.016} = 4.124256 \text{ kJ/kgK} \quad$ same as Table A.5

2.68

A pneumatic cylinder (a piston cylinder with air) must close a door with a force of 500 N. The cylinder cross-sectional area is 5 cm^2. With V = 50 cm^3, T = 20°C, what is the air pressure and its mass?

$$F = PA - P_oA \implies$$

$$P = P_o + F/A = 100 \text{ kPa} + \frac{500 \text{ N}}{0.0005 \text{ m}^2 \times 1000 \text{ N/kN}} = \textbf{1100 kPa}$$

$$m = \frac{P_1V_1}{RT_1} = \frac{1100 \text{ kPa} \times 0.00005 \text{ m}^3}{0.287 \text{ kJ/kgK} \times 293 \text{ K}} = \textbf{0.00065 kg = 0.65 g}$$

Comment: Dependent upon your understanding of the problem you could also have neglected the atmospheric pressure to get 1000 kPa and 0.59 g for answers.

2.69

Is it reasonable to assume that at the given states the substance behaves as an ideal gas?

Solution:

a) Oxygen, O_2 at   30°C, 3 MPa   **Ideal Gas** (T » T_c = 155 K from A.2)

b) Methane, CH_4 at   30°C, 3 MPa   **Ideal Gas** (T » T_c = 190 K from A.2)

c) Water, H_2O at   30°C, 3 MPa   **NO** compressed liquid $P > P_{sat}$ (B.1.1)

d) R-134a   at   30°C, 3 MPa   **NO** compressed liquid $P > P_{sat}$ (B.5.1)

e) R-134a   at   30°C, 100 kPa **Ideal Gas** P is low $< P_{sat}$ (B.5.1)

2.70

Helium in a steel tank is at 250 kPa, 300 K with a volume of 0.1 m^3. It is used to fill a balloon and when the pressure drops to 125 kPa, the flow of helium stops by itself. If all the helium still is at 300 K, how big a balloon did I get?

Solution:

State 1:   m = V/v   assume ideal gas so

$$m = \frac{P_1 V_1}{RT_1} = \frac{250 \text{ kPa} \times 0.1 \text{ m}^3}{2.0771 \text{ kJ/kgK} \times 300 \text{ K}}$$
$$= 0.0401 \text{ kg}$$

State 2:   Same mass so then $(T_2 = T_1)$

$$V_2 = \frac{mRT_2}{P_2} = \frac{P_1 V_1}{RT_1} \frac{RT_2}{P_2} = V_1 \frac{P_1}{P_2} = 0.1 \text{ m}^3 \frac{250}{125} = 0.2 \text{ m}^3$$

The balloon volume is

$$V_{balloon} = V_2 - V_1 = 0.2 - 0.1 = \textbf{0.1 m}^3$$

2.71

A hollow metal sphere of 150-mm inside diameter is weighed on a precision beam balance when evacuated and again after being filled to 875 kPa with an unknown gas. The difference in mass is 0.0025 kg, and the temperature is 25°C. What is the gas, assuming it is a pure substance listed in Table A.5 ?

Solution:

Assume an ideal gas with total volume: $V = \dfrac{\pi}{6}(0.15)^3 = 0.001767 \text{ m}^3$

$$M = \frac{m\overline{R}T}{PV} = \frac{0.0025 \text{ kg} \times 8.3145 \text{ kJ/kmol-K} \times 298.2 \text{ K}}{875 \text{ kPa} \times 0.001767 \text{ m}^3}$$

$$= \textbf{4.009} \text{ kg/kmol} \approx M_{He}$$

$$\Rightarrow \quad \textbf{Helium Gas}$$

2.72

A spherical helium balloon of 10 m in diameter is at ambient T and P, 15$^{\text{o}}$C and 100 kPa. How much helium does it contain? It can lift a total mass that equals the mass of displaced atmospheric air. How much mass of the balloon fabric and cage can then be lifted?

We need to find the masses and the balloon volume

$$V = \frac{\pi}{6} D^3 = \frac{\pi}{6} 10^3 = 523.6 \text{ m}^3$$

$$m_{He} = \rho V = \frac{V}{v} = \frac{PV}{RT} = \frac{100 \text{ kPa} \times 523.6 \text{ m}^3}{2.0771 \text{ kJ/kgK} \times 288 \text{ K}} = \mathbf{87.5 \text{ kg}}$$

$$m_{air} = \frac{PV}{RT} = \frac{100 \text{ kPa} \times 523.6 \text{ m}^3}{0.287 \text{ kJ/kgK} \times 288 \text{ K}}$$
$$= 633 \text{ kg}$$

$$m_{lift} = m_{air} - m_{He} = 633\text{-}87.5 = \mathbf{545.5 \text{ kg}}$$

2.73

A glass is cleaned in 45$^\text{o}$C hot water and placed on the table bottom up. The room air at 20$^\text{o}$C that was trapped in the glass gets heated up to 40$^\text{o}$C and some of it leaks out so the net resulting pressure inside is 2 kPa above ambient pressure of 101 kPa. Now the glass and the air inside cools down to room temperature. What is the pressure inside the glass?

Solution:

1 air: 40$^\text{o}$C, 103 kPa

2 air: 20$^\text{o}$C,   ?

Slight amount of liquid water seals to table top

AIR

Constant Volume: $V_1 = V_2$,

Constant Mass $m_1 = m_2$

Ideal Gas $P_1V_1 = m_1RT_1$ and $P_2V_2 = m_1RT_2$

Take Ratio

$$P_2 = P_1 \frac{T_1}{T_2} = 103 \text{ kPa} \times \frac{20 + 273}{40 + 273} = \textbf{96.4 kPa}$$

This is a vacuum relative to atm pressure so the glass is pressed against table.

2.74

Air in an internal combustion engine has 227°C, 1000 kPa with a volume of 0.1 m^3. Now combustion heats it to 1800 K in a constant volume process. What is the mass of air and how high does the pressure become?

The mass comes from knowledge of state 1 and ideal gas law

$$m = \frac{P_1 V_1}{RT_1} = \frac{1000 \text{ kPa} \times 0.1 \text{ m}^3}{0.287 \text{ kJ/kgK} \times (227 + 273) \text{ K}} = \textbf{0.697 kg}$$

The final pressure is found from the ideal gas law written for state 1 and state 2 and then eliminate the mass, gas constant and volume ($V_2 = V_1$) between the equations

$$P_1 \, V_1 = m \, RT_1 \qquad \text{and} \qquad P_2 \, V_2 = m \, RT_2$$

$$P_2 = P_1 \times T_2/T_1 = 1000 \text{ kPa} \times \frac{1800}{500} = \textbf{3600 kPa}$$

2.75

Air in an automobile tire is initially at $-10°C$ and 190 kPa. After the automobile is driven awhile, the temperature gets up to $10°C$. Find the new pressure. You must make one assumption on your own.

Solution:

Assume constant volume $V_2 = V_1$ and that air is an ideal gas

$$P_1V_1 = mRT_1 \quad \text{and} \quad P_2V_2 = mRT_2$$

so

$$P_2 = P_1 \times T_2/T_1$$
$$= 190 \text{ kPa} \times \frac{283.15}{263.15} = \textbf{204.4 kPa}$$

2.76

A rigid tank of 1 m³ contains nitrogen gas at 600 kPa, 400 K. By mistake someone lets 0.5 kg flow out. If the final temperature is 375 K what is the final pressure?

Solution:

$$m = \frac{PV}{RT} = \frac{600 \text{ kPa} \times 1 \text{ m}^3}{0.2968 \text{ kJ/kgK} \times 400 \text{ K}} = 5.054 \text{ kg}$$

$$m_2 = m - 0.5 = 4.554 \text{ kg}$$

$$P_2 = \frac{m_2 R T_2}{V} = \frac{4.554 \text{ kg} \times 0.2968 \text{ kJ/kgK} \times 375 \text{ K}}{1 \text{ m}^3} = \textbf{506.9 kPa}$$

2.77

Assume we have 3 states of saturated vapor R-134a at +40 ^{o}C, 0 ^{o}C and –40 ^{o}C. Calculate the specific volume at the set of temperatures and corresponding saturated pressure assuming ideal gas behavior. Find the percent relative error = $100(v – v_g)/v_g$ with v_g from the saturated R-134a table.

Solution:

R-134a.   Table values from Table B.5.1 P_{sat}, $v_g(T)$

Ideal gas constant from Table A.5: $R_{R-134a} = 0.08149$ kJ/kg K

T	P_{sat} , kPa	v_g, m^3/kg	$v_{ID.G.} = RT / P_{sat}$	error %
-40 ^{o}C	51.8	0.35696	0.36678	2.75
0 ^{o}C	294	0.06919	0.07571	9.4
40 ^{o}C	1017	0.02002	0.02509	25.3

2.78

Do Problem 2.77 for R-410A.

Solution:

R-410A.   Table values from Table B.4.1 P_{sat}, $v_g(T)$

Ideal gas constant from Table A.5: $R_{R-410a} = 0.1146$ kJ/kg K

T	P_{sat} , kPa	v_g, m^3/kg	$v_{ID.G.} = RT / P_{sat}$	error %
-40 ^{o}C	175.0	0.14291	0.1527	6.8
0 ^{o}C	798.7	0.03267	0.03919	20
40 ^{o}C	2420.7	0.00967	0.01483	53.3

2.79

Do Problem 2.77 for the substance ammonia.

Solution:

NH3.   Table values from Table B.2.1 P_{sat}, $v_g(T)$

Ideal gas constant from Table A.5: $R_{ammonia}$ = 0.4882 kJ/kg K

T	P_{sat} , kPa	v_g, m³/kg	$v_{ID.G.}$ = RT / P_{sat}	error %
-40 °C	71.7	1.5526	1.5875	2.25
0 °C	429.6	0.28929	0.3104	7.3
40 °C	1555	0.08313	0.09832	18.3

2.80

A 1 m^3 rigid tank has propane at 100 kPa, 300 K and connected by a valve to another tank of 0.5 m^3 with propane at 250 kPa, 400 K. The valve is opened and the two tanks come to a uniform state at 325 K. What is the final pressure?

Solution:

Propane is an ideal gas (P << P$_c$) with R = 0.1886 kJ/kgK from Tbl. A.5

$$m_A = \frac{P_A V_A}{R T_A} = \frac{100 \text{ kPa} \times 1 \text{ m}^3}{0.1886 \text{ kJ/kgK} \times 300 \text{ K}} = 1.7674 \text{ kg}$$

$$m_B = \frac{P_B V_B}{R T_B} = \frac{250 \text{ kPa} \times 0.5 \text{ m}^3}{0.1886 \text{ kJ/kgK} \times 400 \text{ K}} = 1.6564 \text{ kg}$$

$$V_2 = V_A + V_B = 1.5 \text{ m}^3$$

$$m_2 = m_A + m_B = 3.4243 \text{ kg}$$
$$P_2 = \frac{m_2 R T_2}{V_2} = \frac{3.4243 \text{ kg} \times 0.1886 \text{ kJ/kgK} \times 325 \text{ K}}{1.5 \text{ m}^3} = \textbf{139.9 kPa}$$

2.81

A vacuum pump is used to evacuate a chamber where some specimens are dried at 50°C. The pump rate of volume displacement is 0.5 m³/s with an inlet pressure of 0.1 kPa and temperature 50°C. How much water vapor has been removed over a 30-min period?

Solution:

Use ideal gas since   P << lowest P in steam tables.

From table A.5 we get   R = 0.46152 kJ/kg K

$m = \dot{m}\,\Delta t$ with mass flow rate as: $\dot{m} = \dot{V}/v = P\dot{V}/RT$ (ideal gas)

$$\Rightarrow \; m = P\dot{V}\Delta t/RT = \frac{0.1 \text{ kPa} \times 0.5 \text{ m}^3/\text{s} \times 30 \text{ min} \times 60 \text{ s/min}}{0.46152 \text{ kJ/kgK} \times 323.15 \text{ K}} = \textbf{0.603 kg}$$

2.82

A 1-m^3 rigid tank with air at 1 MPa, 400 K is connected to an air line as shown in Fig. P2.82. The valve is opened and air flows into the tank until the pressure reaches 5 MPa, at which point the valve is closed and the temperature inside is 450K.

  a. What is the mass of air in the tank before and after the process?

  b. The tank eventually cools to room temperature, 300 K. What is the pressure inside the tank then?

Solution:

P, T known at both states and assume the air behaves as an ideal gas.

$$m_{air1} = \frac{P_1 V}{RT_1} = \frac{1000 \text{ kPa} \times 1 \text{ m}^3}{0.287 \text{ kJ/kgK} \times 400 \text{ K}} = \textbf{8.711 kg}$$

$$m_{air2} = \frac{P_2 V}{RT_2} = \frac{5000 \text{ kPa} \times 1 \text{ m}^3}{0.287 \text{ kJ/kgK} \times 450 \text{ K}} = \textbf{38.715 kg}$$

Process 2 $\rightarrow$ 3 is constant V, constant mass cooling to T$_3$

$$P_3 = P_2 \times (T_3/T_2) = 5000 \text{ kPa} \times (300/450) = \textbf{3.33 MPa}$$

2.83

A cylindrical gas tank 1 m long, inside diameter of 20 cm, is evacuated and then filled with carbon dioxide gas at 20°C. To what pressure should it be charged if there should be 1.2 kg of carbon dioxide?

Solution:

Assume CO_2 is an ideal gas, table A.5:   R = 0.1889 kJ/kg K

$$V_{cyl} = A \times L = \frac{\pi}{4}(0.2)^2 \times 1 = 0.031416 \text{ m}^3$$

$$P \ V = mRT \qquad => \quad P = \frac{mRT}{V}$$

$$\Rightarrow P = \frac{1.2 \text{ kg} \times 0.1889 \text{ kJ/kg K} \times (273.15 + 20) \text{ K}}{0.031416 \text{ m}^3} = \textbf{2115 kPa}$$

2.84

Ammonia in a piston/cylinder arrangement is at 700 kPa, 80°C. It is now cooled at constant pressure to saturated vapor (state 2) at which point the piston is locked with a pin. The cooling continues to −10°C (state 3). Show the processes 1 to 2 and 2 to 3 on both a *P–v* and *T–v* diagram.

Solution:

State 1: T, P   from table B.2.2   this is superheated vapor.

State 2: T, x   from table B.2.1

State 3: T, v   two-phase

Compressibility Factor

Borgnakke and Sonntag

2.85

Find the compressibility factor (Z) for saturated vapor ammonia at 100 kPa and at 2000 kPa.

Table B.2.2: $v_1 = 1.1381$ m^3/kg, $T_1 = -33.6\,^oC$, $P_1 = 100$ kPa

$v_2 = 0.06444$ m^3/kg, $T_2 = 49.37\,^oC$, $P_2 = 2000$ kPa

Table A.5: $R = 0.4882$ kJ/kg K

Extended gas law: $Pv = ZRT$ so we can calculate Z from this

$$Z_1 = \frac{P_1 v_1}{RT_1} = \frac{100 \text{ kPa} \times 1.1381 \text{ m}^3/\text{kg}}{0.4882 \text{ kJ/kg-K} \times (273.15 - 33.6) \text{ K}} = 0.973$$

$$Z_2 = \frac{P_2 v_2}{RT_2} = \frac{2000 \text{ kPa} \times 0.06444 \text{ m}^3/\text{kg}}{0.4882 \text{ kJ/kg-K} \times (273.15 + 49.37) \text{ K}} = 0.8185$$

So state 1 is close to ideal gas and state 2 is not so close.

2.86

Find the compressibility for nitrogen at

a. 2000 kPa, 120 K ;   b. 2000 kPa, 300 K ;   c. 120 K, v = 0.005 m^3/kg

Solution:

Table B.6 has the properties for nitrogen.

a)   B.6.2: v = 0.01260 m^3/kg

$$Z = Pv/RT = \frac{2000 \text{ kPa} \times 0.01260 \text{ m}^3/\text{kg}}{0.2968 \text{ kJ/kg-K} \times 120 \text{ K}} = \mathbf{0.707}$$

b)   B.6.2: v = 0.04440 m^3/kg

$$Z = Pv/RT = \frac{2000 \text{ kPa} \times 0.04440 \text{ m}^3/\text{kg}}{0.2968 \text{ kJ/kg-K} \times 300 \text{ K}} = \mathbf{0.997}$$

c)   B.6.1: $v_f < v < v_g$ so this is a two-phase state.

$$Z = Pv/RT = \frac{2000 \text{ kPa} \times 0.005 \text{ m}^3/\text{kg}}{0.2968 \text{ kJ/kg-K} \times 120 \text{ K}} = \mathbf{0.28}$$

The liquid in this state is incompressible with a low volume and the vapor is very close to the critical point. If you calculate a quality it is x = 0.507. The compressibility for the saturated vapor alone is 0.44.

2.87

Find the compressibility for carbon dioxide at 60°C and 10 MPa using Fig. D.1

Solution:

Table A.2   CO2: $T_c = 304.1$ K $P_c = 7.38$ MPa

$T_r = T/T_c = 333/304.1 = 1.095$

$P_r = P/P_c = 10/7.38 = 1.355$

From Figure D.1:   **Z $\approx$ 0.45**

Compare with table B.3.2:   v = 0.00345 m^3/kg

$$Z = Pv/RT = \frac{10\ 000 \text{ kPa} \times 0.00345 \text{ m}^3/\text{kg}}{0.1889 \text{ kJ/kg-K} \times 333 \text{ K}} = 0.55$$

2.88

What is the percent error in specific volume if the ideal gas model is used to represent the behavior of superheated ammonia at 40°C, 500 kPa? What if the generalized compressibility chart, Fig. D.1, is used instead?

Solution:

NH_3 T = 40°C = 313.15 K, T_c = 405.5 K, P_c = 11.35 MPa from Table A.1

Table B.2.2:   v = **0.2923 m³/kg**

Ideal gas: $v = \dfrac{RT}{P} = \dfrac{0.48819 \text{ kJ/kgK} \times 313 \text{ K}}{500 \text{ kPa}}$ = **0.3056 m³/kg** $\Rightarrow$ **4.5% error**

Figure D.1: $T_r = \dfrac{313.15}{405.5} = 0.772,$ $P_r = \dfrac{0.5}{11.35} = 0.044$ $\Rightarrow Z = $ **0.97**

$\qquad\qquad v = \dfrac{ZRT}{P}$ = **0.2964 m³/kg** $\Rightarrow$ **1.4% error**

2.89

A cylinder fitted with a frictionless piston contains butane at 25°C, 500 kPa. Can the butane reasonably be assumed to behave as an ideal gas at this state ?

Solution

Butane 25°C, 500 kPa, Table A.2: $T_c = 425$ K; $P_c = 3.8$ MPa

$$T_r = \frac{25 + 273}{425} = 0.701; \qquad P_r = \frac{0.5}{3.8} = 0.13$$

Look at generalized chart in Figure D.1

Actual $P_r > P_{r,\ sat} = 0.1$ =>   **liquid!! not a gas**

The pressure should be less than 380 kPa to have a gas at that T.

2.90

Estimate the saturation pressure of chlorine at 300 K.

Solution:

We do not have a table in the B section for Chlorine so we must use the generalized chart.

Table A.2: $P_c = 7.98$ MPa, $T_c = 416.9$ K

$T_r = T/T_c = 300 / 416.9 = 0.7196$

Figure D.1: $P_{r\,sat} = 0.13$ (same estimation from Table D.4)

$P = P_c\, P_{r\,sat} = 7.98 \times 0.13 = $ **1.04 MPa**

If you use the CATT3 program then you will find $P_{r\,sat} = 0.122$ and $P = 973$ kPa

2.91

A bottle with a volume of 0.1 m^3 contains butane with a quality of 75% and a temperature of 300 K. Estimate the total butane mass in the bottle using the generalized compressibility chart.

Solution:

We need to find the property v the mass is: m = V/v

so find v given T_1 and x   as : $v = v_f + x\ v_{fg}$

Table A.2: Butane $T_c = 425.2$ K $P_c = 3.8$ MPa = 3800 kPa

$T_r = 300/425.2 = 0.705$ =>

From Fig. D.1 or table D.4: $Z_f \approx 0.02$; $Z_g \approx 0.9$; $P_{r\ sat} = 0.1$

$P = P_{sat} = P_{r\ sat} \times P_c = 0.1 \times 3.80$ MPa $\times 1000$ kPa/MPa = 380 kPa

$v_f = Z_f RT/P = 0.02 \times 0.14304$ kJ/kgK $\times 300$ K/380 kPa = 0.00226 m^3/kg

$v_g = Z_g RT/P = 0.9 \times 0.14304$ kJ/kgK $\times 300$ K/380 kPa = 0.1016 m^3/kg

$v = 0.00226 + 0.75 \times (0.1016 - 0.00226) = 0.076765$ m^3/kg

$m = \dfrac{V}{v} = \dfrac{0.1}{0.076765} =$ **1.303 kg**

2.92

Find the volume of 2 kg of ethylene at 270 K, 2500 kPa using Z from Fig. D.1

Ethylene Table A.2: $T_c = 282.4$ K, $P_c = 5.04$ MPa
Table A.5: $R = 0.2964$ kJ/kg K

The reduced temperature and pressure are:

$$T_r = \frac{T}{T_c} = \frac{270}{282.4} = 0.956, \quad P_r = \frac{P}{P_c} = \frac{2.5}{5.04} = 0.496$$

Enter the chart with these coordinates and read: $Z = 0.76$

$$V = \frac{mZRT}{P} = \frac{2 \text{ kg} \times 0.76 \times 0.2964 \text{ kJ/kg-K} \times 270 \text{ K}}{2500 \text{ kPa}} = \textbf{0.0487 m}^3$$

2.93

For $T_r = 0.7$, what is the ratio v_g/v_f using Fig. D.1 compared to Table. D.3

For the saturated states we can use Fig. D.1 with the estimates

$$Z_f = 0.02, \qquad Z_g = 0.9$$

so

$$v_g/v_f = (ZRT/P)_g \,/\, (ZRT/P)_f = Z_g/Z_f = \frac{0.02}{0.9} = \mathbf{0.0222}$$

Table D.3 list the entries more accurately than we can read the figure

$$Z_f = 0.017, \qquad Z_g = 0.897$$

so

$$v_g/v_f = (ZRT/P)_g \,/\, (ZRT/P)_f = Z_g/Z_f = \frac{0.017}{0.897} = \mathbf{0.0189}$$

2.94

Argon is kept in a rigid 5 m^3 tank at $-30°C$, 3 MPa. Determine the mass using the compressibility factor. What is the error (%) if the ideal gas model is used?

Solution:

No Argon table, so we use generalized chart Fig. D.1

$T_r = 243.15/150.8 = 1.612$, $P_r = 3000/4870 = 0.616$ $=>$ $Z \cong 0.96$

$$m = \frac{PV}{ZRT} = \frac{3000 \text{ kPa} \times 5 \text{ m}^3}{0.96 \times 0.2081 \text{ kJ/kgK} \times 243.2 \text{ K}} = \textbf{308.75 kg}$$

Ideal gas $Z = 1$

$$m = \frac{PV}{RT} = 296.4 \text{ kg}   \textbf{4\% error}$$

2.95

Refrigerant R-32 is at -10 ^{o}C with a quality of 15%. Find the pressure and specific volume.

Solution:

For R-32 there is no section B table printed. We will use compressibility chart.

From Table A.2: $T_c = 351.3$ K ; $P_c = 5.78$ MPa ;

From Table A.5:   R = 0.1598 kJ/kg K

$T_r = T/T_c = 263/351.3 = 0.749$

From Table D.4 or Figure D.1, $Z_f \approx 0.029$; $Z_g \approx 0.86$; $P_{r\,sat} \approx 0.16$

$P = P_{r\,sat}\,P_c = 0.16 \times 5780 =$ **925 kPa**

$v = v_f + x\,v_{fg} = (Z_f + x \times Z_{fg})\,RT/P$

$= [0.029 + 0.15 \times (0.86 - 0.029)] \times 0.1598$ kJ/kg-K $\times$ 263 K/ 925 kPa

$=$ **0.007 m^3/kg**

2.96

To plan a commercial refrigeration system using R-123 we would like to know how much more volume saturated vapor R-123 occupies per kg at -30 ^oC compared to the saturated liquid state.

Solution:

For R-123 there is no section B table printed. We will use compressibility chart.

   From Table A.2 T_c = 456.9 K ; P_c = 3.66 MPa ;   M = 152.93

   $T_r = T/T_c = 243/456.9 = 0.53$

   $R = \overline{R}/M = 8.31451$ kJ/kmol-K / 152.93 kg/kmol = 0.0544 kJ/kgK

The value of T_r is below the range in Fig. D.1 so use the table D.4

   Table D.4, $Z_g = 0.979$ $Z_f = 0.00222$

   $Z_{fg} = 0.979 – 0.0022 = 0.9768;$ $P_r = P_{r\ sat} = 0.0116$

   $P = P_r \times P_c = 42.5$ kPa

   $v_{fg} = Z_{fg}$ RT/P = 0.9768 × 0.0544 kJ/kgK × 243 K / 42.5 kPa

      = **0.304 m^3/kg**

Comment: If you check with the software the solution is off by a factor of 6. The linear interpolation is poor and so is the approximation for $P_{r\ sat}$ so the real saturation pressure should be 6.75 kPa. Also the very small value of Z_f is inaccurate by itself, minute changes in the curve gives large relative variations.

2.97

A new refrigerant R-125 is stored as a liquid at -20 ^{o}C with a small amount of vapor. For a total of 1.5 kg R-125 find the pressure and the volume.

Solution:

As there is no section B table use compressibility chart.

Table A.2:   R-125 $T_c = 339.2$ K $P_c = 3.62$ MPa

$T_r = T / T_c = 253.15 / 339.2 = 0.746$

We can read from Figure D.1 or a little more accurately interpolate from table D.4 entries:

$P_{r\ sat} = 0.16$; $Z_g = 0.86$; $Z_f = 0.029$

$P = P_{r\ sat}\ P_c = 0.16 \times 3620$ kPa = **579 kPa**

$V_{liq} = Z_f\ m_{liq}\ RT/P$

$= 0.029 \times 1.5$ kg $\times 0.06927$ kJ/kgK $\times 253.15$ K/ 579 kPa

$= $ **0.0013 m^3**

Equations of State

For these problems see appendix D for the equation of state (EOS) and chapter 12.

2.98

Determine the pressure of nitrogen at 160 K, v = 0.00291 m^3/kg using ideal gas, van der Waal Equation of State and the nitrogen table.

Nitrogen from table A.2: T_c = 126.2 K, P_c = 3390 kPa,

Ideal gas: $P = \dfrac{RT}{v} = \dfrac{0.2968 \times 160}{0.00291} \dfrac{\text{kJ/kg-K} \times \text{K}}{m^3/\text{kg}} =$ **16 319 kPa**

For **van der Waal equation of state** from Table D.1 we have

$$b = \frac{1}{8}\frac{RT_c}{P_c} = 0.125 \times \frac{0.2968 \times 126.2}{3390} = 0.001\ 381\ m^3/\text{kg},$$

$$a = 27\ b^2\ P_c = 27 \times (0.001\ 381)^2 \times 3390 = 0.174\ 562\ \text{kPa}\ (m^3/\text{kg})^2$$

The EOS is: $P = \dfrac{RT}{v - b} - \dfrac{a}{v^2} = \dfrac{0.2968 \times 160}{0.00291 - 0.001\ 381} - \dfrac{0.174\ 562}{0.00291^2} =$ **10 444 kPa**

Table B.6.2:   P = **10 000 kPa**.

2.99

Determine the pressure of nitrogen at 160 K, v = 0.00291 m^3/kg using Redlich-Kwong Equation of State and the nitrogen table.

Nitrogen from table A.2: T_c = 126.2 K, P_c = 3390 kPa,

$$T_r = T/T_c = 160/126.2 = 1.26783$$

For **Redlich-Kwong EOS** we have the parameters from Table D.1

$$b = 0.08664 \, \frac{RT_c}{P_c} = 0.08664 \times \frac{0.2968 \times 126.2}{3390} = 0.000\,957\,3 \text{ m}^3/\text{kg},$$

$$a = 0.42748 \, T_r^{-1/2} \, \frac{R^2 T_c^2}{P_c} = 0.42748 \times \frac{0.2968^2 \times 126.2^2}{1.2678^{1/2} \times 3390} = 0.157\,122 \text{ kPa (m}^3/\text{kg)}^2$$

The equation is:

$$P = \frac{RT}{v-b} - \frac{a}{v^2+bv}$$

$$= \frac{0.2968 \times 160}{0.00291 - 0.000\,9573} - \frac{0.157122}{0.00291^2 + 0.000\,9573 \times 0.00291}$$

$$= \mathbf{10\,357 \text{ kPa}}$$

Table B.6.2:   P = **10 000 kPa**.

2.100

Determine the pressure of nitrogen at 160 K, $v = 0.00291$ m^3/kg using Soave Equation of State and the nitrogen table.

Nitrogen from table A.2: $T_c = 126.2$ K, $P_c = 3390$ kPa,
$$T_r = T/T_c = 160/126.2 = 1.26783$$

For **Soave EOS** see Appendix D (very close to Redlich-Kwong)
$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$
where the parameters are from Table D.1 and D.4
$$\omega = 0.039$$
$$f = 0.48 + 1.574\omega - 0.176\omega^2 = 0.54112$$
$$a_o = 0.42748\,[1 + f\,(1 - T_r^{1/2})\,]^{\,2} = 0.371184$$

$$b = 0.08664\,\frac{RT_c}{P_c} = 0.08664 \times \frac{0.2968 \times 126.2}{3390} = 0.000\,957\,3 \text{ m}^3/\text{kg},$$

$$a = 0.371184\,\frac{R^2 T_c^2}{P_c} = 0.371184 \times \frac{0.2968^2 \times 126.2^2}{3390} = 0.153\,616 \text{ kPa } (\text{m}^3/\text{kg})^2$$

$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$

$$= \frac{0.2968 \times 160}{0.00291 - 0.000\,9573} - \frac{0.153\,616}{0.00291^2 + 0.000\,9573 \times 0.00291}$$

$$= \textbf{10 669 kPa}$$

Nitrogen Table B.6.2:   P = **10 000 kPa**.

2.101

Carbon dioxide at 60^oC is pumped at a very high pressure 10 MPa into an oil well to reduce the oil viscosity for better oil flow. We want to find its specific volume from the CO_2 table, ideal gas and van der Waals equation of state by iteration.

Table B.3.2:   v = **0.00345 m^3/kg**.

Ideal gas: $v = \dfrac{RT}{P} = \dfrac{0.1889 \times (60 + 273.15)}{10\ 000}\ \dfrac{\text{kJ/kg-K} \times \text{K}}{\text{kPa}} = $ **0.006 293 m^3/kg**

Carbon dioxide from table A.2: $T_c = 304.1$ K, $P_c = 7380$ kPa,

For **van der Waal equation of state** from Table D.1 we have

$$b = \frac{1}{8}\frac{RT_c}{P_c} = 0.125 \times \frac{0.1889 \times 304.1}{7380} = 0.000\ 972\ 98 \text{ m}^3/\text{kg},$$

$$a = 27\ b^2\ P_c = 27 \times (0.000\ 972\ 98)^2 \times 7380 = 0.188\ 6375 \text{ kPa } (\text{m}^3/\text{kg})^2$$

The EOS is: $P = \dfrac{RT}{v - b} - \dfrac{a}{v^2}$

Since it is nonlinear in v we use trial and error starting with the Table entry.

$v = 0.00345$: $P = \dfrac{0.1889 \times 333.15}{0.00345 - 0.000\ 972\ 98} - \dfrac{0.188\ 6375}{0.00345^2} = 9557.8$ kPa   low

$v = 0.003$ $P = \dfrac{0.1889 \times 333.15}{0.003 - 0.000\ 972\ 98} - \dfrac{0.188\ 6375}{0.003^2} = 10\ 086.8$ kPa   high

$v = 0.00307$: $P = \dfrac{0.1889 \times 333.15}{0.00307 - 0.000\ 972\ 98} - \dfrac{0.188\ 6375}{0.00307^2} = 9995.4$ OK.

$v = $ **0.00307 m^3/kg**

2.102

Solve the previous problem using the Redlich-Kwong equation of state. Notice this becomes trial and error.

Carbon dioxide from table A.2: $T_c = 304.1$ K, $P_c = 7380$ kPa,

$$T_r = T/T_c = 333.15/304.1 = 1.09553$$

For **Redlich-Kwong EOS** we have the parameters from Table D.1

$$b = 0.08664 \frac{RT_c}{P_c} = 0.08664 \times \frac{0.1889 \times 304.1}{7380} = 0.000\ 6744\ m^3/kg,$$

$$a = 0.42748\ T_r^{-1/2} \frac{R^2T_c^2}{P_c} = 0.42748 \times \frac{0.1889^2 \times 304.1^2}{1.09553^{1/2} \times 7380} = 0.18262\ kPa\ (m^3/kg)^2$$

The equation is:

$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$

$$10\ 000\ = \frac{0.1889 \times 333.15}{v - 0.000\ 6744} - \frac{0.18262}{v^2 + 0.000\ 6744 \times v}$$

Trial and error on v (start guided by Table B.3.2):

$v = 0.0035\ m^3/kg$ ⇨ $P = 9772.8$ kPa   so v smaller
$v = 0.0033\ m^3/kg$ ⇨ $P = 10044.7$ kPa   so v larger
$v = 0.0034\ m^3/kg$ ⇨ $P = 9906.5$ kPa

linear interpolation gives

$v = $ **0.00333 m³/kg**   ⇨ $P = 10002.7$ kPa   OK.

2.103

Solve Problem 2.101 using the Soave equation of state, $\omega = 0.239$. Notice this becomes trial and error.

Carbon dioxide from table A.2: $T_c = 304.1$ K, $P_c = 7380$ kPa,

$$T_r = T/T_c = 333.15/304.1 = 1.09553$$

For **Soave EOS** see Appendix D (very close to Redlich-Kwong)

$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$

where the parameters are from Table D.1 and D.4

$$\omega = 0.239$$

$$f = 0.48 + 1.574\omega - 0.176\omega^2 = 0.84613$$

$$a_o = 0.42748\ [1 + f\ (1 - T_r^{1/2})\ ]^{\ 2} = 0.394381$$

$$b = 0.08664\ \frac{RT_c}{P_c} = 0.08664 \times \frac{0.1889 \times 304.1}{7380} = 0.000\ 6744\ \text{m}^3/\text{kg},$$

$$a = 0.394381\ \frac{R^2T_c^2}{P_c} = 0.394381 \times \frac{0.1889^2 \times 304.1^2}{7380} = 0.176\ 342\ \text{kPa (m}^3/\text{kg)}^2$$

The equation is:

$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$

$$10\ 000\ = \frac{0.1889 \times 333.15}{v - 0.000\ 6744} - \frac{0.176\ 342}{v^2 + 0.000\ 6744 \times v}$$

Trial and error on v (start guided by Table B.3.2):

$v = 0.0035$ m^3/kg $\Rightarrow$ $P = 10\ 202$ kPa   so v larger

$v = 0.0036$ m^3/kg $\Rightarrow$ $P = 10\ 051$ kPa   so v close

v = 0.00363 m^3/kg $\Rightarrow$ $P = 10\ 006$ kPa   OK

2.104

A tank contains 8.35 kg methane in 0.1 m^3 at 250 K. Find the pressure using ideal gas, van der Waal EOS and the methane table.

The state is given by (T = 250 K, v = V/m = 0.1/8.35 = 0.011976 m^3/kg)

Table A.2 or B.7.2:
  T_c = 190.4 K,
  P_c = 4600 kPa

$$T_r = \frac{T}{T_c} = \frac{250}{190.4} = 1.313$$

Ideal gas model: $P = \dfrac{RT}{v} = \dfrac{0.5183 \times 250}{0.011976} = 10\ 820$ kPa

For **van der Waal equation of state** from Table D.1 we have

$$b = \frac{1}{8}\frac{RT_c}{P_c} = 0.125 \times \frac{0.5183 \times 190.4}{4600} = 0.002\ 681\ 64 \text{ m}^3/\text{kg},$$

$$a = 27\ b^2\ P_c = 27 \times (0.002\ 681\ 64)^2 \times 4600 = 0.893\ 15 \text{ kPa (m}^3/\text{kg)}^2$$

The EOS is: $P = \dfrac{RT}{v - b} - \dfrac{a}{v^2} = \dfrac{0.5183 \times 250}{0.011976 - 0.002\ 681\ 64} - \dfrac{0.89315}{0.011976^2} = 7714$ kPa

Locating the state in **Table B.7.2:**   P = 8000 kPa, very close

2.105

Do the previous problem using the Redlich-Kwong equation of state.

The state is given by $(T = 250$ K, $v = V/m = 0.1/8.35 = 0.011976$ m^3/kg)

Table A.2 or B.7.2:
$T_c = 190.4$ K,
$P_c = 4600$ kPa

$T_r = \dfrac{T}{T_c} = \dfrac{250}{190.4} = 1.313$

Ideal gas model: $\quad P = \dfrac{RT}{v} = \dfrac{0.5183 \times 250}{0.011976} = 10\,820$ kPa

For **Redlich-Kwong equation of state** we have the parameters from Table D.1

$$b = 0.08664\,\frac{RT_c}{P_c} = 0.08664 \times \frac{0.5183 \times 190.4}{4600} = 0.001\,858\,7 \text{ m}^3/\text{kg},$$

$$a = 0.42748\,T_r^{-1/2}\,\frac{R^2 T_c^2}{P_c} = 0.42748 \times \frac{0.5183^2 \times 190.4^2}{1.313^{1/2} \times 4600} = 0.789809 \text{ kPa (m}^3/\text{kg)}^2$$

The equation is:

$$P = \frac{RT}{v-b} - \frac{a}{v^2 + bv}$$

$$= \frac{0.5183 \times 250}{0.011976 - 0.0018587} - \frac{0.789809}{0.011976^2 + 0.0018587 \times 0.011976}$$

$$= 8040 \text{ kPa}$$

Locating the state in **Table B.7.2:** $\quad$ P = 8000 kPa, very close

2.106

Do Problem 2.104 using the Soave EOS.

A tank contains 8.35 kg methane in 0.1 m^3 at 250 K. Find the pressure using ideal gas, van der Waal EOS and the methane table.

The state is given by (T = 250 K, v = V/m = 0.1/8.35 = 0.011976 m^3/kg)

Table A.2 or B.7.2:

T_c = 190.4 K,

P_c = 4600 kPa

$T_r = \dfrac{T}{T_c} = \dfrac{250}{190.4} = 1.313$

Ideal gas model: $P = \dfrac{RT}{v} = \dfrac{0.5183 \times 250}{0.011976} = 10\ 820$ kPa

For **Soave EOS** we have the parameters from Table D.1 and D.4

$\omega = 0.011$

$f = 0.48 + 1.574\omega - 0.176\omega^2 = 0.49729$

$a_o = 0.42748\ [1 + f\ (1 - T_r^{1/2})\ ]^{\ 2} = 0.367714$

$b = 0.08664\ \dfrac{RT_c}{P_c} = 0.08664 \times \dfrac{0.5183 \times 190.4}{4600} = 0.001\ 8587$ m^3/kg,

$a = 0.367714\ \dfrac{R^2 T_c^2}{P_c} = 0.367714 \times \dfrac{0.5183^2 \times 190.4^2}{4600} = 0.778\ 482$ kPa (m^3/kg)^2

The equation is:

$P = \dfrac{RT}{v - b} - \dfrac{a}{v^2 + bv}$

$= \dfrac{0.5183 \times 250}{0.011976 \text{ - } 0.001\ 8587} - \dfrac{0.778\ 482}{0.011976^2 + 0.0018587 \times 0.011976}$

$= $ **8108.7 kPa**

Locating the state in **Table B.7.2:**   P = 8000 kPa

Review Problems

2.107

Determine the quality (if saturated) or temperature (if superheated) of the
following substances at the given two states:
Solution:

a) Water, H_2O, use Table B.1.1 or B.1.2

1) $120°C$, 1 m^3/kg $v > v_g$ superheated vapor, **T = 120 °C**

2) 10 MPa, 0.01 m^3/kg => two-phase $v < v_g$

$x = (0.01 – 0.001452) / 0.01657 = $ **0.516**

b) Nitrogen, N_2, table B.6

1) 1 MPa, 0.03 m^3/kg => superheated vapor since $v > v_g$

Interpolate between sat. vapor and superheated vapor B.6.2:

$$T \cong 103.73 + (120\text{-}103.73) \times \frac{0.03–0.02416}{0.03117–0.02416} = \textbf{117 K}$$

2) 100 K, 0.03 m^3/kg => sat. liquid + vapor as two-phase $v < v_g$

$v = 0.03 = 0.001452 + x \times 0.029764 \Rightarrow $ **x = 0.959**

States shown are
placed relative to the
two-phase region, not
to each other.

2.108

Give the phase and the missing properties of P, T, v and x.

Solution:

    a. R-410A $T = 10°C$ $v = 0.01$ m^3/kg

            Table B.4.1 $v < v_g = 0.02383$ m^3/kg

            sat. liquid + vapor. $P = P_{sat} = 1085.7$ kPa,

            $x = (v - v_f)/v_{fg} = (0.01 - 0.000886)/0.02295 = 0.2713$

    b. H$_2$O $T = 350°C$ $v = 0.2$ m^3/kg

            Table B.1.1 at given T: $v > v_g = 0.00881$ m^3/kg

            sup. vapor $P \cong 1.40$ MPa, $x =$ undefined

    c. R-410A $T = -5 °C$ $P = 600$ kPa

            sup. vapor $(P < P_g = 678,9$ kPa at $-5°C)$

            Table B.4.2:

            $v = 0.04351$ m^3/kg at $-8.67°C$

            $v = 0.04595$ m^3/kg at $0°C$

                $=>$ $v = 0.04454$ m^3/kg at $-5°C$

    d. R-134a $P = 294$ kPa, $v = 0.05$ m^3/kg

            Table B.5.1: $v < v_g = 0.06919$ m^3/kg

            two-phase $T = T_{sat} = 0°C$

            $x = (v - v_f)/v_{fg} = (0.05 - 0.000773)/0.06842 = 0.7195$

States shown are
placed relative to the
two-phase region, not
to each other.

2.109

Find the phase, quality x if applicable and the missing property P or T.

Solution:

a. H_2O $T = 120°C$ $v = 0.5 \text{ m}^3/\text{kg}$

Table B.1.1 at given T: $v < v_g = 0.89186$

sat. liq. + vap. $P = P_{sat} = $ **198.5 kPa,**

$x = (v - v_f)/v_{fg} = (0.5 - 0.00106)/0.8908 = $ **0.56**

b. H_2O $P = 100 \text{ kPa}$ $v = 1.8 \text{ m}^3/\text{kg}$

Table B.1.2   at given P: $v > v_g = 1.694$

sup. vap., interpolate in Table B.1.3

$$T = \frac{1.8 - 1.694}{1.93636 - 1.694}(150 - 99.62) + 99.62 = \textbf{121.65 °C}$$

c. H_2O $T = 263 \text{ K}$ $v = 0.2 \text{ m}^3/\text{kg}$

Table B.1.5   at given T = -10 °C: $v < v_g = 466.757$

sat. solid + vap., $P = P_{sat} = $ **0.26 kPa,**

$x = (v - v_i)/v_{ig} = (200 - 0.001)/466.756 = $ **0.4285**

States shown are
placed relative to the
two-phase region, not
to each other.

2.110

Find the phase, quality x if applicable and the missing property P or T.

Solution:

a. NH_3 $P = 800$ kPa $v = 0.2$ m^3/kg;

Superheated Vapor ($v > v_g$ at 800 kPa)

Table B 2.2 interpolate between 70°C and 80°C

T = **71.4°C**

b. NH_3 $T = 20^\circ$C $v = 0.1$ m^3/kg

Table B.2.1 at given T: $v < v_g = 0.14922$

sat. liq. + vap. , $P = P_{sat} =$ **857.5 kPa,**

$x = (v - v_f)/v_{fg} = (0.1 - 0.00164)/0.14758 =$ **0.666**

2.111

Give the phase and the missing properties of P, T, v and x. These may be a little more difficult if the appendix tables are used instead of the software.

a) R-410A at $T = 10°C$, $v = 0.02$ m^3/kg: Table B.4.1 $v > v_g$ at $10°C$

    $\Rightarrow$ **sup. vap.** Table B.4.2 interpolate between sat. and sup. both at $10°C$

    $P = 680.7 + (600 - 680.7)\dfrac{0.036\text{-}0.03471}{0.04018\text{-}0.03471} =$ **661.7 kPa**

b) H$_2$O $v = 0.2$ m^3/kg , $x = 0.5$: Table B.1.1

    sat. liq. + vap. $v = (1\text{-}x)\, v_f + x\, v_g$ $\Rightarrow$ $v_f + v_g = 0.4$ m^3/kg

    since v_f is so small we find it approximately where $v_g = 0.4$ m^3/kg.

    $v_f + v_g = 0.39387$ at $150°C$, $v_f + v_g = 0.4474$ at $145°C$.

    An interpolation gives $T \cong$ **149.4°C,** $P \cong$ **468.2 kPa**

c) H$_2$O $T = 60°C$, $v = 0.001016$ m^3/kg: Table B.1.1 $v < v_f = 0.001017$

        $\Rightarrow$ **compr. liq.**   see Table B.1.4

    $v = 0.001015$ at 5 MPa so $P \cong 0.5(5000 + 19.9) =$ **2.51 MPa**

d) NH$_3$ $T = 30°C$, $P = 60$ kPa : Table B.2.1 $P < P_{sat}$

    $\Rightarrow$ **sup. vapor**   interpolate in Table B.2.2   (50 kPa to 100 kPa)

    $v = 2.9458 + (1.4657 - 2.9458)\dfrac{60 \text{ - } 50}{100 \text{ - } 50} =$ **2.65 m^3/kg**

    v is not linearly proportional to P (more like 1/P) so the computer table gives a more accurate value of 2.45 m^3/kg

e) R-134a $v = 0.005$ m^3/kg , $x = 0.5$:   **sat. liq. + vap.**   Table B.5.1

    $v = (1\text{-}x)\, v_f + x\, v_g$ $\Rightarrow$ $v_f + v_g = 0.01$ m^3/kg

    $v_f + v_g = 0.010946$ at $65°C$, $v_f + v_g = 0.009665$ at $70°C$.

    An interpolation gives: $T \cong$ **68.7°C**, $P =$ **2.06 MPa**

States shown are placed relative to the two-phase region, not to each other.

2.112

Refrigerant-410a in a piston/cylinder arrangement is initially at 50°C, $x = 1$. It is then expanded in a process so that $P = Cv^{-1}$ to a pressure of 100 kPa. Find the final temperature and specific volume.

Solution:

State 1: 50°C, x = 1   Table B.4.1: $P_1 = 3065.2$ kPa, $v_1 = 0.00707$ m^3/kg

Process: $Pv = C = P_1v_1$;   => $P_2 = C/v_2 = P_1v_1/v_2$

State 2: 100 kPa and $v_2 = v_1P_1/P_2 =$ **0.2167 m^3/kg**

   $v_2 < v_g$ at 100 kPa, $T_2 \cong$ **-51.65°C** from Table B.4.2

Notice T is **not** constant

The first part of the process may be in the sup. vapor region.

2.113

Consider two tanks, A and B, connected by a valve, as shown in Fig. P2.113. Each has a volume of 200 L and tank A has R-410A at 25°C, 10% liquid and 90% vapor by volume, while tank B is evacuated. The valve is now opened and saturated vapor flows from A to B until the pressure in B has reached that in A, at which point the valve is closed. This process occurs slowly such that all temperatures stay at 25°C throughout the process. How much has the quality changed in tank A during the process?

Solution:

State A1:   Table B.4.1 $v_f = 0.000944$ m^3/kg, $v_g = 0.01514$ m^3/kg

$$m_{A1} = \frac{V_{liq1}}{v_{f\,25°C}} + \frac{V_{vap1}}{v_{g\,25°C}} = \frac{0.1 \times 0.2}{0.000944} + \frac{0.9 \times 0.2}{0.01514}$$

$$= 21.186 + 11.889 = 33.075 \text{ kg}$$

$$x_{A1} = \frac{11.889}{33.075} = 0.3594 \; ;$$

State B2:   Assume A still two-phase so saturated P for given T

$$m_{B2} = \frac{V_B}{v_{g\,25°C}} = \frac{0.2}{0.01514} = 13.210 \text{ kg}$$

State A2:   mass left is $m_{A2} = 33.075 - 13.210 = 19.865$ kg

$$v_{A2} = \frac{0.2}{19.865} = 0.010068 = 0.000944 + x_{A2} \times 0.01420$$

$$x_{A2} = 0.6425 \qquad \qquad \Delta x = \mathbf{0.283}$$

2.114

Water in a piston/cylinder is at 90°C, 100 kPa, and the piston loading is such that pressure is proportional to volume, $P = CV$. Heat is now added until the temperature reaches 200°C. Find the final pressure and also the quality if in the two-phase region.
Solution:

Final state: 200°C , on process line $P = CV$

State 1:   Table B.1.1: $v_1 = 0.001036$ m^3/kg

$P_2 = P_1 v_2/v_1$ from process equation

Check state 2 in Table B.1.1

$v_g(T_2) = 0.12736;$ $P_g(T_2) = 1.5538$ MPa

If $v_2 = v_g(T_2)$ $\Rightarrow$ $P_2 = 12.3$ MPa $> P_g$ not OK

If sat. $P_2 = P_g(T_2) = 1553.8$ kPa $\Rightarrow$ $v_2 = 0.0161$ m^{3}kg $< v_g$ sat. OK,

$P_2 = 1553.8$ kPa, $x_2 = (0.0161 - 0.001156) / 0.1262 = $ **0.118**

2.115

A tank contains 2 kg of nitrogen at 100 K with a quality of 50%. Through a volume flowmeter and valve, 0.5 kg is now removed while the temperature remains constant. Find the final state inside the tank and the volume of nitrogen removed if the valve/meter is located at

      a.   The top of the tank

      b.   The bottom of the tank

Solution

                     Table B.6.1:

$v_1 = 0.001452 + x_1 \times 0.029764 = 0.016334 \ \text{m}^3/\text{kg}$

$V_{tank} = m_1 v_1 = 0.0327 \ \text{m}^3$

$m_2 = m_1 - 0.5 = 1.5 \ \text{kg}$

$v_2 = V_{tank}/m_2 = 0.0218 \ < \ v_g(T)$

$x_2 = \dfrac{0.0218 - 0.001452}{0.031216 - 0.001452} = \ \mathbf{0.6836}$

Top: flow out is sat. vap. $v_g = 0.031216 \ \text{m}^3/\text{kg}, \ \ V_{out} = m_{out}v_g = \mathbf{0.0156 \ m^3}$

Bottom: flow out is sat. liq. $v_f = 0.001452$ $V_{out} = m_{out}v_f = \mathbf{0.000726 \ m^3}$

2.116

A spring-loaded piston/cylinder contains water at 500°C, 3 MPa. The setup is such that pressure is proportional to volume, $P = CV$. It is now cooled until the water becomes saturated vapor. Sketch the P-v diagram and find the final pressure.

Solution:

State 1:   Table B.1.3: $v_1 = 0.11619$ m^3/kg

Process: m is constant and $P = C_0V = C_0m\ v = C\ v$

$P = Cv \implies C = P_1/v_1 = 3000/0.11619 = 25820$ kPa kg/m^3

State 2: $x_2 = 1$ & $P_2 = Cv_2$ (on process line)

Trial & error on T_{2sat} or P_{2sat}:

Here from B.1.2:

at 2 MPa $v_g = 0.09963 \implies C = 20074$ (low)

2.5 MPa $v_g = 0.07998 \implies C = 31258$ (high)

2.25 MPa $v_g = 0.08875 \implies C = 25352$ (low)

Interpolate to get the right C $\implies$ **$P_2 = 2270$ kPa**

2.117

A container with liquid nitrogen at 100 K has a cross sectional area of 0.5 m^2. Due to heat transfer, some of the liquid evaporates and in one hour the liquid level drops 30 mm. The vapor leaving the container passes through a valve and a heater and exits at 500 kPa, 260 K. Calculate the volume rate of flow of nitrogen gas exiting the heater.

Solution:

Properties from table B.6.1 for volume change, exit flow from table B.6.2:

$$\Delta V = A \times \Delta h = 0.5 \times 0.03 = 0.015 \text{ m}^3$$

$$\Delta m_{liq} = -\Delta V/v_f = -0.015/0.001452 = -10.3306 \text{ kg}$$

$$\Delta m_{vap} = \Delta V/v_g = 0.015/0.0312 = 0.4808 \text{ kg}$$

$$m_{out} = 10.3306 - 0.4808 = 9.85 \text{ kg}$$

$$v_{exit} = 0.15385 \text{ m}^3/\text{kg}$$

$$\dot{V} = \dot{m}v_{exit} = (9.85 / 1 \text{ } h) \times 0.15385 \text{ m}^3/\text{kg}$$

$$= 1.5015 \text{ m}^3/\text{h} = \mathbf{0.02526 \text{ m}^3/\text{min}}$$

2.118

For a certain experiment, R-410A vapor is contained in a sealed glass tube at 20°C. It is desired to know the pressure at this condition, but there is no means of measuring it, since the tube is sealed. However, if the tube is cooled to –20°C small droplets of liquid are observed on the glass walls. What is the initial pressure?

Solution:

Control volume: R-410A fixed volume (V) & mass (m) at 20°C

Process: cool to -20°C at constant v, so we assume saturated vapor

State 2: $v_2 = v_{g \text{ at } -20°C} = 0.06480$ m^3/kg

State 1: 20°C, $v_1 = v_2 = 0.06480$ m^3/kg

interpolate between 400 and 500 kPa in Table B.4.2

=>   **P$_1$ = 485 kPa**

2.119

A cylinder/piston arrangement contains water at 105°C, 85% quality with a volume of 1 L. The system is heated, causing the piston to rise and encounter a linear spring as shown in Fig. P2.119. At this point the volume is 1.5 L, piston diameter is 150 mm, and the spring constant is 100 N/mm. The heating continues, so the piston compresses the spring. What is the cylinder temperature when the pressure reaches 200 kPa?

Solution:

$P_1 = 120.8$ kPa, $v_1 = v_f + x\, v_{fg} = 0.001047 + 0.85*1.41831 = 1.20661$

$m = V_1 / v_1 = \dfrac{0.001}{1.20661} = 8.288 \times 10^{-4}$ kg

$v_2 = v_1\,(V_2 / V_1) = 1.20661 \times 1.5 = 1.8099$

& $P = P_1 = 120.8$ kPa   ($T_2 = 203.5$°C)

$P_3 = P_2 + (k_s/A_p^2)\, m(v_3 - v_2)$ linear spring

$A_p = (\pi/4) \times 0.15^2 = 0.01767$ m^2 ; $k_s = 100$ kN/m (matches P in kPa)

$200 = 120.8 + (100/0.01767^{\,2}) \times 8.288 \times 10^{-4}(v_3 - 1.8099)$

$200 = 120.8 + 265.446\ (v_3 - 1.8099) \quad \Rightarrow \quad v_3 = 2.1083$ m^3/kg

$T_3 \cong 600 + 100 \times (2.1083 - 2.01297)/(2.2443 - 2.01297) \cong \mathbf{641°C}$

2.120

Determine the mass of methane gas stored in a 2 m^3 tank at –30°C, 2 MPa. Estimate the percent error in the mass determination if the ideal gas model is used.
Solution:

Table B.7

Methane Table B.7.1 at –30°C = 243.15 K > T_c = 190.6 K,   so superheated

vapor in Table B.7.2. Linear interpolation between 225 and 250 K at 2 MPa.

$$\Rightarrow \quad v \cong 0.05289 + \frac{243.15\text{-}225}{250\text{-}225} \times (0.06059 - 0.05289) = 0.05848 \text{ m}^3/\text{kg}$$

$$m = V/v = 2/0.05848 = 34.2 \text{ kg}$$

Ideal gas assumption

$$v = \frac{RT}{P} = \frac{0.51835 \times 243.15}{2000} = 0.06302 \text{ m}^3/\text{kg}$$

$$m = \frac{V}{v} = \frac{2}{0.06302} = 31.74 \text{ kg}$$

Error:

$$\Delta m = 2.46 \text{ kg} ; \quad \text{ideal gas } \textbf{7.2\% too small}$$

Comment: The compressibility of the methane Z = 0.93.

2.121

A cylinder containing ammonia is fitted with a piston restrained by an external force that is proportional to cylinder volume squared. Initial conditions are 10°C, 90% quality and a volume of 5 L. A valve on the cylinder is opened and additional ammonia flows into the cylinder until the mass inside has doubled. If at this point the pressure is 1.2 MPa, what is the final temperature?

Solution:

State 1 Table B.2.1: $v_1 = 0.0016 + 0.9(0.205525 - 0.0016) = 0.18513 \text{ m}^3/\text{kg}$

$$P_1 = 615 \text{ kPa}; \quad V_1 = 5 \text{ L} = 0.005 \text{ m}^3$$

$$m_1 = V/v = 0.005/0.18513 = 0.027 \text{ kg}$$

State 2: $P_2 = 1.2 \text{ MPa}$, Flow in so: $m_2 = 2 \ m_1 = 0.054 \text{ kg}$

Process: Piston $F_{ext} = KV^2 = PA \quad => \quad P = CV^2 \quad => P_2 = P_1 \ (V_2/V_1)^2$

From the process equation we then get:

$$V_2 = V_1 \ (P_2/P_1)^{1/2} = 0.005 \ (\frac{1200}{615})^{1/2} = 0.006984 \text{ m}^3$$

$$v_2 = V/m = \frac{0.006984}{0.054} = 0.12934 \text{ m}^3/\text{kg}$$

At P_2, v_2: $\quad T_2 = \textbf{70.9°C}$

2.122

A cylinder has a thick piston initially held by a pin as shown in Fig. P2.122. The cylinder contains carbon dioxide at 200 kPa and ambient temperature of 290 K. The metal piston has a density of 8000 kg/m^3 and the atmospheric pressure is 101 kPa. The pin is now removed, allowing the piston to move and after a while the gas returns to ambient temperature. Is the piston against the stops?

Solution:

Force balance on piston determines equilibrium float pressure.

Piston $m_p = A_p \times l \times \rho$ $\qquad\qquad\qquad$ $\rho_{piston} = 8000$ kg/m^3

$$P_{ext \text{ on } CO_2} = P_0 + \frac{m_p g}{A_p} = 101 + \frac{A_p \times 0.1 \times 9.807 \times 8000}{A_p \times 1000} = 108.8 \text{ kPa}$$

Pin released, as $P_1 > P_{ext}$ piston moves up, $T_2 = T_o$ & if piston at stops,

then $\quad V_2 = V_1 \times H_2/H_1 = V_1 \times 150 / 100$

Ideal gas with $T_2 = T_1$ then gives

$$\Rightarrow \quad P_2 = P_1 \times V_1 / V_2 = 200 \times \frac{100}{150} = 133 \text{ kPa} > P_{ext}$$

$$\Rightarrow \quad \textbf{piston is at stops, and } \mathbf{P_2 = 133 \text{ kPa}}$$

2.123

What is the percent error in pressure if the ideal gas model is used to represent the behavior of superheated vapor R-410A at 60°C, 0.03470 m^3/kg? What if the generalized compressibility chart, Fig. D.1, is used instead (iterations needed)?

Solution:

Real gas behavior: P = 1000 kPa   from Table B.4.2

Ideal gas constant: $R = \bar{R}/M = 8.31451/72.585 = 0.1146$ kJ/kg K

$P = RT/v = 0.1146 \times (273.15 + 60) / 0.0347$

$= $ **1100 kPa   which is 10% too high**

Generalized chart Fig D.1 and critical properties from A.2:

$T_r = 333.2/(273.15 + 71.3) = 0.967;$ $P_c = 4901$ kPa

Assume P = 1000 kPa   => $P_r = 0.204$ => $Z \cong 0.92$

$v = ZRT/P = 0.92 \times 0.1146$ kJ/kg-K $\times 333.15$ K /1000 kPa

$= 0.03512$ m^3/kg   too high

Assume P = 1050 kPa   => $P_r = 0.214$ => $Z \cong 0.915$

$v = ZRT/P = 0.915 \times 0.1146$ kJ/kg-K $\times 333.15$ K / 1050 kPa

$= 0.03327$ m^3/kg   too low

$P \cong 1000 + (1050 - 1000) \times \dfrac{0.03470 - 0.03512}{0.03327 - 0.03512} = $ **1011 kPa   1.1 % high**

2.124

An initially deflated and flat balloon is connected by a valve to a 12 m³ storage tank containing helium gas at 2 MPa and ambient temperature, 20°C. The valve is opened and the balloon is inflated at constant pressure, $P_o = 100$ kPa, equal to ambient pressure, until it becomes spherical at $D_1 = 1$ m. If the balloon is larger than this, the balloon material is stretched giving a pressure inside as

$$P = P_0 + C\left(1 - \frac{D_1}{D}\right)\frac{D_1}{D}$$

The balloon is inflated to a final diameter of 4 m, at which point the pressure inside is 400 kPa. The temperature remains constant at 20°C. What is the maximum pressure inside the balloon at any time during this inflation process? What is the pressure inside the helium storage tank at this time?

Solution:

At the end of the process we have D = 4 m so we can get the constant C as

$$P = 400 = P_0 + C\left(1 - \frac{1}{4}\right)\frac{1}{4} = 100 + C \times 3/16 \quad => \quad C = 1600 \text{ kPa}$$

The pressure is: $P = 100 + 1600(1 - X^{-1})X^{-1}$; $X = D / D_1$

Differentiate to find max: $\dfrac{dP}{dD} = C(-X^{-2} + 2X^{-3})/D_1 = 0$

$$=> \quad -X^{-2} + 2X^{-3} = 0 \Rightarrow \quad X = 2$$

at max P => $D = 2D_1 = 2$ m; $V = \dfrac{\pi}{6}D^3 = 4.18 \text{ m}^3$

$$Pmax = 100 + 1600\left(1 - \frac{1}{2}\right)\frac{1}{2} = \textbf{500 kPa}$$

Helium is ideal gas A.5: $m = \dfrac{PV}{RT} = \dfrac{500 \times 4.189}{2.0771 \times 293.15} = 3.44 \text{ kg}$

$m_{TANK, 1} = \dfrac{PV}{RT} = \dfrac{2000 \times 12}{2.0771 \times 293.15} = 39.416 \text{ kg}$

$m_{TANK, 2} = 39.416 - 3.44 = 35.976 \text{ kg}$

$P_{T2} = m_{TANK, 2}\,RT/V = (m_{TANK, 1} / m_{TANK, 2}) \times P_1 = \textbf{1825.5 kPa}$

2.125

A piston/cylinder arrangement, shown in Fig. P2.125, contains air at 250 kPa, 300°C. The 50-kg piston has a diameter of 0.1 m and initially pushes against the stops. The atmosphere is at 100 kPa and 20°C. The cylinder now cools as heat is transferred to the ambient.

a. At what temperature does the piston begin to move down?

b. How far has the piston dropped when the temperature reaches ambient?

Solution:

Piston $A_p = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$

Balance forces when piston floats:

$$P_{float} = P_O + \frac{m_p g}{A_p} = 100 + \frac{50 \times 9.807}{0.00785 \times 1000}$$

$$= 162.5 \text{ kPa} = P_2 = P_3$$

To find temperature at 2 assume ideal gas:

$$T_2 = T_1 \times \frac{P_2}{P_1} = 573.15 \times \frac{162.5}{250} = \mathbf{372.5 \text{ K}}$$

b) Process 2 -> 3 is constant pressure as piston floats to $T_3 = T_O = 293.15$ K

$$V_2 = V_1 = A_p \times H = 0.00785 \times 0.25 = 0.00196 \text{ m}^3 = 1.96 \text{ L}$$

Ideal gas and $P_2 = P_3$ => $V_3 = V_2 \times \frac{T_3}{T_2} = 1.96 \times \frac{293.15}{372.5} = \mathbf{1.54 \text{ L}}$

$$\Delta H = (V_2 - V_3)/A = (1.96-1.54) \times 0.001/0.00785 = \mathbf{0.053 \text{ m} = 5.3 \text{ cm}}$$

Linear Interpolation

2.126

Find the pressure and temperature for saturated vapor R-410A with v = 0.1 m³/kg

Solution:

Table B.4.1 Look at the saturated vapor column v_g and it is found between –35°C and –30°C. We must then do a linear interpolation between these values.

$$T = -35 + [\,-30 - (-35)\,]\,\frac{0.1 - 0.11582}{0.09470 - 0.11582}$$

$$= -35 + 5 \times 0.749 = \mathbf{-31.3°C}$$

$$P = 218.4 + (269.6 - 218.4) \times 0.749 = \mathbf{256.7\ kPa}$$

To understand the interpolation equation look at the smaller and larger triangles formed in the figure. The ratio of the side of the small triangle in v as (0.11582 - 0.1) to the side of the large triangle (0.11582 – 0.09470) is equal to 0.749. This fraction of the total ΔP = 269.6 – 218.4 or ΔT = –30 – (–35) is added to the lower value to get the desired interpolated result.

2.127

Use a linear interpolation to estimate properties of ammonia to fill out the table below

	P [kPa]	T [°C]	v [m³/kg]	x
a)	550			0.75
b)	80	20		
c)		10	0.4	

Solution:

a) Find the pressures in Table B.2.1 that brackets the given pressure.

$$T = 5 + (10 - 5)\frac{550 - 515.9}{615.2 - 515.9} = 5 + 5 \times 0.341 = \textbf{6.7 °C}$$

$$v_f = 0.001583 + (0.0016 - 0.001583)\, 0.341 = 0.001589 \text{ m}^3/\text{kg}$$

$$v_g = 0.24299 + (0.20541 - 0.24299)\, 0.341 = 0.230175 \text{ m}^3/\text{kg}$$

$$v = v_f + xv_{fg} = 0.001589 + 0.75(0.230175 - 0.001589)$$

$$= \textbf{0.1729 m}^3\textbf{/kg}$$

b) Interpolate between 50 and 100 kPa to get properties at 80 kPa

$$v = 2.8466 + (1.4153 - 2.8466)\frac{80 - 50}{100 - 50}$$

$$= 2.8466 + (-1.4313) \times 0.6 = \textbf{1.9878 m}^3\textbf{/kg}$$

x: Undefined

c) Table B.2.1: $v > v_g$ so the state is superheated vapor.

Table B.2.2 locate state between 300 and 400 kPa.

$$P = 300 + (400 - 300)\frac{0.4 - 0.44251}{0.32701 - 0.44251}$$

$$= 300 + 100 \times 0.368 = \textbf{336.8 kPa}$$

x: Undefined

2.128

Use a linear interpolation to estimate Tsat at 900 kPa for nitrogen. Sketch by hand the curve Psat(T) by using a few table entries around 900 kPa from table B.6.1. Is your linear interpolation over or below the actual curve?

Solution:

The 900 kPa in Table B.6.1 is located between 100 and 105 K.

$$T = 100 + (105 - 100) \frac{900 - 779.2}{1084.6 - 779.2}$$
$$= 100 + 5 \times 0.3955 = \textbf{102 K}$$

The actual curve has a positive second derivative (it curves up) so T is slightly underestimated by use of the chord between the 100 K and the 105 K points, as the chord is above the curve.

2.129

Use a double linear interpolation to find the pressure for superheated R-134a at $13°C$ with v = 0.3 m^3/kg.

Solution:

Table B.5.2: Superheated vapor

At $10°C$, 0.3 m^3/kg

$$P = 50 + (100 - 50) \times \frac{0.3 - 0.45608}{0.22527 - 0.45608} = 83.8 \text{ kPa}$$

At $20°C$, 0.3 m^3/kg

$$P = 50 + (100 - 50) \times \frac{0.3 - 0.47287}{0.23392 - 0.47287} = 86.2 \text{ kPa}$$

Interpolating to get $13°C$ between the $10°C$ and $20°C$ P's above,
$$P = 83.8 + (3/10) \times (86.2 - 83.8) = \textbf{84.5 kPa}$$

This could also be interpolated as following:

At $13°C$, 50 kPa,   v = 0.45608 + (3/10) × 0.0168 = 0.4611 m^3/kg
At $13°C$, 100 kPa,   v = 0.22527 + (3/10) × 0.0087 = 0.2279 m^3/kg

Interpolating at 0.3 m^3/kg.

$$P = 50 + (100 - 50) \times \frac{0.3 - 0.4611}{0.2279 - 0.4611} = \textbf{84.5 kPa}$$

2.130

Find the specific volume for CO_2 at $0^O C$ and 625 kPa.

Solution:

The state is superheated vapor in Table B.3.2 between 400 and 800 kPa.

$$v = 0.12552 + (0.06094 - 0.12552)\frac{625 - 400}{800 - 400}$$

$$= 0.12552 + (-0.06458) \times 0.5625 = \mathbf{0.0892\ m^3/kg}$$

Computer Tables

2.131

Use the computer software to find the properties for water at the 4 states in Problem 2.35

    Start the software, click the tab for water as the substance, and click the small calculator icon. Select the proper CASE for the given properties.

	CASE	RESULT
a)	1 (T, P)	Compressed liquid, x = undefined, $v = 0.001002 \ m^3/kg$
b)	5 (P, v)	Two-phase, $T = 151.9°C$, $x = 0.5321$
c)	1 (T, P)	Sup. vapor, x = undefined, $v = 0.143 \ m^3/kg$
d)	4 (T, x)	$P = P_{sat} = 8581 \ kPa$, $v = 0.01762 \ m^3/kg$

2.132

Use the computer software to find the properties for ammonia at the 2 states listed in Problem 2.32

Start the software, click the tab for cryogenic substances, and click the tab for the substance ammonia. Then click the small calculator icon and select the proper CASE for the given properties.

	CASE	RESULT
a)	2 (T, v)	Sup. vapor, x = undefined, P = 1200 kPa
b)	4 (T, x)	Two-phase, P = 2033 kPa, v = 0.03257 m³/kg

2.133
Use the computer software to find the properties for ammonia at the 3 states listed in Problem 2.127

Start the software, click the tab for cryogenic substances, select ammonia and click the small calculator icon. Select the proper CASE for the given properties.

	CASE	RESULT
a)	8 (P, x)	$T = 6.795°C$, $v = 0.1719$ m^3/kg
b)	1 (T, P)	Sup. vapor, x = undefined, $v = 1.773$ m^3/kg
c)	2 (T, v)	Sup. vapor, x = undefined, $P = 330.4$ kPa

2.134

Find the value of the saturated temperature for nitrogen by linear interpolation in table B.6.1 for a pressure of 900 kPa. Compare this to the value given by the computer software.

The 900 kPa in Table B.6.1 is located between 100 and 105 K.

$$T = 100 + (105 - 100)\,\frac{900 - 779.2}{1084.6 - 779.2}$$

$$= 100 + 5 \times 0.3955 = 101.98 \text{ K}$$

The actual curve has a positive second derivative (it curves up) so T is slightly underestimated by use of the chord between the 100 K and the 105 K points, as the chord is above the curve.

From the computer software:

CASE: 8 (P,x) $T = -171°C = 102.15 \text{ K}$

So we notice that the curvature has only a minor effect.

2.135

Use the computer software to sketch the variation of pressure with temperature in Problem 2.44. Extend the curve a little into the single-phase region.

P was found for a number of temperatures. A small table of (P, T) values were entered into a spreadsheet and a graph made as shown below. The superheated vapor region is reached at about $140°$C and the graph shows a small kink at that point.

ENGLISH UNIT PROBLEMS

Borgnakke Sonntag

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CHAPTER 2

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Concept Problems

2.136E

Cabbage needs to be cooked (boiled) at 250 F. What pressure should the pressure cooker be set for?

Solution:

If I need liquid water at 250 F, I must have a pressure that is at least the saturation pressure for this temperature.

Table F.7.1: 250 F P_{sat} = **29.823 psia.**

The pot must have a lid that can be fastened to hold the higher pressure, which is a pressure cooker.

2.137E

   If I have 1 ft^3 of ammonia at 15 psia, 60 F how much mass is that?

   Ammonia Tables F.8:

   F.8.1 P_{sat} = 107.64 psia at 60 F so superheated vapor.

   F.8.2 v = 21.5641 ft^3/lbm   under subheading 15 psia

$$m = \frac{V}{v} = \frac{1 \text{ ft}^3}{21.5641 \text{ ft}^3/\text{lbm}} = \textbf{0.0464 lbm}$$

The P-v (log-log) diagram from CATT3, P in psi and v in ft^3/lbm. Cross-hair indicates the state.

2.138E

For water at 1 atm with a quality of 10% find the volume fraction of vapor.

This is a two-phase state at a given pressure:

Table F.7.2: $v_f = 0.01\ 672\ \text{ft}^3/\text{lbm},\quad v_g = 26.8032\ \text{ft}^3/\text{lbm}$

From the definition of quality we get the masses from total mass, m, as

$$m_f = (1 - x)\ m, \qquad m_g = x\ m$$

The volumes are

$$V_f = m_f\ v_f = (1 - x)\ m\ v_f, \qquad V_g = m_g\ v_g = x\ m\ v_g$$

So the volume fraction of vapor is

$$\text{Fraction} = \frac{V_g}{V} = \frac{V_g}{V_g + V_f} = \frac{x\ m\ v_g}{x\ m\ v_g + (1 - x)m\ v_f}$$

$$= \frac{0.1 \times 26.8032}{0.1 \times 26.8032 + 0.9 \times 0.016\ 72} = \frac{2.68032}{2.69537} = \textbf{0.9944}$$

Notice that the liquid volume is only about 0.5% of the total. We could also have found the overall $v = v_f + xv_{fg}$ and then $V = m\ v$.

2.139E

Locate the state of R-410A at 30 psia, -20 F. Indicate in both the P-v and the T-v diagrams the location of the nearest states listed in the printed table F.9

From F.9:   F.9.1 at -20 F, P_{sat} = 40.923 psi so we have superheated vapor.
          F.9.2 at 30 psi we find the state for -20 F.

Computer-Aided Thermodynamic Tables 3

File   Edit   Tables/Substances   Options   Help

P-V Diagram (Log-Log)

R-410a Properties

T	-20	F
P	30	psia
V	2.035	ft3/lbm
U	105.2	Btu/lbm
H	116.5	Btu/lbm
S	0.2732	Btu/lbm/R
X		

Type	Temp	Pressure	Specific Volume	Internal Energy	Specific Enthalpy	Specific Entropy	Quality
	F	psia	ft3/lbm	Btu/lbm	Btu/lbm	Btu/lbm/R	
1. R-410a	-20	30	2.035	105.2	116.5	0.2732	

Water \ Refrigerants \ Cryogenics \ Air \ Ideal Gases \ Compressibility \ Psychrometrics

2.140E

Calculate the ideal gas constant for argon and hydrogen based on Table F.1 and verify the value with Table F.4

The gas constant for a substance can be found from the universal gas constant from table A.1 and the molecular weight from Table F.1

Argon: $\qquad R = \dfrac{\overline{R}}{M} = \dfrac{1.98589}{39.948} = \mathbf{0.04971} \dfrac{\mathbf{Btu}}{\mathbf{lbm\ R}} = \mathbf{38.683} \dfrac{\mathbf{lbf\text{-}ft}}{\mathbf{lbm\ R}}$

Hydrogen: $\qquad R = \dfrac{\overline{R}}{M} = \dfrac{1.98589}{2.016} = \mathbf{0.98506} \dfrac{\mathbf{Btu}}{\mathbf{lbm\ R}} = \mathbf{766.5} \dfrac{\mathbf{lbf\text{-}ft}}{\mathbf{lbm\ R}}$

Recall from Table A.1:　1 Btu = 778.1693 lbf-ft

Phase Diagrams

2.141E

Water at 80 F can exist in different phases dependent on the pressure. Give the approximate pressure range in lbf/in^2 for water being in each one of the three phases, vapor, liquid or solid.

Solution:

The phases can be seen in Fig. 2.4, a sketch of which is shown to the right.

   T = 80 F = 540 R = 300 K

From Fig. 2.4:

   $P_{VL} \approx 4 \times 10^{-3}$ MPa = 4 kPa = 0.58 psia,

   $P_{LS} = 10^3$ MPa = 145 038 psia

$$
\begin{array}{lll}
0 < P < & 0.58 \text{ psia} & \text{VAPOR} \\
0.58 \text{ psia} < P < 145\ 038 \text{ psia} & & \text{LIQUID} \\
P > 145\ 038 \text{ psia} & & \text{SOLID(ICE)}
\end{array}
$$

2.142E

A substance is at 300 lbf/in.^{2}, 65 F in a rigid tank. Using only the critical properties can the phase of the mass be determined if the substance is oxygen, water or propane?

Solution: Find state relative to the critical point properties, Table F.1

a)	Oxygen	731 lbf/in.^{2}	278.3 R
b)	Water	3208 lbf/in.^{2}	1165.1 R
c)	Propane	616 lbf/in.^{2}	665.6 R

$P < P_c$ for all and $\quad$ T = 65 F = 65 + 459.67 = 525 R

a)	O_2	$T \gg T_c$	Yes gas and $P < P_c$
b)	H_2O	$T \ll T_c$	$P \ll P_c$ so you cannot say
c)	C_3H_8	$T < T_c$	$P < P_c$ you cannot say

General Tables

2.143E

Determine the missing property (of P, T, v, and x if apllicable) for water at

a.   680 lbf/in.^{2}, 0.03 ft^3/lbm
b.   150 lbf/in.^{2}, 320 F
c.   400 F, 3 ft^3/lbm

Solution: All cases can be seen from Table F.7.1

a.   680 lbf/in.^{2}, 0.03 ft^3/lbm

v_g = **0.2183**, v_f = **0.02472** ft^3/lbm,   so **liquid + vapor mixture**

b.   150 lbf/in.^{2}, 320 F: **compressed liquid** $P > P_{sat}(T)$ = **89.6 lbf/in^2**

c.   400 F, 3 ft^3/lbm: **sup. vapor** $v > v_g(T)$ = **2.339** ft^3/lbm

States shown are placed relative to the two-phase region, not to each other.

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2.144E

Determine the phase of the substance at the given state using Appendix F Tables.

Solution:

a. water 200 F, 70 psia:

   F.7.1 at 200 F $P > P_{sat}(T) = $ **11.53 psia**   => **compressed liquid**

b. Ammonia 10 F, 20 lbf/in.^{2}:

   F.8.1 at 10 F $P < P_{sat}(T) = $ **38.508 lbf/in^2** => **sup. vapor**

   F.8.2: Superheated by (20 – (-16.63)) F = 36.6 F, v = 14.7635 ft^3/lbm

c. R-410A 30 F, 50 ft^3/lbm

   F.9.1 at 30 F: $P < P_{sat}(T) = $ **111.8 lbf/in^2** => **sup. vapor,**

   F.9.2: Interpolate to find v

The S-L fusion line goes slightly to the left for water. It tilts slightly to the right for most other substances.

States shown are placed relative to the two-phase region, not to each other.

2.145E

Give the phase and the missing property of P, T, v and x for R-134a at
a.   T = -10 F, P = 18 psia
b.   P = 40 psia, v = 1.3 ft^3/lbm

Solution:

a. Look in Table F.10.1 at –10 F: P > P$_{sat}$ = 16.76 psia

   This state is **compressed liquid** so x is undefined and v = v$_f$ = 0.01173 ft^3/lbm

b. Look in Table F.10.1 close to 50 psia there we see

      v > v$_g$ = 0.95 ft^3/lbm so **superheated vapor**

   Look then in Table F.10.2 under 40 psia and interpolate between the 60 F and 80 F.

      (40 psia, 1.3 ft^3/lbm) : T = 66.6 F

For a better accuracy use the computer software CATT3 which gives T = 67.4 F.

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2.146E

Give the phase and the missing property of P, T, v and x for ammonia at
   a.   T = 120 F, v = 1.876 ft^3/lbm
   b.   T = 120 F, x = 0.5

Solution:

a. Look in Table F.8.1 at 120 F: v > v$_g$ = 1.0456 ft^3/lbm so sup vap

   x = undefined,   F.8.2   P = 175 psia

b. Look in Table F.8.1 at 120 F:

   v = v$_f$ + x v$_{fg}$ = 0.02836 + 0.5 × 1.0172 = 0.53696 ft^3/lbm

2.147E

Give the phase and the specific volume.

Solution:

a. R-410A $T = -25$ F, $P = 30$ lbf/in.^{2} Table F.9.2 $T > T_{sat} = -33.24$ F

$\Rightarrow$ **sup.vap.** $v \cong 1.9534 + \dfrac{-25+33.24}{-20+33.24}\,(2.0347 - 1.9534) = $ **2.004 ft^3/lbm**

b. R-410A $T = -25$ F, $P = 40$ lbf/in.^{2} Table F.9.2 $T < T_{sat} = -21.0$ F

$P > P_{sat} \Rightarrow$ **compresssed Liquid**   F.9.1: $v \cong v_f = $ **0.01246 ft^3/lbm**

c. H_2O $T = 280$ F, $P = 35$ lbf/in.^{2} Table F.7.1 $P < P_{sat} = 49.2$ psia

$\Rightarrow$ **superheated vapor**

$v \cong 21.734 + (10.711 - 21.734) \times (15/20) = $ **1.0669 ft^3/lbm**

d. NH_3 $T = 60$ F, $P = 15$ lbf/in.^{2} Table F.8.1 $P_{sat} = 107.6$ psia

$P < P_{sat} \Rightarrow$ **superheated vapor** $v \cong $ **21.564 ft^3/lbm**

States shown are
placed relative to the
two-phase region, not
to each other.

2.148E

Determine the specific volume for R-410A at these states:

     a. 20 F, 70 psia
     b. 70 F, 150 psia
     c. 70 F, quality 25%

    a. F.9.2 $P < P_{sat} = 93.13$ psia so sup vapor.

$$v = 1.0783 + \frac{70 - 60}{75 - 60}(0.8393 - 1.0783) = \mathbf{0.9190\ ft^3/lbm}$$

    b. F.9.1 $P < P_{sat} = 215.95$ psia   so superheated vapor

       F.9.2: $v = 0.4236 + \frac{70 - 60}{80 - 60}(0.4545 - 0.4236) = \mathbf{0.43905\ ft^3/lbm}$

    c. F.9.1 $v_f = 0.01486\ ft^3/lbm,\ \ v_{fg} = 0.2576\ ft^3/lbm$

                 $v = v_f + x\ v_{fg} = 0.01486 + 0.25 \times 0.2576 = \mathbf{0.07926\ ft^3/lbm}$

States shown are
placed relative to the
two-phase region, not
to each other.

2.149E

Give the missing property of P, T, v and x for

      a. R-410A at 80 F, v = 0.2 ft^3/lbm

      b. R-410A at 60 psia, v = 1.1 ft^3/lbm

      c. Ammonia at 60 F, v = 3.2 ft^3/lbm

a) Table F.9.1 at 80 F: $v < v_g = 0.2308$ ft^3/lbm so we have 2 phase L+V

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.2 - 0.01525}{0.2156} = \mathbf{0.8569}; \quad P = P_{sat} = \mathbf{250.665 \ psia}$$

b) Table F.9.2 at 60 psia: $v > v_g = 1.0038$ ft^3/lbm so we have superheated vapor

    so x is undefined.

      F.9.2 between 20 and 40 F:

$$T = 20 + 20 \times \frac{1.1 - 1.0783}{1.1405 - 1.0783} = \mathbf{26.98 \ F}$$

c) Table F.8.1 at 60 F: $v > v_g = 2.7481$ ft^3/lbm so we have superheated vapor.

      F.8.2 between 90 and 100 psia:

$$P = 90 + 10 \times \frac{3.2 - 3.3503}{2.9831 - 3.3503} = \mathbf{94.09 \ psia}$$

States shown are
placed relative to the
two-phase region, not
to each other.

2.150E

Saturated liquid water at 150 F is put under pressure to decrease the volume by 1% while keeping the temperature constant. To what pressure should it be compressed?

F.7.1: $v = v_f = 0.01634$ ft^3/lbm

New v: $v = 0.99\ v_f = 0.0161766$ ft^3/lbm

look in F.7.3: close to 2000 psia

interpolate between 2000 and 8000 psia   P = **2610 psia**

2.151E

A sealed rigid vessel has volume of 35 ft^3 and contains 2 lbm of water at 200 F. The vessel is now heated. If a safety pressure valve is installed, at what pressure should the valve be set to have a maximum temperature of 400 F?

Solution:

Process: v = V/m = constant

State 1: v_1 = 35/2 = 17.5 ft^3/lbm

from Table F.7.1

it is 2-phase

State 2: 400°F, 17.5 ft^3/lbm

Table F.7.2 between 20

and 40 lbf/in^2 so interpolate

$$P \cong 20 + (40 - 20)\ \frac{17.5 - 25.427}{12.623 - 25.427} = \mathbf{32.4\ lbf/in^2}$$

(The result is 28.97 psia if found by the software, i.e. linear interpolation is not so accurate)

2.152E

You want a pot of water to boil at 220 F. How heavy a lid should you put on the 6 inch diameter pot when $P_{atm} = 14.7$ psia?

Solution:

Table F.7.1 at 220 F : $P_{sat} = 17.189$ psia

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \; 6^2 = 28.274 \text{ in}^2$$

$$F_{net} = (P_{sat} - P_{atm}) \, A = (17.189 - 14.7) \, (\text{lbf/ in}^2) \times 28.274 \text{ in}^2$$
$$= 70.374 \text{ lbf}$$

$$F_{net} = m_{lid} \, g$$

$$m_{lid} = F_{net}/g = \frac{70.374 \text{ lbf}}{32.174 \text{ ft/s}^2} = \frac{70.374 \times 32.174 \text{ lbm ft/s}^2}{32.174 \text{ ft/s}^2} = \mathbf{70.374 \text{ lbm}}$$

Some lids are clamped on, the problem deals with one that stays on due to its weight.

2.153E

Saturated water vapor at 240 F has its pressure decreased to increase the volume by 10%, keeping the temperature constant. To what pressure should it be expanded?

Solution:

Initial state: $v = 16.3257$ ft^3/lbm   from table F.7.1

Final state: $v = 1.1 \times v_g = 1.1 \times 16.3257 = 17.9583$ ft^3/lbm

Interpolate between sat. at 240 F, $P = 24.968$ lbf/in^2 and sup. vapor in Table F.7.2 at 240 F, 20 lbf/in^2

$$P = 24.968 + (20 - 24.968) \frac{17.9583 - 16.3257}{20.475 - 16.3257}$$

$$\cong \textbf{23.0 lbf/in}^2$$

2.154E

A glass jar is filled with saturated water at 300 F and quality 25%, and a tight lid is put on. Now it is cooled to 10 F. What is the mass fraction of solid at this temperature?

Solution:

Constant volume and mass $\Rightarrow v_1 = v_2 = V/m$

From Table F.7.1: $\qquad v_1 = 0.01745 + 0.25 \times 6.4537 = 1.630875 \text{ ft}^3/\text{lbm}$

From Table F.7.4: $\qquad v_2 = 0.01744 + x_2 \times 9043 = v_1 = 1.630875 \text{ ft}^3/\text{lbm}$

$\qquad\qquad\qquad \Rightarrow x_2 = 0.000178$ mass fraction vapor

$x_{solid} = 1 - x_2 = 0.9998$ or **99.98 %**

2.155E

A boiler feed pump delivers 100 ft^3/min of water at 400 F, 3000 lbf/in.^{2}. What is the mass flowrate (lbm/s)? What would be the percent error if the properties of saturated liquid at 400 F were used in the calculation? What if the properties of saturated liquid at 3000 lbf/in.^{2} were used?

Solution:

Table F.7.3: $v = 0.0183$ ft^3/lbm (interpolate 2000-8000 psia)

$$\dot{m} = \frac{\dot{V}}{v} = \frac{100}{60 \times 0.018334} = 91.07 \text{ lbm/s}$$

$v_{f (400 F)} = 0.01864$ ft^3/lbm $\Rightarrow$ $\dot{m} = 89.41$ lbm/s **error 1.8%**

$v_{f (3000 \text{ lbf/in}^2)} = 0.03475$ ft^3/lbm $\Rightarrow$ $\dot{m} = 47.96$ lbm/s **error 47%**

The constant T line is nearly vertical for the liquid phase in the P-v diagram. The state is at so high P, T that the saturated liquid line is not extremely steep.

2.156E

A pressure cooker has the lid screwed on tight. A small opening with A = 0.0075 in^2 is covered with a petcock that can be lifted to let steam escape. How much mass should the petcock have to allow boiling at 250 F with an outside atmosphere at 15 psia?

Solution:

Table F.7.1 at 250 F: P_{sat} = 29.823 psia

$F_{net} = (P_{sat} - P_{atm})$ A = (29.823 - 15) psia × 0.0075 in^2
        = 0.111 lbf

$F_{net} = m_{petcock}$ g

$m_{petcock} = F_{net}/g = \dfrac{0.111 \text{ lbf}}{32.174 \text{ ft/s}^2} = \dfrac{0.111 \times 32.174 \text{ lbm ft/s}^2}{32.174 \text{ ft/s}^2} =$ **0.111 lbm**

Some petcocks are held down by a spring, the problem deals with one that stays on due to its weight.

2.157E

Two tanks are connected together as shown in Fig. P2.52, both containing water. Tank A is at 30 lbf/in.^{2}, $v = 8$ ft^3/lbm, $V = 40$ ft^3 and tank B contains 8 lbm at 80 lbf/in.^{2}, 750 F. The valve is now opened and the two come to a uniform state. Find the final specific volume.

Solution:

Control volume both tanks. Constant total volume and mass process.

State A1: (P, v)   two-phase, $m_A = V_A/v_A = 40$ ft^3 /8 ft^3/lbm = 5 lbm

State B1: (P, T)   Table F.7.2: $v_B = (8.561 + 9.322)/2 = 8.9415$ ft^3/lbm

$\Rightarrow V_B = m_B v_B = 8$ lbm $\times$ 8.9415 ft^3/lbm = 71.532 ft^3

Final state: $m_{tot} = m_A + m_B = 5 + 8 = 13$ lbm

$V_{tot} = V_A + V_B = 111.532$ ft^3

$v_2 = V_{tot}/m_{tot} = 111.532$ ft^3/13 lbm = **8.579 ft^3/lbm**

2.158E

Ammonia at 70 F with a quality of 50% and total mass 4.5 lbm is in a rigid tank with an outlet valve at the bottom. How much liquid (mass) can you take out through the valve assuming the temperature stays constant?

Solution:

The bottom has liquid until the state inside becomes saturated vapor.

$$V = m_1 v_1 = 4.5 \times 0.5 \, (0.02631 + 2.3098) = 5.25625 \text{ ft}^3$$

$$m_2 = V/v_2 = 5.25625 \text{ ft}^3/ \, 2.3098 \text{ ft}^3/\text{lbm} = 2.2756 \text{ lbm}$$

$$m = m_1 - m_2 = 4.5 - 2.2756 = \textbf{2.224 lbm}$$

Ideal Gas

2.159E

Give the phase and the specific volume for each of the following.
Solution:

a. CO_2 $T = 510$ F $P = 75$ lbf/in.^{2} Table F.4

superheated vapor ideal gas

$$v = RT/P = \frac{35.1 \text{ ft-lbf/lbm-R} \times (510 + 459.7) \text{ R}}{75 \text{ lbf/in.}^2 \times 144 \text{ (ft/ in.)}^2}$$

$$= \textbf{3.152 ft}^3\textbf{/lbm}$$

b. Air $T = 68$ F $P = 2$ atm   Table F.4

superheated vapor ideal gas

$$v = RT/P = \frac{53.34 \text{ ft-lbf/lbm-R} \times (68 + 459.7) \text{ R}}{2 \times 14.6 \text{ lbf/in.}^2 \times 144 \text{ (ft/ in.)}^2}$$

$$= \textbf{6.6504 ft}^3\textbf{/lbm}$$

c. Ar $T = 300$ F, $P = 30$ lbf/in.^{2} Table F.4

Ideal gas: $$v = RT/P = \frac{38.68 \text{ ft-lbf/lbm-R} \times (300 + 459.7) \text{ R}}{30 \text{ lbf/in.}^2 \times 144 \text{ (ft/ in.)}^2}$$

$$= \textbf{6.802 ft}^3\textbf{/lbm}$$

2.160E

A cylindrical gas tank 3 ft long, inside diameter of 8 in., is evacuated and then filled with carbon dioxide gas at 77 F. To what pressure should it be charged if there should be 2.6 lbm of carbon dioxide?

Solution:

Assume CO_2 is an ideal gas table F.4: $P = mRT/V$

$$V_{cyl} = A \times L = \frac{\pi}{4}(8 \text{ in})^2 \times 3 \times 12 \text{ in} = 1809.6 \text{ in}^3$$

$$P = \frac{2.6 \text{ lbm} \times 35.1 \text{ ft-lbm/lbm-R} \times (77 + 459.67) \text{ R} \times 12 \text{ in/ft}}{1809.6 \text{ in}^3}$$

$$= \textbf{324.8 lbf/in}^2$$

2.161E

A spherical helium balloon of 30 ft in diameter is at ambient T and P, 60 F and 14.69 psia. How much helium does it contain? It can lift a total mass that equals the mass of displaced atmospheric air. How much mass of the balloon fabric and cage can then be lifted?

We need to find the masses and the balloon volume

$$V = \frac{\pi}{6} D^3 = \frac{\pi}{6} 30^3 = 14\ 137\ \text{ft}^3$$

$$m_{He} = \rho V = \frac{V}{v} = \frac{PV}{RT}$$

$$= \frac{14.69\ \text{psi} \times 14\ 137\ \text{ft}^3 \times 144\ (\text{in/ft})^2}{386.0\ \text{ft-lbf/lbmR} \times 520\ \text{R}}$$

$$= 148.99\ \text{lbm}$$

$$m_{air} = \frac{PV}{RT} = \frac{14.69\ \text{psi} \times 14\ 137\ \text{ft}^3 \times 144\ (\text{in/ft})^2}{53.34\ \text{ft-lbf/lbmR} \times 520\ \text{R}}$$

$$= 1078\ \text{lbm}$$

$$m_{lift} = m_{air} - m_{He} = 1078 - 149 = \textbf{929 lbm}$$

2.162E

Helium in a steel tank is at 36 psia, 540 R with a volume of 4 ft^3. It is used to fill a balloon. When the pressure drops to 18 psia, the flow of helium stops by itself. If all the helium still is at 540 R, how big a balloon is produced?

Solution:

State 1:   m = V/v   assume ideal gas so

$$m = \frac{P_1 V_1}{RT_1} = \frac{36 \text{ psi} \times 4 \text{ ft}^3 \times 144 \text{ (in/ft)}^2}{386 \text{ ft-lbf/lbmR} \times 540 \text{ R}}$$
$$= 0.0995 \text{ lbm}$$

State 2:   Same mass so then ($T_2 = T_1$)

$$V_2 = \frac{mRT_2}{P_2} = \frac{P_1 V_1}{RT_1} \frac{RT_2}{P_2} = V_1 \frac{P_1}{P_2}$$
$$= 4 \frac{36}{18} = 8 \text{ ft}^3$$

The balloon volume is

$$V_{balloon} = V_2 - V_1 = 8 - 4 = \textbf{4 ft}^3$$

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Review Problems

2.163E

A 35 ft^3 rigid tank has propane at 15 psia, 540 R and connected by a valve to another tank of 20 ft^3 with propane at 40 psia, 720 R. The valve is opened and the two tanks come to a uniform state at 600 R. What is the final pressure?

Solution:

Propane is an ideal gas (P << P$_c$) with R = 35.04 ft-lbf/lbm R from Tbl. F.4

$$m_A = \frac{P_A V_A}{RT_A} = \frac{15 \times 35 \times 144}{35.04 \times 540} = 3.995 \text{ lbm}$$

$$m = \frac{P_B V_B}{RT_B} = \frac{40 \times 20 \times 144}{35.04 \times 720} = 4.566 \text{ lbm}$$

$$V_2 = V_A + V_B = 55 \text{ ft}^3$$

$$m_2 = m_A + m_B = 8.561 \text{ lbm}$$
$$P_2 = \frac{m_2 R T_2}{V_2} = \frac{8.561 \times 35.04 \times 600}{55 \times 144} = \textbf{22.726 psia}$$

2.164E

What is the percent error in specific volume if the ideal gas model is used to represent the behavior of superheated ammonia at 100 F, 80 lbf/in.^{2}? What if the generalized compressibility chart, Fig. D.1, is used instead?

Solution:

Ammonia Table F.8.2: $v = 4.186$ ft^3/lbm

Ideal gas $\quad v = \dfrac{RT}{P} = \dfrac{90.72 \times 559.7}{80 \times 144} = 4.4076$ ft^3/lbm $\quad$ **5.3% error**

Generalized compressibility chart and Table D.4

$$T_r = 559.7/729.9 = 0.767, \ \ P_r = 80/1646 = 0.0486$$

$$\Rightarrow \quad Z \cong 0.96$$

$$v = \dfrac{ZRT}{P} = 0.96 \times 4.4076 \text{ ft}^3/\text{lbm} = 4.231 \text{ ft}^3/\text{lbm} \quad \textbf{1.0\% error}$$

2.165E

Air in an internal combustion engine has 440 F, 150 psia with a volume of 2 ft^3. Now combustion heats it to 3000 R in a constant volume process. What is the mass of air and how high does the pressure become?

The mass comes from knowledge of state 1 and ideal gas law

$$m = \frac{P_1 V_1}{RT_1} = \frac{150 \text{ psia} \times 2 \text{ ft}^3 \times 144 \text{ (in/ft)}^2}{53.34 \text{ ft-lbf/lbm-R} \times (440 + 459.67) \text{ R}} = \textbf{0.90 lbm}$$

The final pressure is found from the ideal gas law written for state 1 and state 2 and then eliminate the mass, gas constant and volume ($V_2 = V_1$) between the equations

$$P_1 \, V_1 = m \, RT_1 \qquad \text{and} \qquad P_2 \, V_2 = m \, RT_2$$

$$P_2 = P_1 \times T_2/T_1 = 150 \text{ psia} \times \frac{3000}{440 + 459.67} = \textbf{500.2 psia}$$

2.166E

A 35 ft^3 rigid tank has air at 225 psia and ambient 600 R connected by a valve to a piston cylinder. The piston of area 1 ft^2 requires 40 psia below it to float, Fig. P2.166. The valve is opened and the piston moves slowly 7 ft up and the valve is closed. During the process air temperature remains at 600 R. What is the final pressure in the tank?

$$m_A = \frac{P_A V_A}{R T_A} = \frac{225 \text{ psia} \times 35 \text{ ft}^3 \times 144 \text{ (in/ft)}^2}{53.34 \text{ ft-lbf/lbm-R} \times 600 \text{ R}} = 35.433 \text{ lbm}$$

$$m_{B2} - m_{B1} = \frac{\Delta V_A}{v_B} = \frac{\Delta V_B P_B}{RT} = \frac{40 \text{ psia} \times (1 \times 7) \text{ ft}^3 \times 144 \text{ (in/ft)}^2}{53.34 \text{ ft-lbf/lbm-R} \times 600 \text{ R}}$$
$$= 1.26 \text{ lbm}$$

$$m_{A2} = m_A - (m_{B2} - m_{B1}) = 35.433 - 1.26 = 34.173 \text{ lbm}$$

$$P_{A2} = \frac{m_{A2} RT}{V_A} = \frac{34.173 \times 53.34 \times 600}{35 \times 144} = \textbf{217 psia}$$

2.167E

Give the phase and the missing properties of P, T, v and x. These may be a little more difficult if the appendix tables are used instead of the software.

Solution:

a. R-410A at $T = 50$ F, $v = 0.4$ ft^3/lbm: Table F.9.1: $v > v_g = 0.3818$ ft^3/lbm

sup. vap. F.9.2 interpolate between sat. and sup. vap at 50 F.

Find v at 150 psia, 50 F: $v = 0.4066$

Now interpolate between the sat vap 157.473 psia and the 150 psia.

$P \cong 150 + 7.473\ (0.4 - 0.4066)/(0.3818 - 0.4066) =$ **152 lbf/in^2**

b. H_2O $v = 2$ ft^3/lbm, $x = 0.5$: Table F.7.1

since v_f is so small we find it approximately where $v_g = 4$ ft^3/lbm.

$v_f + v_g = 4.3293$ at 330 F, $v_f + v_g = 3.80997$ at 340 F.

linear interpolation $T \cong$ **336 F, $P \cong$ 113 lbf/in^2**

c. H_2O $T = 150$ F, $v = 0.01632$ ft^3/lbm: Table F.7.1, $v < v_f$

compr. liquid $P \cong$ **500 lbf/in^2**

d. NH_3 $T = 80$ F, $P = 13$ lbf/in.^{2} Table F.8.1 $P < Psat$

sup. vap. interpolate between 10 and 15 psia: $v =$ **26.97 ft^3/lbm**

v is not linear in P (more like 1/P) so computer table is more accurate.

e. R-134a $v = 0.08$ ft^3/lbm, x = 0.5: Table F.10.1

since v_f is so small we find it approximately where $v_g = 0.16$ ft^3/lbm.

$v_f + v_g = 0.1729$ at 150 F, $v_f + v_g = 0.1505$ at 160 F.

linear interpolation $T \cong$ **156 F,** $P \cong$ **300 lbf/in^2**

States shown are placed relative to the two-phase region, not to each other.

2.168E

A pressure cooker (closed tank) contains water at 200 F with the liquid volume being 1/10 of the vapor volume. It is heated until the pressure reaches 300 lbf/in.^{2}. Find the final temperature. Has the final state more or less vapor than the initial state?

Solution:

Process: Constant volume and mass.

$V_f = m_f v_f = V_g/10 = m_g v_g/10;$

Table F.7.1: $v_f = 0.01663$ ft^3/lbm, $v_g = 33.631$ ft^3/lbm

$$x_1 = \frac{m_g}{m_g + m_f} = \frac{10\ m_f v_f / v_g}{m_f + 10\ m_f v_f / v_g} = \frac{10\ v_f}{10\ v_f + v_g} = \frac{0.1663}{0.1663 + 33.631} = 0.00492$$

$$v_2 = v_1 = 0.01663 + x_1 \times 33.615 = 0.1820\ \text{ft}^3/\text{lbm}$$

$$P_2, v_2 \implies \quad T_2 = T_{sat}(300\ \text{psia}) = \textbf{417.43 F}$$

At state 2: $v_2 = v_f + x_2\ v_{fg}$

$0.1820 = 0.01890 + x_2 \times 1.5286$

$\implies \quad x_2 = \textbf{0.107}$

More vapor at final state

2.169E

Refrigerant-410A in a piston/cylinder arrangement is initially at 60 F, x = 1. It is then expanded in a process so that $P = Cv^{-1}$ to a pressure of 30 lbf/in.^{2}. Find the final temperature and specific volume.

Solution:

State 1: $P_1 = 184.98$ lbf/in^2 $v_1 = 0.3221$ ft^3/lbm

Process: $Pv = C = P_1 v_1 = P_2 v_2$

State 2: $P_2 = 30$ lbf/in^2 and on process line (equation).

$$v_2 = \frac{v_1 P_1}{P_2} = 0.\,3221 \times 184.98\,/\,30 = \textbf{1.9861 ft}^3\textbf{/lbm}$$

Table F.9.2 between saturated at -33.24 F and -20 F: $T_2 \cong$ **-27.9 F**

Notice T is **not** constant

Compressiblity Factor

2.170E

A substance is at 70 F, 300 lbf/in.^{2} in a 10 ft^3 tank. Estimate the mass from the compressibility chart if the substance is a) air, b) butane or c) propane.

Solution:

Use Fig. D.1 for compressibility Z and table F.1 for critical properties

$$m = \frac{PV}{ZRT} = \frac{300 \times 144 \times 10}{530 \ ZR} = \frac{815.09}{ZR}$$

Air use nitrogen $P_c = 492$ lbf/in.^{2}; $T_c = 227.2$ R

$P_r = 0.61$; $T_r = 2.33$; $Z = 0.98$

$$m = \frac{PV}{ZRT} = \frac{815.09}{ZR} = \frac{815.09}{0.98 \times 55.15} = \textbf{15.08 lbm}$$

Butane $P_c = 551$ lbf/in.^{2}; $T_c = 765.4$ R

$P_r = 0.544$; $T_r = 0.692$; $Z = 0.09$

$$m = \frac{PV}{ZRT} = \frac{815.09}{ZR} = \frac{815.09}{0.09 \times 26.58} = \textbf{340.7 lbm}$$

Propane $P_c = 616$ lbf/in.^{2}; $T_c = 665.6$ R

$P_r = 0.487$; $T_r = 0.796$; $Z = 0.08$

$$m = \frac{PV}{ZRT} = \frac{815.09}{ZR} = \frac{815.09}{0.08 \times 35.04} = \textbf{290.8 lbm}$$

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2.171E

Determine the mass of an ethane gas stored in a 25 ft^3 tank at 250 F, 440 lbf/in.^{2} using the compressibility chart. Estimate the error (%) if the ideal gas model is used.

Solution

Table F.1: $T_r = (250 + 460) / 549.7 = 1.29$ and $P_r = 440/708 = 0.621$

Figure D.1 $\Rightarrow$ $Z = 0.9$

$$m = \frac{PV}{ZRT} = \frac{440 \text{ psia} \times 25 \text{ ft}^3 \times 144 \text{ (in/ft)}^2}{0.9 \times 51.38 \text{ ft-lbf/lbm-R} \times 710 \text{ R}} = \textbf{48.25 lbm}$$

Ideal gas $Z = 1$

   $\Rightarrow$ $m = PV/RT = 43.21$ lbm   **10% error**

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Equations of State

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2.172E

Determine the pressure of R-410A at 100 F, $v = 0.2$ ft^3/lbm using ideal gas and the van der Waal Equation of State.

From F.1 for R-410A: $M = 72.585$, $T_c = 620.1$ R, $P_c = 711$ psia

For ideal gas we get:

$$R = \bar{R}/M = 1545.36 / 72.585 = 21.29 \text{ lbf-ft/lbm-R}$$

$$P = RT/v = \frac{21.29 \times (100 + 459.67)}{0.2 \times 144} \frac{\text{lbf-ft}}{\text{lbm-R}} \frac{R}{\text{ft}^3/\text{lbm} \times (\text{in./ft})^2}$$

$$= \textbf{413.73 psi}$$

For **van der Waal equation of state** from Table D.1 we have

$$b = \frac{1}{8} \frac{RT_c}{P_c} = 0.125 \times \frac{21.29 \times 620.1}{711 \times 144} = 0.016\ 118 \text{ ft}^3/\text{lbm},$$

$$a = 27\ b^2\ P_c = 27 \times (0.016\ 118)^2 \times 711 = 4.987\ 283 \text{ psi (ft}^3/\text{lbm)}^2$$

The EOS is: $P = \dfrac{RT}{v-b} - \dfrac{a}{v^2} = \dfrac{21.29 \times 559.67/144}{0.2 - 0.016\ 118} - \dfrac{4.987\ 283}{0.2^2} = \textbf{325.3 psi}$

From Table F.9.2 we see it is around 300 psia.

2.173E

Determine the pressure of R-410A at 100 F, $v = 0.2$ ft^3/lbm using ideal gas and the Redlich-Kwong Equation of State.

From F.1 for R-410A: $M = 72.585$, $T_c = 620.1$ R, $P_c = 711$ psia

For ideal gas we get:

$$R = \overline{R}/M = 1545.36 / 72.585 = 21.29 \text{ lbf-ft/lbm-R}$$

$$P = RT/v = \frac{21.29 \times (100 + 459.67)}{0.2 \times 144} \frac{\text{lbf-ft}}{\text{lbm-R}} \frac{R}{\text{ft}^3/\text{lbm} \times (\text{in./ft})^2}$$

$$= \textbf{413.73 psi}$$

For **Redlich-Kwong EOS** we have the parameters from Table D.1

$$T_r = T/T_c = (100 + 459.67) / 620.1 = 0.90255$$

$$b = 0.08664 \frac{RT_c}{P_c} = 0.08664 \times \frac{21.29 \times 620.1}{711 \times 144} = 0.011\ 172 \text{ ft}^3/\text{lbm},$$

$$a = 0.42748\ T_r^{-1/2} \frac{R^2 T_c^2}{P_c} = 0.42748 \times \frac{21.29^2 \times 620.1^2}{0.90255^{1/2} \times 711 \times 144^2}$$

$$= 5.319\ 372 \text{ psi (ft}^3/\text{lbm})^2$$

The equation is:

$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$

$$= \frac{21.29 \times 559.67 / 144}{0.2 - 0.011\ 172} - \frac{5.319\ 372}{0.2^2 + 0.011\ 172 \times 0.2}$$

$$= \ \textbf{312 psia}$$

From Table F.9.2 we see it is around 300 psia.

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 3

8e

Updated June 2013

CONTENT CHAPTER 3

In-Text Concept Questions

3.a

In a complete cycle what is the net change in energy and in volume?

For a complete cycle the substance has no change in energy and therefore no storage, so the net change in energy is zero.

For a complete cycle the substance returns to its beginning state, so it has no change in specific volume and therefore no change in total volume.

3.b

Explain in words what happens with the energy terms for the stone in Example 3.3. What would happen if it were a bouncing ball falling to a hard surface?

In the beginning all the energy is potential energy associated with the gravitational force. As the stone falls the potential energy is turned into kinetic energy and in the impact the kinetic energy is turned into internal energy of the stone and the water. Finally the higher temperature of the stone and water causes a heat transfer to the ambient until ambient temperature is reached.

With a hard ball instead of the stone the impact would be close to elastic transforming the kinetic energy into potential energy (the material acts as a spring) that is then turned into kinetic energy again as the ball bounces back up. Then the ball rises up transforming the kinetic energy into potential energy (mgZ) until zero velocity is reached and it starts to fall down again. The collision with the floor is not perfectly elastic so the ball does not rise exactly up to the original height losing a little energy into internal energy (higher temperature due to internal friction) with every bounce and finally the motion will die out. All the energy eventually is lost by heat transfer to the ambient or sits in lasting deformation (internal energy) of the substance.

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3.c

Make a list of at least 5 systems that store energy, explaining which form of energy.

A spring that is compressed. Potential energy $(1/2) kx^2$

A battery that is charged. Electrical potential energy.   V Amp h

A raised mass (could be water pumped up higher) Potential energy mgH

A cylinder with compressed air. Potential (internal) energy like a spring.

A tank with hot water. Internal energy mu

A fly-wheel. Kinetic energy (rotation) $(1/2) I\omega^2$

A mass in motion. Kinetic energy $(1/2) mV^2$

3.d

A constant mass goes through a process where 100 J of heat transfer comes in and 100 J of work leaves. Does the mass change state?

Yes it does.

As work leaves a control mass its volume must go up, v increases

As heat transfer comes in an amount equal to the work out means u is constant if there are no changes in kinetic or potential energy.

3.e

The electric company charges the customers per kW-hour. What is that in SI units?

Solution:

The unit kW-hour is a rate multiplied with time. For the standard SI units the rate of energy is in W and the time is in seconds. The integration in Eq.3.4 and on page 135 becomes

$$1 \text{ kW-hour} = 1000 \text{ W} \times 60 \frac{\text{min}}{\text{hour}} \text{ hour} \times 60 \frac{\text{s}}{\text{min}} = 3\ 600\ 000 \text{ Ws}$$

$$= 3\ 600\ 000 \text{ J} = \textbf{3.6 MJ}$$

3.f

Torque and energy and work have the same units (N m). Explain the difference.

Solution:

Work = force $\times$ displacement, so units are N $\times$ m. Energy in transfer

Energy is stored, could be from work input   1 J = 1 N m

Torque = force $\times$ arm,   static, no displacement needed

3.g

What is roughly the relative magnitude of the work in the process 1-2c versus the process 1-2a shown in figure 3.15?

By visual inspection the area below the curve 1-2c is roughly 50% of the rectangular area below the curve 1-2a. To see this better draw a straight line from state 1 to point f on the axis. This curve has exactly 50% of the area below it.

3.h

Helium gas expands from 125 kPa, 350 K and 0.25 m^3 to 100 kPa in a polytropic process with n = 1.667. Is the work positive, negative or zero?

The boundary work is: $\qquad W = \int P \, dV$

P drops but does V go up or down?

The process equation is: $\qquad PV^n = C$

so we can solve for P to show it in a P-V diagram

$$P = CV^{-n}$$

as n = 1.667 the curve drops as V goes up we see

$\qquad V_2 > V_1 \quad$ giving $\quad dV > 0$

and the work is then positive.

3.i

An ideal gas goes through an expansion process where the volume doubles. Which process will lead to the larger work output: an isothermal process or a polytropic process with $n = 1.25$?

The process equation is: $PV^n = C$

The polytropic process with $n = 1.25$ drops the pressure faster than the isothermal process with $n = 1$ and the area below the curve is then smaller.

3.j

Water is heated from 100 kPa, 20°C to 1000 kPa, 200°C. In one case pressure is raised at T = C, then T is raised at P = C. In a second case the opposite order is done. Does that make a difference for $_1Q_2$ and $_1W_2$?

Yes it does. Both $_1Q_2$ and $_1W_2$ are process dependent. We can illustrate the work term in a P-v diagram.

In one case the process proceeds from 1 to state "a" along constant T then from "a" to state 2 along constant P.

The other case proceeds from 1 to state "b" along constant P and then from "b" to state 2 along constant T.

3.k

A rigid insulated tank A contains water at 400 kPa, 800°C. A pipe and valve connect this to another rigid insulated tank B of equal volume having saturated water vapor at 100 kPa. The valve is opened and stays open while the water in the two tanks comes to a uniform final state. Which two properties determine the final state?

Continuity eq.: $\quad m_2 - m_{1A} - m_{1B} = 0 \quad \Rightarrow \quad m_2 = m_{1A} + m_{1B}$

Energy eq.: $\qquad m_2 u_2 - m_{1A} u_{1A} - m_{1B} u_{1B} = 0 - 0$

Process: $\qquad$ Insulated: $_1Q_2 = 0$,

$\qquad\qquad$ Rigid: $\quad V_2 = C = V_A + V_B \quad \Rightarrow \qquad _1W_2 = 0$

From continuity eq. and process: $\quad v_2 = V_2/m_2 = \dfrac{m_{1A}}{m_2} v_{1A} + \dfrac{m_{1B}}{m_2} v_{1B}$

From energy eq.: $\qquad\qquad u_2 = \dfrac{m_{1A}}{m_2} u_{1A} + \dfrac{m_{1B}}{m_2} u_{1B}$

Final state 2: (v_2, u_2) both are the mass weighted average of the initial values.

3.l

To determine v or u for some liquid or solid, is it more important that I know P or T?

T is more important, v and u are nearly independent of P in the liquid and solid phases.

3.m

To determine v or u for an ideal gas, is it more important that I know P or T?

For v they are equally important ($v = RT/P$), but for u only T is important. For an ideal gas u is a function of T only (independent of P).

3.n

I heat 1 kg of substance at a constant pressure (200 kPa) 1 degree. How much heat is needed if the substance is water at $10^{o}C$, steel at $25^{o}C$, air at 325 K, or ice at $-10^{o}C$.

Heating at constant pressure gives (recall the analysis in Section 3.9, page 109)
$$_1Q_2 = H_2 - H_1 = m(h_2 - h_1) \approx m\, C_p\, (T_2 - T_1)$$

For all cases: $_1Q_2 = 1 \text{ kg} \times C \times 1 \text{ K}$

Water $10^{o}C$, 200 kPa (liquid) so A.4:   C = 4.18 kJ/kg-K, $_1Q_2 = 4.18$ kJ

Steel $25^{o}C$, 200 kPa (solid)   so A.3:   C = 0.46 kJ/kg-K $_1Q_2 = 0.46$ kJ

Air 325 K, 200 kPa (gas)   so A.5: $C_p = 1.004$ kJ/kg-K $_1Q_2 = 1.004$ kJ

Ice $-10^{o}C$, 200 kPa (solid)   so A.3:   C = 2.04 kJ/kg-K $_1Q_2 = 2.04$ kJ

Comment: For liquid water we could have interpolated $h_2 - h_1$ from Table B.1.1 and for ice we could have used Table B.1.5. For air we could have used Table A.7. If the temperature is very different from those given the tables will provide a more accurate answer.

Borgnakke and Sonntag

Concept Problems

3.1

What is 1 cal in SI units and what is the name given to 1 N-m?

Look in the conversion factor table A.1 under energy:

$$1 \text{ cal (Int.)} = 4.1868 \text{ J} = 4.1868 \text{ Nm} = 4.1868 \text{ kg m}^2/\text{s}^2$$

This was historically defined as the heat transfer needed to bring 1 g of liquid water from 14.5$^\text{o}$C to 15.5$^\text{o}$C, notice the value of the heat capacity of water in Table A.4

$$1 \text{ N-m} = 1 \text{ J} \quad \text{or} \quad \text{Force times displacement = energy in Joule}$$

3.2

A car engine is rated at 110 kW. What is the power in hp?

Solution:

The horsepower is an older unit for power usually used for car engines. The conversion to standard SI units is given in Table A.1

1 hp = 0.7355 kW = 735.5 W

1 hp = 0.7457 kW for the UK horsepower

110 kW = 110 kW / 0.7457 (kW/hp) = **147.5 hp**

3.3

Why do we write ΔE or $E_2 - E_1$ whereas we write $_1Q_2$ and $_1W_2$?

ΔE or $E_2 - E_1$ is the **change** in the stored energy from state 1 to state 2 and depends only on states 1 and 2 not upon the process between 1 and 2.

$_1Q_2$ and $_1W_2$ are amounts of energy **transferred during the process** between 1 and 2 and depend on the process path. The quantities are associated with the process and they are not state properties.

3.4

If a process in a control mass increases energy $E_2 - E_1 > 0$ can you say anything about the sign for $_1Q_2$ and $_1W_2$?

No.

The net balance of the heat transfer and work terms from the energy equation is

$$E_2 - E_1 = {_1Q_2} - {_1W_2} > 0$$

but that does not separate the effect of the two terms.

3.5

In Fig. P3.5, CV A is the mass inside a piston-cylinder, CV B is that plus the piston, outside which is the standard atmosphere. Write the energy equation and work term for the two CVs assuming we have a non-zero Q between state 1 and state 2.

CV A:
$$E_2 - E_1 = m_A(e_2 - e_1) = m_A(u_2 - u_1) = {}_1Q_2 - {}_1WA_2$$
$${}_1WA_2 = \int P \, dV = P(V_2 - V_1)$$

CV B:
$$E_2 - E_1 = m_A(e_2 - e_1) + m_{pist}(e_2 - e_1) = m_A(u_2 - u_1) + m_{pist}g(Z_2 - Z_1)$$
$$= {}_1Q_2 - {}_1WB_2$$
$${}_1WB_2 = \int P_o \, dV = P_o(V_2 - V_1)$$

Notice how the P inside CV A is $P = P_o + m_{pist}g/A_{cyl}$ i.e. the first work term is larger than the second. The difference between the work terms is exactly equal to the potential energy of the piston sitting on the left hand side in the CV B energy Eq. The two equations are mathematically identical.

$${}_1WA_2 = P(V_2 - V_1) = [P_o + m_{pist}g/A_{cyl}] (V_2 - V_1)$$
$$= {}_1WB_2 + m_{pist}g(V_2 - V_1)/A_{cyl}$$
$$= {}_1WB_2 + m_{pist}g(Z_2 - Z_1)$$

3.6

A 500 W electric space heater with a small fan inside heats air by blowing it over a hot electrical wire. For each control volume: a) wire only b) all the room air and c) total room plus the heater, specify the storage, work and heat transfer terms as

+ 500W or -500W or 0 W, neglect any $\dot{Q}$ through the room walls or windows.

	Storage	Work	Heat transfer
Wire	0 W	-500 W	-500 W
Room air	500 W	0 W	500 W
Tot room	500 W	-500 W	0 W

3.7

Two engines provide the same amount of work to lift a hoist. One engine can provide 3 F in a cable and the other 1 F, What can you say about the motion of the point where the force F acts in the two engines?

Since the two work terms are the same we get

$$W = \int F \, dx = 3 \, F \, x_1 = 1 \, F \, x_2$$

$$x_2 = 3 \, x_1$$

so the lower force has a larger displacement.

3.8

Two hydraulic piston/cylinders are connected through a hydraulic line so they have roughly the same pressure. If they have diameters of D_1 and $D_2 = 2D_1$ respectively, what can you say about the piston forces F_1 and F_2?

For each cylinder we have the total force as: $F = PA_{cyl} = P \pi D^2/4$

$$F_1 = PA_{cyl\ 1} = P \pi D_1^2/4$$

$$F_2 = PA_{cyl\ 2} = P \pi D_2^2/4 = P \pi 4 D_1^2/4 = 4 F_1$$

The forces are the total force acting up due to the cylinder pressure. There must be other forces on each piston to have a force balance so the pistons do not move.

3.9

Assume a physical set-up as in Fig. P3.5. We now heat the cylinder. What happens to P, T and v (up, down or constant)? What transfers do we have for Q and W (pos., neg., or zero)?

P_o

m_p

Solution:

Process: $P = P_o + m_p g/A_{cyl} = C$

Heat in so T increases, v increases and Q is positive.

As the volume increases the work is positive. $_1W_2 = \int P\, dV$

3.10

A drag force on an object moving through a medium (like a car through air or a submarine through water) is $F_d = 0.225 \, A \, \rho V^2$. Verify the unit becomes Newton.

Solution:

$$F_d = 0.225 \, A \, \rho V^2$$

$$\text{Units} = m^2 \times (\text{kg/m}^3) \times (m^2/s^2) = \text{kg m} / s^2 = N$$

3.11

The sketch shows three physical situations, show the possible process in a P-v diagram.

a)   b)   c)

P$_O$

m$_p$

R-

P

P$_1$

1

V

V$_{stop}$

P

P$_1$

1

V

V$_1$

P

P$_1$

1

V

V$_1$ V$_{stop}$

3.12

For the indicated physical set-up in a-b and c above write a process equation and the expression for work.

a) $P = P_1$ and $V \geq V_{stop}$ or $V = V_{stop}$ and $P \leq P_1$

$_1W_2 = P_1(V_2 - V_1)$ [$P_1 = P_{float}$]

b) $P = A + BV;$ $_1W_2 = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$

c) $P = P_1$ and $V \leq V_{stop}$ or $V = V_{stop}$ and $P \geq P_1$

$_1W_2 = P_1(V_2 - V_1)$ [$P_1 = P_{float}$]

a)   b)   c)

3.13

Assume the physical situation as in Fig. P3.11b; what is the work term a, b, c or d?

a: $_1w_2 = P_1(v_2 - v_1)$ b: $_1w_2 = v_1(P_2 - P_1)$

c: $_1w_2 = \frac{1}{2}(P_1 + P_2)(v_2 - v_1)$ d: $_1w_2 = \frac{1}{2}(P_1 - P_2)(v_2 + v_1)$

Solution:

work term is formula c, the area under the process curve in a P-v diagram.

The avg. height is $\frac{1}{2}(P_1 + P_2)$

The base is $(v_2 - v_1)$

3.14

The sketch in Fig. P3.14 shows a physical situation; show the possible process in a P-v diagram.

a)   b)   c)

P_o

m_p

R-

Solution:

3.15

What can you say about the beginning state of the R-410A in Fig. P3.11 versus the case in Fig. P3.14 for the same piston-cylinder?

For the case where the piston floats as in Fig. P3.11 the pressure of the R-410A must equal the equilibrium pressure that floats (balance forces on) the piston.

The situation in Fig. P3.14 is possible if the R-410A pressure equals or exceeds the float pressure.

3.16

A piece of steel has a conductivity of k = 15 W/mK and a brick has k = 1 W/mK. How thick a steel wall will provide the same insulation as a 10 cm thick brick?

The heat transfer due to conduction is from Eq. 3.23

$$\dot{Q} = -\,kA\,\frac{dT}{dx} \approx kA\,\frac{\Delta T}{\Delta x}$$

For the same area and temperature difference the heat transfers become the same for equal values of $(k\,/\,\Delta x)$ so

$$(\frac{k}{\Delta x})_{\text{brick}} = (\frac{k}{\Delta x})_{\text{steel}} \qquad =>$$

$$\Delta x_{\text{steel}} = \Delta x_{\text{brick}}\frac{k_{\text{steel}}}{k_{\text{brick}}} = 0.1 \text{ m} \times \frac{15}{1} = \textbf{1.5 m}$$

3.17

A thermopane window, see Fig. 3.38, traps some gas between the two glass panes. Why is this beneficial?

The gas has a very low conductivity relative to a liquid or solid so the heat transfer for a given thickness becomes smaller. The gap is furthermore made so small that possible natural convection motion is reduced to a minimum. It becomes a trade off to minimize the overall heat transfer due to conduction and convection. Typically these windows can be manufactured with an E-glaze to reduce radiation loss (winter) or gain (summer).

multiple glazings
low-E coating

Gas filled space
Spacer and sealer
Window structural frame

3.18

On a chilly 10°C fall day a house, 20°C inside, loses 6 kW by heat transfer. What transfer happens on a 30°C warm summer day assuming everything else is the same?

The heat transfer is $\dot{Q} = CA\ \Delta T$ where the details of the heat transfer is in the factor C. Assuming those details are the same then it is the temperature difference that changes the heat transfer so

$$\dot{Q} = CA\ \Delta T = 6\ kW\ = CA\ (20 - 10)\ K\ \Rightarrow\ CA = 0.6\ \frac{kW}{K}$$

Then

$$\dot{Q} = CA\ \Delta T =\ 0.6\ \frac{kW}{K} \times (20 - 30)\ K = -\ 6\ kW\ \text{ (it goes in)}$$

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3.19

Verify that a surface tension S with units N/m also can be called a *surface energy* with units J/m^2. The latter is useful for consideration of a liquid drop or liquid in small pores (capillary).

Units: $$N/m = Nm/m^2 = J/m^2$$

This is like a potential energy associated with the surface. For water in small pores it tends to keep the water in the pores rather than in a drop on the surface.

3.20

Some liquid water is heated so it becomes superheated vapor. Do I use u or h in the energy equation? Explain.

The energy equation for a control mass is: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

The storage of energy is a change in u (when we neglect kinetic and potential energy changes) and that is always so. To solve for the heat transfer we must know the work in the process and it is for a certain process (P = C) that the work term combines with the change in u to give a change in h. To avoid confusion you should always write the energy equation as shown above and substitute the appropriate expression for the work term when you know the process equation that allows you to evaluate work.

3.21

Some liquid water is heated so it becomes superheated vapor. Can I use specific heat to find the heat transfer? Explain.

NO.

The specific heat cannot give any information about the energy required to do the phase change. The specific heat is useful for single phase state changes only.

3.22

Look at the R-410A value for u_f at -50$^{\text{o}}$C. Can the energy really be negative? Explain.

The absolute value of *u* and *h* are arbitrary. A constant can be added to all *u* and *h* values and the table is still valid. It is customary to select the reference such that *u* for saturated liquid water at the triple point is zero. The standard for refrigerants like R-410A is that *h* is set to zero as saturated liquid at -40$^{\text{o}}$C, other substances as cryogenic substances like nitrogen, methane etc. may have different states at which *h* is set to zero. The ideal gas tables use a zero point for *h* as 25$^{\text{o}}$C or at absolute zero, 0 K.

3.23

A rigid tank with pressurized air is used to a) increase the volume of a linear spring loaded piston cylinder (cylindrical geometry) arrangement and b) to blow up a spherical balloon. Assume that in both cases $P = A + BV$ with the same A and B. What is the expression for the work term in each situation?

The expression is exactly the same; the geometry does not matter as long as we have the same relation between P and V then

$$_1W_2 = \int P \, dV = \int (A + BV) \, dV$$
$$= A(V_2 - V_1) + 0.5 \, B \, (V_2^2 - V_1^2)$$
$$= A(V_2 - V_1) + 0.5 \, B \, (V_2 + V_1) \, (V_2 - V_1)$$
$$= 0.5 \, [\, A + B \, V_2 + A + B \, V_1] \, (V_2 - V_1)$$
$$= 0.5 \, (P_1 + P_2) \, (V_2 - V_1)$$

Notice the last expression directly gives the area below the curve in the P-V diagram.

3.24

An ideal gas in a piston-cylinder is heated with 2 kJ during an isothermal process. How much work is involved?

Energy Eq.: $u_2 - u_1 = {_1}q_2 - {_1}w_2 = 0$ since $u_2 = u_1$ (isothermal)

Then

$$_1W_2 = m \; _1w_2 = {_1}Q_2 = m \; _1q_2 = \textbf{2 kJ}$$

3.25

An ideal gas in a piston-cylinder is heated with 2 kJ during an isobaric process. Is the work pos., neg., or zero?

As the gas is heated u and T increase and since PV = mRT it follows that the volume increase and thus work goes out.

w > 0

3.26

You heat a gas 10 K at P = C. Which one in Table A.5 requires most energy? Why?

A constant pressure process in a control mass gives (recall Section 3.9 and Eq.3.44)

$$_1q_2 = u_2 - u_1 + {}_1w_2 = u_2 - u_1 + P_1(v_2 - v_1)$$
$$= h_2 - h_1 \approx C_p \, \Delta T$$

The one with the highest specific heat is hydrogen, H_2. The hydrogen has the smallest mass, but the same kinetic energy per mol as other molecules and thus the most energy per unit mass is needed to increase the temperature.

3.27

You mix 20°C water with 50°C water in an open container. What do you need to know to determine the final temperature?

The process will take place at constant pressure (atmospheric) and you can assume there will be minimal heat transfer if the process is reasonably fast. The energy equation then becomes

$$U_2 - U_1 = 0 - {}_1W_2 = -P(V_2 - V_1)$$

Which we can write as

$$H_2 - H_1 = 0 = m_2 h_2 - (m_{1\ 20C} h_{1\ 20C} + m_{1\ 50C} h_{1\ 50C})$$

You need the amount of mass at each temperature $m_{1\ 20C}$ and $m_{1\ 50C}$.

Kinetic and Potential Energy

3.28

A piston motion moves a 25 kg hammerhead vertically down 1 m from rest to a velocity of 50 m/s in a stamping machine. What is the change in total energy of the hammerhead?

Solution: C.V. Hammerhead

The hammerhead does not change internal energy (i.e. same P, T), but it does have a change in kinetic and potential energy.

$$E_2 - E_1 = m(u_2 - u_1) + m[(1/2)V_2{}^2 - 0] + mg\,(Z_2 - 0)$$

$$= 0 + 25 \text{ kg} \times (1/2) \times (50 \text{ m/s})^2 + 25 \text{ kg} \times 9.80665 \text{ m/s}^2 \times (-1) \text{ m}$$

$$= 31\ 250 \text{ J} - 245.17 \text{ J} = 31\ 005 \text{ J} = \mathbf{31\ kJ}$$

3.29

A 1200 kg car is accelerated from 30 to 50 km/h in 5 s. How much work is that? If you continue from 50 to 70 km/h in 5 s; is that the same?

The work input is the increase in kinetic energy.

$$E_2 - E_1 = (1/2)m\left[\mathbf{V}_2^2 - \mathbf{V}_1^2\right] = \,_1W_2$$

$$= 0.5 \times 1200 \text{ kg } [50^2 - 30^2]\left(\frac{\text{km}}{\text{h}}\right)^2$$

$$= 600 \text{ kg} \times [\, 2500 - 900 \,]\left(\frac{1000 \text{ m}}{3600 \text{ s}}\right)^2 = 74\ 074 \text{ J} = \mathbf{74.1 \text{ kJ}}$$

The second set of conditions does not become the same

$$E_2 - E_1 = (1/2)m\left[\mathbf{V}_2^2 - \mathbf{V}_1^2\right] = 600 \text{ kg} \times [\, 70^2 - 50^2 \,]\left(\frac{1000 \text{ m}}{3600 \text{ s}}\right)^2 = \mathbf{111 \text{ kJ}}$$

3.30

The rolling resistance of a car depends on its weight as: F = 0.006 mg. How far will a car of 1200 kg roll if the gear is put in neutral when it drives at 90 km/h on a level road without air resistance?

Solution:

The car decreases its kinetic energy to zero due to the force (constant) acting over the distance.

$$m\,(1/2V_2^2 - 1/2V_1^2) = -_1W_2 = -\int F\,dx\ = -FL$$

$$V_2 = 0, \quad V_1 = 90\,\frac{km}{h} = \frac{90 \times 1000}{3600}\,ms^{-1} = 25\,ms^{-1}$$

$$-1/2\,mV_1^2 = -FL = -\,0.006\,mgL$$

$$\Rightarrow \qquad L = \frac{0.5\,V_1^2}{0.006\,g} = \frac{0.5 \times 25^2}{0.006 \times 9.807}\,\frac{m^2/s^2}{m/s^2} = 5311\,m$$

Remark: Over 5 km!   The air resistance is much higher than the rolling resistance so this is not a realistic number by itself.

3.31

A piston of mass 2 kg is lowered 0.5 m in the standard gravitational field. Find the required force and work involved in the process.

Solution:

$$F = ma = 2 \text{ kg} \times 9.80665 \text{ m/s}^2 = \textbf{19.61 N}$$

$$W = \int F \, dx = F \int dx = F \, \Delta x = 19.61 \text{ N} \times 0.5 \text{ m} = \textbf{9.805 J}$$

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3.32

A 1200 kg car accelerates from zero to 100 km/h over a distance of 400 m. The road at the end of the 400 m is at 10 m higher elevation. What is the total increase in the car kinetic and potential energy?

Solution:

$$\Delta KE = \tfrac{1}{2}\, m\, (\mathbf{V}_2^2 - \mathbf{V}_1^2)$$

$$\mathbf{V}_2 = 100 \text{ km/h} = \frac{100 \times 1000}{3600} \text{ m/s}$$

$$= 27.78 \text{ m/s}$$

$$\Delta KE = \tfrac{1}{2} \times 1200 \text{ kg} \times (27.78^2 - 0^2)\ (\text{m/s})^2 = 463\ 037\ \text{J} = \mathbf{463\ kJ}$$

$$\Delta PE = mg(Z_2 - Z_1) = 1200 \text{ kg} \times 9.807 \text{ m/s}^2\ (\ 10 - 0\ )\ \text{m} = 117684\ \text{J}$$
$$= \mathbf{117.7\ kJ}$$

3.33

A hydraulic hoist raises a 1750 kg car 1.8 m in an auto repair shop. The hydraulic pump has a constant pressure of 800 kPa on its piston. What is the increase in potential energy of the car and how much volume should the pump displace to deliver that amount of work?

Solution: C.V. Car.

No change in kinetic or internal energy of the car, neglect hoist mass.

$$E_2 - E_1 = PE_2 - PE_1 = mg\,(Z_2 - Z_1)$$

$$= 1750 \text{ kg} \times 9.80665 \text{ m/s}^2 \times 1.8 \text{ m} = \textbf{30 891 J}$$

The increase in potential energy is work into car from pump at constant P.

$$W = E_2 - E_1 = \int P\,dV = P\,\Delta V \qquad \Rightarrow$$

$$\Delta V = \frac{E_2 - E_1}{P} = \frac{30891 \text{ J}}{800 \times 1000 \text{ Pa}} = \textbf{0.0386 m}^3$$

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3.34

Airplane takeoff from an aircraft carrier is assisted by a steam driven piston/cylinder device with an average pressure of 1250 kPa. A 17500 kg airplane should be accelerated from zero to a speed of 30 m/s with 30% of the energy coming from the steam piston. Find the needed piston displacement volume.

Solution: C.V. Airplane.

No change in internal or potential energy; only kinetic energy is changed.

$$E_2 - E_1 = m (1/2) (\mathbf{V}_2^2 - 0) = 17500 \text{ kg} \times (1/2) \times 30^2 \text{ (m/s)}^2$$

$$= 7\,875\,000 \text{ J} = 7875 \text{ kJ}$$

The work supplied by the piston is 30% of the energy increase.

$$W = \int P \, dV = P_{avg} \, \Delta V = 0.30 \, (E_2 - E_1)$$

$$= 0.30 \times 7875 \text{ kJ} = 2362.5 \text{ kJ}$$

$$\Delta V = \frac{W}{P_{avg}} = \frac{2362.5}{1250} \frac{\text{kJ}}{\text{kPa}} = \mathbf{1.89 \ m^3}$$

3.35

Solve Problem 3.34, but assume the steam pressure in the cylinder starts at 1000 kPa, dropping linearly with volume to reach 100 kPa at the end of the process.

Solution: C.V. Airplane.

$$E_2 - E_1 = m \, (1/2) \, (V_2^2 - 0)$$

$$= 17\,500 \text{ kg} \times (1/2) \times 30^2 \, (\text{m/s})^2$$

$$= 7875\,000 \text{ J} = 7875 \text{ kJ}$$

$$W = 0.30(E_2 - E_1) = 0.30 \times 7875 = 2362.5 \text{ kJ}$$

$$W = \int P \, dV = (1/2)(P_{beg} + P_{end}) \, \Delta V$$

$$\Delta V = \frac{W}{P_{avg}} = \frac{2362.5 \text{ kJ}}{1/2(1000 + 100) \text{ kPa}} = \mathbf{4.29 \text{ m}^3}$$

3.36

A steel ball weighing 5 kg rolls horizontal with 10 m/s. If it rolls up an incline how high up will it be when it comes to rest assuming standard gravitation.

C.V. Steel ball.

Energy Eq.: $E_2 - E_1 = {}_1Q_2 - {}_1W_2 = 0 - 0 = 0$

$$E_1 = mu_1 + mgZ_1 + 0.5\ mV^2$$
$$E_2 = mu_2 + mgZ_2 + 0$$

We assume the steel ball does not change temperature ($u_2 = u_1$) so then the energy equation gives

$$mu_2 + mgZ_2 - mu_1 - mgZ_1 - 0.5\ mV^2 = 0$$
$$mg\ (Z_2 - Z_1) = 0.5\ mV^2$$
$$Z_2 - Z_1 = 0.5\ V^2/g = 0.5 \times 10^2\ (m^2/s^2)\ /\ (9.81\ m/s^2) = \mathbf{5.1\ m}$$

3.37

A hydraulic cylinder of area 0.01 m^2 must push a 1000 kg arm and shovel 0.5 m straight up. What pressure is needed and how much work is done?

$F = mg = 1000 \text{ kg} \times 9.81 \text{ m/s}^2$

$\quad = 9810 \text{ N} = PA$

$P = F/A = 9810 \text{ N}/ 0.01 \text{ m}^2$

$\quad = 981\,000 \text{ Pa} = \textbf{981 kPa}$

$W = \int F \, dx = F \, \Delta x = 9810 \text{ N} \times 0.5 \text{ m} = \textbf{4905 J}$

3.38

A hydraulic cylinder has a piston of cross sectional area 10 cm^2 and a fluid pressure of 2 MPa. If the piston is moved 0.25 m how much work is done?

Solution:

The work is a force with a displacement and force is constant: F = PA

$$W = \int F \, dx \ = \ \int PA \, dx = PA \, \Delta x$$

$$= 2000 \text{ kPa} \times 10 \times 10^{-4} \text{ m}^2 \times 0.25 \text{ m} \ = \textbf{0.5 kJ}$$

Units: kPa m^2 m = kN m^{-2} m^2 m = kN m = kJ

3.39

Two hydraulic piston/cylinders are connected with a line. The master cylinder has an area of 5 cm^2 creating a pressure of 1000 kPa. The slave cylinder has an area of 3 cm^2. If 25 J is the work input to the master cylinder what is the force and displacement of each piston and the work output of the slave cylinder piston?

Solution:

$$W = \int F_x \, dx = \int P \, dv = \int P A \, dx = P A \Delta x$$

$$\Delta x_{master} = \frac{W}{PA} = \frac{25}{1000 \times 5 \times 10^{-4}} \frac{J}{kPa\ m^2} = 0.05 \text{ m}$$

$$A\Delta x = \Delta V = 5 \times 10^{-4} \times 0.05 = 2.5 \times 10^{-5} \text{ m}^3 = \Delta V_{slave} = A \, \Delta x \rightarrow$$

$$\Delta x_{slave} = \Delta V/A = 2.5 \times 10^{-5} \text{ m}^3 / \, 3 \times 10^{-4} \text{ m}^2 = 0.0083\ 33 \text{ m}$$

$$F_{master} = P\ A = 1000 \text{ kPa} \times 5 \times 10^{-4} \text{ m}^2 \times 10^3 \text{ Pa/kPa} = \textbf{500 N}$$

$$F_{slave} = P\ A = 1000 \text{ kPa} \times 10^3 \text{ Pa/kPa} \times 3 \times 10^{-4} \text{ m}^2 = \textbf{300 N}$$

$$W_{slave} = F\ \Delta x = 300 \text{ N} \times 0.08333 \text{ m} = \textbf{25 J}$$

Master   Slave

3.40

The air drag force on a car is 0.225 A ρV^2. Assume air at 290 K, 100 kPa and a car frontal area of 4 m^2 driving at 90 km/h. How much energy is used to overcome the air drag driving for 30 minutes?

The formula involves density and velocity and work involves distance so:

$$\rho = \frac{1}{v} = \frac{P}{RT} = \frac{100}{0.287 \times 290} = 1.2015 \frac{kg}{m^3}$$

$$V = 90 \frac{km}{h} = 90 \times \frac{1000}{3600} \frac{m}{s} = 25 \text{ m/s}$$

$$\Delta x = V \Delta t = 25 \text{ m/s} \times 30 \text{ min} \times 60 \text{ s/min} = 45\ 000 \text{ m}$$

Now

$$F = 0.225 \text{ A } \rho V^2 = 0.225 \times 4 \text{ m}^2 \times 1.2015 \frac{kg}{m^3} \times 25^2 \frac{m^2}{s^2}$$

$$= 675.8 \text{ m}^2 \frac{kg}{m^3} \times \frac{m^2}{s^2} = \mathbf{676 \text{ N}}$$

$$W = F \Delta x = 676 \text{ N} \times 45\ 000 \text{ m} = 30\ 420\ 000 \text{ J} = \mathbf{30.42 \text{ MJ}}$$

3.41

A bulldozer pushes 800 kg of dirt 100 m with a force of 1500 N. It then lifts the dirt 3 m up to put it in a dump truck. How much work did it do in each situation?

Solution:

$$W = \int F\, dx = F\,\Delta x$$
$$= 1500\ N \times 100\ m$$
$$= 150\ 000\ J = \mathbf{150\ kJ}$$

$$W = \int F\, dz = \int mg\, dz = mg\,\Delta Z$$
$$= 800\ kg \times\ 9.807\ m/s^2 \times 3\ m$$
$$= 23\ 537\ J = \mathbf{23.5\ kJ}$$

3.42

Two hydraulic cylinders maintain a pressure of 1200 kPa. One has a cross sectional area of 0.01 m^2 the other 0.03 m^2. To deliver a work of 1 kJ to the piston how large a displacement (V) and piston motion H is needed for each cylinder? Neglect P_{atm}.

Solution:

$$W = \int F \, dx = \int P \, dV = \int PA \, dx = PA * H = P\Delta V$$

$$\Delta V = \frac{W}{P} = \frac{1 \text{ kJ}}{1200 \text{ kPa}} = \textbf{0.000 833 m}^3$$

Both cases the height is   H = ΔV/A

$$H_1 = \frac{0.000833}{0.01} \text{ m} = \textbf{0.0833 m}$$

$$H_2 = \frac{0.000833}{0.03} \text{ m} = \textbf{0.0278 m}$$

3.43

A linear spring, $F = k_s(x - x_0)$, with spring constant $k_s = 500$ N/m, is stretched until it is 100 mm longer. Find the required force and work input.

Solution:

$$F = k_s(x - x_0) = 500 \times 0.1 = \textbf{50 N}$$

$$W = \int F \, dx = \int k_s(x - x_0) d(x - x_0) = k_s(x - x_0)^2/2$$

$$= 500 \, \frac{N}{m} \times (0.1^2/2) \, m^2 = \textbf{2.5 J}$$

3.44

A piston of 2 kg is accelerated to 20 m/s from rest. What constant gas pressure is required if the area is 10 cm^2, the travel 10 cm and the outside pressure is 100 kPa?

C.V. Piston

$$(E_2 - E_1)_{PIST.} = m(u_2 - u_1) + m[(1/2)V_2^2 - 0] + mg\,(0 - 0)$$

$$= (1/2)\,m\,V_2^2 = 0.5 \times 2\ kg \times 20^2\ (m/s)^2 = 400\ J$$

Energy equation for the piston is:

$$(E_2 - E_1)_{PIST.} = W_{gas} - W_{atm} = P_{avg}\ \Delta V_{gas} - P_o\ \Delta V_{gas}$$

$$\Delta V_{gas} = A\ L = 10\ cm^2 \times 10\ cm = 0.0001\ m^3$$

$$P_{avg}\ \Delta V_{gas} = (E_2 - E_1)_{PIST.} + P_o\ \Delta V_{gas}$$

$$P_{avg} = (E_2 - E_1)_{PIST.}\ /\ \Delta V_{gas} + P_o$$

$$= 400\ J\ /\ 0.0001\ m^3 + 100\ kPa$$

$$= 4000\ kPa + 100\ kPa = \textbf{4100 kPa}$$

Boundary work

3.45

A 25 kg piston is above a gas in a long vertical cylinder. Now the piston is released from rest and accelerates up in the cylinder reaching the end 5 m higher at a velocity of 25 m/s. The gas pressure drops during the process so the average is 600 kPa with an outside atmosphere at 100 kPa. Neglect the change in gas kinetic and potential energy, and find the needed change in the gas volume.

Solution:

C.V. Piston

$$(E_2 - E_1)_{PIST.} = m(u_2 - u_1) + m[(1/2)V_2^2 - 0] + mg\ (H_2 - 0)$$

$$= 0 + 25\ kg \times (1/2) \times 25^2 (m/s)^2 + 25\ kg \times 9.80665\ m/s^2 \times 5\ m$$

$$= 7812.5\ J + 1225.8\ J = 9038.3\ J = 9.038\ kJ$$

Energy equation for the piston is:

$$E_2 - E_1\ = W_{gas} - W_{atm} = P_{avg}\ \Delta V_{gas} - P_o\ \Delta V_{gas}$$

(remark $\Delta V_{atm} = - \Delta V_{gas}$ so the two work terms are of opposite sign)

$$\Delta V_{gas} = \frac{9.038}{600 - 100}\ \frac{kJ}{kPa} = \textbf{0.018 m}^{\textbf{3}}$$

3.46

The R-410A in Problem 3.14 c is at 1000 kPa, 50°C with mass 0.1 kg. It is cooled so the volume is reduced to half the initial volume . The piston mass and gravitation is such that a pressure of 400 kPa will float the piston. Find the work in the process.

If the volume is reduced the piston must drop and thus float with P = 400 kPa. The process therefore follows a process curve shown in the P-V diagram.

Table B.4.2: $v_1 = 0.03320$ m^3/kg

$$_1W_2 = \int PdV = \text{area}$$

$$= P_{float}\,(V_2 - V_1) = -P_{float}\,V_1/2$$

$$= -400 \text{ kPa} \times 0.1 \text{ kg} \times 0.0332 \text{ m}^3/\text{kg} /2$$

$$= \mathbf{-\,0.664 \text{ kJ}}$$

3.47

A 400-L tank A, see figure P3.47, contains argon gas at 250 kPa, 30°C. Cylinder B, having a frictionless piston of such mass that a pressure of 150 kPa will float it, is initially empty. The valve is opened and argon flows into B and eventually reaches a uniform state of 150 kPa, 30°C throughout. What is the work done by the argon?

Solution:

Take C.V. as all the argon in both A and B. Boundary movement work done in cylinder B against constant external pressure of 150 kPa. Argon is an ideal gas, so write out that the mass and temperature at state 1 and 2 are the same

$$P_{A1}V_A = m_ART_{A1} = m_ART_2 = P_2(V_A + V_{B2})$$

$$\Rightarrow \quad V_{B2} = (P_{A1}/P_2)V_A - V_A = \frac{250 \times 0.4}{150} - 0.4 = 0.2667 \text{ m}^3$$

$$_1W_2 = \int_1^2 P_{ext}dV = P_{ext}(V_{B2} - V_{B1}) = 150 \text{ kPa} \times (0.2667 - 0) \text{ m}^3 = \textbf{40 kJ}$$

Notice there is a pressure loss in the valve so the pressure in B is always 150 kPa while the piston floats.

3.48

A piston cylinder contains 2 kg of liquid water at 20°C and 300 kPa, as shown in Fig. P3.48. There is a linear spring mounted on the piston such that when the water is heated the pressure reaches 3 MPa with a volume of 0.1 m^3.

  a) Find the final temperature

  b) Plot the process in a P-v diagram.

  c) Find the work in the process.

Solution:

Take CV as the water. This is a constant mass:

$$m_2 = m_1 = m \ ;$$

State 1: Compressed liquid, take saturated liquid at same temperature.

  B.1.1: $v_1 = v_f(20) = 0.001002$ m^3/kg,

State 2: $v_2 = V_2/m = 0.1/2 = 0.05$ m^3/kg and P = 3000 kPa from B.1.2

  => Two-phase: $T_2 =$ **233.9^{o}C**

Work is done while piston moves at linearly varying pressure, so we get:

$$_1W_2 = \int P \ dV = \text{area} = P_{avg} \ (V_2 - V_1) = \tfrac{1}{2} \ (P_1 + P_2)(V_2 - V_1)$$

$$= 0.5 \ (300 + 3000) \ \text{kPa} \times (0.1 - 0.002) \ \text{m}^3 = \textbf{161.7 kJ}$$

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Computer-Aided Thermodynamic Tables 3

File   Edit   Tables/Substances   Options   Help

P-V Diagram (Log-Log)

Water Properties

T	20	C
P	0.3	MPa
V	0.001002	m3/kg
U	83.92	kJ/kg
H	84.22	kJ/kg
S	0.2965	kJ/kg/K
X		

#	Temp	Pressure	Specific Volume	Internal Energy	Specific Enthalpy	Specific Entropy	Quality
	C	MPa	m3/kg	kJ/kg	kJ/kg	kJ/kg/K	
1	20	0.3	0.001002	83.92	84.22	0.2965	
2	20	0.3	0.001002	83.92	84.22	0.2965	
3	233.9	3	0.05	2197	2347	5.285	0.7453

Water / Refrigerants / Cryogenics / Air / Ideal Gases / Compressibility / Psychrometrics /

The process shown by CATT3 in a log-log diagram.

The linear relation $P = A + Bv$,   has $B = 55.1$ MPa/(m^3/kg)

3.49

Air in a spring loaded piston/cylinder has a pressure that is linear with volume, *P = A + BV*. With an initial state of P = 150 kPa, V = 1 L and a final state of 800 kPa and volume 1.5 L it is similar to the setup in Problem 3.48. Find the work done by the air.

Solution:

Knowing the process equation: P = A + BV   giving a linear variation of pressure versus volume the straight line in the P-V diagram is fixed by the two points as state 1 and state 2. The work as the integral of   PdV   equals the area under the process curve in the P-V diagram.

State 1: $P_1 = 150$ kPa $V_1 = 1$ L $= 0.001$ m^3

State 2: $P_2 = 800$ kPa $V_2 = 1.5$ L $= 0.0015$ m^3

Process: P = A + BV   linear in V

$$\Rightarrow \quad {}_1W_2 = \int_1^2 PdV = \left(\frac{P_1 + P_2}{2}\right)(V_2 - V_1)$$

$$= \frac{1}{2}(150 + 800) \text{ kPa} \times (1.5 - 1) \text{ L} \times 0.001 \text{ m}^3/\text{L}$$

$$= \textbf{0.2375 kJ}$$

3.50

Heat transfer to a block of 1.5 kg ice at -10°C melts it to liquid at 10°C in a kitchen. How much work does the water gives out?

Work is done against the atmosphere due to volume change in the process. The pressure is 101 kPa so we approximate the states as saturated

State 1: Compressed solid, B.1.5, $v_1 = 0.0010891$ m^3/kg

State 2: Compressed liquid B.1.1 $v_2 = 0.001000$ m^3/kg

$$_1W_2 = \int PdV = P_o (V_2 - V_1) = P_o\, m\, (v_2 - v_1)$$

$$= 101.325 \text{ kPa} \times 1.5 \text{ kg} \times (0.001 - 0.0010891) \text{ m}^3/\text{kg}$$

$$= -\, \mathbf{0.0135 \; kJ}$$

Notice the work is negative, the volume is reduced!

3.51

A cylinder fitted with a frictionless piston contains 5 kg of superheated refrigerant R-134a vapor at 1000 kPa, 140°C. The setup is cooled at constant pressure until the R-134a reaches a quality of 25%. Calculate the work done in the process.

Solution:

Constant pressure process boundary work. State properties from Table B.5.2

State 1: $v = 0.03150 \ m^3/kg$,

State 2: $v = 0.000871 + 0.25 \times 0.01956 = 0.00576 \ m^3/kg$

Interpolated to be at 1000 kPa, numbers at 1017 kPa could have

been used in which case: $v = 0.00566 \ m^3/kg$

$$_1W_2 = \int P \ dV = P \ (V_2 - V_1) = mP \ (v_2 - v_1)$$

$$= 5 \ kg \times 1000 \ kPa \times (0.00576 - 0.03150) \ m^3/kg = \textbf{-128.7 kJ}$$

Borgnakke and Sonntag

3.52

A piston/cylinder contains 2 kg water at 20$^{\text{o}}$C with volume 0.1 m^3. By mistake someone locks the piston preventing it from moving while we heat the water to saturated vapor. Find the final temperature, volume and the process work.

Solution

     1: v_1 = V/m = 0.1 m^3/2 kg = 0.05 m^3/kg (two-phase state)

     2: Constant volume: $v_2 = v_g = v_1$

$$V_2 = V_1 = \mathbf{0.1\ m^3}$$

$$_1W_2 = \int P\ dV = \mathbf{0}$$

State 2: $(v_2, x_2 = 1)$

$$T_2 = T_{sat} = 250 + 5\,\frac{0.05 - 0.05013}{0.04598 - 0.05013} = \mathbf{250.2°C}$$

Polytropic process

3.53

A nitrogen gas goes through a polytropic process with n = 1.3 in a piston/cylinder. It starts out at 600 K, 600 kPa and ends at 800 K. Is the work positive, negative or zero?

The work is a boundary work so it is

$$W = \int PdV = \int Pm \, dv = AREA$$

so the sign depends on the sign for dV (or dv). The process looks like the following

The actual
process is on a
steeper curve
than n = 1.

$P = C \, v^{-1.3}$

$T = C \, v^{-0.3}$

As the temperature increases we notice the volume decreases so

$$dv < 0 \quad \Rightarrow \quad W < 0$$

Work is **negative** and goes into the nitrogen gas.

3.54

Helium gas expands from 125 kPa, 350 K and 0.25 m^3 to 100 kPa in a polytropic process with n = 1.667. How much work does it give out?

Solution:

Process equation: $PV^n = \text{constant} = P_1V_1^n = P_2V_2^n$

Solve for the volume at state 2

$$V_2 = V_1\,(P_1/P_2)^{1/n} = 0.25 \times \left(\frac{125}{100}\right)^{0.6} = 0.2852 \text{ m}^3$$

Work from Eq.3.21

$$_1W_2 = \frac{P_2V_2 - P_1\,V_1}{1-n} = \frac{100\times 0.2852 - 125\times 0.25}{1 - 1.667} \text{ kPa m}^3 = \textbf{4.09 kJ}$$

The actual process is on a steeper curve than n = 1.

3.55

Air goes through a polytropic process from 125 kPa, 325 K to 300 kPa and 500 K. Find the polytropic exponent n and the specific work in the process.

Solution:

Process: $Pv^n = \text{Const} = P_1 v_1^n = P_2 v_2^n$

Ideal gas $Pv = RT$ so

$$v_1 = \frac{RT}{P} = \frac{0.287 \times 325}{125} = 0.7462 \text{ m}^3/\text{kg}$$

$$v_2 = \frac{RT}{P} = \frac{0.287 \times 500}{300} = 0.47833 \text{ m}^3/\text{kg}$$

From the process equation

$$(P_2/ P_1) = (v_1/ v_2)^n \quad => \quad \ln(P_2/ P_1) = n \ln(v_1/ v_2)$$

$$n = \ln(P_2/ P_1) \big/ \ln(v_1/ v_2) = \frac{\ln 2.4}{\ln 1.56} = \mathbf{1.969}$$

The work is now from Eq.3.21 per unit mass and ideal gas law

$$_1w_2 = \frac{P_2 v_2 - P_1 v_1}{1\text{-}n} = \frac{R(T_2 - T_1)}{1\text{-}n} = \frac{0.287(500 - 325)}{1 - 1.969} \text{ (kJ/kg-K) K}$$

$$= \mathbf{-51.8 \text{ kJ/kg}}$$

3.56

A balloon behaves so the pressure is $P = C_2 \, V^{1/3}$, $C_2 = 100$ kPa/m. The balloon is blown up with air from a starting volume of 1 m^3 to a volume of 4 m^3. Find the final mass of air assuming it is at 25°C and the work done by the air.

Solution:

The process is polytropic with exponent $n = -1/3$.

$$P_1 = C_2 \, V^{1/3} = 100 \times 1^{1/3} = 100 \text{ kPa}$$

$$P_2 = C_2 \, V^{1/3} = 100 \times 4^{1/3} = 158.74 \text{ kPa}$$

$$_1W_2 = \int P \, dV \; = \frac{P_2 V_2 - P_1 V_1}{1 - n} \qquad \text{(Equation 3.18 and 3.21)}$$

$$= \frac{158.74 \times 4 - 100 \times 1}{1 - (-1/3)} \text{ kPa-m}^3 = \mathbf{401.2 \text{ kJ}}$$

$$m_2 = \frac{P_2 V_2}{R T_2} = \frac{158.74 \times 4}{0.287 \times 298} \frac{\text{kPa-m}^3}{\text{kJ/kg}} = \mathbf{7.424 \text{ kg}}$$

3.57

Consider a piston cylinder with 0.5 kg of R-134a as saturated vapor at -10°C. It is now compressed to a pressure of 500 kPa in a polytropic process with n = 1.5. Find the final volume and temperature, and determine the work done during the process.

Solution:

Take CV as the R-134a which is a control mass. $\qquad m_2 = m_1 = m$

Process: $Pv^{1.5} = \text{constant}$ until $P = 500$ kPa

1: (T, x) $v_1 = 0.09921$ m^3/kg, $\quad P = P_{sat} = 201.7$ kPa $\quad$ from Table B.5.1

2: (P, process) $v_2 = v_1 (P_1/P_2)^{(1/1.5)}$

$$= 0.09921 \times (201.7/500)^{2/3} = \textbf{0.05416} \text{ m}^3\text{/kg}$$

Given (P, v) at state 2 from B.5.2 it is superheated vapor at **$T_2 = 79°C$**

Process gives $P = C\,v^{-1.5}$, which is integrated for the work term, Eq.(3.21)

$$_1W_2 = \int P \, dV = \frac{m}{1 - 1.5} (P_2v_2 - P_1v_1)$$

$$= \frac{2}{-0.5} \text{ kg} \times (500 \times 0.05416 - 201.7 \times 0.09921) \text{ kPa-m}^3\text{/kg}$$

$$= \textbf{-7.07 kJ}$$

Heat Transfer rates

3.58

The brake shoe and steel drum on a car continuously absorbs 75 W as the car slows down. Assume a total outside surface area of 0.1 m^2 with a convective heat transfer coefficient of 10 W/m^2 K to the air at 20°C. How hot does the outside brake and drum surface become when steady conditions are reached?

Solution :

Convection heat transfer, Eq.3.24

$$\dot{Q} = hA\Delta T \quad \Rightarrow \quad \Delta T = \frac{\dot{Q}}{hA}$$

$$\Delta T = (T_{BRAKE} - 20) = \frac{75 \text{ W}}{10 \text{ Wm}^{-2}\text{K}^{-1} \times 0.1 \text{ m}^2} = \textbf{75 °C}$$

$$T_{BRAKE} = 20 + 75 = \textbf{95°C}$$

3.59

A water-heater is covered up with insulation boards over a total surface area of 3 m^2. The inside board surface is at 75°C and the outside surface is at 18°C and the board material has a conductivity of 0.08 W/m K. How thick a board should it be to limit the heat transfer loss to 200 W ?

Solution :

Steady state conduction through a single layer board, Eq.3.23.

$$\dot{Q}_{cond} = k \; A \; \frac{\Delta T}{\Delta x} \quad \Rightarrow \quad \Delta x = k \; A \; \Delta T / \dot{Q}$$

$$\Delta x = 0.08 \; \frac{W}{m \; K} \times 3 \; m^2 \times \frac{75 - 18}{200} \; \frac{K}{W}$$

$$= \mathbf{0.068 \; m}$$

3.60

Find the rate of conduction heat transfer through a 1.5 cm thick hardwood board, k = 0.16 W/m K, with a temperature difference between the two sides of 20$^{\text{o}}$C.

One dimensional heat transfer by conduction, we do not know the area so we can find the flux (heat transfer per unit area W/m^2).

$$\dot{q} = \dot{Q}/A = k\ \frac{\Delta T}{\Delta x} = 0.16\ \frac{W}{m\ K} \times \frac{20}{0.015}\ \frac{K}{m} = \textbf{213 W/m}^{\textbf{2}}$$

3.61

A 2 m^2 window has a surface temperature of 15°C and the outside wind is blowing air at 2°C across it with a convection heat transfer coefficient of h = 125 W/m^{2}K. What is the total heat transfer loss?

Solution:

Convection heat transfer, Eq.3.24

$$\dot{Q} = h\ A\ \Delta T = 125\ \text{W/m}^2\text{K} \times 2\ \text{m}^2 \times (15 - 2)\ \text{K} = \mathbf{3250\ W}$$

as a rate of heat transfer out.

15°C

2°C

3.62

Due to a faulty door contact the small light bulb (25 W) inside a refrigerator is kept on and limited insulation lets 50 W of energy from the outside seep into the refrigerated space. How much of a temperature difference to the ambient at 20°C must the refrigerator have in its heat exchanger with an area of 1 m^2 and an average heat transfer coefficient of 15 W/m^2 K to reject the leaks of energy.

Solution :

$$\dot{Q}_{tot} = 25 + 50 = 75 \text{ W to go out}$$

Convection heat transfer, Eq.3.24

$$\dot{Q} = hA\ \Delta T = 15 \times 1 \times \Delta T = 75 \text{ W}$$

$$\Delta T = \frac{\dot{Q}}{hA} = \frac{75 \text{ W}}{15 \text{ W/(m}^2\text{K)} \times 1 \text{ m}^2} = \mathbf{5\ °C}$$

so   T must be at least **25 °C**

3.63

A large condenser (heat exchanger) in a power plant must transfer a total of 100 MW from steam running in a pipe to sea water being pumped through the heat exchanger. Assume the wall separating the steam and seawater is 4 mm of steel, conductivity 15 W/m K and that a maximum of 5°C difference between the two fluids is allowed in the design. Find the required minimum area for the heat transfer neglecting any convective heat transfer in the flows.

Solution :

Steady conduction through the 4 mm steel wall, Eq.3.23.

$$\dot{Q} = k \ A \ \frac{\Delta T}{\Delta x} \quad \Rightarrow \quad A = \dot{Q} \ \Delta x \ / \ k\Delta T$$

$$A = 100 \times 10^6 \ W \times 0.004 \ m \ / \ (15 \ W/mK \times 5 \ K)$$

$$= \textbf{480 m}^2$$

Condensing water		Sea water

3.64

The black grille on the back of a refrigerator has a surface temperature of 35°C with a total surface area of 1 m^2. Heat transfer to the room air at 20°C takes place with an average convective heat transfer coefficient of 15 W/m^2 K. How much energy can be removed during 15 minutes of operation?

Solution :

Convection heat transfer, Eq.3.24

$$\dot{Q} = hA\ \Delta T;\quad Q = \dot{Q}\ \Delta t = \ hA\ \Delta T\ \Delta t$$

$Q = 15$ W/m^2 K $\times$ 1 m^2 $\times$ (35-20) K $\times$ 15 min $\times$ 60 s/min

$= 202\ 500$ J $= $ **202.5 kJ**

3.65

A pot of steel, conductivity 50 W/m K, with a 5 mm thick bottom is filled with 15°C liquid water. The pot has a diameter of 20 cm and is now placed on an electric stove that delivers 500 W as heat transfer. Find the temperature on the outer pot bottom surface assuming the inner surface is at 15°C.

Solution :

Steady conduction, Eq.3.23, through the bottom of the steel pot. Assume the inside surface is at the liquid water temperature.

$$\dot{Q} = k \ A \ \frac{\Delta T}{\Delta x} \ \Rightarrow \ \Delta T = \dot{Q} \ \Delta x \ / \ kA$$

$$\Delta T = 500 \ W \times 0.005 \ m \ /(50 \ W/m\text{-}K \times \frac{\pi}{4} \times 0.2^2 \ m^2) = 1.59 \ K$$

$$T = 15 + 1.59 \cong \mathbf{16.6°C}$$

3.66

A log of burning wood in the fireplace has a surface temperature of 450°C. Assume the emissivity is 1 (perfect black body) and find the radiant emission of energy per unit surface area.

Solution :

Radiation heat transfer, Eq.3.25

$$\dot{Q}/A = 1 \times \sigma \ T^4$$
$$= 5.67 \times 10^{-8} \ W/m^2K^4 \times (273.15 + 450)^4 \ K^4$$
$$= 15 \ 505 \ W/m^2$$
$$= \textbf{15.5 kW/m}^2$$

3.67

A wall surface on a house is at 30°C with an emissivity of $\varepsilon = 0.7$. The surrounding ambient to the house is at 15°C, average emissivity of 0.9. Find the rate of radiation energy from each of those surfaces per unit area.

Solution :

Radiation heat transfer, Eq.3.25

$$\dot{Q}/A = \varepsilon\sigma AT^4, \quad \sigma = 5.67 \times 10^{-8} \ W/m^2K^4$$

a) $\dot{Q}/A = 0.7 \times 5.67 \times 10^{-8} \ W/m^2K^4 \times (\ 273.15 + 30)^4 \ K^4 = $ **335 W/m^2**

b) $\dot{Q}/A = 0.9 \times 5.67 \times 10^{-8} \ W/m^2K^4 \times 288.15^4 \ K^4 = $ **352 W/m^2**

3.68

A radiant heat lamp is a rod, 0.5 m long and 0.5 cm in diameter, through which 400 W of electric energy is deposited. Assume the surface has an emissivity of 0.9 and neglect incoming radiation. What will the rod surface temperature be ?

Solution :

For constant surface temperature outgoing power equals electric power.

Radiation heat transfer, Eq.3.25

$$\dot{Q}_{rad} = \varepsilon\sigma AT^4 = \dot{Q}_{el} \Rightarrow$$

$$T^4 = \dot{Q}_{el} / \varepsilon\sigma A$$

$$= 400 \text{ W}/ (0.9 \times 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \times 0.5 \times \pi \times 0.005 \text{ m}^2)$$

$$= 9.9803 \times 10^{11} \text{ K}^4$$

$$\Rightarrow \quad T \cong \mathbf{1000 \text{ K}} \quad \mathbf{OR} \quad \mathbf{725 \, °C}$$

3.69

A radiant heating lamp has a surface temperature of 1000 K with $\varepsilon = 0.8$. How large a surface area is needed to provide 250 W of radiation heat transfer?

Radiation heat transfer, Eq.3.25. We do not know the ambient so let us find the area for an emitted radiation of 250 W from the surface

$$\dot{Q} = \varepsilon\sigma AT^4$$

$$A = \frac{\dot{Q}}{\varepsilon\sigma T^4} = \frac{250}{0.8 \times 5.67 \times 10^{-8} \times 1000^4} \frac{W}{W/m^2}$$

$$= \mathbf{0.0055}\ \mathbf{m^2}$$

Borgnakke and Sonntag

Properties (u, h) from General Tables

3.70

Determine the phase of the following substances and find the values of the unknown quantities.

      a. Nitrogen, P = 2000 kPa, 120 K,   v = ?, Z = ?
      b. Nitrogen 120 K, v = 0.0050 m³/kg, Z = ?
      c. Air, T = 100 C, v = 0.500 m³/kg, P = ?
      d. R-410A, T = 25°C, v = 0.01 m³/kg, P = ?, h = ?

Solution:

a. B.6.2 at 2000 kPa, 120 K: v = 0.0126 m³/kg;   A.5: R = 0.2968 kJ/kgK

$$Z = \frac{Pv}{RT} = \frac{2000 \times 0.0126}{0.2968 \times 120} \frac{\text{kPa m}^3/\text{kg}}{(\text{kJ/kgK}) \text{ K}} = \textbf{0.7075}$$

b. Table B.6.1: $v_f < v < v_g = 0.00799$ m³/kg so two-phase L + V

$$P = P_{sat} = \textbf{2513 kPa}$$

$$x = (v - v_f)/v_{fg} = \frac{0.005 - 0.001915}{0.00608} = 0.5074$$

$$Z = \frac{Pv}{RT} = \frac{2513 \times 0.005}{0.2968 \times 120} \frac{\text{kPa m}^3/\text{kg}}{(\text{kJ/kgK}) \text{ K}} = \textbf{0.353}$$

c. Ideal gas,   P = RT/v = 0.287 kJ/kgK × 373.15 K / 0.5 m³/kg = **214 kPa**

d. B.4.1 at 25°C, $v_f = 0.000944$, $v_g = 0.01514$, $v_f < v < v_g$: saturated.

$$P = \textbf{1653.6 kPa}, \qquad x = \frac{v - v_f}{v_{fg}} = \frac{0.01 - 0.000944}{0.0142} = 0.63775,$$

$$h = h_f + x\, h_{fg} = 97.59 + 0.63775 \times 186.43 = \textbf{216.486 kJ/kg}$$

States shown are placed relative to the two-phase region, not to each other.

Borgnakke and Sonntag

3.71

Find the phase and the missing properties of T, P, v, u and x for water at:

      a.   500 kPa, 100°C   b.   5000 kPa, u = 800 kJ/kg

      c.   5000 kPa, v = 0.06 m³/kg   d.   -6°C, v = 1 m³/kg

Solution:

a)   Look in Table B.1.2 at 500 kPa

     $T < T_{sat} = 151°C$ =>   compressed liquid

     Table B.1.4:   v = 0.001043 m³/kg, u = 418.8 kJ/kg

b)   Look in Table B.1.2 at 5000 kPa

     $u < u_f = 1147.78$ kJ/kg =>   compressed liquid

     Table B.1.4:   between 180°C and 200°C

$$T = 180 + (200 - 180)\ \frac{800 - 759.62}{848.08 - 759.62} = 180 + 20 \times 0.4567 = 189.1°C$$

$$v = 0.001124 + 0.4567\ (0.001153 - 0.001124) = 0.001137 \ m^3/kg$$

c)   Look in Table B.1.2 at 5000 kPa

     $v > v_g = 0.03944$ m³/kg   =>   superheated vapor

     Table B.1.3:   between 400°C and 450°C.

$$T = 400 + 50 \times \frac{0.06 - 0.05781}{0.0633 - 0.05781} = 400 + 50 \times 0.3989 = 419.95°C$$

$$u = 2906.58 + 0.3989 \times (2999.64 - 2906.58) = 2943.7 \ kJ/kg$$

d)   B.1.5: $v_i < v < v_g = 334.14$ m³/kg =>   2-phase, $P = P_{sat} = 887.6$ kPa,

     $x = (v - v_i) / v_{fg} = (1 - 0.0010898)/334.138 = 0.0029895$

     $u = u_i + x\ u_{fg} = -345.91 + 0.0029895 \times 2712.9 = -337.8 \ kJ/kg$

3.72

States shown are placed relative to the two-phase region, not to each other.

3.73

Find the missing properties and give the phase of the ammonia, NH_3.

     a. $T = 65^oC$, $P = 600$ kPa $u = ?$ $v = ?$

     b. $T = 20^oC$, $P = 100$ kPa $u = ?$ $v = ?$ $x = ?$

     c. $T = 50^oC$, $v = 0.1185$ m^3/kg $u = ?$ $P = ?$ $x = ?$

Solution:

a) Table B.2.1 $P < Psat$ => superheated vapor Table B.2.2:

$$v = 0.5 \times 0.25981 + 0.5 \times 0.26888 = \mathbf{0.2645\ m^3/kg}$$
$$u = 0.5 \times 1425.7 + 0.5 \times 1444.3 = \mathbf{1435\ kJ/kg}$$

b) Table B.2.1: $P < Psat$ => x = **undefined**, **superheated vapor**, from B.2.2:

$$v = \mathbf{1.4153\ m^3/kg}\ ;\quad u = \mathbf{1374.5\ kJ/kg}$$

c) Sup. vap. ($v > v_g$) Table B.2.2. P = **1200 kPa**, x = **undefined**

$$u = \mathbf{1383\ kJ/kg}$$

States shown are placed relative to the two-phase region, not to each other.

Borgnakke and Sonntag

3.74

Find the missing properties of (P, T, v, u, h and x) and indicate the states in a P-v and T-v diagram for

       a. Water at 5000 kPa, u = 1000 kJ/kg (Table B.1 reference)

       b. R-134a at 20°C, u = 300 kJ/kg

       c. Nitrogen at 250 K, 200 kPa

Solution:

a)   Compressed liquid: B.1.4   interpolate between 220°C and 240°C.

      $T = \textbf{233.3}^{o}\textbf{C}$, $v = \textbf{0.001213 m}^{3}\textbf{/kg}$, $x = \textbf{undefined}$

b)   Table B.5.1: $u < u_{g}$ => two-phase liquid and vapor

      $x = (u - u_{f})/u_{fg} = (300 - 227.03)/162.16 = 0.449988 = \textbf{0.45}$

      $v = 0.000817 + 0.45 \times 0.03524 = \textbf{0.01667 m}^{3}\textbf{/kg}$

c)   Table B.6.1: $T > T_{sat}$ (200 kPa) so superheated vapor in Table B.6.2

        $x = \textbf{undefined}$

        $v = 0.5(0.35546 + 0.38535) = \textbf{0.3704 m}^{3}\textbf{/kg}$,

        $u = 0.5(177.23 + 192.14) = \textbf{184.7 kJ/kg}$

States shown are placed relative to the two-phase region, not to each other.

Borgnakke and Sonntag

3.75

Determine the phase and the missing properties

a. H_2O 20°C, v = 0.001000 m³/kg   P = ?   u = ?
b. R-410A   400 kPa, v = 0.075 m³/kg   T = ?, u = ?
c. NH_3 10°C, v = 0.1 m³/kg   P = ? u = ?
d. N_2 101.3 kPa, h = 60 kJ/kg   T = ?   v = ?

a) Enter Table B.1.1 with T and we see at 20°C,   v < v_f so compressed liquid and
Table B.1.4:   **P = 5000 kPa   and   u = 83.64 kJ/kg**.

b) Table B.4.2:   P = 400 kPa   v > v_g so superheated, interpolate

$$T = 0 + 20 \, \frac{0.075 - 0.07227}{0.07916 - 0.7227} = 20 \times 0.3962 = \textbf{7.9°C}$$

$$u = 261.51 + (276.44 - 261.51) \times 0.3962 = \textbf{267.43 kJ/kg},$$

c)   Table B.2.1 at 10°C, v_f = 0.0016 m³/kg, v_g = 0.20541 m³/kg
so two-phase P = P_{sat} = **615.2 kPa**

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.1 - 0.0016}{0.20381} = 0.4828$$

$$u = u_f + x \, u_{fg} = 225.99 + x \times 1099.7 = \textbf{756.93 kJ/kg}$$

d)   Table B.6.1 shows that at 101.3 kPa, h_f < h < h_g = 76.69 kJ/kg,
so saturated two-phase **T = 77.3 K**

$$x = (h - h_f) / h_{fg} = (60 + 122.15)/198.84 = 0.916$$

$$v = v_f + x \, v_{fg} = 0.001240 + 0.916 \times 0.21515 = \textbf{0.1983 m}^3\textbf{/kg}$$

States shown are placed relative to the two-phase region, not to each other.

3.76

Find the missing properties of (u, h, and x)

a. H_2O $T = 120°C$, $v = 0.5 \text{ m}^3/\text{kg}$

b. H_2O $T = 100°C$, $P = 10 \text{ MPa}$

c. N_2 $T = 100 \text{ K}$, $x = 0.75$

d. N_2 $T = 200 \text{ K}$, $P = 200 \text{ kPa}$

e. NH_3 $T = 100°C$, $v = 0.1 \text{ m}^3/\text{kg}$

Solution:

a) Table B.1.1: $v_f < v < v_g$ =>   L+V mixture, P = **198.5 kPa**,

    x = (0.5 - 0.00106)/0.8908 = **0.56**,

    u = 503.48 + 0.56 × 2025.76 = **1637.9 kJ/kg**

b) Table B.1.4: compressed liquid,   v = 0.**001039 m^3/kg**,   u = **416.1 kJ/kg**

c) Table B.6.1:   100 K,   x = 0.75

    v = 0.001452 + 0.75 × 0.02975 = **0.023765 m^3/kg**

    u = -74.33 + 0.75 × 137.5 = **28.8 kJ/kg**

d) Table B.6.2:   200 K, 200 kPa

         v = **0.29551 m^3/kg**   ;   u = **147.37 kJ/kg**

e) Table B.2.1: $v > v_g$ =>   superheated vapor, x = **undefined**

    B.2.2: $P = 1600 + 400 \times \dfrac{0.1 - 0.10539}{0.08248 - 0.10539} = \textbf{1694 kPa}$

States shown are placed relative to the two-phase region, not to each other.

3.77

Determine the phase of the following substances and find the values of the unknown quantities.

   a. R-410A, T = −20°C, u = 220 kJ/kg, P = ?, x = ?

   b. Ammonia, T = −20°C, v = 0.35 m^3/kg, P = ?, u = ?

   c. Water, P = 400 kPa, h = 2800 kJ/kg,   T = ?, v = ?

a. At −20°C in B.4.1 $u < u_g = 245.99$ kJ/kg   so,

$$x = (u − u_f)/u_{fg} = (220 − 27.92)/218.07 = \textbf{0.8808}$$

$$P = P_{sat} = \textbf{399.6 kPa}$$

b. At −20°C, $v_f = 0.001504$, $v_g = 0.62334$ m^3/kg, $v_f < v < v_g$: saturated.

$$P = \textbf{190.2 kPa},$$

$$x = \frac{v − v_f}{v_{fg}} = \frac{0.35 - 0.001504}{0.62184} = 0.56043,$$

$$u = u_f + x\, u_{fg} = 88.76 + 0.56043 \times 1210.7 = 767.27 \text{ kJ/kg}$$

c. B.1.2 at 400 kPa: $h > h_g = 2738.5$ kJ/kg   so superheated vapor.

  we locate it between 150 and 200°C and interpolate

$$y = (2800 − 2752.82)/(2860.51 − 2752.82) = 0.43811$$

$$T = 150 + y\, (200 − 150) = \textbf{171.9°C}$$

$$v = 0.47084 + y\, (0.53422 − 0.47084) = \textbf{0.4986 } \mathbf{m^3}\textbf{/kg}$$

Borgnakke and Sonntag

3.78

Find the missing properties for CO_2 at:

a) 20^oC, 2 MPa   v = ? and h = ?

b) 10^oC, x = 0.5   P = ?, u = ?

c) 1 MPa, v = 0.05 m^3/kg,   T = ?, h = ?

Solution:

a) Table B.3.1 $P < P_{sat} = 5729$ kPa   so superheated vapor.

  Table B.3.2:   v = 0.0245 m^3/kg, h = 368.42 kJ/kg

b) Table B.3.1   since x given it is two-phase

        $P = P_{sat} = 4502$ kPa

        $u = u_f + x\ u_{fg} = 107.6 + 0.5 \times 169.07 = 192.14$ kJ/kg

c) Table B.3.1 $v > v_g \approx 0.0383$ m^3/kg   so superheated vapor

  Table B.3.2:   Between 0 and 20^oC so interpolate.

        $$T = 0 + 20 \times \frac{0.05 - 0.048}{0.0524 - 0.048} = 20 \times 0.4545 = 9.09^oC$$

        $h = 361.14 + (379.63 - 361.14) \times 0.4545 = 369.54$ kJ/kg

Borgnakke and Sonntag

3.79

Find the missing properties among (T, P, v, u, h and x if applicable) and indicate the states in a P-v and a T-v diagram for

     a.   R-410A   P = 500 kPa, h = 300 kJ/kg

     b.   R-410A $T = 10^oC$, u = 200 kJ/kg

     c.   R-134a $T = 40^oC$, h = 400 kJ/kg

Solution:

a) Table B.4.1: $h > h_g$ => **superheated vapor**, look in section 500 kPa and interpolate

$$T = 0 + 20 \times \frac{300 - 287.84}{306.18 - 287.84} = 20 \times 0.66303 = \mathbf{13.26°C},$$

$$v = 0.05651 + 0.66303 \times (0.06231\text{-}0.05651) = \mathbf{0.06036\ m^3/kg},$$

$$u = 259.59 + 0.66303 \times (275.02 - 259.59) = \mathbf{269.82\ kJ/kg}$$

b) Table B.4.1: $u < u_g = 255.9$ kJ/kg => L+V mixture, P = **1085.7 kPa**

$$x = \frac{u - u_f}{u_{fg}} = \frac{200 - 72.24}{183.66} = \mathbf{0.6956},$$

$$v = 0.000886 + 0.6956 \times 0.02295 = \mathbf{0.01685\ m^3/kg},$$

$$h = 73.21 + 0.6956 \times 208.57 = \mathbf{218.3\ kJ/kg}$$

c) Table B.5.1: $h < h_g$ => **two-phase L + V,** look in B.5.1 at 40°C:

$$x = \frac{h - h_f}{h_{fg}} = \frac{400 - 256.5}{163.3} = 0.87875, \qquad P = P_{sat} = \ \mathbf{1017\ kPa},$$

$$v = 0.000\ 873 + 0.87875 \times 0.01915 = \mathbf{0.0177\ m^3/kg}$$

$$u = 255.7 + 0.87875 \times 143.8 = \mathbf{382.1\ kJ/kg}$$

States shown are placed relative to the two-phase region, not to each other.

Borgnakke and Sonntag

3.80

Saturated liquid water at 20°C is compressed to a higher pressure with constant temperature. Find the changes in u and h from the initial state when the final pressure is   a) 500 kPa,   b) 2000 kPa

Solution:

State 1 is located in Table B.1.1 and the states a-c are from Table B.1.4

State	u [kJ/kg]	h [kJ/kg]	$\Delta u = u - u_1$	$\Delta h = h - h_1$	$\Delta(Pv)$
1	83.94	83.94			
a	83.91	84.41	-0.03	0.47	0.5
b	83.82	85.82	-0.12	1.88	2

For these states u stays nearly constant, dropping slightly as P goes up.
h varies with Pv changes.

3.81

Determine the phase of the following substances and find the values of the unknown quantities.

a. Water, P = 500 kPa, u = 2850 kJ/kg, T = ?, v = ?

b. R-134a, T = −10°C, v = 0.08 m^3/kg, P = ?, u = ?

c. Ammonia, T = −20°C, u = 1000 kJ/kg, P = ?, x = ?

a. B.1.2 at 500 kPa: u > u$_g$ = 2561 kJ/kg   so superheated vapor.
   we locate it between 300 and 350°C and interpolate

$$x = (2850 - 2802.9)/(2882.6 - 2802.9) = 0.590966$$
$$T = 300 + x (350 - 300) = 329.55°C$$
$$v = 0.52256 + x (0.57012 - 0.52256) = 0.5507 \text{ m}^3/\text{kg}$$

b. B.5.1 at −10°C, v$_f$ = 0.000755 m^3/kg, v$_g$ = 0.09921 m^3/kg, v$_f$ < v < v$_g$:
   saturated,   P = 201.7 kPa,

$$x = (v - v_f)/v_{fg} = (0.08-0.000755)/0.09845 = 0.805,$$
$$u = u_f + x \, u_{fg} = 186.57 + 0.805 \times 185.7 = 336 \text{ kJ/kg}$$

c. B.2.1 at −20°C: u < u$_g$ = 1299.5 kJ/kg   so,   P = P$_{sat}$ = 190.2 kPa

$$x = (u - u_f)/u_{fg} = (1000 - 88.76)/1210.7 = 0.7526$$

Problem Analysis

3.82

Consider Problem 3.101. Take the whole room as a C.V. and write both conservation of mass and energy equations. Write some equations for the process (two are needed) and use those in the conservation equations. Now specify the four properties that determines initial (2) and final state (2), do you have them all? Count unknowns and match with equations to determine those.

C.V.: Containment room and reactor.

Mass: $m_2 - m_1 = 0$; $m_2 = m_1 = V_{reactor}/v_1$

Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: Room volume constant $V = C$ $\Rightarrow$ ${}_1W_2 = 0$

Room insulated $\Rightarrow$ ${}_1Q_2 = 0$

Using these in the equation for mass and energy gives:

$$m_2 = V_2/v_2 = m_1 ; \qquad m(u_2 - u_1) = 0 - 0 = 0$$

State 1: P_1, T_1 so Table B.1.4 gives v_1, u_1 $\Rightarrow$ m_1

State 2: P_2, ?

We do not know **one** state 2 property and the total room volume

Energy equation then gives $u_2 = u_1$ (a state 2 property)

State 2: P_2, u_2 $\Rightarrow$ v_2

Now we have the room volume as

Continuity Eq.: $m_2 = V/v_2 = m_1$ so $V = m_1 v_2$

Borgnakke and Sonntag

3.83

Consider a steel bottle as a CV. It contains carbon dioxide at –20 ^{o}C, quality 20%. It has a safety valve that opens at 6 MPa. The bottle is now accidentally heated until the safety valve opens. Write the process equation that is valid until the valve opens and plot the P-v diagram for the process.

Solution:

C.V. carbon dioxide, which is a control mass of constant volume.

Energy Eq.3.5 (3.26): $\quad\quad u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process: $\quad\quad$ V = constant and m = constant

$\quad\quad\quad\quad => \textbf{ v = constant }; \quad {}_1w_2 = 0$

State 1: $\quad\quad$ Table B.3.1, $\quad P_1 = P_{sat} = 1970$ kPa

$\quad\quad\quad\quad v_1 = v_f + x\ v_{fg} = \ldots\ldots.$

State 2: $\quad\quad$ 6000 kPa, $v_1 = v_2 = \ldots..$ $\quad => $ Table B.3.1 or B.3.2

$\quad\quad\quad\quad T_2 = \ldots , \ u_2 = \ldots\ldots$

$\quad\quad\quad\quad {}_1q_2 = (u_2 - u_1) = \ldots\ldots$

State 2 could also be in the two-phase region if

$v_1 = v_2 < v_g$ at 6 MPa

3.84

A piston/cylinder contains water with quality 75% at 200 kPa. Slow expansion is performed while there is heat transfer and the water is at constant pressure. The process stops when the volume has doubled. How do you determine the final state and the heat transfer?

CV. Water, this is a control mass, we do not know size so do all per unit mass.

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process: $P = C$; $\quad {}_1w_2 = \int P dv = P(v_2 - v_1)$

State 1: $\quad x_1, P_1$ Table B.1.2 gives T_1, u_1

State 2: $\quad v_2 = 2v_1 = \dots$ & $P_2 = P_1$

Compare v_2 to v_g to determine if sup. vapor or not.

Either find x_2 or interpolate to get T_2 and u_2 in B.1.3

Process: $P = C$ so the work term integral is

$${}_1w_2 = \int P dv = P(v_2 - v_1) = \dots\dots$$

From the energy equation

$${}_1q_2 = (u_2 - u_1) + {}_1w_2 = \dots\dots$$

3.85

Consider Problem 3.173 with the final state given but that you were not told the piston hits the stops and only told $V_{stop} = 2 \, V_1$. Sketch the possible P-v diagram for the process and determine which number(s) you need to uniquely place state 2 in the diagram. There is a kink in the process curve what are the coordinates for that state? Write an expression for the work term.

C.V. R-410A. Control mass goes through process: $1 \to 2 \to 3$

As piston floats pressure is constant ($1 \to 2$) and the volume is constant for the second part ($2 \to 3$). So we have: $v_3 = v_2 = 2 \times v_1$

State 2: $V_2 = V_{stop} \implies v_2 = 2 \times v_1 = v_3$ and $P_2 = P_1 \implies T_2 = \ldots$

State 3: Table B.4.2 (P,T) $v_3 = 0.02249 \, m^3/kg$, $u_3 = 287.91 \, kJ/kg$

Now we can find state 1 $(T_1, v_1 = v_3/2 = 0.011245 \, m^3/kg)$ two-phase

Then state 2 (the kink)

$$v_2 = v_3 = 0.02249 \, m^3/kg$$
$$P_2 = P_1 = 1085.7 \, kPa = P_{sat}$$

$$W = \int P \, dV = P(V_2 - V_1) = Pm \, (v_2 - v_1)$$

3.86

Take problem 3.210 and write the left hand side (storage change) of the conservation equations for mass and energy. How do you write m_1 and Eq. 3.5?

C.V.: Both rooms A and B in tank.

Continuity Eq.: $\quad m_2 - m_{A1} - m_{B1} = 0$;

Energy Eq.: $\quad m_2 u_2 - m_{A1} u_{A1} - m_{B1} u_{B1} = {}_1Q_2 - {}_1W_2$

Notice how the state 1 term split into two terms

$$m_1 = \int \rho \, dV = \int (1/v) \, dV = V_A/v_{A1} + V_B/v_{B1} = m_{A1} + m_{B1}$$

and for energy as

$$m_1 u_1 = \int \rho u \, dV = \int (u/v) \, dV = (u_{A1}/v_{A1})V_A + (u_{B1}/v_{B1})V_B$$

$$= m_{A1} u_{A1} + m_{B1} u_{B1}$$

Formulation continues as:

Process constant total volume: $\quad V_{tot} = V_A + V_B \quad$ and $\quad {}_1W_2 = \emptyset$

$$m_2 = m_{A1} + m_{B1} \quad => \quad v_2 = V_{tot}/m_2$$

etc.

3.87

Two rigid insulated tanks are connected with a pipe and valve. One tank has 0.5 kg air at 200 kPa, 300 K and the other has 0.75 kg air at 100 kPa, 400 K. The valve is opened and the air comes to a single uniform state without any heat transfer. How do you determine the final temperature and pressure?

Solution:

C.V. Total tank. Control mass of constant volume.

Mass and volume: $\qquad m_2 = m_A + m_B; \qquad V = V_A + V_B$

Energy Eq.: $\qquad U_2 - U_1 = m_2 \, u_2 \; - m_A u_{A1} - m_B u_{B1} = {}_1Q_2 - {}_1W_2 = 0$

Process Eq.: $\qquad V = $ constant $\Rightarrow \; {}_1W_2 = 0;$ Insulated $\Rightarrow \; {}_1Q_2 = 0$

Ideal gas at A1: $\quad V_A = m_A R T_{A1}/P_{A1} = \ldots..$

$\qquad\qquad\qquad u_{A1} = \ldots\ldots$ from Table A.7

Ideal gas at B1: $\quad V_B = m_B R T_{B1}/P_{B1} = \ldots\ldots.$

$\qquad\qquad\qquad u_{B1} = \ldots\ldots.$ from Table A.7

State 2: $\qquad\qquad m_2 = m_A + m_B; \quad \Rightarrow \; v_2 = V/m_2 = \ldots..$

Energy Eq.: $\qquad u_2 = \dfrac{m_A u_{A1} + m_B u_{B1}}{m_2} = \ldots..$ $\qquad \Rightarrow$ Table A.7.1: $\mathbf{T_2}$

$\qquad\qquad\qquad P_2 = m_2 \, R T_2 / V = \ldots\ldots..$

3.88

Look at problem 3.183 and plot the P-v diagram for the process. Only T_2 is given, how do you determine the 2nd property of the final state? What do you need to check and does it have an influence on the work term?

Process: $P = \text{constant} = F/A = P_1$ if $V > V_{min}$

$V = \text{constant} = V_{1a} = V_{min}$ if $P < P_1$

State 1: (P, T) $V_1 = mRT_1/P_1 = 0.5 \times 0.287 \times 1000/2000 = 0.07175 \text{ m}^3$

The only possible P-V combinations for this system are shown in the diagram so both state 1 and 2 must be on the two lines. For state 2 we need to know if it is on the horizontal P line segment or the vertical V segment. Let us check state 1a:

State 1a: $P_{1a} = P_1, V_{1a} = V_{min}$,   Ideal gas so $T_{1a} = T_1 \dfrac{V_{1a}}{V_1}$

We see if $T_2 < T_{1a}$ then state 2 must have $V_2 = V_{1a} = V_{min} = 0.03 \text{ m}^3$. So state 2 is known by (T_2, v_2) and $P_2 = P_1 \times \dfrac{T_2}{T_1} \times \dfrac{V_1}{V_2}$

If it was that $T_2 > T_{1a}$ then we know state 2 as: $T_2, P_2 = P_1$ and we then have

$$V_2 = V_1 \times \frac{T_2}{T_1}$$

The work is the area under the process curve in the P-V diagram and so it does make a difference where state 2 is relative to state 1a. For the part of the process that proceeds along the constant volume V_{min} the work is zero, there is only work when the volume changes.

$$_1W_2 = \int_1^2 P \, dV = P_1 (V_{1a} - V_1)$$

Simple processes

3.89

A 100-L rigid tank contains nitrogen (N_2) at 900 K, 3 MPa. The tank is now cooled to 100 K. What are the work and heat transfer for this process?

Solution:

C.V.: Nitrogen in tank. $m_2 = m_1$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant, $v_2 = v_1 = V/m$ => $\mathbf{{}_1W_2 = \emptyset}$

Table B.6.2: State 1: $v_1 = 0.0900 \text{ m}^3/\text{kg}$ => $m = V/v_1 = 1.111 \text{ kg}$

$u_1 = 691.7 \text{ kJ/kg}$

State 2: 100 K, $v_2 = v_1 = V/m$,   look in Table B.6.2 at 100 K

200 kPa: $v = 0.1425 \text{ m}^3/\text{kg}$; $u = 71.7 \text{ kJ/kg}$

400 kPa: $v = 0.0681 \text{ m}^3/\text{kg}$; $u = 69.3 \text{ kJ/kg}$

so a linear interpolation gives:

$P_2 = 200 + 200 \, (0.09 - 0.1425)/(0.0681 - 0.1425) = 341 \text{ kPa}$

$u_2 = 71.7 + (69.3 - 71.7)\dfrac{0.09 - 0.1425}{0.0681 - 0.1425} = 70.0 \text{ kJ/kg}$,

${}_1Q_2 = m(u_2 - u_1) = 1.111 \text{ kg } (70.0 - 691.7) \text{ kJ/kg} = \mathbf{-690.7 \text{ kJ}}$

3.90

A constant pressure piston/cylinder aasembly contains 0.2 kg water as saturated vapor at 400 kPa. It is now cooled so the water occupies half the original volume. Find the work and the heat transfer in the process.

Solution:

Energy Eq.3.5: $\quad m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process: $\ P = C; \quad {_1}W_2 = \int PdV = P(V_2 - V_1)$

Table B.1.2

State 1: $\quad v_1 = 0.46246 \ m^3/kg; \ u_1 = 2553.55 \ kJ/kg$

State 2: $\quad v_2 = v_1/2 = 0.23123 \ m^3/kg = v_f + x \ v_{fg};$

$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{0.23123 - 0.001084}{0.46138} = 0.4988$$

$$u_2 = u_{f2} + x_2 \ u_{fg2} = 604.29 + x_2 \times 1949.26 = 1576.58 \ kJ/kg$$

Process: $\ P = C$ so the work term integral is

$${_1}W_2 = P(V_2\text{-}V_1) = 400 \ kPa \times 0.2 \times (0.23123 - 0.46246) \ m^3$$

$$= \mathbf{-18.5 \ kJ}$$

From the energy equation

$${_1}Q_2 = m(u_2 - u_1) + {_1}W_2 = 0.2 \ (1576.58 - 2553.55) - 18.5$$

$$= \mathbf{-213.9 \ kJ}$$

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3.91

Saturated vapor R-410A at 0^oC in a rigid tank is cooled to -20^oC. Find the specific heat transfer.

Solution:

C.V.: R-410A in tank. $m_2 = m_1$;

Energy Eq.3.5: $(u_2 - u_1) = {_1}q_2 - {_1}w_2$

Process: V = constant, $v_2 = v_1 = V/m$ => ${_1}w_2 = \cancel{0}$

Table B.4.1: State 1: $u_1 = 253.0$ kJ/kg

State 2: -20^oC, $v_2 = v_1 = V/m$,   look in Table B.4.1 at -20^oC

$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{0.03267 - 0.000803}{0.06400} = 0.4979$$

$$u_2 = u_{f2} + x_2\, u_{fg2} = 27.92 + x_2 \times 218.07 = 136.5 \text{ kJ/kg}$$

From the energy equation

$${_1}q_2 = (u_2 - u_1) = (\,136.5 - 253.0\,) = \textbf{-116.5 kJ/kg}$$

3.92

Ammonia at 0°C, quality 60% is contained in a rigid 200-L tank. The tank and ammonia is now heated to a final pressure of 1 MPa. Determine the heat transfer for the process.

Solution:

C.V.: NH_3

Continuity Eq.: $\qquad m_2 = m_1 = m$;

Energy Eq.3.5: $\qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: Constant volume $\Rightarrow \qquad v_2 = v_1$ & $\quad {}_1W_2 = 0$

State 1: Table B.2.1 two-phase state.

$\qquad v_1 = 0.001566 + x_1 \times 0.28763 = 0.17414 \text{ m}^3/\text{kg}$

$\qquad u_1 = 179.69 + 0.6 \times 1138.3 = 862.67 \text{ kJ/kg}$

$\qquad m = V/v_1 = 0.2 \text{ m}^3/0.17414 \text{ m}^3/\text{kg} = 1.148 \text{ kg}$

State 2: P_2 , $v_2 = v_1$ superheated vapor Table B.2.2

$\qquad \Rightarrow T_2 \cong 100°C, \quad u_2 \cong 1490.5 \text{ kJ/kg}$

So solve for heat transfer in the energy equation

$\qquad {}_1Q_2 = m(u_2 - u_1) = 1.148 \text{ kg } (1490.5 - 862.67) \text{ kJ/kg} = \textbf{720.75 kJ}$

3.93

A rigid tank contains 1.5 kg of R-134a at 40°C, 500 kPa. The tank is placed in a refrigerator that brings it to -20°C. Find the process heat transfer and show the process in a *P-v* diagram.

CV the R-134a.

Energy Eq.: $U_2 - U_1 = {_1}Q_2 - {_1}W_2 = {_1}Q_2$,

Process: Rigid tank $V = C$ => $v = $ constant & ${_1}W_2 = \int_1^2 PdV = 0$

State 1: $v_1 = 0.04656$ m^3/kg,

$u_1 = 407.44$ kJ/kg

State 2: **T, v** => two-phase (straight down in P-v diagram from state 1)

$x_2 = (v - v_f)/v_{fg} = (0.04656 - 0.000738)/0.14576 = 0.31437$

$u_2 = u_f + x_2\, u_{fg} = 173.65 + 0.31437 \times 192.85 = 234.28$ kJ/kg

From the energy equation

${_1}Q_2 = m(u_2 - u_1) = 1.5$ kg $(234.28 - 407.44)$ kJ/kg = **-259.7 kJ**

3.94

A piston cylinder contains air at 600 kPa, 290 K and a volume of 0.01 m^3. A constant pressure process gives 54 kJ of work out. Find the final volume, the temperature of the air and the heat transfer.

C.V AIR control mass

Continuity Eq.: $\quad\quad m_2 - m_1 = 0$

Energy Eq.: $\quad\quad\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $\quad\quad P = C \quad$ so $\quad {}_1W_2 = \int P\, dV = P(V_2 - V_1)$

State 1 : P_1 , $T_1, V_1 \quad\quad$ State 2 : $P_1 = P_2$, ?

$$m_1 = P_1 V_1 / RT_1 = \frac{600 \text{ kPa} \times 0.01 \text{ m}^3}{0.287 \text{ kJ/kg-K} \times 290 \text{ K}} = 0.0721 \text{ kg}$$

$$_1W_2 = P(V_2 - V_1) = 54 \text{ kJ} \rightarrow$$

$$V_2 - V_1 = {}_1W_2 / P = 54 \text{ kJ} / 600 \text{ kPa} = 0.09 \text{ m}^3$$

$$V_2 = V_1 + {}_1W_2 / P = 0.01 + 0.09 = 0.1 \text{ m}^3$$

Ideal gas law : $P_2 V_2 = mRT_2$

$$T_2 = P_2 V_2 / mR = \frac{P_2 V_2}{P_1 V_1} T_1 = \frac{0.10}{0.01} \times 290 \text{ K} = \textbf{2900 K}$$

Energy equation with u's from table A.7.1

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

$$= 0.0721 \text{ kg} \times (2563.8 - 207.19) \text{ kJ/kg} + 54 \text{ kJ}$$

$$= \textbf{223.9 kJ}$$

3.95

Two kg water at 120°C with a quality of 25% has its temperature raised 20°C in a constant volume process as in Fig. P3.95. What are the heat transfer and work in the process?

Solution:

C.V. Water. This is a control mass

Energy Eq.: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process : V = constant

$\rightarrow \ {_1}W_2 = \int P\, dV = \mathbf{0}$

State 1: T, x_1 from Table B.1.1

$$v_1 = v_f + x_1\, v_{fg} = 0.00106 + 0.25 \times 0.8908 = 0.22376 \text{ m}^3/\text{kg}$$

$$u_1 = u_f + x_1\, u_{fg} = 503.48 + 0.25 \times 2025.76 = 1009.92 \text{ kJ/kg}$$

State 2: $T_2,\ v_2 = v_1 < v_{g2} = 0.50885$ m^3/kg   so two-phase

$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{0.22376 - 0.00108}{0.50777} = 0.43855$$

$$u_2 = u_{f2} + x_2\, u_{fg2} = 588.72 + x_2 \times 1961.3 = 1448.84 \text{ kJ/kg}$$

From the energy equation

$${_1}Q_2 = m(u_2 - u_1) = 2 \text{ kg} \times (1448.84 - 1009.92) \text{ kJ/kg} = \mathbf{877.8 \text{ kJ}}$$

3.96

A cylinder fitted with a frictionless piston contains 2 kg of superheated refrigerant R-134a vapor at 350 kPa, 100°C. The cylinder is now cooled so the R-134a remains at constant pressure until it reaches a quality of 75%. Calculate the heat transfer in the process.

Solution:

C.V.: R-134a $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = const. $\Rightarrow {}_1W_2 = \int PdV = P\Delta V = P(V_2 - V_1) = Pm(v_2 - v_1)$

State 1: Table B.5.2 $h_1 = (490.48 + 489.52)/2 = 490$ kJ/kg

State 2: Table B.5.1 $h_2 = 206.75 + 0.75 \times 194.57 = 352.7$ kJ/kg (350.9 kPa)

$\qquad {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + Pm(v_2 - v_1) = m(h_2 - h_1)$

$\qquad {}_1Q_2 = 2$ kg $\times$ (352.7 - 490) kJ/kg = **-274.6 kJ**

3.97

A piston cylinder contains 1.5 kg water at 200 kPa, 150$^{\text{o}}$C. It is now heated in a process where pressure is linearly related to volume to a state of 600 kPa, 350$^{\text{o}}$C. Find the final volume, the work and the heat transfer in the process.

Take as CV the 1.5 kg of water.

$$m_2 = m_1 = m \; ;$$

Process Eq.: $P = A + BV$ (linearly in V)

State 1: $(P, T) \implies v_1 = 0.95964 \; m^3/kg,$

$\qquad\qquad u_1 = 2576.87 \; kJ/kg$

State 2: $(P, T) \implies v_2 = 0.47424 \; m^3/kg, \; u_2 = 2881.12 \; kJ/kg$

$$V_2 = mv_2 = \mathbf{0.7114 \; m^3}$$

From process eq.:

$$_1W_2 = \int P \, dV = \text{area} = \frac{m}{2} (P_1 + P_2)(v_2 - v_1)$$

$$= \frac{1.5}{2} \, kg \, (200 + 600) \, kPa \, (0.47424 - 0.95964) \, m^3/kg = \mathbf{-291.24 \; kJ}$$

Notice volume is reduced so work is negative.

Energy Eq.: $\qquad m(u_2 - u_1) = \,_1Q_2 - \,_1W_2$

$$_1Q_2 = m(u_2 - u_1) + \,_1W_2$$

$$= 1.5 \, kg \, (2881.12 - 2576.87) \, kJ/kg - 291.24 \, kJ$$

$$= \mathbf{165.1 \; kJ}$$

3.98

A piston/cylinder contains 50 kg of water at 200 kPa with a volume of 0.1 m³.
Stops in the cylinder are placed to restrict the enclosed volume to a maximum of
0.5 m³. The water is now heated until the piston reaches the stops. Find the
necessary heat transfer.

Solution:

C.V. H_2O m = constant

Energy Eq.3.5: $m(e_2 - e_1) = m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : P = constant (forces on piston constant)

$$\Rightarrow {}_1W_2 = \int P \, dV = P_1 (V_2 - V_1)$$

Properties from Table B.1.1

State 1: $v_1 = 0.1/50 = 0.002$ m³/kg   =>   2-phase as $v_1 < v_g$

$$x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{0.002 - 0.001061}{0.88467} = 0.001061$$

$$h_1 = 504.68 + 0.001061 \times 2201.96 = 507.02 \text{ kJ/kg}$$

State 2: $v_2 = 0.5/50 = 0.01$ m³/kg   also 2-phase same P

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.01 - 0.001061}{0.88467} = 0.01010$$

$$h_2 = 504.68 + 0.01010 \times 2201.96 = 526.92 \text{ kJ/kg}$$

Find the heat transfer from the energy equation as

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

$${}_1Q_2 = 50 \text{ kg} \times (526.92 - 507.02) \text{ kJ/kg} = \textbf{995 kJ}$$

[Notice that ${}_1W_2 = P_1 (V_2 - V_1) = 200$ kPa $\times (0.5 - 0.1)$ m³ = 80 kJ]

3.99

Ammonia (0.5 kg) is in a piston cylinder at 200 kPa, -10°C is heated in a process where the pressure varies linear with the volume to a state of 120°C, 300 kPa. Find the work and the heat transfer for the ammonia in the process.

Solution:

Take CV as the Ammonia, constant mass.

Continuity Eq.: $m_2 = m_1 = m$;

Process: $P = A + BV$ (linear in V)

State 1: Superheated vapor $v_1 = 0.6193$ m^3/kg, $u_1 = 1316.7$ kJ/kg

State 2: Superheated vapor $v_2 = 0.63276$ m^3/kg, $u_2 = 1542.0$ kJ/kg

Work is done while piston moves at increasing pressure, so we get

$$_1W_2 = \int P \, dV = \text{area} = P_{avg} (V_2 - V_1) = \frac{1}{2} (P_1 + P_2)m(v_2 - v_1)$$

$$= \frac{1}{2}(200 + 300) \text{ kPa} \times 0.5 \text{ kg} (0.63276 - 0.6193) \text{ m}^3/\text{kg}$$

$$= \textbf{1.683 kJ}$$

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

$$= 0.5 \text{ kg} (1542.0 - 1316.7) \text{ kJ/kg} + 1.683 \text{ kJ}$$

$$= \textbf{114.3 kJ}$$

3.100

A piston/cylinder contains 1 kg water at 20°C with volume 0.1 m^3. By mistake someone locks the piston preventing it from moving while we heat the water to saturated vapor. Find the final temperature and the amount of heat transfer in the process.

Solution:

C.V. Water. This is a control mass

Energy Eq.: $m\,(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : V = constant → ${}_1W_2 = 0$

State 1: T, $v_1 = V_1/m = 0.1$ m^3/kg $> v_f$ so two-phase

$$x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{0.1 - 0.001002}{57.7887} = 0.0017131$$

$$u_1 = u_f + x_1\,u_{fg} = 83.94 + x_1 \times 2318.98 = 87.913 \text{ kJ/kg}$$

State 2: $v_2 = v_1 = 0.1$ & $x_2 = 1$

→ found in Table B.1.1 between 210°C and 215° C

$$T_2 = 210 + 5 \times \frac{0.1 - 0.10441}{0.09479 - 0.10441} = 210 + 5 \times 0.4584 = 212.3°C$$

$$u_2 = 2599.44 + 0.4584\,(2601.06 - 2599.44) = 2600.2 \text{ kJ/kg}$$

From the energy equation

$${}_1Q_2 = m(u_2 - u_1) = 1 \text{ kg } (2600.2 - 87.913) \text{ kJ/kg} = \textbf{2512.3 kJ}$$

3.101

A water-filled reactor with volume of 1 m^3 is at 20 MPa, 360°C and placed inside a containment room as shown in Fig. P3.101. The room is well insulated and initially evacuated. Due to a failure, the reactor ruptures and the water fills the containment room. Find the minimum room volume so the final pressure does not exceed 200 kPa.

Solution:

C.V.: Containment room and reactor.

Mass: $m_2 = m_1 = V_{reactor}/v_1 = 1/0.001823 = 548.5$ kg

Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0 - 0 = 0$

State 1: Table B.1.4 $v_1 = 0.001823$ m^3/kg, $u_1 = 1702.8$ kJ/kg

Energy equation then gives $u_2 = u_1 = 1702.8$ kJ/kg

State 2: $P_2 = 200$ kPa, $u_2 < u_g$ => Two-phase Table B.1.2

$x_2 = (u_2 - u_f)/u_{fg} = (1702.8 - 504.47)/2025.02 = 0.59176$

$v_2 = 0.001061 + 0.59176 \times 0.88467 = 0.52457$ m^3/kg

$V_2 = m_2 v_2 = 548.5$ kg $\times 0.52457$ m^3/kg = **287.7 m^3**

3.102

A rigid tank holds 0.75 kg ammonia at 70°C as saturated vapor. The tank is now cooled to 20°C by heat transfer to the ambient. Which two properties determine the final state. Determine the amount of work and heat transfer during the process.

C.V. The ammonia, this is a control mass.

Process: Rigid tank $V = C$ $\Rightarrow$ $v = $ constant & $_1W_2 = \int_1^2 PdV = \mathbf{0}$

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2 = {}_1Q_2$,

State 1: $v_1 = 0.03787$ m^3/kg,

$\qquad\quad$ $u_1 = 1338.9$ kJ/kg

State 2:   **T, v** $\Rightarrow$ two-phase (straight down in P-v diagram from state 1)

$x_2 = (v - v_f)/v_{fg} = (0.03787 - 0.001638)/0.14758 = 0.2455$

$u_2 = u_f + x_2\, u_{fg} = 272.89 + 0.2455 \times 1059.3 = 532.95$ kJ/kg

$_1Q_2 = m(u_2 - u_1) = 0.75$ kg $(532.95 - 1338.9)$ kJ/kg $= \mathbf{-604.5\ kJ}$

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3.103

Water in a 150-L closed, rigid tank is at 100°C, 90% quality. The tank is then cooled to −10°C. Calculate the heat transfer during the process.

Solution:

C.V.: Water in tank. $\quad m_2 = m_1$;

Energy Eq.3.5: $\qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant, $\quad v_2 = v_1, \quad {}_1W_2 = 0$

State 1: Two-phase L + V look in Table B.1.1

$\qquad v_1 = 0.001044 + 0.9 \times 1.6719 = 1.5057 \text{ m}^3/\text{kg}$

$\qquad u_1 = 418.94 + 0.9 \times 2087.6 = 2297.8 \text{ kJ/kg}$

State 2: $T_2, v_2 = v_1 \quad \Rightarrow \quad$ mix of saturated solid + vapor Table B.1.5

$\qquad v_2 = 1.5057 = 0.0010891 + x_2 \times 466.7 \quad => \quad x_2 = 0.003224$

$\qquad u_2 = \text{-}354.09 + 0.003224 \times 2715.5 = \text{-}345.34 \text{ kJ/kg}$

$\qquad m = V/v_1 = 0.15/1.5057 = 0.09962 \text{ kg}$

$\quad {}_1Q_2 = m(u_2 - u_1) = 0.09962 \text{ kg } (\text{-}345.34 - 2297.8) \text{ kJ/kg} = \textbf{-263.3 kJ}$

3.104

A 25 kg mass moves with 25 m/s. Now a brake system brings the mass to a complete stop with a constant deceleration over a period of 5 seconds. The brake energy is absorbed by 0.5 kg water initially at 20^oC, 100 kPa. Assume the mass is at constant P and T. Find the energy the brake removes from the mass and the temperature increase of the water, assuming P = C.

Solution:

C.V. The mass in motion.

$$E_2 - E_1 = \Delta E = 0.5\ m\mathbf{V}^2 = 0.5 \times 25 \times 25^2/1000 = \mathbf{7.8125\ kJ}$$

C.V. The mass of water.

$$m(u_2 - u_1)_{H_2O} = \Delta E = 7.8125\ kJ$$

$$\Rightarrow \quad u_2 - u_1 = 7.8125\ kJ/\ 0.5\ kg = 15.63\ kJ/kg$$

$$u_2 = u_1 + 15.63 = 83.94 + 15.63 = 99.565\ kJ/kg$$

Assume $u_2 = u_f$ then from Table B.1.1: $\quad T_2 \cong 23.7^o$C, $\quad \Delta T = \mathbf{3.7^oC}$

We could have used $u_2 - u_1 = C\Delta T$ with C from Table A.4: C = 4.18 kJ/kg K giving $\Delta T = 15.63/4.18 = \mathbf{3.7^oC}$.

3.105

A cylinder having a piston restrained by a linear spring (of spring constant 15 kN/m) contains 0.5 kg of saturated vapor water at 120°C, as shown in Fig. P3.105. Heat is transferred to the water, causing the piston to rise. If the piston cross-sectional area is 0.05 m^2, and the pressure varies linearly with volume until a final pressure of 500 kPa is reached. Find the final temperature in the cylinder and the heat transfer for the process.

Solution:

C.V. Water in cylinder.

Continuity: $\qquad m_2 = m_1 = m$;

Energy Eq.3.5: $\qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: (T, x) Table B.1.1 $\Rightarrow$ $v_1 = 0.89186$ m^3/kg, $\quad u_1 = 2529.2$ kJ/kg

Process: $\quad P_2 = P_1 + \dfrac{k_s m}{A_p^2}(v_2 - v_1) = 198.5 + \dfrac{15 \times 0.5}{(0.05)^2}(v_2 - 0.89186)$

State 2: $\quad P_2 = 500$ kPa and on the process curve (see above equation).

$\qquad \Rightarrow \quad v_2 = 0.89186 + (500 - 198.5) \times (0.05^2/7.5) = 0.9924$ m^3/kg

$\qquad$ (P, v) Table B.1.3 $\Rightarrow$ $T_2 = \textbf{803°C};$ $\quad u_2 = 3668$ kJ/kg

The process equation allows us to evaluate the work

$$
{}_1W_2 = \int P dV = \left(\frac{P_1 + P_2}{2}\right) m(v_2 - v_1)
$$

$$
= \left(\frac{198.5 + 500}{2}\right) \text{kPa} \times 0.5 \text{ kg} \times (0.9924 - 0.89186) \text{ m}^3/\text{kg} = 17.56 \text{ kJ}
$$

Substitute the work into the energy equation and solve for the heat transfer

$$
{}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.5 \text{ kg} \times (3668 - 2529.2) \text{ kJ/kg} + 17.56 \text{ kJ} = \textbf{587 kJ}
$$

3.106

A piston cylinder arrangement with a linear spring similar to Fig. P3.105 contains R-134a at 15°C, x = 0.4 and a volume of 0.02 m³. It is heated to 60°C at which point the specific volume is 0.03002 m³/kg. Find the final pressure, the work and the heat transfer in the process.

Take CV as the R-134a.

$$m_2 = m_1 = m \quad ; \qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

State 1: T_1, x_1 => Two phase so Table B.5.1: $P_1 = P_{sat} = 489.5$ kPa

$v_1 = v_f + x_1 \, v_{fg} = 0.000805 + 0.4 \times 0.04133 = 0.01734 \; m^3/kg$

$u_1 = u_f + x_1 \, u_{fg} = 220.1 + 0.4 \times 166.35 = 286.64 \; kJ/kg$

$m = V_1/v_1 = 0.02 \; m^3/ \, 0.01734 \; m^3/kg = 1.1534 \; kg$

State 2: (T, v) Superheated vapor, Table B.5.2.

$$P_2 = 800 \; kPa, \; v_2 = 0.03002 \; m^3/kg, \; u_2 = 421.2 \; kJ/kg$$

$V_2 = m \, v_2 = 1.1534 \; kg \times 0.03002 \; m^3/kg = 0.03463 \; m^3$

Work is done while piston moves at linearly varying pressure, so we get

$${}_1W_2 = \int P \, dV = area = P_{avg} \, (V_2 - V_1) = 0.5(P_2 + P_1)(V_2 - V_1)$$

$$= 0.5 \times (489.5 + 800) \; kPa \, (0.03463 - 0.02) \; m^3 = \textbf{9.433 kJ}$$

Heat transfer is found from the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.1534 \times (421.2 - 286.64) + 9.43 = \textbf{164.6 kJ}$$

3.107

A 10-m high open cylinder, $A_{cyl} = 0.1$ m^2, contains 20°C water above and 2 kg of 20°C water below a 198.5-kg thin insulated floating piston, shown in Fig. P3.107. Assume standard g, Po. Now heat is added to the water below the piston so that it expands, pushing the piston up, causing the water on top to spill over the edge. This process continues until the piston reaches the top of the cylinder. Find the final state of the water below the piston (T, P, v) and the heat added during the process.

Solution:

C.V. Water below the piston.

Piston force balance at initial state: $F\uparrow = F\downarrow = P_A A = m_p g + m_B g + P_0 A$

State $1_{A,B}$:   Comp. Liq. $\Rightarrow$ $v \cong v_f = 0.001002$ m^3/kg; $u_{1A} = 83.95$ kJ/kg

$V_{A1} = m_A v_{A1} = 0.002$ m^3; $m_{tot} = V_{tot}/v = 1/0.001002 = 998$ kg

mass above the piston $m_{B1} = m_{tot} - m_A =$ **996 kg**

$P_{A1} = P_0 + (m_p + m_B)g/A = 101.325 + \dfrac{(198.5 + 996) \times 9.807}{0.1 \times 1000} =$ **218.5 kPa**

State 2_A: $P_{A2} = P_0 + \dfrac{m_p g}{A} =$ **120.8 kPa** ; $v_{A2} = V_{tot}/m_A =$ **0.5 m^3/kg**

$x_{A2} = (0.5 - 0.001047)/1.4183 = 0.352$; $T_2 =$ **105°C**

$u_{A2} = 440.0 + 0.352 \times 2072.34 = 1169.5$ kJ/kg

Continuity eq. in A: $m_{A2} = m_{A1}$

Energy: $m_A(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P linear in V as m_B is linear with V

${}_1W_2 = \int PdV = \dfrac{1}{2}(218.5 + 120.82)$ kPa (1 - 0.002) m^3

$= $ **169.32 kJ**

${}_1Q_2 = m_A(u_2 - u_1) + {}_1W_2 = 2170.1 + 169.3 =$ **2340.4 kJ**

3.108

Assume the same setup as in Problem 3.101, but the room has a volume of 100 m^3. Show that the final state is two-phase and find the final pressure by trial and error.

C.V.: Containment room and reactor.

Mass: $m_2 = m_1 = V_{reactor}/v_1 = 1/0.001823 = 548.5$ kg

Energy: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2 = 0 - 0 = 0 \Rightarrow u_2 = u_1 = 1702.8$ kJ/kg

Total volume and mass => $v_2 = V_{room}/m_2 = 0.1823$ m^3/kg

State 2: u_2, v_2 Table B.1.1 see Figure.

Note that in the vicinity of $v = 0.1823$ m^3/kg crossing the saturated vapor line the internal energy is about 2585 kJ/kg. However, at the actual state 2, $u = 1702.8$ kJ/kg. Therefore state 2 must be in the two-phase region.

Trial & error $\quad v = v_f + x v_{fg}$; $u = u_f + x u_{fg}$

$\Rightarrow u_2 = 1702.8 = u_f + \dfrac{v_2 - v_f}{v_{fg}} u_{fg}$

Compute RHS for a guessed pressure P_2:

$P_2 = 600$ kPa: RHS $= 669.88 + \dfrac{0.1823 - 0.001101}{0.31457} \times 1897.52 = 1762.9$ too large

$P_2 = 550$ kPa: RHS $= 655.30 + \dfrac{0.1823 - 0.001097}{0.34159} \times 1909.17 = 1668.1$ too small

Linear interpolation to match $u = 1702.8$ gives $P_2 \cong$ **568.5 kPa**

Borgnakke and Sonntag

3.109

A piston cylinder contains carbon dioxide at -20$^\text{o}$C and quality 75%. It is compressed in a process where pressure is linear in volume to a state of 3 MPa and 20$^\text{o}$C. Find the specific heat transfer.

CV Carbon dioxide out to the source, both $_1Q_2$ and $_1W_2$

Energy Eq.3.5: $m(u_2 - u_1) = {_1Q_2} - {_1W_2}$

Process: $P = A + BV$ => $_1W_2 = \int P\ dV = \frac{1}{2}\ m(P_1 + P_2)\ (v_2 - v_1)$

State 1: Table B.3.1 $P = 1969.6$ kPa

$\quad v_1 = 0.000969 + 0.75 \times 0.01837 = 0.01475$ m^3/kg,

$\quad u_1 = 39.64 + 0.75 \times 246.25 = 224.33$ kJ/kg,

State 2: Table B.3 $v_2 = 0.01512$ m^3/kg, $u_2 = 310.21$ kJ/kg,

$\quad _1w_2 = \frac{1}{2}\ (P_1 + P_2)(\ v_2 - v_1)$

$\quad\quad = \frac{1}{2} \times (1969.6 + 3000)$ kPa $\times (\ 0.01512 - 0.01475)$ m^3/kg

$\quad\quad = \textbf{0.92 kJ/kg}$

$_1q_2 = u_2 - u_1 + {_1w_2} = 310.21 - 224.33 + 0.92 = \textbf{86.8 kJ/kg}$

3.110

A rigid steel tank of mass 2.5 kg contains 0.5 kg R-410A at 0°C with specific volume 0.01m/kg. The whole system is now heated to to a room temperature of 25°C.

> a. Find the volume of the tank.
> b. Find the final P.
> c. Find the process heat transfer.

C.V. R-410A and steel tank. Control mass goes through process: 1 -> 2

Continuity Eq.: $m_2 - m_{R410a} - m_{st} = 0$

Energy Eq.: $m_{R410a}(u_2 - u_1) + m_{st}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $V = C$ so ${}_1W_2 = 0$

State 1: $T_1 = 0°C$, $v_1 = 0.01 \ m^3/kg$,

$\qquad V = mv_1 = 0.005 \ m^3$

$\qquad x_1 = (v - v_f)/v_{fg} = (0.01 - 0.000855)/0.03182$

$\qquad = 0.28758$

$\qquad u_1 = u_f + x_1 \ u_{fg} = 57.07 + x_1 \ 195.95$

$\qquad = 113.42 \ kJ/kg$

State 2: $(\ T, v \ ) \ => $ sup-vapor (straight up in T-v diagram from state 1)

B.4.1 at 25°C, $v_f = 0.000944 \ m^3/kg$, $v_g = 0.01514 \ m^3/kg$, $v_f < v < v_g$: saturated.

$\qquad P = 1653.6 \ kPa, \qquad x = \dfrac{v - v_f}{v_{fg}} = \dfrac{0.01 - 0.000944}{0.01420} = 0.63775,$

$\qquad u_2 = u_f + x_2 \ u_{fg} = 96.03 + x_2 \ 162.95 = 199.95 \ kJ/kg$

From the energy Eq.:

$\qquad {}_1Q_2 = m_{R410a}(u_2 - u_1) + m_{st} \ C_{st}(T_2 - T_1)$

$\qquad = 0.5 \ kg \times (199.95 - 113.42) \ kJ/kg + 2.5 \ kg \times 0.46 \ kJ/kgK \times (25–0) \ K$

$\qquad = \textbf{72.0 kJ}$

3.111

The piston/cylinder in Fig.P3.111 contains 0.1 kg water at 500°C, 1000 kPa. The piston has a stop at half the original volume. The water now cools to room temperature 25°C

(a) Sketch the possible water states in a P-v diagram.
(b) Find the final pressure and volume
(c) Find the heat transfer and work in the process

Energy Eq.: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process Eq: $P = C$ if $v > v_{stop}$; $V = C$ if $P < P_{float}$

State 1: $v_1 = 0.35411$ m^3/kg, $u_1 = 3124.34$ kJ/kg

State a: $v_a = v_1/2 = 0.177055$ m^3/kg $< v_{g\ 1000\ kPa}$ so $T_a = T_{sat\ 1000\ kPa} = 179.9$°C

The possible state 2 (P,V) combinations are shown. State "a" is (1000 kPa, v_a) so it is

two-phase with $T_a = 180$°C $> T_2$

$P_2 = P_{sat\ 25\ C} = $ **3.169 kPa**   and $v_2 = v_a$

$x_2 = (v_2 - v_f)/v_{fg} = (0.177 - 0.001003)/43.358$

$\quad = 0.0040604$

$u_2 = u_f + x_2\ u_{fg} = 104.86 + x_2\ 2304.9 = 114.219$ kJ/kg

$V_2 = mv_2 = 0.1$ kg $\times 0.177055$ m^3/kg = **0.0177 m^3**

${_1}W_2 = m \int P\ dv = m\ P_1\ (v_2 - v_1)$ [see area below process curve in figure]

$\quad = 0.1$ kg $\times 1000$ kPa $\times (0.177055 - 0.35411)$ m^3/kg $= -$ **17.706 kJ**

${_1}Q_2 = m(u_2 - u_1) + {_1}W_2 = 0.1$ kg $(114.219 - 3124.34)$ kJ/kg $- 17.706$ kJ

$\quad = $ **–318.72 kJ**

3.112

A spring loaded piston/cylinder assembly contains 1 kg water at 500°C, 3 MPa. The setup is such that the pressure is proportional to volume, P = CV. It is now cooled until the water becomes saturated vapor. Sketch the P-v diagram and find the final state, the work and heat transfer in the process.

Solution :

State 1: Table B.1.3: $v_1 = 0.11619$ m^3/kg, $u_1 = 3107.92$ kJ/kg

Process: m is constant and $P = C_0 V = C_0 m\ v = C\ v$

polytropic process with $n = -1$

$P = Cv \implies C = P_1/v_1 = 3000/0.11619 = 25\ 820$ kPa kg/m^3

State 2: $x_2 = 1$ & $P_2 = Cv_2$ (on process line)

Trial & error on T_{2sat} or P_{2sat}:

Here from B.1.2:

at 2 MPa $v_g = 0.09963 \implies C = P/v_g = 20074$ (low)

2.5 MPa $v_g = 0.07998 \implies C = P/v_g = 31258$ (high)

2.25 MPa $v_g = 0.08875 \implies C = P/v_g = 25352$ (low)

Now interpolate to match the right slope C:

$$P_2 = 2250 + 250\ \frac{25\ 820 - 25\ 352}{31\ 258 - 25\ 352} = 2270 \text{ kPa},$$

$v_2 = P_2/C = 2270/25820 = 0.0879$ m^3/kg, $u_2 = 2602.07$ kJ/kg

P is linear in V so the work becomes (area in P-v diagram)

$$_1W_2 = \int P\ dv = m\ \frac{1}{2}(P_1 + P_2)(v_2 - v_1)$$

$$= 1 \text{ kg} \times \frac{1}{2}\ (3000 + 2270) \text{ kPa } (0.0879 - 0.11619) \text{ m}^3 = \mathbf{-\ 74.5\ kJ}$$

From the energy Eq.:

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1\ (2602.07 - 3107.92) - 74.5$$

$$= \mathbf{-1250.85\ kJ}$$

Borgnakke and Sonntag

3.113

A piston cylinder contains 1.5 kg water at 600 kPa, 350°C. It is now cooled in a process where pressure is linearly related to volume to a state of 200 kPa, 150°C. Plot the P-v diagram for the process and find both the work and the heat transfer in the process.

Take as CV the 1.5 kg of water.

$$m_2 = m_1 = m ;$$

Energy Eq.3.5: $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $\qquad P = A + BV$ (linearly in V)

State 1: (P, T) => $\quad v_1 = 0.47424 \text{ m}^3/\text{kg},$

$\qquad\qquad\qquad u_1 = 2881.12 \text{ kJ/kg}$

State 2: (P, T) => $\quad v_2 = 0.95964 \text{ m}^3/\text{kg}, \ u_2 = 2576.87 \text{ kJ/kg}$

From process eq.: $\quad {}_1W_2 = \int P \ dV = \text{area} = \dfrac{m}{2}(P_1 + P_2)(v_2 - v_1)$

$$= \dfrac{1.5}{2} \text{ kg } (200 + 600) \text{ kPa } (0.95964 - 0.47424) \text{ m}^3/\text{kg}$$

$$= \textbf{291.24 kJ}$$

From energy eq.: $\quad {}_1Q_2 = m(u_2 - u_1) + {}_1W_2$

$$= 1.5 \text{ kg } (2576.87 - 2881.12) \text{ kJ/kg} + 291.24 \text{ kJ}$$

$$= \textbf{-165.14 kJ}$$

3.114

Superheated refrigerant R-134a at 20°C, 0.5 MPa is cooled in a piston/cylinder arrangement at constant temperature to a final two-phase state with quality of 50%. The refrigerant mass is 5 kg, and during this process 500 kJ of heat is removed. Find the initial and final volumes and the necessary work.

Solution:

C.V. R-134a, this is a control mass.

Continuity: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = -500$ kJ - ${}_1W_2$

State 1: T_1 , P_1 Table B.5.2, $v_1 = 0.04226$ m^3/kg ; $u_1 = 390.52$ kJ/kg

$\Rightarrow$ $V_1 = mv_1 =$ **0.211 m^3**

State 2: T_2 , x_2 $\Rightarrow$ Table B.5.1

$u_2 = 227.03 + 0.5 \times 162.16 = 308.11$ kJ/kg,

$v_2 = 0.000817 + 0.5 \times 0.03524 = 0.018437$ m^3/kg $\Rightarrow$ $V_2 = mv_2 =$ **0.0922 m^3**

${}_1W_2 = -500$ kJ - $m(u_2 - u_1) = -500$ kJ $- 5$ kg $\times (308.11 - 390.52)$ kJ/kg

$= $ **-87.9 kJ**

3.115

Two kilograms of nitrogen at 100 K, $x = 0.5$ is heated in a constant pressure process to 300 K in a piston/cylinder arrangement. Find the initial and final volumes and the total heat transfer required.

Solution:

Take CV as the nitrogen.

Continuity Eq.: $\qquad m_2 = m_1 = m$;

Energy Eq.3.5: $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = constant $\quad \Rightarrow \quad {}_1W_2 = \int PdV = Pm(v_2 - v_1)$

State 1: Table B.6.1

$\qquad v_1 = 0.001452 + 0.5 \times 0.02975 = 0.01633 \text{ m}^3/\text{kg}, \quad$ **$V_1 = 0.0327 \text{ m}^3$**

$\qquad\qquad h_1 = -73.20 + 0.5 \times 160.68 = 7.14 \text{ kJ/kg}$

State 2: (P = 779.2 kPa , 300 K) => sup. vapor interpolate in Table B.6.2

$\qquad v_2 = 0.14824 + (0.11115 - 0.14824) \times 179.2/200 = 0.115 \text{ m}^3/\text{kg}, \quad$ **$V_2 = 0.23 \text{ m}^3$**

$\qquad h_2 = 310.06 + (309.62 - 310.06) \times 179.2/200 = 309.66 \text{ kJ/kg}$

Now solve for the heat transfer from the energy equation

$\qquad {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) = 2 \text{ kg} \times (309.66 - 7.14) \text{ kJ/kg} = \textbf{605 kJ}$

Energy Equation: Solids and Liquids

3.116

In a sink 5 liters of water at 70$^\text{o}$C is combined with 1 kg aluminum pots, 1 kg of flatware (steel) and 1 kg of glass all put in at 20$^\text{o}$C. What is the final uniform temperature neglecting any heat loss and work?

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$

For the water: $v_f = 0.001023$ m^3/kg, V = 5 L = 0.005 m^3;   m = V/v = 4.8876 kg

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v\,i}(T_2 - T_1)_i = T_2 \sum m_i C_{v\,i} - \sum m_i C_{v\,i}T_{1\,i}$$

noticing that all masses have the same T$_2$ but not same initial T.

$$\sum m_i C_{v\,i} = 4.8876 \times 4.18 + 1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8 = 22.59 \text{ kJ/K}$$

Energy Eq.: $22.59\, T_2 = 4.8876 \times 4.18 \times 70 + (1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8) \times 20$

$$= 1430.11 + 43.2 = 1473.3 \text{ kJ}$$

$$T_2 = \mathbf{65.2^o C}$$

3.117

A computer CPU chip consists of 50 g silicon, 20 g copper, 50 g polyvinyl chloride (plastic). It heats from 15^oC to 70^oC as the computer is turned on. How much energy does the heating require?

Energy Eq.: $\quad U_2 - U_1 = \sum m_i (u_2 - u_1)_i = {}_1Q_2 - {}_1W_2$

For the solid masses we will use the specific heats, Table A.3, and they all have the same temperature so

$$\sum m_i(u_2 - u_1)_i = \sum m_iC_{v\,i} (T_2 - T_1)_i = (T_2 - T_1)\sum m_iC_{v\,i}$$

$$\sum m_iC_{v\,i} = 0.05 \times 0.7 + 0.02 \times 0.42 + 0.05 \times 0.96 = 0.0914 \text{ kJ/K}$$

$$U_2 - U_1 = 0.0914 \text{ kJ/K} \times (70 - 15) \text{ K} = \textbf{5.03 kJ}$$

3.118

A copper block of volume 1 L is heat treated at 500°C and now cooled in a 200-L oil bath initially at 20°C, shown in Fig. P3.118. Assuming no heat transfer with the surroundings, what is the final temperature?

Solution:

   C.V. Copper block and the oil bath.

   Also assume no change in volume so the work will be zero.

   Energy Eq.: $U_2 - U_1 = m_{met}(u_2 - u_1)_{met} + m_{oil}(u_2 - u_1)_{oil} = {}_1Q_2 - {}_1W_2 = 0$

   Properties from Table A.3 and A.4

   $$m_{met} = V\rho = 0.001 \text{ m}^3 \times 8300 \text{ kg/m}^3 = 8.3 \text{ kg},$$

   $$m_{oil} = V\rho = 0.2 \text{ m}^3 \times 910 \text{ kg/m}^3 = 182 \text{ kg}$$

   Solid and liquid Eq.3.32: $\Delta u \cong C_v \Delta T,$

   Table A.3 and A.4: $C_{v \text{ met}} = 0.42 \dfrac{kJ}{kg \text{ K}}, \ C_{v \text{ oil}} = 1.8 \dfrac{kJ}{kg \text{ K}}$

   The energy equation for the C.V. becomes

   $$m_{met}C_{v \text{ met}}(T_2 - T_{1,met}) + m_{oil}C_{v \text{ oil}}(T_2 - T_{1,oil}) = 0$$

   $$8.3 \text{ kg} \times 0.42 \frac{kJ}{kg \text{ K}} (T_2 - 500 \text{ C}) + 182 \text{ kg} \times 1.8 \frac{kJ}{kg \text{ K}} (T_2 - 20 \text{ C}) = 0$$

   $$331.09 \ T_2 - 1743 - 6552 = 0$$

   $$\Rightarrow \ T_2 = \mathbf{25 \ °C}$$

Borgnakke and Sonntag

3.119

A 1 kg steel pot contains 1 kg liquid water both at 15^oC. It is now put on the stove where it is heated to the boiling point of the water. Neglect any air being heated and find the total amount of energy needed.

Solution:

Energy Eq.: $U_2 - U_1 = {_1}Q_2 - {_1}W_2$

The steel does not change volume and the change for the liquid is minimal, so ${_1}W_2 \cong 0$.

State 2: $T_2 = T_{sat}$ (1atm) $= 100^oC$

   Tbl B.1.1 : $u_1 = 62.98$ kJ/kg, $u_2 = 418.91$ kJ/kg

   Tbl A.3 : $C_{st} = 0.46$ kJ/kg K

Solve for the heat transfer from the energy equation

$${_1}Q_2 = U_2 - U_1 = m_{st} \, (u_2 - u_1)_{st} + m_{H2O} \, (u_2 - u_1)_{H2O}$$
$$= m_{st}C_{st} \ (T_2 - T_1) + m_{H2O} \, (u_2 - u_1)_{H2O}$$

$${_1}Q_2 = 1 \text{ kg} \times 0.46 \, \frac{kJ}{kg \ K} \times (100 - 15) \text{ K} + 1 \text{ kg} \times (418.91 - 62.98) \text{ kJ/kg}$$
$$= 39.1 + 355.93 = \textbf{395 kJ}$$

3.120

I have 2 kg of liquid water at 20°C, 100 kPa. I now add 20 kJ of energy at a constant pressure. How hot does it get if it is heated? How fast does it move if it is pushed by a constant horizontal force? How high does it go if it is raised straight up?

a) Heat at 100 kPa.

Energy equation:

$$E_2 - E_1 = {}_1Q_2 - {}_1W_2 = {}_1Q_2 - P(V_2 - V_1) = H_2 - H_1 = m(h_2 - h_1)$$

$$h_2 = h_1 + {}_1Q_2/m = 83.94 + 20/2 = 94.04 \text{ kJ/kg}$$

Back interpolate in Table B.1.1: $T_2 = $ **22.5^{o}C**

[We could also have used $\Delta T = {}_1Q_2/mC = 20 / (2*4.18) = 2.4^oC$]

b) Push at constant P. It gains kinetic energy.

$$0.5 \text{ m } \mathbf{V}_2^2 = {}_1W_2$$

$$\mathbf{V}_2 = \sqrt{2 \, {}_1W_2/m} = \sqrt{2 \times 20 \times 1000 \text{ J/2 kg}} = \textbf{141.4 m/s}$$

c) Raised in gravitational field

$$\text{m g } Z_2 = {}_1W_2$$

$$Z_2 = {}_1W_2/m \text{ g} = \frac{20\,000 \text{ J}}{2 \text{ kg} \times 9.807 \text{ m/s}^2} = \textbf{1019 m}$$

Comment: Notice how fast (500 km/h) and how high it should be to have the same energy as raising the temperature just 2 degrees. I.e. in most applications we can disregard the kinetic and potential energies unless we have very high **V** or Z.

3.121

A house is being designed to use a thick concrete floor mass as thermal storage material for solar energy heating. The concrete is 30 cm thick and the area exposed to the sun during the daytime is 4 m × 6 m. It is expected that this mass will undergo an average temperature rise of about 3°C during the day. How much energy will be available for heating during the nighttime hours?

Solution:

C.V.: Control mass concrete.

$$V = 4 \text{ m} \times 6 \text{ m} \times 0.3 \text{ m} = 7.2 \text{ m}^3$$

Concrete is a solid with some properties listed in Table A.3

$$m = \rho V = 2200 \text{ kg/m}^3 \times 7.2 \text{ m}^3 = 15 \ 840 \text{ kg}$$

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

The available heat transfer is the change in U. From Eq.3.33 and C from table A.3

$$\Delta U = m \ C \ \Delta T = 15 \ 840 \text{ kg} \times 0.88 \frac{\text{kJ}}{\text{kg K}} \times 3 \text{ K} = 41 \ 818 \text{ kJ} = \textbf{41.82 MJ}$$

3.122

Because a hot water supply must also heat some pipe mass as it is turned on so it does not come out hot right away. Assume 80$^{\text{o}}$C liquid water at 100 kPa is cooled to 45$^{\text{o}}$C as it heats 15 kg of copper pipe from 20 to 45$^{\text{o}}$C. How much mass (kg) of water is needed?

Solution:

C.V. Water and copper pipe. No external heat transfer, no work.

Energy Eq.3.5: $U_2 - U_1 = \Delta U_{cu} + \Delta U_{H_2O} = 0 - 0$

From Eq.3.33 and Table A.3:

$$\Delta U_{cu} = mC\,\Delta T = 15\ \text{kg} \times 0.42\,\frac{\text{kJ}}{\text{kg K}} \times (45 - 20)\ \text{K} = 157.5\ \text{kJ}$$

From the energy equation

$$m_{H_2O} = -\,\Delta U_{cu} / \Delta u_{H_2O}$$

$$m_{H_2O} = \Delta U_{cu} / C_{H_2O}(-\Delta T_{H_2O}) = \frac{157.5}{4.18 \times 35} = \textbf{1.076 kg}$$

or using Table B.1.1 for water

$$m_{H_2O} = \Delta U_{cu} / (u_1 - u_2) = \frac{157.5}{334.84 - 188.41}\frac{\text{kJ}}{\text{kJ/kg}} = \textbf{1.076 kg}$$

Cu pipe

Water

The real problem involves a flow and is not analyzed by this simple process.

3.123

A car with mass 1275 kg drives at 60 km/h when the brakes are applied quickly to decrease its speed to 20 km/h. Assume the brake pads are 0.5 kg mass with heat capacity of 1.1 kJ/kg K and the brake discs/drums are 4.0 kg steel. Further assume both masses are heated uniformly. Find the temperature increase in the brake assembly.

Solution:

C.V. Car. Car loses kinetic energy and brake system gains internal u.

No heat transfer (short time) and no work term.

m = constant;

Energy Eq.3.5: $E_2 - E_1 = 0 - 0 = m_{car} \frac{1}{2}(V_2^2 - V_1^2) + m_{brake}(u_2 - u_1)$

The brake system mass is two different kinds so split it, also use C_v from Table A.3 since we do not have a u table for steel or brake pad material.

$$m_{steel}\, C_v\, \Delta T + m_{pad}\, C_v\, \Delta T = m_{car}\, 0.5\, (60^2 - 20^2) \left(\frac{1000}{3600}\right)^2 \text{ m}^2/\text{s}^2$$

$$(4 \times 0.46 + 0.5 \times 1.1) \frac{\text{kJ}}{\text{K}}\, \Delta T = 1275 \text{ kg} \times 0.5 \times (3200 \times 0.077\ 16) \text{ m}^2/\text{s}^2$$

$$= 157\ 406 \text{ J} = 157.4 \text{ kJ}$$

$$\Rightarrow \Delta T = \textbf{65.9 °C}$$

3.124

A piston cylinder (0.5 kg steel altogether) maintaining a constant pressure has 0.2 kg R-134a as saturated vapor at 150 kPa. It is heated to 40°C and the steel is at the same temperature as the R-134a at any time. Find the work and heat transfer for the process.

C.V. The R-134a plus the steel. Constant total mass

$m_2 = m_1 = m$;

$U_2 - U_1 = m_{R134a}(u_2 - u_1)_{R134a} + m_{steel}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: B.5.2 sat. vapor $v_1 = 0.13139$ m³/kg, $u_1 = 368.06$ kJ/kg

State 2: B.5.2 sup. vapor $v_2 = 0.16592$ m³/kg, $u_2 = 411.59$ kJ/kg

$V_1 = mv_1 = 0.2 \times 0.13139 = 0.02628$ m³

$V_2 = mv_2 = 0.2 \times 0.16592 = 0.03318$ m³

Steel: A.3, $C_{steel} = 0.46$ kJ/kg-K

Process: P = C for the R134a and constant volume for the steel =>

${}_1W_2 = \int P\ dV = P_1(V_2 - V_1) = 150$ kPa $\times (0.03318 - 0.02628)$ m³

= **1.035 kJ**

${}_1Q_2 = m_{R134a}(u_2 - u_1) + m_{steel}(u_2 - u_1) + {}_1W_2$

$= m_{R134a}(u_2 - u_1) + m_{steel}C_{steel}(T_2 - T_1) + {}_1W_2$

$= 0.2 \times (411.59 - 368.06) + 0.5 \times 0.46 \times [40 - (-17.29)] + 1.035$

$= 8.706 + 13.177 + 1.035 = $ **22.92 kJ**

3.125

A 25 kg steel tank initially at –10$^{\text{o}}$C is filled up with 100 kg of milk (assume properties as water) at 30$^{\text{o}}$C. The milk and the steel come to a uniform temperature of +5 $^{\text{o}}$C in a storage room. How much heat transfer is needed for this process?

Solution:

C.V. Steel + Milk. This is a control mass.

Energy Eq.3.5: $U_2 - U_1 = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

Process: V = constant, so there is no work

$\qquad {}_1W_2 = 0.$

Use Eq.3.33 and values from A.3 and A.4 to evaluate changes in u

$${}_1Q_2 = m_{\text{steel}}\,(u_2 \text{ - } u_1)_{\text{steel}} + m_{\text{milk}}(u_2 \text{ - } u_1)_{\text{milk}}$$

$$= 25 \text{ kg} \times 0.46\,\frac{\text{kJ}}{\text{kg K}} \times [5 - (-10)] \text{ K} + 100 \text{ kg} \times 4.18\,\frac{\text{kJ}}{\text{kg K}} \times (5 - 30) \text{ K}$$

$$= \ 172.5 - 10450 \ = -\textbf{10277 kJ}$$

3.126

An engine consists of a 100 kg cast iron block with a 20 kg aluminum head, 20 kg steel parts, 5 kg engine oil and 6 kg glycerine (antifreeze). Everything begins at 5° C and as the engine starts we want to know how hot it becomes if it absorbs a net of 7000 kJ before it reaches a steady uniform temperature.

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

Process: The steel does not change volume and the change for the liquid is minimal, so ${}_1W_2 \cong 0$.

So sum over the various parts of the left hand side in the energy equation

$$m_{Fe}(u_2 - u_1) + m_{Al}(u_2 - u_1)_{Al} + m_{st}(u - u_1)_{st}$$
$$+ m_{oil}(u_2 - u_1)_{oil} + m_{gly}(u_2 - u_1)_{gly} = {}_1Q_2$$

Table A.3 : $C_{Fe} = 0.42$, $C_{Al} = 0.9$, $C_{st} = 0.46$ all units of kJ/kg K

Table A.4 : $C_{oil} = 1.9$, $C_{gly} = 2.42$ all units of kJ/kg K

So now we factor out $T_2 - T_1$ as $u_2 - u_1 = C(T_2 - T_1)$ for each term

$$[m_{Fe}C_{Fe} + m_{Al}C_{Al} + m_{st}C_{st} + m_{oil}C_{oil} + m_{gly}C_{gly}] (T_2 - T_1) = {}_1Q_2$$

$T_2 - T_1 = {}_1Q_2 / \Sigma m_i C_i$

$$= \frac{7000}{100 \times 0.42 + 20 \times 0.9 + 20 \times 0.46 + 5 \times 1.9 + 6 \times 2.42} \frac{kJ}{kJ/K}$$

$$= \frac{7000}{93.22} K = 75 K$$

$T_2 = T_1 + 75^oC = 5 + 75 = \mathbf{80^oC}$

Properties (u, h, C_v and C_p), Ideal Gas

3.127

An ideal gas is heated from 500 to 1500 K. Find the change in enthalpy using constant specific heat from Table A.5 (room temperature value) and discuss the accuracy of the result if the gas is

    a. Argon   b.   Oxygen   c.   Carbon dioxide

Solution:

$$T_1 = 500 \text{ K}, T_2 = 1500 \text{ K}, \quad \Delta h = C_{P0}(T_2 - T_1)$$

a) Ar : $\Delta h = 0.520$ kJ/kg-K $\times$ (1500-500) K = 520 kJ/kg

      Monatomic inert gas very good approximation.

b) O_2 : $\Delta h = 0.922$ kJ/kg-K $\times$ (1500-500) K = 922 kJ/kg

      Diatomic gas approximation is OK with some error.

c) CO_2: $\Delta h = 0.842$ kJ/kg-K $\times$ (1500-500) K = 842 kJ/kg

      Polyatomic gas heat capacity changes, see figure 3.26

      See also appendix C for more explanation.

3.128

Use the ideal gas air table A.7 to evaluate the heat capacity C_p at 300 K as a slope of the curve h(T) by $\Delta h/\Delta T$. How much larger is it at 1000 K and 1500 K.

Solution :

From Eq.3.39:

$$C_p = \frac{dh}{dT} = \frac{\Delta h}{\Delta T} = \frac{h_{320} - h_{290}}{320 - 290} = \textbf{1.005 kJ/kg K}$$

$$1000 \text{ K} \quad C_p = \frac{\Delta h}{\Delta T} = \frac{h_{1050} - h_{950}}{1050 - 950} = \frac{1103.48 - 989.44}{100} = \textbf{1.140 kJ/kg K}$$

$$1500 \text{ K} \quad C_p = \frac{\Delta h}{\Delta T} = \frac{h_{1550} - h_{1450}}{1550 - 1450} = \frac{1696.45 - 1575.4}{100} = \textbf{1.21 kJ/kg K}$$

Notice an increase of 14%, 21% respectively.

3.129

Estimate the constant specific heats for R-134a from Table B.5.2 at 100 kPa and 125$^{\text{o}}$C. Compare this to table A.5 and explain the difference.

Solution:

Using values at 100 kPa for h and u at 120$^{\text{o}}$C and 130$^{\text{o}}$C

from Table B5.2, the approximate specific heats at 125$^{\text{o}}$C are

$$C_p \approx \frac{\Delta h}{\Delta T} = \frac{521.98 - 511.95}{130 - 120} = 1.003 \text{ kJ/kg K}$$

compared with 0.852 kJ/kg K for the ideal-gas value at 25$^{\text{o}}$C
from Table A.5.

$$C_v \approx \frac{\Delta u}{\Delta T} = \frac{489.36 - 480.16}{130 - 120} = 0.920 \text{ kJ/kg K}$$

compared with 0.771 kJ/kg K for the ideal-gas value at 25$^{\text{o}}$C
from Table A.5.

There are two reasons for the differences. First, R-134a is not exactly an ideal gas at the given state, 125$^{\text{o}}$C and 100 kPa. Second and by far the biggest reason for the differences is that R-134a, chemically CF_3CH_2 , is a polyatomic molecule with multiple vibrational mode contributions to the specific heats (see Appendix C), such that they are strongly dependent on temperature. Note that if we repeat the above approximation for C_p in Table B.5.2 at 25$^{\text{o}}$C, the resulting value is 0.851 kJ/kg K.

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3.130

We want to find the change in u for carbon dioxide between 600 K and 1200 K.

a) Find it from a constant C_{vo} from table A.5

b) Find it from a C_{vo} evaluated from equation in A.6 at the average T.

c) Find it from the values of u listed in table A.8

Solution :

a) $\Delta u \cong C_{vo} \Delta T = 0.653$ kJ/kg-K $\times (1200 - 600)$ K = **391.8 kJ/kg**

b) $T_{avg} = \dfrac{1}{2} (1200 + 600) = 900, \qquad \theta = \dfrac{T}{1000} = \dfrac{900}{1000} = 0.9$

$C_{po} = 0.45 + 1.67 \times 0.9 - 1.27 \times 0.9^2 + 0.39 \times 0.9^3 = 1.2086$ kJ/kg K

$C_{vo} = C_{po} - R = 1.2086 - 0.1889 = 1.0197$ kJ/kg K

$\Delta u = 1.0197 \times (1200 - 600) = $ **611.8 kJ/kg**

c) $\Delta u = 996.64 - 392.72 = $ **603.92 kJ/kg**

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3.131

Nitrogen at 300 K, 3 MPa is heated to 500 K. Find the change in enthalpy using a) Table B.6, b) Table A.8, and c) Table A.5.

B.6.2 $h_2 - h_1 = 519.29 - 304.94 = 214.35$ kJ/kg

A.8 $h_2 - h_1 = 520.75 - 311.67 = 209.08$ kJ/kg

A.5 $h_2 - h_1 = C_{po}(T_2 - T_1) = 1.042$ kJ/kg-K $\times (500 - 300)$ K

                $= 208.4$ kJ/kg

Comment: The results are listed in order of accuracy (B.6.2 best).

3.132

We want to find the change in u for carbon dioxide between $50^{\circ}C$ and $200^{\circ}C$ at a pressure of 10 MPa. Find it using ideal gas and Table A.5 and repeat using the B section table.

Solution:

Using the value of C_{vo} for CO_2 from Table A.5,

$$\Delta u = C_{vo} \Delta T = 0.653 \text{ kJ/kg-K} \times (200 - 50) \text{ K} = \textbf{97.95 kJ/kg}$$

Using values of u from Table B3.2 at 10 000 kPa, with linear interpolation between $40^{\circ}C$ and $60^{\circ}C$ for the $50^{\circ}C$ value,

$$\Delta u = u_{200} - u_{50} = 437.6 - 230.9 = \textbf{206.7 kJ/kg}$$

Note: Since the state $50^{\circ}C$, 10 000 kPa is in the dense-fluid supercritical region, a linear interpolation is quite inaccurate. The proper value for u at this state is found from the CATT software to be 245.1 instead of 230.9. This results is

$$\Delta u = u_{200} - u_{50} = 437.6 - 245.1 = \textbf{192.5 kJ/kg}$$

3.133

We want to find the change in u for oxygen gas between 600 K and 1200 K.

a) Find it from a constant C_{vo} from table A.5

b) Find it from a C_{vo} evaluated from equation in A.6 at the average T.

c) Find it from the values of u listed in table A.8

Solution:

a) $\Delta u \cong C_{vo} \, \Delta T = 0.662 \text{ kJ/kg-K} \times (1200 - 600) \text{ K} = \textbf{397.2 kJ/kg}$

b) $T_{avg} = \dfrac{1}{2}(1200 + 600) = 900 \text{ K}, \qquad \theta = \dfrac{T}{1000} = \dfrac{900}{1000} = 0.9$

$C_{po} = 0.88 - 0.0001 \times 0.9 + 0.54 \times 0.9^2 - 0.33 \times 0.9^3 = 1.0767 \text{ kJ/kgK}$

$C_{vo} = C_{po} - R = 1.0767 - 0.2598 = 0.8169 \text{ kJ/kg-K}$

$\Delta u = 0.8169 \times (1200 - 600) = \textbf{490.1 kJ/kg}$

c) $\Delta u = 889.72 - 404.46 = \textbf{485.3 kJ/kg}$

3.134

For a special application we need to evaluate the change in enthalpy for carbon dioxide from 30°C to 1500°C at 100 kPa. Do this using constant specific heat from Table A.5 and repeat using Table A.8. Which is the more accurate one?

Solution:

Using constant specific heat:

$$\Delta h = C_{po}\Delta T = 0.842\ (1500 - 30) = \textbf{1237.7 kJ/kg}$$

Using Table A.8:

$$30°C = 303.15\ K \Rightarrow h = 214.38 + \frac{3.15}{50}(257.9 - 214.38) = 217.12\ kJ/kg$$

$$1500°C = 1773.15\ K \Rightarrow$$

$$h = 1882.43 + \frac{73.15}{100}(2017.67 - 1882.43) = 1981.36\ kJ/kg$$

$$\Delta h = 1981.36 - 217.12 = \textbf{1764.2 kJ/kg}$$

The result from A.8 is best. For large ΔT or small ΔT at high T_{avg}, constant specific heat is poor approximation or it must be evaluated at a higher T (A.5 is at 25°C).

3.135

Water at 150°C, 400 kPa, is brought to 1200°C in a constant pressure process. Find the change in the specific internal energy, using a) the steam tables, b) the ideal gas water table A.8, and c) the specific heat from A.5.

Solution:

a)

State 1: Table B.1.3   Superheated vapor u_1 = 2564.48 kJ/kg

State 2: Table B.1.3 u_2 = 4467.23 kJ/kg

u_2 - u_1 = 4467.23 - 2564.48 = **1902.75 kJ/kg**

b)

Table A.8 at 423.15 K: u_1 = 591.41 kJ/kg

Table A.8 at 1473.15 K: u_2 = 2474.25 kJ/kg

u_2 - u_1 = 2474.25 - 591.41 = **1882.8 kJ/kg**

c)   Table A.5 : C_{vo} = 1.41 kJ/kgK

u_2 - u_1 = 1.41 kJ/kgK × (1200 – 150) K = **1480.5 kJ/kg**

Notice how the average slope from 150°C to 1200 °C is higher than the one at 25°C (= C_{vo})

3.136

Repeat the previous problem but use a constant specific heat at the average temperature from equation in Table A.6 and also integrate the equation in Table A.6 to get the change in enthalpy.

$$T_{ave} = \frac{1}{2} (30 + 1500) + 273.15 = 1038.15 \text{ K}; \quad \theta = T/1000 = 1.0382$$

$$\text{Table A.6} \quad \Rightarrow \quad C_{po} = 1.2513 \text{ kJ/kg-K}$$

$$\Delta h = C_{po,ave} \, \Delta T = 1.2513 \text{ kJ/kg-K} \times 1470 \text{ K} = \textbf{1839 kJ/kg}$$

For the entry to Table A.6:

$$30°C = 303.15 \text{ K} \implies \theta_1 = 0.30315$$

$$1500°C = 1773.15 \text{ K} \implies \theta_2 = 1.77315$$

$$\Delta h = h_2 - h_1 = \int C_{po} \, dT$$

$$= [0.45 \, (\theta_2 - \theta_1) + 1.67 \times \frac{1}{2} \, (\theta_2^2 - \theta_1^2)$$

$$-1.27 \times \frac{1}{3} \, (\theta_2^3 - \theta_1^3) + 0.39 \times \frac{1}{4} \, (\theta_2^4 - \theta_1^4)]$$

$$= \textbf{1762.8 kJ/kg}$$

3.137

Water at 20°C, 100 kPa, is brought to 200 kPa, 1500°C. Find the change in the specific internal energy, using the water table and the ideal gas water table in combination.

Solution:

State 1: Table B.1.1 $u_1 \cong u_f = 83.95$ kJ/kg

State 2: Highest T in Table B.1.3 is 1300°C

Using a Δu from the ideal gas tables, A.8, we get

$$u_{1500} = 3139 \text{ kJ/kg} \qquad u_{1300} = 2690.72 \text{ kJ/kg}$$

$$u_{1500} - u_{1300} = 448.26 \text{ kJ/kg}$$

We now add the ideal gas change at low P to the steam tables, B.1.3, $u_X = 4683.23$ kJ/kg as the reference.

$$u_2 - u_1 = (u_2 - u_X)_{\text{ID.G.}} + (u_X - u_1)$$

$$= 448.28 + 4683.23 - 83.95 = \textbf{5048 kJ/kg}$$

3.138

Reconsider Problem 3.134 and examine if also using Table B.3 would be more accurate and explain.

Table B.3 does include non-ideal gas effects, however at 100 kPa these effects are extremely small so the answer from Table A.8 is accurate.

Table B.3. does not cover the 100 kPa superheated vapor states as the saturation pressure is below the triple point pressure. Secondly Table B.3 does not go up to the high temperatures covered by Table A.8 and A.9 at which states you do have ideal gas behavior. Table B.3 covers the region of states where the carbon dioxide is close to the two-phase region and above the critical point (dense fluid) which are all states where you cannot assume ideal gas.

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Specific Heats Ideal Gas

3.139

Air is heated from 300 to 350 K at V = C. Find $_1q_2$. What if from 1300 to 1350 K?

Process: V = C   → $_1W_2 = \emptyset$

Energy Eq.: $u_2 - u_1 = {}_1q_2 - 0$ → $_1q_2 = u_2 - u_1$

Read the u-values from Table A.7.1

   a) $_1q_2 = u_2 - u_1 = 250.32 - 214.36 =$ **36.0 kJ/kg**

   b) $_1q_2 = u_2 - u_1 = 1067.94 - 1022.75 =$ **45.2 kJ/kg**

   case a) $C_v \approx 36/50 = 0.72$ kJ/kg K , see A.5

   case b) $C_v \approx 45.2/50 = 0.904$ kJ/kg K (25 % higher)

3.140

A rigid container has 2 kg of carbon dioxide gas at 100 kPa, 1200 K that is heated to 1400 K. Solve for the heat transfer using a. the heat capacity from Table A.5 and b. properties from Table A.8

Solution:

C.V. Carbon dioxide, which is a control mass.

Energy Eq.3.5: $U_2 - U_1 = m (u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $\Delta V = 0 \Rightarrow {}_1W_2 = 0$

a) For constant heat capacity we have: $u_2 - u_1 = C_{vo} (T_2 - T_1)$ so

$$ {}_1Q_2 \cong mC_{vo} (T_2 - T_1) = 2 \text{ kg} \times 0.653 \text{ kJ/kg-K} \times (1400 - 1200) \text{ K} = \textbf{261.2 kJ} $$

b) Taking the u values from Table A.8 we get

$$ {}_1Q_2 = m (u_2 - u_1) = 2 \text{ kg} \times (1218.38 - 996.64) \text{ kJ/kg} = \textbf{443.5 kJ} $$

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3.141

Do the previous problem for nitrogen, N_2, gas.

A rigid container has 2 kg of carbon dioxide gas at 100 kPa, 1200 K that is heated to 1400 K. Solve for the heat transfer using a. the heat capacity from Table A.5 and b. properties from Table A.8

Solution:

C.V. Nitrogen gas, which is a control mass.

Energy Eq.3.5: $U_2 - U_1 = m (u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $\Delta V = 0 \implies {}_1W_2 = 0$

a) For constant heat capacity we have: $u_2 - u_1 = C_{vo} (T_2 - T_1)$ so

$${}_1Q_2 \cong mC_{vo} (T_2 - T_1) = 2 \text{ kg} \times 0.745 \text{ kJ/kg-K} \times (1400 - 1200) \text{ K} = \textbf{298 kJ}$$

b) Taking the u values from Table A.8, we get

$${}_1Q_2 = m (u_2 - u_1) = 2 \text{ kg} \times (1141.35 - 957) \text{ kJ/kg} = \textbf{368.7 kJ}$$

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3.142

Air, 3 kg, is in a piston/cylinder similar to Fig. P.3.5 at 27°C, 300 kPa. It is now heated to 500 K. Plot the process path in a P-v diagram, and find the work and heat transfer in the process.

Solution:

CV Air, so this is a control mass.

Energy Eq.3.5: $U_2 - U_1 = m \, (u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process: $P = C$ so ${_1}W_2 = \int P dV = P_1(V_2 - V_1)$

State 1: T_1, P_1 ideal gas so $P_1V_1 = mRT_1$

$V_1 = mR \, T_1 / P_1 = 3 \text{ kg} \times 0.287 \text{ kJ/kg-K} \times 300.15 \text{ K} / 300 \text{ kPa}$

$= 0.86143 \text{ m}^3$

State 2: $T_2, P_2 = P_1$ and ideal gas so $P_2V_2 = mRT_2$

$V_2 = mR \, T_2 / P_2 = 3 \times 0.287 \times 500/300 = 1.435 \text{ m}^3$

From the process

${_1}W_2 = \int P dV = P \, (V_2 - V_1) = 300 \text{ kPa}(1.435 - 0.86143) \text{ m}^3 = \textbf{172.1 kJ}$

From the energy equation

${_1}Q_2 = m \, (u_2 - u_1) + {_1}W_2 = mC_{vo} \, (T_2 - T_1) + {_1}W_2$

$= 3 \text{ kg} \times 0.717 \text{ kJ/kg-K} \times (500 - 300) \text{ K} + 172.1 \text{ kJ}$

$= \textbf{602.3 kJ}$

3.143

A closed rigid container is filled with 1.5 kg water at 100 kPa, 55°C, 1 kg of stainless steel and 0.5 kg of PVC (polyvinyl chloride) both at 20°C and 0.1 kg of air at 400 K, 100 kPa. It is now left alone with no external heat transfer and no water vaporizes. Find the final temperature and air pressure.

CV. Container.

Process: V = constant $\Rightarrow$ $_1W_2 = 0$ and also given $_1Q_2 = 0$

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {_1}Q_2 - {_1}W_2 = 0$

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v\,i}\,(T_2 - T_1)_i = T_2 \sum m_i C_{v\,i} - \sum m_i C_{v\,i} T_{1\,i}$$

noticing that all masses have the same T_2 but not same initial T.

$$\sum m_i C_{v\,i} = 1.5 \times 4.18 + 1 \times 0.46 + 0.5 \times 0.96 + 0.1 \times 0.717 = 7.282 \text{ kJ/K}$$

The T for air must be converted to ^{o}C like the others.

Energy Eq.: $T_2 \sum m_i C_{v\,i} = \sum m_i C_{v\,i} T_{1\,i}$

$$7.282\, T_2 = 1.5 \times 4.18 \times 55 + (1 \times 0.46 + 0.5 \times 0.96) \times 20$$

$$+ 0.1 \times 0.717 \times \ (400 - 273.15) = 372.745 \text{ kJ}$$

$$T_2 = \mathbf{51.2^oC}$$

The volume of the air is constant so from PV = mRT it follows that P varies with T

$$P_2 = P_1\, T_2/T_{1\,air} = 100 \text{ kPa} \times 324.34 \text{ K}/\,400 \text{ K} = \mathbf{81\ kPa}$$

3.144

A 250 L rigid tank contains methane at 500 K, 1500 kPa. It is now cooled down to 300 K. Find the mass of methane and the heat transfer using ideal gas.

Solution:

C.V. Methane gas, which is a control mass.

Energy Eq.3.5: $U_2 - U_1 = m\,(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $\Delta V = 0 \Rightarrow {}_1W_2 = 0$

State 1: Ideal gas so

$$m = P_1V/RT_1 = \frac{1500 \times 0.25}{0.5183 \times 500}\,\frac{\text{kPa m}^3}{(\text{kJ/kg-K}) \times \text{K}} = \textbf{1.447 kg}$$

For constant heat capacity A.5, we have: $u_2 - u_1 = C_{vo}\,(T_2 - T_1)$ so

$$u_2 - u_1 = C_v\,(T_2 - T_1) = 1.736 \text{ kJ/kg-K} \times (300 - 500)\text{ K} = -347.2 \text{ kJ/kg}$$

$${}_1Q_2 = m(u_2 - u_1) = 1.447 \text{ kg }(-347.2)\text{ kJ/kg} = -\textbf{502.4 kJ}$$

3.145

A 10-m high cylinder, cross-sectional area 0.1 m^2, has a massless piston at the bottom with water at 20°C on top of it, shown in Fig. P3.145. Air at 300 K, volume 0.3 m^3, under the piston is heated so that the piston moves up, spilling the water out over the side. Find the total heat transfer to the air when all the water has been pushed out.

Solution:

The water on top is compressed liquid and has volume and mass

$$V_{H_2O} = V_{tot} - V_{air} = 10 \times 0.1 - 0.3 = 0.7 \text{ m}^3$$

$$m_{H_2O} = V_{H_2O}/v_f = 0.7 \text{ m}^3/ 0.001002 \text{ m}^3/\text{kg} = 698.6 \text{ kg}$$

The initial air pressure is then

$$P_1 = P_0 + m_{H_2O}g/A = 101.325 + \frac{698.6 \times 9.807}{0.1 \times 1000} = \textbf{169.84 kPa}$$

and then $m_{air} = PV/RT = \dfrac{169.84 \times 0.3}{0.287 \times 300} \dfrac{\text{kPa m}^3}{\text{(kJ/kg-K)} \times \text{K}} = 0.592 \text{ kg}$

State 2: No liquid over piston: $P_2 = P_0 = 101.325 \text{ kPa}, \quad V_2 = 10 \times 0.1 = 1 \text{ m}^3$

State 2: $P_2, V_2 \quad \Rightarrow \quad T_2 = \dfrac{T_1 P_2 V_2}{P_1 V_1} = \dfrac{300 \times 101.325 \times 1}{169.84 \times 0.3} = 596.59 \text{ K}$

The process line shows the work as an area

$$_1W_2 = \int PdV = \tfrac{1}{2}(P_1 + P_2)(V_2 - V_1) = \tfrac{1}{2}(169.84 + 101.325)(1 - 0.3) = 94.91 \text{ kJ}$$

The energy equation solved for the heat transfer becomes

$$_1Q_2 = m(u_2 - u_1) + {_1W_2} \cong mC_v(T_2 - T_1) + {_1W_2}$$

$$= 0.592 \text{ kg} \times 0.717 \text{ kJ/kg-K} \times (596.59 - 300) \text{ K} + 94.91 \text{ kJ} = \textbf{220.7 kJ}$$

Remark: we could have used u values from Table A.7:

$$u_2 - u_1 = 432.5 - 214.36 = 218.14 \text{ kJ/kg} \quad \text{versus } 212.5 \text{ kJ/kg with } C_v.$$

3.146

A cylinder with a piston restrained by a linear spring contains 2 kg of carbon dioxide at 500 kPa, 400°C. It is cooled to 40°C, at which point the pressure is 300 kPa. Calculate the heat transfer for the process.

Solution:

C.V. The carbon dioxide, which is a control mass.

Continuity Eq.: $m_2 - m_1 = 0$

Energy Eq.: $m\,(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = A + BV$ (linear spring)

$${}_1W_2 = \int PdV = \tfrac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

Equation of state: $PV = mRT$ (ideal gas)

State 1: $V_1 = mRT_1/P_1 = 2 \times 0.18892 \times 673.15 /500 = 0.5087 \ \text{m}^3$

State 2: $V_2 = mRT_2/P_2 = 2 \times 0.18892 \times 313.15 /300 = 0.3944 \ \text{m}^3$

$${}_1W_2 = \tfrac{1}{2}(500 + 300)(0.3944 - 0.5087) = -45.72 \ \text{kJ}$$

To evaluate $u_2 - u_1$ we will use the specific heat at the average temperature.

From Figure 3.26: $C_{po}(T_{avg}) \approx 5.4\ R = 1.02 \implies C_{vo} = 0.83 = C_{po} - R$

For comparison the value from Table A.5 at 300 K is $C_{vo} = 0.653$ kJ/kg K

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = mC_{vo}(T_2 - T_1) + {}_1W_2$$
$$= 2 \times 0.83(40 - 400) - 45.72 = \textbf{-643.3 kJ}$$

Remark: We could also have used the ideal gas table in A.8 to get $u_2 - u_1$.

3.147

Water at 100 kPa, 400 K is heated electrically adding 700 kJ/kg in a constant pressure process. Find the final temperature using

        a) The water tables B.1   b) The ideal gas tables A.8

        c) Constant specific heat from A.5

Solution :

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process: $P = \text{constant}$ $\Rightarrow$ ${}_1w_2 = P (v_2 - v_1)$

Substitute this into the energy equation to get

$$_1q_2 = h_2 - h_1$$

Table B.1:

$$h_1 \cong 2675.46 + \frac{126.85 - 99.62}{150 - 99.62} \times (2776.38 - 2675.46) = 2730.0 \text{ kJ/kg}$$

$$h_2 = h_1 + {}_1q_2 = 2730 + 700 = 3430 \text{ kJ/kg}$$

$$T_2 = 400 + (500 - 400) \times \frac{3430 - 3278.11}{3488.09 - 3278.11} = \textbf{472.3°C}$$

Table A.8:

$$h_2 = h_1 + {}_1q_2 = 742.4 + 700 = 1442.4 \text{ kJ/kg}$$

$$T_2 = 700 + (750 - 700) \times \frac{1442.4 - 1338.56}{1443.43 - 1338.56} = 749.5 \text{ K} = \textbf{476.3°C}$$

Table A.5

$$h_2 - h_1 \cong C_{po} (T_2 - T_1)$$

$$T_2 = T_1 + {}_1q_2 / C_{po} = 400 + 700 / 1.872 = 773.9 \text{K} = \textbf{500.8°C}$$

3.148

A constant pressure container is filled with 1.5 kg water at 100 kPa, 55°C, 1 kg of stainless steel and 0.5 kg of PVC (polyvinyl chloride) both at 20°C and 0.1 kg of air at 400 K, 100 kPa. It is now left alone with no external heat transfer and no water vaporizes. Find the final temperature and process work.

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$

Process: P = constant => ${}_1W_2 = P(V_2 - V_1)$ and given ${}_1Q_2 = 0$

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v\,i}(T_2 - T_1)_i = T_2\sum m_i C_{v\,i} - \sum m_i C_{v\,i}T_{1\,i}$$

noticing that all masses have the same T_2 but not same initial T. Since it is only the air that changes volume then the work term and internal energy change for air combines to give changes in enthalpy as

$$m_{air}(u_2 - u_1) + {}_1W_2 = m_{air}(h_2 - h_1) = m_{air}C_p(T_2 - T_1)$$

$$\sum m_i C_{v\,i} = 1.5 \times 4.18 + 1 \times 0.46 + 0.5 \times 0.96 + 0.1 \times 1.004 = 7.3104 \text{ kJ/K}$$

Energy Eq.: $7.3104\ T_2 = 1.5 \times 4.18 \times 55 + (1 \times 0.46 + 0.5 \times 0.96) \times 20$

$$+ 0.1 \times 1.004 \times (400 - 273.15) = 376.386 \text{ kJ}$$

$$T_2 = \mathbf{51.48^oC}$$

$$V_1 = mRT_1/P_1 = 0.1 \text{ kg} \times 0.287 \text{ kJ/kgK} \times 400 \text{ K} /100 \text{ kPa} = 0.1148 \text{ m}^3$$

$$V_2 = V_1 \times T_2/T_{1air} = 0.1148 \ 324.63 / 400 = 0.09317 \text{ m}^3$$

$${}_1W_2 = P(V_2 - V_1) = 100 \text{ kPa} (0.09317 - 0.1148) \text{ m}^3 = \mathbf{-2.16 \text{ kJ}}$$

3.149

A spring loaded piston/cylinder contains 1.5 kg of air at 27C and 160 kPa. It is now heated to 900 K in a process where the pressure is linear in volume to a final volume of twice the initial volume. Plot the process in a P-v diagram and find the work and heat transfer.

Take CV as the air.

$$m_2 = m_1 = m \quad ; \qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

Process: $P = A + BV \quad => \quad {}_1W_2 = \int P \, dV = \text{area} = 0.5(P_1 + P_2)(V_2 - V_1)$

State 1: Ideal gas. $\quad V_1 = mRT_1/P_1 = 1.5 \times 0.287 \times 300/160 = 0.8072 \text{ m}^3$

 Table A.7 $\qquad u_1 = u(300) = 214.36 \text{ kJ/kg}$

State 2: $P_2V_2 = mRT_2 \qquad$ so ratio it to the initial state properties

$$P_2V_2 / P_1V_1 = P_22 / P_1 = mRT_2 / mRT_1 = T_2 / T_1 \quad =>$$

$$P_2 = P_1 (T_2 / T_1)(1/2) = 160 \times (900/300) \times (1/2) = 240 \text{ kPa}$$

Work is done while piston moves at linearly varying pressure, so we get

$$_1W_2 = 0.5(P_1 + P_2)(V_2 - V_1) = 0.5 \times (160 + 240) \text{ kPa} \times 0.8072 \text{ m}^3 = \textbf{161.4 kJ}$$

Heat transfer is found from energy equation

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.5 \times (674.824 - 214.36) + 161.4 = \textbf{852.1 kJ}$$

3.150

A constant pressure piston cylinder contains 0.5 kg air at 300 K, 400 kPa. Assume the piston cylinder has a total mass of 1 kg steel and is at the same temperature as the air at any time. The system is now heated to 1600 K by heat transfer.

(a) Find the heat transfer using constant specific heats for air.
(b) Find the heat transfer **NOT** using constant specific heats for air.

C.V. Air and Steel.

Energy Eq.: $\qquad U_2 - U_1 = m_{air}(u_2 - u_1) + m_{st}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = C \Rightarrow {}_1W_2 = \int_1^2 PdV = P(V_2 - V_1) = P\, m_{air}(v_2 - v_1)$

$${}_1Q_2 = m_{air}(u_2 - u_1)_{air} + m_{st}(u_2 - u_1)_{st} + {}_1W_2 = m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st}$$

Use A.3: $\quad (u_2 - u_1)_{st} = C(T_2 - T_1) = 0.46 \text{ kJ/kgK} \times (1600 - 300) \text{ K} = 598 \text{ kJ/kg}$

Use A.5: $\quad (h_2 - h_1)_{air} = C_p(T_2 - T_1) = 1.004 \text{ kJ/kgK} \times (1600 - 300) \text{ K} = 1305.2 \text{ kJ/kg}$

$$\begin{aligned}{}_1Q_2 &= m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st} \\ &= 0.5 \text{ kg} \times 1305 \text{ kJ/kg} + 1 \text{ kg} \times 598 \text{ kJ/kg} = \textbf{1250.6 kJ}\end{aligned}$$

Use air tables A.7: $\quad (h_2 - h_1)_{air} = 1757.33 - 300.47 = 1456.86 \text{ kJ/kg}$

$$\begin{aligned}{}_1Q_2 &= m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st} \\ &= 0.5 \text{ kg} \times 1456.86 \text{ kJ/kg} + 1 \text{ kg} \times 598 \text{ kJ/kg} = \textbf{1326.43 kJ}\end{aligned}$$

Comment: we could also have computed the work explicitly

$$\begin{aligned}{}_1W_2 &= P\, m_{air}(v_2 - v_1) = m_{air} R(T_2 - T_1) \\ &= 0.5 \text{ kg} \times 0.287 \text{ kJ/kgK} \times (1600 - 300) \text{ K} = 186.55 \text{ kJ} \\ (u_2 - u_1)_{air} &= 1298.08 - 214.36 = 1083.72 \text{ kJ/kg} \\ {}_1Q_2 &= m_{air}(u_2 - u_1)_{air} + m_{st}(u_2 - u_1)_{st} + {}_1W_2 \\ &= 0.5 \times 1083.72 + 598 + 186.55 = 1139.86 + 186.55 = 1326.4 \text{ kJ}\end{aligned}$$

3.151

An insulated cylinder is divided into two parts of 1 m^3 each by an initially locked piston, as shown in Fig. P3.151. Side A has air at 200 kPa, 300 K, and side B has air at 1.0 MPa, 1000 K. The piston is now unlocked so it is free to move, and it conducts heat so the air comes to a uniform temperature $T_A = T_B$. Find the mass in both A and B, and the final T and P.

C.V. A + B   Force balance on piston: $P_A A = P_B A$

So the final state in A and B is the same.

State 1A: Table A.7 $u_{A1} = 214.364$ kJ/kg,

$m_A = P_{A1}V_{A1}/RT_{A1} = 200$ kPa $\times$ 1 m^3/(0.287 kJ/kg-K $\times$ 300 K) = **2.323 kg**

State 1B: Table A.7 $u_{B1} = 759.189$ kJ/kg,

$m_B = P_{B1}V_{B1}/RT_{B1} = 1000$ kPa $\times$ 1 m^3/(0.287 kJ/kg-K $\times$ 1000 K) = **3.484 kg**

For chosen C.V. $_1Q_2 = 0$, $_1W_2 = 0$ so the energy equation becomes

$m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0$

$(m_A + m_B)u_2 = m_A u_{A1} + m_B u_{B1}$

$= 2.323 \times 214.364 + 3.484 \times 759.189 = 3143$ kJ

$u_2 = 3143$ kJ / (3.484 + 2.323) kg = 541.24 kJ/kg

From interpolation in Table A.7: $\Rightarrow$ **$T_2 =$ 736 K**

$P = (m_A + m_B)RT_2/V_{tot} = 5.807$ kg $\times$ 0.287 $\frac{kJ}{kg\ K} \times$ 736 K/ 2 m^3 = **613 kPa**

Borgnakke and Sonntag

3.152

Air in a piston cylinder is at 1800 K, 7 MPa and expands in a polytropic process with n = 1.5 to a volume 8 times larger. Find the specific work and the specific heat transfer in the process and draw the P-v diagram. Use constant specific heat to solve the problem.

C.V. Air of constant mass $m_2 = m_1 = m$.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^{1.50} =$ constant, $v_2/v_1 = 8$

State 1: $P_1 = 7$ MPa, $T_1 = 1800$ K

State 2: $(v_2 = 8v_1, ?)$ Must be on process curve so

$P_2 = P_1 (v_1/v_2)^n = 7000 (1/8)^{1.50} = 309.36$ kPa

$$T_2 = T_1 \frac{P_2 v_2}{P_1 v_1} = T_1(v_1/v_2)^{n-1} = 1800 (1/8)^{0.5} = 636.4 \text{ K}$$

Work from the process expressed in Eq.3.21

$${}_1w_2 = \int Pdv = \frac{P_2 v_2 - P_1 v_1}{1-n} = \frac{R}{1-n} (T_2 - T_1) = \frac{0.287 (636.4 - 1800)}{1 - 1.5}$$

   $= \textbf{667.9 kJ/kg}$

Heat transfer from the energy equation

   ${}_1q_2 = (u_2 - u_1) + {}_1w_2 = 0.717 (636.4 - 1800) + 667.9 = \textbf{--166.4 kJ/kg}$

Notice:
n = 1.5, k = 1.4

n > k

3.153

Do the previous problem but do **not** use constant specific heats.
Air in a piston cylinder is at 1800 K, 7 MPa and expands in a polytropic process
with n = 1.5 to a volume 8 times larger. Find the specific work and the specific
heat transfer in the process and draw the P-v diagram.

C.V. Air of constant mass $m_2 = m_1 = m$.

Energy Eq.3.5: $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $\quad Pv^{1.50} = $ constant, $\quad v_2/v_1 = 8$

State 1: $\quad P_1 = 7$ MPa, $T_1 = 1800$ K

State 2: $\quad (v_2 = 8v_1, ?)$ Must be on process curve so

$\quad P_2 = P_1 (v_1/v_2)^n = 7000 (1/8)^{1.50} = 309.36$ kPa

$\quad T_2 = T_1 \dfrac{P_2v_2}{P_1v_1} = T_1(v_1/v_2)^{n-1} = 1800 (1/8)^{0.5} = 636.4$ K

Table A.7: $\quad u_1 = 1486.33$ kJ/kg and interpolate $u_2 = 463.06$ kJ/kg

Work from the process expressed in Eq.3.21

$\quad {}_1w_2 = \int Pdv = \dfrac{P_2v_2 - P_1v_1}{1-n} = \dfrac{R}{1-n}(T_2 - T_1) = \dfrac{0.287 (636.4 - 1800)}{1 - 1.5}$

$\quad = $ **667.9 kJ/kg**

Heat transfer from the energy equation

$\quad {}_1q_2 = (u_2 - u_1) + {}_1w_2 = (463.06 - 1486.33) + 667.9 = $ –**355.4 kJ/kg**

Notice:
 n = 1.5, k = 1.4

 n > k

Borgnakke and Sonntag

3.154

Helium gas expands from 125 kPa, 350 K and 0.25 m^3 to 100 kPa in a polytropic process with n = 1.667. How much heat transfer is involved?

Solution:

C.V. Helium gas, this is a control mass.

Energy equation: $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process equation: $\quad PV^n = \text{constant} = P_1 V_1^n = P_2 V_2^n$

Ideal gas (A.5): $\quad m = PV/RT = \dfrac{125 \times 0.25}{2.0771 \times 350} = 0.043 \text{ kg}$

Solve for the volume at state 2

$$V_2 = V_1 (P_1/P_2)^{1/n} = 0.25 \times \left(\dfrac{125}{100}\right)^{0.6} = 0.2852 \text{ m}^3$$

$$T_2 = T_1 P_2 V_2/(P_1 V_1) = 350 \text{ K} \dfrac{100 \times 0.2852}{125 \times 0.25} = 319.4 \text{ K}$$

Work from Eq.3.21

$$_1W_2 = \dfrac{P_2 V_2 - P_1 V_1}{1-n} = \dfrac{100 \times 0.2852 - 125 \times 0.25}{1 - 1.667} \text{ kPa m}^3 = 4.09 \text{ kJ}$$

Use specific heat from Table A.5 to evaluate $u_2 - u_1$, $C_v = 3.116$ kJ/kg K

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m\, C_v\, (T_2 - T_1) + {}_1W_2$$
$$= 0.043 \text{ kg} \times 3.116 \text{ kJ/kg-K} \times (319.4 - 350) \text{ K} + 4.09 \text{ kJ} = \textbf{-0.01 kJ}$$

3.155

A piston cylinder contains 0.1 kg air at 300 K and 100 kPa. The air is now slowly compressed in an isothermal (T = C) process to a final pressure of 250 kPa. Show the process in a P-V diagram and find both the work and heat transfer in the process.

Solution :

Process: T = C & ideal gas $\Rightarrow$ PV = mRT = constant

The work was found as in Eq.3.22

$$_1W_2 = \int PdV = \int \frac{mRT}{V} dV = mRT \ln \frac{V_2}{V_1} = mRT \ln \frac{P_1}{P_2}$$

$$= 0.1 \text{ kg} \times 0.287 \text{ kJ/kg-K} \times 300 \text{ K} \times \ln (100 / 250) = \textbf{-7.89 kJ}$$

since $T_1 = T_2 \Rightarrow u_2 = u_1$

The energy equation thus becomes

$$_1Q_2 = m \times (u_2 - u_1) + {_1W_2} = {_1W_2} = \textbf{-7.89 kJ}$$

3.156

A gasoline engine has a piston/cylinder with 0.1 kg air at 4 MPa, 1527°C after combustion and this is expanded in a polytropic process with n = 1.5 to a volume 10 times larger. Find the expansion work and heat transfer using Table A.5 heat capacity.

Take CV as the air. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process Eq.: $Pv^n = Constant$ (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $P_2 = P_1 \left(v_2 / v_1\right)^{-n} = 4000 \times 10^{-1.5} = 126.5$ kPa

$$T_2 = T_1 \ (P_2v_2 / P_1v_1) = (1527 + 273) \frac{126.5 \times 10}{4000} = 569.3 \text{ K}$$

From process eq.: ${_1}W_2 = \int P \, dV = \frac{m}{1-n} (P_2v_2 - P_1v_1) = \frac{mR}{1-n} (T_2 - T_1)$

$$= \frac{0.1 \times 0.287}{1 - 1.5} (569.3 - 1800) = \textbf{70.64 kJ}$$

From energy eq.: ${_1}Q_2 = m(u_2 - u_1) + {_1}W_2 = mC_v(T_2 - T_1) + {_1}W_2$

$= 0.1 \text{ kg} \times 0.717 \text{ kJ/kg-K} \times (569.3 - 1800) \text{ K} + 70.64 \text{ kJ}$

$= \textbf{−17.6 kJ}$

3.157

Solve the previous problem using Table A.7

Take CV as the air. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $Pv^n = Constant$ (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $P_2 = P_1 \left(v_2 / v_1\right)^{-n} = 4000 \times 10^{-1.5} = 126.5$ kPa

$$T_2 = T_1 \ (P_2v_2 / P_1v_1) = (1527 + 273) \frac{126.5 \times 10}{4000} = 569.3 \text{ K}$$

From process eq.: $\displaystyle {}_1W_2 = \int P \ dV = \frac{m}{1-n} (P_2v_2 - P_1v_1) = \frac{mR}{1-n} (T_2 - T_1)$

$$= \frac{0.1 \times 0.287}{1 - 1.5} (569.3 - 1800) = \textbf{70.64 kJ}$$

From energy eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$

$= 0.1$ kg $(411.78 - 1486.33)$ kJ/kg $+ 70.64$ kJ

$= \textbf{--36.8 kJ}$

The only place where Table A.7 comes in is for values of u_1 and u_2

3.158

Find the specific heat transfer in Problem 3.55.

Air goes through a polytropic process from 125 kPa, 325 K to 300 kPa and 500 K. Find the polytropic exponent n and the specific work in the process.

Solution:

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process: $Pv^n = Const = P_1 v_1^n = P_2 v_2^n$

Ideal gas $Pv = RT$ so

$$v_1 = \frac{RT}{P} = \frac{0.287 \times 325}{125} = 0.7462 \text{ m}^3/\text{kg}$$

$$v_2 = \frac{RT}{P} = \frac{0.287 \times 500}{300} = 0.47833 \text{ m}^3/\text{kg}$$

From the process equation

$$(P_2/P_1) = (v_1/v_2)^n \quad \Rightarrow \quad \ln(P_2/P_1) = n \ln(v_1/v_2)$$

$$n = \ln(P_2/P_1) / \ln(v_1/v_2) = \frac{\ln 2.4}{\ln 1.56} = \mathbf{1.969}$$

The work is now from Eq.3.21 per unit mass and ideal gas law

$${}_1w_2 = \frac{P_2 v_2 - P_1 v_1}{1-n} = \frac{R(T_2 - T_1)}{1-n} = \frac{0.287(500 - 325)}{1-1.969} \text{ (kJ/kg-K) K}$$

$$= \mathbf{-51.8 \text{ kJ/kg}}$$

From the energy equation

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = C_v (T_2 - T_1) + {}_1w_2$$
$$= 0.717 \text{ kJ/kg-K } (500 - 325) \text{ K} - 51.5 \text{ kJ/kg}$$
$$= \mathbf{73.98 \text{ kJ/kg}}$$

3.159

A piston/cylinder has nitrogen gas at 750 K and 1500 kPa. Now it is expanded in a polytropic process with n = 1.2 to P = 750 kPa. Find the final temperature, the specific work and specific heat transfer in the process.

C.V. Nitrogen. This is a control mass going through a polytropic process.

Continuty: $m_2 = m_1$

Energy Eq.3.5: $\qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $\qquad\qquad Pv^n = $ constant

Substance ideal gas: $\qquad Pv = RT$

$$T_2 = T_1 \, (P_2/P_1)^{\frac{n-1}{n}} = 750 \left(\frac{750}{1500}\right)^{\frac{0.2}{1.2}} = 750 \times 0.8909 = \textbf{668 K}$$

The work is integrated as in Eq.3.21

$$_1w_2 = \int Pdv = \frac{1}{1-n}(P_2v_2 - P_1v_1) = \frac{R}{1-n}(T_2 - T_1)$$

$$= \frac{0.2968}{1-1.2} \text{ kJ/kg-K } (668 - 750) \text{ K} = \textbf{121.7 kJ/kg}$$

The energy equation with values of u from Table A.8 is

$$_1q_2 = u_2 - u_1 + {}_1w_2 = 502.8 - 568.45 + 121.7 = \textbf{56.0 kJ/kg}$$

If constant specific heat is used from Table A.5

$$_1q_2 = C_v(T_2 - T_1) + {}_1w_2 = 0.745(668 - 750) + 121.7 = \textbf{60.6 kJ/kg}$$

The actual process is on a steeper curve than n = 1.

3.160

A piston/cylinder has 1 kg propane gas at 700 kPa, 40°C. The piston cross-sectional area is 0.5 m^2, and the total external force restraining the piston is directly proportional to the cylinder volume squared. Heat is transferred to the propane until its temperature reaches 700°C. Determine the final pressure inside the cylinder, the work done by the propane, and the heat transfer during the process.

Solution:

C.V. The 1 kg of propane.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = P_{ext} = CV^2 \Rightarrow PV^{-2} = \text{constant}$, polytropic $n = -2$

Ideal gas: $PV = mRT$, and process yields

$$P_2 = P_1(T_2/T_1)^{\frac{n}{n-1}} = 700\left(\frac{700+273.15}{40+273.15}\right)^{2/3} = \textbf{1490.7 kPa}$$

The work is integrated as Eq.3.21

$$_1W_2 = \int\limits_1^2 PdV = \frac{P_2V_2 - P_1V_1}{1 - n} = \frac{mR(T_2 - T_1)}{1 - n}$$

$$= \frac{1 \times 0.18855 \times (700 - 40)}{1 - (-2)} = \textbf{41.48 kJ}$$

The energy equation with specific heat from Table A.5 becomes

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = mC_v(T_2 - T_1) + {}_1W_2$$

$$= 1 \text{ kg} \times 1.490 \text{ kJ/kg-K} \times (700 - 40) \text{ K} + 41.48 \text{ kJ}$$

$$= \textbf{1024.9 kJ}$$

3.161

A piston/cylinder arrangement of initial volume 0.025 m^3 contains saturated water vapor at 180°C. The steam now expands in a polytropic process with exponent $n = 1$ to a final pressure of 200 kPa, while it does work against the piston. Determine the heat transfer in this process.

Solution:

C.V. Water. This is a control mass.

State 1: Table B.1.1 $P = 1002.2$ kPa, $v_1 = 0.19405$ m^3/kg, $u_1 = 2583.7$ kJ/kg ,

$$m = V/v_1 = 0.025/0.19405 = 0.129 \text{ kg}$$

Process: $Pv = \text{const.} = P_1v_1 = P_2v_2$;   polytropic process $n = 1$.

$$\Rightarrow v_2 = v_1 P_1/P_2 = 0.19405 \times 1002.1/200 = 0.9723 \text{ m}^3/\text{kg}$$

State 2: P_2, v_2 $\Rightarrow$ Table B.1.3 $T_2 \cong 155°C$, $u_2 = 2585$ kJ/kg

$$_1W_2 = \int PdV = P_1V_1 \ln \frac{v_2}{v_1} = 1002.2 \text{ kPa} \times 0.025 \text{ m}^3 \ln \frac{0.9723}{0.19405} = 40.37 \text{ kJ}$$

$$_1Q_2 = m(u_2 - u_1) + {_1W_2} = 0.129(2585 - 2583.7) + 40.37 = \textbf{40.54 kJ}$$

Notice T drops, it is not an ideal gas.

3.162

A piston cylinder contains pure oxygen at ambient conditions 20°C, 100 kPa. The piston is moved to a volume that is 7 times smaller than the initial volume in a polytropic process with exponent n = 1.25. Use constant heat capacity to find the final pressure and temperature, the specific work and the specific heat transfer.

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process Eq: $Pv^n = C;$ $P_2 = P_1 (v_1/v_2)^n = 100 (7)^{1.25} = 1138.6$ kPa

From the ideal gas law and state 2 (P, v) we get

$$T_2 = T_1(P_2/P_1)(v_1/v_2) = 293 \times \frac{1138.6}{100} \times (1/7) = 476.8 \text{ K}$$

We could also combine process eq. and gas law to give: $T_2 = T_1 (v_1/v_2)^{n-1}$

Polytropic work Eq. 3.21: ${}_1w_2 = \frac{1}{1-n} (P_2v_2 - P_1v_1) = \frac{R}{1-n} (T_2 - T_1)$

$${}_1w_2 = \frac{0.2598}{1 - 1.25} \frac{\text{kJ}}{\text{kg K}} \times (476.8 - 293.2) \text{ K} = -\textbf{190.88 kJ/kg}$$

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = C_v (T_2 - T_1) + {}_1w_2$$

$$= 0.662 (476.8 - 293.2) - 190.88 = -\textbf{69.3 kJ/kg}$$

The actual process is on a steeper curve than n = 1.

3.163

A piston/cylinder in a car contains 0.2 L of air at 90 kPa, 20°C, shown in Fig. P3.163. The air is compressed in a quasi-equilibrium polytropic process with polytropic exponent $n = 1.25$ to a final volume six times smaller. Determine the final pressure, temperature, and the heat transfer for the process.

Solution:

C.V. Air. This is a control mass going through a polytropic process.

Continuty: $m_2 = m_1$

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^n = \text{const.}$

$P_1v_1{}^n = P_2v_2{}^n \Rightarrow P_2 = P_1(v_1/v_2)^n = 90 \text{ kPa} \times 6^{1.25} = \mathbf{845.15 \ kPa}$

Substance ideal gas: $Pv = RT$

$T_2 = T_1(P_2v_2/P_1v_1) = 293.15 \text{ K } (845.15/90 \times 6) = \mathbf{458.8 \ K}$

$$m = \frac{PV}{RT} = \frac{90 \times 0.2 \times 10^{-3}}{0.287 \times 293.15} = 2.14 \times 10^{-4} \text{ kg}$$

The work is integrated as in Eq.3.21

$${}_1w_2 = \int Pdv = \frac{1}{1-n}(P_2v_2 - P_1v_1) = \frac{R}{1-n}(T_2 - T_1)$$

$$= \frac{0.287}{1-1.25} \text{ kJ/kg-K } (458.8 - 293.15) \text{ K} = -190.17 \text{ kJ/kg}$$

The energy equation with values of u from Table A.7 is

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = 329.4 - 208.03 - 190.17 = -68.8 \text{ kJ/kg}$$

$${}_1Q_2 = m \ {}_1q_2 = \mathbf{-0.0147 \ kJ} \quad \text{(i.e a heat loss)}$$

3.164

An air pistol contains compressed air in a small cylinder, shown in Fig. P3.164. Assume that the volume is 1 cm^3, pressure is 1 MPa, and the temperature is 27°C when armed. A bullet, $m = 15$ g, acts as a piston initially held by a pin (trigger); when released, the air expands in an isothermal process (T = constant). If the air pressure is 0.1 MPa in the cylinder as the bullet leaves the gun, find

a.   The final volume and the mass of air.

b.   The work done by the air and work done on the atmosphere.

c**.**   The work to the bullet and the bullet exit velocity.

Solution:

C.V. Air.

Air ideal gas: $m_{air} = P_1V_1/RT_1 = 1000 \times 10^{-6}/(0.287 \times 300) = \mathbf{1.17{\times}10^{-5}\ kg}$

Process: $PV = \text{const} = P_1V_1 = P_2V_2 \Rightarrow V_2 = V_1P_1/P_2 = \mathbf{10\ cm^3}$

$$_1W_2 = \int PdV = \int \frac{P_1V_1}{V}\,dV = P_1V_1\ \ln(V_2/V_1) = \mathbf{2.303\ J}$$

$$_1W_{2,ATM} = P_0(V_2 - V_1) = 101 \times (10 - 1) \times 10^{-6}\ kJ = \mathbf{0.909\ J}$$

$$W_{bullet} = {_1W_2} - {_1W_{2,ATM}} = 1.394\ J = \frac{1}{2}\,m_{bullet}(V_{exit})^2$$

$$V_{exit} = (2W_{bullet}/m_B)^{1/2} = (2 \times 1.394/0.015)^{1/2} = \mathbf{13.63\ m/s}$$

3.165

Air goes through a polytropic process with n = 1.3 in a piston/cylinder setup. It starts at 200 kPa, 300 K and ends with a pressure of 2200 kPa. Find the expansion ratio (v_2/v_1), the specific work, and the specific heat transfer.

Take CV as the air. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $Pv^n = \text{Constant}$ (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $(v_2/v_1) = (P_2/P_1)^{-1/n}$ and $Pv = RT$ => $T_2/T_1 = (v_2 / v_1)^{1 - n}$

$$(v_2/v_1) = (P_2/P_1)^{-1/n} = (2200 / 200)^{-1/1.3} = \textbf{0.1581}$$

$$T_2 = T_1 (P_2/P_1)^{(n-1)/n} = 300 \left(\frac{2200}{200}\right)^{0.3/1.3} = \textbf{521.7 K}$$

From process eq.:

$$_1w_2 = \int P \, dv = \text{area} = \frac{1}{1-n} (P_2v_2 - P_1v_1) = \frac{R}{1-n} (T_2 - T_1)$$

$$= \frac{0.287}{1-1.3} \text{ kJ/kg-K} \times (521.7 \text{ - } 300) \text{ K} = \textbf{–212.09 kJ/kg}$$

From the energy equation and constant specific heat from Table A.5

$$_1q_2 = u_2 \text{ - } u_1 + {}_1w_2 = C_v (T_2 - T_1) + {}_1w_2$$

$$= 0.717 \text{ kJ/kg-K} \times (521.7 - 300) \text{ K} - 212.09 \text{ kJ/kg} = \textbf{-53.13 kJ/kg}$$

3.166

Nitrogen gas goes through a polytropic process with n = 1.3 in a piston/cylinder arrangement. It starts out at 600 K and 600 kPa and ends with 800 K. Find the final pressure, the process specific work and heat transfer.

Take CV as the nitrogen. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $Pv^n = \text{Constant}$ (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $P_2 = P_1 \left(v_2 / v_1\right)^{-n}$ and $Pv = RT$ => $T_2/T_1 = (v_2 / v_1)^{1-n}$

$$P_2 = P_1 \left(T_2 / T_1\right)^{n/(n-1)} = 600 \left(\frac{800}{600}\right)^{1.3/0.3} = \textbf{2087 kPa}$$

From process eq.:

$${}_1w_2 = \int P \, dv = \text{area} = \frac{1}{1-n}(P_2v_2 - P_1v_1) = \frac{R}{1-n}(T_2 - T_1)$$

$$= \frac{0.2968}{1-1.3}(800 - 600) = \textbf{--197.9 kJ/kg}$$

From the energy equation and Table A.8

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = (609.41 - 449.16 - 197.9) \text{ kJ/kg} = \textbf{-37.65 kJ/kg}$$

From the energy equation and constant specific heat from Table A.5

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = C_v (T_2 - T_1) + {}_1w_2$$

$$= 0.745 \text{ kJ/kg-K} \times (800 - 600) \text{ K} - 197.9 \text{ kJ/kg} = -48.9 \text{ kJ/kg}$$

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3.167

A piston/cylinder contains pure oxygen at 500 K, 600 kPa. The piston is moved to a volume such that the final temperature is 700 K in a polytropic process with exponent n = 1.25. Use ideal gas approximation and constant heat capacity to find the final pressure, the specific work and heat transfer.

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process Eq: $Pv^n = C$ and ideal gas $Pv = RT$ gives

$$P_2 = P_1(v_1/v_2)^n = P_1(T_2/T_1)^{n/(n-1)} = 600(7/5)^5 = \textbf{3227 kPa}$$

Reversible work Eq. 3.21: ${}_1w_2 = \dfrac{1}{1-n}(P_2v_2 - P_1v_1) = \dfrac{R}{1-n}(T_2 - T_1)$

$${}_1w_2 = \frac{0.2598}{1 - 1.25}\ \frac{kJ}{kg\ K} \times (700 - 500)\ K = -\textbf{207.84 kJ/kg}$$

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = C_v(T_2 - T_1) + {}_1w_2$$

$$= 0.662\ kJ/kg\text{-}K \times (700 - 500)\ K - 207.84\ kJ/kg$$

$$= -\ \textbf{69.3 kJ/kg}$$

The actual process is on a steeper curve than n = 1.

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3.168

Calculate the heat transfer for the process described in Problem 3.57.

Consider a piston cylinder with 0.5 kg of R-134a as saturated vapor at -10°C. It is now compressed to a pressure of 500 kPa in a polytropic process with n = 1.5. Find the final volume and temperature, and determine the work done during the process.

Solution:

Take CV as the R-134a which is a control mass

Continuity: $m_2 = m_1 = m$;   Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^{1.5}$ = constant.   Polytropic process with n = 1.5

1: (T, x)   P = Psat = 201.7 kPa   from Table B.5.1

$v_1 = 0.09921 \ m^3/kg, \quad u_1 = 372.27 \ kJ/kg$

2: (P, process)

$$v_2 = v_1 \ (P_1/P_2)^{(1/1.5)} = 0.09921 \times (201.7/500)^{0.667} = 0.05416 \ m^3/kg$$

=> Table B.5.2   superheated vapor, $T_2 = 79°C$, $V_2 = mv_2 = 0.027 \ m^3$

$u_2 = 440.9 \ kJ/kg$

Process gives $P = C \ v^{(-1.5)}$, which is integrated for the work term, Eq.3.21

${}_1W_2 = \int P \ dV = m(P_2v_2 - P_1v_1)/(1-1.5)$

$= -2 \times 0.5 \ kg \times (500 \times 0.05416 - 201.7 \times 0.09921) \ kJ/kg = \textbf{-7.07 kJ}$

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.5(440.9 - 372.27) + (-7.07) = \textbf{27.25 kJ}$

3.169

A piston cylinder shown in Fig. P3.169 contains 0.5 m^3 of R-410A at 2 MPa, 150°C. The piston mass and atmosphere gives a pressure of 450 kPa that will float the piston. The whole setup cools in a freezer maintained at -20°C. Find the heat transfer and show the P-v diagram for the process when T_2 = -20°C.

C.V.: R-410A. Control mass.

Continuity: m = constant,

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $F\downarrow = F\uparrow = P\ A = P_{air}A + F_{stop}$

if $V < V_{stop} \Rightarrow F_{stop} = \emptyset$

This is illustrated in the P-v diagram shown below.

State 1: $v_1 = 0.02247$ m^3/kg, $u_1 = 373.49$ kJ/kg

$\Rightarrow$ m = V/v = 22.252 kg

State 2: T_2 and on line $\Rightarrow$ compressed liquid, see figure below.

$v_2 \cong v_f = 0.000\ 803$ m^3/kg $\Rightarrow V_2 = 0.01787$ m^3; $u_2 = u_f = 27.92$ kJ/kg

$_1W_2 = \int PdV = P_{lift}(V_2 - V_1) = 450$ kPa $(0.01787 - 0.5)$ m^3 = -217.0 kJ ;

Energy eq. $\Rightarrow$

$_1Q_2 = 22.252$ kg $\times (27.92 - 373.49)$ kJ/kg $- 217.9$ kJ = **-7906.6 kJ**

3.170

A cylinder containing 1 kg of ammonia has an externally loaded piston. Initially the ammonia is at 2 MPa, 180°C and is now cooled to saturated vapor at 40°C, and then further cooled to 20°C, at which point the quality is 50%. Find the total work and the heat transfer for the process, assuming a piecewise linear variation of P versus V.

Solution:

State 1: (T, P)   Table B.2.2

$\quad v_1 = 0.10571 \ \text{m}^3/\text{kg}$

$\quad u_1 = 1630.6 \ \text{kJ/kg}$

State 2: (T, x)   Table B.2.1 sat. vap.

$\quad P_2 = 1555 \ \text{kPa},$

$\quad v_2 = 0.08313 \ \text{m}^3/\text{kg}$

$\quad u_2 = 1341.0 \ \text{kJ/kg}$

State 3: (T, x) $P_3 = 857 \ \text{kPa}, \quad v_3 = (0.001638 + 0.14922)/2 = 0.07543 \ \text{m}^3/\text{kg}$

$\qquad\qquad\qquad\qquad\qquad u_3 = (272.89 + 1332.2)/2 = 802.55 \ \text{kJ/kg}$

Sum the work as two integrals each evaluated by the area in the P-v diagram.

$$_1W_3 = \int_1^3 PdV \approx \left(\frac{P_1 + P_2}{2}\right) m(v_2 - v_1) + \left(\frac{P_2 + P_3}{2}\right) m(v_3 - v_2)$$

$$= \frac{2000 + 1555}{2} \ \text{kPa} \times 1 \ \text{kg} \times (0.08313 - 0.10571) \ \text{m}^3/\text{kg}$$

$$+ \frac{1555 + 857}{2} \ \text{kPa} \times 1 \ \text{kg} \times (0.07543 - 0.08313) \ \text{m}^3/\text{kg}$$

$$= \textbf{-49.4 kJ}$$

From the energy equation

$$_1Q_3 = m(u_3 - u_1) + {}_1W_3 = 1 \ \text{kg} \times (802.55 - 1630.6) \ \text{kJ/kg} - 49.4 \ \text{kJ}$$

$$= \textbf{–877.5 kJ}$$

3.171

10 kg of water in a piston cylinder arrangement exists as saturated liquid/vapor at 100 kPa, with a quality of 50%. It is now heated so the volume triples. The mass of the piston is such that a cylinder pressure of 200 kPa will float it, as in Fig. 3.171. Find the final temperature and the heat transfer in the process.

Solution:

Take CV as the water.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process: v = constant until $P = P_{lift}$, then P is constant.

State 1: Two-phase so look in Table B.1.2 at 100 kPa

$$u_1 = 417.33 + 0.5 \times 2088.72 = 1461.7 \text{ kJ/kg},$$

$$v_1 = 0.001043 + 0.5 \times 1.69296 = 0.8475 \text{ m}^3/\text{kg}$$

State 2: v_2, $\mathbf{P_2 \leq P_{lift}}$ => $v_2 = 3 \times 0.8475 = 2.5425 \text{ m}^3/\text{kg}$;

Interpolate: $\mathbf{T_2 = 829°C}$, $u_2 = 3718.76 \text{ kJ/kg}$

=> $V_2 = mv_2 = 25.425 \text{ m}^3$

From the process equation (see P-V diagram) we get the work as

$${_1}W_2 = P_{lift}(V_2 - V_1) = 200 \text{ kPa} \times 10 \text{ kg} (2.5425 - 0.8475) \text{ m}^3/\text{kg} = 3390 \text{ kJ}$$

From the energy equation we solve for the heat transfer

$${_1}Q_2 = m(u_2 - u_1) + {_1}W_2 = 10 \text{ kg} \times (3718.76 - 1461.7) \text{ kJ/kg} + 3390 \text{ kJ}$$

$$= \mathbf{25\ 961 \text{ kJ}}$$

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3.172

A helium gas is heated at constant volume from a state of 100 kPa, 300 K to 500 K. A following process expands the gas at constant pressure to three times the initial volume. What are the specific work and the specific heat transfer in the combined process?

The two processes are:

1 -> 2: Constant volume $V_2 = V_1$

2 -> 3: Constant pressure $P_3 = P_2$

Use ideal gas approximation for helium.

State 1: T, P $\Rightarrow$ $v_1 = RT_1/P_1$

State 2: $V_2 = V_1$ $\Rightarrow$ $P_2 = P_1 (T_2/T_1)$

State 3: $P_3 = P_2$ $\Rightarrow$ $V_3 = 3V_2$; $T_3 = T_2 \, v_3/v_2 = 500 \times 3 = 1500$ K

We find the work by summing along the process path.

$$_1w_3 = {_1w_2} + {_2w_3} = {_2w_3} = P_3(v_3 - v_2) = R(T_3 - T_2)$$
$$= 2.0771 \text{ kJ/kg-K} \times (1500 - 500) \text{ K} = \mathbf{2077 \text{ kJ/kg}}$$
$$_1q_3 = u_3 - u_1 + {_1w_3} = C_v (T_3 - T_1) + {_1w_3}$$
$$= 3.116 \text{ kJ/kg-K} \times (1500 - 300) \text{ K} + 2077 \text{ kJ/kg} = \mathbf{5816 \text{ kJ/kg}}$$

3.173

A vertical cylinder fitted with a piston contains 5 kg of R-410A at 10°C, shown in Fig. P3.173. Heat is transferred to the system, causing the piston to rise until it reaches a set of stops at which point the volume has doubled. Additional heat is transferred until the temperature inside reaches 50°C, at which point the pressure inside the cylinder is 1.4 MPa.

a.   What is the quality at the initial state?

b.   Calculate the heat transfer for the overall process.

Solution:

C.V. R-410A. Control mass goes through process: 1 -> 2 -> 3

As piston floats pressure is constant (1 -> 2) and the volume is constant for the second part (2 -> 3).   So we have: $v_3 = v_2 = 2 \times v_1$

State 3: Table B.4.2 (P,T) $v_3 = 0.02249$ m³/kg, $u_3 = 287.91$ kJ/kg

So we can then determine state 1 and 2 Table B.4.1:

$v_1 = 0.011245 = 0.000886 + x_1 \times 0.02295$ => $x_1 = \mathbf{0.4514}$

b) $u_1 = 72.24 + 0.4514 \times 183.66 = 155.14$ kJ/kg

State 2: $v_2 = 0.02249$ m³/kg, $P_2 = P_1 = 1086$ kPa   this is still 2-phase.

We get the work from the process equation (see P-V diagram)

$$_1W_3 = {_1}W_2 = \int_1^2 PdV = P_1(V_2 - V_1) = 1086 \text{ kPa} \times 5 \text{ kg} \times 0.011245 \text{ m}^3/\text{kg}$$

$$= 61.1 \text{ kJ}$$

The heat transfer from the energy equation becomes

$_1Q_3 = m(u_3\text{-}u_1) + {_1}W_3 = 5 \text{ kg} \times (287.91 - 155.14) \text{ kJ/kg} + 61.1 \text{ kJ}$

$= \mathbf{725.0 \text{ kJ}}$

3.174

Water in a piston/cylinder (Fig. P3.174) is at 101 kPa, 25°C, and mass 0.5 kg. The piston rests on some stops, and the pressure should be 1000 kPa to float the piston. We now heat the water, so the piston just reaches the end of the cylinder. Find the total heat transfer.

Solution:

Take CV as the water.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process: v = constant until $P = P_{float}$, then P is constant.

Volume does go up so we get $v_2 = 5\ v_1$

State 1: $v_1 = 0.001003$ m³/kg, $u_1 = 104.86$ kJ/kg

State 2: $v_2, P_2 = P_{float}$ so $v_2 = 5 \times 0.001003 = 0.005015$ m³/kg ;

$x_2 = (v_2 - v_f)\ /v_{fg} = (0.005015 - 0.001127)/0.19332 = 0.02011,$

$u_2 = 761.67 + x_2 \times 1821.97 = 798.31$ kJ/kg

From the process equation (see P-V diagram) we get the work as

$_1w_2 = P_{float}(v_2 - v_1) = 1000$ kPa $(0.005015 - 0.001003)$ m³/kg $= 4.012$ kJ/kg

From the energy equation we solve for the heat transfer

$_1Q_2 = m[u_2 - u_1 + {_1}w_2] = 0.5 \times [798.31 - 104.86 + 4.012] =$ **348.7 kJ**

P$_o$

cb

H$_2$O

P

2

P$_2$

P$_1$

1

cb

V

cb

3.175

A setup as in Fig. P3.169 has the R-410A initially at 1000 kPa, 50°C of mass 0.1 kg. The balancing equilibrium pressure is 400 kPa and it is now cooled so the volume is reduced to half the starting volume. Find the work and heat transfer for the process.

Take as CV the 0.1 kg of R-410A.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = P_{float}$ or $v = C = v_1$,

State 1:   (P, T) $=>$ $v_1 = 0.0332$ m^3/kg,

$u_1 = 292.695$ kJ/kg

State 2:   (P, v) $=>$ $v_2 = v_1/2 = 0.0166$ m^3/kg $< v_g$, so it is two-phase.

$x_2 = (v_2 - v_f) /v_{fg} = (0.0166 - 0.000803)/0.064 = 0.2468$

$u_2 = u_f + x_2 u_{fg} = 27.92 + x_2 218.07 = 81.746$ kJ/kg

From process eq.: ${}_1W_2 = \int P\, dV = area = mP_2 (v_2 - v_1)$

$= 0.1$ kg $\times$ 400 kPa (0.0166 – 0.0332) m^3/kg = -0.**664 kJ**

From energy eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.1\times (81.746 - 292.695) - 0.664$

$= $ **-21.8 kJ**

3.176

A piston/cylinder contains 1 kg of liquid water at 20°C and 300 kPa. Initially the piston floats, similar to the setup in Problem 3.173, with a maximum enclosed volume of 0.002 m^3 if the piston touches the stops. Now heat is added so a final pressure of 600 kPa is reached. Find the final volume and the heat transfer in the process.

Solution:

Take CV as the water which is a control mass: $\qquad m_2 = m_1 = m$;

Energy Eq.: $\quad m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Table B.1.1: 20°C => $P_{sat} = 2.34$ kPa

State 1: Compressed liquid $v = v_f(20) = 0.001002$ m^3/kg, u = 83.94 kJ/kg

State 1a: $v_{stop} = 0.002$ m^3/kg , 300 kPa

State 2: Since $P_2 = 600$ kPa $> P_{lift}$ then piston is pressed against the stops

$$v_2 = v_{stop} = 0.002 \ \text{m}^3/\text{kg} \ \text{ and } \ V = \textbf{0.002 m}^3$$

For the given P : $v_f < v < v_g$ so 2-phase $\quad T = T_{sat} = 158.85$ °C

$$x = (v - v)/v = (0.002 - 0.001101)/0.31457 = 0.002858$$
$$u = 669.88 + x \ 1897.52 = 675.3 \ \text{kJ/kg}$$

Work is done while piston moves at P_{lift} = constant = 300 kPa so we get

$${_1}W_2 = \int P \ dV = m \ P_{lift}(v_2 - v_1) = 1 \ \text{kg} \times 300 \ \text{kPa} \times (0.002 - 0.001002) \ \text{m}^3$$
$$= 0.30 \ \text{kJ}$$

The heat transfer is from the energy equation

$${_1}Q_2 = m(u_2 - u_1) + {_1}W_2 = 1 \ \text{kg} \times (675.3 - 83.94) \ \text{kJ/kg} + 0.30 \ \text{kJ}$$
$$= \textbf{591.7 kJ}$$

3.177

A cylinder/piston arrangement contains 5 kg of water at 100°C with $x = 20\%$ and the piston, $m_P = 75$ kg, resting on some stops, similar to Fig. P3.171. The outside pressure is 100 kPa, and the cylinder area is $A = 24.5$ cm^2. Heat is now added until the water reaches a saturated vapor state. Find the initial volume, final pressure, work, and heat transfer terms and show the *P–v* diagram.

Solution:

C.V. The 5 kg water.

Continuty: $m_2 = m_1 = m$;   Energy: $m(u_2 - u_1) = {_1Q_2} - {_1W_2}$

Process: V = constant if $P < P_{lift}$ otherwise $P = P_{lift}$ see P-v diagram.

$$P_3 = P_2 = P_{lift} = P_0 + m_p\, g / A_p = 100 + \frac{75 \times 9.807}{0.00245 \times 1000} = \textbf{400 kPa}$$

State 1: (T,x) Table B.1.1

   $v_1 = 0.001044 + 0.2 \times 1.6719$ m^3/kg, $V_1 = mv_1 = 5 \times 0.3354 = \textbf{1.677 m}^3$

   $u_1 = 418.91 + 0.2 \times 2087.58 = 836.4$ kJ/kg

State 3: (P, x = 1) Table B.1.2   => $v_3 = 0.4625 > v_1$, $u_3 = 2553.6$ kJ/kg

Work is seen in the P-V diagram (if volume changes then $P = P_{lift}$)

   ${_1W_3} = {_2W_3} = P_{ext}m(v_3 - v_2) = 400$ kPa $\times 5$ kg $(0.46246 - 0.3354)$ m^3/kg

       $= \textbf{254.1 kJ}$

Heat transfer is from the energy equation

   ${_1Q_3} = 5$ kg $\times (2553.6 - 836.4)$ kJ/kg $+ 254.1$ kJ $= \textbf{8840 kJ}$

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3.178

A piston cylinder setup similar to Problem 3.171 contains 0.1 kg saturated liquid and vapor water at 100 kPa with quality 25%. The mass of the piston is such that a pressure of 500 kPa will float it. The water is heated to 300°C. Find the final pressure, volume, work, $_1W_2$ and $_1Q_2$.

Solution:

Take CV as the water: $m_2 = m_1 = m$

Process: $v = $ constant until $P = P_{lift}$

To locate state 1: Table B.1.2

$v_1 = 0.001043 + 0.25 \times 1.69296 = 0.42428$ m^3/kg

$u_1 = 417.33 + 0.25 \times 2088.72 = 939.51$ kJ/kg

1a: $v_{1a} = v_1 = 0.42428$ m^3/kg $> v_g$ at 500 kPa,

state 1a is superheated vapor $T_{1a} = 200$°C

State 2 is 300°C so heating continues after state 1a to 2 at constant P =>

2: T_2, $P_2 = P_{lift} = $ **500 kPa** =>

Table B.1.3 $v_2 = 0.52256$ m^3/kg; $u_2 = 2802.91$ kJ/kg

$V_2 = mv_2 = $ **0.05226 m^3**

$_1W_2 = P_{lift} (V_2 - V_1) = 500$ kPa $\times (0.05226 - 0.04243)$ m$^3 = $ **4.91 kJ**

The heat transfer is from the energy equation

$_1Q_2 = m(u_2 - u_1) + {_1W_2} = 0.1$ kg $\times (2802.91 - 939.51)$ kJ/kg + 4.91 kJ

= **191.25 kJ**

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3.179

A piston/cylinder contains 0.1 kg R-410A at 600 kPa, 60$^{\text{o}}$C. It is now cooled, so the volume is reduced to half the initial volume. The piston has upper stops mounted and the piston mass and gravitation is such that a floating pressure is 400 kPa.

a) Find the final temperature

b) How much work is involved.

c) What is the heat transfer in the process.

d) Show the process path in a P-v diagram.

Take as CV the 0.1 kg of R-410A. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = P_{float}$ or $v = C = v_1$,

State 1: (P, T)   => $v_1 = 0.06023$ m^3/kg,

$u_1 = 304.91$ kJ/kg

State 2: (P, v)   => $v_2 = v_1/2 = 0.030115$ m^3/kg < v_g, so it is two-phase.

$x_2 = (v_2 - v_f) / v_{fg} = (0.030115 - 0.000803)/0.064 = 0.458$

$u_2 = u_f + x_2\, u_{fg} = 27.92 + x_2\, 218.07 = 127.8$ kJ/kg

From process eq.: ${}_1W_2 = \int P\, dV = \text{area} = mP_2\,(v_2 - v_1)$

= 0.1 kg × 400 kPa (0.030115 – 0.06023) m^3/kg = - **1.2046 kJ**

From energy eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$

= 0.1 kg × (127.8 – 304.91) kJ/kg - 1.2046 kJ

= **-18.9 kJ**

3.180

A piston cylinder contains air at 1000 kPa, 800 K with a volume of 0.05 m^3. The piston is pressed against the upper stops, see Fig. P3.14c and it will float at a pressure of 750 kPa. Now the air is cooled to 400 K. What is the process work and heat transfer?

CV. Air, this is a control mass

Energy Eq.3.5 $\qquad$ $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process Eq.: $\qquad$ $P = P_{float}$ $\quad$ or $\quad$ $v = C = v_1,$

State 1: $\quad$ u = 592.58 kJ/kg, $\quad$ m = PV/RT = $1000 \times 0.05 /(0.287 \times 800) = 0.2178$ kg

We need to find state 2. Let us see if we proceed past state 1a during the cooling.

$$T_{1a} = T_1 \, P_{float} / P_1 = 800 \times 750 / 100 = 600 \text{ K}$$

so we do cool below T_{1a}. That means the piston is floating. Write the ideal gas law for state 1 and 2 to get

$$V_2 = \frac{mRT_2}{P_2} = \frac{P_1V_1T_2}{P_2T_1} = \frac{1000 \times 0.05 \times 400}{750 \times 800} \text{ m}^3 = 0.0333 \text{ m}^3$$

$${_1}W_2 = {_{1a}}W_2 = \int P \, dV = P_2 \, (V_2 - V_1)$$

$$= 750 \text{ kPa} \times (0.0333 - 0.05) \text{ m}^3 = \textbf{--12.5 kJ}$$

From energy eq.:

$${_1}Q_2 = m(u_2 - u_1) + {_1}W_2$$

$$= 0.2178 \text{ kg} \times (286.49 - 592.58) \text{ kJ/kg} - 12.5 \text{ kJ}$$

$$= \textbf{-79.2 kJ}$$

3.181

The piston/cylinder arrangement in Fig. P3.181 contains 10 g ammonia at 20°C with a volume of 1 L. There are some stops so if the piston is at the stops the volume is 1.4 L. The ammonia is now heated to 200°C. The piston and cylinder is made of 0.5 kg aluminum. Assume that the mass has the same temperature as the ammonia at any time. Find the final volume and the total heat transfer and plot the P-V diagram for the process.

C.V. NH3. Control mass goes through process: 1 -> 2 -> 3

Energy Eq.: $U_3 - U_1 = m_{NH3} (u_3 - u_1) + m_{Alu} (u_3 - u_1) = {}_1Q_3 - {}_1W_3$

As piston floats pressure is constant (1 -> 2) and the volume is constant for the second
   part (2 -> 3) if we go this far. So we have at stop: $v_3 = v_2 = 1.4 \times v_1$

State 1: B.2.1 $v_1 = V/m = 0.001 / 0.01 = 0.1 \ m^3/kg \ < v$ so 2-phase   P = 857.5 kPa

$$x_1 = (v - v_f)/ v_{fg} = (0.1 - 0.001638)/ 0.14758 = 0.6665$$

$$u_1 = u_f + x_1 \ u_{fg} = 272.89 + x_1 \times 1059.3 = 978.91 \ kJ/kg$$

State 2: $v_2 = 1.4 \times v_1 = 0.14 \ m^3/kg$ & P = 857.5 kPa   still 2-phase so $T_2 = 20^oC$

State 3: 200^oC & $v_3 = v_2 = 0.14 \ m^3/kg$, => P = 1600 kPa, V = **1.4 L**, $u_3 = 1676.5 \ kJ/kg$

We get the work from the process equation (see P-V diagram)

$${}_1W_3 = {}_1W_2 = \int_1^2 PdV = P_1(V_2 - V_1) = 857.5 \ kPa \times (0.0014 - 0.001) \ m^3 = 0.343 \ kJ$$

The heat transfer from the energy equation becomes

$${}_1Q_3 = m_{NH3} (u_3 - u_1) + m_{Alu}C_{Alu} (T_3 - T_1) + {}_1W_3$$

$$= 0.01 \ kg (1676.5 - 978.91) \ kJ/kg + 0.5 \ kg \times 0.9 \ kJ/kg\text{-}K \ (200–20) \ K + 0.343 \ kJ$$

$$= \textbf{88.32 kJ}$$

3.182

Air in a piston/cylinder at 200 kPa, 600 K, is expanded in a constant-pressure process to twice the initial volume (state 2), shown in Fig. P3.182. The piston is then locked with a pin and heat is transferred to a final temperature of 600 K. Find P, T, and h for states 2 and 3, and find the work and heat transfer in both processes.

Solution:

C.V. Air. Control mass $m_2 = m_3 = m_1$

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2$;

Process 1 to 2: $P = \text{constant}$ $\Rightarrow$ ${}_1w_2 = \int P\,dv = P_1(v_2 - v_1) = R(T_2 - T_1)$

Ideal gas $Pv = RT$ $\Rightarrow$ $T_2 = T_1 v_2/v_1 = 2T_1 = \mathbf{1200\ K}$

$\quad P_2 = P_1 = 200\text{ kPa},$ ${}_1w_2 = RT_1 = \mathbf{172.2\ kJ/kg}$

Table A.7 $\mathbf{h_2 = 1277.8\ kJ/kg,}$ $\mathbf{h_3 = h_1 = 607.3\ kJ/kg}$

$\quad {}_1q_2 = u_2 - u_1 + {}_1w_2 = h_2 - h_1 = 1277.8 - 607.3 = \mathbf{670.5\ kJ/kg}$

Process $2\rightarrow3$: $v_3 = v_2 = 2v_1$ $\Rightarrow$ $\mathbf{{}_2w_3 = 0,}$

$\quad P_3 = P_2 T_3/T_2 = P_1 T_1/2T_1 = P_1/2 = \mathbf{100\ kPa}$

$\quad\quad {}_2q_3 = u_3 - u_2 = 435.1 - 933.4 = \mathbf{-498.3\ kJ/kg}$

3.183

A piston/cylinder has 0.5 kg air at 2000 kPa, 1000 K as shown in Fig P3.183. The cylinder has stops so $V_{min} = 0.03 \text{ m}^3$. The air now cools to 400 K by heat transfer to the ambient. Find the final volume and pressure of the air (does it hit the stops?) and the work and heat transfer in the process.

We recognize this is a possible two-step process, one of constant P and one of constant V. This behavior is dictated by the construction of the device.

Continuity Eq.: $\qquad m_2 - m_1 = 0$

Energy Eq.3.5: $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $\qquad$ P = constant = F/A = $P_1 \qquad$ if $\quad$ V > Vmin

$\qquad\qquad$ V = constant = $V_{1a} = V_{min} \qquad$ if $\quad$ P < P_1

State 1: (P, T) $\quad V_1 = mRT_1/P_1 = 0.5 \times 0.287 \times 1000/2000 = 0.07175 \text{ m}^3$

The only possible P-V combinations for this system is shown in the diagram so both state 1 and 2 must be on the two lines. For state 2 we need to know if it is on the horizontal P line segment or the vertical V segment. Let us check state 1a:

State 1a: $\qquad P_{1a} = P_1, V_{1a} = V_{min}$

$$\text{Ideal gas so } T_{1a} = T_1 \frac{V_{1a}}{V_1} = 1000 \times \frac{0.03}{0.07175} = 418 \text{ K}$$

We see that $T_2 < T_{1a}$ and state 2 must have $V_2 = V_{1a} = V_{min} = 0.03 \text{ m}^3$.

$$P_2 = P_1 \times \frac{T_2}{T_1} \times \frac{V_1}{V_2} = 2000 \times \frac{400}{1000} \times \frac{0.07175}{0.03} = 1913.3 \text{ kPa}$$

The work is the area under the process curve in the P-V diagram

$${}_1W_2 = \int_1^2 P \, dV = P_1 (V_{1a} - V_1) = 2000 \text{ kPa} (0.03 - 0.07175) \text{ m}^3 = -\textbf{83.5 kJ}$$

Now the heat transfer is found from the energy equation, u's from Table A.7.1,

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.5 (286.49 - 759.19) - 83.5 = \textbf{-319.85 kJ}$$

3.184

Air in a rigid tank is at 100 kPa, 300 K with a volume of 0.75 m^3. The tank is heated to 400 K, state 2. Now one side of the tank acts as a piston letting the air expand slowly at constant temperature to state 3 with a volume of 1.5 m^3. Find the pressures at states 2 and 3, Find the total work and total heat transfer.

State 1: $\qquad m = P_1 V_1 / RT_1 = \dfrac{100 \times 0.75}{0.287 \times 300} \dfrac{\text{kPa m}^3}{\text{kJ/kg}} = 0.871 \text{ kg}$

Process 1 to 2: Constant volume heating, $\quad dV = 0 \quad => \quad {}_1W_2 = 0$

$\qquad P_2 = P_1 \, T_2 \, / \, T_1 \ = 100 \times 400 \, / \, 300 = \textbf{133.3 kPa}$

Process 2 to 3: Isothermal expansion, $\quad dT = 0 \quad => \quad u_3 = u_2$ and

$\qquad P_3 = P_2 \, V_2 \, / \, V_3 = 133.3 \times 0.75 \, / \, 1.5 = \textbf{66.67 kPa}$

$\qquad {}_2W_3 = \displaystyle\int_2^3 P \; dV = P_2 V_2 \ln\left(\dfrac{V_3}{V_2}\right) = 133.3 \times 0.75 \ln(2) = 69.3 \text{ kJ}$

The overall process:

$\qquad {}_1W_3 = {}_1W_2 + {}_2W_3 = {}_2W_3 = \textbf{69.3 kJ}$

From the energy equation

$\qquad {}_1Q_3 = m(u_3 - \ u_1) + {}_1W_3 = m \, C_v \, (T_3 - T_1) + {}_1W_3$

$\qquad\qquad = 0.871 \text{ kg} \times 0.717 \text{ kJ/kg-K} \times (400 - 300) \text{ K} + 69.3 \text{ kJ}$

$\qquad\qquad = \textbf{131.8 kJ}$

3.185

A 100 hp car engine has a drive shaft rotating at 2000 RPM. How much torque is on the shaft for 25% of full power?

Solution:

Power = 0.25×100 hp = 0.25×73.5 kW (if SI hp) = 18.375 kW = Tω

ω = angular velocity (rad/s) = RPM 2 π /60

$$T = Power/\omega = \frac{power \times 60}{RPM \times 2\pi} = \frac{18.375 \text{ kW} \times 60 \text{ s/min}}{2000 \times 2\pi \text{ rad/min}} = \textbf{87.73 Nm}$$

We could also have used UK hp to get 0.25×74.6 kW. then T = 89 Nm.

3.186

A crane use 2 kW to raise a 100 kg box 20 m. How much time does it take?

$$\text{Power} = \dot{W} = F\mathbf{V} = mg\mathbf{V} = mg\frac{L}{t}$$

$$t = \frac{mgL}{\dot{W}} = \frac{100 \text{ kg } 9.807 \text{ m/s}^2 \, 20 \text{ m}}{2000 \text{ W}} = \mathbf{9.81 \ s}$$

3.187

An escalator raises a 100 kg bucket of sand 10 m in 1 minute. Determine the rate of work done during the process.

Solution:

The work is a force with a displacement and force is constant: $F = mg$

$$W = \int F\, dx = F \int dx = F\, \Delta x = 100 \text{ kg} \times 9.80665 \text{ m/s}^2 \times 10 \text{ m} = 9807 \text{ J}$$

The rate of work is work per unit time

$$\dot{W} = \frac{W}{\Delta t} = \frac{9807 \text{ J}}{60 \text{ s}} = \textbf{163 W}$$

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3.188

A piston/cylinder of cross sectional area 0.01 m^2 maintains constant pressure. It contains 1 kg water with a quality of 5% at 150°C. If we heat so 1 g/s liquid turns into vapor what is the rate of work out?

$$V_{vapor} = m_{vapor} \, v_g \, , \quad V_{liq} = m_{liq} \, v_f$$

$$m_{tot} = constant = m_{vapor} \, m_{liq}$$

$$V_{tot} = V_{vapor} + V_{liq}$$

$$\dot{m}_{tot} \qquad = 0 = \dot{m}_{vapor} + \dot{m}_{liq} \quad \Rightarrow \quad \dot{m}_{liq} = -\dot{m}_{vapor}$$

$$\dot{V}_{tot} = \dot{V}_{vapor} + \dot{V}_{liq} = \dot{m}_{vapor} v_g + \dot{m}_{liq} v_f$$

$$= \dot{m}_{vapor} \, (v_g - v_f) = \dot{m}_{vapor} \, v_{fg}$$

$$\dot{W} = P\dot{V} = P \, \dot{m}_{vapor} \, v_{fg}$$

$$= 475.9 \text{ kPa} \times 0.001 \text{ kg/s} \times 0.39169 \text{ m}^3/\text{kg} = \textbf{0.1864 kW}$$

$$= \textbf{186 W}$$

3.189

A pot of water is boiling on a stove supplying 325 W to the water. What is the rate of mass (kg/s) vaporizing assuming a constant pressure process?

To answer this we must assume all the power goes into the water and that the process takes place at atmospheric pressure 101 kPa, so T = 100°C.

Energy equation

$$dQ = dE + dW = dU + PdV = dH = h_{fg}\, dm$$

$$\frac{dQ}{dt} = h_{fg}\,\frac{dm}{dt}$$

$$\frac{dm}{dt} = \frac{\dot{Q}}{h_{fg}} = \frac{325\ W}{2257\ kJ/kg} = \textbf{0.144 g/s}$$

The volume rate of increase is

$$\frac{dV}{dt} = \frac{dm}{dt}\,v_{fg} = 0.144\ g/s \times 1.67185\ m^3/kg$$

$$= 0.24 \times 10^{-3}\ m^3/s = 0.24\ L/s$$

3.190

The heaters in a spacecraft suddenly fail. Heat is lost by radiation at the rate of 100 kJ/h, and the electric instruments generate 75 kJ/h. Initially, the air is at 100 kPa, 25°C with a volume of 10 m^3. How long will it take to reach an air temperature of −20°C?

Solution:

C.M. Air

Continuity Eq: $\frac{dM}{dt} = 0$ $\dot{W} = 0$

Q_{el} C.V. $\dot{Q}_{rad}$

Energy Eq: $\frac{dE}{dt} = \dot{Q}_{el} - \dot{Q}_{rad}$ $\dot{KE} = 0$

$\dot{PE} = 0$

$$\dot{E} = \dot{U} = \dot{Q}_{el} - \dot{Q}_{rad} = \dot{Q}_{net} \Rightarrow U_2 - U_1 = m(u_2 - u_1) = \dot{Q}_{net}(t_2 - t_1)$$

$$\text{Ideal gas: } m = \frac{P_1 V_1}{RT_1} = \frac{100 \times 10}{0.287 \times 298.15} = 11.688 \text{ kg}$$

$$m(u_2 - u_1) = m\, C_{v0}(T_2 - T_1) = 11.688 \times 0.717\,(-20 - 25) = -377.1 \text{ kJ}$$

$$t_2 - t_1 = m\,(u_2 - u_1)/\dot{Q}_{net} = (-377.1 \text{ kJ})\,/(-25 \text{ kJ/h}) = \textbf{15.08 h}$$

3.191

As fresh poured concrete hardens, the chemical transformation releases energy at a rate of 2 W/kg. Assume the center of a poured layer does not have any heat loss and that it has an average specific heat of 0.9 kJ/kg-K. Find the temperature rise during 1 hour of the hardening (curing) process.

Solution:

$$\dot{U} = (m\dot{u}) = mC_V\dot{T} = \dot{Q} = m\dot{q}$$

$$\dot{T} = \dot{q}/C_V = 2{\times}10^{-3}\,kW/\,0.9\,kJ/kg\text{-}K$$

$$= 2.222 \times 10^{-3}\ °C/sec$$

$$\Delta T = \dot{T}\Delta t = 2.222 \times 10^{-3} \times 3600 = \textbf{8 °C}$$

3.192

A pot of 1.2 kg water at 20°C is put on a stove supplying 1250 W to the water. After how long time can I expect it to come to a boil (100°C)?

Energy Equation on a rate form:$\quad \dfrac{dE_{water}}{dt} = \dfrac{dU_{water}}{dt} = \dot{Q} - \dot{W} = \dot{Q} - P\dot{V}$

$$\dot{Q} = \frac{dU_{water}}{dt} + P\dot{V} = \frac{dH_{water}}{dt} = m_{water}C_p \frac{dT_{water}}{dt}$$

Integrate over time

$$Q = \dot{Q}\,\Delta t = \Delta H = m_{water}\,(h_2 - h_1) \approx m_{water}C_p\,(T_2 - T_1)$$

$$\Delta t = m_{water}\,(h_2 - h_1)/\dot{Q} \approx m_{water}C_p\,(T_2 - T_1)/\dot{Q}$$
$$= 1.2\,(419.02 - 83.94)/1.25 \approx 1.2 \times 4.18\,(100 - 20)/1.25$$
$$= \mathbf{321.7\ s} \approx 5.5\ min$$

Comment: Notice how close the two results are, i.e. use of constant C_p is OK.

3.193

A computer in a closed room of volume 200 m^3 dissipates energy at a rate of 10 kW. The room has 50 kg wood, 25 kg steel and air, with all material at 300 K, 100 kPa. Assuming all the mass heats up uniformly, how long will it take to increase the temperature 10°C?

Solution:

C.V. Air, wood and steel. $m_2 = m_1$; no work

Energy Eq.3.5: $U_2 - U_1 = {}_1Q_2 = \dot{Q}\Delta t$

The total volume is nearly all air, but we can find volume of the solids.

$V_{wood} = m/\rho = 50/510 = 0.098$ m^3 ; $V_{steel} = 25/7820 = 0.003$ m^3

$V_{air} = 200 - 0.098 - 0.003 = 199.899$ m^3

$\qquad m_{air} = PV/RT = 101.325 \times 199.899/(0.287 \times 300) = 235.25$ kg

We do not have a u table for steel or wood so use heat capacity from A.3.

$\qquad \Delta U = [m_{air}\, C_v + m_{wood}\, C_v + m_{steel}\, C_v\,]\Delta T$

$\qquad\qquad = (235.25 \times 0.717 + 50 \times 1.38 + 25 \times 0.46)$ kJ/K $\times 10$ K

$\qquad\qquad = 1686.7 + 690 + 115 = 2492$ kJ $= \dot{Q} \times \Delta t = 10$ kW $\times \Delta t$

$\qquad => \qquad \Delta t = 2492$ kJ/10 kW = **249.2 sec = 4.2 minutes**

3.194

The rate of heat transfer to the surroundings from a person at rest is about 400 kJ/h. Suppose that the ventilation system fails in an auditorium containing 100 people. Assume the energy goes into the air of volume 1500 m^3 initially at 300 K and 101 kPa. Find the rate (degrees per minute) of the air temperature change.

Solution:

$$\dot{Q} = n\,\dot{q} = 100 \times 400 = 40\,000 \text{ kJ/h} = 666.7 \text{ kJ/min}$$

$$\frac{dE_{air}}{dt} = \dot{Q} = m_{air}C_v\frac{dT_{air}}{dt}$$

$$m_{air} = PV/RT = 101 \text{ kPa} \times 1500 \text{ m}^3/ (0.287 \text{ kJ/kg-K} \times 300 \text{ K}) = 1759.6 \text{ kg}$$

$$\frac{dT_{air}}{dt} = \dot{Q}/mC_v = 666.7 \text{ kJ/min} / (1759.6 \text{ kg} \times 0.717 \text{ kJ/kg-K})$$

$$= \mathbf{0.53°C/min}$$

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3.195

A steam generating unit heats saturated liquid water at constant pressure of 800 kPa in a piston cylinder. If 1.5 kW of power is added by heat transfer find the rate (kg/s) of saturated vapor that is made.

Solution:

Energy equation on a rate form making saturated vapor from saturated liquid

$$\dot{U} = (\dot{m}u) = \dot{m}\Delta u = \dot{Q} - \dot{W} = \dot{Q} - P\dot{V} = \dot{Q} - P\,\dot{m}\Delta v$$

Rearrange to solve for heat transfer rate

$$\dot{Q} = \dot{m}(\Delta u + \Delta vP) = \dot{m}\,\Delta h = \dot{m}\,h_{fg}$$

So now

$$\dot{m} = \dot{Q}/\,h_{fg} = 1.5\ kW/\ 2048.04\ kJ/kg = \textbf{0.732 g/s}$$

3.196

A 500 Watt heater is used to melt 2 kg of solid ice at -10°C to liquid at $+5^{\circ}$C at a constant pressure of 150 kPa.

    a) Find the change in the total volume of the water.

    b) Find the energy the heater must provide to the water.

    c) Find the time the process will take assuming uniform T in the water.

Solution:

Take CV as the 2 kg of water. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: Compressed solid, take saturated solid at same temperature.

   $v = v_i(-10) = 0.0010891$ m^3/kg, $h = h_i = -354.09$ kJ/kg

State 2: Compressed liquid, take saturated liquid at same temperature

   $v = v_f = 0.001$, $h = h_f = 20.98$ kJ/kg

Change in volume:

   $V_2 - V_1 = m(v_2 - v_1) = 2(0.001 - 0.0010891) = $ **−0.000178 m^3**

Work is done while piston moves at constant pressure, so we get

   ${}_1W_2 = \int P\ dV = \text{area} = P(V_2 - V_1) = -150 \times 0.000178 = -0.027$ kJ $= -27$ J

Heat transfer is found from energy equation

   ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) = 2 \times [20.98 - (-354.09)] = $ **750 kJ**

The elapsed time is found from the heat transfer and the rate of heat transfer

   $t = {}_1Q_2/\dot{Q} = (750$ kJ $/500$ W$) \times 1000$ J/kJ $= 1500$ s $= $ **25 min**

3.197

A drag force on a car, with frontal area A = 2 m^2, driving at 80 km/h in air at 20$^{\text{o}}$ C is $F_d = 0.225$ A $\rho_{\text{air}}\mathbf{V}^2$. How much power is needed and what is the traction force?

$$\dot{W} = F\mathbf{V}$$

$$\mathbf{V} = 80\,\frac{km}{h} = 80 \times \frac{1000}{3600}\,ms^{-1} = 22.22\ ms^{-1}$$

$$\rho_{AIR} = \frac{P}{RT} = \frac{101}{0.287 \times 293} = 1.20\ kg/m^3$$

$$F_d = 0.225\ A\rho\mathbf{V}^2 = 0.225 \times 2 \times 1.2 \times 22.22^2 = \mathbf{266.61\ N}$$

$$\dot{W} = F\mathbf{V} = 266.61\ N \times 22.22\ m/s = 5924\ W = \mathbf{5.92\ kW}$$

3.198

A mass of 3 kg nitrogen gas at 2000 K, V = C, cools with 500 W. What is dT/dt?

Process: V = C $\rightarrow$ $_1W_2 = 0$

$$\frac{dE}{dt} = \frac{dU}{dt} = m\frac{dU}{dt} = mC_v\frac{dT}{dt} = \dot{Q} - W = \dot{Q} = -500 \text{ W}$$

$$C_{v\,2000} = \frac{du}{dT} = \frac{\Delta u}{\Delta T} = \frac{u_{2100} - u_{1900}}{2100-1900} = \frac{1819.08 - 1621.66}{200} = 0.987 \text{ kJ/kg K}$$

$$\frac{dT}{dt} = \frac{\dot{Q}}{mC_v} = \frac{-500 \text{ W}}{3 \times 0.987 \text{ kJ/K}} = \textbf{-0.17} \frac{\textbf{K}}{\textbf{s}}$$

Remark: Specific heat from Table A.5 has $C_{v\,300}$ = 0.745 kJ/kg K which is nearly 25% lower and thus would over-estimate the rate with 25%.

3.199

Consider the pot in Problem 3.119. Assume the stove supplies 1 kW of heat. How much time does the process take?

A 1 kg steel pot contains 1 kg liquid water both at 15$^\text{o}$C. It is now put on the stove where it is heated to the boiling point of the water. Neglect any air being heated and find the total amount of energy needed.

Solution:

Energy Eq.: $U_2 - U_1 = {_1}Q_2 - {_1}W_2$

The steel does not change volume and the change for the liquid is minimal, so ${_1}W_2 \cong 0$.

State 2: $T_2 = T_{sat}$ (1atm) = 100$^\text{o}$C

Tbl B.1.1 : $u_1 = 62.98$ kJ/kg, $u_2 = 418.91$ kJ/kg

Tbl A.3 : $C_{st} = 0.46$ kJ/kg K

Solve for the heat transfer from the energy equation

$${_1}Q_2 = U_2 - U_1 = m_{st}\, (u_2 - u_1)_{st} + m_{H2O}\, (u_2 - u_1)_{H2O}$$

$$= m_{st}C_{st}\ (T_2 - T_1) + m_{H2O}\, (u_2 - u_1)_{H2O}$$

$${_1}Q_2 = 1 \text{ kg} \times 0.46\, \frac{\text{kJ}}{\text{kg K}} \times (100 - 15)\text{ K} + 1 \text{ kg} \times (418.91 - 62.98)\text{ kJ/kg}$$

$$= 39.1 + 355.93 = 395 \text{ kJ}$$

To transfer that amount of heat with a rate of 1 kW we get the relation

$${_1}Q_2 = \int \dot{Q}\ dt = \dot{Q}\ \Delta t \ \Rightarrow\ \Delta t\ = {_1}Q_2\, /\, \dot{Q} = 395 \text{ kJ } /1 \text{ (kJ/s)} = \textbf{395 s}$$

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General work

3.200

Electric power is volts times ampere (P = V i). When a car battery at 12 V is charged with 6 amp for 3 hours how much energy is delivered?

Solution:

Work term integrated as

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t = V\, i\, \Delta t$$
$$= 12\ V \times 6\ Amp \times 3 \times 3600\ s$$
$$= 777\ 600\ J = \textbf{777.6 kJ}$$

Remark: Volt times ampere is also watts, 1 W = 1 V × 1 Amp. = 1 J/s

3.201

A copper wire of diameter 2 mm is 10 m long and stretched out between two posts. The normal stress (pressure) $\sigma = E(L – L_o)/L_o$, depends on the length L versus the un-stretched length L_o and Young's modulus $E = 1.1 \times 10^6$ kPa. The force is $F = A\sigma$ and measured to be 110 N. How much longer is the wire and how much work was put in?

Solution:

$F = As = AE \, \Delta L/ L_o$ and $\Delta L = FL_o /AE$

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times 0.002^2 = 3.142 \times 10^{-6} \, m^2$$

$$\Delta L = \frac{110 \, N \times 10 \, m}{3.142 \times 10^{-6} \, m^2 \times 1.1 \times 10^6 \times 10^3 \, Pa} = \textbf{0.318 m}$$

$$_1W_2 = \int F \, dx = \int A \, s \, dx = \int AE \frac{x}{L_o} \, dx$$

$$= \frac{AE}{L_o} \tfrac{1}{2} x^2 \qquad \text{where } x = L - L_o$$

$$= \frac{3.142 \times 10^{-6} \times 1.1 \times 10^6 \times 10^3}{10} \times \tfrac{1}{2} \times 0.318^2 = \textbf{17.47 J}$$

3.202

A film of ethanol at 20°C has a surface tension of 22.3 mN/m and is maintained on a wire frame as shown in Fig. P3.202. Consider the film with two surfaces as a control mass and find the work done when the wire is moved 10 mm to make the film 20 × 40 mm.

Solution :

Assume a free surface on both sides of the frame, i.e., there are two surfaces 20 × 30 mm

$$W = -\int S \, dA = -22.3 \times 10^{-3} \text{ N/m} \times 2 \, (800 - 600) \times 10^{-6} \text{ m}^2$$

$$= -8.92 \times 10^{-6} \text{ J} = \mathbf{-8.92 \; \mu J}$$

3.203

A 10-L rigid tank contains R-410A at $-10°C$, 80% quality. A 10-A electric current (from a 6-V battery) is passed through a resistor inside the tank for 10 min, after which the R-410A temperature is 40°C. What was the heat transfer to or from the tank during this process?

Solution:

C.V. R-410A in tank. Control mass at constant V.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: Constant V $\Rightarrow$ $v_2 = v_1$

$\Rightarrow$ no boundary work, but electrical work

State 1 from table B.4.1

$v_1 = 0.000827 + 0.8 \times 0.04470 = 0.03659$ m^3/kg

$u_1 = 42.32 + 0.8 \times 207.36 = 208.21$ kJ/kg

$m = V/v = 0.010$ m^3 /0.03659 m^3/kg $= 0.2733$ kg

State 2: Table B.4.2   at 40°C and $v_2 = v_1 = 0.03659$ m^3/kg

$\Rightarrow$ superheated vapor, so use linear interpolation to get

$P_2 = 800 + 200 \times (0.03659 – 0.04074)/(0.03170 – 0.04074)$

$= 800 + 200 \times 0.45907 = 892$ kPa

$u_2 = 286.83 + 0.45907 \times (284.35 – 286.83) = 285.69$ kJ/kg

${}_1W_2 \, _{elec} = –power \times \Delta t = –Amp \times volts \times \Delta t = - \dfrac{10 \times 6 \times 10 \times 60}{1000} = –36$ kJ

${}_1Q_2 = m(u_2 – u_1) + {}_1W_2 = 0.2733$ kg (285.69 – 208.21) kJ/kg – 36 kJ

$= –$**14.8 kJ**

3.204

A battery is well insulated while being charged by 12.3 V at a current of 6 A. Take the battery as a control mass and find the instantaneous rate of work and the total work done over 4 hours.

Solution :

Battery thermally insulated $\Rightarrow$ Q = 0

For constant voltage E and current i,

Power = E i = 12.3 × 6 = **73.8 W** [Units V × A = W]

$$W = \int \text{power dt} = \text{power } \Delta t$$

= 73.8 W × 4 h × 3600 (s/h) = 1 062 720 J = **1062.7 kJ**

3.205

A sheet of rubber is stretched out over a ring of radius 0.25 m. I pour liquid water at 20°C on it, as in Fig. P3.205, so the rubber forms a half sphere (cup). Neglect the rubber mass and find the surface tension near the ring?

Solution:

$$F \uparrow = F \downarrow \quad ; \quad F \uparrow = SL$$

The length is the perimeter, $2\pi r$, and there is two surfaces

$$S \times 2 \times 2\pi r = m_{H2o}\, g = \rho_{H2o}\, Vg = \rho_{H2o} \times \frac{1}{12} \pi\, (2r)^3 g = \rho_{H2o} \times \pi \frac{2}{3} r^3$$

$$S = \rho_{H2o}\, \frac{1}{6} r^2\, g$$

$$= 997 \text{ kg/m}^3 \times \frac{1}{6} \times 0.25^2 \text{ m}^2 \times 9.81 \text{ m/s}^2$$

$$= \textbf{101.9 N/m}$$

3.206

Assume we fill a spherical balloon from a bottle of helium gas. The helium gas provides work $\int P\,dV$ that stretches the balloon material $\int S\,dA$ and pushes back the atmosphere $\int P_o\,dV$. Write the incremental balance for $dW_{helium} = dW_{stretch} + dW_{atm}$ to establish the connection between the helium pressure, the surface tension S and P_o as a function of radius.

$$W_{He} = \int P\,dV = \int S\,dA + \int P_o\,dV$$

$$dW_{He} = P\,dV = S\,dA + P_o\,dV$$

$$dV = d\left(\frac{\pi}{6}D^3\right) = \frac{\pi}{6} \times 3D^2\,dD$$

$$dA = d(2 \times \pi \times D^2) = 2\pi\,(2D)\,dD$$

$$P\frac{\pi}{2}D^2\,dD = S\,(4\pi)D\,dD + P_o\frac{\pi}{2}D^2\,dD$$

Divide by $\frac{\pi}{2}D^2$ to recognize

$$\mathbf{P_{He} = P_o + 8\frac{S}{D} = P_o + 4\frac{S}{r}}$$

3.207

Assume a balloon material with a constant surface tension of S = 2 N/m. What is the work required to stretch a spherical balloon up to a radius of r = 0.5 m? Neglect any effect from atmospheric pressure.

Assume the initial area is small, and that we have 2 surfaces inside and out

$$W = -\int S \, dA = -S \, (A_2 - A_1)$$

$$= -S(A_2) = -S(2\times \pi \, D_2^2)$$

$$= -2 \text{ N/m} \times 2 \times \pi \times 1 \text{ m}^2 = \text{-}12.57 \text{ J}$$

$$W_{in} = -W = \textbf{12.57 J}$$

3.208

A soap bubble has a surface tension of $S = 3 \times 10^{-4}$ N/cm as it sits flat on a rigid ring of diameter 5 cm. You now blow on the film to create a half sphere surface of diameter 5 cm. How much work was done?

$$_1W_2 \quad = \int F \ dx \ = \int S \ dA \ = S \ \Delta A$$

$$= 2 \times S \times (\frac{\pi}{2} D^2 - \frac{\pi}{4} D^{\ 2})$$

$$= 2 \times 3 \times 10^{-4} \text{ N/cm} \times 100 \text{ cm/m} \times \frac{\pi}{2} 0.05^2 \text{ m}^2 \ (\ 1 - 0.5\ )$$

$$= \mathbf{1.18 \times 10^{-4}} \ \mathbf{J}$$

Notice the bubble has 2 surfaces.

$$A_1 = \frac{\pi}{4} D^{\ 2} \ ,$$

$$A_2 \ = \text{½} \ \pi \ D^2$$

3.209

A 0.5-m-long steel rod with a 1-cm diameter is stretched in a tensile test. What is the required work to obtain a relative strain of 0.1%? The modulus of elasticity of steel is 2×10^8 kPa.

Solution :

$$_{-1}W_2 = \frac{AEL_0}{2}\,(e)^2, \quad A = \frac{\pi}{4}\,(0.01)^2 = 78.54 \times 10^{-6}\ m^2$$

$$_{-1}W_2 = \frac{1}{2} \times 78.54 \times 10^{-6}\ m^2 \times 2{\times}10^8\ kPa \times 0.5\ m\ (10^{-3})^2$$

$$= \textbf{3.93 J}$$

More Complex Devices

3.210

A rigid tank is divided into two rooms by a membrane, both containing water, shown in Fig. P3.210. Room A is at 200 kPa, $v = 0.5$ m^3/kg, $V_A = 1$ m^3, and room B contains 3.5 kg at 0.5 MPa, 400°C. The membrane now ruptures and heat transfer takes place so the water comes to a uniform state at 100°C. Find the heat transfer during the process.

Solution:

C.V.: Both rooms A and B in tank.

Continuity Eq.: $m_2 = m_{A1} + m_{B1}$;

Energy Eq.: $m_2 u_2 - m_{A1} u_{A1} - m_{B1} u_{B1} = {}_1Q_2 - {}_1W_2$

State 1A: (P, v) Table B.1.2, $m_{A1} = V_A / v_{A1} = 1/0.5 = 2$ kg

$$x_{A1} = \frac{v - v_f}{v_{fg}} = \frac{0.5 - 0.001061}{0.88467} = 0.564$$

$$u_{A1} = u_f + x\, u_{fg} = 504.47 + 0.564 \times 2025.02 = 1646.6 \text{ kJ/kg}$$

State 1B: Table B.1.3, $v_{B1} = 0.6173$, $u_{B1} = 2963.2$, $V_B = m_{B1} v_{B1} = 2.16$ m^3

Process constant total volume: $V_{tot} = V_A + V_B = 3.16$ m^3 and ${}_1W_2 = 0$

$$m_2 = m_{A1} + m_{B1} = 5.5 \text{ kg} \quad \Rightarrow \quad v_2 = V_{tot}/m_2 = 0.5746 \text{ m}^3/\text{kg}$$

State 2: T_2 , v_2 $\Rightarrow$ Table B.1.1   two-phase as $v_2 < v_g$

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.5746 - 0.001044}{1.67185} = 0.343 \ ,$$

$$u_2 = u_f + x\, u_{fg} = 418.91 + 0.343 \times 2087.58 = 1134.95 \text{ kJ/kg}$$

Heat transfer is from the energy equation

$${}_1Q_2 = m_2 u_2 - m_{A1} u_{A1} - m_{B1} u_{B1}$$
$$= (5.5 \times 1134.95 - 2 \times 1646.6 - 3.5 \times 2963.2) \text{ kg} \times \text{kJ/kg}$$
$$= \textbf{-7421 kJ}$$

3.211

A piston cylinder has a water volume separated in $V_A = 0.2$ m³ and $V_B = 0.3$ m³ by a stiff membrane. The initial state in A is 1000 kPa, $x = 0.75$ and in B it is 1600 kPa and 250°C. Now the membrane ruptures and the water comes to a uniform state at 200°C. What is the final pressure? Find the work and the heat transfer in the process.

Take the water in A and B as CV.

Continuity: $m_2 - m_{1A} - m_{1B} = 0$

Energy: $m_2 u_2 - m_{1A} u_{1A} - m_{1B} u_{1B} = {}_1Q_2 - {}_1W_2$

Process: $P_2 = P_{eq} = constant = P_{1A}$ as piston floats and m_p, P_o do not change

State 1A: Two phase. Table B.1.2

$v_{1A} = 0.001127 + 0.75 \times 0.19332 = 0.146117$ m³/kg,

$u_{1A} = 761.67 + 0.75 \times 1821.97 = 2128.15$ kJ/kg

State 1B: Table B.1.3 $v_{1B} = 0.14184$ m³/kg, $u_{1B} = 2692.26$ kJ/kg

=> $m_{1A} = V_{1A}/v_{1A} = 1.3688$ kg, $m_{1B} = V_{1B}/v_{1B} = 2.115$ kg

State 2: 1000 kPa, 200°C sup. vapor => $v_2 = 0.20596$ m³/kg, $u_2 = 2621.9$ kJ/kg

$m_2 = m_{1A} + m_{1B} = 3.4838$ kg   => $V_2 = m_2 v_2 = 3.4838 \times 0.20596 = 0.7175$ m³

Piston moves at constant pressure

$_1W_2 = \int P\ dV = P_{eq}(V_2 - V_1) = 1000$ kPa $\times (0.7175 - 0.5)$ m³ = **217.5 kJ**

$_1Q_2 = m_2 u_2 - m_{1A} u_{1A} - m_{1B} u_{1B} + {}_1W_2$
     $= 3.4838 \times 2621.9 - 1.3688 \times 2128.15 - 2.115 \times 2692.26 + 217.5 =$ **744 kJ**

3.212

The cylinder volume below the constant loaded piston has two compartments A and B filled with water. A has 0.5 kg at 200 kPa, 150°C and B has 400 kPa with a quality of 50% and a volume of 0.1 m^3. The valve is opened and heat is transferred so the water comes to a uniform state with a total volume of 1.006 m^3.

   a) Find the total mass of water and the total initial volume.
   b) Find the work in the process
   c) Find the process heat transfer.

Solution:

Take the water in A and B as CV.

   Continuity: $m_2 - m_{1A} - m_{1B} = 0$

   Energy: $m_2 u_2 - m_{1A} u_{1A} - m_{1B} u_{1B} = {}_1Q_2 - {}_1W_2$

   Process: P = constant = P_{1A} if piston floats

                   (V_A positive) i.e. if $V_2 > V_B = 0.1$ m^3

   State A1: Sup. vap. Table B.1.3   v = 0.95964 m^3/kg, u = 2576.9 kJ/kg

                    => V = mv = 0.5 kg × 0.95964 m^3/kg = 0.47982

   State B1: Table B.1.2   v = (1-x) × 0.001084 + x × 0.4625 = 0.2318 m^3/kg

                    =>   m = V/v = 0.4314 kg

                 u = 604.29 + 0.5 × 1949.3 = 1578.9 kJ/kg

   State 2: 200 kPa, $v_2 = V_2/m = 1.006/0.9314 = 1.0801$ m^3/kg

           Table B.1.3 => close to $T_2 = 200$°C   and $u_2 = 2654.4$ kJ/kg

   So now

                 $V_1 = 0.47982 + 0.1 =$ **0.5798 m^3**, $m_1 = 0.5 + 0.4314 =$ **0.9314 kg**

Since volume at state 2 is larger than initial volume piston goes up and the pressure then is constant (200 kPa which floats piston).

$${}_1W_2 = \int P\, dV = P_{lift}\,(V_2 - V_1) = 200 \text{ kPa } (1.006 - 0.57982) \text{ m}^3 = \textbf{85.24 kJ}$$

$${}_1Q_2 = m_2 u_2 - m_{1A} u_{1A} - m_{1B} u_{1B} + {}_1W_2$$
$$= 0.9314 × 2654.4 - 0.5 × 2576.9 - 0.4314 × 1578.9 + 85.24 = \textbf{588 kJ}$$

3.213

Water in a tank A is at 250 kPa with a quality of 10% and mass 0.5 kg. It is connected to a piston cylinder holding constant pressure of 200 kPa initially with 0.5 kg water at 400°C. The valve is opened and enough heat transfer takes place to have a final uniform temperature of 150°C. Find the final P and V, the process work and the process heat transfer.

C.V. Water in A and B. Control mass goes through process: 1 -> 2

Continuity Eq.: $m_2 - m_{A1} - m_{B1} = 0$ => $m_2 = m_{A1} + m_{B1} = 0.5 + 0.5 = 1$ kg

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

State A1: $v_{A1} = 0.001067 + x_{A1} \times 0.71765 = 0.072832$; $V_{A1} = mv = 0.036416$ m^3

$$u_{A1} = 535.08 + 0.1 \times 2002.14 = 735.22 \text{ kJ/kg}$$

State B1: $v_{B1} = 1.5493$ m^3/kg; $u_{B1} = 2966.69$ kJ/kg

$$=> \quad V_{B1} = m_{B1}v_{B1} = 0.77465 \text{ m}^3$$

State 2: If $V_2 > V_{A1}$ then $P_2 =$ **200 kPa** that is the piston floats.

For $(T_2, P_2) = (150°C, 200 \text{ kPa})$ => superheated vapor $u_2 = 2576.87$ kJ/kg

$$v_2 = 0.95964 \text{ m}^3\text{/kg} \qquad V_2 = m_2v_2 = \textbf{0.95964 m}^3 > V_{A1} \text{ checks OK.}$$

The possible state 2 (P,V) combinations are shown. State "a" is at 200 kPa and

$$v_a = \frac{V_{A1}}{m_2} = 0.036 \text{ m}^3\text{/kg}$$

and thus two-phase

$$T_a = 120°C < T_2$$

Process: $_1W_2 = P_2 (V_2 - V_1) = 200 \text{ kPa} \times (0.95964 - 0.77465 - 0.036416)$ m^3

$$= \textbf{29.72 kJ}$$

From the energy Eq.:

$$_1Q_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} + {}_1W_2$$

$$= 1 \times 2576.87 - 0.5 \times 735.222 - 0.5 \times 2966.69 + 29.72 = \textbf{755.6 kJ}$$

3.214

Two rigid tanks are filled with water. Tank A is 0.2 m³ at 100 kPa, 150°C and tank B is 0.3 m³ at saturated vapor 300 kPa. The tanks are connected by a pipe with a closed valve. We open the valve and let all the water come to a single uniform state while we transfer enough heat to have a final pressure of 300 kPa. Give the two property values that determine the final state and find the heat transfer.

State A1: $u = 2582.75$ kJ/kg, $v = 1.93636$ m³/kg

$\Rightarrow m_{A1} = V/v = 0.2/1.93636 =$ **0.1033 kg**

State B1: $u = 2543.55$ kJ/kg, $v = 0.60582$ m³/kg

$\Rightarrow m_{B1} = V/v = 0.3 / 0.60582 =$ **0.4952 kg**

The total volume (and mass) is the sum of volumes (mass) for tanks A and B.

$m_2 = m_{A1} + m_{B1} = 0.1033 + 0.4952 = 0.5985$ kg,

$V_2 = V_{A1} + V_{B1} = 0.2 + 0.3 = 0.5$ m³

$\Rightarrow v_2 = V_2/m_2 = 0.5 /0.5985 =$ **0.8354 m³/kg**

State 2: $[P_2, v_2] = [300$ kPa, 0.8354 m³/kg]

$\Rightarrow T_2 = 274.76°C$ and $u_2 = 2767.32$ kJ/kg

The energy equation is (neglecting kinetic and potential energy)

$m_2\, u_2 - m_A u_{A1} - m_B u_{B1} = {_1}Q_2 - {_1}W_2 = {_1}Q_2$

${_1}Q_2 = (0.5985 \times 2767.32 - 0.1033 \times 2582.75 - 0.4952 \times 2543.55)$ kg ×kJ/kg

$=$ **129.9 kJ**

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3.215

A tank has a volume of 1 m^3 with oxygen at 15°C, 300 kPa. Another tank contains 4 kg oxygen at 60°C, 500 kPa. The two tanks are connected by a pipe and valve which is opened allowing the whole system to come to a single equilibrium state with the ambient at 20°C. Find the final pressure and the heat transfer.

C.V. Both tanks of constant volume.

Continuity Eq.: $m_2 - m_{1A} - m_{1B} = 0$

Energy Eq.: $m_2 u_2 - m_{1A} u_{1A} - m_{1B} u_{1B} = {}_1Q_2 - {}_1W_2$

Process Eq.: $V_2 = V_A + V_B = $ constant, ${}_1W_2 = 0$

State 1A: $m_{1A} = \dfrac{P_{1A} V_A}{RT_{1A}} = \dfrac{300 \text{ kPa} \times 1 \text{ m}^3}{0.2598 \text{ kJ/kg-K} \times 288.15 \text{ K}} = 4.007 \text{ kg}$

State 1B: $V_B = \dfrac{m_{1B} RT_{1B}}{P_{1B}} = \dfrac{4 \text{ kg} \times 0.2598 \text{ kJ/kg-K} \times 333.15 \text{ K}}{500 \text{ kPa}} = 0.6924 \text{ m}^3$

State 2: $(T_2, v_2 = V_2/m_2)$ $V_2 = V_A + V_B = 1 + 0.6924 = 1.6924 \text{ m}^3$

$m_2 = m_{1A} + m_{1B} = 4.007 + 4 = 8.007 \text{ kg}$

$P_2 = \dfrac{m_2 RT_2}{V_2} = \dfrac{8.007 \text{ kg} \times 0.2598 \text{ kJ/kg-K} \times 293.15 \text{ K}}{1.6924 \text{ m}^3} = \textbf{360.3 kPa}$

Heat transfer from energy equation

${}_1Q_2 = m_2 u_2 - m_{1A} u_{1A} - m_{1B} u_{1B} = m_{1A}(u_2 - u_{1A}) + m_{1B}(u_2 - u_{1B})$

$= m_{1A} C_v (T_2 - T_{1A}) + m_{1B} C_v (T_2 - T_{1B})$

$= 4.007 \text{ kg} \times 0.662 \text{ kJ/kg-K} \times (20 - 15) \text{ K} + 4 \text{ kg} \times 0.662 \text{ kJ/kg-K} \times (20 - 60) \text{ K}$

$= \textbf{- 92.65 kJ}$

Borgnakke and Sonntag

3.216

A rigid insulated tank is separated into two rooms by a stiff plate. Room A of 0.5 m^3 contains air at 250 kPa, 300 K and room B of 1 m^3 has air at 500 kPa, 1000 K. The plate is removed and the air comes to a uniform state without any heat transfer. Find the final pressure and temperature.

Solution:

C.V. Total tank. Control mass of constant volume.

Mass and volume: $m_2 = m_A + m_B$; $V = V_A + V_B = 1.5 \ m^3$

Energy Eq.: $U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = Q - W = 0$

Process Eq.: $V = constant \Rightarrow W = 0$; Insulated $\Rightarrow Q = 0$

Ideal gas at 1: $m_A = P_{A1}V_A/RT_{A1} = 250 \times 0.5/(0.287 \times 300) = 1.452 \ kg$

$u_{A1} = 214.364 \ kJ/kg$ from Table A.7

Ideal gas at 2:

$m_B = P_{B1}V_B/RT_{B1} = 500 \ kPa \times 1 \ m^3/(0.287 \ kJ/kg\text{-}K \times 1000 \ K) = 1.742 \ kg$

$u_{B1} = 759.189 \ kJ/kg$ from Table A.7

$m_2 = m_A + m_B = 3.194 \ kg$

$$u_2 = \frac{m_A u_{A1} + m_B u_{B1}}{m_2} = \frac{1.452 \times 214.364 + 1.742 \times 759.189}{3.194} = 511.51 \ kJ/kg$$

$\Rightarrow$ Table A.7.1: $\mathbf{T_2 = 698.6 \ K}$

$P_2 = m_2 RT_2 /V = 3.194 \ kg \times 0.287 \ kJ/kg\text{-}K \times 698.6 \ K /1.5 \ m^3 = \mathbf{426.9 \ kPa}$

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3.217

A rigid tank A of volume 0.6 m^3 contains 3 kg water at 120°C and the rigid tank B is 0.4 m^3 with water at 600 kPa, 200°C. They are connected to a piston cylinder initially empty with closed valves. The pressure in the cylinder should be 800 kPa to float the piston. Now the valves are slowly opened and heat is transferred so the water reaches a uniform state at 250°C with the valves open. Find the final volume and pressure and the work and heat transfer in the process.

C.V.: A + B + C.
Only work in C, total mass constant.

$m_2 - m_1 = 0 \quad => \quad m_2 = m_{A1} + m_{B1}$

$U_2 - U_1 = {}_1Q_2 - {}_1W_2$;

${}_1W_2 = \int PdV = P_{lift} (V_2 - V_1)$

1A: $v = 0.6/3 = 0.2$ m^3/kg $=> \quad x_{A1} = (0.2 - 0.00106)/0.8908 = 0.223327$

$u = 503.48 + 0.223327 \times 2025.76 = 955.89$ kJ/kg

1B: $v = 0.35202$ m^3/kg $=> \quad m_{B1} = 0.4/0.35202 = 1.1363$ kg ; $u = 2638.91$ kJ/kg

$m_2 = 3 + 1.1363 = 4.1363$ kg and $V_1 = V_A + V_B = 1$ m^3

$V_2 = V_A + V_B + V_C = 1$ m$^3 + V_C$
Locate state 2: Must be on P-V lines shown
State 1a: 800 kPa,

$v_{1a} = \dfrac{V_A + V_B}{m} = 0.24176$ m^3/kg

800 kPa, $v_{1a} \quad => \quad T = 173°C$ too low.

Assume 800 kPa: 250°C $=> \qquad v = 0.29314$ m^3/kg $> v_{1a}$ OK

$V_2 = m_2 v_2 = 4.1363$ kg $\times 0.29314$ m^3/kg = **1.2125 m^3**

Final state is : **800 kPa**; 250°C $=> \quad u_2 = 2715.46$ kJ/kg

${}_1W_2 = P_{lift} (V_2 - V_1) = 800$ kPa $\times (1.2125 - 1)$ m^3 = **170 kJ**

${}_1Q_2 = m_2 u_2 - m_1 u_1 + {}_1W_2 = m_2 u_2 - m_{A1} u_{A1} - m_{B1} u_{B1} + {}_1W_2$

$= 4.1363 \times 2715.46 - 3 \times 955.89 - 1.1363 \times 2638.91 + 170$

$= 11\ 232 - 2867.7 - 2998.6 + 170 = $ **5536 kJ**

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Review Problems

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3.218

Ten kilograms of water in a piston/cylinder setup with constant pressure is at 450°C and a volume of 0.633 m^3. It is now cooled to 20°C. Show the *P–v* diagram and find the work and heat transfer for the process.

Solution:

C.V. The 10 kg water.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = C $\Rightarrow$ ${}_1W_2 = mP(v_2 - v_1)$

State 1: (T, $v_1 = 0.633/10 = 0.0633$ m^3/kg)   Table B.1.3

$\quad\quad\quad\quad$ $P_1 = 5$ MPa, $h_1 = 3316.2$ kJ/kg

State 2: (P = P = 5 MPa, 20°C) $\Rightarrow$ Table B.1.4

$\quad\quad\quad\quad$ $v_2 = 0.000\ 999\ 5$ m^3/kg ; $h_2 = 88.65$ kJ/kg

The work from the process equation is found as

$\quad$ ${}_1W_2 = 10$ kg $\times 5000$ kPa $\times(0.0009995 - 0.0633)$ m^3/kg = **-3115 kJ**

The heat transfer from the energy equation is

$\quad$ ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$

$\quad$ ${}_1Q_2 = 10$ kg $\times(88.65 - 3316.2)$ kJ/kg = **-32 276 kJ**

3.219

A piston/cylinder (Fig. P3.171) contains 1 kg of water at 20°C with a volume of 0.1 m^3. Initially the piston rests on some stops with the top surface open to the atmosphere, P_o and a mass so a water pressure of 400 kPa will lift it. To what temperature should the water be heated to lift the piston? If it is heated to saturated vapor find the final temperature, volume and the work, $_1W_2$.

Solution:

(a) State to reach lift pressure of P = 400 kPa,   v = V/m = 0.1 m^3/kg

Table B.1.2: $v_f < v < v_g = 0.4625$ m^3/kg

=>   T = T$_{sat}$ = **143.63°C**

(b) State 2 is saturated vapor at 400 kPa since state 1a is two-phase.

$v_2 = v_g = 0.4625$ m^3/kg , $V_2 = m\,v_2 = 0.4625$ m^3,

Pressure is constant as volume increase beyond initial volume.

$_1W_2 = \int P\,dV = P(V_2 - V_1) = P_{lift}(V_2 - V_1)$

$= 400$ kPa $\times (0.4625 - 0.1)$ m$^3 = $ **145 kJ**

3.220

Two kilograms of water is contained in a piston/cylinder (Fig. P3.220) with a massless piston loaded with a linear spring and the outside atmosphere. Initially the spring force is zero and $P_1 = P_o = 100$ kPa with a volume of 0.2 m^3. If the piston just hits the upper stops the volume is 0.8 m^3 and $T = 600°C$. Heat is now added until the pressure reaches 1.2 MPa. Find the final temperature, show the *P–V* diagram and find the work done during the process.

Solution:

State 1: $v_1 = V/m = 0.2 / 2 = 0.1$ m^3/kg

Process: $1 \rightarrow 2 \rightarrow 3$ or $1 \rightarrow 3'$

State at stops: 2 or 2'

$v_2 = V_{stop}/m = 0.4$ m^3/kg & $T_2 = 600°C$

Table B.1.3 $\Rightarrow$ $P_{stop} = 1$ MPa $< P_3$

since $P_{stop} < P_3$ the process is as $1 \rightarrow 2 \rightarrow 3$

State 3: $P_3 = 1.2$ MPa, $v_3 = v_2 = 0.4$ m^3/kg $\Rightarrow$ $T_3 \cong \textbf{770°C}$

$$W_{13} = W_{12} + W_{23} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) + 0$$

$$= \frac{1}{2}(100 + 1000) \text{ kPa} \times (0.8 - 0.2) \text{ m}^3$$

$$= \textbf{330 kJ}$$

3.221

Two springs with same spring constant are installed in a massless piston/cylinder with the outside air at 100 kPa. If the piston is at the bottom, both springs are relaxed and the second spring comes in contact with the piston at $V = 2$ m^3. The cylinder (Fig. P3.221) contains ammonia initially at $-2°C$, $x = 0.13$, $V = 1$ m^3, which is then heated until the pressure finally reaches 1200 kPa. At what pressure will the piston touch the second spring? Find the final temperature, the total work done by the ammonia and the heat transfer.

Solution :

State 1: $P = 399.7$ kPa   Table B.2.1

$v = 0.00156 + 0.13 \times 0.3106 = 0.0419$ m^3/kg

$u = 170.52 + 0.13 \times 1145.78 = 319.47$ kJ/kg

$m = V/v = 1/0.0419 = 23.866$ kg

At bottom state 0:   0 m^3, 100 kPa

State 2: $V = 2$ m^3 and on line   0-1-2

Final state 3: 1200 kPa, on line segment 2.

Slope of line 0-1-2: $\Delta P/\Delta V = (P_1 - P_0)/\Delta V = (399.7-100)/1 = 299.7$ kPa/ m^3

$P_2 = P_1 + (V_2 - V_1)\Delta P/\Delta V = 399.7 + (2-1) \times 299.7 = $ **699.4 kPa**

State 3: Last line segment has twice the slope.

$P_3 = P_2 + (V_3 - V_2)2\Delta P/\Delta V \quad \Rightarrow \quad V_3 = V_2 + (P_3 - P_2)/(2\Delta P/\Delta V)$

$V_3 = 2 + (1200-699.4)/599.4 = 2.835$ m^3

$v_3 = v_1 V_3/V_1 = 0.0419 \times 2.835/1 = 0.1188 \quad \Rightarrow \quad T_3 = $ **51°C**

$u_3 = 1384.39$ kJ/kg $= 1383 + (1404.8-1383)\dfrac{0.1188 - 0.11846}{0.12378 - 0.11846}$ from B.2.2

$_1W_3 = {}_1W_2 + {}_2W_3 = \dfrac{1}{2}(P_1 + P_2)(V_2 - V_1) + \dfrac{1}{2}(P_3 + P_2)(V_3 - V_2)$

$= 549.6 + 793.0 = $ **1342.6 kJ**

The energy equation gives the heat transfer as

$_1Q_3 = m(u_3 - u_1) + {}_1W_3 = 23.866$ kg $(1384.39 - 319.47)$ kJ/kg $+ 1342.6$ kJ

$= $ **26 758 kJ**

3.222

Ammonia, NH_3, is contained in a sealed rigid tank at $0°C$, $x = 50\%$ and is then heated to $100°C$. Find the final state P_2, u_2 and the specific work and heat transfer.

Solution:

Continuity Eq.: $\quad m_2 = m_1$;

Energy Eq.3.5: $\quad E_2 - E_1 = {_1}Q_2$; $\quad \left({_1}W_2 = \emptyset \right)$

Process: $V_2 = V_1 \Rightarrow v_2 = v_1 = 0.001566 + 0.5 \times 0.28783 = 0.14538$ m^3/kg

Table B.2.2: $\quad v_2$ & $T_2 \Rightarrow$ between 1000 kPa and 1200 kPa

$$P_2 = 1000 + 200 \frac{0.14538 - 0.17389}{0.14347 - 0.17389} = \textbf{1187 kPa}$$

$u_2 = 1490.5 + (1485.8 - 1490.5) \times 0.935$

$\quad\quad = 1485.83$ kJ/kg

$u_1 = 179.69 + 0.5 \times 1138.3 = 748.84$ kJ/kg

Process equation gives no displacement: ${_1}w_2 = \textbf{0}$;

The energy equation then gives the heat transfer as

$\quad\quad {_1}q_2 = u_2 - u_1 = 1485.83 - 748.84 = \textbf{737 kJ/kg}$

3.223

A piston/cylinder system contains 50 L of air at 300°C, 100 kPa, with the piston initially on a set of stops as shown. A total external constant force acts on the piston so a balancing pressure inside should be 200 kPa. The cylinder is made of 2 kg of steel initially at 1300°C. The system is insulated so that heat transfer only occurs between the steel cylinder and the air. The system comes to equilibrium. Find the final temperature and the work done by the air in the process and plot the process P-V diagram.

C.V.: Steel + water out to ambient T_0. This is a control mass.

Energy Eq.: $U_2 - U_1 = m_{air}(u_2 - u_1) + m_{st}(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Entropy Eq.: $S_2 - S_1 = m_{air}(s_2 - s_1) + m_{st}(s_2 - s_1) = \int dQ/T + {_1}S_{2\ gen} = {_1}S_{2\ gen}$

Process: ${_1}Q_2 = 0$ and must be on P-V diagram shown

$$m_{air} = P_1V_1 / RT_1 = 100 \times 0.05 /(0.287 \times 573.15) = 0.0304 \text{ kg}$$

Since $V_{1a} = V_1$ then $T_{1a} = T_1(P_{float}/P_1) = 573.15 \times 200/100 = 1146.3$ K

Use constant C_v for air at 900 K: $C_v = \Delta u/\Delta T = 0.833$ kJ/kgK (from A.7)

To reach state 1a: $\Delta U_{air} = mC_v\Delta T = 0.0304 \times 0.833 \times (1146 - 573) = 14.5$ kJ

$$\Delta U_{st} = mC_v\Delta T = 2 \times 0.46 \times (1146 - 1573) = -392.8 \text{ kJ}$$

Conclusion from this is: T_2 is higher than $T_{1a} = 1146$ K, piston lifts, $P_2 = P_{float}$

Write the work as ${_1}W_2 = P_2 (V_2 - V_1)$ and use constant C_v in the energy Eq. as

$$m_{air} C_v (T_2 - T_1) + m_{st} C_{st} (T_2 - T_1) = - P_2m_{air}v_2 + P_2 V_1$$

now $P_2 v_2 = RT_2$ for the air, so isolate T_2 terms as

$$[m_{air} (C_v + R) + mC_{st}] T_2 = m_{air} C_vT_{1\ air} + mC_{st} T_{1\ st} + P_2V_1$$

$$[0.0304 \times 1.12 + 2 \times 0.46] T_2 = 0.0304 \times 0.833 \times 573.15$$

$$+ 2 \times 0.46 \times 1573.15 + 200 \times 0.05$$

Solution gives: $T_2 =$ **1542.7 K**, $V_2 = 0.0304 \times 0.287 \times 1542.7/200 = 0.0673$ m^3

$${_1}W_2 = P_2 (V_2 - V_1) = 200 (0.0673 - 0.05) = \textbf{3.46 kJ}$$

3.224

A piston/cylinder arrangement has the piston loaded with outside atmospheric pressure and the piston mass to a pressure of 150 kPa. It contains water at −2°C, which is then heated until the water becomes saturated vapor. Find the final temperature and specific work and heat transfer for the process.

Solution:

C.V. Water in the piston cylinder.

Continuity: $m_2 = m_1$,

Energy Eq.3.5 per unit mass: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process: P = constant = P_1, => ${}_1w_2 = \int_1^2 P\, dv = P_1(v_2 - v_1)$

State 1: T_1, P_1 => Table B.1.5 compressed solid, take as saturated solid.

$$v_1 = 1.09\times10^{-3} \text{ m}^3/\text{kg}, \quad u_1 = -337.62 \text{ kJ/kg}$$

State 2: x = 1, $P_2 = P_1$ = 150 kPa due to process => Table B.1.2

$$v_2 = v_g(P_2) = 1.1593 \text{ m}^3/\text{kg}, \quad T_2 = \textbf{111.4°C} ; \quad u_2 = 2519.7 \text{ kJ/kg}$$

From the process equation

$${}_1w_2 = P_1(v_2 - v_1) = 150 \text{ kPa } (1.1593 - 1.09\times10^{-3}) \text{ m}^3/\text{kg} = \textbf{173.7 kJ/kg}$$

From the energy equation

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = 2519.7 - (-337.62) + 173.7 = \textbf{3031 kJ/kg}$$

3.225

A piston/cylinder contains 1 kg of ammonia at 20°C with a volume of 0.1 m^3, shown in Fig. P3.225. Initially the piston rests on some stops with the top surface open to the atmosphere, Po, so a pressure of 1400 kPa is required to lift it. To what temperature should the ammonia be heated to lift the piston? If it is heated to saturated vapor find the final temperature, volume, and the heat transfer.

Solution:

C.V. Ammonia which is a control mass.

$$m_2 = m_1 = m ; \qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

State 1: 20°C; $v_1 = 0.10 < v_g \Rightarrow x_1 = (0.1 - 0.001638)/0.14758 = 0.6665$

$$u_1 = u_f + x_1 \, u_{fg} = 272.89 + 0.6665 \times 1059.3 = 978.9 \text{ kJ/kg}$$

Process: Piston starts to lift at state 1a (P_{lift}, v_1)

State 1a: 1400 kPa, v_1 Table B.2.2 (superheated vapor)

$$T_a = 50 + (60 - 50) \frac{0.1 - 0.09942}{0.10423 - 0.09942} = 51.2 \text{ °C}$$

State 2: x = 1.0, $v_2 = v_1 \Rightarrow V_2 = mv_2 = \textbf{0.1 m}^{\textbf{3}}$

$$T_2 = 30 + (0.1 - 0.11049) \times 5/(0.09397 - 0.11049) = \textbf{33.2 °C}$$

$$u_2 = 1338.7 \text{ kJ/kg}; \qquad {}_1W_2 = 0;$$

$${}_1Q_2 = m_1 q_2 = m(u_2 - u_1) = 1 \text{ kg} \times (1338.7 - 978.9) \text{ kJ/kg} = \textbf{359.8 kJ}$$

3.226

A piston held by a pin in an insulated cylinder, shown in Fig. P3.226, contains 2 kg water at 100°C, quality 98%. The piston has a mass of 102 kg, with cross-sectional area of 100 cm^2, and the ambient pressure is 100 kPa. The pin is released, which allows the piston to move. Determine the final state of the water, assuming the process to be adiabatic.

Solution:

C.V. The water. This is a control mass.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process in cylinder: $P = P_{float}$ (if piston not supported by pin)

$$P_2 = P_{float} = P_0 + m_p g/A = 100 + \frac{102 \times 9.807}{100 \times 10^{-4} \times 10^3} = 200 \text{ kPa}$$

We thus need one more property for state 2 and we have one equation namely the energy equation. From the equilibrium pressure the work becomes

$${_1}W_2 = \int P_{float} \, dV = P_2 \, m(v_2 - v_1)$$

With this work the energy equation gives per unit mass

$$u_2 - u_1 = {_1}q_2 - {_1}w_2 = 0 - P_2(v_2 - v_1)$$

or with rearrangement to have the unknowns on the left hand side

$$u_2 + P_2 v_2 = h_2 = u_1 + P_2 v_1$$

$$h_2 = u_1 + P_2 v_1 = 2464.8 \text{ kJ/kg} + 200 \text{ kPa} \times 1.6395 \text{ m}^3/\text{kg} = 2792.7 \text{ kJ/kg}$$

State 2: (P_2 , h_2) Table B.1.3 => $T_2 \cong$ **161.75°C**

3.227

A vertical cylinder (Fig. P3.227) has a 61.18-kg piston locked with a pin trapping 10 L of R-410A at 10°C, 90% quality inside. Atmospheric pressure is 100 kPa, and the cylinder cross-sectional area is 0.006 m^2. The pin is removed, allowing the piston to move and come to rest with a final temperature of 10°C for the R-410A. Find the final pressure, final volume and the work done and the heat transfer for the R-410A.

Solution:

CV R-410A, this is a control mass.

Energy Eq.: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process in cylinder: $P = P_{float}$ (if piston not supported by pin)

State 1: (T, x) from table B.4.1

$\quad v_1 = 0.000886 + 0.9 \times 0.02295 = 0.021541 \ \text{m}^3/\text{kg}$

$\quad u_1 = 72.24 + 0.9 \times 183.66 = 131.64 \ \text{kJ/kg}$

$\quad m = V_1/v_1 = 0.010 \ \text{m}^3/0.021541 \ \text{m}^3/\text{kg} = 0.464 \ \text{kg}$

Force balance on piston gives the equilibrium pressure

$$P_2 = P_0 + m_p g/A_P = 100 + \frac{61.18 \times 9.807}{0.006 \times 1000} = \textbf{200 kPa}$$

State 2: (T,P) in Table B.4.2 $v_2 = (0.1507 + 0.16322)/2 = 0.15696 \ \text{m}^3/\text{kg}$

$\qquad\qquad\qquad\qquad u_2 = (265.06 + 279.13)/2 = 272.095 \ \text{kJ/kg}$

$V_2 = mv_2 = 0.464 \ \text{kg} \times 0.15696 \ \text{m}^3/\text{kg} = 0.0728 \ \text{m}^3 = \textbf{72.8 L}$

${_1}W_2 = \int P_{equil} \ dV = P_2(V_2 - V_1) = 200 \ \text{kPa} \ (0.0728 - 0.010) \ \text{m}^3 = \textbf{12.56 kJ}$

$\qquad {_1}Q_2 = m(u_2 - u_1) + {_1}W_2$

$\qquad\qquad = 0.464 \ \text{kg} \ (272.095 - 131.64) \ \text{kJ/kg} + 12.56 \ \text{kJ} = \textbf{77.73 kJ}$

3.228

A cylinder having an initial volume of 3 m^3 contains 0.1 kg of water at 40°C. The water is then compressed in an isothermal quasi-equilibrium process until it has a quality of 50%. Calculate the work done in the process splitting it into two steps. Assume the water vapor is an ideal gas during the first step of the process.

Solution: C.V. Water

State 2: (40°C, x = 1) Tbl B.1.1 => P_G = 7.384 kPa, v_G = 19.52 m^3/kg

State 1: $v_1 = V_1/m = 3 / 0.1 = 30$ m^3/kg   ($> v_G$)

so $H_2O \sim$ ideal gas from 1-2 so since constant T

$$P_1 = P_G \frac{v_G}{v_1} = 7.384 \times \frac{19.52}{30} = 4.8 \text{ kPa}$$

$$V_2 = mv_2 = 0.1 \times 19.52 = 1.952 \text{ m}^3$$

Process   T = C: and ideal gas gives work from Eq.3.21

$$_1W_2 = \int_1^2 PdV = P_1V_1 \ln\frac{V_2}{V_1} = 4.8 \times 3.0 \times \ln\frac{1.952}{3} = -6.19 \text{ kJ}$$

$v_3 = 0.001008 + 0.5 \times 19.519 = 9.7605$ => $V_3 = mv_3 = 0.976$ m^3

P = C = P_g: This gives a work term as

$$_2W_3 = \int_2^3 PdV = P_g (V_3 - V_2) = 7.384 \text{ kPa} (0.976 - 1.952) \text{ m}^3 = -7.21 \text{ kJ}$$

Total work:

$$_1W_3 = {}_1W_2 + {}_2W_3 = -6.19 - 7.21 = \textbf{-13.4 kJ}$$

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3.229

A spring-loaded piston/cylinder arrangement contains R-134a at 20°C, 24% quality with a volume 50 L. The setup is heated and thus expands, moving the piston. It is noted that when the last drop of liquid disappears the temperature is 40°C. The heating is stopped when $T = 130$°C. Verify the final pressure is about 1200 kPa by iteration and find the work done in the process.

Solution:

      C.V. R-134a. This is a control mass.

State 1: Table B.5.1   =>

$v_1 = 0.000817 + 0.24 \times 0.03524 = 0.009274$

$P_1 = 572.8$ kPa,

$m = V/v_1 = 0.050 / 0.009274 = 5.391$ kg

Process: Linear Spring

    $P = A + Bv$

State 2: $x_2 = 1$, T_2 $\Rightarrow$ $P_2 = 1.017$ MPa, $v_2 = 0.02002$ m^3/kg

Now we have fixed two points on the process line so for final state 3:

$$P_3 = P_1 + \frac{P_2 - P_1}{v_2 - v_1}(v_3 - v_1) = \text{RHS} \qquad \text{Relation between } P_3 \text{ and } v_3$$

State 3: T_3 and on process line $\Rightarrow$ iterate on P_3 given T_3

      at $P_3 = 1.2$ MPa => $v_3 = 0.02504$ => P_3 - RHS = -0.0247

      at $P_3 = 1.4$ MPa => $v_3 = 0.02112$ => P_3 - RHS = 0.3376

Linear interpolation gives :

$$P_3 \cong 1200 + \frac{0.0247}{0.3376 + 0.0247}(1400 - 1200) = 1214 \text{ kPa}$$

$$v_3 = 0.02504 + \frac{0.0247}{0.3376 + 0.0247}(0.02112 - 0.02504) = 0.02478 \text{ m}^3/\text{kg}$$

$$W_{13} = \int P\, dV \;\; = \tfrac{1}{2}(P_1 + P_3)(V_3 - V_1) = \tfrac{1}{2}(P_1 + P_3)\, m\, (v_3 - v_1)$$

$$= \tfrac{1}{2} \times 5.391 \text{ kg } (572.8 + 1214) \text{ kPa} \times (0.02478 - 0.009274) \text{ m}^3/\text{kg}$$

$$= \mathbf{74.7 \text{ kJ}}$$

3.230

Water in a piston/cylinder, similar to Fig. P3.225, is at 100°C, x = 0.5 with mass 1 kg and the piston rests on the stops. The equilibrium pressure that will float the piston is 300 kPa. The water is heated to 300°C by an electrical heater. At what temperature would all the liquid be gone? Find the final (P,v), the work and heat transfer in the process.

C.V. The 1 kg water.

Continuty: $m_2 = m_1 = m$;   Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant if $P < P_{lift}$ otherwise $P = P_{lift}$ see P-v diagram.

State 1: (T,x) Table B.1.1

$\qquad v_1 = 0.001044 + 0.5 \times 1.6719 = 0.83697$ m^3/kg

$\qquad u_1 = 418.91 + 0.5 \times 2087.58 = 1462.7$ kJ/kg

State 1a: (300 kPa, v = $v_1 > v_{g\ 300\ kPa} = 0.6058$ m^3/kg) so superheated vapor

Piston starts to move at state 1a, ${}_1W_{1a} = 0$, $u_{1a} = 2768.82$ kJ/kg

$\qquad {}_1Q_{1a} = m(u - u) = 1$ kg (2768.82 – 1462.7) kJ/kg = 1306.12 kJ

State 1b: reached before state 1a so v = $v_1 = v_g$ see this in B.1.1

$\qquad T_{1b} = 120 + 5 (0.83697 - 0.8908)/(0.76953 - 0.8908) = \textbf{122.2}^{\textbf{o}}\textbf{C}$

State 2: ($T_2 > T_{1a}$) Table B.1.3 => $v_2 = 0.87529$, $u_2 = 2806.69$ kJ/kg

Work is seen in the P-V diagram (when volume changes $P = P_{lift}$)

$\qquad {}_1W_2 = {}_{1a}W_2 = P_2\, m(v_2 - v_1) = 300 \times 1(0.87529 - 0.83697) = \textbf{11.5 kJ}$

Heat transfer is from the energy equation

$\qquad {}_1Q_2 = 1$ kg (2806.69 - 1462.7) kJ/kg + 11.5 kJ = **1355.5 kJ**

3.231

A piston/cylinder arrangement has a linear spring and the outside atmosphere acting on the piston, shown in Fig. P3.231. It contains water at 3 MPa, 400°C with the volume being 0.1 m^3. If the piston is at the bottom, the spring exerts a force such that a pressure of 200 kPa inside is required to balance the forces. The system now cools until the pressure reaches 1 MPa. Find the heat transfer for the process.

Solution:

C.V. Water.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: Table B.1.3

$v_1 = 0.09936$ m^3/kg, $u_1 = 2932.8$ kJ/kg

$m = V/v_1 = 0.1/0.09936 = 1.006$ kg

Process: Linear spring so P linear in v.

$$P = P_0 + (P_1 - P_0)v/v_1$$

$$v_2 = \frac{(P_2 - P_0)v_1}{P_1 - P_0} = \frac{(1000 - 200)0.09936}{3000 - 200} = 0.02839 \text{ m}^3/\text{kg}$$

State 2: P_2 , v_2 $\Rightarrow$ $x_2 = (v_2 - 0.001127)/0.19332 = 0.141$, $T_2 = 179.91$°C,

$u_2 = 761.62 + x_2 \times 1821.97 = 1018.58$ kJ/kg

Process => ${}_1W_2 = \int PdV = \frac{1}{2} m(P_1 + P_2)(v_2 - v_1)$

$= \frac{1}{2}$ 1.006 kg (3000 + 1000) kPa (0.02839 -0.09936) m^3/kg

$= -142.79$ kJ

Heat transfer from the energy equation

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.006$ kg (1018.58 - 2932.8) kJ/kg - 142.79 kJ

$= $ **-2068.5 kJ**

3.232

A 1 m^3 tank containing air at 25°C and 500 kPa is connected through a valve to another tank containing 4 kg of air at 60°C and 200 kPa. Now the valve is opened and the entire system reaches thermal equilibrium with the surroundings at 20°C. Assume constant specific heat at 25°C and determine the final pressure and the heat transfer.

Control volume all the air. Assume air is an ideal gas.

Continuity Eq.: $\quad m_2 - m_{A1} - m_{B1} = 0$

Energy Eq.: $\quad U_2 - U_1 = m_2 u_2 - m_{A1} u_{A1} - m_{B1} u_{B1} = {}_1Q_2 - {}_1W_2$

Process Eq.: $\quad V = \text{constant} \quad \Rightarrow \quad {}_1W_2 = 0$

State 1:

$$m_{A1} = \frac{P_{A1} V_{A1}}{RT_{A1}} = \frac{(500 \text{ kPa})(1 m^3)}{(0.287 \text{ kJ/kgK})(298.2 \text{ K})} = 5.84 \text{ kg}$$

$$V_{B1} = \frac{m_{B1} R T_{B1}}{P_{B1}} = \frac{(4 \text{ kg})(0.287 \text{ kJ/kgK})(333.2 \text{ K})}{(200 \text{ kN/m}^2)} = 1.91 \text{ m}^3$$

State 2: $T_2 = 20°C$, $v_2 = V_2/m_2$

$m_2 = m_{A1} + m_{B1} = 4 + 5.84 = 9.84 \text{ kg}$

$V_2 = V_{A1} + V_{B1} = 1 + 1.91 = 2.91 \text{ m}^3$

$$P_2 = \frac{m_2 R T_2}{V_2} = \frac{(9.84 \text{ kg})(0.287 \text{ kJ/kgK})(293.2 \text{ K})}{2.91 \text{ m}^3} = \textbf{284.5 kPa}$$

Energy Eq.5.5 or 5.11:

$${}_1Q_2 = U_2 - U_1 = m_2 u_2 - m_{A1} u_{A1} - m_{B1} u_{B1}$$
$$= m_{A1}(u_2 - u_{A1}) + m_{B1}(u_2 - u_{B1})$$
$$= m_{A1} C_{v0}(T_2 - T_{A1}) + m_{B1} C_{v0}(T_2 - T_{B1})$$
$$= 5.84 \times 0.717 \,(20 - 25) + 4 \times 0.717 \,(20 - 60) = \textbf{−135.6 kJ}$$

The air gave energy out.

3.233

A rigid container has two rooms filled with water, each 1 m^3 separated by a wall (see Fig. P3.210). Room A has P = 200 kPa with a quality x = 0.80. Room B has P = 2 MPa and T = 400°C. The partition wall is removed and the water comes to a uniform state, which after a while due to heat transfer has a temperature of 200°C. Find the final pressure and the heat transfer in the process.

Solution:

C.V. A + B. Constant total mass and constant total volume.

Continuity: $\quad m_2 - m_{A1} - m_{B1} = 0 ; \quad V_2 = V_A + V_B = 2 \text{ m}^3$

Energy Eq.3.5: $\quad U_2 - U_1 = m_2 u_2 - m_{A1} u_{A1} - m_{A1} u_{A1} = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

Process: $\quad V = V_A + V_B = \text{constant} \qquad => \quad {}_1W_2 = 0$

State 1A: Table B.1.2 $\quad u_{A1} = 504.47 + 0.8 \times 2025.02 = 2124.47 \text{ kJ/kg},$

$\qquad\qquad\qquad v_{A1} = 0.001061 + 0.8 \times 0.88467 = 0.70877 \text{ m}^3/\text{kg}$

State 1B: Table B.1.3 $\quad u_{B1} = 2945.2, \quad v_{B1} = 0.1512$

$\qquad m_{A1} = 1/v_{A1} = 1.411 \text{ kg} \qquad m_{B1} = 1/v_{B1} = 6.614 \text{ kg}$

State 2: $\quad T_2, v_2 = V_2/m_2 = 2/(1.411 + 6.614) = 0.24924 \text{ m}^3/\text{kg}$

$\qquad\qquad$ Table B.1.3 superheated vapor. $\qquad 800 \text{ kPa} < P_2 < 1 \text{ MPa}$

Interpolate to get the proper v_2

$$P_2 \cong 800 + \frac{0.24924 - 0.2608}{0.20596 - 0.2608} \times 200 = \textbf{842 kPa} \qquad u_2 \cong 2628.8 \text{ kJ/kg}$$

From the energy equation

$\qquad {}_1Q_2 = 8.025 \times 2628.8 - 1.411 \times 2124.47 - 6.614 \times 2945.2 = \textbf{- 1381 kJ}$

3.234

Consider the piston/cylinder arrangement shown in Fig. P3.234. A frictionless piston is free to move between two sets of stops. When the piston rests on the lower stops, the enclosed volume is 400 L. When the piston reaches the upper stops, the volume is 600 L. The cylinder initially contains water at 100 kPa, 20% quality. It is heated until the water eventually exists as saturated vapor. The mass of the piston requires 300 kPa pressure to move it against the outside ambient pressure. Determine the final pressure in the cylinder, the heat transfer and the work for the overall process.

Solution:

C.V. Water.   Check to see if piston reaches upper stops.

Energy Eq.3.5: $m(u_4 - u_1) = {}_1Q_4 - {}_1W_4$

Process: If   P < 300 kPa   then   V = 400 L,   line 2-1 and below

            If   P > 300 kPa   then   V = 600 L,   line 3-4 and above

            If   P = 300 kPa   then   400 L < V < 600 L line 2-3

State 1: $v_1 = 0.001043 + 0.2 \times 1.693 = 0.33964$; $m = \dfrac{V_1}{v_1} = \dfrac{0.4}{0.33964} = 1.178$ kg

$u_1 = 417.36 + 0.2 \times 2088.7 = 835.1$ kJ/kg

State 3: $v_3 = \dfrac{0.6}{1.178} = 0.5095 < v_G = 0.6058$ at $P_3 = 300$ kPa

        $\Rightarrow$ Piston does reach upper stops to reach sat. vapor.

State 4: $v_4 = v_3 = 0.5095$ m^3/kg $= v_G$ at P_4 From Table B.1.2

            => $P_4 = \textbf{361 kPa,}$ $u_4 = 2550.0$ kJ/kg

        ${}_1W_4 = {}_1W_2 + {}_2W_3 + {}_3W_4 = 0 + {}_2W_3 + 0$

        ${}_1W_4 = P_2(V_3 - V_2) = 300 \times (0.6 - 0.4) = \textbf{60 kJ}$

        ${}_1Q_4 = m(u_4 - u_1) + {}_1W_4 = 1.178(2550.0 - 835.1) + 60 = \textbf{2080 kJ}$

The three lines for process parts are shown in the P-V diagram, and is dictated by the motion of the piston (force balance).

3.235

Ammonia (2 kg) in a piston/cylinder is at 100 kPa, -20$^{\text{o}}$C and is now heated in a polytropic process with n = 1.3 to a pressure of 200 kPa. Do not use ideal gas approximation and find T_2, the work and heat transfer in the process.

Take CV as the Ammonia, constant mass.

Energy Eq.3.5:　　　$m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process:　　　　　　Pv^n = constant　　(n = 1.3)

State 1: Superheated vapor table B.2.2.

$$v_1 = 1.2101 \text{ m}^3/\text{kg}, \quad u_1 = 1307.8 \text{ kJ/kg}$$

Process gives: $v_2 = v_1 (P_1/P_2)^{1/n} = 1.2101 (100/200)^{1/1.3} = 0.710 \text{ m}^3/\text{kg}$

State 2: Table B.2.2 at 200 kPa interpolate:　$u_2 = 1376.49 \text{ kJ/kg}, \quad T_2 = 24^{\text{o}}\text{C}$

Work is done while piston moves at increasing pressure, so we get

$${}_1W_2 = \frac{m}{1 - n} (P_2 v_2 - P_1 v_1) = \frac{2}{1 - 1.3} \text{ kg} (200 \times 0.71 - 100 \times 1.2101) \text{ kJ/kg}$$

$$= \textbf{-139.9 kJ}$$

Heat transfer is found from the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

$$= 2 (1376.49 - 1307.8) - 139.9$$

$$= \textbf{-2.52 kJ}$$

3.236

A small flexible bag contains 0.1 kg ammonia at -10^oC and 300 kPa. The bag material is such that the pressure inside varies linear with volume. The bag is left in the sun with with an incident radiation of 75 W, loosing energy with an average 25 W to the ambient ground and air. After a while the bag is heated to 30^oC at which time the pressure is 1000 kPa. Find the work and heat transfer in the process and the elapsed time.

Take CV as the Ammonia, constant mass.

Continuity Eq.: $\qquad\qquad m_2 = m_1 = m$;

Energy Eq.3.5: $\qquad m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process: $\qquad\qquad\qquad P = A + BV \qquad$ (linear in V)

State 1: Compressed liquid $P > P_{sat}$, take saturated liquid at same temperature.

$$v_1 = v_{f\text{ -}10} = 0.001534 \text{ m}^3/\text{kg}, \qquad u_1 = u_f = 133.96 \text{ kJ/kg}$$

State 2: Table B.2.1 at 30^oC : $\quad P < P_{sat} \quad$ so superheated vapor

$$v_2 = 0.13206 \text{ m}^3/\text{kg}, \; u_2 = 1347.1 \text{ kJ/kg}, \qquad V_2 = mv_2 = \mathbf{0.0132 \text{ m}^3}$$

Work is done while piston moves at increacing pressure, so we get

$${_1}W_2 = \tfrac{1}{2}(300 + 1000) \text{ kPa}*0.1 \text{ kg}(0.13206 - 0.001534) \text{ m}^3/\text{kg} = \mathbf{8.484 \text{ kJ}}$$

Heat transfer is found from the energy equation

$${_1}Q_2 = m(u_2 - u_1) + {_1}W_2 = 0.1 \text{ kg } (1347.1 - 133.96) \text{ kJ/kg} + 8.484 \text{ kJ}$$

$$= 121.314 + 8.484 = \mathbf{129.8 \text{ kJ}}$$

$$\dot{Q}_{net} = 75 - 25 = 50 \text{ Watts}$$

Assume the constant rate $\dot{Q}_{net} = dQ/dt = {_1}Q_2 / t$, so the time becomes

$$t = {_1}Q_2 / \dot{Q}_{net} = \frac{129800}{50} \text{ (J/W)} = \mathbf{2596 \text{ s}} = \mathbf{43.3 \text{ min}}$$

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3.237

A cylinder/piston arrangement contains 0.1 kg R-410A of quality $x = 0.2534$ and at $-20°C$. Stops are mounted so $V_{stop} = 3V_1$, see Fig. P3.237. The system is now heated to the final temperature of $20°C$. Find the work and heat transfer in the process and draw the P-v diagram.

C.V. The R-410A mass.

Energy Eq.5.11: $\qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $\qquad\qquad P = \text{Constant if } V < V_{stop}; \quad V = V_{stop} \text{ if } P > P_1$

State 1: $\qquad u_1 = 27.92 + x_1\,218.07 = 83.18 \text{ kJ/kg}, \quad P_1 = P_{sat} = 399.6 \text{ kPa}$

$\qquad\qquad v_1 = 0.000803 + x_1\,0.064 = 0.01702 \text{ m}^3/\text{kg}$

State 1a: $\qquad v_{stop} = 3\,v_1 = 0.05106 \text{ m}^3/\text{kg} < v_g \text{ at } P_1$

State 2: $\qquad$ at $20°C > T_1$: $\quad v_{stop} > v_g = 0.01758 \text{ m}^3/\text{kg}$ so superheated vapor.

$\qquad\qquad$ Table B.4.2: Find it at $P_2 = \mathbf{600\ kPa}$, $u_2 = 273.56 \text{ kJ/kg}$

$\qquad\qquad V_1 = mv_1 = 0.0017 \text{ m}^3, \quad V_2 = mv_2 = 0.0051 \text{ m}^3$

$\qquad {}_1W_2 = \int PdV = P_1\,(V_2 - V_1) = 399.6 \text{ kPa } (0.0051 - 0.0017) \text{ m}^3 = \mathbf{1.36\ kJ}$

$\qquad {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.1 \text{ kg } (273.56 - 83.18) \text{ kJ/kg} + 1.36 \text{ kJ}$

$\qquad\qquad = \mathbf{20.398\ kJ}$

See the work term from the process in the P-v diagram

3.238

A piston/cylinder, shown in Fig. P3.238, contains R-410A at $-20°C$, $x = 20\%$. The volume is $0.2\ m^3$. It is known that $V_{stop} = 0.4\ m^3$, and if the piston sits at the bottom, the spring force balances the other loads on the piston. It is now heated up to $20°C$. Find the mass of the fluid and show the P–v diagram. Find the work and heat transfer.

Solution:

C.V. R-410A, this is a control mass. Properties in Table B.4.

Continuity Eq.: $\qquad m_2 = m_1$

Energy Eq.3.5: $\qquad E_2 - E_1 = m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $\quad P = A + BV,\quad V < 0.4\ m^3,\quad A = 0\ (at\ V = 0,\ P = 0)$

State 1: $\ v_1 = 0.000803 + 0.2 \times 0.0640 = 0.0136\ m^3/kg$

$\qquad\qquad u_1 = 27.92 + 0.2 \times 218.07 = 71.5\ kJ/kg$

$\qquad\qquad m = m_1 = V_1/v_1\ = 14.706\ kg$

System: on line

$\qquad V \le V_{stop};$

$\qquad P_1 = 399.6\ kPa$

$\qquad P_{stop} = 2P_1 = 799.2\ kPa$

$\qquad v_{stop}\ = 2v_1 = 0.0272\ m^3/kg$

State stop: $(P,v) \Rightarrow T_{stop} \cong 0°C \qquad$ TWO-PHASE STATE

Since $T_2 > T_{stop} \quad \Rightarrow \quad v_2 = v_{stop} = 0.0272\ m^3/kg$

State 2: $(T_2\ ,\ v_2)$ Table B.4.2: Interpolate between 1000 and 1200 kPa

$\qquad\qquad P_2 = 1035\ kPa\ ;\quad u_2 =\ 366.5\ kJ/kg$

From the process curve, see also area in P-V diagram, the work is

$$_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_{stop})(V_{stop} - V_1) = \frac{1}{2}(399.6 + 799.2)0.2 = \textbf{119.8 kJ}$$

From the energy equation

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 14.706(266.5 - 71.5) + 119.8 = \textbf{2987.5 kJ}$$

3.239

A spherical balloon contains 2 kg of R-410A at 0°C, 30% quality. This system is heated until the pressure in the balloon reaches 1 MPa. For this process, it can be assumed that the pressure in the balloon is directly proportional to the balloon diameter. How does pressure vary with volume and what is the heat transfer for the process?

Solution:

C.V. R-410A which is a control mass.

$$m_2 = m_1 = m \; ;$$

Energy Eq.3.5: $\qquad m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

State 1: 0°C, x = 0.3. Table B.4.1 gives P_1 = 798.7 kPa

$$v_1 = 0.000855 + 0.3 \times 0.03182 = 0.01040 \text{ m}^3/\text{kg}$$

$$u_1 = 57.07 + 0.3 \times 195.95 = 115.86 \text{ kJ/kg}$$

Process: $P \propto D$, $V \propto D^3$ $\Rightarrow$ $PV^{-1/3}$ = constant, polytropic $\quad$ n = -1/3.

$$\Rightarrow \quad V_2 = mv_2 = V_1 \, (P_2 /P_1)^3 = mv_1 \, (P_2 /P_1)^3$$

$$v_2 = v_1 \, (P_2 /P_1)^3 = 0.01040 \times (1000 / 798.7)^3 = 0.02041 \text{ m}^3/\text{kg}$$

State 2: P_2 = 1 MPa, process : v_2 = 0.02041 $\rightarrow$ Table B.4.2, T_2 = 7.25°C (sat)

$$v_f = 0.000877, \; v_{fg} = 0.02508 \text{ m}^3/\text{kg}, \; u_f = 68.02, \; u_{fg} = 187.18 \text{ kJ/kg}$$

$$x_2 = 0.7787, \quad u_2 = 213.7 \text{ kJ/kg},$$

$${_1}W_2 = \int P \, dV = m \, \frac{P_2 v_2 - P_1 v_1}{1 - n} = 2 \, \frac{1000 \times 0.02041 - 798.7 \times 0.0104}{1 - (-1/3)} = 18.16 \text{ kJ}$$

$${_1}Q_2 = m(u_2 - u_1) + {_1}W_2 = 2 \, (213.7 - 115.86) + 18.16 = \textbf{213.8 kJ}$$

Notice: The R-410A is not an ideal gas at any state in this problem.

3.240

A piston/cylinder arrangement B is connected to a 1-m^3 tank A by a line and valve, shown in Fig. P3.240. Initially both contain water, with A at 100 kPa, saturated vapor and B at 400°C, 300 kPa, 1 m^3. The valve is now opened and, the water in both A and B comes to a uniform state.

a.   Find the initial mass in A and B.

b.   If the process results in $T_2 = 200$°C, find the heat transfer and work.

Solution:

   C.V.: A + B. This is a control mass.

   Continuity equation: $m_2 - (m_{A1} + m_{B1}) = 0$;

   Energy: $m_2 u_2 - m_{A1} u_{A1} - m_{B1} u_{B1} = {_1}Q_2 - {_1}W_2$

   System: if $V_B \geq 0$ piston floats $\Rightarrow$ $P_B = P_{B1} = $ const.

   if $V_B = 0$ then $P_2 < P_{B1}$ and $v = V_A/m_{tot}$ see P-V diagram

   ${_1}W_2 = \int P_B dV_B = P_{B1}(V_2 - V_1)_B = P_{B1}(V_2 - V_1)_{tot}$

   State A1: Table B.1.1, x = 1

   $v_{A1} = 1.694$ m^3/kg, $u_{A1} = 2506.1$ kJ/kg

   $m_{A1} = V_A/v_{A1} = $ **0.5903 kg**

   State B1: Table B.1.2 sup. vapor

   $v_{B1} = 1.0315$ m^3/kg, $u_{B1} = 2965.5$ kJ/kg

   $m_{B1} = V_{B1}/v_{B1} = $ **0.9695 kg**

   $m_2 = m_{TOT} = 1.56$ kg

*   At (T_2 , P_{B1}) $v_2 = 0.7163 > v_a = V_A/m_{tot} = 0.641$ so $V_{B2} > 0$

so now state 2: $P_2 = P_{B1} = 300$ kPa, $T_2 = 200$ °C

   $\Rightarrow u_2 = 2650.7$ kJ/kg   and $V_2 = m_2 v_2 = 1.56 \times 0.7163 = 1.117$ m^3

(we could also have checked T_a at: 300 kPa, 0.641 m^3/kg $\Rightarrow$ T = 155 °C)

   ${_1}W_2 = P_{B1}(V_2 - V_1) = $ **-264.82 kJ**

   ${_1}Q_2 = m_2 u_2 - m_{A1} u_{A1} - m_{B1} u_{B1} + {_1}W_2 = $ **-484.7 kJ**

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3.241

Consider the system shown in Fig. P3.241. Tank A has a volume of 100 L and contains saturated vapor R-134a at 30°C. When the valve is cracked open, R-134a flows slowly into cylinder B. The piston mass requires a pressure of 200 kPa in cylinder B to raise the piston. The process ends when the pressure in tank A has fallen to 200 kPa. During this process heat is exchanged with the surroundings such that the R-134a always remains at 30°C. Calculate the heat transfer for the process.

Solution:

C.V. The R-134a. This is a control mass.

Continuity Eq.: $\quad\quad\quad m_2 = m_1 = m$;

Energy Eq.3.5: $\quad m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process in B: If $V_B > 0$ then $\quad P = P_{float}$ (piston must move)

$$\Rightarrow \quad {_1}W_2 = \int P_{float}\, dV = P_{float} m(v_2 - v_1)$$

Work done in B against constant external force (equilibrium P in cyl. B)

State 1: 30°C, x = 1. Table B.5.1: $v_1 = 0.02671$ m^3/kg, $u_1 = 394.48$ kJ/kg

$$m = V/v_1 = 0.1 / 0.02671 = 3.744 \text{ kg}$$

State 2: 30°C, 200 kPa   superheated vapor Table B.5.2

$$v_2 = 0.11889 \text{ m}^3\text{/kg}, \quad u_2 = 403.1 \text{ kJ/kg}$$

From the process equation

$${_1}W_2 = P_{float} m(v_2 - v_1) = 200 \times 3.744 \times (0.11889 - 0.02671) = 69.02 \text{ kJ}$$

From the energy equation

$${_1}Q_2 = m(u_2 - u_1) + {_1}W_2 = 3.744 \times (403.1 - 394.48) + 69.02 = \textbf{101.3 kJ}$$

ENGLISH UNIT PROBLEMS

Borgnakke Sonntag

Fundamentals of
Thermodynamics

SOLUTION MANUAL
CHAPTER 3
English units

8e

Borgnakke and Sonntag

CHAPTER 3

Concept Problems

3.242E

What is 1 cal in english units, what is 1 Btu in ft-lbf?

Look in Table A.1 for the conversion factors under energy

1 Btu = **778.1693 lbf-ft**

$$1 \text{ cal} = 4.1868 \text{ J} = \frac{4.1868}{1055} \text{ Btu} = \textbf{0.00397 Btu}$$
$$= 0.00397 \times 778.1693 \text{ lbf-ft} = \textbf{3.088 lbf-ft}$$

3.243E

Work as F Δx has units of lbf-ft, what is that in Btu?

Look in Table A.1 for the conversion factors under energy

1 lbf-ft = 1 lbf-ft / (778.1693 ft-lbf/Btu) = 1.28507×10^{-3} Btu

3.244E

Work in the expression Eq. 3.18 or Eq. 3.22 involves PV. For P in psia and V in ft^3, how does that become Btu?

Units: $psia \times ft^3 = (lbf/in.^2) \times ft^3 = lbf\text{-}ft \times (ft/in.)^2 = 144 \ lbf\text{-}ft$
   $= (144/778.1693) \ Btu$
   $= 144 \times 1.28507 \times 10^{-3} \ Btu$
   $= 0.18509 \ Btu$

3.245E

Look at the R-410A value for u_f at -60 F. Can the energy really be negative? Explain.

The absolute value of u and h are arbitrary. A constant can be added to all u and h values and the table is still valid. It is customary to select the reference such that u for saturated liquid water at the triple point is zero. The standard for refrigerants like R-410A is that h is set to zero as saturated liquid at -40 F, other substances as cryogenic substances like nitrogen, methane etc. may have different states at which h is set to zero. The ideal gas tables use a zero point for h as 77 F or at absolute zero, 0 R.

3.246E

An ideal gas in a piston-cylinder is heated with 2 Btu during an isothermal process. How much work is involved?

Energy Eq.: $u_2 - u_1 = {_1}q_2 - {_1}w_2 = 0$ since $u_2 = u_1$ (isothermal)

Then

$$_1W_2 = {_1}Q_2 = m\ {_1}w_2 = \textbf{2 Btu}$$

3.247E

You heat a gas 20 R at P = C. Which one in table F.4 requires most energy? Why?

A constant pressure process in a control mass gives (recall p. 109 and Eq. 3.39)

$$_1q_2 = u_2 - u_1 + {_1}w_2 = h_2 - h_1 \approx C_p \, \Delta T$$

The one with the highest specific heat is hydrogen, H_2. The hydrogen has the smallest mass, but the same kinetic energy per mol as other molecules and thus the most energy per unit mass is needed to increase the temperature.

3.248E

The air drag force on a car is 0.225 A $\rho \mathbf{V}^2$. Verify that the unit becomes lbf.

$$F_d = 0.225 \ A \ \rho \mathbf{V}^2$$

Units: $\text{ft}^2 \times (\text{lbm/ft}^3) \times (\text{ft/s})^2 = \text{lbm-ft} / \text{s}^2 = (1/32.174) \ \text{lbf}$

Recall the result from Newtons law p. 9.

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Kinetic and Potential Energy

3.249E

An escalator raises a 200 lbm bucket of sand 30 ft in 1 minute. Determine the amount of work done during the process.

Solution:

$$W = \int F dx = F \int dx = F\,\Delta x = mgH$$

$$= 200 \text{ lbm} \times 32.174 \text{ ft/s}^2 \times 30 \text{ ft}$$

$$= 200 \text{ lbf} \times 30 \text{ ft} = 6000 \text{ ft lbf}$$

$$= (6000/778) \text{ Btu}$$

$$= \textbf{7.71 Btu}$$

3.250E

A hydraulic hoist raises a 3650 lbm car 6 ft in an auto repair shop. The hydraulic pump has a constant pressure of 100 lbf/in.^{2} on its piston. What is the increase in potential energy of the car and how much volume should the pump displace to deliver that amount of work?

Solution: C.V. Car.

No change in kinetic or internal energy of the car, neglect hoist mass.

$$E_2 - E_1 = PE_2 - PE_1 = mg\,(Z_2 - Z_1)$$

$$= \frac{3650 \times 32.174 \times 6}{32.174} = \textbf{21 900 lbf-ft}$$

The increase in potential energy is work into car from pump at constant P.

$$W = E_2 - E_1 = \int P\,dV = P\,\Delta V \qquad \Rightarrow$$

$$\Delta V = \frac{E_2 - E_1}{P} = \frac{21\ 900}{100 \times 144} = \textbf{1.52 ft}^3$$

3.251E

A piston motion moves a 50 lbm hammerhead vertically down 3 ft from rest to a velocity of 150 ft/s in a stamping machine. What is the change in total energy of the hammerhead?

Solution: C.V. Hammerhead

The hammerhead does not change internal energy i.e. same P,T

$$E_2 - E_1 = m(u_2 - u_1) + m(\frac{1}{2}V_2^2 - 0) + mg\,(h_2 - 0)$$

$$= 0 + [\,50 \times (1/2) \times 150^2 + 50 \times 32.174 \times (-3)]\ \text{lbm-(ft/s)}^2$$

$$= [562\ 500 - 4826]/32.174 = 17\ 333\ \text{lbf-ft}$$

$$= (\frac{17\ 333}{778})\ \text{Btu} = \textbf{22.28 Btu}$$

Recall that $1\ \text{lbf-ft} = 32.174\ \text{lbm-(ft/s)}^2$

3.252E

Airplane takeoff from an aircraft carrier is assisted by a steam driven piston/cylinder with an average pressure of 200 psia. A 38 500 lbm airplane should be accelerated from zero to a speed of 100 ft/s with 30% of the energy coming from the steam piston. Find the needed piston displacement volume.

Solution: C.V. Airplane.

No change in internal or potential energy; only kinetic energy is changed.

$$E_2 - E_1 = m \, (1/2) \, (\mathbf{V}_2^2 - 0) = 38 \, 500 \text{ lbm} \times (1/2) \times 100^2 \text{ (ft/s)}^2$$

$$= 192 \, 500 \, 000 \text{ lbm-(ft/s)}^2 = 5 \, 983 \, 092 \text{ lbf-ft}$$

The work supplied by the piston is 30% of the energy increase.

$$W = \int P \, dV = P_{avg} \, \Delta V = 0.30 \, (E_2 - E_1)$$
$$= 0.30 \times 5 \, 983 \, 092 \text{ lbf-ft} = 1 \, 794 \, 928 \text{ lbf-ft}$$

$$\Delta V = \frac{W}{P_{avg}} = \frac{1 \, 794 \, 928}{200} \, \frac{\text{lbf-ft}}{144 \text{ lbf/ft}^2} = \mathbf{62.3 \ ft^3}$$

3.253E

A bulldozer pushes 1000 lbm of dirt 300 ft with a force of 400 lbf. It then lifts the dirt 10 ft up to put it in a dump truck. How much work did it do in each situation?

Solution:

$W = \int F \, dx = F \, \Delta x$
$\quad = 400 \text{ lbf} \times 300 \text{ ft}$
$\quad = 120\,000 \text{ lbf-ft} = \textbf{154 Btu}$

$W = \int F \, dz = \int mg \, dz = mg \, \Delta Z$
$\quad = 1000 \text{ lbm} \times 32.174 \text{ ft/s}^2 \times 10 \text{ ft} / (32.174 \text{ lbm-ft} / \text{s}^2\text{-lbf})$
$\quad = 10\,000 \text{ lbf-ft} = \textbf{12.85 Btu}$

3.254E

Two hydraulic cylinders maintain a pressure of 175 psia. One has a cross sectional area of 0.1 ft^2 the other 0.3 ft^2. To deliver a work of 1 Btu to the piston how large a displacement (V) and piston motion H is needed for each cylinder? Neglect P_{atm}

Solution:

$$W = \int F\, dx = \int P\, dV = \int PA\, dx = PA \times H = P\, \Delta V$$
$$W = 1\ \text{Btu} = 778.17\ \text{lbf-ft}$$
$$\Delta V = \frac{W}{P} = \frac{778.17\ \text{lbf-ft}}{175 \times 144\ \text{lbf/ft}^2} = \textbf{0.030 873 ft}^3$$

Both cases the height is $H = \Delta V/A$

$$H_1 = \frac{0.030873}{0.1} = \textbf{0.3087 ft}$$
$$H_2 = \frac{0.030873}{0.3} = \textbf{0.1029 ft}$$

3.255E

A linear spring, $F = k_s(x - x_0)$, with spring constant $k_s = 35$ lbf/ft, is stretched until it is 2.5 in. longer. Find the required force and work input.

Solution:

$$F = k_s(x - x_0) = 35 \text{ lbf/ft} \times (2.5/12) \text{ ft} = 7.292 \text{ lbf}$$

$$W = \int F dx = \int k_s(x - x_0) d(x - x_0) = \frac{1}{2} k_s(x - x_0)^2$$

$$= \frac{1}{2} \times 35 \text{ lbf/ft} \times (2.5/12)^2 \text{ ft}^2 = 0.76 \text{ lbf-ft}$$

$$= \mathbf{9.76 \times 10^{-4} \text{ Btu}}$$

3.256E

A cylinder fitted with a frictionless piston contains 10 lbm of superheated refrigerant R-134a vapor at 100 lbf/in.^{2}, 300 F. The setup is cooled at constant pressure until the R-134a reaches a quality of 25%. Calculate the work done in the process.

Solution:

Constant pressure process boundary work. State properties from Table F.10

State 1: Table F.10.2 v_1 = 0.76629 ft^3/lbm;

State 2: Table F.10.1 v_2 = 0.013331 + 0.25 × 0.46652 = 0.12996 ft^3/lbm

Interpolated to be at 100 psia, numbers at 101.5 psia could have been used.

$$_1W_2 = \int P \, dV = P \, (V_2 - V_1) = mP \, (v_2 - v_1)$$

$$= 10 \text{ lbm} \times 100 \text{ lbf/in.}^2 \times \frac{144}{778} \frac{(\text{in/ft})^2}{\text{lbf-ft/Btu}} \times (0.12996 - 0.76629) \text{ ft}^3/\text{lbm}$$

$$= \mathbf{-117.78 \ Btu}$$

3.257E

A piston of 4 lbm is accelerated to 60 ft/s from rest. What constant gas pressure is required if the area is 4 in^2, the travel 4 in. and the outside pressure is 15 psia?

C.V. Piston

$$(E_2 - E_1)_{PIST.} = m(u_2 - u_1) + m[(1/2)V_2^2 - 0] + mg\ (0 - 0)$$

$$= (1/2)\ m\ V_2^2 = 0.5 \times 4\ lbm \times 60^2\ (ft/s)^2$$

$$= 7200\ lbm\text{-}ft^2/s^2 = \frac{7200}{32.174}\ ft\text{-}lbf$$

Energy equation for the piston is:

$$(E_2 - E_1)_{PIST.} = W_{gas} - W_{atm} = P_{avg}\ \Delta V_{gas} - P_o\ \Delta V_{gas}$$

$$\Delta V_{gas} = A\ L = 4\ in^2 \times 4\ in = 16\ in^3$$

$$P_{avg}\ \Delta V_{gas} = (E_2 - E_1)_{PIST.} + P_o\ \Delta V_{gas}$$

$$P_{avg} = (E_2 - E_1)_{PIST.}\ /\ \Delta V_{gas} + P_o$$

$$= \frac{7200\ lbf\text{-}ft}{32.174 \times 16\ in^3} + 15\ lbf/in^2$$

$$= 167.84\ psia + 15\ psia = \textbf{182.8 psia}$$

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Properties General Tables

3.258E

Find the missing properties and give the phase of the substance.

a. H_2O $u = 1000$ Btu/lbm, $T = 270$ F $h = ?$ $v = ?$ $x = ?$

b. H_2O $u = 450$ Btu/lbm, $P = 1500$ lbf/in.^{2} $T = ?$ $x = ?$ $v = ?$

c. R-410A $T = 30$ F, $P = 120$ lbf/in.^{2} $h = ?$ $x = ?$

Solution:

a) Table F.7.1: $u_f < u < u_g$ => 2-phase mixture of liquid and vapor

   $x = (u - u_f)/ u_{fg} = (1000 - 238.81)/854.14 = \textbf{0.8912}$

   $v = v_f + x\, v_{fg} = 0.01717 + 0.8912 \times 10.0483 = \textbf{8.972 ft}^3\textbf{/lbm}$

   $h = h_f + x\, h_{fg} = 238.95 + 0.8912 \times 931.95 = \textbf{1069.5 Btu/lbm}$

     $(h = u + Pv = 1000 + 41.848 \times 8.972 \times 144/778)$

b) Table F.7.1: $u < u_f$ so compressed liquid B.1.3, $x = \textbf{undefined}$

  $T = \textbf{471.8 F}$, $v = \textbf{0.019689 ft}^3\textbf{/lbm}$

c) Table F.9.1: $P > P_{sat}$ => $x = \textbf{undef}$, **compr. liquid**

  Approximate as saturated liquid at same T, $h \cong h_f = \textbf{24.11 Btu/lbm}$

States shown are placed relative to the two-phase region, not to each other.

3.259E

Find the missing properties among (P, T, v, u, h) together with x, if applicable, and give the phase of the substance.

a.   R-410A $T = 50$ F, $u = 85$ Btu/lbm

b. H_2O $T = 600$ F, $h = 1322$ Btu/lbm

c.   R-410A $P = 150$ lbf/in.^{2}, $h = 135$ Btu/lbm

Solution:

a) Table F.9.1: $u < u_g$ => L+V mixture,   P = **157.473 lbf/in^2**

               $x = (85 - 31.06)/ 78.96 =$ **0.6831**

               $v = 0.014 + 0.6831 \times 0.3676 =$ **0.2653 ft^3/lbm**

               $h = 31.47 + 0.6831 \times 89.67 =$ **92.72 Btu/lbm**

b) Table F.7.1: $h > h_g$ => superheated vapor   follow 600 F in F.7.2

          $P \cong$ **200 lbf/in^2** ; $v =$ **3.058 ft^3/lbm ;** $u =$ **1208.9 Btu/lbm**

c) Table F.9.1: $h > h_g$ => superheated vapor so in F.9.2

             $T \cong$ **100 F** ; $v =$ **0.483 ft^3/lbm**

             $u =$ **121.25 Btu/lbm**

States shown are placed relative to the two-phase region, not to each other.

3.260E

  Find the missing properties among (P, T, v, u, h) together qwith x if applicable and give the phase of the substance.

  a.   R-134a $T = 140$ F, $h = 185$ Btu/lbm
  b. NH_3 $T = 170$ F, $P = 60$ lbf/in.^{2}
  c.   R-134a $T = 100$ F, $u = 175$ Btu/lbm

Solution:

a) Table F.10.1: $h > h_g$ => x = **undef**, **superheated vapor** F.10.2,

   find it at given T between saturated 243.9 psi and 200 psi to match h:

$$v \cong 0.1836 + (0.2459 - 0.1836) \times \frac{185 - 183.63}{186.82 - 183.63} = \textbf{0.2104 ft}^3\textbf{/lbm}$$

$$P \cong 243.93 + (200 - 243.93) \times \frac{185 - 183.63}{186.82 - 183.63} = \textbf{225 lbf/in}^2$$

b)   Table F.8.1: $P < P_{sat}$ $\Rightarrow$ x = **undef.**   **superheated vapor** F.8.2,

   $v = (6.3456 + 6.5694)/ 2 = \textbf{6.457 ft}^3\textbf{/lbm}$

   $u = h\text{-}Pv = (1/2)(694.59 + 705.64) - 60 \times 6.4575 \times (144/778)$

          $= 700.115 - 71.71 = \textbf{628.405 Btu/lbm}$

c) Table F.10.1: $u > u_g$ => **sup. vapor,**

   Interpolate between 40 and 60 psia tables in F.10.2

          $P = 40 + (40 - 20)(175 - 175.57)/(174.85 - 175.57)$

          $= 40 + 20 \times 0.791667 = \textbf{55.8 lbf/in}^2$;

   $v = 1.4015 + (0.9091 - 1.4015) \times 0.791667 = \textbf{1.0117 ft}^3\textbf{/lbm;}$

   $h = 185.95 + (184.94 - 185.95) \times 0.791667 = \textbf{185.15 Btu/lbm}$

States shown are placed relative to the two-phase region, not to each other.

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Simple Processes

3.261E

Ammonia at 30 F, quality 60% is contained in a rigid 8-ft^3 tank. The tank and ammonia are now heated to a final pressure of 150 lbf/in.^{2}. Determine the heat transfer for the process.

Solution:

C.V.: NH_3

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process: Constant volume $\Rightarrow$ $v_2 = v_1$ & ${_1}W_2 = 0$

State 1: Table F.8.1 two-phase state.

$\qquad v_1 = 0.02502 + 0.6 \times 4.7978 = 2.904$ ft^3/lbm

$\qquad u_1 = 75.06 + 0.6 \times 491.17 = 369.75$ Btu/lbm

$\qquad m = V/v_1 = 8/2.904 = 2.755$ lbm

State 2: P_2, $v_2 = v_1$ superheated vapor Table F.8.2

$\qquad\qquad \Rightarrow$ $T_2 \cong 258$ F, $u_2 \cong 661.42$ Btu/lbm

So solve for heat transfer in the energy equation

$\qquad {_1}Q_2 = 2.755$ lbm $\times$ (661.42 - 369.75) Btu/lbm = **803.6 Btu**

3.262E

Saturated vapor R-410A at 30 F in a rigid tank is cooled to 0 F. Find the specific heat transfer.

Solution:

C.V.: R-410A in tank. $m_2 = m_1$;

Energy Eq.3.5: $(u_2 - u_1) = {}_1q_2 - {}_1w_2$

Process: V = constant, $v_2 = v_1 = V/m$ => $_1w_2 = \cancel{0}$

Table F.9.1: State 1: $v_1 = 0.5426$ ft^3/lbm, $u_1 = 108.63$ Btu/lbm

State 2: 0 F, $v_2 = v_1 = V/m$,   look in Table F.9.1 at 0 F

$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{0.5426 - 0.01295}{0.9448} = 0.560595$$

$$u_2 = u_{f2} + x_2\, u_{fg2} = 13.37 + x_2 \times 92.75 = 65.365 \text{ Btu/lbm}$$

From the energy equation

$$_1q_2 = (u_2 - u_1) = (65.365 - 108.63) = \textbf{-43.26 Btu/lbm}$$

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3.263E

Saturated vapor R-410A at 100 psia in a constant pressure piston cylinder is heated to 70 F. Find the specific heat transfer.

Solution:

C.V. R-410A: $m_2 = m_1 = m$;

Energy Eq.3.5 $(u_2 - u_1) = {_1}q_2 - {_1}w_2$

Process: P = const. $\Rightarrow {_1}w_2 = \int Pdv = P\Delta v = P(v_2 - v_1)$

State 1: Table F.9.2 (or F.9.1) $h_1 = 119.38$ Btu/lbm

State 2: Table F.9.2 $h_2 = 130.44$ Btu/lbm

$${_1}q_2 = (u_2 - u_1) + {_1}w_2 = (u_2 - u_1) + P(v_2 - v_1) = (h_2 - h_1)$$
$${_1}q_2 = 130.44 - 119.38 = \textbf{11.06 Btu/lbm}$$

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3.264E

A rigid tank holds 1.5 lbm R-410A at 100 F as saturated vapor. The tank is now cooled to 60 F by heat transfer to the ambient. Which two properties determine the final state. Determine the amount of work and heat transfer during the process.

C.V. The R-410A, this is a control mass.

Process: Rigid tank $V = C$ $\Rightarrow$ $v = $ constant & $_1W_2 = \int_1^2 PdV = \mathbf{0}$

Energy Eq.: $U_2 - U_1 = {_1Q_2} - {_1W_2} = {_1Q_2}$,

State 1: $v_1 = 0.1657$ ft^3/lbm,

   $u_1 = 111.7$ Btu/lbm

State 2:   **T, v** $\Rightarrow$ two-phase (straight down in P-v diagram from state 1)

$x_2 = (v - v_f)/v_{fg} = (0.1657 - 0.01451)/0.3076 = 0.49151$

$u_2 = u_f + x_2\, u_{fg} = 34.78 + 0.49151 \times 75.82 = 72.046$ Btu/lbm

$_1Q_2 = m(u_2 - u_1) = 1.5$ lbm $(72.046 - 111.7)$ Btu/lbm = **-59.5 Btu**

3.265E

A cylinder fitted with a frictionless piston contains 4 lbm of superheated refrigerant R-134a vapor at 400 lbf/in.^{2}, 200 F. The cylinder is now cooled so the R-134a remains at constant pressure until it reaches a quality of 75%. Calculate the heat transfer in the process.

Solution:

C.V.: R-134a $m_2 = m_1 = m;$

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = const. $\Rightarrow {}_1W_2 = \int PdV = P\Delta V = P(V_2 - V_1) = Pm(v_2 - v_1)$

State 1: Table F.10.2 $h_1 = 192.92$ Btu/lbm

State 2: Table F.10.1 $h_2 = 140.62 + 0.75 \times 43.74 = 173.425$ Btu/lbm

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + Pm(v_2 - v_1) = m(h_2 - h_1)$$

$$= 4 \text{ lbm} \times (173.425 - 192.92) \text{ Btu/lbm} = \textbf{-77.98 Btu}$$

3.266E

A water-filled reactor with volume of 50 ft^3 is at 2000 lbf/in.^{2}, 560 F and placed inside a containment room, as shown in Fig. P3.101. The room is well insulated and initially evacuated. Due to a failure, the reactor ruptures and the water fills the containment room. Find the minimum room volume so the final pressure does not exceed 30 lbf/in.^{2}.

C.V.: Containment room and reactor.

Mass: $m_2 = m_1 = V_{reactor}/v_1 = 50/0.02172 = 2295.7$ lbm

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = \emptyset \implies u_2 = u_1 = 552.5$ Btu/lbm

State 2: 30 lbf/in.^{2}, $u_2 < ug \implies$ 2 phase Table F.7.1

$$u = 552.5 = 218.48 + x_2\, 869.41 \implies x_2 = 0.3842$$

$$v_2 = 0.017 + 0.3842 \times 13.808 = 5.322 \text{ ft}^3/\text{lbm}$$

$$V_2 = mv_2 = 2295.7 \text{ lbm} \times 5.322 \text{ ft}^3/\text{lbm} = \textbf{12 218 ft}^3$$

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3.267E

Ammonia (1 lbm) is in a piston cylinder at 30 psia, 20 F is heated in a process where the pressure varies linear with volume to a state of 240 F, 40 psia. Find the work and the heat transfer in the process.

Solution:

Take CV as the Ammonia, constant mass.

Continuity Eq.: $\qquad m_2 = m_1 = m$;

Process: $\qquad P = A + BV \qquad$ (linear in V)

State 1: Superheated vapor $\quad v_1 = 9.7206$ ft^3/lbm,

$$u_1 = 622.39 - 30 \times 9.7206 \times \frac{144}{778} = 568.4 \text{ Btu/lbm}$$

State 2: Superheated vapor $\quad v_2 = 10.9061$ ft^3/lbm,

$$u_2 = 741.4 - 40 \times 10.9061 \times 144/778 = 660.65 \text{ Btu/lbm}$$

Work is done while piston moves at increasing pressure, so we get

$$_1W_2 = \int P\, dV = \text{area} = P_{avg}\,(V_2 - V_1) = \frac{1}{2}\,(P_1 + P_2)m(v_2 - v_1)$$

$$= \frac{1}{2}(30 + 40) \text{ psi} \times 1 \text{ lbm } (10.9061 - 9.7206) \text{ ft}^3/\text{lbm}$$

$$= 41.493 \text{ psi-ft}^3 = 41.493 \times 144 \text{ lbf-ft} = 5974.92 \text{ lbf-ft } = \textbf{7.68 Btu}$$

(see conversions in A.1, p. 755)

Heat transfer from the energy equation

$$_1Q_2 = m(u_2 - u_1) + {_1W_2} = 1\,(660.65 - 568.4) + 7.68 = \textbf{99.93 Btu}$$

3.268E

A piston cylinder arrangement with a linear spring similar to Fig. P3.105 contains R-134a at 60 F, x = 0.6 and a volume of 0.7 ft^3. It is heated to 140 F at which point the specific volume is 0.4413 ft^3/lbm. Find the final pressure, the work and the heat transfer in the process.

Take CV as the R-134a.

$$m_2 = m_1 = m \quad ; \qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

State 1: T_1, x_1 => Two phase so Table F.10.1: $P_1 = P_{sat} = 72.271$ psia

$v_1 = v_f + x_1\, v_{fg} = 0.01291 + 0.6 \times 0.6503 = 0.40309$ ft^3/lbm

$u_1 = u_f + x_1\, u_{fg} = 98.27 + 0.6 \times 69.31 = 139.856$ Btu/lbm

$m = V_1/v_1 = 0.7$ ft^3/ 0.40309 ft^3/lbm = 1.7366 lbm

State 2: (T, v) Superheated vapor, Table F.10.2.

$P_2 = 125$ psia, $v_2 = 0.4413$ ft^3/lbm, $u_2 = 180.77$ Btu/lbm

$V_2 = m\, v_2 = 1.7366 \times 0.4413 = 0.76636$ ft^3

Work is done while piston moves at linearly varying pressure, so we get

$${}_1W_2 = \int P\, dV = \text{area} = P_{avg}\, (V_2 - V_1) = 0.5(P_2 + P_1)(V_2 - V_1)$$

$$= 0.5 \times (72.271 + 125) \text{ psia } (0.76636 - 0.7) \text{ ft}^3$$

$$= 6.54545 \times (144/778) \text{ Btu} = \mathbf{1.212\ Btu}$$

Heat transfer is found from the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.7366 \times (180.77 - 139.856) + 1.212$$

$$= \mathbf{72.26\ Btu}$$

3.269E

Water in a 6-ft^3 closed, rigid tank is at 200 F, 90% quality. The tank is then cooled to 20 F. Calculate the heat transfer during the process.

Solution:

C.V.: Water in tank. $\quad m_2 = m_1$; $\quad m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process: V = constant, $v_2 = v_1$, $\quad {_1}W_2 = 0$

State 1: Two-phase L + V look in Table F.7.1

$\quad v_1 = 0.01663 + 0.9 \times 33.6146 = 30.27$ ft^3/lbm

$\quad u_1 = 168.03 + 0.9 \times 906.15 = 983.6$ Btu/lbm

State 2: T_2, $v_2 = v_1 \quad \Rightarrow \quad$ mix of sat. solid + vap. Table F.7.4

$\quad v_2 = 30.27 = 0.01744 + x_2 \times 5655 \quad => \quad x_2 = 0.00535$

$\quad u_2 = -149.31 + 0.00535 \times 1166.5 = -143.07$ Btu/lbm

$\quad m = V/v_1 = 6$ ft^3/ 30.27 ft^3/lbm = 0.198 lbm

${_1}Q_2 = m(u_2 - u_1) = 0.198$ lbm (-143.07 - 983.6) Btu/lbm = **-223 Btu**

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3.270E

A constant pressure piston/cylinder has 2 lbm water at 1100 F and 2.26 ft^3. It is now cooled to occupy 1/10 of the original volume. Find the heat transfer in the process.

C.V.: Water $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = const. $\Rightarrow {}_1W_2 = \int PdV = P\Delta V = P(V_2 - V_1) = Pm(v_2 - v_1)$

State 1: Table F.7.2   (T, $v_1 = V/m = 2.26/2 = 1.13$ ft^3/lbm)
$P_1 = 800$ psia, $h_1 = 1567.81$ Btu/lbm

State 2: Table F.7.2   (P, $v_2 = v_1/10 = 0.113$ ft^3/lbm)   two-phase state
$T_2 = 510 + (520 - 510)(800 - 743.53)/(811.48 - 743.53) = 518.3$ F
$x_2 = (v_2 - v_f)/v_{fg} = (0.113 - 0.02087)/0.5488 = 0.1679$
$h_2 = h_f + x_2\, h_{fg} = 509.63 + x_2\, 689.62 = 625.42$ Btu/lbm

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$
$$= 2\,(625.42 - 1567.81) = \textbf{-1884.8 Btu}$$

Solids and Liquids

3.271E

I have 4 lbm of liquid water at 70 F, 15 psia. I now add 20 Btu of energy at a constant pressure. How hot does it get if it is heated? How fast does it move if it is pushed by a constant horizontal force? How high does it go if it is raised straight up?

a) Heat at 15 psia.

Energy equation:

$$E_2 - E_1 = {}_1Q_2 - {}_1W_2 = {}_1Q_2 - P(V_2 - V_1) = H_2 - H_1 = m(h_2 - h_1)$$

$$h_2 = h_1 + {}_1Q_2/m = 38.09 + 20/4 = 43.09 \text{ Btu/lbm}$$

Back interpolate in Table F.7.1: $T_2 = \textbf{75 F}$

(We could also have used $\Delta T = {}_1Q_2/mC = 20 / (4 \times 1.00) = 5$ F)

b) Push at constant P. It gains kinetic energy.

$$0.5 \, m \, \textbf{V}_2^2 = {}_1W_2$$

$$\textbf{V}_2 = \sqrt{2 \, {}_1W_2/m} = \sqrt{2 \times 20 \times 778.17 \text{ lbf-ft/4 lbm}}$$

$$= \sqrt{2 \times 20 \times 778.17 \times 32.174 \text{ lbm-(ft/s)}^2 /4 \text{ lbm}} = \textbf{500 ft/s}$$

c) Raised in gravitational field

$$m \, g \, Z_2 = {}_1W_2$$

$$Z_2 = {}_1W_2/m \, g = \frac{20 \times 778.17 \text{ lbf-ft}}{4 \text{ lbm} \times 32.174 \text{ ft/s}^2} \times 32.174 \frac{\text{lbm-ft/s}^2}{\text{lbf}} = \textbf{3891 ft}$$

Comment: Notice how fast (500 ft/s) and how high it should be to have the same energy as raising the temperature just 5 degrees. I.e. in most applications we can disregard the kinetic and potential energies unless we have very high **V** or Z.

3.272E

A computer cpu chip consists of 0.1 lbm silicon, 0.05 lbm copper, 0.1 lbm polyvinyl chloride (plastic). It now heats from 60 F to 160 F as the computer is turned on. How much energy did the heating require?

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {_1}Q_2 - {_1}W_2$

For the solid masses we will use the specific heats, Table F.2, and they all have the same temperature so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v\,i} (T_2 - T_1)_i = (T_2 - T_1)\sum m_i C_{v\,i}$$

$$\sum m_i C_{v\,i} = 0.1 \times 0.167 + 0.05 \times 0.1 + 0.1 \times 0.229 = 0.0446 \text{ Btu/R}$$

$$U_2 - U_1 = 0.0446 \text{ Btu/R} \times (160 - 60) \text{ R} = \textbf{4.46 Btu}$$

3.273E

A copper block of volume 60 in.^{3} is heat treated at 900 F and now cooled in a 3-ft^3 oil bath initially at 70 F. Assuming no heat transfer with the surroundings, what is the final temperature?

C.V. Copper block and the oil bath.

Also assume no change in volume so the work will be zero.

Energy Eq.: $U_2 - U_1 = m_{met}(u_2 - u_1)_{met} + m_{oil}(u_2 - u_1)_{oil} = {}_1Q_2 - {}_1W_2 = 0$

Solid and liquid $\Delta u \cong C_V \Delta T$, C_V from Table F.2 and F.3

$$m_{met} = V\rho = 60 \times 12^{-3} \text{ ft}^3 \times 518 \text{ lbm/ft}^3 = 17.986 \text{ lbm}$$

$$m_{oil} = V\rho = 3.0 \text{ ft}^3 \times 57 \text{ lbm/ft}^3 = 171 \text{ lbm}$$

$$C_{V\ met} = 0.10 \text{ Btu/lbm-R};\qquad C_{V\ oil} = 0.43 \text{ Btu/lbm-R}$$

Energy equation becomes

$$m_{met}\, C_{V\ met}\, (T_2 - T_{1,met}) + m_{oil}\, C_{V\ oil}\, (T_2 - T_{1,oil}) = 0$$

$$17.986 \text{ lbm} \times 0.10 \text{ Btu/lbm-R } (T_2 - 900 \text{ F})$$

$$+ 171 \text{ lbm} \times 0.43 \text{ Btu/lbm-R } (T_2 - 70 \text{ F}) = 0$$

$$\Rightarrow\ T_2 = \textbf{89.8 F}$$

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Ideal Gas

3.274E

Estimate the constant specific heats for R-134a from Table F.10.2 at 15 psia and 150 F. Compare this to table F.4 and explain the difference.

Solution:

Using values at 15 psia for h and u at 140 F and 160 F from Table F.10.2, the approximate specific heats at 150 F are

$$C_p \approx \frac{\Delta h}{\Delta T} = \frac{199.95 - 195.59}{160 - 140} = 0.218 \text{ Btu/lbm R}$$

compared with 0.203 Btu/lbm-R for the ideal-gas value at 77 F from Table F.4.

$$C_v \approx \frac{\Delta u}{\Delta T} = \frac{188.03 - 184.08}{160 - 140} = 0.198 \text{ Btu/lbm R}$$

compared with 0.184 Btu/lbm-R for the ideal-gas value at 77 F from Table F.4.

There are two reasons for the differences. First, R-134a is not exactly an ideal gas at the given state, 150 F and 15 psia. Second and by far the biggest reason for the differences is that R-134a, chemically CF_3CH_2 , is a polyatomic molecule with multiple vibrational mode contributions to the specific heats (see Appendix C), such that they are strongly dependent on temperature. Note that if we repeat the above approximation for C_p in Table F.10.2 at 77 F, the resulting value is 0.203 Btu/lbm R

3.275E

Air is heated from 540 R to 640 R at V = C. Find $_1q_2$? What if from 2400 to 2500 R?

Process: V = C $\rightarrow$ $_1W_2 = \emptyset$

Energy Eq.: $u_2 - u_1 = {}_1q_2 - 0$ $\rightarrow$ $_1q_2 = u_2 - u_1$

Read the u-values from Table F.5

a) $_1q_2 = u_2 - u_1 = 109.34 - 92.16 = $ **17.18 Btu/lbm**

b) $_1q_2 = u_2 - u_1 = 474.33 - 452.64 = $ **21.7 Btu/lbm**

case a) $C_v \approx 17.18/100 = 0.172$ Btu/lbm-R, see F.4

case b) $C_v \approx 21.7/100 = 0.217$ Btu/lbm-R (26 % higher)

3.276E

A 30-ft high cylinder, cross-sectional area 1 ft^2, has a massless piston at the bottom with water at 70 F on top of it, as shown in Fig. P3.107. Air at 540 R, volume 10 ft^3 under the piston is heated so that the piston moves up, spilling the water out over the side. Find the total heat transfer to the air when all the water has been pushed out.

Solution

The water on top is compressed liquid and has mass

$$V_{H2O} = V_{tot} - V_{air} = 30 \times 1 - 10 = 20 \text{ ft}^3$$
$$m_{H2O} = V_{H2O}/v_f = 20/0.016051 = 1246 \text{ lbm}$$

Initial air pressure is: $P_1 = P_0 + m_{H2O}g/A = 14.7 + \dfrac{g}{1 \times 144} = 23.353 \text{ psia}$

and then $m_{air} = \dfrac{PV}{RT} = \dfrac{23.353 \times 10 \times 144}{53.34 \times 540} = 1.1675 \text{ lbm}$

State 2: $P_2 = P_0 = 14.7 \text{ lbf/in}^2$, $V_2 = 30 \times 1 = 30 \text{ ft}^3$

$$_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

$$= \frac{1}{2}(23.353 + 14.7)(30 - 10) \times 144 = 54\,796 \text{ lbf-ft} = 70.43 \text{ Btu}$$

State 2: $P_2, V_2 \Rightarrow T_2 = \dfrac{T_1P_2V_2}{P_1V_1} = \dfrac{540 \times 14.7 \times 30}{23.353 \times 10} = 1019.7 \text{ R}$

$$_1Q_2 = m(u_2 - u_1) + {_1W_2} = 1.1675 \times 0.171\,(1019.7 - 540) + 70.43$$
$$= \textbf{166.2 Btu}$$

3.277E

A closed rigid container is filled with 3 lbm water at 1 atm, 130 F, 2 lbm of stainless steel and 1 lbm of PVC (polyvinyl chloride) both at 70 F and 0.2 lbm of air at 700 R, 1 atm. It is now left alone with no external heat transfer and no water vaporizes. Find the final temperature and air pressure.

CV. Container.

Process: $V = $ constant $=> {}_1W_2 = 0$ and also given ${}_1Q_2 = 0$

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$

For the liquid and the metal masses we will use the specific heats (Tbl F.3, F.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v\,i} (T_2 - T_1)_i = T_2 \sum m_i C_{v\,i} - \sum m_i C_{v\,i} T_{1\,i}$$

noticing that all masses have the same T_2 but not same initial T.

$$\sum m_i C_{v\,i} = 3 \times 1.0 + 2 \times 0.11 + 1 \times 0.229 + 0.2 \times 0.171 = 3.4832 \text{ Btu/R}$$

The T for air must be converted to F like the others.

Energy Eq.: $3.4832\ T_2 = 3 \times 1.0 \times 130 + (2 \times 0.11 + 1 \times 0.229) \times 70$

$$+ 0.2 \times 0.171 \times (700 - 459.67) = 429.649 \text{ Btu}$$

$$T_2 = \textbf{123.35 F}$$

The volume of the air is constant so from $PV = mRT$ it follows that P varies with T

$$P_2 = P_1\ T_2/T_{1\,air} = 1 \text{ atm} \times (123.35 + 459.67) / 700 = \textbf{0.833 atm}$$

3.278E

An engine consists of a 200 lbm cast iron block with a 40 lbm aluminum head, 40 lbm steel parts, 10 lbm engine oil and 12 lbm glycerine (antifreeze). Everything begins at 40 F and as the engine starts, it absorbs a net of 7000 Btu before it reaches a steady uniform temperature. We want to know how hot it becomes.

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

Process: The steel does not change volume and the change for the liquid is minimal, so ${}_1W_2 \cong 0$.

So sum over the various parts of the left hand side in the energy equation

$$m_{Fe}(u_2 - u_1) + m_{Al}(u_2 - u_1)_{Al} + m_{st}(u - u_1)_{st}$$
$$+ m_{oil}(u_2 - u_1)_{oil} + m_{gly}(u_2 - u_1)_{gly} = {}_1Q_2$$

Tbl F.2 : $C_{Fe} = 0.1$, $C_{Al} = 0.215$, $C_{st} = 0.11$ all units of Btu/lbm-R

Tbl F.3 : $C_{oil} = 0.46$, $C_{gly} = 0.58$ all units of Btu/lbm-R

So now we factor out $T_2 - T_1$ as $u_2 - u_1 = C(T_2 - T_1)$ for each term

$$[\, m_{Fe}C_{Fe} + m_{Al}C_{Al} + m_{st}C_{st} + m_{oil}C_{oil} + m_{gly}C_{gly}\,]\,(T_2 - T_1) = {}_1Q_2$$

$$T_2 - T_1 = {}_1Q_2 \,/\, \Sigma m_i\, C_i$$

$$= \frac{7000}{200 \times 0.1 + 40 \times 0.215 + 40 \times 0.11 + 10 \times 0.46 + 12 \times 0.58}$$

$$= \frac{7000}{44.56}\, R = 157\ R$$

$$T_2 = T_1 + 157\ F = 40 + 157 = \mathbf{197\ F}$$

3.279E

 A car with mass 3250 lbm drives with 60 mi/h when the brakes are applied to quickly decrease its speed to 20 mi/h. Assume the brake pads are 1 lbm mass with heat capacity of 0.2 Btu/lbm R and the brake discs/drums are 8 lbm steel where both masses are heated uniformly. Find the temperature increase in the brake assembly.

C.V. Car. Car looses kinetic energy and brake system gains internal u.

No heat transfer (short time) and no work term.

m = constant;

Energy Eq.3.5: $E_2 - E_1 = 0 - 0 = m_{car} \frac{1}{2}(V_2^2 - V_1^2) + m_{brake}(u_2 - u_1)$

The brake system mass is two different kinds so split it, also use C_v since we do not have a u table for steel or brake pad material.

$$m_{steel} C_v \Delta T + m_{pad} C_v \Delta T = m_{car} \frac{1}{2}(V_2^2 - V_1^2)$$

$(8 \times 0.11 + 1 \times 0.2)$ Btu/R $\times \Delta T$

$= 3250 \times 0.5 \times (3600 - 400) \times 1.46667^2 / (32.174 \times 778)$ Btu

$= 446.9$ Btu

$\Rightarrow \quad \Delta T = \textbf{414 F}$

3.280E

Water at 60 psia is brought from 320 F to 1800 F. Evaluate the change in specific internal energy using a) the steam tables, b) the ideal gas Table F.6, and the specific heat F.4

Solution:

a)

State 1: Table F.7.3   Superheated vapor $u_1 = 1109.46$ Btu/lbm

State 2: Table F.7.3 $u_2 = 1726.69$ Btu/lbm

$$u_2 - u_1 = 1726.69 - 1109.46 = \textbf{617.23 Btu/lbm}$$

b)

Table F.6 at 780 R: $u_1 = 1978.7 - 1.98589 \times 780 = 429.71$ Btu/lbmol

Table F.6 at 2260 R: $u_2 = 15894 - 1.98589 \times 2260 = 11\,406$ Btu/lbmol

$$u_2 - u_1 = (11\,406 - 429.71)/18.013 = \textbf{609.35 Btu/lbm}$$

c)   Table F.4 : $C_{vo} = 0.337$ Btu/lbm-R

$$u_2 - u_1 = 0.337 \text{ Btu/lbm-R } (1800 - 320) \text{ R} = \textbf{498.8 Btu/lbm}$$

Notice how the average slope from 320 F to 1800 F is higher than the one at 77 F ($= C_{vo}$)

3.281E

A 65 gallons rigid tank contains methane gas at 900 R, 200 psia. It is now cooled down to 540 R. Assume ideal gas and find the needed heat transfer.

Solution:

Ideal gas and recall from Table A.1 that 1 gal = 231 in^3,

$$m = P_1V/RT_1 = \frac{200\ \text{psi} \times 65\ \text{gal} \times 231\ \text{in}^3/\text{gal}}{96.35\ \text{lbf-ft/lbm-R} \times 900\ \text{R} \times 12\ \text{in/ft}} = 2.886\ \text{lbm}$$

Process: $V = \text{constant} = V_1$ => $_1W_2 = 0$

Use specific heat from Table F.4

$$u_2 - u_1 = C_v\,(T_2 - T_1) = 0.415\ \text{Btu/lbm-R}\ (900 - 540)\ \text{R}$$
$$= -149.4\ \text{Btu/lbm}$$

Energy Equation

$$_1Q_2 = m(u_2 - u_1) = 2.886\,(-149.4) = \textbf{--431.2 Btu}$$

Polytropic Processes

3.282E

Oxygen at 50 lbf/in.^{2}, 200 F is in a piston/cylinder arrangement with a volume of 4 ft^3. It is now compressed in a polytropic process with exponent, $n = 1.2$, to a final temperature of 400 F. Calculate the heat transfer for the process.

Continuity: $m_2 = m_1$; Energy: $E_2 - E_1 = m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

State 1: T, P and ideal gas, small change in T, so use Table F.4

$$\Rightarrow \quad m = \frac{P_1 V_1}{R T_1} = \frac{50 \text{ psi} \times 4 \text{ ft}^3 \times 144 \text{ in}^2/\text{ft}^2}{48.28 \text{ lbf-ft/lbm-R} \times 659.67 \text{ R}} = 0.9043 \text{ lbm}$$

Process: $PV^n = $ constant

$${_1}W_2 = \frac{1}{1-n}(P_2 V_2 - P_1 V_1) = \frac{mR}{1-n}(T_2 - T_1) = \frac{0.9043 \times 48.28}{1 - 1.2} \times \frac{400 - 200}{778}$$

$$= -56.12 \text{ Btu}$$

$${_1}Q_2 = m(u_2 - u_1) + {_1}W_2 \cong mC_v(T_2 - T_1) + {_1}W_2$$

$$= 0.9043 \times 0.158 (400 - 200) - 56.12 = \textbf{- 27.54 Btu}$$

3.283E

An air pistol contains compressed air in a small cylinder, as shown in Fig. P3.164. Assume that the volume is 1 in.^{3}, pressure is 10 atm, and the temperature is 80 F when armed. A bullet, $m = 0.04$ lbm, acts as a piston initially held by a pin (trigger); when released, the air expands in an isothermal process (T = constant). If the air pressure is 1 atm in the cylinder as the bullet leaves the gun, find

a. The final volume and the mass of air.
b. The work done by the air and work done on the atmosphere.
c. The work to the bullet and the bullet exit velocity.

C.V. Air.   Air ideal gas:

$$m_{air} = \frac{P_1 V_1}{R T_1} = \frac{10 \text{ atm} \times 14.7 \text{ psi/atm} \times 1 \text{ in}^3}{53.34 \text{ lbf-ft/lbm-R} \times 539.67 \text{ R} \times 12 \text{ in/ft}} = \textbf{4.26×10}^{\textbf{-5}} \textbf{ lbm}$$

Process: $PV = const = P_1 V_1 = P_2 V_2 \Rightarrow V_2 = V_1 P_1 / P_2 = \textbf{10 in}^{\textbf{3}}$

$${}_1 W_2 = \int P dV = P_1 V_1 \int (1/V) \, dV = P_1 V_1 \ln(V_2 / V_1)$$

$$= 147 \text{ psi} \times 1 \text{ in}^3 \times \ln(10) = 28.2 \text{ lbf-ft} = \textbf{0.0362 Btu}$$

$${}_1 W_{2,ATM} = P_0 (V_2 - V_1) = 14.7 \text{ psi} \times 9 \text{ in}^3 = 11.03 \text{ lbf-ft} = \textbf{0.0142 Btu}$$

$$W_{bullet} = {}_1 W_2 - {}_1 W_{2,ATM} = 0.022 \text{ Btu} = \tfrac{1}{2} m_{bullet}(V_{ex})^2$$

$$\textbf{V}_{ex} = (2 W_{bullet}/m_B)^{1/2} = (2 \times 0.022 \times 778 \times 32.174 / 0.04)^{1/2} = \textbf{165.9 ft/s}$$

3.284E

Helium gas expands from 20 psia, 600 R and 9 ft^3 to 15 psia in a polytropic process with n = 1.667. How much heat transfer is involved?

Solution:

C.V. Helium gas, this is a control mass.

Energy equation: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process equation: $PV^n = \text{constant} = P_1V_1^n = P_2V_2^n$

Ideal gas (F.4): $m = PV/RT = \dfrac{20 \text{ psi} \times 9 \text{ ft}^3 \times 144 \text{ in}^2/\text{ft}^2}{386 \text{ lbf-ft/lbm-R} \times 600 \text{ R}} = 0.112 \text{ lbm}$

Solve for the volume at state 2

$$V_2 = V_1 \, (P_1/P_2)^{1/n} = 9 \times \left(\frac{20}{15}\right)^{0.6} = 10.696 \text{ ft}^3$$

$$T_2 = T_1 \, P_2V_2/(P_1V_1) = 600 \text{ R} \, \frac{15 \times 10.696}{20 \times 9} = 534.8 \text{ R}$$

Work from Eq.3.21

$$\begin{aligned}
{}_1W_2 &= \frac{P_2V_2 - P_1 V_1}{1\text{-}n} = \frac{15 \times 10.696 - 20 \times 9}{1 - 1.667} \text{ psi ft}^3 = 29.33 \text{ psia ft}^3 \\
&= 4223 \text{ lbf-ft} = 5.43 \text{ Btu}
\end{aligned}$$

Use specific heat from Table F.4 to evaluate $u_2 - u_1$, $C_v = 0.744$ Btu/lbm R

$$\begin{aligned}
{}_1Q_2 &= m(u_2 - u_1) + {}_1W_2 = m \, C_v \, (T_2 - T_1) + {}_1W_2 \\
&= 0.112 \text{ lbm} \times 0.744 \text{ Btu/lbm-R} \times (534.8 - 600) \text{ R} + 5.43 \text{ Btu} \\
&= \textbf{-0.003 Btu}
\end{aligned}$$

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More Complex Devices

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3.285E

Water in a tank A is at 270 F with a quality of 10% and mass 1 lbm. It is connected to a piston cylinder holding constant pressure of 40 psia initially with 1 lbm water at 700 F. The valve is opened and enough heat transfer takes place to have a final uniform temperature of 280 F. Find the final P and V, the process work and the process heat transfer.

Solution:

C.V. Water in A and B. Control mass goes through process: 1 -> 2

Continuity Eq.: $m_2 - m_{A1} - m_{B1} = 0$ => $m_2 = m_{A1} + m_{B1} = 1.0 + 1.0 = 2$ lbm

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

State A1: $v_{A1} = 0.01717 + x_{A1} \times 10.0483 = 1.022$; $V_{A1} = mv = 1.022$ ft^3

$u_{A1} = 238.81 + 0.1 \times 854.14 = 324.22$ Btu/lbm

State B1: $v_{B1} = 17.196$ ft^3/lbm; $u_{B1} = 1255.14$ Btu/lbm

=> $V_{B1} = m_{B1}v_{B1} = 17.196$ ft^3

State 2: If $V_2 > V_{A1}$ then $P_2 =$ **40 psia** that is the piston floats.

For $(T_2, P_2) = (280$ F, 40 psia) => superheated vapor $u_2 = 1097.31$ Btu/lbm

$v_2 = 10.711$ ft^3/lbm $V_2 = m_2v_2 =$ **21.422 ft^3** $> V_{A1}$ checks OK.

The possible state 2 (P,V) combinations are shown. State a is 40 psia, $v_a = V_{A1}/m_2 = 0.511$ and thus two-phase $T_a = 267.3$ F

less than T_2

Process: ${}_1W_2 = P_2 (V_2 - V_1) = 40 (21.422 - 1.022 - 17.196)\frac{144}{778} =$ **23.72 Btu**

From the energy Eq.:

${}_1Q_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} + {}_1W_2$

$= 2 \times 1097.31 - 1.0 \times 324.22 - 1.0 \times 1255.14 + 23.72$

$=$ **638.98 Btu**

3.286E

Two rigid tanks are filled with water as shown in Fig. P.3.214. Tank A is 7 ft^3 at 1 atm, 280 F and tank B is 11 ft^3 at saturated vapor 40 psia. The tanks are connected by a pipe with a closed valve. We open the valve and let all the water come to a single uniform state while we transfer enough heat to have a final pressure of 40 psia. Give the two property values that determine the final state and find the heat transfer.

Solution:

State A1: u = 1102.4 Btu/lbm, v = 29.687 ft^3/lbm

$\qquad$ => m_{A1} = V/v = 7.0/29.687 = 0.236 lbm

State B1: u = 1092.3 Btu/lbm, v = 19.501 ft^3/lbm

$\qquad$ => m_{B1} = V/v = 11.0 / 19.501 = 0.564 lbm

The total volume (and mass) is the sum of volumes (mass) for tanks A and B.

$\qquad m_2 = m_{A1} + m_{B1}$ = 0.236 + 0.564 = 0.800 lbm,

$\qquad V_2 = V_{A1} + V_{B1}$ = 7.0 + 11.0 = 18.0 ft^3

$\qquad\qquad$ => $v_2 = V_2/m_2$ = 18.0 /0.800 = **22.5 ft^3/lbm**

State 2: [P_2, v_2] = [40 psia, 22.5 ft^3/lbm]

$\qquad\qquad$ => T_2 = 1053.5 F and u_2 = 1395.1 Btu/lbm

The energy equation is (neglecting kinetic and potential energy)

$\qquad m_2\, u_2 - m_A u_{A1} - m_B u_{B1} = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

$\qquad {}_1Q_2$ = 0.800 × 1395.1 – 0.236 × 1102.4 – 0.564 × 1092.3

$\qquad$ = **239.9 Btu**

3.287E

A vertical cylinder fitted with a piston contains 10 lbm of R-410A at 50 F, shown in Fig. P3.173. Heat is transferred to the system causing the piston to rise until it reaches a set of stops at which point the volume has doubled. Additional heat is transferred until the temperature inside reaches 120 F, at which point the pressure inside the cylinder is 200 lbf/in.^{2}.

a.   What is the quality at the initial state?

b.   Calculate the heat transfer for the overall process.

Solution:

C.V. R-410A. Control mass goes through process: 1 -> 2 -> 3

As piston floats pressure is constant (1 -> 2) and the volume is constant for the second part (2 -> 3).   So we have: $v_3 = v_2 = 2 \times v_1$

State 3: Table F.9.2 (P,T) $v_3 = 0.3652$ ft^3/lbm, $u_3 = 123.5$ Btu/lbm

So we can determine state 1 and 2 Table F.9.1:

$$v_1 = 0.1826 = 0.01420 + x_1(0.3636) \Rightarrow x_1 = \mathbf{0.463}$$

$$u_1 = 31.06 + 0.463 \times 78.96 = 67.6 \text{ Btu/lbm}$$

State 2: $v_2 = 0.3652$ ft^3/lbm, $P_2 = P_1 = 157.5$ psia,   this is still 2-phase.

We get the work from the process equation (see P-V diagram)

$$_1W_3 = {_1}W_2 = \int_1^2 PdV = P_1(V_2 - V_1)$$

$$= 157.5 \text{ psia} \times 10 \text{ lbm } (0.3652 - 0.1826) \text{ ft}^3/\text{lbm} \times 144 \text{ (in/ft)}^2$$

$$= 41\ 413.7 \text{ lbf-ft} = 53.2 \text{ Btu}$$

The heat transfer from the energy equation becomes

$$_1Q_3 = m(u_3 - u_1) + {_1}W_3 = 10(123.5 - 67.6) + 53.2 = \mathbf{612.2 \text{ Btu}}$$

3.288E

A piston cylinder shown in Fig. P3.169 contains 18 ft^3 of R-410A at 300 psia, 300 F. The piston mass and atmosphere gives a pressure of 70 psia that will float the piston. The whole setup cools in a freezer maintained at 0 F. Find the heat transfer and show the P-v diagram for the process when $T_2 = 0$ F.

Solution:

C.V.: R-410A. Control mass.

Continuity: m = constant,

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $F\downarrow = F\uparrow = P\ A = P_{air}A + F_{stop}$

if $V < V_{stop} \Rightarrow F_{stop} = \emptyset$

This is illustrated in the P-v diagram shown below.

Po

R-410A

State 1: $v_1 = 0.3460$ ft^3/lbm, $u_1 = 159.95$ Btu/lbm

$\Rightarrow$ m = V/v = 52.023 lbm

State 2: T_2 and on line $\Rightarrow$ compressed liquid, see figure below.

$v_2 \cong v_f = 0.01295$ ft^3/lbm $\Rightarrow V_2 = 0.674$ ft^3; $u_2 = u_f = 13.37$ Btu/lbm

${}_1W_2 = \int PdV = P_{lift}(V_2 - V_1) = 70$ psi $(0.674 - 18)$ ft^3 $\times 144$ in^2/ft^2

$= -174\ 646$ lbf-ft $= -224.5$ Btu

Energy eq. $\Rightarrow$

${}_1Q_2 = 52.023\ (13.37 - 159.95) - 224.5 =$ **-7850 Btu**

3.289E

A setup as in Fig. P3.169 has the R-410A initially at 150 psia, 120 F of mass 0.2 lbm. The balancing equilibrium pressure is 60 psia and it is now cooled so the volume is reduced to half the starting volume. Find the heat transfer for the process.

Solution:

Take as CV the 0.2 lbm of R-410A.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = P_{float}$ or $v = C = v_1$,

State 1: (P, T) => $v_1 = 0.5099$ ft^3/lbm

$\quad\quad\quad\quad\quad\quad\quad u_1 = 125.28$ Btu/lbm

State 2: (P, v) => $v_2 = v_1/2 = 0.2550$ ft^3/lbm $< v_g$, so it is two-phase.

$\quad\quad\quad x_2 = (v_2 - v_f) /v_{fg} = (0.255 - 0.0129)/0.9911 = 0.2443$

$\quad\quad\quad u_2 = u_f + x_2\, u_{fg} = 12.57 + x_2\, 93.33 = 35.37$ Btu/lbm

From process eq.: ${}_1W_2 = \int P\, dV = \text{area} = mP_2\, (v_2 - v_1)$

$\quad\quad\quad\quad = 0.2$ lbm $\times 60$ psi $(0.255 - 0.5099)$ ft^3/lbm $\times 144$ in^2/ft^2

$\quad\quad\quad\quad = 440.5$ lbf-ft $= - 0.57$ Btu

From energy eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.2\times (35.37 - 125.28) - 0.57$

$\quad\quad\quad\quad = \textbf{-18.55 Btu}$

3.290E

A piston cylinder contains air at 150 psia, 1400 R with a volume of 1.75 ft^3. The piston is pressed against the upper stops, see Fig. P3.14-c and it will float at a pressure of 110 psia. Now the air is cooled to 700 R. What is the process work and heat transfer?

CV. Air, this is a control mass

Energy Eq.3.5 $\qquad$ $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $\qquad$ $P = P_{float}$ $\quad$ or $\quad$ $v = C = v_1$,

State 1: $\quad$ u = 247.04 Btu/lbm,

$\qquad$ m = PV/RT = 150 psi × 1.75 ft^3/(53.34 lbf-ft/lbm-R × 1400 R) = 0.506 lbm

We need to find state 2. Let us see if we proceed past state 1a during the cooling.

$\qquad$ $T_{1a} = T_1\, P_{float}\, /\, P_1 = 1400\ R × 110\, /\, 150 = 1026.67\ R$

so we do cool below T_{1a}. That means the piston is floating. Write the ideal gas law for state 1 and 2 to get

$$V_2 = \frac{mRT_2}{P_2} = \frac{P_1 V_1 T_2}{P_2 T_1} = \frac{150 × 1.75 × 700}{110 × 1400} = 1.1932\ \text{ft}^3$$

$$ {}_1W_2 = {}_{1a}W_2 = \int P\ dV = P_2\,(V_2 - V_1) $$

$\qquad$ = 110 psia × (1.1932 – 1.75) ft^3 = **–8819.7 lbf-ft = –11.34 Btu**

From the energy equation

$\qquad$ $ {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 $

$\qquad\qquad$ = 0.506 lbm × (119.7 – 247.04) Btu/lbm – 11.34 Btu

$\qquad\qquad$ = **-75.8 kJ**

3.291E

A mass of 6 lbm nitrogen gas at 3600 R, V = C, cools with 1 Btu/s. What is dT/dt?

Process: V = C   → $_1W_2 = 0$

$$\frac{dE}{dt} = \frac{dU}{dt} = m\frac{dU}{dt} = mC_v\frac{dT}{dt} = \dot{Q} - W = \dot{Q} = \text{-1 Btu/s}$$

$$C_{p\ 3600} = \frac{dh}{dT} = \frac{\Delta h}{\Delta T} = \frac{h_{3800} - h_{3400}}{3800 - 3400} = \frac{25857 - 22421}{400 \times 28.013} = 0.3066 \text{ Btu/lbm-R}$$

$$C_{v\ 3600} = C_{p\ 3600} - R = 0.3066 - 55.15 / 778 = 0.2357 \text{ Btu/lbm-R}$$

$$\frac{dT}{dt} = \frac{\dot{Q}}{mC_v} = \frac{\text{-1 Btu/s}}{6 \times 0.2357 \text{ Btu/R}} = \textbf{-0.71} \frac{\textbf{R}}{\textbf{s}}$$

Remark: Specific heat from Table F.4 has $C_{v\ 300} = 0.178$ Btu/lbm-R which is nearly 25% lower and thus would over-estimate the rate with 25%.

3.292E

A crane use 7000 Btu/h to raise a 200 lbm box 60 ft. How much time does it take?

$$\text{Power} = \dot{W} = F\mathbf{V} = mg\mathbf{V} = mg\frac{L}{t}$$

$$F = mg = 200\,\frac{32.174}{32.174}\,\text{lbf} = 200\,\text{lbf}$$

$$t = \frac{FL}{\dot{W}} = \frac{200\,\text{lbf} \times 60\,\text{ft}}{7000\,\text{Btu/h}} = \frac{200 \times 60 \times 3600}{7000 \times 778.17}\,\text{s}$$

$$= \mathbf{7.9\ s}$$

Recall Eq. on page 9: $\quad 1\,\text{lbf} = 32.174\,\text{lbm ft/s}^2, \quad 1\,\text{Btu} = 778.17\,\text{lbf-ft}\ \ (\text{A.1})$

3.293E

A computer in a closed room of volume 5000 ft^3 dissipates energy at a rate of 10 kW. The room has 100 lbm of wood, 50 lbm of steel and air, with all material at 540 R, 1 atm. Assuming all the mass heats up uniformly how long time will it take to increase the temperature 20 F?

C.V. Air, wood and steel. $m_2 = m_1$; no work

Energy Eq.3.5: $U_2 - U_1 = {}_1Q_2 = \dot{Q}\Delta t$

The total volume is nearly all air, but we can find volume of the solids.

$\quad$ $V_{wood} = m/\rho = 100/44.9 = 2.23$ ft^3 ; $V_{steel} = 50/488 = 0.102$ ft^3

$\quad$ $V_{air} = 5000 - 2.23 - 0.102 = 4997.7$ ft^3

$\quad$ $m_{air} = PV/RT = 14.7 \times 4997.7 \times 144/(53.34 \times 540) = 367.3$ lbm

We do not have a u table for steel or wood so use specific heat.

$\quad$ $\Delta U = [m_{air}\, C_v + m_{wood}\, C_v + m_{steel}\, C_v\,]\, \Delta T$

$\quad\quad\quad = (367.3 \times 0.171 + 100 \times 0.3 + 50 \times 0.11)$ Btu/F $\times$ 20 F

$\quad\quad\quad = 1256.2 + 600 + 110 = 1966$ Btu $= \dot{Q} \times \Delta t = (10/1.055) \times \Delta t$

$\quad$ => $\quad$ $\Delta t = [1966/10]\, 1.055 = 207$ sec = **3.5 minutes**

3.294E

Water is in a piston cylinder maintaining constant P at 330 F, quality 90% with a volume of 4 ft^3. A heater is turned on heating the water with 10 000 Btu/h. What is the elapsed time to vaporize all the liquid?

Solution:

Control volume water.

Continuity Eq.: m_{tot} = constant = m_{vapor} + m_{liq}

  on a rate form: $\dot{m}_{tot}$ = 0 = $\dot{m}_{vapor}$ + $\dot{m}_{liq}$ $\Rightarrow$ $\dot{m}_{liq}$ = -$\dot{m}_{vapor}$

Energy equation: $\dot{U} = \dot{Q} - \dot{W} = \dot{m}_{vapor} u_{fg} = \dot{Q} - P \dot{m}_{vapor} v_{fg}$

Rearrange to solve for $\dot{m}_{vapor}$

$$\dot{m}_{vapor} (u_{fg} + Pv_{fg}) = \dot{m}_{vapor} h_{fg} = \dot{Q}$$

From table F.7.1

  h_{fg} = 887.5 Bt/lbm, v_1 = 0.01776 + 0.9 4.2938 = 3.8822 ft^3/lbm

  $m_1 = V_1/v_1$ = 4/3.8822 = 1.0303 lbm, $m_{liq} = (1-x_1)m_1$ = 0.10303 lbm

$$\dot{m}_{vapor} = \dot{Q}/h_{fg} = \frac{10\ 000}{887.5} \frac{\text{Btu/h}}{\text{Btu/lbm}} = 11.2676 \text{ lbm/h} = 0.00313 \text{ lbm/s}$$

$$\Delta t = m_{liq} / \dot{m}_{vapor} = 0.10303 / 0.00313 = \textbf{32.9 s}$$

3.295E

A piston/cylinder has 2 lbm of R-134a at state 1 with 200 F, 90 lbf/in.^{2}, and is then brought to saturated vapor, state 2, by cooling while the piston is locked with a pin. Now the piston is balanced with an additional constant force and the pin is removed. The cooling continues to a state 3 where the R-134a is saturated liquid. Show the processes in a P-V diagram and find the work and heat transfer in each of the two steps, 1 to 2 and 2 to 3.

Solution :

C.V. R-134a   This is a control mass. Properties from table F.10.1 and F.10.2

State 1: (T,P)   =>   v = 0.7239 ft^3/lbm, u = 194.605 Btu/lbm

State 2 given by fixed volume and $x_2 = 1.0$, $v_2 = v_1 = v_g$ => $_1W_2 = 0$

$$T_2 = 50 + 10 \times \frac{0.7239 - 0.7921}{0.6632 - 0.7921} = 55.3 \text{ F},$$

$$P_2 = 60.311 + (72.271 - 60.311) \times 0.5291 = 66.64 \text{ psia}$$

$$u_2 = 164.95 + (166.28 - 164.95) \times 0.5291 = 165.65 \text{ Btu/lbm}$$

From the energy equation $_1Q_2 = m(u_2 - u_1) + _1W_2 = m(u_2 - u_1)$

$$= 2 \ (165.65 - 194.605) = \textbf{-57.91 Btu}$$

State 3 reached at constant P (F = constant) state 3: $P_3 = P_2$ and

$$v_3 = v_f = 0.01271 + (0.01291 - 0.01271) \times 0.5291 = 0.01282 \text{ ft}^3\text{/lbm}$$

$$u_3 = u_f = 91.68 + (94.95 - 91.68) \times 0.5291 = 93.41 \text{ Btu/lbm}$$

$$_1W_3 = _1W_2 + _2W_3 = 0 + _2W_3 = \int P \ dV = \ P(V_3 - V_2) = mP(v_3 - v_2)$$

$$= 2 \times 66.64 \ (0.01282 - 0.7239) \frac{144}{778} = - \ \textbf{17.54 Btu}$$

From the energy equation

$$_2Q_3 = m(u_3 - u_2) + _2W_3 = 2(93.41 - 165.65) - 17.54 = \textbf{–162.02 Btu}$$

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Rates of Work

3.296E

A force of 300 lbf moves a truck with 40 mi/h up a hill. What is the power?

Solution:

$$\dot{W} = F\ \mathbf{V} = 300\ \text{lbf} \times 40\ (\text{mi/h})$$

$$= 12\ 000 \times \frac{1609.3 \times 3.28084}{3600}\ \frac{\text{lbf-ft}}{\text{s}}$$

$$= 17\ 600\ \frac{\text{lbf-ft}}{\text{s}} = \textbf{22.62 Btu/s}$$

Heat Transfer Rates

3.297E

Find the rate of conduction heat transfer through a 1.5 cm thick hardwood board, k = 0.09 Btu/h-ft-R, with a temperature difference between the two sides of 40 F.

One dimensional heat transfer by conduction, we do not know the area so we can find the flux (heat transfer per unit area Btu/ft²h).

$$t = 1.5 \text{ cm} = 0.59 \text{ in.} = 0.0492 \text{ ft}$$

$$\dot{q} = \dot{Q}/A = k\ \frac{\Delta T}{\Delta x}\ = 0.09\ \frac{\text{Btu}}{\text{h-ft-R}} \times \frac{40}{0.0492}\ \frac{\text{R}}{\text{ft}}\ = \textbf{73.2 Btu/ft}^2\textbf{-h}$$

3.298E

A water-heater is covered up with insulation boards over a total surface area of 30 ft^2. The inside board surface is at 175 F and the outside surface is at 70 F and the board material has a conductivity of 0.05 Btu/h ft F. How thick a board should it be to limit the heat transfer loss to 720 Btu/h ?

Solution:

Steady state conduction through a single layer board.

$$\dot{Q}_{cond} = k \; A \; \frac{\Delta T}{\Delta x} \quad \Rightarrow \quad \Delta x = k \; A \; \Delta T / \dot{Q}$$

$$\Delta x = 0.05 \; \frac{Btu}{h\text{-}ft\text{-}R} \times 30 \; ft^2 \; (175\text{-}70) \; R / \; 720 \; Btu/h$$

$$= 0.219 \; ft = \textbf{2.6 in}$$

3.299E

The sun shines on a 1500 ft^2 road surface so it is at 115 F. Below the 2 inch thick asphalt, average conductivity of 0.035 Btu/h-ft-F, is a layer of compacted rubbles at a temperature of 60 F. Find the rate of heat transfer to the rubbles.

Solution:

$$\dot{Q} = k \ A \ \frac{\Delta T}{\Delta x}$$

$$= \ 0.035 \ \frac{Btu}{h\text{-}ft\text{-}R} \times 1500 \ \ ft^2 \times \frac{115 - 60}{2/12} \frac{R}{ft}$$

$$= \mathbf{17325 \ Btu/h}$$

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Review Problems

3.300E

A cylinder having an initial volume of 100 ft^3 contains 0.2 lbm of water at 100 F. The water is then compressed in an isothermal quasi-equilibrium process until it has a quality of 50%. Calculate the work done in the process assuming water vapor is an ideal gas.

Solution:

State 1: T_1, $v_1 = V/m = \dfrac{100}{0.2} = 500$ ft^3/lbm   ($> v_g$)

since $P_g = 0.95$ psia, very low so water is an ideal gas from 1 to 2.

$$P_1 = P_g \times \frac{v_g}{v_1} = 0.950 \times \frac{350}{500} = 0.6652 \text{ lbf/in}^2$$

$$V_2 = mv_2 = 0.2 \text{ lbm} * 350 \text{ ft}^3/\text{lbm} = 70 \text{ ft}^3$$

$$v_3 = 0.01613 + 0.5 \times (350 - 0.01613) = 175.0 \text{ ft}^3/\text{lbm}$$

For ideal gas and constant T the work term follows Eq. 3.21

$$_1W_2 = \int PdV = P_1V_1 \ln \frac{V_2}{V_1} = 0.6652 \times \frac{144}{778} \times 100 \ln \frac{70}{100} = -4.33 \text{ Btu}$$

For the constant pressure part of the process the work becomes

$$_2W_3 = P_2 \, m(v_3 - v_2) = 0.95 \text{ psi} \times 0.2 \text{ lbm} \times (175 - 350) \text{ ft}^3/\text{lbm} \times 144 \text{ in}^2/\text{ft}^2$$

$$= -4788 \text{ lbf-ft} = -6.15 \text{ Btu}$$

$$_1W_3 = -6.15 - 4.33 = -\textbf{10.48 Btu}$$

3.301E

A piston/cylinder contains 2 lbm of water at 70 F with a volume of 0.1 ft^3, shown in Fig. P3.225. Initially the piston rests on some stops with the top surface open to the atmosphere, Po, so a pressure of 40 lbf/in.^{2} is required to lift it. To what temperature should the water be heated to lift the piston? If it is heated to saturated vapor find the final temperature, volume, and the heat transfer.

Solution:

C.V. Water. This is a control mass.

$$m_2 = m_1 = m ; \quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

State 1: 20 C, $v_1 = V/m = 0.1/2 = 0.05$ ft^3/lbm

$x = (0.05 - 0.01605)/867.579 = 0.0003913$

$u_1 = 38.09 + 0.0003913 \times 995.64 = 38.13$ Btu/lbm

To find state 2 check on state 1a:

$P = 40$ psia, $v = v_1 = 0.05$ ft^3/lbm

Table F.7.1: $v_f < v < v_g = 10.501$, $x_{1a} > 0$

State 2 is saturated vapor at 40 psia as state 1a is two-phase. $T_2 = 267.3$ F

$v_2 = vg = 10.501$ ft^3/lbm , $V_2 = m\,v_2 = 21.0$ ft^3, $u_2 = ug = 1092.27$ Btu/lbm

Pressure is constant as volume increase beyond initial volume.

$${}_1W_2 = \int P\, dV = P_{lift} (V_2 - V_1) = 40\,(21.0 - 0.1) \times 144 / 778 = 154.75 \text{ Btu}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 2\,(1092.27 - 38.13) + 154.75 = 2263 \text{ Btu}$$

3.302E

A twenty pound-mass of water in a piston/cylinder with constant pressure is at 1100 F and a volume of 22.6 ft^3. It is now cooled to 100 F. Show the *P–v* diagram and find the work and heat transfer for the process.

Solution:

C.V. Water

Energy Eq.3.5: $\quad\quad {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$

Process Eq.: Constant pressure $\quad \Rightarrow \quad {}_1W_2 = mP(v_2 - v_1)$

Properties from Table F.7.2 and F.7.3

State 1: T_1, $\ v_1 = 22.6/20 = 1.13$ ft^3/lbm, $\ P_1 = 800$ lbf/in^2 , $\ h_1 = 1567.8$ Btu/lbm

State 2: $\ 800$ lbf/in^2, 100 F

$\quad\quad \Rightarrow \quad v_2 = 0.016092$ ft^3/lbm, $\ h_2 = 70.15$ Btu/lbm

The work from the process equation is found as

$$ {}_1W_2 = 20 \text{ lbm} \times 800 \text{ psi} \times (0.016092 - 1.13) \text{ ft}^3/\text{lbm} \times 144 \text{ in}^2/\text{ft}^2 $$

$$ = \text{-2 566 444 lbf-ft} = \textbf{-3299 Btu} $$

The heat transfer from the energy equation is

$$ {}_1Q_2 = 20 \text{ lbm} \times (70.15 - 1567.8) \text{ Btu/lbm} = \textbf{-29 953 Btu} $$

3.303E

A cylinder fitted with a frictionless piston contains R-134a at 100 F, 80% quality, at which point the volume is 3 Gal. The external force on the piston is now varied in such a manner that the R-134a slowly expands in a polytropic process to 50 lbf/in.^{2}, 80 F. Calculate the work and the heat transfer for this process.

Solution:

C.V. The mass of R-134a. Properties in Table F.10.1

$$v_1 = v_f + x_1\ v_{fg} = 0.01387 + 0.8 \times 0.3278 = 0.2761\ \text{ft}^3/\text{lbm}$$

$$u_1 = 108.51 + 0.8 \times 62.77 = 158.73\ \text{Btu/lbm}; \quad P_1 = 138.926\ \text{psia}$$

$$m = V/v_1 = 3 \times 231 \times 12^{-3} / 0.2761 = 0.401/ 0.2761 = 1.4525\ \text{lbm}$$

State 2: $v_2 = 1.1035\ \text{ft}^3/\text{lbm}$ (sup.vap.); $u_2 = 171.32\ \text{Btu/lbm}$

(linear interpolation is not so accurate as v is more like ~1/P)

Process: $n = \ln \dfrac{P_1}{P_2} / \ln \dfrac{V_2}{V_1} = \ln \dfrac{138.926}{50} / \ln \dfrac{1.1035}{0.2761} = 0.7376$

$$_1W_2 = \int P\ dV = \frac{P_2\ V_2 - P_1\ V_1}{1 - n}$$

$$= \frac{50 \times 1.1035 - 138.926 \times 0.2761}{1 - 0.7376} \times 1.4525 \times \frac{144}{778} = \textbf{17.23 Btu}$$

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.4525\ (171.32 - 158.73) + 17.23 = \textbf{35.5 Btu}$$

3.304E

Ammonia, NH_3, is contained in a sealed rigid tank at 30 F, $x = 50\%$ and is then heated to 200 F. Find the final state P_2, u_2 and the specific work and heat transfer.

Solution:

Continuity Eq.: $m_2 = m_1$;

Energy Eq.3.5: $E_2 - E_1 = {}_1Q_2$; $\left({}_1W_2 = 0\right)$

Process: $V_2 = V_1 \Rightarrow v_2 = v_1 = 0.02502 + 0.5 \times 4.7945 = 2.422$ ft^3/lbm

State 1: Table F.8.1, $u_1 = 75.06 + 0.5 \times 491.17 = 320.65$ Btu/lbm

Table F.8.2: v_2 & $T_2 \Rightarrow$ between 150 psia and 175 psia

$P_2 = 163$ lbf/in^2, $u_2 = 633.5$ Btu/lbm

linear interpolation

Process equation gives no displacement: ${}_1w_2 = \mathbf{0}$;

The energy equation then gives the heat transfer as

$${}_1q_2 = u_2 - u_1 = 633.5 - 320.65 = \textbf{312.85 Btu/lbm}$$

3.305E

Water in a piston/cylinder, similar to Fig. P3.225, is at 212 F, x = 0.5 with mass 1 lbm and the piston rests on the stops. The equilibrium pressure that will float the piston is 40 psia. The water is heated to 500 F by an electrical heater. At what temperature would all the liquid be gone? Find the final (P,v), the work and heat transfer in the process.

C.V. The 1 lbm water.

Continuty: $m_2 = m_1 = m$;   Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant if $P < P_{lift}$ otherwise $P = P_{lift}$ see P-v diagram.

State 1: (T,x) Table F.7.1

$v_1 = 0.01672 + 0.5 \times 26.7864 = 13.4099$ ft^3/lbm

$u_1 = 180.09 + 0.5 \times 897.51 = 628.845$ Btu/lbm

State 1a: (40 psia, v = $v_1 > v_{g\ 40\ psia}$ = 10.501 ft^3/lbm) so superheated vapor

Piston starts to move at state 1a, ${}_1W_{1a} = 0$,

State 1b: reached before state 1a so v = $v_1 = v_g$ see this in F.7.1

$T_{1b} = 250 + 10 (13.40992 – 13.8247)/(11.7674 – 13.8247) = $ **252 F**

State 2: ($T_2 > T_{1a}$) Table F.7.2   => $v_2 = 14.164$, $u_2 = 1180.06$ Btu/lbm

Work is seen in the P-V diagram (when volume changes $P = P_{lift}$)

${}_1W_2 = {}_{1a}W_2 = P_2\ m(v_2 - v_1)$

$= 40$ psi $\times 1$ lbm $(14.164 - 13.4099)$ ft^3/lbm $\times 144$ in^2/ft^2

$= 4343.5$ lbf-ft = **5.58 Btu**

Heat transfer is from the energy equation

${}_1Q_2 = 1$ lbm $(1180.06 - 628.845)$ Btu/lbm $+ 5.58$ Btu = **556.8 Btu**

3.306E
An insulated cylinder is divided into two parts of 10 ft³ each by an initially locked piston. Side A has air at 2 atm, 600 R and side B has air at 10 atm, 2000 R as shown in Fig. P3.151. The piston is now unlocked so it is free to move, and it conducts heat so the air comes to a uniform temperature $T_A = T_B$. Find the mass in both A and B and also the final T and P.

C.V. A + B . Then $_1Q_2 = \emptyset$, $_1W_2 = \emptyset$.

Force balance on piston: $P_A A = P_B A$,   so final state in A and B is the same.

State 1A: $u_{A1} = 102.457$ Btu/lbm;

$$m_A = \frac{PV}{RT} = \frac{29.4 \text{ psi} \times 10 \text{ ft}^3 \times 144 \text{ in}^2/\text{ft}^2}{53.34 \text{ lbf-ft/lbm-R} \times 600 \text{ R}} = \textbf{1.323 lbm}$$

State 1B: $u_{B1} = 367.642$ Btu/lbm ;

$$m_B = \frac{PV}{RT} = \frac{147 \text{ psi} \times 10 \text{ ft}^3 \times 144 \text{ in}^2/\text{ft}^2}{53.34 \text{ lbf-ft/lbm-R} \times 2000 \text{ R}} = \textbf{1.984 lbm}$$

For chosen C.V. $_1Q_2 = 0$, $_1W_2 = 0$ so the energy equation becomes

$$m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0$$

$$(m_A + m_B)u_2 = m_A u_{A1} + m_B u_{B1}$$

$$= 1.323 \times 102.457 + 1.984 \times 367.642 = 864.95 \text{ Btu}$$

$$u_2 = 864.95 \text{ Btu}/3.307 \text{ lbm} = 261.55 \text{ Btu/lbm} \quad \Rightarrow \quad T_2 = \textbf{1475 R}$$

$$P = m_{tot}RT_2/V_{tot} = \frac{3.307 \text{ lbm} \times 53.34 \text{ lbf-ft/lbm-R} \times 1475 \text{ R}}{20 \text{ ft}^3 \times 144 \text{ in}^2/\text{ft}^2}$$

$$= \textbf{90.34 lbf/in}^2$$

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 4

8e

Updated June 2013

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CONTENT CHAPTER 4

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In-Text Concept Questions

4.a

A mass flow rate into a control volume requires a normal velocity component. Why?

The tangential velocity component does not bring any substance across the control volume surface as it flows parallel to it, the normal component of velocity brings substance in or out of the control volume according to its sign. The normal component must be into the control volume to bring mass in, just like when you enter a bus (it does not help that you run parallel with the bus side).

V

$V_{tangential}$

V_{normal}

4.b

Can a steady state device have boundary work?

No. Any change in size of the control volume would require either a change in mass inside or a change in state inside, neither of which is possible in a steady-state process.

4.c

Can you say something about changes in $\dot{m}$ and $\dot{V}$ through a steady flow device?

The continuity equation expresses the conservation of mass, so the total amount of $\dot{m}$ entering must be equal to the total amount leaving. For a single flow device the mass flow rate is constant through it, so you have the same mass flow rate across any total cross-section of the device from the inlet to the exit.

The volume flow rate is related to the mass flow rate as

$$\dot{V} = v\,\dot{m}$$

so it can vary if the state changes (then v changes) for a constant mass flow rate.

This also means that the velocity can change (influenced by the area as $\dot{V} = \mathbf{V}A$) and the flow can experience an acceleration (like in a nozzle) or a deceleration (as in a diffuser).

4.d

In a multiple device flow system, I want to determine a state property. Where should I be looking for information—upstream or downstream?

Generally flow is affected more by what happened to it which is upstream than what is in front of it. Only the pressure information can travel upstream and give rise to accelerations (nozzle) or decelerations (stagnation type flow). Heat transfer that can heat or cool a flow can not travel very fast and is easily overpowered by the convection. If the flow velocity exceeds the speed of sound even the pressure information can not travel upstream.

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4.e

How does a nozzle or sprayhead generate kinetic energy?

By accelerating the fluid from a high pressure towards the lower pressure, which is outside the nozzle. The higher pressure pushes harder than the lower pressure so there is a net force on any mass element to accelerate it.

4.f

What is the difference between a nozzle flow and a throttle process?

In both processes a flow moves from a higher to a lower pressure. In the nozzle the pressure drop generates kinetic energy, whereas that does not take place in the throttle process. The pressure drop in the throttle is due to a flow restriction and represents a loss.

4.g

If you throttle a saturated liquid what happens to the fluid state? What if this is done to an ideal gas?

The throttle process is approximated as a constant enthalpy process. Changing the state from saturated liquid to a lower pressure with the same h gives a two-phase state so some of the liquid will vaporize and it becomes colder.

If the same process happens in an ideal gas then same h gives the same temperature (h a function of T only) at the lower pressure.

4.h

A turbine at the bottom of a dam has a flow of liquid water through it. How does that produce power? Which terms in the energy equation are important if the CV is the turbine only? If the CV is the turbine plus the upstream flow up to the top of the lake, which terms in the energy equation are then important?

The water at the bottom of the dam in the turbine inlet is at a high pressure. It runs through a nozzle generating kinetic energy as the pressure drops. This high kinetic energy flow impacts a set of rotating blades or buckets, which converts the kinetic energy to power on the shaft and the flow leaves at low pressure and low velocity.

CV Turbine only.

The high P in and the low P out shows up in the $(h = u + Pv)$ flow terms of the energy equation giving the difference in the flow work terms Pv in and out.

CV Turbine plus upstream flow.

For this CV the pressures in and out are the same (1 atm) so the difference is in the potential energy terms (gz) included in h_{tot}.

4.i

If you compress air the temperature goes up, why? When the hot air, high P flows in long pipes it eventually cools to ambient T. How does that change the flow?

As the air is compressed, volume decreases so work is done on a mass element, its energy and hence temperature goes up. If it flows at nearly constant P and cools, its density increases (v decreases) so it slows down for same mass flow rate ($\dot{m} = \rho A \mathbf{V}$) and flow area.

4.j

A mixing chamber has all flows at the same P, neglecting losses. A heat exchanger has separate flows exchanging energy, but they do not mix. Why have both kinds?

You might allow mixing when you can use the resulting output mixture, say it is the same substance. You may also allow it if you definitely want the outgoing mixture, like water out of a faucet where you mix hot and cold water. Even if it is different substances it may be desirable, say you add water to dry air to make it more moist, typical for a winter time air-conditioning set-up.

In other cases it is different substances that flow at different pressures with one flow heating or cooling the other flow. This could be hot combustion gases heating a flow of water or a primary fluid flow around a nuclear reactor heating a transfer fluid flow. Here the fluid being heated should stay pure so it does not absorb gases or radioactive particles and becomes contaminated. Even when the two flows have the same substance there may be a reason to keep them at separate pressures.

An open mixing chamber   A closed tube in shell heat exchanger.

4.k

An initially empty cylinder is filled with air from 20°C, 100 kPa until it is full. Assuming no heat transfer is the final temperature larger, equal to or smaller than 20°C? Does the final T depend on the size of the cylinder?

This is a transient problem with no heat transfer and no work. The balance equations for the tank as C.V. become

Continuity Eq.: $\quad m_2 - 0 = m_i$

Energy Eq.: $\quad m_2 u_2 - 0 = m_i h_i + Q - W = m_i h_i + 0 - 0$

Final state: $\quad u_2 = h_i = u_i + P_i v_i \quad \& \quad P_2 = P_i$

$T_2 > T_i$ and it does not depend on V

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Concept-Study Guide Problems

4.1

A temperature difference drives a heat transfer. Does a similar concept apply to $\dot{m}$?

Yes.   A pressure difference drives the flow. The fluid is accelerated in the direction of a lower pressure as it is being pushed harder behind it than in front of it. This also means a higher pressure in front can decelerate the flow to a lower velocity which happens at a stagnation point on a wall.

$F_1 = P_1 A$ $F_2 = P_2 A$

4.2

What kind of effect can be felt upstream in a flow?

Only the pressure can be felt upstream in a subsonic flow. In a supersonic flow no information can travel upstream. The temperature information travels by conduction and even small velocities overpowers the conduction with the convection of energy so the temperature at a given location is mainly given by the upstream conditions and influenced very little by the downstream conditions.

4.3

Which one of the properties (P, v, T) can be controlled in a flow? How?

Since the flow is not contained there is no direct control over the volume and thus no control of v. The pressure can be controlled by installation of a pump or compressor if you want to increase it or use a turbine, nozzle or valve through which the pressure will decrease. The temperature can be controlled by heating or cooling the flow in a heat exchanger.

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4.4

Air at 500 kPa is expanded to 100 kPa in two steady flow cases. Case one is a nozzle and case two is a turbine, the exit state is the same for both cases. What can you say about the specific turbine work relative to the specific kinetic energy in the exit flow of the nozzle?

For these single flow devices let us assume they are adiabatic and that the turbine does not have any exit kinetic energy then the energy equations become:

Energy Eq.4.13 nozzle: $\quad h_1 + \frac{1}{2}V_1^2 + gZ_1 = h_2 + \frac{1}{2}V_2^2 + gZ_2$

Energy Eq.4.13 turbine: $\quad h_1 + \frac{1}{2}V_1^2 + gZ_1 = h_2 + gZ_2 + w_T$

Comparing the two we get $\frac{1}{2}V_2^2 = w_T$ so the result is that the nozzle delivers kinetic energy of the same amount as the turbine delivers shaft work.

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4.5

Pipes that carry a hot fluid like steam in a power plant, exhaust pipe for a diesel engine in a ship etc. are often insulated. Is that to reduce the heat loss or is there another purpose?

You definitely want to insulate pipes that carry hot steam from the boiler to the turbines in a power plant and pipes that flows hot water from one location to another. Even if the energy in the flow is unwanted the pipes should be insulated for safety. Any place that people could touch a hot surface (or very cold surface) there is a risk of a burn and that should be avoided.

4.6

A windmill takes a fraction of the wind kinetic energy out as power on a shaft. In what manner does the temperature and wind velocity influence the power? Hint: write the power as mass flow rate times specific work.

The work as a fraction f of the flow of kinetic energy becomes

$$\dot{W} = \dot{m}w = \dot{m}\, f\, \tfrac{1}{2} \mathbf{V}_{in}^2 = \rho A \mathbf{V}_{in}\, f\, \tfrac{1}{2} \mathbf{V}_{in}^2$$

so the power is proportional to the velocity cubed. The temperature enters through the density, so assuming air is ideal gas

$$\rho = 1/v = P/RT$$

and the power is inversely proportional to temperature.

A windmill farm west of Denmark in the North Sea.

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4.7

An underwater turbine extracts a fraction of the kinetic energy from the ocean current. In what manner does the temperature and water velocity influence the power? Hint: write the power as mass flow rate times specific work.

The work as a fraction f of the flow of kinetic energy becomes

$$\dot{W} = \dot{m}w = \dot{m}\, f\, \frac{1}{2}\, \mathbf{V}_{in}^2 = \rho A \mathbf{V}_{in}\, f\, \frac{1}{2}\, \mathbf{V}_{in}^2$$

so the power is proportional to the velocity cubed. The temperature enters through the density, so assuming water is incompressible density is constant and the power does not vary with the temperature.

A proposed underwater tidal flow turbine farm. Each turbine is for 1 MW with a diameter of 11.5 m mounted on the seafloor.

4.8

A liquid water turbine in the bottom of a dam takes energy out as power on a shaft. Which term(s) in the energy equation are changing and important?

The water at the bottom of the dam in the turbine inlet is at a high pressure. In a standard turbine it runs through blade passages like a propeller. In this case the inlet high pressure is used directly to generate the force on the moving blades.

For a Pelton turbine the water runs through a nozzle generating kinetic energy as the pressure drops. The high kinetic energy flow impacts a set of rotating buckets and converts the flow kinetic energy to power on the shaft so the flow leaves at low pressure and low velocity.

From Environmental Energy Technologies Division of the Lawrence Berkeley National Laboratory, and the U.S. Department of Energy

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4.9

You blow a balloon up with air. What kind of work terms including flow work do you see in that case? Where is the energy stored?

As the balloon is blown up mass flow in has flow work associated with it. Also as the balloon grows there is a boundary work done by the inside gas and a smaller boundary work from the outside of the balloon to the atmosphere. The difference between the latter two work terms goes into stretching the balloon material and thus becomes internal energy (or you may call that potential energy) of the balloon material. The work term to the atmosphere is stored in the atmosphere and the part of the flow work that stays in the gas is stored as the gas internal energy.

4.10

A storage tank for natural gas (NG) has a top dome that can move up or down as gas is added or subtracted from the tank maintaining 110 kPa, 290 K inside. A pipeline at 110 kPa, 290 K now supplies some NG to the tank. Does it change state during the filling process? What happens to the flow work?

As the pressure inside the storage tank is the same as in the pipeline the state does not change. However the tank volume goes up and work is done on the moving boundary at the 110 kPa so this work equals the flow work. The net effect is the flow work goes into raising the dome.

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Continuity equation and flow rates

4.11

A large brewery has a pipe of cross sectional area a 0.2 m^2 flowing carbon dioxide at 400 kPa, 10°C with a volume flow rate of 0.3 m^3/s. Find the velocity and the mass flow rate.

$$\dot{m} = A\mathbf{V}/v = \dot{V}/v$$

$$v = RT/P = 0.1889 \text{ kJ/kg-K} \times 283.15 \text{ K}/ 400 \text{ kPa} = 0.1337 \text{ m}^3/\text{kg}$$

$$\dot{m} = \dot{V}/ v = (0.3 \text{ m}^3/\text{s}) \ / (0.1337 \text{ m}^3/\text{kg}) = \textbf{2.24 kg/s}$$

$$\mathbf{V} = \dot{V} / A = 0.1337 \text{ (m}^3/\text{s}) / 0.2 \text{ m}^2 = \textbf{0.67 m/s}$$

4.12

Air at 35°C, 105 kPa, flows in a 100 mm × 150 mm rectangular duct in a heating system. The mass flow rate is 0.015 kg/s. What are the velocity of the air flowing in the duct and the volume flow rate?

Solution:

Assume a constant velocity across the duct area with

$$A = 100 \times 150 \times 10^{-6} \ m^2 = 0.015 \ m^2$$

Ideal gas so:

$$v = \frac{RT}{P} = \frac{0.287 \ kJ/kg\text{-}K \times 308.2 \ K}{105 \ kPa} = 0.8424 \ m^3/kg$$

and the volumetric flow rate from Eq. 4.3,

$$\dot{V} = \dot{m}v = A\mathbf{V} = 0.015 \ kg/s \times 0.8424 \ m^3/kg = \mathbf{0.01264 \ m^3/s}$$

$$\mathbf{V} = \frac{\dot{V}}{A} = \frac{0.01264 \ m^3/s}{0.015 \ m^2} = \mathbf{0.84 \ m/s}$$

4.13

A pool is to be filled with 60 m^3 water from a garden hose of 2.5 cm diameter flowing water at 2 m/s. Find the mass flow rate of water and the time it takes to fill the pool.

Solution:

With constant velocity we have

$$\dot{V} = \dot{m}v = A\mathbf{V} = \pi \times (0.025 \text{ m})^2 \times 2 \text{ m/s} = 0.003927 \text{ m}^3/\text{s}$$

$$\Delta t = V / \dot{V} = 60 \text{ m}^3 / (0.003927 \text{ m}^3/\text{s}) = 15279 \text{ s} = \textbf{4 h 14 min 39 s}$$

From table A.3 we get the water density

$$\dot{m} = \dot{V}/ v = \rho\dot{V} = 997 \text{ kg/ m}^3 \times 0.003927 \text{ m}^3/\text{s} = \textbf{3.9 kg/s}$$

4.14

An empty bathtub has its drain closed and is being filled with water from the faucet at a rate of 10 kg/min. After 10 minutes the drain is opened and 4 kg/min flows out and at the same time the inlet flow is reduced to 2 kg/min. Plot the mass of the water in the bathtub versus time and determine the time from the very beginning when the tub will be empty.

Solution:

During the first 10 minutes we have

$$\frac{dm_{cv}}{dt} = \dot{m}_i = 10 \text{ kg/min}, \qquad \Delta m = \dot{m} \, \Delta t_1 = 10 \times 10 = 100 \text{ kg}$$

So we end up with 100 kg after 10 min. For the remaining period we have

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e = 2 - 4 = -2 \text{ kg/min}$$

$$\Delta m_2 = \dot{m}_{net} \, \Delta t_2 \;\rightarrow\; \Delta t_2 = \frac{\Delta m}{\dot{m}_{net}} = -100/-2 = 50 \text{ min.}$$

So it will take an additional 50 min. to empty

$$\Delta t_{tot} = \Delta t_1 + \Delta t_2 = 10 + 50 = \textbf{60 min.}$$

4.15

A flat channel of depth 1 m has a fully developed laminar flow of air at P_o, T_o with a velocity profile as: $V = 4V_c \, x(H - x)/H^2$, where V_c is the velocity on the center-line and x is the distance across the channel as shown in Fig. P4.15. Find the total mass flow rate and the average velocity both as functions of V_c and H.

$$\dot{m} = AV/v = \dot{V}/v$$

Since the velocity is distributed we need to integrate over the area. From Eq.4.2

$$\dot{V} = \int V_{local} \, dA = \int V(x) \, W \, dx$$

where W is the depth. Substituting the velocity we get

$$\dot{V} = \int 4V_c \, (x/H)(1 - x/H) \, W \, dx$$

$$= 4 \, V_c \, WH \int_0^1 z \, (1 - z) \, dz \qquad (z = x/H)$$

$$= 4 \, V_c \, WH \, (\tfrac{1}{2} z^2 - \tfrac{1}{3} z^3 \,)\Big|_0^1 = \tfrac{2}{3} \, V_c \, WH = \tfrac{2}{3} \, V_c \, A$$

Average velocity: $\qquad\qquad V = \dot{V}/A = \dfrac{2}{3} V_c$

Mass flow rate: $\qquad\qquad \dot{m} = \dot{V}/v = \dfrac{2}{3} V_c \, WH/v$

4.16

Nitrogen gas flowing in a 50-mm diameter pipe at 15°C, 200 kPa, at the rate of 0.05 kg/s, encounters a partially closed valve. If there is a pressure drop of 30 kPa across the valve and essentially no temperature change, what are the velocities upstream and downstream of the valve?

Solution:

Same inlet and exit area: $\qquad A = \dfrac{\pi}{4}(0.050)^2 = 0.001963\ \text{m}^2$

Ideal gas: $\quad v_i = \dfrac{RT_i}{P_i} = \dfrac{0.2968\ \text{kJ/kg-K} \times 288.2\ \text{K}}{200\ \text{kPa}} = 0.4277\ \text{m}^3/\text{kg}$

From Eq.6.3,

$$V_i = \frac{\dot{m}v_i}{A} = \frac{0.05\ \text{kg/s} \times 0.4277\ \text{m}^3/\text{kg}}{0.001963\ \text{m}^2} = \textbf{10.9 m/s}$$

Ideal gas: $\quad v_e = \dfrac{RT_e}{P_e} = \dfrac{0.2968\ \text{kJ/kg-K} \times 288.2\ \text{K}}{170\ \text{kPa}} = 0.5032\ \text{m}^3/\text{kg}$

$$V_e = \frac{\dot{m}v_e}{A} = \frac{0.05\ \text{kg/s} \times 0.5032\ \text{m}^3/\text{kg}}{0.001963\ \text{m}^2} = \textbf{12.8 m/s}$$

4.17

A boiler receives a constant flow of 5000 kg/h liquid water at 5 MPa, 20°C and it heats the flow such that the exit state is 450°C with a pressure of 4.5 MPa. Determine the necessary minimum pipe flow area in both the inlet and exit pipe(s) if there should be no velocities larger than 20 m/s.

Solution:

Mass flow rate from Eq. 4.3, both $\mathbf{V} \leq 20$ m/s

$$\dot{m}_i = \dot{m}_e = (A\mathbf{V}/v)_i = (A\mathbf{V}/v)_e = 5000 \; \frac{1}{3600} \; \text{kg/s}$$

Table B.1.4 $v_i = 0.001$ m^3/kg,

Table B.1.3 $v_e = (0.08003 + 0.0633)/2 = 0.07166$ m^3/kg,

$$A_i \geq v_i \, \dot{m}/\mathbf{V}_i = 0.001 \; \text{m}^3/\text{kg} \times \frac{5000}{3600} \; \text{kg/s} \, / \, 20 \; \text{m/s}$$

$$= 6.94 \times 10^{-5} \; \text{m}^2 = \textbf{0.69 cm}^2$$

$$A_e \geq v_e \, \dot{m}/\mathbf{V}_e = 0.07166 \; \text{m}^3/\text{kg} \times \frac{5000}{3600} \; \text{kg/s} \, / \, 20 \; \text{m/s}$$

$$= 4.98 \times 10^{-3} \; \text{m}^2 = \textbf{50 cm}^2$$

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4.18

A 0.6 m diameter household fan takes air in at 98 kPa, 20$^\text{o}$C and delivers it at 105 kPa, 21$^\text{o}$C with a velocity of 1.5 m/s. What are the mass flow rate (kg/s), the inlet velocity and the outgoing volume flow rate in m^3/s?
Solution:

Continuity Eq. $\dot{m}_i = \dot{m}_e = A\mathbf{V}/v$

Ideal gas $v = RT/P$

Area : $A = \dfrac{\pi}{4} D^2 = \dfrac{\pi}{4} \times 0.6^2 = 0.2827 \ m^2$

$\dot{V}_e = A\mathbf{V}_e = 0.2827 \times 1.5 = \mathbf{0.4241 \ m^3/s}$

$v_e = \dfrac{RT_e}{P_e} = \dfrac{0.287 \times (21 + 273)}{105} = 0.8036 \ m^3/kg$

$\dot{m}_i = \dot{V}_e / v_e = 0.4241/0.8036 = \mathbf{0.528 \ kg/s}$

$A\mathbf{V}_i / v_i = \dot{m}_i = A\mathbf{V}_e / v_e$

$\mathbf{V}_i = \mathbf{V}_e \times (v_i/v_e) = \mathbf{V}_e \times \dfrac{RT_i}{P_i v_e} = 1.5 \times \dfrac{0.287 \ kJ/kg\text{-}K \times (20 + 273) \ K}{98 \ kPa \times 0.8036 \ m^3/kg} = \mathbf{1.6 \ m/s}$

4.19

An airport ventilation system takes 2.5 m^3/s air at 100 kPa, 17^oC into a furnace and heats it to 52^oC and delivers the flow to a duct with cross-sectional area 0.4 m^2 at 110 kPa. Find the mass flow rate and the velocity in the duct?
Solution:

The inflate flow is given by a $\dot{m}_i$

Continuity Eq.: $\dot{m}_i = \dot{V}_i / v_i = \dot{m}_e = A_e \mathbf{V}_e / v_e$

Ideal gas: $v_i = \dfrac{RT_i}{P_i} = \dfrac{0.287 \times 290}{100} = 0.8323 \dfrac{m^3}{kg}$

$$v_e = \dfrac{RT_e}{P_e} = \dfrac{0.287 \times (52 + 273)}{110}$$

$$= 0.8479 \ m^3/kg$$

$$\dot{m}_i = \dot{V}_i/v_i = 2.5 \ (m^3/s) \ /0.8323 \ (m^3/kg) = 3.004 \ kg/s$$

$$\mathbf{V}_e = \dot{m} \ v_e/ A_e = \dfrac{3.004 \times 0.8479}{0.4} \dfrac{m^3/s}{m^2} = \mathbf{6.37 \ m/s}$$

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Single flow single device processes

Nozzles, diffusers

4.20

Liquid water at 15°C flows out of a nozzle straight up 15 m. What is nozzle V_{exit}?

Energy Eq.4.13: $h_{exit} + \frac{1}{2} V_{exit}^2 + gH_{exit} = h_2 + \frac{1}{2} V_2^2 + gH_2$

If the water can flow 15 m up it has specific potential energy of gH_2 which must equal the specific kinetic energy out of the nozzle $V_{exit}^2/2$. The water does not change P or T so h is the same.

$$V_{exit}^2/2 = g(H_2 - H_{exit}) = gH \quad =>$$
$$V_{exit} = \sqrt{2gH} = \sqrt{2 \times 9.807 \times 15 \ m^2/s^2} = \mathbf{17.15 \ m/s}$$

4.21

A nozzle receives an ideal gas flow with a velocity of 25 m/s, and the exit is at 100 kPa, 300 K with a velocity of 250 m/s. Determine the inlet temperature if the gas is argon, helium or nitrogen.

Solution:

C.V. Nozzle: $\dot{m}_i = \dot{m}_e$ & assume no heat transfer $\Rightarrow$

Energy Eq.4.13: $h_i + \frac{1}{2} V_i^2 = \frac{1}{2} V_e^2 + h_e \Rightarrow h_i = h_e + \frac{1}{2} V_e^2 - \frac{1}{2} V_i^2$

$$h_i - h_e \approx C_p (T_i - T_e) = \frac{1}{2} (V_e^2 - V_i^2) = \frac{1}{2} (250^2 - 25^2) \text{ m}^2/\text{s}^2$$

$$= 30937.5 \text{ J/kg} = 30.938 \text{ kJ/kg}$$

Specific heats for ideal gases are from table A.5

Argon $\quad C_p = 0.52$ kJ/kg K; $\quad \Delta T = \dfrac{30.938}{0.52} = 59.5 \quad$ **$T_i = 359.5$ K**

Helium $\quad C_p = 5.913$ kJ/kg K; $\quad \Delta T = \dfrac{30.938}{5.193} = 5.96 \quad$ **$T_i = 306$ K**

Nitrogen $\quad C_p = 1.042$ kJ/kg K; $\quad \Delta T = \dfrac{30.938}{1.042} = 29.7 \quad$ **$T_i = 330$ K**

Inlet   Exit

Low V   Hi V

Hi P, A   Low P, A

4.22

A diffuser receives 0.1 kg/s steam at 500 kPa, 350°C. The exit is at 1 MPa, 400°C with negligible kinetic energy and the flow is adiabatic. Find the diffuser inlet velocity and the inlet area.

Solution:

Energy Eq.4.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$

Process: $Z_1 = Z_2$

State 1: Table B.1.3 $h_1 = 3167.65$ kJ/kg, $v_1 = 0.57012$ m^3/kg

State 2: $\mathbf{V}_2 = 0$, Table B.1.3 $h_2 = 3263.88$ kJ/kg

Then from the energy equation

$$\frac{1}{2}\mathbf{V}_1^2 = h_2 - h_1 = 3263.88 - 3167.65 = 96.23 \text{ kJ/kg}$$

$$\mathbf{V}_1 = \sqrt{2(h_2 - h_1)} = \sqrt{2 \times 96.23 \times 1000} = \mathbf{438.7 \ m/s}$$

The mass flow rate from Eq.4.3

$$\dot{m} = \rho A\mathbf{V} = A\mathbf{V}/v$$

$$A = \dot{m}v/\mathbf{V} = 0.1 \text{ kg/s} \times 0.57012 \text{ m}^3/\text{kg} / 438.7 \text{ m/s} = 0.00013 \text{ m}^2 = \mathbf{1.3 \ cm^2}$$

Inlet

Exit

Hi V

Low V

Low P, A

Hi P, A

4.23

In a jet engine a flow of air at 1000 K, 200 kPa and 30 m/s enters a nozzle, as shown in Fig. P4.23, where the air exits at 850 K, 90 kPa. What is the exit velocity assuming no heat loss?

Solution:

C.V. nozzle. No work, no heat transfer

Continuity $\dot{m}_i = \dot{m}_e = \dot{m}$

Energy : $\dot{m} \, (h_i + \frac{1}{2}V_i^2) = \dot{m}(h_e + \frac{1}{2}V_e^2)$

Due to high T take h from table A.7.1

$$\frac{1}{2}V_e^2 = \frac{1}{2} V_i^2 + h_i - h_e$$
$$= \frac{1}{2000} (30)^2 + 1046.22 - 877.4$$
$$= 0.45 + 168.82 = 169.27 \text{ kJ/kg}$$

$$V_e = (2000 \times 169.27)^{1/2} = \mathbf{581.8 \text{ m/s}}$$

4.24

In a jet engine a flow of air at 1000 K, 200 kPa and 40 m/s enters a nozzle where the air exits at 500 m/s, 90 kPa. What is the exit temperature assuming no heat loss?

Solution:

C.V. nozzle, no work, no heat transfer

Continuity $\quad \dot{m}_i = \dot{m}_e = \dot{m}$

Energy : $\quad \dot{m}\,(h_i + \frac{1}{2}\mathbf{V}_i^2) = \dot{m}(h_e + \frac{1}{2}\mathbf{V}_e^2)$

Due to the high T we take the h value from Table A.7.1

$$h_e = h_i + \frac{1}{2}\,\mathbf{V}_i^2 - \frac{1}{2}\mathbf{V}_e^2$$
$$= 1046.22 \text{ kJ/kg} + 0.5 \times (40^2 - 500^2) \text{ m}^2/\text{s}^2 \times (1/1000) \text{ kJ/J}$$
$$= 1046.22 - 124.2 = 922.02 \text{ kJ/kg}$$

Interpolation in Table A.7.1

$$T_e = 850 + 50\,\frac{922.02 - 877.4}{933.15 - 877.4} = \mathbf{890\ K}$$

40 m/s
200 kPa

500 m/s
90 kPa

4.25

Superheated vapor ammonia enters an insulated nozzle at 30°C, 1000 kPa, shown in Fig. P4.25, with a low velocity and at the steady rate of 0.01 kg/s. The ammonia exits at 300 kPa with a velocity of 450 m/s. Determine the temperature (or quality, if saturated) and the exit area of the nozzle.

Solution:

C.V. Nozzle, steady state, 1 inlet and 1 exit flow, insulated so no heat transfer.

Energy Eq.4.13: $q + h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2,$

Process: $q = 0, \quad \mathbf{V}_i = 0$

Table B.2.2: $h_i = 1479.1 = h_e + 450^2/(2\times1000) \quad \Rightarrow \quad h_e = 1377.85 \text{ kJ/kg}$

Table B.2.1: $P_e = 300 \text{ kPa}$ Sat. state at -9.24°C :

                  $h_e = 1377.85 = 137.89 + x_e \times 1293.82,$

=> $x_e = \mathbf{0.9584},$ $v_e = 0.001536 + x_e \times 0.40566 = 0.3903 \text{ m}^3/\text{kg}$

$$A_e = \dot{m}_e v_e/\mathbf{V}_e = 0.01 \times 0.3903 / 450 = \mathbf{8.67 \times 10^{-6} \ m^2}$$

4.26

The wind is blowing horizontally at 30 m/s in a storm at P_0, 20°C toward a wall, where it comes to a stop (stagnation) and leaves with negligible velocity similar to a diffuser with a very large exit area. Find the stagnation temperature from the energy equation.

Solution:

Energy Eq.:

$$h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$$

Process:

$$Z_1 = Z_2 \qquad \text{and} \qquad V_2 = 0$$

$$h_2 = h_1 + \frac{1}{2}\mathbf{V}_1^2$$

$$T_2 = T_1 + \frac{1}{2}\times(\mathbf{V}_1^2 / C_p)$$

$$= 293 \text{ K} + \frac{1}{2}\times(30 \text{ m/s})^2 / (1.004 \text{ kJ/kg-K} \times 1000 \text{ J/kJ})$$

$$= \mathbf{293.45 \text{ K}}$$

4.27

A sluice gate dams water up 5 m. There is a small hole at the bottom of the gate so liquid water at 20$^{\mathrm{O}}$C comes out of a 1 cm diameter hole. Neglect any changes in internal energy and find the exit velocity and mass flow rate.

Solution:

Energy Eq.4.13: $h_1 + \frac{1}{2}V_1^2 + gZ_1 = h_2 + \frac{1}{2}V_2^2 + gZ_2$

Process: $h_1 = h_2$ both at P = 1 atm

$V_1 = 0$ $Z_1 = Z_2 + 5$ m

$\frac{1}{2}V_2^2 = g(Z_1 - Z_2)$

$V_2 = \sqrt{2g(Z_1 - Z_2)} = \sqrt{2 \times 9.806 \times 5} = \textbf{9.902 m/s}$

$\dot{m} = \rho AV = AV/v = \frac{\pi}{4}D^2 \times (V_2/v)$

$= \frac{\pi}{4} \times (0.01)^2 \text{ m}^2 \times (9.902 \text{ m/s}) / (0.001002 \text{ m}^3/\text{kg}) = \textbf{0.776 kg/s}$

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4.28

A diffuser, shown in Fig. P4.28, has air entering at 100 kPa, 300 K, with a velocity of 200 m/s. The inlet cross-sectional area of the diffuser is 100 mm^2. At the exit, the area is 860 mm^2, and the exit velocity is 20 m/s. Determine the exit pressure and temperature of the air.

Solution:

Continuity Eq.4.3: $\dot{m}_i = A_i \mathbf{V}_i / v_i = \dot{m}_e = A_e \mathbf{V}_e / v_e,$

Energy Eq.(per unit mass flow) 4.13: $h_i + \frac{1}{2}\mathbf{V}_i{}^2 = h_e + \frac{1}{2}\mathbf{V}_e{}^2$

$$h_e - h_i = \frac{1}{2} \times 200^2/1000 - \frac{1}{2} \times 20^2/1000 = 19.8 \text{ kJ/kg}$$

$$T_e = T_i + (h_e - h_i)/C_p = 300 + \frac{19.8 \text{ kJ/kg}}{1.004 \text{ kJ/kg-K}} = \textbf{319.72 K}$$

Now use the continuity equation and the ideal gas law

$$v_e = v_i \left(\frac{A_e \mathbf{V}_e}{A_i \mathbf{V}_i}\right) = (RT_i/P_i) \left(\frac{A_e \mathbf{V}_e}{A_i \mathbf{V}_i}\right) = RT_e/P_e$$

$$P_e = P_i \left(\frac{T_e}{T_i}\right)\left(\frac{A_i \mathbf{V}_i}{A_e \mathbf{V}_e}\right) = 100 \text{ kPa} \left(\frac{319.72}{300}\right)\left(\frac{100 \times 200}{860 \times 20}\right) = \textbf{123.92 kPa}$$

Inlet   Exit

Hi V   Low V

Low P, A   Hi P, A

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4.29

A meteorite hits the upper atmosphere at 3000 m/s, where the pressure is 0.1 atm and the temperature is –40°C. How hot does the air become right in front of the meteorite assuming no heat transfer in this adiabatic stagnation process?

Solution:

Energy Eq.: $\quad h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$

Process: $\quad\quad Z_1 = Z_2 \quad\quad$ and $\quad\quad V_2 = 0$

$$h_2 = h_1 + \frac{1}{2}\mathbf{V}_1^2$$

$$T_2 = T_1 + \frac{1}{2}\times(\mathbf{V}_1^2/C_p) = 233 + \frac{1}{2}\times\frac{3000^2}{1.004\times 1000} = \mathbf{4715\ K}$$

At this high temperature we cannot assume constant specific heats, so use A.7

$$h_2 = h_1 + \frac{1}{2}\mathbf{V}_1^2 = 233.3 + \frac{1}{2}\times\frac{3000^2}{1000} = 4733.3\ \text{kJ/kg}$$

Table A.7 is listed to 3000 K so we have to extrapolate to get

$$T_2 = 3000 + 50\ \frac{4733.3 - 3525.36}{3525.36 - 3460.73} = \mathbf{3935\ K}$$

The value of C_p over 3000 K is 1.293 kJ/kgK from the last two table entries. At this temperature there will be some chemical reactions that should be considered to have a more realistic temperature.

4.30

The front of a jet engine acts similar to a diffuser, receiving air at 900 km/h, −5°C, 50 kPa, bringing it to 80 m/s relative to the engine before entering the compressor. If the flow area is increased to 120% of the inlet area, find the temperature and pressure in the compressor inlet.

Solution:

C.V. Diffuser, Steady state, 1 inlet, 1 exit flow, no q, no w.

Continuity Eq.4.3: $\dot{m}_i = \dot{m}_e = (A\mathbf{V}/v)$

Energy Eq.4.12: $\dot{m} \left(h_i + \frac{1}{2}\mathbf{V}_i^2 \right) = \dot{m} \left(\frac{1}{2}\mathbf{V}_e^2 + h_e \right)$

$$h_e - h_i = C_p \left(T_e - T_i \right) = \frac{1}{2}\mathbf{V}_i^2 - \frac{1}{2}\mathbf{V}_e^2 = \frac{1}{2} \left(\frac{900 \times 1000}{3600} \right)^2 - \frac{1}{2}(80)^2$$

$$= \frac{1}{2}(250)^2 - \frac{1}{2}(80)^2 = 28050 \text{ J/kg} = 28.05 \text{ kJ/kg}$$

$$\Delta T = 28.05/1.004 = 27.9 \quad \Rightarrow \quad T_e = -5 + 27.9 = \mathbf{22.9°C}$$

Now use the continuity eq.:

$$A_i\mathbf{V}_i /v_i = A_e\mathbf{V}_e /v_e \quad \Rightarrow \quad v_e = v_i \left(\frac{A_e\mathbf{V}_e}{A_i\mathbf{V}_i} \right)$$

$$v_e = v_i \times \frac{1.2 \times 80}{1 \times 250} = v_i \times 0.384$$

Ideal gas: $Pv = RT \quad \Rightarrow \quad v_e = RT_e/P_e = RT_i \times 0.384/P_i$

$$P_e = P_i (T_e/T_i)/0.384 = 50 \text{ kPa} \times 296/(268 \times 0.384) = \mathbf{143.8 \text{ kPa}}$$

Fan

Throttle flow

4.31

R-410A at $-5°C$, 700 kPa is throttled, so it becomes cold at $-40°C$. What is the exit P?

C.V. Throttle. Steady state,

Process with: $q = w = 0;$ and $\mathbf{V}_i = \mathbf{V}_e, \ Z_i = Z_e$

Energy Eq.4.13: $h_i = h_e,$

Inlet state: Table B.4.1 $h_i = 50.22$ kJ/kg (slightly compressed liquid)

Exit state: Table B.4.1   since $h < h_g = 262.83$ kJ/kg it is two-phase

                                      $P = Psat = $ **175 kPa**

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4.32

Carbon dioxide is throttled from 20°C, 2 MPa to 800 kPa. Find the exit temperature assuming ideal gas behavior and repeat for real-gas behavior.

C.V. Throttle (valve, restriction), Steady flow, 1 inlet and exit, no q, w

Energy Eq.4.13: $h_i = h_e$

Ideal gas: same h gives $T_i = T_e = 20°C$

Real gas : $\left.\begin{array}{l} h_i = h_e = 368.42 \text{ kJ/kg} \\ P_e = 0.8 \text{ MPa} \end{array}\right\}$ Table B.3.2 $T_e = \textbf{5.3°C} \ (= \textbf{278 K})$

Comment: As you go towards lower P it becomes closer to ideal gas and the constant h curve bends to become horizontal (h becomes fct of T only)

4.33

Saturated liquid R-134a at 25°C is throttled to 300 kPa in a refrigerator. What is the exit temperature? Find the percent increase in the volume flow rate.

Solution:

Steady throttle flow. Assume no heat transfer and no change in kinetic or potential energy.

$$h_e = h_i = h_{f\ 25^oC} = 234.59 \text{ kJ/kg} = h_{f\ e} + x_e\ h_{fg\ e} \quad \text{at 300 kPa}$$

From table B.5.1 we get $T_e = T_{sat}(300 \text{ kPa}) = \mathbf{0.56^oC}$

Let's use 0°C for the following (we could interpolate to get at 0.56^{o}C)

$$x_e = \frac{h_e - h_{f\ e}}{h_{fg\ e}} = \frac{234.59 - 200}{198.36} = 0.1744$$

$$v_e = v_f + x_e\ v_{fg} = 0.000773 + x_e\ 0.06842 = 0.0127 \text{ m}^3\text{/kg}$$

$$v_i = v_{f\ 25^oC} = 0.000829 \text{ m}^3\text{/kg}$$

$\dot{V} = \dot{m}v$ so the ratio becomes

$$\frac{\dot{V}_e}{\dot{V}_i} = \frac{\dot{m}v_e}{\dot{m}v_i} = \frac{v_e}{v_i} = \frac{0.0127}{0.000829} = \mathbf{15.32}$$

So the increase is 14.32 times or **1432 %**

4.34

A supply line has a steady flow of R-410A at 1000 kPa, 60$^{\text{o}}$C from which a flow is taken out through a throttle with an exit flow at 300 kPa. Find the exit temperature.

C.V. Throttle. Steady state,

Process with: $q = w = 0$;   and $\mathbf{V}_i = \mathbf{V}_e$, $Z_i = Z_e$

Energy Eq.4.13: $h_i = h_e$,

Inlet state: Table B.4.2 $h_i = 335.75$ kJ/kg

Exit state: Table B.4.2   since $h > h_g$

Interpolate: $T = 40 + 20\,\dfrac{335.75 - 327.22}{344.81 - 327.22} = \mathbf{49.7^oC}$,

4.35

Carbon dioxide used as a natural refrigerant flows out of a cooler at 10 MPa, 40^oC after which it is throttled to 1.4 MPa. Find the state (T, x) for the exit flow.

C.V. Throttle. Steady state,

Process with: $q = w = 0$; and $\mathbf{V}_i = \mathbf{V}_e$, $Z_i = Z_e$

Energy Eq.4.13: $h_i = h_e$,

Inlet state: Table B.3.2 $h_i = 200.14$ kJ/kg

Exit state: Table B.3.1 since $h < h_g = 323.87$ kJ/kg

Interpolate: $T = $ **-30.6^oC**, $h_f = 19.2$ kJ/kg, $h_{fg} = 304.67$ kJ/kg

$x = (200.14 - 19.2) / 304.67 = $ **0.594**

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4.36

Liquid water at 180^OC, 2000 kPa is throttled into a flash evaporator chamber having a pressure of 500 kPa. Neglect any change in the kinetic energy. What is the fraction of liquid and vapor in the chamber?

Solution:

Energy Eq.4.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$

Process: $Z_1 = Z_2$ and $\mathbf{V}_2 = \mathbf{V}_1$

$\Rightarrow h_2 = h_1 = 763.71$ kJ/kg from Table B.1.4

State 2: $P_2 \& h_2 \Rightarrow$ 2 – phase

$h_2 = h_f + x_2 h_{fg}$

$x_2 = (h_2 - h_f) / h_{fg} = \dfrac{763.71 - 640.21}{2108.47} = 0.0586$

Fraction of   Vapor: $x_2 = 0.0586$ (5.86 %)

                Liquid: $1 - x_2 = 0.941$ (94.1 %)

Two-phase out of the valve. The liquid drops to the bottom.

4.37

Helium is throttled from 1.2 MPa, 20°C, to a pressure of 100 kPa. The diameter of the exit pipe is so much larger than the inlet pipe that the inlet and exit velocities are equal. Find the exit temperature of the helium and the ratio of the pipe diameters.

Solution:

C.V. Throttle. Steady state,

Process with: $\quad$ q = w = 0; $\quad$ and $\quad \mathbf{V}_i = \mathbf{V}_e, \; Z_i = Z_e$

Energy Eq.4.13: $\quad h_i = h_e, \quad$ Ideal gas $\quad => \quad T_i = T_e = \mathbf{20°C}$

$$\dot{m} = \frac{A\mathbf{V}}{RT/P} \quad \text{But } \dot{m}, \mathbf{V}, T \text{ are constant} \quad => \quad P_i A_i = P_e A_e$$

$$\Rightarrow \quad \frac{D_e}{D_i} = \left(\frac{P_i}{P_e}\right)^{1/2} = \left(\frac{1.2}{0.1}\right)^{1/2} = \mathbf{3.464}$$

4.38

Methane at 1 MPa, 300 K is throttled through a valve to 100 kPa. Assume no change in the kinetic energy. What is the exit temperature?

Solution:

Energy Eq. 4.13: $h_1 + \frac{1}{2} \mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2} \mathbf{V}_2^2 + gZ_2$

Process: $Z_1 = Z_2$ and $\mathbf{V}_2 = \mathbf{V}_1$

State 1, Table B.7.2: $h_1 = 618.76$ kJ/kg

Use energy eq.: $\Rightarrow$ $h_2 = h_1 = 618.76$ kJ/kg

State 2: P_2 & h_2 $\Rightarrow$ look in B.7.2 : Superheated vapor

$$T_2 = 275 + 25 \frac{618.76 - 572.36}{627.58 - 572.36} = \textbf{296 K}$$

Notice if it had been ideal gas then $T_2 = T_1$

4.39

R-134a is throttled in a line flowing at 25°C, 750 kPa with negligible kinetic energy to a pressure of 165 kPa. Find the exit temperature and the ratio of exit pipe diameter to that of the inlet pipe (D_{ex}/D_{in}) so the velocity stays constant.

Solution:

Energy Eq.4.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$

Process: $Z_1 = Z_2$ and $\mathbf{V}_2 = \mathbf{V}_1$

State 1, Table B.5.1: $h_1 = 234.59$ kJ/kg, $v_1 = v_f = 0.000829$ m^3/kg

Use energy eq.: $\Rightarrow$ $h_2 = h_1 = 234.59$ kJ/kg

State 2: P_2 & h_2 $\Rightarrow$ 2 – phase   and $T_2 = T_{sat}$ (165 kPa) = **-15°C**

$h_2 = h_f + x_2\, h_{fg} = 234.59$ kJ/kg

$x_2 = (h_2 - h_f\,)\,/\,h_{fg} = (234.59 – 180.19)\,/\,209 = 0.2603$

$v_2 = v_f + x_2 \times v_{fg} = 0.000746 + 0.2603 \times 0.11932 = 0.0318$ m^3/kg

Now the continuity equation with $\mathbf{V}_2 = \mathbf{V}_1$ gives, from Eq.4.3,

$$\dot{m} = \rho A\mathbf{V} = A\mathbf{V}/v = A_1\mathbf{V}_1/v_1 = (A_2\,\mathbf{V}_1)\,/\,v_2$$

$$(A_2\,/\,A_1) = v_2\,/\,v_1\ = (D_2\,/\,D_1)^2$$

$$(D_2/D_1) = (v_2\,/\,v_1)^{0.5} = (0.0318\,/\,0.000829)^{0.5} = \mathbf{6.19}$$

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Turbines, Expanders

4.40

A steam turbine has an inlet of 3 kg/s water at 1200 kPa, 350°C and velocity of 15 m/s. The exit is at 100 kPa, 150°C and very low velocity. Find the specific work and the power produced.

Solution:

Energy Eq.4.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2 + w_T$

Process: $Z_1 = Z_2$ and $\mathbf{V}_2 = 0$

Table B.1.3: $h_1 = 3153.59$ kJ/kg, $h_2 = 2776.38$ kJ/kg

$$w_T = h_1 + \frac{1}{2}\mathbf{V}_1^2 - h_2 = 3153.59 + \frac{15^2}{2000} - 2776.38 = \textbf{377.3 kJ/kg}$$

(remember to convert m^2/s^2 = J/kg to kJ/kg by dividing with 1000)

$\dot{W}_T = \dot{m} \times w_T = 3$ kg/s $\times$ 377.3 kJ/kg

      = **1132 kW**

1

$\dot{W}_T$

2

4.41

Air at 20 m/s, 1500 K, 875 kPa with 5 kg/s flows into a turbine and it flows out at 25 m/s, 850 K, 105 kPa. Find the power output using constant specific heats.

Solution:

Energy Eq.4.13: $h_1 + \frac{1}{2} \mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2} \mathbf{V}_2^2 + gZ_2 + w_T$

Process: $Z_1 = Z_2$ and $\mathbf{V}_2 = 0$

Table A.5: $C_p = 1.004$ kJ/kg-K

$$w_T = h_1 + \frac{1}{2} \mathbf{V}_1^2 - h_2 - \frac{1}{2} \mathbf{V}_2^2 = C_p(T_1 - T_2) + \frac{1}{2} \mathbf{V}_1^2 - \frac{1}{2} \mathbf{V}_2^2$$

$$= 1.004 \text{ kJ/kg-K} \times (1500 - 850) \text{ K} + \frac{20^2 - 25^2}{2000} \text{ (m}^2\text{/s}^2) \times \text{(kJ/J)}$$

$$= \mathbf{652.5 \ kJ/kg}$$

(remember to convert m^2/s^2 = J/kg to kJ/kg by dividing with 1000)

$$\dot{W}_T = \dot{m} \times w_T = 5 \text{ kg/s} \times 652.5 \text{ kJ/kg}$$

$$= \mathbf{3263 \ kW}$$

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4.42

Solve the previous problem using Table A.7.
Air at 20 m/s, 1500 K, 875 kPa with 5 kg/s flows into a turbine and it flows out at
25 m/s, 850 K, 105 kPa. Find the power output using constant specific heats.

Solution:

Energy Eq.4.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2 + w_T$

Process: $\qquad Z_1 = Z_2 \qquad$ and $\qquad \mathbf{V}_2 = 0$

Table A.7.1: $\qquad h_1 = 1635.8$ kJ/kg, $\qquad h_2 = 877.4$ kJ/kg

$$w_T = h_1 - h_2 + \frac{1}{2}\mathbf{V}_1^2 - \frac{1}{2}\mathbf{V}_2^2$$

$$= (1635.8 - 877.4) \text{ kJ/kg} + \frac{20^2 - 25^2}{2000} \text{ (m}^2\text{/s}^2) \times \text{(kJ/J)}$$

$$= \textbf{758.3 kJ/kg}$$

(remember to convert m^2/s^2 = J/kg to kJ/kg by dividing with 1000)

$$\dot{W}_T = \dot{m} \times w_T = 5 \text{ kg/s} \times 758.3 \text{ kJ/kg}$$

$$= \textbf{3792 kW}$$

4.43

A windmill with rotor diameter of 20 m takes 40% of the kinetic energy out as shaft work on a day with 20$^\text{o}$C and wind speed of 35 km/h. What power is produced?

Solution:

Continuity Eq. $\dot{m}_i = \dot{m}_e = \dot{m}$

Energy $\dot{m}\ (h_i + \tfrac{1}{2}\mathbf{V}_i^2 + gZ_i) = \dot{m}(h_e + \tfrac{1}{2}\mathbf{V}_e^2 + gZ_e) + \dot{W}$

Process information: $\dot{W} = \dot{m}\tfrac{1}{2}\mathbf{V}_i^2 \times 0.4$

$\dot{m} = \rho A\mathbf{V} = A\mathbf{V}_i\,/v_i$

$A = \dfrac{\pi}{4}\,D^{\,2} = \dfrac{\pi}{4}\,20^2 = 314.16\ \text{m}^2$

$v_i = RT_i/P_i = \dfrac{0.287 \times 293}{101.3} = 0.8301\ \text{m}^3/\text{kg}$

$\mathbf{V}_i = 35\ \text{km/h} = \dfrac{35 \times 1000}{3600} = 9.7222\ \text{m/s}$

$\dot{m} = A\mathbf{V}_i\,/v_i = \dfrac{314.16 \times 9.7222}{0.8301} = 3679.5\ \text{kg/s}$

$\tfrac{1}{2}\,\mathbf{V}_i^2 = \tfrac{1}{2}\ 9.7222^2\ \text{m}^2/\text{s}^2 = 47.26\ \text{J/kg}$

$\dot{W} = 0.4\ \dot{m}\tfrac{1}{2}\,\mathbf{V}_i^2 = 0.4 \times 3679.5\ \text{kg/s} \times 47.26\ \text{J/kg} = 69\ 557\ \text{W}$

$\qquad\qquad = \textbf{69.56 kW}$

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4.44

A liquid water turbine receives 2 kg/s water at 2000 kPa, 20°C and velocity of 15 m/s. The exit is at 100 kPa, 20°C and very low velocity. Find the specific work and the power produced.

Solution:

Energy Eq.4.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2 + w_T$

Process: $Z_1 = Z_2$ and $\mathbf{V}_2 = 0$

State 1:   Table B.1.4 $h_1 = 85.82$ kJ/kg

State 2:   Table B.1.1 $h_2 = 83.94$ (which is at 2.3 kPa so we

                  should add $\Delta Pv = 97.7 \times 0.001$ to this)

$$w_T = h_1 + \frac{1}{2}\mathbf{V}_1^2 - h_2 = 85.82 + 15^2/2000 - 83.94 = \mathbf{1.9925 \ kJ/kg}$$

$$\dot{W}_T = \dot{m} \times w_T = 2 \text{ kg/s} \times 1.9925 \text{ kJ/kg} = \mathbf{3.985 \ kW}$$

Notice how insignificant the specific kinetic energy is.

4.45

What is the specific work one can expect from the dam in Problem 4.27?

Solution:

Energy Eq.4.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2 + w_T$

Process: $\mathbf{V}_2 = \mathbf{V}_1$; $P_1 = P_2 = P_0$; $h_1 = h_2$

Same velocity if same flow area (incompressible flow) and the water temperature does not change significantly. The specific potential enrgy difference can be extracted

$w_T = gZ_1 - gZ_2 = 9.806 \text{ m/s}^2 \times (5 - 0) \text{ m} = 49 \text{ m}^2/\text{s}^2 = \textbf{49 J/kg}$

This is a very small amount of specific work.

4.46

A small, high-speed turbine operating on compressed air produces a power output of 100 W. The inlet state is 400 kPa, 50°C, and the exit state is 150 kPa, −30°C. Assuming the velocities to be low and the process to be adiabatic, find the required mass flow rate of air through the turbine.

Solution:

C.V. Turbine, no heat transfer, no ΔKE, no ΔPE

Energy Eq.4.13: $h_{in} = h_{ex} + w_T$

Ideal gas so use constant specific heat from Table A.5

$$w_T = h_{in} - h_{ex} \cong C_p(T_{in} - T_{ex})$$
$$= 1.004 \text{ (kJ/kg-K) } [50 - (-30)] \text{ K} = 80.3 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}w_T \quad \Rightarrow \quad \dot{m} = \dot{W}/w_T = 0.1 \text{ kW } /80.3 \text{ kJ/kg} = \textbf{0.00125 kg/s}$$

The dentist's drill has a small air flow and is not really adiabatic.

4.47

Hoover Dam across the Colorado River dams up Lake Mead 200 m higher than the river downstream. The electric generators driven by water-powered turbines deliver 1300 MW of power. If the water is 17.5°C, find the minimum amount of water running through the turbines.

Solution:

C.V.: H_2O pipe + turbines,

Continuity: $\dot{m}_{in} = \dot{m}_{ex}$;

Energy Eq.4.13: $(h+ \mathbf{V}^2/2 + gz)_{in} = (h+ \mathbf{V}^2/2 + gz)_{ex} + w_T$

Water states: $h_{in} \cong h_{ex}$; $v_{in} \cong v_{ex}$

Now the specific turbine work becomes

$$w_T = gz_{in} - gz_{ex} = 9.807 \times 200/1000 = 1.961 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}_T/w_T = \frac{1300 \times 10^3 \text{ kW}}{1.961 \text{ kJ/kg}} = 6.63 \times 10^5 \text{ kg/s}$$

$$\dot{V} = \dot{m}v = 6.63 \times 10^5 \text{ kg/s} \times 0.001001 \text{ m}^3/\text{kg} = \mathbf{664 \ m^3/s}$$

4.48

A small turbine, shown in Fig. P 4.48, is operated at part load by throttling a 0.25 kg/s steam supply at 1.4 MPa, 250°C down to 1.1 MPa before it enters the turbine and the exhaust is at 10 kPa. If the turbine produces 110 kW, find the exhaust temperature (and quality if saturated).

Solution:

C.V. Throttle, Steady, $q = 0$ and $w = 0$. No change in kinetic or potential energy. The energy equation then reduces to

Energy Eq.4.13: $\qquad h_1 = h_2 = 2927.2$ kJ/kg from Table B.1.3

C.V. Turbine, Steady, no heat transfer, specific work: $w = \dfrac{110}{0.25} = 440$ kJ/kg

Energy Eq.: $\quad h_1 = h_2 = h_3 + w = 2927.2$ kJ/kg
$\Rightarrow \quad h_3 = 2927.2 - 440 = 2487.2$ kJ/kg

State 3: (P, h) Table B.1.2 $\qquad h < h_g$
$\qquad 2487.2 = 191.83 + x_3 \times 2392.8$

$\Rightarrow \quad T = \mathbf{45.8°C} , \quad x_3 = \mathbf{0.959}$

4.49

A small expander (a turbine with heat transfer) has 0.05 kg/s helium entering at 1000 kPa, 550 K and it leaves at 250 kPa, 300 K. The power output on the shaft is measured to 55 kW. Find the rate of heat transfer neglecting kinetic energies.
Solution:

C.V. Expander. Steady operation

Cont. $\quad\quad \dot{m}_i = \dot{m}_e = \dot{m}$

Energy $\quad\quad \dot{m}h_i + \dot{Q} = \dot{m}h_e + \dot{W}$

$\dot{Q} = \dot{m}\,(h_e - h_i) + \dot{W}$

Use specific heat from Table A.5: $C_{p\ He} = 5.193$ kJ/kg K

$$\dot{Q} = \dot{m}C_p\,(T_e - T_i) + \dot{W}$$
$$= 0.05 \text{ kg/s} \times 5.193 \text{ kJ/kg-K} \times (300 - 550) \text{ K} + 55 \text{ kW}$$
$$= -64.91 + 55 = \textbf{-9.9 kW}$$

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Compressors, fans

4.50

A compressor in a commercial refrigerator receives R-410A at -25°C, x = 1. The exit is at 1000 kPa, 40°C. Neglect kinetic energies and find the specific work.

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also assume no heat transfer and $Z_i = Z_e$

From Table B.4.1 : h_i = 269.77 kJ/kg

From Table B.4.2 : h_e = 316.05 kJ/kg

Energy Eq.4.13 reduces to

$w_c = h_i - h_e$ = (269.77 – 316.05) kJ/kg = **-46.28 kJ/kg**

4.51

A compressor brings nitrogen from 100 kPa, 290 K to 2000 kPa. The process has a specific work input of 450 kJ/kg and the exit temperature is 450 K. Find the specific heat transfer using constant specific heats.

Solution:

C.V. Compressor. Not adiabatic, neglect kinetic and potential energy changes.

Energy Eq.4.13: $q + h_i + \mathbf{V}_i^2/2 + gZ_i = h_e + \mathbf{V}_e^2/2 + w + gZ_e$

Process: $Z_i = Z_e$; $\mathbf{V}_i^2/2 = \mathbf{V}_e^2/2$

Solve for the heat transfer

$$q = h_e + w - h_i = C_p\,(T_e - T_i) + w$$

$$= 1.042 \text{ kJ/kg-K} \times (450 - 290) \text{ K} - 450 \text{ kJ/kg} = \mathbf{-283.3 \text{ kJ/kg}}$$

Recall standard sign notation is work positive out.

4.52

A portable fan blows 0.3 kg/s room air with a velocity of 15 m/s (see Fig. P4.18). What is the minimum power electric motor that can drive it? Hint: Are there any changes in P or T?

Solution:

C.V. Fan plus space out to near stagnant inlet room air.

Energy Eq.4.13: $\qquad q + h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2 + w$

Here $\ q \cong 0,\ \ \mathbf{V}_i \cong 0\ \$ and $\ h_i = h_e\ \ \ $ same P and T

$-w = \mathbf{V}_e^2/2 = (15^2/2)\ (m/s)^2 = 112.5\ J/kg = 0.1125\ kJ/kg$

$-\dot{W} = -\dot{m}w = 0.3\ kg/s \times 0.1125\ kJ/kg = \mathbf{0.034\ kW}$

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4.53

A refrigerator uses the natural refrigerant carbon dioxide where the compressor brings 0.02 kg/s from 1 MPa, -20°C to 6 MPa using 2 kW of power. Find the compressor exit temperature.

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also assume no heat transfer and $Z_i = Z_e$

From Table B.3.2 : $h_i = 342.31$ kJ/kg

Energy Eq.4.13 reduces to

$$\dot{W} = \dot{m} w_c = \dot{m}\,(h_i - h_e) \Rightarrow \quad h_e = h_i - \dot{W}/\dot{m}$$

$$h_e = 342.31 \text{ kJ/kg} - [-2 \text{ kW}] / (0.02 \text{ kg/s}) = 442.31 \text{ kJ/kg}$$

From Table B.3.2 :

$$T_e = 100 + 20\,(442.31 - 421.69)/(445.02 - 421.69) = \mathbf{117.7\ ^oC}$$

4.54

An air compressor takes in air at 100 kPa, 17°C and delivers it at 1 MPa, 600 K to a constant-pressure cooler, which it exits at 300 K. Find the specific compressor work and the specific heat transfer in the cooler.

Solution

C.V. air compressor q = 0

Continuity Eq.: $\dot{m}_2 = \dot{m}_1$

Energy Eq.4.13: $0 = h_1 + w_{c\ in} - h_2$

Compressor section   Cooler section

Table A.7: $h_1 = 290.19$ kJ/kg, $h_2 = 607.32$ kJ/kg, $h_3 = 300.47$ kJ/kg

$$w_{c\ in} = h_2 - h_1 = 607.32 - 290.19 = \textbf{317.13 kJ/kg}$$

C.V. cooler w = 0

Continuity Eq.: $\dot{m}_3 = \dot{m}_1$

Energy Eq.4.13: $0 = h_2 - q_{out} - h_3$

$$q_{out} = h_2 - h_3 = 607.32 - 300.47 = \textbf{306.85 kJ/kg}$$

Comment: For thess temperatures we could have used constant specific heats from A.5

4.55

A compressor brings R-134a from 150 kPa, -10$^{\text{o}}$C to 1200 kPa, 50$^{\text{o}}$C. It is water cooled with a heat loss estimated as 40 kW and the shaft work input is measured to be 150 kW. How much is the mass flow rate through the compressor?

Solution:

C.V Compressor. Steady flow.
Neglect kinetic and potential energies.

Energy : $\dot{m}\,h_i + \dot{Q} = \dot{m}h_e + \dot{W}$

$\dot{m} = (\dot{Q} - \dot{W})/(h_e - h_i)$

Look in table B.5.2

$h_i = 393.84$ kJ/kg, $h_e = 426.84$ kJ/kg

$\dot{m} = \dfrac{-40 - (-150)}{426.84 - 393.84}\dfrac{\text{kW}}{\text{kJ/kg}} = \textbf{3.333 kg/s}$

4.56

The compressor of a large gas turbine receives air from the ambient at 95 kPa, 20°C, with a low velocity. At the compressor discharge, air exits at 1.52 MPa, 430°C, with velocity of 90 m/s. The power input to the compressor is 5000 kW. Determine the mass flow rate of air through the unit.

Solution:

C.V. Compressor, steady state, single inlet and exit flow.

Energy Eq.4.13: $\quad q + h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2 + w$

Here we assume $q \cong 0$ and $\mathbf{V}_i \cong 0$ so using constant C_{Po} from A.5

$$-w = C_{Po}(T_e - T_i) + \mathbf{V}_e^2/2 = 1.004(430 - 20) + \frac{(90)^2}{2 \times 1000} = 415.5 \text{ kJ/kg}$$

Notice the kinetic energy is 1% of the work and can be neglected in most cases. The mass flow rate is then from the power and the specific work

$$\dot{m} = \frac{\dot{W}_c}{-w} = \frac{5000}{415.5} \frac{\text{kW}}{\text{kJ/kg}} = \mathbf{12.0 \text{ kg/s}}$$

4.57

How much power is needed to run the fan in Problem 4.18?

A household fan of diameter 0.6 m takes air in at 98 kPa, 20°C and delivers it at 105 kPa, 21°C with a velocity of 1.5 m/s. What are the mass flow rate (kg/s), the inlet velocity and the outgoing volume flow rate in m^3/s?

Solution:

Continuity Eq. $\dot{m}_i = \dot{m}_e = A\mathbf{V}/v$

Ideal gas $v = RT/P$

Area : $A = \dfrac{\pi}{4} D^2 = \dfrac{\pi}{4} \times 0.6^2 = 0.2827 \text{ m}^2$

$\dot{V}_e = A\mathbf{V}_e = 0.2827 \times 1.5 = 0.4241 \text{ m}^3/\text{s}$

$v_e = \dfrac{RT_e}{P_e} = \dfrac{0.287 \times (21 + 273)}{105} = 0.8036 \text{ m}^3/\text{kg}$

$\dot{m}_i = \dot{V}_e/v_e = 0.4241/0.8036 = 0.528 \text{ kg/s}$

$A V_i /v_i = \dot{m}_i = A\mathbf{V}_e / v_e$

$\mathbf{V}_i = \mathbf{V}_e \times (v_i/v_e) = \mathbf{V}_e \times \dfrac{RT_i}{P_i v_e} = 1.5 \times \dfrac{0.287 \text{ kJ/kg-K} \times (20 + 273) \text{ K}}{98 \text{ kPa} \times 0.8036 \text{ m}^3/\text{kg}} = 1.6 \text{ m/s}$

$\dot{W} = \dot{m}(h_i + \tfrac{1}{2}V_i^2 - h_e - \tfrac{1}{2}V_e^2) = \dot{m}\,[C_p\,(T_i - T_e) + \tfrac{1}{2}\,V_i^2 - \tfrac{1}{2}V_e^2\,]$

$\quad = 0.528\,[\,1.004\,(-1) + \dfrac{1.6^2 - 1.5^2}{2000}] = 0.528\,[-1.004 + 0.000155]$

$\quad = -\,\textbf{0.53 kW}$

4.58

A compressor in an industrial air-conditioner compresses ammonia from a state of saturated vapor at 200 kPa to a pressure of 1000 kPa. At the exit the temperature is measured to be 100^OC and the mass flow rate is 0.5 kg/s. What is the required motor size (kW) for this compressor?

Solution:

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.4.13: $\qquad\qquad w_C = h_1 - h_2$

States: 1: B.2.2: $\quad h_1 = 1419.6$ kJ/kg

$\qquad$ 2: B.2.2 $\quad h_{2,AC} = 1664.3$ kJ/kg

Energy equation:

$\qquad -w_C = h_2 - h_1 = 1664.3 - 1419.6 = 244.7$ kJ/kg

$$\dot{W} = 0.5 \text{ kg/s} \times 244.7 \text{ kJ/kg} = \mathbf{122\ kW}$$

4.59

An exhaust fan in a building should be able to move 3 kg/s air atmospheric pressure air at 25°C through a 0.5 m diameter vent hole. How high a velocity must it generate and how much power is required to do that?

Solution:

C.V. Fan and vent hole. Steady state with uniform velocity out.

Continuity Eq.: $\dot{m}$ = constant = $\rho A\mathbf{V}$ = $A\mathbf{V}$ / v = $A\mathbf{V}P/RT$

Ideal gas : $Pv = RT$, and area is $A = \frac{\pi}{4} D^2$

Now the velocity is found

$$\mathbf{V} = \dot{m}\, RT/(\frac{\pi}{4} D^2 P)$$

$$= 3 \text{ kg/s} \times 0.287 \text{ kJ/kg-K} \times 293.15 \text{ K} / (\frac{\pi}{4} \times 0.5^2 \text{ m}^2 \times 101 \text{ kPa})$$

$$= \textbf{12.7 m/s}$$

The kinetic energy out is

$$\frac{1}{2}\mathbf{V}_2^2 = \frac{1}{2} \times 12.7^2 / 1000 = 0.08065 \text{ kJ/kg}$$

which is provided by the work (only two terms in energy equation that does not cancel, we assume $\mathbf{V}_1 = 0$)

$$\dot{W}_{in} = \dot{m}\, \frac{1}{2}\mathbf{V}_2^2 = 3 \text{ kg/s} \times 0.08065 \text{ kJ/kg} = \textbf{0.242 kW}$$

4.60

A compressor receives R-410A as saturated vapor R-410A at 400 kPa and brings it to 2000 kPa, 60°C. Then a cooler brings it to saturated liquid at 2000 kPa (see Fig. P4.54). Find the specific compressor work and the specific heat transfer in the cooler?

C.V. Compressor. Assume adiabatic and neglect kinetic, potential energies.

Energy Eq.4.13: $w = h_1 - h_2$

States: 1: B.4.2 $h_1 = 271.9$ kJ/kg

2: B.4.2 $h_2 = 320.62$ kJ/kg

3: B.4.1 $h_3 = 106.14 + \dfrac{2.31}{5}(114.95 - 106.14) = 110.2$ kJ/kg

$-w_C = h_2 - h_1 = 320.62 - 271.9 =$ **48.72 kJ/kg**

C.V. Cooler. No work, neglect kinetic, potential energies.

Energy Eq. 4.13: $q = h_3 - h_2 = 110.2 - 320.62 =$ **–210.4 kJ/kg**

The compressor is inside the unit to the left that also houses a fan and the heat exchanger.

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4.61

An air flow is brought from 20°C, 100 kPa to 1000 kPa, 330°C by an adiabatic compressor driven by a 50-kW motor. What are the mass flow rate and the exit volume flow rate of air?

C.V. Compressor. Assume adiabatic and neglect kinetic, potential energies.

Energy Eq.4.13: $\qquad\qquad$ w = h$_1$ - h$_2$

$$-w_C = h_2 - h_1 \approx C_P (T_2 - T_1) = 1.004 \text{ kJ/kg-K} \times (330 - 20) \text{ K}$$
$$= 311.2 \text{ kJ/kg}$$

The mass flow rate scale the work term so

$$\dot{m} = \dot{W} / -w_C = \frac{50 \text{ kW}}{311.2 \text{ kJ/kg}} = \textbf{0.1606 kg/s}$$

$$\dot{V} = \dot{m} v = \dot{m} \frac{RT}{P} = 0.1606 \text{ kg/s} \frac{0.287 \times (330 + 273) \text{ kJ/kg}}{1000 \text{ kPa}}$$
$$= \textbf{0.0278 m}^3\textbf{/s}$$

Heaters/Coolers

4.62

The air conditioner in a house or a car has a cooler that brings atmospheric air from 30^oC to 10^oC both states at 101 kPa. If the flow rate is 0.75 kg/s find the rate of heat transfer.

Solution:

CV. Cooler. Steady state single flow with heat transfer.

Neglect changes in kinetic and potential energy and no work term.

Energy Eq. 4.13: $\qquad q_{out} = h_i - h_e$

Use constant specific heat from Table A.5 (T is around 300 K)

$$q_{out} = h_i - h_e = C_P (T_i - T_e)$$

$$= 1.004 \frac{kJ}{kg\ K} \times (30 - 10)\ K = 20.1\ kJ/kg$$

$$\dot{Q}_{out} = \dot{m}\ q_{out} = 0.75 \times 20.1 = \mathbf{15\ kW}$$

4.63

A boiler section boils 3 kg/s saturated liquid water at 2000 kPa to saturated vapor in a reversible constant-pressure process. Find the specific heat transfer in the process.

C.V. Boiler. Steady state single inlet and exit flow, neglect potential energy and since velocities are low we neglect kinetic energies.

Energy Eq. 4.13: $q + h_i = h_e$

From Table B.1.2: $h_i = 908.77$ kJ/kg
         Table B.1.3: $h_e = 2799.51$ kJ/kg

$$q = h_e - h_i = 2799.51 - 908.77 = \textbf{1890.74 kJ/kg}$$

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4.64

A condenser (cooler) receives 0.05 kg/s R-410A at 2000 kPa, 80$^{\text{O}}$C and cools it to 10$^{\text{O}}$C. Assume the exit properties are as for saturated liquid with the same T. What cooling capacity (kW) must the condenser have?

Solution:

C.V. R-410A condenser. Steady state single flow, heat transfer out and no work.

Energy Eq. 4.12: $\dot{m}\, h_1 = \dot{m}\, h_2 + \dot{Q}_{out}$

Inlet state: Table B.4.2 $h_1 = 343.22$ kJ/kg,

Exit state: Table B.4.1 $h_2 = 73.21$ kJ/kg (compressed liquid)

To be more accurate we could have added (P-Psat)v to the h_2.

Process: Neglect kinetic and potential energy changes.

Cooling capacity is taken as the heat transfer out i.e. positive out so

$\dot{Q}_{out} = \dot{m}\,(h_1 - h_2) = 0.05$ kg/s $\times$ (343.22 − 73.21) kJ/kg

= **13.5 kW**

4.65

Carbon dioxide enters a steady-state, steady-flow heater at 300 kPa, 300 K, and exits at 275 kPa, 1500 K, as shown in Fig. P4.65. Changes in kinetic and potential energies are negligible. Calculate the required heat transfer per kilogram of carbon dioxide flowing through the heater.

Solution:

C.V. Heater Steady state single inlet and exit flow.

Energy Eq. 4.13: $q + h_i = h_e$

Table A.8: $q = h_e - h_i = 1614.9 - 214.4 = $ **1400.5 kJ/kg**

[If we use C_{p0} from A.5 then $q \cong 0.842$ kJ/kg-K $\times$ (1500 - 300)K = 1010.4 kJ/kg]

Too large ΔT, T_{ave} to use C_{p0} at room temperature.

4.66

Find the heat transfer in Problem 4.17

A boiler receives a constant flow of 5000 kg/h liquid water at 5 MPa, 20°C and it heats the flow such that the exit state is 450°C with a pressure of 4.5 MPa. Determine the necessary minimum pipe flow area in both the inlet and exit pipe(s) if there should be no velocities larger than 20 m/s.

Solution:

C.V. Heater. Steady state single inlet and exit flow, neglect potential energy and since velocities are low we neglect kinetic energies.

Energy Eq. 4.13: $q + h_i = h_e$

From Table B.1.4: $h_i = 88.64$ kJ/kg
      Table B.1.3: $h_e = (3330.23 + 3316.15)/2 = 3323.19$ kJ/kg

$$q = h_e - h_i = 3323.19 - 88.64 = \textbf{3234.6 kJ/kg}$$

$P_e < P_i$

P_i

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4.67

A chiller cools liquid water for air-conditioning purposes. Assume 2.5 kg/s water at 20°C, 100 kPa is cooled to 5°C in a chiller. How much heat transfer (kW) is needed?

Solution:

C.V. Chiller. Steady state single flow with heat transfer. Neglect changes in kinetic and potential energy and no work term.

Energy Eq. 4.13: $\qquad q_{out} = h_i - h_e$

Properties from Table B.1.1:

$\qquad\qquad h_i = 83.94$ kJ/kg $\qquad$ and $\qquad\qquad h_e = 20.98$ kJ/kg

Now the energy equation gives

$\qquad\qquad q_{out} = 83.94 - 20.98 = 62.96$ kJ/kg

$\qquad\qquad \dot{Q}_{out} = \dot{m} \, q_{out} = 2.5 \times 62.96 = \mathbf{157.4\ kW}$

Alternative property treatment since single phase and small ΔT

If we take constant specific heat for the liquid from Table A.4

$\qquad\qquad q_{out} = h_i - h_e \cong C_p (T_i - T_e)$
$\qquad\qquad\qquad = 4.18$ kJ/kg-K $\times (20 - 5)$ K $= 62.7$ kJ/kg

$\qquad\qquad \dot{Q}_{out} = \dot{m} \, q_{out} = 2.5$ kg/s $\times 62.7$ kJ/kg $= \mathbf{156.75\ kW}$

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4.68

Saturated liquid nitrogen at 600 kPa enters a boiler at a rate of 0.008 kg/s and exits as saturated vapor, see Fig. P4.68. It then flows into a super heater also at 600 kPa where it exits at 600 kPa, 280 K. Find the rate of heat transfer in the boiler and the super heater.

Solution:

C.V.: boiler steady single inlet and exit flow, neglect KE, PE energies in flow

Continuity Eq.: $\dot{m}_1 = \dot{m}_2 = \dot{m}_3$

Table B.6.1: $h_1 = -81.469$ kJ/kg, $h_2 = 86.85$ kJ/kg,

Table B.6.2: $h_3 = 289.05$ kJ/kg

Energy Eq. 4.13: $q_{boiler} = h_2 - h_1 = 86.85 - (-81.469) = 168.32$ kJ/kg

$\dot{Q}_{boiler} = \dot{m}_1 q_{boiler} = 0.008$ kg/s $\times$ 168.32 kJ/kg = **1.346 kW**

C.V. Superheater (same approximations as for boiler)

Energy Eq. 4.13: $q_{sup\ heater} = h_3 - h_2 = 289.05 - 86.85 = 202.2$ kJ/kg

$\dot{Q}_{sup\ heater} = \dot{m}_2 q_{sup\ heater} = 0.008$ kg/s $\times$ 202.2 kJ/kg = **1.62 kW**

4.69

Carbon dioxide used as a natural refrigerant flows through a cooler at 10 MPa, which is supercritical so no condensation occurs. The inlet is at 220^OC and the exit is at 50^OC. Find the specific heat transfer.

C.V. Cooler. Steady state single flow with heat transfer. Neglect changes in kinetic and potential energy and no work term.

Energy Eq. 4.13: $\qquad 0 = h_i - h_e + q$

Properties from Table B.3.2:

$\qquad h_i = 542.91$ kJ/kg   and

$\qquad h_e = (200.14 + 312.11)/2 = 256.13$ kJ/kg

Now the energy equation gives

$\qquad q = 256.13 - 542.91 = $ **–286.79 kJ/kg**

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4.70

In a steam generator, compressed liquid water at 10 MPa, 30°C, enters a 30-mm diameter tube at the rate of 3 L/s. Steam at 9 MPa, 400°C exits the tube. Find the rate of heat transfer to the water.

Solution:

C.V. Steam generator. Steady state single inlet and exit flow.

Constant diameter tube: $\quad A_i = A_e = \frac{\pi}{4}(0.03)^2 = 0.0007068 \ m^2$

Table B.1.4 $\quad \dot{m} = \dot{V}_i/v_i = 0.003/0.0010003 = 3.0 \ kg/s$

$\quad \mathbf{V}_i = \dot{V}_i/A_i = 0.003/0.0007068 = 4.24 \ m/s$

Exit state properties from Table B.1.3

$\quad \mathbf{V}_e = \mathbf{V}_i \times v_e/v_i = 4.24 \times 0.02993/0.0010003 = 126.86 \ m/s$

The energy equation Eq. 4.12 is solved for the heat transfer as

$$\dot{Q} = \dot{m}\left[(h_e - h_i) + \left(\mathbf{V}_e^2 - \mathbf{V}_i^2\right)/2\right]$$
$$= 3.0 \ kg/s \times \left[3117.8 - 134.86 + \frac{126.86^2 - 4.24^2}{2 \times 1000}\right] kJ/kg = \mathbf{8973 \ kW}$$

Steam exit

gas out

Typically hot combustion gas in

liquid water in

4.71

An oven has five radiant heaters; each one is rated at 15 kW. It should heat some 2-kg steel plates from 20°C to 800 K. How many of these plates per minute can it heat?

C.V. Oven, steady state operation. A flow of plates in (represents an $\dot{m}$)

Energy Eq.: $\quad 0 = \dot{m}\,(h_i - h_e) + \dot{Q}$

$$\dot{Q} = \dot{m}\,(h_e - h_i) = \dot{m}\,C\,(T_e - T_i)$$

$$\dot{m} = \dot{Q} / C\,(T_e - T_i) = \frac{5 \times 15 \text{ kW}}{0.46\,(800\text{ - }293)\text{ kJ/kg}} = 0.322 \text{ kg/s}$$

$N = \dot{m}/m = (0.322 / 2)\ \ 1/s = 0.161\ 1/s = \textbf{9.65 per min}$

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Front end of oven with steel rollers to bring the plates in
a skirt hangs down to limit heat losses.

4.72

A cryogenic fluid as liquid nitrogen at 90 K, 400 kPa flows into a probe used in cryogenic surgery. In the return line the nitrogen is then at 160 K, 400 kPa. Find the specific heat transfer to the nitrogen. If the return line has a cross sectional area 100 times larger than the inlet line what is the ratio of the return velocity to the inlet velocity?

Solution:

C.V line with nitrogen. No kinetic or potential energy changes

Continuity Eq.: $\dot{m} = \text{constant} = \dot{m}_e = \dot{m}_i = A_e \mathbf{V}_e / v_e = A_i \mathbf{V}_i / v_i$

Energy Eq. 4.13: $q = h_e - h_i$

State i, Table B.6.1: $h_i = -95.58$ kJ/kg, $v_i = 0.001343$ m^3/kg

State e, Table B.6.2: $h_e = 162.96$ kJ/kg, $v_e = 0.11647$ m^3/kg

From the energy equation

$$q = h_e - h_i = 162.96 - (-95.58) = \mathbf{258.5 \ kJ/kg}$$

From the continuity equation

$$\mathbf{V}_e / \mathbf{V}_i = A_i / A_e \ (v_e / v_i) = \frac{1}{100} \ \frac{0.11647}{0.001343} = \mathbf{0.867}$$

4.73

An evaporator has R-410A at -20^oC and quality 20% flowing in, with the exit flow being saturated vapor at -20^oC. Knowing that there is no work, find the specific heat transfer.

C.V. Heater Steady state single inlet and exit flow.

Energy Eq.4.13: $0 = q + h_1 - h_2$

Table B.4: $h_1 = 28.24 + 0.2 \times 243.65 = 76.97$ kJ/kg

$h_2 = 271.89$ kJ/kg

$q = h_2 - h_1 = 271.89 - 76.97 = \textbf{194.92 kJ/kg}$

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4.74

A flow of liquid glycerine flows around an engine, cooling it as it absorbs energy. The glycerine enters the engine at 60°C and receives 19 kW of heat transfer. What is the required mass flow rate if the glycerine should come out at maximum 95° C?

Solution:

C.V. Liquid flow (glycerine is the coolant), steady flow. no work.

Energy Eq.: $\dot{m}h_i + \dot{Q} = \dot{m}h_e$

$$\dot{m} = \dot{Q}/(h_e - h_i) = \frac{\dot{Q}}{C_{gly}(T_e - T_i)}$$

From table A.4 $C_{gly} = 2.42$ kJ/kg-K

$$\dot{m} = \frac{19}{2.42\ (95 - 60)} \frac{kW}{kJ/kg} = \textbf{0.224 kg/s}$$

Air intake filter   Fan Radiator

Shaft power

Atm. air

cb

Exhaust flow

Coolant flow

Pumps, pipe and channel flows

4.75

An irrigation pump takes water from a river at 10^OC, 100 kPa and pumps it up to an open canal, where it flows out 100 m higher at 10^OC. The pipe diameter in and out of the pump is 0.1 m and the motor driving the unit is 5 hp. What is the flow rate neglecting kinetic energy and losses?

C.V. Pump plus pipe to canal.

This is a single steady flow device with atmospheric pressure in and out.

Energy Eq.: $0 = h_i + 0 + gZ_i + w_{p\ in} - h_e - 0 - gZ_e$

$w_{p\ in} = g(Z_e - Z_i) = 9.81 \text{ m/s}^2 \times 100 \text{ (m/s)}^2 = 0.981 \text{ kJ/kg}$

$\dot{W}_{p\ in} = 5 \text{ hp} = 5 \text{ hp} \times 0.746 \text{ kW/hp} = 3.73 \text{ kW}$

Find the flow rate from the work

$\dot{m} = \dot{W}_{p\ in} / w_{p\ in} = 3.73 \text{ kW}/ 0.981 \text{ kJ/kg} = $ **3.8 kg/s**

4.76

A pipe flows water at 15°C from one building to another. In the winter time the pipe loses an estimated 500 W of heat transfer. What is the minimum required mass flow rate that will ensure that the water does not freeze (i.e. reach 0°C)?

Solution:

Energy Eq.: $\dot{m}h_i + \dot{Q} = \dot{m}h_e$

Assume saturated liquid at given T from table B.1.1

$$\dot{m} = \frac{\dot{Q}}{h_e - h_i} = \frac{-500 \times 10^{-3}}{0 - 62.98} \frac{kW}{kJ/kg} = \frac{0.5}{62.98} \ kg/s = \mathbf{0.007\ 94\ kg/s}$$

4.77

A river flowing at 0.5 m/s across a 1-m-high and 10-m-wide area has a dam that creates an elevation difference of 2 m. How much energy could a turbine deliver per day if 80% of the potential energy can be extracted as work?

CV Dam from upstream to downstream 2 m lower elevation

Continuity Eq.: $\dot{m} = \text{constant} = \dot{m}_e = \dot{m}_i = A_e\mathbf{V}_e/v_e = A_i\mathbf{V}_i/v_i$

Find the mass flow rate

$$\dot{m} = A\mathbf{V}_i/v = \rho A\mathbf{V}_i = 997 \text{ kg/m}^3 \times (1 \times 10) \text{ m}^2 \times 0.5 \text{ m/s} = 4985 \text{ kg/s}$$

Energy Eq.: $0 = \dot{m} (h_i + 0.5\mathbf{V}_i^2 + gZ_i) - \dot{m} (h_e + 0.5\mathbf{V}_e^2 + gZ_e) - \dot{W}$

The velocity in and out is the same assuming same flow area and the two enthalpies are also the same since same T and $P = P_0$.

$$\dot{W} = 0.8 \, \dot{m} \, g(Z_i - Z_e) = 0.8 \times 4985 \text{ kg/s} \times 9.807 \text{ m/s}^2 \times 2 \text{ m}$$
$$= 78\,221 \text{ J/s} = 78\,221 \text{ W} = 78.2 \text{ kW}$$

$$W = \dot{W} \, \Delta t = 78.2 \text{ (kJ/s)} \times 24 \times 60 \times 60 \text{ s} = \textbf{6.76 GJ}$$

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4.78

A steam pipe for a 300-m tall building receives superheated steam at 200 kPa at ground level. At the top floor the pressure is 125 kPa and the heat loss in the pipe is 110 kJ/kg. What should the inlet temperature be so that no water will condense inside the pipe?

Solution:

C.V. Pipe from 0 to 300 m, no ΔKE, steady state, single inlet and exit flow.
Neglect any changes in kinetic energy.

Energy Eq. 4.13: $\quad q + h_i = h_e + gZ_e$

No condensation means: Table B.1.2, $\quad h_e = h_g$ at 125 kPa = 2685.4 kJ/kg

$$h_i = h_e + gZ_e - q = 2685.4 + \frac{9.807 \times 300}{1000} - (-110) = 2810.1 \text{ kJ/kg}$$

At 200 kPa: T ~ **170$^{\text{o}}$C** Table B.1.3

4.79

Consider a water pump that receives liquid water at 25^OC, 100 kPa and delivers it to a same diameter short pipe having a nozzle with exit diameter of 2 cm (0.02 m) to the atmosphere 100 kPa. Neglect the kinetic energy in the pipes and assume constant u for the water. Find the exit velocity and the mass flow rate if the pump draws a power of 1 kW.

Solution:

Continuity Eq.: $\dot{m}_i = \dot{m}_e = A\mathbf{V}/v$; $\quad A = \dfrac{\pi}{4}D_e^2 = \dfrac{\pi}{4} \times 0.02^2 = 3.1416 \times 10^{-4} \ m^2$

Energy Eq. 4.13: $\qquad h_i + \dfrac{1}{2}\mathbf{V}_i^2 + gZ_i = h_e + \dfrac{1}{2}\mathbf{V}_e^2 + gZ_e + w$

Properties: $\qquad h_i = u_i + P_i v_i = h_e = u_e + P_e v_e$; $\qquad P_i = P_e$; $\quad v_i = v_e$

$$w = -\dfrac{1}{2}\mathbf{V}_e^2 \quad \Rightarrow \quad -\dot{W} = \dot{m}\,(\dfrac{1}{2}\mathbf{V}_e^2) = A \times \dfrac{1}{2}\mathbf{V}_e^3/v_e$$

$$\mathbf{V}_e = \Big(\dfrac{-2\,\dot{W}\,v_e}{A}\Big)^{1/3} = \Big(\dfrac{2 \times 1000 \times 0.001003}{3.1416 \times 10^{-4}}\Big)^{1/3} = \mathbf{18.55 \ m/s}$$

$$\dot{m} = A\mathbf{V}_e/v_e = 3.1416 \times 10^{-4} \times 18.55 \,/\, 0.001003 = \mathbf{5.81 \ kg/s}$$

4.80

A small water pump is used in an irrigation system. The pump takes water in from a river at $10^{o}C$, 100 kPa at a rate of 5 kg/s. The exit line enters a pipe that goes up to an elevation 20 m above the pump and river, where the water runs into an open channel. Assume the process is adiabatic and that the water stays at $10^{o}C$. Find the required pump work.

Solution:

C.V. pump + pipe. Steady state , 1 inlet, 1 exit flow. Assume same velocity in and out, no heat transfer.

Continuity Eq.: $\dot{m}_{in} = \dot{m}_{ex} = \dot{m}$

Energy Eq. 4.12:

$$\dot{m}(h_{in} + (1/2)\mathbf{V}_{in}^2 + gz_{in}) =$$

$$\dot{m}(h_{ex} + (1/2)\mathbf{V}_{ex}^2 + gz_{ex}) + \dot{W}$$

States: $h_{in} = h_{ex}$ same (T, P)

$$\dot{W} = \dot{m}\, g(z_{in} - z_{ex}) = 5 \text{ kg/s} \times 9.807 \text{ m/s}^2 \times (0 - 20) \text{ m} /(1000 \text{ J/kJ})$$

$$= \mathbf{-0.98 \ kW}$$

I.E. 0.98 kW required input

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4.81

A small stream with water at 15°C runs out over a cliff creating a 50 m tall waterfall. Estimate the downstream temperature when you neglect the horizontal flow velocities upstream and downstream from the waterfall. How fast was the water dropping just before it splashed into the pool at the bottom of the waterfall?

Solution:

CV. Waterfall, steady state. Assume no $\dot{Q}$ nor $\dot{W}$

Energy Eq. 4.13: $h + \frac{1}{2}V^2 + gZ = $ const.

State 1: At the top zero velocity $Z_1 = 100$ m

State 2: At the bottom just before impact, $Z_2 = 0$

State 3: At the bottom after impact in the pool.

$$h_1 + 0 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + 0 = h_3 + 0 + 0$$

Properties: $h_1 \cong h_2$ same T, P

$$\Rightarrow \quad \frac{1}{2}\mathbf{V}_2^2 = gZ_1$$

$$\mathbf{V}_2 = \sqrt{2gZ_1} = \sqrt{2 \times 9.806 \text{ m/s}^2 \times 50 \text{ m}} = \textbf{31.3 m/s}$$

Energy equation from state 1 to state 3

$$h_3 = h_1 + gZ_1$$

use $\Delta h = C_p \Delta T$ with value from Table A.4 (liquid water)

$$T_3 = T_1 + gZ_1 / C_p$$
$$= 20 + 9.806 \text{ m/s}^2 \times 50 \text{ m/ } (4180 \text{ J/kg-K}) = \textbf{20.12 °C}$$

4.82

A cutting tool uses a nozzle that generates a high speed jet of liquid water.

Assume an exit velocity of 500 m/s of 20°C liquid water with a jet diameter of 2 mm (0.002 m). How much mass flow rate is this? What size (power) pump is needed to generate this from a steady supply of 20°C liquid water at 200 kPa?

Solution:

C.V. Nozzle. Steady state, single flow.

Continuity equation with a uniform velocity across A

$$\dot{m} = A\mathbf{V}/v = \frac{\pi}{4} D^2 \, \mathbf{V} / v = \frac{\pi}{4} 0.002^2 \times 500 / 0.001002 = \textbf{1.568 kg/s}$$

Assume $Z_i = Z_e = \emptyset,$ $\quad u_e = u_i$ and $\mathbf{V}_i = 0$ $\quad P_e = 100$ kPa (atmospheric)

Energy Eq.4.13: $\qquad h_i + \emptyset + \emptyset = h_e + \frac{1}{2}\mathbf{V}_e^2 + \emptyset + w$

$$w = h_i - h_e - \frac{1}{2}\mathbf{V}_e^2 = u_i - u_e + P_i \, v_i - P_e \, v_e - \frac{1}{2}\mathbf{V}_e^2$$

$$= (P_i - P_e) \, v_i - \frac{1}{2}\mathbf{V}_e^2$$

$$= 0.001002 \text{ m}^3/\text{kg} \times (200 - 100) \text{ kPa} - 0.5 \times \frac{500^2 \text{ m}^2/\text{s}^2}{1000 \text{ J/kJ}}$$

$$= 0.1002 - 125 \cong -125 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}w = 1.568 \text{ kg/s} \times (-125 \text{ kJ/kg}) = \textbf{-196 kW}$$

4.83

The main waterline into a tall building has a pressure of 600 kPa at 5 m below ground level. A pump brings the pressure up so the water can be delivered at 200 kPa at the top floor 100 m above ground level. Assume a flow rate of 10 kg/s liquid water at 10^OC and neglect any difference in kinetic energy and internal energy u. Find the pump work.

Solution:

C.V. Pipe from inlet at -5 m up to exit at +150 m, 200 kPa.

Energy Eq.4.13: $\quad h_i + \frac{1}{2}V_i^2 + gZ_i = h_e + \frac{1}{2}V_e^2 + gZ_e + w$

With the same u the difference in h's are the Pv terms

$$w = h_i - h_e + \frac{1}{2}(V_i^2 - V_e^2) + g(Z_i - Z_e)$$

$$= P_iv_i - P_ev_e + g(Z_i - Z_e)$$

$$= 600 \times 0.001 - 200 \times 0.001 + 9.806 \times (-5 - 100)/1000$$

$$= 0.4 - 1.03 = -0.63 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}w = 10 \times (-0.63) = \textbf{-6.3 kW}$$

Multiple flow single device processes

Turbines, Compressors, Expanders

4.84

An adiabatic steam turbine in a power plant receives 5 kg/s steam at 3000 kPa, 500°C. Twenty percent of the flow is extracted at 1000 kPa, 350°C to a feedwater heater, and the remainder flows out at 200 kPa, 200°C (see Fig. P4.84). Find the turbine power output.

C.V. Turbine   Steady state, 1 inlet and 2 exit flows.

Continuity Eq.4.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$; => $\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 150$ lbm/s

Energy Eq.4.10: $\dot{m}_1 h_1 = \dot{W}_T + \dot{m}_2 h_2 + \dot{m}_3 h_3$

   Table B.1.2 $h_1 = 3456.48$ kJ/kg,
                   $h_2 = 3157.65$ kJ/kg
   Table B.1.1 : $h_3 = 2870.46$ kJ/kg

From the energy equation, Eq.4.10

   $\dot{W}_T = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = (5 \times 3456.48 - 1 \times 3157.65 - 4 \times 2870.46)$ kJ/s
       = **2643 kW**

4.85

A compressor receives 0.05 kg/s R-410A at 200 kPa, -20°C and 0.1 kg/s R-410A at 400 kPa, 0°C. The exit flow is at 1000 kPa, 60°C as shown in Fig. P4.85. Assume it is adiabatic, neglect kinetic energies and find the required power input.

C.V. whole compressor steady, 2 inlets, 1 exit, no heat transfer $\dot{Q} = 0$

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 0.05 + 0.1 = 0.15$ kg/s

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{W}_C = \dot{m}_3 h_3$

Table B.3.2: $h_1 = 278.72$ kJ/kg,
$h_2 = 290.42$ kJ/kg
Table B.3.2: $h_3 = 335.75$ kJ/kg

$\dot{W}_C = 0.05 \times 278.72 + 0.1 \times 290.42 - 0.15 \times 335.75 = -7.385$ kW

Power input is **7.4 kW**

4.86

Cogeneration is often used where a steam supply is needed for industrial process energy. Assume a supply of 5 kg/s steam at 0.5 MPa is needed. Rather than generating this from a pump and boiler, the setup in Fig. P4.86 is used so the supply is extracted from the high-pressure turbine. Find the power the turbine now cogenerates in this process.

Solution:

C.V. Turbine, steady state, 1 inlet and 2 exit flows, assume adiabatic, $\dot{Q}_{CV} = 0$

Continuity Eq.4.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$

Energy Eq.4.10: $\dot{Q}_{CV} + \dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3 + \dot{W}_T$;

Supply state 1: 20 kg/s at 10 MPa, 500°C

Process steam 2: 5 kg/s, 0.5 MPa, 155°C,

Exit state 3: 20 kPa, x = 0.9

Table B.1.3: $h_1 = 3373.7$, $h_2 = 2755.9$ kJ/kg,

Table B.1.2: $h_3 = 251.4 + 0.9 \times 2358.3$

$= 2373.9$ kJ/kg

$\dot{W}_T = (20 \times 3373.7 - 5 \times 2755.9 - 15 \times 2373.9)$ (kg/s) (kJ/kg)

$= 18\ 084$ kW = **18.084 MW**

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4.87

A steam turbine receives steam from two boilers. One flow is 5 kg/s at 3 MPa, 700°C and the other flow is 10 kg/s at 800 kPa, 500°C. The exit state is 10 kPa, with a quality of 96%. Find the total power out of the adiabatic turbine.

Solution:

C.V. whole turbine steady, 2 inlets, 1 exit, no heat transfer $\dot{Q} = 0$

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 5 + 15 = 20$ kg/s

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{W}_T$

Table B.1.3: $h_1 = 3911.7$ kJ/kg,
$h_2 = 3480.6$ kJ/kg

Table B.1.2: $h_3 = 191.8 + 0.96 \times 2392.8$
$= 2488.9$ kJ/kg

$\dot{W}_T = 5 \times 3911.7 + 10 \times 3480.6 - 15 \times 2488.9 = 17\ 031$ kW = **17 MW**

4.88

A compressor receives 0.1 kg/s R-134a at 150 kPa, -10°C and delivers it at 1000 kPa, 40°C. The power input is measured to be 3 kW. The compressor has heat transfer to air at 100 kPa coming in at 20°C and leaving at 30°C. How much is the mass flow rate of air?

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also have an air flow outside the compressor housing no changes in kenetic or potential energy.

Continuity Eq.: $\dot{m}_2 = \dot{m}_1$

Energy Eq. 4.12: $\dot{m}_1 h_1 + \dot{W}_{in} + \dot{m}_{air} h_3 = \dot{m}_2 h_2 + \dot{m}_{air} h_4$

Ideal gas for air and constant specific heat: $h_4 - h_3 \sim C_{p\ air} (T_4 - T_3)$

$$\dot{m}_{air} = [\dot{m}_1 (h_1 - h_2) + \dot{W}_{in}] / C_{p\ air} (T_4 - T_3)$$

$$= \frac{0.1\ (393.84 - 420.25) + 3}{1.004\ (30 - 20)} = \frac{0.359}{10} \frac{kW}{kJ/kg}$$

$$= \textbf{0.0359 kg/s}$$

4.89

Two steady flows of air enters a control volume, shown in Fig. P4.89. One is 0.025 kg/s flow at 350 kPa, 150°C, state 1, and the other enters at 450 kPa, 15°C, state 2. A single flow of air exits at 100 kPa, –40°C, state 3. The control volume rejects 1 kW heat to the surroundings and produces 4 kW of power. Neglect kinetic energies and determine the mass flow rate at state 2.

Solution:

C.V. Steady device with two inlet and one exit flows, we neglect kinetic energies. Notice here the Q is rejected so it goes out.

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 0.025 + \dot{m}_2$

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{W}_{CV} + \dot{Q}_{loss}$

Substitute the work and heat transfer into the energy equation and use constant specific heat

$$0.025 \times 1.004 \times 423.2 + \dot{m}_2 \times 1.004 \times 288.2$$

$$= (0.025 + \dot{m}_2)\, 1.004 \times 233.2 + 4.0 + 1.0$$

Now solve for $\dot{m}_2$.

$$\dot{m}_2 = \frac{4.0 + 1.0 + 0.025 \times 1.004 \times (233.2 - 423.2)}{1.004\,(288.2 - 233.2)} \text{ kW/(kJ/kg)}$$

$$= \textbf{0.0042 kg/s}$$

4.90

A large expansion engine has two low velocity flows of water entering. High pressure steam enters at point 1 with 2.0 kg/s at 2 MPa, 500°C and 0.5 kg/s cooling water at 120 kPa, 30°C enters at point 2. A single flow exits at point 3 with 150 kPa, 80% quality, through a 0.15 m diameter exhaust pipe. There is a heat loss of 300 kW. Find the exhaust velocity and the power output of the engine.

Solution:

C.V. : Engine (Steady state)

Constant rates of flow, $\dot{Q}_{loss}$ and $\dot{W}$

State 1:   Table B.1.3: $h_1 = 3467.6$ kJ/kg

State 2:   Table B.1.1: $h_2 = 125.77$ kJ/kg

$h_3 = 467.1 + 0.8 \times 2226.5 = 2248.3$ kJ/kg

$v_3 = 0.00105 + 0.8 \times 1.15825 = 0.92765$ m^3/kg

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 2 + 0.5 = 2.5$ kg/s $= (AV/v) = (\pi/4)D^2V/v$

Energy Eq.4.10: $\dot{m}_1h_1 + \dot{m}_2h_2 = \dot{m}_3(h_3 + 0.5\ V^2) + \dot{Q}_{loss} + \dot{W}$

$\mathbf{V} = \dot{m}_3 v_3 / [\frac{\pi}{4}D^2] = 2.5 \times 0.92765 / (0.7854 \times 0.15^2) = \mathbf{131.2\ m/s}$

$0.5\ \mathbf{V}^2 = (0.5 \times 131.2^2\ \text{m}^2/\text{s}^2)/1000\ \text{J/kJ} = 8.6$ kJ/kg

$\dot{W} = 2 \times 3467.6 + 0.5 \times 125.77 - 2.5\ (2248.3 + 8.6) - 300 = \mathbf{1056\ kW}$

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Heat Exchangers

4.91

A condenser (heat exchanger) brings 1 kg/s water flow at 10 kPa quality 95% to saturated liquid at 10 kPa, as shown in Fig. P4.91. The cooling is done by lake water at 20°C that returns to the lake at 30°C. For an insulated condenser, find the flow rate of cooling water.

Solution:

C.V. Heat exchanger

Table B.1.1: $h_{20} = 83.94$ kJ/kg , $h_{30} = 125.77$ kJ/kg

Table B.1.2: $h_{95\%,\ 10kPa} = 191.81 + 0.95 \times 2392.82 = 2465$ kJ/kg

$h_{f,\ 10\ kPa} = 191.81$ kJ/kg

Energy Eq.4.10: $\dot{m}_{cool}h_{20} + \dot{m}_{H_2O}h_{300} = \dot{m}_{cool}h_{30} + \dot{m}_{H_2O}h_{f,\ 10\ kPa}$

$$\dot{m}_{cool} = \dot{m}_{H_2O}\ \frac{h_{95\%} - h_{f,\ 10kPa}}{h_{30} - h_{20}} = 1\ \text{kg/s} \times \frac{2465 - 191.81}{125.77 - 83.94} = \mathbf{54.3\ kg/s}$$

4.92

Air at 600 K flows with 3 kg/s into a heat exchanger and out at 100°C. How much (kg/s) water coming in at 100 kPa, 20°C can the air heat to the boiling point?

C.V. Total heat exchanger. The flows are not mixed so the two flowrates are constant through the device. No external heat transfer and no work.

Energy Eq.4.10: $\dot{m}_{air}h_{air\ in} + \dot{m}_{water}h_{water\ in} = \dot{m}_{air}h_{air\ out} + \dot{m}_{water}h_{water\ out}$

$\dot{m}_{air}[h_{air\ in} - h_{air\ out}] = \dot{m}_{water}[h_{water\ out} - h_{water\ in}]$

Table B.1.2: $h_{water\ out} - h_{water\ in} = 2675.46 - 83.94 = 2591.5$ kJ/kg

Table A.7.1: $h_{air\ in} - h_{air\ out} = 607.32 - 374.14 = 233.18$ kJ/kg

Solve for the flow rate of water from the energy equation

$$\dot{m}_{water} = \dot{m}_{air}\frac{h_{air\ in} - h_{air\ out}}{h_{water\ out} - h_{water\ in}} = 3 \text{ kg/s} \times \frac{233.18}{2591.5} = \textbf{0.27 kg/s}$$

4.93

Steam at 500 kPa, 300°C is used to heat cold water at 15°C to 75°C for domestic hot water supply. How much steam per kg liquid water is needed if the steam should not condense?

Solution:
C.V. Each line separately. No work but there is heat transfer out of the steam flow and into the liquid water flow.

Water line energy Eq.: $\dot{m}_{liq}h_i + \dot{Q} = \dot{m}_{liq}h_e \Rightarrow \dot{Q} = \dot{m}_{liq}(h_e - h_i)$
For the liquid water look in Table B.1.1

$$\Delta h_{liq} = h_e - h_i = 313.91 - 62.98 = 250.93 \text{ kJ/kg}$$
$$(\cong C_p \, \Delta T = 4.18 \, (75 - 15) = 250.8 \text{ kJ/kg })$$

Steam line energy has the same heat transfer but it goes out

Steam Energy Eq.: $\dot{m}_{steam}h_i = \dot{Q} + \dot{m}_{steam}h_e \Rightarrow \dot{Q} = \dot{m}_{steam}(h_i - h_e)$

For the steam look in Table B.1.3 at 500 kPa

$$\Delta h_{steam} = h_i - h_e = 3064.2 - 2748.67 = 315.53 \text{ kJ/kg}$$

Now the heat transfer for the steam is substituted into the energy equation for the water to give

$$\dot{m}_{steam} / \dot{m}_{liq} = \Delta h_{liq} / \Delta h_{steam} = \frac{250.93}{315.53} = \textbf{0.795}$$

4.94

A dual-fluid heat exchanger has 5 kg/s water entering at 40°C, 150 kPa and leaving at 10°C, 150 kPa. The other fluid is glycol, entering at −10°C, 160 kPa and leaving at 10°C, 160 kPa. Find the required mass flow rate of glycol and the rate of internal heat transfer.

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for glycol and water each. The two flows exchange energy with no heat transfer to/from the outside.

Continuity Eqs.: Each line has a constant flow rate through it.

Energy Eq.4.10: $\dot{m}_{H_2O}\, h_1 + \dot{m}_{glycol}\, h_3 = \dot{m}_{H_2O}\, h_2 + \dot{m}_{glycol}\, h_4$

Process: Each line has a constant pressure.

Table B.1: $h_1 = 167.54$ kJ/kg, $h_2 = 41.99$ kJ/kg

Table A.4: $C_{P\ gly} = 2.42$ kJ/kg-K so

$$h_4 - h_3 = C_{P\ gly}\,(T_4 - T_3) = 2.42\,[10 - (-10)] = 48.4 \text{ kJ/kg}$$

$$\dot{m}_{glycol} = \dot{m}_{H_2O}\,\frac{h_1 - h_2}{h_4 - h_3} = 5\,\frac{167.54 - 41.99}{48.4} = \mathbf{12.97\ kg/s}$$

4.95

A heat exchanger, shown in Fig. P4.95, is used to cool an air flow from 800 K to 360 K, both states at 1 MPa. The coolant is a water flow at 15°C, 0.1 MPa. If the water leaves as saturated vapor, find the ratio of the flow rates $\dot{m}_{H_2O}/\dot{m}_{air}$

Solution:

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside.

Continuity Eqs.:   Each line has a constant flow rate through it.

Energy Eq.4.10: $\dot{m}_{air}h_1 + \dot{m}_{H_2O}h_3 = \dot{m}_{air}h_2 + \dot{m}_{H_2O}h_4$

Process: Each line has a constant pressure.

Air states, Table A.7.1: $h_1 = 822.20$ kJ/kg, $h_2 = 360.86$ kJ/kg

Water states, Table B.1.1: $h_3 = 62.98$ kJ/kg (at 15°C),

Table B.1.2: $h_4 = 2675.5$ kJ/kg (at 100 kPa)

$$\dot{m}_{H_2O}/\dot{m}_{air} = \frac{h_1 - h_2}{h_4 - h_3} = \frac{822.20 - 360.86}{2675.5 - 62.99} = \mathbf{0.1766}$$

4.96

A superheater brings 2.5 kg/s saturated water vapor at 2 MPa to 450°C. The energy is provided by hot air at 1200 K flowing outside the steam tube in the opposite direction as the water, which is a counter flowing heat exchanger. Find the smallest possible mass flow rate of the air so the air exit temperature is 20°C larger than the incoming water temperature (so it can heat it).

Solution:

C.V. Superheater. Steady state with no external $\dot{Q}$ or any $\dot{W}$ the two flows exchanges energy inside the box. Neglect kinetic and potential energies at all states.

Energy Eq.4.10:　$\dot{m}_{H_2O}\, h_3 + \dot{m}_{air}\, h_1 = \dot{m}_{H_2O}\, h_4 + \dot{m}_{air}\, h_2$

Process:　Constant pressure in each line.

State 1: Table B.1.2　　$T_3 = 212.42°C, \ h_3 = 2799.51$ kJ/kg

State 2: Table B.1.3　　$h_4 = 3357.48$ kJ/kg

State 3: Table A.7　　$h_1 = 1277.81$ kJ/kg

State 4:　　　$T_2 = T_3 + 20 = 232.42°C = 505.57$ K

A.7 :　$h_2 = 503.36 + \dfrac{5.57}{20}\,(523.98 - 503.36) = 509.1$ kJ/kg

From the energy equation we get

$$\dot{m}_{air} = \dot{m}_{H_2O}\ (h_4 - h_3)/(h_1 - h_2)$$

$$= 2.5\,(3357.48 - 2799.51) / (1277.81 - 509.1) = \textbf{1.815 kg/s}$$

4.97

A two fluid heat exchanger has 2 kg/s liquid ammonia at 20°C, 1003 kPa entering state 3 and exiting at state 4. It is heated by a flow of 1 kg/s nitrogen at 1500 K, state 1, leaving at 600 K, state 2 similar to Fig. P4.95. Find the total rate of heat transfer inside the heat exchanger. Sketch the temperature versus distance for the ammonia and find state 4 (T, v) of the ammonia.

Solution:

CV: Nitrogen flow line, steady rates of flow, $\dot{Q}$ out and $\dot{W} = 0$

Continiuty: $\dot{m}_1 = \dot{m}_2 = 1$ kg/s ;   Energy Eq: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{Q}_{out}$

Tbl. A.8: $h_1 = 1680.7$ kJ/kg; $h_2 = 627.24$ kJ/kg

$\dot{Q}_{out} = \dot{m}_1(h_1 - h_2) = 1 (1680.7 - 627.24) = $ **1053.5 kW**

If Tbl A.5 is used: Cp = 1.042 kJ/kg K

$\dot{Q}_{out} = \dot{m}_1 Cp (T_1 - T_2) = 1 \times 1.042 (1500 - 600) = $ **937.8 kW**

CV The whole heat exchanger: No external $\dot{Q}$, constant pressure in each line.

$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_1 h_2 + \dot{m}_3 h_4$ => $h_4 = h_3 + \dot{m}_1(h_1 - h_2)/\dot{m}_3$

$h_4 = 274.3 + 1053.5 /2 = 801$ kJ/kg < h_g =>   2-phase

$x_4 = (h_4 - h_f)/ h_{fg} = (801 - 298.25) / 1165.2 = 0.43147$

$v_4 = v_f + x_4 v_{fg} = 0.001658 + 0.43147 \times 0.12647 = 0.05623$ m^3/kg

$T_4 = T_{3a} = 25^o$C   This is the boiling temperature for 1003 kPa.

4.98

In a co-flowing (same direction) heat exchanger 1 kg/s air at 500 K flows into one channel and 2 kg/s air flows into the neighboring channel at 300 K. If it is infinitely long what is the exit temperature? Sketch the variation of T in the two flows.

C.V. Heat Exchanger (no $\dot{W}$)

Continuity Eq.: $\qquad \dot{m}_1 = \dot{m}_3$ and $\dot{m}_2 = \dot{m}_4$

Energy Eq.4.10: $\qquad \dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_1 h_3 + \dot{m}_2 h_4$

Same exit T: $\qquad h_3 = h_4 = [\dot{m}_1 h_1 + \dot{m}_2 h_2] / [\dot{m}_1 + \dot{m}_2]$

Using conctant specific heat

$$T_3 = T_4 = \frac{\dot{m}_1}{\dot{m}_1 + \dot{m}_2}\ T_1 + \frac{\dot{m}_2}{\dot{m}_1 + \dot{m}_2}\ T_2 = \tfrac{1}{3} \times 500 + \tfrac{2}{3} \times 300 = \mathbf{367\ K}$$

4.99

An air/water counter flowing heat exchanger has one line with 2 kg/s at 125 kPa, 1000 K entering, and the air is leaving at 100 kPa, 400 K. The other line has 0.5 kg/s water entering at 200 kPa, 20°C and leaving at 200 kPa. What is the exit temperature of the water?

C.V. Heat Exchanger (no $\dot{W}$)

Continuity Eq.: $\dot{m}_1 = \dot{m}_3 = \dot{m}_{H_2O}$ and $\dot{m}_2 = \dot{m}_4 = \dot{m}_{air}$

Energy Eq.4.10: $\dot{m}_{H_2O}h_1 + \dot{m}_{air}h_4 = \dot{m}_{H_2O}h_3 + \dot{m}_{air}h_2$

Flowrates and properties known except for T_1

$$h_1 = h_3 + \dot{m}_{air}\,(h_2 - h_4) / \dot{m}_{H_2O}$$

$$= 83.94 + 2\,(1046.22 - 401.3) / 0.5 = 2663.6 \text{ kJ/kg}$$

$h_1 < h_1 = 2706.6$ kJ/kg at 200 kPa so two-phase   T = **120.23°C**

The water is not completely evaporated at the exit.

Borgnakke and Sonntag

4.100

An automotive radiator has glycerine at 95°C enter and return at 55°C as shown in Fig. P4.100. Air flows in at 20°C and leaves at 25°C. If the radiator should transfer 25 kW what is the mass flow rate of the glycerine and what is the volume flow rate of air in at 100 kPa?

Solution:

If we take a control volume around the whole radiator then there is no external heat transfer - it is all between the glycerin and the air. So we take a control volume around each flow separately.

Glycerine: $\dot{m}h_i + (-\dot{Q}) = \dot{m}h_e$

Table A.4: $\dot{m}_{gly} = \dfrac{-\dot{Q}}{h_e - h_i} = \dfrac{-\dot{Q}}{C_{gly}(T_e - T_i)} = \dfrac{-25}{2.42(55 - 95)} = \textbf{0.258 kg/s}$

Air $\dot{m}h_i + \dot{Q} = \dot{m}h_e$

Table A.5: $\dot{m}_{air} = \dfrac{\dot{Q}}{h_e - h_i} = \dfrac{\dot{Q}}{C_{air}(T_e - T_i)} = \dfrac{25}{1.004(25 - 20)} = 4.98 \text{ kg/s}$

$$\dot{V} = \dot{m}v_i ; \quad v_i = \frac{RT_i}{P_i} = \frac{0.287 \times 293}{100} \frac{\text{kJ/kg-K} \times \text{K}}{\text{kPa}} = 0.8409 \text{ m}^3/\text{kg}$$

$$\dot{V}_{air} = \dot{m}v_i = 4.98 \times 0.8409 = \textbf{4.19 m}^3\textbf{/s}$$

Air intake filter

95°C   Atm. air

Shaft power

Exhaust flow

Coolant flow 55°C

4.101

A cooler in an air conditioner brings 0.5 kg/s air at 35°C to 5°C, both at 101 kPa and it then mix the output with a flow of 0.25 kg/s air at 20°C, 101 kPa sending the combined flow into a duct. Find the total heat transfer in the cooler and the temperature in the duct flow.

Solution:

Cooler section   Mixing section

C.V. Cooler section (no $\dot{W}$)

Energy Eq.4.12: $\dot{m}h_1 = \dot{m}h_2 + \dot{Q}_{cool}$

$\dot{Q}_{cool} = \dot{m}(h_1 - h_2) = \dot{m}\,C_p\,(T_1 - T_2) = 0.5 \times 1.004 \times (35\text{-}5) = 15.06$ kW

C.V. mixing section (no $\dot{W}, \dot{Q}$)

Continuity Eq.: $\dot{m}_2 + \dot{m}_3 = \dot{m}_4$

Energy Eq.4.10: $\dot{m}_2 h_2 + \dot{m}_3 h_3 = \dot{m}_4 h_4$

$\dot{m}_4 = \dot{m}_2 + \dot{m}_3 = 0.5 + 0.25 = 0.75$ kg/s

$\dot{m}_4 h_4 = (\dot{m}_2 + \dot{m}_3)h_4 = \dot{m}_2 h_2 + \dot{m}_3 h_3$

$\dot{m}_2 (h_4 - h_2) + \dot{m}_3 (h_4 - h_3) = \emptyset$

$\dot{m}_2 C_p (T_4 - T_2) + \dot{m}_3 C_p (T_4 - T_3) = \emptyset$

$T_4 = (\dot{m}_2 / \dot{m}_4) T_2 + (\dot{m}_3 / \dot{m}_4) T_3 = 5(0.5/0.75) + 20(0.25/0.75) = \mathbf{10°C}$

4.102

A copper wire has been heat treated to 1000 K and is now pulled into a cooling chamber that has 1.5 kg/s air coming in at 20°C; the air leaves the other end at 60°C. If the wire moves 0.25 kg/s copper, how hot is the copper as it comes out?

Solution:
C.V. Total chamber, no external heat transfer

Energy eq.: $\dot{m}_{cu}\, h_{i\ cu} + \dot{m}_{air}\, h_{i\ air} = \dot{m}_{cu}\, h_{e\ cu} + \dot{m}_{air}\, h_{e\ air}$

$$\dot{m}_{cu}\, (h_e - h_i)_{cu} = \dot{m}_{air}(h_i - h_e)_{air}$$

$$\dot{m}_{cu}\, C_{cu}\, (T_e - T_i)_{cu} = \dot{m}_{air}\, C_{p\ air}(T_e - T_i)_{air}$$

Heat capacities from A.3 for copper and A.5 for air

$$(T_e - T_i)_{cu} = \frac{\dot{m}_{air} C_{p\ air}}{\dot{m}_{cu} C_{cu}} (T_e - T_i)_{air} = \frac{1.5 \times 1.004}{0.25 \times 0.42} (20 - 60)\ K = -573.7\ K$$

$$T_e = T_i - 573.7 = 1000 - 573.7 = \textbf{426.3 K}$$

4.103

A coflowing (same-direction) heat exchanger has one line with 0.25 kg/s oxygen at 17°C, 200 kPa entering and the other line has 0.6 kg/s nitrogen at 150 kPa, 500 K entering. The heat exchanger is very long, so the two flows exit at the same temperature. Use constant heat capacities and find the exit temperature.

C.V. Heat Exchanger (no $\dot{W}$)

Continuity Eq.: $\dot{m}_1 = \dot{m}_3$ and $\dot{m}_2 = \dot{m}_4$

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_1 h_3 + \dot{m}_2 h_4$

Same exit T: $T_3 = T_4 = T_{ex}$

Using conctant specific heat

$$\dot{m}_1 C_{P\,O2} T_1 + \dot{m}_2 C_{P\,N2} T_2 = \dot{m}_1 C_{P\,O2} T_{ex} + \dot{m}_2 C_{P\,N2} T_{ex}$$

$$T_{ex} = \frac{\dot{m}_1 C_{P\,O2}}{\dot{m}_1 C_{P\,O2} + \dot{m}_2 C_{P\,N2}}\, T_1 + \frac{\dot{m}_2 C_{P\,N2}}{\dot{m}_1 C_{P\,O2} + \dot{m}_2 C_{P\,N2}}\, T_2$$

$$= \frac{0.25 \times 0.922 \times 290 + 0.6 \times 1.042 \times 500}{0.25 \times 0.922 + 0.6 \times 1.042} = \textbf{443.4 K}$$

Mixing processes

4.104

Two flows of air are both at 200 kPa; one has 1 kg/s at 400 K and the other has 2 kg/s at 290 K. The two flows are mixed together in an insulated box to produce a single exit flow at 200 kPa. Find the exit temperature.

Solution:

Cont.: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

$$= (\dot{m}_1 + \dot{m}_2) h_3$$

Mixing chamber

Solve for the exit enthalpy

$$h_3 = [\, \dot{m}_1 h_1 + \dot{m}_2 h_2 \,] / (\dot{m}_1 + \dot{m}_2)$$

since the T's are modest use constant specific heats

$$T_3 = [\, \dot{m}_1 T_1 + \dot{m}_2 T_2 \,] / (\dot{m}_1 + \dot{m}_2) = \tfrac{1}{3} \times 400 + \tfrac{2}{3} \times 290 = \mathbf{326.7\ K}$$

4.105

Two air flows are combined to a single flow. Flow one is 1 m^3/s at 20°C and the other is 2 m^3/s at 200°C both at 100 kPa. They mix without any heat transfer to produce an exit flow at 100 kPa. Neglect kinetic energies and find the exit temperature and volume flow rate.

  Solution:

Cont. $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

                  $= (\dot{m}_1 + \dot{m}_2) h_3$

$\dot{m}_1 (h_3 - h_1) + \dot{m}_2 (h_3 - h_2) = 0$

$\dot{m}_1 C_p (T_3 - T_1) + \dot{m}_2 C_p (T_3 - T_2) = 0$

$T_3 = (\dot{m}_i / \dot{m}_3) / T_1 + (\dot{m}_2 / \dot{m}_3) T_2$

We need to find the mass flow rates

$v_1 = RT_1/P_1 = (0.287 \text{ kJ/kg-K} \times 293 \text{ K})/100 \text{ kPa} = 0.8409 \text{ m}^3/\text{kg}$

$v_2 = RT_2/P_2 = (0.287 \text{ kJ/kg-K} \times 473 \text{ K})/100 \text{ kPa} = 1.3575 \text{ m}^3/\text{kg}$

$\dot{m}_1 = \dfrac{\dot{V}_1}{v_1} = \dfrac{1}{0.8409} = 1.1892 \dfrac{\text{kg}}{\text{s}}, \quad \dot{m}_2 = \dfrac{\dot{V}_2}{v_2} = \dfrac{2}{1.3575} = 1.4733 \dfrac{\text{kg}}{\text{s}}$

$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 2.6625 \text{ kg/s}$

$T_3 = \dfrac{1.1892}{2.6625} \times 20 + \dfrac{1.4733}{2.6625} \times 200 = \textbf{119.6}^{\textbf{o}} \textbf{ C}$

$v_3 = \dfrac{RT_3}{P_3} = \dfrac{0.287 \,(119.6 + 273)}{100} = 1.1268 \text{ m}^3/\text{kg}$

$\dot{V}_3 = \dot{m}_3 \, v_3 = 2.6625 \text{ kg/s} \times 1.1268 \text{ m}^3/\text{kg} = \textbf{3.0 m}^{\textbf{3}}\textbf{/s}$

4.106

A flow of water at 2000 kPa, 20°C is mixed with a flow of 2 kg/s water at 2000 kPa, 180°C. What should the flowrate of the first flow be to produce an exit state of 200 kPa and 100°C?

Solution:

C.V. Mixing chamber and valves. Steady state no heat transfer or work terms.

Continuity Eq.4.9:　　　　　　　$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq.4.10:　　　　　　　$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$

Properties　Table B.1.1:　　$h_1 = 85.8$ kJ/kg;　　$h_3 = 419.0$ kJ/kg

　　　　　　Table B.1.4:　　$h_2 = 763.7$ kJ/kg

$$\dot{m}_1 = \dot{m}_2 \times \frac{h_2 - h_3}{h_3 - h_1} = 2 \times \frac{763.7 - 419.0}{419.0 - 85.8} = \textbf{2.069 kg/s}$$

4.107

An open feedwater heater in a powerplant heats 4 kg/s water at 45°C, 100 kPa by mixing it with steam from the turbine at 100 kPa, 250°C. Assume the exit flow is saturated liquid at the given pressure and find the mass flow rate from the turbine.

Solution:

C.V.   Feedwater heater.

No external $\dot{Q}$ or $\dot{W}$

MIXING CHAMBER

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$

State 1:   Table B.1.1 $h = h_f = 188.42$ kJ/kg   at 45°C
State 2:   Table B.1.3 $h_2 = 2974.33$ kJ/kg
State 3:   Table B.1.2 $h_3 = h_f = 417.44$ kJ/kg at 100 kPa

$$\dot{m}_2 = \dot{m}_1 \times \frac{h_1 - h_3}{h_3 - h_2} = 4 \times \frac{188.42 - 417.44}{417.44 - 2974.33} = \mathbf{0.358 \ kg/s}$$

4.108

Two flows are mixed to form a single flow. Flow at state 1 is 1.5 kg/s water at 400 kPa, 200°C and flow at state 2 is 500 kPa, 100°C. Which mass flow rate at state 2 will produce an exit $T_3 = 150^O$C if the exit pressure is kept at 300 kPa?

Solution:

C.V. Mixing chamber and valves. Steady state no heat transfer or work terms.

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$

Properties   Table B.1.3: $h_1 = 2860.51$ kJ/kg; $h_3 = 2760.95$ kJ/kg

Table B.1.4: $h_2 = 419.32$ kJ/kg

$$\dot{m}_2 = \dot{m}_1 \times \frac{h_1 - h_3}{h_3 - h_2} = 1.5 \times \frac{2860.51 - 2760.95}{2760.95 - 419.32} = \textbf{0.0638 kg/s}$$

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4.109

A de-superheater has a flow of ammonia 1.5 kg/s at 1000 kPa, 100°C which is mixed with another flow of ammonia at 25°C and quality 50% in an adiabatic mixing chamber. Find the flow rate of the second flow so the outgoing ammonia is saturated vapor at 1000 kPa.

C.V.   Desuperheater.

No external $\dot{Q}$ or $\dot{W}$

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$

State 1:   Table B.2.2 $h_1 = 1664.3$ kJ/kg
State 2:   Table B.2.1 $h_2 = 298.25 + 0.5 \times 1165.2 = 880.85$ kJ/kg
State 3:   Table B.2.2 $h_3 = 1463.4$ kJ/kg

$$\dot{m}_2 = \dot{m}_1 \times \frac{h_1 - h_3}{h_3 - h_2} = 1.5 \times \frac{1664.3 - 1463.4}{1463.4 - 880.85} = \textbf{0.517 kg/s}$$

4.110

A mixing chamber with heat transfer receives 2 kg/s of R-410A at 1 MPa, 40°C in one line and 1 kg/s of R-410A at 15°C, quality 50% in a line with a valve. The outgoing flow is at 1 MPa, 60°C. Find the rate of heat transfer to the mixing chamber.

Solution:

C.V. Mixing chamber. Steady with 2 flows in and 1 out, heat transfer in.

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$;   => $\dot{m}_3 = 2 + 1 = 3$ kg/s

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{Q} = \dot{m}_3 h_3$

Properties: Table B.4.2: $h_1 = 316.05$ kJ/kg, $h_3 = 335.75$ kJ/kg

Table B.4.1: $h_2 = 81.15 + 0.5 \times 201.64 = 181.97$ kJ/kg

Energy equation then gives the heat transfer as

$\dot{Q} = 3 \times 335.75 - 2 \times 316.05 - 1 \times 181.97 = $ **193.18 kW**

4.111

A geothermal supply of hot water at 500 kPa, 150°C is fed to an insulated flash evaporator at the rate of 1.5 kg/s. A stream of saturated liquid at 200 kPa is drained from the bottom of the chamber, and a stream of saturated vapor at 200 kPa is drawn from the top and fed to a turbine. Find the mass flow rate of the two exit flows.

C.V. Flash chamber. Disregard any kinetic and potential energies

Continuity Eq.: $0 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3$

Energy Eq.: $0 = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$

Process: $\dot{Q} = 0, \ \dot{W} = 0$
$h_1 = 632.2$ kJ/kg   from Table B.1.4 (B.1.1: 632.18)

State 2 and 3:   Saturated vapor $h_2 = 2706.63$ kJ/kg
Saturated liquid $h_3 = 504.68$ kJ/kg

Substitute $\dot{m}_3$ from continuity eq. into the energy eq. to get

$$0 = \dot{m}_1 h_1 - \dot{m}_2 h_2 - (\dot{m}_1 - \dot{m}_2) h_3$$

$$\dot{m}_2 = \dot{m}_1 (h_1 - h_3) / (h_2 - h_3) = \dot{m}_1 (h_1 - h_f) / (h_g - h_f)$$
$$= 1.5 (632.2 - 504.68) /(2706.63 - 504.68) = \mathbf{0.0869 \ kg/s}$$

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 1.5 - 0.0869 = \mathbf{1.413 \ kg/s}$$

The fraction of the flow at state 2 equals the quality x in the flow right after the valve and the fraction going out at state 3 is $(1 - x)$

Two-phase out of the valve. The liquid drops to the bottom.

4.112

An insulated mixing chamber receives 2 kg/s R-134a at 1 MPa, 100°C in a line with low velocity. Another line with R-134a as saturated liquid 60°C flows through a valve to the mixing chamber at 1 MPa after the valve, as shown in Fig. P4.110. The exit flow is saturated vapor at 1 MPa flowing at 20 m/s. Find the flow rate for the second line.

Solution:

C.V. Mixing chamber. Steady state, 2 inlets and 1 exit flow.

   Insulated q = 0,   No shaft or boundary motion   w = 0.

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$;

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 (h_3 + \frac{1}{2} \mathbf{V}_3^2)$

$$\dot{m}_2 (h_2 - h_3 - \frac{1}{2} \mathbf{V}_3^2) = \dot{m}_1 (h_3 + \frac{1}{2} \mathbf{V}_3^2 - h_1)$$

1: Table B.5.2:   1 MPa, 100°C, $h_1 = 483.36$ kJ/kg

2: Table B.5.1: $x = \varnothing$, 60°C, $h_2 = 287.79$ kJ/kg

3: Table B.5.1: $x = 1$, 1 MPa, 20 m/s, $h_3 = 419.54$ kJ/kg

Now solve the energy equation for $\dot{m}_2$

$$\dot{m}_2 = 2 \times [419.54 + \frac{1}{2} 20^2 \times \frac{1}{1000} - 483.36] / [287.79 - 419.54 - \frac{1}{2} \frac{20^2}{1000}]$$

$$= 2 \times [-63.82 + 0.2] / [-131.75 - 0.2] = \textbf{0.964 kg/s}$$

Notice how kinetic energy was insignificant.

4.113

To keep a jet engine cool some intake air bypasses the combustion chamber. Assume 2 kg/s hot air at 2000 K, 500 kPa is mixed with 1.5 kg/s air 500 K, 500 kPa without any external heat transfer. Find the exit temperature by using constant specific heat from Table A.5.

Solution:

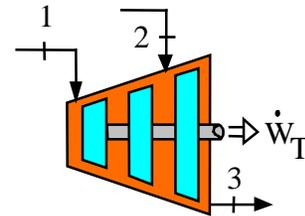C.V.   Mixing Section

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$ => $\dot{m}_3 = 2 + 1.5 = 3.5$ kg/s

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

$$h_3 = (\dot{m}_1 h_1 + \dot{m}_2 h_2) / \dot{m}_3 \; ;$$

For a constant specific heat divide the equation for h_3 with C_p to get

$$T_3 = \frac{\dot{m}_1}{\dot{m}_3} T_1 + \frac{\dot{m}_2}{\dot{m}_3} T_2 = \frac{2}{3.5} 2000 + \frac{1.5}{3.5} 500 = \textbf{1357 K}$$

Mixing section

4.114

Solve the previous problem using values from Table A.7
To keep a jet engine cool some intake air bypasses the combustion chamber.
Assume 2 kg/s hot air at 2000 K, 500 kPa is mixed with 1.5 kg/s air 500 K, 500
kPa without any external heat transfer. Find the exit temperature by using values
from Table A.7.

Solution:

C.V.   Mixing Section

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$ => $\dot{m}_3 = 2 + 1.5 = 3.5$ kg/s

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

$$h_3 = (\dot{m}_1 h_1 + \dot{m}_2 h_2) / \dot{m}_3 \; ;$$

Using A.7 we look up the h at states 1 and 2 to calculate h_3

$$h_3 = \frac{\dot{m}_1}{\dot{m}_3} h_1 + \frac{\dot{m}_2}{\dot{m}_3} h_2 = \frac{2}{3.5} 2251.58 + \frac{1.5}{3.5} 503.36 = 1502 \text{ kJ/kg}$$

Now we can backinterpolate to find at what temperature do we have that h

$$T_3 = 1350 + 50 \frac{1502 - 1455.43}{1515.27 - 1455.43} = \textbf{1389 K}$$

This procedure is the most accurate.

Mixing section

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Multiple Devices, Cycle Processes

4.115

A flow of 5 kg/s water at 100 kPa, 20°C should be delivered as steam at 1000 kPa, 350°C to some application. Consider compressing it to 1000 kPa, 20°C and then heat it at constant 1000 kPa to 350°C. Which devices are needed and find the specific energy transfers in those devices.

To raise the pressure of a liquid flow requires a pump which delivers the difference between the flow work going out and in. Constant pressure heating is a simple heater (or heat exchanger).

Inlet state: B.1.1: $h_1 = 83.94$ kJ/kg, $v_1 = 0.001002$ m^3/kg

State 3: $h_3 = 3157.65$ kJ/kg

The pump delivers the difference between the flow work out and flow work in.

Pump: $-w_p = (P_2 - P_1) \, v_1 = (1000 - 100) \text{ kPa} \times 0.001002 \text{ m}^3/\text{kg}$

                 $= \textbf{0.9 kJ/kg}$.

Heater: $q = h_3 - h_2 = h_3 - (h_1 - w_p)$

               $= 3157.65 - (83.94 + 0.9) = \textbf{3072.8 kJ/kg}$

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4.116

A two-stage compressor takes nitrogen in at 20°C, 150 kPa and compresses it to 600 kPa, 450 K. Then it flows through an intercooler, where it cools to 320 K, and the second stage compresses it to 3000 kPa, 530 K. Find the specific work in each of the two compressor stages and the specific heat transfer in the intercooler.

The setup is like this:

C.V.: Stage 1 nitrogen, Steady flow

Process: adiabatic: q = 0,

Energy Eq. 4.13: $-w_{C1} = h_2 - h_1$,

Assume constant $C_{P0} = 1.042$ from A.5

$w_{C1} = h_1 - h_2 = C_{P0}(T_1 - T_2) = 1.042$ kJ/kgK (293 – 450) K = **-163.6 kJ/kg**

C.V. Intercooler, no work and no changes in kinetic or potential energy.

$q_{23} = h_3 - h_2 = C_{P0}(T_3 - T_2) = 1.042 (320 - 450) =$ **-135.5 kJ/kg**

C.V. Stage 2. Analysis the same as stage 1.

$w_{C2} = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.042 (320 - 530) =$ **-218.8 kJ/kg**

4.117

The intercooler in the previous problem uses cold liquid water to cool the nitrogen. The nitrogen flow is 0.1 kg/s, and the liquid water inlet is 20$^{\text{o}}$C and is set up to flow in the opposite direction from the nitrogen, so the water leaves at 35$^{\text{o}}$C. Find the flow rate of the water.

Solution:

C.V. Heat exchanger, steady 2 flows in and two flows out.

Continuity eq.: $\qquad \dot{m}_1 = \dot{m}_2 = \dot{m}_{H2O}; \qquad \dot{m}_3 = \dot{m}_4 = \dot{m}_{N2}$

Energy eq.: $\qquad 0 = \dot{m}_{H2O}(h_3 - h_4) + \dot{m}_{N2}(h_2 - h_1)$

Due to the lower range of temperature we will use constant specific heats from A.5 and A.4 so the energy equation is approximated as

$$0 = \dot{m}_{H2O}C_{pH2O}(T_3 - T_4) + \dot{m}_{N2}C_p(T_2 - T_1)$$

Now solve for the water flow rate

$$\dot{m}_{H2O} = \dot{m}_{N2}\, C_{pN2}\, (T_2 - T_1) / [C_{pH2O}\, (T_4 - T_3)]$$
$$= 0.1 \text{ kg/s} \times 1.042 \times (450 - 320) / [4.18 \times (35 - 20)]$$
$$= \mathbf{0.216\ kg/s}$$

4.118

The following data are for a simple steam power plant as shown in Fig. P4.118.

State	1	2	3	4	5	6	7
P MPa	6.2	6.1	5.9	5.7	5.5	0.01	0.009
T °C		45	175	500	490		40
h kJ/kg	-	194	744	3426	3404	-	168

State 6 has $x_6 = 0.92$, and velocity of 200 m/s. The rate of steam flow is 25 kg/s, with 300 kW power input to the pump. Piping diameters are 200 mm from steam generator to the turbine and 75 mm from the condenser to the steam generator. Determine the velocity at state 5 and the power output of the turbine.

Solution:

Turbine $A_5 = (\pi/4)(0.2)^2 = 0.031\ 42\ m^2$, $v_5 = 0.06163\ m^3/kg$

$$\mathbf{V}_5 = \dot{m}v_5/A_5 = 25\ kg/s \times 0.061\ 63\ m^3/kg\ /\ 0.031\ 42\ m^2 = \mathbf{49\ m/s}$$

$$h_6 = 191.83 + 0.92 \times 2392.8 = 2393.2\ kJ/kg$$

$$w_T = h_5 - h_6 + \frac{1}{2}(\mathbf{V}_5^2 - \mathbf{V}_6^2)$$

$$= 3404 - 2393.2 + (49^2 - 200^2)/(2 \times 1000) = 992\ kJ/kg$$

$$\dot{W}_T = \dot{m}w_T = 25\ kg/s \times 992\ kJ/kg = \mathbf{24\ 800\ kW}$$

Remark: Notice the kinetic energy change is small relative to enthalpy change.

4.119

For the same steam power plant as shown in Fig. P4.118, assume the cooling water comes from a lake at 15°C and is returned at 25°C. Determine the rate of heat transfer in the condenser and the mass flow rate of cooling water from the lake.

Solution:

Condenser $A_7 = (\pi/4)(0.075)^2 = 0.004\ 418\ m^2$, $v_7 = 0.001\ 008\ m^3/kg$

$$\mathbf{V}_7 = \dot{m}v_7/A_7 = 25 \times 0.001\ 008\ /\ 0.004\ 418 = 5.7\ m/s$$

$$h_6 = 191.83 + 0.92 \times 2392.8 = 2393.2\ kJ/kg$$

$$q_{COND} = h_7 - h_6 + \frac{1}{2}\ (\ \mathbf{V}_7^2 - \mathbf{V}_6^2\ )$$

$$= 168 - 2393.2 + (5.7^2 - 200^2\ )/(2 \times 1000) = -2245.2\ kJ/kg$$

$$\dot{Q}_{COND} = 25\ kg/s \times (-2245.2\ kJ/kg) = \mathbf{-56\ 130\ kW}$$

This rate of heat transfer is carried away by the cooling water so

$$-\dot{Q}_{COND} = \dot{m}_{H_2O}(h_{out} - h_{in})_{H_2O} = 56\ 130\ kW$$

$$=> \qquad \dot{m}_{H_2O} = \frac{56\ 130}{104.9 - 63.0}\ \frac{kW}{kJ/kg} = \mathbf{1339.6\ kg/s}$$

4.120

For the same steam power plant as shown in Fig. P4.118, determine the rate of heat transfer in the economizer, which is a low temperature heat exchanger. Find also the rate of heat transfer needed in the steam generator.

Solution:

Economizer $A_7 = \pi D_7^2/4 = 0.004\ 418\ m^2$, $v_7 = 0.001\ 008\ m^3/kg$

$$\mathbf{V}_2 = \mathbf{V}_7 = \dot{m}v_7/A_7 = 25 \times 0.001\ 008/0.004\ 418 = 5.7\ m/s,$$

$$\mathbf{V}_3 = (v_3/v_2)\mathbf{V}_2 = (0.001\ 118\ /\ 0.001\ 008)\ 5.7 = 6.3\ m/s \approx \mathbf{V}_2$$

so kinetic energy change unimportant

$$q_{ECON} = h_3 - h_2 = 744 - 194 = 550.0\ kJ/kg$$

$$\dot{Q}_{ECON} = \dot{m}q_{ECON} = 25\ kg/s \times 550.0\ kJ/kg = \mathbf{13\ 750\ kW}$$

Generator $A_4 = \pi D_4^2/4 = 0.031\ 42\ m^2$, $v_4 = 0.060\ 23\ m^3/kg$

$$\mathbf{V}_4 = \dot{m}v_4/A_4 = 25\ kg/s \times 0.060\ 23\ m^3/kg\ /0.031\ 42\ m^2 = 47.9\ m/s$$

$$q_{GEN} = 3426 - 744 + (47.9^2 - 6.3^2)/(2\times1000) = 2683\ kJ/kg$$

$$\dot{Q}_{GEN} = \dot{m}q_{GEN} = 25\ kg/s \times 2683\ kJ/kg = \mathbf{67\ 075\ kW}$$

4.121

A somewhat simplified flow diagram for a nuclear power plant is given in Fig. P4.121. Mass flow rates and the various states in the cycle are shown in the accompanying table. The cycle includes a number of heaters in which heat is transferred from steam, taken out of the turbine at some intermediate pressure, to liquid water pumped from the condenser on its way to the steam drum. The heat exchanger in the reactor supplies 157 MW, and it may be assumed that there is no heat transfer in the turbines.

a. Assume the moisture separator has no heat transfer between the two turbinesections, determine the enthalpy and quality (h_4, x_4).

b. Determine the power output of the low-pressure turbine.

c. Determine the power output of the high-pressure turbine.

d. Find the ratio of the total power output of the two turbines to the total power delivered by the reactor.

Solution:

a) Moisture Separator, steady state, no heat transfer, no work

Continuity Eq.: $\dot{m}_3 = \dot{m}_4 + \dot{m}_9$, $\dot{m}_4 = \dot{m}_3 - \dot{m}_9 = 62.874 – 4.662 = 58.212$ kg/s

Energy Eq.: $\dot{m}_3 h_3 = \dot{m}_4 h_4 + \dot{m}_9 h_9$;
From the energy equation we get

$$h_4 = (\dot{m}_3 h_3 - \dot{m}_9 h_9) / \dot{m}_4 = \frac{62.874 \times 2517 - 4.662 \times 558}{58.212} = 2673.9 \text{ kJ/kg}$$

$h_4 = 2673.9 = 566.18 + x_4 \times 2160.6 \Rightarrow x_4 = \mathbf{0.9755}$

b) Low Pressure Turbine, steady state no heat transfer

Continuity Eq.: $\dot{m}_4 = \dot{m}_5 + \dot{m}_8$

Energy Eq.: $\dot{m}_4 h_4 = \dot{m}_5 h_5 + \dot{m}_8 h_8 + \dot{W}_{CV(LP)}$

$\dot{m}_5 = \dot{m}_4 - \dot{m}_8 = 58.212 – 2.772 = 55.44$ kg/s

$\dot{W}_{CV(LP)} = \dot{m}_4 h_4 - \dot{m}_5 h_5 - \dot{m}_8 h_8$

$= 58.212 \times 2673.9 - 55.44 \times 2279 - 2.772 \times 2459$

$= 22\,489$ kW $= \mathbf{22.489}$ **MW**

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c) High Pressure Turbine, steady state no heat transfer

Energy Eq.: $\dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{m}_{12} h_{12} + \dot{m}_{17} h_{17} + \dot{W}_{CV(HP)}$

$\dot{W}_{CV(HP)} = \dot{m}_2 h_2 - \dot{m}_3 h_3 - \dot{m}_{12} h_{12} - \dot{m}_{17} h_{17}$

$\qquad = 75.6 \times 2765 - 62.874 \times 2517 - 8.064 \times 2517 - 4.662 \times 2593$

$\qquad = 18\ 394 \text{ kW} = \textbf{18.394 MW}$

d) $\qquad \eta = (\dot{W}_{HP} + \dot{W}_{LP})/\dot{Q}_{REACT} = (22.489 + 18.394) / 157 = \textbf{0.26}$

4.122

Consider the power plant as described in the previous problem.
a. Determine the quality of the steam leaving the reactor.
b. What is the power to the pump that feeds water to the reactor?

Solution:

a) Reactor: Cont.: $\dot{m}_{20} = \dot{m}_{21}$; $\dot{Q}_{CV} = 157$ MW

Energy Eq.4.12:

$$\dot{Q}_{CV} + \dot{m}_{20}h_{20} = \dot{m}_{21}h_{21}$$

157 000 kW + 1386 kg/s × 1221 kJ/kg = 1386 kg/s × h_{21}

$h_{21} = 1334.3$ kJ/kg = (1282.4 + x_{21} × 1458.3) kJ/kg   => x_{21} = **0.0349**

b) C.V. Reactor feedwater pump

Cont. $\dot{m}_{19} = \dot{m}_{20}$ Energy Eq.: $\dot{m}_{19}h_{19} = \dot{m}_{19}h_{20} + \dot{W}_{Cv,P}$

Table B.1: $h_{19} = h(277°C, 7240$ kPa$) = 1220$ kJ/kg, $h_{20} = 1221$ kJ/kg

$\dot{W}_{Cv,P} = \dot{m}_{19}(h_{19} - h_{20}) = 1386$ kg/s (1220 - 1221) kJ/kg = **-1386 kW**

4.123

A R-410A heat pump cycle shown in Fig. P4.123 has a R-410A flow rate of 0.05 kg/s with 5 kW into the compressor. The following data are given

State	1	2	3	4	5	6
P, kPa	3100	3050	3000	420	400	390
T, °C	120	110	45		-10	-5
h, kJ/kg	377	367	134	-	280	284

Calculate the heat transfer from the compressor, the heat transfer from the R-410A in the condenser and the heat transfer to the R-410A in the evaporator.

Solution:

CV: Compressor

$$\dot{Q}_{COMP} = \dot{m}(h_1 - h_6) + \dot{W}_{COMP} = 0.05\,(377 - 284) - 5.0 = \textbf{-0.35 kW}$$

CV: Condenser

$$\dot{Q}_{COND} = \dot{m}(h_3 - h_2) = 0.05\ \text{kg/s}\,(134 - 367)\ \text{kJ/kg} = \textbf{-11.65 kW}$$

C.V. Valve:

$$h_4 = h_3 = 134\ \text{kJ/kg}$$

CV: Evaporator

$$\dot{Q}_{EVAP} = \dot{m}\,(h_5 - h_4) = 0.05\ \text{kg/s}\,(280 - 134)\ \text{kJ/kg} = \textbf{7.3 kW}$$

4.124

A modern jet engine has a temperature after combustion of about 1500 K at 3200 kPa as it enters the turbine setion, see state 3 Fig. P.4.124. The compressor inlet is 80 kPa, 260 K state 1 and outlet state 2 is 3300 kPa, 780 K; the turbine outlet state 4 into the nozzle is 400 kPa, 900 K and nozzle exit state 5 at 80 kPa, 640 K. Neglect any heat transfer and neglect kinetic energy except out of the nozzle. Find the compressor and turbine specific work terms and the nozzle exit velocity.

Solution:

The compressor, turbine and nozzle are all steady state single flow devices and they are adiabatic.

We will use air properties from table A.7.1:
$h_1 = 260.32$, $h_2 = 800.28$, $h_3 = 1635.80$, $h_4 = 933.15$, $h_5 = 649.53$ kJ/kg

Energy equation for the compressor gives
$$w_{c\ in} = h_2 - h_1 = 800.28 - 260.32 = \textbf{539.36 kJ/kg}$$

Energy equation for the turbine gives
$$w_T = h_3 - h_4 = 1635.80 - 933.15 = \textbf{702.65 kJ/kg}$$

Energy equation for the nozzle gives

$$h_4 = h_5 + \tfrac{1}{2}\,\textbf{V}_5^2$$

$$\tfrac{1}{2}\,\textbf{V}_5^2 = h_4 - h_5 = 933.15 - 649.53 = 283.62 \text{ kJ/kg}$$

$$\textbf{V}_5 = [2(h_4 - h_5)]^{1/2} = (2 \times 283.62 \times 1000)^{1/2} = \textbf{753 m/s}$$

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4.125

A proposal is made to use a geothermal supply of hot water to operate a steam turbine, as shown in Fig. P4.125. The high-pressure water at 1.5 MPa, 180°C, is throttled into a flash evaporator chamber, which forms liquid and vapor at a lower pressure of 400 kPa. The liquid is discarded while the saturated vapor feeds the turbine and exits at 10 kPa, 90% quality. If the turbine should produce 1 MW, find the required mass flow rate of hot geothermal water in kilograms per hour.

Solution:

Separation of phases in flash-evaporator
constant h in the valve flow so

Table B.1.3: $h_1 = 763.5$ kJ/kg

$h_1 = 763.5 = 604.74 + x \times 2133.8$

$\Rightarrow$ $x = 0.07439 = \dot{m}_2/\dot{m}_1$

Table B.1.2: $h_2 = 2738.6$ kJ/kg;

$h_3 = 191.83 + 0.9 \times 2392.8 = 2345.4$ kJ/kg

Energy Eq.4.12 for the turbine

$$\dot{W} = \dot{m}_2(h_2 - h_3) \qquad \Rightarrow \qquad \dot{m}_2 = \frac{1000}{2738.6 - 2345.4}\frac{kW}{kJ/kg} = 2.543 \text{ kg/s}$$

$$\Rightarrow \quad \dot{m}_1 = \dot{m}_2/x = 34.19 \text{ kg/s} = \textbf{123 075 kg/h}$$

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Transient processes

4.126

An initially empty cylinder is filled with air from 20°C, 100 kPa until it is full. Assuming no heat transfer is the final temperature larger, equal to or smaller than 20°C? Does the final T depend on the size of the cylinder?

This is a transient problem with no heat transfer and no work. The balance equations for the tank as C.V. become

Continuity Eq.:　　　$m_2 - 0 = m_i$

Energy Eq.:　　　　$m_2 u_2 - 0 = m_i h_i + Q - W = m_i h_i + 0 - 0$

Final state:　　　　$u_2 = h_i$ &　　$P_2 = P_i$

$T_2 > T_i$　and it does not depend on V

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4.127

An initially empty canister of volume 0.2 m^3 is filled with carbon dioxide from a line at 800 kPa, 400 K. Assume the process runs until it stops by itself and it is adiabatic. Use constant specific heat to find the final temperature in the canister.

C.V. Canister and valve, transient process with no heat transfer or work.

Continuity Eq.4.15: $m_2 - m_1 = m_{in}$;

Energy Eq.4.16: $m_2 u_2 – m_1 u_1 = - m_{in} h_{in} + {}_1Q_2 - {}_1W_2$

Process: ${}_1Q_2 = 0$, V= constant so ${}_1W_2 = 0$

State 1: $m_1 = 0$ => $m_2 = m_{in}$

State 2: $P_2 = P_{line} = 800$ kPa, one more property

Energy Eq.: $u_2 = h_{in} = u_{in} + RT_{in}$ $\Rightarrow$ $C_{vo} T_2 = C_{vo} T_{in} + RT_{in} = C_{Po}T_{in}$

$T_2 = (C_{Po}/C_{vo}) T_{in} = k \times T_{in} = 1.289 \times 400 \text{ K} = \textbf{515.6 K}$

4.128

Repeat the previous problem but use the ideal gas Tables A.8 to solve it.

C.V. Canister and valve, transient process with no heat transfer or work.

Continuity Eq.4.15: $m_2 - m_1 = m_{in}$;

Energy Eq.4.16: $m_2 u_2 - m_1 u_1 = - m_{in} h_{in} + {_1}Q_2 - {_1}W_2$

Process: ${_1}Q_2 = 0$, V= constant so ${_1}W_2 = 0$

State 1: $m_1 = 0$ => $m_2 = m_{in}$

State 2: $P_2 = P_{line} = 800$ kPa,   one more property

Energy Eq.: $u_2 = h_{in} = 303.76$ kJ/kg from A.8

                back interpolate for u_2: $T_2 =$ **495.9 K**

4.129

A tank contains 1 m^3 air at 100 kPa, 300 K. A pipe flowing air at 1000 kPa, 300 K is connected to the tank and it is filled slowly to 1000 kPa. Find the heat transfer to reach a final temperature of 300 K.

C.V. The tank volume and the compressor.

This is a transient problem (filling of tank).

Continuity Eq.4.15: $m_2 - m_1 = m_{in}$

Energy Eq.4.16: $m_2 u_2 - m_1 u_1 = {}_1Q_2 - {}_1W_2 + m_{in}h_{in}$

Process: Constant volume $\quad {}_1W_2 = 0$,

States: $\quad u_1 = u_2 = u_{in} = u_{300}$; $\quad h_{in} = u_{in} + RT_{in}$

$$m_1 = P_1V_1/RT_1 = 100 \text{ kPa} \times 1 \text{ m}^3 /(0.287 \text{ kJ/kg-K} \times 300 \text{ K})$$

$$= 1.1614 \text{ kg}$$

$$m_2 = P_2V_2/RT_2 = 1000 \text{ kPa} \times 1 \text{ m}^3/(0.287 \text{ kJ/kg-K} \times 300 \text{ K})$$

$$= 11.6144 \text{ kg}$$

Heat transfer from the energy equation

$${}_1Q_2 = m_2u_2 - m_1u_1 - m_{in}h_{in} = (m_1 + m_{in}) \, u_1 - m_1u_1 - m_{in}u_{in} - m_{in}RT_{in}$$

$$= m_1u_1 - m_1u_1 + m_{in}u_1 - m_{in}u_{in} - m_{in}RT_{in} = - \, m_{in}RT_{in}$$

$$= (11.6144 - 1.1614) \text{ kg} \times 0.287 \text{ kJ/kg-K} \times 300 \text{ K} = \textbf{–900 kJ}$$

Borgnakke and Sonntag

4.130

A 1-m^3 tank contains ammonia at 150 kPa, 25°C. The tank is attached to a line flowing ammonia at 1200 kPa, 60°C. The valve is opened, and mass flows in until the tank is half full of liquid, by volume at 25°C. Calculate the heat transferred from the tank during this process.

Solution:

C.V. Tank.   Transient process as flow comes in.

State 1 Table B.2.2 interpolate between 20 °C and 30°C:

$v_1 = 0.9552$ m^3/kg; $u_1 = 1380.6$ kJ/kg

$m_1 = V/v_1 = 1/0.9552 = 1.047$ kg

State 2: 0.5 m^3 liquid and 0.5 m^3 vapor from Table B.2.1 at 25°C

$v_f = 0.001658$ m^3/kg; $v_g = 0.12813$ m^3/kg

$m_{LIQ2} = 0.5/0.001658 = 301.57$ kg, $m_{VAP2} = 0.5/0.12813 = 3.902$ kg

$m_2 = 305.47$ kg, $x_2 = m_{VAP2}/m_2 = 0.01277$,

From continuity equation

$m_i = m_2 - m_1 = 304.42$ kg

Table B.2.1: $u_2 = 296.6 + 0.01277 \times 1038.4 = 309.9$ kJ/kg

State inlet: Table B.2.2 $h_i = 1553.3$ kJ/kg

Energy Eq.4.16:

$Q_{CV} + m_i h_i = m_2 u_2 - m_1 u_1$

$Q_{CV} = 305.47 \times 309.9 - 1.047 \times 1380.6 - 304.42 \times 1553.3 =$ **-379 636 kJ**

4.131

A 2.5-L tank initially is empty and we want 10 g of ammonia in it. The ammonia comes from a line with saturated vapor at 25°C. To end up with the desired amount we cool the can while we fill it in a slow process keeping the can and content at 30°C. Find the final pressure to reach before closing the valve and the heat transfer?

Solution:

C.V. Tank:

Continuity Eq.4.15: $m_i = m_2$

Energy Eq.4.16: $m_2 u_2 - 0 = m_i h_i + {}_1Q_2$

State 2: 30°C, $v_2 = V/m_2 = 0.0025 \ m^3/0.010 \ kg = 0.25 \ m^3/kg$

From Table B.2.2 we locate the state between 500 and 600 kPa.

$$P_2 = 500 + (600 - 500)\frac{0.25 - 0.28103}{0.23152 - 0.28103} = \textbf{562.7 kPa}$$

$u_2 = 1370$ kJ/kg,

State i Table B.2.2: $h_i = 1463.5$ kJ/kg

Now use the energy ewquation to solve for the heat transfer

$${}_1Q_2 = m_2 u_2 - m_i h_i = m_2(u_2 - h_i)$$
$$= 0.01 \ kg \times (1370 - 1463.5 \ ) \ kJ/kg = \textbf{--0.935 kJ}$$

Borgnakke and Sonntag

4.132

An evacuated 150-L tank is connected to a line flowing air at room temperature, 25°C, and 8 MPa pressure. The valve is opened allowing air to flow into the tank until the pressure inside is 6 MPa. At this point the valve is closed. This filling process occurs rapidly and is essentially adiabatic. The tank is then placed in storage where it eventually returns to room temperature. What is the final pressure?

Solution:

C.V. Tank:

Continuity Eq.4.15: $m_i = m_2$

Energy Eq.4.16: $m_i h_i = m_2 u_2$ => $u_2 = h_i$

Use constant specific heat C_{Po} from table A.5 then energy equation:

$$T_2 = (C_P/C_v)\, T_i = kT_i = 1.4 \times 298.2 = 417.5 \text{ K}$$

Process: constant volume cooling to T_3:

$$P_3 = P_2 \times T_3/T_2 = 6.0 \text{ MPa} \times 298.15 \text{ K}/\, 417.5 \text{ K} = \textbf{4.29 MPa}$$

4.133

An insulated 2-m^3 tank is to be charged with R- 134a from a line flowing the refrigerant at 3 MPa, 90°C. The tank is initially evacuated, and the valve is closed when the pressure inside the tank reaches 3 MPa. Find the mass in the tank and its final temperature.

C.V. Tank and valve, transient process with no heat transfer or work.

Continuity Eq.4.15: $m_2 - m_1 = m_{in}$;

Energy Eq.4.16: $m_2u_2 - m_1u_1 = - m_{in} h_{in} + {}_1Q_2 - {}_1W_2$

Process: ${}_1Q_2 = 0$, V= constant so ${}_1W_2 = 0$

State 1: $m_1 = 0$ => $m_2 = m_{in}$

State 2: $P_2 = P_{line} = 3$ MPa, one more property

Energy Eq.: $u_2 = h_{in} = 436.19$ kJ/kg

State 2: (P, u) interpolate B.5.2: $T_2 = 100 + 10 \dfrac{436.19 - 433.77}{446.48 - 433.77} = \mathbf{101.9°C}$,

$v_2 = 0.00665 + 0.1904 (0.00734 - 0.00665) = 0.006781$ m^3/kg

$m_2 = V/v_2 = [2$ m$^3/0.006781$ m^3/kg$] = \mathbf{294.9\ kg}$

4.134

Find the final state for the previous problem if the valve is closed when the tank reaches 2 MPa.

C.V. Tank and valve, transient process with no heat transfer or work.

Continuity Eq.4.15: $m_2 - m_1 = m_{in}$;

Energy Eq.4.16: $m_2 u_2 - m_1 u_1 = - m_{in} h_{in} + {}_1Q_2 - {}_1W_2$

Process: ${}_1Q_2 = 0$, V= constant so ${}_1W_2 = 0$

State 1: $m_1 = 0$ => $m_2 = m_{in}$

State 2: $P_2 = 2$ MPa,   one more property

Energy Eq.: $u_2 = h_{in} = 436.19$ kJ/kg

State 2: (P, u) B.5.2: $T_2 = $ **90°C** very close, $v_2 = 0.01137$ m^3/kg

   $m_2 = V/v_2 = [2$ m^3/0.01137 m^3/kg] = **175.9 kg**

4.135

Helium in a steel tank is at 250 kPa, 300 K with a volume of 0.1 m^3. It is used to fill a balloon. When the tank pressure drops to 150 kPa the flow of helium stops by itself. If all the helium still is at 300 K how big a balloon did I get? Assume the pressure in the balloon varies linearly with volume from 100 kPa (V = 0) to the final 150 kPa. How much heat transfer did take place?

Solution:

Take a C.V. of all the helium.
This is a control mass, the tank mass changes density and pressure.

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = 100 + CV$

State 1: P_1, T_1, V_1

State 2: P_2, T_2, $V_2 = ?$

Ideal gas:

$$P_2 V_2 = mRT_2 = mRT_1 = P_1 V_1$$

$V_2 = V_1(P_1/P_2) = 0.1 \times (250/150) = 0.16667$ m^3

$V_{bal} = V_2 - V_1 = 0.16667 - 0.1 = 0.06667$ m^3

$_1W_2 = \int P\, dV = $ AREA $= \frac{1}{2}\,(P_1 + P_2)(V_2 - V_1)$

$\qquad = \frac{1}{2}(250 + 150)$ kPa $\times 0.06667$ m$^3 = 13.334$ kJ

$U_2 - U_1 = {}_1Q_2 - {}_1W_2 = m\,(u_2 - u_1) = mC_v\,(T_2 - T_1) = 0$

so $\quad {}_1Q_2 = {}_1W_2 = $ **13.334 kJ**

Remark: The process is transient, but you only see the flow mass if you select the tank or the balloon as a control volume. That analysis leads to more terms that must be elliminated between the tank control volume and the balloon control volume.

4.136

A 25-L tank, shown in Fig. P4.136, that is initially evacuated is connected by a valve to an air supply line flowing air at 20°C, 800 kPa. The valve is opened, and air flows into the tank until the pressure reaches 600 kPa.Determine the final temperature and mass inside the tank, assuming the process is adiabatic. Develop an expression for the relation between the line temperature and the final temperature using constant specific heats.

Solution:

C.V. Tank:

Continuity Eq.4.15: $m_2 = m_i$

Energy Eq.4.16: $m_2 u_2 = m_i h_i$

Table A.7: $u_2 = h_i = 293.64$ kJ/kg

$$\Rightarrow T_2 = \textbf{410.0 K}$$

$$m_2 = \frac{P_2 V}{RT_2} = \frac{600 \times 0.025}{0.287 \times 410} = \textbf{0.1275 kg}$$

Assuming constant specific heat,

$$h_i = u_i + RT_i = u_2 , \quad RT_i = u_2 - u_i = C_{vo}(T_2 - T_i)$$

$$C_{vo}T_2 = (C_{vo} + R)T_i = C_{Po}T_i \ , \quad T_2 = \left(\frac{C_{Po}}{C_{vo}}\right) T_i = kT_i$$

For $T_i = 293.2$K & constant C_{Po}, $\quad T_2 = 1.40 \times 293.2 = \textbf{410.5 K}$

4.137

A nitrogen line, 300 K and 0.5 MPa, shown in Fig. P4.137, is connected to a turbine that exhausts to a closed initially empty tank of 50 m^3. The turbine operates to a tank pressure of 0.5 MPa, at which point the temperature is 250 K. Assuming the entire process is adiabatic, determine the turbine work.

Solution:

C.V. turbine & tank $\Rightarrow$ Transient process

Conservation of mass Eq.4.15: $m_i = m_2 \Rightarrow$ m

Energy Eq.4.16: $m_i h_i = m_2 u_2 + W_{CV}$; $W_{CV} = m(h_i - u_2)$

Table B.6.2: $P_i = 0.5$ MPa, $T_i = 300$ K,   Nitrogen; $h_i = 310.28$ kJ/kg

2: $P_2 = 0.5$ MPa, $T_2 = 250$ K, $u_2 = 183.89$ kJ/kg, $v_2 = 0.154$ m^3/kg

$m_2 = V/v_2 = 50$ m^3 / (0.154 m^3/kg) = 324.7 kg

$W_{CV} = 324.7$ kg (310.28 - 183.89) kJ/kg = 41 039 kJ = **41.04 MJ**

We could with good accuracy have solved using ideal gas and Table A.5

Borgnakke and Sonntag

4.138

A 1- m^3 rigid tank contains 100 kg R-410A at ambient temperature 15°C. A valve on top of the tank is opened, and saturated vapor is throttled to ambient pressure, 100 kPa, and flows to a collector system. During the process, the temperature inside the tank remains at 15°C. The valve is closed when no more liquid remains inside. Calculate the heat transfer to the tank.

C.V. Tank, no work and neglect kinetic and potential energies.

Continuity Eq.4.15: $m_2 - m_1 = - m_e$

Energy Eq.4.16: $m_2u_2 - m_1u_1 = {}_1Q_2 - m_eh_e$

State 1 has $v_1 = V/m_1 = 1/100 = 0.01$ m^3/kg $< v_g$ so two-phase

B.4.1: $x_1 = (0.01 - 0.000904)/0.01955 = 0.46527$

$u_1 = u_f + x_1u_{fg} = 80.02 + x_1 \times 177.1 = 162.42$ kJ/kg

State 2 is saturated vapor $T_2 = 15$°C

$v_2 = v_g = 0.02045$ m^3/kg, $u_2 = u_g = 257.12$ kJ/kg

$m_2 = V/v_2 = \dfrac{1\ \text{m}^3}{0.02045\ \text{m}^3/\text{kg}} = 48.9$ kg

The exit state e: $h_e = h_g = 282.79$ kJ/kg

$$\begin{aligned}
{}_1Q_2 &= m_2u_2 - m_1u_1 + m_eh_e \\
&= 48.9 \times 257.12 - 100 \times 162.42 + (100 - 48.9) \times 282.79 \\
&= \mathbf{10\ 782\ kJ}
\end{aligned}$$

Borgnakke and Sonntag

4.139

A 200 liter tank initially contains water at 100 kPa and a quality of 1%. Heat is transferred to the water thereby raising its pressure and temperature. At a pressure of 2 MPa a safety valve opens and saturated vapor at 2 MPa flows out. The process continues, maintaining 2 MPa inside until the quality in the tank is 90%, then stops. Determine the total mass of water that flowed out and the total heat transfer.

Solution:

C.V. Tank, no work but heat transfer in and flow out. Denoting State 1: initial state, State 2: valve opens, State 3: final state.

Continuity Eq.: $m_3 - m_1 = - m_e$

Energy Eq.: $m_3 u_3 - m_1 u_1 = - m_e h_e + {}_1Q_3$

State 1 Table B.1.2: $v_1 = v_f + x_1 v_{fg} = 0.001043 + 0.01 \times 1.69296$
$$= 0.01797 \text{ m}^3/\text{kg}$$
$$u_1 = u_f + x_1 u_{fg} = 417.33 + 0.01 \times 2088.72 = 438.22 \text{ kJ/kg}$$
$$m_1 = V/v_1 = 0.2 \text{ m}^3/(0.01797 \text{ m}^3/\text{kg}) = 11.13 \text{ kg}$$

State 3 (2MPa): $v_3 = v_f + x_3 v_{fg} = 0.001177 + 0.9 \times 0.09845 = 0.8978 \text{ m}^3/\text{kg}$
$$u_3 = u_f + x_3 u_{fg} = 906.42 + 0.9 \times 1693.84 = 2430.88 \text{ kJ/kg}$$
$$m_3 = V/v_3 = 0.2 \text{ m}^3/(0.08978 \text{ m}^3/\text{kg}) = 2.23 \text{ kg}$$

Exit state (2MPa): $h_e = h_g = 2799.51 \text{ kJ/kg}$

Hence: $m_e = m_1 - m_3 = 11.13 \text{ kg} - 2.23 \text{ kg} = \textbf{8.90 kg}$

Applying the 1st law between state 1 and state 3
$${}_1Q_3 = m_3 u_3 - m_1 u_1 + m_e h_e$$
$$= 2.23 \times 2430.88 - 11.13 \times 438.22 + 8.90 \times 2799.51$$
$$= 25\ 459 \text{ kJ} = \textbf{25.46 MJ}$$

4.140

A 1-L can of R-134a is at room temperature 20°C with a quality 50%. A leak in the top valve allows vapor to escape and heat transfer from the room takes place so we reach a final state of 5°C with a quality of 100%. Find the mass that escaped and the heat transfer.

Solution:

CV The can of R-134a not including the nozzle/valve out to ambient 20^oC

Continuity Eq.: $\qquad m_2 - m_1 = - m_e$

Energy Eq.: $\quad m_2u_2 - m_1u_1 = - m_eh_e + {}_1Q_2 - {}_1W_2$

Process Eq.: $\quad V = \text{constant} \quad => \quad {}_1W_2 = \int PdV = 0$

State 1: (T,x) $\quad v_1 = v_f + x_1v_{fg} = 0.000817 + 0.5 \times 0.03524 = 0.018437 \text{ m}^3/\text{kg}$

$\qquad\qquad\quad u_1 = u_f + x_1u_{fg} = 227.03 + 0.5 \times 162.16 = 308.11 \text{ kJ/kg}$

$\qquad\qquad\quad m_1 = V / v_1 = 0.001 \text{ m}^3/ 0.018437 \text{ m}^3/\text{kg} = 0.05424 \text{ kg}$

State 2: (T,x)

$\qquad\quad v_2 = v_g = 0.05833 \text{ m}^3/\text{kg}, \ u_2 = u_g = 380.85 \text{ kJ/kg}$

$\qquad\quad m_2 = V/v_2 = 0.001 \text{ m}^3/ 0.05833 \text{ m}^3/\text{kg} = 0.017144 \text{ kg}$

Exit state e: Saturated vapor starting at 20^oC ending at 5^oC so we take an **average**

$\qquad h_e = 0.5(h_{e1} + h_{e2}) = 0.5 (409.84 + 401.32) = 405.58 \text{ kJ/kg}$

$\qquad m_e = m_1 - m_2 = \textbf{0.0371 kg}$

The heat transfer from the energy equation becomes

$\qquad {}_1Q_2 = m_2u_2 - m_1u_1 + m_eh_e = 6.5293 - 16.7119 + 15.047 = \textbf{4.864 kJ}$

4.141

A 2 m tall cylinder has a small hole in the bottom. It is filled with liquid water 1 m high, on top of which is 1 m high air column at atmospheric pressure of 100 kPa. As the liquid water near the hole has a higher P than 100 kPa it runs out. Assume a slow process with constant T. Will the flow ever stop? When?

Solution:

$$P_{bot} = P_{air} + \rho g L_f$$

For the air $PV = mRT$ and the total height is $H = 2$ m

$$P_{air} = mRT/V_{air} \quad ; \quad V_{air} = A\, L_{air} = A\,(\,H - L_f\,)$$

$$P_{bot} = \frac{m_a R_a T_a}{A(H-L_f)} + \rho_f g\, L_f = \frac{P_{a1} V_{a1}}{A(H-L_f)} + \rho_f\, gL_f = \frac{P_{a1} L_{a1}}{H-L_f} + \rho_f\, gL_f \geq P_o$$

Solve for L_f ; $\rho_f = 1/(v_f) = 1/0.0021002 = 998$ kg/m^3

$$P_{a1}\, L_{a1} + \rho_f\, g\, L_f\,(\,H - L_f\,) \geq P_o\,(\,H - L_f\,)$$

$$(\rho_f g H + P_o\,)\, L_f - \rho_f g L^2_{\,f} = P_o\, H - P_{a1}\, L_{a1} \geq 0$$

Put in numbers and solve quadratic eq.

$$L^2_{\,f} - [\,H + (P_o/\rho g)\,]\, L_f + \frac{P_o H - P_{a1} L_{a1}}{\rho g} = 0$$

$$(P_o/\rho g) = \frac{100\ \text{kPa m}^3\ \text{s}^3}{998 \times 9.807\ \text{kg m}} = 10.217\ \text{m}$$

$$\frac{P_o H - P_{a1} L_{a1}}{\rho g} = \frac{100\,(2\text{-}1)}{998 \times 9.807} = 10.217\ \text{m}$$

$$L^2_{\,f} - 12.217\, L_f + 10.217 = 0$$

$$L_f = \frac{12.217}{2} \pm \sqrt{\frac{12.217^2}{4} - \frac{12.217}{4}} = 6.1085 \pm 5.2055$$

$$\Rightarrow 11.314 \quad \text{or} \quad \textbf{0.903 m} \qquad \text{so the second root is the solution}$$

Verify

$$P_{a2} = P_{a1}\cdot \frac{L_{a1}}{H-L_f} = 100\,\frac{1}{2 - 0.903} = 91.158\ \text{kPa}$$

$$\rho g L_f = 998 \times 9.807 \times 0.903 = 8838\ \text{Pa} = 8.838\ \text{kPa}$$

$$P_{bot} = P_{a2} + \rho g L_f = 91.158 + 8.838 = 99.996\ \text{kPa} \quad \text{OK}$$

Review Problems

4.142

A pipe of radius R has a fully developed laminar flow of air at P_o, T_o with a velocity profile as: $V = V_c [1 – (r/R)^2]$, where V_c is the velocity on the center-line and r is the radius as shown in Fig. P4.142. Find the total mass flow rate and the average velocity both as functions of V_c and R.

$$\dot{m} = AV/v = \dot{V}/v$$

Since the velocity is distributed we need to integrate over the area. From Eq.4.2

$$\dot{V} = \int V_{local}\, dA = \int V(r)\, 2\pi r\, dr$$

where W is the depth. Substituting the velocity we get

$$\dot{V} = \int V_c\, 2\pi r\, [1 - (r/R)^2]\, dr$$

$$= 2\pi\, V_c\, R^2 \int_0^1 z\, (1 - z^2)\, dz$$

$$= 2\pi\, V_c\, R^2\, (\tfrac{1}{2} z^2 - \tfrac{1}{4} z^4\, \big|_0^1 = \tfrac{\pi}{2} V_c\, R^2 = \tfrac{1}{2} V_c\, A$$

$$V = \dot{V}/A = \tfrac{1}{2} V_c$$

$$\dot{m} = \dot{V}/v = \tfrac{\pi}{2} V_c\, R^2/v$$

4.143

Steam at 3 MPa, 400°C enters a turbine with a volume flow rate of 5 m^3/s. An extraction of 15% of the inlet mass flow rate exits at 600 kPa, 200°C. The rest exits the turbine at 20 kPa with a quality of 90%, and a velocity of 20 m/s. Determine the volume flow rate of the extraction flow and the total turbine work.

Solution:

Inlet flow (Table B.1.3): $v_1 = 0.09936$ m^3/kg, $h_1 = 3230.82$ kJ/kg

$$\dot{m}_1 = \dot{V}/v_1 = 5/0.09936 = 50.32 \text{ kg/s}$$

Extraction flow : $v_2 = 0.35202$ m^3/kg; $h_2 = 2850.12$ kJ/kg

$$\dot{m}_2 = 0.15\, \dot{m}_1 = 7.55 \text{ kg/ s};$$

$$\dot{V}_2 = \dot{m}_2 v = 7.55 \text{ kg/s} \times 0.35202 \text{ m}^3/\text{kg} = \mathbf{2.658\ m^3/\ s}$$

Exit flow : $\dot{m}_3 = 0.85\, \dot{m}_1 = 42.77$ kg /s

Table B.1.2 $h_3 = 251.38 + 0.9 \times 2358.33 = 2373.88$ kJ/kg

Energy Eq.: $0 = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 - \dot{W}_T$

$$\dot{W}_T = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$
$$= 50.32 \times 3230.82 - 7.55 \times 2850.12 - 42.77 \times 2373.88$$
$$= \mathbf{39\ 525\ kW} = 39.5 \text{ MW}$$

4.144

In a glass factory a 2 m wide sheet of glass at 1500 K comes out of the final rollers that fix the thickness at 5 mm with a speed of 0.5 m/s. Cooling air in the amount of 20 kg/s comes in at 17°C from a slot 2 m wide and flows parallel with the glass. Suppose this setup is very long so the glass and air comes to nearly the same temperature (a co-flowing heat exchanger) what is the exit temperature?

Solution:

Energy Eq.: $\dot{m}_{glass}h_{glass\ 1} + \dot{m}_{air}h_{air\ 2} = \dot{m}_{glass}h_{glass\ 3} + \dot{m}_{air}h_{air\ 4}$

$\dot{m}_{glass} = \rho\dot{V} = \rho AV = 2500\ kg/m^3 \times 2\ m \times 0.005\ m \times 0.5\ m/s = 12.5\ kg/s$

$\dot{m}_{glass}C_{glass}\ (T_3 - T_1) + \dot{m}_{air}\ C_{Pa}\ (T_4 - T_2) = \varnothing$

$T_4 = T_3$, $C_{glass} = 0.80\ kJ/kg\ K$, $C_{Pa} = 1.004\ kJ/kg\ K$

$T_3 = \dfrac{\dot{m}_{glass}C_{glass}\ T_1 + \dot{m}_{air}C_{Pa}\ T_2}{\dot{m}_{glass}C_{glass} + \dot{m}_{air}C_{Pa}} = \dfrac{12.5\times0.80\times1500 + 20\times1.004\times290}{12.5\times0.80 + 20\times1.004}\ K$

$= \textbf{692.3 K}$

We could use table A.7.1 for air, but then it will be trial and error

4.145

Assume a setup similar to the previous problem but the air flows in the opposite direction of the glass, it comes in where the glass goes out. How much air flow at $17^{\circ}C$ is required to cool the glass to 450 K assuming the air must be at least 120 K cooler than the glass at any location?

Solution:

Energy Eq.: $\quad \dot{m}_1 h_1 + \dot{m}_4 h_4 = \dot{m}_3 h_3 + \dot{m}_2 h_2$

$\quad\quad T_4 = 290 \text{ K} \quad \text{and} \quad T_3 = 450 \text{ K}$

$\quad\quad \dot{m}_{glass} = \rho \dot{V} = \rho A V = 2500 \text{ kg/m}^3 \times 2 \text{ m} \times 0.005 \text{ m} \times 0.5 \text{ m/s} = 12.5 \text{ kg/s}$

$\quad\quad T_2 \leq T_1 - 120 \text{ K} = 1380 \text{ K}$

$$\dot{m} = \dot{m}_4 = \dot{m}_2 = \dot{m}_1 \frac{h_1 - h_3}{h_2 - h_4}$$

Let us check the limit and since T is high use table A.7.1 for air.

$\quad\quad h_4 = 290.43 \text{ kJ/kg}, \quad h_2 = 1491.33 \text{ kJ/kg}$

$$\dot{m} = \dot{m}_4 = \dot{m}_2 = \dot{m}_1 \frac{h_1 - h_3}{h_2 - h_4} = \dot{m}_1 \frac{C_{glass}(T_1 - T_3)}{h_2 - h_4}$$

$$\dot{m} = 12.5 \text{ kg/s} \frac{0.8 \ (1500 - 450)}{1491.33 - 290.43} = \textbf{8.743 kg/s}$$

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4.146

Two kg of water at 500 kPa, 20^OC is heated in a constant pressure process to 1700^OC. Find the best estimate for the heat transfer.

Solution:

C.V. Heater; steady state 1 inlet and exit, no work term, no ΔKE, ΔPE .

Continuity Eq.: $\dot{m}_{in} = \dot{m}_{ex} = \dot{m}$,

Energy Eq.4.13: $q + h_{in} = h_{ex} \Rightarrow q = h_{ex} - h_{in}$

steam tables only go up to 1300^oC so use an intermediate state at lowest pressure (closest to ideal gas) $h_X(1300^\mathrm{o}$C, 10 kPa) from Table B.1.3 and table A.8 for the high T change Δh from 1300^oC to 1700^oC.

$$h_{ex} - h_{in} = (h_{ex} - h_X) + (h_X - h_{in})$$
$$= (4515.55 - 3416.77) + (5409.7 - 83.96) = 6424.5 \text{ kJ/kg}$$
$$Q = m(h_{ex} - h_{in}) = 2 \times 6424.5 = \textbf{12 849 kJ}$$

Why is it done this way? The two tables have different offset values for *h*. Once differences in h between two states are taken from the same table the offset goes out.

4.147

A 500-L insulated tank contains air at 40°C, 2 MPa. A valve on the tank is opened, and air escapes until half the original mass is gone, at which point the valve is closed. What is the pressure inside then?

Solution:

State 1: ideal gas $\quad m_1 = P_1 V / R T_1 = \dfrac{2000 \times 0.5}{0.287 \times 313.2} = 11.125$ kg

Continuity eq.4.15: $\quad m_e = m_1 - m_2, \ m_2 = m_1/2 \implies m_e = m_2 = 5.5625$ kg

Energy Eq.4.16: $\quad m_2 u_2 - m_1 u_1 = - m_e h_{e \ avg}$

Substitute constant specific heat from table A.5 and evaluate the exit enthalpy as the average between the beginning and the end values

$$h_{e \ avg} \approx C_P (T_1 + T_2)/2$$

$$5.5625 \times 0.717 \, T_2 - 11.125 \times 0.717 \times 313.2 = - 5.5625 \times 1.004 \, (313.2 + T_2)/2$$

Solving, $T_2 = 239.4$ K

$$P_2 = \frac{m_2 R T_2}{V} = \frac{5.5625 \times 0.287 \times 239.4}{0.5} = \textbf{764 kPa}$$

4.148

Three air flows all at 200 kPa are connected to the same exit duct and mix without external heat transfer. Flow one has 1 kg/s at 400 K, flow two has 3 kg/s at 290 K and flow three has 2 kg/s at 700 K. Neglect kinetic energies and find the volume flow rate in the exit flow.

Solution:

Continuity Eq. $\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = \dot{m}_4 h_4$

Energy Eq.: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{m}_4 h_4$

$\dot{V}_4 = \dot{m} v_4$

$h_4 = \dfrac{\dot{m}_1}{\dot{m}_4} h_1 + \dfrac{\dot{m}_2}{\dot{m}_4} h_2 + \dfrac{\dot{m}_3}{\dot{m}_4} h_3 = \dfrac{1}{6} \times 401.3 + \dfrac{3}{6} \times 290.43 + \dfrac{2}{6} \times 713.56$

$= 449.95$ kJ/kg

$T_4 = 440 + 20 \dfrac{449.95 - 441.93}{462.34 - 441.93} = 447.86$ K

$v_4 = RT_4 / P_4 = 0.287$ kJ/kg-K $\times 447.86$ K/200 kPa = **0.643 m³/kg**

$\dot{V}_4 = \dot{m}_4 v_4 = 6 \times 0.643 = $ **3.858 m³/s**

4.149

Consider the power plant as described in Problem 4.121.

a. Determine the temperature of the water leaving the intermediate pressure heater, T_{13}, assuming no heat transfer to the surroundings.

b. Determine the pump work, between states 13 and 16.

Solution:

a) Intermediate Pressure Heater

Continuity Eq.: $\dot{m}_{11} = \dot{m}_{13}$; $\dot{m}_{12} + \dot{m}_{15} = \dot{m}_{14}$ feedwater line closed

Energy Eq.4.10: $\dot{m}_{11}h_{11} + \dot{m}_{12}h_{12} + \dot{m}_{15}h_{15} = \dot{m}_{13}h_{13} + \dot{m}_{14}h_{14}$

$$\dot{m}_{14} = \dot{m}_{12} + \dot{m}_{15} = 8.064 + 4.662 = 12.726 \text{ kg/s}$$

$$75.6{\times}284.6 + 8.064{\times}2517 + 4.662{\times}584 = 75.6{\times}h_{13} + 12.726{\times}349$$

$$h_{13} = 530.35 \text{ kJ/kg} \qquad \rightarrow T_{13} = \textbf{126.3°C}$$

b) The high pressure pump

Energy Eq.4.12: $\dot{m}_{13}h_{13} = \dot{m}_{16}h_{16} + \dot{W}_{Cv,P}$

$\dot{W}_{Cv,P} = \dot{m}_{13}(h_{13} - h_{16}) = 75.6$ kg/s $(530.35 - 565)$ kJ/kg = **-2620 kW**

4.150

Consider the powerplant as described in Problem 4.121.
 a. Find the power removed in the condenser by the cooling water (not shown).
 b. Find the power to the condensate pump.
 c. Do the energy terms balance for the low pressure heater or is there a
    heat transfer not shown?

Solution:

a) Condenser:

Continuity Eq.: $\dot{m}_5 + \dot{m}_{10} = \dot{m}_6$;   also $\dot{m}_5 = \dot{m}_3 - \dot{m}_9 - \dot{m}_8$

Energy Eq.4.10: $\dot{Q}_{CV} + \dot{m}_5 h_5 + \dot{m}_{10} h_{10} = \dot{m}_6 h_6$

$$\dot{m}_{10} = \dot{m}_6 - \dot{m}_5 = \dot{m}_6 - (\dot{m}_3 - \dot{m}_9 - \dot{m}_8)$$
$$= 75.6 - (62.874 - 4.662 - 2.772) = 75.6 - 55.44 = 20.16 \text{ kg/s}$$

$$\dot{Q}_{CV} + 55.44 \times 2279 + 20.16 \times 142.51 = 75.6 \times 138.3$$

$$\dot{Q}_{CV} = -118\ 765 \text{ kW} = \textbf{-118.77 MW}$$

b) The condensate pump

$$\dot{W}_{CV,P} = \dot{m}_6(h_6 - h_7) = 75.6 \text{ kg/s} (138.31 - 140) \text{ kJ/kg} = \textbf{-127.8 kW}$$

c) Low pressure heater   Assume no heat transfer

$$\dot{m}_{14} h_{14} + \dot{m}_8 h_8 + \dot{m}_7 h_7 + \dot{m}_9 h_9 = \dot{m}_{10} h_{10} + \dot{m}_{11} h_{11}$$
LHS = 12.726×349 + 2.772×2459 + 75.6×140 + 4.662×558 = 24 443 kW
RHS = (12.726 + 2.772 + 4.662) × 142.51 + 75.6 × 284.87 = 24 409 kW
        A slight imbalance, but OK.

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4.151

A 1-m^3, 40-kg rigid steel tank contains air at 500 kPa, and both tank and air are at 20°C. The tank is connected to a line flowing air at 2 MPa, 20°C. The valve is opened, allowing air to flow into the tank until the pressure reaches 1.5 MPa and is then closed. Assume the air and tank are always at the same temperature and the final temperature is 35°C. Find the final air mass and the heat transfer.

Solution:

Control volume: Air and the steel tank.

Continuity Eq.4.15: $m_2 - m_1 = m_i$

Energy Eq.4.16: $(m_2u_2 - m_1u_1)_{AIR} + m_{ST}(u_2 - u_1)_{ST} = m_ih_i + {}_1Q_2$

$$m_{1\ AIR} = \frac{P_1V}{RT_1} = \frac{500 \times 1}{0.287 \times 293.2} = 5.94 \text{ kg}$$

$$m_{2\ AIR} = \frac{P_2V}{RT_2} = \frac{1500 \times 1}{0.287 \times 308.2} = \mathbf{16.96 \text{ kg}}$$

$$m_i = (m_2 - m_1)_{air} = 16.96 - 5.94 = 11.02 \text{ kg}$$

The energy equation now gives

$$\begin{aligned}
{}_1Q_2 &= (m_2u_2 - m_1u_1)_{air} + m_{st}(u_2 - u_1)_{st} - m_ih_i \\
&= m_1(u_2 \text{-} u_1) + m_i(u_2 - u_i - RT_i) + m_{st}C_{st}(T_2 - T_1) \\
&\cong m_1C_v(T_2 - T_1) + m_i[C_v(T_2 - T_i) - RT_i] + m_{st}C_{st}(T_2 - T_1) \\
&= 5.94 \times 0.717(35 - 20) + 11.02[0.717(35 - 20) - 0.287 \times 293.2] \\
&\quad + 40 \times 0.46(35 - 20) \\
&= 63.885 - 808.795 + 276 \\
&= \mathbf{-\ 468.9 \text{ kJ}}
\end{aligned}$$

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4.152

A steam engine based on a turbine is shown in Fig. P4.152. The boiler tank has a volume of 100 L and initially contains saturated liquid with a very small amount of vapor at 100 kPa. Heat is now added by the burner, and the pressure regulator does not open before the boiler pressure reaches 700 kPa, which it keeps constant. The saturated vapor enters the turbine at 700 kPa and is discharged to the atmosphere as saturated vapor at 100 kPa. The burner is turned off when no more liquid is present in the boiler. Find the total turbine work and the total heat transfer to the boiler for this process.

Solution:

C.V. Boiler tank.   Heat transfer, no work and flow out.

Continuity Eq.4.15: $m_2 - m_1 = -m_e$

Energy Eq.4.16: $m_2 u_2 - m_1 u_1 = Q_{CV} - m_e h_e$

State 1: Table B.1.1, 100 kPa $\Rightarrow$ $v_1 = 0.001\ 043$, $u_1 = 417.36$ kJ/kg

$\Rightarrow m_1 = V/v_1 = 0.1/0.001\ 043 = 95.877$ kg

State 2: Table B.1.1, 700 kPa $\Rightarrow$ $v_2 = v_g = 0.2729$, $u_2 = 2572.5$ kJ/kg

$\Rightarrow$ $m_2 = V/v_g = 0.1/0.2729 = 0.366$ kg,

Exit state: Table B.1.1, 700 kPa $\Rightarrow$ $h_e = 2763.5$ kJ/kg

From continuity eq.: $m_e = m_1 - m_2 = 95.511$ kg

$Q_{CV} = m_2 u_2 - m_1 u_1 + m_e h_e$

$= 0.366 \times 2572.5 - 95.877 \times 417.36 + 95.511 \times 2763.5$

$= 224\ 871$ kJ = **224.9 MJ**

C.V. Turbine, steady state, inlet state is boiler tank exit state.

Turbine exit state: Table B.1.1, 100 kPa $\Rightarrow$ $h_e = 2675.5$ kJ/kg

$W_{turb} = m_e (h_{in} - h_{ex}) = 95.511 \times (2763.5 - 2675.5) = $ **8405 kJ**

BOILER   Pressure regulator Adiabatic turbine

4.153

An insulated spring-loaded piston/cylinder, shown in Fig. P4.153, is connected to an air line flowing air at 600 kPa, 700 K by a valve. Initially the cylinder is empty and the spring force is zero. The valve is then opened until the cylinder pressure reaches 300 kPa. By noting that $u_2 = u_{line} + C_V(T_2 - T_{line})$ and $h_{line} - u_{line} = RT_{line}$ find an expression for T_2 as a function of P_2, P_o, T_{line}. With P = 100 kPa, find T_2.

Solution:

C.V. Air in cylinder, insulated so $_1Q_2 = 0$

Continuity Eq.4.15: $\qquad m_2 - m_1 = m_{in}$

Energy Eq.4.16: $\qquad m_2u_2 - m_1u_1 = m_{in}h_{line} - {}_1W_2$

$\qquad m_1 = 0 \implies m_{in} = m_2 \; ; \; m_2u_2 = m_2h_{line} - \frac{1}{2}(P_0 + P_2)m_2v_2$

$$\implies \qquad u_2 + \frac{1}{2}(P_0 + P_2)v_2 = h_{line}$$

Use constant specific heat in the energy equation

$$C_V(T_2 - T_{line}) + u_{line} + \frac{1}{2}(P_0 + P_2)RT_2/P_2 = h_{line}$$

$$\left[\mathbf{C_V} + \frac{\mathbf{1}}{\mathbf{2}} \frac{\mathbf{P_0 + P_2}}{\mathbf{P_2}} \mathbf{R} \right] \mathbf{T_2} = (\mathbf{R} + \mathbf{C_V})\mathbf{T_{line}}$$

with #'s: $T_2 = \dfrac{R + C_V}{\frac{2}{3}R + C_V} T_{line} \; ; \quad C_V/R = 1/(k-1) , \quad k = 1.4$

$$T_2 = \frac{k - 1 + 1}{\frac{2}{3}k - \frac{2}{3} + 1} T_{line} = \frac{3k}{2k + 1} T_{line} = 1.105 \; T_{line} = \mathbf{773.7 \; K}$$

4.154

A mass-loaded piston/cylinder, shown in Fig. P4.154, containing air is at 300 kPa, 17°C with a volume of 0.25 m^3, while at the stops $V = 1$ m^3. An air line, 500 kPa, 600 K, is connected by a valve that is then opened until a final inside pressure of 400 kPa is reached, at which point $T = 350$ K. Find the air mass that enters, the work, and heat transfer.

Solution:

C.V. Cylinder volume.

Continuity Eq.4.15: $m_2 - m_1 = m_{in}$

Energy Eq.4.16: $m_2 u_2 - m_1 u_1 = m_{in} h_{line} + Q_{CV} - {_1}W_2$

Process: P_1 is constant to stops, then constant V to state 2 at P_2

State 1: P_1, T_1 $m_1 = \dfrac{P_1 V}{RT_1} = \dfrac{300 \times 0.25}{0.287 \times 290.2} = 0.90$ kg

State 2:
     Open to $P_2 = 400$ kPa, $T_2 = 350$ K

$m_2 = \dfrac{400 \times 1}{0.287 \times 350} = 3.982$ kg

$m_i = 3.982 - 0.90 = $ **3.082 kg**

Only work while constant P

$_1 W_2 = P_1(V_2 - V_1) = 300(1 - 0.25) = $ **225 kJ**

Energy Eq.: $Q_{CV} + m_i h_i = m_2 u_2 - m_1 u_1 + {_1}W_2$

$Q_{CV} = 3.982 \times 0.717 \times 350 - 0.90 \times 0.717 \times 290.2 + 225$

$- 3.082 \times 1.004 \times 600 = $ **-819.2 kJ**

We could also have used the air tables A.7.1 for the u's and h_i.

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4.155

A 2-m^3 storage tank contains 95% liquid and 5% vapor by volume of liquified natural gas (LNG) at 160 K, as shown in Fig. P4.155. It may be assumed that LNG has the same properties as pure methane. Heat is transferred to the tank and saturated vapor at 160 K flows into a steady flow heater which it leaves at 300 K. The process continues until all the liquid in the storage tank is gone. Calculate the total amount of heat transfer to the tank and the total amount of heat transferred to the heater.

Solution:

CV: Tank, flow out, transient.

Continuity Eq.: $m_2 - m_1 = -m_e$

Energy Eq.:

$Q_{Tank} = m_2 u_2 - m_1 u_1 + m_e h_e$

At 160 K, from Table B.7:

$$m_f = V_f / v_f = \frac{0.95 \times 2}{0.00297} = 639.73 \text{ kg}, \quad m_g = V_g / v_g = \frac{0.05 \times 2}{0.03935} = 2.541 \text{ kg}$$

$m_1 = 642.271$ kg, $\quad m_2 = V/v_{g2} = 2/0.03935 = 50.826$ kg

$m_1 u_1 = 639.73(-106.35) + 2.541(207.7) = -67507$ kJ

$m_e = m_1 - m_2 = 591.445$ kg

$Q_{Tank} = 50.826 \times 207.7 - (-67\ 507) + 591.445 \times 270.3$

$= $ **+237 931 kJ**

CV: Heater, steady flow, $P = P_{G\ 160\ K} = 1593$ kPa

$Q_{Heater} = m_{e\ Tank}(h_e - h_i)_{Heater}$

$= 591.445(612.9 - 270.3) = $ **202 629 kJ**

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CHAPTER 4

Borgnakke and Sonntag

Continuity and Flow Rates

4.156E

Refrigerant R-410A at 100 psia, 60 F flows at 0.1 lbm/s in a 2.5 ft^2 cross sectional area pipe. Find the velocity and the volume flow rate.

$$\dot{m} = A\mathbf{V}/v = \dot{V}/v$$

Table F.9.2: $v = 0.6848$ ft^3/lbm

$\dot{V} = v\,\dot{m} = 0.6848$ ft^3/lbm $\times$ 0.1 lbm/s = **0.06848 ft^3/s**

$\mathbf{V} = \dot{V}\,/\,A = 0.06848$ (ft^3/s) / 2.5 ft^2 = **0.0274 ft/s**

4.157E

Air at 95 F, 16 lbf/in.^{2}, flows in a 4 in. × 6 in. rectangular duct in a heating system. The volumetric flow rate is 30 cfm (ft^3/min). What is the velocity of the air flowing in the duct?

Solution:

Assume a constant velocity across the duct area with

$$A = 4 \times 6 \times \frac{1}{144} = 0.167 \text{ ft}^2$$

and the volumetric flow rate from Eq.4.3,

$$\dot{V} = \dot{m}v = A\mathbf{V}$$

$$\mathbf{V} = \frac{\dot{V}}{A} = \frac{30}{60 \times 0.167} = \textbf{3.0 ft/s}$$

Ideal gas so note:

$$\left(\begin{array}{l} \text{note ideal gas:} \quad v = \dfrac{RT}{P} = \dfrac{53.34 \times 554.7}{16 \times 144} = 12.842 \text{ ft}^3/\text{lbm} \\[2mm] \qquad\quad \dot{m} = \dfrac{\dot{V}}{v} = \dfrac{30}{60 \times 12.842} = 0.0389 \text{ lbm/s} \end{array} \right)$$

4.158E

A pool is to be filled with 2500 ft^3 water from a garden hose of 1 in. diameter flowing water at 6 ft/s. Find the mass flow rate of water and the time it takes to fill the pool.

Solution:

With constant velocity we have

$$\dot{V} = \dot{m}v = A\mathbf{V} = \pi \times (1/12)^2 \text{ ft}^2 \times 6 \text{ ft/s} = 0.1309 \text{ ft}^3/\text{s}$$

$$\Delta t = V / \dot{V} = 2500 \text{ ft}^3 / (0.1309 \text{ ft}^3/\text{s}) = 19\ 098 \text{ s} = \textbf{5 h 18 min 18 s}$$

From table F.3 we get the water density

$$\dot{m} = \dot{V}/v = \rho\dot{V} = 62.2 \text{ lbm/ ft}^3 \times 0.1309 \text{ ft}^3/\text{s} = \textbf{8.14 lbm/s}$$

4.159E

A hot air home heating system takes 500 ft³/min (cfm) air at 14.7 psia, 65 F into a furnace and heats it to 130 F and delivers the flow to a square duct 0.5 ft by 0.5 ft at 15 psia. What is the velocity in the duct?

Solution:

The inflate flow is given by a $\dot{m}_i$

Continuity Eq.: $\dot{m}_i = \dot{V}_i / v_i = \dot{m}_e = A_e \mathbf{V}_e / v_e$

Ideal gas: $v_i = \dfrac{RT_i}{P_i} = \dfrac{53.34 \times 525}{14.7 \times 144} = 13.23 \dfrac{ft^3}{lbm}$

$$v_e = \frac{RT_e}{P_e} = \frac{53.34 \text{ lbf-ft/lbm-R} \times (130 + 460) \text{ R}}{15 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2}$$

$$= 14.57 \text{ ft}^3/ \text{lbm}$$

$$\dot{m}_i = \dot{V}_i / v_i = 500 \text{ (ft}^3/\text{min)} /[60 \text{ (s/min)} \times 13.23 \frac{ft^3}{lbm}] = 0.63 \text{ lbm/s}$$

$$\mathbf{V}_e = \dot{m} \, v_e / A_e = \frac{0.63 \times 14.57}{0.5 \times 0.5} \frac{ft^3/s}{ft^2} = \mathbf{36.7 \ ft/s}$$

4.160E

Liquid water at 60 F flows out of a nozzle straight up 40 ft. What is nozzle $\mathbf{V}_{exit}$?

Energy Eq.4.13: $h_{exit} + \frac{1}{2}\mathbf{V}_{exit}^2 + gH_{exit} = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gH_2$

If the water can flow 40 ft up it has specific potential energy of gH_2 which must equal the specific kinetic energy out of the nozzle $\mathbf{V}_{exit}^2/2$. The water does not change P or T so h is the same.

$$\mathbf{V}_{exit}^2/2 = g(H_2 - H_{exit}) = gH \quad =>$$
$$\mathbf{V}_{exit} = \sqrt{2gH} = \sqrt{2 \times 32.174 \times 40 \text{ ft}^2/\text{s}^2} = \textbf{50.7 ft/s}$$

4.161E

A diffuser receives 0.2 lbm/s steam at 80 psia, 600 F. The exit is at 150 psia, 700 F with negligible kinetic energy and the flow is adiabatic. Find the diffuser inlet velocity and the inlet area.

Solution:

Energy Eq.4.13: $h_1 + \frac{1}{2}V_1^2 + gZ_1 = h_2 + \frac{1}{2}V_2^2 + gZ_2$

Process: $Z_1 = Z_2$

State 1:   Table F.7.2 $h_1 = 1330.66$ Btu/lbm, $v_1 = 7.794$ ft^3/lbm

State 2: $V_2 = 0$,   Table F.7.2 $h_2 = 1376.55$ Btu/lbm

Then from the energy equation

$$\frac{1}{2}V_1^2 = h_2 - h_1 = 1376.55 - 1330.66 = 45.89 \text{ Btu/lbm}$$

$$V_1 = \sqrt{2(h_2 - h_1)} = \sqrt{2 \times 32.174 \times 778 \times 45.89} = \textbf{1515.7 ft/s}$$

The mass flow rate from Eq.4.3

$\dot{m} = \rho A V = AV/v$

$A = \dot{m}v/V = 0.2$ lbm/s $\times$ (7.794 ft^3/lbm) / 1515.7 ft/s

$= 0.000103$ ft^2 = **0.148 in.^{2}**

Inlet

Hi V
Low P, A

Exit

Low V
Hi P, A

4.162E

Saturated vapor R-134a leaves the evaporator in a heat pump at 50 F, with a steady mass flow rate of 0.2 lbm/s. What is the smallest diameter tubing that can be used at this location if the velocity of the refrigerant is not to exceed 20 ft/s?

Solution:

Mass flow rate Eq.4.3: $\dot{m} = \dot{V}/v = A\mathbf{V}/v$

Exit state Table F.10.1: (T = 50 F, x =1) => $v = v_g = 0.792$ ft^3/lbm

The minimum area is associated with the maximum velocity for given $\dot{m}$

$$A_{MIN} = \frac{\dot{m}v_g}{\mathbf{V}_{MAX}} = \frac{0.2 \text{ lbm/s} \times 0.792 \text{ ft}^3/\text{lbm}}{20 \text{ ft/s}} = 0.00792 \text{ ft}^2 = \frac{\pi}{4} D^2_{MIN}$$

$D_{MIN} ==$ **0.1004 ft = 1.205 in**

Exit

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Single Flow Devices

4.163E

In a jet engine a flow of air at 1800 R, 30 psia and 90 ft/s enters a nozzle where the air exits at 1500 R, 13 psia, as shown in Fig. P.4.23. What is the exit velocity assuming no heat loss?

Solution:

C.V. nozzle. No work, no heat transfer

Continuity $\quad \dot{m}_i = \dot{m}_e = \dot{m}$

Energy : $\quad \dot{m}\,(h_i + \tfrac{1}{2}\mathbf{V}_i^2) = \dot{m}(h_e + \tfrac{1}{2}\mathbf{V}_e^2)$

Due to high T take h from table F.5

$$\tfrac{1}{2}\mathbf{V}_e^2 = \tfrac{1}{2}\,\mathbf{V}_i^2 + h_i - h_e$$

$$= \frac{90^2}{2 \times 32.174 \times 778} + 449.79 - 369.28$$

$$= 0.16 + 80.51 = 80.67 \text{ Btu/lbm}$$

$\mathbf{V}_e = (2 \times 32.174 \text{ lbm-ft/s}^2\text{-lbf} \times 778 \text{ lbf-ft/Btu} \times 80.67 \text{ Btu/lbm})^{1/2}$

$\quad = \mathbf{2010 \ ft/s}$

4.164E

A sluice gate dams water up 15 ft. A 0.5 in. diameter hole at the bottom of the gate allows liquid water at 70 F to come out. Neglect any changes in internal energy and find the exit velocity and mass flow rate.

Solution:

Energy Eq.4.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$

Process: $h_1 = h_2$ both at P = 1 atm
            $V_1 = 0$ $Z_1 = Z_2 + 15$ ft

$\frac{1}{2}\mathbf{V}_2^2 = g\,(Z_1 - Z_2)$

$\mathbf{V}_2 = \sqrt{2g(Z_1 - Z_2)} = \sqrt{2 \times 32.174 \times 15} = \mathbf{31.07\ ft/s}$

$\dot{m} = \rho A\mathbf{V} = A\mathbf{V}/v = \frac{\pi}{4}D^2 \times (\mathbf{V}_2/v)$

$= \frac{\pi}{4} \times (0.5/12)^2\ ft^2 \times (31.07\ ft/s) / (0.01605\ ft^3/lbm)$

$= \mathbf{2.639\ lbm/s}$

4.165E

A diffuser shown in Fig. P4.28 has air entering at 14.7 lbf/in.^{2}, 540 R, with a velocity of 600 ft/s. The inlet cross-sectional area of the diffuser is 0.2 in.^{2}. At the exit, the area is 1.75 in.^{2}, and the exit velocity is 60 ft/s. Determine the exit pressure and temperature of the air.

Solution:

Continuity Eq.4.3: $\dot{m}_i = A_i V_i/v_i = \dot{m}_e = A_e V_e/v_e,$

Energy Eq.(per unit mass flow) 4.13: $h_i + \frac{1}{2}V_i^2 = h_e + \frac{1}{2}V_e^2$

$$h_e - h_i = (1/2)\times(600^2 - 60^2)/(32.174 \times 778) = 7.119 \text{ Btu/lbm}$$

$$T_e = T_i + (h_e - h_i)/C_p = 540 \text{ R} + \frac{7.119 \text{ Btu/lbm}}{0.24 \text{ Btu/lbm-R}} = \mathbf{569.7 \ R}$$

Now use the continuity equation and the ideal gas law

$$v_e = v_i \left(\frac{A_e V_e}{A_i V_i}\right) = (RT_i/P_i)\left(\frac{A_e V_e}{A_i V_i}\right) = RT_e/P_e$$

$$P_e = P_i \left(\frac{T_e}{T_i}\right)\left(\frac{A_i V_i}{A_e V_e}\right) = 14.7 \left(\frac{569.7}{540}\right)\left(\frac{0.2 \times 600}{1.75 \times 60}\right) = \mathbf{17.72 \ lbf/in.^2}$$

Inlet
Hi V
Low P, A

Exit
Low V
Hi P, A

4.166E

Nitrogen gas flows into a convergent nozzle at 30 lbf/in.^{2}, 600 R and very low velocity. It flows out of the nozzle at 15 lbf/in.^{2}, 500 R. If the nozzle is insulated find the exit velocity.

Solution:

C.V. Nozzle steady state one inlet and exit flow, insulated so it is adiabatic.

Energy Eq.4.13: $\quad h_1 + 0 = h_2 + \frac{1}{2} \mathbf{V}_2^2$

$\mathbf{V}_2^2 = 2 \, (\, h_1 - h_2 \,) \cong 2 \, C_{PN_2} \, (T_1 - T_2)$

$\quad = 2 \times 0.249 \text{ Btu/lbm-R} \times (600 - 500) \text{ R}$

$\quad = 24.9 \text{ Btu/lbm}$

$\mathbf{V}_2^2 = 2 \times 24.9 \text{ Btu/lbm} \times 778 \text{ lbf-ft/Btu} \times 32.174 \text{ lbm-ft/s}^2\text{-lbf}$

$\quad = 1\,246\,562 \text{ ft}^2 / \text{s}^2$

$\quad\quad \mathbf{V}_2 = \mathbf{1116 \text{ ft/s}}$

4.167E

A meteorite hits the upper atmosphere at 10 000 ft/s, where the pressure is 0.1 atm and the temperature is −40 F. How hot does the air become right in front of the meteorite assuming no heat transfer in this adiabatic stagnation process?

Solution:

Energy Eq.: $\quad h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$

Process: $\quad\quad Z_1 = Z_2 \quad\quad$ and $\quad\quad V_2 = 0$

$$h_2 = h_1 + \frac{1}{2}\mathbf{V}_1^2$$

$$T_2 = T_1 + \frac{1}{2}(\mathbf{V}_1^2/C_p) = 419.67 + \frac{1}{2} \times \frac{10\,000^2}{0.24 \times 778 \times 32.174} = \mathbf{8742\ R}$$

At this high temperature we cannot assume constant specific heats, so use F.5

$$h_2 = h_1 + \frac{1}{2}\mathbf{V}_1^2 = 100.4 + \frac{1}{2} \times \frac{10\,000^2}{778 \times 32.174} = 2097.9 \text{ Btu/lbm}$$

Table F.5 is listed to 5400 R so we have to extrapolate to get

$$T_2 = 5400 + 100\,\frac{2097.9\ -\ 1515.632}{1515.632\ -\ 1484.762} = \mathbf{7286\ R}$$

The value of C_p over 5400 R is 0.309 Btu/lbmR from the last two table entries. At this temperature there will be some chemical reactions that should be considered to have a more realistic temperature.

4.168E

Refrigerant R-410A flows out of a cooler at 70 F, 220 psia after which it is throttled to 77 psia. Find the state (T, x) for the exit flow.

C.V. Throttle. Steady state,

Process with: $q = w = 0$;   and $\mathbf{V}_i = \mathbf{V}_e$, $Z_i = Z_e$

Energy Eq.4.13: $h_i = h_e$,

Inlet state: Table F.9.1 $h_i = 39.17$ Btu/lbm

Exit state: Table F.9.1   since $h < h_g = 118.21$ Btu/lbm

Interpolate: $T = \mathbf{10\ F}$, $h_f = 17.0$ Btu/lbm, $h_{fg} = 101.22$ Btu/lbm

   $x = (39.17 – 17.0) / 101.22 = \mathbf{0.219}$

4.169E

R-410A at 90 F, 300 psia is throttled so it becomes cold at 10 F. What is exit P?

State 1 is slightly compressed liquid so
Table F.9.1: $h = h_f = 47.30$ Btu/lbm

At the lower temperature it becomes two-phase since the throttle flow has constant h and at 10 F: $h_g = 118.21$ Btu/lbm

$$P = P_{sat} = \textbf{76.93 psia}$$

4.170E

Saturated liquid R-410A at 30 F is throttled to 40 psia in a refrigerator. What is the exit temperature? Find the percent increase in the volume flow rate.

Solution:

Steady throttle flow. Assume no heat transfer and no change in kinetic or potential energy.

$$h_e = h_i = h_{f\,30\,F} = 24.11 \text{ Btu/lbm} = h_{f\,e} + x_e\,h_{fg\,e} \quad \text{at 40 psia}$$

From table F.9.1 we get $T_e = T_{sat}$ (40 psia) = **-21 F**

$$x_e = \frac{h_e - h_{f\,e}}{h_{fg\,e}} = \frac{24.11 - 6.354}{108.776} = 0.1632$$

$$v_e = v_f + x_e\,v_{fg} = 0.01253 + x_e\,1.47584 = 0.25338 \text{ ft}^3/\text{lbm}$$

$$v_i = v_{f\,30\,F} = 0.01364 \text{ ft}^3/\text{lbm}$$

$\dot{V} = \dot{m}v$ so the ratio becomes

$$\frac{\dot{V}_e}{\dot{V}_i} = \frac{\dot{m}v_e}{\dot{m}v_i} = \frac{v_e}{v_i} = \frac{0.25338}{0.01364} = \mathbf{18.58}$$

So the increase is 17.58 times or **1758 %**

4.171E

Saturated vapor R-410A at 75 psia is throttled to 15 psia. What is the exit temperature. Repeat the question if you assumed it behaves like an ideal gas.

C.V. Throttle. Steady state,

Process with: $q = w = 0$;   and $\mathbf{V}_i = \mathbf{V}_e$, $Z_i = Z_e$

Energy Eq.4.13: $h_i = h_e$,

Inlet state: Table F.9.2 $h_i = 118.09$ Btu/lbm

Exit state: Table F.9.2   since $h > h_g$

Exit state very close to $T = -20$ **F**,

For an ideal gas h is only a function of T, so $T_e = T_i = $ **8.71 F**

4.172E

Helium is throttled from 175 lbf/in.^{2}, 70 F, to a pressure of 15 lbf/in.^{2}. The. diameter of the exit pipe is so much larger than the inlet pipe that the inlet and exit velocities are equal. Find the exit temperature of the helium and the ratio of the pipe diameters.

C.V. Throttle. Steady state,

Process with: $q = w = 0$; and $\mathbf{V}_i = \mathbf{V}_e,\ \ Z_i = Z_e$

Energy Eq.4.13: $h_i = h_e$, Ideal gas $\Rightarrow$ $T_i = T_e = $ **70 F**

$$\dot{m} = \frac{A\mathbf{V}}{RT/P} \quad \text{But } \dot{m},\ \mathbf{V},\ T \text{ are constant} \quad \Rightarrow \quad P_i A_i = P_e A_e$$

$$\Rightarrow \quad \frac{D_e}{D_i} = \left(\frac{P_i}{P_e}\right)^{1/2} = \left(\frac{175}{15}\right)^{1/2} = \mathbf{3.416}$$

4.173E

A liquid water turbine receives 4 lbm/s water at 300 psia, 77 F with a velocity of 50 ft/s. The exit is at 15 psia, 77 F with very low velocity. Find the specific work and the power produced.

Solution:

Energy Eq.4.13: $h_1 + \frac{1}{2} \mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2} \mathbf{V}_2^2 + gZ_2 + w_T$

Process: $Z_1 = Z_2$ and $\mathbf{V}_2 = 0$

State 1:   Table F.7.1 $h_1 = 45.09$ Btu/lbm

Add $\Delta Pv = (300-0.464) \times 0.01606 \times 32.174/144 = 1.075$ Btu/lbm

State 2:   Table F.7.1 $h_2 = 45.09$

Add $\Delta Pv = (15-0.464) \times 0.01606 \times 32.174/144 = 0.052$ Btu/lbm

$$w_T = h_1 + \frac{1}{2} \mathbf{V}_1^2 - h_2$$

$$= (45.09 + 1.075) + \frac{1}{2} \times \frac{50^2}{778 \times 32.174} - (45.09 + 0.052)$$

$$= \mathbf{1.073\ Btu/lbm}$$

$$\dot{W}_T = \dot{m} \times w_T = 4 \times 1.073 = \mathbf{4.29\ Btu/s}$$

Notice how insignificant the specific kinetic energy is (0.05 Btu/lbm).

The added ΔPv terms are due to P being higher than saturated P, see p.134.

4.174E

Hoover Dam across the Colorado River dams up Lake Mead 600 ft higher than the river downstream, as shown in Fig. P4.47. The electric generators driven by water-powered turbines deliver 1.2×10^6 Btu/s. If the water is 65 F, find the minimum amount of water running through the turbines.

Solution:

C.V.: H_2O pipe + turbines,

Continuity: $\dot{m}_{in} = \dot{m}_{ex}$;

Energy Eq.4.13: $(h + \mathbf{V}^2/2 + gz)_{in} = (h + \mathbf{V}^2/2 + gz)_{ex} + w_T$

Water states: $h_{in} \cong h_{ex}$; $v_{in} \cong v_{ex}$

Now the specific turbine work becomes

$$w_T = gz_{in} - gz_{ex}$$

$$= 32.174 \text{ ft/s}^2 \times 600 \text{ ft}/(778 \text{ lbf-ft/Btu} \times 32.174 \text{ lbm-ft/s}^2\text{-lbf})$$

$$= 0.771 \text{ Btu/lbm}$$

$$\dot{m} = \dot{W}_T/w_T = 1.2 \times 10^6 (\text{Btu/s}) / 0.771 (\text{Btu/lbm}) = 1.556 \times 10^6 \text{ lbm/s}$$

$$\dot{V} = \dot{m}v = 1.556 \times 10^6 \times 0.016043 = \mathbf{24\ 963\ ft^3/s}$$

4.175E

A small expander (a turbine with heat transfer) has 0.1 lbm/s helium entering at 160 psia, 1000 R and it leaves at 40 psia, 540 R. The power output on the shaft is measured to 55 Btu/s. Find the rate of heat transfer neglecting kinetic energies.

Solution:

C.V. Expander. Steady operation

Continuity Eq.: $\dot{m}_i = \dot{m}_e = \dot{m}$

Energy Eq.: $\dot{m}h_i + \dot{Q} = \dot{m}h_e + \dot{W}$

$$\dot{Q} = \dot{m}(h_e - h_i) + \dot{W}$$

Use heat capacity from Table F.4: $C_{p\,He} = 1.24$ Btu/lbm R

$$\dot{Q} = \dot{m}C_p(T_e - T_i) + \dot{W}$$

$$= 0.1 \text{ lbm/s} \times 1.24 \text{ Btu/lbm-R} \times (540 - 1000) \text{ R} + 55 \text{ Btu/s}$$

$$= -57.04 + 55 = \textbf{-2.0 Btu/s}$$

4.176E

A small, high-speed turbine operating on compressed air produces a power output of 0.1 hp. The inlet state is 60 lbf/in.^{2}, 120 F, and the exit state is 14.7 lbf/in.^{2}, −20 F. Assuming the velocities to be low and the process to be adiabatic, find the required mass flow rate of air through the turbine.

Solution:

C.V. Turbine, no heat transfer, no ΔKE, no ΔPE

Energy Eq.4.13: $h_{in} = h_{ex} + w_T$

Ideal gas so use constant specific heat from Table A.5

$$w_T = h_{in} - h_{ex} \cong C_p(T_{in} - T_{ex})$$

$$= 0.24(120 - (-20)) = 33.6 \text{ Btu/lbm}$$

$$\dot{W} = \dot{m}w_T \quad \Rightarrow$$

$$\dot{m} = \dot{W}/w_T = \frac{0.1 \text{ hp} \times 550 \text{ lbf-ft/s-hp}}{778 \text{ lbf-ft/Btu} \times 33.6 \text{ Btu/lbm}} = \textbf{0.0021 lbm/s} = \textbf{7.57 lbm/h}$$

The dentist's drill has a small air flow and is not really adiabatic.

4.177E

A compressor in a commercial refrigerator receives R-410A at -10 F and x = 1. The exit is at 200 psia, 120 F. Neglect kinetic energies and find the specific work.

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also assume no heat transfer and $Z_i = Z_e$

From Table F.9.1 : $\qquad$ $h_i = 116.29$ Btu/lbm

From Table F.9.2 : $\qquad$ $h_e = 137.02$ Btu/lbm

Energy Eq.4.13 reduces to

$$w_c = h_i - h_e = (116.29 - 137.02) = \textbf{-20.73 Btu/lbm}$$

4.178E

A compressor in an industrial air-conditioner compresses ammonia from a state of saturated vapor at 20 psia to a pressure of 125 psia. At the exit the temperature is measured to be 200 F and the mass flow rate is 1 lbm/s. What is the required power input to this compressor?

Solution:

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.4.13: $\qquad w_C = h_1 - h_2$

States: 1: F.8.2: $\quad h_1 = 605.47$ Btu/lbm

$\qquad$ 2: F.8.2 $\quad h_{2,AC} = 710.44$ Btu/lbm

Energy equation:

$$-w_C = h_2 - h_1 = 710.44 - 605.47 = 104.97 \text{ Btu/lbm}$$
$$\dot{W} = 1 \text{ lbm/s} \times 104.97 \text{ Btu/lbm} = \textbf{105 Btu/s}$$

4.179E

An exhaust fan in a building should be able to move 6 lbm/s air at 14.4 psia, 68 F through a 1.4 ft diameter vent hole. How high a velocity must it generate and how much power is required to do that?

Solution:

C.V. Fan and vent hole. Steady state with uniform velocity out.

Continuity Eq.: $\dot{m}$ = constant = $\rho A \mathbf{V}$ = $A\mathbf{V} / v$ = $A\mathbf{V}P/RT$

Ideal gas : $Pv = RT$,   and area is $A = \dfrac{\pi}{4} D^2$

Now the velocity is found

$$\mathbf{V} = \dot{m}\, RT/(\frac{\pi}{4} D^2\, P) = \frac{6 \text{ lbm/s} \times 53.34 \text{ lbf-ft/lbm-R} \times (459.7 + 68) \text{ R}}{\frac{\pi}{4} \times 1.4^2 \times 14.4 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2}$$

$= \mathbf{52.908\ ft/s}$

The kinetic energy out is

$$\frac{1}{2}\mathbf{V}_2^2 = \frac{1}{2} \times 52.908^2 / 32.174 = 43.502 \text{ lbf-ft/lbm}$$

which is provided by the work (only two terms in energy equation that does not cancel, we assume $\mathbf{V}_1 = 0$)

$$\dot{W}_{in} = \dot{m}\,\frac{1}{2}\mathbf{V}_2^2 = 6 \times 43.502 = \mathbf{261\ lbf\text{-}ft/s} = \mathbf{0.335\ Btu/s}$$

4.180E

An air flow is brought from 77 F, 14.7 psia to 150 psia, 620 F by an adiabatic compressor driven by a 50-kW motor. What are the mass flow rate and the exit volume flow rate of air?

C.V. Compressor. Assume adiabatic and neglect kinetic, potential energies.

Energy Eq.4.13: $\qquad w = h_1 - h_2$

$$-w_C = h_2 - h_1 \approx C_P (T_2 - T_1) = 0.24 \text{ Btu/lbm-R} \times (620 - 77) \text{ R}$$
$$= 130.32 \text{ Btu/lbm}$$

The mass flow rate scale the work term so

$$\dot{m} = \dot{W} / -w_C = \frac{(50/1.055) \text{ Btu/s}}{130.32 \text{ Btu/lbm}} = \textbf{0.3637 lbm/s}$$

$$\dot{V} = \dot{m} \, v = \dot{m} \frac{RT}{P} = 0.3637 \text{ lbm/s} \frac{53.34 \times (620 + 459.67) \text{ ft-lbf/lbm}}{150 \text{ lbf/in}^2 \times 144 \text{ (in/ft)}^2}$$
$$= \textbf{2.666 ft}^3\textbf{/s}$$

4.181E

Carbon dioxide gas enters a steady-state, steady-flow heater at 45 lbf/in.^{2} 60 F, and exits at 40 lbf/in.^{2}, 1800 F. It is shown in Fig. P4.65 here changes in kinetic and potential energies are negligible. Calculate the required heat transfer per lbm of carbon dioxide flowing through the heater.

Solution:

C.V. Heater Steady state single inlet and exit flow.

Energy Eq.4.13: $q + h_i = h_e$

Table F.6 $q = h_e - h_i = \dfrac{20470.8 - (-143.4)}{44.01} = $ **468.4 Btu/lbm**

[If we use C_{P0} then $q \cong 0.203$ Btu/lbm-R (1800 - 60) R = 353.2 Btu/lbm]

Too large ΔT, T_{ave} to use C_{P0} at room temperature.

4.182E

A condenser (cooler) receives 0.1 lbm/s of R-410A at 300 psia, 140 F and cools it to 70 F. Assume the exit properties are as for saturated liquid same T. What cooling capacity (Btu/h) must the condenser have?

Solution:

C.V. R-410A condenser. Steady state single flow, heat transfer out and no work.

Energy Eq.4.12: $\dot{m}\,h_1 = \dot{m}\,h_2 + \dot{Q}_{out}$

Inlet state: Table F.9.2 $h_1 = 137.34$ Btu/lbm,

Exit state: Table F.9.1 $h_2 = 39.17$ Btu/lbm (compressed liquid)

Process: Neglect kinetic and potential energy changes.

Cooling capacity is taken as the heat transfer out i.e. positive out so

$\dot{Q}_{out} = \dot{m}\,(h_1 - h_2) = 0.1$ lbm/s (137.34 – 39.17) Btu/lbm

= 9.82 Btu/s

4.183E

In a steam generator, compressed liquid water at 1500 lbf/in.^{2}, 100 F, enters a 1-in. diameter tube at the rate of 5 ft^3/min. Steam at 1250 lbf/in.^{2}, 750 F exits the tube. Find the rate of heat transfer to the water.

Solution:

C.V. Steam generator. Steady state single inlet and exit flow.

Constant diameter tube: $A_i = A_e = \dfrac{\pi}{4}\left(\dfrac{1}{12}\right)^2 = 0.00545 \text{ ft}^2$

Table B.1.4 $\quad \dot{m} = \dot{V}_i/v_i = 5 \times 60/0.016058 = 18\ 682 \text{ lbm/h}$

$\mathbf{V}_i = \dot{V}_i/A_i = 5/(0.00545 \times 60) = 15.3 \text{ ft/s}$

Exit state properties from Table F.7.2 and inlet from F.7.3

$\mathbf{V}_e = \mathbf{V}_i \times v_e/v_i = 15.3 \text{ ft/s} \times (0.526/0.01605) = 501.4 \text{ ft/s}$

The energy equation Eq.4.12 is solved for the heat transfer as

$\dot{Q} = \dot{m}\left[(h_e - h_i) + \left(\mathbf{V}_e^2 - \mathbf{V}_i^2\right)/2\right]$

$= 18\ 682\left[1341.5 - 71.99 + \dfrac{501.4^2 - 15.3^2}{2 \times 32.174 \times 778}\right] = \mathbf{2.381 \times 10^7 \ Btu/h}$

Steam exit

gas out

Typically hot combustion gas in

liquid water in

4.184E

An oven has five radiant heaters; each one is rated at 15 Btu/s. It should heat some 4-lbm steel plates from 77 F to 1400 R. How many of these plates per minute can it heat?

C.V. Oven, steady state operation. A flow of plates in (represents an $\dot{m}$)

Energy Eq.: $0 = \dot{m}\,(h_i - h_e) + \dot{Q}$

$$\dot{Q} = \dot{m}\,(h_e - h_i) = \dot{m}\,C\,(T_e - T_i)$$

$$\dot{m} = \dot{Q} / C\,(T_e - T_i) = \frac{5 \times 15 \text{ Btu/s}}{0.11\,(1400 - 536.7) \text{ Btu/lbm}} = 0.79 \text{ lbm/s}$$

$$N = \dot{m}/m = (0.79 / 4)\ \ 1/s = 0.197 \text{ per s} = \textbf{11.85 per min}$$

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Front end of oven with steel rollers to bring the plates in,
a skirt hangs down to limit heat losses.

4.185E

An evaporator has R-410A at 0 F and quality 20% flowing in, with the exit flow being saturated vapor at 0 F. Knowing that there is no work, find the specific heat transfer.

C.V. Heater Steady state single inlet and exit flow.

Energy Eq.4.13: $0 = q + h_1 - h_2$

Table F.9.1: $h_1 = 13.52 + 0.2 \times 103.76 = 34.27$ Btu/lbm

$h_2 = 117.28$ Btu/lbm

$q = h_2 - h_1 = 117.28 - 34.27 =$ **83.01 Btu/lbm**

Evaporator

1

2

$\dot{Q}_{evap}$

P

63

1   2

v

T

0

1   2

v

4.186E

A flow of liquid glycerine flows around an engine, cooling it as it absorbs energy. The glycerine enters the engine at 140 F and receives 13 hp of heat transfer. What is the required mass flow rate if the glycerine should come out at a maximum 200 F?

Solution:

C.V. Liquid flow (glycerine is the coolant), steady flow. no work.

Energy Eq.: $\dot{m}h_i + \dot{Q} = \dot{m}h_e$

$$\dot{m} = \dot{Q}/(h_e - h_i) = \frac{\dot{Q}}{C_{gly}(T_e - T_i)}$$

From table F.3: $C_{gly} = 0.58$ Btu/lbm R

$$\dot{m} = \frac{13 \text{ hp} \times (2544.4/3600) \text{ btu/s-hp}}{0.58 \text{ btu/lbm-R }(200 - 140) \text{ R}} = \textbf{0.264 lbm/s}$$

Air intake filter   Fan Radiator

Shaft power

cb

Exhaust flow

Atm. air

Coolant flow

Borgnakke and Sonntag

4.187E

A small water pump is used in an irrigation system. The pump takes water in from a river at 50 F, 1 atm at a rate of 10 lbm/s. The exit line enters a pipe that goes up to an elevation 100 ft above the pump and river, where the water runs into an open channel. Assume the process is adiabatic and that the water stays at 50 F. Find the required pump work.

Solution:

C.V. pump + pipe. Steady state , 1 inlet, 1 exit flow. Assume same velocity in and out, no heat transfer.

Continuity Eq.: $\dot{m}_{in} = \dot{m}_{ex} = \dot{m}$

Energy Eq.4.12:

$$\dot{m}(h_{in} + (1/2)\mathbf{V}_{in}^2 + gz_{in}) =$$

$$\dot{m}(h_{ex} + (1/2)\mathbf{V}_{ex}^2 + gz_{ex}) + \dot{W}$$

States: $h_{in} = h_{ex}$ same (T, P)

$$\dot{W} = \dot{m}g(z_{in} - z_{ex}) = 10 \text{ lbm/s} \times \frac{32.174 \text{ ft/s}^2}{32.174 \text{ lbm ft/s}^2 \text{ /lbf}} \times (-100) \text{ ft}$$

$$= \textbf{-1000 lbf-ft/s} = \textbf{-1.285 Btu/s}$$

I.E.   1.285 Btu/s   required input

Multiple Flow Devices

4.188E

A steam turbine receives water at 2000 lbf/in.^{2}, 1200 F at a rate of 200 lbm/s as shown in Fig. P4.84. In the middle section 50 lbm/s is withdrawn at 300 lbf/in.^{2}, 650 F and the rest exits the turbine at 10 lbf/in.^{2}, 95% quality. Assuming no heat transfer and no changes in kinetic energy, find the total turbine work.

C.V. Turbine   Steady state, 1 inlet and 2 exit flows.

Continuity Eq.4.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$;   => $\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 150$ lbm/s

Energy Eq.4.10: $\dot{m}_1 h_1 = \dot{W}_T + \dot{m}_2 h_2 + \dot{m}_3 h_3$

Table F.7.2 $h_1 = 1598.6$ Btu/lbm,

$h_2 = 1341.6$ Btu/lbm

Table F.7.1 : $h_3 = h_f + x_3 h_{fg} = 161.2 + 0.95 \times 982.1$

$= 1094.2$ Btu/lbm

From the energy equation, Eq.4.10

$$\dot{W}_T = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = 200 \times 1598.6 - 50 \times 1341.6 - 150 \times 1094.2$$

$$= \mathbf{8.85 \times 10^4 \ Btu/s}$$

4.189E

A condenser, as the heat exchanger shown in Fig. P4.91, brings 1 lbm/s water flow at 1 lbf/in.^{2} from 500 F to saturated liquid at 1 lbf/in.^{2}. The cooling is done by lake water at 70 F that returns to the lake at 90 F. For an insulated condenser, find the flow rate of cooling water.

Solution:

C.V. Heat exchanger

Energy Eq.4.10: $\quad \dot{m}_{cool}h_{70} + \dot{m}_{H_2O}h_{500} = \dot{m}_{cool}h_{90} + \dot{m}_{H_2O}h_{f\,1}$

Table F.7.1: $h_{70} = 38.09$ Btu/lbm, $h_{90} = 58.07$ Btu/lbm, $h_{f,1} = 69.74$ Btu/lbm

Table F.7.2: $h_{500,1} = 1288.5$ btu/lbm

$$\dot{m}_{cool} = \dot{m}_{H_2O}\frac{h_{500} - h_{f,\,1}}{h_{90} - h_{70}} = 1 \text{ lbm/s} \times \frac{1288.5 - 69.74}{58.07 - 38.09} = \textbf{61 lbm/s}$$

4.190E

A heat exchanger is used to cool an air flow from 1400 to 680 R, both states at 150 lbf/in.^{2}. The coolant is a water flow at 60 F, 15 lbf/in.^{2} and it is shown in Fig. P4.95. If the water leaves as saturated vapor, find the ratio of the flow rates $\dot{m}_{H_2O}/\dot{m}_{air}$.

Solution:

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside.

Continuity Eqs.: Each line has a constant flow rate through it.

Energy Eq.4.10: $\dot{m}_{air}h_1 + \dot{m}_{H_2O}h_3 = \dot{m}_{air}h_2 + \dot{m}_{H_2O}h_4$

Process: Each line has a constant pressure.

Table F.5: $h_1 = 343.016$ Btu/lbm, $h_2 = 162.86$ Btu/lbm

Table F.7: $h_3 = 28.08$ Btu/lbm, $h_4 = 1150.9$ Btu/lbm (at 15 psia)

$$\dot{m}_{H_2O}/\dot{m}_{air} = \frac{h_1 - h_2}{h_4 - h_3} = \frac{343.016 - 162.86}{1150.9 - 28.08} = \mathbf{0.1604}$$

4.191E

A dual-fluid heat exchanger has 10 lbm/s water entering at 100 F, 20 psia and leaving at 50 F, 20 psia. The other fluid is glycol, entering at 10 F, 22 psia and leaving at 50 F, 22 psia. Find the required mass flow rate of glycol and the rate of internal heat transfer.

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for glycol and water each. The two flows exchange energy with no heat transfer to/from the outside.

Continuity Eqs.: Each line has a constant flow rate through it.

Energy Eq.4.10: $\dot{m}_{H_2O}\, h_1 + \dot{m}_{glycol}\, h_3 = \dot{m}_{H_2O}\, h_2 + \dot{m}_{glycol}\, h_4$

Process: Each line has a constant pressure.

Table F.7: $h_1 = 68.04$ Btu/lbm, $h_2 = 18.05$ Btu/lbm

Table F.3: $C_{P\, gly} = 0.58$ Btu/lbm-R so

$$h_4 - h_3 = C_{P\, gly}\,(T_4 - T_3) = 0.58\,[50 - 10] = 23.2 \text{ Btu/lbm}$$

$$\dot{m}_{glycol} = \dot{m}_{H_2O}\,\frac{h_1 - h_2}{h_4 - h_3} = 10 \text{ lbm/s}\,\frac{68.04 - 18.05}{23.2} = \mathbf{21.55 \text{ lbm/s}}$$

4.192E

An automotive radiator has glycerine at 200 F enter and return at 130 F as shown in Fig. P4.100. Air flows in at 68 F and leaves at 77 F. If the radiator should transfer 33 hp what is the mass flow rate of the glycerine and what is the volume flow rate of air in at 15 psia?

Solution:

If we take a control volume around the whole radiator then there is no external heat transfer - it is all between the glycerin and the air. So we take a control volume around each flow separately.

Heat transfer: $\dot{Q} = 33 \text{ hp} = 33 \times 2544.4 / 3600 = 23.324 \text{ Btu/s}$

Glycerine energy: $\dot{m}h_i + (-\dot{Q}) = \dot{m}h_e$

Table F.3: $\dot{m}_{gly} = \dfrac{-\dot{Q}}{h_e - h_i} = \dfrac{-\dot{Q}}{C_{gly}(T_e-T_i)} = \dfrac{-23.324}{0.58(130 - 200)} = \textbf{0.574 lbm/s}$

Air $\dot{m}h_i + \dot{Q} = \dot{m}h_e$

Table F.4: $\dot{m}_{air} = \dfrac{\dot{Q}}{h_e - h_i} = \dfrac{\dot{Q}}{C_{air}(T_e-T_i)} = \dfrac{23.324}{0.24(77 - 68)} = 8.835 \text{ lbm/s}$

$$\dot{V} = \dot{m}v_i \; ; \quad v_i = \frac{RT_i}{P_i} = \frac{53.34 \times 527.7}{15 \times 144} = 13.03 \text{ ft}^3/\text{lbm}$$

$$\dot{V}_{air} = \dot{m}v_i = 8.835 \times 13.03 = \textbf{115 ft}^3\textbf{/s}$$

Air intake filter

Shaft power

cb

200 F

Atm. air

Exhaust flow

Coolant flow 130 F

4.193E

Steam at 80 psia, 600 F is used to heat cold water at 60 F to 170 F for domestic hot water supply. How much steam per lbm liquid water is needed if the steam should not condense?

Solution:

C.V. Each line separately. No work but there is heat transfer out of the steam flow and into the liquid water flow.

Water line energy Eq.: $\dot{m}_{liq}h_i + \dot{Q} = \dot{m}_{liq}h_e \Rightarrow \dot{Q} = \dot{m}_{liq}(h_e - h_i)$

For the liquid water look in Table F.7.1

$$\Delta h_{liq} = h_e - h_i = 137.96 - 28.08 = 109.88 \text{ btu/lbm}$$
$$(\cong C_p \Delta T = 1.0 \,(170 - 60) = 110 \text{ btu/lbm})$$

Steam line energy has the same heat transfer but it goes out

Steam Energy Eq.: $\dot{m}_{steam}h_i = \dot{Q} + \dot{m}_{steam}h_e \Rightarrow \dot{Q} = \dot{m}_{steam}(h_i - h_e)$

For the steam look in Table F.7.2 at 80 psia

$$\Delta h_{steam} = h_i - h_e = 1330.66 - 1183.61 = 147.05 \text{ btu/lbm}$$

Now the heat transfer for the steam is substituted into the energy equation for the water to give

$$\dot{m}_{steam} / \dot{m}_{liq} = \Delta h_{liq} / \Delta h_{steam} = \frac{109.88}{147.05} = \mathbf{0.747}$$

Hot water out

Steam out

Steam in

cb

Cold water in

4.194E

A copper wire has been heat treated to 1800 R and is now pulled into a cooling chamber that has 3 lbm/s air coming in at 70 F; the air leaves the other end at 120 F. If the wire moves 0.5 lbm/s copper, how hot is the copper as it comes out?

Solution:

C.V. Total chamber, no external heat transfer

Energy eq.: $\dot{m}_{cu}\, h_{i\ cu} + \dot{m}_{air}\, h_{i\ air} = \dot{m}_{cu}\, h_{e\ cu} + \dot{m}_{air}\, h_{e\ air}$

$$\dot{m}_{cu}\,(h_e - h_i)_{cu} = \dot{m}_{air}(h_i - h_e)_{air}$$

$$\dot{m}_{cu}\, C_{cu}\,(T_e - T_i)_{cu} = \dot{m}_{air}\, C_{p\ air}(T_e - T_i)_{air}$$

Heat capacities from F.2 for copper and F.4 for air

$$(T_e - T_i)_{cu} = \frac{\dot{m}_{air} C_{p\ air}}{\dot{m}_{cu} C_{cu}}\,(T_e - T_i)_{air}$$

$$= \frac{3 \times 0.24}{0.5 \times 0.1}\,(70 - 120)\ R = -720\ R$$

$$T_e = T_i - 720 = 1800 - 720 = \mathbf{1080\ R}$$

Air

Cu

Air

4.195E

Two flows of air are both at 30 psia; one has 1 lbm/s at 720 R and the other has 2 lbm/s at 520 R. The two flows are mixed together in an insulated box to produce a single exit flow at 30 psia. Find the exit temperature.

Solution:

Cont.: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

$= (\dot{m}_1 + \dot{m}_2) h_3$

Mixing chamber

Solve for the exit enthalpy

$$h_3 = [\dot{m}_1 h_1 + \dot{m}_2 h_2] / (\dot{m}_1 + \dot{m}_2)$$

since the T's are modest use constant specific heats

$$T_3 = [\dot{m}_1 T_1 + \dot{m}_2 T_2] / (\dot{m}_1 + \dot{m}_2) = \frac{1}{3} \times 720 + \frac{2}{3} \times 520 = \textbf{586.7 R}$$

4.196E

A de-superheater has a flow of ammonia 3 lbm/s at 150 psia, 200 F which is mixed with another flow of ammonia at 80 F and quality 25% in an adiabatic mixing chamber. Find the flow rate of the second flow so the outgoing ammonia is saturated vapor at 150 psia.

C.V.   Desuperheater.

No external $\dot{Q}$ or $\dot{W}$

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$

State 1:   Table F.8.2 $h_1 = 707.96$ Btu/lbm
State 2:   Table F.8.1 $h_2 = 131.68 + 0.25 \times 497.94 = 256.17$ Btu/lbm
State 3:   Table F.8.2 $h_3 = 629.45$ Btu/lbm

$$\dot{m}_2 = \dot{m}_1 \times \frac{h_1 - h_3}{h_3 - h_2} = 3 \times \frac{707.96 - 629.45}{629.45 - 256.17} = \mathbf{0.631 \ lbm/s}$$

4.197E

An insulated mixing chamber receives 4 lbm/s R-134a at 150 lbf/in.^{2}, 220 F in a line with low velocity. Another line with R-134a as saturated liquid 130 F flows through a valve to the mixing chamber at 150 lbf/in.^{2} after the valve. The exit flow is saturated vapor at 150 lbf/in.^{2} flowing at 60 ft/s. Find the mass flow rate for the second line.

Solution:

C.V. Mixing chamber. Steady state, 2 inlets and 1 exit flow.

Insulated q = 0,   No shaft or boundary motion   w = 0.

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$;

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 (h_3 + \frac{1}{2} \mathbf{V}_3^2)$

$$\dot{m}_2 (h_2 - h_3 - \frac{1}{2} \mathbf{V}_3^2) = \dot{m}_1 (h_3 + \frac{1}{2} \mathbf{V}_3^2 - h_1)$$

State 1: Table F.10.1:   150 psia, 220 F, $h_1 = 209.63$ Btu/lbm
State 2: Table F.10.1:   x = 0, 130 F, $h_2 = 119.88$ Btu/lbm
State 3: Table F.10.2:   x = 1, 150 psia, $h_3 = 180.61$ Btu/lbm

$$\frac{1}{2} \mathbf{V}_3^2 = \frac{1}{2} \times 60^2 /(32.174 \times 778) = 0.072 \text{ Btu/lbm}$$

$$\dot{m}_2 = \dot{m}_1 (h_3 + \frac{1}{2} \mathbf{V}_3^2 - h_1)/ (h_2 - h_3 - \frac{1}{2} \mathbf{V}_3^2)$$

$$= 4 (180.61 + 0.072 - 209.63)/ (119.88 - 180.61 - 0.072) = \textbf{1.904 lbm/s}$$

Notice how kinetic energy was insignificant.

Multiple Devices, Cycle Processes

4.198E

The following data are for a simple steam power plant as shown in Fig. P4.118.

State	1	2	3	4	5	6	7
P psia	900	890	860	830	800	1.5	1.4
T F		115	350	920	900		110
h Btu/lbm	-	85.3	323	1468	1456	1029	78

State 6 has $x_6 = 0.92$, and velocity of 600 ft/s. The rate of steam flow is 200 000 lbm/h, with 400 hp input to the pump. Piping diameters are 8 in. from steam generator to the turbine and 3 in. from the condenser to the steam generator. Determine the power output of the turbine and the heat transfer rate in the condenser.

Turbine: $A_5 = \pi D_5^2/4 = 0.349 \text{ ft}^2, \quad v_5 = 0.964 \text{ ft}^3/\text{lbm}$

$$\mathbf{V}_5 = \dot{m}v_5/A_5 == \frac{200\ 000 \times 0.964}{3600 \times 0.349} = 153 \text{ ft/s}$$

$$w = (h_5 + 0.5\mathbf{V}_5^2) - (h_6 + 0.5\mathbf{V}_6^2) = 1456 - 1029 - \frac{600^2 - 153^2}{2 \times 25\ 037}$$

$$= 420.2 \text{ Btu/lbm}$$

Recall the conversion 1 Btu/lbm = 25 037 ft^2/s^2, 1 hp = 2544 Btu/h

$$\dot{W}_{TURB} = \frac{420.2 \times 200\ 000}{2544} = \textbf{33 000 hp}$$

Condenser: $A_7 = \pi D_7^2/4 = 0.0491 \text{ ft}^2, \quad v_7 = 0.01617 \text{ ft}^3/\text{lbm}$

$$\mathbf{V}_7 = \dot{m}v_7/A_7 = \frac{200000 \times 0.01617}{3600 \times 0.0491} = 18 \text{ ft/s}$$

$$q = 78.02 - 1028.7 + \frac{18^2 - 600^2}{2 \times 25\ 037} = -957.9 \text{ Btu/lbm}$$

$$\dot{Q}_{COND} = 200\ 000\ (-957.9) = \textbf{-1.916×10}^8 \textbf{ Btu/h}$$

4.199E

For the same steam power plant as shown in Fig. P4.118 and Problem 4.198E determine the rate of heat transfer in the economizer which is a low temperature heat exchanger and the steam generator. Determine also the flow rate of cooling water through the condenser, if the cooling water increases from 55 to 75 F in the condenser.

Condenser: $A_7 = \pi D_7^2/4 = 0.0491\ \text{ft}^2, \quad v_7 = 0.01617\ \text{ft}^3/\text{lbm}$

$$\mathbf{V}_7 = \dot{m}v_7/A_7 = \frac{200000 \times 0.01617}{3600 \times 0.0491} = 18\ \text{ft/s}$$

$$q = 78.02 - 1028.7 + \frac{18^2 - 600^2}{2 \times 25\ 037} = -957.9\ \text{Btu/lbm}$$

$$\dot{Q}_{COND} = 200\ 000\ (-957.9) = \mathbf{-1.916 \times 10^8\ Btu/h}$$

Economizer $\mathbf{V}_3 \approx \mathbf{V}_2$ since liquid v is constant: $v_3 \approx v_2$ and $A_3 = A_2$,

$$q = h_3 - h_2 = 323.0 - 85.3 = 237.7\ \text{Btu/lbm}$$

$$\dot{Q}_{ECON} = 200\ 000\ (237.7) = \mathbf{4.75 \times 10^7\ Btu/h}$$

Generator: $A_4 = \pi D_4^2/4 = 0.349\ \text{ft}^2, \quad v_4 = 0.9595\ \text{ft}^3/\text{lbm}$

$$\mathbf{V}_4 = \dot{m}v_4/A_4 = \frac{200\ 000 \times 0.9505}{3600 \times 0.349} = 151\ \text{ft/s}$$

$$A_3 = \pi D_3^2/4 = 0.349\ \text{ft}^2, \quad v_3 = 0.0491\ \text{ft}^3/\text{lbm}$$

$$\mathbf{V}_3 = \dot{m}v_3/A_3 = \frac{200\ 000 \times 0.0179}{3600 \times 0.0491} = 20\ \text{ft/s}\ ,$$

$$q = 1467.8 - 323.0 + \frac{151^2 - 20^2}{2 \times 25\ 037} = \mathbf{1145.2\ Btu/lbm}$$

$$\dot{Q}_{GEN} = 200\ 000 \times (1145.2) = \mathbf{2.291 \times 10^8\ Btu/h}$$

4.200E

A two-stage compressor takes nitrogen in at 80 F, 20 psia and compresses it to 80 psia, 800 R. Then it flows through an intercooler, where it cools to 580 R, and the second stage compresses it to 400 psia, 1000 R. Find the specific work in each of the two compressor stages and the specific heat transfer in the intercooler.

The setup is like this:

C.V.: Stage 1 nitrogen, Steady flow

Process: adiabatic: $q = 0$,   reversible: $s_{gen} = 0$

Energy Eq. 4.13: $-w_{C1} = h_2 - h_1$,

Assume constant $C_{P0} = 0.249$ from F.4

$$w_{C1} = h_1 - h_2 = C_{P0}(T_1 - T_2) = 0.249 \text{ Btu/lbmR } (540 - 800) \text{ K}$$

$$= \textbf{-64.74 Btu/lbm}$$

C.V. Intercooler, no work and no changes in kinetic or potential energy.

$$q_{23} = h_3 - h_2 = C_{P0}(T_3 - T_2) = 0.249 (800 - 580) = \textbf{-135.5 Btu/lbm}$$

C.V. Stage 2. Analysis the same as stage 1.

$$w_{C2} = h_3 - h_4 = C_{P0}(T_3 - T_4) = 0.249 (580 - 1000) = \textbf{-104.6 Btu/lbm}$$

4.201E

The intercooler in the previous problem uses cold liquid water to cool the nitrogen. The nitrogen flow is 0.1 lbm/s, and the liquid water inlet is 77 F and is set up to flow in the opposite direction from the nitrogen, so the water leaves at 105 F. Find the flow rate of the water.

Solution:

C.V. Heat exchanger, steady 2 flows in and two flows out.

Continuity eq.: $\dot{m}_1 = \dot{m}_2 = \dot{m}_{H2O}$; $\qquad \dot{m}_3 = \dot{m}_4 = \dot{m}_{N2}$

Energy eq.: $\qquad 0 = \dot{m}_{H2O}(h_3 - h_4) + \dot{m}_{N2}(h_2 - h_1)$

Due to the lower range of temperature we will use constant specific heats from F.4 and F.3 so the energy equation is approximated as

$$0 = \dot{m}_{H2O}C_{pH2O}(T_3 - T_4) + \dot{m}_{N2}C_p(T_2 - T_1)$$

Now solve for the water flow rate

$$\dot{m}_{H2O} = \dot{m}_{N2}\,C_{pN2}(T_2 - T_1)\,/\,[C_{pH2O}(T_4 - T_3)]$$

$$= 0.1 \text{ lbm/s} \times \frac{0.249 \text{ Btu/lbm-R} \times (800 - 580)\text{ R}}{1.0 \text{ Btu/lbm-R} \times (105 - 77)\text{ R}}$$

$$= \textbf{0.196 lbm/s}$$

4.202E

A R-410A heat pump cycle shown in Fig. P4.123 has a R-410A flow rate of 0.1 lbm/s with 4 Btu/s into the compressor. The following data are given

State	1	2	3	4	5	6
P, psia	410	405	400	62	60	58
T, F	220	200	110		10	14
h, Btu/lbm	154	150	56	-	120	122

Calculate the heat transfer from the compressor, the heat transfer from the R-410A in the condenser and the heat transfer to the R-410A in the evaporator.

Solution:

CV: Compressor

$$\dot{Q}_{COMP} = \dot{m}(h_1 - h_6) + \dot{W}_{COMP} = 0.1\,(154 - 122) - 4.0 = \textbf{-0.8 Btu/s}$$

CV: Condenser

$$\dot{Q}_{COND} = \dot{m}(h_3 - h_2) = 0.1\ \text{lbm/s}\,(56 - 150)\ \text{Btu/lbm} = \textbf{-9.4 Btu/s}$$

C.V. Valve:

$$h_4 = h_3 = 56\ \text{Btu/lbm}$$

CV: Evaporator

$$\dot{Q}_{EVAP} = \dot{m}\,(h_5 - h_4) = 0.1\ \text{lbm/s}\,(120 - 56)\ \text{Btu/lbm} = \textbf{6.4 Btu/s}$$

4.203E

A proposal is made to use a geothermal supply of hot water to operate a steam turbine, as shown in Fig. P4.125. The high pressure water at 200 lbf/in.^{2}, 350 F, is throttled into a flash evaporator chamber, which forms liquid and vapor at a lower pressure of 60 lbf/in.^{2}. The liquid is discarded while the saturated vapor feeds the turbine and exits at 1 lbf/in.^{2}, 90% quality. If the turbine should produce 1000 hp, find the required mass flow rate of hot geothermal water in pound-mass per hour.

Solution:

Separation of phases in flash-evaporator
constant h in the valve flow so

Table F.7.3: h_1 = 321.8 Btu/lbm

h_1 = 321.8 = 262.25 + x × 915.8

⇒   x = 0.06503 = $\dot{m}_2/\dot{m}_1$

Table F.7.2: h_2 = 1178.0 Btu/lbm;

Table F.7.1: h_3 = 69.74 + 0.9 × 1036 = 1002.1 Btu/lbm

$$\dot{W} = \dot{m}_2(h_2 - h_3) \quad => \quad \dot{m}_2 = \frac{1000 \times 2545}{1178.0 - 1002.1} = 14\ 472 \text{ lbm/h}$$

⇒ $\dot{m}_1$ = **222 539 lbm/h**

Notice conversion   1 hp = 2445 Btu/h from Table A.1

Transient Processes

4.204E

A 1 Gallon tank initially is empty and we want to have 0.03 lbm of R-410A in it. The R-410A comes from a line with saturated vapor at 20 F. To end up with the desired amount we cool the can while we fill it in a slow process keeping the can and content at 20 F. Find the final pressure to reach before closing the valve and the heat transfer.

Solution:

C.V. Tank:

Continuity Eq.4.20: $m_2 - 0 = m_i$

Energy Eq.4.21: $m_2 u_2 - 0 = m_i h_i + {}_1Q_2$

State 2: 20 F, $v_2 = V/m_2 = 1\ (231/12^3)/0.030 = 4.456\ \text{ft}^3/\text{lbm}$

From Table F.9.2 we locate the state between 15 and 20 psia.

$$P_2 = 15 + (20 - 15)\frac{4.456 - 4.6305}{3.4479 - 4.6305} = \textbf{15.74 psia}$$

$u_2 = 112.68\ \text{Btu/lbm},$

State i Table F.9.1: $h_i = 119.07\ \text{Btu/lbm}$

Now use the energy equation to solve for the heat transfer

$${}_1Q_2 = m_2 u_2 - m_i h_i = m_2(u_2 - h_i)$$
$$= 0.03 \times (112.68 - 119.07) = \textbf{-0.192 Btu}$$

4.205E

An initially empty cylinder is filled with air from 70 F, 15 psia until it is full. Assuming no heat transfer is the final temperature larger, equal to or smaller than 70 F? Does the final T depends on the size of the cylinder?

This is a transient problem with no heat transfer and no work. The balance equations for the tank as C.V. become

Continuity Eq.: $\qquad m_2 - 0 = m_i$

Energy Eq.: $\qquad m_2 u_2 - 0 = m_i h_i + Q - W = m_i h_i + 0 - 0$

Final state: $\qquad u_2 = h_i \ \& \qquad P_2 = P_i$

$T_2 > T_i$ and it does not depend on V

4.206E

A tank contains 10 ft^3 air at 15 psia, 540 R. A pipe flowing air at 150 psia, 540 R is connected to the tank and it is filled slowly to 150 psia. Find the heat transfer to reach a final temperature of 540 R.

C.V. The tank volume and the compressor.

This is a transient problem (filling of tank).

Continuity Eq.4.20: $m_2 - m_1 = m_{in}$

Energy Eq.4.21: $m_2u_2 - m_1u_1 = {}_1Q_2 - {}_1W_2 + m_{in}h_{in}$

Process: Constant volume ${}_1W_2 = 0$,

States: $u_1 = u_2 = u_{in} = u_{540}$; $h_{in} = u_{in} + RT_{in}$

$m_1 = P_1V_1/RT_1 = 15 \times 144 \times 10/(53.34 \times 540) = 0.75$ lbm

$m_2 = P_2V_2/RT_2 = 150 \times 144 \times 10/(53.34 \times 540) = 7.499$ lbm

Heat transfer from the energy equation

$_1Q_2 = m_2u_2 - m_1u_1 - m_{in}h_{in} = (m_1 + m_{in}) u_1 - m_1u_1 - m_{in}u_{in} - m_{in}RT_{in}$

$= m_1u_1 - m_1u_1 + m_{in}u_1 - m_{in}u_{in} - m_{in}RT_{in} = -m_{in}RT_{in}$

$= -(7.499 - 0.75)$ lbm $\times 53.34$ ft-lbf/lbm-R $\times 540$ R

$= \mathbf{-194\ 395\ ft\text{-}lbf = -250\ Btu}$

4.207E

A 1-ft^3 tank, shown in Fig. P4.136, that is initially evacuated is connected by a valve to an air supply line flowing air at 70 F, 120 lbf/in.^{2}. The valve is opened, and air flows into the tank until the pressure reaches 90 lbf/in.^{2}. Determine the final temperature and mass inside the tank, assuming the process is adiabatic. Develop an expression for the relation between the line temperature and the final temperature using constant specific heats.

Solution:

C.V. Tank:

Continuity Eq.4.20: $m_2 = m_i$

Energy Eq.4.21: $m_2 u_2 = m_i h_i$

Table F.5: $u_2 = h_i = 126.78$ Btu/lbm

$\Rightarrow T_2 =$ **740 R**

$$m_2 = \frac{P_2 V}{R T_2} = \frac{90 \times 144 \times 1}{53.34 \times 740} = \textbf{0.3283 lbm}$$

Assuming constant specific heat,

$$h_i = u_i + R T_i = u_2, \quad R T_i = u_2 - u_i = C_{Vo}(T_2 - T_i)$$

$$C_{Vo} T_2 = (C_{Vo} + R) T_i = C_{Po} T_i,$$

$$T_2 = \left(\frac{C_{Po}}{C_{vo}}\right) T_i = k T_i$$

For $T_i = 529.7$ R & constant C_{PO},

$$T_2 = 1.40 \times 529.7 = \textbf{741.6 R}$$

Borgnakke and Sonntag

4.208E

Helium in a steel tank is at 40 psia, 540 R with a volume of 4 ft^3. It is used to fill a balloon. When the tank pressure drops to 24 psia the flow of helium stops by itself. If all the helium still is at 540 R how big a balloon did I get? Assume the pressure in the balloon varies linearly with volume from 14.7 psia (V = 0) to the final 24 psia. How much heat transfer did take place?

Solution:

Take a C.V. of all the helium.

This is a control mass, the tank mass
       changes density and pressure.

Energy Eq.: $U_2 - U_1 = {_1}Q_2 - {_1}W_2$

Process Eq.: $P = 14.7 + CV$

State 1: P_1, T_1, V_1

State 2: $P_2, T_2, V_2 = ?$

Ideal gas:

$$P_2 V_2 = mRT_2 = mRT_1 = P_1V_1$$

$$V_2 = V_1(P_1/P_2) = 4 \times (40/24) = 6.6667 \text{ ft}^3$$

$$V_{bal} = V_2 - V_1 = 6.6667 - 4 = 2.6667 \text{ ft}^3$$

$${_1}W_2 = \int P \, dV = AREA = \tfrac{1}{2} (P_1 + P_2)(V_2 - V_1)$$

$$= \tfrac{1}{2}(40 + 24) \times 2.6667 \times 144 = 12\ 288 \text{ lbf-ft} = 15.791 \text{ Btu}$$

$$U_2 - U_1 = {_1}Q_2 - {_1}W_2 = m (u_2 - u_1) = mC_v (T_2 - T_1) = 0$$

so $\quad {_1}Q_2 = {_1}W_2 = $ **15.79 Btu**

Remark: The process is transient, but you only see the flow mass if you select the tank or the balloon as a control volume. That analysis leads to more terms that must be elliminated between the tank control volume and the balloon control volume.

4.209E

A 20-ft^3 tank contains ammonia at 20 lbf/in.^{2}, 80 F. The tank is attached to a line flowing ammonia at 180 lbf/in.^{2}, 140 F. The valve is opened, and mass flows in until the tank is half full of liquid, by volume at 80 F. Calculate the heat transferred from the tank during this process.

Solution:

C.V. Tank.   Transient process as flow comes in.

$$m_1 = V/v_1 = 20/16.765 = 1.193 \text{ lbm}$$
$$m_{f2} = V_{f2}/v_{f2} = 10/0.026677 = 374.855 \text{ lbm},$$
$$m_{g2} = V_{g2}/v_{g2} = 10/1.9531 = 5.120 \text{ lbm}$$
$$m_2 = m_{f2} + m_{g2} = 379.975 \text{ lbm} \quad => \quad x_2 = m_{g2}/m_2 = 0.013475$$
$$\text{Table F.8.1,} \quad u_2 = 130.9 + 0.013475 \times 443.4 = 136.9 \text{ Btu/lbm}$$
$$u_1 = 595.0 \text{ Btu/lbm}, \quad h_i = 667.0 \text{ Btu/lbm}$$

Continuity Eq,: $\quad m_i = m_2 - m_1 = 378.782 \text{ lbm}$,

From continuity equation

$$m_i = m_2 - m_1 = 379.975 - 1.193 = 378.782 \text{ lbm}$$

Energy eq.: $\quad Q_{CV} + m_i h_i = m_2 u_2 - m_1 u_1$

$$Q_{CV} = 379.975 \times 136.9 - 1.193 \times 595.0 - 378.782 \times 667.0$$
$$= \textbf{-201 339 Btu}$$

4.210E

A nitrogen line, 540 R, and 75 lbf/in.^{2}, is connected to a turbine that exhausts to a closed initially empty tank of 2000 ft^3, as shown in Fig. P4.137. The turbine operates to a tank pressure of 75 lbf/in.^{2}, at which point the temperature is 450 R. Assuming the entire process is adiabatic, determine the turbine work.

C.V. turbine & tank $\Rightarrow$ Transient problem

Conservation of mass: $m_i = m_2 = m$

Energy Eq.: $m_i h_i = m_2 u_2 + W_{CV}$; $W_{CV} = m(h_i - u_2)$

Inlet state: $P_i = 75$ lbf/in^2, $T_i = 540$ R

Final state 2: $P_2 = 75$ lbf/in^2, $T_2 = 450$ R

$$v_2 = RT_2/P_2 = 55.15 \times 450/(75 \times 144) = 2.298 \text{ ft}^3/\text{lbm}$$

$$m_2 = V/v_2 = 2000/2.298 = 870.32 \text{ lbm}$$

$$h_i - u_2 = u_i + RT_i - u_2 = RT_i + C_v (T_i - T_2)$$

$$= \frac{55.15}{778.17} 540 + 0.178 (540 - 450) = 38.27 + 16.02 = 54.29 \frac{\text{Btu}}{\text{lbm}}$$

$$W_{CV} = 870.32 \times 54.29 = \textbf{47 250 Btu}$$

4.211E

A mass-loaded piston/cylinder containing air is at 45 lbf/in.^{2}, 60 F with a volume of 9 ft^3, while at the stops $V = 36$ ft^3. An air line, 75 lbf/in.^{2}, 1100 R, is connected by a valve, as shown in Fig. P4.154. The valve is then opened until a final inside pressure of 60 lbf/in.^{2} is reached, at which point T = 630 R. Find the air mass that enters, the work, and heat transfer.

Solution:

C.V. Cylinder volume.

Continuity Eq.4.20: $m_2 - m_1 = m_{in}$

Energy Eq.4.21: $m_2 u_2 - m_1 u_1 = m_{in} h_{line} + {}_1 Q_2 - {}_1 W_2$

Process: P_1 is constant to stops, then constant V to state 2 at P_2

State 1: P_1, T_1 $m_1 = \dfrac{P_1 V}{RT_1} = \dfrac{45 \times 9 \times 144}{53.34 \times 519.7} = 2.104$ lbm

State 2:   Open to: $P_2 = 60$ lbf/in^2, $T_2 = 630$ R
Table F.5:

$h_i = 266.13$ btu/lbm

$u_1 = 88.68$ Btu/lbm

$u_2 = 107.62$ Btu/lbm

$_1 W_2 = \int P dV = P_1 (V_{stop} - V_1) = 45 \times (36 - 9) \dfrac{144}{778} = \mathbf{224.9\ Btu}$

$m_2 = P_2 V_2 / RT_2 = 60 \times 36 \times 144\ /(53.34 \times 630) = 9.256$ lbm

$m_i = m_2 - m_1 = 9.256 - 2.104 = 7.152$ lbm

$_1 Q_2 = m_2 u_2 - m_1 u_1 - m_i h_i + {}_1 W_2$

$= 9.256 \times 107.62 - 2.104 \times 88.68 - 7.152 \times 266.13 + 224.9$

$= \mathbf{-868.9\ Btu}$

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 5

8e

Updated June 2013

CONTENT CHAPTER 5

The clip-art used in the solution manual is from **Microsoft**.

In-Text Concept Questions

5.a

Electrical appliances (TV, stereo) use electric power as input. What happens to the power? Are those heat engines? What does the second law say about those devices?

Most electric appliances such as TV, VCR, stereo and clocks dissipate power in electrical circuits into internal energy (they get warm) some power goes into light and some power into mechanical energy. The light is absorbed by the room walls, furniture etc. and the mechanical energy is dissipated by friction so all the power eventually ends up as internal energy in the room mass of air and other substances.

These are not heat engines, just the opposite happens, namely electrical power is turned into internal energy and redistributed by heat transfer. These are irreversible processes.

5.b

Geothermal underground hot water or steam can be used to generate electric power. Does that violate the second law?

No.
Since the earth is not uniform we consider the hot water or steam supply as coming from one energy source (the high T) and we must reject heat to a low temperature reservoir as the ocean, a lake or the atmosphere which is another energy reservoir.

Iceland uses a significant amount of steam to heat buildings and to generate electricity. Microsoft clipart.

5.c

A windmill produces power on a shaft taking kinetic energy out of the wind. Is it a heat engine? Is it a perpetual machine? Explain.

Since the wind is generated by a complex system driven by solar heat input and radiation out to space it is a kind of heat engine.

Microsoft clipart.

Within our lifetime it looks like it is perpetual. However with a different time scale the climate will change, the sun will grow to engulf the earth as it burns out of fuel. There is a storage effect and a non-uniform distribution of states in the system that drives this.

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5.d

heat engines and heat pumps (refrigerators) are energy conversion devices altering amounts of energy transfer between Q and W. Which conversion direction (Q → W or W → Q) is limited and which is unlimited according to the second law.

The work output of a heat engine is limited (Q to W).
You can transform W to Q unlimited (a heat pump that does not work well or you may think about heat generated by friction).

5.e

Ice cubes in a glass of liquid water will eventually melt and all the water approach room temperature. Is this a reversible process? Why?

There is heat transfer from the warmer ambient to the water as long as there is a temperature difference. Eventually the temperatures approach each other and there is no more heat transfer. This is irreversible, as we cannot make ice-cubes out of the water unless we run a refrigerator and that requires a work from the surroundings, which does not leave the surroundings unchanged.

Microsoft clipart.

5.f

Does a process become more or less reversible with respect to heat transfer if it is

fast rather than slow? *Hint*: Recall from Chapter 3 that $\dot{Q} = CA \, \Delta T$.

If the higher heat transfer rate is caused by a larger ΔT then the process is more irreversible so as the process would be slower due to a lower ΔT then it approaches a reversible process. If the rate of heat transfer is altered due to the factor CA with the same ΔT then it is irreversible to the same degree.

5.g

If you generated hydrogen from, say, solar power, which of these would be more efficient: (1) transport it and then burn it in an engine or (2) convert the solar power to electricity and transport that? What else would you need to know in order to give a definite answer?

Case (1):

First there is a certain efficiency when converting solar power to hydrogen. Then the transport and packaging of hydrogen has some energy expenditures associated with it. The hydrogen could be compressed to a high pressure (typically 70 MPa) which is expensive in terms of work input and then stored in a tank. One alternative would be to cool it down to become a liquid to have a much smaller volume but the temperature at which this happens is very low so the cooling and continued cooling under transport requires a significant work input also. Certain materials like metal-hydrides, boron salt slurries and nano-carbon fibers allows storage of hydrogen at more modest pressures and are all currently under investigation as other alternative storage methods. After the hydrogen is transported to an engine then the engine efficiency determines the work output.

Case (2):

If the solar power is located where there is access to electrical transmission lines then it can be used in solar panels, solar heating of water or other substance to run a heat engine cycle like a power plant to generate electricity. All of these processes have a certain efficiency that must be evaluated to estimate the overall efficiency. To make new transmission lines is costly and has an impact on the environment that must be considered.

You also need to look at the time of day/year at which the power is required and when it is available. The end use also presents some limitations like if the power should be used for a car then the energy must be stored temporarily like in a battery.

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Concept Problems

5.1

Two heat engines operate between the same two energy reservoirs and both receives the same Q_H. One engine is reversible and the other is not. What can you say about the two Q_L's?

The reversible heat engine can produce more work (has a higher efficiency) than the irreversible heat engine and due to the energy conservation it then gives out a smaller Q_L compared to the irreversible heat engine.

$$W_{rev} = Q_H - Q_{L\ rev} \quad > \quad W_{irrev} = Q_H - Q_{L\ irrev}$$

$$\Rightarrow \quad Q_{L\ rev} < Q_{L\ irrev}$$

5.2

Compare two domestic heat pumps (A and B) running with the same work input. If A is better than B which one heats the house most?

The statement that A is better means it has a higher COP and since

$$Q_{H\,A} = COP_A\ W\quad >\quad Q_{H\,B} = COP_B\ W$$

it can thus provide more heat to the house. The higher heat comes from the higher Q_L it is able to draw in.

5.3

Suppose we forget the model for heat transfer as $\dot{Q} = CA\,\Delta T$, can we draw some information about direction of Q from the second law?

One of the classical statements of the second law is the Clausius statement saying that you cannot have heat transfer from a lower temperature domain to a higher temperature domain without work input.

The opposite, namely a transfer of heat from a high temperature domain towards a lower temperature domain can happen (which is a heat engine with zero efficiency). That is the only direction the heat transfer can have namely from the high T towards the low T environment.

5.4

A combination of two heat engines is shown in Fig. P5.4. Find the overall thermal efficiency as a function of the two individual efficiencies.

The overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \dot{Q}_M = \eta_2 (1 - \eta_1) \dot{Q}_H$$

so the final result is

$$\eta_{TH} = \eta_1 + \eta_2 (1 - \eta_1)$$

Figure P5.4
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5.5

Compare two heat engines receiving the same Q, one at 1200 K and the other at 1800 K; they both reject heat at 500 K. Which one is better?

The maximum efficiency for the engines are given by the Carnot heat engine efficiency as

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = 1 - \frac{T_L}{T_H}$$

Since they have the same low temperature the one with the highest T_H will have a higher efficiency and thus presumably better.

5.6

A car engine takes atmospheric air in at 20$^{\text{o}}$C, no fuel, and exhausts the air at –20$^{\text{o}}$C producing work in the process. What do the first and the second laws say about that?

Energy Eq.: $W = Q_H - Q_L$ = change in energy of air.   **OK**

2$^{\text{nd}}$ law: Exchange energy with only one reservoir. **NOT OK**.
                 This is a violation of the statement of Kelvin-Planck.

Remark: You cannot create and maintain your own energy reservoir.

5.7

A combination of two refrigerator cycles is shown in Fig. P5.7. Find the overall COP as a function of COP_1 and COP_2.

The overall COP becomes

$$COP = \beta = \frac{\dot{Q}_L}{\dot{W}_{tot}} = \frac{\dot{Q}_L}{\dot{W}_1} \frac{\dot{W}_1}{\dot{W}_{tot}} = COP_1 \frac{\dot{W}_1}{\dot{W}_{tot}} = COP_1 \frac{1}{1 + \dot{W}_2/\dot{W}_1}$$

where we used $\dot{W}_{tot} = \dot{W}_1 + \dot{W}_2$. Use definition of COP_2 and energy equation for

refrigerator 1 to eliminate $\dot{Q}_M$ as

$$COP_2 = \dot{Q}_M / \dot{W}_2 \quad \text{and} \quad \dot{Q}_M = \dot{W}_1 + \dot{Q}_L$$

so we have

$$\dot{W}_2 = \dot{Q}_M / COP_2 = (\dot{W}_1 + \dot{Q}_L) / COP_2$$

and then

$$\dot{W}_2 / \dot{W}_1 = (1 + \dot{Q}_L/\dot{W}_1) / COP_2 = (1 + COP_1) / COP_2$$

Finally substitute into the first equation and rearrange a little to get

$$COP = \beta = \frac{COP_1 \, COP_2}{COP_1 + COP_2 + 1}$$

Figure P5.7
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5.8

After you have returned from a car trip the car engine has cooled down and is thus back to the state in which it started. What happened to all the energy released in the burning of the gasoline? What happened to all the work the engine gave out?

Solution:

All the energy from the fuel generates heat and work out of the engine. The heat is directly dissipated in the atmosphere and the work is turned into kinetic energy and internal energy by all the frictional forces (wind resistance, rolling resistance, brake action). Eventually the kinetic energy is lost by braking the car so in the end all the energy is absorbed by the environment increasing its internal energy.

Microsoft clipart.

5.9

Does a reversible heat engine burning coal (which, in practice, cannot be done reversibly) have impacts on our world other than depletion of the coal reserve?

Solution:

When you burn coal you form carbon dioxide CO_2 which is a greenhouse gas. It absorbs energy over a wide spectrum of wavelengths and thus traps energy in the atmosphere that otherwise would go out into space.

Coal from various locations also has sulfur and other substances like heavy metals in it. The sulfur generates sulfuric acid (resulting in acid rain) in the atmosphere and can damage the forests.

Microsoft clipart.

Flue gas desulfurization (FGD) limestone wet scrubber

clean flue gas to stack

reheater

mist eliminator

mist-eliminator washwater

scrubbing slurry

bleed stream

scrubber

sludge removal system

sludge to disposal

clear liquid return

ground limestone slurry

flue gas containing sulfur dioxide

effluent hold tank

5.10

If the efficiency of a power plant goes up as the low temperature drops, why do power plants not just reject energy at say –40°C?

In order to reject heat the ambient must be at the low temperature. Only if we moved the plant to the North Pole would we see such a low T.

Remark: You cannot create and maintain your own energy reservoir.

Microsoft clipart.

5.11

If the efficiency of a power plant goes up as the low temperature drops why not let the heat rejection go to a refrigerator at, say, -10^oC instead of ambient 20^oC?

The refrigerator must pump the heat up to 20^oC to reject it to the ambient. The refrigerator must then have a work input that will exactly offset the increased work output of the power plant, if they are both ideal. As we can not build ideal devices the actual refrigerator will require more work than the power plant will produce extra.

5.12

A coal-fired power plant operates with a high T of 600°C whereas a jet engine has about 1400 K. Does that mean we should replace all power plants with jet engines?

The thermal efficiency is limited by the Carnot heat engine efficiency.

That is, the low temperature is also important. Here the power plant has a much lower T in the condenser than the jet engine has in the exhaust flow so the jet engine does not necessarily have a higher efficiency than the power plant.

Gas-turbines are used in power plants where they can cover peak power demands needed for shorter time periods and their high temperature exhaust can be used to boil additional water for the steam cycle.

$\dot{Q}_H$ from coal

$\dot{W}_{P, in}$

$\dot{W}_T$

$\dot{Q}_L$ to ambient

Microsoft clipart.

5.13

A heat transfer requires a temperature difference, see chapter 3, to push the $\dot{Q}$. What implications does that have for a real heat engine? A refrigerator?

This means that there are temperature differences between the source of energy and the working substance so T_H is smaller than the source temperature. This lowers the maximum possible efficiency. As heat is rejected the working

substance must have a higher temperature T_L than the ambient receiving the $\dot{Q}_L$, which lowers the efficiency further.

For a refrigerator the high temperature must be higher than the ambient to

which the $\dot{Q}_H$ is moved. Likewise the low temperature must be lower than the cold space temperature in order to have heat transfer from the cold space to the cycle substance. So the net effect is the cycle temperature difference is larger than the reservoir temperature difference and thus the COP is lower than that estimated from the cold space and ambient temperatures.

Both of these situations and statements are illustrated in Fig.5.27.

5.14

Hot combustion gases (air) at 1500 K are used as the heat source in a heat engine where the gas is cooled to 750 K and the ambient is at 300 K. This is not a constant T source. How does that affect the efficiency?

Solution:

If the efficiency is written as

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = 1 - \frac{T_L}{T_H}$$

then T_H is somewhere between 1500 K and 750 K and it is not a linear average.

After studying chapter 6 and 7 we can solve this problem and find the proper average high temperature based on properties at states 1 and 2.

Heat Engines and Refrigerators

Heat Engines and Refrigerators

5.15

A window mounted air conditioner removes 3.5 kJ from the inside of a home using 1.75 kJ work input. How much energy is released outside and what is its coefficient of performance?

C.V. A/C unit. The energy Q_H goes into the outside air.

Energy Eq.: $\qquad Q_H = W + Q_L = 1.75 + 3.5 = \textbf{5.25 kJ}$

COP: $\qquad \beta = \dfrac{Q_L}{W} = 3.5 / 1.75 = \textbf{2}$

T_{amb}

Q_H

REF

$W = 1.75$ kJ

$Q_L = 3.5$ kJ

T_L

5.16

A lawnmower tractor engine produces 18 hp using 40 kW of heat transfer from burning fuel. Find the thermal efficiency and the rate of heat transfer rejected to the ambient?

Conversion Table A.1: $18 \text{ hp} = 18 \times 0.7457 \text{ kW} = 13.423 \text{ kW}$

Efficiency: $\eta_{TH} = \dot{W}_{out}/\dot{Q}_H = \dfrac{13.423}{40} = \mathbf{0.33}$

Energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 40 - 13.423 = \mathbf{26.6 \text{ kW}}$

$\dot{Q}_H$ $\dot{Q}_L$

$\dot{W}_{out}$

Microsoft clipart.

5.17

Calculate the thermal efficiency of the steam power plant cycle described in Example 4.9.

Solution:

From solution to Example 4.9,

$$w_{net} = w_t + w_p = 640.7 - 4$$

$$= 636.7 \text{ kJ/kg}$$

$$q_H = q_b = 2831 \text{ kJ/kg}$$

$$\eta_{TH} = w_{net}/q_H = \frac{636.7}{2831} = \textbf{0.225}$$

Notice we cannot write $w_{net} = q_H - q_L$ as there is an extra heat transfer $_1\dot{Q}_2$ as a loss in the line. This needs to be accounted for in the overall energy equation.

5.18

Assume we have a refrigerator operating at steady state using 500 W of electric power with a COP of 2.5. What is the net effect on the kitchen air?

Take a C.V. around the whole kitchen. The only energy term that crosses the control surface is the work input $\dot{W}$ apart from energy exchanged with the kitchen surroundings. That is the kitchen is being heated with a rate of $\dot{W}$.

Remark: The two heat transfer rates are both internal to the kitchen. $\dot{Q}_H$ goes into the kitchen air and $\dot{Q}_L$ actually leaks from the kitchen into the refrigerated space, which is the reason we need to drive it out again.

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5.19

A room is heated with a 1500 W electric heater. How much power can be saved if a heat pump with a COP of 2.5 is used instead?

Assume the heat pump has to deliver 1500 W as the $\dot{Q}_H$.

Heat pump: $\beta' = \dot{Q}_H/\dot{W}_{IN}$

$$\dot{W}_{IN} = \dot{Q}_H/\beta' = \frac{1500}{2.5} = 600 \text{ W}$$

So the heat pump requires an input of 600 W thus saving the difference

$$\dot{W}_{saved} = 1500 \text{ W} - 600 \text{ W} = \textbf{900 W}$$

Room

$\dot{Q}_H$

HP $\dot{W}_{in}$

cb

$\dot{Q}_L$

T_L

Microsoft clipart.

5.20

Calculate the coefficient of performance of the R-134a refrigerator given in Example 4.8.

Solution:

From the definition

$$\beta = Q_L/W_{IN} = \frac{14.54}{5} = 2.91$$

Notice we cannot write $W_{IN} = Q_H - Q_L$ as there is a small Q in the compressor. This needs to be accounted for in the overall energy equation.

5.21

Calculate the thermal efficiency of the steam power plant cycle described in Problem 4.118.

From solution to Problem 4.118,

Turbine $\quad A_5 = (\pi/4)(0.2)^2 = 0.03142 \ m^2$

$$\mathbf{V}_5 = \dot{m}v_5/A_5 = 25 \ kg/s \times 0.06163 \ m^3/kg \ / \ 0.03142 \ m^2 = 49 \ m/s$$

$$h_6 = 191.83 + 0.92 \times 2392.8 = 2393.2 \ kJ/kg$$

$$w_T = 3404 - 2393.2 - (200^2 - 49^2)/(2 \times 1000) = 992 \ kJ/kg$$

$$\dot{W}_T = \dot{m}w_T = 25 \times 992 = 24 \ 800 \ kW$$

$$\dot{W}_{NET} = 24800 - 300 = 24 \ 500 \ kW$$

From the solution to Problem 4.120

Economizer $\quad A_7 = \pi D_7^2/4 = 0.004 \ 418 \ m^2, \quad v_7 = 0.001 \ 008 \ m^3/kg$

$$\mathbf{V}_2 = \mathbf{V}_7 = \dot{m}v/A_7 = 25 \times 0.001008 \ / \ 0.004418 = 5.7 \ m/s,$$

$$\mathbf{V}_3 = (v_3/v_2)\mathbf{V}_2 = (0.001 \ 118 \ / \ 0.001 \ 008) \ 5.7 = 6.3 \ m/s \approx \mathbf{V}_2$$

so kinetic energy change is unimportant

$$q_{ECON} = h_3 - h_2 = 744 - 194 = 550.0 \ kJ/kg$$

$$\dot{Q}_{ECON} = \dot{m}q_{ECON} = 25 \ (550.0) = 13 \ 750 \ kW$$

Generator $\quad A_4 = \pi D_4^2/4 = 0.031 \ 42 \ m^2, \quad v_4 = 0.060 \ 23 \ m^3/kg$

$$\mathbf{V}_4 = \dot{m}v_4/A_4 = 25 \times 0.060 \ 23/0.031 \ 42 = 47.9 \ m/s$$

$$q_{GEN} = 3426 - 744 + (47.9^2 - 6.3^2)/(2 \times 1000) = 2683 \ kJ/kg$$

$$\dot{Q}_{GEN} = 25 \times (2683) = 67 \ 075 \ kW$$

The total added heat transfer is

$$\dot{Q}_H = 13 \ 758 + 67 \ 075 = 80 \ 833 \ kW$$

$$\Rightarrow \eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = \frac{24500}{80833} = \mathbf{0.303}$$

5.22

A large coal fired power plant has an efficiency of 45% and produces net 1,500 MW of electricity. Coal releases 25 000 kJ/kg as it burns so how much coal is used per hour?

From the definition of the thermal efficiency and the energy release by the combustion called heating value HV we get

$$\dot{W} = \eta \, \dot{Q}_H = \eta \cdot \dot{m} \cdot HV$$

then

$$\dot{m} = \frac{\dot{W}}{\eta \times HV} = \frac{1500 \text{ MW}}{0.45 \times 25000 \text{ kJ/kg}} = \frac{1500 \times 1000 \text{ kJ/s}}{0.45 \times 25000 \text{ kJ/kg}}$$

$$= 133.33 \text{ kg/s} = \textbf{480 000 kg/h}$$

5.23

A window air-conditioner discards 1.7 kW to the ambient with a power input of 500 W. Find the rate of cooling and the coefficient of performance.

Solution:

In this case $\dot{Q}_H$ = 1.7 kW goes to the ambient so

Energy Eq. : $\dot{Q}_L = \dot{Q}_H - \dot{W}$ = 1.7 – 0.5 = **1.2 kW**

$$\beta_{REFRIG} = \frac{\dot{Q}_L}{\dot{W}} = \frac{1.2}{0.5} = \textbf{2.4}$$

T_{amb}

$\dot{Q}_H$ = 1.7 kW

REF

$\dot{W}$ = 0.5 kW

$\dot{Q}_L$

T_L

Microsoft clipart.

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5.24

An industrial machine is being cooled by 0.4 kg/s water at 15$^\text{o}$C that is chilled from 35$^\text{o}$C by a refrigeration unit with a COP of 3. Find the rate of cooling required and the power input to the unit.

Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) = \dot{m}\, C_P\, (T_1 - T_2)$$

$$= 0.4 \text{ kg/s} \times 4.18 \text{ kJ/kg-K} \times (35 - 15) \text{ K}$$

$$= 33.44 \text{ kW}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow$$

$$\dot{W} = \dot{Q}_L / \beta = 33.44 / 3 = \textbf{11.15 kW}$$

Comment: An outside cooling tower is often used for this, see Chapter 11.

5.25

Calculate the COP of the R-410A heat pump cycle described in Problem 4.123.

The cycle is given by the following states:

Where

$$\dot{Q}_H = \dot{m}\,(h_2 - h_3) = 0.05 \text{ kg/s}\,(367 - 134)\text{ kJ/k} = 11.65 \text{ kW}$$

The COP is

$$\beta' = \text{COP} = \dot{Q}_H / \dot{W}_{IN} = \frac{11.65 \text{ kW}}{5 \text{ kW}} = \mathbf{2.33}$$

5.26

A window air-conditioner unit is placed on a laboratory bench and tested in cooling mode using 750 W of electric power with a COP of 1.75. What is the cooling power capacity and what is the net effect on the laboratory?

Definition of COP: $\beta = \dot{Q}_L / \dot{W}$

Cooling capacity: $\dot{Q}_L = \beta\,\dot{W} = 1.75 \times 750 = \mathbf{1313\ W}$

For steady state operation the $\dot{Q}_L$ comes from the laboratory and $\dot{Q}_H$ goes to the laboratory giving a net to the lab of $\dot{W} = \dot{Q}_H - \dot{Q}_L = 750$ W, that is heating it.

5.27

A farmer runs a heat pump with a 2 kW motor. It should keep a chicken hatchery at 30°C, which loses energy at a rate of 10 kW to the colder ambient T_{amb}. What is the minimum coefficient of performance that will be acceptable for the heat pump?

Solution:

Power input: $\dot{W} = 2$ kW

Energy Eq. for hatchery: $\dot{Q}_H = \dot{Q}_{Loss} = 10$ kW

Definition of COP: $\beta = COP = \dfrac{\dot{Q}_H}{\dot{W}} = \dfrac{10}{2} = \mathbf{5}$

5.28

A sports car engine delivers 100 hp to the driveshaft with a thermal efficiency of 25%. The fuel has a heating value of 40 000 kJ/kg. Find the rate of fuel consumption and the combined power rejected through the radiator and exhaust.

Solution:

Heating value (HV): $\quad \dot{Q}_H = \dot{m} \cdot HV$

From the definition of the thermal efficiency

$$\dot{W} = \eta \, \dot{Q}_H = \eta \cdot \dot{m} \cdot HV$$

$$\dot{m} = \frac{\dot{W}}{\eta \cdot HV} = \frac{100 \times 0.7355}{0.25 \times 40\ 000} = 0.00736 \text{ kg/s} = \textbf{7.36 g/s}$$

Conversion of power from hp to kW in Table A.1.

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = (\dot{W}/\eta - \dot{W}) = (\frac{1}{\eta} - 1)\dot{W}$$

$$= (\frac{1}{0.25} - 1)\ 100 \text{ hp} \times 0.7355 \text{ kW/hp} = \textbf{221 kW}$$

Air intake filter Fuel line
Fan
Shaft power
Exhaust flow

5.29

R-410A enters the evaporator (the cold heat exchanger) in an A/C unit at -20°C, x = 28% and leaves at -20°C, x = 1. The COP of the refrigerator is 1.5 and the mass flow rate is 0.003 kg/s. Find the net work input to the cycle.

Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_2 - h_1) = \dot{m}[h_g - (h_f + x_1 \, h_{fg})]$$

$$= \dot{m}[h_{fg} - x_1 \, h_{fg}] = \dot{m} \, (1 - x_1)h_{fg}$$

$$= 0.003 \text{ kg/s} \times 0.72 \times 243.65 \text{ kJ/kg} = 0.5263 \text{ kW}$$

$$\beta = \text{COP} = \dot{Q}_L \, / \, \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L \, / \, \beta = 0.5263 \, / \, 1.5 = \textbf{0.35 kW}$$

5.30

In a Rankine cycle steam power plant 0.9 MW is taken out in the condenser, 0.63 MW is taken out in the turbine, and the pump work is 0.03 MW. Find the plant thermal efficiency. If everything could be reversed find the COP as a refrigerator.

Solution:

CV. Total plant:

Energy Eq.:

$$\dot{Q}_H + \dot{W}_{P,in} = \dot{W}_T + \dot{Q}_L$$

$$\dot{Q}_H = \dot{W}_T + \dot{Q}_L - \dot{W}_{P,in}$$

$$= 0.63 + 0.9 - 0.03 = 1.5 \text{ MW}$$

$$\eta_{TH} = \frac{\dot{W}_T - \dot{W}_{P,in}}{\dot{Q}_H} = \frac{0.63 - 0.03}{1.5} = \mathbf{0.40}$$

$$\beta = \frac{\dot{Q}_L}{\dot{W}_T - \dot{W}_{P,in}} = \frac{0.9}{0.63 - 0.03} = \mathbf{1.5}$$

5.31

An experimental power plant generates 130 MW of electrical power. It uses a supply of 1200 MW from a geothermal source and rejects energy to the atmosphere. Find the power to the air and how much air should be flowed to the cooling tower (kg/s) if its temperature cannot be increased more than 12°C.

Solution:

C.V. Total power plant.

Energy equation gives the amount of heat rejection to the atmosphere as

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 1200 - 130 = \textbf{1070 MW}$$

The energy equation for the air flow that absorbs the energy is

$$\dot{Q}_L = \dot{m}_{air}\,\Delta h = \dot{m}_{air}\,C_p\,\Delta T$$

$$\dot{m}_{air} = \frac{\dot{Q}_L}{C_p \Delta T} = \frac{1070 \times 1000}{1.004 \times 12}\,\frac{MW}{(kJ/kg\text{-}K) \times K} = \textbf{88 811 kg/s}$$

This is too large to make, so some cooling by liquid water or evaporative cooling should be used, see chapter 11.

Microsoft clipart.

5.32

A water cooler for drinking water should cool 25 L/h water from 18$^{\text{o}}$C to 10$^{\text{o}}$C while the water reservoir also gains 60 W from heat transfer. Assume a small refrigeration unit with a COP of 2.5 does the cooling. Find the total rate of cooling required and the power input to the unit.

The mass flow rate is

$$\dot{m} = \rho \dot{V} = \frac{25 \times 10^{-3}}{0.001002} \frac{1}{3600} \text{ kg/s} = 6.93 \text{ g/s}$$

Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) + \dot{Q}_{H\,TR}$$

$$= \dot{m} \, C_P \, (T_1 - T_2) + \dot{Q}_{H\,TR}$$

$$= 6.93 \times 10^{-3} \text{ kg/s} \times 4.18 \text{ kJ/kg-K} \times (18 - 10) \text{ K} + 60 \text{ W}$$

$$= 291.8 \text{ W}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 291.8 / 2.5 = \textbf{116.7 W}$$

Comment: The unit does not operate continuously so the instantaneous power is higher during the periods it does operate.

5.33

A large stationary diesel engine produces 5 MW with a thermal efficiency of 40%. The exhaust gas, which we assume is air, flows out at 800 K and the intake air is 290 K. How large a mass flow rate is that, assuming this is the only way we reject heat? Can the exhaust flow energy be used?

Heat engine: $\dot{Q}_H = \dot{W}_{out}/\eta_{TH} = \dfrac{5}{0.4} = 12.5$ MW

Energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 12.5 - 5 = 7.5$ MW

Exhaust flow: $\dot{Q}_L = \dot{m}_{air}(h_{800} - h_{290})$

$$\dot{m}_{air} = \frac{\dot{Q}_L}{h_{800} - h_{290}} = \frac{7.5 \times 1000}{822.2 - 290.43} \frac{\text{kW}}{\text{kJ/kg}} = \textbf{14.1 kg/s}$$

The flow of hot gases can be used to heat a building or it can be used to heat water in a steam power plant since that operates at lower temperatures.

5.34

For each of the cases below determine if the heat engine satisfies the first law (energy equation) and if it violates the second law.

    a. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 4$ kW, $\dot{W} = 2$ kW

    b. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

    c. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 2$ kW, $\dot{W} = 5$ kW

    d. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW

Solution:

	1st. law	2nd law
a	Yes	Yes (possible)
b	Yes	No, impossible Kelvin - Planck
c	No	Yes, but energy not conserved
d	Yes	Yes (Irreversible $\dot{Q}$ over ΔT)

5.35

For each of the cases in problem 5.34 determine if a heat pump satisfies the first law (energy equation) and if it violates the second law.

    a. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 4$ kW, $\dot{W} = 2$ kW

    b. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

    c. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 2$ kW, $\dot{W} = 5$ kW

    d. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW

Solution:

	1st. law	2nd law
a	Satisfied	Does not violate
b	Satisfied	Does not violate
c	Violated	Does not violate, but 1st law
d	Satisfied	Does violate, Clausius

5.36

Calculate the amount of work input a refrigerator needs to make ice cubes out of a tray of 0.25 kg liquid water at 10^oC. Assume the refrigerator has $\beta = 3.5$ and a motor-compressor of 750 W. How much time does it take if this is the only cooling load?

C.V. Water in tray. We neglect tray mass.

Energy Eq.: $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : $\quad P = \text{constant} = P_o$

$\qquad {}_1W_2 = \int P\, dV = P_o m(v_2 - v_1)$

$\qquad {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$

Tbl. B.1.1 : $h_1 = 41.99$ kJ/kg, Tbl. B.1.5 : $h_2 = -333.6$ kJ/kg

$\qquad {}_1Q_2 = 0.25(-333.4 - 41.99) = -93.848$ kJ

Consider now refrigerator

$\qquad \beta = Q_L/W$

$\qquad W = Q_L/\beta = - {}_1Q_2/\beta = 93.848/3.5 = \mathbf{26.81\ kJ}$

For the motor to transfer that amount of energy the time is found as

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t$$

$$\Delta t = W/\dot{W} = (26.81 \times 1000)\text{J}\,/750\ \text{W} = \mathbf{35.75\ s}$$

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Comment: We neglected a baseload of the refrigerator so not all the 750 W are available to make ice, also our coefficient of performance is very optimistic and finally the heat transfer is a transient process. All this means that it will take much more time to make ice-cubes.

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Second Law and Processes

5.37

Prove that a cyclic device that violates the Kelvin–Planck statement of the second law also violates the Clausius statement of the second law.

Solution: Proof very similar to the proof in section 5.2.

H.E. violating Kelvin receives Q_H from T_H and produces net $W = Q_H$.

This W input to H.P. receiving Q_L from T_L. H.P. discharges $Q_H + Q_L$ to T_H.

Net Q to T_H is : $-Q_H + Q_H + Q_L = Q_L$.

H.E. + H.P. together transfers Q_L from T_L to T_H with no W thus violates Clausius.

5.38

Discuss the factors that would make the power plant cycle described in Problem 4.118 an irreversible cycle.

Solution:

General discussion, but here are a few of the most significant factors.

1. Combustion process that generates the hot source of energy.

2. Heat transfer over finite temperature difference in boiler.

3. Flow resistance and friction in turbine results in less work out.

4. Flow friction and heat loss to/from ambient in all pipes.

5. Heat transfer over finite temperature difference in condenser.

5.39

Discuss the factors that would make the heat pump described in Problem 4.123 an irreversible cycle.

Solution:

General discussion but here are a few of the most significant factors.

1. Unwanted heat transfer in the compressor.

2. Pressure loss (back flow leak) in compressor

3. Heat transfer and pressure drop in line 1 => 2.

4. Pressure drop in all lines.

5. Throttle process 3 => 4.

5.40

Assume a cyclic machine that exchanges 6 kW with a 250°C reservoir and has

    a. $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

    b. $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW

and $\dot{Q}_L$ is exchanged with a 30°C ambient. What can you say about the processes in the two cases a and b if the machine is a heat engine? Repeat the question for the case of a heat pump.

Solution:

    Heat engine

        a.   Since $\dot{Q}_L = 0$ impossible Kelvin – Planck

        b.   Possible, irreversible, $\eta_{eng} = 0$

    Heat pump

        a.   Possible, irreversible   (like an electric heater)

        b.   Impossible, $\beta \rightarrow \infty$, Clausius

5.41

Consider a heat engine and heat pump connected as shown in figure P5.41. Assume $T_{H1} = T_{H2} > T_{amb}$ and determine for each of the three cases if the setup satisfy the first law and/or violates the 2nd law.

	$\dot{Q}_{H1}$	$\dot{Q}_{L1}$	$\dot{W}_1$	$\dot{Q}_{H2}$	$\dot{Q}_{L2}$	$\dot{W}_2$
a	6	4	2	3	2	1
b	6	4	2	5	4	1
c	3	2	1	4	3	1

Solution:

	1st. law	2nd law
a	Yes	Yes (possible)
b	Yes	No, combine Kelvin - Planck
c	Yes	No, combination clausius

5.42

Consider the four cases of a heat engine in problem 5.34 and determine if any of those are perpetual machines of the first or second kind.

a. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 4$ kW, $\dot{W} = 2$ kW

b. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

c. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 2$ kW, $\dot{W} = 5$ kW

d. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW

Solution:

	1st. law	2nd law
a	Yes	Yes (possible)
b	Yes	No, impossible Kelvin - Planck
		Perpetual machine second kind
		It violates the 2nd law converts all $\dot{Q}$ to $\dot{W}$
c	No	Yes, but energy not conserved
		Perpetual machine first kind
		It generates energy inside
d	Yes	Yes (Irreversible $\dot{Q}$ over ΔT)

5.43

The simple refrigeration cycle is shown in Problem 5.23 and in Fig. 5.6. Mention a few of the processes that are expected to be irreversible.

The throttling process is highly irreversible.

Both of the heat transfer processes are externally irreversible, large ΔT between the working substance and the source or sink energy reservoir.

Both of the heat transfer processes are also internally irreversible, smaller ΔT in the substance so there is not a single uniform T across the flow cross sectional area. This is necessary to redistribute the energy uniformly in the working substance.

The compressor has friction and flow losses so not all of the shaft work goes into raising the substance pressure. Such an effect is described by a device efficiency in chapter 7.

Carnot Cycles and Absolute Temperature

5.44

Calculate the thermal efficiency of a Carnot cycle heat engine operating between reservoirs at 300°C and 45°C. Compare the result to that of Example 4.7.

Solution:

$$\eta_{TH} = W_{net} / Q_H = 1 - \frac{T_L}{T_H} = 1 - \frac{45 + 273}{300 + 273} = \mathbf{0.445} \ \ (\text{Carnot})$$

$$\eta_{EX\ 4.7} = w_{net} / q_H \ = \frac{640.8 - 4}{2831.1} = 0.225$$

(efficiency about ½ of the Carnot)

5.45

An ideal (Carnot) heat engine has an efficiency of 40%. If the high temperature is raised 15% what is the new efficiency keeping the same low temperature?

Solution:

$$\eta_{TH} = W_{net} / Q_H = 1 - \frac{T_L}{T_H} = 0.4 \qquad \Rightarrow \qquad \frac{T_L}{T_H} = 0.6$$

so if T_H is raised 15% the new ratio becomes

$$\frac{T_L}{T_{H\,new}} = 0.6 / 1.15 = 0.5217 \quad \Rightarrow \qquad \eta_{TH\,new} = 1 - 0.5217 = \textbf{0.478}$$

5.46

At a few places where the air is very cold in the winter, like –30°C it is possible to find a temperature of 13°C down below ground. What efficiency will a heat engine have operating between these two thermal reservoirs?

Solution:

$$\eta_{TH} = 1 - \frac{T_L}{T_H}$$

The ground becomes the hot source and the atmosphere becomes the cold side of the heat engine

$$\eta_{TH} = 1 - \frac{273 - 30}{273 + 13} = 1 - \frac{243}{286} = \mathbf{0.15}$$

This is low because of the modest temperature difference.

Microsoft clipart.

5.47

Consider the combination of a heat engine and a heat pump, as in Problem 5.41, with a low temperature of 400 K. What should the high temperature be so that the heat engine is reversible? For that temperature what is the COP for a reversible heat pump?

For all three cases of the heat engine the ratio between the heat transfers and the work term is the same as:

$$\dot{Q}_H : \dot{Q}_L : \dot{W} \quad = \quad 6{:}4{:}2 \ = \ 3{:}2{:}1$$

For a reversible heat engine we must have the heat transfer ratio equal to the temperature ratio so

$$\frac{\dot{Q}_H}{\dot{Q}_L} = \frac{T_H}{T_L} = \frac{3}{2} = \frac{T_H}{400 \text{ K}} \quad \Rightarrow \quad T_H = (3/2)\ 400 \text{ K} = \textbf{600 K}$$

The COP is

$$\text{COP}_{HP} = \dot{Q}_H / \dot{W} = \frac{3}{1} = \textbf{3} \quad (= \frac{T_H}{T_H - T_L} = \frac{600}{600 - 400})$$

5.48

Find the power output and the low T heat rejection rate for a Carnot cycle heat engine that receives 6 kW at 250°C and rejects heat at 30°C as in Problem 5.40.

Solution:

From the definition of the absolute temperature Eq. 5.4

$$\eta_{carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{303}{523} = 0.42$$

Definition of the heat engine efficiency gives the work as

$$\dot{W} = \eta \, \dot{Q}_H = 0.42 \times 6 = \textbf{2.52 kW}$$

Apply the energy equation

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 6 - 2.52 = \textbf{3.48 kW}$$

5.49

A large heat pump should upgrade 4 MW of heat at 65°C to be delivered as heat at 145°C. What is the minimum amount of work (power) input that will drive this?

For the minimum work we assume a Carnot heat pump and $\dot{Q}_L$ = 4 MW.

$$\beta_{HP} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{T_H}{T_H - T_L} = \frac{273.15 + 145}{145 - 65} = 5.227$$

$$\beta_{REF} = \beta_{HP} - 1 = \frac{\dot{Q}_L}{\dot{W}_{in}} = 4.227$$

Now we can solve for the work

$$\dot{W}_{in} = \dot{Q}_L / \beta_{REF} = 4/4.227 = \textbf{0.946 MW}$$

This is a domestic or small office building size A/C unit, much smaller than the 4 MW in this problem.

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5.50

A temperature of about 0.01 K can be achieved by magnetic cooling. In this process a strong magnetic field is imposed on a paramagnetic salt, maintained at 1 K by transfer of energy to liquid helium boiling at low pressure. The salt is then thermally isolated from the helium, the magnetic field is removed, and the salt temperature drops. Assume that 1 mJ is removed at an average temperature of 0.1 K to the helium by a Carnot-cycle heat pump. Find the work input to the heat pump and the coefficient of performance with an ambient at 300 K.

Solution:

$$\beta = \frac{Q_L}{W_{in}} = \frac{T_L}{T_H - T_L} = \frac{0.1}{299.9} = \mathbf{0.00033}$$

$$W_{in} = \frac{Q_L}{\beta} = \frac{1 \times 10^{-3}}{0.00033} = \mathbf{3\ J}$$

Remark: This is an extremely large temperature difference for a heat pump. A real one is built as a refrigerator within a refrigerator etc.

5.51

The lowest temperature that has been achieved is about 1×10^{-6} K. To achieve this an additional stage of cooling is required beyond that described in the previous problem, namely nuclear cooling. This process is similar to magnetic cooling, but it involves the magnetic moment associated with the nucleus rather than that associated with certain ions in the paramagnetic salt. Suppose that 10 μJ is to be removed from a specimen at an average temperature of 10^{-5} K (ten micro-joules is about the potential energy loss of a pin dropping 3 mm). Find the work input to a Carnot heat pump and its coefficient of performance to do this assuming the ambient is at 300 K.

Solution:

The heat removed from the cold space is

$$Q_L = 10 \ \mu J = 10 \times 10^{-6} \ J \qquad \text{at } T_L = 10^{-5} \ K$$

Carnot heat pump satisfies Eq.7.4

$$\Rightarrow Q_H = Q_L \times \frac{T_H}{T_L} = 10 \times 10^{-6} \ J \times \frac{300}{10^{-5}} = 300 \ J$$

From the energy equation for the heat pump

$$W_{in} = Q_H - Q_L = 300 - 10 \times 10^{-6} \cong \mathbf{300 \ J}$$

$$\beta = \frac{Q_L}{W_{in}} = \frac{10 \times 10^{-6}}{300} = \mathbf{3.33 \times 10^{-8}}$$

5.52

Consider the setup with two stacked (temperature wise) heat engines as in Fig. P5.4. Let $T_H = 850$ K, $T_M = 600$ K and $T_L = 350$ K. Find the two heat engine efficiencies and the combined overall efficiency assuming Carnot cycles.

The individual efficiencies

$$\eta_1 = 1 - \frac{T_M}{T_H} = 1 - \frac{600}{850} = \mathbf{0.294}$$

$$\eta_2 = 1 - \frac{T_L}{T_M} = 1 - \frac{350}{600} = \mathbf{0.417}$$

The overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \dot{Q}_M = \eta_2 (1 - \eta_1) \dot{Q}_H$$

so the final result is

$$\eta_{TH} = \eta_1 + \eta_2 (1 - \eta_1) = 0.294 + 0.417 (1 - 0.294) = \mathbf{0.588}$$

Comment: It matches a single heat engine $\eta_{TH} = 1 - \frac{T_L}{T_H} = 1 - \frac{350}{850} = 0.588$

5.53

Find the maximum coefficient of performance for the refrigerator in your kitchen, assuming it runs in a Carnot cycle.

Solution:

The refrigerator coefficient of performance is

$$\beta = Q_L/W = Q_L/(Q_H - Q_L) = T_L/(T_H - T_L)$$

Assuming $T_L \sim 0°C,$ $T_H \sim 35°C,$

$$\beta \leq \frac{273.15}{35 - 0} = \mathbf{7.8}$$

Actual working fluid temperatures must be such that

$$T_L < T_{refrigerator} \quad and \quad T_H > T_{room}$$

A refrigerator does not operate in a Carnot cycle. The actual vapor compression cycle is examined in Chapter 9.

5.54

A car engine burns 5 kg fuel (equivalent to addition of Q_H) at 1500 K and rejects energy to the radiator and the exhaust at an average temperature of 750 K. If the fuel provides 40 000 kJ/kg what is the maximum amount of work the engine can provide?

Solution:

A heat engine $Q_H = m \, q_{fuel} = 5 \text{ kg} \times 40\ 000 \text{ kJ/kg} = 200\ 000 \text{ kJ}$

Assume a Carnot efficiency (maximum theoretical work)

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{750}{1500} = 0.5$$

$$W = \eta \, Q_H = \textbf{100 000 kJ}$$

5.55

An air-conditioner provides 1 kg/s of air at 15°C cooled from outside atmospheric air at 35°C. Estimate the amount of power needed to operate the air-conditioner. Clearly state all assumptions made.

Solution:

Consider the cooling of air which needs a heat transfer as

$$\dot{Q}_{air} = \dot{m}\ \Delta h \cong \dot{m}\ C_p \Delta T = 1\ kg/s \times 1.004\ kJ/kg\ K \times 20\ K = 20\ kW$$

Assume Carnot cycle refrigerator

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) \cong \frac{T_L}{T_H - T_L} = \frac{273 + 15}{35 - 15} = 14.4$$

$$\dot{W} = \dot{Q}_L / \beta = \frac{20.0}{14.4} = \mathbf{1.39\ kW}$$

This estimate is the theoretical maximum performance. To do the required heat transfer $T_L \cong 5°C$ and $T_H = 45°C$ are more likely; secondly

$$\beta < \beta_{carnot}$$

5.56

A refrigerator should remove 400 kJ from some food. Assume the refrigerator works in a Carnot cycle between –15$^{\mathrm{o}}$C and 45$^{\mathrm{o}}$C with a motor-compressor of 400 W. How much time does it take if this is the only cooling load?

Assume Carnot cycle refrigerator

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) \cong \frac{T_L}{T_H - T_L} = \frac{273 - 15}{45 - (-15)} = 4.3$$

This gives the relation between the low T heat transfer and the work as

$$\dot{Q}_L = \frac{Q}{t} = \beta\,\dot{W} = 4.3\,\dot{W}$$

$$t = \frac{Q}{\beta\,\dot{W}} = \frac{400 \times 1000}{4.3 \times 400}\ \frac{J}{W} = \textbf{233 s}$$

5.57

Calculate the amount of work input a refrigerator needs to make ice cubes out of a tray of 0.25 kg liquid water at 10°C. Assume the refrigerator works in a Carnot cycle between –8°C and 35°C with a motor-compressor of 600 W. How much time does it take if this is the only cooling load?

Solution:

C.V. Water in tray. We neglect tray mass.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : $P = \text{constant} + P_o$

$${}_1W_2 = \int P \, dV = P_o m(v_2 - v_1)$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

Tbl. B.1.1 : $h_1 = 41.99$ kJ/kg, Tbl. B.1.5 : $h_2 = -333.6$ kJ/kg

$${}_1Q_2 = 0.25 \text{ kg } (-333.4 - 41.99) \text{ kJ/kg} = -93.848 \text{ kJ}$$

Consider now refrigerator

$$\beta = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L} = \frac{273 - 8}{35 - (-8)} = 6.16$$

$$W = \frac{Q_L}{\beta} = -\frac{{}_1Q_2}{\beta} = \frac{93.848}{6.16} = 15.24 \text{ kJ}$$

For the motor to transfer that amount of energy the time is found as

$$W = \int \dot{W} \, dt = \dot{W} \, \Delta t$$

$$\Delta t = \frac{W}{\dot{W}} = \frac{15.24 \times 1000}{600} \frac{J}{W} = \mathbf{25.4 \text{ s}}$$

Comment: We neglected a baseload of the refrigerator so not all the 600 W are available to make ice, also our coefficient of performance is very optimistic and finally the heat transfer is a transient process. All this means that it will take much more time to make ice-cubes.

5.58

We propose to heat a house in the winter with a heat pump. The house is to be maintained at 20°C at all times. When the ambient temperature outside drops to −10°C, the rate at which heat is lost from the house is estimated to be 25 kW. What is the minimum electrical power required to drive the heat pump?

Solution:

Minimum power if we assume a Carnot cycle

Energy equation for the house (steady state): $\dot{Q}_H = \dot{Q}_{leak} = 25$ kW

$$\beta' = \frac{\dot{Q}_H}{\dot{W}_{IN}} = \frac{T_H}{T_H\text{-}T_L} = \frac{293.2}{20 - (-10)} = 9.773 \implies \dot{W}_{IN} = \frac{25}{9.773} = \textbf{2.56 kW}$$

5.59

A household freezer operates in a room at 20°C. Heat must be transferred from the cold space at a rate of 2 kW to maintain its temperature at −30°C. What is the theoretically smallest (power) motor required to operate this freezer?

  Solution:

Assume a Carnot cycle between T_L = -30°C and T_H = 20°C:

$$\beta = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{T_L}{T_H - T_L} = \frac{273.15 - 30}{20 - (-30)} = 4.86$$

$$\dot{W}_{in} = \dot{Q}_L / \beta = 2/4.86 = \mathbf{0.41\ kW}$$

This is the theoretical minimum power input.
Any actual machine requires a larger input.

5.60

A thermal storage is made with a rock (granite) bed of 2 m^3 which is heated to 400 K using solar energy. A heat engine receives a Q_H from the bed and rejects heat to the ambient at 290 K. The rock bed therefore cools down and as it reaches 290 K the process stops. Find the energy the rock bed can give out. What is the heat engine efficiency at the beginning of the process and what is it at the end of the process?

Solution:

Assume the whole setup is reversible and that the heat engine operates in a Carnot cycle. The total change in the energy of the rock bed is

$u_2 - u_1 = q = C\,\Delta T = 0.89$ kJ/kgK $\times$ (400 - 290) K = 97.9 kJ/kg

$m = \rho V = 2750$ kg/m^3 $\times$ 2 m^3 = 5500 kg ,

$Q = mq = 5500$ kg $\times$ 97.9 kJ/kg = **538 450 kJ**

To get the efficiency use the CARNOT cycle result as

$\eta = 1 - T_o/T_H = 1 - 290/400 =$ **0.275** at the beginning of process

$\eta = 1 - T_o/T_H = 1 - 290/290 =$ **0.0** at the end of process

Borgnakke and Sonntag

5.61

It is proposed to build a 1000-MW electric power plant with steam as the working fluid. The condensers are to be cooled with river water (see Fig. P5.61). The maximum steam temperature is 550°C, and the pressure in the condensers will be 10 kPa. Estimate the temperature rise of the river downstream from the power plant.

Solution:

$$\dot{W}_{NET} = 10^6 \text{ kW}, \quad T_H = 550°C = 823.3 \text{ K}$$

$$P_{COND} = 10 \text{ kPa} \rightarrow T_L = T_G \text{ (P = 10 kPa)} = 45.8°C = 319 \text{ K}$$

$$\eta_{TH\ CARNOT} = \frac{T_H - T_L}{T_H} = \frac{823.2 - 319}{823.2} = 0.6125$$

$$\Rightarrow \dot{Q}_{L\ MIN} = 10^6 \left(\frac{1 - 0.6125}{0.6125} \right) = 0.6327 \times 10^6 \text{ kW}$$

But $\dot{m}_{H_2O} = \dfrac{60 \times 8 \times 10/60}{0.001} = 80\ 000 \text{ kg/s}$ having an energy flow of

$$\dot{Q}_{L\ MIN} = \dot{m}_{H_2O}\ \Delta h = \dot{m}_{H_2O}\ C_{P\ LIQ\ H_2O}\ \Delta T_{H_2O\ MIN}$$

$$\Rightarrow \Delta T_{H_2O\ MIN} = \frac{\dot{Q}_{L\ MIN}}{\dot{m}_{H_2O} C_{P\ LIQ\ H_2O}} = \frac{0.6327 \times 10^6}{80000 \times 4.184} \frac{\text{kW}}{\text{kg/s} \times \text{kJ/kg-K}}$$

$$= \mathbf{1.9°C}$$

5.62

A certain solar-energy collector produces a maximum temperature of 100°C. The energy is used in a cyclic heat engine that operates in a 10°C environment. What is the maximum thermal efficiency? If the collector is redesigned to focus the incoming light, what should the maximum temperature be to produce a 25% improvement in engine efficiency?

Solution:

$$\text{For } T_H = 100°C = 373.2 \text{ K} \quad \& \quad T_L = 283.2 \text{ K}$$

$$\eta_{th\ max} = \frac{T_H - T_L}{T_H} = \frac{90}{373.2} = \mathbf{0.241}$$

The improved efficiency is

$$\eta_{th\ max} = 0.241 \times 1.25 = 0.301$$

With the Carnot cycle efficiency

$$\eta_{th\ max} = \frac{T_H - T_L}{T_H} = 1 - \frac{T_L}{T_H} = 0.301$$

Then

$$T_H = T_L / (1 - 0.301) = 405 \text{ K} = \mathbf{132°C}$$

5.63

A constant temperature of $-125°C$ must be obtained in a cryogenic experiment, although it gains 120 W due to heat transfer. What is the smallest motor you would need for a heat pump absorbing heat from the container and rejecting heat to the room at $20°C$?

Solution:

We do not know the actual device so find the work for a Carnot cycle

$$\beta_{REF} = \dot{Q}_L / \dot{W} = \frac{T_L}{T_H - T_L} = \frac{148.15}{20 - (-125)} = 1.022$$

$$\Rightarrow \qquad \dot{W} = \dot{Q}_L / \beta_{REF} = 120 \text{ W}/1.022 = \textbf{117. 4 W}$$

Borgnakke and Sonntag

5.64

Helium has the lowest normal boiling point of any of the elements at 4.2 K. At this temperature the enthalpy of evaporation is 83.3 kJ/kmol. A Carnot refrigeration cycle is analyzed for the production of 1 kmol of liquid helium at 4.2 K from saturated vapor at the same temperature. What is the work input to the refrigerator and the coefficient of performance for the cycle with an ambient at 300 K?

Solution:

For the Carnot cycle the ratio of the heat transfers is the ratio of temperatures

$$Q_L = n\,\overline{h}_{fg} = 1 \text{ kmol} \times 83.3 \text{ kJ/kmol} = 83.3 \text{ kJ}$$

$$Q_H = Q_L \times \frac{T_H}{T_L} = 83.3 \times \frac{300}{4.2} = 5950 \text{ kJ}$$

$$W_{IN} = Q_H - Q_L = 5950 - 83.3 = \textbf{5886.7 kJ}$$

$$\beta = \frac{Q_L}{W_{IN}} = \frac{83.3}{5886.7} = \textbf{0.0142} \qquad [\ = \frac{T_L}{T_H - T_L}\ ]$$

5.65

R-134a fills a 0.1-m^3 capsule at 20°C, 200 kPa. It is placed in a deep freezer, where it is cooled to −10°C. The deep freezer sits in a room with ambient temperature of 20°C and has an inside temperature of −10°C. Find the amount of energy the freezer must remove from the R-134a and the extra amount of work input to the freezer to perform the process.

Solution:

C.V. R-134a out to the -10°C space.

Energy equation: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : V = Const   => $v_2 = v_1$ => ${}_1W_2 = 0$

Table B.5.2: $v_1 = 0.11436$ m^3/kg, $u_1 = 395.27$ kJ/kg

$m = V/ v_1 = 0.87443$ kg

State 2: $v_2 = v_1 > v_g = 0.09921$ m^3/kg   Table B.5.1 => sup vap

(T,v)   interpolate between 150 kPa and 200 kPa in B.5.2

$$P_2 = 150 + 50 \frac{0.11436 - 0.13602}{0.10013 - 0.13602} = 150 + 50 \times 0.6035 = 180 \text{ kPa}$$

$u_2 = 373.44 + 0.6035 \times (372.31 - 373.44) = 372.76$ kJ/kg

${}_1Q_2 = m(u_2 - u_1) = 0.87443$ kg $\times$ (372.76 - 395.27) kJ/kg

$= $ **- 19.68 kJ**

Consider the freezer and assume Carnot cycle

$$\beta = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L} = \frac{273 - 10}{20 - (-10)} = 8.767$$

$W_{in} = Q_L / \beta = 19.68$ kJ/ 8.767 = **2.245 kJ**

5.66

A heat engine has a solar collector receiving 0.2 kW per square meter inside which a transfer media is heated to 450 K. The collected energy powers a heat engine which rejects heat at 40°C. If the heat engine should deliver 2.5 kW what is the minimum size (area) solar collector?

Solution:

$$T_H = 450 \text{ K} \qquad T_L = 40°C = 313.15 \text{ K}$$

$$\eta_{HE} = 1 - \frac{T_L}{T_H} = 1 - \frac{313.15}{450} = 0.304$$

$$\dot{W} = \eta \, \dot{Q}_H \quad => \quad \dot{Q}_H = \frac{\dot{W}}{\eta} = \frac{2.5}{0.304} \text{ kW} = 8.224 \text{ kW}$$

$$\dot{Q}_H = 0.2 \, (\text{kW/m}^2) \, A \quad => \quad A = \frac{\dot{Q}_H}{0.2} = \textbf{41 m}^2$$

5.67

A heat pump is driven by the work output of a heat engine as shown in figure P5.67. If we assume ideal devices find the ratio of the total power $\dot{Q}_{L1} + \dot{Q}_{H2}$ that heats the house to the power from the hot energy source $\dot{Q}_{H1}$ in terms of the temperatures.

$$\beta_{HP} = \dot{Q}_{H2}/\dot{W} = \dot{Q}_{H2}/(\dot{Q}_{H2} - \dot{Q}_{L2}) = \frac{T_{room}}{T_{room} - T_{amb}}$$

$$\dot{W} = \eta_{HE} \cdot \dot{Q}_{H1} = (1 - \frac{T_{room}}{T_H}) \, \dot{Q}_{H1}$$

$$\dot{W} = \dot{Q}_{H2}/\beta_{HP} = \frac{T_{room}}{T_{room} - T_{amb}} \dot{Q}_{H2}$$

$$\dot{Q}_{L1} = \dot{Q}_{H1} - \dot{W} = [1 - 1 + \frac{T_{room}}{T_H}] \, \dot{Q}_{H1}$$

$$\frac{\dot{Q}_{H2} + \dot{Q}_{L1}}{\dot{Q}_{H1}} = 1 - 1 + \frac{T_{room}}{T_H} + \frac{1 - \dfrac{T_{room}}{T_H}}{\dfrac{T_{room} - T_{amb}}{T_{room}}} = \frac{T_{room}}{T_H} + \frac{T_{room} - T^2_{room}/T_H}{T_{room} - T_{amb}}$$

$$= T_{room} [\frac{1}{T_H} + \frac{1 - \dfrac{T_{room}}{T_H}}{T_{room} - T_{amb}}] = \frac{T_{room}}{T_H} [1 + \frac{T_H - T_{room}}{T_{room} - T_{amb}}]$$

$$= \mathbf{\frac{T_{room}}{T_H} \left[\frac{T_H - T_{amb}}{T_{room} - T_{amb}} \right]}$$

House T_{room}

5.68

Sixty kilograms per hour of water runs through a heat exchanger, entering as saturated liquid at 200 kPa and leaving as saturated vapor. The heat is supplied by a Carnot heat pump operating from a low-temperature reservoir at 16°C with a COP of half that of a similar Carnot unit. Find the rate of work into the heat pump.

Solution:

C.V. Heat exchanger

$\dot{m}_1 = \dot{m}_2$; $\qquad \dot{m}_1 h_1 + \dot{Q}_H = \dot{m}_1 h_2$

Table B.1.2: $h_1 = 504.7$ kJ/kg,

$\qquad\qquad h_2 = 2706.7$ kJ/kg

$T_H = T_{sat}(P) = 120.93 + 273.15$

$\qquad = 394.08$ K

$\dot{Q}_H = \dfrac{60}{3600}(2706.7 - 504.7) = 36.7$ kW

First find the COP of a Carnot heat pump.

$\beta' = \dot{Q}_H/\dot{W} = T_H / (T_H - T_L) = 394.08 / (394.08 - 289.15) = 3.76$

Now we can do the actual one as $\beta'_H = 3.76/2 = 1.88$

$\dot{W} = \dot{Q}_H/\beta'_H = 36.7$ kW $/1.88 = $ **19.52 kW**

5.69

A power plant with a thermal efficiency of 40% is located on a river similar to Fig. P5.61. With a total river mass flow rate of 1×10^5 kg/s at 15°C find the maximum power production allowed if the river water should not be heated more than 1 degree.

The maximum heating allowed determines the maximum $\dot{Q}_L$ as

$$\dot{Q}_L = \dot{m}_{H_2O} \, \Delta h = \dot{m}_{H_2O} \, C_{P \, LIQ \, H_2O} \, \Delta T_{H_2O}$$

$$= 1 \times 10^5 \text{ kg/s} \times 4.18 \text{ kJ/kg-K} \times 1 \text{ K} = 418 \text{ MW}$$

$$= \dot{W}_{NET}(1/\eta_{TH \, ac} - 1)$$

$$\dot{W}_{NET} = \dot{Q}_L \, / \, (1/\eta_{TH \, ac} - 1) = \dot{Q}_L \, \frac{\eta_{TH \, ac}}{1 - \eta_{TH \, ac}}$$

$$= 418 \text{ MW} \times \frac{0.4}{1 - 0.4} = \textbf{279 MW}$$

5.70

Liquid sodium leaves a nuclear reactor at 800°C and is used as the energy source in a steam power plant. The condenser cooling water comes from a cooling tower at 15°C. Determine the maximum thermal efficiency of the power plant. Is it misleading to use the temperatures given to calculate this value?

Solution:

$$T_H = 800°C = 1073.2 \text{ K}, \quad T_L = 15°C = 288.2 \text{ K}$$

$$\eta_{TH\,MAX} = \frac{T_H - T_L}{T_H} = \frac{1073.2 - 288.2}{1073.2} = \mathbf{0.731}$$

It might be misleading to use 800°C as the value for T_H, since there is not a supply of energy available at a constant temperature of 800°C (liquid Na is cooled to a lower temperature in the heat exchanger).

$\Rightarrow$ The Na cannot be used to boil H_2O at 800°C.

Similarly, the H_2O leaves the cooling tower and enters the condenser at 15°C, and leaves the condenser at some higher temperature.

$\Rightarrow$ The water does not provide for condensing steam at a constant temperature of 15°C.

5.71

The management of a large factory cannot decide which of two fuels to purchase. The selected fuel will be used in a heat engine operating between the fuel burning temperature and a low exhaust temperature. of. Fuel A burns at 2200 K and exhausts at 450 K, delivering 30 000 kJ/kg, and costs $1.50/kg. Fuel B burns at 1200 K and exhausts at 350 K, delivering 40 000 kJ/kg and costs $1.30/kg. Which fuel would you buy and why?

Solution:

Fuel A: $\eta_{TH,A} = 1 - \dfrac{T_L}{T_H} = 1 - \dfrac{450}{2200} = 0.795$

$W_A = \eta_{TH,A} \times Q_A = 0.795 \times 30\ 000 = 23\ 850$ kJ/kg

$W_A/\$_A = 23\ 850/1.5 = 15\ 900$ kJ/$

Fuel B: $\eta_{TH,B} = 1 - \dfrac{T_L}{T_H} = 1 - \dfrac{350}{1200} = 0.708$

$W_B = \eta_{TH,B} \times Q_B = 0.708 \times 40\ 000 = 28\ 320$ kJ/kg

$W_B/\$_B = 28\ 320/1.3 = 21\ 785$ kJ/$

Select fuel B for more work per dollar though it has a lower thermal efficiency.

5.72

A sales person selling refrigerators and deep freezers will guarantee a minimum coefficient of performance of 4.5 year round. How would you evaluate that? Are they all the same?

Solution:

Assume a high temperature of 35°C. If a freezer compartment is included $T_L \sim$ -20°C (deep freezer) and fluid temperature is then $T_L \sim$ -30°C

$$\beta_{\text{deep freezer}} \le \frac{T_L}{T_H - T_L} = \frac{273.15 - 30}{35 - (-30)} = 3.74$$

A hot summer day may require a higher T_H to push Q_H out into the room, so even lower β.

Claim is possible for a refrigerator, but not for a deep freezer.

5.73

A cyclic machine, shown in Fig. P5.73, receives 325 kJ from a 1000 K energy reservoir. It rejects 125 kJ to a 400 K energy reservoir and the cycle produces 200 kJ of work as output. Is this cycle reversible, irreversible, or impossible?

Solution:

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1000} = 0.6$$

$$\eta_{eng} = \frac{W}{Q_H} = \frac{200}{325} = 0.615 > \eta_{Carnot}$$

This is **impossible.**

$T_H = 1000\ K$

$Q_H = 325$ kJ

HE cb

$W = 200$ kJ

$Q_L = 125$ kJ

$T_L = 400\ K$

5.74

Consider the previous problem and assume the temperatures and heat input are as given. If the actual machine has an efficiency that is half that of the corresponding Carnot cycle, find the work out and the rejected heat transfer.

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1000} = 0.6$$

$$\eta_{eng} = \eta_{Carnot}/2 = 0.3 = \frac{W}{Q_H}$$

$$W = 0.3\, Q_H = 0.3 \times 325 = \textbf{97.5 kJ}$$

$T_H = 1000\ K$

$Q_H = 325\ kJ$

HE
cb

$W = ?$

$Q_L = ?$

$T_L = 400\ K$

$$Q_L = Q_H - W = (325 - 97.5)\ kJ = \textbf{227.5 kJ}$$

5.75

Repeat problem 5.61 using a more realistic thermal efficiency of 45%.

$$\dot{W}_{NET} = 10^6 \text{ kW} = \eta_{TH\ ac}\ \dot{Q}_H, \qquad \eta_{TH\ ac} = 0.45$$

$$\Rightarrow \dot{Q}_L = \dot{Q}_H - \dot{W}_{NET} = \dot{W}_{NET}/\eta_{TH\ ac} - \dot{W}_{NET} = \dot{W}_{NET}(1/\eta_{TH\ ac} - 1)$$

$$= 10^6 \text{ kW } \left(\frac{1 - 0.45}{0.45}\right) = 1.222 \times 10^6 \text{ kW}$$

But $\dot{m}_{H_2O} = \dfrac{60 \times 8 \times 10/60}{0.001} = 80\ 000$ kg/s having an energy flow of

$$\dot{Q}_L = \dot{m}_{H_2O}\ \Delta h = \dot{m}_{H_2O}\ C_{P\ LIQ\ H_2O}\ \Delta T_{H_2O}$$

$$\Rightarrow \Delta T_{H_2O} = \frac{\dot{Q}_L}{\dot{m}_{H_2O} C_{P\ LIQ\ H_2O}} = \frac{1.222 \times 10^6}{80\ 000 \times 4.18}\frac{\text{kW}}{\text{kg/s} \times \text{kJ/kg-K}} = \mathbf{3.65°C}$$

5.76

An inventor has developed a refrigeration unit that maintains the cold space at −10°C, while operating in a 25°C room. A coefficient of performance of 8.5 is claimed. How do you evaluate this?

Solution:

$$\beta_{Carnot} = \frac{Q_L}{W_{in}} = \frac{T_L}{T_H - T_L} = \frac{263.15}{25 - (-10)} = 7.52$$

$8.5 > \beta_{Carnot} \Rightarrow$ **impossible claim**

$T_H = 25C$

$\dot{Q}_H$

REF $\Leftarrow \dot{W}$

$\dot{Q}_L$

$T_L = -10C$

5.77

A heat pump receives energy from a source at $80^{o}C$ and delivers energy to a boiler that operates at 350 kPa. The boiler input is saturated liquid water and the exit is saturated vapor both at 350 kPa. The heat pump is driven by a 2.5 MW motor and has a COP that is 60% of a Carnot heat pump COP. What is the maximum mass flow rate of water the system can deliver?

$$T_H = T_{sat} = 138.88^{o}C = 412 \text{ K}, \quad h_{fg} = 2148.1 \text{ kJ/kg}$$

$$\beta_{HP \; Carnot} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{T_H}{T_H - T_L} = \frac{412}{138.88 - 80} = 7$$

$$\beta_{HP \; ac} = 0.6 \times 7 = 4.2 = \dot{Q}_H / \dot{W}_{in}$$

$$\dot{Q}_H = 4.2 \; \dot{W}_{in} = 4.2 \times 2.5 \text{ MW} = 10.5 \text{ MW} = \dot{m} \; h_{fg}$$

$$\dot{m} = \dot{Q}_H / h_{fg} = 10\;500 \text{ kW} / 2148.1 \text{ kJ/kg} = \textbf{4.89 kg/s}$$

5.78

In a remote location, you run a heat engine to provide power to run a refrigerator. The input to the heat engine is 800 K and the low T is 400 K; it has an actual efficiency equal to half of that of the corresponding Carnot unit. The refrigerator has $T_L = -10^oC$ and $T_H = 35^oC$, with a cop that is one-third that of the corresponding Carnot unit. Assume a cooling capacity of 2 kW is needed and find the rate of heat input to the heat engine.

Heat engine: $\eta_{Carnot} = 1 - \dfrac{T_L}{T_H} = 1 - \dfrac{400}{800} = 0.5;$ $\eta_{ac} = 0.25$

Refrigerator: $\beta_{ref.\ Carnot} = \dfrac{T_L}{T_H - T_L} = \dfrac{273 - 10}{35 - (-10)} = 5.844;$ $\beta_{ref.\ ac} = 1.95$

Cooling capacity: $\dot{Q}_L = 2\ kW = \beta_{ref.\ ac}\ \dot{W};$ $\dot{W} = 2\ kW/1.95 = 1.026\ kW$

This work must be provided by the heat engine $\dot{W} = \eta_{ac}\ \dot{Q}_H$

$$\dot{Q}_H = \dot{W} / \eta_{ac} = 1.026\ kW\ /\ 0.25 = \textbf{4.1 kW}$$

5.79

A car engine with a thermal efficiency of 33% drives the air-conditioner unit (a refrigerator) besides powering the car and other auxiliary equipment. On a hot (35°C) summer day the A/C takes outside air in and cools it to 5°C sending it into a duct using 2 kW of power input and it is assumed to be half as good as a Carnot refrigeration unit. Find the rate of fuel (kW) being burned extra just to drive the A/C unit and its COP. Find the flow rate of cold air the A/C unit can provide.

$$\dot{W}_{extra} = \eta \, \dot{Q}_{H\,extra} \quad \Rightarrow \quad \dot{Q}_{H\,extra} = \dot{W}_{extra} \, / \, \eta = 2 \text{ kW} \, / \, 0.33 = \textbf{6 kW}$$

$$\beta = \frac{Q_L}{W_{IN}} = 0.5 \, \beta_{Carnot} = 0.5 \, \frac{T_L}{T_H - T_L} = 0.5 \, \frac{5 + 273.15}{35 - 5} = 4.636$$

$$\dot{Q}_L = \beta \, \dot{W} = 4.636 \times 2 \text{ kW} = 9.272 \text{ kW} = \dot{m}_{air} \, C_{P\,air} \, \Delta T_{air}$$

$$\dot{m}_{air} = \dot{Q}_L \, / \, [C_{P\,air} \, \Delta T_{air} \,] = \frac{9.272 \text{ kW}}{1.004 \text{ kJ/kg-K} \times (35 - 5) \text{ K}} = \textbf{0.308 kg/s}$$

5.80

A large heat pump should upgrade 5 MW of heat at 85°C to be delivered as heat at 150°C. Suppose the actual heat pump has a COP of 2.5 how much power is required to drive the unit. For the same COP how high a high temperature would a Carnot heat pump have assuming the same low T?

This is an actual COP for the heat pump as

$$\beta_{HP} = COP = \dot{Q}_H/\dot{W}_{in} = 2.5 \quad \Rightarrow \quad \dot{Q}_L/\dot{W}_{in} = 1.5$$

$$\dot{W}_{in} = \dot{Q}_L / 1.5 = 5 / 1.5 = \textbf{3.333 MW}$$

The Carnot heat pump has a COP given by the temperatures as

$$\beta_{HP} = \dot{Q}_H/\dot{W}_{in} = \frac{T_H}{T_H - T_L} = 2.5 \quad \Rightarrow \quad T_H = 2.5\ T_H - 2.5\ T_L$$

$$\Rightarrow \quad T_H = \frac{2.5}{1.5} T_L = \frac{5}{3}(85 + 273.15) = \textbf{597 K}$$

Finite ΔT Heat Transfer

5.81

A refrigerator keeping 5°C inside is located in a 30°C room. It must have a high temperature ΔT above room temperature and a low temperature ΔT below the refrigerated space in the cycle to actually transfer the heat. For a ΔT of 0, 5 and 10°C respectively calculate the COP assuming a Carnot cycle.

Solution:

From the definition of COP and assuming Carnot cycle

$$\beta = \frac{Q_L}{W_{IN}} = \frac{T_L}{T_H - T_L} \quad \text{when T's are absolute temperatures}$$

	ΔT	T_H	T_H	T_L	T_L	β
		°C	K	°C	K	
a	0	30	303	5	278	11.1
b	5	35	308	0	273	7.8
c	10	40	313	-5	268	5.96

Notice how the COP drops significantly with the increase in ΔT.

5.82

The ocean near Havaii has 20$^{\text{o}}$C near the surface and 5$^{\text{o}}$C at some depth. A power plant based on this temperature difference is being planned. How large an efficiency could it have? If the two heat transfer terms (Q_H and Q_L) both require a 2 degree difference to operate what is the maximum efficiency then?

Solution:

$$T_H = 20°C = 293.2 \text{ K}; \qquad T_L = 5°C = 278.2 \text{ K}$$

$$\eta_{\text{TH MAX}} = \frac{T_H - T_L}{T_H} = \frac{293.2 - 278.2}{293.2} = \textbf{0.051}$$

$$\eta_{\text{TH mod}} = \frac{T_{H'} - T_{L'}}{T_{H'}} = \frac{291.2 - 280.2}{291.2} = \textbf{0.038}$$

This is a very low efficiency so it has to be done on a very large scale to be economically feasible and then it will have some environmental impact.

5.83

A house is cooled by a heat pump driven by an electric motor using the inside as the low-temperature reservoir. The house gains energy in direct proportion to the temperature difference as $\dot{Q}_{gain} = K(T_H - T_L)$. Determine the minimum electric power to drive the heat pump as a function of the two temperatures.

Solution:

Refrigerator COP: $\quad \beta = \dot{Q}_L / \dot{W}_{in} \leq T_L/(T_H - T_L)$;

Heat gain must be removed: $\quad \dot{Q}_L = \dot{Q}_{gain} = K(T_H - T_L)$

Solve for required work and substitute in for β

$$\dot{W}_{in} = \dot{Q}_L/\beta \geq K(T_H - T_L) \times (T_H - T_L)/T_L$$

$$\dot{W}_{in} \geq K(T_H - T_L)^2/T_L$$

5.84

An air conditioner in a very hot region uses a power input of 2.5 kW to cool a 5°C space with the high temperature in the cycle as 40°C. The Q_H is pushed to the ambient air at 30°C in a heat exchanger where the transfer coefficient is 50 W/m^{2}K. Find the required minimum heat transfer area.

Solution:

$$\dot{W} = 2.5 \text{ kW} = \dot{Q}_H / \beta_{HP}; \qquad \beta_{HP} = T_H/(T_H - T_L) = \frac{273 + 40}{40 - 5} = 8.943$$

$$\dot{Q}_H = \dot{W} \times \beta_{HP} = 2.5 \times 8.943 = 22.36 \text{ kW} = h \text{ A } \Delta T$$

$$A = \frac{\dot{Q}_H}{h \text{ } \Delta T} = \frac{22.36 \times 10^3}{50 \times (40 - 30)} = \mathbf{44.72 \text{ m}^2}$$

5.85

A small house is kept at 20°C inside loses 12 kW to the outside ambient at 0°C. A heat pump is used to help heat the house together with possible electric heat. The heat pump is driven by a motor of 2.5 kW and it has a COP that is ¼ of a Carnot heat pump unit. Find the actual COP for the heat pump and the amount of electric heat that must be used (if any) to maintain the house temperature.

CV. House   Energy: $0 = \dot{Q}_H + \dot{W}_{el.} - \dot{Q}_{Loss}$

Definition of COP: $\beta' = COP_{HP} = \dfrac{\dot{Q}_H}{\dot{W}} = \dfrac{1}{4}\dfrac{T_H}{T_H - T_L} = \dfrac{1}{4}\dfrac{293.15}{20 - 0} = \textbf{3.664}$

$\dot{Q}_H = COP_{HP}\ \dot{W}_{HP} = 3.664 \times 2.5\ kW = 9.16\ kW$

$\dot{W}_{el.} = \dot{Q}_{Loss} - \dot{Q}_H = 12 - 9.16 = \textbf{2.84 kW}$

C.V. Total   Energy: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{HP} = 9.16 - 2.5 = 6.66\ kW$

Entropy: $0 = \dot{Q}_L/T_L - \dot{Q}_{loss}/T_L + \dot{S}_{gen}$

$\dot{S}_{gen} = (\dot{Q}_{loss} - \dot{Q}_L)\ /T_L = \dfrac{12 - 6.66}{273.15} = 0.0195\ kW/K$

5.86

Consider a room at 20°C that is cooled by an air conditioner with a COP of 3.2 using a power input of 2 kW and the outside is at 35°C. What is the constant in the heat transfer Eq. 5.14 for the heat transfer from the outside into the room?

Definition of the coefficient of performance, Eq.5.2 and Eq.5.14

$$\dot{Q}_L = \beta_{AC}\dot{W} = 3.2 \times 2 \text{ kW} = 6.4 \text{ kW} = \dot{Q}_{\text{leak in}} = CA \, \Delta T$$

$$CA = \dot{Q}_L / \Delta T = \frac{6.4 \text{ kW}}{(35 - 20) \text{ K}} = \textbf{0.427 kW/K}$$

Here:

$$T_L = T_{\text{house}} = 20°C$$

$$T_H = T_{\text{amb}} = 35°C$$

5.87

A car engine operates with a thermal efficiency of 35%. Assume the air-conditioner has a coefficient of performance of $\beta = 3$ working as a refrigerator cooling the inside using engine shaft work to drive it. How much fuel energy should be spend extra to remove 1 kJ from the inside?

Solution:

Car engine: $\qquad W = \eta_{eng} \, Q_{fuel}$

Air conditioner: $\qquad \beta = \dfrac{Q_L}{W}$

$$W = \eta_{eng} \, Q_{fuel} = \frac{Q_L}{\beta}$$

$$Q_{fuel} = Q_L \, / \, (\eta_{eng} \, \beta) = \frac{1}{0.35 \times 3} = \mathbf{0.952 \ kJ}$$

5.88

Arctic explorers are unsure if they can use a 5-kW motor driven heat pump to stay warm. It should keep their shelter at 15°C. The shelter losses energy at a rate of 0.5 kW per degree difference to the colder ambient. The heat pump has a COP that is 50% of a Carnot heat pump. If the ambient temperature can fall to -25°C at night, would you recommend this heat pump to the explorers?

CV Heat pump.
The heat pump should deliver a rate of heating that equals the heat loss to the ambient for steady inside temperature.

$$COP = \beta' = \dot{Q}_H / \dot{W} = 0.5 \, \beta'_{Carnot} = \frac{1}{2} \times \frac{T_H}{T_H - T_L} = \frac{1}{2} \times \frac{273 + 15}{15 - (-25)} = 3.6$$

The heat pump can then provide a heating capacity of

$$\dot{Q}_H = \beta' \, \dot{W} = 3.6 \times 5 \text{ kW} = 18 \text{ kW}$$

The heat loss is

$$\dot{Q}_{leak \; out} = CA \, \Delta T = 0.5 \text{ kW/K} \times [15 - (-25)] \text{ K} = 20 \text{ kW}$$

The heat pump is not sufficient to cover the loss and **not recommended**.

5.89

Using the given heat pump in the previous problem, how warm would it make the shelter in the arctic night?

The high is now an unknown so both the heat loss and the heat pump performance depends on that. The energy balance around the shelter then gives

$$\dot{Q}_H = \beta' \, \dot{W} = \dot{Q}_{leak\ out} = CA \, \Delta T$$

Substitute the expression for β' and $CA \, \Delta T$ to give

$$\frac{1}{2} \times \frac{T_H}{T_H - T_L} \, \dot{W} = 0.5 \text{ kW/K} \times [T_H - T_L]$$

Multiply with the temperature difference, factor 2 and divide by the work to get

$$T_H = \frac{0.5 \times 2}{5 \text{ K}} [T_H - T_L]^2 = \frac{0.2}{K} [T_H - T_L]^2$$

Solve this equation like $0.2 \, x^2 - x - T_L = 0$, with $x = T_H - T_L$ and $T_L = 248.15$ K

$$x = T_H - T_L = 37.81 \text{ K} \qquad \text{(negative root discarded)}$$

$$T_H = x + T_L = 37.81 - 25 = \textbf{12.8°C}$$

5.90

An air conditioner cools a house at $T_L = 20°C$ with a maximum of 1.2 kW power input. The house gains 0.6 kW per degree temperature difference to the ambient and the refrigeration COP is $\beta = 0.6\ \beta_{Carnot}$. Find the maximum outside temperature, T_H, for which the air conditioner provides sufficient cooling.

Solution:

Here:

$T_L = T_{house}$

$T_H = T_{amb}$

In this setup the low temperature space is the house and the high temperature space is the ambient. The heat pump must remove the gain or leak heat transfer to keep it at a constant temperature.

$\dot{Q}_{leak} = 0.6\ (T_{amb} - T_{house}) = \dot{Q}_L$ which must be removed by the heat pump.

$\beta = \dot{Q}_L / \dot{W} = 0.6\ \beta_{carnot} = 0.6\ T_{house} / (T_{amb} - T_{house})$

Substitute in for $\dot{Q}_L$ and multiply with $(T_{amb} - T_{house})\dot{W}$:

$0.6\ (T_{amb} - T_{house})^2 = 0.6\ T_{house}\ \dot{W}$

Since $T_{house} = 293.15\ K$ and $\dot{W} = 1.2\ kW$ it follows

$(T_{amb} - T_{house})^2 = T_{house}\ \dot{W} = 293.15 \times 1.2 = 351.78\ K^2$

Solving $\Rightarrow (T_{amb} - T_{house}) = 18.76 \quad \Rightarrow \quad T_{amb} = $ **311.9 K = 38.8 °C**

Comment: We did assume here that $\beta = 0.6\ \beta_{carnot}$, the statement could also have been understood as $\beta' = 0.6\ \beta'_{carnot}$ which would lead to a slightly different result.

5.91

A house is cooled by an electric heat pump using the outside as the high-temperature reservoir. For several different summer outdoor temperatures, estimate the percent savings in electricity if the house is kept at 25°C instead of 20°C. Assume that the house is gaining energy from the outside directly proportional to the temperature difference as in Eq. 5.14.

Solution:

Air-conditioner (Refrigerator) $\dot{Q}_{LEAK} \propto (T_H - T_L)$

Max Perf. $\dfrac{\dot{Q}_L}{\dot{W}_{IN}} = \dfrac{T_L}{T_H - T_L} = \dfrac{K(T_H - T_L)}{\dot{W}_{IN}}, \quad \dot{W}_{IN} = \dfrac{K(T_H - T_L)^2}{T_L}$

A: $T_{L_A} = 20°C = 293.2$ K   B: $T_{L_B} = 25°C = 298.2$ K

T_H,°C	$\dot{W}_{IN_A}$/K	$\dot{W}_{IN_B}$/K	% saving
45	2.132	1.341	37.1 %
40	1.364	0.755	44.6 %
35	0.767	0.335	56.3 %

5.92

A heat pump has a coefficient of performance that is 50% of the theoretical maximum. It maintains a house at 20°C, which leaks energy of 0.6 kW per degree temperature difference to the ambient. For a maximum of 1.0 kW power input find the minimum outside temperature for which the heat pump is a sufficient heat source.

Solution:

C.V. House. For constant 20°C the heat pump must provide $\dot{Q}_{leak} = 0.6\ \Delta T$

$$\dot{Q}_H = \dot{Q}_{leak} = 0.6\ (T_H - T_L) = \beta'\ \dot{W}$$

C.V. Heat pump. Definition of the coefficient of performance and the fact that the maximum is for a Carnot heat pump.

$$\beta' = \frac{\dot{Q}_H}{\dot{W}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} = 0.5\ \beta'_{Carnot} = 0.5 \times \frac{T_H}{T_H - T_L}$$

Substitute into the first equation to get

$$0.6\ (T_H - T_L) = [\ 0.5 \times T_H\ /\ (T_H - T_L)\ ]\ 1 \quad =>$$

$$(T_H - T_L)^2 = (0.5\ /\ 0.6)\ T_H \times 1 = 0.5\ /\ 0.6 \times 293.15 = 244.29$$

$$T_H - T_L = 15.63 \quad => \quad T_L = 20 - 15.63 = \textbf{4.4 °C}$$

5.93

The room in problem 5.90 has a combined thermal mass of 2000 kg wood, 250 kg steel, and 500 kg drywall (gypsum). Estimate how quickly the room heats up if the air conditioner is turned off on a day when it is 35°C outside.

Without the air-conditioner the house gains heat and the energy equation for the house becomes

$$m \, C \frac{dT}{dt} = \dot{Q}_{in}$$

The gain is due to the temperature difference as

$$\dot{Q}_{in} = 0.6 \, (T_H - T_L) = 0.6 \text{ kW/K } (35 - 20) \text{ K} = 9 \text{ kW}$$

The combined (mC) is using an estimate C for gypsum as 1 kJ/kg-K

$$mC = [2000 \times 1.38 + 250 \times 0.46 + 500 \times 1] \text{ kJ/K} = 3375 \text{ kJ/K}$$

$$\frac{dT}{dt} = \dot{Q}_{in} / mC = 9 \text{ kW}/ \, 3375 \text{ kJ/K} = \mathbf{0.00267 \text{ K/s}}$$

5.94

A window air conditioner cools a room at $T_L = 22^oC$, with a maximum of 1.2 kW power input possible. The room gains 0.6 kW per degree temperature difference to the ambient, and the refrigeration COP is $\beta = 0.6\ \beta_{Carnot}$. Find the actual power required on a day when the temperature is 30^oC outside.

COP refrigerator: $\beta = \dfrac{Q_L}{W_{IN}} = 0.6\ \beta_{Carnot} = 0.6\ \dfrac{T_L}{T_H - T_L} = \dfrac{273.15 + 22}{30 \text{ - } 22} = 22.1$

Heat gain: $\dot{Q}_L = 0.6\ (T_H \text{ - } T_L) = 0.6 \text{ kW/K} \times (30 - 22) \text{ K} = 4.8 \text{ kW}$

$$\dot{W} = \dot{Q}_L / \beta = 4.8 \text{ kW}/22.1 = \mathbf{0.217\ kW}$$

5.95

On a cold (-10°C) winter day a heat pump provides 20 kW to heat a house maintained at 20°C and it has a COP$_{\text{HP}}$ of 4. How much power does the heat pump require? The next day a winter storm brings the outside to -15°C, assuming the same COP and the same house heat transfer coefficient for the heat loss to the outside air. How much power does the heat pump require then?

If we look at the heat loss for the house we have

$$\dot{Q}_{\text{loss}} = 20 \text{ kW} = \text{CA } \Delta\text{T} \qquad \Rightarrow \qquad \text{CA} = \frac{20 \text{ kW}}{20 \text{ - (-10) K}} = 0.667 \text{ kW/K}$$

So now with the new outdoor temperature we get

$$\dot{Q}_{\text{loss}} = \text{CA } \Delta\text{T} = 0.667 \text{ kW/K} \times [20 - (-15)] \text{ K} = 23.3 \text{ kW}$$

$$\dot{Q}_{\text{loss}} = \dot{Q}_{\text{H}} = \text{COP } \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_{\text{loss}} / \text{COP} = \frac{23.3 \text{ kW}}{4} = \mathbf{5.83 \text{ kW}}$$

5.96

In the previous problem, it was assumed that the COP will be the same when the outside temperature drops. Given the temperatures and the actual COP at the -10° C winter day, give an estimate for a more realistic COP for the outside at -15^oC case.

As the outside T drops the temperature in the low temperature heat exchanger drops and it will be harder for the heat pump. A reasonable assumption is then that the reduced COP will follow the ideal (Carnot cycle) COP.

$$\text{At} = -10^oC: \quad \beta_{Carnot} = \frac{T_H}{T_H - T_L} = \frac{293.15}{20 - (-10)} = 9.772; \quad COP = 4$$

$$\text{At} = -15^oC: \quad \beta_{Carnot} = \frac{T_H}{T_H - T_L} = \frac{293.15}{20 - (-15)} = 8.376$$

$$COP = \frac{8.376}{9.772} \times 4 = \mathbf{3.43}$$

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Ideal Gas Carnot Cycles

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5.97

Hydrogen gas is used in a Carnot cycle having an efficiency of 60% with a low temperature of 300 K. During the heat rejection the pressure changes from 90 kPa to 120 kPa. Find the high and low temperature heat transfer and the net cycle work per unit mass of hydrogen.

Solution:

As the efficiency is known, the high temperature is found as

$$\eta = 0.6 = 1 - \frac{T_L}{T_H} \quad => T_H = T_L /(1 - 0.6) = 750 \text{ K}$$

Now the volume ratio needed for the heat transfer, $T_3 = T_4 = T_L$, is

$$v_3 / v_4 = (RT_3 / P_3) / (RT_4 / P_4) = P_4 / P_3 = 120 / 90 = 1.333$$

so from Eq.5.9 we have with $R = 4.1243$ kJ/kg-K from Table A.5

$$q_L = RT_L \ln (v_3/v_4) = \textbf{355.95 kJ/kg}$$

Using the efficiency from Eq.5.5 then

$$q_H = q_L / (1 - 0.6) = \textbf{889.9 kJ/kg}$$

The net work equals the net heat transfer

$$w = q_H - q_L = \textbf{533.9 kJ/kg}$$

5.98

Carbon dioxide is used in an ideal gas refrigeration cycle, reverse of Fig. 5.24. Heat absorption is at 250 K and heat rejection is at 325 K where the pressure changes from 1200 kPa to 2400 kPa. Find the refrigeration COP and the specific heat transfer at the low temperature.

The analysis is the same as for the heat engine except the signs are opposite so the heat transfers move in the opposite direction.

$$\beta = \dot{Q}_L / \dot{W} = \beta_{carnot} = T_L / (T_H - T_L) = \frac{250}{325 - 250} = \mathbf{3.33}$$

$$q_H = RT_H \ln(v_2/v_1) = RT_H \ln\left(\frac{P_1}{P_2}\right) = 0.1889 \text{ kJ/kg-K} \times 325 \text{ K} \ln\left(\frac{2400}{1200}\right)$$

$$= 42.55 \text{ kJ/kg}$$

$$q_L = q_H \, T_L / T_H = 42.55 \text{ kJ/kg} \times 250 / 325 = \mathbf{32.73 \ kJ/kg}$$

5.99

An ideal gas Carnot cycle with air in a piston cylinder has a high temperature of 1000 K and a heat rejection at 400 K. During the heat addition the volume triples. Find the two specific heat transfers (q) in the cycle and the overall cycle efficiency.

Solution:

The P-v diagram of the cycle is shown to the right.
From the integration along the process curves done in the main text we have Eq.5.7

$$q_H = R \, T_H \, \ln(v_2/v_1)$$
$$= 0.287 \text{ kJ/kg} \times 1000 \ln(3)$$
$$= \textbf{315.3 kJ/kg}$$

Since it is a Carnot cycle the knowledge of the temperatures gives the cycle efficiency as

$$\eta_{TH} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1000} = \textbf{0.6}$$

from which we can get the other heat transfer from Eq.5.4

$$q_L = q_H \, T_L / T_H = 315.3 \times 400 / 1000 = \textbf{126.1 kJ/kg}$$

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5.100

Air in a piston/cylinder goes through a Carnot cycle with the P-v diagram shown in Fig. 5.24. The high and low temperatures are 600 K and 300 K respectively. The heat added at the high temperature is 250 kJ/kg and the lowest pressure in the cycle is 75 kPa. Find the specific volume and pressure after heat rejection and the net work per unit mass.

Solution:

$q_H = 250$ kJ/kg , $T_H = 600$ K, $T_L = 300$ K, $P_3 = 75$ kPa

The states as shown in figure 5.24

1: 600 K , 2: 600 K, 3: 75 kPa, 300 K 4: 300 K

Since this is a Carnot cycle and we know the temperatures the efficiency is

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{600} = 0.5$$

and the net work becomes

$$w_{NET} = \eta q_H = 0.5 \times 250$$
$$= \textbf{125 kJ/kg}$$

The heat rejected is

$$q_L = q_H - w_{NET} = 125 \text{ kJ/kg}$$

After heat rejection is state 4. From equation 5.9

$3 \rightarrow 4$ Eq.5.9 : $q_L = RT_L \ln (v_3/v_4)$

$v_3 = RT_3 / P_3 = 0.287$ kJ/kg-K $\times 300$ K $/ 75$ kPa $= 1.148$ m^3/kg

$v_4 = v_3 \exp(-q_L/RT_L) = 1.148$ m^3/kg $\exp(-125/0.287 \times 300)$

$= \textbf{0.2688 m}^3\textbf{/kg}$

$P_4 = RT_4 / v_4 = 0.287$ kJ/kg-K $\times 300$ K$/ 0.2688$ m^3/kg $= \textbf{320 kPa}$

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Review Problems

5.101

A 4L jug of milk at 25°C is placed in your refrigerator where it is cooled down to 5°C. The high temperature in the Carnot refrigeration cycle is 45°C, the low temperature is -5°C and the properties of milk are the same as for liquid water. Find the amount of energy that must be removed from the milk and the additional work needed to drive the refrigerator.

Solution:

C.V milk + out to the 5 °C refrigerator space

Energy Eq.: $m(u_2 − u_1) = {_1Q_2} − {_1W_2}$

Process : P = constant = 1 atm => $_1W_2 = Pm (v_2 - v_1)$

State 1: Table B.1.1, $v_1 \cong v_f = 0.001003$ m^3/kg, $h_1 \cong h_f = 104.87$ kJ/kg

$\qquad m_2 = m_1 = V_1/v_1 = 0.004 / 0.001003 = 3.988$ kg

State 2: Table B.1.1, $h_2 \cong h_f = 20.98$ kJ/kg

$\qquad {_1Q_2} = m(u_2 − u_1) + {_1W_2} = m(u_2 − u_1) + Pm (v_2 - v_1) = m(h_2 − h_1)$

$\qquad {_1Q_2} = 3.998 (20.98 - 104.87) = -3.988 \times 83.89 =$ **- 334.55 kJ**

C.V. Refrigeration cycle $T_L = -5$ °C ; $T_H = 45$ °C, assume Carnot

$\qquad$ Ideal : $\beta = Q_L / W = Q_L / (Q_H - Q_L) = T_L/ (T_H − T_L)$

$\qquad\qquad = 278.15 / [45 – (-5)] = 5.563$

$\qquad W = Q_L / \beta = 334.55$ kJ/ $5.563 =$ **60.14 kJ**

5 °C

AIR

MILK

Remark: If you calculate the work term $_1W_2$ you will find that it is very small, the volume does not change (liquid). The heat transfer could then have been done as $m(u_2 − u_1)$ without any change in the numbers.

5.102

Consider the combination of the two heat engines as in Fig. P5.4. How should the intermediate temperature be selected so the two heat engines have the same efficiency assuming Carnot cycle heat engines.

Heat engine 1: $\qquad \eta_{TH\,1} = 1 - \dfrac{T_M}{T_H}$

Heat engine 2: $\qquad \eta_{TH\,2} = 1 - \dfrac{T_L}{T_M}$

$$\eta_{TH\,1} = \eta_{TH\,2} \quad \Rightarrow \quad 1 - \frac{T_M}{T_H} = 1 - \frac{T_L}{T_M} \quad \Rightarrow \quad \frac{T_M}{T_H} = \frac{T_L}{T_M}$$

$$\Rightarrow \quad T_M = \sqrt{T_L T_H}$$

5.103

Consider a combination of a gas turbine power plant and a steam power plant as shown in Fig. P5.4. The gas turbine operates at higher temperatures (thus called a topping cycle) than the steam power plant (then called a bottom cycle). Assume both cycles have a thermal efficiency of 32%. What is the efficiency of the overall combination assuming Q_L in the gas turbine equals Q_H to the steam power plant?

Let the gas turbine be heat engine number 1 and the steam power plant the heat engine number 2. Then the overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \dot{Q}_M = \eta_2 (1 - \eta_1) \dot{Q}_H$$

so the final result is

$$\eta_{TH} = \eta_1 + \eta_2 (1 - \eta_1)$$
$$= 0.32 + 0.32(1 - 0.32) = \mathbf{0.538}$$

5.104

We wish to produce refrigeration at −30°C. A reservoir, shown in Fig. P5.104, is available at 200°C and the ambient temperature is 30°C. Thus, work can be done by a cyclic heat engine operating between the 200°C reservoir and the ambient. This work is used to drive the refrigerator. Determine the ratio of the heat transferred from the 200°C reservoir to the heat transferred from the −30°C reservoir, assuming all processes are reversible.

Solution:

Equate the work from the heat engine to the refrigerator.

$$\dot{W} = \dot{Q}_{H1}\left(\frac{T_H - T_0}{T_H}\right)$$

also

$$\dot{W} = \dot{Q}_{L2}\left(\frac{T_0 - T_L}{T_L}\right)$$

$$\dot{Q}_{H1} / \dot{Q}_{L2} = \left(\frac{T_o - T_L}{T_L}\right)\left(\frac{T_H}{T_H - T_o}\right) = \left(\frac{60}{243.2}\right)\left(\frac{473.2}{170}\right) = \textbf{0.687}$$

5.105

Redo the previous problem, assuming the actual devices both have a performance that is 60% of the theoretical maximum.

For the heat engine this means:

$$\dot{W} = 0.6 \, \dot{Q}_{H1} \left(\frac{T_H - T_0}{T_H} \right)$$

For the refrigerator it means:

$$\dot{W} = \dot{Q}_{L2} \left(\frac{T_0 - T_L}{T_L} \right) / 0.6$$

The ratio of the two heat transfers becomes

$$\dot{Q}_{H1} / \dot{Q}_{L2} = \left(\frac{T_0 - T_L}{T_L} \right) \times \frac{1}{0.6} \times \left(\frac{T_H}{T_H - T_0} \right) \times \frac{1}{0.6}$$

$$= \left(\frac{60}{243.2} \right) \left(\frac{473.2}{170} \right) \frac{1}{0.36} = \mathbf{1.91}$$

As the heat engine delivers less work and the refrigerator requires more work energy from the high T source must increase significantly.

5.106

A house should be heated by a heat pump, $\beta' = 2.2$, and maintained at 20^oC at all times. It is estimated that it loses 0.8 kW per degree the ambient is lower than the inside. Assume an outside temperature of -10^oC and find the needed power to drive the heat pump?

Solution : Ambient $T_L = -10^oC$

Heat pump : $\qquad \beta' = \dot{Q}_H/\dot{W}$

House : $\qquad\qquad \dot{Q}_H = \dot{Q}_{leak} = 0.8\,(\,T_H - T_L)$

$\qquad \dot{W} = \dot{Q}_H/\beta' = \dot{Q}_{leak}\,/\,\beta' = 0.8\,(\,T_H - T_L)\,/\,\beta'$

$\qquad\quad = 0.8[20 - (-10)]\,/2.2 = \textbf{10.91 kW}$

5.107

Give an estimate for the COP in the previous problem and the power needed to drive the heat pump when the outside temperature drops to -15°C.

Solution:

Minimum power if we assume a Carnot cycle

We assume the heat transfer coefficient stays the same

$$\dot{Q}_H = \dot{Q}_{leak} = 25 \text{ kW} = CA\,\Delta T = CA\,[20-(-10)] \quad \Rightarrow \quad CA = \frac{5}{6} \text{ kW/K}$$

$$\dot{Q}_{leak\ new} = CA\,\Delta T = \frac{5}{6}[20-(-15)] = 29.167 \text{ kW}$$

$$\beta' = \frac{\dot{Q}_H}{\dot{W}_{IN}} = \frac{T_H}{T_H - T_L} = \frac{293.15}{35} = 8.3757 \quad \Rightarrow \quad \dot{W}_{IN} = \frac{29.167}{8.3757} = \mathbf{3.48\ kW}$$

Comment. Leak heat transfer increases and COP is lower when T outside drops.

5.108

A farmer runs a heat pump with a motor of 2 kW. It should keep a chicken hatchery at 30°C which loses energy at a rate of 0.5 kW per degree difference to the colder ambient. The heat pump has a coefficient of performance that is 50% of a Carnot heat pump. What is the minimum ambient temperature for which the heat pump is sufficient?

Solution:

C.V. Hatchery, steady state.

To have steady state at 30°C for the hatchery

Energy Eq.: $\dot{Q}_H = \dot{Q}_{Loss} = \beta_{AC}\dot{W}$

Process Eq.: $\dot{Q}_{Loss} = 0.5\,(T_H - T_{amb})$; $\beta_{AC} = \frac{1}{2}\,\beta_{CARNOT}$

COP for the reference Carnot heat pump

$$\beta_{CARNOT} = \frac{\dot{Q}_H}{\dot{W}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} = \frac{T_H}{T_H - T_L} = \frac{T_H}{T_H - T_{amb}}$$

Substitute the process equations and this β_{CARNOT} into the energy Eq.

$$0.5 \text{ kW/K } (T_H - T_{amb}) = \frac{1}{2}\,\frac{T_H}{T_H - T_{amb}}\,\dot{W}$$

$$(T_H - T_{amb})^2 = \frac{1}{2}\,T_H\dot{W}/0.5 \text{ kW/K} = T_H\dot{W} = (273 + 30) \text{ K} \times 2 \text{ K} = 606 \text{ K}^2$$

$$T_H - T_{amb} = 24.62 \text{ K}$$

$$T_{amb} = 30 - 24.62 = \mathbf{5.38°C}$$

Comment: That of course is not a very low temperature and the size of the system is not adequate for most locations.

5.109

An air-conditioner with a power input of 1.2 kW is working as a refrigerator (β = 3) or as a heat pump (β' = 4). It maintains an office at 20°C year round which exchanges 0.5 kW per degree temperature difference with the atmosphere. Find the maximum and minimum outside temperature for which this unit is sufficient.

Solution:

Analyze the unit in heat pump mode

Replacement heat transfer equals the loss: $\dot{Q} = 0.5$ kW/K (T_H - T_{amb})

$$\dot{W} = \frac{\dot{Q}_H}{\beta'} = 0.5 \text{ kW/K } \frac{T_H - T_{amb}}{4}$$

$$T_H - T_{amb} = 4\frac{\dot{W}}{0.5} = 9.6 \text{ K}$$

Heat pump mode: Minimum T_{amb} = 20 - 9.6 = **10.4 °C**

The unit as a refrigerator must cool with rate: $\dot{Q} = 0.5$ (T_{amb} - T_{house})

$$\dot{W} = \frac{\dot{Q}_L}{\beta}\textbf{Error! Bookmark not defined.} = 0.5 \text{ kW/K } (T_{amb} - T_{house}) / 3$$

$$T_{amb} - T_{house} = 3\frac{\dot{W}}{0.5} = 7.2 \text{ K}$$

Refrigerator mode: Maximum T_{amb} = 20 + 7.2 = **27.2 °C**

5.110

An air-conditioner on a hot summer day removes 8 kW of energy from a house at 21°C and pushes energy to the outside which is at at 31°C. The house has 15 000 kg mass with an average specific heat of 0.95 kJ/kgK. In order to do this the cold side of the air-conditioner is at 5°C and the hot side is at 40°C. The air-conditioner (refrigerator) has a COP that is 60% of a corresponding Carnot refrigerator. Find the actual COP of the air-conditioner and the power required to run it.

A steady state refrigerator definition of COP

$$\text{COP} = \beta_{REF} = \dot{Q}_L \, / \, \dot{W} = \dot{Q}_L \, / \, (\dot{Q}_H - \dot{Q}_L) = 0.6 \, \beta_{Carnot}$$

Carnot: $\qquad \beta_{Carnot} = \dfrac{T_L}{T_H - T_L} = \dfrac{5 + 273.15}{40 - 5} = 7.95$

$$\beta_{REF} = \ 0.6 \times 7.95 = \textbf{4.77}$$

$\Rightarrow \qquad \dot{W} = \dot{Q}_L / \, \beta_{REF} = 8 \text{ kW} / \, 4.77 = \textbf{1.68 kW}$

Any heat transfer must go from a higher to a lower T domain.

5.111

The air-conditioner in the previous problem is turned off. How quickly does the house heat up in degrees per second (°C/s)?

Once the A/C unit is turned off we do not cool the house so heat leaks in from the atm. at a rate of 8 kW (that is what we had to remove to keep steady state).

Energy Eq.: $\qquad \dot{E}_{CV} = \dot{Q}_{leak} = 8 \text{ kW} = m_{house} \, C_P \dfrac{dT}{dt}$

$$\dfrac{dT}{dt} = \dot{Q}_{leak} \,/\, m_{house} \, C_P$$

$$= \dfrac{8 \text{ kW}}{15\ 000 \times 0.95 \text{ kJ/K}} = \mathbf{0.56 \times 10^{-3} \text{ K/s}}$$

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5.112

Air in a rigid 1 m^3 box is at 300 K, 200 kPa. It is heated to 600 K by heat transfer from a reversible heat pump that receives energy from the ambient at 300 K besides the work input. Use constant specific heat at 300 K. Since the coefficient of performance changes write $dQ = m_{air} C_v dT$ and find dW. Integrate dW with temperature to find the required heat pump work.

Solution:

COP:
$$\beta' = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} \cong \frac{T_H}{T_H - T_L}$$

$$m_{air} = P_1 V_1 / RT_1 = 200 \times 1 / 0.287 \times 300 = 2.322 \text{ kg}$$

$$dQ_H = m_{air} C_v dT_H = \beta' dW \cong \frac{T_H}{T_H - T_L} dW$$

$$\Rightarrow \quad dW = m_{air} C_v [\frac{T_H}{T_H - T_L}] dT_H$$

$$_1W_2 = \int m_{air} C_v (1 - \frac{T_L}{T}) dT = m_{air} C_v \int (1 - \frac{T_L}{T}) dT$$

$$= m_{air} C_v [T_2 - T_1 - T_L \ln \frac{T_2}{T_1}]$$

$$= 2.322 \text{ kg} \times 0.717 \text{ kJ/kg-K} [600 - 300 - 300 \ln \frac{600}{300}] \text{ K} = \textbf{153.1 kJ}$$

5.113

A Carnot heat engine, shown in Fig. P5.113, receives energy from a reservoir at T_{res} through a heat exchanger where the heat transferred is proportional to the temperature difference as $\dot{Q}_H = K(T_{res} - T_H)$. It rejects heat at a given low temperature T_L. To design the heat engine for maximum work output show that the high temperature, T_H, in the cycle should be selected as $T_H = \sqrt{T_{res} T_L}$

Solution:

$$W = \eta_{TH} Q_H = \frac{T_H - T_L}{T_H} \times K(T_{res} - T_H) ; \quad \text{maximize } W(T_H) \Rightarrow \frac{\delta W}{\delta T_H} = 0$$

$$\frac{\delta W}{\delta T_H} = K(T_{res} - T_H)T_L T_H^{-2} - K(1 - T_L/T_H) = 0$$

$$\Rightarrow \mathbf{T_H = \sqrt{T_{res} T_L}}$$

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5.114

A combination of a heat engine driving a heat pump (see Fig. P5.114) takes waste energy at 50°C as a source Q_{w1} to the heat engine rejecting heat at 30°C. The remainder Q_{w2} goes into the heat pump that delivers a Q_H at 150°C. If the total waste energy is 5 MW find the rate of energy delivered at the high temperature.

Solution:

Waste supply: $\dot{Q}_{w1} + \dot{Q}_{w2} = 5$ MW

Heat Engine:

$\dot{W} = \eta\,\dot{Q}_{w1} = (1 - T_{L1} / T_{H1})\,\dot{Q}_{w1}$

Heat pump:

$\dot{W} = \dot{Q}_H / \beta_{HP} = \dot{Q}_{W2} / \beta'$

$= \dot{Q}_{w2} / [T_{H1} / (T_H - T_{H1})]$

Equate the two work terms:

$$(1 - T_{L1} / T_{H1})\,\dot{Q}_{w1} = \dot{Q}_{w2} \times (T_H - T_{H1}) / T_{H1}$$

Substitute $\dot{Q}_{w1} = 5$ MW $- \dot{Q}_{w2}$

$$(1 - 303.15/323.15)(5 - \dot{Q}_{w2}) = \dot{Q}_{w2} \times (150 - 50) / 323.15$$

$20\,(5 - \dot{Q}_{w2}) = \dot{Q}_{w2} \times 100$ => $\dot{Q}_{w2} = 0.8333$ MW

$\dot{Q}_{w1} = 5 - 0.8333 = 4.1667$ MW

$\dot{W} = \eta\,\dot{Q}_{w1} = 0.06189 \times 4.1667 = 0.258$ MW

$\dot{Q}_H = \dot{Q}_{w2} + \dot{W} = \textbf{1.09 MW}$

(For the heat pump $\beta' = 423.15 / 100 = 4.23$)

5.115

A furnace, shown in Fig. P5.115, can deliver heat, Q_{H1} at T_{H1} and it is proposed to use this to drive a heat engine with a rejection at T_{atm} instead of direct room heating. The heat engine drives a heat pump that delivers Q_{H2} at T_{room} using the atmosphere as the cold reservoir. Find the ratio Q_{H2}/Q_{H1} as a function of the temperatures. Is this a better set-up than direct room heating from the furnace?

Solution:

C.V.: Heat Eng.: $\dot{W}_{HE} = \eta \dot{Q}_{H1}$ where $\eta = 1 - T_{atm}/T_{H1}$

C.V.: Heat Pump: $\dot{W}_{HP} = \dot{Q}_{H2}/\beta'$ where $\beta' = T_{rm}/(T_{rm} - T_{atm})$

Work from heat engine goes into heat pump so we have

$$\dot{Q}_{H2} = \beta' \dot{W}_{HP} = \beta' \eta \dot{Q}_{H1}$$

and we may substitute T's for β', η. If furnace is used directly $\dot{Q}_{H2} = \dot{Q}_{H1}$, so if $\beta'\eta > 1$ this proposed setup is better. Is it? For $T_{H1} > T_{atm}$ formula shows that it is good for Carnot cycles. In actual devices it depends whether $\beta'\eta > 1$ is obtained.

5.116

Consider the rock bed thermal storage in Problem 5.60. Use the specific heat so you can write dQ_H in terms of dT_{rock} and find the expression for dW out of the heat engine. Integrate this expression over temperature and find the total heat engine work output.

Solution:

The rock provides the heat Q_H

$$dQ_H = -dU_{rock} = -mC \ dT_{rock}$$

$$dW = \eta dQ_H = -(1 - \frac{T_o}{T_{rock}}) \ mC \ dT_{rock}$$

$$m = \rho V = 2750 \ kg/m^3 \times 2 \ m^3 = 5500 \ kg$$

$${}_1W_2 = \int -(1 - \frac{T_o}{T_{rock}}) \ mC \ dT_{rock} = -mC \ [T_2 - T_1 - T_o \ln \frac{T_2}{T_1}]$$

$$= -5500 \ kg \times 0.89 \ kJ/kg\text{-}K \ [\ 290 - 400 - 290 \ln \frac{290}{400} \ ] \ K$$

$$= \textbf{81 945 kJ}$$

5.117

Consider a Carnot cycle heat engine operating in outer space. Heat can be rejected from this engine only by thermal radiation, which is proportional to the radiator area and the fourth power of absolute temperature, $\dot{Q}_{rad} \sim KAT^4$. Show that for a given engine work output and given T_H, the radiator area will be minimum when the ratio $T_L/T_H = 3/4$.

Solution:

$$\dot{W}_{NET} = \dot{Q}_H \left(\frac{T_H - T_L}{T_H} \right) = \dot{Q}_L \left(\frac{T_H - T_L}{T_L} \right); \qquad \text{also} \quad \dot{Q}_L = KAT_L^4$$

$$\frac{\dot{W}_{NET}}{KT_H^4} = \frac{AT_L^4}{T_H^4} \left(\frac{T_H}{T_L} - 1 \right) = A \left[\left(\frac{T_L}{T_H} \right)^3 - \left(\frac{T_L}{T_H} \right)^4 \right] = \text{const}$$

Differentiating,

$$dA \left[\left(\frac{T_L}{T_H} \right)^3 - \left(\frac{T_L}{T_H} \right)^4 \right] + A \left[3 \left(\frac{T_L}{T_H} \right)^2 - 4 \left(\frac{T_L}{T_H} \right)^3 \right] d \left(\frac{T_L}{T_H} \right) = 0$$

$$\frac{dA}{d(T_L/T_H)} = - A \left[3 \left(\frac{T_L}{T_H} \right)^2 - 4 \left(\frac{T_L}{T_H} \right)^3 \right] \Big/ \left[\left(\frac{T_L}{T_H} \right)^3 - \left(\frac{T_L}{T_H} \right)^4 \right] = 0$$

$$\frac{T_L}{T_H} = \frac{3}{4} \quad \text{for min. A}$$

Check that it is minimum and not maximum with the 2nd derivative > 0.

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5.118

A Carnot heat engine operating between a high T_H and low T_L energy reservoirs has an efficiency given by the temperatures. Compare this to two combined heat engines one operating between T_H and an intermediate temperature T_M giving out work W_A and the other operating between T_M and T_L giving out W_B. The combination must have the same efficiency as the single heat engine so the heat transfer ratio $Q_H/Q_L = \psi(T_H,T_L) = [Q_H/Q_M] [Q_M/Q_L]$. The last two heat transfer ratios can be expressed by the same function $\psi()$ involving also the temperature T_M. Use this to show a condition the function $\psi()$ must satisfy.

The overall heat engine is a Carnot heat engine so

$$\dot{Q}_H / \dot{Q}_L = \frac{T_H}{T_L} = \psi(T_H,T_L)$$

The individual heat engines

$$\dot{Q}_H / \dot{Q}_M = \psi(T_H,T_M) \quad \text{and} \quad \dot{Q}_M / \dot{Q}_L = \psi(T_M,T_L)$$

Since an identity is

$$\dot{Q}_H / \dot{Q}_L = [\dot{Q}_H / \dot{Q}_M] \ [\dot{Q}_M / \dot{Q}_L] = \psi(T_H,T_L)$$

it follows that we have

$$\psi(T_H,T_L) = \psi(T_H,T_M) \times \ \psi(T_M,T_L)$$

Notice here that the product of the two functions must cancel the intermediate temperature T_M, this shows a condition the function $\psi()$ must satisfy. The Kelvin and Rankine temperature scales are determined by the choice of the function

$$\psi(T_H,T_L) = \ T_H / T_L = \dot{Q}_H / \dot{Q}_L$$

satisfying the above restriction.

5.119

On a cold (-10°C) winter day a heat pump provides 20 kW to heat a house maintained at 20°C and it has a COP$_{HP}$ of 4 using the maximum power available.

The next day a winter storm brings the outside to -15°C, assuming that the COP$_{HP}$ changes by the same percentage as a Carnot unit and that the house loses heat to the outside air. How cold is the house then?

If we look at the heat loss for the house we have

$$\dot{Q}_{loss} = 20 \text{ kW} = \text{ CA } \Delta T \qquad \Rightarrow \qquad CA = \frac{20 \text{ kW}}{20 \text{ - (-10) K}} = 0.667 \text{ kW/K}$$

$$\dot{Q}_{loss} = \dot{Q}_H = \text{COP } \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_{loss} / \text{COP} = \frac{20 \text{ kW}}{4} = 5 \text{ kW}$$

$$\text{At } = \text{-10}^o\text{C:} \qquad \beta'_{\text{Carnot 1}} = \frac{T_H}{T_H - T_{L1}} = \frac{293.15}{20 - (-10)} = 9.772$$

With a changed COP we get

$$\text{COP}_{HP} = 4 \, \beta'_{\text{Carnot 2}} / \beta'_{\text{Carnot 1}} = (4/9.772) \, \beta'_{\text{Carnot 2}}$$

At the new unknown temperature of the house, T_H, we have

$$\beta'_{\text{Carnot 2}} = \frac{T_H}{T_H - T_{L2}} ; \qquad \dot{Q}_{loss} = \text{CA } (T_H - T_{L2})$$

The energy equation for the house becomes $\dot{Q}_{loss} = \dot{Q}_H$ and substitution gives

$$0.667 \text{ kW/K } (T_H - T_{L2}) = \text{COP}_{HP} \, \dot{W} = (4/9.772) \frac{T_H}{T_H - T_{L2}} \, 5 \text{ kW}$$

or $\qquad (T_H - T_{L2})^2 = \dfrac{4 \times 5 \text{ kW}}{9.772 \times 0.667 \text{ kW/K}} \, T_H = 3.07 \text{ K} \times T_H$

Solve with $x = T_H - T_{L2}$, $x^2 - 3.07 \, x - 3.07(273.15 - 15) = 0$,

$$x = 3.07/2 + [(3.07/2)^2 + 792.52]^{1/2} = 29.729$$

$$T_H = x + T_{L2} = 29.729 - 15 = \textbf{14.7}^\textbf{o}\textbf{C}$$

5.120

A 10-m^3 tank of air at 500 kPa, 600 K acts as the high-temperature reservoir for a Carnot heat engine that rejects heat at 300 K. A temperature difference of 25°C between the air tank and the Carnot cycle high temperature is needed to transfer the heat. The heat engine runs until the air temperature has dropped to 400 K and then stops. Assume constant specific heat capacities for air and find how much work is given out by the heat engine.

Solution:

AIR

Q_H W

HE

Q_L

300 K

$$T_H = T_{air} - 25°C, \quad T_L = 300 \text{ K}$$

$$m_{air} = \frac{P_1 V}{RT_1} = \frac{500 \times 10}{0.287 \times 600} = 29.04 \text{ kg}$$

$$dW = \eta dQ_H = \left(1 - \frac{T_L}{T_{air} - 25}\right) dQ_H$$

$$dQ_H = -m_{air} du = -m_{air} C_v dT_{air}$$

$$W = \int dW = -m_{air} C_v \int \left[1 - \frac{T_L}{T_a - 25}\right] dT_a$$

$$= -m_{air} C_v \left[T_{a2} - T_{a1} - T_L \ln \frac{T_{a2} - 25}{T_{a1} - 25}\right]$$

$$= -29.04 \text{ kg} \times 0.717 \text{ kJ/kg-K} \times \left[400 - 600 - 300 \ln \frac{375}{575}\right] \text{K}$$

$$= \textbf{1494.3 kJ}$$

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Fundamentals of Thermodynamics

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CHAPTER 5

Heat Engines and Refrigerators

5.121E

A window mounted air-conditioner removes 3.5 Btu from the inside of a home using 1.75 Btu work input. How much energy is released outside and what is its coefficient of performance?

C.V. Refrigerator. The energy Q_H goes into the kitchen air.

Energy Eq.: $\qquad Q_H = W + Q_L = 1.75 + 3.5 = \textbf{5.25 btu}$

COP: $\qquad\qquad \beta = \dfrac{Q_L}{W} = 3.5 / 1.75 = \textbf{2}$

5.122E

A lawnmower tractor engine produces 18 hp using 40 Btu/s of heat transfer from burning fuel. Find the thermal efficiency and the rate of heat transfer rejected to the ambient?

Conversion Table A.1:   18 hp = 18 × (2544.4/3600) Btu/s = 12.722 Btu/s

Efficiency: $\eta_{TH} = \dot{W}_{out}/\dot{Q}_H = \dfrac{12.722}{40} =$ **0.318**

Energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{out}$ = 40 – 12.72 = **27.3 Btu/s**

$\dot{Q}_H$ $\dot{Q}_L$

$\dot{W}_{out}$

5.123E

Calculate the thermal efficiency of the steam power plant cycle described in Problem 4.198.

Solution:

From solution to problems 4.198, 199

$$\dot{W}_{NET} = 33\ 000 - 400 = 32\ 600 \text{ hp} = 8.3 \times 10^7 \text{ Btu/h}$$

$$\dot{Q}_{H,tot} = \dot{Q}_{econ} + \dot{Q}_{gen}$$

$$= 4.75 \times 10^7 + 2.291 \times 10^8 = 2.766 \times 10^8 \text{ Btu/h};$$

$$\eta = \frac{\dot{W}}{\dot{Q}_H} = \frac{8.3 \times 10^7}{2.766 \times 10^8} = \mathbf{0.30}$$

5.124E

A large coal fired power plant has an efficiency of 45% and produces net 1500 MW of electricity. Coal releases 12 500 Btu/lbm as it burns so how much coal is used per hour?

From the definition of the thermal efficiency and the energy release by the combustion called heating value HV we get

$$\dot{W} = \eta \cdot \dot{Q}_H = \eta \cdot \dot{m} \cdot HV$$

then

$$\dot{m} = \frac{\dot{W}}{\eta \times HV} = \frac{1500 \text{ MW}}{0.45 \times 12\,500 \text{ btu/lbm}} = \frac{1500 \times (1000/1.055) \text{ Btu/s}}{0.45 \times 12\,500 \text{ Btu/lbm}}$$
$$= 252.765 \text{ lbm/s} = \mathbf{909\,950 \text{ lbm/h}}$$

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5.125E

An industrial machine is being cooled by 0.8 lbm/s water at 60 F which is chilled from 95 F by a refrigeration unit with a COP of 3. Find the rate of cooling required and the power input to the unit.

Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) = \dot{m}\, C_P\, (T_1 - T_2)$$

$$= 0.8 \text{ lbm/s} \times 1 \text{ Btu/lbm-R} \times (95 - 60) \text{ R}$$

$$= 28 \text{ Btu/s}$$

$$\beta = COP = \dot{Q}_L / \dot{W} \quad \Rightarrow$$

$$\dot{W} = \dot{Q}_L / \beta = 28 / 3 = \textbf{9.33 Btu/s}$$

Comment: An outside cooling tower is often used for this, see Chapter 11.

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5.126E

A water cooler for drinking water should cool 10 gal/h water from 65 F to 50 F using a small refrigeration unit with a COP of 2.5. Find the rate of cooling required and the power input to the unit.

The mass flow rate is

$$\dot{m} = \rho \dot{V} = \frac{10 \times 231 /12^3}{0.01603} \frac{1}{3600} \text{ lbm/s} = 0.0232 \text{ lbm/s}$$

Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) = \dot{m} \, C_P \, (T_1 - T_2)$$

$$= 0.0232 \times 1.0 \times (65 - 50) = 0.348 \text{ Btu/s}$$

$$\beta = COP = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 0.348 \text{ (Btu/s)} / 2.5 = \textbf{0.139 Btu/s}$$

Comment: The unit does not operate continuously.

5.127E

A window air-conditioner unit is placed on a laboratory bench and tested in cooling mode using 0.75 Btu/s of electric power with a COP of 1.75. What is the cooling power capacity and what is the net effect on the laboratory?

Definition of COP: $\beta = \dot{Q}_L / \dot{W}$

Cooling capacity: $\dot{Q}_L = \beta \, \dot{W} = 1.75 \times 0.75 = \mathbf{1.313 \ Btu/s}$

For steady state operation the $\dot{Q}_L$ comes from the laboratory and $\dot{Q}_H$ goes to the laboratory giving a net to the lab of $\dot{W} = \dot{Q}_H - \dot{Q}_L = 0.75$ Btu/s, that is heating it.

5.128E

A farmer runs a heat pump with a 2 kW motor. It should keep a chicken hatchery at 90 F, which loses energy at a rate of 10 Btu/s to the colder ambient T_{amb}. What is the minimum coefficient of performance that will be acceptable for the heat pump?

Power input: $\dot{W} = 2 \text{ kW} = 2 \times 2544.4 / 3600 = 1.414 \text{ Btu/s}$

Energy Eq. for hatchery: $\dot{Q}_H = \dot{Q}_{Loss} = 10 \text{ Btu/s}$

Definition of COP: $\beta = COP = \dfrac{\dot{Q}_H}{\dot{W}} = \dfrac{10}{1.414} = \mathbf{7.07}$

5.129E

R-410A enters the evaporator (the cold heat exchanger) in an A/C unit at 0 F, x = 28% and leaves at 0 F, x = 1. The COP of the refrigerator is 1.5 and the mass flow rate is 0.006 lbm/s. Find the net work input to the cycle.

Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_2 - h_1) = \dot{m}[h_g - (h_f + x_1\, h_{fg})]$$

$$= \dot{m}[h_{fg} - x_1\, h_{fg}] = \dot{m}\,(1 - x_1)h_{fg}$$

$$= 0.006 \text{ lbm/s} \times 0.72 \times 103.76 \text{ Btu/lbm} = 0.448 \text{ Btu/s}$$

$$\beta = COP = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 0.448 / 1.5 = \textbf{0.3 Btu/s}$$

5.130E

A large stationary diesel engine produces 2 000 hp with a thermal efficiency of 40%. The exhaust gas, which we assume is air, flows out at 1400 R and the intake is 520 R. How large a mass flow rate is that if that accounts for half the $\dot{Q}_L$? Can the exhaust flow energy be used?

Power   2 000 hp = 2 000 × 2544.4 / 3600 = 1413.6 Btu/s

Heat engine: $\dot{Q}_H = \dot{W}_{out}/\eta_{TH} = \dfrac{14\ 136}{0.4} = 3534$ Btu/s

Energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 3534 - 1413.6 = 2120.4$ Btu/s

Exhaust flow: $\frac{1}{2}\dot{Q}_L = \dot{m}_{air}(h_{1400} - h_{520})$

$$\dot{m}_{air} = \frac{1}{2}\frac{\dot{Q}_L}{h_{1400} - h_{520}} = \frac{1}{2}\frac{2120.4}{343.02 - 124.38} = \textbf{4.85 lbm/s}$$

The flow of hot gases can be used to heat a building or it can be used to heat water in a steam power plant since it operates at lower temperatures.

5.131E

Calculate the amount of work input a refrigerator needs to make ice cubes out of a tray of 0.5 lbm liquid water at 50 F. Assume the refrigerator has $\beta = 3.5$ and a motor-compressor of 750 W. How much time does it take if this is the only cooling load?

Solution:

C.V. Water in tray. We neglect tray mass.

Energy Eq.: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process : $P = constant = P_o$

$${_1}W_2 = \int P\, dV = P_o m(v_2 - v_1)$$

$${_1}Q_2 = m(u_2 - u_1) + {_1}W_2 = m(h_2 - h_1)$$

Tbl. F.7.1 : $h_1 = 18.05$ Btu/lbm, Tbl. F.7.4 : $h_2 = -143.34$ Btu/lbm

$${_1}Q_2 = 0.5(-143.34 - 18.05) = -80.695 \text{ Btu}$$

Consider now refrigerator

$$\beta = Q_L/W$$

$$W = Q_L/\beta = -{_1}Q_2/\beta = 80.695/3.5 = \textbf{23.06 Btu}$$

For the motor to transfer that amount of energy the time is found as

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t$$

$$\Delta t = W/\dot{W} = (23.06 \times 1055)/750 = \textbf{32.4 s}$$

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Comment: We neglected a baseload of the refrigerator so not all the 750 W are available to make ice, also our coefficient of performance is very optimistic and finally the heat transfer is a transient process. All this means that it will take much more time to make ice-cubes.

Borgnakke and Sonntag

Carnot Cycles and Absolute T

7.132E

Calculate the thermal efficiency of a Carnot-cycle heat engine operating between reservoirs at 920 F and 110 F. Compare the result with that of Problem 5.123.

Solution:

$$T_H = 920 \text{ F}, \quad T_L = 110 \text{ F}$$

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{110 + 459.67}{920 + 459.67} = \mathbf{0.587} \quad \text{(about twice 5.123: 0.3)}$$

5.133E

A steam power plant has 1200 F in the boiler, 630 Btu/s work out of turbine, 900 Btu/s is taken out at 100 F in the condenser and the pump work is 30 Btu/s. Find the plant thermal efficiency. Assume the same pump work and heat transfer to the boiler, how much is the turbine power if the plant is running in a Carnot cycle?

Solution:

CV. Total plant:

Energy Eq.:

$$\dot{Q}_H + \dot{W}_{P,in} = \dot{W}_T + \dot{Q}_L$$

$$\dot{Q}_H = 630 + 900 - 30 = 1500 \text{ Btu/s}$$

$$\eta_{TH} = \frac{\dot{W}_T - \dot{W}_{P,in}}{\dot{Q}_H}$$ **Error! Bookmark not defined.** $$= \frac{600}{1500} = \textbf{0.40}$$

$$\eta_{carnot} = \dot{W}_{net} / \dot{Q}_H = 1 - T_L/T_H = 1 - \frac{100 + 459.67}{1200 + 459.67} = 0.663$$

$$\dot{W}_T - \dot{W}_{P,in} = \eta_{carnot}\dot{Q}_H = 0.663 \times 1500 \text{ Btu/s} = 995 \text{ Btu/s}$$

$$\Rightarrow \quad \dot{W}_T = 995 + 30 = \textbf{1025} \frac{\textbf{Btu}}{\textbf{s}}$$

5.134E

A large heat pump should upgrade 4000 Btu/s of heat at 175 F to be delivered as heat at 280 F. What is the minimum amount of work (power) input that will drive this?

For the minimum work we assume a Carnot heat pump and $\dot{Q}_L$ = 4000 Btu/s.

$$\beta_{HP} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{T_H}{T_H - T_L} = \frac{459.7 + 280}{280 - 175} = 7.04$$

$$\beta_{REF} = \beta_{HP} - 1 \; = \frac{\dot{Q}_L}{\dot{W}_{in}} = 6.04$$

Now we can solve for the work

$$\dot{W}_{in} = \dot{Q}_L / \beta_{REF} = 4000/6.04 = \textbf{662 Btu/s}$$

This is a domestic or small office building size A/C unit, much smaller than the 4000 Btu/s in this problem.

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5.135E

A car engine burns 10 lbm of fuel (equivalent to addition of Q_H) at 2600 R and rejects energy to the radiator and the exhaust at an average temperature of 1300 R. If the fuel provides 17 200 Btu/lbm what is the maximum amount of work the engine can provide?

Solution:

A heat engine $\quad Q_H = m\, q_{fuel} = 10 \times 17200 = 170\ 200$ Btu

Assume a Carnot efficiency (maximum theoretical work)

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{1300}{2600} = 0.5$$

$$W = \eta\, Q_H = 0.5 \times 170\ 200 = \mathbf{85\ 100\ Btu}$$

Air intake filter   Fuel line

Fan

Shaft power

cb

Exhaust flow

5.136E

Consider the combination of a heat engine and a heat pump as given in problem 5.41 with a low temperature of 720 R. What should the high temperature be so the heat engine is reversible? For that temperature what is the COP for a reversible heat pump?

For all three cases of the heat engine the ratio between the heat transfers and the work term is the same as:

$$\dot{Q}_H : \dot{Q}_L : \dot{W} \quad = \quad 6{:}4{:}2 \ = \ 3{:}2{:}1$$

For a reversible heat engine we must have the heat transfer ratio equal to the temperature ratio so

$$\frac{\dot{Q}_H}{\dot{Q}_L} = \frac{T_H}{T_L} = \frac{3}{2} = \frac{T_H}{720 \text{ R}} \quad => \quad T_H = (3/2)\,720 \text{ R} = \textbf{1080 R}$$

The COP is

$$\text{COP}_{HP} = \dot{Q}_H / \dot{W} = \frac{3}{1} = \textbf{3} \quad (= \frac{T_H}{T_H - T_L} = \frac{1080}{1080 - 720})$$

5.137E

An air-conditioner provides 1 lbm/s of air at 60 F cooled from outside atmospheric air at 95 F. Estimate the amount of power needed to operate the air-conditioner. Clearly state all assumptions made.

Solution:

Consider the cooling of air which needs a heat transfer as

$$\dot{Q}_{air} = \dot{m}\,\Delta h \cong \dot{m}\,C_p\,\Delta T = 1\ \text{lbm/s} \times 0.24\ \text{Btu/lbm-R} \times (95 - 60)\ R$$

$$= 8.4\ \text{Btu/s}$$

Assume Carnot cycle refrigerator

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) \cong \frac{T_L}{T_H - T_L} = \frac{60 + 459.67}{95 - 60} = 14.8$$

$$\dot{W} = \dot{Q}_L / \beta = \frac{8.4}{14.8} = \textbf{0.57 Btu/s}$$

This estimate is the theoretical maximum performance. To do the required heat transfer $T_L \cong 40\ F$ and $T_H = 110\ F$ are more likely; secondly

$$\beta < \beta_{carnot}$$

5.138E

We propose to heat a house in the winter with a heat pump. The house is to be maintained at 68 F at all times. When the ambient temperature outside drops to 15 F, the rate at which heat is lost from the house is estimated to be 80 000 Btu/h. What is the minimum electrical power required to drive the heat pump?

Solution:

Minimum power if we
assume a Carnot cycle
$\dot{Q}_H = \dot{Q}_{leak} = 80\ 000$ Btu/h

$$\beta' = \frac{\dot{Q}_H}{\dot{W}_{IN}} = \frac{T_H}{T_H - T_L} = \frac{527.7}{68 - 15} = 9.957$$

$$\Rightarrow \dot{W}_{IN} = 80\ 000\ /\ 9.957 = 8035\ \text{Btu/h} = \textbf{2.355 kW}$$

5.139E

Consider the setup with two stacked (temperature wise) heat engines as in Fig. P5.4. Let T_H = 1500 R, T_M = 1000 R and T_L = 650 R. Find the two heat engine efficiencies and the combined overall efficiency assuming Carnot cycles.

The individual efficiencies

$$\eta_1 = 1 - \frac{T_M}{T_H} = 1 - \frac{1000}{1500} = \mathbf{0.333}$$

$$\eta_2 = 1 - \frac{T_L}{T_M} = 1 - \frac{650}{1000} = \mathbf{0.35}$$

The overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \dot{Q}_M = \eta_2 (1 - \eta_1) \dot{Q}_H$$

so the final result is

$$\eta_{TH} = \eta_1 + \eta_2 (1 - \eta_1) = 0.333 + 0.35 (1 - 0.333) = \mathbf{0.566}$$

Comment: It matches a single heat engine $\eta_{TH} = 1 - \dfrac{T_L}{T_H} = 1 - \dfrac{650}{1500} = 0.567$

5.140E

A thermal storage is made with a rock (granite) bed of 70 ft^3 which is heated to 720 R using solar energy. A heat engine receives a Q_H from the bed and rejects heat to the ambient at 520 R. The rock bed therefore cools down and as it reaches 520 R the process stops. Find the energy the rock bed can give out. What is the heat engine efficiency at the beginning of the process and what is it at the end of the process?

Solution:

Assume the whole setup is reversible and that the heat engine operates in a Carnot cycle. The total change in the energy of the rock bed is

$u_2 - u_1 = q = C \, \Delta T = 0.21$ Btu/lbm-R (720 - 520) R = 42 Btu/lbm

$m = \rho V = 172$ lbm/ft$^3 \times 70$ ft$^3 = 12040$ lbm;

$Q = mq = 12040$ lbm $\times 42$ Btu/lbm = **505 680 Btu**

To get the efficiency assume a Carnot cycle device

$\eta = 1 - T_o / T_H = 1 - 520/720 = \mathbf{0.28}$ at the beginning of process

$\eta = 1 - T_o / T_H = 1 - 520/520 = \mathbf{0}$ at the end of process

5.141E

A heat engine has a solar collector receiving 600 Btu/h per square foot inside which a transfer media is heated to 800 R. The collected energy powers a heat engine which rejects heat at 100 F. If the heat engine should deliver 8500 Btu/h what is the minimum size (area) solar collector?

Solution:

$$T_H = 800 \text{ R} \qquad T_L = 100 + 459.67 = 560 \text{ R}$$

$$\eta_{HE} = 1 - \frac{T_L}{T_H} = 1 - \frac{560}{800} = 0.30$$

$$\dot{W} = \eta \, \dot{Q}_H \quad \Rightarrow \quad \dot{Q}_H = \frac{\dot{W}}{\eta} = \frac{8500}{0.30} = 28\,333 \text{ Btu/h}$$

$$\dot{Q}_H = 600 \text{ A} \quad \Rightarrow \quad A = \frac{\dot{Q}_H}{600} = \textbf{47 ft}^2$$

5.142E

Six-hundred pound-mass per hour of water runs through a heat exchanger, entering as saturated liquid at 250 F and leaving as saturated vapor. The heat is supplied by a Carnot heat pump operating from a low-temperature reservoir at 60 F with a COP of half of the similar carnot unit. Find the rate of work into the heat pump.

Solution:

C.V. Heat exchanger

$$\dot{m}_1 = \dot{m}_2 ; \qquad \dot{m}_1 h_1 + \dot{Q}_H = \dot{m}_1 h_2$$

Table F.7.1: $h_1 = 218.58$ Btu/lbm

$h_2 = 1164.19$ Btu/lbm

$$\dot{Q}_H = \frac{600}{3600}(1164.19 - 218.58) = 157.6 \text{ Btu/s}$$

For the Carnot heat pump, $T_H = 250 \text{ F} = 710 \text{ R}$.

$$\beta = \dot{Q}_H/\dot{W} = \frac{T_H}{T_H - T_L} = \frac{710}{190} = 3.737$$

$$\beta_{ac} = 3.737/2 = 1.87$$

$$\dot{W} = \dot{Q}_H/\beta_{ac} = 157.6/1.87 = \textbf{84.3 Btu/s}$$

5.143E

A power plant with a thermal efficiency of 40% is located on a river similar to Fig. P5.61. With a total river mass flow rate of 2×10^5 lbm/s at 60 F find the maximum power production allowed if the river water should not be heated more than 2 F.

The maximum heating allowed determines the maximum $\dot{Q}_L$ as

$$\dot{Q}_L = \dot{m}_{H_2O} \, \Delta h = \dot{m}_{H_2O} \, C_{P \, LIQ \, H_2O} \, \Delta T_{H_2O}$$

$$= 2 \times 10^5 \text{ lbm/s} \times 1.0 \text{ Btu/lbm-R} \times 2 \text{ R} = 4 \times 10^5 \text{ Btu/s}$$

$$= \dot{W}_{NET}(1/\eta_{TH \, ac} - 1)$$

$$\dot{W}_{NET} = \dot{Q}_L \, / \, (1/\eta_{TH \, ac} - 1) = \dot{Q}_L \, \frac{\eta_{TH \, ac}}{1 - \eta_{TH \, ac}}$$

$$= 4 \times 10^5 \text{ Btu/s} \times \frac{0.4}{1 - 0.4} = \mathbf{2.67 \times 10^5 \text{ Btu/s}}$$

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5.144E

A nuclear reactor provides a flow of liquid sodium at 1500 F, which is used as the energy source in a steam power plant. The condenser cooling water comes from a cooling tower at 60 F. Determine the maximum thermal efficiency of the power plant. Is it misleading to use the temperatures given to calculate this value?

Solution:

$$T_H = 1500\ F = 1960\ R, \quad T_L = 60\ F = 520\ R$$

$$\eta_{TH\ MAX} = \frac{T_H - T_L}{T_H} = \frac{1960 - 520}{19860} = 0.735$$

It might be misleading to use 1500 F as the value for T_H, since there is not a supply of energy available at a constant temperature of 1500 F (liquid Na is cooled to a lower temperature in the heat exchanger).

$\Rightarrow$ The Na cannot be used to boil H_2O at 1500 F.

Similarly, the H_2O leaves the cooling tower and enters the condenser at 60 F, and leaves the condenser at some higher temperature.

$\Rightarrow$ The water does not provide for condensing steam at a constant temperature of 60 F.

Borgnakke and Sonntag

5.145E

An inventor has developed a refrigeration unit that maintains the cold space at 14 F, while operating in a 77 F room. A coefficient of performance of 8.5 is claimed. How do you evaluate this?

Solution:

Assume Carnot cycle then

$$\beta_{Carnot} = \frac{Q_L}{W_{in}} = \frac{T_L}{T_H - T_L} = \frac{14 + 459.67}{77 - 14} = 7.5$$

$8.5 > \beta_{Carnot} \Rightarrow$ **impossible claim**

$T_H = 77\ F$

$\uparrow \dot{Q}_H$

REF $\Leftarrow \dot{W}$

$\uparrow \dot{Q}_L$

$T_L = 14\ F$

Finite ΔT Heat Transfer

5.146E

A car engine operates with a thermal efficiency of 35%. Assume the air-conditioner has a coefficient of performance that is one third of the theoretical maximum and it is mechanically pulled by the engine. How much fuel energy should you spend extra to remove 1 Btu at 60 F when the ambient is at 95 F?

Solution:

Air conditioner

$$\beta = \frac{Q_L}{W} = \frac{T_L}{T_H - T_L} = \frac{60 + 459.67}{95 - 60} = 14.8$$

$\beta_{actual} = \beta / 3 = 4.93$

$W = Q_L / \beta = 1 / 4.93 = 0.203 \text{ Btu}$

Work from engine

$W = \eta_{eng} \, Q_{fuel} = 0.203 \text{ Btu}$

$$Q_{fuel} = W / \eta_{eng} = \frac{0.203}{0.35} = \textbf{0.58 Btu}$$

5.147E

In a remote location you run a heat engine to provide the power to run a refrigerator. The input to the heat engine is at 1450 R and the low T is 700 R, it has an actual efficiency equal to ½ of the corresponding Carnot unit. The refrigerator has a T_L = 15 F and T_H = 95 F with a COP that is 1/3 of the corresponding Carnot unit. Assume a cooling capacity of 7000 Btu/h is needed and find the rate of heat input to the heat engine.

Heat engine: $\eta_{Carnot} = 1 - \dfrac{T_L}{T_H} = 1 - \dfrac{700}{1450} = 0.517;$ $\eta_{ac} = 0.259$

Refrigerator: $\beta_{ref.\ Carnot} = \dfrac{T_L}{T_H - T_L} = \dfrac{459.7 + 15}{95 - 15} = 5.934;$ $\beta_{ref.\ ac} = 1.978$

Cooling capacity: $\dot{Q}_L = 7000 \text{ Btu/h} = \beta_{ref.\ ac}\ \dot{W};$

$$\dot{W} = 7000 \text{ Btu/h} /1.978 = 3538.9 \text{ Btu/h}$$

This work must be provided by the heat engine $\dot{W} = \eta_{ac}\ \dot{Q}_H$

$$\dot{Q}_H = \dot{W} / \eta_{ac} = 3538.9 \text{ (Btu/h)} / 0.259 = 13\ 664 \text{ Btu/h}$$

$$= \mathbf{3.796 \text{ Btu/s}}$$

5.148E

A heat pump cools a house at 70 F with a maximum of 4000 Btu/h power input. The house gains 2000 Btu/h per degree temperature difference to the ambient and the heat pump coefficient of performance is 60% of the theoretical maximum. Find the maximum outside temperature for which the heat pump provides sufficient cooling.

Solution:

Here:

$T_L = T_{house}$

$T_H = T_{amb}$

In this setup the low temperature space is the house and the high temperature space is the ambient. The heat pump must remove the gain or leak heat transfer to keep it at a constant temperature.

$$\dot{Q}_{leak} = 2000 \, (T_{amb} - T_{house}) = \dot{Q}_L$$

which must be removed by the heat pump.

$\beta' = \dot{Q}_H / \dot{W} = 1 + \dot{Q}_L / \dot{W} = 0.6 \, \beta'_{carnot} = 0.6 \, T_{amb} / (T_{amb} - T_{house})$

Substitute in for $\dot{Q}_L$ and multiply with $(T_{amb} - T_{house})$:

$$(T_{amb} - T_{house}) + 2000 \, (T_{amb} - T_{house})^2 / \dot{W} = 0.6 \, T_{amb}$$

Since $T_{house} = 529.7$ R and $\dot{W} = 4000$ Btu/h it follows

$$T^2_{amb} - 1058.6 \, T_{amb} + 279522.7 = 0$$

Solving => $T_{amb} = $ **554.5 R = 94.8 F**

Comment: We did assume here that $\beta' = 0.6 \, \beta'_{carnot}$, the statement could also have been understood as $\beta = 0.6 \, \beta_{carnot}$ which would lead to a slightly different result.

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5.149E

A small house is kept at 77 F inside looses 12 Btu/s to the outside ambient at 32 F. A heat pump is used to help heat the house together with possible electric heat. The heat pump is driven by a motor of 2.5 kW and it has a COP that is ¼ of a Carnot heat pump unit. Find the actual COP for the heat pump and the amount of electric heat that must be used (if any) to maintain the house temperature.

CV. House   Energy: $0 = \dot{Q}_H + \dot{W}_{el.} - \dot{Q}_{Loss}$

Definition of COP: $\beta' = COP_{HP} = \dfrac{\dot{Q}_H}{\dot{W}} = \dfrac{1}{4}\dfrac{T_H}{T_H - T_L} = \dfrac{1}{4}\dfrac{536.7}{77 - 32} =$ **2.982**

$\dot{Q}_H = COP_{HP}\,\dot{W}_{HP} = 2.982 \times (2.5/1.055)$ Btu/s = 7.066 Btu/s

$\dot{W}_{el.} = \dot{Q}_{Loss} - \dot{Q}_H = 12 - 7.066 =$ **4.934 Btu/s**

5.150E

A house is cooled by an electric heat pump using the outside as the high-temperature reservoir. For several different summer outdoor temperatures estimate the percent savings in electricity if the house is kept at 77 F instead of 68 F. Assume that the house is gaining energy from the outside directly proportional to the temperature difference.

Solution:

Air-conditioner (Refrigerator) $\dot{Q}_{LEAK} \propto (T_H - T_L)$

Max Perf. $\dfrac{\dot{Q}_L}{\dot{W}_{in}} = \dfrac{T_L}{T_H - T_L} = \dfrac{K(T_H - T_L)}{\dot{W}_{in}}$, $\dot{W}_{in} = \dfrac{K(T_H - T_L)^2}{T_L}$

A: $T_{L_A} = 68\ F = 527.7\ R$ B: $T_{L_B} = 77\ F = 536.7\ R$

T_H, F	$\dot{W}_{IN_A}/K$	$\dot{W}_{IN_B}/K$	% saving
115	4.186	2.691	35.7 %
105	2.594	1.461	43.7 %
95	1.381	0.604	56.3 %

5.151E

Arctic explorers are unsure if they can use a 5 kW motor driven heat pump to stay warm. It should keep their shelter at 60 F which loses energy at a rate of 0.3 Btu/s per degree difference to the colder ambient. The heat pump has a COP that is 50% of a Carnot heat pump. If the ambient temperature can fall to -10 F at night, would you recommend this heat pump to the explorers?

CV Heat pump.
The heat pump should deliver a rate of heating that equals the heat loss to the ambient for steady inside temperature.

$$\text{COP} = \beta' = \dot{Q}_H/\dot{W} = 0.5\ \beta'_{\text{Carnot}} = \frac{1}{2} \times \frac{T_H}{T_H - T_L} = \frac{1}{2} \times \frac{459.7 + 60}{60 - (-10)} = 3.7$$

The heat pump can then provide a heating capacity of

$$\dot{Q}_H = \beta'\ \dot{W} = 3.7 \times 5\ \text{kW} = 18.5\ \text{kW} = 17.53\ \text{Btu/s}$$

The heat loss is

$$\dot{Q}_{\text{leak out}} = \text{CA}\ \Delta T = 0.3\ \text{Btu/s-R} \times [60 - (-10)]\ \text{R} = 21\ \text{Btu/s}$$

The heat pump is not sufficient to cover the loss and **not recommended**.

5.152E

Using the given heat pump in the previous problem how warm could it make the shelter in the arctic night?

The high is now an unknown so both the heat loss and the heat pump performance depends on that. The energy balance around the shelter then gives

$$\dot{Q}_H = \beta' \, \dot{W} = \dot{Q}_{leak \; out} = CA \, \Delta T$$

Substitute the expression for β' and $CA \, \Delta T$ to give

$$\frac{1}{2} \times \frac{T_H}{T_H - T_L} \, \dot{W} = 0.3 \text{ Btu/s-R} \times [T_H - T_L]$$

Multiply with the temperature difference, factor 2 and divide by the work, 5 kW = (5/1.055) Btu/s, so we get

$$T_H = \frac{0.3 \times 2}{5/1.055 \text{ R}} [T_H - T_L]^2 = \frac{0.1137}{R} [T_H - T_L]^2$$

Solve this equation like $0.1137 \, x^2 - x - T_L = 0$, with $x = T_H - T_L$ and $T_L = 459.7 - 10 = 449.7$ R

$$x = T_H - T_L = 67.44 \text{ R} \qquad \text{(negative root discarded)}$$

$$T_H = x + T_L = 67.44 - 10 = \mathbf{57.4 \; F}$$

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Ideal Gas Garnot Cycle

5.153E

Carbon dioxide is used in an ideal gas refrigeration cycle, reverse of Fig. 5.24. Heat absorption is at 450 R and heat rejection is at 585 R where the pressure changes from 180 psia to 360 psia. Find the refrigeration COP and the specific heat transfer at the low temperature.

The analysis is the same as for the heat engine except the signs are opposite so the heat transfers move in the opposite direction.

$$\beta = \dot{Q}_L / \dot{W} = \beta_{carnot} = T_L / (T_H - T_L) = \frac{450}{585 - 450} = \textbf{3.33}$$

$$q_H = RT_H \ln(v_2/v_1) = RT_H \ln \left(\frac{P_1}{P_2} \right) = 35.1 \text{ ft-lbf/lbm-R} \times 585 \text{ R} \times \ln(\frac{360}{180})$$

$$= 14\ 232.7 \text{ ft-lbf/lbm} = 18.29 \text{ Btu/lbm}$$

$$q_L = q_H\ T_L / T_H = 18.29 \times 450 / 585 = \textbf{14.07 Btu/lbm}$$

5.154E

Air in a piston/cylinder goes through a Carnot cycle with the P-v diagram shown in Fig. 5.24. The high and low temperatures are 1200 R and 600 R respectively. The heat added at the high temperature is 100 Btu/lbm and the lowest pressure in the cycle is 10 lbf/in.^{2}. Find the specific volume and pressure at all 4 states in the cycle assuming constant specific heats at 80 F.

Solution:

$q_H = 100$ Btu/lbm,　$T_H = 1200$ R,　$T_L = 600$ R,　$P_3 = 10$ lbf/in.^{2}

$C_v = 0.171$ Btu/lbm R ;　　　$R = 53.34$ ft-lbf/lbm-R

The states as shown in figure 5.24

1: 1200 R ,　　2: 1200 R,　　3: 10 psi, 600 R　　4: 600 R

As we know state 3 we can work backwards towards state 1

$v_3 = RT_3 / P_3 = 53.34 \times 600 /(10 \times 144) = 22.225$ ft^3/lbm

Process　2→3　from Eq.5.8 & C_v = constant

$=> C_v \ln (T_L / T_H) + R \ln (v_3/v_2) = 0$

$=> \ln (v_3/v_2) = - (C_v / R) \ln (T_L / T_H)$

$= - (0.171/53.34) \ln (600/1200) = 1.7288$

$=> v_2 = v_3 / \exp (1.7288) = 22.225/5.6339 = 3.9449$ ft^3/lbm

Process　1→2　and Eq.5.7:　　$q_H = RT_H \ln (v_2 / v_1)$

$\ln (v_2 / v_1) = q_H /RT_H = 100 \times 778/(53.34 \times 1200) = 1.21547$

$v_1 = v_2 / \exp (1.21547) = 1.1699$ ft^3/lbm

$v_4 = v_1 \times v_3 / v_2 = 1.1699 \times 22.225/3.9449 = 6.591$ ft^3/lbm

$P_1 = RT_1 / v_1 = 53.34 \times 1200/(1.1699 \times 144) = 379.9$ psia

$P_2 = RT_2 / v_2 = 53.34 \times 1200/(3.9449 \times 144) = 112.7$ psia

$P_4 = RT_4 / v_4 = 53.34 \times 600/(6.591 \times 144) = 33.7$ psia

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Review Problems

Borgnakke and Sonntag

5.155E

We wish to produce refrigeration at −20 F. A reservoir is available at 400 F and the ambient temperature is 80 F, as shown in Fig. P5.104. Thus, work can be done by a cyclic heat engine operating between the 400 F reservoir and the ambient. This work is used to drive the refrigerator. Determine the ratio of the heat transferred from the 400 F reservoir to the heat transferred from the −20 F reservoir, assuming all processes are reversible.

Solution: Equate the work from the heat engine to the refrigerator.

$$\dot{W} = \dot{Q}_{H1}\left(\frac{T_H - T_0}{T_H}\right)$$

also

$$\dot{W} = \dot{Q}_{L2}\left(\frac{T_0 - T_L}{T_L}\right)$$

$T_H = 860\ R$ $T_0 = 540\ R$

$\dot{Q}_{H1}$ $\dot{Q}_{H2}$

$\dot{W}$

(HE)   (REF)

$\dot{Q}_{L1}$ $\dot{Q}_{L2}$

$T_0 = 540\ R$ $T_L = 440\ R$

$$\dot{Q}_{H1} / \dot{Q}_{L2} = \left(\frac{T_O - T_L}{T_L}\right)\left(\frac{T_H}{T_H - T_O}\right) = \frac{100}{440} \times \frac{860}{320} = \mathbf{0.611}$$

5.156E

An air-conditioner on a hot summer day removes 8 Btu/s of energy from a house at 70 F and pushes energy to the outside which is at 88 F. The house has 30 000 lbm mass with an average specific heat of 0.23 Btu/lbm-R. In order to do this the cold side of the air-conditioner is at 40 F and the hot side is 100 F. The air conditioner (refrigerator) has a COP that is 60% of a corresponding Carnot refrigerator. Find the actual air-conditioner COP and the required power to run it.

A steady state refrigerator definition of COP

$$COP = \beta_{REF} = \dot{Q}_L / \dot{W} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) = 0.6\ \beta_{Carnot}$$

Carnot: $\beta_{Carnot} = \dfrac{T_L}{T_H - T_L} = \dfrac{40 + 459.7}{100 - 40} = 8.328$

$\beta_{REF} = 0.6 \times 8.328 = \mathbf{5}$

$\Rightarrow \quad \dot{W} = \dot{Q}_L / \beta_{REF} = 8\ \text{Btu/s} / 5 = \mathbf{1.6\ Btu/s}$

Any heat transfer must go from a higher to a lower T domain.

5.157E

The air conditioner in the previous problem is turned off. How fast does the house heat up in degrees per second (F/s)?

Once the A/C unit is turned off we do not cool the house so heat leaks in from the atm. at a rate of 8 Btu/s (that is what we had to remove to keep steady state).

Energy Eq.:
$$\dot{E}_{CV} = \dot{Q}_{leak} = 8 \text{ Btu/s} = m_{house}\, C_P \frac{dT}{dt}$$

$$\frac{dT}{dt} = \dot{Q}_{leak} / m_{house}\, C_P$$
$$= \frac{8 \text{ Btu/s}}{30\,000 \times 0.23 \text{ Btu/R}} = \mathbf{1.16 \times 10^{-3} \text{ R/s}}$$

5.158E

A window air conditioner cools a room at $T_L = 68$ F with a maximum of 1.2 kW power input. The room gains 0.33 Btu/s per degree temperature difference to the ambient and the refrigeration COP is $\beta = 0.6\ \beta_{Carnot}$. Find the maximum outside temperature, T_H, for which the air conditioner provides sufficient cooling.

Solution:

Here:

$T_L = T_{house}$

$T_H = T_{amb}$

In this setup the low temperature space is the house and the high temperature space is the ambient. The heat pump must remove the gain or leak heat transfer to keep it at a constant temperature.

$$\dot{Q}_{leak} = 0.33\ (T_{amb} - T_{house}) = \dot{Q}_L$$

which must be removed by the heat pump.

$$\beta = \dot{Q}_L / \dot{W} = 0.6\ \beta_{carnot} = 0.6\ T_{house} / (T_{amb} - T_{house})$$

Substitute in for $\dot{Q}_L$ and multiply with $(T_{amb} - T_{house})\dot{W}$:

$$0.33\ (T_{amb} - T_{house})^2 = 0.6\ T_{house}\ \dot{W}$$

Since $T_{house} = 459.7 + 68 = 527.7$ R and $\dot{W} = 1.2$ kW $= 1.1374$ Btu/s, it follows

$$(T_{amb} - T_{house})^2 = (0.6/0.33) \times 527.7 \times 1.1374 = 1091.28\ R^2$$

Solving $\Rightarrow (T_{amb} - T_{house}) = 33.03 \Rightarrow T_{amb} = \mathbf{560.73\ R = 101\ F}$

Comment: We did assume here that $\beta = 0.6\ \beta_{carnot}$, the statement could also have been understood as $\beta' = 0.6\ \beta'_{carnot}$ which would lead to a slightly different result.

5.159E

The room in problem 5.158E has a combined thermal mass of 4 000 lbm wood, 500 lbm steel and 1000 lbm plaster board. Estimate how fast the room heats up if the air-conditioner is turned off on a day it is 95 F outside.

Without the air-conditioner the house gains heat and the energy equation for the house becomes

$$m\,C\,\frac{dT}{dt} = \dot{Q}_{in}$$

The gain is due to the temperature difference as

$$\dot{Q}_{in} = 0.33\,(T_H - T_L) = 0.33\ \text{Btu/s-R}\,(95 - 68)\ R = 8.91\ \text{Btu/s}$$

The combined (mC) is using an estimate C for gypsum as 0.24 Btu/lbm-R

$$mC = [4000 \times 0.33 + 500 \times 0.11 + 1000 \times 0.24\,]\ \text{Btu/R} = 1615\ \text{Btu/R}$$

$$\frac{dT}{dt} = \dot{Q}_{in}\,/mC = 8.91\ \text{(Btu/s)}\,/\,1615\ \text{Btu/R} = \textbf{0.0055 R/s}$$

5.160E

A 350-ft^3 tank of air at 80 lbf/in.^{2}, 1080 R acts as the high-temperature reservoir for a Carnot heat engine that rejects heat at 540 R. A temperature difference of 45 F between the air tank and the Carnot cycle high temperature is needed to transfer the heat. The heat engine runs until the air temperature has dropped to 700 R and then stops. Assume constant specific heat capacities for air and find how much work is given out by the heat engine.

Solution:

$T_H = T_{air} - 45$, $T_L = 540$ R

$$m_{air} = \frac{P_1 V}{R T_1} = \frac{80 \times 350 \times 144}{53.34 \times 1080} = 69.991 \text{ lbm}$$

$$dW = \eta dQ_H = \left(1 - \frac{T_L}{T_{air} - 45} \right) dQ_H$$

$$dQ_H = -m_{air} du = -m_{air} C_V dT_{air}$$

$$W = \int dW = -m_{air} C_V \int \left[1 - \frac{T_L}{T_a - 45} \right] dT_a$$

$$= -m_{air} C_V \left[T_{a2} - T_{a1} - T_L \ln \frac{T_{a2} - 45}{T_{a1} - 45} \right]$$

$$= -69.991 \text{ lbm} \times 0.171 \text{ Btu/lbm-R} \times \left[700 - 1080 - 540 \ln \frac{655}{1035} \right] \text{R}$$

$$= \textbf{1591 Btu}$$

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5.161E

Air in a rigid 40 ft^3 box is at 540 R, 30 lbf/in.^{2}. It is heated to 1100 R by heat transfer from a reversible heat pump that receives energy from the ambient at 540 R besides the work input. Use constant specific heat at 540 R. Since the coefficient of performance changes write $dQ = m_{air} C_v dT$ and find dW. Integrate dW with temperature to find the required heat pump work.

Solution:

$$\text{COP:} \quad \beta' = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} \cong \frac{T_H}{T_H - T_L}$$

$$m_{air} = P_1 V_1 / RT_1 = (30 \times 40 \times 144) / (540 \times 53.34) = 6.0 \text{ lbm}$$

$$dQ_H = m_{air} C_v dT_H = \beta' dW \cong \frac{T_H}{T_H - T_L} dW$$

$$\Rightarrow \quad dW = m_{air} C_v [\frac{T_H}{T_H - T_L}] dT_H$$

$$_1W_2 = \int m_{air} C_v (1 - \frac{T_L}{T}) dT = m_{air} C_v \int (1 - \frac{T_L}{T}) dT$$

$$= m_{air} C_v [T_2 - T_1 - T_L \ln \frac{T_2}{T_1}]$$

$$= 6.0 \text{ lbm} \times 0.171 \text{ Btu/lbm-R} \times [1100 - 540 - 540 \ln (\frac{1100}{540})] \text{ R}$$

$$= \textbf{180.4 Btu}$$

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 6

8e

Updated June 2013

Borgnakke and Sonntag

CONTENT CHAPTER 6

The clip-art included in the solution manual is from **Microsoft**.

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In-Text Concept Questions

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6.a

Does Clausius say anything about the sign for $\oint dQ$?

No.

The total (net) heat transfer can be coming in like in a heat engine ($W_{out} = Q_H - Q_L$) in which case it is positive. It can also be net going out like in a refrigerator or heat pump ($W_{in} = Q_H - Q_L$) in which case the sign is negative. Finally if you look at a transmission gearbox there could be no heat transfer (first approximation) in which case the integral is zero.

6.b

Does the statement of Clausius require a constant T for the heat transfer as in a Carnot cycle?

No.

The statement for a cycle involves an integral of dQ/T so T can vary, which it does during most processes in actual devices. This just means that you cannot that easily get a closed expression for the integral.

6.c

How can you change s of a substance going through a reversible process?

From the definition of entropy

$$ds = \frac{dq}{T}$$

for a reversible process. Thus only heat transfer gives a change in s, expansion/compression involving work does not give such a contribution.

6.d

A reversible process adds heat to a substance. If T is varying does that influence the change in s?

Yes.

$$\text{Reversible:} \quad ds = \frac{dq}{T}$$

So if T goes up it means that s changes less per unit of dq, and the opposite if T decreases then s changes more per unit of dq.

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6.e

Water at 100 kPa, 150°C receives 75 kJ/kg in a reversible process by heat transfer. Which process changes s the most: constant T, constant v or constant P?

$$ds = \frac{dq}{T}$$

Look at the constant property lines in a T-s diagram, Fig. 6.5. The constant v line has a higher slope than the constant P line also at positive slope. Thus both the constant P and v processes have an increase in T. As T goes up the change in s is smaller for the same area (heat transfer) under the process curve in the T-s diagram as compared with the constant T process.

The constant T (isothermal) process therefore changes s the most.

In a reversible process the area below the process curve in the T-s diagram is the heat transfer.

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6.f

A liquid is compressed in a reversible adiabatic process. What is the change in T?

If the process is reversible then s is constant, $ds = \dfrac{dq}{T} = 0$

Change in s for a liquid (an incompressible substance) is

Eq. 6.10: $\quad ds = \dfrac{C}{T}\,dT$

From this it follows that if ds = 0 then **T is constant**.

6.g

An ideal gas goes through a constant T reversible heat addition process. How do the properties (v, u, h, s, P) change (up, down or constant)?

Ideal gas: u(T), h(T)   so they are both constant

Eq. 6.2 gives:	$ds = dq/T > 0$	so s goes up by q/T
Eq. 6.12 gives:	$ds = (R/v)\,dv$	so v increases
Eq. 6.14 gives:	$ds = -(R/P)\,dP$	so P decreases

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6.h

Carbon dioxide is compressed to a smaller volume in a polytropic process with n = 1.2. How do the properties (u, h, s, P, T) change (up, down or constant)?

For carbon dioxide Table A.5   k = 1.289 so we have n < k and the process curve can be recognized in Figure 8.13. From this we see a smaller volume means moving to the left in the P-v diagram and thus also up.

From P-v diagram: P up, T up
From T-s diagram   Since T is up then s down.
As T is up so is h and u.

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6.i

A substance has heat transfer out. Can you say anything about changes in s if the process is reversible? If it is irreversible?

Reversible: $\quad ds = \dfrac{dq}{T} < 0 \qquad$ since $\quad dq < 0$

Irreversible: $\quad ds = \dfrac{dq}{T} + ds_{gen} = ? \quad dq < 0 \quad$ but $\quad ds_{gen} > 0$

You cannot say, ds depends on the magnitude of dq/T versus ds_{gen}

6.j

A substance is compressed adiabatically so P and T go up. Does that change s?

If the process is reversible then s is constant, $\quad ds = \dfrac{dq}{T} = 0$

If the process is irreversible then s goes up, $\quad ds = \dfrac{dq}{T} + ds_{gen} = ds_{gen} > 0$

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Concept Problems

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6.1

When a substance has completed a cycle, v, u, h, and s are unchanged. Did anything happen? Explain.

Yes.

During various parts of the cycle work and heat transfer may be transferred. That happens at different P and T. The net work out equals the net heat transfer in (energy conservation) so dependent upon the sign it is a heat engine or a heat pump (refrigerator). The net effect is thus a conversion of energy from one storage location to another and it may also change nature, some Q was changed to W or the opposite.

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6.2

Assume a heat engine with a given Q_H. Can you say anything about Q_L if the engine is reversible? If it is irreversible?

For a reversible heat engine it must be that:

$$\oint \frac{dQ}{T} = 0 = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} \qquad \text{or integrals if T not constant}$$

So as T_L is lower than T_H then Q_L must be correspondingly lower than Q_H to obtain the net zero integral.

For an irreversible heat engine we have

$$\oint \frac{dQ}{T} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} < 0$$

This means that Q_L is larger than before (given Q_H and the T's). The irreversible heat engine rejects more energy and thus gives less out as work.

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6.3

CV A is the mass inside a piston/cylinder, CV B is that plus part of the wall out to a source of $_1Q_2$ at T_s. Write the entropy equation for the two control volumes assuming no change of state of the piston mass or walls.

Fig. P6.3

The general entropy equation for a control mass is Eq.6.37

$$S_2 - S_1 = \int_1^2 \frac{dQ}{T} + {}_1S_{2\ gen}$$

The left hand side is storage so that depends of what is inside the C.V. and the integral is summing the dQ/T that crosses the control volume surface while the process proceeds from 1 to 2.

C.V. A: $m_A (s_2 - s_1) = \int_1^2 \frac{dQ}{T_A} + {}_1S_{2\ gen\ CV\ A}$

C.V. B: $m_A (s_2 - s_1) = \int_1^2 \frac{dQ}{T_s} + {}_1S_{2\ gen\ CV\ B}$

In the first equation the temperature is that of mass m_A which possibly changes from 1 to 2 whereas in the second equation it is the reservoir temperature T_s. The two entropy generation terms are also different the second one includes the first one plus any s generated in the walls that separate the mass m_A from the reservoir and there is a Q over a finite temperature difference. When the storage effect in the walls are neglected the left hand sides of the two equations are equal.

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6.4

Consider the previous set-up with the mass m_A and the piston cylinder of mass m_p starting out at two different temperatures. After a while the temperature becomes uniform without any external heat transfer. Write the entropy equation storage term ($S_2 - S_1$) for the total mass.

In this case the storage effect must be summed up over all the mass inside the control volume.

$$S_2 - S_1 = m_A \, (s_2 - s_1)_A + m_P \, (s_2 - s_1)_P$$
$$\approx m_A \, (s_2 - s_1)_A + m_P \, C_P \ln \left(\frac{T_2}{T_{1\,P}} \right)$$

The last equation assumed a constant specific heat for the solid material of the piston, a common assumption. There is only a single temperature T_2, but there are two different temperatures for state 1: $T_{1\,A}$ and $T_{1\,P}$. The temperature T_2 would be found from the energy equation.

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6.5

Water at 100^oC, quality 50% in a rigid box is heated to 110^oC. How do the properties (P, v, x, u and s) change? (increase, stay about the same, or decrease)

A fixed mass in a rigid box give a constant v process. So

P goes up (in the two-phase region $P = P_{sat}$ at given T)

v stays constant.

x goes up (we get closer to the saturated vapor state see P-v diagram)

u goes up (Q in and no work)

s goes up (Q in)

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6.6

Liquid water at 20°C, 100 kPa is compressed in a piston/cylinder without any heat transfer to a pressure of 200 kPa. How do the properties (T, v, u, and s) change? (increase, stay about the same, or decrease)

Adiabatic dq = 0:   dq = T ds = 0;
Incompressible dv = 0:   dw = P dv = 0

(T, v, u, and s)   they are all constant.
Only the pressure and enthalpy goes up.

In the T-s
diagram the
two states are
in the same
location as T
does not go
up for v
constant.

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6.7

A reversible process in a piston/cylinder is
shown in Fig. P6.7. Indicate the storage
change $u_2 - u_1$ and transfers $_1w_2$ and $_1q_2$ as
positive, zero, or negative

$$_1w_2 = \int P \, dv > 0 \, ; \quad _1q_2 = \int T \, ds > 0$$

$u_2 - u_1 > 0$ from general shape of the constant u curves. Further out in the ideal
gas region the constant u curve become horizontal ($u = fct(T)$ only).

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6.8

A reversible process in a piston/cylinder
is shown in Fig. P6.8. Indicate the
storage change $u_2 - u_1$ and transfers $_1w_2$
and $_1q_2$ as positive, zero, or negative

$$_1w_2 = \int P \, dv > 0 \; ; \qquad _1q_2 = \int T \, ds = 0$$

$$u_2 - u_1 = {_1q_2} - {_1w_2} < 0$$

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6.9

Air at 290 K, 100 kPa in a rigid box is heated to 325 K. How do the properties (P, v, u and s) change? (increase, stay about the same, or decrease)

Rigid box: v = constant,   (P, u, and s) all increases.

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6.10

Air at 20°C, 100 kPa is compressed in a piston/cylinder without any heat transfer to a pressure of 200 kPa. How do the properties (T, v, u and s) change? (increase, about the same or decrease)

T goes up,
v goes down
u goes up (work in, q = 0)
s = constant

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6.11

Carbon dioxide is compressed to a smaller volume in a polytropic process with n = 1.4. How do the properties (u, h, s, P, T) change (up, down or constant)?

For carbon dioxide Table A.5   k = 1.289   so we have n > k and the process curve can be recognized in Figure 6.13. From this we see a smaller volume means moving to the left in the P-v diagram and thus also up.

P up, T up.   As T is up so is h and u.

From the T-s diagram as n > k then we move to larger T means s is up.

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6.12

Process A: Air at 300 K, 100 kPa is heated to 310 K at constant pressure.
Process B: Heat air at 1300 K to 1310 K at constant 100 kPa.
Use the table below to compare the property changes.

	Property	$\Delta_A > \Delta_B$	$\Delta_A \approx \Delta_B$	$\Delta_A < \Delta_B$
a	$\Delta = v_2 - v_1$		√	
b	$\Delta = h_2 - h_1$			√
c	$\Delta = s_2 - s_1$	√		

a. Ideal gas $Pv = RT$ so v goes with absolute T
$\Delta v = (R/P)\,\Delta T$ thus the same

b. Since $dh = C_P\,dT$ and C_P increases with T

c. At constant P: $ds = (C_P/T)\,dT$
C_P is only 15% higher at 1300 K compared to 300 K (see Fig. 3.11)

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6.13

Why do we write ΔS or $S_2 - S_1$ whereas we write $\int dQ/T$ and $_1S_{2\,gen}$?

This is similar to the terms in the continuity equation $m_2 - m_1$ versus m_i, m_e and the energy equation $E_2 - E_1$ versus $_1Q_2$, $_1W_2$. The first part is the **change in the storage** $S_2 - S_1$ of entropy in the mass and the second part is the transfer or generation during the process from 1 to 2. The storage terms correspond to the left hand side of the balance equation for entropy. The integral $\int dQ/T$ represents a **transfer** of entropy **across the control volume surface** during the process from 1 to 2 and the $_1S_{2\,gen}$ expresses the total amount of entropy being generated **inside** the control volume and both are on the right hand side of the balance equation.

6.14

A reversible heat pump has a flux of s entering as $\dot{Q}_L/T_L$. What can you say about the exit flux of s at T_H?

For the entropy equation 6.3 and 6.42, the rate of storage is zero and the generation term is zero. Thus the entropy equation becomes

$$0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} + 0$$

So: $\dfrac{\dot{Q}_L}{T_L} = \dfrac{\dot{Q}_H}{T_H} = \text{flux of s}$

We have the same flux of s in as out matching the result in chapter 5.

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6.15

An electric baseboard heater receives 1500 W of electrical power that heats the room air which loses the same amount out through the walls and windows. Specify exactly where entropy is generated in that process.

      a Electrical heating wire (electrical work turned into internal energy,
        leaving as heat transfer).
    b Heat transfer from hot wire to cooler room air, i.e. in the wire coverings
    c Room air to walls
    d Inside walls and windows, heat transfer over a finite ΔT
    e from outside wall surface to ambient T

The electric wire is inside the pipe and surrounded by sand to electrically isolate it. The pipe has fins mounted to increase the surface area.

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6.16

A 500 W electric space heater with a small fan inside heats air by blowing it over a hot electrical wire. For each control volume: a) wire at T_{wire} only b) all the room air at T_{room} and c) total room plus the heater, specify the storage, entropy transfer terms and entropy generation as rates (neglect any $\dot{Q}$ through the room walls or windows).

	Storage	$\dot{Q}/T$	$\dot{S}_{gen}$
C.V. Wire only	0	$\dfrac{-500 \text{ W}}{T_{wire}}$	$\dfrac{500 \text{ W}}{T_{wire}}$
C.V. Room air	$\dfrac{500 \text{ W}}{T_{room}}$	$\dfrac{500 \text{ W}}{T_{wire}}$	$500 \text{ W} \left(\dfrac{1}{T_{room}} - \dfrac{1}{T_{wire}}\right)$
C.V. Total room	$\dfrac{500 \text{ W}}{T_{room}}$	0	$\dfrac{500 \text{ W}}{T_{room}}$

Remark: Room only receives the electrical power input of 500 W.

Some of the heaters can be radiant heaters in which case the fan is not needed.

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Inequality of Clausius

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6.17

Consider the steam power plant in Example 4.7 and assume an average T in the line between 1 and 2. Show that this cycle satisfies the inequality of Clausius.

Solution:

Show Clausius: $\displaystyle\oint \frac{dQ}{T} \leq 0$

For this problem we have three heat transfer terms:

$q_b = 2831$ kJ/kg, $q_{loss} = 21$ kJ/kg, $q_c = 2173.3$ kJ/kg

$$\oint \frac{dq}{T} = \frac{q_b}{T_b} - \frac{q_{loss}}{T_{avg\ 1\text{-}2}} - \frac{q_c}{T_c}$$
$$= \frac{2831}{573} - \frac{21}{568} - \frac{2173.3}{318}$$
$$= -1.93 \text{ kJ/kg K} < 0 \quad \textbf{OK}$$

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6.18

A heat engine receives 6 kW from a 250$^\text{o}$C source and rejects heat at 30$^\text{o}$C. Examine each of three cases with respect to the inequality of Clausius.

a. $\dot{W} = 6$ kW   b. $\dot{W} = 0$ kW   c. Carnot cycle

Solution:

$T_H = 250 + 273 = 523$ K ; $T_L = 30 + 273 = 303$ K

Case a) $\int \frac{d\dot{Q}}{T} = \frac{6000}{523} - \frac{0}{303} = 11.47$ kW/K > 0 **Impossible**

b) $\int \frac{d\dot{Q}}{T} = \frac{6000}{523} - \frac{6000}{303} = -8.33$ kW/K < 0 **OK**

c) $\int \frac{d\dot{Q}}{T} = 0 = \frac{6000}{523} - \frac{\dot{Q}_L}{303}$ $\Rightarrow$

$\dot{Q}_L = \frac{303}{523} \times 6$ kW $= 3.476$ kW

$\dot{W} = \dot{Q}_H - \dot{Q}_L = 2.529$ kW

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6.19

Use the inequality of Clausius to show that heat transfer from a warm space towards a colder space without work is a possible process i.e. a heat engine with no work output.

$$\text{Clausius:} \quad \int \frac{dQ}{T} \leq 0 \quad \text{or} \quad \int \frac{d\dot{Q}}{T} \leq 0$$

Take C.V. as the space separating the warm and cold space. It is the same Q that crosses each surface (from energy equation) so

$$\int \frac{dQ}{T} = \frac{Q}{T_{warm}} - \frac{Q}{T_{cold}} = Q \left(\frac{1}{T_{warm}} - \frac{1}{T_{cold}} \right) < 0 \qquad \text{OK}$$

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6.20

Use the inequality of Clausius to show that heat transfer from a cold space towards a warmer space without work is an impossible process i.e. a heat pump with no work input.

$$\text{Clausius:} \quad \int \frac{dQ}{T} \leq 0 \quad \text{or} \quad \int \frac{d\dot{Q}}{T} \leq 0$$

Take C.V. as the space separating the warm and cold space. It is the same Q that crosses each surface (from energy equation) so

$$\int \frac{dQ}{T} = \frac{Q}{T_{cold}} - \frac{Q}{T_{warm}} = Q\left(\frac{1}{T_{cold}} - \frac{1}{T_{warm}}\right) > 0 \quad \text{Impossible!}$$

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6.21

Assume the heat engine in Problem 5.34 has a high temperature of 1000 K and a low temperature of 400 K. What does the inequality of Clausius say about each of the four cases?

Solution:

Cases a) $\int \dfrac{d\dot{Q}}{T} = \dfrac{6}{1000} - \dfrac{4}{400} = -0.004 \text{ kW/K} < 0$ **OK**

  b) $\int \dfrac{d\dot{Q}}{T} = \dfrac{6}{1000} - \dfrac{0}{400} = 0.006 \text{ kW/K} > 0$ **Impossible**

  c) $\int \dfrac{d\dot{Q}}{T} = \dfrac{6}{1000} - \dfrac{2}{400} = 0.001 \text{ kW/K} > 0$ **Impossible**

  d) $\int \dfrac{d\dot{Q}}{T} = \dfrac{6}{1000} - \dfrac{6}{400} = -0.009 \text{ kW/K} < 0$ **OK**

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6.22

Let the steam power plant in Problem 5.30 have 700$^\text{o}$C in the boiler and 40$^\text{o}$C during the heat rejection in the condenser. Does that satisfy the inequality of Clausius? Repeat the question for the cycle operated in reverse as a refrigerator.

Solution:

$$\dot{Q}_H = 1 \text{ MW} \qquad\qquad \dot{Q}_L = 0.58 \text{ MW}$$

$$\int \frac{d\dot{Q}}{T} = \frac{1000}{973} - \frac{580}{313} = -0.82 \text{ kW/K} < 0 \qquad \textbf{OK}$$

Refrigerator

$$\int \frac{d\dot{Q}}{T} = \frac{580}{313} - \frac{1000}{973} = 0.82 > 0 \qquad \textbf{Cannot be possible}$$

Borgnakke and Sonntag

6.23

Examine the heat engine given in Problem 5.74 to see if it satisfies the inequality of Clausius.

Solution:

$$Q_H = \quad 325 \text{ kJ} \quad \text{at} \quad T_H = 1000 \text{ K}$$
$$Q_L = \quad 125 \text{ kJ} \quad \text{at} \quad T_L = \quad 400 \text{ K}$$

$$\int \frac{dQ}{T} = \frac{325}{1000} - \frac{125}{400} = 0.0125 \text{ kJ/K} > 0 \qquad \textbf{Impossible}$$

Borgnakke and Sonntag

Entropy of a pure substance

Borgnakke and Sonntag

6.24

Determine the entropy for these states:
  a. Nitrogen, P = 2000 kPa, 120 K
  b. Nitrogen, 120 K, v = 0.005 m³/kg
  c. R-410A, T = 25ºC, v = 0.01 m³/kg

Solution:

a) Table B.6.1: $P < P_{sat}$ = 2513 kPa so superheated vapor.

B.6.2 at 2000 kPa: s = **4.8116 kJ/kg-K**

b) Table B.6.1: $v_f < v < v_g$ = 0.00799 m³/kg so two-phase L + V

$$x = (v - v_f)/v_{fg} = \frac{0.005 - 0.001915}{0.00608} = 0.5074$$

$$s = s_f + x\, s_{fg} = 3.8536 + x\, 0.7659 = \textbf{4.2422 kJ/kg K}$$

c) Table B.4.1 $v_f < v < v_g$ = 0.01514 m³/kg so two-phase L + V

$$x = (v - v_f)/v_{fg} = \frac{0.01 - 0.000944}{0.01420} = 0.63775$$

$$s = s_f + x\, s_{fg} = 0.3631 + x\, 0.6253 = \textbf{0.7619 kJ/kg K}$$

Borgnakke and Sonntag

6.25

Determine the missing property among P, T, s, and x for R-410A at:

      a. $T = -20^\circ C$, $v = 0.1377$ m^3/kg

      b. $T = 20^\circ C$, $v = 0.01377$ m^3/kg

      c. $P = 200$ kPa, $s = 1.409$ kJ/kgK

    a)   B.4.1: $v > v_g = 0.0648$ m^3/kg =>

             B.4.2   superheated vapor so x is undefined

             very close to 200 kPa, $s = 1.1783$ kJ/kg-K

    b)   B.4.1: $0.000923 = v_f < v < v_g = 0.01758$ m^3/kg => Two-phase

             $P = P_{sat} = 1444.2$ kPa

$$x = (v - v_f)/v_{fg} = \frac{0.01377 - 0.000923}{0.01666} = 0.77113$$

             $s = s_f + x\, s_{fg} = 0.3357 + 0.77113 \times 0.6627 = 0.8467$ kJ/kg-K

    c)   Table B.4.2 at 200 kPa, $s > s_g$ so superheated vapor,

          x is undefined, and we find the state at $T = 60^\circ C$.

Borgnakke and Sonntag

6.26

Find the missing properties of *P, v, s* and *x* for ammonia (NH_3) at

a. $T = 65°C$, $P = 600$ kPa
b. $T = 20°C$, $u = 800$ kJ/kg
c. $T = 50°C$, $v = 0.1185$ m^3/kg

a) B.2.2 average between 60°C and 70°C
$v = (0.25981 + 0.26999)/2 = 0.26435$ m^3/kg
$s = (5.6383 + 5.7094)/2 = 5.6739$ kJ/kgK

b) B.2.1: $u < u_g = 1332.2$ kJ/kg $\Rightarrow$ $P = P_{sat} = 857.5$ kPa
$x = (u - u_f)/u_{fg} = \dfrac{800 - 272.89}{1059.3} = 0.49666$
$v = 0.001638 + x \times 0.14758 = 0.07494$ m^3/kg,
$s = 1.0408 + x \times 4.0452 = 3.04989$ kJ/kg-K

c) B.2.1: $v > v_g = 0.06337$ m^3/kg $\Rightarrow$
B.2.2 superheated vapor so x is undefined
very close to 1200 kPa, $s = 5.1497$ kJ/kgK

Borgnakke and Sonntag

6.27

Find the entropy for the following water states and indicate each state on a T-s diagram relative to the two-phase region.

   a. $250^{\circ}C$, $v = 0.02 \ m^3/kg$

   b. $250^{\circ}C$, 2000 kPa

   c. $-2^{\circ}C$, 100 kPa

Solution:

a)   Table B.1.1: $0.001251 = v_f < v < v_g = 0.05013 \ m^3/kg$ => Two-phase

$$x = \frac{0.02 - 0.001251}{0.04887} = 0.38365$$

$$s = s_f + x \ s_{fg} = 2.7927 + 0.38365 \times 3.2802 = \textbf{4.05 kJ/kg K}$$

b)   Table B.1.1: $P < P_{sat} = 3973$ kPa => superheated vapor B.1.3

$$s = \textbf{6.5452 kJ/kg K}$$

c)   Table B.1.1 $T < T_{tripple} = 0.01^{\circ}C$ so goto B.1.5

Table B.1.5: $P > P_{sat} = 0.5177$ kPa so compressed solid

$$s = \textbf{-1.2369 kJ/kg K}$$

Borgnakke and Sonntag

6.28

Determine the missing property among P, v, s, and x for CO_2 and indicate each state on a T-s diagram relative to the two-phase region.

   a. $T = -20^\circ C$, $P = 2000$ kPa
   b. $T = 20^\circ C$, $s = 1.49$ kJ/kgK
   c. $T = -10^\circ C$, $s = 1$ kJ/kgK

a) Table B.3.1 at $-20^\circ C$: $P > P_{sat} = 1969.6$ kPa   => Compressed liquid

$$v = v_f = 0.000969 \text{ m}^3/\text{kg}$$
$$s = s_f = 0.1672 \text{ kJ/kg-K}, \quad \text{x is undefined}$$

b) Table B.3.1: $s > s_g = 1.0406$ kJ/kg-K => superheated vapor

Table B.3.2 located between 1400 kPa and 2000 kPa

$$P = 1400 + 600 \frac{1.49 - 1.5283}{1.4438 - 1.5283} = 1400 + 600 \times 0.4532 = 1672 \text{ kPa}$$
$$v = 0.03648 + (0.02453 - 0.03648)\, 0.4532 = 0.03106 \text{ m}^3/\text{kg}$$

c) Table B.3.1: $0.2501 = s_f < s < s_g = 1.2328$ kJ/kgK   => Two-phase

$$x = \frac{s - s_f}{s_{fg}} = \frac{1 - 0.2501}{0.9828} = 0.763; \quad P = P_{sat} = 2648.7 \text{ kPa}$$

$$v = v_f + x\, v_{fg} = 0.001017 + 0.763 \times 0.01303 = 0.01096 \text{ m}^3/\text{kg}$$

Borgnakke and Sonntag

6.29

Determine the missing property among (T, P, v, s)

        a. H_2O 20^oC, v = 0.01000 m^3/kg

        b.   R-410A   400 kPa, s = 1.17 kJ/kgK

        c. NH_3 10^oC, v = 0.1 m^3/kg

        d. N_2 101.3 kPa,   s = 3.5 kJ/kg-K

a)   Table B.1.1 at 20^oC: $v < v_f = 0.001002$ m^3/kg => Compr. Liquid

       B.1.4:   at about **5000 kPa,**   s = **0.2955 kJ/kg-K**

b) Table B.4.1: $s > s_g = 1.0779$ kJ/kg-K => Sup. vapor

   Table B.4.2: $T = 0 + 20 \times \dfrac{1.17 - 1.1483}{1.2108 - 1.1483} = 20 \times 0.3472 = \mathbf{6.94^oC}$

              $v = 0.07227 + (0.07916 - 0.07227)\,0.3472 = \mathbf{0.07466\ m^3/kg}$

c) Table B.2.1: $v_f < v < v_g = 0.20541$ so two-phase L + V, P = **615.2 kPa**

           $x = \dfrac{v - v_f}{v_{fg}} = \dfrac{0.1 - 0.0016}{0.20381} = 0.4828$

           $s = s_f + x\,s_{fg} = 0.8778 + x \times 4.3266 = \mathbf{2.9667\ kJ/kg\text{-}K}$

d) Table B.6.1: $s_f < s < s_g = 5.4033$ kJ/kgK so two-phase, T = **77.3 K**

           $x = \dfrac{s - s_f}{s_{fg}} = \dfrac{3.5 - 2.8326}{2.5707} = 0.25962$

           $v = v_f + x\,v_{fg} = 0.00124 + x \times 0.21515 = \mathbf{0.0571\ m^3/kg}$

Borgnakke and Sonntag

6.30

Two kg water at 120°C with a quality of 25% has its temperature raised 20°C in a constant volume process. What are the new quality and specific entropy?

Solution:

State 1 from Table B.1.1 at 120°C

$$v = v_f + x\, v_{fg} = 0.001060 + 0.25 \times 0.8908 = 0.22376 \text{ m}^3/\text{kg}$$

State 2 has same v at 140°C also from Table B.1.1

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.22376 - 0.00108}{0.50777} = \mathbf{0.4385}$$

$$s = s_f + x\, s_{fg} = 1.739 + 0.4385 \times 5.1908 = \mathbf{4.015 \text{ kJ/kg K}}$$

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6.31

Two kg water at 400 kPa with a quality of 25% has its temperature raised 20°C in a constant pressure process. What is the change in entropy?

Solution:

State 1 from Table B.1.2 at 400 kPa

$$s = s_f + x\, s_{fg} = 1.7766 + 0.25 \times 5.1193 = 3.0564 \text{ kJ/kg}$$

State 2 has same P from Table B.1.2 at 400 kPa

$$T_2 = T_{sat} + 20 = 143.63 + 20 = 163.63^{O}C$$

so state is superheated vapor look in B.1.3 and interpolate between 150 and 200 C in the 400 kPa superheated vapor table.

$$s_2 = 6.9299 + (7.1706 - 6.9299)\, \frac{163.63 - 150}{200 - 150} = 6.9955 \text{ kJ/kgK}$$

$$s_2 - s_1 = 6.9955 - 3.0564 = \textbf{3.9391 kJ/kgK}$$

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6.32

Saturated liquid water at 20°C is compressed to a higher pressure with constant temperature. Find the changes in u and s when the final pressure is
a. 500 kPa   b. 2000 kPa   c. 20 000 kPa

Solution:

		kJ/kg		kJ/kg K		
B.1.1:	$u_1 = 83.94$		$s_1 = 0.2966$			
B.1.4:	$u_a = 83.91$		$s_a = 0.2965$		$\Delta u = -0.03$	$\Delta s = -0.0001$
B.1.4:	$u_b = 83.82$		$s_b = 0.2962$		$\Delta u = -0.12$	$\Delta s = -0.0004$
B.1.4:	$u_c = 82.75$		$s_c = 0.2922$		$\Delta u = -1.19$	$\Delta s = -0.0044$

Nearly constant u and s, incompressible media

Borgnakke and Sonntag

6.33

Saturated vapor water at 250°C is expanded to a lower pressure with constant temperature. Find the changes in u and s when the final pressure is

a. 100 kPa   b. 50 kPa   c. 10 kPa

Solution:

Table B.1.1 for the first state then B.1.3 for the a, b and c states.

kJ/kg	kJ/kg K	kJ/kg	kJ/kg K
$u_1 = 2602.37$	$s_1 = 6.0729$		
$u_a = 2733.73$	$s_a = 8.0332$	$\Delta u = 131.36$	$\Delta s = 1.9603$
$u_b = 2734.97$	$s_b = 8.3555$	$\Delta u = 132.6$	$\Delta s = 2.2826$
$u_c = 2735.95$	$s_c = 9.1002$	$\Delta u = 133.58$	$\Delta s = 3.0273$

Remark: You approach ideal gas as P drops so u is u(T) but s is still s(T,P).

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6.34

Determine the missing property among *P, T, s, x* for the following states:

    a. Ammonia 25°C, $v = 0.10$ m^3/kg

    b. Ammonia 1000 kPa, s = 5.2 kJ/kg K

    c. R-410A 500 kPa, s = 1.4 kJ/kg K

    d. R-410A 50°C, s = 0.8 kJ/kg K

Solution:

	Table	P kPa	T $^{\circ}$C	s kJ/kg K	x
a)	B2.1	1003	25	4.1601	0.7776
b)	B2.2	1000	42.53	5.2	-----
c)	B4.2	500	100	1.4	-----
d)	B4.1	3065	50	0.8	0.6973

a) $x = (0.1 - 0.001658)/0.12647 =$ **0.7776**

       $s = s_f + x \, s_{fg} = 1.121 + x \times 3.9083 =$ **4.1601 kJ/kg K**

b) $T = 40 + 10 \times (5.2 - 5.1778)/(5.2654 - 5.1778) =$ **42.53°C**

           superheated vapor so x is undefined

c) $s > s_g = 1.0647$ so superheated vapor found close to 100°C

d) $s_f < s < s_g$ so two-phase $P = P_{sat} = 3065.2$ kPa

       $x = (0.8 - 0.5067)/0.4206 = 0.69734$

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Reversible processes

Borgnakke and Sonntag

6.35

In a Carnot engine with ammonia as the working fluid, the high temperature is 60°C and as Q_H is received, the ammonia changes from saturated liquid to saturated vapor. The ammonia pressure at the low temperature is 190 kPa. Find T_L, the cycle thermal efficiency, the heat added per kilogram, and the entropy, *s*, at the beginning of the heat rejection process.

Solution:

Constant T $\Rightarrow$ constant P from 1 to 2, Table B.2.1

$$q_H = \int T ds = T (s_2 - s_1) = T\, s_{fg}$$

$$= h_2 - h_1 = h_{fg} = \textbf{997.0 kJ/kg}$$

States 3 & 4 are two-phase, Table B.2.1

$$\Rightarrow \quad T_L = T_3 = T_4 = T_{sat}(P) = \textbf{–20°C}$$

$$\eta_{cycle} = 1 - \frac{T_L}{T_H} = 1 - \frac{253.2}{333.2} = \textbf{0.24}$$

Table B.2.1: $\quad s_3 = s_2 = s_g(60°C) = \textbf{4.6577 kJ/kg K}$

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6.36
　　Consider a Carnot-cycle heat pump with R-410A as the working fluid. Heat is rejected from the R-410A at 35°C, during which process the R-410A changes from saturated vapor to saturated liquid. The heat is transferred to the R-410A at 0°C.
　　a. Show the cycle on a *T–s* diagram.
　　b. Find the quality of the R-410A at the beginning and end of the isothermal heat addition process at 0°C.
　　c. Determine the COP for the cycle.

Solution:

a)

b) From Table B.4.1, state 3 is saturated liquid

$$s_4 = s_3 = 0.4189 \text{ kJ/kg K}$$
$$= 0.2264 + x_4(0.8104)$$
$$\Rightarrow \quad x_4 = \mathbf{0.2375}$$

State 2 is saturated vapor so from Table B.4.1
$$s_1 = s_2 = 0.9671 \text{ kJ/kg K} = 0.2264 + x_1(0.8104)$$

$$\Rightarrow \quad x_1 = \mathbf{0.914}$$

c)
$$\beta' = \frac{q_H}{w_{IN}} = \frac{T_H}{T_H - T_L} = \frac{308.15}{35} = \mathbf{8.8}$$

T-s diagram
from CATT3
for R-410A

6.37
Do Problem 6.36 using refrigerant R-134a instead of R-410A.

Consider a Carnot-cycle heat pump with R-410A as the working fluid. Heat is rejected from the R-410A at 35°C, during which process the R-410A changes from saturated vapor to saturated liquid. The heat is transferred to the R-410A at 0°C.
  a. Show the cycle on a *T–s* diagram.
        b. Find the quality of the R-410A at the beginning and end of the
              isothermal heat addition process at 0°C.
  c. Determine the coefficient of performance for the cycle.

Solution:

a)

b) From Table B.5.1, state 3 is
       saturated liquid

$$s_4 = s_3 = 1.1673 \text{ kJ/kg K}$$
$$= 1.0 + x_4(0.7262)$$

$$=> \qquad x_4 = \mathbf{0.2303}$$

State 2 is saturated vapor so from Table B.5.1
$$s_1 = s_2 = 1.7139 \text{ kJ/kg K} = 1.0 + x_1(0.7262)$$
$$=> \qquad x_1 = \mathbf{0.983}$$

c) $$\beta' = \frac{q_H}{w_{IN}} = \frac{T_H}{T_H - T_L} = \frac{308.15}{35} = \mathbf{8.8}$$

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6.38

Water is used as the working fluid in a Carnot cycle heat engine, where it changes from saturated liquid to saturated vapor at 200°C as heat is added. Heat is rejected in a constant pressure process (also constant T) at 20 kPa. The heat engine powers a Carnot cycle refrigerator that operates between –15°C and +20°C. Find the heat added to the water per kg water. How much heat should be added to the water in the heat engine so the refrigerator can remove 1 kJ from the cold space?

Solution:

Carnot cycle heat engine:

Constant T $\Rightarrow$ constant P from 1 to 2, Table B.2.1

$$q_H = \int T ds = T\,(s_2 - s_1) = T\,s_{fg} = h_{fg}$$

$$= 473.15\,(4.1014) = \textbf{1940 kJ/kg}$$

States 3 & 4 are two-phase, Table B.2.1

$$\Rightarrow \quad T_L = T_3 = T_4 = T_{sat}(P) = 60.06\,^oC$$

Carnot cycle refrigerator (T_L and T_H are different from above):

$$\beta_{ref} = \frac{Q_L}{W} = \frac{T_L}{T_H - T_L} = \frac{273 - 15}{20 - (-15)} = \frac{258}{35} = 7.37$$

$$W = \frac{Q_L}{\beta} = \frac{1}{7.37} = 0.136 \text{ kJ}$$

The needed work comes from the heat engine

$$W = \eta_{HE}\, Q_{H\,H2O} \; ; \qquad \eta_{HE} = 1 - \frac{T_L}{T_H} = 1 - \frac{333}{473} = 0.296$$

$$Q_{H\,H2O} = \frac{W}{\eta_{HE}} = \frac{0.136}{0.296} = \textbf{0.46 kJ}$$

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6.39

Water at 1 MPa, 250°C is expanded in a piston/cylinder to 200 kPa, x = 1.0 in a reversible process. Find the sign for the work and the sign for the heat transfer.

Solution:

The process is not specified, but the beginning and end states are and we assume a unidirectional process so v keeps decreasing.

State 1: Table B.1.3:

$$v_1 = 0.23268 \text{ m}^3/\text{kg}; \quad u_1 = 2709.9 \text{ kJ/kg}; \quad s_1 = 6.9246 \text{ kJ/kg K}$$

State 2: Table B.1.1:

$$v_2 = 0.8857 \text{ m}^3/\text{kg}; \quad u_2 = 2529.5 \text{ kJ/kg}; \quad s_2 = 7.1271 \text{ kJ/kg K}$$

Reversible process: $dw = P \, dv$ $\qquad dq = T \, ds$

$$v_2 > v_1 \quad => \qquad {}_1w_2 = \int P \, dv \; > 0$$

$$s_2 > s_1 \quad => \qquad {}_1q_2 = \int T \, ds \; > 0$$

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6.40

R-410A at 1 MPa and 60°C is expanded in a piston cylinder to 500 kPa, 40°C in a reversible process. Find the sign for both the work and the heat transfer for this process.

Solution:

The process is not specified, but the beginning and end states are and we assume a unidirectional process so v keeps increasing.

$$_1w_2 = \int P \, dv \qquad \text{so sign } dv$$

$$_1q_2 = \int T \, ds \qquad \text{so sign } ds$$

State 1: B.4.2 $v_1 = 0.03470 \text{ m}^3/\text{kg}$ $s_1 = 1.2019 \text{ kJ/kg K}$

State 2: B.4.2 $v_2 = 0.06775 \text{ m}^3/\text{kg}$ $s_2 = 1.2398 \text{ kJ/kg K}$

      $dv > 0$ =>   **w is positive**

      $ds > 0$ =>   **q is positive**

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6.41

A piston/cylinder compressor takes R-410A as saturated vapor 500 kPa and compresses it in a reversible adiabatic process to 3000 kPa. Find the final temperature and the specific compression work.

CV R-410A, this is a control mass

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.6.3: $s_2 - s_1 = \int dq/T = {}_1q_2 /T$

Process: Adiabatic and reversible $\Rightarrow$ ${}_1q_2 = 0$ so then $s_2 = s_1$

State 1: P_1, x = 1, $u_1 = 248.29$ kJ/kg, $s_1 = 1.0647$ kJ/kgK

State 2: P_2, $s_2 = s_1$

$$T_2 = 60 + 20 \times \frac{1.0647 - 0.9933}{1.0762 - 0.9933} = 60 + 20 \times 0.8613 = \textbf{77.2}^\textbf{o}\textbf{C}$$

$$u_2 = 274.96 + (298.38 - 274.96) \times 0.8613 = 295.13 \text{ kJ/kg}$$

Now the work becomes

$${}_1w_2 = u_1 - u_2 = 248.29 - 295.13 = \textbf{-46.84 kJ/kg}$$

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6.42

A piston/cylinder receives R-410A at 500 kPa and compresses it in a reversible adiabatic process to 1800 kPa, 60$^{\text{o}}$C. Find the initial temperature.

CV R-410A, this is a control mass

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.6.3: $s_2 - s_1 = \int dq/T = {}_1q_2/T$

Process: Adiabatic and reversible $\Rightarrow$ $_1q_2 = 0$ so then $s_2 = s_1$

State 1: P_1, $s_1 = s_2 = 1.1076$ kJ/kgK =>

$$T_1 = -13.89 + 13.89 \times \frac{1.1076 - 1.0647}{1.1155 - 1.0647} = \mathbf{-2.16^oC}$$

Borgnakke and Sonntag

6.43

Compression and heat transfer brings carbon dioxide in a piston/cylinder from 1400 kPa, 20°C to saturated vapor in an isothermal process. Find the specific heat transfer and the specific work.

Solution:

m = constant

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.6.3: $s_2 - s_1 = \int dq/T = {}_1q_2 /T$

Process: T = C and assume reversible $\Rightarrow$ ${}_1q_2 = T (s_2 - s_1)$

State 1:   Table B.4.2:
   $u_1 = 259.18$ kJ/kg,
   $s_1 = 1.0057$ kJ/kg K

State 2:   Table B.4.1
   $u_2 = 258.16$ kJ/kg,
   $s_2 = 0.9984$ kJ/kg K

${}_1q_2 = (273 + 20) \times (0.9984 - 1.0057) = $ **–2.14 kJ/kg**

${}_1w_2 = {}_1q_2 + u_1 - u_2 = -2.14 + 259.18 - 258.16$
           $= $ **–1.12 kJ/kg**

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6.44

A piston cylinder maintaining constant pressure contains 0.1 kg saturated liquid water at 100°C. It is now boiled to become saturated vapor in a reversible process. Find the work term and then the heat transfer from the energy equation. Find the heat transfer from the entropy equation, is it the same?

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m(s_2 - s_1) = \int \dfrac{dq}{T} + 0 = \dfrac{{}_1Q_2}{T}$

Process: $P = C \Rightarrow {}_1W_2 = m\,P(v_2 - v_1) = m\,P\,v_{fg}$

$$= 0.1 \text{ kg} \times 101.3 \text{ kPa} \times 1.67185 \text{ m}^3/\text{kg} = \mathbf{16.936 \text{ kJ}}$$

From the energy equation we get

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m\,u_{fg} + {}_1W_2$$

$$= 0.1 \times 2087.58 + 16.936 = 225.7 \text{ kJ}$$

$$\text{or} \quad = m(h_2 - h_1) = m\,h_{fg} = 0.1 \times 2257.03 = \mathbf{225.7 \text{ kJ}}$$

From the entropy equation we can get

$${}_1Q_2 = mT(s_2 - s_1) = m\,T\,s_{fg} = 0.1 \times 373.15 \times 6.048 = \mathbf{225.68 \text{ kJ}}$$

So they are equal to within round off errors.

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6.45

A piston cylinder contains 0.25 kg of R-134a at 100 kPa. It will be compressed in an adiabatic reversible process to 400 kPa and should be 70°C. What should the initial temperature be?

C.V. R-134a which is a control mass.

Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T = 0$

State 2: $s_2 = s_1 = 1.9051$ kJ/kgK

Work backwards from state 2 to state 1

State 1: 100 kPa & s_1 => $T_1 = $ **26.4°C**

Borgnakke and Sonntag

6.46

A piston/cylinder contains 0.5 kg of water at 200 kPa, 300°C, and it now cools to 150°C in an isobaric process. The heat goes into a heat engine that rejects heat to the ambient at 25°C (shown in Fig. P.6.46), and the whole process is assumed to be reversible. Find the heat transfer out of the water and the work given out by the heat engine.

C.V. H_2O

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T + 0$

Process: $P = C \Rightarrow W = \int P \, dV = P(V_2 - V_1) = m \, P \, (v_2 - v_1)$

State 1: B.1.3 $s_1 = 7.8926$ kJ/kg-K, $h_1 = 3071.79$ kJ/kg

State 2: B.1.3 $s_2 = 7.2795$ kJ/kg K, $h_2 = 2768.8$ kJ/kg

From the process equation and the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) = 0.5(2768.8 - 3071.79)$$

$$= -\textbf{151.495 kJ}$$

CV Total

Energy Eq.3.5: $m(u_2 - u_1) = - Q_L - {}_1W_2 - W_{HE}$

Entropy Eq.6.3: $m(s_2 - s_1) = - Q_L/T_{amb} + 0$

$$Q_L = mT_{amb}(s_1 - s_2) = 0.5 \text{ kg } 298.15 \text{ K } (7.8926 - 7.2795) \text{ kJ/kgK}$$

$$= 91.398 \text{ kJ}$$

Now the energy equation for the heat engine gives

$$W_{HE} = - {}_1Q_2 - Q_L = 151.495 - 91.398 = \textbf{60.1 kJ}$$

Borgnakke and Sonntag

6.47

A cylinder fitted with a piston contains ammonia at 50°C, 20% quality with a volume of 1 L. The ammonia expands slowly, and during this process heat is transferred to maintain a constant temperature. The process continues until all the liquid is gone. Determine the work and heat transfer for this process.

Solution:

C.V. Ammonia in the cylinder.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T + 0$

Process: T = constant to $x_2 = 1.0$, P = constant = 2.033 MPa

=> $W = \int P\, dV = P(V_2 - V_1) = m\, P\, (v_2 - v_1)$

=> $\int dQ/T = {}_1Q_2 / T$

Table B.2.1: $T_1 = 50°C$, $x_1 = 0.20$, $V_1 = 1$ L

$v_1 = 0.001777 + 0.2 \times 0.06159 = 0.014095$ m^3/kg

$s_1 = 1.5121 + 0.2 \times 3.2493 = 2.1620$ kJ/kg K

$m = V_1/v_1 = 0.001/0.014095 = 0.071$ kg

$v_2 = v_g = 0.06336$ m^3/kg,

$s_2 = s_g = 4.7613$ kJ/kg K

${}_1W_2 = Pm(v_2 - v_1) = 2033 \times 0.071 \times (0.06336 - 0.014095) = $ **7.11 kJ**

From the entropy equation

${}_1Q_2 = Tm (s_2 - s_1) = 323.2$ K $\times 0.071$ kg $(4.7613 - 2.1620)$ kJ/kg-K

= **59.65 kJ**

or ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$

$h_1 = 421.48 + 0.2 \times 1050.01 = 631.48$ kJ/kg, $h_2 = 1471.49$ kJ/kg

${}_1Q_2 = 0.071$ kg $(1471.49 - 631.48)$ kJ/kg = **59.65 kJ**

Borgnakke and Sonntag

6.48

Water in a piston/cylinder at 400$^{\text{o}}$C, 2000 kPa is expanded in a reversible adiabatic process. The specific work is measured to be 415.72 kJ/kg out. Find the final P and T and show the P-v and the T-s diagram for the process.

Solution:

C.V. Water, which is a control mass. Adiabatic so: $_1q_2 = 0$

Energy Eq.3.5: $u_2 - u_1 = {_1q_2} - {_1w_2} = -{_1w_2}$

Entropy Eq.6.3: $s_2 - s_1 = \int dq/T = 0$ (= since reversible)

State 1: Table B.1.3 $u_1 = 2945.21$ kJ/kg; $s_1 = 7.127$ kJ/kg K

State 2:   (s, u): $u_2 = u_1 - {_1w_2} = 2945.21 - 415.72 = 2529.49$ kJ/kg

=> **sat. vapor 200 kPa,   T = 120.23°C**

Borgnakke and Sonntag

6.49

A piston/cylinder has 2 kg water at 1000 kPa, 200°C which is now cooled with a constant loading on the piston. This isobaric process ends when the water has reached a state of saturated liquid. Find the work and heat transfer and sketch the process in both a P-v and a T-s diagram.

Solution:

C.V. H_2O

Energy Eq.3.5: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T$

Process: $P = C \implies W = \int P \, dV = P(V_2 - V_1)$

State 1: B.1.3 $v_1 = 0.20596 \text{ m}^3/\text{kg}$, $s_1 = 6.6939 \text{ kJ/kg-K}$, $u_1 = 2621.90 \text{ kJ/kg}$

State 2: B.1.2 $v_2 = 0.001127 \text{ m}^3/\text{kg}$, $s_2 = 2.1386 \text{ kJ/kg K}$, $u_2 = 761.67 \text{ kJ/kg}$

From the process equation

$${_1}W_2 = m \, P \, (v_2 - v_1) = 2 \times 1000 \, (0.001127 - 0.20596) = \textbf{-409.7 kJ}$$

From the energy equation we get

$${_1}Q_2 = m(u_2 - u_1) + {_1}W_2 = 2 \, (761.67 - 2621.90) - 409.7 = \textbf{-4130.2 kJ}$$

Borgnakke and Sonntag

6.50

One kilogram of water at 300°C expands against a piston in a cylinder until it reaches ambient pressure, 100 kPa, at which point the water has a quality of 90.2%. It may be assumed that the expansion is reversible and adiabatic. What was the initial pressure in the cylinder and how much work is done by the water?

Solution:

C.V. Water. Process: Rev., $Q = 0$

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = -{}_1W_2$

Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T$

Process: Adiabatic $Q = 0$ and reversible $=>$ $s_2 = s_1$

State 2: $P_2 = 100$ kPa, $x_2 = 0.902$ from Table B.1.2

$s_2 = 1.3026 + 0.902 \times 6.0568 = 6.7658$ kJ/kg K

$u_2 = 417.36 + 0.902 \times 2088.7 = 2301.4$ kJ/kg

State 1   At $T_1 = 300°C$, $s_1 = 6.7658$ Find it in Table B.1.3

$\Rightarrow$ $P_1 = \mathbf{2000\ kPa,}$ $u_1 = 2772.6$ kJ/kg

From the energy equation

$_1W_2 = m(u_1 - u_2) = 1(2772.6 - 2301.4) = \mathbf{471.2\ kJ}$

Borgnakke and Sonntag

6.51

Water at 1000 kPa, 250°C is brought to saturated vapor in a rigid container, shown in Fig. P8.54. Find the final T and the specific heat transfer in this isometric process.

Solution:

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.6.3: $s_2 - s_1 = \int dq/T$

Process: $v = $ constant $\Rightarrow$ ${}_1w_2 = 0$

State 1: (T, P) Table B.1.3 $u_1 = 2709.91$ kJ/kg, $v_1 = 0.23268$ m^3/kg

State 2: $x = 1$ and $v_2 = v_1$ so from Table B.1.1 we see $P_2 \cong 800$ kPa

$\quad T_2 = 170 + 5 \times (0.23268 - 0.24283)/(0.2168 - 0.24283)$

$\qquad = 170 + 5 \times 0.38993 = $ **171.95°C**

$u_2 = 2576.46 + 0.38993 \times (2580.19 - 2576.46) = 2577.9$ kJ/kg

From the energy equation

$\quad {}_1q_2 = u_2 - u_1 = 2577.9 - 2709.91 = $ **−132 kJ/kg**

Notice to get ${}_1q_2 = \int T\,ds$ we must know the function T(s) which we do not readily have for this process.

Borgnakke and Sonntag

6.52

Estimate the specific heat transfer from the area in the T-s diagram and compare it to the correct value for the states and process in Problem 6.51.

Solution:

Energy Eq.3.5: $u_2 - u_1 = {_1}q_2 - {_1}w_2$

Entropy Eq.6.3: $s_2 - s_1 = \int dq/T$

Process: $v = $ constant $\Rightarrow$ ${_1}w_2 = 0$

State 1: (T, P) Table B.1.3 $u_1 = 2709.91$ kJ/kg, $v_1 = 0.23268$ m^3/kg,

$s_1 = 6.9246$ kJ/kg K

State 2: $x = 1$ and $v_2 = v_1$ so from Table B.1.1 we see $P_2 \cong 800$ kPa

$T_2 = 170 + 5 \times (0.23268 - 0.24283)/(0.2168 - 0.24283)$

$= 170 + 5 \times 0.38993 = $ **171.95°C**

$u_2 = 2576.46 + 0.38993 \times (2580.19 - 2576.46) = 2577.9$ kJ/kg

$s_2 = 6.6663 + 0.38993 (6.6256 - 6.6663) = 6.6504$ kJ/kg K

From the energy equation

${_1}q_{2 \text{ actual}} = u_2 - u_1 = 2577.9 - 2709.91 = $ **−132 kJ/kg**

Assume a linear variation of T versus s.

$_1q_2 = \int T\, ds = \text{area} \cong \frac{1}{2}(T_1 + T_2)(s_2 - s_1)$

$= \frac{1}{2}(171.95 + (2 \times 273.15) + 250)$ K $(6.6504 - 6.9246)$ kJ/kg-K

$= $ **-132.74 kJ/kg**

very close i.e. the $v = C$ curve is close to a straight line in the T-s diagram. Look at the constant v curves in Fig. E.1. In the two-phase region they curve slightly and more so in the region above the critical point.

Borgnakke and Sonntag

6.53

A closed tank, $V = 10$ L, containing 5 kg of water initially at 25°C, is heated to 150°C by a heat pump that is receiving heat from the surroundings at 25°C. Assume that this process is reversible. Find the heat transfer to the water and the change in entropy.

Solution:

C.V.: Water from state 1 to state 2.

Energy Eq.3.5: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T$

Process: constant volume (reversible isometric)   so ${_1}W_2 = 0$

State 1: $v_1 = V/m = 0.002$ from Table B.1.1

$x_1 = (0.002 - 0.001003)/43.358 = 0.000023$

$u_1 = 104.86 + 0.000023 \times 2304.9 = 104.93$ kJ/kg

$s_1 = 0.3673 + 0.000023 \times 8.1905 = 0.36759$ kJ/kg K

Continuity eq. (same mass) and $V = C$ fixes v_2

State 2: $T_2, v_2 = v_1$ so from Table B.1.1

$x_2 = (0.002 - 0.001090)/0.39169 = 0.0023233$

$u_2 = 631.66 + 0.0023233 \times 1927.87 = 636.14$ kJ/kg

$s_2 = 1.8417 + 0.0023233 \times 4.9960 = 1.8533$ kJ/kg K

Energy eq. has $W = 0$, thus provides heat transfer as

${_1}Q_2 = m(u_2 - u_1) = $ **2656.05 kJ**

The entropy change becomes

$m(s_2 - s_1) = 5(1.8533 - 0.36759) = $ **7.4286 kJ/K**

Notice we do not perform the integration $\int dQ/T$ to find change in s as the equation for the dQ as a function of T is not known.

Borgnakke and Sonntag

6.54

A piston/cylinder has 2 kg of R-410A at 60°C, 100 kPa which is compressed to 1000 kPa. The process happens so slowly that the temperature is constant. Find the heat transfer and work for the process assuming it to be reversible.

Solution:

CV : R-410A   Control Mass

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$;

Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T$

Process: T = constant and assume reversible process

1: (T,P), Table B.4.2: $v_1 = 0.37833 \text{ m}^3/\text{kg}$, $u_1 = 309.4 \text{ kJ/kg}$,

$s_1 = 1.4910 \text{ kJ/kg K}$

2: (T,P), Table B.4.2: $v_2 = 0.03470 \text{ m}^3/\text{kg}$, $u_2 = 301.04 \text{ kJ/kg}$,

$s_2 = 1.2019 \text{ kJ/kg K}$

From the entropy equation (2nd law)

$${}_1Q_2 = mT(s_2 - s_1) = 2 \text{ kg} \times 333.15 \text{ K} \times (1.2019 - 1.4910) \text{ kJ/kg-K}$$

$$= -\textbf{192.63 kJ}$$

From the energy equation

$${}_1W_2 = {}_1Q_2 - m(u_2 - u_1) = -192.63 \text{ kJ} - 2 \text{ kg} \times (301.04 - 309.4) \text{ kJ/kg}$$

$$= -\textbf{175.9 kJ}$$

Borgnakke and Sonntag

6.55

A heavily insulated cylinder/piston contains ammonia at 1200 kPa, 60°C. The piston is moved, expanding the ammonia in a reversible process until the temperature is −20°C. During the process 200 kJ of work is given out by the ammonia. What was the initial volume of the cylinder?

C.V. ammonia. Control mass with no heat transfer.

State 1: Table B.2.2 $v_1 = 0.1238$ m^3/kg, $s_1 = 5.2357$ kJ/kg K

$u_1 = 1404.8$ kJ/kg

Entropy Eq.: $m(s_2 - s_1) = \int dQ/T + \,_1S_{2\,gen}$

Process: reversible ($_1S_{2\,gen} = 0$) and adiabatic (dQ = 0)   => $s_2 = s_1$

State 2: T_2, s_2 $\Rightarrow$ $x_2 = (5.2357 - 0.3657)/5.2498 = 0.928$

$u_2 = 88.76 + 0.928 \times 1210.7 = 1211.95$ kJ/kg

$_1Q_2 = 0 = m(u_2 - u_1) + \,_1W_2 = m(1211.95 - 1404.8) + 200$

$\Rightarrow$ m = 1.037 kg

$V_1 = mv_1 = 1.037 \times 0.1238 = $ **0.1284 m^3**

Borgnakke and Sonntag

6.56

Water at 1000 kPa, 250°C is brought to saturated vapor in a piston/cylinder with an isothermal process. Find the specific work and heat transfer. Estimate the specific work from the area in the P-v diagram and compare it to the correct value.

Solution:

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5 $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.3: $\quad m(s_2 - s_1) = \int dQ/T$

Process: T = constant, reversible

State 1: Table B.1.3:

$\quad v_1 = 0.23268 \ \text{m}^3/\text{kg}; \quad u_1 = 2709.91 \ \text{kJ/kg}; \quad s_1 = 6.9246 \ \text{kJ/kg K}$

State 2: (T, x) Table B.1.1 $\quad P_2 = 3973 \ \text{kPa}$

$\quad v_2 = 0.05013 \ \text{m}^3/\text{kg}, \quad u_2 = 2602.37 \ \text{kJ/kg}, \quad s_2 = 6.0729 \ \text{kJ/kg K}$

From the entropy equation

$${}_1q_2 = \int T \ ds = T(s_2 - s_1) = (250 + 273) \ (6.0729 - 6.9246) = \textbf{-445.6 kJ/kg}$$

From the energy equation

$${}_1w_2 = {}_1q_2 + u_1 - u_2 = -445.6 + 2709.91 - 2602.37 = \textbf{-338 kJ/kg}$$

Estimation of the work term from the area in the P-v diagram

$${}_1w_{2 \ area} \cong \frac{1}{2} (P_1 + P_2)(v_2 - v_1) = \frac{1}{2}(1000 + 3973)(0.05013 - 0.23268)$$

$$= -454 \ \text{kJ/kg}$$

Not extremely accurate estimate; P-v curve not linear more like Pv = constant as curve has positive curvature the linear variation over-estimates area.

Borgnakke and Sonntag

6.57

A rigid, insulated vessel contains superheated vapor steam at 3 MPa, 400°C. A valve on the vessel is opened, allowing steam to escape. The overall process is irreversible, but the steam remaining inside the vessel goes through a reversible adiabatic expansion. Determine the fraction of steam that has escaped, when the final state inside is saturated vapor.

C.V.: steam remaining inside tank. Rev. & Adiabatic (inside only)

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Entropy Eq.: $m(s_2 - s_1) = \int dQ/T + {_1}S_{2\ gen}$

Rev $({_1}S_{2\ gen} = 0)$ Adiabatic $(Q = 0)$ $\Rightarrow$ $s_2 = s_1 = 6.9212 = s_G$ at T_2

$\Rightarrow$ $T_2 = 141°C$, $v_2 = v_{g\ at\ T_2} = 0.4972\ m^3/kg$

$$\frac{m_e}{m_1} = \frac{m_1 - m_2}{m_1} = 1 - \frac{m_2}{m_1} = 1 - \frac{v_1}{v_2} = 1 - \frac{0.09936}{0.4972} = \mathbf{0.80}$$

Borgnakke and Sonntag

6.58

Water at 100 kPa, 25°C is brought to the boiling point in a piston/cylinder with an isobaric process. The heat is supplied by a heat pump with the cold side at the ambient temperature of 25°C. Assume that the whole process is reversible and find the work input to the heat pump per kg of water.

C.V. H_2O

Energy Eq.3.5: $\qquad u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.6.3: $\qquad s_2 - s_1 = \int dq/T \ + 0$

Process: $\qquad\qquad P = C \ \Rightarrow \ w = \int P \ dv \ = P(v_2 - v_1)$

State 1: B.1.1 $\quad s_1 = 0.3673$ kJ/kg-K, $h_1 = 104.87$ kJ/kg

State 2: B.1.2 $\quad s_2 = 1.3025$ kJ/kg K, $h_2 = 417.44$ kJ/kg

From the process equation and the energy equation

$\qquad {}_1q_2 = u_2 - u_1 + {}_1w_2 = h_2 - h_1 = 417.44 - 104.87 = 312.57$ kJ/kg

CV Total

Energy Eq.3.5: $\qquad u_2 - u_1 = q_L - {}_1w_2 + w_{HP}$

Entropy Eq.6.3: $\qquad s_2 - s_1 = \ q_L/T_{amb} \ + 0$

$\qquad\qquad q_L = T_{amb}(s_2 - s_1) = 298.15$ K $(1.3025 - 0.3673)$ kJ/kgK

$\qquad\qquad = 278.83$ kJ/kg

Now the energy equation for the heat engine gives

$\qquad\qquad w_{HP} = {}_1q_2 - q_L = 312.57 - 278.83 = $ **33.74 kJ/kg**

6.59

Water at 1000 kPa, 200°C is brought to saturated vapor in a piston/cylinder with an isobaric process. Find the specific work and heat transfer. Estimate the specific heat transfer from the area in the T-s diagram and compare it to the correct value.

Solution:

C.V. H_2O

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.6.3: $s_2 - s_1 = \int dq/T$

Process: $P = C$ => $w = \int P\, dv = P(v_2 - v_1)$

State 1: B.1.3 $v_1 = 0.20596$ m^3/kg, $s_1 = 6.6939$ kJ/kg-K, $u_1 = 2621.90$ kJ/kg

State 2: B1.3 $v_2 = 0.19444$ m^3/kg, $s_2 = 6.5864$ kJ/kg K, $u_2 = 2583.64$ kJ/kg

$T_2 = 179.91$°C

From the process equation

$_1w_2 = P\,(v_2 - v_1) = 1000$ kPa $(0.19444 - 0.20596)$ m^3/kg = **-11.52 kJ/kg**

From the energy equation

$_1q_2 = u_2 - u_1 + {}_1w_2 = 2583.64 - 2621.90 - 11.52 =$ **-49.78 kJ/kg**

Now estimate the heat transfer from the T-s diagram.

$_1q_2 = \int T\, ds = \text{AREA} \cong \frac{1}{2}(T_1 + T_2)(s_2 - s_1)$

$= \frac{1}{2}(200 + 179.91 + 2 \times 273.15)$ K $\times (6.5864 - 6.6939)$ kJ/kg-K

$= 463.105 \times (-0.1075) =$ **-49.78 kJ/kg**

very close approximation. The P = C curve in the T-s diagram is nearly a straight line. Look at the constant P curves on Fig.E.1. Up over the critical point they curve significantly.

Borgnakke and Sonntag

Entropy of a liquid or a solid

Borgnakke and Sonntag

6.60

Two 5 kg blocks of steel, one at 250°C the other at 25°C, come in thermal contact. Find the final temperature and the change in entropy of the steel?

C.V. Both blocks, no external heat transfer, C from Table A.3.

Energy Eq.: $U_2 - U_1 = m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0 - 0$
$$= m_A C(T_2 - T_{A1}) + m_B C(T_2 - T_{B1})$$

$$T_2 = \frac{m_A T_{A1} + m_B T_{B1}}{m_A + m_B} = \frac{1}{2} T_{A1} + \frac{1}{2} T_{B1} = \mathbf{137.5°C}$$

Entropy Eq.6.37: $\quad S_2 - S_1 = m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B = {}_1S_{2\ gen}$

Entropy changes from Eq.6.11

$$S_2 - S_1 = m_A C \ln \frac{T_2}{T_{A1}} + m_B C \ln \frac{T_2}{T_{B1}}$$

$$= 5 \times 0.46 \ln \frac{137.5 + 273.15}{250 + 273.15} + 5 \times 0.46 \ln \frac{137.5 + 273.15}{298.15}$$

$$= \text{-}0.5569 + 0.7363 = \mathbf{0.1794\ kJ/K}$$

B

A

Heat transfer over a finite temperature difference is an irreversible process

Borgnakke and Sonntag

6.61

A rigid tank of 1.2 kg steel contains 1.5 kg of R-134a at 40°C, 500 kPa. The tank is placed in a refrigerator that brings it to -20°C. Find the process heat transfer and the combined steel and R-134a change in entropy.

C.V. The steel tank and the R-134a.

The energy equation, Eq. 3.5 now becomes (summing over the mass)

$$m_{st} (u_2 - u_1)_{st} + m_{R134a} (u_2 - u_1)_{R134a} = {}_1Q_2 - 0$$

Process: No change in volume so no work as used above.

Use specific heat from Table A.3 for steel and Table B.5 for R-134a

R-134a: $v_1 = 0.04656$ m^3/kg, $u_1 = 407.44$ kJ/kg, $s_1 = 1.7971$ kJ/kgK

State 2: $v_2 = v_1 < v_g$ => $x_2 = (v_2 - v_f)/v_{fg} = \dfrac{0.04656 - 0.000738}{0.14576} = 0.314366$

$u_2 = u_f + x_2\, u_{fg} = 173.65 + x_2 \times 192.85 = 234.275$ kJ/kg

$s_2 = s_f + x_2\, s_{fg} = 0.9007 + x_2 \times 0.8388 = 1.16439$ kJ/kg-K

Now the heat transfer from the energy equation

$${}_1Q_2 = m_{R134a}(u_2 - u_1)_{R134a} + m_{st}\, C_{st}\, (T_2 - T_1)$$

$$= 1.5 \times (234.275 - 407.44) + 1.2 \times 0.46\,(-20 - 40) = \textbf{-292.87 kJ}$$

Steel: $m_{st}(s_2 - s_1)_{st} = m_{st}C_{st} \ln (T_2/T_1) = 1.2 \times 0.46 \ln \dfrac{253.15}{313.15} = -0.11741$ kJ/K

Entropy change for the total control volume steel and R-134a

$$S_2 - S_1 = m_{st}\, (s_2 - s_1)_{st} + m_{R134a}\, (s_2 - s_1)_{R134a}$$

$$= -0.11741 + 1.5(1.16439 - 1.7971) = - \textbf{1.066 kJ/K}$$

Borgnakke and Sonntag

6.62

A large slab of concrete, $5 \times 8 \times 0.3$ m, is used as a thermal storage mass in a solar-heated house. If the slab cools overnight from 23°C to 18°C in an 18°C house, what is the net entropy change associated with this process?

Solution:

C.V.: Control mass concrete.

$$V = 5 \times 8 \times 0.3 = 12 \text{ m}^3$$
$$m = \rho V = 2200 \times 12 = 26\ 400 \text{ kg}$$

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m(s_2 - s_1) = \dfrac{{}_1Q_2}{T_0} + {}_1S_{2\ gen}$

Process: V = constant   so ${}_1W_2 = 0$

Use heat capacity (Table A.3) for change in u of the slab

$\quad {}_1Q_2 = mC\Delta T = 26400 \text{ kg} \times 0.88 \text{ kJ/kg-K} \times (-5) \text{ K} = -116\ 160 \text{ kJ}$

We add all the storage changes as in Eq.6.39:

$$\Delta S_{slab} = m(s_2 - s_1) = m\ C \ln \frac{T_2}{T_1}$$

$$= 26400 \text{ kg} \times 0.88 \text{ kJ/kg-K} \times \ln \frac{291.2}{296.2} = \textbf{-395.5 kJ/K}$$

Borgnakke and Sonntag

6.63

A foundry form box with 25 kg of 200°C hot sand is dumped into a bucket with 50 L water at 15°C. Assuming no heat transfer with the surroundings and no boiling away of liquid water, calculate the net entropy change for the mass.

Solution:

C.V. Sand and water, constant pressure process

$$m_{sand}(u_2 - u_1)_{sand} + m_{H_2O}(u_2 - u_1)_{H_2O} = -P(V_2 - V_1)$$

$$\Rightarrow m_{sand}\Delta h_{sand} + m_{H_2O}\Delta h_{H_2O} = 0$$

For this problem we could also have said that the work is nearly zero as the solid sand and the liquid water will not change volume to any measurable extent. Now we get changes in u's instead of h's. For these phases $C_V = C_P = C$ which is a consequence of the incompressibility. Now the energy equation becomes

$$m_{sand}C_{sand}\Delta T_{sand} + m_{H_2O}C_{H_2O}\Delta T_{H_2O} = 0$$

25 kg × 0.8 kJ/kg-K ×(T_2 - 200) K

   + (50×10^{-3} m^3 / 0.001001 m^3/kg) × 4.184 kJ/kg-K × (T_2 - 15) K = 0

   T_2 = 31.2°C

$S_2 - S_1 = m_{sand}(s_2 - s_1) + m_{H2O} (s_2 - s_1)$

   $= m_{sand} C_{sand} \ln(T_2/T_1) + m_{H2O} C_{H2O} \ln(T_2/T_1)$

   $= 25 \times 0.8 \ln\left(\dfrac{304.3}{473.15}\right) + 49.95 \times 4.184 \ln\left(\dfrac{304.3}{288.15}\right) = \mathbf{2.57\ kJ/K}$

Box holds the sand for
form of the cast part

Borgnakke and Sonntag

6.64

Heat transfer to a block of 1.5 kg ice at -10$^{\text{o}}$C melts it to liquid at 10$^{\text{o}}$C in a kitchen. Find the entropy change of the water.

Water changes state from nearly saturated solid to nearly saturated liquid. The pressure is 101 kPa but we approximate the state properties with saturated state at the same temperature.

State 1: Compressed (saturated) solid, B.1.5, $s_1 = -1.2995$ kJ/kg-K

State 2: Compressed (saturated) liquid B.1.1 $s_2 = 0.1510$ kJ/kg-K

The entropy change is

$$\Delta s = s_2 - s_1 = 0.151 - (-1.2995) = \textbf{1.4505 kJ/kg-K}$$

Borgnakke and Sonntag

6.65

In a sink 5 liters of water at 70°C is combined with 1 kg aluminum pots, 1 kg of flatware (steel) and 1 kg of glass all put in at 20°C. What is the final uniform temperature and change in stored entropy neglecting any heat loss and work?

Energy Eq.: $\quad U_2 - U_1 = \sum m_i (u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$

Entropy Eq.: $\quad S_2 - S_1 = \int dQ/T + {}_1S_{2\text{ gen}}$

For the water: $v_f = 0.001023 \text{ m}^3/\text{kg}, \text{ V} = 5 \text{ L} = 0.005 \text{ m}^3; \quad m = V/v = 4.8876 \text{ kg}$

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i (u_2 - u_1)_i = \sum m_i C_{v\,i} (T_2 - T_1)_i = T_2 \sum m_i C_{v\,i} - \sum m_i C_{v\,i} T_{1\,i}$$

noticing that all masses have the same T_2 but not same initial T.

$$\sum m_i C_{v\,i} = 4.8876 \times 4.18 + 1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8 = 22.59 \text{ kJ/K}$$

Energy Eq.: $\quad 22.59 \text{ T}_2 = 4.8876 \times 4.18 \times 70 + (1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8) \times 20$

$$= 1430.11 + 43.2$$

$$T_2 = \mathbf{65.2°C}$$

$$S_2 - S_1 = \sum m_i (s_2 - s_1)_i = \sum m_i \, C_i \ln \frac{T_2}{T_{i1}}$$

$$= 4.8876 \times 4.18 \times \ln \frac{65.22 + 273.15}{70 + 273.15}$$

$$+ 1 \times (0.9 + 0.46 + 0.8) \ln \frac{65.22 + 273.15}{20 + 273.15}$$

$$= -0.28659 + 0.30986 = \mathbf{0.02327 \text{ kJ/K}}$$

Borgnakke and Sonntag

6.66

A piston cylinder has constant pressure of 2000 kPa with water at 20$^\text{o}$C. It is now heated up to 100$^\text{o}$C. Find the heat transfer and the entropy change using the steam tables. Repeat the calculation using constant heat capacity and incompressibility.

Solution:

C.V. Water. Constant pressure heating.

Energy Eq.3.5: $\quad\quad\quad u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.6.37: $\quad\quad s_2 - s_1 = {}_1q_2 / T_{SOURCE} + {}_1s_{2\,gen}$

Process: $\quad\quad\quad P = P_1 \implies {}_1w_2 = P(v_2 - v_1)$

The energy equation then gives the heat transfer as

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = h_2 - h_1$$

Steam Tables B.1.4: $\quad h_1 = 85.82$ kJ/kg; $\quad s_1 = 0.2962$ kJ/kg K

$\quad\quad\quad\quad\quad\quad\quad h_2 = 420.45$ kJ/kg; $\quad s_2 = 1.3053$ kJ/kg K

$${}_1q_2 = h_2 - h_1 = -85.82 + 420.45 = \textbf{334.63 kJ/kg}$$

$$s_2 - s_1 = 1.3053 - 0.2962 = \textbf{1.0091 kJ/kg K}$$

Now using values from Table A.4: Liquid water $\quad C_p = 4.18$ kJ/kg K

$$h_2 - h_1 \cong C_p(T_2 - T_1) = 4.18 \times 80 = \textbf{334.4 kJ/kg}$$

$$s_2 - s_1 \cong C_p \ln(T_2/T_1) = 4.18 \ln \frac{373.15}{293.15} = \textbf{1.0086 kJ/kg K}$$

Approximations are very good

Borgnakke and Sonntag

6.67

A 4 L jug of milk at 25°C is placed in your refrigerator where it is cooled down to the refrigerators inside constant temperature of 5°C. Assume the milk has the property of liquid water and find the entropy change of the milk.

Solution:

C.V. Jug of milk. Control mass at constant pressure.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

State 1: Table B.1.1: $v_1 \cong v_f = 0.001003$ m^3/kg, $s_f = 0.3673$ kJ/kg K

$$m = V/v = 0.004 \text{ m}^3/ 0.001003 \text{ (m}^3\text{/kg)} = 3.988 \text{ kg}$$

State 2: Table B.1.1: $s = s_f = 0.0761$ kJ/kg K

The change of entropy becomes

$$S_2 - S_1 = m(s_2 - s_1) = 3.988 \text{ kg } (0.0761 - 0.3673) \text{ kJ/kg-K}$$

$$= -\ \textbf{1.1613 kJ/K}$$

Borgnakke and Sonntag

6.68

A constant pressure container of 1.2 kg steel contains 1.5 kg of R-134a at 40$^{\text{o}}$C, 500 kPa. The container is placed in a refrigerator that brings it to -20$^{\text{o}}$C. Find the process heat transfer and the combined steel and R-134a change in entropy.

C.V. The steel container and the R-134a.

The energy equation, Eq. 3.5 now becomes (summing over the mass)

$$m_{st} (u_2 - u_1)_{st} + m_{R134a} (u_2 - u_1)_{R134a} = {}_1Q_2 - {}_1W_2$$

Process: P = C so ${}_1W_2 = P (V_2 - V_1) = P\, m_{R134a} (v_2 - v_1)_{R134a}$

Substitute the work into the energy eqution and we combine the R-134a terms

$${}_1Q_2 = m_{R134a}(h_2 - h_1)_{R134a} + m_{st}\, C_{st} (T_2 - T_1)$$

Use specific heat from Table A.3 for steel and Table B.5 for R-134a

R-134a: $v_1 = 0.04656$ m^3/kg, $h_1 = 430.72$ kJ/kg, $s_1 = 1.7971$ kJ/kgK

State 2:   500 kPa, -20$^{\text{o}}$C   compressed liquid.

$$v_2 = 0.000738, \ \ s_2 = s_f = 0.9007 \text{ kJ/kg-K}$$

$$h_2 = h_f + \Delta Pv = 173.74 + (500\text{-}133.7) \times 0.000738 = 174.0 \text{ kJ/kg}$$

notice how the correction for P higher than Psat is small

Now the heat transfer from the energy equation

$${}_1Q_2 = m_{R134a}(h_2 - h_1)_{R134a} + m_{st}\, C_{st} (T_2 - T_1)$$
$$= 1.5 \times (174.0 - 430.72) + 1.2 \times 0.46\,(-20 - 40) = -\textbf{418.2 kJ}$$

Steel: $m_{st}(s_2 - s_1)_{st} = m_{st}C_{st} \ln (T_2/T_1) = 1.2 \times 0.46 \ln \dfrac{253.15}{313.15} = -0.11741$ kJ/kgK

Entropy change for the total control volume steel and R-134a

$$S_2 - S_1 = m_{st} (s_2 - s_1)_{st} + m_{R134a} (s_2 - s_1)_{R134a}$$
$$= -0.11741 + 1.5(0.9007 - 1.7971) = -\textbf{1.462 kJ/K}$$

Borgnakke and Sonntag

6.69

A 10-kg steel container is cured at 500°C. An amount of liquid water at 15°C, 100 kPa is added to the container so a final uniform temperature of the steel and the water becomes 50°C. Neglect any water that might evaporate during the process and any air in the container. How much water should be added and how much was the entropy changed?

CV. The steel and the water no external heat transfer nor any work.

Energy Eq.: $\quad m_{H2O}(u_2 - u_1)_{H2O} + m_{st}(u_2 - u_1) = 0$

$\qquad\qquad m_{H2o}(209.3 - 62.98) + m_{st}C(T_2 - T_1) = 0$

$\qquad m_{H2O} \times 146.32$ kJ/kg $+ 10 \times 0.46 \times (50 - 500)$ kJ $= 0$

$\qquad m_{H2O} = 2070/146.32 = $ **14.147 kg**

Entropy Eq. 6.37: $\quad m_{H2O}(s_2 - s_1) + m_{st}(s_2 - s_1) = \varnothing + {_1}S_{2\ gen}$

$$S_2 - S_1 = 14.147 \text{ kg } (0.7037 - 0.2245) \text{ kJ/kg-K} + 10\text{kg} \times 0.46 \text{ kJ/kg-K} \times \ln\frac{50 + 273}{773}$$

$\qquad = 6.7792 - 4.0141 = $ **2.7651 kJ/K**

Borgnakke and Sonntag

6.70

A pan in an autoshop contains 5 L of engine oil at 20°C, 100 kPa. Now 3 L of hot 100°C oil is mixed into the pan. Neglect any work term and find the final temperature and the entropy change.

Solution:

Since we have no information about the oil density, we assume the same for both from Table A.4: $\rho = 885 \text{ kg/m}^3$

Energy Eq.: $m_2 u_2 - m_A u_A - m_B u_B \cong 0 - 0$

$\Delta u \cong C_v \Delta T$ so same $C_v = 1.9$ kJ/kg K for all oil states.

$$T_2 = \frac{m_A}{m_2} T_A + \frac{m_B}{m_2} T_B = \frac{5}{8} \times 20 + \frac{3}{8} \times 100 = 50.0^{\circ}\text{C} = \textbf{323.15 K}$$

$$S_2 - S_1 = m_2 s_2 - m_A s_A - m_B s_B = m_A(s_2 - s_A) + m_B(s_2 - s_B)$$

$$= 0.005 \times 885 \times 1.9 \ln \frac{323.15}{293.15} + 0.003 \times 885 \times 1.9 \ln \frac{323.15}{373.15}$$

$$= 0.8192 - 0.7257 = + \textbf{0.0935 kJ/K}$$

Entropy generation is the total change in S, recall Eq.6.39, no external Q

 Oils shown before mixed to final uniform state.

Borgnakke and Sonntag

6.71

A computer CPU chip consists of 50 g silicon, 20 g copper, 50 g polyvinyl chloride (plastic). It heats from 15°C to 75°C as the computer is turned on. How much did the entropy increase?

C.V. CPU chip. The process has electrical work input and no heat transfer.

Entropy Eq.: $S_2 - S_1 = \sum m_i(s_2 - s_1)_i = \int dQ/T + {}_1S_2\text{ gen} = {}_1S_2\text{ gen}$

For the solid masses we will use the specific heats, Table A.3, and they all have the same temperature so

$$\sum m_i(s_2 - s_1)_i = \sum m_iC_i \ln(T_2 / T_1)_i = \ln(T_2/T_1) \sum m_iC_i$$

$$\sum m_iC_i = 0.05 \times 0.7 + 0.02 \times 0.42 + 0.05 \times 0.96 = 0.0914 \text{ kJ/K}$$

$$S_2 - S_1 = 0.0914 \text{ kJ/K} \times \ln(348.15 / 288.15) = \textbf{0.0173 kJ/K}$$

Borgnakke and Sonntag

6.72

A 5-kg aluminum radiator holds 2 kg of liquid R-134a at -10°C. The setup is brought indoors and heated with 220 kJ. Find the final temperature and the change in entropy of all the mass.

Solution:

C.V. The aluminum radiator and the R-134a.

Energy Eq.3.5: $m_2u_2 - m_1u_1 = {_1}Q_2 - 0$

Process: No change in volume so no work as used above.

The energy equation now becomes (summing over the mass)

$$m_{al} (u_2 - u_1)_{al} + m_{R134a} (u_2 - u_1)_{R134a} = {_1}Q_2$$

Use specific heat from Table A.3 and A.4

$$m_{al}C_{al} (T_2 - T_1) + m_{R134a}C_{R134a} \ln (T_2 - T_1) = {_1}Q_2$$

$$T_2 - T_1 = {_1}Q_2 / [m_{al}C_{al} + m_{R134a}C_{R134a}]$$

$$= 220 \text{ kJ}/ [5 \times 0.9 + 2 \times 1.43] \text{ kJ/K} = 29.89^oC$$

$$T_2 = -10 + 29.89 = 19.89^oC$$

Entropy change for solid (A.3) and liquid (A.4) from Eq.6.11

$$S_2 - S_1 = m_{al} (s_2 - s_1)_{al} + m_{R134a} (s_2 - s_1)_{R134a}$$

$$= m_{al}C_{al} \ln (T_2/T_1) + m_{R134a}C_{R134a} \ln (T_2/T_1)$$

$$= (5 \times 0.9 + 2 \times 1.43) \text{ kJ/K } \ln \frac{(19.89 + 273.15)}{-10 + 273.15}$$

$$= \textbf{0.792 kJ/K}$$

Borgnakke and Sonntag

6.73
A 12 kg steel container has 0.2 kg superheated water vapor at 1000 kPa, both at 200°C. The total mass is now cooled to ambient temperature 30°C. How much heat transfer was taken out and what is the steel-water entropy change?

Solution:

C.V.: Steel and the water, control mass of constant volume.

Energy Eq.3.5: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

Process: V = constant => ${}_1W_2 = 0$

State 1: H_2O Table B.1.3: $u_1 = 2621.9$ kJ/kg, $v_1 = 0.20596$ m³/kg,
$s_1 = 6.6939$ kJ/kg K

State 2: H_2O: T_2 , $v_2 = v_1$ => from Table B.1.1

$$x_2 = \frac{v - v_f}{v_{fg}} = \frac{0.20596 - 0.001004}{32.8922} = 0.006231$$

$$u_2 = 125.77 + x_2 \times 2290.81 = 140.04 \text{ kJ/kg}$$

$$s_2 = 0.4369 + x_2 \times 8.0164 = 0.48685 \text{ kJ/kg K}$$

${}_1Q_2 = m(u_2 - u_1) = m_{steel}C_{steel} (T_2 - T_1) + m_{H2O} (u_2 - u_1)_{H2O}$

$= 12$ kg $\times 0.46$ kJ/kg-K (30 – 200) K + 0.2 kg(140.04 –2621.9) kJ/kg

$= \textbf{-1434.8 kJ}$

Entropy changes from Eq.6.11 and the water tables

$$S_2 - S_1 = m_2 s_2 - m_1 s_1 = m_{steel}C_{steel} \ln \left(\frac{T_2}{T_1}\right) + m_{H2O} (s_2 - s_1)_{H2O}$$

$$= 12 \times 0.46 \text{ kJ/K} \times \ln \left(\frac{303.15}{473.15}\right) + 0.2 \text{ kg} (0.48685 - 6.6939) \text{ kJ/kg-K}$$

$$= -2.4574 - 1.2414$$

$$= \textbf{--3.699 kJ/K}$$

Borgnakke and Sonntag

6.74

Find the total work the heat engine can give out as it receives energy from the rock bed as described in Problem 5.60 (see Fig.P 6.74). Hint: write the entropy balance equation for the control volume that is the combination of the rock bed and the heat engine.

Solution:

To get the work we must integrate over the process or do the 2nd law for a control volume around the whole setup out to T_0

C.V. Heat engine plus rock bed out to T_0. W and Q_L goes out.

Energy Eq.3.5: $\qquad (U_2 - U_1)_{rock} = - Q_L - W$

Entropy Eq.6.3, 11: $\qquad (S_2 - S_1)_{rock} = - \dfrac{Q_L}{T_0} = mC \ \ln \left(\dfrac{T_2}{T_1}\right)$

$$= 5500 \times 0.89 \ln \frac{290}{400} = -1574.15 \ \text{kJ/K}$$

$$Q_L = -T_0 \,(S_2 - S_1)_{rock} = -290 \,(-1574.15) = 456\ 504 \ \text{kJ}$$

The energy drop of the rock $-(U_2 - U_1)_{rock}$ equals Q_H into heat engine

$$(U_2 - U_1)_{rock} = mC \,(T_2 - T_1) = 5500 \times 0.89 \,(290 - 400) = -538\ 450 \ \text{kJ}$$

$$W = -(U_2 - U_1)_{rock} - Q_L = 538450 - 456504 = \textbf{81 946 kJ}$$

Borgnakke and Sonntag

6.75

Consider problem 6.60 if the two blocks of steel exchange energy through a heat engine similar to the setup in Problem 6.74. Find the work output of the heat engine.

C.V. Both blocks, no external heat transfer, C from Table A.3.

Energy Eq.: $U_2 - U_1 = m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0 - W$
$$= m_A C(T_2 - T_{A1}) + m_B C(T_2 - T_{B1})$$

Entropy Eq.6.37: $S_2 - S_1 = m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B = {}_1S_{2\ gen}$

Process: Assume reversible then ${}_1S_{2\ gen} = 0$

Entropy changes from Eq.6.11

$$S_2 - S_1 = m_A C \ln \frac{T_2}{T_{A1}} + m_B C \ln \frac{T_2}{T_{B1}} = 0$$

Now solve for T_2 by combining the "ln" terms as

$$0 = \ln \frac{T_2}{T_{A1}} + \ln \left(\frac{T_2}{T_{B1}} \right)^{m_B/m_A} = \ln \left[\frac{T_2}{T_{A1}} \left(\frac{T_2}{T_{B1}} \right)^{m_B/m_A} \right]$$

So the factors inside the ln function equal to one. The rewrite as

$$\frac{T_2}{T_{A1}} \left(\frac{T_2}{T_{B1}} \right)^{m_B/m_A} = 1 \quad => \quad T_2 = T_{A1}^{x_A} T_{B1}^{x_B}$$

$$x_A = m_A / (m_A + m_B) ; \qquad x_B = m_B / (m_A + m_B)$$

In the actual case the two masses are the same so $x_A = x_B = \frac{1}{2}$ and the result is

$$T_2 = \sqrt{T_{A1} T_{B1}} = \sqrt{523.15 \times 298.15} = 394.94 \text{ K} = 121.8^\circ\text{C}$$

The work output comes from the energy equation

$$W = m_A C(T_{A1} - T_2) + m_B C(T_{B1} - T_2)$$
$$= 5 \text{ kg} \times 0.46 \text{ kJ/kg-K} \times [(250 - 121.8) + (25 - 121.8)] \text{ K}$$
$$= \textbf{72.2 kJ}$$

Borgnakke and Sonntag

6.76

Two kg of liquid lead initially at 400°C is poured into a form. It then cools at constant pressure down to room temperature of 20°C as heat is transferred to the room. The melting point of lead is 327°C and the enthalpy change between the phases, h_{if}, is 24.6 kJ/kg. The specific heats are in Tables A.3 and A.4. Calculate the net entropy change for the mass.

Solution:

C.V. Lead, constant pressure process

$$m_{Pb}(u_2 - u_1)_{Pb} = {}_1Q_2 - P(V_2 - V_1)$$

We need to find changes in enthalpy (u + Pv) for each phase separately and then add the enthalpy change for the phase change.

Consider the process in several steps:

Cooling liquid to the melting temperature

Solidification of the liquid to solid, recall $s_{if} = h_{if}/T$, see page 268

Cooling of the solid to the final temperature

$${}_1Q_2 = m_{Pb}(h_2 - h_1) = m_{Pb}(h_2 - h_{327,sol} - h_{if} + h_{327,f} - h_{400})$$

$$= 2 \text{ kg} \times [0.138 \times (20 - 327) - 24.6 + 0.155 \times (327 - 400)] \text{ kJ/kg}$$

$$= -84.732 - 49.2 - 22.63 = -156.56 \text{ kJ}$$

$$S_2 - S_1 = m_{Pb}[C_{p\ sol}\ln(T_2/600) - (h_{if}/600) + C_{P\ liq}\ln(600/T_1)]$$

$$= 2 \times \left[0.138 \ln \frac{293.15}{600} - \frac{24.6}{600} + 0.155 \ln \frac{600}{673.15}\right] = \textbf{-0.315 kJ/K}$$

Borgnakke and Sonntag

Entropy of ideal gases

Borgnakke and Sonntag

6.77

Air inside a rigid tank is heated from 300 to 350 K. Find the entropy increase $s_2 - s_1$? What if it is from 1300 to 1350 K?

Process: $V = C$ → $_1W_2 = \emptyset$

Entropy change from Eq.6.17:

a) $s_2 - s_1 = C_{vo} \ln (\frac{T_2}{T_1}) = 0.717 \ln \left(\frac{350}{300}\right) = \textbf{0.1105 kJ/kgK}$

b) $s_2 - s_1 = C_{vo} \ln (\frac{T_2}{T_1}) = 0.717 \ln \left(\frac{1350}{1300}\right) = \textbf{0.02706 kJ/kgK}$

From A.7:
case a) $C_v \approx \Delta u / \Delta T = 36/50 = 0.72$ kJ/kg K , see A.5

case b) $C_v \approx \Delta u / \Delta T = 45.2/50 = 0.904$ kJ/kg K (25 % higher)
    so result should have been 0.0341 kJ/kgK

Borgnakke and Sonntag

6.78

A rigid tank contains 1 kg methane at 500 K, 1500 kPa. It is now cooled down to 300 K. Find the heat transfer and the change in entropy using ideal gas.

Ideal gas, constant volume so there is no work.

Energy Eq. 3.5: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2 = {_1}Q_2$

Use specific heat from Table A.5

$u_2 - u_1 = C_v (T_2 - T_1) = 1.736 (300 - 500) = -347.2$ kJ/kg

${_1}Q_2 = m(u_2 - u_1) = 1 (-347.2) = $ **–347.2 kJ**

The change in s for an ideal gas, Eqs.6.16-17 and $v_2 = v_1$ gives

$$m(s_2 - s_1) = m [C_{vo} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}] = m C_{vo} \ln \frac{T_2}{T_1}$$

$$= 1 \text{ kg} \times 1.736 \text{ kJ/kg-K } \ln \frac{300}{500} = \textbf{– 0.8868 kJ/K}$$

Borgnakke and Sonntag

6.79

Three kg of air is in a piston/cylinder keeping constant pressure at 27$^{\text{o}}$C, 300 kPa. It is now heated to 500 K. Plot the process path in a T-s diagram and find the heat transfer in the process.

Solution:

CV Air, so this is a control mass.

Energy Eq.3.5: $\qquad U_2 - U_1 = m\,(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Process: $\qquad$ P = C so $\quad {_1}W_2 = \int P dV = P_1(V_2 - V_1) = m\,(Pv_2 - Pv_1)$

State 1: $\qquad T_1, P_1 \quad$ ideal gas so $\quad P_1 V_1 = mRT_1$

State 2: $T_2, P_2 = P_1 \quad$ and ideal gas so $\qquad P_2 V_2 = mRT_2$

From the energy equation

$\qquad {_1}Q_2 = m\,(u_2 - u_1) + m\,(Pv_2 - Pv_1) = m\,(h_2 - h_1) = mC_{Po}\,(T_2 - T_1)$

$\qquad = 3 \text{ kg} \times 1.004 \text{ kJ/kg-K} \times (500 - 300) \text{ K} = \textbf{602.4 kJ}$

If we were to find the change in entropy we get

$\qquad s_2 - s_1 = C_{Po} \ln(T_2 / T_1) = 1.004 \ln (500/300) = 0.5129 \text{ kJ/kgK}$

The process curve in the T-s diagram is: $\quad T = T_1 \exp[(s - s_1)/C_{Po}]$

Borgnakke and Sonntag

6.80

A piston/cylinder setup contains air at 100 kPa, 400 K which is compressed to a final pressure of 1000 kPa. Consider two different processes (i) a reversible adiabatic process and (ii) a reversible isothermal process. Show both processes in P-v and a T-s diagram. Find the final temperature and the specific work for both processes.

Solution:
C.V. Air, control mass of unknown size and mass.

Energy Eq.3.5: $\quad u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.6.3: $\quad s_2 - s_1 = \int dq/T$

$\quad$ i) $\quad dq = 0$ so $\quad {}_1q_2 = 0$

$\quad$ ii) $\quad T = C$ so $\int dq/T = {}_1q_2/T$

i) For this process the entropy equation reduces to:

$$s_2 - s_1 = 0 \quad \text{so we have constant s, an isentropic process.}$$

The relation for an ideal gas, constant s and k becomes Eq.6.23

$$T_2 = T_1 (P_2 / P_1)^{\frac{k-1}{k}} = 400 \left(\frac{1000}{100}\right)^{\frac{0.4}{1.4}} = 400 \times 10^{0.28575} = \textbf{772 K}$$

From the energy equation we get the work term
$$_1w_2 = u_1 - u_2 = C_v(T_1 - T_2) = 0.717(400 - 772) = \textbf{-266.7 kJ/kg}$$

ii) For this process $T_2 = T_1$ so since ideal gas we get

$$u_2 = u_1 \quad \text{also} \quad s_{T2}^{o} = s_{T1}^{o} \quad \Rightarrow \quad \text{Energy Eq.:} \quad {}_1w_2 = {}_1q_2$$

Now from the entropy equation we solve for ${}_1q_2$

$$_1w_2 = {}_1q_2 = T(s_2 - s_1) = T[s_{T2}^{o} - s_{T1}^{o} - R \ln \frac{P_2}{P_1}] = -RT \ln \frac{P_2}{P_1}$$

$$= - 0.287 \times 400 \ln 10 = \textbf{−264 kJ/kg}$$

Borgnakke and Sonntag

6.81

Prove that the two relations for changes in s, Eqs.6.16 and 6.17 are equivalent once we assume constant specific heat. Hint: recall the relation for specific heat in Eq. 3.42.

From Eq. 3.42: $\qquad\qquad C_{po} = C_{vo} + R$

Start with Eq.6.16: $\quad s_2 - s_1 = C_{po} \ln \dfrac{T_2}{T_1} - R \ln \dfrac{P_2}{P_1}$

Now substitute Eq.3.42 to get:

$$s_2 - s_1 = (C_{vo} + R) \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$= C_{vo} \ln \frac{T_2}{T_1} - R \left[\ln \frac{P_2}{P_1} - \ln \frac{T_2}{T_1} \right]$$

$$= C_{vo} \ln \frac{T_2}{T_1} - R \ln \left(\frac{P_2}{P_1} \frac{T_1}{T_2} \right)$$

Use the ideal gas law $Pv = RT$ for both states to get the ratio

$$\frac{P_2 v_2}{R T_2} = \frac{P_1 v_1}{R T_1} \quad \Rightarrow \quad \frac{P_2}{P_1} \frac{T_1}{T_2} = \frac{v_1}{v_2}$$

so then we get to Eq. 6.17 as

$$s_2 - s_1 = C_{vo} \ln \frac{T_2}{T_1} - R \ln \frac{v_1}{v_2}$$

$$= C_{vo} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Borgnakke and Sonntag

6.82

A closed rigid container is filled with 1.5 kg water at 100 kPa, 55°C, 1 kg of stainless steel and 0.5 kg of PVC (polyvinyl chloride) both at 20°C and 0.1 kg of hot air at 400 K, 100 kPa. It is now left alone, with no external heat transfer and no water vaporizes. Find the final temperature and the change in entropy of the masses.

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$

Process: ${}_1Q_2 = 0$; ${}_1W_2 = 0$

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v\,i}(T_2 - T_1)_i = T_2 \sum m_i C_{v\,i} - \sum m_i C_{v\,i} T_{1\,i}$$

noticing that all masses have the same T_2 but not same initial T.

$$\sum m_i C_{v\,i} = 1.5 \times 4.18 + 1 \times 0.46 + 0.5 \times 0.96 + 0.1 \times 0.717 = 7.282 \text{ kJ/K}$$

Energy Eq.: $7.282\, T_2 = 1.5 \times 4.18 \times 55 + (1 \times 0.46 + 0.5 \times 0.96) \times 20$

$$+ \, 0.1 \times 0.717 \times (400 - 273.15) = 372.745 \text{ kJ}$$

$$T_2 = \mathbf{51.2°C} = 324.3 \text{ K}$$

The change in entropy for the solids and liquid follow Eq.6.11 and that for the ideal gas is from Eq.6.16 or 6.17 or 6.19, we use here 6.17 since $v_2 = v_1$

$$S_2 - S_1 = \sum m_i (s_2 - s_1)_i = \sum C_{v\,i}\, m_i\, [\,\ln(T_2 / T_1)_i\,]$$

$$= 1.5 \times 4.18 \ln \frac{324.3}{328.15} + 1 \times 0.46 \ln \frac{324.3}{293.15}$$

$$+ \, 0.5 \times 0.96 \ln \frac{324.3}{293.15} + 0.1 \times 0.717 \ln \frac{324.3}{400}$$

$$= \mathbf{0.00588 \text{ kJ/K}}$$

Borgnakke and Sonntag

6.83

Water at 150°C, 400 kPa, is brought to 1200°C in a constant pressure process. Find the change in the specific entropy, using a) the steam tables, b) the ideal gas water Table A.8, and c) the specific heat from A.5.

Solution:

a)

State 1: Table B.1.3   Superheated vapor $s_1 = 6.9299$ kJ/kgK

State 2: Table B.1.3 $s_2 = 9.7059$ kJ/kgK

$$s_2 - s_1 = 9.7059 - 6.9299 = \textbf{2.776 kJ/kgK}$$

b)

Table A.8 at 423.15 K: $s_{T1}^o = 11.13891$ kJ/kgK

Table A.8 at 1473.15 K: $s_{T2}^o = 13.86383$ kJ/kgK

$$s_2 - s_1 = s_{T2}^o - s_{T1}^o - R \ln \frac{P_2}{P_1} = s_{T2}^o - s_{T1}^o$$

$$s_{T2}^o - s_{T1}^o = 13.86383 - 11.13891 = \textbf{2.72492 kJ/kgK}$$

c)   Table A.5: $C_{po} = 1.872$ kJ/kgK

$$s_2 - s_1 \approx C_{po} \ln \left(\frac{T_2}{T_1} \right) = 1.872 \ln \left(\frac{1473.15}{423.15} \right) = \textbf{2.3352 kJ/kgK}$$

Notice how the average slope from 150°C to 1200°C is higher than the one at 25°C ($= C_{po}$)

Slope at 25°C

Borgnakke and Sonntag

6.84

R-410A at 400 kPa is brought from 20°C to 120°C in a constant pressure process. Evaluate the change in specific entropy using Table B.4 and using ideal gas with C_p = 0.81 kJ/kgK.

Table B.4.2 s_1 = 1.2108 kJ/kg-K, s_2 = 1.4788 kJ/kg-K

$s_2 - s_1$ = 1.4788 – 1.2108 = **0.268 kJ/kg-K**

Eq. 6.16: $s_2 - s_1 \approx C_{po} \ln \dfrac{T_2}{T_1}$ = 0.81 $\ln \dfrac{393.15}{293.15}$ = **0.238 kJ/kg-K**

Two explanations for the difference are as the average temperature is higher than 25°C we could expect a higher value of the specific heat and secondly it is not an ideal gas (if you calculate Z = Pv/RT = 0.94).

6.85

      R-410A at 300 kPa, 20°C is brought to 200°C in a constant volume process. Evaluate the change in specific entropy using Table B.4 and using ideal gas with $C_v = 0.695$ kJ/kgK.

      Table B.4.2 $s_1 = 1.2485$ kJ/kg-K, $v_1 = 0.10720$ m^3/kg

                          $s_2 = 1.6413$ kJ/kg-K, $v_2 = 0.10714$ m^3/kg at 500 kPa

                          (very close, we could have interpolated between 400 and 500 kPa)

                          $s_2 - s_1 = 1.6413 - 1.2485 = $ **0.3928 kJ/kg-K**

Eq. 6.17: $s_2 - s_1 \approx C_{vo} \ln \dfrac{T_2}{T_1} = 0.695 \ln \dfrac{473.15}{293.15} = $ **0.333 kJ/kg-K**

      Two explanations for the difference are as the average temperature is higher than 25°C we could expect a higher value of the specific heat and secondly it is not an ideal gas (if you calculate Z = Pv/RT = 0.96).

Borgnakke and Sonntag

6.86

Consider a small air pistol with a cylinder volume of 1 cm^3 at 250 kPa, 27°C. The bullet acts as a piston initially held by a trigger. The bullet is released so the air expands in an adiabatic process. If the pressure should be 120 kPa as the bullet leaves the cylinder find the final volume and the work done by the air.

Solution:

C.V. Air. Assume a reversible, adiabatic process.

Energy Eq.3.5: $u_2 - u_1 = 0 - {}_1w_2$;

Entropy Eq.6.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\,gen} = \emptyset$

State 1: (T_1, P_1) State 2: $(P_2, ?)$

So we realize that one piece of information is needed to get state 2.

Process: Adiabatic ${}_1q_2 = 0$ Reversible ${}_1s_{2\,gen} = 0$

With these two terms zero we have a zero for the entropy change. So this is a constant s (isentropic) expansion process giving $s_2 = s_1$. From Eq.6.23

$$T_2 = T_1(P_2 / P_1)^{\frac{k-1}{k}} = 300 \left(\frac{120}{250}\right)^{\frac{0.4}{1.4}} = 300 \times 0.48^{0.28575} = 243.24 \text{ K}$$

The ideal gas law $PV = mRT$ at both states leads to

$$V_2 = V_1 P_1 T_2/P_2 T_1 = 1 \times 250 \times 243.24/120 \times 300 = \textbf{1.689 cm}^{\textbf{3}}$$

The work term is from Eq.6.29 or Eq.4.4 with polytropic exponent n = k

$$_1W_2 = \frac{1}{1 - k} (P_2V_2 - P_1V_1) = \frac{1}{1 - 1.4} (120 \times 1.689 - 250 \times 1) \times 10^{-6} \text{ kPa m}^3$$

$$= \textbf{0.118 J}$$

Borgnakke and Sonntag

6.87

Oxygen gas in a piston cylinder at 300 K, 100 kPa with volume 0.1 m^3 is compressed in a reversible adiabatic process to a final temperature of 700 K. Find the final pressure and volume using Table A.5 and repeat the process with Table A.8.

Solution:

C.V. Air. Assume a reversible, adiabatic process.

Energy Eq.3.5: $u_2 - u_1 = 0 - {}_1w_2$;

Entropy Eq.6.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\ gen} = 0$

Process: Adiabatic ${}_1q_2 = 0$ Reversible ${}_1s_{2\ gen} = 0$

Properties: Table A.5: $k = 1.393$

With these two terms zero we have a zero for the entropy change. So this is a constant s (isentropic) expansion process. From Eq.6.23

$$P_2 = P_1(T_2 / T_1)^{\frac{k}{k-1}} = 100 \left(\frac{700}{300}\right)^{\frac{1.393}{1-1.393}} = \textbf{2015 kPa}$$

Using the ideal gas law to eliminate P from this equation leads to Eq.6.24

$$V_2 = V_1(T_2 / T_1)^{\frac{1}{1-k}} = 0.1 \times \left(\frac{700}{300}\right)^{\frac{1}{1-1.393}} = \textbf{0.0116 m}^{\textbf{3}}$$

Using the ideal gas tables A.8 we get

$$s_2 - s_1 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = 0 \quad \text{or} \quad P_2 = P_1 \exp[(s^o_{T2} - s^o_{T1})/R]$$

$$P_2 = 100 \exp[(7.2336 - 6.4168)/0.2598] = \textbf{2319.5 kPa}$$

$$V_2 = V_1 (T_2 / T_1)(P_1/P_2) = 0.1 \text{ m}^3 \times \frac{700}{300} \times \frac{100}{2319.5} = \textbf{0.010 m}^{\textbf{3}}$$

Borgnakke and Sonntag

6.88

Air in a rigid tank is at 100 kPa, 300 K with a volume of 0.75 m^3. The tank is heated to 400 K, state 2. Now one side of the tank acts as a piston, letting the air expand slowly at constant temperature to state 3 with a volume of 1.5 m^3. Find the entropy at states 1, 2 and 3.

Solution:

State 1: (P_1, T_1) $m = P_1 V_1 / RT_1 = \dfrac{100 \times 0.75}{0.287 \times 300} \dfrac{\text{kPa } m^3}{\text{kJ/kg}} = 0.871$ kg

Process 1 to 2: Constant volume heating, $dV = 0$ $\Rightarrow$ $_1W_2 = 0$

State 2: $T_2, P_2 = P_1 T_2 / T_1 = 100 \times 400 / 300 = 133.3$ kPa

Process 2 to 3: Isothermal expansion, $dT = 0$ $\Rightarrow$ $u_3 = u_2$ and

State 3: $T_3, P_3 = P_2 V_2 / V_3 = 133.3 \times 0.75 / 1.5 = 66.67$ kPa

The entropy is generally used as a relative value with different reference states in different tables. We can use here the reference as in Table A.7 so

$$s = s_T^o - R \ln(P/P_0)$$

where s_T^o is absolute entropy from A.7 at T and $P_0 = 100$ kPa

$s_1 = 6.86926$ kJ/kgK   which is at 300 K, 100 kPa

$s_2 = 7.15926 - 0.287 \ln(133.3/100) = 7.07677$ kJ/kgK

$s_3 = 7.15926 - 0.287 \ln(66.67/100) = 7.27561$ kJ/kgK

To get total entropy for the given mass multiply with m = 0.871 kg, which will give (**5.9831, 6.1639, 6.3371**) all in kJ/K.

Borgnakke and Sonntag

6.89

An insulated piston/cylinder setup contains carbon dioxide gas at 800 kPa, 300 K which is then compressed to 6 MPa in a reversible adiabatic process. Calculate the final temperature and the specific work using a) ideal gas tables A.8 and b) using constant specific heats A.5.

Solution:

C.V. CO_2, a control mass undergoing a reversible, adiabatic process.

Energy Eq.3.5: $u_2 - u_1 = 0 - {}_1w_2$;

Entropy Eq.6.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\,gen} = \emptyset$

Process: Adiabatic ${}_1q_2 = 0$ Reversible ${}_1s_{2\,gen} = 0$

State 1: (300 K, 800 kPa)   State 2: (6000 kPa, ?)

With two terms zero in the entropy equation we have a zero for the entropy change. So this is a constant s (isentropic) expansion process, $s_2 = s_1$.

a) Table A.8 for CO_2 and Eq.6.19

$$s_2 - s_1 = 0 = s_{T2}^o - s_{T1}^o - R \ln(P_2/P_1)$$

$$s_{T2}^o = s_{T1}^o + R \ln \frac{P_2}{P_1} = 4.8631 + 0.1889 \ln \frac{6000}{800} = 5.2437 \text{ kJ/kg-K}$$

Now interpolate in A.8 to find T_2

$$T_2 = 450 + 50\,(5.2437 - 5.2325)/(5.3375 - 5.2325) = \textbf{455.3 K}$$

$${}_1w_2 = -(u_2 - u_1) = -(271 - 157.7) = \textbf{–113.3 kJ/kg}$$

b) Table A.5: $k = 1.289$, $C_{Vo} = 0.653$ kJ/kg K and now Eq.6.23

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300 \left(\frac{6000}{800}\right)^{0.224} = \textbf{471.1 K}$$

$${}_1w_2 = -C_{Vo}(T_2 - T_1) = -0.653 \text{ kJ/kg-K}\,(471.1 - 300)\text{K} = \textbf{-111.7 kJ/kg}$$

Borgnakke and Sonntag

6.90

Extend the previous problem to solve using Table B.3.

C.V. CO_2, a control mass undergoing a reversible, adiabatic process.

Energy Eq.3.5: $u_2 - u_1 = 0 - {}_1w_2$;

Entropy Eq.6.37: $s_2 - s_1 = \int dq/T + {}_1s_2{}_{gen} = \emptyset$

Process: Adiabatic ${}_1q_2 = 0$ Reversible ${}_1s_2{}_{gen} = 0$

State 1: (300 K, 800 kPa) State 2: (6000 kPa, ?)

With two terms zero in the entropy equation we have a zero for the entropy change. So this is a constant s (isentropic) expansion process, $s_2 = s_1$.

From Table B.3.2: $s_1 = 1.8240$ kJ/kg-K, $u_1 = 333.5$ kJ/kg

$T_2 = 260.7^oC = \textbf{533.9 K}$, $u_2 = 502.9$ kJ/kg

$${}_1w_2 = -(u_2 - u_1) = -(502.9 - 333.5) = \textbf{–169.4 kJ/kg}$$

Borgnakke and Sonntag

6.91

A handheld pump for a bicycle has a volume of 25 cm^3 when fully extended. You now press the plunger (piston) in while holding your thumb over the exit hole so that an air pressure of 300 kPa is obtained. The outside atmosphere is at P_0, T_0. Consider two cases: (1) it is done quickly (~1 s), and (2) it is done very slowly (~1 h).

      a.   State assumptions about the process for each case.

      b.   Find the final volume and temperature for both cases.

Solution:

C.V. Air in pump. Assume that both cases result in a reversible process.

      State 1: P_0, T_0 State 2: 300 kPa, ?

      One piece of information must resolve the ? for a state 2 property.

Case I) Quickly means no time for heat transfer

      Q = 0, so a reversible adiabatic compression.

$$u_2 - u_1 = -_1w_2 \; ; \quad s_2 - s_1 = \int dq/T + _1s_{2\ gen} = 0$$

With constant s and constant heat capacity we use Eq.6.23

$$T_2 = T_1(P_2 / P_1)^{\frac{k-1}{k}} = 298\left(\frac{300}{101.325}\right)^{\frac{0.4}{1.4}} = \textbf{405.3 K}$$

Use ideal gas law PV = mRT at both states so ratio gives

$$\Rightarrow \quad V_2 = P_1V_1T_2/T_1P_2 = \textbf{11.48 cm}^3$$

Case II)   Slowly, time for heat transfer so T = constant = T_0.

The process is then a reversible isothermal compression.

$$T_2 = T_0 = \textbf{298 K} \quad \Rightarrow \quad V_2 = V_1P_1/P_2 = \textbf{8.44 cm}^3$$

Borgnakke and Sonntag

6.92

A piston/cylinder, shown in Fig. P6.92, contains air at 1380 K, 15 MPa, with $V_1 =$ 10 cm^3, $A_{cyl} = 5$ cm^2. The piston is released, and just before the piston exits the end of the cylinder the pressure inside is 200 kPa. If the cylinder is insulated, what is its length? How much work is done by the air inside?

Solution:

C.V. Air, Cylinder is insulated so adiabatic, $Q = 0$.

Continuity Eq.: $m_2 = m_1 = m$,

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = 0 + {}_1S_{2\,gen}$

State 1: (T_1, P_1) State 2: $(P_2, ?)$

So one piece of information is needed for the ?,   assume reversible process.

$\qquad {}_1S_{2\,gen} = 0 \quad \Rightarrow \quad s_2 - s_1 = 0$ (which is also Eq.6.3)

State 1:   Table A.7: $u_1 = 1095.2$ kJ/kg, $s^o_{T1} = 8.5115$ kJ/kg K

$$m = P_1V_1/RT_1 = \frac{15000 \times 10\times10^{-6}}{0.287 \times 1380} = 0.000379 \text{ kg}$$

State 2: P_2 and from Entropy eq.: $s_2 = s_1$ so from Eq.6.19

$$s^o_{T2} = s^o_{T1} + R \ln \frac{P_2}{P_1} = 8.5115 + 0.287 \ln(\frac{200}{15000}) = 7.2724 \text{ kJ/kg K}$$

Now interpolate in Table A.7 to get T_2

$\qquad T_2 = 440 + 20\,(7.2724 - 7.25607)/(7.30142 - 7.25607) = 447.2$ K

$\qquad u_2 = 315.64 + (330.31 - 315.64)\,0.36 = 320.92$ kJ/kg

$$V_2 = V_1 \frac{T_2\,P_1}{T_1 P_2} = \frac{10 \times 447.2 \times 15000}{1380 \times 200} = 243 \text{ cm}^3$$

$\qquad \Rightarrow \;\; L_2 = V_2/A_{cyl} = 243/5 = \textbf{48.6 cm}$

$\qquad {}_1w_2 = u_1 - u_2 = 774.3$ kJ/kg, ${}_1W_2 = m_1w_2 = \textbf{0.2935 kJ}$

Borgnakke and Sonntag

6.93

Argon in a light bulb is at 90 kPa and 20°C when it is turned on and electric input now heats it to 60°C. Find the entropy increase of the argon gas.

Solution:

C.V. Argon gas. Neglect any heat transfer.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1W_2 \text{ electrical in}$

Entropy Eq.6.37: $s_2 - s_1 = \int dq/T + {}_1s_2 \text{ gen} = {}_1s_2 \text{ gen}$

Process: v = constant and ideal gas   => $P_2/P_1 = T_2/T_1$

Evaluate changes in s from Eq.6.16 or 8.17

$$s_2 - s_1 = C_p \ln (T_2/T_1) - R \ln (P_2/P_1) \qquad \text{Eq.6.16}$$

$$= C_p \ln (T_2/T_1) - R \ln (T_2/T_1) = C_v \ln(T_2/T_1) \qquad \text{Eq.6.17}$$

$$= 0.312 \text{ kJ/kg-K} \times \ln \left[\frac{60 + 273}{20 + 273} \right] = \mathbf{0.04 \text{ kJ/kg K}}$$

Since there was no heat transfer but work input all the change in s is generated by the process (irreversible conversion of W to internal energy)

Borgnakke and Sonntag

6.94

We wish to obtain a supply of cold helium gas by applying the following technique. Helium contained in a cylinder at ambient conditions, 100 kPa, 20°C, is compressed in a reversible isothermal process to 600 kPa, after which the gas is expanded back to 100 kPa in a reversible adiabatic process.
  a. Show the process on a *T–s* diagram.
  b. Calculate the final temperature and the net work per kilogram of helium.

Solution:

a)

b)   The adiabatic reversible expansion gives constant s from the entropy equation Eq.6.37. With ideal gas and constant specific heat this gives relation in Eq.6.23

$$T_3 = T_2(P_3/P_2)^{\frac{k-1}{k}} = 293.15 \,(100/600)^{0.4} = \textbf{143.15 K}$$

The net work is summed up over the two processes. The isothermal process has work as Eq.6.31

$$_1w_2 = -RT_1 \ln(P_2/P_1) = -2.0771 \text{ kJ/kg-K} \times 293.15 \text{ K} \times \ln(600/100)$$

$$= -1091.0 \text{ kJ/kg}$$

The adiabatic process has a work term from energy equation with no q

$$_2w_3 = C_{Vo}(T_2-T_3) = 3.116 \text{ kJ/kg-K} (293.15 - 143.15) \text{ K} = +467.4 \text{ kJ/kg}$$

The net work is the sum

$$w_{NET} = -1091.0 + 467.4 = \textbf{-623.6 kJ/kg}$$

Borgnakke and Sonntag

6.95

A 1-m^3 insulated, rigid tank contains air at 800 kPa, 25°C. A valve on the tank is opened, and the pressure inside quickly drops to 150 kPa, at which point the valve is closed. Assuming that the air remaining inside has undergone a reversible adiabatic expansion, calculate the mass withdrawn during the process.

Solution:

C.V.: Air remaining inside tank, m_2.

Cont.Eq.: $m_2 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {_1}S_{2\ gen}$

Process: adiabatic ${_1}Q_2 = 0$ and reversible ${_1}S_{2\ gen} = 0$

Entropy eq. then gives $s_2 = s_1$ and ideal gas gives the relation in Eq.6.23

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 298.2 \text{ K } (150/800)^{0.286} = \textbf{184.8 K}$$

$$m_1 = P_1V/RT_1 = (800 \text{ kPa} \times 1 \text{ m}^3)/(0.287 \text{ kJ/kgK} \times 298.2 \text{ K}) = 9.35 \text{ kg}$$

$$m_2 = P_2V/RT_2 = (150 \text{ kPa} \times 1 \text{ m}^3)/(0.287 \text{ kJ/kgK} \times 184.8 \text{ K}) = 2.83 \text{ kg}$$

$$m_e = m_1 - m_2 = \textbf{6.52 kg}$$

Borgnakke and Sonntag

6.96

Two rigid, insulated tanks are connected with a pipe and valve. One tank has 0.5 kg air at 200 kPa, 300 K and the other has 0.75 kg air at 100 kPa, 400 K. The valve is opened and the air comes to a single uniform state without any heat transfer. Find the final temperature and the change in entropy of the air.

Solution:

C.V. Total tank. Control mass of constant volume.

Mass and volume: $m_2 = m_A + m_B;$ $V = V_A + V_B$

Energy Eq.: $U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = {}_1Q_2 - {}_1W_2 = 0$

Process Eq.: $V = \text{constant} \Rightarrow {}_1W_2 = 0;$ $\text{Insulated} \Rightarrow {}_1Q_2 = 0$

Ideal gas at A1: $V_A = m_A R T_{A1}/P_{A1} = 0.5 \times 0.287 \times 300 / 200 = 0.2153 \text{ m}^3$

Ideal gas at B1: $V_B = m_B R T_{B1}/ P_{B1} = 0.75 \times 0.287 \times 400 / 100 = 0.861 \text{ m}^3$

State 2: $m_2 = m_A + m_B = 1.25 \text{ kg}; \ V_2 = V_A + V_B = 1.0763 \text{ m}^3$

Energy Eq.: $u_2 = \dfrac{m_A u_{A1} + m_B u_{B1}}{m_2}$ and use constant specific heat

$$T_2 = \frac{m_A}{m_2} T_{A1} + \frac{m_B}{m_2} T_{B1} = \frac{0.5}{1.25} 300 + \frac{0.75}{1.25} 400 = \textbf{360 K}$$

$$P_2 = m_2 R T_2/V = 1.25 \text{ kg} \times 0.287 \text{ kJ/kgK} \times 360 \text{ K}/ 1.0763 \text{ m}^3 = 120 \text{ kPa}$$

$$S_2 - S_1 = m_A[C_P \ln T_2/T_{A1} - R\ln(P_2/P_{A1})] + m_B[C_P \ln T_2/T_{B1} - R\ln(P_2/P_{B1})]$$

$$= 0.5[\ 1.004 \ln\frac{360}{300} - 0.287 \ln\frac{120}{200}\ ] + 0.75[1.004 \ln\frac{360}{400} - 0.287 \ln\frac{120}{100}]$$

$$= 0.5 \times 1.3514 + 0.75 \times (-0.1581) = \textbf{0.5571 kJ/K}$$

Borgnakke and Sonntag

6.97

Two rigid tanks, shown in Fig. P6.97, each contain 10 kg N_2 gas at 1000 K, 500 kPa. They are now thermally connected to a reversible heat pump, which heats one and cools the other with no heat transfer to the surroundings. When one tank is heated to 1500 K the process stops. Find the final (P, T) in both tanks and the work input to the heat pump, assuming constant heat capacities.

Solution:

Control volume of hot tank B,

Process = constant volume & mass   so no work

Energy equation Eq.3.5 and specific heat in Eq.5.20 gives

$U_2 - U_1 \cong mC_v(T_2 - T_1) = {}_1Q_2 = 10\ kg \times 0.7448\ kJ/kg\text{-}K \times 500\ K = 3724\ kJ$

$P_2 = P_1T_2/T_1 = 1.5(P_1) = \textbf{750 kPa}$

State: 1 = initial,
2 = final hot
3 = final cold

To fix temperature in cold tank, C.V.: total

For this CV only W_{HP} cross the control surface no heat transfer. The entropy equation Eq.6.37 for a reversible process becomes

$(S_2 - S_1)_{tot} = 0 = m_{hot}\ (s_2 - s_1) + m_{cold}\ (s_3 - s_1)$

Use specific heats to evaluate the changes in s from Eq.6.16 and division by m

$C_{p,hot}\ \ln(T_2\ /\ T_1) - R\ \ln(P_2\ /\ P_1) + C_{p,cold}\ \ln(T_3\ /\ T_1) - R\ \ln(P_3\ /\ P_1) = 0$

$P_3 = P_1T_3/T_1\quad \text{and}\quad P_2 = P_1T_2/T_1$

Now everything is in terms of T and $C_p = C_v + R$, so

$C_{v,hot}\ \ln(T_2/T_1) + C_{v,cold}\ \ln(T_3/T_1) = 0$

same C_v: $T_3 = T_1(T_1/T_2) = \textbf{667 K},\quad P_3 = \textbf{333 kPa}$

$Q_{cold} = -\ {}_1Q_3 = m\ C_v\ (T_3 - T_1) = \text{-2480 kJ},$

$W_{HP} = {}_1Q_2 + Q_{cold} = {}_1Q_2 - {}_1Q_3 = \textbf{1244 kJ}$

Borgnakke and Sonntag

6.98

A hydrogen gas in a piston/cylinder assembly at 300 K, 100 kPa with a volume of 0.1 m^3 is now slowly compressed to a volume of 0.01 m^3 while cooling it in a reversible isothermal process. What is the final pressure, the heat transfer and the change in entropy?

Solution:

C.V. Hydrogen, control mass.

Energy Eq. 3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: T = constant so with ideal gas   => $u_2 = u_1$

From the process equation $(T_2 = T_1)$ and ideal gas law we get

$$P_2 = P_1 (V_1/V_2) = 10\ P_1 = \textbf{1000 kPa}$$

we can calculate the work term as in Eq.3.22

$${}_1Q_2 = {}_1W_2 = \int PdV = P_1V_1 \ln (V_2/V_1)$$

$$= 100\ \text{kPa} \times 0.1\ \text{m}^3 \times \ln (1/10) = \textbf{-23.0 kJ}$$

The change of entropy from the entropy equation Eq.6.3 is

$$m(s_2 - s_1) = {}_1Q_2/T_1 = \frac{-23}{300} \frac{\text{kJ}}{\text{K}} = \textbf{-0.07667 kJ/K}$$

If instead we use Eq.6.17 we would get

$$\Delta S = m(s_2 - s_1) = m(C_{vo} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}) = m\ R \ln \frac{v_2}{v_1}$$

$$= (P_1V_1/T_1) \ln (\frac{v_2}{v_1}) = {}_1Q_2/T_1$$

consistent with the above result.

Borgnakke and Sonntag

6.99

A rigid tank contains 4 kg air at 300^oC, 4 MPa which acts as the hot energy reservoir for a heat engine with its cold side at 20^oC shown in Fig. P.6.99. Heat transfer to the heat engine cools the air down in a reversible process to a final 20^oC and then stops. Find the final air pressure and the work output of the heat engine.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T$ (T not constant)

Process: $v = $ constant $\Rightarrow$ ${}_1W_2 = 0$

State 2: T_2 and $v_2 = v_1$,

$P_2 = P_1T_2/T_1 = 4000 \times 293.15/573.15 =$ **2045.9 kPa**

From the energy equation

$Q_H = -{}_1Q_2 = -m(u_2 - u_1) = -m\ C_{vo}\ (T_2 - T_1)$

$= -4 \text{ kg} \times 0.717 \text{ kJ/kg-K } (293.15 - 573.15) \text{ K} =$ **803.0 kJ**

Take now CV total as the air plus heat engine out to ambient

Entropy Eq.6.3: $m(s_2 - s_1) = -\dfrac{Q_L}{T_{amb}}$ $\Rightarrow$

$Q_L = -mT_{amb}\ (s_2 - s_1) = -mT_{amb}\ C_{vo}\ \ln\dfrac{T_2}{T_1}$

$= -4 \text{ kg} \times 293.15 \text{ K} \times 0.717 \text{ kJ/kg-K } \ln(293.15/573.15) = 563.7 \text{ kJ}$

Now the CV heat engine can give the engine work from the energy equation

Energy H.E.: $W_{HE} = Q_H - Q_L = 803.0 - 563.7 =$ **239.3 kJ**

Notice to get ${}_1q_2 = \int T\ ds$ we must know the function $T(s)$ which we do not readily have for this process.

Borgnakke and Sonntag

Polytropic processes

6.100

An ideal gas having a constant specific heat undergoes a reversible polytropic expansion with exponent, $n = 1.4$. If the gas is carbon dioxide will the heat transfer for this process be positive, negative, or zero?

Solution:

CO_2: Table A.5 $k = 1.289 < n$

Since $n > k$ and $P_2 < P_1$

it follows that $s_2 < s_1$ and thus Q flows out.

$_1Q_2 < 0$

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6.101

Repeat the previous problem for the gas carbon monoxide, CO.

Solution:

T vs s diagram with points 1 and 2, showing lines labeled n = k, n > k, P = const, and n < k.

CO: Table A.5 $k = 1.399 = n$

Since $n \approx k$ and $P_2 < P_1$

it follows that $s_2 \approx s_1$ and thus adiabatic.

$$_1Q_2 \approx 0$$

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6.102

A nitrogen gas goes through a polytropic process with n = 1.3 in a piston/cylinder arrangement. It starts out at 600 K, 600 kPa and ends at 800 K. Is the heat transfer positive, negative or zero?

N_2: Table A.5   k = 1.40 and n = 1.3 < k

   Since n < k   and $T_2 > T_1$

   process goes up on the n < k curve

   $s_2 < s_1$ and thus q must go out.

   q is negative

6.103

A cylinder/piston contains 1 kg methane gas at 100 kPa, 300 K. The gas is compressed reversibly to a pressure of 800 kPa. Calculate the work required if the process is adiabatic.

Solution:

C.V. Methane gas of constant mass $m_2 = m_1 = m$ and reversible process.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.3 (37): $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = \int dQ/T$

Process: ${}_1Q_2 = 0$ $\Rightarrow$ $s_2 = s_1$

thus isentropic process s = constant and ideal gas gives the relation in Eq.6.23 with k = 1.299 from Table A.5

$$T_2 = T_1 \, (P_2/P_1)^{\frac{k-1}{k}} = 300 \text{ K} \left(\frac{800}{100}\right)^{0.230} = 483.99 \text{ K}$$

$${}_1W_2 = -mC_{V0}(T_2 - T_1) = -1 \text{ kg} \times 1.736 \text{ kJ/kg-K} (483.99 - 293.15) \text{ K}$$

$$= \textbf{-331.3 kJ}$$

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6.104

Do the previous problem but assume the process is isothermal.

Solution:

C.V. Methane gas of constant mass $m_2 = m_1 = m$ and reversible process.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_2{}_{gen} = \int dQ/T$

Process: $T = $ constant. For ideal gas then $u_2 = u_1$ and $s^{\circ}_{T2} = s^{\circ}_{T1}$

Energy eq. gives ${}_1W_2 = {}_1Q_2$ and $\int dQ/T = {}_1Q_2/T$

with the entropy change found from Eq.6.16

$$\Rightarrow \quad {}_1W_2 = {}_1Q_2 = mT(s_2 - s_1) = -mRT \ln(P_2/P_1)$$

$$= -0.5183 \text{ kJ/K} \times 300 \text{ K} \ln\left(\frac{800}{100}\right) = -\textbf{323.3 kJ}$$

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6.105

A piston/cylinder contains air at 300 K, 100 kPa. It is now compressed in a reversible adiabatic process to a volume 7 times as small. Use constant heat capacity and find the final pressure and temperature, the specific work and specific heat transfer for the process.

Solution:

Expansion ratio: $\quad v_2/v_1 = 1/7$

Process eq.: Rev. adiabatic and ideal gas gives $\quad Pv^n = C$, with $n = k$

$$P_2/P_1 = (v_2/v_1)^{-k} = 7^{1.4} = 15.245$$

$$P_2 = P_1 (7^{1.4}) = 100 \times 15.245 = \textbf{1524.5 kPa}$$

$$T_2 = T_1 (v_1/v_2)^{k-1} = 300 \times 7^{0.4} = \textbf{653.4 K}$$

$$_1q_2 = \textbf{0 kJ/kg}$$

Polytropic process work term from Eq.6.29

$$_1w_2 = \frac{R}{1-k}(T_2 - T_1) = \frac{0.287}{-0.4} \text{ kJ/kg-K } (653.4 - 300) \text{ K} = \textbf{-253.6 kJ/kg}$$

Notice: $C_v = R/(k-1)$ so the work term is also the change in u consistent with the energy equation.

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6.106

A piston/cylinder contains pure oxygen at 500 K, 600 kPa. The piston is moved to a volume such that the final temperature is 700 K in a polytropic process with exponent n = 1.25. Use ideal gas approximation and constant heat capacity to find the final pressure, the specific work, and the heat transfer.

Take CV as the oxygen. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $Pv^n = Constant$ (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $P_2 = P_1 \left(v_2 / v_1\right)^{-n}$ and $Pv = RT$ $\Rightarrow$ $T_2/T_1 = \left(v_2 / v_1\right)^{1-n}$

$$P_2 = P_1 \left(T_2 / T_1\right)^{n/(n-1)} = 600 \left(\frac{700}{500}\right)^{1.25/0.25} = \textbf{3226.9 kPa}$$

From process eq.:

$${}_1w_2 = \int P \, dv = area = \frac{1}{1-n}(P_2 v_2 - P_1 v_1) = \frac{R}{1-n}(T_2 - T_1)$$

$$= \frac{0.2598}{1-1.25}(700 - 500) = \textbf{–207.8 kJ/kg}$$

From the energy equation and Table A.8

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = (480.18 - 331.72 - 207.8) \text{ kJ/kg} = \textbf{-59.34 kJ/kg}$$

From the energy equation and constant specific heat from Table A.5

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = C_v (T_2 - T_1) + {}_1w_2$$
$$= 0.662 \text{ kJ/kg-K } (700 - 500) \text{ K} - 207.8 \text{ kJ/kg} = -75.4 \text{ kJ/kg}$$

The function T(s) can be done with constant C_P: $a = C_P - nR/(n-1) = –0.377$

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6.107

Do Problem 6.103 and assume the process is polytropic with n = 1.15

Process: Pv^n = constant with n = 1.15 ;

The T-P relation is given in Eq.6.28

$$T_2 = T_1 \ (P_2/P_1)^{\frac{n-1}{n}} = 300 \left(\frac{800}{100}\right)^{0.130} = 393.1 \ K$$

and the work term is given by Eq.6.29

$$_1W_2 = \int mP \ dv = m(P_2v_2 - P_1v_1)/(1 - n) = mR \ (T_2 - T_1)/(1 - n)$$

$$= 1 \ kg \times 0.5183 \ \frac{kJ}{kg\text{-}K} \times \frac{393.1 - 293.15}{1 - 1.15} \ K = \textbf{-345.4 kJ}$$

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6.108

Hot combustion air at 2000 K expands in a polytropic process to a volume 6 times as large with n = 1.3. Find the specific boundary work and the specific heat transfer.

Energy Eq.: $u_2 - u_1 = {_1}q_2 - {_1}w_2$

Reversible work Eq. 6.29: ${_1}w_2 = \dfrac{1}{1-n}(P_2v_2 - P_1v_1) = \dfrac{R}{1-n}(T_2 - T_1)$

Process Eq: $Pv^n = C;\quad T_2 = T_1(v_1/v_2)^{n-1} = 2000\left(\dfrac{1}{6}\right)^{0.3} = 1168.4\ K$

Properties from Table A.7.1: $u_1 = 1677.52\ kJ/kg,\ u_2 = 905.46\ kJ/kg$

$${_1}w_2 = \dfrac{0.287}{1-1.3}\ kJ/kg\text{-}K\ (1168.4 - 2000)\ K = \textbf{795.6 kJ/kg}$$

$${_1}q_2 = u_2 - u_1 + {_1}w_2 = 905.46 - 1677.52 + 795.6 = \textbf{23.5 kJ/kg}$$

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6.109

      Air in a piston/cylinder is at 1800 K, 7 MPa and expands in a polytropic process with n = 1.5 to a volume eight times larger. Find the specific work and specific heat transfer in the process using Table A.7 and draw the T-s diagram.

C.V. Air of constant mass $m_2 = m_1 = m$.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^{1.50} = $ constant, $v_2/v_1 = 8$

State 1: $P_1 = 7$ MPa, $T_1 = 1800$ K

State 2: $(v_2 = 8v_1, ?)$ Must be on process curve so

$$P_2 = P_1 (v_1/v_2)^n = 7000 (1/8)^{1.50} = 309.36 \text{ kPa}$$

$$T_2 = T_1 \frac{P_2 v_2}{P_1 v_1} = T_1(v_1/v_2)^{n-1} = 1800 (1/8)^{0.5} = 636.4 \text{ K}$$

Table A.7: $u_1 = 1486.33$ kJ/kg   and   interpolate $u_2 = 463.06$ kJ/kg

Work from the process expressed in Eq.3.21, 6.29

$${}_1w_2 = \int P dv = \frac{P_2 v_2 - P_1 v_1}{1 - n} = \frac{R}{1 - n} (T_2 - T_1) = \frac{0.287 (636.4 - 1800)}{1 - 1.5}$$

$$= \textbf{667.9 kJ/kg}$$

Heat transfer from the energy equation

$${}_1q_2 = (u_2 - u_1) + {}_1w_2 = (463.06 - 1486.33) + 667.9 = -\textbf{355.4 kJ/kg}$$

Notice:
n = 1.5, k = 1.4

n > k

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6.110

Helium in a piston/cylinder at 20°C, 100 kPa is brought to 400 K in a reversible polytropic process with exponent n = 1.25. You may assume helium is an ideal gas with constant specific heat. Find the final pressure and both the specific heat transfer and specific work.

Solution:

C.V. Helium

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^n = C$ & $Pv = RT$ $=>$ $Tv^{n-1} = C$

Table A.5: $C_v = 3.116$ kJ/kg K, $R = 2.0771$ kJ/kg K

From the process equation and $T_1 = 293.15$, $T_2 = 400$ K

$\quad T_1 v^{n-1} = T_2 v^{n-1}$ $=>$ $v_2 / v_1 = (T_1 / T_2)^{1/n-1} = 0.2885$

$\quad P_2 / P_1 = (v_1 / v_2)^n = 4.73$ $=> P_2 = $ **473 kPa**

The work is from Eq.6.29 per unit mass

$\quad {}_1w_2 = \int P \, dv = \int C \, v^{-n} \, dv = [\, C / (1-n) \,] \times (v_2^{1-n} - v_1^{1-n})$

$\qquad = \dfrac{1}{1-n} (P_2 v_2 - P_1 v_1) = \dfrac{R}{1-n} (T_2 - T_1)$

$\qquad = \dfrac{2.0771}{1 - 1.25}$ kJ/kg-K $\times (400 - 293.15)$ K = **-887.7 kJ/kg**

The heat transfer follows from the energy equation

$\quad {}_1q_2 = u_2 - u_1 + {}_1w_2 = C_v (T_2 - T_1) + (- 887.7) = $ **-554.8 kJ/kg**

Borgnakke and Sonntag

6.111

The power stroke in an internal combustion engine can be approximated with a polytropic expansion. Consider air in a cylinder volume of 0.2 L at 7 MPa, 1800 K, shown in Fig. P6.111. It now expands in a reversible polytropic process with exponent, $n = 1.5$, through a volume ratio of 10:1. Show this process on P–v and T–s diagrams, and calculate the work and heat transfer for the process.

Solution:

C.V. Air of constant mass $m_2 = m_1 = m$.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.3 (37): $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = \int dQ/T$

Process: $PV^{1.50} = $ constant, $V_2/V_1 = 10$

State 1: $P_1 = 7$ MPa, $T_1 = 1800$ K, $V_1 = 0.2$ L

$$m_1 = \frac{P_1 V_1}{RT_1} = \frac{7000 \times 0.2 \times 10^{-3}}{0.287 \times 1800} = 2.71 \times 10^{-3} \text{ kg}$$

State 2: $(v = V_2/m, ?)$ Must be on process curve so Eq.6.24 gives

$$T_2 = T_1 (V_1/V_2)^{n-1} = 1800\ (1/10)^{0.5} = 569.2 \text{ K}$$

Table A.7: $u_1 = 1486.33$ kJ/kg and interpolate $u_2 = 411.707$ kJ/kg

Notice:
n = 1.5, k = 1.4

n > k

Work from the process expressed in Eq.6.29

$${}_1W_2 = \int PdV = mR(T_2 - T_1)/(1 - n)$$

$$= \frac{2.71 \times 10^{-3} \times 0.287(569.2 - 1800)}{1 - 1.5} = \textbf{1.91 kJ}$$

Heat transfer from the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

$$= 2.71 \times 10^{-3} \text{ kg} \times (411.707 - 1486.33) \text{ kJ/kg} + 1.91 \text{ kJ} = \textbf{-1.00 kJ}$$

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6.112

A cylinder/piston contains saturated vapor R-410A at 10°C; the volume is 10 L. The R-410A is compressed to 2 MPa, 60°C in a reversible (internally) polytropic process. Find the polytropic exponent n and calculate the work and heat transfer.

Solution:

C.V. R-410A of constant mass $m_2 = m_1 = m$ out to ambient.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = {}_1Q_2/T_{amb} + {}_1S_{2\,gen}$

Process: $P_1v_1^n = P_2v_2^n$ Eq.6.36

State 1: (T_1, x_1) Table B.4.1 $P_1 = 1085.7$ kPa, $v_1 = 0.02383$ m^3/kg

$m = V_1/v_1 = 0.01/0.02383 = 0.4196$ kg

State 2: (T_2, P_2) Table B.4.2 $v_2 = 0.01536$ m^3/kg

From process eq. $P_2/P_1 = \dfrac{2000}{1085.7} = \left(\dfrac{0.02383}{0.01536}\right)^n$ => **n = 1.39106**

The work is from Eq.6.29

$${}_1W_2 = \int PdV = m\frac{P_2v_2 - P_1v_1}{1-n} = 0.4196 \frac{2000 \times 0.01536 - 1085.7 \times 0.02383}{1 - 1.39106}$$

$= \mathbf{-5.20}$ **kJ**

Heat transfer from energy equation

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.4196 (289.9 - 255.9) - 5.20 =$ **9.07 kJ**

Notice:
 n = 1.39, k = 1.17

 n > k

The two curves are shown below from the CATT3 process plot function.

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6.113

Air goes through a polytropic process with n = 1.3 in a piston/cylinder setup. It starts at 200 kPa, 300 K and ends with a pressure of 2200 kPa. Find the expansion ratio v_2/v_1, the specific work, and the specific heat transfer.

Take CV as the air. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $Pv^n = \text{Constant}$ (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $(v_2/v_1) = (P_2/P_1)^{-1/n}$ and $Pv = RT \implies T_2/T_1 = (v_2/v_1)^{1-n}$

$$(v_2/v_1) = (P_2/P_1)^{-1/n} = (2200/200)^{-1/1.3} = \textbf{0.1581}$$

$$T_2 = T_1 (P_2/P_1)^{(n-1)/n} = 300 \left(\frac{2200}{200}\right)^{0.3/1.3} = \textbf{521.7 K}$$

From process eq.:

$$_1w_2 = \int P \, dv = \text{area} = \frac{1}{1-n}(P_2v_2 - P_1v_1) = \frac{R}{1-n}(T_2 - T_1)$$

$$= \frac{0.287}{1-1.3}(521.7 - 300) = \textbf{-212.09 kJ/kg}$$

From the energy equation and constant specific heat from Table A.5

$$_1q_2 = u_2 - u_1 + {}_1w_2 = C_v(T_2 - T_1) + {}_1w_2$$
$$= 0.717 \text{ kJ/kg-K } (521.7 - 300) \text{ K} - 212.09 \text{ kJ/kg} = \textbf{-53.13 kJ/kg}$$

From the energy equation and Table A.7

$$_1q_2 = u_2 - u_1 + {}_1w_2$$
$$= 376.0 - 214.36 - 212.09 = \textbf{-50.45 kJ/kg}$$

process plotted from CATT3

Borgnakke and Sonntag

6.114

A cylinder/piston contains air at ambient conditions, 100 kPa and 20°C with a volume of 0.3 m^3. The air is compressed to 800 kPa in a reversible polytropic process with exponent, $n = 1.2$, after which it is expanded back to 100 kPa in a reversible adiabatic process.

      a.   Show the two processes in *P–v* and *T–s* diagrams.

      b.   Determine the final temperature and the net work.

Solution:

a)

$$m = P_1 V_1 / R T_1$$
$$= \frac{100 \times 0.3}{0.287 \times 293.2}$$
$$= 0.3565 \text{ kg}$$

b) The process equation is expressed in Eq.6.28

$$T_2 = T_1 (P_2/P_1)^{\frac{n-1}{n}} = 293.2 \left(\frac{800}{100}\right)^{0.167} = 414.9 \text{ K}$$

The work is from Eq.6.29

$$_1w_2 = \int_1^2 P dv = \frac{P_2 v_2 - P_1 v_1}{1-n} = \frac{R(T_2 - T_1)}{1-n} = \frac{0.287(414.9 - 293.2)}{1 - 1.20} = -174.6 \text{ kJ/kg}$$

Isentropic relation is from Eq.6.23

$$T_3 = T_2 (P_3/P_2)^{\frac{k-1}{k}} = 414.9 \left(\frac{100}{800}\right)^{0.286} = \textbf{228.9 K}$$

With zero heat transfer the energy equation gives the work

$$_2w_3 = C_{V0}(T_2 - T_3) = 0.717(414.9 - 228.9) = +133.3 \text{ kJ/kg}$$

$$w_{NET} = 0.3565(-174.6 + 133.3) = \textbf{-14.7 kJ}$$

Borgnakke and Sonntag

Entropy generation

Borgnakke and Sonntag

6.115
　　　　Consider a heat transfer of 100 kJ from 1500 K hot gases to a steel container at 750 K that has a heat transfer of the 100 kJ out to some air at 375 K. Determine the entropy generation in each of the control volumes indicated in Fig. P6.115.

　　　　There is no change in energy or entropy in the indicated control volumes, so no storage effect. There is a transfer of energy in and out of each CV and an associated transfer of entropy.

Take CV1
Energy Eq.:

$$0 = Q - Q$$

Entropy Eq.:

$$0 = \frac{Q}{T_H} - \frac{Q}{T_M} + S_{gen\ CV1}$$

$$S_{gen\ CV1} = \frac{Q}{T_M} - \frac{Q}{T_H}$$

$$= \frac{100}{750} - \frac{100}{1500} = \mathbf{0.0667\ kJ/K}$$

Take CV2
Energy Eq.:

$$0 = Q - Q$$

Entropy Eq.:

$$0 = \frac{Q}{T_M} - \frac{Q}{T_L} + S_{gen\ CV2}$$

$$S_{gen\ CV2} = \frac{Q}{T_L} - \frac{Q}{T_M}$$

$$= \frac{100}{375} - \frac{100}{750} = \mathbf{0.133\ kJ/K}$$

Figure P6.115
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6.116

A rigid tank has 0.1 kg saturated vapor R-410A at 0^oC that is cooled to -20^oC by a -20^oC heat sink. Show the process in a T-s diagram; find the change in entropy of the R-410A, the heat sink, and the total entropy generation.

Solution:

C.V.: R-410A in tank + out to the sink. $m_2 = m_1$;

Energy Eq.3.5: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Entropy Eq. 6.37: $m(s_2 - s_1) = {_1}Q_2 /T_{sink} + {_1}S_{2\ gen}$

Process: V = constant, $v_2 = v_1 = V/m$ => ${_1}W_2 = 0$

Table B.4.1: State 1: $u_1 = 253.02$ kJ/kg, $s_1 = 1.0368$ kJ/kgK

State 2: -20^oC, $v_2 = v_1 = V/m$,   look in Table B.4.1 at -20^oC

$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{0.03267 - 0.000803}{0.06400} = 0.4979$$

$$u_2 = u_{f2} + x_2\ u_{fg2} = 27.92 + x_2 \times 218.07 = 136.5 \text{ kJ/kg}$$

$$s_2 = s_{f2} + x_2\ s_{fg2} = 0.1154 + x_2 \times 0.9625 = 0.5946 \text{ kJ/kgK}$$

From the energy equation

${_1}Q_2 = m(u_2 - u_1) = 0.1$ kg (136.5 – 253.0) kJ/kg = **-11.65 kJ**

$(S_2 - S_1)_{R-410a} = m(s_2 - s_1) = 0.1 (0.5946 - 1.0368) = $ **–0.0442 kJ/K**

$(S_2 - S_1)_{sink} = -{_1}Q_2 /T_{sink} = 11.65 $ kJ/253.15 K = **0.046 kJ/K**

${_1}S_{2\ gen} = m(s_2 - s_1) - {_1}Q_2 /T_{sink} = -0.0442 + 0.046 = $ **0.0018 kJ/K**

Borgnakke and Sonntag

6.117

One kg water at 500°C and 1 kg saturated water vapor both at 200 kPa are mixed in a constant pressure and adiabatic process. Find the final temperature and the entropy generation for the process.

Solution:

Continuity Eq.: $\quad m_2 - m_A - m_B = 0$

Energy Eq.3.5: $\quad m_2 u_2 - m_A u_A - m_B u_B = -_1 W_2$

Entropy Eq.6.37: $m_2 s_2 - m_A s_A - m_B s_B = \int dQ/T + _1 S_{2\ gen}$

Process: $\qquad P = \text{Constant} \quad => \quad _1 W_2 = \int P dV = P(V_2 - V_1)$

$\qquad\qquad\qquad Q = 0$

Substitute the work term into the energy equation and rearrange to get

$$m_2 u_2 + P_2 V_2 = m_2 h_2 = m_A u_A + m_B u_B + PV_1 = m_A h_A + m_B h_B$$

where the last rewrite used $PV_1 = PV_A + PV_B$.

State A1: Table B.1.3 $\quad h_A = 3487.03$ kJ/kg, $\quad s_A = 8.5132$ kJ/kg K

State B1: Table B.1.2 $\quad h_B = 2706.63$ kJ/kg, $\quad s_B = 7.1271$ kJ/kg K

Energy equation gives:

$$h_2 = \frac{m_A}{m_2} h_A + \frac{m_B}{m_2} h_B = \tfrac{1}{2}\, 3487.03 + \tfrac{1}{2}\, 2706.63 = 3096.83 \text{ kJ/kg}$$

State 2: $\quad P_2, h_2 = 3096.83$ kJ/kg $\quad => \quad s_2 = 7.9328$ kJ/kg K; $\quad T_2 = \textbf{312.2°C}$

With the zero heat transfer we have

$\quad _1 S_{2\ gen} = m_2 s_2 - m_A s_A - m_B s_B$

$\qquad\qquad = 2 \times 7.9328 - 1 \times 8.5132 - 1 \times 7.1271 = \textbf{0.225 kJ/K}$

Borgnakke and Sonntag

6.118

A car uses an average power of 25 hp for a one hour round trip. With a thermal efficiency of 35% how much fuel energy was used? What happened to all the energy? What change in entropy took place if we assume ambient at 20^oC?

Since it is a round trip, there are no changes in storage of energy for the car after it has cooled down again. All the energy is given out to the ambient in the form of exhaust flow (hot air) and heat transfer from the radiator and underhood air flow.

$$W = \int \dot{W} \, dt = 25 \text{ hp} \times 0.7457 \text{ (kW/hp)} \times 3600 \text{ s} = 67 \text{ } 113 \text{ kJ} = \eta \text{ Q}$$

Fuel energy used to deliver the W

$\text{Q} = \text{E} / \eta = 67 \text{ } 113 \text{ kJ} / 0.35 = \textbf{191 751 kJ}$

$\Delta \text{S} = \text{Q} / \text{T} = 191 \text{ } 751 \text{ kJ} / 293.15 \text{ K} = \textbf{654.1 kJ/K}$

All the energy (Q) ends up in the ambient at the ambient temperature.

Borgnakke and Sonntag

6.119

A computer chip dissipates 2 kJ of electric work over time and rejects that as heat transfer from its 50°C surface to 25°C air. How much entropy is generated in the chip? How much if any is generated outside the chip?

C.V.1 Chip with surface at 50°C, we assume chip state is constant.

Energy: $U_2 - U_1 = 0 = {_1}Q_2 - {_1}W_2 = W_{electrical\ in} - Q_{out\ 1}$

Entropy: $S_2 - S_1 = 0 = -\dfrac{Q_{out\ 1}}{T_{surf}} + {_1}S_{2\ gen1}$

$${_1}S_{2\ gen1} = \dfrac{Q_{out\ 1}}{T_{surf}} = \dfrac{W_{electrical\ in}}{T_{surf}} = \dfrac{2\ kJ}{323.15\ K} = \textbf{6.19 J/K}$$

C.V.2 From chip surface at 50°C to air at 25°C, assume constant state.

Energy: $U_2 - U_1 = 0 = {_1}Q_2 - {_1}W_2 = Q_{out\ 1} - Q_{out\ 2}$

Entropy: $S_2 - S_1 = 0 = \dfrac{Q_{out1}}{T_{surf}} - \dfrac{Q_{out2}}{T_{air}} + {_1}S_{2\ gen2}$

$${_1}S_{2\ gen2} = \dfrac{Q_{out2}}{T_{air}} - \dfrac{Q_{out1}}{T_{surf}} = \dfrac{2\ kJ}{298.15\ K} - \dfrac{2\ kJ}{323.15\ K} = \textbf{0.519 J/K}$$

Borgnakke and Sonntag

6.120

An insulated cylinder/piston contains R-134a at 1 MPa, 50°C, with a volume of 100 L. The R-134a expands, moving the piston until the pressure in the cylinder has dropped to 100 kPa. It is claimed that the R-134a does 190 kJ of work against the piston during the process. Is that possible?

Solution:

C.V. R-134a in cylinder. Insulated so assume Q = 0.

State 1: Table B.5.2, $v_1 = 0.02185 \text{ m}^3/\text{kg}$, $u_1 = 409.39 \text{ kJ/kg}$,

$s_1 = 1.7494 \text{ kJ/kg K}$, $m = V_1/v_1 = 0.1/0.02185 = 4.577 \text{ kg}$

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0 - 190 \Rightarrow$

$u_2 = u_1 - {}_1W_2/m = 367.89 \text{ kJ/kg}$

State 2: P_2 , u_2 $\Rightarrow$ Table B.5.2: $T_2 = -19.25°C$; $s_2 = 1.7689 \text{ kJ/kg K}$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2,gen} = {}_1S_{2,gen}$

$_1S_{2,gen} = m(s_2 - s_1) = 0.0893 \text{ kJ/K}$

This is possible since $_1S_{2,gen} > 0$

6.121

A rigid tank holds 0.75 kg ammonia at 70°C as saturated vapor. The tank is now cooled to 20°C by heat transfer to the ambient at 20°C. Determine the amount of entropy generation during the process.

C.V. The ammonia out to 20°C, this is a control mass.

Process: Rigid tank V = C   =>   v = constant & $_1W_2 = \int\limits_1^2 PdV = \mathbf{0}$

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2$,

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2,gen} = {}_1Q_2/T_0 + {}_1S_{2,gen}$

State 1: $v_1 = 0.03787$ m^3/kg,

$u_1 = 1338.9$ kJ/kg, $s_1 = 4.3533$ kJ/kgK

State 2:   **T, v** => two-phase (straight down in P-v diagram from state 1)

$x_2 = (v - v_f)/v_{fg} = (0.03787 - 0.001638)/0.14758 = 0.2455$

$u_2 = u_f + x_2\, u_{fg} = 272.89 + 0.2455 \times 1059.3 = 532.95$ kJ/kg

$s_2 = s_f + x_2\, s_{fg} = 1.0408 + 0.2455 \times 4.0452 = 2.0339$ kJ/kgK

$_1Q_2 = m(u_2 - u_1) = 0.75$ kg $(532.95 - 1338.9)$ kJ/kg = **-604.5 kJ**

$_1S_{2,gen} = m(s_2 - s_1) - {}_1Q_2/T_0 = 0.75(2.0339 - 4.3533) + 604.5/293.15$
    = **0.322 kJ/K**

Borgnakke and Sonntag

6.122

The unrestrained expansion of the reactor water in Problem 3.101 has a final state in the two-phase region. Find the entropy generated in the process.

A water-filled reactor with volume of 1 m^3 is at 20 MPa, 360°C and placed inside a containment room as shown in Fig. P3.101. The room is well insulated and initially evacuated. Due to a failure, the reactor ruptures and the water fills the containment room. Find the minimum room volume so the final pressure does not exceed 200 kPa.

Solution:

C.V.: Containment room and reactor.

Mass: $m_2 = m_1 = V_{reactor}/v_1 = 1/0.001823 = 548.5$ kg

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0 - 0 = 0$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen}$

State 1: (T, P) Table B.1.4 $u_1 = 1702.8$ kJ/kg, $s_1 = 3.877$

Energy equation implies $u_2 = u_1 = 1702.8$ kJ/kg

State 2: $P_2 = 200$ kPa, $u_2 < u_g$ => Two-phase Table B.1.2

$x_2 = (u_2 - u_f)/u_{fg} = (1702.8 - 504.47)/2025.02 = 0.59176$

$v_2 = 0.001061 + 0.59176 \times 0.88467 = 0.52457$ m^3/kg

$s_2 = s_f + x_2 s_{fg} = 1.53 + 0.59176 \times 5.597 = 4.8421$ kJ/kg K

$V_2 = m_2\,v_2 = 548.5 \times 0.52457 = \textbf{287.7 m}^3$

From the entropy equation the generation is

${}_1S_{2\,gen} = m(s_2 - s_1) = 548.5\,(4.8421 - 3.877) = \textbf{529.4 kJ/K}$

Entropy is generated due to the unrestrained expansion. No work was taken out as the volume goes up.

Borgnakke and Sonntag

6.123

Heat transfer from a 20°C kitchen to a block of 1.5 kg ice at -10°C melts it to liquid at 10°C. How much entropy is generated?

Water changes state from nearly saturated solid to nearly saturated liquid. The pressure is 101 kPa but we approximate the state properties with saturated state at the same temperature.

CV Ice out to the 20°C kitchen air

Energy Eq.3.5: $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

Entropy Eq.6.37: $\quad m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = {}_1Q_2/T_0 + {}_1S_{2\,gen}$

State 1: Compressed (saturated) solid, B.1.5,
$\qquad u_1 = -354.09$ kJ/kg, $s_1 = -1.2995$ kJ/kg-K

State 2: Compressed (saturated) liquid B.1.1
$\qquad u_2 = 41.99$ kJ/kg, $s_2 = 0.1510$ kJ/kg-K

Heat transfer from the energy Eq.

$\qquad {}_1Q_2 = m(u_2 - u_1) = 1.5\,[41.99 - (-354.09)] = 594.12$ kJ

From the entropy Eq.

$\qquad {}_1S_{2\,gen} = m(s_2 - s_1) - {}_1Q_2/T_0$

$\qquad\qquad = 1.5\,[0.151 - (-1.2995)] - 594.12/293.15 = \mathbf{0.149\ kJ/K}$

Borgnakke and Sonntag

6.124

Ammonia is contained in a rigid sealed tank unknown quality at 0°C. When heated in boiling water to 100°C its pressure reaches 1200 kPa. Find the initial quality, the heat transfer to the ammonia and the total entropy generation.

Solution:

C.V. Ammonia, which is a control mass of constant volume.

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.6.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\ gen}$

State 2: 1200 kPa, 100°C => Table B.2.2

$s_2 = 5.5325$ kJ/kg K, $v_2 = 0.14347$ m^3/kg, $u_2 = 1485.8$ kJ/kg

State 1: $v_1 = v_2$ => Table B.2.1

$x_1 = (0.14347 - 0.001566)/0.28763 = 0.49336$

$u_1 = 741.28$ kJ/kg, $s_1 = 0.7114 + x_1 \times 4.6195 = 2.9905$ kJ/kg K

Process: V = constant => ${}_1w_2 = 0$

${}_1q_2 = (u_2 - u_1) = 1485.8 - 741.28 = 744.52$ kJ/kg

To get the total entropy generation take the C.V out to the water at 100°C.

${}_1s_{2\ gen} = s_2 - s_1 - {}_1q_2/T = 5.5325 - 2.9905 - 744.52/373.15$

$= \textbf{0.547 kJ/kg K}$

Borgnakke and Sonntag

6.125

Water in a piston/cylinder is at 101 kPa, 25°C, and mass 0.5 kg. The piston rests on some stops, and the pressure should be 1000 kPa to float the piston. We now heat the water from a 200°C reservoir, so the volume becomes 5 times the initial volume. Find the total heat transfer and the entropy generation.

Solution:

Take CV as the water out to the reservoir.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = {}_1Q_2/T_{res} + {}_1S_{2\,gen}$

Process: v = constant, then $P = C = P_{float}$.

Volume does go up so we get $v_2 = 5\,v_1$

State 1: $v_1 = 0.001003$ m³/kg, $u_1 = 104.86$ kJ/kg, $s_1 = 0.3673$ kJ/kgK

State 2: $P_2 = P_{float}$, $v_2 = 5 \times 0.001003 = 0.005015$ m³/kg; $T_2 = 179.91$°C

$x_2 = (v_2 - v_f)/v_{fg} = (0.005015 - 0.001127)/0.19332 = 0.02011$,

$u_2 = 761.67 + x_2 \times 1821.97 = 798.31$ kJ/kg

$s_2 = 2.1386 + x_2 \times 4.4478 = 2.2280$ kJ/kgK

From the process equation (see P-V diagram) we get the work as

$_1w_2 = P_{float}(v_2 - v_1) = 1000$ kPa $(0.005015 - 0.001003)$ m³/kg $= 4.012$ kJ/kg

From the energy equation we solve for the heat transfer

$_1Q_2 = m[u_2 - u_1 + {}_1w_2] = 0.5\times[798.31 - 104.86 + 4.012] =$ **348.7 kJ**

$_1S_{2\,gen} = m(s_2 - s_1) - {}_1Q_2/T_{res} = 0.5(2.2280 - 0.3673) - 348.7/473.15$

$= $ **0.1934 kJ/K**

6.126

Do Problem 6.125 assuming the piston/cylinder is 1.5 kg of steel and has the the same temperature as the water at any time.

Solution:

Take CV as the water and steel out to the reservoir.

Continuity Eq.: $m_2 = m_1 = m_{H2O} + m_{steel}$

Energy Eq.: $m_{H2O} (u_2 - u_1)_{H2O} + m_{steel} (u_2 - u_1)_{steel} = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m_{H2O}(s_2 - s_1)_{H2O} + m_{steel}(s_2 - s_1)_{steel} = {}_1Q_2/T_{res} + {}_1S_{2\ gen}$

Process: v = constant, then $P = C = P_{float}$.

Volume does go up so we get $v_2 = 5\ v_1$

State 1: $v_1 = 0.001003$ m^3/kg, $u_1 = 104.86$ kJ/kg, $s_1 = 0.3673$ kJ/kgK

State 2: $P_2 = P_{float}$, $v_2 = 5 \times 0.001003 = 0.005015$ m^3/kg; $T_2 = 179.91°C$

$x_2 = (v_2 - v_f) /v_{fg} = (0.005015 - 0.001127)/0.19332 = 0.02011$,

$u_2 = 761.67 + x_2 \times 1821.97 = 798.31$ kJ/kg

$s_2 = 2.1386 + x_2 \times 4.4478 = 2.2280$ kJ/kgK

There is only work when piston moves and then $P = P_{float}$ so the work is

${}_1W_2 = P_{float}(V_2 - V_1) = 1000$ kPa $\times 0.5 (0.005015 - 0.001003)$ m^3 = 2.006 kJ

From the energy equation we solve for the heat transfer

${}_1Q_2 = U_2 - U_1 + {}_1W_2$

$= 0.5 \times [798.31 - 104.86] + 1.5 \times 0.46 \times (179.91 - 25) + 2.006$

$= \textbf{455.6 kJ}$

For the entropy change we use B.1.1 for water and A.5 and Eq.6.11 for steel

${}_1S_{2\ gen} = S_2 - S_1 - {}_1Q_2/T_{res}$

$= 0.5(2.2280 - 0.3673) + 1.5 \times 0.46 \times \ln \dfrac{453.06}{298.15} - \dfrac{455.6}{473.15}$

$= \textbf{0.256 kJ/K}$

Borgnakke and Sonntag

6.127

A cylinder fitted with a movable piston contains water at 3 MPa, 50% quality, at which point the volume is 20 L. The water now expands to 1.2 MPa as a result of receiving 600 kJ of heat from a large source at 300°C. It is claimed that the water does 124 kJ of work during this process. Is this possible?

Solution:

C.V.: H_2O in Cylinder

Energy Eq.: $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$;

Entropy Eq.6.37: $\quad m(s_2 - s_1) = \dfrac{{}_1Q_2}{T_{source}} + {}_1S_{2,gen}; \quad T_{source} = 300^oC,$

Process: $\quad {}_1Q_2 = 600$ kJ, ${}_1W_2 = 124$ kJ ?

State 1: 3 MPa, $x_1 = 0.5$, Table B.1.2: $T_1 = 233.9^oC$

$\quad v_1 = v_f + x_1 v_{fg} = 0.001216 + 0.5 \times 0.06546 = 0.033948$ m^3/kg

$\quad u_1 = u_f + x_1 u_{fg} = 1804.5$ kJ/kg, $\quad s_1 = s_f + x_1 s_{fg} = 4.4162$ kJ/kg-K

$\quad m_1 = V_1/v_1 = 0.02$ m^3/ [0.033948 m^3/kg] = 0.589 kg

Now solve for u_2 from the energy equation

$$u_2 = u_1 + ({}_1Q_2 - {}_1W_2)/m_1$$

$$= 1804.5 + (600 - 124)/0.589 = 2612.6 \text{ kJ/kg}$$

State 2: $P_2 = 1.2$ MPa : $u_2 = 2612.6$ kJ/kg $\quad$ Table B.1.3

$$T_2 \cong 200^oC, \quad s_2 = 6.5898 \text{ kJ/kgK}$$

From the entropy equation

$${}_1S_{2,gen} = m(s_2 - s_1) - \dfrac{{}_1Q_2}{T_{source}}$$

$$= 0.589 \text{ kg } (6.5898 - 4.4162) \text{ kJ/kg-K} - \dfrac{600}{300 + 273} \text{ kJ/K}$$

$$= 0.2335 \text{ kJ/K} \geq 0; \quad \textbf{Process is possible}$$

Borgnakke and Sonntag

6.128

A piston/cylinder device keeping a constant pressure has 1 kg water at 20°C and 1 kg of water at 100°C both at 500 kPa separated by a thin membrane. The membrane is broken and the water comes to a uniform state with no external heat transfer. Find the final temperature and the entropy generation for the process.

Solution:

Continuity Eq.: $m_2 - m_A - m_B = 0$

Energy Eq.3.5: $m_2u_2 - m_Au_A - m_Bu_B = -_1W_2$

Entropy Eq.6.37: $m_2s_2 - m_As_A - m_Bs_B = \int dQ/T + _1S_{2\ gen}$

Process: $\qquad$ P = Constant $\Rightarrow$ $\qquad _1W_2 = \int PdV = P(V_2 - V_1)$

$\qquad\qquad\qquad$ Q = 0

Substitute the work term into the energy equation and rearrange to get

$$m_2u_2 + P_2V_2 = m_2h_2 = m_Au_A + m_Bu_B + PV_1 = m_Ah_A + m_Bh_B$$

where the last rewrite used $PV_1 = PV_A + PV_B$.

State A1: Table B.1.4 $\quad h_A = 84.41$ kJ/kg $\qquad s_A = 0.2965$ kJ/kg K

State B1: Table B.1.4 $\quad h_B = 419.32$ kJ/kg $\qquad s_B = 1.3065$ kJ/kg K

Energy equation gives:

$$h_2 = \frac{m_A}{m_2}h_A + \frac{m_B}{m_2}h_B = \frac{1}{2}\,84.41 + \frac{1}{2}\,419.32 = 251.865 \text{ kJ/kg}$$

State 2: $\quad h_2 = 251.865$ kJ/kg & $P_2 = 500$ kPa from Table B.1.4

$\qquad\quad T_2 = 60.085°C, \qquad s_2 = 0.83184$ kJ/kg K

With the zero heat transfer we have

$$_1S_{2\ gen} = m_2s_2 - m_As_A - m_Bs_B$$

$$= 2 \times 0.83184 - 1 \times 0.2965 - 1 \times 1.3065 = \textbf{0.0607 kJ/K}$$

Borgnakke and Sonntag

6.129

Reconsider Problem 3.109 where CO_2 is compressed from -20°C, x = 0.75 to a state of 3 MPa, 20°C in a piston/cylinder where pressure is linear in volume. Assume heat transfer is from a reservoir at 100°C and find the specific entropy generation in the process (external to the CO_2).

CV Carbon dioxide out to the source, both $_1Q_2$ and $_1W_2$

Energy Eq.3.5: $m(u_2 - u_1) = {_1Q_2} - {_1W_2}$

Entropy Eq.6.37: $S_2 - S_1 = \int dQ/T + {_1S_{2,gen}} = {_1Q_2}/T_{res} + {_1S_{2,gen}}$

Process: $P = A + BV$ => $_1W_2 = \int P\ dV = ½\ m(P_1 + P_2)(v_2 - v_1)$

State 1: Table B.3.1   P = 1969.6 kPa
$v_1 = 0.000969 + 0.75 \times 0.01837 = 0.01475$ m^3/kg,
$u_1 = 39.64 + 0.75 \times 246.25 = 224.33$ kJ/kg,
$s_1 = 0.1672 + 0.75 \times 1.1157 = 1.004$ kJ/kg-K

State 2: Table B.3 $v_2 = 0.01512$ m^3/kg, $u_2 = 310.21$ kJ/kg, $s_2 = 1.3344$ kJ/kg-K
$_1w_2 = ½\ (P_1 + P_2)(v_2 - v_1) = ½ \times (1969.6 + 3000)(0.01512 - 0.01475)$
$= 0.92$ kJ/kg
$_1q_2 = u_2 - u_1 + {_1w_2} = 310.21 - 224.33 + 0.92 = 86.8$ kJ/kg

$_1s_{2,gen} = s_2 - s_1 - {_1q_2}/T_{res} = 1.3344 - 1.004 - \dfrac{86.8}{373.15} = \textbf{0.098 kJ/kg-K}$

Borgnakke and Sonntag

6.130

A piston/cylinder contains 1 kg water at 150 kPa, 20°C. The piston is loaded so pressure is linear in volume. Heat is added from a 600°C source until the water is at 1 MPa, 500°C. Find the heat transfer and the total change in entropy.

Solution:

CV H_2O out to the source, both $_1Q_2$ and $_1W_2$

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = {}_1Q_2 / T_{SOURCE} + {}_1S_{2\,gen}$

Process: $P = A + BV$ => $_1W_2 = \int P \, dV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$

State 1: B.1.1 Compressed liquid use saturated liquid at same T:

$v_1 = 0.001002 \text{ m}^3/\text{kg};$ $u_1 = 83.94 \text{ kJ/kg};$ $s_1 = 0.2966 \text{ kJ/kg K}$

State 2: Table B.1.3 sup. vap.

$v_2 = 0.35411 \text{ m}^3/\text{kg}$

$u_2 = 3124.3 \text{ kJ/kg};$

$s_2 = 7.7621 \text{ kJ/kg K}$

$_1W_2 = \frac{1}{2}(1000 + 150) \text{ kPa} \times 1 \text{ kg} (0.35411 - 0.001002) \text{ m}^3/\text{kg} = 203 \text{ kJ}$

$_1Q_2 = 1(3124.3 - 83.94) + 203 = \mathbf{3243.4 \text{ kJ}}$

$m(s_2 - s_1) = 1 \text{ kg} (7.7621 - 0.2968) \text{ kJ/kg-K} = 7.4655 \text{ kJ/K}$

$_1Q_2 / T_{source} = 3.7146 \text{ kJ/K}$ (for source $Q = -{}_1Q_2$)

$_1S_{2\,gen} = m(s_2 - s_1) - {}_1Q_2 / T_{SOURCE}$

$= \Delta S_{H2O} + \Delta S_{source} = 7.4655 - 3.7146 = \mathbf{3.751 \text{ kJ/K}}$

Remark: This is an external irreversible process (delta T to the source)

Borgnakke and Sonntag

6.131

A closed rigid container is filled with 1.5 kg water at 100 kPa, 55°C, 1 kg of stainless steel and 0.5 kg of PVC (polyvinyl chloride) both at 20°C and 0.1 kg of air at 400 K, 100 kPa. It is now left alone with no external heat transfer and no water vaporizes. Find the final temperature and the entropy generation for the process.

CV. Container.

Process: V = constant $\Rightarrow$ $_1W_2 = 0$ and also given $_1Q_2 = 0$

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {_1Q_2} - {_1W_2} = 0$

Entropy Eq.6.37: $S_2 - S_1 = \sum m_i(s_2 - s_1)_i = 0 + {_1S_{2,gen}}$

For the liquid and the metal mass we will use the specific heat (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v\,i}(T_2 - T_1)_i = T_2 \sum m_i C_{v\,i} - \sum m_i C_{v\,i} T_{1\,i}$$

$$\sum m_i(s_2 - s_1)_i = \sum m_i C_{v\,i} \ln(T_2/T_1)_i$$

noticing that all masses have the same T_2 but not same initial T.

$$\sum m_i C_{v\,i} = 1.5 \times 4.18 + 1 \times 0.46 + 0.5 \times 0.96 + 0.1 \times 0.717 = 7.282 \text{ kJ/K}$$

Energy Eq.: $7.282\, T_2 = 1.5 \times 4.18 \times 55 + (1 \times 0.46 + 0.5 \times 0.96) \times 20$

$$+ 0.1 \times 0.717 \times (400-273.15) = 372.745 \text{ kJ}$$

$$T_2 = \mathbf{51.2°C}$$

The volume of the air is constant so entropy change from Eq.6.17 is the same expression as for the solids and liquids given above.

$$_1S_{2,gen} = S_2 - S_1 = \sum m_i(s_2 - s_1)_i = 1.5 \times 4.18 \times \ln(324.35/328.15)$$

$$+ (1 \times 0.46 + 0.5 \times 0.96) \times \ln(\frac{324.35}{293.15}) + 0.1 \times 0.717 \times \ln(\frac{324.35}{400})$$

$$= -0.07304 \text{ kJ/K} + 0.09507 \text{ kJ/K} - 0.01503 \text{ kJ/K} = \mathbf{0.007 \text{ kJ/K}}$$

Borgnakke and Sonntag

6.132

A cylinder/piston contains water at 200 kPa, 200°C with a volume of 20 L. The piston is moved slowly, compressing the water to a pressure of 800 kPa. The loading on the piston is such that the product *PV* is a constant. Assuming that the room temperature is 20°C, show that this process does not violate the second law.

Solution:

C.V.: Water + cylinder out to room at 20°C

Energy Eq.3.5: $\qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $\qquad m(s_2 - s_1) = {}_1Q_2 / T_{room} + {}_1S_{2\ gen}$

Process: $PV = \text{constant} = Pmv \implies v_2 = P_1v_1/P_2$

$$ {}_1w_2 = \int Pdv = P_1v_1 \ln(v_2/v_1) $$

State 1: Table B.1.3, $\quad v_1 = 1.0803 \text{ m}^3/\text{kg}, \quad u_1 = 2654.4 \text{ kJ/kg},$

$$ s_1 = 7.5066 \text{ kJ/kg K} $$

State 2: P_2, $v_2 = P_1v_1/P_2 = 200 \times 1.0803/800 = 0.2701 \text{ m}^3/\text{kg}$

$$ \text{Table B.1.3:} \quad u_2 = 2655.0 \text{ kJ/kg}, \quad s_2 = 6.8822 \text{ kJ/kg K} $$

$$ {}_1w_2 = 200 \times 1.0803 \ln\left(\frac{0.2701}{1.0803}\right) = \text{-299.5 kJ/kg} $$

$$ {}_1q_2 = u_2 - u_1 + {}_1w_2 = 2655.0 - 2654.4 - 299.5 = \text{-298.9 kJ/kg} $$

$$ {}_1s_{2,gen} = s_2 - s_1 - \frac{{}_1q_2}{T_{room}} = 6.8822 - 7.5066 + \frac{298.9}{293.15} $$

$$ = 0.395 \text{ kJ/kg K} > 0 \quad \textbf{satisfies entropy eq.} $$

Borgnakke and Sonntag

6.133

A rigid steel tank of mass 2.5 kg contains 0.5 kg R-410A at 0°C with specific volume 0.01m/kg. The whole system heats up to the room temperature, 25°C. Find the process heat transfer and the entropy generation.

C.V. R-410A and steel tank out to room T. Control mass.

Continuity Eq.: $m_2 - m_{R410a} - m_{st} = 0$

Energy Eq.: $m_{R410a}(u_2 - u_1) + m_{st}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m_{R410a}(s_2 - s_1) + m_{st}(s_2 - s_1) = {}_1Q_2/T_{room} + {}_1S_{2,gen}$

Process: $V = C$ so ${}_1W_2 = 0$

State 1: $T_1 = 0°C$, $v_1 = 0.01$ m^3/kg,

$\qquad V = mv_1 = 0.005$ m^3

$\qquad x_1 = (v - v_f)/v_{fg} = (0.01 - 0.000855)/0.03182$

$\qquad\quad = 0.28758$

$\qquad u_1 = u_f + x_1 u_{fg} = 57.07 + x_1\ 195.95$

$\qquad\quad = 113.42$ kJ/kg

$\qquad s_1 = 0.2264 + x_1\ 0.8104 = 0.45945$ kJ/kgK

State 2:   (**T, v**) => sup-vapor (straight up in T-v diagram from state 1)
 B.4.1 at 25°C, $v_f = 0.000944$ m^3/kg, $v_g = 0.01514$ m^3/kg, $v_f < v < v_g$: saturated.

$\qquad P = 1653.6$ kPa, $x = \dfrac{v - v_f}{v_{fg}} = \dfrac{0.01 - 0.000944}{0.01420} = 0.63775,$

$\qquad u_2 = u_f + x_2 u_{fg} = 96.03 + x_2\ 162.95 = 199.95$ kJ/kg

$\qquad s_2 = s_f + x_2 s_{fg} = 0.3631 + x_2\ 0.6253 = 0.7619$ kJ/kgK

From the energy Eq.:

$\qquad {}_1Q_2 = m_{R410a}(u_2 - u_1) + m_{st} C_{st}(T_2 - T_1)$

$\qquad\quad = 0.5$ kg $\times (199.95 - 113.42)$ kJ/kg + 2.5 kg $\times$ 0.46 kJ/kgK $\times (25{-}0)$ K

$\qquad\quad = $ **72.0 kJ**

$\qquad {}_1S_{2,gen} = m_{R410a}(s_2 - s_1) + m_{st} C_{st} \ln(T_2/T_1) - {}_1Q_2/T_{room}$

$\qquad\quad = 0.5$ kg $\times(0.7619 - 0.45945)\dfrac{kJ}{kgK} + 2.5$ kg $\times 0.46 \dfrac{kJ}{kgK}\times \ln\dfrac{298.15}{273.15}$

$\qquad\qquad - (72/298.15)$ kJ/K

$\qquad\quad = 0.1512$ kJ/K $+ 0.1007$ kJ/K $- 0.2415$ kJ/K = **0.0104 kJ/K**

Borgnakke and Sonntag

6.134

A piston/cylinder has ammonia at 2000 kPa, 80°C with a volume of 0.1 m^3. The piston is loaded with a linear spring and outside ambient is at 20°C, shown in Fig. P6.134. The ammonia now cools down to 20°C at which point it has a quality of 15%. Find the work, the heat transfer and the total entropy generation in the process.

CV Ammonia out to the ambient, both $_1Q_2$ and $_1W_2$

Energy Eq.3.5: $m(u_2 - u_1) = {_1Q_2} - {_1W_2}$

Entropy Eq.6.37: $m(s_2 - s_1) = {_1Q_2} / T_{ambient} + {_1S_{2\ gen}}$

Process: $P = A + BV$ => $_1W_2 = \int P\ dV = \frac{1}{2}\ m(P_1 + P_2)(v_2 - v_1)$

State 1: Table B.2.2

$v_1 = 0.07595$ m^3/kg, $u_1 = 1421.6$ kJ/kg, $s_1 = 5.0707$ kJ/kg K

$m = V_1/v_1 = 0.1/0.07595 = 1.31665$ kg

State 2: Table B.2.1

$v_2 = 0.001638 + 0.15 \times 0.14758 = 0.023775$ m^3/kg

$u_2 = 272.89 + 0.15 \times 1059.3 = 431.785$ kJ/kg

$s_2 = 1.0408 + 0.15 \times 4.0452 = 1.64758$ kJ/kg K

$_1W_2 = \frac{1}{2}\ m(P_1 + P_2)(v_2 - v_1)$

$= \frac{1}{2} \times 1.31665$ kg $(2000 + 857.5)$ kPa $(0.023775 - 0.07595)$ m^3/kg

$= $ **- 98.15 kJ**

$_1Q_2 = m(u_2 - u_1) + {_1W_2} = 1.31665\ (431.785 - 1421.6) - 98.15$

$= $ **- 1401.39 kJ**

$_1S_{2\ gen} = m(s_2 - s_1) - ({_1Q_2}/ T_{amb})$

$= 1.31665\ (1.64758 - 5.0707) - \dfrac{-1401.39}{293.15}$

$= -4.507051 + 5.78045 = $ **0.273 kJ/k**

Borgnakke and Sonntag

6.135

One kilogram of ammonia (NH_3) is contained in a spring-loaded piston/cylinder, Fig. P6.135, as saturated liquid at −20°C. Heat is added from a reservoir at 100°C until a final condition of 800 kPa, 70°C is reached. Find the work, heat transfer, and entropy generation, assuming the process is internally reversible.

Solution:

C.V. = NH_3 out to the reservoir.

Continuity Eq.: $m_2 = m_1 = m$

Energy Eq.3.5: $E_2 - E_1 = m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $S_2 - S_1 = \int dQ/T + {}_1S_{2,gen} = {}_1Q_2/T_{res} + {}_1S_{2,gen}$

Process: $P = A + BV$ linear in V =>

$${}_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)m(v_2 - v_1)$$

State 1: Table B.2.1

  $P_1 = 190.08$ kPa,

  $v_1 = 0.001504$ m^3/kg

  $u_1 = 88.76$ kJ/kg,

  $s_1 = 0.3657$ kJ/kg K

State 2: Table B.2.2 sup. vapor

  $v_2 = 0.199$ m^3/kg, $u_2 = 1438.3$ kJ/kg, $s_2 = 5.5513$ kJ/kg K

  ${}_1W_2 = \frac{1}{2}(190.08 + 800)$ kPa $\times 1$ kg $(0.1990 - 0.001504)$ m^3/kg = **97.768 kJ**

  ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1(1438.3 - 88.76) + 97.768 = $ **1447.3 kJ**

  ${}_1S_{2,gen} = m(s_2 - s_1) - {}_1Q_2/T_{res} = 1(5.5513 - 0.3657) - \dfrac{1447.3}{373.15} = $ **1.307 kJ/K**

Borgnakke and Sonntag

6.136

A 5 kg aluminum radiator holds 2 kg of liquid R-134a both at –10°C. The setup is brought indoors and heated with 220 kJ from a heat source at 100°C. Find the total entropy generation for the process assuming the R-134a remains a liquid.

Solution:

C.V. The aluminum radiator and the R-134a.

Energy Eq.3.5: $m_2u_2 - m_1u_1 = {}_1Q_2 - 0$

Process: No change in volume so no work as used above.

The energy equation now becomes (summing over the mass)

$$m_{al}\,(u_2 - u_1)_{al} + m_{R134a}\,(u_2 - u_1)_{R134a} = {}_1Q_2$$

Use specific heat from Table A.3 and A.4

$$m_{al}C_{al}\,(T_2 - T_1) + m_{R134a}C_{R134a}\ln(T_2 - T_1) = {}_1Q_2$$

$$T_2 - T_1 = {}_1Q_2 / [m_{al}C_{al} + m_{R134a}C_{R134a}]$$

$$= 220 / [5 \times 0.9 + 2 \times 1.43] = 29.89°C$$

$$T_2 = -10 + 29.89 = 19.89°C$$

Entropy generation from Eq.6.37

$${}_1S_{2\,gen} = m(s_2 - s_1) - {}_1Q_2/T$$

$$= m_{al}C_{al}\ln(T_2/T_1) + m_{R134a}C_{R134a}\ln(T_2/T_1) - \frac{{}_1Q_2}{T_{amb}}$$

$$= (5 \times 0.9 + 2 \times 1.43)\ln\frac{(19.89 + 273.15)}{-10 + 273.15} - \frac{220}{373.15}$$

$$= 0.7918 - 0.5896$$

$$= \textbf{0.202 kJ/K}$$

Borgnakke and Sonntag

6.137

A piston/cylinder of total 1 kg steel contains 0.5 kg ammonia at 1600 kPa both masses at 120°C. Some stops are placed so a minimum volume is 0.02 m^3, shown in Fig. P6.137. Now the whole system is cooled down to 30°C by heat transfer to the ambient at 20°C, and during the process the steel keeps same temperature as the ammonia. Find the work, the heat transfer and the total entropy generation in the process.

Energy Eq.3.5: $m(u_2 - u_1) + m_{st}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) + m_{st}(s_2 - s_1) = {}_1Q_2/T_{amb} + {}_1S_{2\ gen}$

State 1 : $v_1 = 0.11265$ m^3/kg, $u_1 = 1516.6$ kJ/kg, $s_1 = 5.5018$ kJ/kg K

$V_1 = mv_1 = 0.05634$ m^3

Stop 1a: $v_{stop} = V/m = 0.02/0.5 = 0.04$ m^3/kg, $P_{stop} = P_1 \Rightarrow$ T ~ 42°C

State 2 : $30°C < T_{stop}$ so $v_2 = v_{stop} = 0.04$ m^3/kg

$$x_2 = \left(\frac{v_2 - v_f}{v_{fg}}\right) = \frac{0.04 - 0.00168}{0.10881} = 0.35217$$

$$u_2 = 320.46 + x_2 \times 1016.9 = 678.58 \text{ kJ/kg}$$

$$s_2 = 1.2005 + x_2 \times 3.7734 = 2.5294 \text{ kJ/kg K}$$

${}_1W_2 = \int P\ dV = P_1 m\ (v_2 - v_1) = 1600 \times 0.5\ (0.004 - 0.11268) = -58.14$ kJ

${}_1Q_2 = m\ (u_2 - u_1) + m_{st}(u_2 - u_1) + {}_1W_2$

$= 0.5(\ 678.58 - 1516.6\ ) + 1 \times 0.46(30 - 120) - 58.14$

$= -419.01 - 41.4 - 58.14 = $ **–518.55 kJ**

${}_1S_{2\ gen} = m(s_2 - s_1) + m_{st}\ (s_2 - s_1) - {}_1Q_2/T_{amb}$

$= 0.5\ (2.5294 - 5.5018) + 1 \times 0.46 \ln \dfrac{273+30}{273+120} - \dfrac{-518.5}{293.15}$

$= -1.4862 - 0.1196 + 1.6277 = $ **0.02186 kJ/K**

Borgnakke and Sonntag

6.138

A piston/cylinder contains 0.1 kg water at 500°C, 1000 kPa. The piston has a stop at half the original volume, similar to Fig. 6.137. The water now cools to room temperature 25°C. Find the heat transfer and the entropy generation.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m(s_2 - s_1) = {}_1Q_2/T_{room} + {}_1S_{2,gen}$

Process Eq: $P = C$ if $v > v_{stop}$; $V = C$ if $P < P_{float}$

State 1: $v_1 = 0.35411 \ m^3/kg$, $u_1 = 3124.34 \ kJ/kg$

$s_1 = 7.7621 \ kJ/kgK$

State a: $v_a = v_1/2 = 0.177055 \ m^3/kg < v_{g \ 1000 \ kPa}$ so $T_a = T_{sat \ 1000 \ kPa} = 179.9°C$

The possible state 2 (P,V) combinations are shown. State "a" is (1000 kPa, v_a) so it is

two-phase with $T_a = 180°C > T_2$

$P_2 = P_{sat \ 25 \ C} = 3.169 \ kPa$ and $v_2 = v_a$

$x_2 = (v_2 - v_f)/v_{fg} = (0.177 - 0.001003)/43.358$

$= 0.0040604$

$u_2 = u_f + x_2 \ u_{fg} = 104.86 + x_2 \ 2304.9 = 114.219 \ kJ/kg$

$s_2 = s_f + x_2 \ s_{fg} = 0.3673 + x_2 \ 8.1905 = 0.40056 \ kJ/kgK$

${}_1W_2 = m \int P \ dv = m \ P_1 \ (v_2 - v_1)$ [see area below process curve in figure]

$= 0.1 \ kg \times 1000 \ kPa \times (0.177055 - 0.35411) \ m^3/kg = -17.706 \ kJ$

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.1 \ kg \ (114.219 - 3124.34) \ kJ/kg - 17.706 \ kJ$

$= \mathbf{-318.72 \ kJ}$

${}_1S_{2,gen} = m \ (s_2 - s_1) - {}_1Q_2/T_{room}$

$= 0.1 \ kg(0.40056 - 7.7621) \ kJ/kgK + 318.72 \ kJ/298.15 \ K$

$= -0.73615 \ kJ/K + 1.06899 \ kJ/K = \mathbf{0.333 \ kJ/K}$

Borgnakke and Sonntag

6.139

A hollow steel sphere with a 0.5-m inside diameter and a 2-mm thick wall contains water at 2 MPa, 250°C. The system (steel plus water) cools to the ambient temperature, 30°C. Calculate the net entropy change of the system and surroundings for this process.

C.V.: Steel + water out to ambient T_0. This is a control mass.

Energy Eq.: $U_2 - U_1 = m_{H_2O}(u_2 - u_1) + m_{steel}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $S_2 - S_1 = \int dQ/T + {}_1S_{2\;gen} = {}_1Q_2/T_0 + {}_1S_{2\;gen}$

Process: $V = $ constant $\Rightarrow {}_1W_2 = 0$

$m_{steel} = (\rho V)_{steel} = 8050 \times (\pi/6)[(0.504)^3 - (0.5)^3] = 12.746$ kg

$V_{H2O} = (\pi/6)(0.5)^3$, $m_{H_2O} = V/v = 6.545 \times 10^{-2}/0.11144 = 0.587$ kg

$v_2 = v_1 = 0.11144 = 0.001004 + x_2 \times 32.889 \Rightarrow x_2 = 3.358 \times 10^{-3}$

$u_2 = 125.78 + 3.358 \times 10^{-3} \times 2290.8 = 133.5$ kJ/kg

$s_2 = 0.4639 + 3.358 \times 10^{-3} \times 8.0164 = 0.4638$ kJ/kg K

$_1Q_2 = m_{H_2O}(u_2 - u_1) + m_{steel}(u_2 - u_1)$

$\quad = 0.587(133.5 - 2679.6) + 12.746 \times 0.48(30 - 250)$

$\quad = -1494.6 + (-1346) = -2840.6$ kJ

$S_2 - S_1 = m_{H_2O}(s_2 - s_1) + m_{steel}(s_2 - s_1) = 0.587(0.4638 - 6.545)$

$\qquad + 12.746 \times 0.48 \ln (303.15 / 523.15) = $ **-6.908 kJ/K**

$\Delta S_{SURR} = -{}_1Q_2/T_0 = +2840.6/303.2 = $ **+9.370 kJ/K**

$_1S_{2\;gen} = S_2 - S_1 - {}_1Q_2/T_0 = -6.908 + 9.370 = $ **+2.462 kJ/K**

6.140

A cylinder/piston arrangement contains 10 g ammonia at 20°C with a volume of 1 L. There are some stops so if the piston is at the stops the volume is 1.4 L. The ammonia is now heated to 200°C by a 240°C source. The piston and cylinder is made of 0.5 kg aluminum, and assume that the mass has the same temperature as the ammonia at any time. Find the total heat transfer and the total entropy generation

C.V. NH3. Control mass, goes out to source

Energy Eq.: $U_3 - U_1 = m_{NH3} (u_3 - u_1) + m_{Alu} (u_3 - u_1) = {}_1Q_3 - {}_1W_3$

Entropy Eq.: $m_{NH3} (s_3 - s_1) + m_{Alu} (s_3 - s_1) = {}_1Q_3/T_{source} + {}_1S_{3,gen}$

State 1: B.2.1 $v_1 = V/m = 0.001 / 0.01 = 0.1 \text{ m}^3/\text{kg} < v$ so 2-phase $P = 857.5 \text{ kPa}$

$x_1 = (v - v_f)/ v_{fg} = (0.1 - 0.001638)/ 0.14758 = 0.6665$

$u_1 = u_f + x_1 u_{fg} = 272.89 + x_1 \times 1059.3 = 978.91 \text{ kJ/kg}$

$s_1 = s_f + x_1 s_{fg} = 1.0408 + x_1 \times 4.0452 = 3.73693 \text{ kJ/kgK}$

State 2: $v_2 = 1.4 \times v_1 = 0.14 \text{ m}^3/\text{kg}$ & $P = 857.5 \text{ kPa}$ still 2-phase so $T_2 = 20^oC$

State 3: 200^oC & $v_3 = v_2, \Rightarrow P = 1600 \text{ kPa}, u_3 = 1676.5 \text{ kJ/kg}, s_3 = 5.9734 \text{ kJ/kgK}$

We get the work from the process equation (see P-V diagram)

$_1W_3 = {}_1W_2 = P_1 m(v_2 - v_1) = 857.5 \text{ kPa} \times 0.01 (0.14 - 0.1) \text{ m}^3 = 0.343 \text{ kJ}$

The energy equation and the entropy equation give heat transfer and entropy generation

$_1Q_3 = m_{NH3} (u_3 - u_1) + m_{Alu} (u_3 - u_1) + {}_1W_3$

$= 0.01 (1676.5 - 978.91) + 0.5 \times 0.9 (200 - 20) + 0.343 = \textbf{88.32 kJ}$

$_1S_{3,gen} = m_{NH3}(s_3 - s_1) + m_{Alu}(s_3 - s_1) - {}_1Q_3/T_{source}$

$= 0.01 (5.9734 - 3.73693) + 0.5 \times 0.9 \ln(\dfrac{473.15}{293.15}) - \dfrac{88.32}{513.15}$

$= 0.02236 + 0.21543 - 0.1721 = \textbf{0.0657 kJ/K}$

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6.141
A cylinder/piston arrangement contains 0.1 kg R-410A of quality x = 0.2534 and at −20°C. Stops are mounted so $V_{stop} = 3V_1$, similar to Fig. P6.140. The system is now heated to the final temperature of 20°C by a 50°C source. Find the total entropy generation.

        C.V. The R-410A mass out to source.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

        Entropy Eq.: $S_2 - S_1 = \int dQ/T + {}_1S_{2\ gen} = {}_1Q_2/T_{source} + {}_1S_{2\ gen}$

Process: $P = $ Constant if $V < V_{stop}$; $V = V_{stop}$ if $P > P_1$

State 1: $u_1 = 27.92 + x_1\ 218.07 = 83.18$ kJ/kg, $P_1 = P_{sat} = 399.6$ kPa

          $v_1 = 0.000803 + x_1\ 0.064 = 0.01702$ m³/kg

          $s_1 = 0.1154 + x_1\ 0.9625 = 0.2439$ kJ/kgK

State 1a: $v_{stop} = 3\ v_1 = 0.05106$ m³/kg $< v_g$ at P_1

State 2:   at 20°C $> T_1$: $v_{stop} > v_g = 0.01758$ m³/kg so superheated vapor.

          Table B.4.2: $P_2 = 600$ kPa, $u_2 = 273.56$ kJ/kg, $s_2 = 1.1543$ kJ/kgK

          ${}_1W_2 = \int PdV = P_1\ m(v_2 - v_1) = 399.6 \times 0.1\ (0.051 - 0.017) = 1.36$ kJ

              ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.1(273.56 - 83.18) + 1.36 = 20.398$ kJ

              ${}_1S_{2,gen} = m\ (s_2 - s_1) - {}_1Q_2/T_{source}$

$$= 0.1\ (1.1543 - 0.2439) - \frac{20.398}{323.15} = \textbf{0.0279 kJ/K}$$

See the work term
from the process in
the P-v diagram

P₀

cb

R-410a

Borgnakke and Sonntag

6.142

One kg of air at 300 K is mixed with 2 kg air at 400 K in a process at a constant 100 kPa and Q = 0. Find the final T and the entropy generation in the process.

C.V. All the air.

Energy Eq.: $U_2 - U_1 = 0 - W$
Entropy Eq.: $S_2 - S_1 = 0 + {}_1S_{2\ gen}$
Process Eq.: $P = C;\quad W = P(V_2 - V_1)$

Substitute W into energy Eq.

$$U_2 - U_1 + W = U_2 - U_1 + P(V_2 - V_1) = H_2 - H_1 = 0$$

Due to the low T let us use constant specific heat

$$H_2 - H_1 = m_A(h_2 - h_1)_A + m_B(h_2 - h_1)_B$$
$$= m_A C_p(T_2 - T_{A1}) + m_B C_p(T_2 - T_{B1}) = 0$$

$$T_2 = \frac{m_A T_{A1} + m_B T_{B1}}{m_A + m_B} = \frac{1}{3} T_{A1} + \frac{2}{3} T_{B1} = \textbf{366.67 K}$$

Entropy change is from Eq. 6.16 with no change in P

$${}_1S_{2\ gen} = S_2 - S_1 = m_A C_p \ln \frac{T_2}{T_{A1}} + m_B C_p \ln \frac{T_2}{T_{B1}}$$
$$= 1 \times 1.004 \ln \frac{366.67}{300} + 2 \times 1.004 \ln \frac{366.67}{400}$$
$$= 0.20148 - 0.17470 = \textbf{0.0268 kJ/K}$$

Remark: If you check, the volume does not change and there is no work.

Borgnakke and Sonntag

6.143

Air in a rigid tank is at 900 K, 500 kPa and it now cools to the ambient temperature of 300 K by heat loss to the ambient. Find the entropy generation.

C.V. Air out to ambient. No size given so do it per unit mass.

Energy Eq.3.5: $u_2 - u_1 = C_v (T_2 - T_1) = {}_1q_2 - {}_1w_2$

Entropy Eq.6.37: $s_2 - s_1 = {}_1q_2/T_{amb} + {}_1s_{2 \text{ gen tot}}$

Process: $V = $ constant $\implies v_2 = v_1$ also ${}_1W_2 = 0$

Ideal gas: $P_2 = P_1 T_2 / T_1 = 500 \times 300/900 = 166.67$ kPa

From Table A.7: $u_1 = 674.82$ kJ/kg; $s_{T1} = 8.01581$ kJ/kg-K;

$u_2 = 214.36$ kJ/kg; $s_{T2} = 6.86926$ kJ/kg-K

$${}_1q_2 = u_2 - u_1 = 214.36 - 674.82 = -460.46 \text{ kJ/kg}$$

$${}_1s_{2 \text{ gen tot}} = s_2 - s_1 - \frac{{}_1q_2}{T_{amb}} = s_{T2} - s_{T1} - R \ln(\frac{P_2}{P_1}) - \frac{{}_1q_2}{T_{amb}}$$

$$= 6.86926 - 8.01581 - 0.287 \ln(\frac{166.67}{500}) - \frac{-460.46}{300}$$

$$= \mathbf{0.661 \text{ kJ/kg-K}}$$

We could also have used constant specific heat being slightly less accurate.

Borgnakke and Sonntag

6.144

Two rigid insulated tanks are connected with a pipe and valve. One tank has 0.5 kg air at 200 kPa, 300 K and the other has 0.75 kg air at 100 kPa, 400 K. The valve is opened and the air comes to a single uniform state without external heat transfer. Find the final T and P and the entropy generation.

Solution:

C.V. Total tank. Control mass of constant volume.

Mass and volume: $m_2 = m_A + m_B;$ $V = V_A + V_B$

Energy Eq.: $U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = {}_1Q_2 - {}_1W_2 = 0$

Entropy Eq.: $S_2 - S_1 = m_2 s_2 - m_A s_{A1} - m_B s_{B1} = {}_1Q_2/T + {}_1S_{2\ gen}$

Process Eq.: $V = $ constant $\Rightarrow {}_1W_2 = 0;$ Insulated $\Rightarrow {}_1Q_2 = 0$

Ideal gas at A1: $V_A = m_A RT_{A1}/P_{A1} = 0.5 \times 0.287 \times 300 / 200 = 0.2153 \text{ m}^3$

Ideal gas at B1: $V_B = m_B RT_{B1}/ P_{B1} = 0.75 \times 0.287 \times 400 / 100 = 0.861 \text{ m}^3$

State 2: $m_2 = m_A + m_B = 1.25 \text{ kg};\ V_2 = V_A + V_B = 1.0763 \text{ m}^3$

Energy Eq.: $u_2 = \dfrac{m_A u_{A1} + m_B u_{B1}}{m_2}$ and use constant specific heat

$$T_2 = \frac{m_A}{m_2} T_{A1} + \frac{m_B}{m_2} T_{B1} = \frac{0.5}{1.25} 300 + \frac{0.75}{1.25} 400 = \textbf{360 K}$$

$$P_2 = m_2 RT_2/V = 1.25 \text{ kg} \times 0.287 \text{ kJ/kgK} \times 360 \text{ K}/ 1.0763 \text{ m}^3 = \textbf{120 kPa}$$

$$S_2 - S_1 = m_A[C_P \ln T_2/T_{A1} - R\ln(P_2/P_{A1})] + m_B[C_P \ln T_2/T_{B1} - R\ln(P_2/P_{B1})]$$

$$= 0.5[\ 1.004 \ln\frac{360}{300} - 0.287 \ln\frac{120}{200}\ ] + 0.75[1.004 \ln\frac{360}{400} - 0.287 \ln\frac{120}{100}]$$

$$= 0.5 \times 1.3514 + 0.75 \times (-0.1581) = 0.5571 \text{ kJ/K}$$

${}_1S_{2\ gen} = S_2 - S_1 = \textbf{0.5571 kJ/K}$

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6.145

One kg of air at 100 kPa is mixed with 2 kg air at 200 kPa, both at 300 K, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process.

C.V. All the air.

Energy Eq.: $U_2 - U_1 = Q - W$

Entropy Eq.: $S_2 - S_1 = Q/T + {}_1S_{2\,gen}$

Process Eqs.: $V = C;\quad W = 0,\ Q = 0$

States A1, B1: $u_{A1} = u_{B1}$

$V_A = m_A R T_1/P_{A1};\quad V_B = m_B R T_1/P_{B1}$

$U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 0 \Rightarrow u_2 = (u_{A1} + 2u_{B1})/3 = u_{A1}$

State 2: $\mathbf{T_2 = T_1 = 300\ K}$ (from u_2); $m_2 = m_A + m_B = 3$ kg;

$V_2 = m_2 R T_1/P_2 = V_A + V_B = m_A R T_1/P_{A1} + m_B R T_1/P_{B1}$

Divide with $m_A R T_1$ and get

$3/P_2 = 1/P_{A1} + 2/P_{B1} = \dfrac{1}{100} + \dfrac{2}{200} = 0.02$ kPa^{-1} $\Rightarrow P_2 = \mathbf{150\ kPa}$

Entropy change from Eq. 6.16 with the same T, so only P changes

$${}_1S_{2\,gen} = S_2 - S_1 = -m_A R \ln \frac{P_2}{P_{A1}} - m_B R \ln \frac{P_2}{P_{B1}}$$

$$= -0.287\, [1 \times \ln \frac{150}{100} + 2 \times \ln \frac{150}{200}]$$

$$= -0.287\,(0.4055 - 0.5754) = \mathbf{0.049\ kJ/K}$$

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6.146

A rigid storage tank of 1.5 m^3 contains 1 kg argon at 30°C. Heat is then transferred to the argon from a furnace operating at 1300°C until the specific entropy of the argon has increased by 0.343 kJ/kg K. Find the total heat transfer and the entropy generated in the process.

Solution:

C.V. Argon out to 1300°C. Control mass. , m = 1 kg

Argon is an ideal gas with constant heat capacity.

Energy Eq.3.5: $m (u_2 - u_1) = m\,C_v (T_2 - T_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = {}_1Q_2/T_{res} + {}_1S_2\,{}_{gen\,tot}$

Process: V = constant   => $v_2 = v_1$ also ${}_1W_2 = 0$

Properties: Table A.5   R = 0.20813, C_v = 0.312 kJ/kg K

State 1:   (T_1, v_1= V/m) $P_1 = mRT_1/V$ = 42.063 kPa

State 2: $s_2 = s_1 + 0.343$,   and change in s from Eq.6.16 or Eq.6.17

$$s_2 - s_1 = C_p \ln (T_2 / T_1) - R \ln (T_2 / T_1) = C_v \ln (T_2 / T_1)$$

$$T_2 / T_1 = \exp[\frac{s_2 - s_1}{C_v}] = \exp[\frac{0.343}{0.312}] = \exp(1.09936) = 3.0$$

$Pv = RT$ => $(P_2 / P_1) (v_2 / v_1) = T_2 / T_1 = P_2 / P_1$

$T_2 = 3.0 \times T_1$ = 909.45 K, $P_2 = 3.0 \times P_1$ = 126.189 kPa

Heat transfer from energy equation

${}_1Q_2 = 1 \times 0.312 (909.45 - 303.15) =$ **189.2 kJ**

Entropy generation from entropy equation

${}_1S_2\,{}_{gen\,tot} = m(s_2 - s_1) - {}_1Q_2/T_{res}$

$= 1 \times 0.343 - 189.2 / (1300 + 273) =$ **0.223 kJ/K**

6.147

Argon in a light bulb is at 110 kPa, 90°C. The light is turned off so the argon cools to the ambient 20°C. Disregard the glass and any other mass and find the specific entropy generation.

Solution:

C.V. Argon gas. Neglect any heat transfer.

Energy Eq.3.5: $\quad m(u_2 - u_1) = {}_1Q_2$

Entropy Eq.6.37: $\quad s_2 - s_1 = \int dq/T + {}_1s_2{}_{\text{ gen}} = {}_1q_2/T_{room} + {}_1s_2{}_{\text{ gen}}$

Process: $\qquad\qquad$ v = constant and ideal gas $\quad =>\qquad$ $P_2/P_1 = T_2/T_1$

$\qquad {}_1q_2 = u_2 - u_1 = C_v (T_2 - T_1) = 0.312 (20 - 90) = -21.84 \text{ kJ/kg}$

Evaluate changes in s from Eq.6.16 or 8.17

$\qquad s_2 - s_1 = C_p \ln (T_2/T_1) - R \ln (P_2/P_1) \qquad\qquad$ Eq.6.16

$\qquad\qquad = C_p \ln (T_2/T_1) - R \ln (T_2/T_1) = C_v \ln(T_2/T_1) \qquad$ Eq.6.17

$\qquad\qquad = 0.312 \ln [(20 + 273)/(90 + 273)] = -0.06684 \text{ kJ/kg K}$

$\qquad {}_1s_2{}_{\text{ gen}} = s_2 - s_1 - {}_1q_2/T_{room} = -0.06684 + 21.84 / 293.15$

$\qquad\qquad = \textbf{0.00766 kJ/kgK}$

Borgnakke and Sonntag

6.148

A rigid tank contains 2 kg of air at 200 kPa and ambient temperature, 20°C. An electric current now passes through a resistor inside the tank. After a total of 100 kJ of electrical work has crossed the boundary, the air temperature inside is 80°C. Is this possible?

Solution:

C.V.: Air in tank out to ambient;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$, ${}_1W_2 = -100$ kJ

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1Q_2/T_{amb} + {}_1S_{2\ gen}$

Process: Constant volume and mass so $v_2 = v_1$

State 1: $T_1 = 20^oC$, $P_1 = 200$ kPa, $m_1 = 2$ kg

State 2: $T_2 = 80^oC$, $v_2 = v_1$

Ideal gas, Table A.5: $R = 0.287$ kJ/kg-K, $C_v = 0.717$ kJ/kg-K

Assume constant specific heat then energy equation gives

$${}_1Q_2 = mC_v(T_2 - T_1) + {}_1W_2 = 2 \times 0.717(80 - 20) - 100 = -14.0 \text{ kJ}$$

Change in s from Eq.6.17 (since second term drops out)

$$s_2 - s_1 = C_v \ln (T_2/T_1) + R\ln \frac{v_2}{v_1} \ ; \quad v_2 = v_1, \ \ \ln \frac{v_2}{v_1} = 0$$

$$s_2 - s_1 = C_v\ln (T_2/T_1) = 0.1336 \text{ kJ/kg-K}$$

Now Eq.6.37

$${}_1S_{2\ gen} = m(s_2 - s_1) - {}_1Q_2/T_{amb} = 2 \times 0.1336 + \frac{14}{293} = 0.315 \text{ kJ/K } \geq 0,$$

Process is Possible

Note: $P_2 = P_1 \dfrac{T_2}{T_1}$ in Eq.6.16 $s_2 - s_1 = C_p \ln\dfrac{T_2}{T_1}$ - R ln $\dfrac{P_2}{P_1}$, results in the same answer as Eq.6.17.

Borgnakke and Sonntag

6.149

A piston/cylinder system contains 50 L of air at 300°C, 100 kPa, with the piston initially on a set of stops. A total external constant force acts on the piston so a balancing pressure inside should be 200 kPa. The cylinder is made of 2 kg of steel initially at 1300°C. The system is insulated so that heat transfer occurs only between the steel cylinder and the air. The system comes to equilibrium. Find the final temperature, and the entropy generation.

C.V.: Steel + water out to ambient T_0. This is a control mass.

Energy Eq.: $U_2 - U_1 = m_{air}(u_2 - u_1) + m_{st}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $S_2 - S_1 = m_{air}(s_2 - s_1) + m_{st}(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1S_{2\ gen}$

Process: ${}_1Q_2 = 0$ and must be on P-V diagram shown

$$m_{air} = P_1V_1 / RT_1 = 100 \times 0.05 /(0.287 \times 573.15) = 0.0304 \text{ kg}$$

Since $V_{1a} = V_1$ then $T_{1a} = T_1(P_{float}/P_1) = 573.15 \times 200/100 = 1146.3$ K

Use constant C_v for air at 900 K: $C_v = \Delta u/\Delta T = 0.833$ kJ/kgK (from A.7)

To reach state 1a: $\Delta U_{air} = mC_v\Delta T = 0.0304 \times 0.833 \times (1146 - 573) = 14.5$ kJ

$$\Delta U_{st} = mC_v\Delta T = 2 \times 0.46 \times (1146 - 1573) = -392.8 \text{ kJ}$$

Conclusion from this is: T_2 is higher than $T_{1a} = 1146$ K, piston lifts, $P_2 = P_{float}$

Write the work as ${}_1W_2 = P_2 (V_2 - V_1)$ and use constant C_v in the energy Eq. as

$$m_{air} C_v (T_2 - T_1) + m_{st} C_{st} (T_2 - T_1) = - P_2 m_{air} v_2 + P_2 V_1$$

now $P_2 v_2 = RT_2$ for the air, so isolate T_2 terms as

$$[m_{air} (C_v + R) + mC_{st}] T_2 = m_{air} C_v T_{1\ air} + mC_{st} T_{1\ st} + P_2V_1$$

$$[0.0304 \times 1.12 + 2 \times 0.46] T_2 = 0.0304 \times 0.833 \times 573.15$$

$$+ 2 \times 0.46 \times 1573.15 + 200 \times 0.05$$

Solution gives: $T_2 =$ **1542.7 K**

$${}_1S_{2\ gen} = S_2 - S_1 = m_{air}(s_2 - s_1) + m_{st}(s_2 - s_1)$$

$$= m_{air} [C_P \ln(T_2/T_{1\ air}) - R \ln(P_2/P_1)] + m_{st} C_{st} \ln(T_2/T_{1\ st})$$

$$= 0.0304[1.12 \ln \frac{1542.7}{573.15} - 0.287 \ln \frac{200}{100}] + 2 \times 0.46 \ln (\frac{1542.7}{1573.15})$$

$$= 0.027665 - 0.017982 = \textbf{0.0097 kJ/K}$$

Borgnakke and Sonntag

6.150

A spring loaded piston cylinder contains 1.5 kg air at 27°C and 160 kPa. It is now heated in a process where pressure is linear in volume, P = A + BV, to twice the initial volume where it reaches 900 K. Find the work, the heat transfer and the total entropy generation assuming a source at 900 K.

Solution:

C.V. Air out to the 900 K source. Since air T is lower than the source

temperature we know that this is an irreversible process.

Continuity Eq.: $m_2 = m_1 = m$,

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1Q_2/T_{source} + {}_1S_{2\ gen}$

Process: $P = A + BV$

State 1: (T_1, P_1) Table A.7 $u_1 = 214.36$ kJ/kg

$V_1 = mRT_1/ P_1 = (1.5 \times 0.287 \times 300)$ kJ / 160 kPa $= 0.8072$ m^3

State 2: $(T_2, v_2 = 2\ v_1)$ Table A.7 $u_2 = 674.824$ kJ/kg

$P_2 = RT_2/ v_2 = RT_2/2v_1 = T_2\ P_1/ 2T_1 = P_1\ T_2/2\ T_1$

$= 160$ kPa $\times 900$ K/ $(2 \times 300$ K$) = 240$ kPa

From the process equation we can express the work as

${}_1W_2 = \int PdV = 0.5 \times (P_1 + P_2)\ (V_2 - V_1) = 0.5 \times (P_1 + P_2)\ V_1$

$= 0.5 \times (160 + 240)$ kPa $\times 0.8072$ m$^3 =$ **161.4 kJ**

${}_1Q_2 = 1.5 \times (674.824 - 214.36) + 161.4 =$ **852.1 kJ**

Change in s from Eq.6.19 and Table A.7 values

$${}_1S_{2\ gen} = m(s^o_{T2} - s^o_{T1} - R\ \ln \frac{P_2}{P_1}) - {}_1Q_2/T_{SOURCE}$$

$$= 1.5 \times [8.0158 - 6.8693 - 0.287\ \ln\ (\frac{240}{160})] - (\frac{852.1}{900})$$

$$= 1.545 - 0.947 = \textbf{0.598 kJ/K}$$

Borgnakke and Sonntag

6.151

A rigid container with volume 200 L is divided into two equal volumes by a partition, shown in Fig. P6.151. Both sides contain nitrogen, one side is at 2 MPa, 200°C, and the other at 200 kPa, 100°C. The partition ruptures, and the nitrogen comes to a uniform state at 70°C. Assume the temperature of the surroundings is 20°C, determine the work done and the net entropy change for the process.

Solution:

C.V. : A + B   no change in volume. $_1W_2 = 0$

$m_{A1} = P_{A1}V_{A1}/RT_{A1} = (2000 \times 0.1)/(0.2968 \times 473.2) = 1.424$ kg

$m_{B1} = P_{B1}V_{B1}/RT_{B1} = (200 \times 0.1)/(0.2968 \times 373.2) = 0.1806$ kg

$P_2 = m_{TOT}RT_2/V_{TOT} = (1.6046 \times 0.2968 \times 343.2)/0.2 = 817$ kPa

From Eq.6.16

$S_2 - S_1 = m_{A1}\,(s_2 - s_1)_{A1}\; + m_{B1}\,(s_2 - s_1)_{B1}$

$= 1.424 \text{ kg} \left[1.042 \ln\frac{343.2}{473.2} - 0.2968 \ln\frac{817}{2000}\right] \text{ kJ/kgK}$

$+ 0.1806 \text{ kg} \left[1.042 \ln\frac{343.2}{373.2} - 0.2968 \ln\frac{817}{200}\right] \text{ kJ/kgK} = -0.1894 \text{ kJ/K}$

$_1Q_2 = U_2 - U_1 = 1.424 \times 0.745(70 - 200) + 0.1806 \times 0.745(70 - 100)$

$= -141.95$ kJ

From Eq.6.37

$_1S_{2\,gen} = S_2 - S_1 - {_1Q_2}/T_0 = -0.1894 \text{ kJ/K} + 141.95 \text{ kJ }/293.2 \text{ K}$

$= -0.1894 + 0.4841 = \mathbf{+0.2947 \text{ kJ/K}}$

Borgnakke and Sonntag

6.152

A constant pressure piston/cylinder contains 0.5 kg air at 300 K, 400 kPa. Assume the piston/cylinder has a total mass of 1 kg steel and is at the same temperature as the air at any time. The system is now heated to 1600 K by heat transfer from a 1700 K source. Find the entropy generation using constant specific heat for air.

C.V. Air and Steel.

Energy Eq.: $U_2 - U_1 = m_{air}(u_2 - u_1) + m_{st}(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Entropy Eq.6.37: $m_{air}(s_2 - s_1) + m_{st}(s_2 - s_1) = {_1}Q_2/T_{source} + {_1}S_{2\ gen}$

Process: $P = C$ => ${_1}W_2 = \int_1^2 PdV = P(V_2 - V_1) = P\, m_{air}(v_2 - v_1)$

${_1}Q_2 = m_{air}(u_2 - u_1)_{air} + m_{st}(u_2 - u_1)_{st} + {_1}W_2 = m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st}$

Use A.3: $(u_2 - u_1)_{st} = C(T_2 - T) = 0.46$ kJ/kgK $\times (1600 - 300)$ K $= 598$ kJ/kg

Use A.5: $(h_2 - h_1)_{air} = C_p(T_2 - T_1) = 1.004$ kJ/kgK $\times (1600 - 300)$ K $= 1305.2$ kJ/kg

$\quad {_1}Q_2 = m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st}$

$\quad\quad = 0.5$ kg $\times 1305$ kJ/kg $+ 1$ kg $\times 598$ kJ/kg $= 1250.6$ kJ

$\quad S_2 - S_1 = m_{air}(s_2 - s_1) + m_{st}(s_2 - s_1)$

$\quad\quad\quad = 0.5$ kg $\times 1.004$ kJ/kgK $\times \ln\dfrac{1600}{300} + 1$ kg $\times 0.46$ kJ/kgK $\times \ln\dfrac{1600}{300}$

$\quad\quad\quad = 1.6104$ kJ/K

$\quad\quad {_1}S_{2\ gen} = S_2 - S_1 - {_1}Q_2/T_{source} = 1.6104 - 1250.6/1700 = \textbf{0.875 kJ/K}$

Borgnakke and Sonntag

6.153

Do Problem 6.152 using Table A.7.

C.V. Air and Steel.

Energy Eq.: $U_2 - U_1 = m_{air}(u_2 - u_1) + m_{st}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m_{air}(s_2 - s_1) + m_{st}(s_2 - s_1) = {}_1Q_2/T_{source} + {}_1S_{2\ gen}$

Process: $P = C$ => ${}_1W_2 = \int\limits_1^2 PdV = P(V_2 - V_1) = P\ m_{air}(v_2 - v_1)$

${}_1Q_2 = m_{air}(u_2 - u_1)_{air} + m_{st}(u_2 - u_1)_{st} + {}_1W_2 = m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st}$

Use air tables A.7: $(h_2 - h_1)_{air} = 1757.33 - 300.47 = 1456.86$ kJ/kg

$(s_2 - s_1)_{air} = 8.69051 - 6.86926 + 0 = 1.82125$ kJ/kgK

No pressure correction as $P_2 = P_1$

Use A.3: $(u_2 - u_1)_{st} = C(T_2 - T_1) = 0.46$ kJ/kgK $\times (1600 - 300)$ K $= 598$ kJ/kg

${}_1Q_2 = m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st}$

$= 0.5$ kg $\times 1456.86$ kJ/kg $+ 1$ kg $\times 598$ kJ/kg $= \mathbf{1326.43\ kJ}$

$S_2 - S_1 = m_{air}(s_2 - s_1) + m_{st}(s_2 - s_1)$

$= 1.82125 + 1$ kg $\times 0.46$ kJ/kgK $\times \ln\dfrac{1600}{300} = 2.59128$ kJ/K

${}_1S_{2\ gen} = S_2 - S_1 - {}_1Q_2/T_{source} = 2.59128 - 1326.43/1700 = \mathbf{1.811\ kJ/K}$

Borgnakke and Sonntag

6.154

Nitrogen at 200°C, 300 kPa is in a piston cylinder, volume 5 L, with the piston locked with a pin. The forces on the piston require a pressure inside of 200 kPa to balance it without the pin. The pin is removed and the piston quickly comes to its equilibrium position without any heat transfer. Find the final P, T and V and the entropy generation due to this partly unrestrained expansion.

Solution:

C.V. Nitrogen gas.

Energy Eq.3.5: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2 = -\int P_{eq}\, dV = -P_2\,(V_2 - V_1)$

Entropy Eq.6.37: $m(s_2 - s_1) = 0 + {_1}S_{2\,gen}$

Process: ${_1}Q_2 = 0$ (already used), $P = P_{eq}$ after pin is out.

State 1: 200 °C, 300 kPa   State 2: $P_2 = P_{eq} =$ **200 kPa**

$m = P_1 V_1 / RT_1 = 300 \times 0.005 / 0.2968 \times 473.15 = 0.01068$ kg

The energy equation becomes

$mu_2 + P_2 V_2 = mu_1 + P_2 V_1 = mh_2$ =>

$h_2 = u_1 + P_2 V_1 / m = u_1 + P_2 V_1\, RT_1 / P_1 V_1 = u_1 + (P_2/P_1)\, RT_1$

Solve using constant C_p, C_v

$C_p T_2 = C_v T_1 + (P_2/P_1)\, RT_1$

$T_2 = T_1\,[C_v + (P_2/P_1)\, R] / C_p$

$\qquad = 473.15\,[0.745 + (200 / 300) \times 0.2368] / 1.042$

$\qquad =$ **428.13 K**

$V_2 = V_1(\,T_2\,/\,T_1) \times (\,P_1/P_2\,) = 0.005 \times \dfrac{428.13}{473.15} \times \dfrac{300}{200}$

$\qquad = 0.00679$ m^3

${_1}S_{2\,gen} = m(s_2 - s_1) \cong m[C_p \ln (T_2/T_1) - R \ln (P_2/ P_1)]$

$\qquad = P_1 V_1 / RT_1\,[C_p \ln (T_2/T_1) - R \ln (P_2/ P_1)]$

$\qquad = 0.01068\,[1.042 \times \ln (428.13/473.15) - 0.2968 \times \ln (200 / 300)]$

$\qquad =$ **0.000173 kJ/K**

Borgnakke and Sonntag

6.155
The air in the tank of Problem 6.88 receives the heat transfer from a reservoir at 450 K. Find the entropy generation due to the process from 1 to 3.

C.V. Air out to reservoir.

Energy eq.: $m(u_3 - u_1) = {}_1Q_3 - {}_1W_3$

Entropy eq.: $m(s_3 - s_1) = {}_1Q_3 / T_{res} + {}_1S_{3\ gen}$

State 1: $m = P_1 V_1 / R T_1 = \dfrac{100 \times 0.75}{0.287 \times 300} \dfrac{kPa\ m^3}{kJ/kg} = 0.871$ kg

Process 1 to 2: Constant volume heating, $dV = 0 \Rightarrow {}_1W_2 = 0$

$P_2 = P_1 T_2 / T_1 = 100 \times 400 / 300 = 133.3$ kPa

Process 2 to 3: Isothermal expansion, $dT = 0 \Rightarrow u_3 = u_2$ and

$P_3 = P_2 V_2 / V_3 = 133.3 \times 0.75 / 1.5 = 66.67$ kPa

$${}_2W_3 = \int_2^3 P\ dV = P_2 V_2 \ln\left(\frac{V_3}{V_2}\right) = 133.3 \times 0.75 \ln(2) = 69.3 \text{ kJ}$$

The overall process:

$${}_1W_3 = {}_1W_2 + {}_2W_3 = {}_2W_3 = 69.3 \text{ kJ}$$

From the energy equation

$${}_1Q_3 = m(u_3 - u_1) + {}_1W_3 = m\ C_v (T_3 - T_1) + {}_1W_3$$
$$= 0.871 \times 0.717 (400 - 300) + 69.3 = 131.8 \text{ kJ}$$

$${}_1S_{3\ gen} = m(s_3 - s_1) - {}_1Q_3 / T_{res}$$
$$= m\ (C_P \ln \frac{T_3}{T_1} - R \ln \frac{P_3}{P_1}) - {}_1Q_3 / T_{res}$$
$$= 0.871\ [\ 1.004 \ln \frac{400}{300} - 0.287 \ln \frac{66.67}{100}\ ] - \frac{131.8}{450}$$
$$= \mathbf{0.060\ kJ/K}$$

6.156

One kg of carbon dioxide at 100 kPa, 400 K is mixed with two kg carbon dioxide at 200 kPa, 2000 K, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process using constant heat capacity from Table A.5.

C.V. All the carbon dioxide.

Continuity: $m_2 = m_A + m_B = 3$ kg

Energy Eq.: $U_2 - U_1 = 0 - 0$

Entropy Eq.: $S_2 - S_1 = 0 + {}_1S_2 \text{ gen}$

Process Eqs.: $V = C; \quad W = 0, \quad Q = 0$

$V_A = m_A RT_1/P_{A1}; \quad V_B = m_B RT_1/P_{B1}$

$U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 0$

$\qquad = m_2 C_v T_2 - m_A C_v T_{A1} - m_B C_v T_{B1}$

$\Rightarrow T_2 = (m_A T_{A1} + m_B T_{B1})/ m_2 = \frac{1}{3} \times 400 + \frac{2}{3} \times 2000 = \textbf{1466.7 K}$

State 2: $V_2 = m_2 RT_2/P_2 = V_A + V_B = m_A RT_{A1}/P_{A1} + m_B RT_{B1}/P_{B1}$

$\qquad = 1 \times R \times 400/100 + 2 \times R \times 2000/200 = 24 R$

Substitute m_2, T_2 and solve for P_2

$\qquad P_2 = 3 R T_2/24 R = 3 \times 1466.7 / 24 = \textbf{183.3 kPa}$

Entropy change from Eq. 6.16

$(s_2 - s_1)_A = C_p \ln \frac{T_2}{T_{A1}} - R \ln \frac{P_2}{P_{A1}} = 0.842 \ln \frac{1466.7}{400} - 0.1889 \ln \frac{183.3}{100}$

$\qquad = 0.97955$ kJ/kgK

$(s_2 - s_1)_B = C_p \ln \frac{T_2}{T_{B1}} - R \ln \frac{P_2}{P_{B1}} = 0.842 \ln \frac{1466.7}{2000} - 0.1889 \ln \frac{183.3}{200}$

$\qquad = -0.24466$ kJ/kgK

${}_1S_2 \text{ gen} = S_2 - S_1 = m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B$

$\qquad = 1 \times 0.97955 + 2 (-0.24466) = \textbf{0.49 kJ/K}$

Borgnakke and Sonntag

6.157

One kg of carbon dioxide at 100 kPa, 400 K is mixed with two kg carbon dioxide at 200 kPa, 2000 K, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process using table A.8.

C.V. All the carbon dioxide.

Continuity: $m_2 = m_A + m_B = 3$ kg

Energy Eq.: $U_2 - U_1 = 0 - 0$

Entropy Eq.: $S_2 - S_1 = 0 + {}_1S_{2\ gen}$

Process Eqs.: $V = C; \quad W = 0, \ Q = 0$

$V_A = m_A R T_1 / P_{A1}; \quad V_B = m_B R T_1 / P_{B1}$

$U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 0$

$u_2 = (m_A u_{A1} + m_B u_{B1}) / m_2 = \dfrac{1}{3} \times 228.19 + \dfrac{2}{3} \times 1912.67 = 1351.18$ kJ/kg

Interpolate in Table A.8: $T_2 = 1517.2$ K, $s^o_{T2} = 6.6542$ kJ/kgK

State 2: $V_2 = m_2 R T_2 / P_2 = V_A + V_B = m_A R T_{A1}/P_{A1} + m_B R T_{B1}/P_{B1}$

$= 1 \times R \times 400/100 + 2 \times R \times 2000/200 = 24\ R$

Substitute m_2, T_2 and solve for P_2

$P_2 = 3\ R\ T_2 / 24\ R = 3 \times 1517.2 / 24 =$ **189.65 kPa**

Entropy change from Eq. 6.19

$(s_2 - s_1)_A = s^o_{T2} - s^o_{T1} - R \ln \dfrac{P_2}{P_{A1}} = 6.6542 - 5.1196 - 0.1889 \ln \dfrac{189.65}{100}$

$= 1.4137$ kJ/kgK

$(s_2 - s_1)_B = s^o_{T2} - s^o_{T1} - R \ln \dfrac{P_2}{P_{B1}} = 6.6542 - 7.0278 - 0.1889 \ln \dfrac{189.65}{200}$

$= -0.36356$ kJ/kgK

${}_1S_{2\ gen} = S_2 - S_1 = m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B$

$= 1 \times 1.4137 + 2\ (-0.36356) =$ **0.687 kJ/K**

Borgnakke and Sonntag

6.158

Nitrogen at 600 kPa, 127°C is in a 0.5 m^3 insulated tank connected to a pipe with a valve to a second insulated initially empty tank of volume 0.25 m^3, shown in Fig. P6.158. The valve is opened and the nitrogen fills both tanks at a uniform state. Find the final pressure and temperature and the entropy generation this process causes. Why is the process irreversible?

Solution:

CV Both tanks + pipe + valve Insulated : Q = 0   Rigid: W = 0

Energy Eq.3.5: $m(u_2 - u_1) = 0 - 0$ => $u_2 = u_1 = u_{a1}$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1S_{2\ gen}$ (dQ = 0)

1: P_1 , T_1 , V_a =>   m = PV/RT = (600 × 0.25)/ (0.2968 × 400) = 1.2635 kg

2: $V_2 = V_a + V_b$; uniform state $v_2 = V_2 / m$; $u_2 = u_{a1}$

Ideal gas u (T) => $u_2 = u_{a1}$ => $T_2 = T_{a1}$ = **400 K**

P_2 = mR T_2 / V_2 = (V_1 / V_2) P_1 = ½ × 600 = **300 kPa**

From entropy equation and Eq.6.19 for entropy change

$S_{gen} = m(s_2 - s_1) = m[s_{T2} - s_{T1} - R \ln(P_2 / P_1)]$

          = m [0 - R ln (P_2 / P_1)] = -1.2635 × 0.2968 ln ½ = **0.26 kJ/K**

Irreversible due to unrestrained expansion in valve P ↓ but no work out.

Borgnakke and Sonntag

6.159

A cylinder/piston contains carbon dioxide at 1 MPa, 300°C with a volume of 200 L. The total external force acting on the piston is proportional to V^3. This system is allowed to cool to room temperature, 20°C. What is the total entropy generation for the process?

Solution:

C.V. Carbon dioxide gas of constant mass $m_2 = m_1 = m$ out to ambient.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = {}_1Q_2/T_{amb} + {}_1S_{2\,gen}$

Process: $P = CV^3$ or $PV^{-3} = $ constant, which is polytropic with $n = -3$

State 1: (T, P) => $m = P_1V_1/RT_1 = \dfrac{1000 \times 0.2}{0.18892 \times 573.2} = 1.847$ kg

State 2: (T, ?) state must be on process curve and ideal gas leads to Eq.6.28

$\Rightarrow P_2 = P_1(T_2/T_1)^{\frac{n}{n-1}} = 1000(293.2/573.2)^{3/4} = 604.8$ kPa

$$V_2 = V_1(T_1/T_2)^{\frac{1}{n-1}} = 0.16914 \text{ m}^3$$

$${}_1W_2 = \int PdV = (P_2V_2 - P_1V_1)/(1-n)$$

$$= [604.8 \times 0.16914 - 1000 \times 0.2] / [1 - (-3)] = -24.4 \text{ kJ}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

$$= 1.847 \times 0.653 (20 - 300) - 24.4 = -362.1 \text{ kJ}$$

From Eq.6.16

$$m(s_2 - s_1) = 1.847\left[0.842 \ln\frac{293.2}{573.2} - 0.18892 \ln\frac{604.8}{1000}\right]$$

$$= 1.847[-0.4694] = -0.87 \text{ kJ/K}$$

$$\Delta S_{SURR} = - {}_1Q_2/T_{amb} = +362.1 / 293.2 = +1.235 \text{ kJ/K}$$

From Eq.6.37 or 8.39

$${}_1S_{2\,gen} = m(s_2 - s_1) - {}_1Q_2/T_{amb} = \Delta S_{NET} = \Delta S_{CO2} + \Delta S_{SURR}$$

$$= -0.87 + 1.235 = \textbf{+0.365 kJ/K}$$

Notice:
n = -3, k = 1.3

n < k

Borgnakke and Sonntag

6.160

The air in the engine cylinder of Problem 3.156 looses the heat to the engine coolant at 100^oC. Find the entropy generation (external to the air) using constant specific heat.

Take CV as the air. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1Q_2/T_{amb} + {}_1S_{2\ gen}$

Process Eq.: $Pv^n = $ Constant   (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $P_2 = P_1 \left(v_2 / v_1\right)^{-n} = 4000 \times 10^{-1.5} = 126.5$ kPa

$$T_2 = T_1 \ (P_2 v_2 / P_1 v_1) = (1527 + 273)\frac{126.5 \times 10}{4000} = 569.3 \text{ K}$$

From process eq.: ${}_1W_2 = \int P \, dV = \frac{m}{1-n}(P_2 v_2 - P_1 v_1) = \frac{mR}{1-n}(T_2 - T_1)$

$$= \frac{0.1 \times 0.287}{1 - 1.5}(569.3 - 1800) = 70.64 \text{ kJ}$$

From energy eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = mC_v(T_2 - T_1) + {}_1W_2$

$$= 0.1 \times 0.717(569.3 - 1800) + 70.64 = -17.6 \text{ kJ}$$

$m(s_2 - s_1) = m\ [C_P \ln(T_2/T_1) - R \ln(P_2/P_1)]$

$$= 0.1\ [\ 1.004 \ln\frac{569.3}{1800} - 0.287 \ln\frac{126.5}{4000}\ ] = -0.01645 \text{ kJ/K}$$

$${}_1S_{2\ gen} = m(s_2 - s_1) - {}_1Q_2/T_{amb}$$
$$= -0.011645 + 17.6/373.15 = \textbf{0.0307 kJ/K}$$

Borgnakke and Sonntag

6.161

     A cylinder/piston contains 100 L of air at 110 kPa, 25°C. The air is compressed in a reversible polytropic process to a final state of 800 kPa, 500 K. Assume the heat transfer is with the ambient at 25°C and determine the polytropic exponent *n* and the final volume of the air. Find the work done by the air, the heat transfer and the total entropy generation for the process.

       Solution:

C.V. Air of constant mass $m_2 = m_1 = m$ out to ambient.

Energy Eq.3.5: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {_1}S_{2\,gen} = {_1}Q_2/T_0 + {_1}S_{2\,gen}$

Process: $Pv_1{^n} = P_2v_2{^n}$ Eq.6.27

State 1: (T_1, P_1) State 2: (T_2, P_2)

Thus the unknown is the exponent   n.

$$m = P_1 V_1 /(RT_1) = 110 \times 0.1/(0.287 \times 298.15) = 0.1286 \text{ kg}$$

The relation from the process and ideal gas is in Eq.6.28

$$T_2/T_1 = (P_2/P_1)^{\frac{n-1}{n}} \quad => \quad \frac{500}{298.15} = \left(\frac{800}{110}\right)^{\frac{n-1}{n}} \quad \Rightarrow \quad \frac{n-1}{n} = 0.260573$$

$$\mathbf{n = 1.3524,} \quad V_2 = V_1(P_1/P_2)^{\frac{1}{n}} = 0.1 \left(\frac{110}{800}\right)^{0.73943} = \mathbf{0.02306 \text{ m}^3}$$

The work is from Eq.6.29

$$_1W_2 = \int PdV = \frac{P_2V_2 - P_1V_1}{1 - n} = \frac{800 \times 0.02306 - 110 \times 0.1}{1 - 1.3524} = \mathbf{-21.135 \text{ kJ}}$$

Heat transfer from the energy equation

$$_1Q_2 = mC_v(T_2 - T_1) + {_1}W_2$$
$$= 0.1286 \times 0.717 \times (500 - 298.15) - 21.135 = \mathbf{-2.523 \text{ kJ}}$$

Entropy change from Eq.6.16

$$s_2 - s_1 = C_{P0} \ln(T_2/T_1) - R \ln(P_2/P_1)$$
$$= 1.004 \ln\left(\frac{500}{298.15}\right) - 0.287 \ln\left(\frac{800}{110}\right) = -0.0504 \frac{\text{kJ}}{\text{kg K}}$$

From the entropy equation (also Eq.6.37)

$$_1S_{2,gen} = m(s_2 - s_1) - {_1}Q_2/T_0$$
$$= 0.1286 \times (-0.0504) + (2.523/298.15) = \mathbf{0.00198 \text{ kJ/K}}$$

Borgnakke and Sonntag

Rates or fluxes of entropy

6.162

A room at 22$^\text{o}$C is heated electrically with 1500 W to keep steady temperature. The outside ambient is at 5$^\text{o}$C. Find the flux of S (= $\dot{Q}$/T) into the room air, into the ambient and the rate of entropy generation.

CV. The room and walls out to the ambient T, we assume steady state

Energy Eq.: $0 = \dot{W}_{el\ in} - \dot{Q}_{out}$ => $\dot{Q}_{out} = \dot{W}_{el\ in} = 1500$ W

Entropy Eq.: $0 = - \dot{Q}_{out}/T + \dot{S}_{gen\ tot}$

      Flux of S into room air at 22$^\text{o}$C: $\dot{Q}/T = 1500 / 295.15 = 5.08$ W/K

      Flux of S into ambient air at 5$^\text{o}$C: $\dot{Q}/T = 1500 / 278.15 = 5.393$ W/K

      Entropy generation: $\dot{S}_{gen\ tot} = \dot{Q}_{out}/T = 1500 / 278.15 =$ **5.393 W/K**

Comment: The flux of S into the outside air is what leaves the control volume and since the control volume did not receive any S it was all made in the process. Notice most of the generation is done in the heater, the room heat loss process generates very little S (5.393 – 5.08 = 0.313 W/K)

Borgnakke and Sonntag

6.163

A mass of 3 kg nitrogen gas at 2000 K, V = C, cools with 500 W. What is dS/dt?

Assume that we do not generate any s in the nitrogen then

Entropy Eq. 6.42: $\quad \dot{S}_{CV} = \dfrac{\dot{Q}}{T} = -\dfrac{500}{2000}\dfrac{W}{K} = \mathbf{-0.25\ W/K}$

6.164

A heat pump, see problem 5.49, should upgrade 5 MW of heat at 85$^\text{o}$C to heat delivered at 150$^\text{o}$C. For a reversible heat pump what are the fluxes of entropy in and out of the heat pump?

C.V.TOT. Assume reversible Carnot cycle.

Energy Eq.: $\dot{Q}_L + \dot{W} = \dot{Q}_H$

Entropy Eq.: $0 = \dfrac{\dot{Q}_L}{T_L} - \dfrac{\dot{Q}_H}{T_H} \implies \dfrac{\dot{Q}_L}{T_L} = \dfrac{\dot{Q}_H}{T_H}$

The fluxes of entropy become the same as

$$\frac{\dot{Q}_H}{T_H} = \frac{\dot{Q}_L}{T_L} = \frac{5}{273.15 + 85}\frac{MW}{K} = \textbf{0.01396 MW/K}$$

This is what constitutes a reversible process (flux of S in = flux out, no generation).

Borgnakke and Sonntag

6.165

Reconsider the heat pump in the previous problem and assume it has a COP of 2.5. What are the fluxes of entropy in and out of the heat pump and the rate of entropy generation inside it?

C.V. TOT.

Energy Eq.: $\dot{Q}_L + \dot{W} = \dot{Q}_H$

Entropy Eq.: $0 = \dfrac{\dot{Q}_L}{T_L} - \dfrac{\dot{Q}_H}{T_H} + \dot{S}_{gen\ tot}$

Definition of COP: $\beta_{HP} = \dfrac{\dot{Q}_H}{\dot{W}_{in}} = 2.5;$ $\beta_{REF} = \beta_{HP} - 1 = \dfrac{\dot{Q}_L}{\dot{W}_{in}} = 1.50$

$$\dot{W}_{in} = \dot{Q}_L / \beta_{REF} = 5/1.50 = 3.333\ \text{MW}$$

$$\dot{Q}_H = \dot{Q}_L + \dot{W} = 5\ \text{MW} + 3.333\ \text{MW} = 8.333\ \text{MW}$$

$$\frac{\dot{Q}_L}{T_L} = \frac{5}{273.15 + 85}\frac{\text{MW}}{\text{K}} = \textbf{0.01396 MW/K}$$

$$\frac{\dot{Q}_H}{T_H} = \frac{8.333}{273.15 + 85}\frac{\text{MW}}{\text{K}} = \textbf{0.01969 MW/K}$$

From the entropy equation

$$\dot{S}_{gen\ tot} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_L}{T_L} = (0.01969 - 0.01396)\frac{\text{MW}}{\text{K}} = \textbf{5.73 kW/K}$$

Borgnakke and Sonntag

6.166

A radiant heating lamp powered by electricity has a surface temperature of 1000 K emitting 500 W. The radiation is absorbed by surfaces at the ambient 18$^{\text{o}}$C. Find the total entropy generation and specify where it is made, including how much.

CV1 Radiant element.

Energy Eq.: $0 = \dot{W}_{\text{el in}} - \dot{Q}_{\text{out}}$ Entropy Eq.: $0 = -\dot{Q}_{\text{out}}/T_{\text{surf}} + \dot{S}_{\text{gen CV1}}$

$$\dot{Q}_{\text{out}} = \dot{W}_{\text{el in}} = 500 \text{ W}; \qquad \dot{S}_{\text{gen CV1}} = \dot{Q}_{\text{out}}/T_{\text{surf}} = \frac{500}{1000} = \textbf{0.5 W/K}$$

CV2 Space between radiant element and 18$^{\text{o}}$C surfaces (this is the room air).

Energy Eq.: $0 = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}$ Entropy Eq.: $0 = \dfrac{\dot{Q}_{\text{in}}}{T_{\text{in}}} - \dfrac{\dot{Q}_{\text{out}}}{T_{\text{amb}}} + \dot{S}_{\text{gen CV2}}$

$$\dot{Q}_{\text{in}} = \dot{Q}_{\text{out}} = 500 \text{ W}; \qquad \dot{S}_{\text{gen CV2}} = \frac{\dot{Q}_{\text{out}}}{T_{\text{amb}}} - \frac{\dot{Q}_{\text{in}}}{T_{\text{in}}} = \frac{500}{291.15} - \frac{500}{1000}$$

$$= \textbf{1.217 W/K}$$

The total entropy generation is the sum of the two contributions above, which also matches with a total control volume that is the element and the room air.

CV Radiant element and space out to ambient 18$^{\text{o}}$C.

Energy Eq.: $0 = \dot{W}_{\text{el in}} - \dot{Q}_{\text{out}}$ Entropy Eq.: $0 = -\dot{Q}_{\text{out}}/T_{\text{amb}} + \dot{S}_{\text{gen CV tot}}$

$$\dot{Q}_{\text{out}} = \dot{W}_{\text{el in}} = 500 \text{ W}; \qquad \dot{S}_{\text{gen CV tot}} = \frac{\dot{Q}_{\text{out}}}{T_{\text{amb}}} = \frac{500}{291.15} = \textbf{1.717 W/K}$$

Borgnakke and Sonntag

6.167

A heat pump with COP = 4 uses 1 kW of power input to heat a 25$^{\text{o}}$C room, drawing energy from the outside at 15$^{\text{o}}$C. Assume the high/low temperatures in the heat pump are 45$^{\text{o}}$C /0$^{\text{o}}$C. Find the total rates of entropy into and out of the heat pump, the rate from the outside at 15$^{\text{o}}$C, and the rate to the room at 25$^{\text{o}}$C?

Solution:

C.V.TOT.

Energy Eq.: $\dot{Q}_L + \dot{W} = \dot{Q}_H$

Entropy Eq.: $0 = \dfrac{\dot{Q}_L}{T_L} - \dfrac{\dot{Q}_H}{T_H} + \dot{S}_{\text{gen CV tot}}$

From definition of COP: $\dot{Q}_H = \text{COP } \dot{W} = 4 \times 1 \text{ kW} = 4 \text{ kW}$

From energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W} = (4 - 1) \text{ kW} = 3 \text{ kW}$

Flux into heat pump at 0$^{\text{o}}$C: $\dfrac{\dot{Q}_L}{T_{\text{L-HP}}} = \dfrac{3}{273.15} \dfrac{\text{kW}}{\text{K}} = \textbf{0.0110 kW/K}$

Flux out of heat pump at 45$^{\text{o}}$C: $\dfrac{\dot{Q}_H}{T_{\text{H-HP}}} = \dfrac{4}{318.15} \dfrac{\text{kW}}{\text{K}} = \textbf{0.0126 kW/K}$

Flux out into room at $T_H = 25^{\text{o}}$C: $\dfrac{\dot{Q}_H}{T_H} = \dfrac{4}{298.15} \dfrac{\text{kW}}{\text{K}} = \textbf{0.0134 kW/K}$

Flux from outside at 15$^{\text{o}}$C: $\dfrac{\dot{Q}_L}{T_L} = \dfrac{3}{288.15} \dfrac{\text{kW}}{\text{K}} = \textbf{0.0104 kW/K}$

Comment: Following the flow of energy notice how the flux from the outside at 15$^{\text{o}}$C grows a little when it arrives at 0$^{\text{o}}$C this is due to entropy generation in the low T heat exchanger. The flux out of the heat pump at 45$^{\text{o}}$C is larger than the flux in which is due to entropy generation in the heat pump cycle (COP is smaller than Carnot COP) and finally this flux increases due to entropy generated in the high T heat exchanger as the energy arrives at room T.

Borgnakke and Sonntag

6.168

A window receives 500 W of heat transfer at the inside surface of 20$^{\text{o}}$C and transmits the 500 W from its outside surface at 2$^{\text{o}}$C continuing to ambient air at –5$^{\text{o}}$C. Find the flux of entropy at all three surfaces and the window's rate of entropy generation.

Flux of entropy: $\quad \dot{S} = \dfrac{\dot{Q}}{T}$

$\dot{S}_{\text{inside}} = \dfrac{500}{293.15} \dfrac{\text{W}}{\text{K}} = 1.7056 \text{ W/K}$

$\dot{S}_{\text{win}} = \dfrac{500}{275.15} \dfrac{\text{W}}{\text{K}} = 1.8172 \text{ W/K}$

$\dot{S}_{\text{amb}} = \dfrac{500}{268.15} \dfrac{\text{W}}{\text{K}} = 1.8646 \text{ W/K}$

Window only: $\dot{S}_{\text{gen win}} = \dot{S}_{\text{win}} - \dot{S}_{\text{inside}} = 1.8172 - 1.7056 = \textbf{0.112 W/K}$

If you want to include the generation in the outside air boundary layer where T changes from 2$^{\text{o}}$C to the ambient –5$^{\text{o}}$C then chose the control volume as CV tot and it becomes

$$\dot{S}_{\text{gen tot}} = \dot{S}_{\text{amb}} - \dot{S}_{\text{inside}} = 1.8646 - 1.7056 = 0.159 \text{ W/K}$$

Borgnakke and Sonntag

6.169

An amount of power, say 1000 kW, comes from a furnace at 800°C going into water vapor at 400°C. From the water the power goes to a solid metal at 200°C and then into some air at 70°C. For each location calculate the flux of s through a surface as $(\dot{Q}/T)$. What makes the flux larger and larger?

Solution:

$$
\begin{array}{cccc}
T_1 & \Rightarrow T_2 & \Rightarrow T_3 & \Rightarrow T_4 \\
\text{furnace} & \text{vapor} & \text{metal} & \text{air}
\end{array}
$$

Flux of s: $F_s = \dot{Q}/T$ with T as absolute temperature.

$F_{s1} = 1000/1073.15 = 0.932$ kW/K, $F_{s2} = 1000/673.15 = 1.486$ kW/K

$F_{s3} = 1000/473.15 = 2.11$ kW/K, $F_{s4} = 1000/343.15 = 2.91$ kW/K

T	800	400	200	70	(°C)
T $_{amb}$	1073	673	476	646	K
Q/T	0.932	1.486	2.114	2.915	kW/K

$_1S_{2\ gen}$ for every change in T

Q over ΔT is an irreversible process

Borgnakke and Sonntag

6.170

Room air at 23$^{\text{o}}$C is heated by a 2000 W space heater with a surface filament temperature of 700 K, shown in Fig. P6.170. The room at steady state looses the power to the outside which is at 7$^{\text{o}}$C. Find the rate(s) of entropy generation and specify where it is made.

Solution:

For any C.V at steady state the entropy equation as a rate form is Eq.6.43

$$\frac{dS_{c.v.}}{dt} = 0 = \int d\dot{Q}/T + \dot{S}_{gen}$$

C.V. Heater Element

$$\dot{S}_{gen} = -\int d\dot{Q}/T = -(-2000/700) = \textbf{2.857 W/K}$$

C.V. Space between heater 700 K and room 23°C

$$\dot{S}_{gen} = -\int d\dot{Q}/T = (-2000 / 700) - [-2000 / (23+273)] = \textbf{3.9 W/K}$$

C.V. Wall between 23°C inside and 7°C outside

$$\dot{S}_{gen} = -\int d\dot{Q}/T = [-2000 / (23+273)] - [2000 / (7 + 273)] = \textbf{0.389 W/K}$$

Notice biggest $\dot{S}_{gen}$ is for the largest change in 1/T.

Borgnakke and Sonntag

6.171

A car engine block receives 2 kW at its surface of 450 K from hot combustion gases at 1500 K. Near the cooling channel the engine block transmits 2 kW out at its 400 K surface to the coolant flowing at 370 K. Finally, in the radiator the coolant at 350 K delivers the 2 kW to air which is at 25 C. Find the rate of entropy generation inside the engine block, inside the coolant and in the radiator/air combination.

For a C.V at steady state we have the entropy equation as a rate form as Eq.6.43

$$\frac{dS_{c.v.}}{dt} = 0 = \int d\dot{Q}/T + \dot{S}_{gen}$$

CV1. Engine block receives 2 kW at 450 K and it leaves at 400 K

$$\dot{S}_{gen1} = -\int d\dot{Q}/T = \frac{-2000}{450} - [\frac{-2000}{400}] = \textbf{0.555 W/K}$$

CV2. The coolant receives 2 kW at 370 K andf gives it out at 350 K

$$\dot{S}_{gen2} = -\int d\dot{Q}/T = [\frac{-2000}{370}] - [\frac{-2000}{350}] = \textbf{0.309 W/K}$$

CV3 Radiator to air heat transfer.

$$\dot{S}_{gen3} = -\int d\dot{Q}/T = [\frac{-2000}{350}] - [\frac{-2000}{298.15}] = \textbf{0.994 W/K}$$

Notice the biggest $\dot{S}_{gen}$ is for the largest change $\Delta[1/T]$

Gases   Steel Glycol   Air flow

CV1 CV2   CV3

Radiator

Remark: The flux of S is $\dot{Q}/T$ flowing across a surface. Notice how this flux increases as the heat transfer flows towards lower and lower T.

T [K]	1500	450	370	298.15
$\dot{Q}/T$ [W/K]	1.33	4.44	5.40	6.71

Borgnakke and Sonntag

6.172

A farmer runs a heat pump using 2 kW of power input. It keeps a chicken hatchery at a constant 30°C while the room loses 10 kW to the colder outside ambient at 10°C. Find the COP of the heat pump, the rate of entropy generated in the heat pump and its heat exchangers, and the rate of entropy generated in the heat loss process?

Solution:

C.V. Hatchery, steady state.
To have steady state at 30°C for the hatchery

Energy Eq.: $\quad 0 = \dot{Q}_H - \dot{Q}_{Loss} \qquad \Rightarrow \qquad \dot{Q}_H = \dot{Q}_{Loss} = 10 \text{ kW}$

COP: $\qquad \text{COP} = \dot{Q}_H / \dot{W} = 10 / 2 = \mathbf{5}$

C.V. Heat pump, steady state

Energy eq.: $\quad 0 = \dot{Q}_L + \dot{W} - \dot{Q}_H \qquad \Rightarrow \qquad \dot{Q}_L = \dot{Q}_H - \dot{W} = 8 \text{ kW}$

Entropy Eq.: $\quad 0 = \dfrac{\dot{Q}_L}{T_L} - \dfrac{\dot{Q}_H}{T_H} + \dot{S}_{\text{gen HP}}$

$$\dot{S}_{\text{gen HP}} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_L}{T_L} = \frac{10}{273 + 30} - \frac{8}{273 + 10} = \mathbf{0.00473 \text{ kW/K}}$$

C.V. From hatchery at 30°C to the ambient 10°C. This is typically the walls and the outer thin boundary layer of air. Through this goes $\dot{Q}_{Loss}$.

Entropy Eq.: $\quad 0 = \dfrac{\dot{Q}_{Loss}}{T_H} - \dfrac{\dot{Q}_{Loss}}{T_{amb}} + \dot{S}_{\text{gen walls}}$

$$\dot{S}_{\text{gen walls}} = \frac{\dot{Q}_{Loss}}{T_{amb}} - \frac{\dot{Q}_{Loss}}{T_H} = \frac{10}{283} - \frac{10}{303} = \mathbf{0.00233 \text{ kW/K}}$$

Borgnakke and Sonntag

Review problems

Borgnakke and Sonntag

6.173

An insulated cylinder/piston has an initial volume of 0.15 m^3 and contains steam at 400 kPa, 200°C. The steam is expanded adiabaticly, and the work output is measured very carefully to be 30 kJ. It is claimed that the final state of the water is in the two-phase (liquid and vapor) region. What is your evaluation of the claim?

Solution:

    C.V. Water.

    Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

    Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T$

    Process: ${}_1Q_2 = 0$ and reversible

    State 1: (T, P)   Table B.1.3

            $v_1 = 0.5342$, $u_1 = 2646.8$, $s_1 = 7.1706$ kJ/kg K

$$m = \frac{V_1}{v_1} = \frac{0.15}{0.5342} = 0.2808 \text{ kg}$$

With the assumed reversible process we have from entropy equation

                $s_2 = s_1 = 7.1706$ kJ/kg K

and from the energy equation

        $u_2 = u_1 - {}_1W_2/m = 2646.8 - \dfrac{30}{0.2808} = 2540.0$ kJ/kg

State 2 given by (u, s)   check Table B.1.1: s_G (at $u_G = 2540$) $= 7.0259 < s_1$

   $\Rightarrow$ **State 2 must be in superheated vapor region.**

Borgnakke and Sonntag

6.174

A piston cylinder has a water volume separated in $V_A = 0.2 \ m^3$ and $V_B = 0.3 \ m^3$ by a stiff membrane. The initial state in A is 1000 kPa, x = 0.75 and in B it is 1600 kPa and 250°C. Now the membrane ruptures and the water comes to a uniform state at 200°C with heat transfer from a 250°C source. Find the work, the heat transfer, and the total entropy generation in the process.

Take the water in A and B as CV.

Continuity: $m_2 - m_{1A} - m_{1B} = 0$

Energy: $m_2 u_2 - m_{1A} u_{1A} - m_{1B} u_{1B} = {}_1 Q_2 - {}_1 W_2$

Entropy Eq.: $m_2 s_2 - m_A s_{A1} - m_B s_{B1} = {}_1 Q_2 / T_{res} + {}_1 S_{2 \ gen}$

Process: $P_2 = P_{eq} = constant = P_{1A}$ as piston floats and m_p, P_o are constant

State 1A: Two phase. Table B.1.2

$v_{1A} = 0.001127 + 0.75 \times 0.19332 = 0.146117 \ m^3/kg$,

$u_{1A} = 761.67 + 0.75 \times 1821.97 = 2128.15 \ kJ/kg$

$s_{1A} = 2.1386 + 0.75 \times 4.4478 = 5.47445 \ kJ/kg\text{-}K$

State 1B: $v_{1B} = 0.14184 \ m^3/kg$, $u_{1B} = 2692.26 \ kJ/kg$, $s_{1B} = 6.6732 \ kJ/kg\text{-}K$

$\Rightarrow \quad m_{1A} = V_{1A}/v_{1A} = 1.3688 \ kg, \quad m_{1B} = V_{1B}/v_{1B} = 2.115 \ kg$

State 2: 1000 kPa, 200°C sup. vapor $\Rightarrow$

$v_2 = 0.20596 \ m^3/kg$, $u_2 = 2621.9 \ kJ/kg$, $s_2 = 6.6939 \ kJ/kgK$

$m_2 = m_{1A} + m_{1B} = 3.4838 \ kg \Rightarrow V_2 = m_2 v_2 = 3.4838 \times 0.20596 = 0.7175 \ m^3$

So now

$$_1 W_2 = \int P \ dV = P_{eq} (V_2 - V_1) = 1000 \ (0.7175 - 0.5)$$
$$= \textbf{217.5 kJ}$$
$$_1 Q_2 = m_2 u_2 - m_{1A} u_{1A} - m_{1B} u_{1B} + {}_1 W_2$$
$$= 3.4838 \times 2621.9 - 1.3688 \times 2128.15$$
$$- 2.115 \times 2692.26 + 217.5 = \textbf{744 kJ}$$

$$_1 S_{2 \ gen} = m_2 s_2 - m_A s_{A1} - m_B s_{B1} - {}_1 Q_2 / T_{res}$$
$$= 3.4838 \times 6.6939 - 1.3688 \times 5.47445 - 2.115 \times 6.6732 - \frac{744}{523.15}$$
$$= \textbf{0.2908 kJ/K}$$

Borgnakke and Sonntag

6.175

The water in the two tanks of Problem 3.214 receives the heat transfer from a reservoir at 300°C. Find the total entropy generation due to this process.

Two rigid tanks are filled with water. Tank A is 0.2 m^3 at 100 kPa, 150°C and tank B is 0.3 m^3 at saturated vapor 300 kPa. The tanks are connected by a pipe with a closed valve. We open the valve and let all the water come to a single uniform state while we transfer enough heat to have a final pressure of 300 kPa. Give the two property values that determine the final state and heat transfer.

Take CV total A + B out to reservoir (neglect kinetic and potential energy)

Energy Eq.: $m_2 u_2 - m_A u_{A1} - m_B u_{B1} = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

Entropy Eq.: $m_2 s_2 - m_A s_{A1} - m_B s_{B1} = {}_1Q_2/T_{res} + {}_1S_{2\ gen}$

State A1: $u = 2582.75$ kJ/kg, $v = 1.93636$ m^3/kg, $s = 7.6133$ kJ/kg-K

$=> m_{A1} = V/v = 0.2/1.93636 = 0.1033$ kg

State B1: $u = 2543.55$ kJ/kg, $v = 0.60582$ m^3/kg, $s = 6.9918$ kJ/kg-K

$=> m_{B1} = V/v = 0.3 / 0.60582 = 0.4952$ kg

The total volume (and mass) is the sum of volumes (mass) for tanks A and B.

$m_2 = m_{A1} + m_{B1} = 0.1033 + 0.4952 = 0.5985$ kg,

$V_2 = V_{A1} + V_{B1} = 0.2 + 0.3 = 0.5$ m^3

$=> v_2 = V_2/m_2 = 0.5 /0.5985 = 0.8354$ m^3/kg

State 2: $[P_2, v_2] = [300$ kPa, 0.8354 m^3/kg$]$

$=> T_2 = 274.76$°C and $u_2 = 2767.32$ kJ/kg, $s = 7.60835$ kJ/kgK

From energy eq.

${}_1Q_2 = 0.5985 \times 2767.32 - 0.1033 \times 2582.75 - 0.4952 \times 2543.55 = 129.9$ kJ

From entropy equation

${}_1S_{2\ gen} = m_2 s_2 - m_A s_{A1} - m_B s_{B1} - {}_1Q_2/T_{res}$
$= 0.5985 \times 7.60835 - 0.1033 \times 7.6133 - 0.4952 \times 6.9918$
$- 129.9 / (273.15 + 300) = \textbf{0.0782 kJ/K}$

Borgnakke and Sonntag

6.176

A steel piston/cylinder of 1 kg contains 2.5 kg ammonia at 50 kPa, -20°C. Now it is heated to 50°C at constant pressure through the bottom of the cylinder from external hot gas at 200°C, and we assume the steel has the same temperature as the ammonia. Find the heat transfer from the hot gas and the total entropy generation.

Solution:

C.V. Ammonia plus space out to the hot gas.

Energy Eq.3.5: $m_{NH3}(u_2 - u_1) + m_{steel}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $S_2 - S_1 = \int dQ/T + {}_1S_{2,gen} = {}_1Q_2/ T_{gas} + {}_1S_{2\ gen}$

$S_2 - S_1 = m_{NH3}(s_2 - s_1) + m_{steel}(s_2 - s_1)$

Process: $P = C \quad => \quad {}_1W_2 = Pm_{NH3}(v_2 - v_1)_{NH3}$

State 1: (B.2.2) $v_1 = 2.4463\ m^3/kg,\ h_1 = 1434.6\ kJ/kg,\ s_1 = 6.3187\ kJ/kg\ K$

State 2: (B.2.2) $v_2 = 3.1435\ m^3/kg,\ h_2 = 1583.5\ kJ/kg,\ s_2 = 6.8379\ kJ/kg\ K$

Substitute the work into the energy equation and solve for the heat transfer

$_1Q_2 = m_{NH3}(h_2 - h_1) + m_{steel}(u_2 - u_1)$

$= 2.5\ (1583.5 - 1434.6) + 1 \times 0.46\ [50 - (-20)] = \textbf{404.45 kJ}$

$_1S_{2\ gen} = m_{NH3}(s_2 - s_1) + m_{steel}(s_2 - s_1) - {}_1Q_2/T_{gas}$

$= 2.5\ (6.8379 - 6.3187) + 1 \times 0.46\ \ln(\dfrac{323.15}{253.15}) - \dfrac{404.45}{473.15}$

$= \textbf{0.555 kJ/K}$

Borgnakke and Sonntag

6.177

Water in a piston/cylinder is at 1 MPa, 500°C. There are two stops, a lower one at which V_{min} = 1 m³ and an upper one at V_{max} = 3 m³. The piston is loaded with a mass and outside atmosphere such that it floats when the pressure is 500 kPa. This setup is now cooled to 100°C by rejecting heat to the surroundings at 20°C. Find the total entropy generated in the process.

C.V. Water.

Initial state: Table B.1.3: v_1 = 0.35411 m³/kg, u_1 = 3124.3, s_1 = 7.7621

m =V/v_1 = 3/0.35411 = 8.472 kg

Final state: 100°C and on line in P-V diagram.

Notice the following: v_g(500 kPa) = 0.3749 > v_1, v_1 = v_g(154°C)

T_{sat}(500 kPa) = 152°C > T_2 , so now piston hits bottom stops.

State 2: v_2 = v_{bot} = V_{bot}/m = 0.118 m³/kg,

x_2 = (0.118 − 0.001044)/1.67185 = 0.0699,

u_2 = 418.91 + 0.0699×2087.58 = 564.98 kJ/kg,

s_2 = 1.3068 + 0.0699×6.048 = 1.73 kJ/kg K

Now we can do the work and then the heat transfer from the energy equation

$_1W_2$ = $\int$PdV = 500(V_2 - V_1) = -1000 kJ   ($_1w_2$ = -118 kJ/kg)

$_1Q_2$ = m(u_2 - u_1) + $_1W_2$ = -22683.4 kJ   ($_1q_2$ = -2677.5 kJ/kg)

Take C.V. total out to where we have 20°C:

m(s_2 - s_1) = $_1Q_2$/T_0 + S_{gen} $\Rightarrow$

S_{gen} = m(s_2 - s_1) − $_1Q_2$/T_0 = 8.472 (1.73 - 7.7621) + 22683 / 293.15

= **26.27 kJ/K**   (= ΔS_{water} + ΔS_{sur})

Borgnakke and Sonntag

6.178

A piston/cylinder contains air at 300 K, 100 kPa. A reversible polytropic process with n = 1.3 brings the air to 500 K. Any heat transfer if it comes in is from a 325°C reservoir and if it goes out it is to the ambient at 300 K. Sketch the process in a P-v and a T-s diagram. Find the specific work and specific heat transfer in the process. Find the specific entropy generation (external to the air) in the process.

Solution:

Process : $Pv^n = C$

$$P_2/P_1 = (T_2/T_1)^{n/(n-1)} = (500/300)^{1.3/0.3} = 9.148$$

$$_1w_2 = \int P\ dv = \left(\frac{P_2v_2-P_1v_1}{1-n}\right) = \frac{R}{1-n}\ (T_2-T_1)$$
$$= \frac{0.287}{1-1.3}\ (500 - 300) = \textbf{-191.3 kJ/kg}$$

Energy equation

$$_1q_2 = u_2 - u_1 + _1w_2 = C_v\ (T_2 - T_1) + _1w_2$$
$$= 0.717\ (500 - 300) - 191.3 = \textbf{-47.93 kJ/kg}$$

The $_1q_2$ is negative and thus goes out. Entropy is generated between the air and ambient.

$$s_2 - s_1 = _1q_2/T_{amb} + _1s_{2\ gen}$$

$$_1s_{2\ gen} = s_2 - s_1 - _1q_2/T_{amb} = C_p \ln (T_2/T_1) - R \ln (P_2/P_1) - _1q_2/T_{amb}$$

$$_1s_{2\ gen} = 1.004 \ln \left(\frac{500}{300}\right) - 0.287 \ln 9.148 - \left(\frac{-47.93}{300}\right)$$
$$= 0.51287 - 0.635285 + 0.15977$$
$$= \textbf{0.03736 kJ/kg K}$$

Notice:
n = 1.3, k = 1.4

n < k

Borgnakke and Sonntag

6.179

Assume the heat transfer in problem 3.213 came from a 200°C reservoir. What is the total entropy generation in the process?

C.V. Water in A and B. Control mass goes through process: 1 -> 2

Continuity Eq.: $m_2 - m_{A1} - m_{B1} = 0$ $=> m_2 = m_{A1} + m_{B1} = 0.5 + 0.5 = 1$ kg

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m_2 s_2 - m_A s_{A1} - m_B s_{B1} = {}_1Q_2/T_{res} + {}_1S_{2\ gen}$

State A1: $v_{A1} = 0.001067 + x_{A1} \times 0.71765 = 0.072832$; $V_{A1} = mv = 0.036416$ m^3

$u_{A1} = 535.08 + 0.1 \times 2002.14 = 735.22$ kJ/kg;

$s_{A1} = 1.6072 + 0.1 \times 5.4455 = 2.15175$ kJ/kgK

State B1: $v_{B1} = 1.5493$ m^3/kg; $u_{B1} = 2966.69$ kJ/kg; $s_{B1} = 8.2217$ kJ/kg-K

$V_{B1} = (mv)_{B1} = 0.77465$ m^3

State 2:   If $V_2 > V_{A1}$ then $P_2 = 200$ kPa that is the piston floats.

For $(T_2, P_2) = (150°C, 200$ kPa$)$ $=>$ superheated vapor

$u_2 = 2576.87$ kJ/kg; $v_2 = 0.95964$ m^3/kg, $s_2 = 7.2795$ kJ/kgK

$V_2 = m_2 v_2 = 0.95964$ m^3 $> V_{A1}$ checks OK.

Process: ${}_1W_2 = P_2 (V_2 - V_1) = 200 (0.95964 - 0.77465 - 0.036416) = 29.715$ kJ

From the energy and entropy equations:

${}_1Q_2 = m_2 u_2 - m_{A1} u_{A1} - m_{B1} u_{B1} + {}_1W_2$

$= 1 \times 2576.87 - 0.5 \times 735.222 - 0.5 \times 2966.69 + 29.715 = 755.63$ kJ

${}_1S_{2\ gen} = m_2 s_2 - m_A s_{A1} - m_B s_{B1} - {}_1Q_2/T_{res}$

$= 1 \times 7.2795 - 0.5 \times 2.15175 - 0.5 \times 8.2217 - 755.63/473.15$

$= \textbf{0.496 kJ/K}$

The possible state 2 (P,V) combinations are shown. State a is 200 kPa, $v_a = V_{A1}/m_2 = 0.036$ and thus two-phase $T_a = 120°C$

less than T_2

Borgnakke and Sonntag

6.180

A closed tank, $V = 10$ L, containing 5 kg of water initially at 25°C, is heated to 150°C by a heat pump that is receiving heat from the surroundings at 25°C. Assume that this process is reversible. Find the heat transfer to the water and the work input to the heat pump.

C.V.: Water from state 1 to state 2.

Process: constant volume (reversible isometric)

State 1: $v_1 = V/m = 0.002 \Rightarrow x_1 = (0.002 - 0.001003)/43.358 = 0.000023$

$u_1 = 104.86 + 0.000023 \times 2304.9 = 104.93$ kJ/kg

$s_1 = 0.3673 + 0.000023 \times 8.1905 = 0.36759$ kJ/kg K

Continuity eq. (same mass) and V = C fixes v_2

2: $T_2, v_2 = v_1 \Rightarrow$

$x_2 = (0.002 - 0.001090)/0.39169 = 0.0023233$

$u_2 = 631.66 + 0.0023233 \times 1927.87 = 636.14$ kJ/kg

$s_2 = 1.8417 + 0.0023233 \times 4.9960 = 1.8533$ kJ/kg K

Energy eq. has W = 0, thus provides heat transfer as

$_1Q_2 = m(u_2 - u_1) = \mathbf{2656}$ **kJ**

Entropy equation for the total (tank plus heat pump) control volume gives for a reversible process:

$m(s_2 - s_1) = Q_L/T_0 \Rightarrow$

$Q_L = mT_0(s_2 - s_1) = 5 \times 298.15 \times (1.8533 - 0.36759) = 2214.8$ kJ

and then the energy equation for the heat pump gives

$W_{HP} = {_1Q_2} - Q_L = 2656 - 2214.8 = \mathbf{441.2}$ **kJ**

Borgnakke and Sonntag

6.181

A resistor in a heating element is a total of 0.5 kg with specific heat of 0.8 kJ/kgK. It is now receiving 500 W of electric power so it heats from 20^oC to 180^oC. Neglect external heat loss and find the time the process took and the entropy generation.

C.V. Heating element.

Energy Eq.: $m(u_2 - u_1) = {}_1W_{2\ in} = \dot{W}_{electrical\ in}\ \Delta t$

Entropy Eq.: $m(s_2 - s_1) = 0 + {}_1S_{2\ gen}$ (no heat transfer)

$\Delta t = m(u_2 - u_1) / \dot{W}_{electrical\ in} = m\ C\ (T_2 - T_1) / \dot{W}_{electrical\ in}$

$= 0.5\ kg \times 800\ J/kg\text{-}K \times (180 - 20)\ K / 500\ (J/s) = \textbf{128 s}$

$${}_1S_{2\ gen} = m(s_2 - s_1) = m\ C\ \ln \frac{T_2}{T_1} = 0.5\ kg \times 0.8\ kJ/kg\text{-}K\ \ln\left(\frac{180 + 273}{20 + 273}\right)$$

$$= \textbf{0.174 kJ/K}$$

Borgnakke and Sonntag

6.182

Two tanks contain steam, and they are both connected to a piston/cylinder as shown in Fig. P6.182. Initially the piston is at the bottom and the mass of the piston is such that a pressure of 1.4 MPa below it will be able to lift it. Steam in A is 4 kg at 7 MPa, 700°C and B has 2 kg at 3 MPa, 350°C. The two valves are opened, and the water comes to a uniform state. Find the final temperature and the total entropy generation, assuming no heat transfer.

Solution:

Control mass: All water $m_A + m_B$.

Continuity Eq.: $m_2 = m_A + m_B = 6$ kg

Energy Eq.3.5: $m_2u_2 - m_Au_{A1} - m_Bu_{B1} = {}_1Q_2 - {}_1W_2 = - {}_1W_2$

Entropy Eq.6.37: $m_2s_2 - m_As_{A1} - m_Bs_{B1} = {}_1S_{2\,gen}$

B.1.3: $v_{A1} = 0.06283$, $u_{A1} = 3448.5$, $s_{A1} = 7.3476$, $V_A = 0.2513$ m^3

B.1.3: $v_{B1} = 0.09053$, $u_{B1} = 2843.7$, $s_{B1} = 6.7428$, $V_B = 0.1811$ m^3

The only possible P, V combinations for state 2 are on the two lines.

Assume $V_2 > V_A + V_B \Rightarrow P_2 = P_{lift}$, ${}_1W_2 = P_2(V_2 - V_A - V_B)$

Substitute into energy equation:

$m_2h_2 = m_Au_{A1} + m_Bu_{B1} + P_2(V_A + V_B)$

$= 4 \times 3448.5 + 2 \times 2843.7 + 1400 \times 0.4324 = 20\,086.8$ kJ

State 2: $h_2 = m_2h_2/m_2 = 20\,086.8$ kJ $/6$ kg $= 3347.8$ kJ/kg, $P_2 = 1400$ kPa,

$v_2 = 0.2323$ m^3/kg, $s_2 = 7.433$ kJ/kg-K, **$T_2 = 441.9$ °C**,

Check assumption: $V_2 = m_2v_2 = 1.394$ m$^3 > V_A + V_B$ **OK.**

${}_1S_{2\,gen} = 6 \times 7.433 - 4 \times 7.3476 - 2 \times 6.7428 =$ **1.722 kJ/K**

Borgnakke and Sonntag

6.183

Assume the heat source in Problem 3.217 is at 300°C in a setup similar to Fig. P6.182. Find the heat transfer and the entropy generation.

C.V.: A + B + C.

Only work in C, total mass constant.

$m_2 - m_1 = 0 \quad \Rightarrow \quad m_2 = m_{A1} + m_{B1}$

$U_2 - U_1 = {}_1Q_2 - {}_1W_2$;

$S_2 - S_1 = {}_1Q_2/T_{res} + {}_1S_{2\,gen}$

${}_1W_2 = \int PdV = P_{lift}(V_2 - V_1)$

1A: $v = 0.6/3 = 0.2 \text{ m}^3/\text{kg} \quad \Rightarrow \quad x_{A1} = (0.2 - 0.00106)/0.8908 = 0.223327$

  $u = 503.48 + x \times 2025.76 = 955.89 \text{ kJ/kg}, s = 1.5275 + x \times 5.602 = 2.7786 \text{ kJ/kgK}$

1B: $v = 0.35202 \text{ m}^3/\text{kg} \quad \Rightarrow \quad ; \quad u = 2638.91 \text{ kJ/kg}, s = 6.9665 \text{ kJ/kgK}$

  $m_{B1} = 0.4/0.35202 = 1.1363 \text{ kg} \quad \text{and} \quad m_2 = 3 + 1.1363 = 4.1363 \text{ kg}$

$V_2 = V_A + V_B + V_C = 1 \text{ m}^3 + V_C$

Locate state 2:   Must be on P-V lines shown

State 1a: 800 kPa,

$$v_{1a} = \frac{V_A + V_B}{m} = 0.24176 \text{ m}^3/\text{kg}$$

800 kPa, $v_{1a} \quad \Rightarrow \quad T = 173°C \quad$ too low.

Assume 800 kPa: 250°C $\Rightarrow \qquad v = 0.29314 \text{ m}^3/\text{kg} > v_{1a}$ OK

$V_2 = m_2 v_2 = 4.1363 \text{ kg} \times 0.29314 \text{ m}^3/\text{kg} = \textbf{1.2125 m}^3$

Final state is : **800 kPa**; 250°C $\Rightarrow \quad u_2 = 2715.46 \text{ kJ/kg}, s_2 = 7.0384 \text{ kJ/kgK}$

${}_1W_2 = P_{lift}(V_2 - V_1) = 800 \text{ kPa} \times (1.2125 - 1) \text{ m}^3 = \textbf{170 kJ}$

${}_1Q_2 = m_2 u_2 - m_1 u_1 + {}_1W_2 = m_2 u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} + {}_1W_2$

  $= 4.1363 \times 2715.46 - 3 \times 955.89 - 1.1363 \times 2638.91 + 170$

  $= 11\,232 - 2867.7 - 2998.6 + 170 = \textbf{5536 kJ}$

${}_1S_{2\,gen} = S_2 - S_1 - {}_1Q_2/T_{res} = m_2 s_2 - m_{A1}s_{A1} - m_{B1}s_{B1} - {}_1Q_2/T_{res}$

  $= 4.1363 \times 7.0384 - 3 \times 2.7786 - 1.1363 \times 6.9665 - 5536/573.15$

  $= \textbf{3.202 kJ/K}$

Borgnakke and Sonntag

6.184

A cylinder fitted with a piston contains 0.5 kg of R-134a at 60°C, with a quality of 50 percent. The R-134a now expands in an internally reversible polytropic process to ambient temperature, 20°C at which point the quality is 100 percent. Any heat transfer is with a constant-temperature source, which is at 60°C. Find the polytropic exponent n and show that this process satisfies the second law of thermodynamics.

Solution:

C.V.: R-134a, Internally Reversible, Polytropic Expansion: $PV^n = $ Const.

Cont.Eq.: $m_2 = m_1 = m$;   Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2 \text{ gen}}$

State 1: $T_1 = 60°C$, $x_1 = 0.5$, Table B.5.1: $P_1 = P_g = 1681.8$ kPa,

$v_1 = v_f + x_1 v_{fg} = 0.000951 + 0.5 \times 0.010511 = 0.006207$ m^3/kg

$s_1 = s_f + x_1 s_{fg} = 1.2857 + 0.5 \times 0.4182 = 1.4948$ kJ/kg K,

$u_1 = u_f + x_1 u_{fg} = 286.19 + 0.5 \times 121.66 = 347.1$ kJ/kg

State 2: $T_2 = 20°C$, $x_2 = 1.0$, $P_2 = P_g = 572.8$ kPa, Table B.5.1

$v_2 = v_g = 0.03606$ m^3/kg, $s_2 = s_g = 1.7183$ kJ/kg-K

$u_2 = u_g = 389.19$ kJ/kg

Process: $PV^n = $ Const. $\Rightarrow$ $\dfrac{P_1}{P_2} = \left(\dfrac{v_2}{v_1}\right)^n$ $\Rightarrow$ $n = \ln \dfrac{P_1}{P_2} \Big/ \ln \dfrac{v_2}{v_1} = \mathbf{0.6122}$

$_1W_2 = \int PdV = \dfrac{P_2V_2 - P_1V_1}{1-n}$

$= 0.5(572.8 \times 0.03606 - 1681.8 \times 0.006207)/(1 - 0.6122) = 13.2$ kJ

2nd Law for C.V.: R-134a plus wall out to source:

$_1S_{2 \text{ gen}} = m(s_2 - s_1) - \dfrac{Q_H}{T_H}$;   Check $\Delta S_{\text{net}} \geq 0$

$Q_H = {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 34.2$ kJ

$_1S_{2 \text{ gen}} = 0.5(1.7183 - 1.4948) - 34.2/333.15 = 0.0092$ kJ/K,

$_1S_{2 \text{ gen}} > 0$ **Process Satisfies 2nd Law**

Borgnakke and Sonntag

6.185

A device brings 2 kg of ammonia from 150 kPa, -20°C to 400 kPa, 80°C in a polytropic process. Find the polytropic exponent, n, the work and the heat transfer. Find the total entropy generated assuming a source at 100°C.

Solution:

C.V. Ammonia of constant mass $m_2 = m_1 = m$ out to source.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1Q_2/T + {}_1S_{2\ gen}$

Process: $P_1v_1^n = P_2v_2^n$ Eq. (8.27)

State 1: Table B.2.2

$v_1 = 0.79774 \text{ m}^3/\text{kg}, \quad s_1 = 5.7465 \text{ kJ/kg K}, \quad u_1 = 1303.3 \text{ kJ/kg}$

State 2: Table B.2.2

$v_2 = 0.4216 \text{ m}^3/\text{kg}, \quad s_2 = 5.9907 \text{ kJ/kg K}, \quad u_2 = 1468.0 \text{ kJ/kg}$

$\ln (P_2/P_1) = \ln (v_1/v_2)^n = n \times \ln (v_1/v_2)$

$\ln \left(\dfrac{400}{150}\right) = n \times \ln \left(\dfrac{0.79774}{0.4216}\right) = 0.98083 = n \times 0.63773$

$\Rightarrow$ n = **1.538**

The work term is integration of PdV as done in text leading to Eq.6.29

$${}_1W_2 = \frac{m}{1-n} (P_2v_2 - P_1v_1)$$

$$= \frac{2}{1-1.538} \times (400 \times 0.4216 - 150 \times 0.79774) = -\textbf{182.08 kJ}$$

Notice we did not use Pv = RT as we used the ammonia tables.

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 2 (1468 - 1303.3) - 182.08 = \textbf{147.3 kJ}$

From Eq.6.37

$${}_1S_{2\ gen} = m(s_2 - s_1) - {}_1Q_2/T = 2 (5.9907 - 5.7465) - \frac{147.3}{373.15}$$

= **0.0936 kJ/K**

Notice:
 n = 1.54, k = 1.3

n > k

Borgnakke and Sonntag

6.186

A rigid tank with 0.5 kg ammonia at 1600 kPa, 160°C is cooled in a reversible process by giving heat to a reversible heat engine that has its cold side at ambient 20°C, shown in Fig. P6.186. The ammonia eventually reaches 20°C and the process stops. Find the heat transfer from the ammonia to the heat engine and the work output of the heat engine.

C.V. Ammonia

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T$ (T not constant)

Process: v = constant $\Rightarrow$ ${}_1W_2 = 0$

State 1: (T, P) Table B.2.2: $u_1 = 1596.1$ kJ/kg, $v_1 = 0.12662$ m^3/kg,

$\qquad\qquad s_1 = 5.7485$ kJ/kgK

State 2: T_2 and $v_2 = v_1$ Table B.2.1 as $v_2 < v_g$

$\qquad\qquad \Rightarrow$ 2-phase, $P_2 = P_{sat} = 857.5$ kPa

$\qquad x_2 = (v_2 - v_f)/v_{fg} = (0.12662 - 0.001638)/0.14758 = 0.846876$

$\qquad u_2 = u_f + x_2\, u_{fg} = 272.89 + 0.846876 \times 1059.3 = 1170$ kJ/kg

$\qquad s_2 = s_f + x_2\, s_{fg} = 1.0408 + 0.846876 \times 4.0452 = 4.4666$ kJ/kgK

From the energy equation

$\qquad Q_H = -{}_1Q_2 = -m(u_2 - u_1) = -0.5(1170 - 1596.1) =$ **213.05 kJ**

Take now CV total ammonia plus heat engine out to ambient

Entropy Eq.6.3: $\qquad m(s_2 - s_1) = -\dfrac{Q_L}{T_{amb}} \qquad \Rightarrow$

$\qquad Q_L = -mT_{amb}\,(s_2 - s_1) = -0.5 \times 293.15\,(4.4666 - 5.7485)$

$\qquad\quad = 187.89$ kJ

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Now the CV heat engine can give the engine work from the energy equation

Energy H.E.: $W_{HE} = Q_H - Q_L = 213.05 - 187.89 = \textbf{25.2 kJ}$

Notice to get $_1q_2 = \int T\,ds$ we must know the function T(s) which we do not readily have for this process.

Borgnakke and Sonntag

6.187

A piston/cylinder with constant loading of piston contains 1 L water at 400 kPa, quality 15%. It has some stops mounted so the maximum possible volume is 11 L. A reversible heat pump extracting heat from the ambient at 300 K, 100 kPa heats the water to 300°C. Find the total work and heat transfer for the water and the work input to the heat pump.

Solution: Take CV around the water and check possible P-V combinations.

State 1: $v_1 = 0.001084 + 0.15 \times 0.46138 = 0.07029$ m^3/kg

$u_1 = 604.29 + 0.15 \times 1949.26 = 896.68$ kJ/kg

$s_1 = 1.7766 + 0.15 \times 5.1193 = 2.5445$ kJ/kg K

$m_1 = V_1/v_1 = 0.001/0.07029 = 0.0142$ kg

State a: $v = 11\, v_1 = 0.77319$ m^3/kg,

400 kPa

=> Sup. vapor $T_a = 400°C > T_2$

State 2: Since $T_2 < T_a$ then piston is not at stops but floating so $P_2 = 400$ kPa.

(T, P) => $v_2 = 0.65484$ m^3/kg => $V_2 = (v_2/v_1) \times V_1 = 9.316$ L

$_1W_2 = \int P\, dV = P(V_2 - V_1) = 400$ kPa $(9.316 - 1) \times 0.001$ m^3 = **3.33 kJ**

$_1Q_2 = m(u_2 - u_1) + {_1W_2} = 0.0142 (2804.8 - 896.68) + 3.33 =$ **30.43 kJ**

Take CV as water plus the heat pump out to the ambient.

$m(s_2 - s_1) = Q_L/T_o$ =>

$Q_L = mT_o (s_2 - s_1) = 0.0142$ kg $\times 300$ K$\times (7.5661 - 2.5445)$ kJ/kg-K

= 21.39 kJ

$W_{HP} = {_1Q_2} - Q_L = 30.43 - 21.39 =$ **9.04 kJ**

Borgnakke and Sonntag

6.188

An un-insulated cylinder fitted with a piston contains air at 500 kPa, 200°C, at which point the volume is 10 L. The external force on the piston is now varied in such a manner that the air expands to 150 kPa, 25 L volume. It is claimed that in this process the air produces 70% of the work that would have resulted from a reversible, adiabatic expansion from the same initial pressure and temperature to the same final pressure. Room temperature is 20°C.

a) What is the amount of work claimed?

b) Is this claim possible?

Solution:

C.V.: Air; $R = 0.287$ kJ/kg-K, $C_p = 1.004$ kJ/kg K, $C_v = 0.717$ kJ/kg K

State 1: $T_1 = 200^oC$, $P_1 = 500$ kPa, $V_1 = 10$ L $= 0.01$ m^3;

$$m_1 = V_1/v_1 = P_1V_1/RT_1 = 0.0368 \text{ kg}$$

State 2: $P_2 = 150$ kPa, $V_2 = 25$ L $= 0.025$ m^3

$\eta_s = 70\%$; Actual Work is 70% of Isentropic Work

a) Assume Reversible and Adiabatic Process; $s_1 = s_{2s}$

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 473.15 \ (150 / 500) = 335.4 \text{ K}$$

Energy Eq.: $\quad _1Q_{2s} = m(u_{2s} - u_1) + {_1}W_{2s}; \quad _1Q_{2s} = 0$

Assume constant specific heat

$$_1W_{2 \ s} = mC_v(T_1 - T_{2s}) = 3.63 \text{ kJ}$$

$$_1W_{2 \ ac} = 0.7 \times {_1}W_{2 \ s} = \textbf{2.54 kJ}$$

b) Use Ideal Gas Law; $T_{2 \ ac} = T_1P_2V_2 / P_1V_1 = 354.9$ K

Energy Eq.: $\quad _1Q_{2 \ ac} = mC_v(T_{2 \ ac} - T_1) + {_1}W_{2 \ ac} = -0.58$ kJ

2nd Law: $\quad _1S_{2 \ gen} = m(s_2 - s_1) - \dfrac{Q_{cv}}{T_O} \ ; \quad Q_{CV} = {_1}Q_{2 \ ac}, \ T_O = 20^oC$

$$s_2 - s_1 = C_p \ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1} = 0.0569 \text{ kJ/kg-K}$$

$$_1S_{2 \ gen} = 0.00406 \text{ kJ/K} \geq 0 \ ; \quad \textbf{Process is Possible}$$

Borgnakke and Sonntag

6.189

A small halogen light bulb receives an electrical power of 50 W. The small filament is at 1000 K and gives out 20% of the power as light and the rest as heat transfer to the gas, which is at 500 K; the glass is at 400 K. All the power is absorbed by the room walls at 25°C. Find the rate of generation of entropy in the filament, in the total bulb including glass and the total room including bulb.

Solution:

$\dot{W}_{el}$ = 50 W

$\dot{Q}_{RAD}$ = 10 W

$\dot{Q}_{COND}$ = 40 W

We will assume steady state and no storage in the bulb, air or room walls.

C.V. Filament steady-state

Energy Eq.3.31: $dE_{c.v.}/dt = 0 = \dot{W}_{el} - \dot{Q}_{RAD} - \dot{Q}_{COND}$

Entropy Eq.6.42: $dS_{c.v.}/dt = 0 = -\dfrac{\dot{Q}_{RAD}}{T_{FILA}} - \dfrac{\dot{Q}_{COND}}{T_{FILA}} + \dot{S}_{gen}$

$\dot{S}_{gen} = (\dot{Q}_{RAD} + \dot{Q}_{COND})/T_{FILA} = \dot{W}_{el}/T_{FILA} = \dfrac{50}{1000} = \textbf{0.05 W/K}$

C.V. Bulb including glass

$\dot{Q}_{RAD}$ leaves at 1000 K $\qquad\qquad$ $\dot{Q}_{COND}$ leaves at 400 K

$\dot{S}_{gen} = \int d\dot{Q}/T = -(-10/1000) - (-40/400) = \textbf{0.11 W/K}$

C.V. Total room. All energy leaves at 25°C

Eq.5.31: $\qquad dE_{c.v.}/dt = 0 = \dot{W}_{el} - \dot{Q}_{RAD} - \dot{Q}_{COND}$

Eq.6.42: $\qquad dS_{c.v.}/dt = 0 = -\dfrac{\dot{Q}_{TOT}}{T_{WALL}} + \dot{S}_{gen}$

$\dot{S}_{gen} = \dfrac{\dot{Q}_{TOT}}{T_{WALL}} = 50/(25+273) = \textbf{0.168 W/K}$

Borgnakke and Sonntag

Solutions using the Pr and vr functions in Table A.7.2

If you would like to see more of these please let me know (claus@umich.edu) and I can prepare more of the problem solutions using these functions.

Borgnakke and Sonntag

6.92   uses P_r function

A piston/cylinder, shown in Fig. P6.92, contains air at 1380 K, 15 MPa, with $V_1 = 10 \text{ cm}^3$, $A_{cyl} = 5 \text{ cm}^2$. The piston is released, and just before the piston exits the end of the cylinder the pressure inside is 200 kPa. If the cylinder is insulated, what is its length? How much work is done by the air inside?

Solution:

C.V. Air, Cylinder is insulated so adiabatic, $Q = 0$.

Continuity Eq.: $m_2 = m_1 = m$,

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = 0 + {}_1S_{2\,gen}$

State 1: (T_1, P_1) State 2: $(P_2, ?)$

So one piece of information is needed for the ?,   assume reversible process.

$\qquad {}_1S_{2\,gen} = 0 \quad \Rightarrow \quad s_2 - s_1 = 0$

State 1:   Table A.7.1: $u_1 = 1095.2$ kJ/kg,

$\qquad$ Table A.7.2: $P_{r1} = 340.53$, $v_{r1} = 2.7024$

$\qquad m = P_1V_1/RT_1 = \dfrac{15000 \times 10 \times 10^{-6}}{0.287 \times 1380} = 0.000379$ kg

State 2: P_2 and from Entropy eq.: $s_2 = s_1$

$\qquad \Rightarrow \quad P_{r2} = P_{r1}P_2/P_1 = 340.53 \times 200/15000 = 4.5404$

$\qquad$ Interpolate in A.7.2 to match the P_{r2} value

$\qquad T_2 = 447$ K, $u_2 = 320.85$ kJ/kg, $v_{r2} = 65.67$

$\Rightarrow \ V_2 = V_1 v_{r2}/v_{r1} = 10 \times 65.67 / 2.7024 = \textbf{243 cm}^3$

$\qquad \Rightarrow \ L_2 = V_2 /A_{cyl} = 243/5 = \textbf{48.6 cm}$

$\Rightarrow \ {}_1w_2 = u_1 - u_2 = 774.4$ kJ/kg, ${}_1W_2 = m_1w_2 = \textbf{0.2935 kJ}$

We could also have done $V_2 = V_1 (T_2P_1/T_1P_2)$ from ideal gas law and thus did not need the vr function for this problem

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6.105 uses v_r function

A piston/cylinder contains air at 300 K, 100 kPa. It is now compressed in a reversible adiabatic process to a volume 7 times as small. Use constant heat capacity and find the final pressure and temperature, the specific work and specific heat transfer for the process.

Solution: Here we use the v_r function from Table A.7.2

Expansion ratio: $v_2/v_1 = 1/7$

Process eq.:   Rev. adiabatic and ideal gas gives $Pv^n = C$, with $n = k$

Since we know the v ratio and s is constant we use the v_r function

$v_{r1} = 179.49$ $=> \ v_{r2} = v_{r1} \ v_2/v_1 = 179.49/7 = 25.641$

Table A.7.2:   Interpolate $T_2 = \mathbf{640.7 \ K}$

$P_2 = P_1 \times (T_2 / T_1) \times (v_1/v_2) = 100 \times (640.7/300) \times 7 = \mathbf{1495 \ kPa}$

Adiabatic: $_1q_2 = \mathbf{0 \ kJ/kg}$

Polytropic process work term from the energy equation

$_1w_2 = -(u_2 - u_1) = -(466.37 - 214.36) = \mathbf{-252.0 \ kJ/kg}$

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6. additional problem uses P_r function

A mass of 1 kg of air contained in a cylinder at 1.5 MPa, 1000 K, expands in a reversible adiabatic process to 100 kPa. Calculate the final temperature and the work done during the process, using

     a.   Constant specific heat, value from Table A.5

     b.   The ideal gas tables, Table A.7

Solution:

C.V. Air.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

Process: ${}_1Q_2 = 0$, ${}_1S_{2\ gen} = 0$ => $s_2 = s_1$

a) Using constant Cp from Table A.5 gives the power relation Eq.6.23.

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 1000\left(\frac{0.1}{1.5}\right)^{0.286} = \mathbf{460.9\ K}$$

$${}_1W_2 = -(U_2 - U_1) = mC_{Vo}(T_1 - T_2)$$

$$= 1\ kg \times 0.717\ kj/kg\text{-}K\ (1000 - 460.9)\ K = \mathbf{386.5\ kJ}$$

b) Use the tabulated **reduced pressure function** that includes variable heat capacity from A.7.2 so since $s_2 = s_1$ we have

$$P_{r2} = P_{r1} \times P_2/P_1 = 91.65 \times \frac{0.1}{1.5} = 6.11$$

Interpolation gives $T_2 = \mathbf{486\ K}$ and $u_2 = 349.4\ kJ/kg$

$${}_1W_2 = m(u_1 - u_2) = 1\ kg\ (759.2 - 349.4)\ kJ/kg = \mathbf{409.8\ kJ}$$

ENGLISH UNIT PROBLEMS

Borgnakke Sonntag

Fundamentals of
Thermodynamics

SOLUTION MANUAL
CHAPTER 6
English units

8e

UPDATED JULY 2013

CHAPTER 6

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Concept Problems

6.190E

Water at 20 psia, 240 F receives 40 Btu/lbm in a reversible process by heat transfer. Which process changes s the most: constant T, constant v or constant P?

$$ds = \frac{dq}{T}$$

Look at the constant property lines in a T-s diagram, Fig. 6.5. The constant v line has a higher slope than the constant P line also at positive slope. Thus both the constant P and v processes have an increase in T. As T goes up the change in s is smaller for the same area (heat transfer) under the process curve in the T-s diagram as compared with the constant T process.

The constant T (isothermal) process therefore changes s the most.

In a reversible process the area below the process curve in the T-s diagram is the heat transfer.

6.191E

Saturated water vapor at 20 psia is compressed to 60 psia in a reversible adiabatic process. Find the change in v and T.

Process adiabatic: $dq = 0$

Process reversible: $ds_{gen} = 0$

Change in s: $ds = dq/T + ds_{gen} = 0 + 0 = 0$ thus s is constant

Table F.7.2: $T_1 = 227.96$ F, $v_1 = 20.091$ ft^3/lbm, $s_1 = 1.732$ Btu/lbm R

Table F.7.2 at 60 psia and $s = s_1 = 1.732$ Btu/lbm R

$$T = 400 + 40\, \frac{1.732 - 1.7134}{1.736 - 1.7134} = 400 + 40 \times 0.823 = \textbf{432.9 F}$$

$$v = 8.353 + (8.775 - 8.353) \times 0.823 = \textbf{8.700 ft}^3\textbf{/lbm}$$

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Entropy, Clausius

6.192E

Consider the steam power plant in Problem 5.133E and show that this cycle satisfies the inequality of Clausius.

Solution:

Show Clausius: $\qquad \int \dfrac{dQ}{T} \leq 0$

For this problem we have two heat transfer terms:

Boiler: $\qquad$ 1000 Btu/s at 1200 F = 1660 R

Condenser: $\qquad$ 580 Btu/s at 100 F = 560 R

$$\int \dfrac{dQ}{T} = \dfrac{Q_H}{T_H} - \dfrac{Q_L}{T_L} = \dfrac{1000}{1660} - \dfrac{580}{560}$$

$$= 0.6024 - 1.0357 = \textbf{-0.433 Btu/s R} < 0 \qquad \textbf{OK}$$

6.193E

Find the missing properties of P, v, s and x for ammonia, NH_3.

 a. T = 190 F, P = 100 psia
 b. T = 80 F, h = 650 Btu/lbm
 c. T = 120 F, v = 1.6117 ft^3/lbm

a) Table F.8.1, P < P_g = 708 psia, superheated vapor so x = undefined,

$$v = (3.8804 + 4.0188)/2 = 3.9496 \text{ ft}^3/\text{lbm}$$
$$s = (1.3658 + 1.3834)/2 = 1.3746 \text{ Btu/lbm R}$$

b) Table F.8.2   h > h_g = 629.62 Btu/lbm so superheated vapor

x = undefined, found between 50 and 60 psia

$$P = 50 + 10 \ (650 - 651.49)/(649.57 - 651.49) = 57.76 \text{ psia}$$

$$v = 6.5573 + 0.77604 \ (5.4217 - 6.5573) = 5.676 \text{ ft}^3/\text{lbm}$$

$$s = 1.3588 + 0.77604 \ (1.3348 - 1.3588) = 1.3402 \text{ Btu/lbm R}$$

c) Table F.8.1:   v > v_g = 1.0456 ft^3/lbm so superheated vapor

Table F.8.2:   x = undefined, P = 200 psia, s = 1.2052 Btu/lbm R

6.194E

Find the missing properties and give the phase of the substance

    a. H_2O $s = 1.75$ Btu/lbm R, $T = 150$ F $h = ?$ $P = ?$ $x = ?$

    b. H_2O $u = 1350$ Btu/lbm, $P = 1500$ lbf/in.^{2} $T = ?$ $x = ?$ $s = ?$

a) Table F.7.1: $s < s_g = 1.8683$ Btu/lbm-R,   so 2 phase

$$P = P_{sat}(T) = 3.722 \text{ lbf/in.}^2$$

$$x = (s - s_f)/s_{fg} = (1.75 - 0.215)/1.6533 = 0.92845$$

$$h = 117.95 + 0.92845 \times 1008.1 = 1053.9 \text{ Btu/lbm}$$

b) Table F.7.2,   found between 1000 F and 1100 F,   x = undefined,

    $T = 1000 + 100 (1350\text{-}1340.43)/(1387.16 – 1340.43) =1020.5$ F,

    $s = 1.6001 + 0.2048 (1.6398 – 1.6001) =1.6083$ Btu/lbm R

6.195E

Determine the missing property among P, T, s, and x for R-410A at:

     a.   T = -20 F, v = 3.1214 ft^3/lbm

     b.   T = 60 F, v = 0.3121 ft^3/lbm

     c.   P = 30 psia, s = 0.3425 Btu/lbm-R

     a)   F.9.1: $v > v_g = 1.4522$ ft^3/lbm =>
               F.9.2   superheated vapor so x is undefined
               very close to 20 psia, s = 0.2862 Btu/lbm-R

     b)   F.9.1: $0.01451 = v_f < v < v_g = 0.3221$ ft^3/lbm => Two-phase
               $P = P_{sat} = 184.98$ psia

$$x = (v - v_f)/v_{fg} = \frac{0.3121 - 0.01451}{0.3076} = 0.96746$$

$$s = s_f + x\, s_{fg} = 0.0744 + 0.96746 \times 0.1662 = 0.2352 \text{ Btu/lbm-R}$$

     c)   Table F.9.2 at 30 psia, $s > s_g$ so superheated vapor,
               x is undefined, and we find the state at T = 160 F.

Reversible Processes

6.196E

In a Carnot engine with water as the working fluid, the high temperature is 500 F and as Q_L is received, the water changes from saturated liquid to saturated vapor. The water pressure at the low temperature is 14.7 lbf/in.^{2}. Find T_L, cycle thermal efficiency, heat added per pound-mass, and entropy, s, at the beginning of the heat rejection process.

Constant T $\implies$ constant P from 1 to 2 Table F.7.1

$$q_H = \int T ds = T (s_2 - s_1) = T\, s_{fg}$$

$$= h_2 - h_1 = h_{fg} = \textbf{714.76 Btu/lbm}$$

States 3 & 4 are two-phase Table F.7.1

$$\implies T_L = T_3 = T_4 = \textbf{212 F}$$

$$\eta_{cycle} = 1 - T_L/T_H = 1 - \frac{212 + 459.67}{500 + 459.67} = \textbf{0.300}$$

Table F.8.1: $s_3 = s_2 = s_g(T_H) = \textbf{1.4335 Btu/lbm R}$

6.197E

Consider a Carnot-cycle heat pump with R-410A as the working fluid. Heat is rejected from the R-410A at 110 F, during which process the R-410A changes from saturated vapor to saturated liquid. The heat is transferred to the R-410A at 30 F.

a. Show the cycle on a *T–s* diagram.

b. Find the quality of the R-410A at the beginning and end of the isothermal heat addition process at 30 F.

c. Determine the coefficient of performance for the cycle.

a)

Table F.9.1

b) State 3 is saturated liquid

$$s_4 = s_3 = 0.1115 \text{ Btu/lbm R}$$
$$= 0.0526 + x_4(0.1955)$$
$$x_4 = \mathbf{0.30128}$$

State 2 is saturated vapor

$$s_1 = s_2 = 0.2261 \text{ Btu/lbm R}$$
$$= 0.0526 + x_1(0.1955)$$
$$x_1 = \mathbf{0.8875}$$

c) $\quad \beta' = \dfrac{q_H}{w_{IN}} = \dfrac{T_H}{T_H - T_L} = \dfrac{569.67}{110 - 30} = \mathbf{7.121}$ (depends only on the temperatures)

6.198E

Do Problem 6.197E using refrigerant R-134a instead of R-410A.

a)

b) Table F.10.1

State 3 is saturated liquid

$s_4 = s_3 = 0.2882$ Btu/lbm R

$= 0.2375 + x_4(0.1749)$

$x_4 = \mathbf{0.2899}$

State 2 is saturated vapor

$s_1 = s_2 = 0.4087$ Btu/lbm R

$= 0.2375 + x_1(0.1749)$

$x_1 = \mathbf{0.9788}$

c) $\beta' = \dfrac{q_H}{w_{IN}} = \dfrac{T_H}{T_H - T_L} = \dfrac{569.67}{110 - 30} = \mathbf{7.121}$

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6.199E

R-410A at 150 psia and 140 F is expanded in a piston cylinder to 75 psia, 80 F in a reversible process. Find the sign for both the work and the heat transfer for this process.

Solution:

$$_1w_2 = \int P \, dv \qquad\qquad \text{so sign } dv$$

$$_1q_2 = \int T \, ds \qquad\qquad \text{so sign } ds$$

F.9.2 $\quad v_1 = 0.5356 \text{ ft}^3/\text{lbm} \qquad s_1 = 0.2859 \text{ Btu/lbm-R}$

F.9.2 $\quad v_2 = 0.9911 \text{ ft}^3/\text{lbm} \qquad s_2 = 0.2857 \text{ Btu/lbm-R}$

$dv > 0 \qquad\qquad => \qquad$ **w is positive**

$ds < 0 \ \ (\approx 0) \quad => \qquad$ **q is negative (nearly zero)**

6.200E

A piston/cylinder receives R-410A at 75 psia and compresses it in a reversible adiabatic process to 300 psia, 160 F. Find the initial temperature.

CV R-410A, this is a control mass

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.6.3: $s_2 - s_1 = \int dq/T = {}_1q_2 /T$

Process: Adiabatic and reversible $\Rightarrow$ ${}_1q_2 = 0$ so then $s_2 = s_1$

State 1: $P_1, s_1 = s_2 = 0.2673$ Btu/lbm-R $\Rightarrow$

$$T_1 = 20 + (40 - 20) \times \frac{0.2673 - 0.2595}{0.2688 - 0.2595} = \mathbf{36.77\ F}$$

6.201E

A piston/cylinder contains 1 lbm of water at 40 psia, 600 F and it now cools to 280 F in an isobaric process. The heat goes into a heat engine which rejects heat to the ambient at 77 F shown in Fig. P6.46 and the whole process is assumed reversible. Find the heat transfer out of the water and the work given out by the heat engine.

C.V. H_2O

Energy Eq.3.5: $\quad\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.3: $\quad\quad m(s_2 - s_1) = \int dQ/T + 0$

Process: $\quad\quad\quad\quad P = C \implies W = \int P\, dV = P(V_2 - V_1) = m\, P\, (v_2 - v_1)$

State 1: F.7.2 $\quad s_1 = 1.8621$ Btu/lbm-R, $h_1 = 1333.43$ Btu/lbm

State 2: F.7.3 $\quad s_2 = 1.6857$ Btu/lbm-R, $h_2 = 1176.59$ Btu/lbm

From the process equation and the energy equation

$\quad\quad {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) = 1$ lbm (1176.59 - 1333.43) Btu/lbm

$\quad\quad\quad = \mathbf{-156.84\ Btu}$

CV Total

Energy Eq.3.5: $\quad\quad m(u_2 - u_1) = - Q_L - {}_1W_2 - W_{HE}$

Entropy Eq.6.3: $\quad\quad m(s_2 - s_1) = - Q_L/T_{amb} + 0$

$\quad\quad Q_L = mT_{amb}(s_1 - s_2) = 1$ lbm $\times$ 536.7 R $\times$ (1.8621 – 1.6857) Btu/lbmR

$\quad\quad\quad = 94.67$ Btu

Now the energy equation for the heat engine gives

$\quad\quad W_{HE} = - {}_1Q_2 - Q_L = 156.84 - 94.67 = \mathbf{62.17\ Btu}$

6.202E

A cylinder fitted with a piston contains ammonia at 120 F, 20% quality with a volume of 60 in.^{3}. The ammonia expands slowly, and during this process heat is transferred to maintain a constant temperature. The process continues until all the liquid is gone. Determine the work and heat transfer for this process.

C.V. Ammonia in the cylinder.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.3: $\quad m(s_2 - s_1) = \int dQ/T + 0$

Process: $\quad$ T = constant to $x_2 = 1.0$, $\quad$ Table F.8.1: $\quad P = 286.5$ lbf/in^2

$\Rightarrow \quad W = \int P\, dV = P(V_2 - V_1) = m\, P\, (v_2 - v_1)$

$\Rightarrow \int dQ/T = {}_1Q_2 / T$

$T_1 = 120$ F, $x_1 = 0.20$, $V_1 = 60$ in^3

$v_1 = 0.02836 + 0.2 \times 1.0171 = 0.2318$ ft^3/lbm

$s_1 = 0.3571 + 0.2 \times 0.7829 = 0.5137$ Btu/lbm R,

$$m = V/v = \frac{60}{1728 \times 0.2318} = 0.15 \text{ lbm}$$

State 2: Saturated vapor,

$\quad v_2 = 1.045$ ft^3/lbm, $\quad s_2 = 1.140$ Btu/lbm R

$${}_1W_2 = \frac{286.5 \times 144}{778} \times 0.15 \times (1.045 - 0.2318) = \textbf{6.47 Btu}$$

From the entropy equation

$\quad {}_1Q_2 = T\, m\, (s_2 - s_1)$

$\quad\quad = 579.7$ R $\times 0.15$ lbm $\times (1.1400 - 0.5137)$ Btu/lbm-R

$\quad\quad = \textbf{54.46 Btu}$

- **or** - $h_1 = 178.79 + 0.2 \times 453.84 = 269.56$ Btu/lbm; $\quad h_2 = 632.63$ Btu/lbm

$\quad {}_1Q_2 = m(h_2 - h_1) = 0.15(632.63 - 269.56) = \textbf{54.46 Btu}$

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6.203E

One pound-mass of water at 600 F expands against a piston in a cylinder until it reaches ambient pressure, 14.7 lbf/in.^{2}, at which point the water has a quality of 90%. It may be assumed that the expansion is reversible and adiabatic.

      a. What was the initial pressure in the cylinder?

      b. How much work is done by the water?

Solution:

C.V. Water. Process: Rev., Q = 0

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = -{}_1W_2$

Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T$

Process: Adiabatic Q = 0 and reversible   => $s_2 = s_1$

State 2: $P_2 = 14.7$ lbf/in^2, $x_2 = 0.90$ from Table F.7.1

        $s_2 = 0.3121 + 0.9 \times 1.4446 = 1.6123$ Btu/lbm R

        $u_2 = 180.1 + 0.9 \times 897.5 = 987.9$ Btu/lbm

State 1   Table F.7.2: at $T_1 = 600$ F, $s_1 = s_2$

          $\Rightarrow P_1 = $ **335 lbf/in^2** $u_1 = 1201.2$ Btu/lbm

From the energy equation

        ${}_1W_2 = m(u_1 - u_2) = 1$ lbm$(1201.2 - 987.9)$ Btu/lbm = **213.3 Btu**

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6.204E

A closed tank, $V = 0.35$ ft^3, containing 10 lbm of water initially at 77 F is heated to 350 F by a heat pump that is receiving heat from the surroundings at 77 F. Assume that this process is reversible. Find the heat transfer to the water and the work input to the heat pump.

C.V.: Water from state 1 to state 2.

Process: constant volume (reversible isometric)

1: $v_1 = V/m = 0.35/10 = 0.035$ ft^3/lbm $\Rightarrow$ $x_1 = 2.692 \times 10^{-5}$

$u_1 = 45.11$ Btu/lbm, $s_1 = 0.08779$ Btu/lbm R

Continuity eq. (same mass) and constant volume fixes v_2

State 2: T_2, $v_2 = v_1 \Rightarrow x_2 = (0.035 - 0.01799) / 3.3279 = 0.00511$

$u_2 = 321.35 + 0.00511 \times 788.45 = 325.38$ Btu/lbm

$s_2 = 0.5033 + 0.00511 \times 1.076 = 0.5088$ Btu/lbm R

Energy eq. has zero work, thus provides heat transfer as

$_1Q_2 = m(u_2 - u_1) = 10(325.38 - 45.11) =$ **2802.7 Btu**

Entropy equation for the total control volume gives for a reversible process:

$$m(s_2 - s_1) = Q_L/T_0$$

$\Rightarrow$

$Q_L = mT_0(s_2 - s_1)$

= 10 lbm (77 + 459.67)R (0.5088 - 0.08779) Btu/lbm-R

= 2259.4 Btu

and the energy equation for the heat pump gives

$W_{HP} = {_1Q_2} - Q_L = 2802.7 - 2259.4 =$ **543.3 Btu**

6.205E

A cylinder containing R-134a at 60 F, 30 lbf/in.^{2}, has an initial volume of 1 ft^3. A piston compresses the R-134a in a reversible, isothermal process until it reaches the saturated vapor state. Calculate the required work and heat transfer to accomplish this process.

Solution:

C.V. R-134a.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.:3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

Process: T = constant, reversible so ${}_1S_{2\ gen} = 0$

State 1: (T, P) Table F.10.2 $u_1 = 168.41$ Btu/lbm, $s_1 = 0.4321$ Btu/lbm R

$m = V/v_1 = 1/1.7367 = 0.5758$ lbm

State 2: (60 F sat. vapor)
Table F.10.1
  $u_2 = 166.28$ Btu/lbm,
  $s_2 = 0.4108$ Btu/lbm R

As T is constant we can find Q by integration as

${}_1Q_2 = \int Tds = mT(s_2 - s_1) = 0.5758$ lbm $\times 519.7$ R $\times(0.4108 - 0.4321)$ Btu/lbm-R

= **-6.374 Btu**

The work is then from the energy equation

${}_1W_2 = m(u_1 - u_2) + {}_1Q_2 = 0.5758$ lbm $\times (168.41 - 166.28)$ Btu/lbm - 6.374 Btu

= **-5.15 Btu**

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6.206E

A rigid, insulated vessel contains superheated vapor steam at 400 lbf/in.^{2}, 700 F. A valve on the vessel is opened, allowing steam to escape. It may be assumed that the steam remaining inside the vessel goes through a reversible adiabatic expansion. Determine the fraction of steam that has escaped, when the final state inside is saturated vapor.

C.V.: steam remaining inside tank. Rev. & Adiabatic (inside only)

Cont.Eq.: $m_2 = m_1 = m$; Energy Eq.: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Entropy Eq.: $m(s_2 - s_1) = \int dQ/T + {_1}S_{2\,gen} = 0 + 0$

State 1: Table F.7.2 $v_1 = 1.6503$ ft^3/lbm, $s_1 = 1.6396$ Btu/lbm R

State 2: Table F.7.1 $s_2 = s_1 = 1.6396$ Btu/lbm R = s_g at P_2

$\Rightarrow$ Interpolate in F.7.1 $P_2 = 63.861$ lbf/in^2, $v_2 = v_g = 6.800$ ft^3/lbm

$$\frac{m_e}{m_1} = \frac{m_1 - m_2}{m_1} = 1 - \frac{m_2}{m_1} = 1 - \frac{v_1}{v_2} = 1 - \frac{1.6503}{6.800} = \textbf{0.757}$$

Entropy of a Liquid or Solid

6.207E

Two 5 lbm blocks of steel, one at 500 F the other at 80 F, come in thermal contact. Find the final temperature and the change in the entropy of the steel.

C.V. Both blocks, no external heat transfer, C = 0.11 Btu/lbm-R from Table F.2.

Energy Eq.: $U_2 - U_1 = m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0 - 0$

$$= m_A C(T_2 - T_{A1}) + m_B C(T_2 - T_{B1})$$

$$T_2 = \frac{m_A T_{A1} + m_B T_{B1}}{m_A + m_B} = \frac{1}{2} T_{A1} + \frac{1}{2} T_{B1} = \textbf{290 F}$$

Entropy Eq.6.37: $S_2 - S_1 = m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B = {}_1S_{2\ gen}$

Entropy changes from Eq.6.11

$$S_2 - S_1 = m_A C \ln \frac{T_2}{T_{A1}} + m_B C \ln \frac{T_2}{T_{B1}}$$

$$= 5 \times 0.11 \ln \frac{290 + 459.67}{500 + 459.67} + 5 \times 0.11 \ln \frac{290 + 459.67}{80 + 459.67}$$

$$= -0.13583 + 0.18077 = \textbf{0.0449 Btu/R}$$

Heat transfer over a finite temperature difference is an irreversible process

6.208E

A foundry form box with 50 lbm of 400 F hot sand is dumped into a bucket with 2 ft^3 water at 60 F. Assuming no heat transfer with the surroundings and no boiling away of liquid water, calculate the net entropy change for the masses.

C.V. Sand and water, P = const.

Energy Eq.: $m_{sand}(u_2 - u_1)_{sand} + m_{H_2O}(u_2 - u_1)_{H_2O} = -P(V_2 - V_1)$

$\Rightarrow m_{sand}\Delta h_{sand} + m_{H_2O}\Delta h_{H_2O} = \emptyset, \quad m_{H_2O} = \dfrac{2}{0.016035} = 124.73$ lbm

50 lbm × 0.19 Btu/lbm-R × $(T_2 - 400\ F)$

+ 124.73 lbm × 1.0 Btu/lbm-R × $(T_2 - 60\ F) = \emptyset,$

$T_2 = 84\ F$

$S_2 - S_1 = m_{sand}(s_2 - s_1) + m_{H2O}(s_2 - s_1)$

$= m_{sand}\ C_{sand}\ \ln(T_2/T_1) + m_{H2O}\ C_{H2O}\ \ln(T_2/T_1)$

$= 50 × 0.19 × \ln\left(\dfrac{544}{860}\right) + 124.73 × 1.0 × \ln\left(\dfrac{544}{520}\right)$

$= \mathbf{1.293\ Btu/R}$

Box holds the sand for
form of the cast part

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6.209E

Heat transfer to a block of 3 lbm ice at 15 F melts it to liquid at 50 F in a kitchen. Find the entropy change of the water.

Water changes state from nearly saturated solid to nearly saturated liquid. The pressure is 1 atm, but we approximate the state properties with saturated state at the same temperature.

State 1: Compressed (saturated) solid, F.7.4, $s_1 = -0.3093$ Btu/lbm-R

State 2: Compressed (saturated) liquid F.7.1 $s_2 = 0.0361$ Btu/lbm-R

The entropy change is

$$\Delta S = m(s_2 - s_1) = 3 \text{ lbm } [0.0361 - (-0.3093)] \text{ Btu/lbm-R}$$
$$= \mathbf{1.0362} \text{ Btu/R}$$

6.210E

A rigid tank of 1.2 lbm steel contains 1.5 lbm of R-134a at 100 F, 80 psia. The tank is placed in a refrigerator that brings it to 0 F. Find the process heat transfer and the combined steel and R-134a change in entropy.

C.V. The steel tank and the R-134a.

The energy equation, Eq. 3.5 now becomes (summing over the mass)

$$m_{st} (u_2 - u_1)_{st} + m_{R134a} (u_2 - u_1)_{R134a} = {}_1Q_2 - 0$$

Process: No change in volume so no work as used above.

Use specific heat from Table F.2 for steel and Table F.10 for R-134a

R-134a: $v_1 = 0.6617$ ft^3/lbm, $u_1 = 174.06$ Btu/lbm, $s_1 = 0.4252$ Btu/lbmR

State 2: $v_2 = v_1 < v_g$ => $x_2 = (v_2 - v_f)/v_{fg} = \dfrac{0.6617 - 0.01187}{2.1340} = 0.304513$

$u_2 = u_f + x_2\, u_{fg} = 75.88 + x_2 \times 82.24 = 100.923$ Btu/lbm

$s_2 = s_f + x_2\, s_{fg} = 0.2178 + x_2 \times 0.1972 = 0.27785$ Btu/lbmR

Now the heat transfer from the energy equation

$${}_1Q_2 = m_{R134a}(u_2 - u_1)_{R134a} + m_{st}\, C_{st}\, (T_2 - T_1)$$

$$= 1.5 \times (100.923 - 174.06) + 1.2 \times 0.11 (0 - 100) = \textbf{--122.91 Btu}$$

Steel: $m_{st}(s_2 - s_1)_{st} = m_{st}C_{st}\, \ln(T_2/T_1) = 1.2 \times 0.11\ \ln\dfrac{459.7}{559.7} = -0.02598$ Btu/lbmR

Entropy change for the total control volume steel and R-134a

$$S_2 - S_1 = m_{st}\, (s_2 - s_1)_{st} + m_{R134a}\, (s_2 - s_1)_{R134a}$$

$$= -0.02598 + 1.5(0.27785 - 0.4252) = -\textbf{0.247 Btu/R}$$

$_1Q_2$ 0 F

6.211E

A 5-lbm aluminum radiator holds 2 lbm of liquid R-134a at 10 F. The setup is brought indoors and heated with 220 Btu. Find the final temperature and the change in entropy of all the mass.

Solution:

C.V. The aluminum radiator and the R-134a.

Energy Eq.3.5: $\quad m_2 u_2 - m_1 u_1 = {_1}Q_2 - 0$

Process: No change in volume so no work as used above.

The energy equation now becomes (summing over the mass)

$$m_{al} (u_2 - u_1)_{al} + m_{R134a} (u_2 - u_1)_{R134a} = {_1}Q_2$$

Use specific heat from Table F.2 and F.3

$$m_{al}C_{al} (T_2 - T_1) + m_{R134a}C_{R134a} \ln (T_2 - T_1) = {_1}Q_2$$

$$T_2 - T_1 = {_1}Q_2 / [m_{al}C_{al} + m_{R134a}C_{R134a}]$$

$$= 220 / [5 \times 0.215 + 2 \times 0.34] = 125 \text{ F}$$

$$T_2 = 10 + 125 = 135 \text{ F}$$

Entropy change for solid (F.2) and liquid (F.3) from Eq.6.11

$$S_2 - S_1 = m_{al} (s_2 - s_1)_{al} + m_{R134a} (s_2 - s_1)_{R134a}$$

$$= m_{al}C_{al} \ln (T_2/T_1) + m_{R134a}C_{R134a} \ln (T_2/T_1)$$

$$= (5 \times 0.215 + 2 \times 0.34) \ln \frac{(135 + 459.67)}{10 + 459.67}$$

$$= \textbf{0.414 Btu/R}$$

6.212E

Four pounds of liquid lead at 750 F are poured into a form. It then cools at constant pressure down to room temperature at 68 F as heat is transferred to the room. The melting point of lead is 620 F and the enthalpy change between the phases h_{if} is 10.6 Btu/lbm. The specific heats are in Table F.2 and F.3. Calculate the net entropy change for the lead.

Solution:

C.V. Lead, constant pressure process

$$m_{Pb}(u_2 - u_1)_{Pb} = {}_1Q_2 - P(V_2 - V_1)$$

We need to find changes in enthalpy (u + Pv) for each phase separately and then add the enthalpy change for the phase change.

$$C_{liq} = 0.038 \text{ Btu/lbm R}, \quad C_{sol} = 0.031 \text{ Btu/lbm R}$$

Consider the process in several steps:

Cooling liquid to the melting temperature

Solidification of the liquid to solid

Cooling of the solid to the final temperature

$${}_1Q_2 = m_{Pb}(h_2 - h_1) = m_{Pb}(h_2 - h_{620,sol} - h_{if} + h_{620,f} - h_{750})$$

$$= 4 \times [0.031 \times (68 - 620) - 10.6 + 0.038 \times (620 - 750)]$$

$$= -68.45 - 42.4 - 19.76 = -130.61 \text{ Btu}$$

$$S_2 - S_1 = m_{Pb}[C_{p\ sol}\ln(T_2/1079.7) - (h_{if}/1079.7) + C_{P\ liq}\ln(1079.7/T_1)]$$

$$= 4 \times \left[0.031 \ln \frac{527.7}{1079.7} - \frac{10.6}{1079.7} + 0.038 \ln \frac{1079.6}{1209.7}\right] = \textbf{-0.145 Btu/R}$$

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Entropy of Ideal Gases

6.213E

Air inside a rigid tank is heated from 550 to 600 R. Find the entropy increase $s_2 - s_1$? What if it is from 2300 to 2350 R?

Process: $V = C$ $\rightarrow {}_1W_2 = \emptyset$

Entropy change from Eq.6.17:

a) $s_2 - s_1 = C_{vo} \ln (\dfrac{T_2}{T_1}) = 0.171 \ln \left(\dfrac{600}{550}\right) =$ **0.015 Btu/lbm-R**

b) $s_2 - s_1 = C_{vo} \ln (\dfrac{T_2}{T_1}) = 0.171 \ln \left(\dfrac{2350}{2300}\right) =$ **0.00368 Btu/lbm-R**

From F.5:

case a) $C_v \approx \Delta u / \Delta T = 6.868/40 = 0.1717$ Btu/lbm-R, see F.4 (0.171)

case b) $C_v \approx \Delta u / \Delta T = 21.53/100 = 0.215$ Btu/lbm-R (25 % higher)
so result should have been 0.00463 Btu/lbm-R

6.214E

R-410A at 60 psia is brought from 60 F to 240 F in a constant pressure process. Evaluate the change in specific entropy using Table F.9 and using ideal gas with $C_p = 0.1935$ Btu/lbmR.

Table F.9.2 $s_1 = 0.2849$ Btu/lbm-R, $s_2 = 0.3496$ Btu/lbm-R

$s_2 - s_1 = 0.3496 - 0.2849 = $ **0.0647 Btu/lbm-R**

Eq. 6.16: $s_2 - s_1 \approx C_{po} \ln \dfrac{T_2}{T_1} = 0.1935 \ln \dfrac{240 + 460}{60 + 460} = $ **0.0575 Btu/lbm-R**

Two explanations for the difference are as the average temperature is higher than 77 F we could expect a higher value of the specific heat and secondly it is not an ideal gas (if you calculate $Z = Pv/RT = 0.94$).

6.215E

Oxygen gas in a piston/cylinder at 500 R and 1 atm with a volume of 1 ft^3 is compressed in a reversible adiabatic process to a final temperature of 1000 R. Find the final pressure and volume using constant heat capacity from Table F.4 and repeat with Table F.6.

Solution:

C.V. Air. Assume a reversible, adiabatic process.

Energy Eq.3.5: $u_2 - u_1 = 0 - {}_1w_2$;

Entropy Eq.6.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\ gen} = 0$

Process: Adiabatic ${}_1q_2 = 0$ Reversible ${}_1s_{2\ gen} = 0$

Properties: Table F.4: $k = 1.393$

With these two terms zero we have a zero for the entropy change. So this is a constant s (isentropic) expansion process. From Eq.6.23

$$P_2 = P_1 (T_2 / T_1)^{\frac{k}{k-1}} = 14.7\ (1000/500)^{3.5445} = \textbf{171.5 psia}$$

Using the ideal gas law to eliminate P from this equation leads to Eq.6.24

$$V_2 = V_1 (T_2 / T_1)^{\frac{1}{1-k}} = 1 \times \left(\frac{1000}{500}\right)^{\frac{1}{1-1.393}} = \textbf{0.171 ft}^3$$

Using the ideal gas tables F.6 we get

$$s_2 - s_1 = s_{T2}^o - s_{T1}^o - R \ln(P_2/P_1) \quad \text{or} \quad P_2 = P_1 \exp[(s_{T2}^o - s_{T1}^o)/R]$$

$$P_2 = 14.7\ \text{psia} \times \exp[(53.475 - 48.4185)/1.98589] = \textbf{187.55 psia}$$

$$V_2 = V_1 (T_2 / T_1)(P_1/P_2) = 1\ \text{ft}^3 \times \frac{1000}{500} \times \frac{14.7}{187.55} = \textbf{0.157 ft}^3$$

6.216E

A handheld pump for a bicycle has a volume of 2 in.^{3} when fully extended. You now press the plunger (piston) in while holding your thumb over the exit hole so an air pressure of 45 lbf/in.^{2} is obtained. The outside atmosphere is at P_o, T_o. Consider two cases: (1) it is done quickly (~1 s), and (2) it is done slowly (~1 h).

   a. State assumptions about the process for each case.

   b. Find the final volume and temperature for both cases.

Solution:

C.V. Air in pump. Assume that both cases result in a reversible process.

   State 1: P_0, T_0 State 2: 45 lbf/in.^{2}, ?

   One piece of information must resolve the ? for a state 2 property.

Case I) Quickly means no time for heat transfer

   $Q = 0$, so a reversible adiabatic compression.

   $u_2 - u_1 = -_1w_2$; $s_2 - s_1 = \int dq/T + {_1}s_{2\ gen} = \emptyset$

With constant s and constant heat capacity we use Eq.6.23

$$T_2 = T_1(P_2 / P_1)^{\frac{k-1}{k}} = 536.7 \left(\frac{45}{14.696}\right)^{\frac{0.4}{1.4}} = \textbf{738.9 R}$$

Use ideal gas law $PV = mRT$ at both states so ratio gives

   $=>$ $V_2 = P_1V_1T_2/T_1P_2 = \textbf{0.899 in}^3$

Case II)   Slowly, time for heat transfer so $T = $ constant $= T_0$.

The process is then a reversible isothermal compression.

   $T_2 = T_0 = \textbf{536.7 R}$ $=>$ $V_2 = V_1P_1/P_2 = \textbf{0.653 in}^3$

6.217E

A piston/cylinder contains air at 2500 R, 2200 lbf/in.^{2}, with $V_1 = 1$ in.^{3}, $A_{cyl} = 1$ in.^{2} as shown in Fig. P6.92. The piston is released and just before the piston exits the end of the cylinder the pressure inside is 30 lbf/in.^{2}. If the cylinder is insulated, what is its length? How much work is done by the air inside?

Solution:

C.V. Air, Cylinder is insulated so adiabatic, $Q = 0$.

Continuity Eq.: $m_2 = m_1 = m$,

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = - {}_1W_2$

Entropy Eq.6.14: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = 0 + {}_1S_{2\ gen}$

State 1: (T_1, P_1) State 2: $(P_2, ?)$

So one piece of information is needed for the ?,   assume reversible process.

$$_1S_{2\ gen} = 0   \Rightarrow   s_2 - s_1 = 0$$

State 1:   Table F.5: $u_1 = 474.33$ Btu/lbm, $s^\circ_{T1} = 2.03391$ Btu/lbm R

$$m = P_1V_1/RT_1 = \frac{2200 \times 1.0}{53.34 \times 2500 \times 12} = 1.375 \times 10^{-3} \text{ lbm}$$

State 2: P_2 and from Entropy eq.: $s_2 = s_1$ so from Eq.6.19

$$s^\circ_{T2} = s^\circ_{T1} + R \ln \frac{P_2}{P_1} = 2.03391 + \frac{53.34}{778} \ln(\frac{30}{2200}) = 1.73944 \text{ Btu/lbm R}$$

Now interpolate in Table F.5 to get T_2

$$T_2 = 840 + 40 (1.73944 - 1.73463)/(1.74653 - 1.73463) = 816.2 \text{ R}$$

$$u_2 = 137.099 + (144.114 - 137.099)\ 0.404 = 139.93 \text{ Btu/lbm}$$

$$V_2 = V_1 \frac{T_2\ P_1}{T_1 P_2} = \frac{1 \times 816.2 \times 2200}{2500 \times 30} = \textbf{23.94 in}^3$$

$$\Rightarrow\ L_2 = V_2 /A_{cyl} = 23.94/1 = \textbf{23.94 in}$$

$$_1W_2 = m(u_1 - u_2) = 1.375 \times 10^{-3}(474.33 - 139.93) = \textbf{0.46 Btu}$$

6.218E

A 25-ft^3 insulated, rigid tank contains air at 110 lbf/in.^{2}, 75 F. A valve on the tank is opened, and the pressure inside quickly drops to 15 lbf/in.^{2}, at which point the valve is closed. Assuming that the air remaining inside has undergone a reversible adiabatic expansion, calculate the mass withdrawn during the process.

C.V.: Air remaining inside tank, m_2.

Cont.Eq.: $m_2 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

Process: adiabatic ${}_1Q_2 = 0$ and reversible ${}_1S_{2\ gen} = 0$

Entropy eq. then gives $s_2 = s_1$ and ideal gas gives the relation in Eq.6.23

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 535 \text{ R } (15/110)^{0.286} = 302.6 \text{ R}$$

$$m_1 = P_1 V/RT_1 = 110 \text{ psia} \times 144\, \frac{\text{in}^2}{\text{ft}^2} \times 25 \text{ ft}^3/(53.34\, \frac{\text{ft-lbf}}{\text{lbm-R}} \times 535 \text{ R})$$

$$= 13.88 \text{ lbm}$$

$$m_2 = P_2 V/RT_2 = 15 \text{ psia} \times 144\, \frac{\text{in}^2}{\text{ft}^2} \times 25 \text{ ft}^3/(53.34\, \frac{\text{ft-lbf}}{\text{lbm-R}} \times 302.6 \text{ R})$$

$$= 3.35 \text{ lbm}$$

$m_e = m_1 - m_2 = $ **10.53 lbm**

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Polytropic Processes

6.219E

A cylinder/piston contains 0.1 lbm methane gas at 15 psia, 70 F. The gas is compressed reversibly to a pressure of 120 psia. Calculate the work required if the process is adiabatic.

Solution:

C.V. Methane gas of constant mass $m_2 = m_1 = m$ and reversible process.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.3 (37): $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = \int dQ/T$

Process: ${}_1Q_2 = 0$ $\Rightarrow$ $s_2 = s_1$

thus isentropic process s = const and ideal gas gives relation in Eq.6.23

$$T_2 = T_1 \, (P_2/P_1)^{\frac{k-1}{k}} = 529.7 \text{ R} \left(\frac{120}{15}\right)^{0.230} = 854.6 \text{ R}$$

$${}_1W_2 = -mC_{V0}(T_2 - T_1) = -0.1 \text{ lbm} \times 0.415 \text{ Btu/lbm-R} (854.6 - 529.7) \text{ R}$$

$$= \textbf{-13.48 Btu}$$

6.220E

Helium in a piston/cylinder at 70 F, 20 psia is brought to 720 R in a reversible polytropic process with exponent n = 1.25. You may assume helium is an ideal gas with constant specific heat. Find the final pressure and both the specific heat transfer and specific work.

Solution:

C.V. Helium, control mass. $C_v = 0.744$ Btu/lbm R,   R = 386 ft lbf/ lbm R

Process $Pv^n = C$ & $Pv = RT$ => $Tv^{n-1} = C$

From the process equation and $T_1 = 70 + 460 = 530$ R, $T_2 = 720$ R

$T_1 v^{n-1} = T_2 v^{n-1}$ => $v_2 / v_1 = (T_1 / T_2)^{1/n-1} = 0.2936$

$P_2 / P_1 = (v_1 / v_2)^n = 4.63$ => $P_2 = $ **69.4 lbf/in.^{2}**

The work is from Eq.6.29 per unit mass

$$_1w_2 = \int P \, dv = \int C \, v^{-n} \, dv = \frac{1}{1-n} (P_2 v_2 - P_1 v_1) = \frac{R}{1-n} (T_2 - T_1)$$

$$= \frac{386}{778 \times (-0.25)} \text{ Btu/lbm-R } (720 - 530) \text{ R} = \textbf{-377 Btu/lbm}$$

The heat transfer follows from the energy equation

$_1q_2 = u_2 - u_1 + {}_1w_2 = C_v (T_2 - T_1) + {}_1w_2$

$= 0.744(720 - 530) + (-377) = $ **-235.6 Btu/lbm**

6.221E

A cylinder/piston contains air at ambient conditions, 14.7 lbf/in.^{2} and 70 F with a volume of 10 ft^3. The air is compressed to 100 lbf/in.^{2} in a reversible polytropic process with exponent, $n = 1.2$, after which it is expanded back to 14.7 lbf/in.^{2} in a reversible adiabatic process.

      a. Show the two processes in *P–v* and *T–s* diagrams.

      b. Determine the final temperature and the net work.

a)

$$m = P_1V_1/RT_1$$
$$= \frac{14.7 \times 144 \times 10}{53.34 \times 529.7}$$
$$= 0.7492 \text{ lbm}$$

b)   The process equation is expressed in Eq.6.28

$$T_2 = T_1(P_2/P_1)^{\frac{n-1}{n}} = 529.7\left(\frac{100}{14.7}\right)^{0.167} = 729.6 \text{ R}$$

The work is from Eq.6.29

$$_1w_2 = \int_1^2 Pdv = \frac{P_2v_2 - P_1v_1}{1 - n} = \frac{R(T_2 - T_1)}{1 - n} = \frac{53.34(729.6 - 529.7)}{778(1 - 1.20)}$$
$$= -68.5 \text{ Btu/lbm}$$

Isentropic relation is from Eq.6.23

$$T_3 = T_2(P_3/P_2)^{\frac{k-1}{k}} = 729.7 \text{ R} \left(\frac{14.7}{100}\right)^{0.286} = \mathbf{421.6 \text{ R}}$$

With zero heat transfer the energy equation gives the work

$$_2w_3 = C_{V0}(T_2 - T_3) = 0.171(729.6 - 421.6) = +52.7 \text{ Btu/lbm}$$
$$w_{NET} = 0.7492(-68.5 + 52.7) = \mathbf{-11.8 \text{ Btu}}$$

Entropy Generation

6.222E

Consider a heat transfer of 100 Btu from 2400 R hot gases to a steel container at 1200 R which has a heat transfer of the 100 Btu out to some air at 600 R. Determine the entropy generation in each of the control volumes indicated in Fig. P6.115.

There is no change in energy or entropy in the indicated control volumes, so no storage effect. There is a transfer of energy in and out of each CV and an associated transfer of entropy.

Take CV1
Energy Eq.:

$0 = Q - Q$

Entropy Eq.:

$$0 = \frac{Q}{T_H} - \frac{Q}{T_M} + S_{gen\ CV1}$$

$$S_{gen\ CV1} = \frac{Q}{T_M} - \frac{Q}{T_H}$$

$$= \frac{100}{1200} - \frac{100}{2400} = \mathbf{0.0417\ Btu/R}$$

Take CV2
Energy Eq.:

$0 = Q - Q$

Entropy Eq.:

$$0 = \frac{Q}{T_M} - \frac{Q}{T_L} + S_{gen\ CV2}$$

$$S_{gen\ CV2} = \frac{Q}{T_L} - \frac{Q}{T_M}$$

$$= \frac{100}{600} - \frac{100}{1200} = \mathbf{0.083\ Btu/R}$$

6.223E

A computer chip dissipates 2 Btu of electric work over time and rejects that as heat transfer from its 125 F surface to 70 F air. How much entropy is generated in the chip? How much if any is generated outside the chip?

C.V.1 Chip with surface at 125 F, we assume chip state is constant.

Energy: $U_2 - U_1 = 0 = {_1}Q_2 - {_1}W_2 = W_{electrical\ in} - Q_{out\ 1}$

Entropy: $S_2 - S_1 = 0 = -\dfrac{Q_{out\ 1}}{T_{surf}} + {_1}S_{2\ gen1}$

$${_1}S_{2\ gen1} = \frac{Q_{out\ 1}}{T_{surf}} = \frac{W_{electrical\ in}}{T_{surf}} = \frac{2\ Btu}{(125+459.7)\ R} = \textbf{0.0034 Btu/R}$$

C.V.2 From chip surface at 125 F to air at 70 F, assume constant state.

Energy: $U_2 - U_1 = 0 = {_1}Q_2 - {_1}W_2 = Q_{out\ 1} - Q_{out\ 2}$

Entropy: $S_2 - S_1 = 0 = \dfrac{Q_{out1}}{T_{surf}} - \dfrac{Q_{out2}}{T_{air}} + {_1}S_{2\ gen2}$

$${_1}S_{2\ gen2} = \frac{Q_{out2}}{T_{air}} - \frac{Q_{out1}}{T_{surf}} = \frac{2\ Btu}{529.7\ R} - \frac{2\ Btu}{584.7\ R} = \textbf{0.000 36 Btu/R}$$

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6.224E

An insulated cylinder/piston contains R-134a at 150 lbf/in.^{2}, 120 F, with a volume of 3.5 ft^3. The R-134a expands, moving the piston until the pressure in the cylinder has dropped to 15 lbf/in.^{2}. It is claimed that the R-134a does 180 Btu of work against the piston during the process. Is that possible?

Solution:

C.V. R-134a in cylinder. Insulated so assume Q = 0.

State 1: Table F.10.2, $v_1 = 0.3332$ ft^3/lbm, $u_1 = 175.33$ Btu/lbm,

$s_1 = 0.41586$ Btu/lbm R, $m = V_1/v_1 = 3.5/0.3332 = 10.504$ lbm

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = \emptyset - 180 \Rightarrow$

$u_2 = u_1 - {}_1W_2/m = 158.194$ Btu/lbm

State 2: P_2 , u_2 $\Rightarrow$ Table F.10.2: $T_2 = -2$ F ; $s_2 = 0.422$ Btu/lbm R

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2,gen} = {}_1S_{2,gen}$

${}_1S_{2,gen} = m(s_2 - s_1) = 10.504 (0.422 - 0.41586) = \mathbf{0.0645}$ **Btu/R**

This is **possible since ${}_1S_{2\ gen} > \emptyset$**

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6.225E

Heat transfer from a 70 F kitchen to a block of 3 lbm ice at 15 F melts it to liquid at 50 F. Find the entropy generation.

Water changes state from nearly saturated solid to nearly saturated liquid. The pressure is 1 atm, but we approximate the state properties with saturated state at the same temperature.

CV Ice out to the 20^oC kitchen air

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1Q_2/T_0 + {}_1S_{2\ gen}$

State 1: Compressed (saturated) solid, F.7.4,

$u_1 = -151.75$ Btu/lbm, $s_1 = -0.3093$ Btu/lbm-R

State 2: Compressed (saturated) liquid F.7.1

$u_2 = 38.09$ Btu/lbm, $s_2 = 0.0746$ Btu/lbm-R

Heat transfer from the energy Eq.

$_1Q_2 = m(u_2 - u_1) = 3\ [38.09 - (-151.75)] = 569.52$ Btu

From the entropy Eq.

$_1S_{2\ gen} = m(s_2 - s_1) - {}_1Q_2/T_0$

$= 3\ [0.0746 - (-0.3093)] - 569.52/529.7 = $ **0.0765 Btu/R**

6.226E

A mass and atmosphere loaded piston/cylinder contains 4 lbm of water at 500 lbf/in.^{2}, 200 F. Heat is added from a reservoir at 1200 F to the water until it reaches 1200 F. Find the work, heat transfer, and total entropy production for the system and surroundings.

Solution:

C.V. Water out to surroundings at 1200 F. This is a control mass.

Energy Eq.3.5: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1Q_2/T_{res} + {}_1S_{2\ gen}$

Process: P = constant so ${}_1W_2 = P(V_2 - V_1) = mP(v_2 - v_1)$

State 1: Table F.7.3, $v_1 = 0.01661$ ft^3/lbm

$h_1 = 169.18$ Btu/lbm, $s_1 = 0.2934$ Btu/lbm R

State 2: Table F.7.2, $v_2 = 1.9518$ ft^3/lbm, $h_2 = 1629.8$ Btu/lbm,

$s_2 = 1.8071$ Btu/lbm R

Work is found from the process (area in P-V diagram)

$${}_1W_2 = mP(v_2 - v_1) = 4 \times 500(1.9518 - 0.01661)\frac{144}{778} = \textbf{716.37 Btu}$$

The heat transfer from the energy equation is

$${}_1Q_2 = U_2 - U_1 + {}_1W_2 = m(u_2 - u_1) + mP(v_2 - v_1) = m(h_2 - h_1)$$

$${}_1Q_2 = 4(1629.8 - 169.18) = \textbf{5842.48 Btu}$$

Entropy generation from entropy equation (Eq.6.37)

$${}_1S_{2\ gen} = m(s_2 - s_1) - \frac{{}_1Q_2}{T_{res}} = 4(1.8071 - 0.2934) - \frac{5842.48}{1659.67} = \textbf{2.535 Btu/R}$$

6.227E

A 1 gallon jug of milk at 75 F is placed in your refrigerator where it is cooled down to the refrigerators inside temperature of 40 F. Assume the milk has the properties of liquid water and find the entropy generated in the cooling process.

Solution:

C.V. Jug of milk. Control mass at constant pressure.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen}$

State 1: Table F.7.1: $v_1 \cong v_f = 0.01606$ ft^3/lbm, $h_1 = h_f = 43.085$ Btu/lbm;

$\qquad\qquad\qquad s_f = 0.08395$ Btu/lbm R

State 2: Table F.7.1: $h_2 = h_f = 8.01$ Btu/lbm, $s_2 = s_f = 0.0162$ Btu/lbm R

Process: P = constant = 14.7 psia $=>$ ${}_1W_2 = mP(v_2 - v_1)$

$\qquad\qquad V_1 = 1$ Gal $= 231$ in^3 $=>$ $m = 231 / 0.01606 \times 12^3 = 8.324$ lbm

Substitute the work into the energy equation and solve for the heat transfer

$\qquad {}_1Q_2 = m(h_2 - h_1) = 8.324$ lbm $(8.01 - 43.085)$ Btu/lbm $= -292$ Btu

The entropy equation gives the generation as

$\qquad {}_1S_{2\,gen} = m(s_2 - s_1) - {}_1Q_2/T_{refrig}$

$\qquad\qquad = 8.324\,(0.0162 - 0.08395) - (-292 / 500)$

$\qquad\qquad = -0.564 + 0.584 = \mathbf{0.02\ Btu/R}$

40 F

AIR

MILK
cb

Borgnakke and Sonntag

6.228E

A cylinder/piston contains water at 30 lbf/in.^{2}, 400 F with a volume of 1 ft^3. The piston is moved slowly, compressing the water to a pressure of 120 lbf/in.^{2}. The loading on the piston is such that the product *PV* is a constant. Assuming that the room temperature is 70 F, show that this process does not violate the second law.

Solution:

C.V.: Water + cylinder out to room at 70 F

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = {}_1Q_2 / T_{room} + {}_1S_{2\ gen}$

Process: $PV = constant = Pmv \Rightarrow v_2 = P_1v_1/P_2$

$$_1w_2 = \int Pdv = P_1v_1 \ln(v_2/v_1)$$

State 1: Table B.1.3, $v_1 = 16.891$ ft^3/lbm, $u_1 = 1144$ Btu/lbm,

$s_1 = 1.7936$ Btu/lbm R

State 2: P_2 , $v_2 = P_1v_1/P_2 = 30 \times 16.891/120 = 4.223$ ft^3/lbm

Table F.7.3: $T_2 = 425.4$ F, $u_2 = 1144.4$ Btu/lbm, $s_2 = 1.6445$ Btu/lbmR

$$_1w_2 = 30 \times 16.891 \times \frac{144}{778} \ln \left(\frac{4.223}{16.891}\right) = -130.0 \text{ Btu}$$

$$_1q_2 = u_2 - u_1 + {}_1w_2 = 1144.4 - 1144 - 130 = -129.6 \text{ Btu/lbm}$$

$$_1s_{2,gen} = s_2 - s_1 - \frac{_1q_2}{T_{room}} = 1.6445 - 1.7936 + \frac{129.6}{529.67}$$

$= 0.0956$ Btu/lbm R $> \emptyset$ **satisfies entropy equation.**

6.229E

One pound mass of ammonia (NH_3) is contained in a linear spring-loaded piston/cylinder as saturated liquid at 0 F. Heat is added from a reservoir at 225 F until a final condition of 200 lbf/in.^{2}, 160 F is reached. Find the work, heat transfer, and entropy generation, assuming the process is internally reversible.

Solution:

C.V. = NH_3 out to the reservoir.

Continuity Eq.: $m_2 = m_1 = m$

Energy Eq.3.5: $E_2 - E_1 = m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Entropy Eq.6.37: $S_2 - S_1 = \int dQ/T + {_1}S_{2,gen} = {_1}Q_2/T_{res} + {_1}S_{2,gen}$

Process: $P = A + BV$ linear in V   =>

$${_1}W_2 = \int P \, dV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)m(v_2 - v_1)$$

State 1: Table F.8.1

$P_1 = 30.4$ psia,

$v_1 = 0.0242$ ft^3/lbm

$u_1 = 42.46$ Btu/lbm,

$s_1 = 0.0967$ Btu/lbm R

State 2: Table F.8.2 sup. vap.

$v_2 = 1.7807$ ft^3/lbm, $u_2 = 677.36$ Btu/lbm,

$s_2 = 1.2514$ Btu/lbm R

$${_1}W_2 = \frac{1}{2}(30.4 + 200)1(1.7807 - 0.0242)\times 144/778 = \textbf{37.45 Btu}$$

$${_1}Q_2 = m(u_2 - u_1) + {_1}W_2 = 1(677.36 - 42.46) + 37.45 = \textbf{672.35 Btu}$$

$$S_{gen} = m(s_2 - s_1) - {_1}Q_2/T_{res} = 1(1.2514 - 0.0967) - \frac{672.35}{684.7} = \textbf{0.173 Btu/R}$$

6.230E

A hollow steel sphere with a 2-ft inside diameter and a 0.1-in. thick wall contains water at 300 lbf/in.^{2}, 500 F. The system (steel plus water) cools to the ambient temperature, 90 F. Calculate the net entropy change of the system and surroundings for this process.

C.V.: Steel + water out to ambient T_0. This is a control mass.

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2 = m_{H_2O}(u_2 - u_1) + m_{steel}(u_2 - u_1)$

Entropy Eq.: $S_2 - S_1 = \int dQ/T + {}_1S_{2\,gen} = {}_1Q_2/T_0 + {}_1S_{2\,gen}$

Process: V = constant $\Rightarrow$ ${}_1W_2 = 0$

$$V_{steel} = \frac{\pi}{6}\left[2.0083^3 - 2^3\right] = 0.0526 \text{ ft}^3$$

$$m_{steel} = (\rho V)_{steel} = 490 \times 0.0526 = 25.763 \text{ lbm}$$

$$V_{H_2O} = \pi/6 \times 2^3 = 4.189 \text{ ft}^3, \qquad m_{H_2O} = V/v = 2.372 \text{ lbm}$$

$$v_2 = v_1 = 1.7662 = 0.016099 + x_2 \times 467.7 \quad \Rightarrow \quad x_2 = 3.74 \times 10^{-3}$$

$$u_2 = 61.745 \text{ Btu/lbm, } s_2 = 0.1187 \text{ Btu/lbm R}$$

$${}_1Q_2 = \Delta U_{steel} + \Delta U_{H_2O} = (mC)_{steel}(T_2 - T_1) + m_{H_2O}(u_2 - u_1)$$

$$= 25.763 \times 0.107(90\text{-}500) + 2.372(61.74 - 1159.5)$$

$$= -1130 - 2603.9 = -3734 \text{ Btu}$$

$$S_2 - S_1 = m_{steel}(s_2 - s_1) + m_{H_2O}(s_2 - s_1) = 25.763 \times 0.107 \times \ln(550/960)$$

$$+ 2.372(0.1187 - 1.5701) = -4.979 \text{ Btu/R}$$

$$\Delta S_{SUR} = -Q_{12}/T_0 = 3734/549.67 = 6.793 \text{ Btu/R}$$

$${}_1S_{2\,gen} = S_2 - S_1 - {}_1Q_2/T_0 = \Delta S_{SYS} + \Delta S_{SUR}$$

$$= -4.979 + 6.793 = \textbf{1.814 Btu/R}$$

6.231E

One lbm of air at 540 R is mixed with two lbm air at 720 R in a process at a constant 15 psia and Q = 0. Find the final T and the entropy generation in the process.

C.V. All the air.

Energy Eq.: $U_2 - U_1 = 0 - W$
Entropy Eq.: $S_2 - S_1 = 0 + {}_1S_{2\ gen}$
Process Eq.: $P = C; \quad W = P(V_2 - V_1)$

Substitute W into energy Eq.

$$U_2 - U_1 + W = U_2 - U_1 + P(V_2 - V_1) = H_2 - H_1 = 0$$

Due to the low T let us use constant specific heat

$$H_2 - H_1 = m_A(h_2 - h_1)_A + m_B(h_2 - h_1)_B$$
$$= m_A C_p(T_2 - T_{A1}) + m_B C_p(T_2 - T_{B1}) = 0$$

$$T_2 = \frac{m_A T_{A1} + m_B T_{B1}}{m_A + m_B} = \frac{1}{3} T_{A1} + \frac{2}{3} T_{B1} = \mathbf{660\ R}$$

Entropy change is from Eq. 6.16 with no change in P and Table F.4 for C_p

$$_1S_{2\ gen} = S_2 - S_1 = m_A C_p \ln \frac{T_2}{T_{A1}} + m_B C_p \ln \frac{T_2}{T_{B1}}$$
$$= 1 \times 0.24 \ln \frac{660}{540} + 2 \times 0.24 \ln \frac{660}{720}$$
$$= 0.0482 - 0.0418 = \mathbf{0.0064\ Btu/R}$$

Remark: If you check the volume does not change and there is no work.

6.232E

One lbm of air at 15 psia is mixed with two lbm air at 30 psia, both at 540 R, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process.

C.V. All the air.

Energy Eq.: $U_2 - U_1 = 0 - 0$

Entropy Eq.: $S_2 - S_1 = 0 + {}_1S_{2\,gen}$

Process Eqs.: $V = C; \quad W = 0, \ Q = 0$

States A1, B1: $u_{A1} = u_{B1}$

$V_A = m_A RT_1/P_{A1}; \quad V_B = m_B RT_1/P_{B1}$

$U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 0 \ \Rightarrow \ u_2 = (u_{A1} + 2u_{B1})/3 = u_{A1}$

State 2: $\mathbf{T_2 = T_1 = 540\ R}$ (from u_2); $\quad m_2 = m_A + m_B = 3$ kg;

$\qquad V_2 = m_2 RT_1/P_2 = V_A + V_B = m_A RT_1/P_{A1} + m_B RT_1/P_{B1}$

Divide with $m_A RT_1$ and get

$$3/P_2 = 1/P_{A1} + 2/P_{B1} = \frac{1}{15} + \frac{2}{30} = 0.133\ \text{psia}^{-1} \ \Rightarrow \ P_2 = \mathbf{22.5\ psia}$$

Entropy change from Eq. 6.16 with the same T, so only P changes

$${}_1S_{2\,gen} = S_2 - S_1 = \ -m_A R \ln\frac{P_2}{P_{A1}} - m_B R \ln\frac{P_2}{P_{B1}}$$

$$= -\,1 \times 53.34 \,[\, \ln\frac{22.5}{15} + 2 \ln\frac{22.5}{30} \,]$$

$$= -53.34 \,(0.4055 - 0.5754) = \mathbf{9.06\ lbf\text{-}ft/R = 0.0116\ Btu/R}$$

6.233E

A rigid container with volume 7 ft^3 is divided into two equal volumes by a partition. Both sides contain nitrogen, one side is at 300 lbf/in.^{2}, 400 F, and the other at 30 lbf/in.^{2}, 200 F. The partition ruptures, and the nitrogen comes to a uniform state at 160 F. Assume the temperature of the surroundings is 68 F, determine the work done and the net entropy change for the process.

Solution:

C.V.: A + B Control mass no change in volume   => $_1W_2 = 0$

$m_{A1} = P_{A1}V_{A1}/RT_{A1} = 300 \times 144 \times 3.5 /(55.15 \times 859.7) = 3.189$ lbm

$m_{B1} = P_{B1}V_{B1}/RT_{B1} = 30 \times 144 \times 3.5 / (55.15 \times 659.7) = 0.416$ lbm

$P_2 = m_{TOT}RT_2/V_{TOT} = 3.605 \times 55.15 \times 619.7/(144 \times 7) = 122.2$ lbf/in^2

From Eq.6.16

$S_2 - S_1 = m_{A1} (s_2 - s_1)_{A1} + m_{B1} (s_2 - s_1)_{B1}$

$$= 3.189 \left[0.249 \ln \frac{619.7}{859.7} - \frac{55.15}{778} \ln \frac{122.2}{300}\right]$$

$$+ 0.416 \left[0.249 \ln \frac{619.7}{659.7} - \frac{55.15}{778} \ln \frac{122.2}{30}\right]$$

$$= -0.0569 - 0.0479 = -0.1048 \text{ Btu/R}$$

$_1Q_2 = m_{A1}(u_2 - u_1) + m_{B1}(u_2 - u_1)$

$$= 3.189 \times 0.178(160 - 400) + 0.416 \times 0.178(160 - 200) = -139.2 \text{ Btu}$$

From Eq.6.37

$_1S_{2 \text{ gen}} = S_2 - S_1 - _1Q_2/T_0 = -0.1048$ Btu/R + 139.2 Btu/ 527.7 R

$$= +\textbf{0.159 Btu/R}$$

6.234E

A constant pressure piston/cyl. is 2 lbm of steel and it contains 1 lbm of air 540 R, 60 psia. The system is now heated to 2600 R by a 2800 R source and the steel has the same temperature as the air. Find the entropy generation using constant specific heats.

C.V. Air and Steel.

Energy Eq.: $U_2 - U_1 = m_{air}(u_2 - u_1) + m_{st}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m_{air}(s_2 - s_1) + m_{st}(s_2 - s_1) = {}_1Q_2/T_{source} + {}_1S_{2\ gen}$

Process: $P = C \Rightarrow {}_1W_2 = \int\limits_1^2 PdV = P(V_2 - V_1) = P\, m_{air}(v_2 - v_1)$

${}_1Q_2 = m_{air}(u_2 - u_1)_{air} + m_{st}(u_2 - u_1)_{st} + {}_1W_2 = m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st}$

Use F.2: $(u_2 - u_1)_{st} = C(T_2 - T_1) = 0.11\ Btu/lbm\text{-}R \times (2600 - 540)\ R$

$= 226.6\ Btu/lbm$

Use F.4: $(h_2 - h_1)_{air} = C_p(T_2 - T_1) = 0.24\ Btu/lbm\text{-}R \times (2600 - 540)\ R$

$= 494.4\ Btu/lbm$

${}_1Q_2 = m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st}$

$= 1\ lbm \times 494.4\ Btu/lbm + 2\ lbm \times 226.6\ Btu/lbm = 947.6\ Btu$

$S_2 - S_1 = m_{air}(s_2 - s_1) + m_{st}(s_2 - s_1)$

$= 1\ lbm \times 0.24\ Btu/lbm\text{-}R \times \ln\dfrac{2600}{540}$

$+ 2\ lbm \times 0.11\ Btu/lbm\text{-}R \times \ln\dfrac{2600}{540} = 0.723\ Btu/R$

${}_1S_{2\ gen} = S_2 - S_1 - {}_1Q_2/T_{source} = 0.723 - 947.6/2800 = \mathbf{0.385\ Btu/R}$

6.235E

Do Problem 6.234E using Table F.5.

C.V. Air and Steel.

Energy Eq.: $U_2 - U_1 = m_{air}(u_2 - u_1) + m_{st}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m_{air}(s_2 - s_1) + m_{st}(s_2 - s_1) = {}_1Q_2/T_{source} + {}_1S_2{}_{gen}$

Process: $P = C \Rightarrow {}_1W_2 = \int_1^2 PdV = P(V_2 - V_1) = P\, m_{air}(v_2 - v_1)$

${}_1Q_2 = m_{air}(u_2 - u_1)_{air} + m_{st}(u_2 - u_1)_{st} + {}_1W_2 = m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st}$

Use air tables F.5: $(h_2 - h_1)_{air} = 674.421 - 129.18 = 545.24$ Btu/lbm

$(s_2 - s_1)_{air} = 2.04517 - 1.63979 + 0 = 0.40538$ Btu/lbmR

No pressure correction as $P_2 = P_1$

Use F.2: $(u_2 - u_1)_{st} = C(T_2 - T_1) = 0.11$ Btu/lbm-R $\times (2600 - 540)$ R

$= 226.6$ Btu/lbm

${}_1Q_2 = m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st}$

$= 1$ lbm $\times 545.24$ Btu/lbm $+ 2$ lbm $\times 226.6$ Btu/lbm $= \mathbf{998.44\ Btu}$

$S_2 - S_1 = m_{air}(s_2 - s_1) + m_{st}(s_2 - s_1)$

$= 0.40538 + 2$ lbm $\times 0.11$ kJ/kgK $\times \ln\dfrac{2600}{540} = 0.75115$ Btu/lbm

${}_1S_2{}_{gen} = S_2 - S_1 - {}_1Q_2/T_{source} = 0.75115 - 998.44/2800 = \mathbf{0.395\ Btu/R}$

6.236E

Nitrogen at 90 lbf/in.^{2}, 260 F is in a 20 ft^3 insulated tank connected to a pipe with a valve to a second insulated initially empty tank of volume 10 ft^3. The valve is opened and the nitrogen fills both tanks. Find the final pressure and temperature and the entropy generation this process causes. Why is the process irreversible?

CV Both tanks + pipe + valve Insulated : Q = 0   Rigid: W = 0

Energy Eq.3.5: $m(u_2 - u_1) = 0 - 0$ => $u_2 = u_1 = u_{a1}$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = {}_1S_{2\,gen}$ (dQ = 0)

State 1: P_1 , T_1 , V_a => Ideal gas

$m = PV/RT = (90 \times 20 \times 144)/ (55.15 \times 720) = 6.528$ lbm

State 2: $V_2 = V_a + V_b$; uniform final state $v_2 = V_2 / m$; $u_2 = u_{a1}$

Ideal gas u (T) => $u_2 = u_{a1}$ => $T_2 = T_{a1} = $ **720 R**

$P_2 = mR\, T_2 / V_2 = (V_1 / V_2) P_1 = (2/3) \times 90 = $ **60 lbf/in.^{2}**

From entropy equation and Eq.6.19 for entropy change

$S_{gen} = m(s_2 - s_1) = m\, (s_{T2}{}^o - s_{T1}{}^o - R \ln (P_2 / P_1)$

$= m\, (0 - R \ln (P_2 / P_1) = -6.528 \times 55.15 \times (1/778) \ln 2/3$

$= $ **0.187 Btu/R**

Irreversible due to unrestrained expansion in valve P $\downarrow$ but no work out.

If not a uniform final state then flow until $P_{2b} = P_{2a}$ and valve is closed.

Assume no Q between A and B

$$m_{a2} + m_{b2} = m_{a1} \; ; \qquad m_{a2}\, v_{a2} + m_{b2}\, v_{b2} = m_{a1} v_{a1}$$

$$m_{a2}\, s_{a2} + m_{b2}\, s_{b2} - m_{a1} s_{a1} = 0 + {}_1S_{2\,gen}$$

Now we must assume m_{a2} went through rev adiabatic expansion

1) $V_2 = m_{a2}\, v_{a2} + m_{b2}\, v_{b2}$;   2) $P_{b2} = P_{a2}$;

3) $s_{a2} = s_{a1}$;   4) Energy eqs.

4 Eqs   4 unknowns : P_2 , T_{a2} , T_{b2} , $x = m_{a2} / m_{a1}$

$$V_2 / m_{a1} = x\, v_{a2} + (1 - x)\, v_{b2} = x \times (R\, T_{a2} / P_2) + (1 - x)\, (R\, T_{b2} / P_2)$$

Energy Eq.: $m_{a2}\, (u_{a2} - u_{a1}) + m_{b2}\, (u_{b2} - u_{a1}) = 0$

$$x\, C_v\, (T_{a2} - T_{a1}) + (1 - x)\, C_v\, (T_{b2} - T_{a1}) = 0$$

$$x\, T_{a2} + (1 - x) T_{b2} = T_{a1}$$

Vol constraint: $P_2\, V_2 / m_{a1}\, R = x\, T_{a2} + (1 - x)\, T_{b2} = T_{a1}$

$$P_2 = m_{a1}\, R\, T_{a1} / V_2 = m_{a1}\, R\, T_{a1} / (1.5 V_{a1}) = 2/3\, P_{a1} = \textbf{60 lbf/in.}^2$$

$s_{a2} = s_{a1} \Rightarrow T_{a2} = T_{a1}\, (P_2 / P_{a1})^{k-1 / k} = 720 \times (2/3)^{0.2857} = \textbf{641.24 R}$

Now we have final state in A

$v_{a2} = R\, T_{a2} / P_2 = 4.0931$ ft/lbm; $m_{a2} = V_a / v_{a2} = 4.886$ lbm

$x = m_{a2} / m_{a1} = 0.7485$ $m_{b2} = m_{a1} - m_{a2} = 1.642$ lbm

Substitute into energy equation

$$T_{b2} = (T_{a1} - x\, T_{a2}) / (1 - x) = 954.4\ R$$

${}_1S_{2\,gen} = m_{b2}\, (s_{b2} - s_{a1}) = m_{b2}\, [\, C_p \ln (T_{b2} / T_{a1}) - R \ln (P_2 / P_{a1})]$

$$= 1.642\, [\, 0.249 \ln (954.4/720) - (55.15/778) \ln (2/3)\,]$$

$$= \textbf{0.1624 Btu/R}$$

Comment: This shows less entropy generation as the mixing part is out compared with the previous solution.

6.237E

A cylinder/piston contains carbon dioxide at 150 lbf/in.^{2}, 600 F with a volume of 7 ft^3. The total external force acting on the piston is proportional to V^3. This system is allowed to cool to room temperature, 70 F. What is the total entropy generation for the process?

C.V. Carbon dioxide gas of constant mass $m_2 = m_1 = m$ out to ambient.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = {}_1Q_2/T_{amb} + {}_1S_{2\,gen}$

Process: $P = CV^3$ or PV^{-3} = constant, which is polytropic with n = -3

State 1: $P_1 = 150$ lbf/in^2, $T_1 = 600$ F = 1060 R, $V_1 = 7$ ft^3 Ideal gas

$$m = \frac{P_1 V_1}{RT_1} = \frac{150 \times 144 \times 7}{35.10 \times 1060} = 4.064 \text{ lbm}$$

Process: $P = CV^3$ or PV^{-3} = const. polytropic with n = -3.

$$P_2 = P_1(T_2/T_1)^{\frac{n}{n-1}} = 150 \left(\frac{530}{1060}\right)^{0.75} = 89.2 \text{ lbf/in}^2$$

$$\& \; V_2 = V_1(T_1/T_2)^{\frac{1}{n-1}} = V_1 \times \frac{P_1}{P_2} \times \frac{T_2}{T_1} = 7 \times \frac{150}{89.2} \times \frac{530}{1060} = 5.886$$

$${}_1W_2 = \int PdV = \frac{P_2V_2 - P_1V_1}{1 - n} = \frac{(89.2 \times 5.886 - 150 \times 7)}{1 + 3} \times \frac{144}{778} = -24.3 \text{ Btu}$$

$${}_1Q_2 = 4.064 \times 0.158 \times (530 - 1060) - 24.3 = -346.6 \text{ Btu}$$

$$m(s_2 - s_1) = 4.064 \times \left[0.203 \times \ln\left(\frac{530}{1060}\right) - \frac{35.10}{778}\ln\left(\frac{89.2}{150}\right)\right] = -0.4765 \text{ Btu/R}$$

$$\Delta S_{SURR} = -{}_1Q_2/T_{amb} = 364.6 / 530 = +0.6879 \text{ Btu/R}$$

From Eq.6.37 or 6.39

$${}_1S_{2\,gen} = m(s_2 - s_1) - {}_1Q_2/T_{amb} = \Delta S_{NET} = \Delta S_{CO2} + \Delta S_{SURR}$$

$$= -0.4765 \text{ Btu/R} + 0.6879 \text{ Btu/R} = \mathbf{+0.2114 \text{ Btu/R}}$$

Notice:
n = -3, k = 1.3

n < k

6.238E

A cylinder/piston contains 4 ft^3 of air at 16 lbf/in.^{2}, 77 F. The air is compressed in a reversible polytropic process to a final state of 120 lbf/in.^{2}, 400 F. Assume the heat transfer is with the ambient at 77 F and determine the polytropic exponent n and the final volume of the air. Find the work done by the air, the heat transfer and the total entropy generation for the process.

Solution:

C.V. Air of constant mass $m_2 = m_1 = m$ out to ambient.

Energy Eq.3.5: $m(u_2 - u_1) = {_1}Q_2 - {_1}W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {_1}S_{2\ gen} = {_1}Q_2/T_0 + {_1}S_{2\ gen}$

Process: $Pv_1{}^n = P_2 v_2{}^n$ Eq.6.27

State 1: (T_1, P_1) State 2: (T_2, P_2)

Thus the unknown is the exponent   n.

$\quad m = (P_1 V_1)/(RT_1) = (16 \times 4 \times 144)/(53.34 \times 537) = 0.322$ lbm

The relation from the process and ideal gas is in Eq.6.28

$$T_2/T_1 = (P_2/P_1)^{\frac{n-1}{n}} \quad \Rightarrow \quad \frac{n-1}{n} = \ln(T_2/T_1)/\ln(P_2/P_1) = 0.2337$$

n = 1.305, $V_2 = V_1(P_1/P_2)^{1/n} = 4 \times (16/20)1/1.305 =$ **0.854 ft^3**

$$_1W_2 = \int PdV = \frac{P_2 V_2 - P_1 V_1}{1 - n}$$

$\quad = [(120 \times 0.854 - 16 \times 4)(144/778)]/(1 - 1.305) =$ **-23.35 Btu / lbm**

$\quad {_1}Q_2 = m(u_2 - u_1) + {_1}W_2 = mC_v(T_2 - T_1) + {_1}W_2$

$\quad\quad = 0.322 \times 0.171 \times (400 - 77) - 23.35 =$ **-5.56 Btu / lbm**

$\quad s_2 - s_1 = C_p \ln(T_2/T_1) - R \ln(P_2/P_1)$

$\quad\quad = 0.24 \ln(860/537) - (53.34/778) \ln(120/16) = -0.0251$ Btu/lbm R

$\quad {_1}S_{2\ gen} = m(s_2 - s_1) - {_1}Q_2/T_0$

$\quad\quad = 0.322 \times (-0.0251) + (5.56/537) =$ **0.00226 Btu/R**

Rates or Fluxes of Entropy

6.239E

A room at **7**2 F is heated electrically with 1500 W to keep steady temperature. The outside ambient is at 40 F. Find the flux of S ($= \dot{Q}/T$) into the room air, into the ambient and the rate of entropy generation.

CV. The room and walls out to the ambient T, we assume steady state

Energy Eq.: $0 = \dot{W}_{el\ in} - \dot{Q}_{out}$ => $\dot{Q}_{out} = \dot{W}_{el\ in} = 1500$ W

Entropy Eq.: $0 = - \dot{Q}_{out}/T + \dot{S}_{gen\ tot}$

   Flux of S into room air at 22°C: $\dot{Q}/T = 1500 / 531.7 = 2.82$ W/R

   Flux of S into ambient air at 5°C: $\dot{Q}/T = 1500 / 499.7 = 3.0$ W/R

   Entropy generation: $\dot{S}_{gen\ tot} = \dot{Q}_{out}/T = 1500 / 499.7 =$ **3.0 W/R**

Comment: The flux of S into the outside air is what leaves the control volume and since the control volume did not receive any S it was all made in the process. Notice most of the generation is done in the heater, the room heat loss process generates very little S (3.0 – 2.82 = 0.18 W/R)

6.240E

A heat pump with COP = 4 uses 1 kW of power input to heat a 78 F room, drawing energy from the outside at 60 F. Assume the high/low temperatures in the HP are 120 F/32 F. Find the total rates of entropy in and out of the heat pump, the rate from the outside at 60 F and to the room at 78 F.

Solution:

C.V.TOT.

Energy Eq.: $\dot{Q}_L + \dot{W} = \dot{Q}_H$

Entropy Eq.: $0 = \dfrac{\dot{Q}_L}{T_L} - \dfrac{\dot{Q}_H}{T_H} + \dot{S}_{gen\ CV\ tot}$

From definition of COP: $\dot{Q}_H = COP\ \dot{W} = 4 \times 1\ kW = 4\ kW$

From energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W} = (4-1)\ kW = 3\ kW$

Flux into heat pump at 32 F: $\dfrac{\dot{Q}_L}{T_{L\text{-}HP}} = \dfrac{3}{491.7}\ \dfrac{kW}{R} = \textbf{0.0061 kW/R}$

Flux out of heat pump at 120 F: $\dfrac{\dot{Q}_H}{T_{H\text{-}HP}} = \dfrac{4}{579.7}\ \dfrac{kW}{R} = \textbf{0.0069 kW/R}$

Flux out into room at $T_H = 78$ F: $\dfrac{\dot{Q}_H}{T_H} = \dfrac{4}{537.7}\ \dfrac{kW}{R} = \textbf{0.00744 kW/R}$

Flux from outside at 60 F: $\dfrac{\dot{Q}_L}{T_L} = \dfrac{3}{519.7}\ \dfrac{kW}{R} = \textbf{0.00577 kW/R}$

Comment: Following the flow of energy notice how the flux from the outside at 60 F grows a little when it arrives at 32 F this is due to entropy generation in the low T heat exchanger. The flux out of the heat pump at 120 F is larger than the flux in which is due to entropy generation in the heat pump cycle (COP is smaller than Carnot COP) and finally this flux increases due to entropy generated in the high T heat exchanger as the energy arrives at room T.

6.241E

A window receives 800 Btu/h of heat transfer at the inside surface of 70 F and transmits the 800 Btu/h from its outside surface at 36 F continuing to ambient air at 23 F. Find the flux of entropy at all three surfaces and the window's rate of entropy generation.

Flux of entropy: $\dot{S} = \dfrac{\dot{Q}}{T}$

$\dot{S}_{inside} = \dfrac{800}{529.7}\dfrac{Btu}{h\text{-}R} = 1.51 \text{ Btu/h-R}$

$\dot{S}_{win} = \dfrac{800}{495.7}\dfrac{Btu}{h\text{-}R} = 1.614 \text{ Btu/h-R}$

$\dot{S}_{amb} = \dfrac{800}{482.7}\dfrac{Btu}{h\text{-}R} = 1.657 \text{ Btu/h-R}$

Window

Inside | Outside

70 F | 36 F | 23 F

Window only: $\dot{S}_{gen\ win} = \dot{S}_{win} - \dot{S}_{inside} = 1.614 - 1.51 = \mathbf{0.104\ Btu/h\text{-}R}$

If you want to include the generation in the outside air boundary layer where T changes from 36 F to the ambient 23 F then it is

$$\dot{S}_{gen\ tot} = \dot{S}_{amb} - \dot{S}_{inside} = 1.657 - 1.51 = 0.147 \text{ Btu/h-R}$$

6.242E

A farmer runs a heat pump using 2.5 hp of power input. It keeps a chicken hatchery at a constant 86 F while the room loses 20 Btu/s to the colder outside ambient at 50 F. What is the rate of entropy generated in the heat pump? What is the rate of entropy generated in the heat loss process?

Solution:

C.V. Hatchery, steady state.

Power: $\dot{W} = 2.5$ hp $= 2.5 \ 2544.4 / 3600 = 1.767$ Btu/s

To have steady state at 86 F for the hatchery

Energy Eq.: $0 = \dot{Q}_H - \dot{Q}_{Loss}$ $\Rightarrow$ $\dot{Q}_H = \dot{Q}_{Loss} = 20$ Btu/s

C.V. Heat pump, steady state

Energy eq.: $0 = \dot{Q}_L + \dot{W} - \dot{Q}_H$ $\Rightarrow$ $\dot{Q}_L = \dot{Q}_H - \dot{W} = 18.233$ Btu/s

Entropy Eq.: $0 = \dfrac{\dot{Q}_L}{T_L} - \dfrac{\dot{Q}_H}{T_H} + \dot{S}_{gen\ HP}$

$$\dot{S}_{gen\ HP} = \dfrac{\dot{Q}_H}{T_H} - \dfrac{\dot{Q}_L}{T_L} = \dfrac{20}{545.7} - \dfrac{18.233}{509.7} = \mathbf{0.000\ 878\ \dfrac{Btu}{s\ R}}$$

C.V. From hatchery at 86 F to the ambient 50 F. This is typically the walls and the outer thin boundary layer of air. Through this goes $\dot{Q}_{Loss}$.

Entropy Eq.: $0 = \dfrac{\dot{Q}_{Loss}}{T_H} - \dfrac{\dot{Q}_{Loss}}{T_{amb}} + \dot{S}_{gen\ walls}$

$$\dot{S}_{gen\ walls} = \dfrac{\dot{Q}_{Loss}}{T_{amb}} - \dfrac{\dot{Q}_{Loss}}{T_H} = \dfrac{20}{509.7} - \dfrac{20}{545.7} = \mathbf{0.00259\ \dfrac{Btu}{s\ R}}$$

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Review Problems

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6.243E

Water in a piston/cylinder is at 150 lbf/in.^{2}, 900 F, as shown in Fig. P6.177. There are two stops, a lower one at which V_{min} = 35 ft^3 and an upper one at V_{max} = 105 ft^3. The piston is loaded with a mass and outside atmosphere such that it floats when the pressure is 75 lbf/in.^{2}. This setup is now cooled to 210 F by rejecting heat to the surroundings at 70 F. Find the total entropy generated in the process.

C.V. Water.

State 1: Table F.7.2 v_1 = 5.3529 ft^3/lbm, u_1 = 1330.2 btu/lbm,

s_1 = 1.8381 Btu/lbm

$m = V/v_1 = 105/5.353 = 19.615$ lbm

State 2: 210 F and on line in P-v diagram.

Notice the following: $v_g(P_{float})$ = 5.818 ft^3/lbm, $v_{bot} = V_{min}/m$ = 1.7843

$T_{sat}(P_{float})$ = 307.6 F, $T_2 < T_{sat}(P_{float})$ $\Rightarrow$ $V_2 = V_{min}$

State 2: 210 F, $v_2 = v_{bot}$ $\Rightarrow$ x_2 = (1.7843 -0.0167)/27.796 = 0.06359

u_2 = 178.1 + 0.06359×898.9 = 235.26 btu/lbm,

s_2 = 0.3091 + 0.06359×1.4507 = 0.4014 btu/lbm R

Now we can do the work and then the heat transfer from the energy equation

$$_1W_2 = \int PdV = P_{float}(V_2 - V_1) = 75(35 - 105)\frac{144}{778} = \text{-971.72 Btu}$$

$$_1Q_2 = m(u_2 - u_1) + {_1}W_2 = 19.615(235.26 - 1330.2) - 971.72 = \text{-22449 Btu}$$

Take C.V. total out to where we have 70 F:

$$m(s_2 - s_1) = {_1}Q_2/T_0 + S_{gen} \Rightarrow$$

$$S_{gen} = m(s_2 - s_1) - {_1}Q_2/T_0 = 19.615(0.4014 - 1.8381) + \frac{22449}{529.67}$$

$$= \textbf{14.20 Btu/R}   (= \Delta S_{water} + \Delta S_{sur})$$

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6.244E

A cylinder/piston contains 5 lbm of water at 80 lbf/in.^{2}, 1000 F. The piston has cross-sectional area of 1 ft^2 and is restrained by a linear spring with spring constant 60 lbf/in. The setup is allowed to cool down to room temperature due to heat transfer to the room at 70 F. Calculate the total (water and surroundings) change in entropy for the process.

State 1: Table F.7.2 $v_1 = 10.831$ ft^3/lbm, $u_1 = 1372.3$ btu/lbm,

$$s_1 = 1.9453 \text{ Btu/lbm R}$$

State 2: T_2 & on line in P-v diagram.

$$P = P_1 + (k_s/A_{cyl}^2)(V - V_1)$$

Assume state 2 is two-phase,

$$\Rightarrow P_2 = P_{sat}(T_2) = 0.3632 \text{ lbf/in}^2$$

$$v_2 = v_1 + (P_2 - P_1)A_{cyl}^2/mk_s = 10.831 + (0.3632 - 80)1\times12/5\times60$$

$$= 7.6455 \text{ ft}^3/\text{lbm} = v_f + x_2 v_{fg} = 0.01605 + x_2\ 867.579$$

$$x_2 = 0.008793, \quad u_2 = 38.1 + 0.008793\times995.64 = 46.85 \text{ btu/lbm},$$

$$s_2 = 0.0746 + 0.008793\times1.9896 = 0.0921 \text{ Btu/lbm R}$$

$$_1W_2 = \frac{1}{2}(P_1 + P_2)m(v_2 - v_1)$$

$$= \frac{5}{2}(80 + 0.3632)(7.6455 - 10.831)\frac{144}{778} = \text{-118.46 Btu}$$

$$_1Q_2 = m(u_2 - u_1) + {_1}W_2 = 5(46.85 - 1372.3) - 118.46 = \text{-6746 Btu}$$

$$\Delta S_{tot} = S_{gen\ tot} = m(s_2 - s_1) - {_1}Q_2/T_{room}$$

$$= 5(0.0921 - 1.9453) + 6746/529.67 = \textbf{3.47 Btu/R}$$

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6.245E

A cylinder with a linear spring-loaded piston contains carbon dioxide gas at 300 lbf/in.^{2} with a volume of 2 ft^3. The device is of aluminum and has a mass of 8 lbm. Everything (Al and gas) is initially at 400 F. By heat transfer the whole system cools to the ambient temperature of 77 F, at which point the gas pressure is 220 lbf/in.^{2}. Find the total entropy generation for the process.

Solution:

CO_2: $m = P_1V_1/RT_1 = 300 \times 2 \times 144/(35.10 \times 860) = 2.862$ lbm

$V_2 = V_1(P_1/P_2)(T_2/T_1) = 2(300/220)(537/860) = 1.703$ ft^3

$_1W_{2\ CO_2} = \int PdV = 0.5(P_1 + P_2)(V_2 - V_1)$

$$= [(300 + 220)/2](1.703 - 2)\frac{144}{778} = -14.29 \text{ Btu}$$

$_1Q_{2\ CO_2} = mC_{V0}(T_2 - T_1) + {}_1W_2 = 0.156 \times 2.862(77 - 400) - 14.29 = -158.5$ Btu

$_1Q_{2\ Al} = mC(T_2 - T_1) = 8 \times 0.21(77 - 400) = -542.6$ Btu

System: CO_2 + Al

Entropy Eq.: $m_{CO_2}(s_2 - s_1)_{CO_2} + m_{AL}(s_2 - s_1)_{AL} = {}_1Q_2/T_0 + S_{\text{gen tot}}$

$_1Q_2 = -542.6 - 158.5 = -701.14$ Btu

$m_{CO_2}(s_2 - s_1)_{CO_2} + m_{AL}(s_2 - s_1)_{AL} =$

$2.862[0.201 \ln(537/860) - (35.10/778)\ln(220/300)]$

$+ 8 \times 0.21 \ln(537/860) = -0.23086 - 0.79117 = -1.022$ Btu/R

$S_{\text{gen tot}} = m_{CO_2}(s_2 - s_1)_{CO_2} + m_{AL}(s_2 - s_1)_{AL} - {}_1Q_2/T_0$

$$= -1.022 + \frac{701.14}{537} = \textbf{+0.2837 Btu/R}$$

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 7

8e

Updated June 2013

CONTENT CHAPTER 7

In-Text Concept Questions

7.a

A reversible adiabatic flow of liquid water in a pump has increasing P. How about T?

Solution:

Steady state single flow: $\quad s_e = s_i + \int_i^e \frac{dq}{T} + s_{gen} = s_i + 0 + 0$

Adiabatic (dq = 0) means integral vanishes and reversible means $s_{gen} = 0$, so s is constant. Properties for liquid (incompressible) gives Eq.6.19

$$ds = \frac{C}{T} dT$$

then constant s gives constant T.

7.b

A reversible adiabatic flow of air in a compressor has increasing P. How about T?

Solution:

Steady state single flow: $\qquad s_e = s_i + \int_i^e \frac{dq}{T} + s_{gen} = s_i + 0 + 0$

so s is constant. Properties for an ideal gas gives Eq.6.15 and for constant specific heat we get Eq.6.16. A higher P means a higher T, which is also the case for a variable specific heat, recall Eq.6.19 using the standard entropy.

7.c

A compressor receives R-134a at –10°C, 200 kPa with an exit of 1200 kPa, 50°C. What can you say about the process?

Solution:

Properties for R-134a are found in Table B.5

Inlet state: s_i = 1.7328 kJ/kg K

Exit state: s_e = 1.7237 kJ/kg K

Steady state single flow: $$s_e = s_i + \int_i^e \frac{dq}{T} + s_{gen}$$

Since s decreases slightly and the generation term can only be positive, it must be that the heat transfer is negative (out) so the integral gives a contribution that is smaller than -s_{gen}.

7.d

A flow of water at some velocity out of a nozzle is used to wash a car. The water then falls to the ground. What happens to the water state in terms of **V**, T and s?

let us follow the water flow. It starts out with kinetic and potential energy of some magnitude at a compressed liquid state P, T. As the water splashes onto the car it looses its kinetic energy (it turns in to internal energy so T goes up by a very small amount). As it drops to the ground it then looses all the potential energy which goes into internal energy. Both of theses processes are irreversible so s goes up.

If the water has a temperature different from the ambient then there will also be some heat transfer to or from the water which will affect both T and s.

7.e

In a steady state single flow s is either constant or it increases. Is that true?
Solution:

No.

Steady state single flow: $\qquad s_e = s_i + \int_i^e \dfrac{dq}{T} + s_{gen}$

Entropy can only go up or stay constant due to s_{gen}, but it can go up or down due to the heat transfer which can be positive or negative. So if the heat transfer is large enough it can overpower any entropy generation and drive s up or down.

7.f

If a flow device has the same inlet and exit pressure, can shaft work be done?

The reversible work is given by Eq.7.14

$$w = -\int v\, dP + (\mathbf{V}_i^2 - \mathbf{V}_e^2) + g\,(Z_i - Z_e)$$

For a constant pressure the first term drops out but the other two remains. Kinetic energy changes can give work out (windmill) and potential energy changes can give work out (a dam).

7.g

A polytropic flow process with n = 0 might be which device?

As the polytropic process is $Pv^n = C$, then n = 0 is a constant pressure process. This can be a pipe flow, a heat exchanger flow (heater or cooler) or a boiler.

Concept Problems

7.1

If we follow a mass element going through a reversible adiabatic flow process what can we say about the change of state?

Following a mass (this is a control mass)

$$de = dq - dw = 0 - Pdv = - Pdv; \qquad \text{compression/expansion changes e}$$

$$ds = dq/T + ds_{gen} = 0 + 0 \quad \Rightarrow \qquad s = \text{constant, isentropic process.}$$

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7.2

Which process will make the statement in concept question e) on page 330 true?

Solution:

If the process is said to be adiabatic then:
Steady state adiabatic single flow:　　$s_e = s_i + s_{gen} \geq s_i$

7.3

A reversible process in a steady flow with negligible kinetic and potential energy changes is shown in the diagrams. Indicate the change h_e - h_i and transfers w and q as positive, zero or negative

$dw = -\, v\, dP > 0$ P drops so work is positive out.

$dq = T\, ds = 0$ s is constant, and process reversible so adiabatic.

h_e - $h_i = q - w = 0 - w < 0$ so enthalpy drops

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7.4

A reversible process in a steady flow of air with negligible kinetic and potential energy changes is shown in the diagrams. Indicate the change $h_e - h_i$ and transfers w and q as positive, zero or negative

$dw = -v\, dP > 0$ P drops so work is positive out.

$dq = T\, ds > 0$ s is increasing and process reversible so q is positive.

$h_e - h_i = 0$ as they are functions of T and thus the same.

7.5

A reversible steady isobaric flow has 1 kW of heat added with negligible changes in KE and PE, what is the work transfer?

$P = C$:   Shaft work Eq. 7.14:

$$dw = -v\,dP + \Delta KE + \Delta PE - T\,ds_{gen} = 0 + 0 + 0 - 0 = \mathbf{0}$$

7.6

An air compressor has a significant heat transfer out. See example 9.4 for how high T becomes if there is no heat transfer. Is that good, or should it be insulated?

That depends on the use of the compressed air. If there is no need for the high T, say it is used for compressed air tools, then the heat transfer will lower T and result in lower specific volume reducing the work. For those applications the compressor may have fins mounted on its surface to promote the heat transfer. In very high pressure compression it is done in stages between which is a heat exchanger called an intercooler.

This is a small compressor driven by an electric motor. Used to charge air into car tires.

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7.7

Friction in a pipe flow causes a slight pressure decrease and a slight temperature increase. How does that affect entropy?

Solution:

The friction converts flow work (P drops) into internal energy (T up if single phase). This is an irreversible process and s increases.

If liquid: Eq. 6.10 $ds = \dfrac{C}{T} dT$ so s follows T

If ideal gas Eq. 6.14 $ds = C_p \dfrac{dT}{T} - R \dfrac{dP}{P}$ (both terms increase)

7.8

To increase the work out of a turbine for a given inlet and exit pressure how should the inlet state be changed?

$$w = -\int v \, dP + \ldots \qquad \text{Eq.7.14}$$

For a given change in pressure boosting v will result in larger work term. So for **larger inlet T** we get a larger v and thus larger work. That is why we increase T by combustion in a gasturbine before the turbine section.

7.9

An irreversible adiabatic flow of liquid water in a pump has higher P. How about T?

Solution:

Steady state single flow: $\quad s_e = s_i + \int_i^e \dfrac{dq}{T} + s_{gen} = s_i + 0 + s_{gen}$

so s is increasing. Properties for liquid (incompressible) gives Eq.6.10 where an increase in s gives an increase in T.

7.10

The shaft work in a pump to increase the pressure is small compared to the shaft work in an air compressor for the same pressure increase. Why?

The reversible work is given by Eq. 7.14 or 7.15 if reversible and no kinetic or potential energy changes

$$w = -\int v \, dP$$

The liquid has a very small value for v compared to a large value for a gas.

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7.11

Liquid water is sprayed into the hot gases before they enter the turbine section of a large gasturbine power plant. It is claimed that the larger mass flow rate produces more work. Is that the reason?

No. More mass through the turbine does give more work, but the added mass is only a few percent. As the liquid vaporizes the specific volume increases dramatically which gives a much larger volume flow throught the turbine and that gives more work output.

$$\dot{W} = \dot{m}w = -\dot{m}\int v\ dP = -\int \dot{m}v\ dP = -\int \dot{V}\ dP$$

This should be seen relative to the small work required to bring the liquid water up to the higher turbine inlet pressure from the source of water (presumably atmospheric pressure).

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7.12

A tank contains air at 400 kPa, 300 K and a valve opens up for flow out to the outside which is at 100 kPa, 300 K. What happens to the air temperature inside?

As mass flows out of the tank the pressure will drop, the air that remains basically goes through a simple (adiabatic if process is fast enough) expansion process so the temperature also drops. If the flow rate out is very small and the process thus extremely slow, enough heat transfer may take place to keep the temperature constant.

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Steady state reversible processes single flow

7.13

A turbine receives steam at 6 MPa, 600°C with an exit pressure of 600 kPa. Assume the stage is adiabatic and negelect kinetic energies. Find the exit temperature and the specific work.
Solution:

C.V. Stage 1 of turbine.

The stage is adiabatic so $q = 0$ and we will assume reversible so $s_{gen} = 0$

Energy Eq.4.13: $w_T = h_i - h_e$

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Inlet state: B.1.3: $h_i = 3658.40$ kJ/kg, $s_i = 7.1676$ kJ/kg K

Exit state: 600 kPa, $s = s_i$

Table B.1.3 $\Rightarrow$ $T \cong$ **246.7°C,** $h_e = 2950.19$ kJ/kg

$w_T = 3658.4 - 2950.19 =$ **708.2 kJ/kg**

7.14

An evaporator has R-410A at -20°C and quality 80% flowing in with the exit flow being saturated vapor at -20°C. Consider the heating to be a reversible process and find the specific heat transfer from the entropy equation.

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + q/T + 0$

$$q = T\,(s_e - s_i) = T\,(s_g - s_i)$$

Inlet: $s_i = 0.1154 + x_i\,0.9625 = 0.8854$ kJ/kg-K

Exit: $s_g = 1.0779$ kJ/kg-K

$q = (273.15 - 20)\,K \times (1.0779 - 0.8854)$ kJ/kgK = **48.73 kJ/kg**

Remark: It fits with $h_e - h_i = (1 - x_i)\,h_{fg} = 0.2 \times 243.65 = 48.73$ kJ/kg

7.15

Steam enters a turbine at 3 MPa, 450°C, expands in a reversible adiabatic process and exhausts at 50 kPa. Changes in kinetic and potential energies between the inlet and the exit of the turbine are small. The power output of the turbine is 800 kW. What is the mass flow rate of steam through the turbine?

Solution:

C.V. Turbine, Steady single inlet and exit flows. Adiabatic: $\dot{Q} = 0$.

Continuity Eq.4.11: $\dot{m}_i = \dot{m}_e = \dot{m}$,

Energy Eq.4.12: $\dot{m}h_i = \dot{m}h_e + \dot{W}_T$,

Entropy Eq. 7.8: $\dot{m}s_i + \emptyset = \dot{m}s_e$ (Reversible $\dot{S}_{gen} = 0$)

Explanation for the work term is in Sect. 7.3, Eq.7.14

Inlet state: Table B.1.3 $h_i = 3344$ kJ/kg, $s_i = 7.0833$ kJ/kg K

Exit state: P_e , $s_e = s_i$ $\Rightarrow$ Table B.1.2 saturated as $s_e < s_g$

$\quad x_e = (7.0833 - 1.091)/6.5029 = 0.92148$,

$\quad h_e = 340.47 + 0.92148 \times 2305.40 = 2464.85$ kJ/kg

$\dot{m} = \dot{W}_T/w_T = \dot{W}_T/(h_i - h_e) = \dfrac{800}{3344 - 2464.85} \dfrac{\text{kW}}{\text{kJ/kg}} = \textbf{0.91 kg/s}$

7.16

The exit nozzle in a jet engine receives air at 1200 K, 150 kPa with neglible kinetic energy. The exit pressure is 80 kPa and the process is reversible and adiabatic. Use constant heat capacity at 300 K to find the exit velocity.

Solution:

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i = h_e + \mathbf{V}_e^2/2$ ($Z_i = Z_e$)

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Use constant specific heat from Table A.5, $C_{Po} = 1.004 \dfrac{kJ}{kg\ K}$, $k = 1.4$

The isentropic process ($s_e = s_i$) gives Eq.6.23

=> $T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 1200\ K\ (80/150)^{0.2857} = 1002.7\ K$

The energy equation becomes

$$\mathbf{V}_e^2/2 = h_i - h_e \cong C_P(T_i - T_e)$$

$$\mathbf{V}_e = \sqrt{2\ C_P(T_i - T_e)} = \sqrt{2\times1.004(1200-1002.7) \times 1000} = \mathbf{629.4\ m/s}$$

7.17

Do the previous problem using the air tables in A.7

The exit nozzle in a jet engine receives air at 1200 K, 150 kPa with neglible kinetic energy. The exit pressure is 80 kPa and the process is reversible and adiabatic. Use constant heat capacity at 300 K to find the exit velocity.

Solution:

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i = h_e + V_e^2/2$ ($Z_i = Z_e$)

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Process: $q = 0$, $s_{gen} = 0$ as used above leads to $s_e = s_i$

Inlet state: $h_i = 1277.8$ kJ/kg, $s_{Ti}^o = 8.3460$ kJ/kg K

The constant s is rewritten from Eq.6.19 as

$$s_{Te}^o = s_{Ti}^o + R \ln(P_e / P_i) = 8.3460 + 0.287 \ln (80/150) = 8.1656 \text{ kJ/kg K}$$

Interpolate in A.7   =>

$$T_e = 1000 + 50 \frac{8.1656 - 8.1349}{8.1908 - 8.1349} = 1027.46 \text{ K}$$

$$h_e = 1046.2 + (1103.5 - 1046.3) \times \frac{8.1656 - 8.1349}{8.1908 - 8.1349} = 1077.7 \text{ kJ/kg}$$

From the energy equation we have $V_e^2/2 = h_i - h_e$, so then

$$V_e = \sqrt{2 \, (h_i - h_e)} = \sqrt{2(1277.8 - 1077.7) \text{ kJ/kg} \times 1000 \text{ J/kJ}} = \textbf{632.6 m/s}$$

7.18

A reversible adiabatic compressor receives 0.05 kg/s saturated vapor R-410A at 400 kPa and has an exit presure of 1800 kPa. Neglect kinetic energies and find the exit temperature and the minimum power needed to drive the unit.

Solution:

C.V. Compressor, Steady single inlet and exit flows. Adiabatic: $\dot{Q} = 0$.

Continuity Eq.4.11: $\dot{m}_i = \dot{m}_e = \dot{m}$,

Energy Eq.4.12: $\dot{m}h_i = \dot{m}h_e + \dot{W}_C$,

Entropy Eq.7.8: $\dot{m}s_i + \emptyset = \dot{m}s_e$ (Reversible $\dot{S}_{gen} = 0$)

Inlet state: B 4.2.: $h_i = 271.90$ kJ/kg, $s_i = 1.0779$ kJ/kg K

Exit state: P_e , $s_e = s_i$ $\Rightarrow$ Table B.4.2 $h_e = 314.33$ kJ/kg, **$T_e \cong 51.9°C$**

$-w_c = h_e - h_i = 314.33 - 271.90 = 42.43$ kJ/kg

$-\dot{W}_c = $ Power In $= -w_c\dot{m} = 42.43$ kJ/kg $\times$ 0.05kg/s = **2.12 kW**

Explanation for the work term is in Sect. 7.3, Eq.7.14

7.19

In a heat pump that uses R-134a as the working fluid, the R-134a enters the compressor at 150 kPa, $-10°C$ and the R-134a is compressed in an adiabatic process to 1 MPa using 4 kW of power input. Find the mass flow rate it can provide assuming the process is reversible.

Solution:

C.V. Compressor, Steady single inlet and exit flows. Adiabatic: $\dot{Q} = 0$.

Continuity Eq.4.11: $\dot{m}_1 = \dot{m}_2 = \dot{m}$,

Energy Eq.4.12: $\dot{m}h_1 = \dot{m}h_2 + \dot{W}_C$,

Entropy Eq.7.8: $\dot{m}s_1 + \emptyset = \dot{m}s_2$ (Reversible $\dot{S}_{gen} = 0$)

Inlet state: Table B.5.2 $h_1 = 393.84$ kJ/kg, $s_1 = 1.7606$ kJ/kg K

Exit state: $P_2 = 1$ MPa & $s_2 = s_1$ $\Rightarrow$ $h_2 = 434.9$ kJ/kg

$\qquad \dot{m} = \dot{W}_c / w_c = \dot{W}_c /(h_1 - h_2) = -4$ kW / (393.84 - 434.9) kJ/kg

$\qquad\quad = \textbf{0.097 kg/s}$

Explanation for the
work term is in
Sect. 7.3
Eq.7.14

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7.20

Nitrogen gas flowing in a pipe at 500 kPa, 200°C, and at a velocity of 10 m/s, should be expanded in a nozzle to produce a velocity of 300 m/s. Determine the exit pressure and cross-sectional area of the nozzle if the mass flow rate is 0.15 kg/s, and the expansion is reversible and adiabatic.

Solution:

C.V. Nozzle. Steady flow, no work out and no heat transfer.

Energy Eq.4.13: $h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2$

Entropy Eq.7.9: $s_i + \int dq/T + s_{gen} = s_i + 0 + 0 = s_e$

Properties Ideal gas Table A.5:

$$C_{Po} = 1.042 \frac{kJ}{kg\ K}, \ \ R = 0.2968 \frac{kJ}{kg\ K}, \ \ k = 1.40$$

$h_e - h_i = C_{Po}(T_e - T_i) = 1.042(T_e - 473.2) = (10^2 - 300^2)/(2\times1000)$

Solving for exit T: $T_e = 430$ K,

Process: $s_i = s_e$ =>   For ideal gas expressed in Eq.6.23

$$P_e = P_i(T_e/T_i)^{\frac{k}{k-1}} = 500 \text{ kPa} \left(\frac{430}{473.2}\right)^{3.5} = \textbf{357.6 kPa}$$

$v_e = RT_e/P_e = (0.2968 \text{ kJ/kg-K} \times 430 \text{ K})/357.6 \text{ kPa} = 0.35689 \text{ m}^3/\text{kg}$

$$A_e = \dot{m}v_e/\mathbf{V}_e = \frac{0.15 \times 0.35689}{300} \frac{\text{(kg/s) (m}^3/\text{kg)}}{\text{m/s}} = \textbf{1.78} \times \textbf{10}^{\textbf{-4}} \textbf{ m}^2$$

7.21

A reversible isothermal expander (a turbine with heat transfer) has an inlet flow of carbon dioxide at 3 MPa, 80°C and an exit flow at 1 MPa, 80°C. Find the specific heat transfer from the entropy equation and the specific work from the energy equation assuming ideal gas.

CV the expander, control surface at 80°C.

Energy Eq.4.13: $0 = h_i - h_e + q - w$

Entropy Eq.7.9: $0 = s_i - s_e + \int dq/T + s_{gen} = s_i - s_e + q/T + 0$

From entropy equation

$$q = T\,(s_e - s_i) = T\,(C_{Po}\,\ln\frac{T_e}{T_i} - R\,\ln\frac{P_e}{P_i}) = -\,RT\,\ln\frac{P_e}{P_i}$$

$$= -0.1889 \text{ kJ/kg-K} \times 353.15 \text{ K} \times \ln\frac{1}{3} = \textbf{73.29 kJ/kg}$$

From energy equation
$$w = h_i - h_e + q = q = \textbf{73.29 kJ/kg}$$

7.22

Solve the previous Problem using Table B.3

Energy Eq.4.13: $0 = h_i - h_e + q - w$

Entropy Eq.7.9: $0 = s_i - s_e + \int dq/T + s_{gen} = s_i - s_e + q/T + 0$

Inlet state: $h_i = 421.16$ kJ/kg, $s_i = 1.5385$ kJ/kg-K

Exit state: $h_e = 435.23$ kJ/kg, $s_e = 1.775$ kJ/kg-K

From entropy equation

$\qquad q = T\,(s_e - s_i) = 353.15\,(1.775 - 1.5385) = \mathbf{83.52\ kJ/kg}$

From energy equation

$\qquad w = h_i - h_e + q = 421.16 - 435.23 + 83.52 = \mathbf{69.45\ kJ/kg}$

Remark: When it is not an ideal gas h is a fct. of both T and P.

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7.23

A compressor in a commercial refrigerator receives R-410A at -25$^\text{o}$C and unknown quality. The exit is at 2000 kPa, 60$^\text{o}$C and the process assumed reversible and adiabatic. Neglect kinetic energies and find the inlet quality and the specific work.

CV Compressor. q = 0.

Energy Eq.4.13: $w_C = h_i - h_e$

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Exit state: 2000 kPa, 60$^\text{o}$C => $s_e = 1.0878$ kJ/kgK = s_i , $h_e = 320.62$ kJ/kg

Inlet state: T, s Table B.4.1 $\Rightarrow x_i = (1.0878 - 0.0871)/1.0022 =$ **0.9985**

$h_i = 21.08 + x_i\ 248.69 = 269.4$ kJ/kg

$w_C = 269.4 - 320.62 = $ –**51.2 kJ/kg**

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7.24

A boiler section boils 3 kg/s saturated liquid water at 2000 kPa to saturated vapor in a reversible constant pressure process. Assume you do not know that there is no work. Prove that there is no shaftwork using the first and second laws of thermodynamics.

Solution:

C.V. Boiler. Steady, single inlet and single exit flows.

Energy Eq.4.13: $h_i + q = w + h_e$;

Entropy Eq.7.9: $s_i + q/T = s_e$

States: Table B.1.2, $T = T_{sat} = 212.42°C = 485.57$ K

$h_i = h_f = 908.77$ kJ/kg, $\qquad s_i = 2.4473$ kJ/kg K

$h_e = h_g = 2799.51$ kJ/kg, $\qquad s_e = 6.3408$ kJ/kg K

From entropy equation

$q = T(s_e - s_i) = 485.57$ K $(6.3408 - 2.4473)$ kJ/kg-K = **1890.6 kJ/kg**

From energy equation

$w = h_i + q - h_e = 908.77 + 1890.6 - 2799.51 =$ **-0.1 kJ/kg**

It should be zero (non-zero due to round off in values of s, h and T_{sat}).

Often it is a long pipe
and not a chamber

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7.25

A compressor brings a hydrogen gas flow at 280 K, 100 kPa up to a pressure of 1000 kPa in a reversible process. How hot is the exit flow and what is the specific work input?

CV Compressor. Assume q = 0.

Energy Eq.4.13: $w_C = h_i - h_e \approx C_p (T_i - T_e)$

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

So constant s gives the power relation in Eq. 6.23 with k from A.5

$$T_e = T_i (P_e/P_i)^{(k-1)/k} = 280 \text{ K } (1000/100)^{(1.409-1)/1.409} = \textbf{546.3 K}$$

Now the work from the energy equation, C_p from A.5

$$w_C = 14.209 \text{ kJ/kg-K} \times (280 - 546.3) \text{ K} = \textbf{-3783.9 kJ/kg}$$

7.26

Atmospheric air at -45°C, 60 kPa enters the front diffuser of a jet engine with a velocity of 900 km/h and frontal area of 1 m^2. After the adiabatic diffuser the velocity is 20 m/s. Find the diffuser exit temperature and the maximum pressure possible.

Solution:

C.V. Diffuser, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2,$ and $h_e - h_i = C_p(T_e - T_i)$

Entropy Eq.7.9: $s_i + \int dq/T + s_{gen} = s_i + 0 + 0 = s_e$ (Reversible, adiabatic)

Specific heat and ratio of specific heats from Table A.5: $C_{Po} = 1.004 \dfrac{kJ}{kg\ K}$,

$k = 1.4$,   the energy equation then gives:

$\quad 1.004[\ T_e - (-45)] = 0.5[(900 \times 1000/3600)^2 - 20^2\ ]/1000 = 31.05$ kJ/kg

$\qquad \Rightarrow\ T_e = -14.05\ °C = \mathbf{259.1\ K}$

Constant s for an ideal gas is expressed in Eq.6.23 (we need the inverse relation here):

$$P_e = P_i\ (T_e/T_i)^{\frac{k}{k-1}} = 60\ \text{kPa}\ (259.1/228.1)^{3.5} = \mathbf{93.6\ kPa}$$

7.27

A compressor is surrounded by cold R-134a so it works as an isothermal compressor. The inlet state is 0°C, 100 kPa and the exit state is saturated vapor. Find the specific heat transfer and specific work.

Solution:

C.V. Compressor. Steady, single inlet and single exit flows.

Energy Eq.4.13: $h_i + q = w + h_e$;

Entropy Eq.7.9: $s_i + q/T = s_e$

Inlet state: Table B.5.2, $h_i = 403.4$ kJ/kg, $s_i = 1.8281$ kJ/kg K

Exit state: Table B.5.1, $h_e = 398.36$ kJ/kg, $s_e = 1.7262$ kJ/kg K

From entropy equation

$q = T(s_e - s_i) = 273.15$ K $\times (1.7262 - 1.8281)$ kJ/kg-K = **- 27.83 kJ/kg**

From energy equation

$w = 403.4 + (-27.83) - 398.36 =$ **-22.8 kJ/kg**

Explanation for the work term is in Sect. 7.3

Eq. 7.14

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7.28

A flow of 2 kg/s saturated vapor R-410A at 500 kPa is heated at constant pressure to 60°C. The heat is supplied by a heat pump that receives heat from the ambient at 300 K and work input, shown in Fig. P7.28. Assume everything is reversible and find the rate of work input.

Solution:

C.V. Heat exchanger

Continuity Eq.: $\dot{m}_1 = \dot{m}_2$;

Energy Eq.: $\dot{m}_1 h_1 + \dot{Q}_H = \dot{m}_1 h_2$

Table B.4.2:

  $h_1 = 274.33$ kJ/kg, $s_1 = 1.0647$ kJ/kg K

  $h_2 = 342.32$ kJ/kg, $s_2 = 1.2959$ kJ/kg K

Notice we can find $\dot{Q}_H$ but the temperature T_H is not constant making it difficult to evaluate the COP of the heat pump.

C.V. Total setup and assume everything is reversible and steady state.

Energy Eq.: $\dot{m}_1 h_1 + \dot{Q}_L + \dot{W}_{in} = \dot{m}_1 h_2$

Entropy Eq.: $\dot{m}_1 s_1 + \dot{Q}_L/T_L + 0 = \dot{m}_1 s_2$ (T_L is constant, $s_{gen} = 0$)

$\dot{Q}_L = \dot{m}_1 T_L [s_2 - s_1] = 2$ kg/s $\times$ 300 K $\times$ [1. 2959 – 1.0647] kJ/kg-K
    $= 138.72$ kW

$\dot{W}_{in} = \dot{m}_1 [h_2 - h_1] - \dot{Q}_L = 2\ (342.32 - 274.33) - 138.72 = $**–2.74 kW**

Comment: Net work is nearly zero, due to the very low T the flow comes in at so the first heating of the flow actually generates work out and only the heating to above ambient T requires work input.

7.29

A flow of 2 kg/s hot exhaust air at 150°C, 125 kPa supplies heat to a heat engine in a setup similar to the previous problem with the heat engine rejecting heat to the ambient at 290 K and the air leaves the heat exchanger at 50°C. Find the maximum possible power out of the heat engine.

CV. Total setup. Assume everything is reversible.

Energy Eq.: $\dot{m}_1 h_1 = \dot{Q}_L + \dot{W}_{HE} + \dot{m}_1 h_2$

Entropy Eq.: $\dot{m}_1 s_1 + 0 = \dot{Q}_L / T_L + \dot{m}_1 s_2$ (T_L is constant, $s_{gen} = 0$)

$$\dot{Q}_L = \dot{m}_1 T_L (s_1 - s_2) = 2 \text{ kg/s} \times 290 \text{ K} \times 1.004 \text{ kJ/kgK} \times \ln(423/323)$$
$$= \textbf{157 kW}$$

From the energy equation

$$\dot{W}_{HE} = \dot{m}_1(h_1 - h_2) - \dot{Q}_L = \dot{m}_1 C_{Po}(T_1 - T_2) - \dot{Q}_L$$
$$= 2 \text{ kg/s} \times 1.004 \text{ kJ/kgK} \times (150 - 50) \text{ K} - 157 \text{ kW} = \textbf{43.8 kW}$$

7.30

A diffuser is a steady-state device in which a fluid flowing at high velocity is decelerated such that the pressure increases in the process. Air at 120 kPa, 30°C enters a diffuser with velocity 200 m/s and exits with a velocity of 20 m/s. Assuming the process is reversible and adiabatic what are the exit pressure and temperature of the air?

Solution:

C.V. Diffuser, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $\quad h_i + V_i^2/2 = h_e + V_e^2/2, \quad => \quad h_e - h_i = C_{Po}(T_e - T_i)$

Entropy Eq.7.9: $\quad s_i + \int dq/T + s_{gen} = s_i + 0 + 0 = s_e \quad$ (Reversible, adiabatic)

Use constant specific heat from Table A.5, $C_{Po} = 1.004 \dfrac{kJ}{kg\ K}, \quad k = 1.4$

Energy equation then gives:

$C_{Po}(T_e - T_i) = 1.004(T_e - 303.2) = (200^2 - 20^2)/(2 \times 1000) \quad => \quad T_e = \mathbf{322.9\ K}$

The isentropic process ($s_e = s_i$) gives Eq.6.23

$$P_e = P_i(T_e/T_i)^{\frac{k}{k-1}} = 120\ kPa\ (322.9/303.2)^{3.5} = \mathbf{149.6\ kPa}$$

7.31

Air enters a turbine at 800 kPa, 1200 K, and expands in a reversible adiabatic process to 100 kPa. Calculate the exit temperature and the specific work output using Table A.7 and repeat using constant specific heat from Table A.5

Solution:

C.V. Air turbine.

Adiabatic: $q = 0$, reversible: $s_{gen} = 0$

Energy Eq.4.13: $w_T = h_i - h_e$,

Entropy Eq.7.9: $s_e = s_i$

a) Table A.7: $h_i = 1277.8$ kJ/kg, $s^o_{Ti} = 8.34596$ kJ/kg K

The constant s process is written from Eq.6.19 as

$$\Rightarrow s^o_{Te} = s^o_{Ti} + R \ln(\frac{P_e}{P_i}) = 8.34596 + 0.287 \ln\left(\frac{100}{800}\right) = 7.7492 \text{ kJ/kg K}$$

Interpolate in A.7.1 $\Rightarrow T_e =$ **706 K**, $h_e = 719.9$ kJ/kg

$w = h_i - h_e =$ **557.9 kJ/kg**

b) Table A.5: $C_{Po} = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$, then from Eq.6.23

$$T_e = T_i (P_e/P_i)^{\frac{k-1}{k}} = 1200 \text{ K} \left(\frac{100}{800}\right)^{0.286} = \textbf{662.1 K}$$

$w = C_{Po}(T_i - T_e) = 1.004$ kJ/kg-K $(1200 - 662.1)$ K $=$ **539.8 kJ/kg**

7.32

An expander receives 0.5 kg/s air at 2000 kPa, 300 K with an exit state of 400 kPa, 300 K. Assume the process is reversible and isothermal. Find the rates of heat transfer and work neglecting kinetic and potential energy changes.

Solution:

C.V. Expander, single steady flow.

Energy Eq.: $\dot{m}h_i + \dot{Q} = \dot{m}h_e + \dot{W}$

Entropy Eq.: $\dot{m}s_i + \dot{Q}/T + \dot{m}s_{gen} = \dot{m}s_e$

Process: T is constant and $s_{gen} = 0$

Ideal gas and isothermal gives a change in entropy by Eq. 6.15, so we can solve for the heat transfer

$$\dot{Q} = T\dot{m}(s_e - s_i) = -\dot{m}RT \ln \frac{P_e}{P_i}$$

$$= -0.5 \text{ kg/s} \times 300 \text{ K} \times 0.287 \text{ kJ/kg-K} \times \ln \frac{400}{2000} = \textbf{69.3 kW}$$

From the energy equation we get

$$\dot{W} = \dot{m}(h_i - h_e) + \dot{Q} = \dot{Q} = \textbf{69.3 kW}$$

7.33

A highly cooled compressor brings a hydrogen gas flow at 300 K, 100 kPa up to a pressure of 800 kPa in an isothermal process. Find the specific work assuming a reversible process.

CV Compressor. Isothermal $T_i = T_e$ so that ideal gas gives $h_i = h_e$.

Energy Eq.4.13: $w_C = h_i + q - h_e = q$

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + q/T + 0$

$q = T(s_e - s_i) = T [-R \ln(P_e/P_i)]$

$w = q = -4.1243 \text{ kJ/kg-K} \times 300 \text{ K} \times \ln(8) = -\textbf{2573 kJ/kg}$

R is from Table A.5.

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7.34

A compressor receives air at 290 K, 95 kPa and a shaft work of 5.5 kW from a gasoline engine. It should deliver a mass flow rate of 0.01 kg/s air to a pipeline. Find the maximum possible exit pressure of the compressor.

Solution:

C.V. Compressor, Steady single inlet and exit flows. Adiabatic: $\dot{Q} = 0$.

Continuity Eq.4.11: $\dot{m}_i = \dot{m}_e = \dot{m}$,

Energy Eq.4.12: $\dot{m}h_i = \dot{m}h_e + \dot{W}_C$,

Entropy Eq.7.8: $\dot{m}s_i + \dot{S}_{gen} = \dot{m}s_e$ (Reversible $\dot{S}_{gen} = 0$)

$$\dot{W}_c = \dot{m}w_c \implies -w_c = -\dot{W}/\dot{m} = 5.5/0.01 = 550 \text{ kJ/kg}$$

Use constant specific heat from Table A.5, $C_{Po} = 1.004 \text{ kJ/kgK}$, $k = 1.4$

$$h_e = h_i + 550 \implies T_e = T_i + 550 \text{ kJ/kg} /1.004 \text{ kJ/kgK}$$

$$T_e = 290 + 550/1.004 = 837.81 \text{ K}$$

$$s_i = s_e \implies P_e = P_i (T_e/T_i)^{\frac{k}{k-1}} \qquad \text{Eq.6.23}$$

$$P_e = 95 \text{ kPa} \times (837.81/290)^{3.5} = \textbf{3893 kPa}$$

7.35

A reversible steady state device receives a flow of 1 kg/s air at 400 K, 450 kPa and the air leaves at 600 K, 100 kPa. Heat transfer of 900 kW is added from a 1000 K reservoir, 50 kW rejected at 350 K and some heat transfer takes place at 500 K. Find the heat transferred at 500 K and the rate of work produced.

Solution:

C.V. Device, single inlet and exit flows.

Energy equation, Eq.4.12:

$$\dot{m}h_1 + \dot{Q}_3 - \dot{Q}_4 + \dot{Q}_5 = \dot{m}h_2 + \dot{W}$$

Entropy equation with zero generation, Eq.7.8:

$$\dot{m}s_1 + \dot{Q}_3/T_3 - \dot{Q}_4/T_4 + \dot{Q}_5/T_5 = \dot{m}s_2$$

Solve for the unknown heat transfer using Table A.7.1 and Eq. 6.19 for change in s

$$\dot{Q}_5 = T_5 \ [s_2 - s_1] \ \dot{m} + \frac{T_5}{T_4} \dot{Q}_4 \ - \ \frac{T_5}{T_3} \dot{Q}_3$$

$$= 500 \times 1 \ (7.5764 - 7.1593 - 0.287 \ln \frac{100}{450}) + \frac{500}{350} \times 50 - \frac{500}{1000} \times 900$$

$$= 424.4 + 71.4 - 450 = 45.8 \text{ kW}$$

Now the work from the energy equation is

$$\dot{W} = 1 \times (401.3 - 607.3) + 900 - 50 + 45.8 = \textbf{689.8 kW}$$

7.36

A steam turbine in a powerplant receives 5 kg/s steam at 3000 kPa, 500°C. 20% of the flow is extracted at 1000 kPa to a feedwater heater and the remainder flows out at 200 kPa. Find the two exit temperatures and the turbine power output.

C.V. Turbine. Steady flow and adiabatic q = 0.

Continuity Eq.4.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$;

Energy Eq.4.10: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3 + \dot{W}$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3$

State 1: $h_1 = 3456$ kJ/kg, $s_1 = 7.234$ kJ/kgK

We also assume turbine is reversible $\dot{S}_{gen} = 0$ => $s_1 = s_2 = s_3$

State 2: (P,s) **$T_2 = 330.6$°C**, $h_2 = 3116$ kJ/kg

State 3: (P,s) **$T_3 = 140.7$°C**, $h_3 = 2750$ kJ/kg

$$\dot{W} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = 5 \times 3456 - 1 \times 3116 - 4 \times 2750 = \textbf{3164 kW}$$

7.37

A reversible adiabatic compression of an air flow from 20°C, 100 kPa to 200 kPa is followed by an expansion down to 100 kPa in an ideal nozzle. What are the two processes? How hot does the air get? What is the exit velocity?

Solution:

Separate control volumes around compressor and nozzle. For ideal compressor we have
inlet : 1 and exit : 2

Adiabatic : q = 0.
Reversible: $s_{gen} = 0$

Energy Eq.4.13: $h_1 + 0 = w_C + h_2$; $\quad h_2 = h_3 + \frac{1}{2}\mathbf{V}^2$

Entropy Eq.7.9: $s_1 + 0/T + 0 = s_2$; $\quad s_2 + 0/T = s_3$

So both processes are **isentropic**.

$$- w_C = h_2 - h_1 , \quad s_2 = s_1$$

Properties Table A.5 air: $C_{Po} = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$

Process gives constant s (isentropic) which with constant C_{Po} gives Eq.6.23

$$\Rightarrow \quad T_2 = T_1 (P_2/P_1)^{\frac{k-1}{k}} = 293.15 \,(200/100)^{0.2857} = \mathbf{357.4\ K}$$

$$\Rightarrow \quad -w_C = C_{Po}(T_2 - T_1) = 1.004\,(357.4 - 293.2) = 64.457 \text{ kJ/kg}$$

The ideal nozzle then expands back down to P_1 (constant s) so state 3 equals state 1. The energy equation has no work but kinetic energy and gives:

$$\frac{1}{2}\mathbf{V}^2 = h_2 - h_1 = -w_C = 64\ 457 \text{ J/kg} \quad \text{(remember conversion to J)}$$

$$\Rightarrow \quad \mathbf{V}_3 = \sqrt{2 \times 64\ 457} = \mathbf{359\ m/s}$$

7.38

A small turbine delivers 1.5 MW and is supplied with steam at 700°C, 2 MPa. The exhaust passes through a heat exchanger where the pressure is 10 kPa and exits as saturated liquid. The turbine is reversible and adiabatic. Find the specific turbine work, and the heat transfer in the heat exchanger.

Solution:

Continuity Eq.4.11: Steady

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}$$

Turbine: Energy Eq.4.13: $w_T = h_1 - h_2$

Entropy Eq.7.9: $s_2 = s_1 + s_{T\ gen}$

Inlet state: Table B.1.3 $\quad h_1 = 3917.45$ kJ/kg, $\quad s_1 = 7.9487$ kJ/kg K

Ideal turbine $\quad s_{T\ gen} = 0, \quad s_2 = s_1 = 7.9487 = s_{f2} + x\ s_{fg2}$

State 3: P = 10 kPa, $s_2 < s_g$ => saturated 2-phase in Table B.1.2

$\quad\Rightarrow x_{2,s} = (s_1 - s_{f2})/s_{fg2} = (7.9487 - 0.6492)/7.501 = 0.9731$

$\quad\Rightarrow h_{2,s} = h_{f2} + x\ h_{fg2} = 191.8 + 0.9731 \times 2392.8 = 2520.35$ kJ/kg

$\quad w_{T,s} = h_1 - h_{2,s} = $ **1397.05 kJ/kg**

$$\dot{m} = \dot{W} / w_{T,s} = 1500 / 1397 = 1.074 \text{ kg/s}$$

Heat exchanger:

$\qquad$ Energy Eq.4.13: $\qquad q = h_3 - h_2$,

$\qquad$ Entropy Eq.7.9: $\qquad s_3 = s_2 + \int dq/T + s_{He\ gen}$

$q = h_3 - h_{2,s} = 191.83 - 2520.35 = $ **-2328.5 kJ/kg**

$\dot{Q} = \dot{m}\ q = 1.074 \text{ kg/s} \times (-2328.5) \text{ kJ/kg} = $ **- 2500 kW**

Explanation for the work term is in Sect. 7.3, Eq.7.14

7.39

One technique for operating a steam turbine in part-load power output is to throttle the steam to a lower pressure before it enters the turbine, as shown in Fig. P7.39. The steamline conditions are 2 MPa, 400°C, and the turbine exhaust pressure is fixed at 10 kPa. Assuming the expansion inside the turbine is reversible and adiabatic, determine the specific turbine work for no throttling and the specific turbine work (part-load) if it is throttled to 500 kPa. Show both processes in a *T–s* diagram.

C.V Turbine. Full load reversible and adiabatic

Entropy Eq.7.9 reduces to constant s so from Table B.1.3 and B.1.2

$$s_3 = s_1 = 7.1270 = 0.6492 + x_{3a} \times 7.5010$$

$$\Rightarrow \quad x_{3a} = 0.86359$$

$$h_{3a} = 191.81 + 0.86359 \times 2392.82 = 2258.2 \text{ kJ/kg}$$

Energy Eq.4.13 for turbine

$$_1w_{3a} = h_1 - h_{3a} = 3247.6 - 2258.2 = \textbf{989.4 kJ/kg}$$

The energy equation for the part load operation gives the exit h. Notice that we have constant h in the throttle process, $h_2 = h_1$:

2b: P, $h_{2b} = h_1 = 3247.6$ kJ/kg $\Rightarrow$ $s_{2b} = 7.7563$ kJ/kgK

3b: P, $s = s_{2b} = 7.7563$ $\Rightarrow$ $x_{3b} = (7.7563 – 0.6492)/7.501 = 0.9475$

$$h_{3b} = 191.81 + x_{3b} \times 2392.82 = 2459.0 \text{ kJ/kg}$$

$$w_T = 3247.6 – 2459.0 = \textbf{788.6 kJ/kg}$$

7.40

An adiabatic air turbine receives 1 kg/s air at 1500 K, 1.6 MPa and 2 kg/s air at 400 kPa, T_2 in a setup similar to Fig. P4.87 with an exit flow at 100 kPa. What should the temperature T_2 be so the whole process can be reversible?

The process is reversible if we do not generate any entropy. Physically in this problem it means that state 2 must match the state inside the turbine so we do not mix fluid at two different temperatures (we assume the pressure inside is exactly 400 kPa).

For this reason let us select the front end as C.V. and consider the flow from state 1 to the 400 kPa. This is a single flow

Entropy Eq.7.9: $\quad s_1 + 0/T + 0 = s_2$;

Property Eq.6.19 $\quad s_2 - s_1 = 0 = s^o_{T2} - s^o_{T1} - R \ln(P_2 / P_1)$

$$s^o_{T2} = s^o_{T1} + R \ln(P_2 / P_1) = 8.61208 + 0.287 \ln \frac{400}{1600} = 8.2142 \text{ kJ/kg-K}$$

From A.7.1: $\qquad T_2 = \textbf{1071.8 K}$

If we solve with constant specific heats we get from Eq.6.23 and k = 1.4

$$T_2 = T_1 (P_2 / P_1)^{(k-1)/k} = 1500 \text{ K } (400/1600)^{0.2857} = 1009.4 \text{ K}$$

7.41

A turbo charger boosts the inlet air pressure to an automobile engine. It consists of an exhaust gas driven turbine directly connected to an air compressor, as shown in Fig. P7.41. For a certain engine load the conditions are given in the figure. Assume that both the turbine and the compressor are reversible and adiabatic having also the same mass flow rate. Calculate the turbine exit temperature and power output. Find also the compressor exit pressure and temperature.

Solution:

CV: Turbine, Steady single inlet and exit flows,

Process: adiabatic: $q = 0$,

reversible: $s_{gen} = 0$

EnergyEq.4.13: $w_T = h_3 - h_4$,

Entropy Eq.7.8: $s_4 = s_3$

The property relation for ideal gas gives Eq.6.23, k from Table A.5

$$s_4 = s_3 \;\rightarrow\; T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 923.2\ \text{K} \left(\frac{100}{170}\right)^{0.286} = \mathbf{793.2\ K}$$

The energy equation is evaluated with specific heat from Table A.5

$$w_T = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004(923.2 - 793.2) = 130.5\ \text{kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = \mathbf{13.05\ kW}$$

C.V. Compressor, steady 1 inlet and 1 exit, same flow rate as turbine.

Energy Eq.4.13: $-w_C = h_2 - h_1$,

Entropy Eq.7.9: $s_2 = s_1$

Express the energy equation for the shaft and compressor having the turbine power as input with the same mass flow rate so we get

$$-w_C = w_T = 130.5\ \text{kJ/kg} = C_{P0}(T_2 - T_1) = 1.004(T_2 - 303.2)$$
$$T_2 = \mathbf{433.2\ K}$$

The property relation for $s_2 = s_1$ is Eq.6.23 and inverted as

$$P_2 = P_1(T_2/T_1)^{\frac{k}{k-1}} = 100\ \text{kPa} \left(\frac{433.2}{303.2}\right)^{3.5} = \mathbf{348.7\ kPa}$$

7.42

Two flows of air both at 200 kPa, one has 3 kg/s at 400 K and the other has 2 kg/s at 290 K. The two lines exchange energy through a number of ideal heat engines taking energy from the hot line and rejecting it to the colder line. The two flows then leave at the same temperature. Assume the whole setup is reversible and find the exit temperature and the total power out of the heat engines.

Solution:

C.V. Total setup

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_1 h_3 + \dot{m}_2 h_4 + \dot{W}_{TOT}$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} + \int d\dot{Q}/T = \dot{m}_1 s_3 + \dot{m}_2 s_4$

Process: Reversible $\dot{S}_{gen} = 0$ Adiabatic $\dot{Q} = 0$

Assume the exit flow has the same pressure as the inlet flow then the pressure part of the entropy cancels out and we have

Exit same T, P   => $h_3 = h_4 = h_e$; $s_3 = s_4 = s_e$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_{TOT} h_e + \dot{W}_{TOT}$$

$$\dot{m}_1 s_1 + \dot{m}_2 s_2 = \dot{m}_{TOT} s_e$$

$$s_{Te}^o = \frac{\dot{m}_1}{\dot{m}_{TOT}} s_{T1}^o + \frac{\dot{m}_2}{\dot{m}_{TOT}} s_{T2}^o = \frac{3}{5} \times 7.1593 + \frac{2}{5} \times 6.8352 = 7.02966 \text{ kJ/kgK}$$

Table A.7:   => $T_e \cong 351.98$ K; $h_e = 352.78$ kJ/kg

$$\dot{W}_{TOT} = \dot{m}_1(h_1 - h_e) + \dot{m}_2 (h_2 - h_e)$$
$$= 3(401.3 - 352.78) + 2(290.43 - 352.78) = \mathbf{20.86 \text{ kW}}$$

Note: The solution using constant heat capacity writes the entropy equation using Eq.6.16, the pressure terms cancel out so we get

$$\frac{3}{5} C_p \ln(T_e/T_1) + \frac{2}{5} C_p \ln(T_e/T_2) = 0 \quad => \quad \ln T_e = (3 \ln T_1 + 2 \ln T_2)/5$$

7.43

A flow of 5 kg/s water at 100 kPa, 20°C should be delivered as steam at 1000 kPa, 350°C to some application. We have a heat source at constant 500°C. If the process should be reversible how much heat transfer should we have?

CV Around unknown device out to the source surface.

Energy Eq.: $\quad \dot{m}h_i + \dot{Q} = \dot{m}h_e + \dot{W}$

Entropy Eq.: $\quad \dot{m}s_i + \dot{Q}/T_S + 0 = \dot{m}s_e \qquad$ (T_S is constant, $s_{gen} = 0$)

Inlet state: $\quad s_i = 0.2966$ kJ/kgK, Table B.1.1
Exit state: $\quad s_e = 7.301$ kJ/kgK, Table B.1.3

$$\dot{Q} = \dot{m}\, T_S\, (s_e - s) = 5 \text{ kg/s} \times 773.15 \text{ K} \times (7.301 - 0.2966) \text{ kJ/kg-K}$$
$$= \textbf{27.1 MW}$$

The theory does not say exactly how to do it. As the pressure goes up we must have a pump or compressor and since the substance temperature is lower than the source temperature a reversible heat transfer must happen through some kind of heat engine receiving a Q from the source and delivering it to the flow extracting some work in the process.

7.44

A heat-powered portable air compressor consists of three components: (a) an adiabatic compressor; (b) a constant pressure heater (heat supplied from an outside source); and (c) an adiabatic turbine. Ambient air enters the compressor at 100 kPa, 300 K, and is compressed to 600 kPa. All of the power from the turbine goes into the compressor, and the turbine exhaust is the supply of compressed air. If this pressure is required to be 200 kPa, what must the temperature be at the exit of the heater?

Solution:

$P_2 = 600$ kPa, $\quad P_4 = 200$ kPa

Adiabatic and reversible compressor:

Process: $\qquad q = 0$ and $s_{gen} = 0$

Energy Eq.4.13: $\qquad h - w_c = h_2$

Entropy Eq.7.8: $\qquad s_2 = s_1$

For constant specific heat the isentropic relation becomes Eq.8.23

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300 \text{ K } (6)^{0.2857} = 500.8 \text{ K}$$

$$-w_c = C_{P0}(T_2 - T_1) = 1.004(500.8 - 300) = 201.5 \text{ kJ/kg}$$

Adiabatic and reversible turbine: $\qquad q = 0$ and $s_{gen} = 0$

Energy Eq.4.13: $\qquad h_3 = w_T + h_4$; $\qquad$ Entropy Eq.7.8: $\qquad s_4 = s_3$

For constant specific heat the isentropic relation becomes Eq.6.23

$$T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = T_3 (200/600)^{0.2857} = 0.7304 \text{ } T_3$$

Energy Eq. for shaft: $\qquad -w_c = w_T = C_{P0}(T_3 - T_4)$

$\qquad$ 201.5 kJ/kg = 1.004 kJ/kgK $\times$ $T_3(1 - 0.7304)$ $\quad =>$ $\quad T_3 = $ **744.4 K**

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7.45

A two-stage compressor having an interstage cooler takes in air, 300 K, 100 kPa, and compresses it to 2 MPa, as shown in Fig. P7.46. The cooler then cools the air to 340 K, after which it enters the second stage, which has an exit pressure of 15 MPa. Both stages are adiabatic, and reversible. Find q in the cooler, total specific work, and compare this to the work required with no intercooler.

Solution:

C.V.: Stage 1 air, Steady flow

Process: adiabatic: $q = 0$, reversible: $s_{gen} = 0$

Energy Eq.4.13: $-w_{C1} = h_2 - h_1$, Entropy Eq.7.8: $s_2 = s_1$

Assume constant $C_{P0} = 1.004$ from A.5 and isentropic leads to Eq.6.23

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300(2000/100)^{0.286} = 706.7 \text{ K}$$

$$w_{C1} = h_1 - h_2 = C_{P0}(T_1 - T_2) = 1.004 \text{ kJ/kgK} \times (300 - 706.7) \text{ K}$$

$$= \textbf{-408.3 kJ/kg}$$

C.V. Intercooler, no work and no changes in kinetic or potential energy.

$$q_{23} = h_3 - h_2 = C_{P0}(T_3 - T_2) = 1.004(340 - 706.7) = \textbf{-368.2 kJ/kg}$$

C.V. Stage 2. Analysis the same as stage 1. So from Eq.6.23

$$T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 340 \text{ K } (15/2)^{0.286} = 604.6 \text{ K}$$

$$w_{C2} = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004(340 - 604.6) = \textbf{-265.7 kJ/kg}$$

Same flow rate through both stages so the total work is the sum of the two

$$w_{comp} = w_{C1} + w_{C2} = -408.3 - 265.7 = \textbf{-682.8 kJ/kg}$$

For no intercooler ($P_2 = 15.74$ MPa) same analysis as stage 1. So Eq.6.23

$$T_2 = 300 \text{ K } (15000/100)^{0.286} = 1255.6 \text{ K}$$

$$w_{comp} = 1.004 \text{ kJ/kg-K } (300 - 1255.6) \text{ K} = \textbf{-959.4 kJ/kg}$$

7.46

A certain industrial process requires a steady supply of saturated vapor steam at 200 kPa, at a rate of 0.5 kg/s. Also required is a steady supply of compressed air at 500 kPa, at a rate of 0.1 kg/s. Both are to be supplied by the process shown in Fig. P7.46. Steam is expanded in a turbine to supply the power needed to drive the air compressor, and the exhaust steam exits the turbine at the desired state. Air into the compressor is at the ambient conditions, 100 kPa, 20°C. Give the required steam inlet pressure and temperature, assuming that both the turbine and the compressor are reversible and adiabatic.

Solution:

C.V. Each device. Steady flow.
Both adiabatic (q = 0) and
reversible (s_{gen} = 0).

Steam turbine   Air compressor

Compressor: $s_4 = s_3$ => $T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 293.2 \text{ K} \left(\frac{500}{100}\right)^{0.286} = 464.6$ K

$\dot{W}_C = \dot{m}_3(h_3 - h_4) = 0.1 \text{ kg/s} \times 1.004 \text{ kJ/kg-K } (293.2 - 464.6) \text{ K} = -17.2$ kW

Turbine: Energy: $\dot{W}_T = +17.2 \text{ kW} = \dot{m}_1(h_1 - h_2)$;   Entropy: $s_2 = s_1$

Table B.1.2: $P_2 = 200$ kPa, $x_2 = 1$ =>

$h_2 = 2706.6$ kJ/kg, $s_2 = 7.1271$ kJ/kgK

$h_1 = 2706.6 + 17.2/0.5 = 2741.0$ kJ/kg

$s_1 = s_2 = 7.1271$ kJ/kg K,

300 kPa: $s = s_2$ => h = 2783.0 kJ/kg

Interpolate between the 200 and 300 kPa

$P = 200 + (300 - 200) \dfrac{2741 - 2706.63}{2783.0 - 2706.63} = \textbf{245 kPa}$

$T = 120.23 + (160.55 - 120.23) \dfrac{2741 - 2706.63}{2783.0 - 2706.63} = \textbf{138.4°C}$

If you use the software you get: At h_1, $s_1 \rightarrow P_1 = \textbf{242 kPa}, T_1 = \textbf{138.3°C}$

7.47

A certain industrial process requires a steady 0.75 kg/s supply of compressed air at 500 kPa, at a maximum temperature of 30°C. This air is to be supplied by installing a compressor and aftercooler. Local ambient conditions are 100 kPa, 20°C. Using an reversible compressor, determine the power required to drive the compressor and the rate of heat rejection in the aftercooler.

Solution:

Air Table A.5: $R = 0.287$ kJ/kg-K, $C_p = 1.004$ kJ/kg K, $k = 1.4$

State 1: $T_1 = T_O = 20^oC$, $P_1 = P_O = 100$ kPa, $\dot{m} = 0.5$ kg/s

State 2: $P_2 = P_3 = 500$ kPa

State 3: $T_3 = 30^oC$, $P_3 = 500$ kPa

Compressor: Assume Isentropic (adiabatic $q = 0$ and reversible $s_{gen} = 0$) From entropy equation Eq.7.9 this gives constant s which is expressed for an ideal gas in Eq.6.23

$$T_2 = T_1 \, (P_2/P_1)^{\frac{k-1}{k}} = 293.15 \text{ K } (500/100)^{0.2857} = 464.6 \text{ K}$$

Energy Eq.4.13: $\qquad q_c + h_1 = h_2 + w_c; \qquad q_c = 0,$

assume constant specific heat from Table A.5

$\qquad w_c = C_p(T_1 - T_2) = 1.004$ kJ/kgK $(293.15 - 464.6)$ K $= -172.0$ kJ/kg

$\qquad \dot{W}_C = \dot{m}w_C =$ **-129 kW**

Aftercooler Energy Eq.4.13: $\qquad q + h_2 = h_3 + w; \quad w = 0,$

assume constant specific heat

$\qquad \dot{Q} = \dot{m}q = \dot{m}C_p(T_3 - T_2) = 0.75$ kg/s $\times 1.004$ kJ/kg-K $(303.15 - 464.6)$ K

$\qquad = -$**121.6 kW**

Compressor section   Aftercooler section

7.48

Consider a steam turbine power plant operating near critical pressure, as shown in Fig. P7.48. As a first approximation, it may be assumed that the turbine and the pump processes are reversible and adiabatic. Neglecting any changes in kinetic and potential energies, calculate

a.   The specific turbine work output and the turbine exit state

b.   The pump work input and enthalpy at the pump exit state

c.   The thermal efficiency of the cycle

Solution:

$P_1 = P_4 = 20$ MPa
$T_1 = 700$ °C
$P_2 = P_3 = 20$ kPa
$T_3 = 40$ °C

a) State 1: (P, T) Table B.1.3 $h_1 = 3809.1$ kJ/kg, $s_1 = 6.7993$ kJ/kg K

   C.V. Turbine.

   Entropy Eq.7.9: $s_2 = s_1 = 6.7993$ kJ/kg K

   Table B.1.2 $s_2 = 0.8319 + x_2 \times 7.0766$ $\Rightarrow$ $x_2 = 0.8433$

   $h_2 = 251.4 + 0.8433 \times 2358.33 = \mathbf{2240.1}$

   Energy Eq.4.13: $w_T = h_1 - h_2 = \mathbf{1569 \ kJ/kg}$

b) State 3: (P, T) Compressed liquid, take sat. liq. Table B.1.1

   $h_3 = 167.5$ kJ/kg, $v_3 = 0.001008$ m^3/kg

   Property relation v = constant gives work from Eq.7.15 as

   $w_P = - v_3(P_4 - P_3) = -0.001008(20000 - 20) = \mathbf{-20.1 \ kJ/kg}$

   $h_4 = h_3 - w_P = 167.5 + 20.1 = \mathbf{187.6 \ kJ/kg}$

c) The heat transfer in the boiler is from energy Eq.4.13

   $q_{boiler} = h_1 - h_4 = 3809.1 - 187.6 = 3621.5$ kJ/kg

   $w_{net} = 1569 - 20.1 = \mathbf{1548.9 \ kJ/kg}$

   $\eta_{TH} = w_{net}/q_{boiler} = \dfrac{1548.9}{3621.5} = \mathbf{0.428}$

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Transient processes

7.49

A 10 m tall, 2 m^2 cross sectional area water tank is on a tower so the bottom is 5 m up from ground level and the top is open to the atmosphere. It is initially empty and then filled by a pump taking water at ambient T = 17°C, 100 kPa from a small pond at ground level. Assume the process is reversible and find the total pump work.

CV Pump, pipe and water tank.

In this problem there clearly is a potential energy change but we will neglect any kinetic energy. Pressure of the water in and pressure in the tank is the same P_o, which is the pressure where the volume (water surface) expands in the tank.

Continuity Eq.4.15: $m_2 - 0 = m_{in}$

Energy Eq.4.16: $m_2(u_2 + gZ_2) - 0 = 0 - {_1}W_{2\ atm} + m_{in}(h_{in} + gZ_{in}) + W_{pump}$

Entropy Eq.7.13: $m_2s_2 - 0 = \int dQ/T + {_1}S_{2\ gen} + m_{in}s_{in} = m_{in}s_{in}$

Process: Adiabatic ${_1}Q_2 = 0$, Process ideal ${_1}S_{2\ gen} = 0$, $s_2 = s_{in}$ and $Z_{in} = 0$

From the entropy we conclude the T stays the same, recall Eq.6.11 and the work to the atmosphere is

$${_1}W_{2\ atm} = P_o (V_2 - 0) = m_2P_2v_2$$

So this will cancel the the incoming flow work included in $m_{in}h_{in}$. The energy equation becomes with $Z_{in} = 0$

$$m_2(u_2 + gZ_2) = - m_2P_2v_2 + m_{in}(u_{in} + P_{in}v_{in}) + W_{pump}$$

and then since state 2: P_o, T_{in} and state in: P_o, T_{in} then $u_2 = u_{in}$. Left is

$$m_2gZ_2 = W_{pump}$$

The final elevation is the average elevation for the mass $Z_2 = 5m + 5m = 10m$.

$$W_{pump} = m_2g\ Z_2$$
$$= 997\ kg/m^3 \times (10 \times 2)\ m^3 \times 9.807\ m/s^2 \times 10\ m$$
$$= 1\ 955\ 516\ J = \textbf{1 955.5 kJ}$$

7.50

Air in a tank is at 300 kPa, 400 K with a volume of 2 m^3. A valve on the tank is opened to let some air escape to the ambient to a final pressure inside of 200 kPa. Find the final temperature and mass assuming a reversible adiabatic process for the air remaining inside the tank.

Solution:

C.V. Total tank.

Continuity Eq.4.15: $\qquad m_2 - m_1 = -m_{ex}$

Energy Eq.4.16: $\qquad m_2u_2 - m_1u_1 = -m_{ex}h_{ex} + {}_1Q_2 - {}_1W_2$

Entropy Eq.7.12: $\qquad m_2s_2 - m_1s_1 = -m_{ex}s_{ex} + \int dQ/T + {}_1S_2\,{}_{gen}$

Process: $\qquad$ Adiabatic $\;{}_1Q_2 = 0;\;$ rigid tank $\;{}_1W_2 = 0$

This has too many unknowns (we do not know state 2).

C.V. m_2 the mass that remains in the tank. This is a control mass.

Energy Eq.3.5: $\qquad m_2(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.14: $\qquad m_2(s_2 - s_1) = \int dQ/T + {}_1S_2\,{}_{gen}$

Process: $\qquad$ Adiabatic $\;{}_1Q_2 = 0;\;$ Reversible $\;{}_1S_2\,{}_{gen} = 0$

$\Rightarrow \qquad s_2 = s_1$

Ideal gas and process Eq.6.23

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 400 \text{ K } (200/300)^{0.2857} = \textbf{356.25 K}$$

$$m_2 = \frac{P_2V}{RT_2} = \frac{200 \times 2}{0.287 \times 356.25}\frac{\text{kPa m}^3}{\text{(kJ/kg-K)} \times \text{K}} = \textbf{3.912 kg}$$

Notice that the work term is not zero for mass m_2. The work goes into pushing the mass m_{ex} out.

7.51

A 0.5 m^3 tank contains carbon dioxide at 300 K, 150 kPa is now filled from a supply of carbon dioxide at 300 K, 150 kPa by a compressor to a final tank pressure of 450 kPa. Assume the whole process is adiabatic and reversible. Find the final mass and temperature in the tank and the required work to the compressor.

C.V. The tank and the compressor.

Continuity Eq.4.15: $\quad\quad\quad\quad m_2 - m_1 = m_{in}$

Energy Eq.4.16: $\quad\quad\quad\quad m_2 u_2 - m_1 u_1 = {_1}Q_2 - {_1}W_2 + m_{in} h_{in}$

Entropy Eq.7.13: $\quad\quad m_2 s_2 - m_1 s_1 = \int dQ/T + {_1}S_{2\,gen} + m_{in} s_{in}$

Process: Adiabatic $\quad {_1}Q_2 = 0$, Process ideal $\quad {_1}S_{2\,gen} = 0$, $\;s_1 = s_{in}$

$\quad\quad \Rightarrow\; m_2 s_2 = m_1 s_1 + m_{in} s_{in} = (m_1 + m_{in}) s_1 = m_2 s_1 \;\Rightarrow\; s_2 = s_1$

Constant s so since the temperatures are modest use Eq.6.23

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300 \text{ K } (450/150)^{0.2242} = \textbf{383.79 K}$$

$$m_1 = P_1 V_1 / RT_1 = \frac{150 \text{ kPa} \times 0.5 \text{ m}^3}{0.1889 \text{ kJ/kg-K} \times 300 \text{ K}} = 1.3235 \text{ kg}$$

$$m_2 = P_2 V_2 / RT_2 = 450 \text{ kPa} \times 0.5 \text{ m}^3 / (0.1889 \times 383.79) \text{ kJ/kg} = \textbf{3.1035 kg}$$

$$\Rightarrow\; m_{in} = 1.78 \text{ kg}$$

$${_1}W_2 = m_{in} h_{in} + m_1 u_1 - m_2 u_2 = m_{in} RT_{in} + m_1(u_1 - u_{in}) - m_2(u_2 - u_{in})$$

$$= 1.78 \, (0.1889 \times 300) + m_1 \, (0) - 3.1035 \times 0.653 \, (383.79 - 300)$$

$$= \textbf{-68.93 kJ} \quad\quad \text{(work must come in)}$$

Borgnakke and Sonntag

7.52

A tank contains 1 kg of carbon dioxide at 6 MPa, 60°C and it is connected to a turbine with an exhaust at 1000 kPa. The carbon dioxide flows out of the tank and through the turbine to a final state in the tank of saturated vapor is reached. If the process is adiabatic and reversible find the final mass in the tank and the turbine work output.

C.V. The tank and turbine. This is a transient problem.

Continuity Eq.4.15: $m_2 - m_1 = -m_{ex}$

Energy Eq.4.16: $m_2 u_2 - m_1 u_1 = -m_{ex} h_{ex} + {}_1Q_2 - {}_1W_2$

Entropy Eq.7.12: $m_2 s_2 - m_1 s_1 = -m_{ex} s_{ex} + \int dQ/T + {}_1S_{2\ gen}$

Process: Adiabatic ${}_1Q_2 = 0$; reversible ${}_1S_{2\ gen} = 0$

State 1: $v_1 = 0.00801$ m^3/kg, $u_1 = 322.51$ kJ/kg, $s_1 = 1.2789$ kJ/kg-K

State 2: Sat. vapor, 1 property missing

C.V. m_2 the mass that remains in the tank. This is a control mass.

Process: Adiabatic ${}_1Q_2 = 0$; Reversible ${}_1S_{2\ gen} = 0$

Entropy Eq.6.14: $m_2(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = 0 + 0$

$\Rightarrow$ $s_2 = s_1$ this is the missing property.

State 2: $T_2 = -19.19^o$C, $v_2 = 0.018851$ m^3/kg, $u_2 = 285.87$ kJ/kg

State exit: $s_{ex} = s_2 = s_1$ follows from entropy Eq. for first C.V. with the use of the continuity equation. Use 1004.5 kPa for -40°C.

$x_{ex} = (1.2789 - 0)/1.3829 = 0.924796 \implies h_{ex} = 298.17$ kJ/kg

Tank volume constant so $V = m_1 v_1 = m_2 v_2$

$m_2 = m_1\ v_1\ /\ v_2 = 1$ kg $\times\ 0.00801\ /\ 0.018851 =$ **0.4249 kg**

From energy eq.

${}_1W_2 = m_1 u_1 - m_2 u_2 - m_{ex} h_{ex}$

$= 1 \times 322.51 - 0.4249 \times 285.87 - 0.5751 \times 298.17$ [kg kJ/kg]

$=$ **29.57 kJ**

Borgnakke and Sonntag

7.53

Air in a tank is at 300 kPa, 400 K with a volume of 2 m^3. A valve on the tank is opened to let some air escape to the ambient to a final pressure inside of 200 kPa. At the same time the tank is heated so the air remaining has a constant temperature. What is the mass average value of the s leaving assuming this is an internally reversible process?

Solution:

C.V. Tank, emptying process with heat transfer.

Continuity Eq.4.15: $m_2 - m_1 = -m_e$

Energy Eq.4.16: $m_2u_2 - m_1u_1 = -m_eh_e + {}_1Q_2$

Entropy Eq.7.13: $m_2s_2 - m_1s_1 = -m_es_e + {}_1Q_2/T + 0$

Process: $T_2 = T_1$ => ${}_1Q_2$ in at 400 K

                  Reversible ${}_1S_{2\ gen} = 0$

State 1: Ideal gas $m_1 = P_1V/RT_1 = 300 \times 2/0.287 \times 400 = 5.2265$ kg

State 2:   200 kPa, 400 K

$m_2 = P_2V/RT_2 = 200$ kPa $\times 2$ m^3/(0.287 $\times$ 400) kJ/kg = 3.4843 kg

=> $m_e = 1.7422$ kg

From the energy equation:

$${}_1Q_2 = m_2u_2 - m_1u_1 + m_eh_e$$

$$= 3.4843 \times 286.49 - 5.2265 \times 286.49 + 1.7422 \times 401.3$$

$$= 1.7422 \text{ kg } (401.3 - 286.49) \text{ kJ/kg} = 200 \text{ kJ}$$

$$m_es_e = m_1s_1 - m_2s_2 + {}_1Q_2/T$$

$$= 5.2265[7.15926 - 0.287 \ln (300/100)] - 3.4843[7.15926$$

$$- 0.287 \ln (200/100)] + (200/400)$$

$$m_es_e = 35.770 - 24.252 + 0.5 = 12.018 \text{ kJ/K}$$

$$s_e = 12.018/1.7422 = 6.89817 = \textbf{6.8982 kJ/kg K}$$

Note that the exit state e in this process is for the air before it is throttled across the discharge valve. The throttling process from the tank pressure to ambient pressure is a highly irreversible process.

Borgnakke and Sonntag

7.54

An insulated 2 m^3 tank is to be charged with R-134a from a line flowing the refrigerant at 3 MPa. The tank is initially evacuated, and the valve is closed when the pressure inside the tank reaches 2 MPa. The line is supplied by an insulated compressor that takes in R-134a at 5°C, quality of 96.5 %, and compresses it to 3 MPa in a reversible process. Calculate the total work input to the compressor to charge the tank.

Solution:

C.V.: Compressor, R-134a. Steady 1 inlet and 1 exit flow, no heat transfer.

Energy Eq.4.13: $q_c + h_1 = h_1 = h_2 + w_c$

Entropy Eq.7.9: $s_1 + \int dq/T + s_{gen} = s_1 + 0 = s_2$

inlet: $T_1 = 5^oC$, $x_1 = 0.965$ use Table B.5.1

$s_1 = s_f + x_1 s_{fg} = 1.0243 + 0.965 \times 0.6995 = 1.6993$ kJ/kg K,

$h_1 = h_f + x_1 h_{fg} = 206.75 + 0.965 \times 194.57 = 394.51$ kJ/kg

exit: $P_2 = 3$ MPa

From the entropy eq.: $s_2 = s_1 = 1.6993$ kJ/kg K;

$T_2 = 90^oC$, $h_2 = 436.19$ kJ/kg

$w_c = h_1 - h_2 = -41.68$ kJ/kg

C.V.: Tank; $V_T = 2$ m^3, $P_T = 2$ MPa

Energy Eq.4.16: $Q + m_i h_i = m_3 u_3 - m_1 u_1 + m_e h_e + W$;

Process and states have: $Q = 0$, $W = 0$, $m_e = 0$, $m_1 = 0$, $m_3 = m_i$

Final state: $P_3 = 2$ MPa, $u_3 = h_i = h_2 = 436.19$ kJ/kg,

$\rightarrow T_3 = 90^oC$, $v_3 = 0.01137$ m^3/kg, $s_3 = 1.785$ kJ/kgK

$m_3 = V_T/v_3 = 2/0.01137 = 175.9$ kg;

The work term is from the specific compressor work and the total mass

$-W_c = m_T(-w_c) = $ **7 331 kJ**

Comment: The filling process is not reversible note $s_3 > s_2$

7.55

An underground salt mine, 100 000 m^3 in volume, contains air at 290 K, 100 kPa. The mine is used for energy storage so the local power plant pumps it up to 2.1 MPa using outside air at 290 K, 100 kPa. Assume the pump is ideal and the process is adiabatic. Find the final mass and temperature of the air and the required pump work.

Solution:

C.V. The mine volume and the pump

Continuity Eq.4.15: $\qquad m_2 - m_1 = m_{in}$

Energy Eq.4.16: $\qquad m_2u_2 - m_1u_1 = {}_1Q_2 - {}_1W_2 + m_{in}h_{in}$

Entropy Eq.7.13: $\qquad m_2s_2 - m_1s_1 = \int dQ/T + {}_1S_{2\ gen} + m_{in}s_{in}$

Process: Adiabatic $\quad {}_1Q_2 = 0$, Process ideal $\quad {}_1S_{2\ gen} = 0$, $s_1 = s_{in}$

$\qquad \Rightarrow\ m_2s_2 = m_1s_1 + m_{in}s_{in} = (m_1 + m_{in})s_1 = m_2s_1\ \Rightarrow\ s_2 = s_1$

Constant s $\Rightarrow \qquad$ Eq.8.19 $\quad s^o_{T2} = s^o_{Ti} + R\ \ln(P_2\,/\,P_{in})$

$\qquad s^o_{T2} = 6.83521 + 0.287\ \ln(\ 21\ ) = 7.7090$ kJ/kg K

$\qquad$ A.7 $\qquad \Rightarrow\ T_2 = \mathbf{680\ K}$, $u_2 = 496.94$ kJ/kg

$$m_1 = P_1V_1/RT_1 = \frac{100\ \text{kPa} \times 10^5\ \text{m}^3}{0.287\ \text{kJ/kg-K} \times 290\ \text{K}} = 1.20149 \times 10^5\ \text{kg}$$

$m_2 = P_2V_2/RT_2 = 100$ kPa $\times 21 \times 10^5$ m^3/(0.287 × 680) kJ/kg = $\mathbf{10.760 \times 10^5\ kg}$

$$\Rightarrow\ m_{in} = 9.5585 \times 10^5\ \text{kg}$$

$\qquad {}_1W_2 = m_{in}h_{in} + m_1u_1 - m_2u_2$

$\qquad = m_{in}(290.43) + m_1(207.19) - m_2(496.94) = \mathbf{-2.322 \times 10^8\ kJ}$

7.56

R-410A at 120°C, 4 MPa is in an insulated tank and flow is now allowed out to a turbine with a backup pressure of 800 kPa. The flow continue to a final tank pressure of 800 kPa and the process stops. If the initial mass was 1 kg how much mass is left in the tank and what is the turbine work assuming a reversible process?

Solution:

C.V. Total tank and turbine.

Continuity Eq.4.15: $m_2 - m_1 = -m_{ex}$

Energy Eq.4.16: $m_2 u_2 - m_1 u_1 = -m_{ex} h_{ex} + {}_1Q_2 - {}_1W_2$

Entropy Eq.7.12: $m_2 s_2 - m_1 s_1 = -m_{ex} s_{ex} + \int dQ/T + {}_1S_{2\,gen}$

Process: Adiabatic ${}_1Q_2 = 0$;   Reversible ${}_1S_{2\,gen} = 0$

This has too many unknowns (we do not know state 2 only P_2).

C.V. m_2 the mass that remains in the tank. This is a control mass.

Entropy Eq.7.3 (6.37): $m_2(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen}$

Process: Adiabatic ${}_1Q_2 = 0$;   Reversible ${}_1S_{2\,gen} = 0$

$\Rightarrow \quad s_2 = s_1$

State 1: $v_1 = 0.00897$ m³/kg, $u_1 = 331.39$ kJ/kg, $s_1 = 1.1529$ kJ/kg-K

State 2 (P,s): $T_2 = 33.23$°C, $v_2 = 0.37182$ m³/kg, $u_2 = 281.29$ kJ/kg

State exit: $s_{ex} = s_2 = s_1$ follows from entropy Eq. for first C.V. using the continuity eq., this is identical to state 2, $h_{ex} = 312.85$ kJ/kg

Tank volume constant so $V = m_1 v_1 = m_2 v_2$

$m_2 = m_1 v_1 / v_2 = 1$ kg × 0.00897 / 0.37182 = **0.0241 kg**

From energy eq.

${}_1W_2 = m_1 u_1 - m_2 u_2 - m_{ex} h_{ex}$
= 1 × 331.39 − 0.0241 × 281.29 − 0.9759 × 312.85 [kg kJ/kg]
= **19.3 kJ**

Reversible shaft work, Bernoulli equation

7.57

A river flowing at 0.5 m/s across 1 m high and 10 m wide area has a dam that creates an elevation difference of 2 m. How much energy can a turbine deliver per day if 80% of the potential energy can be extracted as work?

CV Dam from upstream to downstream 2 m lower elevation

Continuity Eq.: $\dot{m} = \text{constant} = \dot{m}_e = \dot{m}_i = A_e \mathbf{V}_e / v_e = A_i \mathbf{V}_i / v_i$
Find the mass flow rate

$$\dot{m} = A\mathbf{V}_i/v = \rho A \mathbf{V}_i = 997 \text{ kg/m}^3 \times (1 \times 10) \text{ m}^2 \times 0.5 \text{ m/s} = 4985 \text{ kg/s}$$

Energy Eq.: $0 = \dot{m} (h_i + 0.5\mathbf{V}_i^2 + gZ_i) - \dot{m} (h_e + 0.5\mathbf{V}_e^2 + gZ_e) - \dot{W}$

The velocity in and out is the same assuming same flow area and the two enthalpies are also the same since same T and $P = P_0$.

This is consistent with Eq.7.14 [$w = g(Z_i - Z_e)$ - loss]

$$\dot{W} = 0.8 \, \dot{m} \, g(Z_i - Z_e) = 0.8 \times 4985 \text{ kg/s} \times 9.807 \text{ m/s}^2 \times 2 \text{ m}$$
$$= 78\,221 \text{ J/s} = 78\,221 \text{ W} = 78.2 \text{ kW}$$

$$W = \dot{W} \, \Delta t = 78.2 \text{ (kJ/s)} \times 24 \times 60 \times 60 \text{ s} = \mathbf{6.76 \text{ GJ}}$$

7.58

How much liquid water at 15$^\text{o}$C can be pumped from 100 kPa to 300 kPa with a 3 kW motor?

Incompressible flow (liquid water) and we assume reversible. Then the shaftwork is from Eq.7.15 (7.16)

$$w = -\int v\ dP = -v\ \Delta P = -0.001\ \text{m}^3/\text{kg}\ (300-100)\ \text{kPa}$$
$$= -0.2\ \text{kJ/kg}$$

$$\dot{m} = \frac{\dot{W}}{-w} = \frac{3}{0.2}\ \frac{\text{kW}}{\text{kJ/kg}} = \textbf{15 kg/s}$$

Remark: The pump should also generate the kinetic energy, so the $\dot{m}$ will be smaller.

7.59

A large storage tank contains saturated liquid nitrogen at ambient pressure, 100 kPa; it is to be pumped to 500 kPa and fed to a pipeline at the rate of 0.5 kg/s. How much power input is required for the pump, assuming it to be reversible?

Solution:

C.V. Pump, liquid is assumed to be incompressible.

Table B.6.1 at $P_i = 101.3$ kPa , $v_{Fi} = 0.00124$ m^3/kg

Eq.7.15

$w_{PUMP} = - w_{cv} = \int v dP \approx v_{Fi}(P_e - P_i)$

$= 0.00124(500 - 101) = 0.494$ kJ/kg

liquid nitrogen

$\dot{W}_{PUMP} = \dot{m} w_{PUMP} = 0.5$ kg/s $(0.494$ kJ/kg$) = \textbf{0.247 kW}$

7.60

Liquid water at 300 kPa, 15°C flows in a garden hose with a small ideal nozzle. How high a velocity can be generated? If the water jet is directed straight up how high will it go?

Solution:

Liquid water is incompressible and we will assume process is reversible.

Bernoulli's Eq. across the nozzle Eq.7.16: $\qquad v\Delta P = \Delta(\frac{1}{2}\mathbf{V}^2)$

$$\mathbf{V} = \sqrt{2v\Delta P} = \sqrt{2\times0.001001 \times (300\text{-}101) \times 1000} = \mathbf{19.96 \ m/s}$$

Bernoulli's Eq.7.16 for the column: $\qquad \Delta(\frac{1}{2}\mathbf{V}^2) = \Delta gZ$

$$\Delta Z = \Delta(\frac{1}{2}\mathbf{V}^2)/g = v\Delta P/g$$

$\qquad = 0.001001 \ m^3/kg \times (300 - 101) \ kPa \times 1000 \ (Pa/kPa)/ \ 9.807 \ m/s^2$

$\qquad = \mathbf{20.3 \ m}$

7.61

A wave comes rolling in to the beach at 2 m/s horizontal velocity. Neglect friction and find how high up (elevation) on the beach the wave will reach.

We will assume a steady reversible single flow at constant pressure and temperature for the incompressible liquid water. The water will flow in and up the sloped beach until it stops ($V = 0$) so Bernoulli Eq.7.17 leads to

$$gz_{in} + \frac{1}{2}V^2_{in} = gz_{ex} + 0$$

$$(z_{ex} - z_{in}) = \frac{1}{2g}V^2_{in} = \frac{1}{2 \times 9.807 \text{ m/s}^2} 2^2 \text{ (m/s)}^2 = \textbf{0.204 m}$$

7.62

A small pump takes in water at 20°C, 100 kPa and pumps it to 2.5 MPa at a flow rate of 100 kg/min. Find the required pump power input.

Solution:

C.V. Pump. Assume reversible pump and incompressible flow.

With single steady state flow it leads to the work in Eq.7.15

$$w_p = -\int v\,dP = -v_i(P_e - P_i) = -0.001002 \text{ m}^3/\text{kg} \ (2500 - 100) \text{ kPa} = -2.4 \text{ kJ/kg}$$

$$\dot{W}_p = \dot{m}w_p = \frac{100}{60} \frac{\text{kg/min}}{\text{sec/min}} \ (-2.4 \text{ kJ/kg}) = \textbf{-4.0 kW}$$

7.63

An irrigation pump takes water from a river at 10°C, 100 kPa and pumps it up to an open canal at a 50 m higher elevation. The pipe diameter in and out of the pump is 0.1 m and the motor driving the pump is 5 hp. Neglect kinetic energies and friction, find the maximum possible mass flow rate.

CV the pump. The flow is incompressible and steady flow. The pump work is the difference between the flow work in and out and from Bernoulli's eq. for the pipe that is equal to the potential energy increase sincle pump inlet pressure and pipe outlet pressure are the same.

$$w_p = v \, \Delta P = g \, \Delta Z = 9.81 \times 50 \text{ J/kg} = 0.49 \text{ kJ/kg}$$

The horsepower is converted from Table A.1

$$\dot{W}_{motor} = 5 \text{ hp} = 5 \text{ hp} \times 0.746 \text{ kW/hp} = 3.73 \text{ kW}$$

$$\dot{m} = \dot{W}_{motor} / w_p = 3.73 \text{ kW} / 0.49 \text{ kJ/kg} = \textbf{7.6 kg/s}$$

Comment:

$$\dot{m} = AV/v \quad \Rightarrow \quad V = \frac{\dot{m} v}{A} = \frac{4\dot{m}}{\rho \, \pi \, D^2} = \frac{4 \times 7.6}{997 \times \pi \times 0.1^2} = 0.97 \text{ m/s}$$

The power to generated the kinetic energy is

$$\text{Power} = \dot{m} \, 0.5 \, V^2 = 7.6 \text{ kg/s} \times 0.5 \times 0.97^2 \text{ (m/s)}^2 = 3.57 \text{ W}$$

This is insignificant relative to the power needed for the potential energy increase.

Pump inlet and the pipe exit both have close to atmospheric pressure.

Borgnakke and Sonntag

7.64

A firefighter on a ladder 25 m above ground should be able to spray water an additional 10 m up with the hose nozzle of exit diameter 2.5 cm. Assume a water pump on the ground and a reversible flow (hose, nozzle included) and find the minimum required power.

Solution:

C.V.: pump + hose + water column, total height difference 35 m.

Continuity Eq.4.3, 6.11: $\dot{m}_{in} = \dot{m}_{ex} = (\rho A \mathbf{V})_{nozzle}$

Energy Eq.4.12: $\dot{m}(-w_p) + \dot{m}(h + \mathbf{V}^2/2 + gz)_{in} = \dot{m}(h + \mathbf{V}^2/2 + gz)_{ex}$

Process: $h_{in} \cong h_{ex}$, $\mathbf{V}_{in} \cong \mathbf{V}_{ex} = 0$, $z_{ex} - z_{in} = 35$ m , $\rho = 1/v \cong 1/v_f$

$-w_p = g(z_{ex} - z_{in}) = 9.81 \times (35 - 0) = 343.2$ J/kg

The velocity in the exit nozzle is such that it can rise 10 m. Make that column a C.V. for which Bernoulli Eq.7.17 is:

$$gz_{noz} + \frac{1}{2}\mathbf{V}^2_{noz} = gz_{ex} + 0$$
$$\mathbf{V}_{noz} = \sqrt{2g(z_{ex} - z_{noz})}$$
$$= \sqrt{2 \times 9.81 \times 10} = 14 \text{ m/s}$$

$$\dot{m} = \frac{\pi}{v_f}\left(\frac{D}{2}\right)^2 \mathbf{V}_{noz} = (\pi/4)\, 0.025^2 \times 14 / 0.001 = 6.873 \text{ kg/s}$$

$$-\dot{W}_p = -\dot{m}w_p = 6.873 \text{ kg/s} \times 343.2 \text{ J/kg} = \mathbf{2.36 \text{ kW}}$$

7.65

Saturated R-410A at -10°C is pumped/compressed to a pressure of 2.0 MPa at the rate of 0.5 kg/s in a reversible adiabatic process. Calculate the power required and the exit temperature for the two cases of inlet state of the R-410A:

     a) quality of 100 %.

     b) quality of 0 %.

Solution:

C.V.: Pump/Compressor, $\dot{m} = 0.5$ kg/s, R-410A

a) State 1: Table B.5.1, $T_1 = -10^oC$, $x_1 = 1.0$ Saturated vapor

    $P_1 = P_g = 573.1$ kPa, $h_1 = h_g = 275.78$ kJ/kg, $s_1 = s_g = 1.0567$ kJ/kg-K

    Assume Compressor is isentropic, $s_2 = s_1 = 1.0567$ kJ/kg-K

    $h_2 = 334.18$ kJ/kg, **$T_2 = 52^oC$**

    Energy Eq.4.13: $q_c + h_1 = h_2 + w_c$; $q_c = 0$

    $w_{cs} = h_1 - h_2 = -58.4$ kJ/kg;   => $\dot{W}_C = \dot{m}w_C = $ **-29.2 kW**

b) State 1: $T_1 = -10^oC$, $x_1 = 0$ Saturated liquid. This is a pump.

    $P_1 = 573.1$ kPa, $h_1 = h_f = 42.80$ kJ/kg, $v_1 = v_f = 0.000827$ m^3/kg

    Energy Eq.4.13: $q_p + h_1 = h_2 + w_p$; $q_p = 0$

    Assume Pump is isentropic and the liquid is incompressible, Eq.7.15:

        $w_{ps} = -\int v\ dP = -v_1(P_2 - P_1) = -1.18$ kJ/kg

    $h_2 = h_1 - w_p = 42.80 - (-1.18) = 43.98$ kJ/kg, $P_2 = 2$ MPa

    Assume State 2 is approximately a saturated liquid   =>   **$T_2 \cong -9.2^oC$**

            $\dot{W}_P = \dot{m}w_P = $ **-0.59 kW**

7.66

Liquid water at ambient conditions, 100 kPa, 25°C, enters a pump at the rate of 0.5 kg/s. Power input to the pump is 3 kW. Assuming the pump process to be reversible, determine the pump exit pressure and temperature.

Solution:

C.V. Pump. Steady single inlet and exit flow with no heat transfer.

Energy Eq.4.13: $w = h_i - h_e = \dot{W}/\dot{m} = -3/0.5 = -6.0$ kJ/kg

Using also incompressible media we can use Eq.7.15

$$w = -\int v dP \approx -v_i(P_e - P_i) = -0.001003(P_e - 100)$$

from which we can solve for the exit pressure

$$P_e = 100 + 6.0/0.001003 = 6082 \text{ kPa} = \textbf{6.082 MPa}$$

$-\dot{W} = 3$ kW, $P_i = 100$ kPa

$T_i = 25°C$, $\dot{m} = 0.5$ kg/s

Energy Eq.: $h_e = h_i - w = 104.87 - (-6) = 110.87$ kJ/kg

Use Table B.1.4 at 5 MPa => **$T_e = 25.3°C$**

Remark:

If we use the software we get: $\left.\begin{array}{l} s_i = 0.36736 = s_e \\ \text{At } s_e \text{ \& } P_e \end{array}\right\} \rightarrow T_e = \textbf{25.1°C}$

7.67

The underwater bulb nose of a container ship has a velocity relative to the ocean water as 10 m/s. What is the pressure at the front stagnation point that is 2 m down from the water surface.

Solution:

C.V. A stream line of flow from the freestream to the stagnation point on the front of the bulb nose.

Eq.7.17: $v(P_e - P_i) + \frac{1}{2}(\mathbf{V}_e^2 - \mathbf{V}_i^2) + g(Z_e - Z_i) = 0$

Horizontal so $Z_e = Z_i$ and $\mathbf{V}_e = 0$

$$\Delta P = \frac{1}{2v}\mathbf{V}_i^2 = \frac{10^2}{0.001001 \times 2000} = 49.95 \text{ kPa}$$

$P_i = P_o + gH/v = 101 \text{ kPa} + 9.81 \text{ m/s}^2 \times 2 \text{ m}/(0.001001 \text{ m}^3/\text{kg} \times 1000 \text{ J/kJ})$

   $= 120.6 \text{ kPa}$

$P_e = P_i + \Delta P = 120.6 + 49.95 = \mathbf{170.6 \text{ kPa}}$

Borgnakke and Sonntag

7.68

A small water pump on ground level has an inlet pipe down into a well at a depth H with the water at 100 kPa, 15°C. The pump delivers water at 400 kPa to a building. The absolute pressure of the water must be at least twice the saturation pressure to avoid cavitation. What is the maximum depth this setup will allow?

Solution:

C.V. Pipe in well, no work, no heat transfer

From Table B.1.1

$P_{inlet\ pump} \geq 2\ P_{sat,\ 15C} = 2 \times 1.705 = 3.41$ kPa

Process:

Assume $\Delta KE \approx \varnothing$, $v \approx$ constant. =>

Bernoulli Eq.7.16:

$v\ \Delta P + g\ H = 0$ =>

1000 J/kJ $\times\ 0.001001$ m^3/kg $\times$ ($3.41 - 100$) kPa $+ 9.80665$ m/s$^2 \times H = 0$

$\Rightarrow$ **H = 9.86 m**

Since flow has some kinetic energy and there are losses in the pipe the height is overestimated. Also the start transient would generate a very low inlet pressure due to the necessary dynamic forces (it moves flow by suction).

7.69

A pump/compressor pumps a substance from 150 kPa, 10°C to 1 MPa in a reversible adiabatic process. The exit pipe has a small crack, so that a small amount leaks to the atmosphere at 100 kPa. If the substance is (a) water, (b) R-134a, find the temperature after compression and the temperature of the leak flow as it enters the atmosphere neglecting kinetic energies.

Solution:

C.V.: Compressor, reversible adiabatic

Eq.4.13: $h_1 - w_c = h_2$;   Eq.7.8: $s_1 = s_2$

State 2: P_2, $s_2 = s_1$

C.V.: Crack (Steady throttling process)

Eq.4.13: $h_3 = h_2$;   Eq.7.8: $s_3 = s_2 + s_{gen}$

State 3: P_3, $h_3 = h_2$

a) Water 1:   compressed liquid, Table B.1.1

$$-w_c = + \int v dP = v_{f1}(P_2 - P_1) = 0.001 \text{ m}^3/\text{kg} \times (1000 - 150) \text{ kPa}$$

$$= 0.85 \text{ kJ/kg}$$

$$h_2 = h_1 - w_c = 41.99 + 0.85 = 42.84 \text{ kJ/kg} \implies \mathbf{T_2 = 10.2°C}$$

P_3 , $h_3 = h_2$ $\implies$ compressed liquid at **~10.2°C**

States 1 and 3 are at 150, 100 kPa, and same v.

b) R-134a 1:   superheated vapor, Table B.5.2, $s_1 = 1.8220$ kJ/kg-K

$s_2 = s_1$ & P_2 $\implies$ **$T_2 = 83.1$°C** , $h_2 = 455.57$ kJ/kg

$-w_c = h_2 - h_1 = 455.57 - 410.60 = 44.97$ kJ/kg

P_3 , $h_3 = h_2$ $\implies$ **$T_3 = 60.6$°C**

7.70

Atmospheric air at 100 kPa, 17°C blows at 60 km/h towards the side of a building. Assume the air is nearly incompressible find the pressure and the temperature at the stagnation point (zero velocity) on the wall.

Solution:

C.V. A stream line of flow from the freestream to the wall.

Eq.7.17:

$$v(P_e\text{-}P_i) \; + \frac{1}{2}\,(\mathbf{V}_e^2\text{-}\mathbf{V}_i^2) + g(Z_e \text{ - } Z_i) \; = 0$$

$$\mathbf{V}_i = 60\,\frac{km}{h} \times 1000\,\frac{m}{km} \times \frac{1}{3600}\,\frac{h}{s} = 16.667 \text{ m/s}$$

$$v = \frac{RT_i}{P_i} = \frac{0.287 \times 290.15}{100}\,\frac{kJ/kg}{kPa} = 0.8323\,\frac{m^3}{kg}$$

$$\Delta P = \frac{1}{2v}\,\mathbf{V}_i^2 = \frac{16.667^2}{0.8323 \times 2000}\,\frac{(m/s)^2}{m^3/kg \times J/kJ} = 0.17 \text{ kPa}$$

$$P_e = P_i + \Delta P = 100.17 \text{ kPa}$$

Then Eq.6.23 for an isentropic process:

$$T_e = T_i\,(P_e/P_i)^{0.286} = 290.15 \text{ K} \times 1.0005 = \textbf{290.3 K}$$

Very small effect due to low velocity and air is light (large specific volume)

7.71

A small pump is driven by a 2 kW motor with liquid water at 150 kPa, 10°C entering. Find the maximum water flow rate you can get with an exit pressure of 1 MPa and negligible kinetic energies. The exit flow goes through a small hole in a spray nozzle out to the atmosphere at 100 kPa. Find the spray velocity.

Solution:

C.V. Pump. Liquid water is incompressible so work from Eq.7.15

$$\dot{W} = \dot{m}w = -\dot{m}v(P_e - P_i) \Rightarrow$$

$$\dot{m} = \dot{W}/ [-v(P_e - P_i)] = -2/[-0.001003 \text{ m}^3/\text{kg} (1000 - 150) \text{ kPa}]$$
$$= 2.35 \text{ kg/s}$$

C.V Nozzle. No work, no heat transfer, $v \approx$ constant => Bernoulli Eq.7.17

$$\frac{1}{2}\mathbf{V}_{ex}^2 = v\Delta P = 0.001 \text{ m}^3/\text{kg} (1000 - 100) \text{ kPa} = 0.9 \text{ kJ/kg} = 900 \text{ J/kg}$$

$$\mathbf{V}_{ex} = \sqrt{2 \times 900 \text{ J/kg}} = \textbf{42.4 m/s}$$

7.72

A speed boat has a small hole in the front of the drive with the propeller that sticks down into the water at a water depth of 0.25 m. Assume we have a stagnation point at that hole when the boat is sailing with 40 km/h, what is the total pressure there?

Solution:

C.V. A stream line of flow from the freestream to the wall.

Eq.7.17: $v(P_e-P_i) + \frac{1}{2}(\mathbf{V}_e^2-\mathbf{V}_i^2) + g(Z_e - Z_i) = 0$

$\mathbf{V}_i = 40\,\frac{km}{h} \times 1000\,\frac{m}{km} \times \frac{1}{3600}\,\frac{h}{s} = 11.111 \text{ m/s}$

$\Delta P = \frac{1}{2v}\mathbf{V}_i^2 = \frac{11.111^2}{0.001001 \times 2000} = 61.66 \text{ kPa}$

$P_i = P_o + gH/v = 101 + 9.81 \times 0.25/\,(0.001001 \times 1000) = 103.45 \text{ kPa}$

$P_e = P_i + \Delta P = 103.45 + 61.66 = \textbf{165 kPa}$

Remark: This is fast for a boat

7.73

You drive on the highway with 120 km/h on a day with 17°C, 100 kPa atmosphere. When you put your hand out of the window flat against the wind you feel the force from the air stagnating, i.e. it comes to relative zero velocity on your skin. Assume the air is nearly incompressible and find the air temperature and pressure right on your hand.

Solution:

Energy Eq.4.13:
$$\frac{1}{2}\mathbf{V}^2 + h_o = h_{st}$$

$$T_{st} = T_o + \frac{1}{2}\mathbf{V}^2/C_p = 17 + \frac{1}{2}[(120\times1000)/3600]^2 \times (1/1004)$$

$$= 17 + 555.5/1004 = \mathbf{17.6°C}$$

$$v = RT_o/P_o = 0.287 \times 290/100 = 0.8323 \text{ m}^3/\text{kg}$$

From Bernoulli Eq.7.17:

$$v\Delta P = \frac{1}{2}\mathbf{V}^2$$

$$P_{st} = P_o + \frac{1}{2}\mathbf{V}^2/v = 100 + 555.5/(0.8323 \times 1000) = \mathbf{100.67 \text{ kPa}}$$

Borgnakke and Sonntag

7.74

A small dam has a pipe carrying liquid water at 150 kPa, 20°C with a flow rate of 2000 kg/s in a 0.5 m diameter pipe. The pipe runs to the bottom of the dam 15 m lower into a turbine with pipe diameter 0.35 m. Assume no friction or heat transfer in the pipe and find the pressure of the turbine inlet. If the turbine exhausts to 100 kPa with negligible kinetic energy what is the rate of work?

Solution:

C.V. Pipe. Steady flow no work, no heat transfer.

States: compressed liquid B.1.1 $v_2 \approx v_1 \approx v_f = 0.001002$ m^3/kg

Continuity Eq.4.3: $\dot{m} = \rho\, A\mathbf{V} = A\mathbf{V}/v$

$$\mathbf{V}_1 = \dot{m}v_1/A_1 = 2000 \text{ kg/s} \times 0.001002 \text{ m}^3/\text{kg} / (\tfrac{\pi}{4} 0.5^2 \text{ m}^2) = 10.2 \text{ m s}^{-1}$$

$$\mathbf{V}_2 = \dot{m}v_2/A_2 = 2000 \text{ kg/s} \times 0.001002 \text{ m}^3/\text{kg} /(\tfrac{\pi}{4} 0.35^2 \text{ m}^2) = 20.83 \text{ m s}^{-1}$$

From Bernoulli Eq.7.16 for the pipe (incompressible substance):

$$v(P_2 - P_1) + \tfrac{1}{2}(\mathbf{V}_2^2 - \mathbf{V}_1^2) + g(Z_2 - Z_1) = 0 \Rightarrow$$

$$P_2 = P_1 + [\tfrac{1}{2}(\mathbf{V}_1^2 - \mathbf{V}_2^2) + g(Z_1 - Z_2)]/v$$

$$= 150 \text{ kPa} + \frac{\tfrac{1}{2}\times(10.2^2 - 20.83^2) + 9.80665 \times 15}{1000 \times 0.001002} \frac{\text{m}^2\text{s}^{-2}}{\text{J/kJ} \times \text{m}^3/\text{kg}}$$

$$= 150 - 17.8 = \mathbf{132.2\ kPa}$$

Note that the pressure at the bottom should be higher due to the elevation difference but lower due to the acceleration. Now apply the energy equation Eq.7.13 for the total control volume

$$w = -\int v\, dP + \tfrac{1}{2}(\mathbf{V}_1^2 - \mathbf{V}_3^2) + g(Z_1 - Z_3)$$

$$= -0.001002\,(100 - 150) + [\tfrac{1}{2}\times 10.2^2 + 9.80665 \times 15]/1000 = 0.25 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}w = 2000 \text{ kg/s} \times 0.25 \text{ kJ/kg} = \mathbf{500\ kW}$$

7.75

An air flow at 100 kPa, 290 K, 100 m/s is directed towards a wall. At the wall the flow stagnates (comes to zero velocity) without any heat transfer. Find the stagnation pressure a) assuming incompressible flow b) assume an adiabatic compression. Hint: T comes from the energy equation.

Solution:

Ideal gas: $\quad v = RT_o/P_o = 0.287 \times 290/100 = 0.8323 \ m^3/kg$

Kinetic energy: $\quad \frac{1}{2} \mathbf{V}^2 = \frac{1}{2} (100^2/1000) = 5 \ kJ/kg$

a) Reversible and incompressible gives Bernoulli Eq.7.17:

$$\Delta P = \frac{1}{2} \mathbf{V}^2/v = 5/0.8323$$

$$= 6 \ kPa$$

$$P_{st} = P_o + \Delta P = 106 \ kPa$$

b) adiabatic compression

Energy Eq.4.13: $\quad \frac{1}{2} \mathbf{V}^2 + h_o = h_{st}$

$$h_{st} - h_o = \frac{1}{2} \mathbf{V}^2 = C_p \Delta T$$

$$\Delta T = \frac{1}{2} \mathbf{V}^2/C_p = 5/1.004 = 5°C$$

$$\Rightarrow \ T_{st} = 290 + 5 = 295 \ K$$

Entropy Eq.7.9 assume also reversible process:

$$s_o + s_{gen} + \int (1/T) \ dq = s_{st}$$

as $dq = 0$ and $s_{gen} = 0$ then it follows that $s = $ constant

This relation gives Eq.6.23:

$$P_{st} = P_o \left(\frac{T_{st}}{T_o}\right)^{\frac{k}{k-1}} = 100 \times (295/290)^{3.5} = \mathbf{106 \ kPa}$$

7.76

A flow of air at 100 kPa, 300 K enters a device and goes through a polytropic process with n = 1.3 before it exits at 800 K. Find the exit pressure, the specific work and heat transfer using constant specific heats.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.4.13: $h_1 + q = h_2 + w$ Neglect kinetic, potential energies

Entropy Eq.7.9: $s_1 + \int dq/T + s_{gen} = s_2$

$T_e = 800$ K; $T_i = 300$ K; $P_i = 100$ kPa

Process Eq.6.28: $P_e = P_i (T_e/ T_i)^{\frac{n}{n-1}} = 100 \, (800/300)^{\frac{1.3}{0.3}} = \mathbf{7\ 012\ kPa}$

and the process leads to Eq.7.18 for the work term

$$w = -\frac{n}{n-1} R \, (T_e - T_i) = -(1.3/0.3) \times 0.287 \text{ kJ/kg-K} \times (800 - 300) \text{ K}$$

$$= \mathbf{-621.8\ kJ/kg}$$

$q = h_e - h_i + w = C_P (T_e - T_i) + w$

$= 1.004$ kJ/kg-K $(800 - 300)$ K $- 621.8$ kJ/kg

$= \mathbf{-119.8\ kJ/kg}$

Notice:
dP > 0
so dw < 0

ds < 0
so dq < 0

7.77

Solver the previous problem but use the air tables A.7

Air at 100 kPa, 300 K, flows through a device at steady state with the exit at 1000 K during which it went through a polytropic process with n = 1.3. Find the exit pressure, the specific work and heat transfer.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.4.13: $h_1 + q = h_2 + w$ Neglect kinetic, potential energies

Entropy Eq.7.9: $s_1 + \int dq/T + s_{gen} = s_2$

$$T_e = 800 \text{ K}; \quad T_i = 300 \text{ K}; \quad P_i = 100 \text{ kPa}$$

Process Eq.6.28: $P_e = P_i \, (T_e/ T_i)^{\frac{n}{n-1}} = 100 \, (800/300)^{\frac{1.3}{0.3}} = \textbf{7 012 kPa}$

and the process leads to Eq.7.18 for the work term

$$w = -\frac{n}{n\text{-}1} R \, (T_e - T_i) = -\frac{1.3}{0.3} (1.3/0.3) \times 0.287 \text{ kJ/kg-K} \times (800 - 300) \text{ K}$$

$$= - \textbf{621.8 kJ/kg}$$

$q = h_e - h_i + w = 822.2 - 300.5 - \textbf{621.8}$

$$= -\textbf{100.1 kJ/kg}$$

Notice:
dP > 0
so dw < 0

ds < 0
so dq < 0

7.78

Helium gas enters a steady-flow expander at 800 kPa, 300°C, and exits at 120 kPa. The expansion process can be considered as a reversible polytropic process with exponent, $n = 1.3$. Calculate the mass flow rate for 150 kW power output from the expander.

Solution:

CV: expander, reversible polytropic process.
From Eq.6.28:

$$T_e = T_i \left(\frac{P_e}{P_i}\right)^{\frac{n-1}{n}} = 573.2 \left(\frac{120}{800}\right)^{\frac{0.3}{1.3}} = 370 \text{ K}$$

Work evaluated from Eq.7.18

$$w = -\int v dP = -\frac{nR}{n-1}(T_e - T_i) = \frac{-1.3 \times 2.07703}{0.3} \text{ kJ/kg-K } (370 - 573.2) \text{ K}$$

$$= 1828.9 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}/w = 150 \text{ kW } / 1828.9 \text{ kJ/kg} = \textbf{0.082 kg/s}$$

7.79

A flow of 4 kg/s ammonia goes through a device in a polytropic process with an inlet state of 150 kPa, -20°C and an exit state of 400 kPa, 60°C. Find the polytropic exponent n, the specific work and heat transfer.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.4.13: $h_1 + q = h_2 + w$ Neglect kinetic, potential energies

Entropy Eq.7.9: $s_1 + \int dq/T + s_{gen} = s_2$

Process Eq.6.27: $P_1 v_1^n = P_2 v_2^n$:

State 1: Table B.2.2 $v_1 = 0.79774$, $s_1 = 5.7465$ kJ/kg K, $h_1 = 1422.9$ kJ/kg

State 2: Table B.2.2 $v_2 = 0.3955$, $s_2 = 5.8560$ kJ/kg K, $h_2 = 1590.4$ kJ/kg

$\ln (P_2/P_1) = n \ln (v_1/v_2)$ $\Rightarrow$ $0.98083 = n \times 0.70163$

$n = \ln (P_2/P_1) / \ln (v_1/v_2) = $ **1.3979**

From the process and the integration of v dP gives Eq.7.18.

$$w_{shaft} = -\frac{n}{n-1} (P_2 v_2 - P_1 v_1) = -3.5132 \ (158.2 - 119.66) = \textbf{-135.4 kJ/kg}$$

$q = h_2 + w - h_1 = 1590.4 - 1422.9 - 135.4 = $ **32.1 kJ/kg**

Notice:
dP > 0
so dw < 0

ds > 0
so dq > 0

7.80

Calculate the air temperature and pressure at the stagnation point right in front of a meteorite entering the atmosphere (-50 °C, 50 kPa) with a velocity of 2000 m/s. Do this assuming air is incompressible at the given state and repeat for air being a compressible substance going through an adiabatic compression.

Solution:

Kinetic energy: $\frac{1}{2}\mathbf{V}^2 = \frac{1}{2}(2000)^2/1000 = 2000$ kJ/kg

Ideal gas: $v_{atm} = RT/P = 0.287$ kJ/kg-K $\times$ 223 K/50 kPa $= 1.28$ m^3/kg

a) incompressible

Energy Eq.4.13: $\Delta h = \frac{1}{2}\mathbf{V}^2 = 2000$ kJ/kg

If A.5 $\Delta T = \Delta h/C_p = 1992$ K   unreasonable, too high for that C_p

Use A.7: $h_{st} = h_o + \frac{1}{2}\mathbf{V}^2 = 223.22 + 2000 = 2223.3$ kJ/kg

$T_{st} = 1977$ K

Bernoulli (incompressible) Eq.7.17:

$\Delta P = P_{st} - P_o = \frac{1}{2}\mathbf{V}^2/v = 2000$ (m/s)^2/1.28 m^3/kg $= 1562.5$ kPa

$P_{st} = 1562.5 + 50 = 1612.5$ kPa

b) compressible

$T_{st} = 1977$ K   the same energy equation.

From A.7.1: $s^o_{T\,st} = 8.9517$ kJ/kg K; $s^o_{T\,o} = 6.5712$ kJ/kg K

Eq.6.19:

$P_{st} = P_o \times e^{(s^o_{T\,st} - s^o_{T\,o})/R}$

$= 50 \times \exp\left[\dfrac{8.9517 - 6.5712}{0.287}\right]$

$= \mathbf{200\ 075\ kPa}$

Notice that this is highly compressible, v is not constant.

Borgnakke and Sonntag

7.81

An expansion in a gas turbine can be approximated with a polytropic process with exponent n = 1.25. The inlet air is at 1200 K, 800 kPa and the exit pressure is 125 kPa with a mass flow rate of 0.75 kg/s. Find the turbine heat transfer and power output.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.4.13: $h_i + q = h_e + w$ Neglect kinetic, potential energies

Entropy Eq.7.9: $s_i + \int dq/T + s_{gen} = s_e$

Process Eq.6.28:

$$T_e = T_i \, (P_e/ P_i)^{\frac{n-1}{n}} = 1200 \, (125/800)^{\frac{0.25}{1.25}} = 827.84 \text{ K}$$

so the exit enthalpy is from Table A.7.1

$$h_e = 822.2 + \frac{27.84}{50}(877.4 - 822.2) = 852.94 \text{ kJ/kg}$$

The process leads to Eq.7.18 for the work term

$$\dot{W} = \dot{m}w = -\dot{m}\frac{nR}{n-1}(T_e - T_i) = -0.75 \frac{1.25 \times 0.287}{0.25} \times (827.84 - 1200)$$

$$= \textbf{400.5 kW}$$

Energy equation gives

$$\dot{Q} = \dot{m}q = \dot{m}(h_e - h_i) + \dot{W} = 0.75(852.94 - 1277.81) + 400.5$$

$$= -318.65 + 400.5 = \textbf{81.9 kW}$$

Notice:
dP < 0
so dw > 0

ds > 0
so dq > 0

Notice this process has some heat transfer in during expansion which is unusual. The typical process would have n = 1.5 with a heat loss.

Steady state irreversible processes

7.82

Consider a steam turbine with inlet 2 MPa, 350°C and an exhaust flow as saturated vapor 100 kPa. There is a heat loss of 6 kJ/kg to the ambient. Is the turbine possible?

Solution:

At the given states

Table B.1.3: s_i = 6.9562 kJ/kg K; s_e = 7.3593 kJ/kg K

Do the second law for the turbine, Eq.7.8

$$\dot{m}_e s_e = \dot{m}_i s_i + \int d\dot{Q}/T + \dot{S}_{gen}$$

$$s_e = s_i + \int dq/T + s_{gen}$$

$$s_{gen} = s_e - s_i - \int dq/T = 7.3593 - 6.9562 - 6/298 = 0.383 \text{ kJ/kgK} > 0$$

Entropy goes up even if q goes out. **This is an irreversible process**.

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7.83

A large condenser in a steam power plant dumps 15 MW by condensing saturated water vapor at 45°C to saturated liquid. What is the water flow rate and the entropy generation rate with an ambient at 25°C?

Solution:

This process transfers heat over a finite temperature difference between the water inside the condenser and the outside ambient (cooling water from the sea, lake or river or atmospheric air)

C.V. The Condensing water flow

Energy Eq.: $0 = \dot{m} \, (h_g - h_f) - \dot{Q}_{out}$

$$\dot{m} = \dot{Q}_{out} / h_{fg} = \frac{15\ 000}{2394.77} \frac{kW}{kJ/kg} = \textbf{6.264 kg/s}$$

C.V. The wall that separates the inside 45°C water from the ambient at 25°C.

Entropy Eq. 7.1 for steady state operation:

$$\frac{dS}{dt} = 0 = \sum \frac{\dot{Q}}{T} + \dot{S}_{gen} = \frac{\dot{Q}}{T_{45}} - \frac{\dot{Q}}{T_{25}} + \dot{S}_{gen}$$

$$\dot{S}_{gen} = \frac{15}{25 + 273} \frac{MW}{K} - \frac{15}{45 + 273} \frac{MW}{K} = \textbf{3.17} \frac{kW}{K}$$

7.84

R-410A at -5$^\text{o}$C, 700 kPa is throttled so it becomes cold at –40$^\text{o}$C. What is exit P and the specific entropy generation?

C.V. Throttle. Steady state,

Process with: q = w = 0; and $\mathbf{V}_i = \mathbf{V}_e$, $Z_i = Z_e$

Energy Eq.4.13: $h_i = h_e$,

Inlet state: Table B.4.1 $h_i = 50.22$ kJ/kg, $s_i = 0.1989$ kJ/kg-K

(slightly compressed liquid)

Exit state: Table B.4.1 since $h < h_g = 262.83$ kJ/kg it is two-phase

P = Psat = **175 kPa**

$$x_e = \frac{h_e - h_{f\,e}}{h_{fg\,e}} = \frac{50.22 - 0}{262.83} = 0.19107, \; s_e = 0 + x_e \, 1.1273 = 0.2154 \text{ kJ/kg-K}$$

$s_{gen} = s_e - s_i = 0.2154 - 0.1989 = $ **0.0165 kJ/kg-K**

7.85

Ammonia is throttled from 1.5 MPa, 35$^{\mathrm{o}}$C to a pressure of 291 kPa in a refrigerator system. Find the exit temperature and the specific entropy generation in this process.

The throttle process described in Example 6.5 is an irreversible process. Find the entropy generation per kg ammonia in the throttling process.

Solution:

The process is adiabatic and irreversible. The consideration with the energy given in the example resulted in a constant h and two-phase exit flow.

Table B.2.1: $s_i = 1.2792$ kJ/kg K, $h_i = 346.8$ kJ/kg

Table B.2.1: $x = (h_i - h_f)/h_{fg} = (346.8 - 134.41)/1296.4 = 0.1638$

$$s_e = s_f + x_e\, s_{fg} = 0.5408 + 0.1638 \times 4.9265 = 1.34776 \text{ kJ/kg K}$$

We assumed no heat transfer so the entropy equation Eq.7.9 gives

$$s_{gen} = s_e - s_i - \int dq/T = 1.34776 - 1.2792 - 0 = \textbf{0.0686 kJ/kg K}$$

7.86

A compressor in a commercial refrigerator receives R-410A at -25°C and x = 1. The exit is at 1000 kPa and 20°C. Is this compressor possible?

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also assume no heat transfer and $Z_i = Z_e$

From Table B.4.1 : h_i = 269.77 kJ/kg, s_i = 1.0893 kJ/kg-K

From Table B.4.2 : h_e = 295.49 kJ/kg, s_e = 1.073 kJ/kg-K

Entropy gives

$$s_{gen} = s_e - s_i - \int dq/T = 1.073 – 1.0893 - \int dq/T = negative$$

The result is negative unless dq is negative (it should go out, but T < T ambient) so this compressor is **impossible**

7.87

R-134a at 30$^{\text{o}}$C, 800 kPa is throttled in a steady flow to a lower pressure so it comes out at –10$^{\text{o}}$C. What is the specific entropy generation?

Solution:

The process is adiabatic and irreversible. The consideration of the energy given in example 6.5 resulted in a constant h and two-phase exit flow.

Table B.4.1: $h_i = 241.79$ kJ/kg, $s_i = 1.143$ kJ/kg-K (compressed liquid)

State 2: -10$^{\text{o}}$C, $h_e = h_i < h_g$ so two-phase $x_e = (h_e - h_f)/h_{fg} = 0.267$

Table B.4.1: $s_e = s_f + x_e \, s_{fg} = 0.9507 + 0.267 \times 0.7812 = 1.16$ kJ/kg-K

We assumed no heat transfer so the entropy equation Eq.7.9 gives

$$s_{gen} = s_e - s_i - \int dq/T = 1.16 - 1.143 - 0 = \textbf{0.017 kJ/kg K}$$

7.88

Analyze the steam turbine described in Problem 4.84. Is it possible?

Solution:

C.V. Turbine. Steady flow and adiabatic.

Continuity Eq.4.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$;

Energy Eq.4.10: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3 + \dot{W}$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3$

States from Table B.1.3: $s_1 = 7.2337$, $s_2 = 7.3010$, $s_3 = 7.5066$ kJ/kg-K

$\dot{S}_{gen} = 1 \times 7.301 + 4 \times 7.5066 - 5 \times 7.2337 =$ **1.16 kW/K   > 0**

Since it is positive => **possible.**

Notice the entropy is increasing through the turbine: $s_1 < s_2 < s_3$

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7.89

Two flowstreams of water, one at 0.6 MPa, saturated vapor, and the other at 0.6 MPa, 600°C, mix adiabatically in a steady flow process to produce a single flow out at 0.6 MPa, 400°C. Find the total entropy generation for this process.
Solution:

Continuity Eq.4.9: $\dot{m}_3 = \dot{m}_1 + \dot{m}_2,$

Energy Eq.4.10: $\dot{m}_3 h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2$

1: B.1.2 $h_1 = 2756.8$ kJ/kg, $s_1 = 6.760$ kJ/kg-K
2: B.1.3 $h_2 = 3700.9$ kJ/kg, $s_2 = 8.2674$ kJ/kg-K
3: B.1.3 $h_3 = 3270.3$ kJ/kg, $s_3 = 7.7078$ kJ/kg-K

$$\Rightarrow \dot{m}_1/\dot{m}_3 = (h_3 - h_2) / (h_1 - h_2) = 0.456$$

Entropy Eq.7.7: $\dot{m}_3 s_3 = \dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen}$ $\Rightarrow$

$$\dot{S}_{gen}/\dot{m}_3 = s_3 - (\dot{m}_1/\dot{m}_3) s_1 - (\dot{m}_2/\dot{m}_3) s_2$$
$$= 7.7078 - 0.456 \times 6.760 - 0.544 \times 8.2674 = \mathbf{0.128\ kJ/kg\ K}$$

The mixing process generates entropy. The two inlet flows could have exchanged energy (they have different T) through some heat engines and produced work, the process failed to do that, thus irreversible.

7.90

A geothermal supply of hot water at 500 kPa, 150°C is fed to an insulated flash evaporator at the rate of 1.5 kg/s. A stream of saturated liquid at 200 kPa is drained from the bottom of the chamber and a stream of saturated vapor at 200 kPa is drawn from the top and fed to a turbine. Find the rate of entropy generation in the flash evaporator.

Solution:

Two-phase out of the valve. The liquid drops to the bottom.

Continuity Eq.4.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$

Energy Eq.4.10: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{S}_{gen} + \int d\dot{Q}/T = \dot{m}_2 s_2 + \dot{m}_3 s_3$

Process: $\dot{Q} = 0$, irreversible (throttle)

B.1.1 $h_1 = 632.18$ kJ/kg, $s_1 = 1.8417$ kJ/kg-K

B.1.2 $h_3 = 2706.63$ kJ/kg, $s_3 = 7.1271$ kJ/kg-K,

$h_2 = 504.68$ kJ/kg, $s_2 = 1.53$ kJ/kg-K

From the energy equation we solve for the flow rate

$$\dot{m}_3 = \dot{m}_1(h_1 - h_2)/(h_3 - h_2) = 1.5 \times 0.0579 = 0.08685 \text{ kg/s}$$

Continuity equation gives: $\dot{m}_2 = \dot{m}_1 - \dot{m}_2 = 1.41315$ kg/s

Entropy equation now leads to

$$\dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3 - \dot{m}_1 s_1$$

$$= 1.41315 \times 1.53 + 0.08685 \times 7.127 - 1.5 \times 1.8417$$

$$= \textbf{0.01855 kW/K}$$

7.91

A compressor in a commercial refrigerator receives R-410A at -25$^{\text{o}}$C and x = 1. The exit is at 2000 kPa and 80$^{\text{o}}$C. Neglect kinetic energies and find the specific entropy generation.

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also assume no heat transfer q = 0 and $Z_1 = Z_2$

Entropy Eq.7.9: $s_i + \int dq/T + s_{gen} = s_e = s_i + 0 + s_{gen}$

From Table B.4.1 : $s_i = 1.0893$ kJ/kgK

From Table B.4.2 : $s_e = 1.1537$ kJ/kgK

Entropy generation becomes

$s_{gen} = s_e - s_i = 1.1537 - 1.0893 = \textbf{0.0644 kJ/kgK}$

7.92

A steam turbine has an inlet of 2 kg/s water at 1000 kPa and 400°C with velocity of 15 m/s. The exit is at 100 kPa, 150°C and very low velocity. Find the power produced and the rate of entropy generation.

Solution:

C.V. Turbine. Steady flow and adiabatic.

Continuity Eq.4.9: $\dot{m}_1 = \dot{m}_2$;

Energy Eq.4.10: $\dot{m}_1(h_1 + \frac{1}{2}\mathbf{V}^2) = \dot{m}_2 h_2 + \dot{W}$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2$

States from Table B.1.3: $h_1 = 3263.88$ kJ/kg, $s_1 = 7.4650$ kJ/kgK,

$h_2 = 2776.38$ kJ/kg, $s_2 = 7.6133$ kJ/kgK

$$\dot{W} = \dot{m}_1(h_1 + \frac{1}{2}\mathbf{V}^2 - h_2) = 2 \text{ kg/s } (3263.88 + \frac{1}{2}\frac{15^2}{1000} - 2776.38) \text{ kJ/kg}$$

$= \textbf{975 kW}$

$\dot{S}_{gen} = \dot{m}_1(s_2 - s_1) = 2$ kg/s $(7.6133 - 7.4650)$ kJ/kgK = **0.3 kW/K**

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7.93

A factory generates compressed air from ambient 100 kPa, 17°C, by compression to 1000 kPa, 600 K after which it cools in a constant pressure cooler to 300 K by heat transfer to the ambient. Find the specific entropy generation in the compressor and in the cooler.

Solution
C.V. air compressor q = 0

Continuity Eq.: $\dot{m}_2 = \dot{m}_1$; Energy Eq.4.13: $0 = h_1 + w_{c\ in} - h_2$
Entropy Eq.: $0 = s_1 - s_2 + s_{gen\ comp}$

Table A.7 State 1: $h_1 = 290.19$ kJ/kg, $s^o_{T1} = 6.83521$ kJ/kg-K,

Table A.7 State 2: $h_2 = 607.32$ kJ/kg, $s^o_{T2} = 7.57638$ kJ/kg-K,

Table A.7 State 3: $h_3 = 300.47$ kJ/kg, $s^o_{T3} = 6.86926$ kJ/kg-K,

$$s_{gen\ comp} = s_2 - s_1 = s^o_{T2} - s^o_{T1} - R\ ln(P_2/P_1)$$

$$= 7.57638 - 6.83521 - 0.287\ ln(1000/100) = \textbf{0.080 kJ/kgK}$$

C.V. cooler w = $\emptyset$

Continuity Eq.: $\dot{m}_3 = \dot{m}_1$; Energy Eq.4.13: $0 = h_2 - q_{out} - h_3$
Entropy Eq.: $0 = s_2 - s_3 - q_{out}/T_{amb} + s_{gen\ cool}$

$$q_{out} = h_2 - h_3 = 607.32 - 300.47 = 306.85\ kJ/kg$$

$$s_{gen\ cool} = s_3 - s_2 + q_{out}/T_{amb} = s^o_{T3} - s^o_{T2} + q_{out}/T_{amb}$$

$$= 6.86926 - 7.57638 + \frac{306.85}{290} = \textbf{0.351 kJ/kg}$$

Compressor section   Cooler section

7.94

A mixing chamber receives 5 kg/min ammonia as saturated liquid at −20°C from one line and ammonia at 40°C, 250 kPa from another line through a valve. The chamber also receives 325 kJ/min energy as heat transferred from a 40°C reservoir. This should produce saturated ammonia vapor at −20°C in the exit line. What is the mass flow rate in the second line and what is the total entropy generation in the process?

Solution:

CV: Mixing chamber out to reservoir

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{Q} = \dot{m}_3 h_3$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{Q}/T_{res} + \dot{S}_{gen} = \dot{m}_3 s_3$

From Table B.2.1: $h_1 = 89.05$ kJ/kg, $s_1 = 0.3657$ kJ/kg-K
From Table B.2.2: $h_2 = 1551.7$ kJ/kg, $s_2 = 5.9599$ kJ/kg-K
From Table B.2.1: $h_3 = 1418.05$ kJ/kg, $s_3 = 5.6158$ kJ/kg-K

From the energy equation:

$$\dot{m}_2 = [(\dot{m}_1(h_1 - h_3) + \dot{Q}]/(h_3 - h_2)$$

$$= [5 \times (89.05 - 1418.05) + 325] / (1418.05 - 1551.7)$$

$$= \textbf{47.288 kg/min} \Rightarrow \dot{m}_3 = 52.288 \text{ kg/min}$$

$$\dot{S}_{gen} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{Q}/T_{res}$$

$$= 52.288 \times 5.6158 - 5 \times 0.3657 - 47.288 \times 5.9599 - 325/313.15$$
$$= \textbf{8.94 kJ/K min}$$

7.95

Carbon dioxide at 300 K, 200 kPa is brought through a steady device where it is heated to 600 K by a 700 K reservoir in a constant pressure process. Find the specific work, specific heat transfer and specific entropy generation.

Solution:

C.V. Heater and walls out to the source. Steady single inlet and exit flows.

Since the pressure is constant and there are no changes in kinetic or potential energy between the inlet and exit flows the work is zero.　　**w = 0**

Continuity Eq.4.11: 　　$\dot{m}_i = \dot{m}_e = \dot{m}$

Energy Eq.4.13: 　　　　$h_i + q = h_e$

Entropy Eq.7.9: 　　$s_i + \int dq/T + s_{gen} = s_e = s_i + q/T_{source} + s_{gen}$

Properties are from Table A.8 so the energy equation gives

　　$q = h_e - h_i = 506.07 - 214.38 =$ **291.7 kJ/kg**

From the entropy equation

　　$s_{gen} = s_e - s_i - q/T_{source} = (5.5279 - 4.8631)$ kJ/kgK - 291.7 kJ/kg /700 K

　　　　$= 0.6648 - 0.4167 =$ **0.248 kJ/kg K**

7.96

Methane at 1 MPa, 300 K is throttled through a valve to 100 kPa. Assume no change in the kinetic energy and ideal gas behavior. What is the specific entropy generation?

Continuity Eq.4.11: $\dot{m}_i = \dot{m}_e = \dot{m}$

Energy Eq.4.13: $h_i + 0 = h_e$

Entropy Eq.7.8, 7.9: $s_i + \int dq/T + s_{gen} = s_e = s_i + 0 + s_{gen}$

Properties are from Table B.7.2 so the energy equation gives

$h_e = h_i = 618.76 \text{ kJ/kg} \Rightarrow T_e = 296 \text{ K}, \ s_e = 11.5979 \text{ kJ/kg-K}$

$s_{gen} = s_e - s_i = 11.5979 - 10.4138 = \textbf{1.184 kJ/kgK}$

7.97

A heat exchanger that follows a compressor receives 0.1 kg/s air at 1000 kPa, 500 K and cools it in a constant pressure process to 320 K. The heat is absorbed by ambient ait at 300 K. Find the total rate of entropy generation.

Solution:

C.V. Heat exchanger to ambient, steady constant pressure so no work.

Energy Eq.4.12: $\dot{m}h_i = \dot{m}h_e + \dot{Q}_{out}$

Entropy Eq.7.7: $\dot{m}s_i + \dot{S}_{gen} = \dot{m}s_e + \dot{Q}_{out}/T$

Using Table A.5 and Eq.8.25 for change in s

$$\dot{Q}_{out} = \dot{m}(h_i - h_e) = \dot{m}C_{Po}(T_i - T_e) = 0.1 \times 1.004(500 - 320) = 18.07 \text{ kW}$$

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) + \dot{Q}_{out}/T = \dot{m}C_{Po} \ln(T_e/T_i) + \dot{Q}_{out}/T$$

$$= 0.1 \text{ kg/s} \times 1.004 \text{ kJ/kg-K} \ln(320/500) + 18.07 \text{ kW}/300 \text{ K}$$

$$= \textbf{0.0154 kW/K}$$

Using Table A.7.1 and Eq. 6.19 for change in entropy

$$h_{500} = 503.36 \text{ kJ/kg}, \qquad h_{320} = 320.58 \text{ kJ/kg};$$

$$s_{T_{500}} = 7.38692 \text{ kJ/kg K}, \qquad s_{T_{320}} = 6.93413 \text{ kJ/kg K}$$

$$\dot{Q}_{out} = \dot{m}(h_i - h_e) = 0.1 \text{ kg/s} (503.36 - 320.58) \text{ kJ/kg} = 18.19 \text{ kW}$$

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) + \dot{Q}_{out}/T$$

$$= 0.1 \text{ kg/s} (6.93413 - 7.38692) \text{ kJ/kg-K} + 18.19 \text{ kW}/300 \text{ K}$$

$$= \textbf{0.0156 kW/K}$$

7.98

A dual fluid heat exchanger has 5 kg/s water enter at 40°C, 150 kPa and leaving at 10°C, 150 kPa. The other fluid is glycol coming in at -10°C, 160 kPa and leaves at 10°C, 160 kPa. Find the mass flow rate of glycol and the rate of entropy generation.

Continuity Eqs.:   Each line has a constant flow rate through it.

Energy Eq.4.10: $\dot{m}_{H_2O}\, h_1 + \dot{m}_{glycol}\, h_3 = \dot{m}_{H_2O}\, h_2 + \dot{m}_{glycol}\, h_4$

Entropy Eq.7.7: $0 = \dot{m}_{H_2O}\, s_1 + \dot{m}_{glycol}\, s_3 - \dot{m}_{H_2O}\, s_2 - \dot{m}_{glycol}\, s_4 + \dot{S}_{gen}$

Process: Each line has a constant pressure.

Table B.1: $h_1 = 167.54$ kJ/kg, $h_2 = 41.99$ kJ/kg, $s_1 = 0.5724$, $s_2 = 0.151$ kJ/kg-K

We could have used specific heat for the changes.

Table A.4: $C_{P\ gly} = 2.42$ kJ/kg-K so

$$h_4 - h_3 = C_{P\ gly}\, (T_4 - T_3) = 2.42\ [10 - (-10)] = 48.4 \text{ kJ/kg}$$
$$s_4 - s_3 = C_{P\ gly}\, \ln (T_4/T_3) = 2.42\ \ln(283.15/263.15) = 0.1773 \text{ kJ/kgK}$$

$$\dot{m}_{glycol} = \dot{m}_{H_2O}\, \frac{h_1 - h_2}{h_4 - h_3} = 5\ \frac{167.54 - 41.99}{48.4} = \textbf{12.97 kg/s}$$

$$\dot{S}_{gen} = \dot{m}_{H_2O}\, (s_2 - s_1) + \dot{m}_{glycol}\, (s_4 - s_3)$$

$$= 5 \text{ kg/s } (0.151 - 0.5724) \text{ kJ/kgK} + 12.97 \text{ kg/s} \times 0.1773 \text{ kJ/kgK}$$
$$= \textbf{0.193 kW/K}$$

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for glycol and water each. The two flows exchange energy with no heat transfer to/from the outside.

7.99

Two flows of air both at 200 kPa; one has 2 kg/s at 400 K and the other has 1 kg/s at 290 K. The two flows are mixed together in an insulated box to produce a single exit flow at 200 kPa. Find the exit temperature and the total rate of entropy generation.

Solution:

Continuity Eq.4.9:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 2 + 1 = 3 \text{ kg/s}$$

Energy Eq.4.10:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

Entropy Eq.7.7: $\qquad \dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} = \dot{m}_3 s_3$

Using constant specific heats from A.5 and Eq.6.16 for s change.

Divide the energy equation with $\dot{m}_3 C_{Po}$

$$T_3 = (\dot{m}_1/\dot{m}_3)T_1 + (\dot{m}_2/\dot{m}_3)T_2 = \frac{2}{3} \times 400 + \frac{1}{3} \times 290 = \textbf{363.33 K}$$

$$\dot{S}_{gen} = \dot{m}_1(s_3 - s_1) + \dot{m}_2(s_3 - s_2)$$

$$= [1 \times 1.004 \ln (\frac{363.33}{400}) + 2 \times 1.004 \ln (\frac{363.33}{290})] \text{ kg/s} \times \text{kJ/kg-K}$$

$$= \textbf{0.356 kW/K}$$

Using A.7.1 and Eq.6.19 for change in s.

$$h_3 = (\dot{m}_1/\dot{m}_3)h_1 + (\dot{m}_2/\dot{m}_3)h_2 = \frac{2}{3} \times 401.3 + \frac{1}{3} \times 290.43 = 364.34 \text{ kJ/kg}$$

From A.7.1: $\quad T_3 = \textbf{364.44 K} \qquad\qquad s_{T_3} = 7.06216 \text{ kJ/kg K}$

$$\dot{S}_{gen} = [2 (7.06216 - 7.15926) + 1 (7.06216 - 6.83521)] \text{ kg/s} \times \text{kJ/kgK}$$

$$= \textbf{0.03275 kW/K}$$

The pressure correction part of the entropy terms cancel out as all three states have the same pressure.

7.100

A condenser in a power plant receives 5 kg/s steam at 15 kPa, quality 90% and rejects the heat to cooling water with an average temperature of 17°C. Find the power given to the cooling water in this constant pressure process and the total rate of enropy generation when condenser exit is saturated liquid.

Solution:

C.V. Condenser. Steady state with no shaft work term.

Energy Eq.4.12: $\quad\quad \dot{m}\, h_i + \dot{Q} = \dot{m} h_e$

Entropy Eq.7.8: $\quad\quad \dot{m}\, s_i + \dot{Q}/T + \dot{S}_{gen} = \dot{m}\, s_e$

Properties are from Table B.1.2

$\quad h_i = 225.91 + 0.9 \times 2373.14 = 2361.74$ kJ/kg , $\quad\quad h_e = 225.91$ kJ/kg

$\quad s_i = 0.7548 + 0.9 \times 7.2536 = 7.283$ kJ/kg K, $\;\; s_e = 0.7548$ kJ/kg-K

$\quad \dot{Q}_{out} = -\dot{Q} = \dot{m}\,(h_i - h_e) = 5(2361.74 - 225.91) = \mathbf{10679\ kW}$

$\quad \dot{S}_{gen} = \dot{m}\,(s_e - s_i) + \dot{Q}_{out}/T$

$\quad\quad\quad = 5(0.7548 - 7.283) + 10679/(273 + 17)$

$\quad\quad\quad = -32.641 + 36.824 = \mathbf{4.183\ kW/K}$

Often the cooling media flows inside a long pipe carrying the energy away.

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7.101

A large supply line has a steady flow of R-410A at 1000 kPa, 60°C. It is used in three different adiabatic devices shown in Fig. P7.101, a throttle flow, an ideal nozzle and an ideal turbine. All the exit flows are at 300 kPa. Find the exit temperature and specific entropy generation for each device and the exit velocity of the nozzle.

Inlet state: B.4.2: $h_i = 335.75$ kJ/kg , $s_i = 1.2019$ kJ/kg-K

C.V. Throttle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i = h_e$ ($Z_i = Z_e$ and **V**'s are small)

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + s_{gen}$

Exit state: $h_e = h_i = 335.75$ kJ/kg, $T_e = \mathbf{47.9°C}$, $s_e = 1.332$ kJ/kg-K
                           $s_{gen} = s_e - s_i = 1.332 - 1.2019 = \mathbf{0.2\ kJ/kg\ K}$

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i = h_e + \mathbf{V}_e^2/2$ ($Z_i = Z_e$)

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

The isentropic process ($s_e = s_i$) gives from B.4.2

$$T_e = \mathbf{4.6°C}, \quad s_{gen} = 0, \quad h_e = 296.775 \text{ kJ/kg}$$

The energy equation becomes

$$\mathbf{V}_e^2/2 = h_i - h_e = 335.75 - 296.775 = 38.975 \text{ kJ/kg}$$

$$\mathbf{V}_e = \sqrt{2 \times 38.975 \times 1000} = \mathbf{279.2\ m/s}$$

Turbine:

Process: Reversible and adiabatic $\Rightarrow$ same as for nozzle except w, $\mathbf{V}_e = 0$

Energy Eq.4.13: $h_i = h_e + w$ ($Z_i = Z_e$)

$$T_e = \mathbf{4.6°C}, \quad s_{gen} = 0, \quad h_e = 296.775 \text{ kJ/kg}$$

7.102

A two-stage compressor takes nitrogen in at 20$^{\text{o}}$C, 150 kPa compresses it to 600 kPa, 450 K then it flows through an intercooler where it cools to 320 K and the second stage compresses it to 3000 kPa, 530 K. Find the specific entropy generation in each of the two compressor stages.

The setup is like this:

C.V.: Stage 1 nitrogen, Steady flow

Process: adiabatic: q = 0, s_{gen} = 0

Energy Eq. 4.13: $-w_{C1} = h_2 - h_1$,   Entropy Eq. 7.10: $0 = s_1 - s_2 + s_{gen\ C1}$

Assume constant C_{P0} = 1.042 from A.5 and Eq.6.16 for change in entropy

$$s_{gen\ C1} = s_2 - s_1 = C_{P0} \ln(T_2/T_1) - R \ln(P_2/P_1)$$
$$= 1.042 \ln(450/293.15) - 0.2968 \ln(600/150)$$
$$= \textbf{0.0351 kJ/kgK}$$

$$s_{gen\ C2} = s_4 - s_3 = C_{P0} \ln(T_4/T_3) - R \ln(P_4/P_3)$$
$$= 1.042 \ln(530/320) - 0.2968 \ln(3000/600)$$
$$= \textbf{0.0481 kJ/kgK}$$

7.103

The intercooler in the previous problem uses cold liquid water to cool the nitrogen. The nitrogen flow is 0.1 kg/s and the liquid water inlet is 20°C and setup to flow in the opposite direction as the nitrogen so the water leaves at 35°C. Find the flow rate of the water and the entropy generation in this intercooler.

Solution:

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A hydraulic motor driven compressor with intercooler in small pipe between the two stages.

C.V. Heat exchanger, steady 2 flows in and two flows out.

Continuity eq.: $\dot{m}_1 = \dot{m}_2 = \dot{m}_{H2O};$ $\dot{m}_3 = \dot{m}_4 = \dot{m}_{N2}$

Energy eq.: $0 = \dot{m}_{N2} (h_3 - h_4) + \dot{m}_{H2O} (h_2 - h_1)$

Entropy Eq.7.7: $0 = \dot{m}_{N2} (s_3 - s_4) + \dot{m}_{H_2O} (s_2 - s_1) + \dot{S}_{gen}$

Due to the lower range of temperature we will use constant specific heats from A.5 and A.4 so the energy equation is approximated as

$$0 = \dot{m}_{H2O}C_{pH2O} (T_2 - T_1) + \dot{m}_{N2}C_p (T_3 - T_4)$$

Now solve for the water flow rate

$\dot{m}_{H2O} = \dot{m}_{N2} C_{pN2} (T_3 - T_4) / [C_{pH2O} (T_1 - T_2)]$

$\quad = 0.1 \text{ kg/s} \times 1.042 \text{ kJ/kgK} \times (450 - 320) \text{ K} / [4.18 \times (35 - 20) \text{ kJ/kg}]$

$\quad = \textbf{0.216 kg/s}$

$\dot{S}_{gen} = \dot{m}_{N2} (s_4 - s_3) + \dot{m}_{H_2O} (s_1 - s_2)$

$\quad = \dot{m}_{N2}C_P \ln (T_4/T_3) + \dot{m}_{H_2O}C_P \ln (T_1/T_2)$

$\quad = 0.1 \text{ kg/s} \times 1.042 \text{ kJ/kgK} \ln \dfrac{320}{450} + 0.216 \text{ kg/s} \times 4.18 \text{ kJ/kgK} \ln \dfrac{308}{293}$

$\quad = -0.03552 + 0.04508 = \textbf{0.00956 kW/K}$

7.104

Air at 327°C, 400 kPa with a volume flow 1 m^3/s runs through an adiabatic turbine with exhaust pressure of 100 kPa. Neglect kinetic energies and use constant specific heats. Find the lowest and highest possible exit temperature. For each case find also the rate of work and the rate of entropy generation.

Solution:

C.V Turbine. Steady single inlet and exit flows, q = 0.

Inlet state: (T, P) $v_i = RT_i / P_i = 0.287 \times 600/400 = 0.4305$ m^3/kg

$$\dot{m} = \dot{V}/v_i = 1/0.4305 = 2.323 \text{ kg/s}$$

The lowest exit T is for maximum work out i.e. reversible case

Process: Reversible and adiabatic   => constant s   from Eq.7.9

Eq.8.23: $T_e = T_i (P_e/P_i)^{\frac{k-1}{k}} = 600 \times (100/400)^{0.2857} = $ **403.8 K**

$\Rightarrow$ $w = h_i - h_e = C_{Po}(T_i - T_e) = 1.004 \times (600 - 403.8) = 197$ kJ/kg

$$\dot{W}_T = \dot{m}w = 2.323 \times 197 = \textbf{457.6 kW} \quad \text{and} \quad \dot{S}_{gen} = \textbf{0}$$

Highest exit T occurs when there is no work out, throttling

$q = \varnothing$; **w = $\varnothing$** $\Rightarrow$ $h_i - h_e = 0$ $\Rightarrow$ $T_e = T_i = $ **600 K**

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) = -\dot{m}R \ln\frac{P_e}{P_i} = -2.323 \times 0.287 \ln\frac{100}{400} = \textbf{0.924 kW/K}$$

7.105

A counter flowing heat exchanger has one line with 2 kg/s at 125 kPa, 1000 K entering and the air is leaving at 100 kPa, 400 K. The other line has 0.5 kg/s water coming in at 200 kPa, 20°C and leaving at 200 kPa. What is the exit temperature of the water and the total rate of entropy generation?

Solution:

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside.

Energy Eq.4.10: $\dot{m}_{AIR}\Delta h_{AIR} = \dot{m}_{H2O}\Delta h_{H2O}$

From A.7: $h_1 - h_2 = 1046.22 - 401.3 = 644.92$ kJ/kg

From B.1.2 $h_3 = 83.94$ kJ/kg; $s_3 = 0.2966$ kJ/kg K

$h_4 - h_3 = (\dot{m}_{AIR}/\dot{m}_{H2O})(h_1 - h_2) = (2/0.5)644.92 = 2579.68$ kJ/kg

$h_4 = h_3 + 2579.68 = 2663.62$ kJ/kg $< h_g$ at 200 kPa

$T_4 = T_{sat} = 120.23°C$,

$x_4 = (2663.62 - 504.68)/2201.96 = 0.9805$,

$s_4 = 1.53 + x_4 \, 5.597 = 7.01786$ kJ/kg K

From entropy Eq.7.7

$\dot{S}_{gen} = \dot{m}_{H2O} (s_4 - s_3) + \dot{m}_{AIR}(s_2 - s_1)$

$= 0.5(7.01786 - 0.2966) + 2(7.1593 - 8.1349 - 0.287 \ln (100/125))$

$= 3.3606 - 1.823 = \mathbf{1.54 \ kW/K}$

7.106

A large supply line has a steady air flow at 500 K, 200 kPa. It is used in three different adiabatic devices shown in Fig. P9.85, a throttle flow, an ideal nozzle and an ideal turbine. All the exit flows are at 100 kPa. Find the exit temperature and specific entropy generation for each device and the exit velocity of the nozzle.

C.V. Throttle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $\qquad h_i = h_e \qquad$ ($Z_i = Z_e$ and **V**'s are small)

Entropy Eq.7.8: $\qquad s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + s_{gen}$

Since it is air we have h = h(T) so same h means same $T_e = T_i = $ **500 K**

$$s_{gen} = s_e - s_i = s_{Te}^o - s_{Ti}^o - R \ln(P_e / P_i) = 0 - 0.287 \ln(1/2) = \textbf{0.2 kJ/kg K}$$

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $\qquad h_i = h_e + \textbf{V}_e^2/2 \qquad$ ($Z_i = Z_e$)

Entropy Eq.7.8: $\qquad s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Use constant specific heat from Table A.5, $C_{Po} = 1.004 \dfrac{kJ}{kg\ K}, \quad k = 1.4$

The isentropic process ($s_e = s_i$) gives Eq.8.23

$$\Rightarrow \qquad T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 500\ (100/200)^{\ 0.2857} = \textbf{410 K}$$

The energy equation becomes: $\qquad \textbf{V}_e^2/2 = h_i - h_e \cong C_P(T_i - T_e)$

$$\textbf{V}_e = \sqrt{2\ C_P(T_i - T_e)} = \sqrt{2 \times 1.004\ kJ/kgK\ (500-410)\ K \times 1000\ J/kJ} = \textbf{424}$$

m/s

Turbine:

Process: Reversible and adiabatic $\quad \Rightarrow \quad$ constant s $\quad$ from Eq.7.8

Eq.6.23: $\qquad T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 500 \times (100/200)^{\ 0.2857} = \textbf{410 K}$

$$\Rightarrow \quad w = h_i - h_e = C_{Po}(T_i - T_e) = 1.004 \times (500 - 410) = 90\ kJ/kg$$

7.107

In a heat-driven refrigerator with ammonia as the working fluid, a turbine with inlet conditions of 2.0 MPa, 70°C is used to drive a compressor with inlet saturated vapor at −20°C. The exhausts, both at 1.2 MPa, are then mixed together. The ratio of the mass flow rate to the turbine to the total exit flow was measured to be 0.62. Can this be true?

Solution:

Assume the compressor and the turbine are both adiabatic.

C.V. Total:

Continuity Eq.4.11: $\dot{m}_5 = \dot{m}_1 + \dot{m}_3$

Energy Eq.4.10: $\dot{m}_5 h_5 = \dot{m}_1 h_1 + \dot{m}_3 h_3$

Entropy: $\dot{m}_5 s_5 = \dot{m}_1 s_1 + \dot{m}_3 s_3 + \dot{S}_{C.V.,gen}$

$$s_5 = y s_1 + (1-y)s_3 + \dot{S}_{C.V.,gen}/\dot{m}_5$$

Assume $y = \dot{m}_1/\dot{m}_5 = 0.62$

State 1: Table B.2.2 $h_1 = 1542.7$ kJ/kg, $s_1 = 4.982$ kJ/kg K,

State 3: Table B.2.1 $h_3 = 1418.1$ kJ/kg, $s_3 = 5.616$ kJ/kg K

Solve for exit state 5 in the energy equation

    $h_5 = y h_1 + (1-y)h_3 = 0.62 \times 1542.7 + (1 - 0.62)1418.1 = 1495.4$ kJ/kg

State 5: $h_5 = 1495.4$ kJ/kg, $P_5 = 1200$ kPa $\Rightarrow$ $s_5 = 5.056$ kJ/kg K

Now check the 2nd law, entropy generation

    $\Rightarrow$ $\dot{S}_{C.V.,gen}/\dot{m}_5 = s_5 - y s_1 - (1-y)s_3 =$ **-0.1669 Impossible**

The problem could also have been solved assuming a reversible process and then find the needed flow rate ratio y. Then y would have been found larger than 0.62 so the stated process can not be true.

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7.108

Repeat problem 7.106 for the throttle and the nozzle when the inlet air temperature is 2500 K and use the air tables.

C.V. Throttle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i = h_e$ ($Z_i = Z_e$ and **V**'s are small)

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + s_{gen}$

Since it is air we have h = h(T) so same h means same $T_e = T_i =$ **2500 K**

$$s_{gen} = s_e - s_i = s^o_{Te} - s^o_{Ti} - R \ln(P_e / P_i) = 0 - 0.287 \ln(1/2) = \textbf{0.2 kJ/kg K}$$

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i = h_e + \mathbf{V}_e^2/2$ ($Z_i = Z_e$)

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

The isentropic process ($s_e = s_i$) gives Eq.6.19

$$0 = s_e - s_i = s^o_{Te} - s^o_{Ti} - R \ln(P_e / P_i)$$

=> $s^o_{Te} = s^o_{Ti} + R \ln(P_e / P_i) = 9.24781 + 0.287 \ln (100/200) = 9.04888$

    T = **2136.6 K,** $h_e = 2422.86$ kJ/kg

The energy equation becomes

$$\mathbf{V}_e^2/2 = h_i - h_e = 2883.06 - 2422.86 = 460.2 \text{ kJ/kg}$$

$$\mathbf{V}_e = \sqrt{2 \times 1000 \times 460.2} = \textbf{959 m/s}$$

Borgnakke and Sonntag

7.109

Carbon dioxide used as a natural refrigerant flows through a cooler at 10 MPa, which is supercritical so no condensation occurs. The inlet is at 200$^{\text{o}}$C and the exit is at 40$^{\text{o}}$C. Assume the heat transfer is to the ambient at 20$^{\text{o}}$C and find the specific entropy generation.

C.V. Heat exchanger to ambient, steady constant pressure so no work.

Energy Eq.4.12: $\qquad\qquad$ $h_i = h_e + q_{out}$

Entropy Eq.7.9: $\qquad\qquad$ $s_i + s_{gen} = s_e + q_{out}/T$

Using Table B.3

$$q_{out} = (h_i - h_e) = 519.49 - 200.14 = 319.35 \text{ kJ/kg}$$

$$s_{gen} = s_e - s_i + q_{out}/T$$

$$= (0.6906 - 1.5705) \text{ kJ/kgK} + \frac{319.35 \text{ kJ/kg}}{293.15 \text{ K}} = \textbf{0.2095 kJ/kgK}$$

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7.110

Saturated liquid nitrogen at 600 kPa enters a boiler at a rate of 0.005 kg/s and exits as saturated vapor. It then flows into a super heater also at 600 kPa where it exits at 600 kPa, 280 K. Assume the heat transfer comes from a 300 K source and find the rates of entropy generation in the boiler and the super heater.

Solution:

C.V.: boiler steady single inlet and exit flow, neglect KE, PE energies in flow

Continuity Eq.: $\dot{m}_1 = \dot{m}_2 = \dot{m}_3$

Table B.6.1: $h_1 = -81.53$ kJ/kg, $s_1 = 3.294$ kJ/kgK,

$h_2 = 86.85$ kJ/kg, $s_2 = 5.041$ kJ/kgK

Table B.6.2: $h_3 = 289.05$ kJ/kg, $s_3 = 6.238$ kJ/kgK

Energy Eq.4.13: $q_{boiler} = h_2 - h_1 = 86.85 - (-81.53) = 168.38$ kJ/kg

$\dot{Q}_{boiler} = \dot{m}_1 q_{boiler} = 0.005 \times 168.38 = 0.842$ kW

Entropy Eq.: $\dot{S}_{gen} = \dot{m}_1(s_2 - s_1) - \dot{Q}_{boiler}/T_{source}$
$= 0.005(5.041-3.294) - 0.842/300 =$ **0.0059 kW/K**

C.V. Superheater (same approximations as for boiler)

Energy Eq.4.13: $q_{sup\ heater} = h_3 - h_2 = 289.05 - 86.85 = 202.2$ kJ/kg

$\dot{Q}_{sup\ heater} = \dot{m}_2 q_{sup\ heater} = 0.005$ kg/s $\times 202.2$ kJ/kg $= 1.01$ kW

Entropy Eq.: $\dot{S}_{gen} = \dot{m}_1(s_3 - s_2) - \dot{Q}_{sup\ heater}/T_{source}$
$= 0.005$ kg/s $(6.238-5.041)$kJ/kgK $- 1.01$ kW/300 K
$=$ **0.00262 kW/K**

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7.111

A steam turbine in a power plant receives steam at 3000 kPa, 500°C. The turbine has two exit flows, one is 20% of the flow at 1000 kPa, 350°C to a feedwater heater and the remainder flows out at 200 kPa, 200°C. Find the specific turbine work and the specific entropy generation both per kg flow in.

C.V. Steam turbine (x = 0.2 = extraction fraction)

Energy Eq.4.13: $\quad w = h_1 - xh_2 - (1 - x)h_3$

Entropy Eq.7.8: $\quad s_2 = s_1 + s_{gen\ HP}$ (full flow rate)

Entropy Eq.7.9: $\quad s_3 = s_2 + s_{gen\ LP}$ [flow rate is fraction (1-x)]

Overall entropy gen: $\quad s_{gen\ HP} = s_{gen\ HP} + (1 - x)\ s_{gen\ LP}$

Inlet state: Table B.1.3 $\quad h_1 = 3456.48$ kJ/kg; $\quad s_1 = 7.2337$ kJ/kg K

Extraction state: $\quad h_2 = 3157.65$ kJ/kg, $\quad s_2 = 7.3010$ kJ/kg K

Exit (actual) state: Table B.1.3 $\quad h_3 = 2870.46$; $\quad s_3 = 7.5066$ kJ/kg K

Actual turbine energy equation

$$w = 3456.48 - 0.2 \times 3157.65 - 0.8 \times 2870.46 = \textbf{528.58 kJ/kg}$$

$$s_{gen\ tot} = 7.301 - 7.2337 + 0.8\ (7.5066 - 7.301)$$

$$= \textbf{0.232 kJ/kgK}$$

7.112

One type of feedwater heater for preheating the water before entering a boiler operates on the principle of mixing the water with steam that has been bled from the turbine. For the states as shown in Fig. P7.112, calculate the rate of net entropy increase for the process, assuming the process to be steady flow and adiabatic.

Solution:

CV: Feedwater heater, Steady flow, no external heat transfer.

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq.4.10: $\dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_1) h_2 = \dot{m}_3 h_3$

Properties: All states are given by (P,T) table B.1.1 and B.1.3

$h_1 = 168.42, \ h_2 = 2828, \ h_3 = 675.8 \quad$ all kJ/kg

$s_1 = 0.572, \quad s_2 = 6.694, \quad s_3 = 1.9422 \quad$ all kJ/kg K

Solve for the flow rate from the energy equation

$$\dot{m}_1 = \frac{\dot{m}_3(h_3 - h_2)}{(h_1 - h_2)} = \frac{4(675.8 - 2828)}{(168.42 - 2828)} = 3.237 \text{ kg/s}$$

$$\Rightarrow \quad \dot{m}_2 = 4 - 3.237 = 0.763 \text{ kg/s}$$

The second law for steady flow, $\dot{S}_{CV} = 0$, and no heat transfer, Eq.7.7:

$$\dot{S}_{C.V.,gen} = \dot{S}_{SURR} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$
$$= 4(1.9422) - 3.237(0.572) - 0.763(6.694) = \mathbf{0.8097 \text{ kJ/K s}}$$

7.113

A coflowing (same direction) heat exchanger has one line with 0.5 kg/s oxygen at 17°C, 200 kPa entering and the other line has 0.6 kg/s nitrogen at 150 kPa, 500 K entering. The heat exchanger is very long so the two flows exit at the same temperature. Use constant heat capacities and find the exit temperature and the total rate of entropy generation.

Solution:

C.V. Heat exchanger, steady 2 flows in and two flows out.

Energy Eq.4.10: $\dot{m}_{O2}h_1 + \dot{m}_{N2}h_3 = \dot{m}_{O2}h_2 + \dot{m}_{N2}h_4$

Same exit temperature so $T_4 = T_2$ with values from Table A.5

$$\dot{m}_{O2}C_{P\,O2}T_1 + \dot{m}_{N2}C_{P\,N2}T_3 = (\dot{m}_{O2}C_{P\,O2} + \dot{m}_{N2}C_{P\,N2})T_2$$

$$T_2 = \frac{0.5 \times 0.922 \times 290 + 0.6 \times 1.042 \times 500}{0.5 \times 0.922 + 0.6 \times 1.042} = \frac{446.29}{1.0862}$$

$$= \textbf{410.9 K}$$

Entropy Eq.7.7 gives for the generation

$$\dot{S}_{gen} = \dot{m}_{O2}(s_2 - s_1)   + \dot{m}_{N2}(s_4 - s_3)$$

$$= \dot{m}_{O2}C_P \ln (T_2/T_1) + \dot{m}_{N2}C_P \ln (T_4/T_3)$$

$$= 0.5 \times 0.922 \ln (410.9/290) + 0.6 \times 1.042 \ln (410.9/500)$$

$$= 0.16064 - 0.1227 = \textbf{0.0379 kW/K}$$

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7.114

A supply of 5 kg/s ammonia at 500 kPa, 20°C is needed. Two sources are available one is saturated liquid at 20°C and the other is at 500 kPa and 140°C. Flows from the two sources are fed through valves to an insulated mixing chamber, which then produces the desired output state. Find the two source mass flow rates and the total rate of entropy generation by this setup.

Solution:

C.V. mixing chamber + valve. Steady, no heat transfer, no work.

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$;

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} = \dot{m}_3 s_3$

State 1: Table B.2.1 $h_1 = 273.4$ kJ/kg, $s_1 = 1.0408$ kJ/kg K

State 2: Table B.2.2 $h_2 = 1773.8$ kJ/kg, $s_2 = 6.2422$ kJ/kg K

State 3: Table B.2.2 $h_3 = 1488.3$ kJ/kg, $s_3 = 5.4244$ kJ/kg K

As all states are known the energy equation establishes the ratio of mass flow rates and the entropy equation provides the entropy generation.

$$\dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_2)h_2 = \dot{m}_3 h_3 \quad => \quad \dot{m}_1 = \dot{m}_3 \frac{h_3 - h_2}{h_1 - h_2} = 0.952 \text{ kg/s}$$

$$\dot{m}_2 = \dot{m}_3 - \dot{m}_1 = 4.05 \text{ kg/s}$$

$$\dot{S}_{gen} = (5 \times 5.4244 - 0.95 \times 1.0408 - 4.05 \times 6.2422) \text{ (kg/s)} \times \text{(kJ/kgK)}$$

$$= \textbf{0.852 kW/K}$$

Transient processes

7.115

Calculate the specific entropy generated in the filling process given in Example 4.10.

Solution:

C.V. Cannister filling process where: $_1Q_2 = 0$; $_1W_2 = 0$; $m_1 = 0$

Continuity Eq.4.15: $m_2 - 0 = m_{in}$;

Energy Eq.4.16: $m_2 u_2 - 0 = m_{in} h_{line} + 0 + 0 \quad \Rightarrow \quad u_2 = h_{line}$

Entropy Eq.7.12: $m_2 s_2 - 0 = m_{in} s_{line} + 0 + _1S_{2\ gen}$

Inlet state : 1.4 MPa, 300°C, $h_i = 3040.4$ kJ/kg, $s_i = 6.9533$ kJ/kg K

final state: 1.4 MPa, $u_2 = h_i = 3040.4$ kJ/kg

$\Rightarrow T_2 = 452$°C, $s_2 = 7.45896$ kJ/kg K

$_1S_{2\ gen} = m_2(s_2 - s_i)$

$_1s_{2\ gen} = s_2 - s_i = 7.45896 – 6.9533 = \textbf{0.506 kJ/kg K}$

7.116

A 1-m^3 rigid tank contains 100 kg R-410A at a temperature of 15°C. A valve on top of the tank is opened, and saturated vapor is throttled to ambient pressure, 100 kPa, and flows to a collector system. During the process the temperature inside the tank remains at 15°C by heat transfer from the 20°C ambient. The valve is closed when no more liquid remains inside. Calculate the heat transfer to the tank and total entropy generation in the process.

Solution:

C.V. Tank out to surroundings. Rigid tank so no work term.

Continuity Eq.4.15: $m_2 - m_1 = - m_e$;

Energy Eq.4.16: $m_2u_2 - m_1u_1 = Q_{CV} - m_eh_e$

Entropy Eq.7.12: $m_2s_2 - m_1s_1 = Q_{CV}/T_{SUR} - m_es_e + S_{gen}$

State 1: Table B.4.1, $v_1 = V_1/m_1 = 1/100 = 0.000904 + x_1\,0.01955$

$x_1 = 0.46527,$ $u_1 = 80.02 + 0.46527 \times 177.1 = 162.42$ kJ/kg

$s_1 = 0.3083 + 0.46527\times 0.6998 = 0.6339;$ $h_e = h_g = 282.79$ kJ/kg

State 2: $v_2 = v_g = 0.02045,$ $u_2 = u_g = 257.12,$ $s_2 = 1.0081$ kJ/kg K

Exit state: $h_e = 282.79, P_e = 100$ kPa $\to T_e = -18.65°C,$ $s_e = 1.2917$ kJ/kgK

$m_2 = 1/0.02045 = 48.9$ kg; $m_e = 100 - 48.9 = 51.1$ kg

$Q_{CV} = m_2u_2 - m_1u_1 + m_eh_e$

$= 48.9 \times 257.12 - 100\times 162.42 + 51.1 \times 282.79 =$ **10 782 kJ**

$S_{gen} = m_2s_2 - m_1s_1 + m_es_e - Q_{CV}/T_{SUR}$

$= 48.9 \times 1.0081 - 100 \times 0.6339 + 51.1 \times 1.2917 - 10\,782 / 293.15$

$=$ **15.14 kJ/K**

7.117

A 0.2 m^3 initially empty container is filled with water from a line at 500 kPa, 200 °C until there is no more flow. Assume the process is adiabatic and find the final mass, final temperature and the total entropy generation.

Solution:

C.V. The container volume and any valve out to line.

Continuity Eq.4.15: $\quad m_2 - m_1 = m_2 = m_i$

Energy Eq.4.16: $\quad m_2u_2 - m_1u_1 = m_2u_2 = {}_1Q_2 - {}_1W_2 + m_ih_i = m_ih_i$

Entropy Eq.7.12: $\quad m_2s_2 - m_1s_1 = m_2s_2 = \int dQ/T + {}_1S_{2\ gen} + m_is_i$

Process: Adiabatic ${}_1Q_2 = 0$, Rigid ${}_1W_2 = 0$ Flow stops $P_2 = P_{line}$

State i: $\quad h_i = 2855.37 \text{ kJ/kg};\qquad s_i = 7.0592 \text{ kJ/kg K}$

State 2: $\quad 500 \text{ kPa}, \quad u_2 = h_i = 2855.37 \text{ kJ/kg} \quad \Rightarrow \quad$ Table B.1.3

$\qquad T_2 \cong \textbf{332.9°C} , \quad s_2 = 7.5737 \text{ kJ/kg}, \quad v_2 = 0.55387 \text{ m}^3/\text{kg}$

$\qquad m_2 = V/v_2 = 0.2/0.55387 = \textbf{0.361 kg}$

From the entropy equation

$\qquad {}_1S_{2\ gen} = m_2s_2 - m_2s_i$

$\qquad\qquad = 0.361(7.5737 - 7.0592) = \textbf{0.186 kJ/K}$

7.118

An initially empty 0.1 m^3 cannister is filled with R-410A from a line flowing saturated liquid at $-5°C$. This is done quickly such that the process is adiabatic. Find the final mass, liquid and vapor volumes, if any, in the cannister. Is the process reversible?

Solution:

C.V. Cannister filling process where: $_1Q_2 = 0$; $_1W_2 = 0$; $m_1 = 0$

Continuity Eq.4.15: $m_2 - 0 = m_{in}$;

Energy Eq.4.16: $m_2u_2 - 0 = m_{in}h_{line} + 0 + 0 \quad \Rightarrow \quad u_2 = h_{line}$

1: Table B.4.1 $\quad u_f = 49.65$, $u_{fg} = 201.75$, $h_f = 50.22 \quad$ all kJ/kg

2: $P_2 = P_{line}$; $u_2 = h_{line} \Rightarrow$ 2 phase $u_2 > u_f$; $u_2 = u_f + x_2u_{fg}$

$x_2 = (50.22 - 49.65)/201.75 = 0.002825$

$\Rightarrow v_2 = v_f + x_2v_{fg} = 0.000841 + 0.002825 \times 0.03764 = 0.0009473$ m^3/kg

$\Rightarrow m_2 = V/v_2 = \textbf{105.56 kg}$; $m_f = 106.262$ kg; $m_g = 0.298$ kg

$V_f = m_f\ v_f = \textbf{0.089 m}^3$; $\quad V_g = m_gv_g = \textbf{0.0115 m}^3$

Process is irreversible (throttling) $s_2 > s_f$

7.119

A 1 L can of R-134a is at room temperature 20°C with a quality of 50%. A leak in the top valve allows vapor to escape and heat transfer from the room takes place so we reach final state of 5°C with a quality of 100%. Find the mass that escaped, the heat transfer and the entropy generation not including that made in the valve.

CV The can of R-134a not including the nozzle/valve out to ambient 20°C

Continuity Eq.: $\quad m_2 - m_1 = - m_e$

Energy Eq.: $\quad m_2 u_2 - m_1 u_1 = - m_e h_e + {}_1Q_2 - {}_1W_2$

Entropy Eq.7.12: $\quad m_2 s_2 - m_1 s_1 = - m_e s_e + \int dQ/T + {}_1S_{2\ gen}$

Process Eq.: $\quad V = \text{constant} \quad \Rightarrow \quad {}_1W_2 = \int PdV = 0$

State 1: (T,x) $\quad v_1 = v_f + x_1 v_{fg} = 0.000817 + 0.5 \ 0.03524 = 0.018437 \ \text{m}^3/\text{kg}$

$\qquad\qquad\qquad u_1 = u_f + x_1 u_{fg} = 227.03 + 0.5 \ 162.16 = 308.11 \ \text{kJ/kg}$

$\qquad\qquad\qquad s_1 = s_f + x_1 s_{fg} = 1.0963 + 0.5 \ 0.622 = 1.4073 \ \text{kJ/kg}$

$\qquad\qquad\qquad\qquad m_1 = V \ / \ v_1 = 0.001 \ / \ 0.018437 = 0.05424 \ \text{kg}$

State 2: (T,x)

$\qquad\quad v_2 = v_g = 0.05833 \ \text{m}^3/\text{kg}, \ \ u_2 = u_g = 380.85 \ \text{kJ/kg}, \ \ s_2 = s_g = 1.7239 \ \text{kJ/kg K}$

$\qquad\quad m_2 = V/v_2 = 0.001 \ / \ 0.05833 = 0.017144 \ \text{kg}$

Exit state e: Saturated vapor starting at 20°C ending at 5°C so we take an average

$\qquad\quad h_e = 0.5(h_{e1} + h_{e2}) = 0.5 \ (409.84 + 401.32) = 405.58 \ \text{kJ/kg}$

$\qquad\quad s_e = 0.5(s_{e1} + s_{e2}) = 0.5 \ (1.7183 + 1.7239) = 1.7211 \ \text{kJ/kg K}$

$\qquad\quad m_e = m_1 - m_2 = \mathbf{0.0371 \ kg}$

The heat transfer from the energy equation becomes

$\qquad {}_1Q_2 = m_2 u_2 - m_1 u_1 + m_e h_e = 6.5293 - 16.7119 + 15.047 = \mathbf{4.864 \ kJ}$

${}_1S_{2\ gen} = m_2 s_2 - m_1 s_1 + m_e s_e - {}_1Q_2/T_{amb}$

$\qquad\quad = 0.029555 - 0.076332 + 0.063853 - 0.016592 = \mathbf{0.000484 \ kJ/K}$

7.120

An empty can of 0.002 m^3 is filled with R-134a from a line flowing saturated liquid R-134a at 0°C. The filling is done quickly so it is adiabatic, but after a while in storage the can warms up to room temperature 20°C. Find the final mass in the cannister and the total entropy generation.

Solution:

C.V. Cannister filling process where: $_1Q_2 = 0$; $_1W_2 = 0$; $m_1 = 0$

Continuity Eq.4.15: $m_2 - 0 = m_{in}$;

Energy Eq.4.16: $m_2u_2 - 0 = m_{in}h_{line} + 0 + 0$ $\Rightarrow$ $u_2 = h_{line}$

Inlet state: Table B.5.1 $h_{line} = 200$ kJ/kg, $s_{line} = 1.0$ kJ/kg K

State 2: $P_2 = P_{line}$ and $u_2 = h_{line} = 200$ kJ/kg $> u_f$

$x_2 = (200 - 199.77) / 178.24 = 0.00129$

$v_2 = 0.000773 + x_2\, 0.06842 = 0.000861$ m^3/kg
$m_2 = V / v_2 = 0.002/0.000861 =$ **2.323 kg**

State 3: $T_3 = 20°C$, $v_3 = v_2$ $(m_3 = m_2)$ $=>$

$x_3 = (0.000861 - 0.000817)/0.03524 = 0.0012486$

$u_3 = 227.03 + x\, 162.16 = 227.23$ kJ/kg

$s_3 = 1.0963 + x\, 0.622 = 1.0971$ kJ/kg-K

Energy Eq.4.16: $m_2u_3 - m_2u_2 = {_2Q_3} + 0$ $\Rightarrow$ $_2Q_3 = m_2(u_3 - u_2)$

Entropy Eq.7.12: $m_2s_3 - 0 = m_{in}s_{line} + {_2Q_3}/T_3 + {_1S_3}{_{gen}}$

$_2Q_3 = m_2(u_3 - u_2) = 2.323\,(227.23 - 200) = 63.255$ kJ

$_1S_3{_{gen}} = m_2(s_3 - s_{line}) - {_2Q_3}/T_3$

$= 2.323$kg $(1.0971 - 1)$ kJ/kgK $- 63.255$ kJ $/293.15$ K

$=$ **0.0098 kJ/K**

Borgnakke and Sonntag

7.121

A cook filled a pressure cooker with 3 kg water at 20°C and a small amount of air and forgot about it. The pressure cooker has a vent valve so if P > 200 kPa steam escapes to maintain a pressure of 200 kPa. How much entropy was generated in the throttling of the steam through the vent to 100 kPa when half the original mass has escaped?

Solution:

The pressure cooker goes through a transient process as it heats water up to the boiling temperature at 200 kPa then heats more as saturated vapor at 200 kPa escapes. The throttling process is steady state as it flows from saturated vapor at 200 kPa to 100 kPa which we assume is a constant h process.

C.V. Pressure cooker, no work.

Continuity Eq.6.15: $m_2 - m_1 = -m_e$

Energy Eq.6.16: $m_2 u_2 - m_1 u_1 = -m_e h_e + {}_1Q_2$

Entropy Eq.7.12: $m_2 s_2 - m_1 s_1 = -m_e s_e + \int dQ/T + {}_1S_{2\ gen}$

State 1: $v_1 = v_f = 0.001002$ m^3/kg $V = m_1 v_1 = 0.003006$ m^3

State 2: $m_2 = m_1/2 = 1.5$ kg, $v_2 = V/m_2 = 2v_1$, $P_2 = 200$ kPa

Exit: $h_e = h_g = 2706.63$ kJ/kg, $s_e = s_g = 7.1271$ kJ/kg K

So we can find the needed heat transfer and entropy generation if we know the C.V. surface temperature T. If we assume T for water then ${}_1S_{2\ gen} = 0$, which is an internally reversible externally irreversible process, there is a ΔT between the water and the source.

C.V. Valve, steady flow from state e (200 kPa) to state 3 (at 100 kPa).

Energy Eq.: $h_3 = h_e$

Entropy Eq.: $s_3 = s_e + {}_e s_{3\ gen}$ generation in valve (throttle)

State 3: 100 kPa, $h_3 = 2706.63$ kJ/kg   Table B.1.3 $\Rightarrow$

$$T_3 = 99.62 + (150-99.62)\frac{2706.63 - 2675.46}{2776.38 - 2675.46} = 115.2°C$$

$$s_3 = 7.3593 + (7.6133 - 7.3593)\ 0.30886 = 7.4378 \text{ kJ/kg K}$$

$${}_e S_{3\ gen} = m_e(s_3 - s_e) = 1.5\ (7.4378 - 7.1271) = \textbf{0.466 kJ/K}$$

7.122

A 10 m tall 0.1 m diameter pipe is filled with liquid water at 20$^{\text{o}}$C. It is open at the top to the atmosphere, 100 kPa, and a small nozzle is mounted in the bottom. The water is now let out through the nozzle splashing out to the ground until the pipe is empty. Find the water initial exit velocity, the average kinetic energy in the exit flow and the total entropy generation for the process.

Total mass: $\quad m = \rho AH = \rho \dfrac{\pi}{4} D^2 H = 998 \dfrac{\pi}{4} 0.1^2 \times 10 = 78.383 \text{ kg}$

Bernoulli: $\quad \dfrac{1}{2} \mathbf{V}^2 = gH \quad => \quad \mathbf{V}_1 = \sqrt{2gH_1} = \sqrt{2 \times 9.807 \times 10} = \mathbf{14 \text{ m/s}}$

$$\dfrac{1}{2} \mathbf{V}^2_{\text{avg}} = gH_{\text{avg}} = g \dfrac{1}{2} H_1 = 9.807 \times 5 = \mathbf{49 \text{ m}^2\text{/s}^2} \quad (\text{J/kg})$$

All the energy (average kinetic energy) is dispersed in the ambient at 20$^{\text{o}}$C so

$$S_{\text{gen}} = \dfrac{Q}{T} = \dfrac{m}{2T} \mathbf{V}^2_{\text{avg}} = \dfrac{78.383 \text{ kg} \times 49 \text{ J/kg}}{293.15 \text{ K}} = \mathbf{13.1 \text{ J/K}}$$

7.123

A 200 L insulated tank contains nitrogen gas at 200 kPa, 300 K. A line with nitrogen at 500 K, 500 kPa adds 40% more mass to the tank with a flow through a valve. Use constant specific heats to find the final temperature and the entropy generation.

C.V. Tank, no work and no heat transfer.

Continuity Eq.6.15: $\quad m_2 - m_1 = m_i$

Energy Eq.6.16: $\quad m_2 u_2 - m_1 u_1 = m_i h_i + {}_1Q_2$

Entropy Eq.7.12: $\quad m_2 s_2 - m_1 s_1 = m_i s_i + \int dQ/T + {}_1S_{2\ gen}$

Process Eq.: $\quad {}_1Q_2 = 0, \quad m_2 = 1.4\ m_1; \quad m_i = 0.4\ m_1$

Write $h_i = u_i + RT_i$ then the energy equation contains only u's so we can substitute the u differences with $C_{v0}\ \Delta T$ and divide by $m_2\ C_{v0}$ to get

$$T_2 = \frac{m_1}{m_2}\ T_1 + \frac{m_i}{m_2}T_i + \frac{m_i\ R}{m_2 C_{v0}}\ T_i$$

$$= \frac{1}{1.4}\ 300 + \frac{0.4}{1.4}\ 500 + \frac{0.4}{1.4}\frac{0.2968}{0.745}\ 500 = \textbf{414.0 K}$$

We need the pressure for the entropy

$$P_2 = m_2 RT_2/V = 1.4\ P_1 \times T_2/T_1 = 1.4 \times 200 \times 414/300 = 386.4\ \text{kPa}$$

$$m_1 = \frac{P_1 V_1}{RT_1} = \frac{200 \times 0.2}{0.2968 \times 300} = 0.4492\ \text{kg}; \quad m_2 = 1.4\ m_1 = 0.6289\ \text{kg}$$

The entropy generation, with entropy difference from Eq.6.16, becomes

$${}_1S_{2\ gen} = m_2 s_2 - m_1 s_1 - m_i\ s_i = m_2(s_2 - s_i) - m_1(s_1 - s_i)$$

$$= m_2\ [C_{P0}\ln(\frac{T_2}{T_i}) - R\ \ln(\frac{P_2}{P_i})] - m_1\ [C_{P0}\ \ln(\frac{T_1}{T_i}) - R\ \ln(\frac{P_1}{P_i})]$$

$$= 0.6289[\ 1.042\ \ln(\frac{414}{500}) - 0.2968\ \ln(\frac{386.4}{500})]$$

$$- 0.4492\ [1.042\ \ln(\frac{300}{500}) - 0.2968\ \ln(\frac{200}{500})]$$

$$= \ \textbf{0.0414 kJ/K}$$

Borgnakke and Sonntag

7.124

A 200 L insulated tank contains nitrogen gas at 200 kPa, 300 K. A line with nitrogen at 1500 K, 1000 kPa adds 40% more mass to the tank with a flow through a valve. Use table A.8 to find the final temperature and the entropy generation.

C.V. Tank, no work and no heat transfer.

Continuity Eq.6.15: $\quad m_2 - m_1 = m_i$

Energy Eq.6.16: $\quad m_2 u_2 - m_1 u_1 = m_i h_i + {}_1Q_2$

Entropy Eq.7.12: $\quad m_2 s_2 - m_1 s_1 = m_i s_i + \int dQ/T + {}_1S_{2\ gen}$

Process Eq.: $\quad {}_1Q_2 = 0, \quad m_2 = 1.4\ m_1; \quad m_i = 0.4\ m_1$

The energy equation can be solved for u_2 to get

$$u_2 = \frac{m_1}{m_2} u_1 + \frac{m_i}{m_2} h_i = \frac{1}{1.4}\ 222.63 + \frac{0.4}{1.4}\ 1235.5 = 512.02\ \text{kJ/kg}$$

A.8: $\quad T_2 = 679.8\ \text{K}, \quad s_{T2}^o = 7.7088\ \text{kJ/kgK}, \quad s_{T1}^o = 6.8463, \ s_{Ti}^o = 8.6345\ \text{kJ/kgK}$

We need the pressure for the entropy

$$P_2 = m_2 R T_2 / V = 1.4\ P_1 \times T_2/T_1 = 1.4 \times 200 \times 679.8/300 = 634.5\ \text{kPa}$$

$$m_1 = \frac{P_1 V_1}{R T_1} = \frac{200 \times 0.2}{0.2968 \times 300} = 0.4492\ \text{kg}; \quad m_2 = 1.4\ m_1 = 0.6289\ \text{kg}$$

The entropy generation, with entropy difference from Eq.6.19, becomes

$${}_1S_{2\ gen} = m_2 s_2 - m_1 s_1 - m_i s_i = m_2(s_2 - s_i) - m_1(s_1 - s_i)$$

$$= m_2\ [s_{T2}^o - s_{Ti}^o - R\ \ln(\frac{P_2}{P_i})] - m_1\ [s_{T1}^o - s_{Ti}^o - R\ \ln(\frac{P_1}{P_i})]$$

$$= 0.6289[\ 7.7088 - 8.6345 - 0.2968\ \ln(\frac{634.5}{1000})]$$

$$- 0.4492\ [6.8463 - 8.6345 - 0.2968\ \ln(\frac{200}{1000})]$$

$$= \mathbf{0.0914\ kJ/K}$$

7.125

Air from a line at 12 MPa, 15°C, flows into a 500-L rigid tank that initially contained air at ambient conditions, 100 kPa, 15°C. The process occurs rapidly and is essentially adiabatic. The valve is closed when the pressure inside reaches some value, P_2. The tank eventually cools to room temperature, at which time the pressure inside is 5 MPa. What is the pressure P_2? What is the net entropy change for the overall process?

Solution:

CV: Tank. Mass flows in, so this is transient. Find the mass first

$$m_1 = P_1V/RT_1 = \frac{100 \times 0.5}{0.287 \times 288.2} = 0.604 \text{ kg}$$

Fill to P_2, then cool to $T_3 = 15°C$, $P_3 = 5$ MPa

$$m_3 = m_2 = P_3V/RT_3$$

$$= \frac{5000 \times 0.5}{0.287 \times 288.2} = 30.225 \text{ kg}$$

Mass: $m_i = m_2 - m_1 = 30.225 - 0.604 = 29.621 \text{ kg}$

In the process 1-2 heat transfer = 0

Energy Eq.4.16: $m_ih_i = m_2u_2 - m_1u_1$; $m_iC_{P0}T_i = m_2C_{v0}T_2 - m_1C_{v0}T_1$

$$T_2 = \frac{(29.621 \times 1.004 + 0.604 \times 0.717) \times 288.2}{30.225 \times 0.717} = 401.2 \text{ K}$$

$$P_2 = m_2RT_2/V = (30.225 \times 0.287 \times 401.2)/0.5 = \textbf{6.960 MPa}$$

Consider now the total process from the start to the finish at state 3.

Energy Eq.4.16: $Q_{CV} + m_ih_i = m_2u_3 - m_1u_1 = m_2h_3 - m_1h_1 - (P_3 - P_1)V$

But, since $T_i = T_3 = T_1$, $m_ih_i = m_2h_3 - m_1h_1$

$\Rightarrow Q_{CV} = -(P_3 - P_1)V = -(5000 - 100)0.5 = -2450 \text{ kJ}$

From Eq.7.13 also Eqs.7.24-7.26

$$S_{gen} = m_3s_3 - m_1s_1 - m_is_i - Q_{CV}/T_0 = m_3(s_3 - s_i) - m_1(s_1 - s_i) - Q_{CV}/T_0$$

$$= 30.225\left[0 - 0.287 \ln\frac{5}{12}\right] - 0.604\left[0 - 0.287 \ln\frac{0.1}{12}\right] + (2450 / 288.2)$$

$$= \textbf{15.265 kJ/K}$$

7.126

An insulated piston/cylinder contains 0.1 m^3 air at 250 kPa, 300 K and it maintains constant pressure. More air flows in through a valve from a line at 300 kPa, 400 K so the volume increases 60%. Use constant specific heats to solve for the final temperature and the total entropy generation.

C.V. Piston/cylinder volume, no heat transfer.

Continuity Eq.6.15: $\quad m_2 - m_1 = m_i$

Energy Eq.6.16: $\quad m_2 u_2 - m_1u_1 = m_i h_i - {}_1W_2$

Entropy Eq.7.12: $\quad m_2s_2 - m_1s_1 = m_i s_i + \int dQ/T + {}_1S_{2\ gen}$

Process Eq.: $\quad {}_1Q_2 = 0, \ V_2 = 1.6\ V_1; \ P = C,$

$${}_1W_2 = P(V_2 - V_1) = P_1(1.6 - 1)V_1 = 0.6\ P_1V_1 = 0.6\ m_1RT_1$$

$$m_1 = P_1V_1/RT_1 = \frac{250 \times 0.1}{0.287 \times 300} = 0.2904 \text{ kg};$$

Write $h_i = u_i + RT_i$ then the energy equation contains only u's so we can substitute the u differences with $C_{v0}\ \Delta T$ and get

$$m_2C_{v0}T_2 - m_1C_{v0}T_1 = m_iC_{v0}T_i + m_iRT_i - 0.6\ m_1RT_1$$

$$m_2T_2 = m_1T_1 + m_iT_i\ [1 + (R/C_{v0})\ ] - 0.6\ m_1T_1R/C_{v0}$$

In this equation m_2, T_2 and m_i are unknowns however we also have

$$m_2T_2 = P_2V_2\ /R = 1.6\ P_1V_1/R = 1.6\ m_1T_1$$

so we can solve for the mass m_i.

$$m_iT_i\ [1 + (R/C_{v0})\ ] = 1.6\ m_1T_1 - m_1T_1 + 0.6\ m_1T_1R/C_{v0}$$

$$m_iT_i\ [1+ (R/C_{v0})]\ = 0.6\ [1 + R/C_{v0}]\ m_1T_1$$

$$m_i = 0.6\ m_1\ (T_1/\ T_i) = 0.6 \times 0.2904\ (300/\ 400) = 0.13068 \text{ kg}$$

$$m_2 = m_i + m_1 = 0.42108 \text{ kg}; \quad T_2 = 1.6\ (m_1/\ m_2)\ T_1 = \textbf{331 K}$$

The entropy generation, with entropy difference from Eq.6.16, becomes

$${}_1S_{2\ gen} = m_2s_2 - m_1s_1 - m_i s_i = m_2(s_2 - s_i) - m_1(s_1 - s_i)$$

$$= m_2\ [C_{P0}\ln(\frac{T_2}{T_i}) - R\ \ln(\frac{P_2}{P_i})] - m_1\ [C_{P0}\ \ln(\frac{T_1}{T_i}) - R\ \ln(\frac{P_1}{P_i})]$$

$$= 0.42108[\ 1.004\ \ln(\frac{331}{400}) - 0.287\ \ln(\frac{250}{300})]$$

$$- 0.2904\ [1.004\ \ln(\frac{300}{400}) - 0.287\ \ln(\frac{250}{300})]$$

$$= \ \textbf{0.0107 kJ/K}$$

7.127

A balloon is filled with air from a line at 200 kPa, 300 K to a final state of 110 kPa, 300 K with a mass of 0.1 kg air. Assume the pressure is proportional to the balloon volume as: P = 100 kPa + CV. Find the heat transfer to/from the ambient at 300 K and the total entropy generation.

C.V. Balloon out to the ambient. Assume $m_1 = 0$

Continuity Eq.4.15: $m_2 - 0 = m_{in}$;

Energy Eq.4.16: $m_2 u_2 - 0 = m_{in} h_{in} + {_1}Q_2 - {_1}W_2$

Entropy Eq.7.12: $m_2 s_2 - 0 = m_{in} s_{in} + \int \frac{dQ}{T} + {_1}S_{2\ gen} = m_{in} s_{in} + \frac{{_1}Q_2}{T} + {_1}S_{2\ gen}$

Process Eq.:   P = A + C V,   A = 100 kPa

State 2 (P, T):

$$V_2 = m_2 R T_2 / P_2 = \frac{0.1 \times 0.287 \times 300}{110} = 0.078273 \text{ m}^3$$

$$P_2 = A + CV_2 \quad \Rightarrow \quad C = (P_2 - 100) / V_2 = 127.758 \text{ kPa/m}^3$$

Inlet state: $h_{in} = h_2 = u_2 + P_2 v_2$, $s_{in} = s_2 - R \ln(\frac{P_{in}}{P_2})$

$$
\begin{aligned}
{_1}W_2 &= \int P \, dV = \int A + CV \, dV = A (V_2 - 0) + \tfrac{1}{2} C (V_2^2 - 0) \\
&= 100 \times 0.078273 + \tfrac{1}{2} \, 127.758 \times 0.078273^2 \\
&= 8.219 \text{ kJ} \quad [= \tfrac{1}{2}(P_o + P_2)V_2 = \text{area in P-V diagram}]
\end{aligned}
$$

$$
\begin{aligned}
{_1}Q_2 &= m_2(u_2 - h_{line}) + {_1}W_2 = - P_2 V_2 + {_1}W_2 \\
&= - 110 \times 0.078273 + 8.219 = \textbf{--0.391 kJ}
\end{aligned}
$$

$$
\begin{aligned}
{_1}S_{2\ gen} &= m_2(s_2 - s_{in}) - \frac{{_1}Q_2}{T} = m_2 \, R \, \ln \left(\frac{P_{in}}{P_2}\right) - \frac{{_1}Q_2}{T} \\
&= 0.1 \times 0.287 \ln(\frac{200}{110}) + \frac{0.391}{300} = \textbf{0.0185 kJ/K}
\end{aligned}
$$

Device efficiency

7.128

A steam turbine inlet is at 1200 kPa, 400°C. The exit is at 200 kPa. What is the lowest possible exit temperature? Which efficiency does that correspond to?

We would expect the lowest possible exit temperature when the maximum amount of work is taken out. This happens in a reversible process so if we assume it is adiabatic this becomes an isentropic process.

Exit: 200 kPa, s = s_{in} = 7.3773 kJ/kg K $\Rightarrow$ **T = 171.5°C**

The efficiency from Eq.7.27 measures the turbine relative to an isentropic turbine, so the **efficiency** will be **100%**.

7.129

A steam turbine inlet is at 1200 kPa, 400°C. The exit is at 200 kPa. What is the highest possible exit temperature? Which efficiency does that correspond to?

The highest possible exit temperature would be if we did not get any work out, i.e. the turbine broke down. Now we have a throttle process with constant h assuming we do not have a significant exit velocity.

Exit: 200 kPa, h = h_{in} = 3260.66 kJ/kg $\Rightarrow$ **T = 392°C**

Efficiency: $\eta = \dfrac{w}{w_s} = \mathbf{0}$

Remark: Since process is irreversible there is no area under curve in T-s diagram that correspond to a q, nor is there any area in the P-v diagram corresponding to a shaft work.

7.130

A steam turbine inlet is at 1200 kPa, 400°C. The exit is at 200 kPa, 200°C. What is the isentropic efficiency?

Inlet: h_{in} = 3260.66 kJ/kg, s_{in} = 7.3773 kJ/kg K

Exit: h_{ex} = 2870.46 kJ/kg, s_{ex} = 7.5066 kJ/kg K

Ideal Exit: 200 kPa, s = s_{in} = 7.3773 kJ/kg K $\Rightarrow$ h_s = 2812.6 kJ/kg

w_{ac} = h_{in} - h_{ex} = 3260.66 – 2870.46 = 390.2 kJ/kg

w_s = h_{in} - h_s = 3260.66 – 2812.6 = 448.1 kJ/kg

$$\eta = \frac{w_{ac}}{w_s} = \frac{390.2}{448.1} = \mathbf{0.871}$$

Borgnakke and Sonntag

7.131

A compressor in a commercial refrigerator receives R-410A at -25°C and x = 1.
The exit is at 2000 kPa and 80°C. Neglect kinetic energies and find the isentropic compressor efficiency.

Solution:

C.V. Compressor, steady state, single inlet and
exit flow. For this device we also assume no
heat transfer q = 0 and $Z_1 = Z_2$

Energy Eq.4.13: $h_i + 0 = w_C + h_e$;

Entropy Eq.7.9: $s_i + \int dq/T + s_{gen} = s_e = s_i + 0 + s_{gen}$

From Table B.4.1 : $h_i = 269.77$ kJ/kg, $s_i = 1.0893$ kJ/kgK

From Table B.4.2 : $h_e = 343.22$ kJ/kg, $s_e = 1.1537$ kJ/kgK

Energy equation gives

$w_{C\ ac} = h_i - h_e = 269.77 - 343.22 = $ -73.45 kJ/kgK

The ideal compressor has an exit state e,s: 2000 kPa, 1.0893 kJ/kgK

Table B.4.2 $\Rightarrow$ $T_{e\ s} \cong 60.45°C$, $h_{e\ s} = 321.13$ kJ/kg

$w_{C\ s} = 269.77 - 321.13 = -51.36$ kJ/kg

The isentropic efficiency measures the actual compressor to the ideal one

$\eta = w_{C\ s} / w_{C\ ac} = -51.36 / (-73.45) = $ **0.70**

7.132

A steam turbine has an inlet of 2 kg/s water at 1000 kPa and 400°C with velocity of 15 m/s. The exit is at 100 kPa, 150°C and very low velocity. Find the power produced and the rate of entropy generation.

Solution:

C.V. Turbine. Steady flow and adiabatic.

Continuity Eq.6.9: $\dot{m}_1 = \dot{m}_2$;

Energy Eq.6.10: $\dot{m}_1(h_1 + \frac{1}{2}\mathbf{V}^2) = \dot{m}_2 h_2 + \dot{W}$

Entropy Eq.9.7: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2$

States from Table B.1.3: $h_1 = 3263.88$ kJ/kg, $s_1 = 7.4650$ kJ/kgK,

$\qquad\qquad\qquad\qquad h_2 = 2776.38$ kJ/kg, $s_2 = 7.6133$ kJ/kgK

$$\dot{W} = \dot{m}_1 (h_1 + \frac{1}{2}\mathbf{V}^2 - h_2) = 2 \text{ kg/s} (3263.88 + \frac{1}{2}\frac{15^2}{1000} - 2776.38) \text{ kJ/kg}$$

$$= \mathbf{975\ kW}$$

$$\dot{S}_{gen} = \dot{m}_1(s_2 - s_1) = 2 \text{ kg/s} (7.6133 - 7.465) \text{ kJ/kgK} = \mathbf{0.297\ kW/K}$$

7.133

The exit velocity of a nozzle is 500 m/s. If $\eta_{nozzle} = 0.88$ what is the ideal exit velocity?

The nozzle efficiency is given by Eq. 7.29 and since we have the actual exit velocity we get

$$\mathbf{V}_{e\,s}^2 = \mathbf{V}_{ac}^2/\eta_{nozzle} \Rightarrow$$
$$\mathbf{V}_{e\,s} = \mathbf{V}_{ac}/\sqrt{\eta_{nozzle}} = 500\,/\sqrt{0.88} = \mathbf{533\ m/s}$$

Borgnakke and Sonntag

7.134

An emergency drain pump should be able to pump 0.1 m³/s liquid water at 15°C, 10 m vertically up delivering it with a velocity of 20 m/s. It is estimated that the pump, pipe and nozzle have a combined isentropic efficiency expressed for the pump as 60%. How much power is needed to drive the pump?

Solution:

C.V. Pump, pipe and nozzle together. Steady flow, no heat transfer.

Consider the ideal case first (it is the reference for the efficiency).

Energy Eq.4.12: $\dot{m}_i(h_i + \mathbf{V}^2_i/2 + gZ_i) + \dot{W}_{in} = \dot{m}_e(h_e + \mathbf{V}^2_e/2 + gZ_e)$

Solve for work and use reversible process Eq.7.15

$$\dot{W}_{ins} = \dot{m}\ [h_e - h_i + (\mathbf{V}^2_e - \mathbf{V}^2_i)/2 + g(Z_e - Z_i)]$$

$$= \dot{m}[(P_e - P_i)v + \mathbf{V}^2_e/2 + g\Delta Z]$$

$$\dot{m} = \dot{V}/v = 0.1/0.001001 = 99.9 \text{ kg/s}$$

$$\dot{W}_{ins} = 99.9[0 + (20^2/2) \times (1/1000) + 9.807 \times (10/1000)]$$

$$= 99.9(0.2 + 0.09807) = 29.8 \text{ kW}$$

With the estimated efficiency the actual work, Eq.7.28 is

$$\dot{W}_{in\ actual} = \dot{W}_{in\ s}/\eta = 29.8/0.6 = 49.7 \text{ kW} = \mathbf{50\ kW}$$

7.135

Find the isentropic efficiency of the R-134a compressor in Example 4.8

Solution:

State 1: Table B.5.2 $h_1 = 387.2$ kJ/kg; $s_1 = 1.7665$ kJ/kg K

State 2ac: $h_2 = 435.1$ kJ/kg

State 2s: s = 1.7665 kJ/kg K,   800 kPa   =>   h = 431.8 kJ/kg; T = 46.8°C

   $-w_{c\ s} = h_{2s} - h_1 = 431.8 - 387.2 = 44.6$ kJ/kg

   $-w_{ac} = 5/0.1 = 50$ kJ/kg

   $\eta = w_{c\ s}/\ w_{ac} = 44.6/50 =$ **0.89**

7.136

A gas turbine with air flowing in at 1200 kPa, 1200 K has an exit pressure of 200 kPa and an isentropic efficiency of 87%. Find the exit temperature.

Solution:

C.V. Ideal air turbine.

Adiabatic: $q = 0$, reversible: $s_{gen} = 0$

Energy Eq.4.13: $w_T = h_i - h_e$,

Entropy Eq.7.9: $s_e = s_i$

Table A.7: $h_i = 1277.8$ kJ/kg, $s^o_{Ti} = 8.34596$ kJ/kg K

The constant s process is written from Eq.6.19 as

$$\Rightarrow s^o_{Te} = s^o_{Ti} + R \ln(\frac{P_e}{P_i}) = 8.34596 + 0.287 \ln\left(\frac{200}{1200}\right) = 7.83173 \text{ kJ/kg K}$$

Interpolate in A.7.1 $\Rightarrow T_{e\,s} = 761.9$ K, $h_{e\,s} = 780.52$ kJ/kg

$$w_{T\,s} = h_i - h_{e\,s} = 1277.81 - 780.52 = 497.3 \text{ kJ/kg}$$

The actual turbine then has

$$w_{T\,ac} = \eta_T\, w_{T\,s} = 0.87 \times 497.3 = 432.65 \text{ kJ/kg} = h_i - h_{e\,ac}$$

$$h_{e\,ac} = h_i - w_{T\,ac} = 1277.81 - 432.65 = 845.16 \text{ kJ/kg}$$

Interpolate in A.7.1 $\Rightarrow$ $T_{e\,ac} = \textbf{820.8 K}$

If constant specific heats are used we get

Table A.5: $C_{Po} = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$, then from Eq.6.23

$$T_{e\,s} = T_i\, (P_e/P_i)^{\frac{k-1}{k}} = 1200 \left(\frac{200}{1200}\right)^{0.286} = 719.2 \text{ K}$$

$$w_{T\,s} = C_{Po}(T_i - T_{e\,s}) = 1.004(1200 - 719.2) = 482.72 \text{ kJ/kg}$$

The actual turbine then has

$$w_{T\,ac} = \eta_T\, w_{T\,s} = 0.87 \times 482.72 = 419.97 \text{ kJ/kg} = C_{Po}(T_i - T_{e\,ac})$$

$$T_{e\,ac} = T_i - w_{T\,ac}/ C_{Po} = 1200 - 419.97/1.004 = \textbf{781.7 K}$$

7.137

A gas turbine with air flowing in at 1200 kPa, 1200 K has an exit pressure of 200 kPa. Find the lowest possible exit temperature. Which efficiency does that correspond to?

Solution:

Look at the T-s diagram for the possible processes. We notice that the lowest exit T is for the isentropic process (the ideal turbine)

Table A.7: $h_i = 1277.8$ kJ/kg, $s^o_{Ti} = 8.34596$ kJ/kg K

The constant s process is written from Eq.6.19 as

$$\Rightarrow s^o_{Te} = s^o_{Ti} + R \ln(\frac{P_e}{P_i}) = 8.34596 + 0.287 \ln\left(\frac{200}{1200}\right) = 7.83173 \text{ kJ/kg K}$$

Interpolate in A.7.1 $\Rightarrow T_{e\,s} = $ **761.9 K**

This is an efficiency of **100%**

Borgnakke and Sonntag

7.138

Repeat Problem 7.46 assuming the steam turbine and the air compressor each have an isentropic efficiency of 80%.

A certain industrial process requires a steady supply of saturated vapor steam at 200 kPa, at a rate of 0.5 kg/s. Also required is a steady supply of compressed air at 500 kPa, at a rate of 0.1 kg/s. Both are to be supplied by the process shown in Fig. P9.41. Steam is expanded in a turbine to supply the power needed to drive the air compressor, and the exhaust steam exits the turbine at the desired state. Air into the compressor is at the ambient conditions, 100 kPa, 20°C. Give the required steam inlet pressure and temperature, assuming that both the turbine and the compressor are reversible and adiabatic.

Solution:

C.V. Each device. Steady flow.
Both adiabatic (q = 0) and actual
devices ($s_{gen} > 0$) given by η_{sT}
and η_{sc}.

Steam turbine   Air compressor

Air Eq.6.23, $\quad T_{4s} = T_3(P_4/P_3)^{\frac{k-1}{k}} = 293.2\left(\frac{500}{100}\right)^{0.286} = 464.6$ K

$\dot{W}_{Cs} = \dot{m}_3(h_3 - h_{4s}) = 0.1$ kg/s $\times$ 1.004 kJ/kgK (293.2 - 464.6) K

$\qquad = -17.21$ kW

$\dot{W}_{Cs} = \dot{m}_3(h_3 - h_4) = \dot{W}_{Cs}/\eta_{sc} = -17.2/0.80 = -21.5$ kW

Now the actual turbine must supply the actual compressor work. The actual state 2 is given so we must work backwards to state 1.

$$\dot{W}_T = +21.5 \text{ kW} = \dot{m}_1(h_1 - h_2) = 0.5(h_1 - 2706.6)$$

$$\Rightarrow \quad h_1 = 2749.6 \text{ kJ/kg}$$

Also, $\quad \eta_{sT} = 0.80 = (h_1 - h_2)/(h_1 - h_{2s}) = 43/(2749.6 - h_{2s})$

$$\Rightarrow h_{2s} = 2695.8 \text{ kJ/kg}$$

$$2695.8 = 504.7 + x_{2s}(2706.6 - 504.7) \quad => \quad x_{2s} = 0.9951$$

$$s_{2s} = 1.5301 + 0.9951(7.1271 - 1.5301) = 7.0996 \text{ kJ/kg K}$$

$$(s_1 = s_{2s}, h_1) \rightarrow P_1 = \textbf{269 kPa}, \quad T_1 = \textbf{143.5°C}$$

7.139

Liquid water enters a pump at 15°C, 100 kPa, and exits at a pressure of 5 MPa. If the isentropic efficiency of the pump is 75%, determine the enthalpy (steam table reference) of the water at the pump exit.

Solution:

CV: pump $\dot{Q}_{CV} \approx 0,\;\; \Delta KE \approx 0,\;\; \Delta PE \approx 0$

2nd law, reversible (ideal) process: $s_{es} = s_i \;\Rightarrow$

Eq.7.15 for work term.

$$w_s = -\int_{i}^{es} vdP \approx -v_i(P_e - P_i) = -0.001001 \text{ m}^3/\text{kg } (5000 - 100) \text{ kPa}$$

$$= -4.905 \text{ kJ/kg}$$

Real process Eq.7.28: $w = w_s/\eta_s = -4.905/0.75 = -6.54$ kJ/kg

Energy Eq.4.13: $h_e = h_i - w = 62.99 + 6.54 =$ **69.53 kJ/kg**

7.140

Ammonia is brought from saturated vapor at 300 kPa to 1400 kPa, 140°C in a steady flow adiabatic compressor. Find the compressor specific work, entropy generation and its isentropic efficiency.

C.V. Actual Compressor, assume adiabatic and neglect kinetic energies.

Energy Eq.4.13: $\qquad w_C = h_i - h_e$

Entropy Eq.7.9: $\qquad s_e = s_i + s_{gen}$

States: 1: B.2.2 $\quad h_i = 1431.7$ kJ/kg, $\quad s_i = 5.4565$ kJ/kg-K

$\qquad$ 2: B.2.2 $\quad h_e = 1752.8$ kJ/kg, $\quad s_e = 5.7023$ kJ/kg-K

$$-w_C = h_e - h_i = 1752.8 - 1431.7 = 321.1 \text{ kJ/kg}$$

Ideal compressor. We find the exit state from (P,s).

State 2s: $P_e, s_{es} = s_i = 5.4565$ kJ/kg-K $\quad \Rightarrow \quad h_{es} = 1656.08$ kJ/kg

$$-w_{Cs} = h_{2s} - h_i = 1656.08 - 1431.7 = 224.38 \text{ kJ/kg}$$

$$\eta_C = -w_{Cs} / -w_C = \frac{224.38}{321.1} = \mathbf{0.699}$$

7.141

Find the isentropic efficiency of the nozzle in example 4.4.

Solution:

C.V. adiabatic nozzle with known inlet state and velocity.

Inlet state: B.1.3 $h_i = 2850.1$ kJ/kg; $s_i = 6.9665$ kJ/kg K

Process ideal: adiabatic and reversible   Eq.7.9 gives constant s

ideal exit, (150 kPa, s) ; $x_{es} = (6.9665 - 1.4335)/5.7897 = 0.9557$

$h_{es} = h_f + x_{es}\, h_{fg} = 2594.9$ kJ/kg

$\mathbf{V}_{es}^2/2 = h_i - h_{es} + \mathbf{V}_i^2/2 = 2850.1 - 2594.9 + (50^2)/2000 = 256.45$ kJ/kg

$\mathbf{V}_{es} = 716.2$ m/s

From Eq.7.30,

$$\eta_{noz.} = (\mathbf{V}_e^2/2)/(\mathbf{V}_{es}^2/2) = 180/256.45 = \mathbf{0.70}$$

7.142

A centrifugal compressor takes in ambient air at 100 kPa, 17°C, and discharges it at 450 kPa. The compressor has an isentropic efficiency of 80%. What is your best estimate for the discharge temperature?

Solution:

C.V. Compressor. Assume adiabatic, no kinetic energy is important.

Energy Eq.4.13: $\qquad w = h_1 - h_2$

Entropy Eq.7.9: $\qquad s_2 = s_1 + s_{gen}$

We have two different cases, the ideal and the actual compressor.

We will solve using constant specific heat.

State 2 for the ideal, $s_{gen} = 0$ so $s_2 = s_1$ and it becomes:

Eq.8.23: $\quad T_{2s} = T_1 \left(\dfrac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 290 \, (450 / 100)^{0.2857} = 445.7 \text{ K}$

$\quad w_s = h_1 - h_{2s} = C_p \, (T_1 - T_{2s}) = 1.004 \, (290 - 445.7) = -156.3 \text{ kJ/kg}$

The actual work from definition Eq.7.28 and then energy equation:

$\quad w_{ac} = w_s/\eta = -156.3 / 0.8 = -195.4 \text{ kJ/kg} = h_1 - h_2 = C_p(T_1 - T_2)$

$$\Rightarrow \; T_2 = T_1 - w_{ac} / C_p$$

$$= 290 + 195.4/1.004 = \mathbf{484.6 \text{ K}}$$

Solving using Table A.7.1 instead will give

State 1: Table A.7.1: $\quad s^o_{T1} = 6.83521 \text{ kJ/kg K}$

Now constant s for the ideal is done with Eq.6.19

$$s^o_{T2s} = s^o_{T1} + R \ln(\frac{P_2}{P_1}) = 6.83521 + 0.287 \ln(\frac{450}{100}) = 7.26688 \text{ kJ/kg K}$$

From A.7.1: $\quad T_{2s} = 442.1 \text{ K}$ and $h_{2s} = 446.795 \text{ kJ/kg}$

$w_s = h_1 - h_{2s} = 290.43 - 446.795 = -156.4 \text{ kJ/kg}$

The actual work from definition Eq.7.28 and then energy equation:

$\quad w_{ac} = w_s/\eta = -156.4 / 0.8 = -195.5 \text{ kJ/kg}$

$$\Rightarrow \; h_2 = 195.5 + 290.43 = 485.93, \quad \text{Table A.7.1:} \quad T_2 = \mathbf{483 \text{ K}}$$

The answer is very close to the previous one due to the modest T's.

Borgnakke and Sonntag

7.143

A refrigerator uses carbon dioxide that is brought from 1 MPa, -20$^{\text{o}}$C to 6 MPa using 2 kW power input to the compressor with a flow rate of 0.02 kg/s. Find the compressor exit temperature and its isentropic efficiency.

C.V. Actual Compressor, assume adiabatic and neglect kinetic energies.

Energy Eq.4.13: $\qquad -w_C = h_2 - h_1 = \dfrac{\dot{W}}{\dot{m}} = \dfrac{2 \text{ kW}}{0.02 \text{ kg/s}} = 100 \text{ kJ/kg}$

Entropy Eq.7.9: $\qquad s_2 = s_1 + s_{gen}$

States: 1: B.3.2 $\quad h_1 = 342.31$ kJ/kg, $\quad s_1 = 1.4655$ kJ/kg-K

$\qquad$ 2: B.3.2 $\quad h_2 = h_1 - w_C = 442.31$ kJ/kg $\quad \Rightarrow \quad T_2 = \textbf{117.7}^{\textbf{o}}\textbf{C}$

Ideal compressor. We find the exit state from (P,s).

State 2s: $\quad P_2, s_{2s} = s_1 = 1.4655$ kJ/kg-K $\quad \Rightarrow \quad h_{2s} = 437.55$ kJ/kg

$\qquad\qquad -w_{C\,s} = h_{2s} - h_1 = 437.55 - 342.31 = 95.24$ kJ/kg

$$\eta_C = -w_{C\,s} / -w_C = \frac{95.24}{100} = \textbf{0.952}$$

7.144

The small turbine in Problem 7.33 was ideal. Assume instead the isentropic turbine efficiency is 88%. Find the actual specific turbine work and the entropy generated in the turbine.

Solution:

Continuity Eq.4.11: (Steady)

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}$$

Turbine: Energy Eq.4.13:

$$w_T = h_1 - h_2$$

Entropy Eq.7.9: $s_2 = s_1 + s_{T\,gen}$

Inlet state: Table B.1.3 $h_1 = 3917.45$ kJ/kg, $s_1 = 7.9487$ kJ/kg K

Ideal turbine $s_{T\,gen} = 0$, $s_2 = s_1 = 7.9487 = s_{f2} + x\,s_{fg2}$

State 2: $P = 10$ kPa, $s_2 < s_g$ => saturated 2-phase in Table B.1.2

$\Rightarrow\ x_{2,s} = (s_1 - s_{f2})/s_{fg2} = (7.9487 - 0.6492)/7.501 = 0.9731$

$\Rightarrow\ h_{2,s} = h_{f2} + x \times h_{fg2} = 191.8 + 0.9731 \times 2392.8 = 2520.35$ kJ/kg

$\quad w_{T,s} = h_1 - h_{2,s} = 1397.05$ kJ/kg

Explanation for the reversible work term is in sect. 7.3 Eq.7.14

$$w_{T,AC} = \eta \times w_{T,s} = \textbf{1229.9 kJ/kg}$$

$$= h_1 - h_{2,AC} \Rightarrow h_{2,AC} = h_1 - w_{T,AC} = 2687.5 \text{ kJ/kg}$$

$$\Rightarrow T_{2,AC} = 100°C ,\ s_{2,AC} = 8.4479 \text{ kJ/kg-K}$$

$$s_{T\,gen} = s_{2,AC} - s_1 = \textbf{0.4992 kJ/kg K}$$

7.145

Redo Problem 7.41 assuming the compressor and turbine in the turbocharger both have isentropic efficiency of 85%.

CV: Turbine, Steady single inlet and exit flows,

Process: adiabatic: q = 0,

reversible: $s_{gen} = 0$

EnergyEq.4.13: $w_T = h_3 - h_4$,

Entropy Eq.7.8: $s_4 = s_3$

The property relation for ideal gas gives Eq.6.23, k from Table A.5

$$s_4 = s_3 \;\rightarrow\; T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 923.2 \text{ K} \left(\frac{100}{170}\right)^{0.286} = \textbf{793.2 K}$$

The energy equation is evaluated with specific heat from Table A.5

$$w_{T\,s} = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004(923.2 - 793.2) = 130.5 \text{ kJ/kg}$$

The actual turbine : $w_{T\,ac} = \eta_T\, w_{T\,s} = 130.5 \times 0.85 = 110.9$ kJ/kg

$$\dot{W}_T = \dot{m}w_T = 110.9 \text{ kJ/kg} \times 0.1 \text{ kg/s} = \textbf{11.09 kW}$$

C.V. Compressor, steady 1 inlet and 1 exit, same flow rate as turbine.

Energy Eq.4.13: $-w_C = h_2 - h_1$,

Entropy Eq.7.9: $s_2 = s_1$

Express the energy equation for the shaft and actual compressor having the actual turbine power as input with the same mass flow rate so we get

$$-w_{C\,ac} = -w_{C\,s}/\eta_C = C_{P0}(T_{2\,s} - T_1)/\eta_C = w_{T\,ac} = 110.9 \text{ kJ/kg}$$
$$= 1.004(T_{2\,ac} - 303.2) = 1.004(T_{2\,s} - 303.2)/0.85$$
$$T_{2\,ac} = \textbf{413.66 K,}\quad T_{2\,s} = 397.09 \text{ K}$$

The property relation for $s_2 = s_1$ (only for ideal) is Eq.6.23 and inverted as

$$P_2 = P_1(T_{2\,s}/T_1)^{\frac{k}{k-1}} = 100 \text{ kPa} \left(\frac{397.09}{303.2}\right)^{3.5} = \textbf{257 kPa}$$

Borgnakke and Sonntag

7.146

A pump receives water at 100 kPa, 15°C and a power input of 1.5 kW. The pump has an isentropic efficiency of 75% and it should flow 1.2 kg/s delivered at 30 m/s exit velocity. How high an exit pressure can the pump produce?

Solution:

CV Pump. We will assume the ideal and actual pumps have same exit pressure, then we can analyse the ideal pump.

Specific work: $\qquad$ $w_{ac} = 1.5/1.2 = 1.25$ kJ/kg

Ideal work Eq.7.28: $\qquad$ $w_s = \eta\, w_{ac} = 0.75 \times 1.25 = 0.9375$ kJ/kg

As the water is incompressible (liquid) we get

Energy Eq.7.14:

$$w_s = (P_e - P_i)v + V^2_e/2 = (P_e - P_i)0.001001 + (30^2/2)/1000$$
$$= (P_e - P_i)0.001001 + 0.45$$

Solve for the pressure difference

$$P_e - P_i = (w_s - 0.45)/0.001001 = 487 \text{ kPa}$$

$P_e = 587$ kPa

Water pump from a car

7.147

A turbine receives air at 1500 K, 1000 kPa and expands it to 100 kPa. The turbine has an isentropic efficiency of 85%. Find the actual turbine exit air temperature and the specific entropy increase in the actual turbine using Table A.7.

Solution:

C.V. Turbine. steady single inlet and exit flow.

To analyze the actual turbine we must first do the ideal one (the reference).

Energy Eq.4.13: $w_T = h_1 - h_2$;

Entropy Eq.7.9: $s_2 = s_1 + s_{gen} = s_1$

Entropy change in Eq.6.19 and Table A.7.1:

$$s^o_{T2} = s^o_{T1} + R \ln(P_2/P_1) = 8.61208 + 0.287 \ln(100/1000) = 7.95124$$

Interpolate in A.7 $\Rightarrow$ $T_{2s} = 849.2$, $h_{2s} = 876.56$ $\Rightarrow$

$$w_T = 1635.8 - 876.56 = 759.24 \text{ kJ/kg}$$

Now we can consider the actual turbine from Eq.7.27 and Eq.4.13:

$$w^T_{ac} = \eta_T w_T = 0.85 \times 759.24 = 645.35 = h_1 - h_{2ac}$$

$$\Rightarrow \quad h_{2ac} = h_1 - w^T_{ac} = 990.45 \quad \Rightarrow \quad \mathbf{T_{2ac} = 951\ K}$$

The entropy balance equation is solved for the generation term

$$s_{gen} = s_{2ac} - s_1 = 8.078 - 8.6121 - 0.287 \ln(100/1000) = \mathbf{0.1268\ kJ/kg\ K}$$

7.148

Air enters an insulated turbine at 50°C, and exits the turbine at - 30°C, 100 kPa. The isentropic turbine efficiency is 70% and the inlet volumetric flow rate is 20 L/s. What is the turbine inlet pressure and the turbine power output?

Solution:

C.V.: Turbine, $\eta_S = 0.7$, Insulated

Air table A.5: $C_p = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$

Inlet: $T_i = 50^oC$, $\dot{V}_i = 20$ L/s $= 0.02$ m^3/s ;

Exit (actual): $T_e = -30^oC$, $P_e = 100$ kPa

1st Law Steady state Eq.4.13: $\quad q_T + h_i = h_e + w_T$; $\quad q_T = 0$

Assume Constant Specific Heat

$\quad$ $w_T = h_i - h_e = C_p(T_i - T_e) = 80.3$ kJ/kg

$\quad$ $w_{Ts} = w/\eta = 114.7$ kJ/kg, $\quad$ $w_{Ts} = C_p(T_i - T_{es})$

$\quad$ Solve for $T_{es} = 208.9$ K

Isentropic Process Eq.6.23: $\quad P_e = P_i\,(T_e\,/\,T_i)^{\frac{k}{k-1}}$ $\quad \Rightarrow$ $\quad$ **$P_i = 461$ kPa**

$\quad$ $\dot{m} = P_i\dot{V}/RT_i = (461$ kPa $\times 0.02$ m^3/s$)\,/(0.287$ kJ/kg-K $\times 323.15$ K$)$

$\quad\quad = 0.099$ kg/s

$\quad$ $\dot{W}_T = \dot{m}w_T = 0.099$ kg/s $\times 80.3$ kJ/kg $=$ **7.98 kW**

7.149

Carbon dioxide, CO_2, enters an adiabatic compressor at 100 kPa, 300 K, and exits at 1000 kPa, 520 K. Find the compressor efficiency and the entropy generation for the process.

Solution:

C.V. Ideal compressor. We will assume constant heat capacity.

Energy Eq.4.13: $w_c = h_1 - h_2$,

Entropy Eq.7.9: $s_2 = s_1$: $T_{2s} = T_1\left(\dfrac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300\left(\dfrac{1000}{100}\right)^{0.2242} = 502.7$ K

$\quad w_{cs} = C_p(T_1 - T_{2s}) = 0.842(300-502.7) = -170.67$ kJ/kg

C.V. Actual compressor

$\quad w_{cac} = C_p(T_1 - T_{2ac}) = 0.842(300 - 520) = -185.2$ kJ/kg

$\quad \eta_c = w_{cs}/w_{cac} = -170.67/(-185.2) = \mathbf{0.92}$

Use Eq.6.16 for the change in entropy

$\quad s_{gen} = s_{2ac} - s_1 = C_p \ln (T_{2ac}/T_1) - R \ln (P_2/P_1)$

$\quad\quad = 0.842 \ln(520 / 300) - 0.1889 \ln(1000 / 100) = \mathbf{0.028 \ kJ/kg \ K}$

Constant heat capacity is not the best approximation. It would be more accurate to use Table A.8. Entropy change in Eq.6.19 and Table A.8:

$\quad s_{T2}^o = s_{T1}^o + R \ln(P_2/P_1) = 4.8631 + 0.1889 \ln(1000/100) = 5.29806$

$\quad$ Interpolate in A.8 $\Rightarrow$ $T_{2s} = 481$ K, $h_{2s} = 382.807$ kJ/kg $\Rightarrow$

$-w_{cs} = 382.807 – 214.38 = 168.43$ kJ/kg; $-w_{cac} = 422.12 – 214.38 = 207.74$ kJ/kg

$\quad\quad \eta_c = w_{cs}/w_{cac} = -168.43/(-207.74) = \mathbf{0.81}$

$\quad\quad s_{gen} = s_{2ac} - s_1 = 5.3767 – 4.8631 – 0.1889 \ln(10) = 0.0786$ kJ/kgK

7.150

A small air turbine with an isentropic efficiency of 80% should produce 270 kJ/kg of work. The inlet temperature is 1000 K and it exhausts to the atmosphere. Find the required inlet pressure and the exhaust temperature.

Solution:

C.V. Turbine actual energy Eq.4.13:

$$w = h_i - h_{e,ac} = 270 \text{ kJ/kg}$$

Table A.7: $h_i = 1046.22 \implies h_{e,ac} = 776.22 \text{ kJ/kg},$ **T_e = 757.9 K**

C.V. Ideal turbine, Eq.7.27 and energy Eq.4.13:

$$w_s = w/\eta_s = 270/0.8 = 337.5 = h_i - h_{e,s} \implies h_{e,s} = 708.72 \text{ kJ/kg}$$

From Table A.7: $T_{e,s} = 695.5$ K

Entropy Eq.7.9: $s_i = s_{e,s}$ adiabatic and reversible

To relate the entropy to the pressure use Eq.6.19 inverted and standard entropy from Table A.7.1:

$$P_e/P_i = \exp[(s_{Te}^o - s_{Ti}^o) / R] = \exp[(7.733 - 8.13493)/0.287] = 0.2465$$

$$P_i = P_e / 0.2465 = 101.3/0.2465 = \textbf{411 kPa}$$

If constant heat capacity were used

$$T_e = T_i - w/C_p = 1000 - 270/1.004 = 731 \text{ K}$$

C.V. Ideal turbine, Eq.7.27 and energy Eq.4.13:

$$w_s = w/\eta_s = 270/0.8 = 337.5 \text{ kJ/kg} = h_i - h_{e,s} = C_p(T_i - T_{e,s})$$

$$T_{e,s} = T_i - w_s/C_p = 1000 - 337.5/1.004 = 663.8 \text{ K}$$

Eq.7.9 (adibatic and reversible) gives constant s and relation is Eq.6.23

$$P_e/P_i = (T_e/T_i)^{k/(k-1)} \implies P_i = 101.3 (1000/663.8)^{3.5} = 425 \text{ kPa}$$

7.151

A compressor in an industrial air-conditioner compresses ammonia from a state of saturated vapor at 200 kPa to a pressure 800 kPa. At the exit, the temperature is measured to be 100°C and the mass flow rate is 0.5 kg/s. What is the required motor size for this compressor and what is its isentropic efficiency?

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.4.13: $\qquad\qquad w = h_1 - h_2$

Entropy Eq.7.9: $\qquad\qquad s_2 = s_1 + s_{gen}$

We have two different cases, the ideal and the actual compressor.

States: 1: B.2.2: $\quad h_1 = 1419.6$ kJ/kg, $v_1 = 0.5946$ m^3/kg, $s_1 = 5.5979$ kJ/kg K

$\qquad$ 2ac: B.2.3 $\quad h_{2,AC} = 1670.6$ kJ/kg, $\quad v_{2,AC} = 0.21949$ m^3/kg

$\qquad\quad$ 2s: B.2.3 (P, s = s$_1$) $\quad h_{2,s} = 1613.7$ kJ/kg, $T_{2,s} = 76.6^{\text{o}}\text{C}$

ACTUAL:

$$-w_{C,AC} = h_{2,AC} - h_1 = 1670.6 - 1419.6 = 251 \text{ kJ/kg}$$

$$\dot{W}_{in} = \dot{m}\,(-w_{C,AC}) = 0.5 \text{ kg/s} \times 251 \text{ kJ/kg} = \textbf{125.5 kW}$$

IDEAL:

$$-w_{c,s} = h_{2,s} - h_1 = 1613.7 - 1419.6 = 194.1 \text{ kJ/kg}$$

Definition Eq.7.28: $\qquad\qquad \eta_c = w_{c,s}/w_{c,AC} = \textbf{0.77}$

7.152

Repeat Problem 7.48 assuming the turbine and the pump each have an isentropic efficiency of 85%.

Solution:

$P_1 = P_4 = 20$ MPa
$T_1 = 700$ °C
$P_2 = P_3 = 20$ kPa
$T_3 = 40$ °C
$\eta_P = \eta_T = 85\%$

a) State 1: (P, T) Table B.1.3 $h_1 = 3809.1$ kJ/kg, $s_1 = 6.7993$ kJ/kg K

C.V. Turbine. First we do the ideal, then the actual.

Entropy Eq.7.9: $s_2 = s_1 = 6.7993$ kJ/kg K

Table B.1.2 $s_2 = 0.8319 + x_2 \times 7.0766$ => $x_2 = 0.8433$

$h_{2\,s} = 251.4 + 0.8433 \times 2358.33 = 2240.1$ kJ/kg

Energy Eq.4.13: $w_{T\,s} = h_1 - h_{2\,s} = 1569$ kJ/kg

$w_{T\,AC} = \eta_T w_{T\,s} = \mathbf{1333.65} = h_1 - h_{2\,AC}$

$h_{2\,AC} = h_1 - w_{T\,AC} = 2475.45$ kJ/kg;

$\mathbf{x_{2,AC} = (2475.45 - 251.4)/2358.3 = 0.943}$, $\mathbf{T_{2,AC} = 60.06°C}$

b)

State 3: (P, T) Compressed liquid, take sat. liq. Table B.1.1

$h_3 = 167.54$ kJ/kg, $v_3 = 0.001008$ m^3/kg

$w_{P\,s} = -v_3(P_4 - P_3) = -0.001008(20000 - 20) = -20.1$ kJ/kg

$-w_{P,AC} = -w_{P,s}/\eta_\rho = 20.1/0.85 = \mathbf{23.7} = h_{4,AC} - h_3$

$h_{4,AC} = \mathbf{191.2}$ $T_{4,AC} \cong 45.7°C$

c) The heat transfer in the boiler is from energy Eq.4.13

$q_{boiler} = h_1 - h_4 = 3809.1 - 191.2 = 3617.9$ kJ/kg

$w_{net} = 1333.65 - 23.7 = \mathbf{1310}$ **kJ/kg**

$\eta_{TH} = w_{net}/q_{boiler} = \dfrac{1310}{3617.9} = \mathbf{0.362}$

7.153

Assume an actual compressor has the same exit pressure and specific heat transfer as the ideal isothermal compressor in Problem 7.27 with an isothermal efficiency of 80%. Find the specific work and exit temperature for the actual compressor.

Solution:

C.V. Compressor. Steady, single inlet and single exit flows.

Energy Eq.4.13: $h_i + q = w + h_e$;

Entropy Eq.7.9: $s_i + q/T = s_e$

Inlet state: Table B.5.2, $h_i = 403.4$ kJ/kg, $s_i = 1.8281$ kJ/kg K

Exit state: Table B.5.1, $h_e = 398.36$ kJ/kg, $s_e = 1.7262$ kJ/kg K

$q = T(s_e - s_i) = 273.15(1.7262 - 1.8281) = - 27.83$ kJ/kg

$w = 403.4 + (-27.83) - 398.36 = -22.8$ kJ/kg

From Eq.7.29 for a cooled compressor

$w_{ac} = w_T /\eta = - 22.8/0.8 =$ **28.5 kJ/kg**

Now the energy equation gives

$h_e = h_i + q - w_{ac} = 403.4 + (-27.83) + 28.5 = 404.07$

$T_{e\ ac} \approx$ **6°C** $\qquad\qquad P_e = 294$ kPa

Explanation for the reversible work term is in Sect. 7.3
Eqs. 7.13 and 7.14

Borgnakke and Sonntag

7.154

A nozzle in a high pressure liquid water sprayer has an area of 0.5 cm^2. It receives water at 350 kPa, 20°C and the exit pressure is 100 kPa. Neglect the inlet kinetic energy and assume a nozzle isentropic efficiency of 85%. Find the ideal nozzle exit velocity and the actual nozzle mass flow rate.

Solution:

C.V. Nozzle. Liquid water is incompressible v ≈ constant, no work, no heat transfer => Bernoulli Eq.7.17

$$\frac{1}{2}\mathbf{V}_{ex}^2 - 0 = v(P_i - P_e) = 0.001002\ (350 - 100) = 0.2505 \text{ kJ/kg}$$

$$\mathbf{V}_{ex} = \sqrt{2 \times 0.2505 \times 1000 \text{ J/kg}} = \mathbf{22.38\ m\ s^{-1}}$$

This was the ideal nozzle now we can do the actual nozzle, Eq. 7.30

$$\frac{1}{2}\mathbf{V}_{ex\ ac}^2 = \eta\, \frac{1}{2}\mathbf{V}_{ex}^2 = 0.85 \times 0.2505 = 0.2129 \text{ kJ/kg}$$

$$\mathbf{V}_{ex\ ac} = \sqrt{2 \times 0.2129 \times 1000 \text{ J/kg}} = 20.63 \text{ m s}^{-1}$$

$$\dot{m} = \rho A \mathbf{V}_{ex\ ac} = A\mathbf{V}_{ex\ ac}/v = 0.5 \times 10^{-4} \times 20.63 / 0.001002 = \mathbf{1.03\ kg/s}$$

These are examples of relatively low pressure spray systems.

7.155

Air flows into an insulated nozzle at 1 MPa, 1200 K with 15 m/s and mass flow rate of 2 kg/s. It expands to 650 kPa and exit temperature is 1100 K. Find the exit velocity, and the nozzle efficiency.

Solution:

C.V. Nozzle. Steady 1 inlet and 1 exit flows, no heat transfer, no work.

Energy Eq.4.13: $h_i + (1/2)\mathbf{V}_i^2 = h_e + (1/2)\mathbf{V}_e^2$

Entropy Eq.7.9: $s_i + s_{gen} = s_e$

Ideal nozzle $s_{gen} = 0$ and assume same exit pressure as actual nozzle. Instead of using the standard entropy from Table A.7 and Eq.6.19 let us use a constant heat capacity at the average T and Eq.6.23. First from A.7.1

$$C_{p\ 1150} = \frac{1277.81 - 1161.18}{1200 - 1100} = 1.166 \text{ kJ/kg K};$$

$$C_v = C_{p\ 1150} - R = 1.166 - 0.287 = 0.8793, \quad k = C_{p\ 1150}/C_v = 1.326$$

Notice how they differ from Table A.5 values.

$$T_{e\ s} = T_i\ (P_e/P_i)^{\frac{k-1}{k}} = 1200 \left(\frac{650}{1000}\right)^{0.24585} = 1079.4 \text{ K}$$

$$\frac{1}{2}\mathbf{V}_{e\ s}^2 = \frac{1}{2}\mathbf{V}_i^2 + C(T_i - T_{e\ s}) = \frac{1}{2} \times 15^2 + 1.166(1200 - 1079.4) \times 1000$$

$$= 112.5 + 140619.6 = 140732 \text{ J/kg} \quad \Rightarrow \quad \mathbf{V}_{e\ s} = 530.5 \text{ m/s}$$

Actual nozzle with given exit temperature

$$\frac{1}{2}\mathbf{V}_{e\ ac}^2 = \frac{1}{2}\mathbf{V}_i^2 + h_i - h_{e\ ac} = 112.5 + 1.166(1200 - 1100) \times 1000$$

$$= 116712.5 \text{ J/kg}$$

$$\Rightarrow \quad \mathbf{V}_{e\ ac} = \textbf{483 m/s}$$

$$\eta_{noz} = (\frac{1}{2}\mathbf{V}_{e\ ac}^2 - \frac{1}{2}\mathbf{V}_i^2)/(\frac{1}{2}\mathbf{V}_{e\ s}^2 - \frac{1}{2}\mathbf{V}_i^2) =$$

$$= (h_i - h_{e,\ AC})/(h_i - h_{e,\ s}) = \frac{116600}{140619.6} = \textbf{0.829}$$

7.156

A nozzle is required to produce a flow of air at 200 m/s at 20°C, 100 kPa. It is estimated that the nozzle has an isentropic efficiency of 92%. What nozzle inlet pressure and temperature is required assuming the inlet kinetic energy is negligible?

Solution:

C.V. Air nozzle: P_e, T_e(real), V_e(real), η_s(real)

For the real process: $h_i = h_e + \mathbf{V}_e^2/2$ or

$T_i = T_e + \mathbf{V}_e^2/2C_{P0} = 293.2 + 200^2/2 \times 1000 \times 1.004 = \mathbf{313.1\ K}$

For the ideal process, from Eq.7.30:

$\mathbf{V}_{es}^2/2 = \mathbf{V}_e^2/2\eta_s = 200^2/2 \times 1000 \times 0.92 = 21.74$ kJ/kg

and $h_i = h_{es} + (\mathbf{V}_{es}^2/2)$

$T_{es} = T_i - \mathbf{V}_{es}^2/(2C_{P0}) = 313.1 - 21.74/1.004 = 291.4$ K

The constant s relation in Eq.6.23 gives

$$\Rightarrow \quad P_i = P_e\ (T_i/T_{es})^{\frac{k}{k-1}} = 100\left(\frac{313.1}{291.4}\right)^{3.50} = \mathbf{128.6\ kPa}$$

Borgnakke and Sonntag

7.157

A water-cooled air compressor takes air in at 20°C, 90 kPa and compresses it to 500 kPa. The isothermal efficiency is 88% and the actual compressor has the same heat transfer as the ideal one. Find the specific compressor work and the exit temperature.

Solution:

Ideal isothermal compressor exit 500 kPa, 20°C

Reversible process: $dq = T \, ds \implies q = T(s_e - s_i)$

$$q = T(s_e - s_i) = T[s^o_{Te} - s^o_{T1} - R \ln(P_e / P_i)]$$

$$= - RT \ln (P_e / P_i) = - 0.287 \times 293.15 \ln (500/90) = - 144.3 \text{ kJ/kg}$$

As same temperature for the ideal compressor $h_e = h_i \implies$

$$w = q = -144.3 \text{ kJ/kg} \implies w_{ac} = w / \eta = \textbf{- 163.98 kJ/kg}, \quad q_{ac} = q$$

Now for the actual compressor energy equation becomes

$$q_{ac} + h_i = h_{e \, ac} + w_{ac} \implies$$

$$h_{e \, ac} - h_i = q_{ac} - w_{ac} = - 144.3 - (-163.98) = 19.7 \text{ kJ/kg} \approx C_p (T_{e \, ac} - T_i)$$

$$T_{e \, ac} = T_i + 19.7/1.004 = \textbf{39.6°C}$$

Review Problems

7.158

A flow of saturated liquid R-410A at 200 kPa in an evaporator is brought to a state of superheated vapor at 200 kPa, 20$^\text{o}$C. Assume the process is reversible find the specific heat transfer and specific work.

C.V. Evaporator. From the device we know that potential and kinetic energies are not important (see chapter 6).

Since the pressure is constant and the process is reversible from Eq.7.14

$$w = -\int vdP + 0 + 0 - 0 = \mathbf{0}$$

From energy equation

$$h_i + q = w + h_e = h_e; \qquad q = h_e - h_i$$

State i: $h_i = 4.18$ kJ/kg,   State e: $h_e = 311.78$ kJ/kg

$$q = h_e - h_i = 311.78 - 4.18 = \mathbf{307.6\ kJ/kg}$$

7.159

A flow of R-410A at 2000 kPa, 40°C in an isothermal expander is brought to a state of 1000 kPa in a reversible process. Find the specific heat transfer and work.

C.V. Expander. Steady reversible, single inlet and exit flow. Some q and w

Energy Eq.4.13: $h_i + q = w + h_e$;

Entropy Eq.7.9: $s_i + \int dq/T + s_{gen} = s_e$

Process: T = constant so $\int dq/T = q/T$ and reversible $s_{gen} = 0$

State i: $h_i = 295.49$ kJ/kg, $s_i = 1.0099$ kJ/kg-K

State e: $h_e = 316.05$ kJ/kg, $s_e = 1.1409$ kJ/kg-K

From entropy equation

$q = T (s_e - s_i) = 313.15$ K $(1.1409 – 1.0099)$ kJ/kg-K = **41.023 kJ/kg**

From the energy equation

$w = h_i – h_e + q = 295.49 – 316.05 + 41.023 =$ **330.46 kJ/kg**

7.160

A coflowing heat exchanger has one line with 2 kg/s saturated water vapor at 100 kPa entering. The other line is 1 kg/s air at 200 kPa, 1200 K. The heat exchanger is very long so the two flows exit at the same temperature. Find the exit temperature by trial and error. Calculate the rate of entropy generation.

Solution:

C.V. Heat exchanger, steady 2 flows in and two flows out.
No W, no external Q

Flows: $\dot{m}_1 = \dot{m}_2 = \dot{m}_{H_2O}$; $\dot{m}_3 = \dot{m}_4 = \dot{m}_{air}$

Energy: $\dot{m}_{H_2O} (h_2 - h_1) = \dot{m}_{air} (h_3 - h_4)$

State 1: Table B.1.2 $h_1 = 2675.5$ kJ/kg   State 2: 100 kPa, T_2

State 3: Table A.7 $h_3 = 1277.8$ kJ/kg,   State 4: 200 kPa, T_2

Only one unknown T_2 and one equation the energy equation:

$\quad$ 2(h_2 - 2675.5) = 1(1277.8 - h_4)   =>   1 kg/s × ($2h_2 + h_4$) = 6628.8 kW

At 500 K: $h_2 = 2902.0$, $h_4 = 503.36$ => LHS = 6307   too small

At 700 K: $h_2 = 3334.8$, $h_4 = 713.56$ => LHS = 7383   too large

Linear interpolation $T_2 = 560$ K, $h_2 = 3048.3$, $h_4 = 565.47$ => LHS = 6662

Final states are with **$T_2 = 554.4$ K = 281 °C**

H2O: Table B.1.3, $h_2 = 3036.8$ kJ/kg, $s_2 = 8.1473$, $s_1 = 7.3593$ kJ/kg K

AIR: Table A.7, $h_4 = 559.65$ kJ/kg, $s_{T4} = 7.4936$, $s_{T3} = 8.3460$ kJ/kg K

The entropy balance equation, Eq.7.7, is solved for the generation term:

$\quad \dot{S}_{gen} = \dot{m}_{H_2O} (s_2 - s_1) + \dot{m}_{air} (s_4 - s_3)$

$\qquad\qquad = 2(8.1473 - 7.3593) + 1 (7.4936 - 8.3460) = $ **0.724 kW/K**

No pressure correction is needed as the air pressure for 4 and 3 is the same.

7.161

Air at 100 kPa, 17°C is compressed to 400 kPa after which it is expanded through a nozzle back to the atmosphere. The compressor and the nozzle both have an isentropic efficiency of 90% and are adiabatic. The kinetic energy into and out of the compressor can be neglected. Find the compressor work and its exit temperature and find the nozzle exit velocity.

C.V. Ideal compressor, inlet: 1 exit: 2

Adiabatic : q = 0.
Reversible: $s_{gen} = 0$

Energy Eq.4.13: $h_1 + 0 = w_C + h_2$;

Entropy Eq.7.8: $s_1 + 0/T + 0 = s_2$

$- w_{Cs} = h_2 - h_1$, $s_2 = s_1$

Properties use air Table A.5: $C_{Po} = 1.004 \dfrac{kJ}{kg\ K}$, $R = 0.287 \dfrac{kJ}{kg\ K}$,   k = 1.4,

Process gives constant s (isentropic) which with constant C_{Po} gives Eq.6.23

$$\Rightarrow \quad T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 290\ (400/100)^{0.2857} = \textbf{430.9 K}$$

$$\Rightarrow \quad -w_{Cs} = C_{Po}(T_2 - T_1) = 1.004\ (430.9 - 290) = \textbf{141.46 kJ/kg}$$

The ideal nozzle then expands back down to state 1 (constant s). The actual compressor discharges at state 3 however, so we have:

$$w_C = w_{Cs}/\eta_C = \textbf{-157.18} \quad \Rightarrow \quad T_3 = T_1 - w_C/C_p = \textbf{446.6 K}$$

Nozzle receives air at 3 and exhausts at 5. We must do the ideal (exit at 4) first.

$$s_4 = s_3 \Rightarrow \text{Eq.6.23:} \qquad T_4 = T_3\ (P_4/P_3)^{\frac{k-1}{k}} = 300.5\ K$$

$$\tfrac{1}{2}\mathbf{V}_s^2 = C_p(T_3 - T_4) = 146.68 \Rightarrow \tfrac{1}{2}\mathbf{V}_{ac}^2 = 132\ kJ/kg \Rightarrow V_{ac} = \textbf{513.8 m/s}$$

If we need it, the actual nozzle exit (5) can be found:

$$T_5 = T_3 - \mathbf{V}_{ac}^2/2C_p = 315\ K$$

7.162

A vortex tube has an air inlet flow at 20°C, 200 kPa and two exit flows of 100 kPa, one at 0°C and the other at 40°C. The tube has no external heat transfer and no work and all the flows are steady and have negligible kinetic energy. Find the fraction of the inlet flow that comes out at 0°C. Is this setup possible?

Solution:

C.V. The vortex tube.   Steady, single inlet and two exit flows. No q or w.

Continuity Eq.: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$;   Energy: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$

Entropy: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3$

States all given by temperature and pressure. Use constant heat capacity to

evaluate changes in h and s. Solve for $x = \dot{m}_2/\dot{m}_1$ from the energy equation

$\dot{m}_3/\dot{m}_1 = 1 - x$; $h_1 = x\,h_2 + (1-x)\,h_3$

=> $x = (h_1 - h_3)/(h_2 - h_3) = (T_1 - T_3)/(T_2 - T_3) = (20{-}40)/(0{-}40) = 0.5$

Evaluate the entropy generation (assuming constant specific heat)

$\dot{S}_{gen}/\dot{m}_1 = x\,s_2 + (1-x)s_3 - s_1 = 0.5(s_2 - s_1) + 0.5(s_3 - s_1)$

$= 0.5\,[C_p \ln(T_2/T_1) - R \ln(P_2/P_1)] + 0.5[C_p \ln(T_3/T_1) - R \ln(P_3/P_1)]$

$= 0.5\left[1.004 \ln\left(\dfrac{273.15}{293.15}\right) - 0.287 \ln\left(\dfrac{100}{200}\right)\right]$

$\qquad\quad + 0.5\left[1.004 \ln\left(\dfrac{313.15}{293.15}\right) - 0.287 \ln\left(\dfrac{100}{200}\right)\right]$

$= \mathbf{0.1966\ kJ/kg\ K\quad > 0\qquad}$ **So this is possible**.

7.163

Air enters an insulated turbine at 50°C, and exits the turbine at - 30°C, 100 kPa. The isentropic turbine efficiency is 70% and the inlet volumetric flow rate is 20 L/s. What is the turbine inlet pressure and the turbine power output?

C.V.: Turbine, $\eta_s = 0.7$, Insulated

Air: $C_p = 1.004$ kJ/kg-K, $R = 0.287$ kJ/kg-K, $k = 1.4$

Inlet: $T_i = 50^oC$, $\dot{V}_i = 20$ L/s $= 0.02$ m^3/s

Exit: $T_e = -30^oC$, $P_e = 100$ kPa

a) Energy Eq., steady flow: $q + h_i = h_e + w_T$; $q = 0$

Assume Constant Specific Heat

$w_T = h_i - h_e = C_p(T_i - T_e) = 80.3$ kJ/kg

$w_{Ts} = w/\eta = 114.7$ kJ/kg, $w_{Ts} = C_p(T_i - T_{es})$

Solve for $T_{es} = 208.9$ K

Isentropic Process: $P_e = P_i \, (T_e / T_i)^{\frac{k}{k-1}}$ => **$P_i = 461$ kPa**

b) $\dot{W}_T = \dot{m} w_T$; $\dot{m} = P\dot{V}/RT = 0.099$ kg/s   => $\dot{W}_T = $ **7.98 kW**

7.164

A stream of ammonia enters a steady flow device at 100 kPa, 50°C, at the rate of 1 kg/s. Two streams exit the device at equal mass flow rates; one is at 200 kPa, 50°C, and the other as saturated liquid at 10°C. It is claimed that the device operates in a room at 25°C on an electrical power input of 250 kW. Is this possible?

Solution:

Control volume: Steady device out to ambient 25°C.

Energy Eq.4.10:

$$\dot{m}_1 h_1 + \dot{Q} + \dot{W}_{el} = \dot{m}_2 h_2 + \dot{m}_3 h_3$$

Entropy Eq.7.7:

$$\dot{m}_1 s_1 + \dot{Q}/T_{room} + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3$$

State 1:   Table B.2.2, $h_1 = 1581.2$ kJ/kg, $s_1 = 6.4943$ kJ/kg K

State 2:   Table B.2.2 $h_2 = 1576.6$ kJ/kg, $s_2 = 6.1453$ kJ/kg K

State 3:   Table B.2.1 $h_3 = 226.97$ kJ/kg, $s_3 = 0.8779$ kJ/kg K

From the energy equation

$$\dot{Q} = 0.5 \times 1576.6 + 0.5 \times 226.97 - 1 \times 1581.2 - 250 = -929.4 \text{ kW}$$

From the entropy equation

$$\dot{S}_{gen} = 0.5{\times}6.1453 + 0.5 \times 0.8779 - 1 \times 6.4943 - (-929.4)/298.15$$
$$= 0.1345 \text{ kW/K} > 0$$

since $\dot{S}_{gen} > 0$ this is possible

7.165

In a heat-powered refrigerator, a turbine is used to drive the compressor using the same working fluid. Consider the combination shown in Fig. P9.157 where the turbine produces just enough power to drive the compressor and the two exit flows are mixed together. List any assumptions made and find the ratio of mass

flow rates $\dot{m}_3/\dot{m}_1$ and T_5 (x_5 if in two-phase region) if the turbine and the compressor are reversible and adiabatic

Solution:

CV: compressor

$$s_{2S} = s_1 = 1.0779 \text{ kJ/kg K} \rightarrow h_{2S} = 317.43 \text{ kJ/kg}$$

$$w_{SC} = h_1 - h_{2S} = 271.89 - 317.43 = -45.54 \text{ kJ/kg}$$

CV: turbine

$$s_{4S} = s_3 = 1.0850 \text{ kJ/kgK} \text{ and } P_{4S} \Rightarrow h_{4S} = 319.72 \text{ kJ/kg}$$

$$w_{ST} = h_3 - h_{4S} = 341.29 - 319.72 = 21.57 \text{ kJ/kg}$$

As $\dot{w}_{TURB} = -\dot{w}_{COMP}$, $\dot{m}_3/\dot{m}_1 = -\dfrac{w_{SC}}{w_{ST}} = \dfrac{45.54}{21.57} = \mathbf{2.111}$

CV: mixing portion

$$\dot{m}_1 h_{2S} + \dot{m}_3 h_{4S} = (\dot{m}_1 + \dot{m}_3)h_5$$

$$1 \times 317.43 + 2.111 \times 319.72 = 3.111 \, h_5$$

$$\Rightarrow h_5 = 318.984 \text{ kJ/kg} \quad \Rightarrow \quad T_5 = \mathbf{58.7°C}$$

7.166

A certain industrial process requires a steady 0.5 kg/s supply of compressed air at 500 kPa, at a maximum temperature of 30°C. This air is to be supplied by installing a compressor and aftercooler, see Fig. P9.46. Local ambient conditions are 100 kPa, 20°C. Using an isentropic compressor efficiency of 80%, determine the power required to drive the compressor and the rate of heat rejection in the aftercooler.

Air table A.5: $R = 0.287$ kJ/kg-K, $C_p = 1.004$ kJ/kg-K, $k = 1.4$

State 1: $T_1 = T_o = 20^oC$, $P_1 = P_o = 100$ kPa, $\dot{m} = 0.5$ kg/s

State 2: $P_2 = P_3 = 500$ kPa

State 3: $T_3 = 30^oC$, $P_3 = 500$ kPa

We have $\eta_s = 80\% = w_{Cs}/w_{C\ ac}$

Compressor: First do the ideal (Isentropic)

$$T_{2s} = T_1\ (P_2/P_1)^{\frac{k-1}{k}} = 293.15\ (500/100)^{0.2857} = 464.6\ K$$

Energy Eq.: $\qquad q_c + h_1 = h_2 + w_c$; $\qquad q_c = 0$, assume constant specific heat

$\qquad w_{cs} = C_p(T_1 - T_{2s}) = 1.004\ (293.15 - 464.6) = -172.0$ kJ/kg

$\qquad \eta_s = w_{Cs}/w_{C\ ac}$, $\ w_{C\ ac} = w_{Cs}/\eta_s = -215$, $\ \dot{W}_C = \dot{m}w_C = $ **-107.5 kW**

$\qquad w_{C\ ac} = C_p\ (T_1 - T_2)$, solve for $T_2 = 507.5$ K

Aftercooler:

Energy Eq.: $\ q + h_2 = h_3 + w$; $\ w = 0$, assume constant specific heat

$\qquad q = C_p\ (T_3 - T_2) = 1.004(303.15 - 507.5) = -205$ kJ/kg,

$\qquad \dot{Q} = \dot{m}q = $ **-102.5 kW**

7.167

Carbon dioxide flows through a device entering at 300 K, 200 kPa and leaving at 500 K. The process is steady state polytropic with n = 3.8 and heat transfer comes from a 600 K source. Find the specific work, specific heat transfer and the specific entropy generation due to this process.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.4.13: $h_i + q = h_e + w$ Neglect kinetic, potential energies

Entropy Eq.7.9: $s_i + \int dq/T + s_{gen} = s_e$

Process Eq.6.28:

$$P_e = P_i \, (T_e/ \, T_i)^{\frac{n}{n-1}} \quad = 200(500/300)^{\frac{3.8}{2.8}} = 400 \text{ kPa}$$

and the process leads to Eq.7.18 for the work term

$$w = -\frac{n}{n-1} R \, (T_e - T_i) = -\frac{3.8}{2.8} \times 0.1889 \times (500 - 300) = \textbf{-51.3 kJ/kg}$$

Energy equation gives

$$q = h_e - h_i + w = 401.52 - 214.38 - 51.3 = \textbf{135.8 kJ/kg}$$

Entropy equation gives (CV out to source)

$$s_{gen} = s_e - s_i - q/T_{source} = s_{Te}^o - s_{Ti}^o - R \ln(P_e \, / \, P_i) - q/T_{source}$$

$$= 5.3375 - 4.8631 - 0.1889 \ln (400/200) - (135.8/600)$$

$$= \textbf{0.117 kJ/kg K}$$

Notice:
dP > 0
so dw <0

ds > 0
so dq > 0

Notice process is externally irreversible, ΔT between source and CO_2

7.168

A flow of nitrogen, 0.1 kg/s, comes out of a compressor stage at 500 kPa, 500 K and is now cooled to 310 K in a counterflowing intercooler by liquid water at 125 kPa, 15°C which leaves at 22°C. Find the flowrate of water and the total rate of entropy generation.

Solution:

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A hydraulic motor driven compressor with intercooler in small pipe between the two stages.

Continuity eq.: $\dot{m}_1 = \dot{m}_2 = \dot{m}_{H2O}$; $\dot{m}_3 = \dot{m}_4 = \dot{m}_{N2}$

Energy eq.: $0 = \dot{m}_{N2}\,(h_3 - h_4) + \dot{m}_{H2O}\,(h_2 - h_1)$

Entropy Eq.7.7: $0 = \dot{m}_{N2}\,(s_3 - s_4) + \dot{m}_{H_2O}\,(s_2 - s_1) + \dot{S}_{gen}$

Due to the lower range of temperature we will use constant specific heats from A.5 and A.4 so the energy equation is approximated as

$$0 = \dot{m}_{H2O}C_{pH2O}\,(T_2 - T_1) + \dot{m}_{N2}C_p\,(T_3 - T_4)$$

Now solve for the water flow rate

$$\dot{m}_{H2O} = \dot{m}_{N2}\,C_{pN2}\,(T_3 - T_4) \,/\, [C_{pH2O}\,(T_1 - T_2)]$$

$$= 0.1 \text{ kg/s} \times 1.042 \text{ kJ/kgK} \times (500 - 310) \text{ K} \,/\, [4.18 \times (22 - 15) \text{ kJ/kg}]$$

$$= \mathbf{0.677 \text{ kg/s}}$$

$$\dot{S}_{gen} = \dot{m}_{N2}\,(s_4 - s_3) + \dot{m}_{H_2O}\,(s_1 - s_2)$$

$$= \dot{m}_{N2}C_P \ln\,(T_4/T_3) + \dot{m}_{H_2O}C_P \ln\,(T_1/T_2)$$

$$= 0.1 \text{ kg/s} \times 1.042 \text{ kJ/kgK} \ln \frac{310}{500} + 0.677 \text{ kg/s} \times 4.18 \text{ kJ/kgK} \ln \frac{295}{288}$$

$$= -0.04981 + 0.06796 = \mathbf{0.0182 \text{ kW/K}}$$

7.169

An initially empty spring-loaded piston/cylinder requires 100 kPa to float the piston. A compressor with a line and valve now charges the cylinder with water to a final pressure of 1.4 MPa at which point the volume is 0.6 m^3, state 2. The inlet condition to the reversible adiabatic compressor is saturated vapor at 100 kPa. After charging the valve is closed and the water eventually cools to room temperature, 20°C, state 3. Find the final mass of water, the piston work from 1 to 2, the required compressor work, and the final pressure, P_3.

Solution:

Process 1→2: transient, adiabatic.
for C.V. compressor + cylinder
Assume process is reversible

Continuity: $m_2 - 0 = m_{in}$,   Energy: $m_2 u_2 - \emptyset = (m_{in}h_{in}) - W_c - {}_1W_2$

Entropy Eq.: $m_2 s_2 - \emptyset = m_{in}s_{in} + 0$ $\Rightarrow$ $s_2 = s_{in}$

Inlet state: Table B.1.2, $h_{in} = 2675.5$ kJ/kg, $s_{in} = 7.3594$ kJ/kg K

$$ {}_1W_2 = \int PdV = \frac{1}{2}(P_{float} + P_2)(V_2 - \emptyset) = \frac{1}{2}(100+1400)0.6 = \textbf{450 kJ} $$

State 2: P_2 , $s_2 = s_{in}$ Table B.1.3 $\Rightarrow$ $v_2 = 0.2243$, $u_2 = 2984.4$ kJ/kg

$m_2 = V_2/v_2 = 0.6/0.2243 = \textbf{2.675 kg}$

$W_c = m_{in}h_{in} - m_2u_2 - {}_1W_2 = 2.675 \times (2675.5 - 2984.4) - 450 = \textbf{-1276.3 kJ}$

State 3 must be on line & 20°C

Assume 2-phase $\Rightarrow$ $P_3 = P_{sat}(20°C) = 2.339$ kPa

less than P_{float} so compressed liquid

Table B.1.1: $v_3 \cong v_f(20°C) = 0.001002$ $\Rightarrow$ $V_3 = m_3v_3 = 0.00268$ m^3

On line: $P_3 = 100 + (1400 - 100) \times 0.00268/0.6 = \textbf{105.8 kPa}$

7.170

Consider the scheme shown in Fig. P7.170 for producing fresh water from salt water. The conditions are as shown in the figure. Assume that the properties of salt water are the same as for pure water, and that the pump is reversible and adiabatic.

a.   Determine the ratio $(\dot{m}_7/\dot{m}_1)$, the fraction of salt water purified.

b.   Determine the input quantities, w_P and q_H.

c.   Make a second law analysis of the overall system.

C.V. Flash evaporator: Steady flow, no external q, no work.

Energy Eq.: $\dot{m}_1 h_4 = (\dot{m}_1 - \dot{m}_7)h_5 + \dot{m}_7 h_6$

Table B.1.1   or $632.4 = (1 - (\dot{m}_7/\dot{m}_1))\, 417.46 + (\dot{m}_7/\dot{m}_1)\, 2675.5$

$\Rightarrow \dot{m}_7/\dot{m}_1 = $ **0.0952**

C.V. Pump steady flow, incompressible liq.:

$w_P = -\int v\,dP \approx -v_1(P_2 - P_1) = -0.001001(700 - 100) = $ **-0.6 kJ/kg**

$h_2 = h_1 - w_P = 62.99 + 0.6 = 63.6$ kJ/kg

C.V. Heat exchanger: $h_2 + (\dot{m}_7/\dot{m}_1)h_6 = h_3 + (\dot{m}_7/\dot{m}_1)h_7$

$63.6 + 0.0952 \times 2675.5 = h_3 + 0.0952 \times 146.68 \Rightarrow\ h_3 = 304.3$ kJ/kg

C.V. Heater: $q_H = h_4 - h_3 = 632.4 - 304.3 = $ **328.1 kJ/kg**

CV: entire unit, entropy equation per unit mass flow rate at state 1

$S_{C.V.,gen} = -q_H/T_H + (1 - (\dot{m}_7/\dot{m}_1))s_5 + (\dot{m}_7/\dot{m}_1)s_7 - s_1$

$= (-328.1/473.15) + 0.9048 \times 1.3026 + 0.0952 \times 0.5053 - 0.2245$

$= 0.3088$ kJ/K-kg-m_1

7.171

A rigid 1.0 m^3 tank contains water initially at 120°C, with 50 % liquid and 50% vapor, by volume. A pressure-relief valve on the top of the tank is set to 1.0 MPa (the tank pressure cannot exceed 1.0 MPa - water will be discharged instead). Heat is now transferred to the tank from a 200°C heat source until the tank contains saturated vapor at 1.0 MPa. Calculate the heat transfer to the tank and show that this process does not violate the second law.

Solution:

C.V. Tank and walls out to the source. Neglect storage in walls. There is flow out and no boundary or shaft work.

Continuity Eq.4.15: $\quad m_2 - m_1 = - m_e$

Energy Eq.4.16: $\quad m_2 u_2 - m_1 u_1 = - m_e h_e + {}_1Q_2$

Entropy Eq.7.13: $\quad m_2 s_2 - m_1 s_1 = - m_e s_e + \int dQ/T + {}_1S_{2\ gen}$

State 1: $T_1 = 120^o$C, Table B.1.1

$\quad v_f = 0.00106 \text{ m}^3/\text{kg}, \quad m_{liq} = 0.5V_1/v_f = 471.7 \text{ kg}$

$\quad v_g = 0.8919 \text{ m}^3/\text{kg}, \quad m_g = 0.5V_1/v_g = 0.56 \text{ kg},$

$\quad m_1 = 472.26 \text{ kg}, \quad x_1 = m_g/m_1 = 0.001186$

$\quad u_1 = u_f + x_1 u_{fg} = 503.5 + 0.001186 \times 2025.8 = 505.88 \text{ kJ/kg},$

$\quad s_1 = s_f + x_1 s_{fg} = 1.5275 + 0.001186 \times 5.602 = 1.5341 \text{ kJ/kg-K}$

State 2: $P_2 = 1.0$ MPa, sat. vap. $x_2 = 1.0, \quad V_2 = 1\text{m}^3$

$\quad v_2 = v_g = 0.19444 \text{ m}^3/\text{kg}, \qquad m_2 = V_2/v_2 = 5.14 \text{ kg}$

$\quad u_2 = u_g = 2583.6 \text{ kJ/kg}, \qquad s_2 = s_g = 6.5864 \text{ kJ/kg-K}$

Exit: $P_e = 1.0$ MPa, sat. vap. $x_e = 1.0, \quad h_e = h_g = 2778.1 \text{ kJ/kg},$

$\quad s_e = s_g = 6.5864 \text{ kJ/kg}, \qquad m_e = m_1 - m_2 = 467.12 \text{ kg}$

From the energy equation we get

$$_1Q_2 = m_2 u_2 - m_1 u_1 + m_e h_e = \textbf{1 072 080 kJ}$$

From the entropy Eq.7.13 or Eq.7.24 (with 7.25 and 7.26) we get

$$_1S_{2\ gen} = m_2 s_2 - m_1 s_1 + m_e s_e - \frac{_1Q_2}{T_H}; \qquad T_H = 200^o\text{C} = 473 \text{ K}$$

$$_1S_{2\ gen} = \Delta S_{net} = \textbf{120.4 kJ} \geq \textbf{0} \quad \textbf{Process Satisfies 2}^{\textbf{nd}} \textbf{ Law}$$

7.172

A jet-ejector pump, shown schematically in Fig. P7.172, is a device in which a low-pressure (secondary) fluid is compressed by entrainment in a high-velocity (primary) fluid stream. The compression results from the deceleration in a diffuser. For purposes of analysis this can be considered as equivalent to the turbine-compressor unit shown in Fig. P7.165 with the states 1, 3, and 5 corresponding to those in Fig. P7.172. Consider a steam jet-pump with state 1 as saturated vapor at 35 kPa; state 3 is 300 kPa, 150°C; and the discharge pressure, P_5, is 100 kPa.

a.   Calculate the ideal mass flow ratio, $\dot{m}_1/\dot{m}_3$.

b.   The efficiency of a jet pump is defined as $\eta = (\dot{m}_1/\dot{m}_3)_{actual} / (\dot{m}_1/\dot{m}_3)_{ideal}$ for the same inlet conditions and discharge pressure. Determine the discharge temperature of the jet pump if its efficiency is 10%.

a) ideal processes (isen. comp. & exp.)

$\left.\begin{array}{l}\text{expands } 3\text{-}4s \\ \text{comp} \quad 1\text{-}2s\end{array}\right\}$ then mix at const. P

$s_{4s} = s_3 = 7.0778 = 1.3026 + x_{4s} \times 6.0568 \quad => \quad x_{4s} = 0.9535$

$h_{4s} = 417.46 + 0.9535 \times 2258.0 = 2570.5 \text{ kJ/kg}$

$s_{2s} = s_1 = 7.7193 \rightarrow T_{2s} = 174°C \quad \& \quad h_{2s} = 2823.8 \text{ kJ/kg}$

$\dot{m}_1(h_{2s} - h_1) = \dot{m}_3(h_3 - h_{4s})$

$\Rightarrow (\dot{m}_1/\dot{m}_3)_{IDEAL} = \dfrac{2761.0 - 2570.5}{2823.8 - 2631.1} = \mathbf{0.9886}$

b) real processes with jet pump eff. = 0.10

$\Rightarrow (\dot{m}_1/\dot{m}_3)_{ACTUAL} = 0.10 \times 0.9886 = 0.09886$

Energy Eq.: $\dot{m}_1 h_1 + \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_3)h_5$

$0.09886 \times 2631.1 + 1 \times 2761.0 = 1.09896 \, h_5$

State 5: $h_5 = 2749.3 \text{ kJ/kg}, \, P_5 = 100 \text{ kPa} \quad => \quad T_5 = \mathbf{136.5 \, ^oC}$

7.173

A horizontal, insulated cylinder has a frictionless piston held against stops by an external force of 500 kN. The piston cross-sectional area is 0.5 m^2, and the initial volume is 0.25 m^3. Argon gas in the cylinder is at 200 kPa, 100°C. A valve is now opened to a line flowing argon at 1.2 MPa, 200°C, and gas flows in until the cylinder pressure just balances the external force, at which point the valve is closed. Use constant heat capacity to verify that the final temperature is 645 K and find the total entropy generation.

Solution:

The process has inlet flow, no work (volume constant) and no heat transfer.

Continuity Eq.4.15: $m_2 - m_1 = m_i$

Energy Eq.4.16: $m_2 u_2 - m_1 u_1 = m_i h_i$

$m_1 = P_1 V_1 / R T_1 = 200 \times 0.25 / (0.2081 \times 373.15) = 0.644$ kg

Force balance: $P_2 A = F$ $\Rightarrow$ $P_2 = \dfrac{500}{0.5} = 1000$ kPa

For argon use constant heat capacities so the energy equation is:

$$m_2 \; C_{Vo} \; T_2 - m_1 \; C_{Vo} \; T_1 = (m_2 - m_1) \; C_{Po} \; T_{in}$$

We know P_2 so only 1 unknown for state 2.

Use ideal gas law to write $m_2 T_2 = P_2 V_1 / R$ and $m_1 T_1 = P_1 V_1 / R$

and divide the energy equation with C_{Vo} to solve for the change in mass

$(P_2 V_1 - P_1 V_1)/R = (m_2 - m_1)(C_{Po}/C_{Vo}) T_{in}$

$(m_2 - m_1) = (P_2 - P_1) V_1 / (R \; k \; T_{in})$

$\qquad = (1000 - 200) \times 0.25 / (0.2081 \times 1.667 \times 473.15) = 1.219$ kg

$\qquad m_2 = 1.219 + 0.644 = 1.863$ kg.

$T_2 = P_2 V_1 / (m_2 R) = 1000 \times 0.25 / (1.863 \times 0.2081) = 645$ K **OK**

Entropy Eq.7.12: $m_2 s_2 - m_1 s_1 = m_i s_i + 0 + {}_1S_{2 \; gen}$

${}_1S_{2 \; gen} = m_1 (s_2 - s_1) + (m_2 - m_1)(s_2 - s_i)$

$$= m_1 \Big[C_p \ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1} \Big] + (m_2 - m_1)\Big[C_p \ln\frac{T_2}{T_i} - R \ln\frac{P_2}{P_i} \Big]$$

$$= 0.644 [\; 0.52 \ln\frac{645}{373.15} - 0.2081 \ln\frac{1000}{200}]$$

$$+ 1.219[\; 0.52 \ln\frac{645}{473.15} - 0.2081 \ln\frac{1000}{1200}]$$

$$= - 0.03242 + 0.24265 = \textbf{0.21 kJ/K}$$

7.174

Supercharging of an engine is used to increase the inlet air density so that more fuel can be added, the result of which is an increased power output. Assume that ambient air, 100 kPa and 27°C, enters the supercharger at a rate of 250 L/s. The supercharger (compressor) has an isentropic efficiency of 75%, and uses 20 kW of power input. Assume that the ideal and actual compressor have the same exit pressure. Find the ideal specific work and verify that the exit pressure is 175 kPa. Find the percent increase in air density entering the engine due to the supercharger and the entropy generation.

C.V.: Air in compressor (steady flow)

Cont: $\dot{m}_{in} = \dot{m}_{ex} = \dot{m} = \dot{V}/v_{in} = 0.29$ kg/s

Energy: $\dot{m}h_{in} - \dot{W} = \dot{m}h_{ex}$ Assume: $\dot{Q} = 0$

Entropy: $\dot{m}s_{in} + \dot{S}_{gen} = \dot{m}s_{ex}$

$$v_{in} = \frac{RT_{in}}{P_{in}} = 0.8614 \text{ m}^3/\text{kg}, \quad s^o_{Ti} = 6.86975 \text{ kJ/kg K}, \quad h_{in} = 300.62 \text{ kJ/kg}$$

$$\eta_c = w_{C\,s}/w_{C\,ac} \Rightarrow \quad -\dot{W}_S = -\dot{W}_{AC} \times \eta_c = 15 \text{ kW}$$

$$-w_{C\,s} = -\dot{W}_S/\dot{m} = 51.724 \text{ kJ/kg}, \quad -w_{C\,ac} = 68.966 \text{ kJ/kg}$$

Table A.7: $h_{ex\,s} = h_{in} - w_{C\,s} = 300.62 + 51.724 = 352.3$ kJ/kg

$$\Rightarrow T_{ex\,s} = 351.5 \text{ K}, \quad s^o_{Te} = 7.02830 \text{ kJ/kg K}$$

$$P_{ex} = P_{in} \times e^{(s^o_{T\,ex} - s^o_{T\,in})/R} = 100 \times \exp\left[\frac{7.0283 - 6.86975}{0.287}\right]$$

$$= \mathbf{173.75 \text{ kPa}}$$

The actual exit state is

$$h_{ex\,ac} = h_{in} - w_{C\,ac} = 369.6 \text{ kJ/kg} \Rightarrow T_{ex\,ac} = 368.6 \text{ K}$$

$$v_{ex} = RT_{ex}/P_{ex} = 0.6088 \text{ m}^3/\text{kg}, \quad s^o_{Tex\,ac} = 7.0764$$

$$\rho_{ex}/\rho_{in} = v_{in}/v_{ex} = 0.8614/0.6088 = \mathbf{1.415 \text{ or } 41.5\% \text{ increase}}$$

$$s_{gen} = s_{ex} - s_{in} = 7.0764 - 6.86975 - 0.287 \ln(\frac{173.75}{100}) = \mathbf{0.0481 \text{ kJ/kg K}}$$

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7.175

A rigid steel bottle, $V = 0.25 \text{ m}^3$, contains air at 100 kPa, 300 K. The bottle is now charged with air from a line at 260 K, 6 MPa to a bottle pressure of 5 MPa, state 2, and the valve is closed. Assume that the process is adiabatic, and the charge always is uniform. In storage, the bottle slowly returns to room temperature at 300 K, state 3. Find the final mass, the temperature T_2, the final pressure P_3, the heat transfer $_1Q_3$ and the total entropy generation.

C.V. Bottle. Flow in, no work, no heat transfer.

Continuity Eq.4.15: $\quad m_2 - m_1 = m_{in}$;

Energy Eq.4.16: $\quad m_2u_2 - m_1u_1 = m_{in}h_{in}$

State 1 and inlet: Table A.7, $\quad u_1 = 214.36 \text{ kJ/kg}, \quad h_{in} = 260.32 \text{ kJ/kg}$

$$m_1 = P_1V/RT_1 = (100 \times 0.25)/(0.287 \times 300) = 0.290 \text{ kg}$$

$$m_2 = P_2V/RT_2 = 5000 \times 0.25/(0.287 \times T_2) = 4355.4/T_2$$

Substitute into energy equation : $\quad u_2 + 0.00306 \, T_2 = 260.32 \text{ kJ/kg}$

Now trial and error on T_2

$\quad T_2 = 360 \quad \Rightarrow \quad \text{LHS} = 258.63 \text{ (low)}; \quad T_2 = 370 \quad \Rightarrow \quad \text{LHS} = 265.88 \text{ (high)}$

Interpolation $T_2 = 362.3 \text{ K} \quad (\text{LHS} = 260.3 \text{ OK})$

$\quad m_2 = 4355.4/362.3 = 12.022 \text{ kg} ; \quad P_3 = m_2RT_3/V = \textbf{4140 kPa}$

Now use the energy equation from the beginning to the final state

$\quad _1Q_3 = m_2u_3 - m_1u_1 - m_{in}h_{in} = (12.022 - 0.29) \, 214.36 \ - 11.732 \times 260.32$

$\qquad = \textbf{-539.2 kJ}$

Entropy equation from state 1 to state 3 with change in s from Eq.8.19

$\quad S_{gen} = m_2s_3 - m_1s_1 - m_{in}s_{in} - {_1Q_3}/T = m_2(s_3 - s_{in}) - m_1(s_1 - s_{in}) - {_1Q_3}/T$

$\qquad = 12.022[6.8693 - 6.7256 - R \ln(4140/6000)]$

$\qquad - 0.29[6.8693 - 6.7256 - R \ln(100/6000)] + 539.2/300 = \textbf{4.423 kJ/K}$

Problem could have been solved with constant specific heats from A.5 in which case we would get the energy explicit in T_2 (no iterations).

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7.176

A certain industrial process requires a steady 0.5 kg/s of air at 200 m/s, at the condition of 150 kPa, 300 K. This air is to be the exhaust from a specially designed turbine whose inlet pressure is 400 kPa. The turbine process may be assumed to be reversible and polytropic, with polytropic exponent n = 1.20.
a) What is the turbine inlet temperature?
b) What are the power output and heat transfer rate for the turbine?
c) Calculate the rate of net entropy increase, if the heat transfer comes from a source at a temperature 100°C higher than the turbine inlet temperature.

Solution:

C.V. Turbine, this has heat transfer, $PV^n = $ Constant, n = 1.2

Process polytropic Eq.8.28: $\qquad T_e / T_i = (P_e/P_i)^{\frac{n-1}{n}} \quad => \quad T_i = 353.3 \text{ K}$

Energy Eq.4.12: $\qquad \dot{m}_i(h + V^2/2)_{in} + \dot{Q} = \dot{m}_{ex}(h + V^2/2)_{ex} + \dot{W}_T$

Reversible shaft work in a polytropic process, Eq.7.14 and Eq.7.18:

$$w_T = -\int v \, dP + (V_i^2 - V_e^2)/2 = -\frac{n}{n-1}(P_e v_e - P_i v_i) + (V_i^2 - V_e^2)/2$$

$$= -\frac{n}{n-1}R(T_e - T_i) - V_e^2/2 = 71.8 \text{ kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = \textbf{35.9 kW}$$

Assume constant specific heat in the energy equation

$$\dot{Q} = \dot{m}[C_P(T_e - T_i) + V_e^2/2] + \dot{W}_T = \textbf{19.2 kW}$$

Entropy Eq.7.7 or 9.23 with change in entropy from Eq.8.16:

$$dS_{net}/dt = \dot{S}_{gen} = \dot{m}(s_e - s_i) - \dot{Q}_H/T_H, \qquad T_H = T_i + 100 = 453.3 \text{ K}$$

$$s_e - s_i = C_P \ln(T_e / T_i) - R \ln(P_e / P_i) = 0.1174 \text{ kJ/kg K}$$

$$dS_{net}/dt = 0.5 \times 0.1174 - 19.2/453.3 = 0.0163 \text{ kW/K}$$

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Problems solved with P_r and v_r functions

7.17   uses P_r function

Do the previous problem using the air tables in A.7

The exit nozzle in a jet engine receives air at 1200 K, 150 kPa with neglible kinetic energy. The exit pressure is 80 kPa and the process is reversible and adiabatic. Use constant heat capacity at 300 K to find the exit velocity.

Solution:

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i = h_e + V_e^2/2$ $(Z_i = Z_e)$

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Process: $q = 0$, $s_{gen} = 0$ as used above leads to $s_e = s_i$

Inlet state: $h_i = 1277.8$ kJ/kg, $P_{r\,i} = 191.17$

The constant s is done using the P_r function from A.7.2

$$P_{r\,e} = P_{r\,i}\,(P_e / P_i) = 191.17\,(80/150) = 101.957$$

Interpolate in A.7   =>

$$T_e = 1000 + 50\,\frac{101.957 - 91.651}{111.35 - 91.651} = 1026.16 \text{ K}$$

$$h_e = 1046.2 + 0.5232 \times (1103.5 - 1046.2) = 1076.2 \text{ kJ/kg}$$

From the energy equation we have $V_e^2/2 = h_i - h_e$, so then

$$\mathbf{V_e} = \sqrt{2\,(h_i - h_e)} = \sqrt{2(1277.8 - 1076.2)\,\text{kJ/kg} \times 1000\,\text{J/kJ}} = \mathbf{635\ m/s}$$

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7.31   uses P_r function

Air enters a turbine at 800 kPa, 1200 K, and expands in a reversible adiabatic process to 100 kPa. Calculate the exit temperature and the work output per kilogram of air, using

a.   The ideal gas tables, Table A.7
b.   Constant specific heat, value at 300 K from table A.5

Solution:

C.V. Air turbine.

Adiabatic: $q = 0$, reversible: $s_{gen} = 0$

Energy Eq.4.13: $w_T = h_i - h_e$,

Entropy Eq.7.9: $s_e = s_i$

a) Table A.7: $h_i = 1277.8$ kJ/kg, $P_{r\,i} = 191.17$

The constant s process is done using the P_r function from A.7.2

$$\Rightarrow P_{r\,e} = P_{r\,i} \, (P_e / P_i) = 191.17 \left(\frac{100}{800}\right) = 23.896$$

Interpolate in A.7.1 $\Rightarrow T_e = \mathbf{705.7\ K}$, $h_e = 719.7$ kJ/kg

$$w = h_i - h_e = 1277.8 - 719.7 = \mathbf{558.1\ kJ/kg}$$

b)   Table A.5: $C_{Po} = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$,   then from Eq.8.23

$$T_e = T_i \, (P_e/P_i)^{\frac{k-1}{k}} = 1200 \left(\frac{100}{800}\right)^{0.286} = \mathbf{662.1\ K}$$

$$w = C_{Po}(T_i - T_e) = 1.004(1200 - 662.1) = \mathbf{539.8\ kJ/kg}$$

7.34   uses P_r function

A compressor receives air at 290 K, 95 kPa and a shaft work of 5.5 kW from a gasoline engine. It should deliver a mass flow rate of 0.01 kg/s air to a pipeline. Find the maximum possible exit pressure of the compressor.

Solution:

C.V. Compressor, Steady single inlet and exit flows. Adiabatic: $\dot{Q} = 0$.

Continuity Eq.4.11: $\dot{m}_i = \dot{m}_e = \dot{m}$,

Energy Eq.4.12: $\dot{m}h_i = \dot{m}h_e + \dot{W}_C$,

Entropy Eq.7.8: $\dot{m}s_i + \dot{S}_{gen} = \dot{m}s_e$ (Reversible $\dot{S}_{gen} = 0$)

$$\dot{W}_c = \dot{m}w_c \ \Rightarrow \ -w_c = -\dot{W}/\dot{m} = 5.5/0.01 = 550 \text{ kJ/kg}$$

Use Table A.7, $h_i = 290.43$ kJ/kg, $P_{r\,i} = 0.9899$

$h_e = h_i + (-w_c) = 290.43 + 550 = 840.43$ kJ/kg

A.7 $\Rightarrow$ $T_e = 816.5$ K, $P_{r\,e} = 41.717$

$P_e = P_i (P_{r\,e}/P_{r\,i}) = 95 \times (41.717/0.9899) = $ **4003 kPa**

7.55   uses P_r function

An underground saltmine, 100 000 m^3 in volume, contains air at 290 K, 100 kPa. The mine is used for energy storage so the local power plant pumps it up to 2.1 MPa using outside air at 290 K, 100 kPa. Assume the pump is ideal and the process is adiabatic. Find the final mass and temperature of the air and the required pump work.

Solution:

C.V. The mine volume and the pump

Continuity Eq.4.15: $m_2 - m_1 = m_{in}$

Energy Eq.4.16: $m_2 u_2 - m_1 u_1 = {}_1Q_2 - {}_1W_2 + m_{in} h_{in}$

Entropy Eq.7.12: $m_2 s_2 - m_1 s_1 = \int dQ/T + {}_1S_{2\ gen} + m_{in} s_{in}$

Process: Adiabatic ${}_1Q_2 = 0$, Process ideal ${}_1S_{2\ gen} = 0$, $s_1 = s_{in}$

$\Rightarrow m_2 s_2 = m_1 s_1 + m_{in} s_{in} = (m_1 + m_{in}) s_1 = m_2 s_1 \Rightarrow s_2 = s_1$

Constant s $\Rightarrow$ $P_{r2} = P_{r\ i}\ (P_2\ /\ P_i) = 0.9899 \left(\dfrac{2100}{100}\right) = 20.7879$

A.7.2 $\Rightarrow T_2 = $ **680 K** , $u_2 = 496.94$ kJ/kg

$m_1 = P_1 V_1 / RT_1 = 100 \times 10^5 / (0.287 \times 290) = 1.20149 \times 10^5$ kg

$m_2 = P_2 V_2 / RT_2 = 100 \times 21 \times 10^5 / (0.287 \times 680) = $ **10.760 × 10^5 kg**

$\Rightarrow m_{in} = 9.5585 \times 10^5$ kg

${}_1W_2 = m_{in} h_{in} + m_1 u_1 - m_2 u_2$

$= m_{in}(290.43) + m_1(207.19) - m_2(496.94) = $ **-2.322 × 10^8 kJ**

7.80 **uses P_r function**

Calculate the air temperature and pressure at the stagnation point right in front of a meteorite entering the atmosphere (-50 °C, 50 kPa) with a velocity of 2000 m/s. Do this assuming air is incompressible at the given state and repeat for air being a compressible substance going through an adiabatic compression.

Solution:

Kinetic energy: $\frac{1}{2}\mathbf{V}^2 = \frac{1}{2}(2000)^2/1000 = 2000$ kJ/kg

Ideal gas: $v_{atm} = RT/P = 0.287 \times 223/50 = 1.28$ m^3/kg

a) incompressible

Energy Eq.4.13: $\Delta h = \frac{1}{2}\mathbf{V}^2 = 2000$ kJ/kg

If A.5 $\Delta T = \Delta h/C_p = 1992$ K   unreasonable, too high for that C_p

Use A.7: $h_{st} = h_o + \frac{1}{2}\mathbf{V}^2 = 223.22 + 2000 = 2223.3$ kJ/kg

$T_{st} = 1977$ K

Bernoulli (incompressible) Eq.7.17:

$\Delta P = P_{st} - P_o = \frac{1}{2}\mathbf{V}^2/v = 2000/1.28 = 1562.5$ kPa

$P_{st} = 1562.5 + 50 = 1612.5$ kPa

b) compressible

$T_{st} = 1977$ K   the same energy equation.

From A.7.2:   Stagnation point $P_{r\,st} = 1580.3$;   Free $P_{r\,o} = 0.39809$

$P_{st} = P_o \times \dfrac{P_{r\,st}}{P_{r\,o}} = 50 \times \dfrac{1580.3}{0.39809}$

$= \mathbf{198\ 485\ kPa}$

Notice that this is highly compressible, v is not constant.

7.174 uses P_r function

Supercharging of an engine is used to increase the inlet air density so that more fuel can be added, the result of which is an increased power output. Assume that ambient air, 100 kPa and 27°C, enters the supercharger at a rate of 250 L/s. The supercharger (compressor) has an isentropic efficiency of 75%, and uses 20 kW of power input. Assume that the ideal and actual compressor have the same exit pressure. Find the ideal specific work and verify that the exit pressure is 175 kPa. Find the percent increase in air density entering the engine due to the supercharger and the entropy generation.

C.V.: Air in compressor (steady flow)

Cont: $\dot{m}_{in} = \dot{m}_{ex} = \dot{m} = \dot{V}/v_{in} = 0.29$ kg/s

Energy: $\dot{m}h_{in} - \dot{W} = \dot{m}h_{ex}$ Assume: $\dot{Q} = 0$

Entropy: $\dot{m}s_{in} + \dot{S}_{gen} = \dot{m}s_{ex}$

Inlet state: $v_{in} = RT_{in}/P_{in} = 0.8614$ m^3/kg, $P_{r\,in} = 1.1167$

$$\eta_c = w_{C\,s}/w_{C\,ac} \Rightarrow -\dot{W}_S = -\dot{W}_{AC} \times \eta_c = 15 \text{ kW}$$

$$-w_{C\,s} = -\dot{W}_S/\dot{m} = 51.724 \text{ kJ/kg},   -w_{C\,ac} = 68.966 \text{ kJ/kg}$$

Table A.7: $h_{ex\,s} = h_{in} - w_{C\,s} = 300.62 + 51.724 = 352.3$ kJ/kg

$\Rightarrow T_{ex\,s} = 351.5$ K, $P_{r\,ex} = 1.949$

$P_{ex} = P_{in} \times P_{r\,ex}/P_{r\,in} = 100 \times 1.949 / 1.1167 = $ **174.5 kPa**

The actual exit state is

$h_{ex\,ac} = h_{in} - w_{C\,ac} = 369.6$ kJ/kg $\Rightarrow T_{ex\,ac} = 368.6$ K

$v_{ex} = RT_{ex}/P_{ex} = 0.606$ m^3/kg

$\rho_{ex}/\rho_{in} = v_{in}/v_{ex} = 0.8614/0.606 = $ **1.42 or 42 % increase**

$s_{gen} = s_{ex} - s_{in} = 7.0767 - 6.8693 - 0.287 \ln(174/100)] = $ **0.0484 kJ/kg K**

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 7
English units

8e

UPDATED AUGUST 2013

CHAPTER 7

Borgnakke and Sonntag

Steady Single Flow Devices

7.177E

A compressor receives R-134a at 20 F, 30 psia with an exit of 200 psia, x = 1.
What can you say about the process?
Solution:

Properties for R-134a are found in Table F.10

Inlet state: s_i = 0.4157 Btu/lbm R

Exit state: s_e = 0.4080 Btu/lbm R

Steady state single flow: $s_e = s_i + \int_i^e \frac{dq}{T} + s_{gen}$

Since s decreases slightly and the generation term can only be positive,
it must be that the heat transfer is negative (out) so the integral gives a
contribution that is smaller than $-s_{gen}$.

7.178E

A condenser receives R-410A at 0 F and quality 80% with the exit flow being saturated liquid at 0 F. Consider the cooling to be a reversible process and find the specific heat transfer from the entropy equation.

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + q/T + 0$

$$q = T\,(s_e - s_i) = T\,(s_f - s_i)$$

Inlet: $s_i = 0.0306 + x_i\,0.2257 = 0.21116$ Btu/lbm-R

Exit: $s_f = 0.0306$ Btu/lbm-R

$$q = (459.67 + 0)\ R \times (0.0306 - 0.21116)\ \text{Btu/lbm-R}$$
$$= -\,\textbf{83.0 Btu/lbm}$$

Remark: It fits with $h_e - h_i = -\,(1 - x_i)\,h_{fg} = -0.8 \times 103.76 = -83.0$ Btu/lbm

7.179E
Steam enters a turbine at 450 lbf/in.2, 900 F, expands in a reversible adiabatic process and exhausts at 130 F. Changes in kinetic and potential energies between the inlet and the exit of the turbine are small. The power output of the turbine is 800 Btu/s. What is the mass flow rate of steam through the turbine?

Solution:

C.V. Turbine, Steady single inlet and exit flows. Adiabatic: $\dot{Q} = 0$.

Continuity Eq.4.11: $\dot{m}_i = \dot{m}_e = \dot{m}$,

Energy Eq.4.12: $\dot{m}h_i = \dot{m}h_e + \dot{W}_T$,

Entropy Eq.7.8: $\dot{m}s_i + \emptyset = \dot{m}s_e$ (Reversible $\dot{S}_{gen} = 0$)

Explanation for the work term is in Sect. 7.3, Eq.7.14

Inlet state: Table F.7.2 $h_i = 1468.3$ btu/lbm, $s_i = 1.7113$ btu/lbm R

Exit state: $s_e = 1.7113$ Btu/lbm R, $T_e = 130$ F $\Rightarrow$ saturated

$x_e = (1.7113 - 0.1817)/1.7292 = 0.8846$,

$h_e = 97.97 + x_e\ 1019.78 = 1000$ Btu/lbm

$w = h_i - h_e = 1468.3 - 1000 = 468.3$ Btu/lbm

$\dot{m} = \dot{W} / w = (800\ \text{Btu/s}) / (468.3\ \text{Btu/lbm}) = \textbf{1.708 lbm/s}$

7.180E

The exit nozzle in a jet engine receives air at 2100 R, 20 psia with neglible kinetic energy. The exit pressure is 10 psia and the process is reversible and adiabatic. Use constant heat capacity at 77 F to find the exit velocity.

Solution:

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i = h_e + V_e^2/2$ ($Z_i = Z_e$)

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Use constant specific heat from Table F.4, $C_{Po} = 0.24 \dfrac{Btu}{lbm\ R}$, $k = 1.4$

The isentropic process ($s_e = s_i$) gives Eq.6.23

$$=> \qquad T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 2100\ (10/20)^{0.2857} = 1722.7\ R$$

The energy equation becomes (conversion 1 Btu/lbm = 25 037 ft^2/s^2 in A.1)

$$V_e^2/2 = h_i - h_e \cong C_P(T_i - T_e)$$

$$V_e = \sqrt{2\ C_P(T_i - T_e)} = \sqrt{2 \times 0.24(2100 - 1722.7) \times 25\ 037} = \textbf{2129 ft/s}$$

7.181E

Do the previous problem using Table F.5.

The exit nozzle in a jet engine receives air at 2100 R, 20 psia with neglible kinetic energy. The exit pressure is 10 psia and the process is reversible and adiabatic. Use constant heat capacity at 77 F to find the exit velocity.

Solution:

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i = h_e + \mathbf{V}_e^2/2$ ($Z_i = Z_e$)

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

The constant s is rewritten from Eq.6.19 as

$$s_{Te}^o = s_{Ti}^o + R \ln(P_e / P_i) = 1.98461 + \frac{53.34}{778} \ln (10/20) = 1.937088 \text{ Btu/lbmR}$$

Interpolate in F.5   =>

$$T_e = 1750 + 50 \frac{1.937088 - 1.93444}{1.94209 - 1.93444} = 1750 + 50 \times 0.34608 = 1767.3 \text{ R}$$

$$h_e = 436.205 + (449.794 - 436.205) \times 0.34608 = 440.908 \text{ Btu/lbm}$$

The energy equation becomes (conversion 1 Btu/lbm = 25 037 ft^2/s^2 in A.1)

$$\mathbf{V}_e^2/2 = h_i - h_e = 532.57 - 440.908 = 91.662 \text{ Btu/lbm}$$

$$\mathbf{V}_e = \sqrt{2 (h_i - h_e)} = \sqrt{2 \times 91.662 \times 25\ 037} = \mathbf{2142 \text{ ft/s}}$$

7.182E

In a heat pump that uses R-134a as the working fluid, the R-134a enters the compressor at 30 lbf/in.^{2}, 20 F. In the compressor the R-134a is compressed in an adiabatic process to 150 lbf/in.^{2} using 1.5 Btu/s of power. Find the mass flow rate it can provide assuming the process is reversible.

Solution:

C.V. Compressor, Steady single inlet and exit flows. Adiabatic: $\dot{Q} = 0$.

Continuity Eq.4.11: $\dot{m}_1 = \dot{m}_2 = \dot{m}$,

Energy Eq.4.12: $\dot{m}h_1 = \dot{m}h_2 + \dot{W}_C$,

Entropy Eq.7.8: $\dot{m}s_1 + \emptyset = \dot{m}s_2$ (Reversible $\dot{S}_{gen} = 0$)

Inlet state: Table F.10.2 $h_1 = 169.82$ Btu/lbm, $s_1 = 0.4157$ Btu/lbm R

Exit state: $P_2 = 150$ psia & s_2 $\Rightarrow$ $h_2 = 184.46$ Btu/lbm

$\dot{m} = \dot{W}_c/w_c = \dot{W}_c /(h_1 - h_2) = -1.5 /(169.82 - 184.46) =$ **0.102 lbm/s**

Explanation for the
work term is in
Sect. 7.3
Eq.7.14

7.183E

A compressor in a commercial refrigerator receives R-410A at -10 F and unknown quality. The exit is at 300 psia, 140 F and the process assumed reversible and adiabatic. Neglect kinetic energies and find the inlet quality and the specific work.

CV Compressor. q = 0.

Energy Eq.4.13: $w_C = h_i - h_e$

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Exit state: 300 psia, 140 F $s = s_i$

Table F.9.2 $\Rightarrow$ $h_e = 137.34$ Btu/lbm, $s_e = s_i = 0.2582$ Btu/lbm-R

Inlet state: F.9.1: T, $s_i = s_e$

$x_i = (0.2582 - 0.0231)/0.2362 = 0.99534$

$h_i = 10.08 + x_i \, 106.2 = 115.78$ Btu/lbm

$w_C = 115.78 - 137.34 = $ –**21.56 Btu/lbm**

7.184E

A compressor brings a hydrogen gas flow at 500 R, 1 atm up to a pressure of 10 atm in a reversible process. How hot is the exit flow and what is the specific work input?

CV Compressor. Assume q = 0.

Energy Eq.4.13: $w_C = h_i - h_e \approx C_p (T_i - T_e)$

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Properties from Table F.4 and constant s from Eq.6.23

$T_e = T_i (P_e/P_i)^{(k-1)/k} = 500 \text{ R } (10/1)^{(1.409-1)/1.409} = \mathbf{975.5 \ R}$

Now the work from the energy equation, C_p from F.4

$w_C = 3.394 \text{ Btu/lbm-R} \times (500 - 975.5) \text{ R} = \mathbf{-1613.8 \ Btu/lbm}$

7.185E

A flow of 4 lbm/s saturated vapor R-410A at 100 psia is heated at constant pressure to 140 F. The heat is supplied by a heat pump that receives heat from the ambient at 540 R and work input, shown in Fig. P7.28. Assume everything is reversible and find the rate of work input.

Solution:

C.V. Heat exchanger

Continuity Eq.: $\dot{m}_1 = \dot{m}_2$;

Energy Eq.: $\dot{m}_1 h_1 + \dot{Q}_H = \dot{m}_1 h_2$

Table F.9.2:

  $h_1 = 119.38$ Btu/lbm,

  $s_1 = 0.2498$ Btu/lbm R

  $h_2 = 146.13$ Btu/lbm,

  $s_2 = 0.2994$ Btu/lbm R

Notice we can find $\dot{Q}_H$ but the temperature T_H is not constant making it difficult to evaluate the COP of the heat pump.

C.V. Total setup and assume everything is reversible and steady state.

Energy Eq.: $\dot{m}_1 h_1 + \dot{Q}_L + \dot{W} = \dot{m}_1 h_2$

Entropy Eq.: $\dot{m}_1 s_1 + \dot{Q}_L / T_L + 0 = \dot{m}_1 s_2$ (T_L is constant, $s_{gen} = 0$)

$\dot{Q}_L = \dot{m}_1 T_L [s_2 - s_1] = 4 \times 540 [0.2994 – 0.2498] = 107.14$ Btu/s

$\dot{W} = \dot{m}_1 [h_2 - h_1] - \dot{Q}_L = 4 (146.13 – 119.38) – 107.14 =$ **-0.14 Btu/s**

Comment: Net work is nearly zero, due to the very low inlet T so the first heating of the flow actually generates work out and only the heating to above ambient T requires work input.

7.186E

A diffuser is a steady-state, steady-flow device in which a fluid flowing at high velocity is decelerated such that the pressure increases in the process. Air at 18 lbf/in.^{2}, 90 F enters a diffuser with velocity 600 ft/s and exits with a velocity of 60 ft/s. Assuming the process is reversible and adiabatic what are the exit pressure and temperature of the air?

C.V. Diffuser, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i + \mathbf{V}_i^2/2g_c = h_e + \mathbf{V}_e^2/2g_c$, $=>$ $h_e - h_i = C_{Po}(T_e - T_i)$

Entropy Eq.7.9: $s_i + \int dq/T + s_{gen} = s_i + 0 + 0 = s_e$ (Reversible, adiabatic)

Energy equation then gives (conversion 1 Btu/lbm = 25 037 ft^2/s^2 from A.1):

$$C_{Po}(T_e - T_i) = 0.24(T_e - 549.7) = \frac{600^2 - 60^2}{2 \times 25\ 037}$$

$T_e = \mathbf{579.3\ R}$

The isentropic process ($s_e = s_i$) gives Eq.6.23

$$P_e = P_i(T_e/T_i)^{\frac{k}{k-1}} = 18\left(\frac{579.3}{549.7}\right)^{3.5} = \mathbf{21.6\ lbf/in^2}$$

7.187E

An expander receives 1 lbm/s air at 300 psia, 540 R with an exit state of 60 psia, 540 R. Assume the process is reversible and isothermal. Find the rates of heat transfer and work neglecting kinetic and potential energy changes.

Solution:

C.V. Expander, single steady flow.

Energy Eq.: $\dot{m}h_i + \dot{Q} = \dot{m}h_e + \dot{W}$

Entropy Eq.: $\dot{m}s_i + \dot{Q}/T + \dot{m}s_{gen} = \dot{m}s_e$

Process: T is constant and $s_{gen} = 0$

Ideal gas and isothermal gives a change in entropy by Eq. 6.15, so we can solve for the heat transfer

$$\dot{Q} = T\dot{m}(s_e - s_i) = -\dot{m}RT \ln \frac{P_e}{P_i}$$

$$= - 1 \text{ lbm/s} \times 540 \text{ R} \times \frac{53.34}{778} \text{ Btu/lbm-R} \times \ln \frac{60}{300} = \mathbf{59.6 \text{ Btu/s}}$$

From the energy equation we get

$$\dot{W} = \dot{m}(h_i - h_e) + \dot{Q} = \dot{Q} = \mathbf{59.6 \text{ Btu/s}}$$

7.188E

One technique for operating a steam turbine in part-load power output is to throttle the steam to a lower pressure before it enters the turbine, as shown in Fig. P7.39. The steamline conditions are 200 lbf/in.^{2}, 600 F, and the turbine exhaust pressure is fixed at 1 lbf/in.^{2}. Assuming the expansion inside the turbine to be reversible and adiabatic, determine the specific turbine work for no throttling and the specific turbine work (part-load) if it is throttled to 60 psia. Show both processes in a *T–s* diagram.

C.V. Turbine full-load, reversible.

Entropy Eq.7.9 reduces to constant s so from Table F.7.2 and F.7.1

$$s_{3a} = s_1 = 1.6767 \text{ Btu/lbm R} = 0.132\ 66 + x_{3a} \times 1.8453$$

$$x_{3a} = 0.8367$$

$$h_{3a} = 69.74 + 0.8367 \times 1036.0 = 936.6 \text{ Btu/lbm}$$

$$_1w_{3a} = h_1 - h_{3a} = 1322.05 - 936.6 = \textbf{385.45 Btu/lbm}$$

The energy equation for the part load operation gives the exit h. Notice that we have constant h in the throttle process, $h_2 = h_1$:

2b: P, $h_{2b} = h_1 = 1322.05$ btu/lbm   => $s_{2b} = 1.80656$ Btu/lbmR

3b: P, $s = s_{2b} = 1.80656$ => $x_{3b} = (1.80656 – 0.132\ 66)/1.8453 = 0.90711$

$$h_{3b} = 69.74 + x_{3b} \times 1036.0 = 1009.5 \text{ Btu/lbm}$$

$$w_T = 1322.05 – 1009.5 = \textbf{312.54 Btu/lbm}$$

7.189E

An adiabatic air turbine receives 2 lbm/s air at 2700 R, 240 psia and 4 lbm/s air at 60 psia, T_2 in a setup similar to Fig. P4.87 with an exit flow at 15 psia. What should the temperature T_2 be so the whole process can be reversible?

The process is reversible if we do not generate any entropy. Physically in this problem it means that state 2 must match the state inside the turbine so we do not mix fluid at two different temperatures (we assume the pressure inside is exactly 60 psia).

For this reason let us select the front end as C.V. and consider the flow from state 1 to the 60 psia. This is a single flow

Entropy Eq.7.9: $\qquad s_1 + 0/T + 0 = s_2$;

Property Eq.6.19 $\qquad s_2 - s_1 = 0 = s_{T2}^o - s_{T1}^o - R \ln(P_2 / P_1)$

$\qquad s_{T2}^o = s_{T1}^o + R \ln(P_2 / P_1) = 2.05606 + \dfrac{53.34}{778} \ln \dfrac{60}{240} = 1.9610$ Btu/lbm-R

$\qquad$ From F.5: $\qquad\qquad T_2 = \mathbf{1928.8\ R}$

If we solve with constant specific heats we get from Eq.6.23 and k = 1.4

$$T_2 = T_1\, (P_2 / P_1)^{(k-1)/k} = 2700\ R\ (60/240)^{0.2857} = 1817\ R$$

Borgnakke and Sonntag

7.190E

A 20 ft^3 tank contains carbon dioxide at 540 R, 20 psia is now filled from a supply of carbon dioxide at 540 R, 20 psia by a compressor to a final tank pressure of 60 psia. Assume the whole process is adiabatic and reversible. Find the final mass and temperature in the tank and the required work to the compressor.

C.V. The tank and the compressor.

Continuity Eq.4.15: $\qquad m_2 - m_1 = m_{in}$

Energy Eq.4.16: $\qquad m_2u_2 - m_1u_1 = {_1}Q_2 - {_1}W_2 + m_{in}h_{in}$

Entropy Eq.7.13: $\qquad m_2s_2 - m_1s_1 = \int dQ/T + {_1}S_{2\ gen} + m_{in}s_{in}$

Process: Adiabatic $\quad {_1}Q_2 = 0$, Process ideal $\quad {_1}S_{2\ gen} = 0$, $s_1 = s_{in}$

$\qquad \Rightarrow\ m_2s_2 = m_1s_1 + m_{in}s_{in} = (m_1 + m_{in})s_1 = m_2s_1\ \Rightarrow\ s_2 = s_1$

Constant s so since the temperatures are modest use Eq.6.23

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 540\ R\ (60/20)^{0.2242} = \mathbf{690.8\ R}$$

$$m_1 = P_1V_1/RT_1 = \frac{20 \times 20 \times 144}{35.1 \times 540} = 3.039\ lbm$$

$$m_2 = P_2V_2/RT_2 = 60 \times 20 \times 144/(35.1 \times 690.8) = \mathbf{7.127\ lbm}$$

$$\Rightarrow\ m_{in} = 4.088\ lbm$$

$${_1}W_2 = m_{in}h_{in} + m_1u_1 - m_2u_2 = m_{in}RT_{in} + m_1(u_1 - u_{in}) - m_2(u_2 - u_{in})$$

$$= 4.088\ (35.1 \times 540/778) + m_1\ (0) - 7.127 \times 0.156\ (690.8 - 540)$$

$$= \mathbf{-68.07\ Btu} \qquad \text{(work must come in)}$$

7.191E

An underground saltmine, 3.5×106 ft^3 in volume, contains air at 520 R, 14.7 lbf/in.^{2}. The mine is used for energy storage so the local power plant pumps it up to 310 lbf/in.^{2} using outside air at 520 R, 14.7 lbf/in.^{2}. Assume the pump is ideal and the process is adiabatic. Find the final mass and temperature of the air and the required pump work. Overnight, the air in the mine cools down to 720 R. Find the final pressure and heat transfer.

C.V. The mine volume and the pump

Continuity Eq.4.15: $\qquad m_2 - m_1 = m_{in}$

Energy Eq.4.16: $\qquad m_2u_2 - m_1u_1 = {_1}Q_2 - {_1}W_2 + m_{in}h_{in}$

Entropy Eq.7.13: $\qquad m_2s_2 - m_1s_1 = \int dQ/T + {_1}S_{2\,gen} + m_{in}s_{in}$

Process: Adiabatic $\quad {_1}Q_2 = 0$, Process ideal $\quad {_1}S_{2\,gen} = 0$, $s_1 = s_{in}$

$\qquad \Rightarrow m_2s_2 = m_1s_1 + m_{in}s_{in} = (m_1 + m_{in})s_1 = m_2s_1 \Rightarrow s_2 = s_1$

Constant s $\Rightarrow \qquad$ Eq.6.19 $\qquad s^o_{T2} = s^o_{Ti} + R \ln(P_e / P_i)$

Table F.4 $\Rightarrow s^o_{T2} = 1.63074 + \dfrac{53.34}{778} \ln (\dfrac{310}{14.7}) = 1.83976$ Btu/lbm R

$$\Rightarrow T_2 = \textbf{1221 R} , \; u_2 = 213.13 \text{ Btu/lbm}$$

Now we have the states and can get the masses

$$m_1 = P_1V_1/RT_1 = \frac{14.7 \times 3.5{\times}10^6 \times 144}{53.34 \times 520} = 2.671{\times}10^5 \text{ lbm}$$

$$m_2 = P_2V_2/RT_2 = \frac{310 \times 3.5{\times}10^6 \times 144}{53.34 \times 1221} = \textbf{2.4} \times \textbf{10}^\textbf{6} \textbf{ lbm}$$

$$\Rightarrow m_{in} = m_2 - m_1 = 2.1319{\times}10^6 \text{ lbm}$$

${_1}W_2 = m_{in}h_{in} + m_1u_1 - m_2u_2 = 2.1319{\times}10^6 \times 124.38 + 2.671{\times}10^5$

$\qquad \times 88.73 - 2.4{\times}10^6 \times 213.13 = -2.226 \times 10^8$ Btu = -pump work in

$$W_{pump} = \textbf{2.23} \times \textbf{10}^\textbf{8} \textbf{ Btu}$$

${_2}W_3 = 0, \qquad P_3 = P_2T_3/T_2 = 310{\times}720/1221 = \textbf{182.8 lbf/in}^\textbf{2}$

${_2}Q_3 = m_2(u_3 - u_2) = 2.4{\times}10^6(123.17 - 213.13) = \textbf{-2.16} \times \textbf{10}^\textbf{8} \textbf{ Btu}$

7.192E

R-410A at 240 F, 600 psia is in an insulated tank and flow is now allowed out to a turbine with a backup pressure of 125 psia. The flow continues to a final tank pressure of 125 psia and the process stops. If the initial mass was 1 lbm how much mass is left in the tank and what is the turbine work assuming a reversible process?

Solution:

C.V. Total tank and turbine.

Continuity Eq.4.15: $\qquad m_2 - m_1 = -m_{ex}$

Energy Eq.4.16: $\qquad m_2 u_2 - m_1 u_1 = -m_{ex} h_{ex} + {}_1Q_2 - {}_1W_2$

Entropy Eq.7.12: $\qquad m_2 s_2 - m_1 s_1 = -m_{ex} s_{ex} + \int dQ/T + {}_1S_{2\,gen}$

Process: $\qquad$ Adiabatic ${}_1Q_2 = 0$; Reversible ${}_1S_{2\,gen} = 0$

   This has too many unknowns (we do not know state 2 only P_2).

C.V. m_2 the mass that remains in the tank. This is a control mass.

Entropy Eq.6.3 (6.37): $\qquad m_2(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen}$

Process: $\qquad$ Adiabatic ${}_1Q_2 = 0$; Reversible ${}_1S_{2\,gen} = 0$

$\qquad\qquad \Rightarrow \qquad s_2 = s_1$

State 1: $\quad v_1 = 0.1342$ ft^3/lbm, $\quad u_1 = 139.96$ Btu/lbm, $\quad s_1 = 0.2703$ Btu/lbm-R

State 2 (P,s): $\quad T_2 = 86.05$ F, $\quad v_2 = 0.5722$ ft^3/lbm, $\quad u_2 = 119.45$ Btu/lbm

State exit: $s_{ex} = s_2 = s_1$ follows from entropy Eq. for first C.V. using the
$\qquad\qquad$ continuity eq., this is identical to state 2, $h_{ex} = 132.695$ Btu/lbm

Tank volume constant so $V = m_1 v_1 = m_2 v_2$

$\qquad\qquad m_2 = m_1\,v_1\,/\,v_2 = 1 \times 0.1342\,/\,0.5722 =$ **0.2345 lbm**

From energy eq.
$\qquad\qquad {}_1W_2 = m_1 u_1 - m_2 u_2 - m_{ex} h_{ex}$
$\qquad\qquad\qquad = 1 \times 139.96 - 0.2345 \times 119.45 - 0.7655 \times 132.695$ [Btu]
$\qquad\qquad\qquad =$ **10.37 Btu**

Reversible Shaft Work, Bernoulli

Borgnakke and Sonntag

7.193E

A river flowing at 2 ft/s across 3 ft high and 30 ft wide area has a dam that creates an elevation difference of 7 ft. How much energy can a turbine deliver per day if 80% of the potential energy can be extracted as work?

CV Dam from upstream to downstream 2 m lower elevation

Continuity Eq.: $\dot{m} = \text{constant} = \dot{m}_e = \dot{m}_i = A_e\mathbf{V}_e/v_e = A_i\mathbf{V}_i/v_i$

Find the mass flow rate

$\dot{m} = A\mathbf{V}_i/v = \rho A\mathbf{V}_i = 62.2 \text{ lbm/ft}^3 \times (3 \times 30) \text{ ft}^2 \times 2 \text{ ft/s} = 11\ 196 \text{ lbm/s}$

Energy Eq.: $0 = \dot{m}\ (h_i + 0.5\mathbf{V}_i^2 + gZ_i) - \dot{m}\ (h_e + 0.5\mathbf{V}_e^2 + gZ_e) - \dot{W}$

The velocity in and out is the same assuming same flow area and the two enthalpies are also the same since same T and $P = P_0$.

This is consistent with Eq.7.14 [$w = g(Z_i - Z_e)$ - loss]

$\dot{W} = 0.8\ \dot{m}\ g(Z_i - Z_e) = 0.8 \times 11\ 196 \text{ lbm/s} \times 32.174 \text{ ft/s}^2 \times 7 \text{ ft}$
$= (2\ 017\ 232 / 25\ 037) \text{ Btu/s} = 80.57 \text{ Btu/s}$

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 from A.1

$W = \dot{W}\ \Delta t = 80.57 \text{ (Btu/s)} \times 24 \times 60 \times 60 \text{ s} = \textbf{6.96 MBtu}$

7.194E

How much liquid water at 60 F can be pumped from 14.7 psia to 35 psia with a 3 kW motor?

Incompressible flow (liquid water) and we assume reversible. Then the shaftwork is from Eq.7.15 (7.16)

$$w = -\int v \, dP = -v \, \Delta P = -0.016 \text{ ft}^3/\text{lbm} \ (35 - 14.7) \text{ psia}$$

$$= -46.77 \text{ lbf-ft/lbm} = -0.06 \text{ Btu/lbm}$$

$$\dot{W} = 3 \text{ kW} = 2.844 \text{ Btu/s}$$

$$\dot{m} = \frac{\dot{W}}{-w} = \frac{2.844}{0.06} = \textbf{47.4 lbm/s}$$

7.195E

A wave comes rolling in to the beach at 6 ft/s horizontal velocity. Neglect friction and find how high up (elevation) on the beach the wave will reach.

We will assume a steady reversible single flow at constant pressure and temperature for the incompressible liquid water. The water will flow in and up the sloped beach until it stops ($V = 0$) so Bernoulli Eq.7.17 leads to

$$gz_{in} + \frac{1}{2}V^2_{in} = gz_{ex} + 0$$

$$(z_{ex} - z_{in}) = \frac{1}{2g}V^2_{in} = \frac{1}{2 \times 32.174 \text{ ft/s}^2} 6^2 \text{ (ft/s)}^2 = \textbf{0.56 ft}$$

Borgnakke and Sonntag

7.196E

A small pump takes in water at 70 F, 14.7 lbf/in.^{2} and pumps it to 250 lbf/in.^{2} at a flow rate of 200 lbm/min. Find the required pump power input.

Solution:

C.V. Pump. Assume reversible pump and incompressible flow.

This leads to the work in Eq.7.15

$$w_p = -\int v dP = -v_i(P_e - P_i)$$

$$= -0.01605 \text{ ft}^3/\text{lbm } (250 - 14.7) \text{ psi} \times \frac{144 \text{ in}^2/\text{ft}^2}{778 \text{ lbf-ft}/(\text{Btu/lbm})}$$

$$= -0.7 \text{ Btu/lbm}$$

$$\dot{W}_{p \text{ in}} = \dot{m}(-w_p) = \frac{200}{60} \text{ lbm/s } (0.7 \text{ Btu/lbm}) = \textbf{2.33 Btu/s = 3.3 hp}$$

Borgnakke and Sonntag

7.197E

An irrigation pump takes water from a river at 50 F, 1 atm and pumps it up to an open canal at a 150 ft higher elevation. The pipe diameter in and out of the pump is 0.3 ft and the motor driving the pump is 5 hp. Neglect kinetic energies and friction, find the maximum possible mass flow rate.

CV the pump. The flow is incompressible and steady flow. The pump work is the difference between the flow work in and out and from Bernoulli's eq. for the pipe that is equal to the potential energy increase sincle pump inlet pressure and pipe outlet pressure are the same.

$$w_p = v\ \Delta P = g\ \Delta Z = 32.174\ \text{ft/s}^2 \times 150\ \text{ft}$$

$$= 4826.1\ (\text{ft/s})^2 = (4826.1\ /\ 25037)\ \text{Btu/lbm} = 0.19276\ \text{Btu/lbm}$$

The horsepower is converted from Table A.1

$$\dot{W}_{motor} = 5\ \text{hp} = 5 \times 2544\ \text{Btu/h} = 12\ 720\ \text{btu/h} = 3.533\ \text{Btu/s}$$

$$\dot{m} = \dot{W}_{motor}\ /\ w_p = 3.533\ /\ 0.19276 = \textbf{18.33 lbm/s}$$

Comment:

$$\dot{m} = AV/v \qquad \Rightarrow \qquad V = \frac{\dot{m}v}{A} = \frac{4\dot{m}}{\rho\ \pi\ D^2} = \frac{4 \times 18.33}{62.2 \times \pi \times 0.3^2} = 4.17\ \text{ft/s}$$

The power to generated the kinetic energy is

$$\text{Power} = \dot{m}\ 0.5\ V^2 = 18.33 \times 0.5 \times 4.17^2 /\ 25037 = 0.0064\ \text{Btu/s}$$

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 from A.1
This is insignificant relative to the power needed for the potential energy increase.

Pump inlet and the pipe exit both have close to atmospheric pressure.

7.198E

A fireman on a ladder 80 ft above ground should be able to spray water an additional 30 ft up with the hose nozzle of exit diameter 1 in. Assume a water pump on the ground and a reversible flow (hose, nozzle included) and find the minimum required power.

Solution:

C.V.: pump + hose + water column, total height difference 35 m.

Continuity Eq.4.3, 4.11: $\dot{m}_{in} = \dot{m}_{ex} = (\rho A \mathbf{V})_{nozzle}$

Energy Eq.4.12: $\dot{m}(-w_p) + \dot{m}(h + \mathbf{V}^2/2 + gz)_{in} = \dot{m}(h + \mathbf{V}^2/2 + gz)_{ex}$

Process: $h_{in} \cong h_{ex}$, $\mathbf{V}_{in} \cong \mathbf{V}_{ex} = 0$, $z_{ex} - z_{in} = 110$ ft, $\rho = 1/v \cong 1/v_f$

$-w_p = g(z_{ex} - z_{in}) = 32.174 \times (110 - 0)/25\ 037 = 0.141$ Btu/lbm

Recall the conversion 1 Btu/lbm $= 25\ 037$ ft^2/s^2 from Table A.1. The velocity in the exit nozzle is such that it can rise 30 ft. Make that column a C.V. for which Bernoulli Eq.7.17 is:

$$gz_{noz} + \frac{1}{2}\mathbf{V}^2_{noz} = gz_{ex} + 0$$

$$\mathbf{V}_{noz} = \sqrt{2g(z_{ex} - z_{noz})}$$
$$= \sqrt{2 \times 32.174 \times 30} = 43.94 \text{ ft/s}$$

Assume: $v = v_{F,70F} = 0.01605$ ft^3/lbm

$$\dot{m} = \frac{\pi}{v_f}\left(\frac{D}{2}\right)^2 \mathbf{V}_{noz} = (\pi/4)(1^2/144) \times 43.94 / 0.01605 = 14.92 \text{ lbm/s}$$

$$\dot{W}_{pump} = \dot{m}w_p = 14.92 \times 0.141 \times (3600/2544) = \mathbf{3\ hp}$$

7.199E

Saturated R-410A at 10 F is pumped/compressed to a pressure of 300 lbf/in.^{2} at the rate of 1.0 lbm/s in a reversible adiabatic steady flow process. Calculate the power required and the exit temperature for the two cases of inlet state of the R-410A:

      a) quality of 100 %.

      b) quality of 0 %.

Solution:

C.V.: Pump/Compressor, $\dot{m}$ = 1 lbm/s, R-410A

a) State 1: Table F.10.1, x_1 = 1.0 Saturated vapor, P_1 = P_g = 76.926 psia,

    h_1 = h_g = 118.21 Btu/lbm, s_1 = s_g = 0.2535 Btu/lbm R

    Assume Compressor is isentropic, s_2 = s_1 = 0.2535 Btu/lbm R

        h_2 = 134.54 Btu/lbm, **T_2 = 130.5 F**

    Energy Eq.4.13: q_c + h_1 = h_2 + w_c; q_c = 0

    w_{cs} = h_1 - h_2 = 118.21 – 134.54 = - 16.33 Btu/lbm;

          => $\dot{W}_C$ = $\dot{m}w_C$ = **-16.3 Btu/s = -23.1 hp**

b) State 1: T_1 = 10 F, x_1 = 0 Saturated liquid. This is a pump.

    P_1 = 76.926 psia, h_1 = h_f = 17.0 Btu/lbm, v_1 = v_f = 0.01316 ft^3/lbm

    Energy Eq.4.13: q_p + h_1 = h_2 + w_p; q_p = 0

    Assume Pump is isentropic and the liquid is incompressible, Eq.7.18:

    w_{ps} = - ∫ v dP = -v_1(P_2 - P_1) = -0.01316 (300 – 76.926) 144

        = -422.73 lbf-ft/lbm = - 0.543 Btu/lbm

        h_2 = h_1 - w_p = 17.0 - (- 0.543) = 17.543 Btu/lbm,

    Assume State 2 is approximately a saturated liquid   => **T_2 ≅ 11.5 F**

        $\dot{W}_P$ = $\dot{m}w_P$ = 1 (- 0.543) = **-0.543 Btu/s** = -0.77 hp

7.200E

Liquid water at ambient conditions, 14.7 lbf/in.^{2}, 75 F, enters a pump at the rate of 1 lbm/s. Power input to the pump is 3 Btu/s. Assuming the pump process to be reversible, determine the pump exit pressure and temperature.

Solution:

C.V. Pump. Steady single inlet and exit flow with no heat transfer.

Energy Eq.4.13: $w = h_i - h_e = \dot{W}/\dot{m} = -3/1 = -3.0$ btu/lbm

Using also incompressible media we can use Eq.7.15

$$w_P = -\int v dP \approx -v_i(P_e - P_i) = -0.01606 \text{ ft/lbm}(P_e - 14.7 \text{ psia})$$

from which we can solve for the exit pressure

$$3 \cong 0.01606(P_e - 14.7) \times \frac{144}{778} \ \Rightarrow \ P_e = \textbf{1023.9 lbf/in}^2$$

e

Pump   -$\dot{W}$

i

$-\dot{W} = 3$ Btu/s, $P_i = 14.7$ psia
$T_i = 75$ F $\dot{m} = 1$ lbm/s

Energy Eq.: $h_e = h_i - w_P = 43.09 + 3 = 46.09$ Btu/lbm

Use Table F.7.3 at 1000 psia   => $\textbf{T}_\textbf{e} = \textbf{75.3 F}$

Borgnakke and Sonntag

7.201E

The underwater bulb nose of a container ship has a velocity relative to the ocean water as 30 ft/s. What is the pressure at the front stagnation point that is 6 ft down from the water surface.

Solution:

C.V. A stream line of flow from the freestream to the wall.

Eq.7.17: $v(P_e - P_i) + \frac{1}{2}(\mathbf{V}_e^2 - \mathbf{V}_i^2) + g(Z_e - Z_i) = 0$

$$\Delta P = \frac{1}{2v}\mathbf{V}_i^2 = \frac{30^2}{0.01603 \times 32.174 \times 144 \times 2} = 6.06 \text{ psi}$$

$P_i = P_o + gH/v = 14.695 + 6/(0.01603 \times 144) = 17.29 \text{ psia}$

$P_e = P_i + \Delta P = 17.29 + 6.06 = \textbf{23.35 psia}$

This container-ship is under construction and not loaded. The red line is the water line under normal load.

Borgnakke and Sonntag

7.202E

A speed boat has a small hole in the front of the drive with the propeller that sticks down into the water at a water depth of 15 in. Assume we have a stagnation point at that hole when the boat is sailing with 30 mi/h, what is the total pressure there?

Solution:

C.V. A stream line of flow from the freestream to the wall.

Eq.7.17: $v(P_e - P_i) + \frac{1}{2}(\mathbf{V}_e^2 - \mathbf{V}_i^2) + g(Z_e - Z_i) = 0$

$\mathbf{V}_i = 30\,\frac{mi}{h} \times 1.467\,\frac{ft\text{-}h}{s\text{-}mi} = 44.01 \text{ ft/s}$

$\Delta P = \frac{1}{2v}\mathbf{V}_i^2 = \frac{44.01^2}{0.01603 \times 32.174 \times 144 \times 2} = 13.04 \text{ psi}$

$P_i = P_o + gH/v = 14.695 + \frac{15/12}{0.01603 \times 144} = 15.24 \text{ psia}$

$P_e = P_i + \Delta P = 15.24 + 13.04 = \textbf{28.28 psia}$

Remark: This is fast for a boat

7.203E

Helium gas enters a steady-flow expander at 120 lbf/in.^{2}, 500 F, and exits at 18 lbf/in.^{2}. The mass flow rate is 0.4 lbm/s, and the expansion process can be considered as a reversible polytropic process with exponent, $n = 1.3$. Calculate the power output of the expander.

Solution:

CV: expander, reversible polytropic process.
From Eq.6.28:

$$T_e = T_i \left(\frac{P_e}{P_i}\right)^{\frac{n-1}{n}} = 960 \left(\frac{18}{120}\right)^{\frac{0.3}{1.3}} = 619.6 \text{ R}$$

Table F.4: $R = 386$ lbf-ft/lbm-R

Work evaluated from Eq.7.18

$$w = -\int v dP = -\frac{nR}{n-1}(T_e - T_i) = -\frac{1.3 \times 386}{0.3 \times 778}(619.6 - 960)$$

$$= +731.8 \text{ Btu/lbm}$$

$$\dot{W} = \dot{m}w = 0.4 \times 731.8 \times \frac{3600}{2544} = \textbf{414 hp}$$

7.204E

An expansion in a gas turbine can be approximated with a polytropic process with exponent n = 1.25. The inlet air is at 2100 R, 120 psia and the exit pressure is 18 psia with a mass flow rate of 2 lbm/s. Find the turbine heat transfer and power output.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.4.13: $h_i + q = h_e + w$ Neglect kinetic, potential energies

Entropy Eq.7.9: $s_i + \int dq/T + s_{gen} = s_e$

Process Eq.6.28:

$$T_e = T_i \, (P_e/\, P_i)^{\frac{n-1}{n}} \quad = 2100 \text{ R } (18/120)^{\frac{0.25}{1.25}} = 1436.9 \text{ R}$$

so the exit enthalpy is from Table F.5, $h_i = 532.6$ Btu/lbm

$$h_e = 343.0 + \frac{36.9}{40}(353.5 - 343.0) = 352.7 \text{ Btu/lbm}$$

The process leads to Eq.7.18 for the work term

$$\dot{W} = \dot{m}w = -\dot{m}\frac{nR}{n-1} \, (T_e - T_i) = -2 \, \frac{1.25 \times 53.34}{0.25 \times 778} \times (1436.9 - 2100)$$

$$= \textbf{454.6 Btu/s}$$

Energy equation gives

$$\dot{Q} = \dot{m}q = \dot{m}(h_e - h_i) + \dot{W} = 2(352.7 - 532.6) + 454.6$$

$$= -359.8 + 454.6 = \textbf{94.8 Btu/s}$$

Notice:
dP < 0
so dw > 0

ds > 0
so dq > 0

Notice this process has some heat transfer in during expansion which is unusual. The typical process would have n = 1.5 with a heat loss.

Steady Irreversible Processes

7.205E

Analyse the steam turbine described in Problem 4.188E. Is it possible?

C.V. Turbine. Steady flow and adiabatic.

Continuity Eq.4.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$;

Energy Eq.4.10: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3 + \dot{W}$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3$

States from Table F.7.2: $s_1 = 1.6398$ Btu/lbm R, $s_2 = 1.6516$ Btu/lbm R,

$$s_3 = s_f + x\, s_{fg} = 0.283 + 0.95 \times 1.5089 = 1.71 \text{ Btu/lbm R}$$

$$\dot{S}_{gen} = 40 \times 1.6516 + 160 \times 1.713 - 200 \times 1.6398 = \textbf{12.2 Btu/s} \cdot \textbf{R}$$

Since it is positive => possible.

Notice the entropy is increasing through turbine: $s_1 < s_2 < s_3$

7.206E

A large condenser in a steam power plant dumps 15 000 Btu/s at 115 F with an ambient at 77 F. What is the entropy generation rate?
Solution:

This process transfers heat over a finite temperature difference between the water inside the condenser and the outside ambient (cooling water from the sea, lake or river or atmospheric air)

C.V. The wall that separates the inside 115 F water from the ambient at 77 F.

Entropy Eq. 7.1 for steady state operation:

Condensing water		Sea water
115 F	cb	77 F

$$\frac{dS}{dt} = 0 = \sum \frac{\dot{Q}}{T} + \dot{S}_{gen} = \frac{\dot{Q}}{T_{115}} - \frac{\dot{Q}}{T_{77}} + \dot{S}_{gen}$$

$$\dot{S}_{gen} = \left[\frac{15\ 000}{536.7} - \frac{15\ 000}{115 + 459.7} \right] \frac{Btu}{s\ R} = \mathbf{1.85} \frac{\mathbf{Btu}}{\mathbf{s\ R}}$$

Borgnakke and Sonntag

7.207E

A compressor in a commercial refrigerator receives R-410A at -10 F and x = 1. The exit is at 150 psia and 60 F. Is this compressor possible?

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also assume no heat transfer and $Z_i = Z_e$

From Table F.9.1 : h_i = 116.29 Btu/lbm, s_i = 0.2592 Btu/lbm-R

From Table F.9.2 : h_e = 124.58 Btu/lbm, s_e = 0.2508 Btu/lbm-R

Entropy gives

$$s_{gen} = s_e - s_i - \int dq/T = 0.2508 – 0.2592 - \int dq/T =$$

The result is negative unless dq is negative (it should go out, but T < T ambient) so this compressor is **impossible**

Borgnakke and Sonntag

7.208E

R-134a at 90 F, 125 psia is throttled in a steady flow to a lower pressure so it comes out at 10 F. What is the specific entropy generation?

Solution:

The process is adiabatic and irreversible. The consideration of the energy given in example 6.5 resulted in a constant h and two-phase exit flow.

Table F.10.1: $h_i = 105.34$ Btu/lbm, $s_i = 0.2757$ Btu/lbm-R (compressed liquid)

State 2: 10 F, $h_e = h_i < h_g$ so two-phase $x_e = (h_e - h_f)/h_{fg} = 0.2956$

Table F.10.1: $s_e = s_f + x_e \, s_{fg} = 0.2244 + 0.2956 \times 0.1896 = 0.2804$ Btu/lbm-R

We assumed no heat transfer so the entropy equation Eq.7.9 gives

$$s_{gen} = s_e - s_i - \int dq/T = 0.2804 - 0.2757 - 0 = \textbf{0.0047 Btu/lbm-R}$$

7.209E

Two flowstreams of water, one at 100 lbf/in.^{2}, saturated vapor, and the other at 100 lbf/in.^{2}, 1000 F, mix adiabatically in a steady flow process to produce a single flow out at 100 lbf/in.2, 600 F. Find the total entropy generation for this process.

Solution:

Continuity Eq.4.9: $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$,

Energy Eq.4.10: $\dot{m}_3 h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2$

State properties from Table F.7.2

$h_1 = 1187.8$, $h_2 = 1532.1$, $h_3 = 1329.3$ all in Btu/lbm

$s_1 = 1.6034$, $s_2 = 1.9204$, $s_3 = 1.7582$ all in Btu/lbm R

$$\Rightarrow \quad \dot{m}_1/\dot{m}_3 = (h_3 - h_2) / (h_1 - h_2) = 0.589$$

Entropy Eq.7.7: $\dot{m}_3 s_3 = \dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen}$ $\Rightarrow$

$\dot{S}_{gen}/\dot{m}_3 = s_3 - (\dot{m}_1/\dot{m}_3) s_1 - (\dot{m}_2/\dot{m}_3) s_2$

$$= 1.7582 - 0.589 \times 1.6034 - 0.411 \times 1.9204 = \mathbf{0.0245} \, \frac{\mathbf{Btu}}{\mathbf{lbm \ R}}$$

The mixing process generates entropy. The two inlet flows could have exchanged energy (they have different T) through some heat engines and produced work, the process failed to do that, thus irreversible.

Borgnakke and Sonntag

7.210E

A compressor in a commercial refrigerator receives R-410A at -10 F and x = 1. The exit is at 300 psia and 160 F. Neglect kinetic energies and find the specific entropy generation.

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also assume no heat transfer q = 0 and $Z_1 = Z_2$

Entropy Eq.7.9: $\qquad s_i + \int dq/T + s_{gen} = s_e = s_i + 0 + s_{gen}$

From Table F.9.1 : $\qquad s_i = 0.2592$ Btu/lbm-R

From Table F.9.2 : $\qquad s_e = 0.2673$ Btu/lbm-R

Entropy generation becomes

$$s_{gen} = s_e - s_i = 0.2673 - 0.2592 = \textbf{0.0081 Btu/lbm-R}$$

7.211E

A steam turbine has an inlet of 4 lbm/s water at 150 psia and 550 F with velocity of 50 ft/s. The exit is at 1 atm, 240 F and very low velocity. Find the power produced and the rate of entropy generation.

Solution:

C.V. Turbine. Steady flow and adiabatic.

Continuity Eq.4.9: $\dot{m}_1 = \dot{m}_2$;

Energy Eq.4.10: $\dot{m}_1(h_1 + \frac{1}{2}\mathbf{V}^2) = \dot{m}_2 h_2 + \dot{W}$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2$

States from Table F.7.2: $h_1 = 1300.05$ Btu/lbm, $s_1 = 1.6862$ Btu/lbm-R,

$h_2 = 1164.02$ Btu/lbm, $s_2 = 1.7764$ Btu/lbm-R

$\dot{W} = \dot{m}_1(h_1 + \frac{1}{2}\mathbf{V}^2 - h_2) = 4\,(1300.05 + \frac{1}{2}\frac{50^2}{25037} - 1164.02) = \textbf{546.1 Btu/s}$

$\dot{S}_{gen} = \dot{m}_1(s_2 - s_1) = 4\,(1.7764 - 1.6862) = \textbf{0.361 Btu/s-R}$

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 from A.1

Borgnakke and Sonntag

7.212E

A dual fluid heat exchanger has 10 lbm/s water enter at 104 F, 20 psia and leaving at 50 F, 20 psia. The other fluid is glycol coming in at 14 F, 22 psia and leaves at 50 F, 22 psia. Find the mass flow rate of glycol and the rate of entropy generation.

Continuity Eqs.:   Each line has a constant flow rate through it.

Energy Eq.4.10: $\dot{m}_{H_2O} \, h_1 + \dot{m}_{glycol} \, h_3 = \dot{m}_{H_2O} \, h_2 + \dot{m}_{glycol} \, h_4$

Entropy Eq.7.7: $0 = \dot{m}_{H_2O} \, s_1 + \dot{m}_{glycol} \, s_3 - \dot{m}_{H_2O} \, s_2 - \dot{m}_{glycol} \, s_4 + \dot{S}_{gen}$

Process: Each line has a constant pressure.

Table F.7: $h_1 = 72.03$, $h_2 = 18.05$ Btu/lbm, $s_1 = 0.1367$, $s_2 = 0.0361$ Btu/lbm-R

We could have used specific heat for the changes.

Table F.3: $C_{P \; gly} = 0.58$ Btu/lbm-R so

$$h_4 - h_3 = C_{P \; gly} \, (T_4 - T_3) = 0.58 \, [50 - 14] = 20.88 \; \text{Btu/lbm}$$
$$s_4 - s_3 = C_{P \; gly} \, \ln \, (T_4/T_3) = 0.58 \, \ln(509.7/473.7) = 0.0425 \; \text{Btu/lbm-R}$$

$$\dot{m}_{glycol} = \dot{m}_{H_2O} \, \frac{h_1 - h_2}{h_4 - h_3} = 10 \, \frac{72.03 - 18.05}{20.88} = \textbf{25.85 lbm/s}$$

$$\dot{S}_{gen} = \dot{m}_{H_2O} \, (s_2 - s_1) + \dot{m}_{glycol} \, (s_4 - s_3)$$

$$= 10 \; \text{lbm/s} \, (0.0361 - 0.1367) \; \text{Btu/lbm-R}$$

$$+ \; 25.85 \; \text{lbm/s} \times 0.0425 \; \text{Btu/lbm-R}$$

$$= \; \textbf{0.093 Btu/s-R}$$

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for glycol and water each. The two flows exchange energy with no heat transfer to/from the outside.

Borgnakke and Sonntag

7.213E

A factory generates compressed air from ambient 15 psia, 62 F, by compression to 150 psia, 1080 R after which it cools in a constant pressure cooler to 540 R by heat transfer to the ambient. Find the specific entropy generation in the compressor and in the cooler operation.

Solution

C.V. air compressor $q = 0$

Continuity Eq.: $\dot{m}_2 = \dot{m}_1$; Energy Eq.4.13: $0 = h_1 + w_{c\ in} - h_2$

Entropy Eq.: $0 = s_1 - s_2 + s_{gen\ comp}$

Table F.5 State 1: $h_1 = 124.79$ Btu/lbm, $s^o_{T1} = 1.63151$ Btu/lbm-R,

Table F.5 State 2: $h_2 = 261.099$ Btu/lbm, $s^o_{T2} = 1.80868$ Btu/lbm-R,

Table F.5 State 3: $h_3 = 129.18$ Btu/lbm, $s^o_{T3} = 1.63979$ Btu/lbm-R,

$$s_{gen\ comp} = s_2 - s_1 = s^o_{T2} - s^o_{T1} - R\ \ln(P_2/P_1)$$

$$= 1.80868 - 1.63151 - \frac{53.34}{778}\ \ln(1000/100) = \textbf{0.019 Btu/lbmR}$$

C.V. cooler $w = 0$

Continuity Eq.: $\dot{m}_3 = \dot{m}_1$; Energy Eq.4.13: $0 = h_2 - q_{out} - h_3$

Entropy Eq.: $0 = s_2 - s_3 - q_{out}/T_{amb} + s_{gen\ cool}$

$q_{out} = h_2 - h_3 = 261.099 - 129.18 = 131.919$ Btu/lbm

$$s_{gen\ cool} = s_3 - s_2 + q_{out}/T_{amb} = s^o_{T3} - s^o_{T2} + q_{out}/T_{amb}$$

$$= 1.63979 - 1.80868 + \frac{131.919}{521.7} = \textbf{0.084 Btu/lbm}$$

Compressor section   Cooler section

7.214E

A mixing chamber receives 10 lbm/min ammonia as saturated liquid at 0 F from one line and ammonia at 100 F, 40 lbf/in.^{2} from another line through a valve. The chamber also receives 340 Btu/min energy as heat transferred from a 100-F reservoir. This should produce saturated ammonia vapor at 0 F in the exit line. What is the mass flow rate at state 2 and what is the total entropy generation in the process?

Solution:

CV: Mixing chamber out to reservoir

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{Q} = \dot{m}_3 h_3$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{Q}/T_{res} + \dot{S}_{gen} = \dot{m}_3 s_3$

From Table F.8.1: $h_1 = 42.6$ Btu/lbm, $s_1 = 0.0967$ Btu/lbm R
From Table F.8.2: $h_2 = 664.33$ Btu/lbm, $s_2 = 1.4074$ Btu/lbm R
From Table F.8.1: $h_3 = 610.92$ Btu/lbm, $s_3 = 1.3331$ Btu/lbm R

From the energy equation:

$$\dot{m}_2 = \frac{\dot{m}_1(h_1 - h_3) + \dot{Q}}{h_3 - h_2} = \frac{10(42.6 - 610.92) + 340}{610.92 - 664.33} = \textbf{100.1 lbm/min}$$

$$\Rightarrow \qquad \dot{m}_3 = 110.1 \text{ lbm/min}$$

$$\dot{S}_{gen} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{Q}/T_{res}$$

$$= 110.1 \times 1.3331 - 10 \times 0.0967 - 100.1 \times 1.4074 - \frac{340}{559.67} = \textbf{4.37} \frac{\textbf{Btu}}{\textbf{R min}}$$

7.215E

A condenser in a power plant receives 10 lbm/s steam at 130 F, quality 90% and rejects the heat to cooling water with an average temperature of 62 F. Find the power given to the cooling water in this constant pressure process and the total rate of enropy generation when condenser exit is saturated liquid.

Solution:

C.V. Condenser. Steady state with no shaft work term.

Energy Eq.4.12: $\dot{m} h_i + \dot{Q} = \dot{m} h_e$

Entropy Eq.7.8: $\dot{m} s_i + \dot{Q}/T + \dot{S}_{gen} = \dot{m} s_e$

Properties are from Table F.7.1

$h_i = 98.0 + 0.9 \times 1019.8 = 1015.8$ Btu/lbm, $h_e = 98.0$ Btu/lbm

$s_i = 0.1817 + 0.9 \times 1.7292 = 1.7380$ Btu/lbm R, $s_e = 0.1817$ Btu/lbm R

$\dot{Q}_{out} = -\dot{Q} = \dot{m} (h_i - h_e) = 10(1015.8 - 98.0) = $ **9178 Btu/s**

$\dot{S}_{gen} = \dot{m} (s_e - s_i) + \dot{Q}_{out}/T$

$= 10(0.1817 - 1.738) + 9178/(459.7 + 62)$

$= -15.563 + 17.592 = $ **2.03 Btu/s-R**

Often the cooling media flows inside a long pipe carrying the energy away.

7.216E

A large supply line has a steady flow of R-410A at 175 psia, 140 F. It is used in three different adiabatic devices shown in Fig. P7.101, a throttle flow, an ideal nozzle and an ideal turbine. All the exit flows are at 60 psia. Find the exit temperature and specific entropy generation for each device and the exit velocity of the nozzle.

Inlet state: F.9.2: $h_i = 143.11$ Btu/lbm, $s_i = 0.2804$ Btu/lbm-R

C.V. Throttle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i = h_e$ ($Z_i = Z_e$ and **V**'s are small)

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + s_{gen}$

Exit state: $h_e = h_i = 143.11$ Btu/lbm, $T_e = $ **118.87 F**, $s_e = 0.3076$ kJ/kg-K

$s_{gen} = s_e - s_i = 0.3076 - 0.2804 = $ **0.0272 Btu/lbm-R**

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i = h_e + \mathbf{V}_e^2/2$ ($Z_i = Z_e$)

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

The isentropic process ($s_e = s_i$) gives from F.9.2

$$T_e = \textbf{49.15 F}, \quad \textbf{s}_{gen} = \textbf{0}, \quad h_e = 128.36 \text{ Btu/lbm}$$

The energy equation becomes

$$\mathbf{V}_e^2/2 = h_i - h_e = 143.11 - 128.36 = 14.75 \text{ Btu/lbm}$$

$$\mathbf{V}_e = \sqrt{2 \times 14.75 \times 25037} = \textbf{859 ft/s}$$

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 from A.1

Turbine:

Process: Reversible and adiabatic => same as for nozzle except w, $\mathbf{V}_e = 0$

Energy Eq.4.13: $h_i = h_e + w$ ($Z_i = Z_e$)

$$T_e = \textbf{49.15 F}, \quad \textbf{s}_{gen} = \textbf{0}, \quad h_e = 128.36 \text{ Btu/lbm}$$

7.217E

Air at 540 F, 60 lbf/in.^{2} with a volume flow 40 ft^3/s runs through an adiabatic turbine with exhaust pressure of 15 lbf/in.^{2}. Neglect kinetic energies and use constant specific heats. Find the lowest and highest possible exit temperature. For each case find also the rate of work and the rate of entropy generation.

$T_i = 540\ F = 1000\ R$

$v_i = RT_i /P_i = 53.34 \times 1000/(60 \times 144) = 6.174\ ft^3 / lbm$

$\dot{m} = \dot{V}/v_i = 40\ (ft^3/s)\ /\ 6.174\ ft^3/ lbm = 6.479\ lbm/s$

a. **lowest exit T**, this must be reversible for maximum work out.

Process: Reversible and adiabatic => constant s from Eq.7.9

Eq.6.23: $T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 1000\ R\ (15/60)^{0.286} = \textbf{673 R}$

$w = 0.24\ (1000 - 673) = 78.48\ Btu/lbm$; $\dot{W} = \dot{m}w = \textbf{508.5 Btu/s}$

$\dot{S}_{gen} = \textbf{0}$

b. **Highest exit T**, for no work out. $T_e = T_i = \textbf{1000 R}$

$\dot{W} = \textbf{0}$

$\dot{S}_{gen} = \dot{m}(s_e - s_i) = -\dot{m}R \ln(P_e/P_i)$

$= -6.479 \times \dfrac{53.34}{778} \ln(15/60) = \textbf{0.616 Btu/s·R}$

7.218E

A large supply line has a steady air flow at 900 R, 2 atm. It is used in three different adiabatic devices shown in Fig. P7.101, a throttle flow, an ideal nozzle and an ideal turbine. All the exit flows are at 1 atm. Find the exit temperature and specific entropy generation for each device and the exit velocity of the nozzle.

C.V. Throttle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $\quad\quad h_i = h_e \quad$ ($Z_i = Z_e$ and **V**'s are small)

Entropy Eq.7.8: $\quad\quad s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + s_{gen}$

Since it is air we have $h = h(T)$ so same h means same $T_e = T_i = $ **900 R**

$$s_{gen} = s_e - s_i = s_{Te}^o - s_{Ti}^o - R \ln(P_e / P_i) = 0 - (53.34 / 778) \ln(1/2)$$
$$= \textbf{0.0475 Btu/lbm-R}$$

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $\quad\quad h_i = h_e + \mathbf{V}_e^2/2 \quad$ ($Z_i = Z_e$)

Entropy Eq.7.9: $\quad\quad s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Use constant specific heat from Table F.4, $\;C_{Po} = 0.24$ Btu/lbm-R, $\;k = 1.4$

The isentropic process ($s_e = s_i$) gives Eq.6.23

$=> \quad\quad T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 900 \, (1/2)^{0.2857} = \textbf{738.3 R}$

The energy equation becomes: $\quad\quad \mathbf{V}_e^2/2 = h_i - h_e \cong C_P(T_i - T_e)$

$$\mathbf{V}_e = \sqrt{2 \, C_P(T_i - T_e)} = \sqrt{2 \times 0.24(900 - 738.3) \times 25037} = \textbf{1394 ft/s}$$

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 from A.1

Turbine:

Process: Reversible and adiabatic $\quad =>\quad$ constant s $\quad$ from Eq.7.9

Eq.6.23: $\quad\quad T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 900 \times (1/2)^{0.2857} = \textbf{738.3 R}$

$\Rightarrow \quad w = h_i - h_e = C_{Po}(T_i - T_e) = 0.24 \times (900 - 738.3) = 38.8$ Btu/lbm

7.219E

Repeat the previous problem for the throttle and the nozzle when the inlet air temperature is 4000 R and use the air tables.

C.V. Throttle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i = h_e$ ($Z_i = Z_e$ and **V**'s are small)

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + s_{gen}$

Since it is air we have h = h(T) so same h means same $T_e = T_i =$ **4000 R**

$$s_{gen} = s_e - s_i = s_{Te}^o - s_{Ti}^o - R \ln(P_e / P_i) = 0 - (53.34 / 778) \ln(1/2)$$
$$= \mathbf{0.0475 \ Btu/lbm\text{-}R}$$

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: $h_i = h_e + \mathbf{V}_e^2/2$ ($Z_i = Z_e$)

Entropy Eq.7.9: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

The isentropic process ($s_e = s_i$) gives Eq.6.19

$$0 = s_e - s_i = s_{Te}^o - s_{Ti}^o - R \ln(P_e / P_i)$$

=> $\quad s_{Te}^o = s_{Ti}^o + R \ln(P_e / P_i) = 2.17221 + (53.34/778) \ln (1/2) = 2.12469$

T = **3846 R,** $\quad h_e = 912.2$ Btu/lbm

The energy equation becomes

$$\mathbf{V}_e^2/2 = h_i - h_e = 1087.988 - 912.2 = 175.79 \text{ Btu/lbm}$$

$$\mathbf{V}_e = \sqrt{2 \times 25\ 037 \times 175.79} = \mathbf{2967 \ ft/s}$$

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 from A.1

Borgnakke and Sonntag

7.220E

A supply of 10 lbm/s ammonia at 80 lbf/in.^{2}, 80 F is needed. Two sources are available one is saturated liquid at 80 F and the other is at 80 lbf/in.^{2}, 260 F. Flows from the two sources are fed through valves to an insulated mixing chamber, which then produces the desired output state. Find the two source mass flow rates and the total rate of entropy generation by this setup.

Solution:

C.V. mixing chamber + valve. Steady, no heat transfer, no work.

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$;

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} = \dot{m}_3 s_3$

State 1: Table F.8.1 $h_1 = 131.68$ Btu/lbm, $s_1 = 0.2741$ Btu/lbm R

State 2: Table F.8.2 $h_2 = 748.5$ Btu/lbm, $s_2 = 1.4604$ Btu/lbm R

State 3: Table F.8.2 $h_3 = 645.63$ Btu/lbm, $s_3 = 1.2956$ Btu/lbm R

As all states are known the energy equation establishes the ratio of mass flow rates and the entropy equation provides the entropy generation.

$$\dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_1)h_2 = \dot{m}_3 h_3 \implies \dot{m}_1 = \dot{m}_3 \frac{h_3 - h_2}{h_1 - h_2} = 10 \times \frac{-102.87}{-616.82} = 1.668 \text{ lbm/s}$$

$$\implies \quad \dot{m}_2 = \dot{m}_3 - \dot{m}_1 = 8.332 \text{ lbm/s}$$

$$\dot{S}_{gen} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

$$= 10 \times 1.2956 - 1.668 \times 0.2741 - 8.332 \times 1.46 = \textbf{0.331 Btu/s·R}$$

7.221E

An initially empty 5 ft^3 tank is filled with air from 70 F, 15 psia until it is full. Assume no heat transfer and find the final mass and entropy generation.

Solution:

C.V. Tank + valve out to line. No boundary/shaft work, $m_1 = 0$; $Q = 0$ and we recognize that this is a transient problem..

Continuity Eq.4.15: $\qquad m_2 - 0 = m_i$

Energy Eq.4.16: $\qquad m_2 u_2 - 0 = m_i h_i$

Entropy Eq.7.12: $\qquad m_2 s_2 - 0 = m_i s_i + 0 + {}_1 S_{2 \text{ gen}}$

State 2: $P_2 = P_i$ and $u_2 = h_i = h_{line} = h_2 - RT_2$ (ideal gas)

To reduce or eliminate guess use: $\quad h_2 - h_{line} = C_{Po}(T_2 - T_{line})$

Energy Eq. becomes: $\quad C_{Po}(T_2 - T_{line}) - RT_2 = 0$

$$T_2 = T_{line} C_{Po}/(C_{Po} - R) = T_{line} C_{Po}/C_{Vo} = k\, T_{line}$$

Use F.4: $C_P = 0.24 \dfrac{Btu}{lbm\ R}, \quad k = 1.4 \implies \quad T_2 = 1.4 \times (70 + 460) = \mathbf{742\ R}$

$$m_2 = P_2 V/RT_2 = \frac{15\ \text{psia} \times 5\ \text{ft}^3 \times 144\ (\text{in/ft})^2}{53.34\ (\text{ft lbf/lbm R}) \times 742 R} = \mathbf{0.2729\ lbm}$$

$${}_1 S_{2\ gen} = m_2 (s_2 - s_i) = m_2 [\, C_P \ln(T_2 / T_{line}) - R \ln(P_2 / P_{line})\,]$$
$$= 0.2729[0.24 \times \ln(1.4) - 0] = \mathbf{0.351\ Btu/R}$$

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7.222E

An empty cannister of volume 0.05 ft^3 is filled with R-134a from a line flowing saturated liquid R-134a at 40 F. The filling is done quickly so it is adiabatic. How much mass of R-134a is in the cannister? How much entropy was generated?

Solution:

C.V. Cannister filling process where: $_1Q_2 = 0$; $_1W_2 = 0$; $m_1 = 0$

Continuity Eq.4.15: $m_2 - 0 = m_{in}$;

Energy Eq.4.16: $m_2u_2 - 0 = m_{in}h_{line} + 0 + 0$ $\Rightarrow$ $u_2 = h_{line}$

Entropy Eq.7.13: $m_2s_2 - 0 = m_{in}s_{line} + 0 + {_1S_2}_{gen}$

Inlet state: Table F.10.1 $h_{line} = 88.56$ Btu/lbm, $s_{line} = 0.244$ Btu/lbm R

State 2: $P_2 = P_{line}$ and $u_2 = h_{line} = 88.56$ Btu/lbm $> u_f$

$$x_2 = \frac{u_2 - u_f}{u_{fg}} = \frac{88.56 - 88.45}{75.16} = 0.001464$$

$$v_2 = 0.01253 + x_2\, 0.9395 = 0.013905 \text{ ft}^3/\text{lbm}$$

$$m_2 = V/v_2 = 0.05/0.013905 = \textbf{3.596 lbm}$$

$$s_2 = 0.244 + x_2\, 0.1678 = 0.24425 \text{ Btu/lbm R}$$

$${_1S_2}_{gen} = m_2(s_2 - s_{line}) = 3.596\,(0.24425 - 0.244) = \textbf{0.0009 Btu/R}$$

7.223E

A can of volume 8 ft^3 is empty and filled with R-410A from a line at 200 psia, 100 F. The process is adiabatic and stops at P = 150 psia. Use Table F.9 to find the final temperature and the entropy generation.

Solution:

C.V. Cannister filling process where: $_1Q_2 = \emptyset$; $_1W_2 = \emptyset$; $m_1 = \emptyset$

Continuity Eq.4.15: $m_2 - \emptyset = m_{in}$;

Energy Eq.4.16: $m_2u_2 - \emptyset = m_{in}h_{line} + \emptyset + \emptyset \quad \Rightarrow \quad u_2 = h_{line}$

Entropy Eq.7.12: $\quad m_2s_2 - 0 = m_{in}s_{line} + {_1S_2}_{\,gen}$

1: Table F.9.2 $\quad h_{line} = 131.84$ Btu/lbm, $s_{line} = 0.2578$ Btu/lbm-R

2: $P_2 = 150$ psia ; $u_2 = h_{line} \Rightarrow \quad T_2 = \textbf{152.8 F}, \quad s_2 = 0.29084$ kJ/kg-K

$\qquad v_2 = 0.5515$ ft^3/lbm $\quad \Rightarrow \quad m_2 = V/v_2 = 14.506$ lbm

$\qquad {_1S_2}_{\,gen} = m_2s_2 - m_{in}s_{line} = m_2(s_2 - s_{line})$

$\qquad\qquad = 14.506\,(0.29084 - 0.2578) = \textbf{0.479 Btu/R}$

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7.224E
Air from a line at 1800 lbf/in.^{2}, 60 F, flows into a 20-ft^3 rigid tank that initially contained air at ambient conditions, 14.7 lbf/in.^{2}, 60 F. The process occurs rapidly and is essentially adiabatic. The valve is closed when the pressure inside reaches some value, P_2. The tank eventually cools to room temperature, at which time the pressure inside is 750 lbf/in.^{2}. What is the pressure P_2? What is the net entropy change for the overall process?

CV: Tank. Mass flows in, so this is transient. Find the mass first

$$m_1 = P_1V/RT_1 = \frac{14.7 \times 144 \times 20}{53.34 \times 520} = 1.526 \text{ lbm}$$

Fill to P_2, then cool to $T_3 = 60$ F, $P_3 = 750$ psia

$$m_3 = m_2 = P_3V/RT_3$$

$$= \frac{750 \times 144 \times 20}{53.34 \times 520} = 77.875 \text{ lbm}$$

Cont. Eq.: $m_i = m_2 - m_1 = 77.875 - 1.526 = 76.349$ lbm

The filling process from 1 to 2 ($T_1 = T_i$)

1-2 heat transfer = 0   so Energy Eq.: $m_ih_i = m_2u_2 - m_1u_1$

$$m_iC_{P0}T_i = m_2C_{V0}T_2 - m_1C_{V0}T_1$$

$$T_2 = \frac{76.349 \times 0.24 + 1.526 \times 0.171}{77.875 \times 0.171} \times 520 = 725.7 \text{ R}$$

$$P_2 = m_2RT_2/V = 77.875 \times 53.34 \times 725.7 / (144 \times 20) = \textbf{1047 lbf/in}^2$$

Consider the overall process from 1 to 3

Energy Eq.: $Q_{CV} + m_ih_i = m_2u_3 - m_1u_1 = m_2h_3 - m_1h_1 - (P_3 - P_1)V$

But, since $T_i = T_3 = T_1$, $m_ih_i = m_2h_3 - m_1h_1$

$\Rightarrow Q_{CV} = -(P_3 - P_1)V = -(750 - 14.7) \times 20 \times 144/778 = -2722$ Btu

From Eq.7.13 also Eqs.7.24-7.26

$$_1S_{2 \text{ gen}} = m_3s_3 - m_1s_1 - m_is_i - Q_{CV}/T_0 = m_3(s_3 - s_i) - m_1(s_1 - s_i) - Q_{CV}/T_0$$

$$= 77.875\left[0 - \frac{53.34}{778}\ln\left(\frac{750}{1800}\right)\right] - 1.526\left[0 - \frac{53.34}{778}\ln\left(\frac{14.7}{1800}\right)\right]$$

$$+ 2722/520 = \textbf{9.406 Btu/R}$$

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Device Efficiency

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7.225E

A steam turbine inlet is at 200 psia, 800 F. The exit is at 40 psia. What is the lowest possible exit temperature? Which efficiency does that correspond to?

We would expect the lowest possible exit temperature when the maximum amount of work is taken out. This happens in a reversible process so if we assume it is adiabatic this becomes an isentropic process.

Exit: 40 psia, $s = s_{in} = 1.7659$ Btu/lbm R $\Rightarrow$ **T = 409.6 F**

The efficiency from Eq.7.27 measures the turbine relative to an isentropic turbine, so the **efficiency** will be **100%**.

7.226E

A steam turbine inlet is at 200 psia, 800 F. The exit is at 40 psia. What is the highest possible exit temperature? Which efficiency does that correspond to?

The highest possible exit temperature would be if we did not get any work out, i.e. the turbine broke down. Now we have a throttle process with constant h assuming we do not have a significant exit velocity.

Exit: 40 psia, $h = h_{in} = 1425.31$ Btu/lbm $\Rightarrow$ **T = 786 F**

Efficiency: $\qquad \eta = \dfrac{w}{w_s} = \mathbf{0}$

Remark: Since process is irreversible there is no area under curve in T-s diagram that correspond to a q, nor is there any area in the P-v diagram corresponding to a shaft work.

7.227E

A steam turbine inlet is at 200 psia, 800 F. The exit is at 40 psia, 600 F. What is the isentropic efficiency?

from table F.7.2

Inlet: h_{in} = 1425.31 Btu/lbm, s_{in} = 1.7659 Btu/lbm R

Exit: h_{ex} = 1333.43 Btu/lbm, s_{ex} = 1.8621 Btu/lbm R

Ideal Exit: 40 psia, s = s_{in} = 1.7659 Btu/lbm R $\Rightarrow$ h_s = 1241.05 Btu/lbm

w_{ac} = h_{in} - h_{ex} = 1425.31 – 1333.43 = 91.88 Btu/lbm

w_s = h_{in} - h_s = 1425.31 – 1241.05 = 184.3 Btu/lbm

$$\eta = \frac{w_{ac}}{w_s} = \frac{91.88}{184.3} = \mathbf{0.498}$$

7.228E

The exit velocity of a nozzle is 1500 ft/s. If $\eta_{nozzle} = 0.88$ what is the ideal exit velocity?

The nozzle efficiency is given by Eq. 7.30 and since we have the actual exit velocity we get

$$\mathbf{V}^2_{e\ s} = \mathbf{V}^2_{ac}/\eta_{nozzle} \quad \Rightarrow$$
$$\mathbf{V}_{e\ s} = \mathbf{V}_{ac}/\sqrt{\eta_{nozzle}} = 1500\ (\text{ft/s}) / \sqrt{0.88} = \mathbf{1599\ ft/s}$$

7.229E

A small air turbine with an isentropic efficiency of 80% should produce 120 Btu/lbm of work. The inlet temperature is 1800 R and it exhausts to the atmosphere. Find the required inlet pressure and the exhaust temperature.

Solution:

C.V. Turbine actual energy Eq.4.13:

$$w = h_i - h_{e,ac} = 120$$

Table F.5: $h_i = 449.794$ Btu/lbm

$$\Rightarrow \quad h_{e,ac} = h_i - 120 = 329.794 \text{ Btu/lbm}, \quad \mathbf{T_e = 1349\ R}$$

C.V. Ideal turbine, Eq.7.27 and energy Eq.4.13:

$$w_s = w/\eta_s = 120/0.8 = 150 = h_i - h_{e,s} \quad \Rightarrow \quad h_{e,s} = 299.794 \text{ Btu/lbm}$$

From Table F.5: $T_{e,s} = 1232.7$ R , $s^o_{Te} = 1.84217$ Btu/lbm R

Entropy Eq.7.9: $s_i = s_{e,s}$ adiabatic and reversible

To relate the entropy to the pressure use Eq.6.19 inverted and standard entropy from Table F.5:

$$P_e/P_i = \exp[\ (s^o_{Te} - s^o_{Ti}\ )/R\ ] = \exp[(1.84217 - 1.94209)\frac{778}{53.34}] = 0.2328$$

$$P_i = P_e / 0.2328 = 14.7/0.2328 = \mathbf{63.14\ psia}$$

If constant heat capacity was used

$$T_e = T_i - w/C_p = 1800 - 120/0.24 = 1300 \text{ R}$$

$$T_{e,s} = T_i - w_s/C_p = 1800 - 150/0.24 = 1175 \text{ R}$$

Eq.7.9 (adibatic and reversible) gives constant s and relation is Eq.6.23

$$P_e/P_i = (T_e/T_i)^{k/(k-1)} \quad \Rightarrow \quad P_i = 14.7\ (1800/1175)^{3.5} = 65.4 \text{ psia}$$

7.230E

Redo Problem 7.198 if the water pump has an isentropic efficiency of 85% (hose, nozzle included).

Solution:

C.V.: pump + hose + water column, total height difference 110 ft. Here **V** is velocity, not volume.

Continuity Eq.4.3, 4.11: $\dot{m}_{in} = \dot{m}_{ex} = (\rho A\mathbf{V})_{nozzle}$

Energy Eq.4.12: $\dot{m}(-w_p) + \dot{m}(h + \mathbf{V}^2/2 + gz)_{in} = \dot{m}(h + \mathbf{V}^2/2 + gz)_{ex}$

Process: $h_{in} \cong h_{ex}$, $\mathbf{V}_{in} \cong \mathbf{V}_{ex} = 0$, $z_{ex} - z_{in} = 110$ ft, $\rho = 1/v \cong 1/v_f$

$-w_p = g(z_{ex} - z_{in}) = 32.174 \times (110 - 0)/25\ 037 = 0.141$ Btu/lbm

Recall the conversion 1 Btu/lbm $= 25\ 037$ ft^2/s^2 from Table A.1. The velocity in the exit nozzle is such that it can rise 30 ft. Make that column a C.V. for which Bernoulli Eq.7.17 is:

$$gz_{noz} + \frac{1}{2}\mathbf{V}^2_{noz} = gz_{ex} + 0$$

$$\mathbf{V}_{noz} = \sqrt{2g(z_{ex} - z_{noz})}$$

$$= \sqrt{2 \times 32.174 \times 30} = 43.94 \text{ ft/s}$$

Assume: $v = v_{F,70F} = 0.01605$ ft^3/lbm

$$\dot{m} = \frac{\pi}{v_f}\left(\frac{D}{2}\right)^2\mathbf{V}_{noz} = (\pi/4)\ (1^2/144) \times 43.94 / 0.01605 = 14.92 \text{ lbm/s}$$

$$\dot{W}_{pump} = \dot{m}w_p/\eta = 14.92 \times 0.141 \times (3600/2544)/0.85 = \textbf{3.5 hp}$$

7.231E

Air enters an insulated compressor at ambient conditions, 14.7 lbf/in.^{2}, 70 F, at the rate of 0.1 lbm/s and exits at 400 F. The isentropic efficiency of the compressor is 70%. What is the exit pressure? How much power is required to drive the compressor?

Solution:

C.V. Compressor: P_1, T_1, T_e(real), $\eta_{s\,COMP}$ known, assume constant C_{P0}

Energy Eq.4.13 for real: $-w = C_{P0}(T_e - T_i) = 0.24(400 - 70) = 79.2$ Btu/lbm

Ideal $-w_s = -w \times \eta_s = 79.2 \times 0.7 = 55.4$ Btu/lbm

Energy Eq.4.13 for ideal:

$55.4 = C_{P0}(T_{es} - T_i) = 0.24(T_{es} - 530)$, $T_{es} = 761$ R

Constant entropy for ideal as in Eq.6.23:

$P_e = P_i(T_{es}/T_i)^{\frac{k}{k-1}} = 14.7(761/530)^{3.5} = $ **52.1 lbf/in^2**

$-\dot{W}_{REAL} = \dot{m}(-w) = 0.1 \times 79.2 \times 3600/2544 = $ **11.2 hp**

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7.232E

A nozzle is required to produce a steady stream of R-134a at 790 ft/s at ambient conditions, 15 lbf/in.^{2}, 70 F. The isentropic efficiency may be assumed to be 90%. What pressure and temperature are required in the line upstream of the nozzle?

C.V. Nozzle, steady flow and no heat transfer.

Actual nozzle energy Eq.: $h_1 = h_2 + \mathbf{V}_2^2/2$

State 2 actual: Table F.10.2 $h_2 = 180.975$ Btu/lbm

$$h_1 = h_2 + \mathbf{V}_2^2/2 = 180.975 + \frac{790^2}{2 \times 25\ 037} = 193.44 \text{ Btu/lbm}$$

Recall 1 Btu/lbm = 25 037 ft^2/s^2 from Table A.1.

Ideal nozzle exit: $h_{2s} = h_1 - KE_s = 193.44 - \dfrac{790^2}{2 \times 25\ 037}\big/0.9 = 179.59$ Btu/lbm

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 from A.1

State 2s: $(P_2, h_{2s}) \Rightarrow T_{2s} = 63.16$ F, $s_{2s} = 0.4481$ Btu/lbm R

Entropy Eq. ideal nozzle: $s_1 = s_{2s}$

State 1: $(h_1, s_1 = s_{2s})$ $\Rightarrow$ Double interpolation or use software.

For 40 psia: given h_1 then s = 0.4544 Btu/lbm R,   T = 134.47 F

For 60 psia: given h_1 then s = 0.4469 Btu/lbm R,   T = 138.13 F

Now a linear interpolation to get P and T for proper s

$$P_1 = 40 + 20 \frac{0.4481 - 0.4544}{0.4469 - 0.4544} = \textbf{56.8 psia}$$

$$T_1 = 134.47 + (138.13 - 134.47)\frac{0.4481 - 0.4544}{0.4469 - 0.4544} = \textbf{137.5 F}$$

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Review Problems

7.233E

A watercooled air compressor takes air in at 70 F, 14 lbf/in.^{2} and compresses it to 80 lbf/in.^{2}. The isothermal efficiency is 88% and the actual compressor has the same heat transfer as the ideal one. Find the specific compressor work and the exit temperature.

Solution:

Ideal isothermal compressor exit 80 psia, 70 F

Reversible process: $dq = T\ ds$ $\Rightarrow$ $q = T(s_e - s_i)$

$$q = T(s_e - s_i) = T[s^o_{Te} - s^o_{T1} - R\ \ln(P_e\ /\ P_i)]$$

$$= -\ RT\ \ln\ (P_e\ /\ P_i) = -\ (460 + 70)\ \frac{53.34}{778}\ \ln \frac{80}{14} = -\ 63.3\ \text{Btu/lbm}$$

As same temperature for the ideal compressor $h_e = h_i$ $\Rightarrow$

$$w = q = \text{-63.3 Btu/lbm}   \Rightarrow   w_{ac} = w\ /\eta = \textbf{- 71.93 Btu/lbm},   q_{ac} = q$$

Now for the actual compressor energy equation becomes

$$q_{ac} + h_i = h_{e\ ac} + w_{ac} \Rightarrow$$

$$h_{e\ ac} - h_i = q_{ac} - w_{ac} = -\ 63.3 - (-71.93) = 8.63\ \text{Btu/lbm} \approx C_p\ (T_{e\ ac} - T_i)$$

$$T_{e\ ac} = T_i + 8.63/0.24 = \textbf{105.9 F}$$

7.234E

Repeat Problem 7.199 for a pump/compressor isentropic efficiency of 70%.

Solution:

C.V.: Pump/Compressor, $\dot{m}$ = 1 lbm/s, R-410A

a)   State 1: Table F.9.1, x_1 = 1.0   Saturated vapor, $P_1 = P_g$ = 76.926 psia,

$h_1 = h_g$ = 118.21 Btu/lbm, $s_1 = s_g$ = 0.2535 Btu/lbm R

Ideal Compressor is isentropic, $s_2 = s_1$ = 0.2535 Btu/lbm R

h_2 = 134.54 Btu/lbm, T_2 = 130.5 F

Energy Eq.4.13: $q_c + h_1 = h_2 + w_c$; q_c = 0

$w_{cs} = h_1 - h_2$ = 118.21 – 134.54 = - 16.33 Btu/lbm;

Now the actual compressor

$w_{c, \, AC} = w_{cs}/\eta$ = - 23.33 = $h_1 - h_{2 \, AC}$

$h_{2, \, AC}$ = 134.54 + 23.33 = 157.87 $\Rightarrow$ T_2 = **217 F**

$\Rightarrow$ $\dot{W}_{C \, in}$ = $\dot{m}(-w_C)$ = **23.3 Btu/s = 33 hp**

b)   State 1: T_1 = 10 F, x_1 = 0   Saturated liquid. This is a pump.

P_1 = 76.926 psia, $h_1 = h_f$ = 17.0 Btu/lbm, $v_1 = v_f$ = 0.01316 ft³/lbm

Energy Eq.4.13: $q_p + h_1 = h_2 + w_p$; q_p = 0

Ideal pump is isentropic and the liquid is incompressible, Eq.7.18:

$w_{ps} = - \int v \; dP = -v_1(P_2 - P_1)$ = -0.01316 (300 – 76.926) 144

= -422.73 lbf-ft/lbm = - 0.543 Btu/lbm

Now the actual pump

$w_{c, \, AC} = w_{cs}/\eta$ = - 0.776 = $h_1 - h_{2 \, AC}$

$h_2 = h_1 - w_p$ = 17.0 - (- 0.776) = 17.776 Btu/lbm,

Assume State 2 is approximately a saturated liquid $\Rightarrow$ $T_2 \cong$ **12.2 F**

$\dot{W}_{P \, in}$ = $\dot{m}(-w_P)$ = 1 (0.776) = **0.776 Btu/s = 0.99 hp**

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7.235E

Air at 1 atm, 60 F is compressed to 4 atm, after which it is expanded through a nozzle back to the atmosphere. The compressor and the nozzle both have efficiency of 90% and kinetic energy in/out of the compressor can be neglected. Find the actual compressor work and its exit temperature and find the actual nozzle exit velocity.

Steady state separate control volumes around compressor and nozzle. For ideal compressor we have inlet : 1 and exit : 2

Adiabatic : $q = 0$.

Reversible: $s_{gen} = 0$

Energy Eq.: $h_1 + 0 = w_C + h_2$;

Entropy Eq.: $s_1 + 0/T + 0 = s_2$

Ideal compressor: $w_c = h_1 - h_2$, $\quad s_2 = s_1$

The constant s from Eq. 6.23 gives

$$T_2 = T_1 (P_2/P_1)^{\frac{k-1}{k}} = (459.7 + 60)\ R \times (4/1)^{0.2857} = 772\ R$$

$$\Rightarrow \quad -w_C = h_2 - h_1 = C_P(T_2 - T_1) = 0.24\ (772 - 519.7) = 60.55\ Btu/lbm$$

Actual compressor: $w_{c,AC} = w_{c,s}/\eta_c =$ **-67.3 Btu/lbm** $= h_1 - h_3$

$$\Rightarrow \quad T_3 = T_1 - w_{c,AC}/C_P = 519.7 + 67.3/0.24 = \textbf{800 R}$$

Ideal nozzle: $\quad s_4 = s_3 \quad$ so use Eq.6.23 again

$$\Rightarrow T_4 = T_3 \times (P_4/P_3)^{\frac{k-1}{k}} = 800\ (1/4)^{0.2857} = 538.4\ R$$

$$\mathbf{V}_s^2/2 = h_3 - h_4 = C_P(T_3 - T_4) = 0.24(800 - 538.4) = 62.78\ Btu/lbm$$

$$\mathbf{V}_{AC}^2/2 = \mathbf{V}_s^2 \times \eta_{NOZ}/2 = 62.78 \times 0.9 = 56.5\ Btu/lbm$$

$$\mathbf{V}_{AC} = \sqrt{2 \times 56.5 \times 25\ 037} = \textbf{1682 ft/s}$$

Remember conversion $\quad$ 1 Btu/lbm = 25 037 ft^2/s^2 from Table A.1.

7.236E

A rigid 35 ft^3 tank contains water initially at 250 F, with 50 % liquid and 50% vapor, by volume. A pressure-relief valve on the top of the tank is set to 150 lbf/in.^{2} (the tank pressure cannot exceed 150 lbf/in.^{2} - water will be discharged instead). Heat is now transferred to the tank from a 400 F heat source until the tank contains saturated vapor at 150 lbf/in.^{2}. Calculate the heat transfer to the tank and show that this process does not violate the second law.

C.V. Tank and walls out to the source. Neglect storage in walls. There is flow out and no boundary or shaft work.

Continuity Eq.4.15: $\quad m_2 - m_1 = - m_e$

Energy Eq.4.16: $\quad m_2 u_2 - m_1 u_1 = - m_e h_e + {}_1Q_2$

Entropy Eq.7.12: $\quad m_2 s_2 - m_1 s_1 = - m_e s_e + \int dQ/T + {}_1S_{2\,gen}$

State 1: 250 F, Table F.7.1 $\qquad v_{f1} = 0.017, \quad v_{g1} = 13.8247$ ft^3/lbm

$\quad m_{LIQ} = V_{LIQ} / v_{f1} = 0.5 \times 35/0.017 = 1029.4$ lbm

$\quad m_{VAP} = V_{VAP} / v_{g1} = 0.5 \times 35/13.8247 = 1.266$ lbm

$\quad m = 1030.67$ lbm

$\quad x = m_{VAP} / (m_{LIQ} + m_{VAP}) = 0.001228$

$\quad u = u_{f1} + x\ u_{fg1} = 218.48 + 0.001228 \times 869.41 = 219.55$

$\quad s = s_{f1} + x\ s_{fg1} = 0.3677 + 0.001228 \times 1.3324 = 0.36934$

state 2: $v_2 = v_g = 3.2214$ ft^3/lbm, $\quad u_2 = 1110.31, \quad h_2 = 1193.77$ Btu/lbm

$\qquad s_2 = 1.576$ Btu/lbm-R $\qquad m_2 = V/v_2 = 10.865$ lbm

From the energy equation we get

$\quad {}_1Q_2 = m_2 u_2 - m_1 u_1 + m_e h_e$

$\qquad = 10.865 \times 1110.31 - 1030.67 \times 219.55 + 1019.8 \times 1193.77$

$\qquad = 1\ 003\ 187$ Btu

$\quad {}_1S_{2\,gen} = m_2 s_2 - m_1 s_1 - m_e s_e - {}_1Q_2 / T_{source}$

$\qquad = 10.865 \times 1.576 - 1030.67 \times 0.36934 + 1019.8 \times 1.57 - 1003187/860$

$\qquad = \textbf{77.2 Btu/R}$

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 8

8e

Updated June 2013

Borgnakke and Sonntag

Fundamentals of Thermodynamics 8th Edition
Borgnakke and Sonntag

CONTENT CHAPTER 8

In-Text Concept Questions

8.a

Can I have any energy transfer as heat transfer that is 100% available?

By definition the possible amount of work that can be obtained equals the exergy. The maximum is limited to that out of a reversible heat engine, if constant T then that is the Carnot heat engine

$$W = (1 - \frac{T_o}{T})Q$$

So we get a maximum for an infinite high temperature T, where we approach an efficiency of one. In practice you do not have such a source (the closest would be solar radiation) and secondly no material could contain matter at very high T so a cycle process can proceed (the closest would be a plasma suspended by a magnetic field as in a tokamak).

8.b

Is electrical work 100% available?

Yes. By definition work is 100% exergy.

8.c

A nozzle does not have any actual work involved, how should you then interpret the reversible work?

The purpose of the nozzle is not to make work, but the reversible work can be used to generate kinetic energy in addition to what comes out of the actual nozzle. The reversible work plus the actual kinetic energy would equal the possible total kinetic energy out of the reversible nozzle. The other point is that the reversible nozzle has a heat transfer in (if the actual nozzle is irreversible) also unusual for a nozzle.

8.d

If an actual control volume process is reversible what can you say about the work term?

The work term is the maximum possible. If the work is positive it is the maximum possible work out and if the work is negative it is the minimum work possible that must be supplied.

8.e

Can entropy change in a control volume process that is reversible?

Yes it can. A flow in or out at a state with a different entropy than the average entropy inside or a heat transfer will change the entropy. All the terms that changes the entropy are transfer terms so there is no net increase (generation) of entropy.

8.f

Energy can be stored as internal energy, potential energy or kinetic energy. Are those energy forms all 100% available?

The internal energy is only partly available, a process like an expansion can give out work or if it cools by heat transfer out it is a Q out that is only partly available as work. Potential energy like from gravitation, mgH, or a compressed spring or a charged battery are forms that are close to 100% available with only small losses present. Kinetic energy like in a fly-wheel or motion of a mass can be transferred to work out with losses depending on the mechanical system.

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8.g

We cannot create nor destroy energy, can we create or destroy exergy?

Yes. Every process that is irreversible to some degree destroys exergy. This destruction is directly proportional to the entropy generation. We cannot create exergy at most it can stay constant, which is the case for a reversible process.

8.h

In a turbine what is the source of exergy?

The flow into the turbine is the source, it provides the conditions that allows the turbine to give work out. If the turbine exit flow has useful exergy we can write the net source as the inlet flow exergy minus the exit flow exergy

8.i

In a pump what is the source of exergy?

The shaft work is the input that drives the pump, which in turn pushes on the flow to generate a higher pressure exit flow. The increase in the flow exergy is the desired output, that is expressing the increase in P in terms of exergy.

8.j

In a pump what gains exergy?

The higher pressure exit flow is the desired output. When we express that in terms of exergy it becomes the increase in the flow exergy that is the output, that is expressing the increase in P in terms of exergy.

8.k

In a heat engine what is the source of exergy?

Generally it is the high temperature heat transfer. However, if the rejected heat transfer at the low temperature has any useful exergy we can also count the difference as the source.

8.l

In a heat pump what is the source of exergy?

The work input is what drives the heat pump.

8.m

In Eq.8.39 for the heat engine the source of exergy was written as a heat transfer. How does the expression look like if the source is a flow of hot gas being cooled down as it gives energy to the heat engine?

Look at a heat exchanger that provides such a set-up in Fig. 8.3

C.V. Heat exchanger plus H.E.

Energy Eq.6.12:

$$0 = \dot{m}_1 h_1 - \dot{m}_1 h_2 - \dot{W} - \dot{Q}_L$$

Entropy Eq.7.2:

$$0 = \dot{m}_1 s_1 - \dot{m}_1 s_2 - \dot{Q}_L/T_o + \dot{S}_{gen}$$

Solve for $\dot{Q}_L$ and sustitute into energy Eq.

$$\dot{Q}_L = T_o \dot{m}_1 (s_1 - s_2) + T_o \dot{S}_{gen}$$

$$\dot{W} = \dot{m}_1 [h_1 - h_2 - T_o (s_1 - s_2)] - T_o \dot{S}_{gen}$$

$$= \dot{m}_1 (\psi_1 - \psi_2) - T_o \dot{S}_{gen}$$

$$= \eta_{HE\ II}\ \dot{m}_1 (\psi_1 - \psi_2) = \eta_{HE\ II} \dot{W}^{rev}$$

So the H.E. work is a fraction of the flow exergy.

Concept-Study Guide Problems

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8.1

Why does the reversible C.V. counterpart to the actual C.V. have the same storage and flow terms?

If you want to compare two devices they should be comparable in as many respects as possible to be useful. Specifically when we want to find the possible work output all the aspects that can generate work are important and must be considered. The flows in and out and the storage effect are closely tied to the possible work output and thus should be the same for the two control volumes. Obviously one control volume that receives a high exergy flow or depletes its stored exergy can give out more work than a control volume that does not have such effects.

8.2

Can one of the heat transfers in Eq.8.5-6 be to or from the ambient?

Yes it can, then the reversible heat transfer we find from the ambient to balance the entropy is in addition to the one already listed. In many practical devices there is an actual heat transfer loss to the surroundings because of elevated temperatures inside the device.

8.3

All the energy in the ocean is that available?

No. Since the ocean is at the ambient T (it **is** the ambient) it is not possible to extract any work from it. You can extract wave energy (wind generated kinetic energy) or run turbines from the tide flow of water (moon generated kinetic energy). However, since the ocean temperature is not uniform there are a few locations where cold and warmer water flows close to each other like at different depths. In that case a heat engine can operate due to the temperature difference.

8.4

Does a reversible process change the exergy if there is no work involved?

Yes. There can be heat transfer involved and that has an exergy associated with it, which then equals the change of exergy of the substance. A flow can enter and exit a device with different levels of exergy. What is unchanged is the total (CV plus surroundings) exergy that remains constant, but it may be redistributed.

8.5

Is the reversible work between two states the same as ideal work for the device?

No. It depends on the definition of ideal work. The ideal device does not necessarily have the same exit state as the actual device. An ideal turbine is approximated as a reversible adiabatic device so the ideal work is the isentropic work. The reversible work is between the inlet state and the actual exit state that do not necessarily have the same entropy.

8.6

When is the reversible work the same as the isentropic work?

That happens when the inlet and exit states (or beginning and end states) have the same entropy. For a reversible adiabatic process in a C.V (which then is isentropic) the two work terms become the same.

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8.7

If I heat some cold liquid water to T_o, do I increase its exergy?

No. You decrease its exergy by bringing it closer to T_o, where it has zero exergy, if we neglect pressure effects. Any substance at a T different from ambient (higher or lower) has a positive exergy since you can run a heat engine using the two temperatures as the hot and cold reservoir, respectively. For a T lower than the ambient it means that the ambient is the hot side of the heat engine.

8.8

Are reversible work and exergy connected?

Yes. They are very similar. Reversible work is usually defined as the reversible work that can be obtained between two states, inlet-exit or beginning to end. Exergy is a property of a given state and defined as the reversible work that can be obtained by changing the state of the substance from the given state to the dead state (ambient).

The only difference is the work term exchanged with the ambient if the control volume changes its volume, $W = P_o \, \Delta V$.

8.9

Consider exergy associated with a flow. The total exergy is based on the thermodynamic state, the kinetic and potential energies. Can they all be negative?

No. By virtue of its definition kinetic energy can only be positive. The potential energy is measured from a reference elevation (standard sea level or a local elevation) so it can be negative. The thermodynamic state can only have a positive exergy the smallest it can be is zero if it is the ambient dead state.

8.10

Verify that Eq.8.29 reduces to Eq.8.14 for a steady state process

The expression for the reversible work is

$$\dot{W}^{rev} = \dot{\Phi}_q + \sum \dot{m}_i \, \psi_i - \sum \dot{m}_e \, \psi_e - \dot{\Phi}_{CV} + P_o \, \dot{V} \qquad (10.29)$$

Definition of steady state is no storage effect so:

$$\dot{V} = 0; \qquad \dot{\Phi}_{CV} = 0$$

the last two terms in Eq.8.29 drop out. Eq.8.14 is also written for a single flow so the summations are each a single term.

$$\sum \dot{m}_i \, \psi_i - \sum \dot{m}_e \, \psi_e = \dot{m} \, \psi_i - \dot{m} \, \psi_e = \dot{m} \, (\psi_i - \psi_e)$$

The heat transfer term is from Eq.8.21 and with single flow

$$\dot{\Phi}_q = \sum (1 - \frac{T_o}{T_j}) \, \dot{Q}_j = \dot{m} \sum (1 - \frac{T_o}{T_j}) \, q_j \qquad (10.21)$$

Now we have

$$\dot{W}^{rev} = \dot{m} \sum (1 - \frac{T_o}{T_j}) \, q_j + \dot{m} \, (\psi_i - \psi_e)$$

divide by the mass flow rate to get

$$w^{rev} = \sum (1 - \frac{T_o}{T_j}) \, q_j + \psi_i - \psi_e$$

The last difference in exergies are from Eq.8.23

$$\psi_i - \psi_e = (h_{tot\,i} - T_o s_i) - (h_{tot\,e} - T_o s_e)$$

which when substituted gives Eq.8.14.

8.11

What is the second law efficiency of a Carnot Heat engine?

The Carnot Heat engine is by definition reversible and thus it delivers the maximum amount of work possible. It has a second law eficiency of 100%.

8.12

What is the second law efficiency of a reversible heat engine?

Since the reversible heat engine has no entropy generation it produces the maximum work possible and the actual work is the reversible work so it has a second law efficiency of 100%.

8.13

For a nozzle what is the output and input (source) expressed in exergies?

For the nozzle a high pressure low velocity inlet flow generates a higher velocity in the exit flow at the expense of the pressure. The desired output is the higher velocity expressed as kinetic energy, which is part of exergy. The source is the inlet high pressure expressed in flow exergy.

8.14

Is the exergy equation independent of the energy and entropy equations?

No. The exergy equation is derived from the other balance equations by defining the exergy from the state properties and the reference dead state.

8.15

Use the exergy balance equation to find the efficiency of a steady state Carnot heat engine operating between two fixed temperature reservoirs?

The exergy balance equation, Eq.8.38, for this case looks like

$$0 = \left(1 - \frac{T_o}{T_H}\right)\dot{Q}_H - \left(1 - \frac{T_o}{T_L}\right)\dot{Q}_L - \dot{W} + 0 + 0 - 0 - 0$$

Steady state (LHS = 0) and dV/dt = 0, no mass flow terms, Carnot cycle so reversible and the destruction is then zero. From the energy equation we have

$$0 = \dot{Q}_H - \dot{Q}_L - \dot{W}$$

which we can subtract from the exergy balance equation to get

$$0 = -\frac{T_o}{T_H}\dot{Q}_H + \frac{T_o}{T_L}\dot{Q}_L$$

Solve for one heat transfer in terms of the other

$$\dot{Q}_L = \frac{T_L}{T_H}\dot{Q}_H$$

The work from the energy equation is

$$\dot{W} = \dot{Q}_H - \dot{Q}_L = \dot{Q}_H\left[\,1 - \frac{T_L}{T_H}\,\right]$$

from which we can read the Carnot cycle efficiency as we found in Chapter 7.

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Exergy, Reversible work

8.16

A control mass gives out 10 kJ of energy in the form of
a.   Electrical work from a battery
b.   Mechanical work from a spring
c.   Heat transfer at 500°C
Find the change in exergy of the control mass for each of the three cases.

Solution:

a) Work is exergy $\Delta\Phi = -W_{el} =$ **-10 kJ**

b) Work is exergy $\Delta\Phi = -W_{spring} =$ **-10 kJ**

c) Give the heat transfer to a Carnot heat engine and W is exergy

$$\Delta\Phi = -\left[1 - \frac{T_0}{T_H}\right] Q_{out} = -\left(1 - \frac{298.15}{773.15}\right) 10 = \textbf{-6.14 kJ}$$

8.17

A fraction of some power to a motor (1), 2 kW, is turned into heat transfer at 500 K (2) and then it dissipates in the ambient at 300 K (3). Give the rates of exergy along the process 1-2-3.

Solution:

The exergy of an amount of heat transfer equals the possible work that can be extracted. This is the work out of a Carnot heat engine with heat transfer to the ambient as the other reservoir. The result is from Chapter 5 as also shown in Eq. 8.1 and Eq. 8.38

1: $\quad \dot{\Phi} = \dot{W} = \mathbf{2\ kW}$

2: $\quad \dot{\Phi} = \dot{W}_{rev\ HE} = (1 - \dfrac{T_O}{T})\dot{Q} = (1 - \dfrac{300}{500})\ 2\ kW = \mathbf{0.8\ kW}$

3: $\quad \dot{\Phi} = \dot{W}_{rev\ HE} = (1 - \dfrac{T_O}{T_O})\dot{Q} = (1 - \dfrac{300}{300})\ 2\ kW = \mathbf{0\ kW}$

As the energy transforms from wotrk to heat transfer it loses exergy and as the heat transfer goes to lower and lower T its exergy value drops ending as zero at the ambient T.

8.18

A refrigerator should remove 1.5 kW from the cold space at -10$^{\text{o}}$C while it rejects heat to the kitchen at 25$^{\text{o}}$C. Find the reversible work.

The reversible work is related to the Carnot cycle work as the the two reservoirs are at constant temperatures.

$$\dot{W} = \dot{Q}_L / \beta \; ; \qquad \beta = \dot{Q}_L / \dot{W}_{carnot} = \frac{T_L}{T_H - T_L} = \frac{263.15}{25 - (-10)} = 7.52$$

In general we have defined the reversible work with the standard sign definition

$$\dot{W}^{rev} = - \dot{Q}_L / \beta = - \frac{1.5}{7.52} \text{ kW} = \textbf{–0.2 kW}$$

8.19

A heat engine receives 5 kW at 800 K and 10 kW at 1000 K rejecting energy by heat transfer at 600 K. Assume it is reversible and find the power output. How much power could be produced if it could reject energy at $T_o = 298$ K?

Solution:

C.V. The heat engine, this is in steady state.

Energy Eq.: $\quad 0 = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_L - \dot{W}$

Entropy Eq.: $\quad 0 = \dfrac{\dot{Q}_1}{T_1} + \dfrac{\dot{Q}_2}{T_2} - \dfrac{\dot{Q}_L}{T_L} + 0$

Now solve for $\dot{Q}_L$ from the entropy equation

$$\dot{Q}_L = \frac{T_L}{T_1}\dot{Q}_1 + \frac{T_L}{T_2}\dot{Q}_2 = \frac{600}{800} \times 5 + \frac{600}{1000} \times 10 = 9.75 \text{ kW}$$

Substitute into the energy equation and solve for the work term

$$\dot{W} = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_L = 5 + 10 - 9.75 = \textbf{5.25 kW}$$

For a low temperature of 298 K we can get

$$\dot{Q}_{L2} = \frac{298}{600}\dot{Q}_L = 4.843 \text{ kW}$$

$$\dot{W} = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_L = 5 + 10 - 4.843 = \textbf{10.16 kW}$$

Remark: Notice the large increase in the power output.

8.20

A household refrigerator has a freezer at T_F and a cold space at T_C from which energy is removed and rejected to the ambient at T_A as shown in Fig. P8.20.

Assume that the rate of heat transfer from the cold space, $\dot{Q}_C$, is the same as from the freezer, $\dot{Q}_F$, find an expression for the minimum power into the heat pump.

Evaluate this power when $T_A = 20°C$, $T_C = 5°C$, $T_F = -10°C$, and $\dot{Q}_F = 3$ kW.

Solution:

C.V. Refrigerator (heat pump), Steady, no external flows except heat transfer.

Energy Eq.: $\dot{Q}_F + \dot{Q}_c + \dot{W}_{in} = \dot{Q}_A$

(amount rejected to ambient)

Reversible gives maximum work (minimum work in) as from Eq. 8.1 or 8.11 on rate form (standard sign notation for work).

$$\dot{W}^{rev} = \dot{Q}_F\left[1 - \frac{T_A}{T_F}\right] + \dot{Q}_c\left[1 - \frac{T_A}{T_C}\right] - \dot{Q}_A\left[1 - \frac{T_A}{T_A}\right]$$

$$= 3\left[1 - \frac{293.15}{263.15}\right] + 3\left[1 - \frac{293.15}{278.15}\right] - 0$$

$$= -\textbf{0.504 kW} \quad \text{(negative so work goes in)}$$

8.21

An air compressor takes air in at the state of the surroundings 100 kPa, 300 K. The air exits at 400 kPa, 200°C using 100 kW of power. Determine the minimum compressor work input.

C.V. Compressor, Steady flow, minimum work in is reversible work.

Energy Eq.: $0 = h_1 - h_2 - w_c$ =>

$$- w_c = h_2 - h_1 = 475.8 - 300.47 = 175.33 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}/- w_c = 100 \text{ kW}/175.33 = 0.57 \text{ kg/s}$$

$\psi_1 = 0$ at ambient conditions

Get the properties from the air table A.7.1 and correct standard entropy for the pressure

$$s_0 - s_2 = s_{T_0}^{\circ} - s_{T_2}^{\circ} - R \ln(P_0/P_2)$$

$$= 6.86926 - 7.3303 - 0.287 \ln(100/400) = -0.06317 \text{ kJ/kg K}$$

$$\psi_2 = h_2 - h_0 + T_0(s_0 - s_2) = 475.79 - 300.473 + 300 (-0.06317)$$

$$= 156.365 \text{ kJ/kg}$$

$$\mathbf{-\dot{W}^{REV}} = \dot{m}(\psi_2 - \psi_1) = 0.57 \text{ kg/s } (156.365 - 0) \text{ kJ/kg} = \mathbf{89.1 \text{ kW}} = \dot{W}_{c \text{ min}}$$

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8.22

The compressor in a refrigerator takes refrigerant R-134a in at 100 kPa, $-20°C$ and compresses it to 1 MPa, $60°C$. With the room at $20°C$ find the minimum compressor work.

Solution:

C.V. Compressor out to ambient. Minimum work in is the reversible work.

Steady flow, 1 inlet and 2 exit

Energy Eq.: $\quad w_c = h_1 - h_2 + q^{rev}$

Entropy Eq.: $\quad s_2 = s_1 + \int dq/T + s_{gen} = s_1 + q^{rev}/T_o + 0$

$$\Rightarrow \quad q^{rev} = T_o(s_2 - s_1)$$

$$w_{c\ min} = h_1 - h_2 + T_o(s_2 - s_1)$$

$$= 387.22 - 441.89 + 293.15 \times (1.7818 - 1.7665)$$

$$= -54.67 + 4.4852$$

$$= -\mathbf{50.18\ kJ/kg}$$

8.23

Calculate the reversible work out of the two-stage turbine shown in Problem 4.86, assuming the ambient is at 25°C. Compare this to the actual work which was found to be 18.08 MW.

C.V. Turbine. Steady flow, 1 inlet and 2 exits.

Use Eq. 8.11 for steady flow with $q = 0$ for adiabatic turbine.

Supply state 1:   20 kg/s at 10 MPa, 500°C

Process steam 2:   5 kg/s, 0.5 MPa, 155°C,

Exit state 3:   20 kPa,   x = 0.9

Table B.1.3: $h_1 = 3373.7$, $h_2 = 2755.9$ kJ/kg,

$s_1 = 6.5966$, $s_2 = 6.8382$ kJ/kg K

Table B.1.2: $h_3 = 251.4 + 0.9 \times 2358.3 = 2373.9$ kJ/kg,

$s_3 = 0.8319 + 0.9 \times 7.0766 = 7.2009$ kJ/kg K

$$\dot{W}^{rev} = (\dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3) - T_0(\dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3)$$

$$= 20 \times 3373.7 - 5 \times 2755.9 - 15 \times 2373.9$$

$$- 298.15 (20 \times 6.5966 - 5 \times 6.8382 - 15 \times 7.2009)$$

$$= \mathbf{21.14\ MW} = \dot{W}^{ac} + \dot{Q}^{rev} = 18084\ kW + 3062.7\ kW$$

8.24

Find the specific reversible work for a steam turbine with inlet 4 MPa, 500°C and an actual exit state of 100 kPa, x = 1.0 with a 25°C ambient.

Solution:

Steam turbine $T_o = 25°C = 298.15$ K

Inlet state: Table B.1.3 $h_i = 3445.2$ kJ/kg; $s_i = 7.090$ kJ/kg K

Exit state: Table B.1.2 $h_e = 2675.5$ kJ/kg; $s_e = 7.3593$ kJ/kg K

From Eq.8.14 asuming it to be adiabatic,

$$w^{rev} = (h_i - T_o s_i) - (h_e - T_o s_e) = (h_i - h_e) + T_o(s_e - s_i)$$
$$= (3445.2 - 2675.5) + 298.2(7.3593 - 7.0900)$$
$$= 769.7 + 80.3 = \textbf{850.0 kJ/kg}$$

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8.25

A steam turbine receives steam at 6 MPa, 800°C. It has a heat loss of 49.7 kJ/kg and an isentropic efficiency of 90%. For an exit pressure of 15 kPa and surroundings at 20°C, find the actual work and the reversible work between the inlet and the exit.

C.V. Reversible adiabatic turbine (isentropic)

$$w_T = h_i - h_{e,s} \; ; \qquad s_{e,s} = s_i = 7.6566 \text{ kJ/kg K}, \quad h_i = 4132.7 \text{ kJ/kg}$$

$$x_{e,s} = (7.6566 - 0.7548)/7.2536 = 0.9515,$$

$$h_{e,s} = 225.91 + 0.9515 \times 2373.14 = 2483.9 \text{ kJ/kg}$$

$$w_{T,s} = 4132.7 - 2483.9 = 1648.79 \text{ kJ/kg}$$

C.V. Actual turbine

$$w_{T,ac} = \eta w_{T,s} = \mathbf{1483.91 \text{ kJ/kg}}$$

$$= h_i - h_{e,ac} - q_{loss} \implies$$

$$h_{e,ac} = h_i - q_{loss} - w_{T,ac} = 4132.7 - 49.7 - 1483.91 = 2599.1 \text{ kJ/kg}$$

Actual exit state: P, h $\implies$ saturated vapor, $s_{e,ac} = 8.0085 \text{ kJ/kg K}$

C.V. Reversible process, work from Eq.8.14

$$q^R = T_0(s_{e,ac} - s_i) = 293.15 \times (8.0085 - 7.6566) = 103.16 \frac{\text{kJ}}{\text{kg}}$$

$$w^R = h_i - h_{e,ac} + q^R = 4132.7 - 2599.1 + 103.16 = \mathbf{1636.8 \text{ kJ/kg}}$$

8.26

A compressor in a refrigerator receives R-410A at 150 kPa, -40°C and it brings it up to 600 kPa, using an actual specific work of 58.65 kJ/kg in an adiabatic compression. Find the specific reversible work.

Energy Eq.: $0 = h_1 - h_2 - w$

State 1: B.4.2 $h_1 = 263.99$ kJ/kg, $s_1 = 1.1489$ kJ/kg K

$\qquad\qquad h_2 = h_1 - w = 263.99 + 58.65 = 322.64$ kJ/kg

State 2: B.4.2   P, h_2 => $s_2 = 1.2152$ kJ/kg K

$$w^{rev} = \psi_1 - \psi_2 = h_1 - h_2 - T_0 (s_1 - s_2)$$
$$= -58.65 - 298.15 (1.1489 - 1.2152)$$
$$= -58.65 + 19.767 = \mathbf{-38.9 \ kJ/kg}$$

8.27

Air flows through a constant pressure heating device, shown in Fig. P8.27. It is heated up in a reversible process with a work input of 200 kJ/kg air flowing. The device exchanges heat with the ambient at 300 K. The air enters at 300 K, 400 kPa. Assuming constant specific heat develop an expression for the exit temperature and solve for it by iterations.

C.V. Total out to T_0

Energy Eq.: $\quad h_1 + q_0^{rev} - w^{rev} = h_2$

Entropy Eq.: $\quad s_1 + q_0^{rev}/T_0 = s_2 \quad \Rightarrow \quad q_0^{rev} = T_0(s_2 - s_1)$

$\quad\quad h_2 - h_1 = T_0(s_2 - s_1) - w^{rev} \quad\quad$ (same as from Eq. 8.14)

Constant C_p gives: $\quad C_p(T_2 - T_1) = T_0 C_p \ln(T_2/T_1) + 200$

The energy equation becomes

$$\mathbf{T_2 - T_0 \ln(\frac{T_2}{T_1}) = T_1 + \frac{200}{C_p}}$$

$T_1 = 300$ K, $\quad C_p = 1.004$ kJ/kg K, $\quad T_0 = 300$ K

$$T_2 - 300 \ln\left(\frac{T_2}{300}\right) = 300 + \frac{200}{1.004} = 499.3 \text{ K}$$

Now trial and error on T_2

$\quad\quad$ At 600 K $\quad$ LHS = 392 (too low)

$\quad\quad$ At 800 K $\quad\quad$ LHS = 505.75

Linear interpolation gives $\quad \mathbf{T_2 = 790 \text{ K}} \quad$ (LHS = 499.5 **OK**)

8.28

An adiabatic and reversible air compressor takes air in at 100 kPa, 310 K. The air exits at 600 kPa at the rate of 0.4 kg/s. Determine the minimum compressor work input and repeat for an inlet at 295 K instead. Why is the work less for a lower inlet *T*?

Compressor. Reversible adiabatic, constant s:

Inlet at 310 K: $T_{2S} = T_1 \left(\dfrac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 310 \left(\dfrac{600}{100}\right)^{0.2857} = 517.2 \text{ K}$

The reversible process ensures the minimum work input.

$$\dot{W} = \dot{m}(-w_S) = \dot{m}C_{P0}(T_{2S} - T_1)$$
$$= 0.4 \text{ kg/s} \times 1.004 \text{ kJ/kg-K} \times (517.2 - 310) \text{ K}$$
$$= \textbf{83.2 kW}$$

Inlet at 295 K: $T_{2S} = T_1 \left(\dfrac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 295 \left(\dfrac{600}{100}\right)^{0.2857} = 492.2 \text{ K}$

$$\dot{W} = \dot{m}(-w_S) = \dot{m}C_{P0}(T_{2S} - T_1) = 0.4 \times 1.004 \,(492.2 - 295)$$
$$= 0.4 \text{ kg/s} \times 1.004 \text{ kJ/kg-K} \times (492.2 - 295) \text{ K}$$
$$= \textbf{79.2 kW}$$

The work term is reduced due to the reduced specific volume, v_1, recall Eq. 7.15 for the indicated shaft work.

8.29

An air flow of 5 kg/min at 1500 K, 125 kPa goes through a constant pressure heat exchanger, giving energy to a heat engine shown in Figure P8.29. The air exits at 500 K and the ambient is at 298 K, 100 kPa. Find the rate of heat transfer delivered to the engine and the power the engine can produce.

Solution:

C.V. Heat exchanger

Continuity eq.: $\dot{m}_1 = \dot{m}_2$;

Energy Eq.4.12: $\dot{m}_1 h_1 = \dot{m}_1 h_2 + \dot{Q}_H$

Table A.7.1: $h_1 = 1635.8$ kJ/kg,

$h_2 = 503.36$ kJ/kg, $s_1 = 8.61209$ kJ/kg K

$s_2 = 7.38692$ kJ/kg K

$$\dot{Q}_H = \dot{m}(h_1 - h_2) = \frac{5}{60}\,\frac{kg}{s}\,(1635.8 - 503.36)\,\frac{kJ}{kg} = \mathbf{94.37\ kW}$$

Notice T_H is not constant so we do not know the heat engine efficiency.

C.V. Total system for which we will write the second law.

Entropy Equation 7.8: $\dot{m}\,s_1 + \dot{S}_{gen} = \dot{m}\,s_2 + \dot{Q}_L/T_o$

Process: Assume reversible $\dot{S}_{gen} = 0$, and P = C for air

$$\dot{Q}_L = T_o\,\dot{m}\,(s_1 - s_2) = 298\ K\,\frac{5}{60}\,\frac{kg}{s}\,(8.61209 - 7.38692)\,\frac{kJ}{kg\ K}$$

$$= 30.425\ kW$$

Energy equation for the heat engine gives the work as

$$\dot{W} = \dot{Q}_H - \dot{Q}_L = 94.37 - 30.425 = \mathbf{63.9\ kW}$$

Borgnakke and Sonntag

8.30

Water at 800°C, 15 MPa is flowing through a heat exchanger giving off energy to come out as saturated liquid water at 15 MPa in a steady flow process. Find the specific heat transfer and the specific flow-exergy the water has delivered.

Let us fix the states first from Table B.1.2 and B.1.3:

State 1 (in):   h = 4092.43 kJ/kg,   s = 7.2040 kJ/kg K

State 2 (ex):   h = 1610.45 kJ/kg,   s = 3.6847,   T = 342.24°C = 615.4 K

CV Heat exchanger, no work.

Energy Eq.: $q_{out} = h_{in} - h_{ex} =$ 4092.43 - 1610.45 = **2482 kJ/kg**

Flow exergy: $\Delta\psi = \psi_{in} - \psi_{ex} = (h - T_o s)_{in} - (h - T_o s)_{ex}$

$$= q_{out} - T_o(s_{in} - s_{ex})$$

$$= 2482 - 298.15(7.2040 - 3.6847)$$

$$= \textbf{1432.7 kJ/kg}$$

Borgnakke and Sonntag

8.31

A rock bed consists of 6000 kg granite and is at 70°C. A small house with lumped mass of 12000 kg wood and 1000 kg iron is at 15°C. They are now brought to a uniform final temperature with no external heat transfer by connecting the house and rock bed through some heat engines. If the process is reversible, find the final temperature and the work done in the process.

Solution:

Take C.V. Total (rockbed and heat engine)

Energy Eq.: $\quad m_{rock}(u_2 - u_1) + m_{wood}(u_2 - u_1) + m_{Fe}(u_2 - u_1) = -_1W_2$

Entropy Eq.: $\quad m_{rock}(s_2 - s_1) + m_{wood}(s_2 - s_1) + m_{Fe}(s_2 - s_1) = \emptyset$

$$(mC)_{rock}\ln \frac{T_2}{T_1} + (mC)_{wood}\ln \frac{T_2}{T_1} + (mC)_{Fe}\ln \frac{T_2}{T_1} = \emptyset$$

$6000 \times 0.89 \ln (T_2/343.15) + 12000 \times 1.26 \ln (T_2/288.15)$

$$+ 1000 \times 0.46 \ln (T_2/288.15) = \emptyset$$

$\Rightarrow \quad \textbf{T}_2 = \textbf{301.3 K}$

Now from the energy equation

$-_1W_2 = 6000 \text{ kg} \times 0.89 \text{ kJ/kgK} \times (301.3 - 343.15) \text{ K}$

$$+ (12000 \times 1.26 + 1000 \times 0.46) \text{ kJ/K} \times (301.3 - 288.15) \text{ K}$$

$\Rightarrow {}_1W_2 = \textbf{18 602 kJ}$

8.32

A constant pressure piston/cylinder has 1 kg of saturated liquid water at 100 kPa. A rigid tank contains air at 1200 K, 1000 kPa. They are now thermally connected by a reversible heat engine cooling the air tank and boiling the water to saturated vapor. Find the required amount of air and the work out of the heat engine.

CV. Tank, piston/cyl. and the heat engine.

The minimum amount of air is when T_2 of air equals water T = 99.62 C.

State: 1 = initial air
2 = final air
3 = initial water
4 = final water

For this CV only W_{HE} and W_{pist} cross the control surface no heat transfer. The entropy equation Eq.6.3 and Eq.6.37 for a reversible process becomes

$$(S_2 - S_1)_{tot} = m_{air}(s_2 - s_1) + m_{H2O}(s_4 - s_3) = 0 + 0$$

$$m_{H2O}(s_4 - s_3) = m_{H2O}\, s_{fg} = 1 \text{ kg} \times 6.0568 \text{ kJ/kg-K} = 6.0568 \text{ kJ/K}$$

$$s_2 - s_1 = C_v \ln \frac{T_2}{T_1} + R \ln\frac{v_2}{v_1} = C_v \ln \frac{T_2}{T_1} = 0.717 \ln \frac{372.77}{1200}$$

$$= -0.838256 \text{ kJ/kg-K}$$

Now solve for the air mass from entropy equation

$$m_{air} = -m_{H2O}(s_4 - s_3) / (s_2 - s_1) = -\frac{6.0568}{-0.838256} = \textbf{7.2255 kg}$$

Let us find the two heat transfers

Energy Eq, water: $m_{H2O}(u_4 - u_3) = Q_{H2O} - W_{H2O};$ $W_{H2O} = Pm(v_4 - v_3)$

$$Q_{H2O} = m_{H2O}(u_4 - u_3) + W_{H2O} = m_{H2O}(h_4 - h_3) = m_{H2O}\, h_{fg} = 2258.02 \text{ kJ}$$

Energy Eq. air: $m_{air}(u_2 - u_1) = -Q_{air} - 0 = m_{air} C_v (T_2 - T_1)$

$$Q_{air} = \textbf{7}.2255 \text{ kg } 0.717 (1200 - 372.77) = 4285.6 \text{ kJ}$$

Now the work out of the heat engine is

$$W_{HE} = Q_{air} - Q_{H2O} = 4285.6 - 2258.02 = \textbf{2027.6 kJ}$$

8.33

A piston/cylinder has forces on the piston so it keeps constant pressure. It contains 2 kg of ammonia at 1 MPa, 40°C and is now heated to 100°C by a reversible heat engine that receives heat from a 200°C source. Find the work out of the heat engine.

C.V. Ammonia plus heat engine

Energy: $m_{am}(u_2 - u_1) = {_1}Q_{2,200} - W_{H.E.} - {_1}W_{2,pist}$

Entropy: $m_{am}(s_2 - s_1) = {_1}Q_2/T_{res} + 0$

$\Rightarrow {_1}Q_2 = m_{am}(s_2 - s_1)T_{res}$

Process: $P = $ const. $\Rightarrow {_1}W_2 = P(v_2 - v_1)m_{am}$

Substitute the piston work term and heat transfer into the energy equation

$W_{H.E.} = m_{am}(s_2 - s_1)T_{res} - m_{am}(h_2 - h_1)$

Table B.2.2: $h_1 = 1508.5$ kJ/kg, $s_1 = 5.1778$ kJ/kg K,

$h_2 = 1664.3$ kJ/kg, $s_2 = 5.6342$ kJ/kg K

$W_{H.E.} = 2 \times \left[(5.6342 - 5.1778)473.15 - (1664.3 - 1508.5)\right] = \mathbf{120.3\ kJ}$

8.34

A basement is flooded with 16 m^3 of water at 15°C. It is pumped out with a small pump driven by a 0.75 kW electric motor. The hose can reach 8 m vertically up and to ensure the water can flow over the edge of a dike it should have a velocity of 20 m/s at that point generated by a nozzle, see Fig. P8.34. Find the maximum flow rate you can get and how fast the basement can be emptied.

C.V. Pump plus hose and nozzle, single steady state flow. For maximum flow rate assume reversible process so from Eq.8.14:

$$w^{rev} = T_o \, (s_2 - s_1) - (h_{2 \, tot} - h_{1 \, tot}) + 0$$

Since we have no heat transfer and reversible s is constant and with a liquid flow T is also constant so $h_2 = h_1$. We could also have used Bernoulli equation.

$$w^{rev} = - \left(\frac{1}{2}V_2^2 + gH - 0\right) = - \left(\frac{1}{2} 20^2 + 9.81 \times 8\right) = -278.5 \text{ J/kg}$$

$$\dot{m} = \dot{W}/ (-w^{rev}) = \frac{0.75 \text{ kW}}{0.2785 \text{ kJ/kg}} = \textbf{2.693 kg/s}$$

$$m = V/v = 16 \text{ m}^3 / 0.001 \text{ m}^3/\text{kg} = 16 \, 000 \text{ kg}$$

$$\Delta t = m / \dot{m} = \frac{16 \, 000 \text{ kg}}{2.693 \text{ kg/s}} = 5941 \text{ sec} = \textbf{99 min.}$$

Irreversibility

Borgnakke and Sonntag

8.35

A 20°C room is heated with a 2000 W electric baseboard heater. What is the rate of irreversibility?

Let us not consider any storage in the room or any heat loss but just the effect of the heating process. Electric power comes in to the heater and leaves as heat transfer to the room air.

C.V. heater

Entropy Eq.: $\quad 0 = -\dfrac{\dot{Q}}{T_{air}} + \dot{S}_{gen} \quad \Rightarrow \quad \dot{S}_{gen} = \dfrac{\dot{Q}}{T_{air}}$

Irreversibility: $\quad \dot{I} = T_o\, \dot{S}_{gen} = 293.15 \text{ K} \times \dfrac{2000 \text{ W}}{293.15 \text{ K}} = \mathbf{2000\ W}$

8.36

A refrigerator removes 1.5 kW from the cold space at -10$^\text{o}$C using 750 W of power input while it rejects heat to the kitchen at 25$^\text{o}$C. Find the rate of irreversibility.

C.V. Refrigerator

Energy Eq.: $\quad\quad\quad 0 = \dot{Q}_L + \dot{W} - \dot{Q}_H \quad\quad \Rightarrow \quad \dot{Q}_H = \dot{Q}_L + \dot{W}$

Entropy Eq.: $\quad\quad\quad 0 = \dfrac{\dot{Q}_L}{T_L} - \dfrac{\dot{Q}_H}{T_H} + \dot{S}_{gen} \quad \Rightarrow \quad\quad \dot{S}_{gen} = \dfrac{\dot{Q}_H}{T_H} - \dfrac{\dot{Q}_L}{T_L}$

$$\dot{Q}_H = \dot{Q}_L + \dot{W} = 1.5 \text{ kW} + 0.75 \text{ kW} = 2.25 \text{ kW}$$

$$\dot{S}_{gen} = \frac{2250 \text{ W}}{298.15 \text{ K}} - \frac{1500 \text{ W}}{263.15 \text{ K}} = 1.8464 \text{ W/K}$$

Irreversibility: $\quad\quad\quad \dot{I} = T_o\, \dot{S}_{gen} = 298.15 \text{ K} \times 1.8464 \text{ W/K} = \textbf{550 W}$

8.37

Calculate the irreversibility for the condenser in Problem 7.100 assuming an ambient temperature at 17°C.

Solution:

C.V. Condenser. Steady state with no shaft work term.

Energy Equation 4.12: $\dot{m}\,h_i + \dot{Q} = \dot{m}h_e$

Entropy Equation 7.8: $\dot{m}\,s_i + \dot{Q}/T + \dot{S}_{gen} = \dot{m}\,s_e$

Properties are from Table B.1.2

$h_i = 225.91 + 0.9 \times 2373.14 = 2361.74$ kJ/kg , $\qquad h_e = 225.91$ kJ/kg

$s_i = 0.7548 + 0.9 \times 7.2536 = 7.283$ kJ/kg K, $\qquad s_e = 0.7548$ kJ/kg K

From the energy equation

$$\dot{Q}_{out} = -\dot{Q} = \dot{m}\,(h_i - h_e) = 5(2361.74 - 225.91) = 10679 \text{ kW}$$

From the entropy equation

$$\dot{S}_{gen} = \dot{m}\,(s_e - s_i) + \dot{Q}_{out}/T = 5(0.7548 - 7.283) + 10679/(273.15 + 17)$$
$$= -32.641 + 36.805 = 4.164 \text{ kW/K}$$

From Eq.8.13,

$$\dot{I} = T_o\,\dot{S}_{gen} = 290.15 \times 4.164 = \mathbf{1208 \text{ kW}}$$

Often the cooling media flows inside a long pipe carrying the energy away.

Borgnakke and Sonntag

8.38

A throttle process is an irreversible process. Assume an air flow at 1000 kPa, 400 K runs through a valve out to ambient 100 kPa. Find the reversible work and irreversibility assuming an ambient temperature at 25°C.

Solution:

C.V. Throttle. Steady state, adiabatic $q = 0$ and no shaft work $w = 0$.

Energy Eq.4.13: $h_e = h_i$ so temperature is constant

$$s_e - s_i = s_{Te}^o - s_{Ti}^o - R \ln(P_e/P_i) = -0.287 \ln \frac{1000}{100} = -0.6608 \text{ kJ/kg K}$$

The reversible work is the difference in exergy expressed in Eq.6.11, 8.14 and 8.29 per unit mass flow

$$w^{rev} = \psi_i - \psi_e = (h_i - T_o s_i) - (h_e - T_o s_e) = (h_i - h_e) + T_o(s_e - s_i)$$

$$= 0 + 298.15 \, (-0.6608) = \textbf{197 kJ/kg}$$

$$i = w^{rev} - w = 197 - 0 = \textbf{197 kJ/kg}$$

Process is in the ideal gas region

8.39

A compressor in a refrigerator receives R-410A at 150 kPa, -40°C and it brings it up to 600 kPa, 40°C in an adiabatic compression. Find the specific work, reversible work, entropy generation and irreversibility.

States: 1: B.4.2 h_1 = 263.99 kJ/kg, s_1 = 1.1489 kJ/kg K

          2: B.4.2 h_2 = 322.64 kJ/kg, s_2 = 1.2152 kJ/kg K

C.V. Compressor, steady state, q = 0, no KE, PE energies.

Energy Eq.: $w = h_1 - h_2 = 263.99 - 322.64 =$ **–58.65 kJ/kg**

Entropy Eq.: $0 = s_1 - s_2 + s_{gen}$ ⇨

$$s_{gen} = s_2 - s_1 = 1.2152 - 1.1489 = \textbf{0.0663 kJ/kg-K}$$

$$w^{rev} = \psi_1 - \psi_2 = h_1 - h_2 - T_0 (s_1 - s_2)$$
$$= -58.65 - 298.15 (1.1489 - 1.2152)$$
$$= -58.65 + 19.767 = \textbf{–38.9 kJ/kg}$$

Irreversibility: $i = T_o s_{gen} = 298.15 \text{ K} \times 0.0663 \text{ kJ/kg-K} =$ **19.77 kJ/kg**

8.40

A constant pressure piston/cylinder contains 2 kg of water at 5 MPa and 100°C. Heat is added from a reservoir at 600°C to the water until it reaches 600°C. We want to find the total irreversibility in the process.

Solution:
C.V. Piston cylinder out to the reservoir (incl. the walls).

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m(s_2 - s_1) = {}_1Q_2/T_{res} + {}_1S_{2\ gen}$

State 1: $h_1 = 422.71$ kJ/kg, $s_1 = 1.303$ kJ/kg K

State 2: $h_2 = 3666.47$ kJ/kg, $s_2 = 7.2588$ kJ/kg K

Process: $P = C$ => ${}_1W_2 = P(V_2 - V_1)$

From the energy equation we get

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) = 2(3666.47 - 422.71) = 6487.5 \text{ kJ}$$

From the entropy equation we get

$${}_1S_{2\ gen} = m(s_2 - s_1) - \frac{{}_1Q_2}{T_{res}} = 2(7.2588 - 1.303) - \frac{6487.5}{273 + 600} = 4.4816 \frac{\text{kJ}}{\text{K}}$$

Now the irreversibility is from Eq. 8.18

$${}_1I_2 = m\ {}_1i_2 = T_o\ {}_1S_{2\ gen} = 298.15 \text{ K} \times 4.4816 \frac{\text{kJ}}{\text{K}} = \mathbf{1336\ kJ}$$

8.41

A constant "flow" of steel parts 2 kg/s at 20°C goes into a furnace where they are heat treated to 900°C by a source at an average 1250 K. Find the reversible work and the irreversibility in this process?

Energy Eq.: $0 = \dot{m}(h_i - h_e) + \dot{Q}$

Entropy Eq.: $0 = \dot{m}(s_i - s_e) + \dot{Q} / T_s + \dot{S}_{gen}$

$\dot{Q} = \dot{m}(h_e - h_i) = \dot{m}C_P (T_e - T_i) = 2 \text{ kg/s} \times 0.46 \text{ kJ/kg-K} \times 880 \text{ K}$
$= 809.6 \text{ kW}$

$\dot{S}_{gen} = \dot{m}(s_e - s_i) - \dot{Q} / T_s = \dot{m}C_P \ln(T_e/T_i) - \dot{Q} / T_s$
$= 2 \text{ kg/s} \times 0.46 \text{ kJ/kg-K} \times \ln(1173 / 293) - 809.6 \text{ kW} / 1250 \text{ K}$
$= 0.6285 \text{ kW/K}$

$\dot{I} = T_0\dot{S}_{gen} = 293 \text{ K} \times 0.6285 \text{ kW/K} = \textbf{184.15 kW}$

The reversible work is equal to the irreversibility plus the actual work, Eq.8.12

$\dot{W}^{rev} = \dot{I} + \dot{W}_{ac} = \dot{I} = \textbf{184.15 kW}$

8.42

Calculate the reversible work and irreversibility for the process described in Problem 3.146, assuming that the heat transfer is with the surroundings at 20°C.

Solution:

Linear spring gives

$$_1W_2 = \int PdV = \tfrac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

$$_1Q_2 = m(u_2 - u_1) + {_1W_2}$$

Equation of state: $PV = mRT$

State 1: $V_1 = mRT_1/P_1 = 2 \times 0.1889 \times 673.15 /500 = 0.5087 \text{ m}^3$

State 2: $V_2 = mRT_2/P_2 = 2 \times 0.1889 \times 313.15 /300 = 0.3944 \text{ m}^3$

$$_1W_2 = \tfrac{1}{2}(500 + 300)(0.3944 - 0.5087) = -45.72 \text{ kJ}$$

From Figure 3.26: $C_p(T_{avg}) = 5.25 R = 0.99 \Rightarrow C_v = 0.803 = C_p - R$

For comparison the value from Table A.5 at 300 K is $C_v = 0.653$ kJ/kg K

$$_1Q_2 = mC_v(T_2 - T_1) + {_1W_2} = 2 \times 0.803(40 - 400) - 45.72 = -623.9 \text{ kJ}$$

$$_1W_2^{rev} = T_0(S_2 - S_1) - (U_2 - U_1) + {_1Q_2} (1 - T_0/T_H)$$

$$= T_0m(s_2 - s_1) + {_1W_2^{ac}} - {_1Q_2}\, T_0/T_o$$

$$= T_0m[C_P \ln(T_2 / T_1) - R \ln(P_2 / P_1)] + {_1W_2^{ac}} - {_1Q_2}$$

$$= 293.15 \times 2 \,[\, 0.99 \ln(313/673) - 0.1889 \ln(300/500)] - 45.72 + 623.9$$

$$= -387.8 - 45.72 + 623.9 = \textbf{190.4 kJ}$$

$$_1I_2 = {_1W_2^{rev}} - {_1W_2^{ac}} = 190.4 - (-45.72) = \textbf{236.1 kJ}$$

8.43

An air compressor receives atmospheric air at $T_0 = 17°C$, 100 kPa, and compresses it up to 1400 kPa. The compressor has an isentropic efficiency of 88% and it loses energy by heat transfer to the atmosphere as 10% of the isentropic work. Find the actual exit temperature and the reversible work.

C.V. Compressor

Isentropic: $w_{c,in,s} = h_{e,s} - h_i$; $\quad s_{e,s} = s_i$

From table A.7.1 and entropy equation we get

$$s^o_{Te\,s} = s^o_{Ti} + R \ln (P_e/P_i) = 6.83521 + 0.287 \ln(14) = 7.59262 \text{ kJ/kgK}$$

Back interpolate in Table A.7: $\quad \Rightarrow h_{e,s} = 617.23$ kJ/kg

$$w_{c,in,s} = 617.23 - 290.43 = 326.8 \text{ kJ/kg}$$

Actual: $w_{c,in,ac} = w_{c,in,s}/\eta_c = 371.36$ kJ/kg; $\quad q_{loss} = 32.68$ kJ/kg

$$w_{c,in,ac} + h_i = h_{e,ac} + q_{loss}$$

$$\Rightarrow h_{e,ac} = 290.43 + 371.36 - 32.68 = 629.1 \text{ kJ/kg}$$

$$\Rightarrow T_{e,ac} = \textbf{621 K}$$

Reversible: $\quad w^{rev} = h_i - h_{e,ac} + T_0(s_{e,ac} - s_i)$

$$= 290.43 - 629.1 + 290.15 \times (7.6120 - 6.8357)$$

$$= -338.67 + 225.38 = \textbf{-113.3 kJ/kg}$$

Since q_{loss} is also to the atmosphere it is the net q exchanged with the ambient that explains the change in s.

8.44

Two flows of air both at 200 kPa of equal flow rates mix in an insulated mixing chamber. One flow is 1 kg/s at 1500 K and the other is 2 kg/s at 300 K. Find the irreversibility in the process per kilogram of air flowing out.

C.V. Mixing chamber

Continuity Eq..: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq.: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

Entropy Eq.: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} = \dot{m}_3 s_3$

Properties from Table A.7

$h_3 = (h_1 + 2h_2)/3 = (1635.8 + 2 \times 300.47)/3 = 745.58$ kJ/kg

$\Rightarrow s^{\circ}_{T3} = 7.7848$ kJ/kg K (linear interpolation)

From the entropy equation (same P so no correction for that)

$\dot{S}_{gen}/\dot{m}_3 = s_3 - (s_1 + 2s_2)/3 = 7.7848 - (8.61208 + 2 \times 6.86926)/3$

$= 0.3346$ kJ/kg K

$i = \dot{I}/\dot{m}_3 = T_0 \dot{S}_{gen}/\dot{m}_3 = 298.15 \times 0.3346 = $ **99.76 kJ/kg**

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8.45

Fresh water can be produced from saltwater by evaporation and subsequent condensation. An example is shown in Fig. P8.45, where 150-kg/s saltwater, state 1, comes from the condenser in a large power plant. The water is throttled to the saturated pressure in the flash evaporator and the vapor, state 2, is then condensed by cooling with sea water. As the evaporation takes place below atmospheric pressure, pumps must bring the liquid water flows back up to P_0. Assume that the saltwater has the same properties as pure water, the ambient is at 20°C and that there are no external heat transfers. With the states as shown in the table below find the irreversibility in the throttling valve and in the condenser.

State	1	2	5	7	8
T [°C]	30	25	23	17	20
h [kJ/kg]	125.77	2547.2	96.5	71.37	83.96
s [kJ/kg K]	0.4369	8.558	0.3392	0.2535	0.2966

C.V. Valve. $P_2 = P_{sat}(T_2 = T_3) = 3.169$ kPa

Continuity Eq.: $\dot{m}_1 = \dot{m}_{ex} = \dot{m}_2 + \dot{m}_3$

Energy Eq.: $h_1 = h_e$;   Entropy Eq.: $s_1 + s_{gen} = s_e$

$\quad h_e = h_1 \implies x_e = (125.77 - 104.87)/2442.3 = 0.008558$

$\quad\quad\quad \implies s_e = 0.3673 + 0.008558 \times 8.1905 = 0.4374$ kJ/kg K

$$\dot{m}_2 = (1 - x_e)\dot{m}_1 = 148.716 \text{ kg/s}$$

$$s_{gen} = s_e - s_1 = 0.4374 - 0.4369 = 0.000494 \text{ kJ/kg K}$$

$$\dot{I} = \dot{m}T_0 s_{gen} = 150 \times 293.15 \times 0.000494 = \textbf{21.72 kW}$$

C.V. Condenser.

Energy Eq.: $\dot{m}_2 h_2 + \dot{m}_7 h_7 = \dot{m}_2 h_5 + \dot{m}_7 h_8 \implies$

$$\dot{m}_7 = \dot{m}_2 \times (h_2 - h_5)/(h_8 - h_7) = 148.716 \times \frac{2547.2 - 96.5}{83.96 - 71.37} = 28\,948 \frac{kg}{s}$$

Entropy Eq.: $\dot{m}_2 s_2 + \dot{m}_7 s_7 + \dot{S}_{gen} = \dot{m}_2 s_5 + \dot{m}_7 s_8$

$\dot{I} = T_0 \dot{S}_{gen} = T_0 \left[\dot{m}_2(s_5 - s_2) + \dot{m}_7(s_8 - s_7) \right]$

$\quad = 293.15[148.716(0.3392 - 8.558) + 28948(0.2966 - 0.2535)]$

$\quad = 293.15 \times 25.392 = \textbf{7444 kW}$

8.46

A rock bed consists of 6000 kg granite and is at 70°C. A small house with lumped mass of 12000 kg wood and 1000 kg iron is at 15°C. They are now brought to a uniform final temperature by circulating water between the rock bed and the house. Find the final temperature and the irreversibility of the process, assuming an ambient at 15°C.

C.V. Total Rockbed and house. No work, no Q irreversible process.

Energy Eq.: $(mC)_{rock}(T_2 - 70) + (mC_{wood} + mC_{Fe})(T_2 - 15) = \emptyset$

$T_2 = \mathbf{29.0°C = 302.2\ K}$

Entropy Eq.: $S_2 - S_1 = \sum m_i(s_2 - s_1)_i = 0 + S_{gen}$

$S_{gen} = \sum m_i(s_2 - s_1)_i = 5340\ \ln \dfrac{302.2}{343.15} + 15580\ \ln \dfrac{302.2}{288.15} = 63.13\ kJ/K$

$_1I_2 = (T_0)_1 S_{2,gen} = 288.15 \times 63.13 = \mathbf{18\ 191\ kJ}$

Borgnakke and Sonntag

8.47

A computer CPU chip consists of 50 g silicon, 20 g copper, and 50g polyvinyl chloride (plastic). It now heats from ambient 25°C to 70°C in an adiabatic process as the computer is turned on. Find the amount of irreversibility.

C.V. CPU chip. The process has electrical work input and no heat transfer.

Entropy Eq.: $S_2 - S_1 = \sum m_i(s_2 - s_1)_i = \int dQ/T + {}_1S_{2\ gen} = {}_1S_{2\ gen}$

Irreversibility: ${}_1I_2 = T_0\ {}_1S_{2\ gen}$

For the solid masses we will use the specific heats, Table A.3, and they all have the same temperature so

$$\sum m_i(s_2 - s_1)_i = \sum m_i C_i \ln(T_2 / T_1)_i = \ln(T_2/T_1) \sum m_i C_i$$

$$\sum m_i C_i = 0.05 \times 0.7 + 0.02 \times 0.42 + 0.05 \times 0.96 = 0.0914 \text{ kJ/K}$$

$${}_1S_{2\ gen} = S_2 - S_1 = 0.0914 \times \ln(343.15 / 288.15) = 0.016 \text{ kJ/K}$$

$${}_1I_2 = T_0\ {}_1S_{2\ gen} = 298.15 \text{ K} \times 0.016 \text{ kJ/K} = \textbf{4.77 kJ}$$

8.48

A car air-conditioning unit has a 0.5-kg aluminum storage cylinder that is sealed with a valve and it contains 2 L of refrigerant R-134a at 500 kPa and both are at room temperature 20°C. It is now installed in a car sitting outside where the whole system cools down to ambient temperature at −10°C. What is the irreversibility of this process?

C.V. Aluminum and R-134a

Energy Eq.: $\quad m_{Al}(u_2 - u_1)_{Al} + m_R(u_2 - u_1)_R = {_1}Q_2 - {_1}W_2 \quad ({_1}W_2 = 0)$

Entropy Eq.: $\quad m_{AL}(s_2 - s_1)_{Al} + m_R(s_2 - s_1)_R = {_1}Q_2/T_0 + {_1}S_{2\text{ gen}}$

Table A.3 for specific heat $\quad C_{v,Al} = C_{p,Al} = 0.9$ kJ/kg K

$\quad (u_2 - u_1)_{Al} = C_{v,Al}(T_2 - T_1) = 0.9(-10 - 20) = -27$ kJ/kg

$\quad (s_2 - s_1)_{Al} = C_{p,Al}\ln(T_2/T_1) = 0.9\ln(263.15/293.15) = -0.09716$ kJ/kg K

Table B.5.2: $\quad v_1 = 0.04226$ m^3/kg, $\quad u_1 = 390.5$ kJ/kg,

$\qquad\qquad s_1 = 1.7342$ kJ/kg K, $\qquad\qquad m_{R134a} = V/v_1 = 0.0473$ kg

$\quad v_2 = v_1 = 0.04226$ & $T_2 \quad => \quad x_2 = (0.04226 - 0.000755)/0.09845 = 0.4216$

$\quad u_2 = 186.57 + 0.4216 \times 185.7 = 264.9$ kJ/kg,

$\quad s_2 = 0.9507 + 0.4216 \times 0.7812 = 1.2801$ kJ/kg K

$\quad {_1}Q_2 = 0.5 \times (-27) + 0.0473(264.9 - 390.5) = -19.44$ kJ

$_1S_{2\text{ gen}} = 0.5\,(-0.09716) + 0.0473(1.2801 - 1.7342) + \dfrac{19.44}{263.15} = 0.003815$ kJ/K

$\quad {_1}I_2 = T_0\,({_1}S_{2\text{ gen}}) = 263.15 \times 0.003815 = \textbf{1.0 kJ}$

Borgnakke and Sonntag

8.49

R-134a is filled into an insulated 0.2 m^3 initially empty container from a line at 500 kPa, saturated vapor until the flow stops by itself. Find the final mass and temperature in the container and the total irreversibility in the process.

Solution:

C.V. Cannister filling process where: $_1Q_2 = \emptyset$; $_1W_2 = \emptyset$; $m_1 = \emptyset$

Continuity Eq.4.20: $m_2 - \emptyset = m_{in}$;

Energy Eq.4.21: $m_2u_2 - \emptyset = m_{in}h_{line} + \emptyset + \emptyset \implies u_2 = h_{line}$

Entropy Eq.7.13: $m_2s_2 - \emptyset = m_{in}s_{line} + \emptyset + {}_1S_2{}_{gen}$

Inlet state: Table B.5.1 $h_{line} = 407.45$ kJ/kg, $s_{line} = 1.7198$ kJ/kg K

State 2: $P_2 = P_{line}$ and $u_2 = h_{line} = 407.45$ kJ/kg $> u_g \implies T_2 = $ **40°C**

$v_2 = 0.04656$ m^3/kg, $s_2 = 1.7971$ kJ/kg K

$m_2 = V / v_2 = 0.2/0.04656 = $ **4.296 kg**

$_1S_2{}_{gen} = m_2(s_2 - s_{line}) = 4.296 (1.7971 - 1.7198) = $ **0.332 kJ/K**

Borgnakke and Sonntag

8.50

The water cooler in Problem 5.24 operates steady state. Find the rate of exergy destruction (irreversibility).

Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) = \dot{m}\, C_P\, (T_1 - T_2)$$
$$= 0.4 \text{ kg/s} \times 4.18 \text{ kJ/kg-K} \times (35 - 15) \text{ K}$$
$$= 33.44 \text{ kW}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 33.44 / 3 = 11.15 \text{ kW}$$

Energy equation for the refrigerator

$$\dot{Q}_H = \dot{Q}_L + \dot{W} = 33.44 \text{ kW} + 11.15 \text{ kW} = 44.59 \text{ kW}$$

C.V. Total

Entropy Eq.: $\quad 0 = \dot{m}(s_1 - s_2) - \dot{Q}_H/T_H + \dot{S}_{gen}$

$$\dot{S}_{gen} = \dot{Q}_H/T_H - \dot{m}(s_1 - s_2) = \dot{Q}_H/T_H - \dot{m}C_p \ln(T_1/T_2)$$
$$= \frac{44.59 \text{ kW}}{298.15 \text{ K}} - 0.4 \text{ kg/s} \times 4.18 \text{ kJ/kg-K} \times \ln\frac{308.15}{288.15}$$
$$= 0.0857 \text{ kW/K}$$

$$\dot{I} = T_0\, \dot{S}_{gen} = 298.15 \text{ K} \times 0.0857 \text{ kW/K} = \mathbf{25.55 \text{ kW}}$$

8.51

Air enters the turbocharger compressor (see Fig. P8.51), of an automotive engine at 100 kPa, 30°C, and exits at 200 kPa. The air is cooled by 50°C in an intercooler before entering the engine. The isentropic efficiency of the compressor is 75%. Determine the temperature of the air entering the engine and the irreversibility of the compression-cooling process.

Solution:

a) Compressor. First ideal which is reversible adiabatic, constant s:

$$T_{2S} = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 303.15 \left(\frac{200}{100}\right)^{0.2857} = 369.5 \text{ K}$$

$$w_S = C_{P0}(T_1 - T_{2S}) = 1.004(303.15 - 369.5) = -66.565 \text{ kJ/kg}$$

Now the actual compressor

$$w = w_S/\eta_S = -66.565/0.75 = -88.75 \text{ kJ/kg} = C_P(T_1 - T_2)$$

$$\Rightarrow T_2 = 391.55 \text{ K}$$

$$T_3(\text{to engine}) = T_2 - \Delta T_{\text{INTERCOOLER}} = 391.55 - 50$$

$$= 341.55 \text{ K} = \textbf{68.4°C}$$

b) Irreversibility from Eq.8.15 with rev. work from Eq.8.14, (q = 0 at T_H)

$$s_3 - s_1 = 1.004 \ln\left(\frac{341.55}{303.15}\right) - 0.287 \ln\left(\frac{200}{100}\right) = -0.0792 \frac{\text{kJ}}{\text{kg K}}$$

$$i = T(s_3 - s_1) - (h_3 - h_1) - w \qquad [\ w^{\text{rev}} = (h_1 - Ts_1) - (h_3 - Ts_3)\ ]$$

$$= T(s_3 - s_1) - C_P(T_3 - T_1) - C_P(T_1 - T_2)$$

$$= 303.15(-0.0792) - 1.004(-50) = \textbf{+26.2 kJ/kg}$$

8.52

A constant pressure piston/cylinder has 1 kg of saturated liquid water at 100 kPa. A rigid tank contains air at 1000 K, 1000 kPa. They are now thermally connected by conduction through the walls cooling the air tank and bringing the water to saturated vapor. Find the required amount of air and the irreversibility of the process assuming no external heat transfer.

State: 1 = initial air

2 = final air

3 = initial water

4 = final water we assume $T_4 = T_2$

Take CV as each of the masses. This gives Q from the two energy equations.

$$Q = m_{H2O}(u_4 - u_3) + W_{H2O} = m_{H2O}(h_4 - h_3) = m_{H2O}\, h_{fg} = 2258.02 \text{ kJ}$$

$$Q = -m_{air}(u_2 - u_1) = -m_{air}\, C_v\, (T_2 - T_1)$$

$$m_{air} = m_{H2O}\, h_{fg} / C_v\, (T_1 - T_2) = \frac{2258.02}{0.717\,(1000 - 372.77)} = \textbf{5.0209 kg}$$

The entropy equation Eq.8.37 for an irreversible process becomes

$$(S_2 - S_1)_{tot} = m_{air}(s_2 - s_1) + m_{H2O}(s_4 - s_3) = 0 + {}_1S_{2\ gen}$$

$$m_{H2O}(s_4 - s_3) = m_{H2O}\, s_{fg} = 1 \text{ kg} \times 6.0568 \text{ kJ/kg-K} = 6.0568 \text{ kJ/K}$$

$$s_2 - s_1 = C_v \ln\frac{T_2}{T_1} + R \ln\frac{v_2}{v_1} = C_v \ln\frac{T_2}{T_1} = 0.717 \ln\frac{372.77}{1000} = -0.70753 \frac{\text{kJ}}{\text{kg-K}}$$

Now solve for the entropy generation

$${}_1S_{2\ gen} = m_{air}(s_2 - s_1) + m_{H2O}(s_4 - s_3)$$

$$= 5.0209\,(-0.70753) + 6.0568 = 2.5044 \text{ kJ/K}$$

$${}_1I_2 = T_0\,({}_1S_{2\ gen}) = 298.15 \times 2.5044 = \textbf{746.6 kJ}$$

Exergy

Exergy

8.53

The automatic transmission in a car receives 25 kW shaft work and gives out 23 kW to the drive shaft. The balance is dissipated in the hydraulic fluid and metal casing, all at 45°C, which in turn transmits it to the outer atmosphere at 20°C. Find all the exergy transfer rates.

Solution:

C.V. Total unit. Steady state and surface at 45°C

Exergy transfer:

2 kW

23 kW

25 kW

$$\dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right)\dot{Q}_H; \quad \dot{\Phi}_W = \dot{W};$$

The two shaft works are 100% exergy

$$\dot{\Phi}_W = \dot{W} = 25 \text{ kW} \quad \text{coming in and} \quad \dot{\Phi}_W = \dot{W} = 23 \text{ kW} \quad \text{leaving}$$

The remaingning balance of energy leaves as heat transfer from the 45°C surface

$$\dot{\Phi}_{\text{out with Q}} = \left(1 - \frac{T_o}{T_{\text{Surface}}}\right)\dot{Q}_{\text{out}} = (1 - \frac{293.15}{273.15 + 45}) \, 2 \text{ kW} = \textbf{0.157 kW}$$

Here we used 293.15 for the reference T as it was the ambient. Once the heat transfer is redistributed at the ambient T the exergy transfer is zero.

8.54

A heat engine receives 1 kW heat transfer at 1000 K and gives out 400 W as work with the rest as heat transfer to the ambient. What are the fluxes of exergy in and out?

Exergy flux in: $\dot{\Phi}_H = \left(1 - \dfrac{T_o}{T_H}\right)\dot{Q}_H = \left(1 - \dfrac{298.15}{1000}\right) 1\ kW = \mathbf{0.702\ kW}$

Exergy flux out: $\dot{\Phi}_L = \left(1 - \dfrac{T_o}{T_L}\right)\dot{Q}_L = \mathbf{0}$ $(\,T_L = T_o\,)$

The other exergy flux out is the power $\dot{\Phi}_{out} = \dot{W} = \mathbf{0.4\ kW}$

8.55

In a refrigerator 1 kW is removed from the -10°C cold space and 1.3 kW is moved into the 30°C warm space. Find the exergy fluxes including direction associated with the two heat transfers.

$$\dot{\Phi}_H = (1 - \frac{T_o}{T_H})\, \dot{Q}_H = (1 - \frac{298.15}{303.15})\, 1.3 \text{ kW} = \textbf{0.021 kW}$$

$$\dot{\Phi}_L = (1 - \frac{T_o}{T_L})\, \dot{Q}_L = (1 - \frac{298.15}{263.15})\, 1 \text{ kW} = \textbf{--0.133 kW}$$

I.e. the flux goes into the cold space!   Why?   As you cool it $T < T_o$ and you increase its exergy, it is further away from the ambient.

The refrigerator gives 0.133 kW of exergy to the cold space and 0.021 kW to the hot space in terms of exergy driven by a work

input of $\dot{W} = (1.3 - 1) = 0.3$ kW that is a second law efficiency of about 50%.

8.56

A steady stream of R-410A at ambient temperature, 20°C and 800 kPa enters a solar collector and it exits at 80°C, 600 kPa. Calculate the change in exergy of the R410a.

Solution:

Inlet (T,P) Table B.4.2: $h_i = 300.02$ kJ/kg, $s_i = 1.1105$ kJ/kg K

Exit (T,P) Table B.4.2: $h_e = 359.67$ kJ/kg, $s_e = 1.3265$ kJ/kg K

From Eq.8.23 or flow terms in Eq.8.38

$$\Delta\psi_{ie} = \psi_e - \psi_i = (h_e - h_i) - T_0(s_e - s_i) = (359.67 - 300.02)$$

$$- 293.15(1.3265 - 1.1105) = \textbf{-3.67 kJ/kg}$$

Remark: it is negative due to the pressure loss.

8.57

A heat pump has a coefficient of performance of 2 using a power input of 3 kW. Its low temperature is T_o and the high temperature is 80°C, with an ambient at T_o. Find the fluxes of exergy associated with the energy fluxes in and out.

First let us do the energies in and out

$$COP = \beta = \frac{\dot{Q}_H}{\dot{W}} \quad => \quad \dot{Q}_H = \beta \dot{W} = 2 \times 3 \text{ kW} = 6 \text{ kW}$$

Energy Eq.: $\quad \dot{Q}_L = \dot{Q}_H - \dot{W} = 6 - 3 = 3 \text{ kW}$

Exergy flux in: $\quad \dot{\Phi}_L = \left(1 - \frac{T_o}{T_L}\right) \dot{Q}_L = \mathbf{0} \quad\quad (T_L = T_o)$

Exergy flux in: $\quad \dot{\Phi}_W = \dot{W} = \mathbf{3 \text{ kW}}$

Exergy flux out: $\quad \dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right) \dot{Q}_H = \left(1 - \frac{298.15}{353.15}\right) 6 \text{ kW} = \mathbf{0.935 \text{ kW}}$

Remark: The process then destroys (3 – 0.935) kW of exergy.

Borgnakke and Sonntag

8.58

A flow of air at 1000 kPa, 300 K is throttled to 500 kPa. What is the irreversibility? What is the drop in flow exergy?

A throttle process is constant enthalpy if we neglect kinetic energies.

Process: $h_e = h_i$ so ideal gas $=>$ $T_e = T_i$

Entropy Eq.: $s_e - s_i = s_{gen} = s_{Te}^o - s_{Ti}^o - R \ln \dfrac{P_e}{P_i} = 0 - R \ln \dfrac{P_e}{P_i}$

$s_{gen} = -0.287 \ln (500 / 1000) = 0.2$ kJ/kg K

Eq.8.15: $i = T_o\, s_{gen} = $ 298 0.2 = **59.6 kJ/kg**

The drop in exergy is exergy destruction, which is the irreversibility

$$\Delta\psi = i = \textbf{59.6 kJ/kg}$$

8.59

A power plant has an overall thermal efficiency of 40% receiving 100 MW of heat transfer from hot gases at an average of 1300 K and rejects heat transfer at 50^oC from the condenser to a river at ambient temperature 20^oC. Find the rate of both energy and exergy a) from the hot gases and b) from the condenser.

C.V. High T heat exchanger.

Energy flux in: $\dot{Q}_H = 100 \text{ MW}$

Exergy flux in: $\dot{\Phi}_H = \left(1 - \dfrac{T_o}{T_H}\right)\dot{Q}_H = \left(1 - \dfrac{293.15}{1300}\right) 100 \text{ MW} = \mathbf{77.45 \ MW}$

C.V. condenser.

Overall power plant: $\dot{W} = \eta\dot{Q}_H$ so from energy equation

Energy flux out $\dot{Q}_L = \dot{Q}_H - \dot{W} = (1 - \eta)\,\dot{Q}_H = 0.6 \times 100 \text{ MW} = \mathbf{60 \ MW}$

Exergy flux out: $\dot{\Phi}_L = \left(1 - \dfrac{T_o}{T_L}\right)\dot{Q}_L = \left(1 - \dfrac{293.15}{323.15}\right) 60 \text{ MW} = \mathbf{5.57 \ MW}$

Borgnakke and Sonntag

8.60

Find the change in exergy from inlet to exit of the condenser in Problem 7.48.

Solution:

Condenser of Prob. 7.48 has inlet equal to turbine exit.

State 2: $P_2 = 20$ kPa; $s_2 = s_1 = 6.7993$ kJ/kg K

$$\Rightarrow \; x_2 = (6.7993 - 0.8319)/7.0766 = 0.8433$$

$$h_2 = 2240.1 \text{ kJ/kg}$$

State 3: $P_2 = P_3; T_3 = 40°C$; Compressed liquid assume sat.liq. same T

Table B.1.1 $h_3 = 167.5$ kJ/kg; $s_3 = 0.5724$ kJ/kg K

From Eq.8.23 or from flow terms in Eq.8.38

$$\psi_3 - \psi_2 = (h_3 - T_o s_3) - (h_2 - T_o s_2)$$

$$= (h_3 - h_2) - T_o(s_3 - s_2)$$

$$= (167.5 - 2240.1) - 298.2(0.5724 - 6.7993\,)$$

$$= -2072.6 + 1856.9 = \textbf{-215.7 kJ/kg}$$

8.61

Calculate the change in exergy (kW) of the two flows in Problem 7.105.

Solution:

The two flows in the heat exchanger
exchanges energy and thus also exergy
(exergy). Fist find state 4

Air A.7: $h_1 = 1046.22$, $h_2 = 401.3$ kJ/kg,

$s_{T1}^o = 8.1349$, $s_{T2}^o = 7.1593$ kJ/kg K

Water B.1.1: $h_3 = 83.94$ kJ/kg, $s_3 = 0.2966$ kJ/kg K

Energy Eq.6.10: $\dot{m}_{AIR}\Delta h_{AIR} = \dot{m}_{H2O}\Delta h_{H2O}$

$h_4 - h_3 = (\dot{m}_{AIR}/\dot{m}_{H2O})(h_1 - h_2) = (2/0.5)644.92 = 2579.68$ kJ/kg

$h_4 = h_3 + 2579.68 = 2663.62 \quad < h_g \quad$ at 200 kPa

$T_4 = T_{sat} = 120.23°C,$

$\quad x_4 = (2663.62 - 504.68)/2201.96 = 0.9805,$

$\quad s_4 = 1.53 + x_4\, 5.597 = 7.01786$ kJ/kg K

We consider each flow separately and for each flow exergy is Eq.8.23, include
mass flow rate as in Eq.8.38, use $T_O = 20$ C

For the air flow:

$\dot{m}_1(\psi_1 - \psi_2) = \dot{m}_1[\, h_1 - h_2 - T_O(s_1 - s_2)\,]$

$\qquad = 2\,[\, 1046.22 - 401.3 - 293.2(8.1349 - 7.1593 - 0.287\ln\frac{125}{100})]$

$\qquad = 2\,(644.92 - 267.22) = \textbf{755.4 kW}$

For the water flow:

$\dot{m}_3(\psi_4 - \psi_3) = \dot{m}_3[\, h_4 - h_3 - T_O(s_4 - s_3)\,]$

$\qquad = 0.5\,[\, 2663.62 - 83.94 - 293.2(7.01786 - 0.2966)]$

$\qquad = 0.5[\, 2579.68 - 1970.7\,] = \textbf{304.7 kW}$

8.62

A steady-flow device receives R-410A at 40^oC, 800 kPa and it exits at 40^oC, 100 kPa. Assume a reversible isothermal process. Find the change in specific exergy.

C.V. The steady flow device assume $T_0 = 25^oC$

Inlet (T,P) Table B.4.2: $h_i = 319.42$ kJ/kg, $s_i = 1.1746$ kJ/kg K

Exit (T,P) Table B.4.2: $h_e = 330.12$ kJ/kg, $s_e = 1.4380$ kJ/kg K

From Eq.8.22 or 10.23

$$\Delta\psi_{ie} = \psi_e - \psi_i = (h_e - h_i) - T_0(s_e - s_i) = (330.12 - 319.42)$$

$$- 298.2(1.4380 - 1.1746) = -\mathbf{67.85 \ kJ/kg}$$

The device is like an expander, P drops so work is out and s increases so heat transfer comes in.

Borgnakke and Sonntag

8.63

Consider the springtime melting of ice in the mountains, which gives cold water running in a river at 2°C while the air temperature is 20°C. What is the exergy of the water relative to the temperature of the ambient?

Solution:

$$\psi = h_1 - h_0 - T_0(s_1 - s_0) \quad \text{flow exergy from Eq.8.22}$$

Approximate both states as saturated liquid from Table B.1.1

$$\psi = 8.392 - 83.96 - 293.15(0.03044 - 0.2966) = \textbf{2.457 kJ/kg}$$

Why is it positive? As the water is brought to 20°C it can be heated with q_L from a heat engine using q_H from atmosphere $T_H = T_0$ thus giving out work.

Clip-art from Microsoft.

8.64

Nitrogen flows in a pipe with velocity 300 m/s at 500 kPa, 300°C. What is its exergy with respect to an ambient at 100 kPa, 20°C?

Solution:

From the exergy or exergy in Eq.8.22

$$\psi = h_1 - h_0 + (1/2)\mathbf{V}_1^2 - T_0(s_1 - s_0)$$

$$= C_p(T_1 - T_0) + (1/2)\mathbf{V}_1^2 - T_0 \left[C_p \ln\left(\frac{T_1}{T_0}\right) - R \ln\left(\frac{P_1}{P_0}\right) \right]$$

$$= 1.042(300 - 20) + \frac{300^2}{2000} - 293.15 \left(1.042 \ln\left(\frac{573.15}{293.15}\right) - 0.2968 \ln\left(\frac{500}{100}\right) \right)$$

$$= \textbf{272 kJ/kg}$$

Notice that the high velocity does give a significant contribution.

8.65

Compressed air for machines and tools in a plant is generated by a central compressor receiving air at 100 kPa, 300 K, 1 kg/s delivering it at 600 kPa to a buffer tank and a distribution pipe. After flowing through the tank and pipe the air is at the ambient 300 K at its point of use. Assume a reversible adiabatic compressor and find the compressor exit temperature and the increase in air exergy through the compressor.

C.V. Compressor

Energy: $0 = h_1 - h_2 + w_C$;   Entropy: $0 = s_1 - s_2 + 0$

$$s_2 - s_1 = s_{T2}^0 - s_{T1}^0 - R \ln(P_2/P_1) \quad \text{find } s_{T2}^0 \text{ and then } T_2$$

or with constant specific heat

$$T_2 = T_1 \Big(\frac{P_2}{P_1}\Big)^{\frac{k-1}{k}} = 300 \Big(\frac{600}{100}\Big)^{0.2857} = \textbf{500.5 K}$$

Exergy increase through the compressor matches with the compressor work

$$\psi_2 - \psi_1 = (h_2 - h_1) - T_0(s_2 - s_1) = h_2 - h_1 = C_P (T_2 - T_1) = w_C$$
$$= 1.004 \text{ kJ/kgK} (500.5 - 300) \text{ K} = \textbf{201.3 kJ/kg}$$

8.66

For the air system in the previous problem find the increase in the air exergy from the inlet to the point of use. How much exergy was lost in the flow after the compressor exit?

From inlet, state 1 to final point of use state 3.

$$\psi_3 - \psi_1 = (h_3 - h_1) - T_0(s_3 - s_1) = 0 - T_0[0 - R \ln(P_3/P_1)]$$

$$= T_0 \, R \, \ln(P_3/P_1) = 298.15 \text{ K} \times 0.287 \text{ kJ/kgK} \times \ln(600/100)$$

$$= \textbf{153.3 kJ/kg}$$

Compressor exit, reversible adiabatic, constant s process: $s_2 = s_1$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300 \left(\frac{600}{100}\right)^{0.2857} = 500.5 \text{ K}$$

So then

$$\psi_{lost} = \psi_2 - \psi_3 = -(\psi_3 - \psi_1) + (\psi_2 - \psi_1) = -(\psi_3 - \psi_1) + h_2 - h_1$$

$$= -(\psi_3 - \psi_1) + C_{P0}(T_2 - T_1)$$

$$= -153.3 \text{ kJ/kg} + 1.004 \text{ kJ/kgK} \times (500.5 - 300) \text{ K}$$

$$= \textbf{48 kJ/kg}$$

8.67

Calculate the exergy of the water at the initial and final states of Problem 6.130, and the irreversibility of the process.

State properties

1: $u_1 = 83.94$ kJ/kg, $s_1 = 0.2966$ kJ/kg K, , $v_1 = 0.001$ m^3/kg

2: $u_2 = 3124.3$ kJ/kg, $s_2 = 7.7621$ kJ/kg K, $v_2 = 0.354$ m^3/kg

0: $u_o = 104.86$ kJ/kg, $s_o = 0.3673$ kJ/kg K, $v_o = 0.001003$ m^3/kg

Process transfers: $_1W_2^{ac} = 203$ kJ, $_1Q_2^{ac} = 3243.4$ kJ, $T_H = 873.15$ K

$\phi = (u - T_o s) - (u_o - T_o s_o) + P_o(v - v_o)$

$\phi_1 = (83.94 - 298.15 \times 0.2966) - (104.86 - 298.15 \times 0.3673)$

$+ 100 (0.001002 - 0.001003) = 0.159$ kJ/kg

$\phi_2 = (3124.3 - 298.15 \times 7.7621) - (104.86 - 298.15 \times 0.3673)$

$+ 100 (0.35411 - 0.001003) = 850$ kJ/kg

$_1I_2 = m(\phi_1 - \phi_2) + [1 - (T_o/T_H)]_1Q_2^{ac} - _1W_2^{ac} + P_o(V_2 - V_1)$

$= -849.84 + (1 - \dfrac{298.15}{873.15}) \, 3243.4 - 203 + 100 (0.3541 - 0.001)$

$= -849.84 + 2135.9 - 203 + 35.31 = \textbf{1118 kJ}$

$[(S_{gen} = 3.75$ kJ/K; $T_o S_{gen} = 1118$ kJ so OK]

8.68

A geothermal source provides 10 kg/s of hot water at 500 kPa, 145°C flowing into a flash evaporator that separates vapor and liquid at 200 kPa. Find the three fluxes of exergy (inlet and two outlets) and the irreversibility rate.

C.V. Flash evaporator chamber. Steady flow with no work or heat transfer.

Cont. Eq.: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$;

Energy Eq.: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$

Entropy Eq.: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3$

B.1.1: $h_O = 104.87$, $s_O = 0.3673$, B.1.4: $h_1 = 610.7$, $s_1 = 1.7902$

B.1.2: $h_2 = 2706.63$, $s_2 = 7.1271$, $h_3 = 504.68$, $s_3 = 1.530$

$$h_1 = x h_2 + (1 - x) h_3 \implies x = \dot{m}_2/\dot{m}_1 = \frac{h_1 - h_3}{h_2 - h_3} = 0.04815$$

$$\dot{m}_2 = x \dot{m}_1 = 0.4815 \text{ kg/s} \qquad \dot{m}_3 = (1-x)\dot{m}_1 = 9.5185 \text{ kg/s}$$

$$\dot{S}_{gen} = 0.4815 \times 7.1271 + 9.5185 \times 1.53 - 10 \times 1.7902 = 0.093 \text{ kW/K}$$

Flow exergy Eq.8.22: $\psi = (h - T_O s) - (h_O - T_O s_O) = h - h_O - T_O(s - s_O)$

$$\psi_1 = 610.7 - 104.87 - 298.15 (1.7902 - 0.3673) = 81.59 \text{ kJ/kg}$$

$$\psi_2 = 2706.63 - 104.87 - 298.15 (7.1271 - 0.3673) = 586.33 \text{ kJ/kg}$$

$$\psi_3 = 504.68 - 104.87 - 298.15 (1.53 - 0.3673) = 53.15 \text{ kJ/kg}$$

$$\dot{m}_1 \psi_1 = \textbf{815.92 kW} \qquad \dot{m}_2 \psi_2 = \textbf{282.3 kW} \qquad \dot{m}_3 \psi_3 = \textbf{505.9 kW}$$

Balance of flow terms in Eq.8.38

$$\dot{I} = \dot{m}_1 \psi_1 - \dot{m}_2 \psi_2 - \dot{m}_3 \psi_3 = \textbf{27.7 kW} \quad (= T_O \dot{S}_{gen})$$

Borgnakke and Sonntag

8.69

An air compressor is used to charge an initially empty 200-L tank with air up to 5 MPa. The air inlet to the compressor is at 100 kPa, 17°C and the compressor isentropic efficiency is 80%. Find the total compressor work and the change in exergy of the air.

C.V. Tank + compressor Transient process with constant inlet conditions, no heat transfer.

Continuity: $m_2 - m_1 = m_{in}$ ($m_1 = 0$)   Energy: $m_2 u_2 = m_{in} h_{in} - {}_1W_2$

Entropy: $m_2 s_2 = m_{in} s_{in} + {}_1S_2$ gen

Reversible compressor: ${}_1S_2$ GEN $= 0$ $\Rightarrow$ $s_2 = s_{in}$

State 1: $v_1 = RT_1/P_1 = 0.8323$ m^3/kg,   the ambient state

State inlet, Table A.7.1: $h_{in} = 290.43$ kJ/kg, $s^o_{Tin} = 6.83521$ kJ/kg K

Eq.6.19: $s^o_{T2} = s^o_{Tin} + R \ln (\dfrac{P_2}{P_{in}}) = 6.83521 + 0.287 \ln (\dfrac{5000}{100}) = 7.95796$

Table A.7.1 $\Rightarrow$ $T_{2,s} = 854.6$ K, $u_{2,s} = 637.25$ kJ/kg

$\Rightarrow {}_1w_{2,s} = h_{in} - u_{2,s} = 290.43 - 637.25 = -346.82$ kJ/kg

Actual compressor: ${}_1w_{2,AC} = {}_1w_{2,s}/\eta_c = -433.53$ kJ/kg

$u_{2,AC} = h_{in} - {}_1w_{2,AC} = 290.43 - (-433.53) = 723.96$ kJ/kg

$\Rightarrow T_{2,AC} = 958.5$ K, s^o_{T2} ac $= 8.08655$ kJ/kg K

State 2 $\boxed{u, P}$ $v_2 = RT_2/P_2 = 0.05502$ m^3/kg so $m_2 = V_2/v_2 = 3.635$ kg

$\Rightarrow {}_1W_2 = m_2 ({}_1w_{2,AC}) = $ **-1575.9 kJ**

$m_2(\phi_2 - \phi_1) = m_2[u_2 - u_1 + P_0(v_2 - v_1) - T_0(s_2 - s_1)]$

$= 3.635 [723.96 - 207.19 + 100(0.05502 - 0.8323) - 290[8.08655 -$

$6.83521 - 0.287 \ln(5000/100)] = $ **1460.4 kJ**

8.70

Find the exergy at all 4 states in the power plant of Problem 7.48 with an ambient at 298 K.

Solution:

Flow exergy from Eq.8.22 neglecting kinetic and potential energy is:

$$\psi = h - h_0 - T_0(s - s_0)$$

so we need (h,s) for all four states.

$P_1 = P_4 = 20$ MPa, $T_1 = 700$ °C
$h_1 = 3809.1$ kJ/kg,
$s_1 = 6.7993$ kJ/kg K
$P_2 = P_3 = 20$ kPa, $T_3 = 40$ °C
State 3: (P, T) Comp. liquid,
take sat. liquid Table B.1.1
$h_3 = 167.5$ kJ/kg,
$v_3 = 0.001008$ m^3/kg

C.V. Turbine.

Entropy Eq.7.8: $s_2 = s_1 = 6.7993$ kJ/kg K

Table B.1.2 $s_2 = 0.8319 + x_2 \times 7.0766$ => $x_2 = 0.8433$

$h_2 = 251.4 + 0.8433 \times 2358.33 = 2240.1$ kJ/kg

$w_T = h_1 - h_2 = 3809.1 - 2240.1 = 1569$ kJ/kg

CV. Pump, property relation v = constant gives work from Eq.7.15 as

$w_P = - v_3(P_4 - P_3) = -0.001008(20000 - 20) = -20.1$ kJ/kg

$h_4 = h_3 - w_P = 167.5 + 20.1 = 187.6$ kJ/kg

Flow exergy from Eq.8.23 and notice that since turbine work and pump work are reversible they represent also change in avalability.

$\psi_1 = h_1 - h_0 - T_0(s_1 - s_0) = 3809.1 - 104.87 - 298 (6.7993 - 0.3673)$

   $= \textbf{1787.5 kJ/kg}$

$\psi_2 = h_2 - h_0 - T_0(s_2 - s_0) = \psi_1 - w_T = 1787.5 - 1569 = \textbf{218.5 kJ/kg}$

$\psi_3 = h_3 - h_0 - T_0(s_3 - s_0) = 167.5 - 104.87 - 298(0.5724 - 0.3673)$

   $= \textbf{1.51 kJ/kg}$

$\psi_4 = h_4 - h_0 - T_0(s_4 - s_0) = \psi_3 - w_P = 1.51 + 20.1 = \textbf{21.61 kJ/kg}$

Borgnakke and Sonntag

8.71

An electric stove has one heating element at 300°C getting 750 W of electric power. It transfers 90% of the power to 1 kg water in a kettle initially at 20°C, 100 kPa, the rest 10% leaks to the room air. The water at a uniform T is brought to the boiling point. At the start of the process what is the rate of exergy transfer by: a) electrical input   b) from heating element   and c) into the water at T_{water}.

a) Work is exergy $\dot{\Phi} = \dot{W} =$ **750 W**

b) Heat transfer at 300°C is only partly exergy

$$\dot{\Phi} = \left(1 - \frac{T_o}{T_H}\right)\dot{Q} = \left(1 - \frac{293.15}{273.15 + 300}\right)750 = \textbf{366 W}$$

c) Water receives heat transfer at 20°C as 90% of 750 W = 675 W

$$\dot{\Phi} = \left(1 - \frac{T_o}{T_{water}}\right)\dot{Q} = \left(1 - \frac{293.15}{273.15 + 20}\right)675 = \textbf{0 W}$$

750 W at 300°C

8.72

Air flows at 1500 K, 100 kPa through a constant pressure heat exchanger giving energy to a heat engine and comes out at 500 K. What is the constant temperature the same heat transfer should be delivered at to provide the same exergy?

Solution:

C.V. Heat exchanger

Continuity eq.: $\dot{m}_1 = \dot{m}_2$;

Energy Eq.4.12: $\dot{m}_1 h_1 = \dot{m}_1 h_2 + \dot{Q}_H$

Table A.7.1: $h_1 = 1635.8$ kJ/kg,

$h_2 = 503.36$ kJ/kg, $s_1 = 8.61209$ kJ/kg K

$s_2 = 7.38692$ kJ/kg K

$q_{out} = h_1 - h_2 = 1635.8 - 503.36 = 1132.4$ kJ/kg

Exergy from heat transfer at T: $\Delta\psi = (1 - \dfrac{T_o}{T_H}) \, q_{out} = \psi_1 - \psi_2$

Eq.8.23 and the flow terms per unit mass flow in Eq.8.38:

$$\psi_1 - \psi_2 = h_1 - h_2 - T_o \, (s_1 - s_2)$$
$$= 1132.4 - 298.15 \,(8.6121 - 7.38692)$$
$$= 1132.4 - 356.3 = 767.1 \text{ kJ/kg}$$

$$1 - \dfrac{T_o}{T_H} = (\psi_1 - \psi_2) \,/ \, q_{out} = 767.1 \,/\, 1132.4 = 0.6774$$

$$\dfrac{T_o}{T_H} = 0.3226 \implies \quad \mathbf{T_H = 924 \text{ K}}$$

Borgnakke and Sonntag

8.73

A wooden bucket (2 kg) with 10 kg hot liquid water, both at 85°C, is lowered 400 m down into a mineshaft. What is the exergy of the bucket and water with respect to the surface ambient at 20°C?

C.V. Bucket and water. Both thermal exergy and potential energy terms.

$v_1 \approx v_0$ for both wood and water so work to atm. is zero.

Use constant heat capacity table A.3 for wood and table B.1.1 (sat. liq.) for water.
From Eq.8.28

$$\phi_1 - \phi_0 = m_{wood}[u_1 - u_0 - T_0(s_1 - s_0)] + m_{H_2O}[u_1 - u_0 - T_0(s_1 - s_0)] + m_{tot}g(z_1 - z_0)$$

$$= 2 \text{ kg } [1.26 \frac{kJ}{kg\ K} (85 - 20)\ K - 293.15\ K \times 1.26 \frac{kJ}{kg\ K} \times \ln\frac{273.15 + 85}{293.15}]$$

$$+ 10 \text{ kg } [355.82 - 83.94 - 293(1.1342 - 0.2966)] \text{ kJ/kg}$$

$$+ 12 \text{ kg} \times 9.807 \text{ (m/s}^2) \times (-400) \text{ m /1000 (J/kJ)}$$

$$= 15.85 + 263.38 - 47.07 = \textbf{232.2 kJ}$$

8.74

A flow of 0.1 kg/s hot water at 70°C is mixed with a flow of 0.2 kg/s cold water at 20°C in a shower fixture. What is the rate of exergy destruction (irreversibility) for this process?

Solution:

Continuity Eq.4.9:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$
$$= 0.1 + 0.2 = 0.3 \text{ kg/s}$$

Energy Eq.4.10:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} = \dot{m}_3 s_3$

Use specific heat from A.4 and Eq.6.11 for s change or Table B.1.1 saturated liquid values. Divide the energy equation with $\dot{m}_3 C_P$

$$T_3 = (\dot{m}_1/\dot{m}_3)T_1 + (\dot{m}_2/\dot{m}_3)T_2 = \frac{1}{3} \times 70 + \frac{2}{3} \times 20 = 36.67^\circ C$$

$$\dot{S}_{gen} = \dot{m}_1(s_3 - s_1) + \dot{m}_2(s_3 - s_2)$$
$$= 0.1 \times 4.18 \ln(309.82/343.15) + 0.2 \times 4.18 \ln(309.82/293.15)$$
$$= 0.00353 \text{ kW/K}$$

$$\dot{\Phi}_{Destruction} = \dot{I} = T_o \dot{S}_{gen} = 298.15 \times 0.00353 = \textbf{1.05 kW}$$

You could also have decided to use 20°C for the ambient T in this problem.

Borgnakke and Sonntag

8.75

A 1 kg block of copper at 350°C is quenched in a 10 kg oil bath initially at ambient temperature of 20°C. Calculate the final uniform temperature (no heat transfer to/from ambient) and the change of exergy of the system (copper and oil).
Solution:

C.V. Copper and oil. $\quad C_{co} = 0.42$ kJ/kg K, $\quad C_{oil} = 1.8$ kJ/kg K

$$m_2 u_2 - m_1 u_1 = {}_1Q_2 - {}_1W_2 = 0 = m_{co} C_{co}(T_2 - T_1)_{co} + (mC)_{oil}(T_2 - T_1)_{oil}$$

$$1 \times 0.42 \ (T_2 - 350) + 10 \times 1.8 \ (T_2 - 20) = 0$$

$$18.42 \ T_2 = 507 \qquad \Rightarrow \qquad T = 27.5°C = 300.65 \text{ K}$$

For each mass copper and oil, we neglect work term (v = C) so Eq.8.28 is

$$(\phi_2 - \phi_1) = u_2 - u_1 - T_0(s_2 - s_1) = mC \ [(T_2 - T_1) - T_0 \ln (T_2 / T_1) \ ]$$

$$m_{cv}(\phi_2 - \phi_1)_{cv} + m_{oil} \ (\phi_2 - \phi_1)_{oil} =$$

$$= 0.42 \times [(-322.5) - 293.15 \ln \frac{300.65}{623.15}] + 10 \times 1.8 \ [7.5 - 293.15 \ln \frac{300.65}{293.15}]$$

$$= - 45.713 + 1.698 = \textbf{- 44.0 kJ}$$

8.76

A water kettle has 1 kg of saturated liquid water at P_o. It is on an electric stove that heats it from a hot surface at 500 K. Water vapor escapes from the kettle and when the last liquid drop disappears the stove is turned off. Find the destruction of exergy two places: a) between the hot surface and the water and b) between the electrical wire input and the hot surface.

C.V. The water, this is a control mass. We bring it from saturated liquid to saturated vapor same pressure.

Energy eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 \Rightarrow {}_1Q_2 = m (h_2 - h_1) = mh_{fg} = 2257$ kJ

C.V. Kettle bottom, consider the hot surface to water heat transfer.

$$\psi_{in} = \left(1 - \frac{T_o}{T_{H\ surface}}\right) {}_1Q_2 = \left(1 - \frac{298.15}{500}\right) 2257 \text{ kJ} = 911 \text{ kJ}$$

$$\psi_{out} = \left(1 - \frac{T_o}{T_{water}}\right) {}_1Q_2 = \left(1 - \frac{298.15}{373.15}\right) 2257 \text{ kJ} = 453.6 \text{ kJ}$$

The destruction is the drop in exergy

$$\psi_{destruction} = \psi_{in} - \psi_{out} = 911 - 453.6 = \textbf{457.4 kJ}$$

C.V. Wire and hot plate.

$$\psi_{in} = W_{electrical} = {}_1Q_2 = 2257 \text{ kJ}$$

$$\psi_{out} = \left(1 - \frac{T_o}{T_{H\ surface}}\right) {}_1Q_2 = \left(1 - \frac{298.15}{500}\right) 2257 \text{ kJ} = 911 \text{ kJ}$$

$$\psi_{destruction} = \psi_{in} - \psi_{out} = 2257 - 911 = \textbf{1346 kJ}$$

Comment: We could also have found the destruction of exergy by finding the entropy generation terms and then $\psi_{destruction} = T_o S_{gen}$

8.77

A 200 L insulated tank contains nitrogen gas at 200 kPa, 300 K. A line with nitrogen at 500 K, 500 kPa adds 40% more mass to the tank with a flow through a valve. Use constant specific heats to find the final temperature and the exergy destruction.

C.V. Tank, no work and no heat transfer.

Continuity Eq.4.20: $\qquad m_2 - m_1 = m_i$

Energy Eq.4.21: $\qquad m_2 u_2 - m_1 u_1 = m_i h_i + {}_1Q_2$

Entropy Eq.7.12: $\qquad m_2 s_2 - m_1 s_1 = m_i s_i + \int dQ/T + {}_1S_{2\ gen}$

Process Eq.: $\qquad {}_1Q_2 = 0, \quad m_2 = 1.4\ m_1; \quad m_i = 0.4\ m_1$

Write $h_i = u_i + RT_i$ then the energy equation contains only u's so we can substitute the u differences with $C_{v0}\ \Delta T$ and divide by $m_2\ C_{v0}$ to get

$$T_2 = \frac{m_1}{m_2}\ T_1 + \frac{m_i}{m_2}T_i + \frac{m_i\ R}{m_2 C_{v0}}\ T_i$$

$$= \frac{1}{1.4}\ 300 + \frac{0.4}{1.4}\ 500 + \frac{0.4}{1.4}\ \frac{0.2968}{0.745}\ 500 = \textbf{414.0 K}$$

We need the pressure for the entropy

$$P_2 = m_2 RT_2/V = 1.4\ P_1 \times T_2/T_1 = 1.4 \times 200 \times 414/300 = 386.4\ kPa$$

$$m_1 = \frac{P_1 V_1}{RT_1} = \frac{200 \times 0.2}{0.2968 \times 300} = 0.4492\ kg; \quad m_2 = 1.4\ m_1 = 0.6289\ kg$$

The entropy generation, with entropy difference from Eq.6.16, becomes

$${}_1S_{2\ gen} = m_2 s_2 - m_1 s_1 - m_i\ s_i = m_2(s_2 - s_i) - m_1(s_1 - s_i)$$

$$= m_2\ [C_{P0}\ln(\frac{T_2}{T_i}) - R\ \ln(\frac{P_2}{P_i})] - m_1\ [C_{P0}\ \ln(\frac{T_1}{T_i}) - R\ \ln(\frac{P_1}{P_i})]$$

$$= 0.6289[\ 1.042\ \ln(\frac{414}{500}) - 0.2968\ \ln(\frac{386.4}{500})]$$

$$- 0.4492\ [1.042\ \ln(\frac{300}{500}) - 0.2968\ \ln(\frac{200}{500})] = 0.0414\ kJ/K$$

$$\Phi_{destruction} = T_o\ {}_1S_{2\ gen} = 298.15 \times 0.0414\ kJ = \textbf{12.34 kJ}$$

8.78

A 10-kg iron disk brake on a car is initially at 10°C. Suddenly the brake pad hangs up, increasing the brake temperature by friction to 110°C while the car maintains constant speed. Find the change in exergy of the disk and the energy depletion of the car's gas tank due to this process alone. Assume that the engine has a thermal efficiency of 35%.

Solution:

All the friction work is turned into internal energy of the disk brake.

Energy eq.: $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 \Rightarrow {}_1Q_2 = m_{Fe}C_{Fe}(T_2 - T_1)$

$$_1Q_2 = 10 \times 0.45 \times (110 - 10) = 450 \text{ kJ}$$

Neglect the work to the surroundings at P_0, so change in exergy is from Eq.8.27

$$\Delta\phi = m(u_2 - u_1) - T_0 m(s_2 - s_1)$$

Change in s for a solid, Eq.6.11

$$m(s_2 - s_1) = mC \ln(T_2/T_1) = 10 \times 0.45 \times \ln\left(\frac{383.15}{283.15}\right) = 1.361 \text{ kJ/K}$$

$$\Delta\phi = 450 - 283.15 \times 1.361 = \mathbf{64.63 \text{ kJ}}$$

$$W_{engine} = \eta_{th}Q_{gas} = {}_1Q_2 = \text{Friction work}$$

$$Q_{gas} = {}_1Q_2/\eta_{th} = 450/0.35 = \mathbf{1285.7 \text{ kJ}}$$

8.79

Water as saturated liquid at 200 kPa goes through a constant pressure heat exchanger as shown in Fig. P8.79. The heat input is supplied from a reversible heat pump extracting heat from the surroundings at 17°C. The water flow rate is 2 kg/min and the whole process is reversible, that is, there is no overall net entropy change. If the heat pump receives 40 kW of work find the water exit state and the increase in exergy of the water.

C.V. Heat exchanger + heat pump.

$$\dot{m}_1 = \dot{m}_2 = 2 \text{ kg/min}, \quad \dot{m}_1 h_1 + \dot{Q}_0 + \dot{W}_{in} = \dot{m}_1 h_2, \quad \dot{m}_1 s_1 + \dot{Q}_0/T_0 = \dot{m}_1 s_2$$

Substitute $\dot{Q}_0$ into energy equation and divide by $\dot{m}_1$

$$h_1 - T_0 s_1 + w_{in} = h_2 - T_0 s_2$$

LHS = 504.7 - 290.15 × 1.5301 + 40×60/2 = 1260.7 kJ/kg

State 2: P_2 , $h_2 - T_0 s_2 = 1260.7$ kJ/kg

At sat. vap. $h_g - T_0 s_g = 638.8$ so state 2 is superheated vapor at 200 kPa.

At 600°C: $h_2 - T_0 s_2 = 3703.96 - 290.15 \times 8.7769 = 1157.34$ kJ/kg

At 700°C: $h_2 - T_0 s_2 = 3927.66 - 290.15 \times 9.0194 = 1310.68$ kJ/kg

Linear interpolation $\Rightarrow$ **$T_2 = 667°C$**

$$\Delta\psi = (h_2 - T_0 s_2) - (h_1 - T_0 s_1) = w_{in} = \textbf{1200 kJ/kg}$$
$$= 1260.7 - 504.7 + 290.15 \times 1.5301 \approx \textbf{1200 kJ/kg}$$

8.80

Ammonia, 2 kg, at 400 kPa, 40$^\text{o}$C is in a piston/cylinder together with an unknown mass of saturated liquid ammonia at 400 kPa. The piston is loaded so it keeps constant pressure and the two masses are allowed to mix to a final uniform state of saturated vapor without external heat transfer. Find the total exergy destruction in the process.

C.V. All the ammonia.

Continuity Eq.: $\quad m_2 - m_A - m_B = 0$

Energy Eq.: $\quad m_2 u_2 - m_A u_A - m_B u_B = Q - W$

Entropy Eq.: $\quad m_2 s_2 - m_A s_A - m_B s_B = \int dQ/T + {}_1S_{2\ gen}$

Process: $\quad Q = 0, \ P = C \ $ so $W = P(V_2 - V_A - V_B)$

State A: $\quad h_A = 1543.6$ kJ/kg, $\ s_A = 5.7111$ kJ/kg-K, $\ m_A = 2$ kg

State B: $\quad h_B = 171.65$ kJ/kg, $\ s_B = 0.6793$ kJ/kg-K, $\ m_B = ?$ kg

State 2: $\quad h_2 = 1440.2$ kJ/kg, $\ s_2 = 5.3559$ kJ/kg-K, $\ m_2 = m_A + m_B$

Insert the process equation into the energy equation

$$m_2 h_2 - m_A h_A - m_B h_B = 0$$

Now solve the energy equation for the unknown mass m_B

$$m_B = m_A \frac{h_2 - h_A}{h_B - h_2} = 2 \text{ kg} \frac{1440.2 - 1543.6}{171.65 - 1440.2} = 0.163 \text{ kg}$$

Solve for entropy generation

$${}_1S_{2\ gen} = m_2 s_2 - m_A s_A - m_B s_B$$
$$= 2.163 \times 5.3559 - 2 \times 5.7111 - 0.163 \times 0.6793$$
$$= 0.051886 \text{ kJ/K}$$

Exergy destruction

$$\psi_{destruction} = T_o\, S_{gen} = 298.15 \times 0.051886 = \textbf{15.47 kJ}$$

Exergy Balance Equation

8.81

Apply the exergy equation to solve Problem 8.18

A refrigerator should remove 1.5 kW from the cold space at -10°C while it rejects heat to the kitchen at 25°C. Find the reversible work.

The exergy balance equation is Eq.8.38

$$\frac{d\Phi}{dt} = \left(1 - \frac{T_o}{T}\right)\dot{Q}_{cv} - \dot{W}_{cv} + P_o\frac{dV}{dt} + \dot{m}_i\psi_i - \dot{m}_e\psi_e - T_o\dot{S}_{gen}$$

$$0 = \left(1 - \frac{T_o}{T_L}\right)\dot{Q}_L - \left(1 - \frac{T_o}{T_H}\right)\dot{Q}_H - \dot{W}_{cv} + 0 + 0 - 0 - 0$$

The two d/dt terms are zero due to steady state, no flow and reversible give zero for the last three terms so we get the equation we can solve for the work

$$\dot{W}_{cv} = \left(1 - \frac{T_o}{T_L}\right)\dot{Q}_L - \left(1 - \frac{T_o}{T_H}\right)\dot{Q}_H$$

To solve we need the heat transfer $\dot{Q}_H$ as we do have $\dot{Q}_L$ = 1.5 kW. We need to use the entropy equation to get

$$\dot{Q}_H = (T_H/T_L)\,\dot{Q}_L = \frac{298.15}{263.15}\,1.5 \text{ kW} = 1.6995 \text{ kW}$$

$$\dot{W}_{cv} = \left(1 - \frac{T_o}{T_L}\right)\dot{Q}_L - \left(1 - \frac{T_o}{T_H}\right)\dot{Q}_H = \dot{Q}_L - \dot{Q}_H + \frac{T_o}{T_H}\dot{Q}_H - \frac{T_o}{T_L}\dot{Q}_L$$

$$= \dot{Q}_L - \dot{Q}_H = 1.5 - 1.6995 = -0.1995 \text{ kW} = \mathbf{-0.2 \text{ kW}}$$

8.82

Apply the exergy equation to solve Problem 8.35 with $T_o = 20^oC$.

A 20^oC room is heated with a 2000 W electric baseboard heater. What is the rate of irreversibility?

The exergy balance equation is Eq.8.38

$$\frac{d\Phi}{dt} = (1 - \frac{T_o}{T}) \dot{Q}_{cv} - \dot{W}_{cv} + P_o\frac{dV}{dt} + \dot{m}_i\psi_i - \dot{m}_e\psi_e - T_o\dot{S}_{gen}$$

For the heater we have steady state (d/dt terms are zero), one $\dot{Q}_{cv}$ out at T_o, electrical work in and no flow terms leaving us with

$$0 = (1 - \frac{T_o}{T_o}) \times \dot{Q}_{cv} + \dot{W}_{electrical} - T_o\dot{S}_{gen}$$

$$\dot{I} = T_o \dot{S}_{gen} = \dot{W}_{electrical} = \textbf{2000 W}$$

This assumes that $T_o = 20^oC$.

8.83

Estimate some reasonable temperatures to use and find all the fluxes of exergy in the refrigerator given in Example 5.2

We will assume the following temperatures:

Ambient: $T = 20^oC$ usually it is the kitchen air.

Low T: $T = 5^oC$ (refrigerator) $T = -10^oC$ (freezer)

$$\dot\Phi_W = \dot W = \textbf{150 W}$$

$$\dot\Phi_H = (1 - \frac{T_o}{T_H}) \, \dot Q_H = (1 - \frac{T_{amb}}{T_{amb}}) \, \dot Q_H = \textbf{0}$$

$$\dot\Phi_L = (1 - \frac{T_o}{T_L}) \, \dot Q_L = (1 - \frac{293}{278}) \, 250 = \textbf{-13.5 W}$$

I.e. the flux goes into the cold space! Why? As you cool it $T < T_o$ and you increase its exergy, it is further away from the ambient.

8.84

Find the specific flow exergy in and out of the steam turbine in Example 7.1 assuming an ambient at 293 K. Use the exergy balance equation to find the reversible specific work. Does this calculation of specific work depend on T_o?

Solution:

The specific flow exergy is from Eq. 8.22

$$\psi_i = h_i + \frac{1}{2}\mathbf{V}_i^2 - T_o s_i - (h_o - T_o s_o)$$

Reference state: $h_o = 83.94$ kJ/kg, $s_o = 0.2966$ kJ/kg K,

$$h_o - T_o s_o = -2.9638 \text{ kJ/kg}$$

The properties are listed in Example 7.1 so the specific flow exergies are

$$\psi_i = 3051.2 + 1.25 - 293 \times 7.1228 - (-2.9638) = \textbf{968.43 kJ/kg}$$

$$\psi_e = 2655.0 + 20 - 293 \times 7.1228 - (-2.9638) = \textbf{590.98 kJ/kg}$$

The reversible work is from Eq.8.29, with q = 0 and steady state, so

$$w = \psi_i - \psi_e = 968.43 - 590.98 = \textbf{377.45 kJ/kg}$$

The offset T_o terms drop out as we take the difference and also $(s_i = s_e)$

$$\psi_i - \psi_e = h_i - h_e - T_o(s_i - s_e) = h_i - h_e$$

Notice since the turbine is reversible we get the same as in Example 7.1

8.85

Apply the exergy equation to solve Problem 8.36.

A refrigerator removes 1.5 kW from the cold space at -10°C using 750 W of power input while it rejects heat to the kitchen at 25°C. Find the rate of irreversibility.

The exergy balance equation is Eq.8.38

$$\frac{d\Phi}{dt} = \left(1 - \frac{T_o}{T}\right) \dot{Q}_{cv} - \dot{W}_{cv} + P_o\frac{dV}{dt} + \dot{m}_i\psi_i - \dot{m}_e\psi_e - T_o\dot{S}_{gen}$$

$$0 = (1 - \frac{T_o}{T_L}) \dot{Q}_L - (1 - \frac{T_o}{T_H}) \dot{Q}_H - \dot{W}_{cv} + 0 + 0 - 0 - T_o\dot{S}_{gen}$$

The two d/dt terms are zero due to steady state and no flow. Now solve for the irreversibility, notice the $\dot{Q}_H$ term drops out since $T_H = T_o$.

$$\dot{I} = T_o \dot{S}_{gen} = (1 - \frac{T_o}{T_L}) \dot{Q}_L - \dot{W}_{cv}$$

$$= (1 - \frac{298.15}{263.15}) 1.5 \text{ kW} - (-0.75) \text{ kW}$$

$$= -0.2 \text{ kW} + 0.75 \text{ kW} = \textbf{0.55 kW}$$

8.86

Evaluate the steady state exergy fluxes due to a heat transfer of 50 W through a wall with 600 K on one side and 400 K on the other side. What is the exergy destruction in the wall.

Solution:

Exergy flux due to a $\dot{Q}$ term Eq.8.36:

$$\dot{\Phi}_Q = (1 - \frac{T_o}{T}) \dot{Q}$$

$$\dot{\Phi}_1 = (1 - \frac{T_o}{T_1}) \dot{Q} = (1 - \frac{298}{600}) 50 = 25.2 \text{ W}$$

$$\dot{\Phi}_2 = (1 - \frac{T_o}{T_2}) \dot{Q} = (1 - \frac{298}{400}) 50 = 12.75 \text{ W}$$

1 | 600 K | 2 | 400 K | 250 W

Steady state state so no storage and Eq.8.38 is

$$0 = \dot{\Phi}_1 - \dot{\Phi}_2 - \dot{\Phi}_{destr.}$$

$$\dot{\Phi}_{destr.} = \dot{\Phi}_1 - \dot{\Phi}_2 = 25.2 - 12.75 = \textbf{12.45 W}$$

8.87

A counterflowing heat exchanger cools air at 600 K, 400 kPa to 320 K using a supply of water at 20°C, 200 kPa. The water flow rate is 0.1 kg/s and the air flow rate is 1 kg/s. Assume this can be done in a reversible process by the use of heat engines and neglect kinetic energy changes. Find the water exit temperature and the power out of the heat engine(s).

C.V. Total

Energy eq.: $\dot{m}_a h_1 + \dot{m}_{H_2O} h_3 = \dot{m}_a h_2 + \dot{m}_{H_2O} h_4 + \dot{W}$

Entropy Eq.: $\dot{m}_a s_1 + \dot{m}_{H_2O} s_3 = \dot{m}_a s_2 + \dot{m}_{H_2O} s_4$ $\qquad (s_{gen} = 0)$

Table A.7: $h_1 = 607.316$ kJ/kg, $\quad s^\circ_{T1} = 7.57638$ kJ/kg K

Table A.7: $h_2 = 320.576$ kJ/kg, $\quad s^\circ_{T2} = 6.93413$ kJ/kg K,

Table B.1.1: $h_3 = 83.96$ kJ/kg, $\quad s_3 = 0.2966$ kJ/kg K

From the entropy equation we first find state 4

$\quad s_4 = (\dot{m}_a/\dot{m}_{H_2O})(s_1 - s_2) + s_3 = (1/0.1)(7.57638 - 6.93413) + 0.2966 = 6.7191$

4: $P_4 = P_3$, $s_4 \Rightarrow$ Table B.1.2: $\quad x_4 = (6.7191-1.530)/5.597 = 0.9271$,

$\quad h_4 = 504.68 + 0.9271 \times 2201.96 = 2546.1$ kJ/kg, $\quad T_4 = \mathbf{120.20°C}$

From the energy equation

$\quad \dot{W} = \dot{m}_a(h_1 - h_2) + \dot{m}_{H_2O}(h_3 - h_4)$

$\quad\quad = 1(607.32 - 320.58) + 0.1(83.96 - 2546.1) = \mathbf{40.53\ kW}$

The work could be found from the exergy balance equation (requires state 4) as

$\quad \dot{W} = \dot{m}_a(\psi_1 - \psi_2) + \dot{m}_{H_2O}(\psi_3 - \psi_4)$

Since the entropies (including $\dot{m}$'s) balance out it is the same as the energy Eq.

8.88

Consider the condenser in Problem 7.48. Find the specific energy and exergy that are given out, assuming an ambient at $20^{\circ}C$. Find also the specific exergy destruction in the process.

Solution:

Condenser from state 2 to state 3

$P_2 = P_3 = 20$ kPa

$T_3 = 40$ °C

State 1: (P, T) Table B.1.3

$h_1 = 3809.1$ kJ/kg, $s_1 = 6.7993$ kJ/kg K

C.V. Turbine.

      Entropy Eq.7.8: $s_2 = s_1 = 6.7993$ kJ/kg K

      Table B.1.2 $s_2 = 0.8319 + x_2 \times 7.0766$ => $x_2 = 0.8433$

      $h_2 = 251.4 + 0.8433 \times 2358.33 = 2240.1$ kJ/kg

State 3: (P, T) Compressed liquid, take sat. liq. Table B.1.1

         $h_3 = 167.54$ kJ/kg, $s_3 = 0.5724$ kJ/kg K

  C.V. Condenser

Energy Eq.: $q_L = h_2 - h_3 = 2240.1 - 167.54 =$ **2072.56 kJ/kg**

Exergy Eq.: $\Delta\psi = \psi_2 - \psi_3 = h_2 - h_3 - T_o(s_2 - s_3)$

               $= 2072.56 - 293.15(6.7993 - 0.5724)$

               $=$ **247.1 kJ/kg** going out

    Since all the exergy that goes out ends up at the ambient where it has zero exergy, the destruction equals the outgoing exergy.

$$\psi_{destr} = \Delta\psi = \textbf{247.1 kJ/kg}$$

Notice the condenser gives out a large amount of energy but little exergy.

8.89

Apply the exergy equation to find the exergy destruction for Problem 8.54

Exergy flux in: $\dot{\Phi}_H = \left(1 - \dfrac{T_o}{T_H}\right)\dot{Q}_H = \left(1 - \dfrac{298.15}{1000}\right) 1 \text{ kW} = 0.702 \text{ kW}$

Exergy flux out: $\dot{\Phi}_L = \left(1 - \dfrac{T_o}{T_L}\right)\dot{Q}_L = 0 \qquad (T_L = T_o)$

The other exergy flux out is the power $\dot{\Phi}_{out} = \dot{W} = 0.6 \text{ kW}$

The exergy balance equation Eq.8.38 for the HE becomes (steady state)

$0 = \dot{\Phi}_H - \dot{\Phi}_L - \dot{W} - \dot{\Phi}_{destruction}$

$\dot{\Phi}_{destruction} = \dot{\Phi}_H - \dot{\Phi}_L - \dot{W} = 702 \text{ W} - 0 - 600 \text{ W} = \textbf{102 W}$

8.90

A flow of 1 kg/s of air at 300 K is mixed with a flow of 2 kg/s air at 1500 K in an insulated pipe junction at a pressure of 100 kPa. Find the exit temperature and the exergy destruction.

Solution:

Continuity Eq.4.9:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 1 + 2 = 3 \text{ kg/s}$$

Energy Eq.4.10:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} = \dot{m}_3 s_3$

Due to high T at state 2 use A.7 and Eq.6.19 for s change.

$$h_3 = (\dot{m}_1/\dot{m}_3)h_1 + (\dot{m}_2/\dot{m}_3)h_2 = \frac{1}{3} \times 300.47 + \frac{2}{3} \times 1635.8 = 1190.69 \text{ kJ/kg}$$

From A.7.1: $T_3 = \mathbf{1125.4 \text{ K}}$ $s_{T_3} = 8.27073 \text{ kJ/kg K}$

$$\dot{S}_{gen} = 2(8.27073 - 8.61208) + 1(8.27073 - 6.86926)$$
$$= 0.71877 \text{ kW/K}$$

$$\dot{\Phi}_{destruction} = T_o \dot{S}_{gen} = 298.15 \times 0.71877 = \mathbf{214.3 \text{ kW}}$$

The pressure correction part of the entropy terms cancel out as all three states have the same pressure.

8.91

The condenser in a power plant cools 10 kg/s water at 10 kPa, quality 90% so it comes out as saturated liquid at 10 kPa. The cooling is done by ocean-water coming in at ambient 15°C and returned to the ocean at 20°C. Find the transfer out of the water and the transfer into the ocean-water of both energy and exergy (4 terms).

Solution:
C.V. Water line. No work but heat transfer out.

Energy Eq.: $\dot{Q}_{out} = \dot{m}\,(h_1 - h_2) = 10(2345.35 - 191.81) =$ **21 535 kW**

C.V. Ocean water line. No work but heat transfer in equals water heattransfer out

Energy Eq.: $q = h_4 - h_3 = 83.94 - 62.98 = 20.96$ kJ/kg

$$\dot{m}_{ocean} = \dot{Q}_{out}/q = 21\ 535 / 20.96 = 1027.4 \text{ kg/s}$$

Exergy out of the water follows from a flow term in Eq.8.38

$$\dot{\Phi}_{out} = \dot{m}(\psi_1 - \psi_2) = \dot{m}\,[\,h_1 - h_2 - T_O\,(s_1 - s_2)\,]$$
$$= 10\,[\,2345.35 - 191.81 - 288.15(7.4001 - 0.6492)]$$
$$= \textbf{2082.3 kW}$$

Exergy into the ocean water

$$\dot{\Phi}_{ocean} = \dot{m}_{ocean}(\psi_4 - \psi_3) = \dot{m}_{ocean}\,[\,h_4 - h_3 - T_o(s_4 - s_3)]$$
$$= 1027.4\,[\,20.96 - 288.15(0.2966 - 0.2245)]$$
$$= \textbf{189.4 kW}$$

Notice there is a large amount of energy exchanged but very little exergy.

Often the cooling media flows inside a long pipe carrying the energy away.

8.92

Consider the car engine in Example 5.1 and assume the fuel energy is delivered at a constant 1500 K. The 70% of the energy that is lost is 40% exhaust flow at 900 K and the remainder 30% heat transfer to the walls at 450 K goes on to the coolant fluid at 370 K, finally ending up in atmospheric air at ambient 20°C. Find all the energy and exergy flows for this heat engine. Find also the exergy destruction and where that is done.

From the example in the text we get: $\dot{Q}_L = 0.7\,\dot{Q}_H = 233$ kW

This is separated into two fluxes:

$$\dot{Q}_{L1} = 0.4\,\dot{Q}_H = 133 \text{ kW} \quad @900 \text{ K}$$

$$\dot{Q}_{L2} = 0.3\,\dot{Q}_H = 100 \text{ kW} \quad @450 \text{ K}$$

$$= \dot{Q}_{L3} = 100 \text{ kW} \quad @370 \text{ K}$$

$$= \dot{Q}_{L4} = 100 \text{ kW} \quad @293 \text{ K}$$

Gases	Steel	Glycol	Air flow
1500 K	450 K	370 K	293 K

Radiator

Assume all the fuel energy is delivered at 1500 K then that has an exergy of

$$\dot{\Phi}_{QH} = (1 - \frac{T_o}{T_H})\,\dot{Q}_H = (1 - \frac{293}{1500})\,333 = 267.9 \text{ kW}$$

Work is exergy

$$\dot{\Phi}_W = \dot{W} = 100 \text{ kW}$$

The exhaust flow exergy becomes

$$\dot{\Phi}_{QL1} = (1 - \frac{T_o}{T_{L1}})\,\dot{Q}_H = (1 - \frac{293}{900})\,133 = 89.7 \text{ kW}$$

The heat transfer through the coolant keeps dropping in temperature and are

$$\dot{\Phi}_{QL2} = (1 - \frac{T_o}{T_{L2}}) \, \dot{Q}_{QL2} = (1 - \frac{293}{450}) \, 100 = 34.9 \text{ kW}$$

$$\dot{\Phi}_{QL3} = (1 - \frac{T_o}{T_{L3}}) \, \dot{Q}_{QL3} = (1 - \frac{293}{370}) \, 100 = 20.8 \text{ kW}$$

$$\dot{\Phi}_{QL4} = (1 - \frac{T_o}{T_{L4}}) \, \dot{Q}_{QL4} = (1 - \frac{293}{293}) \, 100 = 0 \text{ kW}$$

The destruction terms are the drops in exergy fluxes. For the inside of the cylinder

$$\dot{\Phi}_{\text{Destruction inside}} = \dot{\Phi}_{QH} - \dot{W} - \dot{\Phi}_{QL1} - \dot{\Phi}_{QL2} = 43.3 \text{ kW}$$

Outside the cylinder, the wall that separates the gas from the coolant gives

$$\dot{\Phi}_{\text{Destruction wall}} = \dot{\Phi}_{QL2} - \dot{\Phi}_{QL3} = 14.1 \text{ kW}$$

The wall in the radiator separating the coolant from the air gives

$$\dot{\Phi}_{\text{Destruction radiator}} = \dot{\Phi}_{QL3} - \dot{\Phi}_{QL4} = 20.8 \text{ kW}$$

8.93

A piston/cylinder has forces on the piston so it keeps constant pressure. It contains 2 kg of ammonia at 1 MPa, 40°C and is now heated to 100°C by a reversible heat engine that receives heat from a 200°C source. Find the work out of the heat engine using the exergy balance equation.

Solution:

To evaluate it we need the change in exergy Eq.8.28

$$\Phi_2 - \Phi_1 = m_{am}(u_2 - u_1) + P_o(V_2 - V_1) - m_{am}T_o(s_2 - s_1)$$

The work ($W = W_{H.E.} + {}_1W_{2,pist}$) is from the integrated exergy Eq.8.38

for a reversible process with no flow terms and use of Eq.8.28

$$W = P_o(V_2 - V_1) + (1 - \frac{T_o}{T_H}){}_1Q_2 - (\Phi_2 - \Phi_1) - 0$$

$$= (1 - \frac{T_o}{T_H}){}_1Q_2 - m_{am}(u_2 - u_1) + m_{am}T_o(s_2 - s_1)$$

Now we must evaluate the three terms on the RHS and the work ${}_1W_{2,pist}$.

State 1: $u_1 = 1369.8$ kJ/kg, $v_1 = 0.13868$ m³/kg, $s_1 = 5.1778$ kJ/kg K

State 2: $u_2 = 1490.5$ kJ/kg, $v_2 = 0.17389$ m³/kg, $s_2 = 5.6342$ kJ/kg K

$${}_1W_{2,pist} = m_{am}P(v_2 - v_1) = 2 \times 1000 (0.17389 - 0.13868) = 70.42 \text{ kJ}$$

C.V. Heat engine and ammonia (otherwise we involve another Q)

Entropy: $m_{am}(s_2 - s_1) = {}_1Q_2/T_H + 0$

$\Rightarrow {}_1Q_2 = T_H m_{am}(s_2 - s_1)$

$= 473.15 \times 2 (5.6342 - 5.1778)$

$= 431.89$ kJ

Substitute this heat transfer into the work term

$$W = (1 - \frac{298.15}{473.15}) \, 431.89 - 2(1490.5{-}1369.8) + 2{\times}298.15(5.6342{-}5.1778)$$

$$= 159.74 - 241.4 + 272.15 = 190.49 \text{ kJ}$$

$$W_{H.E.} = W - {}_1W_{2,pist} = 190.49 - 70.42 = \textbf{120.0 kJ}$$

Borgnakke and Sonntag

8.94

An air-conditioner on a hot summer day removes 8 kW of energy from a house at 21°C and pushes energy to the outside which is at at 31°C. The house has 15 000 kg mass with an average specific heat of 0.95 kJ/kgK. In order to do this the cold side of the air-conditioner is at 5°C and the hot side is at 40°C. The air-conditioner (refrigerator) has a COP that is 60% of a corresponding Carnot refrigerator. Find the actual air-conditioner COP, the power required to run the air-conditioner, the rate of exergy destruction **inside** the air conditioner and the total rate of exergy destruction due to the air-conditioner and house.

Any heat transfer must go from a higher to a lower T domain.

Q leak from atm. 31 C

A steady state refrigerator definition of COP

$$\text{COP} = \beta_{REF} = \dot{Q}_L / \dot{W} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) = 0.6\ \beta_{Carnot}$$

Carnot: $\beta_{Carnot} = \dfrac{T_L}{T_H - T_L} = \dfrac{5 + 273.15}{40 - 5} = 7.95$; $\beta_{REF} = 0.6 \times 7.95 = \mathbf{4.77}$

$$\Rightarrow \qquad \dot{W} = \dot{Q}_L / \beta_{REF} = 8\ \text{kW} / 4.77 = \mathbf{1.68\ kW}$$

$$\dot{Q}_H = \dot{Q}_L + \dot{W} = 9.68\ \text{kW}$$

C.V. Refrigerator (A/C unit).

Energy Eq.: $\qquad \dot{E}_{CV} = 0 = \dot{Q}_L + \dot{W} - \dot{Q}_H$

Entropy Eq.: $\qquad \dot{S}_{CV} = 0 = \dfrac{\dot{Q}_L}{T_L} - \dfrac{\dot{Q}_H}{T_H} + \dot{S}_{gen\ a/c} \qquad \Rightarrow$

$$\dot{\Phi}_{destr.} = T_o\ \dot{S}_{gen\ a/c} = 298.15\ [\ \frac{9.68}{40 + 273.15} - \frac{8}{5 + 273.15}\ ] = \mathbf{0.64\ kW}$$

CV. Total: house + A/C unit, $\dot{Q}_{Leak}$ and $\dot{Q}_H$ comes from and leaves to atm.

Energy Eq.: $\qquad \dot{E}_{CV} = 0 = \dot{Q}_{leak} + \dot{W} - \dot{Q}_H$

Entropy Eq.: $\qquad \dot{S}_{CV} = 0 = \dfrac{\dot{Q}_{leak}}{T_{atm}} - \dfrac{\dot{Q}_H}{T_{atm}} + \dot{S}_{gen\ tot} \qquad \Rightarrow \dot{S}_{gen\ tot} = \dfrac{\dot{W}}{T_{atm}}$

$$\dot{\Phi}_{destr.} = T_o\ \dot{S}_{gen\ tot} = T_o \frac{\dot{W}}{T_{atm}} = 298.15\ \text{K}\ \frac{1.68\ \text{kW}}{304.15\ \text{K}} = \mathbf{1.65\ kW}$$

8.95

If the air-conditioner in the previous problem is turned off how fast does the house heat up in K/sec?

Once the A/C unit is turned off we do not cool the house so heat leaks in from the atm. at a rate of 8 kW (that is what we had to remove to keep steady state).

Energy Eq.:
$$\dot{E}_{CV} = \dot{Q}_{leak} = 8 \text{ kW} = m_{house} \, C_P \frac{dT}{dt}$$

$$\frac{dT}{dt} = \dot{Q}_{leak} / m_{house} \, C_P = \frac{8 \text{ kW}}{15\,000 \times 0.95 \text{ kJ/K}} = \mathbf{0.56 \times 10^{-3} \text{ K/s}}$$

8.96

A disk brake of 2 kg steel and 1 kg brake pads is at 20°C. The brakes are now stopping a car so they dissipate energy by friction and heats up to $T_2 = 200$^{o}C. Assume the brake pads have specific heat of 0.6 kJ/kg-K. After this process the disk and pads now slowly cools to the ambient 20°C $= T_3$. Find the exergy destruction in the braking process (1 to 2) and that in the cooling process (2 to 3).

Control volume around the disk brake assembly.
Energy equation

$$U_2 - U_1 = {}_1Q_2 - {}_1W_2 = - {}_1W_2 = m_{st}C_{st}\,(T_2 - T_1) + m_{pad}C_{pad}\,(T_2 - T_1)$$

$$= (2 \times 0.46 + 1 \times 0.6)\ kJ/K \times (200 - 20)\ K = 273.6\ kJ$$

Entropy equation

$$S_2 - S_1 = 0 + {}_1S_{2\ gen} = {}_1S_{2\ gen} = m_{st}C_{st}\,\ln(T_2/T_1) + m_{pad}C_{pad}\,\ln(T_2/T_1)$$

$$= (2 \times 0.46 + 1 \times 0.6)\,\ln\frac{473}{293} = 0.7277\ kJ/K$$

$${}_1\Phi_{2\ destruction} = T_o\ {}_1S_{2\ gen} = 298.15\ K \times 0.7277\ kJ/K = \mathbf{217\ kJ}$$

The cooling process 2 to 3, extend CV out to ambient T_o.

$$U_3 - U_2 = {}_2Q_3 = - 273.6\ kJ \quad [\ = - (U_2 - U_1)\ ]$$

$${}_2S_{3\ gen} = S_3 - S_2 - {}_2Q_3\,/\,T_o = - 0.7277 - \frac{-273.6}{293} = 0.2056\ kJ/K$$

$${}_2\Phi_{3\ destruction} = T_o\ {}_2S_{3\ gen} = 298.15\ K \times 0.2056\ kJ/K = \mathbf{61.3\ kJ}$$

Brake pads

Hydraulic line, fluid acts on piston that pushes the brake pad.

Disk

Clip-art from Microsoft.

8.97

A small house kept at 20$^{\text{o}}$C inside loses 12 kW to the outside ambient at 0$^{\text{o}}$C. A heat pump is used to help heat the house together with possible electrical heat. The heat pump is driven by a motor of 2.5 kW and it has a COP that is one quarter of a Carnot heat pump unit. Find the actual heat pump COP and the exergy destruction in the whole process.

CV. House

Energy: $0 = \dot{Q}_H + \dot{W}_{el.} - \dot{Q}_{Loss}$

Definition of COP: $\beta' = COP_{HP} = \dfrac{\dot{Q}_H}{\dot{W}} = \dfrac{1}{4}\dfrac{T_H}{T_H - T_L} = \dfrac{1}{4}\dfrac{293.15}{20-0} = \textbf{3.664}$

$$\dot{Q}_H = COP_{HP}\ \dot{W}_{HP} = 3.664 \times 2.5\ \text{kW} = 9.16\ \text{kW}$$

$$\dot{W}_{el.} = \dot{Q}_{Loss} - \dot{Q}_H = 12 - 9.16 = 2.84\ \text{kW}$$

C.V. Total   Energy: $0 = \dot{Q}_L + \dot{W}_{HP} + \dot{W}_{el.} - \dot{Q}_{Loss}$

$$\dot{Q}_{Loss} - \dot{Q}_L = \dot{W}_{HP} + \dot{W}_{el.} = 2.5 + 2.84 = 5.34\ \text{kW}$$

Entropy: $0 = \dot{Q}_L/T_L - \dot{Q}_{loss}/T_L + \dot{S}_{gen}$

$\dot{\Phi}_{destr.} = T_o\ \dot{S}_{gen} = T_o\ (\dot{Q}_{loss} - \dot{Q}_L)\ /T_L = 273.15\ \dfrac{5.34}{273.15} = \textbf{5.34 kW}$

Comment: We must use $T_o = 273.15$ K here as the $\dot{Q}_L$ and $\dot{Q}_{loss}$ were exchanged with the ambient. Otherwise the destruction becomes larger than the supplied work terms because the exergy exchanged with the ambient is non-zero

8.98

A farmer runs a heat pump using 2 kW of power input. It keeps a chicken hatchery at a constant 30°C while the room loses 10 kW to the colder outside ambient at 10°C. Find the COP of the heat pump, the rate of exergy destruction in the heat pump and its heat exchangers, and the rate of exergy destruction in the heat loss process.

C.V. Hatchery, steady state at 30°C for the hatchery

Energy Eq.: $\quad 0 = \dot{Q}_H - \dot{Q}_{Loss} \quad\quad \Rightarrow \quad\quad \dot{Q}_H = \dot{Q}_{Loss} = 10 \text{ kW}$

$$COP = \dot{Q}_H / \dot{W} = 10\text{kW} / 2 \text{ kW} = \mathbf{5}$$

C.V. Heat pump, steady state

Energy eq.: $\quad 0 = \dot{Q}_L + \dot{W} - \dot{Q}_H \quad \Rightarrow \quad \dot{Q}_L = \dot{Q}_H - \dot{W} = 8 \text{ kW}$

Entropy Eq.: $\quad 0 = \dfrac{\dot{Q}_L}{T_L} - \dfrac{\dot{Q}_H}{T_H} + \dot{S}_{gen \, HP}$

$$\dot{S}_{gen \, HP} = \dfrac{\dot{Q}_H}{T_H} - \dfrac{\dot{Q}_L}{T_L} = \dfrac{10}{273+30} - \dfrac{8}{273+10} = 0.00473 \text{ kW/K}$$

$$\dot{\Phi}_{destr. \, HP} = T_o \, \dot{S}_{gen \, HP} = 283.15 \text{ K} \times 0.00473 \text{ kW/K} = \mathbf{1.34 \text{ kW}}$$

C.V. From hatchery at 30°C to the ambient 10°C. This is typically the walls and the outer thin boundary layer of air. Through this goes $\dot{Q}_{Loss}$.

Entropy Eq.: $\quad 0 = \dfrac{\dot{Q}_{Loss}}{T_H} - \dfrac{\dot{Q}_{Loss}}{T_{amb}} + \dot{S}_{gen \, walls}$

$$\dot{S}_{gen \, walls} = \dfrac{\dot{Q}_{Loss}}{T_{amb}} - \dfrac{\dot{Q}_{Loss}}{T_H} = \dfrac{10}{283} - \dfrac{10}{303} = 0.00233 \text{ kW/K}$$

$$\dot{\Phi}_{destr. \, walls} = T_o \, \dot{S}_{gen \, HP} = 283.15 \text{ K} \times 0.00233 \text{ kW/K} = \mathbf{0.66 \text{ kW}}$$

Comment: Notice the two exergy destruction terms exactly equals work input. It was necessary to use T_o as ambient T otherwise the flux of exergy associated with the heat transfers $\dot{Q}_L$ and $\dot{Q}_{Loss}$ would be non-zero and a storage effect present.

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$\dot{W} = 2$ kW

$\dot{Q}_L$

HP

$\dot{Q}_H$

$\dot{Q}_{loss}$

cb

Device Second-Law Efficiency

8.99

A heat engine receives 1 kW heat transfer at 1000 K and gives out 400 W as work with the rest as heat transfer to the ambient. Find its first and second law efficiencies.

First law efficiency is based on the energies

$$\eta_I = \dot{W}/\dot{Q}_H = \frac{0.4}{1} = \mathbf{0.4}$$

The second law efficiency is based on work out versus exergy in

Exergy flux in: $\dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right)\dot{Q}_H = \left(1 - \frac{298.15}{1000}\right) 1 \text{ kW} = 0.702 \text{ kW}$

$$\eta_{II} = \frac{\dot{W}}{\dot{\Phi}_H} = \frac{0.4}{0.702} = \mathbf{0.57}$$

Notice the exergy flux in is equal to the Carnot heat engine power output given 1 kW at 1000 K and rejecting energy to the ambient.

8.100

A heat exchanger increases the exergy of 3 kg/s water by 1650 kJ/kg using 10 kg/s air coming in at 1400 K and leaving with 600 kJ/kg less exergy. What are the irreversibility and the second law efficiency?

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside.

The irreversibility is the destruction of exergy so

$$\dot{I} = \dot{\Phi}_{destruction} = \dot{\Phi}_{in} - \dot{\Phi}_{out} = 10 \times 600 - 3 \times 1650 = \textbf{1050 kW}$$

The second law efficiency, Eq.8.32

$$\eta_{II} = \dot{\Phi}_{out} / \dot{\Phi}_{in} = \frac{3 \times 1650}{10 \times 600} = \textbf{0.825}$$

8.101

Find the second law efficiency of the heat pump in problem 8.57.

The second law efficiency is a ratio of exergies namely what we want out divided by what we have to put in. Exergy from first term on RHS Eq. 8.38

$$\dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right)\dot{Q}_H; \qquad \dot{Q}_H = \beta\,\dot{W} = 2 \times 2 \text{ kW} = 4 \text{ kW}$$

$$\eta_{II} = \frac{\dot{\Phi}_H}{\dot{W}} = \left(1 - \frac{T_o}{T_H}\right)\frac{\dot{Q}_H}{\dot{W}} = \left(1 - \frac{298.15}{353.15}\right)\frac{4}{2} = \mathbf{0.31}$$

80 °C

$\dot{Q}_H$

HP

$\dot{W} = 2$ kW

$\dot{Q}_L$

T_o

8.102

A steam turbine inlet is at 1200 kPa, 500°C. The actual exit is at 300 kPa with an actual work of 407 kJ/kg. What is its second law efficiency?

The second law efficiency is the actual work out measured relative to the reversible work out, Eq. 8.29.

Steam turbine $\quad T_o = 25°C = 298.15$ K

Inlet state: Table B.1.3 $\quad h_i = 3476.28$ kJ/kg; $\quad s_i = 7.6758$ kJ/kg K

Actual turbine energy Eq.: $\quad h_e = h_i - w_{ac} = 3476.28 - 407 = 3069.28$ kJ/kg

Actual exit state: Table B.1.3 $\quad T_e = 300°C; \quad s_e = 7.7022$ kJ/kg K

From Eq.8.29 for steady state and adiabatic process,

$$w^{rev} = (h_i - T_o s_i) - (h_e - T_o s_e) = (h_i - h_e) + T_o(s_e - s_i)$$
$$= (3476.28 - 3069.28) + 298.15(7.7022 - 7.6758)$$
$$= 407 + 7.87 = 414.9 \text{ kJ/kg}$$

$$\eta_{II} = w_{ac}/w^{rev} = 407 / 414.9 = \mathbf{0.98}$$

8.103

Find the second law efficiency for the compressor in problem 8.21.
An air compressor takes air in at the state of the surroundings 100 kPa, 300 K. The air exits at 400 kPa, 200°C using 100 kW of power. Determine the minimum compressor work input.

C.V. Compressor, Steady flow, minimum work in is reversible work.

Energy Eq.: $\quad 0 = h_1 - h_2 - w_c \quad =>$

$$- w_c = h_2 - h_1 = 475.8 - 300.47 = 175.33 \text{ kJ/kg}$$

$\psi_1 = 0$ at ambient conditions

Get the properties from the air table A.7.1 and correct standard entropy for the pressure

$$s_0 - s_2 = s_{T_0}^o - s_{T_2}^o - R \ln(P_0/P_2)$$

$$= 6.86926 - 7.3303 - 0.287 \ln(100/400) = -0.06317 \text{ kJ/kg K}$$

$$\psi_2 = h_2 - h_0 + T_0(s_0 - s_2) = 475.79 - 300.473 + 300\,(-0.06317)$$

$$= 156.365 \text{ kJ/kg}$$

$$-w^{REV} = (\psi_2 - \psi_1) = (156.365 - 0)\text{ kJ/kg} = 156.365 \text{ kJ/kg}$$

2nd law efficiency:

$$\eta_{II} = \frac{w_{rev}}{w_{ac}} = \frac{-156.365}{-175.33} = \mathbf{0.892} \quad (\,= \frac{w_{ac} + i}{w_{ac}}\,)$$

8.104

Find the isentropic efficiency and the second law efficiency for the compressor in Problem 8.26.

A compressor in a refrigerator receives R-410A at 150 kPa, -40°C and it brings it up to 600 kPa, using an actual specific shaft work of 58.65 kJ/kg in an adiabatic compression. Find the specific reversible work.

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.: $\quad 0 = h_1 - h_2 - w$

Entropy Eq.7.8: $\quad\quad s_2 = s_1 + s_{gen}$

State 1: B.4.2 $\quad h_1 = 263.99$ kJ/kg, $\quad s_1 = 1.1489$ kJ/kg K

$\quad\quad\quad\quad\quad h_{2,AC} = h_1 - w = 263.99 + 58.65 = 322.64$ kJ/kg

State 2ac: B.4.2 $\quad$ P, h_2 => $\quad s_2 = 1.2152$ kJ/kg K

State 2s: $\quad$ B.4.2 $\quad$ (P, s = s_1) $\quad h_{2,s} = 302.67$ kJ/kg

We have two different compressors:

    IDEAL: ACTUAL:

    $-w_{c,s} = h_{2,s} - h_1 = 38.68$ kJ/kg $\quad\quad\quad -w_{C,AC} = h_{2,AC} - h_1 = 58.65$ kJ/kg

Definition Eq.7.27: $\quad\quad\quad\quad\quad \eta_c = w_{c,s}/w_{c,AC} = \textbf{0.659}$

Rev. work Eq.8.29: $\quad w_{rev} = \psi_1 - \psi_{2,AC} = h_1 - h_{2,AC} - T_0(s_1 - s_{2,AC})$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad = -58.65 - 298.15\,(1.1489 - 1.2152)$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad = -58.65 + 19.767 = -38.9$ kJ/kg

2nd law efficiency: $\quad\quad\quad \eta_{II} = \dfrac{w_{rev}}{w_{ac}} = \dfrac{-38.9}{-58.65} = \textbf{0.663} \quad (= \dfrac{w_{ac} + i}{w_{ac}})$

8.105

A steam turbine has inlet at 4 MPa, 500°C and actual exit of 100 kPa, x = 1.0.
Find its first law (isentropic) and its second law efficiencies.

Solution:

C.V. Steam turbine

Energy Eq.4.13: $w = h_i - h_e$

Entropy Eq.7.9: $s_e = s_i + s_{gen}$

Inlet state: Table B.1.3 $h_i = 3445.2$ kJ/kg; $s_i = 7.0900$ kJ/kg K

Exit (actual) state: Table B.1.2 $h_e = 2675.5$; $s_e = 7.3593$ kJ/kg K

Actual turbine energy equation

$$w = h_i - h_e = 769.7 \text{ kJ/kg}$$

Ideal turbine reversible process so $s_{gen} = 0$ giving

$$s_{es} = s_i = 70900 = 1.3025 + x_{es} \times 6.0568$$

$$x_{es} = 0.9555, \quad h_{es} = 417.4 + 0.9555 \times 2258.0 = 2575.0 \text{ kJ/kg}$$

The energy equation for the ideal gives

$$w_s = h_i - h_{es} = 870.2 \text{ kJ/kg}$$

The first law efficiency is the ratio of the two work terms

$$\eta_s = w/w_s = \mathbf{0.885}$$

The reversible work for the actual turbine states is, Eq.8.29

$$w^{rev} = (h_i - h_e) + T_o(s_e - s_i)$$

$$= 769.7 + 298.2(7.3593 - 7.0900)$$

$$= 769.7 + 80.3 = 850.0 \text{ kJ/kg}$$

Second law efficiency Eq.8.31

$$\eta_{2^{nd} \text{ Law}} = w/w_{rev} = 769.7/850.0 = \mathbf{0.906}$$

8.106

Find the second law efficiency for the compressed air system in Problem 8.65. Consider the total system from the inlet to the final point of use.

In the total system the exergy increase in the air from inlet to final exit point is the gain and the source is the compressor work input.

Compressor. Reversible adiabatic, constant s: $s_2 = s_1$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300 \left(\frac{600}{100}\right)^{0.2857} = 500.5 \text{ K}$$

$$\psi_2 - \psi_1 = (h_2 - h_1) - T_0(s_2 - s_1) = h_2 - h_1 = C_{P0}(T_2 - T_1)$$
$$= 1.004 \,(500.5 - 300) = 201.3 \text{ kJ/kg} = -w_C$$

From inlet, state 1 to final point of use state 3.

$$\psi_3 - \psi_1 = (h_3 - h_1) - T_0(s_3 - s_1) = 0 - T_0[0 - R \ln(P_3/P_1)]$$
$$= T_0 \, R \, \ln(P_3/P_1) = 298.15 \times 0.287 \times \ln (600/100)$$
$$= 153.3 \text{ kJ/kg}$$
$$\eta_{II} = \frac{\psi_3 - \psi_1}{\psi_2 - \psi_1} = \frac{153.3}{201.3} = \mathbf{0.76}$$

8.107

A turbine receives steam at 3000 kPa, 500°C and has two exit flows, one at 1000 kPa, 350°C with 20% of the flow and the remainder at 200 kPa, 200°C. Find the isentropic and the second law efficiencies.

C.V. Steam turbine (x = 0.2 = extraction fraction)

Energy Eq.4.13: $\quad\quad w = h_1 - xh_2 - (1 - x)h_3$

Entropy Eq.7.9: $\quad\quad s_e = s_1 + s_{gen}$

Inlet state: Table B.1.3 $\quad\quad h_1 = 3456.48$ kJ/kg; $\quad s_1 = 7.2337$ kJ/kg K

Extraction state: $\quad\quad h_2 = 3157.65$ kJ/kg, $\quad s_2 = 7.3010$ kJ/kg K

Exit (actual) state: Table B.1.3 $\quad\quad h_3 = 2870.46$; $\quad s_3 = 7.5066$ kJ/kg K

Actual turbine energy equation

$$w = 3456.48 - 0.2 \times 3157.65 - 0.8 \times 2870.46 = 528.58 \text{ kJ/kg}$$

Ideal turbine reversible process so $s_{gen} = 0$ giving

$$s_{2s} = s_1 = 7.2337 => h_{2s} = 3117.43 \text{ kJ/kg}$$

$$s_{3s} = s_1 = 7.2337 => h_{3s} = 2750.12 \text{ kJ/kg}$$

The energy equation for the ideal turbine gives

$$w_s = h_1 - xh_{2s} - (1 - x)h_{3s}$$

$$= 3456.48 - 0.2 \times 3117.43 - 0.8 \times 2750.12 = 632.9 \text{ kJ/kg}$$

The first law efficiency: $\quad\quad \eta_s = w/w_s = \mathbf{0.835}$

The reversible work for the actual turbine states is, Eq.8.29

$$w^{rev} = [h_1 - xh_2 - (1 - x)h_3] - T_o[s_1 - xs_2 - (1 - x)s_3]$$

$$= 528.58 - 298.2(7.2337 - 0.2 \times 7.301 - 0.8 \times 7.5066)$$

$$= 528.58 + 69.1 = 597.68 \text{ kJ/kg}$$

Second law efficiency Eq.8.31

$$\eta_{II} = w/w_{rev} = 528.58/597.68 = \mathbf{0.884}$$

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8.108

Steam enters a turbine at 25 MPa, 550°C and exits at 5 MPa, 325°C at a flow rate of 70 kg/s. Determine the total power output of the turbine, its isentropic efficiency and the second law efficiency.

Solution:

$$h_i = 3335.6 \text{ kJ/kg}, \quad s_i = 6.1765 \text{ kJ/kg K},$$

$$h_e = 2996.5 \text{ kJ/kg}, \quad s_e = 6.3289 \text{ kJ/kg K}$$

Actual turbine: $w_{T,ac} = h_i - h_e = 339.1 \text{ kJ/kg}$

$$\dot{W} = \dot{m}\, w_{T,ac} = 70 \times 339.1 = \mathbf{23\ 740\ kW}$$

Isentropic turbine has a different exit state: $s_{e,s} = s_i \implies h_{e,s} = 2906.6 \text{ kJ/kg}$

$$w_{T,s} = h_i - h_{e,s} = 429 \text{ kJ/kg}$$

Rev. turbine has the same exit state as the actual turbine:

$$w_{rev} = w_{T,ac} + T_0(s_e - s_i) = 339.1 + 45.44 = 384.54 \text{ kJ/kg}$$

Eq.7.27: $\eta_T = w_{T,ac}/w_{T,s} = 339.1/429 = \mathbf{0.79}$

Eq.8.31: $\eta_{II} = w_{T,ac}/w_{rev} = 339.1/384.54 = \mathbf{0.88}$

8.109

A heat engine operating with an evironment at 298 K produces 5 kW of power output with a first law efficiency of 50%. It has a second law efficiency of 80% and T_L = 310 K. Find all the energy and exergy transfers in and out.

Solution:

From the definition of the first law efficiency (conversion efficiency)

$$\dot{Q}_H = \dot{W} / \eta_I = \frac{5}{0.5} = \textbf{10 kW}$$

Energy Eq.: $\quad \dot{Q}_L = \dot{Q}_H - \dot{W} = 10 - 5 = \textbf{5 kW}$

Exergy output: $\qquad \dot{\Phi}_W = \dot{W} = \textbf{5 kW}$

From the definition of the second law efficiency $\eta_{II} = \dot{W}/\dot{\Phi}_H$, this requires that we assume the exergy rejected at 310 K is lost and not counted, otherwise the

efficiency should be $\eta_{II} = \dot{W}/(\dot{\Phi}_H - \dot{\Phi}_L)$.

Exergy from source: $\qquad \dot{\Phi}_H = (1 - \dfrac{T_o}{T_H}) \, \dot{Q}_H = \dot{W}/\eta_{II} = \dfrac{5}{0.8} = \textbf{6.25 kW}$

Exergy rejected: $\qquad \dot{\Phi}_L = (1 - \dfrac{T_o}{T_L}) \, \dot{Q}_L = (1 - \dfrac{298}{310}) \, 5 = \textbf{0.194 kW}$

Notice from the $\dot{\Phi}_H$ form we could find the single characteristic T_H as

$$(1 - \frac{T_o}{T_H}) = 6.25 \text{ kW} / \dot{Q}_H = 0.625 \quad => \quad T_H = 795 \text{ K}$$

8.110

A flow of nitrogen, 0.1 kg/s, comes out of a compressor stage at 500 kPa, 500 K and is now cooled to 310 K in a counterflowing intercooler by liquid water at 125 kPa, 15^oC which leaves at 22^oC. Find the flowrate of water and the second law efficiency for the heat exchanger.

Solution:

Continuity eq.: $\dot{m}_1 = \dot{m}_2 = \dot{m}_{H2O};$ $\dot{m}_3 = \dot{m}_4 = \dot{m}_{N2}$

Energy eq.: $0 = \dot{m}_{N2} (h_3 - h_4) + \dot{m}_{H2O} (h_2 - h_1)$

Entropy Eq.7.7: $0 = \dot{m}_{N2} (s_3 - s_4) + \dot{m}_{H_2O} (s_2 - s_1) + \dot{S}_{gen}$

Due to the lower range of temperature we will use constant specific heats from A.5 and A.4 so the energy equation is approximated as

$$0 = \dot{m}_{H2O}C_{pH2O} (T_2 - T_1) + \dot{m}_{N2}C_p (T_3 - T_4)$$

Now solve for the water flow rate

$\dot{m}_{H2O} = \dot{m}_{N2} C_{pN2} (T_3 - T_4) / [C_{pH2O} (T_1 - T_2)]$

$= 0.1$ kg/s $\times 1.042$ kJ/kgK $\times (500 - 310)$ K $/ [4.18 \times (22 - 15)$ kJ/kg]

$= \textbf{0.677 kg/s}$

The second law efficiency is the exergy gain over the exergy source so we need exergy change in both flows.

$\dot{m}_{N2} (\psi_4 - \psi_3) = \dot{m}_{N2}C_P [T_4 - T_3 - T_0 \ln (T_4/T_3)]$

$= 0.1$ kg/s $\times 1.042$ kJ/kgK $\times [310 - 500 - 298.15 \ln \dfrac{310}{500}]$ K $= -4.95$ kW

$\dot{m}_{H_2O} (\psi_1 - \psi_2) = \dot{m}_{H_2O}C_P [T_1 - T_2 - T_0 \ln (T_1/T_2)]$

$= 0.677$ kg/s $\times 4.18$ kJ/kgK $[22 - 15 - 298.15 \ln(\dfrac{295}{288})]$ K $= -0.45$ kW

The nitrogen loses exergy as it cools, but the water also loses why? The water is brought closer to T_0 as it is being heated up which is closer to zero exergy. Both exergies drops and equals total exergy destruction $(T_0 S_{gen} = 5.4$ kW$)$ $\eta_{II} < 0$!.

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A hydraulic motor driven compressor, intercooler in pipe between the two stages.

8.111

Air flows into a heat engine at ambient conditions 100 kPa, 300 K, as shown in Fig. P8.111. Energy is supplied as 1200 kJ per kg air from a 1500 K source and in some part of the process a heat transfer loss of 300 kJ/kg air happens at 750 K. The air leaves the engine at 100 kPa, 800 K. Find the first and the second law efficiencies.

C.V. Engine out to reservoirs

$$h_i + q_{1500} = q_{750} + h_e + w$$

$$w_{ac} = h_i + q_{1500} - q_{750} - h_e$$

$$= 300.47 + 1200 - 300 - 822.20 = 378.27 \text{ kJ/kg}$$

$$\eta_{TH} = w/q_{1500} = \textbf{0.3152}$$

For second law efficiency also a q to/from ambient

$$s_i + (q_{1500}/T_H) + (q_0/T_0) = (q_{750}/T_m) + s_e$$

$$q_0 = T_0(s_e - s_i) + (T_0/T_m)q_{750} - (T_0/T_H)q_{1500}$$

$$= 300\left(7.88514 - 6.86925 - 0.287 \ln\frac{100}{100}\right) + \frac{300}{750} 300$$

$$-(300/1500)\ 1200 = 184.764 \text{ kJ/kg}$$

$$w^{rev} = h_i - h_e + q_{1500} - q_{750} + q_0 = w_{ac} + q_0 = 563.03 \text{ kJ/kg}$$

$$\eta_{II} = w_{ac}/w_{rev} = 378.27/563.03 = \textbf{0.672}$$

Comment: You could also have defined the second law efficiency as

Exergy in: $\varphi_H = (1-T_0/T_H)\ q_{1500} = 0.80 \times 1200 = 960 \text{ kJ/kg}$

$$\eta_{II} = w_{ac}/\varphi_H = 378.27 / 960 = 0.394$$

This measure places zero value of the loss at 750 K, zero exergy value for the two flow terms.

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8.112

Air enters a compressor at ambient conditions, 100 kPa, 300 K, and exits at 800 kPa. If the isentropic compressor efficiency is 85%, what is the second-law efficiency of the compressor process?

Solution:

Ideal (isentropic, Eq.6.23)

$$T_{2s} = 300(8)^{0.286} = 543.8 \text{ K}$$
$$-w_s = 1.004(543.8 - 300) = 244.6 \text{ kJ/kg}$$
$$-w = \frac{-w_s}{\eta_s} = \frac{244.6}{0.85} = 287.8 \text{ kJ/kg}$$

Actual exit temperature

$$T_2 = T_1 + \frac{-w}{C_{P0}} = 300 + \frac{287.8}{1.004} = 586.8 \text{ K}$$

Eq.6.16: $s_2 - s_1 = 1.004 \ln(586.8/300) - 0.287 \ln 8 = 0.07645 \text{ kJ/kgK}$

Exergy, Eq.8.23

$\psi_2 - \psi_1 = (h_2 - h_1) - T_0(s_2 - s_1) = 287.8 - 300(0.07645) = 264.9 \text{ kJ/kg}$

2nd law efficiency, Eq.8.32 or 8.34 (but for a compressor):

$$\eta_{II} = \frac{\psi_2 - \psi_1}{-w} = \frac{264.9}{287.8} = \mathbf{0.92}$$

8.113

Consider that the air in the previous problem after the compressor flows in a pipe to an air tool and at that point the temperature has dropped to ambient 300 K and an air pressure of 750 kPa. What is the second law efficiency for the total system?

C.V. Compressor

Energy: $0 = h_1 - h_2 + w_C$;   Entropy: $0 = s_1 - s_2 + 0$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300 \left(\frac{800}{100}\right)^{0.2857} = 543.4 \text{ K}$$

Exergy increase through the compressor matches with the ideal compressor work

$$w_{C\,s} = h_2 - h_1 = C_P \, (T_2 - T_1) = 1.004 \, (543.4 - 300) \text{ kJ/kg} = 244.4 \text{ kJ/kg}$$

The actual compressor work is

$$w_{C\,ac} = w_{C\,s} \, / \, \eta_{C\,s} = 244.4/0.85 = 287.5 \text{ kJ/kg}$$

From inlet, state 1 to final point of use state 3.

$$\psi_3 - \psi_1 = (h_3 - h_1) - T_0(s_3 - s_1) = 0 - T_0[0 - R \ln(P_3/P_1)]$$

$$= T_0 \, R \ln(P_3/P_1) = 298.15 \text{ K} \times 0.287 \text{ kJ/kgK} \times \ln (750/100)$$

$$= 172.4 \text{ kJ/kg}$$

So then the second law efficiency is the gain $(\psi_3 - \psi_1)$ over the source $w_{C\,ac}$ as

$$\eta_{II} = \frac{\psi_3 - \psi_1}{w_{C\,ac}} = \frac{172.4}{287.5} = \mathbf{0.6}$$

Borgnakke and Sonntag

8.114

A compressor is used to bring saturated water vapor at 1 MPa up to 15 MPa, where the actual exit temperature is 650°C. Find the irreversibility and the second-law efficiency.

Solution:

Inlet state: Table B.1.2 $h_i = 2778.08$ kJ/kg, $s_i = 6.5864$ kJ/kg K

Actual compressor Table B.1.3: $h_{e,ac} = 3712.32$ kJ/kg, $s_{e,ac} = 6.8223$ kJ/kg K

Energy Eq. Actual compressor: $-w_{c,ac} = h_{e,ac} - h_i = 934.2$ kJ/kg

From Eq.8.13: $i = T_0(s_{e,ac} - s_i) = 298.15 (6.8223 - 6.5864) = $ **70.33 kJ/kg**

From Eq.8.15: $w_{rev} = i + w_{c,ac} = 70.33 - 934.2 = -863.87$ kJ/kg

$\eta_{II} = -w_{rev}/w_{c,ac} = 863.87/934.2 = $ **0.925**

8.115

Use the exergy equation to analyze the compressor in Example 4.8 to find its second law efficiency assuming an ambient at 20$^\text{o}$C.

C.V. The R-134a compressor. Steady flow. We need to find the reversible work and compare that to the actual work.

Notice the heat loss goes out to ambient T and has thus 0 exergy.

Exergy eq.: 8.38: $0 = \dot{m}(\psi_1 - \psi_2) + (-\dot{W}_{comp}^{rev}) + 0$

$$-\dot{W}_{comp}^{rev} = \dot{m} \, [\, h_2 - h_1 - T_O \, (\, s_2 - s_1 \,) \,]$$

$$= 0.1 \text{ kg/s } [435.1 - 387.2 \, - 293.15 \text{ K} \times (1.7768 - 1.7665) \frac{\text{kJ}}{\text{kg K}}]$$

$$= 4.5 \text{ kW}$$

$$\eta_{II} = -\dot{W}_{comp}^{rev} / -\dot{W}_{comp}^{ac} = \frac{4.5}{5} = \textbf{0.90}$$

For a real device this is a little high.

8.116

Calculate the second law efficiency of the coflowing heat exchanger in Problem 7.113 with an ambient at 17°C.

Solution:

C.V. Heat exchanger, steady 2 flows in and two flows out.

First solve for the exit temperature in Problem 7.113

C.V. Heat exchanger, steady 2 flows in and two flows out.

Energy Eq.4.10: $\dot{m}_{O2}h_1 + \dot{m}_{N2}h_3 = \dot{m}_{O2}h_2 + \dot{m}_{N2}h_4$

Same exit tempearture so $T_4 = T_2$ with values from Table A.5

$$\dot{m}_{O2}C_{P\,O2}T_1 + \dot{m}_{N2}C_{P\,N2}T_3 = (\dot{m}_{O2}C_{P\,O2} + \dot{m}_{N2}C_{P\,N2})T_2$$

$$T_2 = \frac{0.5 \times 0.922 \times 290 + 0.6 \times 1.042 \times 500}{0.5 \times 0.922 + 0.6 \times 1.042} = \frac{446.29}{1.0862} = 410.9 \text{ K}$$

The second law efficiency for a heat exchanger is the ratio of the exergy gain by one fluid divided by the exergy drop in the other fluid. We thus have to find the change of exergy in both flows. For each flow exergy in Eq.8.23 include mass flow rate as in Eq.8.38

For the oxygen flow:

$$\dot{m}_{O2}(\psi_2 - \psi_1) = \dot{m}_{O2}[\,h_2 - h_1 - T_o(\,s_2 - s_1\,)\,]$$

$$= \dot{m}_{O2}[\,C_P(T_2 - T_1) - T_o[\,C_P\ln(T_2/T_1) - R\ln(P_2/P_1)\,]$$

$$= \dot{m}_{O2}C_P[\,T_2 - T_1 - T_o\ln(T_2/T_1)\,]$$

$$= 0.5 \times 0.922[\,410.9 - 290 - 290\ln(410.9/290)\,] = 9.148 \text{ kW}$$

For the nitrogen flow

$$\dot{m}_{N2}(\psi_3 - \psi_4) = \dot{m}_{N2}C_P[\,T_3 - T_4 - T_o\ln(T_3/T_4)\,]$$

$$= 0.6 \times 1.042[\,500 - 410.9 - 290\ln(500/410.9)\,] = 20.122 \text{ kW}$$

From Eq.8.32

$$\eta_{2^{nd}\,Law} = \frac{\dot{m}_{O2}(\psi_1 - \psi_2)}{\dot{m}_{N2}(\psi_3 - \psi_4)} = \frac{9.148}{20.122} = \mathbf{0.513}$$

Borgnakke and Sonntag

8.117

An air-compressor receives air at 290 K, 100 kPa and brings it up to a higher pressure in an adiabatic process. The actual specific work is 210 kJ/kg and the isentropic efficiency is 82%. Find the exit pressure and the second law efficiency.

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.4.13: $\qquad w = h_1 - h_2$

Entropy Eq.7.9: $\qquad s_2 = s_1 + s_{gen}$

We have two different cases, the ideal and the actual compressor.

$$-w_{C,AC} = h_{2,AC} - h_1 = 210 \text{ kJ/kg} = w_{c,s} / \eta_c$$

$$T_{2,AC} = T_1 - w_{C,AC} / C_P = 290 + 210 / 1.004 = 499.2 \text{ K}$$

For the isentropic compressor we get

$$-w_{c,s} = -\eta_c \, w_{C,AC} = 0.82 \times 210 = 172.2 \text{ kJ/kg} = h_{2,s} - h_1$$

$$T_{2,s} = T_1 - w_{c,s}/ C_P = 290 + 172.2/1.004 = 461.5 \text{ K}$$

$$P_2 = P_1 \, (T_{2,s} / T_1)^{k/k-1} = 100 \, (461.5/290)^{3.5} = \mathbf{508.4 \text{ kPa}}$$

Now the

$$w^{rev} = (\psi_1 - \psi_{2,AC}) = (h_1 - h_{2,AC}) + T_o(s_{2,AC} - s_1)$$

$$= w_{C,AC} + T_o[\, C_P \ln(T_{2,ac} / T_1) - R \ln (P_2 / P_1)]$$

$$= -210 + 298.15 \, [\, 1.004 \ln(\frac{499.2}{290}) - 0.287 \ln(\frac{508.4}{100}) \,]$$

$$= -210 + 23.4 = - 186.6 \text{ kJ/kg}$$

$$\eta_{II} = w^{rev} / w_{C,AC} = \frac{-186.6}{-210} = \mathbf{0.88}$$

8.118

A flow of 2 kg/s water at 1000 kPa, 80°C goes into a constant pressure boiler where the water is heated to 400°C. Assume the hot gases that heats the water is air coming in at 1200 K and leaving at 900 K as in a counter flowing heat exchanger. Find the total rate of irreversibility in the process and the second law efficiency of the boiler setup.

Solution:

C.V. Heat exchanger, steady flow 1 inlet and
1 exit for air and water each. The two flows
exchange energy with no heat transfer
to/from the outside. We need to find the air
mass flow rate.

Energy Eq.: $\dot{m}_{H2O}(h_2 - h_1) = \dot{m}_{air}(h_3 - h_4)$

$$\dot{m}_{air} = \dot{m}_{H2O} \frac{h_2 - h_1}{h_3 - h_4} = 2 \frac{3263.88 - 335.64}{1277.81 - 933.15} = 16.992 \text{ kg/s}$$

Exergy increase of the water flow

$$\dot{m}_{H2O}(\psi_2 - \psi_1) = \dot{m}_{H2O}[h_2 - h_1 - T_o(s_2 - s_1)]$$
$$= 2 [3263.88 - 335.64 - 298.15(7.465 - 1.0746)]$$
$$= 2 [2928.24 - 1905.3] = 2045.9 \text{ kW}$$

Exergy decrease of the air flow

$$\dot{m}_{air}(\psi_3 - \psi_4) = \dot{m}_{air}[h_3 - h_4 - T_o(s_3 - s_4)]$$
$$= 16.992 [1277.81 - 933.15 - 298.15(8.34596 - 8.01581)]$$
$$= 4183.9 \text{ kW}$$

The irreversibility is the destruction of exergy

$$\dot{I} = \dot{\Phi}_{destruction} = \dot{\Phi}_{in} - \dot{\Phi}_{out} = 4183.9 \text{ kW} - 2045.9 \text{ kW} = \textbf{2138 kW}$$

$$\eta_{2^{nd} \text{ Law}} = \frac{\dot{m}_{H2O}(\psi_2 - \psi_1)}{\dot{m}_{air}(\psi_3 - \psi_4)} = \frac{2045.9}{4183.9} = \textbf{0.489}$$

8.119

A heat exchanger brings 10 kg/s water from 100^oC to 500^oC at 2000 kPa using air coming in at 1400 K and leaving at 460 K. What is the second law efficiency?

Solution:

C.V. Heat exchanger, steady flow 1 inlet and
1 exit for air and water each. The two flows
exchange energy with no heat transfer
to/from the outside. We need to find the air
mass flow rate.

Energy Eq.: $\dot{m}_{H2O}(h_2 - h_1) = \dot{m}_{air}(h_3 - h_4)$

$$\dot{m}_{air} = \dot{m}_{H2O}\frac{h_2 - h_1}{h_3 - h_4} = 10\frac{3467.55 - 420.45}{1515.27 - 462.34} = 28.939 \text{ kg/s}$$

Exergy increase of the water flow

$$\dot{m}_{H2O}(\psi_2 - \psi_1) = \dot{m}_{H2O}[h_2 - h_1 - T_o(s_2 - s_1)]$$
$$= 10 \, [\, 3467.55 - 420.45 - 298.15(7.4316 - 1.3053)]$$
$$= 10 \, [\, 3047.1 - 1826.56 \,] = 12 \, 205 \text{ kW}$$

Exergy decrease of the air flow

$$\dot{m}_{air}(\psi_3 - \psi_4) = \dot{m}_{air}[h_3 - h_4 - T_o(s_3 - s_4)]$$
$$= 28.939 \, [1515.27 - 462.34 - 298.15(8.52891 - 7.30142)]$$
$$= 19 \, 880 \text{ kW}$$

$$\eta_{2^{nd} \, Law} = \frac{\dot{m}_{H2O}(\psi_2 - \psi_1)}{\dot{m}_{air}(\psi_3 - \psi_4)} = \frac{12 \, 205}{19 \, 880} = \textbf{0.614}$$

Borgnakke and Sonntag

8.120

Calculate the second law efficiency of the counter flowing heat exchanger in Problem 7.105 with an ambient at 20°C.

Solution:

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside.

Heat exchanger Prob 7.105 with $T_o = 20°C$ solve first for state 4.

Energy Eq.4.10: $\dot{m}_{AIR}\Delta h_{AIR} = \dot{m}_{H2O}\Delta h_{H2O}$

From A.7: $h_1 - h_2 = 1046.22 - 401.3 = 644.92$ kJ/kg

From B.1.2 $h_3 = 83.94$ kJ/kg; $s_3 = 0.2966$ kJ/kg K

$h_4 - h_3 = (\dot{m}_{AIR}/\dot{m}_{H2O})(h_1 - h_2) = (2/0.5)644.92 = 2579.68$ kJ/kg

$h_4 = h_3 + 2579.68 = 2663.62 < h_g$ at 200 kPa

$T_4 = T_{sat} = 120.23°C,$

$x_4 = (2663.62 - 504.68)/2201.96 = 0.9805,$

$s_4 = 1.53 + x_4\ 5.597 = 7.01786$ kJ/kg K

We need the change in exergy for each flow from Eq.8.23

$(\psi_1 - \psi_2) = (h_1 - h_2) + T_o(s_2 - s_1)$

$= (1046.2 - 401.3) + 293.2(7.1593 - 8.1349 - 0.287 \ln(100/125)$

$= 644.9 + 293.2(-0.91156) = 377.6$ kJ/kg

$(\psi_4 - \psi_3) = (h_4 - h_3) + T_o(s_4 - s_3)$

$= (2663.6 - 83.9) - 293.2(7.0179 - 0.2966)$

$= 2579.9 - 1970.7 = 609.0$

Efficiency from Eq.8.32

$$\eta_{2^{nd}\ Law} = [\dot{m}_w(\psi_4 - \psi_3)]/[\dot{m}_A(\psi_1 - \psi_2)]$$

$$= (0.5 \times 609.0)/(2 \times 377.6) = \mathbf{0.403}$$

8.121

A steam turbine receives 5 kg/s steam at $400^{\circ}C$, 10 MPa. One flow of 0.8 kg/s is extracted at 3 MPa as saturated vapor and the remainder runs out at 1500 kPa with a quality of 0.975. Find the second law efficiency of the turbine.

C.V. Turbine. Steady flow and adiabatic q = 0.

Continuity Eq.4.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$;

Energy Eq.4.10: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3 + \dot{W}$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3$

State 1: $h_1 = 3096.46$ kJ/kg, $s_1 = 6.2119$ kJ/kg-K
State 2: $h_2 = 2804.14$ kJ/kg, $s_2 = 6.1869$ kJ/kg-K
State 3: $h_3 = 844.87 + 0.975 \times 1947.28 = 2743.5$ kJ/kg,
$s_3 = 2.315 + 0.975 \times 4.1298 = 6.34156$ kJ/kg-K

$$\dot{W} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = 5 \times 3096.46 - 0.8 \times 2804.14 - 4.2 \times 2743.5$$
$$= 1716 \text{ kW}$$

This work is now compared to the reversible work possible

$$\dot{W}^{rev} = \dot{m}_1 \psi_1 - \dot{m}_2 \psi_2 - \dot{m}_3 \psi_3$$
$$= \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 - T_o [\dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3]$$
$$= 1716 \text{ kW} - 298.15 [5 \times 6.2119 - 0.8 \times 6.1869 - 4.2 \times 6.34156] \text{ kW}$$
$$= 1716 \text{ kW} + 156.4 \text{ kW} = 1872 \text{ kW}$$

$$\eta_{2^{nd} \text{ Law}} = \dot{W} / \dot{W}^{rev} = \frac{1716}{1872} = \mathbf{0.917}$$

Review Problems

8.122

Calculate the irreversibility for the process described in Problem 4.154, assuming that heat transfer is with the surroundings at 17°C.

Solution:

C.V. Cylinder volume out to $T_O = 17\ ^O C$.

Continuity Eq.4.15: $\qquad m_2 - m_1 = m_{in}$

Energy Eq.4.16: $\qquad m_2 u_2 - m_1 u_1 = m_{in} h_{line} + {_1}Q_2 - {_1}W_2$

Entropy Eq.7.12: $\qquad m_2\ s_2 - m_1 s_1 = m_i s_i + {_1}Q_2 / T_O + {_1}S_{2\ gen}$

Process: $\quad P_1$ is constant to stops, then constant V to state 2 at P_2

State 1: $P_1, T_1 \qquad m_1 = \dfrac{P_1 V}{RT_1} = \dfrac{300 \times 0.25}{0.287 \times 290.2} = 0.90$ kg

State 2:

Open to $P_2 = 400$ kPa, $T_2 = 350$ K

$m_2 = \dfrac{400 \times 1}{0.287 \times 350} = 3.982$ kg

$m_i = 3.982 - 0.90 = 3.082$ kg

Only work while constant P

$\qquad {_1}W_2 = P_1(V_2 - V_1) = 300(1 - 0.25) = 225$ kJ

Energy eq.:

$\quad {_1}Q_2 = m_2 u_2 - m_1 u_1 + {_1}W_2 - m_i h_i$

$\qquad = 3.982 \times 0.717 \times 350 - 0.90 \times 0.717 \times 290.2 + 225$

$\qquad - 3.082 \times 1.004 \times 600 = -819.2$ kJ

Entropy eq. gives

$\quad T_O\ {_1}S_{2\ gen} = I = T_O\ [\ m_1\ (s_2 - s_1) + m_i\ (s_2 - s_i)] - {_1}Q_2$

$\qquad = 290.15[0.9(C_p \ln \dfrac{350}{290} - R \ln \dfrac{400}{300}) + 3.082(C_p \ln \dfrac{350}{600} - R \ln \dfrac{400}{500})]$

$\qquad\qquad - (\ -819.2\ kJ)$

$\qquad = 290.15\ (0.0956 - 1.4705) + 819.2$

$\qquad = \textbf{420.3 kJ}$

8.123

The high-temperature heat source for a cyclic heat engine is a steady flow heat exchanger where R-134a enters at 80°C, saturated vapor, and exits at 80°C, saturated liquid at a flow rate of 5 kg/s. Heat is rejected from the heat engine to a steady flow heat exchanger where air enters at 150 kPa and ambient temperature 20°C, and exits at 125 kPa, 70°C. The rate of irreversibility for the overall process is 175 kW. Calculate the mass flow rate of the air and the thermal efficiency of the heat engine.

C.V. R-134a Heat Exchanger,

$\dot{m}_{R134a}$ = 5 kg/s, Table B.5.1

Inlet: $T_1 = 80^O C$, sat. vapor $x_1 = 1.0$,

 $h_1 = h_g = 429.189$ kJ/kg,

 $s_1 = s_g = 1.6862$ kJ/kg-K

Exit: $T_2 = 80^O C$, sat. liquid $x_2 = 0.0$

 $h_2 = h_f = 322.794$ kJ/kg,

 $s_2 = s_f = 1.3849$ kJ/kg-K

C.V. Air Heat Exchanger, C_p = 1.004 kJ/kg-K, R = 0.287 kJ/kg-K

Inlet: $T_3 = 20^O C$, $P_3 = 150$ kPa   Exit: $T_4 = 70^O C$, $P_4 = 125$ kPa

 $s_4 - s_3 = C_p \ln (T_4 / T_3) - R \ln(P_4 / P_3) = 0.2103$ kJ/kg-K

Entropy Eq. for the total system as control volume (since we know $\dot{I}$):

$$\dot{I} / T_o = \dot{S}_{net} = \dot{m}_{R134a} (s_2 - s_1) + \dot{m}_{air}(s_4 - s_3)$$

$$\dot{m}_{air} = \frac{\dot{I}/T_o - \dot{m}_{R134a}(s_2 - s_1)}{s_4 - s_3} = \frac{(175/293.15) - 5 (1.3849 - 1.6862)}{0.2103} \frac{kW/K}{kJ/kg\text{-}K}$$

 = **10.0 kg/s**

Energy Eq. for each line: $\dot{Q} + \dot{m}h_{in} = \dot{m}h_{ex} + \dot{W}$; $\dot{W} = 0$

R-134a: $_1\dot{Q}_2 = -\dot{Q}_H = \dot{m}_{R134a}(h_2 - h_1)$ = -532 kW

Air: $\dot{Q}_L = {}_3\dot{Q}_4 = \dot{m}_{air}(h_4 - h_3) = \dot{m}_{air} C_p(T_4 - T_3) = 501.8$ kW

Control volume heat engine

$$\dot{W}_{net} = \dot{Q}_H - \dot{Q}_L = 532 - 501.8 = 30.2 \text{ kW};$$

$$\eta_{th} = \dot{W}_{net} / \dot{Q}_H = \mathbf{0.057,} \quad \textbf{or } \mathbf{5.7\%}$$

Borgnakke and Sonntag

8.124

A cylinder with a linear spring-loaded piston contains carbon dioxide gas at 2 MPa with a volume of 50 L. The device is of aluminum and has a mass of 4 kg. Everything (Al and gas) is initially at 200°C. By heat transfer the whole system cools to the ambient temperature of 25°C, at which point the gas pressure is 1.5 MPa. Find the exergy at the initial and final states and the destruction of exergy in the process.

State 1: $\qquad T_1 = 200\ ^oC, \quad v_1 = V_1/m = 0.05\ /\ 1.1186 = 0.0447\ m^3/kg$

State 2: $\qquad v_2 = v_1 \times (2\ /\ 1.5) \times (298.15\ /\ 473.15) = 0.03756\ m^3/kg$

The metal does not change volume, so the combined is using Eq.8.27 as

$$\phi_1 = m_{gas}\phi_{gas} + m_{Al}\phi_{Al}$$

$$= m_{gas}[u_1 - u_o - T_o(s_1 - s_o)]_{cv} + m_{gas}P_o(v_1 - v_o) + m_{Al}[u_1 - u_o - T_o(s_1 - s_o)]_{Al}$$

$$= m_{gas}C_v\ (T_1 - T_o) - m_{gas}T_o\ [C_p \ln \frac{T_1}{T_o} - R \ln \frac{P_1}{P_o}] + m_{gas}P_o\ (v_1 - v_o)$$

$$+ m_{Al}\ [C\ (T_1 - T_o) - T_oC \ln (T_1/T_o)\ ]_{Al}$$

$$\phi_1 = 1.1186\ [\ 0.653(200-25) - 298.15\ (0.842 \ln \frac{473.15}{298.15} - 0.18892 \ln \frac{2000}{100}\ )$$

$$+ 100\ (0.0447 - 0.5633\ )\ ] + 4 \times 0.90\ [\ 200 - 25 - 298.15 \ln \frac{473.15}{298.15}\ ]$$

$$= 128.88 + 134.3 = \textbf{263.2 kJ}$$

$$\phi_2 = 1.1186\ [\ 0.653(25 - 25) - 298.15\ (0.842 \ln \frac{298.15}{298.15} - 0.18892 \ln \frac{1500}{100}\ )$$

$$+ 100\ (0.03756 - 0.5633\ )\ ] + 4 \times 0.9\ [\ 25 - 25 - 298.15 \ln \frac{298.15}{298.15}\ ]$$

$$= 111.82 + 0 = \textbf{111.82 kJ}$$

The irreversibility is as in Eq.8.38 integrated over time (no flow terms)

$$_1I_2 = \phi_1 - \phi_2 + [1 - (T_0/T_H)]\ _1Q_2 - _1W_2^{AC} + P_om(\ V_2 - V_1)$$

$$= 263.2 - 111.82 + 0 - (-14) + 100 \times 1.1186\ (0.03756 - 0.0447) = 164.58\ kJ$$

$$[(_1S_{s\ gen} = 0.552\ kJ/K; \qquad T_{O}\ _1S_{s\ gen} = 164.58\ kJ \quad so\ OK\ ]$$

Borgnakke and Sonntag

8.125

A two-stage compressor takes nitrogen in at 20°C, 150 kPa compresses it to 600 kPa, 450 K then it flows through an intercooler where it cools to 320 K and the second stage compresses it to 3000 kPa, 530 K. Find the specific exergy increase and the specific exergy destruction in each of the two compressor stages.

The setup is like this:

C.V.: Stage 1 nitrogen, Steady flow

Process: adiabatic: $q = 0$, $s_{gen} = 0$

Energy Eq. 4.13: $-w_{C1} = h_2 - h_1$,   Entropy Eq. 7.10: $0 = s_1 - s_2 + s_{gen\ C1}$

Assume constant $C_{P0} = 1.042$ from A.5 and Eq.6.16 for change in entropy

$$s_{gen\ C1} = s_2 - s_1 = C_{P0} \ln(T_2/T_1) - R \ln (P_2/P_1)$$

$$= 1.042 \ln(450/293.15) - 0.2968 \ln(600/150) = 0.0351 \text{ kJ/kgK}$$

$$\psi_{destruction\ C1} = T_o\, s_{gen\ C1} = 298.15 \text{ K} \times 0.0351 \text{ kJ/kgK} = \textbf{10.46 kJ/kg}$$

$$\psi_2 - \psi_1 = h_2 - h_1 - T_o (s_2 - s_1) = C_{P0}(T_2 - T_1) - T_o (s_2 - s_1)$$

$$= 1.042(450 - 293.15) - 298.15 \times 0.0351 = \textbf{152.98 kJ/kg}$$

$$s_{gen\ C2} = s_4 - s_3 = C_{P0} \ln(T_4/T_3) - R \ln (P_4/P_3)$$

$$= 1.042 \ln(530/320) - 0.2968 \ln(3000/600) = 0.0481 \text{ kJ/kgK}$$

$$\psi_{destruction\ C2} = T_o\, s_{gen\ C2} = 298.15 \text{ K} \times 0.0481 \text{ kJ/kgK} = \textbf{14.34 kJ/kg}$$

$$\psi_4 - \psi_3 = h_4 - h_3 - T_o (s_4 - s_3) = C_{P0}(T_4 - T_3) - T_o (s_4 - s_3)$$

$$= 1.042(530 - 320) - 298.15 \times 0.0481 = \textbf{204.48 kJ/kg}$$

Comment: Notice how the exergy increases are the enthalpy increase minus the exergy destruction.

8.126

The intercooler in the previous problem uses cold liquid water to cool the nitrogen. The nitrogen flow is 0.1 kg/s and the liquid water inlet is 20°C and setup to flow in the opposite direction as the nitrogen so the water leaves at 35°C. Find the flow rate of the water and the exergy destruction in this intercooler.

C.V. Heat exchanger, steady 2 flows in and two flows out.

Continuity eq.: $\dot{m}_1 = \dot{m}_2 = \dot{m}_{H2O}$; $\dot{m}_3 = \dot{m}_4 = \dot{m}_{N2}$

Energy eq.: $0 = \dot{m}_{N2} (h_3 - h_4) + \dot{m}_{H2O} (h_2 - h_1)$

Entropy Eq.7.7: $0 = \dot{m}_{N2} (s_3 - s_4) + \dot{m}_{H2O} (s_2 - s_1) + \dot{S}_{gen}$

Due to the lower range of temperature we will use constant specific heats from A.5 and A.4 so the energy equation is approximated as

$$0 = \dot{m}_{H2O}C_{pH2O} (T_2 - T_1) + \dot{m}_{N2}C_p (T_3 - T_4)$$

Now solve for the water flow rate

$\dot{m}_{H2O} = \dot{m}_{N2} C_{pN2} (T_3 - T_4) / [C_{pH2O} (T_1 - T_2)]$

      $= 0.1$ kg/s $\times 1.042$ kJ/kgK $\times (450 - 320)$ K $/ [4.18 \times (35 - 20)$ kJ/kg]

      $= \textbf{0.216 kg/s}$

$\dot{S}_{gen} = \dot{m}_{N2} (s_4 - s_3) + \dot{m}_{H2O} (s_1 - s_2)$

      $= \dot{m}_{N2}C_P \ln (T_4/T_3) + \dot{m}_{H2O}C_P \ln (T_1/T_2)$

      $= 0.1$ kg/s $\times 1.042$ kJ/kgK $\ln \dfrac{320}{450} + 0.216$ kg/s $\times 4.18$ kJ/kgK $\ln \dfrac{308}{293}$

      $= -0.03552 + 0.04508 = 0.00956$ kW/K

$\dot{\Phi}_{destruction} = \dot{I} = T_o \dot{S}_{gen} = 298.15$ K $\times 0.00956$ kW/K $= \textbf{2.85 kW}$

A hydraulic motor driven compressor with intercooler in small pipe between the two stages.

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8.127

Find the irreversibility in the cooling process of the glass plate in Problem 4.144.

In a glass factory a 2 m wide sheet of glass at 1500 K comes out of the final rollers that fix the thickness at 5 mm with a speed of 0.5 m/s. Cooling air in the amount of 20 kg/s comes in at 17°C from a slot 2 m wide and flows parallel with the glass. Suppose this setup is very long so the glass and air comes to nearly the same temperature (a co-flowing heat exchanger) what is the exit temperature?

Energy Eq.: $\dot{m}_{glass}h_{glass\ 1} + \dot{m}_{air}h_{air\ 2} = \dot{m}_{glass}h_{glass\ 3} + \dot{m}_{air}h_{air\ 4}$

$$\dot{m}_{glass} = \rho\dot{V} = \rho AV = 2500 \times 2 \times 0.005 \times 0.5 = 12.5 \text{ kg/s}$$

$$\dot{m}_{glass}C_{glass}(T_3 - T_1) + \dot{m}_{air}C_{Pa}(T_4 - T_2) = \varnothing$$
$$T_4 = T_3 \text{ , } C_{glass} = 0.80 \text{ kJ/kg K, } C_{Pa} = 1.004 \text{ kJ/kg K}$$

$$T_3 = \frac{\dot{m}_{glass}C_{glass}\ T_1 + \dot{m}_{air}C_{Pa}\ T_2}{\dot{m}_{glass}C_{glass} + \dot{m}_{air}C_{Pa}} = \frac{12.5 \times 0.80 \times 1500 + 20 \times 1.004 \times 290}{12.5 \times 0.80 + 20 \times 1.004}$$

$$= 692.3 \text{ K}$$

To find the irreversibility we need the entropy generation term

Entropy Eq.: $\dot{m}_{glass}s_{glass\ 1} + \dot{m}_{air}s_{air\ 2} + \dot{S}_{gen} = \dot{m}_{glass}s_{glass\ 3} + \dot{m}_{air}s_{air\ 4}$

$$\dot{S}_{gen} = \dot{m}_{glass}(s_3 - s_1)_{glass} + \dot{m}_{air}(s_4 - s_2)_{air}$$

$$= \dot{m}_{glass}\ C_{glass}\ \ln(T_3/T_1) + \dot{m}_{air}\ C_{Pa}\ \ln(T_4/T_2)$$
$$= 12.5 \times 0.8 \times \ln(692.3/1500) + 20 \times 1.004 \times \ln(692.3/290)$$
$$= -7.732 \text{ kW/K} + 17.472 \text{ kW/K} = 9.74 \text{ kW/K}$$

$$\dot{I} = \dot{\Phi}_{destruction} = T_o\dot{S}_{gen} = 298.15 \text{ K} \times 9.74 \text{ kW/K} = \textbf{2904 kW}$$

8.128

Air in a piston/cylinder arrangement is at 110 kPa, 25°C, with a volume of 50 L. It goes through a reversible polytropic process to a final state of 700 kPa, 500 K, and exchanges heat with the ambient at 25°C through a reversible device. Find the total work (including the external device) and the heat transfer from the ambient.

C.V. Total out to ambient

Energy: $m_a(u_2 - u_1) = {}_1Q_2 - {}_1W_{2,tot}$,

Entropy: $m_a(s_2 - s_1) = {}_1Q_2/T_0 + 0$

$m_a = 110 \times 0.05/0.287 \times 298.15 = 0.0643$ kg

$${}_1Q_2 = T_0 m_a(s_2 - s_1) = 298.15 \times 0.0643 \big[7.3869 - 6.8631$$

$$- 0.287 \ln (700/110) \big] = -0.14 \text{ kJ}$$

$${}_1W_{2,tot} = {}_1Q_2 - m_a(u_2 - u_1)$$

$$= -0.14 - 0.0643 \times (359.844 - 213.037) = \textbf{-9.58 kJ}$$

This is identical to the reversible work in Eq.8.17 $(T_j = T_0)$.

8.129

A rigid container with volume 200 L is divided into two equal volumes by a partition. Both sides contains nitrogen, one side is at 2 MPa, 300°C, and the other at 1 MPa, 50°C. The partition ruptures, and the nitrogen comes to a uniform state at 100°C. Assuming the surroundings are at 25°C find the actual heat transfer and the irreversibility in the process.

Solution:

C.V. Total container

Continuity Eq.: $\qquad m_2 - m_A - m_B = 0$

Energy Eq.: $\qquad m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $\qquad m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B = {}_1Q_2/T_{sur} + {}_1S_{s\ gen}$

Process: $\qquad V = C \quad => \quad {}_1W_2 = 0$

From the initial state we get the mass as

$$m_2 = m_A + m_B = \frac{P_{A1}V_A}{RT_{A1}} + \frac{P_{B1}V_B}{RT_{B1}}$$

$$= \frac{2000 \times 0.1}{0.2968 \times 573.15} + \frac{1000 \times 0.1}{0.2968 \times 323.15} = 1.176 + 1.043 = 2.219 \text{ kg}$$

$$P_2 = m_2RT_2/V_{tot} = 2.219 \times 0.2968 \times 373.15/0.2 = 1228.8 \text{ kPa}$$

From the energy equation we get the heat transfer as the change in U

$${}_1Q_2 = m_AC_v(T_2 - T_1)_A + m_BC_v(T_2 - T_1)_B$$

$$= 1.176 \times 0.745 \times (100 - 300) + 1.043 \times 0.745 \times (100 - 50)$$

$$= \textbf{-136.4 kJ}$$

The entropy changes are found from Eq.8.16

$$(s_2 - s_1)_A = 1.042 \times \ln\frac{373.15}{573.15} - 0.2968 \times \ln\frac{1228.8}{2000} = -0.09356 \text{ kJ/kg K}$$

$$(s_2 - s_1)_B = 1.042 \times \ln\frac{373.15}{323.15} - 0.2968 \times \ln\frac{1228.8}{1000} = 0.0887 \text{ kJ/kg K}$$

The entropy generation follows from the entropy equation

$${}_1S_{2,gen} = 1.176 \times (-0.09356) + 1.043 \times 0.0887 + 136.4/298.15 = 0.4396 \text{ kJ/K}$$

Now the irreversibility comes from Eq. 10.13

$${}_1I_2 = T_0 \times {}_1S_{2,gen} = \textbf{131.08 kJ}$$

8.130

Consider the irreversible process in Problem 6.182. Assume that the process could be done reversibly by adding heat engines/pumps between tanks A and B and the cylinder. The total system is insulated, so there is no heat transfer to or from the ambient. Find the final state, the work given out to the piston and the total work to or from the heat engines/pumps.

C.V. Water $m_A + m_B$ + heat engines. No $Q_{external}$, only $_1W_{2,cyl} + W_{HE}$

Continuity Eq.: $m_2 = m_{A1} + m_{B1} = 6$ kg,

Energy Eq.: $m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = -_1W_{2,cyl} - W_{HE}$

Entropy Eq.: $m_2s_2 - m_{A1}s_{A1} - m_{B1}s_{B1} = \emptyset + \emptyset$

   $v_{A1} = 0.06283$ m^3/kg, $u_{A1} = 3448.5$ kJ/kg, $s_{A1} = 7.3476$ kJ/kg-K,

   $v_{B1} = 0.09053$ m^3/kg, $u_{B1} = 2843.7$ kJ/kg, $s_{B1} = 6.7428$ kJ/kg-K

   $V_A = m_{A1}\, v_{A1} = 0.2513$ m^3; $V_B = m_{B1}v_{B1} = 0.1811$ m^3

   $m_2s_2 = 4 \times 7.3476 + 2 \times 6.7428 = 42.876$ kJ/K $\Rightarrow$ $s_2 = 7.146$ kJ/kg K

If $P_2 < P_{lift} = 1.4$ MPa then

   $V_{2'} = V_A + V_B = 0.4324$ m^3 , $v_{2'} = 0.07207$ m^3/kg

   $(P_{lift}, s_2) \Rightarrow v_2 = 0.20135 \Rightarrow V_2 = 1.208$ m$^3 > V_{2'}$ OK

                    $\Rightarrow P_2 = P_{lift} = $ **1.4 MPa** $u_2 = 2874.2$ kJ/kg

   $_1W_{2,cyl} = P_{lift}(V_2 - V_A - V_B) = 1400 \times (1.208 - 0.4324) = $ **1085.84 kJ**

   $W_{HE} = m_{A1}u_{A1} + m_{B1}u_{B1} - m_2u_2 - {}_1W_{2,cyl}$

     $= 4 \times 3447.8 + 2 \times 2843.7 - 6 \times 2874.2 - 1085.84 = $ **1147.6 kJ**

8.131

Consider the heat engine in Problem 8.111. The exit temperature was given as 800 K, but what are the theoretical limits for this temperature? Find the lowest and the highest, assuming the heat transfers are as given. For each case give the first and second law efficiency.

The **lowest exhaust temperature** will occur when the maximum amount of work is delivered which is a reversible process. Assume no other heat transfers then

Energy Eq.: $\quad 0 = h_i + q_{1500} - q_{750} - h_e - w$

Entropy Eq.: $\quad 0 = s_i + q_H/T_H + \emptyset - s_e - q_m/T_m$

$$s_e - s_i = q_H/T_H - q_m/T_m = s^\circ_{Te} - s^\circ_{Ti} - R \ln(P_e/P_i)$$

$$s^\circ_{Te} = s^\circ_{Ti} + R \ln(P_e/P_i) + q_H/T_H - q_m/T_m$$

$$= 6.86926 + 0.287 \ln(100/100) + 1200/1500 - 300/750$$

$$= 7.26926 \text{ kJ/kg K}$$

Table A.7.1 $\quad\Rightarrow\quad$ $T_{e,min} = $ **446 K**, $\quad h_e = 447.9$ kJ/kg

$$w_{rev} = h_i + q_{1500} - q_{750} - h_e = 300.47 + 1200 - 300 - 447.9$$

$$= 752.57 \text{ kJ/kg}$$

$$\eta_I = \eta_{TH} = \frac{w_{rev}}{q_{1500}} = \frac{752.57}{1200} = \textbf{0.627}$$

The second law efficiency measures the work relative to the source of exergy and not q_{1500}. So

$$\eta_{II} = \frac{w_{rev}}{(1 - T_o/T_H)q_{1500}} = \frac{752.57}{(1 - 300/1500)1200} = \frac{752.57}{960} = \textbf{0.784}$$

Comment: This measure places zero value of the loss at 750 K, zero exergy value for the two flow terms.

The **maximum exhaust temperature** occurs with no work out

$$h_e = h_i + q_H - q_m \quad\Rightarrow\quad h_e = 300.473 + 1200 - 300 = 1200.5 \text{ kJ/kg}$$

Table A.7.1 $\quad\Rightarrow\quad$ $T_{e,max} = $ **1134 K**

Now : $\quad w_{ac} = 0 \quad$ so $\quad \eta_I = \eta_{II} = $ **0**

8.132

A small air gun has 1 cm^3 air at 250 kPa, 27°C. The piston is a bullet of mass 20 g. What is the potential highest velocity with which the bullet can leave?

Solution:
The exergy of the air can give the bullet kinetic energy expressed in the exergy change Eq.8.28 (no heat transfer and reversible),

$$\Phi_2 - \Phi_1 = m(u_2 - u_1) + P_o(V_2 - V_1) - mT_o(s_2 - s_1) = {}_1W_2 + P_o(V_2 - V_1)$$

Ideal gas so: $\quad m = PV/RT = \dfrac{250 \times 1 \times 10^{-6}}{0.287 \times 300} = 2.9 \times 10^{-6}$ kg

The second state with the lowest exergy to give maximum velocity is the dead state and we take $T_o = 20^o$C. Now solve for the exergy change

$$\Phi_2 - \Phi_1 = -m(u_2 - u_1) + mT_o(s_2 - s_1)$$

$$= mC_v(T_1 - T_2) + mT_o \left[C_p \ln(\frac{T_2}{T_1}) - R \ln(\frac{P_2}{P_1}) \right]$$

$$= 2.9 \times 10^{-6} \left[0.717(27 - 20) + 293.15 (1.004 \ln\frac{293}{300} - 0.287 \ln\frac{100}{250}) \right]$$

$$= 0.0002180 \text{ kJ} = 0.218 \text{ J} = \frac{1}{2} m_{bullet}V_{ex}^2$$

$$V_{ex} = \sqrt{2 \times 0.218/0.020} = \textbf{4.67 m/s}$$

Comment: Notice that an isentropic expansion from 250 kPa to 100 kPa will give the final air temperature as 230.9 K but less work out. The above process is not adiabatic but Q is transferred from ambient at T_o.

8.133

Consider the nozzle in Problem 7.154. What is the second law efficiency for the nozzle?

A nozzle in a high pressure liquid water sprayer has an area of 0.5 cm^2. It receives water at 350 kPa, 20°C and the exit pressure is 100 kPa. Neglect the inlet kinetic energy and assume a nozzle isentropic efficiency of 85%. Find the ideal nozzle exit velocity and the actual nozzle mass flow rate.

Solution:

C.V. Nozzle. Liquid water is incompressible v ≈ constant, no work, no heat transfer   => Bernoulli Eq.7.17

$$\frac{1}{2}V_{ex}^2 - 0 = v(P_i - P_e) = 0.001002\,(350 - 100) = 0.2505 \text{ kJ/kg}$$

$$V_{ex} = \sqrt{2 \times 0.2505 \times 1000 \text{ J/kg}} = 22.38 \text{ m s}^{-1}$$

This was the ideal nozzle now we can do the actual nozzle, Eq. 7.30

$$\frac{1}{2}V_{ex\,ac}^2 = \eta\,\frac{1}{2}V_{ex}^2 = 0.85 \times 0.2505 = 0.2129 \text{ kJ/kg}$$

The second law efficiency is the actual nozzle compared to a reversible process between the inlet and actual exit state. However no work so the reversible exit state (with P_e, $T_{ex\,ac}$) then must have the reversible possible kinetic energy.

Energy actual nozzle: $h_i + 0 = h_e + \frac{1}{2}V_{ex\,ac}^2$ same Z, no q and no w.

The reversible process has zero change in exergies from Eq.8.38 as

$$\psi_i = \psi_e = h_i + 0 - T_o\,s_i = h_e + \frac{1}{2}V_{ex\,rev}^2 - T_o s_e$$

$$\frac{1}{2}V_{ex\,rev}^2 = h_i - h_e + T_o\,(s_e - s_i) = \frac{1}{2}V_{ex\,ac}^2 + T_o\,s_{gen}$$

We can not get properties for these states accurately enough by interpolation so use constant specific heat and incompressibility.

$$h_{ex\,ac} = h_i - \frac{1}{2}V_{ex\,ac}^2 = u_{ex\,ac} + P_e v = u_i + P_i v - \frac{1}{2}V_{ex\,ac}^2$$

$$u_{ex\,ac} - u_i = P_i v - P_e v - \frac{1}{2}V_{ex\,ac}^2 = C(T_{ex\,ac} - T_i)$$

$$T_{ex\,ac} = T_i + (P_i v - P_e v - \frac{1}{2}V_{ex\,ac}^2)/C$$
$$= 20 + [(350 - 100)\,0.001002 - 0.2129]\,/\,4.18 = 20.01°C$$

$$\frac{1}{2}V_{ex\,rev}^2 = \frac{1}{2}V_{ex\,ac}^2 + T_o\,s_{gen} = 0.2129 + 293.15 \times 4.18 \ln\frac{293.16}{293.15}$$
$$= 0.2547 \text{ kJ/kg}$$

$$\eta_{II} = 0.2129/0.2547 = \textbf{0.84}$$

8.134

Consider the light bulb in Problem 6.189. What are the fluxes of exergy at the various locations mentioned? What are the exergy destruction in the filament, the entire bulb including the glass and the entire room including the bulb? The light does not affect the gas or the glass in the bulb but it gets absorbed on the room walls.

A small halogen light bulb receives an electrical power of 50 W. The small filament is at 1000 K and gives out 20% of the power as light and the rest as heat transfer to the gas, which is at 500 K; the glass is at 400 K. All the power is absorbed by the room walls at 25°C. Find the rate of generation of entropy in the filament, in the total bulb including glass and the total room including bulb.

Solution:

$\dot{W}_{el} = 50$ W

$\dot{Q}_{RAD} = 10$ W

$\dot{Q}_{COND} = 40$ W

We will assume steady state and no storage in the bulb, air or room walls.

C.V. Filament steady-state

Energy Eq.3.3: $\quad dE_{c.v.}/dt = 0 = \dot{W}_{el} - \dot{Q}_{RAD} - \dot{Q}_{COND}$

Entropy Eq.6.42: $\quad dS_{c.v.}/dt = 0 = -\dfrac{\dot{Q}_{RAD}}{T_{FILA}} - \dfrac{\dot{Q}_{COND}}{T_{FILA}} + \dot{S}_{gen}$

$\dot{I} = T_o \dot{S}_{gen} = (\dot{Q}_{RAD} + \dot{Q}_{COND})\dfrac{T_o}{T_{FILA}} = T_o\dfrac{\dot{W}_{el}}{T_{FILA}} = 298\,\dfrac{50}{1000} = \textbf{14.9 W}$

C.V. Bulb including glass

$\dot{Q}_{RAD}$ leaves at 1000 K $\qquad\qquad \dot{Q}_{COND}$ leaves at 400 K

$\dot{I} = T_o\dot{S}_{gen} = T_o\!\int d\dot{Q}/T = 298[-(-10/1000) - (-40/400)] = \textbf{32.8 W}$

C.V. Total room. All energy leaves at 25°C

Eq.3.3: $\quad dE_{c.v.}/dt = 0 = \dot{W}_{el} - \dot{Q}_{RAD} - \dot{Q}_{COND}$

Eq.6.42: $\quad dS_{c.v.}/dt = 0 = -\dfrac{\dot{Q}_{TOT}}{T_{WALL}} + \dot{S}_{gen}$

$$\dot{I} = T_o \dot{S}_{gen} = T_o \frac{\dot{Q}_{TOT}}{T_{WALL}} = 298 \frac{50}{25+273} = \textbf{50 W}$$

Since the room is at T_o then all the incoming exergy is destroyed.

Flux into filament: Flux $= \dot{W}_{el} = \textbf{50 W}$

Flux leaving the filament: Flux $= (\dot{Q}_{RAD} + \dot{Q}_{COND})(1 - \frac{T_o}{T_{FILA}})$

$$= 50 \text{ W} - 14.9 \text{ W} = \textbf{35.1 W}$$

Flux leaving the glass: Flux $= \dot{Q}_{RAD}(1 - \frac{T_o}{T_{FILA}}) + \dot{Q}_{COND}(1 - \frac{T_o}{T_{glass}})$

$$= 10 \times (1 - \frac{298}{1000}) \text{ W} + 40 \times (1 - \frac{298}{400}) \text{ W}$$

$$= 7.02 \text{ W} + 10.2 \text{ W} = \textbf{17.22 W}$$

Flux at walls: Flux $= 0$ (wall is at T_o)

8.135

Air in a piston/cylinder arrangement, shown in Fig. P8.135, is at 200 kPa, 300 K with a volume of 0.5 m^3. If the piston is at the stops, the volume is 1 m^3 and a pressure of 400 kPa is required. The air is then heated from the initial state to 1500 K by a 1700 K reservoir. Find the total irreversibility in the process assuming surroundings are at 20°C.

Solution:

Energy Eq.: $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq,: $\quad m(s_2 - s_1) = \int dQ/T + {}_1S_2\,_{gen}$

Process: $\quad P = P_0 + \alpha(V-V_0) \quad$ if $\quad V \le V_{stop}$

Information: $\quad P_{stop} = P_0 + \alpha(V_{stop}-V_0)$

Eq. of state $\Rightarrow \quad T_{stop} = T_1 P_{stop} V_{stop}/P_1 V_1 = 1200 < T_2$

So the piston will hit the stops => $V_2 = V_{stop}$

$$P_2 = (T_2/T_{stop})\, P_{stop} = (1500/1200)\, 400 = 500 \text{ kPa} = 2.5\, P_1$$

State 1:

$$m_2 = m_1 = \frac{P_1 V_1}{RT_1}$$

$$= \frac{200 \times 0.5}{0.287 \times 300}$$

$$= 1.161 \text{ kg}$$

$${}_1W_2 = \tfrac{1}{2}(P_1 + P_{stop})(V_{stop} - V_1) = \tfrac{1}{2}(200 + 400)(1 - 0.5) = 150 \text{ kJ}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.161(1205.25 - 214.36) + 150 = 1301 \text{ kJ}$$

$$s_2 - s_1 = s_{T2}^o - s_{T1}^o - R \ln(P_2/P_1) = 8.6121 - 6.8693 - 0.287 \ln 2.5 = 1.48 \text{ kJ/kg K}$$

Take control volume as total out to reservoir at T_{RES}

$${}_1S_2\,_{gen\ tot} = m(s_2 - s_2) - {}_1Q_2/T_{RES} = 1.161 \times 1.48 - 1301/1700 = 0.953 \text{ kJ/K}$$

$${}_1I_2 = T_0({}_1S_2\,_{gen}) = 293.15 \times 0.953 = \textbf{279 kJ}$$

8.136

Air enters a steady-flow turbine at 1600 K and exhausts to the atmosphere at 1000 K. The second law efficiency is 85%. What is the turbine inlet pressure?

C.V.: Turbine, exits to atmosphere so assume P_e = 100 kPa

Inlet: T_i = 1600 K, Table A.7: h_i = 1757.3 kJ/kg, s_i^o = 8. 69051 kJ/kg K

Exit: T_e = 1000 K, h_e = 1046.2 kJ/kg, s_e^o = 8. 13493 kJ/kg K

Energy Eq.: $q + h_i = h_e + w$; $q = 0$ => $w = (h_i - h_e) = 711.1$ kJ/kg

Second law efficiency expresses work as a fraction of exergy change

$$\psi_i - \psi_e = w/\eta_{II} = 711.11/0.85 = 836.6 \text{ kJ/kg} = (h_i - h_e) - T_o(s_i - s_e)$$

$h_i - h_e = w = 711.11$ kJ/kg,   assume $T_O = 25^O C$ →

$$s_i - s_e = (711.11 - 836.6) / 298.15 = -0.420896 \text{ kJ/kg-K}$$

$$s_i - s_e = s_i^o - s_e^o - R \ln(P_i/P_e) = -0.4209 \text{ kJ/kg K}$$

=> $R \ln(P_i/P_e) = 8.69051 - 8.13493 + 0.420896 = 0.976476$ kJ/kg-K

$P_i/P_e = 30.035$; P_i = **3003 kPa**

8.137

A 1 kg rigid steel tank contains 1.2 kg of R-134a at 20°C and 500 kPa. Now the set-up is placed in a freezer that brings it to -20°C. The freezer operates in a 20°C kitchen and has a coefficient of performance that is half that of a Carnot refrigerator. Find the heat transfer out of the R-134a, the extra work input to the refrigerator due to this process and the total irreversibility including the refrigerator.

C.V. The R-134a and the steel.

Energy Eq.: $m_R(u_2 - u_1)_R + m_{st}(u_2 - u_1)_{st} = {}_1Q_2 - {}_1W_2$

Entropy Eq,: $m_R(s_2 - s_1)_R + m_{st}(s_2 - s_1)_{st} = \int dQ/T + {}_1S_2{}_{gen}$

Process: $\qquad V = C \quad so \quad {}_1W_2 = 0$

State 1: $\qquad v_1 = 0.04226 \ m^3/kg, \quad u_1 = 390.52 \ kJ/kg, \quad s_1 = 1.7342 \ kJ/kg\text{-}K$

State 2: $\qquad v_2 = v_1, \ -20^{\circ}C \ \Rightarrow \ 2 \ phase \ state$

$\qquad x_2 = (v_2 - v_f)/v_{fg} = (0.04226 - 0.000738)/0.14576 = 0.28486$

$\qquad u_2 = 173.65 + x_2 \times 192.85 = 228.59 \ kJ/kg,$

$\qquad s_2 = 0.9007 + x_2 \times 0.8388 = 1.13964 \ kJ/kg\text{-}K$

$\qquad {}_1Q_2 = m_R(u_2 - u_1)_R + m_{st}(u_2 - u_1)_{st}$

$\qquad \qquad = 1.2 \ (228.59 - 390.52) + 1 \times 0.46(\text{-}20 - 20) = \mathbf{-212.7 \ kJ}$

Refrigerator: $\quad \beta = 0.5 \ \beta_{Carnot} = 0.5 \ \dfrac{T_L}{T_H - T_L} = 0.5 \ \dfrac{253.15}{20 - (\text{-}20)} = 3.164$

$\qquad \qquad W_{ref} = Q_L/\beta = 212.7 \ kJ/ \ 3.164 = \mathbf{67.23 \ kJ}$

C.V. Total incl. refrigerator. Storage effect in R-134a and steel, otherwise transfer terms.

Energy Eq.: $m_R(u_2 - u_1)_R + m_{st}(u_2 - u_1)_{st} = W_{ref} - Q_H$

Entropy Eq,: $m_R(s_2 - s_1)_R + m_{st}(s_2 - s_1)_{st} = -Q_H/T_0 + {}_1S_2{}_{gen}$

$\qquad Q_H = W_{ref} - m(u_2 - u_1) = 67.23 - (-212.7) = 279.93 \ kJ$

$\qquad {}_1S_2{}_{gen} = m_R(s_2 - s_1)_R + m_{st}(s_2 - s_1)_{st} + \dfrac{Q_H}{T_0}$

$\qquad \qquad = 1.2(1.13964\text{-}1.7342) + 1 \times 0.46 \ln(\dfrac{253.15}{293.15}) + \dfrac{279.93}{293.15}$

$\qquad \qquad = -0.713472 - 0.06748 + 0.954904 = 0.17395 \ kJ/K$

$\qquad {}_1I_2 = T_0 \left({}_1S_2{}_{gen}\right) = 293.15 \times 0.17395 = \mathbf{50.99 \ kJ}$

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8.138

A piston/cylinder arrangement has a load on the piston so it maintains constant pressure. It contains 1 kg of steam at 500 kPa, 50% quality. Heat from a reservoir at 700°C brings the steam to 600°C. Find the second-law efficiency for this process. Note that no formula is given for this particular case so determine a reasonable expression for it.

Solution:

1: Table B.1.2 $P_1, x_1 \implies v_1 = 0.001093 + 0.5 \times 0.3738 = 0.188$ m^3/kg,

$h_1 = 640.21 + 0.5 \times 2108.47 = 1694.5$ kJ/kg,

$s_1 = 1.8606 + 0.5 \times 4.9606 = 4.341$ kJ/kg K

2: $P_2 = P_1, T_2 \implies v_2 = 0.8041, \quad h_2 = 3701.7$ kJ/kg, $\quad s_2 = 8.3521$ kJ/kg K

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2 - P(V_2 - V_1)$

$_1Q_2 = m(u_2 - u_1) + Pm(v_2 - v_1) = m(h_2 - h_1) = 2007.2$ kJ

$_1W_2 = Pm(v_2 - v_1) = 308.05$ kJ

$_1W_{2\text{ to atm}} = P_0 m(v_2 - v_1) = 61.61$ kJ

Useful work out = $_1W_2 - {}_1W_{2\text{ to atm}} = 246.44$ kJ

$\Delta\phi_{\text{reservoir}} = (1 - T_0/T_{\text{res}})_1Q_2 = \left(1 - \dfrac{298.15}{973.15}\right) 2007.2 = 1392.2$ kJ

$\eta_{\text{II}} = W_{\text{net}}/\Delta\phi = \mathbf{0.177}$

Remark: You could argue that the stored exergy (exergy) should be accounted for in the second law efficiency, but it is not available from this device alone.

8.139

A jet of air at 200 m/s flows at 25$^{\text{o}}$C, 100 kPa towards a wall where the jet flow stagnates and leaves at very low velocity. Consider the process to be adiabatic and reversible. Use the exergy equation and the second law to find the stagnation temperature and pressure.

Solution:

C.V. From free flow to stagnation point. Reversible adiabatic steady flow.

Exergy Eq.8.38: $0 = \dot{m}\psi_i - \dot{m}\psi_e - \dot{\Phi}_{destr.}$

Entropy Eq.: $0 = \dot{m}s_i - \dot{m}s_e + \int \dot{m}dq/T + \dot{m}s_{gen} = \dot{m}s_i - \dot{m}s_e + 0 + 0$

Process: Reversible $\dot{\Phi}_{destr.} = 0$, $s_{gen} = 0$, adiabatic $q = 0$

From exergy Eq.: $\psi_e - \psi_i = 0 = h_e - T_o s_e - h_i + T_o s_i - \frac{1}{2}\mathbf{V}_i^2$

From entropy Eq.: $s_e = s_i$,   so entropy terms drop out

Exergy eq. now leads to: $h_e = h_i + \frac{1}{2}\mathbf{V}_i^2$ $\Rightarrow$ $T_e = T_i + \frac{1}{2}\mathbf{V}_i^2 /C_p$

$$T_e = 25 + \frac{1}{2}\frac{200^2 \text{ J/kg}}{1004 \text{ J/kg K}} = \textbf{44.92}^{\textbf{o}}\textbf{C}$$

Eq.8.23: $P_e = P_i \left(T_e/T_e \right)^{\frac{k}{k-1}} = 100 \left(\frac{273 + 44.92}{273 + 25}\right)^{1.4 / 0.4} = \textbf{125.4 kPa}$

State i is the free stream state.

State e is the stagnation state.

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Problems Solved Using Pr and vr Functions

8.43 Solved using the isentropic P_r function in A.7.2

An air compressor receives atmospheric air at $T_0 = 17°C$, 100 kPa, and compresses it up to 1400 kPa. The compressor has an isentropic efficiency of 88% and it loses energy by heat transfer to the atmosphere as 10% of the isentropic work. Find the actual exit temperature and the reversible work.

C.V. Compressor

Isentropic: $w_{c,in,s} = h_{e,s} - h_i$; $s_{e,s} = s_i$

Table A.7: $P_{r,e,s} = P_{r,i} \times (P_e/P_i) = 0.9917 \times 14 = 13.884$

$\Rightarrow h_{e,s} = 617.51$ kJ/kg

$w_{c,in,s} = 617.51 - 290.58 = 326.93$ kJ/kg

Actual: $w_{c,in,ac} = w_{c,in,s}/\eta_c = 371.51$; $q_{loss} = 32.693$ kJ/kg

$w_{c,in,ac} + h_i = h_{e,ac} + q_{loss}$

$\Rightarrow h_{e,ac} = 290.58 + 371.51 - 32.693 = 629.4$ kJ/kg

$\Rightarrow T_{e,ac} = 621$ K

Reversible: $w^{rev} = h_i - h_{e,ac} + T_0(s_{e,ac} - s_i)$

$= 290.58 - 629.4 + 290.15 \times (7.6121 - 6.8357)$

$= -338.82 + 225.42 = $ **-113.4 kJ/kg**

Since q_{loss} is also to the atmosphere the above term is the net heat transfer and the reversible work requires the heat transfer to be reversible.

8.69 Solved using the isentropic P_r function in A.7.2

An air compressor is used to charge an initially empty 200-L tank with air up to 5 MPa. The air inlet to the compressor is at 100 kPa, 17°C and the compressor isentropic efficiency is 80%. Find the total compressor work and the change in exergy of the air.

Solution:

C.V. Tank + compressor Transient process with constant inlet conditions, no heat transfer.

Continuity: $m_2 - m_1 = m_{in}$ ($m_1 = 0$)   Energy: $m_2 u_2 = m_{in} h_{in} - {}_1W_2$

Entropy: $m_2 s_2 = m_{in} s_{in} + {}_1S_{2\ gen}$

Reversible compressor: ${}_1S_{2\ GEN} = 0 \Rightarrow s_2 = s_{in}$

State 1: $v_1 = RT_1/P_1 = 0.8323$ m^3/kg,

State inlet, Table A.7.1: $h_{in} = 290.43$ kJ/kg, $s^o_{Tin} = 6.8352$ kJ/kg K

Table A.7.2 $P_{rin} = 0.9899$ used for constant s process

Table A.7.2 $\Rightarrow$ $P_{r2} = P_{rin}(P_2/P_{in}) = 0.9899 \times (5000/100) = 49.495$

$\Rightarrow$ $T_{2,s} = 855$ K, $u_{2,s} = 637.2$ kJ/kg

$\Rightarrow {}_1w_{2,s} = h_{in} - u_{2,s} = 290.43 - 637.2 = -346.77$ kJ/kg

Actual compressor: ${}_1w_{2,AC} = {}_1w_{2,s}/\eta_c = -433.46$ kJ/kg

$u_{2,AC} = h_{in} - {}_1w_{2,AC} = 290.43 - (-433.46) = 723.89$ kJ/kg

Backinterpolate in Table A.7.1 $\Rightarrow$ $T_{2,AC} = 958$ K, $s^o_{T2,AC} = 8.0867$ kJ/kg K

$\Rightarrow$ $v_2 = RT_2/P_2 = 0.055$ m^3/kg

State 2 $\boxed{u, P}$ $m_2 = V_2/v_2 = 3.636$ kg $\Rightarrow {}_1W_2 = m_2({}_1w_{2,AC}) =$ **-1576 kJ**

$m_2(\phi_2 - \phi_1) = m_2[u_2 - u_1 + P_0(v_2 - v_1) - T_0(s_2 - s_1)]$

$= 3.636\ [723.89 - 207.19 + 100(0.055 - 0.8323) - 290[8.0867 -$

$6.8352 - 0.287 \ln(5000/100)] =$ **1460.3 kJ**

Here we used Eq.6.19 for the change in entropy.

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 8
English Units

8e

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CHAPTER 8

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Exergy, Reversible work

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8.140E

A control mass gives out 1000 Btu of energy in the form of
- a.   Electrical work from a battery
- b.   Mechanical work from a spring
- c.   Heat transfer at 700 F

Find the change in exergy of the control mass for each of the three cases.

Solution:

a) Work is exergy $\Delta\Phi = -W_{el} = \mathbf{-1000\ Btu}$

b) Work is exergy $\Delta\Phi = -W_{spring} = \mathbf{-1000\ Btu}$

c) Give the heat transfer to a Carnot heat engine and W is exergy

$$\Delta\Phi = -\left(1 - \frac{T_0}{T_H}\right)Q_{out} = -\left(1 - \frac{537}{1160}\right)1000\ Btu = \mathbf{-537\ Btu}$$

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8.141E

A fraction of some power to a motor (1), 2 kW, is turned into heat transfer at 800 R (2) and then it dissipates in the ambient at 540 R (3). Give the rates of exergy along the process 1-2-3.

Solution:

The exergy of an amount of heat transfer equals the possible work that can be extracted. This is the work out of a Carnot heat engine with heat transfer to the ambient as the other reservoir. The result is from Chapter 5 as also shown in Eq. 8.1 and Eq. 8.38

1: $\dot{\Phi} = \dot{W} = $ **2 kW = 1.896 Btu/s**

2: $\dot{\Phi} = \dot{W}_{rev\ HE} = (1 - \dfrac{T_o}{T})\dot{Q} = (1 - \dfrac{540}{800})\ 2\ kW = $ **0.65 kW = 0.616 Btu/s**

3: $\dot{\Phi} = \dot{W}_{rev\ HE} = (1 - \dfrac{T_o}{T_o})\dot{Q} = (1 - \dfrac{540}{540})\ 2\ kW = $ **0 kW = 0 Btu/s**

As the energy transforms from wotrk to heat transfer it loses exergy and as the heat transfer goes to lower and lower T its exergy value drops ending as zero at the ambient T.

8.142E

A refrigerator should remove 1.5 Btu/s from the cold space at 15 F while it rejects heat to the kitchen at 77 F. Find the reversible work.

The reversible work is related to the Carnot cycle work as the the two reservoirs are at constant temperatures.

$$\dot{W} = \dot{Q}_L / \beta \ ; \qquad \beta = \dot{Q}_L/\dot{W}_{carnot} = \frac{T_L}{T_H - T_L} = \frac{15 + 459.7}{77 - 15} = 7.65$$

In general we have defined the reversible work with the standard sign definition

$$\dot{W}^{rev} = - \dot{Q}_L / \beta = - \frac{1.5}{7.65} \text{ Btu/s} = \textbf{–0.2 Btu/s}$$

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8.143E

A heat engine receives 15 000 btu/h at 1400 R and 30 000 btu/h at 1800 R rejecting energy by heat transfer at 900 R. Assume it is reversible and find the power output. How much power could be produced if it could reject energy at $T_o = 540$ R?

Solution:

C.V. The heat engine, this is in steady state.

Energy Eq.: $0 = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_L - \dot{W}$

Entropy Eq.: $0 = \dfrac{\dot{Q}_1}{T_1} + \dfrac{\dot{Q}_2}{T_2} - \dfrac{\dot{Q}_L}{T_L} + 0$

Now solve for $\dot{Q}_L$ from the entropy equation

$$\dot{Q}_L = \frac{T_L}{T_1}\dot{Q}_1 + \frac{T_L}{T_2}\dot{Q}_2 = \frac{900}{1400} \times 15\ 000 + \frac{900}{1800} \times 30\ 000 = 24\ 643 \text{ Btu/h}$$

Substitue into the enrgy equation and solve for the work term

$$\dot{W} = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_L = 15\ 000 + 30\ 000 - 24\ 643 = \textbf{20 357 Btu/h}$$

For a low temperature of 540 R we can get

$$\dot{Q}_{L2} = \frac{540}{900}\dot{Q}_L = 14\ 786 \text{ btu/h}$$

$$\dot{W} = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_L = 15\ 000 + 30\ 000 - 14\ 786 = \textbf{30 214 Btu/h}$$

Remark: Notice the large increase in the power output.

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8.144E

The compressor in a refrigerator takes refrigerant R-134a in at 15 lbf/in.^{2}, 0 F and compresses it to 125 lbf/in.^{2}, 100 F. With the room at 70 F find the reversible heat transfer and the minimum compressor work.

Solution:

C.V. Compressor out to ambient. Minimum work in is the reversible work.

Steady flow, 1 inlet and 2 exit

Energy Eq.: $\quad w_c = h_1 - h_2 + q^{rev}$

Entropy Eq.: $\quad s_2 = s_1 + \int dq/T + s_{gen} = s_1 + q^{rev}/T_o + 0$

$\Rightarrow \quad q^{rev} = T_o(s_2 - s_1)$

$q^{rev} = 529.67 \text{ R} \times (0.41262 - 0.42288)$ Btu/lbm-R = **-5.43 Btu/lbm**

$w_{c\ min} = h_1 - h_2 + T_o(s_2 - s_1) = 167.193 - 181.059 - 5.43 = -$**19.3 Btu/lbm**

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8.145E

A compressor in a refrigerator receives R-410A at 20 psia, -40 F and it brings it up to 100 psia, using an actual specific work of 23.48 Btu/lbm in an adiabatic compression. Find the specific reversible work.

Energy Eq.: $\quad 0 = h_1 - h_2 - w$

State 1: F.9.2 $\quad h_1 = 113.73$ Btu/lbm, $\quad s_1 = 0.2772$ Btu/lbm-R

$\qquad\qquad h_2 = h_1 - w = 113.73 + 23.48 = 137.21$ kJ/kg

State 2: F.9.2 $\quad$ P, $h_2 = 137.21$ Btu/lbm, $T_2 = 100$ F, $s_2 = 0.2840$ Btu/lbm-R

$w^{rev} = \psi_1 - \psi_2 = h_1 - h_2 - T_0(s_1 - s_2)$

$\qquad = 113.73 - 137.21 - 536.7(0.2772 - 0.2840)$

$\qquad = -23.48 + 3.65 = \textbf{-19.83 Btu/lbm}$

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8.146E

        Air flows through a constant pressure heating device as shown in Fig. P8.27. It is heated up in a reversible process with a work input of 85 Btu/lbm air flowing. The device exchanges heat with the ambient at 540 R. The air enters at 540 R, 60 lbf/in.^{2}. Assuming constant specific heat develop an expression for the exit temperature and solve for it.

        C.V. Total out to T_0

        Energy Eq.: $h_1 + q_0^{rev} - w^{rev} = h_2$

        Entropy Eq.: $s_1 + q_0^{rev}/T_0 = s_2$ $\Rightarrow$ $q_0^{rev} = T_0(s_2 - s_1)$

           $h_2 - h_1 = T_0(s_2 - s_1) - w^{rev}$ (same as from Eq. 8.14)

        Constant C_p gives: $C_p(T_2 - T_1) = T_0 C_p \ln (T_2/T_1) + 85$

        The energy equation becomes

$$T_2 - T_0 \ln(T_2/T_1) = T_1 + 85/C_p$$

     $T_1 = 540$ R, $C_p = 0.24$ Btu/lbm R, $T_0 = 540$ R

          $T_2 - 540 \ln (T_2/540) = 540 + (85/0.24) = 894.17$ R

        Now trial and error on T_2

           At 1400 R   LHS = 885.56 R(too low)

           At 1420 R   LHS = 897.9 R

    Interpolate to get   **T_2 = 1414 R**   (LHS = 894.19 R **OK**)

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8.147E

An adiabatic and reversible air compressor takes air in at 15 psia, 560 R. The air exits at 90 psia at the rate of 0.8 lbm/s. Determine the minimum compressor work input and repeat for an inlet at 530 R instead. Why is the work less for a lower inlet *T*?

Compressor. Reversible adiabatic, constant s:

Inlet at 560 R:
$$T_{2S} = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 560 \text{ R} \left(\frac{90}{15}\right)^{0.2857} = 934.3 \text{ R}$$

The reversible process ensures the minimum work input.

$$\dot{W} = \dot{m}(-w_S) = \dot{m}C_{P0}(T_{2S} - T_1) = 0.8 \times 0.24 \,(934.3 - 560)$$
$$= \textbf{71.86 Btu/s}$$

Inlet at 530 R:
$$T_{2S} = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 530 \text{ R} \left(\frac{90}{15}\right)^{0.2857} = 884.3 \text{ R}$$

$$\dot{W} = \dot{m}(-w_S) = \dot{m}C_{P0}(T_{2S} - T_1) = 0.8 \times 0.24 \,(884.3 - 530)$$
$$= \textbf{68.0 Btu/s}$$

The work term is reduced due to the reduced specific volume, v_1, recall Eq. 7.15 for the indicated shaft work.

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8.148E

A rock bed consists of 12 000 lbm granite and is at 160 F. A small house with lumped mass of 24 000 lbm wood and 2000 lbm iron is at 60 F. They are now brought to a uniform final temperature with no external heat transfer by connecting the house and rock bed through some heat engines. If the process is reversible, find the final temperature and the work done during the process.

Take C.V. Total (rockbed and heat engine)

Energy Eq.: $\quad m_{rock}(u_2 - u_1) + m_{wood}(u_2 - u_1) + m_{Fe}(u_2 - u_1) = -_1W_2$

Entropy Eq.: $\quad m_{rock}(s_2 - s_1) + m_{wood}(s_2 - s_1) + m_{Fe}(s_2 - s_1) = 0$

$$(mC)_{rock}\ln\frac{T_2}{T_1} + (mC)_{wood}\ln\frac{T_2}{T_1} + (mC)_{Fe}\ln\frac{T_2}{T_1} = 0$$

$12000 \times 0.212 \ln (T_2/619.67) + 24000 \times 0.33 \ln (T_2/519.67)$

$$+ 2000 \times 0.11 \ln (T_2/519.67) = 0$$

$\Rightarrow$ **T_2 = 541.9 R**

Now from the energy equation

$-_1W_2 = 12\,000 \times 0.212(541.9 - 619.67)$

$$+ (24\,000 \times 0.33 + 2000 \times 0.11)(541.9 - 519.67)$$

$\Rightarrow$ $_1W_2$ = **16 895 Btu**

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8.149E

A basement is flooded with 250 ft^3 of water at 60 F. It is pumped out with a small pump driven by a 0.75 kW electric motor. The hose can reach 25 ft vertically up and to ensure the water can flow over the edge of a dike it should have a velocity of 45 ft/s at that point generated by a nozzle, see Fig. P8.34. Find the maximum flow rate you can get and how fast the basement can be emptied.

C.V. Pump plus hose and nozzle, single steady state flow. For maximum flow rate assume reversible process so from Eq.8.14:

$$w^{rev} = T_o\,(s_2 - s_1) - (h_{2\,tot} - h_{1\,tot}) + 0$$

Since we have no heat transfer and reversible s is constant and with a liquid flow T is also constant so $h_2 = h_1$. We could also have used Bernoulli equation.

$$w^{rev} = -\left(\frac{1}{2}V_2^2 + gH - 0\right) = -\left(\frac{1}{2}\,45^2/25\,037 + 1 \times 25/778\right)$$

$$= -(0.04044 + 0.032134) = -0.07257\ \text{Btu/lbm}$$

Recall 1 Btu/lbm = 25 037 ft^2/s^2 from conversion in Table A.1

$$\dot{m} = \dot{W}/(-w^{rev}) = \frac{(0.75/1.055)\ \text{Btu/s}}{0.07257\ \text{Btu/lbm}} = \textbf{9.796 lbm/s}$$

$$m = V/v = 250\ \text{ft}^3\ /\ 0.01603\ \text{ft}^3/\text{lbm} = 15\ 596\ \text{lbm}$$

$$\Delta t = m\ /\ \dot{m} = \frac{15\ 596\ \text{lbm}}{9.796\ \text{lbm/s}} = 1592\ \text{sec} = \textbf{26.5 min.}$$

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8.150E

A heat engine receives 3500 Btu/h heat transfer at 1800 R and gives out 2000 Btu/h as work with the rest as heat transfer to the ambient. What are the fluxes of exergy in and out?

Exergy flux in: $\dot{\Phi}_H = \left(1 - \dfrac{T_o}{T_H}\right)\dot{Q}_H = \left(1 - \dfrac{536.7}{1800}\right)3500 \text{ Btu/h} = $ **2456 btu/h**

Exergy flux out: $\dot{\Phi}_L = \left(1 - \dfrac{T_o}{T_L}\right)\dot{Q}_L = \mathbf{0}$ $(T_L = T_o)$

The other exergy flux out is the power $\dot{\Phi}_{out} = \dot{W} = $ **2000 Btu/h**

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8.151E

A flow of air at 150 psia, 540 R is throttled to 75 psia. What is the irreversibility? What is the drop in flow exergy assuming an ambient at 77 F?

A throttle process is constant enthalpy if we neglect kinetic energies.

Process: $h_e = h_i$ so ideal gas   => $T_e = T_i$

Entropy Eq.: $s_e - s_i = s_{gen} = s^o_{Te} - s^o_{Ti} - R \ln \dfrac{P_e}{P_i} = 0 - R \ln \dfrac{P_e}{P_i}$

$$s_{gen} = -\frac{53.34}{778} \ln \left(\frac{75}{150}\right) = 0.0475 \text{ Btu/lbm R}$$

Eq.8.15: $i = T_o\, s_{gen} = 536.7 \times 0.0475 = \textbf{25.49 Btu/lbm}$

The drop in exergy is exergy destruction, which is the irreversibility

$$\Delta\psi = i = \textbf{25.49 Btu/lbm}$$

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Irreversibility

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8.152E

A compressor in a refrigerator receives R-410A at 20 psia, -40 F and it brings it up to 100 psia, 100 F in an adiabatic compression. Find the specific work, entropy generation and irreversibility.

States: 1: F.9.2 $h_1 = 113.73$ Btu/lbm, $s_1 = 0.2772$ Btu/lbm-R

2: F.9.2 $h_2 = 137.21$ Btu/lbm, $s_2 = 0.2840$ Btu/lbm-R

C.V. Compressor, steady state, q = 0, no KE, PE energies.

Energy Eq.: $w = h_1 - h_2 = 113.73 - 137.21 = $ **–23.48 Btu/lbm**

Entropy Eq.: $0 = s_1 - s_2 + s_{gen}$ ⇨

$s_{gen} = s_2 - s_1 = 0.2840 - 0.2772 = $ **0.0068 Btu/lbm-R**

Irreversibility: $i = T_o\, s_{gen} = 536.7\ R \times 0.0068$ Btu/lbm-R = **3.65 Btu/lbm**

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8.153E

A constant pressure piston/cylinder contains 4 lbm of water at 1000 psia and 200 F. Heat is added from a reservoir at 1200 F to the water until it reaches 1200 F. We want to find the total irreversibility in the process.

Solution:

C.V. Piston cylinder out to the reservoir (incl. the walls).

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m(s_2 - s_1) = {}_1Q_2/T_{res} + {}_1S_{2\ gen}$

State 1: $h_1 = 168.07$ Btu/lbm, $s_1 = 0.294$ Btu/lbm R

State 2: $h_2 = 1619.67$ Btu/lbm, $s_2 = 1.726$ Btu/lbm R

Process: $P = C$ $=>$ ${}_1W_2 = P(V_2 - V_1)$

From the energy equation we get

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

$$= 4(1619.67 - 168.07) = 5806.4 \text{ Btu}$$

From the entropy equation we get

$${}_1S_{2\ gen} = m(s_2 - s_1) - \frac{{}_1Q_2}{T_{res}} = 4(1.726 - 0.294) - \frac{5806.4}{459.7 + 1200}$$

$$= 2.2295 \text{ Btu/R}$$

Now the irreversibility is from Eq. 8.18

$${}_1I_2 = m\ {}_1i_2 = T_o\ {}_1S_{2\ gen} = 536.7 \text{ R} \times 2.2295 \text{ Btu/R} = \textbf{1197 Btu}$$

8.154E

A constant "flow" of steel parts 4 lbm/s at 77 F goes into a furnace where they are heat treated to 1600 F by a source at an average 2100 R. Find the reversible work and the irreversibility in this process?

Energy Eq.: $0 = \dot{m}(h_i - h_e) + \dot{Q}$

Entropy Eq.: $0 = \dot{m}(s_i - s_e) + \dot{Q} / T_s + \dot{S}_{gen}$

$$\dot{Q} = \dot{m}(h_e - h_i) = \dot{m}C_P (T_e - T_i) = 4 \text{ lbm/s} \times 0.11 \text{ Btu/lbm-R} \times 1523 \text{ R}$$
$$= 670 \text{ Btu/s}$$

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) - \dot{Q} / T_s = \dot{m}C_P \ln(T_e/T_i) - \dot{Q} / T_s$$
$$= 4 \text{ lbm/s} \times 0.11 \text{ Btu/lbm-R} \times \ln(2060/537)$$
$$- (670 / 2100) \text{ Btu/s-R}$$
$$= 0.2725 \text{ Btu/s-R}$$

$$\dot{I} = T_0\dot{S}_{gen} = 537 \text{ R} \times 0.2725 \text{ Btu/s-R} = \textbf{146.3 Btu/s}$$

The reversible work is equal to the irreversibility plus the actual work, Eq.8.12

$$\dot{W}^{rev} = \dot{I} + \dot{W}_{ac} = \dot{I} = \textbf{146.3 Btu/s}$$

Borgnakke and Sonntag

8.155E

A cylinder with a piston restrained by a linear spring contains 4 lbm of carbon dioxide at 70 psia, 750 F. It is cooled to 75 F, at which point the pressure is 45 psia. Find the reversible work and the irreversibility assuming the heat transfer is with surroundings at 68 F.

Linear spring gives

$$_1W_2 = \int PdV = \tfrac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

$$_1Q_2 = m(u_2 - u_1) + {_1}W_2$$

Equation of state: $PV = mRT$

State 1: $V_1 = mRT_1/P_1 = 4 \times 35.1 \times 1209.7 /(70 \times 144) = 16.849$ ft^3

State 2: $V_2 = mRT_2/P_2 = 4 \times 35.1 \times 534.7 /(45 \times 144) = 11.585$ ft^3

$$_1W_2 = \tfrac{1}{2}(70 + 45) \text{ psi } (11.585 - 16.849) \text{ ft}^3 \times \frac{144 \text{ in}^2/\text{ft}^2}{778 \text{ lbf-ft/Btu}}$$

$$= -56.02 \text{ Btu}$$

Table F.4 at 77 F is $C_V = 0.156$ Btu/lbm-R

$$_1Q_2 = mC_V(T_2 - T_1) + {_1}W_2 = 4 \times 0.156 (75 - 750) - 56.02 = -477.2 \text{ Btu}$$

$$_1W_2^{rev} = T_o(S_2 - S_1) - (U_2 - U_1) + {_1}Q_2 (1 - T_o/T_H)$$

$$= T_o m(s_2 - s_1) + {_1}W_2^{ac} - {_1}Q_2 \, T_o/T_o$$

$$= T_o m[C_P \ln(T_2 / T_1) - R \ln(P_2 / P_1)] + {_1}W_2^{ac} - {_1}Q_2$$

$$= 527.7 \times 4 \, [\, 0.201 \ln \frac{534.7}{1209.7} - \frac{35.1}{778} \ln \frac{45}{70} \,] - 56.02 + 477.2$$

$$= -304.3 - 56.02 + 477.2 = \mathbf{116.9 \text{ Btu}}$$

$$_1I_2 = {_1}W_2^{rev} - {_1}W_2^{ac} = 116.9 - (-56.02) = \mathbf{172.9 \text{ Btu}}$$

Borgnakke and Sonntag

8.156E

Fresh water can be produced from saltwater by evaporation and subsequent condensation. An example is shown in Fig. P8.45 where 300-lbm/s saltwater, state 1, comes from the condenser in a large power plant. The water is throttled to the saturated pressure in the flash evaporator and the vapor, state 2, is then condensed by cooling with sea water. As the evaporation takes place below atmospheric pressure, pumps must bring the liquid water flows back up to P_0. Assume that the saltwater has the same properties as pure water, the ambient is at 68 F, and that there are no external heat transfers. With the states as shown in the table below find the irreversibility in the throttling valve and in the condenser.

State	1	2	5	7	8
T [F]	86	77	74	63	68
h [Btu/lbm]	54.08	1095.1	42.09	31.08	36.09
s [Btu/lbm R]	0.1043	2.044	0.0821	0.0613	0.0708

C.V. Valve: $\dot{m}_1 = \dot{m}_{ex} = \dot{m}_2 + \dot{m}_3$,

Energy Eq.: $h_1 = h_e$; Entropy Eq.: $s_i + s_{gen} = s_e$

$\qquad P_2 = P_{sat}(T_2 = T_3) = 0.4641$ psia

$\qquad h_e = h_1 \Rightarrow x_e = (54.08 - 45.08)/1050.0 = 0.008571$

$\qquad \Rightarrow s_e = 0.08769 + 0.008571 \times 1.9565 = 0.1045$ Btu/lbm R

$\qquad\qquad \dot{m}_2 = (1 - x_e)\, \dot{m}_i = (1 - 0.008571)\, 300 = 297.44$ lbm/s

$\qquad s_{gen} = s_e - s_i = 0.1045 - 0.1043 = 0.0002$ Btu/lbm R

$\qquad \dot{I} = \dot{m}T_0 s_{gen} = 300 \text{ lbm/s} \times 528 \text{ R} \times 0.0002 \text{ Btu/lbm-R} = 31.68$ Btu/s

C.V. Condenser.

Energy Eq.: $\dot{m}_2 h_2 + \dot{m}_7 h_7 = \dot{m}_2 h_5 + \dot{m}_7 h_8 \Rightarrow$

$\qquad \dot{m}_7 = \dot{m}_2 \times (h_2 - h_5)/(h_8 - h_7) = 297.44 \dfrac{1095.1 - 42.09}{36.09 - 31.08} = 62516 \dfrac{\text{lbm}}{\text{s}}$

Entropy Eq.: $\dot{m}_2 s_2 + \dot{m}_7 s_7 + \dot{S}_{gen} = \dot{m}_2 s_5 + \dot{m}_7 s_8$

$\qquad \dot{I} = T_0 \dot{S}_{gen} = T_0 \left[\dot{m}_2(s_5 - s_2) + \dot{m}_7(s_8 - s_7) \right]$

$\qquad = 528 \left[297.44(0.0821 - 2.044) + 62516(0.0708 - 0.0613) \right]$

$\qquad = 528 \times 10.354 = \textbf{5467 Btu/s}$

Borgnakke and Sonntag

8.157E

A rock bed consists of 12 000 lbm granite and is at 160 F. A small house with lumped mass of 24 000 lbm wood and 2000 lbm iron is at 60 F. They are now brought to a uniform final temperature by circulating water between the rock bed and the house. Find the final temperature and the irreversibility in the process assuming an ambient at 60 F.

C.V. Total Rockbed and house. No work, no Q irreversible process.

Energy eq.: $m_{rock}(u_2 - u_1) + m_{wood}(u_2 - u_1) + m_{Fe}(u_2 - u_1) = 0$

Entropy Eq.: $m_{rock}(s_2 - s_1) + m_{wood}(s_2 - s_1) + m_{Fe}(s_2 - s_1) = S_{gen}$

$$(mC)_{rock}\ln\frac{T_2}{T_1} + (mC)_{wood}\ln\frac{T_2}{T_1} + (mC)_{Fe}\ln\frac{T_2}{T_1} = S_{gen}$$

Energy eq.: $(mC)_{rock}(T_2 - 160) + (mC_{wood} + mC_{Fe})(T_2 - 60) = 0$

$$12\ 000 \times 0.212\ (T_2 - 160) + (24\ 000 \times 0.33 + 2000 \times 0.11)(T_2 - 60) = 0$$

$$2544\ (T_2 - 160) + (7920 + 220)(T_2 - 60) = 0$$

$$T_2 = \textbf{83.8 F = 543.5 R}$$

$$S_{gen} = \sum m_i(s_2 - s_1)_i = (mC)_{rock}\ln\frac{T_2}{T_1} + (mC)_{wood}\ln\frac{T_2}{T_1} + (mC)_{Fe}\ln\frac{T_2}{T_1}$$

$$= 2544\ \ln\frac{543.5}{619.67} + (7920 + 220)\ \ln\frac{543.5}{519.67} = 35.26\ \text{Btu/R}$$

$$_1I_2 = T_0\ S_{gen} = 519.67 \times 35.26 = \textbf{18 324 Btu}$$

Borgnakke and Sonntag

8.158E

Air enters the turbocharger compressor of an automotive engine at 14.7 lbf/in^2, 90 F, and exits at 25 lbf/in^2, as shown in Fig. P8.51. The air is cooled by 90 F in an intercooler before entering the engine. The isentropic efficiency of the compressor is 75%. Determine the temperature of the air entering the engine and the irreversibility of the compression-cooling process.

Solution:

a) Compressor. First ideal which is reversible adiabatic, constant s:

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 550 \text{ R} \left(\frac{25}{14.7}\right)^{0.286} = 640.2 \text{ R}$$

$$-w_s = C_{P0}(T_{2s} - T_1) = 0.24(640.2 - 550) = 21.65 \text{ Btu/lbm}$$

Now the actual compressor

$$-w = -w_s/\eta_s = 21.65/0.75 = 28.87 = C_{P0}(T_2 - T_1) = 0.24(T_2 - 550)$$

$$\Rightarrow \quad T_2 = 670.3 \text{ R}$$

Cool down 90 F $\quad \Rightarrow$ T$_3$ = 670.3 - 90 = **580.3 R**

b) Irreversibility from Eq.8.15 with rev. work from Eq.8.14, (q = 0 at T$_H$)

$$s_3 - s_1 = 0.24 \ln\frac{580.3}{550} - \frac{53.34}{778} \ln\frac{25}{14.7} = -0.0235 \text{ Btu/lbm R}$$

$$i = T(s_3 - s_1) - (h_3 - h_1) - w = T(s_3 - s_1) - C_P(T_3 - T_1) - C_P(T_1 - T_2)$$

$$= 550(-0.0235) - 0.24(-90) = +\textbf{8.7 Btu/lbm}$$

Borgnakke and Sonntag

Exergy

Borgnakke and Sonntag

8.159E

A heat pump has a coefficient of performance of 2 using a power input of 15000 Btu/h. Its low temperature is T_o, and the high temperature is 180 F, with ambient at T_o. Find the fluxes of exergy associated with the energy fluxes in and out.

First let us do the energies in and out

$$COP = \beta = \frac{\dot{Q}_H}{\dot{W}} \quad => \quad \dot{Q}_H = \beta\, \dot{W} = 2 \times 15\,000 \text{ Btu/h} = 30\,000 \text{ Btu/h}$$

Energy Eq.: $\quad \dot{Q}_L = \dot{Q}_H - \dot{W} = 30\,000 - 15\,000 = 15\,000 \text{ Btu/h}$

Exergy flux in: $\quad \dot{\Phi}_L = \left(1 - \frac{T_o}{T_L}\right)\dot{Q}_L = \mathbf{0} \qquad (T_L = T_o)$

Exergy flux in: $\quad \dot{\Phi}_W = \dot{W} = \mathbf{15\,000\ Btu/h}$

Exergy flux out: $\quad \dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right)\dot{Q}_H = \left(1 - \frac{536.7}{639.7}\right)30\,000 = \mathbf{4830\ Btu/h}$

Remark: It destroys $15\,000 - 4830 = 10\,170$ Btu/h of exergy.

Borgnakke and Sonntag

8.160E

A steady flow device receives R-410A at 125 psia, 100 F, and it exits at 15 psia, 100 F. Assume a reversible isothermal process. Find the change in specific exergy.

Solution:

Inlet Table F.9.2 : $h_i = 135.96$ Btu/lbm, $s_i = 0.2763$ Btu/lbm R

Exit Table F.9.2 : $h_e = 141.10$ Btu/lbm, $s_e = 0.3411$ Btu/lbm R

From Eq.8.23 or flow terms in Eq.8.38

$$\Delta\psi_{ie} = \psi_e - \psi_i = (h_e - h_i) - T_0(s_e - s_i)$$
$$= (141.10 - 135.96) - 536.7\,(0.3411 - 0.2763)$$
$$= -\textbf{29.6 Btu/lbm}$$

Remark: it is negative due to the pressure loss.

Borgnakke and Sonntag

8.161E

Consider the springtime melting of ice in the mountains, which gives cold water running in a river at 34 F while the air temperature is 68 F. What is the exergy of the water relative to the temperature of the ambient?

$$\psi = h_1 - h_0 - T_0(s_1 - s_0) \qquad \text{flow exergy from Eq.8.22}$$

Approximate both states as saturated liquid

$$\psi = 1.9973 - 36.088 - 527.67 \times (0.00405 - 0.07081) = \textbf{1.136 Btu/lbm}$$

Why is it positive? As the water is brought to 68 F it can be heated with q_L from a heat engine using q_H from atmosphere $T_H = T_0$ thus giving out work.

Borgnakke and Sonntag

8.162E

        Compressed air for machines and tools in a plant is generated by a central compressor receiving air at 15 psia, 540 R, 1 lbm/s delivering it at 90 psia to a buffer tank and a distribution pipe. After flowing through the tank and pipe the air is at the ambient 540 R at its point of use. Assume a reversible adiabatic compressor and find the compressor exit temperature and the increase in air exergy through the compressor.

C.V. Compressor

Energy: $0 = h_1 - h_2 + w_C$;   Entropy: $0 = s_1 - s_2 + 0$

$$s_2 - s_1 = s_{T2}^0 - s_{T1}^0 - R \ln(P_2/P_1) \quad \text{find } s_{T2}^0 \text{ and then } T_2$$

or with constant specific heat

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 540 \text{ R} \left(\frac{90}{15}\right)^{0.2857} = \textbf{901 R}$$

Exergy increase through the compressor matches with the compressor work

$$\psi_2 - \psi_1 = (h_2 - h_1) - T_0(s_2 - s_1) = h_2 - h_1 = C_P (T_2 - T_1) = w_C$$

$$= 0.24 \text{ Btu/lbm-R } (901 - 540) \text{ R} = \textbf{86.6 Btu/lbm}$$

Borgnakke and Sonntag

8.163E

For the air system in the previous problem find the increase in the air exergy from the inlet to the point of use. How much exergy was lost in the flow after the compressor exit?

From inlet, state 1 to final point of use state 3.

$$\psi_3 - \psi_1 = (h_3 - h_1) - T_0(s_3 - s_1) = 0 - T_0[0 - R \ln(P_3/P_1)]$$

$$= T_0 R \ln(P_3/P_1) = 537 \text{ R} \times (53.34/778) \text{ Btu/lbm-R} \times \ln(90/15)$$

$$= \textbf{65.97 Btu/lbm}$$

Compressor exit, reversible adiabatic, constant s process: $s_2 = s_1$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 540 \text{ R} \left(\frac{90}{15}\right)^{0.2857} = 901 \text{ R}$$

So then

$$\psi_{\text{lost}} = \psi_2 - \psi_3 = -(\psi_3 - \psi_1) + (\psi_2 - \psi_1) = -(\psi_3 - \psi_1) + h_2 - h_1$$

$$= -(\psi_3 - \psi_1) + C_{P0}(T_2 - T_1)$$

$$= -65.97 \text{ kJ/kg} + 0.24 \text{ Btu/lbm-R} (901 - 540) \text{ R}$$

$$= \textbf{20.67 btu/lbm}$$

Borgnakke and Sonntag

8.164E

A geothermal source provides 20 lbm/s of hot water at 80 lbf/in.^{2}, 280 F flowing into a flash evaporator that separates vapor and liquid at 30 lbf/in.^{2}. Find the three fluxes of exergy (inlet and two outlets) and the irreversibility rate.

C.V. Flash evaporator chamber. Steady flow with no work or heat transfer.

Cont. Eq.: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$;

Energy Eq.: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$

Entropy Eq.: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3$

Properties from Table F.7.1

$h_O = 45.08$, $h_1 = 249.17$, $h_2 = 1164.3$, $h_3 = 218.9$ Btu/lbm

$s_O = 0.08769$, $s_1 = 0.4098$, $s_2 = 1.6997$, $s_3 = 0.36815$ Btu/lbm R

$h_1 = x h_2 + (1 - x) h_3 \implies x = \dot{m}_2/\dot{m}_1 = \dfrac{h_1 - h_3}{h_2 - h_3} = 0.03202$

$\dot{m}_2 = x \dot{m}_1 = 0.64$ lbm/s $\qquad \dot{m}_3 = (1-x)\dot{m}_1 = 19.36$ lbm/s

$\dot{S}_{gen} = 0.64 \times 1.6997 + 19.36 \times 0.36815 - 20 \times 0.4098 = 0.0192$ Btu/s-R

Flow exergy Eq.8.22: $\psi = (h - T_O s) - (h_O - T_O s_O) = h - h_O - T_O(s - s_O)$

$\psi_1 = 249.17 - 45.08 - 537 \times (0.4098 - 0.08769) = 31.117$ Btu/lbm

$\psi_2 = 1164.3 - 45.08 - 537 \times (1.6997 - 0.08769) = 253.57$ Btu/lbm

$\psi_3 = 218.9 - -45.08 - 537 \times (0.36815 - 0.08769) = 23.21$ Btu/lbm

$\dot{m}_1 \psi_1 = \textbf{622.3 Btu/s}$ $\dot{m}_2 \psi_2 = \textbf{162.3 Btu/s}$ $\dot{m}_3 \psi_3 = \textbf{449.3 Btu/s}$

Balance of flow terms in Eq.8.38

$\dot{I} = \dot{m}_1 \psi_1 - \dot{m}_2 \psi_2 - \dot{m}_3 \psi_3 = \textbf{10.7 Btu/s}$ $(= T_O \dot{S}_{gen})$

Borgnakke and Sonntag

8.165E

An air compressor is used to charge an initially empty 7-ft^3 tank with air up to 750 lbf/in.^{2}. The air inlet to the compressor is at 14.7 lbf/in.^{2}, 60 F and the compressor isentropic efficiency is 80%. Find the total compressor work and the change in energy of the air.

C.V. Tank + compressor (constant inlet conditions)

Continuity: $m_2 - 0 = m_{in}$ Energy: $m_2 u_2 = m_{in} h_{in} - {}_1 W_2$

Entropy: $m_2 s_2 = m_{in} s_{in} + {}_1 S_{2\ GEN}$

To use isentropic efficiency we must calc. ideal device

State 1: $v_1 = RT_1/P_1 = 13.103$ ft^3/lbm, the ambient state

Reversible compressor: ${}_1 S_{2\ GEN} = 0 \implies s_2 = s_{in}$

$$\implies s^o_{T2} = s^o_{Tin} + R \ln(\frac{P_2}{P_{in}}) = 1.6307 + \frac{53.34}{778} \times \ln(\frac{750}{14.7}) = 1.9003\ \frac{Btu}{lbm\ R}$$

$$\implies T_{2,s} = 1541\ R \quad u_{2,s} = 274.49\ Btu/lbm$$

$$\implies {}_1 w_{2,s} = h_{in} - u_{2,s} = 124.38 - 274.49 = -150.11\ Btu/lbm$$

Actual compressor: ${}_1 w_{2,AC} = {}_1 w_{2,s}/\eta_c = -187.64$ Btu/lbm

$u_{2,AC} = h_{in} - {}_1 w_{2,AC} = 312 \implies T_{2,AC} = 1729\ R$

Final state 2 $\boxed{u, P} \implies v_2 = RT_2/P_2 = 0.854$ ft^3/lbm

$m_2 = V_2/v_2 = 8.2$ lbm

$$\implies {}_1 W_2 = m_2({}_1 W_{2,AC}) = \textbf{-1539 Btu}$$

$m_2(\phi_2 - \phi_1) = m_2 [u_2 - u_1 + P_0(v_2 - v_1) - T_0(s_2 - s_1)]$

$$= 8.2 \left[312 - 88.733 + 14.7(0.854 - 13.103)\frac{144}{778} - 520\left(1.9311\right.\right.$$

$$\left.\left. - 1.63074 - \frac{53.34}{778} \ln\frac{750}{14.7}\right)\right] = 8.2 \times 173.94 = \textbf{1426.3 Btu}$$

Borgnakke and Sonntag

8.166E
An electric stove has one heating element at 600 F getting 750 W of electric power. It transfers 90% of the power to 2 lbm water in a kettle initially at 70 F, 1 atm, the rest 10% leaks to the room air. The water at a uniform T is brought to the boiling point. At the start of the process what is the rate of exergy transfer by: a) electrical input   b) from heating element   and c) into the water at T_{water}.

We take here the reference T to be the room 70 F = 529.67 R

a) Work is exergy $\dot{\Phi} = \dot{W} = $ **750 W**

b) Heat transfer at 600 F is only partly exergy

$$\dot{\Phi} = \left(1 - \frac{T_o}{T_H}\right)\dot{Q} = \left(1 - \frac{529.67}{459.67 + 600}\right)750 = \textbf{375 W}$$

c) Water receives heat transfer at 70 F as 90% of 750 W = 675 W

$$\dot{\Phi} = \left(1 - \frac{T_o}{T_{water}}\right)\dot{Q} = \left(1 - \frac{529.67}{459.67 + 70}\right)675 = \textbf{0 W}$$

750 W at 600 F

Borgnakke and Sonntag

8.167E

A wood bucket (4 lbm) with 20 lbm hot liquid water, both at 180 F, is lowered 1300 ft down into a mineshaft. What is the exergy of the bucket and water with respect to the surface ambient at 70 F?

C.V. Bucket and water. Both thermal exergy and potential energy terms.

$v_1 \approx v_0$ for both wood and water so work to atm. is zero.

Use constant heat capacity table F.2 for wood and table F.7.1 (sat. liquid) for water.
From Eq.8.28

$$\phi_1 - \phi_0 = m_{wood}[u_1 - u_0 - T_0(s_1 - s_0)] + m_{H_2O}[u_1 - u_0 - T_0(s_1 - s_0)] + m_{tot}g(z_1 - z_0)$$

$$= 4[0.3(180 - 70) - 0.3 \times 530 \ln \frac{640}{530}] + 20[147.76 - 38.09$$

$$- 530(0.263 - 0.074)] + 24 \times 32.174 \times (-1300 / 25\ 037)$$

$$= 12.05 + 199.3 - 40.1 = \textbf{171.25 Btu}$$

Recall 1 Btu/lbm = 25 037 ft^2/s^2

Borgnakke and Sonntag

8.168E

A 20-lbm iron disk brake on a car is at 50 F. Suddenly the brake pad hangs up, increasing the brake temperature by friction to 230 F while the car maintains constant speed. Find the change in exergy of the disk and the energy depletion of the car's gas tank due to this process alone. Assume that the engine has a thermal efficiency of 35%.

All the friction work is turned into internal energy of the disk brake.

$$m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 \Rightarrow {}_1Q_2 = m_{Fe}C_{Fe}(T_2 - T_1)$$

$${}_1Q_2 = 20 \times 0.107 \times (230 - 50) = 385.2 \text{ Btu}$$

Change in s for a solid, Eq.8.11

$$m(s_2 - s_1) = mC \ln(T_2 / T_1) = 20 \times 0.107 \times \ln\left(\frac{690}{510}\right) = 0.6469 \text{ Btu/R}$$

No change in kinetic or potential energy, no volume change and neglect the work to the surroundings at P_0, so change in exergy is from Eq.8.27

$$\Delta\phi = m(u_2 - u_1) - T_0m(s_2 - s_1) = 385.2 - 510 \times 0.6469 = \textbf{55.28 Btu}$$

$$W_{engine} = \eta_{th}Q_{gas} = {}_1Q_2 = \text{Friction work}$$

$$Q_{gas} = {}_1Q_2/\eta_{th} = 385.2/0.35 = \textbf{1100 Btu}$$

8.169E

Apply the exergy equation to find the exergy destruction in Problem 8.150E

Exergy flux in: $\dot{\Phi}_H = \left(1 - \dfrac{T_o}{T_H}\right)\dot{Q}_H = \left(1 - \dfrac{536.7}{1800}\right)3500 \text{ Btu/h} = 2456 \text{ Btu/h}$

Exergy flux out: $\dot{\Phi}_L = \left(1 - \dfrac{T_o}{T_L}\right)\dot{Q}_L = 0 \qquad (T_L = T_o)$

The other exergy flux out is the power $\dot{\Phi}_{out} = \dot{W} = 2000 \text{ Btu/h}$

The exergy balance equation Eq.8.38 for the HE becomes (steady state)

$$0 = \dot{\Phi}_H - \dot{\Phi}_L - \dot{W} - \dot{\Phi}_{destruction}$$

$$\dot{\Phi}_{destruction} = \dot{\Phi}_H - \dot{\Phi}_L - \dot{W} = 2456 \text{ Btu/h} - 0 - 2000 \text{ Btu/h}$$
$$= \textbf{456 Btu/h}$$

Borgnakke and Sonntag

8.170E

The condenser in a power plant cools 20 lbm/s water at 120 F, quality 90% so it comes out as saturated liquid at 120 F. The cooling is done by ocean-water coming in at 60 F and returned to the ocean at 68 F. Find the transfer out of the water and the transfer into the ocean-water of both energy and exergy (4 terms).

Solution:

C.V. Water line. No work but heat transfer out.

Energy Eq.: $\dot{Q}_{out} = \dot{m}\,(h_1 - h_2) = 20(1010.99 - 87.99) = \textbf{18 460 Btu/s}$

C.V. Ocean water line. No work but heat transfer in equals water heattransfer out

Energy Eq.: $q = h_4 - h_3 = 36.09 - 28.08 = 8.0$ Btu/lbm

$$\dot{m}_{ocean} = \dot{Q}_{out}/q = 18\,460 / 8.0 = 2308 \text{ kg/s}$$

Exergy out of the water follows from flow term Eq.8.38 (we will use $T_O = 60$ F)

$$\dot{\Phi}_{out} = \dot{m}(\psi_1 - \dot{m}\psi_2) = \dot{m}\,[\,h_1 - h_2 - T_O\,(\,s_1 - s_2)\,]$$
$$= 20\,[\,1010.99 - 87.99 - 519.7(1.7567 - 0.1646)]$$
$$= \textbf{1912 Btu/s}$$

Exergy into the ocean water

$$\dot{\Phi}_{ocean} = \dot{m}_{ocean}(\psi_4 - \psi_3) = \dot{m}_{ocean}\,[\,h_4 - h_3 - T_o(s_4 - s_3)]$$
$$= 2308\,[\,8.0 - 519.7(0.0708 - 0.0555)]$$
$$= \textbf{112 Btu/s}$$

Notice there is a large amount of energy exchanged, but very little exergy.

Often the cooling media flows inside a long pipe carrying the energy away.

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Device 2nd Law Efficiency

Borgnakke and Sonntag

8.171E

A heat engine receives 3500 Btu/h heat transfer at 1800 R and gives out 1400 Btu/h as work with the rest as heat transfer to the ambient. Find its first and second law efficiencies.

First law efficiency is based on the energies

$$\eta_I = \dot{W}/\dot{Q}_H = \frac{1400}{3500} = \mathbf{0.4}$$

The second law efficiency is based on work out versus exergy in

Exergy flux in: $\dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right)\dot{Q}_H = \left(1 - \frac{536.7}{1800}\right)3500$ Btu/h = **2456 btu/h**

$$\eta_{II} = \frac{\dot{W}}{\dot{\Phi}_H} = \frac{1400}{2456} = \mathbf{0.57}$$

Notice the exergy flux in is equal to the Carnot heat engine power output given 3500 Btu/h at 1800 R and rejecting energy to the ambient.

Energy terms   Exergy terms

Borgnakke and Sonntag

8.172E

A heat exchanger increases the exergy of 6 lbm/s water by 800 btu/lbm using 20 lbm/s air coming in at 2500 R and leaving with 250 Btu/lbm less exergy. What are the irreversibility and the second law efficiency?

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside.

The irreversibility is the destruction of exergy (exergy) so

$$\dot{I} = \dot{\Phi}_{destruction} = \dot{\Phi}_{in} - \dot{\Phi}_{out} = 20 \times 250 - 6 \times 800 = \textbf{200 Btu/s}$$

The second law efficiency, Eq.8.32

$$\eta_{II} = \dot{\Phi}_{out} / \dot{\Phi}_{in} = \frac{6 \times 800}{20 \times 250} = \textbf{0.96}$$

Borgnakke and Sonntag

8.173E

Find the second law efficiency of the heat pump in problem 8.159E.

The second law efficiency is a ratio of exergies namely what we want out divided by what we have to put in. Exergy from first term on RHS Eq. 8.38

$$\dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right)\dot{Q}_H;$$

$$\dot{Q}_H = \beta\,\dot{W} = 2 \times 15\ 000\ \text{Btu/h} = 30\ 000\ \text{Btu/h}$$

$$\eta_{II} = \frac{\dot{\Phi}_H}{\dot{W}} = \left(1 - \frac{T_o}{T_H}\right)\frac{\dot{Q}_H}{\dot{W}} = \left(1 - \frac{536.7}{639.7}\right)\frac{30\ 000}{15\ 000} = \mathbf{0.32}$$

180 F

$\dot{Q}_H$

HP
cb

$\dot{W} = 15\ 000\ \text{Btu/h}$

$\dot{Q}_L$

T_o

Borgnakke and Sonntag

8.174E

Find the isentropic efficiency and the second law efficiency for the compressor in Problem 8.145

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.4.13: $w = h_1 - h_2$

Entropy Eq.7.8: $s_2 = s_1 + s_{gen}$

We have two different cases, the ideal and the actual compressor.

State 1: F.9.2 $h_1 = 113.73$ Btu/lbm, $s_1 = 0.2772$ Btu/lbm-R

$h_2 = h_1 - w = 113.73 + 23.48 = 137.21$ kJ/kg

State 2: F.9.2 P, $h_2 = 137.21$ Btu/lbm, $T_2 = 100$ F, $s_2 = 0.2840$ Btu/lbm-R

State 2s: B.4.2 (P, s = s_1) $h_{2,s} = 133.44$ Btu/lbm

Actual compressor: $-w_{C,AC} = h_{2,AC} - h_1 = 137.21 - 113.73 = 23.48$ Btu/lbm
Ideal work: $-w_{c,s} = h_{2,s} - h_1 = 133.44 - 113.73 = 19.71$ Btu/lbm

Definition Eq.7.27: $\eta_c = w_{c,s}/w_{c,AC} = \mathbf{0.839}$

Rev. work Eq.8.29

$$w^{rev} = \psi_1 - \psi_2 = h_1 - h_2 - T_0(s_1 - s_2)$$
$$= 113.73 - 137.21 - 536.7(0.2772 - 0.2840)$$
$$= -23.48 + 3.65 = -19.83 \text{ Btu/lbm}$$

2nd law efficiency: $\eta_{II} = \dfrac{w_{rev}}{w_{ac}} = \dfrac{-19.83}{-23.48} = \mathbf{0.8445}$ $\left(= \dfrac{w_{ac} + i}{w_{ac}} \right)$

Borgnakke and Sonntag

8.175E

      A steam turbine has an inlet at 600 psia, and 900 F and actual exit of 1 atm with x = 1.0. Find its first law (isentropic) and the second-law efficiencies

C.V. Steam turbine

Energy Eq.4.13: $w = h_i - h_e$

Entropy Eq.7.9: $s_e = s_i + s_{gen}$

Inlet state: Table F.7.2 $h_i = 1462.92$ Btu/lbm; $s_i = 1.6766$ Btu/lbm R

Exit (actual) state: F.7.2 $h_e = 1150.49$ Btu/lbm; $s_e = 1.7567$ Btu/lbm R

Actual turbine energy equation

$$w = h_i - h_e = 312.43 \text{ Btu/lbm}$$

Ideal turbine reversible process so $s_{gen} = 0$ giving

$$s_{es} = s_i = 1.6766 = 0.3121 + x_{es} \times 1.4446$$

$$x_{es} = 0.94455, \; h_{es} = 180.13 + 0.94455 \times 970.35 = 1096.67$$

The energy equation for the ideal gives

$$w_s = h_i - h_{es} = 366.25 \text{ Btu/lbm}$$

The first law efficiency is the ratio of the two work terms

$$\eta_s = w/w_s = 312.43/366.25 = \mathbf{0.853}$$

The reversible work for the actual turbine states is, Eq.8.29

$$w^{rev} = (h_i - h_e) + T_o(s_e - s_i)$$

$$= 312.43 + 536.7(1.7567 - 1.6766)$$

$$= 312.43 + 42.99 = 355.4 \text{ Btu/lbm}$$

Second law efficiency Eq.8.31

$$\eta_{2^{nd} \text{ Law}} = w/w_{rev} = 312.43/355.4 = \mathbf{0.879}$$

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8.176E
A heat engine operating with an evironment at 540 R produces 17 000 Btu/h of power output with a first law efficiency of 50%. It has a second law efficiency of 80% and $T_L = 560$ R. Find all the energy and exergy transfers in and out.

Solution:
From the definition of the first law efficiency

$$\dot{Q}_H = \dot{W} / \eta = \frac{17\ 000}{0.5} = \textbf{34 000 Btu/h}$$

Energy Eq.: $\quad \dot{Q}_L = \dot{Q}_H - \dot{W} = 34\ 000 - 17\ 000 = \textbf{17 000 Btu/h}$

$$\dot{\Phi}_W = \dot{W} = \textbf{17 000 Btu/h}$$

From the definition of the second law efficiency $\eta_{II} = \dot{W}/\dot{\Phi}_H$, this requires that we assume the exergy rejected at 560 R is lost and not counted, otherwise the

efficiency should be $\eta_{II} = \dot{W}/(\dot{\Phi}_H - \dot{\Phi}_L)$.

Exergy from source: $\qquad \dot{\Phi}_H = (1 - \frac{T_o}{T_H})\ \dot{Q}_H = \frac{17\ 000}{0.8} = \textbf{21 250 Btu/h}$

Exergy rejected: $\qquad \dot{\Phi}_L = (1 - \frac{T_o}{T_L})\ \dot{Q}_L = (1 - \frac{540}{560})\ 17\ 000 = \textbf{607 Btu/h}$

Notice from the $\dot{\Phi}_H$ form we could find the single characteristic T_H as

$$(1 - \frac{T_o}{T_H}) = 21\ 250\ \text{Btu/h} / \dot{Q}_H = 0.625 \quad => \quad T_H = 1440\ \text{R}$$

8.177E

Air flows into a heat engine at ambient conditions 14.7 lbf/in.^{2}, 540 R, as shown in Fig. P8.111. Energy is supplied as 540 Btu per lbm air from a 2700 R source and in some part of the process a heat transfer loss of 135 Btu per lbm air happens at 1350 R. The air leaves the engine at 14.7 lbf/in.^{2}, 1440 R. Find the first- and the second-law efficiencies.

C.V. Engine out to reservoirs

$$h_i + q_H = q_L + h_e + w$$

Table F.5: $h_i = 129.18$ Btu/lbm, $s_{Ti}^o = 1.63979$ Btu/lbm R

$h_e = 353.483$ Btu/lbm, $s_{Te}^o = 1.88243$ Btu/lbm R

$w_{ac} = 129.18 + 540 - 135 - 353.483 = 180.7$ Btu/lbm

$\eta_{TH} = w/q_H = 180.7/540 = \textbf{0.335}$

For second law efficiency also a q to/from ambient

$$s_i + (q_H/T_H) + (q_0/T_0) = (q_{loss}/T_m) + s_e$$

$$q_0 = T_0\big[s_e - s_i + (q_{loss}/T_m) - (q_H/T_H)\big]$$

$$= 540\left(1.88243 - 1.63979 + \frac{135}{1350} - \frac{540}{2700}\right) = 77.02 \text{ Btu/lbm}$$

$$w_{rev} = h_i - h_e + q_H - q_{loss} + q_0 = w_{ac} + q_0 = 257.7 \text{ Btu/lbm}$$

$$\eta_{II} = w_{ac}/w_{rev} = 180.7/257.7 = \textbf{0.70}$$

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8.178E

Air enters a compressor at ambient conditions, 15 psia, 540 R, and exits at 120 psia. If the isentropic compressor efficiency is 85%, what is the second-law efficiency of the compressor process?

Ideal (isentropic, Eq.6.23)

$$T_{2s} = 540 \ (8)^{0.2857} = 978.1 \ R$$

$$-w_s = 0.24 \ (978.1 - 540) = 105.14 \ \text{Btu/lbm}$$

$$-w = \frac{-w_s}{\eta_s} = \frac{105.14}{0.85} = 123.7 \ \text{Btu/lbm}$$

Actual exit temperature

$$T_2 = T_1 + \frac{-w}{C_{P0}} = 540 + \frac{123.7}{0.24} = 1055 \ R$$

Eq.6.16: $s_2 - s_1 = 0.24 \ln(1055/540) - (53.34/778) \ln 8 = 0.01817 \ \text{Btu/lbm-R}$

Exergy, Eq.8.23

$\psi_2 - \psi_1 = (h_2 - h_1) - T_0(s_2 - s_1) = 123.7 - 537 \ (0.01817) = 113.94 \ \text{Btu/lbm}$

2nd law efficiency, Eq.8.32 or 8.34 (but for a compressor):

$$\eta_{II} = \frac{\psi_2 - \psi_1}{-w} = \frac{113.94}{123.7} = \mathbf{0.92}$$

Borgnakke and Sonntag

8.179E

Consider that the air in the previous problem after the compressor flows in a pipe to an air tool and at that point the temperature has dropped to ambient 540 R and an air pressure of 110 psia. What is the second law efficiency for the total system?

C.V. Compressor

Energy: $0 = h_1 - h_2 + w_C$; Entropy: $0 = s_1 - s_2 + 0$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 540 \left(\frac{120}{15}\right)^{0.2857} = 978.1 \text{ R}$$

Exergy increase through the compressor matches with the ideal compressor work

$$w_{C\,s} = h_2 - h_1 = C_P(T_2 - T_1) = 0.24(978.1 - 540) \text{ Btu/lbm} = 105.14$$
Btu/lbm

The actual compressor work is

$$w_{C\,ac} = w_{C\,s} / \eta_{C\,s} = 105.14/0.85 = 123.7 \text{ Btu/lbm}$$

From inlet, state 1 to final point of use state 3.

$$\psi_3 - \psi_1 = (h_3 - h_1) - T_0(s_3 - s_1) = 0 - T_0[0 - R \ln(P_3/P_1)]$$

$$= T_0\, R\, \ln(P_3/P_1) = 537 \text{ R} \times (53.34/778) \text{ Btu/lbmR} \times \ln (110/15)$$

$$= 73.36 \text{ Btu/lbm}$$

So then the second law efficiency is the gain ($\psi_3 - \psi_1$) over the source $w_{C\,ac}$ as

$$\eta_{II} = \frac{\psi_3 - \psi_1}{w_{C\,ac}} = \frac{73.36}{123.7} = \mathbf{0.59}$$

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8.180E

A compressor is used to bring saturated water vapor at 103 lbf/in.^{2} up to 2000 lbf/in.^{2}, where the actual exit temperature is 1200 F. Find the irreversibility and the second law efficiency.

Inlet state: Table F.7.1 h_i = 1188.4 Btu/lbm, s_i = 1.601 Btu/lbm R

Actual compressor F.7.2: h_e = 1598.6 Btu/lbm, s_e = 1.6398 Btu/lbm R

Energy Eq. actual compressor: $-w_{c,ac} = h_e - h_i$ = 410.2 Btu/lbm

Eq.8.13: $i = T_0(s_e - s_i)$ = 536.67 × (1.6398 - 1.601) = **20.82 Btu/lbm**

Eq.8.15: $w_{rev} = i + w_{c,ac}$ = 20.82 + (-410.2) = -389.4 Btu/lbm

$\eta_{II} = -w_{rev}/(-w_{c,ac})$ = 389.4 / 410.2 = **0.949**

Borgnakke and Sonntag

8.181E

A coflowing (same direction) heat exchanger has one line with 0.5 lbm/s oxygen at 68 F and 30 psia entering and the other line has 1.2 lbm/s nitrogen at 20 psia and 900 R entering. The heat exchanger is long enough so that the two flows exit at the same temperature. Use constant heat capacities and find the exit temperature and the second law efficiency for the heat exchanger assuming ambient at 68 F.

Solution:

C.V. Heat exchanger, steady 2 flows in and two flows out.

Energy Eq.4.10: $\dot{m}_{O2}h_1 + \dot{m}_{N2}h_3 = \dot{m}_{O2}h_2 + \dot{m}_{N2}h_4$

Same exit tempearture so $T_4 = T_2$ with values from Table F.4

$$\dot{m}_{O2}C_{P\,O2}T_1 + \dot{m}_{N2}C_{P\,N2}T_3 = (\dot{m}_{O2}C_{P\,O2} + \dot{m}_{N2}C_{P\,N2})T_2$$

$$T_2 = \frac{0.5 \times 0.22 \times 527.7 + 1.2 \times 0.249 \times 900}{0.5 \times 0.22 + 1.2 \times 0.249} = \frac{326.97}{0.4088} = \textbf{800 R}$$

The second law efficiency for a heat exchanger is the ratio of the exergy gain by one fluid divided by the exergy drop in the other fluid. For each flow exergy is Eq.8.23 include mass flow rate as in Eq.8.38

For the oxygen flow:

$$\dot{m}_{O2}(\psi_2 - \psi_1) = \dot{m}_{O2}\,[\,h_2 - h_1 - T_O\,(\,s_2 - s_1\,)\,]$$

$$= \dot{m}_{O2}\,[\,C_P(T_2 - T_1) - T_O\,[\,C_P\ln(T_2\,/\,T_1) - R\ln(P_2\,/\,P_1)\,]\,]$$

$$= \dot{m}_{O2}C_P\,[\,T_2 - T_1 - T_O\ln(T_2\,/\,T_1)\,]$$

$$= 0.5 \times 0.22\,[\,800 - 527.7 - 536.7\ln(800/527.7)\,]\ = 5.389 \text{ Btu/s}$$

For the nitrogen flow

$$\dot{m}_{N2}(\psi_3 - \psi_4) = \dot{m}_{N2}C_P\,[\,T_3 - T_4 - T_O\ln(T_3\,/\,T_4)\,]$$

$$= 1.2 \times 0.249\,[\,900 - 800 - 536.7\ln(900/800)\,] = 10.992 \text{ Btu/s}$$

From Eq.8.32

$$\eta_{2^{nd}\,Law} = \frac{\dot{m}_{O2}(\psi_1 - \psi_2)}{\dot{m}_{N2}(\psi_3 - \psi_4)} = \frac{5.389}{10.992} = \textbf{0.49}$$

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Review Problems

Borgnakke and Sonntag

8.182E

Calculate the irreversibility for the process described in Problem 4.211E, assuming that the heat transfer is with the surroundings at 61 F.

C.V. Cylinder volume out to $T_O = 61$ F.

Continuity Eq.4.15: $\quad m_2 - m_1 = m_{in}$

Energy Eq.4.16: $\quad m_2 u_2 - m_1 u_1 = m_{in} h_{line} + {}_1Q_2 - {}_1W_2$

Entropy Eq.7.12: $\quad m_2 s_2 - m_1 s_1 = m_i s_i + {}_1Q_2 / T_O + {}_1S_{2\,gen}$

Process: $\quad P_1$ is constant to stops, then constant V to state 2 at P_2

State 1: $P_1, T_1 \quad m_1 = \dfrac{P_1 V}{RT_1} = \dfrac{45 \times 9 \times 144}{53.34 \times 519.7} = 2.104$ lbm

State 2:

Open to 60 lbf/in^2, $T_2 = 630$ R

Table F.5:

$h_i = 266.13$ btu/lbm

$u_1 = 88.68$ Btu/lbm

$u_2 = 107.62$ Btu/lbm

Only work as piston moves (V changes) while $P = P_1$ until $V = V_{stop}$

$${}_1W_2 = \int PdV = P_1(V_{stop} - V_1) = 45 \times (36 - 9)\dfrac{144}{778} = 224.9 \text{ Btu}$$

$$m_2 = P_2 V_2 / RT_2 = \dfrac{60 \times 36 \times 144}{53.34 \times 630} = 9.256 \text{ lbm}, \quad m_i = 7.152 \text{ lbm}$$

$${}_1Q_2 = m_2 u_2 - m_1 u_1 - m_i h_i + {}_1W_2$$

$$= 9.256 \times 107.62 - 2.104 \times 88.68 - 7.152 \times 266.13 + 224.9$$

$$= -868.9 \text{ Btu}$$

Use from table F.4: $\quad C_p = 0.24, \ R = 53.34 / 778 = 0.06856$ Btu/lbm R,

$$I = T_O \, {}_1S_{2\,gen} = T_O \, [\, m_1 (s_2 - s_1) + m_i (s_2 - s_i)] - {}_1Q_2$$

$$= 520.7[2.104(C_p \ln \dfrac{630}{519.7} - R \ln \dfrac{60}{45}) + 7.152(C_p \ln \dfrac{630}{1100} - R \ln \dfrac{60}{75})]$$

$$- (- 868.9)$$

$$= 520.7 (0.05569 - 0.8473) + 868.9$$

$$= \textbf{456.7 Btu}$$

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8.183E

Calculate the exergy of the system (aluminum plus gas) at the initial and final states of Problem 6.245E, and also the irreversibility.

State 1: $T_1 = 400$ F $v_1 = 2/2.862 = 0.6988$ $P_1 = 300$ psi

Ideal gas $v_2 = v_1(300 / 220)(537 / 860) = 0.595$; $v_O = 8.904 = RT_O / P_O$

The metal does not change volume so the terms as Eq.8.22 are added

$\phi_1 = m_{gas}\phi_{gas} + m_{Al}\phi_{Al\ 1}$

$$= m_{gas}C_v (T_1 - T_O) - m_{gas}T_O [C_p \ln \frac{T_1}{T_O} - R \ln \frac{P_1}{P_O}] + m_{gas}P_O (v_1 - v_O)$$

$$+ m_{Al} [C (T_1 - T_O) - T_O C \ln (T_1 / T_O)]_{Al}$$

$$\phi_1 = 2.862[0.156(400-77) - 537(0.201 \ln \frac{860}{537} - \frac{35.1}{778} \ln \frac{300}{14.7})$$

$$+ 14.7 (0.6988 - 8.904)(\frac{144}{778})] + 8 \times 0.21 [400 - 77 - 537 \ln \frac{860}{537}]$$

$$= 143.96 + 117.78 = \mathbf{261.74\ Btu}$$

$$\phi_2 = 2.862[0.156(77 - 77) - 537 (0.201 \ln \frac{537}{537} - \frac{35.1}{778} \ln \frac{220}{14.7})$$

$$+ 14.7(0.595 - 8.904)(\frac{144}{778})] + 8 \times 0.21 [77 - 77 - 537 \ln \frac{537}{537}]$$

$$= 122.91 + 0 = \mathbf{122.91\ Btu}$$

$$_1W_2\ _{CO_2} = \int PdV = 0.5(P_1 + P_2)(V_2 - V_1) = [(300 + 220)/2] (1.703 - 2) \frac{144}{778}$$

$$= -14.29\ Btu$$

$$_1I_2 = \phi_1 - \phi_2 + (1 - T_O/T_H)\ _1Q_2 - _1W_2^{AC} + P_O m(V_2 - V_1)$$

$$= 261.74 - 122.91 + 0 - (-14.29) + 14.7 \times 2.862 \times \frac{144}{778}(0.595 - 0.6988)$$

$$= \mathbf{152.3\ Btu}$$

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8.184E

Air in a piston/cylinder arrangement, shown in Fig. P8.135, is at 30 lbf/in.^{2}, 540 R with a volume of 20 ft^3. If the piston is at the stops the volume is 40 ft^3 and a pressure of 60 lbf/in.^{2} is required. The air is then heated from the initial state to 2700 R by a 3400 R reservoir. Find the total irreversibility in the process assuming surroundings are at 70 F.

Solution:

Energy Eq.: $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq,: $\quad m(s_2 - s_1) = \int dQ/T + {}_1S_2 \text{ gen}$

Process: $\quad P = P_0 + \alpha(V - V_0) \quad \text{if} \quad V \leq V_\text{stop}$

Information: $\quad P_\text{stop} = P_0 + \alpha(V_\text{stop} - V_0)$

Eq. of state $\Rightarrow \quad T_\text{stop} = T_1 P_\text{stop} V_\text{stop}/P_1 V_1 = 2160 < T_2$

So the piston will hit the stops $\quad \Rightarrow \quad V_2 = V_\text{stop}$

$$P_2 = (T_2/T_\text{stop}) P_\text{stop} = (2700/2160)\,60 = 75 \text{ psia} = 2.5\, P_1$$

State 1:

$$m_2 = m_1 = \frac{P_1 V_1}{RT_1}$$

$$= \frac{30 \times 20 \times 144}{53.34 \times 540}$$

$$= 3.0 \text{ lbm}$$

$${}_1W_2 = \frac{1}{2}(P_1 + P_\text{stop})(V_\text{stop} - V_1) = \frac{1}{2}(30 + 60)(40 - 20) = 166.6 \text{ Btu}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 3(518.165 - 92.16) + 166.6 = 1444.6 \text{ Btu}$$

$$s_2 - s_1 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1)$$

$$= 2.0561 - 1.6398 - (53.34/778) \ln (2.5) = 0.3535 \text{ Btu/lbm R}$$

Take control volume as total out to reservoir at T_RES

$${}_1S_2 \text{ gen tot} = m(s_2 - s_2) - {}_1Q_2/T_\text{RES} = \textbf{0.6356 Btu/R}$$

$${}_1I_2 = T_0({}_1S_2 \text{ gen}) = 530 \times 0.6356 = \textbf{337 Btu}$$

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8.185E

A piston/cylinder arrangement has a load on the piston so it maintains constant pressure. It contains 1 lbm of steam at 80 lbf/in.^{2}, 50% quality. Heat from a reservoir at 1300 F brings the steam to 1000 F. Find the second-law efficiency for this process. Note that no formula is given for this particular case, so determine a reasonable expression for it.

$1: P_1, x_1 \Rightarrow v_1 = 2.7458$ ft^3/lbm, $h_1 = 732.905$ Btu/lbm,

$$s_1 = 1.0374 \text{ Btu/lbm R}$$

$2: P_2 = P_1, T_2 \Rightarrow v_2 = 10.831$ ft^3/lbm, $h_2 = 1532.6$ Btu/lbm,

$$s_2 = 1.9453 \text{ Btu/lbm R}$$

$$m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2 - P(V_2 - V_1)$$

$${}_1Q_2 = m(u_2 - u_1) + Pm(v_2 - v_1) = m(h_2 - h_1) = 799.7 \text{ Btu}$$

$${}_1W_2 = Pm(v_2 - v_1) = 119.72 \text{ Btu}$$

$${}_1W_{2 \text{ to atm}} = P_0 m(v_2 - v_1) = 22 \text{ Btu}$$

Useful work out $= {}_1W_2 - {}_1W_{2 \text{ to atm}} = 119.72 - 22 = 97.72$ Btu

$$\Delta\phi_{\text{reservoir}} = \left(1 - \frac{T_0}{T_{\text{res}}}\right) {}_1Q_2 = \left(1 - \frac{536.67}{1759.67}\right) 799.7 = 556 \text{ Btu}$$

$$n_{II} = W_{\text{net}}/\Delta\phi = \frac{97.72}{556} = \textbf{0.176}$$

Remark: You could argue that the stored exergy (exergy) should be accounted for in the second law efficiency, but it is not available from this device alone.

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8.186E

The exit nozzle in a jet engine receives air at 2100 R, 20 psia with negligible kinetic energy. The exit pressure is 10 psia and the actual exit temperature is 1780 R. What is the actual exit velocity and the second law efficiency?

Solution:

C.V. Nozzle with air has no work, no heat transfer.

Energy eq.: $\qquad h_i = h_e + \dfrac{1}{2}V_{ex}^2$

Entropy Eq.: $\qquad s_i + s_{gen} = s_e$

$$\dfrac{1}{2}V_{ex}^2 = h_i - h_e = 532.57 - 444.36 = 88.21 \text{ Btu/lbm}$$

$$V_{ex} = \sqrt{2 \times 88.21 \times 25\,037} = \textbf{2102 ft s}^{\textbf{-1}}$$

Recall 1 Btu/lbm = 25 037 ft^2/s^2. This was the actual nozzle. Now we can do the reversible nozzle, which then must have a q.

Energy eq.: $\qquad h_i + q = h_e + \dfrac{1}{2}V_{ex\ rev}^2$

Entropy Eq.: $\qquad s_i + q/T_o = s_e \qquad \Rightarrow \quad q = T_o\,(s_e - s_i)$

$$q = T_o[\ C \ln \dfrac{T_e}{T_i} - R \ln \dfrac{P_e}{P_i}\ ] = 536.7\ \Big[0.24 \ln \dfrac{1780}{2100} - \dfrac{53.34}{778.17} \ln \dfrac{10}{20}\Big]$$

$$= 4.198 \text{ Btu/lbm}$$

$$\dfrac{1}{2}V_{ex\ rev}^2 = h_i + q - h_e = 88.21 + 4.198 = 92.408 \text{ Btu/lbm}$$

$$\eta_{II} = \dfrac{1}{2}V_{ex}^2 / \dfrac{1}{2}V_{ex\ rev}^2 = 88.21 / 92.408 = \textbf{0.95}$$

Notice the reversible nozzle is not isentropic (there is a heat transfer).

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 9

8e

Updated June 2013

Borgnakke and Sonntag

CONTENT CHAPTER 9

In-Text Concept Questions

9.a

Consider a Rankine cycle without superheat. How many single properties are needed to determine the cycle? Repeat the answer for a cycle with superheat.

a. No superheat. Two single properties.
High pressure (or temperature) and **low pressure** (or **temperature**). This assumes the condenser output is saturated liquid and the boiler output is saturated vapor. Physically the high pressure is determined by the pump and the low temperature is determined by the cooling medium and the heat echanger.

b. Superheat. Three single properties.
High pressure and **temperature** and **low pressure** (or **temperature**). This assumes the condenser output is saturated liquid. Physically the high pressure is determined by the pump and the high temperature by the heat transfer from the hot source. The low temperature is determined by the cooling medium and the heat echanger.

9.b

Which component determines the high pressure in a Rankine cycle? What determines the low pressure?

The high pressure in the Rankine cycle is determined by the pump. The low pressure is determined as the saturation pressure for the temperature you can cool to in the condenser.

9.c

What is the difference between an open and a closed feedwater heater?

The open feedwater heater mixes the two flows at the extraction pressure and thus requires two feedwater pumps.

The closed feedwater heater does not mix the flows but let them exchange energy (it is a two fluid heat exchanger). The flows do not have to be at the same pressure. The condensing source flow is dumped into the next lower pressure feedwater heater or the condenser or it is pumped up to line pressure by a drip pump and added to the feedwater line.

9.d

In a cogenerating power plant, what is cogenerated?

The electricity is cogenerated. The main product is a steam supply.
Such an arrangement is also known as Combined Heat and Power CHP.

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9.e

A refrigerator in my 20$^{\text{o}}$C kitchen uses R-134a and I want to make ice cubes at –5$^{\text{o}}$C. What is the minimum high P and the maximum low P it can use?

Since the R-134a must give heat transfer out to the kitchen air at 20$^{\text{o}}$C, it must at least be that hot at state 3.

From Table B.5.1:　　$P_3 = P_2 = P_{sat} =$ **573 kPa**　is minimum high P.

Since the R-134a must absorb heat transfer at the freezers –5$^{\text{o}}$C, it must at least be that cold at state 4.

From Table B.5.1:　　$P_1 = P_4 = P_{sat} =$ **245 kPa**　is maximum low P.

9.f

How many parameters are needed to completely determine a standard vapor compression refrigeration cycle?

Two parameters: The high pressure and the low pressure. This assumes the exit of the condenser is saturated liquid and the exit of the evaporator is saturated vapor.

Concept-Study Guide Problems

9.1

Is a steam power plant running in a Carnot cycle? Name the four processes.

No. It runs in a Rankine cycle.

1-2:	An isentropic compression (constant s)	Pump
2-3:	An isobaric heating (constant P)	Boiler
3-4:	An isentropic expansion (constant s)	Turbine
4-1:	An isobaric cooling, heat rejection (constant P)	Condenser

9.2

Raising the boiler pressure in a Rankine cycle for fixed superheat and condenser temperatures in what direction do these change: turbine work, pump work and turbine exit T or x.

Turbine work: about the same P up, but v down
Turbine exit T: same if it was two-phase, down if sup. vapor
Turbine exit x: down
Pump work: up

9.3

For other properties fixed in a Rankine cycle raising the condenser temperature causes changes in which work and heat transfer terms?

This results in less turbine work out.
An increase in heat rejection.
A small reduction in pump work.
A small reduction in boiler heat addition.

9.4

Mention two benefits of a reheat cycle.

The reheat raises the average temperature at which you add heat.

The reheat process brings the states at the lower pressure further out in the superheated vapor region and thus raises the quality (if two-phase) in the last turbine section.

9.5

What is the benefit of the moisture separator in the powerplant of Problem 4.121?

You avoid larger droplets in the turbine and raise the quality for the later stages.

9.6

Instead of the moisture separator in Problem 4.121 what could have been done to remove any liquid in the flow?

A reheat could be done to re-boil the liquid and even superheat it.

9.7

Can the energy removed in a power plant condenser be useful?

Yes.

In some applications it can be used for heating buildings locally or as district heating. Other uses could be to heat greenhouses or as general process steam in a food process or paper mill. These applications are all based on economics and scale. The condenser then has to operate at a higher temperature than it otherwise would.

9.8

If the district heating, see Fig.1.1, should supply hot water at 90°C what is the lowest possible condenser pressure with water as the working substance?

The condenser temperature must be higher than 90°C for which the saturation pressure is 70.14 kPa.

P > 70.14 kPa

Pipes in the power plant

Pipes in the ground for distribution

9.9

What is the mass flow rate through the condensate pump in Fig. 9.14?

We need to check the continuity equation around several CVs.

Do control volume around HP turbine:

Number in 1000 kg/h: $0 = + 320 - 28 - 28 - 12 -$ out to LP turbine
                               out to LP turbine $= 252\ 000$ kg/h

which matches with Fig.: 227 000 in condenser + 25 000 from trap

Condensate pump (main) has **252 000 kg/h**

9.10

A heat pump for a 20$^{\text{o}}$C house uses R-410A and the outside is at –5$^{\text{o}}$C. What is the minimum high P and the maximum low P it can use?

As the heat pump must be able to heat at 20$^{\text{o}}$C that becomes the smallest possible condensing temperature and thus P > P$_{\text{sat}}$ = **1444 kPa**.

It must absorb heat from –5$^{\text{o}}$C and thus must be colder in the evaporation process so P < P$_{\text{sat}}$ = **679 kPa.**

9.11

A heat pump uses carbon dioxide and it is required that it condenses at a minimum of 22$^{\text{o}}$C and receives energy from the outside on a winter day at -10$^{\text{o}}$C. What restrictions does that place on the operating pressures?

The high pressure P > P$_{\text{sat}}$ = 6003 kPa, close to critical P = 7377 kPa

The low pressure P < P$_{\text{sat}}$ = 2649 kPa

Notice for carbon dioxide that the low pressure is fairly high.

9.12

Since any heat transfer is driven by a temperature difference, how does that affect all the real cycles relative to the ideal cycles?

Heat transfers are given as $\dot{Q} = CA\,\Delta T$ so to have a reasonable rate the area and the temperature difference must be large. The working substance then must have a different temperature than the ambient it exchanges energy with. This gives a smaller temperature difference for a heat engine with a lower efficiency as a result. The refrigerator or heat pump must have the working substance with a higher temperature difference than the reservoirs and thus a lower coefficient of performance (COP).

The smaller CA is, the larger ΔT must be for a certain magnitude of the heat transfer rate. This can be a design problem, think about the front end air intake grill for a modern car which is very small compared to a car 20 years ago.

Simple Rankine cycles

9.13

A steam power plant as shown in Fig. 9.3 operating in a Rankine cycle has saturated vapor at 3.0 MPa leaving the boiler. The turbine exhausts to the condenser operating at 10 kPa. Find the specific work and heat transfer in each of the ideal components and the cycle efficiency.

Solution:

Cycle is determined by: $(P_3, x_3, P_1 = P_4,$ assume $x_1 = 0)$

C.V. Pump Reversible and adiabatic.

Energy: $w_p = h_2 - h_1$;   Entropy: $s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.00101 (3000 - 10) = 3.02$ kJ/kg

$\Rightarrow h_2 = h_1 + w_p = 191.81 + 3.02 = 194.83$ kJ/kg

C.V. Boiler : $q_H = h_3 - h_2 = 2804.14 - 194.83 = $ **2609.3 kJ/kg**

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$s_4 = s_3 = 6.1869 = 0.6492 + x_4 (7.501) \Rightarrow x_4 = 0.7383$

$\Rightarrow h_4 = 191.81 + 0.7383 (2392.82) = 1958.34$ kJ/kg

$w_T = 2804.14 - 1958.34 = $ **845.8 kJ/kg**

C.V. Condenser : $q_L = h_4 - h_1 = 1958.34 - 191.81 = $ **1766.5 kJ/kg**

$\eta_{cycle} = w_{net} / q_H = (w_T + w_p) / q_H = (845.8 - 3.0) / 2609.3 = $ **0.323**

9.14

Consider a solar-energy-powered ideal Rankine cycle that uses water as the working fluid. Saturated vapor leaves the solar collector at 1000 kPa, and the condenser pressure is 10 kPa. Determine the thermal efficiency of this cycle.

Solution:

C.V. H_2O ideal Rankine cycle

Cycle is determined by: $(P_3, x_3, P_1 = P_4,$ assume $x_1 = 0)$

State 3: 1000 kPa, sat vap, $h_3 = 2778.08$ kJ/kg, $s_3 = 6.5864$ kJ/kg K

CV Turbine adiabatic and reversible so second law gives

$$s_4 = s_3 = 6.5864 = 0.6492 + x_4 \times 7.5010 \qquad => \quad x_4 = 0.7915$$

$$h_4 = 191.81 + 0.7915 \times 2392.82 = 2085.73 \text{ kJ/kg}$$

The energy equation gives

$$w_T = h_3 - h_4 = 2778.08 - 2085.73 = 692.35 \text{ kJ/kg}$$

C.V. pump and incompressible liquid gives work into pump

$$w_P = v_1(P_2 - P_1) = 0.00101(1000 - 10) = 1.0 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 191.81 + 1.0 = 192.81 \text{ kJ/kg}$$

C.V. boiler gives the heat transfer from the energy equation as

$$q_H = h_3 - h_2 = 2778.08 - 192.81 = 2585.3 \text{ kJ/kg}$$

The cycle net work and efficiency are found as

$$w_{NET} = w_T - w_P = 692.35 - 1.0 = 691.35 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 691.35/2585.3 = \textbf{0.267}$$

9.15

The power plant in the previous problem is augmented with a natural gas burner to superheat the water to 300°C before entering the turbine. Find the cycle efficiency with this configuration and the specific heat transfer added by the natural gas burner.

Solution:

C.V. H_2O ideal Rankine cycle

Cycle is determined by: $(P_3, T_3, P_1 = P_4$, assume $x_1 = 0)$

State 3: 1000 kPa, 300°C, $h_3 = 3051.15$ kJ/kg, $s_3 = 7.1228$ kJ/kg K

CV Turbine adiabatic and reversible so second law gives

$\qquad s_4 = s_3 = 7.1228 = 0.6492 + x_4 \times 7.5010 \qquad => \quad x_4 = 0.86303$

$\qquad h_4 = 191.81 + 0.86303 \times 2392.82 = 2256.88$ kJ/kg

The energy equation gives

$\qquad w_T = h_3 - h_4 = 3051.15 - 2256.88 = 794.27$ kJ/kg

C.V. pump and incompressible liquid gives work into pump

$\qquad w_P = v_1(P_2 - P_1) = 0.00101(1000 - 10) = 1.0$ kJ/kg

$\qquad h_2 = h_1 + w_P = 191.81 + 1.0 = 192.81$ kJ/kg

C.V. boiler gives the heat transfer from the energy equation as

$\qquad q_H = h_3 - h_2 = 3051.15 - 192.81 = 2858.3$ kJ/kg

$\qquad q_{H\ gas} = h_3 - g_{1000\ kPa} = 3051.15 - 2778.08 = \textbf{273.07 kJ/kg}$

The cycle net work and efficiency are found as

$\qquad w_{NET} = w_T - w_P = 794.27 - 1.0 = 793.27$ kJ/kg

$\qquad \eta_{TH} = w_{NET}/q_H = 793.27/2858.3 = \textbf{0.277}$

9.16

A utility runs a Rankine cycle with a water boiler at 3.0 MPa and the cycle has the highest and lowest temperatures of 450°C and 60°C respectively. Find the plant efficiency and the efficiency of a Carnot cycle with the same temperatures.

Solution:

The states properties from Tables B.1.1 and B.1.3

$\qquad$ 1: 60°C, x = 0 => $h_1 = 251.11$, $v_1 = 0.001017$ m^3/kg, $P_{sat} = 19.94$ kPa

$\qquad$ 3: 3.0 MPa , 450°C => $h_3 = 3344$ kJ/kg, $s_3 = 7.0833$ kJ/kg K

C.V. Pump Reversible and adiabatic.

Energy: $\qquad w_p = h_2 - h_1$; Entropy: $\qquad s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$\qquad w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.001017 (3000 - 19.94) = 3.03$ kJ/kg

$\qquad\qquad => h_2 = h_1 + w_p = 251.11 + 3.03 = 254.14$ kJ/kg

C.V. Boiler : $\quad q_H = h_3 - h_2 = 3344 - 254.14 = 3089.86$ kJ/kg

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$\qquad\qquad s_4 = s_3 = 7.0833 = 0.8311 + x_4 (7.0784)$ => $\quad x_4 = 0.8833$

$\qquad\qquad\qquad => h_4 = 251.11 + 0.8833 (2358.48) = 2334.36$ kJ/kg

$\qquad\qquad w_T = 3344 - 2334.36 = 1009.6$ kJ/kg

C.V. Condenser : $q_L = h_4 - h_1 = 2334.36 - 251.11 = 2083.25$ kJ/kg

$\qquad \eta_{cycle} = w_{net} / q_H = (w_T + w_p) / q_H = (1009.6 - 3.03) / 3089.86 = \mathbf{0.326}$

$$\eta_{carnot} = 1 - T_L / T_H = 1 - \frac{273.15 + 60}{273.15 + 450} = \mathbf{0.54}$$

9.17

The power plant in the previous problem has a too low quality in the low pressure turbine section so the plant wants to increase the superheat. What should the superheat be so the quality of the water in the turbine stays above 92%.

Cycle is determined by: $(P_3, T_1, x_4,$ assume $x_1 = 0)$

Consider the turbine and its exhaust at condenser 60°C, 92% quality.

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$, $x_4 = 0.92$

$$s_4 = s_3 = 0.8311 + x_4 (7.0784) = 7.34323 \text{ kJ/kgK}$$

Look in B.1.3, 3 MPa match s:

$$T_3 = 500 + 100 \frac{7.34323 - 7.2337}{7.5084 - 7.2337} = \mathbf{539.9°C}$$

9.18

A power plant for a polar expedition uses ammonia which is heated to 80$^{\text{o}}$C at 1000 kPa in the boiler and the condenser is maintained at -15$^{\text{o}}$C. Find the cycle efficiency.

Solution:

Standard Rankine cycle with superheat. From the listed information we get from Table B.2.2

State 1: $h_1 = 111.66$ kJ/kg, $v_1 = 0.001519$ m^3/kg,

$P_1 = 236.3$ kPa, $s = 0.4538$ kJ/kgK

State 3: $h_3 = 1614.6$ kJ/kg, $s_3 = 5.4971$ kJ/kgK

C.V. Tubine: Energy: $w_{T,s} = h_3 - h_4$;

Entropy: $s_4 = s_3 = 5.4971$ kJ/kg K

$$\Rightarrow x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{5.4971 - 0.4538}{5.0859} = 0.9916 ;$$

$h_4 = 111.66 + 0.9916 \times 1312.9 = 1413.56$ kJ/kg

$w_{T,s} = 1614.6 - 1413.56 = 201.04$ kJ/kg

C.V. Pump: $w_P = \int v\ dP = v_1(P_2 - P_1) = 0.001519(1000 - 236.3) = 1.16$ kJ/kg

$\Rightarrow h_2 = h_1 + w_P = 111.66 + 1.16 = 112.8$ kJ/kg

C.V. Boiler: $q_H = h_3 - h_2 = 1614.6 - 112.8 = 1501.8$ kJ/kg

$$\eta_{CYCLE} = w_{NET}/q_H = \frac{201.04 - 1.16}{1501.8} = \mathbf{0.133}$$

Comment: The cycle efficiency is low due to the low high temperature.

9.19

A Rankine cycle with R-410A has the boiler at 3 MPa superheating to 180°C and the condenser operates at 800 kPa. Find all four energy transfers and the cycle efficiency.

Standard Rankine cycle given by ($P_2 = P_3$, T_3, P_1, assume $x_1 = 0$)

From Table B4:

State 1 (P, x) : $v_1 = 0.000855$ m^3/kg, $h_1 = 57.76$ kJ/kg   (at 0°C)

State 3 (P, T): $h_3 = 445.09$ kJ/kg, $s_3 = 1.3661$ kJ/kg-K

State 4 (800 kPa, s = s_3): $h_4 = 385.97$ kJ/kg interpolated superheated vapor.

C.V. Pump: $w_P = \int v\ dP = v_1(P_2 - P_1) = 0.000855\ (3000 - 800) =$ **1.881 kJ/kg**

$\Rightarrow\ h_2 = h_1 + w_P = 57.76 + 1.881 = 59.64$ kJ/kg

C.V. Boiler: $q_H = h_3 - h_2 = 445.09 - 59.64 =$ **385.45 kJ/kg**

C.V. Tubine: Energy: $w_{T,s} = h_3 - h_4 = 445.09 - 385.97 =$ **59.12 kJ/kg**

C.V. Condenser: $q_L = h_4 - h_1 = 385.97 - 57.76 =$ **328.21 kJ/kg**

$$\eta_{CYCLE} = w_{NET}/q_H = \frac{59.12 - 1.881}{385.45} = \textbf{0.148}$$

9.20

A steam power plant has a high pressure of 3 MPa and it maintains 60°C in the condenser. A condensing turbine is used, but the quality should not be lower than 90% at any state in the turbine. For a turbine power output of 8 MW find the work and heat transfer in all components and the cycle efficiency.

Solution:

Based on the standard Rankine cycle and Table B.1.

State 1: Sat. liquid. $P_1 = 19.94$ kPa, $h_1 = 251.11$ kJ/kg, $v_1 = 0.001017$ m^3/kg

Consider C.V. pump

Energy: $h_2 - h_1 = w_p = v_1 (P_2 - P_1) = 0.001017 (3000 - 19.94) = 3.03$ kJ/kg

State 2: $P_2 = 3000$ kPa, $h_2 = h_1 + w_p = 251.11 + 3.03 = 254.1$ kJ/kg

State 4: $P_4 = P_1 = 19.94$ kPa,   x = 0.9

$$s_4 = s_f + x_4 s_{fg} = 0.8311 + 0.9 \times 7.0784 = 7.20166 \text{ kJ/kg-K}$$

$$h_4 = h_f + x_4 h_{fg} = 251.11 + 0.9 \times 2358.48 = 2373.74 \text{ kJ/kg}$$

Consider the turbine for which $s_4 = s_3$.

State 3: Table B.2.2   3000 kPa, $s_3 = 7.20166$ kJ/kg K => $h_3 = 3432.5$ kJ/kg

Turbine: $w_T = h_3 - h_4 = 3432.5 - 2373.74 = 1058.8$ kJ/kg

Boiler: $q_H = h_3 - h_2 = 3432.5 - 254.1 = 3178.4$ kJ/kg

Condenser: $q_L = h_4 - h = 2373.74 - 251.1 = 2122.6$ kJ/kg

Efficiency: $\eta_{TH} = w_{NET}/q_H = (w_T - w_P)/q_H = \dfrac{1058.8 - 3.03}{3178.4} = \mathbf{0.332}$

Scaling: $\dot{m} = \dot{W}_T/w_T = 8\ 000$ kW / 1058.8 kJ/kg = 7.555 kg/s

$$\dot{W}_p = \dot{m}\ w_p = \mathbf{22.9\ kW};\ \ \dot{Q}_H = \dot{m}\ q_H = \mathbf{24\ MW},\ \ \dot{Q}_L = \dot{m}\ q_L = \mathbf{16\ MW}$$

9.21

A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85°C, and the condenser temperature is 40°C. Calculate the thermal efficiency of this cycle.

Solution:

CV: Pump   (use R-134a Table B.5)

$$w_P = h_2 - h_1 = \int_1^2 v\,dP \approx v_1(P_2\text{-}P_1)$$

$$= 0.000873(2926.2 - 1017.0) = 1.67 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 256.54 + 1.67 = 258.21 \text{ kJ/kg}$$

CV: Boiler

$$q_H = h_3 - h_2 = 428.10 - 258.21 = 169.89 \text{ kJ/kg}$$

CV: Turbine

$$s_4 = s_3 = 1.6782 = 1.1909 + x_4 \times 0.5214 \quad => \quad x_4 = 0.9346$$

$$h_4 = 256.54 + 0.9346 \times 163.28 = 409.14 \text{ kJ/kg}$$

Energy Eq.: $w_T = h_3 - h_4 = 428.1 - 409.14 = 18.96 \text{ kJ/kg}$

$$w_{NET} = w_T - w_P = 18.96 - 1.67 = 17.29 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 17.29/169.89 = \mathbf{0.102}$$

9.22

Do Problem 9.21 with R-410A as the working fluid and boiler exit at 4000 kPa, 70°C.

A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85°C, and the condenser temperature is 40°C. Calculate the thermal efficiency of this cycle.

Solution:

CV: Pump   (use R-410A Table B.4)

$$w_P = h_2 - h_1 = \int_1^2 v\,dP \approx v_1(P_2\text{-}P_1) = 0.001025(4000 - 2420.7) = 1.619 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 124.09 + 1.619 = 125.71 \text{ kJ/kg}$$

CV: Boiler: $q_H = h_3 - h_2 = 287.88 - 125.71 = 162.17$ kJ/kg

CV: Turbine

$$s_4 = s_3 = 0.93396 = 0.4473 + x_4 \times 0.5079, \quad => \quad x_4 = 0.9582$$

$$h_4 = 124.09 + 0.9582 \times 159.04 = 276.48 \text{ kJ/kg}$$

$$w_T = h_3 - h_4 = 287.88 - 276.48 = 11.4 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = (11.4 - 1.62)/162.17 = \textbf{0.060}$$

9.23

A low temperature power plant operates with R-410A maintaining -20$^{\text{o}}$C in the condenser, a high pressure of 3 MPa with superheat. Find the temperature out of the boiler/superheater so the turbine exit temperature is 60$^{\text{o}}$C and find the overall cycle efficiency.

State 1: $P_1 = 399.6$ kPa, $v_1 = 0.000803$ m^3/kg, $h_1 = 28.24$ kJ/kg

State 4: $P_4 = P_1 \approx 400$ kPa, $h_4 = 343.58$ kJ/kg, $s_4 = 1.3242$ kJ/kg-K

State 3:   3 MPa, s = s_4 $\Rightarrow$ $h_3 = 426.56$ kJ/kg, $T_3 = \mathbf{163.6^{o}C}$

Pump: $w_p = v_1 (P_2 - P_1) = 0.000803$ m^3/kg (3000 – 399.6) kPa = 2.09 kJ/kg

Boiler: $q_H = h_3 - h_2 = 426.56 - (28.24 + 2.09) = 396.23$ kJ/kg

Turbine: $w_T = h_3 - h_4 = 426.56 - 343.58 = 82.98$ kJ/kg

Efficiency: $\eta_{TH} = w_{NET}/q_H = (w_T - w_P)/q_H = \dfrac{82.98 - 2.09}{396.23} = \mathbf{0.204}$

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9.24

A low temperature power plant should produce a turbine work of 2.5 MW. It operates with carbon dioxide maintaining $-10^{\circ}C$ in the condenser, a high pressure of 6 MPa and it superheats to $100^{\circ}C$. Find the turbine exit temperature, the mass flow rate required and the overall cycle efficiency.

State 1: $v_1 = 0.001017$ m^3/kg, $h_1 = 63.62$ kJ/kg, $P_1 = 2648.7$ kPa

State 3: $h_3 = 421.69$ kJ/kg, $s_3 = 1.4241$ kJ/kg-K

State 4:   (2648.7 kPa, $s = s_3$)   Do a double linear interpolation in B.3.2.

At 2000 kPa: $14.775^{\circ}C$, h = 362.84;   At 3000 kPa: $44.096^{\circ}C$, h = 382.98

Then at 2648.7 kPa: $T_4 = \mathbf{33.79^{o}C}$, $h_4 = 375.9$ kJ/kg

[use CATT3 (entry P, T) for accuracy then $T = 34.5^{\circ}C$, h = 376.3 kJ/kg]

C.V. Pump: $w_P = \int v \, dP = v_1(P_2 - P_1) = 0.001017 \, (6000 - 2648.7) = 3.408$ kJ/kg

$$\Rightarrow h_2 = h_1 + w_P = 63.62 + 3.408 = 67.03 \text{ kJ/kg}$$

C.V. Boiler: $q_H = h_3 - h_2 = 421.69 - 67.03 = 354.66$ kJ/kg

C.V. Tubine: Energy: $w_{T,s} = h_3 - h_4 = 421.69 - 375.9 = \mathbf{45.79 \ kJ/kg}$

Scaling: $\dot{m} = \dot{W}_T/w_T = 2\,500 \text{ kW} / 45.79 \text{ kJ/kg} = \mathbf{54.6 \ kg/s}$

$$\eta_{CYCLE} = w_{NET}/q_H = \frac{45.79 - 3.408}{354.66} = \mathbf{0.129}$$

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Here are the CATT3 entries I used for locating state 4.:

Start out at P = 2.648.7 kPa = 2.6487 MPa, T = 34°C the s is then too low

For some reason the entry (P,s) does not work (it iterates out of bounce, to be fixed in next edition of the CATT program)

Computer-Aided Thermodynamic Tables 3

File   Edit   Tables/Substances   Options   Help

T-S Diagram

CO2 Properties

T	34.5	C
P	2.649	MPa
V	0.0191	m3/kg
U	325.8	kJ/kg
H	376.3	kJ/kg
S	1.424	kJ/kg/K
X		

Type	Temp	Pressure	Specific Volume	Internal Energy	Specific Enthalpy	Specific Entropy	Quality
	C	MPa	m3/kg	kJ/kg	kJ/kg	kJ/kg/K	
1. CO2	34	2.649	0.01905	325.3	375.8	1.422	
2. CO2	34.2	2.649	0.01907	325.5	376	1.423	
3. CO2	34.4	2.649	0.01909	325.7	376.2	1.423	
4. CO2	34.5	2.649	0.0191	325.8	376.3	1.424	

Water \ Refrigerants \ Cryogenics \ Air \ Ideal Gases \ Compressibility \ Psychrometrics

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9.25

Do Problem 9.21 with ammonia as the working fluid.

A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85°C, and the condenser temperature is 40°C. Calculate the thermal efficiency of this cycle.

Solution:

CV: Pump   (use Ammonia Table B.2)

$$w_P = h_2 - h_1 = \int_1^2 v\,dP = v_1(P_2 - P_1)$$

$$= 0.001725(4608.6 - 1554.9) = 5.27 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 371.43 + 5.27 = 376.7 \text{ kJ/kg}$$

CV: Boiler

$$q_H = h_3 - h_2 = 1447.8 - 376.7 = 1071.1 \text{ kJ/kg}$$

CV: Turbine

$$s_4 = s_3 = 4.3901 = 1.3574 + x_4 \times 3.5088 \qquad => \qquad x_4 = 0.8643$$

$$h_4 = 371.43 + 0.8643 \times 1098.8 = 1321.13 \text{ kJ/kg}$$

Energy Eq.:

$$w_T = h_3 - h_4 = 1447.8 - 1321.13 = 126.67 \text{ kJ/kg}$$

$$w_{NET} = w_T - w_P = 126.67 - 5.27 = 121.4 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 121.4/1071.1 = \textbf{0.113}$$

9.26

Geothermal water can be used directly as a source to a steam turbine. Consider 10 kg/s water at 500 kPa, 150°C brought to a flash chamber where it is throttled to 200 kPa as shown in Fig. P9.26. From the chamber saturated vapor at 200 kPa flows to the turbine with an exit at 10 kPa. From state 4 it is cooled in a condenser and pumped back into the ground. Determine quality at the turbine exit and the power that can be obtained from the turbine.

Solution:

Two-phase out of the valve. The liquid drops to the bottom.

Continuity Eq.4.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$

Energy Eq.4.10: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{S}_{gen} + \int d\dot{Q}/T = \dot{m}_2 s_2 + \dot{m}_3 s_3$

Process: $\dot{Q} = 0$, irreversible (throttle)

B.1.1 $h_1 = 632.18$ kJ/kg, $s_1 = 1.8417$ kJ/kg K

B.1.2 $h_3 = 2706.63$ kJ/kg, $s_3 = 7.1271$ kJ/kg K, $h_2 = 504.68$ kJ/kg,

From the energy equation we solve for the flow rate out to turbine

$$\dot{m}_3 = \dot{m}_1 (h_1 - h_2)/(h_3 - h_2) = 10 \times 0.0579 = 0.579 \text{ kg/s}$$

The flow through the turbine is rev. and adiabatic thus isentropic

State 4: $s_4 = s_3 = 7.1271 = 0.6492 + x_4 \times 7.5010 \implies x_4 = \textbf{0.8636}$

$h_4 = 191.81 + x_4 \times 2392.82 = 2258.25$ kJ/kg

$w_T = h_3 - h_4 = 2706.63 - 2258.25 = 448.38$ kJ/kg

$\dot{W}_T = \dot{m}_3 \, w_T = 0.579 \text{ kg/s} \times 448.38 \text{ kJ/kg} = \textbf{259.6 kW}$

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9.27

Some geothermal location have higher pressure and temperature so assume the geothermal power plant as in problem 9.26 but with a supply as saturated liquid at 250°C. For this case the flash chamber operates at 1000 kPa. For a supply of 10 kg/s determine the mass flow that goes through the turbine and the power output.

Solution:

Two-phase out of the valve. The liquid drops to the bottom.

Continuity Eq.4.9: $\quad \dot{m}_1 = \dot{m}_2 + \dot{m}_3$

Energy Eq.4.10: $\quad \dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$

Entropy Eq.7.7: $\quad \dot{m}_1 s_1 + \dot{S}_{gen} + \int d\dot{Q}/T = \dot{m}_2 s_2 + \dot{m}_3 s_3$

Process: $\quad \dot{Q} = 0,\quad$ irreversible (throttle)

B.1.1 $h_1 = 1085.34$ kJ/kg

B.1.2 $h_3 = 2778.08$ kJ/kg, $h_{fg} = 2015.29$ kJ/kg, $h_2 = 762.79$ kJ/kg,

From the energy equation we solve for the flow rate out to turbine

$$\dot{m}_3 = \dot{m}_1 \frac{h_1 - h_2}{h_3 - h_2} = 10 \frac{1085.34 - 762.79}{2015.29} = 10 \times 0.16 = \textbf{1.6 kg/s}$$

The flow through the turbine is rev. and adiabatic thus isentropic

State 4: $\quad s_4 = s_3 = 6.5864 = 0.6492 + x_4 \times 7.5010 \quad \Rightarrow \quad x_4 = 0.79152$

$\quad\quad\quad\quad h_4 = 191.81 + x_4 \times 2392.82 = 2085.775$ kJ/kg

$\quad\quad\quad\quad w_T = h_3 - h_4 = 2778.08 - 2085.775 = 692.3$ kJ/kg

$\quad\quad\quad\quad \dot{W}_T = \dot{m}_3\, w_T = 1.6$ kg/s $\times 692.3$ kJ/kg $= \textbf{1107.7 kW}$

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9.28

With a higher supply pressure and temperature of the geothermal source it is possible to have two flash evaporators as shown in Fig. P9.28. Assume the supply is saturated water at 250^oC and the first chamber flashes to 2000 kPa and saturated liquid at state 2 is flashed to 500 kPa with the saturated vapor out added to the turbine which has an exit state of 10 kPa with quality 78%. For a supply of 10 kg/s determine the mass flow that goes through the turbine and the power output.

The enthalpies in kJ/kg for the states in the flow diagram are:

$h_1 = 1085.34,\ \ h_2 = 908.77,\ \ \ h_3 = 2799.51,\ \ h_{fg\ 2000\ kPa} = 1890.74,$

$h_6 = 640.21,\ \ \ h_5 = 2748.67,\ h_{fg\ 500\ kPa} = 2108.47$

$h_4 = 191.81 + 0.78 \times 2392.82 = 2058.21$

To determine the mass flow rates we need the split in the two chambers. The fractions of the vapor output flows equals the quality in the flow right after the valve.

Flash Chamber 1: $\dot{m}_1 h_1 = \dot{m}_3 h_3 + (\dot{m}_1 - \dot{m}_3) h_2$ =>

$$x_{1a} = \dot{m}_3/\dot{m}_1 = (h_1 - h_2)/h_{fg\ 2000\ kPa} = \frac{1085.34 - 908.77}{1890.74} = 0.093387$$

$$\dot{m}_3 = x_{1a} \dot{m}_1 = 0.093387 \times 10\ kg/s = \textbf{0.93387 kg/s}$$

Flash Chamber 2: $\dot{m}_2 h_2 = \dot{m}_5 h_5 + (\dot{m}_2 - \dot{m}_5) h_6$ =>

$$x_{2a} = \dot{m}_5/\dot{m}_2 = (h_2 - h_6)/h_{fg\ 500\ kPa} = \frac{908.77 - 640.21}{2108.47} = 0.127372$$

$$\dot{m}_5 = x_{2a} \dot{m}_2 = 0.127372 \times (10 - 0.93387)\ kg/s = \textbf{1.15477 kg/s}$$

CV Turbine. Energy Eq. gives power out put

$$\dot{W}_T = \dot{m}_3 h_3 + \dot{m}_5 h_5 - (\dot{m}_3 + \dot{m}_5) h_4$$
$$= 0.93387 \times 2799.51 + 1.15477 \times 2748.67 - 2.08864 \times 2058.21$$
$$= \textbf{1489.6 kW}$$

9.29

A smaller power plant produces 25 kg/s steam at 3 MPa, 600°C in the boiler. It cools the condenser with ocean water coming in at 15°C and returned at 18°C so the condenser exit is at 50°C. Find the net power output and the required mass flow rate of ocean water.

Solution:

The states properties from Tables B.1.1 and B.1.3

$\quad$ 1: 45°C, $x = 0$: $h_1 = 188.42$ kJ/kg, $v_1 = 0.00101$ m³/kg, $P_{sat} = 9.59$ kPa

$\quad$ 3: 3.0 MPa, 600°C: $\quad h_3 = 3682.34$ kJ/kg, $\quad s_3 = 7.5084$ kJ/kg K

C.V. Pump Reversible and adiabatic.

$\quad$ Energy: $\qquad w_p = h_2 - h_1$; $\quad$ Entropy: $\qquad s_2 = s_1$

$\quad$ since incompressible it is easier to find work (positive in) as

$\quad w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.00101$ m³/kg $(3000 - 9.6)$ kPa $= 3.02$ kJ/kg

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$\qquad s_4 = s_3 = 7.5084 = 0.6386 + x_4 (7.5261) \quad => \quad x_4 = 0.9128$

$\qquad => h_4 = 188.42 + 0.9128 (2394.77) = 2374.4$ kJ/kg

$\qquad w_T = 3682.34 - 2374.4 = 1307.94$ kJ/kg

$$\dot{W}_{NET} = \dot{m}(w_T - w_p) = 25 (1307.94 - 3.02) = \textbf{32.6 MW}$$

C.V. Condenser : $q_L = h_4 - h_1 = 2374.4 - 188.42 = 2186$ kJ/kg

$$\dot{Q}_L = \dot{m}q_L = 25 \text{ kg/s} \times 2186 \text{ kJ/kg} = 54.65 \text{ MW} = \dot{m}_{ocean}\, C_p\, \Delta T$$

$$\dot{m}_{ocean} = \dot{Q}_L / C_p\, \Delta T = \frac{54\,650}{4.18 \times (18 - 15)} = \textbf{4358 kg/s}$$

9.30

Consider an ideal Rankine cycle using water with a high-pressure side of the cycle at a supercritical pressure. Such a cycle has a potential advantage of minimizing local temperature differences between the fluids in the steam generator, such as the instance in which the high-temperature energy source is the hot exhaust gas from a gas-turbine engine. Calculate the thermal efficiency of the cycle if the state entering the turbine is 30 MPa, 550°C, and the condenser pressure is 10 kPa. What is the steam quality at the turbine exit?

Solution:

For the efficiency we need the net work and steam generator heat transfer.

C.V. Pump. For this high exit pressure we use Table B.1.4 compressed liquid

State 1: $s_1 = 0.6492$ kJ/kg K, $h_1 = 191.81$ kJ/kg

Entropy Eq.: $s_2 = s_1$ => $h_2 = 222.5$ kJ/kg

$$w_p = h_2 - h_1 = 30.69 \text{ kJ/kg}$$

C.V. Turbine. Assume reversible and adiabatic.

Entropy Eq.: $s_4 = s_3 = 6.0342 = 0.6492 + x_4 \times 7.501$

$$x_4 = \textbf{0.7179} \quad \text{Very low for a turbine exhaust}$$

$$h_4 = 191.81 + x_4 \times 2392.82 = 1909.63, \quad h_3 = 3275.36 \text{ kJ/kg}$$

$$w_T = h_3 - h_4 = 1365.7 \text{ kJ/kg}$$

Steam generator: $q_H = h_3 - h_2 = 3052.9$ kJ/kg

$$w_{NET} = w_T - w_p = 1365.7 - 30.69 = 1335 \text{ kJ/kg}$$

$$\eta = w_{NET}/q_H = 1335 / 3052.9 = \textbf{0.437}$$

9.31

Consider the ammonia Rankine-cycle power plant shown in Fig. P9.31. The plant was designed to operate in a location where the ocean water temperature is 25°C near the surface and 5°C at some greater depth. The mass flow rate of the working fluid is 1000 kg/s.

    a. Determine the turbine power output and the pump power input for the cycle.

    b. Determine the mass flow rate of water through each heat exchanger.

    c. What is the thermal efficiency of this power plant?

Solution:

a) C.V. Turbine. Assume reversible and adiabatic.

$$s_2 = s_1 = 5.0863 = 0.8779 + x_2 \times 4.3269 \quad => \quad x_2 = 0.9726$$

$$h_2 = 227.08 + 0.9726 \times 1225.09 = 1418.6 \text{ kJ/kg}$$

$$w_T = h_1 - h_2 = 1460.29 - 1418.6 = 41.69 \text{ kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = 1000 \times 41.69 = \mathbf{41\ 690\ kW}$$

Pump: $w_P \approx v_3(P_4 - P_3) = 0.0016(857 - 615) = 0.387 \text{ kJ/kg}$

$$\dot{W}_P = \dot{m}w_P = 1000 \times 0.387 = \mathbf{387\ kW}$$

b)   Consider condenser heat transfer to the low T water

$$\dot{Q}_{\text{to low T H2O}} = 1000(1418.6 - 227.08) = 1.1915 \times 10^6 \text{ kW}$$

$$\dot{m}_{\text{low T H2O}} = \frac{1.1915 \times 10^6}{29.38 - 20.98} = \mathbf{141\ 850\ kg/s}$$

$$h_4 = h_3 + w_P = 227.08 + 0.39 = 227.47 \text{ kJ/kg}$$

Now consider the boiler heat transfer from the high T water

$$\dot{Q}_{\text{from high T H2O}} = 1000(1460.29 - 227.47) = 1.2328 \times 10^6 \text{ kW}$$

$$\dot{m}_{\text{high T H2O}} = \frac{1.2328 \times 10^6}{104.87 - 96.50} = \mathbf{147\ 290\ kg/s}$$

c) $\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = \dfrac{41\ 690 - 387}{1.2328 \times 10^6} = \mathbf{0.033}$

9.32

Do problem 9.31 with carbon dioxide as the working fluid.

Consider the ammonia Rankine-cycle power plant shown in Fig. P9.31. The plant was designed to operate in a location where the ocean water temperature is 25°C near the surface and 5°C at some greater depth. The mass flow rate of the working fluid is 1000 kg/s.

   a.   Determine the turbine power output and the pump power input for the cycle.
   b.   Determine the mass flow rate of water through each heat exchanger.
   c.   What is the thermal efficiency of this power plant?

a) C.V. Turbine. Assume reversible and adiabatic.

$$s_2 = s_1 = 1.0406 = 0.4228 + x_2 \times 0.6963 \quad => \quad x_2 = 0.88726$$

$$h_2 = 112.83 + 0.88726 \times 197.15 = 287.75 \text{ kJ/kg}$$

$$w_T = h_1 - h_2 = 294.96 - 287.75 = 7.206 \text{ kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = 1000 \times 7.206 = \textbf{7 206 kW}$$

Pump: $w_P \approx v_3(P_4 - P_3) = 0.001161(5729 - 4502) = 1.425$ kJ/kg

$$\dot{W}_P = \dot{m}w_P = 1000 \times 1.425 = \textbf{1425 kW}$$

b)   Consider condenser heat transfer to the low T water

$$\dot{Q}_{\text{to low T H2O}} = 1000(287.75 - 112.83) = 0.2749 \times 10^6 \text{ kW}$$

$$\dot{m}_{\text{low T H2O}} = \frac{0.2749 \times 10^6}{29.38 - 20.98} = \textbf{32 728 kg/s}$$

$$h_4 = h_3 + w_P = 112.83 + 1.425 = 114.26 \text{ kJ/kg}$$

Now consider the boiler heat transfer from the high T water

$$\dot{Q}_{\text{from high T H2O}} = 1000(294.96 - 114.26) = 0.1807 \times 10^6 \text{ kW}$$

$$\dot{m}_{\text{high T H2O}} = \frac{0.1807 \times 10^6}{104.87 - 96.50} = \textbf{21 589 kg/s}$$

c) $\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = \dfrac{7\,206 - 1425}{0.1807 \times 10^6} = \textbf{0.032}$

Reheat Cycles

9.33

The power plant in problem 9.16 has a too low quality in the low pressure turbine section so the plant wants to apply reheat at 800 kPa. What should the superheat for the reheat be so the turbine quality stays above 92%.

The reheat cycle has expansion in the turbine from state 3 to 800 kPa then reheat to state 5 and then expansion in the turbine to the new exhaust state 4'. We can then relate states 4' and 5 as follows

C.V. Turbine : $w_{T2} = h_5 - h_{4'}$; $s_{4'} = s_5$ and $x_{4'} = 0.92$

$$s_{4'} = s_5 = 0.8311 + x_{4'}\,(7.0784) = 7.34323 \text{ kJ/kgK}$$

State 5: 800 kPa, s = 7.34323 kJ/kgK => From Table B.1.3 at 800 kPa

$$T_5 = 300 + 50 \frac{7.34323 - 7.2327}{7.4088 - 7.2327} = \mathbf{331.4^oC}$$

9.34

Consider the supercritical cycle in problem 9.30 and assume the turbine first expands to 3 MPa then a reheat to 500°C with a further expansion in the low pressure turbine to 10 kPa. Find the combined specific turbine work and the total specific heat transfer in the boiler.

For the efficiency we need the net work and steam generator heat transfer.

C.V. Pump. For this high exit pressure we use Table B.1.4

State 1: $s_1 = 0.6492$ kJ/kg K, $h_1 = 191.81$ kJ/kg

Entropy Eq.: $s_2 = s_1 \implies h_2 = 222.5$ kJ/kg

State 3: $h_3 = 3275.36$ kJ/kg, $s_3 = 6.0342$ kJ/kg-K

C.V. Turbine section 1. Assume reversible and adiabatic.

Entropy Eq.: $s_4 = s_3 = 6.0342 = 2.6456 + x_4 \times 3.5412$, $x_4 = 0.956907$

$\quad\quad\quad\quad h_4 = 1008.41 + x_4 \times 1795.73 = 2726.76$ kJ/kg,

State 5: $\quad\quad h_5 = 3456.48$ kJ/kg, $s_5 = 7.2337$ kJ/kg-K

C.V. Turbine section 2. Assume reversible and adiabatic.

Entropy Eq.: $s_6 = s_5 = 7.2337 = 0.6492 + x_6 \times 7.501$, $x_6 = 0.87782$

$\quad\quad h_6 = 191.81 + x_6 \times 2392.82 = 2292.27$ kJ/kg

Steam generator: $\quad q_H = h_3 - h_2 + h_5 - h_4$

$\quad\quad\quad\quad\quad\quad = 3275.36 - 222.5 + 3456.48 - 2726.76$

$\quad\quad\quad\quad\quad\quad = 3052.86 + 729.72 = \textbf{3782.6 kJ/kg}$

Turbine: $\quad\quad w_T = h_3 - h_4 + h_5 - h_6$

$\quad\quad\quad\quad\quad\quad = 3275.36 - 2726.76 + 3456.48 - 2292.27$

$\quad\quad\quad\quad\quad\quad = 548.6 + 1164.21 = \textbf{1712.8 kJ/kg}$

9.35

A smaller power plant produces steam at 3 MPa, 600°C in the boiler. It keeps the condenser at 45°C by transfer of 10 MW out as heat transfer. The first turbine section expands to 500 kPa and then flow is reheated followed by the expansion in the low pressure turbine. Find the reheat temperature so the turbine output is saturated vapor. For this reheat find the total turbine power output and the boiler heat transfer.

The states properties from Tables B.1.1 and B.1.3

1: 45°C, $x = 0$: $h_1 = 188.42$ kJ/kg, $v_1 = 0.00101$ m³/kg, $P_{sat} = 9.59$ kPa

3: 3.0 MPa, 600°C: $h_3 = 3682.34$ kJ/kg, $s_3 = 7.5084$ kJ/kg K

6: 45°C, $x = 1$: $h_6 = 2583.19$ kJ/kg, $s_6 = 8.1647$ kJ/kg K

C.V. Pump Reversible and adiabatic.

Energy: $w_p = h_2 - h_1$; Entropy: $s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.00101 (3000 - 9.59) = 3.02$ kJ/kg

$h_2 = h_1 + w_p = 188.42 + 3.02 = 191.44$ kJ/kg

C.V. HP Turbine section

Entropy Eq.: $s_4 = s_3$ => $h_4 = 3093.26$ kJ/kg; $T_4 = 314$°C

C.V. LP Turbine section

Entropy Eq.: $s_6 = s_5 = 8.1647$ kJ/kg K => state 5

State 5: 500 kPa, s_5 => $h_5 = 3547.55$ kJ/kg, $T_5 = $ **529°C**

C.V. Condenser.

Energy Eq.: $\quad q_L = h_6 - h_1 = h_{fg} = 2394.77$ kJ/kg

$$\dot{m} = \dot{Q}_L / q_L = 10\ 000 / 2394.77 = 4.176 \text{ kg/s}$$

Both turbine sections

$$\dot{W}_{T,tot} = \dot{m} w_{T,tot} = \dot{m}(h_3 - h_4 + h_5 - h_6)$$
$$= 4.176\ (3682.34 - 3093.26 + 3547.55 - 2583.19) = \textbf{6487 kW}$$

Both boiler sections

$$\dot{Q}_H = \dot{m}(h_3 - h_2 + h_5 - h_4)$$
$$= 4.176\ (3682.34 - 191.44 + 3547.55 - 3093.26) = \textbf{16 475 kW}$$

9.36

A smaller power plant produces 25 kg/s steam at 3 MPa, 600°C in the boiler. It cools the condenser with ocean water so the condenser exit is at 50°C. There is a reheat done at 500 kPa up to 400°C and then expansion in the low pressure turbine. Find the net power output and the total heat transfer in the boiler.

Solution:

The states properties from Tables B.1.1 and B.1.3

> 1: 50°C, x = 0, h_1 = 209.31 kJ/kg, v_1 = 0.001012 m^3/kg, P_{sat} = 12.35 kPa
>
> 3: 3.0 MPa, 600°C: h_3 = 3682.34 kJ/kg, s_3 = 7.5084 kJ/kg K
>
> 5: 500 kPa, 400°C: h_5 = 3271.83 kJ/kg, s_5 = 7.7937 kJ/kg K

C.V. Pump Reversible and adiabatic. Incompressible flow so

> Energy: w_p = h_2-h_1 = v_1(P_2 - P_1) = 0.001012 (3000 - 12.35) = 3.02 kJ/kg

C.V. LP Turbine section

> Entropy Eq.: s_6 = s_5 = 7.7937 kJ/kg K => two-phase state
>
> $$x_6 = (s_6 - s_f)/s_{fg} = \frac{7.7937 - 0.7037}{7.3725} = 0.9617$$
>
> h_6 = 209.31 + 0.9617 × 2382.75 = 2500.8 kJ/kg

Both turbine sections

> $w_{T,tot}$ = h_3 - h_4 + h_5 - h_6
>
> = 3682.34 - 3093.26 + 3271.83 – 2500.8 = 1360.11 kJ/kg

$$\dot{W}_{net} = \dot{W}_T - \dot{W}_p = \dot{m}(w_{T,tot} - w_p) = 25 (1360.11 – 3.02) = \textbf{33 927 kW}$$

Both boiler sections

> $\dot{Q}_H$ = $\dot{m}$(h_3 - h_2 + h_5 - h_4)
>
> = 25 (3682.34 – 191.44 + 3271.83 - 3093.26) = **91 737 kW**

9.37

Consider an ideal steam reheat cycle where steam enters the high-pressure turbine at 3.0 MPa, 400°C, and then expands to 0.8 MPa. It is then reheated to 400°C and expands to 10 kPa in the low-pressure turbine. Calculate the cycle thermal efficiency and the moisture content of the steam leaving the low-pressure turbine.

Solution:

C.V. Pump reversible, adiabatic and assume incompressible flow

$$w_P = v_1(P_2 - P_1) = 0.00101 \text{ m}^3/\text{kg} \times (3000 - 10) \text{ kPa} = 3.02 \text{ kJ/kg},$$

$$h_2 = h_1 + w_P = 191.81 + 3.02 = 194.83 \text{ kJ/kg}$$

C.V. HP Turbine section

$$P_3 = 3 \text{ MPa}, T_3 = 400^\circ\text{C} \implies h_3 = 3230.82 \text{ kJ/kg}, \ s_3 = 6.9211 \text{ kJ/kg K}$$

$$s_4 = s_3 \implies h_4 = 2891.6 \text{ kJ/kg};$$

C.V. LP Turbine section

State 5: $400^\circ\text{C}, 0.8 \text{ MPa} \implies h_5 = 3267.1 \text{ kJ/kg}, \ s_5 = 7.5715 \text{ kJ/kg K}$

Entropy Eq.: $s_6 = s_5 = 7.5715 \text{ kJ/kg K} \implies$ two-phase state

$$x_6 = \frac{s_6 - s_f}{s_{fg}} = \frac{7.5715 - 0.6492}{7.501} = 0.92285 = \mathbf{0.923}$$

$$h_6 = 191.81 + 0.92285 \times 2392.82 = 2400 \text{ kJ/kg}$$

$$w_{T,tot} = h_3 - h_4 + h_5 - h_6 = 3230.82 - 2891.6 + 3267.1 - 2400 = 1237.8 \text{ kJ/kg}$$

$$q_{H1} = h_3 - h_2 = 3230.82 - 194.83 = 3036 \text{ kJ/kg}$$

$$q_H = q_{H1} + h_5 - h_4 = 3036 + 3267.1 - 2891.6 = 3411.5 \text{ kJ/kg}$$

$$\eta_{CYCLE} = (w_{T,tot} - w_P)/q_H = (1237.8 - 3.02)/3411.5 = \mathbf{0.362}$$

9.38

The reheat pressure affects the operating variables and thus turbine performance. Repeat Problem 9.35 twice, using 0.6 and 1.0 MPa for the reheat pressure.

Solution

C.V. Pump reversible, adiabatic and assume incompressible flow

$$w_P = v_1(P_2 - P_1) = 0.00101 \text{ m}^3/\text{kg} \times (3000 - 9.59) \text{ kPa} = 3.02 \text{ kJ/kg},$$

$$h_2 = h_1 + w_P = 188.42 + 3.02 = 191.44 \text{ kJ/kg}$$

State 3: 3 MPa, 400°C => $h_3 = 3230.82$ kJ/kg, $s_3 = 6.9211$ kJ/kg K

Low T boiler section: $q_{H1} = h_3 - h_2 = 3230.82 - 191.44 = 3039.38$ kJ/kg

State 4: P_4, $s_4 = s_3$

For $P_4 = 1$ MPa: $h_4 = 2940.85$ kJ/kg state 4 is sup. vapor

State 5: 400°C, $P_5 = P_4$ => $h_5 = 3263.9$ kJ/kg, $s_5 = 7.465$ kJ/kg K,

For $P_4 = 0.6$ MPa: $h_4 = 2793.2$ kJ/kg state 4 is sup. vapor

State 5: 400°C, $P_5 = P_4$ => $h_5 = 3270.3$ kJ/kg, $s_5 = 7.7078$ kJ/kg K,

State 6: 45°C, $s_6 = s_5$ => $x_6 = (s_6 - s_f)/s_{fg}$

Total turbine work: $w_{T,tot} = h_3 - h_4 + h_5 - h_6$

Total boiler H.Tr.: $q_H = q_{H1} + h_5 - h_4$

Cycle efficiency: $\eta_{CYCLE} = (w_{T,tot} - w_P)/q_H$

$P_4=P_5$	x_6	h_6	w_T	q_H	η_{CYCLE}
1	0.907	2360.5	1193.4	3362.4	0.3540
0.6	0.9393	2437.8	1270.1	3516.5	0.3603

Notice the small change in efficiency.

9.39

The effect of a number of reheat stages on the ideal steam reheat cycle is to be studied. Repeat Problem 9.35 using two reheat stages, one stage at 1.2 MPa and the second at 0.2 MPa, instead of the single reheat stage at 0.8 MPa.

C.V. Pump reversible, adiabatic and assume incompressible flow, work in

$$w_P = v_1(P_2 - P_1) = 0.00101 \text{ m}^3/\text{kg} \times (3000 - 9.593) \text{ kPa} = 3.02 \text{ kJ/kg},$$

$$h_2 = h_1 + w_P = 188.42 + 3.02 = 191.44 \text{ kJ/kg}$$

$P_4 = P_5 = 1.2 \text{ MPa}, \ P_6 = P_7 = 0.2 \text{ MPa}$

3: $h_3 = 3230.82 \text{ kJ/kg}, \ s_3 = 6.9211 \text{ kJ/kg K}$

4: $P_4, s_4 = s_3 \Rightarrow$ sup. vap. $h_4 = 2985.3$

5: $h_5 = 3260.7 \text{ kJ/kg}, \ s_5 = 7.3773 \text{ kJ/kg K}$

6: $P_6, s_6 = s_5 \Rightarrow$ sup. vapor

$h_6 = 2811.2 \text{ kJ/kg}$

7: $h_7 = 3276.5 \text{ kJ/kg}, \ s_7 = 8.2217 \text{ kJ/kg K}$

8: $P_8, s_8 = s_7 \Rightarrow$ sup. vapor $\ h_8 = 2602 \text{ kJ/kg}$ (used CATT3)

Total turbine work, same flow rate through all sections

$$w_T = (h_3 - h_4) + (h_5 - h_6) + (h_7 - h_8) = 245.5 + 449.5 + 674.5 = 1369.5 \text{ kJ/kg}$$

Total heat transfer in boiler, same flow rate through all sections

$$q_H = (h_3 - h_2) + (h_5 - h_4) + (h_7 - h_6) = 3039 + 319.8 + 465.3 = 3824.5 \text{ kJ/kg}$$

Cycle efficiency: $\qquad \eta_{TH} = \dfrac{w_T - w_P}{q_H} = \dfrac{1369.5 - 3.02}{3824.5} = \mathbf{0.357}$

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Open Feedwater Heaters

9.40

An open FWH receives steam at 1 MPa, 200°C from the turbine and 1 MPa, 100°C water from the feed water line. Find the required fraction of the extraction flow in the turbine.

The setup follows Fig.9.12

State enthalpies: $h_2 = 419.02,$ $h_6 = 2827.86$ all kJ/kg

$h_3 = 762.79$ (x = 0 at 1 MPa)

Analyze the FWH which leads to Eq.9.5:

$$y = \dot{m}_6 / \dot{m}_5 \quad \text{the extraction fraction}$$

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.79 - 419.02}{2827.86 - 419.02} = \mathbf{0.1427}$$

9.41

A power plant for a polar expedition uses ammonia and the boiler exit is 80°C, 1000 kPa and the condenser operates at -15°C. A single open "feed water" heater operates at 400 kPa with an exit state of saturated liquid. Find the mass fraction extracted in the turbine.

CV Feedwater heater.

States given and fixed from knowing Fig. 9.12:

5: $h_5 = 1614.6$ kJ/kg, $s_5 = 5.4971$ kJ/kgK

3: $h_3 = 171.226$ kJ/kg

1: $h_1 = 111.66$ kJ/kg, $v_1 = 0.001519$ m^3/kg

Analyze the pump: $h_2 = h_1 + w_{P1} = h_1 + v_1 (P_2 - P_1)$

$= 111.66 + 0.001519$ m^3/kg $\times (400 - 236.3)$ kPa

$= 111.91$ kJ/kg

Analyze the turbine: 6: 400 kPa, $s_6 = s_1$ => $h_6 = 1479.6$ kJ/kg

Analyze the FWH leads to Eq.9.5:

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{171.226 - 111.91}{1479.6 - 111.91} = \mathbf{0.0401}$$

9.42

Find the cycle efficiency for the cycle in Problem 9.41
CV Feedwater heater.

States given and fixed from knowing Fig. 9.12:

5: $h_5 = 1614.6$ kJ/kg, $s_5 = 5.4971$ kJ/kgK

3: $h_3 = 171.226$ kJ/kg

1: $h_1 = 111.66$ kJ/kg, $v_1 = 0.001519$ m^3/kg

Analyze the pump: $w_{P1} = v_1 (P_2 - P_1) = 0.001519$ m^3/kg $\times (400 - 236.3)$ kPa
$= 0.249$ kJ/kg
$h_2 = h_1 + w_{P1} = 111.66 + 0.249 = 111.909$ kJ/kg

Analyze the turbine: 6: 400 kPa, $s_6 = s_5$ => $h_6 = 1479.6$ kJ/kg
7: -15 C, $s_7 = s_5$ => $h_7 = 1413.56$ kJ/kg

Analyze the FWH leads to Eq.9.5:

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{171.226 - 111.909}{1479.6 - 11.909} = 0.0434$$

$$w_T = h_5 - y\, h_6 - (1 - y)\, h_7$$
$$= 1614.6 - 0.0434 \times 1479.6 - (1 - 0.0434) \times 1413.56$$
$$= 198.15 \text{ kJ/kg}$$

Pump 2 gives: $w_{P2} = v_3 (P_4 - P_3) = 0.00156$ m^3/kg $\times (1000 - 400)$ kPa
$= 0.936$ kJ/kg

Net work: $w_{net} = w_T - w_{P2} - (1 - y)\, w_{P1} = 198.15 - 0.936 - (1-0.0434)\, 0.249$
$= 196.98$ kJ/kg

Boiler: $q_H = h_5 - h_4 = h_5 - (h_3 + w_{P2}) = 1614.6 - 171.65 - 0.936$
$= 1442$ kJ/kg

Cycle efficiency: $\eta = w_{net} / q_H = 196.98 / 1442 = \textbf{0.1366}$

9.43

A steam power plant has high and low pressures of 20 MPa and 10 kPa, and one open feedwater heater operating at 1 MPa with the exit as saturated liquid. The maximum temperature is 800°C and the turbine has a total power output of 5 MW. Find the fraction of the flow for extraction to the feedwater and the total condenser heat transfer rate.

The physical components and the T-s diagram is as shown in Fig. 9.12 in the main text for one open feedwater heater. The same state numbering is used. From the Steam Tables:

State 5: (P, T) $h_5 = 4069.8$ kJ/kg, $s_5 = 7.0544$ kJ/kg K,

State 1: (P, x = 0) $h_1 = 191.81$ kJ/kg, $v_1 = 0.00101$ m^3/kg

State 3: (P, x = 0) $h_3 = 762.8$ kJ/kg, $v_3 = 0.001127$ m^3/kg

Pump P1: $w_{P1} = v_1(P_2 - P_1) = 0.00101 \times 990 = 1$ kJ/kg

$h_2 = h_1 + w_{P1} = 192.81$ kJ/kg

Turbine 5-6: $s_6 = s_5 \Rightarrow h_6 = 3013.7$ kJ/kg

$w_{T56} = h_5 - h_6 = 4069.8 - 3013.7 = 1056.1$ kJ/kg

Feedwater Heater ($\dot{m}_{TOT} = \dot{m}_5$): $y\,\dot{m}_5 h_6 + (1 - y)\,\dot{m}_5 h_2 = \dot{m}_5 h_3$

$\Rightarrow y = \dfrac{h_3 - h_2}{h_6 - h_2} = \dfrac{762.8 - 192.81}{3013.7 - 192.81} = \textbf{0.2021}$

To get state 7 into condenser consider turbine.

$s_7 = s_6 = s_5 \Rightarrow x_7 = (7.0544 - 0.6493)/7.5009 = 0.85391$

$h_7 = 191.81 + 0.85391 \times 2392.82 = 2235.1$ kJ/kg

Find specific turbine work to get total flow rate

$$\dot{W}_T = \dot{m}_{TOT}h_5 - y\dot{m}_{TOT}h_6 - (1 - y)\dot{m}_{TOT}h_7 =$$

$$= \dot{m}_{TOT} \times [h_5 - yh_6 - (1 - y) h_7] = \dot{m}_{TOT} \times 1677.3$$

$$\dot{m}_{TOT} = 5000/1677.3 = 2.98 \text{ kg/s}$$

$$\dot{Q}_L = \dot{m}_{TOT} (1-y) (h_7-h_1) = 2.98 \times 0.7979(2235.1 - 191.81) = \textbf{4858 kW}$$

9.44

A low temperature power plant operates with R-410A maintaining -20°C in the condenser, a high pressure of 3 MPa with superheat to 80°C. There is one open "feed water" heater operating at 800 kPa with an exit as saturated liquid at 0°C. Find the extraction fraction of the flow out of the turbine and the turbine work per unit mass flowing through the boiler.

State 1: $x_1 = 0$, $h_1 = 28.24$ kJ/kg, $v_1 = 0.000803$ m³/kg

State 3: $x_3 = 0$, $h_3 = 57.76$ kJ/kg, $v_3 = 0.000855$ m³/kg

State 5: $h_5 = 329.1$ kJ/kg, $s_5 = 1.0762$ kJ/kg K

State 6: $s_6 = s_5$ => $T_6 = 10.2$°C, $h_6 = 290.3$ kJ/kg

State 7: $s_7 = s_5$ => $x_7 = (s_7 - s_f)/s_{fg} = 0.9982$, $h_7 = 271.5$ kJ/kg

C.V Pump   P1

$\qquad w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.000803 (800 - 400) = 0.32$ kJ/kg

$\qquad\qquad$ => $h_2 = h_1 + w_{P1} = 28.24 + 0.32 = 28.56$ kJ/kg

C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = y$ (the extraction fraction)

$\qquad$ Energy Eq.: $(1 - y) h_2 + y h_6 = 1 h_3$

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{57.76 - 28.56}{290.3 - 28.56} = \mathbf{0.1116}$$

C.V. Turbine

$$\dot{W}_T = \dot{m}_{TOT} h_5 - y \dot{m}_{TOT} h_6 - (1 - y) \dot{m}_{TOT} h_7$$
$$w_T = h_5 - y h_6 - (1 - y) h_7$$
$$= 329.1 - 0.1116 \times 290.3 - (1 - 0.1116) \times 271.5 = \mathbf{55.52 \ kJ/kg}$$

9.45

A Rankine cycle operating with ammonia is heated by some low temperature source so the highest T is 120^oC at a pressure of 5000 kPa. Its low pressure is 1003 kPa and it operates with one open feedwater heater at 2033 kPa. The total flow rate is 5 kg/s. Find the extraction flow rate to the feedwater heater assuming its outlet state is saturated liquid at 2033 kPa. Find the total power to the two pumps.

State 1: $x_1 = 0$, $h_1 = 298.25$ kJ/kg, $v_1 = 0.001658$ m^3/kg

State 3: $x_3 = 0$, $h_3 = 421.48$ kJ/kg, $v_3 = 0.001777$ m^3/kg

State 5: $h_5 = 421.48$ kJ/kg, $s_5 = 4.7306$ kJ/kg K

State 6: $s_6 = s_5$ $=>$ $x_6 = (s_6 - s_f)/s_{fg} = 0.99052$, $h_6 = 1461.53$ kJ/kg

C.V Pump   P1

$\qquad w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.001658(2033 - 1003) = 1.708$ kJ/kg

$\qquad\qquad => h_2 = h_1 + w_{P1} = 298.25 + 1.708 = 299.96$ kJ/kg

C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = y$ (the extraction fraction)

$\qquad$ Energy Eq.: $(1 - y) h_2 + y h_6 = 1 h_3$

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{421.48 - 299.96}{1461.53 - 299.96} = \mathbf{0.1046}$$

$$\dot{m}_{extr} = y\, \dot{m}_{tot} = 0.1046 \times 5 = \mathbf{0.523\ kg/s}$$

$$\dot{m}_1 = (1 - y)\, \dot{m}_{tot} = (1 - 0.1046)\, 5 = 4.477\ kg/s$$

C.V Pump P2

$\qquad w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001777\,(5000 - 2033) = 5.272$ kJ/kg

Total pump work

$$\dot{W}_p = \dot{m}_1 w_{P1} + \dot{m}_{tot}\, w_{P2} = 4.477 \times 1.708 + 5 \times 5.272 = \mathbf{34\ kW}$$

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9.46

A power plant with one open feedwater heater has a condenser temperature of 45°C, a maximum pressure of 5 MPa, and boiler exit temperature of 900°C. Extraction steam at 1 MPa to the feedwater heater is mixed with the feedwater line so the exit is saturated liquid into the second pump. Find the fraction of extraction steam flow, the two specific pump work inputs and the turbine work per unit mass flow in the boiler.

Solution:

The complete diagram is as in Figure 9.12 in the main text.

State out of boiler 5: $h_5 = 4378.82$ kJ/kg, $s_5 = 7.9593$ kJ/kg K

C.V. Turbine reversible, adiabatic: $s_7 = s_6 = s_5$

State 6: P_6 , s_6 $=>$ $h_6 = 3640.6$ kJ/kg, $T_6 = 574^o$C

State 7: $s_7 = s_5$ $=>$ $x_7 = (s_7 - s_f)/s_{fg} = 0.97271$, $h_7 = 2517.84$ kJ/kg

C.V Pump   P1

$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(1000 - 9.6) =$ **1.0 kJ/kg**

$=> h_2 = h_1 + w_{P1} = 188.42 + 1.0 = 189.42$ kJ/kg

C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = y$ (the extraction fraction)

Energy Eq.: $(1 - y) h_2 + y h_6 = 1 h_3$

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.79 - 189.42}{3640.6 - 189.42} = \textbf{0.1661}$$

C.V Pump P2

$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001127(5000 - 1000) =$ **4.5 kJ/kg**

C.V. Turbine

$$\dot{W}_T = \dot{m}_{TOT} h_5 - y \, \dot{m}_{TOT} h_6 - (1 - y) \, \dot{m}_{TOT} h_7$$

$w_T = h_5 - y h_6 - (1 - y) h_7$

$= 4378.82 - 0.1661 \times 3640.6 - (1 - 0.1661) \times 2517.84$

$= \textbf{1674.5 kJ/kg}$

9.47

A steam power plant operates with a boiler output of 20 kg/s steam at 2 MPa, 600°C. The condenser operates at 50°C dumping energy to a river that has an average temperature of 20°C. There is one open feedwater heater with extraction from the turbine at 600 kPa and its exit is saturated liquid. Find the mass flow rate of the extraction flow. If the river water should not be heated more than 5°C how much water should be pumped from the river to the heat exchanger (condenser)?

Solution:

The setup is as shown in Fig. 9.12.

CONDENSER

1: 50°C sat liq. $v_1 = 0.001012$ m³/kg,

$\qquad\qquad h_1 = 209.31$ kJ/kg

2: 600 kPa $\qquad\qquad s_2 = s_1$

3: 600 kPa, sat liq. $\qquad h_3 = h_f = 670.54$ kJ/kg

5: (P, T) $h_5 = 3690.1$ kJ/kg,

$\qquad\qquad s_5 = 7.7023$ kJ/kg K

6: 600 kPa, $s_6 = s_5$ $\qquad \Rightarrow\ h_6 = 3270.0$ kJ/kg

CV P1

$\qquad w_{P1} = v_1(P_2 - P_1) = 0.001012$ m³/kg (600 - 12.35) kPa = 0.595 kJ/kg

$\qquad h_2 = h_1 + w_{P1} = 209.9$ kJ/kg

C.V FWH

$\qquad\qquad y\, h_6 + (1 - y)\, h_2 = h_3$

$\qquad\qquad y = \dfrac{h_3 - h_2}{h_6 - h_2} = \dfrac{670.54 - 209.9}{3270.0 - 209.9} = 0.1505$

$\qquad\qquad \dot{m}_6 = y\, \dot{m}_5 = 0.1505 \times 20 = \textbf{3 kg/s}$

CV Turbine: $s_7 = s_6 = s_5$ $\quad \Rightarrow \quad x_7 = 0.9493$, $h_7 = 2471.17$ kJ/kg

CV Condenser

$\qquad\qquad q_L = h_7 - h_1 = 2471.17 - 209.31 = 2261.86$ kJ/kg

The heat transfer out of the water from 7 to 1 goes into the river water

$\qquad\qquad \dot{Q}_L = (1 - y)\, \dot{m}q_L = 0.85 \times 20 \times 2261.86 = 38\ 429$ kW

$\qquad\qquad = \dot{m}_{H2O}\, \Delta h_{H2O} = \dot{m}_{H2O}\, (h_{f25} - h_{f20}) = \dot{m}\ (20.93)$

$\qquad\qquad \dot{m} = 38\ 429 / 20.93 = \textbf{1836 kg/s}$

9.48

In one type of nuclear power plant, heat is transferred in the nuclear reactor to liquid sodium. The liquid sodium is then pumped through a heat exchanger where heat is transferred to boiling water. Saturated vapor steam at 5 MPa exits this heat exchanger and is then superheated to 600°C in an external gas-fired superheater. The steam enters the turbine, which has one (open-type) feedwater extraction at 0.4 MPa. The condenser pressure is 10 kPa. Determine the heat transfer in the reactor and in the superheater to produce a net power output of 5 MW.

Solution:

The complete cycle diagram is similar to Figure 9.12 except the boiler is separated into a section heated by the reactor and a super heater section.

CV. Pump P1

$$w_{P1} = 0.001008(400 - 7.5) = 0.4 \text{ kJ/kg} ;$$

$$h_2 = h_1 + w_{P1} = 168.8 + 0.4 = 169.2 \text{ kJ/kg}$$

CV. Pump P2

$$w_{P2} = 0.001084(5000 - 400) = 5.0 \text{ kJ/kg}$$

$$h_4 = h_3 + w_{P2} = 604.7 + 5.0 = 609.7 \text{ kJ/kg}$$

C.V. Turbine (to get exit state properties)

$$s_7 = s_6 = 7.2589, \quad P_7 = 0.4 \text{ MPa} \implies T_7 = 221.2^{\text{o}}\text{C}, \quad h_7 = 2904.5 \text{ kJ/kg}$$

$$s_8 = s_6 = 7.2589 = 0.6492 + x_8 \times 7.501 \qquad x_8 = 0.88118$$

$$h_8 = 191.81 + 0.88118 \times 2392.82 = 2300.3 \text{ kJ/kg}$$

CV: Feedwater heater FWH (to get the extraction fraction x_7)

Divide the equations with the total mass flow rate $\dot{m}_3 = \dot{m}_4 = \dot{m}_5 = \dot{m}_6$

Continuity: $x_2 + x_7 = x_3 = 1.0$,   Energy Eq.: $x_2 h_2 + x_7 h_7 = h_3$

$$x_7 = (604.7 - 169.2)/(2904.5 - 169.2) = 0.1592$$

CV: Turbine (to get the total specific work)

Full flow from 6 to 7 and the fraction $(1 - x_7)$ from 7 to 8.

$$w_T = (h_6 - h_7) + (1 - x_7)(h_7 - h_8)$$
$$= 3666.5\text{-}2904.5 + 0.8408(2904.5\text{-}2300.3) = 1270.0 \text{ kJ/kg}$$

CV: Pumps (P1 has $x_1 = 1 - x_7$,   P2 has the full flow $x_3 = 1$)

$$w_P = x_1 w_{P1} + x_3 w_{P2} = 0.8408 \times 0.4 + 1 \times 5.0 = 5.3 \text{ kJ/kg}$$

$$w_{NET} = 1270.0 - 5.3 = 1264.7 \quad \Rightarrow \quad \dot{m} = 5000/1264.7 = 3.954 \text{ kg/s}$$

CV: Reactor (this has the full flow)

$$\dot{Q}_{REACT} = \dot{m}(h_5 - h_4) = 3.954 \, (2794.3 - 609.7) = \textbf{8638 kW}$$

CV: Superheater (this has the full flow)

$$\dot{Q}_{SUP} = \dot{m}(h_6 - h_5) = 3.954 \, (3666.5 - 2794.3) = \textbf{3449 kW}$$

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9.49

Consider an ideal steam regenerative cycle in which steam enters the turbine at 3.0 MPa, 400°C, and exhausts to the condenser at 10 kPa. Steam is extracted from the turbine at 0.8 MPa for an open feedwater heater. The feedwater leaves the heater as saturated liquid. The appropriate pumps are used for the water leaving the condenser and the feedwater heater. Calculate the thermal efficiency of the cycle and the net work per kilogram of steam.

Solution:

This is a standard Rankine cycle with an open FWH as shown in Fig.9.12

C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(800 - 10) = 0.798 \text{ kJ/kg}$$

$$=> \ h_2 = h_1 + w_{P1} = 191.81 + 0.798 = 192.61 \text{ kJ/kg}$$

C.V. FWH $\quad\quad$ Call $\dot{m}_6 / \dot{m}_{tot} = y$ (the extraction fraction)

$$(1 - y) h_2 + y h_6 = 1 h_3$$

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{721.1 - 192.61}{2891.6 - 192.61} = 0.1958$$

C.V Pump P2

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001115(3000 - 800) = 2.45 \text{ kJ/kg}$$

$$h_4 = h_3 + w_{P2} = 721.1 + 2.45 = 723.55 \text{ kJ/kg}$$

CV Boiler: $\quad q_H = h_5 - h_4 = 3230.82 - 723.55 = 2507.3 \text{ kJ/kg}$

CV Turbine

Entropy Eq.: $\quad s_7 = s_6 = s_5 = 6.9211 \text{ kJ/kg K}$

$$P_6 , s_6 \ => \ h_6 = 2891.6 \text{ kJ/kg (superheated vapor)}$$

$$s_7 = s_6 = s_5 = 6.9211 \quad => \quad x_7 = \frac{6.9211 - 0.6492}{7.501} = 0.83614$$

$$=> \ h_7 = 191.81 + x_7 \, 2392.82 = 2192.55 \text{ kJ/kg}$$

Turbine has full flow in HP section and fraction 1-y in LP section

$$\dot{W}_T / \dot{m}_5 = \ h_5 - h_6 + (1 - y) (h_6 - h_7)$$

$$w_T = 3230.82 – 2891.6 + (1 - 0.1988) (2891.6 – 2192.55) = 899.3 \text{ kJ/kg}$$

P2 has the full flow and P1 has the fraction 1-y of the flow

$$w_{net} = w_T - (1 - y) w_{P1} - w_{P2}$$

$$= 899.3 - (1 - 0.1988)0.798 – 2.45 = \textbf{896.2 kJ/kg}$$

$$\eta_{cycle} = w_{net} / q_H = 896.2 / 2507.3 = \textbf{0.357}$$

Closed Feedwater Heaters

9.50

Write the analysis (continuity and energy equations) for the closed feed water heater with a drip pump as shown in Fig.9.13. Take the control volume to have state 4 out so it includes the drip pump. Find the equation for the extraction fraction.

CV Feedwater heater plus drip pump.

Continuity Eq.: $\dot{m}_6 + \dot{m}_2 = \dot{m}_4$

Energy Eq.: $\dot{m}_6 h_6 + \dot{m}_2 h_2 + \dot{m}_6 \, w_{P \, drip} = \dot{m}_4 h_4$

CV drip pump

$$w_{P \, drip} = v_6 \, (P_4 - P_6) = h_{6b} - h_{6a} \; ; \quad h_{6a} = h_{f \, at \, P6}$$

Divide the energy equation with the full flow rate ($\dot{m}_4$) to get

Energy Eq.: $y \, h_6 + (1 - y) \, h_2 + y \, w_{P \, drip} = h_4$

Now solve for the fraction $y = \dot{m}_6 / \dot{m}_4$

$$y = \frac{h_4 - h_2}{h_6 - h_2 + w_{P \, drip}}$$

So to use this expression we assume we know states 2, 4 and 6 and have analyzed the drip pump. Of course if you assume state 3 is at the temperature of the condensing steam at state 6a then take CV to exclude the drip pump and the expression for y is different.

9.51

A closed FWH in a regenerative steam power cycle heats 20 kg/s of water from 100°C, 20 MPa to 200°C, 20 MPa. The extraction steam from the turbine enters the heater at 4 MPa, 275°C, and leaves as saturated liquid. What is the required mass flow rate of the extraction steam?

Solution:

The schematic from Figure 9.13 has the feedwater from the pump coming at state 2 being heated by the extraction flow coming from the turbine state 6 so the feedwater leaves as saturated liquid state 4 and the extraction flow leaves as condensate state 6a.

From table B.1	h kJ/kg
B.1.4: 100°C, 20 MPa	$h_2 = 434.04$
B.1.4: 200°C, 20 MPa	$h_4 = 860.47$
B.1.3: 4 MPa, 275°C	$h_6 = 2880.39$
B.1.2: 4 MPa, sat. liq.	$h_{6a} = 1087.29$

C.V. Feedwater Heater

Energy Eq.: $\dot{m}_2 h_2 + \dot{m}_6 h_6 = \dot{m}_2 h_4 + \dot{m}_6 h_{6a}$

Since all four states are known we can solve for the extraction flow rate

$$\dot{m}_6 = \dot{m}_2 \frac{h_2 - h_4}{h_{6a} - h_6}$$

$$= 20 \frac{434.04 - 860.47}{1087.29 - 2880.39} \text{ kg/s} = \textbf{4.756 kg/s}$$

9.52

Find the specific turbine work from the extraction flow for the cycle in Problem 9.51 and the specific heat transfer in the boiler operating at 20 MPa, assuming the extraction flow goes through a drip pump and added to the feedwater line.

Solution:

The schematic from Figure 9.13 has the feedwater from the pump coming at state 2 (20 MPa, 100°C) being heated by the extraction flow coming from the turbine state 6 (4 MPa, 275°C) so the feedwater leaves at state 3 (20 MPa, 200°C) and the extraction flow leaves as condensate state 6a (4 MPa, x = 0) goes through the drip pump and added to the feedwater line to produce a state 4 (20 MPa, T = ?) going to the boiler.

From B.1 (h in kJ/kg):

$h_2 = 434.04$, $h_3 = 860.47$,
$h_6 = 2880.39$, $h_{6a} = 1087.29$
$v_{6a} = v_f = 0.001252 \text{ m}^3/\text{kg}$
$s_6 = s_5 = 6.2145 \text{ kJ/kg-K}$

C.V. Feedwater Heater

Energy Eq.: $\dot{m}_2 h_2 + \dot{m}_6 h_6 = \dot{m}_2 h_3 + \dot{m}_6 h_{6a}$

Since all four states are known we can solve for the extraction flow rate

$$\dot{m}_6 = \dot{m}_2 \frac{h_2 - h_3}{h_{6a} - h_6} = 20 \frac{434.04 - 860.47}{1087.29 - 2880.39} \text{ kg/s} = 4.756 \text{ kg/s}$$

CV drip pump: $w_{P2} = v_{6a} (P_{6b} - P_{6a}) = 0.001252 (20\,000 - 4000) = 20.03 \text{ kJ/kg}$
$h_{6b} = h_{6a} + w_{P2} = 1087.29 + 20.03 = 1107.32 \text{ kJ/kg}$

CV Junction 6b, 3, 4

$$h_4 = \frac{\dot{m}_2 h_3 + \dot{m}_6 h_{6b}}{\dot{m}_2 + \dot{m}_6} = \frac{20 \times 860.47 + 4.756 \times 1107.32}{20 + 4.756} = 907.89 \text{ kJ/kg}$$

We must assume state 5 is 20 MPa, $s = s_6$ so $h_5 = 3297.59 \text{ kJ/kg}$, then

$w_{T \text{ ext flow}} = h_5 - h_6 = 3297.59 - 2880.39 = \textbf{417.2 kJ/kg}$
$q_{H \text{ ext flow}} = h_5 - h_4 = 3297.59 - 907.89 = \textbf{2389.7 kJ/kg}$

9.53

Assume the cycle in problem 9.51 has a condenser operating at 20 kPa, what would the quality at the turbine exit be? Find the reheat temperature if reheat is at the 4 MPa extraction pressure and the turbine should have a minimum quality of 90%.

Assume rev. adiabatic turbine so 4 MPa, 275°C: $s_5 = s_6 = s_7 = 6.2145$ kJ/kg-K

$$\Rightarrow \quad x_7 = \frac{6.2145 - 0.8319}{7.0766} = \mathbf{0.7606}$$

If we should have the new state 7 as $x_7 = 0.9$ then the entropy is:

$$s_7 = 0.8319 + 0.9 \times 7.0766 = 7.20084 \text{ kJ/kg-K}$$

this is then also entropy after reheat at 4 MPa, $s_8 = s_7$ so then

$$T_8 = 500 + 100 \frac{7.20084 - 7.09}{7.3688 - 7.09} = \mathbf{539.8°C}$$

9.54

A power plant with one closed FWH has a condenser temperature of 45°C, a maximum pressure of 5 MPa, and boiler exit temperature of 900°C. Extraction steam at 1 MPa to the feedwater heater condenses and is pumped up to the 5 MPa feedwater line where all the water goes to the boiler at 200°C. Find the fraction of extraction steam flow and the two specific pump work inputs.

Solution:

$s_1 = 0.6387$ kJ/kg K,
$h_1 = 188.42$ kJ/kg
$v_1 = 0.00101$ m^3/kg,
$s_4 = 2.1386$ kJ/kg K,
$h_4 = 762.79$ kJ/kg
$v_4 = 0.001127$ m^3/kg
$T_6 => h_6 = 853.85$ kJ/kg

From turbine   3   Pump 1   1

6   5   From condenser

7   2

Pump 2   4

C.V. Turbine: Reversible, adiabatic so constant s from inlet to extraction point

$$s_3 = s_{IN} = 7.9593 \text{ kJ/kg K} \quad => \quad T_3 = \textbf{573.8}, \quad h_3 = 3640.6 \text{ kJ/kg}$$

C.V. P1: $w_{P1} = v_1(P_2 - P_1) = 0.00101$ m^3/kg $\times (5000 - 9.6)$ kPa = **5.04 kJ/kg**

$\Rightarrow h_2 = h_1 + w_{P1} = 188.42 + 5.04 = 193.46$ kJ/kg

C.V. P2: $w_{P2} = v_4(P_7 - P_4) = 0.001127$ m^3/kg $\times (5 - 1)$ MPa = **4.508 kJ/kg**

$\Rightarrow h_7 = h_4 + w_{P2} = 762.79 + 4.51 = 767.30$ kJ/kg

C.V. FWH and pump 2:

The extraction fraction is: $y = \dot{m}_3/\dot{m}_6$

Continuity Eq.: $\dot{m}_6 = \dot{m}_2 + \dot{m}_3$, $1 = (1 - y) + y$

Energy: $(1 - y)h_2 + y\, h_3 + y\, w_{P2} = h_6$

$$y = \frac{h_6 - h_2}{h_3 + w_{P2} - h_2} = \frac{853.85 - 193.46}{3640.6 + 4.508 - 193.46} = 0.1913$$

$$\dot{m}_3/\dot{m}_6 = y = \textbf{0.1913}$$

9.55

Do Problem 9.43 with a closed feedwater heater instead of an open and a drip pump to add the extraction flow to the feed water line at 20 MPa. Assume the temperature is 175°C after the drip pump flow is added to the line. One main pump brings the water to 20 MPa from the condenser.

Solution:

$v_1 = 0.00101$ m^3/kg,

$h_1 = 191.81$ kJ/kg

$T_4 = 175^o$C; $h_4 = 751.66$ kJ/kg

$h_{6a} = h_{f\ 1MPa} = 762.79$ kJ/kg,

$v_{6a} = 0.001127$ m^3/kg

Turbine section 1: $s_6 = s_5 = 7.0544$ kJ/kg K

$P_6 = 1$ MPa $\Rightarrow$ $h_6 = 3013.7$ kJ/kg

C.V Pump 1

$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(20\ 000 - 10) = 20.19$ kJ/kg

$\Rightarrow$ $h_2 = h_1 + w_{P1} = 191.81 + 20.19 = 212.0$ kJ/kg

C.V Pump 2

$w_{P2} = h_{6b} - h_{6a} = v_{6a}(P_{6b} - P_{6a}) = 0.001127(20\ 000 - 1000) = 21.41$ kJ/kg

C.V FWH + P2   select the extraction fraction to be $y = \dot{m}_6 / \dot{m}_4$

$y\ h_6 + (1 - y)\ h_2 + y\ (w_{P2}) = h_4$

$y = \dfrac{h_4 - h_2}{h_6 - h_2 + w_{P2}} = \dfrac{751.66 - 212.0}{3013.7 - 212.0 + 21.41} = \mathbf{0.191}$

Turbine: $s_7 = s_6 = s_5$ & $P_7 = 10$ kPa

$\Rightarrow$ $x_7 = \dfrac{7.0544 - 0.6493}{7.5009} = 0.85391$

$h_7 = 191.81 + 0.85391 \times 2392.82 = 2235.1$ kJ/kg

$w_T = [\ h_5 - h_6 + (1 - y)\ (h_6 - h_7)\ ]$

$= [\ 4069.8 - 3013.7 + 0.809\ (3013.7 - 2235.1)] = 1686$ kJ/kg

$\dot{W}_T = 5000$ kW $= \dot{m}_5 \times w_T = \dot{m}_5 \times 1686$ kJ/kg $\Rightarrow$ $\dot{m}_5 = 2.966$ kg/s

$\dot{Q}_L = \dot{m}_5(1 - y)\ (h_7 - h_1) = 2.966 \times 0.809\ (2235.1 - 191.81) = \mathbf{4903\ kW}$

9.56

Repeat Problem 9.49, but assume a closed instead of an open feedwater heater. A single pump is used to pump the water leaving the condenser up to the boiler pressure of 3.0 MPa. Condensate from the feedwater heater is going through a drip pump and added to the feedwater line so state 4 is at T_6.

Solution:

C.V. Turbine, 2nd law:

$s_5 = s_6 = s_7 = 6.9211$ kJ/kg K

$h_5 = 3230.82$, $h_6 = 2891.6$

$\Rightarrow x_7 = (6.9211 - 0.6492)/7.501$

$\qquad = 0.83614$

$h_7 = 191.81 + x_7\ 2392.82$

$\qquad = 2192.55$ kJ/kg

Assume feedwater heater exit state 4 at the T of the condensing steam

C.V Pump 1

$\qquad w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02$ kJ/kg

$\qquad h_2 = h_1 + w_{P1} = 191.81 + 3.02 = 194.83$ kJ/kg

$\qquad T_4 = T_{sat}\ (P_6) = 170.43°C, \quad h_4 \approx h_f = h_{6a} = 721.1$ kJ/kg

C.V Pump 2 (the drip pump)

$\qquad w_{P2} = h_{6b} - h_{6a} = v_{6a}(P_{6b} - P_{6a}) = 0.001115(3000 - 800) = 2.45$ kJ/kg

C.V FWH + P2   select the extraction fraction to be $y = \dot{m}_6 / \dot{m}_4$

$\qquad y\ h_6 + (1 - y)\ h_2 + y\ (w_{P2}) = h_4$

$\qquad y = \dfrac{h_4 - h_2}{h_6 - h_2 + w_{P2}} = \dfrac{721.1 - 194.83}{2891.6 - 194.83 + 2.45} = \mathbf{0.195}$

Turbine work with full flow from 5 to 6 fraction 1– y flows from 6 to 7

$\qquad w_T = [\ h_5 - h_6 + (1 - y)\ (h_6 - h_7)\ ]$

$\qquad\quad = [\ 3230.82 - 2891.6 + 0.805\ (2891.6 - 2192.55)] = 901.95$ kJ/kg

$\qquad w_{net} = w_T - (1-y)w_{P1} - xw_{P2} = 901.95 - 0.805 \times 3.02 - 0.195 \times 2.45$

$\qquad\quad = \mathbf{899.0\ kJ/kg}$

$\qquad q_H = h_5 - h_4 = 3230.82 - 721.1 = 2509.7$ kJ/kg

$\qquad\qquad \eta_{cycle} = w_{net} / q_H = 899.0 / 2509.7 = \mathbf{0.358}$

9.57

Repeat Problem 9.49, but assume a closed instead of an open feedwater heater. A single pump is used to pump the water leaving the condenser up to the boiler pressure of 3.0 MPa. Condensate from the feedwater heater is drained through a trap to the condenser.

Solution:

C.V. Turbine, 2nd law:

$s_4 = s_5 = s_6 = 6.9211$ kJ/kg K

$h_4 = 3230.82$, $h_5 = 2891.6$

$\Rightarrow x_6 = (6.9211 - 0.6492)/7.501$

$= 0.83614$

$h_6 = 191.81 + x_6 \, 2392.82$

$= 2192.55$ kJ/kg

Assume feedwater heater exit at the T of the condensing steam

C.V Pump

$$w_P = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 191.81 + 3.02 = 194.83 \text{ kJ/kg}$$

$$T_3 = T_{sat}(P_5) = 170.43°C, \quad h_3 = h_f = h_7 = 721.1 \text{ kJ/kg}$$

C.V FWH

$$\dot{m}_5 / \dot{m}_3 = y, \quad \text{Energy Eq.:} \quad h_2 + y \, h_5 = h_3 + h_7 \, y$$

$$y = \frac{h_3 - h_2}{h_5 - h_{f\,800}} = \frac{721.1 - 194.83}{2891.6 - 721.1} = 0.2425$$

Turbine work with full flow from 4 to 5 fraction 1-y flows from 5 to 6

$$w_T = h_4 - h_5 + (1 - y)(h_5 - h_6)$$

$$= 3230.82 - 2891.6 + 0.7575 \, (2891.6 - 2192.55)$$

$$= 868.75 \text{ kJ/kg}$$

$$w_{net} = w_T - w_P = 868.75 - 3.02 = \textbf{865.7 kJ/kg}$$

$$q_H = h_4 - h_3 = 3230.82 - 721.1 = 2509.7 \text{ kJ/kg}$$

$$\eta_{cycle} = w_{net} / q_H = 865.7 / 2509.7 = \textbf{0.345}$$

9.58

A Rankine cycle feeds 5 kg/s ammonia at 2 MPa, 140°C to the turbine, which has an extraction point at 800 kPa. The condenser is at -20°C and a closed feed water heater has an exit state (3) at the temperature of the condensing extraction flow and it has a drip pump. The source for the boiler is at constant 180°C. Find the extraction flow rate and state 4 into the boiler.

$P_1 = 190.2$ kPa,
$h_1 = 89.05$ kJ/kg
$v_1 = 0.001504$ m³/kg,
$s_5 = 5.5022$ kJ/kg K,
$h_5 = 1738.2$ kJ/kg
$T_{6a} = T_{sat\ 800\ kPa} = 17.85$°C
$\Rightarrow h_{6a} = 264.18$ kJ/kg

C.V. Turbine: Reversible, adiabatic so constant s from inlet to extraction point

$$s_6 = s_{IN} = 5.5022 \text{ kJ/kg K} \Rightarrow T_6 = 63.4°C, \quad h_6 = 1580.89 \text{ kJ/kg}$$

C.V. P1: $w_{P1} = v_1(P_2 - P_1) = 0.001504(2000 - 190.2) = 2.722$ kJ/kg

$$\Rightarrow h_2 = h_1 + w_{P1} = 91.772 \text{ kJ/kg}$$

C.V. P2: $w_{P2} = v_{6a}(P_4 - P_6) = 0.0016108\ (2000 - 800) = 1.933$ kJ/kg

$$\Rightarrow h_{6b} = h_{6a} + w_{P2} = 266.11 \text{ kJ/kg}$$

C.V. Total FWH and pump (notice $h_3 = h_{6a}$ as we do not have table for this state)

The extraction fraction is: $y = \dot{m}_6/\dot{m}_4$

Energy: $(1 - y)h_2 + yh_6 = (1 - y)h_3 + yh_{6a}$

$$y = \frac{h_3 - h_2}{h_3 - h_2 + h_6 - h_{6a}} = \frac{264.18 - 91.772}{264.18 - 91.772 + 1580.89 - 264.18} = 0.1158$$

$$\dot{m}_6 = y\ \dot{m}_4 = 0.1158 \times 5 = \textbf{0.5789 kg/s}$$

C.V. The junction after FWH and pump 2.

$$h_4 = (1-y)h_3 + y\ h_{6b} = (1 - 0.1158) \times 264.18 + 0.1158 \times 266.11 = 264.4 \text{ kJ/kg}$$

Interpolate in Table B.2.1: $T_4 = \textbf{17.92°C}$

9.59

Assume the powerplant in Problem 9.45 has one closed feedwater heater instead of the open FWH. The extraction flow out of the FWH is saturated liquid at 2033 kPa being dumped into the condenser and the feedwater is heated to 50°C. Find the extraction flow rate and the total turbine power output.

State 1: $x_1 = 0$, $h_1 = 298.25$ kJ/kg, $v_1 = 0.001658$ m^3/kg

State 3: $h_3 = h_f + (P_3 - P_{sat})v_f = 421.48 + (5000-2033)0.001777 = 426.75$ kJ/kg

State 5: $h_5 = 421.48$ kJ/kg, $s_5 = 4.7306$ kJ/kg K

State 6: $s_6 = s_5$ => $x_6 = (s_6 - s_f)/s_{fg} = 0.99052$, $h_6 = 1461.53$ kJ/kg

State 6a: $x_{6a} = 0$ => $h_{6a} = 421.48$ kJ/kg

State 7: $s_7 = s_5$ => $x_7 = (s_7 - s_f)/s_{fg} = 0.9236$, $h_7 = 1374.43$ kJ/kg

C.V Pump   P1

$\qquad w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.001658(5000 - 1003) = 6.627$ kJ/kg

$\qquad\qquad => h_2 = h_1 + w_{P1} = 298.25 + 6.627 = 304.88$ kJ/kg

C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = y$ (the extraction fraction)

$\qquad$ Energy Eq.: $h_2 + y\, h_6 = 1\, h_3 + y\, h_{6a}$

$$y = \frac{h_3 - h_2}{h_6 - h_{6a}} = \frac{426.75 - 304.88}{1461.53 - 421.48} = \mathbf{0.1172}$$

$$\dot{m}_{extr} = y\, \dot{m}_{tot} = 0.1172 \times 5 = \mathbf{0.586\ kg/s}$$

Total turbine work

$\qquad \dot{W}_T = \dot{m}_{tot}(h_5 - h_6) + (1 - y)\dot{m}_{tot}\,(h_6 - h_7)$

$\qquad\qquad = 5(1586.3 - 1461.53) + (5 - 0.586)(1461.53 - 1374.43) = \mathbf{1008\ kW}$

9.60

Assume a variation of the cycle in Problem 9.49 with a closed FWH at 0.8 MPa and one open FWH at 100 kPa. A pump is used to bring the water leaving the condenser up to 100 kPa for an open FWH and a second pump brings the feedwater up to 3.0 MPa as shown in Fig. P9.60. Condensate from the closed FWH is drained through a trap to the open FWH. Calculate the thermal efficiency of the cycle and the specific net work.

Solution:

This is a Rankine cycle with an open FWH and a closed FWH shown in Fig. P9.60.

Here state 6 is out of boiler, state 8 is extraction at 100 kPa, state 9 is out of turbine.

States   1:   10 kPa, $x = 0$: $h_1 = 191.81$ kJ/kg, $v_1 = 0.001010$ m^3/kg

                3: 100 kPa, $x = 0$: $h_3 = 417.44$ kJ/kg, $v_3 = 0.001043$ m^3/kg

                5: 3 MPa, $T = T_{7a} = 170.43^oC$: $h_5 \approx 721.1$ kJ/kg $(= h_{7a})$

                6: 3 MPa, 400^oC: $h_6 = 3230.82$ kJ/kg, $s_6 = 6.9211$ kJ/kg-K

                7: 800 kPa, $s = s_6$: $h_7 = 2891.62$ kJ/kg (sup. vapor)

                7a: 800 kPa, $x = 0$: $h_{7a} = h_f = 721.1$ kJ/kg

                8: 100 kPa, $s = s_6$: $x_8 = (6.9211 - 1.3025)/6.0568 = 0.92765$

                      $h_8 = 417.44 + x_8\, 2258.02 = 2512.09$ kJ/kg

                9: 10 kPa, $s = s_6$: $x_9 = (6.9211 - 0.6492)/7.501 = 0.83614$

                      $h_9 = 191.81 + x_9\, 2392.82 = 2192.55$ kJ/kg

C.V Pump P1

                     $w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(100 - 10) = 0.0909$ kJ/kg

                     $=> h_2 = h_1 + w_{P1} = 191.81 + 0.0909 = 191.9$ kJ/kg

C.V Pump P2

                     $w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001043(3000 - 100) = 3.025$ kJ/kg

                     $h_4 = h_3 + w_{P2} = 417.44 + 3.025 = 420.465$ kJ/kg

Since the open FWH has a flow in from the trap we must do that first.

C.V. Closed FWH   Call $\dot{m}_7 / \dot{m}_{tot} = y_1$ (the extraction fraction)

Energy Eq.: $0 = h_4 - h_5 + y_1(h_7 - h_{7a})$ $=> y_1 = \dfrac{h_5 - h_4}{h_7 - h_{7a}} = \dfrac{721.1 - 420.465}{2891.62 - 721.1} = 0.1385$

CV Open FWH (3 flows in, 1 flow out)

Energy Eq.: $\quad 0 = y_2 h_8 + y_1 h_{7a} + (1 - y_1 - y_2) h_2 - h_3$

$$y_2 = \frac{h_3 - y_1 h_{7a} - (1-y_1) h_2}{h_8 - h_2} = \frac{417.44 - y_1 \times 721.1 - (1- y_1)\, 191.9}{2512.09 - 191.9} = 0.0656$$

CV Boiler: $\quad q_H = h_6 - h_5 = 3230.82 - 721.1 = 2509.7 \text{ kJ/kg}$

CV Turbine $\quad w_T = h_6 - y_1 h_7 - y_2 h_8 - (1 - y_1 - y_2) h_9$

$\qquad\qquad = 3230.82 - 0.1385 \times 2891.62 - 0.0656 \times 2512.09 - 0.7959 \times 2192.55$

$\qquad\qquad = 920.487 \text{ kJ/kg}$

P2 has the full flow and P1 has the fraction $1 - y_1 - y_2$ of the flow

$\qquad\quad w_{net} = w_T - (1 - y_1 - y_2) w_{P1} - w_{P2}$

$\qquad\qquad = 920.487 - 0.7959 \times 0.0909 - 3.025 = \textbf{917.39 kJ/kg}$

$\qquad \eta_{cycle} = w_{net} / q_H = 917.39 / 2509.7 = \textbf{0.366}$

Nonideal Cycles

9.61

A steam power cycle has a high pressure of 3.0 MPa and a condenser exit temperature of 45°C. The turbine efficiency is 85%, and other cycle components are ideal. If the boiler superheats to 800°C, find the cycle thermal efficiency.

Solution:

Basic Rankine cycle as shown in Figure 9.3 in the main text.

C.V. Turbine: $\quad w_T = h_3 - h_4, \quad s_4 = s_3 + s_{T,GEN}$

Ideal: Table B.1.3: $s_4 = s_3 = 7.9862$ kJ/kg K

$\Rightarrow x_{4s} = (7.9862 - 0.6386)/7.5261 = 0.9763$

$h_{4s} = h_f + x\, h_{fg} = 188.42 + 0.9763 \times 2394.77 = 2526.4$ kJ/kg

$w_{Ts} = h_3 - h_{4s} = 4146 - 2526.4 = 1619.6$ kJ/kg

Actual: $w_{T,AC} = \eta \times w_{T,S} = 0.85 \times 1619.6 = 1376.66$ kJ/kg

C.V. Pump: $\quad w_P = \int v\, dP \approx v_1(P_2 - P_1) = 0.00101\,(3000 - 9.6) = 3.02$ kJ/kg

$h_2 = h_1 + w_P = 188.42 + 3.02 = 191.44$ kJ/kg

C.V. Boiler: $\quad q_H = h_3 - h_2 = 4146 - 191.44 = 3954.6$ kJ/kg

$\eta = (w_{T,AC} - w_P)/q_H = (1376.66 - 3.02)/3954.6 = \mathbf{0.347}$

9.62

A Rankine cycle with water superheats to 500$^{\text{o}}$C at 3 MPa in the boiler and the condenser operates at 100$^{\text{o}}$C. All components are ideal except the turbine which has an exit state measured to be saturated vapor at 100$^{\text{o}}$C. Find the cycle efficiency with a) an ideal turbine and b) the actual turbine.

Standard Rankine cycle 1-2-3-4s for ideal turbine.
Modified Rankine cycle 1-2-3-4ac for actual turbine

Table B.1.1: $v_1 = 0.001044$ m^3/kg, $h_1 = 419.02$ kJ/kg
  $h_3 = 3456.48$ kJ/kg, $s_3 = 7.2337$ kJ/kg-K

State 4s: $s_{4s} = s_3 = 7.2337 = s_f + x_{4s}\, s_{fg} = 1.3068 + x_{4s}\, 6.0480 \Rightarrow x_{4s} = 0.97998$
  $\Rightarrow h_{4s} = h_f + x_{4s} h_{fg} = 419.02 + x_{4s}\, 2257.03 = 2630.9$ kJ/kg

State 4ac: $h_{4ac} = h_g = 2676.05$ kJ/kg

C.V. Pump: Assume adiabatic, reversible and incompressible flow

  $w_{ps} = \int v\, dP = v_1(P_2 - P_1) = 3.03$ kJ/kg

  $h_2 = h_1 + w_p = 419.02 + 3.03 = 422.05$ kJ/kg

C.V. Boiler $q_B = h_3 - h_2 = 3456.48 - 422.05 = 3034.4$ kJ/kg

C.V. Turbine; $w_{Ts} = h_3 - h_{4s} = 3456.48 - 2630.9 = 825.58$ kJ/kg

Efficiency: $\eta_{th} = w_{net} / q_B = (825.58 - 3.03)/3034.4 = \mathbf{0.271}$

Actual turbine: $w_{Tac} = h_3 - h_{4ac} = 3456.48 - 2676.05 = 780.43$ kJ/kg

Efficiency: $\eta_{th} = w_{net} / q_B = (780.43 - 3.03)/3034.4 = \mathbf{0.256}$

9.63

For the steam power plant described in Problem 9.13, assume the isentropic efficiencies of the turbine and pump are 85% and 80%, respectively. Find the component specific work and heat transfers and the cycle efficiency.

Solution:

This is a standard Rankine cycle with actual non-ideal turbine and pump.

CV Pump, Rev & Adiabatic:

$$w_{Ps} = h_{2s} - h_1 = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02 \text{ kJ/kg}; \quad s_{2s} = s_1$$

$$w_{Pac} = w_{Ps} / \eta_P = 3.02/0.8 = \textbf{3.775} \text{ kJ/kg} = h_{2a} - h_1$$

$$h_{2a} = w_{Pac} + h_1 = 3.775 + 191.81 = 195.58 \text{ kJ/kg}$$

CV Boiler: $q_H = h_3 - h_{2a} = 2804.14 - 195.58 = \textbf{2608.56 kJ/kg}$

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$$s_4 = s_3 = 6.1869 = 0.6492 + x_4 (7.501) \implies x_4 = 0.7383$$

$$\implies h_4 = 191.81 + 0.7383 (2392.82) = 1958.34 \text{ kJ/kg}$$

$$w_{Ts} = 2804.14 - 1958.34 = 845.8 \text{ kJ/kg}$$

$$w_{Tac} = w_{Ts} \times \eta_T = \textbf{718.9} = h_3 - h_{4a}$$

$$h_{4a} = h_3 - w_{Tac} = 2804.14 - 718.9 = 2085.24 \text{ kJ/kg}$$

CV Condenser: $q_L = h_{4a} - h_1 = 2085.24 - 191.81 = \textbf{1893.4 kJ/kg}$

$$\eta_{cycle} = \frac{w_{Tac} - w_{Pac}}{q_H} = \frac{718.9 - 3.78}{2608.56} = \textbf{0.274}$$

This compares to 0.32 for the ideal case.

state 2s and 2ac nearly the same

9.64

Steam enters the turbine of a power plant at 5 MPa and 400°C, and exhausts to the condenser at 10 kPa. The turbine produces a power output of 20 000 kW with an isentropic efficiency of 85%. What is the mass flow rate of steam around the cycle and the rate of heat rejection in the condenser? Find the thermal efficiency of the power plant and how does this compare with a Carnot cycle.

Solution: $\dot{W}_T = 20\ 000$ kW and $\eta_{Ts} = 85$ %

State 3: $h_3 = 3195.6$ kJ/kg , $s_3 = 6.6458$ kJ/kgK

State 1: $P_1 = P_4 = 10$ kPa , sat liq , $x_1 = 0$

$\qquad T_1 = 45.8°C$, $h_1 = h_f = 191.8$ kJ/kg , $v_1 = v_f = 0.00101$ m^3/kg

C.V Turbine energy Eq.: $q_T + h_3 = h_4 + w_T$; $q_T = 0$

$\qquad w_T = h_3 - h_4$, Assume Turbine is isentropic

$\qquad s_{4s} = s_3 = 6.6458$ kJ/kgK , $s_{4s} = s_f + x_{4s}\ s_{fg}$, solve for $x_{4s} = 0.7994$

$\qquad h_{4s} = h_f + x_{4s}h_{fg} = 191.81 + x_{4s}\ 2392.82 = 2104.6$ kJ/kg

$\qquad w_{Ts} = h_3 - h_{4s} = 1091$ kJ/kg , $w_T = \eta_{Ts}w_{Ts} = 927.3$ kJ/kg

$$\dot{m} = \frac{\dot{W}_T}{w_T} = \mathbf{21.568\ kg/s} , \qquad h_4 = h_3 - w_T = 2268.3\ \text{kJ/kg}$$

C.V. Condenser: energy Eq.: $h_4 = h_1 + q_c + w_c$; $w_c = 0$

$$q_c = h_4 - h_1 = 2076.5\ \text{kJ/kg} , \qquad \dot{Q}_c = \dot{m}\ q_c = \mathbf{44\ 786\ kW}$$

C.V. Pump: Assume adiabatic, reversible and incompressible flow

$\qquad w_{ps} = \int v\ dP = v_1(P_2 - P_1) = 5.04$ kJ/kg

$\qquad$ energy Eq.: $h_2 = h_1 + w_p = 196.8$ kJ/kg

C.V Boiler : energy Eq.: $q_B + h_2 = h_3 + w_B$; $w_B = 0$

$\qquad q_B = h_3 - h_2 = 2998.8$ kJ/kg

$\qquad w_{net} = w_T - w_P = 922.3$ kJ/kg

$\qquad \eta_{th} = w_{net} / q_B = \mathbf{0.307}$

Carnot cycle : $T_H = T_3 = 400°C$, $T_L = T_1 = 45.8°C$

$$\eta_{th} = \frac{T_H - T_L}{T_H} = \mathbf{0.526}$$

9.65

Consider the boiler in Problem 9.21 where the geothermal hot water brings the R-134a to saturated vapor. Assume a counter flowing heat exchanger arrangement. The geothermal water temperature should be equal to or greater than the R-134a temperature at any location inside the heat exchanger. The point with the smallest temperature difference between the source and the working fluid is called the pinch point. If 2 kg/s of geothermal water is available at 95°C, what is the maximum power output of this cycle for R-134a as the working fluid? (hint: split the heat exchanger C.V. into two so the pinch point with $\Delta T = 0$, $T = 85°C$ appears).

2 kg/s of water is available at 95 °C for the boiler. The restrictive factor is the boiling temperature of 85° C. Therefore, break the process up from 2-3 into two parts as shown in the diagram.

CV Pump: $w_P = v_1(P_2-P_1) = 0.000873(2926.2 - 1017.0) = 1.67$ kJ/kg

$h_2 = h_1 + w_P = 256.54 + 1.67 = 258.21$ kJ/kg

Write the energy equation for the first section A-B and D-3:

$$-\dot{Q}_{AB} = \dot{m}_{H2O}(h_A - h_B) = 2(397.94 - 355.88) = 84.12 \text{ kW}$$

$$= \dot{m}_{R134A}(428.1 - 332.65) \Rightarrow \dot{m}_{R134A} = 0.8813 \text{ kg/s}$$

To be sure that the boiling temp. is the restrictive factor, calculate T_C from the energy equation for the remaining section:

$$-\dot{Q}_{AC} = 0.8813(332.65 - 258.21) = 65.60 \text{ kW} = 2(355.88 - h_C)$$

$$\Rightarrow h_C = 323.1 \text{ kJ/kg}, \ T_C = 77.2°C > T_2 = 40°C \ \textbf{OK}$$

CV: Turbine: $s_4 = s_3 = 1.6782 = 1.1909 + x_4 \times 0.5214 \Rightarrow x_4 = 0.9346$

$h_4 = 256.54 + 0.9346 \times 163.28 = 409.14$ kJ/kg

Energy Eq.: $w_T = h_3 - h_4 = 428.1 - 409.14 = 18.96$ kJ/kg

Cycle: $w_{NET} = w_T - w_P = 18.96 - 1.67 = 17.29$ kJ/kg

$$\dot{W}_{NET} = \dot{m}_{R134A}w_{NET} = 0.8813 \times 17.29 = \textbf{15.24 kW}$$

9.66

Do the previous problem with ammonia as the working fluid.

A flow with 2 kg/s of water is available at 95°C for the boiler. The restrictive factor is the boiling temperature of 85°C. Therefore, break the process up from 2-3 into two parts as shown in the diagram.

State 1: 40°C, 1554.9 kPa, $v_1 = 0.001725$ m^3/kg

CV Pump: $w_P = v_1(P_2 - P_1) = 0.001725(4608.6 - 1554.9) = 5.27$ kJ/kg

$h_2 = h_1 + w_P = 371.43 + 5.27 = 376.7$ kJ/kg

$-\dot{Q}_{AB} = \dot{m}_{H2O}(h_A - h_B) = 2(397.94 - 355.88) = 84.12$ kW

$= \dot{m}_{amn}(1447.8 - 609.21) \Rightarrow \dot{m}_{amn} = 0.100$ kg/s

To verify that $T_D = T_3$ is the restrictive factor, find T_C.

$-\dot{Q}_{AC} = 0.100(609.21 - 376.7) = 23.25 = 2.0(355.88 - h_C)$

$h_C = 344.25$ kJ/kg $\Rightarrow T_C = 82.5^o$C $> T_2 = 40^o$C **OK**

CV: Turbine

$s_4 = s_3 = 4.3901 = 1.3574 + x_4 \times 3.5088$ => $x_4 = 0.8643$

$h_4 = 371.43 + 0.8643 \times 1098.8 = 1321.13$ kJ/kg

Energy Eq.:

$w_T = h_3 - h_4 = 1447.8 - 1321.13 = 126.67$ kJ/kg

$w_{NET} = w_T - w_P = 126.67 - 5.27 = 121.4$ kJ/kg

$\dot{W}_{NET} = \dot{m}_{amn}w_{NET} = 0.1 \times 121.4 =$ **12.14 kW**

9.67

A **concentrated solar power** plant receives the energy from molten salt coming in at 560°C and leaving at 300°C in a counterflow heat exchanger where the water comes in at 3 MPa, 60°C and leaves at 450°C, 3 MPa. The molten salt has 5 kg/s flow with $C_p = 1.5$ kJ/kgK. What is the possible water flow rate, the rate of energy transfer and rate of entropy generation.

Continuity Eqs.:   Each line has a constant flow rate through it.

Energy Eq.4.10: $0 = \dot{m}_{H_2O} (h_1 - h_2) + \dot{m}_{salt} (h_3 - h_4)$

Entropy Eq.7.7: $0 = \dot{m}_{H_2O} (s_1 - s_2) + \dot{m}_{salt} (s_3 - s_4) + \dot{S}_{gen}$

Process: Each line has a constant pressure.

Table B.1: $h_1 = 253.61$ kJ/kg, $h_2 = 3344$ kJ/kg, $s_1 = 0.8295$, $s_2 = 7.0833$ kJ/kg-K

From the energy leaving the salt we get

$$\dot{Q} = \dot{m}_{salt} (h_3 - h_4) = \dot{m}_{salt} C_{P \, salt} (T_3 - T_4)$$

$$= 5 \times 1.5 [560 - 300] = \textbf{1950 kW}$$

$$\dot{m}_{H_2O} = \dot{m}_{salt} \frac{h_3 - h_4}{h_2 - h_1} = \frac{\dot{Q}}{h_2 - h_1} = \frac{1950 \text{ kW}}{(3344 - 253.61) \text{ kJ/kg}} = \textbf{0.631 kg/s}$$

$s_4 - s_3 = C_{P \, salt} \ln (T_4/T_3) = 1.5 \ln(573.15/833.15) = -0.561$ kJ/kgK

$$\dot{S}_{gen} = \dot{m}_{H_2O} (s_2 - s_1) + \dot{m}_{salt} (s_4 - s_3)$$

$$= 0.631 \text{ kg/s} (7.0833 - 0.8295) \text{ kJ/kgK} + 5 \text{ kg/s} \times (-0.561 \text{ kJ/kgK})$$

$$= \textbf{1.14 kW/K}$$

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for salt and water each. The two flows exchange energy with no heat transfer to/from the outside.

9.68

Look at the pinch point for the heat exchanger in Problem 9.67 the same way it is explained in Problem 9.65 and determine the maximum possible water flow rate for a pinch point $\Delta T = 10$ K.

C.V. Part of the heat exchanger from the salt inlet to state 3a for the salt and then from state 1a for the water to its outlet at state 2.

Pinch point: $T_{3a} = T_{1a} + 10 = 243.9°C$

Energy Eq.: $0 = \dot{m}_{H_2O} (h_{1a} - h_2) + \dot{m}_{salt} (h_3 - h_{3a})$

$$\dot{Q} = \dot{m}_{salt} (h_3 - h_{3a}) = \dot{m}_{salt} \, C_{P\,salt} (T_3 - T_{3a})$$
$$= 5 \times 1.5 \times (560 - 243.9) = 2370.75 \text{ kW}$$

$$\dot{m}_{H_2O} = \frac{\dot{Q}}{h_2 - h_1} = \frac{2370.75 \text{ kW}}{(3344 - 1008.41) \text{ kJ/kg}} = \textbf{1.015 kg/s}$$

Check also for the salt outlet condition, take the missing CV

Energy Eq.: $\dot{m}_{H_2O} (h_{1a} - h_1) = \dot{m}_{salt} \, C_{P\,salt} (T_{3a} - T_4)$

          $1.015 \, (1008.41 - 253.61) = 5 \times 1.5 \times (243.9 - T_4)$

Solve for T_4 : $T_4 = 243.9 - 102 = 141.7°C$

This temperature is too low, the salt would solidify so the pinch point ΔT should be much higher.

9.69

A steam power plant operates with a high pressure of 4 MPa and has a boiler exit of 600^oC receiving heat from a 750^oC source. The ambient at 20^oC provides cooling to maintain the condenser at 60^oC, all components are ideal except for the turbine which has an isentropic efficiency of 92%. Find the ideal and the actual turbine exit qualities. Find the actual specific work and specific heat transfer in all four components.

Solution:

A standard Rankine cycle with an actual non-ideal turbine.

Boiler exit: $h_3 = 3674.44$ kJ/kg, $s_3 = 7.3688$ kJ/kg K

Condenser exit: $h_1 = 251.11$ kJ/kg,

Ideal Turbine: 4s: 50°C, s = s_3 => $x_{4s} = (7.3688 – 0.8311)/7.0784 =$ **0.9236**,

$h_{4s} = 251.11 + 0.9236 \times 2358.48 = 2429.43$ kJ/kg

=> $w_{Ts} = h_3 - h_{4s} = 1245.01$ kJ/kg

Actual turbine: $w_{Tac} = \eta_{T\,s} w_{Ts} = 0.92 \times 1245.01 =$ **1145.4 kJ/kg** $= h_3 - h_{4ac}$

$h_{4ac} = h_3 – w_{Tac} = 3674.44 – 1145.4 = 2529.04$ kJ/kg

$x_{4ac} = (2529.04 – 251.11)/2358.48 =$ **0.96585**

Pump: $w_P = v_1(P_2 - P_1) = 0.001017(4000 – 19.94) =$ **4.05 kJ/kg**

$h_2 = h_1 + w_P = 251.11 + 4.05 = 255.16$ kJ/kg

$q_H = h_3 – h_2 = 3674.44 – 255.16 =$ **3419.3 kJ/kg**

$q_L = h_{4ac} – h_1 = 2529.04 – 251.11 =$ **2277.9 kJ/kg**

9.70

For the previous Problem find also the specific entropy generation in the boiler heat source setup.

CV. Boiler out to the source.

Entropy Eq.: $\qquad s_2 + \dfrac{q_H}{T_{source}} + s_{gen} = s_3$

State 1: $\qquad$ Sat. liquid $\quad h_1 = 251.11$ kJ/kg, $\quad s_1 = 0.8311$ kJ/kgK

Pump: $\qquad w_P = v_1(P_2 - P_1) = 0.001017(4000 - 19.94) = 4.05$ kJ/kg

State 2: $\qquad h_2 = h_1 + w_P = 251.11 + 4.05 = 255.16$ kJ/kg, $\quad s_2 = s_1$

State 3: $\qquad h_3 = 3674.44$ kJ/kg, $\quad s_3 = 7.3688$ kJ/kgK

Boiler: $\qquad q_H = h_3 - h_2 = 3674.44 - 255.16 = 3419.3$ kJ/kg

$$s_{gen} = s_3 - s_2 - \frac{q_H}{T_{source}} = 7.3688 - 0.8311 - \frac{3419.3}{750 + 273} = \textbf{3.195 kJ/kgK}$$

9.71

Consider the power plant in Problem 9.41. Assume the high temperature source is a flow of liquid water at 120^oC into a heat exchanger at constant pressure 300 kPa and that the water leaves at 90^oC. Assume the condenser rejects heat to the ambient which is at -20^oC. List all the places that have entropy generation and find the entropy generated in the boiler heat exchanger per kg ammonia flowing.

Solution:

a)   The hot water/ammonia boiler.

b)   The condensing ammonia (-15^oC) to the ambient (-20^oC) heat transfer.

c)   The "feedwater" heater has mixing of a flow at state 6 with a flow at state 2.

State 3: $x_3 = 0$, $h_3 = 171.65$ kJ/kg, $v_3 = 0.00156$ m^3/kg, $s_3 = 0.6793$ kJ/kgK

State 5: $h_5 = 1614.6$ kJ/kg, $s_5 = 5.4971$ kJ/kg K

C.V Pump P2   (rev. and adiabatic so $s_4 = s_3$).

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.00156(1000 - 400) = 0.936 \text{ kJ/kg}$$

$$\Rightarrow h_5 - h_4 = h_5 - h_3 - w_{P2} = 1614.6 - 171.65 - 0.936 = 1442 \text{ kJ/kg}$$

C.V. Boiler: Energy Eq.: $0 = \dot{m}_{amm} (h_5 - h_4) + \dot{m}_{H2O} (h_{in} - h_{ex})$

$$\dot{m}_{H2O} / \dot{m}_{amm} = \frac{h_5 - h_4}{h_{in} - h_{ex}} = \frac{1442}{503.69 - 376.9} = 11.373$$

Entropy Eq.: $0 = \dot{m}_{amm} (s_4 - s_5) + \dot{m}_{H2O} (s_{in} - s_{ex}) + \dot{S}_{gen}$

$$s_{amm} = (s_5 - s_4) - (\dot{m}_{H2O} / \dot{m}_{amm})(s_{in} - s_{ex})$$
$$= 5.4971 - 0.6793 - 11.373(1.5275 - 1.1924) = \textbf{1.007 kJ/kgK}$$

9.72

Repeat Problem 9.43 assuming the turbine has an isentropic efficiency of 85%.

The physical components and the T-s diagram is as shown in Fig. 9.12 in the main text for one open feedwater heater. The same state numbering is used. From the Steam Tables:

State 5:   (P, T) $h_5 = 4069.8$ kJ/kg, $s_5 = 7.0544$ kJ/kg K,

State 1:   (P, x = 0) $h_1 = 191.81$ kJ/kg, $v_1 = 0.00101$ m^3/kg

State 3:   (P, x = 0) $h_3 = 762.8$ kJ/kg, $v_3 = 0.001127$ m^3/kg

Pump P1: $w_{P1} = v_1(P_2 - P_1) = 0.00101 \times 990 = 1$ kJ/kg

$\qquad\qquad h_2 = h_1 + w_{P1} = 192.81$ kJ/kg

Turbine 5-6: $s_6 = s_5$ => $h_6 = 3013.7$ kJ/kg

$\qquad\qquad w_{T56,s} = h_5 - h_6 = 4069.8 - 3013.7 = 1056.1$ kJ/kg

$\qquad\qquad w_{T56,AC} = 1056.1 \times 0.85 = 897.69$ kJ/kg

$\qquad\qquad w_{T56,AC} = h_5 - h_{6AC}$ => $h_{6AC} = h_5 - w_{T56,AC}$

$\qquad\qquad\qquad = 4069.8 - 897.69 = 3172.11$ kJ/kg

Feedwater Heater $(\dot{m}_{TOT} = \dot{m}_5)$: $x\dot{m}_5 h_{6AC} + (1 - x)\dot{m}_5 h_2 = \dot{m}_5 h_3$

$$\Rightarrow \quad x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.8 - 192.81}{3172.11 - 192.81} = \textbf{0.1913}$$

To get the turbine work apply the efficiency to the whole turbine. (i.e. the first section should be slightly different).

$\qquad\quad s_{7s} = s_{6s} = s_5$ => $x_{7s} = (7.0544 - 0.6493)/7.5009 = 0.85391,$

$\qquad\quad h_{7s} = 191.81 + 0.85391 \times 2392.82 = 2235.1$ kJ/kg

$\qquad\quad w_{T57,s} = h_5 - h_{7s} = 4069.8 - 2235.1 = 1834.7$ kJ/kg

$\qquad\quad w_{T57,AC} = w_{T57,s} \, \eta_T = 1559.5 = h_5 - h_{7AC}$ => $h_{7AC} = 2510.3$ kJ/kg

Find specific turbine work to get total flow rate

$$\dot{m}_{TOT} = \frac{\dot{W}_T}{x w_{T56} + (1-x)w_{T57}} = \frac{5000}{0.1913 \times 897.69 + 0.8087 \times 1559.5} = \textbf{3.489 kg/s}$$

$$\dot{Q}_L = \dot{m}_{TOT}(1 - x)(h_7 - h_1) = 3.489 \times 0.8087(2510.3 - 191.81) = \textbf{6542 kW}$$

9.73

Steam leaves a power plant steam generator at 3.5 MPa, 400°C (h_1 = 3222.3 kJ/kg, s_1 = 6.8405 kJ/kg K), and enters the turbine at 3.4 MPa, 375°C (h_2 = 3165.7 kJ/kg, s_2 = 6.7675 kJ/kg K). The isentropic turbine efficiency is 88%, and the turbine exhaust pressure is 10 kPa. Condensate leaves the condenser and enters the pump at 35°C, 10 kPa. The isentropic pump efficiency is 80%, and the discharge pressure is 3.7 MPa. The feedwater enters the steam generator at 3.6 MPa, 30°C (h = 129.0 kJ/kg). Calculate the thermal efficiency of the cycle and the entropy generation for the process in the line between the steam generator exit and the turbine inlet, assuming an ambient temperature of 25°C.

1: h_1 = 3222.3 kJ/kg, s_1 = 6.8405 kJ/kg K,

2: h_2 = 3165.7 kJ/kg, s_2 = 6.7675 kJ/kg K

3s: $s_{3S} = s_2 \implies x_{3S} = 0.8157$, h_{3S} = 2143.6 kJ/kg

$\quad\quad w_{T,s} = h_2 - h_{3s} = 3165.7 - 2143.6 = 1022.1$ kJ/kg

$\quad\quad w_{T,ac} = \eta_{T,s}\, w_{T,s} = 899.4$ kJ/kg,   3ac: $h_3 = h_2 - w_{T,ac} = 2266.3$ kJ/kg

$\quad\quad w_{P,s} = v_f(P_5 - P_4) = 0.001006(3700 - 10) = 3.7$ kJ/kg

$\quad\quad w_{P,ac} = w_{P,s}/\eta_{P,s} = 4.6$ kJ/kg

$\quad\quad q_H = h_1 - h_6 = 3222.3 - 129.0 = 3093.3$ kJ/kg

$\quad\quad \eta = w_{NET}/q_H = (899.4 - 4.6)/3093.3 = \mathbf{0.289}$

C.V. Line from 1 to 2:   w = 0,

$\quad\quad$ Energy Eq.: $q = h_2 - h_1 = 3165.7 - 3222.3 = -56.6$ kJ/kg

$\quad\quad$ Entropy Eq.: $s_1 + s_{gen} + q/T_0 = s_2 \implies$

$\quad\quad\quad s_{gen} = s_2 - s_1 - q/T_0 = 6.7675 - 6.8405 - (-56.6/298.15) = \mathbf{0.117\ kJ/kg\ K}$

9.74

Find the entropy generation per unit mass leaving the open FWH in Problem 9.40

The setup follows Fig.9.12

States : $h_2 = 419.02$, $h_6 = 2827.86$, $h_3 = 762.79$ all kJ/kg

$s_2 = 1.3068$, $s_6 = 6.6939$, $s_3 = 2.1386$ (x = 0 at 1 MPa) all kJ/kg-K

Analyze the FWH which leads to Eq.9.5:

$$y = \dot{m}_6 / \dot{m}_5 \quad \text{the extraction fraction}$$

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.79 - 419.02}{2827.86 - 419.02} = 0.1427$$

Entropy eq. scaled to flow at 3: $0 = y\, s_6 + (1 - y)\, s_2 - s_3 + s_{gen}$

$$s_{gen} = s_3 - y\, s_6 - (1 - y)\, s_2$$
$$= 2.1386 - 0.1427 \times 6.6939 - (1 - 0.1427) \times 1.3068$$
$$= \mathbf{0.063\ kJ/kgK}$$

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9.75

Find the rate of entropy generation in the closed FWH in Problem 9.51

The schematic from Figure 9.13 has the feedwater from the pump coming at state 2 being heated by the extraction flow coming from the turbine state 6 so the feedwater leaves as saturated liquid state 4 and the extraction flow leaves as condensate state 6a.

From table B.1	h kJ/kg	s kJ/kg-K
B.1.4: 100°C, 20 MPa	$h_2 = 434.04$	1.2917
B.1.4: 200°C, 20 MPa	$h_4 = 860.47$	2.3031
B.1.3: 4 MPa, 275°C	$h_6 = 2880.39$	6.2145
B.1.2: 4 MPa, sat. liq.	$h_{6a} = 1087.29$	2.7963

C.V. Feedwater Heater

Energy Eq.: $\dot{m}_2 h_2 + \dot{m}_6 h_6 = \dot{m}_2 h_4 + \dot{m}_6 h_{6a}$

Entropy Eq.: $0 = \dot{m}_2 s_2 + \dot{m}_6 s_6 - \dot{m}_2 s_4 - \dot{m}_6 s_{6a} + \dot{S}_{gen}$

Since all four states are known we can solve for the extraction flow rate

$$\dot{m}_6 = \dot{m}_2 \frac{h_2 - h_4}{h_{6a} - h_6} = 20 \frac{434.04 - 860.47}{1087.29 - 2880.39} \text{ kg/s} = 4.756 \text{ kg/s}$$

$$\dot{S}_{gen} = \dot{m}_2(s_4 - s_2) + \dot{m}_6 (s_{6a} - s_6)$$
$$= 20 (2.3031 - 1.2917) + 4.756 (2.7963 - 6.2145)$$
$$= 20.228 - 16.257 = \textbf{3.971 kW/K}$$

Cogeneration

9.76

A cogenerating steam power plant, as in Fig. 9.19, operates with a boiler output of 25 kg/s steam at 7 MPa, 500°C. The condenser operates at 7.5 kPa and the process heat is extracted as 5 kg/s from the turbine at 500 kPa, state 6 and after use is returned as saturated liquid at 100 kPa, state 8. Assume all components are ideal and find the temperature after pump 1, the total turbine output and the total process heat transfer.

Solution:

Pump 1:   Inlet state is saturated liquid: $h_1 = 168.79$ kJ/kg, $v_1 = 0.001008$ m³/kg

$$w_{P1} = \int v \, dP = v_1 \, (P_2 - P_1) = 0.001008(100 - 7.5) = 0.093 \text{ kJ/kg}$$

$$w_{P1} = h_2 - h_1 \Rightarrow h_2 = h_1 + w_{P1} = 168.88 \text{ kJ/kg}, \quad T_2 = \textbf{40.3°C}$$

Turbine: $h_5 = 3410.3$ kJ/kg, $s_5 = 6.7974$ kJ/kg K

$P_6, s_6 = s_5 \Rightarrow x_6 = 0.9952, \quad h_6 = 2738.6$ kJ/kg

$P_7, s_7 = s_5 \Rightarrow x_7 = 0.8106, \quad h_7 = 2119.0$ kJ/kg

From the continuity equation we have the full flow from 5 to 6 and the remainder after the extraction flow is taken out flows from 6 to 7.

$$\dot{W}_T = \dot{m}_5 \, (h_5 - h_6) + 0.80\dot{m}_5 \, (h_6 - h_7) = 25 \, (3410.3 - 2738.6)$$

$$+ 20 \, (2738.6 - 2119) = 16\ 792.5 + 12\ 392 = \textbf{29.185 MW}$$

$$\dot{Q}_{proc} = \dot{m}_6(h_6 - h_8) = 5(2738.6 - 417.46) = \textbf{11.606 MW}$$

9.77

A steam power plant has 4 MPa, 500°C into the turbine and to have the condenser itself deliver the process heat it is run at 101 kPa. How much net power as work is produced for a process heat of 13 MW.

Solution:

From the Rankine cycle we have the states:

1: 101 kPa, $x = 0$, $v_1 = 0.001043$ m^3/kg, $h_1 = 418.6$ kJ/kg

3: 4 MPa, 500°C, $h_3 = 3445.2$ kJ/kg, $s_3 = 7.090$ kJ/kg K

C.V. Turbine: $s_4 = s_3$ $\Rightarrow$ $x_4 = (7.090 - 1.3068)/6.048 = 0.9562$,

$$h_4 = 419.02 + 0.9562 \times 2257.03 = 2577.2 \text{ kJ/kg}$$

$$w_T = h_3 - h_4 = 3445.2 - 2577.2 = 868 \text{ kJ/kg}$$

C.V. Pump: $w_P = v_1(P_2 - P_1) = 0.001043(4000 - 101) = 4.07$ kJ/kg

$$w_P = h_2 - h_1 \Rightarrow h_2 = 419.02 + 4.07 = 423.09 \text{ kJ/kg}$$

C.V. Condenser: $q_{L,out} = h_4 - h_1 = 2577.2 - 419.02 = 2158.2$ kJ/kg

$$\dot{m} = \dot{Q}_{proc}/q_{L,out} = 13\ 000 \text{ kW} / 2158.2 \text{ kJ/kg} = 6.023 \text{ kg/s}$$

$$\dot{W}_T = \dot{m}\ (w_T - w_P) = 6.023\ (868 - 4.07) = \textbf{5203 kW}$$

9.78

Steam must be generated at 150°C for process heat in a food production facility. This is done in a combined heat and power system as extraction steam from the turbine. Assume the standard cycle has a turbine inlet of 3 MPa, 450°C and 60°C in the condenser. What pressure should be used for the extraction so there is a maximum of process heat at 150°C available and the least amount of turbine work lost?

Most energy will be available if the extraction flow can condense at 150°C. Since the flow through the turbine follows from 5 to 6 to 7 the extraction point is selected as the saturation pressure for the 150°C.

$$P_6 = P_{sat\ 150C} = \textbf{476 kPa}$$

As $s_5 = 7.0833 > s_{g\ 150C} = 6.8378$ kJ/kg-K, state 6 is superheated vapor.

9.79

A 15 kg/s steady supply of saturated-vapor steam at 500 kPa is required for drying a wood pulp slurry in a paper mill. It is decided to supply this steam by cogeneration, that is, the steam supply will be the exhaust from a steam turbine. Water at 20°C, 100 kPa, is pumped to a pressure of 5 MPa and then fed to a steam generator with an exit at 400°C. What is the additional heat transfer rate to the steam generator beyond what would have been required to produce only the desired steam supply? What is the difference in net power?

Solution:

Desired exit State 4: P_4 = 500 kPa, sat. vap. => x_4 = 1.0, T_4 = 151.9°C

$$h_4 = h_g = 2748.7 \text{ kJ/kg}, \quad s_4 = s_g = 6.8212 \text{ kJ/kg-K}$$

Inlet State: 20°C, 100 kPa $h_1 = h_f$ = 83.94 kJ/kg, $v_1 = v_f$ = 0.001002 m^3/kg

Without Cogeneration; The water is pumped up to 500 kPa and then heated in the steam generator to the desired exit T.

C.V. Pump: $w_{Pw/o} = v_1(P_4 - P_1) = 0.4$ kJ/kg

$$h_2 = h_1 + w_{Pw/o} = 84.3 \text{ kJ/kg}$$

C.V. Steam Generator: $q_{w/o} = h_4 - h_2 = 2664.4$ kJ/kg

With Cogeneration; The water is pumped to 5 MPa, heated in the steam generator to 400°C and then flows through the turbine with desired exit state.

C.V. Pump: $w_{Pw} = \int vdP = v_1(P_2 - P_1) = 4.91$ kJ/kg

$$h_2 = h_1 + w_{Pw} = 88.85 \text{ kJ/kg}$$

C.V. Steam Generator: Exit 400°C, 5 MPa   => h_3 = 3195.64 kJ/kg

$$q_w = h_3 - h_2 = 3195.64 - 88.85 = 3106.8 \text{ kJ/kg}$$

C.V.: Turbine, Inlet and exit states given

$$w_t = h_3 - h_4 = 3195.64 - 2748.7 = 446.94 \text{ kJ/kg}$$

Comparison

Additional Heat Transfer: $q_w - q_{w/o}$ = 3106.8 - 2664.4 = 442.4 kJ/kg

$$\dot{Q}_{extra} = \dot{m}(q_w - q_{w/o}) = \textbf{6636 kW}$$

Difference in Net Power: $w_{diff} = (w_t - w_{Pw}) + w_{Pw/o}$,

$$w_{diff} = 446.94 - 4.91 + 0.4 = 442.4 \text{ kJ/kg}$$

$$\dot{W}_{diff} = \dot{m}w_{diff} = \textbf{6636 kW}$$

By adding the extra heat transfer at the higher pressure and a turbine all the extra heat transfer can come out as work (it appears as a 100% efficiency)

9.80

In a cogenerating steam power plant the turbine receives steam from a high-pressure steam drum (h = 3445.9 kJ/kg, s = 6.9108 kJ/kgK) and a low-pressure steam drum (h = 2855.4 kJ/kg, s = 7.0592 kJ/kgK) as shown in Fig. P9.80. For the turbine calculation assume a mass weighted average entropy and neglect entropy generation by mixing. The condenser is made as two closed heat exchangers used to heat water running in a separate loop for district heating. The high-temperature heater adds 30 MW and the low-temperature heater adds 31 MW to the district heating water flow. Find the power cogenerated by the turbine and the temperature in the return line to the deaerator.

Solution:

Inlet states from Table B.1.3

$h_1 = 3445.9$ kJ/kg, $s_1 = 6.9108$ kJ/kg K

$h_2 = 2855.4$ kJ/kg, $s_2 = 7.0592$ kJ/kg K

$\dot{m}_{TOT} = \dot{m}_1 + \dot{m}_2 = 27$ kg/s

Assume a reversible turbine and the

two flows can mix without s generation.

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{m}_4 h_4 + \dot{W}_T$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 = \dot{m}_{TOT} s_{mix}$ $\Rightarrow$ $s_{MIX} = 6.9383$ kJ/kg K

State 3: $s_3 = s_{MIX}$ $\Rightarrow$ $h_3 = 2632.4$ kJ/kg, $x_3 = 0.966$

State 4: $s_4 = s_{MIX}$ $\Rightarrow$ $h_4 = 2413.5$ kJ/kg, $x_4 = 0.899$

$$\dot{W}_T = 22 \times 3445.9 + 5 \times 2855.4 - 13 \times 2632.4 - 14 \times 2413.5$$

$$= 22\ 077 \text{ kW} = \textbf{22 MW}$$

District heating line $\dot{Q}_{TOT} = \dot{m}(h_{95} - h_{60}) = 60\ 935$ kW

OK, this matches close enough

C.V. Both heaters: $\dot{m}_3 h_3 + \dot{m}_4 h_4 - \dot{Q}_{TOT} = \dot{m}_{TOT} h_{EX}$

$13 \times 2632.4 - 14 \times 2413.5 - 60\ 935 = 7075.2 = 27 \times h_{EX}$

$h_{EX} = 262 \approx h_f$ $\Rightarrow$ $T_{EX} = \textbf{62.5°C}$

Remark: We could have computed the expansion from state 1 to P_2 followed by a mixing process to find a proper state 2a from which we expand down to P_3 and P_4

9.81

A boiler delivers steam at 10 MPa, 550°C to a two-stage turbine as shown in Fig. 9.19. After the first stage, 25% of the steam is extracted at 1.4 MPa for a process application and returned at 1 MPa, 90°C to the feedwater line. The remainder of the steam continues through the low-pressure turbine stage, which exhausts to the condenser at 10 kPa. One pump brings the feedwater to 1 MPa and a second pump brings it to 10 MPa. Assume all components are ideal. If the process application requires 5 MW of power, how much power can then be cogenerated by the turbine?

Solution:

5: $h_5 = 3500.9$, $s_5 = 6.7561$ kJ/kg K

First ideal turbine T1

6: $s_6 = s_5 \Rightarrow h_6 = 2932.4$ kJ/kg

$w_{T1} = h_5 - h_6 = 568.5$ kJ/kg

Ideal turbine T2

State 7: $s_7 = s_6 = s_5$

$x = \dfrac{6.7561 - 0.6492}{7.501} = 0.8141$

$\Rightarrow \quad h_7 = 2139.9$ kJ/kg

$w_{T2} = h_6 - h_7 = 2932.4 - 2139.9 = 792.5$ kJ/kg

Now do the process heat requirement

8: $h_8 = 377.3$ kJ/kg, approx. from the compressed liq. Table at 500 kPa

$q_{PROC} = h_6 - h_8 = 2932.4 - 377.3 = 2555.1$ kJ/kg

$\dot{m}_6 = \dot{Q}/q_{PROC} = 5000 / 2555.1 = 1.9569$ kg/s $= 0.25\ \dot{m}_{TOT}$

$\Rightarrow \quad \dot{m}_{TOT} = \dot{m}_5 = 7.8275$ kg/s, $\dot{m}_7 = \dot{m}_5 - \dot{m}_6 = 5.8706$ kg/s

$\dot{W}_T = \dot{m}_5 h_5 - \dot{m}_6 h_6 - \dot{m}_7 h_7$

$= 7.8275 \times 3500.9 - 1.9569 \times 2932.4 - 5.8706 \times 2139.9$

$= \textbf{9102 kW}$

9.82

A smaller power plant produces 25 kg/s steam at 3 MPa, 600°C, in the boiler. It cools the condenser to an exit of 45°C and the cycle is shown in Fig. P9.82. An extraction is done at 500 kPa to an open feedwater heater, in addition a steam supply of 5 kg/s is taken out and not returned. The missing 5 kg/s water is added to the feedwater heater from a 20°C, 500 kPa source. Find the needed extraction flow rate to cover both the feedwater heater and the steam supply. Find the total turbine power output.

Solution:

The states properties from Tables B.1.1 and B.1.3

$1: 45^oC, \ x = 0: \ h_1 = 188.42 \ kJ/kg, \ v_1 = 0.00101 \ m^3/kg, \ P_{sat} = 9.59 \ kPa$

$5: 3.0 \ MPa, \ 600^oC: \quad h_5 = 3682.34 \ kJ/kg, \quad s_5 = 7.5084 \ kJ/kg \ K$

$3: 500 \ kPa, \ x = 0: \quad h_3 = 640.21 \ kJ/kg \qquad 8: \ h_8 = 84.41 \ kJ/kg$

$6: 500 \ kPa, \ s_6 = s_5 \ \text{from HP turbine}, \ h_6 = 3093.26 \ kJ/kg$

C.V. Pump 1. Reversible and adiabatic. Incompressible so v = constant

Energy: $\quad w_{p1} = h_2 - h_1 = \int v \ dP = v_1(P_2 - P_1)$

$$= 0.00101 \ (500 - 9.6) = 0.495 \ kJ/kg$$

$$h_2 = h_1 + w_{p1} = 188.42 + 0.495 = 188.915 \ kJ/kg$$

C.V. Turbine sections

Entropy Eq.: $\quad s_7 = s_5 = 7.5084 \ kJ/kg \ K \ => \quad$ two-phase state

$$s_7 = 7.5084 = 0.6386 + x_7 \times 7.5261 \ \Rightarrow \quad x_7 = 0.9128$$

$$h_7 = 188.42 + 0.9128 \times 2394.77 = 2374.4 \ kJ/kg$$

C.V. Feedwater heater, including the make-up water flow, $y = \dot{m}_6/\dot{m}_5$.

Energy eq.: $\quad \dot{m}_8 h_8 + (\dot{m}_5 - \dot{m}_6)h_2 + (\dot{m}_6 - \dot{m}_8)h_6 = \dot{m}_5 h_3$

Divide by $\dot{m}_5$ and solve for y

$$y = \frac{h_3 - h_2 + (h_6 - h_8) \ \dot{m}_8/ \dot{m}_5}{h_6 - h_2} = \frac{640.21 - 188.915 + (3093.26 - 84.41)5/25}{3093.26 - 188.915}$$

$$= 0.3626$$

$$\dot{m}_6 = y \ \dot{m}_5 = 0.3626 \times 25 = \textbf{9.065 kg/s}$$

C.V. Turbine energy equation

$$\dot{W}_T = \dot{m}_5 h_5 - \dot{m}_6 h_6 - \dot{m}_7 h_7$$
$$= 25 \times 3682.34 - 9.065 \times 3093.26 - 16.935 \times 2374.4$$
$$= \textbf{26 182 kW}$$

Refrigeration cycles

9.83

A refrigeration cycle as in Fig. 9.23 can be used for cooling or for heating purposes using one of the two heat exchangers. Suppose a refrigerator should cool meat at -10°C in a 30°C hot kitchen what are the minimum high and maximum low pressures in the cycle if the working substance is a) R-134a or b) R-410A.

Ideal refrigeration cycle
$T_{cond} = 30°C$
$T_{evap} = -10°C = T_1$

Properties from Tbl. B.4 + B.5

The condensing temperature, T_3, must be minimum 30°C to move a heat transfer to the kitchen air. The low temperature in the evaporator must at most -10°C to be able to extract energy from the cold space.

R-134a, B.5:
$$P_{hi} \geq P_3 = P_{sat}(30°C) = 771 \text{ kPa}$$
$$P_{low} \leq P_1 = P_{sat}(-10°C) = 201.7 \text{ kPa}$$

R-410A, B.4:
$$P_{hi} \geq P_3 = P_{sat}(30°C) = 1885 \text{ kPa}$$
$$P_{low} \leq P_1 = P_{sat}(-10°C) = 573 \text{ kPa}$$

9.84

A refrigerator with R-134a as the working fluid has a minimum temperature of $-10°C$ and a maximum pressure of 1 MPa. Assume an ideal refrigeration cycle as in Fig. 9.23. Find the specific heat transfer from the cold space and that to the hot space, and the coefficient of performance.

Solution:

Exit evaporator sat. vapor $-10°C$ from B.5.1: $h_1 = 392.28$, $s_1 = 1.7319$ kJ/kgK

Exit condenser sat. liquid 1 MPa from B.5.1: $h_3 = 255.60$ kJ/kg

Compressor: $s_2 = s_1$ & P_2 from B.5.2 $\Rightarrow$ $h_2 \approx 425.68$ kJ/kg

Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 392.28 - 255.60 =$ **136.7 kJ/kg**

Condenser: $q_H = h_2 - h_3 = 425.68 - 255.60 =$ **170.1 kJ/kg**

COP: $\beta = q_L/w_c = q_L/(q_H - q_L) =$ **4.09**

Ideal refrigeration cycle
$P_{cond} = P_3 = P_2 = 1$ MPa
$T_{evap} = -10^oC = T_1$

Properties from Table B.5

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9.85

Repeat the previous Problem with R-410A as the working fluid. Will that work in an ordinary kitchen?

Solution:

Exit evaporator sat. vapor $-10°C$ from B.4.1: $h_1 = 275.78$, $s_1 = 1.0567$ kJ/kgK

Exit condenser sat. liquid 1 MPa from B.4.1: $h_3 = 68.92$ kJ/kg

Compressor: $s_2 = s_1$ & P_2 from B.4.2 $\Rightarrow$ $h_2 \approx 290.81$ kJ/kg

Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 275.78 - 68.92 =$ **206.9 kJ/kg**

Condenser: $q_H = h_2 - h_3 = 290.81 - 68.92 =$ **221.9 kJ/kg**

COP: $\beta = q_L/w_c = q_L/(q_H - q_L) =$ **13.8**

Ideal refrigeration cycle
$P_{cond} = P_3 = P_2 = 1$ MPa
$T_{evap} = -10^oC = T_1$

Properties from Table B.4

The 1 MPa is too small, the condensing temperature is 7.25°C and the q_H in the condenser can not be rejected to a kitchen normally at 20°C.

9.86

The natural refrigerant carbon dioxide has a fairly low critical temperature. Find the high temperature, the condensing temperature and the COP if it is used in a standard cycle with high and low pressures of 6 MPa and 3 MPa.

Exit evaporator x = 1 and 3 MPa from B.3.2:

$$h_1 = 320.71 \text{ kJ/kg}, \quad s_1 = 1.2098 \text{ kJ/kgK}$$

Exit condenser saturated liquid 6 MPa from B.3.1:

$$T_3 = \textbf{22°C}, \ h_3 = 150 \text{ kJ/kg}$$

Exit compressor: 6 MPa, s = s_1, so interpolate in B.3.2

$$T_2 = \textbf{45.9°C}, \quad h_2 = 348.24 \text{ kJ/kg}$$

COP: $$\beta = \frac{q_L}{w_c} = \frac{h_1 - h_3}{h_2 - h_1} = \textbf{6.2}$$

Remark: The condensing T is too low for a standard refrigerator it can not push the heat transfer to the room which may be hotter than that.

9.87

Consider an ideal refrigeration cycle that has a condenser temperature of 45°C and an evaporator temperature of −15°C. Determine the coefficient of performance of this refrigerator for the working fluids R-134a and R-410A.

Solution:

Ideal refrigeration cycle
$T_{cond} = 45^oC = T_3$
$T_{evap} = -15^oC = T_1$

Property for:	R-134a, B.5	R-410A, B.4
h_1, kJ/kg	389.2	273.9
$s_2 = s_1$, kJ/kg K	1.7354	1.0671
P_2, MPa	1.16	2.7283
T_2, ^oC	51.8*	71.5**
h_2, kJ/kg	429.9*	322.72**
$w_C = h_2 - h_1$	40.7	48.82
$h_3 = h_4$, kJ/kg	264.11	133.61
$q_L = h_1 - h_4$	125.1	140.29
$\beta = q_L/w_C$	**3.07**	**2.87**

Compressor — h_1, $s_2=s_1$, P_2, T_2, h_2, $w_C = h_2 - h_1$
Exp. valve — $h_3 = h_4$
Evaporator — $q_L = h_1 - h_4$

* For state 2 an interpolation between 1 and 1.2 MPa is needed for 1.16 MPa:

At 1 MPa, s = 1.7354 :   T = 45.9 °C and h = 426.8 kJ/kg

At 1.2 MPa, s = 1.7354 : T = 53.3 °C and h = 430.7 kJ/kg

** For state 2 an interpolation between 2 and 3 MPa is needed for 2.728 MPa:

At 2 MPa, s = 1.0671 : T = 54.68 °C and h = 313.942 kJ/kg

At 3 MPa, s = 1.0671 : T = 77.80 °C and h = 326.0 kJ/kg

It would make more sense to use the CATT3 program.

9.88

Do problem 9.87 with ammonia as the working fluid.

Solution:

Ideal refrigeration cycle

$T_{cond} = 45^oC = T_3$

$T_{evap} = -15^oC = T_1$

State 1: $h_1 = 1424.6$ kJ/kg, $s_1 = 5.5397$ kJ/kg-K

State 2: $s = s_1$, $P_2 = 1782$ kPa, $T_2 = 135.1^oC$, $h_2 = 1731.3$ kJ/kg

For state 2 an interpolation between 1.6 and 2 MPa is needed for 1.782 MPa:

At 1.6 MPa, s = 5.5397 :   T = 126.0 °C   and   h = 1712.2 kJ/kg

At 2.0 MPa, s = 5.5397 : T = 146.1°C   and   h = 1754.1 kJ/kg

It would make more sense to use the CATT3 program.

$$w_C = h_2 - h_1 = 1731.3 – 1424.6 = 306.7 \text{ kJ/kg}$$

State 3-4: $h_4 = h_3 = 396.3$ kJ/kg $q_L = h_1 - h_4 = 1028.3$ kJ/kg

$$\beta = q_L/w_C = \textbf{3.353}$$

9.89

A refrigerator receives 500 W of electrical power to the compressor driving the cycle flow of R-134a. The refrigerator operates with a condensing temperature of 40^oC and a low temperature of -5^oC. Find the COP for the cycle and its cooling capacity.

Solution:

Ideal refrigeration cycle
$T_{cond} = 40^oC = T_3$
$T_{evap} = -5^oC = T_1$

Properties from Table B.5

Exit evaporator sat. vapor $-5°C$ from B.5.1: $h_1 = 395.3$ kJ/kg, $s_1 = 1.7288$ kJ/kgK

Compressor: $s_2 = s_1$ & P_2 from B.5.2 $\Rightarrow$ $T_2 = 45^oC$, $h_2 \approx 425$ kJ/kg

$\qquad\qquad w_C = h_2 - h_1 = 425 - 395.3 = 29.7$ kJ/kg

Exit condenser sat. liquid $P_2 = P_3 = 1017$ kPa from B.5.1: $h_3 = 256.50$ kJ/kg

Evaporator: $\quad q_L = h_1 - h_4 = h_1 - h_3 = 395.3 - 256.50 = 138.8$ kJ/kg

Condenser: $\quad q_H = h_2 - h_3 = 425 - 256.5 = 168.5$ kJ/kg

COP: $\qquad\quad \beta = q_L/w_c = q_L/(q_H - q_L) = 138.8 / 29.7 = \textbf{4.67}$

$$\dot{Q}_L = \beta \dot{W} = 4.67 \times 500 \text{ W} = 2335 \text{ W} = \textbf{2.3 kW}$$

9.90

     A new air-conditioner using R-410A is used in heat pump mode. The high pressure is 2000 kPa and the low pressure is 400 kPa. It warms a house at 20°C driven by an electric power input of 2 kW in an ambient at -5°C. Find the COP, the heating rate and the rate of entropy generation for the heat pump.

Exit evaporator sat. vapor 400 kPa from B.4.2: $h_1 = 271.9$, $s_1 = 1.0779$ kJ/kgK

Exit condenser sat. liquid 2 MPa (32.31°C) from B.4.1: $h_3 = 110.21$ kJ/kg

Compressor: $s_2 = s_1$ & P_2 from B.4.2 $\Rightarrow$ $h_2 \approx 317.43$ kJ/kg

Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 271.9 - 110.21 = 161.69$ kJ/kg

Condenser: $q_H = h_2 - h_3 = 317.43 - 110.21 = 207.22$ kJ/kg

Compressor: $w_c = h_2 - h_1 = 317.43 - 271.9 = 45.53$ kJ/kg

COP: $\beta = q_H/w_c = 207.22/45.53 =$ **4.55**

Heating rate: $\dot{Q}_H = \beta \dot{W} = 4.55 \times 2 =$ **9.1 kW**

CV: The A/C unit including the two heat exchangers.

Energy: $0 = q_L - q_H + w_c$

Entropy: $0 = q_L/T_{amb} - q_H/T_{house} + s_{gen}$

$$\dot{S}_{gen} = \dot{Q}_H/T_{house} - \dot{Q}_L/T_{amb} = \frac{9.1 \text{ kW}}{293.15 \text{ K}} - \frac{(9.1 - 2) \text{ kW}}{268.15 \text{ K}} = \textbf{0.00456 kW/K}$$

Ideal refrigeration cycle
$P_{cond} = P_3 = P_2 = 2$ MPa
$T_{evap} = -19.98$°C $= T_1$

Properties from Table B.4

Comment: The entropy generation does not include the heat loss from the house to the ambient (this is why we heat a house) if that was included then q_L comes in at T_{amb} and q_H leaves at T_{amb} so the total entropy generation becomes w/T_{amb}.

9.91

Consider the heat pump in the previous problem. Repeat the questions for a system with an actual compressor with an exit state of 2000 kPa, 65°C.

Exit evaporator sat. vapor 400 kPa from B.4.2: $h_1 = 271.9$, $s_1 = 1.0779$ kJ/kgK

Ideal Compressor: $s_2 = s_1$ & P_2 from B.4.2 $\Rightarrow$ $h_2 \approx 317.43$ kJ/kg

$$w_c = h_2 - h_1 = 317.43 - 271.9 = 45.53 \text{ kJ/kg}$$

Actual compressor exit: $h_{2\ ac} = 326.27$ kJ/kg, $s_2 = 1.1043$ kJ/kg-K

Actual Compressor: $w_{c\ ac} = h_{2\ ac} - h_1 = 326.27 - 271.9 = 54.37$ kJ/kg

Exit condenser sat. liquid 2 MPa (32.31^{o}C) from B.4.1: $h_3 = 110.21$ kJ/kg

Condenser: $q_H = h_{2\ ac} - h_3 = 326.27 - 110.21 = 216.06$ kJ/kg

Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 271.9 - 110.21 = 161.69$ kJ/kg

COP: $\beta = q_H/w_{c\ ac} = 216.06/54.37 =$ **3.974**

Heating rate: $\dot{Q} = \beta \dot{W} = 3.974 \times 2 \text{ kW} =$ **7.95 kW**

CV: The A/C unit including the two heat exchangers.

Energy: $0 = q_L - q_H + w_c$

Entropy: $0 = q_L/T_{amb} - q_H/T_{house} + s_{gen}$

$$\dot{S}_{gen} = \dot{Q}_H/T_{house} - \dot{Q}_L/T_{amb} = \frac{7.95 \text{ kW}}{293.15 \text{ K}} - \frac{(7.95 - 2) \text{ kW}}{268.15 \text{ K}} = \textbf{0.00493 kW/K}$$

Actual refrigeration cycle
$P_{cond} = P_3 = P_2 = 2$ MPa
$T_{evap} = -19.98^o$C $= T_1$

Properties from Table B.4

Comment: The entropy generation does not include the heat loss from the house to the ambient (this is why we heat a house) if that was included then q_L comes in at T_{amb} and q_H leaves at T_{amb} so the total entropy generation becomes w/T_{amb}.

9.92

A heat pump for heat upgrade uses ammonia with a low temperature of 25^oC and a high pressure of 5000 kPa. If it receives 1 MW of shaft work what is the rate of heat transfer at the high temperature?

State 1: Table B.2.1 $h_1 = 1463.5$ kJ/kg, $s_1 = 5.0293$ kJ/kgK

State 3: Table B.2.1 $h_3 = h_f = 631.9$ kJ/kg

Entropy compressor: $s_2 = s_1$ => $T_2 = 156^oC$, $h_2 = 1709.1$ kJ/kg

Energy eq. compressor: $w_C = h_2 - h_1 = 245.6$ kJ/kg

Energy condenser: $q_H = h_2 - h_3 = 1077.2$ kJ/kg

Scaling to power input: $\dot{Q}_H = q_H \dfrac{\dot{W}_{IN}}{w_C}$

$$= 1077.2 \text{ kJ/kg} \times \frac{1000}{245.6} \text{ kg/s}$$

$$= \mathbf{4386 \ kW}$$

9.93

Reconsider the heat pump in the previous problem. Assume the compressor is split into two, first compress to 2000 kPa, then take heat transfer out at constant P to reach saturated vapor then compress to the 5000 kPa. Find the two rates of heat transfer, at 2000 kPa and at 5000 kPa for a total of 1 MW shaft work input.

Ideal heat pump

$T_{evap} = 25^o C = T_1$

State 1: Table B.2.1 $h_1 = 1463.5$ kJ/kg, $s_1 = 5.0293$ kJ/kgK

State 3: Table B.2.1 $h_3 = h_f = 631.9$ kJ/kg

Entropy compressor 1: $s_{2a} = s_1$ => $T_{2a} = 75.3^o C$, $h_{2a} = 1559.1$ kJ/kg

Energy eq. compressor 1: $w_{C1} = h_{2a} - h_1 = 1559.1 - 1463.5 = 95.6$ kJ/kg

Exit heat exchanger 1: $h_{2b} = 1471.5$ kJ/kg, $s_{2b} = 4.768$ kJ/kgK

Entropy compressor 2: $s_{2c} = s_{2b}$ => $T_{2c} = 124.3^o C$, $h_{2c} = 1601.37$ kJ/kg

Energy eq. compressor 2: $w_{C2} = h_{2c} - h_{2b} = 1601.37 - 1471.5 = 129.87$ kJ/kg

Total power input: $\dot{W}_{IN} = \dot{m}\,(w_{C1} + w_{C2})$ $\Rightarrow$

$$\dot{m} = \frac{\dot{W}_{IN}}{w_{C1} + w_{C2}} = \frac{1000}{95.6 + 129.87} = 4.4352 \text{ kg/s}$$

Heat exchanger 1: $\dot{Q}_{H1} = \dot{m}(h_{2a} - h_{2b}) = 4.4352(1559.1\text{-}1471.5) = $ **388.5 kW**

Heat exchanger 2: $\dot{Q}_{H2} = \dot{m}(h_{2c} - h_3) = 4.4352(1601.37\text{-}631.9) = $ **4299.8 kW**

Borgnakke and Sonntag

9.94

A refrigerator with R-134a as the working fluid has a minimum temperature of $-10°C$ and a maximum pressure of 1 MPa. The actual adiabatic compressor exit temperature is 50°C. Assume no pressure loss in the heat exchangers. Find the specific heat transfer from the cold space and that to the hot space, the coefficient of performance and the isentropic efficiency of the compressor.

Solution:

State 1: Inlet to compressor, sat. vapor -10°C,

$h_1 = 392.28$, $s_1 = 1.7319$ kJ/kgK

State 2: Actual compressor exit, $h_{2AC} = 431.24$ kJ/kg

State 3: Exit condenser, sat. liquid 1 MPa, $h_3 = 255.60$ kJ/kg

State 4: Exit valve, $h_4 = h_3$

C.V. Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 392.28 - 255.60 =$ **136.7 kJ/kg**

C.V. Ideal Compressor: $w_{C,S} = h_{2,S} - h_1$, $s_{2,S} = s_1$

State 2s: 1 MPa, s = 1.7319 kJ/kg K; $T_{2,S} = 44.9°C$, $h_{2,S} = 425.7$ kJ /kg

$w_{C,S} = h_{2,S} - h_1 = 33.42$ kJ/kg

C.V. Actual Compressor: $w_C = h_{2,AC} - h_1 = 38.96$ kJ/kg

$\beta = \dfrac{q_L}{w_C} =$ **3.51,** $\qquad \eta_C = w_{C,S}/w_C =$ **0.858**

C.V. Condenser: $q_H = h_{2,AC} - h_3 =$ **186.3 kJ/kg**

Ideal refrigeration cycle
with actual compressor
$P_{cond} = P_3 = P_2 = 1$ MPa
$T_{2AC} = 50^oC$
$T_{evap} = -10^oC = T_1$
Properties from Table B.5

Borgnakke and Sonntag

9.95

An air-conditioner in the airport of Timbuktu runs a cooling system using R-410A with a high pressure of 1800 kPa and a low pressure of 200 kPa. It should cool the desert air at 45°C down to 15°C. Find the cycle COP. Will the system work?

Solution:

Ideal refrigeration cycle
$P_{cond} = P_2 = P_3 = 1800$ kPa
$P_{evap} = P_1$

State 1: $T_1 = -37.0^oC$, $h_1 = 264.27$ kJ/kg, $s_1 = 1.1192$ kJ/kg-K

State 2: $P_2 = 1800$ kPa, $s_2 = s_1$: $T_2 = 63.6^oC$, $h_2 = 327.9$ kJ/kg

$w_C = h_2 - h_1 = 327.9 - 264.27 = 63.63$ kJ/kg

State 3-4: $P_4 = P_3$, $x_3 = 0$: $T_3 = 28.22^oC$, $h_4 = h_3 = 103.096$ kJ/kg

$q_L = h_1 - h_4 = 264.27 - 103.096 = 161.17$ kJ/kg

$\beta = q_L/w_C = 161.17/63.63 = \textbf{2.53}$

The heat rejection from 2-3 to ambient at 45°C has $T_3 = 28.22^oC$ not hot enough so the system will **not work**. The high pressure must be so also $T_3 > 45^oC$.

9.96

A heat pump uses R410a with a high pressure of 3000 kPa and an evaporator operating at -10$^{\text{o}}$C so it can absorb energy from underground water layers at 4$^{\text{o}}$C. Find the COP and the temperature it can deliver energy at.

Solution:

Ideal refrigeration cycle
R-410A
$P_{cond} = P_2 = P_3 = 3000$ kPa
$T_{evap} = -10^{\text{o}}C = T_1$

State 1: $\quad h_1 = 275.78$ kJ/kg, $\quad s_1 = 1.0567$ kJ/kg-K

State 2: $\quad P_2, s_2 = s_1$: $\quad T_2 = 75.3^{\text{o}}C,\ h_2 = 322.43$ kJ/kg

$\qquad\qquad w_C = h_2 - h_1 = 322.43 - 275.78 = 46.65$ kJ/kg

State 3-4: $\quad h_3 = h_4, x_3 = 0$: $\quad T_3 = 49.07^{\text{o}}C,\ h_4 = h_3 = 141.78$ kJ/kg

$\qquad\qquad q_H = h_2 - h_3 = 322.43 - 141.78 = 180.65$ kJ/kg

$$\beta = q_H/w_C = \frac{180.65}{46.65} = \textbf{3.87}$$

It can deliver heat at about 49$^{\text{o}}$C = T_3 (minus a ΔT for the heat transfer rate) a smaller amount of heat is delivered between T_2 and T_3.

Borgnakke and Sonntag

9.97

Consider an ideal heat pump that has a condenser temperature of 50°C and an evaporator temperature of 0°C. Determine the coefficient of performance of this heat pump for the working fluids R-134a and ammonia.

Solution:

Ideal heat pump

$T_{cond} = 50^oC = T_3$

$T_{evap} = 0^oC = T_1$

C.V.	Property for: From Table:	R-134a B.5	NH_3 B.2
	h_1, kJ/kg	398.36	1442.32
Compressor	$s_2 = s_1$, kJ/kgK	1.7262	5.3313
	P_2, MPa	1.3181	2.0333
	T_2, ^oC	55.1	115.6
	h_2, kJ/kg	429.55	1672.84
	$w_C = h_2 - h_1$	31.19	230.52
Exp. valve	$h_3 = h_4$, kJ/kg	271.83	421.58
Condenser	$q_H = h_2 - h_3$	157.72	1251.26
	$\beta' = q_H/w_C$	**5.06**	**5.428**

Borgnakke and Sonntag

9.98

A refrigerator in a meat warehouse must keep a low temperature of -15°C and the outside temperature is 20°C. It uses ammonia as the refrigerant which must remove 5 kW from the cold space. Find the flow rate of the ammonia needed assuming a standard vapor compression refrigeration cycle with a condenser at 20°C.

Solution:

Basic refrigeration cycle: $T_1 = T_4 = -15°C$, $T_3 = 20°C$

Table B.3: $h_4 = h_3 = 274.3$ kJ/kg; $h_1 = h_g = 1424.6$ kJ/kg

$$\dot{Q}_L = \dot{m}_{amm} \times q_L = \dot{m}_{amm} (h_1 - h_4)$$
$$q_L = 1424.6 - 274.3 = 1150.3 \text{ kJ/kg}$$

$$\dot{m}_{amm} = 5.0 \text{ kW}/ 1150.3 \text{ kJ/kg} = \textbf{0.00435 kg/s}$$

Ideal refrigeration cycle

$T_{cond} = 20^oC$

$T_{evap} = -15^oC = T_1$

Properties from Table B.2

9.99

A refrigerator has a steady flow of R-410A as saturated vapor at –20°C into the adiabatic compressor that brings it to 1400 kPa. After the compressor, the temperature is measured to be 60°C. Find the actual compressor work and the actual cycle coefficient of performance.

Solution:

Table B.4.1: $h_1 = 271.89$ kJ/kg, $s_1 = 1.0779$ kJ/kg K

$P_2 = P_3 = 1400$ kPa, $T_3 = 18.88°C$, $h_4 = h_3 = h_f = 87.45$ kJ/kg

$h_{2\ ac} = 330.07$ kJ/kg

C.V. Compressor (actual)

Energy Eq.: $w_{C\ ac} = h_{2\ ac} - h_1 = 330.07 – 271.89 = $ **58.18 kJ/kg**

C.V. Evaporator

Energy Eq.: $q_L = h_1 - h_4 = h_1 - h_3 = 271.89 – 87.45 = 184.44$ kJ/kg

$$\beta = \frac{q_L}{w_{C\ ac}} = \frac{184.44}{58.18} = \textbf{3.17}$$

Ideal refrigeration cycle
with actual compressor
$T_{cond} = 18.88^oC = T_{sat\ 1400\ kPa}$
$T_2 = 60^oC$
$T_{evap} = -20^oC = T_1$
Properties from Table B.4

9.100

The air conditioner in a car uses R-134a and the compressor power input is 1.5 kW bringing the R-134a from 201.7 kPa to 1200 kPa by compression. The cold space is a heat exchanger that cools atmospheric air from the outside 30°C down to 10°C and blows it into the car. What is the mass flow rate of the R-134a and what is the low temperature heat transfer rate. How much is the mass flow rate of air at 10°C?

Standard Refrigeration Cycle

Table B.5: $h_1 = 392.28$ kJ/kg; $s_1 = 1.7319$ kJ/kg K; $h_4 = h_3 = 266$ kJ/kg

C.V. Compressor (assume ideal)

$$\dot{m}_1 = \dot{m}_2 \qquad w_C = h_2 - h_1; \quad s_2 = s_1 + s_{gen}$$

$$P_2, s = s_1 \implies h_2 = 429.5 \text{ kJ/kg} \implies w_C = 37.2 \text{ kJ/kg}$$

$$\dot{m}\, w_C = \dot{W}_C \implies \dot{m} = 1.5 / 37.2 = \textbf{0.0403 kg/s}$$

C.V. Evaporator

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = 0.0405(392.28 - 266) = \textbf{5.21 kW}$$

C.V. Air Cooler

$$\dot{m}_{air}\Delta h_{air} = \dot{Q}_L \approx \dot{m}_{air}C_p\Delta T$$

$$\dot{m}_{air} = \dot{Q}_L / (C_p\Delta T) = 5.21 / (1.004\times20) = \textbf{0.26 kg / s}$$

Ideal refrigeration cycle

$P_{cond} = 1200$ kPa $= P_3$
$P_{evap} = 201.7$ kPa $= P_1$

9.101

A refrigerator in a laboratory uses R-134a as the working substance. The high pressure is 1200 kPa, the low pressure is 165 kPa, and the compressor is reversible. It should remove 500 W from a specimen currently at –10°C (not equal to T_L in the cycle) that is inside the refrigerated space. Find the cycle COP and the electrical power required.

Solution:

State 1: 165 kPa, x = 1, Table B.5.1: h_1 = 389.2 kJ/kg, s_1 = 1.7354 kJ/kg K

State 3: 1200 kPa, 46.31°C, x = 0, Table B.5.1: h_3 = 266.13 kJ/kg

C.V. Compressor

Energy Eq.: $w_C = h_2 - h_1$

Entropy Eq.: $s_2 = s_1 + s_{gen} = s_1$

State 2: 1.2 MPa , $s_2 = s_1$ = 1.7354 kJ/kg, $T_2 \approx 53.4$°C, h_2 = 430.67 kJ/kg

$$w_C = h_2 - h_1 = 430.67 - 389.2 = 41.47 \text{ kJ/kg}$$

Energy Eq. evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 389.2 - 266.13 = 123.07 \text{ kJ/kg}$

COP Refrigerator: $\beta = \dfrac{q_L}{w_C} = \dfrac{123.07}{41.47} = \textbf{2.97}$

Power: $\dot{W}_{IN} = \dot{Q}_L / \beta = 500 \text{ W} / 2.97 = \textbf{168.3 W}$

9.102

Consider the previous problem and find the two rates of entropy generation in the process and where they occur.

Solution:

State 1: 165 kPa, x = 1, Table B.5.1: h_1 = 389.2 kJ/kg, s_1 = 1.7354 kJ/kg K

State 3: 1200 kPa, x = 0, Table B.5.1: h_3 = 266.13 kJ/kg, s_3 = 1.2204 kJ/kg K

Energy Eq. evaporator: $q_L = h_1 - h_4 = h_1 - h_3$ = 389.2 − 266.13 = 123.07 kJ/kg

Mass flow rate: $\dot{m} = \dot{Q}_L / q_L$ = 0.5 / 123.07 = 0.00406 kg/s

C.V. Valve

Energy Eq.: $h_4 = h_3$ = 266.13 kJ/kg   => $x_4 = (h_4 - h_f)/h_{fg}$

$$x_4 = \frac{266.13 - 180.19}{209.0} = 0.4112$$

$s_4 = s_f + x_4\, s_{fg}$ = 0.9258 + x_4 × 0.8096 = 1.2587 kJ/kg K

Entropy Eq.: $s_{gen} = s_4 - s_3$ = 1.2587 − 1.2204 = 0.0383 kJ/kg K

$$\dot{S}_{gen\ valve} = \dot{m}s_{gen} = 0.00406 \times 0.0383 \times 1000 = \textbf{0.155 W/K}$$

There is also entropy generation in the heat transfer process from the specimen at –10°C to the refrigerant T_1 = -15°C = T_{sat} (165 kPa).

$$\dot{S}_{gen\ inside} = \dot{Q}_L \left[\frac{1}{T_{specimen}} - \frac{1}{T_L} \right] = 500\,(\frac{1}{258.15} - \frac{1}{263.15}) = \textbf{0.0368 W/K}$$

Borgnakke and Sonntag

9.103

A refrigerator using R-134a is located in a 20°C room. Consider the cycle to be ideal, except that the compressor is neither adiabatic nor reversible. Saturated vapor at -20°C enters the compressor, and the R-134a exits the compressor at 50°C. The condenser temperature is 40°C. The mass flow rate of refrigerant around the cycle is 0.2 kg/s, and the coefficient of performance is measured and found to be 2.3. Find the power input to the compressor and the rate of entropy generation in the compressor process.

Solution:

Table B.5: $P_2 = P_3 = P_{sat\ 40C} = 1017$ kPa, $h_4 = h_3 = 256.54$ kJ/kg

$s_2 \approx 1.7472$ kJ/kg K, $h_2 \approx 430.87$ kJ/kg;

$s_1 = 1.7395$ kJ/kg K, $h_1 = 386.08$ kJ/kg

$\beta = q_L / w_C$ -> $w_C = q_L / \beta = (h_1 - h_4) / \beta = (386.08 - 256.54) / 2.3 = 56.32$

$$\dot{W}_C = \dot{m}\ w_C = \textbf{11.26 kW}$$

C.V. Compressor $h_1 + w_C + q = h_2$ ->

$q_{in} = h_2 - h_1 - w_C = 430.87 - 386.08 - 56.32 = -11.53$ kJ/kg   i.e. a heat loss

$s_1 + \int dQ/T + s_{gen} = s_2$

$s_{gen} = s_2 - s_1 - q / T_o = 1.7472 - 1.7395 + (11.53 / 293.15) = 0.047$ kJ/kg K

$$\dot{S}_{gen} = \dot{m}\ s_{gen} = 0.2 \times 0.047 = \textbf{0.0094 kW / K}$$

Ideal refrigeration cycle
with actual compressor
$T_{cond} = 40^oC$
$T_2 = 50^oC$
$T_{evap} = -20^oC = T_1$
Properties from Table B.5

Borgnakke and Sonntag

9.104

A small heat pump unit is used to heat water for a hot-water supply. Assume that the unit uses ammonia and operates on the ideal refrigeration cycle. The evaporator temperature is 15°C and the condenser temperature is 60°C. If the amount of hot water needed is 0.1 kg/s, determine the amount of energy saved by using the heat pump instead of directly heating the water from 15 to 60°C.

Solution:

Ideal ammonia heat pump

$T_1 = 15^oC, \quad T_3 = 60^oC$
From Table B.2.1

$h_1 = 1456.3$ kJ/kg, $\quad s_2 = s_1 = 5.1444$ kJ/kg K
$P_2 = P_3 = 2.614$ MPa, $\quad h_3 = 472.8$ kJ/kg

Entropy compressor: $s_2 = s_1 \quad => \quad T_2 = 111.6^oC, \quad h_2 = 1643$ kJ/kg

Energy eq. compressor: $\quad w_C = h_2 - h_1 = 186.7$ kJ/kg

Energy condenser: $\quad q_H = h_2 - h_3 = 1170.2$ kJ/kg

To heat 0.1 kg/s of water from 15^oC to 60^oC,

$$\dot{Q}_{H2O} = \dot{m}(\Delta h) = 0.1(251.11 - 62.98) = 18.81 \text{ kW}$$

Using the heat pump

$$\dot{W}_{IN} = \dot{Q}_{H2O}(w_C/q_H) = 18.81(186.7 / 1170.2) = 3.0 \text{ kW}$$

a saving of **15.8 kW**

9.105

In an actual refrigeration cycle using R-134a as the working fluid, the refrigerant flow rate is 0.05 kg/s. Vapor enters the compressor at 150 kPa, -10°C (h_1 = 394.2 kJ/kg, s_1 = 1.739 kJ/kg-K), and leaves at 1.2 MPa, 75°C (h_2 = 454.2 kJ/kg, s_2 = 1.805 kJ/kg-K). The power input to the non-adiabatic compressor is measured and found be 2.4 kW. The refrigerant enters the expansion valve at 1.15 MPa, 40°C (h = 256.4 kJ/kg), and leaves the evaporator at 160 kPa, -15°C (h = 389.8 kJ/kg). Determine the entropy generation in the compression process, the refrigeration capacity and the coefficient of performance for this cycle.

Solution:

Actual refrigeration cycle

1: compressor inlet T_1 = -10°C, P_1 = 150 kPa

2: compressor exit T_2 = 75°C, P_2 = 1.2 MPa

3: Expansion valve inlet T_3 = 40°C
P_3 = 1.15 MPa

5: evaporator exit T_5 = -15°C, P_5 = 160 kPa

Table B.5 (CATT3) h_1 = 394.2 kJ/kg, s_1 = 1.739 kJ/kg-K,

h_2 = 454.2 kJ/kg, s_2 = 1.805 kJ/kg-K

CV Compressor: $h_1 + q_{COMP} + w_{COMP} = h_2$; $s_1 + \int dq/T + s_{gen} = s_2$

$$w_{COMP} = \dot{W}_{COMP}/\dot{m} = 2.4/0.05 = 48.0 \text{ kJ/kg}$$

$$q_{COMP} = h_2 - w_{COMP} - h_1 = 454.2 - 48.0 - 394.2 = 12 \text{ kJ/kg}$$

$$s_{gen} = s_2 - s_1 - q / T_o = 1.805 - 1.739 - 12/298.15 = \textbf{0.0258 kJ/kg-K}$$

C.V. Evaporator

$$q_L = h_5 - h_4 = 389.8 - 256.4 = 133.4 \text{ kJ/kg}$$

$$\Rightarrow \dot{Q}_L = \dot{m}q_L = 0.05 \times 133.4 = \textbf{6.67 kW}$$

COP: $\beta = q\dfrac{q_L}{w_{COMP}} = \dfrac{133.4}{48.0} = \textbf{2.78}$

Extended refrigeration cycles

9.106

One means of improving the performance of a refrigeration system that operates over a wide temperature range is to use a two-stage compressor. Consider an ideal refrigeration system of this type that uses R-410A as the working fluid, as shown in Fig. 9.25. Saturated liquid leaves the condenser at 40°C and is throttled to -20°C. The liquid and vapor at this temperature are separated, and the liquid is throttled to the evaporator temperature, -50°C. Vapor leaving the evaporator is compressed to the saturation pressure corresponding to -20°C, after which it is mixed with the vapor leaving the flash chamber. It may be assumed that both the flash chamber and the mixing chamber are well insulated to prevent heat transfer from the ambient. Vapor leaving the mixing chamber is compressed in the second stage of the compressor to the saturation pressure corresponding to the condenser temperature, 40°C. Determine the COP of the system. Compare to the COP of a simple ideal refrigeration cycle operating over the same condenser and evaporator ranges as those of the two-stage compressor unit studied in this problem (T_2 = 90.7 ^{o}C, h_2 = 350.35 kJ/kg).

R-410A refrigerator with 2-stage compression

CV: expansion valve, upper loop

$h_2 = h_1 = 124.09 = 28.24 + x_2 \times 243.65;$ $x_2 = 0.3934$

$m_3 = x_2 m_2 = x_2 m_1 = 0.3934$ kg (for m_1=1 kg)

$m_6 = m_1 - m_3 = 0.6066$ kg

CV: expansion valve, lower loop

$h_7 = h_6 = 28.24 = -13.8 + x_7 \times 271.6, \quad x_7 = 0.15478$

$Q_L = m_6 (h_8 - h_7) = 0.6066 (257.80 - 28.24) = 139.25$ kJ

$q_L = Q_L / m_1 = 139.25$ kJ/kg-m_1

CV: 1st stage compressor

$s_8 = s_9 = 1.1568$ kJ/kg-K, $\quad P_9 = P_{SAT\ -20\ ^oC} = 0.3996$ MPa

$\Rightarrow T_9 = 2.7\ ^oC, \quad h_9 = 292.82$ kJ/kg

CV: mixing chamber (assume constant pressure mixing)

Energy Eq.: $\quad m_6 h_9 + m_3 h_3 = m_1 h_4$

or $h_4 = 0.6066 \times 292.82 + 0.3934 \times 271.89 = 284.6$ kJ/kg

$h_4, P_4 \approx 400$ kPa $\Rightarrow T_4 = -6.3\ ^oC, \quad s_4 = 1.1261$ kJ/kg K

CV: 2nd stage compressor $\quad P_4 = 400$ kPa $= P_9 = P_3$

$P_5 = P_{sat\ 40^oC} = 2.4207$ MPa, $s_5 = s_4 \Rightarrow T_5 = 81.3^oC, h_5 = 339.42$ kJ/kg

CV: condenser

Energy Eq.: $q_H = h_5 - h_1 = 339.42 - 124.09 = 215.33$ kJ/kg

$\beta_{2\ stage} = q_L/(q_H - q_L) = 139.25/(215.33 - 139.25) = $ **1.83**

b) 1 stage compression

$h_3 = h_4 = 124.09$ kJ/kg
$h_1 = 257.80$ kJ/kg
$q_L = h_1 - h_4 = 133.7$ kJ/kg
$s_1 = s_2 = 1.1568$ ⎫
$P_2 = 2.4207$ MPa ⎭
$\Rightarrow T_2 = 90.7$ $^oC, h_2 = 350.35$

$q_H = h_2 - h_3 = 350.35 - 124.09 = 226.26$ kJ/kg

$\beta_{1\ stage} = q_L/(q_H - q_L) = 133.7/(226.26 - 133.7) = $ **1.44**

9.107

A cascade system with one refrigeration cycle operating with R-410A has an evaporator at -40°C and a high pressure of 1200 kPa. The high temperature cycle uses R-134a with an evaporator at 0°C and a high pressure of 1200 kPa. Find the ratio of the two cycles mass flow rates and the overall COP.

R-134a cycle   **R-410A cycle**

	T,°C	P	h	s		T,°C	P	h	s
1′	0	294	398.36	1.7262	1	-40	175	262.83	1.1273
2′	46.99	1200	423.296	1.7262	2	45.05	1200	317.65	1.1273
3′	46.31	1200	266.13		3	13.43	1200	78.66	
4′	0		266.13		4	-40	175	78.66	

$$\dot{m}/\dot{m}' = \frac{h_1' - h_4'}{h_2 - h_3} = \frac{398.36 - 266.13}{317.65 - 78.66} = \mathbf{0.5533}$$

$$q_L = h_1 - h_4 = 262.83 - 78.66 = 184.17 \text{ kJ/kg}$$

$$-\dot{W}_{TOT}/\dot{m} = (h_2 - h_1) + (\dot{m}'/\dot{m})(h_2' - h_1')$$

$$= 317.65 - 262.83 + \frac{1}{0.5533}(423.296 - 398.36) = 99.89 \frac{\text{kJ}}{\text{kg}}$$

$$\beta = \dot{Q}_L/(-\dot{W}_{TOT}) = 184.17/99.89 = \mathbf{1.844}$$

9.108

A cascade system is composed of two ideal refrigeration cycles, as shown in Fig. 9.28. The high-temperature cycle uses R-410A. Saturated liquid leaves the condenser at 40°C, and saturated vapor leaves the heat exchanger at –20°C. The low-temperature cycle uses a different refrigerant, R-23. Saturated vapor leaves the evaporator at –80°C, h = 330 kJ/kg, and saturated liquid leaves the heat exchanger at –10°C, h = 185 kJ/kg. R-23 out of the compressor has h = 405 kJ/kg. Calculate the ratio of the mass flow rates through the two cycles and the COP of the system.

R-410A	T,°C	P	h	s		T,°C	P	h	s
1′	-20	0.400	271.89	1.0779	1	-80	0.12	330	1.76
2′	71	2.421	322.61	1.0779	2	50	1.90	405	1.76
3′	40	2.421	124.09		3	-10	1.90	185	
4′	-20		124.09		4	-80	0.12	185	

$$\dot{m}/\dot{m}' = \frac{h'_1 - h'_4}{h_2 - h_3} = \frac{271.89 - 124.09}{405 - 185} = \mathbf{0.6718}$$

$$q_L = h_1 - h_4 = 330 - 185 = 145 \text{ kJ/kg}$$

$$-\dot{W}_{TOT}/\dot{m} = (h_2 - h_1) + (\dot{m}'/\dot{m})(h'_2 - h'_1) = (405-330) + \frac{1}{0.6718}(322.61–271.89) = 150.5 \frac{\text{kJ}}{\text{kg}}$$

$$\beta = \dot{Q}_L/(-\dot{W}_{TOT}) = 145/150.5 = \mathbf{0.96}$$

9.109

A split evaporator is used to provide cooling of the refrigerator section and separate cooling of the freezer section as shown in Fig. P9.109. Assume constant pressure in the two evaporators. How does the COP $= (Q_{L1} + Q_{L2})/W$ compare to a refrigerator with a single evaporator at the lowest temperature?

Throttle processes: $h_4 = h_3$; $h_5 = h_6$

Refrigerator: $q_{L\,R} = h_5 - h_4$

freezer: $q_{L\,F} = h_1 - h_6$

Add the two heat transfers: $q_{L\,R} + q_{L\,F} = h_5 - h_4 + h_1 - h_6 = h_1 - h_4$

which is the **same** as for the standard cycle expanding to the lowest pressure.

$$COP_{split} = COP_{std} = (h_1 - h_4) / (h_2 - h_1)$$

9.110

A refrigerator using R-410A is powered by a small natural gas fired heat engine with a thermal efficiency of 25%, as shown in Fig. P9.110. The R-410A condenses at 40°C and it evaporates at −20°C and the cycle is standard. Find the two specific heat transfers in the refrigeration cycle. What is the overall coefficient of performance as Q_L/Q_1?

Solution:

Evaporator: Inlet State is saturated liq-vap with $h_4 = h_3 = 124.09$ kJ/kg

The exit state is saturated vapor with $h_1 = 271.89$ kJ/kg

$q_L = h_1 - h_4 = h_1 - h_3 =$ **147.8 kJ/kg**

Compressor: Inlet State 1 and Exit State 2 about 2.42 MPa

$w_C = h_2 - h_1$; $s_2 = s_1 = 1.0779$ kJ/kgK

2:       $T_2 \approx 70°C$ $h_2 = 322.6$ kJ/kg

$w_C = h_2 - h_1 = 50.71$ kJ/kg

Condenser: Brings it to saturated liquid at state 3

$q_H = h_2 - h_3 = 322.6 - 124.09 =$ **198.5 kJ/kg**

Overall Refrigerator:

$\beta = q_L / w_C = 147.8 / 50.71 = 2.915$

Heat Engine:

$$\dot{W}_{HE} = \eta_{HE}\dot{Q}_1 = \dot{W}_C = \dot{Q}_L / \beta$$

$$\dot{Q}_L / \dot{Q}_1 = \eta\beta = 0.25 \times 2.915 = \textbf{0.729}$$

Ideal refrigeration cycle
$T_{cond} = 40°C = T_3$
$T_{evap} = -20°C = T_1$
Properties from Table B.4

Ammonia absorption cycles

9.111

Notice the configuration in Fig.9.29 has the left hand side column of devices substitute for a compressor in the standard cycle. What is an expression for the equivalent work output from the left hand side devices, assuming they are reversible and the high and low temperatures are constant, as a function of the pump work W and the two temperatures.

The left hand side devices works like a combination of a heat engine with some additional shaft work input. We can analyze this with a control volume around all the devices that substitute for the compressor in the standard cycle.

C.V. Pump, absorber, heat exchanger and generator.
This C.V. has an inlet flow at state 1 and exit flow at state 2 with numbers as in the standard cycle.

Energy Eq.: $0 = h_1 + w + q'_H - q'_L - h_2$ all per mass flow at 1 and 2.

Entropy Eq.: $0 = s_1 + \dfrac{q'_H}{T_{H'}} - \dfrac{q'_L}{T_{L'}} - s_2 + 0$

Now solve for q'_L from the entropy equation and substitute into the energy equation

$$q'_L = T_{L'} (s_1 - s_2) + (T_{L'} / T_{H'}) \, q'_H$$

$$w + [1 - \frac{T_{L'}}{T_{H'}}] \, q'_H = (h_2 - h_1) - T_{L'} (s_2 - s_1)$$

The high T heat transfer acts as if it was delivered to a Carnot heat engine and the Carnot heat engine work output was added to the shaft work w. That sum gives the increase in exergy from state 1 to state 2. Notice in the standard cycle, $s_2 = s_1$, and the last term is zero.

9.112

As explained in the previous Problem the ammonia absorption cycle is very similar to the set-up sketched in Problem 9.110. Assume the heat engine has an efficiency of 30% and the COP of the refrigeration cycle is 3.0 what is then the ratio of the cooling to the heating heat transfer Q_L/Q_1?

Heat Engine: $\quad W = \eta Q_1 = 0.3\ Q_1$

Refrigerator.: $\quad \beta = Q_L/W \quad \Rightarrow \quad Q_L = \beta W = \beta \eta Q_1$

So now

$$Q_L/Q_1 = \beta\ \eta = 3 \times 0.3 = \mathbf{0.9}$$

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9.113

Give an estimate for the overall COP' of an ammonia absorption cycle used as a chiller to cool water to 5$^\text{o}$C in an 25$^\text{o}$C ambient when the small pump work is neglected. A heat source is available at 100$^\text{o}$C. Find also the efficiency of the heat engine part and the COP of the refrigeration cycle part.

The heat engine operates between the 100°C source and the 25°C ambient so

$$\eta_{HE} = 1 - T_o/T_s = 1 - \frac{298.15}{373.15} = \mathbf{0.20}$$

To estimate the COP of the basic ammonia cycle consider the high/low temperatures to be as $T_2 = 25$°C and $T_4 = 5$°C. From B.2.1 we get

$P_2 = P_{\text{sat 25C}} = 1003$ kPa, $\ h_3 = h_2 = 298.25$ kJ/kg, $\ h_4 = 1447.3$ kJ/kg,

Assume $T_1 = 100$°C, $P_1 = P_2$, $\ h_1 = 1664.3$ kJ/kg

$$COP = q_L / w_C = (h_4 - h_2)/(h_1 - h_4) = \frac{1447.3 - 298.25}{1664.3 - 1447.3} = \mathbf{5.3}$$

Neglecting the small pump work we get

$$COP' = COP \times \eta_{HE} = 5.3 \times 0.2 = \mathbf{1.06}$$

9.114

Consider a small ammonia absorption refrigeration cycle that is powered by solar energy and is to be used as an air conditioner. Saturated vapor ammonia leaves the generator at 50°C, and saturated vapor leaves the evaporator at 10°C. If 3000 kJ of heat is required in the generator (solar collector) per kilogram of ammonia vapor generated, determine the overall performance of this system.

Solution;

NH_3 absorption cycle:

sat. vapor at 50°C exits the generator

sat. vapor at 10°C exits the evaporator

$q_H = q_{GEN} = 3000$ kJ/kg NH_3 out of gen.

C.V. Evaporator

$$q_L = h_2 - h_1 = h_{g\ 10°C} - h_{f\ 50°C} = 1452.2 - 421.6 = 1030.6 \text{ kJ/kg}$$

COP $\Rightarrow$ $q_L/q_H = 1030.6/3000 = \mathbf{0.34}$

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9.115

The performance of an ammonia absorption cycle refrigerator is to be compared with that of a similar vapor-compression system. Consider an absorption system having an evaporator temperature of −10°C and a condenser temperature of 50°C. The generator temperature in this system is 150°C. In this cycle 0.42 kJ is transferred to the ammonia in the evaporator for each kilojoule transferred from the high-temperature source to the ammonia solution in the generator. To make the comparison, assume that a reservoir is available at 150°C, and that heat is transferred from this reservoir to a reversible engine that rejects heat to the surroundings at 25°C. This work is then used to drive an ideal vapor-compression system with ammonia as the refrigerant. Compare the amount of refrigeration that can be achieved per kilojoule from the high-temperature source with the 0.42 kJ that can be achieved in the absorption system.

Solution:

$T_1 = -10\ ^\circ C$

$h_1 = 1430.8$ kJ/kg,

$s_1 = 5.4673$ kJ/kg-K

$h_4 = h_3 = 421.48$ kJ/kg

For the rev. heat engine: $\quad \eta_{TH} = 1 - T'_L/T'_H = 1 - \dfrac{298.2}{423.2} = 0.295$

$\Rightarrow \qquad W_C = \eta_{TH}\, Q'_H = 0.295$ kJ

For the NH_3 refrig. cycle: $\quad P_2 = P_3 = 2033$ kPa , $\quad$ Use 2000 kPa Table

$s_2 = s_1 = 5.4673$ kJ/kg-K $\qquad \Rightarrow \qquad T_2 \approx 135°C, \ \ h_2 \approx 1724$ kJ/kg

$w_C = h_2 - h_1 = 1724 - 1430.8 = 293.2$ kJ/kg

$q_L = h_1 - h_4 = 1430.8 - 421.48 = 1009.3$ kJ/kg

$\beta = q_L/w_C = 1009.3 / 293.2 = 3.44$

$\Rightarrow Q_L = \beta w_C = 3.44 \times 0.295 = \textbf{1.015 kJ}$

This is based on the assumption of an ideal heat engine & refrigeration cycle.

Exergy Concepts

9.116

If we neglect the external irreversibilities due to the heat transfers over finite temperature differences in a power plant how would you define its second law efficiency?

The first law efficiency is a conversion efficiency as

$$\eta_I = \frac{w_{net}}{q_H} = \frac{w_{net}}{h_3 - h_2}$$

The second law efficiency is the same ratio but expressed in exergy

$$\eta_{II} = \frac{output}{source} = \frac{w_{net}}{\varphi_H} = \frac{w_{net}}{\varphi_3 - \varphi_2} \quad or \quad = \frac{w_{net}}{\varphi_H - \varphi_L}$$

The last expression must be used if the heat rejection at the low T is assigned any exergy value (normally not).

9.117

Find the exergy of the water at all four states in the Rankine cycle described in Problem 9.16. Assume that the high-temperature source is 500°C and the low-temperature reservoir is at 25°C. Determine the flow of exergy in or out of the reservoirs per kilogram of steam flowing in the cycle. What is the overall cycle second law efficiency?

Solution (States and properties are from solution to problem 9.16):

Reference State: 100 kPa, 25°C, $s_o = 0.3673$ kJ/kg K, $h_o = 104.87$ kJ/kg

$$\psi_1 = h_1 - h_o - T_o(s_1 - s_o)$$

$$= 251.11 - 104.87 - 298.15(0.8311 - 0.3673) = 7.958 \text{ kJ/kg}$$

$$\psi_2 = 254.14 - 104.87 - 298.15(0.8311 - 0.3673) = \psi_1 + 3.03 = 10.99 \text{ kJ/kg}$$

$$\psi_3 = 3344 - 104.87 - 298.15(7.0833 - 0.3673) = 1236.75 \text{ kJ/kg}$$

$$\psi_4 = \psi_3 - w_{T,s} = 1236.75 - 1009.6 = 227.1 \text{ kJ/kg}$$

$$\Delta\psi_H = (1 - T_o/T_H)q_H = 0.6144 \times 3089.86 = 1898.4 \text{ kJ/kg}$$

$$\Delta\psi_L = (1 - T_o/T_o)q_C = 0 \text{ kJ/kg}$$

$$\eta_{II} = w_{NET}/\Delta\psi_H = (1009.6 - 3.03)/1898.4 = \textbf{0.53}$$

Notice— $T_H > T_3$, $T_L < T_4 = T_1$ so cycle is externally irreversible. Both q_H and q_C are over finite ΔT.

Energy Transfers (kJ/kg):　　　　　　　　Exergy Transfers (kJ/kg):

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9.118

A condenser is maintained at 60°C by cooling it with atmospheric air coming in at 20°C and leaving at 35 °C. The condenser must reject 25 MW from the water to the air. Find the flow rate of air and the second law efficiency of the heat exchanger.

C.V. Condenser, a dual steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside.

Energy Eq.: $\qquad 0 = \dot{m}_{H2O}\,(h_4 - h_1) + \dot{m}_{air}\,(h_a - h_b)$

$$\dot{m}_{H2O}\,(h_4 - h_1) = \dot{Q} = \dot{m}_{air}\,(h_b - h_a) = \dot{m}_{air}\,C_p\,(T_b - T_a)$$

$$\dot{m}_{air} = \dot{Q} / C_p\,(T_b - T_a) = \frac{25\,000\ \text{kW}}{1.004 \times 15\ \text{kJ/kg}} = \textbf{1660 kg/s}$$

Since the water is at constant T the change in entropy equals $\dot{Q}$/T and the exergy terms for the water and air becomes

$$\Delta\dot{\Phi}_{water} = \dot{Q}\,(1 - \frac{T_0}{T_{H2O}}) \;= 25\ \text{MW}\ (\,1 - \frac{293.15}{333.15}) = 3001.6\ \text{kW}$$

$$s_b - s_a = C_p\,\ln(\,T_b/T_a\,) = 1.004\,\ln\,(308.15/293.15) = 0.0501\ \text{kJ/kgK}$$

$$\Delta\dot{\Phi}_{air} = \dot{m}_{air}\,[h_b - h_a - T_0\,(s_b - s_a)] = \dot{Q} - \dot{m}_{air}T_0\,(s_b - s_a)$$

$$= 25\,000 - 1660 \times 293.15 \times 0.0501 = 619.89\ \text{kW}$$

$$\eta_{II} = \Delta\dot{\Phi}_{air} / \Delta\dot{\Phi}_{water} = \frac{619.89}{3001.6} = \textbf{0.2065}$$

Comment: We used $T_0 = 20$°C = 293.15 K, since the air comes in at that T, in order not to straddle the T_0 with the air temperatures. Recall exergy is positive for T's higher and lower than T_0.

9.119

Find the flows and fluxes of exergy in the condenser of Problem 9.29. Use those to determine the second law efficiency, $T_o = 15°C$.

For this case we select $T_o = 15°C = 288$ K, the ocean water temperature.

The states properties from Tables B.1.1 and B.1.3

1: $45°C$, $x = 0$: $h_1 = 188.42$ kJ/kg,

3: 3.0 MPa, $600°C$: $s_3 = 7.5084$ kJ/kg K

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$$s_4 = s_3 = 7.5084 = 0.6386 + x_4 (7.5261) \quad => \quad x_4 = 0.9128$$

$$=> h_4 = 188.42 + 0.9128 (2394.77) = 2374.4 \text{ kJ/kg}$$

C.V. Condenser : $q_L = h_4 - h_1 = 2374.4 - 188.42 = 2186$ kJ/kg

$$\dot{Q}_L = \dot{m}q_L = 25 \times 2186 = 54.65 \text{ MW} = \dot{m}_{ocean} \, C_p \, \Delta T$$

$$\dot{m}_{ocean} = \dot{Q}_L / C_p \, \Delta T = 54\,650 / (4.18 \times 3) = 4358 \text{ kg/s}$$

The net drop in exergy of the water is

$$\dot{\Phi}_{water} = \dot{m}_{water} [h_4 - h_1 - T_o(s_4 - s_1)]$$

$$= 25 [2374.4 - 188.4 - 285 (7.5084 - 0.6386)]$$

$$= 54\,650 - 48\,947 = \mathbf{5703 \text{ kW}}$$

The net gain in exergy of the ocean water is

$$\dot{\Phi}_{ocean} = \dot{m}_{ocean}[h_6 - h_5 - T_o(s_6 - s_5)]$$

$$= \dot{m}_{ocean}[C_p(T_6 - T_5) - T_oC_p \ln(\frac{T_6}{T_5})]$$

$$= 4358 [4.18(15 - 12) - 285 \times 4.18 \ln \frac{273 + 15}{273 + 12}]$$

$$= 54\,650 - 54\,364 = \mathbf{286 \text{ kW}}$$

The second law efficiency is

$$\eta_{II} = \dot{\Phi}_{ocean} / \dot{\Phi}_{water} = \frac{286}{5703} = \mathbf{0.05}$$

In reality all the exergy in the ocean water is destroyed as the 15°C water mixes with the ocean water at 12°C after it flows back out into the ocean and the efficiency does not have any significance. Notice the small rate of exergy relative to the large rates of energy being transferred.

9.120

Find the second law efficiency for the open FWH in Problem 9.40

The setup follows Fig.9.12

State enthalpies [kJ/kg]: $h_2 = 419.02$, $h_6 = 2827.86$, $h_3 = 762.79$ (x = 0 at 1 MPa)

State entropies [kJ/kgK]: $s_2 = 1.3068$, $s_6 = 6.6939$, $s_3 = 2.1386$

Analyze the FWH which leads to Eq.9.5:

$$y = \dot{m}_6 / \dot{m}_5 \quad \text{the extraction fraction} \ (\dot{m}_5 = \dot{m}_3)$$

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.79 - 419.02}{2827.86 - 419.02} = 0.1427$$

The second law efficiency for this FWH can be done similar to a co-flowing heat exchanger. The extraction flow, fraction y of total flow, provides the source of exergy as it goes from state 6 to state 3. The feedwater flow, with fraction (1 – y), increase its exergy from state 2 to state 3.

$$\psi_3 - \psi_2 = h_3 - h_2 - T_o(s_3 - s_2)$$
$$= 762.79 - 419.02 - 298.15(2.1386 - 1.3068)$$
$$= 95.769 \text{ kJ/kg}$$
$$\psi_6 - \psi_3 = h_6 - h_3 - T_o(s_6 - s_3)$$
$$= 2827.86 - 762.79 - 298.15(6.6939 - 2.1386)$$
$$= 706.907 \text{ kJ/kg}$$

$$\eta_{II} = \frac{(1 - y)(\psi_3 - \psi_2)}{y(\psi_6 - \psi_3)} = \frac{0.8573 \times 95.769}{0.1427 \times 706.907} = \mathbf{0.814}$$

9.121

Find the second law efficiency for the closed FWH in Problem 9.51

From table B.1	h kJ/kg	s kJ/kg-K
B.1.4: 100°C, 20 MPa	$h_2 = 434.04$	1.2917
B.1.4: 200°C, 20 MPa	$h_4 = 860.47$	2.3031
B.1.3: 4 MPa, 275°C	$h_6 = 2880.39$	6.2145
B.1.2: 4 MPa, sat. liq.	$h_{6a} = 1087.29$	2.7963

C.V. Feedwater Heater

Energy Eq.: $\dot{m}_2 h_2 + \dot{m}_6 h_6 = \dot{m}_2 h_4 + \dot{m}_6 h_{6a}$

Since all four states are known we can solve for the extraction flow rate

$$\dot{m}_6 = \dot{m}_2 \frac{h_2 - h_4}{h_{6a} - h_6} = 20 \frac{434.04 - 860.47}{1087.29 - 2880.39} \text{ kg/s} = 4.756 \text{ kg/s}$$

The feedwater from state 2 to state 4 gains exergy from the extraction flow which provides the source as it goes from state 6 to state 6a.

$$\psi_4 - \psi_2 = h_4 - h_2 - T_o(s_4 - s_2)$$
$$= 860.47 - 434.04 - 298.15(2.3031 - 1.2917)$$
$$= 124.881 \text{ kJ/kg}$$
$$\psi_6 - \psi_{6a} = h_6 - h_{6a} - T_o(s_6 - s_{6a})$$
$$= 2880.39 - 1087.29 - 298.15(6.2145 - 2.7963)$$
$$= 773.964 \text{ kJ/kg}$$

$$\eta_{II} = \frac{\dot{m}_2 (\psi_4 - \psi_2)}{\dot{m}_6 (\psi_6 - \psi_{6a})} = \frac{20 \times 124.881}{4.756 \times 773.964} = \mathbf{0.679}$$

9.122

For problem 9.58 consider the boiler/super-heater. Find the exergy destruction in this setup and the second law efficiency for the boiler-source set-up.

A Rankine cycle feeds 5 kg/s ammonia at 2 MPa, 140^oC to the turbine, which has an extraction point at 800 kPa. The condenser is at -20^oC and a closed feed water heater has an exit state (3) at the temperature of the condensing extraction flow and it has a drip pump. The source for the boiler is at constant 180^oC. Find the extraction flow rate and state 4 into the boiler.

The boiler has flow in at state 4 and out at state 5 with the source providing a q at 180^oC.

Assume state 4 is saturated liquid at T_4 so $h_4 = h_{f4}$ => $T_4 = 17.92^oC$

State 4: $h_4 = 264.4$ kJ/kg $= h_{f4}$, $s_4 = s_{f4} = 1.00705$ kJ/kgK

State 5: $h_5 = 1738.2$ kJ/kg, $s_5 = 5.5022$ kJ/kg K

Energy Eq. $q_H = h_5 - h_4 = 1738.2 - 264.4 = 1473.8$ kJ/kg

Entropy Eq. $s_4 + q_H/T_H + s_{gen} = s_5$ => $s_{gen} = s_5 - s_4 - q_H/T_H$

$$\dot{S}_{gen} = \dot{m}s_{gen} = 5 [5.5022 - 1.00705 - \frac{1473.8}{180 + 273.15}]$$
$$= 5 \times 1.2428 = \textbf{6.21 kW/K}$$

The flow increase in exergy is:

$$\psi_5 - \psi_4 = h_5 - h_4 - T_o (s_5 - s_4) = 1473.8 - 298.15 (5.5022 - 1.00705)$$
$$= 133.57 \text{ kJ/kg}$$

The exergy provided by the source is:

$$\phi_H = (1 - T_o/ T_H) q_H = (1 - \frac{298.15}{180 + 273.15}) 1473.8 = 504.1 \text{ kJ/kg}$$

Second law efficiency is

$$\eta_{II} = \frac{\text{gain in exergy}}{\text{source exergy input}} = \frac{133.57}{504.1} = \textbf{0.265}$$

$$(= 1 - T_o s_{gen}/\phi_H = 1 - 298.15 \times 1.2428 / 504.1)$$

9.123

The power plant using ammonia in Problem 9.71 has a flow of liquid water at 120^oC, 300 kPa as a heat source, the water leaves the heat exchanger at 90^oC. Find the second law efficiency of this heat exchanger.

C.V. The liquid water source - ammonia boiler heat exchanger.

$\Delta h_{liq} = 503.69 - 376.9 = 126.79$ kJ/kg   (sat. liquid at 120^oC and 90^oC)

$\Delta s_{liq} = 1.5275 - 1.1924 = 0.3351$ kJ/kgK

The energy equation establishes the ratio of the mass flow rates

$$\dot{Q}_H = \dot{m}_{water}\, \Delta h_{liq} = \dot{m}_{NH3}\, q_H$$

$$\Rightarrow \quad \dot{m}_{water} / \dot{m}_{NH3} = q_H / \Delta h_{liq} = 1442/126.79 = 11.373$$

Now the second law efficiency is the ratio of exergy pick-up over exergy source

$$\eta_{II} = \frac{\dot{m}_{NH3}\, \Delta\psi}{\dot{m}_{water}\, \Delta\psi} = \frac{\dot{m}_{NH3}\, [q_H - T_o(s_5 - s_3)]}{\dot{m}_{water}\, (\Delta h_{liq} - T_o\Delta s_{liq})}$$

From the power plant cycle we have state 3:

State 3: $x_3 = 0$, $h_3 = 171.65$ kJ/kg, $s_3 = 0.6793$ kJ/kgK, $v_3 = 0.00156$ m^3/kg

State 5: $h_5 = 1614.6$ kJ/kg, $s_5 = 5.4971$ kJ/kg K

C.V Pump P2: $w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.00156(1000 - 400) = 0.936$ kJ/kg

$$q_H = h_5 - h_4 = 1614.6 - (171.65 + 0.936) = 1442 \text{ kJ/kg}$$
$$s_5 - s_3 = 5.4971 - 0.6793 = 4.81775 \text{ kJ/kgK}$$

$$\eta_{II} = \frac{1}{11.373}\ \frac{1442 - 253.15 \times 4.81775}{126.79 - 253.15 \times 0.3351} = \mathbf{0.466}$$

9.124

A concentrated solar power plant receives the energy from molten salt coming in at 560°C and leaving at 300°C in counterflow heat exchanger where the water comes in at 3 MPa, 60°C and leaves at 450°C, 3 MPa. The molten salt has 5 kg/s flow with $C_{P\,salt}$ = 1.5 kJ/kg-K. What is the possible water flow rate and the rate of energy transfer. Find the second law efficiency of this heat exchanger.

Continuity Eqs.:   Each line has a constant flow rate through it.

Energy Eq.4.10: $0 = \dot{m}_{H_2O}\,(h_1 - h_2) + \dot{m}_{salt}\,(h_3 - h_4)$

Entropy Eq.7.7: $0 = \dot{m}_{H_2O}\,(s_1 - s_2) + \dot{m}_{salt}\,(s_3 - s_4) + \dot{S}_{gen}$

Process: Each line has a constant pressure.

Table B.1: $h_1 = 253.61$ kJ/kg, $h_2 = 3344$ kJ/kg, $s_1 = 0.8295$, $s_2 = 7.0833$ kJ/kg-K

$\psi_2 - \psi_1 = [h_2 - h_1 - T_0(s_2 - s_1)]$

$= [3344 - 253.61 - 298.15\,(7.0833 - 0.8295)] = 1225.82$ kJ/kg

For the salt we get

$h_3 - h_4 = C_{P\,salt}\,(T_3 - T_4) = 1.5\,(560 - 300) = 390$ kJ/kg

$s_3 - s_4 = C_{P\,salt}\,\ln\,(T_3/T_4) = 1.5\,\ln(833.15/573.15) = 0.561$ kJ/kgK

$\psi_3 - \psi_4 = [h_3 - h_4 - T_0(s_3 - s_4)] = 390 - 298.15 \times 0.561 = 222.74$ kJ/kgK

$$\dot{m}_{H_2O} = \dot{m}_{salt}\,\frac{h_3 - h_4}{h_2 - h_1} = \frac{390}{3344 - 253.61} = 0.631 \text{ kg/s}$$

$$\eta_{II} = \frac{\dot{m}_{H2O}(\psi_2 - \psi_1)}{\dot{m}_{salt}(\psi_3 - \psi_4)} = \frac{0.631 \times 1225.82}{5 \times 222.74} = \textbf{0.69}$$

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for salt and water each. The two flows exchange energy with no heat transfer to/from the outside.

9.125

What is the second law efficiency of the heat pump in Problem 9.92?

A heat pump for heat upgrade uses ammonia with a low temperature of 25°C and a high pressure of 5000 kPa. If it receives 1 MW of shaft work what is the rate of heat transfer at the high temperature?

State 1: $h_1 = 1463.5$ kJ/kg, $s_1 = 5.0293$ kJ/kgK

State 3: $h_3 = h_f = 631.9$ kJ/kg, $s_3 = 2.1100$ kJ/kg-K

Entropy compressor: $s_2 = s_1$ => $T_2 = 156^oC$, $h_2 = 1709.1$ kJ/kg

Energy eq. compressor: $w_C = h_2 - h_1 = 245.6$ kJ/kg

Energy condenser: $q_H = h_2 - h_3 = 1077.2$ kJ/kg

Exergy output: $\Delta\psi_H = \psi_2 - \psi_3 = h_2 - h_3 - T_0(s_2 - s_3)$
$$= 1077.2 - 298\,(5.0293 - 2.1100)$$
$$= 207.25 \text{ kJ/kg}$$

$$\eta_{II} = \frac{\text{output}}{\text{source}} = \frac{\Delta\psi_H}{w_{C\,ac}} = \frac{207.25}{245.6} = \mathbf{0.844}$$

9.126

Steam is supplied in a line at 3 MPa, 700°C. A turbine with an isentropic efficiency of 85% is connected to the line by a valve and it exhausts to the atmosphere at 100 kPa. If the steam is throttled down to 2 MPa before entering the turbine find the actual turbine specific work. Find the change in exergy through the valve and the second law efficiency of the turbine.

Take C.V. as valve and a C.V. as the turbine.

Valve: $h_2 = h_1 = 3911.7$ kJ/kg, $s_2 > s_1 = 7.7571$ kJ/kg K,

$h_2, P_2 \implies s_2 = 7.9425$ kJ/kg K

$\psi_1 - \psi_2 = h_1 - h_2 - T_0(s_1-s_2) = 0 - 298.15(7.7571-7.9425) = $ **55.3 kJ/kg**

So some potential work is lost in the throttling process.

Ideal turbine: $s_3 = s_2 \implies h_{3s} = 2929.13$ kJ/kg $w_{T,s} = 982.57$ kJ/kg

$w_{T,ac} = h_2 - h_{3ac} = \eta w_{T,s} = $ **835.2 kJ/kg**

$h_{3ac} = 3911.7 - 835.2 = 3076.5 \implies s_{3ac} = 8.219$ kJ/kg K

$w^{rev} = \psi_2 - \psi_{3ac} = h_2 - h_{3ac} - T_0(s_2 - s_{3ac}) = 835.2 - 298.15(7.9425 - 8.219)$

$= 917.63$ kJ/kg $\implies$

$\eta_{II} = w_{T,ac} / w^{rev} = 835.2/917.63 = $ **0.91**

9.127

The condenser in a refrigerator receives R-134a at 700 kPa, 50°C and it exits as saturated liquid at 25°C. The flowrate is 0.1 kg/s and the condenser has air flowing in at ambient 15°C and leaving at 35°C. Find the minimum flow rate of air and the heat exchanger second-law efficiency.

C.V. Total heat exchanger.
Energy Eq.6.10

$$\dot{m}_1 h_1 + \dot{m}_a h_3 = \dot{m}_1 h_2 + \dot{m}_a h_4$$

$$\Rightarrow \quad \dot{m}_a = \dot{m}_1 \times \frac{h_1 - h_2}{h_4 - h_3}$$

$$= 0.1 \times \frac{436.89 - 234.59}{1.004(35 - 15)} = \textbf{1.007 kg/s}$$

Availability from Eq.8.24

$$\psi_1 - \psi_2 = h_1 - h_2 - T_0(s_1 - s_2) = 436.89 - 234.59$$

$$- 288.15(1.7919 - 1.1201) = 8.7208 \text{ kJ/kg}$$

$$\psi_4 - \psi_3 = h_4 - h_3 - T_0(s_4 - s_3)$$

$$= 1.004(35 - 15) - 288.15 \times 1.004 \times \ln \frac{308.15}{288.15} = +0.666 \text{ kJ/kg}$$

Efficiency from Eq.8.30

$$\eta_{II} = \frac{\dot{m}_a \ (\psi_4 - \psi_3)}{\dot{m}_1 \ (\psi_1 - \psi_2)} = \frac{1.007 \times 0.666}{0.1 \ \times 8.7208} = \textbf{0.77}$$

Borgnakke and Sonntag

9.128

A flow of steam at 10 MPa, 550°C goes through a two-stage turbine. The pressure between the stages is 2 MPa and the second stage has an exit at 50 kPa. Assume both stages have an isentropic efficiency of 85%. Find the second law efficiencies for both stages of the turbine.

CV: T1, $h_1 = 3500.9$ kJ/kg, $s_1 = 6.7561$ kJ/kg K

Isentropic $s_{2s} = s_1 \Rightarrow h_{2s} = 3017.9$ kJ/kg

$w_{T1,s} = h_1 - h_{2s} = 483$ kJ/kg

Actual T1: $w_{T1,ac} = \eta_{T1}\, w_{T1,s} = 410.55 = h_1 - h_{2ac}$

$h_{2ac} = h_1 - w_{T1,ac} = 3090.35$ kJ/kg, $s_{2ac} = 6.8782$ kJ/kg K

CV: T2, $s_{3s} = s_{2ac} = 6.8782 \Rightarrow x_{3s} = (6.8782-1.091)/6.5029 = 0.8899$,

$h_{3s} = 340.47 + 0.8899 \times 2305.4 = 2392.2$ kJ/kg

$w_{T2,s} = h_{2ac} - h_{3s} = 698.15 \Rightarrow w_{T2,ac} = \eta_{T2}\, w_{T2,s} = 593.4$ kJ/kg

$\Rightarrow h_{3ac} = 2496.9$, $x_{3ac} = (2496.9 - 340.47)/2305.4 = 0.9354$,

$s_{3ac} = 1.091 + 0.9354 \times 6.5029 = 7.1736$ kJ/kg K

Actual T1: $i_{T1,ac} = T_0(s_{2ac}-s_1) = 298.15(6.8782 - 6.7561) = 36.4$ kJ/kg

$\Rightarrow w^R_{T1} = w_{T1,ac} + i = 447$ kJ/kg, $\eta_{II} = w_{T1,ac}/w^R_{T1} = \mathbf{0.918}$

Actual T2: $i_{T2,ac} = T_0(s_{3ac}-s_{2ac}) = 298.15(7.1736 - 6.8782) = 88.07$ kJ/kg

$\Rightarrow w^R_{T2} = w_{T2,ac} + i_{T2,ac} = 681.5$, $\eta_{II} = w_{T2,ac}/w^R_{T2} = \mathbf{0.871}$

9.129

A new air-conditioner using R-410A is used in heat pump mode. The high pressure is 2000 kPa and the low pressure is 400 kPa. It warms a house at 20°C driven by an electric power input of 2 kW in an ambient at -5°C. Find the destruction of exergy 4 places: 1) inside the heat pump, 2) in the high T heat exchanger, 3) in the low T heat exchanger, and 4) in the house walls/windows etc. that separates the inside from the outside of the house.

Exit evaporator x = 1, 400 kPa from B.4.2: $h_1 = 271.9$, $s_1 = 1.0779$ kJ/kgK

Exit condenser x = 0, 2 MPa from B.4.1: $h_3 = 110.21$ kJ/kg, $s_3 = 0.40378$ kJ/kgK

Exit valve: $h_3 = h_4 = 28.24 + x_4\ 243.66 \Rightarrow x_4 = 0.33638$, $s_4 = 0.43917$

Compressor: $s_2 = s_1$ & P_2 from B.4.2 $\Rightarrow$ $h_2 \approx 317.43$ kJ/kg

Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 271.9 - 110.21 = 161.69$ kJ/kg

Condenser: $q_H = h_2 - h_3 = 317.43 - 110.21 = 207.22$ kJ/kg

Compressor: $w_c = h_2 - h_1 = 317.43 - 271.9 = 45.53$ kJ/kg,

$$\dot{m} = \dot{W}/w_c = 2\ \text{kW}/45.53\ \text{kJ/kg} = 0.04393\ \text{kg/s}$$

COP: $\beta = q_H/w_c = 207.22/45.53 = 4.55$

Heat transfers: $\dot{Q}_H = \beta\ \dot{W} = 4.55 \times 2 = 9.1$ kW, $\dot{Q}_L = 9.1 - 2 = 7.1$ kW

CV: The A/C unit, entropy only made in valve.

$$\dot{\Phi}_{\text{destr. HP}} = T_0 \dot{S}_{\text{gen}} = \dot{m}\ T_0\ (s_4 - s_3)$$
$$= 0.04393\ \text{kg/s} \times 298.15\ \text{K}\ (0.43917 - 0.40378)\ \text{kJ/kg-K} = \mathbf{0.46\ kW}$$

CV High T heat exchanger with Q from T_2, $T_3 = 32^o$C to room 20°C

Entropy: $0 = \dot{m}\ (s_2 - s_3) - \dot{Q}_H/T_{\text{house}} + \dot{S}_{\text{gen}} \Rightarrow \dot{S}_{\text{gen}} = \dot{Q}_H/T_{\text{house}} - \dot{m}\ (s_2 - s_3)$

$$\dot{\Phi}_{\text{destr. Hi T}} = T_0 \dot{S}_{\text{gen}} = 298.15\ [\ \frac{9.1}{293.15} - 0.04393 \times (1.0779 - 0.40378)]$$
$$= \mathbf{0.426\ kW}$$

CV Low T heat exchanger, here $T_1 = -19.98^o$C is constant as heat is absorbed, the term $\dot{Q}_L/T_1$ is also equal to $\dot{m}\ (s_1 - s_4)$

$$\dot{\Phi}_{\text{destr. Low T}} = T_0 \dot{S}_{\text{gen}} = T_0[\dot{Q}_L/T_1 - \dot{Q}_L/T_{\text{amb}}] = 298.15(\frac{7.1}{253.17} - \frac{7.1}{268.15})$$
$$= \mathbf{0.467\ kW}$$

CV House walls that leaks heat out from 20^oC to ambient 5^oC.

$$\dot{\Phi}_{\text{destr. walls}} = T_0\,\dot{S}_{\text{gen}} = T_0\,[\dot{Q}_H/T_{\text{amb}} - \dot{Q}_H/T_{\text{house}}]$$

$$= 298.15(\frac{9.1}{268.15} - \frac{9.1}{293.15}) = \textbf{0.863 kW}$$

9.130

An air-conditioner using R-410A is used in cooling mode. The high pressure is 3000 kPa and the low pressure is 800 kPa. It cools a house at 20^oC with a rate of 12 kW and the outside ambient is at 35^oC. Find the destruction of exergy 4 places: 1) inside the refrigerator, 2) in the high T heat exchanger, 3) in the low T heat exchanger, and 4) in the house walls/windows etc. that separates the inside from the outside of the house.

The exergy destruction can be found from the exergy balance or from the entropy generation for each control volume

The control volumes 1-4 correspond to the exergy destruction locations. To answer the first one and to find the COP and the high T heat transfer we must analyze the refrigeration cycle.

State 1 (800 kPa, x = 1): $h_1 = 279.14$ kJ/kg, $s_1 = 1.0367$ kJ/kgK, $T_1 = 0.05^oC$

State 2 (3000 kPa, s = s_1): $h_2 = 315.58$ kJ/kg, $T_2 = 70.5\ ^oC$

State 3 (3000 kPa, x = 0): $h_3 = 141.78$ kJ/kg, $s_3 = 0.5011$ kJ/kgK, $T_3 = 49.07\ ^oC$

State 4 (800 kPa, h = h_3): $x_4 = (141.78 - 57.8365)/221.304 = 0.37931$

$$s_4 = 0.22667 + x_4\, 0.81003 = 0.53392 \text{ kJ/kgK}$$

Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 279.14 - 141.78 = 137.36$ kJ/kg

Condenser: $q_H = h_2 - h_3 = 315.58 - 141.78 = 173.8$ kJ/kg

Compressor: $w_c = h_2 - h_1 = 315.58 - 279.14 = 36.44$ kJ/kg,

$$\dot{m} = \dot{Q}_L / q_L = 12 \text{ kW}/137.36 \text{ kJ/kg} = 0.08736 \text{ kg/s}$$

COP: $\beta = q_L/w_c = 137.36/36.44 = 3.7695$

$$\dot{W} = \dot{Q}_L /\beta = 12 \text{ kW}/3.7695 = 3.183 \text{ kW}; \qquad \dot{Q}_H = \dot{W} + \dot{Q}_L = 15.183 \text{ kW}$$

CV1. The only entropy generation is in the valve:

$$\dot{\Phi}_{\text{destr. ref}} = T_0\, \dot{S}_{\text{gen}} = \dot{m}\, T_0\, (s_4 - s_3)$$

$$= 0.08736 \text{ kg/s} \times 298.15 \text{ K } (0.53392 - 0.5011) \text{ kJ/kg-K} = \textbf{0.855 kW}$$

CV2. The heat transfer goes to the ambient from the 2-3 condenser process.

Entropy Eq.: $0 = \dot{m}(s_2 - s_3) - \dot{Q}_H/T_{amb} + \dot{S}_{gen}$

$$\dot{\Phi}_{destr. \ Hi \ T} = T_0 \dot{S}_{gen} = T_0[\ \dot{Q}_H/T_{amb} - \dot{m}(s_2 - s_3)]$$
$$= 298.15[\ 15.183/308.15 - 0.08736(1.0367 - 0.5011)]$$
$$= \textbf{0.74 kW}$$

CV3. Low T heat exchanger, here $T_1 = 0.05^oC$ is constant as heat is absorbed,

the term $\dot{Q}_L/T_1$ is also equal to $\dot{m}(s_1 - s_4)$

$$\dot{\Phi}_{destr. \ Low \ T} = T_0 \dot{S}_{gen} = T_0[\dot{Q}_L/T_1 - \dot{Q}_L/T_{house}] = 298.15(\frac{12}{273.2} - \frac{12}{293.15})$$
$$= \textbf{0.891 kW}$$

CV House walls that leaks heat from ambient 35^oC to house 20^oC.

$$\dot{\Phi}_{destr. \ walls} = T_0 \dot{S}_{gen} = T_0[\dot{Q}_L/T_{house} - \dot{Q}_L/T_{amb}]$$
$$= 298.15(\frac{12}{293.15} - \frac{12}{308.15}) = \textbf{0.594 kW}$$

The 4 destruction terms do not quite add up to the work input (3.183 kW) as the reference T =298.15 K = 25C, had we used 308.15 K instead they would add to the work term exactly.

9.131

Assume the house in the previous problem has a combined 12000 kg hard wood, 2500 kg gypsum plates (Cp = 1 kJ/kg-K) and 750 kg steel all of which is at 20°C. If the air conditioner is turned off how fast does the house heat up (°C/s)?

Since the A/C cools the house at a rate of 12 kW it means there must be a leak into the house of 12 kW coming from the warmer ambient. As the A/C is truned off it then heats with a rate of 12 kW. As the house T goes up that rate drops exponentially in time until the house reaches the ambient 35C.

Energy Eq. house: $\quad\quad\quad\dfrac{dE}{dt} = \dfrac{dU}{dt} = m\ C\dfrac{dT}{dt} = \dot{Q}_{leak}$

The thermal capacity is

$\quad\quad m\ C = (mC)_{wood} + (mC)_{gypsum} + (mC)_{steel}$

$\quad\quad\quad\quad = 12\ 000 \times 1.26 + 2500 \times 1 + 750 \times 0.46 = 17\ 965\ kJ/K$

Then

$\quad\quad \dfrac{dT}{dt} = \dot{Q}_{leak}/\ m\ C = 12\ kW\ /\ 17\ 965\ kJ/K = \mathbf{0.000668\ K/s}$

9.132

Assume the house in problem 9.129 has a combined 12000 kg hard wood, 2500 kg gypsum plates (C_p = 1 kJ/kg-K) and 750 kg steel all of which is at 20°C. If the heat pump is turned off how fast does the house cool down (^{o}C/s)

Since the heat pump heats the house at a rate of 9.1 kW it means there must be a leak out of the house of 9.1 kW going to the colder ambient. As the heat pump is truned off the house then cools with a rate of 9.1 kW. As the house T drops the rate drops exponentially in time until the house reaches the ambient -5C.

Energy Eq. house:
$$\frac{dE}{dt} = \frac{dU}{dt} = m\ C\frac{dT}{dt} = -\dot{Q}_{leak}$$

The thermal capacity is

$$m\ C = (mC)_{wood} + (mC)_{gypsum} + (mC)_{steel}$$
$$= 12\ 000 \times 1.26 + 2500 \times 1 + 750 \times 0.46 = 17\ 965\ kJ/K$$

Then

$$\frac{dT}{dt} = -\dot{Q}_{leak}/\ m\ C = \text{-9.1 kW / 17 965 kJ/K} = \textbf{-0.000506 K/s}$$

Combined Cycles

9.133

A binary system power plant uses mercury for the high-temperature cycle and water for the low-temperature cycle, as shown in Fig. 12.20. The temperatures and pressures are shown in the corresponding T–s diagram. The maximum temperature in the steam cycle is where the steam leaves the superheater at point 4 where it is 500°C. Determine the ratio of the mass flow rate of mercury to the mass flow rate of water in the heat exchanger that condenses mercury and boils the water and the thermal efficiency of this ideal cycle.

The following saturation properties for mercury are known

P, MPa	T_g, °C	h_f, kJ/kg	h_g, kJ/kg	s_f kJ/kgK	s_g, kJ/kgK
0.04	309	42.21	335.64	0.1034	0.6073
1.60	562	75.37	364.04	0.1498	0.4954

Solution:

For the mercury cycle:

$$s_d = s_c = 0.4954 = 0.1034 + x_d \times 0.5039, \quad x_d = 0.7779$$

$$h_b = h_a - w_{P\ HG} \approx h_a \ (\text{since } v_F \text{ is very small})$$

$$q_H = h_c - h_a = 364.04 - 42.21 = 321.83 \text{ kJ/kg}$$

$$q_L = h_d - h_a = 270.48 - 42.21 = 228.27 \text{ kJ/kg}$$

For the steam cycle:

$$s_5 = s_4 = 7.0097 = 0.6493 + x_5 \times 7.5009, \quad x_5 = 0.8480$$

$$h_5 = 191.83 + 0.848 \times 2392.8 = 2220.8$$

$$w_P \approx v_1(P_2 - P_1) = 0.00101(4688 - 10) = 4.7 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 191.8 + 4.7 = 196.5$$

$$q_H \text{ (from Hg)} = h_3 - h_2 = 2769.9 - 196.5 = 2600.4$$

$$q_H \text{ (ext. source)} = h_4 - h_3 = 3437.4 - 2796.9 = 640.5$$

CV: Hg condenser - H_2O boiler: 1st law: $m_{Hg}(h_d - h_a) = m_{H2O}(h_3 - h_2)$

$$m_{Hg}/m_{H2O} = \frac{2796.9 - 196.5}{270.48 - 42.21} = \textbf{11.392}$$

$$q_{H\ TOTAL} = (m_{Hg}/m_{H2O})(h_c - h_b) + (h_4 - h_3) \ \text{(for 1 kg } H_2O)$$

$$= 11.392 \times 321.83 + 640.5 = 4306.8 \text{ kJ}$$

All q_L is from the H_2O condenser:

$$q_L = h_5 - h_1 = 2220.8 - 191.8 = 2029.0 \text{ kJ}$$

$$w_{NET} = q_H - q_L = 4306.8 - 2029.0 = 2277.8 \text{ kJ}$$

$$\eta_{TH} = w_{NET}/q_H = 2277.8/4306.8 = \textbf{0.529}$$

9.134

A Rankine steam power plant should operate with a high pressure of 3 MPa, a low pressure of 10 kPa, and the boiler exit temperature should be 500°C. The available high-temperature source is the exhaust of 175 kg/s air at 600°C from a gas turbine. If the boiler operates as a counter flowing heat exchanger where the temperature difference at the pinch point is 20°C, find the maximum water mass flow rate possible and the air exit temperature.

Solution:

C.V. Pump

$w_P = h_2 - h_1 = v_1(P_2 - P_1)$

$\quad = 0.00101(3000 - 10) = 3.02$ kJ/kg

$h_2 = h_1 + w_P = 191.83 + 3.02 = 194.85$ kJ/kg

Heat exchanger water states

State 2a: $T_{2a} = T_{SAT} = 233.9$ °C

$\quad\quad\quad h_{2a} = 1008.42$ kJ/kg

State 3: $h_3 = 3456.5$ kJ/kg

Heat exchanger air states

inlet: $h_{air,in} = 903.16$ kJ/kg

State 2a: $h_{air}(T_{2a} + 20) = 531.28$ kJ/kg

Air temperature should be 253.9°C at the point where the water is at state 2a.

C.V. Section 2a-3, i-a

$$\dot{m}_{H_2O}(h_3 - h_{2a}) = \dot{m}_{air}(h_i - h_a)$$

$$\dot{m}_{H_2O} = 175 \; \frac{903.16 - 531.28}{3456.5 - 1008.42} \; = \textbf{26.584 kg/s}$$

Take C.V. Total: $\dot{m}_{H_2O}(h_3 - h_2) = \dot{m}_{air}(h_i - h_e)$

$\Rightarrow \; h_e = h_i - \dot{m}_{H_2O}(h_3 - h_2)/\dot{m}_{air}$

$\quad\quad = 903.6 - 26.584(3456.5 - 194.85)/175 = 408.13$ kJ/kg

$\Rightarrow \; T_e = 406.7$ K $= \textbf{133.6 °C}$, $T_e > T_2 = 46.5$ °C OK.

9.135

Consider an ideal dual-loop heat-powered refrigeration cycle using R-134a as the working fluid, as shown in Fig. P9.135. Saturated vapor at 90°C leaves the boiler and expands in the turbine to the condenser pressure. Saturated vapor at −15°C leaves the evaporator and is compressed to the condenser pressure. The ratio of the flows through the two loops is such that the turbine produces just enough power to drive the compressor. The two exiting streams mix together and enter the condenser. Saturated liquid leaving the condenser at 45°C is then separated into two streams in the necessary proportions. Determine the ratio of mass flow rate through the power loop to that through the refrigeration loop. Find also the

performance of the cycle, in terms of the ratio $\dot{Q}_L / \dot{Q}_H$.

Solution:

$T_1 = -15\ ^{\circ}C$ sat. vap.

Table B.5.1 $T_6 = 90^{\circ}C$ sat. vapor $=>$ $P_5 = P_6 = 3.2445$ MPa

Table B.5.1 $T_3 = 45^{\circ}C$ sat. liquid $=>$ $P_2 = P_3 = P_7 = 1.1602$ MPa

$$h_1 = 389.2;\ \ h_3 = h_4 = 264.11;\ \ h_6 = 425.7\ \text{kJ/kg}$$

C.V. Turbine

$$s_7 = s_6 = 1.6671 = 1.2145 + x_7 \times 0.4962;\qquad x_7 = 0.9121$$

$$h_7 = 264.11 + 0.9121 \times 157.85 = 408.08\ \text{kJ/kg}$$

C.V. Compressor   (computer tables are used for this due to value of P or you do a double linear interpolation between 1000 and 1200 kPa to get 1160.2 kPa)

$$s_2 = s_1 = 1.7354,\ P_2\ => \quad T_2 = 52.27^{\circ}C,\quad h_2 = 429.9\ \text{kJ/kg}$$

CV: turbine + compressor

Continuity Eq.: $\dot{m}_1 = \dot{m}_2,\ \ \dot{m}_6 = \dot{m}_7$;

Energy Eq.: $\dot{m}_1 h_1 + \dot{m}_6 h_6 = \dot{m}_1 h_2 + \dot{m}_6 h_7$

$$\dot{m}_6/\dot{m}_1 = \frac{h_2 - h_1}{h_6 - h_7} = \frac{429.9 - 389.2}{425.7 - 408.08} = \textbf{2.31}$$

CV: pump

$$w_P = v_3(P_5 - P_3) = 0.00089(3244.5 - 1160.2) = 1.855 \text{ kJ/kg}$$

$$h_5 = h_3 + w_P = 264.11 + 1.855 = 265.96 \text{ kJ/kg}$$

CV: evaporator $\Rightarrow$ $\dot{Q}_L = \dot{m}_1 (h_1 - h_4)$

CV: boiler $\Rightarrow$ $\dot{Q}_H = \dot{m}_6 (h_6 - h_5)$

$$\beta = \frac{\dot{Q}_L}{\dot{Q}_H} = \frac{\dot{m}_1(h_1 - h_4)}{\dot{m}_6(h_6 - h_5)} = \frac{389.2 - 264.11}{2.31 \ (425.7 - 265.96)} = \textbf{0.34}$$

9.136

For a cryogenic experiment heat should be removed from a space at 75 K to a reservoir at 180 K. A heat pump is designed to use nitrogen and methane in a cascade arrangement (see Fig. 9.28), where the high temperature of the nitrogen condensation is at 10 K higher than the low-temperature evaporation of the methane. The two other phase changes take place at the listed reservoir temperatures. Find the saturation temperatures in the heat exchanger between the two cycles that gives the best coefficient of performance for the overall system.

The nitrogen cycle is the bottom cycle and the methane cycle is the top cycle. Both are standard refrigeration cycles.

$$T_{Hm} = 180 \text{ K} = T_{3m}, \quad T_{LN} = 75 \text{ K} = T_{4N} = T_{1N}$$

$$T_{Lm} = T_{4m} = T_{1m} = T_{3N} - 10, \quad \text{Trial and error on } T_{3N} \text{ or } T_{Lm}.$$

For each cycle we have,

$$-w_C = h_2 - h_1, \quad s_2 = s_1, \quad -q_H = h_2 - h_3, \quad q_L = h_1 - h_4 = h_1 - h_3$$

Nitrogen: $T_4 = T_1 = 75 \text{ K} \implies h_1 = 74.867 \text{ kJ/kg}, \quad s_1 = 5.4609 \text{ kJ/kg K}$

N_2	T_3	h_3	P_2	h_2	$-w_c$	$-q_H$	q_L
a)	120	-17.605	2.5125	202.96	128.1	220.57	92.47
b)	115	-34.308	1.9388	188.35	113.5	222.66	109.18
c)	110	-48.446	1.4672	173.88	99.0	222.33	123.31

Methane: $T_3 = 180 \text{ K} \implies h_3 = -0.5 \text{ kJ/kg}, \quad P_2 = 3.28655 \text{ MPa}$

CH_4	T_4	h_1	s_1	h_2	$-w_c$	$-q_H$	q_L
a)	110	221	9.548	540.3	319.3	540.8	221.5
b)	105	212.2	9.691	581.1	368.9	581.6	212.7
c)	100	202.9	9.851	629.7	426.8	630.2	203.4

The heat exchanger that connects the cycles transfers a Q

$$\dot{Q}_{Hn} = q_{Hn} \dot{m}_n = \dot{Q}_{Lm} = q_{Lm} \dot{m}_m \implies \dot{m}_m/\dot{m}_n = q_{Hn}/q_{Lm}$$

The overall unit then has

$$\dot{Q}_{L\ 75\ K} = \dot{m}_n q_{Ln}; \quad \dot{W}_{tot\ in} = -(\dot{m}_n w_{cn} + \dot{m}_m w_{cm})$$

$$\beta = \dot{Q}_{L\ 75\ K}/\dot{W}_{tot\ in} = q_{Ln}/[-w_{cn} - (\dot{m}_m/\dot{m}_n)w_{cm}]$$

Case	$\dot{m}_m/\dot{m}_n$	$w_{cn}+(\dot{m}_m/\dot{m}_n)w_{cm}$	β
a)	0.996	446.06	0.207
b)	1.047	499.65	0.219
c)	1.093	565.49	0.218

A maximum coefficient of performance is between case b) and c).

9.137

For Problem 9.134, determine the change of exergy of the water flow and that of the air flow. Use these to determine a second law efficiency for the boiler heat exchanger.

From solution to 9.134:

$$\dot{m}_{H2O} = 26.584 \text{ kg/s}, \quad h_2 = 194.85 \text{ kJ/kg}, \quad s_2 = 0.6587 \text{ kJ/kg K}$$

$$h_3 = 3456.5 \text{ kJ/kg}, \quad s_3 = 7.2338, \quad s^\circ_{Ti} = 7.9820, \quad s^\circ_{Te} = 7.1762 \text{ kJ/kg K}$$

$$h_i = 903.16 \text{ kJ/kg}, \quad h_e = 408.13 \text{ kJ/kg}$$

$$\psi_3 - \psi_2 = h_3 - h_2 - T_0(s_3 - s_2) = 1301.28 \text{ kJ/kg}$$

$$\psi_i - \psi_e = h_i - h_e - T_0(s^\circ_{Ti} - s^\circ_{Te}) = 254.78 \text{ kJ/kg}$$

$$\eta_{II} = \frac{(\psi_3 - \psi_2)\,\dot{m}_{H2O}}{(\psi_i - \psi_e)\,\dot{m}_{air}} = \frac{1301.28 \times 26.584}{254.78 \times 175} = \mathbf{0.776}$$

Review Problems

9.138

Do Problem 9.31 with R-134a as the working fluid in the Rankine cycle.

Consider the ammonia Rankine-cycle power plant shown in Fig. P9.31, a plant that was designed to operate in a location where the ocean water temperature is 25°C near the surface and 5°C at some greater depth. The mass flow rate of the working fluid is 1000 kg/s.

  a. Determine the turbine power output and the pump power input for the cycle.
  b. Determine the mass flow rate of water through each heat exchanger.
  c. What is the thermal efficiency of this power plant?

Solution:

a) Turbine

$$s_2 = s_1 = 1.7183 = 1.0485 + x_2 \times 0.6733 \qquad => \qquad x_2 = 0.9948$$

$$h_2 = 213.58 + 0.9948 \times 190.65 = 403.24 \text{ kJ/kg}$$

$$w_T = h_1 - h_2 = 409.84 - 403.24 = 6.6 \text{ kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = 6600 \text{ kW}$$

Pump: $w_P \approx v_3(P_4 - P_3) = 0.000794(572.8 - 415.8) = 0.125 \text{ kJ/kg}$

$$w_P = w_P /\eta_S = 0.125 \quad => \quad \dot{W}_P = \dot{m}w_P = \mathbf{125 \text{ kW}}$$

b)   Consider the condenser heat transfer to the low T water

$$\dot{Q}_{\text{to low T H2O}} = 1000(403.24 - 213.58) = 189\ 660 \text{ kW}$$

$$\dot{m}_{\text{low T H2O}} = \frac{189660}{29.38 - 20.98} = \mathbf{22\ 579 \text{ kg/s}}$$

$$h_4 = h_3 - w_P = 213.58 + 0.125 = 213.71 \text{ kJ/kg}$$

Now consider the boiler heat transfer from the high T water

$$\dot{Q}_{\text{from high T H2O}} = 1000(409.84 - 213.71) = 196\ 130 \text{ kW}$$

$$\dot{m}_{\text{high T H2O}} = \frac{196130}{104.87 - 96.50} = \mathbf{23\ 432 \text{ kg/s}}$$

c) $\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = \dfrac{6600 - 125}{196130} = \mathbf{0.033}$

9.139

A simple steam power plant is said to have the four states as listed: 1: (20^oC, 100 kPa), 2: (25^oC, 1 MPa), 3: (1000^oC, 1 MPa), 4: (250^oC, 100 kPa) with an energy source at 1100^oC and it rejects energy to a 0^oC ambient. Is this cycle possible? Are any of the devices impossible?

Solution:

The cycle should be like Figure 9.3 for an ideal or Fig.9.15 for an actual pump and turbine in the cycle. We look the properties up in Table B.1:

State 1: $h_1 = 83.94$, $s_1 = 0.2966$ State 2: $h_2 = 104.87$, $s_2 = 0.3673$

State 3: $h_3 = 4637.6$, $s_3 = 8.9119$ State 4: $h_4 = 2974.3$, $s_4 = 8.0332$

We may check the overall cycle performance

> Boiler: $q_H = h_3 - h_2 = 4637.6 - 104.87 = 4532.7$ kJ/kg
>
> Condenser: $q_L = h_4 - h_1 = 2974.3 - 83.94 = 2890.4$ kJ/kg
>
> $\eta_{cycle} = q_{net} / q_H = (q_H - q_L) / q_H = 1642.3 / 4532.7 = 0.362$
>
> $\eta_{carnot} = 1 - T_L / T_H = 1 - \dfrac{273.15}{273.15 + 1100} = \textbf{0.80} > \eta_{cycle}$ **OK**

Check the second law for the individual devices:

C.V. Boiler plus wall to reservoir

> $s_{gen} = s_3 - s_2 - \dfrac{q_H}{T_{res}} = 8.9119 - 0.3673 - \dfrac{4532.7}{1373} = 5.24$ kJ/kg K > 0 **OK**

C.V. Condenser plus wall to reservoir

> $s_{gen} = s_1 - s_4 + \dfrac{q_L}{T_{res}} = 0.2966 - 8.0332 + \dfrac{2890.4}{273} = 2.845$ kJ/kg K > 0 **OK**

C.V. Pump: $w_p = h_2 - h_1 = 20.93$ kJ/kg ;

> $s_{gen} = s_2 - s_1 = 0.3673 - 0.2966 = 0.0707$ kJ/kg K > 0 **OK**

C.V. Turbine: $w_T = h_3 - h_4 = 4637.6 - 2974.3 = 1663.3$ kJ/kg

> $s_{gen} = s_4 - s_3 = 8.0332 - 8.9119 = - 0.8787$ kJ/kg K

> > $s_{gen} < 0$ **NOT POSSIBLE**

9.140

A supercritical power plant has a high pressure of 30 MPa and the boiler heats the water to 500°C with 45°C in the condenser. To avoid a quality in the turbine of less than 92% determine the pressure(s) at which to make reheat (look only at the P's listed in Table B.1 do not interpolate between them) if the reheat takes it up to 400°C.

Let us work backwards from the lowest towards the highest pressures. What state should the steam come from if the final exit state is at 45°C and x = 0.92?

State 45°C and x = 0.92: s = 0.6386 + 0.92 × 7.5261 = 7.5626 kJ/kgK

The reheat should then produce 400°C, s ≥ 7.5626 kJ/kgK ,
B.1.2:   At **P = 800 kPa**, 400°C,   s = 7.5715 kJ/kgK.
          At 800 kPa, x = 0.92:   s = 2.0461 + 0.92 × 4.6166 = 6.2934 kJ/kgK

The next reheat should then produce 400°C, s ≥ 6.2934 kJ/kgK ,
B.1.2:   At **P = 8000 kPa**, 400°C,   s = 6.3633 kJ/kgK.
          At 8000 kPa, x = 0.92:   s = 3.2067 + 0.92 × 2.5365 = 5.5403 kJ/kgK

At 30 MPa, 500°C :   s = 5.7904 kJ/kgK so this is fine.

Comment: In reality you do not want any two-phase states in the high pressure sections of the turbine, so if that is a condition you expand from the inlet to a state the closest to sat. vapor (8000 kPa) reheat to 400 C, then expand again to near sat. vapor giving about 2000 kPa, reheat to 400 C and expand again for a final reheat at 500 kPa.

The processes for the different steps are shown generated by CATT3 using the plot process from the Options menu drop down select plot proces.

Starting from the 45°C and x = 0.92 a process is plotted with constant s to 400°C. Then starting with 400°C, 800 kPa a constant pressure process is plotted to x = 0.92 followed by another constant s process up to 400°C. Again a constant pressure process from 400°C, 8000 kPa to x = 0.92 followed by a constant s process from the original 30 MPa, 500°C down to 8000 kPa to verify the final step. The curves do not match precisely as the table entries were chosen to match the hand calculations.

The curves are shown on the following page.

Computer-Aided Thermodynamic Tables 3

File   Edit   Tables/Substances   Options   Help

Water Properties

T	500	C
P	30	MPa
V	0.008679	m3/kg
U	2821	kJ/kg
H	3081	kJ/kg
S	5.79	kJ/kg/K
X		

T-S Diagram

#	Temp	Pressure	Specific Volume	Internal Energy	Specific Enthalpy	Specific Entropy	Quality
	C	MPa	m3/kg	kJ/kg	kJ/kg	kJ/kg/K	
1	45	0.009593	14.04	2257	2392	7.563	0.92
2	400	0.815	0.3771	2959	3267	7.563	
3	400	0.8	0.3843	2960	3267	7.571	
4	170.4	0.8	0.2213	2428	2605	6.293	0.92
5	170.4	0.8	0.2213	2428	2605	6.293	0.92
6	400	8.893	0.03036	2850	3120	6.293	
7	400	8	0.03432	2864	3138	6.363	
8	295.1	8	0.02175	2469	2643	5.54	0.92
9	500	30	0.008679	2821	3081	5.79	
10	300	8	0.02425	2591	2785	5.79	

Water / Refrigerants / Cryogenics / Air / Ideal Gases / Compressibility / Psychrometrics /

Values: S = 0.1567; T = 802.33

P-V Diagram (Log-Log)

Notice how close to the saturated vapor curve the x = 0.92 states are.

9.141

A dairy farmer needs to heat a 0.1 kg/s flow of milk from room temperature 20^oC to 60^oC to pasteurize it and then send the flow through a cooler bringing it to 10° C for storage. He wants to by a heat pump to do this and finds one using R-134a with a high pressure of 3 MPa and a low pressure of 300 kPa. The farmer is very clever and uses the heat pump to heat the milk in a heat exchanger after which the milk flows through the evaporator heat exchanger to cool it. Find the power required to run the heat pump so it can do both the heating and the cooling assuming milk has the properties of water. Is there any excess heating or cooling capacity? What is the total rate of exergy destruction by running this system?

Do the refrigeration cycle.

State 1 (300 kPa, x = 1): $h_1 = 398.69$ kJ/kg, $s_1 = 1.7259$ kJ/kgK, $T_1 = 0.56^oC$

State 2 (3000 kPa, s = s_1): $h_2 = 445.90$ kJ/kg, $T_2 = 95.5$ ^oC

State 3 (3000 kPa, x = 0): $h_3 = 335.225$ kJ/kg, $s_3 = 1.4186$ kJ/kgK, $T_3 = 86.2^oC$

Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 398.69 - 335.225 = 63.465$ kJ/kg

Condenser: $q_H = h_2 - h_3 = 445.90 - 335.225 = 110.675$ kJ/kg

Compressor: $w_c = h_2 - h_1 = 445.90 - 398.69 = 47.21$ kJ/kg,

COP: $\beta = q_L/w_c = 63.465 / 47.21 = 1.344$

Since we need to cool more than we need to heat, the heat pump needs to be sized to the cooling. From the definition of COP and the milk heat transfer required:

$$\dot{Q}_L = \dot{m}C_p(T_2 - T_3) = 0.1 \times 4.18\ (60 - 10) = 20.9\ kW = \beta\ \dot{W}$$

$$\dot{Q}_{H\ milk} = \dot{m}C_p(T_2 - T_1) = 0.1 \times 4.18\ (60 - 20) = 16.72\ kW$$

$$\dot{W} = \dot{Q}_L /\beta = 20.9\ kW/1.344 = \textbf{15.55 kW}; \quad \dot{Q}_H = \dot{W} + \dot{Q}_L = 36.45\ kW$$

$$\dot{Q}_{H\ extra} = \dot{Q}_H - \dot{Q}_{H\ milk} = 36.45 - 16.72 = \textbf{19.73 kW}$$

For exergy we have work in and flow in with zero (T_o), heat dumped into the ambient also assumed zero and then the cold flow out with a positive exergy.

$$\dot{\Phi}_{flow\ out} = \dot{m}\ [\ h_3 - h_1 - T_o(s_3 - s_1)] = \dot{m}C_p[T_3 - T_1 - T_o\ln(T_3/T_1)]$$
$$= 0.1 \times 4.18\ [-10 - 293\ln(283/293)] = 0.073\ kW$$

$$\dot{\Phi}_{destr} = \dot{W} - \dot{\Phi}_{flow\ out} = 15.55 - 0.073 = \textbf{15.48 kW}$$

If we take control volume total (as shown) then the s generation can be found

$$0 = \dot{m}\,(\,s_1 - s_3\,) - (\dot{Q}_H - \dot{Q}_{H\,milk})/T_o + \dot{S}_{gen}$$

and then $\dot{\Phi}_{destr} = T_o\,\dot{S}_{gen}$ giving the same result.

Comment: Clearly the extra heat out at 86°C could be useful and the cycle can be run with a slightly lower high pressure as it only need to heat the milk to 60°C.

9.142

Consider an ideal combined reheat and regenerative cycle in which steam enters the high-pressure turbine at 3.0 MPa, 400°C, and is extracted to an open feedwater heater at 0.8 MPa with exit as saturated liquid. The remainder of the steam is reheated to 400°C at this pressure, 0.8 MPa, and is fed to the low-pressure turbine. The condenser pressure is 10 kPa. Calculate the thermal efficiency of the cycle and the net work per kilogram of steam.

Solution:

In this setup the flow is separated into fractions x and 1-x after coming out of T1. The two flows are recombined in the FWH.

C.V. T1 $\qquad s_6 = s_5 = 6.9211 \text{ kJ/kg K} \quad => \quad h_6 = 2891.6 \text{ kJ/kg}$

$$w_{T1} = h_5 - h_6 = 3230.82 - 2891.6 = 339.22 \text{ kJ/kg}$$

C.V. Pump 1:

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(800 - 10) = 0.798 \text{ kJ/kg}$$

$$=> \quad h_2 = h_1 + w_{P1} = 191.81 + 0.798 = 192.61 \text{ kJ/kg}$$

C.V. FWH, $\quad h_3 = h_f = 721.1$

Energy equation per unit mass flow exit at 3:

$$x \, h_6 + (1 - x) \, h_2 = h_3 \qquad => \quad x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{721.1 - 192.61}{2891.6 - 192.61} = 0.1958$$

C.V. Pump 2

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001115(3000 - 800) = 2.45 \text{ kJ/kg}$$

$$=> \quad h_4 = h_3 + w_{P2} = 721.1 + 2.45 = 723.55 \text{ kJ/kg}$$

C.V. Boiler/steam generator including reheater.

Total flow from 4 to 5 only fraction 1-x from 6 to 7

$$q_H = h_5 - h_4 + (1 - x)(h_7 - h_6) = 2507.3 + 301.95 = 2809.3 \text{ kJ/kg}$$

C.V. Turbine 2

$$s_8 = s_7 = 7.5715 \text{ kJ/kg K} \implies x_8 = (7.5715 - 0.6492)/7.501 = 0.92285$$

$$h_8 = h_f + x_8 \, h_{fg} = 191.81 + 0.92285 \times 2392.82 = 2400.0 \text{ kJ/kg}$$

$$w_{T2} = h_7 - h_8 = 3267.07 - 2400.02 = 867.05 \text{ kJ/kg}$$

Sum the work terms to get net work. Total flow through T1 only fraction 1-x through T2 and P1 and after FWH we have the total flow through P2.

$$w_{net} = w_{T1} + (1 - x) \, w_{T2} - (1 - x) \, w_{P1} - w_{P2}$$

$$= 339.2 + 697.3 - 0.64 - 2.45 = \textbf{1033.41 kJ/kg}$$

$$\eta_{cycle} = w_{net} / q_H = 1033.41 / 2809.3 = \textbf{0.368}$$

9.143

An ideal steam power plant is designed to operate on the combined reheat and regenerative cycle and to produce a net power output of 10 MW. Steam enters the high-pressure turbine at 8 MPa, 550°C, and is expanded to 0.6 MPa, at which pressure some of the steam is fed to an open feedwater heater, and the remainder is reheated to 550°C. The reheated steam is then expanded in the low-pressure turbine to 10 kPa. Determine the steam flow rate to the high-pressure turbine and the power required to drive each of the pumps.

a)

b) $-w_{P12} = 0.00101(600 - 10) = 0.6$ kJ/kg

$h_2 = h_1 - w_{P12} = 191.8 + 0.6 = 192.4$ kJ/kg

$-w_{P34} = 0.00101(8000 - 600) = 8.1$ kJ/kg

$h_4 = h_3 - w_{P34} = 670.6 + 8.1 = 678.7$; $h_5 = 3521.0$ kJ/kg,

$s_6 = s_5 = 6.8778 \Rightarrow T_6 = 182.32\ °C \quad h_6 = 2810.0$ kJ/kg,

$h_7 = 3591.9, \quad s_8 = s_7 = 8.1348 = 0.6493 + x_8 \times 7.5009 \Rightarrow x_8 = 0.9979$

$h_8 = 191.83 + 0.9979 \times 2392.8 = 2579.7$ kJ/kg

CV: heater

Cont: $m_{6a} + m_2 = m_3 = 1$ kg,   Energy Eq: $m_{6a}h_6 + m_2h_2 = m_3h_3$

$$m_{6a} = \frac{670.6 - 192.4}{2810.0 - 192.4} = 0.1827, \quad m_2 = m_7 = 1 - m_{6a} = 0.8173$$

CV: turbine $w_T = (h_5 - h_6) + (1 - m_{6a})(h_7 - h_8)$

$= 3521 - 2810 + 0.8173(3591.9 - 2579.7) = 1538.2$ kJ/kg

CV: pumps

$w_P = m_2w_{P12} + m_4w_{P34} = 0.8214\times(-0.6) + 1\times(-8.1) = -8.6$ kJ/kg

$w_{Net} = 1538.2 - 8.6 = 1529.6$ kJ/kg (m_5)

$\dot{m}_5 = \dot{W}_{Net}/w_{Net} = 10000/1529.6 =$ **6.53 kg/s**

9.144

An industrial application has the following steam requirement: one 10-kg/s stream at a pressure of 0.5 MPa and one 5-kg/s stream at 1.4 MPa (both saturated or slightly superheated vapor). It is obtained by cogeneration, whereby a high-pressure boiler supplies steam at 10 MPa, 500°C to a reversible turbine. The required amount is withdrawn at 1.4 MPa, and the remainder is expanded in the low-pressure end of the turbine to 0.5 MPa providing the second required steam flow.

a. Determine the power output of the turbine and the heat transfer rate in the boiler.

b. Compute the rates needed were the steam generated in a low-pressure boiler without cogeneration. Assume that for each, 20°C liquid water is pumped to the required pressure and fed to a boiler.

Solution:

a) **With cogeneration**

high-pressure turbine.

$$s_4 = s_3 = 6.5966 \text{ kJ/kg K} \quad \Rightarrow \quad T_4 = 219.9 \text{ °C}, \ h_4 = 2852.6 \text{ kJ/kg}$$

$$w_{S\,HPT} = h_3 - h_4 = 3373.7 - 2852.6 = 521.1 \text{ kJ/kg}$$

low-pressure turbine

$$s_5 = s_4 = 6.5966 = 1.8607 + x_5 \times 4.9606, \ x_5 = 0.9547$$

$$h_5 = 640.23 + 0.9547 \times 2108.5 = 2653.2 \text{ kJ/kg}$$

$$w_{S\,LPT} = h_4 - h_5 = 2852.6 - 2653.2 = 199.4 \text{ kJ/kg}$$

$$\dot{W}_{TURB} = 15 \times 521.1 + 10 \times 199.4 = \mathbf{9810 \ kW}$$

$$\dot{W}_P = 15 \times 0.001002 \ (10\,000 - 2.3) = 150.3 \text{ kW}$$

$$h_2 = h_1 + w_P = 83.96 + 10.02 = 94.0 \text{ kJ/kg}$$

$$\dot{Q}_H = \dot{m}_1(h_3 - h_2) = 15(3373.7 - 94.0) = \textbf{49 196 kW}$$

b) **Without cogeneration**

This is to be compared to the amount of heat required to supply 5 kg/s of 1.4 MPa sat. vap. plus 10 kg/s of 0.5 MPa sat. vap. from 20°C water.

Pump 1 and boiler 1

$w_P = 0.001002(1400 - 2.3) = 14.0$ kJ/kg,

$h_2 = h_1 + w_P = 83.96 + 14.0 = 85.4$ kJ/kg

$_2\dot{Q}_3 = \dot{m}_1(h_3 - h_2) = 5(2790.0 - 85.4) = 13\ 523$ kW

$\dot{W}_{P1} = 5 \times 14.0 = 7$ kW

Pump 2 and boiler 2

$h_5 = h_4 + w_{P2} = 83.96 + 0.001002(500 - 2.3) = 84.5$ kJ/kg

$_5\dot{Q}_6 = \dot{m}_4(h_6 - h_5) = 10(2748.7 - 84.5) = 26\ 642$ kW

$\dot{W}_{P2} = 10 \times 0.5 = 5$ kW

Total $\dot{Q}_H = 13523 + 26642 = \textbf{40 165 kW}$

Notice here that the extra heat transfer is about 9000 kW to run the turbines but that provides 9800 kW of work for electricity (a 100% conversion of the extra Q to W).

9.145

The effect of a number of open feedwater heaters on the thermal efficiency of an ideal cycle is to be studied. Steam leaves the steam generator at 20 MPa, 600°C, and the cycle has a condenser pressure of 10 kPa. Determine the thermal efficiency for each of the following cases. **A:** No feedwater heater. **B**: One feedwater heater operating at 1 MPa. **C:** Two feedwater heaters, one operating at 3 MPa and the other at 0.2 MPa.

a) no feed water heater

$$w_P = \int_1^2 vdP$$

$$\approx 0.00101(20000 - 10)$$
$$= 20.2 \text{ kJ/kg}$$
$$h_2 = h_1 + w_P = 191.8 + 20.2 = 212.0$$
$$s_4 = s_3 = 6.5048$$
$$= 0.6493 + x_4 \times 7.5009$$
$$x_4 = 0.78064$$
$$h_4 = 191.83 + 0.780\ 64 \times 2392.8$$
$$= 2059.7$$
$$w_T = h_3 - h_4 = 3537.6 - 2059.7$$
$$= 1477.9 \text{ kJ/kg}$$
$$w_N = w_T - w_P = 1477.9 - 20.2 = 1457.7$$
$$q_H = h_3 - h_2 = 3537.6 - 212.0 = 3325.6$$

$$\eta_{TH} = \frac{w_N}{q_H} = \frac{1457.7}{3325.6} = \mathbf{0.438}$$

b) one feedwater heater

$$w_{P12} = 0.00101(1000 - 10)$$
$$= 1.0 \text{ kJ/kg}$$
$$h_2 = h_1 + w_{P12} = 191.8 + 1.0 = 192.8$$
$$w_{P34} = 0.001127\ (20000 - 1000)$$
$$= 21.4 \text{ kJ/kg}$$
$$h_4 = h_3 + w_{P34} = 762.8 + 21.4 = 784.2$$
$$s_6 = s_5 = 6.5048$$
$$= 2.1387 + x_6 \times 4.4478$$

$x_6 = 0.9816$

$h_6 = 762.8 + 0.9816 \times 2015.3 = 2741.1$

CV: heater

const: $m_3 = m_6 + m_2 = 1.0$ kg

1st law: $m_6 h_6 + m_2 h_2 = m_3 h_3$

$m_6 = \dfrac{762.8 - 192.8}{2741.1 - 192.8} = 0.2237$

(T-s diagram: 20 MPa, 600°C, 1 MPa, 10 kPa; points 5, 4, 6, 3, 2, 1, 7)

$m_2 = 0.7763, \quad h_7 = 2059.7 \ \ (= h_4 \text{ of part a))}$

CV: turbine $\qquad w_T = (h_5 - h_6) + m_2(h_6 - h_7)$

$= (3537.6 - 2741.1) + 0.7763(2741.1 - 2059.7) \ = 1325.5$ kJ/kg

CV: pumps

$w_P = m_1 w_{P12} + m_3 w_{P34} = 0.7763(1.0) + 1(21.4) = 22.2$ kJ/kg

$w_N = 1325.5 - 22.2 = 1303.3$ kJ/kg

CV: steam generator

$q_H = h_5 - h_4 = 3537.6 - 784.2 = 2753.4$ kJ/kg

$\eta_{TH} = w_N/q_H = 1303.3/2753.4 = \mathbf{0.473}$

c) two feedwater heaters

$w_{P12} = 0.00101 \times$
$\qquad (200 - 10)$
$\qquad = 0.2$ kJ/kg

$h_2 = w_{P12} + h_1$
$\qquad = 191.8 + 0.2$
$\qquad = 192.0$

$w_{P34} = 0.001061 \times$
$\qquad (3000 - 200)$
$\qquad = 3.0$ kJ/kg

$h_4 = h_3 + w_{P34}$
$\qquad = 504.7 + 3.0$
$\qquad = 507.7$

(Diagram: ST. GEN., TURBINE, HP HTR, LP HTR, COND, pumps P; stream points 1–10)

$w_{P56} = 0.001217(20000 - 3000)$

$\qquad = 20.7 \text{ kJ/kg}$

$h_6 = h_5 + w_{P56} = 1008.4 + 20.7 = 1029.1$

$s_8 = s_7 = 6.5048 \left.\begin{array}{l}\\\\\end{array}\right\} T_8 = 293.2 \,^{\circ}\text{C}$

at $P_8 = 3$ MPa $\quad h_8 = 2974.8$

$s_9 = s_8 = 6.5048 = 1.5301 + x_9 \times 5.5970$

$x_9 = 0.8888 \implies h_9 = 504.7 + 0.888 \times 2201.9 = 2461.8 \text{ kJ/kg}$

CV: high pressure heater

$\quad$ cont: $\quad m_5 = m_4 + m_8 = 1.0 \text{ kg}$; $\quad$ 1st law: $m_5 h_5 = m_4 h_4 + m_8 h_8$

$\quad m_8 = \dfrac{1008.4 - 507.7}{2974.8 - 507.7} = 0.2030 \qquad m_4 = 0.7970$

CV: low pressure heater

$\quad$ cont: $\quad m_9 + m_2 = m_3 = m_4$; $\quad$ 1st law: $m_9 h_9 + m_2 h_2 = m_3 h_3$

$\quad m_9 = \dfrac{0.7970(504.7 - 192.0)}{2461.8 - 192.0} = 0.1098$

$\quad m_2 = 0.7970 - 0.1098 = 0.6872$

CV: turbine

$\quad w_T = (h_7 - h_8) + (1 - m_8)(h_8 - h_9) + (1 - m_8 - m_9)(h_9 - h_{10})$

$\quad\quad = (3537.6 - 2974.8) + 0.797(2974.8 - 2461.8)$

$\quad\quad\quad + 0.6872(2461.8 - 2059.7) = 1248.0 \text{ kJ/kg}$

CV: pumps

$\quad w_P = m_1 w_{P12} + m_3 w_{P34} + m_5 w_{P56}$

$\quad\quad = 0.6872(0.2) + 0.797(3.0) + 1(20.7) = 23.2 \text{ kJ/kg}$

$\quad w_N = 1248.0 - 23.2 = 1224.8 \text{ kJ/kg}$

CV: steam generator

$\quad q_H = h_7 - h_6 = 3537.6 - 1029.1 = 2508.5 \text{ kJ/kg}$

$\quad \eta_{TH} = w_N/q_H = 1224.8/2508.5 = \mathbf{0.488}$

9.146

A jet ejector, a device with no moving parts, functions as the equivalent of a coupled turbine-compressor unit (see Problems 7.165 and 7.172). Thus, the turbine-compressor in the dual-loop cycle of Fig. P9.135 could be replaced by a jet ejector. The primary stream of the jet ejector enters from the boiler, the secondary stream enters from the evaporator, and the discharge flows to the condenser. Alternatively, a jet ejector may be used with water as the working fluid. The purpose of the device is to chill water, usually for an air-conditioning system. In this application the physical setup is as shown in Fig. P9.146. Using the data given on the diagram, evaluate the performance of this cycle in terms of the ratio Q_L/Q_H.

a. Assume an ideal cycle.
b. Assume an ejector efficiency of 20% (see Problem 7.172).

$T_1 = T_7 = 10\ ^oC$

$T_2 = 150\ ^oC$

$T_4 = 30\ ^oC$

$T_9 = 20\ ^oC$

Assume $T_5 = T_{10}$

(from mixing streams 4 & 9).

$P_3 = P_4 = P_5 = P_8 = P_9 = P_{10} = P_{G\ 30\ ^oC} = 4.246$ kPa

$P_{11} = P_2 = P_{G\ 150^oC} = 475.8$ kPa, $\qquad P_1 = P_6 = P_7 = P_{G\ 10^oC} = 1.2276$ kPa

Cont: $\dot{m}_1 + \dot{m}_9 = \dot{m}_5 + \dot{m}_{10}, \quad \dot{m}_5 = \dot{m}_6 = \dot{m}_7 + \dot{m}_1$

$\dot{m}_7 = \dot{m}_8 = \dot{m}_9, \quad \dot{m}_{10} = \dot{m}_{11} = \dot{m}_2, \quad \dot{m}_3 = \dot{m}_4$

a) $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$; ideal jet ejector

$s_1' = s_1 \ \& \ s_2' = s_2$ (1' & 2' at $P_3 = P_4$)

then, $\dot{m}_1(h_1' - h_1) = \dot{m}_2(h_2 - h_2')$

From $s_2' = s_2 = 0.4369 + x_2' \times 8.0164$; $x_2' = 0.7985$

$h_2' = 125.79 + 0.7985 \times 2430.5 = 2066.5$ kJ/kg

From $s_1' = s_1 = 8.9008 \Rightarrow T_1' = 112 \,°C$, $h_1' = 2710.4$ kJ/kg

$$\Rightarrow \dot{m}_1/\dot{m}_2 = \frac{2746.5 - 2066.5}{2710.4 - 2519.8} = 3.5677$$

Also $h_4 = 125.79$ kJ/kg, $h_7 = 42.01$ kJ/kg, $h_9 = 83.96$ kJ/kg

Mixing of streams 4 & 9 $\Rightarrow$ 5 & 10:

$$(\dot{m}_1 + \dot{m}_2)h_4 + \dot{m}_7 h_9 = (\dot{m}_7 + \dot{m}_1 + \dot{m}_2)h_{5\,=\,10}$$

Flash chamber (since $h_6 = h_5$): $(\dot{m}_7 + \dot{m}_1)h_{5\,=\,10} = \dot{m}_1 h_1 + \dot{m}_7 h_1$

$\Rightarrow$ using the primary stream $\dot{m}_2 = 1$ kg/s:

$4.5677 \times 125.79 + \dot{m}_7 \times 83.96 = (\dot{m}_7 + 4.5677)h_5$

& $(\dot{m}_7 + 3.5677)h_5 = 3.5677 \times 2519.8 + \dot{m}_7 \times 42.01$

Solving, $\dot{m}_7 = 202.627$ & $h_5 = 84.88$ kJ/kg

LP pump: $-w_{LP\,P} = 0.0010(4.246 - 1.2276) = 0.003$ kJ/kg

$h_8 = h_7 - w_{LP\,P} = 42.01 + 0.003 = 42.01$ kJ/kg

Chiller: $\dot{Q}_L = \dot{m}_7(h_9 - h_8) = 202.627(83.96 - 42.01) = 8500$ kW   (for $\dot{m}_2 = 1$)

HP pump: $-w_{HP\,P} = 0.001002(475.8 - 4.246) = 0.47$ kJ/kg

$h_{11} = h_{10} - w_{HP\,P} = 84.88 + 0.47 = 85.35$ kJ/kg

Boiler: $\dot{Q}_{11} = \dot{m}_{11}(h_2 - h_{11}) = 1(2746.5 - 85.35) = 2661.1$ kW

$\Rightarrow \dot{Q}_L/\dot{Q}_H = 8500/2661.1 = \mathbf{3.194}$

b) Jet eject. eff. $= (\dot{m}_1/\dot{m}_2)_{ACT}/(\dot{m}_1/\dot{m}_2)_{IDEAL} = 0.20$

$$\Rightarrow \left(\dot{m}_1/\dot{m}_2\right)_{ACT} = 0.2 \times 3.5677 = 0.7135$$

using $\dot{m}_2 = 1$ kg/s: $1.7135 \times 125.79 + \dot{m}_7 \times 83.96 = (\dot{m}_7 + 1.7135)h_5$

& $(\dot{m}_7 + 0.7135)h_5 = 0.7135 \times 2519.8 + \dot{m}_7 \times 42.01$

Solving, $\dot{m}_7 = 39.762$ & $h_5 = h_{10} = 85.69$ kJ/kg

Then, $\dot{Q}_L = 39.762(83.96 - 42.01) = 1668$ kW

$h_{11} = 85.69 + 0.47 = 86.16$ kJ/kg

$\dot{Q}_H = 1(2746.5 - 86.16) = 2660.3$ kW

& $\dot{Q}_L / \dot{Q}_H = 1668/2660.3 = \textbf{0.627}$

Computer Problems

Borgnakke and Sonntag

9.192 a

A refrigerator with R-12 as the working fluid has a minimum temperature of −10°C and a maximum pressure of 1 MPa. Assume an ideal refrigeration cycle as in Fig. 9.23. Find the specific heat transfer from the cold space and that to the hot space, and the coefficient of performance.

Solution:

Exit evaporator sat. vapor −10°C from CATT3: $h_1 = 183.19$, $s_1 = 0.7019$ kJ/kgK

Exit condenser sat. liquid 1 MPa from CATT3: $h_3 = 76.22$ kJ/kg

Compressor: $s_2 = s_1$ & P_2 from CATT3 $\Rightarrow$ $h_2 = 210.1$ kJ/kg

Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 183.19 - 76.22 =$ **107 kJ/kg**

Condenser: $q_H = h_2 - h_3 = 210.1 - 76.22 =$ **133.9 kJ/kg**

COP: $\beta = q_L/w_c = q_L/(q_H - q_L) =$ **3.98**

Ideal refrigeration cycle
$P_{cond} = P_3 = P_2 = 1$ MPa
$T_{evap} = -10^oC = T_1$

Properties from CATT3

Borgnakke and Sonntag

9.192 b

Consider an ideal refrigeration cycle that has a condenser temperature of 45°C and an evaporator temperature of −15°C. Determine the coefficient of performance of this refrigerator for the working fluid R-12.

Solution:

Ideal refrigeration cycle
$T_{cond} = 45^oC = T_3$
$T_{evap} = -15^oC = T_1$

Property for:	R-12
h_1, kJ/kg	180.97
$s_2 = s_1$, kJ/kg K	0.7051
P_2, MPa	1.0843
T_2, ^oC	54.7
h_2, kJ/kg	212.63
$w_C = h_2 - h_1$	31.66
$h_3 = h_4$, kJ/kg	79.71
$q_L = h_1 - h_4$	101.26
$\beta = q_L/w_C$	**3.198**

Compressor

Exp. valve
Evaporator

The properties are from the computer program CATT3.

9.192 c

A refrigerator in a meat warehouse must keep a low temperature of -15°C and the outside temperature is 20°C. It uses R-12 as the refrigerant which must remove 5 kW from the cold space. Find the flow rate of the R-12 needed assuming a standard vapor compression refrigeration cycle with a condenser at 20°C.

Solution:

Basic refrigeration cycle: $T_1 = T_4 = -15°C$, $T_3 = 20°C$

Computer Tables CATT3: $h_4 = h_3 = 54.87$ kJ/kg; $h_1 = h_g = 180.97$ kJ/kg

$$\dot{Q}_L = \dot{m}_{R-12} \times q_L = \dot{m}_{R-12}(h_1 - h_4)$$

$$q_L = 180.97 - 54.87 = 126.1 \text{ kJ/kg}$$

$$\dot{m}_{R-12} = 5.0 / 126.1 = \mathbf{0.03965 \text{ kg/s}}$$

Ideal refrigeration cycle
$T_{cond} = 20^oC$
$T_{evap} = -15^oC = T_1$

9.192 d

In an actual refrigeration cycle using R-12 as the working fluid, the refrigerant flow rate is 0.05 kg/s. Vapor enters the compressor at 150 kPa, −10°C, and leaves at 1.2 MPa, 75°C. The power input to the compressor is measured and found be 2.4 kW. The refrigerant enters the expansion valve at 1.15 MPa, 40°C, and leaves the evaporator at 175 kPa, −15°C. Determine the entropy generation in the compression process, the refrigeration capacity and the coefficient of performance for this cycle.

Solution:

Actual refrigeration cycle

1: compressor inlet $T_1 = -10^oC$, $P_1 = 150$ kPa

2: compressor exit $T_2 = 75^oC$, $P_2 = 1.2$ MPa

3: Expansion valve inlet $T_3 = 40^oC$
   $P_3 = 1.15$ MPa

5: evaporator exit $T_5 = -15^oC$, $P_5 = 175$ kPa

CATT3: $h_1 = 184.8$, $s_1 = 0.7324$, $h_2 = 226.7$, $s_2 = 0.741$

CV Compressor: $h_1 + q_{COMP} + w_{COMP} = h_2$; $s_1 + \int dq/T + s_{gen} = s_2$

$$w_{COMP} = \dot{W}_{COMP}/\dot{m} = 2.4/0.05 = 48.0 \text{ kJ/kg}$$

$$q_{COMP} = h_2 - w_{COMP} - h_1 = 226.7 - 48.0 - 184.8 = -6.1 \text{ kJ/kg}$$

$$s_{gen} = s_2 - s_1 - q / T_o = 0.741 - 0.7324 + 6.1/298.15 = \textbf{0.029 kJ / kg K}$$

C.V. Evaporator

$$q_L = h_5 - h_4 = 181.5 - 74.59 = 106.9 \text{ kJ/kg}$$

$$\Rightarrow \dot{Q}_L = \dot{m}q_L = 0.05 \times 106.9 = \textbf{5.346 kW}$$

COP: $\beta = q_L/w_{COMP} = 106.9/48.0 = \textbf{2.23}$

9.193 a

Do Problem 9.21 with R-22 as the working fluid.

A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85°C, and the condenser temperature is 40°C. Calculate the thermal efficiency of this cycle.

Solution:

CV: Pump   (use R-22 Computer Table in CATT3)

$$w_P = h_2 - h_1 = \int\limits_1^2 v dP \approx v_1(P_2\text{-}P_1) = 0.000884(4037 - 1534) = 2.21 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 94.27 + 2.21 = 96.48 \text{ kJ/kg}$$

CV: Boiler: $q_H = h_3 - h_2 = 253.69 - 96.48 = 157.21$ kJ/kg

CV: Turbine

$$s_4 = s_3 = 0.7918 = 0.3417 + x_4 \times 0.5329, \quad \Rightarrow \quad x_4 = 0.8446$$

$$h_4 = 94.27 + 0.8446 \times 166.88 = 235.22 \text{ kJ/kg}$$

$$w_T = h_3 - h_4 = 253.69 - 235.22 = 18.47 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = (18.47 - 2.21)/157.21 = \textbf{0.1034}$$

9.193 b

Do problem 9.65 with R-22 as the working fluid.

A flow with 2 kg/s of water is available at 95°C for the boiler. The restrictive factor is the boiling temperature of 85°C. Therefore, break the process up from 2-3 into two parts as shown in the diagram.

Properties from CATT3.

$$-\dot{Q}_{AB} = \dot{m}_{H2O}(h_A - h_B) = 2(397.94 - 355.88) = 84.12 \text{ kW}$$

$$= \dot{m}_{R-22}(253.69 - 165.09) \implies \dot{m}_{R-22} = 0.949 \text{ kg/s}$$

To verify that $T_D = T_3$ is the restrictive factor, find T_C.

$$-\dot{Q}_{AC} = 0.949(165.09 - 96.48) = 65.11 = 2.0(355.88 - h_C)$$

$$h_C = 323.32 \text{ kJ/kg} \implies T_C = 77.2^o\text{C} \quad \textbf{OK}$$

State 1:   40°C,   1533.5 kPa, $v_1 = 0.000884$ m^3/kg

CV Pump: $w_P = v_1(P_2 - P_1) = 0.000884(4036.8 - 1533.5) = 2.21$ kJ/kg

CV: Turbine

$$s_4 = s_3 = 0.7918 = 0.3417 + x_4 \times 0.5329 \implies x_4 = 0.8446$$

$$h_4 = 94.27 + 0.8446 \times 166.88 = 235.22 \text{ kJ/kg}$$

Energy Eq.: $w_T = h_3 - h_4 = 253.69 - 235.22 = 18.47$ kJ/kg

Cycle: $w_{NET} = w_T - w_P = 18.47 - 2.21 = 16.26$ kJ/kg

$$\dot{W}_{NET} = \dot{m}_{R22}w_{NET} = 0.949 \times 16.26 = \textbf{15.43 kW}$$

9.193 c

Consider an ideal refrigeration cycle that has a condenser temperature of 45°C and an evaporator temperature of −15°C. Determine the coefficient of performance of this refrigerator for the working fluid R-22.

Solution:

Ideal refrigeration cycle
$T_{cond} = 45^oC = T_3$
$T_{evap} = -15^oC = T_1$

Property for:	R-22
h_1, kJ/kg	244.13
$s_2 = s_1$, kJ/kg K	0.9505
P_2, MPa	1.729
T_2, ^oC	74.4
h_2, kJ/kg	289.26
$w_C = h_2 - h_1$	45.13
$h_3 = h_4$, kJ/kg	100.98
$q_L = h_1 - h_4$	143.15
$\beta = q_L/w_C$	**3.172**

Compressor — h_1 through $w_C = h_2 - h_1$
Exp. valve — $h_3 = h_4$
Evaporator — $q_L = h_1 - h_4$

The properties are from CATT3.

9.193 SI version of d)

The refrigerant R-22 is used as the working fluid in a conventional heat pump cycle. Saturated vapor enters the compressor of this unit at 10°C; its exit temperature from the compressor is measured and found to be 85°C. If the compressor exit is at 2 MPa what is the compressor isentropic efficiency and the cycle COP?

Solution:

R-22 heat pump:
Computer Table
State 1: $T_{EVAP} = 10^oC$, $x = 1$
$h_1 = 253.42$ kJ/kg, $s_1 = 0.9129$ kJ/kg K

State 2: T_2, P_2: $h_2 = 295.17$ kJ/kg

C.V. Compressor

Energy Eq.: $w_{C\,ac} = h_2 - h_1 = 295.17 - 253.42 = $ **41.75 kJ/kg**

State 2s: 2 MPa , $s_{2S} = s_1 = 0.9129$ kJ/kg $T_{2S} = 69^oC$, $h_{2S} = 280.2$ kJ/kg

Efficiency: $\eta = \dfrac{w_{C\,s}}{w_{C\,ac}} = \dfrac{h_{2S} - h_1}{h_2 - h_1} = \dfrac{280.2 - 253.42}{295.17 - 253.42} = $ **0.6414**

C.V. Condenser

Energy Eq.: $q_H = h_2 - h_3 = 295.17 - 109.6 = 185.57$ kJ/kg

COP Heat pump: $\beta = \dfrac{q_H}{w_{C\,ac}} = \dfrac{185.57}{41.75} = $ **4.44**

ENGLISH UNIT PROBLEMS

Borgnakke Sonntag

Fundamentals of
Thermodynamics

SOLUTION MANUAL
CHAPTER 9
English Units

8e

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CHAPTER 9

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Rankine cycles

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9.147E

A steam power plant, as shown in Fig. 9.3, operating in a Rankine cycle has saturated vapor at 600 lbf/in.^{2} leaving the boiler. The turbine exhausts to the condenser operating at 2.23 lbf/in.^{2}. Find the specific work and heat transfer in each of the ideal components and the cycle efficiency.

Solution:

For the cycle as given:

1: $h_1 = 97.97$ Btu/lbm, $v_1 = 0.01625$ ft^3/lbm,

3: $h_3 = h_g = 1204.06$ Btu/lbm, $s_3 = s_g = 1.4464$ Btu/lbm R

C.V. Pump Reversible and adiabatic.

Energy: $w_p = h_2 - h_1$; Entropy: $s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$w_P = \int v \, dP = v_1(P_2 - P_1) = 0.01625$ ft^3/lbm $\times (600 - 2.2)$ psi $\times 144$ in^2/ft^2

$= 1398.85$ lbf-ft $= $ **1.8 Btu/lbm**

$h_2 = h_1 + w_P = 97.97 + 1.8 = 99.77$ Btu/lbm

C.V. Boiler: $q_H = h_3 - h_2 = 1204.06 - 99.77 = $ **1104.3 Btu/lbm**

C.V. Tubine: $w_T = h_3 - h_4$, $s_4 = s_3$

$s_4 = s_3 = 1.4464 = 0.1817 + x_4 \times 1.7292$ $\Rightarrow$ $x_4 = 0.7314$,

$h_4 = 97.97 + 0.7314 \times 1019.78 = 843.84$ Btu/lbm

$w_T = 1204.06 - 843.84 = $ **360.22 Btu/lbm**

$\eta_{CYCLE} = (w_T - w_P)/q_H = (360.22 - 1.8)/1104.3 = $ **0.325**

C.V. Condenser: $q_L = h_4 - h_1 = 843.84 - 97.97 = $ **745.9 Btu/lbm**

Borgnakke and Sonntag

9.148E

Consider a solar-energy-powered ideal Rankine cycle that uses water as the working fluid. Saturated vapor leaves the solar collector at 150 psia, and the condenser pressure is 0.95 lbf/in.^{2}. Determine the thermal efficiency of this cycle.

H_2O ideal Rankine cycle

CV: turbine

State 3:   Table F.7.1 $h_3 = 1194.91$ Btu/lbm, $s_3 = 1.5704$ Btu/lbm R

CV Turbine adiabatic and reversible so second law gives

$$s_4 = s_3 = 1.5704 = 0.1296 + x_4 \times 1.8526 \qquad => \qquad x_4 = 0.77772$$

$$h_4 = 68.04 + 0.77772 \times 1036.98 = 874.52 \text{ Btu/lbm}$$

The energy equation gives

$$w_T = h_3 - h_4 = 1194.91 - 874.52 = 320.39 \text{ Btu/lbm}$$

C.V. pump and incompressible liquid gives work into pump

$$w_P = \int vdP \approx v_1(P_2 - P_1) = 0.01613 \text{ ft}^3/\text{lbm} \times (150 - 0.95) \text{ psi} \times 144 \text{ in}^2/\text{ft}^2$$

$$= 346.2 \text{ lbf-ft} = 0.44 \text{ Btu/lbm}$$

$$h_2 = h_1 + w_P = 68.04 + 0.44 = 68.48 \text{ Btu/lbm}$$

C.V. boiler gives the heat transfer from the energy equation as

$$q_H = h_3 - h_2 = 1194.91 - 68.48 = 1126.43 \text{ Btu/lbm}$$

The cycle net work and efficiency are found as

$$w_{NET} = w_T - w_P = 320.39 - 0.44 = 319.95 \text{ Btu/lbm}$$

$$\eta_{TH} = w_{NET}/q_H = 319.95/1126.43 = \textbf{0.284}$$

Borgnakke and Sonntag

9.149E

The power plant in the previous problem is augmented with a natural gas burner to superheat the water to 600 F before entering the turbine. Find the cycle efficiency with this configuration and the specific heat transfer added by the natural gas burner.

Solution:

C.V. H_2O ideal Rankine cycle

State 3: 150 psia, 600 F, $h_3 = 1325.69$ Btu/lbm, $s_3 = 1.7110$ Btu/lbm R

CV Turbine adiabatic and reversible so second law gives

$$s_4 = s_3 = 1.7110 = 0.1296 + x_4 \times 1.8526 \qquad => \qquad x_4 = 0.85361$$

$$h_4 = 68.04 + 0.85361 \times 1036.98 = 953.22 \text{ Btu/lbm}$$

The energy equation gives

$$w_T = h_3 - h_4 = 1325.69 - 953.22 = 372.47 \text{ Btu/lbm}$$

C.V. pump and incompressible liquid gives work into pump

$$w_P = \int vdP \approx v_1(P_2 - P_1) = 0.01613 \text{ ft}^3/\text{lbm} \times (150 - 0.95) \text{ psi} \times 144 \text{ in}^2/\text{ft}^2$$

$$= 346.2 \text{ lbf-ft} = 0.44 \text{ Btu/lbm}$$

$$h_2 = h_1 + w_P = 68.04 + 0.44 = 68.48 \text{ Btu/lbm}$$

C.V. boiler gives the heat transfer from the energy equation as

$$q_H = h_3 - h_2 = 1325.69 - 68.48 = 1257.21 \text{ Btu/lbm}$$

The cycle net work and efficiency are found as

$$w_{NET} = w_T - w_P = 372.47 - 0.44 = 372.03 \text{ Btu/lbm}$$

$$\eta_{TH} = w_{NET}/q_H = 372.03/1257.21 = \mathbf{0.296}$$

Borgnakke and Sonntag

9.150E

A Rankine cycle with R-410A has the boiler at 600 psia superheating to 340 F and the condenser operates at 100 psia. Find all four energy transfers and the cycle efficiency.

State 1: $v_1 = 0.01349$ ft^3/lbm, $h_1 = 21.9$ Btu/lbm   (at 23.84 F)

State 3: $h_3 = 183.9$ Btu/lbm, $s_3 = 0.3091$ Btu/lbm-R

State 4: (100 psia, $s = s_3$) $h_4 = 152.0$ Btu/lbm   interpolated sup. vap.

C.V. Pump: $w_P = \int v \, dP = v_1(P_2 - P_1)$

$\qquad\qquad = 0.01349$ ft^3/lbm $\times$ (600 – 100) psi $\times 144$ in^2/ft^2 = 971.3 lbf-ft

$\qquad\qquad$ = **1.25 Btu/lbm**

$\qquad \Rightarrow h_2 = h_1 + w_P = 21.9 + 1.25 = 23.15$ Btu/lbm

C.V. Boiler: $q_H = h_3 - h_2 = 183.9 – 23.15 = $ **160.75 Btu/lbm**

C.V. Tubine energy: $w_{T,s} = h_3 - h_4 = 183.9 - 152.0 = $ **31.9 Btu/lbm**

C.V. Condenser: $q_L = h_4 - h_1 = 152.0 – 21.9 = $ **130.1 Btu/lbm**

$$\eta_{CYCLE} = w_{NET}/q_H = \frac{31.9 - 1.25}{160.75} = \mathbf{0.191}$$

Borgnakke and Sonntag

9.151E

A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 180 F, and the condenser temperature is 100 F. Calculate the thermal efficiency of this cycle.

Solution:

CV: Pump   (use R-134a Table F.10)

$P_1 = 138.93$ psia, $P_2 = P_3 = 400.4$ psia

$h_3 = 184.36$ Btu/lbm, $s_3 = 0.402$ Btu/lbm R

$h_1 = 108.86$ Btu/lbm, $v_1 = 0.01387$ ft^3/lbm

$$w_P = h_2 - h_1 = \int_1^2 vdP \approx v_1(P_2 - P_1)$$

$$= 0.01387(400.4 - 138.93)\frac{144}{778} = 0.671 \text{ Btu/lbm}$$

$$h_2 = h_1 + w_P = 108.86 + 0.671 = 109.53 \text{ Btu/lbm}$$

CV: Boiler

$$q_H = h_3 - h_2 = 184.36 - 109.53 = 74.83 \text{ Btu/lbm}$$

CV: Turbine

$$s_4 = s_3 = 0.402 \implies x_4 = (0.402 - 0.2819)/0.1272 = 0.9442$$

$$h_4 = 176.08 \text{ Btu/lbm,}$$

Energy Eq.: $w_T = h_3 - h_4 = 8.276$ Btu/lbm

$$w_{NET} = w_T - w_P = 8.276 - 0.671 = 7.605 \text{ Btu/lbm}$$

$$\eta_{TH} = w_{NET} / q_H = 7.605/74.83 = \textbf{0.102}$$

Borgnakke and Sonntag

9.152E

Do Problem 9.151E with R-410A as the working fluid, boiler exit at 160 F, 600 psia.

Standard Rankine cycle with properties from the R-410A tables,

$$h_1 = 51.58 \text{ Btu/lbm}, \quad v_1 = 0.01619 \text{ ft}^3/\text{lbm}, \quad P_1 = 332.54 \text{ psia},$$

$$P_2 = P_3 = 600 \text{ psia}, \quad h_3 = 125.06 \text{ Btu/lbm}, \quad s_3 = 0.2248 \text{ Btu/lbm R}$$

CV: Pump $\quad w_P = v_1(P_2-P_1) = 0.01619 (600 - 332.54) \dfrac{144}{778} = 0.80 \text{ Btu/lbm}$

$$h_2 = h_1 + w_P = 51.58 + 0.80 = 52.38 \text{ Btu/lbm}$$

CV: Turbine $\quad s_4 = s_3$

$$\Rightarrow x_4 = (0.2248 - 0.1038)/0.1256 = 0.9634$$

$$h_4 = 119.32 \text{ Btu/lbm}, \quad w_T = h_3 - h_4 = 5.74 \text{ Btu/lbm}$$

CV: Boiler

$$q_H = h_3 - h_2 = 125.06 - 52.38 = 72.68 \text{ Btu/lbm}$$

$$\eta_{TH} = (w_T - w_P)/q_H = (5.74 - 0.80)/72.68 = \mathbf{0.068}$$

Borgnakke and Sonntag

9.153E

A low temperature power plant operates with R-410A maintaining 60 psia in the condenser, a high pressure of 400 psia with superheat. Find the temperature out of the boiler/superheater so the turbine exit temperature is 20 F and find the overall cycle efficiency.

State 1: $P_1 = 60$ psia, $v_1 = 0.0129$ ft^3/lbm, $h_1 = 12.7$ Btu/lbm

State 4: $P_4 = P_1 = 60$ psia, $h_4 = 343.58$ Btu/lbm, $s_4 = 1.3242$ Btu/lbm-R

State 3:   400 psia, $s = s_4$ $\Rightarrow$ $h_3 = 147.2$ Btu/lbm, $T_3 = \textbf{190.3 F}$

Pump: $w_p = v_1 (P_2 - P_1) = 0.0129 (400 - 60) \dfrac{144}{778} = 2.09$ Btu/lbm

Boiler: $q_H = h_3 - h_2 = 426.56 - (12.7 + 0.8) = 133.7$ Btu/lbm

Turbine: $w_T = h_3 - h_4 = 147.2 - 122.1 = 25.1$ Btu/lbm

Efficiency: $\eta_{TH} = w_{NET}/q_H = (w_T - w_P)/q_H = \dfrac{25.1 - 0.8}{133.7} = \textbf{0.182}$

Borgnakke and Sonntag

9.154E

A smaller power plant produces 50 lbm/s steam at 400 psia, 1100 F, in the boiler. It cools the condenser with ocean water coming in at 60 F and returned at 65 F so that the condenser exit is at 110 F. Find the net power output and the required mass flow rate of the ocean water.

Solution:

The states properties from Tables F.7.1 and F.7.2

1: 110 F, x = 0: $h_1 = 78.01$ Btu/lbm, $v_1 = 0.01617$ ft^3/lbm, $P_{sat} = 1.28$ psia

3: 400 psia, 1100 F: $h_3 = 1577.44$ Btu/lbm, $s_3 = 1.7989$ Btu/lbm R

C.V. Pump Reversible and adiabatic.

Energy: $w_p = h_2 - h_1$; Entropy: $s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$$w_p = \int v \, dP = v_1 (P_2 - P_1) = 0.01617 \, (400 - 1.3)\frac{144}{778} = 1.19 \text{ Btu/lbm}$$

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$$s_4 = s_3 = 1.7989 = 0.1473 + x_4 (1.8101) \quad => \quad x_4 = 0.9124$$

$$=> h_4 = 78.01 + 0.9124 \,(1031.28) = 1018.95 \text{ Btu/lbm}$$

$$w_T = 1577.44 - 1018.95 = 558.5 \text{ Btu/lbm}$$

$$\dot{W}_{NET} = \dot{m}(w_T - w_p) = 50 \,(558.5 - 1.19) = \textbf{27 866 Btu/s}$$

C.V. Condenser : $q_L = h_4 - h_1 = 1018.95 - 78.01 = 940.94$ Btu/lbm

$$\dot{Q}_L = \dot{m}q_L = 50 \times 940.94 = 47 \, 047 \text{ Btu/s} = \dot{m}_{ocean} \, C_p \, \Delta T$$

$$\dot{m}_{ocean} = \dot{Q}_L \,/\, C_p \, \Delta T = 47 \, 047 \,/\, (1.0 \times 5) = \textbf{9409 lbm/s}$$

Borgnakke and Sonntag

9.155E

Consider a simple ideal Rankine cycle using water at a supercritical pressure. Such a cycle has a potential advantage of minimizing local temperature differences between the fluids in the steam generator, such as the instance in which the high-temperature energy source is the hot exhaust gas from a gas-turbine engine. Calculate the thermal efficiency of the cycle if the state entering the turbine is 8000 lbf/in.^{2}, 1300 F, and the condenser pressure is 0.95 lbf/in.^{2}. What is the steam quality at the turbine exit?

Solution:

For the efficiency we need the net work and steam generator heat transfer.

State 1: $s_1 = 0.1296$ Btu/lbm R, $h_1 = 68.04$ Btu/lbm

State 3: $h_3 = 1547.5$ Btu/lbm, $s_3 = 1.4718$ Btu/lbm R

C.V. Pump. For this high exit pressure we use Table F.7.3 to get state 2.

Entropy Eq.: $s_2 = s_1$ => $h_2 = 91.69$ Btu/lbm

$$w_p = h_2 - h_1 = 91.69 - 68.04 = 23.65 \text{ Btu/lbm}$$

C.V. Turbine. Assume reversible and adiabatic.

Entropy Eq.: $s_4 = s_3 = 1.4718 = 0.1296 + x_4 \times 1.8526$

$x_4 = \mathbf{0.7245}$ Very low for a turbine exhaust

$$h_4 = 68.04 + x_4 \times 1036.98 = 751.29 \text{ Btu/lbm},$$

$$w_T = h_3 - h_4 = 796.2 \text{ Btu/lbm}$$

Steam generator: $q_H = h_3 - h_2 = 1547.5 - 91.69 = 1455.8$ Btu/lbm

$$w_{NET} = w_T - w_p = 796.2 - 23.65 = 772.6 \text{ Btu/lbm}$$

$$\eta = w_{NET}/q_H = 772.6 / 1455.8 = \mathbf{0.53}$$

Borgnakke and Sonntag

9.156E

A Rankine cycle uses ammonia as the working substance and powered by solar energy. It heats the ammonia to 320 F at 800 psia in the boiler/superheater. The condenser is water cooled, and the exit is kept at 70 F. Find (T, P, and x if applicable) for all four states in the cycle.

NH_3 ideal Rankine cycle

State 1: Table F.8.1, T = 70 F,   x = 0, P_1 = 128.85 psia,

$\qquad$ h_1 = 120.21 Btu/lbm, v_1 = 0.2631 ft^3/lbm

CV Pump:

$\qquad$ $w_P = h_2 - h_1 \approx v_1(P_2 - P_1) = 0.02631$ ft^3/lbm (800 – 128.85) psi $\times 144$ in^2/ft^2

$\qquad$ $= 2542.75$ lbf-ft $= 3.27$ Btu/lbm

$\qquad$ $h_2 = h_1 + w_P = 120.21 + 3.27 = 123.48$ Btu/lbm $\approx h_f + (P_2 - P_{sat})v_f$

$\qquad\qquad$ => $T_2 \approx T_1 = 70$ F   [with the CATT3 we do $(P_2, s_2 = s_1)$]

State 3: 320 F, 800 psia : superheated vapor, s_3 = 1.1915 Btu/lbm

CV: turbine

$\qquad$ $s_4 = s_3 = 1.1915 = 0.2529 + x_4 \times 0.9589$ $\qquad$ => $\qquad$ $x_4 = 0.9788$

$\qquad$ $P_4 = P_1 = 128.85$ psia, $T_4 = T_1 = 70$ F

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9.157E

Assume the powerplant in Problem 9.156E should deliver 1000 Btu/s. What is the mass flow rate of ammonia?

State 3: 320 F, 800 psia: $h_3 = 734.7$ Btu/lbm, $s_3 = 1.1915$ Btu/lbm

CV: turbine

Energy: $w_{T,s} = h_3 - h_4$; Entropy: $0 = s_3 - s_4 + 0$

$s_4 = s_3 = 1.1915 = 0.2529 + x_4 \times 0.9589 \Rightarrow x_4 = 0.9788$

$h_4 = 120.21 + 0.9788 \; 507.89 = 617.3$ Btu/lbm

$w_{T,s} = h_3 - h_4 = 734.7 - 617.3 = 117.4$ Btu/lbm

$\dot{W}_T = \dot{m} \, w_{T,s}$;

$\dot{m} = \dot{W}_T / w_{T,s} = \dfrac{1000 \text{ Btu/s}}{117.4 \text{ Btu/lbm}} = \textbf{8.518 lbm/s}$

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9.158E

Consider an ideal steam reheat cycle in which the steam enters the high-pressure turbine at 600 lbf/in.^{2}, 700 F, and then expands to 150 lbf/in.^{2}. It is then reheated to 700 F and expands to 2.225 lbf/in.^{2} in the low-pressure turbine. Calculate the thermal efficiency of the cycle and the moisture content of the steam leaving the low-pressure turbine.

Solution:

Basic Rankine cycle with a reheat section. For this cycle from Table F.7

State 3: Superheated vapor $\quad h_3 = 1350.62$ Btu/lbm, $s_3 = 1.5871$ Btu/lbm R,

State 1: Saturated liquid $\quad h_1 = 97.97$ Btu/lbm, $v_1 = 0.01625$ ft^3/lbm

C.V. Pump: Adiabatic and reversible. Use incompressible fluid so

$$w_P = \int v \, dP = v_1(P_2 - P_1) = 0.01625(600 - 2.2)\frac{144}{778} = 1.8 \text{ Btu/lbm}$$

$$h_2 = h_1 + w_P = 97.97 + 1.8 = 99.77 \text{ Btu/lbm}$$

C.V. HP Tubine section: $\quad w_{T1} = h_3 - h_4, \quad s_4 = s_3 \Rightarrow h_4 = 1208.93$ Btu/lbm

$$w_{T1} = 1350.62 - 1208.93 = 141.69 \text{ Btu/lbm}$$

C.V. LP Tubine section: $\quad w_{T2} = h_5 - h_6, \quad s_6 = s_5$

State 5: $\quad h_5 = 1376.55$ Btu/lbm, $s_5 = 1.7568$ Btu/lbm R

State 6: $\quad s_6 = s_5 = 1.7568 = 0.1817 + x_6 \times 1.7292 \Rightarrow x_6 = 0.9109$

$$h_6 = 97.97 + 0.9109 \times 1019.78 = 1026.89 \text{ Btu/lbm}$$

$$w_{T2} = 1376.55 - 1026.89 = 349.66 \text{ Btu/lbm}$$

C.V. Boiler: $\quad q_{H1} = h_3 - h_2 = 1350.62 - 99.77 = 1250.85$ Btu/lbm

Total cycle

$$q_H = q_{H1} + h_5 - h_4 = 1250.85 + 1376.55 - 1208.93 = 1418.5 \text{ Btu/lbm}$$

$$w_{T,tot} = w_{T1} + w_{T2} = 141.69 + 349.66 = 491.35 \text{ Btu/lbm}$$

Overall efficiency

$$\eta_{CYCLE} = (w_{T,tot} - w_P)/q_H = (491.35 - 1.8)/1420.3 = \textbf{0.345}$$

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9.159E

An open FWH receives steam at 150 psia, 400 F from the turbine and 150 psia, 200 F water from the feed water line. Find the required fraction of the extraction flow in the turbine.

The setup follows Fig.9.12

State enthalpies: $h_2 = 168.07$, $h_6 = 1219.51$ all Btu/lbm

$h_3 = 330.74$ (x = 0 at 150 psia, 358.47 F)

Analyze the FWH which leads to Eq.9.5:

$$y = \dot{m}_6 / \dot{m}_5 \quad \text{the extraction fraction}$$

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{330.74 - 168.07}{1219.51 - 168.07} = \mathbf{0.1547}$$

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9.160E

Consider an ideal steam regenerative cycle in which steam enters the turbine at 600 lbf/in.^{2}, 700 F, and exhausts to the condenser at 2.225 lbf/in.^{2}. Steam is extracted from the turbine at 150 lbf/in.^{2} for an open feedwater heater. The feedwater leaves the heater as saturated liquid. The appropriate pumps are used for the water leaving the condenser and the feedwater heater. Calculate the thermal efficiency of the cycle and the net work per pound-mass of steam.

From Table F.7.2

$h_5 = 1350.62$ Btu/lbm,

$s_5 = 1.5871$ Btu/lbm R

$h_1 = 97.97$ Btu/lbm,

$v_1 = 0.01625$ ft^3/lbm

Interpolate to get

$h_3 = 330.67$ Btu/lbm,

$v_3 = 0.01809$ ft^3/lbm

C.V. Pump1:

$$w_{P12} = 0.01625(150 - 2.2)\frac{144}{778}$$
$$= 0.44 \text{ Btu/lbm} = h_2 - h_1$$
$$h_2 = h_1 + w_{P12} = 98.41 \text{ Btu/lbm}$$

C.V. Pump2:

$$w_{P34} = 0.01809(600 - 150)144/778 = 1.507 \text{ Btu/lbm}$$
$$\Rightarrow \quad h_4 = h_3 + w_{P34} = 332.18 \text{ Btu/lbm}$$

C.V. Turbine (high pressure section)

Entropy Eq.: $s_6 = s_5 = 1.5871$ Btu/lbm R $\quad => \quad h_6 = 1208.93$ Btu/lbm

CV: feedwater heater, call the extraction fraction $y = \dot{m}_6/\dot{m}_3$

Continuity Eq.: $\dot{m}_3 = \dot{m}_6 + \dot{m}_2$, Energy Eq.: $\dot{m}_6 h_6 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

$$yh_6 + (1 - y)h_2 = h_3 \quad \Rightarrow \quad y_6 = (h_3 - h_2)/(h_6 - h_2)$$
$$\Rightarrow \quad y = (330.67 - 98.41)/(1208.93 - 98.41) = 0.2091$$

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CV: Turbine from 5 to 7

$$s_7 = s_5 \quad \Rightarrow \quad x_7 = (1.5871 - 0.1817)/1.7292 = 0.8127$$

$$h_7 = 97.97 + 0.8127 \times 1019.78 = 926.75 \text{ Btu/lbm}$$

The HP turbine section has the full flow, the LP section has fraction (1-y)

$$w_T = (h_5 - h_6) + (1 - y)(h_6 - h_7)$$

$$= (1350.62 - 1208.93) + 0.7909(1208.93 - 926.75) = 364.87 \text{ Btu/lbm}$$

CV: pumps

P2 has the full flow and P1 has the fraction 1- y of the flow

$$w_P = (1 - y)w_{P12} + w_{P34} = 0.7909 \times 0.44 + 1 \times 1.507 = 1.855 \text{ Btu/lbm}$$

$$w_{NET} = w_T - w_P = 364.87 - 1.855 = \textbf{363.0 Btu/lbm}$$

CV: steam generator

$$q_H = h_5 - h_4 = 1350.62 - 332.18 = 1018.44 \text{ Btu/lbm}$$

$$\eta_{TH} = w_{NET}/q_H = 363/1018.44 = \textbf{0.356}$$

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9.161E

A closed feedwater heater in a regenerative steam power cycle heats 40 lbm/s of water from 200 F, 2000 lbf/in.^{2} to 450 F, 2000 lbf/in.^{2}. The extraction steam from the turbine enters the heater at 600 lbf/in.^{2}, 550 F and leaves as saturated liquid. What is the required mass flow rate of the extraction steam?

Solution:

The schematic from Figure 9.13 has the feedwater from the pump coming at state 2 being heated by the extraction flow coming from the turbine state 6 so the feedwater leaves as saturated liquid state 4 and the extraction flow leaves as condensate state 6a.

From the steam tables F.7:

F.7.3: $h_2 = 172.6$ Btu/lbm

F.7.3: $h_4 = 431.13$ Btu/lbm

F.7.2: $h_6 = 1255.36$ Btu/lbm

Interpolate for this state

F.7.1: $h_{6a} = 471.56$ Btu/lbm

C.V. Feedwater Heater

Energy Eq.: $\dot{m}_2 h_2 + \dot{m}_6 h_6 = \dot{m}_2 h_4 + \dot{m}_6 h_{6a}$

Since all four state are known we can solve for the extraction flow rate

$$\dot{m}_6 = \dot{m}_2 \frac{h_2 - h_4}{h_{6a} - h_6} = 40 \text{ lbm/s} \frac{172.6 - 431.13}{471.56 - 1255.36} = \textbf{13.2} \frac{\textbf{lbm}}{\textbf{s}}$$

9.162E

A Rankine cycle feeds 10 lbm/s ammonia at 300 psia, 280 F to the turbine, which has an extraction point at 125 psia. The condenser is at 0 F, and a closed feed water heater has an exit state (3) at the temperature of the condensing extraction flow and it has a drip pump. The source for the boiler is at constant 350 F. Find the extraction flow rate and state 4 into the boiler.

$P_1 = 30.415$ psia,
$h_1 = 42.6$ Btu/lbm
$v_1 = 0.0242$ ft^3/lbm,
$s_5 = 1.3062$ Btu/lbm-R,
$h_5 = 744.07$ Btu/lbm
$T_{6a} = T_{sat\ 125\ psia} = 68.28$ F
$=> h_{6a} = 118.25$ Btu/lbm

C.V. Turbine: Reversible, adiabatic so constant s from inlet to extraction point

$s_6 = s_5 = 1.3062$ Btu/lbm-R $=>$ $T_6 = 148.1$ F, $h_6 = 679.8$ Btu/lbm

C.V. P1: $w_{P1} = v_1(P_2 - P_1) = 0.0242(300 – 30.415)\ 144/778 = 1.21$ Btu/lbm

$\Rightarrow$ $h_2 = h_1 + w_{P1} = 43.81$ Btu/lbm

C.V. P2: $w_{P2} = v_{6a}(P_4 - P_6) = 0.02625\ (300 – 125)144/778 = 0.85$ Btu/lbm

$\Rightarrow$ $h_{6b} = h_{6a} + w_{P2} = 119.1$ Btu/lbm

C.V. Total FWH and pump (notice $h_3 = h_{6a} + \Delta Pv = h_{6b}$, no table for this state)

State 3: $h_3 = h_f + (P_3 – P_{sat})v_f = 118.25 + 0.02625\ (300–125)\ 144/778 = 119.1$ Btu/lbm

The extraction fraction is: $y = \dot{m}_6/\dot{m}_4$

Energy: $(1 - y)h_2 + yh_6 = (1 - y)h_3 + yh_{6a}$

$$y = \frac{h_3 - h_2}{h_3 - h_2 + h_6 - h_{6a}} = \frac{119.1 - 43.81}{119.1 - 43.81 + 679.8 - 118.25} = 0.1182$$

$$\dot{m}_6 = y\,\dot{m}_4 = 0.1182 \times 10 = \textbf{1.182 lbm/s}$$

C.V. The junction after FWH and pump 2.

$$h_4 = (1–y)h_3 + y\,h_{6b} = (1– 0.1182) \times 119.1 + 0.1182 \times 119.1 = \textbf{119.1 Btu/lbm}$$

9.163E

Do the previous problem with a closed FWH that has a trap and drain to the condenser for the extraction flow.

State 1: $x_1 = 0$, $h_1 = 42.6$ Btu/lbm, $v_1 = 0.0242$ ft^3/lbm

State 3: $h_3 = h_f + (P_3 - P_{sat})v_f = 118.25 + 0.02625 (300-125) 144/778 = 119.1$ Btu/lbm

State 5: $h_5 = 744.07$ Btu/lbm, $s_5 = 1.3062$ Btu/lbm-R

State 6: $s_6 = s_5$ => $h_6 = 679.83$ Btu/lbm

State 6a: $x_{6a} = 0$ => $h_{6a} = 118.25$ Btu/lbm

C.V. P1: $w_{P1} = v_1(P_2 - P_1) = 0.0242(300 - 30.415) 144/778 = 1.21$ Btu/lbm

$$\Rightarrow \quad h_2 = h_1 + w_{P1} = 43.81 \text{ Btu/lbm}$$

C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = y$ (the extraction fraction)

Energy Eq.: $h_2 + y\,h_6 = 1\,h_3 + y\,h_{6a}$

$$y = \frac{h_3 - h_2}{h_6 - h_{6a}} = \frac{119.1 - 43.81}{679.83 - 118.25} = 0.1341$$

$$\dot{m}_{extr} = y\,\dot{m}_{tot} = 0.1341 \times 10 = \textbf{1.341 lbm/s}$$

$$h_4 = h_3 = \textbf{119.1 Btu/lbm}$$

Borgnakke and Sonntag

9.164E

The steam power cycle in Problem 9.147E has an isentropic efficiency of the turbine of 85% and that for the pump it is 80%. Find the cycle efficiency and the specific work and heat transfer in the components.

States numbered as in fig 9.3 of text.

CV Pump: $w_{P,S} = v_1(P_2 - P_1) = 0.01625(600 - 2.2)144/778 = 1.8$ Btu/lbm

$$\Rightarrow w_{P,AC} = 1.8/0.8 = \textbf{2.245 Btu/lbm}$$

$$h_2 = h_1 + w_{P,AC} = 97.97 + 2.245 = 100.2 \text{ Btu/lbm}$$

CV Turbine: $w_{T,S} = h_3 - h_{4s}$, $s_4 = s_3 = 1.4464$ Btu/lbm R

$$s_4 = s_3 = 1.4464 = 0.1817 + x_4 \times 1.7292 \Rightarrow x_4 = 0.7314,$$

$$h_4 = 97.97 + 0.7314 \times 1019.78 = 843.84 \text{ Btu/lbm}$$

$$\Rightarrow w_{T,S} = 1204.06 - 843.84 = 360.22 \text{ Btu/lbm}$$

$$w_{T,AC} = h_3 - h_{4AC} = 360.22 \times 0.85 = \textbf{306.2}$$

$$\Rightarrow h_{4AC} = 897.86 \text{ Btu/lbm (still two-phase)}$$

CV Boiler: $q_H = h_3 - h_2 = 1204.06 - 100.2 = \textbf{1103.9 Btu/lbm}$

$$q_L = h_{4AC} - h_1 = 897.86 - 97.97 = \textbf{799.9 Btu/lbm}$$

$$\eta_{CYCLE} = (w_T - w_P)/q_H = (306.2 - 2.245)/1103.9 = \textbf{0.275}$$

Compared to $(360.22-1.8)/1104.3 = 0.325$ in the ideal case.

state 2s and 2ac nearly the same

9.165E

A **concentrated solar power** plant receives the energy from molten salt coming in at 1000 F and leaving at 600 F in a counterflow heat exchanger where the water comes in at 400 psia, 140 F and leaves at 800 F, 400 psia. The molten salt has 10 lbm/s flow with Cp = 0.36 Btu/lbm-R. What is the possible water flow rate, the rate of energy transfer and rate of entropy generation.

Continuity Eqs.: Each line has a constant flow rate through it.

Energy Eq.4.10: $0 = \dot{m}_{H_2O} (h_1 - h_2) + \dot{m}_{salt} (h_3 - h_4)$

Entropy Eq.7.7: $0 = \dot{m}_{H_2O} (s_1 - s_2) + \dot{m}_{salt} (s_3 - s_4) + \dot{S}_{gen}$

Process: Each line has a constant pressure.

Table F.7: $h_2 = 1416.59$ Btu/lbm, $s_1 = 0.1985$, $s_2 = 1.6844$ Btu/lbm-R

$h_1 = h_f + (P_3 - P_{sat})v_f = 107.96 + (400 - 2.9) \, 0.01629 \times 144/778 = 109.15$ Btu/lbm

From the energy leaving the salt we get

$$\dot{Q} = \dot{m}_{salt} (h_3 - h_4) = \dot{m}_{salt} C_{P\,salt} (T_3 - T_4)$$

$$= 10 \times 0.36 \, [1000 - 600] = \textbf{1440 Btu/s}$$

$$\dot{m}_{H_2O} = \dot{m}_{salt} \frac{h_3 - h_4}{h_2 - h_1} = \frac{\dot{Q}}{h_2 - h_1} = \frac{1440 \text{ Btu/s}}{(1416.59 - 109.15) \text{ Btu/lbm}} = \textbf{1.1 lbm/s}$$

$$s_4 - s_3 = C_{P\,salt} \ln (T_4/T_3) = 0.36 \ln(1059.7/1459.7) = -0.115 \text{ Btu/lbm-R}$$

$$\dot{S}_{gen} = \dot{m}_{H_2O} (s_2 - s_1) + \dot{m}_{salt} (s_4 - s_3)$$

$$= 1.1 \text{ lbm/s } (1.6844 - 0.1985) \frac{\text{Btu}}{\text{lbm-R}} - 10 \text{ lbm/s} \times 0.115 \frac{\text{Btu}}{\text{lbm-R}}$$

$$= \textbf{0.48 Btu/s-R}$$

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for salt and water each. The two flows exchange energy with no heat transfer to/from the outside.

Borgnakke and Sonntag

9.166E

Steam leaves a power plant steam generator at 500 lbf/in.^{2}, 650 F, and enters the turbine at 490 lbf/in.^{2}, 625 F (h = 1314 Btu/lbm, s = 1.5752 Btu/lbm-R). The isentropic turbine efficiency is 88%, and the turbine exhaust pressure is 1.7 lbf/in.^{2}. Condensate leaves the condenser and enters the pump at 110 F, 1.7 lbf/in.^{2}. The isentropic pump efficiency is 80%, and the discharge pressure is 520 lbf/in.^{2}. The feedwater enters the steam generator at 510 lbf/in.^{2}, 100 F (h = 68.1 Btu/lbm). Calculate the thermal efficiency of the cycle and the entropy generation of the flow in the line between the steam generator exit and the turbine inlet, assuming an ambient temperature of 77 F.

$h_1 = 1328$ Btu/lbm, $s_1 = 1.562$ Btu/lbm-R

$$s_{3S} = s_2 = 1.5752 = 0.16483 + x_{3S} \times 1.7686 \quad => \quad x_{3S} = 0.79745$$

$$h_{3S} = 88.1 + 0.797\,45 \times 1025.4 = 905.8 \text{ Btu/lbm}$$

$$w_{T,s} = h_2 - h_{3S} = 1314.0 - 905.8 = 408.2 \text{ Btu/lbm}$$

$$w_{T,ac} = \eta_{T,s}\, w_{T,s} = 0.88 \times 408.2 = 359.2 \text{ Btu/lbm}$$

$$h_3 = h_2 - w_{T,ac} = 1314.0 - 359.2 = 954.8 \text{ Btu/lbm}$$

$$w_{P,s} = 0.016166\,(520 - 1.7)\,144/778 = 1.55 \text{ Btu/lbm}$$

$$w_{P,ac} = w_{P,s}/\eta_{P,s} = 1.55/0.80 = 1.94 \text{ Btu/lbm}$$

$$q_H = h_1 - h_6 = 1328.0 - 68.1 = 1259.9 \text{ Btu/lbm}$$

$$\eta_{TH} = w_{NET}/q_H = (359.2 - 1.94)/1259.9 = \mathbf{0.284}$$

C.V. Line from 1 to 2:   w = Ø,

Energy Eq.: $q = h_2 - h_1 = 1314 - 1328 = -14$ Btu/lbm

Entropy Eq.: $s_1 + s_{gen} + q/T_0 = s_2 \quad =>$

$$s_{gen} = s_2 - s_1 - q/T_0 = 1.5752 - 1.586 - (-14/536.7) = \mathbf{0.0153 \text{ Btu/lbm R}}$$

9.167E

A boiler delivers steam at 1500 lbf/in.^{2}, 1000 F to a reversible two-stage turbine as shown in Fig. 9.19. After the first stage, 25% of the steam is extracted at 200 lbf/in.^{2} for a process application and returned at 150 lbf/in.^{2}, 190 F to the feedwater line. The remainder of the steam continues through the low-pressure turbine stage, which exhausts to the condenser at 2.225 lbf/in.^{2}. One pump brings the feedwater to 150 lbf/in.^{2} and a second pump brings it to 1500 lbf/in.^{2}. If the process application requires 5000 Btu/s of power, how much power can then be cogenerated by the turbine?

C.V. Turbine T1

5: $h_5 = 1490.32$, $s_5 = 1.6001$ Btu/lbmR

6: Rev and adiabatic $\quad s_6 = s_5 \Rightarrow$

Table F.7.2 Sup. vapor

$\quad h_6 = 1246.6$ Btu/lbm

$\quad w_{T1} = h_5 - h_6 = 243.7$ Btu/lbm

State 7: $\quad s_7 = s_6 = s_5 \Rightarrow$

$\quad x_7 = \dfrac{1.6001 - 0.1817}{1.7292} = 0.82026$

$\quad h_7 = 97.97 + 0.82026 \times 1019.78 = 934.45$ Btu/lbm

$$w_{T2} = h_6 - h_7 = 1246.6 - 934.45 = 312.1 \text{ Btu/lbm}$$

8: Compressed liquid use sat. liq. same T: $\quad h_8 = 158.02$ Btu/lbm;

C.V. process unit. Assume no work only heat out.

$\quad q_{PROC} = h_6 - h_8 = 1246.6 - 158.02 = 1088.6$ Btu/lbm

$\quad \dot{m}_6 = \dot{Q}/q_{PROC} = 5000 \text{ Btu/s} / 1088.6 \text{ Btu/lbm} = 4.593 \text{ lbm/s} = 0.25 \, \dot{m}_{TOT}$

$\Rightarrow \dot{m}_{TOT} = \dot{m}_5 = 18.372 \text{ lbm/s}, \quad \dot{m}_7 = \dot{m}_5 - \dot{m}_6 = 13.779 \text{ lbm/s}$

C.V. Total turbine

$\quad \dot{W}_T = \dot{m}_5 h_5 - \dot{m}_6 h_6 - \dot{m}_7 h_7$

$\quad = 18.372 \times 1490.32 - 4.593 \times 1246.6 - 13.779 \times 934.45 \, [\dfrac{\text{lbm}}{\text{s}} \dfrac{\text{Btu}}{\text{lbm}}]$

$\quad = \textbf{8779 Btu/s}$

Borgnakke and Sonntag

Refrigeration Cycles

Borgnakke and Sonntag

9.168E

A car air-conditioner (refrigerator) in 70 F ambient uses R-134a and I want to have cold air at 20 F produced. What is the minimum high P and the maximum low P it can use?

Since the R-134a must give heat transfer out to the ambient at 70 F, it must at least be that hot at state 3.

Ideal Ref. Cycle
$T_{cond} = 70 \text{ F} = T_3$
$T_{evap} = 20 \text{ F}$

Use Table F.10 for R-134a

From Table F.10.1: $P_3 = P_2 = P_{sat} =$ **85.95 psia** is minimum high P.

Since the R-134a must absorb heat transfer at the cold air 20 F, it must at least be that cold at state 4.

From Table F.10.1: $P_1 = P_4 = P_{sat} =$ **33.29 psia** is maximum low P.

Borgnakke and Sonntag

9.169E

Consider an ideal refrigeration cycle that has a condenser temperature of 110 F and an evaporator temperature of 5 F. Determine the coefficient of performance of this refrigerator for the working fluids R-134a and R-410A.

Ideal Ref. Cycle
$T_{cond} = 110\ F = T_3$
$T_{evap} = 5\ F$

Use Table F.10 for R-134a
Use Table F.9 for R-410A

Property for:	R-134a	R-410A
h_1, Btu/lbm	167.5	117.8
$s_2 = s_1$, Btu/lbm-R	0.4149	0.2549
P_2, lbf/in^2	161.1	380.4
T_2, F	123.7	157.3
h_2, Btu/lbm	184.7	138.4
$h_3 = h_4$, Btu/lbm	112.4	56.1
$w_C = h_2 - h_1$	17.2	20.6
$q_L = h_1 - h_4$	55.1	61.7
$\beta = q_L/(-w_C)$	3.20	3.0

Borgnakke and Sonntag

9.170E

Find the high temperature, the condensing temperature, and the COP if ammonia is used in a standard refrigeration cycle with high and low pressures of 800 psia and 300 psia, respectively.

Exit evaporator $x = 1$ and 300 psia from F.8.2:

$$T_1 = 123.2 \text{ F}, \quad h_1 = 632.63 \text{ Btu/lbm}, \quad s_1 = 1.1356 \text{ Btu/lbm-R}$$

Exit condenser saturated liquid 800 psia from F.8.1:

$$T_3 = \textbf{200.65 F}, \quad h_3 = 284.17 \text{ Btu/lbm}$$

Exit compressor: 800 psia, $s = s_1$, so interpolate in F.8.2

$$T_2 = \textbf{269 F}, \quad h_2 = 692.65 \text{ Btu/lbm}$$

$$w_C = h_2 - h_1 = 60.02 \text{ Btu/lbm}$$
$$q_L = h_1 - h_4 = h_1 - h_3 = 348.46 \text{ Btu/lbm}$$

COP:
$$\beta = \frac{q_L}{w_c} = \frac{h_1 - h_3}{h_2 - h_1} = \frac{348.46}{60.02} = \textbf{6.2}$$

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9.171E

A refrigerator receives 500 W of electrical power to the compressor driving the cycle flow of R-134a. The refrigerator operates with a condensing temperature of 100 F and a low temperature of -10 F. Find the COP for the cycle and its cooling capacity.

Solution:

Ideal refrigeration cycle
$T_{cond} = 100\ F = T_3$
$T_{evap} = -10\ F = T_1$

Use Table F.10

State 1: $h_1 = 165.09$ Btu/lbm, $s_1 = 0.4162$ Btu/lbm-R

State 2: $s = s_1$, $P_2 = P_3 = 138.926$ psia, $h_2 = 184.02$ Btu/lbm
$w_C = h_2 - h_1 = 184.02 - 165.09 = 18.934$ Btu/lbm

State 3-4: $h_4 = h_3 = 108.86$ Btu/lbm
$q_L = h_1 - h_4 = 165.09 - 108.86 = 56.23$ Btu/lbm
$COP = \beta = q_L/w_C =$ **2.97**

The cooling capacity is the rate at which it can cool

$$\dot{Q}_L = COP \times \dot{W} = 2.97 \times 500\ W = \textbf{1485 W} = \textbf{1.407 Btu/s}$$

Borgnakke and Sonntag

9.172E

Consider an ideal heat pump that has a condenser temperature of 120 F and an evaporator temperature of 30 F. Determine the coefficient of performance of this heat pump for the working fluids R-410A and ammonia.

Ideal Heat Pump
T_{cond} = 120 F
T_{evap} = 30 F
Use Table F.8 for NH_3
Use Table F.9 for R-410A

Property:	R-410A		NH_3
h_1, Btu/lbm	119.9		619.5
$s_2 = s_1$	0.2482		1.2768
P_2, lbf/in^2	433.3		286.5
T_2, F	158.2		239.3
h_2, Btu/lbm	135.9		719.4
$h_3=h_4$, Btu/lbm	60.8		178.8
$w_C = h_2-h_1$	16.0		99.9
$q_H = h_2-h_3$	75.1		540.6
$\beta' = q_H/w_C$	4.69		5.41

Borgnakke and Sonntag

9.173E

The refrigerant R-134a is used as the working fluid in a conventional heat pump cycle. Saturated vapor enters the compressor of this unit at 50 F; its exit temperature from the compressor is measured and found to be 185 F. If the compressor exit is 300 psia, what is the isentropic efficiency of the compressor and the coefficient of performance of the heat pump?

R-134a heat pump: $T_2 = 185$ F

$\qquad\qquad\qquad T_{EVAP} = 50$ F

State 1: Table F.9.1

$\quad h_1 = 173.8$ Btu/lbm,

$\quad s_1 = 0.4113$ Btu/lbm R

State 2: $\quad h_2 = 193.8$ Btu/lbm

Compressor work: $\quad w_C = h_2 - h_1 = 193.8 - 173.8 = 20.0$ Btu/lbm

Isentropic compressor: $\quad s_{2S} = s_1 = 0.4113$ Btu/lbm R

State 2s: $(P_2, s)\quad T_{2S} = 166.9$ F, $\quad h_{2S} = 188.0$ Btu/lbm

Ideal compressor work: $\quad w_{C\,s} = h_{2S} - h_1 = 188.0 - 173.8 = 14.2$ Btu/lbm

The efficiency is the ratio of the two work terms

$$\eta_{S\ COMP} = \frac{w_{C\,s}}{w_C} = \frac{14.2}{20.0} = \mathbf{0.71}$$

The condenser has heat transfer as ($h_3 = h_f$ at 300 psia)

$$q_H = h_2 - h_3 = 193.8 - 130.1 = 63.7 \text{ Btu/lbm}$$

and a coefficient of performance of

$$\beta' = q_H/w_C = \mathbf{3.19}$$

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Exergy and Combined Cycles

Borgnakke and Sonntag

9.174E

A Rankine cycle maintains 130 F in the condenser which is cooled by a 70 F reservoir. The steam out of the boiler is at 600 psia, 700 F being heated from a 900 F source. Determine the flux of exergy in or out of the reservoirs per unit mass flowing in the cycle. Find the overall cycle second law efficiency.

Solution (Cycle from problem 9.181E):

State 3: Superheated vapor $h_3 = 1350.62$ Btu/lbm, $s_3 = 1.5871$ Btu/lbm R,

State 1: Sat. liq. $h_1 = 97.97$ Btu/lbm, $v_1 = 0.01625$ ft^3/lbm, s = 0.1817 Btu/lbm R

C.V. Pump: Adiabatic and reversible. Use incompressible fluid so

$$w_P = \int v \, dP = v_1(P_2 - P_1) = 0.01625 \text{ ft}^3/\text{lbm} \times (600 - 2.2) \text{ psi} \times 144 \text{ in}^2/\text{ft}^2$$

$$= 1398.85 \text{ lbf-ft} = 1.8 \text{ Btu/lbm}$$

$$h_2 = h_1 + w_P = 97.97 + 1.8 = 99.77 \text{ Btu/lbm}$$

C.V. Boiler: $q_H = h_3 - h_2 = 1350.62 - 99.77 = 1250.85$ Btu/lbm

C.V. Tubine: $s_4 = s_3 = 1.5871$ Btu/lbm R $= 0.1817 + x_4 \, 1.7292 \Rightarrow x_4 = 0.8127,$

$$h_4 = 97.97 + 0.8127 \times 1019.78 = 926.75 \text{ Btu/lbm}$$

$$w_T = h_3 - h_4 = 1350.62 - 926.75 = 423.87 \text{ Btu/lbm}$$

C.V. Condenser: $q_L = h_4 - h_1 = 926.75 - 97.97 = 828.8$ Btu/lbm

We can do the fluxes of exergy evaluarted at the reservoir T's

$$\Delta\psi_H = (1 - T_0/T_H) \, q_H = (1 - \frac{536.67}{1359.67}) \times 1250.85 = \textbf{757.13 Btu/lbm}$$

$$\Delta\psi_L = (1 - T_0/T_L) \, q_L = (1 - \frac{536.67}{529.67}) \, 828.8 = \textbf{-10.95 Btu/lbm}$$

(as $T_L < T_0$ you decrease ψ)

The overall second law efficiency is based on the exergy delivered from the high T source

$$\eta_{II} = \frac{w_{net}}{\Delta\psi_H} = \frac{w_T - w_P}{\Delta\psi_H} = \frac{423.87 - 1.8}{757.13} = \textbf{0.557}$$

Notice $T_H > T_3$, $T_L < T_4 = T_1$, so cycle is externally irreversible. Both q_H and q_L are over finite ΔT.

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Energy Transfers (Btu/lbm):   Exergy Transfers (Btu/lbm):

T_H $q_H = 1251$ $w = 1.8$ $w = 424$ H.E   cb $q_L = 829$ T_L

T_H $\phi_H = 757$ $\phi = 497$ $w = 1.8$ $w = 424$ H.E   cb $\phi = 74.5$ $\phi_L = -11$ T_L

Comment: The exergy into the cycle from the high temperature source must be found from the flow exergies in the water as the temperature is varying.

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9.175E

Find the flows and fluxes of exergy in the condenser of Problem 9.154E. Use those to determine the 2nd law efficiency.

A smaller power plant produces 50 lbm/s steam at 400 psia, 1100 F, in the boiler. It cools the condenser with ocean water coming in at 60 F and returned at 65 F so that the condenser exit is at 110 F. Find the net power output and the required mass flow rate of the ocean water.

Solution:

The states properties from Tables F.7.1

State 1: 110 F, x = 0: $h_1 = 78.01$ Btu/lbm, $s_1 = 0.1473$ Btu/lbm R,

State 3: 400 psia, 1100 F: $h_3 = 1577.44$ Btu/lbm, $s_3 = 1.7989$ Btu/lbm R

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$$s_4 = s_3 = 1.7989 = 0.1473 + x_4 (1.8101) \quad => \quad x_4 = 0.9124$$

$$=> h_4 = 78.01 + 0.9124 (1031.28) = 1018.95 \text{ Btu/lbm}$$

C.V. Condenser : $q_L = h_4 - h_1 = 1018.95 - 78.01 = 940.94$ Btu/lbm

$$\dot{Q}_L = \dot{m}q_L = 50 \times 940.94 = 47\ 047 \text{ Btu/s} = \dot{m}_{ocean} \, C_p \, \Delta T$$

$$\dot{m}_{ocean} = \dot{Q}_L / C_p \, \Delta T = 47\ 047 / (1.0 \times 5) = 9409 \text{ lbm/s}$$

The specific flow exergy for the two states are from Eq.10.24 neglecting kinetic and potential energy

$$\psi_4 = h_4 - h_0 - T_0(s_4 - s_0), \quad \psi_1 = h_1 - h_0 - T_0(s_1 - s_0)$$

The net drop in exergy of the water is

$$\dot{\Phi}_{water} = \dot{m}_{water} \, [h_4 - h_1 - T_o(s_4 - s_1)]$$

$$= 50 \, [1018.95 - 78.01 - 514.7 (1.7989 - 0.1473)]$$

$$= 47\ 047 - 42\ 504 = \textbf{4543 Btu/s}$$

Notice that the reference values drops out.

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The net gain in exergy of the ocean water is

$$\dot{\Phi}_{ocean} = \dot{m}_{ocean}[h_6 - h_5 - T_o(s_6 - s_5)]$$

$$= \dot{m}_{ocean}[C_p(T_6 - T_5) - T_oC_p \ln\left(\frac{T_6}{T_5}\right)]$$

$$= 9409 [1.0 (65 - 60) - 514.7 \times 1.0 \ln \frac{459.7 + 65}{459.7 + 60}]$$

$$= 47\ 047 - 46\ 370 = \textbf{677 Btu/s}$$

The second law efficiency is

$$\eta_{II} = \dot{\Phi}_{ocean} / \dot{\Phi}_{water} = \frac{677}{4543} = \textbf{0.15}$$

In reality all the exergy in the ocean water is destroyed as the 65 F water mixes with the ocean water at 60 F after it flows back out into the ocean and the efficiency does not have any significance. Notice the small rate of exergy relative to the large rates of energy being transferred.

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9.176E

Find the flows of exergy into and out of the feedwater heater in Problem 9.160E.

State 1: $x_1 = 0$, $h_1 = 97.97$ Btu/lbm, $v_1 = 0.01625$ ft^3/lbm, s = 0.1817

State 3: $x_3 = 0$, $h_3 = 330.7$ Btu/lbm, $s_3 = 0.5142$ Btu/lbm R

State 5: $h_5 = 1350.52$ Btu/lbm, $s_5 = 1.5871$ Btu/lbm R

State 6: $s_6 = s_5 = 1.5871$ Btu/lbm R $\Rightarrow$ $h_6 = 1208.93$ Btu/lbm

C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.01625(150 - 2.225)\frac{144}{778} = 0.44 \text{ Btu/lbm}$$

$$\Rightarrow h_2 = h_1 + w_{P1} = 97.97 + 0.44 = 98.41 \text{ Btu/lbm}$$

$$s_2 = s_1 = 0.1817 \text{ Btu/lbm R}$$

C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = x$ (the extraction fraction)

Energy Eq.: $(1 - x) h_2 + x h_6 = 1 h_3$

$$x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{330.7 - 98.41}{1208.93 - 98.41} = 0.2092$$

Ref. State: 14.7 psia, 77 F, $s_o = 0.08774$ Btu/lbm R, $h_o = 45.08$ Btu/lbm

$$\psi_2 = h_2 - h_o - T_o(s_2 - s_o)$$

$$= 98.41 - 45.08 - 536.67(0.1817 - 0.08774) = 2.90 \text{ Btu/lbm}$$

$$\psi_6 = 1208.93 - 45.08 - 536.67(1.5871 - 0.08774) = 359.2 \text{ Btu/lbm}$$

$$\psi_3 = 330.7 - 45.08 - 536.67(0.5142 - 0.08774) = 56.75 \text{ Btu/lbm}$$

The rate of exergy flow scaled with maximum flow rate is then

$$\dot{\Phi}_2/\dot{m}_3 = (1 - x) \psi_2 = 0.7908 \times 2.90 = \textbf{2.297 Btu/lbm}$$

$$\dot{\Phi}_6/\dot{m}_3 = x\psi_6 = 0.2092 \times 359.2 = \textbf{75.144 Btu/lbm}$$

$$\dot{\Phi}_3/\dot{m}_3 = \psi_3 = \textbf{56.75 Btu/lbm}$$

The mixing is destroying $2.297 + 75.144 - 56.75 = 20.7$ Btu/lbm of exergy

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9.177E

For Problem 9.162E, consider the boiler/superheater. Find the exergy destruction and the second law efficiency for the boiler-source setup.

The boiler has flow in at state 4 and out at state 5 with the source providing a q at 350 F.

Assume state 4 is saturated liquid at T_4 so $h_4 = h_{f\,4}$ => $T_4 = 68.4$ F

State 4: $h_4 = 118.4$ Btu/lbm $= h_{f\,4}$, $s_4 = s_{f\,4} = 0.2494$ Btu/lbm-R

State 5: $h_5 = 744.1$ Btu/lbm, $s_5 = 1.3062$ Btu/lbm-R

Energy Eq. $q_H = h_5 - h_4 = 744.1 - 118.4 = 625.7$ Btu/lbm

Entropy Eq. $s_4 + q_H/T_H + s_{gen} = s_5$ => $s_{gen} = s_5 - s_4 - q_H/T_H$

$$\dot{S}_{gen} = \dot{m}s_{gen} = 10\,[1.3062 - 0.2494 - \frac{625.7}{350 + 459.7}\,]$$

$$= 10 \times 0.2841 = \mathbf{2.841\ Btu/s\text{-}R}$$

The flow increase in exergy is:

$$\psi_5 - \psi_4 = h_5 - h_4 - T_o\,(s_5 - s_4) = 625.7 - 536.7\,(1.3062 - 0.2494)$$
$$= 58.62\ \text{Btu/lbm}$$

The exergy provided by the source is:

$$\phi_H = (\,1 - T_o/\,T_H)\,q_H = (1 - \frac{536.7}{350 + 459.7})\,625.7 = 210.96\ \text{Btu/lbm}$$

Second law efficiency is

$$\eta_{II} = \frac{\text{gain in exergy}}{\text{source exergy input}} = \frac{58.62}{210.96} = \mathbf{0.265}$$

$$(\,= 1 - T_o s_{gen}/\phi_H = 1 - 536.7 \times 0.2841\,/\,210.96\,)$$

9.178E

Steam is supplied in a line at 400 lbf/in.^{2}, 1200 F. A turbine with an isentropic efficiency of 85% is connected to the line by a valve and it exhausts to the atmosphere at 14.7 lbf/in.^{2}. If the steam is throttled down to 300 lbf/in.^{2} before entering the turbine find the actual turbine specific work. Find the change in exergy through the valve and the second law efficiency of the turbine.

C.V. Valve: Energy Eq.: $h_2 = h_1 = 1631.79$ Btu/lbm,

Entropy Eq.: $s_2 > s_1 = 1.8327$ Btu/lbm R,

State 2: $h_2, P_2 \Rightarrow s_2 = 1.86407$ Btu/lbm R

Ideal turbine: $s_3 = s_2 \Rightarrow h_{3s} = 1212.28$ Btu/lbm

$w_{T,s} = h_2 - h_{3s} = 419.51$ Btu/lbm

Actual turbine: $w_{T,ac} = \eta_T w_{T,s} = $ **356.58 Btu/lbm**

$h_{3ac} = h_2 - w_{Tac} = 1275.21$ Btu/lbm $\Rightarrow s_{3ac} = 1.9132$ Btu/lbm R

$\psi_2 - \psi_1 = h_2 - h_1 - T_0(s_2 - s_1)$

$= 0 - 536.67(1.86407 - 1.8327) = -16.835$ Btu/lbm

$w^{rev} = \psi_2 - \psi_3 = 1631.79 - 1275.21 - 536.67(1.86407 - 1.9132)$

$= 382.95$ Btu/lbm

$\eta_{II} = w_{ac}/w^{rev} = 356.58/382.95 = $ **0.931**

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9.179E

Find the two heat transfer rates, the total cycle exergy destruction, and the second law efficiency for the refrigerator in problem 9.171E.

A refrigerator receives 500 W of electrical power to the compressor driving the cycle flow of R-134a. The refrigerator operates with a condensing temperature of 100 F and a low temperature of -10 F. Find the COP for the cycle.

State 1: $h_1 = 165.09$ Btu/lbm, $s_1 = 0.4162$ Btu/lbm-R

State 2: $s_2 = s_1$, $P_2 = P_3 = 138.9$ psia, $h_2 = 184.02$ Btu/lbm

$$w_C = h_2 - h_1 = 184.02 - 165.09 = 18.93 \text{ Btu/lbm}$$

State 3: $h_3 = 108.86$ Btu/lbm

State 4: $h_4 = h_3 = 108.86$ Btu/lbm,

$$q_H = h_2 - h_3 = 184.02 - 108.86 = 75.16 \text{ Btu/lbm}$$

$$q_L = h_1 - h_4 = 56.23 \text{ Btu/lbm}$$

$$\dot{m} = \dot{W}/w_C = 0.5 \times 3412 / 18.93 = 90.12 \text{ lbm/hr}$$

$$\dot{Q}_H = 90.12 \times 75.16 = 6773 \text{ Btu/hr}; \quad \dot{Q}_L = 90.12 \times 56.23 = 5067 \text{ Btu/hr}$$

Entropy Eq. for total C.V.: $\quad 0 = \dfrac{q_L}{T_L} - \dfrac{q_H}{T_H} + s_{gen}$

$$i = T_o \, s_{gen} = T_o \left(\dfrac{q_H}{T_H} - \dfrac{q_L}{T_L} \right) = 536.7 \left(\dfrac{75.16}{559.7} - \dfrac{56.23}{449.7} \right) = 4.96 \text{ Btu/lbm}$$

$$I = \dot{m} \, i = 90.12 \text{ lbm/hr} \times 4.96 \text{ Btu/lbm} = \textbf{447 Btu/hr}$$

The cold space gain in availability

$$\Delta\psi_L = (1 - \dfrac{T_o}{T_L})(-q_L) = (1 - \dfrac{536.7}{449.7})(-56.23) = 10.88 \text{ Btu/lbm}$$

$$\eta_{II} = \dfrac{\text{output}}{\text{source}} = \dfrac{\Delta\psi_L}{w_C} = \dfrac{10.88}{18.93} = \textbf{0.575}$$

Remark: The high temperature reservoir also gains availability so

$$\Delta\psi_H = (1 - \dfrac{T_o}{T_H}) q_L = (1 - \dfrac{536.7}{559.7}) 75.16 = 3.09 \text{ Btu/lbm}$$

the total balance is : $\quad w_C = \Delta\psi_H + \Delta\psi_L + i = 3.09 + 10.88 + 4.96 = 18.93$ OK.

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9.180E

Consider an ideal dual-loop heat-powered refrigeration cycle using R-134a as the working fluid, as shown in Fig. P9.135. Saturated vapor at 200 F leaves the boiler and expands in the turbine to the condenser pressure. Saturated vapor at 0 F leaves the evaporator and is compressed to the condenser pressure. The ratio of the flows through the two loops is such that the turbine produces just enough power to drive the compressor. The two exiting streams mix together and enter the condenser. Saturated liquid leaving the condenser at 110 F is then separated into two streams in the necessary proportions. Determine the ratio of mass flow rate through the power loop to that through the refrigeration loop. Find also the performance of the cycle, in terms of the ratio Q_L/Q_H.

	T	P	h	s
	F	lbf/in^2	Btu/lbm	Btu/lbm R
1	0	22.2	166.8	0.4154
2	-	161.1		0.4154
3	110	161.1	112.4	0.067 45
4	0	21.2	112.4	
5	-	503.6		
6	200	503.6	181.7	0.3955
7	110	161.1		0.3955

Computer tables for properties.

$P_2=P_3=P_{SAT}$ at 110 F

$P_5=P_6=P_{SAT}$ at 200 F

$s_2=s_1=0.4154$

$h_2 = 185.0$

Pump work:

$w_P = h_5-h_3$

$\approx v_3(P_5-P_3)$

$$w_P = 0.01415(503.6 - 161.1)\frac{144}{778} = 0.897 \text{ Btu/lbm}$$

$$h_5 = 112.4 + 0.897 = 113.3 \text{ Btu/lbm}$$

$$s_7 = s_6 = 0.3955 = 0.2881 + x_7(0.1203) \implies x_7 = 0.8928$$

$$h_7 = 112.4 + 0.8928 (68.5) = 173.6 \text{ Btu/lbm}$$

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CV: turbine + compressor

Continuity Eq.: $\dot{m}_1 = \dot{m}_2, \quad \dot{m}_6 = \dot{m}_7$

Energy Eq.: $\dot{m}_1 h_1 + \dot{m}_6 h_6 = \dot{m}_2 h_2 + \dot{m}_7 h_7$

$$\dot{m}_6 / \dot{m}_1 = \frac{h_2 - h_1}{h_6 - h_7} = \frac{185.0 - 166.8}{181.7 - 173.6} = \mathbf{2.247}$$

CV evaporator: $\dot{Q}_L = \dot{m}_1 (h_1 - h_4),$ CV boiler: $\dot{Q}_H = \dot{m}_6 (h_6 - h_5)$

$$\Rightarrow \beta = \dot{Q}_L / \dot{Q}_H = \frac{\dot{m}_1 (h_1 - h_4)}{\dot{m}_6 (h_6 - h_5)} = \frac{166.8 - 112.4}{2.247\,(181.7 - 113.3)} = \mathbf{0.354}$$

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Review Problems

9.181E

The power plant in Problem 9.147E is modified to have a superheater section following the boiler so the steam leaves the super heater at 600 lbf/in.^{2}, 700 F. Find the specific work and heat transfer in each of the ideal components and the cycle efficiency.

Solution:

For this cycle from Table F.7

State 3: Superheated vapor $\quad$ h_3 = 1350.62 Btu/lbm, s_3 = 1.5871 Btu/lbm R,

State 1: Saturated liquid $\quad$ h_1 = 97.97 Btu/lbm, v_1 = 0.01625 ft^3/lbm

C.V. Pump: Adiabatic and reversible. Use incompressible fluid so

$$w_P = \int v \, dP = v_1(P_2 - P_1) = 0.01625(600 - 2.2)\frac{144}{778} = \textbf{1.8 Btu/lbm}$$

$$h_2 = h_1 + w_P = 97.97 + 1.8 = 99.77 \text{ Btu/lbm}$$

C.V. Boiler: $\quad$ $q_H = h_3 - h_2 = 1350.62 - 99.77 = \textbf{1250.85 Btu/lbm}$

C.V. Tubine: $\quad$ $w_T = h_3 - h_4, \quad s_4 = s_3$

$\quad$ $s_4 = s_3 = 1.5871$ Btu/lbm R = $0.1817 + x_4 \, 1.7292 \;\Rightarrow\; x_4 = 0.8127,$

$\quad\quad$ $h_4 = 97.97 + 0.8127 \times 1019.78 = 926.75$ Btu/lbm

$w_T = 1350.62 - 926.75 = \textbf{423.87 Btu/lbm}$

$\eta_{CYCLE} = (w_T - w_P)/q_H = (423.87 - 1.8)/1250.85 = \textbf{0.337}$

C.V. Condenser:

$\quad$ $q_L = h_4 - h_1 = 926.75 - 97.97 = \textbf{828.8 Btu/lbm}$

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9.182E

Consider a small ammonia absorption refrigeration cycle that is powered by solar energy and is to be used as an air conditioner. Saturated vapor ammonia leaves the generator at 120 F, and saturated vapor leaves the evaporator at 50 F. If 3000 Btu of heat is required in the generator (solar collector) per pound-mass of ammonia vapor generated, determine the overall performance of this system.

NH_3 absorption cycle:

sat. vapor at 120 F exits the generator.
Sat. vapor at 50 F exits the evaporator

$q_H = q_{GEN} = 3000$ Btu/lbm NH_3

out of generator.

$q_L = h_2 - h_1 = h_{G\,50\,F} - h_{F\,120\,F} = 624.28 - 178.79$

$= 445.49$ Btu/lbm $\Rightarrow$ $q_L/q_H = 445.49/3000 = $ **0.1485**

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9.183E

Consider an ideal combined reheat and regenerative cycle in which steam enters the high-pressure turbine at 500 lbf/in.^{2}, 700 F, and is extracted to an open feedwater heater at 120 lbf/in.^{2} with exit as saturated liquid. The remainder of the steam is reheated to 700 F at this pressure, 120 lbf/in.^{2}, and is fed to the low-pressure turbine. The condenser pressure is 2.225 lbf/in.^{2}. Calculate the thermal efficiency of the cycle and the net work per pound-mass of steam.

5: $h_5 = 1356.66$, $s_5 = 1.6112$

7: $h_7 = 1378.17$, $s_7 = 1.7825$

3: $h_3 = h_f = 312.59$, $v_3 = 0.01788$

C.V. T1

  $s_5 = s_6$ $\Rightarrow h_6 = 1209.76$

$w_{T1} = h_5 - h_6 = 1356.66 - 1209.76$

  $= 146.9$ Btu/lbm

C.V. Pump 1

$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1)$
    $= 0.01623(120 - 2) = 0.354$

 $\Rightarrow h_2 = h_1 + w_{P1} = 93.73 + 0.354 = 94.08$ Btu/lbm

C.V. FWH

  $x\, h_6 + (1 - x)\, h_2 = h_3$

$x = \dfrac{h_3 - h_2}{h_6 - h_2} = \dfrac{312.59 - 94.08}{1209.76 - 94.08} = 0.1958$

C.V. Pump 2

    $w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.01788(500 - 120)(144/778) = 1.26$ Btu/lbm

    $\Rightarrow h_4 = h_3 + w_{P2} = 312.59 + 1.26 = 313.85$ Btu/lbm

    $q_H = h_5 - h_4 + (1 - x)(h_7 - h_6) = 1042.81 + 135.43 = 1178.2$ Btu/lbm

  C.V. Turbine 2

        $s_7 = s_8$ $\Rightarrow x_8 = (1.7825 - 0.1817)/1.7292 = 0.9257$

        $h_8 = h_f + x_8\, h_{fg} = 97.97 + 0.9257 \times 1019.78 = 1041.98$

        $w_{T2} = h_7 - h_8 = 1378.17 - 1041.98 = 336.19$

        $w_{net} = w_{T1} + (1 - x)\, w_{T2} - (1 - x)\, w_{P1} - w_{P2}$

            $= 146.9 + 270.36 - 0.285 - 1.26 = \textbf{415.72 kJ/kg}$

        $\eta_{cycle} = w_{net} / q_H = 415.72 / 1178.2 = \textbf{0.353}$

9.184E

In one type of nuclear power plant, heat is transferred in the nuclear reactor to liquid sodium. The liquid sodium is then pumped through a heat exchanger where heat is transferred to boiling water. Saturated vapor steam at 700 lbf/in.^{2} exits this heat exchanger and is then superheated to 1100 F in an external gas-fired superheater. The steam enters the turbine, which has one (open-type) feedwater extraction at 60 lbf/in.^{2}. The isentropic turbine efficiency is 87%, and the condenser pressure is 0.95 lbf/in.^{2}. Determine the heat transfer in the reactor and in the superheater to produce a net power output of 1000 Btu/s.

$\dot{W}_{NET} = 1000$ Btu/s, $\eta_{ST} = 0.87$

CV Pump 1

$w_{P12} = 0.016136(60 - 1)144/778 = 0.18$ Btu/lbm

$h_2 = h_1 + w_{P12} = 69.73 + 0.18 = 69.91$ Btu/lbm

CV Pump 2

$w_{P34} = 0.017378(700 - 60)144/778 = 2.06$ Btu/lbm

$h_4 = h_3 + w_{P34} = 262.24 + 2.06 = 264.3$ Btu/lbm

CV HP Turbine section

$s_{7S} = s_6 = 1.7682,$ P_7 => $T_{7S} = 500.8$ F, $h_{7S} = 1283.4$ Btu/lbm

$h_7 = h_6 - \eta_{ST}(h_6 - h_{7S}) = 1625.8 - 0.87(1625.8 - 1283.4) = 1327.9$

CV LP Turbine section

$s_{8S} = s_6 = 1.7682 = 0.1296 + x_{8S} \times 1.8526$ => $x_{8S} = 0.8845$

$h_{8S} = 68.04 + 0.8845 \times 1036.98 = 985.25$ Btu/lbm

$h_8 = h_6 - \eta_{ST}(h_6 - h_{8S}) = 1625.8 - 0.87(1625.8 - 985.25) = 1068.5$ Btu/lbm

CV Feedwater heater: Continuity Eq.: $m_2 + m_7 = m_3 = 1.0$ lbm,

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Energy Eq.: $m_2h_2 + m_7h_7 = m_3h_3$

$m_7 = (262.24\text{-}69.91) / (1327.9\text{-}69.91) = 0.1529$

CV: turbine: $w_T = (h_6 - h_7) + (1 - m_7)(h_7 - h_8)$

$= 1625.8\text{-}1327.9 + 0.8471(1327.9\text{-}1068.5) = 517.6 \text{ Btu/lbm}$

CV pumps: $w_P = m_1w_{P12} + m_3w_{P34} = 0.8471{\times}0.18 + 1{\times}2.06 = 2.2 \text{ Btu/lbm}$

$w_{NET} = 517.6 - 2.2 = 515.4 \text{ Btu/lbm}$ $\Rightarrow$ $\dot{m} = 1000/515.4 = 1.940 \text{ lbm/s}$

CV: reactor $\dot{Q}_{REACT} = \dot{m}(h_5 - h_4) = 1.94 (1202 - 264.3) =$ **1819 Btu/s**

CV: superheater $\dot{Q}_{SUP} = \dot{m}(h_6 - h_5) = 1.94 (1625.8 - 1202) =$ **822 Btu/s**

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9.192-eE

Consider an ideal heat pump that has a condenser temperature of 120 F and an evaporator temperature of 30 F. Determine the coefficient of performance of this heat pump for the working fluid R-12.

Ideal Heat Pump
T_{cond} = 120 F
T_{evap} = 30 F

Use computer table for R-12

Property/parameter	R-12
h_1, Btu/lbm	80.42
$s_2 = s_1$, Btu/lbm-R	0.1665
P_2, lbf/in^2	172.3
T_2, F	132.2
h_2, Btu/lbm	91.0
$h_3 = h_4$, Btu/lbm	36.011
$-w_C = h_2 - h_1$	10.58
$q_H = h_2 - h_3$	54.995
$\beta' = q_H/(-w_C)$	5.198

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9.193-dE

Do Problem 9.151 with R-22 as the working fluid.

Standard Rankine cycle with properties from the computer R-22 tables,

$$h_1 = 39.267 \text{ Btu/lbm}, \ v_1 = 0.01404 \text{ ft}^3/\text{lbm}, \ P_1 = 210.6 \text{ psia},$$

$$P_2 = P_3 = 554.8 \text{ psia}, \ h_3 = 110.07 \text{ Btu/lbm}, \ s_3 = 0.1913 \text{ Btu/lbm R}$$

CV: Pump $\quad w_P = v_1(P_2\text{-}P_1) = 0.01404 \ (554.8\text{-}210.6)\dfrac{144}{778} = 0.894 \text{ Btu/lbm}$

$$h_2 = h_1 + w_P = 39.267 + 0.894 = 40.16 \text{ Btu/lbm}$$

CV: Turbine $\quad s_4 = s_3$

$$\Rightarrow x_4 = (0.1913 - 0.07942)/0.13014 = 0.9442$$

$$h_4 = 101.885 \text{ Btu/lbm}, \ w_T = h_3 - h_4 = 8.185 \text{ Btu/lbm}$$

CV: Boiler

$$q_H = h_3 - h_2 = 110.07 - 40.16 = 69.91 \text{ Btu/lbm}$$

$$\eta_{TH} = (w_T - w_P)/q_H = (8.185 - 0.894)/157.21 = \mathbf{0.104}$$

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9.195E

The simple steam power plant in Problem 4.180E, shown in Fig P4.121 has a turbine with given inlet and exit states. Find the exergy at the turbine exit, state 6. Find the second law efficiency for the turbine, neglecting kinetic energy at state 5. Use CATT3 to solve.

Properties from Problem 4.180E and s values from F.7.2.

$h_6 = 1029$, $h_5 = 1455.6$, $h_O = 45.08$ all in Btu/lbm

$s_5 = 1.6408$, $s_6 = 1.8053$, $s_O = 0.08769$ all in Btu/lbm R

Kinetic energy at state 6: $KE_6 = 0.5V_6^2 = 600^2 /(2 \times 25\ 037) = 7.19$ Btu/lbm

Recall 1 Btu/lbm = 25 037 ft^2/s^2

$$\psi_6 = h_6 + KE_6 - h_O - T_O\ (\ s_6 - s_O\ )\ = 61.26 \text{ Btu/lbm}$$

$$w^{rev} = \psi_5 - \psi_6 =\ h_5 - h_6 - T_O(\ s_5 - s_6\ )\ = 515.2 \text{ Btu/lbm}$$

$$w^{AC} = h_5 - h_6 = 426.9 \text{ Btu/lbm}$$

$$\eta_{II} =\ w^{ac} / w^{rev} = 426.9 / 515.2 = \textbf{0.829}$$

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Fundamentals of
Thermodynamics

SOLUTION MANUAL
CHAPTER 10

8e

Updated June 2013

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CONTENT CHAPTER 10

In-Text Concept Questions

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10.a

The Brayton cycle has the same 4 processes as the Rankine cycle, but the T-s and P-v diagrams look very different; why is that?

The Brayton cycle have all processes in the superheated vapor (close to ideal gas) region. The Rankine cycle crosses in over the two-phase region.

10.b

Is it always possible to add a regenerator to the Brayton cycle? What happens when the pressure ratio is increased?

No. When the pressure ratio is high, the temperature after compression is higher than the temperature after expansion. The exhaust flow can then not heat the flow into the combustor.

10.c

Why would you use an intercooler between compressor stages?

The cooler provides two effects. It reduces the specific volume and thus reduces the work in the following compressor stage. It also reduces the temperature into the combustor and thus lowers the peak temperature. This makes the control of the combustion process easier (no autoignition or uncontrollable flame spread), it reduces the formation of NOx that takes place at high temperatures and lowers the cooling requirements for the chamber walls.

10.d

The jet engine does not produce shaft work; how is power produced?

The turbine produces just enough shaft work to drive the compressor and it makes a little electric power for the aircraft. The power is produced as thrust of the engine. In order to exhaust the gases at high speed they must be accelerated so the high pressure in the turbine exit provides that force (high P relative to ambient). The high P into the turbine is made by the compressor, which pushes the flow backwards, and thus has a net resulting force forwards on the blades transmitted to the shaft and the aircraft. The outer housing also has a higher pressure inside that gives a net component in the forward direction.

10.e

How is the compression in the Otto cycle different from the Brayton cycle?

The compression in an Otto cycle is a volume reduction dictated by the piston motion. The physical handles are the volumes V_1 and V_2.

The compression in a Brayton cycle is the compressor pushing on the flow so it determines the pressure. The physical control is the pressure P_2 determined by how much torque you drive the shaft with.

10.f

How many parameters do you need to know to completely describe the Otto cycle? How about the Diesel cycle?

Otto cycle. State 1 (2 parameters) and the compression ratio CR and the energy release per unit mass in the combustion, a total of **4 parameters**. With that information you can draw the diagrams in Figure 10.2. Another way of looking at it is four states (8 properties) minus the four process equations ($s_2 = s_1$, $v_3 = v_2$, $s_4 = s_3$ and $v_4 = v_1$) gives 4 unknowns.

Diesel cycle. Same as for the Otto cycle namely **4 parameters**. The only difference is that one constant v process is changed to a constant P process.

10.g

The exhaust and inlet flow processes are not included in the Otto or Diesel cycles. How do these necessary processes affect the cycle performance?

Due to the pressure loss in the intake system and the dynamic flow process we will not have as much mass in the cylinder or as high a P as in a reversible process. The exhaust flow requires a slightly higher pressure to push the flow out through the catalytic converter and the muffler (higher back pressure) and the pressure loss in the valve so again there is a loss relative to a reversible process. Both of these processes subtract a pumping work from the net work out of the engine and a lower charge mass gives less power (not necessarily lower efficiency) than otherwise could be obtained.

Concept-Study Guide Problems

10.1

Is a Brayton cycle the same as a Carnot cycle? Name the four processes.

No. The Brayton cycle approximates a gas turbine.

1-2:	An isentropic compression (constant s)	Compressor
2-3:	An isobaric heating (constant P)	Combustor
3-4:	An isentropic expansion (constant s)	Turbine
4-1:	An isobaric cooling, heat rejection (constant P)	Heat exchanger

Comment: This cycle is the same as the Rankine cycle 4 processes, but it takes place in the ideal gas region of states. The last process does not exist in the real gas turbine which is an open cycle.

10.2

Why is the back work ratio in the Brayton cycle much higher than in the Rankine cycle?

Recall the expression for shaft work in a steady flow device

$$w = - \int v \, dP \ + \ ....$$

The specific volume in the compressor is not so much smaller than the specific volume in the turbine of the Brayton cycle as it is in the pump (liquid) compared to turbine (superheated vapor) in the Rankine cycle.

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10.3

For a given Brayton cycle the cold air approximation gave a formula for the efficiency. If we use the specific heats at the average temperature for each change in enthalpy will that give a higher or lower efficiency?

The specific heats are increasing functions of temperature. As the expression for the efficiency is from p.464

$$\eta = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{C_{P\,4\text{-}1}}{C_{P\,3\text{-}2}} \frac{T_4 - T_1}{T_3 - T_2}$$

The average T from 4 to 1 is lower than the average T from 2 to 3 and therefore the ratio of the two specific heats is lower than one yielding a **higher efficiency** than the cold air approximation gives for a given set of temperatures. However, the temperature ratios are different from the cold air calculation for the same compression ratio so the final result is a little more complex.

10.4

Does the efficiency of a jet engine change with altitude since the density varies?

No, just like the standard Brayton cycle the simple model performance depends only on the compression ratio.

10.5

Why are the two turbines in Figures 10.8-9 not connected to the same shaft?

Such a configuration gives a little more flexibility in the control of the cycle under varying loads. The two turbines would then not have to run at the same speed for various power output levels.

10.6

Why is an air refrigeration cycle not common for a household refrigerator?

The capacity of the air cycle per mass flowing through the system is very small compared with the vapor compression cycle. The cycle also includes the expander which is one more piece of equipment that will add cost and maintenance requirements to the system.

10.7

Does the inlet state (P_1, T_1) have any influence on the Otto cycle efficiency? How about the power produced by a real car engine?

Very little. The efficiency for the ideal cycle only depends on compression ratio when we assume cold air properties. The u's are slightly non-linear in T so there will be a small effect.

In a real engine there are several effects. The inlet state determines the density and thus the total mass in the chamber. The more mass the more energy is released when the fuel burns, the peak P and T will also change which affects the heat transfer loss to the walls and the formation of NOx (sensitive to T). The combustion process may become uncontrollable if T is too high (knocking). Some increase in P_1 like that done by a turbo-charger or super-charger increases the power output and if high, it must be followed by an intercooler to reduce T_1. If P_1 is too high the losses starts to be more than the gain so there is an optimum level.

10.8

For a given compression ratio does an Otto cycle have higher or lower efficiency than a diesel cycle?

This is actually not clear from the formula's so we need to refer to the T-s diagram where we see that the average T at which the heat is added is higher for the Otto cycle than the diesel cycle for the same compression ratio. However since the diesel cycle runs with much higher compressions ratios than the Otto cycle most typical diesel cycles have higher efficiency than typical Otto cycles.

10.9

How many parameters do you need to know to completely describe the Atkinson cycle? How about the Miller cycle?

Four parameters for the Atkinson cycle. A total of 8 properties minus four known process equations give 4 unknowns.

The Miller cycle has one additional process so that requires one more parameter for a total of **five**. That is 10 properties with 5 known process equations leaving 5 unknowns.

10.10

Why would one consider a combined cycle system for a power plant? For a heat pump or refrigerator?

Dual cycle or combined cycle systems have the advantage of a smaller difference between the high and low ranges for P and T. The heat can be added at several different temperatures reducing the difference between the energy source T and the working substance T. The working substance vapor pressure at the desired T can be reduced from a high value by adding a topping cycle with a different substance or have a higher low pressure at very low temperatures.

10.11

Can the exhaust flow from a gas turbine be useful?

Usually the temperature in the exhaust flow is fairly high compared to ambient conditions so we could use the hot exhaust flow energy. It could be used directly for heating purposes in process applications or heating of buildings. As a topping cycle it can be used to heat/boil water for use in a Rankine cycle.

10.12

Where may a heat engine driven refrigerator be useful?

Any remote location where electricity is not available. Since a rotating shaft is available in a car engine the car A/C unit is driven by a belt engaged with a magnetic clutch so you do not have to have an electric motor.

10.13
Since any heat transfer is driven by a temperature difference, how does that affect all the real cycles relative to the ideal cycles?

Heat transfers are given as $\dot{Q} = CA\,\Delta T$ so to have a reasonable rate the area and the temperature difference must be large. The working substance then must have a different temperature than the ambient it exchanges energy with. This gives a smaller temperature difference for a heat engine with a lower efficiency as a result. The refrigerator or heat pump must have the working substance with a higher temperature difference than the reservoirs and thus a lower coefficient of performance (COP).

The smaller CA is, the larger ΔT must be for a certain magnitude of the heat transfer rate. This can be a design problem, think about the front end air intake grill for a modern car which is very small compared to a car 20 years ago.

10.14

In an Otto cycle the cranking mechanism dictates the volume given the crank position. Can you say something similar for the Brayton cycle?

Yes, in the Brayton cycle the compressor determines the high pressure, the more compressor work input there is, the harder it pushes on the gas leading to a higher exit pressure. So in the Brayton cycle the pressure ratio is one of the design parameters just like the volume compression ratio is in a piston cylinder type of cycle.

10.15

For all the gas cycles it is assumed the ideal compression and expansions are isentropic. This is approximated with a polytropic process having n = k. The expansion after combustion will have some heat loss due to high temperature so what does that imply for the value of n?

In the expansion of the hot gases we can expect a heat loss to the cooler walls so a q out means a decrease in entropy. The polytropic processes as plotted in Fig. 6.13 shows what happens. In an expansion volume goes up so P and T drops then if entropy must decrease it means n > k.

10.16

For all the gas cycles it is assumed the ideal compression and expansions are isentropic. This is approximated with a polytropic process having n = k. The compression in a diesel engine leads to high temperatures and thus will have some heat loss so what does that imply for the value of n?

In the compression of air P and T go up, see Fig. 6.13. The start of the process may have n = k as the temperature of the air is not too different from the wall temperature. As the air gets hotter it will lose heat to the walls and thus entropy will decrease. Moving up and to the left in the T-s diagram corresponds to a process with n < k.

10.17

If we compute the efficiency of an Otto or Diesel cycle we get something like 60% for a compression ratio of 10:1. Is a real engine close to this?

Not at all. A gasoline engine has an efficiency of about 35% pushing it to 40% for a hybrid configuration. Diesel engines can be about 40-45% with bigger two stroke engines, like for ships, having the higher efficiencies. Scale helps in that heat loss and flow losses scale with area whereas power scales with volume. This also means that very small engines have the smallest efficiency as the losses become significant.

10.18

A hybrid powertrain couples a battery/motor with an internal combustion engine. Mention a few factors that make this combination a little more efficient.

1. The combustion engine can be operated in a narrow range of RPM and load (throttle) conditions. This means it can be optimized for that narrow range.

2. The combustion engine does not have to accelerate and go through a transient period with extra fuel or decelerate and idle with no useful power output.

3. The combustion engine can be smaller and operate at full throttle cutting pumping losses and fully use all components that do not need to be designed for a larger capacity. In a standard car all systems must be designed for the maximum expected load, which is used 2% of the time if ever, thus adding mass, increasing warm up time etc.

4. With electric motors driving the wheels regenerative braking can be used. This means the motors act as electrical generators and charge the battery during braking.

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Brayton Cycles, Gas Turbines

10.19

In a Brayton cycle the inlet is at 300 K, 100 kPa and the combustion adds 800 kJ/kg. The maximum temperature is 1400 K due to material considerations. Find the maximum permissible compression ratio and for that the cycle efficiency using cold air properties.

Solution:

Combustion: $h_3 = h_2 + q_H$; $\quad _2w_3 = 0 \quad$ and $\quad T_{max} = T_3 = 1400$ K

$\quad\quad T_2 = T_3 - q_H/C_P = 1400$ K $– 800$ kJ/kg $/1.004$ kJ/kg-K $= 603.2$ K

Reversible adiabatic compression leads to constant s, from Eq.6.23:

$$P_2 / P_1 = (T_2/T_1)^{\frac{k}{k-1}} = (603.2/300)^{3.5} = \textbf{11.53}$$

Reversible adiabatic expansion leads to constant s, from Eq.6.23

$$T_4 = T_3 (P_4/P_3)^{\frac{k-1}{k}} = T_3\, T_1/T_2 = 1400\, \frac{300}{603.2} = 696.3 \text{ K}$$

For net work we get

$$w_T = C_P (T_3 - T_4) = 1.004(1400 – 696.3) = 706.5 \text{ kJ/kg}$$

$$w_C = C_P (T_2 - T_1) = 1.004(603.2 – 300) = 304.4 \text{ kJ/kg}$$

$$w_{net} = w_T - w_C = 706.5 - 304.4 = 402.1 \text{ kJ/kg}$$

$$\eta = w_{net} / q_H = 402.1 / 800 = \textbf{0.503}$$

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10.20

A Brayton cycle has compression ratio of 15:1 with a high temperature of 1600 K and the inlet at 290 K, 100 kPa. Use cold air properties and find the specific heat addition and specific net work output.

Brayton cycle so this means:

Minimum T: $T_1 = 290$ K

Maximum T: $T_3 = 1600$ K

Pressure ratio: $P_2/P_1 = 15$

Compression in compressor: $s_2 = s_1$

$\Rightarrow$ Implemented in Eq.6.23

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 290 \text{ K } (15)^{0.286} = 628.65 \text{ K}$$

Energy input is from the combustor

$$q_H = C_{P0}(T_3 - T_2) = 1.004 \ (1600 - 628.65) = \textbf{975.2 kJ/kg}$$

Do the overall cycle efficiency and the net work

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 15^{-0.4/1.4} = 0.5387$$

$$w_{NET} = \eta \ q_H = 0.5387 \times 975.2 = \textbf{525.34 kJ/kg}$$

10.21

A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 300 K, and the exhaust temperature is 750 K. The minimum pressure in the cycle is 100 kPa, and the compressor pressure ratio is 14 to 1. Calculate the power output of the turbine. What fraction of the turbine output is required to drive the compressor? What is the thermal efficiency of the cycle?

Solution:

Brayton cycle so this means:
  Minimum T: $T_1 = 300$ K
  Exhaust T: $T_4 = 750$ K
  Pressure ratio: $P_2/P_1 = 14$

Solve using constant C_{P0}

Compression in compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.6.23

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300 \text{ K} \times (14)^{0.286} = 638.1 \text{ K}$$

$$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004 (638.1 - 300) = 339.5 \text{ kJ/kg}$$

Expansion in turbine: $s_4 = s_3 \Rightarrow$ Implemented in Eq.6.23

$$T_3 = T_4 (P_3/P_4)^{\frac{k-1}{k}} = 750 \text{ K} \times (14)^{0.286} = 1594 \text{ K}$$

$$w_T = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004 (1594 - 750) = 847.4 \text{ kJ/kg}$$

$$w_{NET} = 847.4 - 339.5 = 507.9 \text{ kJ/kg}$$

Do the overall net and cycle efficiency

$$\dot{m} = \dot{W}_{NET}/w_{NET} = 100\ 000 \text{ kW}/507.9 \text{ kJ/kg} = 196.9 \text{ kg/s}$$

$$\dot{W}_T = \dot{m}w_T = 196.9 \text{ kg/s} \times 847.4 \text{ kJ/kg} = \textbf{166.85 MW}$$

$$w_C/w_T = 339.5/847.4 = \textbf{0.40}$$

Energy input is from the combustor

$$q_H = C_{P0}(T_3 - T_2) = 1.004 (1594 - 638.1) = 959.7 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 507.9/959.7 = \textbf{0.529}$$

10.22

Consider an ideal air-standard Brayton cycle in which the air into the compressor is at 100 kPa, 20°C, and the pressure ratio across the compressor is 12:1. The maximum temperature in the cycle is 1100°C, and the air flow rate is 10 kg/s. Assume constant specific heat for the air, value from Table A.5. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

Solution:

Compression ratio
$$\frac{P_2}{P_1} = 12$$
Max temperature
$$T_3 = 1100^oC$$
$$\dot{m} = 10 \text{ kg/s}$$

The compression is reversible and adiabatic so constant s. From Eq.6.23

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 293.2 \text{ K} \times (12)^{0.286} = 596.8 \text{ K}$$

Energy equation with compressor work in

$$w_C = -_1w_2 = C_{P0}(T_2 - T_1) = 1.004(596.8 - 293.2) = 304.8 \text{ kJ/kg}$$

The expansion is reversible and adiabatic so constant s. From Eq.6.23

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1373.2 \text{ K} \left(\frac{1}{12}\right)^{0.286} = 674.7 \text{ K}$$

Energy equation with turbine work out

$$w_T = C_{P0}(T_3 - T_4) = 1.004(1373.2 - 674.7) = 701.3 \text{ kJ/kg}$$

Scale the work with the mass flow rate

$$\dot{W}_C = \dot{m}w_C = \textbf{3048 kW}, \qquad \dot{W}_T = \dot{m}w_T = \textbf{7013 kW}$$

Energy added by the combustion process

$$q_H = C_{P0}(T_3 - T_2) = 1.004(1373.2 - 596.8) = 779.5 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = (701.3 - 304.8)/779.5 = \textbf{0.509}$$

10.23

A Brayton cycle has air into the compressor at 95 kPa, 290 K and has an efficiency of 50%. The exhaust temperature is 675 K. Find the pressure ratio and the specific heat addition by the combustion for this cycle.

The efficiency is given by the compression ratio so we get

$$\eta = 1 - r_p^{-(k-1)/k} = 0.5 \quad => \quad r_p = P_2/P_1 = (1/0.5)^{\frac{k}{k-1}} = 2^{3.5} = \mathbf{11.314}$$

The compression is reversible and adiabatic so constant s. From Eq.6.23

$$T_2 = T_1 \, r_p^{(k-1)/k} = 290 \text{ K} \times (11.314)^{0.4/1.4} = 580 \text{ K}$$

The expansion process gives

$$T_3 = T_4 \, r_p^{(k-1)/k} = 675 \text{ K} \times (11.314)^{0.4/1.4} = 1350 \text{ K}$$

Energy added by the combustion process

$$q_H = C_{P0}(T_3 - T_2) = 1.004 \text{ kJ/kg-K} \times (1350 - 580) \text{ K} = \mathbf{773 \text{ kJ/kg}}$$

10.24

A Brayton cycle has inlet at 290 K, 90 kPa and the combustion adds 1000 kJ/kg. How high can the compression ratio be so the highest temperature is below 1700 K? Use cold air properties to solve.

Compression ratio
$r_p = P_2 / P_1$
Max temperature
$T_3 = 1700$ K

Combustion adds
$_2q_3 = h_3 - h_2$

Let us work back from state 3 to 2 as

$$_2q_3 = h_3 - h_2 = C_{P0}(T_3 - T_2) \Rightarrow$$

$$T_2 = T_3 - {_2q_3}/ C_{P0} = 1700 \text{ K} - 1000 \text{ kJ/kg} /1.004 \text{ kJ/kg-K} = 704 \text{ K}$$

The compression is reversible and adiabatic so constant s. From Eq.6.23

$$T_2 = T_1 \, r_p^{(k-1)/k} \Rightarrow \qquad r_p = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\frac{704}{290}\right)^{3.5} = \textbf{22.29}$$

10.25

Assume a state of 1400 kPa, 2100 K into the turbine section of a Brayton cycle with an adiabatic expansion to 100 kPa and the compressor inlet temperature is 300 K. Find the missing temperatures in the cycle using Table A.7 and then give the average value of k (ratio of specific heats) for the compression and expansion processes.

Compression process:

$$s^o_{T2} = s^o_{T1} + R \ln(P_2/P_1) = 6.86926 + 0.287 \ln(14) = 7.62667 \text{ kJ/kg-K}$$

$$T_2 = \textbf{629.4 K} \quad \text{and} \quad h_2 = 638.3 \text{ kJ/kg}$$

Expansion process:

$$s^o_{T4} = s^o_{T3} + R \ln(P_4/P_3) = 9.02721 + 0.287 \ln(1/14) = 8.26980 \text{ kJ/kg-K}$$

$$T_4 = \textbf{1124.5 K} \quad \text{and} \quad h_4 = 1189.6 \text{ kJ/kg}$$

In the compression process

$$C_P = (h_2 - h_1) / (T_2 - T_1) = \frac{638.3 - 300.47}{629.4 - 300} = 1.0256 \text{ kJ/kg-K}$$

$$k = C_P/(C_P - R) = \frac{1.0256}{1.0256 - 0.287} = \textbf{1.3886}$$

In the expansion process

$$C_P = (h_3 - h_4) / (T_3 - T_4) = \frac{2376.82 - 1189.6}{2100 - 1124.5} = 1.217 \text{ kJ/kg-K}$$

$$k = C_P/(C_P - R) = \frac{1.217}{1.217 - 0.287} = \textbf{1.308}$$

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10.26

Repeat Problem 10.24 using Table A.7.

Compression ratio
P_2 / P_1

Max temperature
$T_3 = 1700$ K

Combustion adds

$_2q_3 = h_3 - h_2$

Let us work back from state 3 to 2 as

A.7: $h_3 = 1879.76$ kJ/kg

$_2q_3 = h_3 - h_2 \quad \Rightarrow$

$h_2 = h_3 - {_2q_3} = 1879.76 - 1000 = 879.76$ kJ/kg

From A.7: $s^o_{T2} = 7.95477$ kJ/kg-K

The compression is reversible and adiabatic so constant s. From Eq.6.19

$s^o_{T2} = s^o_{T1} + R \ln(P_2/P_1)$ so

$P_2/P_1 = \exp [(s^o_{T2} - s^o_{T1})/R] = \exp [\dfrac{7.95477 - 6.83521}{0.287}] = \textbf{49.4}$

This is too high for a real compressor.

10.27

A Brayton cycle produces net 50 MW with an inlet state of 17°C, 100 kPa and the pressure ratio is 14:1. The highest cycle temperature is 1600 K. Find the thermal efficiency of the cycle and the mass flow rate of air using cold air properties.

Inlet state is state 1, $P_2/P_1 = 14$ and $T_3 = 1600$ K.
Compression: Reversible and adiabatic so constant s from Eq.6.23

$$T_2 = T_1(P_2/P_1)^{(k-1)/k} = 290 \text{ K} \times (14)^{0.2857} = 616.38 \text{ K}$$

$$\eta_{CYCLE} = w_{NET}/q_H = 1 - T_1/T_2 = 1 - \frac{290}{616.38} = \mathbf{0.5295}$$

Combustion: constant pressure

$$q_H = h_3 - h_2 = C_{P0} (T_3 - T_2) = 1.004 \text{ kJ/kg-K} \times (1600 - 616.38) \text{ K}$$
$$= 987.56 \text{ kJ/kg}$$

$$w_{NET} = \eta_{CYCLE} \, q_H = 0.5295 \times 987.55 \text{ kJ/kg} = 522.9 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}/ w_{NET} = 50\,000 \text{ kW} / 522.9 \text{ kJ/kg} = \mathbf{95.62 \text{ kg/s}}$$

10.28

A Brayton cycle produces 14 MW with an inlet state of 17°C, 100 kPa, and a compression ratio of 16:1. The heat added in the combustion is 1160 kJ/kg. What are the highest temperature and the mass flow rate of air, assuming cold air properties?

Solution:

Temperature after compression is

$$T_2 = T_1 \, r_p^{(k-1)/k} = 290 \text{ K} \times 16^{0.4/1.4} = 640.35 \text{ K}$$

The highest temperature is after combustion

$$T_3 = T_2 + q_H/C_p = 640.35 \text{ K} + \frac{1160 \text{ kJ/kg}}{1.004 \text{ kJ/kg-K}} = \mathbf{1795.7 \text{ K}}$$

Efficiency is from Eq.10.1

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 16^{-0.4/1.4} = 0.547$$

From the required power we can find the needed heat transfer

$$\dot{Q}_H = \dot{W}_{net} / \eta = \frac{14\,000}{0.547} \text{ kW} = 25\,594 \text{ kW}$$

$$\dot{m} = \dot{Q}_H / q_H = 25\,594 \text{ kW}/ 1160 \text{ kJ/kg} = \mathbf{22.06 \text{ kg/s}}$$

10.29

Do Problem 10.23 using Table A.7, this becomes trial and error

Solution:

We have states 1 and 4 which mean we can find q_L and since we have efficiency $\eta = 1 - q_L/q_H$, we can find q_H.

From A.7.1: $h_1 = 290.43$ kJ/kg, $s_{T1}^o = 6.83521$ kJ/kg K

$h_4 = 686.785$ kJ/kg, $s_{T4}^o = 7.701065$ kJ/kg-K

The heat rejection becomes $q_L = h_4 - h_1 = 686.785 - 290.43 = 396.355$ kJ/kg

$$q_H = q_L/(1 - \eta) = 2\ q_L = \mathbf{792.71\ kJ/kg} = h_3 - h_2$$

With a known compression ratio we can get states 2 and 4 from states 1 and 3. The compression is reversible and adiabatic so constant s. From Eq.6.19

$$s_2 = s_1 \implies s_{T2}^o = s_{T1}^o + R\ln(P_2/P_1) = 6.83521 + 0.287 \ln r_p$$

We get a similar relation for the expansion process

$$s_4 = s_3 \implies s_{T3}^o = s_{T4}^o + R\ln(P_3/P_4) = 7.701065 + 0.287\ln r_p$$

From the constant specific heat ($\eta = 1 - r_p^{-(k-1)/k}$ => $r_p = 11.3$) so guess $r_p = 12$

$$s_{T2}^o = 6.83521 + 0.287 \ln 12 = 7.54838 \implies h_2 = 590.8 \text{ kJ/kg}$$

$$s_{T3}^o = 7.701065 + 0.287 \ln 12 = 8.41423 \implies h_3 = 1362.44 \text{ kJ/kg}$$

$\implies h_3 - h_2 = 771.6$ kJ/kg   which is too low (should be 792.71 kJ/kg)

Guess $r_p = 14$ and repeat the calculations

$$s_{T2}^o = 6.83521 + 0.287 \ln 14 = 7.59262 \implies h_2 = 617.28 \text{ kJ/kg}$$

$$s_{T3}^o = 7.701065 + 0.287 \ln 14 = 8.458474 \implies h_3 = 1419.76 \text{ kJ/kg}$$

$\implies h_3 - h_2 = 802.48$ kJ/kg   which is too high (should be 792.71 kJ/kg)

Linear interpolation:

$$r_p = 12 + (14 - 12)\frac{792.71 - 771.6}{802.48 - 771.6} = \mathbf{13.37}$$

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Regenerators, Intercoolers, and Non-ideal Cycles

10.30

Would it be better to add an ideal regenerator to the Brayton cycle in problem 10.28?

Inlet state is state 1, $P_2/P_1 = 16$ and $q_H = 1160$ kJ/kg.

Compression: Reversible and adiabatic so constant s from Eq.6.23

$$T_2 = T_1(P_2/P_1)^{(k-1)/k} = 290 \text{ K} \times (16)^{0.2857} = 640.34 \text{ K}$$

Get the high temperature

$$T_3 = T_2 + q_H / C_{P0} = 640.34 \text{ K} + 1160 \text{ kJ/kg} /1.004 \text{ kJ/kg-K} = 1795.7 \text{ K}$$

Expansion: isentropic

$$T_4 = T_3 / (P_3/P_4)^{(k-1)/k} = 1795.7 \text{ K} /16^{0.2857} = 813.2 \text{ K} \; > T_2$$

Since $T_4 > T_2$ some **regeneration can be done**.

We could find (not needed), see p.471,

$$\eta_{CYCLE} = 1 - \frac{T_1}{T_3} (P_2/P_1)^{(k-1)/k} = 1 - T_2 / T_3 = 0.643$$

This is better than the regular cycle $\eta_{CYCLE} = 1 - r_p^{-(k-1)/k} = 0.547$

10.31

A Brayton cycle with an ideal regenerator has inlet at 290 K, 90 kPa with the highest P, T as 1170 kPa, 1700 K. Find the specific heat transfer and the cycle efficiency using cold air properties.

Compression ratio
P_2 / P_1

Max temperature
$T_3 = 1700$ K

Combustion adds

$q_H = h_3 - h_x$

Let us work from state 1 to 2 as constant s, Eq.6.23

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 290 \text{ K} \times (1170/90)^{0.2857} = 603.5 \text{ K}$$

Energy equation with compressor work in

$$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004(603.5 - 290) = 313.5 \text{ kJ/kg} = q_L$$

The expansion is reversible and adiabatic so constant s. From Eq.6.23

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1700 \text{ K} \times \left(\frac{90}{1170}\right)^{0.2857} \textbf{= 816.9 K } > T_2$$

Since the exhaust $T_4 > T_2$ a regenerator can be used. Now the heat transfer added is

$$q_H = h_3 - h_x = C_{P0}(T_3 - T_x) = 1.004(1700 - 816.9) = \textbf{886.6 kJ/kg} = w_T$$

$$\eta_{TH} = w_{NET}/q_H = (886.6 - 313.5)/\ 886.6 = \textbf{0.646}$$

We could also have found efficiency, see p.471, as

$$\eta_{TH} = 1 - \frac{T_1}{T_3}\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 1 - \frac{T_2}{T_3} = 1 - \frac{603.5}{1700} = 0.645$$

10.32

An ideal regenerator is incorporated into the ideal air-standard Brayton cycle of Problem 10.22. Find the thermal efficiency of the cycle with this modification.

Consider an ideal air-standard Brayton cycle in which the air into the compressor is at 100 kPa, 20°C, and the pressure ratio across the compressor is 12:1. The maximum temperature in the cycle is 1100°C, and the air flow rate is 10 kg/s. Assume constant specific heat for the air, value from Table A.5. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

Solution:

Compression ratio
$$\frac{P_2}{P_1} = 12$$

Max temperature
$$T_3 = 1100^{\circ}C$$

$$\dot{m} = 10 \text{ kg/s}$$

The compression is reversible and adiabatic so constant s. From Eq.6.23

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 293.2 \text{ K} \times (12)^{0.286} = 596.8 \text{ K}$$

Energy equation with compressor work in

$$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004(596.8 - 293.2) = 304.8 \text{ kJ/kg}$$

The expansion is reversible and adiabatic so constant s. From Eq.6.23

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1373.2 \text{ K} \times \left(\frac{1}{12}\right)^{0.286} = 674.7 \text{ K}$$

Energy equation with turbine work out

$$w_T = C_{P0}(T_3 - T_4) = 1.004(1373.2 - 674.7) = 701.3 \text{ kJ/kg}$$

Ideal regenerator: $T_X = T_4 = 674.7$ K

$$q_H = h_3 - h_X = 1.004(1373.2 - 674.7) = 701.3 \text{ kJ/kg} = w_T$$

$$\eta_{TH} = w_{NET}/q_H = (701.3 - 304.8)/701.3 = \textbf{0.565}$$

We could also have found efficiency, see p.471, as

$$\eta_{TH} = 1 - \frac{T_1}{T_3}\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 1 - \frac{T_2}{T_3}$$

10.33

Consider an ideal gas-turbine cycle with a pressure ratio across the compressor of 12 to 1. The compressor inlet is at 300 K and 100 kPa, and the cycle has a maximum temperature of 1600 K. An ideal regenerator is also incorporated into the cycle. Find the thermal efficiency of the cycle using cold air (298 K) properties.

Solution:

Compression ratio

$$\frac{P_2}{P_1} = 12$$

Max temperature

$$T_3 = 1600 \text{ K}$$

The efficiency with an ideal regenerator ($T_4 = T_x$) is from page 471

$$\eta_{TH} = 1 - \frac{T_1}{T_3}\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 1 - \frac{300}{1600} \times 12^{0.2857} = \textbf{0.619}$$

10.34

Assume the compressor in Problem 10.28 has an intercooler that cools the air to 330 K operating at 500 kPa followed by a second stage of compression to 1600 kPa. Find the specific heat transfer in the intercooler and the total combined work required.

Solution:

C.V. Stage 1: $1 \Rightarrow 2$

Reversible and adiabatic gives constant s which from Eq.6.23 gives:

$$T_2 = T_1 (P_2/P_1)^{(k-1)/k} = 290 \text{ K} \times (500/100)^{0.2857} = 459.3 \text{ K}$$

$$w_{c1in} = C_P(T_2 - T_1) = 1.004(459.3 - 290) = 187.0 \text{ kJ/kg}$$

CV. Intercooler $2 \Rightarrow 3$

Constant pressure cooling

$$q_{out} = h_2 - h_3 = C_P (T_2 - T_3) = 1.004 (459.3 - 330) = \textbf{129.8 kJ/kg}$$

C.V. Stage 2: $3 \Rightarrow 4$

Reversible and adiabatic gives constant s which from Eq.6.23 gives:

$$T_4 = T_3 (P_4/P_3)^{(k-1)/k} = 330 \text{ K} \times (1600/500)^{0.2857} = 460.1 \text{ K}$$

$$w_{c2in} = C_P(T_4 - T_3) = 1.004(460.1 - 330) = 130.6 \text{ kJ/kg}$$

$$w_{tot} = w_{c1} + w_{c2} = 187 + 130.6 = \textbf{318 kJ/kg}$$

The intercooler reduces the work for stage 2 as T is lower and so is specific volume. The reduction in work due to the intercooler is shaded in the P-v diagram.

10.35

A two-stage air compressor has an intercooler between the two stages as shown in Fig. P10.35. The inlet state is 100 kPa, 290 K, and the final exit pressure is 1.6 MPa. Assume that the constant pressure intercooler cools the air to the inlet temperature, $T_3 = T_1$. It can be shown that the optimal pressure, $P_2 = (P_1 P_4)^{1/2}$, for minimum total compressor work. Find the specific compressor works and the intercooler heat transfer for the optimal P_2.

Solution:

Optimal intercooler pressure $P_2 = \sqrt{100 \times 1600} = 400$ kPa

> 1: $\ h_1 = 290.43$ kJ/kg, $\ s_{T1}^o = 6.83521$ kJ/kg K

C.V. C1: $\ w_{C1} = h_2 - h_1$, $\ s_2 = s_1$ $\quad$ leading to Eq.6.19

$$\Rightarrow \ \ s_{T2}^o = s_{T1}^o + R \ln(P_2/P_1) = 6.83521 + 0.287 \ln 4 = 7.2331 \text{ kJ/kg K}$$

$$\Rightarrow T_2 = 430.3 \text{ K}, \ \ h_2 = 432.05 \text{ kJ/kg}$$

$$w_{C1} = 432.05 - 290.43 = \textbf{141.6 kJ/kg}$$

C.V. Cooler: $\ T_3 = T_1 \ \Rightarrow \ h_3 = h_1$

$$q_{OUT} = h_2 - h_3 = h_2 - h_1 = w_{C1} = \textbf{141.6 kJ/kg}$$

C.V. C2: $\ T_3 = T_1$, $\ s_4 = s_3$ $\quad$ and since $\ s_{T3}^o = s_{T1}^o$, $\ P_4/P_3 = P_2/P_1$

$$\Rightarrow \ \ s_{T4}^o = s_{T3}^o + R \ln(P_4/P_3) = s_{T2}^o, \ \text{ so we have } \ T_4 = T_2$$

Thus we get $\ \ w_{C2} = w_{C1} = \textbf{141.6 kJ/kg}$

10.36

The gas-turbine cycle shown in Fig. P10.36 is used as an automotive engine. In the first turbine, the gas expands to pressure P_5, just low enough for this turbine to drive the compressor. The gas is then expanded through the second turbine connected to the drive wheels. The data for the engine are shown in the figure and assume that all processes are ideal. Determine the intermediate pressure P_5, the net specific work output of the engine, and the mass flow rate through the engine. Find also the air temperature entering the burner T_3, and the thermal efficiency of the engine.

a)   Consider the compressor

$$s_2 = s_1 \Rightarrow T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300\text{ K} \times (6)^{0.286} = 500.8\text{ K}$$

$$-w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.004(500.8 - 300) = 201.6\text{ kJ/kg}$$

Consider then the first turbine work

$$w_{T1} = -w_C = 201.6\text{ kJ/kg} = C_{P0}(T_4 - T_5) = 1.004\text{ kJ/kg-K }(1600\text{ K} - T_5)$$

$$\Rightarrow T_5 = 1399.2\text{ K}$$

$$s_5 = s_4 \Rightarrow \qquad P_5 = P_4\left(\frac{T_5}{T_4}\right)^{\frac{k}{k-1}} = 600\text{ kPa} \times \left(\frac{1399.2}{1600}\right)^{3.5} = \textbf{375 kPa}$$

b) $s_6 = s_5 \Rightarrow T_6 = T_5\left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} = 1399.2\text{ K} \times \left(\frac{100}{375}\right)^{0.286} = 958.8\text{ K}$

The second turbine gives the net work out

$$w_{T2} = C_{P0}(T_5 - T_6) = 1.004(1399.2 - 958.8) = 442.2\text{ kJ/kg}$$

$$\dot{m} = \dot{W}_{NET}/w_{T2} = 150/442.2 = \textbf{0.339 kg/s}$$

c) Ideal regenerator $\Rightarrow T_3 = T_6 = \textbf{958.8 K}$

$$q_H = C_{P0}(T_4 - T_3) = 1.004\text{ kJ/kg-K }(1600 - 958.8)\text{ K} = 643.8\text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 442.2/643.8 = \textbf{0.687}$$

10.37

Repeat Problem 10.35 when the intercooler brings the air to $T_3 = 320$ K. The corrected formula for the optimal pressure is $P_2 = [\,P_1 P_4\,(T_3/T_1)^{n/(n-1)}]^{1/2}$ see Problem 7.245, where n is the exponent in the assumed polytropic process.

Solution:

The polytropic process has n = k (isentropic) so n/(n - 1) = 1.4/0.4 = 3.5

$$P_2 = [\,P_1 P_4\,(T_3/T_1)^{n/(n-1)}]^{1/2}$$

$$= \sqrt{100 \times 1600}\ \text{kPa}\ \sqrt{(320/290)^{3.5}} = 475.2\ \text{kPa}$$

C.V. C1: $s_2 = s_1 \;\Rightarrow\; T_2 = T_1\,(P_2/P_1)^{\frac{k-1}{k}} = 290\ \text{K} \times (475.2/100)^{0.2857} = 452.67\ \text{K}$

$\qquad w_{C1} = h_2 - h_1 = C_p(T_2 - T_1) = 1.004\ \text{kJ/kg-K} \times (452.67 - 290)\ \text{K}$

$\qquad\quad = \mathbf{163.3\ kJ/kg}$

C.V. Cooler: $\quad q_{OUT} = h_2 - h_3 = 1.004\ \text{kJ/kg-K}\ (452.67 - 320)\ \text{K} = \mathbf{133.2\ kJ/kg}$

C.V. C2: $s_4 = s_3 \;\Rightarrow\; T_4 = T_3\,(P_4/P_3)^{\frac{k-1}{k}} = 320\ \text{K} \times (1600/475.2)^{0.2857} = 452.67\ \text{K}$

$\qquad w_{C2} = h_4 - h_3 = C_p(T_2 - T_1) = 1.004\ \text{kJ/kg-K}\ (452.67 - 320)\ \text{K}$

$\qquad\quad = \mathbf{133.2\ kJ/kg}$

10.38

Repeat Problem 10.21, but include a regenerator with 75% efficiency in the cycle.

A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 300 K, and the exhaust temperature is 750 K. The minimum pressure in the cycle is 100 kPa, and the compressor pressure ratio is 14 to 1. Calculate the power output of the turbine. What fraction of the turbine output is required to drive the compressor? What is the thermal efficiency of the cycle?

Solution:

Both compressor and turbine are reversible and adiabatic so constant s, Eq.6.23 relates then T to P assuming constant heat capacity.

Compressor: $\Rightarrow T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300 \text{ K } (14)^{0.286} = 638.1 \text{ K}$

$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004 \ (638.1 - 300) = 339.5 \text{ kJ/kg}$

Turbine $\quad s_4 = s_3 \Rightarrow T_3 = T_4 \ (P_3/P_4)^{\frac{k-1}{k}} = 750 \text{ K} \times (14)^{0.286} = 1594 \text{ K}$

$w_T = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004 \ (1594 - 750) = 847.4 \text{ kJ/kg}$

$w_{NET} = 847.4 - 339.5 = 507.9 \text{ kJ/kg}$

$\dot{m} = \dot{W}_{NET}/w_{NET} = 100\ 000 \text{ kW } /507.9 \text{ kJ/kg} = 196.9 \text{ kg/s}$

$\dot{W}_T = \dot{m}w_T = 196.9 \text{ kg/s} \times 847.4 \text{ kJ/kg} = \textbf{166.85 MW}$

$w_C/w_T = 339.5/847.4 = \textbf{0.40}$

For the regenerator

$\eta_{REG} = 0.75 = \dfrac{h_X - h_2}{h_{X'} - h_2} = \dfrac{T_X - T_2}{T_4 - T_2} = \dfrac{T_X - 638.1}{750 - 638.1}$

$\Rightarrow T_X = 722.0 \text{ K}$

Turbine and compressor work not affected by regenerator.

Combustor needs to add less energy with the regenerator as

$q_H = C_{P0}(T_3 - T_X) = 1.004(1594 - 722) = 875.5 \text{ kJ/kg}$

$\eta_{TH} = w_{NET}/q_H = 507.9/875.5 = \textbf{0.58}$

10.39

An air compressor has inlet of 100 kPa, 290 K and brings it to 500 kPa after which the air is cooled in an intercooler to 340 K by heat transfer to the ambient 290 K. Assume this first compressor stage has an isentropic efficiency of 85% and it is adiabatic. Using constant specific heat and find the compressor exit temperature and the specific entropy generation in the process.

C.V.: Stage 1 air, Steady flow

Isentropic compressor is done first.

Process: adiabatic: $q = 0$, reversible: $s_{gen} = 0$

Energy Eq.6.13: $-w_{C1} = h_2 - h_1$, Entropy Eq.9.8: $s_2 = s_1$

Assume constant $C_{P0} = 1.004$ from A.5 and isentropic leads to Eq.6.23

$$T_{2s} = T_1(P_2/P_1)^{\frac{k-1}{k}} = 290 \text{ K} \times (500/100)^{0.286} = 459.3 \text{ K}$$

$$w_{C1s} = h_1 - h_{2s} = C_{Po}(T_1 - T_2) = 1.004(290 - 459.3) = -170 \text{ kJ/kg}$$

Now the **actual compressor** work becomes

$$w_{C1\ ac} = w_{C1s}/\eta_{C1s} = -170/0.85 = -200 \text{ kJ/kg} = h_1 - h_{2ac} = C_{Po}(T_1 - T_{2ac})$$

$$T_{2ac} = T_1 - w_{C1\ ac}/C_{Po} = 290 + 200/1.004 = \textbf{489.2 K}$$

$$s_{gen} = s_{2ac} - s_1 = C_{Po} \ln(\frac{T_{2ac}}{T_1}) - R \ln(\frac{P_2}{P_1})$$

$$= 1.004 \ln(489.2/290) - 0.287(500/100) = \textbf{0.0631 kJ/kg-K}$$

10.40

A two-stage compressor in a gas turbine brings atmospheric air at 100 kPa, 17°C to 500 kPa, then cools it in an intercooler to 27°C at constant P. The second stage brings the air to 2500 kPa. Assume both stages are adiabatic and reversible. Find the combined specific work to the compressor stages. Compare that to the specific work for the case of no intercooler (i.e. one compressor from 100 to 1000 kPa).

Solution:

C.V. Stage 1: 1 => 2

Reversible and adiabatic gives constant s which from Eq.6.23 gives:

$$T_2 = T_1 (P_2/P_1)^{(k-1)/k} = 290 \text{ K } (500/100)^{0.2857} = 459.3 \text{ K}$$

$$w_{c1in} = C_P(T_2 - T_1) = 1.004(459.3 - 290) = 170.0 \text{ kJ/kg}$$

C.V. Stage 2: 3 => 4

Reversible and adiabatic gives constant s which from Eq.6.23 gives:

$$T_4 = T_3 (P_4/P_3)^{(k-1)/k} = 300 \text{ K } (2500/500)^{0.2857} = 475.1 \text{ K}$$

$$w_{c2in} = C_P(T_4 - T_3) = 1.004 \text{ kJ/kg-K } (475.1 - 300) \text{ K} = 175.8 \text{ kJ/kg}$$

$$w_{tot} = w_{c1} + w_{c2} = 170 + 175.8 = \textbf{345.8 kJ/kg}$$

The intercooler reduces the work for stage 2 as T is lower and so is specific volume.

C.V. One compressor   1 => 5

Reversible and adiabatic gives constant s which from Eq.6.23 gives:

$$T_5 = T_1 (P_5/P_1)^{(k-1)/k} = 290 \text{ K } (2500/100)^{0.2857} = 727.4 \text{ K}$$

$$w_{in} = C_P(T_5 - T_1) = 1.004 \text{ kJ/kg-K } (727.4 - 290) \text{ K} = \textbf{439 kJ/kg}$$

The reduction in work due to the intercooler is shaded in the P-v diagram.

10.41

Repeat Problem 10.21, but assume that the compressor has an isentropic efficiency of 85% and the turbine an isentropic efficiency of 88%.

Solution:

The original cycle was reversible
Brayton cycle so this means:

Minimum T: $T_1 = 300$ K

Exhaust T: $T_{4s} = 750$ K

Pressure ratio: $P_2/P_1 = 14$

Solve using constant C_{P0}

Ideal compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.6.23

$$T_{2s} = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300 \text{ K } (14)^{0.286} = 638.1 \text{ K}$$

$$w_{Cs} = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004 \ (638.1 - 300) = 339.5 \text{ kJ/kg}$$

Actual compressor

$$\Rightarrow w_C = w_{SC}/\eta_{SC} = 339.5/0.85 = 399.4 \text{ kJ/kg} = C_{P0}(T_2-T_1)$$

$$\Rightarrow T_2 = T_1 + w_c/C_{P0} = 300 + 399.4/1.004 = 697.8 \text{ K}$$

Ideal turbine: $s_4 = s_3 \Rightarrow$ Implemented in Eq.6.23

$$T_3 = T_{4s} \ (P_3/P_{4s})^{\frac{k-1}{k}} = 750 \text{ K } (14/1)^{0.286} = 1594 \text{ K}$$

$$w_{Ts} = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004 \ (1594 - 750) = 847.4 \text{ kJ/kg}$$

Actual turbine

$$\Rightarrow w_T = \eta_{ST} \ w_{ST} = 0.88 \times 847.4 = 745.7 \text{ kJ/kg} = C_{P0}(T_3-T_4)$$

$$\Rightarrow T_4 = T_3 - w_T/C_{P0} = 1594 - 745.7/1.004 = 851.3 \text{ K}$$

Do the overall net and cycle efficiency

$$w_{NET} = w_T - w_C = 745.7 - 399.4 = 346.3 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}_{NET}/w_{NET} = 100000 \text{ kW } /346.3 \text{ kJ/kg} = 288.8 \text{ kg/s}$$

$$\dot{W}_T = \dot{m}w_T = 288.8 \text{ kg/s} \times 745.7 \text{ kJ/kg} = \textbf{215.36 MW}$$

$$w_C/w_T = 399.4/745.7 = \textbf{0.536}$$

Energy input is from the combustor

$$q_H = C_{P0}(T_3 - T_2) = 1.004 \text{ kJ/kg-K } (1594 - 697.8) \text{ K} = 899.8 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 346.3/899.8 = \textbf{0.385}$$

10.42

A gas turbine with air as the working fluid has two ideal turbine sections, as shown in Fig. P10.42, the first of which drives the ideal compressor, with the second producing the power output. The compressor input is at 290 K, 100 kPa, and the exit is at 450 kPa. A fraction of flow, x, bypasses the burner and the rest $(1 - x)$ goes through the burner where 1200 kJ/kg is added by combustion. The two flows then mix before entering the first turbine and continue through the second turbine, with exhaust at 100 kPa. If the mixing should result in a temperature of 1000 K into the first turbine find the fraction x. Find the required pressure and temperature into the second turbine and its specific power output.

C.V.Comp.: $-w_C = h_2 - h_1$; $s_2 = s_1$

Reversible and adiabatic gives constant s which from Eq.6.23 gives:

$$T_2 = T_1 (P_2/P_1)^{(k-1)/k} = 290 \text{ K } (450/100)^{0.2857} = 445.7 \text{ K}$$

$$h_2 = 447.75 \text{ kJ/kg}, \quad -w_C = 447.75 - 290.43 = 157.3 \text{ kJ/kg}$$

C.V.Burner: $h_3 = h_2 + q_H = 447.75 + 1200 = 1647.75$ kJ/kg

$$\Rightarrow T_3 = 1510 \text{ K}$$

C.V.Mixing chamber: $(1 - x)h_3 + xh_2 = h_{MIX} = 1046.22$ kJ/kg

$$x = \frac{h_3 - h_{MIX}}{h_3 - h_2} = \frac{1647.75 - 1046.22}{1647.75 - 447.75} = \mathbf{0.5013}$$

$$\dot{W}_{T1} = \dot{W}_{C,in} \Rightarrow w_{T1} = -w_C = 157.3 = h_3 - h_4$$

$$h_4 = 1046.22 - 157.3 = 888.9 \text{ kJ/kg} \Rightarrow T_4 = \mathbf{860 \text{ K}}$$

$$P_4 = P_{MIX}(T_4/T_{MIX})^{k/(k-1)} = 450 \text{ kPa} \times (860/1000)^{3.5} = \mathbf{265 \text{ kPa}}$$

$$s_4 = s_5 \Rightarrow T_5 = T_4 (P_5/P_4)^{(k-1)/k} = 860 \text{ K } (100/265)^{0.2857} = 651 \text{ K}$$

$$h_5 = 661.2 \text{ kJ/kg}$$

$$w_{T2} = h_4 - h_5 = 888.9 - 661.2 = \mathbf{227.7 \text{ kJ/kg}}$$

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10.43

A gas turbine cycle has two stages of compression, with an intercooler between the stages. Air enters the first stage at 100 kPa, 300 K. The pressure ratio across each compressor stage is 4 to 1, and each stage has an isentropic efficiency of 82%. Air exits the intercooler at 330 K. Calculate the temperature at the exit of each compressor stage and the total specific work required.

Solution:

State 1: $P_1 = 100$ kPa, $T_1 = 300$ K

State 3: $T_3 = 330$ K

$\qquad P_2 = 4\, P_1 = 400$ kPa; $\quad P_4 = 4\, P_3 = 1600$ kPa

Energy Eq.: $w_{c1} + h_1 = h_2$ => $w_{c1} = h_2 - h_1 \approx C_P(T_2 - T_1)$

Ideal C1 constant s, Eq.6.23: $T_{2s} = T_1\,(P_2/P_1)^{(k-1)/k} = 445.8$ K

$\qquad w_{c1\,s} = h_{2s} - h_1 \approx C_P(T_{2s} - T_1) = 146.4$ kJ/kg,

Actual Eq.7.28: $w_{c1} = w_{c1\,s}/\eta = 146.4/0.82 = 178.5$ kJ/kg

$\qquad T_2 = T_1 + w_{c1}/C_P =$ **477.8 K**

Ideal C2 constant s, Eq.6.23: $T_{4s} = T_3\,(P_4/P_3)^{(k-1)/k} = 490.4$ K

$\qquad w_{c2\,s} = h_{4s} - h_3 \approx C_P(T_{4s} - T_3) = 161$ kJ/kg;

Actual Eq.7.28: $w_{c2} = w_{c2\,s}/\eta = 196.4$ kJ/kg

$\qquad T_4 = T_3 + w_{c2}/C_P =$ **525.6 K**

Total work in:

$\qquad w = w_{c1} + w_{c2} = 178.5 + 196.4 =$ **374.9 kJ/kg**

Ericsson Cycles

10.44

Consider an ideal air-standard Ericsson cycle that has an ideal regenerator as shown in Fig. P10.44. The high pressure is 1.5 MPa and the cycle efficiency is 60%. Heat is rejected in the cycle at a temperature of 350 K, and the cycle pressure at the beginning of the isothermal compression process is 150 kPa. Determine the high temperature, the compressor work, and the turbine work per kilogram of air.

$P_2 = P_3 = 1.5$ MPa

$T_1 = T_2 = 350$ K

$P_1 = 150$ kPa

$_2q_3 = -_4q_1$ (ideal reg.)

$\Rightarrow q_H = {}_3q_4$ & $w_T = q_H$

$r_p = P_2/P_1 = 10$

$$\eta_{TH} = \eta_{CARNOT\ TH.} = 1 - T_L/T_H = 0.6 \Rightarrow T_3 = T_4 = T_H = \textbf{875 K}$$

$$q_L = w_C = \int v\ dP = RT_1 \ln\left(\frac{P_2}{P_1}\right) = 0.287 \times 350 \times \ln\left(\frac{1500}{150}\right) = \textbf{231.3 kJ/kg}$$

$$w_T = q_H = -\int v\ dP = -RT_3 \ln(P_4/P_3) = \textbf{578.2 kJ/kg}$$

10.45

An air-standard Ericsson cycle has an ideal regenerator. Heat is supplied at 1000°C and heat is rejected at 80°C. Pressure at the beginning of the isothermal compression process is 70 kPa. The heat added is 700 kJ/kg. Find the compressor work, the turbine work, and the cycle efficiency.

Solution:

Identify the states

Heat supplied at high temperature $\quad T_3 = T_4 = 1000°C = 1273.15$ K

Heat rejected at low temperature $\quad T_1 = T_2 = 80°C = 353.15$ K

Beginning of the compression: $\quad P_1 = 70$ kPa

Ideal regenerator: $\quad _2q_3 = -_4q_1 \quad \Rightarrow \quad q_H = _3q_4 = 700$ kJ/kg

$$\Rightarrow \quad w_T = q_H = \mathbf{700 \ kJ/kg}$$

$$\eta_{TH} = \eta_{CARNOT} = 1 - \frac{353.15}{1273.15} = \mathbf{0.7226}$$

$$w_{NET} = \eta_{TH} \ q_H = 0.7226 \times 700 = 505.8 \ \text{kJ/kg}$$

$$w_C = q_L = q_H - w_{NET} = 700 - 505.8 = \mathbf{194.2 \ kJ/kg}$$

Borgnakke and Sonntag

Jet Engine Cycles

10.46

The Brayton cycle in Problem 10.21 is changed to be a jet engine. Find the exit velocity using cold air properties.

A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 300 K, and the exhaust temperature is 750 K. The minimum pressure in the cycle is 100 kPa, and the compressor pressure ratio is 14 to 1. Calculate the power output of the turbine. What fraction of the turbine output is required to drive the compressor? What is the thermal efficiency of the cycle?

Solution:

Brayton cycle so this means:
     Minimum T: $T_1 = 300$ K
     Exhaust T: $T_5 = 750$ K
     Pressure ratio: $P_2/P_1 = 14$

Solve using constant C_{P0}

Compression in compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.6.23

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300 \text{ K } (14)^{0.286} = 638.1 \text{ K}$$

$$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004 \text{ kJ/kg-K } (638.1 - 300) \text{ K} = 339.5 \text{ kJ/kg}$$

Turbine ($w_T = w_C$) and nozzle: $s_5 = s_4 = s_3 \Rightarrow$ Implemented in Eq.6.23

$$T_3 = T_5 (P_3/P_5)^{\frac{k-1}{k}} = 750 \text{ K } (14/1)^{0.286} = 1594 \text{ K}$$

$$h_3 - h_5 = C_{P0}(T_3 - T_5) = 1.004 \text{ kJ/kg-K } (1594 - 750) \text{ K} = 847.4 \text{ kJ/kg}$$

$$1/2 \mathbf{V}_5^2 = h_3 - h_5 - w_C = 847.4 - 339.5 = 507.9 \text{ kJ/kg}$$

$$\mathbf{V}_5 = \sqrt{2 \times 1000 \text{ J/kJ} \times 507.9 \text{ kJ/kg}} = \mathbf{1008 \text{ m/s}}$$

10.47

Consider an ideal air-standard cycle for a gas-turbine, jet propulsion unit, such as that shown in Fig. 10.9. The pressure and temperature entering the compressor are 90 kPa, 290 K. The pressure ratio across the compressor is 14 to 1, and the turbine inlet temperature is 1500 K. When the air leaves the turbine, it enters the nozzle and expands to 90 kPa. Determine the velocity of the air leaving the nozzle.

Solution:

C.V. Compressor: Reversible and adiabatic $\quad s_2 = s_1 \quad$ From Eq.6.20, 6.23

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 290 \text{ K } (14)^{0.2857} = 616.4 \text{ K}$$

$$w_C = h_2 - h_1 \approx C_{P0} (T_2 - T_1) = 1.004 \text{ kJ/kg-K} \times (616.4 - 290) \text{ K} = 327.7 \text{ kJ/kg}$$

C.V. Turbine: $\quad w_T = h_3 - h_4 = w_C \quad$ and $\quad s_4 = s_3 \Rightarrow$

$$T_4 = T_3 - w_C/C_{P0} = 1500 - 327.7/1.004 = 1173.6 \text{ K}$$

C.V. Nozzle: $\quad s_5 = s_4 = s_3 \quad$ so from Eq.6.23

$$T_5 = T_3\left(\frac{P_5}{P_3}\right)^{\frac{k-1}{k}} = 1500 \text{ K} \times \left(\frac{90}{1260}\right)^{0.2857} = 705.7 \text{ K}$$

Now the energy equation

$$(1/2)\mathbf{V}_5^2 = h_4 - h_5 \approx C_{P0} (T_4 - T_5) = 1.004 (1173.6 - 705.7) = 469.77 \text{ kJ/kg}$$

$$\Rightarrow \quad \mathbf{V}_5 = \sqrt{2 \times 1000 \text{ J/kJ} \times 469.77 \text{ kJ/kg}} = \textbf{969 m/s}$$

10.48

The turbine section in a jet engine receives gas (assume air) at 1200 K, 800 kPa with an ambient atmosphere at 80 kPa. The turbine is followed by a nozzle open to the atmosphere and all the turbine work drives a compressor. Find the turbine exit pressure so the nozzle has an exit velocity of 800 m/s. Hint: take the CV around both turbine and nozzle.

Solution:

C.V. Reversible and adiabatic turbine and nozzle. This gives constant s, from Eq.6.23 we can relate the T's and P's

State 3: 1200 K, 800 kPa   State 5: 80 kPa; $s_5 = s_3$

Eq.6.23: $T_5 = T_3 (P_5/P_3)^{(k-1)/k} = 1200 \text{ K } (80/800)^{0.2857} = 621.56 \text{ K}$

Energy: $h_3 + 0 = h_5 + (1/2)\mathbf{V}_5^2 + w_T = h_4 + w_T$

$$w_T = h_3 - h_5 - (1/2)\mathbf{V}_5^2 \cong C_P(T_3 - T_5) - (1/2)\mathbf{V}_3^2$$

$\qquad = 1.004 \text{ kJ/kg-K } (1200 - 621.56) \text{ K} - (1/2) \times 800^2 (\text{J/kg}) / (1000 \text{ J/kJ})$

$\qquad = 580.75 - 320 = 260.75 \text{ kJ/kg}$

C.V. Nozzle alone to establish state 4 (same s as state 5 and 3).

$\qquad h_4 = h_5 + (1/2)\mathbf{V}_5^2 = h_3 - w_T$

$\qquad T_4 = T_5 + (1/2)\mathbf{V}_5^2/C_P = 621.56 + 320/1.004 = 940.29 \text{ K}$

$\qquad P_4 = P_3 (T_4/T_3)^{k/(k-1)} = 800 \text{ kPa} \times (940.29/1200)^{3.5} = \textbf{340.7 kPa}$

10.49

Given the conditions in the previous problem what pressure could an ideal compressor generate (not the 800 kPa but higher).

C.V. Reversible and adiabatic turbine and nozzle. This gives constant s, from Eq.6.23 we can relate the T's and P's

State 3: 1200 K, 800 kPa State 5: 80 kPa; $s_5 = s_3$

Eq.6.23: $T_5 = T_3 (P_5/P_3)^{(k-1)/k} = 1200$ K $(80/800)^{0.2857} = 621.56$ K

Energy: $h_3 + 0 = h_5 + (1/2)\mathbf{V}_5^2 + w_T = h_4 + w_T$

$w_T = h_3 - h_5 - (1/2)\mathbf{V}_5^2 \cong C_P(T_3 - T_5) - (1/2)\mathbf{V}_3^2$

$= 1.004(1200 - 621.56) - (1/2) \times 800^2/1000$

$= 580.75 - 320 = 260.75$ kJ/kg

C.V. Compressor

$w_c = h_2 - h_1 = w_T = 260.75$ kJ/kg

$T_2 = T_1 + w_c/ C_P = 270$ K $+ 260.75$ kJ/kg $/1.004$ kJ/kg-K $= 529.71$ K

Reversible adiabatic compressor, constant s gives relation in Eq.6.23

$P_2 = P_1 \times (T_2/T_1)^{k/(k-1)} = 85$ kPa $\times (529.71/270)^{3.5} = $ **899 kPa**

10.50

Consider a turboprop engine where the turbine powers the compressor and a propeller. Assume the same cycle as in Problem 10.46 with a turbine exit temperature of 900 K. Find the specific work to the propeller and the exit velocity.

Compression in compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.6.23

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300 \text{ K} \times (14)^{0.286} = 638.1 \text{ K}$$

$$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004 \text{ kJ/kg-K} (638.1 - 300) \text{ K} = 339.5 \text{ kJ/kg}$$

C.V. Turbine: $w_T = h_3 - h_4 = w_C + w_{prop}$ and $s_4 = s_3 = s_5 \Rightarrow$

$$T_3 = T_5 (P_3/P_5)^{\frac{k-1}{k}} = 750 \text{ K} \times (14/1)^{0.286} = 1594 \text{ K}$$

$$w_{prop} = C_{P0} (T_3 - T_4) - w_C = 1.004(1594 - 900) - 339.5 = \textbf{357.3 kJ/kg}$$

C.V. Nozzle: $s_5 = s_4 = s_3$ so from Eq.6.23

Now the energy equation

$$(1/2)V_5^2 = h_4 - h_5 \approx C_{P0} (T_4 - T_5) = 1.004 (900 - 750) = 150.6 \text{ kJ/kg}$$

$$\Rightarrow \quad V_5 = \sqrt{2 \times 1000 \times 150.6} = \textbf{549 m/s}$$

10.51

Consider an air standard jet engine cycle operating in a 280 K, 100 kPa environment. The compressor requires a shaft power input of 4000 kW. Air enters the turbine state 3 at 1600 K, 2 MPa, at the rate of 9 kg/s, and the isentropic efficiency of the turbine is 85%. Determine the pressure and temperature entering the nozzle at state 4. If the nozzle efficiency is 95%, determine the temperature and velocity exiting the nozzle at state 5.

Solution:

C.V. Shaft: $\dot{W}_T = \dot{m}(h_3 - h_4) = \dot{W}_C$

CV Turbine: $h_3 - h_4 = \dot{W}_C / \dot{m}$ = 4000 kW /9 kg/s = 444.4 kJ/kg

$T_4 = T_3 - w_{Ta}/ C_p = 1600 - 444.4/1.004 = 1157.4$ K

Work back to the ideal turbine conditions

Eq.7.27: $w_{Ts} = w_{Ta} / \eta = 444.4/0.85 = 522.82 = h_3 - h_{4s} = C_p(T_3 - T_{4s})$

$\Rightarrow$ $T_{4s} = 1600 - 522.82/1.004 = 1079.3$ K,

$P_4 = P_3 (T_{4s}/T_3)^{k/(k-1)} = 2000$ kPa $\times (1079.3/1600)^{3.5} =$ **504.2 kPa**

C.V. Nozzle: First the ideal (reversible and adiabatic i.e. constant s, Eq.6.23)

$\Rightarrow T_{5s} = T_4 (P_5/P_4)^{(k-1)/k} = 1157.4$ K $\times (100/504.2)^{(k-1)/k} = 729$ K

Energy Eq.: $(1/2)\mathbf{V}_{5s}^2 = h_4 - h_{5s} = C_p(T_4 - T_{5s})$

$= 1.004$ kJ/kg-K $\times (1157.4 - 729)$ K $= 430.1$ kJ/kg

Now consider the actual nozzle

Eq.7.30: $0.5\mathbf{V}_{5a}^2 = \eta(0.5\mathbf{V}_{5s}^2) = 408.6$ kJ/kg

$\Rightarrow$ $\mathbf{V}_{5a} = \sqrt{2 \times 1000 \text{ J/kJ} \times 408.6 \text{ kJ/kg}} =$ **904 m/s**

$T_{5a} = T_4 - 0.5\mathbf{V}_{5a}^2/C_p = 1157.4 - 408.6 / 1.004 =$ **750 K**

10.52

Solve the previous problem using the air tables.

Solution done with the air tables A.7:

C.V. Shaft: $\dot{W}_T = \dot{m}(h_3 - h_4) = \dot{W}_C$

CV Turbine: $h_3 - h_4 = \dot{W}_C / \dot{m} = 4000/9 = 444.4$ kJ/kg

$\qquad\qquad h_4 = 1757.3 - 444.4 = 1312.9$ kJ/kg

Work back to the ideal turbine conditions

Eq.7.27: $w_{Ta} = w_C = 444.4 \quad \Rightarrow \quad w_{Ts} = w_{Ta} / \eta = 522.82 = h_3 - h_{4s}$

$\qquad\qquad h_{4s} = 1234.5 \quad \Rightarrow \quad T_{4s} \approx 1163$ K, $s^o_{T4s} = 8.3091$ kJ/kg K

$\qquad\qquad s_{4s} - s_3 = 0 = s^o_{T4s} - s^o_{T3} - R \ln(P_4/P_3)$

$\qquad\qquad 0 = 8.3091 - 8.6905 - 0.287 \ln(P_4/2000) \quad \Rightarrow \quad \mathbf{P_4 = 530\ kPa}$

State 4 from A.7.1: $h_4 = 1312.9$, $T_4 = 1229.8$ K, $s^o_{T4} = 8.3746$ kJ/kg K

First consider the reversible adiabatic (isentropic) nozzle so from Eq.6.19

$\qquad\qquad s_{5s} - s_4 = 0 = s^o_{T5s} - s^o_{T4} - R \ln(P_5/P_4)$

$\qquad\qquad s^o_{T5s} = 8.3746 + 0.287 \ln(100/530) = 7.8960$ kJ/kg K

$\qquad$ Table A.7.1: $T_{5s} = 808.1$ K, $h_{5s} = 831.0$ kJ/kg

$\qquad \Rightarrow \quad 0.5V^2_{5s} = h_4 - h_{5s} = 1312.9 - 831.0 = 481.9$ kJ/kg

Now consider the actual nozzle

$\qquad\qquad$ Eq.7.30: $0.5V^2_{5a} = \eta(0.5V^2_{5s}) = 457.81$ kJ/kg $\Rightarrow$ $\mathbf{V_{5a} = 957\ m/s}$

$\qquad\qquad h_{5a} = h_4 - 0.5V^2_{5a} = 1312.9 - 457.81 = 855.1$ kJ/kg

$\qquad\qquad \Rightarrow \quad T_{5a} \approx \mathbf{830\ K}$

10.53

A jet aircraft is flying at an altitude of 4900 m, where the ambient pressure is approximately 50 kPa and the ambient temperature is –20°C. The velocity of the aircraft is 280 m/s, the pressure ratio across the compressor is 14:1 and the cycle maximum temperature is 1450 K. Assume the inlet flow goes through a diffuser to zero relative velocity at state 1. Find the temperature and pressure at state 1.

Solution:

Ambient

T_X = -20°C = 253.15 K, P_X = 50 kPa = P_5

also $\mathbf{V}_X$ = 280 m/s

Assume that the air at this state is reversibly decelerated to zero velocity and then enters the compressor at 1.

$$P_2/P_1 = 14 \quad \& \quad T_3 = 1450 \text{ K}$$

C.V. Diffuser section, state 1 is the stagnation state.

Energy Eq.:
$$T_1 = T_X + \frac{V_X^2}{2 \times 1000 \times C_p} = 253.15 + \frac{(280)^2}{2 \times 1000 \times 1.004} = \mathbf{292.2 \text{ K}}$$

Eq.6.23:
$$P_1 = P_X \left(\frac{T_1}{T_X}\right)^{\frac{k}{k-1}} = 50 \text{ kPa} \left(\frac{292.2}{253.15}\right)^{3.5} = \mathbf{82.6 \text{ kPa}}$$

10.54

The turbine in a jet engine receives air at 1250 K, 1.5 MPa. It exhausts to a nozzle at 250 kPa, which in turn exhausts to the atmosphere at 100 kPa. The isentropic efficiency of the turbine is 85% and the nozzle efficiency is 95%. Find the nozzle inlet temperature and the nozzle exit velocity. Assume negligible kinetic energy out of the turbine.

C.V. Turbine: First the ideal (reversible and adiabatic i.e. constant s, Eq.6.23)

$$T_{es} = T_i \, (P_e/P_i) = 1250 \text{ K} \times (250/1500)^{(k-1)/k} = 749.2 \text{ K}$$

Energy Eq.: $w_{T,s} = h_i - h_{es} = C_p \, (T_i - T_{es}) = 1.004 \, (1250 - 749.2) = 502.8 \text{ kJ/kg}$

Eq.7.27: $w_{T,AC} = w_{T,s} \times \eta_T = 427.4 \text{ kJ/kg} = h_i - h_{e,AC} = C_p \, (T_i - T_{e,AC})$

$$\Rightarrow \ T_{e,AC} = 1250 - 427.4 / 1.004 = \mathbf{824.3 \text{ K}},$$

C.V. Nozzle: First the ideal (reversible and adiabatic i.e. constant s, Eq.6.23)

$$\Rightarrow T_{es} = T_i \, (P_e/P_i)^{(k-1)/k} = 824.3 \text{ K} \times (100/250)^{(k-1)/k} = 634.4 \text{ K}$$

Energy Eq.: $(1/2)V_{e,s}^2 = h_i - h_{e,s} = 1.004(824.3 - 634.4) = 190.66 \text{ kJ/kg}$

Eq.7.30: $(1/2)V_{e,AC}^2 = (1/2)V_{e,s}^2 \times \eta_{NOZ} = 181.13 \text{ kJ/kg}$

$$V_{e,AC} = \sqrt{2 \times 1000 \text{ J/kJ} \times 181.13 \text{ kJ/kg}} = \mathbf{602 \text{ m/s}}$$

10.55

Solve the previous problem using the air tables.

Solution using air tables A.7:

C.V. Turbine: $h_i = 1336.7$, $s^o_{Ti} = 8.3940$, $s_{es} = s_i$ then from Eq.6.19

$\Rightarrow s^o_{Tes} = s^o_{Ti} + R \ln(P_e/P_i) = 8.3940 + 0.287 \ln(250/1500) = 7.8798$ kJ/kg K

Table A.7.1 $T_{es} = 796$ K, $h_{es} = 817.9$ kJ/kg,

Energy Eq.: $w_{T,s} = h_i - h_{es} = 1336.7 - 817.9 = 518.8$ kJ/kg

Eq.7.27: $w_{T,AC} = w_{T,s} \times \eta_T = 441$ kJ/kg $= h_i - h_{e,AC}$

$\Rightarrow h_{e,AC} = 895.7 \Rightarrow$ **$T_{e,AC}$ = 866 K**, $s^o_{Te} = 7.9730$ kJ/kg K

C.V. Nozzle: $h_i = 895.7$ kJ/kg, $s^o_{Ti} = 7.9730$ kJ/kgK, $s_{es} = s_i$ then from Eq.6.19

$\Rightarrow s^o_{Tes} = s^o_{Ti} + R \ln(P_e/P_i) = 7.9730 + 0.287 \ln(100/250) = 7.7100$ kJ/kgK

Table A.7.1 $\Rightarrow T_{e,s} = 681$ K, $h_{e,s} = 693.1$ kJ/kg

Energy Eq.: $(1/2)V^2_{e,s} = h_i - h_{e,s} = 895.7 - 693.1 = 202.6$ kJ/kg

Eq.7.30: $(1/2)V^2_{e,AC} = (1/2)V^2_{e,s} \times \eta_{NOZ} = 192.47$ kJ/kg

$V_{e,AC} = \sqrt{2 \times 1000 \text{ J/kJ} \times 192.47 \text{ kJ/kg}} =$ **620 m/s**

10.56

An afterburner in a jet engine adds fuel after the turbine thus raising the pressure and temperature due to the energy of combustion. Assume a standard condition of 800 K, 250 kPa after the turbine into the nozzle that exhausts at 95 kPa. Assume the afterburner adds 450 kJ/kg to that state with a rise in pressure for same specific volume, and neglect any upstream effects on the turbine. Find the nozzle exit velocity before and after the afterburner is turned on.

Solution:

Before afterburner is on: 1: 800 K; 250 kPa   and   2: 95 kPa

After afterburner is on: 3: $v = v_1$ and   4: 95 kPa

Assume reversible adiabatic nozzle flow, then constant s from Eq.6.23

$$T_2 = T_1 (P_2/P_1)^{(k-1)/k} = 800 \text{ K} \times (95/250)^{0.2857} = 606.8 \text{ K}$$

Energy Eq.: $(1/2)\mathbf{V}_2^2 = C_P(T_1 - T_2)$

$$\mathbf{V}_2 = \sqrt{2\, C_P(T_1 - T_2)} = \sqrt{2 \times 1004 \text{ J/kg-K } (800 - 606.8) \text{ K}} = \mathbf{622.8 \text{ m/s}}$$

Add the q_{AB} at assumed constant volume then energy equation gives

$$T_3 = T_1 + q_{AB}/C_v = 800 \text{ K} + 450 \text{ kJ/kg}/0.717 \text{ kJ/kg-K} = 1427.6 \text{ K}$$

$$v_3 = v_1 \implies P_3 = P_1(T_3/T_1) = 250 \text{ kPa} \times 1427.6/800 = 446.1 \text{ kPa}$$

Reversible adiabatic expansion, again from Eq.6.23

$$T_4 = T_3 (P_4/P_3)^{(k-1)/k} = 1427.6 \text{ K} \times (95/446.1)^{0.2857} = 917.7 \text{ K}$$

$$\mathbf{V}_2 = \sqrt{2\, C_P(T_3 - T_4)} = \sqrt{2 \times 1004 \text{ J/kg-K } (1427.6 - 917.7) \text{ K}} = \mathbf{1012 \text{ m/s}}$$

Comment: The real process adds some fuel that burns releasing energy so the temperature goes up and due to the confined space then pressure goes up. As the pressure goes up the exit velocity increases altering the mass flow rate through the nozzle. As in most problems the real device is more complicated than we can describe with our simple analysis.

Air-standard refrigeration cycles

10.57

An air standard refrigeration cycle has air into the compressor at 100 kPa, 270 K with a compression ratio of 3:1. The temperature after heat rejection is 300 K. Find the COP and the highest cycle temperature.

From the isentropic compression/expansion processes

$$T_2 / T_1 = (P_2/P_1)^{\frac{k-1}{k}} = 3^{0.2857} = 1.36874 = T_3/T_4$$

$$T_2 = T_1 \times 1.36874 = 270 \text{ K} \times 1.36874 = \mathbf{369.6\ K}$$

The COP, Eq.10.5, is

$$\beta = \frac{1}{\dfrac{T_2}{T_1} - 1} = [\ 1.36874 - 1]^{-1} = \mathbf{2.712}$$

10.58

A standard air refrigeration cycle has -10$^{\text{o}}$C, 100 kPa into the compressor, and the ambient cools the air down to 35$^{\text{o}}$C at 400 kPa. Find the lowest temperature in the cycle, the low T specific heat transfer and the specific compressor work.

Solution:

State 3: 35$^{\text{o}}$C = 308.2 K, 400 kPa
The lowest T is at state 4 which we can relate to state 3, constant s Eq.6.23

$$T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 308.2 \text{ K} \times \left(\frac{100}{400}\right)^{0.2857} = 204.9 \text{ K} = \textbf{–68.3}^{\text{o}}\textbf{C}$$

Now the heat rejected is

$$_4q_1 = h_1 - h_4 = C_p (T_1 - T_4) = 1.004 \text{ kJ/kg-K} \times (-10 + 68.3) \text{ K}$$
$$= \textbf{58.5 kJ/kg}$$

The isentropic compression

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 263.2 \text{ K} \times \left(\frac{400}{100}\right)^{0.2857} = 391.3 \text{ K}$$

from which we get the compressor work

$$w_c = h_1 - h_2 = C_p (T_1 - T_2) = 1.004 \text{ kJ/kg-K} \times (263.2 - 391.3) \text{ K}$$
$$= \textbf{–128.6 kJ/kg}$$

10.59

The formula for the COP assuming cold air properties is given for the standard refrigeration cycle in Eq.10.5. Develop the similar formula for the cycle variation with a heat exchanger as shown in Fig. 10.12.

Definition of COP:
$$\beta = \frac{q_L}{w_{net}} = \frac{q_L}{q_H - q_L} = \frac{1}{\dfrac{q_H}{q_L} - 1}$$

Assuming an ideal heat exchanger $h_4 = h_6$ and $h_1 = h_3$ so $q_L = w_E$

From the refrigeration cycle we get the ratio of the heat transfers as
$$\frac{q_H}{q_L} = \frac{C_p(T_2 - T_3)}{C_p(T_6 - T_5)} = \frac{T_2 - T_1}{T_4 - T_5} = \frac{T_1}{T_5}\frac{(T_2/T_1) - 1}{(T_4/T_5) - 1}$$

The pressure ratios are the same and we have isentropic compression/expansion
$$\frac{P_2}{P_1} = \frac{P_4}{P_5} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{T_4}{T_5}\right)^{k/(k-1)}$$

so now we get
$$\frac{T_2}{T_1} = \frac{T_4}{T_5} \qquad \text{so} \qquad \frac{q_H}{q_L} = \frac{T_1}{T_5} = \frac{T_1}{T_4}\frac{T_4}{T_5} = \frac{T_1}{T_4} r_P^{(k-1)/k}$$

and the COP reduces to
$$\beta = \frac{1}{\dfrac{T_1}{T_5} - 1} = \frac{1}{\dfrac{T_1}{T_4} r_P^{(k-1)/k} - 1}$$

10.60

Assume a refrigeration cycle as shown in Fig. 10.12 with reversible adiabatic compressor and expander. For this cycle the low pressure is 100 kPa and the high pressure is 1.4 MPa with constant pressure heat exchangers, see Fig. 10.12 T-s diagram. The temperatures are $T_4 = T_6 = -50^oC$ and $T_1 = T_3 = 15^oC$. Find the COP for this refrigeration cycle.

Solution:

Standard air refrigeration cycle with $P_1 = 100$ kPa, $P_2 = 1.4$ MPa

We will solve the problem with cold air properties.

Compressor, isentropic $s_2 = s_1$ so from Eq.6.23

$$\Rightarrow T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 288.2 \text{ K} \times (1400/100)^{0.286} = 613 \text{ K}$$

$$w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.004 \text{ kJ/kg-K } (613 - 288.2) \text{ K} = 326 \text{ kJ/kg}$$

Expansion in expander (turbine)

$$s_5 = s_4 \Rightarrow T_5 = T_4(P_5/P_4)^{\frac{k-1}{k}} = 223.2 \text{ K} \times (100/1400)^{0.286} = 104.9 \text{ K}$$

$$w_E = C_{P0}(T_4 - T_5) = 1.004 \text{ kJ/kg-K } (223.2 - 104.9) \text{ K} = 118.7 \text{ kJ/kg}$$

Net cycle work

$$w_{NET} = w_E - w_C = 118.7 - 326.0 = -207.3 \text{ kJ/kg}$$

$$q_L = C_{P0}(T_6 - T_5) = w_E = 118.7 \text{ kJ/kg}$$

Overall cycle performance, COP

$$\beta = q_L/w_{NET} = 118.7 / 207.3 = \textbf{0.573}$$

Borgnakke and Sonntag

10.61

Repeat Problems 10.60, but assume that helium is the cycle working fluid instead of air. Discuss the significance of the results.

A heat exchanger is incorporated into an ideal air-standard refrigeration cycle, as shown in Fig. P10.60. It may be assumed that both the compression and the expansion are reversible adiabatic processes in this ideal case. Determine the coefficient of performance for the cycle.

Solution:

Standard air refrigeration cycle with helium and states as

$$T_1 = T_3 = 15\ ^oC = 288.2\ K, \quad P_1 = 100\ kPa, \quad P_2 = 1.4\ MPa$$

$$T_4 = T_6 = -50\ ^oC = 223.2\ K$$

Compressor, isentropic $s_2 = s_1$ so from Eq.6.23

$$\Rightarrow\ T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 288.2\left(\frac{1400}{100}\right)^{0.40} = 828.2\ K$$

$$w_C = C_{P0}(T_2 - T_1) = 5.193\ kJ/kg\text{-}K\ (828.2 - 288.2)\ K = 2804.1\ kJ/kg$$

Expansion in expander (turbine)

$$s_5 = s_4 \quad \Rightarrow \quad T_5 = T_4(P_5/P_4)^{\frac{k-1}{k}} = 223.2\ K\left(\frac{100}{1400}\right)^{0.40} = 77.7\ K$$

$$w_E = C_{P0}(T_4 - T_5) = 15.193\ kJ/kg\text{-}K\ (223.2 - 77.7)\ K = 755.5\ kJ/kg$$

Net cycle work

$$w_{NET} = 755.5 - 2804.1 = -2048.6\ kJ/kg$$

$$q_L = C_{P0}(T_6 - T_5) = 5.193\ kJ/kg\text{-}K\ (223.2 - 77.7)\ K = 755.5\ kJ/kg$$

Overall cycle performance, COP

$$\beta = q_L/w_{NET} = 755.5/2048.6 = \textbf{0.369}$$

Notice that the low temperature is lower and work terms higher than with air. It is due to the higher heat capacity C_{P0} and ratio of specific heats (k = 1 2/3). The expense is a lower COP requiring more work input per kJ cooling.

Borgnakke and Sonntag

10.62

Repeat Problem 10.60, but assume an isentropic efficiency of 75% for both the compressor and the expander.

Standard air refrigeration cycle with

$$T_1 = T_3 = 15\ ^{o}C = 288.2\ K, \quad P_1 = 100\ kPa, \quad P_2 = 1.4\ MPa$$

$$T_4 = T_6 = -50\ ^{o}C = 223.2\ K$$

We will solve the problem with cold air properties.

Ideal compressor, isentropic $\ \ s_{2S} = s_1 \ \ $ so from Eq.6.23

$$\Rightarrow\ T_{2S} = T_1(P_2/P_1)^{\frac{k-1}{k}} = 288.2(1400/100)^{0.286} = 613\ K$$

$$w_{SC} = -w_{12} = C_{P0}(T_{2S} - T_1) = 1.004(613 - 288.2) = 326\ kJ/kg$$

The actual compressor

$$w_C = w_{SC}\ /\ \eta_{SC} = 326/0.75 = 434.6\ kJ/kg$$

Expansion in ideal expander (turbine)

$$s_5 = s_4 \Rightarrow\ T_{5S} = T_4(P_5/P_4)^{\frac{k-1}{k}} = 223.2(100/1400)^{0.286} = 104.9\ K$$

$$w_E = C_{P0}(T_4 - T_5) = 1.004(223.2 - 104.9) = 118.7\ kJ/kg$$

The actual expander (turbine)

$$w_E = \eta_{SE} \times w_{SE} = 0.75 \times 118.7 = 89.0\ kJ/kg$$
$$= C_{P0}(T_4 - T_5) = 1.004(223.2 - T_5)$$

$$\Rightarrow\ T_5 = 134.5\ K$$

$$w_{NET} = 89.0 - 434.6 = -345.6\ kJ/kg$$

$$q_L = C_{P0}(T_6 - T_5) = 1.004(223.2 - 134.5) = 89.0\ kJ/kg$$

$$\beta = q_L/(-w_{NET}) = 89.0/345.6 = \mathbf{0.258}$$

Otto Cycles

10.63

The mean effective pressure scales with the net work and thus efficiency. Assume the heat transfer per unit mass is a given (it depends on the fuel-air mixture) how does the total power output then vary with the inlet conditions (P_1, T_1)?

The power output comes from speed and displacement in Eq. 10.11

$$\dot{W} = P_{meff}V_{displ} \frac{RPM}{60} \frac{1}{2} = m\ w_{net} \frac{RPM}{60} \frac{1}{2} = m\ \eta\ q_H \frac{RPM}{60} \frac{1}{2}$$

From the last expression we notice that the power scales with mass and efficiency. From our simple analysis we know that the efficiency only depends on compression ratio CR and not on state 1, which then leaves the mass as affected by state 1. The mass of fresh charge is assumed proportional to the mass of air at ambient conditions (close to state 1) in the volume V_{displ}. The constant of proportionality is called the volumetric efficiency and depends on the throttling and other restrictions in the intake system which then leads to state 1.

$$m \approx V_{displ} / v_1 = V_{displ}\ P_1/RT_1$$

A cold charge (T_1 low) with high pressure P_1 (a super- or turbo-charger) will give the highest mass and thus the most power.

10.64

A 4 stroke gasoline engine runs at 1800 RPM with a total displacement of 3L and a compression ratio of 10:1. The intake is at 290 K, 75 kPa with a mean effective pressure of 600 kPa. Find the cycle efficiency and power output.

Efficiency from the compression ratio

$$\eta = 1 - CR^{1-k} = 1 - 10^{-0.4} = \textbf{0.60}$$

The power output comes from speed and displacement in Eq. 10.11

$$\dot{W} = P_{meff}V_{displ}\frac{RPM}{60}\frac{1}{2} = 600 \text{ kPa} \times 0.003 \text{ m}^3 \times \frac{1800}{60} \text{ (1/s)} \times \frac{1}{2}$$

$$= \textbf{27 kW}$$

10.65

Find the missing pressures and temperatures in the previous cycle.

Efficiency from the compression ratio

$$\eta = 1 - CR^{1-k} = 1 - 10^{-0.4} = 0.60$$

$$v_1 = RT_1/P_1 = 0.287 \text{ kJ/kg-K} \times 290 \text{ K}/75 \text{ kPa} = 1.10973 \text{ m}^3/\text{kg}$$

Compression: $T_2 = T_1 (CR)^{k-1} = 290 \text{ K} \times (10)^{0.4} = \textbf{728.45 K}$

$$P_2 = P_1 (CR)^k = 75 \text{ kPa} \times (10)^{1.4} = \textbf{1883.9 kPa}$$

$$q_H = u_3 - u_2 = C_v(T_3 - T_2) = w_{net}/\eta = P_{meff} (v_1 - v_2)/\eta$$

$$= 600 \times 1.10973(1 - 1/CR)/0.60 = 998.76 \text{ kJ/kg}$$

$$T_3 = T_2 + q_H/ C_v = 728.45 + 998.76/0.717 = \textbf{2121.4 K}$$

$$P_3 = P_2 \times (T_3/T_2) = 1883.9 (2121.4 / 728.45) = \textbf{5486 kPa}$$

$$T_4 = T_3/CR^{0.4} = 2121.4 \text{ K} /10^{0.4} = \textbf{844.5 K}$$

$$P_4 = P_3/CR^{1.4} = 5486 \text{ kPa}/10^{1.4} = \textbf{218.4 kPa}$$

Borgnakke and Sonntag

10.66

Air flows into a gasoline engine at 95 kPa, 300 K. The air is then compressed with a volumetric compression ratio of 10:1. In the combustion process 1300 kJ/kg of energy is released as the fuel burns. Find the temperature and pressure after combustion using cold air properties.

Solution:

Solve the problem with constant specific heat.

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq.6.24 and Eq.6.25

$$T_2 = T_1 (v_1/v_2)^{k-1} = 300 \times 10^{0.4} = 753.6 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 95 \times 10^{1.4} = 2386 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v = 753.6 + 1300/0.717 = \textbf{2567 K}$$

$$P_3 = P_2 \times (T_3/T_2) = 2386 \ (2567 \ / \ 753.6) = \textbf{8127 kPa}$$

10.67

A 4 stroke gasoline 4.2 L engine running at 2000 RPM has inlet state of 85 kPa, 280 K and after combustion it is 2000 K and the highest pressure is 5 MPa. Find the compression ratio, the cycle efficiency and the exhaust temperature.

Solution:

Combustion $v_3 = v_2$. Highest T and P are after combustion.

$$CR = v_1/v_2 = v_1/v_3 = (T_1 P_3 / T_3 P_1) = \frac{280 \times 5000}{2000 \times 85} = \mathbf{8.235}$$

Efficiency from the compression ratio

$$\eta = 1 - CR^{1-k} = 1 - 8.235^{-0.4} = \mathbf{0.57}$$

Expansion

$$T_4 = T_3 \, (v_3/v_4)^{k-1} = T_3 \, [\, 1/\, CR \,]^{k-1} = 2000 \times (1/\, 8.235)^{0.4} = \mathbf{860.5 \ K}$$

10.68

Find the power from the engine in Problem 10.67

Solution:

Combustion $v_3 = v_2$. Highest T and P are after combustion.

$$CR = v_1/v_2 = v_1/v_3 = (T_1P_3/T_3P_1) = \frac{280 \times 5000}{2000 \times 85} = 8.235$$

Efficiency from the compression ratio

$$\eta = 1 - CR^{1-k} = 1 - 8.235^{-0.4} = 0.57$$

Compression: $T_2 = T_1(CR)^{k-1} = 280 \text{ K} \times (8.235)^{0.4} = 650.8 \text{ K}$

$q_H = u_3 - u_2 = C_v(T_3 - T_2) = 0.717 \text{ kJ/kg-K } (2000 - 650.8) \text{ K} = 967.4 \text{ kJ/kg}$

$v_1 = RT_1/P_1 = 0.287 \text{ kJ/kg-K} \times 280 \text{ K}/85 \text{ kPa} = 0.9454 \text{ m}^3/\text{kg}$

Displacement and then mep from net work

$v_1 - v_2 = v_1 - v_1/CR = v_1[1 - (1/CR)] = 0.8306 \text{ m}^3/\text{kg}$

$P_{meff} = w_{net}/(v_1 - v_2) = \eta \, q_H/(v_1 - v_2)$

$\qquad = 0.57 \times 967.4/0.8306 = 663.9 \text{ kPa}$

Total power from Eq. 10.11 and 4 stroke cycle

$$\dot{W} = P_{meff}V_{displ} \frac{RPM}{60} \frac{1}{2} = 663.9 \times 0.0042 \times \frac{2000}{60} \times \frac{1}{2} = \mathbf{46.45 \ kW}$$

10.69

A 4 stroke 2.4 L gasoline engine runs at 2500 RPM has an efficiency of 60%. The state before compression is 40 kPa, 280 K and after combustion it is at 2400 K. Find the highest T and P in the cycle, the specific heat transfer added, the cycle mep and the total power produced.

This is a basic 4-stroke Otto cycle.
Compression ratio from the efficiency

$$\eta = 1 - CR^{1-k} \quad => \quad CR = (1 - \eta)^{-1/0.4} = 9.882$$

Compression: $T_2 = T_1(CR)^{k-1} = 280 \times (9.882)^{0.4} = 700 \text{ K}$

$$P_2 = P_1\, CR^k = 40 \times 9.882^{1.4} = 988.2 \text{ kPa}$$

Combustion $v_3 = v_2$. Highest T and P are after combustion.

$$T_3 = \textbf{2400 K}, \quad P_3 = P_2\, T_3 / T_2 = \frac{988.2 \times 2400}{700} = \textbf{3388 kPa}$$

$$q_H = u_3 - u_2 = C_v(T_3 - T_2) = 0.717\,(2400 - 700) = \textbf{1218.9 kJ/kg}$$

$$w_{net} = \eta_{TH} \times q_H = 0.6 \times 1218.9 = 731.34 \text{ kJ/kg}$$

Displacement and P_{meff}

$$v_1 = RT_1/P_1 = (0.287 \text{ kJ/kg-K} \times 280 \text{ K})/40 \text{ kPa} = 2.009 \text{ m}^3/\text{kg}$$

$$v_2 = (1/9.882)\, v_1 = 0.2033 \text{ m}^3/\text{kg}$$

$$P_{meff} = \frac{w_{NET}}{v_1\text{-}v_2} = \frac{731.34}{2.009 - 0.2033} = \textbf{405 kPa}$$

Now we can find the power from Eq.10.11

$$\dot{W} = P_{meff}\, V_{displ} \frac{RPM}{60}\frac{1}{2} = 405 \times 0.0024 \times \frac{2500}{60} \times \frac{1}{2} = \textbf{20.3 kW}$$

10.70

Suppose we reconsider the previous problem and instead of the standard ideal cycle we assume the expansion is a polytropic process with $n = 1.5$. What are the exhaust temperature and the expansion specific work?

This is a modified 4-stroke Otto cycle. Assume the same state 1-2-3 so only state 4 is changed and thus also the efficiency is less than the original 60%.

Compression ratio from the original efficiency assuming the standard cycle

$$\eta = 1 - CR^{1-k} \quad => \quad CR = (1 - \eta)^{-1/0.4} = 9.882$$

If a polytropic expansion with $n = 1.5$ instead then

$$T_{4mod} = T_3 \, CR^{1-n} = 2400 \times 9.882^{-0.5} = \textbf{763.5 K}$$

This means a heat loss out and thus less work (see diagrams)

$$w = \frac{R}{1-n} (T_{4mod} - T_3) = \frac{0.287}{1-1.5} (763.5 - 2400) = \textbf{939.35 kJ/kg}$$

Comments:

The heat loss is

$$_3q_4 = u_4 - u_3 + {}_3w_4 = C_v(T_4 - T_3) + {}_3w_4$$

$$= 0.717(763.5 - 2400) + 939.35 = -234 \text{ kJ/kg}$$

The original cycle has expansion work as

$$T_4 = T_3 \, CR^{1-k} = 2400 \times 9.882^{-0.4} = 960 \text{ K}$$

$$_3w_4 = C_v(T_3 - T_4) = 0.717 \, (2400 - 960) = 1032.5 \text{ kJ/kg}$$

10.71

Air flows into a gasoline engine at 95 kPa, 300 K. The air is then compressed with a volumetric compression ratio of 8:1. In the combustion process 1300 kJ/kg of energy is released as the fuel burns. Find the temperature and pressure after combustion using cold air properties.

Solution:

Compression. Isentropic so we use Eqs.6.24-6.25

$$P_2 = P_1(v_1/v_2)^k = 95 \text{ kPa } (8)^{1.4} = 1746 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 300 \text{ K } (8)^{0.4} = 689.2 \text{ K}$$

Combustion 2 to 3:

$$T_3 = T_2 + q_H/C_v = 689.2 + 1300/0.717 = 2502.3 \text{ kJ/kg}$$

$$T_3 = \textbf{2502 K}$$

$$P_3 = P_2 \times (T_3/T_2) = 1746 \times \frac{2502}{689.2} = \textbf{6338 kPa}$$

10.72

A gasoline engine has a volumetric compression ratio of 8 and before compression has air at 280 K, 85 kPa. The combustion generates a peak pressure of 5500 kPa. Find the peak temperature, the energy added by the combustion process and the exhaust temperature.

Solution:

Solve the problem with cold air properties.

Compression. Isentropic so we use Eqs.6.24-6.25

$$P_2 = P_1(v_1/v_2)^k = 85 \text{ kPa } (8)^{1.4} = 1562 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 280 \text{ K } (8)^{0.4} = 643.3 \text{ K}$$

Combustion. Constant volume

$$T_3 = T_2 (P_3/P_2) = 643.3 \text{ K} \times 5500/1562 = \textbf{2265 K}$$

$$q_H = u_3 - u_2 \approx C_v(T_3 - T_2)$$

$$= 0.717 \text{ kJ/kg-K } (2265 - 643.3) \text{ K} = \textbf{1163 kJ/kg}$$

Exhaust. Isentropic expansion so from Eq.6.24

$$T_4 = T_3/8^{0.4} = 2265 \text{ K } /2.2974 = \textbf{986 K}$$

10.73

To approximate an actual spark-ignition engine consider an air-standard Otto cycle that has a heat addition of 1800 kJ/kg of air, a compression ratio of 7, and a pressure and temperature at the beginning of the compression process of 90 kPa, 10°C. Assuming constant specific heat, with the value from Table A.5, determine the maximum pressure and temperature of the cycle, the thermal efficiency of the cycle and the mean effective pressure.

Solution:

Compression: Reversible and adiabatic so constant s from Eq.6.24-25

$$P_2 = P_1(v_1/v_2)^k = 90 \text{ kPa } (7)^{1.4} = 1372 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 283.2 \text{ K} \times (7)^{0.4} = 616.6 \text{ K}$$

Combustion: constant volume

$$T_3 = T_2 + q_H/C_{V0} = 616.6 + 1800/0.717 = \textbf{3127 K}$$

$$P_3 = P_2T_3/T_2 = 1372 \text{ kPa} \times 3127 / 616.6 = \textbf{6958 kPa}$$

Efficiency and net work

$$\eta_{TH} = 1 - T_1/T_2 = 1 - 283.2/616.5 = \textbf{0.541}$$

$$w_{net} = \eta_{TH} \times q_H = 0.541 \times 1800 = 973.8 \text{ kJ/kg}$$

Displacement and P_{meff}

$$v_1 = RT_1/P_1 = (0.287 \text{ kJ/kg-K} \times 283.2 \text{ K})/90 \text{ kPa} = 0.9029 \text{ m}^3/\text{kg}$$

$$v_2 = (1/7) \, v_1 = 0.1290 \text{ m}^3/\text{kg}$$

$$P_{meff} = \frac{w_{NET}}{v_1 - v_2} = \frac{973.8}{0.9029 - 0.129} = \textbf{1258 kPa}$$

Borgnakke and Sonntag

10.74

A 3.3 L minivan engine runs at 2000 RPM with a compression ratio of 10:1. The intake is at 50 kPa, 280 K and after expansion it is at 750 K. Find the highest T in the cycle, the specific heat transfer added by combustion and the mean effective pressure.

Compression: Reversible and adiabatic so constant s from Eq.6.24-25

$$T_2 = T_1(v_1/v_2)^{k-1} = 280 \times (10)^{0.4} = 703.3 \text{ K}$$

Expansion: isentropic, T_3 is the highest T and T_4 was given

$$T_3 = T_4 \, CR^{k-1} = 750 \text{ K} \times 10^{0.4} = \textbf{1883.9 K}$$

$$q_H = u_3 - u_2 = C_{V0} (T_3 - T_2) = 0.717 (1883.9 - 703.3) = \textbf{846.5 kJ/kg}$$

Efficiency and net work

$$\eta_{TH} = 1 - T_1/T_2 = 1 - 280/703.3 = 0.602$$

$$w_{net} = \eta_{TH} \times q_H = 0.602 \times 846.5 = 509.6 \text{ kJ/kg}$$

Displacement and P_{meff}

$$v_1 = RT_1/P_1 = (0.287 \text{ kJ/kg-K} \times 280 \text{ K})/50 \text{ kPa} = 1.6072 \text{ m}^3/\text{kg}$$

$$v_2 = (1/10) v_1 = 0.16072 \text{ m}^3/\text{kg}$$

$$P_{meff} = \frac{w_{NET}}{v_1 - v_2} = \frac{509.6}{1.6072 - 0.16072} = \textbf{352.3 kPa}$$

10.75

A gasoline engine takes air in at 290 K, 90 kPa and then compresses it. The combustion adds 1000 kJ/kg to the air after which the temperature is 2050 K. Use cold air properties (i.e. constant heat capacities at 300 K) and find the compression ratio, the compression specific work and the highest pressure in the cycle.

Solution:

Standard Otto Cycle

Combustion process: $T_3 = 2050$ K; $u_2 = u_3 - q_H$

$$T_2 = T_3 - q_H / C_{vo} = 2050 - 1000 / 0.717 = 655.3 \text{ K}$$

Compression process

$$P_2 = P_1(T_2 / T_1)^{k/(k-1)} = 90(655.3/290)^{3.5} = 1561 \text{ kPa}$$

$$CR = v_1 / v_2 = (T_2 / T_1)^{1/(k-1)} = (655.3 / 290)^{2.5} = \textbf{7.67}$$

$$-_1w_2 = u_2 - u_1 = C_{vo}(T_2 - T_1) = 0.717 \text{ kJ/kg-K } (655.3 - 290) \text{ K} = \textbf{262 kJ/kg}$$

Highest pressure is after the combustion where $v_3 = v_2$ so we get

$$P_3 = P_2T_3 / T_2 = 1561 \text{ kPa} \times 2050 / 655.3 = \textbf{4883 kPa}$$

10.76

Answer the same three questions for the previous problem, but use variable heat capacities (use table A.7).

A gasoline engine takes air in at 290 K, 90 kPa and then compresses it. The combustion adds 1000 kJ/kg to the air after which the temperature is 2050 K. Use the cold air properties (i.e. constant heat capacities at 300 K) and find the compression ratio, the compression specific work and the highest pressure in the cycle.

Solution:

Standard Otto cycle, solve using Table A.7.1

Combustion process: $T_3 = 2050$ K ; $u_3 = 1725.7$ kJ/kg

$$u_2 = u_3 - q_H = 1725.7 - 1000 = 725.7 \text{ kJ/kg}$$

$$\Rightarrow T_2 = 960.5 \text{ K} ; \qquad s^o_{T2} = 8.0889 \text{ kJ/kg K}$$

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq.6.19

$$0 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = s^o_{T2} - s^o_{T1} - R \ln(T_2 v_1/T_1 v_2)$$

$$= 8.0889 - 6.8352 - 0.287 \ln(960.5/290) - 0.287 \ln(v_1/v_2)$$

Solving $=>$ $v_1 / v_2 =$ **23.78**

Comment: This is much too high for an actual Otto cycle.

$$_{-1}w_2 = u_2 - u_1 = 725.7 - 207.2 = \textbf{518.5 kJ/kg}$$

Highest pressure is after combustion where $v_3 = v_2$ so we get

$$P_3 = P_2 T_3 / T_2 = P_1(T_3 / T_1)(v_1 / v_3)$$

$$= 90 \times (2050 / 290) \times 23.78 = \textbf{15 129 kPa}$$

Borgnakke and Sonntag

10.77

Redo the previous problem but assume the combustion adds 1225 kJ/kg.

Standard Otto cycle, solve using Table A.7.1

Combustion process: $T_3 = 2050$ K ; $u_3 = 1725.7$ kJ/kg

$$u_2 = u_3 - q_H = 1725.7 - 1225 = 500.7 \text{ kJ/kg}$$

$$\Rightarrow T_2 = 684.8 \text{ K} ; \qquad s^o_{T2} = 7.71647 \text{ kJ/kg K}$$

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From Eq.6.19

$$0 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = s^o_{T2} - s^o_{T1} - R \ln(T_2 v_1/T_1 v_2)$$

$$= 7.71647 - 6.83521 - 0.287 \ln(684.8/290) - 0.287 \ln(v_1/v_2)$$

Solving   => $v_1 / v_2 = \textbf{9.13}$

$$-_1 w_2 = u_2 - u_1 = 500.7 - 207.2 = \textbf{293.5 kJ/kg}$$

Highest pressure is after combustion where $v_3 = v_2$ so we get

$$P_3 = P_2 T_3 / T_2 = P_1(T_3 / T_1)(v_1 / v_3)$$

$$= 90 \times (2050 / 290) \times 9.13 = \textbf{5 809 kPa}$$

10.78

A four stroke gasoline engine has a compression ratio of 10:1 with 4 cylinders of total displacement 2.3 L. the inlet state is 280 K, 70 kPa and the engine is running at 2100 RPM with the fuel adding 1400 kJ/kg in the combustion process. What is the net work in the cycle and how much power is produced?

Solution:

Overall cycle efficiency is from Eq.10.12, $r_v = v_1/v_2 = 10$

$$\eta_{TH} = 1 - r_v^{1-k} = 1 - 10^{-0.4} = 0.602$$

$$w_{net} = \eta_{TH} \times q_H = 0.602 \times 1400 = \textbf{842.8 kJ/kg}$$

We also need specific volume to evaluate Eqs.10.9 to 10.11

$$v_1 = RT_1 / P_1 = 0.287 \times 280 / 70 = 1.148 \text{ m}^3/\text{kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{w_{net}}{v_1 \left(1 - \dfrac{1}{r_v}\right)} = \frac{842.8}{1.148 \times 0.9} = 815.7 \text{ kPa}$$

Now we can find the power from Eq.10.11

$$\dot{W} = P_{meff} \, V_{displ} \, \frac{RPM}{60} \, \frac{1}{2} = 815.7 \times 0.0023 \times \frac{2100}{60} \times \frac{1}{2} = \textbf{32.8 kW}$$

10.79

A gasoline engine has a volumetric compression ratio of 10 and before compression has air at 290 K, 85 kPa in the cylinder. The combustion peak pressure is 6000 kPa. Assume cold air properties. What is the highest temperature in the cycle? Find the temperature at the beginning of the exhaust (heat rejection) and the overall cycle efficiency.

Solution:

Compression. Isentropic so we use Eqs.6.24-25

$$P_2 = P_1(v_1/v_2)^k = 85 \text{ kPa } (10)^{1.4} = 2135.1 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 290 \text{ K } (10)^{0.4} = 728.45 \text{ K}$$

Combustion. Constant volume

$$T_3 = T_2 \, (P_3/P_2) = 728.45 \text{ K} \times 6000/2135.1 = \mathbf{2047 \ K}$$

Exhaust is after the isentropic expansion so from Eq.6.24

$$T_4 = T_3 \, / \, (v_1/v_2)^{k-1} = T_3 \, / \, 10^{0.4} = 2047 \, / \, 2.5119 = \mathbf{814.9 \ K}$$

Overall cycle efficiency is from Eq.10.12 $r_v = v_1/v_2$

$$\eta = 1 - r_v^{1-k} = 1 - 10^{-0.4} = \mathbf{0.602}$$

Comment: No actual gasoline engine has an efficiency that high, maybe 35%.

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10.80

Repeat Problem 10.73, but assume variable specific heat. The ideal gas air tables, Table A.7, are recommended for this calculation (or the specific heat from Fig. 3.26 at high temperature).

Solution:

Table A.7: $T_1 = 283.2$ K, $u_1 = 202.3$ kJ/kg, $s^o_{T1} = 6.8113$ kJ/kg K

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq.6.19

$$0 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = s^o_{T2} - s^o_{T1} - R \ln(T_2 v_1/T_1 v_2)$$

$$s^o_{T2} - R \ln(T_2/T_1) = s^o_{T1} + R \ln(v_1/v_2) = 6.8113 + 0.287 \ln 7 = 7.3698$$

This becomes trial and error so estimate first at 600 K and use A.7.1.

$\text{LHS}_{600} = 7.5764 - 0.287 \ln(600/283.2) = 7.3609$ (too low)

$\text{LHS}_{620} = 7.6109 - 0.287 \ln(620/283.2) = 7.3860$ (too high)

Interpolate to get: $T_2 = 607.1$ K, $u_2 = 440.5$ kJ/kg

$\Rightarrow$ $-_1w_2 = u_2 - u_1 = 238.2$ kJ/kg,

$u_3 = 440.5 + 1800 = 2240.5$ $\Rightarrow$ $T_3 = \textbf{2575.8 K}$, $s^o_{T3} = 9.2859$ kJ/kgK

$P_3 = P_1 (v_1/v_3) T_3/T_1 = 90$ kPa $\times 7 \times 2575.8 / 283.2 = \textbf{5730 kPa}$

Expansion 3 to 4: $s_4 = s_3$ $\Rightarrow$ From Eq.6.19 as before

$$s^o_{T4} - R \ln(T_4/T_3) = s^o_{T3} + R \ln(v_3/v_4) = 9.2859 + 0.287 \ln(1/7) = 8.7274$$

This becomes trial and error so estimate first at 1400 K and use A.7.1.

$\text{LHS}_{1400} = 8.5289 - 0.287 \ln(1400/2575.8) = 8.7039$ (too low)

$\text{LHS}_{1450} = 8.5711 - 0.287 \ln(1450/2575.8) = 8.7360$ (too high)

 Interpolation $\Rightarrow$ $T_4 = 1436.6$ K, $u_4 = 1146.9$ kJ/kg

$_3w_4 = u_3 - u_4 = 2240.5 - 1146.9 = 1093.6$ kJ/kg

Net work, efficiency and mep

$$w_{net} = {}_3w_4 + {}_1w_2 = 1093.6 - 238.2 = 855.4 \text{ kJ/kg}$$

$$\eta_{TH} = w_{net} / q_H = 855.4 / 1800 = \textbf{0.475}$$

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 283.2}{90} = 0.9029 \text{ m}^3/\text{kg}; \quad v_2 = \frac{v_1}{CR} = 0.1290 \text{ m}^3/\text{kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = 855.4 / (0.9029 - 0.129) = \textbf{1105 kPa}$$

Comment: Iterations are avoided by the use of the v_r function in A.7.2

10.81

Assume a state of 5000 kPa, 2100 K after combustion in an Otto cycle with a compression ratio of 10:1; the intake temperature is 300 K. Find the missing temperatures in the cycle using Table A.7 and then give the average value of k (ratio of specific heats) for the compression and expansion processes.

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq.6.19

$$0 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = s^o_{T2} - s^o_{T1} - R \ln(T_2 v_1/T_1 v_2)$$

$$s^o_{T2} - R \ln(T_2/T_1) = s^o_{T1} + R \ln(v_1/v_2) = 6.86926 + 0.287 \ln 10 = 7.5301$$

This becomes trial and error so estimate first at 600 K and use A.7.1.

$$LHS_{700} = 7.7401 - 0.287 \ln(700/300) = 7.49692 \text{ (too low)}$$

$$LHS_{740} = 7.80008 - 0.287 \ln(740/300) = 7.54096 \text{ (too high)}$$

Interpolate to get: $T_2 = \textbf{730.1 K}$, $u_2 = 536.5$ kJ/kg

Expansion 3 to 4: $s_4 = s_3$ $\Rightarrow$ From Eq.6.19 as before

$$s^o_{T4} - R \ln(T_4/T_3) = s^o_{T3} + R \ln(v_3/v_4) = 9.02721 - 0.287 \ln 10 = 8.36637$$

This becomes trial and error so estimate first at 1400 K and use A.7.1.

$$LHS_{1100} = 8.24449 - 0.287 \ln(1100/2100) = 8.43007 \text{ (too high)}$$

$$LHS_{1050} = 8.19081 - 0.287 \ln(1050/2100) = 8.389743 \text{ (still too high)}$$

$$LHS_{1000} = 8.13493 - 0.287 \ln(1000/2100) = 8.347866 \text{ (too low)}$$

Interpolation $\Rightarrow$ $T_4 = \textbf{1022.1 K}$, $u_4 = 778.15$ kJ/kg

Compression process

$$C_v = \frac{u_2 - u_1}{T_2 - T_2} = \frac{536.5 - 214.36}{730.1 - 300} = 0.749; \ \ k = (C_v + R)/C_v = \textbf{1.38}$$

Expansion process

$$C_v = \frac{u_3 - u_4}{T_3 - T_4} = \frac{1774.06 - 778.15}{2100 - 1022.1} = 0.9239; \ \ k = (C_v + R)/C_v = \textbf{1.31}$$

10.82

An Otto cycle has the lowest T as 290 K and the lowest P is 150 kPa, the highest T is 2400 K and combustion adds 1200 kJ/kg as heat transfer. Find the compression ratio and the mean effective pressure.

Solution:

Identify states: $T_1 = 290$ K, $P_1 = 150$ kPa, $T_3 = 2400$ K, $q_H = 1200$ kJ/kg

Combustion: $q_H = u_3 - u_2 = C_{vo}(T_3 - T_2) = 1200$ kJ/kg

$$T_2 = T_3 - q_H / C_{vo} = 2400 - 1200/0.717 = 726.36 \text{ K}$$

Compression: $CR = r_v = v_1/v_2 = (T_2/T_1)^{1/(k-1)} = \left(\dfrac{726.36}{290}\right)^{2.5} = \textbf{9.93}$

Overall cycle efficiency is from Eq.10.12, $r_v = v_1/v_2 = 9.93$

$$\eta_{TH} = 1 - r_v^{1-k} = 1 - T_1/T_2 = 1 - 290/726.36 = 0.601$$

$$w_{net} = \eta_{TH} \times q_H = 0.601 \times 1200 = 721.2 \text{ kJ/kg}$$

We also need specific volume to evaluate Eqs.10.9 to 10.11

$$v_1 = RT_1 / P_1 = 0.287 \times 290 / 150 = 0.55487 \text{ m}^3/\text{kg}$$

$$v_2 = v_1/CR = 0.05588 \text{ m}^3/\text{kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{721.2}{0.55487 - 0.05588} \frac{\text{kJ/kg}}{\text{m}^3/\text{kg}} = \textbf{1445 kPa}$$

10.83

The cycle in the previous problem is used in a 2.4 L engine running 1800 RPM. How much power does it produce?

Identify states: $T_1 = 290$ K, $P_1 = 85$ kPa, $T_3 = 2400$ K, $q_H = 1200$ kJ/kg

Combustion: $q_H = u_3 - u_2 = C_{vo}(T_3 - T_2) = 1200$ kJ/kg

$$T_2 = T_3 - q_H / C_{vo} = 2400 - 1200/0.717 = 726.36 \text{ K}$$

Compression: $CR = r_v = v_1/v_2 = (T_2/T_1)^{1/(k-1)} = \left(\dfrac{726.36}{290}\right)^{2.5} = 9.93$

Overall cycle efficiency is from Eq.10.12, $r_v = v_1/v_2 = 9.93$

$$\eta_{TH} = 1 - r_v^{1-k} = 1 - T_1/T_2 = 1 - 290/726.36 = 0.601$$

$$w_{net} = \eta_{TH} \times q_H = 0.601 \times 1200 = 721.2 \text{ kJ/kg}$$

We also need specific volume to evaluate Eqs.10.9 to 10.11

$$v_1 = RT_1 / P_1 = 0.287 \times 290 / 150 = 0.55487 \text{ m}^3/\text{kg}$$

$$v_2 = v_1/CR = 0.05588 \text{ m}^3/\text{kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{721.2}{0.55487 - 0.05588} \frac{\text{kJ/kg}}{\text{m}^3/\text{kg}} = 1445 \text{ kPa}$$

Now we can find the power from Eq.10.11

$$\dot{W} = P_{meff} \, V_{displ} \frac{RPM}{60} \frac{1}{2} = 1445 \times 0.0024 \times \frac{1800}{60} \times \frac{1}{2} = \textbf{52 kW}$$

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10.84

When methanol produced from coal is considered as an alternative fuel to gasoline for automotive engines, it is recognized that the engine can be designed with a higher compression ratio, say 10 instead of 7, but that the energy release with combustion for a stoichiometric mixture with air is slightly smaller, about 1700 kJ/kg. Repeat Problem 10.73 using these values.

Solution:

Compression: Reversible and adiabatic so constant s from Eq.6.24-25

$$P_2 = P_1(v_1/v_2)^k = 90 \text{ kPa } (10)^{1.4} = 2260.7 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 283.15 \text{ K } (10)^{0.4} = 711.2 \text{ K}$$

Combustion: constant volume

$$T_3 = T_2 + q_H / C_{vo} = 711.2 + 1700 / 0.717 = \textbf{3082 K}$$

$$P_3 = P_2(T_3 / T_2) = 2260.7 \text{ kPa } \times 3082 / 711.2 = \textbf{9797 kPa}$$

Efficiency, net work, displacement and P_{meff}

$$\eta_{TH} = 1 - T_1/T_2 = 1 - 283.15/711.2 = \textbf{0.602}$$

$$w_{net} = \eta_{TH} \times q_H = 0.6 \times 1700 = 1023.4 \text{ kJ/kg}$$

$$v_1 = RT_1/P_1 = 0.287 \text{ kJ/kg-K} \times 283.15 \text{ K}/90 \text{ kPa} = 0.9029 \text{ m}^3/\text{kg},$$

$$v_2 = v_1/10 = 0.0903 \text{ m}^3/\text{kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = 1023.4 / (0.9029 - 0.0903) = \textbf{1255 kPa}$$

10.85

It is found experimentally that the power stroke expansion in an internal combustion engine can be approximated with a polytropic process with a value of the polytropic exponent *n* somewhat larger than the specific heat ratio *k*. Repeat Problem 10.73 but assume that the expansion process is reversible and polytropic (instead of the isentropic expansion in the Otto cycle) with *n* equal to 1.50. From the average temperature during expansion estimate the actual average *k*.

See solution to 10.73 except for process 3 to 4.

$T_3 = $ **3127 K**, $P_3 = $ **6.958 MPa**

$v_3 = RT_3/P_3 = v_2 = 0.129 \text{ m}^3/\text{kg}, \quad v_4 = v_1 = 0.9029 \text{ m}^3/\text{kg}$

Process: $Pv^{1.5} = $ constant.

$P_4 = P_3(v_3/v_4)^{1.5} = 6958\ (1/7)^{1.5} = 375.7 \text{ kPa}$

$T_4 = T_3(v_3/v_4)^{0.5} = 3127(1/7)^{0.5} = 1181.9 \text{ K}$

$_1w_2 = \int Pdv = \dfrac{R}{1\text{-}1.4}(T_2 - T_1) = \dfrac{0.287}{\text{-}0.4}(606.6 \text{ -}283.15) = \text{-}239.3 \text{ kJ/kg}$

$_3w_4 = \int Pdv = R(T_4 - T_3)/(1 - 1.5)$

$= \text{-}0.287(1181.9 - 3127)/0.5 = 1116.5 \text{ kJ/kg}$

$w_{NET} = 1116.5 - 239.3 = 877.2 \text{ kJ/kg}$

$\eta_{CYCLE} = w_{NET}/q_H = 877.2/1800 = $ **0.487**

$P_{meff} = \dfrac{w_{net}}{v_1 - v_2} = 877.2/(0.9029 - 0.129) = $ **1133 kPa**

Note a smaller w_{NET}, η_{CYCLE}, P_{meff} compared to an ideal cycle.

The T changes from 3 to 4 (3127 K to 1181.9 K) average is about 2150 K.

$C_{P0} = \dfrac{h_{2200} - h_{2100}}{2200 - 2100} = \dfrac{2502.63 - 2376.82}{2200 - 2100} = 1.258 \text{ kJ/kg-K}$

$k = C_{P0} /(C_{P0} - R) = 1.258/(1.258 - 0.287) = $ **1.296**

10.86

In the Otto cycle all the heat transfer q_H occurs at constant volume. It is more realistic to assume that part of q_H occurs after the piston has started its downward motion in the expansion stroke. Therefore, consider a cycle identical to the Otto cycle, except that the first two-thirds of the total q_H occurs at constant volume and the last one-third occurs at constant pressure. Assume that the total q_H is 2100 kJ/kg, that the state at the beginning of the compression process is 90 kPa, 20°C, and that the compression ratio is 9. Calculate the maximum pressure and temperature and the thermal efficiency of this cycle. Compare the results with those of a conventional Otto cycle having the same given variables.

$P_1 = 90$ kPa, $T_1 = 20^oC$

$r_V = v_1/v_2 = 9$

$q_{23} = (2/3) \times 2100$

$\qquad = 1400$ kJ/kg;

$q_{34} = 2100/3 = 700$ kJ/kg

Constant s compression, Eqs.6.24-25

$$P_2 = P_1(v_1/v_2)^k = 90 \text{ kPa } (9)^{1.4} = 1951 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 293.15 \text{ K } (9)^{0.4} = 706 \text{ K}$$

Constant v combustion

$$T_3 = T_2 + q_{23}/C_{V0} = 706 + 1400/0.717 = \textbf{2660 K}$$

$$P_3 = P_2 T_3/T_2 = 1951 \text{ kPa } (2660/706) = \textbf{7350.8 kPa} = P_4$$

Constant P combustion

$$T_4 = T_3 + q_{34}/C_{P0} = 2660 + 700/1.004 = 3357 \text{ K}$$

Remaining expansion: $\qquad \dfrac{v_5}{v_4} = \dfrac{v_1}{v_4} = \dfrac{P_4}{P_1} \times \dfrac{T_1}{T_4} = \dfrac{7350.8}{90} \times \dfrac{293.15}{3357} = 7.131$

$$T_5 = T_4(v_4/v_5)^{k-1} = 3357 \text{ K } (1/7.131)^{0.4} = 1530 \text{ K}$$

$$q_L = C_{V0}(T_5\text{-}T_1) = 0.717 \text{ kJ/kg-K } (1530 - 293.15) \text{ K} = 886.2 \text{ kJ/kg}$$

$$\eta_{TH} = 1 - q_L/q_H = 1 - 886.2/2100 = \textbf{0.578}$$

Std. Otto Cycle: $\quad \eta_{TH} = 1 - (9)^{-0.4} = \textbf{0.585},$ small difference

10.87

A gasoline engine has a volumetric compression ratio of 9. The state before compression is 290 K, 90 kPa, and the peak cycle temperature is 1800 K. Find the pressure after expansion, the cycle net work and the cycle efficiency using properties from Table A.7.2.

Compression 1 to 2: $s_2 = s_1 \implies v_{r2} = v_{r1}(v_2/v_1)$

$v_{r2} = 196.37/9 = 21.819 \implies T_2 \cong 680\ K, \quad P_{r2} \cong 20.784, \quad u_2 = 496.94$

$P_2 = P_1(P_{r2}/P_{r1}) = 90\ kPa\ (20.784\ /\ 0.995) = 1880\ kPa$

$_1w_2 = u_1 - u_2 = 207.19 - 496.94 = -289.75\ kJ/kg$

Combustion 2 to 3:

$q_H = u_3 - u_2 = 1486.33 - 496.94 = 989.39\ kJ/kg$

$P_3 = P_2(T_3/T_2) = 1880\ kPa\ (1800\ /\ 680) = 4976\ kPa$

Expansion 3 to 4:

$s_4 = s_3 \implies v_{r4} = v_{r3} \times 9 = 1.143 \times 9 = 10.278$

$\implies T_4 = 889\ K, \quad P_{r4} = 57.773, \quad u_4 = 665.8\ kJ/kg$

$P_4 = P_3(P_{r4}/P_{r3}) = 4976\ kPa\ (57.773\ /\ 1051) = \mathbf{273.5\ kPa}$

$_3w_4 = u_3 - u_4 = 1486.33 - 665.8 = 820.5\ kJ/kg$

$w_{NET} = {_3w_4} + {_1w_2} = 820.5 - 289.75 = \mathbf{530.8\ kJ/kg}$

$\eta = w_{NET}/q_H = 530.8/989.39 = \mathbf{0.536}$

10.88

Solve Problem 10.76 using the P_r and v_r functions from Table A7.2

A gasoline engine takes air in at 290 K, 90 kPa and then compresses it. The combustion adds 1000 kJ/kg to the air after which the temperature is 2050 K. Use the cold air properties (i.e. constant heat capacities at 300 K) and find the compression ratio, the compression specific work and the highest pressure in the cycle.

Solution:

Standard Otto cycle, solve using Table A.7.1 and Table A.7.2

Combustion process: $T_3 = 2050$ K ; $u_3 = 1725.7$ kJ/kg

$$u_2 = u_3 - q_H = 1725.7 - 1000 = 725.7 \text{ kJ/kg}$$

$$\Rightarrow T_2 = 960.5 \text{ K} ; \qquad v_{r2} = 8.2166$$

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From the v_r function

$$v_1/v_2 = v_{r1}/v_{r2} = \frac{195.36}{8.2166} = \mathbf{23.78}$$

Comment: This is much too high for an actual Otto cycle.

$$-_1w_2 = u_2 - u_1 = 725.7 - 207.2 = \mathbf{518.5 \text{ kJ/kg}}$$

Highest pressure is after combustion

$$P_3 = P_2 T_3 / T_2 = P_1(T_3 / T_1)(v_1 / v_3)$$

$$= 90 \text{ kPa} \times (2050 / 290) \times 23.78 = \mathbf{15\ 129 \text{ kPa}}$$

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Diesel Cycles

10.89

A diesel engine has an inlet at 95 kPa, 300 K and a compression ratio of 20:1. The combustion releases 1300 kJ/kg. Find the temperature after combustion using cold air properties.

Solution:

Compression process (isentropic) from Eqs.6.24-25

$$T_2 = T_1(v_1 / v_2)^{k-1} = 300 \times 20^{0.4} = 994.3 \text{ K}$$

Combustion at constant P which is the maximum pressure

$$_2w_3 = P (v_3 - v_2)$$

$$_2q_3 = u_3 - u_2 + {_2}w_3 = h_3 - h_2 = C_{po}(T_3 - T_2)$$

$$T_3 = T_2 + {_2}q_3 / C_{po} = 994.3 + 1300 /1.004 = \textbf{2289 K}$$

10.90

A diesel engine has a state before compression of 95 kPa, 290 K, and a peak pressure of 6000 kPa, a maximum temperature of 2400 K. Find the volumetric compression ratio and the thermal efficiency.

Solution:

Standard Diesel cycle and we will use cold air properties.

Compression process (isentropic) from Eqs.6.24-25:

$$(P_2/P_1) = (v_1/v_2)^k = CR^{1.4}$$

$$CR = v_1/v_2 = (P_2/P_1)^{1/k} = (6000/95)^{1/1.4} = \textbf{19.32}$$

$$T_2 = T_1(P_2/P_1)^{k-1/k} = 290 \times (6000/95)^{0.2857} = 947.9 \text{ K}$$

Combustion and expansion volumes

$$v_3 = v_2 \times T_3/T_2 = v_1\, T_3/(T_2 \times CR) ; \quad v_4 = v_1$$

Expansion process, isentropic from Eq.6.23

$$T_4 = T_3\,(v_3/v_4)^{k-1} = T_3\,[T_3/\,(CR \times T_2)]^{k-1}$$

$$= 2400 \text{ K} \times [\,2400/(19.32 \times 947.9)\,]^{0.4} = 1064.6 \text{ K}$$

Efficiency from Eq.10.13

$$\eta = 1 - \frac{1}{k}\frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{1.4}\frac{1064.6 - 290}{2400 - 947.9} = \textbf{0.619}$$

10.91

Find the cycle efficiency and mean effective pressure for the cycle in Problem 10.89

Solution:

Compression process (isentropic) from Eqs.6.24-25

$$T_2 = T_1(v_1 / v_2)^{k-1} = 300 \times 20^{0.4} = 994.3 \text{ K}$$

Combustion at constant P which is the maximum pressure

$$_2w_3 = P (v_3 - v_2)$$
$$_2q_3 = u_3 - u_2 + {}_2w_3 = h_3 - h_2 = C_{po}(T_3 - T_2)$$
$$T_3 = T_2 + {}_2q_3 / C_{po} = 994.3 + 1300 /1.004 = 2289 \text{ K}$$

Combustion and expansion volumes

$$v_3 = v_2 \times T_3/T_2 = v_1 T_3/(T_2 \times CR) ; \quad v_4 = v_1$$

Expansion process, isentropic from Eq.6.23

$$T_4 = T_3 (v_3/v_4)^{k-1} = T_3 [T_3/ (CR \times T_2)]^{k-1}$$
$$= 2289 \times [2289/(20 \times 994.3)]^{0.4} = 964 \text{ K}$$

Efficiency from Eq.10.13

$$\eta = 1 - \frac{1}{k} \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{1.4} \frac{964 - 300}{2289 - 994.3} = \mathbf{0.634}$$

$$w_{net} = \eta q_H = 0.634 \times 1300 = 824.2 \text{ kJ/kg}$$
$$v_1 = RT_1/P_1 = 0.9063 \text{ m}^3/\text{kg}$$

$$P_{meff} = \frac{w_{net}}{v_{max} - v_{min}} = \frac{w_{net}}{v_1 - v_1/CR}$$
$$= \frac{824.2}{0.9063 (1 - 1/20)} \frac{\text{kJ/kg}}{\text{m}^3/\text{kg}} = \mathbf{957 \text{ kPa}}$$

10.92

The **cut off ratio** is the ratio of v_3/v_2, see Fig. 10.18, which is the expansion while combustion occurs at constant pressure. Determine this ratio for the cycle in Problem 10.89.

Solution:

Compression process (isentropic) from Eqs.6.24-25

$$T_2 = T_1(v_1 / v_2)^{k-1} = 300 \times 20^{0.4} = 994.3 \text{ K}$$

Combustion at constant $P_2 = P_3$ which is the maximum pressure

$$_2w_3 = P (v_3 - v_2)$$

$$_2q_3 = u_3 - u_2 + {}_2w_3 = h_3 - h_2 = C_{po}(T_3 - T_2)$$

$$T_3 = T_2 + {}_2q_3 / C_{po} = 994.3 + 1300 /1.004 = 2289 \text{ K}$$

Cut-off ratio: $v_3 / v_2 = T_3 /T_2 = 2289 / 994.3 = \mathbf{2.302}$

10.93

A diesel engine has a compression ratio of 20:1 with an inlet of 95 kPa, 290 K, state 1, with volume 0.5 L. The maximum cycle temperature is 1800 K. Find the maximum pressure, the net specific work, the cut off ratio, see problem 10.92, and the thermal efficiency.

Solution:

Compression process (isentropic) from Eqs. 6.24-25

$$T_2 = T_1(v_1 / v_2)^{k-1} = 290 \times 20^{0.4} = 961 \text{ K}$$

$$P_2 = 95 \times (20)^{1.4} = 6297.5 \text{ kPa} ;$$

$$v_2 = v_1/20 = RT_1/(20 \ P_1) = 0.043805 \text{ m}^3/\text{kg}$$

$$_{-1}w_2 = u_2 - u_1 \approx C_{vo}(T_2 - T_1) = 0.717 \ (961 - 290) = 481.1 \text{ kJ/kg}$$

Combustion at constant P which is the maximum pressure

$$P_3 = P_2 = \textbf{6298 kPa} ;$$

$$v_3 = v_2 \ T_3 /T_2 = 0.043805 \times 1800/961 = 0.08205 \text{ m}^3/\text{kg}$$

Cut-off ratio: $v_3 / v_2 = T_3 /T_2 = 1800 / 961 = \textbf{1.873}$

$$_2w_3 = P \ (v_3 - v_2) = 6298 \times (0.08215 - 0.043805) = 241.5 \text{ kJ/kg}$$

$$_2q_3 = u_3 - u_2 + {_2}w_3 = h_3 - h_2 = C_{po}(T_3 - T_2)$$

$$= 1.004(1800 - 961) = 842.4 \text{ kJ/kg}$$

Expansion process (isentropic) from Eq.6.24

$$T_4 = T_3(v_3 / v_4)^{0.4} = 1800 \ (0.08205 / 0.8761)^{0.4} = 698 \text{ K}$$

$$_3w_4 = u_3 - u_4 \approx C_{vo}(T_3 - T_4) = 0.717 \ (1800 - 698) = 790.1 \text{ kJ/kg}$$

Cycle net work and efficiency

$$w_{net} = {_2}w_3 + {_3}w_4 + {_1}w_2 = 241.5 + 790.1 - 481.1 = \textbf{550.5 kJ/kg}$$

$$\eta = w_{net} / q_H = 550.5/ 842.4 = \textbf{0.653}$$

Borgnakke and Sonntag

10.94

A diesel engine has a bore of 0.1 m, a stroke of 0.11 m and a compression ratio of 19:1 running at 2000 RPM (revolutions per minute). Each cycle takes two revolutions and has a mean effective pressure of 1400 kPa. With a total of 6 cylinders find the engine power in kW and horsepower, hp.

Solution:

Work from mean effective pressure, Eq.10.9.

$$P_{meff} = \frac{w_{net}}{v_{max} - v_{min}} \quad => \quad w_{net} = P_{meff}\,(v_{max} - v_{min})$$

The displacement is

$$\Delta V = \pi Bore^2 \times 0.25 \times S = \pi \times 0.1^2 \times 0.25 \times 0.11 = 0.000864 \text{ m}^3$$

Work per cylinder per power stroke, Eq.10.10

$$W = P_{meff}(V_{max} - V_{min}) = 1400 \times 0.000864 \text{ kPa m}^3 = 1.2096 \text{ kJ/cycle}$$

Only every second revolution has a power stroke so we can find the power, see also Eq.10.11

$$\dot{W} = W \times N_{cyl} \times RPM \times 0.5 \text{ (cycles / min)} \times \text{(min / 60 s)} \times \text{(kJ / cycle)}$$

$$= 1.2096 \times 6 \times 2000 \times 0.5 \times (1/60) = \textbf{121 kW} = \textbf{162 hp}$$

The conversion factor from kW to hp is from Table A.1 under power.

10.95

A super charger is used for a two stroke 10 L diesel engine so intake is 200 kPa, 320 K and the cycle has compression ratio of 18:1 and mean effective pressure of 830 kPa. The engine is 10 L running at 200 RPM find the power output.

Solution:

The power is from Eq.10.11

$$\dot{W} = m\, w_{net} \times N_{cyl} \times RPM \times 1/60$$
$$= m\, P_{meff}\,(v_1 - v_2) \times N_{cyl} \times RPM/60 = P_{meff}\, V_{displ}\, RPM/60$$
$$= 830\ kPa \times 0.010\ m^3 \times 200\ (rev/min)\ /\ 60\ (s/min) = \mathbf{27.7\ kW}$$

10.96

A diesel engine has a state before compression of 95 kPa, 290 K, and a peak pressure of 6000 kPa, a maximum temperature of 2400 K. Use the air tables to find the volumetric compression ratio and the thermal efficiency.

Solution:

Compression: $s_2 = s_1 \Rightarrow$ from Eq.6.19

$$s_{T2}^o = s_{T1}^o + R \ln(P_2 / P_1) = 6.8352 + 0.287 \ln(6000/95) = 8.025 \text{ kJ/kg K}$$

A.7.1 $\Rightarrow T_2 = 907.6 \text{ K}; \ h_2 = 941.16 \text{ kJ/kg};$

$$h_3 = 2755.8 \text{ kJ/kg}; \ s_{T3}^o = 9.19586 \text{ kJ/kg K}$$

$$q_H = h_3 - h_2 = 2755.8 - 941.16 = 1814.2 \text{ kJ/kg}$$

$$CR = v_1/v_2 = (T_1/T_2)(P_2/P_1) = (290/907.6) \times (6000/ 95) = \mathbf{20.18}$$

Expansion process

$$s_{T4}^o = s_{T3}^o + R \ln(P_4 / P_3) = s_{T3}^o + R \ln(T_4 / T_3) + R \ln(v_3/v_4)$$

$$v_3/v_4 = v_3/v_1 = (v_2/v_1) \times (T_3/T_2) = (T_3/T_2) (1/CR)$$

$$= (2400/907.6) (1/20.18) = 0.13104$$

$$s_{T4}^o - R \ln(T_4 / T_3) = s_{T3}^o + R \ln(v_3/v_4) = 9.1958 + 0.287 \ln 0.13104 = 8.61254$$

Trial and error on T_4 since it appears both in s_{T4}^o and the ln function

$T_4 = 1300 \text{ K}$ LHS $= 8.4405 - 0.287 \ln (1300/2400) = 8.616$

$T_4 = 1250 \text{ K}$ LHS $= 8.3940 - 0.287 \ln (1250/2400) = 8.5812$

Now Linear interpolation $T_4 = 1295 \text{ K}, \quad u_4 = 1018.26 \text{ kJ/kg}$

$$q_L = u_4 - u_1 = 1018.26 - 207.19 = 811.08 \text{ kJ/kg}$$

$$\eta = 1 - (q_L/ q_H) = 1 - (811.08/1814.2) = \mathbf{0.553}$$

10.97

At the beginning of compression in a diesel cycle $T = 300$ K, $P = 200$ kPa and after combustion (heat addition) is complete $T = 1500$ K and $P = 7.0$ MPa. Find the compression ratio, the thermal efficiency and the mean effective pressure.

Solution:

Standard Diesel cycle. See P-v and T-s diagrams for state numbers.

Compression process (isentropic) from Eqs.6.24-8.25

$$P_2 = P_3 = 7000 \text{ kPa} \implies v_1 / v_2 = (P_2/P_1)^{1/k} = (7000 / 200)^{0.7143} = \mathbf{12.67}$$

$$T_2 = T_1(P_2 / P_1)^{(k-1)/k} = 300 \text{ K } (7000 / 200)^{0.2857} = 828.4 \text{ K}$$

Expansion process (isentropic) first get the volume ratios

$$v_3 / v_2 = T_3 / T_2 = 1500 / 828.4 = 1.81$$

$$v_4 / v_3 = v_1 / v_3 = (v_1 / v_2)(v_2 / v_3) = 12.67 / 1.81 = 7$$

The exhaust temperature follows from Eq.6.24

$$T_4 = T_3(v_3 / v_4)^{k-1} = 1500 \text{ K } (1 / 7)^{0.4} = 688.7 \text{ K}$$

$$q_L = C_{vo}(T_4 - T_1) = 0.717 \text{ kJ/kg-K } (688.7 - 300) \text{ K} = 278.5 \text{ kJ/kg}$$

$$q_H = h_3 - h_2 \approx C_{po}(T_3 - T_2) = 1.004 \text{ kJ/kg-K } (1500 - 828.4) \text{ K} = 674 \text{ kJ/kg}$$

Overall performance

$$\eta = 1 - q_L / q_H = 1 - 278.5 / 674 = \mathbf{0.587}$$

$$w_{net} = q_{net} = q_H - q_L = 674 - 278.5 = 395.5 \text{ kJ/kg}$$

$$v_{max} = v_1 = R \, T_1 / P_1 = 0.287 \text{ kJ/kg-K} \times 300 \text{ K/ } 200 \text{ kPa} = 0.4305 \text{ m}^3/\text{kg}$$

$$v_{min} = v_{max} / (v_1 / v_2) = 0.4305 / 12.67 = 0.034 \text{ m}^3/\text{kg}$$

$$P_{meff} = \frac{w_{net}}{v_{max} - v_{min}} = 395.5 / (0.4305 - 0.034) = \mathbf{997 \text{ kPa}}$$

Remark: This is a too low compression ratio for a practical diesel cycle.

Borgnakke and Sonntag

10.98

The worlds largest diesel engine has displacement of 25 m^3 running at 200 RPM in a two stroke cycle producing 100 000 hp. Assume an inlet state of 200 kPa, 300 K and a compression ratio of 20:1. What is the mean effective pressure and the flow of air to the engine?

We have 3 parameters for the cycle: T_1, P_1 and CR we need one more, so this comes from the total rate of work.

$$\dot{W} = m\ w_{net} \times N_{cyl} \times RPM \times 1/60$$
$$= m\ P_{meff}\ (v_1 - v_2) \times N_{cyl} \times RPM/60 = P_{meff}\ V_{displ}\ RPM/60$$

$$P_{meff} = \dot{W}\ 60\ /\ V_{displ}\ RPM = \frac{100\ 000 \times 0.746\ kW \times 60\ s}{25\ m^3 \times 200} = \textbf{895 kPa}$$

With this information we could now get all the cycle states etc.
Assume the displacement volume is filled with air at state 1, then

$$m = \frac{P_1 V_{displ}}{RT_1} = \frac{200\ kPa \times 25\ m^3}{0.287\ kJ/kg\text{-}K \times 300\ K} = 58.07\ kg$$

This is repeated 200 times per minute so the rate is

$$\dot{m} = m \times RPM \times 1/60 = 58.07\ 200\ /\ 60 = \textbf{193.57 kg}/s$$

A similar engine is in this container ship shown at dock. The engine was build up inside the ship as it is too large to put in finished. The cranes are on the port dock.

10.99

A diesel engine has air before compression at 280 K, 85 kPa. The highest temperature is 2200 K and the highest pressure is 6 MPa. Find the volumetric compression ratio and the mean effective pressure using cold air properties at 300 K.

Solution:

Compression $(P_2/P_1) = (v_1/v_2)^k = CR^k$

$$CR = v_1/v_2 = (P_2/P_1)^{1/k} = (6000/85)^{1/1.4} = \mathbf{20.92}$$

$$T_2 = T_1(P_2/P_1)^{k-1/k} = 280 \times (6000/85)^{0.2857} = 944.8 \text{ K}$$

Combustion. Highest temperature is after combustion.

$$h_2 = C_P(T_3 - T_2) = 1.004 \text{ kJ/kg-K} (2200 - 944.8) \text{ K} = 1260.2 \text{ kJ/kg}$$

Expansion ($v_4 = CR \times v_2$ and $P_2 = P_3$)

$$T_4 = T_3 (v_3/v_4)^{k-1} = T_3 [v_3 / (CR \times v_2)]^{k-1} = T_3 [T_3/ (CR \times T_2)]^{k-1}$$

$$= 2200 \text{ K} \times (2200/20.92 \times 944.8)^{0.4} = 914.2 \text{ K}$$

$$q_L = u_4 - u_1 = C_V(T_4 - T_1) = 0.717 \text{ kJ/kg-K} (914.2 - 280) \text{ K} = 454.7 \text{ kJ/kg}$$

$$v_1 = RT_1/P_1 = 0.287 \text{ kJ/kg-K} \times 280 \text{ K}/85 \text{ kPa} = 0.9454 \text{ m}^3/\text{kg}$$

Displacement and mep from net work

$$v_1 - v_2 = v_1 - v_1/CR = v_1[1 - (1/CR)] = 0.9002 \text{ m}^3/\text{kg}$$

$$P_{meff} = w_{net}/(v_1 - v_2) = (q_H - q_L)/(v_1 - v_2)$$

$$= (1260.2 - 454.7)/0.9002 = \mathbf{894.8 \text{ kPa}}$$

10.100

Consider an ideal air-standard diesel cycle in which the state before the compression process is 95 kPa, 290 K, and the compression ratio is 20. Find the thermal efficiency for a maximum temperature of 2000 K

Solution:

Diesel cycle: P_1 = 95 kPa, T_1 = 290 K,   CR = v_1/v_2 = 20,

$$v_1 = 0.287 \text{ kJ/kg-K} \times 290 \text{ K}/95 \text{ kPa} = 0.8761 \text{ m}^3/\text{kg} = v_4 = \text{CR } v_2,$$

Compression process (isentropic) from Eqs.6.24-25

$$T_2 = T_1(v_1 / v_2)^{k-1} = 290 \text{ K} \times 20^{0.4} = 961.2 \text{ K}$$

Combustion at constant P which is the maximum pressure

$$v_3 = v_2 \, T_3 /T_2 = \frac{0.8761}{20} \text{ m}^3/\text{kg} \times \frac{2000}{961.2} = 0.09115 \text{ m}^3/\text{kg}$$

Expansion process (isentropic) from Eq.6.24

$$T_4 = T_3(v_3 / v_4)^{0.4} = 2000 \text{ K } (0.09115 / 0.8761)^{0.4} = 808.9 \text{ K}$$

Cycle net work and efficiency

$$\eta_{TH} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{808.9 - 290}{1.4 \, (2000 - 961.2)} = \mathbf{0.643}$$

10.101

Solve Problem 10.90 using the P_r and v_r functions from Table A7.2

A diesel engine has a state before compression of 95 kPa, 290 K, and a peak pressure of 6000 kPa, a maximum temperature of 2400 K. Find the volumetric compression ratio and the thermal efficiency.

Solution:

Compression: $s_2 = s_1$ => From definition of the P_r function

$$P_{r2} = P_{r1} (P_2/P_1) = 0.9899 (6000/95) = 62.52$$

A.7.1 => $T_2 = 907$ K; $h_2 = 941.0$ kJ/kg;

$$h_3 = 2755.8 \text{ kJ/kg}; \quad v_{r3} = 0.43338$$

$q_H = h_3 - h_2 = 2755.8 - 941.0 = 1814.8$ kJ/kg

$CR = v_1/v_2 = (T_1/T_2)(P_2/P_1) = (290/907) \times (6000/ 95) = \mathbf{20.19}$

Expansion process

$$v_{r4} = v_{r3} (v_4 / v_3) = v_{r3} (v_1 / v_3) = v_{r3} (v_1 / v_2) \times (T_2/T_3)$$

$$= v_{r3} \, CR \times (T_2/T_3) = 0.43338 \times 20.19 \times (907/2400) = 3.30675$$

Linear interpolation $T_4 = 1294.8$ K, $u_4 = 1018.1$ kJ/kg

$q_L = u_4 - u_1 = 1018.1 - 207.2 = 810.9$ kJ/kg

$\eta = 1 - (q_L / q_H) = 1 - (810.9/1814.8) = \mathbf{0.553}$

Stirling and Carnot Cycles

10.102

Consider an ideal Stirling-cycle engine in which the state at the beginning of the isothermal compression process is 100 kPa, 25°C, the compression ratio is 6, and the maximum temperature in the cycle is 1100°C. Calculate the maximum cycle pressure and the thermal efficiency of the cycle with and without regenerators.

Ideal Stirling cycle
$T_1 = T_2 = 25 \ {}^{\circ}C$
$P_1 = 100$ kPa
$CR = v_1/v_2 = 6$
$T_3 = T_4 = 1100 \ {}^{\circ}C$

Isothermal compression (heat goes out)

$$T_1 = T_2 \Rightarrow P_2 = P_1(v_1/v_2) = 100 \text{ kPa} \times 6 = 600 \text{ kPa}$$

$$_1w_2 = {}_1q_2 = -RT_1 \ln(v_1/v_2) = -0.287 \times 298.2 \ln(6) = -153.3 \text{ kJ/kg}$$

Constant volume heat addition

$$V_2 = V_3 \Rightarrow P_3 = P_2 T_3/T_2 = 600 \times 1373.2/298.2 = \textbf{2763 kPa}$$

$$q_{23} = u_3 - u_2 = C_{v\,o}(T_3 - T_2) = 0.717 \ (1100 - 25) = 770.8 \text{ kJ/kg}$$

Isothermal expansion (heat comes in)

$$w_{34} = q_{34} = RT_3 \ln(v_4/v_3) = 0.287 \times 1373.2 \times \ln 6 = 706.1 \text{ kJ/kg}$$

$$w_{net} = 706.1 - 153.3 = 552.8 \text{ kJ/kg}$$

Efficiency without regenerator, (q_{23} and q_{34} are coming in from source)

$$\eta_{\text{NO REGEN}} = \frac{w_{net}}{q_{23} + q_{34}} = \frac{552.8}{770.8 + 706.1} = \textbf{0.374,}$$

Efficiency with regenerator, (Now only q_{34} is coming in from source)

$$\eta_{\text{WITH REGEN}} = \frac{w_{net}}{q_{34}} = \frac{552.8}{706.1} = \textbf{0.783}$$

10.103

An air-standard Stirling cycle uses helium as the working fluid. The isothermal compression brings helium from 100 kPa, 37°C to 600 kPa. The expansion takes place at 1200 K and there is no regenerator. Find the work and heat transfer in all of the 4 processes per kg helium and the thermal cycle efficiency.

Helium table A.5: $R = 2.077$ kJ/kg K, $C_{vo} = 3.1156$ kJ/kg K

Compression/expansion: $v_4 / v_3 = v_1 / v_2 = P_2 / P_1 = 600 / 100 = 6$

$1 \rightarrow 2$ $_{-1}w_2 = -q_{12} = \int P\ dv = R\ T_1 \ln(v_1 / v_2) = RT_1 \ln(P_2 / P_1)$

$\qquad\qquad = 2.077$ kJ/kg-K $\times 310$ K $\times \ln 6 =$ **1153.7 kJ/kg**

$2 \rightarrow 3$: $\quad _2w_3 = \mathbf{0}$; $\quad q_{23} = C_{vo}(T_3 - T_2) = 3.1156(1200 - 310) =$ **2773 kJ/kg**

$3 \rightarrow 4$: $\quad _3w_4 = q_{34} = R\ T_3 \ln\dfrac{v_4}{v_3} = 2.077 \times 1200\ \ln 6 =$ **4465.8 kJ/kg**

$4 \rightarrow 1$ $\quad _4w_1 = \mathbf{0}$; $\quad q_{41} = C_{vo}(T_4 - T_1) =$ **-2773 kJ/kg**

$$\eta_{cycle} = \frac{_1w_2 + {_3}w_4}{q_{23} + q_{34}} = \frac{-1153.7 + 4465.8}{2773\ + 4465.8} = \mathbf{0.458}$$

10.104

Consider an ideal air-standard Stirling cycle with an ideal regenerator. The minimum pressure and temperature in the cycle are 100 kPa, 25°C, the compression ratio is 10, and the maximum temperature in the cycle is 1000°C. Analyze each of the four processes in this cycle for work and heat transfer, and determine the overall performance of the engine.

Ideal Stirling cycle diagram as in Fig. 10.19, with

$$P_1 = 100 \text{ kPa}, \quad T_1 = T_2 = 25^\circ\text{C}, \quad v_1/v_2 = 10, \quad T_3 = T_4 = 1000^\circ\text{C}$$

From 1-2 at const T: $\quad _1w_2 = {_1}q_2 = T_1(s_2 - s_1)$

$$= -RT_1\ln(v_1/v_2) = -0.287 \times 298.2 \times \ln(10) = -197.1 \text{ kJ/kg}$$

From 2-3 at const V: $\quad _2w_3 = \cancel{0}$

$$q_{23} = C_{V0}(T_3 - T_2) = 0.717\ (1000 - 25) = \textbf{699 kJ/kg}$$

From 3-4 at const T; $\quad _3w_4 = {_3}q_4 = T_3(s_4 - s_3)$

$$= +RT_3 \times \ln\frac{v_4}{v_3} = 0.287 \times 1237.2 \times \ln(10) = \textbf{841.4 kJ/kg}$$

From 4-1 at const V; $\quad _4w_1 = \cancel{0}$

$$q_{41} = C_{V0}(T_1 - T_4) = 0.717\ (25 - 1000) = \textbf{-699 kJ/kg}$$

$$w_{NET} = -197.1 + 0 + 841.4 + 0 = 644.3 \text{ kJ/kg}$$

Since q_{23} is supplied by $-q_{41}$ (regenerator)

$$q_H = q_{34} = 841.4 \text{ kJ/kg}, \quad \eta_{TH} = \frac{w_{NET}}{q_H} = \frac{644.3}{841.4} = \textbf{0.766}$$

NOTE: $\quad q_H = q_{34} = RT_3 \times \ln(10), \quad q_L = {_{-1}}q_2 = RT_1 \times \ln(10)$

$$\eta_{TH} = \frac{q_H - q_L}{q_H} \text{ \textbf{Error! Bookmark not defined.}} = \frac{T_3 - T_1}{T_3} = \frac{975}{1273.2} = 0.766$$

= Carnot efficiency

10.105

The air-standard Carnot cycle was not shown in the text; show the *T–s* diagram for this cycle. In an air-standard Carnot cycle the low temperature is 320 K and the efficiency is 50%. If the pressure before compression and after heat rejection is 100 kPa, find the high temperature and the pressure just before heat addition.

Solution:

Carnot cycle efficiency from Eq.7.5

$$\eta = 0.5 = 1 - T_H/T_L$$

$$\Rightarrow T_H = T_L/0.5 = \mathbf{640\ K}$$

Just before heat addition is state 2 and after heat rejection is state 1 so $P_1 = 100$ kPa and the isentropic compression is from Eq.6.23

$$P_2 = P_1\ (T_H\ /\ T_L)^{\frac{k}{k-1}} = \mathbf{1131\ kPa}$$

10.106

Air in a piston/cylinder goes through a Carnot cycle in which $T_L = 26.8°C$ and the total cycle efficiency is $\eta = 2/3$. Find T_H, the specific work and volume ratio in the adiabatic expansion for constant C_P, C_v.

Solution:

Carnot cycle efficiency Eq.5.5:

$$\eta = 1 - T_L/T_H = 2/3 \implies T_H = 3 \times T_L = 3 \times 300 = \textbf{900 K}$$

Adiabatic expansion 3 to 4: Pv^k = constant, work from Eq.6.29 (n = k)

$$_3w_4 = (P_4v_4 - P_3v_3)/(1 - k) = \frac{R}{1 - k}(T_4 - T_3) = u_3 - u_4$$

$$= C_v(T_3 - T_4) = 0.717 \text{ kJ/kg-K } (900 - 300) \text{ K} = \textbf{429.9 kJ/kg}$$

$$v_4/v_3 = (T_3/T_4)^{1/(k - 1)} = 3^{2.5} = \textbf{15.6}$$

10.107

Do the problem 10.105 using values from Table A.7.1.

The air-standard Carnot cycle was not shown in the text; show the *T–s* diagram for this cycle. In an air-standard Carnot cycle the low temperature is 320 K and the efficiency is 50%. If the pressure before compression and after heat rejection is 100 kPa, find the high temperature and the pressure just before heat addition.

Solution:

Carnot cycle efficiency Eq.7.5:

$$\eta = 1 - T_L/T_H = 1/2 \implies T_H = 2 \times T_L = 2 \times 320 = \mathbf{640\ K}$$

From A.7.1: $s^{\circ}_{T1} = 6.93413$ kJ/kg K, $s^{\circ}_{T2} = 7.64448$ kJ/kg K

From the entropy $s_1 = s_2$ and Eq. 6.19 we get

$$P_2 = P_1 \exp[\ (s^{\circ}_{T2} - s^{\circ}_{T1})/R] = 100 \text{ kPa } \exp[\ (7.64448 - 6.93413)/0.287]$$

$$= 1188 \text{ kPa}$$

10.108

Do Problem 10.105 using the P_r, v_r functions in Table A.7.2

The air-standard Carnot cycle was not shown in the text; show the *T–s* diagram for this cycle. In an air-standard Carnot cycle the low temperature is 320 K and the efficiency is 50%. If the pressure before compression and after heat rejection is 100 kPa, find the high temperature and the pressure just before heat addition.

Solution:

Carnot cycle efficiency Eq.7.5:

$$\eta = 1 - T_L/T_H = 1/2 \ \Rightarrow \ T_H = 2 \times T_L = 2 \times 320 = 640 \text{ K}$$

From A.7.2: $P_{r1} = 1.3972$, $P_{r2} = 16.598$

From the entropy $s_1 = s_2$ and equation below Table A.7.2 we get

$$P_2 = P_1 \, P_{r2} / P_{r1} = 100 \text{ kPa} \times 16.598 / 1.3972 = \mathbf{1188 \text{ kPa}}$$

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Atkinson and Miller cycles

10.109

An Atkinson cycle has state 1 as 150 kPa, 300 K, compression ratio of 9 and a heat release of 1000 kJ/kg. Find the needed expansion ratio.

Solution:

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq.6.24 and Eq.6.25

$$T_2 = T_1 \, (v_1/v_2)^{k-1} = 300 \text{ K} \times 9^{\,0.4} = 722.5 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^{k} = 150 \text{ kPa} \times 9^{\,1.4} = 3251 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v = 722.5 \text{ K} + 1000 \text{ kJ/kg} /0.717 \text{ kJ/kg-K} = 2117 \text{ K}$$

$$P_3 = P_2 \times (T_3/T_2) = 3251 \text{ kPa} \, (2117 / 722.5) = 9526 \text{ kPa}$$

The expansion should now bring pressure down so $P_4 = P_1$

$$P_4 = P_1 = P_3 \times (v_3/v_4)^{k} = P_3 \times CR^{-k} \;\Rightarrow\; CR = (P_3 /P_4)^{\,1/k}$$

$$CR = (9526 / 150)^{0.71429} = \textbf{19.4}$$

10.110

An Atkinson cycle has state 1 as 150 kPa, 300 K, compression ratio of 9 and expansion ratio of 14. Find the needed heat release in the combustion.

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1 \implies$ From Eq.6.24 and Eq.6.25

$$T_2 = T_1 \, (v_1/v_2)^{k-1} = 300 \text{ K} \times 9^{\,0.4} = 722.5 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 150 \text{ kPa} \times 9^{\,1.4} = 3251 \text{ kPa}$$

The expansion backwards from $P_4 = P_1$ gives

$$P_3 = P_4 \times (v_4/v_3)^k = P_4 \times CR^k = 150 \text{ kPa} \times 14^{1.4} = 6034.9 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $v_3 = v_2 \implies P_3 = P_2 \times (T_3/T_2)$

Energy Eq.: $u_3 = u_2 + q_H \implies q_H = C_v \, (T_3 - T_2)$

Solve for T_3 from process equation

$$T_3 = T_2 \times (P_3 / P_2) = 722.5 \text{ K} \times (6034.9 / 3251) = 1341.2 \text{ K}$$

Solve for q_H from the energy equation

$$q_H = C_v \, (T_3 - T_2) = 0.717 \, (1341.2 - 722.5) = \textbf{443.6 kJ/kg}$$

10.111

Assume we change the Otto cycle in Problem 10.66 to an Atkinson cycle by keeping the same conditions and only increase the expansion to give a different state 4. Find the expansion ratio and the cycle efficiency.

Solution:

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq.6.24 and Eq.6.25

$$T_2 = T_1\,(v_1/v_2)^{k-1} = 300 \times 10^{0.4} = 753.6 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 95 \times 10^{1.4} = 2386 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v = 753.6 + 1300/0.717 = 2567 \text{ K}$$

$$P_3 = P_2 \times (T_3/T_2) = 2386\,(2567\,/\,753.6) = 8127 \text{ kPa}$$

The expansion should now bring pressure down so $P_4 = P_1$

$$P_4 = P_1 = P_3 \times (v_3/v_4)^k = P_3 \times CR^{-k} \implies CR = (P_3\,/P_4)^{1/k}$$

$$CR = (8127\,/\,95)^{0.71429} = \mathbf{24}$$

So now the efficiency becomes

$$\eta = 1 - k\,\frac{CR - CR_1}{CR^k - CR_1^{\,k}} = 1 - 1.4\,\frac{24 - 10}{24^{\,k} - 10^{\,k}} = \mathbf{0.675}$$

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10.112

Repeat Problem 10.73 assuming we change the Otto cycle to an Atkinson cycle by keeping the same conditions and only increase the expansion to give a different state 4.

Solution:

Compression: Reversible and adiabatic so constant s from Eq.6.24-25

$$P_2 = P_1(v_1/v_2)^k = 90 \text{ kPa } (7)^{1.4} = 1372 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 283.2 \text{ K} \times (7)^{0.4} = 616.6 \text{ K}$$
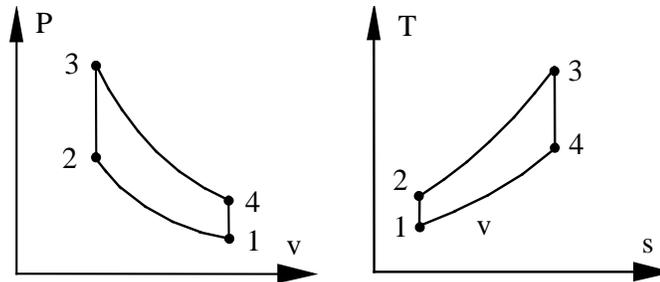
Combustion: constant volume

$$T_3 = T_2 + q_H/C_v = 616.6 + 1800/0.717 = \textbf{3127 K}$$

$$P_3 = P_2T_3/T_2 = 1372 \times 3127 / 616.6 = \textbf{6958 kPa}$$

The expansion should now bring pressure down so $P_4 = P_1$

$$P_4 = P_1 = P_3 \times (v_3/v_4)^k = P_3 \times CR^{-k} \implies CR = (P_3/P_4)^{1/k}$$

$$CR = (6958 / 90)^{0.71429} = \textbf{22.32}$$

So now the efficiency and net work become

$$\eta_{TH} = 1 - k\,\frac{CR - CR_1}{CR^k - CR_1^{\,k}} = 1 - 1.4\,\frac{22.32 - 7}{22.32^{\,k} - 7^{\,k}} = \textbf{0.654}$$

$$w_{net} = \eta_{TH} \times q_H = 0.654 \times 1800 \text{ kJ/kg} = 1177 \text{ kJ/kg}$$

Displacement (we take this as $v_4 - v_2$) and P_{meff}

$$v_1 = RT_1/P_1 = (0.287 \times 283.2)/90 = 0.9029 \text{ m}^3/\text{kg}$$

$$v_2 = (1/7)\,v_1 = 0.1290 \text{ m}^3/\text{kg}; \quad v_4 = 22.32\,v_2 = 2.879 \text{ m}^3/\text{kg}$$

$$P_{meff} = \frac{w_{NET}}{v_4 - v_2} = \frac{1177}{2.879 - 0.129} = \textbf{428 kPa}$$

Comment: The ratio $CR / CR_1 = 22.32/7$ is unrealistic large.

10.113

An Atkinson cycle has state 1 as 150 kPa, 300 K, compression ratio of 9 and expansion ratio of 14. Find the mean effective pressure.

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1$ ⇒ From Eq.6.24 and Eq.6.25

$$T_2 = T_1 (v_1/v_2)^{k-1} = 300 \text{ K} \times 9^{0.4} = 722.5 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 150 \text{ kPa} \times 9^{1.4} = 3251 \text{ kPa}$$

The expansion backwards from $P_4 = P_1$ gives

$$P_3 = P_4 \times (v_4/v_3)^k = P_4 \times CR^k = 150 \text{ kPa} \times 14^{1.4} = 6034.9 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $v_3 = v_2$ ⇒ $P_3 = P_2 \times (T_3/T_2)$

Energy Eq.: $u_3 = u_2 + q_H$ ⇒ $q_H = C_v (T_3 - T_2)$

Solve for T_3 from process equation

$$T_3 = T_2 \times (P_3 / P_2) = 722.5 \text{ K} \times (6034.9 / 3251) = 1341.2 \text{ K}$$

Solve for q_H from the energy equation

$$q_H = C_v (T_3 - T_2) = 0.717 \text{ kJ/kg-K} (1341.2 - 722.5) \text{ K} = 443.6 \text{ kJ/kg}$$

The efficiency and net work become

$$\eta_{TH} = 1 - k \frac{CR - CR_1}{CR^k - CR_1^k} = 1 - 1.4 \frac{14 - 9}{14^k - 9^k} = 0.623$$

$$w_{net} = \eta_{TH} \times q_H = 0.623 \times 443.6 \text{ kJ/kg} = 276.4 \text{ kJ/kg}$$

Displacement (we take this as $v_4 - v_2$) and P_{meff}

$$v_1 = RT_1/P_1 = (0.287 \times 300)/150 = 0.574 \text{ m}^3/\text{kg}$$

$$v_2 = (1/9) v_1 = 0.06378 \text{ m}^3/\text{kg}; \quad v_4 = 14 v_2 = 0.8929 \text{ m}^3/\text{kg}$$

$$P_{meff} = \frac{w_{NET}}{v_4 - v_2} = \frac{276.4}{0.8929 - 0.06378} = \textbf{333 kPa}$$

10.114

A Miller cycle has state 1 as 150 kPa, 300 K, compression ratio of 9 and expansion ratio of 14. If P_4 is 250 kPa find the heat release in the combustion.

Solution:

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq.6.24 and Eq.6.25

$$T_2 = T_1 (v_1/v_2)^{k-1} = 300 \text{ K} \times 9^{0.4} = 722.5 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 150 \text{ kPa} \times 9^{1.4} = 3251 \text{ kPa}$$

The expansion backwards from P_4 gives

$$P_3 = P_4 \times (v_4/v_3)^k = P_4 \times CR^k = 250 \text{ kPa} \times 14^{1.4} = 10\ 058 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v \quad \text{and} \quad P_3 = P_2 \times (T_3/T_2)$$

$$T_3 = T_2 \times (P_3 / P_2) = 722.5 \text{ K} \times (10\ 058 / 3251) = 2235.3 \text{ K}$$

$$q_H = C_v (T_3 - T_2) = 0.717 \text{ kJ/kg-K } (2235.3 - 722.5) \text{ K} = \textbf{1085 kJ/kg}$$

10.115

A Miller cycle has state 1 as 150 kPa, 300 K, compression ratio of 9 and a heat release of 1000 kJ/kg. Find the needed expansion ratio so P_4 is 250 kPa.

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq.6.24 and Eq.6.25

$$T_2 = T_1 (v_1/v_2)^{k-1} = 300 \text{ K} \times 9^{0.4} = 722.5 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 150 \text{ kPa} \times 9^{1.4} = 3251 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v = 722.5 + 1000/0.717 = 2117 \text{ K}$$

$$P_3 = P_2 \times (T_3/T_2) = 3251 \text{ kPa} (2117 / 722.5) = 9526 \text{ kPa}$$

The expansion should now bring pressure down to P_4

$$P_4 = P_3 \times (v_3/v_4)^k = P_3 \times CR^{-k} \implies CR = (P_3 /P_4)^{1/k}$$

$$CR = (9526 / 250)^{0.71429} = \textbf{13.5}$$

10.116

In a Miller cycle assume we know state 1 (intake state) compression ratios CR_1 and CR. Find an expression for the minimum allowable heat release so $P_4 = P_5$ that is, it becomes an Atkinson cycle.

Compression: Reversible and adiabatic so constant s from Eq.6.24-25

$$T_2 = T_1(v_1/v_2)^{k-1} = T_1 CR_1^{k-1}$$

Combustion: constant volume

$$T_3 = T_2 + q_H/C_v \qquad \text{and} \qquad P_3 = P_2 T_3/T_2$$

The expansion should now bring pressure down so $P_4 = P_1$

$$P_4 = P_1 = P_3 \times (v_3/v_4)^k = P_3 \times CR^{-k}$$

From the ideal gas law and the above pressure relation we have

$$P_3 v_3 / P_1 v_1 = T_3 / T_1 = (P_3 / P_1)CR_1 = CR^k \, CR_1$$

so now substitute

$$T_3 = T_2 + q_H/C_v = T_1 CR_1^{k-1} + q_H/C_v$$

to get

$$T_1 CR_1^{k-1} + q_H/C_v = T_1 \, CR^k \, CR_1$$

and now solve for q_H

$$q_H = C_v \, T_1 \, [CR^k \, CR_1 - CR_1^{k-1}] = \mathbf{C_v \, T_1 \, [\, CR^k - CR_1^{\,k} \,] / CR_1}$$

Combined Cycles

10.117

A Rankine steam power plant should operate with a high pressure of 3 MPa, a low pressure of 10 kPa, and the boiler exit temperature should be 500°C. The available high-temperature source is the exhaust of 175 kg/s air at 600°C from a gas turbine. If the boiler operates as a counterflowing heat exchanger where the temperature difference at the pinch point is 20°C, find the maximum water mass flow rate possible and the air exit temperature.

Solution:

C.V. Pump

$w_P = h_2 - h_1 = v_1(P_2 - P_1)$

$\quad = 0.00101(3000 - 10) = 3.02$ kJ/kg

$h_2 = h_1 + w_P = 191.83 + 3.02 = 194.85$ kJ/kg

Heat exchanger water states

State 2a:　$T_{2a} = T_{SAT} = 233.9$ °C

$\qquad h_{2a} = 1008.42$ kJ/kg

State 3:　$h_3 = 3456.5$ kJ/kg

Heat exchanger air states

inlet:　$h_{air,in} = 903.16$ kJ/kg

State 2a:　$h_{air}(T_{2a} + 20) = 531.28$ kJ/kg

Air temperature should be 253.9°C at the point where the water is at state 2a.

C.V. Section 2a-3, i-a

$$\dot{m}_{H_2O}(h_3 - h_{2a}) = \dot{m}_{air}(h_i - h_a)$$

$$\dot{m}_{H_2O} = 175 \, \frac{903.16 - 531.28}{3456.5 - 1008.42} = \textbf{26.584 kg/s}$$

Take C.V. Total:　$\dot{m}_{H_2O}(h_3 - h_2) = \dot{m}_{air}(h_i - h_e)$

$\Rightarrow \quad h_e = h_i - \dot{m}_{H_2O}(h_3 - h_2)/\dot{m}_{air}$

$\qquad = 903.6 - 26.584(3456.5 - 194.85)/175 = 408.13$ kJ/kg

$\Rightarrow \quad T_e = 406.7$ K $= \textbf{133.6 °C}, \quad T_e > T_2 = 46.5$ °C　OK.

10.118

A simple Rankine cycle with R-410A as the working fluid is to be used as a bottoming cycle for an electrical generating facility driven by the exhaust gas from a Diesel engine as the high temperature energy source in the R-410A boiler. Diesel inlet conditions are 100 kPa, 20°C, the compression ratio is 20, and the maximum temperature in the cycle is 2800 K. The R-410A leaves the bottoming cycle boiler at 80°C, 4 MPa and the condenser pressure is 1800 kPa. The power output of the Diesel engine is 1 MW. Assuming ideal cycles throughout, determine

a.   The flow rate required in the diesel engine.

b.   The power output of the bottoming cycle, assuming that the diesel exhaust is cooled to 200°C in the R-410A boiler.

Diesel cycle information given means: $\dot{W}_{DIESEL} = 1$ MW

$P_1 = 100$ kPa, $T_1 = 20$ °C, CR $= v_1/v_2 = 20$, $T_3 = 2800$ K

Consider the Diesel cycle

$T_2 = T_1(v_1/v_2)^{k-1} = 293.2$ K $(20)^{0.4} = 971.8$ K

$P_2 = P_1(v_1/v_2)^k = 100$ kPa $(20)^{1.4} = 6629$ kPa

$q_H = C_{P0}(T_3 - T_2) = 1.004$ kJ/kg-K$(2800 - 971.8)$ K $= 1835.5$ kJ/kg

$v_1 = \dfrac{0.287 \times 293.2}{100} = 0.8415$ m^3/kg, $v_2 = \dfrac{0.8415}{20} = 0.04208$ m^3/kg

$v_3 = v_2(T_3/T_2) = 0.04208$ m^3/kg $(2800/971.8) = 0.12124$ m^3/kg

$T_4 = T_3\left(\dfrac{v_3}{v_4}\right)^{k-1} = 2800$ K $\left(\dfrac{0.12124}{0.8415}\right)^{0.4} = 1290$ K

$q_L = 0.717(293.15 - 1290) = -714.7$ kJ/kg

$w_{NET} = 1835.5 - 714.7 = 1120.76$ kJ/kg

$\dot{m}_{AIR} = \dot{W}_{NET}/w_{NET} = 1000$ kW/1120.76 kJ/kg $= \mathbf{0.892}$ **kg/s**

Rankine cycle information given means:

Boiler exit state: $T_7 = 80^oC$, $P_7 = 4$ MPa,

Condenser: $P_8 = P_5 = 1800$ kPa, $T_5 = 28.22^oC$, $h_5 = h_f = 103.1$ kJ/kg

Consider the Rankine cycle

$s_8 = s_7 = 1.0028 \Rightarrow$ Sup. vapor $T_8 = 32.92^oC$, $h_8 = 290.6$ kJ/kg

$w_T = h_7 - h_8 = 311.48 - 290.6 = 20.88$ kJ/kg

$-w_P = v_5(P_6 - P_5) = 0.0009595$ m^3/kg (4000 - 1800) kPa = 2.11 kJ/kg

$h_6 = h_5 - w_P = 103.1 + 2.11 = 105.21$ kJ/kg

$q_H = h_7 - h_6 = 311.48 – 105.21 = 206.27$ kJ/kg

Connecting the two cycles.

$\dot{Q}_H$ available from Diesel exhaust flow cooled to 200 ^oC:

$\dot{Q}_H = 0.892$ kg/s $\times$ 1.004 kJ/kg-K (1290 - 473.2) K = 731 kW

$\Rightarrow \dot{m}_{R-410a} = \dot{Q}_H/q_H = 731$ kW /206.27 kJ/kg = 3.544 kg/s

$\dot{W}_{R-410a} = 3.544$ kg/s (20.88 - 2.11) kJ/kg = **66.5 kW**

10.119

A small utility gasoline engine of 250 cc runs at 1500 RPM with a compression ratio of 7:1. The inlet state is 75 kPa, 17^oC and the combustion adds 1500 kJ/kg to the charge. This engine runs a heat pump using R-410A with high pressure of 4 MPa and an evaporator operating at 0^oC. Find the rate of heating the heat pump can deliver.

Overall cycle efficiency is from Eq.10.12, $r_v = v_1/v_2 = 7$

$$\eta_{TH} = 1 - r_{1-k} = 1 - 7^{-0.4} = 0.5408$$

$$w_{net} = \eta_{TH} \times q_H = 0.5408 \times 1500 = 811.27 \text{ kJ/kg}$$

We also need specific volume to evaluate Eqs.10.9 to 10.11

$$v_1 = RT_1 / P_1 = 0.287 \text{ kJ/kg-K} \times 290 \text{ K} / 75 \text{ kPa} = 1.1097 \text{ m}^3/\text{kg}$$

$$v_2 = v_1/CR = 0.15853 \text{ m}^3/\text{kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{811.27}{1.097 - 0.15853} \frac{\text{kJ/kg}}{\text{m}^3/\text{kg}} = 852.9 \text{ kPa}$$

Now we can find the power from Eq.10.11 (assume 4 stroke engine)

$$\dot{W} = P_{meff} V_{displ} \frac{RPM}{60} \frac{1}{2} = 852.9 \times 2.5 \times 10^{-4} \times \frac{1500}{60} \times \frac{1}{2} = 2.665 \text{ kW}$$

For the refrigeration cycle we have:

State 1: $h_1 = 279.12$ kJ/kg, $s_1 = 1.0368$ kJ/kgK

State 2:   4 MPa, $s_2 = s_1$ interpolate $\Rightarrow h_2 = 323.81$ kJ/kg

State 3: 4 MPa interpolate $h_3 = 171.62$ kJ/kg

$$\beta_{HP} = \frac{q_H}{w_C} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{323.81 - 171.62}{323.81 - 279.12} = 3.405$$

The work out of the heat engine equals the input to the heat pump

$$\dot{Q}_H = \beta_{HP} \dot{W} = 3.405 \times 2.665 \text{ kW} = \textbf{9.07 kW}$$

10.120

Can the combined cycles in the previous problem deliver more heat than what comes from the R-410A? Find any amounts if so by assuming some conditions.

The hot exhaust gases from the gasoline engine can also be used and as the heat pump delivers heat at a temperature that equals the saturation T at 4 MPa which is 61.9^{o}C, the exhaust gas can be cooled to that. Not all the heat out of the heat engine is exhaust gas, part of $\dot{Q}_L$ is heat transfer rejected by the cooling (air or coolant). So as the engine produces 2.66 kW with an efficiency of 54% we must have:

$$\dot{Q}_{H\ fuel} = \dot{W} / \eta_{TH} = 2.66 \text{ kW} / 0.54 = 4.93 \text{ kW} \quad \text{input so}$$

$$\dot{Q}_{L\ engine} = \dot{Q}_{H\ fuel} - \dot{W} = 4.93 - 2.66 = 2.3 \text{ kW}$$

Generally half of this is exhaust gas and the other half heat transfer. So we can potentially get an additional 1 kW to heat with from the exhaust gas alone in addition to the heat pump output.

10.121

The power plant shown in Fig. 10.24 combines a gas-turbine cycle and a steam-turbine cycle. The following data are known for the gas-turbine cycle. Air enters the compressor at 100 kPa, 25°C, the compressor pressure ratio is 14, the heater input rate is 60 MW; the turbine inlet temperature is 1250°C, the exhaust pressure is 100 kPa; the cycle exhaust temperature from the heat exchanger is 200°C. The following data are known for the steam-turbine cycle. The pump inlet state is saturated liquid at 10 kPa, the pump exit pressure is 12.5 MPa; turbine inlet temperature is 500°C. Determine

a. The mass flow rate of air in the gas-turbine cycle.
b. The mass flow rate of water in the steam cycle.
c. The overall thermal efficiency of the combined cycle.

a) From Air Tables, A.7: P_{r1} = 1.0913, h_1 = 298.66, h_5 = 475.84 kJ/kg

$$s_2 = s_1 \Rightarrow P_{r2S} = P_{r1}(P_2/P_1) = 1.0913 \times 14 = 15.2782$$

$$T_2 = 629 \text{ K}, \quad h_2 = 634.48 \text{ kJ/kg}$$

$$w_C = h_1 - h_2 = 298.66 - 634.48 = -335.82 \text{ kJ/kg}$$

At T_3 = 1523.2 K: P_{r3} = 515.493, h_3 = 1663.91 kJ/kg

$$\dot{m}_{AIR} = \dot{Q}_H/(h_3 - h_2) = \frac{60\,000}{1663.91 - 634.48} = \textbf{58.28 kg/s}$$

b) $\quad P_{r4S} = P_{r3}(P_4/P_3) = 515.493(1/14) = 36.8209$

$$\Rightarrow T_4 = 791 \text{ K}, \quad h_4 = 812.68 \text{ kJ/kg}$$

$$w_{T\,gas} = h_3 - h_4 = 1663.91 - 812.68 = 851.23 \text{ kJ/kg}$$

Steam cycle: $-w_P \approx 0.00101 \text{ m}^3/\text{kg} (12500 - 10) \text{ kPa} = 12.615 \text{ kJ/kg}$

$$h_6 = h_9 - w_P = 191.83 + 12.615 = 204.45 \text{ kJ/kg}$$

At 12.5 MPa, 500 °C: h_7 = 3341.1 kJ/kg, s_7 = 6.4704 kJ/kg K

$$\dot{m}_{H_2O} = \dot{m}_{AIR} \frac{h_4 - h_5}{h_7 - h_6} = 58.28 \frac{812.68 - 475.84}{3341.1 - 204.45} = \textbf{6.259 kg/s}$$

c) $\quad s_8 = s_7 = 6.4704 = 0.6492 + x_8 \times 7.501, \quad x_8 = 0.7761$

$$h_8 = 191.81 + 0.7761 \times 2392.8 = 2048.9 \text{ kJ/kg}$$

$$w_{T\,steam} = h_7 - h_8 = 3341.1 - 2048.9 = 1292.2 \text{ kJ/kg}$$

$$\dot{W}_{NET} = \left[\dot{m}(w_T + w_C)\right]_{AIR} + \left[\dot{m}(w_T + w_P)\right]_{H_2O}$$

$$= 58.28 \, (851.23 - 335.82) + 6.259 \, (1292.2 - 12.615)$$

$$= 30\,038 + 8009 = 38\,047 \text{ kW} = 38.05 \text{ MW}$$

$$\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = 38.047/60 = \mathbf{0.634}$$

Figure 10.24
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Exergy Concepts

10.122

Consider the Brayton cycle in Problem 10.28. Find all the flows and fluxes of exergy and find the overall cycle second-law efficiency. Assume the heat transfers are internally reversible processes, and we neglect any external irreversibility.

Solution:

Efficiency is from Eq.10.1

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 16^{-0.4/1.4} = 0.547$$

from the required power we can find the needed heat transfer

$$\dot{Q}_H = \dot{W}_{net} / \eta = \frac{14\,000}{0.547} = 25\,594 \text{ kW}$$

$$\dot{m} = \dot{Q}_H / q_H = 25\,594 \text{ kW} / 1160 \text{ kJ/kg} = \textbf{22.064 kg/s}$$

Temperature after compression is

$$T_2 = T_1 r_p^{(k-1)/k} = 290 \times 16^{0.4/1.4} = 640.35 \text{ K}$$

The highest temperature is after combustion

$$T_3 = T_2 + q_H/C_p = 640.35 + \frac{1160}{1.004} = 1795.7 \text{ K}$$

For the exit flow I need the exhaust temperature

$$T_4 = T_3 \ r_p^{-\frac{k-1}{k}} = 1795.7 \times 16^{-0.2857} = 813.25 \text{ K}$$

$$\eta_{II} = \dot{W}_{NET}/\dot{\Phi}_H \qquad \text{since the low T exergy flow out is lost}$$

The high T exergy input from combustion is

$$\dot{\Phi}_H = \dot{m}(\psi_3 - \psi_2) = \dot{m}[h_3 - h_2 - T(s_3 - s_2)]$$

$$= 22.064\,[1160 - 298 \times 1.004 \ln(\frac{1795.7}{640.35})] = \textbf{18 784 kW}$$

$$\eta_{II} = \dot{W}_{NET}/\dot{\Phi}_H = 14\,000 / 18\,784 = \textbf{0.745}$$

$$\dot{\Phi}_{flow\ in} = \dot{m}(\psi_1 - \psi_o) = \dot{m}[h_1 - h_o - T_o(s_1 - s_o)]$$

$$= 22.064\,[\,1.004(17 - 25) - 298 \times 1.004 \ln(\frac{290}{298})\,] = \textbf{2.5 kW}$$

$$\dot{\Phi}_{flow\ out} = \dot{m}(\psi_4 - \psi_o) = \dot{m}[h_4 - h_o - T(s_4 - s_o)]$$

$$= 22.064\,[\,1.004(813 - 298) - 298 \times 1.004 \ln(\frac{813}{298})\,] = \textbf{4545 kW}$$

10.123

A Brayton cycle has a compression ratio of 15:1 with a high temperature of 1600 K and an inlet state of 290 K, 100 kPa. Use cold air properties to find the specific net work output and the second law efficiency if we neglect the "value" of the exhaust flow.

Brayton cycle so this means:

  Minimum T: $T_1 = 290$ K
  Maximum T: $T_3 = 1600$ K
  Pressure ratio: $P_2/P_1 = 15$
Solve using constant C_{P0}

Compression in compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.6.23

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 290 \text{ K } (15)^{0.286} = 628.65 \text{ K}$$

Energy input is from the combustor

$$q_H = h_3 - h_2 = C_{P0}(T_3 - T_2) = 1.004 \text{ kJ/kg-K } (1600 - 628.65) \text{ K}$$
$$= 975.2 \text{ kJ/kg}$$

Do the overall cycle efficiency and net work

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 15^{-0.4/1.4} = \mathbf{0.5387}$$

$$w_{NET} = \eta \, q_H = 0.5387 \times 975.2 = 525.34 \text{ kJ/kg}$$

Notice the q_H does not come at a single T so neglecting external irreversibility we get

$$\Phi_{qH} = \text{increase in flow exergy} = \psi_3 - \psi_2 = h_3 - h_2 - T_o(s_3 - s_2)$$
$$= q_H - T_o \, C_P \ln(T_3/T_2) = 975.2 - 298.15 \times 1.004 \ln \left(\frac{1600}{628.65} \right)$$
$$= 695.56 \text{ kJ/kg}$$

$$\eta_{II} = \frac{w_{net}}{\psi_3 - \psi_2} = \frac{525.34}{695.56} = \mathbf{0.755}$$

10.124

Reconsider the previous problem and find the second law efficiency if you do consider the "value" of the exhaust flow.

Brayton cycle so this means:

Minimum T: $T_1 = 290$ K

Maximum T: $T_3 = 1600$ K

Pressure ratio: $P_2/P_1 = 15$

Solve using constant C_{P0}

Compression in compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.6.23

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 290(15)^{0.286} = 628.65 \text{ K}$$

Energy input is from the combustor

$$q_H = h_3 - h_2 = C_{P0}(T_3 - T_2) = 1.004\ (1600 - 628.65) = 975.2 \text{ kJ/kg}$$

Do the overall cycle thermal efficiency and net work

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 15^{-0.4/1.4} = \mathbf{0.5387}$$

$$w_{NET} = \eta\ q_H = 0.5387 \times 975.2 = 525.34 \text{ kJ/kg}$$

Notice the q_H does not come at a single T so neglecting external irreversibility we get

$$\Phi_{qH} = \text{increase in flow exergy} = \psi_3 - \psi_2 = h_3 - h_2 - T_o(s_3 - s_2)$$

$$= q_H - T_o\ C_P \ln(T_3/T_2) = 975.2 - 298.15 \times 1.004 \ln\left(\frac{1600}{628.65}\right)$$

$$= 695.56 \text{ kJ/kg}$$

$$T_4 = T_3\ (P_4/P_3)^{\frac{k-1}{k}} = 1600 \text{ K} \times (1/15)^{0.286} = 738 \text{ K}$$

$$\Phi_{qL} = \psi_4 - \psi_1 = h_4 - h_1 - T_o(s_4 - s_1) = C_P(T_4 - T_1) - T_o\ C_P \ln\frac{T_4}{T_1}$$

$$= 1.004 \text{ kJ/kg-K} \times [\ 738 - 290 - 298 \ln(\frac{738}{290})\ ] \text{ K} = 170.33 \text{ kJ/kg}$$

$$\eta_{II} = \frac{w_{net}}{\Phi_{qH} - \Phi_{qL}} = \frac{525.34}{695.56 - 170.33} = \mathbf{1}$$

Why is it 1?, the cycle is reversible so we could have said that right away.

10.125

For Problem 10.117, determine the change of exergy of the water flow and that of the air flow. Use these to determine a second law efficiency for the boiler heat exchanger.

From solution to 10.117:

$\dot{m}_{H2O} = 26.584$ kg/s, $h_2 = 194.85$ kJ/kg, $s_2 = 0.6587$ kJ/kg K

$h_3 = 3456.5$ kJ/kg, $s_3 = 7.2338$, $s^{\circ}_{Ti} = 7.9820$, $s^{\circ}_{Te} = 7.1762$ kJ/kg K

$h_i = 903.16$ kJ/kg, $h_e = 408.13$ kJ/kg

$\psi_3 - \psi_2 = h_3 - h_2 - T_0(s_3 - s_2) = 1301.28$ kJ/kg

$\psi_i - \psi_e = h_i - h_e - T_0(s^{\circ}_{Ti} - s^{\circ}_{Te}) = 254.78$ kJ/kg

$$\eta_{II} = \frac{(\psi_3 - \psi_2)\dot{m}_{H2O}}{(\psi_i - \psi_e)\dot{m}_{air}} = \frac{1301.28 \times 26.584}{254.78 \times 175} = \mathbf{0.776}$$

10.126

Determine the second law efficiency of an ideal regenerator in the Brayton cycle.

The ideal regenerator has no temperature difference between the two flows and is therefore reversible. Its second law efficiency is **100%**.

10.127

Assume a regenerator in a Brayton cycle has an efficiency of 75%. Find an expression for the second law efficiency.

To do this, assume the pressure is constant in the two flows so we do not have any pressure loss. The change in enthalpy and entropy is then only due to changes in temperature for the two flows.

$$\eta_{II} = \frac{\psi_x - \psi_2}{\psi_4 - \psi_2} = \frac{h_x - h_2 - T_o(s_x - s_2)}{h_4 - h_2 - T_o(s_4 - s_2)}$$

$$= \frac{C_P(T_x - T_2) - T_o\, C_P \ln(T_x/T_2)}{C_P(T_4 - T_2) - T_o\, C_P \ln(T_4/T_2)}$$

$$= \frac{T_x - T_2 - T_o\, \ln(T_x/T_2)}{T_4 - T_2 - T_o\, \ln(T_4/T_2)}$$

Finally we have

$$\eta_{II} = \frac{\eta_{reg} - a\, \ln(T_x/T_2)}{1 - a\, \ln(T_4/T_2)}$$

where a is defined as

$$a = \frac{T_o}{T_4 - T_2}$$

10.128

The Brayton cycle in Problem 10.19 had a heat addition of 800 kJ/kg. What is the exergy increase in the heat addition process?

Solution:

Combustion: $h_3 = h_2 + q_H$;　　$_2w_3 = 0$　and　$T_{max} = T_3 = 1400$ K

$T_2 = T_3 - q_H/C_P = 1400$ K $- 800$ kJ/kg $/1.004$ kJ/kg-K $= 603.2$ K

Now the exergy increase from 2 to 3 becomes　$(P_3 = P_2)$

$$\psi_3 - \psi_2 = h_3 - h_2 - T_0(s_3 - s_2) = q_H - T_0 \, C_P \, \ln(T_3/T_2)$$
$$= 800 - 298.15 \times 1.004 \, \ln \left(\frac{1400}{603.2} \right)$$
$$= \textbf{547.96 kJ/kg}$$

10.129

The conversion efficiency of the Brayton cycle in Eq.10.1 was done with cold air properties. Find a similar formula for the second law efficiency assuming the low T heat rejection is assigned zero exergy value.

The thermal efficiency (first law) is Eq.10.1

$$\eta_{TH} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k}$$

The corresponding 2nd law efficiency is

$$\eta_{II} = \frac{w_{net}}{\Phi_H} = \frac{h_3 - h_2 - (h_4 - h_1)}{h_3 - h_2 - T_O(s_3 - s_2)}$$

where we used Φ_H = increase in flow exergy = $\psi_3 - \psi_2 = h_3 - h_2 - T_o(s_3 - s_2)$. Now divide the difference $h_3 - h_2$ out to get

$$\eta_{II} = \frac{1 - \dfrac{h_4 - h_1}{h_3 - h_2}}{1 - T_O(s_3 - s_2)/(h_3 - h_2)} = \frac{\eta_{TH}}{1 - T_O(s_3 - s_2)/(h_3 - h_2)}$$

$$= \frac{\eta_{TH}}{1 - T_o \dfrac{C_P \ln(T_3 / T_2)}{C_P (T_3 - T_2)}}$$

$$= \frac{\eta_{TH}}{1 - T_o \dfrac{\ln(T_3 / T_2)}{T_3 - T_2}}$$

Comment: Due to the temperature sensitivity of Φ_H the temperatures do not reduce out from the expression.

10.130

Redo the previous problem for a large stationary Brayton cycle where the low T heat rejection is used in a process application and thus has a non-zero exergy.

The thermal efficiency (first law) is Eq.10.1

$$\eta_{TH} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k}$$

The corresponding 2nd law efficiency is

$$\eta_{II} = \frac{w_{net}}{\Phi_H - \Phi_L} = \frac{h_3 - h_2 - (h_4 - h_1)}{h_3 - h_2 - T_o(s_3 - s_2)}$$

where we used

Φ_H = increase in flow exergy = $\psi_3 - \psi_2 = h_3 - h_2 - T_o(s_3 - s_2)$.

Φ_L = rejection of flow exergy = $\psi_4 - \psi_1 = h_4 - h_1 - T_o(s_4 - s_1)$.

Now divide all terms with the difference $h_3 - h_2$ to get

$w_{net} / (h_3 - h_2) = \eta_{TH}$;

$\Phi_H/(h_3 - h_2) = 1 - T_o(s_3 - s_2)(h_3 - h_2) = 1 - T_o \ln(T_3 /T_2) (T_3 - T_2)^{-1}$

$\Phi_L/(h_3 - h_2) = \frac{h_4 - h_1}{h_3 - h_2} [1 - T_o \frac{s_4 - s_1}{h_4 - h_1}] = (1 - \eta_{TH})[1 - T_o \frac{\ln T_4/T_1}{T_4 - T_1}]$

Substitute all terms to get

$$\eta_{II} = \frac{\eta_{TH}}{1 - T_o\dfrac{\ln(T_3 / T_2)}{T_3 - T_2} - (1 - \eta_{TH})[1 - T_o \dfrac{\ln(T_4 / T_1)}{T_4 - T_1}]}$$

Comment: Due to the temperature sensitivity of Φ_H and Φ_L the temperatures do not reduce out from the expression.

Review problems

10.131

Solve Problem 10.19 with variable specific heats using Table A.7.

A Brayton cycle inlet is at 300 K, 100 kPa and the combustion adds 800 kJ/kg. The maximum temperature is 1400 K due to material considerations. What is the maximum allowed compression ratio? For this compression ratio calculate the net work and cycle efficiency using cold air properties.

Solution:

Combustion: $h_3 = h_2 + q_H$; $\quad {}_2w_3 = 0$ and $T_{max} = T_3 = 1400$ K

$\qquad h_2 = h_3 - q_H = 1515.27 - 800 = 715.27$ kJ/kg

From Table A.7.1

$\qquad T_2 \approx 701.6$ K; $\;s^o_{T2} = 7.74251$; $\quad T_1 = 300$ K; $\;s^o_{T1} = 6.86926$ kJ/kg K

Reversible adiabatic compression leads to constant s, from Eq.6.19:

$\qquad P_2 / P_1 = \exp[\;(s^o_{T2} - s^o_{T1})/R\;] = \exp(3.04268) = \mathbf{20.96}$

Reversible adiabatic expansion leads to constant s, from Eq.6.19

$\qquad s^o_{T4} = s^o_{T3} + R\;\ln(P_4 / P_3) = 8.52891 + 0.287\;\ln\,(1/20.96) = 7.65568$ kJ/kgK

From Table A.7.1 by linear interpolation $\qquad T_4 \approx 646.85$ K, $\;h_4 = 656.81$ kJ/kg

$\qquad\qquad w_T = h_3 - h_4 = 1515.27 - 656.81 = 858.46$ kJ/kg

$\qquad\qquad w_C = h_2 - h_1 = 715.27 - 300.47 = 414.8$ kJ/kg

$\qquad\qquad w_{net} = w_T - w_C = 858.46 - 414.8 = \mathbf{443.7\;kJ/kg}$

$\qquad\qquad \eta = w_{net} / q_H = 443.7 / 800 = \mathbf{0.55}$

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10.132

Do Problem 10.28 with properties from table A.7.1 instead of cold air properties.

Solution:

With the variable specific heat we must go through the processes one by one to get net work and the highest temperature T_3.

From A.7.1: $h_1 = 290.43$ kJ/kg, $s^o_{T1} = 6.83521$ kJ/kg K

The compression is reversible and adiabatic so constant s. From Eq.6.19

$$s_2 = s_1 \implies s^o_{T2} = s^o_{T1} + R\ln(P_2/P_1) = 6.83521 + 0.287 \ln 16 = 7.63094$$

$$\implies T_2 = 631.9 \text{ K}, \quad h_2 = 641 \text{ kJ/kg}$$

Energy equation with compressor work in

$$w_C = -_1w_2 = h_2 - h_1 = 641 - 290.43 = 350.57 \text{ kJ/kg}$$

Energy Eq. combustor: $h_3 = h_2 + q_H = 641 + 1160 = 1801$ kJ/kg

State 3: (P, h):   **$T_3 = 1635.7$ K**, $s^o_{T3} = 8.71738$ kJ/kg K

The expansion is reversible and adiabatic so constant s. From Eq.6.19

$$s_4 = s_3 \implies s^o_{T4} = s^o_{T3} + R\ln(P_4/P_3) = 8.71738 + 0.287\ln(1/16) = 7.92165$$

$$\implies T_4 = 827.3 \text{ K}, \quad h_4 = 852.31 \text{ kJ/kg}$$

Energy equation with turbine work out

$$w_T = h_3 - h_4 = 1801 - 852.31 = 948.69 \text{ kJ/kg}$$

Now the net work is

$$w_{net} = w_T - w_C = 948.69 - 350.57 = 598.12 \text{ kJ/kg}$$

The total required power requires a mass flow rate as

$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{14\ 000}{598.12} \frac{\text{kW}}{\text{kJ/kg}} = \textbf{23.41 kg/s}$$

10.133

Repeat Problem 10.36, but assume that the compressor has an efficiency of 82%, that both turbines have efficiencies of 87%, and that the regenerator efficiency is 70%.

a) From solution 10.36: $T_2 = T_1\left(\dfrac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300 \text{ K } (6)^{0.286} = 500.8 \text{ K}$

$-w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.004 \text{ kJ/kg-K } (500.8 - 300) \text{ K} = 201.6 \text{ kJ/kg}$

$-w_C = -w_{SC}/\eta_{SC} = 201.6/0.82 = 245.8 \text{ kJ/kg} = w_{T1}$

$= C_{P0}(T_4 - T_5) = 1.004 \text{ kJ/kg-K } (1600 \text{ K} - T_5) \quad \Rightarrow \quad T_5 = 1355.2 \text{ K}$

$w_{ST1} = w_{T1}/\eta_{ST1} = 245.8/0.87 = 282.5 \text{ kJ/kg}$

$= C_{P0}(T_4 - T_{5S}) = 1.004(1600 - T_{5S}) \quad \Rightarrow \quad T_{5S} = 1318.6 \text{ K}$

$s_{5S} = s_4 \Rightarrow P_5 = P_4(T_{5S}/T_4)^{\frac{k}{k-1}} = 600\left(\dfrac{1318.6}{1600}\right)^{3.5} = \textbf{304.9 kPa}$

b) $P_6 = 100 \text{ kPa}, \quad s_{6S} = s_5$

$T_{6S} = T_5\left(\dfrac{P_6}{P_5}\right)^{\frac{k-1}{k}} = 1355.2\left(\dfrac{100}{304.9}\right)^{0.286} = 985.2 \text{K}$

$w_{ST2} = C_{P0}(T_5 - T_{6S}) = 1.004(1355.2 - 985.2) = 371.5 \text{ kJ/kg}$

$w_{T2} = \eta_{ST2} \times w_{ST2} = 0.87 \times 371.5 = \textbf{323.2 kJ/kg}$

$323.2 = C_{P0}(T_5 - T_6) = 1.004(1355.2 - T_6) \Rightarrow T_6 = 1033.3 \text{K}$

$\dot{m} = \dot{W}_{NET}/w_{NET} = 150/323.2 = \textbf{0.464 kg/s}$

c) $w_C = 245.8 = C_{P0}(T_2 - T_1) = 1.004(T_2 - 300) \Rightarrow T_2 = 544.8 \text{ K}$

$\eta_{REG} = \dfrac{h_3 - h_2}{h_6 - h_2} = \dfrac{T_3 - T_2}{T_6 - T_2} = \dfrac{T_3 - 544.8}{1033.3 - 544.8} = 0.7$

$\Rightarrow T_3 = 886.8 \text{ K}$

$q_H = C_{P0}(T_4 - T_3) = 1.004(1600 - 886.8) = 716 \text{ kJ/kg}$

$\eta_{TH} = w_{NET}/q_H = 323.2/716 = \textbf{0.451}$

10.134

Consider a gas turbine cycle with two stages of compression and two stages of expansion. The pressure ratio across each compressor stage and each turbine stage is 8 to 1. The pressure at the entrance of the first compressor is 100 kPa, the temperature entering each compressor is 20^oC, and the temperature entering each turbine is 1100^oC. A regenerator is also incorporated into the cycle and it has an efficiency of 70%. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

$P_2/P_1 = P_4/P_3 = P_6/P_7 = P_8/P_9 = 8.0$

$P_1 = 100$ kPa

$T_1 = T_3 = 20\ ^oC, \quad T_6 = T_8 = 1100\ ^oC$

Cold air and $s_2 = s_1$ and $s_4 = s_3$

$T_4 = T_2 = T_1 \left(\dfrac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 293.15(8)^{0.286} = 531$ K

Total $-w_C = 2 \times (-w_{12}) = 2C_{P0}(T_2 - T_1) = 2 \times 1.004(531 - 293.15) =$ **477.6 kJ/kg**

Also $s_6 = s_7$ and $s_8 = s_9$: $\Rightarrow$ $T_7 = T_9 = T_6 \left(\dfrac{P_7}{P_6}\right)^{\frac{k-1}{k}} = 1373.15\left(\dfrac{1}{8}\right)^{0.286} = 758$ K

Total $w_T = 2 \times w_{67} = 2C_{P0}(T_6 - T_7) = 2 \times 1.004(1373.15 - 758) = 1235.2$ kJ/kg

$\quad w_{NET} = 1235.2 - 477.6 = 757.6$ kJ/kg

Ideal regenerator: $T_5 = T_9$, $T_{10} = T_4$ so the actual one has

$$\eta_{REG} = \frac{h_5 - h_4}{h_9 - h_4} = \frac{T_5 - T_4}{T_9 - T_4} = \frac{T_5 - 531}{758 - 531} = 0.7 \quad \Rightarrow T_5 = 689.9 \text{ K}$$

$\Rightarrow q_H = (h_6 - h_5) + (h_8 - h_7) = C_{P0}(T_6 - T_5) + C_{P0}(T_8 - T_7)$

$\qquad = 1.004(1373.15 - 689.9) + 1.004(1373.15 - 758)$

$\qquad = 1303.6$ kJ/kg

$\quad \eta_{TH} = w_{NET}/q_H = 757.6/1303.6 =$ **0.581**

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10.135

A gas turbine cycle has two stages of compression, with an intercooler between the stages. Air enters the first stage at 100 kPa, 300 K. The pressure ratio across each compressor stage is 5 to 1, and each stage has an isentropic efficiency of 82%. Air exits the intercooler at 330 K. The maximum cycle temperature is 1500 K, and the cycle has a single turbine stage with an isentropic efficiency of 86%. The cycle also includes a regenerator with an efficiency of 80%. Calculate the temperature at the exit of each compressor stage, the second-law efficiency of the turbine and the cycle thermal efficiency.

State 1: $P_1 = 100$ kPa, $T_1 = 300$ K   State 7: $P_7 = P_o = 100$ kPa

State 3: $T_3 = 330$ K;   State 6: $T_6 = 1500$ K, $P_6 = P_4$

$\qquad P_2 = 5 P_1 = 500$ kPa; $P_4 = 5 P_3 = 2500$ kPa

Ideal compression $T_{2s} = T_1 (P_2/P_1)^{(k-1)/k} = 475.4$ K

1st Law: $q + h_i = h_e + w$; $q = 0$ $\Rightarrow$ $w_{c1} = h_1 - h_2 = C_P(T_1 - T_2)$

$\qquad w_{c1\,s} = C_P(T_1 - T_{2s}) = -176.0$ kJ/kg, $w_{c1} = w_{c1\,s} / \eta = -214.6$

$\qquad T_2 = T_1 - w_{c1}/C_P = \textbf{513.9 K}$

$\qquad T_{4s} = T_3 (P_4/P_3)^{(k-1)/k} = 475.4$ K

$\qquad w_{c2\,s} = C_P(T_3 - T_{4s}) = -193.6$ kJ/kg; $w_{c2} = -236.1$ kJ/kg

$\qquad T_4 = T_3 - w_{c2} / C_P = \textbf{565.2 K}$

Ideal Turbine (reversible and adiabatic)

$\quad T_{7s} = T_6(P_7/P_6)^{(k-1)/k} = 597.4$ K $\Rightarrow$ $w_{Ts} = C_P(T_6 - T_{7s}) = 905.8$ kJ/kg

1st Law Turbine: $q + h_6 = h_7 + w$; $q = 0$

$\quad w_T = h_6 - h_7 = C_P(T_6 - T_7) = \eta_{Ts}\, w_{Ts} = 0.86 \times 905.8 = 779.0$ kJ/kg

$\quad T_7 = T_6 - w_T / C_P = 1500 - 779/1.004 = 723.7$ K

$$s_6 - s_7 = C_P \ln \frac{T_6}{T_7} - R \ln \frac{P_6}{P_7} = -0.1925 \text{ kJ/kg K}$$

$\psi_6 - \psi_7 = (h_6 - h_7) - T_o(s_6 - s_7) = 779.0 - 298.15(-0.1925) = 836.8$ kJ/kg

$$\eta_{2nd\ Law} = \frac{w_T}{\psi_6 - \psi_7} = 779.0 / 836.8 = \textbf{0.931}$$

d) $\eta_{th} = q_H / w_{net}$; $w_{net} = w_T + w_{c1} + w_{c2} = 328.3$ kJ/kg

1st Law Combustor: $q + h_i = h_e + w$; $w = 0$

$\qquad q_c = h_6 - h_5 = C_P(T_6 - T_5)$

Regenerator: $\eta_{reg} = \dfrac{T_5 - T_4}{T_7 - T_4} = 0.8$ $\rightarrow$ $T_5 = 692.1$ K

$\qquad q_H = q_c = 810.7$ kJ/kg; $\eta_{th} = \textbf{0.405}$

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10.136

Repeat the questions in Problem 10.36 when we assume that friction causes pressure drops in the burner and on both sides of the regenerator. In each case, the pressure drop is estimated to be 2% of the inlet pressure to that component of the system, so $P_3 = 588$ kPa, $P_4 = 0.98\ P_3$ and $P_6 = 102$ kPa.

Solution:

From solution Problem 10.36: $T_2 = T_1\left(\dfrac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300$ K $(6)^{0.286} = 500.8$ K

$w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.004$ kJ/kg-K $(500.8 - 300)$ K $= 201.6$ kJ/kg

$P_3 = 0.98 \times 600 = 588$ kPa, $P_4 = 0.98 \times 588 = 576.2$ kPa

$s_5 = s_4 \Rightarrow\quad P_5 = P_4(T_{5S}/T_4)^{\frac{k}{k-1}} = 576.2$ kPa $\left(\dfrac{1399.2}{1600}\right)^{3.5} = \mathbf{360.4\ kPa}$

$P_6 = 100/0.98 = 102$ kPa, $s_{6S} = s_5$

$T_6 = T_5\left(\dfrac{P_6}{P_5}\right)^{\frac{k-1}{k}} = 1399.2\left(\dfrac{102}{292.8}\right)^{0.286} = 975.2$ K

$w_{ST2} = C_{P0}(T_5 - T_6) = 1.004(1399.2 - 975.2) = 425.7$ kJ/kg

$\dot{m} = \dot{W}_{NET}/w_{NET} = 150$ kW $/425.7$ kJ/kg $= \mathbf{0.352\ kg/s}$

$T_3 = T_6 = 975.2$ K

$q_H = C_{P0}(T_4 - T_3) = 1.004\ (1600 - 975.2) = 627.3$ kJ/kg

$\eta_{TH} = w_{NET}/q_H = 425.7/627.3 = \mathbf{0.678}$

10.137

A gasoline engine has a volumetric compression ratio of 9. The state before compression is 290 K, 90 kPa, and the peak cycle temperature is 1800 K. Find the pressure after expansion, the cycle net work and the cycle efficiency using properties from Table A.7.

Use table A.7.1 and interpolation.

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq.6.19

$$0 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = s^o_{T2} - s^o_{T1} - R \ln(T_2 v_1/T_1 v_2)$$

$$s^o_{T2} - R \ln(T_2/T_1) = s^o_{T1} + R \ln(v_1/v_2) = 6.83521 + 0.287 \ln 9 = 7.4658$$

This becomes trial and error so estimate first at 680 K and use A.7.1.

$LHS_{680} = 7.7090 - 0.287 \ln(680/290) = 7.4644$ (too low)

$LHS_{700} = 7.7401 - 0.287 \ln(700/290) = 7.4872$ (too high)

Interpolate to get: $T_2 = 681.23$ K, $u_2 = 497.9$ kJ/kg

$P_2 = P_1 (T_2/T_1) (v_1/v_2) = 90 (681.23 / 290) \times 9 = 1902.7$ kPa

$_1w_2 = u_1 - u_2 = 207.2 - 497.9 = -290.7$ kJ/kg

Combustion 2 to 3: constant volume $v_3 = v_2$

$q_H = u_3 - u_2 = 1486.3 - 497.9 = 988.4$ kJ/kg

$P_3 = P_2(T_3/T_2) = 1902.7 (1800/681.2) = 5028$ kPa

Expansion 3 to 4: $s_4 = s_3$ $\Rightarrow$ From Eq.6.19 as before

$$s^o_{T4} - R \ln(T_4/T_3) = s^o_{T3} + R \ln(v_3/v_4) = 8.8352 + 0.287 \ln(1/9) = 8.2046$$

This becomes trial and error so estimate first at 850 K and use A.7.1.

$LHS_{850} = 7.7090 - 0.287 \ln(850/1800) = 8.1674$ (too low)

$LHS_{900} = 7.7401 - 0.287 \ln(900/1800) = 8.2147$ (too high)

Interpolation $\Rightarrow$ $T_4 = 889.3$ K, $u_4 = 666$ kJ/kg

$P_4 = P_3(T_4/T_3)(v_3/v_4) = 5028 (889.3/1800) (1/9) = $ **276 kPa**

$_3w_4 = u_3 - u_4 = 1486.3 - 666.0 = 820.3$ kJ/kg

Net work and overall efficiency

$w_{NET} = {_3w_4} + {_1w_2} = 820.3 - 290.7 = $ **529.6 kJ/kg**

$\eta = w_{NET}/q_H = 529.6/988.4 = $ **0.536**

10.138

Consider an ideal air-standard diesel cycle in which the state before the compression process is 95 kPa, 290 K, and the compression ratio is 20. Find the maximum temperature (by iteration) in the cycle to have a thermal efficiency of 60%?

Solution:

Diesel cycle: $P_1 = 95$ kPa, $T_1 = 290$ K, $v_1/v_2 = 20$, $\eta_{TH} = 0.6$

Since the efficiency depends on T_3 and T_4, which are connected through the expansion process in a nonlinear manner we have an iterative problem.

$$T_2 = T_1(v_1/v_2)^{k-1} = 290 \text{ K } (20)^{0.4} = 961.2 \text{ K}$$

$$v_1 = 0.287 \text{ kJ/kg-K} \times 290 \text{ K}/95 \text{ kPa} = 0.876 \text{ m}^3/\text{kg} = v_4,$$

$$v_2 = v_1/CR = 0.876 / 20 = 0.0438 \text{ m}^3/\text{kg}$$

$$v_3 = v_2(T_3/T_2) = 0.0438 \text{ m}^3/\text{kg } (T_3/961.2) = 0.0000456 \, T_3$$

$$T_3 = T_4 \, (v_4/v_3)^{k-1} = \left(\frac{0.876}{0.0000456 \, T_3}\right)^{0.4} \Rightarrow T_4 = 0.019345 \, T_3^{1.4}$$

Now substitute this into the formula for the efficiency

$$\eta_{TH} = 0.60 = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{0.019345 \times T_3^{1.4} - 290}{1.4(T_3 - 961.2)}$$

$$\Rightarrow 0.019345 \times T_3^{1.4} - 0.56 \times T_3 + 248.272 = 0$$

Trial and error on this non-linear equation in T_3

   3050 K: LHS = +1.06   3040 K: LHS = -0.036,

   Linear interpolation $T_3 = \mathbf{3040 \text{ K}}$

10.139

Find the temperature after combustion and the specific energy release by combustion in Problem 10.98 using cold air properties. This is a difficult problem and it requires iterations.

The worlds largest diesel engine has displacement of 25 m^3 running at 200 RPM in a two stroke cycle producing 100 000 hp. Assume an inlet state of 200 kPa, 300 K and a compression ratio of 20:1. What is the mean effective pressure?

We have 3 parameters for the cycle: T_1, P_1 and CR we need one more, so this comes from the total rate of work Eq.10.11.

$$\dot{W} = m\, P_{meff}\, (v_1 - v_2) \times N_{cyl} \times RPM/60 = P_{meff}\, V_{displ}\, RPM/60$$

$$P_{meff} = \dot{W}\, 60\, /\, V_{displ}\, RPM = \frac{100\,000 \times 0.746\ kW \times 60\ s}{25\ m^3 \times 200} = 895\ kPa$$

$$v_1 = RT/P = 0.287\ kJ/kg\text{-}K \times 300\ K/\ 200\ kPa = 0.4305\ m^3/kg;$$

$$w_{net} = P_{meff}\, (v_1 - v_2) = 895 \times 0.4305\ (\ 1 - \frac{1}{20}\ ) = 366.03\ kJ/kg$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 300\ K\ (20)^{0.4} = 994.3\ K$$

Combustion and expansion volumes

$$v_3 = v_2 \times T_3/T_2 = v_1\, T_3/(T_2 \times CR)\ ;\quad v_4 = v_1$$

Expansion process, isentropic from Eq.6.23

$$T_4 = T_3\, (v_3/v_4)^{k-1} = T_3\, [T_3/\ (CR \times T_2)]^{k-1} = T_3^k\, (CR \times T_2)^{1-k} = 0.01908\ T_3^k$$

The net work is also given by the heat transfers

$$w_{net} = q_H - q_L = C_P(T_3 - T_2) - C_v(T_4 - T_1)$$

Substitute w_{net} and T_4 into this equation and we get

$$366.03 = 1.004(T_3 - 994.3) - 0.717(0.01908\ T_3^k - 300)$$

divide by 1.004 and combine terms to get

$$1144.63 = T_3 - 0.013626\ T_3^k$$

Trial and error, guess T_3 $\Rightarrow$ $RHS_{1400} = 1054.1$; $RHS_{1600} = 1183$

Interpolate: $T_3 = \mathbf{1540.5\ K}$

$$q_H = C_P(T_3 - T_2) = 1.004(1540.5 - 994.3) = \mathbf{548\ kJ/kg}$$

10.140

Reevaluate the combined Brayton and Rankine cycles in Problem 10.121. For a more realistic case assume the air compressor, the air turbine, the steam turbine and the pump all have an isentropic efficiency of 87%.

a) From Air Tables, A.7: $P_{r1} = 1.0913$, $h_1 = 298.66$, $h_5 = 475.84$ kJ/kg

$$s_2 = s_1 \implies P_{r2S} = P_{r1}(P_2/P_1) = 1.0913 \times 14 = 15.2782$$

$$T_{2S} = 629 \text{ K}, \quad h_{2S} = 634.48$$

$$w_{SC} = h_1 - h_{2S} = 298.66 - 634.48 = -335.82 \text{ kJ/kg}$$

$$w_C = w_{SC}/\eta_{SC} = -335.82/0.87 = -386 = h_1 - h_2 \implies h_2 = 684.66 \text{ kJ/kg}$$

At $T_3 = 1523.2$ K: $P_{r3} = 515.493$, $h_3 = 1663.91$ kJ/kg

$$\dot{m}_{AIR} = \dot{Q}_H/(h_3 - h_2) = \frac{60\,000}{1663.91 - 684.66} = \textbf{61.27 kg/s}$$

b) $P_{r4S} = P_{r3}(P_4/P_3) = 515.493(1/14) = 36.8209$

$$\implies T_{4S} = 791 \text{ K}, \quad h_{4S} = 812.68 \text{ kJ/kg}$$

$$w_{ST} = h_3 - h_{4S} = 1663.91 - 812.68 = 851.23 \text{ kJ/kg}$$

$$w_T = \eta_{ST} \times w_{ST} = 0.87 \times 851.23 = 740.57 = h_3 - h_4 \implies h_4 = 923.34 \text{ kJ/kg}$$

Steam cycle: $\quad -w_{SP} \approx 0.00101(12500 - 10) = 12.615 \text{ kJ/kg}$

$$-w_P = -w_{SP}/\eta_{SP} = 12.615/0.85 = 14.84 \text{ kJ/kg}$$

$$h_6 = h_9 - w_P = 191.83 + 14.84 = 206.67 \text{ kJ/kg}$$

At 12.5 MPa, 500 °C: $\quad h_7 = 3341.7 \text{ kJ/kg}, \quad s_7 = 6.4617 \text{ kJ/kg K}$

$$\dot{m}_{H_2O} = \dot{m}_{AIR} \frac{h_4 - h_5}{h_7 - h_6} = 61.27 \frac{923.34 - 475.84}{3341.7 - 206.67} = \mathbf{8.746 \text{ kg/s}}$$

c) $\quad s_{8S} = s_7 = 6.4617 = 0.6492 + x_{8S} \times 7.501, \quad x_{8S} = 0.7749$

$\quad h_{8S} = 191.81 + 0.7749 \times 2392.8 = 2046.0 \text{ kJ/kg}$

$\quad w_{ST} = h_7 - h_{8S} = 3341.7 - 2046.0 = 1295.7 \text{ kJ/kg}$

$\quad w_T = \eta_{ST} \times w_{ST} = 0.87 \times 1295.7 = 1127.3 \text{ kJ/kg}$

$$\dot{W}_{NET} = \left[\dot{m}(w_T + w_C) \right]_{AIR} + \left[\dot{m}(w_T + w_P) \right]_{H_2O}$$

$$= 61.27(740.57 - 386.0) + 8.746(1127.3 - 14.84)$$

$$= 21725 + 9730 = 31455 \text{ kW} = 31.455 \text{ MW}$$

$$\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = 31.455/60 = \mathbf{0.524}$$

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Problems solved using Table A.7.2

10.37

Repeat Problem 10.35 when the intercooler brings the air to $T_3 = 320$ K. The corrected formula for the optimal pressure is $P_2 = [\, P_1 P_4\, (T_3/T_1)^{n/(n-1)}]^{1/2}$ see Problem 7.245, where n is the exponent in the assumed polytropic process.

Solution:

The polytropic process has n = k (isentropic) so $n/(n - 1) = 1.4/0.4 = 3.5$

$$P_2 = 400 \text{ kPa } \sqrt{(320/290)^{3.5}} = 475.2 \text{ kPa}$$

C.V. C1: $s_2 = s_1 \Rightarrow P_{r2} = P_{r1}(P_2/P_1) = 0.9899 \times (475.2/100)$

$\qquad\qquad = 4.704 \Rightarrow T_2 = 452$ K, $h_2 = 453.75$

$\qquad -w_{C1} = h_2 - h_1 = 453.75 - 290.43 = \textbf{163.3 kJ/kg}$

C.V. Cooler: $q_{OUT} = h_2 - h_3 = 453.75 - 320.576 = \textbf{133.2 kJ/kg}$

C.V. C2: $s_4 = s_3 \Rightarrow P_{r4} = P_{r3}(P_4/P_3) = 1.3972 \times (1600/475.2) = 4.704$

$\qquad\qquad \Rightarrow T_4 = T_2 = 452$ K, $h_4 = 453.75$

$\qquad -w_{C2} = h_4 - h_3 = 453.75 - 320.576 = \textbf{133.2 kJ/kg}$

10.42

A gas turbine with air as the working fluid has two ideal turbine sections, as shown in Fig. P10.42, the first of which drives the ideal compressor, with the second producing the power output. The compressor input is at 290 K, 100 kPa, and the exit is at 450 kPa. A fraction of flow, x, bypasses the burner and the rest $(1 - x)$ goes through the burner where 1200 kJ/kg is added by combustion. The two flows then mix before entering the first turbine and continue through the second turbine, with exhaust at 100 kPa. If the mixing should result in a temperature of 1000 K into the first turbine find the fraction x. Find the required pressure and temperature into the second turbine and its specific power output.

C.V.Comp.: $-w_C = h_2 - h_1$; $s_2 = s_1$

$\qquad P_{r2} = P_{r1}\ (P_2/P_1) = 0.9899 \times (450/100) = 4.4545,$

$\qquad$ A.7.2 interpolate: $\quad T_2 = 445$ K, $\quad h_2 = 446.74$ kJ/kg,

$\qquad\qquad -w_C = 446.74 - 290.43 = 156.3$ kJ/kg

C.V.Burner: $h_3 = h_2 + q_H = 446.74 + 1200 = 1646.74$ kJ/kg

$\qquad \Rightarrow T_3 = 1509$ K

C.V.Mixing chamber: $(1 - x)h_3 + xh_2 = h_{MIX} = 1046.22$ kJ/kg

$$x = \frac{h_3 - h_{MIX}}{h_3 - h_2} = \frac{1646.74 - 1046.22}{1646.74 - 446.74} = \mathbf{0.50}$$

$\qquad \dot{W}_{T1} = \dot{W}_{C,in} \ \Rightarrow\ \dot{w}_{T1} = -w_C = 156.3 \text{ kJ/kg} = h_3 - h_4$

$\qquad h_4 = 1046.22 - 156.3 = 889.9 \text{ kJ/kg} \ \Rightarrow\ T_4 = \mathbf{861\ K}$

$\qquad P_4 = (P_{r4}/P_{rMIX})\ P_{MIX} = (51/91.65) \times 450 = \mathbf{250.4\ kPa}$

$\qquad s_4 = s_5 \ \Rightarrow\ P_{r5} = P_{r4}\ (P_5/P_4) = 51 \times (100/250.4) = 20.367$

$\qquad$ A.7.2 interpolate: $\quad h_5 = 688.2 \text{ kJ/kg}, \quad T_5 = 676$ K

$\qquad w_{T2} = h_4 - h_5 = 889.9 - 688.2 = \mathbf{201.7\ kJ/kg}$

10.80

Repeat Problem 10.73, but assume variable specific heat. The ideal gas air tables, Table A.7, are recommended for this calculation (and the specific heat from Fig. 3.26 at high temperature).

Solution:

Table A.7 is used with interpolation.

$T_1 = 283.2$ K, $u_1 = 202.3$ kJ/kg, $v_{r1} = 210.44$

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From definition of the v_r function

$v_{r2} = v_{r1} (v_2/v_1) = 210.4 (1/7) = 30.063$

Interpolate to get: $T_2 = 603.9$ K, $u_2 = 438.1$ kJ/kg

$\Rightarrow$ $-_1w_2 = u_2 - u_1 = 235.8$ kJ/kg,

$u_3 = 438.1 + 1800 = 2238.1$ $\Rightarrow$ $T_3 = \mathbf{2573.4\ K}$, $v_{r3} = 0.34118$

$P_3 = 90 \times 7 \times 2573.4 / 283.2 = \mathbf{5725\ kPa}$

Expansion 3 to 4: $s_4 = s_3$ $\Rightarrow$ From the v_r function as before

$v_{r4} = v_{r3} (v_4/v_3) = 0.34118 (7) = 2.3883$

Interpolation $\Rightarrow$ $T_4 = 1435.4$ K, $u_4 = 1145.8$ kJ/kg

$_3w_4 = u_3 - u_4 = 2238.1 - 1145.8 = 1092.3$ kJ/kg

Net work, efficiency and mep

➔ $w_{net} = {}_3w_4 + {}_1w_2 = 1092.3 - 235.8 = 856.5$ kJ/kg

$\eta_{TH} = w_{net} / q_H = 856.5 / 1800 = \mathbf{0.476}$

$v_1 = RT_1/P_1 = (0.287 \times 283.2)/90 = 0.9029$ m^3/kg

$v_2 = (1/7)\, v_1 = 0.1290$ m^3/kg

$P_{meff} = \dfrac{w_{net}}{v_1 - v_2} = 856.5 / (0.9029 - 0.129) = \mathbf{1107\ kPa}$

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Fundamentals of Thermodynamic

SOLUTION MANUAL
CHAPTER 10
English Units

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CHAPTER 10

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Brayton Cycles

10.141E

In a Brayton cycle the inlet is at 540 R, 14 psia and the combustion adds 350 Btu/lbm. The maximum temperature is 2500 R due to material considerations. Find the maximum permissible compression ratio and for that the cycle efficiency using cold air properties.

Solution:

Combustion: $h_3 = h_2 + q_H;$ $_2w_3 = 0$ and $T_{max} = T_3 = 2500$ R

$T_2 = T_3 - q_H/C_P = 2500 - 350/0.24 = 1041.67$ R

Reversible adiabatic compression leads to constant s, from Eq.6.23:

$$P_2 / P_1 = (T_2/T_1)^{\frac{k}{k-1}} = (1041.67/540)^{3.5} = \mathbf{9.9696}$$

We can find the efficiency from the compression ratio, Eq.10.1

$$\eta = 1 - T_1/T_2 = 1 - 540/1041.67 = \mathbf{0.482}$$

We can also compute all process work terms.

Reversible adiabatic expansion leads to constant s, from Eq.6.23

$$T_4 = T_3 (P_4/P_3)^{\frac{k-1}{k}} = T_3 T_1/T_2 = 2500 \text{ R} \frac{540}{1041.67} = 1296 \text{ R}$$

For net work we get

$$w_T = C_P (T_3 - T_4) = 0.24 (2500 - 1296) = 288.96 \text{ Btu/lbm}$$

$$w_C = C_P (T_2 - T_1) = 0.24 (1041.67 - 540) = 120.4 \text{ Btu/lbm}$$

$$w_{net} = w_T - w_C = 288.96 - 120.4 = 168.56 \text{ Btu/lbm}$$

$$\eta = w_{net} / q_H = 168.56 / 350 = 0.482$$

10.142E

A Brayton cycle has compression ratio of 15:1 with a high temperature of 2900 R and the inlet at 520 R, 14 psia. Use cold air properties and find the specific heat transfer input and the specific net work output.

Brayton cycle so this means:

Minimum T: $T_1 = 520$ R

Maximum T: $T_3 = 2900$ R

Pressure ratio: $P_2/P_1 = 15$

Compression in compressor: $s_2 = s_1$

$\Rightarrow$ Implemented in Eq.6.23

$$T_2 = T_1 (P_2/P_1)^{\frac{k-1}{k}} = 520 (15)^{0.286} = 1128.1 \text{ R}$$

Energy input is from the combustor

$$q_H = C_{P0}(T_3 - T_2) = 0.24 (2900 - 1128.1) = \textbf{425.3 Btu/lbm}$$

Do the overall cycle efficiency and the net work

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 15^{-0.4/1.4} = 0.5387$$

$$w_{NET} = \eta \, q_H = 0.5387 \times 425.3 = \textbf{229.1 Btu/lbm}$$

10.143E

A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 000 hp to an electric generator. The minimum temperature in the cycle is 540 R, and the exhaust temperature is 1350 R. The minimum pressure in the cycle is 1 atm, and the compressor pressure ratio is 14 to 1. Calculate the power output of the turbine, the fraction of the turbine output required to drive the compressor and the thermal efficiency of the cycle?

Brayton:

$\dot{w}_{NET}$ = 100 000 hp

P_1 = 1 atm, T_1 = 540 R

P_2/P_1 = 14, T_3 = 2900 R

Solve using constant C_{P0}:

Compression in compressor: $s_2 = s_1$ $\Rightarrow$ Implemented in Eq.6.23

$$\rightarrow T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 540(14)^{0.2857} = 1148.6 \text{ R}$$

$$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 0.24 (1148.6 - 540) = 146.1 \text{ Btu/lbm}$$

Expansion in turbine: $s_4 = s_3$ $\Rightarrow$ Implemented in Eq.6.23

$$T_3 = T_4 \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} = 1350 (14)^{0.2857} = 2869.3 \text{ R}$$

$$w_T = h_3 - h_4 = C_{P0}(T_3 - T_4) = 0.24(2869.3 - 1350) = 364.6 \text{ Btu/lbm}$$

$$w_{NET} = w_T - w_C = 364.6 - 146.1 = 218.5 \text{ Btu/lbm}$$

$$\dot{m} = \dot{W}_{NET}/w_{NET} = 100\ 000 \times 2544/218.5 = 1\ 164\ 302 \text{ lbm/h}$$

$$\dot{W}_T = \dot{m}w_T = \textbf{166 865 hp,} \qquad w_C/w_T = \textbf{0.40}$$

Energy input is from the combustor

$$q_H = C_{P0}(T_3 - T_2) = 0.24(2869.3 - 1148.6) = 413 \text{ Btu/lbm}$$

$$\eta_{TH} = w_{NET}/q_H = 218.5/413 = \textbf{0.529}$$

10.144E

A Brayton cycle produces 14 000 Btu/s with an inlet state of 60 F, 14.7 psia, and a compression ratio of 16:1. The heat added in the combustion is 480 Btu/lbm. What are the highest temperature and the mass flow rate of air, assuming cold air properties?

Solution:

Efficiency is from Eq.10.1

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 16^{-0.4/1.4} = 0.547$$

from the required power we can find the needed heat transfer

$$\dot{Q}_H = \dot{W}_{net} / \eta = \frac{14\ 000}{0.547} = 25\ 594 \text{ Btu/s}$$

$$\dot{m} = \dot{Q}_H / q_H = \frac{25\ 594 \text{ Btu/s}}{480 \text{ Btu/lbm}} = \textbf{53.32 lbm/s}$$

Temperature after compression is (Eq.6.23)
$$T_2 = T_1\ r_p^{(k-1)/k} = 519.7 \times 16^{0.4/1.4} = 1148 \text{ R}$$

The highest temperature is after combustion
$$T_3 = T_2 + q_H/C_p = 1148 + \frac{480}{0.24} = \textbf{3148 R}$$

10.145E

Do the previous problem with properties from table F.5 instead of cold air properties.

Solution:

With the variable specific heat we must go through the processes one by one to get net work and the highest temperature T_3.

From F.5: $h_1 = 124.38$ btu/lbm, $s^o_{T1} = 1.63074$ Btu/lbm R

The compression is reversible and adiabatic so constant s. From Eq.6.19

$$s_2 = s_1 \Rightarrow s^o_{T2} = s^o_{T1} + R \ln \left(\frac{P_2}{P_1}\right) = 1.63074 + \frac{53.34}{778} \ln 16$$

$$= 1.82083 \text{ Btu/lbm R}$$

back interpolate in F.5 $\Rightarrow$ $T_2 = 1133.5$ R, $h_2 = 274.58$ Btu/lbm

Energy equation with compressor work in

$$w_C = -_1w_2 = h_2 - h_1 = 274.58 - 124.383 = 150.2 \text{ Btu/lbm}$$

Energy Eq. combustor: $h_3 = h_2 + q_H = 274.58 + 480 = 754.58$ Btu/lbm

State 3: (P, h): **$T_3 = 2876.7$ R**, $s^o_{T3} = 2.07443$ Btu/lbm R

The expansion is reversible and adiabatic so constant s. From Eq.6.19

$$s_4 = s_3 \Rightarrow s^o_{T4} = s^o_{T3} + R\ln(P_4/P_3) = 2.07443 + \frac{53.34}{778} \ln(1/16) = 1.88434$$

$$\Rightarrow T_4 = 1450.6 \text{ R}, h_4 = 356.27 \text{ Btu/lbm}$$

Energy equation with turbine work out

$$w_T = h_3 - h_4 = 754.58 - 356.27 = 398.31 \text{ Btu/lbm}$$

Now the net work is

$$w_{net} = w_T - w_C = 398.31 - 150.2 = 248.11 \text{ Btu/lbm}$$

The total required power requires a mass flow rate as

$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{14\ 000}{248.11} \frac{\text{Btu/s}}{\text{Btu/lbm}} = \textbf{56.4 lbm/s}$$

10.146E

Solve Problem 10.142E using the air tables F.5 instead of cold air properties.

Brayton cycle so this means:

    Minimum T: $T_1 = 520$ R

    Maximum T: $T_3 = 2900$ R

    Pressure ratio: $P_2/P_1 = 15$

Compression in compressor: $s_2 = s_1$

    $\Rightarrow$ Implemented in Eq.6.19

$$s_2 = s_1 \quad \Rightarrow \quad s^o_{T2} = s^o_{T1} + R\ln(P_2/P_1) = 1.6307 + \frac{53.34}{778}\ln 15 = 1.8164$$

    $\Rightarrow T_2 = 1113.8$ R, $h_2 = 269.6$ Btu/lbm

$$w_C = h_2 - h_1 = 269.6 - 124.4 = 145.2 \text{ Btu/lbm}$$

Energy input is from the combustor

$$q_H = h_3 - h_2 = 761.4 - 269.6 = \textbf{491.8 Btu/lbm}$$

The expansion is reversible and adiabatic so constant s. From Eq.6.19

$$s_4 = s_3 \quad \Rightarrow \quad s^o_{T4} = s^o_{T3} + R\ln(P_4/P_3) = 2.0768 + \frac{53.34}{778}\ln(1/15) = 1.8911$$

    $\Rightarrow T_4 = 1488.5$ R, $h_4 = 366.2$ Btu/lbm

Energy equation with turbine work out

$$w_T = h_3 - h_4 = 761.3 - 366.2 = 395.1 \text{ Btu/lbm}$$

Now the net work is

$$w_{net} = w_T - w_C = 395.1 - 145.2 = \textbf{249.9 Btu/lbm}$$

[Comment: Cycle efficiency $\eta = \dfrac{\dot{W}_{net}}{\dot{Q}_H} = \dfrac{w_{net}}{q_H} = \dfrac{249.9}{491.8} = 0.508$]

10.147E

An ideal regenerator is incorporated into the ideal air-standard Brayton cycle of Problem 10.143E. Calculate the cycle thermal efficiency with this modification.

Solution:

Compression ratio

$$\frac{P_2}{P_1} = 14$$

Exhaust temperature

$$T_4 = 1350 \text{ R}$$

Solving for the cycle work terms and heat transfer from problem 10.143E:

The compression is reversible and adiabatic so constant s. From Eq.6.23

$$\rightarrow T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 540 \ (14)^{\ 0.2857} = 1148.6 \text{ R}$$

$$w_C = h_2 - h_1 = C_{P0}(T_2\text{-}T_1) = 0.24 \ (1148.6 - 540) = 146.1 \text{ Btu/lbm}$$

Expansion in turbine: $\quad s_4 = s_3 \quad \Rightarrow \quad$ Implemented in Eq.6.23

$$T_3 = T_4 \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} = 1350 \ (14)^{0.2857} = 2869.3 \text{ R}$$

$$w_T = h_3 - h_4 = C_{P0}(T_3\text{-}T_4) = 0.24(2869.3 - 1350) = 364.6 \text{ Btu/lbm}$$

$$w_{NET} = w_T - w_C = 364.6 - 146.1 = 218.5 \text{ Btu/lbm}$$

Ideal regenerator: $\quad T_X = T_4 = 1350 \text{ R}$

$$q_H = h_3 - h_X = 0.24(2869.3 - 1350) = 364.6 \text{ Btu/lbm} = w_T$$

$$\eta_{TH} = w_{NET}/q_H = 218.5/364.6 = \textbf{0.60}$$

The short answer from the efficiency of an ideal regenerator ($T_4 = T_x$) from p.471

$$\eta_{TH} = 1 - \frac{T_1}{T_3}\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 1 - \frac{T_2}{T_3} = 1 - \frac{1148.6}{2869.3} = \textbf{0.60}$$

10.148E

An air-standard Ericsson cycle has an ideal regenerator as shown in Fig. P10.44. Heat is supplied at 1800 F and heat is rejected at 150 F. Pressure at the beginning of the isothermal compression process is 10 lbf/in.^{2}. The heat added is 300 Btu/lbm. Find the compressor work, the turbine work, and the cycle efficiency.

Identify the states

Heat supplied at high temperature $\quad T_4 = T_3 = 1800\ F = 2349.7\ R$

Heat rejected at low temperature $\quad T_1 = T_2 = 150\ F = 609.7\ R$

Beginning of the compression: $\quad P_1 = 10\ lbf/in^2$

Ideal regenerator: $\quad _2q_3 = -_4q_1 \quad \Rightarrow \quad q_H = {}_3q_4 \quad \Rightarrow$

$$w_T = q_H = \textbf{300 Btu/lbm}$$

$\eta_{TH} = \eta_{CARNOT\ TH.} = 1 - T_L/T_H = 1 - 609.7/2349.7 = \textbf{0.7405}$

$w_{net} = \eta_{TH}\ q_H = 0.7405 \times 300\ Btu/lbm = 222\ Btu/lbm$

$q_L = -w_C = 300 - 222 = \textbf{78 Btu/lbm}$

10.149E

The turbine in a jet engine receives air at 2200 R, 220 lbf/in.^{2}. It exhausts to a nozzle at 35 lbf/in.^{2}, which in turn exhausts to the atmosphere at 14.7 lbf/in.^{2}. Find the nozzle inlet temperature and the nozzle exit velocity. Assume negligible kinetic energy out of the turbine and reversible processes.

Solution:

C.V. Turbine: $h_i = 560.588$ Btu/lbm, $s^o_{Ti} = 1.99765$ Btu/lbm R, $s_{es} = s_i$

Then from Eq.6.19

$$\Rightarrow s^o_{Tes} = s^o_{Ti} + R \ln(P_e/P_i) = 1.99765 + \frac{53.34}{778} \ln (35/220) = 1.8716 \frac{Btu}{lbm\ R}$$

Table F.5 $T_{es} = 1382$ R, $h_{es} = 338.27$ Btu/lbm,

Energy eq.: $w_{T,s} = h_i - h_{es} = 560.588 - 338.27 = 222.3$ Btu/lbm

C.V. Nozzle: $h_i = 338.27$ Btu/lbm, $s^o_{Ti} = 1.8716$ Btu/lbm R, $s_{es} = s_i$

Then from Eq.6.19

$$\Rightarrow s^o_{Tes} = s^o_{Ti} + R \ln(P_e/P_i) = 1.8716 + \frac{53.34}{778} \ln (\frac{14.7}{35}) = 1.81212 \frac{Btu}{lbm\ R}$$

Table F.5 $\Rightarrow T_{e,s} = 1095.0$ R, $h_{e,s} = 264.9$ Btu/lbm

Energy Eq.: $(1/2)V^2_{e,s} = h_i - h_{e,s} = 338.27 - 264.9 = 73.37$ Btu/lbm

$$V_{e,AC} = \sqrt{2 \times 25037 \times 73.37} = \textbf{1917 ft/s}$$

Recall 1 Btu/lbm = 25 037 ft^2/s^2

10.150E

An air standard refrigeration cycle has air into the compressor at 14 psia, 500 R with a compression ratio of 3:1. The temperature after heat rejection is 540 R. Find the COP and the highest cycle temperature.

From the isentropic compression/expansion processes, Eq.6.23

$$T_2/T_1 = (P_2/P_1)^{\frac{k-1}{k}} = 3^{0.2857} = 1.36874 = T_3/T_4$$

$$T_2 = 1.36874 \ T_1 = \textbf{684.4 R}$$

The COP, Eq.10.5, is

$$\beta = \frac{1}{\dfrac{T_2}{T_1} - 1} = [\ 1.36874 - 1]^{-1} = \textbf{2.712}$$

Borgnakke and Sonntag

Otto, Diesel, Stirling and Carnot Cycles

12.151E

A 4 stroke gasoline engine runs at 1800 RPM with a total displacement of 150 in.^{3} and a compression ratio of 10:1. The intake is at 520 R, 10 psia with a mean effective pressure of 90 psia. Find the cycle efficiency and power output.

Efficiency from the compression ratio

$$\eta = 1 - CR^{1-k} = 1 - 10^{-0.4} = \textbf{0.60}$$

The power output comes from speed and displacement in Eq. 10.11

$$\dot{W} = P_{meff}V_{displ}\frac{RPM}{60}\frac{1}{2} = 90\ \frac{lbf}{in^2} \times 150\ in.^3 \times \frac{1800}{60}\ (1/s) \times \frac{1}{2}$$

$$= \textbf{16 875 lbf-ft/s = 30.7 hp = 21.7 Btu/s}$$

10.152E

Air flows into a gasoline engine at 14 lbf/in.^{2}, 540 R. The air is then compressed with a volumetric compression ratio of 10:1. In the combustion process 560 Btu/lbm of energy is released as the fuel burns. Find the temperature and pressure after combustion.

Solution:

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq.6.24 and 6.25

$T_2 = T_1 (v_1/v_2)^{k-1} = 540 \text{ R} \times 10^{0.4} = 1356.4$ R

$P_2 = P_1 \times (v_1/v_2)^k = 14 \text{ lbf/in}^2 \times 10^{1.4} = 351.7$ lbf/in^2

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$T_3 = T_2 + q_H/C_v = 1356.4 + 560/0.171 = \textbf{4631 R}$

$P_3 = P_2 \times (T_3/T_2) = 351.7 \text{ lbf/in}^2 \times (4631 / 1356.4) = \textbf{1201 lbf/in}^2$

10.153E

Find the missing pressures and temperatures in the previous cycle.

Compression 1 to 2: $s_2 = s_1 \implies$ From Eq.6.24 and 6.25

$$T_2 = T_1 (v_1/v_2)^{k-1} = 540 \text{ R} \times 10^{0.4} = \textbf{1356.4 R}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 14 \text{ lbf/in}^2 \times 10^{1.4} = \textbf{351.7 lbf/in}^2$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v = 1356.4 + 560/0.171 = \textbf{4631 R}$$

$$P_3 = P_2 \times (T_3/T_2) = 351.7 \text{ lbf/in}^2 \times (4631 / 1356.4) = \textbf{1201 lbf/in}^2$$

$$T_4 = T_3 / (v_1/v_2)^{k-1} = 4631 \text{ R} \times 10^{-0.4} = \textbf{1843.6 R}$$

$$P_4 = P_3 / (v_1/v_2)^k = 1201 \text{ psi} \times 10^{-1.4} = \textbf{47.8 lbf/in}^2$$

Borgnakke and Sonntag

10.154E

To approximate an actual spark-ignition engine consider an air-standard Otto cycle that has a heat addition of 800 Btu/lbm of air, a compression ratio of 7, and a pressure and temperature at the beginning of the compression process of 13 lbf/in.^{2}, 50 F. Assuming constant specific heat, with the value from Table F.4, determine the maximum pressure and temperature of the cycle, the thermal efficiency of the cycle and the mean effective pressure.

Solution:

State 1: $v_1 = RT_1/P_1 = \dfrac{53.34 \times 510}{13 \times 144} = 14.532$ ft^3/lbm, $v_2 = v_1/7 = 2.076$ ft^3/lbm

The compression process, reversible adiabatic so then isentropic. The constant s is implemented with Eq.6.25 and Eq.6.24

$$P_2 = P_1(v_1/v_2)^k = 13(7)^{1.4} = 198.2 \text{ lbf/in}^2$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 510(7)^{0.4} = 1110.7 \text{ R}$$

The combustion process with constant volume, $q_H = 800$ Btu/lbm

$$T_3 = T_2 + q_H/C_{V0} = 1110.7 + 800/0.171 = \textbf{5789 R}$$

$$P_3 = P_2 T_3/T_2 = 198.2 \times 5789/1110.7 = \textbf{1033 lbf/in}^\textbf{2}$$

Cycle efficiency from the ideal cycle as in Eq.10.12

$$\eta_{TH} = 1 - (T_1/T_2) = 1 - 510/1110.7 = \textbf{0.541}$$

To get the mean effective pressure we need the net work

$$w_{NET} = \eta_{TH} \times q_H = 0.541 \times 800 = 432.8 \text{ Btu/lbm}$$

$$P_{meff} = \frac{w_{NET}}{v_1 - v_2} = \frac{432.8 \times 778}{(14.532-2.076) \times 144} = \textbf{188 lbf/in}^\textbf{2}$$

10.155E

A 4 stroke 85 in^3. gasoline engine runs at 2500 RPM has an efficiency of 60%. The state before compression is 6 psia, 500 R and after combustion it is at 4400 R. Find the highest T and P in the cycle, the specific heat transfer added, the cycle mep and the total power produced.

This is a basic 4-stroke Otto cycle.
Compression ratio from the efficiency

$$\eta = 1 - CR^{1-k} \quad => \quad CR = (1 - \eta)^{-1/0.4} = 9.882$$

Compression: $T_2 = T_1(CR)^{k-1} = 500 \times (9.882)^{0.4} = 1250$ R

$$P_2 = P_1 \, CR^k = 6 \times 9.882^{1.4} = 148.2 \text{ psia}$$

Combustion $v_3 = v_2$. Highest T and P are after combustion.

$$T_3 = \textbf{4400 R}, \quad P_3 = P_2 \, T_3 / T_2 = \frac{148.2 \times 4400}{1250} = \textbf{521.7 psia}$$

$$q_H = u_3 - u_2 = C_v(T_3 - T_2) = 0.171 \, (4400 - 1250) = \textbf{538.65 Btu/lbm}$$

$$w_{net} = \eta_{TH} \times q_H = 0.6 \times 538.65 = 323.19 \text{ Btu/lbm}$$

Displacement and P_{meff}

$$v_1 = RT_1/P_1 = 53.34 \text{ ft-lbf/lbm-R} \times 500 \text{ R}/(144 \times 6 \text{ psia}) = 30.868 \text{ ft}^3/\text{lbm}$$

$$v_2 = (1/9.882) \, v_1 = 3.124 \text{ ft}^3/\text{lbm}$$

$$P_{meff} = \frac{w_{NET}}{v_1 - v_2} = \frac{323.19 \times 778}{(30.868 - 3.124) \, 144} = \textbf{62.937 psia}$$

Now we can find the power from Eq.10.11

$$\dot{W} = P_{meff} \, V_{displ} \frac{RPM}{60} \frac{1}{2} = 62.937 \times \frac{85}{12} \times \frac{2500}{60} \times \frac{1}{2} = \textbf{9287.6 lbf-ft/s}$$

$$= \textbf{16.9 hp}$$

Recall 1 hp = 550 lbf-ft/s.

10.156E

A for stroke gasoline engine has a compression ratio of 10:1 with 4 cylinders of total displacement 75 in^3. the inlet state is 500 R, 10 psia and the engine is running at 2100 RPM with the fuel adding 600 Btu/lbm in the combustion process. What is the net work in the cycle and how much power is produced?

Solution:

Overall cycle efficiency is from Eq.10.12, $r_v = v_1/v_2$

$$\eta_{TH} = 1 - r_v^{1-k} = 1 - 10^{-0.4} = \textbf{0.602}$$

$$w_{net} = \eta_{TH} \times q_H = 0.602 \times 600 = 361.2 \text{ Btu/lbm}$$

We also need specific volume to evaluate Eqs.10.9 to 10.11

$$v_1 = RT_1 / P_1 = 53.34 \times 500 / (10 \times 144) = 18.52 \text{ ft}^3/\text{lbm}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{w_{net}}{v_1 \left(1 - \dfrac{1}{r_v}\right)} = \frac{361.2}{18.52 \times 0.9} \frac{778}{144} = 117.1 \text{ psia}$$

Now we can find the power from Eq.10.11

$$\dot{W} = P_{meff} \, V_{displ} \frac{RPM}{60} \frac{1}{2} = 117.1 \times \frac{75}{12} \times \frac{2100}{60} \times \frac{1}{2} = \textbf{12 808 lbf-ft/s}$$

$$= \textbf{23 hp}$$

Recall 1 hp = 550 lbf-ft/s.

10.157E

An Otto cycle has the lowest T as 520 R and the lowest P as 22 psia, the highest T is 4500 R and combustion adds 500 Btu/lbm as heat transfer. Find the compression ratio and the mean effective pressure.

Solution:

Identify states: $T_1 = 520$ R, $P_1 = 22$ psia, $T_3 = 4500$ R, $q_H = 500$ Btu/lbm

Combustion: $q_H = u_3 - u_2 = C_{vo}(T_3 - T_2) = 500$ Btu/lbm

$$T_2 = T_3 - q_H / C_{vo} = 4500 - 500/0.171 = 1576 \text{ R}$$

Compression: $CR = r_v = v_1/v_2 = (T_2/T_1)^{1/(k-1)} = \left(\dfrac{1576}{520}\right)^{2.5} = \mathbf{15.99}$

Overall cycle efficiency is from Eq.10.12, $r_v = v_1/v_2 = 15.99$

$$\eta_{TH} = 1 - r_v^{1-k} = 1 - T_1/T_2 = 1 - 520/1576 = 0.670$$

$$w_{net} = \eta_{TH} \times q_H = 0.670 \times 500 = 335 \text{ Btu/lbm}$$

We also need specific volume to evaluate Eqs.10.9 to 10.11

$$v_1 = RT_1 / P_1 = (53.34/144) \times 520 / 22 = 8.755 \text{ ft}^3/\text{lbm}$$

$$v_2 = v_1/CR = 0.5475 \text{ ft}^3/\text{lbm}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{335}{8.755 - 0.5475} \frac{\text{Btu/lbm}}{\text{ft}^3/\text{lbm}} \times \frac{778 \text{ lbf-ft/Btu}}{144 \text{ (in/ft)}^2}$$

$$= \mathbf{220.5 \text{ psia}}$$

Comment: This is too high a compression ratio for a standard Otto cycle.

10.158E

A gasoline engine has a volumetric compression ratio of 10 and before compression has air at 520 R, 12.2 psia in the cylinder. The combustion peak pressure is 900 psia. Assume cold air properties. What is the highest temperature in the cycle? Find the temperature at the beginning of the exhaust (heat rejection) and the overall cycle efficiency.

Solution:

Compression. Isentropic so we use Eqs.6.24-6.25

$$P_2 = P_1(v_1/v_2)^k = 12.2\ (10)^{1.4} = 306.45 \text{ psia}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 520\ (10)^{0.4} = 1306.2 \text{ R}$$

Combustion. Constant volume

$$T_3 = T_2\ (P_3/P_2) = 1306.2 \times 900/306.45 = \textbf{3836 R}$$

Exhaust. Isentropic expansion so from Eq.6.23

$$T_4 = T_3 / (v_1/v_2)^{k-1} = T_3 / 10^{0.4} = 3836 / 2.5119 = \textbf{1527 R}$$

Overall cycle efficiency is from Eq.10.12, $r_v = v_1/v_2$

$$\eta = 1 - r_v^{1-k} = 1 - 10^{-0.4} = \textbf{0.602}$$

Comment: No actual gasoline engine has an efficiency that high, maybe 35%.

Borgnakke and Sonntag

10.159E

The cycle in Problem 10.157E is used in a 150 in.^{3} engine running at 1800 RPM. How much power does it produce?

Identify states: $T_1 = 520$ R, $P_1 = 12$ psia, $T_3 = 4500$ R, $q_H = 500$ Btu/lbm

Combustion: $q_H = u_3 - u_2 = C_{vo}(T_3 - T_2) = 500$ Btu/lbm

$$T_2 = T_3 - q_H / C_{vo} = 4500 - 500/0.171 = 1576 \text{ R}$$

Compression: $CR = r_v = v_1/v_2 = (T_2/T_1)^{1/(k-1)} = \left(\dfrac{1576}{520}\right)^{2.5} = \mathbf{15.99}$

Overall cycle efficiency is from Eq.10.12, $r_v = v_1/v_2 = 15.99$

$$\eta_{TH} = 1 - r_v^{1-k} = 1 - T_1/T_2 = 1 - 520/1576 = 0.670$$

$$w_{net} = \eta_{TH} \times q_H = 0.670 \times 500 = 335 \text{ Btu/lbm}$$

We also need specific volume to evaluate Eqs.10.9 to 10.11

$$v_1 = RT_1 / P_1 = (53.34/144) \times 520 / 22 = 8.755 \text{ ft}^3/\text{lbm}$$

$$v_2 = v_1/CR = 0.5475 \text{ ft}^3/\text{lbm}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{335}{8.755 - 0.5475} \frac{\text{Btu/lbm}}{\text{ft}^3/\text{lbm}} \times \frac{778 \text{ lbf-ft/Btu}}{144 \text{ (in/ft)}^2}$$

$$= 220.5 \text{ psia}$$

Now we can find the power from Eq.10.11

$$\dot{W} = P_{meff} V_{displ} \frac{RPM}{60} \frac{1}{2} = 220.5 \frac{\text{lbf}}{\text{in}^2} \times 150 \text{ in.}^3 \times \frac{1800}{60} (1/s) \times \frac{1}{2}$$

$$= \mathbf{41\ 344\ lbf\text{-}ft/s = 75\ hp = 53\ Btu/s}$$

Borgnakke and Sonntag

10.160E

It is found experimentally that the power stroke expansion in an internal combustion engine can be approximated with a polytropic process with a value of the polytropic exponent n somewhat larger than the specific heat ratio k. Repeat Problem 10.154E but assume the expansion process is reversible and polytropic (instead of the isentropic expansion in the Otto cycle) with n equal to 1.50.

First find states 2 and 3. Based on the inlet state we get

$$v_4 = v_1 = RT_1/P_1 = 53.34 \times 510/13 \times 144 = 14.532 \text{ ft}^3/\text{lbm}$$

$$v_3 = v_2 = v_1/7 = 2.076 \text{ ft}^3/\text{lbm}$$

After compression we have constant s leads to Eq.6.24 and Eq.6.25

$$P_2 = P_1(v_1/v_2)^k = 13(7)^{1.4} = 198.2 \text{ lbf/in}^2$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 510(7)^{0.4} = 1110.7 \text{ R}$$

Constant volume combustion

$$T_3 = T_2 + q_H/C_{V0} = 1110.7 + 800/0.171 = 5789 \text{ R}$$

$$P_3 = P_2T_3/T_2 = 198.2 \times 5789/1110.7 = 1033 \text{ lbf/in}^2$$

Process 3 to 4: $Pv^{1.5} = $ constant.

$$P_4 = P_3(v_3/v_4)^{1.5} = 1033(1/7)^{1.5} = 55.78 \text{ lbf/in}^2$$

$$T_4 = T_3(v_3/v_4)^{0.5} = 5789(1/7)^{0.5} = 2188 \text{ R}$$

For the mean effective pressure we need the net work and therefore the induvidual process work terms

$$_1w_2 = \int P \, dv = R(T_2 - T_1)/(1 - 1.4)$$

$$= -53.34(1110.7 - 510)/(0.4 \times 778) = -102.96 \text{ Btu/lbm}$$

$$_3w_4 = \int P \, dv = R(T_4 - T_3)/(1 - 1.5)$$

$$= -53.34(2188 - 5789)/(0.5 \times 778) = 493.8 \text{ Btu/lbm}$$

$$w_{NET} = 493.8 - 102.96 = 390.84 \text{ Btu/lbm}$$

$$\eta_{CYCLE} = w_{NET}/q_H = 390.84/700 = \textbf{0.488}$$

$$P_{meff} = w_{NET}/(v_1 - v_2) = 390.84 \times 778/(14.532 - 2.076) = \textbf{169.5 lbf/in}^2$$

Notice a smaller w_{NET}, η_{CYCLE}, P_{meff} compared to ideal cycle.

10.161E

In the Otto cycle all the heat transfer q_H occurs at constant volume. It is more realistic to assume that part of q_H occurs after the piston has started its downwards motion in the expansion stroke. Therefore consider a cycle identical to the Otto cycle, except that the first two-thirds of the total q_H occurs at constant volume and the last one-third occurs at constant pressure. Assume the total q_H is 700 Btu/lbm, that the state at the beginning of the compression process is 13 lbf/in.^{2}, 68 F, and that the compression ratio is 9. Calculate the maximum pressure and temperature and the thermal efficiency of this cycle. Compare the results with those of a conventional Otto cycle having the same given variables.

$P_1 = 13$ psia, $T_1 = 527.67$ R

$r_V = v_1/v_2 = 7$

$q_{23} = \dfrac{2}{3} \times 700 = 466.7 \dfrac{\text{Btu}}{\text{lbm}}$

$q_{34} = \dfrac{1}{3} \times 700 = 233.3 \dfrac{\text{Btu}}{\text{lbm}}$

Constant s compression, Eqs.6.24-25

$$P_2 = P_1(v_1/v_2)^k = 13(9)^{1.4} = 281.8 \text{ lbf/in}^2$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 527.67(9)^{0.4} = 1270.7 \text{ R}$$

Constant v combustion

$$T_3 = T_2 + q_{23}/C_{V0} = 1270.7 + 466.7/0.171 = 4000 \text{ R}$$

$$P_3 = P_2(T_3/T_2) = 281.8 \times 4000/1270.7 = \textbf{887.1 lbf/in}^2 = P_4$$

Constant P combustion

$$T_4 = T_3 + q_{34}/C_{P0} = 4000 + 233.3/0.24 = \textbf{4972 R}$$

Remaining expansion: $\dfrac{v_5}{v_4} = \dfrac{v_1}{v_4} = (P_4/P_1) \times (T_1/T_4) = \dfrac{88.1}{13} \times \dfrac{527.67}{4972} = 7.242$

$$T_5 = T_4(v_4/v_5)^{k-1} = 4972(1/7.242)^{0.4} = 2252 \text{ R}$$

$$q_L = C_{V0}(T_5 - T_1) = 0.171(2252 - 527.67) = 294.9 \text{ Btu/lbm}$$

$$\eta_{TH} = 1 - q_L/q_H = 1 - 294.9/700 = \textbf{0.579}$$

Standard Otto cycle: $\eta_{TH} = 1 - (9)^{-0.4} = \textbf{0.585}$

10.162E

A diesel engine has a bore of 4 in., a stroke of 4.3 in. and a compression ratio of 19:1 running at 2000 RPM (revolutions per minute). Each cycle takes two revolutions and has a mean effective pressure of 200 lbf/in.^{2}. With a total of 6 cylinders find the engine power in Btu/s and horsepower, hp.

Solution:

Work from mean effective pressure, Eq.10.9.

$$P_{meff} = w_{net} / (v_{max} - v_{min}) \quad -> \quad w_{net} = P_{meff} (v_{max} - v_{min})$$

The displacement is

$$\Delta V = \pi Bore^2 \times 0.25 \times S = \pi \times 4^2 \times 0.25 \times 4.3 = 54.035 \text{ in}^3$$

Work per cylinder per power stroke, Eq.10.10

$$W = P_{meff}(V_{max} - V_{min}) = 200 \times 54.035 / (12 \times 778) = 1.1575 \text{ Btu/cycle}$$

Only every second revolution has a power stroke so we can find the power, see also Eq.10.11

$$\dot{W} = W \times N_{cyl} \times RPM \times 0.5 \ (\frac{cycles}{min}) \ (\frac{min}{60 \text{ s}}) \ (\frac{Btu}{cycle})$$

$$= 1.1575 \times 6 \times 2000 \times 0.5 \times (1/60) = 115.75 \text{ Btu/s}$$

$$= 115.75 \times 3600/2544.43 \text{ hp} = \textbf{164 hp}$$

The conversion factor from Btu/s to hp is from Table A.1 under power.

10.163E

A super charger is used for a diesel engine so intake is 30 psia, 580 R. The cycle has compression ratio of 18:1 and the mean effective pressure is 120 psia. If the engine is 600 in.^{3} running at 200 RPM find the power output.

Solution:

The power is from Eq.10.11

$\dot{W} = m\, w_{net} \times N_{cyl} \times RPM \times 1/60$

$\quad = m\, P_{meff}\, (v_1 - v_2) \times N_{cyl} \times RPM/60 = P_{meff}\, V_{displ}\, RPM/60$

$\quad = 120\, \dfrac{lbf}{in^2} \times 600\ in.^3 \times (200\,/\,60)\ (1/s)$

$\quad = $ **20 000 lbf-ft/s = 25.7 Btu/s = 36.6 hp**

Comment: multiply with ½ if this is a four stroke engine

10.164E

At the beginning of compression in a diesel cycle $T = 540$ R, $P = 30$ lbf/in.^{2} and the state after combustion (heat addition) is 2600 R and 1000 lbf/in.^{2}. Find the compression ratio, the thermal efficiency and the mean effective pressure.

Solution:

Compression process (isentropic) from Eq.6.28 or p295

$$P_2 = P_3 = 1000 \text{ lbf/in}^2 \implies v_1/v_2 = (P_2/P_1)^{1/k} = (1000/30)^{0.7143} = \textbf{12.24}$$

$$T_2 = T_1(P_2/P_1)^{(k-1)/k} = 540(1000/30)^{0.2857} = 1470.6 \text{ R}$$

Expansion process (isentropic)   first get the volume ratios

$$v_3/v_2 = T_3/T_2 = 2600/1470.6 = 1.768$$

$$v_4/v_3 = v_1/v_3 = (v_1/v_2)(v_2/v_3) = 12.24/1.768 = 6.923$$

The exhaust temperature follows from state 3 and constant s, Eq.6.24

$$T_4 = T_3(v_3/v_4)^{k-1} = 2600*6.923^{-0.4} = 1199 \text{ R}$$

$$q_L = C_V(T_4 - T_1) = 0.171(1199-540) = 112.7 \text{ Btu/lbm}$$

$$q_H = h_3 - h_2 = C_P(T_3 - T_2) = 0.24(2600 - 1470.6) = 271.1 \text{ Btu/lbm}$$

Overall performance

$$\eta = 1 - q_L/q_H = 1 - 112.7 / 271.1 = \textbf{0.5843}$$

$$w_{net} = q_{net} = 271.1 - 112.7 = 158.4 \text{ Btu/lbm}$$

$$v_{max} = v_1 = RT_1/P_1 = 53.34 \times 540/(30 \times 144) = 6.6675 \text{ ft}^3/\text{lbm}$$

$$v_{min} = v_{max}(v_1/v_2) = 6.6675 / 12.24 = 0.545 \text{ ft}^3/\text{lbm}$$

$$P_{meff} = \frac{w_{net}}{v_{max} - v_{min}} = \frac{158.4}{6.6675 - 0.545} \times \frac{778}{144} = \textbf{139.8 lbf/in}^2$$

Remark: This is a too low compression ratio for a practical diesel cycle.

10.165E

The **cut off ratio** is the ratio of v_3/v_2, see Fig. 10.18, which is the expansion while combustion occurs at constant pressure. Determine this ratio for the cycle in Problem 10.164E.

Compression process (isentropic) from Eqs.6.23-25

$$T_2 = T_1(P_2/P_1)^{(k-1)/k} = 540(1000/30)^{0.2857} = 1470.6 \text{ R}$$

Cut-off ratio:　$v_3 / v_2 = T_3 /T_2 = 2600/1470.6 = \mathbf{1.768}$

10.166E

A diesel cycle has state 1 as 14 lbf/in.^{2}, 63 F and a compression ratio of 20. For a maximum temperature of 4000 R find the cycle efficiency.

Diesel cycle: P_1 = 14 psia, T_1 = 522.67 R, CR = v_1/v_2 = 20

From the inlet state and the compression we get

$$T_2 = T_1(v_1/v_2)^{k-1} = 522.67 \text{ R } (20)^{0.4} = 1732.4 \text{ R}$$

$$v_1 = \frac{53.34 \times 522.67}{14 \times 144} = 13.829 \text{ ft}^3/\text{lbm}, \qquad v_2 = \frac{13.829}{20} = 0.6915 \text{ ft}^3/\text{lbm}$$

Constant pressure combustion relates v_3 and T_3

$$v_3 = v_2 \times T_3/T_2 = 0.6915 \times 4000/1732.4 = 1.5966 \text{ ft}^3/\text{lbm}$$

Expansion process (isentropic and $v_4 = v_1$) from Eq.6.24

$$T_4 = T_3(v_3 / v_4)^{0.4} = 4000 \text{ R } (1.5966 / 13.829)^{0.4} = 1686.6 \text{ R}$$

Cycle net work and efficiency

$$\eta_{TH} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{1686.6 - 522.67}{1.4 \ (4000 - 1732.4)} = \mathbf{0.633}$$

10.167E

Consider an ideal Stirling-cycle engine in which the pressure and temperature at the beginning of the isothermal compression process are 14.7 lbf/in.^{2}, 80 F, the compression ratio is 6, and the maximum temperature in the cycle is 2000 F. Calculate the maximum pressure in the cycle and the thermal efficiency of the cycle with and without regenerators.

Ideal Stirling cycle
$T_1 = T_2 = 80$ F
$P_1 = 14.7$ lbf/in^2
$\dfrac{v_1}{v_2} = 6$
$T_3 = T_4 = 2000$ F

Isothermal compression (heat goes out)

$$T_1 = T_2 \rightarrow \quad P_2 = P_1 \times v_1/v_2 = 14.7 \times 6 = 88.2$$

$$w_{12} = q_{12} = -RT_1 \ln \frac{v_1}{v_2} = -\frac{53.34}{778} \times 540 \ln 6 = -66.3 \text{ Btu/lbm}$$

Constant volume heat addition

$$V_2 = V_3 \rightarrow P_3 = P_2 \times T_3/T_2 = 88.2 \times \frac{2460}{540} = \textbf{401.8 lbf/in}^2$$

$$q_{23} = C_{V0}(T_3 - T_2) = 0.171 (2460 - 540) = 328.3 \text{ Btu/lbm}$$

Isothermal expansion (heat comes in)

$$w_{34} = q_{34} = RT_3 \ln (v_4/v_3)$$

$$= (53.34/778) \times 2460 \ln 6 = 302.2 \text{ Btu/lbm}$$

$$w_{NET} = 302.2 - 66.3 = 235.9 \text{ Btu/lbm}$$

Efficiency without regenerator, (q_{23} and q_{34} are coming in from source)

$$\eta_{NO\ REGEN} = \frac{235.9}{302.2 + 328.3} = \textbf{0.374,}$$

Efficiency with regenerator, (Now only q_{34} is coming in from source)

$$\eta_{WITH\ REGEN} = \frac{235.9}{302.2} = \textbf{0.781}$$

10.168E

Air in a piston/cylinder goes through a Carnot cycle in which $T_L = 560$ R and the total cycle efficiency is $\eta = 2/3$. Find T_H, the specific work and volume ratio in the adiabatic expansion for constant C_p, C_v.

Carnot cycle efficiency from Eq.5.5

$$\eta = 1 - T_L/T_H = 2/3 \quad \Rightarrow \quad T_H = 3 \times T_L = 3 \times 560 = \textbf{1680 R}$$

Adiabatic expansion 3 to 4: $Pv^k = $ constant, work from Eq.6.29 (n = k)

$$_3w_4 = (P_4v_4 - P_3v_3)/(1 - k) = [R/(1-k)](T_4 - T_3) = u_3 - u_4$$

$$= C_v(T_3 - T_4) = 0.171 \text{ Btu/lbm-R } (1680 - 540) \text{ R} = \textbf{194.94 Btu/lbm}$$

$$v_4/v_3 = (T_3/T_4)^{1/(k-1)} = 3^{2.5} = \textbf{15.6}$$

10.169E

Do the previous problem 10.168E using Table F.5.

Air in a piston/cylinder goes through a Carnot cycle in which $T_L = 560$ R and the total cycle efficiency is $\eta = 2/3$. Find T_H, the specific work and volume ratio in the adiabatic expansion for constant C_p, C_v.

Carnot cycle:

$$\eta = 1 - T_L/T_H = 2/3 \implies T_H = 3 \times T_L = 3 \times 560 = \mathbf{1680\ R}$$

$$_3w_4 = u_3 - u_4 = 302.124 - 95.589 = \mathbf{206.535\ Btu/lbm}$$

Adiabatic expansion 3 to 4: $\quad s_4 = s_3 \implies$ Eq.6.19

$$s^o_{T4} = s^o_{T3} + R \ln \frac{P_4}{P_3} \implies \quad \text{Table F.5 for standard entropy}$$

$$\frac{P_4}{P_3} = \exp\left[(s^o_{T4} - s^o_{T3})/R\right] = \exp\left[\frac{1.64852 - 1.92338}{53.34/778}\right] = 0.018151$$

Ideal gas law then gives

$$\frac{v_4}{v_3} = \frac{T_4}{T_3} \times \frac{P_3}{P_4} = \frac{560}{1680} \times \frac{1}{0.018151} = \mathbf{18.36}$$

Atkinson and Miller cycles

10.170E

An Atkinson cycle has state 1 as 20 psia, 540 R, compression ratio of 9 and expansion ratio of 14. Find the needed heat release in the combustion.

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq. 6.24 and Eq. 6.25

$$T_2 = T_1 \, (v_1/v_2)^{k-1} = 540 \times 9^{\,0.4} = 1300.4 \text{ R}$$

$$P_2 = P_1 \times (v_1/v_2)^{k} = 20 \times 9^{\,1.4} = 433.5 \text{ psi}$$

The expansion backwards from $P_4 = P_1$ gives

$$P_3 = P_4 \times (v_4/v_3)^{k} = P_4 \times CR^{k} = 20 \times 14^{1.4} = 804.65 \text{ psi}$$

Combustion 2 to 3 at constant volume: $v_3 = v_2$ $\Rightarrow$ $P_3 = P_2 \times (T_3/T_2)$

Energy Eq.: $u_3 = u_2 + q_H$ $\Rightarrow$ $q_H = C_v \, (T_3 - T_2)$

Solve for T_3 from process equation

$$T_3 = T_2 \times (P_3 \, / \, P_2) = 1300.4 \text{ R} \times (804.65 \, / \, 433.5) = 2413.8 \text{ R}$$

Solve for q_H from the energy equation

$$q_H = C_v \, (T_3 - T_2) = 0.171 \, (2413.8 - 1300.4) = \textbf{190.4 Btu/lbm}$$

10.171E

Assume we change the Otto cycle in Problem 10.152E to an Atkinson cycle by keeping the same conditions and only increase the expansion to give a different state 4. Find the expansion ratio and the cycle efficiency.

Solution:

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq.6.24 and Eq.6.25

$$T_2 = T_1 (v_1/v_2)^{k-1} = 540 \times 10^{0.4} = 1356.4 \text{ R}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 14 \times 10^{1.4} = 351.7 \text{ psi}$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v = 1356.4 + 560/0.171 = \textbf{4631 R}$$

$$P_3 = P_2 \times (T_3/T_2) = 351.7 \text{ lbf/in}^2 \times (4631 / 1356.4) = \textbf{1201 lbf/in}^2$$

The expansion should now bring pressure down so $P_4 = P_1$

$$P_4 = P_1 = P_3 \times (v_3/v_4)^k = P_3 \times CR^{-k} \implies CR = (P_3 /P_4)^{1/k}$$

$$CR = (1201 / 14)^{0.71429} = \textbf{24.04}$$

So now the efficiency becomes

$$\eta = 1 - k \frac{CR - CR_1}{CR^k - CR_1^k} = 1 - 1.4 \frac{24.04 - 10}{24.04^{\ k} - 10^{\ k}} = \textbf{0.676}$$

10.172E

An Atkinson cycle has state 1 as 20 psia, 540 R, compression ratio of 9 and expansion ratio of 14. Find the mean effective pressure.

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq.6.24 and Eq.6.25

$$T_2 = T_1 \, (v_1/v_2)^{k-1} = 540 \times 9^{\,0.4} = 1300.4 \text{ R}$$

$$P_2 = P_1 \times (v_1/v_2)^{k} = 20 \times 9^{\,1.4} = 433.5 \text{ psia}$$

The expansion backwards from $P_4 = P_1$ gives

$$P_3 = P_4 \times (v_4/v_3)^{k} = P_4 \times CR^{k} = 20 \times 14^{1.4} = 804.65 \text{ psia}$$

Combustion 2 to 3 at constant volume: $v_3 = v_2 \Rightarrow P_3 = P_2 \times (T_3/T_2)$

Energy Eq.: $u_3 = u_2 + q_H$ $\Rightarrow$ $q_H = C_v \, (T_3 - T_2)$

Solve for T_3 from process equation

$$T_3 = T_2 \times (P_3 / P_2) = 1300.4 \times (804.65 / 433.5) = 2413.8 \text{ R}$$

Solve for q_H from the energy equation

$$q_H = C_v \, (T_3 - T_2) = 0.171 \, (2413.8 - 1300.4) = 190.4 \text{ Btu/lbm}$$

The efficiency and net work become

$$\eta_{TH} = 1 - \; k \, \frac{CR - CR_1}{CR^{k} - CR_1^{k}} = 1 - 1.4 \, \frac{14 - 9}{14^{\,k} - 9^{\,k}} = 0.623$$

$$w_{net} = \eta_{TH} \times q_H = 0.623 \times 190.4 = 118.6 \text{ Btu/lbm}$$

Displacement (we take this as $v_4 - v_2$) and P_{meff}

$$v_1 = RT_1/P_1 = 53.34 \times 540/(20 \times 144) = 10.00 \text{ ft}^3/\text{lbm}$$

$$v_2 = (1/9) \, v_1 = 1.111 \text{ ft}^3/\text{lbm}; \; v_4 = 14 \, v_2 = 15.5575 \text{ ft}^3/\text{lbm}$$

$$P_{meff} = \frac{w_{NET}}{v_4 - v_2} = \frac{118.6}{15.5575 - 1.111} \frac{778}{144} = \textbf{44.35 psia}$$

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10.173E

A Miller cycle has state 1 as 20 psia, 540 R, compression ratio of 9 and expansion ratio of 14. If P_4 is 30 psia find the heat release in the combustion.

Solution:

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1$ $\Rightarrow$ From Eq.6.24 and Eq.6.25

$$T_2 = T_1 (v_1/v_2)^{k-1} = 540 \times 9^{0.4} = 1300.4 \text{ R}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 20 \times 9^{1.4} = 433.5 \text{ psia}$$

The expansion backwards from P_4 gives

$$P_3 = P_4 \times (v_4/v_3)^k = P_4 \times CR^k = 30 \times 14^{1.4} = 1207 \text{ psia}$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v \quad \text{and} \quad P_3 = P_2 \times (T_3/T_2)$$

$$T_3 = T_2 \times (P_3 / P_2) = 1300.4 \times (1207 / 433.5) = 3620.7 \text{ R}$$

$$q_H = C_v (T_3 - T_2) = 0.171 (3620.7 - 1300.4) = \mathbf{396.8 \text{ Btu/lbm}}$$

10.174E

A Miller cycle has state 1 as 20 psia, 540 R, compression ratio of 9 and a heat release of 430 Btu/lbm. Find the needed expansion ratio so P_4 is 30 psia.

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1 \implies$ From Eq.6.24 and Eq.6.25

$$T_2 = T_1 (v_1/v_2)^{k-1} = 540 \times 9^{0.4} = 1300.4 \text{ R}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 20 \times 9^{1.4} = 433.5 \text{ psia}$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v = 1300.4 + 430 / 0.171 = 3815 \text{ R}$$

$$P_3 = P_2 \times (T_3/T_2) = 433.5 (3815 / 1300.4) = 1271.8 \text{ psia}$$

The expansion should now bring pressure down to P_4

$$P_4 = P_3 \times (v_3/v_4)^k = P_3 \times CR^{-k} \implies CR = (P_3 /P_4)^{1/k}$$

$$CR = (1271.8 / 30)^{0.71429} = \mathbf{14.53}$$

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Exergy, Combined Cycles and Review

Borgnakke and Sonntag

10.175E

Consider the Brayton cycle in problem 10.144E. Find all the flows and fluxes of exergy and find the overall cycle second-law efficiency. Assume the heat transfers are internally reversible processes and we then neglect any external irreversibility.

Solution:

Efficiency is from Eq.10.1

$$\eta = \dot{W}_{NET}/\dot{Q}_H = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 16^{-0.4/1.4} = \textbf{0.547}$$

from the required power we can find the needed heat transfer

$$\dot{Q}_H = \dot{W}_{net} / \eta = 14\ 000 / 0.547 = 25\ 594\ \text{Btu/s}$$

$$\dot{m} = \dot{Q}_H / q_H = 25\ 594\ \text{(Btu/s)} / 400\ \text{Btu/lbm} = \textbf{63.99 lbm/s}$$

Temperature after compression is

$$T_2 = T_1\ r_p^{(k-1)/k} = 519.67 \times 16^{0.4/1.4} = 1148\ \text{R}$$

The highest temperature is after combustion

$$T_3 = T_2 + q_H/C_p = 1148 + \frac{400}{0.24} = \textbf{2815 R}$$

For the exit flow I need the exhaust temperature

$$T_4 = T_3\ r_p^{-(k-1)/k} = 2815 \times 16^{-0.2857} = 1274.8\ \text{R}$$

The high T exergy input from combustion is

$$\dot{\Phi}_H = \dot{m}(\psi_3 - \psi_2) = \dot{m}[h_3 - h_2 - T(s_3 - s_2)]$$

$$= 63.99\ [400 - 536.67 \times 0.24\ \ln\ (\frac{2815}{1148})] = \textbf{17 895 Btu/s}$$

Since the low T exergy flow out is lost the second law efficiency is

$$\eta_{II} = \dot{W}_{NET}/\dot{\Phi}_H = 14\ 000 / 17\ 895 = \textbf{0.782}$$

$$\dot{\Phi}_{\text{flow out}} = \dot{m}(\psi_4 - \psi_o) = \dot{m}[h_4 - h_o - T(s_4 - s_o)]$$

$$= 63.99\ [\ 0.24(1274.8 - 536.7) - 536.7 \times 0.24\ \ln\ (\frac{1274.8}{536.7})\ ] = \textbf{4205 Btu/s}$$

$$\dot{\Phi}_{\text{flow in}} = \dot{m}(\psi_1 - \psi_o) = \dot{m}[h_1 - h_o - T(s_1 - s_o)]$$

$$= 63.99\ [\ 0.24(60 - 77) - 536.7 \times 0.24\ \ln\ (\frac{519.7}{536.7})\ ] = \textbf{4.2 Btu/s}$$

10.176E

The Brayton cycle in problem 10.141E has a heat addition of 350 Btu/lbm. What is the exergy increase in this process?

Solution:

Combustion: $h_3 = h_2 + q_H$; $\quad {}_2w_3 = 0$ and $T_{max} = T_3 = 2500$ R

$\qquad T_2 = T_3 - q_H/C_P = 2500 - 350/0.24 = 1041.67$ R

Now the exergy increase from 2 to 3 becomes $(P_3 = P_2)$

$$\psi_3 - \psi_2 = h_3 - h_2 - T_0(s_3 - s_2) = q_H - T_0\, C_P \ln(T_3/T_2)$$

$$= 350 - 536.7 \times 0.24 \ln\left(\frac{2500}{1041.67}\right)$$

$$= \mathbf{237.2\ Btu/lbm}$$

10.177E

Solve Problem 10.149E assuming an isentropic turbine efficiency of 85% and a nozzle efficiency of 95%.

Solution:

C.V. Turbine: $h_i = 560.588$ Btu/lbm, $s_{Ti}^o = 1.99765$ Btu/lbm R, $s_{es} = s_i$

Then from Eq.6.19

$$\Rightarrow\ s_{Tes}^o = s_{Ti}^o + R\ \ln(P_e/P_i) = 1.99765 + \frac{53.34}{778}\ \ln\ (35/220) = 1.8716\ \frac{Btu}{lbm\ R}$$

Table F.5 $T_{es} = 1382$ R, $h_{es} = 338.27$ Btu/lbm,

Energy eq.: $w_{T,s} = h_i - h_{es} = 560.588 - 338.27 = 222.3$ Btu/lbm

Eq.7.27: $w_{T,AC} = w_{T,s} \times \eta_T = 188.96 = h_i - h_{e,AC}$ $\Rightarrow h_{e,AC} = 371.6$

Table F.5 $\Rightarrow$ $\mathbf{T_{e,AC} = 1509\ R}$, $s_{Te}^o = 1.8947$ Btu/lbm R

C.V. Nozzle: $h_i = 371.6$ Btu/lbm, $s_{Ti}^o = 1.8947$ Btu/lbm R, $s_{es} = s_i$

Then from Eq.6.19

$$\Rightarrow\ \ \ s_{Tes}^o = s_{Ti}^o + R\ \ln(P_e/P_i) = 1.8947 + \frac{53.34}{778}\ \ln\ (\frac{14.7}{35}) = 1.8352\ \frac{Btu}{lbm\ R}$$

Table F.5 $\Rightarrow$ $T_{e,s} = 1199.6$ R, $h_{e,s} = 291.3$ Btu/lbm

Energy Eq.: $(1/2)\mathbf{V}_{e,s}^2 = h_i - h_{e,s} = 371.6 - 291.3 = 80.3$ Btu/lbm

Eq.7.30: $(1/2)\mathbf{V}_{e,AC}^2 = (1/2)\mathbf{V}_{e,s}^2 \times \eta_{NOZ} = 76.29$ Btu/lbm

$$\mathbf{V}_{e,AC} = \sqrt{2 \times 25037 \times 76.29} = \mathbf{1954\ ft/s}$$

Recall 1 Btu/lbm = 25 037 ft^2/s^2

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10.178E
Solve Problem 10.141E with variable specific heats using Table F.5.

In a Brayton cycle the inlet is at 540 R, 14 psia and the combustion adds 350 Btu/lbm. The maximum temperature is 2500 R due to material considerations. Find the maximum permissible compression ratio and for that the cycle efficiency using cold air properties.

Solution:

Combustion: $h_3 = h_2 + q_H$; $\quad _2w_3 = 0$ and $T_{max} = T_3 = 2500$ R

$\qquad h_2 = h_3 - q_H = 645.721 - 350 = 295.72$ Btu/lbm

From Table F.5 find T_2 from h_2

$\qquad T_2 = 1216.8$ R; $s^o_{T2} = 1.83883$; $T_1 = 540$ R; $s^o_{T1} = 1.63979$ Btu/lbm-R

Reversible adiabatic compression leads to constant s, from Eq.6.19:

$\qquad P_2 / P_1 = \exp[\ (s^o_{T2} - s^o_{T1})/R\ ] = \exp(2.90313) = \mathbf{18.231}$

Reversible adiabatic expansion leads to constant s, from Eq.6.19

$\qquad s^o_{T4} = s^o_{T3} + R\ \ln(P_4/P_3) = 2.03391 + \dfrac{53.34}{778} \ln \dfrac{1}{18.231} = 1.83487$ Btu/lbm-R

From Table F.5.1 by linear interpolation $\quad T_4 = 1197.9$ R, $\ h_4 = 290.905$ Btu/lbm

$\qquad\qquad w_T = h_3 - h_4 = 645.721 - 290.905 = 354.82$ Btu/lbm

$\qquad\qquad w_C = h_2 - h_1 = 295.72 - 129.18 = 166.54$ Btu/lbm

$\qquad\qquad w_{net} = w_T - w_C = 354.82 - 166.54 = \mathbf{188.28}$ Btu/lbm

$\qquad\qquad \eta = w_{net} / q_H = 188.28 / 350 = \mathbf{0.538}$

10.179E

Consider an ideal gas-turbine cycle with two stages of compression and two stages of expansion. The pressure ratio across each compressor stage and each turbine stage is 8 to 1. The pressure at the entrance to the first compressor is 14 lbf/in.^{2}, the temperature entering each compressor is 70 F, and the temperature entering each turbine is 2000 F. An ideal regenerator is also incorporated into the cycle. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

This is as in Fig. 10.6

$P_2/P_1 = P_4/P_3 = P_6/P_7 = P_8/P_9 = 8.0$

$P_1 = 14$ lbf/in^2

$T_1 = T_3 = 70$ F, $T_6 = T_8 = 2000$ F

Assume const. specific heat

$s_2 = s_1$ and $s_4 = s_3$

$T_4 = T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 529.67(8)^{0.2857} = 959.4$ R

Total compressor work

$-w_C = 2 \times (-w_{12}) = 2C_{P0}(T_2 - T_1) = 2 \times 0.24(959.4 - 529.67) =$ **206.3 Btu/lbm**

Also $s_6 = s_7$ and $s_8 = s_9$

$\Rightarrow T_7 = T_9 = T_6\left(\frac{P_7}{P_6}\right)^{\frac{k-1}{k}} = 2459.67\left(\frac{1}{8}\right)^{0.2857} = 1357.9$ R

Total turbine work

$w_T = 2 \times w_{67} = 2C_{P0}(T_6 - T_7) = 2 \times 0.24(2459.67 - 1357.9) = 528.85$ Btu/lbm

$w_{NET} = 528.85 - 206.3 = 322.55$ Btu/lbm

Ideal regenerator: $T_5 = T_9$, $T_{10} = T_4$

$$\Rightarrow q_H = (h_6 - h_5) + (h_8 - h_7) = 2C_{P0}(T_6 - T_5)$$

$$= 2 \times 0.24(2459.67 - 1357.9) = w_T = 528.85 \text{ Btu/lbm}$$

$$\eta_{TH} = w_{NET}/q_H = 322.55/528.85 = \mathbf{0.61}$$

10.180E

Consider an ideal air-standard diesel cycle where the state before the compression process is 14 lbf/in.^{2}, 63 F and the compression ratio is 20. Find the maximum temperature (by iteration) in the cycle to have a thermal efficiency of 60%.

Diesel cycle: $P_1 = 14$, $T_1 = 522.67$ R, $v_1/v_2 = 20$, $\eta_{TH} = 0.60$

From the inlet state and the compression we get

$$T_2 = T_1(v_1/v_2)^{k-1} = 522.67(20)^{0.4} = 1732.4 \text{ R}$$

$$v_1 = \frac{53.34 \times 522.67}{14 \times 144} = 13.829 \text{ ft}^3/\text{lbm}, \qquad v_2 = \frac{13.829}{20} = 0.6915 \text{ ft}^3/\text{lbm}$$

Constant pressure combustion relates v_3 and T_3

$$v_3 = v_2 \times T_3/T_2 = 0.6915 \times T_3/1732.4 = 0.000399 \, T_3$$

The expansion then gives T_4 in terms of T_3

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{k-1} = \left(\frac{13.829}{0.000399 \, T_3}\right)^{0.4} \rightarrow \qquad T_4 = 0.0153 \, T_3^{1.4}$$

Now these T's relate to the given efficiency

$$\eta_{TH} = 0.60 = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{0.0153 \, T_3^{1.4} - 522.67}{1.4(T_3 - 1732.4)}$$

$$\Rightarrow \quad 0.0153 \, T_3^{1.4} - 0.56 \, T_3 + 447.5 = 0$$

Trial and error on this non-linear equation

5100 R: LHS = -35.54,   5500 R: LHS = 5.04,   5450 R: LHS = -0.5

Linear interpolation, $T_3 = $ **5455 R**

10.181E

Repeat Problem 10.179E, but assume that each compressor stage and each turbine stage has an isentropic efficiency of 85%. Also assume that the regenerator has an efficiency of 70%.

$T_{4S} = T_{2S} = 959.4$ R, $-w_{CS} = 206.3$

$T_{7S} = T_{9S} = 1357.9$ R, $w_{TS} = 528.85$

$\Rightarrow$ $w_C = w_{SC}/\eta_{SC} = 242.7$ Btu/lbm

$w_{12} = w_{34} = 242.7/2 = 121.35$ Btu/lbm

$T_2 = T_4 = T_1 + w_{12}/C_{P0}$

$\quad = 529.67 + 121.35/0.24 = 1035.3$ R

$w_T = \eta_T \, w_{TS} = 0.85 \times 528.85 = 449.5$ Btu/lbm

Energy Eq. for actual turbines

$\quad T_7 = T_9 = T_6 - w_{67}/C_{P0} = 2459.67 - 449.5/2 \times 0.24 = 1523$ R

Regenerator efficiency

$$\eta_{REG} = \frac{h_5 - h_4}{h_9 - h_4} = \frac{T_5 - T_4}{T_9 - T_4} = \frac{T_5 - 1035.3}{1523 - 1035.3} = 0.7 \quad \Rightarrow \; T_5 = 1376.7 \text{ R}$$

The two combustors, heat addition

$\quad q_H = C_{P0}(T_6 - T_5) + C_{P0}(T_8 - T_7)$

$\quad\quad = 0.24(2459.67 - 1376.7) + 0.24(2459.67 - 1523)$

$\quad\quad = 484.7$ Btu/lbm

$\quad w_{NET} = w_T + w_C = 449.5 - 242.7 = 206.8$ Btu/lbm

$\quad \eta_{TH} = w_{NET}/q_H = 206.8/484.7 = \mathbf{0.427}$

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 11

8e

Updated June 2013

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CONTENT

In-Text Concept Questions

11.a

Are the mass and mole fractions for a mixture ever the same?

Generally not. If the components all had the same molecular mass the mass and mole fractions would be the same.

11.b

For a mixture how many component concentrations are needed?

A total of N-1 concentrations are needed, N equals total number of components, whether mass or mole fractions. They must sum up to one so the last one is by default.

11.c

Are any of the properties (P, T, v) for oxygen and nitrogen in air the same?

In any mixture under equilibrium T is the same for all components.
Each species has its own pressure equal to its partial pressure P_i.
The partial volume for a component is: $v_i = V/m_i$ and V is the same for all components so v_i is not.

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11.d

If I want to heat a flow of a 4 component mixture from 300 to 310 K at constant P, how many properties and which ones do I need to know to find the heat transfer?

You need to know the flow rate, the four mass fractions, and the component specific heat values (or the h values at both temperatures).

11.e

To evaluate the change in entropy between two states at different T and P values for a given mixture, do I need to find the partial pressures?

Not necessarily provided it is an ideal gas. If the mixture composition does not change then the mixture can be treated as a pure substance where each of the partial pressures is a constant fraction of the total pressure, Eq.11.10 and the changes in u, h and s can be evaluated with the mixture properties as in Eqs. 11.20-24.

If constant specific heat is an inappropriate model to use then u, h and a standard entropy must be evaluated from expressions as in Eqs.11.11-12 and 11.16, this is precisely what is done to make the air tables A.7 from the nitrogen, oxygen and argon properties.

If the substance is not an ideal gas mixture then the properties will depend on the partial pressures.

11.f

What happens to relative and absolute humidity when moist air is heated?

Relative humidity decreases, while absolute humidity remains constant.
See Figs. 11.8 and 11.9.

11.g

If I cool moist air, do I reach the dew first in a constant-P or constant-V process?

The constant-volume line is steeper than the constant-pressure line,
see Fig. 11.3. Saturation in the constant-P process is at a higher T.

11.h

What happens to relative and absolute humidity when moist air is cooled?

Relative humidity increase, while absolute humidity remains constant until
we reach the dew point. See Figs. 11.8 and 11.9. If we cool below the dew point
the relative humidity stays at 100% and the absolute humidity (humidity ratio)
drops as water condenses to liquid (or freezes to solid) and drops out of the gas
mixture.

11.i

Explain in words what the absolute and relative humidity expresses?

Absolute humidity is the ratio of the mass of vapor to the mass of dry air. It says how much water is there per unit mass of dry air.
Relative humidity is the ratio of the mole fraction of vapor to that in a saturated mixture at the same T and P. It expresses how close to the saturated state the water is.

11.j

An adiabatic saturation process changes Φ, ω and T. In which direction?

Relative humidity and absolute humidity increase, and temperature decreases. Why does the temperature decrease? The energy to evaporate some liquid water to go into the gas mixture comes from the immediate surroundings to the liquid water surface where water evaporates, look at the dashed curve in Fig. 11.9. The moist air and the liquid water both cool down.

Concept-study Guide Problems

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11.1

Equal masses of argon and helium are mixed. Is the molecular mass of the mixture the linear average of the two individual ones?

No. The individual molecular masses must be combined using the mole fractions as in:

$$M_{mix} = \sum y_j M_j$$

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11.2

A constant flow of pure argon and pure helium are mixed to produce a flow of mixture mole fractions 0.25 and 0.75 respectively. Explain how to meter the inlet flows to ensure the proper ratio assuming inlet pressures are equal to the total exit pressure and all temperatures are the same.

The inlet flow rate in terms of mass or moles is the same as the exit rate for each component in the mixture. Since the inlet P for each component is the same as the total exit P (which is the sum of the partial pressures if ideal gas) then the volume flow rates in and out are different for each species.

$$P \dot{V}_i = \dot{m}_i R_i T = \dot{n}_i \overline{R}T$$

$$P \dot{V}_{tot} = \dot{n}_{tot} \overline{R}T$$

We can therefore meter the volume flow rate $\dot{V}_i$ to be proportional to $\dot{n}_i$ for each line of the inlet flows. From these two equations we can get the ratio as

$$\dot{V}_i / \dot{V}_{tot} = \dot{n}_i / \dot{n}_{tot} = y_i$$

11.3

For a gas mixture in a tank are the partial pressures important?

Yes. The sum of the partial pressures equals the total pressure and if they are ideal gases the partial pressures are equal to the mole fraction times the total pressure so

$$P_i = y_i P \qquad \text{and} \qquad \sum P_i = \sum y_i P = P$$

11.4

An ideal mixture at T, P is made from ideal gases at T, P by charging them into a steel tank. Assume heat is transferred so T stays the same as the supply. How do the properties (P, v and u) for each component change up, down or constant?

Solution:

Ideal gas: $u = u(T)$ so **constant**

P **drops** from P to partial P_i

v **increases** from v at P to v at P_i same T

11.5

An ideal mixture at T, P is made from ideal gases at T, P by flow into a mixing chamber without any external heat transfer and an exit at P. How do the properties (P, v and h) for each component change up, down or constant?

Solution:

Ideal gas: $h_{mix} = \Sigma\,(c_i h_i)_{out} = \Sigma\,(c_i h_i)_{in}$ same function of T

so **constant** T and then also **constant** h_i

P **drops** from P to partial P_i

v **increases** from v at P to v at P_i same T

11.6

If a certain mixture is used in a number of different processes do I need to consider partial pressures?

No. If the mixture composition stays the same, the pressure for each component, which is a partial pressure, is the same fraction of the total pressure, thus any variation follows the total pressure. Recall air is a mixture and we can deal with most processes involving air without knowledge about its composition. However, to make the air properties we do need to deal with the composition but only once.

11.7

Why is it that I can use a set of tables for air, which is a mixture, without dealing with its composition?

As long as the composition is fixed any property is a fixed weighted average of the components properties and thus only varies with T and total P.

A process that will cool air to saturation and condensation can not be handled by the air tables. In such a process the composition of the liquid and vapor mixtures are different.

11.8

Develop a formula to show how the mass fraction of water vapor is connected to the humidity ratio.

By definition the mass concentration is

$$c = \frac{m_v}{m_a + m_v} = \frac{m_v/m_a}{1 + m_v/m_a} = \frac{\omega}{1 + \omega}$$

and since ω is small then $1 + \omega \approx 1$ and c is close to ω (but not equal to).

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11.9

For air at 110^oC and 100 kPa is there any limit on the amount of water it can hold?

No. Since P_g = 143.3 kPa at 110^oC and P_v < 100 kPa ω can be infinity.

$$\omega = 0.622 \frac{P_v}{P_a} = 0.622 \frac{P_v}{P - P_v}$$

As P_v approaches P, w goes towards infinity.

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11.10

Can moist air below the freezing point, say –5$^{\text{o}}$C, have a dew point?

Yes. At the dew point, water would begin to appear as a solid. It snows. Since it is frost forming on surfaces rather than dew, you can call it frost point.

The contrails after the jets are tiny ice particles formed due to the very low (-40 C) temperature at high altitudes.

11.11

Why does a car with an air-conditioner running often have water dripping out?

The cold evaporator that cools down an air flow brings it below the dew point temperature and thus condenses water out.

11.12

Moist air at 35$^{\text{o}}$C, ω = 0.0175 and Φ = 50% should be brought to a state of 20$^{\text{o}}$C, ω = 0.01 and Φ = 70%. Do I need to add or subtract water?

The humidity ratio (absolute humidity) expresses how much water vapor is present in the mixture

$$\omega = m_v / m_a$$

so to decrease ω we must **subtract water** from the mixture.
The relative humidity expresses how close to the saturated state the vapor is as

$$\Phi = P_v / P_g$$

and not about how much water there is.

Mixture composition and properties

11.13

If oxygen is 21% by mole of air, what is the oxygen state (P, T, v) in a room at 300 K, 100 kPa of total volume 60 m^3?

The temperature is 300 K,

The partial pressure is $P = yP_{tot} = 0.21 \times 100 = 21$ kPa.

At this T, P:

$$v_{O2} = RT/P_{O2} = 0.2598 \text{ kJ/kg-K} \times 300 \text{ K} / 21 \text{ kPa} = 3.711 \text{ m}^3/\text{kg}$$

Remark: If we found the oxygen mass then $\quad m_{O2}v_{O2} = V = 60 \text{ m}^3$

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11.14

A 3 L liquid mixture is 1/3 of each of water, ammonia and ethanol by volume. Find the mass fractions and total mass of the mixture.

Each component has a partial volume of 1 L = 0.001 m^3

$$m_{water} = V/v_f = 0.001 \text{ m}^3 / (0.001 \text{ m}^3/\text{kg}) = 1 \text{ kg}$$
$$m_{amm} = V/v_f = 0.001 \text{ m}^3 \times 604 \text{ kg/m}^3 = 0.604 \text{ kg}$$
$$m_{ethanol} = V/v_f = 0.001 \text{ m}^3 \times 783 \text{ kg/m}^3 = 0.783 \text{ kg}$$

The v_f for water is from Table B.1.1 and $v_f = 1/\rho$ is from A.4

Total mass is: m = 1 + 0.604 + 0.783 = 2.387 kg

$$c_{water} = m_{water}/m = 1 / 2.387 = 0.419$$
$$c_{amm} = m_{amm}/m = 0.604 / 2.387 = 0.253$$
$$c_{ethanol} = m_{ethanol}/m = 0.783 / 2.387 = 0.328$$

11.15

A flow of oxygen and one of nitrogen, both 300 K, are mixed to produce 1 kg/s air at 300 K, 100 kPa. What are the mass and volume flow rates of each line?

For the mixture, M = 0.21×32 + 0.79×28.013 = 28.85

For O_2 ,   c = 0.21 × 32 / 28.85 = 0.2329

For N_2 ,   c = 0.79 × 28.013 / 28.85 = 0.7671

Since the total flow out is 1 kg/s, these are the component flows in kg/s.

Volume flow of O_2 in is

$$\dot{V} = c\dot{m}\,v = c\dot{m}\,\frac{RT}{P} = 0.2329 \text{ kg/s } \frac{0.2598 \text{ kJ/kg-K} \times 300 \text{ K}}{100 \text{ kPa}} = \textbf{0.1815 m}^3\textbf{/s}$$

Volume flow of N_2 in is

$$\dot{V} = c\dot{m}\,v = c\dot{m}\,\frac{RT}{P} = 0.7671 \text{ kg/s } \frac{0.2968 \text{ kJ/kg-K} \times 300 \text{ K}}{100 \text{ kPa}} = \textbf{0.6830 m}^3\textbf{/s}$$

11.16

A gas mixture at 20°C, 125 kPa is 50% N_2, 30% H_2O and 20% O_2 on a mole basis. Find the mass fractions, the mixture gas constant and the volume for 5 kg of mixture.

Solution:

The conversion follows the definitions and identities:

From Eq.11.3: $c_i = y_i M_i / \sum y_j M_j$

From Eq.11.5:

$M_{mix} = \sum y_j M_j = 0.5 \times 28.013 + 0.3 \times 18.015 + 0.2 \times 31.999$

$= 14.0065 + 5.4045 + 6.3998 = 25.811$

$c_{N2} = 14.0065 / 25.811 = 0.5427, \quad c_{H2O} = 5.4045 / 25.811 = 0.2094$

$c_{O2} = 6.3998 / 25.811 = 0.2479, \quad$ sums to 1   OK

From Eq.11.14:

$R_{mix} = \bar{R}/M_{mix} = 8.3145 \text{ kJ/kmol-K} / 25.811 \text{ kg/kmol} = \textbf{0.3221 kJ/kg K}$

$V = mR_{mix} T/P = 5 \text{ kg} \times 0.3221 \text{ kJ/kg-K} \times 393.15 \text{ K}/125 \text{ kPa} = \textbf{5.065 m}^\textbf{3}$

11.17

A mixture of 60% N_2, 30% Ar and 10% O_2 on a mass basis is in a cylinder at 250 kPa, 310 K and volume 0.5 m^3. Find the mole and the mass fractions and the mass of argon.

Solution:

Total mixture $PV = m R_{mix}T$

Mixture composition: c's for $(N_2, Ar, O_2) = (0.6, 0.3, 0.1)$

From Eq.11.15:

$R_{mix} = \sum c_i R_i = 0.6 \times 0.2968 + 0.3 \times 0.2081 + 0.1 \times 0.2598$
$= 0.26629$ kJ/kg K

$m = PV/R_{mix}T = \dfrac{250 \text{ kPa} \times 0.5 \text{ m}^3}{0.26649 \text{ kJ/kg-K} \times 310 \text{ K}} = 1.513$ kg

$m_{ar} = 0.3$ m = **0.454 kg**

From Eq.11.4: $y_i = (c_i / M_i) / \sum c_j/M_j$

	c_i	M_i	c_i/M_i	$\mathbf{y_i}$
N_2	0.6	28.013	0.02141	**0.668**
Ar	0.3	39.948	0.00751	**0.234**
O_2	0.1	31.999	0.003125	**0.098** round up
			0.032055	

11.18

A slightly oxygenated air mixture is 69% N_2, 1% Ar and 30% O_2 on a mole basis. Assume a total pressure of 101 kPa and find the mass fraction of oxygen and its partial pressure.

Solution:

From Eq. 11.3: $c_i = y_i M_i / \sum y_j M_j$

Eq.11.5:

$M_{mix} = \sum y_j M_j = 0.69 \times 28.013 + 0.01 \times 39.948 + 0.3 \times 31.999 = 29.328$

$c_{N2} = (0.69 \times 28.013) / 29.328 = 0.659$

$c_{Ar} = (0.01 \times 39.948) / 29.328 = 0.014$

$c_{O2} = (0.3 \times 31.999) / 29.328 =$ **0.327**,   sums to 1   OK

From Eq.11.10:

$$P_{O2} = y_{O2} P = 0.3 \times 101 = \textbf{30.3 kPa}$$

$R_{mix} = \bar{R}/M_{MIX} = 8.3145$ kJ/kmol-K / 29.328 kg/kmol = 0.2835 kJ/kg K

11.19

A new refrigerant R-407 is a mixture of 23% R-32, 25% R-125 and 52% R-134a on a mass basis. Find the mole fractions, the mixture gas constant and the mixture heat capacities for this new refrigerant.

Solution:

From the conversion in Eq.11.4 we get:

	c_i	M_i	c_i/M_i	y_i
R-32	0.23	52.024	0.004421	0.381
R-125	0.25	120.022	0.002083	0.180
R-134a	0.52	102.03	0.0050965	0.439
			0.0116005	

Eq.11.15:

$$R_{mix} = \sum c_i R_i = 0.23 \times 0.1598 + 0.25 \times 0.06927 + 0.52 \times 0.08149$$
$$= \mathbf{0.09645\ kJ/kg\ K}$$

Eq.11.23:

$$C_{P\ mix} = \sum c_i\ C_{P\ i} = 0.23 \times 0.822 + 0.25 \times 0.791 + 0.52 \times 0.852$$
$$= \mathbf{0.8298\ kJ/kg\ K}$$

Eq.11.21:

$$C_{v\ mix} = \sum c_i C_{v\ i} = 0.23 \times 0.662 + 0.25 \times 0.721 + 0.52 \times 0.771$$
$$= \mathbf{0.7334\ kJ/kg\ K}\ \ (= C_{P\ mix} - R_{mix})$$

11.20

In a car engine gasoline (assume octane C_8H_{18}) is evaporated and then mixed with air in a ratio of 1:15 by mass. In the cylinder the mixture is at 750 kPa, 650 K when the spark fires. For that time find the partial pressure of the octane and the specific volume of the mixture.

Assuming ideal gas the partial pressure is $P_i = y_i P$ and $c_{C8H18} = 1/16 = 0.0625$

From Eq. 11.4: $y_i = [c_i /M_i] / \sum c_j/M_j$

$$y_{C8H18} = \frac{0.0625/114.232}{0.0625/114.232 + 0.9375/28.97} = 0.008186$$

$P_{C8H18} = 0.008186 \times 750 = \mathbf{6.14 \ kPa}$

The gas constant from Eq.11.15:

$$R_{mix} = \sum c_i R_i = 0.0625 \times 0.07279 + 0.9375 \times 0.287 = 0.27361 \ kJ/kg \ K$$

$$v_{mix} = R_{mix}T/P = 0.27361 \times 650/750 = \mathbf{0.2371 \ m^3/kg}$$

11.21

A 100 m^3 storage tank with fuel gases is at 20°C, 100 kPa containing a mixture of acetylene C_2H_2, propane C_3H_8 and butane C_4H_{10}. A test shows the partial pressure of the C_2H_2 is 15 kPa and that of C_3H_8 is 65 kPa. How much mass is there of each component?

Solution:

Assume ideal gases, then the ratio of partial to total pressure is the mole fraction, $y = P/P_{tot}$

$$y_{C2H2} = 15/100 = 0.15, \quad y_{C3H8} = 65/100 = 0.65, \quad y_{C4H10} = 20/100 = 0.20$$

$$n_{tot} = \frac{PV}{\bar{R}T} = \frac{100 \text{ kPa} \times 100 \text{ m}^3}{8.31451 \text{ kJ/kmol-K} \times 293.15 \text{ K}} = 4.1027 \text{ kmol}$$

$$m_{C2H2} = (nM)_{C2H2} = y_{C2H2} \, n_{tot} \, M_{C2H2}$$
$$= 0.15 \times 4.1027 \text{ kmol} \times 26.038 \text{ kg/kmol} = \textbf{16.024 kg}$$

$$m_{C3H8} = (nM)_{C3H8} = y_{C3H8} \, n_{tot} \, M_{C3H8}$$
$$= 0.65 \times 4.1027 \text{ kmol} \times 44.097 \text{ kg/kmol} = \textbf{117.597 kg}$$

$$m_{C4H10} = (nM)_{C4H10} = y_{C4H10} \, n_{tot} \, M_{C4H10}$$
$$= 0.20 \times 4.1027 \text{ kmol} \times 58.124 \text{ kg/kmol} = \textbf{47.693 kg}$$

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11.22

A 2 kg mixture of 25% N_2, 50% O_2 and 25% CO_2 by mass is at 150 kPa and 300 K. Find the mixture gas constant and the total volume.

Solution:

From Eq.11.15:

$$R_{mix} = \sum c_i R_i = 0.25 \times 0.2968 + 0.5 \times 0.2598 + 0.25 \times 0.1889$$
$$= \textbf{0.2513 kJ/kg K}$$

Ideal gas law: $PV = mR_{mix}T$

$V = mR_{mix}T/P = 2 \text{ kg} \times 0.2513 \text{ kJ/kg-K} \times 300 \text{ K} /150 \text{ kPa} = \textbf{1.005 m}^3$

11.23

A diesel engine sprays fuel (assume n-Dodecane $C_{12}H_{26}$, M = 170.34 kg/kmol) into the combustion chamber so it becomes filled with an amount of 1 mol fuel per 88 mol air. Find the fuel fraction on a mass basis and the fuel mass for a chamber that is 0.5 L at 800 K and total pressure of 4000 kPa.

From Eq. 11.3: $c_i = y_i M_i / \sum y_j M_j$

$$c_{fuel} = \frac{(1/89) \times 170.34}{(1/89) \times 170.34 + (88/89)\,28.97} = \mathbf{0.06263}$$

Use ideal gas for the fuel vapor

$$m_{fuel} = \frac{P_{fuel}V}{R_{fuel}T} = \frac{(1/89) \times 4000 \text{ kPa} \times 0.0005 \text{ m}^3}{(8.3145/170.34) \text{ kJ/kg-K} \times 800 \text{ K}} = \mathbf{0.575\ g}$$

We could also have done the total mass and then used the mass fraction

Eq.11.5: $M_{mix} = \sum y_j M_j = (1/89) \times 170.34 + (88/89) \times 28.97 = 30.558$

$R_{mix} = \bar{R}/M_{mix}$ b= 8.3145 / 30.558 = 0.27209

$m = PV / R_{mix}T = 4000 \times 0.0005 /(0.27209 \times 800) = 0.009188$ kg

$m_{fuel} = c_{fuel}\, m = 0.06263 \times 0.009188$ kg = 0.000575 kg = 0.575 g

11.24

A new refrigerant R-410A is a mixture of R-32 and R-125 in a 1:1 mass ratio. What are the overall molecular mass, the gas constant and the ratio of specific heats for such a mixture?

Eq.11.5:

$$M = \sum y_j M_j = 1 / \sum (c_j / M_j) = \frac{1}{\dfrac{0.5}{52.024} + \dfrac{0.5}{120.022}} = \mathbf{72.586}$$

Eq.11.15:

$$R_{mix} = \sum c_i R_i = 0.5 \times 0.1598 + 0.5 \times 0.06927 = \mathbf{0.1145\ kJ/kg\ K}$$

$$= \bar{R}/M_{MIX} = \frac{8.3145\ kJ/kmol\text{-}K}{72.586\ kg/kmol} = same \quad (from\ Eq.11.14)$$

Eq.11.23:

$$C_{P\ mix} = \sum c_i\, C_{P\ i} = 0.5 \times 0.822 + 0.5 \times 0.791 = 0.8065\ kJ/kg\ K$$

Eq.11.21:

$$C_{V\ mix} = \sum c_i C_{V\ i} = 0.5 \times 0.662 + 0.5 \times 0.722 = 0.692\ kJ/kg\ K$$
$$(= C_{P\ mix} - R_{mix})$$

$$k_{mix} = C_{P\ mix} / C_{V\ mix} = 0.8065 / 0.692 = \mathbf{1.1655}$$

11.25

Do Problem 11.24 for R-507a which is 1:1 mass ratio of R-125 and R-143a. The refrigerant R-143a has molecular mass of 84.041 kg/kmol and $C_p = 0.929$ kJ/kg-K.

Refrigerant R-143a is not in Table A.5 so:

$$R = \overline{R}/M = 8.3145 \text{ kJ/kmol-K} / 84.041 \text{ kg/kmol} = 0.098934 \text{ kJ/kg-K}$$
$$C_V = C_p - R = 0.929 - 0.098934 = 0.8301 \text{ kJ/kg-K}$$

Eq.11.5:

$$M = \sum y_j M_j = = 1 / \sum (c_j / M_j) = \frac{1}{\frac{0.5}{120.022} + \frac{0.5}{84.041}} = \mathbf{98.859}$$

Eq.11.15:

$$R_{mix} = \sum c_i R_i = 0.5 \times 0.06927 + 0.5 \times 0.098934 = \mathbf{0.0841 \text{ kJ/kg K}}$$
$$= \overline{R}/M_{MIX} = \frac{8.3145 \text{ kJ/kmol-K}}{98.859 \text{ kg/kmol}} = \text{same} \quad (\text{this is from Eq.11.14})$$

Eq.11.23:

$$C_{P \, mix} = \sum c_i C_{P \, i} = 0.5 \times 0.791 + 0.5 \times 0.929 = 0.86 \text{ kJ/kg K}$$

Eq.11.21:

$$C_{V \, mix} = \sum c_i C_{V \, i} = 0.5 \times 0.722 + 0.5 \times 0.8301 = 0.776 \text{ kJ/kg K}$$
$$(= C_{P \, mix} - R_{mix})$$

$$k_{mix} = C_{P \, mix} / C_{V \, mix} = 0.86 / 0.776 = \mathbf{1.108}$$

Simple processes

11.26

A rigid container has 1 kg CO_2 at 300 K and 1 kg argon at 400 K both at 150 kPa. Now they are allowed to mix without any heat transfer. What is final T, P?

No Q, No W   so the energy equation gives constant U

Energy Eq.:

$$U_2 - U_1 = 0 = m_{CO2}(u_2 - u_1)_{CO2} + m_{Ar}(u_2 - u_1)_{Ar}$$

$$= m_{CO2}C_{v\ CO2}(T_2 - T_1)_{CO2} + m_{Ar}C_{v\ Ar}(T_2 - T_1)_{Ar}$$

$$= (1{\times}0.653 + 1{\times}0.312)\ kJ/K \times T_2 - (1{\times}0.653{\times}300 + 1{\times}0.312{\times}400)\ kJ$$

$$T_2 = \textbf{332.3 K},$$

$$V = V_1 = V_{CO2} + V_{Ar} = m_{CO2}R_{CO2}T_{CO2}/P + m_{Ar}R_{Ar}T_{Ar}/P$$

$$= (1{\times}0.1889{\times}300 + 1{\times}0.2081{\times}400)\ kJ\ /150\ kPa = 0.932\ 73\ m^3$$

Pressure from ideal gas law and Eq.11.15 for R

$$mR = 1\ kg \times 0.1889\ kJ/kg\text{-}K + 1\ kg \times 0.2081\ kJ/kg\text{-}K = 0.397\ kJ/K$$

$$P_2 = mRT/V = 0.397\ kJ/K \times 332.3\ K/\ 0.932\ 73\ m^3 = \textbf{141.4 kPa}$$

11.27

At a certain point in a coal gasification process, a sample of the gas is taken and stored in a 1-L cylinder. An analysis of the mixture yields the following results:

Component	H_2	CO	CO_2	N_2
Percent by mass	2	45	28	25

Determine the mole fractions and total mass in the cylinder at 100 kPa, 20°C. How much heat transfer must be transferred to heat the sample at constant volume from the initial state to 100°C?

Solution:

Determine mole fractions from Eq.11.4: $y_i = (c_i / M_i) / \sum c_j/M_j$

$\sum c_j / M_j = 0.02 / 2.016 + 0.45 / 28.01 + 0.28 / 44.01 + 0.25 / 28.013$

$\qquad = 0.009921 + 0.016065 + 0.006362 + 0.00892 = 0.041268$ kmol/kg

$M_{mix} = 1 / \sum c_j/M_j = 1/0.041268 = 24.232$ kg/kmol

From Eq.11.4

$y_{H2} = 0.009921 \times 24.232 = 0.2404 \qquad y_{CO} = 0.016065 \times 24.232 = 0.3893$

$y_{CO2} = 0.006362 \times 24.232 = 0.1542 \qquad y_{N2} = 0.00892 \times 24.232 = 0.2161$

$R_{mix} = \bar{R}/M_{mix} = 8.3145$ kJ/kmol-K $/24.232$ kg/kmol $= 0.34312$ kJ/kg-K

$$m = \frac{PV}{RT} = \frac{100 \text{ kPa} \times 10^{-3} \text{m}^3}{0.34312 \text{ kJ/kg-K} \times 293.15 \text{ K}} = \textbf{9.942×10}^{\textbf{-4}} \textbf{ kg}$$

$C_{V0\ MIX} = \sum c_i\ C_{V0\ i} = 0.02 \times 10.085 + 0.45 \times 0.744$

$\qquad + 0.28 \times 0.653 + 0.25 \times 0.745 = 0.9056$ kJ/kg K

$_1Q_2 = U_2 - U_1 = mC_{V0}(T_2\text{-}T_1)$

$\qquad = 9.942 \times 10^{-4}$ kg $\times 0.9056$ kJ/kg-K $\times(100\text{-}20)$ K $= \textbf{0.0720 kJ}$

11.28

The mixture in Problem 11.22 is heated to 500 K with constant volume. Find the final pressure and the total heat transfer needed using Table A.5.

Solution:

C.V. Mixture of constant volume.

Process: $V = \text{constant} \Rightarrow {}_1W_2 = \int P \, dV = 0$

Energy Eq.: ${}_1Q_2 = m(u_2 - u_1) \cong m \, C_{Vmix} \, (T_2 - T_1)$

Ideal gas: $PV = mRT \Rightarrow P_2 = P_1(T_2 / T_1)(V_1/V_2)$

$P_2 = P_1 T_2/T_1 = 150 \text{ kPa} \times 500/300 = \textbf{250 kPa}$

From Eq.11.21:

$C_{Vmix} = \sum c_i C_{V\,i} = 0.25 \times 0.745 + 0.5 \times 0.662 + 0.25 \times 0.653$

$\qquad\quad = 0.6805 \text{ kJ/kg K}$

${}_1Q_2 = 2 \text{ kg} \times 0.6805 \text{ kJ/kg-K} (500 - 300) \text{ K} = \textbf{272.2 kJ}$

11.29

The mixture in Problem 11.22 is heated up to 500 K in a constant pressure process. Find the final volume and the total heat transfer using Table A.5.

Solution:

C.V. Mixture

Process: P = constant   => $_1W_2 = \int P\, dV = P(V_2 - V_1)$

Energy Eq.: $_1Q_2 = m(u_2 - u_1) + {_1}W_2 = m(u_2 - u_1) + Pm(v_2 - v_1)$

$$= m(h_2 - h_1) \cong m\, C_{P\ mix}(T_2 - T_1)$$

From Eq.11.15:

$$R_{mix} = \sum c_i R_i = 0.25 \times 0.2968 + 0.5 \times 0.2598 + 0.25 \times 0.1889$$

$$= 0.2513 \text{ kJ/kg K}$$

From Eq.11.23:

$$C_{P\ mix} = \sum c_i\, C_{P\ i} = 0.25 \times 1.042 + 0.5 \times 0.922 + 0.25 \times 0.842$$

$$= 0.932 \text{ kJ/kg K}$$

$$V_2 = m\, R_{mix}\, T_2/P_2$$

$$= 2 \text{ kg} \times 0.2513 \text{ kJ/kg-K} \times 500 \text{ K}/150 \text{ kPa} = \textbf{1.675 m}^{\textbf{3}}$$

$$_1Q_2 = 2 \text{ kg} \times 0.932 \text{ kJ/kg-K } (500 - 300) \text{ K} = \textbf{372.8 kJ}$$

11.30

A flow of 1 kg/s argon at 300 K and another flow of 1 kg/s CO_2 at 1600 K both at 150 kPa are mixed without any heat transfer. What is the exit T, P?

No work implies no pressure change for a simple flow.

P_e = **150 kPa**

The energy equation becomes

$$\dot{m}h_i = \dot{m}h_e = (\dot{m}h_i)_{Ar} + (\dot{m}h_i)_{CO2} = (\dot{m}h_e)_{Ar} + (\dot{m}h_e)_{CO2}$$

$\Rightarrow \qquad \dot{m}_{CO2}C_{p\ CO2}(T_e - T_i)_{CO2} + \dot{m}_{Ar}C_{p\ Ar}(T_e - T_i)_{Ar} = 0$

$\Rightarrow \qquad \dot{m}_{Ar}C_{p\ Ar}T_i + \dot{m}_{CO2}C_{p\ CO2}T_i = [\dot{m}_{Ar}C_{p\ Ar} + \dot{m}_{CO2}C_{p\ CO2}]\ T_e$

$(1 \times 0.520 \times 300 + 1 \times 0.842 \times 1600)$ kW $= (1 \times 0.520 + 1 \times 0.842)$ kW/K $\times T_e$

$T_e = $ **1103.7 K**,

11.31

A flow of 1 kg/s argon at 300 K and another flow of 1 kg/s CO_2 at 1600 K both at 150 kPa are mixed without any heat transfer. Find the exit T, P using variable specific heats.

No work implies no pressure change for a simple flow.

P_e = **150 kPa**

The energy equation becomes

$$\dot{m}h_i = \dot{m}h_e = (\dot{m}h_i)_{Ar} + (\dot{m}h_i)_{CO2} = (\dot{m}h_e)_{Ar} + (\dot{m}h_e)_{CO2}$$

$\Rightarrow \qquad \dot{m}_{CO2}(h_e - h_i)_{CO2} + \dot{m}_{Ar}C_{p\,Ar}(T_e - T_i)_{Ar} = 0$

$\Rightarrow \quad$ 1 kg/s $\times (h_e - 1748.12$ kJ/kg$) + (1 \times 0.52)$ kW/K $\times (T_e - 300$ K$) = 0$

$h_{e\,CO2} + 0.52\,T_e = 1748.12 + 0.52 \times 300 = 1904.12$ kJ/kg

Trial and error on T_e using Table A.8 for $h_{e\,CO2}$

$T_e = 1100$ K: LHS $= 1096.36 + 0.52 \times 1100 = 1668.36$ too small

$T_e = 1300$ K: LHS $= 1352.28 + 0.52 \times 1300 = 2028.28$ too large

$T_e = 1200$ K: LHS $= 1223.34 + 0.52 \times 1200 = 1847.34$ too small

Final interpolation

$$T_e = 1200 + 100\,\frac{1904.12 - 1847.34}{2028.28 - 1847.34} = \mathbf{1231.4\ K},$$

11.32

A pipe flows 0.1 kg/s of a mixture with mass fractions of 40% CO_2 and 60% N_2 at 400 kPa, 300 K. Heating tape is wrapped around a section of pipe with insulation added and 2 kW electrical power is heating the pipe flow. Find the mixture exit temperature.

Solution:

C.V. Pipe heating section. Assume no heat loss to the outside, ideal gases.

Energy Eq.:　$\dot{Q} = \dot{m}(h_e - h_i) = \dot{m}C_{P\ mix}(T_e - T_i)$

From Eq.11.23

　　$C_{P\ mix} = \sum c_i\ C_{P\ i} = 0.4 \times 0.842 + 0.6 \times 1.042 = 0.962$ kJ/kg K

Substitute into energy equation and solve for exit temperature

　　$T_e = T_i + \dot{Q} / \dot{m}C_{P\ mix} = 300 + 2$ kW/ (0.1×0.962) kW/K = **320.8 K**

11.33

A steady flow of 0.1 kg/s carbon dioxide at 1000 K in one line is mixed with 0.2 kg/s of nitrogen at 400 K from another line, both at 100 kPa. The mixing chamber is insulated and has constant pressure of 100 kPa. Use constant heat capacity to find the mixing chamber exit temperature.

Take CV around the mixing chamber

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$;

Concentrations: $c_{CO_2} = \dot{m}_1/\dot{m}_3 = 1/3$; $c_{N_2} = \dot{m}_2/\dot{m}_3 = 2/3$

$C_{P\ mix} = \sum c_i\ C_{P\ i} = (1/3) \times 0.842 + (2/3) \times 1.042 = 0.97533$ kJ/kg

$R_{mix} = \sum c_i R_i = (1/3) \times 0.1889 + (2/3) \times 0.2968 = 0.2608$ kJ/kg

Energy Eq.: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = \dot{m}_1 h_{3\ CO_2} + \dot{m}_2 h_{3\ N_2}$

Divide this equation with $\dot{m}_3$ and take differences in h as $C_P \Delta T$

$$\frac{1}{3} C_{P\ CO_2} T_1 + \frac{2}{3} C_{P\ N_2} T_2 = [\frac{1}{3} C_{P\ CO_2} + \frac{2}{3} C_{P\ N_2}] T_3 = C_{P\ mix} T_3$$

$$T_3 = [\frac{1}{3} \times 0.842 \times 1000 + \frac{2}{3} \times 1.042 \times 400] / 0.97533 = \textbf{572.7 K}$$

11.34

An insulated gas turbine receives a mixture of 10% CO_2, 10% H_2O and 80% N_2 on a mass basis at 1000 K, 500 kPa. The volume flow rate is 2 m^3/s and its exhaust is at 700 K, 100 kPa. Find the power output in kW using constant specific heat from A.5 at 300 K.

Solution:

C.V. Turbine, Steady, 1 inlet, 1 exit flow with an ideal gas mixture, q = 0.

Energy Eq.: $\dot{W}_T = \dot{m}(h_i - h_e) = \dot{m}C_{P\,mix}(T_i - T_e)$

Properties: From Eqs.11.15 and 11.23

$R_{mix} = \sum c_i R_i = 0.1 \times 0.1889 + 0.1 \times 0.4615 + 0.8 \times 0.2968$
$\qquad = 0.30248$ kJ/kg K

$C_{P\,mix} = \sum c_i\, C_{P\,i} = 0.1 \times 0.842 + 0.1 \times 1.872 + 0.8 \times 1.042$
$\qquad = 1.105$ kJ/kg K

$PV = mR_{mix}T \quad => \quad \dot{m} = P\dot{V}\,/\,R_{mix}T$

$\qquad \dot{m} = 500$ kPa $\times 2$ m^3/s /(0.30248×1000) kJ/kg = 3.306 kg/s

$\dot{W}_T = 3.306$ kg/s $\times 1.105$ kJ/kg-K (1000 − 700) K = **1096 kW**

11.35

Solve Problem 11.34 using the values of enthalpy from Table A.8.
An insulated gas turbine receives a mixture of 10% CO_2, 10% H_2O and 80% N_2

on a mass basis at 1000 K, 500 kPa. The volume flow rate is 2 m^3/s and its exhaust is at 700 K, 100 kPa. Find the power output in kW using constant specific heat from A.5 at 300 K.

Solution:

C.V. Turbine, Steady, 1 inlet, 1 exit flow with an ideal gas mixture, q = 0.

Energy Eq.: $\dot{W}_T = \dot{m}(h_i - h_e) = \dot{m} \sum c_j (h_i - h_e)_j$

Properties: From Eqs.11.15 and 11.23

$\quad R_{mix} = \sum c_i R_i = 0.1 \times 0.1889 + 0.1 \times 0.4615 + 0.8 \times 0.2968$

$\qquad = 0.30248$ kJ/kg K

$\quad PV = mR_{mix}T \quad \Rightarrow \quad \dot{m} = P\dot{V} / R_{mix}T$

$\qquad \dot{m} = 500$ kPa $\times$ 2 m^3/s /(0.30248 $\times$ 1000) kJ/kg = 3.306 kg/s

Now get the h values from Table A.8 (all in kJ/kg)

$\quad \dot{W}_T = 3.306$ kg/s [0.1 (971.67 - 616.22) + 0.1 (1994.13 - 1338.56)

$\qquad + 0.8 (1075.91 - 735.86)]$ kJ/kg

$\quad = \textbf{1234 kW}$

11.36

Solve Problem 11.33 assuming the flows are in kmol/s.
A steady flow of 0.1 kg/s carbon dioxide at 1000 K in one line is mixed with 0.2 kg/s of nitrogen at 400 K from another line, both at 100 kPa. The mixing chamber is insulated and has constant pressure of 100 kPa. Use constant heat capacity to find the mixing chamber exit temperature.

Take CV around the mixing chamber

Continuity Eq. mole basis: $\dot{n}_1 + \dot{n}_2 = \dot{n}_3$;

Concentrations: $y_{CO_2} = \dot{n}_1/\dot{n}_3 = 1/3$; $y_{N_2} = \dot{n}_2/\dot{n}_3 = 2/3$

$$\bar{C}_{P\,mix} = \sum y_i \bar{C}_{P\,i} = \frac{1}{3} \times 0.842 \times 44.01 + \frac{2}{3} \times 1.042 \times 28.013 = 31.81 \frac{kJ}{kmol}$$

$$M_{mix} = \sum y_i M_i = (1/3) \times 44.01 + (2/3) \times 28.013 = 33.345 \text{ kg/kmol}$$

Energy Eq. Mole basis: $\dot{n}_1 h_1 + \dot{n}_2 \bar{h}_2 = \dot{n}_3 \bar{h}_3 = \dot{n}_1 \bar{h}_{3\,CO_2} + \dot{n}_2 \bar{h}_{3\,N_2}$

Divide this equation with $\dot{n}_3$ and take differences in $\bar{h}$ as $\bar{C}_P \Delta T$

$$\frac{1}{3} \bar{C}_{P\,CO_2} T_1 + \frac{2}{3} \bar{C}_{P\,N_2} T_2 = [\frac{1}{3} \bar{C}_{P\,CO_2} + \frac{2}{3} \bar{C}_{P\,N_2}] T_3 = \bar{C}_{P\,mix} T_3$$

$$T_3 = [\frac{1}{3} \times 37.056 \times 1000 + \frac{2}{3} \times 29.189 \times 400] / 31.81 = \textbf{633 K}$$

11.37

Solve Problem 11.34 with the percentages on a mole basis and use Table A.9.
An insulated gas turbine receives a mixture of 10% CO_2, 10% H_2O and 80% N_2
on a mole basis at 1000 K, 500 kPa. The volume flow rate is 2 m^3/s and its
exhaust is at 700 K, 100 kPa. Find the power output in kW using constant specific
heat from A.5 at 300 K.

C.V. Turbine, Steady flow, 1 inlet, 1 exit flow with an ideal gas mixture, and no
heat transfer so q = 0.

Energy Eq.: $\dot{W}_T = \dot{m}\,(h_i - h_e) = \dot{n}\,(\bar{h}_i - \bar{h}_e) = \dot{n}\left[\sum y_j\,(\bar{h}_i - \bar{h}_e)_j\right]$

Ideal gas law: $PV = n\bar{R}T$ =>

$$\dot{n} = \frac{P\dot{V}}{\bar{R}T} = \frac{500 \text{ kPa} \times 2 \text{ m}^3/\text{s}}{8.3145 \text{ kJ/kmol-K} \times 1000 \text{ K}} = 0.1203 \text{ kmol/s}$$

Read the enthalpies from Table A.9 (they are all in kJ/kmol)

$$\dot{W}_T = 0.1203\left[0.1(33397 - 17754) + 0.1(26000 - 14190) + 0.8(21463 - 11937)\right]$$
$$= \mathbf{1247\ kW}$$

11.38

A mixture of 0.5 kg nitrogen and 0.5 kg oxygen is at 100 kPa, 300 K in a piston cylinder keeping constant pressure. Now 800 kJ is added by heating. Find the final temperature and the increase in entropy of the mixture using Table A.5 values.

Solution:

C.V. Mixture in the piston cylinder.

Energy Eq.: $\quad\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $\quad$ P = constant $\quad$ => $\quad\quad {}_1W_2 = \int P\ dV = P\ (V_2 - V_1)$

$\quad {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + mP(v_2 - v_1) = m(h_2 - h_1)$

$\quad h_2 - h_1 = {}_1Q_2/m \cong C_{P\ mix}\ (T_2 - T_1)$

From Eq.11.23 and Table A.5:

$\quad C_{P\ mix} = (1/2) \times 0.922 + (1/2) \times 1.042 = 0.982$ kJ/kg K

$\quad T_2 = T_1 + {}_1Q_2/mC_{P\ mix}$

$\quad\quad\quad = 300$ K $+ 800$ kJ$/(1 \times 0.982)$ kJ/K $= \textbf{1114.7 K}$

From Eq.11.24:

$\quad m(s_2 - s_1) = m[C_{P\ mix}\ \ln(T_2\ /\ T_1) - R\ \ln(P_2\ /\ P_1)]$

$\quad\quad\quad = 1$ kg $\times 0.982$ kJ/kg-K $\times \ln(1114.7/300) = \textbf{1.29 kJ/K}$

11.39

A rigid insulated vessel contains 12 kg of oxygen at 200 kPa, 280 K separated by a membrane from 26 kg carbon dioxide at 400 kPa, 360 K. The membrane is removed and the mixture comes to a uniform state. Find the final temperature and pressure of the mixture.

Solution:

C.V. Total vessel. Control mass with two different initial states.

Mass: $m = m_{O2} + m_{CO2} = 12 + 26 = 38$ kg

Process: V = constant (rigid)   => W = 0,   insulated   => Q = 0

Energy: $U_2 - U_1 = 0 - 0 = m_{O2} C_{V\ O2}(T_2 - T_{1\ O2}) + m_{CO2}C_{V\ CO2}(T_2 - T_{1\ CO2})$

Initial state from ideal gas Table A.5

$$C_{V\ O2} = 0.662 \text{ kJ/kg}, \quad C_{V\ CO2} = 0.653 \text{ kJ/kg K}$$

O_2 : $V_{O2} = mRT_1/P = 12 \times 0.2598 \times 280/200 = 4.3646 \text{ m}^3$,

CO_2 : $V_{CO2} = mRT_1/P = 26 \times 0.1889 \times 360/400 = 4.4203 \text{ m}^3$

Final state mixture

$$R_{mix} = \sum c_i R_i = [12 \times 0.2598 + 26 \times 0.1889\ ]/38 = 0.2113 \text{ kJ/kg K}$$

The energy equation becomes

$$m_{O2} C_{V\ O2} T_2 + m_{CO2}C_{V\ CO2} T_2$$
$$= m_{O2} C_{V\ O2} T_{1\ O2} + m_{CO2}C_{V\ CO2} T_{1\ CO2}$$
$$(7.944 + 16.978\ ) \text{ kJ/K} \times T_2 = 2224.32 + 6112.08 = 8336.4 \text{ kJ}$$
$$=> T_2 = \textbf{334.5 K}$$

From mixture gas constant and total volume

$$V_2 = V_{O2} + V_{CO2} = 4.3646 \text{ m}^3 + 4.4203 \text{ m}^3 = 8.7849 \text{ m}^3$$

$$P_2 = mR_{mix}T_2/V_2 = 38 \text{ kg} \times 0.2113 \text{ kJ/kg-K} \times 334.5 \text{ K}/ 8.7849 \text{ m}^3$$
$$= \textbf{305.7 kPa}$$

11.40

A mixture of 0.5 kg nitrogen and 0.5 kg oxygen is at 100 kPa, 300 K in a piston cylinder keeping constant pressure. Now 1200 kJ is added by heating. Find the final temperature and the increase in entropy of the mixture using Table A.8 values.

Solution:

C.V. Mixture in the piston cylinder.

Energy Eq.:　　　$m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process:　　$P = \text{constant}$　$=>$　　${}_1W_2 = \int P\, dV = P\,(V_2 - V_1)$

$\quad {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + mP(v_2 - v_1)$

$\qquad\qquad = m(h_2 - h_1)$

$\quad h_2 - h_1 = {}_1Q_2/m = 1200 \text{ kJ} / 1 \text{ kg} = 1200 \text{ kJ/kg}$

Since T_2 is high we use Table A.8 values guessing a T_2

$$(h_2 - h_1)_{1400K} = \frac{1}{2}(1556.87 - 311.67) + \frac{1}{2}(1426.44 - 273.15)$$

$$= 1199.245 \text{ kJ/kg} \qquad \text{low}$$

$$(h_2 - h_1)_{1500K} = \frac{1}{2}(1680.70 - 311.67) + \frac{1}{2}(1540.23 - 273.15)$$

$$= 1318.06 \text{ kJ/kg} \qquad \text{too high}$$

$$T_2 = 1400 + 100[(1200 - 1199.245)/(1318.06 - 1199.245)] = \textbf{1400.6 K}$$

From Eqs.11.16 and 11.18:

$$s_2 - s_1 = \frac{1}{2}(s^{\circ}_{T2} - s^{\circ}_{T1})_{N2} + \frac{1}{2}(s^{\circ}_{T2} - s^{\circ}_{T1})_{O2}$$

$$= \frac{1}{2}(8.5495 - 6.8463) + \frac{1}{2}(7.9869 - 6.4168)$$

$$= \textbf{1.6367 kJ/kg K}$$

Notice that the pressure term drop out, composition does not change and total pressure is constant.

11.41

New refrigerant R-410A is a mixture of R-32 and R-125 in a 1:1 mass ratio. A process brings 0.5 kg R-410A from 270 K to 320 K at a constant pressure 250 kPa in a piston cylinder. Find the work and heat transfer.

Solution:

C.V. R-410A

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2 - P(V_2 - V_1)$

Process: $P = $ constant $\quad {}_1W_2 = P(V_2 - V_1) = mR(T_2 - T_1)$

$$ {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) $$

From Eq.11.15:

$$ R_{mix} = \sum c_i R_i = \frac{1}{2} \times 0.1598 + \frac{1}{2} \times 0.06927 = 0.1145 \text{ kJ/kg K} $$

FromEq.11.23:

$$ C_{P\ mix} = \frac{1}{2} \times 0.822 + \frac{1}{2} \times 0.791 = 0.8065 \text{ kJ/kg K} $$

From the process equation

$\quad {}_1W_2 = 0.5 \text{ kg} \times 0.1145 \text{ kJ/kg-K } (320 - 270) \text{ K} = \textbf{2.863 kJ}$

From the energy equation

$\quad {}_1Q_2 = m \, C_{P\ mix} (T_2 - T_1) = 0.5 \text{ kg} \times 0.8065 \text{ kJ/kg-K } (320 - 270) \text{ K}$

$\quad = \textbf{20.16 kJ}$

11.42

A piston/cylinder device contains 0.1 kg of a mixture of 40 % methane and 60 % propane gases by mass at 300 K and 100 kPa. The gas is now slowly compressed in an isothermal (T = constant) process to a final pressure of 250 kPa. Show the process in a P-V diagram and find both the work and heat transfer in the process.

Solution:

C.V. Mixture of methane and propane, this is a control mass.

Assume methane & propane are ideal gases at these conditions.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Property from Eq.11.15

   $R_{mix} = 0.4\ R_{CH4} + 0.6\ R_{C3H8}$

   $= 0.4 \times 0.5183 + 0.6 \times 0.1886 = 0.3205$ kJ/kg K

Process: T = constant & ideal gas =>

   ${}_1W_2 = \int P\ dV = mR_{mix}T \int (1/V)dV = mR_{mix}T\ \ln(V_2/V_1)$

      $= mR_{mix}T\ \ln(P_1/P_2)$

      $= 0.1$ kg $\times 0.3205$ kJ/kg-K $\times 300$ K $\ln(100/250) =$ **-8.81 kJ**

Now heat transfer from the energy equation where we notice that u is a constant (ideal gas and constant T) so

   ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = {}_1W_2 =$ **-8.81 kJ**

11.43

The substance R-410A, see Problem 11.41 is at 100 kPa, 290 K. It is now brought to 250 kPa, 400 K in a reversible polytropic process. Find the change in specific volume, specific enthalpy and specific entropy for the process.

Solution:

Eq.11.15: $R_{mix} = \Sigma c_i R_i = \frac{1}{2} \times 0.1598 + \frac{1}{2} \times 0.06927 = 0.1145$ kJ/kg K

Eq.11.23: $C_{Pmix} = \Sigma c_i C_{Pi} = \frac{1}{2} \times 0.822 + \frac{1}{2} \times 0.791 = 0.8065$ kJ/kg K

$v_1 = RT_1/P_1 = 0.1145 \times 290/100 = 0.33205$ m^3/kg

$v_2 = RT_2/P_2 = 0.1145 \times 400/250 = 0.1832$ m^3/kg

$v_2 - v_1 = 0.1832 - 0.33205 =$ **-0.14885 m^3/kg**

$h_2 - h_1 = C_{Pmix} (T_2 - T_1) = 0.8065$ kJ/kg-K $(400 - 290)$ K $=$ **88.72 kJ/kg**

From Eq.11.24

$s_2 - s_1 = C_{Pmix} \ln(T_2 / T_1) - R_{mix} \ln(P_2 / P_1)$

$= 0.8065 \ln (400/290) - 0.1145 \ln (250/100) =$ **0.154 kJ/kg K**

11.44

Natural gas as a mixture of 75% methane and 25% ethane by mass is flowing to a compressor at 17°C, 100 kPa. The reversible adiabatic compressor brings the flow to 350 kPa. Find the exit temperature and the needed work per kg flow.

Solution:

C.V. Compressor. Steady, adiabatic q = 0, reversible $s_{gen} = 0$

Energy Eq.4.13: $-w = h_{ex} - h_{in}$;   Entropy Eq.7.8: $s_i + s_{gen} = s_e$

Process: reversible $\Rightarrow$ $s_{gen} = 0$ $\Rightarrow$ $s_e = s_i$

Assume ideal gas mixture and constant heat capacity, so we need k and C_P

From Eq.11.15 and 11.23:

   $R_{mix} = \sum c_i R_i = 0.75 \times 0.5183 + 0.25 \times 0.2765 = 0.45785$ kJ/kg K

   $C_{P\ mix} = \sum c_i C_{Pi} = 0.75 \times 2.254 + 0.25 \times 1.766 = 2.132$ kJ/kg K

   $C_V = C_{P\ mix} - R_{mix} = 2.132 - 0.45785 = 1.6742$ kJ/kg K

Ratio of specific heats: $k = C_p/ C_v = 1.2734$

The isentropic process gives Eq.6.23, from Eq.11.24 with $s_e = s_i$

   $T_e = T_i\ (P_e/ P_i)^{(k-1)/k} = 290\ (350/100)^{0.2147} = $ **379.5 K**

Work from the energy equation:

   $w_{c\ in} = C_P\ (T_e - T_i) = 2.132$ kJ/kg-K (379.5 – 290) K = **190.8 kJ/kg**

11.45

A compressor brings R-410A (see problem 11.41) from –10 $^{\circ}$C, 125 kPa up to 500 kPa in an adiabatic reversible compression. Assume ideal gas behavior and find the exit temperature and the specific work.

Solution:

C.V. Compressor

Process: q = 0 ; adiabatic and reversible.

Energy Eq.4.13: $w = h_i - h_e$;

Entropy Eq.7.8: $s_e = s_i + s_{gen} + \int dq/T = s_i + 0 + 0 = s_i$

From Eq.11.15:

$$R_{mix} = \sum c_i R_i = \frac{1}{2} \times 0.1598 + \frac{1}{2} \times 0.06927 = 0.1145 \text{ kJ/kg K}$$

FromEq.11.23:

$$C_{P\,mix} = \frac{1}{2} \times 0.822 + \frac{1}{2} \times 0.791 = 0.8065 \text{ kJ/kg K}$$

$$R_{mix}/\,C_{P\,mix} = 0.1145/0.8065 = 0.14197$$

For constant s, ideal gas and use constant specific heat as in Eq.6.23

$$T_e/T_i = (P_e/P_i)^{R/Cp}$$

$$T_e = 263.15 \text{ K} \times (500/125)^{0.14197} = \textbf{320.39 K}$$

$$w \cong C_{P\,mix}(\,T_i - T_e) = 0.8065 \text{ kJ/kg-K } (263.15 - 320.39) \text{ K}$$

$$= \textbf{-46.164 kJ/kg}$$

11.46

Two insulated tanks A and B are connected by a valve. Tank A has a volume of 1 m^3 and initially contains argon at 300 kPa, 10°C. Tank B has a volume of 2 m^3 and initially contains ethane at 200 kPa, 50°C. The valve is opened and remains open until the resulting gas mixture comes to a uniform state. Determine the final pressure and temperature.

Solution:

C.V. Tanks A + B. Control mass no W, no Q.

Energy Eq.3.5 and 3.36:

$$U_2 - U_1 = 0 = m_{Ar}C_{V0}(T_2 - T_{A1}) + m_{C_2H_6}C_{VO}(T_2 - T_{B1})$$

$$m_{Ar} = P_{A1}V_A/RT_{A1} = (300 \times 1) / (0.2081 \times 283.15) = 5.0913 \text{ kg}$$

$$m_{C_2H_6} = P_{B1}V_B/RT_{B1} = (200 \times 2) / (0.2765 \times 323.15) = 4.4767 \text{ kg}$$

Continuity Eq.: $m_2 = m_{Ar} + m_{C_2H_6} = 9.568$ kg

Energy Eq.: $5.0913 \times 0.312 \ (T_2 - 283.2)$
$+ \ 4.4767 \times 1.490 \ (T_2 - 323.2) = 0$

Solving, $T_2 = $ **315.5 K**

$$R_{mix} = \Sigma \ c_iR_i \ = \frac{5.0913}{9.568} \times 0.2081 + \frac{4.4767}{9.568} \times 0.2765 = 0.2401 \text{ kJ/kg K}$$

$$P_2 = m_2RT_2/(V_A+V_B) = 9.568 \text{ kg} \times 0.2401 \text{ kJ/kg-K} \times 315.5 \text{ K} /3 \text{ m}^3$$
$$= \textbf{242 kPa}$$

Borgnakke and Sonntag

11.47

A steady flow of 0.1 kmol/s carbon dioxide at 1000 K in one line is mixed with 0.2 kmol/s of nitrogen at 400 K from another line, both at 100 kPa. The exit mixture at 100 kPa is compressed by a reversible adiabatic compressor to 500 kPa. Use constant specific heat to find the mixing chamber exit temperature and the needed compressor power.

Take CV around the mixing chamber

Continuity Eq. mole basis: $\dot{n}_1 + \dot{n}_2 = \dot{n}_3$;

Concentrations: $y_{CO_2} = \dot{n}_1/\dot{n}_3 = 1/3; \quad y_{N_2} = \dot{n}_2/\dot{n}_3 = 2/3$

$$\bar{C}_{P\,mix} = \sum y_i\bar{C}_{P\,i} = \frac{1}{3}\times 0.842\times44.01 + \frac{2}{3}\times 1.042\times28.013 = 31.81\,\frac{kJ}{kmol}$$

$$R_{mix}/C_{P\,mix} = \bar{R}/\bar{C}_{P\,mix} = 8.3145\,/\,31.81 = 0.2614$$

Energy Eq. Mole basis: $\dot{n}_1 h_1 + \dot{n}_2\bar{h}_2 = \dot{n}_3\bar{h}_3 = \dot{n}_1\bar{h}_{3\,CO_2} + \dot{n}_2\bar{h}_{3\,N_2}$

Divide this equation with $\dot{n}_3$ and take differences in $\bar{h}$ as $\bar{C}_P\,\Delta T$

$$\frac{1}{3}\bar{C}_{P\,CO_2}T_1 + \frac{2}{3}\bar{C}_{P\,N_2}T_2 = [\frac{1}{3}\bar{C}_{P\,CO_2} + \frac{2}{3}\bar{C}_{P\,N_2}]\,T_3 = \bar{C}_{P\,mix}\,T_3$$

$$T_3 = [\frac{1}{3}\times 37.056\times 1000 + \frac{2}{3}\times 29.189\times 400\,]\,/\,31.81 = \textbf{633 K}$$

Now we can do the adiabatic compression

$$T_4 = T_3\,(P_4\,/\,P_3)^{R/Cp} = 633\times 5^{0.2614} = 964.1\text{ K}$$

$$\bar{w}_C = \bar{C}_{P\,mix}\,(T_4 - T_3) = 31.81\text{ kJ/kmol-K }(964.1 - 633)\text{ K}$$
$$= 10\,532\text{ kJ/kmol}$$

$$\dot{W} = \dot{m}\,\bar{w}_C = 0.3\text{ kmol/s}\times 10\,532\text{ kJ/kmol} = \textbf{3160 kW}$$

11.48

A mixture of 2 kg oxygen and 2 kg of argon is in an insulated piston cylinder arrangement at 100 kPa, 300 K. The piston now compresses the mixture to half its initial volume. Find the final pressure, temperature and the piston work.

Solution:

C.V. Mixture. Control mass, boundary work and no Q, assume reversible.

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2 = - {}_1w_2$;

Entropy Eq.6.37: $s_2 - s_1 = 0 + 0 = 0$

Process: constant s $\Rightarrow$ Pv^k = constant, $v_2 = v_1/2$,

Assume ideal gases ($T_1 \gg T_C$) and use k_{mix} and $C_{v\,mix}$ for properties.

Eq.11.15: $R_{mix} = \Sigma c_i R_i = 0.5 \times 0.25983 + 0.5 \times 0.20813 = 0.234$ kJ/kg K

Eq.11.23 $C_{Pmix} = \Sigma c_i C_{Pi} = 0.5 \times 0.9216 + 0.5 \times 0.5203 = 0.721$ kJ/kg K

$C_{vmix} = C_{Pmix} - R_{mix} = 0.487$ kJ/kg K

Ratio of specific heats: $k_{mix} = C_{Pmix}/C_{vmix} = 1.4805$

The relations for the polytropic process

Eq.6.25: $P_2 = P_1(v_1/v_2)^k = P_1(2)^k = 100(2)^{1.4805} =$ **279 kPa**

Eq.6.24: $T_2 = T_1(v_1/v_2)^{k-1} = T_1(2)^{k-1} = 300(2)^{0.4805} =$ **418.6 K**

Work from the energy equation

${}_1W_2 = m_{tot} (u_1 - u_2) = m_{tot} C_v(T_1 - T_2)$

$= 4$ kg $\times 0.487$ kJ/kg-K $(300 - 418.6)$ K = **-231 kJ**

11.49

The gas mixture from Problem 11.27 is compressed in a reversible adiabatic process from the initial state in the sample cylinder to a volume of 0.2 L. Determine the final temperature of the mixture and the work done during the process.

Solution:

From Eq.11.15

$$R_{mix} = \sum c_i R_i = 0.02 \times 4.1243 + 0.45 \times 0.2968 + 0.28 \times 0.1889$$

$$+ 0.25 \times 0.2968 = 0.34314 \text{ kJ/kg K}$$

$$m = PV/R_{mix}T = 100 \times 10^{-3}/(0.34314 \times 293.15) = \mathbf{9.941 \times 10^{-4} \ kg}$$

$$C_{V0 \ MIX} = \sum c_i \ C_{V0 \ i} = 0.02 \times 10.085 + 0.45 \times 0.744$$

$$+ 0.28 \times 0.653 + 0.25 \times 0.745 = 0.9056 \text{ kJ/kg K}$$

$$C_{P0 \ MIX} = C_{V0 \ MIX} + R_{mix} = 0.9056 + 0.34314 = 1.2487 \text{ kJ/kg K}$$

$$\rightarrow k = C_{P0}/C_{V0} = 1.2487/0.9056 = 1.379$$

The process (adiabatic and reversible) is isentropic expressed in Eq.6.32

$$\rightarrow T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = 293.15\left(\frac{1}{0.2}\right)^{0.379} = \mathbf{539.5 \ K}$$

$$_1W_2 = -\Delta U_{12} = -mC_{V0}(T_2-T_1)$$

$$= -9.941 \times 10^{-4} \text{ kg} \times 0.9056 \text{ kJ/kg-K} \times (539.5 - 293.15) \text{ K} = \mathbf{-0.22 \ kJ}$$

11.50

A piston cylinder has 0.1 kg mixture of 25% argon, 25% nitrogen and 50% carbon dioxide by mass at total pressure 100 kPa and 290 K. Now the piston compresses the gases to a volume 7 times smaller in a polytropic process with n = 1.3. Find the final pressure and temperature, the work, and the heat transfer for the process.

Solution:

Expansion ratio: $v_2/v_1 = 1/7$

Mixture properties:

$$R_{mix} = \Sigma\, c_i R_i = 0.25 \times 0.2081 + 0.25 \times 0.2968 + 0.5 \times 0.1889$$
$$= 0.220675 \text{ kJ/kg K}$$

$$C_{v\,mix} = \Sigma\, c_i\, C_{vi} = 0.25 \times 0.312 + 0.25 \times 0.745 + 0.5 \times 0.653$$
$$= 0.59075 \text{ kJ/kg K}$$

Process eq.: Rev. adiabatic and ideal gas gives $Pv^n = C$, with n = 1.3

$$P_2 = P_1\, (v_1/v_2)^n = 100 \times 7^{1.3} = \textbf{1254.9 kPa}$$

$$T_2 = T_1\, (v_1/v_2)^{n-1} = 290 \times 7^{0.3} = \textbf{519.9 K}$$

Polytropic process work term from Eq.3.21 and ideal gas law

$$_1W_2 = \frac{mR}{1-n}\,(T_2 - T_1) = \frac{0.1 \times 0.220675}{-0.3} \text{ kJ/K } (519.9 - 290)\text{ K} = \textbf{-16.91 kJ}$$

Energy Eq.: $_1Q_2 = U_2 - U_1 + {}_1W_2 = m\, C_{v\,mix}\,(T_2 - T_1) + {}_1W_2$

$$= 0.1 \text{ kg} \times 0.59075 \text{ kJ/kg-K } (519.9 - 290)\text{ K} - 16.91 \text{ kJ}$$
$$= \textbf{–3.33 kJ}$$

Entropy generation

Borgnakke and Sonntag

11.51

A flow of gas A and a flow of gas B are mixed in a 1:2 mole ratio with the same T. What is the entropy generation per kmole flow out?

For this case the total flow is 3 mol units so,

$$y_A = n_A/n_{tot} = 1/3; \qquad y_B = n_B/n_{tot} = 2/3$$

Eq. 11.19: $\Delta S = - \bar{R}[(1/3) \ln 1/3 + (2/3) \ln 2/3] = + 0.6365 \bar{R}$
$$= \textbf{5.292 kJ/kmol K}$$

Borgnakke and Sonntag

11.52

A rigid container has 1 kg argon at 300 K and 1 kg argon at 400 K both at 150 kPa. Now they are allowed to mix without any external heat transfer. What is final T, P? Is any s generated?

Energy Eq.: $U_2 - U_1 = 0 = 2mu_2 - mu_{1a} - mu_{1b} = mC_v(2T_2 - T_{1a} - T_{1b})$

$$T_2 = (T_{1a} + T_{1b})/2 = \textbf{350 K},$$

Process Eq.: V = constant =>

$$P_2V = 2mRT_2 = mR(T_{1a} + T_{1b}) = P_1V_{1a} + P_1V_{1b} = P_1V$$

$$P_2 = P_1 = \textbf{150 kPa},$$

ΔS due to temperature changes only, not P, internally we have a Q over a ΔT

$$\Delta S = m\,(s_2 - s_{1a}) + m\,(s_2 - s_{1b}) = mC_p\,[\,\ln\,(T_2/T_{1a}) + \ln\,(T_2/T_{1b})\,]$$

$$= 1\text{ kg} \times 0.520 \text{ kJ/kg-K}\,\Big[\ln\frac{350}{300} + \ln\frac{350}{400}\Big] = \textbf{0.0107 kJ/K}$$

Why did we not account for partial pressures? Since we mix argon with argon there is no entropy generation due to mixing and no partial pressure.

Borgnakke and Sonntag

11.53

What is the entropy generation in problem 11.26?

No Q, No W   so the energy equation gives constant U

Energy Eq.: $U_2 - U_1 = 0 = m_{CO2}(u_2 - u_1)_{CO2} + m_{Ar}(u_2 - u_1)_{Ar}$

$$= m_{CO2}C_{v\ CO2}(T_2 - T_1)_{CO2} + m_{Ar}C_{v\ Ar}(T_2 - T_1)_{Ar}$$

$$= (1 \times 0.653 + 1 \times 0.312) \times T_2 - 1 \times 0.653 \times 300 - 1 \times 0.312 \times 400$$

$T_2 = \mathbf{332.3\ K}$,

$$V = V_1 = V_{CO2} + V_{Ar} = m_{CO2}R_{CO2}T_{CO2}/P + m_{Ar}R_{Ar}T_{Ar}/P$$

$$= 1 \times 0.1889 \times 300/150 + 1 \times 0.2081 \times 400/150 = 0.932\ 73\ m^3$$

Pressure from ideal gas law and Eq.11.15 for R

$$P_2 = (1 \times 0.1889 + 1 \times 0.2081) \times 332.3/0.932\ 73 = \mathbf{141.4\ kPa}$$

$$S_2 - S_1 = 0 +\ _1S_{2\ gen} = m_{CO2}(s_2 - s_1)_{CO2} + m_{Ar}(s_2 - s_1)_{Ar}$$

For each component: $s_2 - s_1 = C_P \ln \dfrac{T_2}{T_1} - R \ln \dfrac{yP_2}{P_1}$ [P's are total pressure]

$$y_{CO2} = \frac{c_{CO2}/M_{CO2}}{c_{CO2}/M_{CO2} + c_{Ar}/M_{Ar}} = \frac{0.5\ /\ 44.01}{0.5\ /\ 39.948 + 0.5\ /\ 44.01} = 0.4758$$

$$y_{Ar} = 1 - y_{CO2} = 0.5242$$

$$_1S_{2\ gen} = 1\ kg \times [0.520 \ln(\frac{332.3}{400}) - 0.2081 \ln (\frac{0.5242 \times 141.4}{150})]\ kJ/kg\text{-}K$$

$$+ 1\ kg \times [0.842 \ln(\frac{332.3}{300}) - 0.1889 \ln (\frac{0.4758 \times 141.4}{150})]\ kJ/kg\text{-}K$$

$$= 0.05027 + 0.23756 = \mathbf{0.2878\ kJ/K}$$

11.54

A flow of 2 kg/s mixture of 50% CO_2 and 50% O_2 by mass is heated in a constant pressure heat exchanger from 400 K to 1000 K by a radiation source at 1400 K. Find the rate of heat transfer and the entropy generation in the process shown in Fig. P11.54.

Solution:

C.V. Heat exchanger   w = 0

Energy Eq.4.12: $\dot{Q}_{in} = \dot{m}(h_e - h_i)$

Values from Table A.8 due to the high T.

$$\dot{Q}_{in} = 2\,[\tfrac{1}{2} \times (971.67 - 303.76) + \tfrac{1}{2} \times (980.95 - 366.03)] = \textbf{1282.8 kW}$$

Entropy Eq.7.8: $\dot{m}_e s_e = \dot{m}_i s_i + \dot{Q}/T_s + \dot{S}_{gen}$

As the pressure is constant the pressure correction in Eq.6.19 drops out to give the generation as

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) - \dot{Q}/T_s$$

$$= 2 \text{ kg/s } [\tfrac{1}{2} \times (6.119 - 5.1196) + \tfrac{1}{2} \times (7.6121 - 6.6838)] \text{ kJ/kg-K}$$

$$- 1282.8 \text{ kW } /1400 \text{ K}$$

$$= \textbf{1.01 kW/K}$$

11.55

A flow of 1.8 kg/s steam at 400 kPa, 400°C is mixed with 3.2 kg/s oxygen at 400 kPa, 400 K in a steady flow mixing-chamber without any heat transfer. Find the exit temperature and the rate of entropy generation.

C.V. Mixing chamber, steady flow, no work, no heat transfer. To do the entropies we need the mole fractions.

$$\dot{n}_{H2O} = \frac{\dot{m}_{H2O}}{M_{H2O}} = \frac{1.8}{18.015} = 0.1 \text{ kmol/s}; \quad \dot{n}_{O2} = \frac{\dot{m}_{O2}}{M_{O2}} = \frac{3.2}{31.999} = 0.1 \text{ kmol/s}$$

$$y_{H2O} = y_{O2} = 0.5$$

Energy Eq.: $\dot{m}_{H2O} h_1 + \dot{m}_{O2} h_2 = \dot{m}_{H2O} h_{3\,H2O} + \dot{m}_{O2} h_{3\,O2}$

Entropy Eq.: $\dot{m}_{H2O} s_1 + \dot{m}_{O2} s_2 + \dot{S}_{gen} = \dot{m}_{H2O} s_{3\,H2O} + \dot{m}_{O2} s_{3\,O2}$

Solve for T from the energy equation

$$\dot{m}_{H2O} (h_{3\,H2O} - h_1) + \dot{m}_{O2} (h_{3\,O2} - h_2) = 0$$

$$\dot{m}_{H2O} C_{P\,H2O}(T_3 - T_1) + \dot{m}_{O2} C_{P\,O2}(T_3 - T_2) = 0$$

$$1.8 \times 1.872 (T_3 - 400 - 273.15) + 3.2 \times 0.922(T_3 - 400) = 0$$

$$T_3 = \textbf{545.6 K}$$

$$\dot{S}_{gen} = \dot{m}_{H2O} (s_{3\,H2O} - s_1) + \dot{m}_{O2} (s_{3\,O2} - s_2)$$

$$= \dot{m}_{H2O} [C_{P\,H2O} \ln \frac{T_3}{T_1} - R \ln y_{H2O}] + \dot{m}_{O2} [C_{P\,O2} \ln \frac{T_3}{T_2} - R \ln y_{O2}]$$

$$= 1.8 \text{ kg/s} [1.872 \ln \frac{545.6}{673.15} - 0.4615 \ln 0.5] \text{ kJ/kg-K}$$

$$+ 3.2 \text{ kg/s} [0.922 \ln \frac{545.6}{400} - 0.2598 \ln 0.5] \text{ kJ/kg-K}$$

$$= - 0.132 + 1.492 = \textbf{1.36 kW/K}$$

11.56

Carbon dioxide gas at 320 K is mixed with nitrogen at 280 K in an insulated mixing chamber. Both flows are at 100 kPa and the mass ratio of carbon dioxide to nitrogen is 2:1. Find the exit temperature and the total entropy generation per kg of the exit mixture.

Solution:

CV mixing chamber. The inlet ratio is so $\dot{m}_{CO_2} = 2\,\dot{m}_{N_2}$ and assume no external heat transfer, no work involved.

Continuity Eq.6.9: $\dot{m}_{N_2} + 2\dot{m}_{N_2} = \dot{m}_{ex} = 3\dot{m}_{N_2}$;

Energy Eq.6.10: $\dot{m}_{N_2}(h_{N_2} + 2\,h_{CO_2}) = 3\dot{m}_{N_2}h_{mix\ ex}$

Take 300 K as reference and write $h = h_{300} + C_{Pmix}(T - 300)$.

$C_{P\,N_2}(T_{i\,N_2} - 300) + 2C_{P\,CO_2}(T_{i\,CO_2} - 300) = 3C_{P\,mix}(T_{mix\ ex} - 300)$

$C_{P\,mix} = \sum c_i C_{P\,i} = \dfrac{2}{3} \times 0.842 + \dfrac{1}{3} \times 1.042 = 0.9087$ kJ/kg K

$3C_{P\,mix}T_{mix\ ex} = C_{P\,N_2}T_{i\,N_2} + 2C_{P\,CO_2}T_{i\,CO_2} = 830.64$ kJ/kg

$T_{mix\ ex} = $ **304.7 K**;

To find the entropies we need the partial pressures, which assuming ideal gas are equal to the mole fractions times the total pressure:

$$y_i = [c_i/\,M_i] / \sum c_j/M_j$$

$$y_{N_2} = [0.3333 / 28.013] / [\dfrac{0.3333}{28.013} + \dfrac{0.6666}{44.01}] = 0.44$$

$$y_{CO_2} = 1 - y_{N_2} = 0.56$$

$$\dot{S}_{gen} = \dot{m}_{ex}s_{ex} - (\dot{m}s)_{iCO_2} - (\dot{m}s)_{iN_2} = \dot{m}_{N_2}(s_e - s_i)_{N_2} + 2\dot{m}_{N_2}(s_e - s_i)_{CO_2}$$

$$\dfrac{\dot{S}_{gen}}{3\dot{m}_{N_2}} = \dfrac{1}{3}\,[C_{PN_2}\ln\dfrac{T_{ex}}{T_{iN_2}} - R_{N_2}\ln y_{N_2}\,] + \dfrac{2}{3}\,[C_{PCO_2}\ln\dfrac{T_{ex}}{T_{iCO_2}} - R_{CO_2}\ln y_{CO_2}]$$

$$= \dfrac{1}{3}\,[\ 1.042\ \ln(\dfrac{304.7}{280}) - 0.2968\ \ln 0.44\ ]$$

$$+ \dfrac{2}{3}\,[\ 0.842\ \ln(\dfrac{304.7}{320}) - 0.1889\ \ln 0.56\ ]$$

$$=\ \ 0.110585 + 0.068275 = \textbf{0.1789 kJ/kg mix K}$$

11.57

Carbon dioxide gas at 320 K is mixed with nitrogen at 280 K in an insulated mixing chamber. Both flows are coming in at 100 kPa and the mole ratio of carbon dioxide to nitrogen is 2:1. Find the exit temperature and the total entropy generation per kmole of the exit mixture.

CV mixing chamber, steady flow. The inlet ratio is $\dot{n}_{CO_2} = 2\,\dot{n}_{N_2}$ and assume no external heat transfer, no work involved.

Continuity: $\quad \dot{n}_{CO_2} + 2\dot{n}_{N_2} = \dot{n}_{ex} = 3\dot{n}_{N_2}$;

Energy Eq.: $\quad \dot{n}_{N_2}(\bar{h}_{N_2} + 2\bar{h}_{CO_2}) = 3\dot{n}_{N_2}\bar{h}_{mix\ ex}$

Take 300 K as reference and write $\bar{h} = \bar{h}_{300} + \bar{C}_{Pmix}(T - 300)$.

$\bar{C}_{P\,N_2}(T_{i\,N_2} - 300) + 2\bar{C}_{P\,CO_2}(T_{i\,CO_2} - 300) = 3\bar{C}_{P\,mix}(T_{mix\ ex} - 300)$

Find the specific heats in Table A.5 to get

$\bar{C}_{P\,mix} = \sum y_i\bar{C}_{P\,i} = (1.042 \times 28.013 + 2 \times 0.842 \times 44.01)/3$

$\qquad\qquad = 34.43$ kJ/kmol K

$3\bar{C}_{P\,mix}T_{mix\ ex} = \bar{C}_{P\,N_2}T_{i\,N_2} + 2\bar{C}_{P\,CO_2}T_{i\,CO_2} = 31889$ kJ/kmol

$\qquad\qquad T_{mix\ ex} = $ **308.7 K**

Partial pressures are total pressure times molefraction

$\qquad\qquad P_{ex\,N_2} = P_{tot}/3; \quad P_{ex\,CO_2} = 2P_{tot}/3$

$\dot{S}_{gen} = \dot{n}_{ex}\bar{s}_{ex} - (\dot{n}\bar{s})_{iCO_2} - (\dot{n}\bar{s})_{iN_2} = \dot{n}_{N_2}(\bar{s}_e - \bar{s}_i)_{N_2} + 2\dot{n}_{N_2}(\bar{s}_e - \bar{s}_i)_{CO_2}$

$\dot{S}_{gen}/3\dot{n}_{N_2} = [\bar{C}_{PN_2}\ln\dfrac{T_{ex}}{T_{iN_2}} - \bar{R}\ln y_{N_2} + 2\bar{C}_{PCO_2}\ln\dfrac{T_{ex}}{T_{iCO_2}} - 2\bar{R}\ln y_{CO_2}]/3$

$\qquad\qquad = [2.8485 + 9.1343 - 2.6607 + 6.742]/3 = $ **5.35 kJ/kmol mix K**

1 N$_2$

2 CO$_2$

MIXING CHAMBER $\dot{S}_{ge}$

3 Mix

11.58

A flow of 1 kg/s carbon dioxide at 1600 K, 100 kPa is mixed with a flow of 2 kg/s water at 800 K, 100 kPa and after the mixing it goes through a heat exchanger where it is cooled to 500 K by a 400 K ambient. How much heat transfer is taken out in the heat exchanger? What is the entropy generation rate for the whole process?

Solution:

C.V. Total mixing section and heat exchanger. Steady flow and no work. To do the entropy at the partial pressures we need the mole fractions.

$$\dot{n}_{H2O} = \dot{m}_{H2O}/M_{H2O} = 2 / 18.015 = 0.11102 \text{ kmol/s}$$

$$\dot{n}_{CO2} = \dot{m}_{CO2}/M_{CO2} = 1 / 44.01 = 0.022722 \text{ kmol/s}$$

$$y_{H2O} = \frac{0.11102}{0.11102 + 0.022722} = 0.8301, \quad y_{CO2} = 1 - y_{H2O} = 0.1699$$

Energy Eq.: $\dot{m}_{H2O} h_1 + \dot{m}_{CO2} h_2 = \dot{Q}_{cool} + \dot{m}_{H2O} h_{4\,H2O} + \dot{m}_{CO2} h_{4\,CO2}$

Entropy Eq.: $\dot{m}_{H2O} s_1 + \dot{m}_{CO2} s_2 + \dot{S}_{gen} = \dfrac{\dot{Q}_{cool}}{T_{amb}} + \dot{m}_{H2O} s_{4\,H2O} + \dot{m}_{CO2} s_{4\,CO2}$

As T is fairly high we use Table A.8 for properties on a mass basis.

	1	2	4 H2O	4 CO2
h [kJ/kg]	1550.13	1748.12	935.12	401.52
s^o_T [kJ/kg K]	12.4244	6.7254	11.4644	5.3375

$$\dot{Q}_{cool} = \dot{m}_{H2O} (h_1 - h_{4\,H2O}) + \dot{m}_{CO2} (h_2 - h_{4\,CO2})$$
$$= 2 (1550.13 - 935.12) + 1 (1748.12 - 401.52) = \textbf{2577 kW}$$

$$\dot{S}_{gen} = \dot{m}_{H2O} (s_{4\,H2O} - s_1) + \dot{m}_{CO2} (s_{4\,CO2} - s_2) + \dot{Q}_{cool}/T_{amb}$$
$$= 2 \text{ kg/s } [11.4644 - 12.4244 - 0.4615 \ln(0.8301)] \text{ kJ/kg-K}$$
$$+ 1 \text{ kg/s } [5.3375 - 6.7254 - 0.1889 \ln(0.1699)] \text{ kJ/kg-K} + \frac{2577}{400} \text{ kW/K}$$
$$= -1.74813 - 1.05307 + 6.4415 = \textbf{3.64 kW/K}$$

11.59

The only known sources of helium are the atmosphere (mole fraction approximately 5×10^{-6}) and natural gas. A large unit is being constructed to separate 100 m^3/s of natural gas, assumed to be 0.001 He mole fraction and 0.999 CH$_4$. The gas enters the unit at 150 kPa, 10°C. Pure helium exits at 100 kPa, 20°C, and pure methane exits at 150 kPa, 30°C. Any heat transfer is with the surroundings at 20°C. Is an electrical power input of 3000 kW sufficient to drive this unit?

0.999 CH$_4$
0.001 He
at 150 kPa, 10 °C

$\dot{V}_1 = 100$ m^3/s

$-\dot{W}_{CV} = 3000$ kW

He
CH4

$P_2 = 100$ kPa
$T_2 = 20$ °C

$P_3 = 140$ kPa
$T_3 = 30$ °C

$$\dot{n}_1 = P_1\dot{V}_1/RT_1 = 150 \text{ kPa} \times 100 \text{ m}^3/\text{s} /(8.3145 \times 283.2) \text{ kJ/kmol} = 6.37 \text{ kmol/s}$$

$$\Rightarrow \quad \dot{n}_2 = 0.001; \quad \dot{n}_1 = 0.006\ 37; \quad \dot{n}_3 = 6.3636 \text{ kmol/s}$$

$$\bar{C}_{P\ He} = 4.003 \text{ kg/kmol} \times 5.193 \text{kJ/kg-K} = 20.7876 \text{ kJ/kmol K},$$

$$\bar{C}_{P\ CH_4} = 16.043 \text{ kg/kmol} \times 2.254 \text{ kJ/kg-K} = 36.1609 \text{ kJ/kmol K}$$

Energy Eq.:

$$\dot{Q}_{CV} = \dot{n}_2\bar{h}_2 + \dot{n}_3\bar{h}_3 - \dot{n}_1\bar{h}_1 + \dot{W}_{CV} = \dot{n}_2\bar{C}_{P0\ He}(T_2-T_1) + \dot{n}_3\bar{C}_{P0\ CH_4}(T_3-T_1) + \dot{W}_{CV}$$

$$= 0.00637 \times 20.7876(20 - 10) + 6.3636 \times 36.1609(30 - 10) + (-3000)$$

$$= +1600 \text{ kW}$$

Entropy Eq.:

$$\dot{S}_{gen} = \dot{n}_2\bar{s}_2 + \dot{n}_3\bar{s}_3 - \dot{n}_1\bar{s}_1 - \dot{Q}_{CV}/T_0$$

$$= 0.00637\left[20.7876 \ln\frac{293.2}{283.2} - 8.3145 \ln\frac{100}{0.001 \times 150}\right]$$

$$+ 6.3636\left[36.1609 \ln\frac{303.2}{283.2} - 8.3145 \ln\frac{140}{0.999 \times 150}\right] - 1600/293.2$$

$$= \textbf{+13.5 kW/K} \ \textbf{> 0} \qquad \text{Since positive, this is possible}$$

11.60

Repeat Problem 11.44 for an isentropic compressor efficiency of 82%.

Solution:

C.V. Compressor. Steady, adiabatic q = 0, reversible $s_{gen} = 0$

Energy Eq.4.13: $-w = h_{ex} - h_{in}$;   Entropy Eq.7.8: $s_i + s_{gen} = s_i = s_e$

Process: reversible $\Rightarrow$ $s_{gen} = 0$ $\Rightarrow$ $s_e = s_i$

Assume ideal gas mixture and constant heat capacity, so we need k and C_P

From Eq.11.15 and 11.23:

$R_{mix} = \sum c_i R_i = 0.75 \times 0.5183 + 0.25 \times 0.2765 = 0.45785$ kJ/kg K

$C_{P\ mix} = \sum c_i C_{Pi} = 0.75 \times 2.254 + 0.25 \times 1.766 = 2.132$ kJ/kg K

$C_V = C_{P\ mix} - R_{mix} = 2.132 - 0.45785 = 1.6742$ kJ/kg K

Ratio of specific heats: $k = C_p/C_v = 1.2734$

The isentropic process gives Eq.6.23, from Eq.11.24 with $s_e = s_i$

$T_e = T_i (P_e/P_i)^{(k-1)/k} = 290 (350/100)^{0.2147} = \textbf{379.5 K}$

Isentropic work from the energy equation:

$w_{c\ in} = C_P (T_e - T_i) = 2.132$ kJ/kg-K $(379.5 - 290)$ K = 190.8 kJ/kg

The actual compressor requires more work

$w_{c\ actual} = w_{c\ in}/\eta = 190.8/0.82 = \textbf{232.7 kJ/kg} = C_p (T_{e\ actual} - T_i)$

$T_{e\ actual} = T_i + w_{c\ actual}/C_P$

$= 290$ K $+ 232.7$ kJ/kg $/ 2.132$ kJ/kg-K $= \textbf{399.1 K}$

11.61

A steady flow 0.3 kg/s of 60% carbon dioxide and 40% water mixture by mass at 1200 K and 200 kPa is used in a constant pressure heat exchanger where 300 kW is extracted from the flow. Find the exit temperature and rate of change in entropy using Table A.5

Solution:

C.V. Heat exchanger, Steady, 1 inlet, 1 exit, no work.

Continuity Eq.: $\qquad c_{CO2} = c_{H2O} = 0.5$

Energy Eq.: $\quad \dot{Q} = \dot{m}(h_e - h_i) \approx \dot{m} \, C_P \, (T_e - T_i) \quad => \quad T_e = T_i + \dot{Q}/\dot{m} \, C_P$

Inlet state: Table A.5

$\qquad C_P = 0.6 \times 0.842 + 0.4 \times 1.872 = 1.254$ kJ/kg-K

Exit state: $T_e = T_i + \dot{Q}/\dot{m} \, C_P = 1200$ K $- 300$ kW$/(0.3 \times 1.254$ kW/K$)$

$\qquad = \textbf{402.6 K}$

The rate of change of entropy for the flow is (P is assumed constant)

$$\dot{m}(s_e - s_i) = \dot{m}[s^o_{Te} - s^o_{Ti} - R \ln(P_e/P_i)] = \dot{m}[s^o_{Te} - s^o_{Ti}) \approx \dot{m} \, C_P \ln(T_e/T_i)$$

$$= 0.3 \times 1.254 \ln(402.6 \, / \, 1200) = \textbf{--0.411 kW/K}$$

The entropy generation rate cannot be estimated unless the average T at which the heat transfer leaves the control volume is known.

11.62

A steady flow of 0.3 kg/s of 60% carbon dioxide and 40% water by mass at 1200 K and 200 kPa is used in a heat exchanger where 300 kW is extracted from the flow. Find the flow exit temperature and the rate of change of entropy using Table A.8.

Solution:

C.V. Heat exchanger, Steady, 1 inlet, 1 exit, no work.

Continuity Eq.: $c_{CO2} = 0.6, \quad c_{H2O} = 0.4$

Energy Eq.: $\dot{Q} = \dot{m}(h_e - h_i) \quad => \quad h_e = h_i + \dot{Q}/\dot{m}$

Inlet state: Table A.8 $\quad h_i = 0.6 \times 1223.34 + 0.4 \times 2466.25 = 1720.5$ kJ/kg

Exit state: $h_e = h_i + \dot{Q}/\dot{m} = 1720.5 - 300/0.3 = 720.5$ kJ/kg

Trial and error for T with h values from Table A.8

@500 K $\quad h_e = 0.6 \times 401.52 + 0.4 \times 935.12 = 614.96$ kJ/kg

@600 K $\quad h_e = 0.6 \times 506.07 + 0.4 \times 1133.67 = 757.11$ kJ/kg

@550 K $\quad h_e = 0.6 \times 453.03 + 0.4 \times 1033.63 = 685.27$ kJ/kg

Interpolate to have the right h: **T = 575 K**

Entropy Eq.7.8: $\quad \dot{m}s_e = \dot{m}s_i + \dot{Q}/T + \dot{S}_{gen}$

The rate of change of entropy for the flow is (P is assumed constant)

$\dot{m}(s_e - s_i) = \dot{m}(s_{Te}^o - s_{Ti}^o)$

$= 0.3[\; 0.6(5.48175 - 6.3483) + 0.4(11.73925 - 13.3492)\;]$

$= $ **–0.349 kW/K**

The entropy generation rate cannot be estimated unless the integral dQ/T can be evaluated. If the dQ leaves at the local mixture T (assumed uniform over the flow cross section) the process is internally reversible and then externally irreversible.

11.63

A mixture of 60% helium and 40% nitrogen by mass enters a turbine at 1 MPa, 800 K at a rate of 2 kg/s. The adiabatic turbine has an exit pressure of 100 kPa and an isentropic efficiency of 85%. Find the turbine work.

Solution:

Assume ideal gas mixture and take CV as turbine.

Energy Eq.4.13: $\quad w_{T\,s} = h_i - h_{es}$,

Entropy Eq.7.8: $\quad s_{es} = s_i$, adiabatic and reversible

Process Eq.6.23: $\quad T_{es} = T_i(P_e/P_i)^{(k-1)/k}$

Properties from Eq.11.23, 11.15 and 6.23

$\quad C_{P\,mix} = 0.6\times 5.193 + 0.4\times 1.042 = 3.5326$ kJ/kg-K

$\quad R_{mix} = 0.6\times 2.0771 + 0.4\times 0.2968 = 1.365$ kJ/kg-K

$\quad (k-1)/k = R/C_{P\,mix} = 1.365/3.5326 = 0.3864$

$\quad T_{es} = 800$ K $(100/1000)^{0.3864} = 328.6$ K

$\quad w_{Ts} = C_P(T_i - T_{es}) = 3.5326$ kJ/kg-K $\times (800 - 328.6)$ K $= 1665$ kJ/kg

$\quad w_{T\,ac} = \eta w_{Ts} = 0.85 \times 1665$ kJ/kg $= 1415.5$ kJ/kg

$\quad \dot{W}_{T\,ac} = \dot{m}w_{T\,ac} = 2$ kg/s $\times 1415.5$ kJ/kg $= \textbf{2831 kW}$

11.64

Three steady flows are mixed in an adiabatic chamber at 150 kPa. Flow one is 2 kg/s of O_2 at 340 K, flow two is 4 kg/s of N_2 at 280 K and flow three is 3 kg/s of CO_2 at 310 K. All flows are at 150 kPa the same as the total exit pressure. Find the exit temperature and the rate of entropy generation in the process.

Solution:

C.V. Mixing chamber, no heat transfer, no work.

Continuity Eq.4.9: $\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = \dot{m}_4$

Energy Eq.4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{m}_3 h_3 = \dot{m}_4 h_4$

Entropy Eq.7.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{m}_3 s_3 + \dot{S}_{gen} = \dot{m}_4 s_4$

Assume ideal gases and since T is close to 300 K use heat capacity from A.5 in the energy equation as

$$\dot{m}_1 C_{P\,O2}(T_1 - T_4) + \dot{m}_2 C_{P\,N2}(T_2 - T_4) + \dot{m}_3 C_{P\,CO2}(T_3 - T_4) = 0$$

$$(2 \times 0.922 \times 340 + 4 \times 1.042 \times 280 + 3 \times 0.842 \times 310)\ kW$$

$$= (2 \times 0.922 + 4 \times 1.042 + 3 \times 0.842\ )\ kW/K \times T_4$$

$$\Rightarrow \quad 2577.06\ K = 8.538\ T_4 \qquad \Rightarrow \quad \mathbf{T_4 = 301.83\ K}$$

State 4 is a mixture so the component exit pressure is the partial pressure. For each component $s_e - s_i = C_P \ln(T_e / T_i) - R \ln(P_e / P_i)$ and the pressure ratio is $P_e / P_i = y\, P_4 / P_i = y$ for each.

$$n = \sum \frac{m}{M} = \frac{2}{32} + \frac{4}{28.013} + \frac{3}{44.01} = 0.0625 + 0.1428 + 0.06817 = 0.2735$$

$$y_{O2} = \frac{0.0625}{0.2735} = 0.2285, \quad y_{N2} = \frac{0.1428}{0.2735} = 0.5222, \quad y_{CO2} = \frac{0.06817}{0.2735} = 0.2493$$

The entropy generation becomes

$$\dot{S}_{gen} = \dot{m}_1(s_4 - s_1) + \dot{m}_2(s_4 - s_2) + \dot{m}_3(s_4 - s_3)$$

$$= \quad 2\ [\ 0.922 \ln(301.83/340) - 0.2598 \ln(0.2285)]$$

$$+ 4\ [\ 1.042 \ln(301.83/280) - 0.2968 \ln(0.5222)]$$

$$+ 3\ [\ 0.842 \ln(301.83/310) - 0.1889 \ln(0.2493)]$$

$$= 0.5475 + 1.084 + 0.2399 = \mathbf{1.871\ kW/K}$$

11.65

A tank has two sides initially separated by a diaphragm. Side A contains 1 kg of water and side B contains 1.2 kg of air, both at 20°C, 100 kPa. The diaphragm is now broken and the whole tank is heated to 600°C by a 700°C reservoir. Find the final total pressure, heat transfer and total entropy generation.

C.V. Total tank out to reservoir.

Energy Eq.3.5: $U_2 - U_1 = m_a(u_2 - u_1)_a + m_v(u_2 - u_1)_v = {_1}Q_2$

Entropy Eq.6.37:

$$S_2 - S_1 = m_a(s_2 - s_1)_a + m_v(s_2 - s_1)_v = {_1}Q_2/T_{res} + S_{gen}$$

Volume: $V_2 = V_A + V_B = m_v v_{v1} + m_a v_{a1} = 0.001 + 1.009 = 1.01 \text{ m}^3$

$v_{v2} = V_2/m_v = 1.01 \text{ m}^3/\text{kg}, \ T_2 \ \Rightarrow \ P_{2v} = 400 \text{ kPa}$

$v_{a2} = V_2/m_a = 0.8417 \text{ m}^3/\text{kg}, \ T_2 \ \Rightarrow \ P_{2a} = mRT_2/V_2 = 297.7 \text{ kPa}$

$P_{2tot} = P_{2v} + P_{2a} = \textbf{697.7 kPa}$

Water table B.1: $u_1 = 83.95 \text{ kJ/kg}, \ u_2 = 3300 \text{ kJ/kg},$

$s_1 = 0.2966 \text{ kJ/kg K}, \ s_2 = 8.4558 \text{ kJ/kg K}$

Air table A.7: $u_1 = 293 \text{ kJ/kg}, \ u_2 = 652.3 \text{ kJ/kg},$

$s_{T1} = 2.492 \text{ kJ/kg K}, \ s_{T2} = 3.628 \text{ kJ/kg K}$

From energy equation we have

${_1}Q_2 = 1(3300 - 83.95) + 1.2(652.6 - 209.4) = \textbf{3747.9 kJ}$

From the entropy equation we have

$S_{gen} = 1(8.4557 - 0.2966) + 1.2 \ [7.9816 - 6.846 - 0.287 \times \ln(297.7/100)]$

$- 3747.9 / 973.2 = \textbf{5.3 kJ/K}$

A   B   700 C

11.66

Reconsider the Problem 11.46, but let the tanks have a small amount of heat transfer so the final mixture is at 400 K. Find the final pressure, the heat transfer and the entropy change for the process.

C.V. Both tanks. Control mass with mixing and heating of two ideal gases.

$$n_{Ar} = P_{A1}V_A/\bar{R}T_{A1} = \frac{300\times1}{8.3145\times283.2} = 0.1274 \text{ kmol}$$

$$n_{C_2H_6} = P_{B1}V_B/\bar{R}T_{B1} = \frac{200\times2}{8.3145\times323.2} = 0.1489 \text{ kmol}$$

Continuity Eq.: $\quad n_2 = n_{Ar} + n_{C_2H_6} = 0.2763 \text{ kmol}$

Energy Eq.: $\quad U_2 - U_1 = n_{Ar}\bar{C}_{V0}(T_2 - T_{A1}) + n_{C_2H_6}\bar{C}_{VO}(T_2 - T_{B1}) = {_1}Q_2$

$$P_2 = n_2\bar{R}T_2/(V_A + V_B) = 0.2763\times8.3145\times400 / 3 = \textbf{306.3 kPa}$$

$${_1}Q_2 = 0.1274\times39.948\times0.312(400 - 283.15)$$

$$+ 0.1489\times30.07\times1.49(400 - 323.15) = \textbf{698.3 kJ}$$

$$\Delta S_{SURR} = -{_1}Q_2/T_{SURR}; \qquad \Delta S_{SYS} = n_{Ar}\Delta\bar{S}_{Ar} + n_{C_2H_6}\Delta\bar{S}_{C_2H_6}$$

$$y_{Ar} = 0.1274/0.2763 = 0.4611$$

$$\Delta\bar{S}_{Ar} = \bar{C}_{P\ Ar}\ln\frac{T_2}{T_{A1}} - \bar{R}\ln\frac{y_{Ar}P_2}{P_{A1}}$$

$$= 39.948\times0.520\ln\frac{400}{283.15} - 8.3145\ln\frac{0.4611\times306.3}{300}$$

$$= 13.445 \text{ kJ/kmol K}$$

$$\Delta\bar{S}_{C_2H_6} = \bar{C}_{C_2H_6}\ln\frac{T_2}{T_{B1}} - \bar{R}\ln\frac{y_{C_2H_6}P_2}{P_{B1}}$$

$$= 30.07\times1.766\ln\frac{400}{323.15} - 8.3145\ln\frac{0.5389\times306.3}{200}$$

$$= 12.9270 \text{ kJ/kmol K}$$

Assume the surroundings are at 400 K (it heats the gas)

$$\Delta S_{NET} = n_{Ar}\Delta\bar{S}_{Ar} + n_{C_2H_6}\Delta\bar{S}_{C_2H_6} + \Delta S_{SURR}$$

$$= 0.1274\times13.445 + 0.1489\times12.9270 - 698.3/400$$

$$= \textbf{1.892 kJ/K}$$

Air- water vapor mixtures

11.67

Atmospheric air is at 100 kPa, 25°C and relative humidity 65%. Find the absolute humidity and the dew point of the mixture. If the mixture is heated to 30°C what is the new relative humidity?

Solution:

Eq.11.25: $P_v = \phi P_g = 0.65 \times 3.169 = 2.06$ kPa

Eq.11.28: $w = 0.622 P_v/(P_{tot} - P_v) = 0.622 \times 2.06/(100 - 2.06) = \mathbf{0.01308}$

T_{dew} is the T such that $P_g(T) = P_v = 2.06$ kPa

B.1.1 => $T_{dew} \cong \mathbf{17.9\ °C}$

Heating => w is constant => P_v is constant

From Table B.1.1: $P_g(30°C) = 4.246$ kPa

$\phi = P_v/P_g = 2.06/4.246 = \mathbf{0.485\ or\ 49\ \%}$

11.68

A flow of 1 kg/s saturated moist air (relative humidity 100%) at 100 kPa, 10$^{\text{o}}$C goes through a heat exchanger and comes out at 25$^{\text{o}}$C. What is the exit relative humidity and how much power is needed?

Solution:

State 1 : $\quad \phi_1 = 1$; $\qquad P_v = P_g = 1.2276$ kPa

Eq.11.28: $\quad w = 0.622\, P_v/P_a = 0.622 \times 1.2276/(100 - 1.2276) = 0.00773$

State 2 : $\quad$ No water added $\quad \Rightarrow \quad w_2 = w_1 \quad \Rightarrow \quad P_{v2} = P_{v1}$

$\quad \phi_2 = P_{v2}/P_{g2} = 1.2276/3.169 = $ **0.387 or 39 %**

Energy Eq.6.10

$$\dot{Q} = \dot{m}_2 h_2 - \dot{m}_1 h_1 = \dot{m}_a (\, h_2 - h_1)_{air} + w\dot{m}_a (\, h_2 - h_1)_{vapor}$$

$$\dot{m}_{tot} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1 + w_1)$$

Energy equation with $C_{P\ air}$ from A.5 and h's from B.1.1

$$\dot{Q} = \frac{\dot{m}_{tot}}{1 + w_1}\, C_{P\ air}\,(25 - 10) + \frac{\dot{m}_{tot}}{1 + w_1}\, w\,(h_{g2} - h_{g1})$$

$$= \frac{1}{1.00773} \times 1.004(25 - 10) + \frac{1 \times 0.00773}{1.00773}\,(2547.17 - 2519.74)$$

$$= 14.9445 + 0.210407 = \textbf{15.15 kW}$$

11.69

If I have air at 100 kPa and a) -10^oC b) 45^oC and c) 110^oC what is the maximum absolute humidity I can have?

Humidity is related to relative humidity (max 100%) and the pressures as in Eq.11.28 where from Eq.11.25 $P_v = \Phi P_g$ and $P_a = P_{tot} - P_v$.

$$\omega = 0.622 \frac{P_v}{P_a} = 0.622 \frac{\Phi P_g}{P_{tot} - \Phi P_g}$$

a) $P_g = 0.2601$ kPa, $\omega = 0.622 \times \dfrac{0.2601}{100 - 0.26} = \mathbf{0.001\ 62}$

b) $P_g = 9.593$ kPa, $\omega = 0.622 \times \dfrac{9.593}{100 - 9.593} = \mathbf{0.0660}$

c) $P_g = 143.3$ kPa,   **no limit** on ω for $P_{tot} = 100$ kPa

11.70

A new high-efficiency home heating system includes an air-to-air heat exchanger which uses energy from outgoing stale air to heat the fresh incoming air. If the outside ambient temperature is −10°C and the relative humidity is 50%, how much water will have to be added to the incoming air, if it flows in at the rate of 1 m^3/s and must eventually be conditioned to 20°C and 45% relative humidity?

Solution:

Outside ambient air: $P_{v1} = \phi_1 P_{g1} = 0.50 \times 0.2602 = 0.1301$ kPa

Assuming $P_1 = P_2 = 100$ kPa,   => $P_{A1} = 100 - 0.1301 = 99.87$ kPa

$$\dot{m}_A = \frac{P_{A1}\dot{V}_1}{R_A T_1} = \frac{99.87 \times 1}{0.287 \times 263.2} = 1.3224 \text{ kg/s}$$

From Eq.11.28: $w_1 = 0.622 \times \dfrac{0.1301}{99.87} = 0.00081$

Conditioned to : $T_2 = 20\ ^{\text{o}}C,\quad \phi_2 = 0.45$

Eq.11.25: $P_{v2} = \phi_2 P_{g2} = 0.45 \times 2.339$ kPa $= 1.0526$ kPa =>

Eq.11.28: $w_2 = 0.622 \times \dfrac{1.0526}{100 - 1.0526} = 0.00662$

Continuity equation for water,

$\dot{m}_{\text{liq in}} = \dot{m}_A(w_2 - w_1) = 1.3224\,(0.00662 - 0.00081)$

$= 0.007683$ kg/s = **27.66 kg/h**

11.71

Consider 100 m^3 of atmospheric air which is an air–water vapor mixture at 100 kPa, 15°C, and 40% relative humidity. Find the mass of water and the humidity ratio. What is the dew point of the mixture?

Solution:

Air-vapor   P = 100 kPa, T = 15 $^{\text{o}}$C, ϕ = 40%

Use Table B.1.1 and then Eq.11.25

$P_g = P_{sat15} = 1.705$ kPa   => $P_v = \phi\, P_g = 0.4 \times 1.705 = 0.682$ kPa

$$m_v = \frac{P_v V}{R_v T} = \frac{0.682 \times 100}{0.461 \times 288.15} = \textbf{0.513 kg}$$

$P_a = P_{tot} - P_{v1} = 100 - 0.682 = 99.32$ kPa

$$m_a = \frac{P_a V}{R_a T} = \frac{99.32 \times 100}{0.287 \times 288.15} = 120.1 \text{ kg}$$

$$w_1 = \frac{m_v}{m_a} = \frac{0.513}{120.1} = \textbf{0.0043}$$

T_{dew} is T when $P_v = P_g = 0.682$ kPa;

Table B.1.2   gives   T = **1.4 $^{\text{o}}$C**

Borgnakke and Sonntag

11.72

A flow of 2 kg/s completely dry air at T_1, 100 kPa is cooled down to 10°C by spraying liquid water at 10°C, 100 kPa into it so it becomes saturated moist air at 10°C. The process is steady state with no external heat transfer or work. Find the exit moist air humidity ratio and the flow rate of liquid water. Find also the dry air inlet temperature T_1.

Solution:

2: saturated $\qquad P_v = P_g = 1.2276$ kPa $\quad$ and $\quad h_{fg}$ (10°C)= 2477.7 kJ/kg

Eq.11.25: $\qquad w_2 = 0.622 \times 1.2276/ (100 - 1.2276) = \mathbf{0.00773}$

C.V. Box

Continuity Eq.: $\quad \dot{m}_a + \dot{m}_{liq} = \dot{m}_a(1 + w_2) \qquad =>$

$$\dot{m}_{liq} = w_2\, \dot{m}_a = \mathbf{0.0155\ kg/s}$$

Energy Eq.: $\qquad \dot{m}_a\, h_{a1} + \dot{m}_{liq}\, h_f = \dot{m}_a\, (h_{a2} + w_2\, h_{g2})$

$\qquad h_{a1} - h_{a2} = C_{pa}\, (T_1 - T_2) = w_2\, h_{g2} - w_2\, h_f = w_2\, h_{fg}$

$\qquad\qquad = 0.0073$ kg/kg dry air $\times$ 2477.75 kJ/kg = 9.15 kJ/kg dry air

$\qquad T_1 = T_2 + (h_{a1} - h_{a2})/C_{pa} = 10 + 9.15/1.004 = \mathbf{29.1°C}$

Borgnakke and Sonntag

11.73

The products of combustion are flowing through a heat exchanger with 12% CO_2, 13% H_2O and 75% N_2 on a volume basis at the rate 0.1 kg/s and 100 kPa. What is the dew-point temperature? If the mixture is cooled 10°C below the dew-point temperature, how long will it take to collect 10 kg of liquid water?

Solution:

Volume basis is the same as mole fraction

$$y_{H_2O} = 0.13; \quad P_{H_2O} = y_{H_2O} \, P = 0.13 \times 100 \text{ kPa} = 13 \text{ kPa},$$

Table B.1.2 $T_{DEW} = \mathbf{50.95 \, {}^oC}$

Cool to 40.95 ^oC < T_{DEW} so saturated   → $P_G = 7.805$ kPa

$$y_{H_2O} = P_G/P = 7.805/100 = n_{H_2O(v)}/(n_{H_2O(v)} + 0.87)$$

$$n_{H_2O(v)} = 0.07365 \text{ per kmol mix in}$$

$$\rightarrow n_{LIQ} = 0.13 - 0.07365 = 0.05635$$

Eq.11.5: $M_{MIX \ IN} = 0.12 \times 44.01 + 0.13 \times 18.015 + 0.75 \times 28.013 = 28.63$ kg/kmol

$$\dot{n}_{MIX \ IN} = \dot{m}_{TOTAL}/M_{MIX \ IN} = \frac{0.1 \text{ kg/s}}{28.63 \text{ kg/kmol}} = 0.003493 \text{ kmol/s}$$

$$\dot{n}_{LIQ \ COND} = 0.003 \ 493 \times 0.05635 = 0.000 \ 197 \text{ kmol/s}$$

$$\text{or } \dot{m}_{LIQ \ COND} = 0.000 \ 197 \text{ kmol/s} \times 18.015 \text{ kg/kmol} = 0.003 \ 55 \text{ kg/s}$$

For 10 kg, it takes: $\Delta t = m/\dot{m} = \dfrac{10 \text{ kg}}{0.00355 \text{ kg/s}}$ ~ **47 minutes**

11.74

Consider a 1 m^3/s flow of atmospheric air at 100 kPa, 25°C, and 80% relative humidity. Assume this flows into a basement room where it cools to 15°C, 100 kPa. Find the rate of water condensing out and the exit mixture volume flow rate.

Solution:

State 1:　　$P_g = P_{sat25} = 3.169$ kPa　=>　$P_v = \phi\, P_g = 0.8 \times 3.169 = 2.535$ kPa

$$\dot{m}_{v1} = \frac{P_v \dot{V}}{R_v T} = \frac{2.535 \times 1}{0.461 \times 298.15} = 0.0184 \text{ kg/s}$$

$$w_1 = \frac{\dot{m}_{v1}}{\dot{m}_{A1}} = 0.622\,\frac{P_{v1}}{P_{A1}} = 0.622\,\frac{2.535}{100 - 2.535} = 0.0162$$

$$\dot{m}_{A1} = \frac{\dot{m}_{v1}}{w_1} = \frac{0.0184}{0.0162} = 1.136 \text{ kg/s} = \dot{m}_{A2} \qquad \text{(continuity for air)}$$

Check for state 2:

$P_{g15°C} = 1.705$ kPa $< P_{v1}$

so liquid water out.

State 2 is saturated　$\phi_2 = 100\%$,　$P_{v2} = P_{g2} = 1.705$ kPa

$$w_2 = 0.622\,\frac{P_{v2}}{P_{A2}} = 0.622\,\frac{1.705}{100 - 1.705} = 0.0108$$

$$\dot{m}_{v2} = w_2 \dot{m}_{A2} = 0.0108 \times 1.136 = 0.0123 \text{ kg/s}$$

$$\dot{m}_{liq} = \dot{m}_{v1} - \dot{m}_{v2} = 0.0184 - 0.0123 = \textbf{0.0061 kg/s}$$

$$\dot{V} = \frac{\dot{m}_{A2}\,R_a T_2}{P_{a2}} = \frac{\dot{m}_{A2}\,R_a T_2}{P - P_{v2}} = \frac{1.136 \times 0.287 \times 288.15}{100 - 1.705} = \textbf{0.956 m}^3\textbf{/s}$$

Note that the given volume flow rate at the inlet is not that at the exit. The mass flow rate of dry air is the quantity that is the same at the inlet and exit.

11.75

Ambient moist air enters a steady-flow air-conditioning unit at 102 kPa, 30°C, with a 60% relative humidity. The volume flow rate entering the unit is 100 L/s. The moist air leaves the unit at 95 kPa, 15°C, with a relative humidity of 100%. Liquid also leaves the unit at 15°C. Determine the rate of heat transfer for this process.

Solution:

State 1: $P_{V1} = \phi_1 P_{G1} = 0.60 \times 4.246 = 2.5476$

$w_1 = 0.622 \times 2.5476/(102 - 2.5476) = 0.01593$

$$\dot{m}_A = \frac{P_{A1}\dot{V}_1}{R_A T_1} = \frac{99.45 \times 0.1}{0.287 \times 303.2} = 0.1143 \text{ kg/s}$$

$P_{v2} = P_{g2} = 1.705 \text{ kPa,} \qquad w_2 = 0.622 \times 1.705/(95 - 1.705) = 0.01137$

Energy Eq.4.10: $\dot{Q}_{CV} + \dot{m}_A h_{A1} + \dot{m}_{V1} h_{V1} = \dot{m}_A h_{A2} + \dot{m}_{V2} h_{A2} + \dot{m}_3 h_{L3}$

$\dot{Q}_{CV}/\dot{m}_A = C_{P0A}(T_2-T_1) + w_2 h_{V2} - w_1 h_{V1} + (w_1-w_2)h_{L3}$

$\quad = 1.004(15-30) + 0.01137 \times 2528.9 - 0.01593 \times 2556.2$

$\quad + 0.00456 \times 63.0 = -26.732 \text{ kJ/kg air}$

$\dot{Q}_{CV} = 0.1143(-26.73) = \textbf{-3.055 kW}$

11.76

A room with air at 40% relative humidity, 20°C having 50 kg of dry air is made moist by boiling water to a final state of 20°C and 80% humidity. How much water was added to the air?

The water content is expressed by the absolute humidity (humidity ratio) from Eq.11.28 and 11.25

$$w_1 = 0.622 \times \frac{0.4 \times 2.339}{101.325 - 0.4 \times 2.339} = 0.005797$$

$$w_2 = 0.622 \times \frac{0.8 \times 2.339}{101.325 - 0.8 \times 2.339} = 0.011703$$

$$m_{water} = m_a \, (w_2 - w_1) = 50 \text{ kg } (0.011703 - 0.005797) = \textbf{0.295 kg}$$

11.77

Consider a 500-L rigid tank containing an air–water vapor mixture at 100 kPa, 35°C, with a 70% relative humidity. The system is cooled until the water just begins to condense. Determine the final temperature in the tank and the heat transfer for the process.

Solution:

$P_{v1} = \phi P_{G1} = 0.7 \times 5.628 = 3.9396$ kPa

Since m_v = const & V = const & also $P_v = P_{G2}$:

$P_{G2} = P_{v1} \times T_2/T_1 = 3.9396 \times T_2/308.2 = 0.01278\ T_2$

Assume $T_2 = 30^oC$: $0.01278 \times 303.2 = 3.875 \neq 4.246 = P_{G\ 30C}$

Assume $T_2 = 25^oC$: $0.01278 \times 298.2 = 3.811 \neq 3.169 = P_{G\ 25C}$

interpolating $\rightarrow T_2 = $ **28.2 ^oC**

$w_2 = w_1 = 0.622 \dfrac{3.9396}{(100\text{-}3.9369)} = 0.025\ 51$

$m_a = P_{a1}V/R_aT_1 = (100\text{-}3.94) \times 0.5/(0.287 \times 308.2) = 0.543$ kg

Energy Eq.:

$_1Q_2 = U_2\text{-}U_1 = m_a(u_{a2}\text{-}u_{a1}) + m_v(u_{v2}\text{-}u_{v1})$

$= m_a[\ C_v\ (T_{a2}\text{-}T_{a1}) + w_1\ (u_{v2}\text{-}u_{v1})\ ]$

$= 0.543\ [0.717(28.2 \text{ - } 35) + 0.02551\ (2414.2 \text{ - } 2423.4)\ ]$

$= 0.543$ kg $(\text{-}5.11$ kJ/kg$) = $ **-2.77 kJ**

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11.78

A saturated air-water vapor mixture at 20 ^{o}C, 100 kPa, is contained in a 5-m^3 closed tank in equilibrium with 1 kg of liquid water. The tank is heated to 80°C. Is there any liquid water in the final state? Find the heat transfer for the process.

AIR
+
VAP
LIQ

Q_{12}

a) Since $V_{liq} = m_{liq}v_F \approx 0.001$ m^3, $V_{GAS} \approx V$

$\phi_1 = 1.00 \rightarrow P_{v1} = P_{G1} = 2.339$ kPa

$w_1 = 0.622 \times 2.339 / (100 - 2.339) = 0.0149$

$$m_a = \frac{P_{a1}V}{R_a T_1} = \frac{97.661 \times 4.999}{0.287 \times 293.2} = 5.802 \text{ kg} \quad => \quad m_{v1} = w_1 m_a = 0.086 \text{ kg}$$

At state 2: $\quad P_{a2} = 97.661 \text{ kPa} \times \dfrac{353.2}{293.2} \times \dfrac{4.999}{5} = 117.623$ kPa

$w_{MAX\ 2} = 0.622 \times 47.39 / 117.623 = 0.2506$

But $\quad w_{2\ ACTUAL} = \dfrac{0.086 + 1.0}{5.802} = 0.1872 < w_{MAX\ 2} \quad \rightarrow$ **No liquid at 2**

$m_{v2} = m_{v1} + m_{liq} = 0.086$ kg $+ 1$ kg $= 1.086$ kg

$Q_{12} = m_a (u_{a2} - u_{a1}) + m_{v2}\ u_{v2} - m_{v1}u_{v1} - m_{liq\ 1}u_{liq\ 1}$

$\quad = m_a\ C_v\ (T_{a2} - T_{a1}) + m_{v2}\ u_{v2} - m_{v1}u_{v1} - m_{liq\ 1}u_{liq\ 1}$

$\quad = 5.802 \times 0.717(80 - 20) + 1.086 \times 2482.2 - 0.086 \times 2402.9 - 1 \times 84.0$

$\quad = 249.6 + 2695.7 - 206.65 - 84 =$ **2655 kJ**

11.79

A flow of 0.2 kg/s liquid water at 80°C is sprayed into a chamber together with 16 kg/s dry air at 60°C. All the water evaporates and the air leaves at 40°C. What is the exit relative humidity and the heat transfer.

CV. Chamber.

Continuity Eq. water: $\dot{m}_{liq} = w_{ex}\, \dot{m}_a$

Energy Eq.: $\dot{m}_{liq}\, h_{liq} + \dot{m}_a\, h_{a\,i} + \dot{Q} = \dot{m}_a\, (wh_v + h_a)_{ex}$

$w_{ex} = \dot{m}_{liq} / \dot{m}_a = 0.2 / 16 = 0.0125$

From Eq.11.25 and 11.28 you can get

$$\phi_{ex} = P_v / P_g = \frac{w}{0.622 + w}\frac{P}{P_g} = \frac{0.0125}{0.622 + 0.0125} \times \frac{100}{7.384} = \textbf{0.267} = \textbf{27\%}$$

$$\dot{Q} = \dot{m}_a\, (wh_v + h_a)_{ex} - \dot{m}_{liq}\, h_{liq} + \dot{m}_a\, h_{a\,i} = \dot{m}_a(\,h_{a\,ex} - h_{a\,i}) + \dot{m}_{liq}(h_v - h_{liq})$$

$$= \dot{m}_a\, C_{p\,a}\, (T_{ex} - T_{in}) + \dot{m}_{liq}\, (h_{v\,40} - h_{f\,80})$$
$$= 16 \times 1.004\, (40 - 60) + 0.2\, (2574.26 - 334.88)$$
$$= -\,321.28 + 447.88 = \textbf{126.6 kW}$$

11.80

A rigid container, 10 m^3 in volume, contains moist air at 45°C, 100 kPa, $\phi = 40\%$. The container is now cooled to 5°C. Neglect the volume of any liquid that might be present and find the final mass of water vapor, final total pressure and the heat transfer.

Solution:

CV container. $m_2 = m_1$; $m_2 u_2 - m_1 u_1 = {}_1Q_2$

State 1: 45°C, $\phi = 40\%$ => $w_1 = 0.0236$, $T_{dew} = 27.7°C$

Final state $T_2 < T_{dew}$ so condensation, $\phi_2 = 100\%$

$P_{v1} = 0.4\ P_g = 0.4 \times 9.593 = 3.837$ kPa, $P_{a1} = P_{tot} - P_{v1} = 96.163$ kPa

$m_a = P_{a1} V / RT_1 = 10.532$ kg, $m_{v1} = w_1\ m_a = 0.248$ kg

$P_{v2} = P_{g2} = 0.8721$ kPa, $P_{a2} = P_{a1} T_2 / T_1 = 84.073$ kPa

$P_2 = P_{a2} + P_{v2} = $ **84.95 kPa**

$m_{v2} = P_{v2} V / R_v T_2 = $ **0.06794 kg**   (= $V/v_g = 0.06797$ steam table)

$m_{f2} = m_{v1} - m_{v2} = 0.180$ kg

The heat transfer from the energy equation becomes

$${}_1Q_2 = m_a(u_2-u_1)_a + m_{v2} u_{g2} + m_{f2} u_{f2} - m_{v1} u_{g1}$$

$$= m_a\ C_v(T_2 - T_1) + m_{v2}\ 2382.3 + m_{f2}\ 20.97 - m_{v1}\ 2436.8$$

$$= -302.06 + 161.853 + 3.775 - 604.33 = -\ \textbf{740.8 kJ}$$

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11.81

A water-filled reactor of 1 m^3 is at 20 MPa, 360°C and located inside an insulated containment room of 100 m^3 that contains air at 100 kPa and 25°C. Due to a failure the reactor ruptures and the water fills the containment room. Find the final pressure.

CV Total container.

$$m_v (u_2 - u_1) + m_a (u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0$$

Initial water: $v_1 = 0.0018226$, $u_1 = 1702.8$ kJ/kg, $m_v = V/v = 548.67$ kg

Initial air: $m_a = \dfrac{PV}{RT} = \dfrac{100 \times 99}{0.287 \times 298.2} = 115.7$ kg

Substitute into energy equation

$548.67\,(u_2 - 1702.8) + 115.7 \times 0.717\,(T_2 - 25) = 0$

$u_2 + 0.1511\,T_2 = 1706.6$ kJ/kg   & $v_2 = V_2/m_v = 0.18226$ m^3/kg

Trial and error 2-phase   (T$_{guess}$, v_2 => x_2 => u_2 => LHS)

T = 150°C LHS = 1546   T = 160°C LHS = 1820.2

T = 155°C LHS = 1678.1 => T = 156°C LHS = 1705.7 OK

$x_2 = 0.5372$, $P_{sat} = 557.5$ kPa

$P_{a2} = P_{a1}V_1T_2/V_2T_1 = 100 \times 99 \times 429.15 / (100 \times 298.15) = 142.5$ kPa

=> $P_2 = P_{a2} + P_{sat} = $ **700 kPa.**

100 m^3

1 m^3

11.82

In the production of ethanol from corn the solids left after fermentation are dried in a continuous flow oven. This process generates a flow of 15 kg/s moist air, 90°C with 70% relative humidity, which contains some volatile organic compounds and some particles. To remove the organic gasses and the particles, the flow is send to a **thermal oxidicer** where natural gas flames brings the mixture to 800°C. Find the rate of heating by the natural gas burners.

For this problem we will just consider the heating of the gas mixture and due to the exit temperature we will use the ideal gas tables A7 and A8.

Eq.11.25: $P_v = \phi\, P_g = 0.70 \times 70.14 = 49.098$ kPa

Eq.11.28: $\omega = 0.622\, \dfrac{P_v}{P_{tot} \text{-} P_v} = 0.622 \times \dfrac{49.098}{100 - 49.098} = 0.60$

Flow: $\dot{m}_{tot} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1 + \omega)$ so

$$\dot{m}_a = \frac{\dot{m}_{tot}}{1 + \omega} = 9.375 \text{ kg/s}; \quad \dot{m}_v = \dot{m}_{tot}\,\frac{\omega}{1 + \omega} = 5.625 \text{ kg/s}$$

Process: Heating   => ω is constant   => P_v is constant

For ideal gas the enthalpy does not depend on pressure so the energy equation gives the heat transfer as

$$\dot{Q} = \dot{m}_a\,(h_2 - h_1)_a + \dot{m}_v\,(h_2 - h_1)_v$$
$$= 9.375\,(1130.2 - 364.0) + 5.625\,(2164.3 \text{ - } 672.75)$$
$$= \mathbf{15\ 573\ kW}$$

11.83

To save on natural gas use in the previous problem it is suggested to take and cool the mixture and condense out some water before heating it back up. So the flow is cooled from 90°C to 50°C and the now dryer mixture is heated to 800°C. Find the amount of water condensed out and the rate of heating by the natural gas burners for this case.

Eq.11.25: $\quad P_{v1} = \phi_1 P_g = 0.70 \times 70.14 = 49.098 \text{ kPa}$

Eq.11.28: $\quad \omega_1 = 0.622 \dfrac{P_v}{P_{tot} - P_v} = 0.622 \times \dfrac{49.098}{100 - 49.098} = 0.60$

Flow: $\quad \dot{m}_{tot} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1 + \omega) \quad$ so

$$\dot{m}_a = \frac{\dot{m}_{tot}}{1 + \omega} = 9.375 \text{ kg/s}; \quad \dot{m}_{v1} = \dot{m}_{tot} \frac{\omega}{1 + \omega} = 5.625 \text{ kg/s}$$

Now cool to 50°C: $\quad P_g = 12.35 \text{ kPa} < P_{v1} \quad$ so $\quad \phi_2 = 100\%, \ P_{v2} = 12.35 \text{ kPa}$

$$\omega_2 = 0.622 \frac{P_v}{P_{tot} - P_v} = 0.622 \times \frac{12.35}{100 - 12.35} = 0.0876$$

$$\dot{m}_{liq} = \dot{m}_a (\omega_1 - \omega_2) = 9.375 (0.6 - 0.0876) = \textbf{4.80375 kg/s}$$

The air flow is not changed so the water vapor flow for heating is

$$\dot{m}_{v2} = \dot{m}_{v1} - \dot{m}_{liq} = 5.625 - 4.80375 = 0.82125 \text{ kg/s}$$

Now the energy equation becomes

$$\dot{Q} = \dot{m}_a (h_3 - h_2)_a + \dot{m}_{v2} (h_3 - h_2)_v$$
$$= 9.375 (1130.2 - 323.75) + 0.82125 (2164.3 - 597.65)$$
$$= \textbf{8847 kW}$$

Comment: If you solve the previous problem you find this is only 57% of the heat for the case of no water removal.

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Tables and formulas or psychrometric chart

11.84

I want to bring air at 35$^\text{o}$C, Φ = 40% to a state of 25$^\text{o}$C, ω = 0.015 do I need to add or subtract water?

The humidity ratio (absolute humidity) expresses how much water vapor is present in the mixture

$$\omega = m_v / m_a$$

Assuming P = 100 kPa,

$$\omega = 0.622 \frac{P_v}{P_a} = 0.622 \frac{P_v}{P - P_v} \quad \text{and} \quad P_v = \Phi P_g$$

At 35°C, 40 % :

$$\omega = 0.622 \times \frac{0.40 \times 5.628}{100 - 0.40 \times 5.628} = 0.014\ 32$$

To get to ω = 0.015 , it is necessary to **add water**.

11.85

A flow moist air at 100 kPa, 40°C, 40% relative humidity is cooled to 15°C in a constant pressure device. Find the humidity ratio of the inlet and the exit flow, and the heat transfer in the device per kg dry air.

Solution:

C.V. Cooler. $\dot{m}_{v1} = \dot{m}_{liq} + \dot{m}_{v2}$

Tables: $P_{g1} = 7.384$ kPa, $P_{v1} = \phi P_g = 0.4 \times 7.384 = 2.954$ kPa,

$\omega_1 = 0.622 \times 2.954 / (100 - 2.954) = 0.0189$

$T_2 < T_{dew}$ [from $P_g(T_{dew}) = 2.954$] $\Rightarrow$ $P_{v2} = 1.705$ kPa $= P_{g2}$ $\Rightarrow$

$\omega_2 = 0.622 \times 1.705 / (100 - 1.705) = 0.0108$

$h_{v1} = 2574.3$ kJ/kg, $h_{v2} = 2528.9$ kJ/kg, $h_f = 62.98$ kJ/kg

$\tilde{q}_{out} = C_P(T_1 - T_2) + \omega_1 h_{v1} - \omega_2 h_{v2} - (\omega_1 - \omega_2) h_f$

$= 1.004(40 - 15) + 0.0189 \times 2574.3 - 0.0108 \times 2528.9 - 0.0073 \times 62.98$

$= $ **45.98 kJ/kg dry air**

Psychrometric chart: State 2: $T < T_{dew} = 23°C$ $\Rightarrow$ $\phi_2 = 100\%$

$\dot{m}_{v1}/\dot{m}_a = \omega_1 = 0.018$, $\tilde{h}_1 = 106$; $\dot{m}_{v2}/\dot{m}_a = \omega_2 = 0.0107$, $\tilde{h}_2 = 62$

$\dot{m}_{liq}/\dot{m}_a = \omega_1 - \omega_2 = 0.0073$, $h_f = 62.98$ kJ/kg

$\dot{m}_a \tilde{q}_{out} = \dot{m}_a \tilde{h}_1 - \dot{m}_{liq} h_f - \dot{m}_a \tilde{h}_2 \Rightarrow$

$\tilde{q}_{out} = \tilde{h}_1 - (\omega_1 - \omega_2) h_f - \tilde{h}_2 = 106 - 0.0073 \times 62.98 - 62$

$= $ **43.54 kJ/kg-dry air**

11.86

Use the formulas and the steam tables to find the missing property of: ϕ, ω, and T_{dry}, total pressure is 100 kPa; repeat the answers using the psychrometric chart

a. $\phi = 50\%$, $\omega = 0.010$ b. $T_{dry} = 25°C$, $T_{wet} = 21°C$

Solution:

a. From Eq.11.28 with $P_a = P - P_v$ solve for P_v:

$$P_v = P \omega /(0.622 + \omega) = 100 \times 0.01/0.632 = 1.582 \text{ kPa}$$

From Eq.11.25 $P_g = P_v/\phi = 1.582/0.5 = 3.165 \text{ kPa}$ => **T = 25°C**

b. At 21°C:: $P_g = 2.505$ => $\omega_2 = 0.622 \times 2.505/(100 - 2.505) = 0.016$

From the steam tables B.1.1

$h_{f2} = 88.126$ and $h_{fg2} = 2451.76 \text{ kJ/kg}$, $h_{v1} = 2547.17$

From Eq.11.30:

$$\omega_1 = [C_p(T_2\text{-}T_1) + \omega_2 \, h_{fg2} \,]/(h_{v1} - h_{f2}) = \mathbf{0.0143}$$

From Eq.11.28 with $P_a = P - P_v$ solve for P_v:

$$P_v = P \omega /(0.622 + \omega) = 2.247 \text{ kPa},$$

From Eq.11.25: $\phi = 2.247/3.169 = \mathbf{0.71}$

Using the psychrometric chart E.4:

a: $\boldsymbol{T_{dry} = 25.3\ °C}$ b: $\boldsymbol{\omega = 0.0141,\quad \phi = 71\text{-}72\%}$

11.87

The discharge moist air from a clothes dryer is at 40°C, 80% relative humidity. The flow is guided through a pipe up through the roof and a vent to the atmosphere. Due to heat transfer in the pipe the flow is cooled to 24°C by the time it reaches the vent. Find the humidity ratio in the flow out of the clothes dryer and at the vent. Find the heat transfer and any amount of liquid that may be forming per kg dry air for the flow.

Solution:

State 1: Just outside chart

$$P_a = 0.8 \times 7.384 = 5.907 \text{ kPa}$$

$$\omega_1 = 0.622 \times 5.907 \,/(100 - 5.907)$$

$$= \textbf{0.0390},$$

$$T_{dew} = 35.8°C$$

State 2: $24°C < T_{dew}$ so it is saturated.

$$\omega_2 = \textbf{0.019},$$

$$\dot{m}_{liq}/\dot{m}_a = \omega_1 - \omega_2 = \textbf{0.0099 kg/kg dry air}$$

Energy Eq.:

$$\dot{Q}/\dot{m}_a = (h_{a2} + \omega_2\, h_{v2}) - (h_{a1} + \omega_1 h_{v1}) + (\omega_1 - \omega_2)\, h_f$$

$$= C_{pa}\, (T_2 - T_1) + \omega_2\, h_{v2} - \omega_1 h_{v1} + (\omega_1 - \omega_2)\, h_f$$

$$= 1.004(24 - 40) + 0.019 \times 2545.1 - 0.039 \times 2574.3 + 0.0099 \times 100.7$$

$$= \textbf{–67 kJ/kg dry air}$$

Borgnakke and Sonntag

11.88

A flow, 0.2 kg/s dry air, of moist air at 40°C, 50% relative humidity flows from the outside state 1 down into a basement where it cools to 16°C, state 2. Then it flows up to the living room where it is heated to 25°C, state 3. Find the dew point for state 1, any amount of liquid that may appear, the heat transfer that takes place in the basement and the relative humidity in the living room at state 3.

Solve using psychrometric chart:

a) $T_{dew} = 27.2$ $(w = w_1, \phi = 100\%)$ $w_1 = 0.0232$, $\tilde{h}_1 = 118.2$ kJ/kg air

b) $T_2 < T_{dew}$ so we have $\phi_2 = 100\%$ liquid water appear in the basement.

$\Rightarrow$ $w_2 = 0.0114$ $\tilde{h}_2 = 64.4$ and from steam tbl. $h_f = 67.17$

$\dot{m}_{liq} = \dot{m}_{air}(w_1-w_2) = 0.2(0.0232-0.0114) = 0.00236$ kg/s

c) Energy equation: $\dot{m}_{air}\,\tilde{h}_1 = \dot{m}_{liq}\,h_f + \dot{m}_{air}\,\tilde{h}_2 + \dot{Q}_{out}$

$\dot{Q}_{out} = 0.2[118.2 - 64.4 - 0.0118{\times}67.17] = 10.6$ kW

d) $w_3 = w_2 = 0.0114$ & 25°C $\Rightarrow$ $\phi_3 = 58\%$.

If you solve by the formulas and the tables the numbers are:

$P_{g40} = 7.384$ kPa; $P_{v1} = \phi\,P_{g40} = 0.5 \times 7.384 = 3.692$ kPa

$w_1 = 0.622 \times 3.692 / (100 - 3.692) = 0.02384$

$P_{v1} = P_g\,(T_{dew})$ $\Rightarrow$ $T_{dew\,1} = 27.5$ °C

2: $\phi = 100\%$, $P_{v2} = P_{g2} = 1.832$ kPa, $w_2 = 0.622{\times}1.832/98.168 = 0.0116$

$\dot{m}_{liq} = \dot{m}_{air}\,(w_1-w_2) = 0.2{\times}0.01223 = 0.00245$ kg/s

3:    $w_3 = w_2$ $\Rightarrow$ $P_{v3} = P_{v2} = 1.832$ kPa   & $P_{g3} = 3.169$ kPa

$\phi_3 = P_v/P_g = 1.832/3.169 = 57.8\%$

Borgnakke and Sonntag

11.89

A steady supply of 1.0 m^3/s air at 25°C, 100 kPa, 50% relative humidity is needed to heat a building in the winter. The outdoor ambient is at 10°C, 100 kPa, 50% relative humidity. What are the required liquid water input and heat transfer rates for this purpose?

Solution:

Air: $R_a = 0.287$ kJ/kg K, $C_p = 1.004$ kJ/kg-K

State 1: $T_1 = 10°C$, $\phi_1 = 50\%$, $P_1 = 100$ kPa

   $P_{g1} = 1.2276$ kPa, $P_{v1} = \phi_1 P_{g1} = 0.6138$ kPa,

   $P_{a1} = P_1 - P_{v1} = 99.39$ kPa $=>$ $\omega_1 = 0.622\ P_{v1}/P_{a1} = 0.003841$

State 2: $T_2 = 25°C$, $P_2 = 100$ kPa, $\phi_2 = 50\%$, $\dot{V}_2 = 1$ m^3/s

   $P_{g2} = 3.169$ kPa, $P_{v2} = \phi_2 P_{g2} = 1.5845$ kPa,

  $P_{a2} = P_2 - P_{v2} = 98.415$ kPa, $\omega_2 = 0.622\ P_{v2}/P_{a2} = 0.010014$

   $\dot{m}_{a2} = P_{a2}\ \dot{V}_2/R_a T_2 = 98.415 \times 1/(0.287 \times 298.15) = 1.15$ kg/s

Steam tables B.1.1: $h_{v1} = 2519.7$ kJ/kg, $h_{v2} = 2547.2$ kJ/kg

State 3:   Assume liq. water at $T_3 = 25°C$, $h_{f3} = 104.9$ kJ/kg

 Conservation of Mass: $\dot{m}_{a1} = \dot{m}_{a2}$, $\dot{m}_{f3} = \dot{m}_{v2} - \dot{m}_{v1}$

   $\dot{m}_{f3} = \dot{m}_{a2}(\omega_2 - \omega_1) = 1.15 \times 0.006173 = $ **0.0071 kg/s**

 Energy Eq.: $\dot{Q} + \dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{f3}h_{f3} = \dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2}$

   $\tilde{q} = \dfrac{\dot{Q}}{\dot{m}_a} = C_p(T_2 - T_1) + \omega_2 h_{v2} - \omega_1 h_{v1} - \dfrac{\dot{m}_{f3}}{\dot{m}_a}h_{f3}$

   $= 1.004(25 - 10) + 0.010014 \times 2547.2 - 0.003841 \times 2519.7$
   $\qquad\qquad - 0.006173 \times 104.9$
   $= 30.24$ kJ/kg dry air

   $=>$ $\dot{Q} = \dot{m}_{a1}\ \tilde{q} = 1.15$ kg/s $\times 30.24$ kJ/kg $= $ **34.78 kW**

11.90

In a ventilation system inside air at 34°C and 70% relative humidity is blown through a channel where it cools to 25°C with a flow rate of 0.75 kg/s dry air. Find the dew point of the inside air, the relative humidity at the end of the channel, and the heat transfer in the channel.

C.V. Cooler. $\dot{m}_{v1} = \dot{m}_{liq} + \dot{m}_{v2}$

Tables: $P_{g1} = 5.352$ kPa, $P_{v1} = \phi\, P_g = 0.7 \times 5.352 = 3.75$ kPa,

$\omega_1 = 0.622 \times 3.75\,/(101 - 3.75) = 0.024$

$T_2 < T_{dew} = 27C$ [from $P_g(T_{dew}) = 3.75$] $=>$ $P_{v2} = 3.169$ kPa $= P_{g2}$

$\omega_2 = 0.622 \times 3.169\,/(101 - 3.169) = 0.02015$

$h_{v1} = 2563.47$ kJ/kg, $h_{v2} = 2547.17$ kJ/kg, $h_f = 104.87$ kJ/kg

$\bar{q}_{out} = C_P(T_1 - T_2) + \omega_1 h_{v1} - \omega_2\, h_{v2} - (\omega_1 - \omega_2)\, h_f$

$= 1.004(34 - 25) + 0.024 \times 2563.47 - 0.02015 \times 2547.17 - 0.00385 \times 104.87$

$= 18.83$ kJ/kg dry air

$\dot{Q} = \dot{m}_a\ \bar{q}_{out} = 0.75$ kg/s $\times 18.83$ kJ/kg $= \textbf{14.1 kW}$

Psychrometric chart: State 2: $T < T_{dew} = 27.5°C$ $=>$ $\phi_2 = 100\%$

$\dot{m}_{v1}/\dot{m}_a = \omega_1 = 0.0234$, $\tilde{h}_1 = 113.7$ kJ/kg air;

$\dot{m}_{v2}/\dot{m}_a = \omega_2 = 0.0202$, $\tilde{h}_2 = 96$ kJ/kg air

$\dot{m}_{liq}/\dot{m}_a = \omega_1 - \omega_2 = 0.0032$, $h_f = 104.87$ kJ/kg

$\dot{m}_a\ \bar{q}_{out} = \dot{m}_a \tilde{h}_1 - \dot{m}_{liq}\, h_f - \dot{m}_a\ \tilde{h}_2 =>$

$\tilde{q}_{out} = \tilde{h}_1 - (\omega_1 - \omega_2)\, h_f - \tilde{h}_2 = 113.7 - 0.0032 \times 104.87 - 96 = 17.36$ kJ/kg-air

$\dot{Q} = \dot{m}_a\ \tilde{q}_{out} = \textbf{13.0 kW}$

11.91

Two moist air streams with 85% relative humidity, both flowing at a rate of 0.1 kg/s of dry air are mixed in a steady setup. One inlet flowstream is at 32.5°C and the other at 16°C. Find the exit relative humidity.

Solution:

CV mixing chamber.

Continuity Eq. water: $\dot{m}_{air} w_1 + \dot{m}_{air} w_2 = 2\dot{m}_{air} w_{ex}$;

Energy Eq.: $\dot{m}_{air} \tilde{h}_1 + \dot{m}_{air} \tilde{h}_2 = 2\dot{m}_{air} \tilde{h}_{ex}$

Properties from the tables and formulas

$P_{g32.5} = 4.937$ kPa; $P_{v1} = 0.85 \times 4.937$ kPa $= 4.196$ kPa

$w_1 = 0.622 \times 4.196 / (100 - 4.196) = 0.0272$

$P_{g16} = 1.831$ kPa; $P_{v2} = 0.85 \times 1.831$ kPa $= 1.556$ kPa

$w_2 = 0.622 \times 1.556 / (100 - 1.556) = 0.00983$

Continuity Eq. water: $w_{ex} = (w_1 + w_2)/2 = 0.0185$;

For the energy equation we have $\tilde{h} = h_a + wh_v$ so:

$2\tilde{h}_{ex} - \tilde{h}_1 - \tilde{h}_2 = 0 = 2h_{a\ ex} - h_{a\ 1} - h_{a\ 2} + 2w_{ex}h_{v\ ex} - w_1h_{v\ 1} - wh_{v\ 2}$

we will use constant specific heat to avoid an iteration on T_{ex}.

$C_{p\ air}(2T_{ex} - T_1 - T_2) + C_{p\ H2O}(2w_{ex}T_{ex} - w_1T_1 - w_2T_2) = 0$

$T_{ex} = [\ C_{p\ air}(T_1 + T_2) + C_{p\ H2O}(w_1T_1 + w_2T_2)\ ]/\ [2C_{p\ air} + 2w_{ex}C_{p\ H2O}]$

$\qquad = [\ 1.004\ (32.5 + 16) + 1.872(0.0272 \times 32.5 + 0.00983 \times 16]/2.0773$

$\qquad = 24.4°C$

$P_{v\ ex} = \dfrac{w_{ex}}{0.622 + w_{ex}} P_{tot} = \dfrac{0.0185}{0.622 + 0.0185} 100 = 2.888$ kPa,

$P_{g\ ex} = 3.069$ kPa $\quad => \quad \phi = 2.888 / 3.069 =$ **0.94 or 94%**

Properties taken from the psychrometric chart

State 1: $w_1 = 0.0266$, $\tilde{h}_1 = 120$ State 2: $w_2 = 0.0094$, $\tilde{h}_2 = 60$

Continuity Eq. water: $w_{ex} = (w_1 + w_2)/2 = 0.018$;

Energy Eq.: $\tilde{h}_{ex} = (\tilde{h}_1 + \tilde{h}_2)/2 = 90$ kJ/kg dry air

exit: w_{ex}, $\tilde{h}_{ex} \quad => \quad T_{ex} =$ **24.5°C**, $\phi =$ **94%**

Notice how the energy in terms of temperature is close to the average of the two flows but the relative humidity is not.

Borgnakke and Sonntag

11.92

A combination air cooler and dehumidification unit receives outside ambient air at 35°C, 100 kPa, 90% relative humidity. The moist air is first cooled to a low temperature T_2 to condense the proper amount of water, assume all the liquid leaves at T_2. The moist air is then heated and leaves the unit at 20°C, 100 kPa, relative humidity 30% with volume flow rate of 0.01 m^3/s. Find the temperature T_2, the mass of liquid per kilogram of dry air and the overall heat transfer rate.

Solution:

$$P_{v1} = \phi_1 P_{G1} = 0.9 \times 5.628 = 5.0652 \text{ kPa}$$

$$w_1 = 0.622 \times \frac{5.0652}{100\text{-}5.0652} = 0.033\ 19$$

$$P_{v3} = \phi_3 P_{G3} = 0.3 \times 2.339 = 0.7017 \text{ kPa}$$

$$w_2 = w_3 = 0.622 \times \frac{0.7017}{100\text{-}0.7017} = 0.0044$$

$$\dot{m}_{LIQ\ 2'}/\dot{m}_a = w_1 - w_2 = 0.033\ 19 - 0.0044 = \textbf{0.028 79 kg/kg air}$$

$$P_{G2} = P_{v3} = 0.7017 \text{ kPa} \quad \rightarrow \quad T_2 = \textbf{1.7 }^\textbf{o}\textbf{C}$$

For a C.V. around the entire unit

$$\dot{Q}_{CV} = \dot{Q}_H + \dot{Q}_C$$

Net heat transfer, Energy Eq.:

$$\dot{Q}_{CV}/\dot{m}_a = (h_{a3}\text{-}h_{a1}) + w_3 h_{v3} - w_1 h_{v1} + \dot{m}_{L2'}\ h_{L2'}/\dot{m}_a$$
$$= 1.004(20\text{-}35) + 0.0044 \times 2538.1 - 0.033\ 19 \times 2565.3 + 0.028\ 79 \times 7.28$$
$$= \text{-}88.82 \text{ kJ/kg air}$$

$$\dot{m}_a = \frac{P_{a3}\dot{V}_3}{R_a T_3} = \frac{(100\text{-}0.7017) \times 0.01}{0.287 \times 293.2} = 0.0118 \text{ kg/s}$$

$$\dot{Q}_{CV} = 0.0118 \text{ kg/s} \times (\text{-}88.82) \text{ kJ/kg air} = \textbf{-1.05 kW}$$

11.93

To make dry coffee powder we spray 0.2 kg/s coffee (assume liquid water) at 80° C into a chamber where we add 10 kg/s dry air at T. All the water should evaporate and the air should leave with a minimum 40^oC and we neglect the powder. How high should T in the inlet air flow be?

CV. Chamber. We assume it is adiabatic.

Continuity Eq. water alone: $\dot{m}_{liq} = w_{ex}\,\dot{m}_a$

Energy Eq.: $\dot{m}_{liq}\,h_{f\,80} + \dot{m}_a\,h_{a\,Ti} = \dot{m}_{liq}\,h_{v\,40} + \dot{m}_a\,h_{a\,40}$

$$w_{ex} = \dot{m}_{liq} / \dot{m}_a = 0.2 / 8 = 0.025$$

From the energy equation you get

$$h_{a\,Ti} - h_{a\,40} = C_{p\,a}\,(T_{in} - T_{ex}) = \dot{m}_{liq}\,(h_{v\,40} - h_{f\,80}) / \dot{m}_a$$
$$1.004 \text{ kJ/kg-K } \Delta T = 0.2\,(2574.26 - 334.88) / 10 = 44.788 \text{ kJ/kg}$$
$$\Delta T = 44.6^oC \qquad \Rightarrow \qquad T_{in} = 40 + 44.6 = \mathbf{84.6^oC}$$

11.94

An insulated tank has an air inlet, $\omega_1 = 0.0084$, and an outlet, $T_2 = 22°C$, $\phi_2 = 90\%$ both at 100 kPa. A third line sprays 0.25 kg/s of water at 80°C, 100 kPa. For steady operation find the outlet specific humidity, the mass flow rate of air needed and the required air inlet temperature, T_1.

Solution:

Take CV tank in steady state. Continuity and energy equations are:

Continuity Eq. water: $\dot{m}_3 + \dot{m}_a\,w_1 = \dot{m}_a\,w_2$

Energy Eq.: $\dot{m}_3 h_f + \dot{m}_a\,\tilde{h}_1 = \dot{m}_a\,\tilde{h}_2$

All state properties are known except T_1.

From the psychrometric chart we get

State 2: $w_2 = \mathbf{0.015}$, $\tilde{h}_2 = 79.5$ State 3: $h_f = 334.91$ (steam tbl)

$\dot{m}_a = \dot{m}_3/(w_2 - w_1) = 0.25/(0.015\text{-}0.0084) = \mathbf{37.88\ kg/s}$

$\tilde{h}_1 = \tilde{h}_2 - (w_2 - w_1)h_f = 79.5 - 0.0066 \times 334.91 = 77.3$

Chart $(w_1, \tilde{h}_1)$ => $T_1 = \mathbf{36.5°C}$

Using the tables and formulas we get

State 2: $P_{g2} = 2.671$; $P_{v2} = 0.9 \times 2.671 = 2.4039$ kPa

$w_2 = 0.622 \times 2.4039 / (100 - 2.4039) = \mathbf{0.0153}$

$\dot{m}_a = \dot{m}_3/(w_2 - w_1) = 0.25/(0.0153 - 0.0084) = \mathbf{36.23\ kg/s}$

To avoid iterations on T_1 we use specific heat values also for water vapor by writing $h_{v1} = h_{v2} + C_{p\ h2o}(T_1 - T_2)$ so the energy equation is

$C_{p\ a}\,T_1 + w_1 C_{p\ h2o}(T_1 - T_2) + w_1 h_{v2} = C_{p\ a}\,T_2 + w_2 h_{v2} - (w_2 - w_1)\,h_f$

The equation now becomes

$(1.004 + 0.0084 \times 1.872)T_1 = (0.0084 \times 1.872 + 1.004)\,22$

$+ (0.0153 - 0.0084)(2541.7 - 334.91) = 37.219$

$T_1 = \mathbf{36.5°C}$

11.95

An air flow 2 kg/s at 30°C, and relative humidity 80% is conditioned by taking half the flow, cooling it and mix it with the other half. Assume the outlet flow should have a water content that is 75% of the original flow. Find the temperature to cool to, the rate of cooling and the final exit flow temperature.

FIGURE P11.95

CV Total setup

Continuity water: $0 = \dot{m}_a \omega_1 - \dot{m}_{liq} - \dot{m}_a \omega_3$; $\dot{m}_{liq} = \dot{m}_a (\omega_1 - \omega_3)$

  State 1: $P_{g1} = 4.246$; $P_{v1} = 0.8 \times 4.246 = 3.3968$ kPa

     $\omega_1 = 0.622 \times 3.3968 / (100 - 3.3968) = 0.02187$

CV Junction

Continuity water: $0 = 0.5 \dot{m}_a \omega_1 + 0.5 \dot{m}_a \omega_2 - \dot{m}_a \omega_3$; $\omega_3 = 0.75 \omega_1$

Energy Eq.: $0 = 0.5 \dot{m}_a(h_{a1} + \omega_1 h_{v1}) + 0.5 \dot{m}_a(h_{a2} + \omega_2 h_{v2}) - \dot{m}_a(h_{a3} + \omega_3 h_{v3})$

Solve for ω_2: $\omega_2 = 2 \omega_3 - \omega_1 = 1.5 \omega_1 - \omega_1 = 0.5 \omega_1 = 0.010935$

State 2: $\phi_2 = 100\%$, $\omega_2 = 0.010935$ => $P_{v2} = 100 \dfrac{\omega_2}{0.622 + \omega_2} = 1.728$ kPa

     $P_{v2} = P_{g2}$ => $T_2 = $ **15.2°C**; $h_f = 63.74$ kJ/kg

CV Cooler

$$\dot{Q} = 0.5 \dot{m}_a (h_2 - h_1)_a + 0.5 \dot{m}_a (\omega_2 h_{v2} - \omega_1 h_{v1}) + \dot{m}_{liq} h_f$$

$$= 0.5 \dot{m}_a [C_{p\,a} (T_2 - T_1) + \omega_2 h_{v2} - \omega_1 h_{v1} + 2(\omega_1 - \omega_3) h_f]$$

$$= 0.5 \frac{2}{1 + \omega_1} \text{ kg/s } [\ 1.004 \ (15.2 - 30) + 0.010935 \times 2529.3$$

$$- 0.02187 \times 2556.25 + 2 \times 0.25 \times 0.02187 \times 63.74] \text{ kJ/kg}$$

$$= -\ \mathbf{41.5 \ kW}$$

From the energy equation for the junction

     $h_{a3} + \omega_3 h_{v3} = 0.5 [h_{a1} + \omega_1 h_{v1} + h_{a2} + \omega_2 h_{v2}] = 0.5[1.004 \ (30 + 15.2)$

             $+ 0.02187 \times 2556.25 + 0.010935 \times 2529.3] = 64.472$ kJ/kg

This is now trial and error on T_3.

     $T_3 = 20C$:   LHS $= 1.004 \times 20 + 0.016403 \times 2538.06 = 61.71$ kJ/kg

     $T_3 = 25C$:   LHS $= 1.004 \times 25 + 0.016403 \times 2547.17 = 66.88$ kJ/kg

     Interpolate for final ans.: $T_3 = $ **22.7°C**

11.96

A water-cooling tower for a power plant cools 45°C liquid water by evaporation. The tower receives air at 19.5°C, $\phi = 30\%$, 100 kPa that is blown through/over the water such that it leaves the tower at 25°C, $\phi = 70\%$. The remaining liquid water flows back to the condenser at 30°C having given off 1 MW. Find the mass flow rate of air, and the amount of water that evaporates.

Solution:

CV Total cooling tower, steady state.

Continuity Eq. for water in air: $w_{in} + \dot{m}_{evap}/\dot{m}_a = w_{ex}$

Energy Eq.: $\dot{m}_a\, \tilde{h}_{in} + \dot{m}_1\, h_{45} = \dot{m}_a\, \tilde{h}_{ex} + (\dot{m}_1 - \dot{m}_{evap})\, h_{30}$

Inlet: 19.5°C, 30% rel hum => $w_{in} = 0.0041$, $\tilde{h}_{in} = 50$ kJ/kg dry air

Exit : 25°C, 70% rel hum => $w_{ex} = 0.0138$, $\tilde{h}_{ex} = 80$ kJ/kg dry air

Take the two water flow difference to mean the 1 MW

$$\dot{Q} = \dot{m}_1\, h_{45} - (\dot{m}_1 - \dot{m}_{evap})\, h_{30} = 1 \text{ MW}$$

Substitute this into the energy equation above and we get

$$\dot{m}_a(\tilde{h}_{ex} - \tilde{h}_{in}) = \dot{m}_a(80 - 50) = 1000 \text{ kW} \quad => \quad \dot{m}_a = \textbf{33.33 kg/s}$$

$$\dot{m}_{evap} = (w_{ex} - w_{in})\, \dot{m}_a = 0.0097 \times 33.33 = \textbf{0.323 kg/s}$$

The needed make-up water flow could be added to give a slightly different meaning to the 1 MW.

11.97

Moist air at 31°C and 50% relative humidity flows over a large surface of liquid water. Find the adiabatic saturation temperature by trial and error. Hint: it is around 22.5^{o}C.

For adiabatic saturation (Φ_2 = 1 and assume P = 100 kPa), energy Eq.11.30

$$\omega_1 (h_{v1} - h_{f2}) = C_p(T_2 - T_1) + \omega_2 h_{fg2}$$

State 1: $\quad \omega_1 = 0.622 \times \dfrac{\Phi P_{g1}}{P_1 - \Phi P_{g1}} = 0.622 \times \dfrac{0.5 \times 4.5}{100 - 0.5 \times 4.5} = 0.01432$

$$\phi_2 = 1 \quad \& \quad \omega_2 = 0.622 \times P_{g2}/(P_2 - P_{g2})$$

Only one unknown in energy Eq.: T_2 . Trial and error on energy equation:

$$C_p T_2 + \omega_2 h_{fg2} + \omega_1 h_{f2} = C_p T_1 + \omega_1 h_{v1}$$
$$= 1.004 \times 31 + 0.01432 \times 2558 = 67.7546 \text{ kJ/kg}$$

$T_2 = 20$ ^{o}C: $\quad P_{g2} = 2.339$ kPa, $\quad h_{f2} = 83.94$ kJ/kg, $\quad h_{fg2} = 2454.12$ kJ/kg

$$\Rightarrow \quad \omega_2 = 0.622 \times 2.339/ 97.661 = 0.0149$$

$$\text{LHS} = 1.004 \times 20 + 0.0149 \times 2454.12 + 0.01432 \times 83.94 = 57.848$$

$T_2 = 25$ ^{o}C: $\quad P_{g2} = 3.169$ kPa, $\quad h_{f2} = 104.87$ kJ/kg, $\quad h_{fg2} = 2442.3$ kJ/kg

$$\Rightarrow \quad \omega_2 = 0.622 \times 3.169/ 96.831 = 0.02036$$

$$\text{LHS} = 1.004 \times 25 + 0.02036 \times 2442.3 + 0.01432 \times 104.87 = 76.327$$

Linear interpolation to match RHS = 67.7546:

$$T_2 = 20 + (25 - 20) \frac{67.7546 - 57.848}{76.327 - 57.848} = \textbf{22.7 }^o\textbf{C}$$

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11.98

A flow of air at 10°C, $\phi = 90\%$, is brought into a house, where it is conditioned to 25°C, 60% relative humidity. This is done with a combined heater-evaporator where any liquid water is at 10°C. Find any flow of liquid, and the necessary heat transfer, both per kilogram dry air flowing. Find the dew point for the final mixture.

CV heater and evaporator. Use psychrometric chart.

Inlet: $\omega_1 = 0.0069$, $\tilde{h}_1 = 47$ kJ/kg dry air, $h_f = 41.99$ kJ/kg

Exit: $\omega_2 = 0.0118$, $\tilde{h}_2 = 75$ kJ/kg dry air, $\mathbf{T_{dew} = 16.5°C}$

From these numbers we see that water and heat must be added.

Continuity eq. for water

$$\dot{m}_{LIQ\ IN}/\dot{m}_A = \omega_2 - \omega_1 = \mathbf{0.0049\ kg/kg\ dry\ air}$$

Energy equation per kg dry air

$$\tilde{q} = \tilde{h}_2 - \tilde{h}_1 - (\omega_2 - \omega_1)\, h_f = \mathbf{27.8\ kJ/kg\ dry\ air}$$

11.99

An air conditioner for an airport receives desert air at 45$^{\text{o}}$C, 10% relative humidity and must deliver this to the buildings at 20$^{\text{o}}$C, 50% relative humidity. They have a cooling system with R-410A running with high pressure of 3000 kPa and low pressure of 1000 kPa and their tap water is 18$^{\text{o}}$C. What should be done to the air? Find the needed heating/cooling per kg dry air.

Check out the psychrometric chart:

State 1: $w_1 = 0.0056$, $\tilde{h}_1 = 79$; State 2: $w_2 = 0.0072$, $\tilde{h}_2 = 58$ kJ/kg

Liquid tap: $h_{liq} = 75.556$ kJ/kg from B.1.1

Now we know the following:

We must **add water** $(w_2 > w_1)$ and then **cool** $(T_{wet\ 1} > 20^{\text{o}}C)$

Water continuity equation: $\dot{m}_{liq} = \dot{m}_{air}\ (w_2 - w_1)$

Energy equation: $\tilde{h}_1 + (w_2 - w_1)\ h_{liq} + \tilde{q} = \tilde{h}_2$

$$\begin{aligned}
\tilde{q} &= \tilde{h}_2 - \tilde{h}_1 - (w_2 - w_1)\ h_{liq} \\
&= 58 - 79 - (0.0072 - 0.0056)\ 75.556 \\
&= -21.12 \text{ kJ/kg dry air}
\end{aligned}$$

For the refrigeration cycle we can find from table B.3.1

$P_{low} = 200$ kPa $\Rightarrow$ $T_{evaporator} = -12^{\text{o}}C$, which is cold enough

$P_{high} = 1500$ kPa $\Rightarrow$ $T_{condenser} = 59^{\text{o}}C > 45^{\text{o}}C$ so it is hot enough.

No absolute scaling was provided (the mass flow rates or $\dot{W}$) so we do not know if the motor/compressor combination is big enough.

11.100

A flow of moist air from a domestic furnace, state 1, is at 45°C, 10% relative humidity with a flow rate of 0.05 kg/s dry air. A small electric heater adds steam at 100°C, 100 kPa generated from tap water at 15°C. Up in the living room the flow comes out at state 4: 30°C, 60% relative humidity. Find the power needed for the electric heater and the heat transfer to the flow from state 1 to state 4.

Properties from the psychrometric chart

State 1: $w_1 = 0.0056$, $\tilde{h}_1 = 79$ kJ/kg dry air
State 4: $w_4 = 0.0160$, $\tilde{h}_4 = 90.5$ kJ/kg dry air

Continuity equation for water from 1 to 4

$$\dot{m}_{liq} = \dot{m}_a \, (\omega_4 - \omega_1) = 0.05 \, (0.016 - 0.0056) = 0.00052 \text{ kg/s}$$

Energy Eq. for heater:

$$\dot{Q}_{heater} = \dot{m}_{liq} \, (h_{out} - h_{in}) = 0.00052 \, (2676.05 - 62.98) = \textbf{1.36 kW}$$

Energy Eq. for line:

$$\dot{Q}_{line} = \dot{m}_a \, (\tilde{h}_4 - \tilde{h}_1) - \dot{m}_{liq} \, h_{vap}$$
$$= 0.05(90.5 - 79) - 0.00052 \times 2676.05$$
$$= \textbf{--0.816 kW}$$

11.101

One means of air-conditioning hot summer air is by evaporative cooling, which is a process similar to the adiabatic saturation process. Consider outdoor ambient air at 35°C, 100 kPa, 30% relative humidity. What is the maximum amount of cooling that can be achieved by such a technique? What disadvantage is there to this approach? Solve the problem using a first law analysis and repeat it using the psychrometric chart, Fig. E.4.

$P_1 = P_2 = 100$ kPa

$P_{v1} = \phi_1 P_{g1} = 0.30 \times 5.628 = 1.6884$

$\omega_1 = 0.622 \times 1.6884/98.31 = 0.01068$

For adiabatic saturation (Max. cooling is for $\phi_2 = 1$),

Energy Eq. Eq.11.30: $\omega_1 (h_{v1} - h_{f2}) = C_p(T_2 - T_1) + \omega_2 h_{fg2}$

$\phi_2 = 1$ & $\omega_2 = 0.622 \times P_{G2}/(P_2 - P_{G2})$

Only one unknown: T_2 . Trial and error on energy equation:

$$C_p T_2 + \omega_2 h_{fg2} + \omega_1 h_{f2} = C_p T_1 + \omega_1 h_{v1}$$
$$= 1.004 \times 35 + 0.01068 \times 2565.3 = 62.537 \text{ kJ/kg}$$

$T_2 = 20\ ^oC$: $P_{G2} = 2.339$ kPa, $h_{f2} = 83.94$, $h_{fg2} = 2454.12$ kJ/kg

$\Rightarrow$ $\omega_2 = 0.622 \times 2.339/ 97.661 = 0.0149$

LHS $= 1.004 \times 20 + 0.0149 \times 2454.1 + 0.01068 \times 83.94 = 57.543$ kJ/kg

$T_2 = 25\ ^oC$: $P_{G2} = 3.169$ kPa, $h_{f2} = 104.87$, $h_{fg2} = 2442.3$ kJ/kg

$\Rightarrow$ $\omega_2 = 0.622 \times 3.169/ 96.831 = 0.02036$

LHS $= 1.004 \times 25 + 0.02036 \times 2442.3 + 0.01068 \times 104.87 = 75.945$ kJ/kg

linear interpolation: $T_2 = \mathbf{21.4\ ^oC}$

This method lowers the temperature but the relative and absolute humidity becomes very high and the slightest cooling like on a wall results in condensation.

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Solution 11.101 Continued.

b)   chart E.4 :   Adiabatic saturation $T_2 \approx$ WetBulbTemperature $\approx$ **21.5 $^{\mathbf{o}}$C**

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11.102

A flow out of a clothes dryer of 0.05 kg/s dry air is at 40^oC and relative humidity 60%. It flows through a heat exchanger where it exits at 20^oC. After heat exchanger the flow combines with another flow of 0.03 kg/s dry air at 30^oC and relative humidity 30%. Find the dew point of state 1, see Fig. P11.102, the heat transfer per kg dry air and the final exit state humidity ratio and relative humidity.

Use the psychrometric chart to solve the problem.

State 1: $w = 0.0286$, $\tilde{h}_1 = 131.5$, $T_{dew} = 31^oC$

State 2: $20^oC < T_{dew}$ so it is saturated.

$w_2 = 0.0148$, $\tilde{h}_2 = 77.8$ kJ/kg air

State 3: $w_3 = 0.0078$, $\tilde{h}_3 = 69.8$ kJ/kg air,

$\dot{m}_{liq}/\dot{m}_a = \omega_1 - \omega_2 = 0.0138$ kg/kg dry air

Energy Eq.:

$\dot{Q}/\dot{m}_a = \tilde{h}_1 - \tilde{h}_2 - (\omega_1 - \omega_2)\, h_f$
$= 131.5 - 77.8 - 0.0138 \times 83.94$
$= 52.5$ kJ/kg dry air

Do CV around the junction where flow 2 and 3 combines to give exit at 4.

Continuity water: $\dot{m}_{a1}\, \omega_2 + \dot{m}_{a3}\, \omega_3 = \dot{m}_{a4}\, \omega_4$

$\omega_4 = (\dot{m}_{a1}/\dot{m}_{a4})\, \omega_2 + (\dot{m}_{a3}/\dot{m}_{a4})\, \omega_3$

$= \dfrac{0.05}{0.08} \times 0.0148 + \dfrac{0.03}{0.08} \times 0.0078 = 0.012175$

Energy Equation: $\dot{m}_{a1}\, \tilde{h}_2 + \dot{m}_{a3}\, \tilde{h}_3 = \dot{m}_{a4}\, \tilde{h}_4$

$\tilde{h}_4 = (\dot{m}_{a1}/\dot{m}_{a4})\, \tilde{h}_2 + (\dot{m}_{a3}/\dot{m}_{a4})\, \tilde{h}_3$

$= \dfrac{0.05}{0.08} \times 77.8 + \dfrac{0.03}{0.08} \times 69.8 = 74.8$ kJ/kg air

From chart given $(\omega_4, \tilde{h}_4)$ we get: $\Phi_4 = 65\%$ and $T_4 = 24^oC$

11.103

Atmospheric air at 35°C, relative humidity of 10%, is too warm and also too dry. An air conditioner should deliver air at 21°C and 50% relative humidity in the amount of 3600 m^3 per hour. Sketch a setup to accomplish this, find any amount of liquid (at 20°C) that is needed or discarded and any heat transfer.

CV air conditioner. First we must check if water should be added or subtracted. We can know this from the absolute humidity ratio.

Properties from the tables and formulas

State 1: P_{g35} = 5.628 kPa ; P_{v1} = 0.10×5.628 kPa = 0.5628 kPa

w_1 = 0.622 × 0.5628 / (101.325 - 0.5628) = 0.003474

State 2: P_{g21} = 2.505 kPa; P_{v2} = 0.5×2.505 kPa = 1.253 kPa

w_2 = 0.622 × 1.253 / (101.325 - 1.253) = 0.007785

As w goes up we must add liquid water. Now we get

Continuity Eq.: $\dot{m}_{air} (1 + w_1) + \dot{m}_{liq} = \dot{m}_{air} (1 + w_2)$

Energy Eq.: $\dot{m}_{air}\tilde{h}_1 + \dot{m}_{liq}h_f + \dot{Q}_{CV} = \dot{m}_{air}\tilde{h}_2$

For the liquid flow we need the air mass flowrate out, 3600 m^3/h = 1 m^3/s

$\dot{m}_{air}$ = $P_{a\,2}\dot{V}/RT$ = (101.325 - 1.253)1/0.287×294.15 = 1.185 kg/s

$\dot{m}_{liq}$ = $\dot{m}_{air}$ (w_2 - w_1) = 0.00511 kg/s = **18.4 kg/h**

$\dot{Q}_{CV}$ = $\dot{m}_{air}$ [$C_{p\,a}(T_2 - T_1)$ + $w_2 h_{v2}$ - $w_1 h_{v1}$] - $\dot{m}_{liq}h_f$

   = 1.185 [1.004 (21 - 35) + 0.007785 × 2539.9 - 0.003474 × 2565.3]
   - 0.00511 × 83.96 = **- 4.21 kW**

If from psychrometric chart.

Inlet: w_1 = 0.0030, $\tilde{h}_1$ = 63.0, $h_{f,20}$ = 83.96 kJ/kg

Exit: w_2 = 0.0076, $\tilde{h}_2$ = 60.2 kJ/kg dry air, P_{v2} , $\dot{m}_{air}$ same as above

$\dot{Q}_{CV}$ = $\dot{m}_{air}$ ($\tilde{h}_2$ - $\tilde{h}_1$) - $\dot{m}_{liq}h_f$ = 1.185(60.2 - 63) - 0.00511 × 83.96
   = - 3.74 kW

11.104

In a car's defrost/defog system atmospheric air, 21°C, relative humidity 80%, is taken in and cooled such that liquid water drips out. The now dryer air is heated to 41°C and then blown onto the windshield, where it should have a maximum of 10% relative humidity to remove water from the windshield. Find the dew point of the atmospheric air, specific humidity of air onto the windshield, the lowest temperature and the specific heat transfer in the cooler.

Solution:

Solve using the psychrometric chart

Air inlet: 21°C, $\phi = 80\%$
=> $w_1 = 0.0124$,
$T_{dew} = \mathbf{17.3°C}$
$\tilde{h}_1 = 72$
Air exit: 41°C, $\phi = 10\%$
=> $w_3 = \mathbf{0.0044}$,
$T_{dew} = \mathbf{1.9°C}$

To remove enough water we must cool to the exit T_{dew}, followed by heating to T_{ex}. The enthalpy from chart $\tilde{h}_2 = 32.5$ and from B.1.1, $h_f(1.9°C) = 8$ kJ/kg

CV cooler:

$$\dot{m}_{liq}/\dot{m}_{air} = w_1 - w_3 = 0.0124 - 0.0044 = 0.008 \text{ kg liq/kg air}$$

$$q = \dot{Q}_{CV}/\dot{m}_{air} = \tilde{h}_2 + (w_1 - w_3) h_f - \tilde{h}_1$$

$$= 32.5 + 0.008 \times 8 - 72 = \mathbf{-39.4 \text{ kJ/kg dry air}}$$

If the steam and air tables are used the numbers are

State 1: $P_{g1} = 2.505$ kPa, $P_{v1} = 2.004$ kPa => $w_1 = 0.01259$

$h_{g1} = 2539.9$, $h_{a1} = 294.3$ => $\tilde{h}_1 = 326.3$ kJ/kg

State 3: $P_{g3} = 7.826$, $P_{v3} = 0.783$ => $w_3 = \mathbf{0.00486}$

State 2: $w_{g3} = w_3$ => $T_2 = T_{3dew} = 3.3°C$, $h_{f2} = 13.77$ kJ/kg

$h_{g2} = 2507.4$, $h_{a2} = 276.56$ => $\tilde{h}_2 = 288.75$ kJ/kg

$$\dot{m}_{liq}/\dot{m}_{air} = 0.00773, \quad q = 288.75 + 0.00773 \times 13.77 - 326.3 = \mathbf{-37.45 \text{ kJ/kg air}}$$

11.105

A commercial laundry runs a dryer that has an exit flow of 0.5 kg/s moist air at 48 $^{\circ}$C, 70% relative humidity. To save on the heating cost a counterflow **stack heat exchanger** is used to heat a cold water line at 10°C for the washers with the exit flow, as shown in Fig. P11.105. Assume the outgoing flow can be cooled to 25°C. Is there a missing flow in the figure? Find the rate of energy recovered by this heat exchanger and the amount of cold water that can be heated to 30°C.

The flow out of the dryer is outside the range for the psychrometric chart so the solution is done with the tables and the formula's.

Dryer oulet, 1: 48°C, Φ = 70%

$\quad \Rightarrow h_{g1} = 2588.51$ kJ/kg

$\quad\quad P_{g1} = 11.247$ kPa,

$P_{v1} = 0.7 \times 11.247$ kPa $= 7.873$ kPa

$\omega_1 = \dfrac{0.622 \times 7.873}{101.325 - 7.873} = 0.05241$

State 2: $T_2 < T_{dew\,1} \approx 42^{\circ}$C so $\Phi_2 = 100\%$. $P_{g2} = 3.169$ kPa, $h_{g2} = 2547.17$ kJ/kg,

$\quad\quad h_{f2} = 104.87$ kJ/kg, $\omega_2 = 0.622 \times 3.169/(101.325 - 3.169) = 0.02008$

Continuity Eq. water 1-2 line: $\dot{m}_a \omega_1 = \dot{m}_a \omega_2 + \dot{m}_{liq}$; $\dot{m}_{liq} = \dot{m}_a(\omega_1 - \omega_2)$

The mass flow rate of dry air is

$$\dot{m}_a = \dot{m}_{moist\ air} /(1 + \omega_1) = 0.5 \text{ kg/s} /(1 + 0.05241) = 0.4751 \text{ kg/s}$$

The heat out of the exhaust air which also equals the energy recovered becomes

$$\dot{Q}_{CV} = \dot{m}_a [C_{p\,a}(T_1 - T_2) + \omega_1 h_{g1} - (\omega_1 - \omega_2) h_{f2} - \omega_2 h_{g2}]$$
$$= 0.4751 [1.004(48 - 25) + 0.05241 \times 2588.51 - 0.03233 \times 104.87$$
$$\quad - \quad 0.02008 \times 2547.17]$$
$$= 0.4751 \text{ kg/s} \times 104.2 \text{ kJ/kg} = \textbf{49.51 kW}$$

$$\dot{m}_{liq} = \dot{Q}_{CV} / (C_{P\,liq} \Delta T_{liq}) = 49.51 / [4.18 (30 - 10)] = \textbf{0.592 kg/s}$$

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11.106

A flow of moist air at 45°C, 10% relative humidity with a flow rate of 0.2 kg/s dry air is mixed with a flow of moist air at 25°C, and absolute humidity of w = 0.018 with a rate of 0.3 kg/s dry air. The mixing takes place in an air duct at 100 kPa and there is no significant heat transfer. After the mixing there is heat transfer to a final temperature of 25°C. Find the temperature and relative humidity after mixing. Find the heat transfer and the final exit relative humidity.

Solution:

C.V : Total Setup
state 3 is internal to CV.

Continuity Eq.: $\dot{m}_{a1}\,w_1 + \dot{m}_{a2}\,w_2 = (\dot{m}_{a1} + \dot{m}_{a2})\,w_3 = (\dot{m}_{a1} + \dot{m}_{a2})\,w_4$

Energy Eq. $\dot{m}_{a1}\,\tilde{h}_1 + \dot{m}_{a2}\,\tilde{h}_2 = (\dot{m}_{a1} + \dot{m}_{a2})\,\tilde{h}_3$

State 1: From Psychrometric chart $w_1 = 0.0056$, $\tilde{h}_1 = 79$ kJ/kg dry air

State 2: From Psychrometric chart $\Phi_2 = 90\%$, $\tilde{h}_2 = 90.5$ kJ/kg dry air

$$w_3 = w_4 = \frac{\dot{m}_{a1}w_1 + \dot{m}_{a2}w_2}{\dot{m}_{a1} + \dot{m}_{a2}} = \frac{0.2}{0.5}\,0.0056 + \frac{0.3}{0.5}\,0.018 = 0.01304$$

$$\tilde{h}_3 = \frac{\dot{m}_{a1}\tilde{h}_1 + \dot{m}_{a2}\tilde{h}_2}{\dot{m}_{a1} + \dot{m}_{a2}} = \frac{0.2}{0.5}\times 79 + \frac{0.3}{0.5}\times 90.5 = 85.9 \text{ kJ/kg dry air}$$

State 3: From Psychrometric chart $T_3 = 32.5$°C, $\Phi_3 = 45\%$

State 4: 25°C, $w_4 = 0.01304$ Read from Psychrometric chart

$$\tilde{h}_4 = 78, \quad \mathbf{\Phi_4 = 66\%}$$

Now do the energy equation for the whole setup

Energy Eq. $\dot{m}_{a1}\,\tilde{h}_1 + \dot{m}_{a2}\,\tilde{h}_2 + \dot{Q} = (\dot{m}_{a1} + \dot{m}_{a2})\,\tilde{h}_4$

$$\dot{Q} = (\dot{m}_{a1} + \dot{m}_{a2})\,\tilde{h}_4 - \dot{m}_{a1}\,\tilde{h}_1 + \dot{m}_{a2}\,\tilde{h}_2$$
$$= 0.5\times 78 - 0.2\times 79 - 0.3\times 90.5 = \mathbf{-3.95\ kW}$$

11.107

An indoor pool evaporates 1.512 kg/h of water, which is removed by a dehumidifier to maintain 21°C, $\phi = 70\%$ in the room. The dehumidifier, shown in Fig. P11.107, is a refrigeration cycle in which air flowing over the evaporator cools such that liquid water drops out, and the air continues flowing over the condenser. For an air flow rate of 0.1 kg/s the unit requires 1.4 kW input to a motor driving a fan and the compressor and it has a coefficient of performance, $\beta = Q_L/W_c = 2.0$. Find the state of the air as it returns to the room and the compressor work input.

Solution:

The unit must remove 1.512 kg/h liquid to keep steady state in the room. As water condenses out state 2 is saturated.

State 1:   21°C, 70%   => $w_1 = 0.0108$, $\tilde{h}_1 = 68.5$

CV 1 to 2: $\dot{m}_{liq} = \dot{m}_a(w_1 - w_2)$ => $w_2 = w_1 - \dot{m}_{liq}/\dot{m}_a$

$\qquad q_L = \tilde{h}_1 - \tilde{h}_2 - (w_1 - w_2) \, h_{f2}$

$\qquad w_2 = 0.0108 - 1.512/3600 \times 0.1 = 0.0066$

State 2: w_2, 100% => $T_2 = 8°C$, $\tilde{h}_2 = 45$, $h_{f2} = 33.6$

$\qquad q_L = 68.5 - 45 - 0.0042 \times 33.6 = 23.36$ kJ/kg dry air

CV Total system : $\tilde{h}_3 = \tilde{h}_1 + \dot{W}_{el}/\dot{m}_a - (w_1 - w_2) \, h_f$

$\qquad\qquad\qquad\qquad = 68.5 + 14 - 0.14 = 82.36$ kJ/kg dry air

State 3: $w_3 = w_2$, $\tilde{h}_3$ => $T_3 = \textbf{46°C}$, $\phi_3 = \textbf{11-12\%}$

$\qquad \dot{W}_c = \dot{m}_a \, q_L/ \beta = \textbf{1.165 kW}$

11.108

A moist air flow of 5 kg/min at 30°C, $\Phi = 60\%$, 100 kPa goes through a dehumidifier in a setup shown in Problem 11.107. The air is cooled down to 15°C and then blown over the condenser. The refrigeration cycle runs with R-134a with a low pressure of 200 kPa and a high pressure of 1000 kPa. Find the COP of the refrigeration cycle, the ratio $\dot{m}_{R\text{-}134a}/\dot{m}_{air}$ and the outgoing T_3 and Φ_3.

Standard Refrigeration Cycle

Table B.5: $h_1 = 392.15$ kJ/kg; $s_1 = 1.732$ kJ/kg K; $h_4 = h_3 = 255.56$ kJ/kg

C.V. Compressor (assume ideal)

$$\dot{m}_1 = \dot{m}_2 \qquad w_C = h_2 - h_1; \quad s_2 = s_1 + s_{gen}$$

$$P_2, s = s_1 \implies h_2 = 425.71 \text{ kJ/kg} \implies w_C = 33.56 \text{ kJ/kg}$$

C.V. Evaporator: $q_L = h_1 - h_4 = 392.15 - 255.56 = 136.59$ kJ/kg

C.V. Condenser: $q_H = h_2 - h_3 = 425.71 - 255.56 = 170.15$ kJ/kg

$$\text{COP} = \beta = q_L \,/\, w_C = \frac{136.59}{33.56} = 4.07$$

$P_1 = 200$ kPa, $P_2 = 1000$ kPa

For the air processes let us use the psychrometric chart.

Air inlet: $w_{in} = 0.016$, $\tilde{h}_{in} = 90.5$ kJ/kg dry air, $T_{dew} = 21^o$C $> 15^o$C

Air 15°C: $\phi = 100\%$, $w_7 = 0.0107$, $\tilde{h}_7 = 62$, $h_f = 62.98$ (B.1.1)

Now do the continuity (for water) and energy equations for the cooling process

$$\dot{m}_{liq}/\dot{m}_{air} = w_{in} - w_7 = 0.016 - 0.0107 = 0.0053 \text{ kg/kg air}$$

$$q_{cool} = \tilde{h}_{in} - \tilde{h}_7 - \dot{m}_{liq}h_f/\dot{m}_{air} = 90.5 - 62 - 0.0053 \times 62.98 = 28.17 \text{ kJ/kg air}$$

Solution 11.108 continued

Now the cooling of the air is done by the R-134a so

$$\dot{Q}_{cool} = \dot{m}_{air}\, q_{cool} = \dot{m}_{R134a}\, q_L \quad \Rightarrow \quad \dot{m}_{R134a}/\dot{m}_{air} = \frac{q_{cool}}{q_L} = \frac{28.17}{136.59} = \mathbf{0.2062}$$

Energy eq. for the air flow being heated

$$\dot{Q}_{heat} = \dot{m}_{air}(\tilde{h}_{ex} - \tilde{h}_7) \quad \Rightarrow \quad \tilde{h}_{ex} = \tilde{h}_7 + \dot{Q}_{heat}/\dot{m}_{air} = \tilde{h}_7 + q_H \times \dot{m}_{R134a}/\dot{m}_{air}$$

$$\tilde{h}_{ex} = 62 + 170.15 \times 0.2062 = 97.08 \text{ kJ/kg dry air} \quad \text{and} \quad w_{ex} = w_7$$

Locate state in the psychrometric chart [just outside edge of chart]

$$T_{ex} = 49.3^\text{o}\text{C} \quad \text{and} \quad \phi_{ex} = 15\%$$

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Psychrometric chart only

11.109

Use the psychrometric chart to find the missing property of: ϕ, ω, T_{wet}, T_{dry}

a. $T_{dry} = 25°C$, $\phi = 80\%$ b. $T_{dry} = 15°C$, $\phi = 100\%$

c. $T_{dry} = 20°C$, and $\omega = 0.010$ d. $T_{dry} = 25°C$, $T_{wet} = 23°C$

Solution:

a. $25°C$, $\phi = 80\%$ => $\omega = 0.016$; $T_{wet} = 22.3°C$

b. $15°C$, $\phi = 100\%$ => $\omega = 0.0106$; $T_{wet} = 15°C$

c. $20°C$, $\omega = 0.010$ => $\phi = 70\%$; $T_{wet} = 16.3°C$

d. $25°C$, $T_{wet} = 23°C$ => $\omega = 0.017$; $\phi = 86\%$

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11.110

Use the psychrometric chart to find the missing property of: ϕ, ω, T_{wet}, T_{dry}

      a. $\phi = 50\%$, $\omega = 0.014$ b. $T_{wet} = 15°C$, $\phi = 60\%$.

      c. $\omega = 0.008$ and $T_{wet} = 15°C$ d. $T_{dry} = 10°C$, $\omega = 0.006$

Solution:

    a. $\phi = 50\%$, $\omega = 0.014$ => $T_{dry} = 31°C$, $T_{wet} = 22.6\ °C$

    b. $T_{wet} = 15°C$, $\phi = 60\%$ => $T_{dry} = 20.2°C$, $\omega = 0.0086$

    c. $\omega = 0.008$, $T_{wet} = 15°C$ => $T_{dry} = 21.8°C$, $\phi = 50\%$

    d. $T_{dry} = 10°C$, $\omega = 0.006$ => $\phi = 80\%$, $T_{wet} = 8.2°C$

11.111

For each of the states in Problem 11.110 find the dew point temperature.

Solution:

The dew point is the state with the same humidity ratio (abs humidity ω) and completely saturated $\phi = 100\%$. From psychrometric chart:

a. $T_{dew} = 19.2°C$ c. $T_{dew} = 10.8°C$

b. $T_{dew} = 12°C$ d. $T_{dew} = 6.5°C$

Finding the solution from the tables is done for cases a,c and d as

Eq.11.28 solve: $P_v = P_g = \omega P_{tot}/[\omega + 0.622] = P_g(T_{dew})$ in B.1.1

For case b use energy Eq. 13.30 to find ω_1 first from $T_{ad\ sat} = T_{wet}$.

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11.112

Use the formulas and the steam tables to find the missing property of: ϕ, ω, and T_{dry}, total pressure is 100 kPa; repeat the answers using the psychrometric chart

a. $\phi = 50\%$, $\omega = 0.010$ b. $T_{wet} = 15°C$, $\phi = 50\%$ c. $T_{dry} = 25°C$, $T_{wet} = 21°C$

a. From Eq.11.28 with $P_a = P - P_v$ solve for P_v:

$$P_v = P\, \omega\, /(0.622 + \omega) = 100 \times 0.01/0.632 = 1.582 \text{ kPa}$$

From Eq.11.25: $P_g = P_v/\phi = 1.582/0.5 = 3.165 \text{ kPa} \quad => \quad T = 25°C$

b. Assume T_{wet} is adiabatic saturation T and use energy Eq.11.30

At 15°C: $P_g = 1.705 \text{ kPa} \quad => \quad \omega = 0.622 \times \dfrac{1.705}{100 - 1.705} = 0.01079$

$LHS = \omega_1\, (h_{v1} - h_{f2}) + C_p T_1 = RHS = C_p T_2 + \omega_2\, h_{fg2}$

$RHS = 1.004 \times 15 + 0.01079 \times 2465.93 = 41.667 \text{ kJ/kg}$

$\omega_1 = 0.622\, \phi P_g/(100 - \phi P_g)$ where P_g is at T_1.   Trial and error.

$LHS_{25C} = 49.98$, $LHS_{20C} = 38.3 \quad => \quad T = 21.4°C$, $\omega_1 = 0.008$

c. At 21°C: $P_g = 2.505 \text{ kPa} \quad => \quad \omega_2 = 0.622 \times \dfrac{2.505}{100 - 2.505} = 0.016$

$h_{f2} = 88.126$ and $h_{fg2} = 2451.76 \text{ kJ/kg}$, $h_{v1} = 2547.17 \text{ kJ/kg}$

From Eq.11.30: $\omega_1 = [C_p(T_2 - T_1) + \omega_2\, h_{fg2}\,]/(h_{v1} - h_{f2}) = 0.0143$

$P_v = P\, \omega\, /(0.622 + \omega) = 2.247$, $\phi = 2.247/3.169 = 0.71$

Using the psychrometric chart E.4:

a: $T_{dry} = 25.3 °C$ b. $T_{dry} = 21.6°C$, $\omega = 0.008$

c: $\omega = 0.0141$, $\phi = 71\text{-}72\%$

11.113

An air-conditioner should cool a flow of ambient moist air at 40°C, 40% relative humidity with 0.2 kg/s flow of dry air. The exit temperature should be 20°C and the pressure is 100 kPa. Find the rate of heat transfer needed and check for the formation of liquid water.

Solution:

Before we know if we should have a liquid water flow term we need to check for condensation (the dew point).

Using the psychrometric chart. i: $\omega_i = 0.018$, $\tilde{h}_i = 106$ kJ/kg air, $T_{dew} = 23°C$

Since $T_{dew} > T_e$ then **condensation** occurs and the exit is $\phi = 100\%$

Exit state: 20°C, $\tilde{h}_e = 77.5$ kJ/kg air, $\omega_e = 0.0148$, $h_f = 83.94$ kJ/kg

CV heat exchanger:

Water: $\dot{m}_a \omega_i = \dot{m}_{liq} + \dot{m}_a \omega_e$ $\Rightarrow$ $\dot{m}_{liq} = \dot{m}_a (\omega_i - \omega_e)$

Energy: $(h_a + \omega h_v)_i + \tilde{q} = (h_a + \omega h_v)_e + (\omega_i - \omega_e) h_f$,

$\tilde{q} = \tilde{h}_e - \tilde{h}_i + (\omega_i - \omega_e) h_f = 77.5 - 106 + (0.018 - 0.0148)\ 83.94$

$= -28.23$ kJ/kg dry air

$\dot{Q} = \dot{m}_a \tilde{q} = 0.2$ kg/s $\times (-28.23$ kJ/kg$) = $ **−5.65 kW**   (it goes out)

11.114

A flow of moist air at 21°C, 60% relative humidity should be produced from mixing of two different moist air flows. Flow 1 is at 10°C, relative humidity 80% and flow 2 is at 32°C and has $T_{wet} = 27°C$. The mixing chamber can be followed by a heater or a cooler. No liquid water is added and $P = 100$ kPa. Find the two controls one is the ratio of the two mass flow rates $\dot{m}_{a1}/\dot{m}_{a2}$ and the other is the heat transfer in the heater/cooler per kg dry air.

Solution:

C.V : Total Setup
state 3 is internal to CV.

Continuity Eq.: $\qquad \dot{m}_{a1}\, w_1 + \dot{m}_{a2}\, w_2 = (\dot{m}_{a1} + \dot{m}_{a2})\, w_4$

Energy Eq. $\qquad \dot{m}_{a1}\, \tilde{h}_1 + \dot{m}_{a2}\, \tilde{h}_2 + \dot{Q}_{a1} = (\dot{m}_{a1} + \dot{m}_{a2})\, \tilde{h}_4$

Define $\quad x = \dot{m}_{a1}/\dot{m}_{a2}\quad$ and substitute into continuity equation

$$\Rightarrow \quad x\, w_1 + w_2 = (1+x)\, w_4 \quad \Rightarrow \quad x = \frac{w_4 - w_2}{w_1 - w_4} = \mathbf{3.773}$$

Energy equation scaled to total flow of dry air

$$\tilde{q} = \dot{Q}_{a1}/(\dot{m}_{a1} + \dot{m}_{a2}) = \tilde{h}_4 - [x/(1+x)]\, \tilde{h}_1 - [1/(1+x)]\, \tilde{h}_2$$

$$= 64 - 0.7905 \times 45 - 0.2095 \times 105$$

$$= \mathbf{6.43\ kJ/kg\text{-}dry\ air}$$

State 1:
$\quad w_1 = 0.006$, $\tilde{h}_1 = 45$

State 2:
$\quad w_2 = 0.0208$, $\tilde{h}_2 = 105$

State 4:
$\quad w_4 = 0.0091$, $\tilde{h}_4 = 64$,
$\quad T_{dew\,4} = 12.5°C$

11.115

Consider a mixing process as in Fig. P11.114 where flow 1 comes in as cold and moist, 10°C, $\phi = 90\%$. It is mixed with a flow 2 at 42°C, $T_{wet} = 31$°C. If the exit flow should be at 21°C, $\phi = 50\%$ find the ratio $\dot{m}_{a1}/\dot{m}_{a2}$ and the heat transfer per kg dry air out.

C.V : Total Setup
state 3 is internal to CV.

Assume no liquid flow then the balance equations are

Continuity Eq.: $\dot{m}_{a1}\,w_1 + \dot{m}_{a2}\,w_2 = (\dot{m}_{a1} + \dot{m}_{a2})\,w_4$

Energy Eq. $\dot{m}_{a1}\,\tilde{h}_1 + \dot{m}_{a2}\,\tilde{h}_2 + \dot{Q} = (\dot{m}_{a1} + \dot{m}_{a2})\,\tilde{h}_4$

The continuity equation is solved for the ratio $x = \dot{m}_{a1}/\dot{m}_{a2}$

$$x = \dot{m}_{a1}/\dot{m}_{a2} = \frac{w_2 - w_4}{w_4 - w_1} = \frac{0.0245 - 0.0076}{0.0076 - 0.0067} = 18.7778$$

Then solve the energy eq. for the heat transfer

$$\dot{Q}/\dot{m}_{a4} = \tilde{h}_4 - \frac{x}{1+x}\tilde{h}_1 - \frac{1}{1+x}\tilde{h}_2 = 60 - \frac{18.7778}{19.7778}\,47 - \frac{1}{19.7778}\,124$$

$$= \textbf{9.107 kJ/kg air out}$$

State 1:
  $w_1 = 0.0067$, $\tilde{h}_1 = 47$

State 2:
  $w_2 = 0.0245$, $\tilde{h}_2 = 124$

State 4:
  $w_4 = 0.0076$, $\tilde{h}_4 = 60$,

11.116

In a hot and dry climate, air enters an air-conditioner unit at 100 kPa, 40°C, and 5% relative humidity, at the steady rate of 1.0 m^3/s. Liquid water at 20°C is sprayed into the air in the AC unit at the rate 20 kg/hour, and heat is rejected from the unit at at the rate 20 kW. The exit pressure is 100 kPa. What are the exit temperature and relative humidity?

State 1: $T_1 = 40°C$, $P_1 = 100$ kPa, $\phi_1 = 5\%$, $\dot{V}_{a1} = 1$ m^3/s

$P_{g1} = 7.3837$ kPa, $P_{v1} = \phi_1 P_{g1} = 0.369$ kPa, $P_{a1} = P - P_{v1} = 99.63$ kPa

$$\omega_1 = 0.622 \frac{P_{v1}}{P_{a1}} = 0.0023, \quad \dot{m}_{a1} = \frac{P_{a1}\dot{V}_{a1}}{RT_{a1}} = 1.108 \text{ kg/s}, \quad h_{v1} = 2574.3 \text{ kJ/kg}$$

State 2 : Liq. Water. 20°C, $\dot{m}_{f2} = 20$ kg/hr $= 0.00556$ kg/s, $h_{f2} = 83.9$ kJ/kg

Conservation of Mass: $\dot{m}_{a1} = \dot{m}_{a3}$, $\dot{m}_{v1} + \dot{m}_{12} = \dot{m}_{v3}$

$\omega_3 = (\dot{m}_{f2} / \dot{m}_{a1}) + \omega_1 = (0.00556/1.108) + 0.0023 = 0.0073$

State 3 : $P_3 = 100$ kPa and $P_{v3} = P_3\omega_3/(0.622 + \omega_3) = 1.16$ kPa

Energy Eq. with $\dot{Q} = -20$ kW :

$$\dot{Q} + \dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{f2}h_{f2} = \dot{m}_{a3}h_{a3} + \dot{m}_{v3}h_{v3};$$

$(h_{a3}-h_{a1}) + \omega_3 h_{v3} = \omega_1 h_{v1} + (\dot{m}_{f2}h_{f2} + \dot{Q})/\dot{m}_{a1}$

$= 0.0023 \times 2574.3 + (0.00556 \times 83.9 - 20)/1.108 = -11.7$

Unknowns: h_{a3}, h_{v3} implicitly given by a single unknown: T_3

Trial and Error for T_3; $T_3 = \mathbf{10°C}$, $P_{g3} = 1.23$ kPa, $\phi_3 = \dfrac{P_{v3}}{P_{g3}} = \mathbf{0.94}$

If we solved with the psychrometric chart we would get:

State 1: $\dot{m}_{v1}/\dot{m}_a = \omega_1 = 0.002$, $\tilde{h}_1 = 65$ kJ/kg dry air;

State 3: $\omega_3 = (\dot{m}_{f2} / \dot{m}_{a1}) + \omega_1 = (0.00556/1.108) + 0.002 = 0.007$

Now the energy equation becomes

$\tilde{h}_3 = \tilde{h}_1 + (\dot{m}_{f2}h_{f2} + \dot{Q})/\dot{m}_{a1} = 65 + (0.00556 \times 83.9 - 20)/1.108 = 47.4$

Given ω_3 we find the state around $\mathbf{10°C}$ and $\phi_3 = \mathbf{90\%}$

11.117

Compare the weather two places where it is cloudy and breezy. At beach A it is 20°C, 103.5 kPa, relative humidity 90% and beach B has 25°C, 99 kPa, relative humidity 40%. Suppose you just took a swim and came out of the water. Where would you feel more comfortable and why?

Solution:

As your skin is wet and air is flowing over it you will feel T_{wet}. With the small difference in pressure from 100 kPa we can use the psychrometric chart.

A: 20°C, $\phi = 90\%$ => $T_{wet} = 18.7°C$

B: 25°C, $\phi = 40\%$ => $T_{wet} = 16°C$

At beach A it is comfortable, at beach B it feels chilly.

11.118

Ambient air at 100 kPa, 30°C, 40% relative humidity goes through a constant pressure heat exchanger as a steady flow. In one case it is heated to 45°C and in another case it is cooled until it reaches saturation. For both cases find the exit relative humidity and the amount of heat transfer per kilogram dry air.

Solution:

CV heat exchanger: $\dot{m}_{Ai} = \dot{m}_{Ae}, \quad \dot{m}_{vi} = \dot{m}_{ve}, \quad w_e = w_i$

$$(h_a + wh_v)_i + q = (h_a + wh_v)_e = \tilde{h}_e, \quad q = \tilde{h}_e - \tilde{h}_i$$

Using the psychrometric chart: i: $w_i = 0.0104, \quad \tilde{h}_i = 76$

Case I) e: $T_e = 45\ ^oC, w_e = w_i => \tilde{h}_e = 92,$

$\phi_e = 17\%, \quad q = 92-76 = $ **16 kJ/kg dry air**

Case II) e: $w_e = w_i, \quad \phi_e = 100\% \Rightarrow \tilde{h}_e = 61, \ T_e = 14.5^oC$

$q = 61-76 = $ **-15 kJ/kg dry air**

11.119

A flow of moist air at 100 kPa, 35°C, 40% relative humidity is cooled by adiabatic evaporation of liquid 20°C water to reach a saturated state. Find the amount of water added per kg dry air and the exit temperature.

Since the liquid is not necessarily at the adiabatic saturation temperature the exit may be close to but not exactly the wet bulb temperature. We will use that as a good guess and check the energy equation.

Chart E4: $T_{wet} = 23.6°C$, $w_1 = 0.0138$, $\tilde{h}_{1\ mix} = 90$ kJ/kg

$w_2 = 0.0186$, $\tilde{h}_{2mix} = 90.6$ kJ/kg

Continuity Eq.: $\dot{m}_A(1 + w_1) + \dot{m}_{liq} = \dot{m}_A(1 + w_2)$

Energy Eq. (q = 0): $\dot{m}_A\tilde{h}_{1mix} + \dot{m}_{liq}h_f = \dot{m}_A\tilde{h}_{2mix}$

Divide the energy equation by the mass flow rate of air

$\dot{m}_{liq}/\dot{m}_A = w_2 - w_1 = 0.0186 - 0.0138 = 0.0048$ kg water/kg air

so

$\tilde{h}_{1mix} + (w_2 - w_1)\ h_f = \tilde{h}_{2mix}$
LHS = 90 + 0.0048 × 83.94 = 90.4; RHS = 90.6

The temperature should be a little lower which will lower w_2 also so

$T_2 = \mathbf{23.5°C}$, $w_2 = 0.0185$ => $\dot{m}_{liq}/\dot{m}_A = \mathbf{0.0047}$ **kg water/kg air**

This is close to the accuracy by which we can read the chart and the first answer is nearly as good.

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11.120

A flow out of a clothes dryer of 0.1 kg/s dry air is at 60°C and relative humidity 60%. It flows through a heat exchanger where it exits at 20°C. After the heat exchanger the flow combines with another flow of 0.03 kg/s dry air at 30°C and relative humidity 40%. Find the dew point of state 1, see Fig. P11.120, the heat transfer per kg dry air and the final exit state humidity ratio and relative humidity.

Use the psychrometric chart to solve the problem except for state 1.

State 1: $P_{g1} = 19.941$ kPa,

$\qquad P_{v1} = \Phi_1 \, P_{g1} = 11.965$ kPa

$\qquad T_{dew} = T_{sat} \, (P_{v1}) = \textbf{49.3}^\circ\textbf{C}$

$\omega_1 = 0.622 \dfrac{11.965}{100 - 11.965} = 0.084534$

$\tilde{h}_1 = h_{a1} - h_{a\,-20C} + \omega_1 h_{a1}$
$\qquad = 333.81 - 253.45 + 0.084534 \times 2609.59$
$\qquad = 301$ kJ/kg air,

State 2: 20°C $< T_{dew}$ so it is saturated

$\qquad \omega_2 = 0.0148, \; \tilde{h}_2 = 77.8$ kJ/kg air

State 3: $\omega_3 = 0.0078, \tilde{h}_3 = 69.8$ kJ/kg air,

$$\dot{m}_{liq}/\dot{m}_a = \omega_1 - \omega_2 = 0.06973 \text{ kg/kg dry air}$$

Energy Eq.: $\dot{Q}/\dot{m}_a = \tilde{h}_1 - \tilde{h}_2 - (\omega_1 - \omega_2) \, h_f$
$\qquad\qquad\qquad = 301 - 77.8 - 0.06973 \times 83.94 = \textbf{217.35 kJ/kg dry air}$

Do CV around the junction where flow 2 and 3 combines to give exit at 4.

Continuity water: $\dot{m}_{a1} \, \omega_2 + \dot{m}_{a3} \, \omega_3 = \dot{m}_{a4} \, \omega_4$

$$\omega_4 = (\dot{m}_{a1}/\dot{m}_{a4}) \, \omega_2 + (\dot{m}_{a3}/\dot{m}_{a4}) \, \omega_3$$

$$= \frac{0.05}{0.08} \times 0.0148 + \frac{0.03}{0.08} \times 0.0078 = \textbf{0.012175}$$

Energy Equation: $\dot{m}_{a1} \, \tilde{h}_2 + \dot{m}_{a3} \, \tilde{h}_3 = \dot{m}_{a4} \, \tilde{h}_4$

$$\tilde{h}_4 = (\dot{m}_{a1}/\dot{m}_{a4}) \, \tilde{h}_2 + (\dot{m}_{a3}/\dot{m}_{a4}) \, \tilde{h}_3$$

$$= \frac{0.05}{0.08} \times 77.8 + \frac{0.03}{0.08} \times 69.8 = 74.8 \text{ kJ/kg air}$$

From chart given $(\omega_4, \tilde{h}_4)$ we get: $\Phi_4 = \textbf{65\%}$ and $T_4 = 24^\circ$C

11.121

Consider two states of atmospheric air. (1) 40°C, $\phi = 50\%$ and (2) 25°C, $T_{wet} =$ 16°C. Suggest a system of devices that will allow air in a steady flow process to change from (1) to (2) and from (2) to (1). Heaters, coolers (de)humidifiers, liquid traps etc. are available and any liquid/solid flowing is assumed to be at the lowest temperature seen in the process. Find the specific and relative humidity for state 1, dew point for state 2 and the heat transfer per kilogram dry air in each component in the systems.

Use the psychrometric chart E.4

1: $w_1 = 0.0232$, $\tilde{h}_1 = 118.5$, $\Phi_1 = 50\%$, $T_{dew} = 27.2$°C, $\tilde{h}_{dew} = 106$

2: $w_2 = 0.0077$, $\tilde{h}_2 = 65$, $\Phi_2 = 40\%$, $T_{dew} = 10.2$°C, $\tilde{h}_{dew} = 50$

Since $w_2 < w_1$ water must be removed in process I to II and added in the process II to I. Water can only be removed by cooling below dew point temperature so

I to II: Cool to dew point, 3, then cool to state 4 (dew pt. for state 2) while liquid water is removed, then heat to state 2.

II to I: Heat while adding water, once w_2 reached only heat.

$$\dot{m}_{liq}/\dot{m}_a = \omega_1 - \omega_2 = 0.0232 - 0.0077 = 0.0155$$

I to II: $q_{cool} = \tilde{h}_1 - \tilde{h}_4 - (\omega_1 - \omega_2) \, h_{f4}$

$= 118.5 - 50 - 0.0155 \times 42.83 = $ **67.8 kJ/kg air**

$q_{heat} = \tilde{h}_2 - \tilde{h}_4 = 65 - 50 = $ **15 kJ/kg air**

II to I: $q_{heat} = \tilde{h}_1 - \tilde{h}_2 - (\omega_1 - \omega_2) \, h_{f2}$

$= 118.5 - 65 - 0.0155 \times 104.87 = $ **51.9 kJ/kg air**

11.122

To refresh air in a room, a counterflow heat exchanger, see Fig. P11.122, is mounted in the wall, drawing in outside air at 0.5°C, 80% relative humidity and pushing out room air, 40°C, 50% relative humidity. Assume an exchange of 3 kg/min dry air in a steady flow device, and also that the room air exits the heat exchanger at 23°C to the outside. Find the net amount of water removed from the room, any liquid flow in the heat exchanger and (T, ϕ) for the fresh air entering the room.

We will use the psychrometric chart to solve this problem.

State 3 (room): $w_3 = 0.0232$, $\tilde{h}_3 = 119.2$, $T_{dew\ 3} = 27°C$

State 1(outside): 0.5°C, $\phi = 80\%$ => $w_1 = 0.0032$, $\tilde{h}_1 = 29.2$ kJ/kg dry air

CV room: $\dot{m}_{v,out} = \dot{m}_a (w_3 - w_2) = \dot{m}_a (w_3 - w_1)$

$$= 3(0.0232-0.0032) = \textbf{0.06 kg/min}$$

The room air is cooled to 23°C $< T_{dew\ 3}$ so liquid will form in the exit flow channel and state 4 is saturated.

State 4: 23°C, $\phi = 100\%$ => $w_4 = 0.0178$, $\tilde{h}_4 = 88$, $h_{f4} = 96.52$ kJ/kg

CV 3 to 4 (flow cooled below $T_{dew\ 3}$ so liquid forms):

$$\dot{m}_{liq\ 4} = \dot{m}_a (w_3 - w_4) = 3 (0.0232 - 0.0178) = \textbf{0.0162 kg/min}$$

CV Heat exchanger with no external heat transfer:

$$m_a(\tilde{h}_2 - \tilde{h}_1) = m_a(\tilde{h}_3 - \tilde{h}_4) - m_{liq}h_{f4}$$

$$\tilde{h}_2 = \tilde{h}_1 + \tilde{h}_3 - \tilde{h}_4 - (w_3-w_4) h_{f4} = 29.2 + 119.2 - 88 - 0.0054\times96.52$$

$$= 59.9 \text{ kJ/kg dry air}$$

State 2: $w_2 = w_1$, $\tilde{h}_2$ => $T_2 = \textbf{32.5°C}$, $\phi = \textbf{12\%}$

Exergy in mixtures

11.123

Consider several flow processes with ideal gasses: 1) A compression of a gas mixture from 100 kPa to 200 kPa; 2) Cooling a gas mixture from 50°C to ambient 20°C using ambient air; 3) Mixing two different gasses at 100 kPa; 4) Throttle a gas mixture from 125 kPa to 100 kPa. For each case explain: What happens to the exergy, is there any exergy destruction and is the composition needed?

1. Exergy increases, mixture specific heat and gas constant needed.
2. Exergy decreases of mixture and some exergy is destroyed due to heat transfer over ΔT. Specific heat for mixture needed.
3. Composition needed, exergy is destroyed.
4. Exergy is destroyed, mixture gas constant needed

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11.124

Find the second law efficiency of the heat exchanger in Problem 11.54.
A flow of 2 kg/s mixture of 50% CO_2 and 50% O_2 by mass is heated in a constant pressure heat exchanger from 400 K to 1000 K by a radiation source at 1400 K. Find the rate of heat transfer and the entropy generation in the process.

Solution:

The second law efficiency follows Eq.11.32 where the wanted term is the flow increase of exergy and the source is the radiation.

$$\dot{\Phi}_{flow} = \dot{m}(\psi_{ex} - \psi_{in}); \qquad \dot{\Phi}_{source} = \dot{Q}_{in} (1 - \frac{T_o}{T_{source}})$$

Heat exchanger Energy Eq.4.12: $\dot{Q}_{in} = \dot{m}(h_e - h_i)$

Values from Table A.8 due to the high T.

$$\dot{Q}_{in} = 2 [\frac{1}{2} \times (971.67 - 303.76) + \frac{1}{2} \times (980.95 - 366.03)] = 1282.8 \text{ kW}$$

$$\dot{\Phi}_{source} = \dot{Q}_{in} (1 - \frac{T_o}{T_{source}}) = 1282.8 (1 - \frac{298.15}{1400}) = 1009.6 \text{ kW}$$

Entropy Eq.7.8: $\dot{m}_e s_e = \dot{m}_i s_i + \dot{Q}/T_s + \dot{S}_{gen}$

As P = C, the pressure correction in Eq.6.28 drops out to give generation as

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) - \dot{Q}/T_s$$
$$= 2 [0.5 \times(6.119 - 5.1196) + 0.5 \times(7.6121 - 6.6838)] - 1282.8/1400$$
$$= 1.01 \text{ kW/K}$$

$$\dot{\Phi}_{flow} = \dot{\Phi}_{source} - \dot{\Phi}_{destruction} = \dot{\Phi}_{source} - T \dot{S}_{gen}$$
$$= 1009.6 - 298.15 \times 1.01 = 708.5$$

$$\eta = \frac{\dot{\Phi}_{flow}}{\dot{\Phi}_{source}} = \frac{708.5}{1009.6} = \mathbf{0.70}$$

Remark: We could also explicitly have found the flow exergy increase.

1400 K
Radiation

i   e

11.125

Consider the mixing of a steam flow with an oxygen flow in Problem 11.55. Find the rate of total inflowing exergy and the rate of exergy destruction in the process.

A flow of 1.8 kg/s steam at 400 kPa, 400°C is mixed with 3.2 kg/s oxygen at 400 kPa, 400 K in a steady flow mixing-chamber without any heat transfer. Find the exit temperature and the rate of entropy generation.

Exergy Flow: $\dot{\Phi}_{in} = \dot{m}\,\psi_{in} = \dot{m}_{H2O}\,\psi_1 + \dot{m}_{O2}\,\psi_2$

$$\psi_1 = h_1 - h_o - T_o(s_1 - s_o)$$
$$= C_{P\,H2O}(T_1 - T_o) - T_o\,[C_{P\,H2O}\,\ln(T_1/T_o) - R\,\ln(P_1/P_o)\,]$$
$$= 1.872\,(400 - 25) - 298.15[\,1.872\,\ln\frac{673.15}{298.15} - 0.4615\,\ln\frac{400}{100}\,]$$
$$= 702 - 298.15\,(1.5245 - 0.63978) = 438.2 \text{ kJ/kg}$$
$$\psi_2 = h_2 - h_o - T_o(s_2 - s_o)$$
$$= C_{P\,O2}(T_2 - T_o) - T_o\,[C_{P\,O2}\,\ln(T_2/T_o) - R\,\ln(P_2/P_o)\,]$$
$$= 0.922(126.85 - 25) - 298.15[0.922\,\ln\frac{400}{298.15} - 0.2598\,\ln\frac{400}{100}\,]$$
$$= 93.906 - 298.15\,(0.27095 - 0.36016) = 120.5 \text{ kJ/kg}$$

$$\dot{\Phi}_{in} = \dot{m}_{H2O}\,\psi_1 + \dot{m}_{O2}\,\psi_2 = 1.8\;438.2 + 3.2\;120.5 = \textbf{1174.4 kW}$$

C.V. Mixing chamber, steady flow, no work, no heat transfer. To do the entropies we need the mole fractions.

$$\dot{n}_{H2O} = \frac{\dot{m}_{H2O}}{M_{H2O}} = \frac{1.8}{18.015} = 0.1 \text{ kmol/s}; \quad \dot{n}_{O2} = \frac{\dot{m}_{O2}}{M_{O2}} = \frac{3.2}{31.999} = 0.1 \text{ kmol/s}$$

$$y_{H2O} = y_{O2} = 0.5$$

Energy Eq.: $\dot{m}_{H2O}\,h_1 + \dot{m}_{O2}\,h_2 = \dot{m}_{H2O}\,h_{3\,H2O} + \dot{m}_{O2}\,h_{3\,O2}$

Entropy Eq.: $\dot{m}_{H2O}\,s_1 + \dot{m}_{O2}\,s_2 + \dot{S}_{gen} = \dot{m}_{H2O}\,s_{3\,H2O} + \dot{m}_{O2}\,s_{3\,O2}$

Solve for T from the energy equation

$$\dot{m}_{H2O}\,(h_{3\,H2O} - h_1) + \dot{m}_{O2}\,(h_{3\,O2} - h_2) = 0$$

$$\dot{m}_{H2O}\,C_{P\,H2O}(T_3 - T_1) + \dot{m}_{O2}\,C_{P\,O2}(T_3 - T_2) = 0$$
$$1.8 \times 1.872\,(T_3 - 400 - 273.15) + 3.2 \times 0.922(T_3 - 400) = 0$$

$$T_3 = \textbf{545.6 K}$$

$$\dot{S}_{gen} = \dot{m}_{H2O} (s_{3\ H2O} - s_1) + \dot{m}_{O2} (s_{3\ O2} - s_2)$$

$$= \dot{m}_{H2O} \left[C_{P\ H2O} \ln \frac{T_3}{T_1} - R \ln y_{H2O} \right] + \dot{m}_{O2} \left[C_{P\ O2} \ln \frac{T_3}{T_2} - R \ln y_{O2} \right]$$

$$= 1.8 \left[1.872 \ln \frac{545.6}{673.15} - 0.4615 \ln 0.5 \right]$$

$$+ 3.2 \left[0.922 \ln \frac{545.6}{400} - 0.2598 \ln 0.5 \right]$$

$$= -0.132 + 1.492 = \textbf{1.36 kW/K}$$

The exergy destruction is proportional to the entropy generation

$$\dot{\Phi}_{in} = T_o \dot{S}_{gen} = 298.15 \times 1.36 = \textbf{405.5 kW}$$

Borgnakke and Sonntag

11.126

A mixture of 75% carbon dioxide and 25% water by mole is flowing at 1600 K, 100 kPa into a heat exchanger where it is used to deliver energy to a heat engine. The mixture leave the heat exchanger at 500 K with a mass flow rate of 2 kg/min. Find the rate of energy and the rate of exergy delivered to the heat engine.

C.V. Heat exchanger, steady flow and no work.

From Table A.8:

$$CO2: \quad h_{in} = 1748.12 \text{ kJ/kg}, \quad s^o_{T \, in} = 6.7254 \text{ kJ/kg K}$$

$$CO2: \quad h_{ex} = 401.52 \text{ kJ/kg}, \quad s^o_{T \, ex} = 5.3375 \text{ kJ/kg K}$$

$$H2O: \quad h_{in} = 3487.69 \text{ kJ/kg}, \quad s^o_{T \, in} = 14.0822 \text{ kJ/kg K}$$

$$H2O: \quad h_{ex} = 935.12 \text{ kJ/kg}, \quad s^o_{T \, ex} = 11.4644 \text{ kJ/kg K}$$

Energy Eq.: $\quad \dot{Q} = \dot{m} \, (h_{in} - h_{ex}) = \dot{m} \sum c_i \, (h_{in} - h_{ex})_i$

$$= \frac{2}{60} \, [0.75 \, (1748.12 - 401.52) + 0.25(3487.69 - 935.12)]$$

$$= \frac{1}{30} \, [\, 1009.95 + 638.14 \,] = \textbf{54.94 kW}$$

Entropy change (P does not change so partial pressures are constant):

$$s_{in} - s_{ex} = 0.75(6.7254 - 5.3375) + 0.25(14.0822 - 11.9644)$$
$$= 1.6954 \text{ kJ/kg K}$$

Exergy Flux:

$$\dot{\Phi} = \dot{m} \, (\psi_{in} - \psi_{ex}) = \dot{Q} - T_o \, \dot{m} \, (s_{in} - s_{ex})$$

$$= 54.94 - 298.15 \times \frac{1}{30} \times 1.6954$$

$$= \textbf{38.09 kW}$$

11.127

For flows with moist air where the water content is changed either by evaporation or by condensation what happens to the exergy? Is the water vapor in air flowing over a lake in equilibrium with the liquid water?

If the liquid water and the water vapor in the air were in thermodynamic equilibrium then there would be no tendency to move from one phase to the other. The water vapor is at its partial pressure and the air mixture temperature whereas the liquid water is at 1 atm pressure and at the liquid water temperature. In such situations equilibrium is attained when the partial Gibbs function ($g = h - Ts$) of the water is the same in the two phases, see Chapter 14. Notice if $T = T_o$ the Gibbs function becomes the same expression as for the flow exergy. The water will transport from the higher Gibbs function (exergy) towards the lower Gibbs function (exergy) if there is a difference between the two.

The difference then determines if evaporation or condensation occurs. If the temperature of the liquid and vapor is the same then the vapor should be at the saturated pressure P_g (plus a very small amount ε) to be in equilibrium with the liquid. When the conditions are close to equilibrium the exergy of the water in the two phases are nearly equal and there is then no difference in the exergy.

Review problems

Borgnakke and Sonntag

11.128

A semi-permeable membrane is used for the partial removal of oxygen from air that is blown through a grain elevator storage facility. Ambient air (79% nitrogen, 21% oxygen on a mole basis) is compressed to an appropriate pressure, cooled to ambient temperature 25°C, and then fed through a bundle of hollow polymer fibers that selectively absorb oxygen, so the mixture leaving at 120 kPa, 25°C, contains only 5% oxygen. The absorbed oxygen is bled off through the fiber walls at 40 kPa, 25°C, to a vacuum pump. Assume the process to be reversible and adiabatic and determine the minimum inlet air pressure to the fiber bundle.

State 1 is the air inlet, state 2 is the 5% [= 0.0416/(0.79 + 0.0416)] oxygen exit mix, state 3 pure O_2 exit.

$$0.79\ N_2 + 0.21\ O_2 \xrightarrow{1}\ \boxed{}\ \xrightarrow{2}\ 0.79\ N_2 + 0.0416\ O_2$$
$$\xrightarrow{3}\ 0.1684\ O_2$$

Let reference be at state 1 so $s = 0$ at $T = 25^oC$ & P_1

$$\bar{s}_{MIX\ 1} = 0 + 0 - y_{A1}\bar{R}\ln y_{A1} - y_{B1}\bar{R}\ln y_{B1}$$

$$\bar{s}_{MIX\ 2} = 0 + 0 - \bar{R}\ln (P_2/P_1) - y_{A2}\bar{R}\ln y_{A2} - y_{B2}\bar{R}\ln y_{B2}$$

Pure O_2: $\bar{s}_3 = 0 - \bar{R}\ln (P_3/P_1)$

Entropy equation: $\dot{n}_1\bar{s}_1 = \dot{n}_2\bar{s}_2 + \dot{n}_3\bar{s}_3$

All T's the same, so only partial pressure terms

$$\bar{R}\Big[-0.8316\ln (P_2/P_1) - 0.79\ln 0.95 - 0.0416\ln 0.05$$

$$- 0.1684\ln (P_3/P_1) + 0.79\ln 0.79 + 0.21\ln 0.21\Big] = 0$$

$$0.8316\ln (P_2/P_1) + 0.1684\ln (P_3/P_1) = -0.3488 + 4.6025 - \ln P_1 = -0.3488$$

$P_{1\ min} = \mathbf{141\ kPa}$

For $P_1 > P_{1\ min}$ we would have entropy generation $\Delta\dot{S} > 0$

11.129

Weighing of masses gives a mixture at 60°C, 225 kPa with 0.5 kg O_2, 1.5 kg N_2 and 0.5 kg CH_4. Find the partial pressures of each component, the mixture specific volume (mass basis), mixture molecular weight and the total volume.

Solution:

From Eq.11.4: $y_i = (m_i /M_i) / \sum m_j/M_j$

$n_{tot} = \sum m_j/M_j = (0.5/31.999) + (1.5/28.013) + (0.5/16.04)$

$\quad = 0.015625 + 0.053546 + 0.031172 = 0.100343$

$y_{O2} = 0.015625/0.100343 = 0.1557,$

$y_{N2} = 0.053546/0.100343 = 0.5336,$

$y_{CH4} = 0.031172/0.100343 = 0.3107$

From Eq.11.10:

$P_{O2} = y_{O2} P_{tot} = 0.1557 \times 225 = 35$ kPa,

$P_{N2} = y_{N2} P_{tot} = 0.5336 \times 225 = 120$ kPa

$P_{CH4} = y_{CH4} P_{tot} = 0.3107 \times 225 = 70$ kPa

$V_{tot} = n_{tot} \bar{R}T/P = 0.100343 \times 8.31451 \times 333.15 / 225 =$ **1.235 m^3**

$v = V_{tot}/m_{tot} = 1.235 / (0.5 + 1.5 + 0.5) =$ **0.494 m^3/kg**

From Eq.11.5:

$M_{mix} = \sum y_jM_j = m_{tot}/n_{tot} = 2.5/0.100343 =$ **24.914**

11.130

A carbureted internal combustion engine is converted to run on methane gas (natural gas). The air-fuel ratio in the cylinder is to be 20 to 1 on a mass basis. How many moles of oxygen per mole of methane are there in the cylinder?

Solution:

The mass ratio $m_{AIR}/m_{CH_4} = 20$, so relate mass and mole $\quad n = m/M$

$$\frac{n_{AIR}}{n_{CH_4}} = \left(\frac{m_{AIR}}{m_{CH_4}}\right) \times M_{CH_4}/M_{AIR} = 20 \times 16.04/28.97 = 11.0735$$

$$\rightarrow \frac{n_{O_2}}{n_{CH_4}} = \frac{n_{O_2}}{n_{AIR}} \times \frac{n_{AIR}}{n_{CH_4}} = 0.21 \times 11.0735 = \textbf{2.325 mole O}_\textbf{2}\textbf{/mole CH}_\textbf{4}$$

11.131

A mixture of 50% carbon dioxide and 50% water by mass is brought from 1500 K, 1 MPa to 500 K, 200 kPa in a polytropic process through a steady state device. Find the necessary heat transfer and work involved using values from Table A.5.

Solution:

Process $Pv^n = $ constant   leading to

$$n \ln(v_2/v_1) = \ln(P_1/P_2); \quad v = RT/P$$

$$n = \ln\left(\frac{1000}{200}\right) / \ln\left(\frac{500 \times 1000}{200 \times 1500}\right) = 3.1507$$

Eq.11.15: $R_{mix} = \Sigma\, c_i R_i = 0.5 \times 0.1889 + 0.5 \times 0.4615 = 0.3252$ kJ/kg K

Eq.11.23: $C_{P\,mix} = \Sigma\, c_i C_{Pi} = 0.5 \times 0.8418 + 0.5 \times 1.872 = 1.3569$ kJ/kg K

Work is from Eq.7.18:

$$w = -\int v dP = -\frac{n}{n-1}(P_e v_e - P_i v_i) = -\frac{nR}{n-1}(T_e - T_i) = \textbf{476.4 kJ/kg}$$

Heat transfer from the energy equation

$$q = h_e - h_i + w = C_P(T_e - T_i) + w = \textbf{-880.5 kJ/kg}$$

11.132

The accuracy of calculations can be improved by using a better estimate for the specific heat. Reconsider the previous problem and use $C_P = \Delta h/\Delta T$, from Table A.8 centered at 1000 K.

A mixture of 50% carbon dioxide and 50% water by mass is brought from 1500 K, 1 MPa to 500 K, 200 kPa in a polytropic process through a steady state device. Find the necessary heat transfer and work involved using values from Table A.5.

Solution:

Using values from Table A.8 we estimate the heat capacities

$$C_{P\,CO_2} = \frac{1096.36 - 849.72}{1100 - 900} = 1.2332 \text{ kJ/kg K}$$

$$C_{P\,H_2O} = \frac{2226.73 - 1768.6}{1100 - 900} = 2.2906 \text{ kJ/kg K}$$

Eq.11.23: $C_{P\,mix} = \Sigma\ c_i C_{Pi} = 0.5 \times 1.2332 + 0.5 \times 2.2906 = 1.7619 \text{ kJ/kg K}$

Eq.11.15: $R_{mix} = \Sigma\ c_i R_i = 0.5 \times 0.1889 + 0.5 \times 0.4615 = 0.3252 \text{ kJ/kg K}$

Process $Pv^n = C$ => $n = \ln(P_1/P_2) / \ln(v_2/v_1)$ and use $Pv = RT$

$$n = \ln\left(\frac{1000}{200}\right) / \ln\left(\frac{500 \times 1000}{200 \times 1500}\right) = 3.1507$$

Work is from Eq.7.18

$$w = -\int v dP = -\frac{n}{n-1}\ (P_e v_e - P_i v_i) = -\frac{nR}{n-1}\ (T_e - T_i) = \textbf{476.4 kJ/kg}$$

Heat transfer from energy equation

$$q = h_e - h_i + w = 1.7619(500 - 1500) + 476.4 = \textbf{-1285.5 kJ/kg}$$

11.133

A large air separation plant takes in ambient air (79% N_2, 21% O_2 by mole) at 100 kPa, 20°C, at a rate of 25 kg/s. It discharges a stream of pure O_2 gas at 200 kPa, 100°C, and a stream of pure N_2 gas at 100 kPa, 20°C. The plant operates on an electrical power input of 2000 kW. Calculate the net rate of entropy change for the process.

Air 79 % N_2 1   2 $P_2 = 200$ kPa

   21 % O_2 pure O_2 $T_2 = 100$ °C

$P_1 = 100$ kPa   pure N_2

$T_1 = 20$ °C   3 $P_3 = 100$ kPa

                     $T_3 = 20$ °C

$\dot{m}_1 = 25$ kg/s $-\dot{W}_{IN} = 2000$ kW

Solution:

To have the flow terms on a mass basis let us find the mass fractions

From Eq. 11.3: $c_i = y_i M_i / \sum y_j M_j$

$c_{O_2} = 0.21 \times 32 / [0.21 \times 32 + 0.79 \times 28.013] = 0.23293$;

$c_{N_2} = 1 - c_{O_2} = 0.76707$

$\dot{m}_2 = c_{O_2}\dot{m}_1 = 5.823$ kg/s ; $\dot{m}_3 = c_{N_2}\dot{m}_1 = 19.177$ kg/s

The energy equation, Eq.4.10 gives the heat transfer rate as

$$\dot{Q}_{CV} = \Sigma\, \dot{m}\Delta h_i + \dot{W}_{CV} = \dot{m}_{O_2}C_{P0\,O_2}(T_2 - T_1) + \dot{m}_{N_2}C_{P0\,N_2}(T_3 - T_1) + \dot{W}_{CV}$$

$$= 5.823 \times 0.922 \times(100-20) + 0 - 2000 = -1570.5 \text{ kW}$$

The entropy equation, Eq.7.7 gives the generation rates as

$$\dot{S}_{gen} = \Sigma\, \dot{m}_i \Delta s_i - \dot{Q}_{CV}/T_0 = (\dot{m}_2 s_2 + \dot{m}_3 s_3 - \dot{m}_1 s_1) - \dot{Q}_{CV}/T_0$$

Use Eq.6.16 for the entropy change

$$\Sigma\, \dot{m}_i \Delta s_i = 5.823 \left[0.922 \ln \frac{373.2}{293.2} - 0.2598 \ln \frac{200}{21} \right]$$

$$+ 19.177\, [0 - 0.2968 \ln (100/79)] = -3.456 \text{ kW/K}$$

$$\dot{S}_{gen} = 1570.5/293.2 - 3.456 = \textbf{1.90 kW/K}$$

11.134

Take Problem 11.57 with inlet temperature of 1400 K for the carbon dioxide and 300 K for the nitrogen. First estimate the exit temperature with the specific heats from Table A.5 and use this to start iterations using A.9 to find the exit temperature.

CV mixing chamber, steady flow. The inlet ratio is $\dot{n}_{CO_2} = 2\,\dot{n}_{N_2}$ and assume no external heat transfer, no work involved.

$\bar{C}_{P\,CO_2} = 44.01 \times 0.842 = 37.06$; $\bar{C}_{P\,N_2} = 28.013 \times 1.042 = 29.189$ kJ/kmol K

Continuity Equation: $0 = \Sigma\dot{n}_{in} - \Sigma\dot{n}_{ex}$;

Energy Equation: $0 = \Sigma\dot{n}_{in}\,\bar{h}_{in} - \Sigma\dot{n}_{ex}\,\bar{h}_{ex}$

$$0 = 2\dot{n}_{N_2}\,\bar{C}_{P\,CO_2}(T_{in} - T_{ex})_{CO_2} + \dot{n}_{N_2}\,\bar{C}_{P\,N_2}\,(T_{in} - T_{ex})_{N_2}$$
$$0 = 2 \times 37.06 \times (1400 - T_{ex}) + 29.189 \times (300 - T_{ex})$$
$$0 = 103768 + 8756.7 - 103.309\,T_{ex} \qquad \Rightarrow \quad \textbf{T}_{ex} = \textbf{1089 K}$$

From Table A.9: $\Sigma\dot{n}_{in}\,\bar{h}_{in} = \dot{n}_{N_2}\,[2 \times 55895 + 1 \times 54] = \dot{n}_{N_2} \times 111844$

@ 1000K   : $\Sigma\dot{n}_{ex}\,\bar{h}_{ex} = \dot{n}_{N_2}\,[2 \times 33397 + 21463] = \dot{n}_{N_2} \times 88257$

@ 1100K   : $\Sigma\dot{n}_{ex}\,\bar{h}_{ex} = \dot{n}_{N_2}\,[2 \times 38885 + 24760] = \dot{n}_{N_2} \times 102530$

@ 1200K   : $\Sigma\dot{n}_{ex}\,\bar{h}_{ex} = \dot{n}_{N_2}\,[2 \times 44473 + 28109] = \dot{n}_{N_2} \times 117055$

Now linear interpolation between 1100 K and 1200 K

$$T_{ex} = 1100 + 100 \times \frac{111844 - 102530}{117055 - 102530} = \textbf{1164 K}$$

11.135

A piston/cylinder has 100 kg of saturated moist air at 100 kPa, 5°C. If it is heated to 45°C in an isobaric process, find $_1Q_2$ and the final relative humidity. If it is compressed from the initial state to 200 kPa in an isothermal process, find the mass of water condensing.

Solution:

Energy Eq.: $\qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2,$

Initial state 1: $\phi_1 = 100\%,$ Table B.1.1: $P_{v1} = 0.8721$ kPa, $h_{v1} = 2510.54$

$\qquad$ Eq.11.28 $\qquad w_1 = 0.622 \dfrac{P_{v1}}{P_{tot} - P_{v1}} = \dfrac{0.8721}{100 - 0.8721} = 0.005472$

$\qquad$ Eq.11.26 with $\quad m_a = m_{tot} - m_{v1} = m_{tot} - w_1 m_a \quad$ gives

$\qquad\qquad\qquad m_a = m_{tot}/(1 + w_1) = 99.456$ kg,

$\qquad$ Eq.11.26 $\qquad m_{v1} = w_1 m_a = 0.544$ kg

Case a: $\quad P = $ constant $\quad => \quad {}_1W_2 = mP(v_2 - v_1) \quad =>$

$\qquad {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) = m_a C_p(T_2 - T_1) + m(h_{v2} - h_{v1})$

$\qquad$ State 2: $\quad w_2 = w_1, \quad T_2 \quad => \quad P_{v2} = P_{v1} \quad$ and

$\qquad\qquad$ Table B.1.1: $\quad h_{v2} = 2583.19$ kJ/kg, $\quad P_{g2} = 9.593$ kPa

$\qquad$ Eq.11.25 $\qquad \phi_2 = \dfrac{P_{v2}}{P_{g2}} = \dfrac{0.8721}{9.593} = 0.091 \quad$ or $\quad \phi_2 = \mathbf{9.1\%}$

$\qquad$ From the energy equation

$\qquad {}_1Q_2 = 99.456 \times 1.004(45 - 5) + 0.544(2583.19 - 2510.54) = \mathbf{4034\ kJ}$

Case b: $\quad$ As P is raised $P_v = y_v P$ would be higher than $P_g \quad =>$ condensation.

$\qquad\qquad T = $ constant $\&\ \phi_2 = 100\% \quad => \quad P_v = P_g = 0.8721$ kPa

$\qquad w_2 = 0.622 \dfrac{P_{v2}}{P_{a2}} = 0.622 \dfrac{P_{v2}}{P_{tot2} - P_{v2}} = \dfrac{0.8721}{200 - 0.8721} = 0.002724$

$\qquad m_{v2} = w_2 m_a = 0.271$ kg, $\qquad m_{liq} = m_{v1} - m_{v2} = \mathbf{0.273\ kg}$

11.136

A piston/cylinder contains helium at 110 kPa at ambient temperature 20°C, and initial volume of 20 L as shown in Fig. P11.136. The stops are mounted to give a maximum volume of 25 L and the nitrogen line conditions are 300 kPa, 30°C. The valve is now opened which allows nitrogen to flow in and mix with the helium. The valve is closed when the pressure inside reaches 200 kPa, at which point the temperature inside is 40°C. Is this process consistent with the second law of thermodynamics?

$P_1 = 110$ kPa, $T_1 = 20\ ^oC$, $V_1 = 20$ L, $V_{max} = 25$ L $= V_2$

$P_2 = 200$ kPa, $T_2 = 40\ ^oC$, $P_i = 300$ kPa, $T_i = 30\ ^oC$

Constant P to stops, then constant $V = V_{max}$ $=>$ $W_{cv} = P_1(V_2 - V_1)$

$Q_{cv} = U_2 - U_1 + W_{cv} - n_i\bar{h}_i$,

$\quad = n_2\bar{h}_2 - n_1\bar{h}_1 - n_i\bar{h}_i - (P_2 - P_1)V_2$

$\quad = n_A(\bar{h}_{A2} - \bar{h}_{Ai}) + n_B(\bar{h}_{B2} - \bar{h}_{B1}) - (P_2 - P_1)V_2$

$n_B = n_1 = P_1V_1/\bar{R}T_1 = 110\times0.02/8.3145\times293.2 = 0.0009$ kmol

$n_2 = n_A + n_B = P_2V_2/\bar{R}T_2 = 200\times0.025/8.3145\times313.2 = 0.00192$ kmol,

$n_A = n_2 - n_B = 0.00102$ kmol

Mole fractions: $y_{A2} = 0.00102/0.00192 = 0.5313$, $y_{B2} = 0.4687$

$Q_{cv} = 0.00102\times28.013\times1.042(40 - 30) + 0.0009\times4.003\times5.193(40 - 20)$

$\quad\quad - (200 - 110)\ 0.025$

$\quad = 0.298 + 0.374 - 2.25 = -1.578$ kJ

$S_{gen} = n_2\bar{s}_2 - n_1\bar{s}_1 - n_i\bar{s}_i - Q_{cv}/T_0$

$\quad\quad = n_A(\bar{s}_{A2} - \bar{s}_{Ai}) + n_B(\bar{s}_{B2} - \bar{s}_{B1}) - Q_{cv}/T_0$

$\bar{s}_{A2} - \bar{s}_{Ai} = 29.189 \ln\dfrac{313.2}{303.2} - \bar{R} \ln\dfrac{0.5313*200}{300} = 9.5763$

$\bar{s}_{B2} - \bar{s}_{B1} = 20.7876 \ln\dfrac{313.2}{293.2} - \bar{R} \ln\dfrac{0.4687*200}{110} = 2.7015$

$S_{gen} = 0.00102\times9.5763 + 0.0009\times2.7015 + 1.578/293.2$

$\quad = \mathbf{0.0176\ kJ/K\ > 0}$ **Satisfies 2nd law.**

Borgnakke and Sonntag

11.137

A spherical balloon has an initial diameter of 1 m and contains argon gas at 200 kPa, 40°C. The balloon is connected by a valve to a 500-L rigid tank containing carbon dioxide at 100 kPa, 100°C. The valve is opened, and eventually the balloon and tank reach a uniform state in which the pressure is 185 kPa. The balloon pressure is directly proportional to its diameter. Take the balloon and tank as a control volume, and calculate the final temperature and the heat transfer for the process.

$$V_{A1} = \frac{\pi}{6}1^3 = 0.5236,$$

$$m_{A1} = \frac{P_{A1}V_{A1}}{RT_{A1}} = \frac{200 \times 0.5236}{0.208\ 13 \times 313.2} = 1.606 \text{ kg}$$

$$m_{B1} = P_{B1}V_{B1}/RT_{B1} = 100 \times 0.50/0.18892 \times 373.2 = 0.709 \text{ kg}$$

Process: $P = C\ D = CV^{1/3} \Rightarrow$ polytropic $n = -1/3 \Rightarrow$

$$V_{A2} = V_{A1}\left(\frac{P_2}{P_{A1}}\right)^3 = 0.5236\left(\frac{185}{200}\right)^3 = 0.4144 \text{ m}^3$$

2: Uniform ideal gas mixture :
$$P_2(V_{A2} + V_B) = (m_A R_A + m_B R_B)T_2$$

$$T_2 = P_2(V_{A2} + V_B) / (m_A R_A + m_B R_B)$$

$$= 185\ (0.4144 + 0.50) / (1.606 \times 0.20813 + 0.709 \times 0.18892) = \textbf{361.3 K}$$

Boundary work in a polytropic process from Eq.6.29

$$W_{12} = \frac{P_2 V_{A2} - P_{A1}V_{A1}}{1 - (-1/3)} = \frac{185 \times 0.4144 - 200 \times 0.5236}{(4/3)} \text{ kPa m}^3$$

$$= -21.0 \text{ kJ}$$

The heat transfer from the energy equation

$$Q = m_A C_{V0A}(T_2 - T_{A1}) + m_B C_{V0B}(T_2 - T_{B1}) + W_{12}$$

$$= 1.606 \times 0.312(361.3 - 313.2) + 0.709 \times 0.653(361.3 - 373.2) - 21.0$$

$$= 18.6 - 21.0 = \textbf{-2.4 kJ}$$

11.138

An insulated rigid 2 m^3 tank A contains CO_2 gas at 200°C, 1MPa. An uninsulated rigid 1 m^3 tank B contains ethane, C_2H_6, gas at 200 kPa, room temperature 20°C. The two are connected by a one-way check valve that will allow gas from A to B, but not from B to A. The valve is opened and gas flows from A to B until the pressure in B reaches 500 kPa and the valve is closed. The mixture in B is kept at room temperature due to heat transfer. Find the total number of moles and the ethane mole fraction at the final state in B. Find the final temperature and pressure in tank A and the heat transfer to/from tank B.

Solution:

Tank A: $V_A = 2$ m^3, state A_1 : CO_2, $T_{A1} = 200°C = 473.2$ K, $P_{A1} = 1$ MPa

$\bar{C}_{v0\ CO_2} = 0.653 \times 44.01 = 28.74$, $\bar{C}_{P0\ CO_2} = 0.842 \times 44.01 = 37.06$ kJ/kmol K

Tank B: $V_B = 1$ m^3, state B_1: C_2H_6, $T_{B1} = 20°C = 293.2$ K, $P_{B1} = 200$ kPa

Slow Flow A to B to $P_{B2} = 500$ kPa and assume $T_{B2} = T_{B1} = T_0$

Total moles scales to pressure, so with same V and T we have

$$P_{B1}V_B = n_{B1}\bar{R}T_{B1}, \qquad P_{B2}V_B = n_{B2\ mix}\bar{R}T_{B2}$$

Mole fraction: $\qquad y_{C_2H_6\ B_2} = \dfrac{n_{B1}}{n_{B2}} = \dfrac{P_{B1}}{P_{B2}} = \dfrac{200}{500} = \mathbf{0.400}$

$$n_{B1} = \frac{P_{B1}V_B}{RT_{B1}} = \frac{200 \times 1}{R \times 293.2} = 0.08204 \text{ kmol}$$

$$n_{B2\ mix} = \frac{P_{B2}V_B}{RT_{B2}} = \frac{500 \times 1}{R \times 293.2} = 0.2051 \text{ kmol}$$

$$n_{CO_2\ B2} = 0.2051 - 0.08201 = 0.12306 \text{ kmol}$$

Now we can work backwards to final state in A

$$n_{A1} = \frac{P_{A1}V_A}{RT_{A1}} = \frac{1000 \times 2}{R \times 473.2} = 0.50833 \text{ kmol}; \ n_{A2} = n_{A1} - n_{CO_2\ B2} = 0.38527 \text{ kmol}$$

C.V. A: All CO_2 Transient with flow out and adiabatic.

Energy Eq.: $Q_{CV\,A} = 0 = n_{A2}\, \bar{u}_{A2} - n_{A1}\, \bar{u}_{A1} + n_{ave}\, \bar{h}_{ave}$

$0 = n_{A2}\, \bar{C}_{v0\,CO_2}T_{A2} - n_{A1}\, \bar{C}_{v0\,CO_2}T_{A1} + n_{ave}\, \bar{C}_{P0\,CO_2}\,(\,T_{A1} + T_{A2})/2$

$0 = 28.74(\,0.38527\times T_{A2} - 0.50833\times473.2) + 0.12306 \times 37.06(473.2 + T_{A2})/2$

$\Rightarrow \quad T_{A2} = \textbf{436.9 K}$

$$P_{A2} = \frac{n_{A2}\,RT_{A2}}{V_A} = \frac{0.38527 \times 8.3145 \times 436.9}{2} = \textbf{700 kPa}$$

C.V. B: Transient with flow in and non-adiabatic.

$Q_{CV\,B} + n_{Bi}\, \bar{h}_{Bi\,ave} = (n\bar{u})_{B2} - (n\bar{u})_{B1} = (n\bar{u})_{CO_2\,B2} + (n\bar{u})_{C_2H_6\,B2} - (n\bar{u})_{C_2H_6\,B1}$

$Q_{CV\,B} = 0.12306 \times 28.74 \times 293.2 + 0\ - 0.12306 \times 37.06\,(473.2 + 436.9)/2$

$= \textbf{-1038 kJ}$

11.139

You have just washed your hair and now blow dry it in a room with 23°C, $\phi =$ 60%, (1). The dryer, 500 W, heats the air to 49°C, (2), blows it through your hair where the air becomes saturated (3), and then flows on to hit a window where it cools to 15°C (4). Find the relative humidity at state 2, the heat transfer per kilogram of dry air in the dryer, the air flow rate, and the amount of water condensed on the window, if any.

The blowdryer heats the air at constant specific humidity to 2 and it then goes through an adiabatic saturation process to state 3, finally cooling to 4.

$\quad$ 1: 23°C, 60% rel hum $\Rightarrow$ $w_1 = 0.0104$, $\tilde{h}_1 = 69$ kJ/kg dry air

$\quad$ 2: $w_2 = w_1$, T_2 $\Rightarrow$ $\phi_2 = 15\%$, $\tilde{h}_2 = 95$ kJ/kg dry air

CV. 1 to 2: $\quad w_2 = w_1$; $\quad q = \tilde{h}_2 - \tilde{h}_1 = 95 - 69 =$ **26 kJ/kg dry air**

$$\dot{m}_a = Q/q = 0.5/26 = \textbf{0.01923 kg/s}$$

CV. 2 to 3: $\quad w_3 - w_2 = \dot{m}_{liq}/\dot{m}_a$; $\quad \dot{m}_a \tilde{h}_2 + \dot{m}_{liq} h_f = \dot{m}_a \tilde{h}_3$

$\quad$ 3: $\phi = 100\%$ $\Rightarrow$ $T_3 = \textbf{T}_{\textbf{wet,2}} = \textbf{24.8°C}$, $w_3 = \textbf{0.0198}$

$\quad$ 4: $\phi = 100\%$, $T_4 \Rightarrow w_4 = 0.01065$

$$\dot{m}_{liq} = (w_3 - w_4)\,\dot{m}_a = (0.0198 - 0.01065) \times 0.01923 = \textbf{0.176 g/s}$$

If the steam **tables and formula's** are used then we get

$\quad$ $h_{g1} = 2543.5$, $\quad h_{g2} = 2590.3$, $\quad P_{g1} = 2.837$ kPa, $\quad P_{v1} = 1.7022$ kPa,

$\quad$ $P_{g2} = 11.8$ kPa, $\quad w_1 = 0.01077$, $\quad w_2 = w_1$, $\quad P_{v2} = P_{v1}$

$\quad$ $\phi_2 = P_{v2}/P_{g2} = 14.4\%$, $\quad h_{f3} = 114$ kJ/kg,

Trial and error for adiabatic saturation temperature.

$\quad$ $T_3 = 25°C$, $w_3 = 0.02$, $P_{v4} = P_{g4} = 1.705$ kPa,

$\quad$ $w_4 = 0.622 \times 1.705/(100 - 1.705) = 0.0108$

11.140

A 0.2 m^3 insulated, rigid vessel is divided into two equal parts A and B by an insulated partition, as shown in Fig. P11.140. The partition will support a pressure difference of 400 kPa before breaking. Side A contains methane and side B contains carbon dioxide. Both sides are initially at 1 MPa, 30°C. A valve on side B is opened, and carbon dioxide flows out. The carbon dioxide that remains in B is assumed to undergo a reversible adiabatic expansion while there is flow out. Eventually the partition breaks, and the valve is closed. Calculate the net entropy change for the process that begins when the valve is closed.

$\Delta P_{MAX} = 400$ kPa, $\quad P_{A1} = P_{B1} = 1$ MPa

$V_{A1} = V_{B1} = 0.1$ m^3

$T_{A1} = T_{B1} = 30^{o}C = 303.2$ K

CO_2 inside B: $s_{B2} = s_{B1}$ to $P_{B2} = 600$ kPa ($P_{A2} = 1000$ kPa)

For CO_2, k = 1.289 $\Rightarrow$ $T_{B2} = 303.2\left(\dfrac{600}{1000}\right)^{\frac{0.289}{1.289}} = 270.4$ K

$n_{B2} = P_{B2}V_{B2}/\bar{R}T_{B2} = 600\times0.1/8.3145\times270.4 = 0.026\,688$

$n_{A2} = n_{A1} = 1000\times0.1/8.3145\times303.2 = 0.039\,668$ kmol

The process 2 to 3 is adiabatic but irreversible with no work.

$Q_{23} = 0 = n_3\bar{u}_3 - \sum_i n_{i2}\bar{u}_{i2} + 0 = n_{A2}\bar{C}_{vo\,A}(T_3\text{-}T_{A2}) + n_{B2}\bar{C}_{vo\,B}(T_3\text{-}T_{B2}) = 0$

$0.039\,668\times16.04\times1.736(T_3\text{-}303.2) + 0.026\,688\times44.01\times0.653(T_3\text{-}270.4) = 0$

Solve $\quad T_3 = 289.8$ K

Get total and partial pressures for the entropy change

$P_3 = n\bar{R}T/V = 0.066356\times8.3145\times289.8/0.2 = 799.4$ kPa

$P_{A3} = 0.5978\times799.4 = 477.9$ kPa , $\quad P_{B3} = P_3 - P_{A3} = 321.5$ kPa

$\bar{s}_{A3} - \bar{s}_{A2} = 16.04\times2.254\ln(\dfrac{289.8}{303.2}) - 8.3145\ln\dfrac{477.9}{1000} = 4.505$ kJ/kmol K

$\bar{s}_{B3} - \bar{s}_{B2} = 44.01\times0.842\ln(\dfrac{289.8}{270.4}) - 8.3145\ln\dfrac{321.5}{600} = 7.7546$ kJ/kmol K

$\Delta S_{NET} = n_{A2}(\bar{s}_{A3} - \bar{s}_{A2}) + n_{B2}(\bar{s}_{B3} - \bar{s}_{B2})$

$= 0.039668 \times 4.505 + 0.026688 \times 7.7546 = $ **+0.3857 kJ/K**

11.141

Ambient air is at a condition of 100 kPa, 35°C, 50% relative humidity. A steady stream of air at 100 kPa, 23°C, 70% relative humidity, is to be produced by first cooling one stream to an appropriate temperature to condense out the proper amount of water and then mix this stream adiabatically with the second one at ambient conditions. What is the ratio of the two flow rates? To what temperature must the first stream be cooled?

$P_1 = P_2 = 100$ kPa
$T_1 = T_2 = 35\ ^oC$
$\phi_1 = \phi_2 = 0.50,\ \phi_4 = 1.0$
$P_5 = 100,\ T_5 = 23\ ^oC$
$\phi_5 = 0.70$

$$P_{v1} = P_{v2} = 0.5 \times 5.628 = 2.814\ kPa \quad => \quad w_1 = w_2 = 0.622 \times \frac{2.814}{100\text{-}2.814} = 0.0180$$

$$P_{v5} = 0.7 \times 2.837 = 1.9859\ kPa \quad => \quad w_5 = 0.622 \times \frac{1.9859}{100\text{-}1.9859} = 0.0126$$

C.V.: Mixing chamber: Call the mass flow ratio $r = m_{a2}/m_{a1}$

cons. mass: $w_1 + r\,w_4 = (1+r)w_5$

Energy Eq.: $h_{a1} + w_1 h_{v1} + r h_{a4} + r w_4 h_{v4} = (1+r)h_{a5} + (1+r)w_5 h_{v5}$

$\rightarrow 0.018 + r w_4 = (1+r)\ 0.0126$

or $r = \dfrac{m_{a2}}{m_{a1}} = \dfrac{0.018\text{-}0.0126}{0.0126\text{-}w_4}$,   with $w_4 = 0.622 \times \dfrac{P_{G4}}{100\text{-}P_{G4}}$

$1.004 \times 308.2 + 0.018 \times 2565.3 + r \times 1.004 \times T_4 + r\,w_4 h_{v4}$

$\qquad = (1+r) \times 1.004 \times 296.2 + (1+r) \times 0.0126 \times 2543.6$

or $r\left[1.004 \times T_4 + w_4 h_{G4} - 329.3\right] + 26.2 = 0$

Assume $T_4 = \textbf{5}\ ^o\textbf{C} \quad \rightarrow P_{G4} = 0.8721,\ h_{G4} = 2510.5$

$w_4 = 0.622 \times 0.8721/(100\text{-}0.8721) = 0.0055$

$$r = m_{a2}/m_{a1} = \frac{0.018\text{-}0.0126}{0.0126\text{-}0.0055} = \textbf{0.7606}$$

$0.7606[1.004 \times 278.2 + 0.0055 \times 2510.5 - 329.6] + 26.2 = -1.42 \approx 0 \quad$ OK

11.142

An air-water vapor mixture enters a steady flow heater humidifier unit at state 1: 10°C, 10% relative humidity, at the rate of 1 m^3/s. A second air-vapor stream enters the unit at state 2: 20°C, 20% relative humidity, at the rate of 2 m^3/s. Liquid water enters at state 3: 10°C, at the rate of 400 kg per hour. A single air-vapor flow exits the unit at state 4: 40°C, as shown in Fig. P11.142. Calculate the relative humidity of the exit flow and the rate of heat transfer to the unit.

State 1: $T_1 = 10°C$, $\phi_1 = 10\%$, $\dot{V}_{a1} = 1$ m^3/s

$P_{g1} = 1.2276$ kPa, $P_{v1} = \phi_1 P_{g1} = 0.1228$ kPa, $P_{a1} = P - P_{v1} = 99.877$ kPa

$$\omega_1 = 0.622 \frac{P_{v1}}{P_{a1}} = 0.000765, \quad \dot{m}_{a1} = \frac{P_{a1}\dot{V}_{a1}}{RT_{a1}} = 1.2288 \text{ kg/s}$$

$$\dot{m}_{v1} = \omega_1 \dot{m}_{a1} = 0.00094 \text{ kg/s}, \quad h_{v1} = h_{g1} = 2519.7 \text{ kJ/kg}$$

State 2: $T_2 = 20°C$, $\phi_2 = 20\%$, $\dot{V}_{a2} = 2$ m^3/s

$P_{g2} = 2.3385$ kPa, $P_{v2} = \phi P_{g2} = 0.4677$ kPa, $P_{a2} = P - P_{v2} = 99.532$ kPa

$$\omega_2 = 0.622 \frac{P_{v2}}{P_{a2}} = 0.002923, \quad \dot{m}_{a2} = \frac{P_{a2}\dot{V}_{a2}}{RT_{a2}} = 2.3656 \text{ kg/s}$$

$$\dot{m}_{v2} = \omega_2 \dot{m}_{a2} = 0.00691 \text{ kg/s}, \quad h_{v2} = h_{g2} = 2538.1 \text{ kJ/kg}$$

State 3: Liquid. $T_3 = 10°C$, $\dot{m}_{f3} = 400$ kg/hr $= 0.1111$ kg/s, $h_{f3} = 42$ kJ/kg

Continuity Eq. air: $\dot{m}_{a4} = \dot{m}_{a2} + \dot{m}_{a1} = 3.5944$ kg/s,

Continuity Eq. water: $\dot{m}_{v4} = \dot{m}_{v1} + \dot{m}_{v2} + \dot{m}_{f3} = 0.11896$ kg/s

$$\omega_4 = \dot{m}_{v4}/\dot{m}_{a4} = 0.0331 = 0.622 \frac{P_{v4}}{P - P_{v4}} \quad \rightarrow \quad P_{v4} = 5.052 \text{ kPa}$$

$P_{g4} = 7.384$ kPa, $\phi_4 = P_{v4}/P_{g4} = \mathbf{0.684}$, $h_{v4} = h_{g4} = 2574.3$ kJ/kg

Energy Eq.: $\dot{Q} + \dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2} + \dot{m}_{f3}h_{f3} = \dot{m}_{a4}h_{a4} + \dot{m}_{v4}h_{v4}$

$$\dot{Q} = 1.004(3.5944 \times 40 - 1.2288 \times 10 - 2.3656 \times 20) + 0.11896 \times 2574.3$$
$$- 0.00094 \times 2519.7 - 0.00691 \times 2538.1 - 0.1111 \times 42.0$$
$$= \mathbf{366 \text{ kW}}$$

11.143

A dehumidifier receives a flow of 0.25 kg/s dry air at 35°C, 90% relative humidity as shown in figure P11.107. It is cooled down to 20°C as it flows over the evaporator and then heated up again as it flows over the condenser. The standard refrigeration cycle uses R-410A with an evaporator temperature of –5°C and a condensation pressure of 3000 kPa. Find the amount of liquid water removed and the heat transfer in the cooling process. How much compressor work is needed? What is the final air exit temperature and relative humidity?

Solution:
This set-up has a standard refrigeration cycle with R-410A. This cycle and the air flow interacts through the two heat transfer processes. The cooling of the air is provided by the refrigeration cycle and thus requires an amount of work that depends on the cycle COP.

Refrigeration cycle:

State 1: $x = 1$ $h_1 = 277.53$ kJ/kg, $s_1 = 1.0466$ kJ/kg K

State 2: $s_2 = s_1$, $h_2 = 318.97$ kJ/kg, $T_2 = 72.9$°C

State 3: $x_3 = 0.0$, $h_3 = h_f = 141.78$ kJ/kg,   ($T_3 = 49.07$°C)

State 4: $h_4 = h_3$ and $P_4 = P_1$

$T_1 = -5$°C, $P_2 = 3000$ kPa

Now for the cycle we get

$$w_C = h_2 - h_1 = 318.97 - 277.53 = 41.44 \text{ kJ/kg}$$

$$q_H = h_2 - h_3 = 318.97 - 141.78 = 177.19 \text{ kJ/kg}$$

$$q_L = h_1 - h_4 = 277.53 - 141.78 = 135.75 \text{ kJ/kg}$$

For the air processes let us use the psychrometric chart.

Air inlet: $w_{in} = 0.019$, $\tilde{h}_{in} = 96$ kJ/kg dry air, $T_{dew} = 24^oC > 15^oC$

Air 15: $\phi = 100\%$, $w_{20} = 0.0148$, $\tilde{h}_{20} = 77.5$, $h_f = 83.94$ (B.1.1)

Now do the continuity (for water) and energy equations for the cooling process

$$\dot{m}_{liq} = \dot{m}_{air}(w_{in} - w_{20}) = 0.25(0.019 - 0.0148) = \textbf{0.00105 kg/s}$$

$$\dot{Q}_{cool} = \dot{m}_{air}(\tilde{h}_{in} - \tilde{h}_{20}) - \dot{m}_{liq}h_f = 0.25(96 - 77.5) - 0.00105 \times 83.94$$
$$= \textbf{4.537 kW}$$

Now the heater from the R-410A cycle has

$$\dot{Q}_{heat} = \dot{Q}_{cool}(q_H / q_L) = 4.537(177.19 / 135.75) = 5.922 \text{ kW}$$

so the compressor work is the balance of the two

$$\dot{W}_C = \dot{Q}_{heat} - \dot{Q}_{cool} = 5.922 - 4.537 = \textbf{1.385 kW}$$

Energy eq. for the air flow being heated

$$\dot{Q}_{heat} = \dot{m}_{air}(\tilde{h}_{ex} - \tilde{h}_{20}) \implies \tilde{h}_{ex} = \tilde{h}_{20} + \dot{Q}_{heat} / \dot{m}_{air}$$

$\tilde{h}_{ex} = 77.5 + 5.922 / 0.25 = 101.2$ kJ/kg dry air   and $w_{ex} = w_{20}$

Locate state in the psychrometric chart

$T = 43.5^oC$ and $\phi = 27\%$

11.144

The air-conditioning by evaporative cooling in Problem 11.101 is modified by adding a dehumidification process before the water spray cooling process. This dehumidification is achieved as shown in Fig. P11.144 by using a desiccant material, which absorbs water on one side of a rotating drum heat exchanger. The desiccant is regenerated by heating on the other side of the drum to drive the water out. The pressure is 100 kPa everywhere and other properties are on the diagram. Calculate the relative humidity of the cool air supplied to the room at state 4, and the heat transfer per unit mass of air that needs to be supplied to the heater unit.

States as noted on Fig. P11.144, text page 552.

At state 1, 35 °C: $P_{v1} = \phi_1 P_{g1} = 0.30 \times 5.628 = 1.6884$ kPa

$$w_1 = 0.622 \times 1.6884/98.31 = 0.010\ 68$$

At $T_3 = 25$ °C: $w_3 = w_2 = w_1/2 = 0.00534$

Evaporative cooling process to state 4, where $T_4 = 20$°C

Energy as in Eq.11.30: $w_3(h_{v3} - h_{f4}) = C_{P0A}(T_4 - T_3) + w_4 h_{fg4}$

$$0.005\ 34\ (2547.2 - 83.9) = 1.004(20 - 25) + w_4 \times 2454.2$$

$$w_4 = 0.0074 = 0.622 \times P_{v4} / (100 - P_{v4})$$

$$P_{v4} = 1.176 \text{ kPa}, \quad \phi_4 = 1.176 / 2.339 = \mathbf{0.503}$$

Following now the flow back we have $T_5 = 25$ °C, $w_5 = w_4 = 0.0074$

Evaporative cooling process to state 6, where $T_6 = 20$°C

$$w_5(h_{v5} - h_{f6}) = C_{P0A}(T_6 - T_5) + w_6 h_{fg6}$$

$$0.0074(2547.2 - 83.9) = 1.004(20 - 25) + w_6 \times 2454.2$$

$$\Rightarrow \quad w_6 = 0.009\ 47$$

For adiabatic heat exchanger,

$$\dot{m}_{A2} = \dot{m}_{A3} = \dot{m}_{A6} = \dot{m}_{A7} = \dot{m}_A, \quad \text{Also } w_2 = w_3, \quad w_6 = w_7$$

So now only T_7 is unknown in the energy equation

$$h_{A2} + w_2 h_{v2} + h_{A6} + w_6 h_{v6} = h_{A3} + w_3 h_{v3} + h_{A7} + w_7 h_{v7}$$

or

$$C_{P0A}T_7 + w_6(h_{v7} - h_{v6}) = C_{P0A}(T_2 + T_6 - T_3) + w_2(h_{v2} - h_{v3})$$

$$1.004\ T_7 + 0.009\ 47(h_{v7} - 2538.1) = 1.004(60 + 20 - 25)$$

$$+ 0.005\ 34(2609.6 - 2547.2) = 55.526$$

By trial and error, $T_7 = 54.7$ °C, $h_{v7} = 2600.3$ kJ/kg

For the heater 7-8, $\quad w_8 = w_7$,

$$\dot{Q}/\dot{m}_A = C_{P0A}(T_8 - T_7) + w_7(h_{v8} - h_{v7})$$
$$= 1.004(80 - 54.7) + 0.009\ 47(2643.7 - 2600.3)\ = \textbf{25.8 kJ/kg dry air}$$

ENGLISH UNIT PROBLEMS

Borgnakke Sonntag

Fundamentals of
Thermodynamics

SOLUTION MANUAL
CHAPTER 11
English Units

8e

Updated August 2013

CHAPTER 11

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Concept Problems

11.145E

If oxygen is 21% by mole of air, what is the oxygen state (P, T, v) in a room at 540 R, 15 psia of total volume 2000 ft^3?

The temperature is 540 R,

The partial pressure is $P_{O2} = yP_{tot} = 0.21 \times 15$ psia = 3.15 psia.

At this T, P:

$$v = RT/P = \frac{48.28 \times 540}{3.15 \times 144} \frac{\text{(ft-lbf/lbm R)} \times R}{\text{(lbf/in}^2\text{) (in/ft)}^2} = 57.48 \text{ ft}^3/\text{lbm}$$

Remark: If we found the oxygen mass then $m_{O2}v_{O2} = V = 2000$ ft^3

Borgnakke and Sonntag

Mixture Composition and Properties

Borgnakke and Sonntag

11.146E

A flow of oxygen and one of nitrogen, both 540 R, are mixed to produce 1 lbm/s air at 540 R, 15 psia. What are the mass and volume flow rates of each line?

We assume air has mole fraction of oxygen 0.21 and nitrogen 0.79.

For the mixture, M = 0.21 × 32 + 0.79 × 28.013 = 28.85

For O_2,   c = 0.21 × 32 / 28.85 = 0.2329

For N_2,   c = 0.79 × 28.013 / 28.85 = 0.7671

Since the total flow out is 1 lbm/s, these are the component flows in lbm/s.
Volume flow of O_2 in is

$$\dot{V} = c\dot{m}v = c\dot{m}\frac{RT}{P} = 0.2329 \times 1 \text{ lbm/s} \times \frac{48.28 \text{ lbf-ft/lbm-R} \times 540 \text{ R}}{15 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2}$$

$$= \textbf{2.81 ft}^\textbf{3}\textbf{/s}$$

Volume flow of N_2 in is

$$\dot{V} = c\dot{m}v = c\dot{m}\frac{RT}{P} = 0.7671 \times 1 \text{ lbm/s} \times \frac{55.15 \text{ lbf-ft/lbm-R} \times 540 \text{ R}}{15 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2}$$

$$= \textbf{10.58 ft}^\textbf{3}\textbf{/s}$$

11.147E

A gas mixture at 250 F, 18 lbf/in.^{2} is 50% N_2, 30% H_2O and 20% O_2 on a mole basis. Find the mass fractions, the mixture gas constant and the volume for 10 lbm of mixture.

Solution:

The conversion follows the definitions and identities:

From Eq.11.3: $c_i = y_i M_i / \sum y_j M_j$

From Eq.11.5:

$M_{mix} = \sum y_j M_j = 0.5 \times 28.013 + 0.3 \times 18.015 + 0.2 \times 31.999$

$\qquad = 14.0065 + 5.4045 + 6.3998 = 25.811$

$c_{N2} = 14.0065 / 25.811 = 0.5427, \quad c_{H2O} = 5.4045 / 25.811 = 0.2094$

$c_{O2} = 6.3998 / 25.811 = 0.2479, \quad$ sums to 1   OK

From Eq.11.14 and $\overline{R} = 1545.36$ lbf-ft/lbmol-R:

$R_{mix} = \overline{R}/M_{mix} = 1545.36 / 25.811 = $ **59.87 lbf ft/lbm R**

$V = mR_{mix} T/P = 10 \text{ lbm} \times \dfrac{59.87 \text{ lbf-ft/lbm-R} \times 710 \text{ R}}{18 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2}$

$\qquad = $ **164 ft^3**

11.148E

In a car engine gasoline (assume octane C_8H_{18}) is evaporated and then mixed with air in a ratio of 1:15 by mass. In the cylinder the mixture is at 110 psia, 1200 R when the spark fires. For that time find the partial pressure of the octane and the specific volume of the mixture.

Assuming ideal gas the partial pressure is $P_i = y_i P$ and $c_{C8H18} = 1/16 = 0.0625$

From Eq. 11.4: $y_i = [c_i / M_i] / \sum c_j / M_j$

$$y_{C8H18} = \frac{0.0625/114.232}{0.0625/114.232 + 0.9375/28.97} = 0.008186$$

$P_{C8H18} = 0.008186 \times 110 = \textbf{0.90 psia}$

The gas constant from Eq.11.15:

$$R_{mix} = \sum c_i R_i = 0.0625 \times 13.53 + 0.9375 \times 53.34 = 50.852 \text{ ft-lbf/lbm-R}$$

$$v_{mix} = R_{mix}T/P = \frac{50.852 \text{ lbf-ft/lbm-R} \times 1200 \text{ R}}{110 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2} = \textbf{3.852 ft}^3\textbf{/lbm}$$

11.149E

A diesel engine sprays fuel (assume n-Dodecane $C_{12}H_{26}$, M = 170.34 lbm/lbmol) into the combustion chamber already filled with in an amount of 1 mol fuel per 88 mol air. Find the fuel fraction on a mass basis and the fuel mass for a chamber that is 0.07 ft^3 at 1400 R and total pressure of 600 psia.

From Eq. 11.3: $\quad c_i = y_i M_i / \sum y_j M_j$

$$c_{fuel} = \frac{(1/89) \times 170.34}{(1/89) \times 170.34 + (88/89)\, 28.97} = \mathbf{0.06263}$$

Use ideal gas for the fuel vapor

$$m_{fuel} = \frac{P_{fuel}V}{R_{fuel}T} = \frac{(1/89) \times 600\ \text{psi} \times 0.07\ \text{ft}^3 \times 144\ (\text{in/ft})^2}{(1545.4/170.34)\ \text{ft-lbf/lbm-R} \times 1400\ \text{R}} = \mathbf{0.00535\ lbm}$$

We could also have done the total mass and then used the mass fraction

Eq.11.5: $\quad M_{mix} = \sum y_j M_j = (1/89) \times 170.34 + (88/89) \times 28.97 = 30.558$

$R_{mix} = \overline{R}/M_{mix} = 1545.4 / 30.558 = 50.5727$ ft-lbf/lbm-R

$$m = PV / R_{mix}T = \frac{600\ \text{psi} \times 0.07\ \text{ft}^3 \times 144\ (\text{in/ft})^2}{50.5727\ \text{ft-lbf/lbm-R} \times 1400\ \text{R}} = 0.08542\ \text{lbm}$$

$m_{fuel} = c_{fuel}\ m = 0.06263 \times 0.08542\ \text{lbm} = 0.00535\ \text{lbm}$

11.150E

A new refrigerant R-410A is a mixture of R-32 and R-125 in a 1:1 mass ratio. What is the overall molecular weight, the gas constant and the ratio of specific heats for such a mixture?

Eq.11.15:

$$R_{mix} = \sum c_i R_i = 0.5 \times 29.7 + 0.5 \times 12.87 = \textbf{21.285 ft-lbf/lbm R}$$

Eq.11.23:

$$C_{P\,mix} = \sum c_i\, C_{P\,i} = 0.5 \times 0.196 + 0.5 \times 0.189 = 0.1925 \text{ Btu/lbm R}$$

Eq.11.21:

$$C_{V\,mix} = \sum c_i C_{V\,i} = 0.5 \times 0.158 + 0.5 \times 0.172 = 0.165 \text{ Btu/lbm R}$$
$$(= C_{P\,mix} - R_{mix})$$

$$k_{mix} = C_{P\,mix} / C_{V\,mix} = 0.1925 / 0.165 = \textbf{1.1667}$$

$$M = \sum y_j M_j = = 1 / \sum (c_j / M_j) = \frac{1}{\dfrac{0.5}{52.024} + \dfrac{0.5}{120.022}} = \textbf{72.586}$$

Borgnakke and Sonntag

11.151E

Do the previous problem for R-507a, which is a 1:1 mass ratio of R-125 and R-143a. The refrigerant R-143a has molecular mass of 84.041 lbm/lbmol, and $C_P = 0.222$ Btu/lbm-R.

For R-143a: $R = 1545.36/84.041 = 18.388$ ft-lbf/lbm-R
$$C_V = C_P - R = 0.222 - 18.388/778.17 = 0.1984 \text{ Btu/lbm-R}$$

Eq.11.15:

$$R_{mix} = \sum c_i R_i = 0.5 \times 12.87 + 0.5 \times 18.388 = \textbf{15.629 ft-lbf/lbm R}$$

Eq.11.23:

$$C_{P \; mix} = \sum c_i \, C_{P \; i} = 0.5 \times 0.189 + 0.5 \times 0.222 = 0.2055 \text{ Btu/lbm R}$$

Eq.11.21:

$$C_{V \; mix} = \sum c_i C_{V \; i} = 0.5 \times 0.172 + 0.5 \times 0.1984 = 0.1852 \text{ Btu/lbm R}$$
$$(= C_{P \; mix} - R_{mix})$$

$$k_{mix} = C_{P \; mix} / C_{V \; mix} = 0.2055 / 0.1852 = \textbf{1.1096}$$

$$M = \sum y_j M_j = = 1 / \sum (c_j / M_j) = \frac{1}{\dfrac{0.5}{84.041} + \dfrac{0.5}{120.022}} = \textbf{98.86}$$

Borgnakke and Sonntag

Simple Processes

11.152E

A rigid container has 1 lbm CO_2 at 540 R and 1 lbm argon at 720 R both at 20 psia. Now they are allowed to mix without any heat transfer. What is final T, P?

No Q, No W   so the energy equation gives constant U

Energy Eq.:

$$U_2 - U_1 = 0 = m_{CO2}(u_2 - u_1)_{CO2} + m_{Ar}(u_2 - u_1)_{Ar}$$

$$= m_{CO2}C_{v\ CO2}(T_2 - T_1)_{CO2} + m_{Ar}C_{v\ Ar}(T_2 - T_1)_{Ar}$$

$$= (1 \times 0.201 + 1 \times 0.124)\ \text{Btu/R} \times T_2 - (1 \times 0.201 \times 540 + 1 \times 0.124 \times 720)\ \text{Btu}$$

$$T_2 = \textbf{608.7 R},$$

Volume from the beginning state

$$V = V_1 = V_{CO2} + V_{Ar} = m_{CO2}R_{CO2}T_{CO2}/P + m_{Ar}R_{Ar}T_{Ar}/P$$

$$= [1 \times 35.10 \times 540/20 + 1 \times 38.68 \times 720/20\ ]/144 = 16.25\ \text{ft}^3$$

Pressure from ideal gas law and Eq.11.15 for R

$$P_2 = mRT/V$$

$$= (1 \times 35.10 + 1 \times 38.68) \times 608.7/(16.25 \times 144) = \textbf{19.2 psia}$$

11.153E

A flow of 1 lbm/s argon at 540 R and another flow of 1 lbm/s CO_2 at 2800 R both at 20 psia are mixed without any heat transfer. What is the exit T, P?

No work implies no pressure change for a simple flow.

P_e = **20 psia**

The energy equation becomes

$$\dot{m}h_i = \dot{m}h_e = (\dot{m}h_i)_{Ar} + (\dot{m}h_i)_{CO2} = (\dot{m}h_e)_{Ar} + (\dot{m}h_e)_{CO2}$$

$\Rightarrow \qquad \dot{m}_{CO2}C_{p\ CO2}(T_e - T_i)_{CO2} + \dot{m}_{Ar}C_{p\ Ar}(T_e - T_i)_{Ar} = 0$

$\Rightarrow \qquad \dot{m}_{Ar}C_{p\ Ar}T_i + \dot{m}_{CO2}C_{p\ CO2}T_i = [\dot{m}_{Ar}C_{p\ Ar} + \dot{m}_{CO2}C_{p\ CO2}]\ T_e$

$(1 \times 0.124 \times 540 + 1 \times 0.201 \times 2800)$ Btu/s
$$= (1 \times 0.124 + 1 \times 0.201)\ \text{Btu/s-R} \times T_e$$

T_e = **1937.7 R**

11.154E

Repeat the previous problem using variable specific heats.

A flow of 1 lbm/s argon at 540 R and another flow of 1 lbm/s CO_2 at 2800 R both at 20 psia are mixed without any heat transfer. What is the exit T, P?

No work implies no pressure change for a simple flow.

$P_e =$ **20 psia**

The energy equation becomes

$$\dot{m}h_i = \dot{m}h_e = (\dot{m}h_i)_{Ar} + (\dot{m}h_i)_{CO2} = (\dot{m}h_e)_{Ar} + (\dot{m}h_e)_{CO2}$$

$$\Rightarrow \quad \dot{m}_{CO2}(h_e - h_i)_{CO2} + \dot{m}_{Ar}C_{p\,Ar}(T_e - T_i)_{Ar} = 0$$

$$\Rightarrow \quad \dot{m}_{Ar}C_{p\,Ar}T_i + \dot{m}_{CO2}h_i = \dot{m}_{Ar}C_{p\,Ar}T_e + \dot{m}_{CO2}h_{eCO2}$$

$$1 \times 0.124 \times 540 + 1 \times 27\,926/44.01 = 701.5 \text{ Btu/s}$$
$$= 1 \text{ lbm/s} \times (0.124 \text{ Btu/lbm-R } T_e + h_{eCO2})$$

Trial and error on RHS using Table F.6 for h_{eCO2}

$T_e = 2000$ R: RHS $= 16982/44.01 + 0.124 \times 2000 = 633.87$ too small

$T_e = 2200$ R: RHS $= 19659/44.01 + 0.124 \times 2200 = 719.494$ too large

Final interpolation

$$T_e = 2000 + 200\,\frac{701.5 \text{ - } 633.87}{719.494 \text{ - } 633.87} = \textbf{2158 R} \;;$$

11.155E

A pipe flows 0.1 lbm/s mixture with mass fractions of 40% CO_2 and 60% N_2 at 60 lbf/in.^{2}, 540 R. Heating tape is wrapped around a section of pipe with insulation added and 2 Btu/s electrical power is heating the pipe flow. Find the mixture exit temperature.

Solution:

C.V. Pipe heating section. Assume no heat loss to the outside, ideal gases.

Energy Eq.: $\dot{Q} = \dot{m}(h_e - h_i) = \dot{m}C_{P\,mix}(T_e - T_i)$

From Eq.11.23

$C_{P\,mix} = \sum c_i\, C_{P\,i} = 0.4 \times 0.201 + 0.6 \times 0.249 = 0.23$ Btu/lbm R

Substitute into energy equation and solve for exit temperature

$$T_e = T_i + \dot{Q}/\dot{m}C_{P\,MIX} = 540\ R + \frac{2\ \text{Btu/s}}{0.1\ \text{lbm/s} \times 0.23\ \text{Btu/lbm R}} = \mathbf{626.9\ R}$$

11.156E

An insulated gas turbine receives a mixture of 10% CO_2, 10% H_2O and 80% N_2 on a mass basis at 1800 R, 75 lbf/in.^{2}. The volume flow rate is 70 ft^3/s and its exhaust is at 1300 R, 15 lbf/in.^{2}. Find the power output in Btu/s using constant specific heat from F.4 at 540 R.

C.V. Turbine, Steady, 1 inlet, 1 exit flow with an ideal gas mixture, q = 0.

Energy Eq.: $\dot{W}_T = \dot{m}(h_i - h_e) = \dot{n}(\bar{h}_i - \bar{h}_e) = \dot{n}\bar{C}_{P\,mix}(T_i - T_e)$

$$PV = n\bar{R}T \quad => \quad \dot{n} = \frac{P\dot{V}}{\bar{R}T} = \frac{75 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2 \times 70 \text{ ft}^3/\text{s}}{1545.4 \text{ lbf-ft/lbmol-R} \times 1800 \text{ R}} = 0.272 \text{ lbmol/s}$$

$$\bar{C}_{P\,mix} = \sum y_i\,\bar{C}_i = 0.1 \times 44.01 \times 0.201 + 0.1 \times 18.015 \times 0.447$$
$$+ 0.8 \times 28.013 \times 0.249 = 7.27 \text{ Btu/lbmol R}$$

$\dot{W}_T = 0.272$ lbmol/s $\times$ 7.27 Btu/lbmol-R $\times$ (1800 − 1300) R = **988.7 Btu/s**

11.157E

Solve Problem 11.156 using the values of enthalpy from Table F.6

C.V. Turbine, Steady, 1 inlet, 1 exit flow with an ideal gas mixture, q = 0.

Energy Eq.: $\dot{W}_T = \dot{m}(h_i - h_e) = \dot{n}(\bar{h}_i - \bar{h}_e)$

$$PV = n\bar{R}T \quad \Rightarrow \quad \dot{n} = \frac{P\dot{V}}{\bar{R}T} = \frac{75 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2 \times 70 \text{ ft}^3/\text{s}}{1545.4 \text{ lbf-ft/lbmol-R} \times 1800 \text{ R}} = 0.272 \text{ lbmol/s}$$

$\dot{W}_T = 0.272 \text{ lbmol/s} \times [0.1(14\ 358 - 8121) + 0.1(11\ 178 - 6468.5)$

$\qquad\qquad\qquad + 0.8(9227 - 5431)] \text{ Btu/lbmol}$

$\qquad = \textbf{1123.7 Btu/s}$

11.158E

A piston cylinder device contains 0.3 lbm of a mixture of 40% methane and 60% propane by mass at 540 R and 15 psia. The gas is now slowly compressed in an isothermal (T = constant) process to a final pressure of 40 psia. Show the process in a P-V diagram and find both the work and heat transfer in the process.

Solution:

C.V. Mixture of methane and propane, this is a control mass.

Assume methane & propane are ideal gases at these conditions.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Property from Eq.11.15

$R_{mix} = 0.4\, R_{CH4} + 0.6\, R_{C3H8}$

$$= 0.4 \times 96.35 + 0.6 \times 35.04 = 59.564\, \frac{\text{ft-lbf}}{\text{lbm R}} = 0.07656\, \frac{\text{Btu}}{\text{lbm R}}$$

Process: T = constant & ideal gas =>

$${}_1W_2 = \int P\, dV = mR_{mix}T \int (1/V)dV = mR_{mix}T \ln (V_2/V_1)$$

$$= mR_{mix}T \ln (P_1/P_2)$$

$$= 0.3 \text{ lbm} \times 0.07656\, \frac{\text{Btu}}{\text{lbm R}} \times 540 \text{ R} \ln (15/40) = \textbf{-12.16 Btu}$$

Now heat transfer from the energy equation where we notice that u is a constant (ideal gas and constant T) so

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = {}_1W_2 = \textbf{-12.16 Btu}$$

11.159E

Two insulated tanks A and B are connected by a valve. Tank A has a volume of 30 ft^3 and initially contains argon at 50 lbf/in.^{2}, 50 F. Tank B has a volume of 60 ft^3 and initially contains ethane at 30 lbf/in.^{2}, 120 F. The valve is opened and remains open until the resulting gas mixture comes to a uniform state. Find the final pressure and temperature.

Energy eq.: $U_2 - U_1 = 0 = n_{Ar} \bar{C}_{V0}(T_2 - T_{A1}) + n_{C_2H_6} \bar{C}_{VO}(T_2 - T_{B1})$

$$n_{Ar} = P_{A1}V_A/\bar{R}T_{A1} = \frac{50 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2 \times 30 \text{ ft}^3}{1545 \text{ lbf-ft/lbmol-R} \times 509.7 \text{ R}} = 0.2743 \text{ lbmol}$$

$$n_{C_2H_6} = P_{B1}V_B/\bar{R}T_{B1} = \frac{30 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2 \times 60 \text{ ft}^3}{1545 \text{ lbf-ft/lbmol-R} \times 579.7 \text{ R}} = 0.2894 \text{ lbmol}$$

$$n_2 = n_{Ar} + n_{C_2H_6} = 0.5637 \text{ lbmol}$$

Substitute this into the energy equation

$$0.2743 \times 39.948 \times 0.0756 \, (T_2 - 509.7)$$

$$+ \, 0.2894 \times 30.07 \times 0.361 \, (T_2 - 509.7) = 0$$

Solving, $T_2 = $ **565.1 R**

$$P_2 = n_2 \bar{R} T_2/(V_A + V_B) = \frac{0.5637 \times 1545 \times 565.1}{90 \times 144} = \textbf{38 lbf/in}^2$$

Borgnakke and Sonntag

11.160E

A mixture of 4 lbm oxygen and 4 lbm of argon is in an insulated piston cylinder arrangement at 14.7 lbf/in.^{2}, 540 R. The piston now compresses the mixture to half its initial volume. Find the final pressure, temperature and the piston work.

Since $T_1 \gg T_C$ assume ideal gases.

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2 = - {}_1w_2$;   Entropy Eq.: $s_2 - s_1 = 0$

Process Eq.: $Pv^k = $ constant, $v_2 = v_1/2$

$$P_2 = P_1(v_1/v_2)^k = P_1(2)^k; \ T_2 = T_1(v_1/v_2)^{k-1} = T_1(2)^{k-1}$$

Find k_{mix} to get P_2, T_2 and $C_{v \, mix}$ for $u_2 - u_1$

$R_{mix} = \Sigma c_i R_i = (0.5 \times 48.28 + 0.5 \times 38.68)/778 = 0.055887$ Btu/lbm R

$C_{Pmix} = \Sigma c_i C_{Pi} = 0.5 \times 0.219 + 0.5 \times 0.1253 = 0.17215$ Btu/lbm R

$C_{vmix} = C_{Pmix} - R_{mix} = 0.11626$ Btu/lbm-R, $k_{mix} = C_{Pmix}/C_{vmix} = 1.4807$

Eq.6.25: $P_2 = 14.7(2)^{1.4805} = $ **41.03 lbf/in^2**,

Eq.6.24: $T_2 = 540 \times 2^{0.4805} = $ **753.5 R**

Work from the energy equation

$${}_1w_2 = u_1 - u_2 = C_v(T_1 - T_2) = 0.11626 (540 - 753.5)$$
$$= -24.82 \text{ Btu/lbm}$$
$${}_1W_2 = m_{tot} \, {}_1w_2 = 8 (-24.82) = \textbf{-198.6 Btu}$$

Borgnakke and Sonntag

11.161E

A flow of gas A and a flow of gas B are mixed in a 1:2 mole ratio with same T. What is the entropy generation per lbmole flow out?

For this the mole fractions are,
$$y_A = n_A/n_{tot} = 1/3; \quad y_B = n_B/n_{tot} = 2/3$$

Eq. 11.19: $\quad \Delta S = -\bar{R}[(1/3)\ln(1/3) + (2/3)\ln(2/3)] = +0.63651\ \bar{R}$

$$= 0.63651 \times 1.98589 = \mathbf{1.264\ Btu/lbmol\text{-}R}$$

11.162E

A rigid container has 1 lbm argon at 540 R and 1 lbm argon at 720 R both at 20 psia. Now they are allowed to mix without any external heat transfer. What is final T, P? Is any s generated?

Energy Eq.: $U_2 - U_1 = 0 = 2mu_2 - mu_{1a} - mu_{1b} = mC_v(2T_2 - T_{1a} - T_{1b})$

$$T_2 = (T_{1a} + T_{1b})/2 = \textbf{630 R},$$

Process Eq.: V = constant =>

$$P_2V = 2mRT_2 = mR(T_{1a} + T_{1b}) = P_1V_{1a} + P_1V_{1b} = P_1V$$

$$P_2 = P_1 = \textbf{20 psia}$$

ΔS due to temp changes only , not P

$$\Delta S = m (s_2 - s_{1a}) + m (s_2 - s_{1b}) = mC [\ln (T_2/T_{1a}) + \ln (T_2/T_{1b})]$$

$$= 1 \times 0.124 \left[\ln \frac{630}{540} + \ln \frac{630}{720} \right] = \textbf{0.00256 Btu/R}$$

Why did we not account for partial pressures? Since we mix argon with argon there is no entropy generation due to mixing and no partial pressure.

11.163E

What is the rate of entropy increase in problem 11.153E?

Using Eq. 11.4, the mole fraction of CO_2 in the mixture is

$$y_{CO2} = \frac{c_{CO2}/M_{CO2}}{c_{CO2}/M_{CO2} + c_{Ar}/M_{Ar}} = \frac{0.5 / 44.01}{0.5 / 39.948 + 0.5 / 44.01} = 0.4758$$

$$y_{Ar} = 1 - y_{CO2} = 0.5242$$

The energy equation becomes

$$\dot{m}h_i = \dot{m}h_e = (\dot{m}h_i)_{Ar} + (\dot{m}h_i)_{CO2} = (\dot{m}h_e)_{Ar} + (\dot{m}h_e)_{CO2}$$

$$\Rightarrow \quad \dot{m}_{CO2}C_{p\ CO2}(T_e - T_i)_{CO2} + \dot{m}_{Ar}C_{p\ Ar}(T_e - T_i)_{Ar} = 0$$

$$\Rightarrow \quad \dot{m}_{Ar}C_{p\ Ar}T_i + \dot{m}_{CO2}C_{p\ CO2}T_i = [\dot{m}_{Ar}C_{p\ Ar} + \dot{m}_{CO2}C_{p\ CO2}]\ T_e$$

$(1 \times 0.124 \times 540 + 1 \times 0.201 \times 2800)$ Btu/s

$$= (1 \times 0.124 + 1 \times 0.201)\ \text{Btu/s-R} \times T_e$$

$$T_e = 1937.7\ R$$

From Eqs. 11.16 and 11.17, from the two inlet states to state 2,

$$\dot{m}(s_e - s_i) = 1 \times [0.124\ \ln(\frac{1937.7}{540}) - \frac{38.68}{778}\ \ln(\frac{0.5242 \times 20}{20})]$$

$$+1 \times [0.201\ \ln(\frac{1937.7}{2800}) - \frac{35.10}{778}\ \ln(\frac{0.4758 \times 20}{20})] = \textbf{0.15 Btu/s R}$$

The increase is also the entropy generation as there is no heat transfer.

Entropy Generation

11.164E

Find the entropy generation for the process in Problem 11.159E.

Energy eq. $\qquad U_2 - U_1 = 0 = n_{Ar}\bar{C}_{V0}(T_2 - T_{A1}) + n_{C_2H_6}\bar{C}_{VO}(T_2 - T_{B1})$

$$n_{Ar} = P_{A1}V_A/\bar{R}T_{A1} = \frac{50 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2 \times 30 \text{ ft}^3}{1545 \text{ lbf-ft/lbmol-R} \times 509.7 \text{ R}} = 0.2743 \text{ lbmol}$$

$$n_{C_2H_6} = P_{B1}V_B/\bar{R}T_{B1} = \frac{30 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2 \times 60 \text{ ft}^3}{1545 \text{ lbf-ft/lbmol-R} \times 579.7 \text{ R}} = 0.2894 \text{ lbmol}$$

$$n_2 = n_{Ar} + n_{C_2H_6} = 0.5637 \text{ lbmol}$$

Substitute into energy equation

$\qquad 0.2743 \times 39.948 \times 0.0756 \ (T_2 - 509.7)$

$\qquad\qquad + 0.2894 \times 30.07 \times 0.361 \ (T_2 - 509.7) = 0$

Solving, $T_2 = 565.1$ R

$$P_2 = n_2\bar{R}T_2/(V_A + V_B) = \frac{0.5637 \times 1545 \times 565.1}{90 \times 144} = 38 \text{ lbf/in}^2$$

$$\Delta S_{SURR} = 0 \qquad \rightarrow \Delta S_{gen} = \Delta S_{SYS} = n_{Ar}\Delta\bar{S}_{Ar} + n_{C_2H_6}\Delta\bar{S}_{C_2H_6}$$

$$y_{Ar} = 0.2743/0.5637 = 0.4866$$

$$\Delta\bar{S}_{Ar} = \bar{C}_{P \ Ar} \ln\frac{T_2}{T_{A1}} - \bar{R} \ln\frac{y_{Ar}P_2}{P_{A1}}$$

$$= 39.948 \times 0.1253 \ln\frac{565.1}{509.7} - \frac{1545}{778}\ln\frac{0.4866 \times 38}{50} = 2.4919 \text{ Btu/lbmol R}$$

$$\Delta\bar{S}_{C_2H_6} = \bar{C}_{C_2H_6} \ln\frac{T_2}{T_{B1}} - \bar{R} \ln\frac{y_{C_2H_6}P_2}{P_{B1}}$$

$$= 30.07 \times 0.427 \ln\frac{565.1}{579.7} - \frac{1545}{778}\ln\frac{0.5134 \times 38}{30}$$

$$= 0.5270 \text{ Btu/lbmol R}$$

$$\Delta S_{gen} = 0.2743 \times 2.4919 + 0.2894 \times 0.5270 = \textbf{0.836 Btu/R}$$

Borgnakke and Sonntag

11.165E

Carbon dioxide gas at 580 R is mixed with nitrogen at 500 R in an insulated mixing chamber. Both flows are at 14.7 lbf/in.^{2} and the mole ratio of carbon dioxide to nitrogen is 2:1. Find the exit temperature and the total entropy generation per mole of the exit mixture.

CV mixing chamber, steady flow. The inlet ratio is $\dot{n}_{CO_2} = 2\,\dot{n}_{N_2}$ and assume no external heat transfer, no work involved.

Continuity: $\dot{n}_{CO_2} + 2\dot{n}_{N_2} = \dot{n}_{ex} = 3\dot{n}_{N_2}$;

Energy Eq.: $\dot{n}_{N_2}(\bar{h}_{N_2} + 2\bar{h}_{CO_2}) = 3\dot{n}_{N_2}\bar{h}_{mix\ ex}$

Take 540 R as reference and write $\bar{h} = \bar{h}_{540} + \bar{C}_{Pmix}(T-540)$.

$$\bar{C}_{P\,N_2}(T_{i\,N_2}-540) + 2\bar{C}_{P\,CO_2}(T_{i\,CO_2}-540) = 3\bar{C}_{P\,mix}(T_{mix\ ex}-540)$$

$$\bar{C}_{P\,mix} = \sum y_i \bar{C}_{P\,i} = (29.178 + 2\times37.05)/3 = 8.2718\ \text{Btu/lbmol R}$$

$$3\bar{C}_{P\,mix}T_{mix\ ex} = \bar{C}_{P\,N_2}T_{i\,N_2} + 2\bar{C}_{P\,CO_2}T_{i\,CO_2} = 13\ 837\ \text{Btu/lbmol}$$

$$T_{mix\ ex} = 557.6\ \text{R};$$

Partial pressures are total pressure times molefraction

$$P_{ex\,N_2} = P_{tot}/3; \quad P_{ex\,CO_2} = 2P_{tot}/3$$

$$\dot{S}_{gen} = \dot{n}_{ex}\bar{s}_{ex} - (\dot{n}\bar{s})_{iCO_2} - (\dot{n}\bar{s})_{iN_2} = \dot{n}_{N_2}(\bar{s}_e - \bar{s}_i)_{N_2} + 2\dot{n}_{N_2}(\bar{s}_e - \bar{s}_i)_{CO_2}$$

$$\dot{S}_{gen}/\dot{n}_{N_2} = \bar{C}_{PN_2}\ln\frac{T_{ex}}{T_{iN_2}} - \bar{R}\ln y_{N_2} + 2\bar{C}_{PCO_2}\ln\frac{T_{ex}}{T_{iCO_2}} - 2\bar{R}\ln y_{CO_2}$$

$$= 0.7575 + 2.1817 - 0.7038 + 1.6104 = \textbf{3.846 Btu/lbmol N}_2\ \textbf{R}$$

11.166E

A steady flow 0.6 lbm/s of 60% carbon dioxide and 40% water mixture by mass at 2200 R and 30 psia is used in a constant pressure heat exchanger where 300 Btu/s is extracted from the flow. Find the exit temperature and rate of change in entropy using Table F.4

Solution:

C.V. Heat exchanger, Steady, 1 inlet, 1 exit, no work.

Continuity Eq.: $c_{CO2} = 0.6;$ $c_{H2O} = 0.4$

Energy Eq.: $\dot{Q} = \dot{m}(h_e - h_i) \approx \dot{m} \, C_P \, (T_e - T_i) \implies T_e = T_i + \dot{Q}/\dot{m} \, C_P$

Inlet state: Table F.4

$$C_P = 0.6 \times 0.201 + 0.4 \times 0.447 = 0.2994 \text{ Btu/lbm-R}$$

Exit state: $T_e = T_i + \dot{Q}/\dot{m} \, C_P = 2200 \text{ R} - \dfrac{300 \text{ Btu/s}}{0.6 \text{ lbm/s} \times 0.2994 \text{ Btu/lbm-R}}$

$$= \textbf{530 R}$$

The rate of change of entropy for the flow is (P is assumed constant)

$$\dot{m}(s_e - s_i) = \dot{m}[s^o_{Te} - s^o_{Ti} - R \ln(P_e/P_i)] = \dot{m}[s^o_{Te} - s^o_{Ti}] \approx \dot{m} \, C_P \ln(T_e/T_i)$$

$$= 0.6 \times 0.2994 \ln(530 \, / \, 2200) = \textbf{–0.256 Btu/s-R}$$

The entropy generation rate cannot be estimated unless the average T at which the heat transfer leaves the control volume is known.

11.167E

A steady flow 0.6 lbm/s of 60% carbon dioxide and 40% water mixture by mass at 2200 R and 30 psia is used in a constant pressure heat exchanger where 300 Btu/s is extracted from the flow. Find the exit temperature and rate of change in entropy using Table F.6

Solution:

C.V. Heat exchanger, Steady, 1 inlet, 1 exit, no work.

Continuity Eq.: $c_{CO2} = 0.6$; $c_{H2O} = 0.4$

Energy Eq.: $\dot{Q} = \dot{m}(h_e - h_i)$ $\Rightarrow$ $h_e = h_i + \dot{Q}/\dot{m}$

Inlet state: Table F.6

   $h_i = 0.6 \times 19659/44.01 + 0.4 \times 15254/18.015 = 606.71$ Btu/lbm

Exit state: $h_e = h_i + \dot{Q}/\dot{m} = 606.71 - 300/0.6 = 106.71$ Btu/lbm

Trial and error for T with h values from Table F.6

   @800 R $h_e = 0.6 \times 2525/44.01 + 0.4 \times 2142/18.015 = 81.984$ Btu/lbm

   @1000 R $h_e = 0.6 \times 4655/44.01 + 0.4 \times 3824/18.015 = 148.37$ Btu/lbm

      Interpolate to have the right h: **T = 874.5 R**

Entropy Eq.7.8: $\dot{m}s_e = \dot{m}s_i + \dot{Q}/T + \dot{S}_{gen}$

The rate of change of entropy for the flow is (P is assumed constant)

   $\dot{m}(s_e - s_i) = \dot{m}(s^o_{Te} - s^o_{Ti})$

      $= 0.6[\ 0.6(55.732 - 66.952)/44.01$

            $+ 0.4(49.019 - 57.605)/18.015\ ]$

   $= $ **–0.344 Btu/s-R**

The entropy generation rate cannot be estimated unless the average T at which the heat transfer leaves the control volume is known.

11.168E

A mixture of 60% helium and 40% nitrogen by mole enters a turbine at 150 lbf/in.^{2}, 1500 R at a rate of 4 lbm/s. The adiabatic turbine has an exit pressure of 15 lbf/in.^{2} and an isentropic efficiency of 85%. Find the turbine work.

Assume ideal gas mixture and take CV as turbine.

Energy Eq. ideal turbine: $w_{T\,s} = h_i - h_{es}$,

Entropy Eq. ideal turbine: $s_{es} = s_i$ $\Rightarrow$ $T_{es} = T_i(P_e/P_i)^{(k-1)/k}$

Properties from Eq.11.23, 11.15 and 6.23

$\bar{C}_{P\,mix} = 0.6 \times 1.25 \times 4.003 + 0.4 \times 0.248 \times 28.013 = 5.7811$ Btu/lbmol R

$(k-1)/k = \bar{R}/\bar{C}_{P\,mix} = 1545/(5.7811 \times 778) = 0.3435$

$M_{mix} = 0.6 \times 4.003 + 0.4 \times 28.013 = 13.607$,

$C_P = \bar{C}_P/M_{mix} = 0.4249$ Btu/lbm R

$T_{es} = 1500$ R $(15/150)^{0.3435} = 680$ R,

$w_{Ts} = C_P(T_i - T_{es}) = 348.4$ Btu/lbm

Then do the actual turbine

$w_{T\,ac} = \eta w_{Ts} = 296.1$ Btu/lbm; $\dot{W} = \dot{m}w_{Ts} =$ **1184 Btu/s**

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11.169E

A tank has two sides initially separated by a diaphragm. Side A contains 2 lbm of water and side B contains 2.4 lbm of air, both at 68 F, 14.7 lbf/in.^{2}. The diaphragm is now broken and the whole tank is heated to 1100 F by a 1300 F reservoir. Find the final total pressure, heat transfer, and total entropy generation.

C.V. Total tank out to reservoir.

Energy Eq.3.5: $U_2 - U_1 = m_a(u_2 - u_1)_a + m_v(u_2 - u_1)_v = {}_1Q_2$

Entropy Eq.6.37:

$$S_2 - S_1 = m_a(s_2 - s_1)_a + m_v(s_2 - s_1)_v = {}_1Q_2/T_{res} + S_{gen}$$

$$V_2 = V_A + V_B = m_v v_{v1} + m_a v_{a1} = 0.0321 + 31.911 = 31.944 \text{ ft}^3$$

Water: $u_1 = 36.08$ Btu/lbm, $s_1 = 0.0708$ Btu.lbm R,

$v_{v2} = V_2/m_v = 15.9718$, $T_2 =>$ Table F.7.2 $P_{2v} = 58.7$ lbf/in^2

$u_2 = 1414.3$ Btu/lbm, $s_2 = 2.011$ Btu/lbm R

Air: $u_1 = 90.05$ Btu/lbm, $u_2 = 278.23$ Btu/lbm,

$s_{T1} = 1.6342$ Btu/lbm R, $s_{T2} = 1.9036$ Btu/lbm R

$v_{a2} = V_2/m_a = 13.3098$, $T_2 =>$ $P_{2a} = mRT_2/V_2 = 43.415$ lbf/in^2

$$P_{2tot} = P_{2v} + P_{2a} = \textbf{102 lbf/in}^2$$

${}_1Q_2 = 2(1414.3 - 36.08) + 2.4(278.23 - 90.05) = \textbf{3208 Btu}$

$S_{gen} = 2(2.011 - 0.0708) + 2.4[1.9036 - 1.6342$

$$- \frac{53.34}{778} \times \ln(\frac{43.415}{14.7})] - \frac{3208}{1760}$$

$= 3.8804 + 0.4684 - 1.823 = \textbf{2.526 Btu/R}$

Air Water vapor Mixtures

11.170E

A 1 lbm/s flow of saturated moist air (relative humidity 100%) at 14.7 psia and 50 F goes through a heat exchanger and comes out at 77 F. What is the exit relative humidity and the how much power is needed?

Solution:

State 1 : $\phi_1 = 1$; $\qquad P_v = P_g = 0.178$ psia

Eq.11.28: $w = 0.622\, P_v/P_a = 0.622 \times 0.178/(14.7 - 0.178) = 0.00762$

State 2 : No water added $\Rightarrow w_2 = w_1 \Rightarrow P_{v2} = P_{v1}$

$\qquad \phi_2 = P_{v2}/P_{g2} = 0.178/0.464 = \mathbf{0.384 \ or \ 38 \ \%}$

Energy Eq.4.10

$$\dot{Q} = \dot{m}_2 h_2 - \dot{m}_1 h_1 = \dot{m}_a(h_2 - h_1)_{air} + w\dot{m}_a(h_2 - h_1)_{vapor}$$

$$\dot{m}_{tot} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1 + w_1)$$

Energy equation with $C_{P\ air}$ from F.4 and h's from F.7.1

$$\dot{Q} = \frac{\dot{m}_{tot}}{1 + w_1} C_{P\ air}\,(77 - 50) + \frac{\dot{m}_{tot}}{1 + w_1}\,w\,(h_{g2} - h_{g1})$$

$$= \frac{1}{1.00762} \times 0.24\,(77 - 50) + \frac{1 \times 0.00762}{1.00762}\,(1090.73 - 1083.29)$$

$$= 6.431 + 0.0563 = \mathbf{6.49 \ Btu/s}$$

11.171E

If I have air at 14.7 psia and a) 15 F b) 115 F and c) 230 F what is the maximum absolute humidity I can have?

Humidity is related to relative humidity (max 100%) and the pressures as in Eq.11.28 where from Eq.11.25 $P_v = \Phi P_g$ and $P_a = P_{tot} - P_v$.

$$\omega = 0.622 \, \frac{P_v}{P_a} = 0.622 \, \frac{\Phi P_g}{P_{tot} - \Phi P_g}$$

a) $P_g = 0.03963$ psia, $\omega = 0.622 \times \dfrac{0.03963}{14.7 - 0.03963} = \mathbf{0.001\ 68}$

b) $P_g = 1.4855$ psia , $\omega = 0.622 \times \dfrac{1.4855}{14.7 - 1.4855} = \mathbf{0.070}$

c) $P_g = 20.781$ psia, **no limit** on ω for $P > 14.7$ psia

11.172E

Consider a volume of 2000 ft^3 that contains an air-water vapor mixture at 14.7 lbf/in.^{2}, 60 F, and 40% relative humidity. Find the mass of water and the humidity ratio. What is the dew point of the mixture?

Air-vap   P = 14.7 lbf/in.^{2}, T = 60 F, ϕ = 40%

$P_g = P_{sat60} = 0.256$ lbf/in.^{2}

$P_v = \phi\, P_g = 0.4 \times 0.256 = 0.1024$ lbf/in.^{2}

$$m_{v1} = \frac{P_v V}{R_v T} = \frac{0.1024 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2 \times 2000 \text{ ft}^3}{85.76 \text{ lbf-ft/lbm-R} \times 520 \text{ R}} = \textbf{0.661 lbm}$$

$P_a = P_{tot} - P_{v1} = 14.7 - 0.1024 = 14.598$ lbf/in.^{2}

$$m_a = \frac{P_a V}{R_a T} = \frac{14.598 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2 \times 2000 \text{ ft}^3}{53.34 \text{ lbf-ft/lbm-R} \times 520 \text{ R}} = 151.576 \text{ lbm}$$

$$w_1 = \frac{m_v}{m_a} = \frac{0.661}{151.576} = \textbf{0.00436}$$

T_{dew} is T when $P_g(T_{dew}) = 0.1024$ lbf/in.^{2};   T = **35.5 F**

11.173E

Consider a 35 ft^3/s flow of atmospheric air at 14.7 psia, 77 F and 80% relative humidity. Assume this flows into a basement room where it cools to 60 F at 14.7 psia. How much liquid will condense out?

Solution:

State 1: $P_g = P_{sat77} = 0.464$ psia $\Rightarrow$ $P_v = \phi\, P_g = 0.8 \times 0.464 = 0.371$ psia

$$\dot{m}_{v1} = \frac{P_v \dot{V}}{R_v T} = \frac{0.371 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2 \times 35 \text{ ft}^3/\text{s}}{85.76 \text{ lbf-ft/lbm-R} \times 536.67 \text{ R}} = 0.0406 \text{ lbm/s}$$

$$w_1 = \frac{\dot{m}_{v1}}{\dot{m}_{A1}} = 0.622 \frac{P_{v1}}{P_{A1}} = 0.622 \frac{0.371}{14.7 \text{ - } 0.371} = 0.0161$$

$$\dot{m}_{A1} = \frac{\dot{m}_{v1}}{w_1} = \frac{0.0406}{0.0161} = 2.522 \text{ lbm/s} = \dot{m}_{A2} \qquad \text{(continuity for air)}$$

Check for state 2:

$P_{g60F} = 0.256$ psia $< P_{v1}$

so liquid water out.

State 2 is saturated $\phi_2 = 100\%$, $P_{v2} = P_{g2} = 0.256$ psia

$$w_2 = 0.622 \frac{P_{v2}}{P_{A2}} = 0.622 \frac{0.256}{14.7 \text{ - } 0.256} = 0.0110$$

$$\dot{m}_{v2} = w_2 \dot{m}_{A2} = 0.0110 \times 2.522 = 0.0277 \text{ lbm/s}$$

$$\dot{m}_{liq} = \dot{m}_{v1} \text{ - } \dot{m}_{v2} = 0.0406 - 0.0277 = \mathbf{0.0129 \text{ lbm/s}}$$

Note that the given volume flow rate at the inlet is not that at the exit. The mass flow rate of dry air is the quantity that is the same at the inlet and exit.

11.174E

Consider a 10-ft^3 rigid tank containing an air-water vapor mixture at 14.7 lbf/in.^{2}, 90 F, with a 70% relative humidity. The system is cooled until the water just begins to condense. Determine the final temperature in the tank and the heat transfer for the process.

$$P_{v1} = \phi P_{G1} = 0.7 \times 0.6988 = 0.489 \text{ lbf/in}^2$$

Since m_v = const & V = const & also $P_v = P_{G2}$:

$$P_{G2} = P_{v1} \times T_2/T_1 = 0.489 \times T_2/549.7$$

For T_2 = 80 F: $0.489 \times 539.7/549.7 = 0.4801 \neq 0.5073$ ($= P_G$ at 80 F)

For T_2 = 70 F: $0.489 \times 529.7/549.7 = 0.4712 \neq 0.3632$ ($= P_G$ at 70 F)

interpolating $\rightarrow T_2 =$ **78.0 F**

$$w_2 = w_1 = 0.622 \frac{0.489}{(14.7\text{-}0.489)} = 0.0214$$

$$m_a = \frac{P_{a1}V}{R_a T_1} = \frac{14.211 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2 \times 10 \text{ ft}^3}{53.34 \text{ lbf-ft/lbm-R} \times 549.7 \text{ R}} = 0.698 \text{ lbm}$$

Energy Eq.:

$$_1Q_2 \quad = U_2 - U_1 = m_a(u_{a2} - u_{a1}) + m_v(u_{v2} - u_{v1})$$

$$= m_a[\ C_v\ (T_{a2} - T_{a1}) + w_1\ (u_{v2} - u_{v1})\ ]$$

$$= 0.698 \text{ lbm } [0.171(78 - 90) + 0.0214(1036.3 - 1040.2)] \text{ Btu/lbm}$$

$$= 0.698 \text{ lbm } (-2.135 \text{ Btu/lbm}) = -\textbf{1.49 Btu}$$

11.175E

A water-filled reactor of 50 ft^3 is at 2000 lbf/in.^{2}, 550 F and located inside an insulated containment room of 5000 ft^3 that has air at 1 atm. and 77 F. Due to a failure the reactor ruptures and the water fills the containment room. Find the final pressure.

CV Total container.

Energy: $m_v(u_2 - u_1) + m_a(u_2 - u_1) = {_1}Q_2 - {_1}W_2 = 0$

Initial water: $v_1 = 0.021407$ ft^3/lbm, $u_1 = 539.24$, $m_v = V/v = 2335.7$ lbm

Initial air: $m_a = PV/RT = 14.7 \times 4950 \times 144/53.34 \times 536.67 = 366.04$ lbm

Substitute into energy equation

$\quad$ 2335.7 $(u_2 - 539.24) + 366.04 \times 0.171 (T_2 - 77) = 0$

$\quad u_2 + 0.0268 T_2 = 541.3 \qquad$ & $\qquad v_2 = V_2/m_v = 2.1407$ ft^3/lbm

Trial and error 2-phase $(T_{guess}, v_2 \Rightarrow x_2 \Rightarrow u_2 \Rightarrow$ LHS)

$\quad$ T = 300 $x_2 = (2.1407 - 0.01745)/6.4537 = 0.329$, $u_2 = 542.73$ Btu/lbm

$\qquad\qquad$ LHS = 550.789 Btu/lbm too large

$\quad$ T = 290 $x_2 = (2.1407 - 0.01735)/7.4486 = 0.28507$, $u_2 = 498.27$ Btu/lbm

$\qquad\qquad$ LHS = 506.05 Btu/lbm too small

$\quad T_2 = 298$ F, $x_2 = 0.3198$, $P_{sat} = 65$ lbf/in^2, LHS = 541.5 OK

$\quad P_{a2} = P_{a1}V_1T_2/V_2T_1 = 14.7$ psi $\times \dfrac{4950 \times 757.7}{5000 \times 536.67} = 20.55$ lbf/in^2

$\qquad \Rightarrow P_2 = P_{a2} + P_{sat} =$ **85.55 lbf/in^2**

11.176E

In the production of ethanol from corn the solids left after fermentation are dried in a continuous flow oven. This process generates a flow of 35 lbm/s moist air, 200 F with 70% relative humidity, which contains some volatile organic compounds and some particles. To remove the organic gasses and the particles, the flow is send to a **thermal oxidicer,** see Fig. P11.82, where natural gas flames brings the mixture to 1500 F. Find the rate of heating by the natural gas burners.

For this problem we will just consider the heating of the gas mixture and due to the exit temperature we will use the ideal gas tables F5 and F6.

Eq.11.25: $P_v = \phi P_g = 0.70 \times 11.53$ psia $= 8.07$ psia

Eq.11.28: $\omega = 0.622 \dfrac{P_v}{P_{tot} - P_v} = 0.622 \times \dfrac{8.07}{14.7 - 8.07} = 0.757$

Flow: $\dot{m}_{tot} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1 + \omega)$ so

$$\dot{m}_a = \frac{\dot{m}_{tot}}{1 + \omega} = 19.92 \text{ lbm/s}; \quad \dot{m}_v = \dot{m}_{tot}\frac{\omega}{1 + \omega} = 15.08 \text{ lbm/s}$$

Process: Heating   => ω is constant   => P_v is constant

For ideal gas the enthalpy does not depend on pressure so the energy equation gives the heat transfer as

$$\dot{Q} = \dot{m}_a (h_2 - h_1)_a + \dot{m}_v (h_2 - h_1)_v$$
$$= 19.92 (493.61 - 157.96) + 15.08 (12779 - 996.45)/18.015$$
$$= 6686 + 9863 = \textbf{16 549 Btu/s}$$

Comment: Notice how much energy is spend on heating the water vapor.

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11.177E

To save on natural gas use in the previous problem it is suggested to take and cool the mixture and condense out some water before heating it back up. So the flow is cooled from 200 F to 120 F as shown in Fig. P11.83 and the now dryer mixture is heated to 1500 F. Find the amount of water condensed out and the rate of heating by the natural gas burners for this case.

Eq.11.25: $P_v = \phi\, P_g = 0.70 \times 11.53$ psia $= 8.07$ psia

Eq.11.28: $\omega = 0.622\, \dfrac{P_v}{P_{tot} - P_v} = 0.622 \times \dfrac{8.07}{14.7 - 8.07} = 0.757$

Flow: $\dot m_{tot} = \dot m_a + \dot m_v = \dot m_a(1 + \omega)$ so

$$\dot m_a = \frac{\dot m_{tot}}{1 + \omega} = 19.92 \text{ lbm/s}; \quad \dot m_v = \dot m_{tot}\, \frac{\omega}{1 + \omega} = 15.08 \text{ lbm/s}$$

Now cool to 120 F: $P_g = 1.695$ psia $< P_{v1}$ so $\phi_2 = 100\%,\ P_{v2} = 1.695$ psia

$$\omega_2 = 0.622\, \frac{P_v}{P_{tot} - P_v} = 0.622 \times \frac{1.695}{14.7 - 1.695} = 0.08107$$

$$\dot m_{liq} = \dot m_a\, (\,\omega_1 - \omega_2) = 19.92\, (0.757 - 0.08107) = 13.464 \text{ lbm/s}$$

The air flow is not changed so the water vapor flow for heating is

$$\dot m_{v2} = \dot m_{v1} - \dot m_{liq} = 15.08 - 13.464 = 1.616 \text{ lbm/s}$$

Now the energy equation becomes

$$\dot Q = \dot m_a\, (h_3 - h_2)_a + \dot m_{v2}\, (h_3 - h_2)_v$$
$$= 19.92\, (493.61 - 157.96) + 1.616\, (12779 - 996.45)/18.015$$
$$= \textbf{7743 Btu/s}$$

Comment: If you solve the previous problem you find this is only 47% of the heat for the case of no water removal.

11.178E

Two moist air streams with 85% relative humidity, both flowing at a rate of 0.2 lbm/s of dry air are mixed in a steady flow setup. One inlet flowstream is at 90 F and the other at 61 F. Find the exit relative humidity.

Solution:

CV mixing chamber.

Continuity Eq. water: $\dot{m}_{air} w_1 + \dot{m}_{air} w_2 = 2\dot{m}_{air} w_{ex}$;

Energy Eq.: $\dot{m}_{air} \tilde{h}_1 + \dot{m}_{air} \tilde{h}_2 = 2\dot{m}_{air} \tilde{h}_{ex}$

Properties from the tables and formulas

$P_{g90} = 0.699$; $P_{v1} = 0.85 \times 0.699 = 0.594$ psia

$w_1 = 0.622 \times 0.594 / (14.7 - 0.594) = 0.0262$

$P_{g61} = 0.2667$; $P_{v2} = 0.85 \times 0.2667 = 0.2267$ psia

$w_2 = 0.622 \times 0.2267 / (14.7 - 0.2267) = 0.00974$

Continuity Eq. water: $w_{ex} = (w_1 + w_2)/2 = 0.018$;

For the energy equation we have $\tilde{h} = h_a + wh_v$ so:

$2\tilde{h}_{ex} - \tilde{h}_1 - \tilde{h}_2 = 0 = 2h_{a\ ex} - h_{a\ 1} - h_{a\ 2} + 2w_{ex}h_{v\ ex} - w_1 h_{v\ 1} - wh_{v\ 2}$

we will use constant heat capacity to avoid an iteration on T_{ex}.

$C_{p\ air}(2T_{ex} - T_1 - T_2) + C_{p\ H2O}(2w_{ex}T_{ex} - w_1 T_1 - w_2 T_2) = 0$

$T_{ex} = [\ C_{p\ air}(T_1 + T_2) + C_{p\ H2O}(w_1 T_1 + w_2 T_2)\ ]/\ [2C_{p\ air} + 2w_{ex}C_{p\ H2O}]$

$= [\ 0.24\ (90 + 61) + 0.447(0.0262 \times 90 + 0.00974 \times 61]/0.4961$

$= 75.7$ F

$P_{v\ ex} = \dfrac{w_{ex}}{0.622 + w_{ex}} P_{tot} = \dfrac{0.018}{0.622 + 0.018}\ 14.7 = 0.413$ psia,

$P_{g\ ex} = 0.445$ psia $\quad=>\quad \phi = 0.413\ /\ 0.445 =$ **0.93 or 93%**

11.179E

A flow of moist air from a domestic furnace, state 1 in Figure P11.100, is at 120 F, 10% relative humidity with a flow rate of 0.1 lbm/s dry air. A small electric heater adds steam at 212 F, 14.7 psia generated from tap water at 60 F. Up in the living room the flow comes out at state 4: 90 F, 60% relative humidity. Find the power needed for the electric heater and the heat transfer to the flow from state 1 to state 4.

State 1: F.7.1: P_{g1} = 1.695 psia, h_{g1} = 1113.54 Btu/lbm

$$P_{v1} = \phi\, P_{g1} = 0.1 \times 1.695 = 0.1695 \text{ psia}$$

$$w_1 = 0.622 \frac{P_{v1}}{P_{tot} - P_{v1}} = 0.622 \frac{0.1695}{14.7 - 0.1695} = 0.00726$$

Starte 2: h_f = 28.08 Btu/lbm ;   State 2a: $h_{g\,212}$ = 1150.49 Btu/lbm

State 4: P_{g4} = 0.699 psia, h_{g4} = 1100.72 Btu/lbm

$$P_{v4} = \phi\, P_{g4} = 0.6 \times 0.699 = 0.4194 \text{ psia}$$

$$w_4 = 0.622 \frac{P_{v4}}{P_{tot} - P_{v4}} = 0.622 \frac{0.4194}{14.7 - 0.4194} = 0.0183$$

$$\dot{m}_{liq} = \dot{m}_a\,(\omega_1 - \omega_4) = 0.1\,(0.0183 - 0.00726) = 0.0011 \text{ lbm/s}$$

Energy Eq. for heater:

$$\dot{Q}_{heater} = \dot{m}_{liq}\,(h_{out} - h_{in}) = 0.0011\,(1150.49 - 28.08)$$

$$= \textbf{1.235 Btu/s} = \textbf{1.17 kW}$$

Energy Eq. for line (excluding the heater):

$$\dot{Q}_{line} = \dot{m}_a\,(h_{a4} + w_4 h_{g4} - h_{a1} - w_1 h_{g1}) - \dot{m}_{liq}\, h_{g\,212}$$

$$= 0.1[\,0.24(90 - 120) + 0.0183 \times 1100.72 - 0.00726 \times 1113.54\,]$$

$$- 0.0011 \times 1150.49$$

$$= \textbf{--0.78 Btu/s}$$

11.180E

Atmospheric air at 95 F, relative humidity of 10%, is too warm and also too dry. An air conditioner should deliver air at 70 F and 50% relative humidity in the amount of 3600 ft^3 per hour. Sketch a setup to accomplish this, find any amount of liquid (at 68 F) that is needed or discarded and any heat transfer.

CV air conditioner. Check first the two states, inlet 1, exit 2.

In: $P_{g1} = 0.8246$ psia, $h_{g1} = 1102.9$ Btu/lbm, $h_{f,68} = 36.08$ Btu/lbm,

$P_{v1} = \phi_1 P_{g1} = 0.08246$ psia, $w_1 = 0.622 P_{v1}/(P_{tot}-P_{v1}) = 0.0035$

Ex: $P_{g2} = 0.36324$ psia, $h_{g2} = 1092$ Btu/lbm

$P_{v2} = \phi_2 P_{g2} = 0.1816$ psia, $w_2 = 0.622 P_{v2}/(P_{tot}-P_{v2}) = 0.00778$

Water must be added ($w_2 > w_1$). Continuity and energy equations

$$\dot{m}_A(1 + w_1) + \dot{m}_{liq} = \dot{m}_A(1 + w_2) \quad \& \quad \dot{m}_A h_{1mix} + \dot{m}_{liq}h_f + \dot{Q}_{CV} = \dot{m}_A h_{2mix}$$

$$\dot{m}_{tot} = P\dot{V}_{tot}/RT = 14.7 \times 3600 \times 144/53.34 \times 529.67 = 270 \text{ lbm/h}$$

$$\dot{m}_A = \dot{m}_{tot}/(1 + w_2) = 267.91 \text{ lbm/h}$$

$$\dot{m}_{liq} = \dot{m}_A(w_2 - w_1) = 267.91(0.00778 - 0.0035) = \textbf{1.147 lbm/h}$$

$$\dot{Q}_{CV} = \dot{m}_A[C_{p\,a} (T_2 – T_1) + w_2 h_{g2} - w_1 h_{g1}] - \dot{m}_{liq}h_{f,68}$$
$$= 267.91 [0.24(70 - 95) + 0.00778 \times 1092 – 0.0035 \times 1102.9]$$
$$- 1.147 \times 36.08$$
$$= \textbf{- 406.8 Btu/h}$$

11.181E

A commercial laundry runs a dryer that has an exit flow of 1 lbm/s moist air at 120 F, 70% relative humidity. To save on the heating cost a counterflow stack heat exchanger is used to heat a cold water line at 50 F for the washers with the exit flow, as shown in Fig. P11.105. Assume the outgoing flow can be cooled to 70 F. Is there a missing flow in the figure? Find the rate of energy recovered by this heat exchanger and the amount of cold water that can be heated to 85 F.

Dryer oulet, 1: 120 F, $\Phi = 70\%$

$\quad => h_{g1} = 1113.54$ Btu/lbm

$\quad P_{g1} = 1.695$ psia,

$P_{v1} = 0.7 \times 1.695$ psia $= 1.1865$ psia

$\omega_1 = \dfrac{0.622 \times 1.1865}{14.7 - 1.1865} = 0.0546$

State 2: $T_2 < T_{dew\,1} \approx 107$ F so $\Phi_2 = 100\%$. $P_{g2} = 0.363$ psia, $h_{g2} = 1092.04$ Btu/lbm

$\qquad h_{f2} = 38.09$ Btu/lbm, $\omega_2 = 0.622 \times 0.363/(14.7 - 0.363) = 0.01575$

Continuity Eq. water 1-2 line: $\dot{m}_a\,\omega_1 = \dot{m}_a\,\omega_2 + \dot{m}_{liq}$; $\dot{m}_{liq} = \dot{m}_a(\omega_1 - \omega_2)$

The mass flow rate of dry air is

$$\dot{m}_a = \dot{m}_{moist\ air}\,/(1 + \omega_1) = 1 \text{ lbm/s } /(1 + 0.0546) = 0.948 \text{ lbm/s}$$

The heat out of the exhaust air which also equals the energy recovered becomes

$$\dot{Q}_{CV} = \dot{m}_a\,[\,C_{p\ a}(T_1 - T_2) + \omega_1\,h_{g1} - (\omega_1 - \omega_2)\,h_{f2} - \omega_2\,h_{g2}\,]$$

$$= 0.948\,[0.24(120 - 70) + 0.0546 \times 1113.54 - 0.03885 \times 38.09$$

$$-\quad 0.01575 \times 1092.04]$$

$$= 0.948 \text{ lbm/s} \times 54.12 \text{ Btu/lbm} = \textbf{51.3 Btu/s}$$

$$\dot{m}_{liq} = \dot{Q}_{CV}\,/\,(\,C_{P\ liq}\,\Delta T_{liq}) = 51.3\,/\,[1.0\,(85 - 50)] = \textbf{1.466 lbm/s}$$

11.182E

An indoor pool evaporates 3 lbm/h of water, which is removed by a dehumidifier to maintain 70 F, $\Phi = 70\%$ in the room. The dehumidifier is a refrigeration cycle in which air flowing over the evaporator cools such that liquid water drops out, and the air continues flowing over the condenser, as shown in Fig. P11.107. For an air flow rate of 0.2 lbm/s the unit requires 1.2 Btu/s input to a motor driving a fan and the compressor and it has a coefficient of performance, $\beta = Q_L/W_C = 2.0$. Find the state of the air after the evaporator, T_2, ω_2, Φ_2 and the heat rejected. Find the state of the air as it returns to the room and the compressor work input.

The unit must remove 3 lbm/h liquid to keep steady state in the room. As water condenses out state 2 is saturated.

1: 70 F, 70% => $P_{g1} = 0.363$ psia, $h_{g1} = 1092.0$ Btu/lbm,

$$P_{v1} = \phi_1 \, P_{g1} = 0.2541 \text{ psia}, \quad w_1 = 0.622 \, P_{v1}/(P_{tot}-P_{v1}) = 0.01094$$

CV 1 to 2: $\dot{m}_{liq} = \dot{m}_a(w_1 - w_2)$ => $w_2 = w_1 - \dot{m}_{liq}/\dot{m}_a$

$$q_L = h_1 - h_2 - (w_1 - w_2) \, h_{f2}$$

$$w_2 = 0.01094 - 3/(3600 \times 0.2) = \mathbf{0.006774}$$

$$P_{v2} = P_{g2} = P_{tot} \, w_2 /(0.622 + w_2) = \frac{14.7 \times 0.006774}{0.628774} = 0.1584 \text{ psia}$$

Table F.7.1: $T_2 = \mathbf{46.8\ F}$ $h_{f2} = 14.88$ btu/lbm, $h_{g2} = 1081.905$ Btu/lbm

$$q_L = 0.24(70 - 46.8) + 0.01094 \times 1092 - 0.006774 \times 1081.905$$

$$- 0.00417 \times 14.88 = 10.12 \text{ Btu/lbm dry air}$$

$$\dot{W}_c = \dot{m}_a \, q_L / \beta = \mathbf{1\ Btu/s}$$

CV Total system :

$$\tilde{h}_3 - \tilde{h}_1 = \dot{W}_{el}/\dot{m}_a - (w_1-w_2) \, h_f = 1.2/0.2 - 0.062 = 5.938 \text{ Btu/lbm dry air}$$

$$= C_{p\,a} (T_3 - T_1) + w_2 h_{v3} - w_1 h_{v1}$$

Trial and error on T_3

3: $w_3 = w_2$, h_3 => $T_3 = \mathbf{112\ F}$, $P_{g3} = 1.36$ psia, $P_{v3} = P_{v2} = 0.1584$

$$\phi_3 = P_{v3}/P_{g3} = 0.12 \quad \text{or} \quad \phi_3 = \mathbf{12\%}$$

11.183E

To refresh air in a room, a counterflow heat exchanger is mounted in the wall, as shown in Fig. P11.122. It draws in outside air at 33 F, 80% relative humidity and draws room air, 104 F, 50% relative humidity, out. Assume an exchange of 6 lbm/min dry air in a steady flow device, and also that the room air exits the heat exchanger to the atmosphere at 72 F. Find the net amount of water removed from room, any liquid flow in the heat exchanger and (T, ϕ) for the fresh air entering the room.

State 3: $P_{g3} = 1.0804$ psia, $h_{g3} = 1106.73$ Btu/lbm,

$P_{v3} = \phi_3 P_{g3} = 0.5402$, $w_3 = 0.622 P_{v3}/(P_{tot}-P_{v3}) = 0.02373$

The room air is cooled to 72 F $< T_{dew1} = 82$ F so liquid will form in the exit flow channel and state 4 is saturated.

4: 72 F, $\phi = 100\%$ => $P_{g4} = 0.3918$ psia, $h_{g4} = 1092.91$ Btu/lbm,

$w_4 = 0.017$, $h_{f4} = 40.09$ Btu/lbm

1: 33 F, $\phi = 80\%$ => $P_{g1} = 0.0925$ psia, $h_{g1} = 1075.83$ Btu/lbm,

$P_{v1} = \phi_1 P_{g1} = 0.074$ psia, $w_1 = 0.00315$

CV 3 to 4: $\dot{m}_{liq,4} = \dot{m}_a (w_3 - w_4) = 6 (0.02373 - 0.017) = $ **0.04 lbm/min**

CV room: $\dot{m}_{v,out} = \dot{m}_a (w_3 - w_2) = \dot{m}_a (w_3 - w_1)$

$= 6(0.02373 - 0.00315) = $ **0.1235 lbm/min**

CV Heat exchanger: $\dot{m}_a(\tilde{h}_2 - \tilde{h}_1) = \dot{m}_a(\tilde{h}_3 - \tilde{h}_4) - \dot{m}_{liq}h_{f4}$

$C_{p\,a}(T_2–T_1) + w_2h_{v2} – w_1h_{v1} = C_{p\,a}(T_3–T_4) + w_3h_{v3} - w_4h_{v4} - (w_3-w_4) h_{f4}$

$0.24(T_2–33) + w_2h_{v2} – 3.3888 = 0.24(104-72) + 26.2627 – 18.5795 – 0.2698$

$0.24 T_2 + 0.00315 h_{v2} = 26.402$ btu/lbm

Trial and error on T_2: $T_2 = $ **95.5 F**, $P_{g2} = 0.837$ psia, $P_{v2} = P_{v1}$

$\phi = P_{v2} / P_{g2} = 0.074 / 0.837 = 0.088$ or $\phi = $ **9%**

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Review Problems

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11.184E

Weighing of masses gives a mixture at 80 F, 35 lbf/in.^{2} with 1 lbm O_2, 3 lbm N_2 and 1 lbm CH_4. Find the partial pressures of each component, the mixture specific volume (mass basis), mixture molecular weight and the total volume.

From Eq. 11.4: $y_i = (m_i /M_i) / \sum m_j/M_j$

$n_{tot} = \sum m_j/M_j = (1/31.999) + (3/28.013) + (1/16.04)$

$= 0.031251 + 0.107093 + 0.062344 = 0.200688$

$y_{O2} = 0.031251/0.200688 = 0.1557,$ $y_{N2} = 0.107093/0.200688 = 0.5336,$

$y_{CH4} = 0.062344/0.200688 = 0.3107$

From Eq.11.10:

$P_{O2} = y_{O2} P_{tot} = 0.1557 \times 35 =$ **5.45 lbf/in.^{2}**,

$P_{N2} = y_{N2} P_{tot} = 0.5336 \times 35 =$ **18.676 lbf/in.^{2}**,

$P_{CH4} = y_{CH4} P_{tot} = 0.3107 \times 35 =$ **10.875 lbf/in.^{2}**

$V_{tot} = n_{tot} \bar{R}T/P = 0.200688 \times 1545 \times 539.7 / (35 \times 144) =$ **33.2 ft^3**

$v = V_{tot}/m_{tot} = 33.2 / (1 + 3 + 1) =$ **6.64 ft^3/lbm**

From Eq.11.5:

$M_{mix} = \sum y_jM_j = m_{tot}/n_{tot} = 5/0.200688 =$ **24.914 lbm/lbmol**

11.185E

A mixture of 50% carbon dioxide and 50% water by mass is brought from 2800 R, 150 lbf/in.^{2} to 900 R, 30 lbf/in.^{2} in a polytropic process through a steady flow device. Find the necessary heat transfer and work involved using values from F.4.

Process $Pv^n = $ constant   leading to

$$n \ln(v_2/v_1) = \ln(P_1/P_2); \quad v = RT/P$$

$$n = \ln(150/30) \, / \, \ln(900 \times 150/30 \times 2800) = 3.3922$$

$$R_{mix} = \Sigma c_i R_i = (0.5 \times 35.1 + 0.5 \times 85.76)/778 = 0.07767 \text{ Btu/lbm R}$$

$$C_{P\,mix} = \Sigma c_i C_{Pi} = 0.5 \times 0.203 + 0.5 \times 0.445 = 0.324 \text{ Btu/lbm R}$$

Work is from Eq.7.18:

$$w = -\int v dP = -\frac{n}{n\text{-}1}\,(P_e v_e - P_i v_i) = -\frac{nR}{n\text{-}1}\,(T_e - T_i)$$

$$= -\frac{3.3922 \times 0.07767}{2.3922}\,(900 - 2800) = \mathbf{209.3}\,\frac{\textbf{Btu}}{\textbf{lbm}}$$

Heat transfer from the energy equation

$$q = h_e - h_i + w = C_P(T_e - T_i) + w = \textbf{-406.3 Btu/lbm}$$

11.186E

A large air separation plant takes in ambient air (79% N_2, 21% O_2 by volume) at 14.7 lbf/in.^{2}, 70 F, at a rate of 2 lb mol/s. It discharges a stream of pure O_2 gas at 30 lbf/in.^{2}, 200 F, and a stream of pure N_2 gas at 14.7 lbf/in.^{2}, 70 F. The plant operates on an electrical power input of 2000 kW. Calculate the net rate of entropy change for the process.

Air 79 % N_2
21 % O_2
P_1 = 14.7 psia
T_1 = 70 F

$\dot{n}_1$ = 2 lbmol/s

1   2   pure O_2 P_2 = 30 psia
          T_2 = 200 F
pure N_2 P_3 = 14.7 psia
3   T_3 = 70 F

$-\dot{W}_{IN}$ = 2000 kW

$$\dot{S}_{gen} = -\frac{\dot{Q}_{CV}}{T_0} + \sum_i \dot{n}_i \Delta \bar{s}_i = -\frac{\dot{Q}_{CV}}{T_0} + (\dot{n}_2 \bar{s}_2 + \dot{n}_3 \bar{s}_3 - \dot{n}_1 \bar{s}_1)$$

The energy equation, Eq.4.10 gives the heat transfer rate as

$$\dot{Q}_{CV} = \Sigma \dot{n} \Delta \bar{h}_i + \dot{W}_{CV} = \dot{n}_{O_2} \bar{C}_{P0\,O_2}(T_2 - T_1) + \dot{n}_{N_2} \bar{C}_{P0\,N_2}(T_3 - T_1) + \dot{W}_{CV}$$

$$= 0.21 \times 2 \times [32 \times 0.213 \times (200 - 70)] + 0 - 2000 \times 3412/3600$$

$$= +382.6 - 1895.6 = -1513 \text{ Btu/s}$$

Use Eq.6.16 (see page 295 also) for the entropy change

$$\Sigma \dot{n}_i \Delta \bar{s}_i = 0.21 \times 2 \left[32 \times 0.219 \ln \frac{660}{530} - \frac{1545}{778} \ln \frac{30}{0.21 \times 14.7} \right]$$

$$+ 0.79 \times 2 \left[0 - \frac{1545}{778} \ln \frac{14.7}{0.79 \times 14.7} \right]$$

$$= -1.9906 \text{ Btu/R s}$$

$$\dot{S}_{gen} = +\frac{1513}{530} - 1.9906 = \mathbf{0.864 \text{ Btu/R s}}$$

11.187E

Ambient air is at a condition of 14.7 lbf/in.^{2}, 95 F, 50% relative humidity. A steady stream of air at 14.7 lbf/in.^{2}, 73 F, 70% relative humidity, is to be produced by first cooling one stream to an appropriate temperature to condense out the proper amount of water and then mix this stream adiabatically with the second one at ambient conditions. What is the ratio of the two flow rates? To what temperature must the first stream be cooled?

$$P_1 = P_2 = P_5 = 14.7 \text{ lbf/in}^2$$
$$T_1 = T_2 = 95 \text{ F}$$
$$\phi_1 = \phi_2 = 0.50, \quad \phi_4 = 1.0$$
$$T_5 = 73 \text{ F}, \phi_5 = 0.70$$

$$P_{v1} = P_{v2} = 0.5 \times 0.8246 = 0.4123, \quad w_1 = w_2 = 0.622 \times \frac{0.4123}{14.7 - 0.4123} = 0.0179$$

$$P_{v5} = 0.7 \times 0.4064 = 0.2845 \quad => \quad w_5 = 0.622 \times \frac{0.2845}{14.7 - 0.2845} = 0.0123$$

MIX: Call the mass flow ratio $r = m_{a2}/m_{a1}$

Conservation of water mass: $w_1 + r\,w_4 = (1 + r)w_5$

Energy Eq.: $h_{a1} + w_1 h_{v1} + r h_{a4} + r w_4 h_{v4} = (1 + r)h_{a5} + (1 + r)w_5 h_{v5}$

$$\rightarrow \quad 0.0179 + r w_4 = (1 + r) \, 0.0123$$

or $r = \dfrac{0.0179 - 0.0123}{0.0123 - w_4}$, with $w_4 = 0.622 \times \dfrac{P_{G4}}{14.7 - P_{G4}}$

$$0.24 \times 555 + 0.0179 \times 1107.2 + r \times 0.24 \times T_4 + r w_4 h_{v4}$$
$$= (1 + r) \times 0.24 \times 533 \; + (1 + r) \times 0.0123 \times 1093.3$$

or $r\left[0.24 \times T_4 + w_4 h_{G4} - 141.4 \right] + 11.66 = 0$

Assume $T_4 = 40$ F $\rightarrow P_{G4} = 0.121\ 66$ psia, $h_{G4} = 1078.9$ Btu/lbm

$$w_4 = 0.622 \times \frac{0.121\ 66}{14.7 - 0.121\ 66} = 0.0052$$

$$\frac{m_{a2}}{m_{a1}} = \frac{0.0179 - 0.0123}{0.0123 - 0.0052} = \mathbf{0.7887}$$

$$0.7887[0.24 \times 500 + 0.0052 \times 1078.9 - 141.4] + 11.66 = -0.29 \approx 0 \quad \text{OK}$$
$$=> \quad T_4 = \mathbf{40\ F}$$

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Fundamentals of
Thermodynamics

SOLUTION MANUAL
CHAPTER 12

8e

Updated June 2013

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Fundamentals of Thermodynamics 8th Edition
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CONTENT CHAPTER 12

The following table gives the values for the compressibility, enthalpy departure and the entropy departure along the saturated liquid-vapor boundary. These are used for all the problems using generalized charts as the figures are very difficult to read accurately (consistently) along the saturated liquid line. It is suggested that the instructor hands out copies of this page or let the students use the computer for homework solutions.

T_r	P_r	Z_f	Z_g	$d(h/RT)_f$	$d(h/RT)_g$	$d(s/R)_f$	$d(s/R)_g$
0.96	0.78	0.14	0.54	3.65	1.39	3.45	1.10
0.94	0.69	0.12	0.59	3.81	1.19	3.74	0.94
0.92	0.61	0.10	0.64	3.95	1.03	4.00	0.82
0.90	0.53	0.09	0.67	4.07	0.90	4.25	0.72
0.88	0.46	0.08	0.70	4.17	0.78	4.49	0.64
0.86	0.40	0.07	0.73	4.26	0.69	4.73	0.57
0.84	0.35	0.06	0.76	4.35	0.60	4.97	0.50
0.82	0.30	0.05	0.79	4.43	0.52	5.22	0.45
0.80	0.25	0.04	0.81	4.51	0.46	5.46	0.39
0.78	0.21	0.035	0.83	4.58	0.40	5.72	0.35
0.76	0.18	0.03	0.85	4.65	0.34	5.98	0.31
0.74	0.15	0.025	0.87	4.72	0.29	6.26	0.27
0.72	0.12	0.02	0.88	4.79	0.25	6.54	0.23
0.70	0.10	0.017	0.90	4.85	0.21	6.83	0.20
0.68	0.08	0.014	0.91	4.92	0.18	7.14	0.17
0.66	0.06	0.01	0.92	4.98	0.15	7.47	0.15
0.64	0.05	0.009	0.94	5.04	0.12	7.81	0.12
0.60	0.03	0.005	0.95	5.16	0.08	8.56	0.08
0.58	0.02	0.004	0.96	5.22	0.06	8.97	0.07
0.54	0.01	0.002	0.98	5.34	0.03	9.87	0.04
0.52	0.0007	0.0014	0.98	5.41	0.02	10.38	0.03

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In-Text Concept Questions

12.a

Mention two uses of the Clapeyron equation.

If you have experimental information about saturation properties down to a certain temperature Clapeyron equation will allow you to make an intelligent curve extrapolation of the saturated pressure versus temperature function $P_{sat}(T)$ for lower temperatures.

From Clapeyrons equation we can calculate a heat of evaporation, heat of sublimation or heat of fusion based on measurable properties P, T and v. The similar changes in entropy are also obtained since

$$h_{fg} = Ts_{fg}; \qquad h_{if} = Ts_{if}; \qquad h_{ig} = Ts_{ig}$$

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12.b

If I raise the temperature in a constant pressure process, does g go up or down?

From the definition and variation in Gibbs function, see Eq.12.15 and Maxwells relation Eq.12.21 last one, we get

$$dg = -s \, dT$$

so Gibbs function decreases as temperature increases.

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12.c

If I raise the pressure in an isentropic process, does *h* go up or down? Is that independent upon the phase?

Tds = 0 = dh – vdP , so h increases as P increases, for any phase. The magnitude is proportional to v (i.e. large for vapor and small for liquid and solid phases)

12.d

If I raise the pressure in a solid at constant *T*, does *s* go up or down?

In Example 12.4, it is found that change in s with P at constant T is negatively related to volume expansivity (a positive value for a solid),

$$ds_T = - v \, \alpha_P \, dP_T$$

so raising P decreases s.

12.e

What does it imply if the compressibility factor is larger than 1?

A compressibility factor that is greater than one comes from domination of intermolecular forces of repulsion (short range) over forces of attraction (long range) – either high temperature or very high density. This implies that the density is lower than what is predicted by the ideal gas law, the ideal gas law assumes the molecules (atoms) can be pressed closer together.

12.f

What is the benefit of the generalized charts? Which properties must be known besides the charts themselves?

The generalized charts allow for the approximate calculations of enthalpy and entropy changes (and P,v,T behavior), for processes in cases where specific data or equation of state are not known. They also allow for approximate phase boundary determinations.

It is necessary to know the critical pressure and temperature, as well as ideal-gas specific heat.

Concept-Study Guide Problems

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12.1

The slope dP/dT of the vaporization line is finite as you approach the critical point, yet h_{fg} and v_{fg} both approach zero. How can that be?

The slope is $\left(\dfrac{dP}{dT}\right)_{sat} = \dfrac{h_{fg}}{Tv_{fg}}$

Recall the math problem what is the limit of $f(x)/g(x)$ when x goes towards a point where both functions f and g goes towards zero. A finite limit for the ratio is obtained if both first derivatives are different from zero so we have

$$dP/dT \rightarrow [dh_{fg}/dT] \Big/ d(Tv_{fg})/dT \quad \text{as} \quad T \rightarrow T_c$$

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12.2

In view of Clapeyron's equation and Fig. 2.4, is there something special about ice I versus the other forms of ice?

Yes. The slope of the phase boundary dP/dT is negative for ice I to liquid whereas it is positive for all the other ice to liquid interphases. This also means that these other forms of ice are all heavier than liquid water. The pressure must be more than 200 MPa ≈ 2000 atm so even the deepest ocean cannot reach that pressure (recall about 1 atm per 10 meters down).

12.3

If we take a derivative as $(\partial P/\partial T)_v$ in the two-phase region, see Figs. 2.7 and 2.8, does it matter what v is? How about T?

In the two-phase region, P is a function only of T, and not dependent on v. The slope is the same at a given T regardless of v. The slope becomes higher with higher T and generally is the highest near the critical point.

12.4

Sketch on a P-T diagram how a constant v line behaves in the compressed liquid region, the two-phase L-V region and the superheated vapor region?

The figures are for water where liquid is denser than solid.

12.5

If I raise the pressure in an isothermal process does h go up or down for a liquid or solid? What do you need to know if it is a gas phase?

Eq. 12.25: $\left(\dfrac{\partial h}{\partial P}\right)_T = v - T \left(\dfrac{\partial v}{\partial T}\right)_P = v[1 - T\alpha_P]$

Liquid or solid, α_P is very small, h increases with P ;

For a gas, we need to know the equation of state.

12.6

The equation of state in Example 12.3 was used as explicit in v. Is it explicit in P?

Yes, the equation can be written explicitly in P.

$$P = \frac{RT}{v + \dfrac{C}{T^3}}$$

12.7

Over what range of states are the various coefficients in Section 12.5 most useful?

For solids or liquids, where the coefficients are essentially constant over a wide range of P's and T's.

12.8

For a liquid or a solid is v more sensitive to T or P? How about an ideal gas?

For a liquid or solid, v is much more sensitive to T than P.
For an ideal gas, v = RT/P , varies directly with T, inversely with P.

12.9

Most equations of state are developed to cover which range of states?

Most equations of state are developed to cover the gaseous phase, from low to moderate densities. Many cover high-density regions as well, including the compressed liquid region. To cover a wider region the EOS must be more complex and usually has many terms so it is only useful on a computer.

12.10

Is an equation of state valid in the two-phase regions?

No. In a two-phase region, P depends only on T. There is a discontinuity at each phase boundary. It is actually difficult to determine the phase boundary from the EOS.

12.11

As $P \rightarrow 0$, the specific volume $v \rightarrow \infty$. For $P \rightarrow \infty$, does $v \rightarrow 0$?

At very low P, the substance will be essentially an ideal gas, $Pv = RT$, so that v becomes very large. However at very high P, the substance eventually must become a solid, which cannot be compressed to a volume approaching zero.

12.12

Must an equation of state satisfy the two conditions in Eqs. 12.49 and 12.50?

It has been observed from experimental measurements that substances do behave in that manner. If an equation of state is to be accurate in the near-critical region, it would have to satisfy these two conditions.

If the equation is simple it may be overly restrictive to impose these as it may lead to larger inaccuracies in other regions.

12.13

At which states are the departure terms for h and s small? What is Z there?

Departure terms for h and s are small at very low pressure or at very high temperature. In both cases, Z is close to 1 and this is the ideal gas region.

12.14

The departure functions for h and s as defined are always positive. What does that imply for the real substance h and s values relative to ideal gas values?

Real-substance h and s are less than the corresponding ideal-gas values. This is true for the range shown in the figures, $P_r < 10$. For higher P the isotherms do bend down and negative values are possible.

Generally this means that there are slightly attractive forces between the molecules leading to some binding energy that shows up as a negative potential energy (thus smaller h and u) and it also give a little less chaos (smaller s) due to the stronger binding as compared to ideal gas. When the pressure becomes large the molecules are so close together that the attractive forces turns into repulsive forces and the departure terms are negative.

12.15

What is the benefit of Kay's rule versus a mixture equation of state?

Kay's rule for a mixture is not nearly as accurate as an equation of state for the mixture, but it is very simple to use and it is general. For common mixtures new mixture EOS are becoming available in which case they are preferable for a greater accuracy in the calculation.

Clapeyron Equation

12.16

An approximation for the saturation pressure can be $\ln P_{sat} = A - B/T$, where A and B are constants. Which phase transition is that suitable for, and what kind of property variations are assumed?

Clapeyron Equation expressed for the three phase transitions are shown in Eqs. 12.5-12.7. The last two leads to a natural log function if integrated and ideal gas for the vapor is assumed.

$$\frac{dP_{sat}}{dT} = P_{sat}\frac{h_{evap}}{RT^2}$$

where h_{evap} is either h_{fg} or h_{ig}. Separate the variables and integrate

$$P_{sat}^{-1}\, dP_{sat} = h_{evap}\, R^{-1}\, T^{-2}\, dT$$

$$\ln P_{sat} = A - B/T \; ; \qquad B = h_{evap}\, R^{-1}$$

if we also assume h_{evap} is constant and A is an integration constant. The function then applies to the liquid-vapor and the solid-vapor interphases with different values of A and B. As h_{evap} is not excactly constant over a wide interval in T it means that the equation cannot be used for the total domain.

12.17

Verify that Clapeyron's equation is satisfied for R-410A at 10$^\text{o}$C in Table B.4.

Clapeyron Eq.: $\qquad\qquad \dfrac{dP_{sat}}{dT} = \dfrac{dP_g}{dT} = \dfrac{h_{fg}}{Tv_{fg}}$

B.4: $\qquad$ P = 1085.7 kPa, $\quad$ h$_{fg}$ = 208.57 kJ/kg, $\quad$ v$_{fg}$ = 0.02295 m^3/kg

Slope around 10$^\text{o}$C best approximated by cord from 5$^\text{o}$C to 15$^\text{o}$C

$$\dfrac{dP_g}{dT} = \dfrac{1255.4 - 933.9}{15 - 5} = 32.15 \text{ kPa/K,}$$

$$\dfrac{h_{fg}}{Tv_{fg}} = \dfrac{208.57}{283.15 \times 0.02295} = 32.096 \text{ kPa/K}$$

This fits very well. Use CATT3 to do from 9 to 11$^\text{o}$C for better approximation as the saturated pressure is very non-linear in T.

12.18

In a Carnot heat engine, the heat addition changes the working fluid from saturated liquid to saturated vapor at T, P. The heat rejection process occurs at lower temperature and pressure $(T - \Delta T)$, $(P - \Delta P)$. The cycle takes place in a piston cylinder arrangement where the work is boundary work. Apply both the first and second law with simple approximations for the integral equal to work. Then show that the relation between ΔP and ΔT results in the Clapeyron equation in the limit $\Delta T \to dT$.

$$q_H = Ts_{fg}; \quad q_L = (T-\Delta T)s_{fg} ; \qquad w_{net} = q_H - q_L = \Delta Ts_{fg}$$

The boundary movement work, $\quad w = \int P dv$

$$w_{NET} = P(v_2-v_1) + \int_2^3 Pdv + (P - \Delta P)(v_4 - v_3) + \int_1^4 Pdv$$

Approximating,

$$\int_2^3 Pdv \approx \left(P - \frac{\Delta P}{2}\right)(v_3 - v_2); \qquad \int_1^4 Pdv \approx \left(P - \frac{\Delta P}{2}\right)(v_1 - v_4)$$

Collecting terms: $\quad w_{NET} \approx \Delta P\left[\left(\frac{v_2+v_3}{2}\right) - \left(\frac{v_1+v_4}{2}\right)\right]$

(the smaller the ΔP, the better the approximation)

$$\Rightarrow \frac{\Delta P}{\Delta T} \approx \frac{s_{fg}}{\frac{1}{2}(v_2 + v_3) - \frac{1}{2}(v_1 + v_4)}$$

In the limit as $\quad \Delta T \to 0$: $\quad v_3 \to v_2 = v_g ,\quad v_4 \to v_1 = v_f$

$$\& \lim_{\Delta T \to 0} \frac{\Delta P}{\Delta T} = \frac{dP_{sat}}{dT} = \frac{s_{fg}}{v_{fg}}$$

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12.19

Verify that Clapeyrons equation is satisfied for carbon dioxide at 6^oC in Table B.3.

Clapeyron Eq.:
$$\frac{dP_{sat}}{dT} = \frac{dP_g}{dT} = \frac{h_{fg}}{Tv_{fg}}$$

B.3: $P = 4072.0$ kPa, $h_{fg} = 211.59$ kJ/kg, $v_{fg} = 0.00732$ m^3/kg

Slope around 6^oC best approximated by cord from 4^oC to 8^oC

$$\frac{dP_g}{dT} = \frac{4283.1 - 3868.8}{8 - 4} = 103.575 \text{ kPa/K},$$

$$\frac{h_{fg}}{Tv_{fg}} = \frac{211.59}{279.15 \times 0.00732} \frac{\text{kJ/kg}}{\text{K m}^3\text{/kg}} = 103.549 \text{ kPa/K}$$

This fits very well.

12.20

Use the approximation given in problem 12.16 and Table B.1 to determine A and B for steam from properties at 25°C only. Use the equation to predict the saturation pressure at 30°C and compare to table value.

$$\ln P_{sat} = A - B/T \qquad \Rightarrow \qquad \frac{dP_{sat}}{dT} = P_{sat}\,(-B)(-T^{-2})$$

so we notice from Eq.12.7 and Table values from B.1.1 and A.5 that

$$B = \frac{h_{fg}}{R} = \frac{2442.3}{0.4615}\frac{kJ/kg}{kJ/kg\text{-}K} = 5292 \text{ K}$$

Now the constant A comes from the saturation pressure as

$$A = \ln P_{sat} + B/T = \ln 3.169 + \frac{5292}{273.15 + 25} = 18.9032$$

Use the equation to predict the saturation pressure at 30°C as

$$\ln P_{sat} = A - B/T = 18.9032 - \frac{5292}{273.15 + 30} = 1.4462$$
$$P_{sat} = 4.2469 \text{ kPa}$$

compare this with the table value of P_{sat} = 4.246 kPa and we have a very accurate approximation.

12.21

A certain refrigerant vapor enters a steady flow constant pressure condenser at 150 kPa, 70°C, at a rate of 1.5 kg/s, and it exits as saturated liquid. Calculate the rate of heat transfer from the condenser. It may be assumed that the vapor is an ideal gas, and also that at saturation, $v_f << v_g$. The following quantities are known for this refrigerant:

$$\ln P_g = 8.15 - 1000/T ; \qquad C_P = 0.7 \text{ kJ/kg K}$$

with pressure in kPa and temperature in K. The molecular mass is 100.

Solution:

Refrigerant: State 1 $T_1 = 70^oC$ $P_1 = 150$ kPa

State 2 $P_2 = 150$ kPa $x_2 = 1.0$ State 3 $P_3 = 150$ kPa $x_3 = 0.0$

Energy Eq.: $_1q_3 = h_3 - h_1 = (h_3 - h_2) + (h_2 - h_1) = - h_{fg\ T3} + C_{P0}(T_2 - T_1)$

Get the saturation temperature at the given pressure

$$\ln (150) = 8.15 - 1000/T_2 \ => \ T_2 = 318.5 \text{ K} = 45.3^oC = T_3$$

Now get the enthalpy of evaporation $h_{fg\ T3}$

$$\frac{dP_g}{dT} = \frac{h_{fg}}{Tv_{fg}}, \qquad v_{fg} \approx v_g = \frac{RT}{P_g}, \qquad \frac{dP_g}{dT} = P_g \frac{d \ln P_g}{dT} = \frac{h_{fg}}{RT^2} P_g$$

$$\frac{d \ln P_g}{dT} = +1000/T^2 = h_{fg}/RT^2$$

$$h_{fg} = 1000 \times R = 1000 \times 8.3145/100 = 83.15 \text{ kJ/kg}$$

Substitute into the energy equation

$$_1q_3 = -83.15 + 0.7(45.3 - 70) = -100.44 \text{ kJ/kg}$$

$$\dot{Q}_{COND} = 1.5(-100.44) = \textbf{-150.6 kW}$$

12.22

Calculate the values h_{fg} and s_{fg} for nitrogen at 70 K and at 110 K from the Clapeyron equation, using the necessary pressure and specific volume values from Table B.6.1.

Clapeyron equation Eq.12.7: $\qquad \dfrac{dP_g}{dT} = \dfrac{h_{fg}}{Tv_{fg}} = \dfrac{s_{fg}}{v_{fg}}$

For N$_2$ at 70 K, using values for P_g from Table B.6 at 75 K and 65 K, and also v_{fg} at 70 K,

$$h_{fg} \approx T(v_g\text{-}v_f)\frac{\Delta P_g}{\Delta T} = 70(0.525\ 015)\left(\frac{76.1\text{-}17.41}{75\text{-}65}\right) = 215.7 \text{ kJ/kg}$$

$s_{fg} = h_{fg}/T = 3.081$ kJ/kg K

Table B.6.1: $\quad h_{fg} = 207.8$ kJ/kg $\quad$ and $\quad s_{fg} = 2.97$ kJ/kg K

Comparison not very close because P_g not linear function of T. Using 71 K & 69 K from the software, we can then get

$$h_{fg} = 70(0.525\ 015)\left(\frac{44.56\text{-}33.24}{71\text{-}69}\right) = 208.0 \text{ kJ/kg}$$

At 110 K,

$$h_{fg} \approx 110(0.014\ 342)\left(\frac{1938.8\text{-}1084.2}{115\text{-}105}\right) = 134.82 \text{ kJ/kg}$$

$$s_{fg} = h_{fg}/T = \frac{134.82}{110} = 1.226 \text{ kJ/kg K}$$

Table B.6.1: $\quad h_{fg} = 134.17$ kJ/kg $\quad$ and $\quad s_{fg} = 1.22$ kJ/kg K

12.23

Find the saturation pressure for refrigerant R-410A at -80°C assuming it is higher than the triple point temperature.

The lowest temperature in Table B.4 for R-410A is -60°C, so it must be extended to -80°C using the Clapeyron Eq. 12.7 integrated as in example 12.1

Table B.4 at $T_1 = -60^{\circ}$C = 213.15 K:

$$P_1 = 64.1 \text{ kPa}, \quad R = 0.1145 \text{ kJ/kg-K}$$

$$\ln\frac{P}{P_1} = \frac{h_{fg}}{R}\frac{(T - T_1)}{T \times T_1} = \frac{279.96}{0.1145}\frac{(193.15 - 213.15)}{193.15 \times 213.15} = -1.1878$$

$$P = 64.1 \exp(-1.1878) = \textbf{19.54 kPa}$$

12.24

Ammonia at –70°C is used in a special application at a quality of 50%. Assume the only table available is B.2 that goes down to –50°C. To size a tank to hold 0.5 kg with x = 0.5, give your best estimate for the saturated pressure and the tank volume.

To size the tank we need the volume and thus the specific volume. If we do not have the table values for v_f and v_g we must estimate those at the lower T. We therefore use Clapeyron equation to extrapolate from –50°C to –70°C to get the saturation pressure and thus v_g assuming ideal gas for the vapor.

The values for v_f and h_{fg} do not change significantly so we estimate

Between -50°C and –70°C: $h_{fg} = 1430$ kJ/kg

and at –70°C we get: $v_f = 0.001375$ m³/kg

The integration of Eq.12.7 is the same as in Example 12.1 so we get

$$\ln \frac{P_2}{P_1} = \frac{h_{fg}}{R} \left(\frac{T_2 - T_1}{T_2 T_1} \right) = \frac{1430}{0.4882} \frac{-70 + 50}{203.15 \times 223.15} = -1.2923$$

$$P_2 = P_1 \exp(-1.2923) = 40.9 \exp(-1.2923) = 11.2 \text{ kPa}$$

$$v_g = RT_2/P_2 = \frac{0.4882 \times 203.15}{11.2} = 8.855 \text{ m}^3/\text{kg}$$

$$v_2 = (1\text{-}x) \, v_f + x \, v_g = 0.5 \times 0.001375 + 0.5 \times 8.855 = 4.428 \text{ m}^3/\text{kg}$$

$$V_2 = mv_2 = \textbf{2.214 m}^3$$

A straight line extrapolation will give a negative pressure.

12.25

Use the approximation given in problem 12.16 and Table B.4 to determine A and B for refrigerant R-410A from properties at 0^oC only. Use the equation to predict the saturation pressure at 5^oC and compare to table value.

$$\ln P_{sat} = A - B/T \qquad \Rightarrow \qquad \frac{dP_{sat}}{dT} = P_{sat}(-B)(-T^{-2})$$

so we notice from Eq.12.7 and Table values from B.4.1 and A.5 that

$$B = \frac{h_{fg}}{R} = \frac{221.37}{0.1145}\frac{kJ/kg}{kJ/kg\text{-}K} = 1933.4 \text{ K}$$

Now the constant A comes from the saturation pressure as

$$A = \ln P_{sat} + B/T = \ln 798.7 + \frac{1933.4}{273.15} = 13.7611$$

Use the equation to predict the saturation pressure at 5^oC as

$$\ln P_{sat} = A - B/T = 13.7611 - \frac{1933.4}{273.15 + 5} = 6.8102$$

$$P_{sat} = 907 \text{ kPa}$$

compare this with the table value of $P_{sat} = 933.9$ kPa and we have an approximation 3% low. Notice h_{fg} decreases so we could have used a lower value for the average in the interval.

12.26

The triple point of CO_2 is -56.4^{o}C. Predict the saturation pressure at that point using Table B.3.

The lowest temperature in Table B.3 for CO_2 is -50°C, so it must be extended to -56.4^{o}C = 216.75 K using the Clapeyron Eq. 12.7 integrated as in Ex. 12.1

Table B.3: at T_1 = -50°C = 223.15 K, P_1 = 682.3 kPa, h_{fg} = 339.73 kJ/kg

Table A.5:   R = 0.1889 kJ/kg-K

$$\ln \frac{P}{P_1} = \frac{h_{fg}}{R} \frac{(T - T_1)}{T \times T_1} = \frac{339.73}{0.1889} \frac{(216.75 - 223.15)}{216.75 \times 223.15} = \text{-0.23797}$$

$$P = 682.3 \exp(\text{-0.23797}) = \textbf{537.8 kPa}$$

Notice from Table 3.2: P = 520.8 kPa so we are 3% high. As h_{fg} becomes larger for lower T's we could have estimated a more suitable value for the interval from -50 to -56.4^{o}C

12.27

Helium boils at 4.22 K at atmospheric pressure, 101.3 kPa, with $h_{fg} = 83.3$ kJ/kmol. By pumping a vacuum over liquid helium, the pressure can be lowered, and it may then boil at a lower temperature. Estimate the necessary pressure to produce a boiling temperature of 1 K and one of 0.5 K.

Solution:

Helium at 4.22 K: $P_1 = 0.1013$ MPa, $\bar{h}_{FG} = 83.3$ kJ/kmol

$$\frac{dP_{SAT}}{dT} = \frac{h_{FG}}{Tv_{FG}} \approx \frac{h_{FG}P_{SAT}}{RT^2} \quad \Rightarrow \quad \ln\frac{P_2}{P_1} = \frac{h_{FG}}{R}\Big[\frac{1}{T_1} - \frac{1}{T_2}\Big]$$

For $T_2 = 1.0$ K:

$$\ln\frac{P_2}{101.3} = \frac{83.3}{8.3145}\Big[\frac{1}{4.22} - \frac{1}{1.0}\Big] \quad \Rightarrow \quad P_2 = 0.048 \text{ kPa} = \mathbf{48\ Pa}$$

For $T_2 = 0.5$ K:

$$\ln\frac{P_2}{101.3} = \frac{83.3}{8.3145}\Big[\frac{1}{4.22} - \frac{1}{0.5}\Big]$$

$$P_2 = 2.1601 \times 10^{-6} \text{ kPa} = \mathbf{2.1601 \times 10^{-3}\ Pa}$$

12.28

Using the properties of water at the triple point, develop an equation for the saturation pressure along the fusion line as a function of temperature.

Solution:

The fusion line is shown in Fig. 2.3 as the S-L interphase. From Eq.12.5 we have

$$\frac{dP_{fusion}}{dT} = \frac{h_{if}}{Tv_{if}}$$

Assume h_{if} and v_{if} are constant over a range of T's. We do not have any simple models for these as function of T other than curve fitting. Then we can integrate the above equation from the triple point (T_1, P_1) to get the pressure P(T) as

$$P - P_1 = \frac{h_{if}}{v_{if}} \ln \frac{T}{T_1}$$

Now take the properties at the triple point from B.1.1 and B.1.5

$$P_1 = 0.6113 \text{ kPa}, \quad T_1 = 273.16 \text{ K}$$

$$v_{if} = v_f - v_i = 0.001 - 0.0010908 = -9.08 \times 10^{-5} \text{ m}^3/\text{kg}$$

$$h_{if} = h_f - h_i = 0.0 - (-333.4) = 333.4 \text{ kJ/kg}$$

The function that approximates the pressure becomes

$$P = 0.6113 - 3.672 \times 10^6 \ln \frac{T}{T_1} \qquad [\text{kPa}]$$

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12.29

Using thermodynamic data for water from Tables B.1.1 and B.1.5, estimate the freezing temperature of liquid water at a pressure of 30 MPa.

$$H_2O \quad \frac{dP_{if}}{dT} = \frac{h_{if}}{Tv_{if}} \approx const$$

At the triple point,

$v_{if} = v_f - v_i = 0.001\ 000 - 0.001\ 090\ 8 = -0.000\ 090\ 8\ m^3/kg$

$h_{if} = h_f - h_i = 0.01 - (-333.40) = 333.41\ kJ/kg$

$$\frac{dP_{if}}{dT} = \frac{333.41}{273.16(-0.000\ 090\ 8)} = -13\ 442\ kPa/K$$

$\Rightarrow$ at P = 30 MPa,

$$T \approx 0.01 + \frac{(30\ 000-0.6)}{(-13\ 442)} = \textbf{-2.2\ }^{\textbf{o}}\textbf{C}$$

12.30

Ice (solid water) at $-3°C$, 100 kPa, is compressed isothermally until it becomes liquid. Find the required pressure.

Water, triple point $T = 0.01^OC$, $P = 0.6113$ kPa

Table B.1.1: $v_f = 0.001$ m^3/kg, $h_f = 0.01$ kJ/kg,

Tabel B.1.5: $v_i = 0.001\ 0908$ m^3/kg, $h_i = -333.4$ kJ/kg

Clapeyron $\dfrac{dP_{if}}{dT} = \dfrac{h_f - h_i}{(v_f - v_i)T} = \dfrac{333.4}{-0.0000908 \times 273.16} = -13\ 442$ kPa/K

$\Delta P \approx \dfrac{dP_{if}}{dT}\ \Delta T = -13\ 442(-3 - 0.01) = 40\ 460$ kPa

$P = P_{tp} + \Delta P =$ **40 461 kPa**

12.31

From the phase diagram for carbon dioxide in Fig. 2.5 and 2.4 for water what can you infer for the specific volume change during melting assuming the liquid has a higher h than the solid phase for those two substances.

The saturated pressure versus temperature has a positive slope for carbon dioxide and a negative slope for water.

$$\text{Clapeyron} \qquad \frac{dP_{if}}{dT} = \frac{h_f - h_i}{(v_f - v_i)T}$$

So if we assume $h_f - h_i > 0$ then we notice that the volume change in the melting gives

Water: $\qquad v_f - v_i < 0 \quad$ so $\quad v_f < v_i$

Carbon dioxide: $\qquad v_f - v_i > 0 \quad$ so $\quad v_f > v_i$

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12.32

A container has a double wall where the wall cavity is filled with carbon dioxide at room temperature and pressure. When the container is filled with a cryogenic liquid at 100 K the carbon dioxide will freeze so that the wall cavity has a mixture of solid and vapor carbon dioxide at the sublimation pressure. Assume that we do not have data for CO_2 at 100 K, but it is known that at $-90°C$: $P_{sat} = 38.1$ kPa, $h_{IG} = 574.5$ kJ/kg. Estimate the pressure in the wall cavity at 100 K.

Solution:

For CO_2 space: at $T_1 = -90\ ^oC = 183.2$ K , $P_1 = 38.1$ kPa, $h_{IG} = 574.5$ kJ/kg

For $T_2 = T_{cO2} = 100$ K: Clapeyron $\qquad \dfrac{dP_{SUB}}{dT} = \dfrac{h_{IG}}{Tv_{IG}} \approx \dfrac{h_{IG}P_{SUB}}{RT^2}$

$$\ln\dfrac{P_2}{P_1} = \dfrac{h_{IG}}{R}\left[\dfrac{1}{183.2} - \dfrac{1}{100}\right] = \dfrac{574.5}{0.188\ 92}\left[\dfrac{1}{183.2} - \dfrac{1}{100}\right] = -13.81$$

or $\quad P_2 = P_1 \times 1.005 \times 10^{-6} \quad \Rightarrow \quad P_2 = 3.83 \times 10^{-5}$ kPa $= \mathbf{3.83 \times 10^{-2}}$ **Pa**

12.33

Small solid particles formed in combustion should be investigated. We would like to know the sublimation pressure as a function of temperature. The only information available is T, h_{FG} for boiling at 101.3 kPa and T, h_{IF} for melting at 101.3 kPa. Develop a procedure that will allow a determination of the sublimation pressure, $P_{sat}(T)$.

T_{NBP} = normal boiling pt T.

T_{NMP} = normal melting pt T.

T_{TP} = triple point T.

1) $T_{TP} \approx T_{NMP}$

2) $$\int_{0.1013\,\text{MPa}}^{P_{TP}} (1/P_{SAT})\, dP_{SAT} \approx \int_{T_{NMP}}^{T_{TP}} \frac{h_{FG}}{RT^2}\, dT$$

Since $h_{FG} \approx const \approx h_{FG\,NBP}$ the integral over temperature becomes

$$\ln \frac{P_{TP}}{0.1013} \approx \frac{h_{FG\,NBP}}{R}\left[\frac{1}{T_{NBP}} - \frac{1}{T_{TP}}\right] \quad \rightarrow \quad \text{get } P_{TP}$$

3) $h_{IG\,at\,TP} = h_G - h_I = (h_G - h_F) + (h_F - h_I) \approx h_{FG\,NBP} + h_{IF\,NMP}$

Assume $h_{IG} \approx const.$ again we can evaluate the integral

$$\ln \frac{P_{SUB}}{P_{TP}} = \int_{P_{TP}}^{P_{SUB}} (1/P_{SUB})\, dP_{SUB} \approx \int_{T_{TP}}^{T} \frac{h_{IG}}{RT^2}\, dT \approx \frac{h_{IG}}{R}\left[\frac{1}{T_{TP}} - \frac{1}{T}\right]$$

or $P_{SUB} = fn(T)$

Property Relations

12.34

Use Gibbs relation $du = Tds - Pdv$ and one of Maxwell's relations to find an expression for $(\partial u/\partial P)_T$ that only has properties P, v and T involved. What is the value of that partial derivative if you have an ideal gas?

$du = Tds - Pdv$ divide this by dP   so we get

$$\left(\frac{\partial u}{\partial P}\right)_T = T\left(\frac{\partial s}{\partial P}\right)_T - P\left(\frac{\partial v}{\partial P}\right)_T = -T\left(\frac{\partial v}{\partial T}\right)_P - P\left(\frac{\partial v}{\partial P}\right)_T$$

where we have used Maxwell Eq.12.19. Now for an ideal gas we get

Ideal gas: $Pv = RT$ $\Rightarrow$ $v = \dfrac{RT}{P}$

then the derivatives are

$$\left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P} \qquad \text{and} \qquad \left(\frac{\partial v}{\partial P}\right)_T = -RTP^{-2}$$

and the derivative of u is

$$\left(\frac{\partial u}{\partial P}\right)_T = -T\left(\frac{\partial v}{\partial T}\right)_P - P\left(\frac{\partial v}{\partial P}\right)_T = -T\frac{R}{P} - P(-RTP^{-2}) = 0$$

This confirms that u is not sensitive to P and only a function of T.

12.35

The Joule-Thomson coefficient μ_J is a measure of the direction and magnitude of the temperature change with pressure in a throttling process. For any three properties x,y,z use the mathematical relation

$$\left(\frac{\partial x}{\partial y}\right)_Z \left(\frac{\partial y}{\partial z}\right)_X \left(\frac{\partial z}{\partial x}\right)_y = -1$$

to show the following relations for the Joule-Thomson coefficient:

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T\left(\frac{\partial v}{\partial T}\right)_P - v}{C_P} = \frac{RT^2}{PC_P}\left(\frac{\partial Z}{\partial T}\right)_P$$

Let x = T, y = P and z = h and substitute into the relations as:

$$\left(\frac{\partial T}{\partial P}\right)_h \left(\frac{\partial P}{\partial h}\right)_T \left(\frac{\partial h}{\partial T}\right)_P = -1$$

Then we have the definition of specific heat as $C_P = \left(\frac{\partial h}{\partial T}\right)_P$ so solve for the first term

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = -\frac{1}{C_P} \Big/ \left(\frac{\partial P}{\partial h}\right)_T = -\frac{1}{C_P}\left(\frac{\partial h}{\partial P}\right)_T$$

The last derivative is substituted with Eq.12.25 so we get

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T\left(\frac{\partial v}{\partial T}\right)_P - v}{C_P}$$

If we use the compressibility factor then we get

$$Pv = ZRT \qquad \Rightarrow \qquad \left(\frac{\partial v}{\partial T}\right)_P = \frac{ZR}{P} + \frac{RT}{P}\left(\frac{\partial Z}{\partial T}\right)_P = \frac{v}{T} + \frac{RT}{P}\left(\frac{\partial Z}{\partial T}\right)_P$$

so then

$$T\left(\frac{\partial v}{\partial T}\right)_P - v = v + \frac{RT^2}{P}\left(\frac{\partial Z}{\partial T}\right)_P - v = \frac{RT^2}{P}\left(\frac{\partial Z}{\partial T}\right)_P$$

and we have shown the last expression also.

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T\left(\frac{\partial v}{\partial T}\right)_P - v}{C_P} = \frac{RT^2}{PC_P}\left(\frac{\partial Z}{\partial T}\right)_P$$

12.36

Find the Joule-Thomson coefficient for an ideal gas from the expression given in Problem 12.35

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T\left(\frac{\partial v}{\partial T}\right)_P - v}{C_P} = \frac{RT^2}{PC_P}\left(\frac{\partial Z}{\partial T}\right)_P$$

For an ideal gas: $v = RT/P$ so then the partial derivative

$$\left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P} \qquad \Rightarrow \qquad T\left(\frac{\partial v}{\partial T}\right)_P - v = \frac{RT}{P} - v = v - v = 0$$

For an ideal gas $Z = 1$ so the very last derivative of Z is also zero.

12.37

Start from Gibbs relation dh = Tds + vdP and use one of Maxwell's equation to get $(\partial h/\partial v)_T$ in terms of properties P, v and T. Then use Eq.12.24 to also find an expression for $(\partial h/\partial T)_v$.

Find $\left(\dfrac{\partial h}{\partial v}\right)_T$ and $\left(\dfrac{\partial h}{\partial T}\right)_v$

dh = Tds + vdP   and use Eq.12.18

$\Rightarrow$ $\left(\dfrac{\partial h}{\partial v}\right)_T = T\left(\dfrac{\partial s}{\partial v}\right)_T + v\left(\dfrac{\partial P}{\partial v}\right)_T = T\left(\dfrac{\partial P}{\partial T}\right)_v + v\left(\dfrac{\partial P}{\partial v}\right)_T$

Also for the second first derivative use Eq.12.24

$\left(\dfrac{\partial h}{\partial T}\right)_v = T\left(\dfrac{\partial s}{\partial T}\right)_v + v\left(\dfrac{\partial P}{\partial T}\right)_v = C_v + v\left(\dfrac{\partial P}{\partial T}\right)_v$

12.38

From Eqs. 12.23 and 12.24 and the knowledge that $C_p > C_v$ what can you conclude about the slopes of constant v and constant P curves in a T-s diagram? Notice that we are looking at functions T(s, P or v given).

Solution:

The functions and their slopes are:

$$\text{Constant v:} \quad T(s) \quad \text{at that v} \quad \text{with slope} \quad \left(\frac{\partial T}{\partial s}\right)_v$$

$$\text{Constant P:} \quad T(s) \quad \text{at that P} \quad \text{with slope} \quad \left(\frac{\partial T}{\partial s}\right)_P$$

Slopes of these functions are now evaluated using Eq.12.23 and Eq.12.24 as

$$\left(\frac{\partial T}{\partial s}\right)_P = \left(\left(\frac{\partial s}{\partial T}\right)_P\right)^{-1} = \frac{T}{C_p}$$

$$\left(\frac{\partial T}{\partial s}\right)_v = \left(\left(\frac{\partial s}{\partial T}\right)_v\right)^{-1} = \frac{T}{C_v}$$

Since we know $C_p > C_v$ then it follows that $T/C_v > T/C_p$ and therefore

$$\left(\frac{\partial T}{\partial s}\right)_v \;>\; \left(\frac{\partial T}{\partial s}\right)_P$$

which means that constant v-lines are steeper than constant P lines in a T-s diagram.

12.39

Derive expressions for $(\partial T/\partial v)_u$ and for $(\partial h/\partial s)_v$ that do not contain the properties h, u, or s. Use Eq. 12.30 with du = 0.

$$\left(\frac{\partial T}{\partial v}\right)_u = -\left(\frac{\partial u}{\partial v}\right)_T \Big/ \left(\frac{\partial u}{\partial T}\right)_v = \frac{P - T\left(\frac{\partial P}{\partial T}\right)_v}{C_v} \qquad \text{(see Eqs. 12.33 and 12.34)}$$

As dh = Tds + vdP $\Rightarrow \left(\frac{\partial h}{\partial s}\right)_v = T + v\left(\frac{\partial P}{\partial s}\right)_v = T - v\left(\frac{\partial T}{\partial v}\right)_s$ (Eq.12.20)

But $\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial s}{\partial v}\right)_T \Big/ \left(\frac{\partial s}{\partial T}\right)_v = -\frac{T\left(\frac{\partial P}{\partial T}\right)_v}{C_v}$ (Eq.12.22)

$\Rightarrow \left(\frac{\partial h}{\partial s}\right)_v = T + \frac{vT}{C_v}\left(\frac{\partial P}{\partial T}\right)_v$

12.40

Evaluate the isothermal changes in the internal energy, the enthalpy and the entropy for an ideal gas. Confirm the results in Chapters 3 and 6.

We need to evaluate du_T, dh_T and ds_T for an ideal gas: $P = RT/v$.

From Eq.12.31 we get

$$du_T = [\ T\ \left(\frac{\partial P}{\partial T}\right)_v - P\ ]\ dv_T = [\ T\ (\frac{R}{v})\ - P\ ]\ dv_T = [\ P - P]\ dv_T = 0$$

From Eq.12.27 we get using $v = RT/P$

$$dh_T = [\ v - T\ \left(\frac{\partial v}{\partial T}\right)_P\ ]\ dP_T = [\ v - T\ (\frac{R}{P})\ ]\ dP_T = [\ v - v\ ]\ dP_T = 0$$

These two equations confirms the statements in chapter 5 that u and h are functions of T only for an ideal gas.

From Eq.12.32 or Eq.12.34 we get

$$ds_T = -\left(\frac{\partial v}{\partial T}\right)_P dP_T\ =\ \left(\frac{\partial P}{\partial T}\right)_v dv_T$$

$$= -\frac{R}{P}\ dP_T\ =\ \frac{R}{v}\ dv_T$$

so the change in s can be integrated to find

$$s_2 - s_1 = -R\ \ln\frac{P_2}{P_1} = R\ \ln\frac{v_2}{v_1} \qquad \text{when}\ \ T_2 = T_1$$

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12.41

Develop an expression for the variation in temperature with pressure in a constant entropy process, $(\partial T/\partial P)_s$, that only includes the properties P–v–T and the specific heat, C_p. Follow the development for Eq.12.32.

$$\left(\frac{\partial T}{\partial P}\right)_s = -\frac{\left(\frac{\partial s}{\partial P}\right)_T}{\left(\frac{\partial s}{\partial T}\right)_P} = -\frac{-\left(\frac{\partial v}{\partial T}\right)_P}{(C_P/T)} = \frac{T}{C_P}\left(\frac{\partial v}{\partial T}\right)_P$$

$$\left\{ \left(\frac{\partial s}{\partial P}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_P, \text{ Maxwell relation Eq. 12.23 and the other is Eq.12.27} \right\}$$

12.42

Use Eq. 12.34 to get an expression for the derivative $(\partial T/\partial v)_s$. What is the general shape of a constant s process curve in a T-v diagram? For an ideal gas can you say a little more about the shape?

Equation 12.34 says

$$ds = C_v \frac{dT}{T} + \left(\frac{\partial P}{\partial T}\right)_v dv$$

so then in a constant s process we have ds = 0 and we find

$$\left(\frac{\partial T}{\partial v}\right)_s = -\frac{T}{C_v}\left(\frac{\partial P}{\partial T}\right)_v$$

As T is higher the slope is steeper (but negative) unless the last term $(\partial P/\partial T)_v$ counteracts. If we have an ideal gas this last term can be determined

$$P = RT/v \qquad \Rightarrow \qquad \left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v}$$

$$\left(\frac{\partial T}{\partial v}\right)_s = -\frac{T}{C_v}\frac{R}{v} = -\frac{P}{C_v}$$

and we see the slope is steeper for higher P and a little lower for higher T as C_v is an increasing function of T.

12.43

Show that the P-v-T relation as $P(v - b) = RT$ satisfies the mathematical relation in Problem 12.35.

$$\left(\frac{\partial x}{\partial y}\right)_Z \left(\frac{\partial y}{\partial z}\right)_X \left(\frac{\partial z}{\partial x}\right)_y = -1$$

Let (x, y, z) be (P, v, T) so we have

$$\left(\frac{\partial P}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$$

The first derivative becomes, $P = RT/(v - b)$

$$\left(\frac{\partial P}{\partial v}\right)_T = -RT (v - b)^{-2} = -P/(v - b)$$

The second derivative, $v = b + RT/P$

$$\left(\frac{\partial v}{\partial T}\right)_P = R/P$$

The third derivative, $T = (P/R)(v - b)$

$$\left(\frac{\partial T}{\partial P}\right)_V = (v - b)/R$$

Substitute all three derivatives into the relation

$$\left(\frac{\partial P}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -\frac{RT}{(v - b)^2} \times \frac{R}{P} \times \frac{v - b}{R} = -\frac{RT}{(v - b)} \times \frac{1}{P} = -1$$

with the last one recognized as a rewrite of the original EOS.

Volume Expansivity and Compressibility

12.44

What are the volume expansivity α_p, the isothermal compressibility β_T, and the adiabatic compressibility β_s for an ideal gas?

The volume expansivity from Eq.12.37 and ideal gas $v = RT/P$ gives

$$\alpha_p = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = \frac{1}{v}\left(\frac{R}{P}\right) = \frac{1}{T}$$

The isothermal compressibility from Eq.12.38 and ideal gas gives

$$\beta_T = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T = -\frac{1}{v}\left(-RT\,P^{-2}\right) = \frac{1}{P}$$

The adiabatic compressibility β_s from Eq.12.40 and ideal gas

$$\beta_s = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_s$$

From Eq.12.32 we get for constant s (ds = 0)

$$\left(\frac{\partial T}{\partial P}\right)_s = \frac{T}{C_p}\left(\frac{\partial v}{\partial T}\right)_P = \frac{T}{C_p}\frac{R}{P} = \frac{v}{C_p}$$

and from Eq.12.34 we get

$$\left(\frac{\partial v}{\partial T}\right)_s = -\frac{C_v}{T}\left(\frac{\partial P}{\partial T}\right)_v = -\frac{C_v}{T}\frac{v}{R} = -\frac{C_v}{P}$$

Finally we can form the desired derivative

$$\left(\frac{\partial v}{\partial P}\right)_s = \left(\frac{\partial v}{\partial T}\right)_s\left(\frac{\partial T}{\partial P}\right)_s = -\frac{C_v}{P}\frac{v}{C_p} = -\frac{v}{kP}$$

$$\beta_s = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_s = \left(-\frac{1}{v}\right)\left(-\frac{v}{kP}\right) = \frac{1}{kP} = \frac{1}{k}\beta_T$$

12.45

Assume a substance has uniform properties in all directions with $V = L_x L_y L_z$ and show that volume expansivity $\alpha_p = 3\delta_T$. Hint: differentiate with respect to T and divide by V.

$$V = L_x L_y L_z$$

From Eq.12.37

$$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = \frac{1}{L_x L_y L_z}\left(\frac{\partial\, L_x L_y L_z}{\partial T}\right)_P$$

$$= \frac{L_y L_z}{L_x L_y L_z}\left(\frac{\partial\, L_x}{\partial T}\right)_P + \frac{L_x L_z}{L_x L_y L_z}\left(\frac{\partial\, L_y}{\partial T}\right)_P + \frac{L_x L_y}{L_x L_y L_z}\left(\frac{\partial\, L_z}{\partial T}\right)_P$$

$$= \frac{1}{L_x}\left(\frac{\partial\, L_x}{\partial T}\right)_P + \frac{1}{L_y}\left(\frac{\partial\, L_y}{\partial T}\right)_P + \frac{1}{L_z}\left(\frac{\partial\, L_z}{\partial T}\right)_P$$

$$= 3\,\delta_T$$

This of course assumes isotropic properties (the same in all directions).

12.46

Determine the volume expansivity, α_P, and the isothermal compressibility, β_T, for water at 20°C, 5 MPa and at 300°C, and 15 MPa using the steam tables.

Water at 20°C, 5 MPa (compressed liquid)

$$\alpha_P = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P \approx \frac{1}{v}\left(\frac{\Delta v}{\Delta T}\right)_P \; ; \qquad \beta_T = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T \approx -\frac{1}{v}\left(\frac{\Delta v}{\Delta P}\right)_T$$

Estimate by finite difference using values at 0°C, 20°C and 40°C,

$$\alpha_P \approx \frac{1}{0.000\ 9995}\frac{0.001\ 0056 - 0.000\ 9977}{40 - 0} = \mathbf{0.000\ 1976\ ^oC^{-1}}$$

Using values at saturation, 5 MPa and 10 MPa,

$$\beta_T \approx -\frac{1}{0.000\ 9995}\frac{0.000\ 9972 - 0.001\ 0022}{10 - 0.0023} = \mathbf{0.000\ 50\ MPa^{-1}}$$

Water at 300°C, 15 MPa (compressed liquid)

$$\alpha_P \approx \frac{1}{0.001\ 377}\frac{0.001\ 4724 - 0.001\ 3084}{320 - 280} = \mathbf{0.002\ 977\ ^oC^{-1}}$$

$$\beta_T \approx -\frac{1}{0.001\ 377}\frac{0.001\ 3596 - 0.001\ 3972}{20 - 10} = \mathbf{0.002\ 731\ MPa^{-1}}$$

12.47

Use the CATT3 software to solve the previous problem.

The benefit of the software to solve for the partial derivatives is that we can narrow the interval over which we determine the slope.

Water at 20°C, 5 MPa (compressed liquid)

$$\alpha_P = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P \approx \frac{1}{v}\left(\frac{\Delta v}{\Delta T}\right)_P \ ; \qquad \beta_T = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T \approx -\frac{1}{v}\left(\frac{\Delta v}{\Delta P}\right)_T$$

Estimate by finite difference using values at 19°C, 20°C and 21°C,

$$\alpha_P \approx \frac{1}{0.000\ 9995}\frac{0.000\ 9997 - 0.000\ 9993}{21 - 19} = \mathbf{0.000\ 40\ ^oC^{-1}}$$

Using values at saturation, 4.5 MPa and 5.5 MPa,

$$\beta_T \approx -\frac{1}{0.000\ 9995}\frac{0.000\ 9993 - 0.000\ 9997}{5.5 - 4.5} = \mathbf{0.000\ 40\ MPa^{-1}}$$

Water at 300°C, 15 MPa (compressed liquid)

$$\alpha_P \approx \frac{1}{0.001\ 377}\frac{0.001\ 385 - 0.001\ 369}{302 - 298} = \mathbf{0.011\ 619\ ^oC^{-1}}$$

$$\beta_T \approx -\frac{1}{0.001\ 377}\frac{0.001\ 373 - 0.001\ 381}{16 - 14} = \mathbf{0.002\ 905\ MPa^{-1}}$$

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12.48

A cylinder fitted with a piston contains liquid methanol at 20°C, 100 kPa and volume 10 L. The piston is moved, compressing the methanol to 20 MPa at constant temperature. Calculate the work required for this process. The isothermal compressibility of liquid methanol at 20°C is 1.22×10^{-9} m^2/N.

$$_1w_2 = \int\limits_1^2 Pdv = \int P\left(\frac{\partial v}{\partial P}\right)_T dP_T = -\int\limits_1^2 v\beta_T \, PdP_T$$

For $v \approx$ constant & $\beta_T \approx$ constant the integral can be evaluated

$$_1w_2 = -\frac{v\beta_T}{2}\left(P_2^2 - P_1^2\right)$$

For liquid methanol, from Table A.4: $\rho = 787$ m^3/kg

$V_1 = 10$ L, $m = 0.01 \times 787 = 7.87$ kg

$$_1W_2 = \frac{0.01\times1220}{2}\left[(20)^2 - (0.1)^2\right] = 2440 \text{ J} = \textbf{2.44 kJ}$$

12.49

For commercial copper at 25$^\mathrm{o}$C (see table A.3) the speed of sound is about 4800 m/s. What is the adiabatic compressibility β_s?

From Eq.12.41 and Eq.12.40

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s = \frac{1}{-\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_s \rho} = \frac{1}{\beta_s \rho}$$

Then we get using density from Table A.3

$$\beta_s = \frac{1}{c^2\rho} = \frac{1}{4800^2 \times 8300}\ \frac{s^2\ m^3}{m^2\ kg} = \frac{1000}{4800^2 \times 8300}\ \frac{1}{kPa}$$

$$= \mathbf{5.23 \times 10^{-9}\ kPa^{-1}}$$

12.50

Use Eq. 12.32 to solve for $(\partial T/\partial P)_s$ in terms of T, v, C_p and α_p. How large a temperature change does 25°C water ($\alpha_p = 2.1 \times 10^{-4}$ K^{-1}) have, when compressed from 100 kPa to 1000 kPa in an isentropic process?

From Eq.12.32 we get for constant s (ds = 0) and Eq.12.37

$$\left(\frac{\partial T}{\partial P}\right)_s = \frac{T}{C_p}\left(\frac{\partial v}{\partial T}\right)_P = \frac{T}{C_p}\,\alpha_p\, v$$

Assuming the derivative is constant for the isentropic compression we estimate with heat capacity from Table A.3 and v from B.1.1

$$\Delta T_s = \left(\frac{\partial T}{\partial P}\right)_s \Delta P_s = \frac{T}{C_p}\,\alpha_p\, v\, \Delta P_s$$

$$= \frac{273.15 + 25}{4.18} \times 2.1 \times 10^{-4} \times 0.001003 \times (1000 - 100)$$

$$= \mathbf{0.013\ K} \qquad \text{barely measurable.}$$

12.51

Sound waves propagate through a media as pressure waves that cause the media to go through isentropic compression and expansion processes. The speed of sound c is defined by $c^2 = (\partial P / \partial \rho)_s$ and it can be related to the adiabatic

compressibility, which for liquid ethanol at 20°C is 9.4×10^{-10} m^2/N. Find the speed of sound at this temperature.

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s = \frac{1}{-\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_s \rho} = \frac{1}{\beta_s \rho}$$

From Table A.4 for ethanol, $\rho = 783$ kg/m^3

$\Rightarrow$ $c = \left(\dfrac{1}{940 \times 10^{-12} \times 783}\right)^{1/2} = $ **1166 m/s**

12.52

Use Table B.3 to find the speed of sound for carbon dioxide at 2500 kPa near 100$^{\text{o}}$C. Approximate the partial derivative numerically.

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2 \left(\frac{\partial P}{\partial v}\right)_s$$

We will use the 2000 kPa and 3000 kPa table entries. We need to find the change in v between two states with the same s at those two pressures.

At 100$^{\text{o}}$C, 2500 kPa: s = (1.6843 + 1.5954)/2 = 1.63985 kJ/kg-K,
    v = (0.03359 + 0.02182)/2 = 0.027705 m^3/kg

    2000 kPa, s = 1.63985 kJ/kg-K: v = 0.031822 m^3/kg
    3000 kPa, s = 1.63985 kJ/kg-K: v = 0.0230556 m^3/kg

$$c^2 \approx -v^2 \left(\frac{\Delta P}{\Delta v}\right)_s = -0.027705^2 \frac{3000 - 2000}{0.0230556 - 0.031822} \frac{\text{kJ}}{\text{kg}} = 87\ 557.8 \frac{\text{J}}{\text{kg}}$$

$$c = \sqrt{87\ 557.8} = \textbf{295.9 m/s}$$

12.53

Use the CATT3 software to solve the previous problem.

At 100°C, 2500 kPa: s = 1.636 kJ/kg-K,   v = 0.02653 m^3/kg

$\quad$ 101°C, s = 1.636 kJ/kg-K: v = 0.02627 m^3/kg, P = 2.531 MPa

$\quad$ 99°C, s = 1.636 kJ/kg-K: v = 0.02679 m^3/kg, P = 2.469 MPa

$$c^2 \approx -v^2\left(\frac{\Delta P}{\Delta v}\right)_s = -0.02653^2 \, \frac{2531 - 2469}{0.02627 - 0.02679} \, \frac{\text{kJ}}{\text{kg}} = 83\ 919.5 \, \frac{\text{J}}{\text{kg}}$$

$$c = \sqrt{83\ 919.5} = \textbf{289.7 m/s}$$

12.54

Consider the speed of sound as defined in Eq. 12.41. Calculate the speed of sound for liquid water at 20°C, 2.5 MPa, and for water vapor at 200°C, 300 kPa, using the steam tables.

From Eq. 12.41: $\quad c^2 = \left(\dfrac{\partial P}{\partial \rho}\right)_s = -v^2\left(\dfrac{\partial P}{\partial v}\right)_s$

Liquid water at 20°C, 2.5 MPa, assume

$$\left(\frac{\partial P}{\partial v}\right)_s \approx \left(\frac{\Delta P}{\Delta v}\right)_T$$

Using saturated liquid at 20°C and compressed liquid at 20°C, 5 MPa,

$$c^2 = -\left(\frac{0.001\ 002+0.000\ 9995}{2}\right)^2\left(\frac{5-0.0023}{0.000\ 9995-0.001\ 002}\right)\frac{\text{MJ}}{\text{kg}}$$

$$= 2.002\times10^6\ \frac{\text{J}}{\text{kg}}$$

$$\Rightarrow \quad c = \textbf{1415 m/s}$$

Superheated vapor water at 200°C, 300 kPa

$$v = 0.7163\ \text{m}^3\text{/kg}, \quad s = 7.3115\ \text{kJ/kg K}$$

At P = 200 kPa & s = 7.3115 kJ/kg K: T = 157°C,   v = 0.9766 m^3/kg

At P = 400 kPa & s = 7.3115 kJ/kg K: T = 233.8^{o}C, v = 0.5754 m^3/kg

$$c^2 = -(0.7163)^2\left(\frac{0.400-0.200}{0.5754-0.9766}\right)\frac{\text{MJ}}{\text{kg}} = 0.2558 \times 10^6\ \text{m}^2\text{/s}^2$$

$$\Rightarrow \quad c = \textbf{506 m/s}$$

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12.55

Use the CATT3 software to solve the previous problem.

From Eq. 12.41: $\quad c^2 = \left(\dfrac{\partial P}{\partial \rho}\right)_s = -v^2\left(\dfrac{\partial P}{\partial v}\right)_s$

Liquid water at 20$^\text{o}$C, 2.5 MPa, assume $\left(\dfrac{\partial P}{\partial v}\right)_s \approx \left(\dfrac{\Delta P}{\Delta v}\right)_s$

and   CATT3:   v = 0.001001 m^3/kg,   s = 0.2961 kJ/kg-K

Using liquid at 3 MPa and 2 MPa at the same s = 0.2961 kJ/kg-K,

$$c^2 = -\,0.001001^2\ \left(\frac{3 - 2}{0.001 - 0.001\ 001}\right) \frac{\text{MJ}}{\text{kg}}$$

$$= 1.002{\times}10^6\,\frac{\text{J}}{\text{kg}}$$

$$\Rightarrow \qquad c = \textbf{1001 m/s}$$

Superheated vapor water at 200$^\text{o}$C, 300 kPa

CATT3:   v = 0.7163 m^3/kg,   s = 7.311 kJ/kg K

At P = 290 kPa & s = 7.311 kJ/kg K: T = 196.2$^\text{o}$C,   v = 0.7351 m^3/kg

At P = 310 kPa & s = 7.311 kJ/kg K: T = 203.7$^\text{o}$C,   v = 0.6986 m^3/kg

$$c^2 = -(0.7163)^2 \left(\frac{0.310 - 0.290}{0.6986 - 0.7351}\right) \frac{\text{MJ}}{\text{kg}} = 0.28114 \times 10^6 \text{ m}^2/\text{s}^2$$

$$\Rightarrow \qquad c = \textbf{530 m/s}$$

12.56

Soft rubber is used as a part of a motor mounting. Its adiabatic bulk modulus is B_s = 2.82×10^6 kPa, and the volume expansivity is $\alpha_p = 4.86 \times 10^{-4}$ K^{-1}. What is the speed of sound vibrations through the rubber, and what is the relative volume change for a pressure change of 1 MPa?

From Eq.12.41 and Eq.12.40

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s = \frac{1}{-\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_s \rho} = \frac{1}{\beta_s \rho} = \frac{B_s}{\rho}$$

$$= 2.82 \times 10^6 \times 1000 \text{ Pa} / 1100 \text{ kg/m}^3 = 2.564 \times 10^6 \ \text{m}^2/\text{s}^2$$

c = 1601 m/s

If the volume change is fast it is isentropic and if it is slow it is isothermal. We will assume it is isentropic

$$\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_s = -\beta_s = -\frac{1}{B_s}$$

then

$$\frac{\Delta V}{V} = -\frac{\Delta P}{B_s} = -\frac{1000}{2.82 \times 10^6} = \mathbf{-3.55 \times 10^{-4}}$$

12.57

Liquid methanol at 25°C has an adiabatic compressibility of 1.05×10^{-9} m^2/N. What is the speed of sound? If it is compressed from 100 kPa to 10 MPa in an insulated piston/cylinder, what is the specific work?

From Eq.12.41 and Eq.12.40 and the density from table A.4

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s = \frac{1}{\beta_s \rho} = \frac{1}{1.05 \times 10^{-9} \times 787}$$

$$= 1.210 \times 10^6 \text{ m}^2/\text{s}^2$$

$$c = \mathbf{1100 \text{ m/s}}$$

The specific work becomes

$$w = \int P \, dv = \int P \, (-\beta_s v) \, dP = -\int \beta_s v \, P \, dP = -\beta_s v \int_1^2 P \, dP$$

$$= -\beta_s v \, 0.5 \, (P_2^2 - P_1^2)$$

$$= -1.05 \times 10^{-9} \text{ m}^2/\text{N} \times \frac{0.5}{787} \text{ m}^3/\text{kg} \times (10\,000^2 - 100^2) \times 1000^2 \text{ Pa}^2$$

$$= \mathbf{-66.7 \text{ J/kg}}$$

12.58

Use Eq. 12.32 to solve for $(\partial T/\partial P)_s$ in terms of T, v, C_p and α_p. How much higher does the temperature become for the compression of the methanol in Problem 12.57? Use $\alpha_p = 2.4 \times 10^{-4}$ K^{-1} for methanol at 25°C.

From Eq.12.32 we get for constant s (ds = 0) and Eq.12.37

$$\left(\frac{\partial T}{\partial P}\right)_s = \frac{T}{C_p}\left(\frac{\partial v}{\partial T}\right)_P = \frac{T}{C_p}\,\alpha_p\,v$$

Assuming the derivative is constant for the isentropic compression we estimate with heat capacity and density (v = 1/ρ) from Table A.4

$$\Delta T_s = \left(\frac{\partial T}{\partial P}\right)_s \Delta P_s = \frac{T}{C_p}\,\alpha_p\,v\,\Delta P_s$$

$$= \frac{298.15}{2.55}\frac{\text{K kg K}}{\text{kJ}} \times 2.4 \times 10^{-4}\,\text{K}^{-1} \times \frac{1}{787}\frac{\text{m}^3}{\text{kg}} \times (10\ 000 - 100)\ \text{kPa}$$

$$= \textbf{0.353 K}$$

12.59

Find the speed of sound for air at 20°C, 100 kPa using the definition in Eq. 12.41 and relations for polytropic processes in ideal gases.

From problem 12.41 : $\quad c^2 = \left(\dfrac{\partial P}{\partial \rho}\right)_s = -v^2\left(\dfrac{\partial P}{\partial v}\right)_s$

For ideal gas and isentropic process, $Pv^k = $ constant

$P = Cv^{-k} \quad \Rightarrow \dfrac{\partial P}{\partial v} = -kCv^{-k-1} = -kPv^{-1}$

$c^2 = -v^2(-kPv^{-1}) = kPv = kRT$

$c = \sqrt{kRT} = \sqrt{1.4 \times 0.287 \times 293.15 \times 1000} = \mathbf{343.2 \ m/s}$

For every 3 seconds after the lightning the sound travels about 1 km.

Equations of State

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12.60

Use Table B3 and find the compressibility of carbon dioxide at the critical point.

$$Pv = Z\,RT$$

At the critical point from B.3:

P = 7377.3 kPa, T = 31°C = 304.15 K, v = 0.002139 m^3/kg

from A.5:   R = 0.1889 kJ/kg-K

$$Z = \frac{Pv}{RT} = \frac{7377.3 \times 0.002139}{0.1889 \times 304.15} = \mathbf{0.27}$$

12.61

Use the equation of state as shown in Example 12.3 where changes in enthalpy and entropy were found. Find the isothermal change in internal energy in a similar fashion; do not compute it from enthalpy.

The equation of state is

$$\frac{Pv}{RT} = 1 - C'\frac{P}{T^4}$$

and to integrate for changes in u from Eq.12.31 we make it explicit in P as

$$P = T^4 \left(\frac{v}{R}T^3 + C'\right)^{-1}$$

Now perform the partial derivative of P

$$\left(\frac{\partial P}{\partial T}\right)_v = 4\,T^3\left(\frac{v}{R}T^3 + C'\right)^{-1} - T^4\left(\frac{v}{R}T^3 + C'\right)^{-2} 3\frac{v}{R}T^2$$

$$= 4\frac{P}{T} - \frac{P^2}{T^4}\,3\frac{v}{R}T^2 = 4\frac{P}{T} - 3\frac{P}{T}\times\frac{Pv}{RT} = \frac{P}{T}\left[\,4 - 3\frac{Pv}{RT}\,\right]$$

Substitute into Eq.12.31

$$du_T = \left[\,T\,\left(\frac{\partial P}{\partial T}\right)_v - P\,\right] dv_T = \left[\,P\left(\,4 - 3\frac{Pv}{RT}\right) - P\,\right] dv_T$$

$$= 3\,P\left(\,1 - \frac{Pv}{RT}\right)\,dv_T = 3\,P\,C'\frac{P}{T^4}\,dv_T$$

The P must be eliminated in terms of v or the opposite, we do the latter as from the equation of state

$$v = \frac{RT}{P} - C'\,R\,\frac{1}{T^3}\qquad \Rightarrow\qquad dv_T = -\frac{RT}{P^2}\,dP_T$$

so now

$$du_T = 3\,C'\frac{P^2}{T^4}\,dv_T = -\,3\,C'\,R\,\frac{1}{T^3}\,dP_T$$

and the integration becomes

$$u_2 - u_1 = -\,3\,C'\,R\,T^{-3}\,(P_2 - P_1)$$

12.62

Use Table B.4 to find the compressibility of R-410A at $60^{\circ}C$ and a) saturated liquid b) saturated vapor and c) 3000 kPa.

Table A.2: $R = 8.31451 / 72.585 = 0.1145$ kJ/kg-K

a) Table B.4.1: $P = 3836.9$ kPa, $v = 0.001227$ m^3/kg

$$Z = \frac{Pv}{RT} = \frac{3836.9 \times 0.001227}{0.1145 \times 333.15} = \textbf{0.1234}$$

b) Table B.4.1: $P = 3836.9$ kPa, $v = 0.00497$ m^3/kg

$$Z = \frac{Pv}{RT} = \frac{3836.9 \times 0.00497}{0.1145 \times 333.15} = \textbf{0.5}$$

c) Table B.4.2: $P = 3000$ kPa, $v = 0.00858$ m^3/kg

$$Z = \frac{Pv}{RT} = \frac{3000 \times 0.00858}{0.1145 \times 333.15} = \textbf{0.675}$$

The R-410A is not an ideal gas at any of these states.

12.63

Use a truncated virial EOS that includes the term with B for carbon dioxide at 20° C, 1 MPa for which B = –0.128 m^3/kmol and T(dB/dT) = 0.266 m^3/kmol. Find the difference between the ideal-gas value and the real-gas value of the internal energy.

virial eq.:
$$P = \frac{RT}{v} + \frac{BRT}{v^2} \; ; \qquad \left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v} + \frac{BR}{v^2} + \frac{RT}{v^2}\left(\frac{dB}{dT}\right)$$

$$u - u^* = -\int_\infty^v \left[\left(\frac{\partial P}{\partial T}\right)_v - P\right]dv = -\int_\infty^v \left[\frac{RT^2}{v^2}\left(\frac{dB}{dT}\right)\right]dv = -\frac{RT}{v}\left[T\left(\frac{dB}{dT}\right)\right]$$

Solution of virial equation (quadratic formula):

$$\bar{v} = \frac{1}{2}\frac{\bar{R}T}{P}\left[1 + \sqrt{1 + 4BP/\bar{R}T}\right] \qquad \text{where: } \frac{\bar{R}T}{P} = \frac{8.3145\times293.15}{1000} = 2.43737$$

$$\bar{v} = \frac{1}{2} \times 2.43737 \left[1 + \sqrt{1 + 4(-0.128)/2.43737}\right] = 2.3018 \ m^3/\text{kmol}$$

Using the minus-sign root of the quadratic formula results in a compressibility factor < 0.5, which is not consistent with such a truncated equation of state.

$$u - u^* = \frac{-8.3145 \times 293.15}{2.3018}\left[0.266\right]/ 44.01 = -\ \textbf{6.4 kJ/kg}$$

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12.64

Solve the previous Problem with Table B.3 values and find the compressibility of the carbon dioxide at that state.

B.3: $v = 0.05236$ m^3/kg, $u = 327.27$ kJ/kg,   A5: $R = 0.1889$ kJ/kg-K

$$Z = \frac{Pv}{RT} = \frac{1000 \times 0.05236}{0.1889 \times 293.15} = 0.9455 \qquad \text{close to ideal gas}$$

To get u^* let us look at the lowest pressure 400 kPa, 20°C: $v = 0.13551$ m^3/kg and $u = 331.57$ kJ/kg.

$$Z = Pv/RT = 400 \times 0.13551/(0.1889 \times 293.15) = 0.97883$$

It is not very close to ideal gas but this is the lowest P in the printed table.

$$u - u^* = 327.27 - 331.57 = \textbf{--4.3 kJ/kg}$$

12.65

A gas is represented by the virial EOS with the first two terms, B and C. Find an expression for the work in an isothermal expansion process in a piston-cylinder.

Virial EOS:
$$P = \frac{RT}{v} + \frac{B(T)\ RT}{v^2} + \frac{C(T)\ RT}{v^3} + \ldots$$

$$= RT[\ v^{-1} + B(T)\ v^{-2} + C(T)\ v^{-3} + ..\ ]$$

The work is $w = \int P\ dv = RT \int [\ v^{-1} + B(T)\ v^{-2} + C(T)\ v^{-3} + ..\ ]\ dv$

With just the first two terms we get

$$w = RT\ [\ \ln\frac{v_2}{v_1} - B(T)\ (v_2^{-1} - v_1^{-1}) - \frac{1}{2}\ C(T)\ (v_2^{-2} - v_1^{-2})\ ]$$

12.66

Extend problem 12.63 to find the difference between the ideal-gas value and the real-gas value of the entropy and compare to table B.3.

Calculate the difference in entropy of the ideal-gas value and the real-gas value for carbon dioxide at the state 20°C, 1 MPa, as determined using the virial equation of state. Use numerical values given in Problem 12.63.

CO_2 at T = 20°C, P = 1 MPa

$$s^*_{P*} - s_P = \int_{v(P)}^{RT/P*} \left(\frac{\partial P}{\partial T}\right)_v dv \; ; \qquad \text{ID Gas:} \quad s^*_{P*} - s_P = \int_{v(P)}^{RT/P*} \frac{R}{v} dv = R \ln \frac{P}{P^*}$$

Therefore, at P: $\quad s^*_P - s_P = -R \ln \dfrac{P}{P^*} + \displaystyle\int_{v(P)}^{RT/P*} \left(\frac{\partial P}{\partial T}\right)_v dv$

virial: $\quad P = \dfrac{RT}{v} + \dfrac{BRT}{v^2} \quad$ and $\qquad \left(\dfrac{\partial P}{\partial T}\right)_v = \dfrac{R}{v} + \dfrac{BR}{v^2} + \dfrac{RT}{v^2}\left(\dfrac{dB}{dT}\right)$

Integrating,

$$s^*_P - s_P = -R \ln \frac{P}{P^*} + R \ln \frac{RT}{P^* v} + R\left[B + T\left(\frac{dB}{dT}\right)\right]\left(\frac{1}{v} - \frac{P^*}{RT}\right)$$

$$= R\left[\ln \frac{RT}{Pv} + \left(B + T\left(\frac{dB}{dT}\right)\right)\frac{1}{v}\right]$$

Using values for CO_2 from solution 12.63 and R = 0.1889 kJ/kgK

$$s^*_P - s_P = 0.1889\left[\ln \frac{2.437\ 37}{2.3018} + \left(-0.128 + 0.266\right)\frac{1}{2.3018}\right]$$
$$= \textbf{0.02214 kJ/kg K}$$

From Table B.3 take the ideal as the lowest P = 400 kPa:

$$s^*_P - s_P = 1.7904 - 1.6025 + 0.1889 \ln(400/1000) = \textbf{0.0148 kJ/kgK}$$

The lowest P = 400 kPa in B3 is not exactly ideal gas (Z = Pv/RT = 0.9788)

12.67

Two uninsulated tanks of equal volume are connected by a valve. One tank contains a gas at a moderate pressure P_1, and the other tank is evacuated. The valve is opened and remains open for a long time. Is the final pressure P_2 greater than, equal to, or less than $P_1/2$? Hint: Recall Fig. 12.5.

Assume the temperature stays constant then for an ideal gas the pressure will be reduced to half the original pressure. For the real gas the compressibility factor maybe different from 1 and then changes towards one as the pressure drops.

$$V_A = V_B \quad \Rightarrow \quad V_2 = 2V_1, \quad T_2 = T_1 = T$$

$$\frac{P_2}{P_1} = \frac{V_1}{V_2}\frac{Z_2}{Z_1}\frac{mRT}{mRT} = \frac{1}{2}\frac{Z_2}{Z_1}$$

$$\text{If } T < T_B, \quad Z_2 > Z_1 \quad \Rightarrow \quad \frac{P_2}{P_1} > \frac{1}{2}$$

$$\text{If } T > T_B, \quad Z_2 < Z_1 \quad \Rightarrow \quad \frac{P_2}{P_1} < \frac{1}{2}$$

12.68

Show how to get the constants in Eq.12.52 for van der Waals EOS.

van der Waals EOS: $\qquad P = \dfrac{RT}{v-b} - \dfrac{a}{v^2}$

The conditions at the critical point relate to these derivatives

$$\left(\frac{\partial P}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3} \qquad ; \qquad \left(\frac{\partial^2 P}{\partial v^2}\right)_T = \frac{2RT}{(v-b)^3} - \frac{6a}{v^4}$$

Set both derivatives to zero at the critical point

$$-\frac{RT_c}{(v_c-b)^2} + \frac{2a}{v_c^3} = 0 \quad (1) \; ; \qquad \frac{2RT_c}{(v_c-b)^3} - \frac{6a}{v_c^4} = 0 \qquad (2)$$

we also have from the EOS

$$P_c = \frac{RT_c}{v_c - b} - \frac{a}{v_c^2} \qquad\qquad (3)$$

Now we need to solve theses three equations for v_c, a and b. Solve the first equation for a and substitute into the second equation to give

$$\frac{2a}{v_c^3} = \frac{RT_c}{(v_c-b)^2} \quad => \quad \frac{6a}{v_c^4} = \frac{3RT_c}{v_c\,(v_c-b)^2} \qquad \text{substitute into Eq.(2)}$$

$$\frac{3RT_c}{v_c\,(v_c-b)^2} = \frac{2RT_c}{(v_c-b)^3} \qquad\qquad \text{now solve to get} \qquad v_c = 3b$$

Substitute back into the first equation to get

$$2\,a = \frac{RT_c}{(v_c-b)^2}\,v_c^3 = RT_c\,\frac{27}{4}\,b$$

Now finally substitute a and v_c into the EOS Eq.(3) to get b.

$$P_c = \frac{RT_c}{v_c-b} - \frac{a}{v_c^2} = \frac{RT_c}{2b} - \frac{RT_c\,\frac{27}{8}\,b}{9\,b^2} \;=\; RT_c\,(0.5 - 3/8)\,/b$$

The result is as in Eq.12.52.

12.69

Show that the van der Waals equation can be written as a cubic equation in the compressibility factor involving the reduced pressure and reduced temperature as

$$Z^3 - \left(\frac{P_r}{8T_r} + 1\right) Z^2 + \left(\frac{27\, P_r}{64\, T_r^2}\right) Z - \frac{27\, P_r^2}{512\, T_r^{\,3}} = 0$$

van der Waals equation, Eq.12.55: $\qquad P = \dfrac{RT}{v\text{-}b} - \dfrac{a}{v^2}$

$$a = \frac{27}{64}\frac{R^2 T_C^{\,2}}{P_C} \qquad\qquad\qquad b = \frac{RT_C}{8P_C}$$

multiply equation by $\dfrac{v^2(v\text{-}b)}{P}$

Get: $\qquad v^3 - (b + \dfrac{RT}{P})\, v^2 + (\dfrac{a}{P})\, v - \dfrac{ab}{P} = 0$

Multiply by $\quad \dfrac{P^3}{R^3\,T^3}\quad$ and substitute $\;Z = \dfrac{Pv}{RT}$

Get: $\qquad Z^3 - (\dfrac{bP}{RT} + 1)\, Z^2 + (\dfrac{aP}{R^2T^2})\, Z - (\dfrac{abP^2}{R^3\,T^3}) = 0$

Substitute for a and b, get:

$$Z^3 - \left(\frac{P_r}{8T_r} + 1\right) Z^2 + \left(\frac{27\, P_r}{64\, T_r^2}\right) Z - \frac{27\, P_r^2}{512\, T_r^{\,3}} = 0$$

Where $\qquad P_r = \dfrac{P}{P_c}, \qquad\quad T_r = \dfrac{T}{T_c}$

12.70

Evaluate changes in an isothermal process for u, h and s for a gas with an equation of state as $P(v - b) = RT$.

From Eq.12.31 we get

$$du_T = \left[\, T\, \left(\frac{\partial P}{\partial T}\right)_v - P\,\right] dv_T = \left[\, T\left(\frac{R}{v-b}\right) - P\,\right] dv_T = [\, P - P]\, dv_T = 0$$

From Eq.12.27 we get using $v = b + RT/P$

$$dh_T = \left[\, v - T\left(\frac{\partial v}{\partial T}\right)_P\,\right] dP_T = \left[\, v - T\left(\frac{R}{P}\right)\,\right] dP_T = b\, dP_T$$

From Eq.12.32 or Eq.12.34 we get

$$ds_T = -\left(\frac{\partial v}{\partial T}\right)_P dP_T = \left(\frac{\partial P}{\partial T}\right)_v dv_T$$

$$= -\frac{R}{P}\, dP_T = \frac{R}{v-b}\, dv_T$$

Now the changes in u, h and s can be integrated to find

$$u_2 - u_1 = 0$$

$$h_2 - h_1 = \int b\, dP = b(P_2 - P_1)$$

$$s_2 - s_1 = -R\, \ln\frac{P_2}{P_1} = R\, \ln\frac{v_2 - b}{v_1 - b}$$

12.71

Develop expressions for isothermal changes in internal energy, enthalpy and entropy for a gas obeying the van der Waals equation of state.

van der Waals equation of state: $\quad P = \dfrac{RT}{v\text{-}b} - \dfrac{a}{v^2}$

$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v\text{-}b}$$

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_v - P = \frac{RT}{v\text{-}b} - \frac{RT}{v\text{-}b} + \frac{a}{v^2}$$

$$(u_2\text{-}u_1)_T = \int_1^2 [T\left(\frac{\partial P}{\partial T}\right)_v - P]dv = \int_1^2 \frac{a}{v^2}dv = a(\frac{1}{v_1} - \frac{1}{v_2})$$

$$(h_2\text{-}h_1)_T = (u_2\text{-}u_1)_T + P_2v_2 - P_1v_1 = P_2v_2 - P_1v_1 + a(\frac{1}{v_1} - \frac{1}{v_2})$$

$$(s_2\text{-}s_1)_T = \int_1^2 \left(\frac{\partial P}{\partial T}\right)_v dv = \int_1^2 \frac{R}{v\text{-}b} dv = R\ln\left(\frac{v_2\text{-}b}{v_1\text{-}b}\right)$$

12.72

Consider the following equation of state, expressed in terms of reduced pressure and temperature: $Z = 1 + (P_r/14T_r)[1 - 6T_r^{-2}]$. What does this predict for the reduced Boyle temperature?

$$Z = \frac{Pv}{RT} = 1 + \frac{P_r}{14\,T_r}\left(1 - \frac{6}{T_r^{\,2}}\right)$$

$$\left(\frac{\partial Z}{\partial P}\right)_T = \frac{1}{14P_cT_r}\left(1 - \frac{6}{T_r^{\,2}}\right)$$

To find the Boyle temperature we must have

$$\underset{P\to 0}{\text{Lim}}\left(\frac{\partial Z}{\partial P}\right)_T = 0 \ \text{ at } T_{boyle}$$

The above derivative is then zero if:

$$\left(1 - \frac{6}{T_r^{\,2}}\right) = 0 \quad \rightarrow \quad T_r = \sqrt{6} = \textbf{2.45}$$

12.73

Use the result of Problem 12.35 to find the reduced temperature at which the Joule-Thomson coefficient is zero, for a gas that follows the EOS given in Problem 12.72

$$Z = \frac{Pv}{RT} = 1 + \frac{P_r}{14\,T_r}\left(1 - \frac{6}{T_r^2}\right)$$

From Problem 12.35

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T\left(\frac{\partial v}{\partial T}\right)_P - v}{C_P} = \frac{RT^2}{PC_P}\left(\frac{\partial Z}{\partial T}\right)_P$$

$$\left(\frac{\partial Z}{\partial T}\right)_P = -\frac{P_r}{14T_c}\,T_r^{-2}\left(1 - 6\,T_r^{-2}\right) + \frac{P_r}{14\,T_r}\left(12\,T_r^{-3}/T_c\right)$$

$$= \frac{P_r}{14T_c}\,T_r^{-2}\left(18\,T_r^{-2} - 1\right)$$

So this is zero for $18\,T_r^{-2} = 1$ or

$$T_r = \sqrt{18}$$

12.74

What is the Boyle temperature for the following equation of state: $P = \dfrac{RT}{v\text{-}b} - \dfrac{a}{v^2 T}$

where a and b are constants.

$$P = \frac{RT}{v\text{-}b} - \frac{a}{v^2 T}$$

Multiplying by $\dfrac{v\text{-}b}{P}$ gives: $\quad v - b = \dfrac{RT}{P} - \dfrac{a(1\text{-}b/v)}{PvT}$

Using for T_{Boyle}:

$$\lim_{P \to 0}\left(\frac{\partial Z}{\partial P}\right)_T = \lim_{P \to 0}\frac{Z\text{-}1}{P\text{-}0} = \frac{1}{RT}\lim_{P \to 0}\left(v - \frac{RT}{P}\right)$$

$$\lim_{P \to 0}\left(v - \frac{RT}{P}\right) = b - \frac{a(1\text{-}0)}{RT \times T} = b - \frac{a}{RT^2} = 0 \text{ at } T_{Boyle}$$

or $\quad T_{Boyle} = \sqrt{\dfrac{a}{Rb}} = \sqrt{\dfrac{27}{64}\dfrac{R^2 T_C^3}{P_C}\dfrac{1}{R}\dfrac{8P_C}{RT_C}} = \sqrt{\dfrac{27}{8}}\,T_C$

12.75

Determine the reduced Boyle temperature as predicted by an equation of state (the experimentally observed value for most substances is about 2.5), using the van der Waals equation and the Redlich–Kwong equation. Note: It is helpful to use Eqs. 12.47 and 12.48 in addition to Eq. 12.46

The Boyle temp. is that T at which $\displaystyle\lim_{P \to 0}\left(\frac{\partial Z}{\partial P}\right)_T = 0$

But $\displaystyle\lim_{P \to 0}\left(\frac{\partial Z}{\partial P}\right)_T = \lim_{P \to 0}\frac{Z\text{-}1}{P\text{-}0} = \frac{1}{RT}\lim_{P \to 0}\left(v - \frac{RT}{P}\right)$

van der Waals: $\quad P = \dfrac{RT}{v\text{-}b} - \dfrac{a}{v^2}$

multiply by $\dfrac{v\text{-}b}{P}$, get

$v\text{-}b = \dfrac{RT}{P} - \dfrac{a(v\text{-}b)}{Pv^2}\quad$ or $\quad v - \dfrac{RT}{P} = b - \dfrac{a(1\text{-}b/v)}{Pv}$

& $\quad RT \times \displaystyle\lim_{P \to 0}\left(\frac{\partial Z}{\partial P}\right)_T = b - \frac{a(1\text{-}0)}{RT} = 0\quad$ only at T_{Boyle}

or $\quad T_{Boyle} = \dfrac{a}{Rb} = \dfrac{27}{8}T_C = \mathbf{3.375\ T_C}$

Redlich-Kwong: $\quad P = \dfrac{RT}{v\text{-}b} - \dfrac{a}{v(v+b)T^{1/2}}$

as in the first part, get

$v - \dfrac{RT}{P} = b - \dfrac{a(1\text{-}b/v)}{Pv(1+b/v)T^{1/2}}$

& $RT \times \displaystyle\lim_{P \to 0}\left(\frac{\partial Z}{\partial P}\right)_T = b - \frac{a(1\text{-}0)}{Pv(1+0)T^{1/2}} = 0\quad$ only at T_{Boyle}

or $\quad T_{Boyle}^{3/2} = \dfrac{a}{Rb} = \dfrac{0.427\ 48\ R^2\ T_C^{5/2}}{RP_C} \times \dfrac{P_C}{0.08\ 664\ R\ T_C}$

$T_{Boyle} = \left(\dfrac{0.427\ 48}{0.086\ 64}\right)^{2/3} T_C = \mathbf{2.9\ T_C}$

12.76

One early attempt to improve on the van der Waals equation of state was an expression of the form

$$P = \frac{RT}{v-b} - \frac{a}{v^2 T}$$

Solve for the constants a, b, and v_C using the same procedure as for the van der Waals equation.

From the equation of state take the first two derivatives of P with v:

$$\left(\frac{\partial P}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3 T} \quad \text{and} \quad \left(\frac{\partial^2 P}{\partial v^2}\right)_T = -\frac{2RT}{(v-b)^3} - \frac{6a}{v^4 T}$$

Since both these derivatives are zero at the critical point:

$$-\frac{RT}{(v-b)^2} + \frac{2a}{v^3 T} = 0 \quad \text{and} \quad -\frac{2RT}{(v-b)^3} - \frac{6a}{v^4 T} = 0$$

Also, $$P_C = \frac{RT_C}{v_C - b} - \frac{a}{v_C^2 T_C}$$

solving these three equations:

$$v_C = 3b, \quad a = \frac{27}{64}\frac{R^2 T_C^3}{P_C}, \quad b = \frac{RT_C}{8P_C}$$

12.77

Develop expressions for isothermal changes in internal energy, enthalpy and entropy for a gas obeying Redlich-Kwong equation of state.

Redlich-Kwong equation of state: $\quad P = \dfrac{RT}{v - b} - \dfrac{a}{v(v + b)T^{1/2}}$

$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v - b} + \frac{a}{2v(v + b)T^{3/2}}$$

From Eq.12.31

$$(u_2 - u_1)_T = \int_1^2 \frac{3a}{2v(v + b)T^{1/2}}\, dv = \frac{-3a}{2bT^{1/2}} \ln\left[\left(\frac{v_2 + b}{v_2}\right)\left(\frac{v_1}{v_1 + b}\right)\right]$$

We find change in h from change in u, so we do not do the derivative in Eq.12.27. This is due to the form of the EOS.

$$(h_2 - h_1)_T = P_2 v_2 - P_1 v_1 - \frac{3a}{2bT^{1/2}} \ln\left[\left(\frac{v_2 + b}{v_2}\right)\left(\frac{v_1}{v_1 + b}\right)\right]$$

Entropy follows from Eq.12.35

$$(s_2 - s_1)_T = \int_1^2 \left[\frac{R}{v - b} + \frac{a/2}{v(v + b)T^{3/2}}\right] dv$$

$$= R \ln\left(\frac{v_2 - b}{v_1 - b}\right) - \frac{a}{2bT^{3/2}} \ln\left[\left(\frac{v_2 + b}{v_2}\right)\left(\frac{v_1}{v_1 + b}\right)\right]$$

12.78

Determine the second virial coefficient B(T) using the van der Waals equation of state. Also find its value at the critical temperature where the experimentally observed value is about $-0.34\ RT_c/P_c$.

From Eq.12.48: $\quad B(T) = -\lim\limits_{P\to 0} \alpha \qquad\qquad$ where Eq. 12.44: $\alpha = \dfrac{RT}{P} - v$

From Eq. 12.51:

van der Waals: $\quad P = \dfrac{RT}{v\text{-}b} - \dfrac{a}{v^2} \qquad$ which we can multiply by $\dfrac{v\text{-}b}{P}$, get

$$v - b = \frac{RT}{P} - \frac{a(v\text{-}b)}{Pv^2} \quad\text{or}\quad v - \frac{RT}{P} = b - \frac{a(1\text{-}b/v)}{Pv}$$

Taking the limit for P -> 0 then (Pv -> RT and v -> ∞) we get :

$$B(T) = b - a/RT = \frac{RT_C}{P_C}\left(\frac{1}{8} - \frac{27\ T_C}{64\ T}\right)$$

where a,b are from Eq.12.52. At $T = T_C$ then we have

$$B(T_C) = \frac{RT_C}{P_C}\left(-\frac{19}{64}\right) = -0.297\ \frac{RT_C}{P_C}$$

12.79

Determine the second virial coefficient B(T) using the Redlich-Kwong equation of state. Also find its value at the critical temperature where the experimentally observed value is about –0.34 RT_c/P_c.

From Eq.12.48: $B(T) = - \lim_{P\to 0} \alpha$ where Eq.12.44: $\alpha = \dfrac{RT}{P} - v$

For Redlich Kwong the result becomes

$$v - \frac{RT}{P} = b - \frac{a(1- b/v)}{Pv(1 + b/v)\, T^{1/2}}$$

Taking the limit for P -> 0 then (Pv -> RT and   v -> ∞) we get :

$$=> \quad B(T) = b - \frac{a}{RT^{3/2}}$$

Now substitute Eqs. 12.54 and 12.55 for a and b,

$$B(T) = \frac{RT_C}{P_C} \left[0.08664 - 0.42748 \left(\frac{T_C}{T}\right)^{3/2} \right]$$

and evaluated at T_C it becomes

$$B(T_C) = \frac{RT_C}{P_C} \left[0.08664 - 0.42748 \right] = -0.341 \frac{RT_C}{P_C}$$

12.80

Oxygen in a rigid tank with 1 kg is at 160 K, 4 MPa. Find the volume of the tank by iterations using the Redlich-Kwong EOS. Compare the result with the ideal gas law.

For the ideal gas law: $Pv = RT$ so $v = RT/P$

$v = 0.2598 \times 160 / 4000 = 0.0104 \text{ m}^3/\text{kg}$; $V = mv = \textbf{0.0104 m}^3$

For Redlich-Kwong, Eq.12.53 and oxygen

$P_c = 5040 \text{ kPa}$; $T_c = 154.6 \text{ K}$; $R = 0.2598 \text{ kJ/kg K}$

$b = 0.08664 \dfrac{RT_c}{P_c} = 0.08664 \times \dfrac{0.2598 \times 154.6}{5040} = 0.000\,690\,5 \text{ m}^3/\text{kg}$

$a = 0.427\,48 \dfrac{R^2 T_c^{5/2}}{P_c} = 0.427\,48 \times \dfrac{0.2598^2 \times 154.6^{5/2}}{5040} = 1.7013$

$P = \dfrac{RT}{v - b} - \dfrac{a}{v(v + b)T^{1/2}}$ trial and error to get v due to nonlinearity

$v = 0.01 \text{ m}^3/\text{kg} \Rightarrow$ $P = 4465.1 - 1279.9 = 3185.2 \text{ kPa}$ too low

$v = 0.008 \text{ m}^3/\text{kg} \Rightarrow$ $P = 5686.85 - 1968.1 = 3718.8 \text{ kPa}$ too low

$v = 0.0075 \text{ m}^3/\text{kg} \Rightarrow$ $P = 6104.41 - 2227.43 = 3876.98 \text{ kPa}$

$v = 0.007 \text{ m}^3/\text{kg} \Rightarrow$ $P = 6588.16 - 2541.70 = 4046.46 \text{ kPa}$

Now we interpolate between the last two entries and check

$v = 0.00714 \text{ m}^3/\text{kg} \Rightarrow P = 6445.15 - 2447.3 = 3997.8 \text{ kPa}$ OK

$V = mv = \textbf{0.00714 m}^3$ (69% of the ideal gas value)

12.81

A flow of oxygen at 230 K, 5 MPa is throttled to 100 kPa in a steady flow process. Find the exit temperature and the specific entropy generation using Redlich-Kwong equation of state and ideal gas heat capacity. Notice this becomes iterative due to the non-linearity coupling *h, P, v* and *T*.

C.V. Throttle. Steady single flow, no heat transfer and no work.

Energy eq.: $h_1 + 0 = h_2 + 0$ so constant h

Entropy Eq.: $s_1 + s_{gen} = s_2$ so entropy generation

Find the change in h from Eq.12.26 assuming C_p is constant.

Redlich-Kwong equation of state: $P = \dfrac{RT}{v-b} - \dfrac{a}{v(v+b)T^{1/2}}$

$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v-b} + \frac{a}{2v(v+b)T^{3/2}}$$

From Eq.12.31

$$(u_2 - u_1)_T = \int_1^2 \frac{3a}{2v(v+b)T^{1/2}}\,dv = \frac{-3a}{2bT^{1/2}} \ln\left[\left(\frac{v_2+b}{v_2}\right)\left(\frac{v_1}{v_1+b}\right)\right]$$

We find change in h from change in u, so we do not do the derivative in Eq.12.27. This is due to the form of the EOS.

$$(h_2 - h_1)_T = P_2v_2 - P_1v_1 - \frac{3a}{2bT^{1/2}} \ln\left[\left(\frac{v_2+b}{v_2}\right)\left(\frac{v_1}{v_1+b}\right)\right]$$

Entropy follows from Eq.12.35

$$(s_2 - s_1)_T = \int_1^2 \left[\frac{R}{v-b} + \frac{a/2}{v(v+b)T^{3/2}}\right]dv$$

$$= R \ln\left(\frac{v_2-b}{v_1-b}\right) - \frac{a}{2bT^{3/2}} \ln\left[\left(\frac{v_2+b}{v_2}\right)\left(\frac{v_1}{v_1+b}\right)\right]$$

$P_c = 5040$ kPa; $T_c = 154.6$ K; $R = 0.2598$ kJ/kg K

$b = 0.08664 \dfrac{RT_c}{P_c} = 0.08664 \times \dfrac{0.2598 \times 154.6}{5040} = 0.000\,690\,5$ m³/kg

$a = 0.427\,48 \dfrac{R^2T_c^{5/2}}{P_c} = 0.427\,48 \times \dfrac{0.2598^2 \times 154.6^{5/2}}{5040} = 1.7013$

We need to find T_2 so the energy equation is satisfied

$$h_2 - h_1 = h_2 - h_x + h_x - h_1 = C_p(T_2 - T_1) + (h_2 - h_1)_T = 0$$

and we will evaluate it similar to Fig. 12.4, where the first term is done from state x to 2 and the second term is done from state 1 to state x (at $T_1 = 230$ K). We do this as we assume state 2 is close to ideal gas, but we do not know T_2.

We first need to find v_1 from the EOS, so guess v and find P

$$v_1 = 0.011 \text{ m}^3/\text{kg} \quad \Rightarrow \quad P = 5796.0 - 872.35 = 4924 \quad \text{too low}$$

$$v_1 = 0.01082 \text{ m}^3/\text{kg} \quad \Rightarrow \quad P = 5899.0 - 900.7 = 4998.3 \quad \text{OK}$$

Now evaluate the change in h along the 230 K from state 1 to state x, that requires a value for v_x. Guess ideal gas at $T_x = 230$ K,

$$v_x = RT_x/P_2 = 0.2598 \times 230/100 = 0.59754 \text{ m}^3/\text{kg}$$

From the EOS: $\qquad P_2 = 100.1157 - 0.3138 = 99.802$ kPa (close)

A few more guesses and adjustments gives

$$v_x = 0.59635 \text{ m}^3/\text{kg}; \quad P_2 = 100.3157 - 0.3151 = 100.0006 \text{ kPa} \quad \text{OK}$$

$$(h_x - h_1)_T = P_x v_x - P_1 v_1 - \frac{3a}{2bT^{1/2}} \ln\left[\left(\frac{v_x + b}{v_x}\right)\left(\frac{v_1}{v_1 + b}\right)\right]$$

$$= 59.635 - 5000 \times 0.01082 - 243.694 \ln\left[\frac{0.59704}{0.59635} \times \frac{0.01082}{0.01151}\right]$$

$$= 59.635 - 54.1 + 14.78335 = 20.318 \text{ kJ/kg}$$

From energy eq.: $\quad T_2 = T_1 - (h_x - h_1)_T/C_p = 230 - 20.318 / 0.922 = \textbf{208 K}$

Now the change in s is done in a similar fashion,

$$s_{gen} = s_2 - s_1 = (s_x - s_1)_T + s_2 - s_x$$

$$= R \ln\left(\frac{v_x - b}{v_1 - b}\right) - \frac{a}{2bT^{3/2}} \ln\left[\left(\frac{v_x + b}{v_x}\right)\left(\frac{v_1}{v_1 + b}\right)\right] + C_p \ln\frac{T_2}{T_x}$$

$$= 0.2598 \ln\left(\frac{0.59566}{0.0101295}\right) - 0.35318 \ln (0.94114) + 0.922 \ln\left(\frac{208}{230}\right)$$

$$= 1.05848 + 0.021425 - 0.092699$$

$$= \textbf{0.987 kJ/kg K}$$

Generalized Charts

12.82

How low should the pressure be so that nitrous oxide (N_2O) gas at 278.6 K can be treated as an ideal gas with 5% accuracy or better?

From Table A.2: $T_c = 309.6$ K, $P_c = 7.24$ MPa $\Rightarrow$ $T_{r1} = \dfrac{278.6}{309.6} = 0.9$

Look in Fig. D.1 following the curve $T_{r1} = 0.9$ to the point where Z = 0.95

$$P_{r1} = 0.125$$

so

$$P < 0.125 \times 7.24 \text{ MPa} = 0.9 \text{ MPA} = \textbf{900 kPa}$$

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12.83

Nitrous oxide (N_2O) at 278.6 K is at a pressure so that it can be in a two-phase state. Find the generalized enthalpy departure for the two saturated states of liquid and vapor.

From Table A.2: $T_c = 309.6$ K, $P_c = 7.24$ MPa $=>$ $T_{r1} = \dfrac{278.6}{309.6} = 0.9$

From Fig. D.2:

   Saturated liquid: $(h^* - h_f)/RT_c = 4.08;$

   Saturated vapor: $(h^* - h_g)/RT_c = 0.875$

12.84

Find the heat of evaporation, h_{fg}, for R-134a at 0°C from the generalized charts and compare to the value in Table B.5.

From Table A.2: $T_c = 374.2$ K, $P_c = 4.06$ MPa $=>$ $T_{r1} = \dfrac{273.15}{374.2} = 0.73$

From Fig. D.2:

Saturated liquid: $(h^* - h_f)/RT_c = 4.7$;

Saturated vapor: $(h^* - h_g)/RT_c = 0.3$

$h_{fg} = h_g - h_f = RT_c [-0.3 - (-4.7)] = 4.4\ RT_c$

$= 4.4 \times 0.08149 \times 374.2 = $ **134.2 kJ/kg**

Table B.5.1: $h_{fg} = $ **198.36 kJ/kg**

The approximation is not very good and can be improved by using the accentric factor.

12.85

A 200-L rigid tank contains propane at 9 MPa, 280°C. The propane is then allowed to cool to 50°C as heat is transferred with the surroundings. Determine the quality at the final state and the mass of liquid in the tank, using the generalized compressibility chart, Fig. D.1.

Propane C_3H_8: $V = 0.2 \text{ m}^3$, $P_1 = 9$ MPa, $T_1 = 280^oC = 553.2$ K

cool to $T_2 = 50\ ^oC = 323.2$ K

From Table A.2: $T_c = 369.8$ K, $P_c = 4.25$ MPa

$$P_{r1} = \frac{9}{4.25} = 2.118, \quad T_{r1} = \frac{553.2}{369.8} = 1.496 \quad \text{From Fig. D.1:} \quad Z_1 = 0.825$$

$$v_2 = v_1 = \frac{Z_1 R T_1}{P_1} = \frac{0.825 \times 0.188\ 55 \times 553.2}{9\ 000} = 0.00956 \text{ m}^3/\text{kg}$$

From Fig. D.1 at $T_{r2} = 0.874$,

$P_{G2} = 0.45 \times 4250 = 1912$ kPa

$v_{G2} = 0.71 \times 0.188\ 55 \times 323.2/1912 = 0.02263 \text{ m}^3/\text{kg}$

$v_{F2} = 0.075 \times 0.188\ 55 \times 323.2/1912 = 0.00239 \text{ m}^3/\text{kg}$

$0.00956 = 0.002\ 39 + x_2(0.02263 - 0.00239) \quad => \quad x_2 = \textbf{0.354}$

$m_{LIQ\ 2} = (1-0.354) \times 0.2/0.00956 = \textbf{13.51 kg}$

These tanks contain liquid propane.

12.86

A rigid tank contains 5 kg of ethylene at 3 MPa, 30°C. It is cooled until the ethylene reaches the saturated vapor curve. What is the final temperature?

$\boxed{C_2H_4}$

$V = \text{const} \quad m = 5 \text{ kg}$

$P_1 = 3 \text{ MPa} \quad T_1 = 30 \text{ °C} = 303.2 \text{ K}$

cool to $x_2 = 1.0$

$P_{r1} = \dfrac{3}{5.04} = 0.595, \quad T_{r1} = \dfrac{303.2}{282.4} = 1.074$

Fig. D.1: $\quad Z_1 = 0.82$

Final state: $x = 1$, $v_2 = v_1$

$Pv = ZRT$ so take the ratio between the two states

$$P_{r2} = P_{r1} \frac{Z_2 T_{r2}}{Z_1 T_{r1}} = 0.595 \frac{Z_{G2} T_{r2}}{0.82 \times 1.074} = 0.6756 \, Z_{G2} T_{r2}$$

Trial & error, Table D.4 may be easier to use than Fig. D.1:

T_{r2}	Z_{G2}	P_{r2}	$P_{r2 \, CALC}$		
0.866	0.72	0.42	0.421	~ OK	=> $T_2 = $ **244.6 K**

12.87

The new refrigerant R-152a is used in a refrigerator with an evaporator temperature of -20^oC and a condensing temperature of 30^oC. What are the high and low pressures in this cycle?

Since we do not have the printed tables for R-152a we will use generalized charts. The critical properties are: $T_c = 386.4$ K, $P_c = 4.52$ MPa.

Evaporator:

$$T_{r1} = T/T_c = (273.15 - 20)/386.4 = 0.655$$

Fig. D.1: $\qquad P_{G\,T1} = P_{r1\,sat}\,P_c = 0.06 \times 4.52 = \textbf{0.271 MPa}$

Condenser:

$$T_{r2} = T/T_c = (273.15 + 30)/386.4 = 0.785$$

Fig. D.1: $\qquad P_{G\,T2} = P_{r2\,sat}\,P_c = 0.22 \times 4.52 = \textbf{0.994 MPa}$

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12.88

A 4-m^3 storage tank contains ethane gas at 10 MPa, 100°C. Use Lee-Kesler EOS and find the mass of the ethane.

The Lee-Kesler EOS is shown as the generalized charts.

Table A.2: $T_c = 305.4$ K, $P_c = 4.88$ MPa,   Table A.5: R = 0.2765 kJ/kg-K
The reduced properties are:

$$P_{r1} = \frac{10}{4.88} = 2.05, \quad T_{r1} = \frac{373.15}{305.4} = 1.22 \quad \Rightarrow \quad \text{Fig. D.1: } Z = 0.56$$

$$m = \frac{PV}{ZRT} = \frac{10\ 000 \times 4}{0.56 \times 0.2765 \times 373.15} = \textbf{692.3 kg}$$

12.89

The ethane gas in the storage tank from the previous problem is cooled to 0^oC. Find the new pressure.

The new final state is given by: $(T_2, v_2 = v_1)$ $\qquad$ $T_{r2} = \dfrac{273.15}{305.4} = 0.8944$

Since Z and P are unknown this becomes trial and error solution.

$\qquad$ $P_2 / Z_2 = mRT_2 / V = 692.3 \times 0.2765 \times 273.15 / 4 = 13071.7$ kPa

Assume it is saturated $\quad$ $P_{r2} = 0.53$ (see Fig. D.1), $Z_g = 0.67$, $Z_f = 0.09$

$\qquad$ $P_2 = 0.53 \times 4880 = 2586$ kPa $\quad$ and

$\qquad$ $Z_2 = P_2 / 13071.7 = 0.198$ (two phase OK)

$\qquad$ Ans.: $P_2 = $ **2586 kPa**

12.90

Use CATT3 to solve the previous two problems when the acentric factor is used to improve the accuracy.

Problem 12.88:

The Lee-Kesler EOS is shown as the generalized charts.

Table A.2: $T_c = 305.4$ K, $P_c = 4.88$ MPa,   Table A.5: $R = 0.2765$ kJ/kg-K

Table D.4: $\omega = 0.099$

The reduced properties are:

$$P_{r1} = \frac{10}{4.88} = 2.05, \quad T_{r1} = \frac{373.15}{305.4} = 1.22 \quad \Rightarrow \quad \text{CATT3:} \quad Z = 0.605$$

$$m = \frac{PV}{ZRT} = \frac{10\,000 \times 4}{0.605 \times 0.2765 \times 373.15} = \textbf{640.8 kg}$$

Problem 12.89:

The new final state is given by: $(T_2, v_2 = v_1)$ $\qquad T_{r2} = \frac{273.15}{305.4} = 0.8944$

Since Z and P are unknown this becomes trial and error solution.

$$P_2 / Z_2 = mRT_2 / V = 640.8 \times 0.2765 \times 273.15 / 4 = 12099.3 \text{ kPa}$$

Assume it is saturated vapor $P_{r2} = 0.483$ (CATT3), $Z_g = 0.69$, $Z_f = 0.078$

$\qquad P_2 = 0.483 \times 4880 = 2357$ kPa and

$\qquad Z_2 = P_2 / 12\,099.3 = 0.1948$ (two phase OK)

$\qquad$ Ans.: $P_2 = \textbf{2357 kPa}$

12.91

A geothermal power plant uses butane as saturated vapor $80^{\circ}C$ into the turbine and the condenser operates at $30^{\circ}C$. Find the reversible specific turbine work.

C_4H_{10} cycle

$T_1 = 80\ ^{\circ}C$, $x_1 = 1.0$; $T_3 = 30^{\circ}C$, $x_3 = 0.0$

$T_{r1} = \dfrac{353.2}{425.2} = 0.831$

From D.1, D.2 and D.3:

$P_1 = 0.325 \times 3800 = 1235$ kPa

$(h_1^* - h_1) = 0.143\ 04 \times 425.2 \times 0.56 = 34.1$

$(s_1^* - s_1) = 0.143\ 04 \times 0.475 = 0.0680$

$T_{r3} = \dfrac{303.2}{425.2} = 0.713$

From D.1, D.2 and D.3: $P_3 = 0.113 \times 3800 = 429$ kPa

sat. liq.: $(h^* - h_f) = RT_c \times 4.81 = 292.5$; $(s^* - s_f) = R \times 6.64 = 0.950$

sat. vap.: $(h^* - h_g) = RT_c \times 0.235 = 14.3$; $(s^* - s_g) = R \times 0.22 = 0.031$

Because of the combination of properties of C_4H_{10} (particularly the large C_{P0}/R), s_1 is larger than s_g at T_3. To demonstrate,

$(s_1^* - s_{g3}^*) = 1.7164 \ln \dfrac{353.2}{303.2} - 0.143\ 04 \ln \dfrac{1235}{429} = 0.1107$

$(s_1 - s_{g3}) = -0.0680 + 0.1107 + 0.031 = +0.0737$ kJ/kg K

so that T_{2S} will be $> T_3$, as shown in the T-s diagram. A number of other heavy hydrocarbons also exhibit this behavior.

Solution 12.91 Continued.

Assume $T_{2S} = 315$ K, $\Rightarrow$ $T_{r2S} = 315 / 425.2 = 0.741$

From D.2 and D.3:

$$(h_{2S}^* - h_{2S}) = RT_c \times 0.21 = 12.8 \quad \text{and} \quad (s_{2S}^* - s_{2S}) = R \times 0.19 = 0.027$$

$$(s_1^* - s_{2S}^*) = 1.7164 \ln \frac{353.2}{315} - 0.143\ 04 \ln \frac{1235}{429} = +0.0453$$

$$(s_1 - s_{2S}) = -0.0680 + 0.0453 + 0.027 \approx 0$$

$$\Rightarrow T_{2S} = 315 \text{ K}$$

$$(h_1^* - h_{2S}^*) = 1.7164(353.2-315) = 65.6$$

$$w_{ST} = h_1 - h_{2S} = -34.1 + 65.6 + 12.8 = \mathbf{44.3 \ kJ/kg}$$

12.92

Consider the following EOS, expressed in terms of reduced pressure and temperature:

$$Z = 1 + \frac{P_r}{14\,T_r}\left(1 - \frac{6}{T_r^2}\right)$$

What does this equation predict for enthalpy departure from the ideal gas value at the state $P_r = 0.4$, $T_r = 0.9$? What is it from the generalized charts?

$$Z = \frac{Pv}{RT} = 1 + \frac{P_r}{14\,T_r}\left(1 - \frac{6}{T_r^2}\right)$$

$$v = \frac{RT}{P} + \frac{RT_c}{14P_c}\left(1 - \frac{6T_c^2}{T^2}\right) ; \quad \left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{P} + \frac{12RT_c^3}{14P_c T^3}$$

$$v - T\left(\frac{\partial v}{\partial T}\right)_p = \frac{RT_c}{14P_c} - \frac{18RT_c^3}{14P_c T^2}$$

Now Eq.12.27 is integrated with limits similar to Eq.12.62

$$h - h^* = \int_0^P \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dP = \frac{RT_c}{14}\left(1 - \frac{18}{T_r^2}\right) P_r = -\textbf{0.606 RT}_\textbf{c}$$

Evaluate at $P_r = 0.4$, $T_r = 0.9$ from Fig. D2 to get

$$h - h^* = -(h^* - h) \approx -0.6\,RT_c$$

This result matched with in the accuracy the figure can be read.

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12.93

Consider the following equation of state, expressed in terms of reduced pressure and temperature:

$$Z = 1 + \frac{P_r}{14\,T_r}\left(1 - \frac{6}{T_r^{\,2}}\right)$$

What does this equation predict for entropy departure from the ideal gas value at the state $P_r = 0.4$, $T_r = 0.9$?

The entropy departure is the change in s for a real gas minus the change in s for an ideal gas, so from Eq.12.32 and eq.6.14 we get

$$d(s - s^*) = C_p\frac{dT}{T} - \left(\frac{\partial v}{\partial T}\right)_p dP - \left[\;C_p\frac{dT}{T} - \frac{R}{P}\,dP\right] = \left[\frac{R}{P} - \left(\frac{\partial v}{\partial T}\right)_p\right] dP$$

Solve now for v from the compressibility factor ($Z = Pv/RT$) to get

$$Z = \frac{Pv}{RT} = 1 + \frac{P_r}{14\,T_r}\left(1 - \frac{6}{T_r^{\,2}}\right)$$

$$v = \frac{RT}{P} + \frac{RT_c}{14P_c}\left(1 - \frac{6T_c^{\,2}}{T^2}\right)\;;\quad \left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{P} + \frac{12RT_c^{\,3}}{14P_cT^3}$$

$$s - s^* = \int\limits_0^P \left[\frac{R}{P} - \left(\frac{\partial v}{\partial T}\right)_p\right] dP = \int\limits_0^P \left[-\frac{12RT_c^{\,3}}{14P_cT^3}\right] dP = -\frac{6}{7}R\frac{P_r}{T_r^{\,3}}$$

Evaluate at $P_r = 0.4$, $T_r = 0.9$ to get

$$s - s^* = -\textbf{0.4703 R}$$

12.94

A very low temperature refrigerator uses neon. From the compressor the neon at 1.5 MPa, 80 K goes through the condenser and comes out at saturated liquid 40 K. Find the specific heat transfer using generalized charts.

State 1: 80 K, 1.5 MPa : $T_{r1} = \dfrac{80}{44.4} = 1.802, \quad P_{r1} = \dfrac{1.5}{2.76} = 0.543$

State 2: 40 K, x = 0: $T_{r2} = 0.90, \ P_{r2} = 0.532$

The enthalpy departure chart Fig. D.2:

$$(h^* - h)_1 = 0.22 \ RT_c , \qquad (h^* - h)_2 = 4.10 \ RT_c$$

$$h_2^* - h_1^* = 1.03 \ (40 - 80) = -41.2 \text{ kJ/kg}$$

$$h_2 - h_1 = h_2^* - h_1^* - (h^* - h)_2 + (h^* - h)_1$$

$$= -41.2 + 0.412 \times 44.4 \ (-4.10 + 0.22) \ = -112.2 \text{ kJ/kg}$$

$$q = h_2 - h_1 = \textbf{-112.2 kJ/kg}$$

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12.95
Repeat the previous problem using CATT3 software for the neon properties.

From CATT3: $h_1 = 138.3$ kJ/kg, $h_2 = 30.24$ kJ/kg (P = 1.46 MPa)

$q = h_2 - h_1 = 30.24 - 138.3 =$ **-108.1 kJ/kg**

12.96

A piston/cylinder contains 5 kg of butane gas at 500 K, 5 MPa. The butane expands in a reversible polytropic process to 3 MPa, 460 K. Determine the polytropic exponent n and the work done during the process.

C_4H_{10} $m = 5$ kg $T_1 = 500$ K $P_1 = 5$ MPa

Rev. polytropic process: $P_1 V_1^n = P_2 V_2^n$

$T_{r1} = \dfrac{500}{425.2} = 1.176$, $P_{r1} = \dfrac{5}{3.8} = 1.316$ From Fig. D.1: $Z_1 = 0.68$

$T_{r2} = \dfrac{460}{425.2} = 1.082$, $P_{r2} = \dfrac{3}{3.8} = 0.789$ From Fig. D.1: $Z_2 = 0.74$

$V_1 = \dfrac{mZRT}{P} = \dfrac{5 \times 0.68 \times 0.1430 \times 500}{5000} = 0.0486 \text{ m}^3$

$V_2 = \dfrac{mZRT}{P} = \dfrac{5 \times 0.74 \times 0.1430 \times 460}{3000} = 0.0811 \text{ m}^3$

Solve for the polytropic exponent, n, as

$n = \ln(P_1/P_2) \,/\, \ln(V_2/V_1) = \ln\left(\dfrac{5}{3}\right) / \ln\left(\dfrac{0.0811}{0.0486}\right) = \textbf{0.9976}$

$_1W_2 = \int\limits_1^2 PdV = \dfrac{P_2V_2 - P_1V_1}{1-n} = \dfrac{3000 \times 0.0811 - 5000 \times 0.0486}{1 - 0.9976} = \textbf{125 kJ}$

12.97

Calculate the heat transfer during the process described in Problem 12.96.

From solution 12.96,

$$V_1 = 0.0486 \text{ m}^3, \quad V_2 = 0.0811 \text{ m}^3, \quad _1W_2 = 125 \text{ kJ}$$

$$T_{r1} = \frac{500}{425.2} = 1.176, \quad P_{r1} = \frac{5}{3.8} = 1.316 \quad \text{From Fig. D.1:} \quad Z_1 = 0.68$$

$$T_{r2} = 1.082, \quad P_{r2} = 0.789, \quad T_2 = 460 \text{ K}$$

From Fig. D.2: $(h^* - h)_1 = 1.30 \text{ RT}_C, \quad (h^* - h)_2 = 0.90 \text{ RT}_C$

$$h_2^* - h_1^* = 1.716(460 - 500) = -83.1 \text{ kJ/kg}$$

$$h_2 - h_1 = -83.1 + \frac{8.3145 \times 425.2}{58.124}(-0.90 + 1.30) = -58.8 \text{ kJ/kg}$$

$$U_2 - U_1 = m(h_2 - h_1) - P_2V_2 + P_1V_1$$

$$= 5(-58.8) - 3000 \times 0.0811 + 5000 \times 0.0486 = -288.3 \text{ kJ}$$

$$_1Q_2 = U_2 - U_1 + _1W_2 = \textbf{-174.3 kJ}$$

12.98

An ordinary lighter is nearly full of liquid propane with a small amount of vapor, the volume is 5 cm^3, and temperature is 23°C. The propane is now discharged slowly such that heat transfer keeps the propane and valve flow at 23°C. Find the initial pressure and mass of propane and the total heat transfer to empty the lighter.

Propane C_3H_8 $T_1 = 23°C = 296.2$ K = constant, $x_1 = 0.0$

$V_1 = 5$ cm$^3 = 5×10^{-6}$ m^3, $T_{r1} = 296.2/369.8 = 0.804$

From Figs. D.1 and D.2,

$P_1 = P_{G\,T1} = 0.25×4.25 = $ **1.063 MPa**, $Z_1 = 0.04$

$(h_1^* - h_1) = 0.188\,55×369.8×4.51 = 314.5$

$m_1 = \dfrac{P_1 V_1}{Z_1 R T_1} = \dfrac{1063×5×10^{-6}}{0.04×0.188\,55×296.2} = $ **0.00238 kg**

State 2: Assume vapor at 100 kPa, 23°C   ideal gas so no corrections

Therefore, m_2 much smaller than m_1 ($\sim 9.0 × 10^{-6}$ kg)

$$Q_{CV} = m_2 u_2 - m_1 u_1 + m_e h_e$$
$$= m_2 h_2 - m_1 h_1 - (P_2 - P_1)V + (m_1 - m_2)h_e$$
$$= m_2(h_2 - h_e) + m_1(h_e - h_1) - (P_2 - P_1)V$$

$(h_e - h_1) = 0 + 0 + 314.5$

$Q_{CV} = \approx 0 + 0.00238(314.5) - (100 - 1063)×5×10^{-6} = $ **0.753 kJ**

Actual lighters use butane and some propane.

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12.99

250-L tank contains propane at 30°C, 90% quality. The tank is heated to 300°C. Calculate the heat transfer during the process.

$$V = 250 \text{ L} = 0.25 \text{ m}^3$$
$$T_1 = 30 \text{ }^{\circ}\text{C} = 303.2 \text{ K}, \quad x_1 = 0.90$$
Heat to $T_2 = 300 \text{ }^{\circ}\text{C} = 573.2 \text{ K}$
$$M = 44.094, \quad T_c = 369.8 \text{ K}, \quad P_c = 4.25 \text{ MPa}$$
$$R = 0.188\,55, \quad C_{P0} = 1.6794$$

$T_{r1} = 0.82 \rightarrow$ Fig. D.1:

$$Z_1 = (1 - x_1)\,Z_{f1} + x_1\,Z_{g1} = 0.1 \times 0.05 + 0.9 \times 0.785 = 0.711$$

Fig D.2: $\quad \dfrac{h_1^* - h_1}{RT_c} = 0.1 \times 4.43 + 0.9 \times 0.52 = 0.911$

$$P_r^{SAT} = 0.30 \quad P_1^{SAT} = 1.275 \text{ MPa}$$

$$m = \frac{1275 \times 0.25}{0.711 \times 0.188\,55 \times 303.2} = 7.842 \text{ kg}$$

$$P_{r2} = \frac{7.842 \times Z_2 \times 0.188\,55 \times 573.2}{0.25 \times 4250} = \frac{Z_2}{1.254}$$

at $T_{r2} = 1.55$ Trial and error on P_{r2}

$\quad P_{r2} = 0.743 \implies P_2 = 3.158 \text{ MPa}, \quad Z_2 = 0.94, \quad (h^* - h)_2 = 0.35\,RT_C$

$(h_2^* - h_1^*) = 1.6794(300 - 30) \qquad = 453.4 \text{ kJ/kg}$

$(h_1^* - h_1) = 0.911 \times 0.188\,55 \times 369.8 = 63.5 \text{ kJ/kg}$

$(h_2^* - h_2) = 0.35 \times 0.188\,55 \times 369.8 = 24.4 \text{ kJ/kg}$

$Q_{12} = m(h_2 - h_1) - (P_2 - P_1)V = 7.842(-24.4 + 453.4 + 63.5) - (3158 - 1275) \times 0.25$

$\qquad = +3862 - 471 = \textbf{3391 kJ}$

12.100

Find the heat of evaporation, h_{fg}, for isobutane ($T_c = 408.2$ K, $P_c = 3.65$ MPa, M = 58.124) at 12.6°C from the generalized charts and compare to the values in the CATT3 computerized tables.

To read the charts we need the reduced temperature

$$T_{r1} = T/T_c = (12.6 + 273.15)/408.2 = 0.70$$

$$(h^* - h)_g = 0.2\ RT_c \ ; \ \ (h^* - h)_f = 4.85\ RT_c$$

$$h_{fg} = h_g - h_f = -(h^* - h)_g - (h^* - h)_f = (-0.2 + 4.85)\ RT_c$$

$$= 4.65\ RT_c = 4.65 \times (8.3145/58.124) \times 408.2$$

$$= \textbf{271.5 kJ/kg}$$

CATT3: $h_{fg} = -25.62 + 368.7 = \textbf{343 kJ/kg}$

The generalized charts are not super accurate, some improvement can be done using the accentric factor.

12.101

A cylinder contains ethylene, C_2H_4, at 1.536 MPa, $-13°C$. It is now compressed isothermally in a reversible process to 5.12 MPa. Find the specific work and heat transfer.

Ethylene C_2H_4 $P_1 = 1.536$ MPa , $T_2 = T_1 = -13°C = 260.2$ K

$T_{r2} = T_{r1} = 260.2 / 282.4 = 0.921$, $P_{r1} = 1.536 / 5.04 = 0.305$

From D.1, D.2 and D.3: $Z_1 = 0.85$

$(h_1^*-h_1) = 0.2964×282.4×0.40 = 33.5$ and $(s_1^*-s_1) = 0.2964×0.30 = 0.0889$

From D.1, D.2 and D.3: $Z_2 = 0.17$, $P_{r2} = 5.12/5.04 = 1.016$ (comp. liquid)

$(h_2^*-h_2) = 0.2964×282.4×4.0 = 334.8$ and $(s_2^*-s_2) = 0.2964×3.6 = 1.067$

Ideal gas: $(h_2^*-h_1^*) = 0$ and $(s_2^*-s_1^*) = 0 - 0.2964 \ln \dfrac{5.12}{1.536} = -0.3568$

$_1q_2 = T(s_2-s_1) = 260.2(-1.067 - 0.3568 + 0.0889) = $ **-347.3 kJ/kg**

$(h_2 - h_1) = -334.8 + 0 + 33.5 = -301.3$ kJ/kg

$(u_2 - u_1) = (h_2-h_1) - RT(Z_2-Z_1) = -301.3 - 0.2964×260.2(0.17-0.85) = -248.9$

$_1w_2 = {_1q_2} - (u_2 - u_1) = -347.3 + 248.9 = $ **-98.4 kJ/kg**

12.102

Saturated vapor R-410A at 30°C is throttled to 200 kPa in a steady flow process. Calculate the exit temperature assuming no changes in the kinetic energy, using the generalized charts, Fig. D.2 and the R-410A tables, Table B.4.

R-410A throttling process

Energy Eq.: $h_2 - h_1 = 0 = (h_2 - h_2^*) + (h_2^* - h_1^*) + (h_1^* - h_1)$

Generalized Chart, Fig. D.2, $\quad R = 8.31451/72.585 = 0.11455$ kJ/kg-K

$$T_{r1} = \frac{303.2}{344.5} = 0.88 \quad \Rightarrow \quad (h_1^* - h_1) = 0.11455 \times 344.5 \ (0.85) = 33.54 \text{ kJ/kg}$$

For C_{P0}, use h values from Table B.4 at low pressure.

$$C_{P0} \approx (330.83 - 314.40) / (40 - 20) = 0.8215 \text{ kJ/kg K}$$

Substituting: $(h_2 - h_2^*) + 0.8215 \ (T_2 - 30) + 33.54 = 0$

at $P_{r2} = 200/4900 = 0.041$

Assume $T_2 = -10^{\circ}$C $\Rightarrow$ $T_{r2} = 263.2/344.5 = 0.764$

$(h_2^* - h_2) = RT \times 0.1 = 0.11455 \times 344.5 \ (0.1) = 3.95$

Substituting : $\quad -3.95 + 0.8215 \ (-10 - 30) + 33.54 = -3.27$

Assume $T_2 = -5^{\circ}$C $\Rightarrow$ $T_{r2} = 268.2/344.5 = 0.778$

$(h_2^* - h_2) = RT \times 0.1 = 0.11455 \times 344.5 \ (0.1) = 3.95$

Substituting : $\quad -3.95 + 0.8215 \ (-5 - 30) + 33.54 = 0.84$

$\Rightarrow T_2 = \textbf{-6.0 }^{\textbf{o}}\textbf{C}$

R-410A tables, B.4: at $T_1 = 30^{\circ}$C, $x_1 = 1.0$ $\Rightarrow$ $h_1 = 284.16$ kJ/kg

$h_2 = h_1 = 284.16$, $P_2 = 0.2$ MPa $\Rightarrow$ $T_2 = \textbf{-13.4 }^{\textbf{o}}\textbf{C}$

12.103

Repeat Problem 12.91 using CATT3 and include the acentric factor for butane to improve the accuracy.

C_4H_{10} cycle

$T_1 = 80\ ^oC,\ x_1 = 1.0\ ;\ T_3 = 30^oC,\ x_3 = 0.0$

$T_{r1} = \dfrac{353.2}{425.2} = 0.831$

From CATT3 with $\omega = 0.199$:

$P_1 = 0.2646 \times 3800 = 1005$ kPa

$(h_1^* - h_1) = 0.1430 \times 425.2 \times 0.5685 = 34.6$

$(s_1^* - s_1) = 0.1430 \times 0.4996 = 0.0714$

$T_{r3} = \dfrac{303.2}{425.2} = 0.713$

From CATT3 with $\omega = 0.199$: $P_3 = 0.07443 \times 3800 = 282.8$ kPa

sat. liq.: $(h^* - h_f) = RT_c \times 6.048 = 367.74$; $(s^* - s_f) = R \times 8.399 = 1.201$

sat. vap.: $(h^* - h_g) = RT_c \times 0.202 = 12.28$; $(s^* - s_g) = R \times 0.201 = 0.0287$

Because of the combination of properties of C_4H_{10} (particularly the large C_{P0}/R), s_1 is larger than s_g at T_3. To demonstrate,

$(s_1^* - s_{g3}^*) = 1.716 \ln \dfrac{353.2}{303.2} - 0.1430 \ln \dfrac{1005}{282.8} = 0.0806$

$(s_1 - s_{g3}) = -0.0714 + 0.0806 + 0.0287 = +0.0379$ kJ/kg K

so that T_{2S} will be $> T_3$, as shown in the T-s diagram. A number of other heavy hydrocarbons also exhibit this behavior.

Solution 12.103 continued.

Assume $T_{2S} = 315$ K, => $T_{r2S} = 315 / 425.2 = 0.741$

From CATT3:

$(h_{2S}^* - h_{2S}) = RT_c \times 0.183 = 11.13$ and $(s_{2S}^* - s_{2S}) = R \times 0.1746 = 0.025$

$(s_1^* - s_{2S}^*) = 1.716 \ln \dfrac{353.2}{315} - 0.1430 \ln \dfrac{1005}{282.8} = +0.01509$

$(s_1 - s_{2S}) = -0.0714 + 0.01509 + 0.025 = -0.031$

Repeat at $T_{2S} = 310$ K to get $T_{r2S} = 0.729$,

$(h_{2S}^* - h_{2S}) = RT_c \times 0.1907 = 11.595$ and $(s_{2S}^* - s_{2S}) = R \times 0.1853 = 0.0265$

$(s_1^* - s_{2S}^*) = 1.716 \ln \dfrac{353.2}{310} - 0.1430 \ln \dfrac{1005}{282.8} = +0.04255$

$(s_1 - s_{2S}) = -0.0714 + 0.04255 + 0.0265 = -0.0023$ very close to 0, OK

$\Rightarrow T_{2S} = 310$ K

$(h_1^* - h_{2S}^*) = 1.716 (353.2 - 310) = 74.13$

$w_{ST} = h_1 - h_{2S} = -34.6 + 74.13 + 11.6 =$ **51.1 kJ/kg**

12.104

A cylinder contains ethylene, C_2H_4, at 1.536 MPa, $-13°C$. It is now compressed in a reversible isobaric (constant *P*) process to saturated liquid. Find the specific work and heat transfer.

Ethylene C_2H_4 ; $P_1 = 1.536$ MPa $= P_2$, $T_1 = -13^oC = 260.2$ K

State 2: saturated liquid, $x_2 = 0.0$

$$T_{r1} = \frac{260.2}{282.4} = 0.921 \quad P_{r1} = P_{r2} = \frac{1.536}{5.04} = 0.305$$

From Figs. D.1, D.2: $Z_1 = 0.85$, $(h_1^* - h_1)/RT_c = 0.40$

$$v_1 = \frac{Z_1RT_1}{P_1} = \frac{0.85 \times 0.29637 \times 260.2}{1536} = 0.042675$$

$$(h_1^* - h_1) = 0.296\,37 \times 282.4 \times 0.40 = 33.5$$

From Figs. D.1, D.2: $T_2 = 0.824 \times 282.4 = 232.7$ K

$$Z_2 = 0.05 \ , \quad (h_2^* - h_2)/RT_c = 4.42$$

$$v_2 = \frac{Z_2RT_2}{P_2} = \frac{0.05 \times 0.29637 \times 232.7}{1536} = 0.002245 \text{ m}^3/\text{kg}$$

$$(h_2^* - h_2) = 0.296\,37 \times 282.4 \times 4.42 = 369.9$$

$$(h_2^* - h_1^*) = C_{P0}(T_2 - T_1) = 1.5482(232.7 - 260.2) = -42.6$$

$$w_{12} = \int Pdv = P(v_2 - v_1) = 1536 \text{ kPa } (0.002\,245 - 0.042\,675) \text{ m}^3/\text{kg}$$

$$= \textbf{-62.1 kJ/kg}$$

$$q_{12} = (u_2 - u_1) + w_{12} = (h_2 - h_1) = -369.9 \ - 42.6 + 33.5$$

$$= \textbf{-379 kJ/kg}$$

12.105

Refrigerant-123, dichlorotrifluoroethane, which is currently under development as a potential replacement for environmentally hazardous refrigerants, undergoes an isothermal steady flow process in which the R-123 enters a heat exchanger as saturated liquid at 40°C and exits at 100 kPa. Calculate the heat transfer per kilogram of R-123, using the generalized charts, Fig. D.2

R-123: $M = 152.93$, $T_C = 456.9$ K, $P_C = 3.67$ MPa

$T_1 = T_2 = 40\ ^oC,\quad x_1 = 0$
$P_2 = 100$ kPa

$T_{r1} = T_{r2} = 313.2/456.9 = 0.685,\qquad P_{r2} = 0.1/3.67 = 0.027$

From Fig. D.2: $P_{r1} = 0.084,\quad (h^* - h)_1/RT_C = 4.9$

From D.1:   saturated $P_1 = 0.084 \times 3670 = 308$ kPa

$P_2 < P_1$ with no work done, so process is irreversibel.

Energy Eq.: $q + h_1 = h_2$,   Entropy Eq.: $s_1 + \int dq/T + s_{gen} = s_2,\quad s_{gen} > 0$

From Fig. D.2: $(h^* - h)_2/RT_C = 0.056$

$q = h_2 - h_1 = 8.3145 \times 456.9 \left[-0.056 + 0 + 4.90 \right]/152.93 = \mathbf{120.4\ kJ/kg}$

12.106

Carbon dioxide collected from a fermentation process at 5°C, 100 kPa should be brought to 243 K, 4 MPa in a steady flow process. Find the minimum amount of work required and the heat transfer. What devices are needed to accomplish this change of state?

$$T_{ri} = \frac{278.2}{304.1} = 0.915, \quad P_{ri} = \frac{100}{7380} = 0.0136$$

From D.2 and D.3 : $(h^*-h)_{ri}/RT_C = 0.02, \quad (s^*-s)_{ri}/R = 0.01$

$$T_{re} = \frac{243}{304.1} = 0.80, \quad P_{re} = \frac{4}{7.38} = 0.542$$

From D.2 and D.3: $(h^*-h)_{re}/RT_C = 4.5 , \quad (s^*-s)_{re}/R = 4.74$

$$(h_i-h_e) = - (h_i^*-h_i) + (h_i^*-h_e^*) + (h_e^*-h_e)$$

$$= - 0.188\ 92 \times 304.1 \times 0.01 + 0.8418(278.2-243)$$

$$+ 0.188\ 92 \times 304.1 \times 4.5 \quad = 287.6 \text{ kJ/kg}$$

$$(s_i-s_e) = - (s_i^*-s_i) + (s_i^*-s_e^*) + (s_e^*-s_e)$$

$$= - 0.188\ 92 \times 0.01 + 0.8418 \ln(278.2/243)$$

$$- 0.188\ 92 \ln(0.1/4) + 0.188\ 92 \times 4.74 \ = 1.7044 \text{ kJ/kg K}$$

$$w^{rev} = (h_i-h_e) -T_0(s_i-s_e) \ = 287.6 - 278.2(1.7044) = \textbf{-186.6 kJ/kg}$$

$$q^{rev} = (h_e-h_i) + w^{rev} \ = -287.6 -186.6 = \textbf{-474.2 kJ/kg}$$

We need a compressor to bring the pressure up and a cooler to bring the temperature down. Cooling it before compression and intercooling between stages in the compressor lowers the compressor work. In an actual set-up we require more work than the above reversible limit.

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12.107

Determine how accurate the generalized chart is for the carbon dioxide process in Problem 12.106 by using the CATT3 software for the carbon dioxide properties.

From the CATT3 software:

Inlet superheated vapor $h_i = 376.1$ kJ/kg   and $s_i = 2.015$ kJ/kg-K

Exit compressed liquid $h_e = 20.21$ kJ/kg and $s_e = 0.07349$ kJ/kg-K

$w^{rev} = (h_i - h_e) - T_0(s_i - s_e) = 376.1 - 20.21 - 278.2 (2.015 - 0.07349)$

   $= $ **-184.1 kJ/kg**

$q^{rev} = (h_e - h_i) + w^{rev} = (20.21 - 376.1) - 184.1 = $ **-540 kJ/kg**

The work is very accurate but the heat transfer a little less so. The enthalpy and entropy differences are both under-estimated by the generalized charts.

12.108

A geothermal power plant on the Raft River uses isobutane as the working fluid. The fluid enters the reversible adiabatic turbine at 160°C, 5.475 MPa, and the condenser exit condition is saturated liquid at 33°C. Isobutane has the properties T_c= 408.14 K, P_c= 3.65 MPa, C_{P0}= 1.664 kJ/kg K and ratio of specific heats $k = 1.094$ with a molecular weight as 58.124. Find the specific turbine work and the specific pump work.

Turbine inlet: $T_1 = 160^oC$, $P_1 = 5.475$ MPa

Condenser exit: $T_3 = 33^oC$, $x_3 = 0.0$, $T_{r3} = 306.2 / 408.1 = 0.75$

From Fig. D.1:

$P_{r3} = 0.16$, $Z_3 = 0.03$ => $P_2 = P_3 = 0.16 \times 3.65 = 0.584$ MPa

$T_{r1} = 433.2 / 408.1 = 1.061$, $P_{r1} = 5.475 / 3.65 = 1.50$

From Fig. D.2 & D.3:

$(h_1^* - h_1) = 0.143\ 05 \times 408.1 \times 2.84 = 165.8$

$(s_1^* - s_1) = 0.143\ 05 \times 2.15 = 0.3076$

$(s_2^* - s_1^*) = 1.664 \ln \dfrac{306.2}{433.2} - 0.143\ 05 \ln \dfrac{0.584}{5.475} = -0.2572$

$(s_2^* - s_2) = (s_2^* - s_{F2}) - x_2\ s_{FG2}$

$\qquad = 0.143\ 05 \times 6.12 - x_2 \times 0.143\ 05(6.12-0.29) = 0.8755 - x_2 \times 0.8340$

$(s_2 - s_1) = 0 = -0.8755 + x_2 \times 0.8340 - 0.2572 + 0.3076$ => $x_2 = 0.99$

$(h_2^* - h_1^*) = C_{P0}(T_2 - T_1) = 1.664(306.2 - 433.2) = -211.3$

From Fig. D.2:,

$(h_2^* - h_2) = (h_2^* - h_{F2}) - x_2 h_{FG2} = 0.143\ 05 \times 408.1[4.69-0.99(4.69-0.32)]$

$\qquad = 273.8 - 0.99 \times 255.1 = 21.3$

Turbine: $\quad w_T = (h_1 - h_2) = -165.8 + 211.3 + 21.3 =$ **66.8 kJ/kg**

Pump: $\qquad v_{F3} = \dfrac{Z_{F3}RT_3}{P_3} = \dfrac{0.03 \times 0.143\ 05 \times 306.2}{584} = 0.00225$

$w_P = - \int v\ dP \approx v_{F3}(P_4 - P_3) = -0.00225\ (5475-584) =$ **-11.0 kJ/kg**

12.109

A flow of oxygen at 230 K, 5 MPa is throttled to 100 kPa in a steady flow process. Find the exit temperature and the entropy generation.

Process: Throttling

Small surface area: $\dot{Q} = 0$;

No shaft: $\dot{W} = 0$

Irreversible: $\dot{S}_{gen} > 0$

We will solve the problem using generalized charts.

$$T_{ri} = \frac{230}{154.6} = 1.488, \quad P_{ri} = \frac{5}{5.04} = 0.992, \quad P_{re} = \frac{0.1}{5.04} = 0.02$$

From D.2: $(h_i^* - h_i) = RT_c \, \Delta h = 0.2598 \times 154.6 \times 0.50 = 20.1$ kJ/kg

Energy Eq.: $(h_e - h_i) = 0 = -(h_e^* - h_e) + (h_e^* - h_i^*) + (h_i^* - h_i)$

Assume $T_e = 208$ K , $T_{re} = 1.345$:

$(h_e^* - h_i^*) = C_p (T_e - T_i) = 0.922 (208 - 230) = -20.3$ kJ/kg

From D.2: $(h_e^* - h_e) = RT_c \, \Delta h = 0.2598 \times 154.6 \times 0.01 = 0.4$

Check first law $(h_e - h_i) = -0.4 - 20.3 + 20.1 \approx 0$ OK $\Rightarrow$ $T_e = $ **208 K**

From D.3,

$(s_i^* - s_i) = 0.2598 \times 0.25 = 0.0649$ and $(s_e^* - s_e) = 0.2598 \times 0.01 = 0.0026$

$(s_e^* - s_i^*) = 0.9216 \ln \frac{208}{230} - 0.2598 \ln \frac{0.1}{5} = 0.9238$ kJ/kg K

$s_{gen} = (s_e - s_i) = -0.0026 + 0.9238 + 0.0649 = $ **0.9861 kJ/kg K**

12.110

An uninsulated piston/cylinder contains propene, C_3H_6, at ambient temperature, 19°C, with a quality of 50% and a volume of 10 L. The propene now expands very slowly until the pressure in the cylinder drops to 460 kPa. Calculate the mass of propene, the work, and heat transfer for this process.

Propene C_3H_6: $T_1 = 19^oC = 292.2$ K, $x_1 = 0.50$, $V_1 = 10$ L

From Fig. D.1: $T_{r1} = 292.2/364.9 = 0.80$,

$P_{r1} = P_{r\,sat} = 0.25$, $P_1 = 0.25 \times 4.6 = 1.15$ MPa

From D.1: $Z_1 = 0.5 \times 0.04 + 0.5 \times 0.805 = 0.4225$

$$m = \frac{P_1 V_1}{Z_1 R T_1} = \frac{1150 \times 0.010}{0.4225 \times 0.197\,58 \times 292.2} = \mathbf{0.471\ kg}$$

Assume reversible and isothermal process (slow, no friction, not insulated)

$$_1Q_2 = m(u_2 - u_1) + {_1W_2}$$

$$_1W_2 = \int_1^2 PdV \text{ (cannot integrate);} \qquad {_1Q_2} = \int_1^2 TdS = Tm(s_2 - s_1)$$

From Figs. D.2 and D.3:

$$h_1^* - h_1 = 0.19758 \times 364.9(0.5 \times 4.51 + 0.5 \times 0.46) = 179.2 \text{ kJ/kg}$$

$$(s_1^* - s_1) = 0.197\,58\,(0.5 \times 5.46 + 0.5 \times 0.39) = 0.5779 \text{ kJ/kg K}$$

The ideal gas change in h and s are

$$(h_2^* - h_1^*) = 0 \text{ and } \qquad (s_2^* - s_1^*) = 0 - 0.197\,58 \ln \frac{460}{1161} = +0.1829 \text{ kJ/kg K}$$

At $T_{r2} = 0.80$, $P_{r2} = 0.10$, from D.1, D.2 and D.3, $Z_2 = 0.93$

$$(h_2^* - h_2) = 0.197\,58 \times 364.9 \times 0.16 = 11.5 \text{ kJ/kg}$$

$$(s_2^* - s_2) = 0.197\,58 \times 0.13 = 0.0257 \text{ kJ/kg K}$$

Now we can do the change in s and h from state 1 to state 2

$$(s_2 - s_1) = \ -(s_2^* - s_2) + (s_2^* - s_1^*) + (s_1^* - s_1)$$

$$= -0.0257 + 0.1829 + 0.5779 = 0.7351 \text{ kJ/kg K}$$

$$(h_2 - h_1) = -\,(h_2^* - h_2) + (h_2^* - h_1^*) + \ h_1^* - h_1$$

$$= -11.5 + 0 + 179.2 = 167.7 \text{ kJ/kg}$$

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Solution 12.110 continued.

The heat transfer is found from the second law

$$_1q_2 = 292.2 \times 0.7351 = 214.8 \text{ kJ/kg} \quad => \quad _1Q_2 = m\,_1q_2 = \textbf{101.2 kJ}$$

We need the internal energy in the energy equation

$$u_2 - u_1 = (h_2 - h_1) + RT(Z_1 - Z_2) = 167.7 + 0.197\,58 \times 292.2\,(0.4225 - 0.93)$$

$$= 138.4 \text{ kJ/kg}$$

$$_1w_2 = {}_1q_2 - (u_2 - u_1) = 214.8 - 138.4 = 76.4 \text{ kJ/kg}$$

$$_1W_2 = m\,_1w_2 = \textbf{36.0 kJ}$$

12.111

An alternative energy power plant has carbon dioxide at 6 MPa, 100°C flowing into a turbine with an exit as saturated vapor at 1 MPa. Find the specific turbine work using generalized charts and repeat using Table B.3.

From Table A.5:   R = 0.1889 kJ/kgK, C_P = 0.842 kJ/kg

From Table A.2: T_C = 304.1 K, P_C = 7.38 MPa

$$P_{ri} = \frac{6}{7.38} = 0.813, \quad T_{ri} = \frac{373.2}{304.1} = 1.227 \quad \Rightarrow \quad \Delta \tilde{h} = 0.70$$

From D.2 and D.3,

$$(h_i^* - h_i) = RT_C \times \Delta \tilde{h} = 0.1889 \times 304.1 \times 0.70 = 46.8 \text{ kJ/kg};$$

$$P_{re} = \frac{1}{7.38} = 0.1355, \ x = 1 \ \text{ so } \ T_{re} = 0.73, \ T_e = 0.73 \times 304.1 = 222 \text{ K}$$

From D.2 and D.3,

$$(h_e^* - h_e) = RT_C \times \Delta \tilde{h} = 0.1889 \times 304.1 \times 0.25 = 46.8 \text{ kJ/kg};$$

$$w = h_i - h_e = h_i^* - h_e^* - (h_i^* - h_i) + (h_e^* - h_e)$$

$$= C_P(h_i - h_e) - RT_C (\Delta \tilde{h}_i - \Delta \tilde{h}_e)$$
$$= 0.842 (373.15 - 222) - 0.1889 \times 0304.1 (0.7 - 0.25)$$
$$= \textbf{101.45 kJ/kg}$$

From Table B.3

$$w = h_i - h_e = 421.69 - 322.39 = \textbf{99.3 kJ/kg}$$

12.112

A distributor of bottled propane, C_3H_8, needs to bring propane from 350 K, 100 kPa to saturated liquid at 290 K in a steady flow process. If this should be accomplished in a reversible setup given the surroundings at 300 K, find the ratio of the volume flow rates $\dot{V}_{in}/\dot{V}_{out}$, the heat transfer and the work involved in the process.

From Table A.2: $T_{ri} = \dfrac{350}{369.8} = 0.946$, $P_{ri} = \dfrac{0.1}{4.25} = 0.024$

From D.1, D.2 and D.3,

$\quad Z_i = 0.99$

$\quad (h_i^* - h_i) = 0.1886 \times 369.8 \times 0.03 = 2.1 \text{ kJ/kg}$

$\quad (s_i^* - s_i) = 0.1886 \times 0.02 = 0.0038 \text{ kJ/kg K}$

$\quad T_{re} = \dfrac{290}{369.8} = 0.784, \text{ and } x = 0$

From D.1, D.2 and D.3,

$\quad P_{re} = 0.22$, $P_e = 0.22 \times 4.25 = 0.935 \text{ MPa}$ and $Z_e = 0.036$

$\quad (h_e^* - h_e) = 0.1886 \times 369.8 \times 4.57 = 318.6 \text{ kJ/kg}$

$\quad (s_e^* - s_e) = 0.1886 \times 5.66 = 1.0672 \text{ kJ/kg K}$

$\quad (h_e^* - h_i^*) = 1.679(290 - 350) = -100.8 \text{ kJ/kg}$

$\quad (s_e^* - s_i^*) \quad = 1.679 \ln \dfrac{290}{350} - 0.1886 \ln \dfrac{0.935}{0.1} = -0.7373 \text{ kJ/kg K}$

$\quad (h_e - h_i) = -318.6 - 100.8 + 2.1 = -417.3 \text{ kJ/kg}$

$\quad (s_e - s_i) = -1.0672 - 0.7373 + 0.0038 = -1.8007 \text{ kJ/kg K}$

$$\frac{\dot{V}_{in}}{\dot{V}_{out}} = \frac{Z_i T_i / P_i}{Z_e T_e / P_e} = \frac{0.99}{0.036} \times \frac{350}{290} \times \frac{0.935}{0.1} = \textbf{310.3}$$

$w^{rev} = (h_i - h_e) - T_0(s_i - s_e) = 417.3 - 300(1.8007) = \textbf{-122.9 kJ/kg}$

$q^{rev} = (h_e - h_i) + w^{rev} = -417.3 - 122.9 = \textbf{-540.2 kJ/kg}$

12.113

An insulated cylinder fitted with a frictionless piston contains saturated-vapor carbon dioxide at $0^{\circ}C$, at which point the cylinder volume is 20 L. The external force on the piston is now slowly decreased, allowing the carbon dioxide to expand until the temperature reaches - $30^{\circ}C$. Calculate the work done by the CO_2 during this process.

CO_2: $T_C = 304.1$ K, $P_c = 7.38$ MPa, $C_p = 0.842$ kJ/kg-K, $R = 0.1889$ kJ/kg K

State 1: $T_1 = 0^{\circ}C$, sat. vap., $x_1 = 1.0$, $V_1 = 20$ L

$T_{r1} = 0.9$, $P_1 = P_{r1}P_C = 0.53 \times 7380 = 3911$ kPa, $Z_1 = Z_g = 0.67$

$(h_1^* - h_1)_g = 0.9\ RT_C$, $(s_1^* - s_1)_g/R = 0.72$, $m = \dfrac{P_1V_1}{Z_1RT_1} = 2.262$ kg

State 2: $T_2 = -30^{\circ}C$

$T_{r2} = 0.8$, $P_2 = P_{r2}P_c = 0.25 \times 7380 = 1845$ kPa

Entropy Eq.: $S_{gen} = m(s_2 - s_1) - {}_1Q_2/T$; ${}_1Q_2 = 0$, $S_{gen} = 0$

$s_2 - s_1 = (s_2 - s_2^*) + (s_2^* - s_1^*) + (s_1^* - s_1) = 0$

$s_2^* - s_1^* = C_P \ln\dfrac{T_2}{T_1} - R \ln\dfrac{P_2}{P_1} = 0.044$ kJ/kg-K, $s_1^* - s_1 = 0.136$ kJ/kg-K

$s_2^* - s_2 = 0.180$ kJ/kg K, $(s_2^* - s_2)_f = 5.46$ R, $(s_2^* - s_2)_g = 0.39$ R

$(s_2^* - s_2) = (1-x_2)(s_2^* - s_2)_f + x_2(s_2^* - s_2)_g$ → $x_2 = 0.889$

Energy Eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$; ${}_1Q_2 = 0$, $u = h - Pv$

$Z_2 = (1 - x_2)Z_f + x_2 Z_g = 0.111 \times 0.04 + 0.889 \times 0.81 = 0.725$;

$(h_2 - h_1) = (h_2 - h_2^*) + (h_2^* - h_1^*) + (h_1^* - h_1)$

$h_2^* - h_1^* = C_p(T_2 - T_1) = -25.3$ kJ/kg, $(h_1^* - h_1) = 51.7$ kJ/kg

$(h_2^* - h_2)_f = 4.51\ RT_C$, $(h_2^* - h_2)_g = 0.46\ RT_C$

$(h_2^* - h_2) = (1 - x_2)(h_2^* - h_2)_f + x_2(h_2^* - h_2)_g = 52.2$ kJ/kg

$h_2 - h_1 = -52.2 - 25.3 + 51.7 = -25.8$ kJ/kg

$u_2 - u_1 = (h_2 - h_1) - Z_2RT_2 + Z_1RT_1 = -25.8 - 0.725 \times 0.18892 \times 243.2$

$\qquad\qquad + 0.67 \times 0.18892 \times 273.2 = -24.5$ kJ/kg

${}_1W_2 = \mathbf{55.4\ kJ}$

12.114

A control mass of 10 kg butane gas initially at 80°C, 500 kPa, is compressed in a reversible isothermal process to one-fifth of its initial volume. What is the heat transfer in the process?

Butane C_4H_{10}:   m = 10 kg, $T_1 = 80$ °C, $P_1 = 500$ kPa

Compressed, reversible T = const, to $V_2 = V_1/5$

$$T_{r1} = \frac{353.2}{425.2} = 0.831, \; P_{r1} = \frac{500}{3800} = 0.132$$

From D.1 and D.3: $Z_1 = 0.92$, $(s_1^* - s_1) = 0.143 \times 0.16 = 0.0230$

$$v_1 = \frac{Z_1 R T_1}{P_1} = \frac{0.92 \times 0.143 \times 353.2}{500} = 0.09296 \text{ m}^3/\text{kg}$$

State 2: $v_2 = v_1/5 = 0.01859$ m^3/kg; $T_{r2} = T_{r1} = 0.831$

From D.1: $P_G = 0.325 \times 3800 = 1235$ kPa

sat. liq.: $Z_F = 0.05$, $(s^* - s_F) = R \times 5.08 = 0.7266$

sat. vap.: $Z_G = 0.775$, $(s^* - s_G) = R \times 0.475 = 0.0680$

Therefore

$$v_F = \frac{0.05 \times 0.143 \times 353.2}{1235} = 0.00205 \text{ m}^3/\text{kg}$$

$$v_G = \frac{0.775 \times 0.143 \times 353.2}{1235} = 0.0317 \text{ m}^3/\text{kg}$$

Since $v_F < v_2 < v_G \rightarrow x_2 = (v_2 - v_F)/(v_G - v_F) = 0.5578$

$$(s_2^* - s_2) = (1 - x_2)(s_2^* - s_{F2}) + x_2(s_2^* - s_{G2})$$

$$= 0.4422 \times 0.7266 + 0.5578 \times 0.0680 = 0.3592 \text{ kJ/kg K}$$

$$(s_2^* - s_1^*) = C_{P0} \ln (T_2/T_1) - R \ln (P_2/P_1) = 0 - 0.143 \ln \frac{1235}{500} = -0.1293$$

$$(s_2 - s_1) = -0.3592 - 0.1293 + 0.0230 = -0.4655 \text{ kJ/kg K}$$

$$_1Q_2 = Tm(s_2 - s_1) = 353.2 \times 10 \, (-0.4655) = \textbf{-1644 kJ}$$

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12.115

A line with a steady supply of octane, C_8H_{18}, is at 400°C, 3 MPa. What is your best estimate for the availability in a steady flow setup where changes in potential and kinetic energies may be neglected?

Availability of Octane at $\quad T_i = 400\ ^oC, P_i = 3$ MPa

From Table A.5: $\quad R = 0.07279$ kJ/kgK, $\quad C_P = 1.711$ kJ/kg

From Table A.2: $\quad T_C = 568.8$ K, $\quad P_C = 2.49$ MPa

$$P_{ri} = \frac{3}{2.49} = 1.205, \quad T_{ri} = \frac{673.2}{568.8} = 1.184$$

From D.2 and D.3,

$$(h_i^* - h_i) = RT_C \times \Delta\tilde{h} = 0.072\ 79 \times 568.8 \times 1.13 = 46.8 \text{ kJ/kg};$$

$$(s_i^* - s_i) = R\ \Delta\tilde{s} = 0.072\ 79 \times 0.69 = 0.05 \text{ kJ/kgK}$$

This is relative to the dead ambient state, assume $T_0 = 298.2$ K, $P_0 = 100$ kPa

$$T_{r0} = \frac{298.2}{568.8} = 0.524, \quad P_{r0} = \frac{0.1}{2.49} = 0.040$$

From D.2 and D.3, The s correction is outside chart (extrapolate or use CATT3)

$$(h_0^* - h_0) = RT_C \times 5.4 = 223.6 \quad \text{and} \quad (s_0^* - s_0) = R \times 9 = 0.655 \text{ kJ/kgK}$$

$$(h_i^* - h_0^*) = C_P(T_i - T_0) = 1.711\ (673.2 - 298.2) = 641.7 \text{ kJ/kg}$$

$$(s_i^* - s_0^*) = 1.711 \times \ln\frac{673.2}{298.2} - 0.072\ 79 \times \ln\frac{3}{0.1} = 1.1459 \text{ kJ/kgK}$$

$$(h_i - h_0) = -46.8 + 641.7 + 223.6 = 818.5 \text{ kJ/kg}$$

$$(s_i - s_0) = -0.05 + 1.1459 + 0.655 = 1.7509 \text{ kJ/kgK}$$

$$\varphi_i = w^{rev} = (h_i - h_0) - T_0(s_i - s_0) = 818.5 - 298.2(1.7509) = \textbf{296.5 kJ/kg}$$

12.116

The environmentally safe refrigerant R-152a is to be evaluated as the working fluid for a heat pump system that will heat a house. It uses an evaporator temperature of –20°C and a condensing temperature of 30°C. Assume all processes are ideal and R-152a has a heat capacity of C_p = 0.996 kJ/kg K. Determine the cycle coefficient of performance.

Ideal Heat Pump T_H = 30 ^{o}C

From A.2:   M = 66.05, R = 0.125 88, T_C = 386.4 K, P_C = 4.52 MPa

$$T_{r3} = \frac{303.2}{386.4} = 0.785$$

$$P_{r3} = P_{r2} = 0.22 \quad => \quad P_3 = P_2 = 994 \text{ kPa}$$

Sat.liq.: $h_3^* - h_3 = 4.56 \times RT_C = 221.8$

T_1 = -20 ^{o}C = 253.2 K, T_{r1} = 0.655, P_{r1} = 0.058 → P_1 = 262 kPa

$h_1^* - h_1 = 0.14 \times RT_C = 6.8$ and $s_1^* - s_1 = 0.14 \times R = 0.0176$

Assume T_2 = 307 K, T_{r2} = 0.795 given P_{r2} = 0.22

From D.2, D.3: $s_2^* - s_2 = 0.34 \times R = 0.0428$; $h_2^* - h_2 = 0.40 \times RT_c = 19.5$

$$s_2^* - s_1^* = 0.996 \ln \frac{307}{253.2} - 0.125\,88 \ln \frac{994}{262} = 0.0241$$

$$s_2 - s_1 = -0.0428 + 0.0241 + 0.0176 = -0.001 \approx 0 \text{ OK}$$

$$\Rightarrow h_2 - h_1 = -19.5 + 0.996(307 - 253.2) + 6.8 = 40.9$$

$$h_2 - h_3 = -19.5 + 0.996(307 - 303.2) + 221.8 = 206.1$$

$$\beta = \frac{q_H}{w_{IN}} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{206.1}{40.9} = \mathbf{5.04}$$

12.117

Rework the previous problem using an evaporator temperature of 0^oC.

Ideal Heat Pump $T_H = 30\ ^oC$

From A.2: $M = 66.05$, $R = 0.125\ 88$, $T_C = 386.4$ K, $P_C = 4.52$ MPa

$T_{r3} = \dfrac{303.2}{386.4} = 0.785$

$P_{r3} = P_{r2} = 0.22$ $=>$ $P_3 = P_2 = 994$ kPa

Sat.liq.: $h_3^* - h_3 = 4.56 \times RT_C = 221.8$

$T_1 = 0\ ^oC = 273.2$ K, $T_{r1} = 0.707$ $=>$ $P_{r1} = 0.106$, $P_1 = 479$ kPa

$h_1^* - h_1 = 0.22 \times RT_C = 10.7$ and $s_1^* - s_1 = 0.21 \times R = 0.0264$

Assume $T_2 = 305$ K, $T_{r2} = 0.789$

$s_2^* - s_2 = 0.35 \times R = 0.0441$ and $h_2^* - h_2 = 0.38 \times RT_C = 18.5$

$s_2^* - s_1^* = 0.996 \ln \dfrac{305.0}{273.2} - 0.125\ 88 \ln \dfrac{994}{479} = 0.0178$

$s_2 - s_1 = -0.0441 + 0.0178 + 0.0264 = 0.0001 \approx 0$ OK

$h_2 - h_1 = -18.5 + 0.996(305.0-273.2) + 10.7 = 23.9$

$h_2 - h_3 = -18.5 + 0.996(305.0-303.2) + 221.8 = 205.1$

$\beta = \dfrac{h_2 - h_3}{h_2 - h_1} = \dfrac{205.1}{23.9} = \textbf{8.58}$

12.118

An uninsulated compressor delivers ethylene, C_2H_4, to a pipe, $D = 10$ cm, at 10.24 MPa, 94°C and velocity 30 m/s. The ethylene enters the compressor at 6.4 MPa, 20.5°C and the work input required is 300 kJ/kg. Find the mass flow rate, the total heat transfer and entropy generation, assuming the surroundings are at 25°C.

$$T_{ri} = \frac{293.7}{282.4} = 1.040 \; , \; P_{ri} = \frac{6.4}{5.04} = 1.270$$

From D.2 and D.3,

$$(h_i^* - h_i) = 0.296\,37 \times 282.4 \times 2.65 = 221.8 \text{ kJ/kg}$$

$$(s_i^* - s_i) = 0.296\,37 \times 2.08 = 0.6164 \text{ kJ/kg K}$$

$$T_{re} = \frac{367.2}{282.4} = 1.30 \; , \; P_{re} = \frac{10.24}{5.04} = 2.032 \; \Rightarrow \; \text{From D.1:} \quad Z_e = 0.69$$

$$v_e = \frac{Z_e R T_e}{P_e} = \frac{0.69 \times 0.296\,37 \times 367.2}{10\,240} = 0.0073 \text{ m}^3\text{/kg}$$

$$A_e = \frac{\pi}{4} D_e^2 = 0.007\,85 \text{ m}^2 \quad \Rightarrow \quad \dot{m} = \frac{A_e V_e}{v_e} = \frac{0.007\,85 \times 30}{0.0073} = \textbf{32.26 kg/s}$$

From D.2 and D.3,

$$(h_e^* - h_e) = 0.296\,37 \times 282.4 \times 1.6 = 133.9 \text{ kJ/kg}$$

$$(s_e^* - s_e) = 0.296\,37 \times 0.90 = 0.2667 \text{ kJ/kg K}$$

$$(h_e^* - h_i^*) = 1.5482(367.2 - 293.7) = 113.8$$

$$(s_e^* - s_i^*) = 1.5482 \ln \frac{367.2}{293.7} - 0.296\,37 \ln \frac{10.24}{6.4} = 0.2065$$

$$(h_e - h_i) = -133.9 + 113.8 + 221.8 = 201.7 \text{ kJ/kg}$$

$$(s_e - s_i) = -0.2667 + 0.2065 + 0.6164 = 0.5562 \text{ kJ/kg K}$$

Energy Eq.:

$$q = (h_e - h_i) + KE_e + w = 201.7 + \frac{30^2}{2 \times 1000} - 300 = -97.9 \text{ kJ/kg}$$

$$\dot{Q}_{cv} = \dot{m}q = 32.26(-97.9) = \textbf{-3158 kW}$$

$$\dot{S}_{gen} = -\frac{\dot{Q}_{cv}}{T_o} + \dot{m}(s_e - s_i) = +\frac{3158}{298.2} + 32.26(0.5562) = \textbf{28.53 kW/K}$$

12.119

The new refrigerant fluid R-123 (see Table A.2) is used in a refrigeration system that operates in the ideal refrigeration cycle, except the compressor is neither reversible nor adiabatic. Saturated vapor at -26.5°C enters the compressor and superheated vapor exits at 65°C. Heat is rejected from the compressor as 1 kW, and the R-123 flow rate is 0.1 kg/s. Saturated liquid exits the condenser at 37.5°C. Specific heat for R-123 is $C_P = 0.6$ kJ/kg. Find the coefficient of performance.

R-123: $T_C = 456.9$ K, $P_C = 3.67$ MPa, M = 152.93 kg/kmol, R = 0.05438 kJ/kg K

State 1: $T_1 = -26.5^OC = 246.7$ K, sat vap., $x_1 = 1.0$

$T_{r1} = 0.54$, Fig D.1, $P_{r1} = 0.01$, $P_1 = P_{r1}P_C = 37$ kPa

Fig. D.2, $h_1^* - h_1 = 0.03$ $RT_C = 0.8$ kJ/kg

State 2: $T_2 = 65^OC = 338.2$ K

State 3: $T_3 = 37.5^OC = 310.7$ K, sat. liq., $x_3 = 0$

$T_{r3} = 0.68$, Fig. D.1: $P_{r3} = 0.08$, $P_3 = P_{r3}P_C = 294$ kPa

$P_2 = P_3 = 294$ kPa, $P_{r2} = 0.080$, $T_{r2} = 0.74$,

Fig. D.2: $h_2^* - h_2 = 0.25$ $RT_C = 6.2$ kJ/kg

$h_3^* - h_3 = 4.92$ $RT_C = 122.2$ kJ/kg

State 4: $T_4 = T_1 = 246.7$ K, $h_4 = h_3$

Energy Eq. Evaporator: $q_L + h_4 = h_1 + w$; w = 0, $h_4 = h_3$

$q_L = h_1 - h_3 = \left(h_1 - h_1^*\right) + \left(h_1^* - h_3^*\right) + \left(h_3^* - h_3\right)$

$h_1^* - h_3^* = C_P(T_1 - T_3) = -38.4$ kJ/kg, $q_L = -0.8 - 38.4 + 122.2 = 83.0$ kJ/kg

Energy Eq. Compressor: $q + h_1 = h_2 + w_c$; $\dot{Q} = -1.0$ kW, $\dot{m} = 0.1$ kg/s

$w_c = h_1 - h_2 + q$; $h_1 - h_2 = \left(h_1 - h_1^*\right) + \left(h_1^* - h_2^*\right) + \left(h_2^* - h_2\right)$

$h_1^* - h_2^* = C_P(T_1 - T_2) = -54.9$ kJ/kg,

$w_c = -0.8 - 54.9 + 6.2 - 10.0 = -59.5$ kJ/kg

$\beta = q_L/w_c = 83.0/59.5 = $ **1.395**

12.120

An evacuated 100-L rigid tank is connected to a line flowing R-142b gas, chlorodifluoroethane, at 2 MPa, 100°C. The valve is opened, allowing the gas to flow into the tank for a period of time, and then it is closed. Eventually, the tank cools to ambient temperature, 20°C, at which point it contains 50% liquid, 50% vapor, by volume. Calculate the quality at the final state and the heat transfer for the process. The ideal-gas specific heat of R-142b is $C_p = 0.787$ kJ/kg K.

Rigid tank V = 100 L, $m_1 = 0$ Line: R-142b CH_3CClF_2

$M = 100.495$, $T_C = 410.3$ K, $P_C = 4.25$ MPa, $C_{P0} = 0.787$ kJ/kg K

$R = \bar{R}/M = 8.31451 / 100.495 = 0.082\ 73$ kJ/kg K

Line $P_i = 2$ MPa, $T_i = 100\ ^oC$,   Flow in to $T_2 = T_0 = 20^oC$

$V_{LIQ\ 2} = V_{VAP\ 2} = 50$ L

Continuity: $m_i = m_2$; Energy: $Q_{CV} + m_i h_i = m_2 u_2 = m_2 h_2 - P_2 V$

From D.2 at i: $P_{ri} = 2 / 4.25 = 0.471$, $T_{ri} = 373.15 / 410.3 = 0.91$

$(h_i^* - h_i) = 0.082\ 73 \times 410.3 \times 0.72 = 24.4$

$(h_2^* - h_i^*) = C_{P0}(T_2 - T_i) = 0.787(20-100) = -63.0$

From D.2: $T_{r2} = \dfrac{293.2}{410.3} = 0.715$ => $P_2 = 0.115 \times 4250 = 489$ kPa

sat. liq.: $Z_F = 0.02$, $(h^* - h_F) = RT_C \times 4.85 = 164.6$

sat. vap.: $Z_G = 0.88$, $(h^* - h_G) = RT_C \times 0.25 = 8.5$

$$m_{LIQ\ 2} = \frac{P_2 V_{LIQ\ 2}}{Z_F RT_2} = \frac{489 \times 0.050}{0.02 \times 0.082\ 73 \times 293.2} = 50.4 \text{ kg}$$

$$m_{VAP\ 2} = \frac{P_2 V_{VAP\ 2}}{Z_G RT_2} = 1.15 \text{ kg}, \qquad m_2 = 51.55 \text{ kg}$$

$x_2 = m_{VAP\ 2}/m_2 = 0.0223$

$(h_2^* - h_2) = (1-x_2)(h_2^* - h_{F2}) + x_2(h_2^* - h_{G2}) = 0.9777 \times 164.6 + 0.0223 \times 8.5 = 161.1$

$Q_{CV} = m_2(h_2 - h_i) - P_2 V = 51.55(-161.1 - 63.0 + 24.4) - 489 \times 0.10$

$= \mathbf{-10\ 343\ kJ}$

Mixtures

12.121

A 2 kg mixture of 50% argon and 50% nitrogen by mole is in a tank at 2 MPa, 180 K. How large is the volume using a model of (a) ideal gas and (b) Kays rule with generalized compressibility charts.

a) Ideal gas mixture

Eq.11.5: $M_{mix} = \sum y_i M_i = 0.5 \times 39.948 + 0.5 \times 28.013 = 33.981$

$$V = \frac{m\bar{R}T}{M_{mix}P} = \frac{2 \times 8.3145 \times 180}{33.981 \times 2000} = \mathbf{0.044 \ m^3}$$

b) Kay's rule Eq.12.84

$P_{c\ mix} = 0.5 \times 4.87 + 0.5 \times 3.39 = 4.13$ MPa

$T_{c\ mix} = 0.5 \times 150.8 + 0.5 \times 126.2 = 138.5$ K

Reduced properties: $P_r = \frac{2}{4.13} = 0.484, \quad T_r = \frac{180}{138.5} = 1.30$

Fig. D.1: $Z = 0.925$

$$V = Z\frac{m\bar{R}T}{M_{mix}P} = 0.925 \times 0.044 = \mathbf{0.0407 \ m^3}$$

12.122

    A 2 kg mixture of 50% argon and 50% nitrogen by mass is in a tank at 2 MPa, 180 K. How large is the volume using a model of (a) ideal gas and (b) van der Waals equation of state with a, b for a mixture?

a) Ideal gas mixture

Eq.11.15: $R_{mix} = \sum c_i\, R_i = 0.5 \times 0.2081 + 0.5 \times 0.2968 = 0.25245$ kJ/kg K

$$V = \frac{mR_{mix}T}{P} = \frac{2 \times 0.25245 \times 180}{2000} = \mathbf{0.0454\ m^3}$$

b) van der Waals equation of state. before we can do the parameters a, b for the mixture we need the individual component parameters.

$$a_{Ar} = \frac{27}{64}\frac{R^2 T_c^2}{P_c} = \frac{27}{64}\frac{(0.2081 \times 150.8)^2}{4870} = 0.08531$$

$$a_{N2} = \frac{27}{64}\frac{R^2 T_c^2}{P_c} = \frac{27}{64}\frac{(0.2968 \times 126.2)^2}{3390} = 0.17459$$

$$b_{Ar} = \frac{RT_c}{8P_c} = \frac{0.2081 \times 150.8}{8 \times 4870} = 0.000\ 805$$

$$b_{N2} = \frac{RT_c}{8P_c} = \frac{0.2968 \times 126.2}{8 \times 3390} = 0.001\ 381$$

Now the mixture parameters are from Eq.12.84

$$a_{mix} = \left(\sum c_i\, a_i^{1/2}\right)^2 = (0.5 \times \sqrt{0.08531} + 0.5 \times \sqrt{0.17459})^2 = 0.126$$

$$b_{mix} = \sum c_i\, b_i = 0.5 \times 0.000\ 805 + 0.5 \times 0.001\ 381 = 0.001\ 093$$

Using now Eq.12.52: $P = \dfrac{RT}{v-b} - \dfrac{a}{v^2}$

$$2000 = \frac{0.25245 \times 180}{v - 0.001\ 093} - \frac{0.126}{v^2}$$

By trial and error we find the specific volume, $v = 0.02097$ m^3/kg

$$V = mv = \mathbf{0.04194\ m^3}$$

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12.123

R-410A is a 1:1 mass ratio mixture of R-32 and R-125. Find the specific volume at 20$^\text{o}$C, 1200 kPa using Kays rule and the generalized charts and compare to Table B.4

Kay's rule Eq.12.83

$P_{c\ mix} = 0.5 \times 5.78 + 0.5 \times 3.62 = 4.70$ MPa

$T_{c\ mix} = 0.5 \times 351.3 + 0.5 \times 339.2 = 345.25$ K

Reduced properties: $P_r = \dfrac{1.2}{4.70} = 0.255,\quad T_r = \dfrac{293.15}{345.25} = 0.849$

Table A.5: $R = 0.1145$ kJ/kg-K   or compute from mix

Fig. D.1: $Z = 0.85$

$v = ZRT/P = 0.85 \times 0.1145 \times 293.15\ /\ 1200 = \textbf{0.0238 m}^\textbf{3}\textbf{/kg}$

Table B.4: $v = \textbf{0.02260 m}^\textbf{3}\textbf{/kg}$

12.124

The R-410A in Problem 12.123 is flowing through a heat exchanger with an exit at 120°C, 1200 kPa. Find the specific heat transfer using Kays rule and the generalized charts and compare to solution using Table B.4

$R_{mix} = 0.5 \times 0.1598 + 0.5 \times 0.06927 = 0.1145$ kJ/kg-K,

$C_{P\ mix} = \sum c_i\ C_{P\ i} = 0.5 \times 0.822 + 0.5 \times 0.791 = 0.8065$ kJ/kg K

Kay's rule Eq.12.84

$P_{c\ mix} = 0.5 \times 5.78 + 0.5 \times 3.62 = 4.70$ MPa

$T_{c\ mix} = 0.5 \times 351.3 + 0.5 \times 339.2 = 345.25$ K

Reduced properties 1: $P_{r1} = \dfrac{1.2}{4.70} = 0.255,\quad T_{r1} = \dfrac{293.15}{345.25} = 0.849$

Fig. D.1: $\left(h_1^* - h_1\right) = 0.4 \times RT_c = 0.4 \times 0.1145 \times 345.25 = 15.81$ kJ/kg

Reduced properties 1: $P_{r2} = \dfrac{1.2}{4.70} = 0.255,\quad T_{r2} = \dfrac{393.15}{345.25} = 1.139$

Fig. D.1: $\left(h_2^* - h_2\right) = 0.2 \times RT_c = 0.2 \times 0.1145 \times 345.25 = 7.906$ kJ/kg

The energy equation gives

$_1q_2 = (h_2 - h_1) = \left(h_2 - h_2^*\right) + \left(h_2^* - h_1^*\right) + \left(h_1^* - h_1\right)$

$= -7.906 + 0.8065\,(120 - 20) + 15.81$

$= \textbf{88.55 kJ/kg mix}$

Table B.4.2: $q = h_2 - h_1 = 393.13 - 290.51 = \textbf{102.62 kJ/kg}$

The main difference is in the value of specific heat, about 1 kJ/kg-K at the avg. T, whereas it is 0.8 kJ/kg-K at 25°C.

12.125

A 2 kg mixture of 50% argon and 50% nitrogen by mass is in a tank at 2 MPa, 180 K. How large is the volume using a model of (a) ideal gas and (b) Redlich Kwong equation of state with a, b for a mixture.

a) Ideal gas mixture

Eq.11.15: $R_{mix} = \sum c_i R_i = 0.5 \times 0.2081 + 0.5 \times 0.2968 = 0.25245$ kJ/kg K

$$V = \frac{mR_{mix}T}{P} = \frac{2 \times 0.25245 \times 180}{2000} = \textbf{0.0454 m}^3$$

b) Redlich Kwong equation of state. Before we can do the parameters a, b for the mixture we need the individual component parameters, Eq.12.54, 13.55.

$$a_{Ar} = 0.42748 \frac{R^2 T_c^{5/2}}{P_c} = 0.42748 \frac{0.2081^2 \times 150.8^{2.5}}{4870} = 1.06154$$

$$a_{N2} = 0.42748 \frac{R^2 T_c^{5/2}}{P_c} = 0.42748 \frac{0.2968^2 \times 126.2^{2.5}}{3390} = 1.98743$$

$$b_{Ar} = 0.08664 \frac{R T_c}{P_c} = 0.08664 \frac{0.2081 \times 150.8}{4870} = 0.000\ 558$$

$$b_{N2} = 0.08664 \frac{R T_c}{P_c} = 0.08664 \frac{0.2968 \times 126.2}{3390} = 0.000\ 957$$

Now the mixture parameters are from Eq.12.84

$$a_{mix} = \left(\sum c_i a_i^{1/2} \right)^2 = (0.5 \times \sqrt{1.06154} + 0.5 \times \sqrt{1.98743})^2 = 1.4885$$

$$b_{mix} = \sum c_i b_i = 0.5 \times 0.000\ 558 + 0.5 \times 0.000\ 957 = 0.000\ 758$$

Using now Eq.12.53: $$P = \frac{RT}{v - b} - \frac{a}{v(v + b)T^{1/2}}$$

$$2000 = \frac{0.25245 \times 180}{v - 0.000\ 758} - \frac{1.4885}{v(v + 0.000\ 758)\ 180^{1/2}}$$

By trial and error we find the specific volume, v = 0.02102 m³/kg

$$V = mv = \textbf{0.04204 m}^3$$

12.126

A modern jet engine operates so that the fuel is sprayed into air at a P, T higher than the fuel critical point. Assume we have a rich mixture of 50% n-octane and 50% air by mole at 500 K and 3.5 MPa near the nozzle exit. Do I need to treat this as a real gas mixture or is an ideal gas assumption reasonable? To answer find Z and the enthalpy departure for the mixture assuming Kay's rule and the generalized charts.

The mole fractions are:

$y_{C8H18} = 0.5$, $y_{N2} = 0.5 \times 0.79 = 0.395$, $y_{O2} = 0.5 \times 0.21 = 0.105$

Eq.11.5:

$M_{mix} = \sum y_i M_i = 0.5 \times 114.232 + 0.395 \times 28.013 + 0.105 \times 31.999$

$= 71.541$

Kay's rule Eq.12.84

$P_{c\ mix} = 0.5 \times 2.49 + 0.395 \times 3.39 + 0.105 \times 5.04 = 3.113$ MPa

$T_{c\ mix} = 0.5 \times 568.8 + 0.395 \times 126.2 + 0.105 \times 154.6 = 350.5$ K

Reduced properties: $P_r = \dfrac{3.5}{3.113} = 1.124$, $T_r = \dfrac{500}{350.5} = 1.427$

Fig. D.1:   **Z = 0.87**   I must treat it as a real gas mixture.

Fig. D.2 $h^* - h = 0.70 \times RT_c = 0.70 \times \dfrac{8.3145}{71.541} \times 350.5 =$ **28.51 kJ/kg**

12.127

A mixture of 60% ethylene and 40% acetylene by moles is at 6 MPa, 300 K. The mixture flows through a preheater where it is heated to 400 K at constant P. Using the Redlich Kwong equation of state with a, b for a mixture find the inlet specific volume. Repeat using Kays rule and the generalized charts.

To do the EOS we need the gas constant, so from Eq.11.5 we get

$$M_{mix} = \sum y_i M_i = 0.6 \times 28.054 + 0.4 \times 26.068 = 27.26$$
$$R_{mix} = 8.3145/27.26 = 0.305 \text{ kJ/kg K}$$

Redlich Kwong EOS the individual component parameters, Eq.12.54, 12.55.

$$a_{C2H4} = 0.42748 \frac{R^2 T_c^{5/2}}{P_c} = 0.42748 \frac{0.2964^2 \times 282.4^{2.5}}{5040} = 9.9863$$

$$a_{C2H2} = 0.42748 \frac{R^2 T_c^{5/2}}{P_c} = 0.42748 \frac{0.3193^2 \times 308.3^{2.5}}{6140} = 11.8462$$

$$b_{C2H4} = 0.08664 \frac{RT_c}{P_c} = 0.08664 \frac{0.2964 \times 282.4}{5040} = 0.001\ 439$$

$$b_{C2H2} = 0.08664 \frac{RT_c}{P_c} = 0.08664 \frac{0.3193 \times 308.3}{6140} = 0.001\ 389$$

Now the mixture parameters are from Eq.12.84 so we need the mass fractions

$$c_{C2H4} = \frac{y\,M}{M_{mix}} = \frac{0.6 \times 28.054}{27.26} = 0.6175, \qquad c_{C2H4} = 1 - c_{C2H4} = 0.3825$$

$$a_{mix} = \left(\sum c_i a_i^{1/2}\right)^2 = (0.6175 \times \sqrt{9.9863} + 0.3825 \times \sqrt{11.8462})^2 = 10.679$$

$$b_{mix} = \sum c_i b_i = 0.6175 \times 0.001\ 439 + 0.3825 \times 0.001\ 389 = 0.001\ 42$$

Using now Eq.12.53:
$$P = \frac{RT}{v - b} - \frac{a}{v(v + b)T^{1/2}}$$

$$6000 = \frac{0.305 \times 300}{v - 0.001\ 42} - \frac{10.679}{v(v + 0.001\ 42)\ 300^{1/2}}$$

By trial and error we find the specific volume, v = **0.006683 m³/kg**

Kay's rule Eq.12.84

$$P_{c\ mix} = 0.6 \times 5.04 + 0.4 \times 6.14 = 5.48 \text{ MPa}$$
$$T_{c\ mix} = 0.6 \times 282.4 + 0.4 \times 308.3 = 292.8 \text{ K}$$

Reduced properties: $P_r = \frac{6}{5.48} = 1.095, \quad T_r = \frac{300}{292.8} = 1.025$

Fig. D.1: Z = 0.4 (difficult to read)

$$v = ZRT/P = 0.4 \times 0.305 \times 300 / 6000 = \textbf{0.0061 m}^3\textbf{/kg}$$

12.128

For the previous problem, find the specific heat transfer using Kay's rule and the generalized charts.

To do the EOS we need the gas constant, so from Eq.11.5 we get

$M_{mix} = \sum y_i M_i = 0.6 \times 28.054 + 0.4 \times 26.068 = 27.26$

$R_{mix} = 8.3145/27.26 = 0.305$ kJ/kg K

$c_{C2H4} = \dfrac{y\,M}{M_{mix}} = \dfrac{0.6 \times 28.054}{27.26} = 0.6175, \qquad c_{C2H4} = 1 - c_{C2H4} = 0.3825$

$C_{P\,mix} = \sum c_i\, C_{P\,i} = 0.6175 \times 1.548 + 0.3825 \times 1.699 = 1.606$ kJ/kg K

Kay's rule Eq.12.84

$P_{c\,mix} = 0.6 \times 5.04 + 0.4 \times 6.14 = 5.48$ MPa

$T_{c\,mix} = 0.6 \times 282.4 + 0.4 \times 308.3 = 292.8$ K

Reduced properties 1: $\qquad P_{r1} = \dfrac{6}{5.48} = 1.095, \quad T_{r1} = \dfrac{300}{292.8} = 1.025$

Fig. D.1: $\quad \left(h_1^* - h_1\right) = 2.1 \times RT_c = 2.1 \times 0.305 \times 292.8 = 187.5$ kJ/kg

Reduced properties 2: $\qquad P_{r2} = \dfrac{6}{5.48} = 1.095, \quad T_{r2} = \dfrac{400}{292.8} = 1.366$

Fig. D.1: $\quad \left(h_2^* - h_2\right) = 0.7 \times RT_c = 0.7 \times 0.305 \times 292.8 = 62.5$ kJ/kg

The energy equation gives

$_1q_2 = (h_2 - h_1) = \left(h_2 - h_2^*\right) + \left(h_2^* - h_1^*\right) + \left(h_1^* - h_1\right)$

$\qquad = -62.5 + 1.606\,(400 - 300) + 187.5$

$\qquad = \textbf{285.6 kJ/kg mix}$

12.129

A gas mixture of a known composition is frequently required for different purposes, e.g., in the calibration of gas analyzers. It is desired to prepare a gas mixture of 80% ethylene and 20% carbon dioxide (mole basis) at 10 MPa, 25°C in an uninsulated, rigid 50-L tank. The tank is initially to contain CO2 at 25°C and some pressure $P1$. The valve to a line flowing C2H4 at 25°C, 10 MPa, is now opened slightly, and remains open until the tank reaches 10 MPa, at which point the temperature can be assumed to be 25°C. Assume that the gas mixture so prepared can be represented by Kay's rule and the generalized charts. Given the desired final state, what is the initial pressure of the carbon dioxide, $P1$?

$A = C_2H_4$, $B = CO_2$ $T_1 = 25\ ^oC$ $P_2 = 10$ MPa, $T_2 = 25\ ^oC$ $y_{A2} = 0.8$, $y_{B2} = 0.2$	$P_i = 10$ MPa ─ A $T_i = 25\ ^oC$ ⊗ $V = 0.05\ m^3$ ▢ B

Mixture at 2 :

$$P_{C2} = 0.8 \times 5.04 + 0.2 \times 7.38 = 5.508 \text{ MPa}$$

$$T_{C2} = 0.8 \times 282.4 + 0.2 \times 304.1 = 286.7 \text{ K}$$

$$T_{r2} = 298.15/286.7 = 1.040; \quad P_{r2} = 10/5.508 = 1.816$$

$$D.1: \quad Z_2 = 0.32$$

$$n_2 = \frac{P_2 V}{Z_2 \bar{R} T_2} = \frac{10\,000 \times 0.05}{0.32 \times 8.3145 \times 298.2} = 0.6302 \text{ kmol}$$

$$n_{A2} = n_i = 0.8\ n_2 = 0.5042 \text{ kmol } C_2H_4$$

$$n_{B2} = n_1 = 0.2\ n_2 = 0.1260 \text{ kmol } CO_2$$

$$T_{r1} = \frac{298.2}{304.1} = 0.981$$

$$P_{r1} = \frac{n_1 Z_{B1} \bar{R} T_1}{P_{CB} V} = \frac{0.126\ Z_{B1} \times 8.3145 \times 298.2}{7380 \times 0.05} = 0.8466\ Z_{B1}$$

By trial & error: $P_{r1} = 0.618$ & $Z_{B1} = 0.73$

$$\Rightarrow P_1 = 0.618 \times 7.38 = \textbf{4.56 MPa}$$

12.130

One kmol/s of saturated liquid methane, CH_4, at 1 MPa and 2 kmol/s of ethane, C_2H_6, at 250°C, 1 MPa are fed to a mixing chamber with the resultant mixture exiting at 50°C, 1 MPa. Assume that Kay's rule applies to the mixture and determine the heat transfer in the process.

Control volume the mixing chamber, inlet CH_4 is 1, inlet C_2H_6 is 2 and the exit state is 3. Energy equation is

$$\dot{Q}_{CV} = \dot{n}_3 \, \bar{h}_3 - \dot{n}_1 \, \bar{h}_1 - \dot{n}_2 \, \bar{h}_2$$

Select the ideal gas reference temperature to be T_3 and use the generalized charts for all three states.

$P_{r1} = P_{rsat} = 1/4.60 = 0.2174 \implies \quad T_{rsat} = 0.783,$

$T_1 = 0.783 \times 190.4 = 149.1 \text{ K}, \quad \Delta h_1 = 4.57$

$P_{r2} = 1/4.88 = 0.205, \quad T_{r2} = 523/305.4 = 1.713, \quad \Delta h_2 = 0.08$

$\bar{h}_1 = \bar{C}_1(T_1 - T_3) - \Delta h_1 \, \bar{R} T_c = 36.15(149.1 - 323.2) - 4.57 \times 8.3145 \times 190.4$
$\quad = -13528 \text{ kJ/kmol}$

$\bar{h}_2 = \bar{C}_2(T_2 - T_3) - \Delta h_2 \, \bar{R} T_c = 53.11(250 - 50) - 0.08 \times 8.3145 \times 305.4$
$\quad = 10\,419 \text{ kJ/kmol}$

Kay's rule Eq.12.84

$T_{cmix} = (1 \times 190.4 + 2 \times 305.4)/3 = 267.1 \text{ K}$

$P_{cmix} = (1 \times 4.60 + 2 \times 4.88)/3 = 4.79 \text{ MPa}$

$T_{r3} = 323.2/267.1 = 1.21 \ , \quad P_{r3} = 1/4.79 = 0.21, \quad \Delta h_3 = 0.15$

$\bar{h}_3 = 0 - 0.15 \times 267.1 \times 8.3145 = -333 \text{ kJ/kmol}$

$\dot{Q}_{CV} = 3(-333) - 1(-13528) - 2(10\,419) = \mathbf{-\ 8309 \ kW}$

12.131

Saturated-liquid ethane at $T_1 = 14°C$ is throttled into a steady flow mixing chamber at the rate of 0.25 kmol/s. Argon gas at $T_2 = 25°C$, $P_2 = 800$ kPa, enters the chamber at the rate of 0.75 kmol/s. Heat is transferred to the chamber from a heat source at a constant temperature of 150°C at a rate such that a gas mixture exits the chamber at $T_3 = 120°C$, $P_3 = 800$ kPa. Find the rate of heat transfer and the rate of entropy generation.

Argon, $T_{a2} = 25°C$, $P_2 = 800$ kPa, $\dot{n}_2 = 0.75$ kmol/s

$T_{ca} = 150$ K, $P_{ca} = 4.87$ MPa, $M_a = 39.948$ kg/kmol, $C_{pa} = 0.52$ kJ/kg K

$\bar{h}_{a3} - \bar{h}_{a2} = M_a C_{pa}(T_3 - T_{a2}) = 1973.4$ kJ/kmol

Inlet: Ethane, $T_{b1} = 14°C$, sat. liq., $x_{b1} = 0$, $\dot{n}_1 = 0.25$ kmol/s

$T_{cb} = 305.4$ K, $P_{cb} = 4.88$ MPa, $M_b = 30.07$ kg/kmol, $C_{pb} = 1.766$ kJ/kg-K

$T_{r1} = 0.94$, $P_{b1} = P_{r1}P_{cb} = 0.69 \times 4880 = 3367$ kPa

$\bar{h}_{b1}^* - \bar{h}_{b1} = 3.81\,\bar{R}T_{cb} = 9674.5$ kJ/kmol, $\quad \bar{s}_{b1}^* - \bar{s}_{b1} = 3.74\,\bar{R} = 31.1$

$\bar{h}_{b3}^* - \bar{h}_{b1}^* = M_b C_{pb}(T_3 - T_{b1}) = 5629.6$ kJ/kmol

Exit: Mix, $T_3 = 120°C$, $P_3 = 800$ kPa   consider this an ideal gas mixture.

Energy Eq.: $\dot{n}_1\bar{h}_{b1} + \dot{n}_2\bar{h}_{a2} + \dot{Q} = \dot{n}_3\bar{h}_3 = \dot{n}_1\bar{h}_{b3} + \dot{n}_2\bar{h}_{a3}$

$\dot{Q} = \dot{n}_1(\bar{h}_{b3} - \bar{h}_{b1}) + \dot{n}_2(\bar{h}_{a3} - \bar{h}_{a2}) = 0.25\,(5629.6 + 9674.5) + 0.75(1973.4)$
$\qquad = \mathbf{5306\ kW}$

Entropy Eq.: $\dot{S}_{gen} = \dot{n}_1(\bar{s}_{b3} - \bar{s}_{b1}) + \dot{n}_2(\bar{s}_{a3} - \bar{s}_{a2}) - \dot{Q}/T_H$; $\quad T_H = 150°C$

$y_a = \dot{n}_2/\dot{n}_{tot} = 0.75$; $\quad y_b = \dot{n}_1/\dot{n}_{tot} = 0.25$

$\bar{s}_{a3} - \bar{s}_{a2} = M_a C_{pa}\ln\dfrac{T_3}{T_{a2}} - \bar{R}\ln\dfrac{y_a P_3}{P_{a2}} = 8.14$ kJ/kmol-K

$\bar{s}_{b3} - \bar{s}_{b1} = M_b C_{pb}\ln\dfrac{T_3}{T_{b1}} - \bar{R}\ln\dfrac{y_b P_3}{P_{b1}} + \bar{s}_{b1}^* - \bar{s}_{b1} =$
$\qquad\qquad = 40.172 + 31.1 = 71.27$ kJ/kmol K

$\dot{S}_{gen} = 0.25 \times 71.27 + 0.75 \times 8.14 - 5306 / 423 = \mathbf{11.38\ kW/K}$

12.132

A cylinder/piston contains a gas mixture, 50% CO2 and 50% C2H6 (mole basis) at 700 kPa, 35°C, at which point the cylinder volume is 5 L. The mixture is now compressed to 5.5 MPa in a reversible isothermal process. Calculate the heat transfer and work for the process, using the following model for the gas mixture:

a. Ideal gas mixture.

b. Kay's rule and the generalized charts.

a) Ideal gas mixture

$$U_2 - U_1 = mC_{v\,mix}(T_2 - T_1) = 0$$

$$Q_{12} = W_{12} = \int P\,dV = P_1V_1 \ln(V_2/V_1) = - P_1V_1 \ln(P_2/P_1)$$

$$= - 700 \times 0.005 \ln(5500/700) = \textbf{-7.71 kJ}$$

b) Kay's rule

$$T_{cmix} = 0.5 \times 304.1 + 0.5 \times 305.4 = 304.75 \text{ K}$$

$$P_{cmix} = 0.5 \times 7.38 + 0.5 \times 4.88 = 6.13 \text{ MPa}$$

$$T_{r1} = 308.15/304.75 = 1.011, \quad P_{r1} = 0.7/6.13 = 0.1142$$

$$Z_1 = 0.96, \quad \Delta h_1 = 0.12, \quad \Delta s_1 = 0.08$$

$$n = P_1V_1/Z_1\bar{R}\,T_1 = \frac{700*0.005}{0.962*8.3145*308.15} = 0.00142 \text{ kmol}$$

$$T_{r2} = T_{r1}, \quad P_{r2} = 5.5/6.13 = 0.897,$$

$$Z_2 = 0.58, \quad \Delta h_2 = 1.35, \quad \Delta s_2 = 1.0$$

$$\bar{h}_2 - \bar{h}_1 = (\bar{h}_2 - \bar{h}_1) - \bar{R}\,T_c(\Delta h_2 - \Delta h_1)$$

$$= 0 - 8.3145 \times 304.75(1.35 - 0.12) = - 3117$$

$$\bar{u}_2 - \bar{u}_1 = \bar{h}_2 - \bar{h}_1 + \bar{R}T(Z_1 - Z_2) = - 3117$$

$$+ 8.3145 \times 308.15(0.96 - 0.58) = -2143 \text{ kJ/kmol}$$

$$Q_{12} = nT(\bar{s}_2 - \bar{s}_1)_T = 0.00142 \times 308.15 \times 8.3145[\, 0 - \ln(5.5/0.7) -1.0$$

$$+ 0.08\,] = \textbf{- 10.85 kJ}$$

$$W_{12} = Q_{12} - n(\bar{u}_2 - \bar{u}_1) = -10.85 - 0.00142(-2143) = \textbf{- 7.81 kJ}$$

12.133

A cylinder/piston contains a gas mixture, 50% CO_2 and 50% C_2H_6 (mole basis) at 700 kPa, 35°C, at which point the cylinder volume is 5 L. The mixture is now compressed to 5.5 MPa in a reversible isothermal process. Calculate the heat transfer and work for the process, using the following model for the gas mixture:

  a. Ideal gas mixture.   b. The van der Waals equation of state.

   a) Ideal gas mixture

$$U_2 - U_1 = mC_{v\ mix}(T_2 - T_1) = 0$$

$$Q_{12} = W_{12} = \int P\ dV = P_1V_1 \ln(V_2/V_1) = -P_1V_1 \ln(P_2/P_1)$$

$$= -700 \times 0.005 \ln(5500/700) = \textbf{-7.71 kJ}$$

  b) van der waal's equation

    For CO_2 : $b = \bar{R}\ T_c/8P_c = 8.3145 \times 304.1/8 \times 7380 = 0.04282$

$$a = 27\ P_c\ b^2 = 27 \times 7380 \times 0.04282^2 = 365.45$$

    For C_2H_6 : $b = \bar{R}\ T_c/8P_c = 8.3145 \times 305.4/8 \times 4880 = 0.06504$

$$a = 27\ P_c\ b^2 = 27 \times 4880 \times 0.06504^2 = 557.41$$

$$a_{mix} = (0.5\sqrt{365.45} + 0.5\sqrt{557.41})^2 = 456.384$$

$$b_{mix} = 0.5 \times 0.04282 + 0.5 \times 0.06504 = 0.05393$$

$$\frac{8.3145*308.2}{\bar{v}_1 - 0.05393} - \frac{456.384}{\bar{v}_1^2} - 700 = 0$$

  By trial and error: $\bar{v}_1 = 3.5329\ m^3/kmol$

$$\frac{8.3145*308.2}{\bar{v}_2 - 0.05393} - \frac{456.384}{\bar{v}_2^2} - 5500 = 0$$

  By trial and error: $\bar{v}_2 = 0.2815\ m^3/kmol$

$$n = V_1/\bar{v}_1 = 0.005/3.5329 = 0.00142$$

$$Q_{12} = nT(\bar{s}_2 - \bar{s}_1)_T = n\ \bar{R}\ T \ln\frac{\bar{v}_2 - b}{\bar{v}_1 - b}$$

$$= 0.00142 \times 8.3145 \times 308.2 \ln\frac{0.2815 - 0.05392}{3.5329 - 0.05392} = \textbf{- 9.93 kJ}$$

$$U_2 - U_1 = 0.00142 \times 456.39(3.5329^{-1} - 0.2815^{-1}) = -2.12\ kJ$$

$$Q_{12} = U_2 - U_1 + W_{12} \implies W_{12} = -9.93 -(-2.12) = \textbf{-7.81 kJ}$$

Helmholtz EOS

12.134

Verify that the ideal gas part of Helmholtz function substituted in Eq.12.89 does lead to the ideal gas law as in note after Eq.12.97.

The ideal gas Helmholtz function is from its definition, see Eq.12.12,
$$a^* = u^* - Ts^* = h^* - RT - T\,s^*$$
We have

$$h^* = h_o^* + \int C_{Po}\, dT$$

$$s^* = s_o^* + \int \frac{C_{Po}}{T}\, dT - R \ln\left(\frac{\rho T}{\rho_o T_o}\right)$$

Now from Eq.12.89 we need to look at $\left(\dfrac{\partial a^*}{\partial \rho}\right)_T$ so following 12.92

$$\left(\frac{\partial a^*}{\partial \rho}\right)_T = \left(\frac{\partial h^*}{\partial \rho}\right)_T - \left(\frac{\partial RT}{\partial \rho}\right)_T - T\left(\frac{\partial s^*}{\partial \rho}\right)_T$$

$$= 0 - 0 + TR\,\frac{\partial}{\partial \rho}\left(\ln \frac{\rho T}{\rho_o T_o}\right)_T$$

$$= RT / \rho$$

So then

$$\rho^2\left(\frac{\partial a^*}{\partial \rho}\right)_T = P = \rho RT \qquad \text{Ideal gas OK}$$

12.135

Gases like argon and neon have constant specific heats. Develop an expression for the ideal gas contribution to Helmholtz function in Eq.12.92 for these cases.

The ideal gas Helmholtz function is from its definition, see Eq.12.92,

$$a^* = u^* - Ts^* = h^* - RT - T\,s^*$$

We have

$$h^* = h_o^* + \int C_{Po}\,dT = h_o^* + C_{Po}(T - T_o)$$

$$s^* = s_o^* + \int \frac{C_{Po}}{T}\,dT - R\,\ln\!\left(\frac{\rho T}{\rho_o T_o}\right)$$

$$= s_o^* + C_{Po}\,\ln(T/T_o) - R\,\ln\!\left(\frac{\rho T}{\rho_o T_o}\right)$$

So now we get

$$a^* = h_o^* + C_{Po}(T - T_o) - RT - T\,s^*$$

$$= h_o^* - T\,s_o^* + C_{Po}(T - T_o) - C_{Po}T\,\ln\left(\frac{T}{T_o}\right) - RT + RT\,\ln\!\left(\frac{\rho T}{\rho_o T_o}\right)$$

$$= C_o + C_1\,T - C_2\,T\,\ln\left(\frac{T}{T_o}\right) + RT\,\ln\!\left(\frac{\rho}{\rho_o}\right)$$

where

$$C_o = h_o^* - C_{Po}T_o\,; \qquad C_1 = C_{Po} - R - s_o^*\,; \qquad C_2 = C_{Po} - R$$

12.136

Find an expression for the change in Helmholtz function for a gas with an EOS as $P(v - b) = RT$.

From Eq.12.31 we get

$$du_T = [\ T\ \left(\frac{\partial P}{\partial T}\right)_v - P\ ]\ dv_T = [\ T\ (\frac{R}{v - b}) - P\ ]\ dv_T = [\ P - P]\ dv_T = 0$$

From Eq.12.32 or Eq.12.34 we get

$$ds_T = -\left(\frac{\partial v}{\partial T}\right)_P dP_T\ =\ \left(\frac{\partial P}{\partial T}\right)_v dv_T$$

$$= -\frac{R}{P}\ dP_T\ =\ \frac{R}{v - b}\ dv_T$$

Now the changes in u and s can be integrated to find

$$(u_2 - u_1)_T = 0$$

$$(s_2 - s_1)_T = -R\ \ln\frac{P_2}{P_1} = R\ \ln\frac{v_2 - b}{v_1 - b}$$

to this we now need to add the variation due to T. For this we get

$$(u_2 - u_1)_v = \int_1^2 C_v\ dT \qquad \text{and} \qquad (s_2 - s_1)_v = \int_1^2 \frac{C_v}{T}\ dT$$

Finally since the Helmholtz function contains the product Ts we need the absolute value of the entropy so

$$s_1 = s_o + \int_o^1 \frac{C_v}{T}\ dT + R\ \ln\frac{v_1 - b}{v_o - b}$$

Then the change in Helmholtz function becomes

$$a_2 - a_1 = u_2 - u_1\ - T_2 s_2 + T_1 s_1 = u_2 - u_1\ - T_2(s_2 - s_1)\ + (T_1 - T_2)\ s_1$$

$$= \int_1^2 C_v\ dT - T_2[\ \int_1^2 \frac{C_v}{T}\ dT + R\ \ln\frac{v_2 - b}{v_1 - b}\ ] + (T_1 - T_2)\ s_1$$

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12.137

Use the equation of state in Example 12.3 and find an expression for isothermal changes in Helmholtz function between two states.

The EOS is

$$\frac{Pv}{RT} = 1 - C'\frac{P}{T^4} \qquad \text{or} \qquad v = \frac{RT}{P} - \frac{C}{T^3}$$

and we use the isothermal changes found in Ex.14.3 as

$$(h_2 - h_1)_T = -\frac{4C}{T^3}(P_2 - P_1)_T$$

$$(s_2 - s_1)_T = -R\ln\left(\frac{P_2}{P_1}\right)_T - \frac{3C}{T^4}(P_2 - P_1)_T$$

As Helmholtz function is: $a = u - Ts$; we get

$$(a_2 - a_1)_T = (u_2 - u_1)_T - T(s_2 - s_1)_T$$

$$= (h_2 - h_1)_T - (P_2v_2 - P_1v_1) - T(s_2 - s_1)_T$$

$$= -\frac{4C}{T^3}(P_2 - P_1)_T - [RT_2 - RT_1 - \frac{C}{T^3}(P_2 - P_1)_T]$$

$$+ RT\ln\left(\frac{P_2}{P_1}\right)_T + \frac{3C}{T^3}(P_2 - P_1)_T$$

$$= -\frac{4C}{T^3}(P_2 - P_1)_T + \frac{C}{T^3}(P_2 - P_1)_T + RT\ln\left(\frac{P_2}{P_1}\right)_T + \frac{3C}{T^3}(P_2 - P_1)_T$$

This now reduces to the final answer

$$(a_2 - a_1)_T = RT\ln\left(\frac{P_2}{P_1}\right)_T$$

12.138

Assume a Helmholtz equation as

$$a^* = C_o + C_1 T - C_2 T \ln \left(\frac{T}{T_o} \right) + RT \ln \left(\frac{\rho}{\rho_o} \right)$$

where C_o, C_1, C_2 are constants and T_o and ρ_o reference values for temperature and density, see Eqs. 12.92 - 95. Find the properties P, u and s from this expression. Is anything assumed for this particular form.

Given Helmholtz function we can find the pressure and entropy from Eq. 12.21 and then u from the definition: a = u – Ts.

$$a^* = C_o + C_1 T - C_2 T \ln \left(\frac{T}{T_o} \right) - RT \ln \left(\frac{v}{v_o} \right)$$

$$\left(\frac{\partial a^*}{\partial v} \right)_T = - RT \frac{\partial}{\partial v} \ln \left(\frac{v}{v_o} \right) = - RT / v$$

$$P = - \left(\frac{\partial a^*}{\partial v} \right)_T = RT / v \qquad \text{i.e. Ideal gas.}$$

$$s = - \left(\frac{\partial a^*}{\partial T} \right)_v = - C_1 + C_2 \ln \left(\frac{T}{T_o} \right) + C_2 + R \ln \left(\frac{v}{v_o} \right)$$

Notice how it looks like Eq.6.17.

$$u = a + Ts = C_o + C_1 T - C_2 T \ln \left(\frac{T}{T_o} \right) - RT \ln \left(\frac{v}{v_o} \right)$$

$$- C_1 T + C_2 T \ln \left(\frac{T}{T_o} \right) + C_2 T + RT \ln \left(\frac{v}{v_o} \right)$$

$$= C_o + C_2 T$$

We find that u is linear in T.

Not only is it ideal gas but it also has constant specific heats.

Review Problems

12.139

Saturated liquid ethane at 2.44 MPa enters a heat exchanger and is brought to 611 K at constant pressure, after which it enters a reversible adiabatic turbine where it expands to 100 kPa. Find the heat transfer in the heat exchanger, the turbine exit temperature and turbine work.

From D.2,

$P_{r1} = 2.44/4.88 = 0.50$, $T_{r1} = 0.89$, $T_1 = 0.89 \times 305.4 = 271.8$ K

$(h_1^* - h_1) = 0.2765 \times 305.4 \times 4.12 = 347.9$

$(h_2^* - h_1^*) = 1.766 \ (611 - 271.8) = 599.0$

$P_{r2} = 0.50$, $T_{r2} = 611/305.4 = 2.00$

From D.2: $(h_2^* - h_2) = RT_c \times 0.14 = 0.2765 \times 305.4 \times 0.14 = 11.8$

$q = (h_2 - h_1) = -11.8 + 599.0 + 347.9 =$ **935.1 kJ/kg**

From D.3,

$(s_2^* - s_2) = 0.2765 \times 0.05 = 0.0138$

$(s_3^* - s_2^*) = 1.766 \ln \dfrac{T_3}{611} - 0.2765 \ln \dfrac{100}{2440}$

Assume $T_3 = 368$ K , $T_{r3} = 1.205$

at $P_{r3} = 0.020$

$(s_3^* - s_2^*) = -0.8954 + 0.8833 = -0.0121$

From D.3,

$(s_3^* - s_3) = 0.2765 \times 0.01 = 0.0028$

$(s_3 - s_2) = -0.0028 - 0.0121 + 0.0138 \approx 0$ OK

Therefore, $T_3 =$ **368 K**

From D.2,

$(h_3^* - h_3) = 0.2765 \times 305.4 \times 0.01 = 0.8$

$w = (h_2 - h_3) = -11.8 + 1.766 \ (611 - 368) + 0.8 =$ **418.1 kJ/kg**

12.140

A piston/cylinder initially contains propane at T = -7°C, quality 50%, and volume 10L. A valve connecting the cylinder to a line flowing nitrogen gas at T = 20°C, P = 1 MPa is opened and nitrogen flows in. When the valve is closed, the cylinder contains a gas mixture of 50% nitrogen, 50% propane on a mole basis at T = 20°C, P = 500 kPa. What is the cylinder volume at the final state, and how much heat transfer took place?

State 1: Propane, $T_1 = -7^O C$, $x_1 = 0.5$, $V_1 = 10$ L

$T_c = 369.8$ K, $P_c = 4.25$ kPa, $C_P = 1.679$ kJ/kg-K, $M = 44.097$ kg/kmol

Fig. D.1: $T_{r1} = 0.72$, $P_{r1} = 0.12$, $P_1 = P_{r1}P_c = 510$ kPa

Fig. D.1: $Z_{f1} = 0.020$, $Z_{g1} = 0.88$, $Z_1 = (1 - x_1)Z_{f1} + x_1 Z_{g1} = 0.45$

$n_1 = P_1 V_1/(Z_1 \bar{R} T_1) = 510 \times 0.01/(0.45 \times 8.3145 \times 266.2) = 0.00512$ kmol

$\bar{h}_1 = \bar{h}_{1o}^* + \bar{C}_P(T_1 - T_o) + (\bar{h}_1 - \bar{h}_1^*)$; $\bar{h}_{1o}^* = 0$,

$(\bar{h}_1^* - \bar{h}_1)_f /\bar{R}T_c = 4.79$, $(\bar{h}_1^* - \bar{h}_1)_g /\bar{R}T_c = 0.25$

$\bar{h}_1^* - \bar{h}_1 = (1 - x_1)(\bar{h}_1^* - \bar{h}_1)_f + x_1 (\bar{h}_1^* - \bar{h}_1)_g = 7748$ kJ/kmol

$\bar{h}_1 = 0 + 1.679 \times 44.094(-7 - 20) - 7748 = -9747$ kJ/kmol

Inlet: Nitrogen, $T_i = 20^O C$, $P_i = 1.0$ MPa,

$T_c = 126.2$ K, $P_c = 3.39$ MPa, $C_{pn} = 1.042$ kJ/kg-K, $M = 28.013$ kg/kmol

$T_{ri} = 2.323$, $P_{ri} = 0.295$, $\bar{h}_i^* - \bar{h}_i = 0.06 \times 8.3145 \times 126.2 = 62.96$ kJ/kmol

$\bar{h}_i = \bar{h}_{io}^* + \bar{C}_{Pn}(T_i - T_o) + (\bar{h}_i - \bar{h}_i^*)$; $\bar{h}_{io}^* = 0$, $T_i - T_o = 0$

State 2: 50% Propane, 50% Nitrogen by mol, $T_2 = 20^O C$, $P_2 = 500$ kPa

$T_{cmix} = \sum y_i T_{ci} = 248$ K, $P_{cmix} = \sum y_i P_{ci} = 3.82$ MPa

$T_{r2} = 1.182$, $P_{r2} = 0.131$, $Z_2 = 0.97$, $(\bar{h}_2^* - \bar{h}_2)/\bar{R}T_c = 0.06$

$\bar{h}_2 = \bar{h}_{2o}^* + \bar{C}_{Pmix}(T_2 - T_o) + (\bar{h}_2 - \bar{h}_2^*)$; $\bar{h}_{2o}^* = 0$, $T_2 - T_o = 0$

a) $n_i = n_1$ => $n_2 = n_1 + n_i = 0.1024$,

$$V_2 = n_2 Z_2 \bar{R} T_2/P_2 = \textbf{0.0484 m}^3$$

b) Energy Eq.: $\quad Q_{cv} + n_i\bar{h}_i = n_2\bar{u}_2 - n_{21}\bar{u}_{21} + W_{cv}; \qquad \bar{u} = \bar{h} - P\bar{v}$

$W_{cv} = (P_1 + P_2)(V_2 - V_1)/2 = 19.88 \text{ kJ}$

$Q_{cv} = n_2\bar{h}_2 - n_1\bar{h}_1 - n_i\bar{h}_i - P_2V_2 + P_1V_1 + W_{cv}$

$\bar{h}_i = -62.96 \text{ kJ/kmol}, \bar{h}_2 = -123.7 \text{ kJ/kmol}, Q_{cv} = \textbf{50.03 kJ}$

12.141

A newly developed compound is being considered for use as the working fluid in a small Rankine-cycle power plant driven by a supply of waste heat. Assume the cycle is ideal, with saturated vapor at 200°C entering the turbine and saturated liquid at 20°C exiting the condenser. The only properties known for this compound are molecular weight of 80 kg/kmol, ideal gas heat capacity $C_{PO} = 0.80$ kJ/kg K and $T_C = 500$ K, $P_C = 5$ MPa. Calculate the work input, per kilogram, to the pump and the cycle thermal efficiency.

$T_1 = 200^oC = 473.2$ K, $x_1 = 1.0$

$T_3 = 20^oC = 293.2$ K, $x_3 = 0.0$

Properties known:

$M = 80$, $C_{PO} = 0.8$ kJ/kg K

$T_C = 500$ K, $P_C = 5.0$ MPa

$T_{r1} = \dfrac{473.2}{500} = 0.946$, $T_{r3} = \dfrac{293.2}{500} = 0.586$

$R = \overline{R}/M = 8.31451/80 = 0.10393$ kJ/kg K

From Fig. D.1,

$P_{r1} = 0.72$, $P_1 = 0.72 \times 5 = 3.6$ MPa $= P_4$

$P_{r3} = 0.023$, $P_3 = 0.115$ MPa $= P_2$, $Z_{F3} = 0.004$

$v_{F3} = \dfrac{Z_{F3}RT_3}{P_3} = \dfrac{0.004 \times 0.10393 \times 293.2}{115} = 0.00106$ m^3/kg

$w_P = - \int\limits_{3}^{4} v\,dP \approx v_{F3}(P_4 - P_3) = -0.00106(3600-115) = \textbf{-3.7 kJ/kg}$

$q_H + h_4 = h_1$, but $h_3 = h_4 + w_P$ => $q_H = (h_1 - h_3) + w_P$

From Fig. D.2:

$(h_1^* - h_1) = RT_C \times 1.25 = 0.103\,93 \times 500 \times 1.25 = 64.9$ kJ/kg

$(h_3^* - h_3) = 0.103\,93 \times 500 \times 5.2 = 270.2$ kJ/kg

$(h_1^* - h_3^*) = C_{P0}(T_1 - T_3) = 0.80(200-20) = 144.0$ kJ/kg

$(h_1 - h_3) = -64.9 + 144.0 + 270.2 = 349.3$ kJ/kg

$q_H = 349.3 + (-3.7) = 345.6$ kJ/kg

Turbine, $(s_2 - s_1) = 0 = -(s_2^* - s_2) + (s_2^* - s_1^*) + (s_1^* - s_1)$

From Fig. D.3,

$\quad (s_1^* - s_1) = 0.10393 \times 0.99 = 0.1029$ kJ/kg K

$\quad (s_2^* - s_1^*) = 0.80 \ln \dfrac{293.2}{473.2} - 0.103\,93 \ln \dfrac{115}{3600} = -0.0250$

Substituting,

$\quad s_2^* - s_2 = +0.1029 - 0.0250 = 0.0779 = (s_2^* - s_{F2}) - x_2 s_{FG2}$

$\quad 0.0779 = 0.103\,93 \times 8.85 - x_2 \times 0.103\,93(8.85 - 0.06) \quad \Rightarrow \quad x_2 = 0.922$

$\quad (h_2^* - h_2) = (h_2^* - h_{F2}) - x_2 h_{FG2}$

From Fig. D.2,

$\quad h_{FG2} = 0.10393 \times 500\,(5.2 - 0.07) = 266.6$

$\quad (h_2^* - h_2) = 270.2 - 0.922 \times 266.6 = 25.0$

$\quad w_T = (h_1 - h_2) = -64.9 + 144.0 + 25.0 = 104.1$ kJ/kg

$$\eta_{TH} = \frac{w_{NET}}{q_H} = \frac{104.1 - 3.7}{345.6} = \mathbf{0.29}$$

12.142

A 200-L rigid tank contains propane at 400 K, 3.5 MPa. A valve is opened, and propane flows out until half the initial mass has escaped, at which point the valve is closed. During this process the mass remaining inside the tank expands according to the relation $Pv^{1.4}$ = constant. Calculate the heat transfer to the tank during the process.

C_3H_8: V = 200 L, T_1 = 400 K, P_1 = 3.5 MPa

Flow out to $m_2 = m_1/2$; $Pv^{1.4}$ = const inside

$T_{r1} = \dfrac{400}{369.8} = 1.082$, $P_{r1} = \dfrac{3.5}{4.25} = 0.824$ Fig D.1: Z_1 = 0.74

$v_1 = \dfrac{0.74 \times 0.188\,55 \times 400}{3500} = 0.01594$, $v_2 = 2v_1 = 0.03188$

$m_1 = \dfrac{0.2}{0.015\,94} = 12.55$ kg, $\quad m_2 = \dfrac{1}{2}m_1 = 6.275$ kg,

$P_2 = P_1\left(\dfrac{v_1}{v_2}\right)^{1.4} = \dfrac{3500}{2^{1.4}} = 1326$ kPa

$\left.\begin{array}{l} P_{r2} = \dfrac{1.326}{4.25} = 0.312 \\[2mm] P_2v_2 = Z_2RT_2 \end{array}\right\}$ Trial & error: saturated with
$T_2 = 0.826 \times 369.8 = 305.5$ K &
$Z_2 = \dfrac{1326 \times 0.03188}{0.188\,55 \times 305.5} = 0.734$

$Z_2 = Z_{F2} + x_2(Z_{G2} - Z_{F2}) = 0.734 = 0.05 + x_2(0.78-0.05)$ => $x_2 = 0.937$

$(h_1^* - h_1) = 0.188\,55 \times 369.8(0.9) = 62.8$

$(h_2^* - h_1^*) = 1.6794(305.5 - 400) = -158.7$

$(h_2^* - h_2) = (h_2^* - h_{F2}) - x_2 h_{FG2} = 0.188\,55 \times 369.8\left[4.41 - 0.937(4.41 - 0.55)\right]$
$\qquad = 55.3$

Energy Eq.: $\qquad Q_{CV} = m_2h_2 - m_1h_1 + (P_1 - P_2)V + m_e h_{e\,AVE}$

Let $h_1^* = 0$ then $\quad h_1 = 0 + (h_1 - h_1^*) = -62.8$

$h_2 = h_1^* + (h_2^* - h_1^*) + (h_2 - h_2^*) = 0 - 158.7 - 55.3 = -214.0$

$h_{e\,AVE} = (h_1 + h_2)/2 = -138.4$

$Q_{CV} = 6.275(-214.0) - 12.55(-62.8)$

$\qquad + (3500-1326) \times 0.2 + 6.275(-138.4) = $ **-981.4 kJ**

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12.143

One kilogram per second water enters a solar collector at 40°C and exits at 190°C, as shown in Fig. P12.143. The hot water is sprayed into a direct-contact heat exchanger (no mixing of the two fluids) used to boil the liquid butane. Pure saturated-vapor butane exits at the top at 80°C and is fed to the turbine. If the butane condenser temperature is 30°C and the turbine and pump isentropic efficiencies are each 80%, determine the net power output of the cycle.

H_2O cycle: solar energy input raises 1 kg/s of liquid H_2O from 40^oC to 190^oC.

Therefore, corresponding heat input to the butane in the heat exchanger is

$$\dot{Q}_H = \dot{m}(h_{F\,190\,C} - h_{F\,40\,C})_{H2O} = 1(807.62 - 167.57) = 640.05 \text{ kW}$$

C_4H_{10} cycle

$T_1 = 80\ ^oC,\ x_1 = 1.0\ ;\ T_3 = 30\ ^oC,\ x_3 = 0.0$

$\eta_{ST} = \eta_{SP} = 0.80$

$T_{r1} = \dfrac{353.2}{425.2} = 0.831$

From D.1, D.2 and D.3:

$P_1 = 0.325 \times 3800 = 1235 \text{ kPa}$

$(h_1^* - h_1) = 0.143\,04 \times 425.2 \times 0.56 = 34.1$

$(s_1^* - s_1) = 0.143\,04 \times 0.475 = 0.0680$

$T_{r3} = \dfrac{303.2}{425.2} = 0.713$

From D.1, D.2 and D.3: $P_3 = 0.113 \times 3800 = 429 \text{ kPa}$

sat. liq.: $(h^* - h_F) = RT_C \times 4.81 = 292.5$; $(s^* - s_F) = R \times 6.64 = 0.950$

sat. vap.: $(h^* - h_G) = RT_C \times 0.235 = 14.3$; $(s^* - s_G) = R \times 0.22 = 0.031$

Because of the combination of properties of C_4H_{10} (particularly the large C_{P0}/R), s_1 is larger than s_G at T_3. To demonstrate,

$(s_1^* - s_{G3}^*) = 1.7164 \ln \dfrac{353.2}{303.2} - 0.143\,04 \ln \dfrac{1235}{429} = 0.1107$

$(s_1 - s_{G3}) = -0.0680 + 0.1107 + 0.031 = +0.0737 \text{ kJ/kg K}$

so that T_{2S} will be $> T_3$, as shown in the T-s diagram. A number of other heavy hydrocarbons also exhibit this behavior.

Assume T_{2S} = 315 K, T_{r2S} = 0.741

From D.2 and D.3:

$(h_{2S}^*-h_{2S}) = RT_C \times 0.21 = 12.8$ and $(s_{2S}^*-s_{2S}) = R \times 0.19 = 0.027$

$(s_1^*-s_{2S}^*) = 1.7164 \ln \dfrac{353.2}{315} - 0.143\,04 \ln \dfrac{1235}{429} = +0.0453$

$(s_1-s_{2S}) = -0.0680 + 0.0453 + 0.027 \approx 0$

$\Rightarrow T_{2S} = 315$ K

$(h_1^*-h_{2S}^*) = 1.7164(353.2-315) = 65.6$

$w_{ST} = h_1-h_{2S} = -34.1 + 65.6 + 12.8 = 44.3$ kJ/kg

$w_T = \eta_S \times w_{ST} = 0.80 \times 44.3 = 35.4$ kJ/kg

At state 3,

$v_3 = \dfrac{0.019 \times 0.143\,04 \times 303.2}{429} = 0.001\,92$ m^3/kg

$-w_{SP} \approx v_3(P_4-P_3) = 0.001\,92(1235-429) = 1.55$ kJ/kg

$-w_P = \dfrac{-w_{SP}}{\eta_{SP}} = \dfrac{1.55}{0.8} = 1.94$ kJ/kg

$w_{NET} = w_T + w_P = 35.4 - 1.94 = 33.46$ kJ/kg

For the heat exchanger,

$\dot{Q}_H = 640.05 = \dot{m}_{C4H10}(h_1-h_4)$

But $h_1-h_4 = h_1-h_3+w_P$

$h_1-h_3 = (h_1-h_1^*) + (h_1^*-h_3^*) + (h_3^*-h_3)$

$\quad = -34.1 + 1.716(80 - 30) + 292.5 = 344.2$ kJ/kg

Therefore,

$$\dot{m}_{C4H10} = \dfrac{640.05}{344.2-1.94} = 1.87 \text{ kg/s}$$

$$\dot{W}_{NET} = \dot{m}_{C4H10} w_{NET} = 1.87 \times 33.46 = \textbf{62.57 kW}$$

12.144

A piston/cylinder contains ethane gas, initially at 500 kPa, 100 L, and at ambient temperature, 0°C. The piston is now moved, compressing the ethane until it is at 20°C, with a quality of 50%. The work required is 25% more than would have been required for a reversible polytropic process between the same initial and final states. Calculate the heat transfer and the entropy generation for the process.

Ethane: $T_c = 305.4$ K, $P_c = 4.88$ MPa,

$R = 0.2765$ kJ/kg-K, $C_p = 1.766$ kJ/kg K

State 1: $T_{r1} = 0.895$, $P_{r1} = 0.102$ → $Z_1 = 0.95$

$v_1 = Z_1 R T_1 / P_1 = 0.1435$ m^3/kg, $m_1 = V_1 / v_1 = 0.697$ kg

$(h_1^* - h_1) = 0.13 R T_c = 11.0$ kJ/kg, $(s_1^* - s_1) = 0.09 R = 0.025$ kJ/kg K

State 2: $T_2 = 20$°C, $x_2 = 0.5$, $_1W_2 = 1.25 W_{rev}$

$T_{r2} = 0.96$, $P_{r2} = 0.78$, $P_2 = P_{r2} P_c = 3806$ kPa

$Z_{f2} = 0.14$, $Z_{g2} = 0.54$, $Z_2 = (1 - x_2)Z_f + x_2 Z_g = 0.34$

$(h_2^* - h_2) = (1 - x_2)\, 3.65\, R T_c + x_2\, (1.39\, R T_c) = 212.8$ kJ/kg

$(s_2^* - s_2) = (1 - x_2)\, 3.45\, R + x_2 \times 1.10\, R = 0.629$ kJ/kg K

$v_2 = Z_2 R T_2 / P_2 = 0.0072$ m^3/kg, $V_2 = m v_2 = 0.005$ m^3

Polytropic process

$$P_1 V_1^n = P_2 V_2^n, \quad \ln \frac{P_2}{P_1} = n \ln \frac{V_1}{V_2} \quad \rightarrow \quad n = 0.6783$$

$$W_{rev} = \int P\, dV = \frac{P_2 V_2 - P_1 V_1}{1 - n} = -96.3 \text{ kJ},$$

$$_1W_2 = 1.25 W_{rev} = -120.4 \text{ kJ}$$

a)   Energy Eq.: $_1Q_2 = m(u_2 - u_1) + {}_1W_2$; $u = h - Pv$

$h_2 - h_1 = (h_2 - h_2^*) + (h_2^* - h_1^*) + (h_1^* - h_1)$

$\qquad\qquad = -212.8 + 1.766(20 - 0) + 11.0 = -166.5$ kJ/kg

$u_2 - u_1 = (h_2 - h_1) - (P_2 v_2 - P_1 v_1) = -122.2$ kJ/kg

$_1Q_2 = 0.697$ kg (-122.2) kJ/kg $- 120.4$ kJ $= $ **-205.6 kJ**

b) Entropy Eq.: $S_{gen} = m(s_2 - s_1) - {}_1Q_2 / T_o$; $T_o = 0^O C = 273.15$ K

$$s_2 - s_1 = (s_2 - s_2^*) + (s_2^* - s_1^*) + (s_1^* - s_1)$$

$$(s_2^* - s_1^*) = C_p \ln(T_2 / T_1) - R \ln(P_2 / P_1) = -0.436 \text{ kJ/kg K},$$

$$S_{gen} = 0.697(-0.629 - 0.436 + 0.025) + \frac{205.6}{273.15} = \textbf{0.028 kJ/K}$$

12.145

Carbon dioxide gas enters a turbine at 5 MPa, 100°C, and exits at 1 MPa. If the isentropic efficiency of the turbine is 75%, determine the exit temperature and the second-law efficiency using the generalized charts.

CO_2 turbine: $\eta_S = w/w_S = 0.75$

inlet: $T_1 = 100^oC$, $P_1 = 5$ MPa, exhaust: $P_2 = 1$ MPa

a) $P_{r1} = \dfrac{5}{7.38} = 0.678$, $T_{r1} = \dfrac{373.2}{304.1} = 1.227$, $P_{r2} = \dfrac{1}{7.38} = 0.136$

From D.2 and D.3,

$(h_1^* - h_1) = 0.188\ 92 \times 304.1 \times 0.52 = 29.9$

$(s_1^* - s_1) = 0.188\ 92 \times 0.30 = 0.0567$

Assume $T_{2S} = 253$ K, $T_{r2S} = 0.832$

From D.2 and D.3: $(h_{2S}^* - h_{2S}) = RT_C \times 0.20 = 11.5$

$(s_{2S}^* - s_{2S}) = R \times 0.17 = 0.0321$

$(s_{2S}^* - s_1^*) = 0.8418 \ln \dfrac{253}{373.2} - 0.188\ 92 \ln \dfrac{1}{5} = -0.0232$

$(s_{2S} - s_1) = -0.0321 - 0.0232 + 0.0567 \approx 0$

$$\Rightarrow T_{2S} = 253 \text{ K}$$

$(h_{2S}^* - h_1^*) = 0.8418(253 - 373.2) = -101.2$

$w_S = (h_1 - h_{2S}) = -29.9 + 101.2 + 11.5 = 82.8$ kJ/kg

$w = \eta_S \times w_S = 0.75 \times 82.8 = 62.1$ kJ/kg $= (h_1 - h_1^*) + (h_1^* - h_2^*) + (h_2^* - h_2)$

Assume $T_2 = 275$ K, $T_{r2} = 0.904$

$(h_1^* - h_2^*) = 0.8418 (373.2 - 275) = 82.7$

From D.2 and D.3,

$(h_2^* - h_2) = RT_C \times 0.17 = 9.8$; $(s_2^* - s_2) = R \times 0.13 = 0.0245$

Substituting,

$w = -29.9 + 82.7 + 9.8 = 62.7 \approx 62.1$ $\Rightarrow T_2 = \textbf{275 K}$

For the exergies we need the entropy changes

$$(s_2^* - s_1^*) = 0.8418 \ln \frac{275}{373.2} - 0.188\,92 \ln \frac{1}{5} = +0.0470$$

$$(s_2 - s_1) = -0.0245 + 0.0470 + 0.0567 = +0.0792$$

Assuming $T_0 = 25\ ^{\circ}C$,

$$(\varphi_1 - \varphi_2) = (h_1 - h_2) - T_0(s_1 - s_2) = 62.1 + 298.2(0.0792) = 85.7 \text{ kJ/kg}$$

$$\eta_{2\text{nd Law}} = \frac{w}{\varphi_1 - \varphi_2} = \frac{62.1}{85.7} = \mathbf{0.725}$$

12.146

A 10- m^3 storage tank contains methane at low temperature. The pressure inside is 700 kPa, and the tank contains 25% liquid and 75% vapor, on a volume basis. The tank warms very slowly because heat is transferred from the ambient.

a. What is the temperature of the methane when the pressure reaches 10 MPa?

b. Calculate the heat transferred in the process, using the generalized charts.

c. Repeat parts (a) and (b), using the methane tables, Table B.7. Discuss the differences in the results.

CH_4: V = 10 m^3, P$_1$ = 700 kPa

$V_{LIQ\ 1}$ = 2.5 m^3, $V_{VAP\ 1}$ = 7.5 m^3

a) $P_{r1} = \dfrac{0.70}{4.60} = 0.152$, $P_{r2} = \dfrac{10}{4.60} = 2.174$

From D.1: Z_{F1} = 0.025, Z_{G1} = 0.87 &

$T_1 = 0.74 \times 190.4 = 140.9$ K

$v_{F1} = \dfrac{0.025 \times 0.5183 \times 140.9}{700} = 0.00261$

$v_{G1} = \dfrac{0.87 \times 0.5183 \times 140.9}{700} = 0.0908$

$m_{LIQ\ 1} = \dfrac{2.5}{0.00261} = 957.9$ kg, $m_{VAP\ 1} = \dfrac{7.5}{0.0908} = 82.6$ kg

Total m = 1040.3 kg

$v_2 = v_1 = \dfrac{V}{m} = \dfrac{10}{1040.5} = 0.00961 = \dfrac{Z_2 \times 0.5183 \times 190.4 \times T_{r2}}{10\ 000}$

or $Z_2 T_{r2} = 0.9737$ at $P_{r2} = 2.174$

By trial and error

$T_{r2} = 1.334$ & $Z_2 = 0.73$, $T_2 = 1.334 \times 190.4 =$ **254.0 K**

b) Energy Eq.:

$Q_{12} = m(u_2-u_1) = m(h_2-h_1) - V(P_2-P_1)$

Using D.2 & $x_1 = \dfrac{82.6}{1040.5} = 0.0794$

$(h_1^* - h_1) = (h_1^* - h_{f1}) - x_1 h_{fg1}$

$= 0.518\ 35 \times 190.4\left[4.72 - 0.0794(4.72-0.29)\right] = 431.1$

$(h_2^* - h_1^*) = C_P (T_2 - T_1) = 2.254\ (254.0 - 140.9) = 254.9$

$(h_2^* - h_2) = 0.5183 \times 190.4(1.47) = 145.1$

$(h_2 - h_1) = -145.1 + 254.9 + 431.1 = 540.9 \text{ kJ/kg}$

$Q_{12} = 1040.5 \text{ kg } (540.9) \text{ kJ/kg} - 10 \text{ m}^3 (10\,000-700) \text{ kPa}$

$\quad = \textbf{469 806 kJ}$

c) Using Table B.7 for CH_4

$T_1 = T_{SAT\,1} = 141.7 \text{ K}, \quad v_{F1} = 0.002\,675, \quad u_{F1} = -178.47$

$v_{G1} = 0.090\,45 \,, u_{G1} = 199.84$

$m_{LIQ\,1} = \dfrac{2.5}{0.002\,675} = 934.6, \qquad m_{VAP\,1} = \dfrac{7.5}{0.090\,45} = 82.9$

Total mass $\quad m = 1017.5 \text{ kg} \quad$ and $\quad v_2 = \dfrac{10}{1017.5} = 0.009\,828 \text{ m}^3/\text{kg}$

At v_2 & $P_2 = 10 \text{ MPa} \rightarrow \begin{cases} T_2 = 259.1 \text{ K} \\ u_2 = 296.11 \end{cases}$

$Q_{12} = m(u_2 - u_1) = 1017.5 \times 296.11 - 934.6(-178.47) - 82.9(199.84)$

$\quad = \textbf{451 523 kJ}$

12.147

Consider the following reference state conditions: the entropy of real saturated liquid methane at −100°C is to be taken as 100 kJ/kmol K, and the entropy of hypothetical ideal gas ethane at −100°C is to be taken as 200 kJ/kmol K. Calculate the entropy per kmol of a real gas mixture of 50% methane, 50% ethane (mole basis) at 20°C, 4 MPa, in terms of the specified reference state values, and assuming Kay's rule for the real mixture behavior.

CH_4: $T_0 = -100\ ^{\circ}C$, $\bar{s}_{LIQ\ 0} = 100$ kJ/kmol K

C_2H_6: $T_0 = -100\ ^{\circ}C$, $P_0 = 1$ MPa, $\bar{s}_0^* = 200$ kJ/kmol K

Also for CH_4: $T_C = 190.4$ K, $P_C = 4.60$ MPa

For a 50% mixture Kay's rule Eq.12.84:

$T_{cmix} = 0.5 \times 190.4 + 0.5 \times 305.4 = 247.9$ K

$P_{cmix} = 0.5 \times 4.60 + 0.5 \times 4.88 = 4.74$ MPa

IG MIX at $T_0 (=-100\ ^{\circ}C)$, $P_0 (=1$ MPa):

CH_4: $T_{r0} = 0.91$, $P_G = 0.57 \times 4.60 = 2.622$ MPa

$\bar{s}_{0\ CH4}^* = \bar{s}_{LIQ\ 0\ P_G} + (\bar{s}^* - \bar{s}_{LIQ})_{at\ P_G} - \bar{R} \ln (P_0/P_G)$

$= 100 + 4.01 \times 8.3145 - 8.3145 \ln (1/2.622) = 141.36$

$\bar{s}_{0\ MIX}^* = 0.5 \times 141.36 + 0.5 \times 200 - 8.3145(0.5 \ln 0.5 + 0.5 \ln 0.5) = 176.44$

$\bar{C}_{P0\ MIX} = 0.5 \times 16.04 \times 2.254 + 0.5 \times 30.07 \times 1.766 = 44.629$

$\bar{s}_{TP\ MIX}^* = 176.44 + 44.629 \ln \dfrac{293.2}{173.2} - 8.3145 \ln \dfrac{4}{1} = 188.41$ kJ/kmol K

For the mixture at T, P: $T_r = 1.183$, $P_r = 0.844$

Entropy departure $\bar{s}_{TP\ MIX}^* - \bar{s}_{TP\ MIX} = 0.4363 \times 8.3145 = 3.63$ kJ/kmol K

Therefore,

$\bar{s}_{TP\ MIX} = 188.41 - 3.63 = $ **184.78 kJ/kmol K**

An alternative is to form the ideal gas mixture at T, P instead of at T_0, P_0 :

$$\bar{s}^*_{TP\ CH4} = \bar{s}_{LIQ\ 0} + (\bar{s}^* - \bar{s}_{LIQ}) + \bar{C}_{P0\ CH4} \ln\frac{T}{T_0} - \bar{R} \ln\frac{P}{P_G}$$

$$\quad P_G, T_0 \quad at\ P_G, T_0$$

$$= 100 + 33.34 + 16.04\times2.254 \ln\frac{293.2}{173.2} - 8.3145 \ln\frac{4}{2.6}$$

$$= 100 + 33.34 + 19.03 - 3.53 = 148.84\ kJ/kmol\ K$$

$$\bar{s}^*_{TP\ C2H6} = 200 + 30.07\times1.766 \ln\frac{293.2}{173.2} - 8.3145 \ln\frac{4}{1}$$

$$= 200 + 27.96 - 11.53 = 216.43\ \ kJ/kmol\ K$$

$$\bar{s}^*_{TP\ MIX} = 0.5\times148.84 + 0.5\times216.43$$

$$\quad - 8.3145(0.5 \ln 0.5 + 0.5 \ln 0.5) = 188.41\ \ kJ/kmol\ K$$

$$\bar{s}_{TP\ MIX} = 188.41 - 3.63 = \textbf{184.78 kJ/kmol K}$$

12.148

Determine the heat transfer and the net entropy change in problem 12.129. Use the initial pressure of the carbon dioxide to be 4.56 MPa before the ethylene is flowing into the tank.

A gas mixture of a known composition is frequently required for different purposes, e.g., in the calibration of gas analyzers. It is desired to prepare a gas mixture of 80% ethylene and 20% carbon dioxide (mole basis) at 10 MPa, 25°C in an uninsulated, rigid 50-L tank. The tank is initially to contain CO2 at 25°C and some pressure P1. The valve to a line flowing C2H4 at 25°C, 10 MPa, is now opened slightly, and remains open until the tank reaches 10 MPa, at which point the temperature can be assumed to be 25°C. Assume that the gas mixture so prepared can be represented by Kay's rule and the generalized charts. Given the desired final state, what is the initial pressure of the carbon dioxide, P1?

$A = C_2H_4$, $B = CO_2$

$T_1 = 25\ ^oC$

$P_2 = 10$ MPa, $T_2 = 25\ ^oC$

$y_{A2} = 0.8$, $y_{B2} = 0.2$

$P_i = 10$ MPa $_A$

$T_i = 25^o\ C$

$V = 0.05\ m^3$ B

Mixture at 2 :

$P_{C2} = 0.8 \times 5.04 + 0.2 \times 7.38 = 5.508$ MPa

$T_{C2} = 0.8 \times 282.4 + 0.2 \times 304.1 = 286.7$ K

$T_{r2} = 298.15/286.7 = 1.040$; $P_{r2} = 10/5.508 = 1.816$

D.1 : $Z_2 = 0.32$

$$n_2 = \frac{P_2 V}{Z_2 \bar{R} T_2} = \frac{10\ 000 \times 0.05}{0.32 \times 8.3145 \times 298.2} = 0.6302\ \text{kmol}$$

$n_{A2} = n_i = 0.8\ n_2 = 0.5042$ kmol C_2H_4

$n_{B2} = n_1 = 0.2\ n_2 = 0.1260$ kmol CO_2

$T_{r1} = \dfrac{298.2}{304.1} = 0.981$ and $P_{r1} = \dfrac{4560}{7380} = 0.618$

Energy Eq.: $Q_{CV} + n_i \bar{h}_i = n_2 \bar{u}_2 - n_1 \bar{u}_1 = n_2 \bar{h}_2 - n_1 \bar{h}_1 - (P_2 - P_1)V$

or $Q_{CV} = n_2(\bar{h}_2 - \bar{h}_2^*) - n_1(\bar{h}_1 - \bar{h}_1^*) - n_i(\bar{h}_i - \bar{h}_i^*) - (P_2 - P_1)V$

(since $T_i = T_1 = T_2$, $\bar{h}_i^* = \bar{h}_1^* = \bar{h}_2^*$)

$(\bar{h}_1^* - \bar{h}_1) = 0.83 \times 8.3145 \times 304.1 = 2099$ kJ/kmol

$(\bar{h}_2^* - \bar{h}_2) = 3.40 \times 8.3145 \times 286.7 = 8105$ kJ/kmol

$T_{ri} = \dfrac{298.2}{282.4} = 1.056, \; P_{ri} = \dfrac{10}{5.04} = 1.984$

$(\bar{h}_i^* - \bar{h}_i) = 3.35 \times 8.3145 \times 282.4 = 7866$ kJ/kmol

$Q_{CV} = 0.6302(-8105) - 0.126(-2099) - 0.5042(-7866) - (10\,000 - 4560) \times 0.05$

$\qquad = \mathbf{-1149 \; kJ}$

$\Delta S_{CV} = n_2 \bar{s}_2 - n_1 \bar{s}_1 , \quad \Delta S_{SURR} = -Q_{CV}/T_0 - n_i \bar{s}_i$

$\Delta S_{gen} = n_2 \bar{s}_2 - n_1 \bar{s}_1 - Q_{CV}/T_0 - n_i \bar{s}_i$

Let $\bar{s}_{A0}^* = \bar{s}_{B0}^* = 0$ at $T_0 = 25\,^{\circ}$C, $P_0 = 0.1$ MPa

Then $\bar{s}_{MIX\,0}^* = -8.3145\,(0.8 \ln 0.8 + 0.2 \ln 0.2) = 4.161$ kJ/kmol K

$\bar{s}_1 = \bar{s}_{B0}^* + (\bar{s}_{P1\,T1}^* - \bar{s}_{P0\,T0}^*)_B + (\bar{s}_1 - \bar{s}_{P1\,T1}^*)_B$

$\quad = 0 + (0 - 8.3145 \ln \dfrac{4.56}{0.1}) - 0.60 \times 8.3145 = -36.75$ kJ/kmol K

$\bar{s}_i = \bar{s}_{A0}^* + (\bar{s}_{Pi\,Ti}^* - \bar{s}_{P0\,T0}^*)_A + (\bar{s}_i - \bar{s}_{Pi\,Ti}^*)_A$

$\quad = 0 + (0 - 8.3145 \ln \dfrac{10}{0.1}) - 2.44 \times 8.3145 = -58.58$ kJ/kmol K

$\bar{s}_2 = \bar{s}_{MIX\,0}^* + (\bar{s}_{P2\,T2}^* - \bar{s}_{P0\,T0}^*)_{MIX} + (\bar{s}_2 - \bar{s}_{P2\,T2}^*)_{MIX}$

$\quad = 4.161 + (0 - 8.3145 \ln \dfrac{10}{0.1}) - 2.551 \times 8.3145 = -55.34$ kJ/kmol K

$S_{gen} = 0.6302(-55.33) - 0.126(-36.75) - 0.5042(-58.58) + 1149/298.2$

$\qquad = \mathbf{+3.15 \; kJ/K}$

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CHAPTER 12

CONTENT CHAPTER 12

Clapeyron Equation

Borgnakke and Sonntag

12.149E

Verify that Clapeyrons equation is satisfied for R-410A at 30 F in Table F.9.

Clapeyron Eq.:
$$\frac{dP_{sat}}{dT} = \frac{dP_g}{dT} = \frac{h_{fg}}{Tv_{fg}}$$

F.9:　　P = 111.796 psia,　　h_{fg} = 95.75 Btu/lbm,　　v_{fg} = 0.5289 ft³/lbm

Slope around 30 F best approximated by cord from 20 F to 40 F
$$\frac{dP_g}{dT} = \frac{133.163 - 93.128}{40 - 20} = 2.00 \text{ psia/R},$$
$$\frac{h_{fg}}{Tv_{fg}} = \frac{95.75}{489.7 \times 0.5289} \frac{778}{144} = 1.997 \text{ psia/R}$$

This fits very well.

12.150E

Use the approximation given in Problem 12.16 and Table F.7 to determine A and B for steam from properties at 70 F only. Use the equation to predict the saturation pressure at 80 F and compare to table value.

$$\ln P_{sat} = A - B/T \qquad \Rightarrow \qquad \frac{dP_{sat}}{dT} = P_{sat}\,(-B)(-T^{-2})$$

so we notice from Eq.12.7 and Table values from F.7.1 and F.4 that

$$B = \frac{h_{fg}}{R} = \frac{1053.95}{85.76\,/\,778} = 9561.3 \text{ R}$$

Now the constant A comes from the saturation pressure as

$$A = \ln P_{sat} + B/T = \ln 0.363 + \frac{9561.3}{459.67 + 70} = 17.038$$

Use the equation to predict the saturation pressure at 80 F as

$$\ln P_{sat} = A - B/T = 17.038 - \frac{9561.3}{459.67 + 80} = -0.6789$$

$$P_{sat} = 0.5071 \text{ psia}$$

compare this with the table value of $P_{sat} = 0.507$ psia and we have a very accurate approximation.

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12.151E

Find the saturation pressure for refrigerant R-410A at –100 F assuming it is higher than the triple point temperature.

The lowest temperature in Table F.9 for R-410A is -80 F, so it must be extended to -100 F using the Clapeyron Eq. 12.7 integrated as in example 12.1

At T_1 = -80 F = 379.7 R ,

$$P_1 = 8.196 \text{ lbf/in.}^2 \quad \text{and} \quad R = \frac{1.9859}{72.585} = 0.027\ 36 \text{ Btu/lbm R}$$

$$\ln \frac{P}{P_1} = \frac{h_{fg}}{R} \frac{(T-T_1)}{T \times T_1} = \frac{121.14}{0.027\ 36} \frac{(359.7\text{-}379.7)}{359.7 \times 379.7} = -0.6484$$

$$P = P_1 \exp(-0.6484) = \textbf{4.286 lbf/in.}^2$$

12.152E

Using thermodynamic data for water from Tables F.7.1 and F.7.3, estimate the freezing temperature of liquid water at a pressure of 5000 lbf/in.^{2}.

H_2O $\dfrac{dP_{if}}{dT} = \dfrac{h_{if}}{Tv_{if}} \approx$ constant

At the triple point,

$v_{if} = v_f - v_i = 0.016\ 022 - 0.017\ 473$

$= -0.001\ 451\ ft^3/lbm$

$h_{if} = h_f - h_i = 0.0 - (-143.34) = 143.34\ Btu/lbm$

$\dfrac{dP_{if}}{dT} = \dfrac{143.34}{491.69(-0.001\ 451)} \times \dfrac{778.2}{144} = -1085.8\ lbf/in.^2\ R$

$\Rightarrow$ at P = 5000 lbf/in^2,

$T \approx 32.02 + \dfrac{(5000-0.09)}{(-1085.8)} = \mathbf{27.4\ F}$

12.153E

Ice (solid water) at 27 F, 1 atm is compressed isothermally until it becomes liquid. Find the required pressure.

Water, triple point T = 32.02 F = 491.69 R, P = 0.088 67 lbf/in.^{2}

v_f = 0.016 022 ft^3/lbm, v_i = 0.017 473 ft^3/lbm

h_f = 0.00 Btu/lbm h_i = -143.34 Btu/lbm

Clapeyron $\dfrac{dP_{if}}{dT} = \dfrac{h_f - h_i}{(v_f - v_i)T} = \dfrac{143.34 \times 778.2}{-0.001\ 451 \times 491.69 \times 144}$ = -1085.8 psia/R

$\Delta P \approx \dfrac{dP_{if}}{dT} \Delta T$ = -1085.8 (27 - 32.02) = 5450.7 lbf/in.^{2}

P = P$_{tp}$ + ΔP = **5451 lbf/in.^{2}**

Volume Expansivity and Compressibility

12.154E

Determine the volume expansivity, α_P, and the isothermal compressibility, β_T, for water at 50 F, 500 lbf/in.^{2} and at 400 F, 1500 lbf/in.^{2} using the steam tables.

Water at 50 F, 500 lbf/in.^{2} (compressed liquid)

$$\alpha_P = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P \approx \frac{1}{v}\left(\frac{\Delta v}{\Delta T}\right)_P$$

Using values at 32 F, 50 F and 100 F

$$\alpha_P \approx \frac{1}{0.015\ 99}\frac{0.016\ 1 - 0.015\ 99}{100 - 32} = \mathbf{0.000\ 101\ F^{-1}}$$

$$\beta_T = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T \approx -\frac{1}{v}\left(\frac{\Delta v}{\Delta P}\right)_T$$

Using values at saturation, 500 and 1000 lbf/in.^{2}

$$\beta_T \approx -\frac{1}{0.015\ 99}\frac{0.015\ 99 - 0.016\ 02}{1000 - 0.178} = \mathbf{0.000\ 0019\ in.^2/lbf}$$

Water at 400 F, 1500 lbf/in.^{2} (compressed liquid)

$$\alpha_P \approx \frac{1}{0.018\ 45}\frac{0.019\ 25 - 0.017\ 85}{450 - 350} = \mathbf{0.000\ 76\ F^{-1}}$$

$$\beta_T \approx -\frac{1}{0.018\ 45}\frac{0.0184 - 0.0185}{2000 - 1000} = \mathbf{0.000\ 0054\ in.^2/lbf}$$

12.155E

Use the CATT3 software to solve the previous problem.

The benefit of the software to solve for the partial derivatives is that we can narrow the interval over which we determine the slope.

Water at 50 F, 500 lbf/in.^{2} (compressed liquid)

$$\alpha_P = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P \approx \frac{1}{v}\left(\frac{\Delta v}{\Delta T}\right)_P \; ; \quad \beta_T = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T \approx -\frac{1}{v}\left(\frac{\Delta v}{\Delta P}\right)_T$$

Estimate by finite difference using values at 40 F, 50 F and 60 F,

$$\alpha_P \approx \frac{1}{0.01600}\frac{0.01601 - 0.01599}{60 - 40} = \textbf{0.000 0625 F}^{\textbf{-1}}$$

Using values at saturation, 500 psia and 1000 psia,

$$\beta_T \approx -\frac{1}{0.0160}\frac{0.01597 - 0.01602}{1000 - 0.178} = \textbf{0.000 0031 psi}^{\textbf{-1}}$$

Water at 400 F, 1500 lbf/in.^{2} (compressed liquid)

Estimate by finite difference using values at 350 F, 400 F and 450 F,

$$\alpha_P \approx \frac{1}{0.01\ 849}\frac{0.019\ 26 - 0.017\ 86}{450 - 350} = \textbf{0.000 757 F}^{\textbf{-1}}$$

Using values at 1000 psia, 1500 psia and 2000 psia,

$$\beta_T \approx -\frac{1}{0.018\ 49}\frac{0.01\ 844 - 0.01\ 855}{2000 - 1000} = \textbf{0.000 0059 psi}^{\textbf{-1}}$$

12.156E

A cylinder fitted with a piston contains liquid methanol at 70 F, 15 lbf/in.^{2} and volume 1 ft^3. The piston is moved, compressing the methanol to 3000 lbf/in.^{2} at constant temperature. Calculate the work required for this process. The isothermal compressibility of liquid methanol at 70 F is 8.3×10^{-6} in.^{2}/lbf.

$$_1w_2 = \int_1^2 Pdv = \int P\left(\frac{\partial v}{\partial P}\right)_T dP_T = -\int_1^2 v\beta_T \, PdP_T$$

For $v \approx$ const $\& \; \beta_T \approx$ const. $\quad => \quad _1w_2 = -\frac{v\beta_T}{2}\left(P_2^2 - P_1^2\right)$

For liquid methanol, from Table F.3 : $\rho = 49.1$ lbm/ft^3

$V_1 = 1.0$ ft^3, $\quad$ m $= 1.0 \times 49.1 = 49.1$ lbm

$$_1W_2 = \frac{1.0 \times 8.3 \times 10^{-6}}{2}\left[(3000)^2 - (15)^2\right] \times 144 = 5378.4 \text{ ft lbf} = \textbf{6.9 Btu}$$

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12.157E

Sound waves propagate through a media as pressure waves that causes the media to go through isentropic compression and expansion processes. The speed of sound c is defined by $c^2 = (\partial P/\partial \rho)_s$ and it can be related to the adiabatic compressibility, which for liquid ethanol at 70 F is 6.4×10^{-6} in.^{2}/lbf. Find the speed of sound at this temperature.

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s = \frac{1}{-\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_s \rho} = \frac{1}{\beta_s \rho}$$

From Table F.3 for ethanol, $\rho = 48.9 \text{ lbm/ft}^3$

$$\Rightarrow c = \left(\frac{32.174 \times 144}{6.4 \times 10^{-6} \times 48.9}\right)^{1/2} = \textbf{3848 ft/s}$$

12.158E

Consider the speed of sound as defined in Eq. 12.42. Calculate the speed of sound for liquid water at 50 F, 250 lbf/in.^{2} and for water vapor at 400 F, 80 lbf/in.^{2} using the steam tables.

From Eq 12.42 : $c^2 = \left(\dfrac{\partial P}{\partial \rho}\right)_s = -v^2\left(\dfrac{\partial P}{\partial v}\right)_s$

Liquid water at 50 F, 250 lbf/in.^{2}

Assume $\left(\dfrac{\partial P}{\partial v}\right)_s \approx \left(\dfrac{\Delta P}{\Delta v}\right)_T$

Using saturated liquid at 50 F and compressed liquid at 50 F, 500 lbf/in.^{2},

$c^2 = -\left(\dfrac{0.016024+0.015998}{2}\right)^2\left(\dfrac{(500-0.18)\times144\times32.174}{0.015998-0.016024}\right) = 22.832\times10^6$

c = **4778 ft/s**

Superheated vapor water at 400 F, 80 lbf/in.^{2}

$v = 6.217$ ft^3/lbm, $s = 1.6790$ Btu/lbm R

At P = 60 lbf/in.^{2} & s = 1.6790: T = 343.8 F, v = 7.7471 ft^3/lbm

At P = 100 lbf/in.^{2} & s = 1.6790: T = 446.2 F, v = 5.2394 ft^3/lbm

$c^2 = -(6.217)^2\left(\dfrac{(100-60)\times144\times32.174}{5.2394-7.7471}\right) = 2.856 \times 10^6$

c = **1690 ft/s**

12.159E

Liquid methanol at 77 F has an adiabatic compressibility of 7.1×10^{-6} in^2/lbf. What is the speed of sound? If it is compressed from 15 psia to 1500 psia in an insulated piston/cylinder, what is the specific work?

From Eq.12.41 and Eq.12.42 and the density from table F.3

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2 \left(\frac{\partial P}{\partial v}\right)_s = \frac{1}{\beta_s \rho} = \frac{1}{7.1 \times 10^{-6} \times 49.1} \, 144 \times 32.174 \text{ ft}^2/\text{s}^2$$

$$= 13.290 \times 10^6 \text{ ft}^2/\text{s}^2$$

c = **3645 ft/s**

The specific work becomes

$$w = \int P \, dv = \int P \, (-\beta_s v) \, dP = - \int \beta_s v \, P \, dP = -\beta_s \, v \int_1^2 P \, dP$$

$$= -\beta_s \, v \, 0.5 \, (P_2^2 - P_1^2)$$

$$= -7.1 \times 10^{-6} \text{ in}^2/\text{lbf} \times \frac{0.5}{49.1} \text{ ft}^3/\text{lbm} \times (1500^2 - 15^2) \text{ psi}^2$$

$$= -0.163 \, (\text{ft-lbf/lbm}) \, (\text{ft/in})^2 = \textbf{-23.4 ft-lbf/lbm = -0.03 Btu/lbm}$$

Equations of State

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12.160E

Use Table F.9 to find the compressibility of R-410A at 140 F and a) saturated liquid   b) saturated vapor and   c) 400 psia

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Table F.1:   R = 1545.36 / 72.585 = 21.29 lbf-ft/lbm-R

a)   Table F.9.1:   P = 556.488 psia, v = 0.01966 ft^3/lbm

$$Z = \frac{Pv}{RT} = \frac{556.488 \ \times 0.01966 \times 144}{21.29 \ \times 599.7} = \textbf{0.1234}$$

b)   Table F.9.1:   P = 556.488 psia, v = 0.0796 ft^3/lbm

$$Z = \frac{Pv}{RT} = \frac{556.488 \ \times 0.0796 \times 144}{21.29 \ \times 599.7} = \textbf{0.5}$$

c)   Table F.9.2:   P = 400 psia, v = 0.1574 ft^3/lbm

$$Z = \frac{Pv}{RT} = \frac{400 \times 0.1574 \times 144}{21.29 \times 599.7} = \textbf{0.71}$$

The R-410A is not an ideal gas at any of these states.

12.161E

Calculate the difference in internal energy of the ideal-gas value and the real-gas value for carbon dioxide at the state 70 F, 150 lbf/in.^{2}, as determined using the virial EOS. At this state B = -2.036 ft^3/lb mol, T(dB/dT) = 4.236 ft^3/lb mol.

Solution: CO_2 at 70 F, 150 lbf/in^2

virial: $\quad P = \dfrac{RT}{v} + \dfrac{BRT}{v^2}$; $\qquad \left(\dfrac{\partial P}{\partial T}\right)_v = \dfrac{R}{v} + \dfrac{BR}{v^2} + \dfrac{RT}{v^2}\left(\dfrac{dB}{dT}\right)$

$$u - u^* = - \int_v^\infty \left[T\left(\dfrac{\partial P}{\partial T}\right)_v - P \right] dv = - \int_v^\infty \dfrac{RT^2}{v^2}\left(\dfrac{dB}{dT}\right) dv = - \dfrac{RT^2}{v}\dfrac{dB}{dT}$$

$\qquad$ B = -2.036 ft^3/lbmol $\qquad\qquad$ $T\left(\dfrac{dB}{dT}\right)$ = 4.236 ft^3/lbmol

Solution of virial equation (quadratic formula):

$$\bar{v} = \dfrac{1}{2}\dfrac{\bar{R}T}{P}\left[1 + \sqrt{1 + 4BP/\bar{R}T} \right]$$

But $\quad \dfrac{\bar{R}T}{P} = \dfrac{1545 \times 529.7}{150 \times 144} = 37.8883$

$$\bar{v} = 0.5 \times 37.8883\left[1 + \sqrt{1 + 4(-2.036)/37.8883} \right] = 35.7294 \text{ ft}^3/\text{lbmol}$$

Using the minus-sign root of the quadratic formula results in a compressibility factor < 0.5, which is not consistent with such a truncated equation of state.

$$\bar{u} - \bar{u}^* = - \dfrac{RT}{v} T \dfrac{dB}{dT} = \dfrac{-1.9859 \times 529.7}{35.7294} 4.236 = \textbf{-123.9 Btu/lbmol}$$

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Generalized Charts

12.162E

How low should the pressure be so that nitrous oxide (N_2O) gas at 501.6 R can be treated as an ideal gas with 5% accuracy or better?

From Table F.1: $T_c = 557.3$ R, $P_c = 1050$ psi

$$\Rightarrow\ T_{r1} = \frac{501.6}{557.3} = 0.9$$

Look in Fig. D.1 following the curve $T_{r1} = 0.9$ to the point where $Z = 0.95$

$P_{r1} = 0.125$ so $P < 0.125 \times 1050$ psi = **131 psi**

12.163E

Nitrous oxide (N_2O) at 501.6 R is at a pressure so that it can be in a two-phase state. Find the generalized enthalpy departure for the two saturated states of liquid and vapor.

From Table F.1: $T_c = 557.3$ R, $P_c = 1050$ psi

$$\Rightarrow \quad T_{r1} = \frac{501.6}{557.3} = 0.9$$

From Fig. D.2:

Saturated liquid: $(h^* - h_f)/RT_c = 4.08;$

Saturated vapor: $(h^* - h_g)/RT_c = 0.875$

12.164E

Find the heat of evaporation, h_{fg}, for R-134a at 30 F from the generalized charts and compare to the value in Table F.10.

From Table F.1: $T_c = 673.6$ R, $P_c = 589$ psia $=>$ $T_{r1} = \dfrac{489.7}{673.6} = 0.727$

$R = 1.98589/102.03 = 0.01946$ Btu/lbm-R

From Fig. D.2:

Saturated liquid: $(h^* - h_f)/RT_c = 4.7$;

Saturated vapor: $(h^* - h_g)/RT_c = 0.3$

$h_{fg} = h_g - h_f = RT_c\, [-0.3 - (-4.7)] = 4.4\ RT_c$

$= 4.4 \times 0.01946 \times 673.6 =$ **57.67 Btu/lbm**

Table F. 10: $h_{fg} =$ **85.63 Btu/lbm**

The approximation is not very good and can be improved by using the accentric factor.

12.165E

A 7-ft^3 rigid tank contains propane at 1300 lbf/in.^{2}, 540 F. The propane is then allowed to cool to 120 F as heat is transferred with the surroundings. Determine the quality at the final state and the mass of liquid in the tank, using the generalized compressibility charts.

Propane C_3H_8:

$V = 7.0$ ft^3, $P_1 = 1300$ lbf/in.^{2}, $T_1 = 540$ F $= 1000$ R

cool to $T_2 = 120$ F $= 580$ R

From Table F.1 : $T_C = 665.6$ R, $P_C = 616$ lbf/in.^{2}

$$P_{r1} = \frac{1300}{616} = 2.110, \quad T_{r1} = \frac{1000}{665.6} = 1.502$$

From D.1: $Z_1 = 0.83$

$$v_2 = v_1 = \frac{Z_1 R T_1}{P_1} = \frac{0.83 \times 35.04 \times 1000}{1300 \times 144} = 0.1554 \text{ ft}^3/\text{lbm}$$

From D.1 at $T_{r2} = 0.871$, saturated $\Rightarrow$ $P_{G2} = 0.43 \times 616 = 265$ lbf/in.^{2}

$$v_{G2} = \frac{0.715 \times 35.04 \times 580}{265 \times 144} = 0.3808 \text{ ft}^3/\text{lbm}$$

$$v_{F2} = \frac{0.075 \times 35.04 \times 580}{265 \times 144} = 0.0399 \text{ ft}^3/\text{lbm}$$

$0.1554 = 0.0399 + x_2(0.3781\text{-}0.0399)$ $\Rightarrow$ $x_2 = \mathbf{0.3388}$

$$m_{\text{LIQ 2}} = (1 - 0.3388)\frac{7.0}{0.1554} = \mathbf{29.8 \text{ lbm}}$$

These tanks
contain liquid
propane.

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12.166E

A rigid tank contains 5 lbm of ethylene at 450 lbf/in.^{2}, 90 F. It is cooled until the ethylene reaches the saturated vapor curve. What is the final temperature?

C_2H_4 m = 5 lbm

$P_1 = 450$ lbf/in^2, $T_1 = 90$ F $= 249.7$ R

$P_{r1} = \dfrac{450}{731} = 0.616$, $T_{r1} = \dfrac{549.7}{508.3} = 1.082$

Fig. D.1 $\Rightarrow$ $Z_1 = 0.82$

$$P_{r2} = P_{r1} \frac{Z_2 T_{r2}}{Z_1 T_{r1}} = 0.616 \frac{Z_{G2} T_{r2}}{0.82 \times 1.082} = 0.6943\, Z_{G2} T_{r2}$$

Trial & error to match a saturated P_{r2}, T_{r2} and the Z_{G2} so Eq. is satisfied. Guess a T_{r2} and find the rest and compare with computed P_{r2} from Eq.

T_{r2}	Z_{G2}	P_{r2}	$P_{r2\,CALC}$		
0.871	0.715	0.43	0.432	~ OK	$\Rightarrow$ $T_2 = \mathbf{442.7\ R}$

12.167E

A piston/cylinder contains 10 lbm of butane gas at 900 R, 750 lbf/in.^{2}. The butane expands in a reversible polytropic process with polytropic exponent, $n = 1.05$, until the final pressure is 450 lbf/in.^{2}. Determine the final temperature and the work done during the process.

C_4H_{10} , m = 10 lbm , T_1 = 900 R , P_1 = 750 lbf/in.^{2}

Rev. polytropic process (n = 1.05), P_2 = 450 lbf/in.^{2}

$$T_{r1} = \frac{900}{765.4} = 1.176, \quad P_{r1} = \frac{750}{551} = 1.361 \quad \Rightarrow \quad \text{From Fig. D.1}: \quad Z_1 = 0.67$$

$$V_1 = mZRT/P = \frac{10 \times 0.67 \times 26.58 \times 900}{750 \times 144} = 1.484 \text{ ft}^3$$

$$P_1 V_1^n = P_2 V_2^n \rightarrow V_2 = 1.484 \left(\frac{750}{450}\right)^{\frac{1}{1.05}} = 2.414 \text{ ft}^3$$

$$Z_2 T_{r2} = \frac{P_2 V_2}{mRT_C} = \frac{450 \times 144 \times 2.414}{10 \times 26.58 \times 765.4} = 0.7688$$

at $P_{r2} = 450/551 = 0.817$

Trial & error: $\quad T_{r2} = 1.068, \quad Z_2 = 0.72 \quad \Rightarrow \quad T_2 = \textbf{817.4 R}$

$${}_1W_2 = \int_1^2 PdV = \frac{P_2 V_2 - P_1 V_1}{1-n} = \left(\frac{450 \times 2.414 - 750 \times 1.484}{1 - 1.05}\right) \times \frac{144}{778} = \textbf{98.8 Btu}$$

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12.168E

Calculate the heat transfer during the process described in Problem 12.167E.

From solution 12.167,

$V_1 = 1.473$ ft^3, $V_2 = 2.396$ ft^3, $_1W_2 = 98.8$ Btu

$T_{r1} = 1.176$, $P_{r1} = 1.361$, $T_{r2} = 1.068$, $P_{r2} = 0.817$, $T_2 = 817.4$ R

From D.1: $\left(\dfrac{h^*-h}{RT_C}\right)_1 = 1.36$, $\left(\dfrac{h^*-h}{RT_C}\right)_2 = 0.95$

$h_2^* - h_1^* = 0.415 \,(817.4 - 900) = -34.3$ Btu/lbm

$h_2 - h_1 = -34.3 + \dfrac{26.58 \times 765.4}{778}\,(-0.95 + 1.36)\; = -23.6$ Btu/lbm

$U_2 - U_1 = m(h_2 - h_1) - P_2 V_2 + P_1 V_1$

$\qquad = 10(-23.6) - \dfrac{450 \times 144 \times 2.414}{778} + \dfrac{750 \times 144 \times 1.484}{778}\; = -231.1$ Btu

$_1Q_2 = U_2 - U_1 + {_1W_2} = \mathbf{-132.3\ Btu}$

12.169E

The new refrigerant R-152a is used in a refrigerator with an evaporator temperature of -10 F and a condensing temperature of 90 F. What are the high and low pressures in this cycle?

Since we do not have the printed tables for R-152a we will use generalized charts. The critical properties are: $T_c = 695.5$ R, $P_c = 656$ psi.

$T_{r1} = T/T_c = (459.67 - 10)/ 695.5 = 0.646$

Fig. D.1: $\quad P_{G\ T1} = P_{r1\ sat}\ P_c = 0.06 \times 656 = \textbf{39.4 psi}$

$T_{r2} = T/T_c = (459.67 + 90)/ 695.5 = 0.79$

Fig. D.1: $\quad P_{G\ T2} = P_{r2\ sat}\ P_c = 0.25 \times 656 = \textbf{164 psi}$

12.170E

A cylinder contains ethylene, C_2H_4, at 222.6 lbf/in.^{2}, 8 F. It is now compressed in a reversible isobaric (constant P) process to saturated liquid. Find the specific work and heat transfer.

Ethylene C_2H_4 P_1 = 222.6 lbf/in.^{2} = P_2, T_1 = 8 F = 467.7 R

State 2: saturated liquid, $x_2 = 0.0$

R = 55.07 ft lbf/lbm R = 0.070 78 Btu/lbm R

$$T_{r1} = \frac{467.7}{508.3} = 0.920 \quad P_{r1} = P_{r2} = \frac{222.6}{731} = 0.305$$

From D.1 and D.2: $\quad Z_1 = 0.85$, $\quad \left(\frac{h^*-h}{RT_C}\right)_1 = 0.40$

$$v_1 = \frac{Z_1 RT_1}{P_1} = \frac{0.85 \times 55.07 \times 467.7}{222.6 \times 144} = 0.683$$

$(h_1^*-h_1) = 0.070\ 78 \times 508.3 \times 0.40 = 14.4$

From D.1 and D.2: $\quad T_2 = 0.822 \times 508.3 = 417.8$ R

$$Z_2 = 0.05 , \quad \left(\frac{h^*-h}{RT_C}\right)_2 = 4.42$$

$$v_2 = \frac{Z_2 RT_2}{P_2} = \frac{0.05 \times 55.07 \times 417.8}{222.6 \times 144} = 0.035\ 89 \ \text{ft}^3/\text{lbm}$$

$(h_2^*-h_2) = 0.070\ 78 \times 508.3 \times 4.42 = 159.0$ Btu/lbm

$(h_2^*-h_1^*) = C_{P0}(T_2-T_1) = 0.411(417.8 - 467.7) = -20.5$ Btu/lbm

$$_1w_2 = \int_1^2 Pdv = P(v_2-v_1) = 222.6(0.035\ 89 - 0.683) \times \frac{144}{778} = \textbf{-26.7 Btu/lbm}$$

$_1q_2 = (u_2 - u_1) + _1w_2 = (h_2-h_1) = -159.0 - 20.5 + 14.4 = \textbf{-165.1 Btu/lbm}$

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12.171E

Saturated vapor R-410A at 80 F is throttled to 30 lbf/in.^{2} in a steady flow process. Find the exit temperature, neglecting kinetic energy, using the generalized charts, Fig. D.2 and repeat using Table F.9.

R-410A throttling process, R = 1.98589/72.585 = 0.02736 Btu/lbm-R

T_1 = 80 F, x_1 = 1.00, P_2 = 30 lbf/in.^{2},

Energy Eq.: h_2-h_1 = (h_2-h_2^*) + (h_2^*-h_1^*) + (h_1^*-h_1) = 0

Generalized charts, $T_{r1} = \dfrac{539.7}{620.1} = 0.87$

From D.2: (h_1^*-h_1) = 0.02736 × 620.1 × 0.75 = 12.72 Btu/lbm

To get C_{P0}, use h values from Table F.9 at low pressure (5 psia).

$$C_{P0} \approx \dfrac{137.6\text{-}133.75}{80\text{-}60} = 0.1925 \text{ Btu/lbm R}$$

Substituting into energy Eq.: (h_2-h_2^*) + 0.1925 (T_2 - 30) + 12.72 = 0

at $P_{r2} = \dfrac{30}{711} = 0.042$

Assume T_2 = 10 F = 469.7 R => $T_{r2} = \dfrac{469.7}{620.1} = 0.757$

(h_2^*-h_2) = 0.02736 × 620.1 × 0.07 = 1.19

Substituting,

-1.19 + 0.1925(10 - 80) + 12.72 = -1.945

Guess T_2 = 20 F = 479.7 R => $T_{r2} = \dfrac{479.7}{620.1} = 0.774$

(h_2^*-h_2) = 0.02736 × 620.1 × 0.07 = 1.19

Substituting,

-1.19 + 0.1925(20 - 80) + 12.72 = -0.02   nearly OK

$\Rightarrow T_2$ = **20 F** (maybe re-estimate C_{P0} at avg T)

R-410A tables, F.9: T_1 = 80 F, x_1 = 1.0 => h_1 = 122.14 Btu/lbm

h_2 = h_1 = 122.14 Btu/lbm, P_2 = 30 lbf/in.^{2} => T_2 = **8.15 F**

12.172E

Find the heat of evaporation, h_{fg}, for isobutane (T_c = 735.8 R, P_c = 529.4 psia, M = 58.124) at 54.7 F from the generalized charts and compare to the values in the CATT3 computerized tables.

To read the charts we need the reduced temperature

$$T_{r1} = T/T_c = (54.7 + 459.7)/735.8 = 0.70$$

$$R = 1.98589 / 58.124 = 0.03417 \text{ Btu/lbm-R}$$

From Fig. D.2: $\qquad (h^* - h)_g = 0.2 \, RT_c \; ; \quad (h^* - h)_f = 4.85 \, RT_c$

$$h_{fg} = h_g - h_f = -(h^* - h)_g - (h^* - h)_f = (-0.2 + 4.85) \, RT_c$$

$$= 4.65 \, RT_c = 4.65 \times 0.03417 \text{ Btu/lbm-R} \times 735.8 \text{ R}$$

$$= \textbf{116.9 kJ/kg}$$

CATT3: $\quad h_{fg} = -11.01 - (-158.5) = \textbf{147.5 Btu/lbm}$

The generalized charts are not super accurate, some improvement can be done using the accentric factor.

Computer-Aided Thermodynamic Tables 3

File Edit Tables/Substances Options Help

T-S Diagram

Iso-Butane Properties

T	54.7	F
P	34.75	psia
V	2.526	ft3/lbm
U	-27.25	Btu/lbm
H	-11.01	Btu/lbm
S	-0.03152	Btu/lbm/R
X	1	
Phase	Saturated Vapor	

Type	Temp	Pressure	Specific Volume	Internal Energy	Specific Enthalpy	Specific Entropy	Quality	Phase
	F	psia	ft3/lbm	Btu/lbm	Btu/lbm	Btu/lbm/R		
1.	54.7	34.75	0.02831	-158.7	-158.5	-0.3183	0	Saturated Liquid
2.	54.7	34.75	2.526	-27.25	-11.01	-0.03152	1	Saturated Vapor

Water \ Refrigerants \ Cryogenics \ Air \ Ideal Gases \ Compressibility \ Psychrometrics

Values: S = -0.6254; T = 586.51

Borgnakke and Sonntag

12.173E

A 10-ft^3 tank contains propane at 90 F, 90% quality. The tank is heated to 600 F. Calculate the heat transfer during the process.

$V = 10$ ft^3

$T_1 = 90$ F $= 549.7$ R, $x_1 = 0.90$

Heat to $T_2 = 600$ F $= 1059.7$ R

$M = 44.094$, $T_C = 665.6$ R

$P_C = 616$ lbf/in.^{2}

$R = 35.04$, $C_{P0} = 0.407$ Btu/lbm R

$T_{r1} = 0.826$ Figs. D.1 and D.2 $\rightarrow$ $Z_1 = 0.1 \times 0.053 + 0.9 \times 0.78 = 0.707$,

$$\frac{h_1^*-h_1}{RT_c} = 0.1 \times 4.4 + 0.9 \times 0.55 = 0.935$$

$P_r^{SAT} = 0.31$; $P_1^{SAT} = 0.31 \times 616 = 191$ lbf/in.^{2}

$$m = \frac{PV}{ZRT} = \frac{191 \times 144 \times 10}{0.707 \times 35.04 \times 549.7} = 20.2 \text{ lbm}$$

$$P_{r2} = \frac{20.2 \times Z_2 \times 35.04 \times 1059.7}{616 \times 144 \times 10} = \frac{Z_2}{1.183} \qquad \text{at } T_{r2} = 1.592$$

Trial & error: $\begin{cases} P_{r2} = 0.79 \text{ and } P_2 = 490 \text{ lbf/in.}^2 \\[2mm] Z_2 = 0.94 \text{ and } \dfrac{h_2^*-h_2}{RT_c} = 0.36 \end{cases}$

$(h_2^*-h_1^*) = 0.407(600-90) = 207.6$ Btu/lbm

$(h_1^*-h_1) = 0.935 \times 35.04 \times 665.9/ 778 = 28.0$

$(h_2^*-h_2) = 0.36 \times 35.04 \times 665.9/ 778 = 10.8$

From the energy equation

$Q_{12} = m(h_2-h_1) - (P_2-P_1)V$

$= 20.2 (-10.8 + 207.6 + 28.0) - (490 - 191) \times \dfrac{144 \times 10}{778}$

$= +4541 - 553 = \textbf{3988 Btu}$

12.174E

Carbon dioxide collected from a fermentation process at 40 F, 15 lbf/in.[2] should be brought to 438 R, 590 lbf/in.[2] in a steady flow process. Find the minimum amount of work required and the heat transfer. What devices are needed to accomplish this change of state?

$$R = \frac{35.1}{778} = 0.045\ 12 \text{ Btu/lbm R}$$

$$T_{ri} = \frac{500}{547.4} = 0.913\ , \quad P_{ri} = \frac{15}{1070} = 0.014$$

From D.2 and D.3: $\quad (\frac{h^*\text{-}h}{RT_C}) = 0.02, \quad (\frac{s^*\text{-}s}{R}) = 0.01\ R$

$$T_{re} = \frac{438}{547.4} = 0.80, \quad P_{re} = \frac{590}{1070} = 0.551$$

From D.2 and D.3: $\quad h_e^*\text{-}h_e = 4.50\ RT_c, \qquad s_e^*\text{-}s_e = 4.70\ R$

$(h_i\text{-}h_e) = - (h_i^*\text{-}h_i) + (h_i^*\text{-}h_e^*) + (h_e^*\text{-}h_e)$

$\quad = - 0.045\ 12 \times 547.4 \times 0.02 + 0.203(500 - 438) + 0.045\ 12 \times 547.4 \times 4.50$

$\quad\quad = 123.2 \text{ Btu/lbm}$

$(s_i\text{-}s_e) = - (s_i^*\text{-}s_i) + (s_i^*\text{-}s_e^*) + (s_e^*\text{-}s_e)$

$\quad = - 0.045\ 12 \times 0.01 + 0.203 \ln \frac{500}{438} - 0.045\ 12 \ln \frac{15}{590} + 0.045\ 12 \times 4.70$

$\quad\quad = 0.4042 \text{ Btu/lbm R}$

$w^{rev} = (h_i\text{-}h_e) - T_0(s_i\text{-}s_e)\ = 123.2 - 500(0.4042) = $ **-78.4 Btu/lbm**

$q^{rev} = (h_e\text{-}h_i) + w^{rev}\ = -123.2 - 78.9 = $ **-202.1 Btu/lbm**

12.175E

A cylinder contains ethylene, C_2H_4, at 222.6 lbf/in.^{2}, 8 F. It is now compressed isothermally in a reversible process to 742 lbf/in.^{2}. Find the specific work and heat transfer.

Ethylene C_2H_4 , $\quad$ R = 55.07 ft lbf/lbm R = 0.070 78 Btu/lbm R

State 1: P_1 = 222.6 lbf/in.^{2}, $\quad T_2 = T_1$ = 8 F = 467.7 R

State 2: P_2 = 742 lbf/in.^{2}

$$T_{r2} = T_{r1} = \frac{467.7}{508.3} = 0.920; \quad P_{r1} = \frac{222.6}{731} = 0.305$$

From D.1, D.2 and D.3: $\quad Z_1 = 0.85$, $\left(\frac{h^*-h}{RT_C}\right)_1 = 0.40$

$(h_1^*-h_1) = 0.070\ 78 \times 508.3 \times 0.40 = 14.4$ Btu/lbm

$(s_1^*-s_1) = 0.070\ 78 \times 0.30$ $\qquad\qquad$ $\boxed{\ \ } = 0.0212$ Btu/lbm R

$$P_{r2} = \frac{742}{731} = 1.015 \text{ (comp. liquid)}$$

From D.1, D.2 and D.3: $\quad Z_2 = 0.17$

$(h_2^*-h_2) = 0.070\ 78 \times 508.3 \times 4.0 = 143.9$

$(s_2^*-s_2) = 0.070\ 78 \times 3.6 = 0.2548$

$(h_2^*-h_1^*) = 0$

$(s_2^*-s_1^*) = 0 - 0.070\ 78 \ln\dfrac{742}{222.6} = -0.0852$

$_1q_2 = T(s_2-s_1) = 467.7(-0.2548 - 0.0852 + 0.0212) = \textbf{-149.1 Btu/lbm}$

$(h_2-h_1) = -143.9 + 0 + 14.4 = -129.5$

$(u_2-u_1) = (h_2-h_1) - RT(Z_2-Z_1)$

$\qquad\qquad = -129.5 - 0.070\ 78 \times 467.7\ (0.17 - 0.85) = -107.0$

$_1w_2 = {}_1q_2 - (u_2-u_1) = -149.1 + 107.0 = \textbf{-42.1 Btu/lbm}$

12.176E

A geothermal power plant on the Raft river uses isobutane as the working fluid. The fluid enters the reversible adiabatic turbine at 320 F, 805 lbf/in.^{2} and the condenser exit condition is saturated liquid at 91 F. Isobutane has the properties Tc = 734.65 R, Pc = 537 lbf/in.^{2}, Cpo = 0.3974 Btu/lbm R and ratio of specific heats k = 1.094 with a molecular weight as 58.124. Find the specific turbine work and the specific pump work.

$$R = 26.58 \text{ ft lbf/lbm R} = 0.034\ 166 \text{ Btu/lbm R}$$

Turbine inlet: $T_{r1} = 779.7 / 734.7 = 1.061$, $P_{r1} = 805 / 537 = 1.499$

Condenser exit: $T_3 = 91 \text{ F}$, $x_3 = 0.0$; $T_{r3} = 550.7 / 734.7 = 0.75$

From D.1 : $P_{r3} = 0.165$, $Z_3 = 0.0275$

$$P_2 = P_3 = 0.165 \times 537 = 88.6 \text{ lbf/in.}^2$$

From D.2 and D.3,

$$(h_1^* - h_1) = 0.034\ 166 \times 734.7 \times 2.85 = 71.5 \text{ Btu/lbm}$$

$$(s_1^* - s_1) = 0.034\ 166 \times 2.15 = 0.0735 \text{ Btu/lbm R}$$

$$(s_2^* - s_1^*) = 0.3974 \ln \frac{550.7}{779.7} - 0.034\ 166 \ln \frac{88.6}{805} = -0.0628 \text{ Btu/lbm R}$$

$$(s_2^* - s_2) = (s_2^* - s_{F2}) - x_2 s_{FG2}\ \ = 0.034\ 166 \times 6.12 - x_2 \times 0.034\ 166(6.12 - 0.29)$$

$$= 0.2090 - x_2 \times 0.1992$$

$$(s_2 - s_1) = 0 = -0.2090 + x_2 \times 0.1992 - 0.0628 + 0.0735 \quad => \quad x_2 = 0.9955$$

$$(h_2^* - h_1^*) = C_{P0}(T_2 - T_1) = 0.3974(550.7 - 779.7) = -91.0 \text{ Btu/lbm}$$

From D.2,

$$(h_2^* - h_2) = (h_2^* - h_{F2}) - x_2 h_{FG2} = 0.034\ 166 \times 734.7[4.69 - 0.9955(4.69 - 0.32)]$$

$$= 117.7 - 0.9955 \times 109.7 = 8.5 \text{ Btu/lbm}$$

Turbine: $w_T = (h_1 - h_2) = -71.5 + 91.0 + 8.5 =$ **28.0 Btu/lbm**

Pump $v_{F3} = \dfrac{Z_{F3} R T_3}{P_3} = \dfrac{0.0275 \times 26.58 \times 550.7}{88.6 \times 144} = 0.031\ 55 \text{ ft}^3/\text{lbm}$

$$w_P = -\int_3^4 v dP \approx v_{F3}(P_4 - P_3) = -0.031\ 55(805 - 88.6) \times \frac{144}{778} = \textbf{-4.2 Btu/lbm}$$

12.177E

A line with a steady supply of octane, C_8H_{18}, is at 750 F, 440 lbf/in.^{2}. What is your best estimate for the availability in an steady flow setup where changes in potential and kinetic energies may be neglected?

Availability of Octane at $\qquad$ $T_i = 750$ F, $P_i = 440$ lbf/in.^{2}

$R = 13.53$ ft lbf/lbm-R $= 0.017\ 39$ Btu/lbm-R

$P_{ri} = \dfrac{440}{361} = 1.219, \quad T_{ri} = \dfrac{1209.7}{1023.8} = 1.182$

From D.2 and D.3:

$(h_1^* \text{-} h_1) = 0.017\ 39 \times 1023.8 \times 1.15 = 20.5$ Btu/lbm

$(s_1^* \text{-} s_1) = 0.017\ 39 \times 0.71 = 0.0123$ Btu/lbm R

Exit state in equilibrium with the surroundings

Assume $T_0 = 77$ F, $P_0 = 14.7$ lbf/in.^{2}

$T_{r0} = \dfrac{536.7}{1023.8} = 0.524 \ , \ P_{r0} = \dfrac{14.7}{361} = 0.041$

From D.2 and D.3:

$(h_0^* \text{-} h_0) = RT_C \times 5.41 = 96.3 \quad$ and $\qquad (s_0^* \text{-} s_0) = R \times 10.38 = 0.1805$

$(h_i^* \text{-} h_0^*) = 0.409(1209.7 - 536.7) = 275.3$ Btu/lbm

$(s_i^* \text{-} s_0^*) = 0.409 \ln \dfrac{1209.7}{536.7} - 0.017\ 39 \ln \dfrac{440}{14.7} = 0.2733$ Btu/lbm R

$(h_i \text{-} h_0) = -20.5 + 275.3 + 96.3 = 351.1$ Btu/lbm

$(s_i \text{-} s_0) = -0.0123 + 0.2733 + 0.1805 = 0.4415$ Btu/lbm R

$\psi_i = w^{rev} = (h_i \text{-} h_0) - T_0(s_i \text{-} s_0) = 351.1 - 536.7\ (0.4415) = \textbf{114.1 Btu/lbm}$

12.178E

A distributor of bottled propane, C_3H_8, needs to bring propane from 630 R, 14.7 lbf/in.^{2} to saturated liquid at 520 R in a steady flow process. If this should be accomplished in a reversible setup given the surroundings at 540 R, find the ratio of the volume flow rates V_{in}/V_{out}, the heat transfer and the work involved in the process.

$R = 35.04/778 = 0.045\ 04$ Btu/lbm R

$T_{ri} = \dfrac{630}{665.6} = 0.947 \quad P_{ri} = \dfrac{14.7}{616} = 0.024$

From D.1, D.2 and D.3 : $\quad Z_i = 0.99$

$(h_i^* - h_i) = 0.045\ 04 \times 665.6 \times 0.03 = 0.9$ Btu/lbm

$(s_i^* - s_i) = 0.045\ 04 \times 0.02 = 0.0009$ Btu/lbm R

$T_{re} = 520/665.6 = 0.781,$

From D.1, D.2 and D.3 :

$P_{re} = = 0.21 , \quad P_e = 0.21 \times 616 = 129$ lbf/in.$^2 , \quad Z_e = 0.035$

$(h_e^* - h_e) = 0.045\ 04 \times 665.6 \times 4.58 = 137.3$ Btu/lbm

$(s_e^* - s_e) = 0.045\ 04 \times 5.72 = 0.2576$ Btu/lbm R

$(h_e^* - h_i^*) = 0.407\ (520 - 630) = -44.8$ Btu/lbm

$(s_e^* - s_i^*) = 0.407 \ln \dfrac{520}{630} - 0.045\ 04 \ln \dfrac{132}{14.7} = -0.1770$ Btu/lbm R

$(h_e - h_i) = -137.3 - 44.8 + 0.9 = -181.2$ Btu/lbm

$(s_e - s_i) = -0.2576 - 0.1759 + 0.0009 = -0.4326$ Btu/lbm R

$$\frac{\dot{V}_{in}}{\dot{V}_{out}} = \frac{Z_i T_i/P_i}{Z_e T_e/P_e} = \frac{0.99}{0.035} \times \frac{630}{520} \times \frac{129}{14.7} = \mathbf{300.7}$$

$w^{rev} = (h_i - h_e) - T_0(s_i - s_e) = 181.2 - 540(0.4326) = \mathbf{-52.4\ Btu/lbm}$

$q^{rev} = (h_e - h_i) + w^{rev} = -181.2 - 52.4 = \mathbf{-233.6\ Btu/lbm}$

Mixtures

12.179E

A 4 lbm mixture of 50% argon and 50% nitrogen by mole is in a tank at 300 psia, 320 R. How large is the volume using a model of (a) ideal gas and (b) Kays rule with generalized compressibility charts.

a) Ideal gas mixture

Eq.11.5: $M_{mix} = \sum y_i M_i = 0.5 \times 39.948 + 0.5 \times 28.013 = 33.981$

$R = 1545.36 / 33.981 = 45.477$ lbf-ft/lbm-R

$$V = \frac{mRT}{P} = \frac{4 \times 45.477 \times 320}{300 \times 144} \frac{\text{lbf-ft}}{\text{psi} \times \text{in}^2/\text{ft}^2} = \mathbf{1.347 \ ft^3}$$

b) Kay's rule   Eq.12.84

$P_{c \ mix} = 0.5 \times 706 + 0.5 \times 492 = 599$ psia

$T_{c \ mix} = 0.5 \times 271.4 + 0.5 \times 227.2 = 249.3$ R

Reduced properties: $P_r = \frac{300}{599} = 0.50, \quad T_r = \frac{320}{249.3} = 1.284$

Fig. D.1: $Z = 0.92$

$$V = Z \frac{m\bar{R}T}{M_{mix}P} = Z \ V_{ID \ gas} = 0.92 \times 1.347 = \mathbf{1.24 \ ft^3}$$

12.180E

R-410A is a 1:1 mass ratio mixture of R-32 and R-125. Find the specific volume at 80 F, 200 psia using Kays rule and the generalized charts and compare to Table F.9

Kay's rule Eq.12.84

$P_{c\ mix} = 0.5 \times 838 + 0.5 \times 525 = 681.5$ psia

$T_{c\ mix} = 0.5 \times 632.3 + 0.5 \times 610.6 = 621.45$ R

Reduced properties: $P_r = \dfrac{200}{681.5} = 0.293, \quad T_r = \dfrac{539.67}{621.45} = 0.868$

Table F.1:   R = 1545.36 / 72.585 = 21.29 lbf-ft/lbm-R

Fig. D.1:   Z = 0.82

$$v = \frac{ZRT}{P} = \frac{0.82 \times 21.29 \times 539.67}{200 \times 144} \frac{\text{lbf-ft/lbm}}{\text{psi} \times \text{in}^2/\text{ft}^2} = \mathbf{0.327\ ft^3/lbm}$$

Table F.9: $v = \mathbf{0.3174\ ft^3/lbm}$

12.181E

The R-410A in Problem 12.180E flows through a heat exchanger and exits at 280 F, 200 psia. Find the specific heat transfer using Kay's rule and the generalized charts and compare this to solution found using Table F.9.

Kay's rule Eq.12.84

$P_{c \, mix} = 0.5 \times 838 + 0.5 \times 525 = 681.5$ psia

$T_{c \, mix} = 0.5 \times 632.3 + 0.5 \times 610.6 = 621.45$ R

Table F.1: $R = 1545.36 / 72.585 = 21.29$ lbf-ft/lbm-R $= 0.02736$ Btu/lbm-R

$C_{P \, mix} = \sum c_i \, C_{P \, i} = 0.5 \times 0.196 + 0.5 \times 0.189 = 0.1925$ Btu/lbm-R

Reduced properties 1: $P_{r1} = \dfrac{200}{681.5} = 0.293, \quad T_{r1} = \dfrac{539.67}{621.45} = 0.868$

Fig. D.1: $\left(h_1^* - h_1\right) = 0.45 \times RT_c = 0.45 \times 0.02736 \times 621.45 = 7.65$ Btu/lbm

Reduced properties 2: $P_{r2} = \dfrac{200}{681.5} = 0.293, \quad T_{r2} = \dfrac{739.67}{621.45} = 1.19$

Fig. D.1: $\left(h_2^* - h_2\right) = 0.25 \times RT_c = 0.25 \times 0.02736 \times 621.45 = 4.25$ Btu/lbm

The energy equation gives

$$_1q_2 = (h_2 - h_1) = \left(h_2 - h_2^*\right) + \left(h_2^* - h_1^*\right) + \left(h_1^* - h_1\right)$$

$$= -4.25 + 0.1925 \, (280 - 80) + 7.65$$

$$= \textbf{41.9 Btu/lbm mix}$$

Table F.9.2: $q = h_2 - h_1 = 176.26 - 126.34 = \textbf{49.92 Btu/lbm}$

The main difference is in the value of specific heat, about 0.25 Btu/lbm-R at the avg. T, whereas it is 0.1925 Btu/lbm-R at 77 F.

Review Problem

12.182E

A new compound is used in an ideal Rankine cycle where saturated vapor at 400 F enters the turbine and saturated liquid at 70 F exits the condenser. The only properties known for this compound are a molecular mass of 80 lbm/lbmol, an ideal gas heat capacity of $Cpo = 0.20$ Btu/lbm-R and $Tc = 900$ R, $Pc = 750$ psia. Find the specific work input to the pump and the cycle thermal efficiency using the generalized charts.

$T_1 = 400$ F $= 860$ R

$x_1 = 1.0$

$T_3 = 70$ F $= 530$ R

$x_3 = 0.0$

Properties known:

$M = 80$, $C_{PO} = 0.2$ Btu/lbm R

$T_C = 900$ R, $P_C = 750$ lbf/in.^{2}

$T_{r1} = \dfrac{860}{900} = 0.956$, $T_{r3} = \dfrac{530}{900} = 0.589$

From D.1: $P_{r1} = 0.76$, $P_1 = 0.76 \times 750 = 570$ lbf/in.$^2 = P_4$

$P_{r3} = 0.025$, $P_3 = 19$ lbf/in.$^2 = P_2$, $Z_{f3} = 0.0045$

$v_{f3} = Z_{f3}RT_3/P_3 = \dfrac{0.0045 \times 1545 \times 530}{19 \times 144 \times 80} = 0.0168$ ft^3/lbm

$w_P = - \int_3^4 vdP \approx v_{f3}(P_4 - P_3) = -0.0168\,(570 - 19) \times \dfrac{144}{778} = $ **-1.71 Btu/lbm**

$q_H + h_4 = h_1$, but $h_3 = h_4 + w_P$ $\Rightarrow$ $q_H = (h_1 - h_3) + w_P$

From D.2,

$(h_1^* - h_1) = (1.9859/80) \times 900 \times 1.34 = 30.0$ Btu/lbm

$(h_3^* - h_3) = (1.9859/80) \times 900 \times 5.2 = 116.1$ Btu/lbm

$(h_1^* - h_3^*) = C_{P0}(T_1 - T_3) = 0.2(400 - 70) = 66.0$ Btu/lbm

$(h_1 - h_3) = -30.0 + 66.0 + 116.1 = 152.1$ Btu/lbm

$q_H = 152.1 + (-1.71) = 150.4$ Btu/lbm

Turbine, $(s_2 - s_1) = 0 = -(s_2^* - s_2) + (s_2^* - s_1^*) + (s_1^* - s_1)$

From D.3,

$(s_1^* - s_1) = (1.9859/80) \times 1.06 = 0.0263$ Btu/lbm R

$(s_2^* - s_1^*) = 0.20 \ln \dfrac{530}{860} - 0.024\,82 \ln \dfrac{19}{570} = -0.0124$ Btu/lbm R

Substituting,

$(s_2^* - s_2) = -0.0124 + 0.0263 = 0.0139 = (s_2^* - s_{f2}) - x_2 s_{fg2}$

$0.0139 = 0.024\,82 \times 8.77 - x_2 \times 0.024\,82\,(8.77 - 0.075)$

$\Rightarrow x_2 = 0.9444$

$(h_2^* - h_2) = (h_2^* - h_{f2}) - x_2 h_{fg2}$

From D.2,

$h_{fg2} = 0.024\,82 \times 900\,(5.2 - 0.07) = 114.6$ Btu/lbm

$(h_2^* - h_2) = 116.1 - 0.9444 \times 114.6 = 7.9$ Btu/lbm

$w_T = (h_1 - h_2) = -30.0 + 66.0 + 7.9 = 43.9$ Btu/lbm

$\eta_{TH} = \dfrac{w_{NET}}{q_H} = \dfrac{43.9 - 1.7}{150.4} = \mathbf{0.281}$

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Fundamentals of
Thermodynamics

SOLUTION MANUAL
CHAPTER 13

8e

Updated June 2013

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CONTENT CHAPTER 13

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In-Text Concept Questions

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13.a

How many kmol of air are needed to burn 1 kmol of carbon?

Combustion Eq.: $C + O_2 \Rightarrow 1\ CO_2$

One kmol of O_2 is required to form one kmol CO_2. Since air is 21 % O_2, this means 4.76 kmol of air.

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13.b

If I burn 1 kmol of hydrogen H_2 with 6 kmol of air what is the air-fuel ratio on a mole basis and what is the percent theoretical air?

Combustion Eq. stoichiometric:
$$H_2 + \nu_{O2}(O_2 + 3.76\ N_2) \Rightarrow 1\ H_2O + 3.76\ \nu_{O2}\ N_2$$
$$\nu_{O2} = 0.5\ ; \quad (A/F)_S = \nu_{O2} \times (1 + 3.76)\ /\ 1 = 2.38$$

Six kmol of air is: $\quad 1.26\ O_2 + 4.74\ N_2$.

The A/F mole ratio is 6, so the percent theoretical air is
$$\%\,\text{Theoretical air} = \frac{(A/F)_{ac}}{(A/F)_S} \times 100 = \frac{6}{2.38} \times 100 = 252\ \%$$

Hydrogen flames are nearly colorless, but even small traces of other substances can give color.

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13.c

For the 110% theoretical air in Eq.13.8 what is the equivalence ratio? Is that mixture rich or lean?

110% Theoretical air means also $AF = 1.1\ AF_S$

so from the definition in Eq.13.6

$$\Phi = \frac{AF_S}{AF} = \frac{1}{1.10} = 0.909 \qquad \text{a lean mixture}$$

13.d

In most cases combustion products are exhausted above the dew point. Why?

If any water in the products condenses, it will be acidic due to the other gases in the products. There are always minute amounts of unburned or partially burned fuel and intermediate species in the products that can combine with water and create a very corrosive mixture.

Corrosion on a metal surface.

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13.e

How is a fuel enthalpy of combustion connected to its enthalpy of formation?

The enthalpy of combustion of a fuel is the difference in enthalpy of the products and reactants for the combustion involving the fuel, these enthalpies include the various enthalpies of formation.

At reference condition: $\overset{\circ}{H}_{RP} = \overset{\circ}{H}_P - \overset{\circ}{H}_R = \overset{\circ}{H}_P - \bar{h}^0_{f\ fuel\ vap\ or\ liq}$

The value is negative so talk about the heating value as $HV = -\overset{\circ}{H}_{RP}$

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13.f

What is the higher and lower heating value HHV, LHV of n-butane?

The heating value is the negative of the enthalpy of combustion. From Table 13.3, the HHV of gaseous n-butane at $25°C$ is

HHV = 49 500 kJ/kg,

and the corresponding LHV is

LHV = 45 714 kJ/kg.

Notice the table is on a mass basis (per kg fuel).

A butane flame

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15.g

What is the value of h_{fg} for n-octane?

This can be obtained from two places. From Table A.10 we get
$$h_{fg} = (\bar{h}_{f\ vap}^0 - \bar{h}_{f\ liq}^0) / M = [(-208\ 600 - (-250\ 105)] / 114.232 = \textbf{363 kJ/kg}$$

The h_{fg} of a fuel listed in Table 13.3 is the difference between the first two columns in the table (or the third and fourth). For n-octane, this value is

$$h_{fg} = -47\ 893 - (-48\ 256) = \textbf{363 kJ/kg}$$

To see this remember

$$\overset{\circ}{H}_{RP} = \overset{\circ}{H}_P - \overset{\circ}{H}_R = \overset{\circ}{H}_P - \bar{h}_{f\ fuel\ vap\ or\ liq}^0$$

so when we take the difference between fuel as gas and fuel as liquid all other terms will cancel out leaving h_{fg} for the fuel when numbers are on mass basis.

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13.h

What happens to the adiabatic flame temperature if I burn rich or lean?

The higher the percent theoretical air used in combustion (the leaner), the larger the number of moles of products, especially nitrogen, which decreases the adiabatic flame temperature. Burning rich causes incomplete combustion, however, with a smaller release of energy.

Experimentally the highest temperature is reached for slightly rich.

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Heavy molecules show up as yellow. Oxygen diffuses in from the air and the fuel evaporates from the wick. As air mixes in, the flame cools.

13.i

Is the irreversibility in a combustion process significant? Why is that?

A combustion process is highly irreversible with a large increase in entropy. It takes place at a rapid rate, due to large driving forces, and results in stable products of combustion that have little or no tendency to return to their former constituents and states.

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13.j

If the air-fuel ratio is larger than stoichiometric is it more or less reversible?

Less reversible, more irreversible. The excess oxidizer (air) is being heated up, Q over a finite temperature difference is an irreversible process. The same is true for A/F smaller than one where the excess fuel is heated up.

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13.k

What makes the fuel cell attractive from a power-generating point of view?

Fuel cells are attractive for power generation because their direct output is electrical energy. They also have a much higher power density as power per unit volume or power per unit mass and thus can be used in mobile applications.

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Concept-Study Guide Problems

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13.1

Is mass conserved in combustion? Is the number of moles constant?

Yes mass is conserved as we do not consider the quantum physic effect of mass and energy conversion.

Moles are not conserved, what is conserved are the atoms so each atom in the reactants must be found in the products.

13.2

Does all combustion take place with air?

Most combustion takes place with air as the oxidizer, special cases do occur where a different oxidizer is used. To reach very high temperatures a gas welder uses a bottle of pure oxygen (and one with the fuel) instead of air and then the flame can cut through steel.

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13.3

Why would I sometimes need A/F on a mole basis? on a mass basis?

If you want to meter (measure) the fuel and air flows it can be done as a volume flowrate which is proportional to moles ($P\dot{V} = \dot{n}\overline{R}T$) in which case concentrations on a mole basis are needed.

The fuel and air flows can also be measured with a method that measures mass flow rate $\dot{m}$ or if you are filling up tanks to store the fuel and oxidizer as in a rocket in both cases the concentrations on a mass basis are needed.

13.4

Why is there no significant difference between the number of moles of reactants versus products in combustion of hydrocarbon fuels with air?

In most hydrocarbon fuels, there are approximately twice as many hydrogen atoms as carbon atoms, so the numbers of moles of CO_2 and H_2O in the products are roughly equal, the total of which is not too different from the O_2 required in the reactants. The number of excess O_2 is the same in reactants and products. The total number of moles is dominated by the N_2 in each, especially with excess air.

13.5

Why are products measured on a dry basis?

Combustion products have traditionally been measured by passing the gas mixture through a series of solutions that selectively absorb the components one-by-one and measuring the resulting gas volume decreases. The water component is condensed out in these processes, leaving the others – that is, a dry basis. Other and newer instruments measure the concentrations by optical means and these are sensitive to moisture content, which can corrode the surfaces and destroy the sensors. If the water stays in the mixture it typically has to stay hot to prevent condensation at undesirable locations where that would alter the concentrations of the remaining gas mixture components.

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13.6

What is the dew point of hydrogen burned with stoichiometric pure oxygen? air?

For H_2 burned with pure oxygen

$$H_2 + 0.5\,O_2 \quad \Rightarrow \quad 1\,H_2O$$

with the only product as water, so the dew-point at 100 kPa pressure is **99.6°C**.
For H_2 burned with stoichiometric air

$$H_2 + 0.5\,(O_2 + 3.76\,N_2) \quad \Rightarrow \quad 1\,H_2O + 1.88\,N_2$$

the product mixture is water and nitrogen. The partial pressure of the water at a total pressure of 100 kPa is

$$P_v = P_{tot}\,y_v = 100 \times \frac{1}{1 + 1.88} = 34.7 \text{ kPa},$$

corresponding to a dew-point of **72.3°C** .

A hydrogen flame is nearly
colorless. If other species are
present they can give rise to some
color of the flame.

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13.7

How does the dew point change as equivalence ratio goes from 0.9 to 1 to 1.1?

For a given amount of water in the products, the smaller the total number of moles of other gases is (as Φ increases), the higher the partial pressure of the water and therefore the dew-point temperature. As Φ becomes greater than 1.0, there will be incomplete combustion, and the resulting composition will be affected to have some unburned fuel and therefore relative less water. The relative maximum amount of water is then at a stoichiometric mixture $\Phi = 1$ and this is also maximum dew point temperature.

13.8

Why does combustion contribute to global warming?

Any combustion of a hydrocarbon fuel produces carbon dioxide, which in the atmosphere is a contributing factor to global warming. Carbon dioxide absorbs radiation over a wide spectrum and thus heats the atmosphere. This is not just man-made, but nature has forest fires and volcanic action that liberate gases into the atmosphere.

Natural gas, mainly methane CH_4, has the highest hydrogen to carbon ratio of the hydrocarbons we find on earth (very little hydrogen can be found).

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13.9

What is the enthalpy of formation for oxygen as O_2? If O? For CO_2?

The enthalpy of formation of O_2 is zero, by choice of the reference base. Relative to this base, the value for the monatomic form O is

$$\bar{h}^{\circ}_{f\,O} = +249\ 170 \text{ kJ/kmol (Table A.9)},$$

and the value for CO_2 is

$$\bar{h}^{\circ}_{f\,CO2} = -393\ 522 \text{ kJ/kmol (Table A.9 or A.10).}$$

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13.10

If the nitrogen content of air could be lowered will the adiabatic flame temperature increase or decrease?

Since that would reduce the number of nitrogen molecules in the products it will increase the adiabatic flame temperature, more energy per molecule.

Energy Eq.: $\Delta H_P = HV + \Delta H_R = $ given

$$\Delta H_P = \nu_{CO_2} \Delta \bar{h}_{CO_2} + \nu_{H_2O} \Delta \bar{h}_{H_2O} + \nu_{N_2} \Delta \bar{h}_{N_2} \quad \text{at Tad}$$

so a lower ν_{N_2} means the $\Delta \bar{h}$'s must be larger.

13.11

Does the enthalpy of combustion depend on AF?

NO.

$H_{RP} = H_P - H_R$ depends on T, P in general.

Assuming ideal gas it does not depend on P.

It is assumed the number is scaled to 1 kmol or 1 kg of fuel burned.

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13.12

Why do some fuels not have entries for liquid fuel in Table 13.3?

Those fuels cannot exist as liquids at 25°C (above their critical temperature).

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13.13

Is a heating value a fixed number for a fuel?

Yes.

The heating value is the negative of the enthalpy of combustion and is only useful if it is a well defined number. It is fixed, but there are normally 4 values quoted as the **higher** (water liquid) and **lower** (water vapor) **heating value** for the fuel as a liquid or vapor if applicable. It should also be clear whether it is the enthalpies or the internal energies that are used, i.e. do you have a flow or a non flow situation. So these qualifiers must accompany the number for it to be unique.

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13.14

Is an adiabatic flame temperature a fixed number for a fuel?

Qualified **Yes.**

It is a single number if we assume reactants are supplied at the reference conditions in a stoichiometric ratio and we know the phase of the fuel (liquid or gas) and the type of oxidizer (air or pure oxygen). Also you need to know if the combustion takes place at constant pressure (the most common) or it may take place at constant volume (unusual). If anyone of these conditions is changed the adiabatic flame temperature is changed. This is less fixed than the heating value, more variables can change the adiabatic flame temperature, say, adding an inert gas will lower the flame temperature, but keep the heating value the same. Chemical equilibrium reactions and incomplete combustion makes the real flame temperature lower than the theoretical adiabatic flame temperature.

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13.15

Does it make a difference for the enthalpy of combustion whether I burn with pure oxygen or air? What about the adiabatic flame temperature?

No difference in the enthalpy of combustion – the nitrogen in the air is the same in the reactants and products, and its enthalpy cancels out (nitrogen and oxygen formation enthalpy are both zero).

The adiabatic flame temperature is much lower for combustion with air, because a significant part of the energy release from the combustion goes into heating the nitrogen (as well as the other products) to the flame temperature.

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13.16

A welder uses a bottle with acetylene and a bottle with oxygen. Why does he use oxygen instead of air?

The oxygen in the air comes with nitrogen that is also being heated up to the flame temperature. The heating value is being shared among all the product components so when they increase (having the nitrogen also) the adiabatic flame temperature drops correspondingly.

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13.17

Some gas-welding is done using bottles of fuel, oxygen and argon. Why do you think argon is used?

Argon is used to shield the welding spot from the oxygen in the surrounding air. If oxygen is on the surface of the liquid metal it may form metal-oxides creating a weak location in the weld.

13.18

Is combustion a reversible process?

NO. It is a highly irreversible process.

13.19

Is combustion with more than 100% theoretical air more or less reversible?

It is **less** reversible. Any deviation from a stoichiometric mixture (lean or rich) will increase the irreversibility. You may also think of the adiabatic flame temperature which is highest for a stoichiometric mixture (like adding the heating value at that T). So with any other mixture the adiabatic flame temperature is lower so the heating value is added at a lower T (a more irreversible process).

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Fuels and the Combustion Process

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13.20

In a picnic grill gaseous propane is fed to a burner together with stoichiometric air. Find the air-fuel ratio on a mass basis and the total amount of reactant mass for 1 kg propane burned.

The reaction equation for stoichiometric mixture is:

$$C_3H_8 + \nu_{O2}\,(O_2 + 3.76\,N_2) \;\rightarrow\; a\,H_2O + b\,CO_2 + c\,N_2$$

C balance: $3 = b$;

H balance: $8 = 2a$ $\Rightarrow$ $a = 4$

O balance: $2\,\nu_{O2} = a + 2b = 4 + 2 \times 3$ $\Rightarrow$ $\nu_{O2} = 5$

N$_2$ balance: $3.76\,\nu_{O2} = c = 18.8$

The reaction is

$$C_3H_8 + 5\,(O_2 + 3.76\,N_2) \;\rightarrow\; 4\,H_2O + 3\,CO_2 + 18.8\,N_2$$

$$AF_{mol} = \frac{5 + 18.8}{1} = 23.8$$

$$AF_{mass} = AF_{mol}\,\frac{M_{air}}{M_{fuel}} = 23.8 \times \frac{28.97}{44.097} = \mathbf{15.64}$$

Total mass of reactants $= 1 + 15.64 = \mathbf{16.64\ kg}$

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13.21

A mixture of fuels is E85, which is 85% ethanol and 15% gasoline (assume octane) by mass. Find the A/F ratio on a mass basis for stoichiometric combustion.

Since the fuel mixture is specified on a mass basis we need to find the mole fractions for the combustion equation. From Eq.13.4 we get

$$y_{ethanol} = (0.85/46.069) / [0.85/46.069 + 0.15/114.232] = 0.93356$$

$$y_{octane} = 1 - y_{ethanol} = 0.06644$$

The reaction equation is

$$0.93356\,C_2H_5OH + 0.06644\,C_8H_{18} + \nu_{O2}\,(O_2 + 3.76\,N_2)$$

$$\rightarrow \nu_{CO2}\,CO_2 + \nu_{H2O}H_2O + 3.76\,\nu_{O2}\,N_2$$

C balance: $2 \times 0.93356 + 8 \times 0.06644 = \nu_{CO2} = 2.39864$

H balance: $6 \times 0.93356 + 18 \times 0.06644 = 2\nu_{H2O} = 6.79728;\ \nu_{H2O} = 3.39864$

O balance: $0.93356 + 2\,\nu_{O2} = 2\,\nu_{CO2} + \nu_{H2O} = 8.19592 \Rightarrow \nu_{O2} = 3.63118$

The A/F ratio is

$$A/F = 4.76\,\nu_{O2} \times 28.97 / [0.93356 \times 46.069 + 0.06644 \times 114.232]$$

$$= \mathbf{9.896}$$

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13.22

Calculate the theoretical air-fuel ratio on a mass and mole basis for the combustion of ethanol, C_2H_5OH.

Reaction Eq.: $C_2H_5OH + \nu_{O2}(O_2 + 3.76N_2) \Rightarrow aCO_2 + bH_2O + cN_2$

Do the atom balance

Balance C: $2 = a$

Balance H: $6 = 2b \quad \Rightarrow \quad b = 3$

Balance O: $1 + 2\nu_{O2} = 2a + b = 4 + 3 = 7 \qquad \Rightarrow \quad \nu_{O2} = 3$

$(A/F)_{mol} = \nu_{O2}(1 + 3.76)/1 = 3 \times 4.76 = \mathbf{14.28}$

$(A/F)_{mass} = (\nu_{O2}M_{O2} + \nu_{N2} M_{N2})/M_{Fuel}$

$\qquad = (3 \times 31.999 + 11.28 \times 28.013) / 46.069 = \mathbf{8.943}$

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13.23
Methane is burned with 200% theoretical air. Find the composition and the dew point of the products.

The reaction equation for stoichiometric mixture is:

$$CH_4 + v_{O2} (O_2 + 3.76 \, N_2) \rightarrow a \, H_2O + b \, CO_2 + c \, N_2$$

C balance: $1 = b$;   H balance: $4 = 2a$

O balance: $2 \, v_{O2} = a + 2b = 2 + 2 \times 1$ $=> \quad v_{O2} = 2$

N_2 balance: $3.76 \, v_{O2} = c = 7.52$

200% theoretical air: $v_{O2} = 2 \times 2 = 4$ so now more O_2 and N_2

$$CH_4 + v_{O2} (O_2 + 3.76 \, N_2) \rightarrow a \, H_2O + b \, CO_2 + c \, N_2 + d \, O_2$$

N_2 balance: $3.76 \, v_{O2} = c = 15.04$

Extra oxygen: $d = 4 - 1 - 1 = 2$

Products: $2 \, H_2O + 1 \, CO_2 + 15.04 \, N_2 + 2 \, O_2$

Water vapor mole fraction: $y_v = \dfrac{2}{1 + 2 + 2 + 15.04} = 0.0998$

Partial water vapor pressure: $P_v = y_v \, P_o = 0.0998 \times 101 = 9.98$ kPa

$$P_g(T_{dew}) = P_v = 9.98 \text{ kPa} \quad \Rightarrow \quad T_{dew} = \textbf{45.8}°\textbf{C}$$

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13.24

Natural gas B from Table 13.2 is burned with 20% excess air. Determine the composition of the products.

The reaction equation (stoichiometric and complete combustion) with the fuel composition is:

$$60.1\ CH_4 + 14.8\ C_2H_6 + 13.4\ C_3H_8 + 4.2\ C_4H_{10} + 7.5\ N_2$$

$$+\ v_{O2}\ (O_2 + 3.76\ N_2) \rightarrow a\ H_2O + b\ CO_2 + c\ N_2$$

C balance: $60.1 + 2 \times 14.8 + 3 \times 13.4 + 4 \times 4.2 = b = 146.7$

H balance: $4 \times 60.1 + 6 \times 14.8 + 8 \times 13.4 + 10 \times 4.2 = 2a = 478.4 \Rightarrow a = 239.2$

O balance: $2\ v_{O2} = a + 2b = 239.2 + 2 \times 146.7 \Rightarrow v_{O2} = 266.3$

N$_2$ balance: $7.5 + 3.76\ v_{O2} = c = 1008.8$

20% excess air: $v_{O2} = 1.2 \times 266.3 = 319.56$ so now more O_2 and N_2

Extra oxygen: $d = 319.56 - 266.3 = 53.26$, $c = 7.5 + 3.76 \times 319.56 = 1209$

Products: $239.2\ H_2O + 146.7\ CO_2 + 1209\ N_2 + 53.26\ O_2$

To the expert the color of the flame can tell about the composition. It can tell about specific gases present if they have distinct color emission.

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13.25

A certain fuel oil has the composition $C_{10}H_{22}$. If this fuel is burned with 150% theoretical air, what is the composition of the products of combustion?

$$C_{10}H_{22} + (1/\phi)\, \nu_{O_2}\, (O_2 + 3.76\, N_2) \rightarrow a\, H_2O + b\, CO_2 + c\, N_2 + d\, O_2$$

Stoichiometric combustion: $\phi = 1$, $d = 0$,

   C balance: $b = 10$

   H balance: $a = 22/2 = 11$,

   O balance: $2\, \nu_{O_2} = a + 2b = 11 + 20 = 31$ $=>$ $\nu_{O2} = 15.5$

Actual case: $1/\phi = 1.5$ $=>$ $\nu_{O2} = 1.5 \times 15.5 = 23.25$

   H balance: $a = 11$,   C balance: $b = 10$,

   N balance: $c = 23.25 \times 3.76 = 87.42$

   O_2 balance: $d = 23.25 - 10 - 11/2 = 7.75$ (excess oxygen)

The combustion process is

$$C_{10}H_{22} + 23.25\, (O_2 + 3.76\, N_2)$$
$$\rightarrow 11\, H_2O + 10\, CO_2 + 87.42\, N_2 + 7.75\, O_2$$

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13.26

A Pennsylvania coal contains 74.2% C, 5.1% H, 6.7% O, (dry basis, mass percent) plus ash and small percentages of N and S. This coal is fed into a gasifier along with oxygen and steam, as shown in Fig. P13.26. The exiting product gas composition is measured on a mole basis to: 39.9% CO, 30.8% H_2, 11.4% CO_2, 16.4% H_2O plus small percentages of CH_4, N_2, and H_2S. How many kilograms of coal are required to produce 100 kmol of product gas? How much oxygen and steam are required?

Convert the mass concentrations to number of kmol per 100 kg coal:

C : $n = 74.2/12.01 = 6.178$ H_2: $n = 5.1/2.016 = 2.530$

O_2: $n = 6.7/31.999 = 0.209$

Now the combustion equation reads

$$x(6.178\, C + 2.53\, H_2 + 0.209\, O_2) + y\, H_2O + z\, O_2 \text{ in and}$$

$$39.9\, CO + 30.8\, H_2 + 11.4\, CO_2 + 16.4\, H_2O \text{ out}$$

in 100 kmol of mix out

Now we can do the atom balance to find (x, y, z)

C balance: $6.178\, x = 39.9 + 11.4$ $\rightarrow$ $x = 8.304$

H_2 balance: $2.53 \times 8.304 + y = 30.8 + 16.4$ $\rightarrow$ $y = 26.191$

O_2 balance: $0.209 \times 8.304 + \dfrac{26.191}{2} + z = \dfrac{39.9}{2} + 11.4 + \dfrac{16.4}{2}$ $\rightarrow z = 24.719$

Therefore, for 100 kmol of mixture out

require: **830.4 kg** of coal

26.191 kmol of steam

24.719 kmol of oxygen

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13.27

For complete stoichiometric combustion of gasoline, C_7H_{17}, determine the fuel molecular weight, the combustion products, and the mass of carbon dioxide produced per kg of fuel burned.

Stoichiometric combustion:

$$C_7H_{17} + \nu_{O_2}\left(O_2 + 3.76\,N_2\right) \;\Rightarrow\; a\,H_2O + b\,CO_2 + c\,N_2$$

C balance: $\qquad 7 = b$

H balance: $\qquad 17 = 2a, \qquad \Rightarrow \quad a = 8.5$

O balance: $\qquad 2\,\nu_{O_2} = a + 2b = 8.5 + 14 = 22.5 \quad \Rightarrow \quad \nu_{O_2} = 11.25$

N balance: $\qquad c = 3.76\ \nu_{O_2} = 3.76 \times 11.25 = 42.3$

$$M_{FUEL} = 7\,M_C + 17\,M_H = 7 \times 12.011 + 17 \times 1.008 = \mathbf{101.213}$$

$$\frac{m_{CO2}}{m_{FUEL}} = \frac{7\,M_{CO2}}{M_{FUEL}} = \frac{7 \times 44.01}{101.213} = \textbf{3.044 kg CO}_2 \textbf{ per kg fuel}$$

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13.28

 A sample of pine bark has the following ultimate analysis on a dry basis, percent by mass: 5.6% H, 53.4% C, 0.1% S, 0.1% N, 37.9% O and 2.9% ash. This bark will be used as a fuel by burning it with 100% theoretical air in a furnace. Determine the air–fuel ratio on a mass basis.

 Converting the Bark Analysis from a mass basis:

Substance	S	H_2	C	O_2	N_2
c/M =	0.1/32	5.6/2	53.4/12	37.9/32	0.1/28
kmol / 100 kg coal	0.003	2.80	4.45	1.184	0.004
Product	SO_2	H_2O	CO_2		
oxygen required	0.003	1.40	4.45	--	--

Combustion requires: $0.003 + 1.40 + 4.45 = 5.853$ kmol O_2 there is in the bark 1.184 kmol O_2 so the net from air is 4.669 kmol O_2

$$AF = (4.669 + 4.669 \times 3.76) \times \frac{28.97}{100} = \mathbf{6.44} \, \frac{\textbf{kg air}}{\textbf{kg bark}}$$

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13.29

Liquid propane is burned with dry air. A volumetric analysis of the products of combustion yields the following volume percent composition on a dry basis: 8.6% CO_2, 0.6% CO, 7.2% O_2 and 83.6% N_2. Determine the percent of theoretical air used in this combustion process.

$$a\,C_3H_8 + b\,O_2 + c\,N_2 \rightarrow 8.6\,CO_2 + 0.6\,CO + d\,H_2O + 7.2\,O_2 + 83.6\,N_2$$

C balance: $3a = 8.6 + 0.6 = 9.2 \Rightarrow a = 3.067$

H_2 balance: $4a = d \Rightarrow d = 12.267$

N_2 balance: $c = 83.6$

O_2 balance: $b = 8.6 + \dfrac{0.6}{2} + \dfrac{12.267}{2} + 7.2 = 22.234$

Air-Fuel ratio $= \dfrac{22.234 + 83.6}{3.067} = 34.51$

Theoretical:

$$C_3H_8 + 5\,O_2 + 18.8\,N_2 \rightarrow 3\,CO_2 + 4\,H_2O + 18.8\,N_2$$

$$\Rightarrow \text{theo. A-F ratio} = \frac{5 + 18.8}{1} = 23.8$$

$$\% \text{ theoretical air} = \frac{34.51}{23.8} \times 100\,\% = \mathbf{145\,\%}$$

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13.30

The coal gasifier in an integrated gasification combined cycle (IGCC) power plant produces a gas mixture with the following volumetric percent composition:

Product	CH_4	H_2	CO	CO_2	N_2	H_2O	H_2S	NH_3
% vol.	0.3	29.6	41.0	10.0	0.8	17.0	1.1	0.2

This gas is cooled to 40°C, 3 MPa, and the H_2S and NH_3 are removed in water scrubbers. Assuming that the resulting mixture, which is sent to the combustors, is saturated with water, determine the mixture composition and the theoretical air–fuel ratio in the combustors.

CH_4	H_2	CO	CO_2	N_2	n
0.3	29.6	41.0	10.0	0.8	81.7

$$y_{H2O} = \frac{n_v}{n_v + 81.7}, \quad \text{where} \quad n_v = \text{number of moles of water vapor}$$

Cool to 40°C $P_G = 7.384$, $P = 3000$ kPa

$$y_{H2O\ MAX} = \frac{7.384}{3000} = \frac{n_v}{n_v + 81.7} \quad \rightarrow \quad n_v = 0.2016$$

a) Mixture composition:

CH_4	H_2	CO	CO_2	N_2	$H_2O(v)$
0.3 kmol	29.6	41.0	10.0	0.8	0.2016

81.9016 kmol (from 100 kmol of the original gas mixture)

$$0.3\, CH_4 + 0.6\, O_2 \rightarrow 0.3\, CO_2 + 0.6\, H_2O$$

$$29.6\, H_2 + 14.8\, O_2 \rightarrow 29.6\, H_2O$$

$$41\, CO + 20.5\, O_2 \rightarrow 41\, CO_2$$

$$\Rightarrow \text{Number of moles of } O_2 = 0.6 + 14.8 + 20.5 = 35.9$$

Number of moles of air = $35.9 + 3.76 \times 35.9$ (N_2)

$$A/F = \frac{28.97(35.9 + 3.76(35.9))}{0.3(16) + 29.6(2) + 41(28) + 10(44) + 0.8(28) + 0.2016(18)}$$

$$= \textbf{2.95 kg air/kg fuel}$$

Borgnakke and Sonntag

13.31

In a combustion process with decane, $C_{10}H_{22}$, and air, the dry product mole fractions are 83.61% N_2, 4.91% O_2, 10.56% CO_2 and 0.92% CO. Find the equivalence ratio and the percent theoretical air of the reactants.

$$x\, C_{10}H_{22} + (1/\phi)\, \nu_{O_2}\, (O_2 + 3.76\, N_2) \rightarrow a\, H_2O + b\, CO_2 + c\, CO + d\, N_2 + e\, O_2$$

Stoichiometric combustion: $\phi = 1$, $c = 0$, $e = 0$,

C balance: $b = 10x$

H balance: $a = 22x/2 = 11x$,

O balance: $2\,\nu_{O_2} = a + 2b = 11x + 20x = 31x$

$\nu_{O_2} = 15.5x$, $\nu_{N_2} = 58.28x$ $\Rightarrow$ $(A/F)_s = (\nu_{O_2} + \nu_{N_2})/x = \mathbf{73.78}$

Actual combustion: $d = 83.61$ $\rightarrow$

N balance: $(1/\phi)\,\nu_{O_2} \times 3.76 = 83.61$ $\rightarrow (1/\phi)\,\nu_{O_2} = 22.24$

C balance: $10x = 10.56 + 0.92 = 11.48$ $\Rightarrow$ $x = 1.148$

$(A/F)_{ac} = (1/\phi)\,\nu_{O_2} \times 4.76/1.148 = \mathbf{92.215}$

$\phi = (F/A)_{ac} / (F/A)_s = (A/F)_s / (A/F)_{ac} = 73.78 / 92.215 = \mathbf{0.80}$ **or** $\phi = \mathbf{0.8}$

Percent theoretical air $= 100\,(1/\phi) = \mathbf{125\%}$

Borgnakke and Sonntag

13.32

The output gas mixture of a certain air–blown coal gasifier has the composition of producer gas as listed in Table 13.2. Consider the combustion of this gas with 120% theoretical air at 100 kPa pressure. Determine the dew point of the products and find how many kilograms of water will be condensed per kilogram of fuel if the products are cooled 10°C below the dew-point temperature.

$$\{3\,CH_4 + 14\,H_2 + 50.9\,N_2 + 0.6\,O_2 + 27\,CO + 4.5\,CO_2\}$$

$$+\, 31.1\,O_2 + 116.9\,N_2 \rightarrow 34.5\,CO_2 + 20\,H_2O + 5.2\,O_2 + 167.8\,N_2$$

Products:

$$y_{H2O} = y_{H2O\ MAX} = P_G/100 = \frac{20}{34.5 + 20 + 5.2 + 167.8}$$

$$\Rightarrow P_G = 8.79\ kPa \quad \rightarrow \quad T_{DEW\ PT} = \textbf{43.2°C}$$

At $T = 33.2°C$, $P_G = 5.13\ kPa$

$$y_{H2O\ max} = \frac{5.13}{100} = \frac{n_{H2O}}{n_{H2O}+34.5+5.2+167.8} \quad \rightarrow \quad n_{H2O} = 11.22$$

$$m_{H2O\ LIQ} = \frac{8.78(18)}{3(16) +14(2) +50.9(28) +0.6(32) +27(28) +4.5(44)} = \textbf{0.0639 kg/kg fuel}$$

13.33

Methanol, CH_3OH, is burned with 200% theoretical air in an engine and the products are brought to 100 kPa, 30°C. How much water is condensed per kilogram of fuel?

$$CH_3OH + \nu_{O_2}\{O_2 + 3.76\ N_2\} \rightarrow CO_2 + 2\ H_2O + 3.76\ \nu_{O_2}N_2$$

Stoichiometric $\quad \nu_{O_2\ S} = 1.5 \quad \Rightarrow \quad \nu_{O_2\ AC} = 3$

Actual products: $CO_2 + 2\ H_2O + 1.5\ O_2 + 11.28\ N_2$

$P_{sat}(30°C) = 4.246$ kPa

$$\Rightarrow y_{H2O} = 0.04246 = \frac{\nu_{H2O}}{1 + \nu_{H2O} + 1.5 + 11.28}$$

$$\Rightarrow \nu_{H2O} = 0.611 \quad \Rightarrow \quad \Delta\nu_{H2O\ cond} = 2 - 0.611 = 1.389$$

$M_{Fu} = 32.042 \qquad \dfrac{\Delta M_{H2O}}{M_{Fu}} = \dfrac{1.389 \times 18}{32.042} = \mathbf{0.781}\ \dfrac{\mathbf{kg\ H_2O}}{\mathbf{kg\ fuel}}$

Borgnakke and Sonntag

13.34

Butane is burned with dry air at 40°C, 100 kPa with AF = 22 on a mass basis. For complete combustion find the equivalence ratio, % theoretical air and the dew point of the products. How much water (kg/kg fuel) is condensed out, if any, when the products are cooled down to ambient temperature, 40°C?

Solution:

$$C_4H_{10} + \nu_{O_2}\{O_2 + 3.76\ N_2\} \rightarrow 4\ CO_2 + 5\ H_2O + 3.76\ \nu_{O_2}N_2$$

Stoichiometric $\quad \nu_{O_2\ S} = 4 + 5/2 = 6.5; \qquad 3.76\ \nu_{O_2} = 24.44$

$$(A/F)_S = 6.5(31.999 + 3.76 \times 28.013)/58.124 = 15.3574$$

Actual: $\qquad \nu_{O_2ac} = \dfrac{(A/F)_{ac}}{(A/F)_s}\ \nu_{O_2\ S} = \dfrac{22}{15.3574}\ 6.5 = 9.31$

$$\text{\% Theoretical air} = \frac{22}{15.3574}\ 100 = \textbf{143.3\%}$$

Equivalence ratio $\quad \Phi = 1/1.4325 = \textbf{0.7}$

Actual products: $\qquad 4\ CO_2 + 5\ H_2O + 2.81\ O_2 + 35.0\ N_2$

The water partial pressure becomes

$$P_v = y_v\ P_{tot} = \frac{5}{4 + 5 + 2.81 + 35.0}\ 100 = 10.68\ \text{kPa}$$

$$T_{dew} = \textbf{46.9}^{\textbf{o}}\textbf{C}$$

$$P_{g\ 40} = 7.348\ \text{kPa} \quad \Rightarrow \quad y_{v\ max} = \frac{7.384}{100} = \frac{\nu_{H2O}}{4 + \nu_{H2O} + 2.81 + 35.0}$$

Solve for $\nu_{H2O\ vap}$:

$\nu_{H2O\ vap} = 3.333 \quad$ still vapor,

$\nu_{H2O\ LIQ} = 5 - 3.333 = 1.667 \quad$ is liquid

$$\frac{m_{H2O\ LIQ}}{m_{Fuel}} = \frac{1.667 \times 18.015}{58.124} = \textbf{0.517}$$

Borgnakke and Sonntag

13.35

The hot exhaust gas from an internal combustion engine is analyzed and found to have the following percent composition on a volumetric basis at the engine exhaust manifold. 10% CO_2, 2% CO, 13% H_2O, 3% O_2 and 72% N_2. This gas is fed to an exhaust gas reactor and mixed with a certain amount of air to eliminate the carbon monoxide, as shown in Fig. P13.35. It has been determined that a mole fraction of 10% oxygen in the mixture at state 3 will ensure that no CO remains. What must the ratio of flows be entering the reactor?

Exhaust gas at state 1: CO_2 10 %, H_2O 13%,
　　　　　　　　　　　CO 2%, O_2 3%, N_2 72%

Exhaust gas at state 3:   CO = 0 %, O_2 = 10 %

```
1 Exh. gas              3 gas
  ┼──►┌────────┐──►
     │ Reactor │  out
  ┼──►└────────┘
2 Air
```

Reaction equation for the carbon monoxide

$$\Rightarrow 0.02\,CO + x\,O_2 + 3.76x\,N_2 \rightarrow 0.02\,CO_2 + (x\text{-}0.01)\,O_2 + 3.76x\,N_2$$

At 3:　　　$\nu_{CO_2} = 0.10 + 0.02 = 0.12,$　　　　　$\nu_{H_2O} = 0.13$

　　　　　　$\nu_{O_2} = (x\text{-}0.01) + 0.03 = x + 0.02$　　$\nu_{N_2} = 0.72 + 3.76x$

or

　　　　　　$n_{TOT} = 0.12 + 0.13 + x + 0.02 + 0.72 + 3.76x = 0.99 + 4.76x$

$y_{O2} = 0.10 = \dfrac{x + 0.02}{0.99 + 4.76x}$　　　$\rightarrow$　　$x = 0.151$

or　　$\dfrac{\text{air } 2}{\text{Exh. Gas } 1} = \dfrac{4.76x}{1} = \mathbf{0.718}\ \dfrac{\textbf{kmol air}}{\textbf{kmol Exh. gas}}$

Borgnakke and Sonntag

Energy Equation, Enthalpy of Formation

Borgnakke and Sonntag

13.36

Acetylene gas C_2H_2 is burned with stoichiometric air in a torch. The reactants are supplied at the reference conditions Po, To. The products come out from the flame at 2500 K after some heat loss by radiation. Find the heat loss per kmol fuel.

The reaction equation is

$$C_2H_2 + 2.5 \, (O_2 + 3.76 \, N_2) \;\rightarrow\; 2 \, CO_2 + 1 \, H_2O + 9.4 \, N_2$$

Energy Eq.: $H_P + Q_{out} = H_R = H_P^o + \Delta H_P + Q_{out} = H_R^o$

$$\Delta H_P = 2 \, \Delta \bar{h}_{CO2} + \Delta \bar{h}_{H2O} + 9.4 \, \Delta \bar{h}_{N2}$$
$$= 2 \times 121 \, 926.5 + 99 \, 130.5 + 9.4 \times 74 \, 301.5 = 1 \, 041 \, 418 \text{ kJ/kmol}$$
$$Q_{out} = H_R^o - H_P^o - \Delta H_P = \bar{h}_{f \, fuel}^{\circ} - \bar{h}_{f \, H2O}^{\circ} - 2 \, \bar{h}_{f \, CO_2}^{\circ} - \Delta H_P$$
$$Q_{out} = 226 \, 731 - (-241 \, 826) - 2(-393 \, 522) - 1 \, 041 \, 418$$
$$= \textbf{214 183 kJ/kmol fuel}$$

Borgnakke and Sonntag

13.37

        Hydrogen is burned with stoichiometric air in a steady flow process where the reactants are supplied at 100 kPa, 298 K. The products are cooled to 800 K in a heat exchanger. Find the heat transfer per kmol hydrogen.

        The combustion equation is

$$H_2 +\ 0.5 \times [\ O_2 + 3.76\,N_2\ ]\ \rightarrow 1\,H_2O + 1.88\,N_2$$

The reactants are at 298 K

$$H_R = 0 + 0 + 0 = 0$$

The products are at 800 K so water is gaseous and from A.9

$$H_P = 1[-241\ 826 + 18\ 002)]\ +\ 1.88\ (0 + 15\ 046)\ = -195\ 538\ kJ/kmol\ fuel$$

$$Q_{CV} = H_P - H_R = \textbf{-195 538 kJ/kmol fuel}$$

Borgnakke and Sonntag

13.38

Natural gas (methane) is burned with stoichiometric air, reactants supplied at reference P_o, T_o in a steady flow burner. The products come out at 800 K, if the burner should deliver 10 kW what is the need flowrate of natural gas in kg/s?

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2} \left(O_2 + 3.76\,N_2\right) \rightarrow 2\,H_2O + 1\,CO_2 + c\,N_2$$

O balance: $2\,\nu_{O2} = 2 + 2 \quad \Rightarrow \quad \nu_{O2} = 2 \quad \Rightarrow \quad c = 2 \times 3.76 = 7.52$

The products are cooled to 800 K (so we do not consider condensation) and the energy equation is

Energy Eq.: $H_R + Q = H_P = H_P^\circ + \Delta H_P = H_R^\circ + Q$

$$Q = H_P^\circ + \Delta H_P - H_R^\circ = 2\,\bar{h}_{f\,H_2O}^\circ + \bar{h}_{f\,CO_2}^\circ + \Delta H_P - \bar{h}_{f\,fuel}^\circ$$

$$\Delta H_P = 2\,\Delta\bar{h}_{H2O} + \Delta\bar{h}_{CO2} + 7.52\,\Delta\bar{h}_{N2}$$
$$= 2 \times 18\,002 + 22\,806 + 7.52 \times 15\,046 = 171\,956$$
$$Q = 2(-241\,826) - 393\,522 + 171\,956 - (-74\,873)$$
$$= -630\,345 \text{ kJ/kmol fuel}$$

$$\dot{Q} = \dot{n}\,Q_{out} = Q_{out}\,\dot{m}/M \qquad \text{so}$$

$$\dot{m} = \dot{Q}M/Q_{out} = 10 \text{ kW} \times 16.043 \text{ (kg/kmol)} / 630\,345 \text{ kJ/kmol}$$
$$= \mathbf{0.0002545\ kg/s} = 0.916 \text{ kg/h}$$

Borgnakke and Sonntag

13.39

Butane gas and 200% theoretical air, both at 25°C, enter a steady flow combustor. The products of combustion exits at 1000 K. Calculate the heat transfer from the combustor per kmol of butane burned.

$$C_4H_{10} + (1/\phi)\,\nu_{O2}(O_2 + 3.76\,N_2) \rightarrow a\,CO_2 + b\,H_2O + c\,N_2 + d\,O_2$$

First we need to find the stoichiometric air ($\phi = 1$, $d = 0$)

C balance: $4 = a$, \qquad H balance: $10 = 2b \Rightarrow b = 5$

O balance: $2\nu_{O2} = 2a + b = 8 + 5 = 13 \Rightarrow \nu_{O2} = 6.5$

Now we can do the actual air: $(1/\phi) = 2 \Rightarrow \nu_{O2} = 2 \times 6.5 = 13$

N balance: $c = 3.76\,\nu_{O2} = 48.88$, \quad O balance: $d = 13 - 6.5 = 6.5$

Energy Eq.: \qquad $q = H_R - H_P = H_R^o - H_P^o - \Delta H_P$

Table A.10: $H_R^o = -126\,200 + 0 + 0 = -126\,200$ kJ/kmol fuel

$H_P^o = 4\,(-393\,522) + 5(-241\,826) + 0 + 0 = -2\,783\,218$ kJ/kmol fuel

The rest of the values are from Table A.9 at 1000 K

$\Delta\bar{h}_{CO2} = 33397$, \quad $\Delta\bar{h}_{N2} = 21463$, \quad $\Delta\bar{h}_{O2} = 22703$, \quad $\Delta\bar{h}_{H2O} = 26000$ kJ/kmol

$\Delta H_P = 4 \times 33\,397 + 5 \times 26\,000 + 48.88 \times 21\,463 + 6.5 \times 22\,703$

$= 1\,460\,269$ kJ/kmol fuel

From the energy equation we get

$q = -126\,200 - (-2\,783\,218) - 1\,460\,269 =$ **1 196 749 kJ/kmol butane**

Borgnakke and Sonntag

13.40

One alternative to using petroleum or natural gas as fuels is ethanol (C_2H_5OH), which is commonly produced from grain by fermentation. Consider a combustion process in which liquid ethanol is burned with 110% theoretical air in a steady flow process. The reactants enter the combustion chamber at 25°C, and the products exit at 60°C, 100 kPa. Calculate the heat transfer per kilomole of ethanol.

$$C_2H_5OH + 1.1 \times 3 \,(O_2 + 3.76\,N_2) \rightarrow 2CO_2 + 3H_2O + 0.3O_2 + 12.408N_2$$

Fuel: $\bar{h}_f^0$ = -277 380 kJ/kmol for liquid from Table A.10,

Products at 60°C, 100 kPa, check for condensation of water

$$y_{H2O\ MIX} = \frac{19.94}{100} = \frac{n_{V\ MAX}}{n_{V\ MAX} + 2 + 0.3 + 12.408}$$

$$\Rightarrow \quad n_{V\ MAX} = 3.66 \; > 3 \quad \Rightarrow \; \text{No liquid}$$

H_R = 1(-277 380) + 0 + 0 = -277 380 kJ/kmol fuel

H_P = 2(-393 522 + 1373) + 3(-241 826 + 1185)

$\qquad$ + 0.3(0 + 1039.5) + 12.408(0 + 1021) = -1 493 241 kJ/kmol fuel

Q_{CV} = H_P - H_R = **-1 215 860 kJ/kmol fuel**

Borgnakke and Sonntag

13.41

Do the previous problem with the ethanol fuel delivered as a vapor.
One alternative to using petroleum or natural gas as fuels is ethanol (C_2H_5OH), which is commonly produced from grain by fermentation. Consider a combustion process in which liquid ethanol is burned with 120% theoretical air in a steady flow process. The reactants enter the combustion chamber at 25°C, and the products exit at 60°C, 100 kPa. Calculate the heat transfer per kilomole of ethanol.

$$C_2H_5OH + 1.1 \times 3 \ (O_2 + 3.76 \ N_2) \quad \rightarrow \quad 2CO_2 + 3H_2O + 0.3O_2 + 12.408N_2$$

Fuel: $\bar{h}_f^0$ = -235 000 kJ/kmol for IG from Table A.10

Products at 60°C, 100 kPa, check for condensation of water

$$y_{H2O \ MIX} = \frac{19.94}{100} = \frac{n_{V \ MAX}}{n_{V \ MAX} + 2 + 0.3 + 12.408}$$

$$\Rightarrow \ n_{V \ MAX} = 3.66 \ > 3 \quad \Rightarrow \ \text{No liquid}$$

The mixture can then hold 3.66 mol of vapor > 3 present $\quad \Rightarrow$ No liquid.

H_R = 1(-235 000) + 0 + 0 = -235 000 kJ/kmol fuel

H_P = 2(-393 522 + 1373) + 3(-241 826 + 1185)

$\qquad$ + 0.3(0 + 1039.5) + 12.408(0 + 1021) = -1 493 241 kJ/kmol fuel

$Q_{CV} = H_P - H_R =$ **-1 258 241 kJ/kmol fuel**

Borgnakke and Sonntag

13.42

As an alternative fuel consider liquid methanol burned with stoichiometric air both supplied at P_0, T_0 in a constant pressure process exhausting the products at 900 K. What is the heat transfer per kmol of fuel.

$$CH_3OH + 1.5\,(O_2 + 3.76\,N_2) \rightarrow 1\,CO_2 + 2\,H_2O + 5.64\,N_2$$

Reactants at 25°C, products are at 900 K, 100 kPa.

CH_3OH: $\bar{h}_f^o = -239\,220$ kJ/kmol from table A.10 for the liquid state

$H_R = 1\,\bar{h}_{LIQ} = -239\,220$ kJ/kmol fuel

$H_P = 1(-393\,522 + 28\,030) + 2(-241\,826 + 21\,937) + 5.64(18\,223)$

$\quad = -702\,492$ kJ/kmol fuel

$Q = H_P - H_R = -702\,492 - (-239\,220) = $ **–463 272 kJ/kmol fuel**

Borgnakke and Sonntag

13.43

The combustion of heptane C_7H_{16} takes place in a steady flow burner where fuel and air are added as gases at P_0, T_0. The mixture has 125% theoretical air and the products are going through a heat exchanger where they are cooled to 600 K. Find the heat transfer from the heat exchanger per kmol of heptane burned.

The reaction equation for stoichiometric ratio is:

$$C_7H_{16} + v_{O2} (O_2 + 3.76 N_2) \Rightarrow 7CO_2 + 8 H_2O + v_{O2} \times 3.76 N_2$$

So the balance (C and H was done in equation) of oxygen gives

$$v_{O2} = 7 + 4 = 11, \qquad v_{O2\ ac} = 1.25 \times 11 = 13.75.$$

Now the actual reaction equation is:

$$C_7H_{16} + 13.75 O_2 + 51.7 N_2 \Rightarrow 7CO_2 + 8 H_2O + 51.7 N_2 + 2.75 O_2$$

To find the heat transfer, take a control volume as combustion chamber and heat exchanger

$$H_R + Q = H_P \quad \Rightarrow \quad Q = H_P^o + \Delta H_P - H_R^o$$

Take the enthalpies from Tables A.9 for the products and A.10 for the fuel

$$Q = 7(-393\ 522 + 12\ 906) + 8 (-241\ 826 + 10\ 499) + 51.7(8894)$$
$$+ 2.75(9245) - (-187\ 900)$$
$$= \textbf{- 3 841 784 kJ/kmol fuel}$$

Borgnakke and Sonntag

13.44

Pentene, C_5H_{10} is burned with stoichiometric pure oxygen in a steady flow process. After giving some heat out the products are at 700 K and used in a heat exchanger, where they are cooled to 35°C. Find the specific heat transfer in the heat exchanger.

$$C_5H_{10} + \nu_{O_2}O_2 \rightarrow 5\ CO_2 + 5\ H_2O \qquad \Rightarrow \qquad \nu_{O_2} = \textbf{7.5}$$

Check for condensation amount

Find x: $y_{H_2O\ max} = \dfrac{P_g(35°)}{P_{tot}} = 0.0555 = \dfrac{x}{5+x} \qquad \Rightarrow \qquad x = \textbf{0.2938}$

Out of the 5 H_2O only 0.2938 are still vapor at the 35°C exit temperature. The heat exchanger cools the products so the energy equation is

$$5\ \dot{n}_F\ \bar{h}_{CO_2} + 5\ \dot{n}_F\ \bar{h}_{H_2O} + \dot{Q} = 5\ \dot{n}_F\ \bar{h}_{CO_2}$$

$$+ (5 - x)\ \dot{n}_F\ \bar{h}_{liq\ H_2O} + (x)\ \dot{n}_F\ \bar{h}_{vap\ H_2O}$$

Find the enthalpy at 700 K all gas A.9 and at 35°C = 308.15 K part of water is a gas in table A.9 and use B.1.1 and A.10 for the liquid water.

$$\bar{h}_{liq\ H_2O} = \bar{h}^°_{f\ liq} + \Delta\bar{h}_{308} = \text{-285 830} + 18.015(146.66 - 104.87) = -285\ 077$$

$$\bar{h}_{vap\ H_2O} = \bar{h}^°_{f\ vap} + \Delta\bar{h} = -241\ 826 + \Delta\bar{h}$$

$$\frac{\dot{Q}}{\dot{n}_F} = 5\ (\Delta\bar{h}_{308} - \Delta\bar{h}_{700})_{CO_2} + (5-x)[\ \bar{h}_{liq\ H_2O} - (\bar{h}^°_{f\ vap} + \Delta\bar{h}_{700})]\ _{H_2O}$$

$$+ x\ [\bar{h}^°_{f\ vap} + \Delta\bar{h}_{308} - (\bar{h}^°_{f\ vap} + \Delta\bar{h}_{700})]\ _{H_2O\ vapor}$$

$$= 5(389.6 - 17\ 754) + 4.706\ [-285\ 077 - (-241\ 826 + 14\ 190)]$$

$$+ 0.2938\ [(-241\ 826 + 338.1) - (-241\ 826 + 14\ 190)]$$

$$= -\textbf{361 209 kJ/kmol Fu}$$

Borgnakke and Sonntag

13.45

Another alternative fuel to be seriously considered is hydrogen. It can be produced from water by various techniques that are under extensive study. Its biggest problem at the present time are cost, storage, and safety. Repeat Problem 13.40 using hydrogen gas as the fuel instead of ethanol.

$$H_2 + 1.1 \times 0.5\,O_2 + 1.1 \times 3.76 \times 0.5\,N_2 \rightarrow 1\,H_2O + 0.05\,O_2 + 2.068\,N_2$$

Products at 60°C, 100 kPa, check for condensation of water

$$y_{H2O\ MAX} = \frac{19.94}{100} = \frac{n_{V\ MAX}}{n_{V\ MAX} + 0.05 + 2.068}$$

Solving, $n_{V\ MAX} = 0.5275 < 1 \Rightarrow n_V = 0.5275, \quad n_{LIQ} = 0.4725$

$$H_R = 0 + 0 + 0 = 0$$

Notice the products are at 60°C so add for water liquid from steam tables

$$H_P = 0.4725\,[\text{-285 830} + 18.015\,(251.11 - 104.87)]$$

$$+ 0.5275\,(\text{-241 826} + 1185) + 0.05(0 + 1039.5) + 2.068(0 + 1021)$$

$$= \text{-258 585 kJ}$$

$$Q_{CV} = H_P - H_R = \textbf{-258 585 kJ/kmol fuel}$$

Borgnakke and Sonntag

13.46

In a new high-efficiency furnace, natural gas, assumed to be 90% methane and 10% ethane (by volume) and 110% theoretical air each enter at 25°C, 100 kPa, and the products (assumed to be 100% gaseous) exit the furnace at 40°C, 100 kPa. What is the heat transfer for this process? Compare this to an older furnace where the products exit at 250°C, 100 kPa.

$$0.9\,CH_4 + 0.1\,C_2H_6 + 1.1 \times 2.15\,O_2 + 3.76 \times 2.365\,N_2$$

$$\rightarrow 1.1\,CO_2 + 2.1\,H_2O + 0.215\,O_2 + 8.892\,N_2$$

Fuel values from table A.10 and the rest from Table A.9

$H_R = 0.9(\text{-74 873}) + 0.1(\text{-84 740}) = \text{-75 860 kJ/kmol fuel}$

$H_P = 1.1(\text{-393 522} + 586) + 2.1(\text{-241 826} + 507.5) + 0.215(445)$

$\qquad + 8.892(437.6)$

$\quad = \text{-935 012 kJ/kmol fuel}$ assuming all gas

$Q_{CV} = H_P - H_R = $ **-859 152 kJ/kmol fuel**

b) $T_P = 250\ ^oC$

$\quad H_P = 1.1(\text{-393 522} + 9370) + 2.1(\text{-241 826} + 7750)$

$\qquad + 0.215(6817) + 8.892(6602) = \text{-853 956 kJ}$

$\quad Q_{CV} = H_P - H_R = $ **-778 096 kJ/kmol fuel**

Borgnakke and Sonntag

13.47

Repeat the previous problem, but take into account the actual phase behavior of the products exiting the furnace.

$$0.9\ CH_4 + 0.1\ C_2H_6 + 1.1 \times 2.15\ O_2 + 3.76 \times 2.365\ N_2$$
$$\rightarrow 1.1\ CO_2 + 2.1\ H_2O + 0.215\ O_2 + 8.892\ N_2$$

Same as 15.41, except check products for saturation at 40°C, 100 kPa

$$y_{v\ max} = \frac{7.384}{100} = \frac{n_{V\ MAX}}{n_{V\ MAX}+10.207} \quad \Rightarrow \quad \text{Solving, } n_{v\ max} = 0.814$$

$$n_V = 0.814, \quad n_{liq} = 2.1 - 0.814 = 1.286$$

Fuel values from table A.10 and the rest from Table A.9

$$H_R = 0.9(\text{-74 873}) + 0.1(\text{-84 740}) = \text{-75 860 kJ/kmol fuel}$$

For the liquid water add difference (40°C – 25°C) from steam tables

$$H_{liq} = 1.286\big[\text{-285 830} + 18.015(167.6 - 104.9)\big] = \text{-366 125 kJ/kmol fuel}$$

$$H_{GAS} = 1.1(\text{-393 522} + 586) + 0.814(\text{-241 826} + 507.5)$$
$$+ 0.215(445) + 8.892(437.6) = \text{-624 676 kJ/kmol fuel}$$

$$Q_{CV} = H_P - H_R = (\text{-366 125} - \text{624 676}) + 75\ 860$$
$$= \textbf{-914 941 kJ/kmol fuel}$$

b) $T_P = 250\ ^oC$

$$H_P = 1.1(\text{-393 522} + 9370) + 2.1(\text{-241 826} + 7750)$$
$$+ 0.215(6817) + 8.892(6602)$$
$$= \text{-853 956 kJ/kmol}$$

$$Q_{CV} = H_P - H_R = \textbf{-778 096 kJ/kmol fuel}$$

Borgnakke and Sonntag

13.48

Methane, CH_4, is burned in a steady adiabatic flow process with two different oxidizers: Case **A**:   Pure oxygen, O_2 and case **B**: A mixture of $O_2 + x$ Ar. The reactants are supplied at T_0, P_0 and the products for both cases should be at 2100 K. Find the required equivalence ratio in case (A) and the amount of Argon, x, for a stoichiometric ratio in case (B).

a) Stoichiometric has $\nu = 2$, actual has:

$$CH_4 + \nu O_2 \rightarrow CO_2 + 2H_2O + (\nu - 2)O_2$$

Energy eq.: $H_R^\circ = H_P^\circ + \Delta H_{P\ 2100}$

$$\Delta H_{P\ 2100} = H_R^\circ - H_P^\circ = \bar{h}_{f\ fuel}^{\,o} + 0 - \bar{h}_{f\ CO_2}^{\,o} - 2\bar{h}_{f\ H_2O}^{\,o} - 0$$

RHS $= -74\ 873 - (-393\ 522) - 2(-241\ 826) = 802\ 301$ kJ/kmol

$\Delta\bar{h}_{CO_2} = 97\ 500$, $\Delta\bar{h}_{H_2O} = 77\ 970$, $\Delta\bar{h}_{O_2} = 62\ 973$ all in kJ/kmol

$$\Delta H_{P\ 2100} = \Delta\bar{h}_{CO_2} + 2\Delta\bar{h}_{H_2O} + (\nu - 2)\,\Delta\bar{h}_{O_2}$$

$$= 97\ 500 + 2 \times 77\ 970 + (\nu - 2) \times 62\ 973$$

$$= 127\ 494 + \nu\ 62\ 973 = 802\ 301 \text{ kJ/kmol fuel}$$

$$\Rightarrow \quad \mathbf{\nu = 10.72}, \quad \Phi = \frac{AF_S}{AF} = \frac{2}{10.72} = \mathbf{0.1866}$$

b) $CH_4 + 2\ O_2 + 2x\ Ar \rightarrow CO_2 + 2H_2O + 2x\ Ar$

$$\Delta\bar{h}_{Ar} = \bar{C}_{P\ Ar}\ \Delta T = C_{P\ Ar}\ M_{Ar}\ \Delta T \quad \text{from table A.5}$$

$$\Delta H_{P\ 2100} = 97\ 500 + 2 \times 77\ 970$$

$$+ 2x \times 0.52 \times 39.948 \times (2100 - 298) = 253\ 440 + x\ 74\ 866$$

Now the energy equation becomes

$$802\ 301 = 253\ 440 + x\ 74\ 866 \quad \Rightarrow \quad x = \mathbf{7.33}$$

Borgnakke and Sonntag

13.49

A rigid container has a 1:1 mole ratio of propane and butane gas together with a stoichiometric ratio of air at P_O, T_O. The charge burns and there is heat transfer to a final temperature of 1000 K. Find the final pressure and the heat transfer per kmol of fuel mixture.

$$C_3H_8 + C_4H_{10} + \nu_{O_2}(O_2 + 3.76\ N_2) \rightarrow 7\ CO_2 + 9\ H_2O + 3.76\ \nu_{O_2}\ N_2$$

The carbon and hydrogen balance has been used so oxygen gives

$$\nu_{O_2} = 7 + 9/2 = 11.5\ ;\quad \nu_{N_2} = 43.24$$

The energy equation is

$$U_P - U_R = Q - 0 = (H_P - H_R) - n_P\bar{R}T_P + n_R\bar{R}T_R$$

$$= H_P^o + \Delta H_P - H_R^o - n_P\bar{R}T_P + n_R\bar{R}T_R$$

$$n_R = 1 + 1 + 11.5 \times 4.76 = 56.74,\qquad n_P = 7 + 9 + 43.24 = 59.24$$

From Table A.9

$$\Delta H_P = 7\ (33\ 397) + 9\ (26\ 000) + 43.24\ (21\ 463) = 1\ 395\ 839\ kJ$$

From Table 13.3 (convert kJ/kg to kJ/kmol with M)

$$H_P^o - H_R^o = 44.094\ (-46\ 352) + 58.124\ (-45\ 714) = -\ 4\ 700\ 926\ kJ$$

Substitute into energy equation

$$Q = -\ 4\ 700\ 926 + 1\ 395\ 839 - 8.3145(59.24 \times 1000 - 56.74 \times 298.15)$$

$$= -\ 3\ 656\ 981\ kJ/2\ kmol\ fuel$$

$$= \textbf{-\ 1\ 828\ 490\ kJ/kmol\ fuel}$$

Borgnakke and Sonntag

13.50

A closed, insulated container is charged with a stoichiometric ratio of oxygen and hydrogen at 25°C and 150 kPa. After combustion, liquid water at 25°C is sprayed in such that the final temperature is 1200 K. What is the final pressure?

Combustion reaction: $\qquad$ $H_2 + \frac{1}{2}(O_2) \rightarrow H_2O$

Products: $\qquad$ $1\ H_2O + x_i H_2O$

Energy eq. per 1 kmol hydrogen (remember flow in):

$$U_2 - U_1 = x_i \bar{h}_i = x_i \bar{h}^{\circ}_{f\ liq} = (1 + x_i)H_P - H_R - (1 + x_i)\bar{R}T_P + \frac{3}{2}\bar{R}T_R$$

Solve for x_i: $\qquad$ $x_i\ [\ \bar{h}^{\circ}_{f\ liq} - H_P + \bar{R}T_P\ ] = H_P - H_R - \bar{R}T_P + \frac{3}{2}\bar{R}T_R$

From Table A.9: $\quad H_R = 0,\quad H_P = -241\ 826 + 34\ 506 = -207\ 320$ kJ/kmol

From Table A.10: $\bar{h}^{\circ}_{f\ liq} = -285\ 830$ kJ/kmol

Substitute into the energy equation

$$x_i\ (-285830 + 207320 + 8.3145 \times 1200) =$$

$$-207\ 320 - 8.3145\left(\ 1200 - \frac{3}{2}\times298.15\right) = -213\ 579$$

$$x_i = 3.116$$

Volume is constant: $\qquad$ $P_1V_1 = n_R\bar{R}T_1,\quad P_2V_1 = n_p\bar{R}T_p$

$n_R = 1 + \frac{1}{2} = 1.5\ ;\qquad n_p = (1 + x_i)$

$$P_2 = \frac{P_1(1 + x_i)T_P}{\frac{3}{2}(T_1)} = \frac{150 \times 4.116 \times 1200}{\frac{3}{2} \times 298.15} = \textbf{1657 kPa}$$

13.51

In a gas turbine natural gas (methane) and stoichiometric air flows into the combustion chamber at 1000 kPa, 500 K. Secondary air, see Fig.P13.51, also at 1000 kPa, 500 K is added right after the combustion to result in a product mixture temperature of 1500 K. Find the A/F ratio mass basis for the primary reactant flow and the ratio of the secondary air to the primary air (mass flow rates ratio).

$$CH_4 + \nu(O_2 + 3.76\ N_2) \rightarrow$$
$$CO_2 + 2H_2O + 3.76\ \nu\ N_2$$

O balance: $\nu = 2, \quad \nu_{N2} = 7.52$

For the primary flow: $(A/F)_{mass} = \dfrac{2 \times 32 + 7.52 \times 28.013}{16.043} = \mathbf{17.12}$

Do a C.V. around the entire set-up, then the energy equation becomes

Energy Eq.: $H_{R1} + H_{R2} = H_{P\ 1500} = \overset{\circ}{H_P} + \Delta H_{P\ 1500}$

$$H_{R1} = \overset{\circ}{H_R} + \Delta H_{R1} = \bar{h}^o_{f\ fuel} + \Delta H_{Fuel} + \Delta H_{air1}$$

$\Delta H_{air1} = 2(\Delta H_{O2} + 3.76\ \Delta H_{N2}) = 2(6086 + 3.76 \times 5911) = 56\ 623$ kJ/kmol

$H_{R2} = 0 + \Delta H_{air2} = \nu_{O2\ add}\ (\Delta H_{O2} + 3.76\ \Delta H_{N2}) = \nu_{O2\ add}\ 28\ 311$ kJ/kmol

$\Delta H_{Fuel} \approx M\ C_P\ \Delta T = 16.043 \times 2.254\ (500 - 298) = 7304.5$ kJ/kmol

$\bar{h}^o_{f\ fuel} - \overset{\circ}{H_P} = HV = 16.043 \times 50\ 010 = 802\ 310$ kJ/kmol

$\Delta H_{P\ 1500} = \Delta\bar{h}_{CO_2} + 2\Delta\bar{h}_{H_2O} + 7.52\ \Delta\bar{h}_{N_2} + \nu_{O2\ add}\ (\Delta\bar{h}_{O_2} + 3.76\ \Delta\bar{h}_{N_2})$

$= 61\ 705 + 2 \times 48\ 149 + 7.52 \times 38\ 405 + \nu_{O2\ add}\ (40\ 600$

$+ 3.76 \times 38\ 405) = 446\ 809 + \nu_{O2\ add}\ 185\ 003$ kJ/kmol

Now substitute everything into energy equation

$802\ 310 + 7304.5 + 56\ 623 + \nu_{O2\ add}\ 28\ 311 = 446\ 809 + \nu_{O2\ add}\ 185\ 003$

$$\nu_{O2\ add} = 2.677$$
$$m_{air2}/m_{air1} = n_{air2}/n_{air1} = \nu_{O2\ add}/\nu = 2.677/2 = \mathbf{1.34}$$

Borgnakke and Sonntag

13.52

A rigid vessel initially contains 2 kmol of carbon and 2 kmol of oxygen at 25°C, 200 kPa. Combustion occurs, and the resulting products consist of 1 kmol of carbon dioxide, 1 kmol of carbon monoxide, and excess oxygen at a temperature of 1000 K. Determine the final pressure in the vessel and the heat transfer from the vessel during the process.

$$2\,C + 2\,O_2 \ \to 1\,CO_2 + 1\,CO + \tfrac{1}{2}O_2$$

Process   V = constant,   C: solid, $n_{1(GAS)} = 2$, $n_{2(GAS)} = 2.5$

$$P_2 = P_1 \times \frac{n_2 T_2}{n_1 T_1} = 200 \times \frac{2.5 \times 1000}{2 \times 298.2} = \textbf{838.4 kPa}$$

$H_1 = 0$ and we neglect the Pv for carbon when doing the U_1.

$H_2 = 1(-393\ 522 + 33\ 397) + 1(-110\ 527 + 21\ 686)$

$\qquad\qquad + (1/2)(0 + 22\ 703) = \textbf{-437 615 kJ}$

Now find heat transfer from energy equation

$$_1Q_2 = (U_2 - U_1) = (H_2 - H_1) - n_2\bar{R}T_2 + n_1\bar{R}T_1$$

$\qquad = (-437\ 615 - 0) - 8.3145(2.5 \times 1000 - 2 \times 298.2)$

$\qquad = \textbf{-453 442 kJ}$

Borgnakke and Sonntag

13.53

Gaseous propane mixes with air, both supplied at 500 K, 0.1 MPa. The mixture goes into a combustion chamber and products of combustion exit at 1300 K, 0.1 MPa. The products analyzed on a dry basis are 11.42% CO_2, 0.79% CO, 2.68% O_2, and 85.11% N_2 on a volume basis. Find the equivalence ratio and the heat transfer per kmol of fuel.

$$C_3H_8 + \alpha\ O_2 + 3.76\ \alpha\ N_2 \rightarrow \beta\ CO_2 + \gamma\ H_2O + 3.76\ \alpha\ N_2$$

$$\beta = 3, \quad \gamma = 4, \quad \alpha = \beta + \gamma/2 = 5, \quad (A/F)_S = 4.76\alpha = 23.8$$

The actual combustion reaction is

$$xC_3H_8 + \alpha\ O_2 + 3.76\ \alpha\ N_2 \rightarrow$$

$$11.42\ CO_2 + y\ H_2O + 85.11\ N_2 + 0.79\ CO + 2.68\ O_2$$

C balance: $\quad 3x = 11.42 + 0.79 \quad => \quad x = 4.07$

H balance: $\quad 8x = 2y \quad\quad\quad\quad => \quad y = 4x = 16.28$

O balance: $\quad 2\alpha = 2 \times 11.42 + y + 0.79 + 2 \times 2.68 = 45.27 \quad => \quad \alpha = 22.635$

N balance: $\quad 3.76\ \alpha = 85.11 \quad => \quad \alpha = 22.6356 \quad$ checks close enough

Rescale the equation by dividing with x to give

$$C_3H_8 + 5.5614\ (O_2 + 3.76\ N_2) \rightarrow$$

$$2.806\ CO_2 + 4\ H_2O + 20.91\ N_2 + 0.194\ CO + 0.6584\ O_2$$

$$A/F = 5.5614\ (1 + 3.76) / 1 = 26.472$$

$$\phi = (A/F)_S/(A/F) = 23.8 / 26.472 = \mathbf{0.899}, \quad \%\text{Theo. air} = 1/\phi = \mathbf{111\%}$$

$$q = h_P - h_R = h_P^o + \sum v_i \Delta h(1300\ K) - h_R$$

$$h_R = h_{f\ fuel}^o + \Delta h_{fuel} + 5.5614\ \Delta h_{O2} + 20.91\ \Delta h_{N2}$$

$$= -103\ 900 + 1.679 \times 44.094\ (500 - 298) + 5.5614\ (6086)$$

$$+ 20.91\ (5911) = 68\ 500\ \text{kJ/kmol fuel}$$

$$h_P = h_P^o + \sum v_i \Delta h(1300\ K)$$

$$= 2.806\ (-393\ 522 + 50\ 148) + 4(-241\ 826 + 38\ 941) + 20.91\ (0$$

$$+ 31\ 503) + 0.194\ (-110\ 527 + 31\ 867) + 0.6584\ (0 + 33\ 345)$$

$$= -1\ 109\ 625\ \text{kJ/kmol fuel}$$

$$q = -1\ 109\ 625 - 68\ 500 = \mathbf{-1\ 178\ 125\ kJ/kmol\ fuel}$$

Borgnakke and Sonntag

Enthalpy of Combustion and Heating Value

Borgnakke and Sonntag

13.54

Find the enthalpy of combustion and the heating value for pure carbon.

Reaction: $\quad C + \nu_{O_2}[\,O_2 + 3.76\,N_2\,] \rightarrow \; CO_2 + \nu_{O_2}3.76\,N_2$

oxygen balance: $\quad \nu_{O_2} = 1$

$\overset{\circ}{H}_{RP} = H_P^o - H_R^o = \; \bar{h}_{f\,CO_2}^{\circ} - \bar{h}_{f\,C}^{\circ} = -393\;522 - 0$

$\quad\quad = \mathbf{-393\;522\;kJ/kmol} = \mathbf{-32\;791\;kJ/kg} \quad\quad (M = 12.011)$

$HV = 393\;522\;kJ/kmol = \mathbf{32\;791\;kJ/kg}$

Borgnakke and Sonntag

13.55

Phenol has an entry in Table 13.3, but it does not have a corresponding value of the enthalpy of formation in Table A.10. Can you calculate it?

$$C_6H_5OH + \nu_{O_2}[\, O_2 + 3.76\, N_2 \,] \rightarrow \; 3\, H_2O + 6\, CO_2 + 3.76\nu_{O_2}\, N_2$$

The C and H balance was introduced (6 C's and 6 H's). At the reference condition the oxygen and nitrogen have zero enthalpy of formation.

Energy Eq.: $\quad H_P = H_R = H_P^o = H_R^o \quad$ since ref. T is assumed.

$$H_{RP}^{\circ} = H_P - H_R = H_P^o - H_R^o = \; 3\, \bar{h}_{f\,H_2O}^{\circ} + 6\, \bar{h}_{f\,CO_2}^{\circ} - \bar{h}_{f\,fuel}^{\circ}$$

Table 13.3 is on a mass basis and let us chose liquid fuel, so we get the molecular weight from the composition

$$M = 6 \times 12.011 + 3 \times 2.016 \; + 16 = 94.114$$

$$H_{RP}^{\circ} = 94.114\,(-31\,117) = -2\,928\,545 \text{ kJ/kmol}$$

Solve the energy equation for fuel formation enthalpy

$$\bar{h}_{f\,fuel}^{\circ} = \; 3\, \bar{h}_{f\,H_2O}^{\circ} + 6\, \bar{h}_{f\,CO_2}^{\circ} - H_{RP}^{\circ}$$

$$= \; 3\,(-241\,826) + 6(-393\,522) - (-2\,928\,545)$$

$$= -\textbf{158 065 kJ/kmol}$$

For fuel as vapor we get

$$H_{RP}^{\circ} = 94.114\,(-31\,774) = -2\,990\,378 \text{ kJ/kmol}$$

$$\bar{h}_{f\,fuel}^{\circ} = \; 3\, \bar{h}_{f\,H_2O}^{\circ} + 6\, \bar{h}_{f\,CO_2}^{\circ} - H_{RP}^{\circ}$$

$$= \; 3\,(-241\,826) + 6(-393\,522) - (-2\,990\,378)$$

$$= -\textbf{96 232 kJ/kmol}$$

Notice if I took liquid water in products to do H_{RP}° then I must use liquid value for $\bar{h}_{f\,H_2O}^{\circ} = -285\,830$ kJ/kmol and the final result is the same.

Borgnakke and Sonntag

13.56

Acetylene gas, C_2H_2, is burned with stoichiometric air in a torch. The reactants are supplied at the reference conditions P_o, T_o. The products come out from the flame at 2800 K after a heat loss by radiation. Find the lower heating value for the fuel, as it is not listed in Table 13.3, and the heat loss per kmol of fuel.

The reaction equation is

$$C_2H_2 + 2.5\ (O_2 + 3.76\ N_2)\ \rightarrow\ 2\ CO_2 + 1\ H_2O + 9.4\ N_2$$

Definition of the heating value

$$LHV = -\overset{\circ}{H}_{RP} = H_R^o - H_P^o = \overset{\circ}{\bar{h}}_{f\ fuel} - \overset{\circ}{\bar{h}}_{f\ H_2O} - 2\ \overset{\circ}{\bar{h}}_{f\ CO_2}$$
$$= 226\ 731 - (-241\ 826) - 2(-393\ 522)$$
$$= 1\ 255\ 601\ kJ/kmol = \mathbf{48\ 222\ kJ/kg}$$

Energy Eq.:
$$H_P + Q_{out} = H_R = H_P^o + \Delta H_P + Q_{out} = H_R^o$$
$$Q_{out} = H_R^o - H_P^o - \Delta H_P = LHV - \Delta H_P$$

$$\Delta H_P = 2 \times 140\ 435 + 115\ 463 + 9.4 \times 85\ 323 = 1\ 198\ 369\ kJ/kmol$$
$$Q_{out} = 1\ 255\ 601 - 1\ 198\ 369 = \mathbf{57\ 232\ kJ/kmol\ fuel}$$

Borgnakke and Sonntag

13.57

Some type of wood can be characterized as $C_1H_{1.5}O_{0.7}$ with a lower heating value of 19 500 kJ/kg. Find its formation enthalpy.

$$C_1H_{1.5}O_{0.7} + \nu_{O_2}[\, O_2 + 3.76\, N_2\,] \rightarrow \quad CO_2 + 0.75\, H_2O + 3.76\nu_{O_2}\, N_2$$

O balance: $0.7 + 2\nu_{O_2} = 2 + 0.75$ $\Rightarrow$ $\nu_{O_2} = 1.025,\ \nu_{N2} = 3.854$

$$M = 1 \times 12.0111 + 1.5 \times 1.008 + 0.7 \times 16 = 24.712$$

$$H^{\circ}_{RP} = H^{o}_{P} - H^{o}_{R} = -M \times HV = -24.712 \times 19\,500 = -481\,884 \text{ kJ/kmol}$$

$$= 0.75\, \bar{h}^{\circ}_{f\,H_2O} + 1\, \bar{h}^{\circ}_{f\,CO_2} - \bar{h}^{\circ}_{f\,fuel}$$

$$\bar{h}^{\circ}_{f\,fuel} = 0.75\, \bar{h}^{\circ}_{f\,H_2O} + 1\, \bar{h}^{\circ}_{f\,CO_2} + M \times HV$$

$$= 0.75\,(-241\,826) + (-393\,522) + 481\,884$$

$$= \textbf{-93 008 kJ/kmol}$$

Borgnakke and Sonntag

13.58

Do problem 13.40 using table 13.3 instead of Table A.10 for the solution.
One alternative to using petroleum or natural gas as fuels is ethanol (C_2H_5OH), which is commonly produced from grain by fermentation. Consider a combustion process in which liquid ethanol is burned with 120% theoretical air in a steady flow process. The reactants enter the combustion chamber at 25°C, and the products exit at 60°C, 100 kPa. Calculate the heat transfer per kilomole of ethanol.

$$C_2H_5OH + 1.1 \times 3\ (O_2 + 3.76\ N_2)\ \rightarrow\ 2\ CO_2 + 3\ H_2O + 0.3\ O_2 + 12.408\ N_2$$

Products at 60°C, 100 kPa, so check for condensation of water

$$y_{H2O\ MIX} = \frac{19.94}{100} = \frac{n_{V\ MAX}}{n_{V\ MAX} + 2 + 0.3 + 12.408}$$

$$\Rightarrow\ n_{V\ MAX} = 3.66\ > 3\ \Rightarrow\ \text{No liquid}$$

Fuel: table 13.3 select (liquid fuel, water vapor) and convert to mole basis

$$\overset{\circ}{H}_{RP} = 46.069\ (\text{-26 811}) = \text{-1 235 156 kJ/kmol}$$

Since the reactants enter at the reference state the energy equation becomes

$$Q_{CV} = H_P - H_R = H_P^o + \Delta H_P - H_R^o = \overset{\circ}{H}_{RP} + \Delta H_P$$

$$\Delta H_P =\ 2\ \Delta\bar{h}_{CO_2} + 3\ \Delta\bar{h}_{H_2O} + 0.3\ \Delta\bar{h}_{O_2} + 12.408\ \Delta\bar{h}_{N_2}$$

$$= 2(1373) + 3(1185) + 0.3(1039.5) + 12.408(1021) = 19\ 281\ \text{kJ/kmol}$$

$$Q_{CV} = \text{-1 235 156} + 19\ 281 = \textbf{-1 215 874 kJ/kmol fuel}$$

Borgnakke and Sonntag

13.59

Agriculturally derived butanol, $C_4H_{10}O$, molecular mass 74.12, also called bio-butanol has a lower heating value LHV = 33 075 kJ/kg for liquid fuel. Find its formation enthalpy.

The reaction equation for stiochiometric ratio is:

$$C_4H_{10}O \; + \; \nu_{O2} \, (O_2 + 3.76 \, N_2) \quad => \; 4CO_2 + 5 \, H_2O + \nu_{O2} \times 3.76 \, N_2$$

where the carbon and hydrogen balance have been done. Now the oxygen

$$\text{O:} \quad 1 + 2\nu_{O2} = 4 \times 2 + 5 \quad => \quad \nu_{O2} = 6; \quad \nu_{N2} = 22.56$$

The lower heating value per kmol fuel is

$$LHV = - \, \overset{\circ}{H}_{RP} = H_R^0 - H_P^0 = \bar{h}^{\circ}_{f \, fuel} - 4 \, \bar{h}^{\circ}_{f \, CO_2} - 5 \, \bar{h}^{\circ}_{f \, H_2O}$$

so the fuel formation enthalpy becomes

$$\bar{h}^{\circ}_{f \, fuel} = LHV + 4 \, \bar{h}^{\circ}_{f \, CO_2} + 5 \, \bar{h}^{\circ}_{f \, H_2O}$$

$$= 74.12 \times 33 \, 075 + 4 \, (-393 \, 522 \,) + 5(-241 \, 826)$$

$$= 2 \, 451 \, 519 - 1 \, 574 \, 088 - 1 \, 209 \, 130$$

$$= \; \textbf{–331 699 kJ/kmol}$$

Borgnakke and Sonntag

13.60

In a picnic grill, gaseous propane and stoichiometric air are mixed and fed to the burner both at ambient P_o and T_o. After combustion the products cool down and at some point exit at 500 K. How much heat transfer was given out for 1 kg of propane?

The reaction equation for stoichiometric mixture is:

$$C_3H_8 + \nu_{O2}\,(O_2 + 3.76\,N_2) \;\rightarrow\; a\,H_2O + b\,CO_2 + c\,N_2$$

C balance: $3 = b$;

H balance: $8 = 2a \quad \Rightarrow \quad a = 4$

O balance: $2\,\nu_{O2} = a + 2b = 4 + 2 \times 3$

$\qquad\qquad \Rightarrow \quad \nu_{O2} = 5$

N_2 balance: $3.76\,\nu_{O2} = c = 18.8$

$A/F = 5 \times 4.76 \times 28.97 / 44.094 = 15.637$

The reaction is

$$C_3H_8 + 5\,(O_2 + 3.76\,N_2) \;\rightarrow\; 4\,H_2O + 3\,CO_2 + 18.8\,N_2$$

All the water is in the vapor phase (500 K products).

Energy Eq.: $H_R = Q_{out} + H_P \quad \Rightarrow \quad H_R^o = Q_{out} + H_P^o + \Delta H_P$

A9: $\Delta H_P = 4 \times 6922 + 3 \times 8305 + 18.8 \times 5911 = 174\,682$ kJ/kmol fuel

$\qquad = (174\,682 / 44.094)$ kJ/kg $= 3961.6$ kJ/kg

Table 13.3: $-H_{RP}^{\circ} = HV = 46\,352$ kJ/kg

$\qquad Q_{out} = H_R^o - H_P^o - \Delta H_P = -H_{RP}^{\circ} - \Delta H_P = HV - \Delta H_P$

$\qquad\qquad = (46\,352 - 3961.6)$ kJ/kg fuel

$\qquad\qquad = \mathbf{42\,390}$ **kJ/kg fuel**

Borgnakke and Sonntag

13.61

Propylbenzene, C_9H_{12}, is listed in Table 13.3, but not in table A.10. No molecular weight is listed in the book. Find the molecular weight, the enthalpy of formation for the liquid fuel and the enthalpy of evaporation.

$$C_9H_{12} + 12\ O_2 \rightarrow\ 9\ CO_2 + 6\ H_2O$$

$$\hat{M} = 9 \times 12.011 + 6 \times 2.016 = \mathbf{120.195}$$

$$\bar{h}^{\circ}_{RP} = = H^{o}_{P} - H^{o}_{R} = \sum_{P} v_i \bar{h}^{\circ}_{f_i} - \bar{h}^{\circ}_{f\,Fu} \quad \Rightarrow \quad \bar{h}^{\circ}_{f\,Fu} = \sum_{P} v_i \bar{h}^{\circ}_{f_i} - \bar{h}^{\circ}_{RP}$$

Formation enthalpies from Table A.10 and enthalpy of combustion from Table 13.3

$$\bar{h}^{\circ}_{f\,Fu} = 9\bar{h}^{\circ}_{f\,CO_2} + 6\bar{h}^{\circ}_{f\,H_2O\ g} - \hat{M}(\text{-41 219})_{\text{liq Fu }H_2O\text{ vap}}$$

$$= 9(\text{-393 522}) + 6(\text{-241 826}) - 120.195(\text{-41 219})$$

$$= \mathbf{\text{-38 336 kJ/kmol}}$$

Take the enthalpy of combustion from Table 13.3 for fuel as a gas and as a vapor, the difference is the enthalpy of evaporation

$$h_{fg} = -(\bar{h}^{\circ}_{RP\ gas} - \bar{h}^{\circ}_{RP\ liq}) = 41\ 603 - 41\ 219 = \mathbf{384\ kJ/kg}$$

Borgnakke and Sonntag

13.62

Liquid pentane is burned with dry air and the products are measured on a dry basis as: 10.1% CO_2, 0.2% CO, 5.9% O_2 remainder N_2. Find the enthalpy of formation for the fuel and the actual equivalence ratio.

$$\nu_{Fu}C_5H_{12} + \nu_{O_2}O_2 + 3.76\ \nu_{O_2}N_2 \rightarrow$$

$$x\ H_2O + 10.1\ CO_2 + 0.2\ CO + 5.9\ O_2 + 83.8\ N_2$$

Balance of C: $5\ \nu_{Fu} = 10.1 + 0.2 \Rightarrow \nu_{Fu} = \textbf{2.06}$

Balance of H: $12\ \nu_{Fu} = 2\ x \Rightarrow x = 6\ \nu_{Fu} = \textbf{12.36}$

Balance of O: $2\ \nu_{O_2} = x + 20.2 + 0.2 + 2 \times 5.9 \Rightarrow \nu_{O_2} = \textbf{22.28}$

Balance of N: $2 \times 3.76\ \nu_{O_2} = 83.8 \times 2 \Rightarrow \nu_{O_2} = \textbf{22.287} \Rightarrow \textbf{OK}$

ν_{O_2} for 1 kmol fuel = 10.816

$\phi = 1,\ C_5H_{12} + 8\ O_2 + 8 \times 3.76\ N_2 \rightarrow 6\ H_2O + 5\ CO_2 + 30.08\ N_2$

$H^{\circ}_{RP} = H^{\circ}_P - H^{\circ}_R = 6\ \bar{h}^{\circ}_{f\ H_2O} + 5\ \bar{h}^{\circ}_{f\ CO_2} - \bar{h}^{\circ}_{f\ fuel}$

Table 13.3: $H^{\circ}_{RP} = -44\ 983 \times 72.151 = -3\ 245\ 568$ kJ/kmol fuel

$\Rightarrow \quad \bar{h}^{\circ}_{f\ fuel} = 6\ \bar{h}^{\circ}_{f\ H_2O} + 5\ \bar{h}^{\circ}_{f\ CO_2} - H^{\circ}_{RP}$

$= 6\ (-241\ 826) + 5\ (-393\ 522) + 3\ 245\ 568$

$= \textbf{−172 998 kJ/kmol}$

$\phi = AFs\ /\ AF = \nu_{O_2\ stoich}/\nu_{O_2\ AC} = 8/10.816 = \textbf{0.74}$

Borgnakke and Sonntag

13.63

Wet biomass waste from a food-processing plant is fed to a catalytic reactor, where in a steady flow process it is converted into a low-energy fuel gas suitable for firing the processing plant boilers. The fuel gas has a composition of 50% methane, 45% carbon dioxide, and 5% hydrogen on a volumetric basis. Determine the lower heating value of this fuel gas mixture per unit volume.

For 1 kmol fuel gas,

$$0.5\ CH_4 + 0.45\ CO_2 + 0.05\ H_2 + 1.025\ O_2$$
$$\rightarrow (0.5 + 0.45)\ CO_2 + 1.05\ H_2O$$

The lower heating value is with water vapor in the products. Since the $0.45\ CO_2$ cancels,

$$\bar{h}_{RP} = 0.5(\text{-}393\ 522) + 1.05(\text{-}241\ 826) - 0.5(\text{-}74\ 873) - 0.05(0)$$
$$= \text{-}413\ 242\ \text{kJ/kmol fuel gas}$$

With $\quad \dfrac{n}{V} = P/\bar{R}T = \dfrac{100}{8.3145 \times 298.2} = 0.04033\ \text{kmol/m}^3$

$$LHV = +413\ 242 \times 0.04033 = \mathbf{16\ 666\ kJ/m^3}$$

Borgnakke and Sonntag

13.64

Determine the lower heating value of the gas generated from coal as described in Problem 13.30. Do not include the components removed by the water scrubbers.

The gas from problem 13.30 is saturated with water vapor. Lower heating value LHV has water as vapor.

$$LHV = -\overset{\circ}{H}_{RP} = \overset{\circ}{H}_P - \overset{\circ}{H}_R$$

Only CH_4, H_2 and CO contributes, find gas mixture after the scrubbers:

CH_4	H_2	CO	CO_2	N_2	n
0.3	29.6	41.0	10.0	0.8	81.7

Cool to 40°C $P_G = 7.384$, $P = 3000$ kPa

$$y_{H2O\ MAX} = \frac{7.384}{3000} = \frac{n_v}{n_v + 81.7} \quad \rightarrow \quad n_v = 0.2016$$

Mixture composition:

CH_4	H_2	CO	CO_2	N_2	$H_2O(v)$
0.3 kmol	29.6	41.0	10.0	0.8	0.2016

81.9016 kmol (from 100 kmol of the original gas mixture)

$$0.3\ CH_4 + 29.6\ H_2 + 41\ CO + 10\ CO_2 + 0.8\ N_2 + 0.2016\ H_2O$$

$$LHV = -[0.3\bar{H}^{\circ}_{RPCH4} + 29.6\bar{H}^{\circ}_{RPH2} + 41\bar{H}^{\circ}_{RPCO}]/81.9$$

$$= -[0.3(-50\ 010 \times 16.043) + 29.6(-241\ 826)$$

$$+ 41(-393\ 522 + 110\ 527)]/81.9$$

$$= \mathbf{232\ 009\ \frac{kJ}{kmol\ gas}}$$

Borgnakke and Sonntag

13.65

Do Problem 13.42 using Table 13.3 instead of Table A.10 for the solution.

As an alternative fuel consider liquid methanol burned with stoichiometric air both supplied at P_o, T_o in a constant pressure process exhausting the products at 900 K. What is the heat transfer per kmol of fuel.

$$CH_3OH + 1.5\,(O_2 + 3.76\,N_2) \rightarrow 1\,CO_2 + 2\,H_2O + 5.64\,N_2$$

Reactants at 25°C, products are at 900 K, 100 kPa.

CH_3OH: From table 13.3 for liquid fuel, water vapor

$$\overset{\circ}{H}_{RP} = -19\,910 \text{ kJ/kg } = -19\,910 \times 32.042 = -637\,956 \text{ kJ/kmol fuel}$$

A9: $\quad \Delta H_P = 1 \times 28\,030 + 2 \times 21\,937 + 5.64 \times 18\,223 = 174\,682 \text{ kJ/kmol fuel}$

$$Q = H_{Po} + \Delta H_P - H_{Ro} = \overset{\circ}{H}_{RP} + \Delta H_P = -637\,956 + 174\,682$$
$$= \mathbf{-463\,274 \text{ kJ/kmol fuel}}$$

Various species added to the methanol changes the flame color

Borgnakke and Sonntag

13.66

Do problem 13.43 using table 13.3 instead of Table A.10 for the solution.

The reaction equation for stiochiometric ratio is:

$$C_7H_{16} + v_{O2}(O_2 + 3.76 N_2) \Rightarrow 7CO_2 + 8 H_2O + v_{O2} \times 3.76 N_2$$

So the balance (C and H was done in equation) of oxygen gives $v_{O2} = 7 + 4 = 11$, and actual one is $11 \times 1.25 = 13.75$. Now the actual reaction equation is:

$$C_7H_{16} + 13.75 O_2 + 51.7 N_2 \Rightarrow 7CO_2 + 8 H_2O + 51.7 N_2 + 2.75 O_2$$

To find the heat transfer take a control volume as combustion chamber and heat exchanger

$$H_R + Q = H_P \Rightarrow Q = H_P^o + \Delta H_P - H_R^o = \overset{\circ}{H}_{RP} + \Delta H_P$$

Now we get the enthalpy of combustion from table 13.3, which is per kg, so scale it with the molecular weight for the fuel. Add all the ΔH_P from A.9

$$\Delta H_P = 7 \Delta \bar{h}_{CO_2} + 8 \Delta \bar{h}_{H_2O} + 51.7 \Delta \bar{h}_{N_2} + 2.75 \Delta \bar{h}_{O_2}$$

$$= 7(12\,906) + 8(10\,499) + 51.7(8894) + 2.75(9245) = 659\,578 \frac{kJ}{kmol}$$

$$Q = M\, \overset{\circ}{H}_{RP} + \Delta H_P = 100.205(-44\,922) + 659\,578$$

$$= \textbf{- 3 841 831 kJ/kmol fuel}$$

Borgnakke and Sonntag

13.67

E85 is a liquid mixture of 85% ethanol and 15% gasoline (assume octane) by mass. Find the lower heating value for this blend.

The heating value of the blend becomes

$$\text{LHV} = 0.85\ \text{LHV}_{\text{ethanol}} + 0.15\ \text{LHV}_{\text{octane}}$$
$$= 0.85\ (26\ 811) + 0.15\ (44\ 425)\ \ \text{kJ/kg}$$
$$= \mathbf{29\ 453\ kJ/kg}$$

As this is lower than gasoline a larger amount of fuel is being used for the same energy release in the engine. If the mixture was given on a mole basis we would have used LHV on a mole basis instead (multiplying Table 13.3 entries by M).

Borgnakke and Sonntag

13.68

Assume the products of combustion in Problem 13.67 are sent out of the tailpipe and cool to ambient 20°C. Find the fraction of the product water that will condense.

Since the fuel mixture is specified on a mass basis we need to find the mole fractions for the combustion equation. From Eq.13.4 we get

$$y_{ethanol} = (0.85/46.069) / [0.85/46.069 + 0.15/114.232] = 0.93356$$

$$y_{octane} = 1 - y_{ethanol} = 0.06644$$

The reaction equation is

$$0.93356\ C_2H_5OH + 0.06644\ C_8H_{18} + \nu_{O2}\ (O_2 + 3.76\ N_2)$$

$$\rightarrow \nu_{CO2}\ CO_2 + \nu_{H2O}H_2O + 3.76\ \nu_{O2}\ N_2$$

C balance: $2 \times 0.93356 + 8 \times 0.06644 = \nu_{CO2} = 2.39864$

H_2 balance: $3 \times 0.93356 + 9 \times 0.06644 = \nu_{H2O} = 3.39864$

O balance: $0.93356 + 2\ \nu_{O2} = 2\ \nu_{CO2} + \nu_{H2O} = 8.19592$ $\Rightarrow \nu_{O2} = 3.63118$

Now the products are:

$$2.39864\ CO_2 + 3.39864\ H_2O + 13.65324\ N_2$$

Check for condensation amount

Find x: $y_{H_2O\ max} = \dfrac{P_g(20°)}{P_{tot}} = \dfrac{2.339}{101.325} = 0.023084 = \dfrac{x}{2.39864 + x + 13.65324}$

$\Rightarrow$ x = 0.3793 mol of water can stay vapor.

Condensing water = 3.39864 – 0.3793 = 3.01934 or **89%**

Borgnakke and Sonntag

13.69

Gaseous propane and stoichiometric air are mixed and fed to a burner both at ambient P_O and T_O. After combustion the products cool down and eventually reach ambient T_O. How much heat transfer was given out for 1 kg of propane?

The reaction equation for stoichiometric mixture is:

$$C_3H_8 + v_{O2}\,(O_2 + 3.76\,N_2) \rightarrow a\,H_2O + b\,CO_2 + c\,N_2$$

C balance: $3 = b$;

H balance: $8 = 2a \Rightarrow a = 4$

O balance: $2\,v_{O2} = a + 2b = 4 + 2 \times 3$

$\Rightarrow v_{O2} = 5$

N$_2$ balance: $3.76\,v_{O2} = c = 18.8$

A/F $= 5 \times 4.76 \times 28.97 / 44.094 = 15.637$

The reaction is

$$C_3H_8 + 5\,(O_2 + 3.76\,N_2) \rightarrow 4\,H_2O + 3\,CO_2 + 18.8\,N_2$$

First we solve the problem assuming all the water is in the vapor phase and then we can solve with the proper split of water into liquid and vapor masses.

Energy Eq.: $\quad H_R = Q_{out} + H_P \quad \Rightarrow \quad H_R^o = Q_{out} + H_P^o$

$$Q_{out} = H_R^o - H_P^o = -H_{RP}^\circ = HV = 46\,352 \text{ kJ/kg fuel}$$

At T_O we need to check for condensation of water partial P limited to P_g.

$$y_{v\,max} = \frac{P_g}{P} = \frac{3.169}{100} = \frac{n_{v\,max}}{n_{v\,max} + 3 + 18.8}$$

$\Rightarrow \quad n_{v\,max} = 0.7135$ and then $\quad n_{liq} = 4 - n_{v\,max} = 3.2865$

So now we know the split of the 4 moles of water into vapor and liquid. The change in the heating value becomes ($HV_{liq\,water} = 50\,343$ kJ/kg fuel)

$$Q_{out} = HV = \frac{0.7135}{4}\,46\,352 + \frac{3.2865}{4}\,50\,343 = \textbf{49 631 kJ/kg fuel}$$

Borgnakke and Sonntag

13.70

In an experiment a 1:1 mole ratio propane and butane is burned in a steady flow with stoichiometric air. Both fuels and air are supplied as gases at 298 K and 100 kPa. The products are cooled to 1000 K as they give heat to some application. Find the lower heating value (per kg fuel mixture) and the total heat transfer for 1 kmol of fuel mixture used.

Combustion: $C_3H_8 + C_4H_{10} + 11.5(O_2 + 3.76\ N) \rightarrow 7\ CO_2 + 9\ H_2O + 43.24\ N_2$

$\nu_{O2} = 7 + 9/2 = 11.5$ $=>$ $\nu_{N2} = 3.76 \times 11.5 = 43.24$

The enthalpy of combustion for the 2 kmol of fuel becomes from Tbl 13.3

$H_P^o - H_R^o = (-46\ 352)\ 44.094 + (-45\ 714)\ 58.124 = -\ 4\ 700\ 925.6$ kJ/2 kmol fuel

$HV = -H_{RP}^{\circ} = \dfrac{4\ 700\ 925.6}{44.094 + 58.124} = \textbf{45 989 kJ/kg fuel}$ Convert to mass basis

$\Delta H_P = 7 \times 33\ 397 + 9 \times 26\ 000 + 43.24 \times 21\ 463 = 1\ 395\ 839$ kJ/2 kmol fuel

The heat transfer for the two kmol of fuel is

$Q = H_R - H_P = -H_{RP}^{\circ} - \Delta H_P = 4\ 700\ 925.6 - 1\ 395\ 839$
      $= 3\ 305\ 087$ kJ/2 kmol fuel
      $= \textbf{1 652 544 kJ/kmol fuel}$

Borgnakke and Sonntag

13.71

Blast furnace gas in a steel mill is available at 250°C to be burned for the generation of steam. The composition of this gas is, on a volumetric basis,

Component	CH_4	H_2	CO	CO_2	N_2	H_2O
Percent by volume	0.1	2.4	23.3	14.4	56.4	3.4

Find the lower heating value (kJ/m^3) of this gas at 250°C and ambient pressure.

Of the six components in the gas mixture, only the first 3 contribute to the heating value. These are, per kmol of mixture:

$$0.024\,H_2, \quad 0.001\,CH_4, \quad 0.233\,CO$$

For these components,

$$0.024\,H_2 + 0.001\,CH_4 + 0.233\,CO + 0.1305\,O_2$$
$$\rightarrow 0.026\,H_2O + 0.234\,CO_2$$

The remainder need not be included in the calculation, as the contributions to reactants and products cancel. For the lower HV(water as vapor) at 250°C

$$\begin{aligned}
\bar{h}_{RP} = \ & 0.026(-241\ 826 + 7750) + 0.234(-393\ 522 + 9370) \\
& - 0.024(0 + 6558) - 0.001(-74\ 873 + 2.254 \times 16.04(250-25)) \\
& - 0.233(-110\ 527 + 6629) - 0.1305(0 + 6817) \\
= \ & \text{-72 749}\ \frac{kJ}{kmol\ fuel}
\end{aligned}$$

$$\bar{v}_0 = \bar{R}\,T_o/P_o = 8.3145 \times 523.2/100 = 43.5015\ m^3/kmol$$

$$LHV = +72\ 749 / 43.5015 = \mathbf{1672\ kJ/m^3}$$

Borgnakke and Sonntag

13.72

Consider natural gas A which are listed in Table 13.2. Calculate the enthalpy of combustion of each gas at 25°C, assuming that the products include vapor water. Repeat the answer for liquid water in the products.

Natural Gas A

$$0.939 \, CH_4 + 0.036 \, C_2H_6 + 0.012 \, C_3H_8 + 0.013 \, C_4H_{10}$$

$$+ \, 2.1485 \, (O_2 + 3.76 \, N_2) \rightarrow 1.099 \, CO_2 + 2.099 \, H_2O + 8.0784 \, N_2$$

$$H_R = 0.939(-74\ 878) + 0.036(-84\ 740) + 0.012(-103\ 900)$$

$$+ \, 0.013(-126\ 200) = -76244 \ kJ$$

a) vapor H_2O

$$H_P = 1.099(-393\ 522) + 2.099(-241\ 826) = -940\ 074 \ kJ/kmol$$

$$\bar{h}_{RP} = H_P - H_R = \textbf{-863 830 kJ/kmol}$$

b) Liq. H_2O

$$H_P = 1.099(-393\ 522) + 2.099(-285\ 830) = -1\ 032\ 438$$

$$\bar{h}_{RP} = \textbf{-956 194 kJ/kmol}$$

These values can be compared to the pure methane

Borgnakke and Sonntag

13.73

Consider natural gas D which are listed in Table 13.2. Calculate the enthalpy of combustion of each gas at 25°C, assuming that the products include vapor water. Repeat the answer for liquid water in the products.

Natural Gas D:

$$0.543 \, CH_4 + 0.163 \, C_2H_6 + 0.162 \, C_3H_8 + 0.074 \, C_4H_{10}$$
$$+ \, 0.058 \, N_2 + v_{O2} \, (O_2 + 3.76 \, N_2)$$
$$\rightarrow \, 1.651 \, CO_2 + 2.593 \, H_2O + v_{O2} \times 3.76 \, N_2$$
$$H_R = 0.543(-74 \, 873) + 0.163(-84 \, 740) + 0.162(-130 \, 900)$$
$$+ \, 0.074(-126 \, 200) = -80 \, 639 \, kJ$$

a) vapor H_2O

$$H_P = 1.651(-393 \, 522) + 2.593(-241 \, 826) = -1 \, 276 \, 760 \, kJ$$

$$\bar{h}_{RP} = \textbf{-1 196 121 kJ/kmol}$$

b) Liq. H_2O

$$H_P = 1.651(-393 \, 522) + 2.593(-285 \, 830) = -1 \, 390 \, 862 \, kJ$$

$$\bar{h}_{RP} = \textbf{-1 310 223 kJ/kmol}$$

Borgnakke and Sonntag

13.74

A burner receives a mixture of two fuels with mass fraction 40% n-butane and 60% methanol, both vapor. The fuel is burned with stoichiometric air. Find the product composition and the lower heating value of this fuel mixture (kJ/kg fuel mix).

Since the fuel mixture is specified on a mass basis we need to find the mole fractions for the combustion equation. From Eq.13.4 we get

$$y_{butane} = (0.4/58.124) / [0.4/58.124 + 0.6/32.042] = 0.26875$$

$$y_{methanol} = 1 - y_{butane} = 0.73125$$

The reaction equation is

$$0.73125 \, CH_3OH + 0.26875 \, C_4H_{10} + \nu_{O2} \, (O_2 + 3.76 \, N_2)$$

$$\rightarrow \nu_{CO2} \, CO_2 + \nu_{H2O} H_2O + 3.76 \, \nu_{O2} \, N_2$$

C balance: $0.73125 + 4 \times 0.26875 = \nu_{CO2} = 1.80625$

H_2 balance: $2 \times 0.73125 + 5 \times 0.26875 = \nu_{H2O} = 2.80625$

O balance: $0.73125 + 2 \, \nu_{O2} = 2 \, \nu_{CO2} + \nu_{H2O} = 6.41875 \implies \nu_{O2} = 2.84375$

Now the products are:

$$1.80625 \, CO_2 + 2.80625 \, H_2O + 10.6925 \, N_2$$

Since the enthalpy of combustion is on a mass basis in table 13.3 (this is also the negative of the heating value) we get

LHV $= 0.4 \times 45\ 714 + 0.6 \times 21\ 093 = $ **30 941 kJ/kg fuel mixture**
Notice we took fuel vapor and water as vapor (lower heating value).

Borgnakke and Sonntag

13.75

Natural gas, we assume methane, is burned with 200% theoretical air and the reactants are supplied as gases at the reference temperature and pressure. The products are flowing through a heat exchanger where they give off energy to some water flowing in at $20^{\circ}C$, 500 kPa and out at $700^{\circ}C$, 500 kPa. The products exit at 400 K to the chimney. How much energy per kmole fuel can the products deliver and how many kg water per kg fuel can they heat?

The reaction equation for stoichiometric mixture is:

$$CH_4 + v_{O2}\left(O_2 + 3.76\,N_2\right) \rightarrow 2\,H_2O + 1\,CO_2 + c\,N_2$$

O balance: $2\,v_{O2} = 2 + 2 \quad \Rightarrow \quad v_{O2} = 2$

200% theoretical air: $v_{O2} = 2 \times 2 = 4$ so now more O_2 and N_2

$$CH_4 + 4\left(O_2 + 3.76\,N_2\right) \rightarrow 2\,H_2O + 1\,CO_2 + 15.04\,N_2 + 2\,O_2$$

The products are cooled to 400 K (so we do not consider condensation) and the energy equation is

Energy Eq.: $H_R + Q = H_P = H_P^{\circ} + \Delta H_P = H_R^{\circ} + Q$

$$Q = H_P^{\circ} - H_R^{\circ} + \Delta H_P = H_{RP}^{\circ} + \Delta H_P$$

From Table 13.3: $H_{RP}^{\circ} = 16.04\,(-50\,010) = -802\,160$ kJ/kmol

$$\Delta H_P = \Delta\bar{h}_{CO2}^* + 2\,\Delta\bar{h}_{H2O}^* + 2\,\Delta\bar{h}_{O2}^* + 15.04\,\Delta\bar{h}_{N2}^*$$

From Table A.9

$\Delta H_{P\,400} = 4003 + 2 \times 3450 + 2 \times 3027 + 15.04 \times 2971 = 61\,641$ kJ/kmol

$$Q = H_{RP}^{\circ} + \Delta H_P = -802\,160 + 61\,641 = \textbf{-740 519 kJ/kmol}$$

$q_{prod} = -Q\,/\,M = 740\,519\,/\,16.04 = 46\,167$ kJ/kg fuel

The water flow has a required heat transfer, using B.1.3 and B.1.4 as

$q_{H2O} = h_{out} - h_{in} = 3925.97 - 83.81 = 3842.2$ kJ/kg water

The mass of water becomes

$$m_{H2O}\,/\,m_{fuel} = q_{prod}\,/\,q_{H2O} = \textbf{12.0 kg water / kg fuel}$$

Borgnakke and Sonntag

13.76

　　　Liquid nitromethane is added to the air in a carburetor to make a stoichiometric mixture where both fuel and air are added at 298 K, 100 kPa. After combustion a constant pressure heat exchanger brings the products to 600 K before being exhausted. Assume the nitrogen in the fuel becomes N_2 gas. Find the total heat transfer per kmole fuel in the whole process.

$$CH_3NO_2 + \nu_{O2}(O_2 + 3.76\ N_2) \rightarrow 1.5\ H_2O + 1\ CO_2 + a\ N_2$$

C and H balances done in equation. The remaining

　　　O balance:　$2 + 2\ \nu_{O2} = 1.5 + 2$　　=>　　$\nu_{O2} = 0.75$

　　　N balance:　$1 + 3.76\ \nu_{O2} \times 2 = 2a$　　=>　　$a = 3.32$

Energy eq.:　　$H_R + Q = H_P$　　=>　　$Q = H_P - H_R = \overset{\circ}{H_P} - \overset{\circ}{H_R} + \Delta H_P - \Delta H_R$

The reactants enter at the reference state, $\Delta H_R = 0$, and the products at 600 K from table A.9

$$\Delta H_P = 1.5\ \Delta\bar{h}_{H2O} + \Delta\bar{h}_{CO2} + 3.32\ \Delta\bar{h}_{N2}$$
$$= 1.5\ (10\ 499) + 1\ (12\ 906) + 3.32\ (8894) = 58\ 183\ \text{kJ/kmol fuel}$$
$$\overset{\circ}{H_P} - \overset{\circ}{H_R} = \overset{\circ}{H_{RP}} = 61.04\ (-10\ 537) = -643\ 178\ \text{kJ/kmol}$$
$$Q = -643\ 178 + 58\ 183 = \textbf{-584 995 kJ/kmol fuel}$$

Borgnakke and Sonntag

13.77

Gasoline, C_7H_{17}, is burned in a steady state burner with stoichiometric air at P_o, T_o. The gasoline is flowing as a liquid at T_o to a carburetor where it is mixed with air to produce a fuel air gas mixture at T_o. The carburetor takes some heat transfer from the hot products to do the heating. After the combustion the products go through a heat exchanger, which they leave at 600 K. The gasoline consumption is 10 kg per hour. How much power is given out in the heat exchanger and how much power does the carburetor need?

Stoichiometric combustion:

$$C_7H_{17} + \nu_{O_2} \left(O_2 + 3.76\ N_2\right) \rightarrow\ 8.5\ H_2O + 7\ CO_2 + c\ N_2$$

O balance: $2\ \nu_{O_2} = 8.5 + 14 = 22.5 \quad \Rightarrow \quad \nu_{O_2} = 11.25$

N balance: $c = 3.76\ \nu_{O_2} = 3.76 \times 11.25 = 42.3$

$$M_{FUEL} = 7\ M_C + 17\ M_H = 7 \times 12.011 + 8.5 \times 2.016 = 101.213$$

C.V. Total, heat exchanger and carburetor included, Q out.

Energy Eq.: $H_R = \overset{\circ}{H_R} = \overset{\circ}{H_P} + \Delta H_P + Q_{out}$

From Table A.9

$\Delta H_P = 8.5 \times 10\ 499 + 7 \times 12\ 906 + 42.3 \times 8894 = 555\ 800$ kJ/kmol

From energy equation and Table 13.3

$$Q_{out} = \overset{\circ}{H_R} - \overset{\circ}{H_P} - \Delta H_P = -\overset{\circ}{H_{RP}} - \Delta H_P$$

$$= 101.213\ (44\ 506) - 555\ 800 = 3\ 948\ 786\ \text{kJ/kmol}$$

Now the power output is

$$\dot{Q} = \dot{n}\ Q_{out} = Q_{out}\ \dot{m}/M = 3\ 948\ 786 \times \frac{10}{3600} / 101.213 = \textbf{108.4 kW}$$

The carburetor air comes in and leaves at the same T so no change in energy, all we need is to evaporate the fuel, h_{fg} so

$$\dot{Q} = \dot{m}\ h_{fg} = \frac{10}{3600}\ (44\ 886 - 44\ 506) = \frac{1}{360} \times 380 = \textbf{1.06 kW}$$

Here we used Table 13.3 for fuel liquid and fuel vapor to get h_{fg} and any phase of the water as long as it is the same for the two.

Borgnakke and Sonntag

13.78

An isobaric combustion process receives gaseous benzene C_6H_6 and air in a stoichiometric ratio at P_0, T_0. To limit the product temperature to 2000 K, liquid water is sprayed in after the combustion. Find the kmol of liquid water added per kmol of fuel and the dew point of the combined products.

The reaction equation for stoichiometric mixture with C and H balance done is:

$$C_6H_6 + v_{O_2}\left(O_2 + 3.76\,N_2\right) \rightarrow 3\,H_2O + 6\,CO_2 + c\,N_2$$

O balance: $2\,v_{O_2} = 3 + 6 \times 2 = 15 \implies v_{O_2} = 7.5$

N balance: $c = 3.76\ v_{O_2} = 3.76 \times 7.5 = 28.2$

With x kmol of water added per kmol fuel the products are
Products: $(3 + x)\,H_2O + 6\,CO_2 + 28.2\,N_2$

Energy Eq.: $H_R = \overset{\circ}{H_R} + x\,\bar{h}^{\circ}_{f\,H_2O\,liq} = \overset{\circ}{H_P} + \Delta H_P$

$$= \overset{\circ}{H_P} + x\,\bar{h}^{\circ}_{f\,H_2O\,vap} + (3 + x)\,\Delta\bar{h}_{H_2O} + 6\,\Delta\bar{h}_{CO_2} + 28.2\,\Delta\bar{h}_{N_2}$$

Where the extra water is shown explicitly. Rearrange to get

$$\overset{\circ}{H_R} - \overset{\circ}{H_P} - 6\Delta\bar{h}_{CO_2} - 28.2\Delta\bar{h}_{N_2} - 3\Delta\bar{h}_{H_2O}$$

$$= x\,(\bar{h}^{\circ}_{f\,H_2O\,vap} - \bar{h}^{\circ}_{f\,H_2O\,liq} + \Delta\bar{h}_{H_2O})$$

$$40\,576 \times 78.114 - 6 \times 91\,439 - 28.2 \times 56\,137 - 3 \times 72\,788$$
$$= x\,[\text{-}241\,826 - (\text{--}285\,830) + 72\,788\,]$$
$$819\,493 = x\,(116\,792) \implies \qquad x = \textbf{7.017 kmol/kmol fuel}$$

Dew point: $y_v = \dfrac{3 + x}{6 + 28.2 + 3 + x} = 0.2265$

$$\Rightarrow P_v = y_v P = 0.2265 \times 101.325 = 22.95 \text{ kPa}$$
$$\text{B.1.2: } T_{dew} = \textbf{63}^{\textbf{o}}\textbf{C}$$

Borgnakke and Sonntag

13.79

A liquid fuel similar to diesel, n-dodecane $C_{12}H_{26}$, is sent into a carburetor, where it is vaporized and mixed with air in a stoichiometric ratio. The liquid fuel and air are supplied at T_O and 100 kPa, and the heat to the carburetor, Q_1, is taken from the combustion products, as in Fig. P13.77. After mixing, the reactant gas goes through a combustor, and after complete combustion the products enter a heat exchanger, where they deliver Q_1 to the carburetor and Q_2 to an application, after which the products exit at 800 K. Find the heat transfer to the carburetor per kmol fuel, Q_1, and the second heat transfer, Q_2, both in kJ/kmol fuel..

First do the combustion equation for n-Dodecane $C_{12}H_{26}$

$$C_{12}H_{26} + \nu_{O_2}(O_2 + 3.76\,N_2) \Rightarrow 12\,CO_2 + 13\,H_2O + 69.56\,N_2$$

O balance: $\quad 2\nu_{O_2} = 12 \times 2 + 13 = 37 \quad \Rightarrow \quad \nu_{O_2} = 37/2 = 18.5$

N_2 balance: $\quad \nu_{O_2} = 3.76 \; \nu_{O_2} = 3.76 \times 18.5 = 69.56$

$$M_{FUEL} = 12\,M_C + 26\,M_H = 12 \times 12.011 + 26 \times 1.008 = 170.34$$

CV Carburetor, here the air is unchanged, liquid fuel vaporizes

$$Q_1 = HV_{f\,vap} - HV_{f\,liq} = 170.34(44\,467 - 44\,109) = 60\,982 \text{ kJ/kmol}$$

CV Total system (notice Q_1 is internal to this CV).

Energy Eq.: $\qquad\qquad 0 = H_{R1} - H_{P2} - Q_2$

Table 13.3: $\quad HV = -\overset{\circ}{H}_{RP} = -170.34\,(-44\,109) = 7\,513\,527 \text{ kJ/kmol}$

Table A.9: $\quad \Delta H_{P2} = 12\,\Delta\overline{h}_{CO2} + 13\,\Delta\overline{h}_{H2O} + 69.56\,\Delta\overline{h}_{N2}$

$$= 12 \times 22\,806 + 13 \times 18\,002 + 69.56 \times 15\,046 = 1\,554\,298 \text{ kJ/kmol}$$

$$Q_2 = H_{R1} - H_{P2} = H_R^o - H_P^o - \Delta H_{P2} = HV - \Delta H_{P2}$$

$$= 7\,513\,527 - 1\,554\,298 = \textbf{5 959 229 kJ/kmol}$$

Borgnakke and Sonntag

13.80
 A mixture of fuels is E85, which is 85% ethanol and 15% gasoline (assume octane) by mass. Assume we put the fuel and air, both at T_o, P_o, into a carburetor and vaporize the fuel as we mix it with stoichiometric air before it flows to an engine. Assume the engine has an efficiency as work divided by the lower heating value of 30%, and we want it to deliver 40 kW. We use heat from the exhaust flow (500 K) for the carburetor. Find the lower heating value of this fuel (kJ/kg), the rate of fuel consumption, the heating rate needed in the carburetor, and the rate of entropy generation in the carburetor.

The heating value of the liquid fuel blend becomes
$$\text{LHV} = 0.85\,\text{LHV}_{\text{ethanol}} + 0.15\,\text{LHV}_{\text{octane}}$$
$$= 0.85 \times 26\,811 + 0.15 \times 44\,425 \quad \text{kJ/kg} = \textbf{29 453 kJ/kg}$$

$$\dot{W} = 0.3\,\dot{m}_{\text{fuel}}\,\text{LHV}$$

$$\dot{m}_{\text{fuel}} = \dot{W}/(0.3 \times \text{LHV}) = 40\ \text{kW}/\ (0.3 \times 29\,453\ \text{kJ/kg})$$
$$= \textbf{0.004527 kg/s = 16.3 kg/h}$$

The difference in heating value based on liquid versus vapor fuel is
$$\Delta\text{LHV} = 0.85\,(27\,731 - 26\,811) + 0.15\,(44\,788 - 44\,425)$$
$$= 0.85 \times 920 + 0.15 \times 363 = 836.45\ \text{kJ/kg}$$

$$\dot{Q} = \dot{m}_{\text{fuel}}\,\Delta\text{LHV} = 0.004527 \times 836.45 = 3.79\ \text{kW}$$

Entropy generation is from vaporizing the fuel by transfer of heat from 500 K exhaust gas to the intake system at To and then mixing it with air. The mole fractions are

Ethanol: $M = 46.069$; Octane: $M = 114.232$
$$\text{DIV} = [0.85\,/\,46.069] + [0.15\,/114.232] = 0.01845 + 0.001313 = 0.019764$$
$$y_{\text{ethanol}} = 0.01845\,/\,0.019764 = 0.9336$$
$$y_{\text{octane}} = 1 - y_{\text{ethanol}} = 0.06644$$

$$0.93356\ C_2H_5OH + 0.06644\ C_8H_{18} + 3.63118\ (O_2 + 3.76\ N_2) \Rightarrow$$
$$2.39864\ CO_2 + 3.39864\ H_2O + 13.65324\ N_2$$

C balance: $0.93356 \times 2 + 0.06644 \times 8 = 2.39864 = v_{CO2}$
H balance: $0.93356 \times 6 + 0.06644 \times 18 = 6.79728 = 2v_{H2O}$

So the A/F ratio on a mole and mass basis are

A/F-mole $= 3.63118 \times 4.76\,/\,1 = 17.2844$;
A/F-mass $= 17.2844 \times 28.97/50.5996 = 9.896$

Borgnakke and Sonntag

The fuel was mixed before the carburetor so afterwards we have a 1: 17.2844 mix.

$$s_{fg\ ethanol} = (282.444 - 160.554)/46.069 = 2.6458 \text{ kJ/kg-K}$$
$$s_{fg\ octane} = (466.514 - 360.575)/114.232 = 0.927402 \text{ kJ/kg-K}$$
$$s_{fg\ fuel\ mix} = 0.85 \times 2.6458 + 0.15 \times 0.927 = 2.38798 \text{ kJ/kg-K}$$
$$M_{fuel} = 0.9336 \times 46.069 + 0.06644 \times 114.232 = 50.5996$$

Entropy Eq.: $\dot{S}_{gen} = \text{out} - \text{in} = \dot{m}_{fuel}\ [s_{fg} - R\ \ln(y_{fuel})] + \dot{m}_{air}\ [- R\ \ln(y_{air})] - \dot{Q}/T_{exhaust}$

$$= 0.004527 \times \left[2.38798 - \frac{8.3145}{50.5996}\ \ln(\frac{1}{18.2844})\right]$$

$$- 0.004527 \times 9.896 \times 0.287\ \ln(\frac{17.2844}{18.2844}) - \frac{3.79}{500} \text{ kW/K}$$

$$= 0.01297 + 0.000723 - 0.00758 = \mathbf{0.0061 \text{ kW/K}}$$

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Adiabatic Flame Temperature

Borgnakke and Sonntag

13.81

In a rocket, hydrogen is burned with air, both reactants supplied as gases at P_o, T_o. The combustion is adiabatic and the mixture is stoichiometeric (100% theoretical air). Find the products dew point and the adiabatic flame temperature (~2500 K).

The reaction equation is:

$$H_2 + v_{O2}\,(O_2 + 3.76\,N_2) \implies H_2O + 3.76\,v_{O2}\,N_2$$

The balance of hydrogen is done, now for oxygen we need $v_{O2} = 0.5$ and thus we have 1.88 for nitrogen.

$$y_v = 1/(1+1.88) = 0.3472 \implies P_v = 101.325 \times 0.3472 = 35.18\ kPa = P_g$$

Table B.1.2:   **$T_{dew} = 72.6$ C.**

Energy Eq:

$$H_R = H_P \implies 0 = -241\ 826 + \Delta h_{water} + 1.88\ \Delta h_{nitrogen}$$

$$\Delta H_P = \Delta h_{water} + 1.88\ \Delta h_{nitrogen} = 241\ 826\ kJ/kmol\ fuel$$

Find now from table A.9 the two enthalpy terms

At 2400 K : $\Delta H_P = 93741 + 1.88 \times 70640 = 226544\ kJ/kmol\ fuel$

At 2600 K : $\Delta H_P = 104520 + 1.88 \times 77963 = 251090\ kJ/kmol\ fuel$

Then interpolate to hit 241 826 to give **T = 2525 K**

Borgnakke and Sonntag

13.82

Hydrogen gas is burned with pure oxygen in a steady flow burner where both reactants are supplied in a stoichiometric ratio at the reference pressure and temperature. What is the adiabatic flame temperature?

The reaction equation is:

$$H_2 + \nu_{O2} \, O_2 \quad \Rightarrow \quad H_2O$$

The balance of hydrogen is done, now for oxygen we need $\nu_{O2} = 0.5$.

Energy Eq.: $H_R = H_P \Rightarrow 0 = -241\ 826 + \Delta \bar{h}_{H2O}$

$$\Rightarrow \quad \Delta \bar{h}_{H2O} = 241\ 826 \text{ kJ/kmol}$$

Interpolate now in table A.9 for the temperature to give this enthalpy

T = 4991 K

Borgnakke and Sonntag

13.83

Some type of wood can be characterized as $C_1H_{1.5}O_{0.7}$ with a lower heating value of 19 500 kJ/kg. Find its adiabatic flame temperature when burned with stoichiometric air at 100 kPa, 298 K.

$$C_1H_{1.5}O_{0.7} + \nu_{O_2}[\ O_2 + 3.76\ N_2\ ] \rightarrow \quad CO_2 + 0.75\ H_2O + 3.76\nu_{O_2}\ N_2$$

O balance: $\quad 0.7 + 2\nu_{O_2} = 2 + 0.75 \quad \Rightarrow \quad \nu_{O_2} = 1.025, \ \nu_{N2} = 3.854$

$$M = 1 \times 12.0111 + 1.5 \times 1.008 + 0.7 \times 16 = 24.712$$

$$\overset{\circ}{H}_{RP} = H_P^O - H_R^O = -M \times HV = -24.712 \times 19\ 500 = -481\ 884\ \text{kJ/kmol}$$

Energy Eq.: $\quad H_P = \overset{\circ}{H}_P + \Delta H_P = H_R = \overset{\circ}{H}_R \quad \Rightarrow \quad \Delta H_P = \overset{\circ}{H}_R - \overset{\circ}{H}_P = -\overset{\circ}{H}_{RP}$

$$\Delta H_P = \Delta \bar{h}_{CO2} + 0.75\ \Delta \bar{h}_{H2O} + 3.854\ \Delta \bar{h}_{N2} = 481\ 884\ \text{kJ/kmol}$$

at 2400 K: $\quad \Delta H_P = 115\ 779 + 0.75 \times 93\ 741 + 3.854 \times 70\ 640 = 458\ 331\ \text{kJ}$

at 2600 K: $\quad \Delta H_P = 128\ 074 + 0.75 \times 104\ 520 + 3.854 \times 77\ 963 = 506\ 933\ \text{kJ}$

$$T = 2400 + 200\ \frac{481\ 884 - 458\ 331}{506\ 933 - 458\ 331} = \textbf{2497 K}$$

Comment: Most wood has some water and some non-combustible solids material so the actual flame temperature will be much lower.

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13.84

A gas turbine burns methane with 200% theoretical air. The air and fuel comes in through two separate compressors bringing them from 100 kPa, 298 K to 1400 kPa and after mixing enters the combustion chamber at 600 K. Find the adiabatic flame temperature using constant specific heat for the ΔH_P terms.

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2} (O_2 + 3.76 \, N_2) \rightarrow 2 \, H_2O + 1 \, CO_2 + c \, N_2$$

O balance: $2 \, \nu_{O2} = 2 + 2$ $\Rightarrow$ $\nu_{O2} = 2$

200% theoretical air: $\nu_{O2} = 2 \times 2 = 4$ so now more O_2 and N_2

$$CH_4 + 4 \, (O_2 + 3.76 \, N_2) \rightarrow 2 \, H_2O + 1 \, CO_2 + 15.04 \, N_2 + 2 \, O_2$$

The energy equation around the combustion chamber becomes

Energy Eq.: $H_P - H_R = 0$ $\Rightarrow$ $\Delta H_P = H_R^{\circ} + \Delta H_R - H_P^{\circ} = -H_{RP}^{\circ} + \Delta H_R$

$\Delta H_R = \Delta H_{Fuel} + \Delta H_{air} = M \, C_P \, \Delta T + 4(\Delta \bar{h}_{O2} + 3.76 \, \Delta \bar{h}_{N2})$

   $= 16.043 \times 2.254 \, (600{-}298) + 4(9245 + 3.76 \times 8894) = 181 \, 666$ kJ/kmol

$-H_{RP}^{\circ} = 16.043 \times 50 \, 010 = 802 \, 310$ kJ/kmol

$\Delta H_P = \Delta \bar{h}_{CO2} + 2 \, \Delta \bar{h}_{H2O} + 15.04 \, \Delta \bar{h}_{N2} + 2 \, \Delta \bar{h}_{O2} \approx \Delta T \sum \nu_i \bar{C}_{Pi}$

   $= 802 \, 310 + 181 \, 666 = 983 \, 976$ kJ/kmol   (from energy Eq.)

$\sum \nu_i \bar{C}_{Pi} = 0.842 \times 44.01 + 2 \times 1.872 \times 18.015 + 15.04 \times 1.042 \times 28.013$

$+ \, 2 \times 0.922 \times 31.999 = 602.52$ kJ/kmol-K

$\Delta T = \Delta H_P / \sum \nu_i \bar{C}_{Pi} = 983 \, 976 \, / \, 602.52 = 1633.1$ K

$T = 298 + 1633 = \textbf{1931 K}$

Borgnakke and Sonntag

13.85

Extend the solution to the previous problem by using Table A.9 for the ΔH_P terms.

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2}\,(O_2 + 3.76\,N_2) \;\rightarrow\; 2\,H_2O + 1\,CO_2 + c\,N_2$$

O balance: $2\,\nu_{O2} = 2 + 2$ $\Rightarrow$ $\nu_{O2} = 2$

200% theoretical air: $\nu_{O2} = 2 \times 2 = 4$ so now more O_2 and N_2

$$CH_4 + 4\,(O_2 + 3.76\,N_2) \;\rightarrow\; 2\,H_2O + 1\,CO_2 + 15.04\,N_2 + 2\,O_2$$

The energy equation around the combustion chamber becomes

Energy Eq.: $H_P - H_R = 0$ $\Rightarrow$ $\Delta H_P = \overset{\circ}{H}_R + \Delta H_R - \overset{\circ}{H}_P = -\overset{\circ}{H}_{RP} + \Delta H_R$

$$\Delta H_R = \Delta H_{Fuel} + \Delta H_{air} = M\,C_P\,\Delta T + 4(\Delta\bar{h}_{O2} + 3.76\,\Delta\bar{h}_{N2})$$

$$= 16.043 \times 2.254\,(600\text{–}298) + 4(9245 + 3.76 \times 8894) = 181\,666 \text{ kJ/kmol}$$

$$-\overset{\circ}{H}_{RP} = 16.043 \times 50\,010 = 802\,310 \text{ kJ/kmol}$$

$$\Delta H_P = \Delta\bar{h}_{CO2} + 2\,\Delta\bar{h}_{H2O} + 15.04\,\Delta\bar{h}_{N2} + 2\,\Delta\bar{h}_{O2}$$

$$= 802\,310 + 181\,666 = 983\,976 \text{ kJ/kmol}\quad \text{(from energy Eq.)}$$

Trial and error with $\Delta\bar{h}$ from Table A.9

At 1800 K
   $\Delta H_P = 79\,432 + 2 \times 62\,693 + 15.04 \times 48\,979 + 2 \times 51\,674 = 1\,044\,810 \text{ kJ/kmol}$
At 1700 K
   $\Delta H_P = 73\,480 + 2 \times 57\,757 + 15.04 \times 45\,430 + 2 \times 47\,959 = 968\,179 \text{ kJ/kmol}$

Linear interpolation: $T = 1700 + 100\,\dfrac{983\,976\ -\ 968\,179}{1\,044\,810\ -\ 968\,179} = \mathbf{1721\ K}$

Borgnakke and Sonntag

13.86

Carbon is burned with air in a furnace with 150% theoretical air and both reactants are supplied at the reference pressure and temperature. What is the adiabatic flame temperature?

$$C + \nu_{O_2}O_2 + 3.76\ \nu_{O_2}N_2 \rightarrow 1\ CO_2 + 3.76\ \nu_{O_2}N_2$$

From this we find $\nu_{O_2} = 1$ and the actual combustion reaction is

$$C + 1.5\ O_2 + 5.64\ N_2 \rightarrow 1\ CO_2 + 5.64\ N_2 + 0.5\ O_2$$

$$H_P = H_P^° + \Delta H_P = H_R = H_R^° \quad \Rightarrow$$

$$\Delta H_P = H_R^° - H_P^° = 0 - (-393\ 522) = 393\ 522\ kJ/kmol$$

$$\Delta H_P = \Delta \bar{h}_{CO2} + 5.64\ \Delta \bar{h}_{N2} + 0.5\ \Delta \bar{h}_{O2}$$

Find T so ΔH_P takes on the required value. To start guessing assume all products are nitrogen $(1 + 5.64 + 0.5 = 7.14)$ that gives $1900 < T < 2000$ K from Table A.9.

$$\Delta H_{P\ 1900} = 85\ 420 + 5.64 \times 52\ 549 + 0.5 \times 55\ 414 = 409\ 503 \quad \text{too high}$$

$$\Delta H_{P\ 1800} = 79\ 432 + 5.64 \times 48\ 979 + 0.5 \times 51\ 674 = 381\ 511$$

Linear interpolation to find

$$T = 1800 + 100\ \frac{393\ 522 - 381\ 511}{409\ 503 - 381\ 511} = \mathbf{1843\ K}$$

Borgnakke and Sonntag

13.87

Acetylene gas at 25°C, 100 kPa is fed to the head of a cutting torch. Calculate the adiabatic flame temperature if the acetylene is burned with

a. 100% theoretical air at 25°C.

b. 100% theoretical oxygen at 25°C.

a) $$C_2H_2 + 2.5\,O_2 + 2.5 \times 3.76\,N_2 \rightarrow 2\,CO_2 + 1\,H_2O + 9.4\,N_2$$

$$H_R = \bar{h}^o_{f\,C2H2} = +226\,731 \text{ kJ/kmol} \quad \text{from table A.10}$$

$$H_P = 2(-393\,522 + \Delta\bar{h}^*_{CO2}) + 1(-241\,826 + \Delta\bar{h}^*_{H2O}) + 9.4\,\Delta\bar{h}^*_{N2}$$

$$Q_{CV} = H_P - H_R = 0 \quad \Rightarrow \quad 2\,\Delta\bar{h}^*_{CO2} + 1\,\Delta\bar{h}^*_{H2O} + 9.4\,\Delta\bar{h}^*_{N2} = 1\,255\,601 \text{ kJ}$$

Trial and Error A.9: $\text{LHS}_{2800} = 1\,198\,369$, $\text{LHS}_{3000} = 1\,303\,775$

Linear interpolation: $T_{PROD} = \textbf{2909 K}$

b) $$C_2H_2 + 2.5\,O_2 \rightarrow 2\,CO_2 + H_2O$$

$$H_R = +226\,731 \text{ kJ} ; \quad H_P = 2(-393\,522 + \Delta\bar{h}^*_{CO2}) + 1(-241\,826 + \Delta\bar{h}^*_{H2O})$$

$$\Rightarrow \quad 2\,\Delta\bar{h}^*_{CO2} + 1\,\Delta\bar{h}^*_{H2O} = 1\,255\,601 \text{ kJ/kmol fuel}$$

At 6000 K (limit of A.9) $2 \times 343\,782 + 302\,295 = 989\,859$

At 5600 K $2 \times 317\,870 + 278\,161 = 913\,901$

Slope   75 958/400 K change

Extrapolate to cover the difference above 989 859 kJ/kmol fuel

$$T_{PROD} \approx 6000 + 400(265\,742/75\,958) \approx \textbf{7400 K}$$

Borgnakke and Sonntag

13.88

Hydrogen gas is burned with 200% theoretical air in a steady flow burner where both reactants are supplied at the reference pressure and temperature. What is the adiabatic flame temperature?

The stoichiometric reaction equation is:

$$H_2 + \nu_{O2} (O_2 + 3.76 N_2) \implies H_2O + 3.76 \nu_{O2} N_2$$

The balance of hydrogen is done, now for oxygen we need $\nu_{O2} = 0.5$ and thus we have for the actual mixture $\nu_{O2} = 1$. The actual reaction is

$$H_2 + 1 (O_2 + 3.76 N_2) \implies 1 H_2O + 3.76 N_2 + 0.5 O_2$$

The energy equation with formation enthalpy from A.9 or A.10 for water is

$$H_R = H_P \implies 0 = -241\ 826 + \Delta H_P$$

$$\Delta H_P = \Delta h_{H2O} + 3.76 \Delta h_{N2} + 0.5 \Delta h_{O2} = +241\ 826 \text{ kJ/kmol}$$

Find now from table A.9 the three enthalpy terms

At 2000 K : $\Delta H_P = 72\ 788 + 3.76 \times 56\ 137 + 0.5 \times 59\ 176 = 313\ 451$

At 1800 K : $\Delta H_P = 62\ 693 + 3.76 \times 48\ 979 + 0.5 \times 51\ 674 = 272\ 691$

At 1600 K : $\Delta H_P = 52\ 907 + 3.76 \times 41\ 904 + 0.5 \times 44\ 267 = 232\ 600$

At 1700 K : $\Delta H_P = 57\ 757 + 3.76 \times 45\ 430 + 0.5 \times 47\ 959 = 252\ 553$

Then interpolate to match 241 826 to give

$$T = 1600 + 100 \frac{241\ 826 - 232\ 600}{252\ 553 - 232\ 600} = \textbf{1646 K}$$

Borgnakke and Sonntag

13.89

Butane gas at 25°C is mixed with 150% theoretical air at 600 K and is burned in an adiabatic steady flow combustor. What is the temperature of the products exiting the combustor?

$$C_4H_{10} + 1.5 \times 6.5 \left(O_2 + 3.76\,N_2\right) \rightarrow 4\,CO_2 + 5\,H_2O + 3.25\,O_2 + 36.66\,N_2$$

Energy Eq.: $H_P - H_R = 0 \quad \Rightarrow \quad \Delta H_P = H_R^{\circ} + \Delta H_R - H_P^{\circ}$

Reactants: $\Delta H_R = 9.75(9245) + 36.66(8894) = 416\ 193$ kJ/kmol ;

$H_R^{\circ} = \bar{h}_{C4H10} = \bar{h}_{f\ IG}^{o} = -126\ 200$ kJ/kmol $\Rightarrow \quad H_R = +289\ 993$ kJ/kmol

$H_P^{\circ} = 4(-393522) + 5(-241826) = -2\ 783\ 218$ kJ/kmol

$\Delta H_P = 4\,\Delta\bar{h}_{CO2}^{*} + 5\,\Delta\bar{h}_{H2O}^{*} + 3.25\,\Delta\bar{h}_{O2}^{*} + 36.66\,\Delta\bar{h}_{N2}^{*}$

From the energy equation we then get

$\Delta H_P = -126\ 200 + 416\ 193 - (-2\ 783\ 218) = 3\ 073\ 211$ kJ/kmol

Trial and Error:

$LHS_{2000\ K} = 2\ 980\ 000,$
$LHS_{2200\ K} = 3\ 369\ 866$

Linear interpolation to match RHS
$\Rightarrow \quad T_P = \textbf{2048 K}$

Borgnakke and Sonntag

13.90

A stoichiometric mixture of benzene, C_6H_6, and air is mixed from the reactants flowing at 25°C, 100 kPa. Find the adiabatic flame temperature. What is the error if constant specific heat at T_0 for the products from Table A.5 are used?

$$C_6H_6 + \nu_{O_2}O_2 + 3.76\ \nu_{O_2}N_2 \rightarrow 6CO_2 + 3H_2O + 3.76\ \nu_{O_2}N_2$$

$$\nu_{O_2} = 6 + 3/2 = 7.5 \Rightarrow \nu_{N_2} = 28.2$$

$$H_P = H_P^{\circ} + \Delta H_P = H_R = H_R^{\circ} \Rightarrow$$

$$\Delta H_P = -H_{RP}^{\circ} = 40576 \times 78.114 = 3\ 169\ 554\ \text{kJ/kmol}$$

$$\Delta H_P = 6\ \Delta\bar{h}_{CO2} + 3\ \Delta\bar{h}_{H2O} + 28.2\ \Delta\bar{h}_{N2}$$

$$\Delta H_{P\ 2600K} = 6(128074) + 3(104\ 520) + 28.2(77\ 963) = 3\ 280\ 600,$$

$$\Delta H_{P\ 2400K} = 6(115\ 779) + 3(93\ 741) + 28.2(70\ 640) = 2\ 968\ 000$$

Linear interpolation $\Rightarrow$ $T_{AD} = $ **2529 K**

$$\sum\nu_i\bar{C}_{Pi} = 6 \times 0.842 \times 44.01 + 3 \times 1.872 \times 18.015 + 28.2 \times 1.042 \times 28.013$$

$$= 1146.66\ \text{kJ/kmol K}$$

$$\Delta T = \Delta H_P \Big/ \sum\nu_i\bar{C}_{Pi} = 3\ 169\ 554\ /\ 1146.66 = 2764$$

$$\Rightarrow T_{AD} = 3062\ \text{K}, \quad \textbf{21\% high}$$

Borgnakke and Sonntag

13.91

What is the adiabatic flame temperature before the secondary air is added in Problem 13.51

$$CH_4 + \nu(O_2 + 3.76\ N_2) \rightarrow$$
$$CO_2 + 2H_2O + 3.76\ \nu\ N_2$$

Fuel

T_{ad}

1500 K

Air1

Air2

O balance: $\nu = 2$, $\nu_{N2} = 7.52$

Do a C.V. around the combustion chamber, then the energy equation becomes

Energy Eq.: $\quad H_{R1} = H_{P\ 1} = H_P^\circ + \Delta H_{P\ 1}$

$$H_{R1} = H_R^\circ + \Delta H_{R1} = \bar{h}_{f\ fuel}^o + \Delta H_{Fuel} + \Delta H_{air1}$$
$$\Delta H_{air1} = 2(\Delta H_{O2} + 3.76\ \Delta H_{N2}) = 2(6086 + 3.76 \times 5911) = 56\ 623\ kJ/kmol$$
$$\Delta H_{Fuel} \approx M\ C_P\ \Delta T = 16.043 \times 2.254\ (500 - 298) = 7304.5\ kJ/kmol$$
$$\bar{h}_{f\ fuel}^o - H_P^\circ = HV = 16.043 \times 50\ 010 = 802\ 310\ kJ/kmol$$

$$\Delta H_{P\ 1} = \Delta\bar{h}_{CO2} + 0.75\ \Delta\bar{h}_{H2O} + 3.854\ \Delta\bar{h}_{N2}$$
$$= \bar{h}_{f\ fuel}^o + \Delta H_{Fuel} + \Delta H_{air1} - H_P^\circ \quad \text{(from energy Eq.)}$$
$$= 802\ 310 + 7304.5 + 56\ 623 = 866\ 237\ kJ/kmol\ fuel$$

at 2400 K: $\quad \Delta H_{P\ 1} = 115\ 779 + 2 \times 93\ 741 + 7.52 \times 70\ 640 = 834\ 474\ kJ/kmol$
at 2600 K: $\quad \Delta H_{P\ 1} = 128\ 074 + 2 \times 104\ 520 + 7.52 \times 77\ 963 = 923\ 396\ kJ/kmol$

$$T = 2400 + 200\ \frac{866\ 237\ -\ 834\ 474}{923\ 396\ -\ 834\ 474} = \textbf{2471 K}$$

Borgnakke and Sonntag

13.92

Ethene, C_2H_4, burns with 150% theoretical air in a steady flow constant-pressure process with reactants entering at P_0, T_0. Find the adiabatic flame temperature.

Stoichiometric

$$C_2H_4 + 3(O_2 + 3.76N_2) \rightarrow 2CO_2 + 2H_2O + 11.28N_2$$

Actual

$$C_2H_4 + 4.5(O_2 + 3.76N_2) \rightarrow 2CO_2 + 2H_2O + 1.5\ O_2 + 16.92N_2$$

$$H_P = H_P^{\circ} + 2\Delta\bar{h}_{CO_2} + 2\Delta\bar{h}_{H2O} + 1.5\Delta\bar{h}_{O_2} + 16.92\Delta\bar{h}_{N2}$$

$$H_R^{\circ} = \bar{h}_{f\ Fu}^{\circ} \qquad\qquad \Delta H_P + H_P^{\circ} = H_R^{\circ}$$

$$\Rightarrow \Delta H_P = -H_{RP}^{\circ} = 28.054 \times 47\ 158 = 1\ 322\ 970.5\ \tfrac{kJ}{kmol\ Fu}$$

$$\Delta H_P = 2\Delta\bar{h}_{CO_2} + 2\Delta\bar{h}_{H2O} + 1.5\Delta\bar{h}_{O_2} + 16.92\Delta\bar{h}_{N2}$$

Initial guess based on $(2+2+1.5+16.92)\ N_2$ from A.9: $\quad T_1 = 2100$ K

$$\Delta H_P(2000) = 1\ 366\ 982, \quad \Delta H_P(1900) = 1\ 278\ 398$$

$$\Rightarrow\ \ \mathbf{T_{AD} \cong 1950\ K}$$

Borgnakke and Sonntag

13.93

A special coal burner uses a stoichiometric mixture of coal and an oxygen - argon mixture (1:1 mole ratio) with the reactants supplied at Po, To. Find the adiabatic flame temperature assuming complete combustion.

Combustion of carbon:

$$C + O_2 + Ar \rightarrow CO_2 + Ar$$

C.V. Combustion chamber.

Energy Eq.:

$$H_R = \overset{\circ}{H_R} = H_P = \overset{\circ}{H_P} + \Delta H_P = \bar{h}_{f\,CO_2} + \Delta \bar{h}_{CO_2} + \Delta \bar{h}_{Ar}$$

Table A.9 or A.10: $\bar{h}_{f\,CO_2}$ = -393 522 kJ/kmol, reference $\overset{\circ}{H_R} = 0$

Table A.5: $\bar{C}_{P\,Ar} = 0.52 \times 39.948 = 20.773$ kJ/kmol-K

$$\Delta H_P = \overset{\circ}{H_R} - \overset{\circ}{H_P} = 0 - \bar{h}_{f\,CO_2} = 393\ 522 \text{ kJ/kmol fuel}$$

$$\Delta H_{P\ 5200} = 292\ 112 + 20.773 \times (5200 - 298) = 393\ 938 \text{ kJ/kmol fuel,}$$

$$\Delta H_{P\ 4800} = 266\ 488 + 20.773 \times (4800 - 298) = 360\ 005 \text{ kJ/kmol fuel,}$$

$$\text{interpolate} \quad \Rightarrow \quad T_4 = T_{ad.flame} = \textbf{5195 K}$$

Comment: At this temperature some chemical equilibrium reactions will be important, see Chapter 14.

13.94

A gas-turbine burns natural gas (assume methane) where the air is supplied to the combustor at 1000 kPa, 500 K and the fuel is at 298 K, 1000 kPa. What is the equivalence ratio and the percent theoretical air if the adiabatic flame temperature should be limited to 1800 K?

The reaction equation for a mixture with excess air is:

$$CH_4 + \nu_{O2}\left(O_2 + 3.76\,N_2\right) \rightarrow 2\,H_2O + 1\,CO_2 + 3.76\nu_{O2}\,N_2 + (\nu_{O2} - 2)O_2$$

$$H_P = H_P^\circ + \Delta H_P = H_R = H_R^\circ + \Delta H_R$$

From table A.9 at 500 K (notice fuel is at 298 K)

$$\Delta H_R = 0 + \nu_{O2}(\Delta h_{O2} + 3.76\,\Delta h_{N2}) = \nu_{O2}(6086 + 3.76 \times 5911) = 28\,311.4\,\nu_{O2}$$

From table A.9 at 1800 K:

$$\Delta H_P = 2\,\Delta h_{H2O} + \Delta h_{CO2} + 3.76\,\nu_{O2}\,\Delta h_{N2} + (\nu_{O2} - 2)\,\Delta h_{O2}$$

$$= 2 \times 62\,693 + 79432 + 3.76\,\nu_{O2} \times 48\,979 + (\nu_{O2} - 2)\,51\,674$$

$$= 101\,470 + 235\,835\,\nu_{O2}$$

From table 13.3: $\quad H_P^\circ - H_R^\circ = H_{RP}^\circ = 16.04(\text{-}50\,010) = \text{-}802\,160$ kJ/kmol

Now substitute all terms into the energy equation

$$\text{-}802\,160 + 101\,470 + 235\,835\,\nu_{O2} = 28\,311.4\,\nu_{O2}$$

Solve for ν_{O2}

$$\nu_{O2} = \frac{802\,160 - 101\,470}{235\,835 - 28\,311.4} = 3.376$$

%Theoretical air $= 100\,(3.376 / 2) = \textbf{168.8 \%}$

$\Phi = AFs / AF = 2 / 3.376 = \textbf{0.592}$

Borgnakke and Sonntag

13.95

Liquid butane at 25°C is mixed with 150% theoretical air at 600 K and is burned in a steady flow burner. Use the enthalpy of combustion from Table 13.3 to find the adiabatic flame temperature out of the burner.

$$C_4H_{10} + 1.5 \times 6.5\,(O_2 + 3.76\,N_2) \rightarrow 4\,CO_2 + 5\,H_2O + 3.25\,O_2 + 36.66\,N_2$$

Energy Eq.: $H_P - H_R = 0 \implies \Delta H_P = \overset{\circ}{H}_R + \Delta H_R - \overset{\circ}{H}_P = -\overset{\circ}{H}_{RP} + \Delta H_R$

Reactants: $\Delta H_R = 9.75(9245) + 36.66(8894) = 416\ 193$ kJ/kmol;

$$\overset{\circ}{H}_{RP} = M \times HV = 58.124 \times (-45\ 344) = -2\ 635\ 575 \text{ kJ/kmol}$$

$$\Delta H_P = 4\Delta\bar{h}^*_{CO2} + 5\Delta\bar{h}^*_{H2O} + 3.25\,\Delta\bar{h}^*_{O2} + 36.66\,\Delta\bar{h}^*_{N2}$$

So the energy equation becomes

$$\Delta H_P = 2\ 635\ 575 + 416\ 193 = 3\ 051\ 768 \text{ kJ/kmol}$$

Trial and Error:

$$\Delta H_{P\ 2000\ K} = 4 \times 91439 + 5 \times 72788 + 3.25 \times 59176 + 36.66 \times 56137$$
$$= 2\ 980\ 000,$$
$$\Delta H_{P\ 2200\ K} = 4 \times 103562 + 5 \times 83153 + 3.25 \times 66770 + 36.66 \times 63362$$
$$= 3\ 369\ 866$$

Linear interpolation to match RHS $\implies T_P = \mathbf{2037\ K}$

Borgnakke and Sonntag

13.96

Natural gas, we assume methane, is burned with 200% theoretical air and the reactants are supplied as gases at the reference temperature and pressure. The products are flowing through a heat exchanger and then out the exhaust, as in Fig. P13.96. What is the adiabatic flame temperature right after combustion before the heat exchanger?

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2}\,(O_2 + 3.76\,N_2) \;\rightarrow\; 1\,CO_2 + 2\,H_2O + c\,N_2$$

O balance: $2\,\nu_{O2} = 2 + 2 \quad \Rightarrow \quad \nu_{O2} = 2$

200% theoretical air: $\nu_{O2} = 2 \times 2 = 4$ so now more O_2 and N_2

$$CH_4 + 4\,(O_2 + 3.76\,N_2) \;\rightarrow\; 1\,CO_2 + 2\,H_2O + 15.04\,N_2 + 2\,O_2$$

Energy Eq.: $H_{air} + H_{fuel} = H_R = H_P$

$$H_P^{\circ} + \Delta H_P = H_R^{\circ} + \Delta H_R \quad \Rightarrow \quad \Delta H_P = H_R^{\circ} + \Delta H_R - H_P^{\circ} = -H_{RP}^{\circ} + 0$$

From Table 13.3: $-H_{RP}^{\circ} = -16.04\,(-50\,010) = 802\,160$ kJ/kmol

$$\Delta H_P = \Delta \bar{h}_{CO2}^{*} + 2\,\Delta \bar{h}_{H2O}^{*} + 2\,\Delta \bar{h}_{O2}^{*} + 15.04\,\Delta \bar{h}_{N2}^{*}$$

From Table A.9

$\Delta H_{P\;1600} = 67\,659 + 2 \times 52\,907 + 2 \times 44\,267 + 15.04 \times 41\,904 = 892\,243$

$\Delta H_{P\;1500} = 61\,705 + 2 \times 48\,149 + 2 \times 40\,600 + 15.04 \times 38\,405 = 816\,814$

$\Delta H_{P\;1400} = 55\,895 + 2 \times 43\,491 + 2 \times 36\,958 + 15.04 \times 34\,936 = 742\,230$

Linear interpolation to get 802 160

$$T = 1400 + 100\,\frac{802\,160 - 742\,230}{816\,814 - 742\,230} = \mathbf{1480\ K}$$

Borgnakke and Sonntag

13.97

Solid carbon is burned with stoichiometric air in a steady flow process. The reactants at T_0, P_0 are heated in a preheater to $T_2 = 500$ K as shown in Fig. P13.97, with the energy given by the product gases before flowing to a second heat exchanger, which they leave at T_0. Find the temperature of the products T_4, and the heat transfer per kmol of fuel (4 to 5) in the second heat exchanger.

Combustion of carbon:

$$C + O_2 + 3.76\,N_2 \rightarrow CO_2 + 3.76\,N_2$$

C.V. Combustion chamber and preheater from 1 to 4, no external Q. For this CV states 2 and 3 are internal and do not appear in equations.

Energy Eq.:

$$H_R = \overset{\circ}{H_R} = H_{P_4} = \overset{\circ}{H_P} + \Delta H_{P_4} = \bar{h}_{f\,CO_2} + \Delta\bar{h}_{CO_2} + 3.76\Delta\bar{h}_{N_2}$$

Table A.9 or A.10: $\bar{h}_{f\,CO_2} = $ -393 522 kJ/kmol, reference $\overset{\circ}{H_R} = 0$

$$\Delta H_{P_4} = \overset{\circ}{H_R} - \overset{\circ}{H_P} = 0 - \bar{h}_{f\,CO_2} = 393\,522 \text{ kJ/kmol fuel}$$

$$\Delta H_{P_4\,2400} = 115\,779 + 3.76 \times 70\,640 = 381\,385 \text{ kJ/kmol fuel,}$$

$$\Delta H_{P_4\,2600} = 128\,074 + 3.76 \times 77\,963 = 421\,215 \text{ kJ/kmol fuel}$$

interpolate $\Rightarrow$ $T_4 = T_{ad.flame} = $ **2461 K**

Control volume: Total. Then energy equation:

$$\overset{\circ}{H_R} + \bar{Q} = \overset{\circ}{H_P}$$

$$\bar{Q} = \bar{H}^{\circ}_{RP} = \bar{h}^{\circ}_{f\,CO_2} - 0 = \textbf{-393 522} \frac{\textbf{kJ}}{\textbf{kmol fuel}}$$

Borgnakke and Sonntag

13.98

Gaseous ethanol, C_2H_5OH, is burned with pure oxygen in a constant volume combustion bomb. The reactants are charged in a stoichiometric ratio at the reference condition. Assume no heat transfer and find the final temperature (> 5000 K).

$$C_2H_5OH + 3\ O_2 \rightarrow 2\ CO_2 + 3\ H_2O$$

Energy Eq.:

$$U_P = U_R = H_R^{\circ} + \Delta H_R - n_R \bar{R} T_R = H_P^{\circ} + \Delta H_P - n_P \bar{R} T_P$$

Solve for the properties that depends on T_P and recall $\Delta H_R = 0$

$$\Delta H_P - n_P \bar{R} T_P = H_R^{\circ} - H_P^{\circ} - n_R \bar{R} T_R = \bar{h}_{f\ fuel}^0 - 2\ \bar{h}_{f\ CO2}^0 - 3\ \bar{h}_{f\ H2O}^0 - 4\bar{R}T_R$$

Fuel: $\bar{h}_{f\ fuel}^0 = $ -235 000 kJ/kmol for IG from Table A.10 so

$$\Delta H_P - n_P \bar{R} T_P = \text{-235 000} - 2(\text{-393 522}) - 3(\text{-241 826})$$

$$- 4 \times 8.31451 \times 298.15 = 1\ 267\ 606 \text{ kJ/kmol}$$

$$\text{LHS} = \Delta H_P - n_P \bar{R} T_P = 2\ \Delta\bar{h}_{CO_2} + 3\ \Delta\bar{h}_{H_2O} - 5 \times 8.31451 \times T_P$$

From Table A.9 we find

$$\text{LHS}_{5600} = 2 \times 317\ 870 + 3 \times 278\ 161 - 41.5726 \times 5600 = 1\ 237\ 417$$

$$\text{LHS}_{6000} = 2 \times 343\ 782 + 3 \times 302\ 295 - 41.5726 \times 6000 = 1\ 345\ 014$$

$$T_{ad.flame} = \textbf{5712 K}$$

Borgnakke and Sonntag

13.99

Liquid *n*-butane at T_0, is sprayed into a gas turbine with primary air flowing at 1.0 MPa, 400 K in a stoichiometric ratio. After complete combustion, the products are at the adiabatic flame temperature, which is too high, so secondary air at 1.0 MPa, 400 K is added, with the resulting mixture being at 1400 K. Show that $T_{ad} > 1400$ K and find the ratio of secondary to primary air flow.

C.V. Combustion Chamber.

$$C_4H_{10} + 6.5 \, O_2 + 6.5 \times 3.76 \, N_2 \rightarrow 5 \, H_2O + 4 \, CO_2 + 24.44 \, N_2$$

Energy Eq.: $H_{air} + H_{fuel} = H_R = H_P$

$$H_P^\circ + \Delta H_P = H_R^\circ + \Delta H_R \quad \Rightarrow \quad \Delta H_P = H_R^\circ + \Delta H_R - H_P^\circ = -H_{RP}^\circ + \Delta H_R$$

$$\Delta H_P = 45344 \times 58.124 + 6.5(3.76 \times 2971 + 3027) = \textbf{2 727 861 kJ/kmol}$$

$$\Delta H_{P \, 1400} = 5 \times 43491 + 4 \times 55895 + 24.44 \times 34936 = \textbf{1 294 871} < \boldsymbol{\Delta H_P}$$

Remark: Try $T_{AD} > 1400$:

$$\Delta H_P = 2 \, 658 \, 263 \, @2400 \, K, \quad \Delta H_P = 2 \, 940 \, 312 \, @2600 \, K$$

C.V. Mixing Chamber. \qquad Air Second: $v_{O2 \, s}O_2 + 3.76 \, N_2$

$$\Delta H_P + v_{O2 \, second} \Delta H_{air} = \Delta H_{P \, 1400} + v_{O2 \, second} \Delta H_{air \, 1400}$$

$$\Rightarrow v_{O2 \, second} = \frac{\Delta H_P - \Delta H_{P \, 1400}}{\Delta H_{air \, 1400} - \Delta H_{air \, 400}} \qquad = \frac{1432990}{168317 - 14198} = \textbf{9.3}$$

ratio $= v_{O2 \, sec}/v_{O2 \, prim} = 9.3/6.5 = \textbf{1.43}$

Borgnakke and Sonntag

13.100

The enthalpy of formation of magnesium oxide, MgO(s), is −601 827 kJ/kmol at 25°C. The melting point of magnesium oxide is approximately 3000 K, and the increase in enthalpy between 298 and 3000 K is 128 449 kJ/kmol. The enthalpy of sublimation at 3000 K is estimated at 418 000 kJ/kmol, and the specific heat of magnesium oxide vapor above 3000 K is estimated at 37.24 kJ/kmol K.

- a. Determine the enthalpy of combustion per kilogram of magnesium.
- b. Estimate the adiabatic flame temperature when magnesium is burned with theoretical oxygen.

a) $\quad Mg + \dfrac{1}{2}O_2 \;\;\rightarrow\;\; MgO(s)$

$$\Delta h_{COMB} = \frac{\Delta \bar{h}_{COMB}}{M} = \frac{\bar{h}_f^{\circ}}{M} = \frac{-601\ 827}{24.32} = \textbf{-24 746 kJ/kg}$$

b) assume $T_R = 25°C$ and also that $T_P > 3000$ K,   (MgO = vapor phase)

1st law: $Q_{CV} = H_P - H_R = 0,\quad$ but $H_R = 0$

$$\Rightarrow H_P = \bar{h}_f^{\circ} + (\bar{h}_{3000} - \bar{h}_{298})_{SOL} + \Delta\bar{h}_{SUB} + \bar{C}_{P\ VAP}(T_P - 3000)$$

$$= -601\ 827 + 128\ 449 + 418\ 000 + 37.24(T_P - 3000) = 0$$

Solving, $T_P = \textbf{4487 K}$

Borgnakke and Sonntag

13.101

In a test of rocket propellant performance, liquid hydrazine (N_2H_4) at 100 kPa, 25°C, and oxygen gas at 100 kPa, 25°C, are fed to a combustion chamber in the ratio of 0.5 kg O_2/kg N_2H_4. The heat transfer from the chamber to the surroundings is estimated to be 100 kJ/kg N_2H_4. Determine the temperature of the products exiting the chamber. Assume that only H_2O, H_2, and N_2 are present. The enthalpy of formation of liquid hydrazine is +50 417 kJ/kmol.

$$\text{Liq. } N_2H_4: \ 100 \text{ kPa}, 25^oC \longrightarrow$$
$$\text{Gas } O_2: \ 100 \text{ kPa}, 25^oC \longrightarrow$$

Comb. chamber $\longrightarrow$ Products

$$\dot{m}_{O2}/\dot{m}_{N2H4} = 0.5 = 32\dot{n}_{O2}/32\dot{n}_{N2H4} \quad \text{and} \quad \dot{Q}/\dot{m}_{N2H4} = -100 \text{ kJ/kg}$$

Energy Eq.: $\quad Q_{CV} = H_P - H_R = -100 \times 32.045 = -3205 \text{ kJ/kmol fuel}$

Combustion eq.: $\quad 1\,N_2H_4 + \frac{1}{2}O_2 \rightarrow H_2O + H_2 + N_2$

$H_R = 1(50417) + \frac{1}{2}(0) = 50417 \text{ kJ}$

$H_P = -241\,826 + \Delta\bar{h}_{H2O} + \Delta\bar{h}_{H2} + \Delta\bar{h}_{N2}$

Energy Eq. now reads

$$H_P = H_R + Q_{CV} \ = \ H_P^o + \ \Delta H_P$$

$$\Delta H_P = \Delta\bar{h}_{H2O} + \Delta\bar{h}_{H2} + \Delta\bar{h}_{N2} = -H_P^o + \ H_R + Q_{CV}$$
$$= 241\,826 + 50\,417 - 3205 = 289\,038 \text{ kJ/kmol fuel}$$

Table A.9 : Guess T and read for water, hydrogen and nitrogen

2800 K: $\ \Delta H_P = 115\,463 + 81\,355 + 85\,323 = 282\,141$ too low

3000 K: $\ \Delta H_P = 126\,548 + 88\,725 + 92\,715 = 307\,988$ too high

Interpolate to get $\quad T_P = \textbf{2854 K}$

Borgnakke and Sonntag

Second Law for the Combustion Process

Borgnakke and Sonntag

13.102

Consider the combustion of hydrogen with pure oxygen in a stoichiometric ratio under steady flow adiabatic conditions. The reactants enter separately at 298 K, 100 kPa and the product(s) exit at a pressure of 100 kPa. What is the exit temperature and what is the irreversibility?

The reaction equation is:

$$H_2 + \nu_{O2}\, O_2 \Rightarrow H_2O$$

The balance of hydrogen is done, now for oxygen we need $\nu_{O2} = 0.5$.

Energy Eq.: $\qquad H_R = H_P \Rightarrow 0 = -241\ 826 + \Delta\bar{h}_{H2O}$

$$\Rightarrow \quad \Delta\bar{h}_{H2O} = 241\ 826 \text{ kJ/kmol}$$

Interpolate now in table A.9 for the temperature to give this enthalpy

T = 4991 K

For this temperature we find from Table A.9, in this case $P = P_o$, so we do not need any pressure correction for the entropy

$$S_P = S_P^\circ = \bar{s}_{H2O}^\circ = 315.848 \text{ kJ/kmol K}$$

For the reactants we have (again no pressure correction)

$$S_R = \bar{s}_{H2}^\circ + 0.5\ \bar{s}_{O2}^\circ = 130.678 + 0.5 \times 205.148 = 233.252 \text{ kJ/kmol K}$$

$$S_{gen} = S_P - S_R = 315.848 - 233.252 = 82.596 \text{ kJ/kmol } H_2 \text{ K}$$

$$I = T_o\, S_{gen} = 298.15 \times 82.596 = \textbf{24 626 kJ/kmol } \mathbf{H_2}$$

Borgnakke and Sonntag

13.103

Consider the combustion of methanol, CH_3OH, with 25% excess air. The combustion products are passed through a heat exchanger and exit at 200 kPa, 400 K. Calculate the absolute entropy of the products exiting the heat exchanger assuming all the water is vapor.

$$CH_3OH + 1.25 \times 1.5 \left(O_2 + 3.76\,N_2\right) \rightarrow CO_2 + 2\,H_2O + 0.375\,O_2 + 7.05\,N_2$$

We need to find the mole fractions to do the partial pressures,

$$n = 1 + 2 + 0.375 + 7.05 = 10.425 \quad => \quad y_i = n_i / n$$

Gas mixture:

	n_i	y_i	$\bar{s}_i^\circ$	$-\bar{R}\ln\dfrac{y_i P}{P_0}$	$\bar{S}_i$
CO_2	1.0	0.0959	225.314	+13.730	239.044
H_2O	2	0.1918	198.787	+7.967	206.754
O_2	0.375	0.0360	213.873	+20.876	234.749
N_2	7.05	0.6763	200.181	-2.511	197.670

$$S_{GAS\ MIX} = \sum n_i \bar{S}_i = \textbf{2134.5 kJ/K kmol fuel}$$

Borgnakke and Sonntag

13.104

Consider the combustion of methanol, CH_3OH, with 25% excess air. The combustion products are passed through a heat exchanger and exit at 200 kPa, 40°C. Calculate the absolute entropy of the products exiting the heat exchanger per kilomole of methanol burned, using the proper amounts of liquid and vapor for the water.

$$CH_3OH + 1.25 \times 1.5\,(O_2 + 3.76\,N_2) \rightarrow CO_2 + 2\,H_2O + 0.375\,O_2 + 7.05\,N_2$$

Products exit at 40 °C, 200 kPa, check for saturation:

$$y_{V\,MAX} = \frac{P_G}{P} = \frac{7.384}{200} = \frac{n_{V\,MAX}}{n_{V\,MAX} + 1 + 0.375 + 7.05}$$

$$n_V = n_{V\,MAX} = 0.323 \quad n_{LIQ} = 1.677$$

Gas mixture:

	n_i	y_i	$\bar{s}_i^\circ$	$-\bar{R}\ln\dfrac{y_i P}{P_0}$	$\bar{S}_i$
CO_2	1.0	0.1143	215.633	+12.270	227.903
H_2O	0.323	0.0369	190.485	+21.671	212.156
O_2	0.375	0.0429	206.592	+20.418	227.01
N_2	7.05	0.8059	193.039	-3.969	189.07

$$S_{GAS\,MIX} = \sum n_i \bar{S}_i = 1714.50 \text{ kJ/K kmol fuel}$$

$$\bar{s}_{LIQ} = 69.950 + 18.015(0.5725 - 0.3674) = 73.645 \text{ kJ/kmol}$$

$$S_{LIQ} = 1.677 \times 73.645 = 123.50 \text{ kJ/K kmol fuel}$$

$$S_{PROD} = 1714.50 + 123.50 = \textbf{1838 kJ/K kmol fuel}$$

Borgnakke and Sonntag

13.105

An inventor claims to have built a device that will take 0.001 kg/s of water from the faucet at 10°C, 100 kPa, and produce separate streams of hydrogen and oxygen gas, each at 400 K, 175 kPa. It is stated that this device operates in a 25°C room on 10-kW electrical power input. How do you evaluate this claim?

Liq H_2O

10°C, 100 kPa

0.001 kg/s

H$_2$ gas

O$_2$ gas

$\left\{ \begin{array}{l} \text{each at} \\ \text{400 K} \\ \text{175 kPa} \end{array} \right.$

$T_0 = 25\ ^oC$

$-\dot{W}_{CV} = 10$ kW

$$H_2O \rightarrow H_2 + \frac{1}{2}O_2$$

$$H_i - H_e = [-285830 + 18.015(42.01 - 104.89)] - 2961 - \frac{1}{2}(3027)$$

$$= -291\ 437\ \text{kJ/kmol}$$

$$(S_i - S_e) = [69.950 + 18.015(0.151 - 0.3674)] - (139.219 - 8.3145 \ln 1.75)$$

$$- \frac{1}{2}(213.873 - 8.3145 \ln 1.75) = -173.124\ \text{kJ/kmol K}$$

$$W_{REV} = (H_i - H_e) - T_0(S_i - S_e) = -291\ 437 - 298.15(-173.124)$$

$$= -239820\ \text{kJ/kmol}$$

$$\dot{W}_{REV} = (0.001/18.015)(-239\ 820) = -13.31\ \text{kW}$$

$$\dot{I} = \dot{W}_{REV} - \dot{W}_{CV} = -13.31 - (-10) < 0 \qquad \textbf{Impossible}$$

Borgnakke and Sonntag

13.106

Propene, C_3H_6, is burned with air in a steady flow burner with reactants at P_O, T_O. The mixture is lean so the adiabatic flame temperature is 1800 K. Find the entropy generation per kmol fuel neglecting all the partial pressure corrections.

The reaction equation for a mixture with excess air is:

$$C_3H_6 + \nu_{O2}\left(O_2 + 3.76\,N_2\right) \rightarrow 3\,H_2O + 3\,CO_2 + 3.76\nu_{O2}\,N_2 + (\nu_{O2} - 4.5)O_2$$

Energy Eq.: $\qquad H_R = \overset{\circ}{H}_R + \Delta H_R = \overset{\circ}{H}_R = H_P = \overset{\circ}{H}_P + \Delta H_P$

The entropy equation: $\qquad S_R + S_{gen} = S_P \quad \Rightarrow \quad S_{gen} = S_P - S_R = S_P - \overset{\circ}{S}_R$

From table A.9 at reference T

$$\Delta H_R = \Delta h_{Fu} + \nu_{O2}(\Delta h_{O2} + 3.76\,\Delta h_{N2}) = 0$$

From table A.9 at 1800 K:

$$\Delta H_P = 3\,\Delta h_{H2O} + 3\,\Delta h_{CO2} + 3.76\,\nu_{O2}\,\Delta h_{N2} + (\nu_{O2} - 4.5)\,\Delta h_{O2}$$

$$= 3 \times 62\,693 + 3 \times 79432 + 3.76\,\nu_{O2} \times 48\,979 + (\nu_{O2} - 4.5)\,51\,674$$

$$= 193\,842 + 235\,835\,\nu_{O2}$$

From table 13.3: $\quad \overset{\circ}{H}_P - \overset{\circ}{H}_R = \overset{\circ}{H}_{RP} = 42.081(-45\,780) = -1\,926\,468$ kJ/kmol

Now substitute all terms into the energy equation

$$-1\,926\,468 + 193\,842 + 235\,835\,\nu_{O2} = 0$$

Solve for ν_{O2}: $\qquad \nu_{O2} = \dfrac{1\,926\,468 - 193\,842}{235\,835} = 7.3468, \quad \nu_{N2} = 27.624$

Table A.9-10 contains the entropies at 100 kPa so we get:

$$S_P = 3 \times 259.452 + 3 \times 302.969 + (7.3468 - 4.5)\,264.797 + 27.624 \times 248.304$$

$$= 9300.24 \text{ kJ/kmol-K}$$

$$S_R = 267.066 + 7.3468 \times 205.148 + 27.624 \times 191.609 = 7067.25 \text{ kJ/kmol K}$$

$$S_{gen} = 9300.24 - 7067.25 = \textbf{2233 kJ/kmol-K}$$

13.107

Hydrogen peroxide, H_2O_2, enters a gas generator at 25°C, 500 kPa at the rate of 0.1 kg/s and is decomposed to steam and oxygen exiting at 800 K, 500 kPa. The resulting mixture is expanded through a turbine to atmospheric pressure, 100 kPa, as shown in Fig. P13.107. Determine the power output of the turbine, and the heat transfer rate in the gas generator. The enthalpy of formation of liquid H_2O_2 is $-187\,583$ kJ/kmol.

$$H_2O_2 \rightarrow H_2O + \frac{1}{2}O_2 \qquad \dot{n}_{H2O2} = \frac{\dot{m}_{H2O2}}{M} = \frac{0.1}{34.015} = 0.00294 \text{ kmol/s}$$

$$\dot{n}_{ex,mix} = \dot{n}_{H2O2} \times 1.5 = 0.00441 \text{ kmol/s}$$

$$\bar{C}_{p\,mix} = \frac{2}{3} \times 1.872 \times 18.015 + \frac{1}{3} \times 0.922 \times 31.999 = 32.317$$

$$\bar{C}_{v\,mix} = 32.317 - 8.3145 = 24.0 \implies k_{mix} = 32.317/24.0 = 1.3464$$

CV: turbine.   Assume reversible $\rightarrow s_3 = s_2$

$$T_3 = T_2\left(\frac{P_3}{P_2}\right)^{\frac{k-1}{k}} = 800\left(\frac{100}{500}\right)^{0.2573} = 528.8 \text{ K}$$

$$w = \bar{C}_{p\,mix}(T_2 - T_3) = 32.317(800 - 528.8) = 8765 \text{ kJ/kmol}$$

$$\dot{W}_{CV} = \dot{n}_{ex,mix}\, w = 0.00441 \times 8765 = \textbf{38.66 kW}$$

CV: Gas Generator

$$\dot{H}_1 = 0.00294(-187\,583 + 0) = -551.49$$

$$\dot{H}_2 = 0.00294(-241\,826 + 18002) + 0.00147(0 + 15836) = -634.76$$

$$\dot{Q}_{CV} = \dot{H}_2 - \dot{H}_1 = -634.76 + 551.49 = \textbf{-83.27 kW}$$

Borgnakke and Sonntag

13.108

Graphite, C, at P_0, T_0 is burned with air coming in at P_0, 500 K in a ratio so the products exit at P_0, 1200 K. Find the equivalence ratio, the percent theoretical air, and the total irreversibility.

$$C + (1/\phi)b(O_2 + 3.76\ N_2) \rightarrow CO_2 + ((1/\phi) - 1)O_2 + 3.76\ (1/\phi)\ N_2$$

Stoichiometric: $b = 1$

Energy Eq.: $H_P = H_R \Rightarrow \Delta H_{P\ 1200} - \Delta H_R = H_R^\circ - H_P^\circ$

$44\ 473 + ((1/\phi) - 1)29\ 761 + 3.76(1/\phi)28\ 109$

$\qquad - (1/\phi)(6086 + 3.76 \times 5911) = 0 - (-393\ 522) \Rightarrow (1/\phi) = 3.536$

$$S_{gen} = S_P - S_R = \sum_{P-R} v\left(\bar{s}^\circ - \bar{R}\ \ln(y)\right)$$

Reactants: $y_{O_2} = 0.21,\ y_{N_2} = 0.79$ (carbon is solid)

Products: $y_{O_2} = 0.1507,\ y_{N_2} = 0.79,\ y_{CO_2} = 0.0593$

$S_P^\circ = 279.39 + 2.536 \times 250.011 + 13.295 \times 234.227 = 4027.5$

$S_R^\circ = 5.74 + 3.536(220.693 + 3.76 \times 206.74) = 3534.8$

For the pressure correction the term with the nitrogen drops out (same y).

$\bar{R} \sum_{P-R} -v\ \ln(y) = \bar{R}\ [-\ln(y_{CO_2}) - 2.536\ \ln(y_{O_2\ prod}) + 3.536\ \ln(y_{O_2\ reac})]$

$\qquad = \bar{R}\ [\ -\ln(0.0593) - 2.536\ \ln(0.1507) + 3.536\ \ln(0.21)\ ]$

$\qquad = 8.3145(2.8235 + 2.536 \times 1.8927 - 3.536 \times 1.5606) = 17.503$

$S_{gen} = 4027.5 - 3534.8 + 17.503 = 510.2$ kJ/kmol carbon-K

$I = T_0\ S_{gen} = \mathbf{152\ 117\ \dfrac{kJ}{kmol\ C}}$

Borgnakke and Sonntag

13.109

Calculate the irreversibility for the process described in Problem 13.52.

$$2\,C + 2\,O_2 \;\rightarrow 1\,CO_2 + 1\,CO + \tfrac{1}{2}O_2$$

Process   V = constant,   C: solid, $n_{1(GAS)} = 2$, $n_{2(GAS)} = 2.5$

$$P_2 = P_1 \times \frac{n_2 T_2}{n_1 T_1} = 200 \times \frac{2.5 \times 1000}{2 \times 298.2} = 838.4 \text{ kPa}$$

$$H_1 = H_R = 0$$

$$H_2 = H_P = 1(-393522 + 33397) + 1(-110527 + 21686)$$

$$+ (1/2)(0 + 22703) = -437\,615 \text{ kJ}$$

$$_1Q_2 = (U_2 - U_1) = (H_2 - H_1) - n_2\bar{R}T_2 + n_1\bar{R}T_1$$

$$= (-437\,615 - 0) - 8.3145(2.5 \times 1000 - 2 \times 298.2) = -453\,442 \text{ kJ}$$

Reactants:

$$S_R = 2(5.740) + 2(205.148 - 8.31451 \ln\frac{200}{100}) = 410.250 \text{ kJ/K}$$

Products:

	n_i	y_i	$\bar{s}^{\circ}_i$	$-\bar{R}\ln\frac{y_i P}{P_0}$	S_i
CO_2	1.0	0.40	269.299	-10.061	259.238
CO	1.0	0.40	234.538	-10.061	224.477
O_2	0.5	0.20	243.579	-4.298	239.281

$$S_P = 1.0(259.238) + 1.0(224.477) + 0.5(239.281) = 603.355 \text{ kJ/K}$$

$$I = T_0(S_P - S_R) - {}_1Q_2$$

$$= 298.15(603.355 - 410.250) - (-453\,442) = \textbf{+511 016 kJ}$$

Borgnakke and Sonntag

13.110

Two kilomoles of ammonia are burned in a steady flow process with x kmol of oxygen. The products, consisting of H_2O, N_2, and the excess O_2, exit at 200°C, 7 MPa.

    a. Calculate x if half the water in the products is condensed.

    b. Calculate the absolute entropy of the products at the exit conditions.

$$2NH_3 + xO_2 \rightarrow 3H_2O + N_2 + (x - 1.5)O_2$$

Products at 200 °C, 7 MPa with $n_{H2O\ LIQ} = n_{H2O\ VAP} = 1.5$

a) $y_{H2O\ VAP} = P_G/P = \dfrac{1.5538}{7} = \dfrac{1.5}{1.5 + 1 + x - 1.5}$ => $x = \mathbf{5.757}$

b) $S_{PROD} = S_{GAS\ MIX} + S_{H2O\ LIQ}$

Gas mixture:	n_i	y_i	$\bar{s}_i^\circ$	$-\bar{R}\ln(y_iP/P_0)$	S_i
H_2O	1.5	0.222	204.595	-22.810	181.785
O_2	4.257	0.630	218.985	-31.482	187.503
N_2	1.0	0.148	205.110	-19.439	185.671

$S_{GAS\ MIX} = 1.5(181.785) + 4.257(187.503) + 1.0(185.67) = 1256.55$ kJ/K

$S_{H2O\ LIQ} = 1.5[69.950 + 18.015(2.3223 - 0.3674)] = 157.75$ kJ/K

$S_{PROD} = 1256.55 + 157.75 = \mathbf{1414.3\ kJ/K}$

Borgnakke and Sonntag

13.111

A flow of 0.02 kmol/s methane CH_4 and 200% theoretical air both at reference conditions are compressed separately to $P_3 = P_4 = 2$ MPa, then mixed and then burned in a steady flow setup (like a gasturbine). After combustion, state 6, heat transfer goes out so the exhaust, state 7, is at 600 K.

a) Find T_3, T_4 and T_5

b) Find the total rate of irreversibility from inlet to state 5.

c) Find the rate of heat transfer minus the work terms ($\dot{Q} - \dot{W}_1 - \dot{W}_2$)

The reaction equation for stoichiometric ratio is:

$$CH_4 + \nu_{O2} (O_2 + 3.76 N_2) \Rightarrow CO_2 + 2 H_2O + \nu_{O2} \times 3.76 N_2$$

where the carbon and hydrogen balance have been done. Now the oxygen

O: $2\nu_{O2} = 2 + 2 \Rightarrow \nu_{O2} = 2; \quad \nu_{N2} = 7.52$

The actual combustion becomes $\nu_{O2} = 2 \times 2 = 4; \quad \nu_{N2} = 15.04$

$$CH_4 + 4 (O_2 + 3.76 N_2) \Rightarrow CO_2 + 2 H_2O + 15.04 N_2 + 2 O_2$$

The two compressors assumed adiabatic and reversible

Air: $\quad T_3 = T_1 \left(P_3/P_1\right)^{\frac{k-1}{k}} = 298.15 \, (2000/100)^{\frac{1.4-1}{1.4}} = \mathbf{701.7 \ K}$

CH_4: $\quad T_4 = T_2 \left(P_4/P_2\right)^{\frac{k-1}{k}} = 298.15 \, (2000/100)^{\frac{1.299-1}{1.299}} = \mathbf{594.2 \ K}$

$\dot{n}_{air} = 4 \times 4.76 \, \dot{n}_{Fu} = 19.04 \, \dot{n}_{Fu} = 0.3808$ kmol/s

$\bar{C}_{Pair} = 28.97 \times 1.004 = 29.0859$ kJ/kmol-K,

$\bar{C}_{PFu} = 16.043 \times 2.254 = 36.1609$ kJ/kmol-K

Borgnakke and Sonntag

Energy Eq. mixing chamber :

$$\dot{n}_{air}\, \bar{h}_3 + \dot{n}_{Fu}\, \bar{h}_4 = \dot{n}_{air}\, \bar{h}_{5\ air} + \dot{n}_{Fu}\, \bar{h}_{5\ Fu}$$

use constant specific heats to evaluate and divide with $\dot{n}_{Fu}$ to get

$$19.04 \times 29.0859 \times 701.7 + 36.1609 \times 594.2 =$$
$$(19.04 \times 29.0859 + 36.1609)\, T_5$$

solve $\qquad T_5 = \mathbf{695.1\ K}$

C.V. From the inlet 1,2 to the exit state 5, no $\dot{Q}$ so $\ y_{5\ air} = \dfrac{19.04}{20.04}$, $y_{5\ Fu} = \dfrac{1}{20.04}$

$$\dot{S}_{gen} = \dot{n}_{air}(\bar{s}_{5\ air} - \bar{s}_1) + \dot{n}_{Fu}\, (\bar{s}_{5\ Fu} - \bar{s}_2)$$

$$= 0.3808\, [29.0859 \ln(\frac{695.1}{298.15}) - 8.3145 \ln(\frac{19.04}{20.04}\frac{2000}{100})]$$

$$+ 0.02\, [36.1609 \ln(\frac{695.1}{298.15}) - 8.3145 \ln(\frac{1}{20.04}\frac{2000}{100})\ ]$$

$$= 0.3808 \times 0.1376 + 0.02 \times 30.625 = 0.6649\ \text{kW/K}$$

$$\dot{I} = T_o\, \dot{S}_{gen} = 298.15 \times 0.6649\ =\ \mathbf{198.2\ kW}$$

C.V. Total control volume from inlet 1, 2 to final exit at state 7

$$\dot{Q} - \dot{W}_1 - \dot{W}_2 = \dot{n}_{air}\, \bar{h}_1 + \dot{n}_{Fu}\, \bar{h}_2 - \dot{n}_P\, \bar{h}_7 = \dot{n}_{Fu}\, [H_R - H_P]$$

$$H_R - H_P = H_R^o - H_P^o - \Delta H_P = -\, \overset{\circ}{H}_{RP} - \Delta H_P$$

The lower heating value per kmol fuel is

$$\text{LHV} = -\, \overset{\circ}{H}_{RP} = H_R^o - H_P^o = 16.043 \times 50\,010 = 802\,310.4\ \text{kJ/kmol}$$

$$\Delta H_P = \Delta \bar{h}_{CO_2} + 2\, \Delta \bar{h}_{H_2O} + 15.04\, \Delta \bar{h}_{N_2} + 2\, \Delta \bar{h}_{O_2}$$

$$= 12\,906 + 2 \times 10\,499 + 15.04 \times 8894 + 2 \times 9245$$

$$= 186\,159.8\ \text{kJ/kmol fuel}$$

$$\dot{Q} - \dot{W}_1 - \dot{W}_2 = 0.02 \times (802\,310.4 - 186\,159.8) = 12\,323\ \text{kW} = \mathbf{12.3\ MW}$$

Borgnakke and Sonntag

13.112

A flow of hydrogen gas is mixed with a flow of oxygen in a stoichiometric ratio, both at 298 K and 50 kPa. The mixture burns without any heat transfer in complete combustion. Find the adiabatic flame temperature and the amount of entropy generated per kmole hydrogen in the process.

The reaction equation is:

$$H_2 + v_{O2} \, O_2 \quad \Rightarrow \quad H_2O$$

The balance of hydrogen is done, now for oxygen we need $v_{O2} = 0.5$.

Energy Eq.: $\qquad H_R = H_P \Rightarrow 0 = -241\ 826 + \Delta\bar{h}_{H2O}$

$$\Rightarrow \quad \Delta\bar{h}_{H2O} = 241\ 826 \text{ kJ/kmol}$$

Interpolate now in table A.9 for the temperature to give this enthalpy

T = 4991 K

For this temperature we find from Table A.9

$$S_P = \bar{s}^{\circ}_{H2O} - \bar{R} \ln(P/P_o) = 315.848 - 8.31451 \ln(0.5) = 321.611 \text{ kJ/kmol K}$$

For the reactants we have

$$S_R = \bar{s}^{\circ}_{H2} - \bar{R} \ln(P/P_o) + 0.5\,[\bar{s}^{\circ}_{O2} - \bar{R} \ln(P/P_o)\,]$$

$$= 130.678 + 0.5 \times 205.148 \ - 1.5 \times 8.31451 \ln(0.5)$$

$$= 241.897 \text{ kJ/kmol K}$$

$$S_{gen} = S_P - S_R = 321.611 - 241.897 = \textbf{79.714 kJ/kmol } \mathbf{H_2} \textbf{ K}$$

Recall that this entropy generation includes the mixing process, we did not use the partial pressures after mixing but the total pressure (50 kPa) before mixing.

Borgnakke and Sonntag

13.113

Methane is burned with air, both of which are supplied at the reference conditions. There is enough excess air to give a flame temperature of 1800 K. What are the percent theoretical air and the irreversibility in the process?

The combustion equation with X times theoretical air is

$$CH_4 + 2X(O_2 + 3.76\ N_2) \rightarrow CO_2 + 2H_2O + 2(X-1)O_2 + 7.52X\ N_2$$

Energy Eq.: $\quad H_{air} + H_{fuel} = H_R = H_P = \overset{\circ}{H_P} + \Delta H_P = \overset{\circ}{H_R} + \Delta H_R$

$$\Rightarrow \quad \Delta H_P = \overset{\circ}{H_R} + \Delta H_R - \overset{\circ}{H_P} = -\overset{\circ}{H_{RP}} + 0$$

From Table 13.3: $\quad -\overset{\circ}{H_{RP}} = -16.04\ (-50\ 010) = 802\ 160$ kJ/kmol

$$\Delta H_P = \Delta\bar{h}^*_{CO2} + 2\ \Delta\bar{h}^*_{H2O} + 2(X-1)\ \Delta\bar{h}^*_{O2} + 7.52X\ \Delta\bar{h}^*_{N2}$$

From Table A.9 and the energy equation

$$\Delta H_{P\ 1800} = 79\ 432 + 2 \times 62\ 693 + 2(X-1)\ 51\ 674 + 7.52X \times 48\ 979 = 802\ 160$$

so

$$101\ 470 + 471\ 670\ X = 802\ 160 \quad \Rightarrow \quad X = 1.4856$$
$$\%\text{Theoretical air} = \mathbf{148.6\%}$$

The products are

Products: $\quad CO_2 + 2H_2O + 0.9712\ O_2 + 11.172\ N_2$

The second law

$$S_{gen} = S_P - S_R \quad \text{and} \quad I = T_O\ S_{gen}$$

Reactants: $P_i = 100$ kPa, $P_O = 100$ kPa, $\bar{s}^o_f$ from Table A.9

	n_i	y_i	$\bar{s}^o_f$	$-\bar{R}\ln\dfrac{y_iP_i}{P_O}$	$\bar{S}_i\ \dfrac{kJ}{kmol\ K}$
CH_4	1	1	186.251	0	186.251
O_2	2X	0.21	205.148	12.976	218.124
N_2	7.52 X	0.79	191.609	1.96	193.569

$$S_R = \sum n_i\bar{S}_i = 2996.84 \text{ kJ/K kmol fuel}$$

Borgnakke and Sonntag

Products: $P_e = 100$ kPa, $P_O = 100$ kPa

	n_i	y_i	$\bar{s}^o_{1800}$	$-\bar{R} \ln \dfrac{y_i P_e}{P_o}$	$\bar{S}_i \dfrac{kJ}{kmol\ K}$
CO_2	1	0.06604	302.969	22.595	325.564
H_2O	2	0.13208	259.452	16.831	276.283
O_2	0.9712	0.06413	264.797	22.838	287.635
N_2	11.172	0.73775	248.304	2.529	250.833

$S_P = \sum n_i \bar{S}_i = 3959.72$ kJ/K kmol fuel;

$I = T_O(S_P - S_R) = 298.15(3959.72 - 2996.84) =$ **287 MJ/kmol fuel**

Borgnakke and Sonntag

13.114

Pentane gas at 25°C, 150 kPa enters an insulated steady flow combustion chamber. Sufficient excess air to hold the combustion products temperature to 1800 K enters separately at 500 K, 150 kPa. Calculate the percent theoretical air required and the irreversibility of the process per kmol of pentane burned.

$$C_5H_{12} + 8X(O_2 + 3.76\ N_2) \rightarrow 5\ CO_2 + 6\ H_2O + 8(X\text{-}1)\ O_2 + 30.08X\ N_2$$

Energy Eq.: $Q_{CV} + H_R = H_P + W_{CV}$; $W_{CV} = 0$, $Q_{CV} = 0$

Reactants: C_5H_{12} : $\overline{h}_f^o$ from A.9   and $\Delta\overline{h}_{500}$ for O_2 and N_2 from A.9

$$H_R = \left(\overline{h}_f^o\right)_{C5H12} + 8X\ \Delta\overline{h}_{O2} + 30.08X\ \Delta\overline{h}_{N2}$$

$$= \text{-}146\ 500 + 8X\ 6086 + 30.08\ X\ 5911 = 226\ 491\ X\ \text{-}\ 146\ 500$$

$$H_P = 5\left(\overline{h}_f^o + \Delta\overline{h}\right)_{CO2} + 6\left(\overline{h}_f^o + \Delta\overline{h}\right)_{H2O} + 8(X\text{-}1)\ \Delta\overline{h}_{O2} + 30.08\ X\ \Delta\overline{h}_{N2}$$

$$= 5(\text{-}393\ 522 + 79\ 432) + 6(\text{-}241\ 826 + 62\ 693) + 8(X\text{-}1)\ 51\ 674$$

$$+ 30.08\ X\ 48\ 979 = 1\ 886\ 680\ X\ \text{-}\ 3\ 058\ 640$$

Energy Eq. solve for X;

$$H_R = H_P = 226\ 491\ X\ \text{-}\ 146\ 500 = 1\ 886\ 680\ X\ \text{-}\ 3\ 058\ 640$$

$$\Rightarrow\quad X = \mathbf{1.754}$$

b) Reactants: $P_{in} = 150$ kPa, $P_O = 100$ kPa, $\overline{s}_f^o$

	n_i	y_i	$\overline{s}_f^o, \overline{s}_{500}^o$	$-\overline{R}\ln\dfrac{y_iP_{in}}{P_o}$	$\overline{S}_i\ \dfrac{kJ}{kmol\ K}$
C_5H_{12}	1	1	348.945	-3.371	345.574
O_2	8X	0.21	220.693	9.605	230.298
N_2	30.08 X	0.79	206.74	-1.411	205.329

$$S_R = \sum n_i\overline{S}_i = 14410.34\ kJ/K\ kmol\ fuel$$

Borgnakke and Sonntag

Products: $P_e = 150$ kPa, $P_O = 100$ kPa

	n_i	y_i	$\bar{s}^o_{1800}$	$-\bar{R} \ln \dfrac{y_i P_e}{P_o}$	$\bar{S}_i \dfrac{kJ}{kmol\ K}$
CO_2	5	0.0716	302.969	18.550	321.519
H_2O	6	0.086	259.452	17.027	276.479
O_2	8(X-1)	0.0864	264.797	16.988	281.785
N_2	30.08X	0.756	248.304	-1.045	247.259

$$S_P = \sum n_i \bar{S}_i = 17\ 732.073 \ \text{kJ/K kmol fuel};$$

$$I = T_O(S_P - S_R) = 298.15(17\ 732.07 - 14\ 410.34)$$

$$= \textbf{990 MJ/kmol fuel}$$

Borgnakke and Sonntag

13.115

A closed rigid container is charged with propene, C_3H_6, and 150% theoretical air at 100 kPa, 298 K. The mixture is ignited and burns with complete combustion. Heat is transferred to a reservoir at 500 K so the final temperature of the products is 700 K. Find the final pressure, the heat transfer per kmole fuel and the total entropy generated per kmol fuel in the process.

$$C_3H_6 + \nu_{O_2}(O_2 + 3.76\ N_2) \rightarrow 3\ CO_2 + 3\ H_2O + x\ N_2$$

Oxygen, O_2 , balance: $\quad 2\ \nu_{O_2} = 6 + 3 = 9 \quad \Rightarrow \quad \nu_{O_2} = 4.5$

Actual Combustion: $\quad \phi = 1.5 \quad \Rightarrow \quad \nu_{O_2\ ac} = 1.5 \times 4.5 = 6.75$

$$C_3H_6 + 6.75\ O_2 + 25.38\ N_2 \rightarrow 3\ CO_2 + 3\ H_2O + 25.38\ N_2 + 2.25\ O_2$$

$$P_2 = P_1 \frac{n_p T_2}{n_R T_1} = 100 \times \frac{33.63 \times 700}{33.13 \times 298.15} = \mathbf{238.3\ kPa}$$

Enthalpies from Table A.9

$$\Delta H_{P\ 700} = 3 \times 17\ 754 + 3 \times 14\ 190 + 25.38 \times 11\ 937 + 2.25 \times 12\ 499$$

$$= 426\ 916\ \tfrac{kJ}{kmol\ fuel}$$

Enthalpy of combustion from table 13.3 converted to mole basis

$$H_{RP}^{\circ} = -45\ 780 \times 42.081 = -1\ 926\ 468\ kJ/kmol\ fuel$$

$$U_2 - U_1 = {}_1Q_2 - 0 = H_2 - H_1 - n_2\overline{R}T_2 + n_1\overline{R}T_1$$

$${}_1Q_2 = H_{RP}^{\circ} + \Delta H_{P\ 700} - n_P\overline{R}T_2 + n_1\overline{R}T_1$$

$$= -1\ 926\ 468 + 426\ 916 - 33.63 \times 8.3145 \times 700$$

$$+ 33.13 \times 8.3145 \times 298.15 = \mathbf{-1.613 \times 10^6}\ \tfrac{\mathbf{kJ}}{\mathbf{kmol\ fuel}}$$

Entropies from Table A.9 and pressure correction

Reactants:	n_i	y_i	$\bar{s}_i^{\circ}$	$-\bar{R}\ln(y_iP/P_0)$	S_i
C_3H_8	1.0	0.0302	267.066	29.104	296.17
O_2	6.75	0.2037	205.143	13.228	218.376
N_2	25.38	0.7661	191.609	2.216	189.393

$$S_1 = 296.17 + 6.75 \times 218.376 + 25.38 \times 189.393 = 6577\ \tfrac{kJ}{kmol\ fuel\ K}$$

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Products:	n_i	y_i	$\bar{s}_i^\circ$	$-\bar{R}\ln(y_iP/P_0)$	S_i
CO_2	3	0.0892	250.752	+12.875	263.627
H_2O	3	0.0892	218.739	+12.875	231.614
O_2	2.25	0.0669	231.465	+15.266	246.731
N_2	25.38	0.7547	216.865	- 4.88	211.985

$$S_2 = 3(263.627 + 231.614) + 2.25 \times 246.731 + 25.38 \times 211.985$$
$$= 7421 \text{ kJ/kmol fuel K}$$

$$_1S_{2\,gen} = S_2 - S_1 - {}_1Q_2/T_{res} = 7421 - 6577 + \frac{1.613 \times 10^6}{500} = \mathbf{4070} \; \frac{\mathbf{kJ}}{\mathbf{kmol\ fuel\ K}}$$

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Problems Involving Generalized Charts or Real Mixtures

Borgnakke and Sonntag

13.116

Liquid butane at 25°C is mixed with 150% theoretical air at 600 K and is burned in an adiabatic steady state combustor. Use the generalized charts for the liquid fuel and find the temperature of the products exiting the combustor.

```
25°C LIQ. C₄H₁₀     ┌──────────┐   Prod.        T_C = 425.2 K
─────────────────►  │ Adiab.   │  ────────►      T_r = 0.701
150% Air            │ Comb.    │   at T_p
600 K               └──────────┘
                         ↘ Q_CV = 0
```

$$\bar{h}_{C4H10} = \bar{h}^o_{f\,IG} + (\bar{h}_{LIQ} - \bar{h}^*) \qquad \text{see Fig. D.2}$$

$$= -126200 + (-4.85 \times 8.3145 \times 425.2) = -143\,346 \text{ kJ}$$

$$C_4H_{10} + 1.5 \times 6.5\,O_2 + 3.76 \times 9.75\,N_2$$

$$\rightarrow 4\,CO_2 + 5\,H_2O + 3.25\,O_2 + 36.66\,N_2$$

$$\bar{h}_{AIR} = 9.75(9245) + 36.66(8894) = 416\,193 \text{ kJ}$$

$$\Rightarrow \quad H_R = 416\,193 - 143\,346 = +272\,847 \text{ kJ}$$

$$H_P = 4(-393522 + \Delta\bar{h}^*_{CO2}) + 5(-241826 + \Delta\bar{h}^*_{H2O}) + 3.25\,\Delta\bar{h}^*_{O2} + 36.66\,\Delta\bar{h}^*_{N2}$$

Energy Eq.: $\qquad H_P - H_R = 0$

$$4\,\Delta\bar{h}^*_{CO2} + 5\,\Delta\bar{h}^*_{H2O} + 3.25\,\Delta\bar{h}^*_{O2} + 36.66\,\Delta\bar{h}^*_{N2} = 3\,056\,065$$

Trial and Error: $\quad \text{LHS}_{2000\,K} = 2\,980\,000, \qquad \text{LHS}_{2200\,K} = 3\,369\,866$

Linear interpolation to match RHS $\quad \Rightarrow \quad T_P = \textbf{2039 K}$

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13.117

Repeat Problem 13.101, but assume that saturated-liquid oxygen at 90 K is used instead of 25°C oxygen gas in the combustion process. Use the generalized charts to determine the properties of liquid oxygen.

Problem same as 13.101, except oxygen enters at 2 as saturated liquid at 90 K.

$$\dot{m}_{O2}/\dot{m}_{N2H4} = 0.5 = 32\dot{n}_{O2}/32\dot{n}_{N2H4} \quad \text{and} \quad \dot{Q}/\dot{m}_{N2H4} = -100 \text{ kJ/kg}$$

Energy Eq.: $Q_{CV} = H_P - H_R = -100 \times 32.045 = -3205$ kJ/kmol fuel

Reaction equation: $1\,N_2H_4 + \frac{1}{2}O_2 \rightarrow H_2O + H_2 + N_2$

At 90 K, $T_{r2} = 90/154.6 = 0.582 \Rightarrow \Delta\tilde{h}_f = 5.2$

Figure D.2, $(\bar{h}^* - \bar{h}) = 8.3145 \times 154.6 \times 5.2 = 6684$ kJ/kmol

$$\Delta\bar{h}_{AT\,2} = -6684 + 0.922 \times 32 \times (90 - 298.15) = -12825 \text{ kJ/kmol}$$

$$H_R = 50417 + \tfrac{1}{2}(0 - 12825) = 44005 \text{ kJ}, \quad H_P^\circ = -241826$$

Energy Eq: $\Delta\bar{h}_P = \Delta\bar{h}_{H_2O} + \Delta\bar{h}_{H_2} + \Delta\bar{h}_{N_2} = Q_{cv} + H_R - H_P^\circ = 282\,626$ kJ/kmol

From Table A.9, $\Delta H_{P\,2800K} = 282\,141$, $\Delta H_{P\,3000K} = 307\,988$ kJ/kmol

Therefore, $T_P = \mathbf{2804\ K}$

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13.118

A gas mixture of 50% ethane and 50% propane by volume enters a combustion chamber at 350 K, 10 MPa. Determine the enthalpy per kilomole of this mixture relative to the thermochemical base of enthalpy using Kay's rule.

$$\bar{h}^*_{MIX\ O} = 0.5(-84740) + 0.5(-103900) = -94320 \text{ kJ/kmol}$$

$$\bar{C}_{P0\ MIX} = 0.5 \times 30.07 \times 1.7662 + 0.5 \times 44.097 \times 1.67 = 63.583$$

$$\bar{h}^*_{350} - \bar{h}^*_{298} = 63.583 \times (350 - 298.2) = 3294 \text{ kJ/kmol}$$

Kay's rule: $T_{C\ MIX} = 0.5 \times 305.4 + 0.5 \times 369.8 = 337.6 \text{ K}$

$\qquad\qquad\qquad P_{C\ MIX} = 0.5 \times 4.88 + 0.5 \times 4.25 = 4.565 \text{ MPa}$

$\qquad\qquad T_r = 350/337.6 = 1.037, \quad P_r = 10/4.565 = 2.19$

From Fig. D.2: $\bar{h}^* - \bar{h} = 8.3145 \times 337.6 \times 3.53 = 9909 \text{ kJ/kmol}$

$\bar{h}_{MIX\ 350K,10MPa} = -94320 + 3294 - 9909 =$ **-100 935 kJ/kmol**

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13.119

A mixture of 80% ethane and 20% methane on a mole basis is throttled from 10 MPa, 65°C, to 100 kPa and is fed to a combustion chamber where it undergoes complete combustion with air, which enters at 100 kPa, 600 K. The amount of air is such that the products of combustion exit at 100 kPa, 1200 K. Assume that the combustion process is adiabatic and that all components behave as ideal gases except the fuel mixture, which behaves according to the generalized charts, with Kay's rule for the pseudocritical constants. Determine the percentage of theoretical air used in the process and the dew-point temperature of the products.

Reaction equation:

Fuel mix: $\bar{h}_{f\,FUEL}^0 = 0.2(-74873) + 0.8(-84740) = -82767$ kJ/kmol

$\bar{C}_{P0\,FUEL} = 0.2 \times 2.2537 \times 16.04 + 0.8 \times 1.7662 \times 30.07 = 49.718$

$\Delta\bar{h}_{FUEL}^* = 49.718(65 - 25) = 1989$ kJ/kmol

$T_{CA} = 305.4$ K, $T_{CB} = 190.4$ K $\Rightarrow$ $T_{c\,mix} = 282.4$ K

$P_{CA} = 4.88$, $P_{CB} = 4.60$ $\Rightarrow$ $P_{c\,mix} = 4.824$ MPa

$T_r = 338.2/282.4 = 1.198$, $P_r = 10/4.824 = 2.073$

$(\bar{h}^* - \bar{h})_{FUEL\,IN} = 8.31451 \times 282.4 \times 2.18 = 5119$

$\Rightarrow \bar{h}_{FUEL\,IN} = -82767 + 1989 - 5119 = -85897\ \frac{kJ}{kmol}$

Energy Eq.:

$1.8(-393522 + 44473) + 2.8(-241826 + 34506)$

$+ 3.2(x - 1)(29761) + (12.03x)(28109)$

$+ 85897 - (3.2x)(9245) - (12.03x)(8894) = 0$

a) $x = 4.104$ or **410.4 %**

b) $n_P = 1.8 + 2.8 + 3.2(4.104 - 1) + 12.03 \times 4.104 = 63.904$

$y_{H2O} = 2.8/63.904 = P_V/100$; $P_V = 4.38$ kPa, $T = $ **30.5°C**

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13.120

Saturated liquid butane enters an insulated constant pressure combustion chamber at 25°C, and x times theoretical oxygen gas enters at the same P and T. The combustion products exit at 3400 K. With complete combustion find x. What is the pressure at the chamber exit? and what is the irreversibility of the process?

Butane: $T_1 = T_o = 25^oC$, sat liq., $x_1 = 0$, $T_c = 425.2$ K, $P_c = 3.8$ MPa

Do the properties from the generalized charts

Fig. D.1: $T_{r1} = 0.7$, $P_{r1} = 0.1$, $P_1 = P_{r1}P_c = 380$ kPa

Figs. D.2 and D.3: $\left(\overline{h}_1^* - \overline{h}_1\right)_f = 4.85\ \overline{R}T_c$, $(\overline{s}_1^* - \overline{s}_1)_f = 6.8\ \overline{R}$

Oxygen: $T_2 = T_o = 25^oC$, X - Theoretical O_2

Products: $T_3 = 3400$ K, Assumes complete combustion

$$C_4H_{10} + 6.5X\ O_2 \rightarrow 4\ CO_2 + 5\ H_2O + 6.5(X\text{-}1)\ O_2$$

Energy Eq.: $Q_{cv} + H_R = H_P + W_{cv}$; $Q_{cv} = 0$, $W_{cv} = 0$

$$H_R = n\left(\overline{h}_f^o + \Delta\overline{h}\right)_{C4H10} = 1(\text{-}126\ 200 + \text{-}17\ 146) = \text{-}143\ 346\ kJ$$

Products: CO_2 $n\left(\overline{h}_f^o + \Delta\overline{h}\right)_{CO2} = 4(\text{-}393\ 522 + 177\ 836) = \text{-}862\ 744\ kJ$

H_2O $n\left(\overline{h}_f^o + \Delta\overline{h}\right)_{H2O} = 5(\text{-}241\ 826 + 149\ 073) = \text{-}463\ 765\ kJ$

O_2 $n\left(\overline{h}_f^o + \Delta\overline{h}\right)_{O2} = 6.5(X\text{-}1)(0 + 114\ 101) = (X\text{-}1)741\ 657\ kJ$

$H_P = \sum n_i \left(\overline{h}_f^o + \Delta\overline{h}\right)_i = 741\ 657X - 2\ 068\ 166$

Energy Eq.: $H_P = H_R$ solve for X; **X = 2.594**

Assume that the exit pressure equals the inlet pressure: $P_e = P_i = 380$ kPa

$$\overline{s}_{C_4H_{10}} = [\overline{s}_f^o - \overline{R}\ \ln\frac{P_1}{P_o} - (\overline{s}_1^* - \overline{s}_1)_f]\ ;\ \ \overline{s}_{O_2} = [\overline{s}^o - \overline{R}\ \ln\frac{P_1}{P_o}]$$

$S_R = S_{C4H10} + S_{O2} = [306.647 - 11.10 - 56.539]$

$+ [205.48 - 11.10] \times 6.5 \times 2.594\ = 3516.45$ kJ/K

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Products:

	n_i	y_i	$\overline{s}_i^o$	$-\overline{R}\ln\dfrac{y_iP_e}{P_o}$	$\overline{S}_i \dfrac{kJ}{kmol\ K}$
CO_2	4	0.2065	341.988	2.016	344.004
H_2O	5	0.2582	293.550	0.158	293.708
O_2	10.368	0.5353	289.499	-5.904	283.595

$S_P = \sum n_i\overline{S}_i = 5784.87$ kJ/K;

$I = T_O(S_P - S_R) = 298.15\ (5784.87 - 3516.45) = \mathbf{676\ 329\ kJ}$

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13.121

Liquid hexane enters a combustion chamber at 31°C, 200 kPa, at the rate 1 kmol/s 200% theoretical air enters separately at 500 K, 200 kPa, and the combustion products exit at 1000 K, 200 kPa. The specific heat of ideal gas hexane is $C_p = 143$ kJ/kmol K. Calculate the rate of irreversibility of the process.

Hexane: $T_c = 507.5$ K, $P_c = 3010$ kPa

$T_{r1} = 0.6$, Fig. D.1: $P_{rg} = 0.028$, $P_{g1} = P_{r1}P_c = 84.47$ kPa

Figs D.2 and D.3: $(\bar{h}_1^* - \bar{h}_1)_f = 5.16\, \bar{R}T_c$, $(\bar{s}_1^* - \bar{s}_1)_f = 8.56\, \bar{R}$

Air: $T_2 = 500$ K, $P_2 = 200$ kPa, 200% theoretical air

Products: $T_3 = 1000$ K, $P_3 = 200$ kPa

a) $\bar{h}_{C6H14} = \bar{h}_f^o - (\bar{h}_1^* - \bar{h}_1)_f + (\bar{h}_1^* - \bar{h}_0^*) + (\bar{h}_0^* - \bar{h}_0)$

$\bar{h}_0^* - \bar{h}_0 = 0$, $\bar{h}_1^* - \bar{h}_0^* = \bar{C}_P(T_1 - T_o) = 858$ kJ/kmol, $\bar{h}_f^o = -167300$ kJ/kmol

$\bar{h}_1^* - \bar{h}_1 = 5.16 \times 8.3145 \times 507.5 = 21773$ kJ/kmol, $\bar{h}_{C6H14} = $ **-188215 kJ/kmol**

$\bar{s}_{C6H14} = \bar{s}_{T_o}^o + \bar{C}_P \ln\frac{T_1}{T_o} - \bar{R}\ln\frac{P_1}{P_o} + (\bar{s}_1 - \bar{s}_1^*)$

$\bar{s}_{T_o}^o + \bar{C}_P\ln\frac{T_1}{T_o} - \bar{R}\ln\frac{P_1}{P_o} = 387.979 + 2.85 - 5.763 = 385.066$ kJ/kmol-K

$\bar{s}_1^* - \bar{s}_1 = 8.56 \times 8.3145 = 71.172$ kJ/kmol-K, $\bar{s}_{C6H14} = $ **313.894 kJ/kmol-K**

b) $C_6H_{14} + 19O_2 + 71.44N_2 \rightarrow 6CO_2 + 7H_2O + 9.5O_2 + 71.44N_2$

$T_{c\,prod} = \sum y_i T_{ci} = 179.3$ K, $T_{r3} = \dfrac{T_3}{T_{c\,prod}} = $ **5.58 → Ideal Gas**

c) 1st Law: $Q + H_R = H_P + W$; $W = 0$ => $Q = H_P - H_R$

$H_R = (\bar{h})_{C6H14} + 19\Delta\bar{h}_{O2} + 71.44\,\Delta\bar{h}_{N2}$

$= -188\,215 + 19\ 6086 + 71.44\ 5911 = 349701$ kJ/kmol fuel

$H_P = 6(\bar{h}_f^o + \Delta\bar{h})_{CO2} = 7(\bar{h}_f^o + \Delta\bar{h})_{H2O} + 9.5(\bar{h}_f^o + \Delta\bar{h})_{O2} + 71.44(\bar{h}_f^o + \Delta\bar{h})_{N2}$

CO_2 - $(\bar{h}_f^o + \Delta\bar{h}) = (-393522 + 33397) = -360125$ kJ/kmol

H_2O - $(\bar{h}_f^o + \Delta\bar{h}) = (-241826 + 26000) = -215826$ kJ/kmol

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$$O_2 \quad - \quad \left(\bar{h}_f^o + \Delta\bar{h}\right) = (0 + 22703) = 22703 \text{ kJ/kmol}$$

$$N_2 \quad - \quad \left(\bar{h}_f^o + \Delta\bar{h}\right) = (0 + 21463) = 21463 \text{ kJ/kmol}$$

$H_P = -1\ 922\ 537$ kJ; $\dot{Q} = \textbf{-2 272 238 kW}$

d) $\dot{I} = T_O\ \dot{n}\ (S_P - S_R) - \dot{Q}$; $T_O = 25^O C$

$$S_R = (\bar{s})_{C6H14} + 19\ (\bar{s}_{500}^o - \bar{R}\ln\frac{y_{O2}P_2}{P_o})_{O2} + 71.44(\bar{s}_{500}^o - \bar{R}\ln\frac{y_{N2}P_2}{P_o})_{N2}$$

$(\bar{s})_{C6H14} = 313.894$ kJ/kmol K, $(\bar{s}_{500}^o)_{O2} = 220.693$ kJ/kmol K

$(\bar{s}_{500}^o)_{N2} = 206.740$ kJ/kmol K, $y_{O2} = 0.21$, $y_{N2} = 0.79$

$\dot{S}_R = 19141.9$ kW/K

Products:

	n_i	y_i	$\bar{s}_i^o$	$-\bar{R}\ln\dfrac{y_iP_e}{P_o}$	$\bar{S}_i$ (kJ/kmol-K)
CO_2	6	0.0639	269.299	17.105	286.404
H_2O	7	0.0745	232.739	15.829	248.568
O_2	9.5	0.1011	243.579	13.291	256.87
N_2	71.44	0.7605	228.171	-3.487	224.684

$S_P = \sum n_i\bar{s}_i = 21950.1$ kJ/K;

$\dot{I} = T_O\ \dot{n}\ (S_P - S_R) - \dot{Q} = \textbf{3 109 628 kW}$

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Fuel Cells

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13.122

In Example 13.15, a basic hydrogen–oxygen fuel cell reaction was analyzed at 25°C, 100 kPa. Repeat this calculation, assuming that the fuel cell operates on air at 25°C, 100 kPa, instead of on pure oxygen at this state.

Anode: $2\,H_2 \rightarrow 4\,e^- + 4\,H^+$

Cathode: $4\,H^+ + 4\,e^- \rightarrow 1\,O_2 + 2\,H_2O$

Overall: $2\,H_2 + 1\,O_2 \rightarrow 2\,H_2O$

Example 13.16: $\Delta G_{25°C} = -474\ 283$ kJ (for pure O_2)

For $P_{O2} = 0.21 \times 0.1$ MPa:

$\bar{S}_{O2} = 205.148 - 8.3145 \ln 0.21 = 218.124$ kJ/kmol

$\Delta S = 2(69.950) - 2(130.678) - 1(218.124) = -339.58$ kJ/kmol K

$\Delta G_{25°C} = -571\ 660 - 298.15(-339.58) = -470\ 414$ kJ/kmol

$$E^\circ = \frac{470414}{96487 \times 4} = \mathbf{1.219\ V}$$

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13.123

Assume that the basic hydrogen-oxygen fuel cell operates at 600 K instead of 298 K as in example 13.15. Find the change in the Gibbs function and the reversible EMF it can generate.

Reaction: $2 H_2 + O_2 \Rightarrow 2 H_2O$

At a temperature of 600 K the water formed is in a vapor state. We can thus find the change in the enthalpy as

$$\Delta H^0_{600\ K} = 2\left(\overline{h}^o_f + \Delta \overline{h}\right)_{H2O\ g} - 2\left(\overline{h}^o_f + \Delta \overline{h}\right)_{H2} - \left(\overline{h}^o_f + \Delta \overline{h}\right)_{O2}$$

$$= 2(-241\ 826 + 10\ 499) - 2(0 + 8799) - 0 - 9245$$

$$= -489\ 497 \text{ kJ/4 kmol e-}$$

$$\Delta S^0_{600\ K} = 2\ \overline{s}^o_{f\ H2O\ g} - 2\ \overline{s}^o_{f\ H2} - \overline{s}^o_{f\ O2}$$

$$= 2 \times 213.051 - 2 \times 151.078 - 226.45$$

$$= -102.504 \text{ kJ/4 kmol e- K}$$

$$\Delta G^0_{600\ K} = \Delta H^0_{600\ K} - T\Delta S^0_{600\ K} = -489\ 497 - 600(-102.504)$$

$$= -427\ 995 \text{ kJ/4 kmol e-}$$

$$W^{rev} = -\Delta \bar{G}^0 = 427\ 995 \text{ kJ/4 kmol e-}$$

$$E^0 = \frac{-\Delta \bar{G}^0}{96485 \times 8} = \frac{427\ 995}{96\ 485 \times 4} = \mathbf{1.109\ V}$$

13.124

A reversible fuel cell operating with hydrogen and pure oxygen produces water at the reference conditions P_o, T_o; this is described in Example 13.15. Find the work output and any heat transfer, both per kmol of hydrogen. Assume an actual fuel cell operates with a second-law efficiency of 70%, and enough heat transfer takes place to keep it at 25°C. How much heat transfer is that per kmol of hydrogen?

The basic fuel cell analysis from Example 13.15.

$$\text{Reaction:} \qquad 2\,H_2 + O_2 \;\Rightarrow\; 2\,H_2O$$

$$W^{rev} = -\,\Delta G = 474\,283 \text{ kJ/2 kmol} = \mathbf{237\ 142\ kJ/kmol}$$

This assumed equilibrium with the surroundings so the heat transfer must be

$$Q = T_o\,(S_P - S_R) = 298.15\,(-326.6\,/\,2) = \mathbf{-48\ 688\ kJ/kmol}$$

that is the heat transfer goes out.

A 70% 2nd law efficiency must mean the actual work is

$$W = 0.7\,W^{rev} = 165\,999 \text{ kJ/kmol}$$

From the energy equation we get

$$Q = H_P - H_R + W = (-571\,660/2) + 165\,999 = \mathbf{-\ 119\ 831\ kJ/kmol}$$

All this heat transfer means that the fuel cell will have a hard time staying at the room temperature it would tend to be warmer.

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13.125

Consider a methane-oxygen fuel cell in which the reaction at the anode is

$$CH_4 + 2H_2O \rightarrow CO_2 + 8e^- + 8H^+$$

The electrons produced by the reaction flow through the external load, and the positive ions migrate through the electrolyte to the cathode, where the reaction is

$$8\ e^- + 8\ H^+ + 2\ O_2 \rightarrow 4\ H_2O$$

Calculate the reversible work and the reversible EMF for the fuel cell operating at 25°C, 100 kPa.

$$CH_4 + 2H_2O \rightarrow CO_2 + 8e^- + 8H^+$$

$$\text{and} \quad 8e^- + 8H^+ + 2CO_2 \rightarrow 4H_2O$$

$$\text{Overall} \quad CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

a) 25 °C assume all liquid H_2O and all comp. at 100 kPa

$$\Delta H^0_{25\,C} = -393\ 522 + 2(-285\ 830) - (-74\ 873) - 0 = -890\ 309 \text{ kJ}$$

$$\Delta S^0_{25\,C} = 213.795 + 2(69.950) - 186.251 - 2(205.148) = -242.852 \text{ kJ/K}$$

$$\Delta G^0_{25\,C} = -890\ 309 - 298.15(-242.852) = -817\ 903 \text{ kJ}$$

$$W^{rev} = -\Delta \bar{G}^0 = +817903 \text{ kJ}$$

$$E^0 = \frac{-\Delta \bar{G}^0}{96485 \times 8} = \frac{+817903}{96485 \times 8} = \textbf{1.06 V}$$

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13.126

Consider a methane-oxygen fuel cell in which the reaction at the anode is

$$CH_4 + 2H_2O \rightarrow CO_2 + 8e^- + 8H^+$$

The electrons produced by the reaction flow through the external load, and the positive ions migrate through the electrolyte to the cathode, where the reaction is

$$8\ e^- + 8\ H^+ + 2\ O_2 \rightarrow 4\ H_2O$$

Assume that the fuel cell operates at 1200 K instead of at room temperature.

$$CH_4 + 2H_2O \rightarrow CO_2 + 8e^- + 8H^+$$

$$\text{and} \quad 8e^- + 8H^+ + 2CO_2 \rightarrow 4H_2O$$

$$\text{Overall} \quad CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

At 1200 K assume all gas H_2O and all comp. at 100 kPa

$$\Delta H^0_{1200\ K} = 1(-393522 + 44473) + 2(-241826 + 34506) - 2(0 + 29761)$$
$$- 1[-74873 + 16.043 \times 2.254(1200 - 298.2)] = -780\ 948\ kJ$$

$$\Delta S^0_{1200\ K} = 1(279.390) + 2(240.485)$$

$$- 1(186.251 + 16.043 \times 2.254 \ln \frac{1200}{298.2}) - 2(250.011)$$

$$= 23.7397\ kJ/K$$

$$\Delta G^0_{1200\ K} = \Delta H^0_{1200\ K} - T\Delta S^0_{1200\ K} = -780\ 948 - 1200(23.7397)$$

$$= -809\ 436\ kJ$$

$$\mathbf{W^{rev} = +809\ 436\ kJ} \qquad E^0 = \frac{+809\ 436}{96\ 485 \times 8} = \mathbf{1.049\ V}$$

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13.127

For a PEC fuel cell operating at 350 K the constants in Eq.13.29 are: i_{leak} = 0.01, i_L = 2, i_o = 0.013 all A/cm^2, b = 0.08 V, c = 0.1 V, ASR = 0.01 Ω cm^2 and EMF = 1.22 V. Find the voltage and the power density for the current density i = 0.25, 0.75 and 1.0 A/cm^2.

The fuel cell output voltage is from Eq.13.29:

$$V = EMF - b \ln \left(\frac{i + i_{leak}}{i_o} \right) - i \, ASR_{ohmic} - c \ln \left(\frac{i_L}{i_L - (i + i_{leak})} \right)$$

i = 0.25:   V = 1.22 – 0.23966 – 0.0025 – 0.01393 = 0.9639 V,
             p = Vi = 0.241 W/cm^2

i = 0.75:   V = 1.22 – 0.32547 – 0.0075 – 0.0478 = 0.8392 V,
             p = Vi = 0.6294 W/cm^2

i = 1.0:   V = 1.22 – 0.34822 – 0.01 – 0.0703 = 0.7915 V,
             p = Vi = 0.7915 W/cm^2

Borgnakke and Sonntag

13.128

Assume the PEC fuel cell in the previous problem. How large an area does the fuel cell have to deliver 1 kW with a current density of 1 A/ cm^2?

The fuel cell output voltage is from Eq.13.29:

$$V = EMF - b \ln \left(\frac{i + i_{leak}}{i_o} \right) - i\, ASR_{ohmic} - c \ln \left(\frac{i_L}{i_L - (i + i_{leak})} \right)$$

i = 1.0: $V = 1.22 - 0.34822 - 0.01 - 0.0703 = 0.7915$ V,

$p = Vi = 0.7915$ W/cm^2

Total power $P = p\, A = 0.7915$ W/cm^2 $\times A = 1000$ W

$A = (1000 / 0.7915)$ cm^2 = **1263 cm^2**

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13.129

A SOC fuel cell at 900 K can be described by an EMF = 1.06 V and the constants in Eq.13.29 as : b = 0 V, c = 0.1 V, ASR = 0.04 Ω cm^2, i_{leak} = 0.01, i_L = 2, i_o = 0.13 all A/cm^2. Find the voltage and the power density for the current density i = 0.25, 0.75 and 1.0 A/cm^2.

The fuel cell output voltage is from Eq.13.29:

$$V = EMF - b \ln \left(\frac{i + i_{leak}}{i_o} \right) - i\, ASR_{ohmic} - c \ln \left(\frac{i_L}{i_L - (i + i_{leak})} \right)$$

i = 0.25: V = 1.06 – 0.0 – 0.01 – 0.01393 = 1.036 V,
p = Vi = 0.259 W/cm^2

i = 0.75: V = 1.06 – 0.0 – 0.03 – 0.0478 = 0.9822 V,
p = Vi = 0.737 W/cm^2

i = 1.0: V = 1.06 – 0.0 – 0.04 – 0.0703 = 0.95 V,
p = Vi = 0.95 W/cm^2

Borgnakke and Sonntag

13.130

Assume the SOC fuel cell in the previous problem. How large an area does the fuel cell have to deliver 1 kW with a current density of 1 A/ cm^2?

The fuel cell output voltage is from Eq.13.29:

$$V = EMF - b \ln \left(\frac{i + i_{leak}}{i_o} \right) - i\ ASR_{ohmic} - c \ln \left(\frac{i_L}{i_L - (i + i_{leak})} \right)$$

$i = 1.0$: $V = 1.06 - 0.0 - 0.04 - 0.0703 = 0.95$ V,

$\qquad\qquad p = Vi = 0.95$ W/cm^2

Total power $P = p\ A = 0.95$ W/cm$^2 \times A = 1000$ W

$\qquad\qquad A = (1000\ /\ 0.95)$ cm$^2 = $ **1053 cm^2**

Borgnakke and Sonntag

13.131

A PEC fuel cell operating at 25$^\text{o}$C, generates 1.0 V that also account for losses. For a total power of 1 kW what is the hydrogen mass flow rate?

The work term for the fuel cell is

$$W = -\Delta G^0_{298\ K} \qquad \text{so} \quad \dot{W} = -\dot{n}\ \Delta G^0_{298\ K}$$

$$\dot{m} = M\dot{n} = \frac{M\dot{W}}{-\Delta G^0_{298\ K}} = \frac{2.016\ \text{kg/kmol} \times 1\ \text{kW}}{0.5 \times 474\ 283\ \text{kJ/kmol}} = 8.5 \times 10^{-6}\ \text{kg/s}$$

Since the actual output was 1 V rather than the ideal 1.229 V we then infer an efficiency of (1/1.229 = 0.814) so

$$\dot{m}_{ac} = \dot{m}\ /\ 0.814 = \mathbf{10.45 \times 10^{-6}\ kg/s}$$

13.132

A basic hydrogen-oxygen fuel cell operates at 600 K instead of 298 K as in example 13.15. For a total power of 5 kW, find the hydrogen mass flow rate and the exergy in the exhaust flow.

Reaction: $2\,H_2 + O_2 \ \Rightarrow \ 2\,H_2O$

At a temperature of 600 K the water formed is in a vapor state. We can thus find the change in the enthalpy as

$$\Delta H^0_{600\,K} = 2\left(\overline{h}^o_f + \Delta\overline{h}\right)_{H2O\,g} - 2\left(\overline{h}^o_f + \Delta\overline{h}\right)_{H2} - \left(\overline{h}^o_f + \Delta\overline{h}\right)_{O2}$$

$$= 2(-241\,826 + 10\,499) - 2(0 + 8799) - 0 - 9245$$

$$= -489\,497 \text{ kJ/4 kmol e-}$$

$$\Delta S^0_{600\,K} = 2\,\overline{s}^o_{f\,H2O\,g} - 2\,\overline{s}^o_{f\,H2} - \overline{s}^o_{f\,O2}$$

$$= 2 \times 213.051 - 2 \times 151.078 - 226.45$$

$$= -102.504 \text{ kJ/4 kmol e- K}$$

$$\Delta G^0_{600\,K} = \Delta H^0_{600\,K} - T\Delta S^0_{600\,K} = -489\,497 - 600(-102.504)$$

$$= -427\,995 \text{ kJ/4 kmol e-}$$

$$W^{rev} = -\Delta\overline{G}^0 = 427\,995 \text{ kJ/4 kmol e-}$$

This was for 2 kmol hydrogen

$$\dot{m} = M\dot{n} = \frac{M\dot{W}}{-\Delta G^0_{600\,K}} = \frac{2.016 \text{ kg/kmol} \times 5 \text{ kW}}{0.5 \times 427\,995 \text{ kJ/kmol}} = \mathbf{47.1 \times 10^{-6} \text{ kg/s}}$$

The exhaust flow is water vapor at 600 K, 100 kPa so use Table B.1

$$\dot{m}\psi = \dot{m}\,[(h - h_o - T_o(s - s_o)]$$

$$= 47.1 \times 10^{-6}\,[3129 - 104.87 - 298.15\,(8.3037 - 0.3673)]$$

$$= \mathbf{0.031 \text{ kW}}$$

Borgnakke and Sonntag

13.133

Consider the fuel cell with methane in Problem 13.125. Find the work output and any heat transfer, both per kmol of methane. Assume an actual fuel cell operates with a second-law efficiency of 75%, and enough heat transfer takes place to keep it at 25°C. How much heat transfer is that per kmol of methane?

$$CH_4 + 2H_2O \rightarrow CO_2 + 8e^- + 8H^+$$

and $\qquad 8e^- + 8H^+ + 2CO_2 \rightarrow 4H_2O$

Overall reaction: $\qquad CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

$T = 25\ ^oC = 298$ K, assume all liquid H_2O and all comp. at 100 kPa

$$\Delta H^0_{25\,C} = -393\ 522 + 2(-285\ 830) - (-74\ 873) - 0 = -890\ 309 \text{ kJ/kmol}$$

$$\Delta S^0_{25\,C} = 213.795 + 2(69.950) - 186.251 - 2(205.148) = -242.852 \text{ kJ/K}$$

$$\Delta G^0_{25\,C} = -890\ 309 - 298.15(-242.852) = -817\ 903 \text{ kJ/kmol}$$

$$W^{rev} = -\Delta \bar{G}^0 = +817\ 903 \text{ kJ/kmol}$$

This assumed equilibrium with the surroundings so the heat transfer must be

$$Q = T_O\ (S_P - S_R) = 298.15\ (-242.852) = \textbf{–72 406 kJ/kmol}$$

A 75% 2nd law efficiency must mean the actual work is

$$W = 0.75\ W^{rev} = 613\ 427 \text{ kJ/kmol}$$

From the energy equation we get

$$Q = H_P - H_R + W = (-890\ 309) + 613\ 427 = \textbf{– 276 881 kJ/kmol}$$

All this heat transfer means that the fuel cell will have a hard time staying at the room temperature it would tend to be warmer.

Borgnakke and Sonntag

Combustion applications and efficiency

Borgnakke and Sonntag

13.134

For the combustion of methane 150% theoretical air is used at 25°C, 100 kPa and relative humidity of 70%. Find the composition and dew point of the products.

The reaction equation for stoichiometric mixture is:

$$CH_4 + v_{O2} \left(O_2 + 3.76\ N_2\right) \rightarrow 2\ H_2O + 1\ CO_2 + 3.76\ v_{O2}\ N_2$$

C balance: $v_{CO2} = 1$, H balance: $2v_{H2O} = 4$, N$_2$ balance: $v_{N2} = 3.76\ v_{O2}$

O balance: $2\ v_{O2} = v_{H2O} + 2v_{CO2} = 2 + 2 \times 1 \Rightarrow v_{O2} = 2$

150% theoretical air: $v_{O2} = 1.5 \times 2 = 3$ so now more O$_2$ and N$_2$

$$CH_4 + 3 \left(O_2 + 3.76\ N_2\right) \rightarrow 2\ H_2O + 1\ CO_2 + 11.28\ N_2 + 1\ O_2$$

Add water to the dry air from Eq.11.28

$$w = 0.622\ \frac{\phi P_g}{P_{tot} - \phi P_g} = 0.622\ \frac{0.7 \times 3.169}{100 - 0.7\ \times 3.169} = 0.0141$$

So the number of moles to add is from Eq.13.32

$$x = 7.655\ w = 7.655 \times 0.0141 = 0.108$$

and the added number of moles is $v_{O2}\ x = 0.324$, the products are then

Products: $2.324\ H_2O + 1\ CO_2 + 11.28\ N_2 + 1\ O_2$

The water partial pressure becomes

$$P_v = y_v\ P_{tot} = \frac{2.324}{2.324 + 1 + 11.28 + 1}\ 100 = 14.894\ kPa$$

$$T_{dew} = \mathbf{53.8^{o}C}$$

13.135

Pentane is burned with 120% theoretical air in a constant pressure process at 100 kPa. The products are cooled to ambient temperature, 20°C. How much mass of water is condensed per kilogram of fuel? Repeat the answer, assuming that the air used in the combustion has a relative humidity of 90%.

$$C_5H_{12} + 1.2 \times 8\,(O_2 + 3.76\,N_2) \rightarrow 5\,CO_2 + 6\,H_2O + 0.96\,O_2 + 36.1\,N_2$$

Products cooled to 20°C, 100 kPa, so for H_2O at 20°C: $P_g = 2.339$ kPa

$$y_{H2O\ MAX} = P_g/P = \frac{2.339}{100} = \frac{n_{H2O\ MAX}}{n_{H2O\ MAX} + 42.06} \implies n_{H2O\ MAX} = 1.007 < n_{H2O}$$

Therefore, $n_{H2O\ VAP} = 1.007$, $n_{H2O\ LIQ} = 6 - 1.007 = 4.993$

$$m_{H2O\ LIQ} = \frac{4.993 \times 18.015}{72.151} = \textbf{1.247 kg/kg fuel}$$

$$P_{v1} = 0.9 \times 2.339 = 2.105 \text{ kPa} \implies w_1 = 0.622 \times \frac{2.105}{97.895} = 0.013375$$

Water from moist air, Eq. 13.32

$$n_{H2O\ IN} = 0.013375 \times \frac{28.97}{18.015} \times 9.6\,(1 + 3.76) = 0.983 \text{ kmol}$$

$$n_{H2O\ OUT} = 0.983 + 6 = 6.983 \implies n_{H2O\ LIQ} = 6.983 - 1.007 = 5.976 \text{ kmol}$$

$$m_{H2O\ LIQ} = \frac{5.976 \times 18.015}{72.151} = \textbf{1.492 kg/kg fuel}$$

13.136

A gas turbine burns methane with 150% theoretical air. Assume the air is 25°C, 100 kPa and relative humidity of 80%. How large a fraction of the product mixture water comes from the moist inlet air?

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2}(O_2 + 3.76 N_2) \rightarrow 2 H_2O + 1 CO_2 + 3.76 \nu_{O2} N_2$$

C balance: $\nu_{CO2} = 1$, H balance: $2\nu_{H2O} = 4$, N$_2$ balance: $\nu_{N2} = 3.76 \nu_{O2}$

O balance: $2 \nu_{O2} = \nu_{H2O} + 2\nu_{CO2} = 2 + 2 \times 1 \quad => \quad \nu_{O2} = 2$

150% theoretical air: $\nu_{O2} = 1.5 \times 2 = 3$ so now more O$_2$ and N$_2$

$$CH_4 + 3(O_2 + 3.76 N_2) \rightarrow 2 H_2O + 1 CO_2 + 11.28 N_2 + 1 O_2$$

Add water to the dry air from Eq.11.28-29

$$w = 0.622 \frac{\phi P_g}{P_{tot} - \phi P_g} = 0.622 \frac{0.7 \times 3.169}{100 - 0.7 \times 3.169} = 0.0141$$

So the number of moles to add is from Eq.13.32

$$x = 7.655 \ w = 7.655 \times 0.0141 = 0.108$$

and the added number of moles is $\nu_{O2} \ x = 0.324$, the products are then

Products: $2.324 H_2O + 1 CO_2 + 11.28 N_2 + 1 O_2$

Fraction of water is $0.324/2.324 = 0.1394$ so **13.9%**

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13.137

In an engine a mixture of liquid octane and ethanol, mole ratio 9:1, and stoichiometric air are taken in at T_0, P_0. In the engine the enthalpy of combustion is used so that 30% goes out as work, 30% goes out as heat loss and the rest goes out the exhaust. Find the work and heat transfer per kilogram of fuel mixture and also the exhaust temperature.

$$0.9\ C_8H_{18} + 0.1\ C_2H_5OH + 11.55\ O_2 + 43.428\ N_2$$
$$\rightarrow 8.4\ H_2O + 7.4\ CO_2 + 43.428\ N_2$$

For 0.9 octane + 0.1 ethanol, convert to mole basis

$$\bar{H}^{\circ}_{RP\ mix} = 0.9\ \overset{\circ}{H}_{RP\ C_8H_{18}} + 0.1\ \overset{\circ}{H}_{RP\ C_2H_5OH}$$

$$= 0.9\ (-44\ 425) \times 114.232 + 0.1\ (-26\ 811) \times 46.069$$

$$= -4\ 690\ 797\ \frac{kJ}{kmol}$$

$$\hat{M}_{mix} = 0.9\ \hat{M}_{oct} + 0.1\ \hat{M}_{alc} = 107.414$$

Energy: $\quad \overset{\circ}{\bar{h}}_{in} + q_{in} = \bar{h}_{ex} + \omega_{ex} = \overset{\circ}{\bar{h}}_{ex} + \Delta\bar{h}_{ex} + \omega_{ex}$

$$\bar{h}^{\circ}_{ex} - \bar{h}^{\circ}_{in} = \bar{H}^{\circ}_{RP\ mix} \quad \Rightarrow \quad \omega_{ex} + \Delta\bar{h}_{ex} - q_{in} = -\bar{H}^{\circ}_{RP\ mix}$$

$$\omega_{ex} = -q_{in} = 0.3\left(-\bar{H}^{\circ}_{RP}\right) = \mathbf{1\ 407\ 239\ \frac{kJ}{kmol}} = \mathbf{13\ 101\ \frac{kJ}{kg\ Fu}}$$

$$\Delta\bar{h}_{prod} = \Delta\bar{h}_{ex} = 0.4\left(-\bar{H}^{\circ}_{RP}\right) = 1\ 876\ 319\ \frac{kJ}{kmol\ Fu}$$

$$\Delta\bar{h}_{prod} = 8.4\ \Delta\bar{h}_{H_2O} + 7.4\Delta\bar{h}_{CO_2} + 43.428\ \Delta\bar{h}_{N_2}$$

$$\Delta\bar{h}_{prod\ 1300} = 8.4 \times 38\ 941 + 7.4 \times 50\ 148 + 43.428 \times 31\ 503 = 2\ 066\ 312$$

$$\Delta\bar{h}_{prod\ 1200} = 8.4 \times 34\ 506 + 7.4 \times 44\ 473 + 43.428 \times 28\ 109 = 1\ 839\ 668$$

Linear interpolation to get the value of $\quad \Delta\bar{h}_{prod} = 1\ 876\ 319$

$$\Rightarrow \quad \text{satisfied for} \quad T = \mathbf{1216\ K}$$

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13.138

The gas turbine cycle in problem 10.28 has a q_H = 1160 kJ/kg air added by combustion. Assume the fuel is methane gas and the q_H equals the heating value at T_o. Find the A/F ratio on a mass basis.

The heating value and the q_H are scaled with the fuel mass and total mixture mass respectively so the relation is, see Eq.13.34,

$$q_H = \frac{HV}{m_{mix}/m_{fuel}} = \frac{HV}{A/F + 1} = 1160 \text{ kJ/kg-mix}$$

$$A/F = \frac{HV}{q_H} - 1 = \frac{50\ 010}{1160} - 1 = \textbf{43.11}$$

Comment: This is about 300% theoretical air.

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13.139

An oven heated by natural gas burners has the combustion take place inside a U-shaped steel pipe, so the heating is done from the outside surface of the pipe by radiation. Each burner delivers 15 kW of radiation, burning 110% theoretical air with methane. The products leave the pipe at 800 K. Find the flow(kg/s) of methane. The burner is now switched to oxygen-enriched air (30% O_2 and 70% N_2), so assume the same conditions as before with the same exit T. Find the new flow (kg/s) of methane needed.

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2} \left(O_2 + 3.76\,N_2\right) \rightarrow 2\,H_2O + 1\,CO_2 + c\,N_2$$

$$\text{O balance:} \quad 2\,\nu_{O2} = 2 + 2 \quad => \quad \nu_{O2} = 2$$

110% theoretical air: $\nu_{O2} = 1.1 \times 2 = 2.2$ so now more O_2 and N_2

$$CH_4 + 2.2 \left(O_2 + 3.76\,N_2\right) \rightarrow 2\,H_2O + 1\,CO_2 + 8.272\,N_2 + 0.2\,O_2$$

Energy Eq. assume reactant enter at reference T:

$$Q_{CV} = H_P - H_R = \overset{\circ}{H_P} + \Delta H_P - \overset{\circ}{H_R} = \overset{\circ}{H_{RP}} + \Delta H_P$$

$$\Delta H_P = 2\,\Delta\bar{h}_{H2O} + \Delta\bar{h}_{CO2} + 8.272\,\Delta\bar{h}_{N2} + 0.2\,\Delta\bar{h}_{O2}$$

$$= 2 \times 18002 + 22806 + 8.272 \times 15046 + 0.2 \times 15836 = 186\,438 \text{ kJ/kmol}$$

$$\overset{\circ}{H_{RP}} = 16.043 \times (-50\,010) = -802\,310 \text{ kJ/kmol}$$

$$Q_{CV} = -802\,310 + 186\,438 = -615\,872 \text{ kJ/kmol,}$$

$$\dot{m} = M\,\dot{Q}_{CV}/-Q_{CV} = 16.043 \times 15\,/615\,872 \text{ kg/s} = \mathbf{0.00039 \text{ kg/s}}$$

The new "air" just changes the nitrogen to oxygen ratio to $70/30 = 2.333$ so the actual reaction is changed to:

$$CH_4 + 2.2 \left(O_2 + 2.333\,N_2\right) \rightarrow 2\,H_2O + 1\,CO_2 + 5.1333\,N_2 + 0.2\,O_2$$

$$\Delta H_P = 2\,\Delta\bar{h}_{H2O} + \Delta\bar{h}_{CO2} + 5.1333\,\Delta\bar{h}_{N2} + 0.2\,\Delta\bar{h}_{O2}$$

$$= 2 \times 18002 + 22806 + 5.1333 \times 15046 + 0.2 \times 15836 = 139\,213 \text{ kJ/kmol}$$

$$Q_{CV} = -802\,310 + 139\,213 = -663\,097 \text{ kJ/kmol,}$$

$$\dot{m} = M\,\dot{Q}_{CV}/-Q_{CV} = 16.043 \times 15\,/663\,097 \text{ kg/s} = \mathbf{0.00036 \text{ kg/s}}$$

This is an 8% reduction in the fuel flow rate.

13.140

A slight disadvantage of the oxygen-enriched air for combustion is an increase in flame temperature, which tends to increase NOx. Find the flame temperature for the previous problem for both cases, standard air and oxygen-enriched air.

Energy Eq. assume reactant enter at reference T:

$$0 = H_P - H_R = H_P^{\circ} + \Delta H_P - H_R^{\circ} = H_{RP}^{\circ} + \Delta H_P$$

$$\Delta H_P = -H_{RP}^{\circ} = HV = 16.043 \times (50\ 010) = 802\ 310 \text{ kJ/kmol}$$

Reaction std. air:

$$CH_4 + 2.2\left(O_2 + 3.76\ N_2\right) \rightarrow 2\ H_2O + 1\ CO_2 + 8.272\ N_2 + 0.2\ O_2$$

$$\Delta H_P = 2\ \Delta\bar{h}_{H2O} + \Delta\bar{h}_{CO_2} + 8.272\ \Delta\bar{h}_{N_2} + 0.2\ \Delta\bar{h}_{O_2}$$

The heating value should be spread over 11.47 moles of products which is about 70 000 each so look in A.9 for oxygen or nitrogen T = 2200 – 2400 K.

$$\Delta H_{P\ 2200K} = 2 \times 83153 + 103562 + 8.272 \times 63362 + 0.2 \times 66770 = 807\ 352$$

$$\Delta H_{P\ 2000K} = 2 \times 72788 + 91\ 439 + 8.272 \times 56137 + 0.2 \times 59176 = 713\ 215$$

Interpolate   T = **2189 K**

Reaction std. air:

$$CH_4 + 2.2\left(O_2 + 2.333\ N_2\right) \rightarrow 2\ H_2O + 1\ CO_2 + 5.1333\ N_2 + 0.2\ O_2$$

$$\Delta H_P = 2\ \Delta\bar{h}_{H2O} + \Delta\bar{h}_{CO_2} + 5.1333\ \Delta\bar{h}_{N_2} + 0.2\ \Delta\bar{h}_{O_2}$$

The heating value should be spread over 8.333 moles of products which is about 96 000 each so look in A.9 for oxygen or nitrogen T ≈ 3000 K.

$$\Delta H_{P\ 3000K} = 2 \times 126\ 548 + 152\ 853 + 5.1333 \times 92715 + 0.2 \times 98\ 013 = 901\ 485$$

$$\Delta H_{P\ 2800K} = 2 \times 115\ 463 + 140\ 435 + 5.1333 \times 85\ 323 + 0.2 \times 90\ 080 = 827\ 366$$

$$\Delta H_{P\ 2600K} = 2 \times 104\ 520 + 128\ 074 + 5.1333 \times 77\ 963 + 0.2 \times 82\ 225 = 753\ 766$$

Interpolate   T = **2732 K**   a significant higher flame temperature.

13.141

A gas turbine burns methane with 200% theoretical air. The air and fuel comes in through two separate compressors bringing them from 100 kPa, 298 K to 1400 kPa and enters a mixing chamber and combustion chamber. What are the specific compressor work and q_H to be used in Brayton cycle calculation? Use constant specific heat to solve.

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2}\,(O_2 + 3.76\,N_2) \;\rightarrow\; 2\,H_2O + 1\,CO_2 + c\,N_2$$

O balance: $2\,\nu_{O2} = 2 + 2 \quad => \quad \nu_{O2} = 2$

200% theoretical air: $\nu_{O2} = 2 \times 2 = 4 \quad$ so now more O_2 and N_2

$$CH_4 + 4\,(O_2 + 3.76\,N_2) \;\rightarrow\; 2\,H_2O + 1\,CO_2 + 15.04\,N_2 + 2\,O_2$$

For 1 kmol fuel: $m = 16.043 + 4 \times 4.76 \times 28.97$

$$= 16.043 + 551.59 = 567.63 \text{ kg reactants}$$

The temperatures after compression are:

$$T_{2\,air} = T_1\,(P_2/P_1)^{(k-1)/k} = 298.15 \times 14^{0.2857} = 633.7 \text{ K} \quad (k = 1.4)$$

$$T_{2\,fuel} = T_1\,(P_2/P_1)^{(k-1)/k} = 298.15 \times 14^{0.230177} = 547.3 \text{ K} \;\; (k = 1.299)$$

$$w_{c\,air} = C_p\,(T_2 - T_1) = 1.004(633.7 - 298.15) = 336.89 \text{ kJ/kg air}$$

$$w_{c\,fuel} = C_p\,(T_2 - T_1) = 2.254(547.3 - 298.15) = 561.68 \text{ kJ/kg fuel}$$

$$w_{c\,total} = \frac{m_{fuel}}{m}\,w_{c\,fuel} + \frac{m_{air}}{m}\,w_{c\,air} = \frac{16.04}{567.63}\,561.58 + \frac{551.59}{567.63}\,336.89$$

$$= \textbf{343.24 kJ/kg mix}$$

$$q = -H^{\circ}_{RP} = HV \text{ but scaled to be per kg reactants}$$

$$q = \frac{50\,010 \times 16.043}{567.63} = \textbf{1413.4 kJ/kg} \quad (= \frac{HV}{A/F + 1})$$

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13.142

Find the equivalent heat transfer q_H to be used in a cycle calculation for constant pressure combustion when the fuel is (a) methane and (b) gaseous octane in both cases use water vapor in the products and a stoichiometric mixture.

Methane combustion becomes

$$CH_4 + \nu_{O2}\,(O_2 + 3.76\,N_2\,) \rightarrow 1\,CO_2 + 2\,H_2O + 7.52\,N_2$$

Oxygen balance: $\quad 2\nu_{O2} = 2 + 2 = 4 \Rightarrow \nu_{O2} = 2, \quad \nu_{N2} = 7.52$

$$A/F = \frac{2 \times 31.999 + 7.52 \times 28.013}{16.043} = 17.12 = m_{air}/m_{fuel}$$

$$q = \frac{HV}{m_{mix}/m_{fuel}} = \frac{HV}{A/F + 1} = \frac{50\,010}{17.12 + 1} = \mathbf{2760\ kJ/kg}$$

Octane combustion, see reaction and A/F in Example 13.1, HV from Table 13.3

$$q = \frac{HV}{m_{mix}/m_{fuel}} = \frac{HV}{A/F + 1} = \frac{44\,788}{15 + 1} = \mathbf{2799\ kJ/kg}$$

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13.143

Consider the steady combustion of propane at 25°C with air at 400 K. The products exit the combustion chamber at 1200 K. It may be assumed that the combustion efficiency is 90%, and that 95% of the carbon in the propane burns to form carbon dioxide; the remaining 5% forms carbon monoxide. Determine the ideal fuel–air ratio and the heat transfer from the combustion chamber.

Ideal combustion process, assumed adiabatic, excess air to keep 1200 K out.

$$C_3H_8 + 5x\ O_2 + 18.8x\ N_2 \rightarrow 3\ CO_2 + 4\ H_2O + 5(x - 1)\ O_2 + 18.8x\ N_2$$

$$H_R = -103900 + 5x(0 + 3027) + 18.8x(0 + 2971) = -103900 + 70990x$$

$$H_P = 3(-393522 + 44473) + 4(-241826 + 34506)$$

$$+ 5(x - 1)(0 + 29761) + 18.8x(0 + 28109) = -2025232 + 677254x$$

Energy Eq.: $H_P - H_R = 0$ Solving, x = 3.169

$$FA_{IDEAL} = 1/(23.8 \times 3.169) = \mathbf{0.01326}$$

$$FA_{ACTUAL} = 0.01326/0.90 = 0.01473$$

$$C_3H_8 + 14.26\ O_2 + 53.62\ N_2$$

$$\rightarrow 2.85\ CO_2 + 0.15\ CO + 4\ H_2O + 9.335\ O_2 + 53.62\ N_2$$

$$H_R = -103900 + 14.26(0 + 3027) + 53.62(0 + 2971) = +98\ 570\ kJ$$

$$H_P = 2.85(-393522 + 44473) + 0.15(-110527 + 28427) + 4(-241826 + 34506)$$

$$+ 9.335(0 + 29761) + 53.62(0 + 28109) = -51\ 361\ kJ$$

$$Q_{CV} = H_P - H_R = \mathbf{-149\ 931\ kJ}$$

Borgnakke and Sonntag

13.144

A gasoline engine is converted to run on propane. Assume the propane enters the engine at 25°C, at the rate 40 kg/h. Only 90% theoretical air enters at 25°C such that 90% of the C burns to form CO_2, and 10% of the C burns to form CO. The combustion products also include H_2O, H_2 and N_2, exit the exhaust at 1000 K. Heat loss from the engine (primarily to the cooling water) is 120 kW. What is the power output of the engine? What is the thermal efficiency?

Propane: $T_1 = 25^OC$, $\dot{m} = 40$ kg/hr, M = 44.094 kg/kmol

Air: $T_2 = 25^OC$, 90% theoretical Air produces 90% CO_2, 10% CO

Products: $T_3 = 1000$ K, CO_2, CO, H_2O, H_2, N_2

$C_3H_8 + 4.5O_2 + 16.92N_2 \rightarrow 2.7\ CO_2 + 0.3CO + 3.3H_2O + 0.7H_2 + 16.92N_2$

$\dot{n}_{C3H8} = \dot{m}/(M\times3600) = 0.000252$ kmol/s

Energy eq.: $\dot{Q} + H_R = H_P + \dot{W}$; $\dot{Q} = $ -120 kW

$H_R = n_{C3H8}\ \overline{h}_f^o = $ -103 900 kJ

Products:

CO$_2$ - $n_{CO2}(\overline{h}_f^o + \Delta\overline{h}) = 2.7(-393522 + 33397) = $ -972337.5 kJ

CO - $n_{CO}(\overline{h}_f^o + \Delta\overline{h}) = 0.3(-110527 + 21686) = $ -26652 kJ

H$_2$O - $n_{H2O}(\overline{h}_f^o + \Delta\overline{h}) = 3.3(-241826 + 26000) = $ -712226 kJ

H$_2$ - $n_{H2}(\overline{h}_f^o + \Delta\overline{h}) = 0.7(0 + 20663) = $ 14464.1 kJ

N$_2$ - $n_{N2}(\overline{h}_f^o + \Delta\overline{h}) = 16.92(0 + 21463) = $ 363154 kJ

$H_P = \sum n_i\ (\overline{h}_f^o + \Delta\overline{h})_i = $ -1 333 598 kJ

$\dot{W} = \dot{Q} + \dot{n}(H_R - H_P) = $ **189.9 kW**

C_3H_8: Table 13.3 $H_{RPo} = $ -50343 kJ/kg

$\dot{HHV} = \dot{n}_{C3H8}\ M(-H_{RPo}) = $ 559.4 kW

$\eta_{th} = \dot{W}/\dot{HHV} = $ **0.339**

Borgnakke and Sonntag

13.145

A small air-cooled gasoline engine is tested, and the output is found to be 1.0 kW. The temperature of the products is measured to 600 K. The products are analyzed on a dry volumetric basis, with the result: 11.4% CO_2, 2.9% CO, 1.6% O_2 and 84.1% N_2. The fuel may be considered to be liquid octane. The fuel and air enter the engine at 25°C, and the flow rate of fuel to the engine is 1.5×10^{-4} kg/s. Determine the rate of heat transfer from the engine and its thermal efficiency.

$$a\,C_8H_{18} + b\,O_2 + 3.76b\,N_2$$
$$\rightarrow 11.4\,CO_2 + 2.9\,CO + c\,H_2O + 1.6\,O_2 + 84.1\,N_2$$
$$b = \frac{84.1}{3.76} = 22.37, \qquad a = \frac{1}{8}(11.4 + 2.9) = 1.788$$
$$c = 9a = 16.088$$
$$C_8H_{18} + 12.5\,O_2 + 47.1\,N_2$$
$$\rightarrow 6.38\,CO_2 + 1.62\,CO + 9\,H_2O + 0.89\,O_2 + 47.1\,N_2$$

$$H_R = \bar{h}^0_{f\,C_8H_{18}} = \text{-250 105 kJ/kmol}$$

$$H_P = 6.38(\text{-393 522} + 15\,788) + 1.62(\text{-110527} + 10\,781)$$
$$+ 9(\text{-241 826} + 12\,700) + 0.89(0 + 11187)$$
$$+ 47.1(0 + 10712) = \text{-4 119 174 kJ/kmol}$$

$$H_P - H_R = \text{-4 119 174} - (\text{-250 105}) = \text{-3 869 069 kJ/kmol}$$

$$\dot{H}_P - \dot{H}_R = (0.00015/114.23)(\text{-3 869 069}) = \text{-5.081 kW} \quad (M_{fuel} = 114.23)$$

$$\dot{Q}_{CV} = \text{-5.081} + 1.0 = \textbf{-4.081 kW}$$

Fuel heating value from table 13.3

$$\dot{Q}_H = \dot{m}\,HV = 0.00015\,(47\,893) = 7.184 \text{ kW}$$

$$\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = 1.0/7.184 = \textbf{0.139}$$

Borgnakke and Sonntag

13.146

A gasoline engine uses liquid octane and air, both supplied at P_0, T_0, in a stoichiometric ratio. The products (complete combustion) flow out of the exhaust valve at 1100 K. Assume that the heat loss carried away by the cooling water, at 100°C, is equal to the work output. Find the efficiency of the engine expressed as (work/lower heating value) and the second law efficiency.

$$C_8H_{18} + \nu_{O_2}(O_2 + 3.76\,N_2) \rightarrow 8\,CO_2 + 9\,H_2O + 47\,N_2$$

$$2\,\nu_{O_2} = 16 + 9 \Rightarrow \nu_{O_2} = 12.5$$

$$\text{LHV} = 44\,425\,\tfrac{kJ}{kg\ fuel} \Rightarrow \text{LHV} = 5.07476{\times}10^6\,\tfrac{kJ}{kmol\ fuel}$$

$$\Delta H_{P\ 1100} = 8 \times 38885 + 9 \times 30190 + 47 \times 24760 = 1746510$$

C.V. Total engine

$$H_{in} = H_{ex} + W + Q_{loss} = H_{ex} + 2\,W$$

$$\Rightarrow 2\,W = H_{in} - H_{ex} = H_R - H_v = -\overset{\circ}{H}_{RP} + \Delta H_R - \Delta H_{P\ 1100}$$

$$= 5.07476{\times}10^6 + 0 - 1746510 = 3328250$$

$$W = 1.664{\times}10^6\,\tfrac{kJ}{kmol\ fuel}$$

$$\eta_{th} = \frac{W}{\text{LHV}} = \frac{1.664{\times}10^6}{5.07476{\times}10^6} = \mathbf{0.328}$$

Find entropies in and out:

inlet:

$$\bar{S}^{\circ}_{Fu} = 360.575$$

$$\bar{S}^{\circ}_{O_2} = 205.148 - 8.3145\ \ln\frac{1}{4.76} = 218.12$$

$$\bar{S}^{\circ}_{N_2} = 191.609 - 8.3145\ \ln\frac{3.76}{4.76} = 193.57$$

$$\bar{S}^{\circ}_{in} = 360.575 + 12.5 \times 218.12 + 47 \times 193.57 = 12185$$

exit: $\bar{S}^{\circ}_{CO_2} = 275.528 - 8.3145\ \ln\frac{8}{64} = 292.82$

$$\bar{S}^{\circ}_{H2O} = 236.732 - 8.3145\ \ln\frac{9}{64} = 253.04$$

Borgnakke and Sonntag

$$\bar{S}_{N_2}^{\circ} = 231.314 - 8.1345 \ln \frac{47}{64} = 233.88$$

$$\bar{S}_{ex}^{\circ} = 8 \times 292.82 + 9 \times 253.04 + 47 \times 233.88 = 15612$$

Assume the same Q_{loss} out to 100°C reservoir in the reversible case and compute Q_0^{rev}:

$$\bar{S}_{in} + Q_0^{rev}/T_0 = \bar{S}_{ex} + Q_{loss}/T_{res}$$

$$\begin{aligned}
Q_0^{rev} \quad &= T_0\left(\bar{S}_{ex} - \bar{S}_{in}\right) + Q_{loss}\, T_0/T_{res} \\
&= 298.15(15612 - 12185) + 1.664 \times 10^6 \times 298.15/373.15 \\
&= 2.351 \times 10^6 \frac{kJ}{kmol\ fuel}
\end{aligned}$$

$$H_{in} + Q_0^{rev} = H_{ex} + W^{rev} + Q_{loss}$$

$$\Rightarrow W^{rev} = H_{in} - H_{ex} - Q_{loss} + Q_0^{rev} = W_{ac} + Q_0^{rev} = 4.015 \times 10^6 \frac{kJ}{kmol\ fuel}$$

$$\eta_{II} = W_{ac}/W^{rev} = 1.664 \times 10^6/4.015 \times 10^6 = \mathbf{0.414}$$

Borgnakke and Sonntag

Review Problems

Borgnakke and Sonntag

13.147

Repeat Problem 13.26 for a certain Utah coal that contains, according to the coal analysis, 68.2% C, 4.8% H, 15.7% O on a mass basis. The exiting product gas contains 30.9% CO, 26.7% H_2, 15.9% CO_2 and 25.7% H_2O on a mole basis.

Convert the mass concentrations to number of kmol per 100 kg coal:

C : 68.2/12.01 = 5.679 H_2: 4.8/2.016 = 2.381

O_2: 15.7/32.00 = 0.491

Now the combustion equation reads

$$x(5.679\,C + 2.381\,H_2 + 0.491\,O_2) + y\,H_2O + z\,O_2 \ \ in$$

$$30.9\,CO + 26.7\,H_2 + 15.9\,CO_2 + 25.7\,H_2O \ \ out$$

in 100 kmol of mix out

Now we can do the atom balance to find (x, y, z)

C : $5.679x = 30.9 + 15.9 \quad \rightarrow \quad x = 8.241$

H_2: $2.381 \times 8.241 + y = 26.7 + 25.7 \quad \rightarrow \quad y = 32.778$

O_2: $0.491 \times 8.241 + \dfrac{32.778}{2} + z = \dfrac{30.9}{2} + 15.9 + \dfrac{25.7}{2}$

$$\rightarrow \quad z = 23.765$$

Therefore, for 100 kmol of mixture out,

require: **824.1 kg** of coal

32.778 kmol of steam

23.765 kmol of oxygen

Borgnakke and Sonntag

13.148

Many coals from the western United States have a high moisture content. Consider the following sample of Wyoming coal, for which the ultimate analysis on an as-received basis is, by mass:

Component	Moisture	H	C	S	N	O	Ash
% mass	28.9	3.5	48.6	0.5	0.7	12.0	5.8

This coal is burned in the steam generator of a large power plant with 150% theoretical air. Determine the air–fuel ratio on a mass basis.

Converting from mass analysis:

Substance	S	H_2	C	O_2	N_2
c/M =	0.5/32	3.5/2	4.86/12	12/32	0.7/28
kmol / 100 kg coal	0.0156	1.75	4.05	0.375	0.025
Product	SO_2	H_2O	CO_2		
oxygen required	0.0156	0.875	4.05	--	--

Combustion requires then oxygen as: $0.0156 + 0.875 + 4.05 = 4.9406$

The coal does include $0.375\ O_2$ so only $4.5656\ O_2$ from air/100 kg coal

$AF = 1.5 \times (4.5656 + 4.5656 \times 3.76) \times 28.97/100 =$ **9.444 kg air/kg coal**

Borgnakke and Sonntag

13.149

A fuel, C_xH_y, is burned with dry air and the product composition is measured on a dry mole basis to be: 9.6% CO_2, 7.3% O_2 and 83.1% N_2. Find the fuel composition (x/y) and the percent theoretical air used.

$$\nu_{Fu}C_xH_y + \nu_{O_2}O_2 + 3.76\nu_{O_2}N_2 \rightarrow 9.6\ CO_2 + 7.3\ O_2 + 83.1\ N_2 + \nu_{H_2O}H_2O$$

N_2 balance: $\quad 3.76\nu_{O_2} = 83.1 \Rightarrow \nu_{O_2} = \mathbf{22.101}$

O_2 balance: $\quad \nu_{O_2} = 9.6 + 7.3 + \frac{1}{2}\nu_{H_2O} \Rightarrow \nu_{H_2O} = \mathbf{10.402}$

H balance: $\quad \nu_{Fu}\,y = 2\,\nu_{H_2O} = \mathbf{20.804}$

C balance: $\quad \nu_{Fu}\,x = \mathbf{9.6}$

Fuel composition ratio = x/y = 9.6/20.804 = **0.461**

Theoretical air $= \dfrac{\nu_{O_2AC}}{\nu_{O_2stoich}}\,100 = \dfrac{22.101}{9.6 + \frac{1}{4}\times 29.804}\,100 = \mathbf{149.3\%}$

Borgnakke and Sonntag

13.150

In an engine liquid octane and ethanol, mole ration 9:1, and stoichiometric air are taken in at 298 K, 100 kPa. After complete combustion, the products run out of the exhaust system where they are cooled to $10^{\circ}C$. Find the dew point of the products and the mass of water condensed per kilogram of fuel mixture.

Reaction equation with 0.9 octane and 0.1 ethanol is

$$0.9\ C_8H_{18} + 0.1\ C_2H_5OH + 11.55\ O_2 + 43.428\ N_2$$
$$\rightarrow 8.4\ H_2O + 7.4\ CO_2 + 43.428\ N_2$$

$$y_{H_2O} = \frac{8.4}{8.4 + 7.4 + 43.428} = 0.1418$$

$$P_{H_2O} = y_{H_2O}P_{tot} = \textbf{14.3 kPa} \qquad \Rightarrow\ T_{dew} = \textbf{52.9 °C}$$

$$10\ ^{\circ}C \Rightarrow P_{H_2O} = 1.2276 \quad \Rightarrow \quad y_{H_2O} = 0.012276 = \frac{x}{x + 7.4 + 43.428}$$

$$\Rightarrow x = \textbf{0.6317} \quad \Rightarrow\ \Delta v_{H_2O} = -7.77\ \tfrac{kmol}{kmol\ Fu\ mix}$$

$$M_{mix} = \sum y_i M_i = 0.9 \times 114.23 + 0.1 \times 46.069 = 107.414$$

$$m_{H_2O\ cond} = \frac{-\Delta v_{H_2O} \times 18.015}{107.414} = \textbf{1.303}\ \tfrac{kg}{kg\ Fu\ mix}$$

Borgnakke and Sonntag

13.151

Find the lower heating value for the fuel blend in problem 15.134 with a scaling as in Table 13.3

Reaction equation with 0.9 octane and 0.1 ethanol is

$$0.9\ C_8H_{18} + 0.1\ C_2H_5OH + 11.55\ O_2 + 43.428\ N_2$$
$$\rightarrow 8.4\ H_2O + 7.4\ CO_2 + 43.428\ N_2$$

We can solve several ways, use Table A.10 to find H°_{RP} and do it on a mole basis and then divide with M for mixture or we can convert the mole fractions to mass fractions and use Table 13.3 values. Let us use Table 13.3.

$$M_{mix} = \sum y_i M_i = 0.9 \times 114.23 + 0.1 \times 46.069 = 107.414$$

$$c_{ethanol} = yM/M_{mix} = \frac{0.1 \times 46.069}{107.414} = 0.042889, \quad c_{octane} = 0.957111$$

$$LHV = c_{ethanol}\ LHV_{ethanol} + c_{octane}\ LHV_{octane}$$
$$= 0.042889\ (26\ 811) + 0.957111\ (44\ 425)\quad kJ/kg$$
$$= \textbf{43 670 kJ/kg}$$

Borgnakke and Sonntag

13.152

Determine the higher heating value of the sample Wyoming coal as specified in Problem 13.148.

The higher heating value is for liquid water in the products. We need the composition of the products.

Converting from mass analysis:

Substance	S	H_2	C	O_2	N_2
c/M =	0.5/32	3.5/2	4.86/12	12/32	0.7/28
kmol / 100 kg coal	0.0156	1.75	4.05	0.375	0.025
Product	SO_2	H_2O	CO_2		

So the combustion equation becomes (for 100 kg coal)

$$\text{Fuel} + \text{Air} \rightarrow 1.75\ H_2O + 4.05\ CO_2 + 0.0156\ SO_2$$

The formation enthalpies are from Table A.10. Therefore,

$$\bar{h}_{RP0} = H_P^o - H_R^o = 4.05(-393\ 522) + 1.75(-285\ 830) + 0.0156(-296\ 842)$$

$$= -2\ 098\ 597\ \text{kJ/100 kg coal}$$

So that HHV = **+20 986 kJ/kg coal**

Borgnakke and Sonntag

13.153

Ethene, C_2H_4, and propane, C_3H_8, in a 1:1 mole ratio as gases are burned with 120% theoretical air in a gas turbine. Fuel is added at 25°C, 1 MPa and the air comes from the atmosphere, 25°C, 100 kPa through a compressor to 1 MPa and mixed with the fuel. The turbine work is such that the exit temperature is 800 K with an exit pressure of 100 kPa. Find the mixture temperature before combustion, and also the work, assuming an adiabatic turbine.

$$\phi = 1: \quad C_2H_4 + C_3H_8 + 8\ O_2 + 30.08\ N_2 \rightarrow 5\ CO_2 + 6\ H_2O + 30.08\ N_2$$

$$\phi = 1.2: \quad C_2H_4 + C_3H_8 + 9.6\ O_2 + 36.096\ N_2$$
$$\rightarrow 5\ CO_2 + 6\ H_2O + 1.6\ O_2 + 36.096\ N_2$$

45.696 kmol air per 2 kmol fuel

C.V. Compressor (air flow)

Energy Eq.: $w_c = h_2 - h_1$ Entropy Eq.: $s_2 = s_1$ $\Rightarrow$

$P_{r2} = P_{r1} \times P_2/P_1 = 13.573$ $\Rightarrow$ $T_{2\ air} =$ **570.8 K**

$w_c = 576.44 - 298.34 =$ **278.1 kJ/kg = 8056.6 kJ/kmol air**

C.V. Mixing Chanber (no change in composition)

$$\dot{n}_{air}\bar{h}_{air\ in} + \dot{n}_{Fu1}\bar{h}_{1\ in} + \dot{n}_{Fu2}\bar{h}_{2\ in} = (SAME)_{exit}$$

$$\left(\bar{C}_{P\ F1} + \bar{C}_{P\ F2}\right)\left(T_{exit} - T_0\right) = 45.696\ \bar{C}_{P\ air}\left(T_{2\ air} - T_{exit}\right)$$

C_2H_4: $\bar{C}_{P\ F1} = 43.43$, C_3H_8: $\bar{C}_{P\ F2} = 74.06$, $\bar{C}_{P\ air} = 29.07$

$$T_{exit} = \frac{45.696\bar{C}_{P\ air}T_2 + (43.43 + 74.06)\ T_0}{43.43 + 74.06 + 45.696\ \bar{C}_{P\ air}} =$$ **548.7 K**

Dew Point Products: $y_{H_2O} = \dfrac{6}{5 + 6 + 1.6 + 36.096} = 0.1232$

$P_{H_2O} = y_{H_2O}P_{tot} = 123.2$ kPa $\Rightarrow T_{dew} =$ **105.5°C**

C.V. Turb. + combustor + mixer + compressor (no Q)

$w_{net} \quad = H_{in} - H_{out} = H_R - H_{P\ 800}$ (800°K out so no liquid H_2O)

$$= \bar{h}^°_{fC2H4} + \bar{h}^°_{fC3H8} - 5\ \bar{h}_{CO_2} - 6\ \bar{h}_{H_2O} - 1.6\ \bar{h}_{O_2} - 36.096\ \bar{h}_{N_2}$$

$$= \textbf{2 576 541}\ \frac{\textbf{kJ}}{\textbf{2 kmol Fu}}$$

$w_T = w_{net} + w_{comp} =$ **2 944 695** $\frac{\textbf{kJ}}{\textbf{2 kmol Fu}}$

Borgnakke and Sonntag

13.154

Phenol C_6H_5OH (M = 94.11 kg/kmol, Cp = 1.7 kJ/kg-K) is used in a combustion process being burned with stoichiometric air. The reactants are supplied at 1.2 MPa, the air at T_o and the phenol as liquid at 100°C. Enough heat transfer Q_1, takes place after the combustion so the products reach 1500 K. After the heat transfer the products are sent through a turbine where they expand to 120 kPa.

  a) Find the heat transfer Q_1 [kJ/kmol fuel].
  b) Find the enthalpy of formation of the fuel as vapor.
  c) Find the specific turbine work output W [kJ/kmol fuel].

First do the combustion equation for Phenol C_6H_5OH

$$C_6H_5OH + \nu_{O_2} (O_2 + 3.76\, N_2) \rightarrow 6\, CO_2 + 3\, H_2O + 26.32\, N_2$$

O balance: $1 + 2\nu_{O_2} = 6 \times 2 + 3 = 15 \Rightarrow \nu_{O_2} = 7$

N_2 balance: $\nu_{N_2} = 3.76\ \nu_{O_2} = 3.76 \times 7 = 26.32$

Energy Eq.: $0 = Q_1 + H_{R1} - H_{P2} = Q_1 - \overset{\circ}{H}_{RP} + \Delta H_{R1} - \Delta H_{P2}$

Table 13.3: $\overset{\circ}{H}_{RP} = 94.11\ (-31\ 117) = -2\ 928\ 421$ kJ/kmol, (fuel as liquid)

$\Delta H_{R1} = M\ Cp\ (T_1 - T_0) = 94.11 \times 1.7 \times (100 - 25) = 11\ 999$ kJ/kmol

Table A.9: $\Delta H_{P2} = 6\ \Delta\overline{h}_{CO2} + 3\ \Delta\overline{h}_{H2O} + 26.32\ \Delta\overline{h}_{N2}$

$= 6 \times 61705 + 3 \times 481549 + 26.32 \times 38405 = 1\ 525\ 497$ kJ/kmol

$Q_1 = \overset{\circ}{H}_{RP} - \Delta H_{R1} + \Delta H_{P2} = -2\ 928\ 421 - 11\ 999 + 1\ 525\ 497$

$= -1\ 414\ 923$ kJ/kmol

$\overset{\circ}{H}_{RP} = 94.11\ (-31\ 774) = -2\ 990\ 251 = 6\ \overline{h}^{\circ}_{f\ CO_2} + 3\ \overline{h}^{\circ}_{f\ H_2O} - \overline{h}^{\circ}_{f\ fuel}$

$\overline{h}^{\circ}_{f\ fuel} = 3\ \overline{h}^{\circ}_{f\ H_2O} + 6\ \overline{h}^{\circ}_{f\ CO_2} - H_{RP}$

$= 3\ (-241\ 826) + 6(-393\ 522) - (-2\ 990\ 251) = \mathbf{-96\ 359\ kJ/kmol}$

For W_T we need exit T and as product is mainly N_2 use air approximation (same k):

Constant C_P, k: $T_3 = T_2\ (P_3/P_2)^{\frac{k-1}{k}} = 1500\ (120/1200)^{0.2857} = 776.95$ K

Variable C_P, k: $P_{r3} = P_{r2}\ (P_3/P_2) = 483.16/10 = 48.316$ $\Rightarrow$ $\mathbf{T_3 = 850\ K}$

See Table A.7.2 for this or use the standard entropy for this calculation

$\Delta H_{P3} = 6 \times 25418 + 3 \times 19969.5 + 26.32 \times 16634.5 = 650\ 237$ kJ/kmol

$W = \Delta H_{P2} - \Delta H_{P3} = 1\ 525\ 497 - 650\ 237 = \mathbf{875\ 260\ kJ/kmol}$

Borgnakke and Sonntag

13.155

The phenol in Problem 13.154 is used in a gasturbine cycle where we need the heating value (or the negative of the enthalpy of combustion at the temperature after the compressor).

a) What is the T after the compressor assuming a simple adiabatic compression from 100 kPa to 1.2 MPa?

b) Find the $H_{RP} = H_P - H_R$ both at that T except phenol is liquid at 100°C?

Assume the reactants are similar to air

Constant C_P, k: $\quad T_2 = T_1 (P_2/P_1)^{\frac{k-1}{k}} = 298.15 (1200/100)^{0.2857} = 606.4$ K

Variable C_P, k: $\quad P_{r2} = P_{r1} (P_2/P_1) = 1.0907 \times 12 = 13.088 \quad \Rightarrow \quad \mathbf{T_3 = 600}$ **K**

See Table A.7.2 for this or use the standard entropy for this calculation
First do the combustion equation for Phenol C_6H_5OH

$$C_6H_5OH + v_{O_2} (O_2 + 3.76 N_2) \rightarrow 6 CO_2 + 3 H_2O + 26.32 N_2$$

O balance: $\quad 1 + 2v_{O_2} = 6 \times 2 + 3 = 15 \quad \Rightarrow \quad v_{O_2} = 7$

N_2 balance: $\quad v_{N_2} = 3.76 \ v_{O_2} = 3.76 \times 7 = 26.32$

For enthalpy of fuel use $C_P \approx 1.7$ kJ/kgK similar to other fuels

$\quad \Delta H_R = \Delta \bar{h}_{fuel} + 7(\Delta \bar{h}_{O2} + 3.76 \Delta \bar{h}_{N2})$
$\quad\quad\quad = 1.7 \times 94.11 \times (600 - 298.15) + 7(9245 + 3.76 \times 8894) = 347\ 097$ kJ/kmol
$\quad \Delta H_P = 6 \times 12906 + 3 \times 10499 + 26.32 \times 8894 = 343\ 023$ kJ/kmol

$\quad \overset{\circ}{H}_{RP} = 94.11 \ (-31\ 774) = -2\ 990\ 251$

Gaseous fuel value at 298 K which is then a hypothetical state as the reference, so since the 600 K is higher than the 100°C then it does not matter.
Now we get

$$\overset{\circ}{H}_{RP} = \overset{\circ}{H}_{RP} + \Delta H_P - \Delta H_R = -2\ 990\ 251 + 343\ 023 - 347\ 097$$
$$= -2\ 994\ 325 \text{ kJ/kmol}$$

For a cycle calculation we would then use $q_H = \overset{\circ}{H}_{RP} / [94.11 \times (1 + AF)]$ in kJ/kg mix

Borgnakke and Sonntag

13.156

Consider the gas mixture fed to the combustors in the integrated gasification combined cycle power plant, as described in Problem 13.30. If the adiabatic flame temperature should be limited to 1500 K, what percent theoretical air should be used in the combustors?

Product	CH_4	H_2	CO	CO_2	N_2	H_2O	H_2S	NH_3
% vol.	0.3	29.6	41.0	10.0	0.8	17.0	1.1	0.2

Mixture may be saturated with water so the gases are (H_2S and NH_3 out)

CH_4	H_2	CO	CO_2	N_2	n
0.3	29.6	41.0	10.0	0.8	81.7

$$y_{v\ max} = 7.384/3000 = n_v/(n_v + 81.7)$$

Solving, $n_v = 0.2$ kmol, the rest is condensed

$$\{0.3\,CH_4 + 29.6\,H_2 + 41.0\,CO + 10.0\,CO_2 + 0.8\,N_2$$

$$+ 0.2\,H_2O + 35.9x\,O_2 + 3.76 \times 35.9x\,N_2\} \rightarrow$$

$$51.3\,CO_2 + 30.4\,H_2O + 35.9(x - 1)\,O_2 + (135.0x + 0.8)\,N_2$$

For the fuel gas mixture at 40°C,

$$n\bar{C}_{P0\ mix} = 0.3 \times 16.04 \times 2.2537 + 29.6 \times 2.016 \times 14.2091$$

$$+ 41.0 \times 28.01 \times 1.0413 + 10.0 \times 44.01 \times 0.8418$$

$$+ 0.8 \times 28.013 \times 1.0416 + 0.2 \times 18.015 \times 1.8723 = 2455.157$$

$$n\bar{h}^0_{f\ mix} = 0.3(-74873) + 29.6(0) + 41.0(-110527)$$

$$+ 10.0(-393522) + 0.8(0) + 0.2(-241826) = -8537654\ kJ$$

$$H_{mix} = -8\,537\,654 + 2455.157(40 - 25) = -8\,500\,827\ kJ$$

Assume air enters at 25°C: $\bar{h}_{air} = 0$

Products at 1500 K:

$$H_P = 51.3(-393522 + 61705) + 30.4(-241826 + 48149)$$

$$+ 35.9(x - 1)(0 + 40600) + (135x + 0.8)(0 + 38405)$$

$$= -24\,336\,806 + 6\,642\,215x$$

Energy Eq.: $\qquad H_P = H_R = H_{mix}$

$$x = \frac{+24336809 - 8500827}{6642215} = 2.384 \quad \text{or} \quad \textbf{238 \% theo. air}$$

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13.157

Carbon monoxide, CO, is burned with 150% theoretical air and both gases are supplied at 150 kPa and 600 K. Find the heating value and the adiabatic flame temperature.

$$CO + \nu_{O_2}(O_2 + 3.76\,N_2) \rightarrow CO_2 + \nu_{N_2}\,N_2$$

O balance: $\quad 1 + 2\nu_{O_2} = 1 \quad \Rightarrow \quad \nu_{O_2} = 0.5 \quad \Rightarrow \quad \nu_{O_2\,actual} = 0.75$

Now the actual reaction equation has excess oxygen as

$$CO + 0.75\,(O_2 + 3.76\,N_2) \rightarrow CO_2 + 2.82\,N_2 + 0.25\,O_2$$

From the definition of heating value (negative of enthalpy of combustion), Eq.13.14 or 13.15

$$HV = -\overset{\circ}{H}_{RP} = -(\overset{\circ}{H}_P - \overset{\circ}{H}_R) = -\bar{h}^o_{f\,CO_2} - 0 + \bar{h}^o_{f\,CO}$$

$$= -(-393\,522) + (-110\,527) = \textbf{282 995 kJ/kmol CO}$$

$$= \textbf{10 103 kJ/kg CO} \qquad \text{(as for Table 13.3)}$$

Actual energy Eq.: $\quad H_R = H_P = \overset{\circ}{H}_P + \Delta H_P = \overset{\circ}{H}_R + \Delta H_R$

$$\Delta H_P = \overset{\circ}{H}_R + \Delta H_R - \overset{\circ}{H}_P = -\overset{\circ}{H}_{RP} + \Delta\bar{h}_{CO} + 0.75\,\Delta\bar{h}_{O_2} + 2.82\,\Delta\bar{h}_{N_2}$$

$$= HV + \Delta\bar{h}_{CO} + 0.75\,\Delta\bar{h}_{O_2} + 2.82\,\Delta\bar{h}_{N_2}$$

$$= 282\,995 + 8942 + 0.75 \times 9245 + 2.82 \times 8894$$

$$= 323\,952 \text{ kJ/kmol}$$

The left hand side is

$$\Delta H_P = \Delta\bar{h}_{CO_2} + 0.25\,\Delta\bar{h}_{O_2} + 2.82\,\Delta\bar{h}_{N_2}$$

$$\Delta H_{P\,2600} = 128\,074 + 0.25 \times 82\,225 + 2.82 \times 77\,963 = 368\,486$$

$$\Delta H_{P\,2400} = 115\,779 + 0.25 \times 74\,453 + 2.82 \times 70\,640 = 333\,597$$

$$\Delta H_{P\,2200} = 103\,562 + 0.25 \times 66\,770 + 2.82 \times 63\,362 = 298\,935$$

Now we can do a linear interpolation for the adiabatic flame temperature

$$T = 2200 + 200\,\frac{323\,952 - 298\,935}{333\,597 - 298\,935} = \textbf{2344 K}$$

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13.158

A rigid container is charged with butene, C_4H_8, and air in a stoichiometric ratio at P_0, T_0. The charge burns in a short time with no heat transfer to state 2. The products then cool with time to 1200 K, state 3. Find the final pressure, P_3, the total heat transfer, $_1Q_3$, and the temperature immediately after combustion, T_2.

The reaction equation is, having used C and H atom balances:

$$C_4H_8 + \nu_{O_2}(O_2 + 3.76\ N_2) \rightarrow 4\ CO_2 + 4\ H_2O + 3.76\ \nu_{O_2}N_2$$

Counting now the oxygen atoms we require $\nu_{O_2} = 6$.

C.V. analysis gives:

$$U_3 - U_1 = Q - W = Q = H_3 - H_1 - P_2V_2 + P_1V_1 = H_3 - H_1 - \bar{R}(n_2T_3 - n_1T_1)$$

$$H_3 - H_1 = H_{P\ 1200} - H_R^\circ = H_P^\circ - H_R^\circ + \Delta H_P = \hat{M}\ H_{RP}^\circ + \Delta H_P$$

$$= 56.108 \times (-45\ 316) + 950055 = -1\ 592\ 535\ \text{kJ/kmol}$$

Where $\hat{M} = 56.108$ and $n_1 = 1 + 6 \times 4.76 = 29.56$, $n_2 = 4 + 4 + 6 \times 3.76 = 30.56$,

Table A.9 at 1200 K: $\Delta h_{CO_2} = 44473$, $\Delta h_{H_2O} = 34506$, $\Delta h_{N_2} = 28109$.

Now solving for the heat transfer:

$$Q = -1592535 - 8.3145(30.56 \times 1200 - 29.56 \times 298.15) = -1824164\ \tfrac{\text{kJ}}{\text{kmol fuel}}$$

To get the pressure, assume ideal gases:

$$P_3 = \frac{n_2\bar{R}T_3}{V_2} = P_1\frac{n_2T_3}{n_1T_1} = 101.325\ \text{kPa} \times \frac{30.56 \times 1200}{29.56 \times 298.15} = 421.6\ \text{kPa}$$

Before heat transfer takes place we have constant U so:

$$U_2 - U_1 = 0 = H_2 - H_1 - n_2\bar{R}T_2 + n_1\bar{R}T_1$$

Now split the enthalpy $H_2 = H_P^\circ + \Delta H_P(T_2)$ and arrange things with the unknowns on LHS and knowns on RHS:

$$\Delta H_P - n_2\bar{R}T_2 = H_R - H_P^\circ - n_1\bar{R}T_1 = 2\ 542\ 590 - 73278 = 2\ 469\ 312$$

Trial and error leads to:

LHS (3000 K) = 3 209 254 - 30.56 × 8.31451 × 3000 = 2 446 980

LHS (3200 K) = 3 471 331 - 30.56 × 8.31451 × 3200 = 2 658 238

linear interpolation   T = **3021 K**

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13.159

Natural gas (approximate it as methane) at a ratio of 0.3 kg/s is burned with 250% theoretical air in a combustor at 1 MPa where the reactants are supplied at T_0. Steam at 1 MPa, 450°C at a rate of 2.5 kg/s is added to the products before they enter an adiabatic turbine with an exhaust pressure of 150 kPa. Determine the turbine inlet temperature and the turbine work assuming the turbine is reversible.

$$CH_4 + \nu_{O_2}(O_2 + 3.76\ N_2) \rightarrow CO_2 + 2\ H_2O + 7.52\ N_2$$

$$2\ \nu_{O_2} = 2 + 2 \implies \nu_{O_2} = 2 \implies \text{Actual } \nu_{O_2} = 2 \times 2.5 = 5$$

$$CH_4 + 5\ O_2 + 18.8\ N_2 \rightarrow CO_2 + 2\ H_2O + 3\ O_2 + 18.8\ N_2$$

C.V. combustor and mixing chamber

$$H_R + n_{H_2O}\bar{h}_{H_2O\ in} = H_{P\ ex}$$

$$n_{H_2O} = \frac{\dot{n}_{H_2O}}{\dot{n}_{Fu}} = \frac{\dot{m}_{H_2O}M_{Fu}}{\dot{m}_{Fu}M_{H_2O}} = \frac{2.5 \times 16.043}{0.3 \times 18.015} = 7.421\ \tfrac{\text{kmol steam}}{\text{kmol fuel}}$$

Energy equation becomes

$$n_{H_2O}\left(\bar{h}_{ex} - \bar{h}_{in}\right)_{H_2O} + \left(\Delta\bar{h}_{CO_2} + 2\Delta\bar{h}_{H_2O} + 3\Delta\bar{h}_{O_2} + 18.8\Delta\bar{h}_{N_2}\right)_{ex}$$

$$= -H^{\circ}_{RP} = 50\ 010 \times 16.043 = 802\ 310$$

$$\left(\bar{h}_{ex} - \bar{h}_{in}\right)_{H_2O} = \Delta\bar{h}_{H_2O\ ex} - 15072.5,\ \text{so then:}$$

$$\left(\Delta\bar{h}_{CO_2} + 9.421\Delta\bar{h}_{H_2O} + 3\Delta\bar{h}_{O_2} + 18.8\Delta\bar{h}_{N_2}\right)_{ex} = 914\ 163\ \tfrac{\text{kJ}}{\text{kmol fuel}}$$

Trial and error on T_{ex}

$$T_{ex} = 1000\ K \implies \text{LHS} = 749\ 956\ ; \qquad T_{ex} = 1100\ K \implies \text{LHS} = 867429$$

$$T_{ex} = 1200\ K \implies \text{LHS} = 987\ 286 \qquad \implies \qquad T_{ex} \cong \mathbf{1139\ K = T_{in\ turbine}}$$

If air then $T_{ex\ turbine} \approx 700\ K$ and $T_{avg} \approx 920\ K$. Find $\bar{C}_{P\ mix}$ between 900 and 1000 K. From Table A.9:

$$\bar{C}_{P\ mix} = \sum n_i \bar{C}_{P_i} \Big/ \sum n_i = \frac{53.67 + 9.421(40.63) + 3(34.62) + 18.8(32.4)}{32.221}$$

$$= 35.673\ \text{kJ/kmol K}$$

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$\bar{C}_{V\,mix} = \bar{C}_{P\,mix} - \bar{R} = 27.3587$ kJ/kmol, $k_{mix} = 1.304$

$T_{ex\,turbine} = 1139\,(150\,/\,1000)^{0.2331} = 732$ K

$\Delta H_{732} = 19370.6 + 9.421(15410) + 3(13567) + 18.8(12932)$

$\qquad = 448\,371$ kJ/kmol

$w_T = H_{in} - H_{ex} = \Delta H_{in} - \Delta H_{ex} = 914\,163 - 448\,371 = 465\,792\,\dfrac{kJ}{kmol\,fuel}$

$\dot{W}_T = \dot{n}_{Fu} w_T = \dot{m}_{Fu} w_T / \bar{M}_{Fu} = (0.3 \times 465\,792)16.043 = \mathbf{8710\ kW}$

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13.160

The turbine in Problem 13.153 is adiabatic. Is it reversible, irreversible, or impossible?

Inlet to the turbine is the exit from the mixing of air and fuel at 1 MPa. From solution to 13.153, we have:

$$\bar{C}_{P\,C_2H_2} = 43.43, \quad \bar{C}_{P\,C_3H_8} = 74.06, \quad T_{turbine,in} = 548.7 \text{ K}$$

$$C_2H_4 + C_3H_8 + 9.6\,O_2 + 36.096\,N_2$$

$$\rightarrow 5\,CO_2 + 6\,H_2O + 1.6\,O_2 + 36.096\,N_2$$

$$S_{ex} - S_{in} = \int dQ/T + S_{gen} = S_{gen} \geq \phi$$

Inlet: 1 MPa, 548.7 K $\bar{S}_{Fu} = \bar{S}_i^{\circ} + \bar{C}_{P\,Fu}\ln(T/T_0)$

	n_i	y_i	$\bar{s}_i^{\circ}$	$-\bar{R}\ln\dfrac{y_iP}{P_0}$	$\bar{S}_i$
C_2H_4	1	0.02097	245.82	12.989	258.809
C_3H_8	1	0.02097	315.09	12.989	328.079
O_2	9.6	0.2013	223.497	-5.816	217.681
N_2	36.096	0.7568	209.388	-16.828	192.56

$$S_{in} = 258.809 + 328.079 + 9.6 \times 217.681 + 36.096 \times 192.56 = 9627.3$$

	n_i	y_i	$\bar{s}_i^{\circ}$	$-\bar{R}\ln\dfrac{y_iP}{P_0}$	S_i
CO_2	5	0.1027	257.496	18.925	276.421
H_2O	6	0.1232	223.826	17.409	241.235
O_2	1.6	0.0329	235.92	28.399	264.319
N_2	36.096	0.7413	221.016	2.489	223.505

$$S_{ex} = 5 \times 276.421 + 6 \times 241.235 \; 1.6 \times 264.319$$

$$+ 36.096 \times 223.505 = 11320 \frac{kJ}{2kmol\ Fu\ K}$$

$$S_{gen} = S_{ex} - S_{in} = \mathbf{1693\ \frac{kJ}{2kmol\ Fu\ K}} > 0$$

Possible, but one should check the state after combustion to account for generation by combustion alone and then the turbine expansion separately.

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13.161

Consider the combustion process described in Problem 13.119.

a. Calculate the absolute entropy of the fuel mixture before it is throttled into the combustion chamber.

b. Calculate the irreversibility for the overall process.

From solution to 13.119, fuel mixture $0.8\, C_2H_6 + 0.2\, CH_4$ at 65°C, 10 MPa

$\bar{C}_{P0\,FUEL} = 49.718$ kJ/kmol K. Using Kay's rule: $T_{r1} = 1.198$, $P_{r1} = 2.073$

and   x = 410.4 % theoretical air

or $13.13\, O_2 + 49.36\, N_2$ in at 600 K, 100 kPa

and $1.8 CO_2 + 2.8 H_2O + 9.93 O_2 + 49.36 N_2$ out at 100 kPa, 1200 K

$\bar{s}^*_{0\,FUEL} = 0.2(186.251) + 0.8(229.597) - 8.3145(0.2 \ln 0.2 + 0.8 \ln 0.8)\ = 225.088$

$$\Delta \bar{s}^*_{TP} = 49.718 \ln \frac{338.2}{298.2} - 8.3145 \ln \frac{10}{0.1} = -32.031$$

From Fig. D.3: $(\bar{s}^* - \bar{s})_{FUEL} = 1.37 \times 8.3145 = 11.391$

$\bar{s}_{FUEL} = 225.088 - 32.031 - 11.391 = \textbf{181.66 kJ/kmol K}$

Air at 600 K, 100 kPa

	n_i	y_i	$\bar{s}^\circ_i$	$-\bar{R}\ln(y_i P/P_0)$	S_i
O_2	13.13	0.21	226.45	+12.976	239.426
N_2	49.36	0.79	212.177	+1.96	214.137

$S_{AIR} = \sum n_i \bar{s}_i = 13713.47$ kJ/K

$S_R = 181.66 + 13713.47 = 13895.1$ kJ/K

Products at 1200 K, 100 kPa

PROD	n_i	y_i	$\bar{s}^o_i$	$-\bar{R}\ln(y_i P/P_0)$	S_i
CO_2	1.8	0.0282	279.390	+29.669	309.059
H_2O	2.8	0.0438	240.485	+26.008	266.493
O_2	9.93	0.1554	250.011	+15.479	265.490
N_2	49.36	0.7726	234.227	+2.145	236.372

$S_P = \sum n_i \bar{s}_i = 15606.1$ kJ/K

$I = T_0(S_P - S_R) - Q_{CV} = 298.15(15\,606.1 - 13\,895.1) + 0\ = \textbf{510 132 kJ}$

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13.162

Liquid acetylene, C_2H_2, is stored in a high-pressure storage tank at ambient temperature, 25°C. The liquid is fed to an insulated combustor/steam boiler at the steady rate of 1 kg/s, along with 140% theoretical oxygen, O_2, which enters at 500 K, as shown in Fig. P13.162. The combustion products exit the unit at 500 kPa, 350 K. Liquid water enters the boiler at 10°C, at the rate of 15 kg/s, and superheated steam exits at 200 kPa.

    a.Calculate the absolute entropy, per kmol, of liquid acetylene at the storage tank state.

    b. Determine the phase(s) of the combustion products exiting the combustor boiler unit, and the amount of each, if more than one.

    c. Determine the temperature of the steam at the boiler exit.

a) C_2H_2: $\bar{S}^{\circ}_{IG\,25°C} = 200.958$ kJ/kmol-K   from A.10

$$T_{R_1} = 298.2/308.3 = 0.967 \implies \text{From Fig. D.1:} \qquad P_{R_1} = 0.82$$

$$P_1 = 0.82 \times 6.14 = 5.03 \text{ MPa}, \quad \left(\bar{S}^* - \bar{S}\right)_1 = 3.33\bar{R} = 27.687 \text{ kJ/kmolK}$$

$$\bar{S}_{liq\,T_1\,P_1} = \bar{S}^{\circ}_{T_0\,P_0} + \Delta T - \bar{R}\ln(P_1/P^{\circ}) + \left(\bar{S} - \bar{S}^*\right)_{P_1\,T_1} = \mathbf{140.695\ \frac{kJ}{kmol\,K}}$$

b) $1\ C_2H_2 + 1.4 \times 2.5\ O_2 \rightarrow 2\ CO_2 + 1\ H_2O + 1\ O_2$

$$H_1 = 226731 + (-3.56 \times \bar{R} \times 308.3) = 217605 \text{ kJ};$$

$$H_2 = 3.5(0 + 6086) = 21301 \text{ kJ}$$

Products $T_3 = 350$ K $= 76.8°C \implies P_G = 41.8$ kPa

$$y_{V\,max} = \frac{P_G}{P} = \frac{41.8}{500} = 0.0836 = \frac{n_{V\,max}}{n_{V\,max} + 2 + 1} \implies n_{V\,max} = 0.2737 = n_{V\,gas\,mix}$$

$$\implies n_{liq} = 1 - 0.2737 = \mathbf{0.7263}$$

Gas Mix $= \mathbf{2\ CO_2 + 0.2737\ H_2O + 1\ O_2}$

c) $H_{liq3} = 0.7263(-285830 + 18.015(321.5 - 104.9)) = -204764$ kJ

$$H_{gas\,mix3} = 2(-393522 + 2036) + 0.2737(-241826 + 1756) + 1541 = -847138 \text{ kJ}$$

$$H_3 = H_{liq3} + H_{gas\,mix3} = -204\,764 - 847\,138 = -1\,051\,902 \text{ kJ}$$

$$H_3 - H_1 - H_2 = -1\,290\,808 \text{ kJ}$$

or $\dot{H}_3 - \dot{H}_1 - \dot{H}_2 = -1\,290\,808/26.038 = -49\,574$ kW $= \dot{m}_{H_2O}\left(h_4 - h_5\right)$

$$h_5 = 42.01 + \frac{49574}{15} = 3346.9 \implies T_5 = \mathbf{433.4°C}$$

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 13
English Units

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CHAPTER 13

CONTENT CHAPTER 13

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Concept Problems

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13.163E

The output gas mixture of a certain air-blown coal gasifier has the composition of producer gas as listed in Table 13.2. Consider the combustion of this gas with 120% theoretical air at 14.7 lbf/in.^{2} pressure. Find the dew point of the products and the mass of water condensed per pound-mass of fuel if the products are cooled 20 F below the dew point temperature?

$$\{3\,CH_4 + 14\,H_2 + 50.9\,N_2 + 0.6\,O_2 + 27\,CO + 4.5\,CO_2\}$$

$$+ 31.1\,O_2 + 116.9\,N_2 \rightarrow 34.5\,CO_2 + 20\,H_2O + 5.2\,O_2 + 167.8\,N_2$$

Products:

$$y_{H2O} = y_{H2O\ MAX} = \frac{P_G}{14.7} = \frac{20}{34.5 + 20 + 5.2 + 167.8}$$

$$\Rightarrow P_G = 1.2923\ lbf/in^2 \rightarrow T_{DEW\ PT} = \mathbf{110.4\ F}$$

At T = 90.4 F, P_G = 0.7089 lbf/in^2

$$y_{H2O\ max} = \frac{0.7089}{14.7} = \frac{n_{H2O}}{n_{H2O} + 34.5 + 5.2 + 167.8} \quad => \quad n_{H2O} = 10.51$$

$$\Rightarrow n_{H2O\ LIQ} = 20 - 10.51 = 9.49\ lb\ mol$$

$$m_{H2O\ LIQ} = \frac{9.49(18)}{3(16)+14(2)+50.9(28)+0.6(32)+27(28)+4.5(44)}$$

$$= \mathbf{0.069\ lbm/lbm\ fuel}$$

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Energy and Enthalpy of Formation

13.164E

What is the enthalpy of formation for oxygen as O_2? If O? For CO_2?

From Table F.6

$$\bar{h}^{\circ}_{f\,O2} = 0$$

$$\bar{h}^{\circ}_{f\,O} = 107\ 124\ \text{Btu/lbmol}$$

$$\bar{h}^{\circ}_{f\,CO2} = -169\ 184\ \text{Btu/lbmol} \quad \text{(or Table F.11)}$$

13.165E

One alternative to using petroleum or natural gas as fuels is ethanol (C_2H_5OH), which is commonly produced from grain by fermentation. Consider a combustion process in which liquid ethanol is burned with 110% theoretical air in a steady flow process. The reactants enter the combustion chamber at 77 F, and the products exit at 140 F, 14.7 lbf/in.^{2}. Calculate the heat transfer per lbmol fuel.

$$C_2H_5OH + 1.1 \times 3\,(O_2 + 3.76\,N_2) \rightarrow 2CO_2 + 3H_2O + 0.3O_2 + 12.408N_2$$

Products at 140 F, 14.7 lbf/in^2, Psat = 2.892 psia

$$y_{H2O} = 2.892/14.7 = n_v/(2 + 0.3 + n_v + 12.408)$$

$$n_v = 3.6023 > 3 \quad \Rightarrow \quad \text{no condensation}$$

$\bar{h}_f^\circ$ = -119 252 Btu/lbmol as liquid from F.11

H_R = -119 252 + 0 + 0 = -119 252 Btu/ lbmol fuel

H_P = 2(-169184 + 570) + 3(-47518 + 506.5)

$\quad\quad$ + 0.3(0 + 443.7) + 12.408(0 + 438.4) = -472 690 Btu/ lbmol fuel

$Q_{CV} = H_P - H_R$ = **-353 438 Btu/lbmol fuel**

An ethanol flame

13.166E

Liquid methanol is burned with stoichiometric air both supplied at P_o, T_o in a constant pressure process and the products exits a heat exchanger at 1600 R. Find the heat transfer per lbmol fuel

$$CH_3OH + 1.5\,(O_2 + 3.76\,N_2) \rightarrow 1\,CO_2 + 2\,H_2O + 5.64\,N_2$$

Reactants at 77 F, products are at 1600 R, 14.7 psia.

CH_3OH: $\bar{h}_f^o = -102\,846$ Btu/lbmol from table F.11 for the liquid state

$H_R = 1\,\bar{h}_{LIQ} + 0 + 0 = -102\,846$ Btu/lbmol fuel

$H_P = 1(-169\,184 + 11\,798) + 2(-103\,966 + 9241) + 5.64(0 + 7681)$

$\quad = -303\,515$ Btu/lbmol fuel

$Q = H_P - H_R = -303\,515 - (-102\,846) = $ **–200 669 Btu/lbmol fuel**

13.167E

In a new high-efficiency furnace, natural gas, assumed to be 90% methane and 10% ethane (by volume) and 110% theoretical air each enter at 77 F, 14.7 lbf/in.^{2}, and the products (assumed to be 100% gaseous) exit the furnace at 100 F, 14.7 lbf/in.^{2}. What is the heat transfer for this process? Compare this to an older furnace where the products exit at 450 F, 14.7 lbf/in.^{2}.

$0.9\ CH_4 + 0.1\ C_2H_6$
77 F, 1 atm

Products
100 F
1 atm

110 % Air
77 F, 1 atm

Heat transfer

$$0.9\ CH_4 + 0.1\ C_2H_6 + 1.1 \times 2.15\ O_2 + 3.76 \times 2.365\ N_2$$

$$\rightarrow 1.1\ CO_2 + 2.1\ H_2O + 0.215\ O_2 + 8.892\ N_2$$

a) $T_P = 100$ F

$H_R = 0.9(-32190) + 0.1(-36432) = -32614$ Btu

$H_P = 1.1(-169184 + 206) + 2.1(-103966 + 185)$

$\quad + 0.215(0 + 162) + 8.892(0 + 160) = -402\ 360$ Btu/lbmol,

assuming all gas

$$Q_{CV} = H_P - H_R = \textbf{-369 746 Btu/lb mol fuel}$$

b) $T_P = 450$ F

$H_P = 1.1(-169184 + 3674) + 2.1(-103966 + 3057)$

$\quad + 0.215(2688) + 8.892(2610) = -370\ 184$ Btu

$Q_{CV} = H_P - H_R = \textbf{-337 570 Btu/lb mol fuel}$

13.168E

Repeat the previous problem, but take into account the actual phase behavior of the products exiting the furnace.

Same as 13.167 , except possible condensation.

$$0.9\,CH_4 + 0.1\,C_2H_6 + 1.1 \times 2.15\,O_2 + 3.76 \times 2.365\,N_2$$
$$\rightarrow 1.1\,CO_2 + 2.1\,H_2O + 0.215\,O_2 + 8.892\,N_2$$

a) 100 F, 14.7 lbf/in^2 Psat = 0.95 psia

$$y_{v\,max} = 0.95 / 14.7 = n_{v\,max}/[n_{v\,max} + 10.207]$$

$$n_{v\,max} = 0.705 \Rightarrow n_v = 0.705; \quad n_{liq} = 2.1 - 0.705 = 1.395$$

$$H_{liq} = 1.395[-122\,885 + 18.015(68.05 - 45.09)] = -170\,847\ \text{Btu/lbmol}$$

$$H_{gas} = 1.1(-169\,184 + 206) + 0.705(-103\,966 + 185)$$

$$+ 0.215(162) + 8.892(160) = -257\,584\ \text{Btu/lbmol}$$

$$H_P = H_{liq} + H_{gas} = -428\,431\ \text{Btu/lbmol}$$

$$Q_{CV} = H_P - H_R = \textbf{-395817 Btu/lbmol fuel}$$

b) $T_P = 450$ F, no condensation

$$H_P = 1.1(-169\,184 + 3674) + 2.1(-103\,966 + 3057)$$

$$+ 0.215(2688) + 8.892(2610)$$

$$= -370\,184\ \text{Btu/lbmol}$$

$$Q_{CV} = H_P - H_R = \textbf{-337 570 Btu/lbmol fuel}$$

13.169E

Pentene, C_5H_{10} is burned with pure oxygen in a steady state process. The products at one point are brought to 1300 R and used in a heat exchanger, where they are cooled to 77 F. Find the specific heat transfer in the heat exchanger.

$$C_5H_{10} + \nu_{O_2}O_2 \rightarrow 5\ CO_2 + 5\ H_2O, \quad \text{stoichiometric: } \nu_{O_2} = 7.5$$

Heat exchanger in at 1300 R, out at 77 F, so some water will condense.

$$5\ H_2O \rightarrow (5 - x)H_2O_{liq} + x\ H_2O_{vap}$$

Check for condensation amount

$$y_{H2O_{max}} = \frac{P_{g\ 77}}{P_{tot}} = \frac{0.464}{14.696} = 0.03158 = \frac{x}{5 + x} \quad \Rightarrow \quad x = 0.163$$

$$q = \frac{Q}{\dot{n}_{fuel}} = 5\left(\bar{h}_{ex} - \bar{h}_{in}\right)_{CO_2} + 5\left(\bar{h}_{ex} - \bar{h}_{in}\right)_{H2O_{vap}} - (5 - x)\bar{h}_{fg\ H2O}$$

$$= 5\ (-8121) + 5\ (-6468.5) - (5 - 0.163)\ 18919$$

$$= \textbf{-164 459} \ \frac{\textbf{Btu}}{\textbf{lb mol fuel}}$$

13.170E

Methane, CH_4, is burned in a steady state process with two different oxidizers: **A**. Pure oxygen, O_2 and **B** a mixture of $O_2 + x$ Ar. The reactants are supplied at T_0, P_0 and the products in are at 3200 R both cases. Find the required equivalence ratio in case **A** and the amount of Argon, x, for a stoichiometric ratio in case **B**.

a) $CH_4 + \nu O_2 \rightarrow CO_2 + 2H_2O + (\nu - 2)O_2$

$\nu_s = 2$ for stoichiometric mixture.

$H_{P\,3200} = H_R^\circ = H_P^\circ + \Delta H_{P\,3200}$

$\Delta \bar{h}_{CO2} = 33\ 579$ Btu/lbmol, $\Delta \bar{h}_{H2O} = 26\ 479$, $\Delta \bar{h}_{O2} = 21\ 860$

$\Delta H_P = H_R^\circ - H_P^\circ = -H_{RP}^\circ = (50\ 010/2.326) \times 16.04 = 344\ 867$ Btu/lbmol

$\quad = 33\ 579 + 2 \times 26\ 479 + (\nu - 2)21\ 860 = 42\ 817 + \nu \times 21\ 860$

$\quad \nu = 13.8175 \qquad \Rightarrow \qquad \phi = \nu_s/\nu = 2/13.8175 = \mathbf{0.1447}$

b) $CH_4 + 2O_2 + 2x\ Ar \rightarrow CO_2 + 2H_2O + 2x\ Ar$

$\Delta \bar{h}_{Ar} = \bar{C}_{P\,Ar}\,\Delta T = C_{P\,Ar}\,M_{Ar}\,\Delta T$

$\Delta H_P = H_R^\circ - H_P^\circ = -H_{RP}^\circ = (50\ 010/2.326) \times 16.04 = 344\ 867$ Btu/lbmol

$\quad = 33579 + 2 \times 26479 + 2 \times 0.1253 \times 39.948 \times (3200 - 536.67)$

Now the energy equation becomes

$\quad 344\ 867 = 86537 + x \times 26662.5 \qquad \Rightarrow \quad \mathbf{x = 9.689}$

13.171E

A rigid vessel initially contains 2 pound mole of carbon and 2 pound mole of oxygen at 77 F, 30 lbf/in.^{2}. Combustion occurs, and the resulting products consist of 1 pound mole of carbon dioxide, 1 pound mole of carbon monoxide, and excess oxygen at a temperature of 1800 R. Determine the final pressure in the vessel and the heat transfer from the vessel during the process.

$$2\,C + 2\,O_2 \ \rightarrow 1\,CO_2 + 1\,CO + \frac{1}{2}O_2$$

V = constant, C: solid, $n_{1(GAS)} = 2$, $n_{2(GAS)} = 2.5$

$$P_2 = P_1 \times \frac{n_2 T_2}{n_1 T_1} = 30 \times \frac{2.5 \times 1800}{2 \times 536.7} = \mathbf{125.8}\ \frac{\mathbf{lbf}}{\mathbf{in^2}}$$

$H_1 = 0$ and we neglect the Pv for carbon when doing the U_1.

$$H_2 = 1(\text{-}169184 + 14358) + 1(\text{-}47518 + 9323) + \frac{1}{2}(0 + 9761) = \text{-}188\ 141\ \text{Btu}$$

$$_1Q_2 = (U_2\text{-}U_1) = (H_2\text{-}H_1) - n_2 \bar{R}T_2 + n_1 \bar{R}T_1$$
$$= (\text{-}188\ 141 - 0) - 1.98589(2.5 \times 1800 - 2 \times 536.67)\ = \mathbf{\text{-}194\ 945\ Btu}$$

13.172E

A closed, insulated container is charged with a stoichiometric ratio of oxygen and hydrogen at 77 F and 20 lbf/in.^{2}. After combustion, liquid water at 77 F is sprayed in such that the final temperature is 2100 R. What is the final pressure?

Combustion reaction: $\qquad$ $H_2 + \frac{1}{2}(O_2) \rightarrow H_2O$

Products: $\qquad$ $1\ H_2O + x_i\ H_2O$

Energy eq. per 1 lbmol hydrogen (remember flow in):

$$U_2 - U_1 = x_i \bar{h}_i = x_i \bar{h}^{\circ}_{f\ liq} = (1 + x_i)H_P - H_R - (1 + x_i)\bar{R}T_P + \tfrac{3}{2}\bar{R}T_R$$

Solve for x_i: $\qquad$ $x_i\ [\ \bar{h}^{\circ}_{f\ liq} - H_P + \bar{R}T_P\ ] = H_P - H_R - \bar{R}T_P + \tfrac{3}{2}\bar{R}T_R$

Table F.6: $\qquad$ $H_R = \phi,\ H_P = -103\ 966 + 14\ 218.5 = -89\ 747.5$ Btu/lbmol,

Table F.11: $\qquad$ $\bar{h}^{\circ}_{f\ liq} = -122\ 885$ Btu/lbmol

Substitute

$$x_i(-122885 + 89747.5 + 1.98588 \times 2100)$$

$$= -89747.5 - 1.98588(2100 - \tfrac{3}{2} \times 536.67) = -92\ 319.2\ \text{Btu/lbmol}$$

$$x_i = 3.187$$

Volume is constant: $\qquad$ $P_1 V_1 = n_R \bar{R} T_1, \quad P_2 V_1 = n_p \bar{R} T_p$

$$n_R = 1 + \tfrac{1}{2} = 1.5\ ; \qquad n_p = (1 + x_i)$$

$$\Rightarrow P_2 = \frac{P_1(1 + x_i)T_P}{1.5\ T_1} = \frac{20(4.187)(2100)}{1.5(536.67)} = \mathbf{218.5\ lbf/in^2}$$

Borgnakke and Sonntag

13.173E

Acetylene gas, C_2H_2, is burned with stoichiometric air in a torch. The reactants are supplied at the reference conditions P_o, T_o. The products come out from the flame at 4500 R after some heat loss by radiation. Find the heat loss per lbmol of fuel.

The reaction equation is

$$C_2H_2 + 2.5 (O_2 + 3.76 N_2) \rightarrow 2 CO_2 + 1 H_2O + 9.4 N_2$$

Energy Eq.: $\qquad H_P + Q_{out} = H_R = H_P^o + \Delta H_P + Q_{out} = H_R^o$

$$Q_{out} = H_R^o - H_P^o - \Delta H_P$$

$$H_R^o - H_P^o = \bar{h}_{f\,fuel}^{\circ} - \bar{h}_{f\,H2O}^{\circ} - 2\,\bar{h}_{f\,CO_2}^{\circ} = 97\,477 - (-103\,966) - 2(-169\,184)$$
$$= 539\,811 \text{ Btu/lbmol}$$

$$\Delta H_P = 2 \times 52\,416.5 + 42\,612 + 9.4 \times 31\,942.5 = 447\,705 \text{ Btu/lbmol}$$

$$Q_{out} = 539\,811 - 447\,705 = \textbf{92 106 Btu/lbmol fuel}$$

13.174E

Natural gas (methane) is burned with stoichiometric air with reactants supplied at the reference conditions Po, To in a steady flow burner. The products come out at 1400 R. If the burner should deliver 10 Btu/s, what is the needed flowrate of natural gas in lbm/s?

The reaction equation for stoichiometric mixture is:

$$CH_4 + v_{O_2} \left(O_2 + 3.76\, N_2\right) \rightarrow 2\, H_2O + 1\, CO_2 + c\, N_2$$

O balance: $\quad 2\, v_{O_2} = 2 + 2 \quad => \quad v_{O_2} = 2 \quad => c = 2 \times 3.76 = 7.52$

The products are cooled to 1400 R (so we do not consider condensation) and the energy equation is

Energy Eq.: $\quad H_R + Q = H_P = \overset{\circ}{H_P} + \Delta H_P = \overset{\circ}{H_R} + Q$

$$Q = \overset{\circ}{H_P} + \Delta H_P - \overset{\circ}{H_R} = 2\, \overset{\circ}{\bar{h}}_{f\, H_2O} + \overset{\circ}{\bar{h}}_{f\, CO_2} + \Delta H_P - \overset{\circ}{\bar{h}}_{f\, fuel}$$

$$\Delta H_P = 2\, \Delta\bar{h}_{H2O} + \Delta\bar{h}_{CO2} + 7.52\, \Delta\bar{h}_{N2}$$

$$= 2 \times 7371 + 9315 + 7.52 \times 6169 = 70\,448 \text{ Btu/lbmol}$$

$$Q = 2(-103\,966) - 169\,184 + 70\,448 - (-32\,190)$$

$$= -274\,478 \text{ Btu/lbmol fuel}$$

$$\dot{Q} = \dot{n}\, Q_{out} = Q_{out}\, \dot{m}/M \qquad so$$

$$\dot{m} = \dot{Q}M/Q_{out} = 10 \text{ Btu/s} \times 16.043 \text{ (lbm/lbmol)} / 274\,478 \text{ Btu/lbmol}$$

$$= \mathbf{0.000584\ lbm/s} = 2.1 \text{ lbm/h}$$

Borgnakke and Sonntag

Enthalpy of combustion and heating value

13.175E

What is the higher heating value, HHV, of n-Butane?

Either convert units from Table 13.3 or compute from the enthalpy of formation.
From Table F.11

$$\bar{h}^{\circ}_{f\,C4H10} = -54\,256 \text{ Btu/lbmol}, \qquad M = 58.124$$

$$\bar{h}^{\circ}_{f\,H2O\,liq} = -122\,885 \text{ Btu/lbmol} \quad \text{(we need liquid for higher heating value)}$$

$$\bar{h}^{\circ}_{f\,CO2} = -169\,184 \text{ Btu/lbmol}$$

$$\text{HHV} = -H^{\circ}_{RP} = \bar{h}^{\circ}_{f\,C4H10} - 4\,\bar{h}^{\circ}_{f\,CO2} - 5\,\bar{h}^{\circ}_{f\,H2O\,liq}$$

$$= -54\,256 - 4(-169\,184) - 5(-122\,885)$$

$$= \mathbf{1\,236\,905 \text{ Btu/lbmol} = 21\,280 \text{ Btu/lbm}}$$

$$= 49\,500 \text{ kJ/kg} = 21\,280 \text{ Btu/lbm} \quad \text{(from Table 13.3)}$$

Borgnakke and Sonntag

13.176E

Find the enthalpy of combustion and heating value for pure carbon.

Reaction: $C + \nu_{O_2}[\, O_2 + 3.76\, N_2\,] \rightarrow CO_2 + \nu_{O_2} 3.76\, N_2$

oxygen balance: $\nu_{O_2} = 1$

$H^{\circ}_{RP} = H^{o}_{P} - H^{o}_{R} = \bar{h}^{\circ}_{f\, CO_2} - \bar{h}^{\circ}_{f\, C} = -169\ 184 - 0$

$\qquad = -\textbf{169 184 Btu/lbmol} = -\textbf{14 086 Btu/lbm}$ $\qquad$ (M = 12.011)

$HV = 169\ 184$ Btu/lbmol $= \textbf{14 086 Btu/lbm}$

13.177E

Acetylene gas, C_2H_2, is burned with stoichiometric air in a torch. The reactants are supplied at the reference conditions P_o, T_o. The products come out from the flame at 5000 R after some heat loss by radiation. Find the lower heating value for the fuel, as it is not listed in Table 13.3, and the heat loss per kmol of fuel.

The reaction equation is

$$C_2H_2 + 2.5\,(O_2 + 3.76\,N_2) \;\rightarrow\; 2\,CO_2 + 1\,H_2O + 9.4\,N_2$$

Definition of the heating value and values from F.11

$$\begin{aligned} LHV &= -H^\circ_{RP} = H^o_R - H^o_P = \bar{h}^\circ_{f\,fuel} - \bar{h}^\circ_{f\,H_2O} - 2\,\bar{h}^\circ_{f\,CO_2}\\ &= 97\,477 - (-103\,966) - 2(-169\,184)\\ &= 539\,811\ \text{Btu/lbmol} = \textbf{20 731 Btu/lbm} \end{aligned}$$

Energy Eq.:
$$H_P + Q_{out} = H_R = H^o_P + \Delta H_P + Q_{out} = H^o_R$$
$$Q_{out} = H^o_R - H^o_P - \Delta H_P = LHV - \Delta H_P$$

$$\Delta H_P = 2 \times 59\,784 + 49\,114 + 9.4 \times 36\,330 = 510\,184\ \text{Btu/lbmol}$$
$$Q_{out} = 539\,811 - 510\,184 = \textbf{29 627 Btu/lbmol fuel}$$

13.178E

Blast furnace gas in a steel mill is available at 500 F to be burned for the generation of steam. The composition of this gas is, on a volumetric basis,

Component	CH_4	H_2	CO	CO_2	N_2	H_2O
Percent by volume	0.1	2.4	23.3	14.4	56.4	3.4

Find the lower heating value (Btu/ft^3) of this gas at 500 F and P_0.

Of the six components in the gas mixture, only the first 3 contribute to the heating value. These are, per lb mol of mixture:

$$0.024 \, H_2, \, 0.001 \, CH_4, \, 0.233 \, CO$$

For these components,

$$0.024 \, H_2 + 0.001 \, CH_4 + 0.233 \, CO + 0.1305 \, O_2 \rightarrow 0.026 \, H_2 + 0.234 \, CO_2$$

The remainder need not be included in the calculation, as the contributions to reactants and products cancel. For the lower HV(water vapor) at 500 F

$$\bar{h}_{RP} = 0.026(-103 \, 966 + 3488) + 0.234(-169 \, 184 + 4229) - 0.024(0 + 2101)$$

$$- 0.001(-32 \, 190 + 0.538 \times 16.04(500\text{-}77)) - 0.233(-47 \, 518 + 2981)$$

$$- 0.1305(0 + 3069) \; = -31 \, 257 \; Btu/lb \, mol \, fuel$$

$$\bar{v}_0 = \frac{\bar{R}T_0}{P_0} = \frac{1545 \times 536.7}{14.7 \times 144} = 391.47 \; ft^3/lb \, mol$$

LHV = 31 680 / 391.47 = **79.85 Btu/ft³**

13.179E

A burner receives a mixture of two fuels with mass fraction 40% n-butane and 60% methanol both vapor. The fuel is burned with stoichiometric air. Find the product composition and the lower heating value of this fuel mixture (Btu/lbm fuel mix).

Since the fuel mixture is specified on a mass basis we need to find the mole fractions for the combustion equation. From Eq.13.4 we get

$$y_{butane} = (0.4/58.124) / [0.4/58.124 + 0.6/32.042] = 0.26875$$

$$y_{methanol} = 1 - y_{butane} = 0.73125$$

The reaction equation is

$$0.73125 \, CH_3OH + 0.26875 \, C_4H_{10} + v_{O2} \, (O_2 + 3.76 \, N_2)$$

$$\rightarrow \, v_{CO2} \, CO_2 + v_{H2O}H_2O + 3.76 \, v_{O2} \, N_2$$

C balance: $\quad 0.73125 + 4 \times 0.26875 = v_{CO2} = 1.80625$

H_2 balance: $\quad 2 \times 0.73125 + 5 \times 0.26875 = v_{H2O} = 2.80625$

O balance: $\quad 0.73125 + 2 \, v_{O2} = 2 \, v_{CO2} + v_{H2O} = 6.41875 \quad => v_{O2} = 2.84375$

Now the products are:

$$1.80625 \, CO_2 + 2.80625 \, H_2O + 10.6925 \, N_2$$

Since the enthalpy of combustion is on a mass basis in table 13.3 (this is also the negative of the heating value) we get

$$LHV = (0.4 \times 45\ 714 + 0.6 \times 21\ 093)/2.326$$

$$= \textbf{13 302 Btu/lbm fuel mixture}$$

Notice we took fuel vapor and water as vapor (lower heating value).

13.180E

A mixture of fuels is E85, which is 85% ethanol and 15% gasoline (assume octane) by mass. Assume we put the fuel and air, both at T_o, P_o, into a carburetor and vaporize the fuel as we mix it with stoichiometric air before it flows to an engine. Assume the engine has an efficiency as work divided by the lower heating value of 30%, and we want it to deliver 55 hp. We use heat from the exhaust flow (900 R) for the carburetor. Find the lower heating value of this fuel (Btu/lbm), the rate of fuel consumption, the heating rate needed in the carburetor, and the rate of entropy generation in the carburetor.

The heating value of the liquid fuel blend (values from Tbl. 13.3 converted)
$$LHV = 0.85 \, LHV_{ethanol} + 0.15 \, LHV_{octane}$$
$$= (0.85 \times 26\,811 + 0.15 \times 44\,425) \, 0.42992 \text{ Btu/lbm} = \mathbf{12\,662 \text{ Btu/lbm}}$$

$$\dot{W} = 0.3 \, \dot{m}_{fuel} \, LHV$$

$$\dot{m}_{fuel} = \dot{W}/(0.3 \times LHV) = 55 \text{ hp} \times 2544 \text{ Btu/h-hp} / (0.3 \times 12\,662 \text{ Btu/lbm})$$
$$= \mathbf{36.835 \text{ lbm/h} = 0.0102 \text{ lbm/s}}$$

The difference in heating value based on liquid versus vapor fuel is
$$\Delta LHV = [0.85 \, (27\,731 - 26\,811) + 0.15 \, (44\,788 - 44\,425)] \, 0.42992$$
$$= (0.85 \times 920 + 0.15 \times 363) \, 0.42992 = 359.6 \text{ Btu/lbm}$$

$$\dot{Q} = \dot{m}_{fuel} \, \Delta LHV = 0.0102 \text{ lbm/s} \times 359.6 \text{ Btu/lbm} = \mathbf{3.68 \text{ Btu/s}}$$

Entropy generation is from vaporizing the fuel by transfer of heat from 900 R exhaust gas to the intake system at To and then mixing it with air. The mole fractions are

Ethanol: M = 46.069; Octane: M = 114.232
$$DIV = [0.85 / 46.069] + [0.15 / 114.232] = 0.01845 + 0.001313 = 0.019764$$
$$y_{ethanol} = 0.01845 / 0.019764 = 0.9336$$
$$y_{octane} = 1 - y_{ethanol} = 0.06644$$

$$0.93356 \, C_2H_5OH + 0.06644 \, C_8H_{18} + 3.63118 \, (O_2 + 3.76 \, N_2) \Rightarrow$$
$$2.39864 \, CO_2 + 3.39864 \, H_2O + 13.65324 \, N_2$$

C balance: $0.93356 \times 2 + 0.06644 \times 8 = 2.39864 = v_{CO2}$
H balance: $0.93356 \times 6 + 0.06644 \times 18 = 6.79728 = 2v_{H2O}$

So the A/F ratio on a mole and mass basis are

A/F-mole = 3.63118 × 4.76 / 1 = 17.2844;
A/F-mass = 17.2844 × 28.97/50.5996 = 9.896

The fuel was mixed before the carburetor so afterwards we have a 1: 17.2844 mix.

$s_{fg\ ethanol}$ = (67.434 – 38.321)/46.069 = 0.63194 Btu/lbm-R

$s_{fg\ octane}$ = (111.399 – 86.122)/114.232 = 0.22128 Btu/lbm-R

$s_{fg\ fuel\ mix}$ = 0.85 × 0.63194 + 0.15 × 0.22128 = 0.57034 Btu/lbm-R

M_{fuel} = 0.9336 × 46.069 + 0.06644 × 114.232 = 50.5996

Entropy Eq.: $\dot{S}_{gen}$ = out – in = $\dot{m}_{fuel}$ [s_{fg} - R ln(y_{fuel})] + $\dot{m}_{air}$ [- R ln(y_{air})] - $\dot{Q}/T_{exhaust}$

$$= 0.0102 \times \left[0.57034 - \frac{1.98589}{50.5996} \ln\left(\frac{1}{18.2844}\right)\right] Btu/s\text{-}R$$

$$- 0.0102 \times 9.896 \times 0.06855 \ln\left(\frac{17.2844}{18.2844}\right) Btu/s\text{-}R - \frac{3.68}{900} Btu/s\text{-}R$$

$$= 0.00698 + 0.000389 - 0.004089 = \mathbf{0.00328\ Btu/s\text{-}R}$$

Adiabatic flame temperature

13.181E

Hydrogen gas is burned with pure oxygen in a steady flow burner where both reactants are supplied in a stoichiometric ratio at the reference pressure and temperature. What is the adiabatic flame temperature?

The reaction equation is:

$$H_2 + \nu_{O2}\, O_2 \quad \Rightarrow\ H_2O$$

The balance of hydrogen is done, now for oxygen we need $\nu_{O2} = 0.5$.

Energy Eq.: $\qquad H_R = H_P \ \Rightarrow\ 0 = -103\ 966 + \Delta\bar{h}_{H2O}$

$$\Rightarrow \quad \Delta\bar{h}_{H2O} = 103\ 966 \text{ Btu/lbmol}$$

Interpolate now in table F.6 for the temperature to give this enthalpy

T = 8985 R

13.182E

Some type of wood can be characterized as $C_1H_{1.5}O_{0.7}$ with a lower heating value of 8380 Btu/lbm. Find its adiabatic flame temperature when burned with stoichiometric air at 1 atm., 77 F.

$$C_1H_{1.5}O_{0.7} + \nu_{O_2}[\ O_2 + 3.76\ N_2\ ] \rightarrow\ \ CO_2 + 0.75\ H_2O + 3.76\nu_{O_2}\ N_2$$

O balance: $0.7 + 2\nu_{O_2} = 2 + 0.75$ $\Rightarrow$ $\nu_{O_2} = 1.025, \ \nu_{N2} = 3.854$

$$M = 1 \times 12.0111 + 1.5 \times 1.008 + 0.7 \times 16 = 24.712$$

$$\overset{\circ}{H}_{RP} = H_P^o - H_R^o = -M \times HV = -24.712 \times 8380 = -207\ 086 \text{ Btu/lbmol}$$

Energy Eq.: $H_P = \overset{\circ}{H}_P + \Delta H_P = H_R = \overset{\circ}{H}_R$ $\Rightarrow$ $\Delta H_P = \overset{\circ}{H}_R - \overset{\circ}{H}_P = -\overset{\circ}{H}_{RP}$

$$\Delta H_P = \Delta\bar{h}_{CO2} + 0.75\ \Delta\bar{h}_{H2O} + 3.854\ \Delta\bar{h}_{N2} = 207\ 086 \text{ Btu/lbmol}$$

at 4400 R: $\Delta H_P = 50\ 948 + 0.75 \times 41\ 325 + 3.854 \times 31\ 068 = 201\ 678$ Btu
at 4600 R: $\Delta H_P = 53\ 885 + 0.75 \times 43\ 899 + 3.854 \times 32\ 817 = 213\ 286$ Btu

$$T = 4400 + 200\ \frac{207\ 086 - 201\ 678}{213\ 286 - 201\ 678} = \textbf{4493 R}$$

Comment: Most wood has some water and some non-combustible solids material so the actual flame temperature will be much lower.

13.183E

Carbon is burned with air in a furnace with 150% theoretical air and both reactants are supplied at the reference pressure and temperature. What is the adiabatic flame temperature?

$$C + \nu_{O_2}O_2 + 3.76\,\nu_{O_2}N_2 \;\rightarrow\; 1\,CO_2 + 3.76\,\nu_{O_2}N_2$$

From this we find $\nu_{O_2} = 1$ and the actual combustion reaction is

$$C + 1.5\,O_2 + 5.64\,N_2 \;\rightarrow\; 1\,CO_2 + 5.64\,N_2 + 0.5\,O_2$$

$$H_P = \overset{\circ}{H}_P + \Delta H_P = H_R = \overset{\circ}{H}_R \quad \Rightarrow$$

$$\Delta H_P = \overset{\circ}{H}_R - \overset{\circ}{H}_P = 0 \;-(-169\,184) = 169\,184 \text{ Btu/lbmol}$$

$$\Delta H_P = \Delta\bar{h}_{CO2} + 5.64\,\Delta\bar{h}_{N2} + 0.5\,\Delta\bar{h}_{O2}$$

Find T so ΔH_P takes on the required value. To start guessing assume all products are nitrogen $(1 + 5.64 + 0.5 = 7.14)$ that gives $3400 < T < 3600$ R from Table F.6.

$$\Delta H_{P\,3400} = 36\,437 + 5.64 \times 22\,421 + 0.5 \times 23\,644 = 174\,713 \quad \text{too high}$$
$$\Delta H_{P\,3200} = 33\,579 + 5.64 \times 20\,717 + 0.5 \times 21\,860 = 161\,353$$

Linear interpolation to find

$$T = 3200 + 200\,\frac{169\,184 - 161\,353}{174\,713 - 161\,353} = \mathbf{3317\ R}$$

13.184E

Acetylene gas at 77 F, 14.7 lbf/in.^{2} is fed to the head of a cutting torch. Calculate the adiabatic flame temperature if the acetylene is burned with 100% theoretical air at 77 F. Repeat the answer for 100% theoretical oxygen at 77 F.

a) $\quad C_2H_2 + 2.5\,O_2 + 2.5 \times 3.76\,N_2 \rightarrow \quad 2\,CO_2 + 1\,H_2O + 9.4\,N_2$

$\quad H_R = \bar{h}^o_{f\,C_2H_2} = 97\,477$ Btu

$\quad H_P = 2(-169\,184 + \Delta\bar{h}^*_{CO_2}) + 1(-103\,966 + \Delta\bar{h}^*_{H_2O}) + 9.4\,\Delta\bar{h}^*_{N_2}$

$\quad Q_{CV} = H_P - H_R = 0$

$\qquad \Rightarrow \quad 2\,\Delta\bar{h}^*_{CO_2} + 1\,\Delta\bar{h}^*_{H_2O} + 9.4\,\Delta\bar{h}^*_{N_2} = 539\,811$ Btu/lbmol

Trial and Error: $T_{PROD} = $ **5236 R**

$\qquad 2 \times 147\,196 + 121\,488 + 9.4 \times 89\,348 = 1\,255\,751$ **OK**

b) $\quad C_2H_2 + 2.5\,O_2 \rightarrow \quad 2\,CO_2 + H_2O$

$\quad H_R = 97\,477$ Btu

$\quad H_P = 2(-169\,184 + \Delta\bar{h}^*_{CO2}) + 1(-103\,966 + \Delta\bar{h}^*_{H2O})$

$\quad \Rightarrow \quad 2\,\Delta\bar{h}^*_{CO2} + 1\,\Delta\bar{h}^*_{H2O} = 539\,811$

At 10 000 R (limit of F.6): $2 \times 135\,426 + 118\,440 = 389\,292$

At 9500 R: $\qquad 2 \times 127\,734 + 111\,289 = 366\,757$

$\qquad$ or 4507/100 R change, $\qquad$ Difference, extrapolating

$T_{PROD} \approx 10\,000 + \dfrac{150519}{45.07} \approx$ **13 340 R**

13.185E

Butane gas at 77 F is mixed with 150% theoretical air at 1000 R and is burned in an adiabatic steady state combustor. What is the temperature of the products exiting the combustor?

$$C_4H_{10} + 1.5 \times 6.5(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 3.25\ O_2 + 36.66N_2$$

$$H_R = H_R^\circ + \Delta H_{air,in}$$

$$H_P = H_P^\circ + 4\Delta\bar{h}_{CO_2} + 5\Delta\bar{h}_{H_2O} + 3.25\Delta\bar{h}_{O_2} + 36.66\Delta\bar{h}_{N_2}$$

$$H_P = H_R \implies \Delta H_P = H_R^\circ - H_P^\circ + \Delta H_{air,in}$$

$$\Delta H_P = -H_{RP}^\circ + \Delta H_{air,in} = \frac{45714}{2.326} \times 58.124 + 9.75 \times 3366 + 36.66 \times 3251$$

$$= 1\ 294\ 339\ \text{Btu/lbmol fuel}$$

$$= \left[4\Delta\bar{h}_{CO2} + 5\Delta\bar{h}_{H2O} + 3.25\Delta\bar{h}_{O2} + 36.66\Delta\bar{h}_{N2} \right] \text{at } T_{ad}$$

Trial and Error:

Find the enthalpies from Table F.6

$$\Delta H_{P,3600R} = 1\ 281\ 185$$
$$\Delta H_{P,3800R} = 1\ 374\ 068$$

Linear interpolation to match LHS
$$\implies\ \textbf{T}_{ad} = \textbf{3628 R}$$

Borgnakke and Sonntag

13.186E

A special coal burner uses a stoichiometric mixture of coal and an oxygen - argon mixture (1:1 mole ratio) with the reactants supplied at Po, To. Find the adiabatic flame temperature assuming complete combustion.

Combustion of carbon:

$$C + O_2 + Ar \rightarrow CO_2 + Ar$$

C.V. Combustion chamber.

Energy Eq.:

$$H_R = \overset{\circ}{H_R} = H_P = \overset{\circ}{H_P} + \Delta H_P = \bar{h}_{f\,CO_2} + \Delta \bar{h}_{CO_2} + \Delta \bar{h}_{Ar}$$

Table F.6 or F.11: $\bar{h}_{f\,CO_2} = $ -169 184 Btu/lbmol, reference $\overset{\circ}{H_R} = 0$

Table F.4: $\bar{C}_{P\,Ar} = 0.124 \times 39.948 = 4.9536$ Btu/lbmol-R

$$\Delta H_P = \overset{\circ}{H_R} - \overset{\circ}{H_P} = 0 - \bar{h}_{f\,CO_2} = 169\ 184 \text{ Btu/lbmol}$$

$$\Delta H_{P\,9000} = 120\ 071 + 4.9536 \times (9000 - 459.7) = 162\ 376 \text{ Btu/lbmol,}$$

$$\Delta H_{P\,9500} = 127\ 734 + 4.9536 \times (9500 - 459.7) = 172\ 516 \text{ Btu/lbmol,}$$

interpolate $\Rightarrow$ $T_4 = T_{ad.flame} = $ **9336 R**

Comment: At this temperature some chemical equilibrium reactions will be important, see Chapter 14.

13.187E

Liquid n-butane at T_0, is sprayed into a gas turbine with primary air flowing at 150 lbf/in.^{2}, 700 R in a stoichiometric ratio. After complete combustion, the products are at the adiabatic flame temperature, which is too high so secondary air at 150 lbf/in.^{2}, 700 R is added, with the resulting mixture being at 2500 R. Show that $T_{ad} > 2500$ R and find the ratio of secondary to primary air flow.

C.V. Combustion Chamber.

$$C_4H_{10} + 6.5\ O_2 + 6.5 \times 3.76\ N_2 \rightarrow 5\ H_2O + 4\ CO_2 + 24.44\ N_2$$

$$H_R = H_{air} + H_{Fu} = H_P = H_P^\circ + \Delta H_P = H_R^\circ + \Delta H_R$$

$$\Delta H_P = H_R^\circ - H_P^\circ + \Delta H_R = -H_{RP}^\circ + \Delta H_R$$

$$= 45344 \times 58.124/2.326 + 6.5(1158 + 3.76 \times 1138)$$

$$= 1\ 168\ 433\ \text{Btu/lbmol fuel}$$

$$\Delta H_{P,2500R} = 4 \times 23755 + 5 \times 18478 + 24.44 \times 14855$$

$$= 550\ 466\ \text{Btu/lbmol fuel}$$

$\Delta H_P > \Delta H_{P,2500R} \implies T_{ad} > 2500$ R (If iteration $T_{ad} \cong 4400$ R)

C.V. Mixing chamber

$$\Delta H_P + \nu_{O2\ 2nd}\Delta H_{air,700} = \Delta H_{P,2500R} + \nu_{O2\ 2nd}\Delta H_{air,2500R}$$

$$\nu_{O2\ 2nd} = \frac{\Delta H_P - \Delta H_{P,2500}}{\Delta H_{air,2500} - \Delta H_{air,700}} = \frac{1168433 - 550466}{71571 - 5437} = 9.344$$

Ratio $= \nu_{O2\ 2nd}/\nu_{O2\ Prim.} = 9.344/6.5 = $ **1.44**

13.188E

Ethene, C_2H_4, burns with 150% theoretical air in a steady flow constant-pressure process with reactants entering at P_0, T_0. Find the adiabatic flame temperature.

Stoichiometric

$$C_2H_4 + 3(O_2 + 3.76N_2) \rightarrow 2CO_2 + 2H_2O + 11.28N_2$$

Actual

$$C_2H_4 + 4.5(O_2 + 3.76N_2) \rightarrow 2\ CO_2 + 2\ H_2O + 1.5\ O_2 + 16.92\ N_2$$

$$H_P = H_R = H_P^\circ + \Delta H_P = H_R^\circ \quad \Rightarrow \quad \text{From Table 13.3 units converted}$$

$$\Delta H_P = H_R^\circ - H_P^\circ = -H_{RP}^\circ = 28.054 \times 47\ 158/2.326 = 568\ 775\ \text{Btu/lbmol}$$

$$\Delta H_P = 2\Delta \bar{h}_{CO2} + 2\Delta \bar{h}_{H2O} + 1.5\Delta \bar{h}_{O2} + 16.92\Delta \bar{h}_{N2}$$

Trial and error on T_{ad}.

$$\Delta H_{P,3400R} = 2 \times 36\ 437 + 2 \times 28\ 867 + 1.5 \times 23\ 644 + 16.92 \times 22\ 421$$

$$= 545\ 437\ \text{Btu/lbmol},$$

$$\Delta H_{P,3600R} = 2 \times 39\ 312 + 2 \times 31\ 293 + 1.5 \times 25\ 441 + 16.92 \times 24\ 135$$

$$= 587\ 736\ \text{Btu/lbmol}$$

Linear interpolation $\qquad$ **$T_{ad} = 3510\ R$**

13.189E

Solid carbon is burned with stoichiometric air in a steady state process, as shown in Fig. P13.97. The reactants at T_0, P_0 are heated in a preheater to $T_2 = 900$ R with the energy given by the products before flowing to a second heat exchanger, which they leave at T_0. Find the temperature of the products T_4, and the heat transfer per lb mol of fuel (4 to 5) in the second heat exchanger.

a) Following the flow we have: Inlet T_1, after preheater T_2, after mixing and combustion chamber T_3, after preheater T_4, after last heat exchanger $T_5 = T_1$.

b) Products out of preheater T_4. Control volume: Total minus last heat exchanger.

$$C + O_2 + 3.76\,N_2 \rightarrow CO_2 + 3.76\,N_2$$

Energy Eq.:

$$H_R = H_R^\circ = H_{P_3} = H_P^\circ + \Delta H_{P_3} = \bar{h}_{f\,CO_2} + \Delta\bar{h}_{CO_2} + 3.76\Delta\bar{h}_{N_2}$$

$\bar{h}_{f\,CO_2}^\circ = $ -169 184 Btu/lbmol,

$$\Delta H_{P_3\,4400} = 50\,948 + 3.76 \times 31068 = 167\,764 \text{ Btu/lbmol},$$

$$\Delta H_{P_3\,4600} = 53\,885 + 3.76 \times 32\,817 = 177\,277 \text{ Btu/lbmol}$$

Linear interpolation $\Rightarrow T_3 = T_{ad.flame} = $ **4430 R**

c) Control volume total. Then energy equation:

$$H_R^\circ + \bar{q} = H_P^\circ$$

$$\bar{q} = H_{RP}^\circ = \bar{h}_{f\,CO_2}^\circ - 0 = \textbf{-169 184 } \frac{\textbf{Btu}}{\textbf{lbmol fuel}}$$

13.190E

In a test of rocket propellant performance, liquid hydrazine (N_2H_4) at 14.7 lbf/in.^{2}, 77 F, and oxygen gas at 14.7 lbf/in.^{2}, 77 F, are fed to a combustion chamber in the ratio of 0.5 lbm O_2/lbm N_2H_4. The heat transfer from the chamber to the surroundings is estimated to be 45 Btu/lbm N_2H_4. Determine the temperature of the products exiting the chamber. Assume that only H_2O, H_2, and N_2 are present. The enthalpy of formation of liquid hydrazine is +21 647 Btu/lb mole.

$$N_2H_4 + \frac{1}{2}O_2 \rightarrow H_2O + H_2 + N_2$$

$$\dot{m}_{O2}/\dot{m}_{N2H4} = 0.5 = 32\dot{n}_{O2}/32\dot{n}_{N2H4} \quad \text{and} \quad \dot{Q}/\dot{m}_{N2H4} = \text{-45 Btu/lbm}$$

$$\Rightarrow \quad Q_{CV} = \text{-45} \times 32.045 = \text{-1442} \frac{\text{Btu}}{\text{lb mol fu}}$$

C.V. combustion chamber: $\quad \dot{n}_{Fu}\bar{h}_1 + \dot{n}_{O2}\bar{h}_2 + \dot{Q}_{CV} = \dot{n}_{tot}\bar{h}_3$

- or - $\quad H_1 + H_2 + Q_{CV} = H_{P3} \quad \Rightarrow \quad H_R^\circ + Q_{CV} = H_P^\circ + \Delta H_{P3}$

$$\Delta H_{P3} = H_R^\circ - H_P^\circ + Q_{CV} = 21\ 647 + 103\ 966 - 1442 = 124\ 171 \frac{\text{Btu}}{\text{lb mol fuel}}$$

Trial and error on T_3:

$$\Delta H_{P\ 5000R} = 49\ 114 + 34\ 627 + 36\ 330 = 120\ 071\ \text{Btu/lbmol,}$$

$$\Delta H_{P\ 5500R} = 55\ 739 + 39\ 032 + 40\ 745 = 135\ 516\ \text{Btu/lbmol,}$$

Interpolate $\quad \Rightarrow T_3 = \textbf{5133 R}$

Second law for the combustion process

13.191E

Two pound moles of ammonia are burned in a steady state process with x lb mol of oxygen. The products, consisting of H_2O, N_2, and the excess O_2, exit at 400 F, 1000 lbf/in.^{2}.

a. Calculate x if half the water in the products is condensed.

b. Calculate the absolute entropy of the products at the exit conditions.

$$2NH_3 + xO_2 \rightarrow 3H_2O + N_2 + (x - 1.5)O_2$$

Products at 400 F, 1000 lbf/in^2 with $n_{H2O\ LIQ} = n_{H2O\ VAP} = 1.5$

a) $y_{H2O\ VAP} = \dfrac{P_G}{P} = \dfrac{247.1}{1000} = \dfrac{1.5}{1.5 + 1 + x - 1.5}$

$x = \textbf{5.070}$

b) $S_{PROD} = S_{GAS\ MIX} + S_{H2O\ LIQ}$

Gas mixture:

	n_i	y_i	$\bar{s}_i^\circ$	$-\bar{R}\ln\dfrac{y_iP}{P_0}$	$\bar{S}_i$
H_2O	1.5	0.2471	48.939	-5.604	43.335
O_2	3.57	0.5881	52.366	-7.326	45.040
N_2	1.0	0.1648	49.049	-4.800	44.249

$S_{GAS\ MIX} = 1.5(43.335) + 3.57(45.040) + 1.0(44.249) = 270.04$ Btu/R

$S_{H2O\ LIQ} = 1.5[16.707 + 18.015(0.5647 - 0.0877)] = 37.95$ Btu/R

$S_{PROD} = 270.04 + 37.95 = \textbf{307.99 Btu/R}$

13.192E

Propene, C_3H_6, is burned with air in a steady flow burner with reactants at P_o, T_o. The mixture is lean so the adiabatic flame temperature is 3200 R. Find the entropy generation per lbmol fuel neglecting all the partial pressure corrections.

The reaction equation for a mixture with excess air is:

$$C_3H_6 + \nu_{O2}\left(O_2 + 3.76\,N_2\right) \rightarrow 3\,H_2O + 3\,CO_2 + 3.76\nu_{O2}\,N_2 + (\nu_{O2} - 4.5)O_2$$

Energy Eq.: $\quad H_R = H_R^{\circ} + \Delta H_R = H_R^{\circ} = H_P = H_P^{\circ} + \Delta H_P$

The entropy equation: $\quad S_R + S_{gen} = S_P \Rightarrow S_{gen} = S_P - S_R = S_P - S_R^{\circ}$

From table F.6 at reference T

$$\Delta H_R = \Delta h_{Fu} + \nu_{O2}(\Delta h_{O2} + 3.76\,\Delta h_{N2}) = 0$$

From table F.6 at 3200 R:

$$\Delta H_P = 3\,\Delta h_{H2O} + 3\,\Delta h_{CO2} + 3.76\,\nu_{O2}\,\Delta h_{N2} + (\nu_{O2} - 4.5)\,\Delta h_{O2}$$
$$= 3 \times 26\,479 + 3 \times 33579 + 3.76\,\nu_{O2} \times 20\,717 + (\nu_{O2} - 4.5)\,21\,860$$
$$= 81\,804 + 99\,756\,\nu_{O2}$$

From table F.11:

$$H_P^{\circ} - H_R^{\circ} = 3(-103966) + 3(-169184) - 8783 = -828\,233 \text{ Btu/lbmol}$$

Now substitute all terms into the energy equation

$$-828\,233 + 81\,804 + 99\,756\,\nu_{O2} = 0$$

Solve for ν_{O2}: $\quad \nu_{O2} = \dfrac{828\,233 - 81\,804}{99\,756} = 7.48255, \quad \nu_{N2} = 28.134$

Table F.6 contains the entropies at 14.7 psia so we get:

$$S_P = 3 \times 61.796 + 3 \times 72.160 + (7.48255 - 4.5)\,63.109 + 28.134 \times 59.175$$
$$= 2254.92 \text{ Btu/lbmol-R}$$
$$S_R = 63.761 + 7.48255 \times 48.973 + 28.134 \times 45.739 = 1717.02 \text{ Btu/lbmol R}$$
$$S_{gen} = 2254.92 - 1717.02 = \textbf{537.9 Btu/lbmol-R}$$

13.193E

Graphite, C, at P_o, T_o is burned with air coming in at P_o, 900 R in a ratio so the products exit at P_o, 2200 R. Find the equivalence ratio, the percent theoretical air and the total irreversibility.

$$C + x(O_2 + 3.76N_2) \rightarrow CO_2 + (x - 1)O_2 + 3.76 \, x \, N_2$$

$$H_P = H_R \Rightarrow \Delta H_{P,2200} - \Delta H_R = H_R^\circ - H_P^\circ = -H_{RP}^\circ$$

$$19659 + (x - 1)13136 + 3.76 \times x \times 12407 - x(2616 + 3.76 \times 2541)$$
$$= 0 - (-169184)$$

$$\Rightarrow 6523 + x \times 47616.2 = 169184 \qquad \Rightarrow \qquad x = 3.416,$$

or **342 % theoretical air**

Equivalence ratio $\phi = 1/x = $ **0.293**

$$S_{gen} = s_P - s_R = \sum_P v_i(s_i^\circ - \bar{R}\ln y_i) - \sum_R v_i(s_i^\circ - \bar{R}\ln y_i)$$

R: $y_{O_2} = 0.21 \qquad y_{N_2} = 0.79$

P: $y_{O_2} = 0.1507 \qquad y_{N_2} = 0.79 \qquad y_{CO_2} = 0.0593$

$$S_{gen} = 66.952 + 5.610 + 2.416(59.844 + 3.758)$$
$$+ 3.76 \times 3.416(56.066 + 0.468) - 1.371$$
$$- 3.416(52.686 + 3.099 + 3.76(49.353 + 0.468))$$
$$= 120.5 \text{ Btu/(lbmol C} \times \text{R)}$$

$$I = T_o S_{gen} = \textbf{64 677 Btu/lbmol C}$$

13.194E

Hydrogen peroxide, H_2O_2, enters a gas generator at 77 F, 75 lbf/in.^{2} at the rate of 0.2 lbm/s and is decomposed to steam and oxygen exiting at 1500 R, 75 lbf/in.^{2}. The resulting mixture is expanded through a turbine to atmospheric pressure, 14.7 lbf/in.^{2}, as shown in Fig. P13.107. Determine the power output of the turbine, and the heat transfer rate in the gas generator. The enthalpy of formation of liquid H_2O_2 is −80 541 Btu/lb mol.

$$H_2O_2 \rightarrow H_2O + \tfrac{1}{2}O_2$$

$$\dot{n}_{Fu} = \dot{m}_{Fu}/M_{Fu} = 0.2/34.015 = 0.00588 \text{ lbmol/s}$$

$$\dot{n}_{ex,mix} = 1.5 \times \dot{n}_{Fu} = 0.00882 \text{ lbmol/s}$$

$$\bar{C}_{p\,mix} = \tfrac{2}{3} \times 0.445 \times 18.015 + \tfrac{1}{3} \times 0.219 \times 31.999 = 7.6804$$

$$\bar{C}_{v\,mix} = \bar{C}_{p\,mix} - 1.98588 = 5.6945 \quad ; \quad k_{mix} = \bar{C}_{p\,mix}/\bar{C}_{v\,mix} = 1.3487$$

Reversible turbine

$$T_3 = T_2 \times (P_3/P_2)^{(k-1)/k} = 1500 \times (14.7/75)^{0.2585} = 984.3 \text{ R}$$

$$\bar{w} = \bar{C}_{p\,mix}(T_2 - T_3) = 7.6804(1500 - 984.3) = 3960.8 \text{ Btu/lbmol}$$

$$\dot{W}_{CV} = \dot{n}_{mix} \times \bar{w} = 0.00882 \times 3960.8 = \textbf{34.9 Btu/s}$$

C.V. Gas generator

$$\dot{Q}_{CV} = \dot{H}_2 - \dot{H}_1$$
$$= 0.00588 \times (-103\,966 + 8306) + 0.00294(7297.5) - 0.00588(-80\,541)$$
$$= \textbf{-67.45 Btu/s}$$

13.195E

Methane is burned with air both supplied at the reference conditions. There is enough excess air to give a flame temperature of 3200 R. What are the percent theoretical air and the irreversibility in the process?

The combustion equation with X times theoretical air is

$$CH_4 + 2X(O_2 + 3.76\ N_2) \rightarrow CO_2 + 2H_2O + 2(X-1)O_2 + 7.52X\ N_2$$

Energy Eq.: $\quad H_{air} + H_{fuel} = H_R = H_P = \overset{\circ}{H}_P + \Delta H_P = \overset{\circ}{H}_R + \Delta H_R$

$$\Rightarrow \quad \Delta H_P = \overset{\circ}{H}_R + \Delta H_R - \overset{\circ}{H}_P = -\overset{\circ}{H}_{RP} + 0$$

From Table 13.3: $\quad -\overset{\circ}{H}_{RP} = -16.04\ (-50\ 010)/2.326 = 344\ 867$ Btu/lbmol

$$\Delta H_P = \Delta\bar{h}^*_{CO2} + 2\ \Delta\bar{h}^*_{H2O} + 2(X-1)\ \Delta\bar{h}^*_{O2} + 7.52X\ \Delta\bar{h}^*_{N2}$$

From Table F.6 and the energy equation

$$\Delta H_{P\ 3200} = 33\ 579 + 2 \times 26\ 479 + 2(X-1)\ 21\ 860 + 7.52X \times 20\ 717 = 344\ 867$$

so

$$42\ 817 + 199\ 512\ X = 344\ 867 \quad \Rightarrow \quad X = 1.514$$

$$\%\text{Theoretical air} = \mathbf{151.4\%}$$

The products are

Products: $\quad CO_2 + 2H_2O + 0.9712\ O_2 + 11.172\ N_2$

The second law

$$S_{gen} = S_P - S_R \quad \text{and} \quad I = T_o\ S_{gen}$$

Reactants: $P_i = 14.7$ psia, $P_o = 14.7$ psia, $\bar{s}^o_f$ from Table F.6 and F.11

	n_i	y_i	$\bar{s}^o_f$	$-\bar{R}\ln\dfrac{y_iP_i}{P_o}$	$\bar{S}_i\ \dfrac{\text{Btu}}{\text{lbmol R}}$
CH_4	1	1	44.459	0	44.459
O_2	2X	0.21	48.973	3.099	52.072
N_2	7.52 X	0.79	45.739	0.468	46.207

$$S_R = \sum n_i\bar{S}_i = 728.21 \text{ Btu/R lbmol fuel}$$

Products: $P_e = 14.7$ psia, $P_O = 14.7$ psia, From Table F.6

	n_i	y_i	$\bar{s}^o_{3200}$	$-\bar{R} \ln \dfrac{y_i P_e}{P_o}$	$\bar{S}_i \dfrac{Btu}{lbmol\ R}$
CO_2	1	0.06604	72.160	5.3966	77.557
H_2O	2	0.13208	61.796	4.0201	65.816
O_2	0.9712	0.06413	61.109	5.4549	66.564
N_2	11.172	0.73775	59.175	0.6040	59.779

$S_P = \sum n_i \bar{S}_i = 941.69$ Btu/R lbmol fuel;

$I = T_O(S_P - S_R) = 536.67(941.69 - 728.21) = $ **114 568 Btu/lbmol fuel**

Borgnakke and Sonntag

Fuel Cells, Efficiency, and Review

13.196E

In Example 13.15, a basic hydrogen–oxygen fuel cell reaction was analyzed at 25°C, 100 kPa. Repeat this calculation, assuming that the fuel cell operates on air at 77 F, 14.7 lbf/in.^{2}, instead of on pure oxygen at this state.

Anode: $2\,H_2 \qquad \rightarrow 4\,e^- + 4\,H^+$

Cathode: $4\,H^+ + 4\,e^- \rightarrow 1\,O_2 + 2\,H_2O$

Overall: $2\,H_2 + 1\,O_2 \rightarrow 2\,H_2O$

Example $\boxed{13.15}$: $\Delta G_{25°C} = $ -474 283 kJ/kmol

$\qquad\qquad$ Or $\Delta G_{77\,F} = $ -203 904 Btu/lbmol

$P_{O_2} = y_{O2} \times P = 0.21 \times 14.7 = 3.087$ lbf/in^2

$\bar{s}_{O_2} = 48.973 - 1.98589 \ln 0.21 = 52.072$

$\Delta S = 2(16.707) - 2(31.186) - 1(52.072) = $ - 81.03 Btu/R

$\Delta H = 2(\text{-122 885}) - 2(0) - 1(0) = $ -245 770 Btu/lbmol

$\Delta G_{77\,F} = $ -245 770 $- 536.67(\text{-81.03}) = $ 202 284 Btu/lbmol

$E^\circ = \text{-}\Delta G/N_0 e\, n_e = $ 202 284 $\times 2.326/(96\,485 \times 4) = $ **1.219 V**

Borgnakke and Sonntag

13.197E

Pentane is burned with 120% theoretical air in a constant pressure process at 14.7 lbf/in². The products are cooled to ambient temperature, 70 F. How much mass of water is condensed per pound-mass of fuel? Repeat the answer, assuming that the air used in the combustion has a relative humidity of 90%.

$$C_5H_{12} + 1.2 \times 8\left(O_2 + 3.76\,N_2\right) \quad \rightarrow \quad 5\,CO_2 + 6\,H_2O + 0.96\,O_2 + 36.1\,N_2$$

Products cooled to 70 F, 14.7 lbf/in²

a) for H_2O at 70 F: $P_G = 0.3632$ lbf/in²

$$y_{H2O\ MAX} = \frac{P_G}{P} = \frac{0.3632}{14.7} = \frac{n_{H2O\ MAX}}{n_{H2O\ MAX} + 42.06}$$

Solving, $n_{H2O\ MAX} = 1.066 < n_{H2O}$

Therefore, $n_{H2O\ VAP} = 1.066$, $n_{H2O\ LIQ} = 6 - 1.066 = 4.934$

$$m_{H2O\ LIQ} = \frac{4.934 \times 18.015}{72.151} = \textbf{1.232 lbm/lbm fuel}$$

b) $P_{v1} = 0.9 \times 0.3632 = 0.3269$ lbf/in²

$$w_1 = 0.622 \times \frac{0.3269}{14.373} = 0.014\ 147$$

$$n_{H2O\ IN} = 0.014147 \times \frac{28.97}{18.015} \times (9.6 + 36.1) = 1.040\ \text{lbmol}$$

$$n_{H2O\ OUT} = 1.04 + 6 = 7.04$$

$$n_{H2O\ LIQ} = 7.04 - 1.066 = 5.974\ \text{lb mol}$$

$$n_{H2O\ LIQ} = \frac{5.974 \times 18.015}{72.151} = \textbf{1.492 lbm/lbm fuel}$$

13.198E

A small air-cooled gasoline engine is tested, and the output is found to be 2.0 hp. The temperature of the products is measured and found to be 730 F. The products are analyzed on a dry volumetric basis, with the following result 11.4% CO_2, 2.9% CO, 1.6% O_2 and 84.1% N_2. The fuel may be considered to be liquid octane. The fuel and air enter the engine at 77 F, and the flow rate of fuel to the engine is 1.8 lbm/h. Determine the rate of heat transfer from the engine and its thermal efficiency.

$$a\,C_8H_{18} + b\,O_2 + 3.76b\,N_2 \rightarrow 11.4\,CO_2 + 2.9\,CO + c\,H_2O + 1.6\,O_2 + 84.1\,N_2$$

$$b = 84.1/3.76 = 22.37, \quad a = (1/8)(11.4 + 2.9) = 1.788, \quad c = 9a = 16.088$$

$$C_8H_{18} + 12.5\,O_2 + 47.1\,N_2$$

$$\rightarrow 6.38\,CO_2 + 1.62\,CO + 9\,H_2O + 0.89\,O_2 + 47.1\,N_2$$

a) $H_R = \bar{h}^{\,\circ}_{f\,C8H18} = $ -107 526 Btu/lbmol

$H_P = 6.38(\text{-169 184} + 6807) + 1.62(\text{-47 518} + 4647)$

$\qquad + 9(\text{-103 966} + 5475) + 0.89(0+4822)$

$\qquad + 47.1(0 + 4617) = $ -1 770 092 Btu/lbmol

$H_P - H_R = \text{-1 770 092} - (\text{-107 526}) = $ -1 662 566 Btu/lbmol

$$\dot{H}_P - \dot{H}_R = \frac{1.8}{114.23}(\text{-1 662 566}) = \text{-26 198 Btu/h} \quad (M_{fuel} = 114.23)$$

$$\dot{Q}_{CV} = \text{-26 198} + 2.0\,(2544) = \textbf{-21 110 Btu/h}$$

b) Fuel heating value from table 15.3 converted to Btu/lbm

$$\dot{Q}_H = 1.8 \times (47\,893/2.326) = 37\,062 \text{ Btu/h}$$

$$\dot{W}_{NET} = 2.0 \times 2544 = 5088 \text{ Btu/h}; \qquad \eta_{TH} = \frac{5088}{37062} = \textbf{0.137}$$

13.199E

A gasoline engine uses liquid octane and air, both supplied at P_o, T_o, in a stoichiometric ratio. The products (complete combustion) flow out of the exhaust valve at 2000 R. Assume that the heat loss carried away by the cooling water, at 200 F, is equal to the work output. Find the efficiency of the engine expressed as (work/lower heating value) and the second law efficiency.

$$C_8H_{18} + 12.5(O_2 + 3.76N_2) \rightarrow 8\ CO_2 + 9\ H_2O + 47\ N_2$$

Table 13.3 LHV convert units kJ/kg to Btu/lbm

$\quad$ LHV = 44 425 × 114.232 / 2.326 = 2 181 753 Btu/lbmol fuel

Table F.6

$\quad$ $\Delta H_{P,2000}$ = 8 × 16 982 + 9 × 13 183 + 47 × 10 804 = 643 470

C.V. Total engine: $H_{in} = H_{ex} + W + Q_{loss} = H_{ex} + 2W$

$\quad$ $W = (H_{in} - H_{ex})/2 = (H_R - H_P)/2 = (-H^{\circ}_{RP} - \Delta H_{P,2000})/2$

$\quad\quad$ = (2 181 753 – 643 470)/2 = 769 142 Btu/lbmol fuel

$\quad$ η_{TH} = W/LHV = 769 142/2 181 753 = **0.353**

For 2nd law efficiency we must find reversible work

$\quad$ $\bar{S}_{in} = \bar{s}_{fuel} + 12.5(\bar{s}_{O_2} + 3.76\bar{s}_{N_2})$

$\quad\quad$ = 86.122 + 12.5$\left[48.973 - 1.98589 \ln (1 / 4.76)\right]$

$\quad\quad\quad\quad\quad\quad$ + 47$\left[45.739 - 1.98589 \ln (3.76 / 4.76)\right]$

$\quad\quad$ = 2908.8 Btu/(lbmol fuel × R)

$\quad$ $\bar{S}_{ex} = 8\bar{s}_{CO_2} + 9\bar{s}_{H_2O} + 47\bar{s}_{N_2} = 8\left[65.677 - 1.98589 \ln (8 / 64)\right]$

$\quad\quad$ + 9$\left[56.619 - 1.98589 \ln (\dfrac{9}{64})\right]$ + 47$\left[55.302 - 1.98589 \ln (\dfrac{47}{64})\right]$

$\quad\quad$ = 3731.1 Btu/(lbmol fuel × R)

Assume the same Q_{loss} out to 200 F = 659.67 R reservoir and compute Q_0^{rev}:

$\quad$ $\bar{S}_{in} + Q_0^{rev}/T_0 = \bar{S}_{ex} + Q_{loss}/T_{res}$

$\quad$ $Q_0^{rev} = T_0(\bar{S}_{ex} - \bar{S}_{in}) + Q_{loss}T_0/T_{res}$

$\quad\quad$ = 536.67(3731.1 - 2908.8) + 769 142*536.67/659.67

$\quad\quad$ = 1 067 034 Btu/lbmol fuel

$$W^{rev} = H_{in} - H_{ex} - Q_{loss} + Q_0^{rev} = W_{ac} + Q_0^{rev}$$

$$= 769\ 142 + 1\ 067\ 034 = 1\ 836\ 176 \text{ Btu/lbmol fuel}$$

$$\eta_{II} = W_{ac}/W^{rev} = 769\ 142/1\ 836\ 176 = \textbf{0.419}$$

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13.200E

Ethene, C_2H_4, and propane, C_3H_8, in a 1:1 mole ratio as gases are burned with 120% theoretical air in a gas turbine. Fuel is added at 77 F, 150 lbf/in.^{2} and the air comes from the atmosphere, 77 F, 15 lbf/in.^{2} through a compressor to 150 lbf/in.^{2} and mixed with the fuel. The turbine work is such that the exit temperature is 1500 R with an exit pressure of 14.7 lbf/in.^{2}. Find the mixture temperature before combustion, and also the work, assuming an adiabatic turbine.

$$C_2H_4 + C_3H_8 + \nu_{O2}(O_2+3.76N_2) \rightarrow 5CO_2 + 6H_2O + \nu_{N2}N_2$$

$$\phi = 1 \Rightarrow \quad \nu_{O2} = 8 \quad \phi = 1/1.2 \quad \Rightarrow \quad \nu_{O2} = 9.6$$

so we have 45.696 lbmol air per 2 lbmol fuel

$$C_2H_4 + C_3H_8 + 9.6(O_2 + 3.76N_2) \rightarrow 5CO_2 + 6H_2O + 1.6\,O_2 + 36.096N_2$$

C.V. Compressor (air flow)

$$w_{c,in} = h_2 - h_1 \quad s_2 = s_1 \Rightarrow \quad P_{r1} = 1.0907$$

$$P_{r2} = P_{r1}P_2/P_1 = 10.907 \Rightarrow \quad T_{2\ air} = 1027.3\ R$$

$$w_{c,in} = 247.81 - 128.38 = 119.53\ Btu/lbm = 3462.4\ Btu/lbmol\ air$$

C.V. Mixing chamber

$$\dot{n}_{air}\bar{h}_{air\ in} + \dot{n}_{fu1}\bar{h}_{fu1} + \dot{n}_{fu2}\bar{h}_{fu2} = (same)_{exit}$$

$$(\bar{C}_{PF1} + \bar{C}_{PF2})(T_{ex} - T_0) = 45.696\,\bar{C}_{P\ air}(T_{2\ air} - T_{ex})$$

$$\bar{C}_{PF1} = 11.53, \quad \bar{C}_{PF2} = 17.95, \quad \bar{C}_{P\ air} = 6.953\ \ Btu/lbmol\text{-}R$$

$$T_{ex} = \frac{45.696\bar{C}_{P\ air}T_2 + (\bar{C}_{PF1} + \bar{C}_{PF2})T_0}{\bar{C}_{PF1} + \bar{C}_{PF2} + 45.696\bar{C}_{air}} = \mathbf{985.6\ R} = T_{in\ combust}$$

Turbine work: take C.V. total and subtract compressor work.

$$W_{total} = H_{in} - H_{out} = H_R - H_{P,1500}$$

$$= \bar{h}^\circ_{f\ F1} + \bar{h}^\circ_{f\ F2} - 5\bar{h}_{CO2} - 6\bar{h}_{H2O} - 36.096\bar{h}_{N2} - 1.6\bar{h}_{O2}$$

$$= 22557 + (-44669) - 5(10557 - 169184)$$

$$\quad - 6(8306 - 103966) - 36.096 \times 6925 - 1.6 \times 7297.5$$

$$= 1\ 083\ 342\ Btu/2\ lbmol\ Fuel$$

$$w_T = w_{tot} + w_{c,in} = 1\ 083\ 342 + 3462.4 \times 45.696$$

$$= \mathbf{1\ 241\ 560\ Btu/2\ lbmol\ fuel}$$

13.201E

Repeat problem 13.190E, but assume that saturated-liquid oxygen at 170 R is used instead of 77 F oxygen gas in the combustion process. Use the generalized charts to determine the properties of liquid oxygen.

Problem the same as 13.190E, except oxygen enters at state 2 as saturated liquid at 170 R.

At 170 R, $T_{r2} = \dfrac{170}{278.6} = 0.61 \Rightarrow \Delta\bar{h}_f = 5.1$

From Fig. D.2:

$(\bar{h}^* - \bar{h}) = 1.98589 \times 278.6 \times 5.1 = 2822$ Btu/lbmol

$\Delta H_{P_3} = H_R^{\circ} + \Delta H_R - H_P^{\circ} + Q_{CV} = 21\ 647 - 0.5(2822)$

$\qquad + 0.5(0.219)(170 - 536.67)(32) + 103\ 966 - 1442$

$\qquad = 121\ 475$ Btu/lbmol

With

$\Delta H_{P\ 5000R} = 49\ 114 + 34\ 627 + 36\ 330 = 120\ 071$ Btu/lbmol,

$\Delta H_{P\ 5500R} = 55\ 739 + 39\ 032 + 40\ 745 = 135\ 516$ Btu/lbmol,

Linear interpolation $\Rightarrow T_3 = \textbf{5045 R}$

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 14

8e

Updated June 2013

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CONTENT CHAPTER 14

In-Text Concept Questions

14.a

For a mixture of O_2 and O the pressure is increased at constant T; what happens to the composition?

An increase in pressure causes the reaction to go toward the side of smaller total number of moles, in this case toward the O_2 .

14.b

For a mixture of O_2 and O the temperature is increased at constant P; what happens to the composition?

A temperature increase causes more O_2 to dissociate to O.

14.c

For a mixture of O_2 and O I add some argon keeping constant T, P; what happens to the moles of O?

Diluting the mixture with a non-reacting gas has the same effect as decreasing the pressure, causing the reaction to shift toward the side of larger total number of moles, in this case the O.

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14.d

When dissociations occur after combustion, does T go up or down?

Dissociation reactions of combustion products lower the temperature.

14.e

For nearly all the dissociations and ionization reactions what happens with the composition when the pressure is raised?

The reactions move towards the side with fewer moles of particles, that is ions and electrons towards the monatomic side and monatomic species combine to form the molecules (diatomic or more)

14.f

How does the time scale for NO formation change when P is lower at same T?

Look at the expression for the time scale in Eq.14.40. As P is lowered the time scale becomes larger. The formation rates drops, and the effect is explained by the larger distance between the molecules/atoms (density lower), the same T essentially means that they have the same characteristic velocity.

14.g

Which atom in air ionizes first as T increases? What is the explanation?

Using Fig. 14.11, we note that as temperature increases, atomic N ionizes to N^+, becoming significant at about 6-8000 K. N has a lower ionization potential compared to O or Ar.

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Concept-Study Guide Problems

14.1

Is the concept of equilibrium limited to thermodynamics?

Equilibrium is a condition in which the driving forces present are balanced, with no tendency for a change to occur spontaneously. This concept applies to many diverse fields of study – one no doubt familiar to the student being that of mechanical equilibrium in statics, or engineering mechanics.

14.2

How does Gibbs function vary with quality as you move from liquid to vapor?

There is no change in Gibbs function between liquid and vapor. For equilibrium we have $g_g = g_f$.

14.3

How is a chemical equilibrium process different from a combustion process?

Chemical equilibrium occurs at a given state, T and P, following a chemical reaction process, possibly a combustion (which is a chemical reaction process) together with several reactions amongst the combustion products. Whereas the combustion is a one-way process (irreversible) the chemical equilibrium is a reversible process that can proceed in both directions.

14.4

Must P and T be held fixed to obtain chemical equilibrium?

No, but we commonly evaluate the condition of chemical equilibrium at a state corresponding to a given temperature and pressure. If T and P changes in a process it means that the chemical composition adjusts towards equilibrium and the composition changes along with the process.

14.5

The change in Gibbs function ΔG^o for a reaction is a function of which property?

The change in Gibbs function for a reaction ΔG is a function of T and P.
The change in standard-state Gibbs function ΔG^o is a function only of T.

14.6

In a steady flow burner T is not controlled, which properties are?

The pressure tends to be constant, only minor pressure changes due to acceleration of the products as density decreases velocity must increase to have the same mass flow rate. The product temperature depends on heat losses (radiation etc.) and any chemical reactions that may take place generally lowering the temperature below the standard adiabatic flame temperature.

14.7

In a closed rigid combustion bomb which properties are held fixed?

The volume is constant. The number of atoms of each element is conserved, although the amounts of various chemical species change. As the products have more internal energy but cannot expand the pressure increases significantly.

14.8

Is the dissociation of water pressure sensitive?

Yes, since the total number of moles on the left and right sides of the reaction equation(s) is not the same.

14.9

At 298 K, K = exp(−184) for the water dissociation, what does that imply?

This is an extremely small number, meaning that the reaction tends to go strongly from right to left – in other words, does not tend to go from left to right (dissociation of water) at all.

14.10

If a reaction is insensitive to pressure prove that it is also insensitive to dilution effects at a given T.

Assume the standard reaction we used to develop the expression for the equilibrium constant: $\nu_A A + \nu_B B \Leftrightarrow \nu_C C + \nu_D D$

let us assume we add an inert component E so the total moles become:

$$n_{tot} = n_A + n_B + n_C + n_D + n_E$$

This will now lower the mole fractions of A, B, C, and D. If the reaction is pressure insensitive then: $\nu_A + \nu_B = \nu_C + \nu_D$ and the equilibrium equation becomes:

$$K = \frac{y_C^{\nu_C} y_D^{\nu_D}}{y_A^{\nu_A} y_B^{\nu_B}} \left(\frac{P}{P^o}\right)^{\nu_C + \nu_D - \nu_A - \nu_B} = \frac{y_C^{\nu_C} y_D^{\nu_D}}{y_A^{\nu_A} y_B^{\nu_B}}$$

Since each $y_i = n_i / n_{tot}$ we get:

$$K = \frac{y_C^{\nu_C} y_D^{\nu_D}}{y_A^{\nu_A} y_B^{\nu_B}} = \frac{(n_C/n_{tot})^{\nu_C} (n_D/n_{tot})^{\nu_D}}{(n_A/n_{tot})^{\nu_A} (n_B/n_{tot})^{\nu_B}}$$

$$= \frac{n_C^{\nu_C} n_D^{\nu_D}}{n_A^{\nu_A} n_B^{\nu_B}} n_{tot}^{\nu_A + \nu_B - \nu_C - \nu_D} = \frac{n_C^{\nu_C} n_D^{\nu_D}}{n_A^{\nu_A} n_B^{\nu_B}}$$

Now we see that the total number of moles that includes n_E does not enter the equation and thus will not affect any progress of the reaction.

14.11

For a pressure sensitive reaction an inert gas is added (dilution), how does the reaction shift?

Assume the standard reaction we used to develop the expression for the equilibrium constant: $\nu_A A + \nu_B B \iff \nu_C C + \nu_D D$

let us assume we add an inert component E so the total moles become:

$$n_{tot} = n_A + n_B + n_C + n_D + n_E$$

This will now lower the mole fractions of A, B, C, and D. If the reaction is pressure sensitive then: $\nu_A + \nu_B \neq \nu_C + \nu_D$ and the equilibrium equation becomes:

$$K = \frac{y_C^{\nu_C}\, y_D^{\nu_D}}{y_A^{\nu_A}\, y_B^{\nu_B}} \left(\frac{P}{P^o}\right)^{\nu_C + \nu_D - \nu_A - \nu_B}$$

Since each $y_i = n_i / n_{tot}$ we get:

$$K = \frac{(n_C/n_{tot})^{\nu_C}\, (n_D/n_{tot})^{\nu_D}}{(n_A/n_{tot})^{\nu_A}\, (n_B/n_{tot})^{\nu_B}} \left(\frac{P}{P^o}\right)^{\nu_C + \nu_D - \nu_A - \nu_B}$$

$$= \frac{n_C^{\nu_C}\, n_D^{\nu_D}}{n_A^{\nu_A}\, n_B^{\nu_B}} \left(\frac{P}{P^o\, n_{tot}}\right)^{\nu_C + \nu_D - \nu_A - \nu_B}$$

As n_{tot} is raised due to n_E it acts as if the pressure P is lowered thus pushing the reaction towards the side with a larger number of moles.

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14.12

In a combustion process is the adiabatic flame temperature affected by reactions?

The adiabatic flame temperature is lower due to dissociation reactions of the products as those products absorb some of the energy, i.e. 2 O atoms have more energy, $h = 2(249\ 170 + \Delta h)$ than a single O_2 molecule, $h = \Delta h$, see Table A.9. The temperature is also influenced by other reactions like the water gas reaction.

14.13

In equilibrium Gibbs function of the reactants and the products is the same; how about the energy?

The chemical equilibrium mixture at a given T, P has a certain total internal energy. There is no restriction on its division among the constituents. The conservation of energy from the reactants to the products will determine the temperature so if it takes place in a fixed volume (combustion bomb) then U is constant whereas if it is in a flow (like a steady flow burner) then H is constant. When this is combined with chemical equilibrium it is actually a lengthy procedure to determine both the composition and the temperature in the actual process.

14.14

Does a dissociation process require energy or does it give out energy?

Dissociation reactions require energy and is thus endothermic. Notice from Table A.9 that all the atoms (N, O, H) has a much higher formation enthalpy than the diatomic molecules (which have formation enthalpy equal to zero).

14.15

If I consider the non-frozen (composition can vary) specific heat, but still assume all components are ideal gases, does that C become a function of temperature? of pressure?

The non-frozen mixture heat capacity will be a function of both T and P, because the mixture composition depends on T and P, while the individual component specific heat capacities depend only on T.

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14.16

What is K for the water gas reaction in Example 14.4 at 1200 K?

Using the result of Example 14.4 and Table A.11

$$\ln K = \frac{1}{2} (\ln K_I - \ln K_{II})$$
$$= 0.5 [-35.736 - (-36.363)] = +0.3135 ,$$

K = 1.3682

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14.17

What would happen to the concentrations of the monatomic species like O, N if the pressure is higher in Fig. 14.11

Since those reaction are pressure sensitive (more moles on RHS than on LHS) the higher pressure will push these reactions to the left and reduce the concentrations of the monatomic species.

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Equilibrium and Phase Equilibrium

14.18

Carbon dioxide at 15 MPa is injected into the top of a 5-km deep well in connection with an enhanced oil-recovery process. The fluid column standing in the well is at a uniform temperature of 40°C. What is the pressure at the bottom of the well assuming ideal gas behavior?

Z_1

Z_2

CO_2

cb

$(Z_1 - Z_2) = 5000 \text{ m}, \quad P_1 = 15 \text{ MPa}$

$T = 40 \; ^{\circ}C = \text{constant}$

Equilibrium at constant T

$-w_{REV} = 0 = \Delta g + \Delta PE$

$= RT \ln (P_2/P_1) + g(Z_2 - Z_1) = 0$

$$\ln (P_2/P_1) = \frac{9.807 \times 5000}{1000 \times 0.188 \; 92 \times 313.2} = 0.8287$$

$$P_2 = 15 \text{ MPa } \exp(0.8287) = \textbf{34.36 MPa}$$

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14.19

Consider a 2-km-deep gas well containing a gas mixture of methane and ethane at a uniform temperature of $30^{\circ}C$. The pressure at the top of the well is 14 MPa, and the composition on a mole basis is 90% methane, 10% ethane. Each component is in equilibrium (top to bottom) with $dG + g\,dZ = 0$ and assume ideal gas, so for each component Eq.14.10 applies. Determine the pressure and composition at the bottom of the well.

$(Z_1 - Z_2) = 2000$ m, Let $A = CH_4$, $B = C_2H_6$

$P_1 = 14$ MPa, $y_{A1} = 0.90$, $y_{B1} = 0.10$

$T = 30\ ^{\circ}C = $ constant

From section 14.1, for A to be at equilibrium between

1 and 2: $W_{REV} = 0 = n_A(\bar{G}_{A1} - \bar{G}_{A2}) + n_A M_A\, g\,(Z_1 - Z_2)$

Similarly, for B: $W_{REV} = 0 = n_B(\bar{G}_{B1} - \bar{G}_{B2}) + n_B M_B\, g\,(Z_1 - Z_2)$

Using eq. 14.10 for A: $\bar{R}T \ln (P_{A2}/P_{A1}) = M_A\, g\,(Z_1 - Z_2)$

with a similar expression for B. Now, ideal gas mixture, $P_{A1} = y_{A1}P$, etc.

Substituting: $\ln \dfrac{y_{A2}P_2}{y_{A1}P_1} = \dfrac{M_A g(Z_1 - Z_2)}{\bar{R}T}$ and $\ln \dfrac{y_{B2}P_2}{y_{B1}P_1} = \dfrac{M_B g(Z_1 - Z_2)}{\bar{R}T}$

$\ln (y_{A2}\,P_2) = \ln(0.9 \times 14) + \dfrac{16.04 \times 9.807(2000)}{1000 \times 8.3145 \times 303.2} = 2.6585$

$\Rightarrow\ y_{A2}\,P_2 = 14.2748$

$\ln (y_{B2}P_2) = \ln(0.1 \times 14) + \dfrac{30.07 \times 9.807(2000)}{1000 \times 8.3145 \times 303.2} = 0.570\,43$

$\Rightarrow\quad y_{B2}\,P_2 = (1 - y_{A2})\,P_2 = 1.76903$

Solving: $P_2 = \mathbf{16.044\ MPa}$ & $y_{A2} = \mathbf{0.8897}$

14.20

A container has liquid water at 20^oC , 100 kPa in equilibrium with a mixture of water vapor and dry air also at 20^oC, 100 kPa. How much is the water vapor pressure and what is the saturated water vapor pressure?

From the steam tables we have for saturated liquid:

$$P_g = \textbf{2.339 kPa}, \quad v_f = 0.001002 \text{ m}^3/\text{kg}$$

The liquid is at 100 kPa so it is compressed liquid still at 20^oC so from Eq.14.15 at constant T

$$g_{liq} - g_f = \int v \, dP = v_f \, (P - P_g)$$

The vapor in the moist air is at the partial pressure P_v also at 20^oC so we assume ideal gas for the vapor

$$g_{vap} - g_g = \int v \, dP = RT \ln \frac{P_v}{P_g}$$

We have the two saturated phases so $g_f = g_g$ ($q = h_{fg} = Ts_{fg}$) and now for equilibrium the two Gibbs function must be the same as

$$g_{vap} = g_{liq} = RT \ln \frac{P_v}{P_g} + g_g = v_f \, (P - P_g) + g_f$$

leaving us with

$$\ln \frac{P_v}{P_g} = v_f \, (P - P_g)/ RT = \frac{0.001002 \, (100 - 2.339)}{0.4615 \times 293.15} = 0.000723$$

$$P_v = P_g \exp(0.000723) = \textbf{2.3407 kPa}.$$

This is only a minute amount above the saturation pressure. For the moist air applications in Chapter 11 we neglected such differences and assumed the partial water vapor pressure at equilibrium (100% relative humidity) is P_g. The pressure has to be much higher for this to be a significant difference.

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14.21

Using the same assumptions as those in developing Eq. d in Example 14.1, develop an expression for pressure at the bottom of a deep column of liquid in terms of the isothermal compressibility, β_T. For liquid water at 20^oC, $\beta_T = 0.0005$ [1/MPa]. Use the result of the first question to estimate the pressure in the Pacific ocean at the depth of 3 km.

$$d\ g_T = v^\circ\ (1 - \beta_T P)\ dP_T \qquad\qquad d\ g_T + g\ dz = 0$$

$$v^\circ\ (1 - \beta_T P)\ dP_T + g\ dz = 0 \quad \text{and integrate} \quad \int v^\circ (1 - \beta_T P)\ dP_T = -\ g \int dz$$

$$\int_{P_0}^{P} (1 - \beta_T P)\ dP_T = +\frac{g}{v^\circ} \int_{0}^{+H} dz \quad \Rightarrow \quad P - P_0 - \beta_T \frac{1}{2}\ [P^2 - P_0^{\ 2}] = \frac{g}{v^\circ}\ H$$

$$P\ (1 - \frac{1}{2}\ \beta_T\ P) = P_0 - \frac{1}{2}\beta_T\ P_0^{\ 2} + \frac{g}{v^\circ}\ H$$

$$v^\circ = v_{f\ 20^\circ C} = 0.001002; \quad H = 3000\ m, \quad g = 9.80665\ m/s^2; \quad \beta_T = 0.0005\ 1/MPa$$

$$P\ (1 - \frac{1}{2} \times 0.0005\ P) = 0.101 - \frac{1}{2} \times 0.0005 \times 0.101^2$$

$$+ [9.80665 \times 3000/0.001002] \times 10^{-6}$$

$$= 29.462\ MPa, \quad \text{which is close to P}$$

Solve by iteration or solve the quadratic equation

$$P = \textbf{29.682 MPa}$$

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Chemical Equilibrium, Equilibrium Constant

Chemical Equilibrium, Equilibrium Constant

14.22

Which of the reactions listed in Table A.11 are pressure sensitive?

Check if: $\nu_A + \nu_B \neq \nu_C + \nu_D$

Reaction	Check	P sensitive?
$H_2 \Leftrightarrow 2H$	$1 < 2$	yes
$O_2 \Leftrightarrow 2O$	$1 < 2$	yes
$N_2 \Leftrightarrow 2N$	$1 < 2$	yes
$2\,H_2O \Leftrightarrow 2\,H_2 + 1\,O_2$	$2 < 3$	yes
$2\,H_2O \Leftrightarrow 2\,H_2 + 2\,OH$	$2 < 4$	yes
$2\,CO_2 \Leftrightarrow 2\,CO + 1\,O_2$	$2 < 3$	yes
$N_2 + O_2 \Leftrightarrow 2\,NO$	$2 = 2$	no
$N_2 + 2O_2 \Leftrightarrow 2\,NO_2$	$3 > 2$	yes

Most of them have more moles on RHS and thus will move towards the RHS if the pressure is lowered. Only the last one has the opposite and will move towards the LHS if the pressure is lowered.

14.23

Calculate the equilibrium constant for the reaction $O_2 \Leftrightarrow 2O$ at temperatures of 298 K and 6000 K. Verify the result with Table A.11.

$$\text{Reaction} \quad O_2 \quad \Leftrightarrow \quad 2O$$

At 25 ^{o}C (298.15 K):

$$\Delta H^0 = 2\bar{h}^0_{f\,O} - 1\bar{h}^0_{f\,O2} = 2(249\ 170) - 1(0) = 498\ 340 \text{ kJ/kmol}$$

$$\Delta S^0 = 2\bar{s}^0_O - 1\bar{s}^0_{O2} = 2(161.059) - 1(205.148) = 116.97 \text{ kJ/kmol K}$$

$$\Delta G^0 = \Delta H^0 - T\Delta S^0 = 498\ 340 - 298.15\times116.97 = 463\ 465 \text{ kJ/kmol}$$

$$\ln K = -\frac{\Delta G^0}{\bar{R}T} = -\frac{463\ 465}{8.3145\times298.15} = -\mathbf{186.961}$$

At 6000 K:

$$\Delta H^0 = 2(249\ 170 + 121\ 264) - (0 + 224\ 210) = 516\ 658 \text{ kJ/kmol}$$

$$\Delta S^0 = 2(224.597) - 1(313.457) = 135.737 \text{ kJ/kmol K}$$

$$\Delta G^0 = 516\ 658 - 6000\times135.737 = -297\ 764 \text{ kJ/kmol}$$

$$\ln K = \frac{+297\ 764}{8.3145\times6000} = +\mathbf{5.969}$$

14.24

Calculate the equilibrium constant for the reaction $H_2 \Leftrightarrow 2H$ at a temperature of 2000 K, using properties from Table A.9. Compare the result with the value listed in Table A.11.

From Table A.9 at 2000 K we find:

$$\Delta \bar{h}_{H_2} = 52\ 942 \text{ kJ/kmol}; \quad \bar{s}_{H_2} = 188.419 \text{ kJ/kmol K}; \quad \bar{h}_f^o = 0$$

$$\Delta \bar{h}_H = 35\ 375 \text{ kJ/kmol}; \quad \bar{s}_H = 154.279 \text{ kJ/kmol K}; \quad \bar{h}_f^o = 217\ 999 \text{ kJ/kmol}$$

$$\Delta G^0 = \Delta H - T\Delta S = H_{RHS} - H_{LHS} - T\ (S^0_{RHS} - S^0_{LHS})$$

$$= 2 \times (35\ 375 + 217\ 999) - 52943 - 2000(2 \times 154.279 - 182.419)$$

$$= 213\ 528 \text{ kJ/kmol}$$

$$\ln K = -\Delta G^0/\bar{R}T = -213\ 528\ /\ (8.3145 \times 2000) = -12.8407$$

Table A.11 $\ln K = -12.841$ OK

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14.25

For the dissociation of oxygen, $O_2 \Leftrightarrow 2O$, around 2000 K we want a mathematical expression for the equilibrium constant K(T). Assume constant heat capacity, at 2000 K, for O_2 and O from Table A.9 and develop the expression from Eqs. 14.12 and 14.15.

From Eq.14.15 the equilibrium constant is

$$K = \exp(-\frac{\Delta G^0}{\bar{R}T}) \; ; \qquad \Delta G^0 = \Delta H^0 - T \, \Delta S^0$$

and the shift is

$$\Delta G^0 = 2 \, \bar{h}_O - \bar{h}_{O2} - T(2\bar{s}_O^o - \bar{s}_{O_2}^o)$$

Substitute the first order approximation to the functions $\bar{h}$ and $\bar{s}^o$ as

$$\bar{h} = \bar{h}_{2000\,K} + \bar{C}_p \, (T - 2000) \; ; \quad \bar{s}^o = \bar{s}_{2000\,K}^o + \bar{C}_p \ln \frac{T}{2000}$$

The properties are from Table A.9 and $\bar{R}$ = 8.3145 kJ/kmol K

Oxygen O_2: $\quad \bar{h}_{2000\,K}$ = 59 176 kJ/kmol, $\quad \bar{s}_{2000\,K}^o$ = 268.748 kJ/kmol K

$$\bar{C}_p = \frac{\bar{h}_{2200\,K} - \bar{h}_{2200\,K}}{2200 - 1800} = \frac{66\,770 - 51\,674}{400} = 37.74 \text{ kJ/kmol K}$$

Oxygen O: $\quad \bar{h}_{2000\,K}$ = 35 713 + 249 170 = 284 883 kJ/kmol,

$\bar{s}_{2000\,K}^o$ = 201.247 kJ/kmol K

$$\bar{C}_p = \frac{\bar{h}_{2200\,K} - \bar{h}_{2200\,K}}{2200 - 1800} = \frac{39\,878 - 31\,547}{400} = 20.8275 \text{ kJ/kmol K}$$

Substitute and collect terms

$$\frac{\Delta G^0}{\bar{R}T} = \frac{\Delta H^0}{\bar{R}T} - \frac{\Delta S^0}{\bar{R}} = \frac{\Delta H_{2000}^0}{\bar{R}T} + \frac{\Delta \bar{C}_{p\,2000}}{\bar{R}} \left[\frac{T - 2000}{T} - \ln \frac{T}{2000} \right] - \frac{\Delta S_{2000}^0}{\bar{R}}$$

Now we have

$$\Delta H_{2000}^0/\bar{R} = (2 \times 284\,883 - 59\,176)/8.3145 = 61\,409.6 \text{ K}$$

$$\Delta \bar{C}_{p\,2000}/\bar{R} = (2 \times 20.8275 - 37.74)/8.3145 = 0.470864$$

$$\Delta S^0_{2000}/\overline{R} = (2 \times 201.247 - 268.748)/8.3145 = 16.08587$$

so we get

$$\frac{\Delta G^0}{\overline{R}T} = \frac{61\,409.6}{T} + 0.470864 \left[\frac{T-2000}{T} - \ln\frac{T}{2000} \right] - 16.08587$$

$$= \frac{60\,467.9}{T} - 15.615 - 0.470864 \ln\frac{T}{2000}$$

Now the equilibrium constant K(T) is approximated as

$$K(T) = \exp\left[15.615 - \frac{60\,467.9}{T} + 0.470864 \ln\frac{T}{2000} \right]$$

Remark: We could have chosen to expand the function $\Delta G^0/\overline{R}T$ as a linear expression instead or even expand the whole $\exp(-\Delta G^0/\overline{R}T)$ in a linear function.

14.26

Find K for: $CO_2 \Leftrightarrow CO + 1/2O_2$ at 3000 K using A.11

The elementary reaction in A.11 is : $2CO_2 \Leftrightarrow 2CO + O_2$

so the wanted reaction is (1/2) times that so

$$K = K_{A.11}^{1/2} = \sqrt{\exp(-2.217)} = \sqrt{0.108935} = 0.33$$

or

$$\ln K = 0.5 \ln K_{A.11} = 0.5\,(-2.217) = -1.1085$$
$$K = \exp(-1.1085) = 0.33$$

14.27

Plot to scale the values of ln K versus $1/T$ for the reaction $2\,CO_2 \Leftrightarrow 2\,CO + O_2$.
Write an equation for ln K as a function of temperature.

$$2\,CO_2 \Leftrightarrow 2\,CO + 1\,O_2$$

T(K)	$10^4 \times \frac{1}{T}$	ln K	T(K)	$10^4 \times \frac{1}{T}$	ln K
2000	5.000	-13.266	4000	2.500	3.204
2400	4.167	-7.715	4500	2.222	4.985
2800	3.571	-3.781	5000	2.000	6.397
3200	3.125	-0.853	5500	1.818	7.542
3600	2.778	1.408	6000	1.667	8.488

For the range
below ~ 5000 K,

$$\ln K \approx A + B/T$$

Using values at
2000 K & 5000 K

A = 19.5056
B = -65 543 K

14.28

Consider the reaction $2\,CO_2 \Leftrightarrow 2\,CO + O_2$ obtained after heating 1 kmol CO_2 to 3000 K. Find the equilibrium constant from the shift in Gibbs function and verify its value with the entry in Table A.11. What is the mole fraction of CO at 3000 K, 100 kPa?

From Table A.9 we get:

$\Delta \bar{h}_{CO} = 93\ 504$ $\qquad \bar{h}^0_{f\ CO} = $ -110 527 $\qquad \bar{s}_{CO} = 273.607$

$\Delta \bar{h}_{CO_2} = 152\ 853$ $\qquad \bar{h}^0_{f\ CO_2} = $ -393 522 $\qquad \bar{s}_{CO_2} = 334.17$

$\Delta \bar{h}_{O_2} = 98\ 013$ $\qquad \bar{s}_{O_2} = 284.466$

$\Delta G^0 = \Delta H - T\Delta S = 2\,H_{CO} + H_{O_2} - 2\,H_{CO_2} - T\,(2\bar{s}_{CO} + \bar{s}_{O_2} - 2\bar{s}_{CO_2})$

$\qquad = 2\,(93\ 504 - 110\ 527) + 98\ 013 + 0 - 2(152\ 853 - 393\ 522)$

$\qquad$ -3000(2×273.607 + 284.466 - 2×334.17) = 55 285

ln K = $-\Delta G^0 / \bar{R}T$ = -55 285/ (8.31451×3000) = -2.2164

Table A.11 ln K = -2.217 OK

At 3000 K, $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $2\,CO_2 \Leftrightarrow 2\,CO + 1\,O_2$

ln K = -2.217	Initial	1	0	0
K = 0.108935	Change	-2z	+2z	+z
	Equil.	1-2z	2z	z

We have $P = P^o = 0.1$ MPa, and $n_{tot} = 1 + z$, so from Eq.14.29

$$K = \frac{y^2_{CO}y_{O_2}}{y^2_{CO_2}}\left(\frac{P}{P^0}\right) = \left(\frac{2z}{1 - 2z}\right)^2 \left(\frac{z}{1 + z}\right)(1) = 0.108935\ ;$$

$\qquad 4\,z^3 = 0.108935\,(1 - 2z)^2(1 + z) \quad => \quad z = 0.22$

$\qquad y_{CO} = 2z\,/\,(1 + z) = \mathbf{0.36}$

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14.29

Carbon dioxide is heated at 100 kPa. What should the temperature be to see a mole fraction of CO as 0.25? For that temperature, what will the mole fraction of CO be if the pressure is 200 kPa?

$$2\,CO_2 \Leftrightarrow 2\,CO + 1\,O_2$$

Initial	1	0	0
Change	-2z	+2z	+z
Equil.	1-2z	2z	z

We have $P = P^o = 0.1$ MPa, and $n_{tot} = 1 + z$, so from Eq.14.29

$$y_{CO} = \frac{2z}{1 + z} = 0.25 \quad => \quad z = 1/7$$

$$K = \frac{y_{CO}^2 y_{O_2}}{y_{CO_2}^2} (\frac{P}{P^0}) = \left(\frac{2z}{1 - 2z}\right)^2 \left(\frac{z}{1 + z}\right)(1) = \left(\frac{2}{7 - 2}\right)^2 \left(\frac{1}{7 + 1}\right) = 0.02$$

$\ln K = -3.912$ from A.11 **T = 2785 K**

$$K = 0.02 = \frac{y_{CO}^2 y_{O_2}}{y_{CO_2}^2} (\frac{P}{P^0}) = \left(\frac{2z}{1 - 2z}\right)^2 \left(\frac{z}{1 + z}\right)(2) \quad =>$$

$$z^3 = 0.0025\,(1 - 2z)^2(1 + z) \quad => \quad z = 0.11776$$

$$y_{CO} = \frac{2z}{1 + z} = \mathbf{0.2107}$$

14.30

Assume a diatomic gas like O_2 or N_2 dissociate at a pressure different from P_o. Find an expression for the fraction of the original gas that has dissociated at any T assuming equilibrium.

Look at initially 1 mol Oxygen and shift reaction with x

$$O_2 \Leftrightarrow 2O$$

Species	O_2	O
Initial	1	0
Change	–x	2x
Equil.	1–x	2x

$n_{tot} = 1 - x + 2x = 1 + x$

$$y_O = \frac{2x}{1 + x} \quad \text{and} \quad y_{O2} = \frac{1 - x}{1 + x}$$

Substitute this into the equilibrium equation as

$$K_T = \frac{y_O^2}{y_{0_2}} \left(\frac{P}{P_o}\right)^{2\text{-}1} = \frac{4x^2}{(1 + x)^2} \frac{1 + x}{1 - x} \left(\frac{P}{P_o}\right) = \frac{4x^2}{1 - x^2} \left(\frac{P}{P_o}\right)$$

Now solve for x as

$$x^2 = (1 - x^2)\ \frac{K_T P_o}{4P}$$

$$x = \sqrt{\frac{K_T}{4(P/P_o) + K_T}}$$

14.31

Hydrogen gas is heated from room temperature to 4000 K, 500 kPa, at which state the diatomic species has partially dissociated to the monatomic form. Determine the equilibrium composition at this state.

$$H_2 \Leftrightarrow 2\,H \qquad\qquad \text{Equil.} \qquad n_{H2} = 1 - x$$
$$-x \quad +2x \qquad\qquad\qquad\qquad\qquad n_H = 0 + 2x$$
$$n = 1 + x$$

$$K = \frac{(2x)^2}{(1-x)(1+x)}\left(\frac{P}{P^0}\right)^{2-1} \qquad \text{at 4000 K:} \quad \ln K = 0.934 \;=>\; K = 2.545$$

$$\frac{2.545}{4\times(500/100)} = 0.127\ 25 = \frac{x^2}{1-x^2} \qquad \text{Solving,} \qquad x = 0.3360$$

$$n_{H2} = 0.664, \quad n_H = 0.672, \quad n_{tot} = 1.336$$

$$y_{H2} = 0.497, \quad y_H = 0.503$$

14.32

Consider the dissociation of oxygen, $O_2 \Leftrightarrow 2\,O$, starting with 1 kmol oxygen at 298 K and heating it at constant pressure 100 kPa. At which temperature will we reach a concentration of monatomic oxygen of 10%?

Look at initially 1 mol Oxygen and shift reaction with x

$$O_2 \Leftrightarrow 2\,O$$

Species	O_2	O
Initial	1	0
Change	-x	2x
Equil.	1-x	2x

$n_{tot} = 1 - x + 2x = 1 + x$

$$y_O = \frac{2x}{1+x} = 0.1 \quad \Rightarrow \quad x = 0.1/(2-0.1) = 0.0526, \quad y_{O2} = 0.9$$

$$K = \frac{y_O^2}{y_{0_2}}\left(\frac{P}{P_o}\right)^{2\text{-}1} = \frac{0.1^2}{0.9}\, 1 = 0.01111 \quad \Rightarrow \quad \ln K = -4.4998$$

Now look in Table A.11: T = **2980 K**

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14.33

Redo Problem 14.32 for a total pressure of 40 kPa.

Look at initially 1 mol Oxygen and shift reaction with x

$$O_2 \Leftrightarrow 2\,O$$

Species O_2 O
Initial   1   0
Change   -x   2x
Equil.   1-x   2x $n_{tot} = 1 - x + 2x = 1 + x$

$$y_O = \frac{2x}{1 + x} = 0.1 \;\; \Rightarrow \;\; x = 0.1/(2 - 0.1) = 0.0526, \;\; y_{O2} = 0.9$$

$$K = \frac{y_O^2}{y_{0_2}} \left(\frac{P}{P_o}\right)^{2\text{-}1} = \frac{0.1^2}{0.9}\,0.4 = 0.004444 \;\;\; \Rightarrow \; \ln K = -5.4161$$

Now look in Table A.11:   T = **2856 K**

14.34

Redo Problem 14.32, but start with 1 kmol oxygen and 1 kmol helium at 298 K, 100 kPa.

Look at initially 1 mol Oxygen and shift reaction with x

$$O_2 \Leftrightarrow 2\,O$$

Species	O_2	O	He	
Initial	1	0	1	
Change	-x	2x		
Equil.	1-x	2x	1	$n_{tot} = 1 - x + 2x + 1 = 2 + x$

$$y_O = \frac{2x}{2 + x} = 0.1 \quad \Rightarrow \quad x = 0.2/(2 - 0.1) = 0.10526,$$

$$y_{O2} = (1 - x)/(2 + x) = 0.425$$

$$K = \frac{y_O^2}{y_{O_2}}\left(\frac{P}{P_o}\right)^{2-1} = \frac{0.1^2}{0.425}\,1 = 0.023529 \quad \Rightarrow \quad \ln K = -3.7495$$

Now look in Table A.11:   T = **3094 K**

14.35

Calculate the equilibrium constant for the reaction: $2CO_2 \Leftrightarrow 2CO + O_2$ at 3000 K using values from Table A.9 and compare the result to Table A.11.

From Table A.9 we get:

kJ/kmol	kJ/kmol	kJ/kmol K

$\Delta \bar{h}_{CO} = 93\ 504$ $\bar{h}^o_{f\ CO} = -110\ 527$ $\bar{s}_{CO} = 273.607$

$\Delta \bar{h}_{CO_2} = 152\ 853$ $\bar{h}^o_{f\ CO_2} = -393\ 522$ $\bar{s}_{CO_2} = 334.17$

$\Delta \bar{h}_{O_2} = 98\ 013$ $\bar{h}^o_{f\ O_2} = 0$ $\bar{s}_{O_2} = 284.466$

$\Delta G^0 = \Delta H - T\Delta S = 2\ H_{CO} + H_{O_2} - 2\ H_{CO_2} - T\ (2\bar{s}_{CO} + \bar{s}_{O_2} - 2\bar{s}_{CO_2})$

$= 2\ (93\ 504 - 110\ 527) + 98\ 013 + 0 - 2(152\ 853 - 393\ 522)$

$-3000(2 \times 273.607 + 284.466 - 2 \times 334.17) = 55\ 285$ kJ/kmol

$\ln K = -\Delta G^0 / \bar{R}T = -55\ 285 / (8.31451 \times 3000) = -2.2164$

Table A.11 $\ln K = -2.217$ OK

14.36

Find the equilibrium constant for: $CO + 1/2 O_2 \Leftrightarrow CO_2$ at 2200 K using Table A.11

The elementary reaction in A.11 is: $2\,CO_2 \Leftrightarrow 2CO + O_2$

The wanted reaction is therefore (-0.5) times that so

$$K = K_{A.11}^{-1/2} = 1 / \sqrt{\exp(\text{-}10.232)} = 1 / \sqrt{0.000036} = 166.67$$

or

$$\ln K = -0.5 \ln K_{A.11} = -0.5\,(-10.232) = 5.116$$
$$K = \exp(5.116) = 166.67$$

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14.37

Pure oxygen is heated from 25°C to 3200 K in an steady flow process at a constant pressure of 200 kPa. Find the exit composition and the heat transfer.

The only reaction will be the dissociation of the oxygen

$O_2 \Leftrightarrow 2O$; From A.11: $K(3200) = \exp(-3.069) = 0.046467$

Look at initially 1 mol Oxygen and shift reaction with x

$$n_{O2} = 1 - x; \quad n_O = 2x; \quad n_{tot} = 1 + x; \quad y_i = n_i/n_{tot}$$

$$K = \frac{y_O^2}{y_{O_2}} \left(\frac{P}{P_o}\right)^{2-1} = \frac{4x^2}{(1 + x)^2} \frac{1 + x}{1 - x} 2 = \frac{8x^2}{1 - x^2}$$

$$x^2 = \frac{K/8}{1 + K/8} \quad \Rightarrow \quad x = 0.07599;$$

$$y_{O_2} = \frac{1 - x}{1 + x} = \textbf{0.859}; \quad y_O = 1 - y_{O_2} = \textbf{0.141}$$

$$\bar{q} = n_{O_2ex}\bar{h}_{O_2ex} + n_{Oex}\bar{h}_{Oex} - \bar{h}_{O_2in} = (1 + x)(y_{O_2}\bar{h}_{O_2} + y_O\bar{h}_O) - 0$$

$$\bar{h}_{O_2} = 106\ 022 \text{ kJ/kmol}; \quad \bar{h}_O = 249\ 170 + 60\ 767 = 309\ 937 \text{ kJ/kmol}$$

$$\Rightarrow \quad \bar{q} = 145\ 015 \text{ kJ/kmol O}_2$$

$$q = \bar{q}/32 = \textbf{4532 kJ/kg} \quad (= 3316.5 \text{ if no reaction})$$

14.38

Nitrogen gas, N_2, is heated to 4000 K, 10 kPa. What fraction of the N_2 is dissociated to N at this state?

$$N_2 \iff 2\,N \qquad @ \; T = 4000 \text{ K}, \quad \ln K = -12.671$$

Initial	1	0	$K = 3.14 \times 10^{-6}$
Change	-x	2x	
Equil.	1-x	2x	$n_{tot} = 1 - x + 2x = 1 + x$

$$y_{N2} = \frac{1 - x}{1 + x}, \qquad y_N = \frac{2x}{1 + x}$$

$$K = \frac{y_N^2}{y_{N2}}\left(\frac{P}{P_o}\right)^{2-1}; \quad \Rightarrow \quad 3.14 \times 10^{-6} = \frac{4x^2}{1 - x^2}\left(\frac{10}{100}\right) \quad \Rightarrow \quad x = 0.0028$$

$$y_{N2} = \frac{1 - x}{1 + x} = \mathbf{0.9944}, \qquad y_N = \frac{2x}{1 + x} = \mathbf{0.0056}$$

14.40

One kilomole Ar and one kilomole O_2 are heated up at a constant pressure of 100 kPa to 3200 K, where it comes to equilibrium. Find the final mole fractions for Ar, O_2, and O.

The only equilibrium reaction listed in the book is dissociation of O_2.

So assuming that, we see in Table A.11: $\ln(K) = -3.072$

$$Ar + O_2 \Rightarrow Ar + (1 - x) O_2 + 2x O$$

The atom balance already shown in above equation can also be done as

Species	Ar	O_2	O
Start	1	1	0
Change	0	-x	2x
Total	1	1-x	2x

The total number of moles is $n_{tot} = 1 + 1-x + 2x = 2 + x$ so

$$y_{Ar} = 1/(2 + x); \quad y_{O_2} = 1 - x/(2 + x); \quad y_O = 2x/(2 + x)$$

and the definition of the equilibrium constant ($P_{tot} = P_o$) becomes

$$K = e^{-3.072} = 0.04633 = \frac{y_O^2}{y_{0_2}} = \frac{4x^2}{(2 + x)(1 - x)}$$

The equation to solve becomes from the last expression

$$(K + 4)x^2 + Kx - 2K = 0$$

If that is solved we get

$$x = -0.0057 \pm 0.1514 = 0.1457; \qquad x \text{ must be positive}$$

$$y_O = 0.1358; \quad y_{0_2} = 0.3981; \quad y_{Ar} = 0.4661$$

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14.41

Air (assumed to be 79% nitrogen and 21% oxygen) is heated in a steady state process at a constant pressure of 100 kPa, and some NO is formed. At what temperature will the mole fraction of NO be 0.001?

$0.79\ N_2 + 0.21\ O_2$ heated at 100 kPa, forms NO

$$N_2 + O_2 \Leftrightarrow 2\ NO \qquad\qquad n_{N2} = 0.79 - x$$
$$-x \quad -x \quad\ +2x \qquad\qquad n_{O2} = 0.21 - x$$
$$n_{NO} = 0\ \ + 2x$$
$$n_{tot}\ = 1.0$$

At exit, $\ y_{NO} = 0.001 = \dfrac{2x}{1.0} \qquad \Rightarrow \qquad x = 0.0005$

$\Rightarrow n_{N2} = 0.7895,\ n_{O2} = 0.2095$

$$K = \frac{y_{NO}^{\ 2}}{y_{N2}y_{O2}}\left(\frac{P}{P^0}\right)^0 = \frac{10^{-6}}{0.7895 \times 0.2095} = 6.046 \times 10^{-6} \qquad \text{or}\ \ \ln K = -12.016$$

From Table A.11, $\quad$ T = **1444 K**

14.42

Pure oxygen is heated from 25°C, 100 kPa to 3200 K in a constant volume container. Find the final pressure, composition, and the heat transfer.

As oxygen is heated it dissociates

$$O_2 \Leftrightarrow 2O \qquad \ln K_{eq} = -3.069 \quad \text{from table A.11}$$

C. V. Heater: $U_2 - U_1 = {}_1Q_2 = H_2 - H_1 - P_2v + P_1v$

Per mole O_2: $\quad {}_1\bar{q}_2 = \bar{h}_2 - \bar{h}_1 + \bar{R}[T_1 - (n_2/n_1)T_2]$

Shift x in reaction 1 to have final composition: $(1 - x)O_2 + 2xO$

$n_1 = 1 \qquad n_2 = 1 - x + 2x = 1 + x$

$y_{O_2} = (1 - x)/(1 + x) ; \quad y_O = 2x/(1 + x)$

Ideal gas and $V_2 = V_1 \Rightarrow P_2 = P_1 n_2 T_2 / n_1 T_1 \Rightarrow P_2/P_o = (1 + x)T_2/T_1$

Substitute the molefractions and the pressure into the equilibrium equation

$$K_{eq} = e^{-3.069} = \frac{y_O^2}{y_{0_2}}\left(\frac{P_2}{P_o}\right) = \left(\frac{2x}{1+x}\right)^2 \left(\frac{1+x}{1-x}\right) \left(\frac{1+x}{1}\right) \left(\frac{T_2}{T_1}\right)$$

$$\Rightarrow \frac{4x^2}{1-x} = \frac{T_1}{T_2} e^{-3.069} = 0.00433 \quad \Rightarrow \quad x = 0.0324$$

The final pressure is then

$$P_2 = P_o(1 + x)\frac{T_2}{T_1} = 100 \,(1 + 0.0324) \times \frac{3200}{298.2} = \textbf{1108 kPa}$$

$(n_{O_2}) = 0.9676, \quad (n_O) = 0.0648, \quad n_2 = 1.0324$

$${}_1\bar{q}_2 = 0.9676 \times 106022 + 0.0648\,(249170 + 60767) - 0$$

$$+ \, 8.3145 \,(298.15 - 1.0324 \times 3200) = \textbf{97681 kJ/kmolO}_2$$

$$y_{O_2} = \frac{0.9676}{1.0324} = \textbf{0.937}; \qquad y_O = \frac{0.0648}{1.0324} = \textbf{0.0628}$$

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14.43

Find the equilibrium constant for the reaction $2NO + O_2 \Leftrightarrow 2NO_2$ from the elementary reactions in Table A.11 to answer which of the nitrogen oxides, NO or NO_2, is the more stable at ambient conditions? What about at 2000 K?

$$2\,NO\ +\ O_2 \Leftrightarrow 2\,NO_2 \quad (1)$$

$$\text{But}\ N_2\ +\ O_2 \Leftrightarrow 2\,NO \quad (2)$$

$$N_2\ + 2\,O_2 \Leftrightarrow 2\,NO_2 \quad (3)$$

Reaction 1 = Reaction 3 - Reaction 2

$$\Rightarrow \quad \Delta G_1^0 = \Delta G_3^0 - \Delta G_2^0 \quad \Rightarrow \quad \ln K_1 = \ln K_3 - \ln K_2$$

At 25 $^{\mathrm{o}}$C, from Table A.11: $\ln K_1 = -41.355 - (-69.868) = +28.513$

$$\text{or} \quad K_1 = 2.416 \times 10^{12}$$

an extremely large number, which means reaction 1 tends to go very strongly from left to right.

At 2000 K: $\ln K_1 = -19.136 - (-7.825) = -11.311$ or $K_1 = 1.224 \times 10^{-5}$

meaning that reaction 1 tends to go quite strongly from right to left.

14.44

Assume the equilibrium mole fractions of oxygen and nitrogen are close to those in air, find the equilibrium mole fraction for NO at 3000 K, 500 kPa disregarding dissociations.

Assume the simple reaction to make NO as:

N_2	+	$O_2 \Leftrightarrow$	2 NO	Ar	$n_{N2} = 0.78 - x$
0.78		0.21	0	0.01	$n_{O2} = 0.21 - x$
-x		-x	+2x		$n_{NO} = 2x, \quad n_{ar} = 0.01$
0.78–x		0.21–x	2x	0.01	$n_{tot} = 1.0$

From A.11 at 3000 K,   ln K = -4.205,   K = 0.014921

$$K = \frac{4x^2}{(0.78 - x)(0.21 - x)} \left(\frac{P}{P^0}\right)^0 \qquad \Rightarrow$$

$$\frac{x^2}{(0.78 - x)(0.21 - x)} = \frac{0.014921}{4} = 0.00373 \quad \text{and} \quad 0 < x < 0.21$$

Solve for x: x = 0.0230 $\Rightarrow$ $y_{NO} = \dfrac{2x}{1.0} = \mathbf{0.046}$

14.45

The combustion products from burning pentane, C_5H_{12}, with pure oxygen in a stoichiometric ratio exists at 2400 K, 100 kPa. Consider the dissociation of only CO_2 and find the equilibrium mole fraction of CO.

$$C_5H_{12} + 8\ O_2 \rightarrow 5\ CO_2 + 6\ H_2O$$

At 2400 K,

ln K = -7.715

K = 4.461×10^{-4}

	$2\ CO_2 \Leftrightarrow 2\ CO + 1\ O_2$		
Initial	5	0	0
Change	-2z	+2z	+z
Equil.	5-2z	2z	z

Assuming $P = P^o = 0.1$ MPa, and $n_{tot} = 5 + z + 6 = 11 + z$

$$K = \frac{y_{CO}^2 y_{O_2}}{y_{CO_2}^2}\left(\frac{P}{P^0}\right) = \left(\frac{2z}{5-2z}\right)^2\left(\frac{z}{11+z}\right)(1) = 4.461 \times 10^{-4}\ ;$$

Trial & Error (compute LHS for various values of z):   z = 0.291

$n_{CO_2} = 4.418;\quad n_{CO} = 0.582;\quad n_{O_2} = 0.291 \quad => \quad y_{CO} = \mathbf{0.0515}$

14.46

A mixture flows with 2 kmol/s CO_2, 1 kmol/s argon and 1 kmol/s CO at 298 K and it is heated to 3000 K at constant 100 kPa. Assume the dissociation of carbon dioxide is the only equilibrium process to be considered. Find the exit equilibrium composition and the heat transfer rate.

Reaction	$2\,CO_2$	$\Leftrightarrow$	$2\,CO$	$+$	O_2	Ar
initial	2		1		0	1
change	-2x		+2x		+x	
equil.	2 - 2x		1 + 2x		x	1

From Table A.11:

$$K = \exp(-2.217) = 0.108935 = \frac{y_{CO}^2\, y_{O2}}{y_{CO2}^2}\left(\frac{P}{P_o}\right)^{3-2}$$

$$= \frac{(1 + 2x)^2}{(4 + x)^2}\,\frac{x}{4 + x}\,\frac{(4 + x)^2}{(2 - 2x)^2}\,(1)^1 = \frac{x}{4}\,\frac{(1 + 2x)^2}{4 + x}\,\frac{1}{(1 - x)^2}$$

then

$$0.43574\,(4 + x)\,(1 - x)^2 = x\,(1 + 2x)^2$$

trial and error solution gives x = 0.32136

The outlet has: $\qquad 1.35728\,CO_2 + 1.64272\,CO + 1\,Ar + 0.32136\,O_2$

Energy equation gives from Table A.9 and A.5 (for argon $C_p\Delta T$)

$$\dot{Q} = \sum \dot{n}_{ex}\,\bar{h}_{ex} - \sum \dot{n}_{in}\,\bar{h}_{in}$$

$$= 1.35728\,(152853 - 393522) + 1.64272\,(93504 - 110527)$$
$$+ 1\,(39.948 \times 0.52 \times (3000 - 298)) + 0.32136\,(98013)$$
$$- 2\,(-393522) - 1(-110527) - 1\,(0)$$
$$= \mathbf{630\ 578\ kW}$$

14.47

A mixture of 1 kmol carbon dioxide, 2 kmol carbon monoxide, and 2 kmol oxygen, at 25°C, 150 kPa, is heated in a constant pressure steady state process to 3000 K. Assuming that only these same substances are present in the exiting chemical equilibrium mixture, determine the composition of that mixture.

initial mix:
1 CO_2, 2 CO,
2 O_2

Constant pressure reactor

$\dot{Q}$

Equil. mix:
CO_2, CO, O_2 at
T = 3000 K,
P = 150 kPa

Reaction	$2\,CO_2$	$\Leftrightarrow$	2 CO	+	O_2
initial	1		2		2
change	-2x		+2x		+x
equil.	(1 - 2x)		(2 + 2x)		(2 + x)

$n_{tot} = 1 - 2x + 2 + 2x + 2 + x = 5 + x$ so $y = n/\,n_{tot}$

From A.11 at 3000 K: $K = \exp(-2.217) = 0.108935$

For each $n > 0$ $\Rightarrow$ $-1 < x < +\frac{1}{2}$

$$K = \frac{y_{CO}^2\, y_{O2}}{y_{CO2}^2}\left(\frac{P}{P^0}\right)^1 = 4\left(\frac{1+x}{1-2x}\right)^2\left(\frac{2+x}{5+x}\right)\left(\frac{150}{100}\right) = 0.108935$$

or $\left(\dfrac{1+x}{1-2x}\right)^2\left(\dfrac{2+x}{5+x}\right) = 0.018\,156,$ Trial & error: x = −0.521

$\begin{array}{ll} n_{CO2} = 2.042 & n_{O2} = 1.479 \\ n_{CO} = 0.958 & n_{tot} = 4.479 \end{array} \Big\}$ $\begin{cases} y_{CO2} = 0.4559 \\ y_{CO} = 0.2139 \\ y_{O2} = 0.3302 \end{cases}$

14.48

Acetylene gas Acetylene gas, C_2H_2, is burned with stoichiometric air in a torch. The reactants are supplied at the reference conditions P_0, T_0. The products come out from the flame at 2800 K after a small heat loss by radiation. Consider the dissociation of CO_2 into CO and O_2 and no others. Find the equilibrium composition of the products. Are there any other reactions that should be considered?

Combustion: $C_2H_2 + 2.5\ (O_2 + 3.76N_2) \rightarrow 2\ CO_2 + 1\ H_2O + 9.4\ N_2$

At 2800 K,

$\qquad\qquad\qquad\qquad\qquad\qquad$ $2\ CO_2 \Leftrightarrow 2\ CO + 1\ O_2 \qquad H_2O \quad N_2$

ln K = -3.781

	$2\ CO_2$	$2\ CO$	$1\ O_2$	H_2O	N_2
Initial	2	0	0	1	9.4
Change	-2z	+2z	+z	0	0
Equil.	2-2z	2z	z	1	9.4

K = 0.0228

Assuming $P = P^o = 0.1$ MPa, and $n_{tot} = 2 + z + 1 + 9.4 = 12.4 + z$

$$K = \frac{y_{CO}^2 y_{O_2}}{y_{CO_2}^2}\ (\frac{P}{P^0}) = \left(\frac{2z}{2 - 2z}\right)^2 \left(\frac{z}{12.4 + z}\right)(1) = 0.0228;$$

Solve: $z^3 = 0.0228\ (1 - z)^2(12.4 + z) \quad => \quad z = 0.4472$

$$y_{CO} = \frac{2z}{12.4 + z} = \textbf{0.0696}; \quad y_{CO2} = \frac{2 - 2z}{12.4 + z} = \textbf{0.086}; \quad y_{O2} = \frac{z}{12.4 + z} = \textbf{0.0348}$$

$$y_{H2O} = \frac{1}{12.4 + z} = \textbf{0.0778}; \quad y_{N2} = \frac{9.4}{12.4 + z} = \textbf{0.7317}$$

Looking in A.11 there will be no dissociation of nitrogen and only a small dissociation of water and oxygen which could be included.

14.49

Consider combustion of methane with pure oxygen forming carbon dioxide and water as the products. Find the equilibrium constant for the reaction at 1000 K. Use an average heat capacity of $C_p = 52$ kJ/kmol K for the fuel and Table A.9 for the other components.

For the reaction equation,

$$CH_4 + 2\,O_2 \Leftrightarrow CO_2 + 2\,H_2O$$

At 1000 K from Table A.9 and A.10 for the fuel at 298 K

$$\Delta H^0_{1000\ K} = 1(-393\ 522 + 33\ 397) + 2(-241\ 826 + 26\ 000)$$
$$- 1[-74\ 873 + 52(1000 - 298.2)] - 2(0 + 22\ 703)$$
$$= -798\ 804 \text{ kJ/kmol}$$

$$\Delta S^0_{1000\ K} = 1 \times 269.299 + 2 \times 232.739 - 1(186.251 + \ln\frac{1000}{298.2}) - 2 \times 243.579$$
$$= 487.158 \text{ kJ/kmol K}$$

$$\Delta G^0_{1000\ K} = \Delta H^0_{1000\ K} - T\ \Delta S^0_{1000\ K}$$
$$= -798\ 804 - 1000 \times 487.158 = -1\ 285\ 962 \text{ kJ/kmol}$$

$$\ln K = -\frac{\Delta G^0}{\bar{R}T} = \frac{+1\ 285\ 962}{8.3145 \times 1000} = +154.665 \quad, \quad \mathbf{K = 1.4796\ E\ 67}$$

This means the reaction is shifted totally to the right.

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14.50

Water from the combustion of hydrogen and pure oxygen is at 3800 K and 50 kPa. Assume we only have H_2O, O_2 and H_2 as gases find the equilibrium composition.

With only the given components we have the reaction

$$2\ H_2O \iff 2H_2 + O_2$$

which at 3800 K has an equilibrium constant from A.11 as $\ln K = -1.906$

Assume we start with 2 kmol water and let it dissociate x to the left then

Species	H_2O	H_2	O_2	
Initial	2	0	0	
Change	-2x	2x	x	
Final	2 − 2x	2x	x	Tot: 2 + x

Then we have

$$K = \exp(-1.906) = \frac{y_{H_2}^2\, y_{O_2}}{y_{H_2O}^2}\left(\frac{P}{P^0}\right)^{2+1-2} = \frac{\left(\frac{2x}{2+x}\right)^2 \frac{x}{2+x}}{\left(\frac{2-2x}{2+x}\right)^2} \frac{50}{100}$$

which reduces to

$$0.148674 = \frac{1}{(1-x)^2}\frac{4x^3}{2+x}\frac{1}{4}\frac{1}{2} \quad \text{or} \quad x^3 = 0.297348\ (1-x)^2\ (2+x)$$

Trial and error to solve for x = 0.54 then the concentrations are

$$y_{H2O} = \frac{2-2x}{2+x} = 0.362; \quad y_{O2} = \frac{x}{2+x} = 0.213; \quad y_{H2} = \frac{2x}{2+x} = 0.425$$

14.51

Repeat problem 14.47 for an initial mixture that also includes 2 kmol of nitrogen, which does not dissociate during the process.

This problem has a dilution of the reactant with nitrogen.

initial mix:
1 CO_2, 2 CO,
2 O_2, 2 N_2

→ Constant pressure reactor ↗ $\dot{Q}$ →

Equilibrium mix:
CO_2, CO, O_2 and N_2
at T = 3000 K, P = 150 kPa

Reaction	2 CO_2	⇔	2 CO	+	O_2
initial	1		2		2
change	-2x		+2x		+x
equil.	(1-2x)		(2+2x)		(2+x)

From A.11 at 3000 K: K = exp(−2.217) = 0.108935

For each n > 0   ⟹ $-1 < x < +\frac{1}{2}$

Equilibrium: $n_{CO2} = (1 - 2x)$, $n_{CO} = (2 + 2x)$, $n_{O2} = (2 + x)$,

$n_{N2} = 2$ so then $n_{tot} = 7 + x$

$$K = \frac{y_{CO}^2 y_{O2}}{y_{CO2}^2} \left(\frac{P}{P^0}\right)^1 = 4 \left(\frac{1+x}{1-2x}\right)^2 \left(\frac{2+x}{7+x}\right) \left(\frac{150}{100}\right) = 0.108935$$

or $\left(\frac{1+x}{1-2x}\right)^2 \left(\frac{2+x}{7+x}\right) = 0.018167$ Trial & error: x = −0.464

$$\left.\begin{array}{ll} n_{CO2} = 1.928 & n_{O2} = 1.536 \\ & n_{N2} = 2.0 \\ n_{CO} = 1.072 & n_{TOT} = 6.536 \end{array}\right\} \quad \left\{\begin{array}{ll} y_{CO2} = 0.295 & y_{O2} = 0.235 \\ y_{CO} = 0.164 & y_{N2} = 0.306 \end{array}\right.$$

14.52

Catalytic gas generators are frequently used to decompose a liquid, providing a desired gas mixture (spacecraft control systems, fuel cell gas supply, and so forth). Consider feeding pure liquid hydrazine, N_2H_4, to a gas generator, from which exits a gas mixture of N_2, H_2, and NH_3 in chemical equilibrium at 100°C, 350 kPa. Calculate the mole fractions of the species in the equilibrium mixture.

Initially, $\qquad 2\,N_2H_4 \rightarrow 1\,N_2 + 1\,H_2 + 2\,NH_3$

Reaction:	N_2	+	$3\,H_2$	$\Leftrightarrow$	$2\,NH_3$	
initial	1		1		2	
change	-x		-3x		+2x	
equil.	(1-x)		(1-3x)		(2+2x)	$n_{TOTAL} = (4-2x)$

$$K = \frac{y_{NH3}^2}{y_{N2}\,y_{H2}^3}\left(\frac{P}{P^0}\right)^{-2} = \frac{(2+2x)^2(4-2x)^2}{(1-x)(1-3x)^3}\left(\frac{350}{100}\right)^{-2}$$

At 100 °C = 373.2 K, for NH_3 use A.5 $\qquad \bar{C}_{P0} = 17.03 \times 2.130 = 36.276$

$$\bar{h}_{NH3}^0 = -45\,720 + 36.276(373.2 - 298.2) = -42\,999 \text{ kJ/kmol}$$

$$\bar{s}_{NH3}^0 = 192.572 + 36.276\ln\frac{373.2}{298.2} = 200.71 \text{ kJ/kmol K}$$

Using A.9,

$$\Delta H_{100\,C}^0 = 2(-42\,999) - 1(0+2188) - 3(0+2179) = -94\,723 \text{ kJ}$$

$$\Delta S_{100\,C}^0 = 2(200.711) - 1(198.155) - 3(137.196) = -208.321 \text{ kJ/K}$$

$$\Delta G_{100\,C}^0 = \Delta H^0 - T\Delta S^0 = -94\,723 - 373.2(-208.321) = -16\,978 \text{ kJ}$$

$$\ln K = -\frac{\Delta G^0}{\bar{R}T} = \frac{+16\,978}{8.3145 \times 373.2} = 5.4716 \quad \Rightarrow \quad K = 237.84$$

Therefore,

$$\left[\frac{(1+x)(2-x)}{(1-3x)}\right]^2 \frac{1}{(1-x)(1-3x)} = \frac{237.84 \times 3.5^2}{16} = 182.096$$

By trial and error, x = 0.226

$$\left.\begin{array}{l} n_{N2} = 0.774 \quad n_{NH3} = 2.452 \\ n_{H2} = 0.322 \quad n_{TOT} = 3.518 \end{array}\right\} \left\{\begin{array}{l} y_{N2} = 0.2181 \\ y_{H2} = 0.0908 \\ y_{NH3} = 0.6911 \end{array}\right.$$

14.53

Complete combustion of hydrogen and pure oxygen in a stoichiometric ratio at P_O, T_O to form water would result in a computed adiabatic flame temperature of 4990 K for a steady state setup. How should the adiabatic flame temperature be found if the equilibrium reaction $2H_2 + O_2 \Leftrightarrow 2H_2O$ is considered? Disregard all other possible reactions (dissociations) and show the final equation(s) to be solved.

$$2H_2 + O_2 \Leftrightarrow 2H_2O$$

Species	H_2	O_2	H_2O
Initial	2	1	0
Shift	−2x	−x	2x
Final	2 − 2x	1 −x	2x

$$Keq = \frac{y_{H_2O}^2}{y_{H_2}^2 y_{O_2}} \left(\frac{P}{P^0}\right)^{-1}, \qquad n_{tot} = 2 - 2x + 1 - x + 2x = 3 - x$$

Energy Eq.: $\qquad H_P = H_R = H_P^o + \Delta H_P = H_R^o = 0$

$$H_P = (1 - x)(2\Delta\bar{h}_{H_2} + \Delta\bar{h}_{O_2}) + 2x(\bar{h}_{fH2O}^o + \Delta\bar{h}_{H2O}) = 0 \qquad (1)$$

Equilibrium constant:

$$K_{eq} = \frac{4x^2}{(3-x)^2} \frac{(3 - x)^2}{(2 - 2x)^2} \frac{3 - x}{1 - x} = \frac{x^2(3 - x)}{(1 - x)^3} = K_T \qquad (2)$$

A.10: $\qquad \bar{h}_{fH2O}^o = -241\ 826$ kJ/kmol; $\qquad$ A.11: $\ln(K_T)$

A.9: $\qquad \Delta\bar{h}_{H_2}(T), \quad \Delta\bar{h}_{O_2}(T), \quad \Delta\bar{h}_{H_2O}(T),$

Trial and error (solve for x, T) using Eqs. (1) and (2).

Guess T = 3600 K: K = exp(1.996) = 7.35956, x = 0.53986

$\qquad\qquad H_P = (1 - 0.53986)(2 \times 111\ 367 + 122\ 245)$

$\qquad\qquad\qquad + 2 \times 0.53986 \times (-241\ 826 + 160\ 484) = 70\ 912$ high

Guess T = 3400 K: K = exp(3.128) = 22.8283, x = 0.648723

$\qquad\qquad H_P = (1 - 0.648723)(2 \times 103\ 736 + 114\ 101)$

$\qquad\qquad\qquad + 2 \times 0.648723 \times (-241\ 826 + 149\ 073) = -7381$ low

Now do a linear interpolation to get $H_P = 0$:

$\qquad\qquad$ T = 3400 + 200 × 7381/(70 912 + 7381) = **3419 K**, K = 20.50

$\qquad\qquad$ x = 0.63905, $y_{O_2} = 0.153$; $y_{H_2} = 0.306$; $y_{H_2O} = 0.541$

14.54

Consider the water gas reaction in Example 14.4. Find the equilibrium constant at 500, 1000, 1200 and 1400 K. What can you infer from the result?

As in Example 14.4, III $H_2 + CO_2 \Leftrightarrow H_2O + CO$

I $2\,CO_2 \Leftrightarrow 2\,CO + O_2$

II $2\,H_2O \Leftrightarrow 2\,H_2 + O_2$

Then, $\ln K_{III} = 0.5\,(\ln K_I - \ln K_{II})$

At 500 K, $\ln K_{III} = 0.5\,(-115.234 - (-105.385)) = -4.9245$,
$K = 0.007\ 266$

At 1000 K, $\ln K_{III} = 0.5\,(-47.052 - (-46.321)) = -0.3655$,
$K = 0.693\ 85$

At 1200 K, $\ln K_{III} = 0.5\,(-35.736 - (-36.363)) = +0.3135$,
$K = 1.3682$

At 1400 K, $\ln K_{III} = 0.5\,(-27.679 - (-29.222)) = +0.7715$,
$K = 2.163$

It is seen that at lower temperature, reaction III tends to go strongly from right to left, but as the temperature increases, the reaction tends to go more strongly from left to right. If the goal of the reaction is to produce more hydrogen, then it is desirable to operate at lower temperature.

14.55

A piston/cylinder contains 0.1 kmol hydrogen and 0.1 kmol Ar gas at 25°C, 200 kPa. It is heated up in a constant pressure process so the mole fraction of atomic hydrogen is 10%. Find the final temperature and the heat transfer needed.

When gas is heated up H_2 splits partly into H as

$$H_2 \iff 2H \qquad \text{and the gas is diluted with Ar}$$

Component	H_2	Ar	H	
Initial	0.1	0.1	0	
Shift	-x	0	2x	
Final	0.1-x	0.1	2x	Total = 0.2 + x

$y_H = 0.1 = 2x /(0.2 + x) \quad \Rightarrow \quad 2x = 0.02 + 0.1x$

$\Rightarrow \quad x = 0.010526 \quad \Rightarrow n_{tot} = 0.21053$

$y_{H2} = 0.425 = [(0.1 - x)/(0.2 + x)]; \quad y_{Ar} = 1 - rest = 0.475$

Do the equilibrium constant:

$$K(T) = \frac{y_H^2}{y_{H2}} \left(\frac{P}{P^0}\right)^{2-1} = \left(\frac{0.01}{0.425}\right) \times \left(\frac{200}{100}\right) = 0.047059$$

$\ln (K) = -3.056$ so from Table A.11 interpolate to get $T = 3110$ K

To do the energy eq., we look up the enthalpies in Table A.9 at 3110 K

$h_{H2} = 92\ 829.1; \quad h_H = h_f + \Delta h = 217\ 999 + 58\ 447.4 = 276\ 445.4$

$h_{Ar} = 0 + C_P(3110 - 298.15) = 20.7863 \times (3110-298.13) = 58\ 447.9$

(same as Δh for H)

Now get the total number of moles to get

$n_H = 0.021053; \quad n_{H2} = n_{tot} \times \frac{1-x}{2+x} = 0.08947; \quad n_{Ar} = 0.1$

Since pressure is constant $W = P\Delta V$ and Q becomes differences in h

$Q = n\Delta h = 0.08947 \times 92\ 829.1 - 0 + 0.021053 \times 276\ 446.4$

$- 0 + 0.1 \times 58\ 447.9$

$= \textbf{19 970 kJ}$

14.56

The van't Hoff equation

$$d \ln K = \frac{\Delta H^o}{\overline{R}T^2} dT_P^o$$

relates the chemical equilibrium constant K to the enthalpy of reaction ΔH^o. From the value of K in Table A.11 for the dissociation of hydrogen at 2000 K and the value of ΔH^o calculated from Table A.9 at 2000 K use van't Hoff equation to predict the constant at 2400 K.

$$H_2 \Leftrightarrow 2H$$

$$\Delta H° = 2 \times (35\ 375 + 217\ 999) - 52\ 942 = 453\ 806 \text{ kJ/kmol}$$

$$\ln K_{2000} = -12.841;$$

Assume $\Delta H°$ is constant and integrate the Van't Hoff equation

$$\ln K_{2400} - \ln K_{2000} = \int\limits_{2400}^{2000} \frac{\Delta H^o}{\overline{R}T^2} dT = -\frac{\Delta H°}{\overline{R}} (\frac{1}{T_{2400}} - \frac{1}{T_{2000}})$$

$$\ln K_{2400} = \ln K_{2000} + \Delta H° (\frac{1}{T_{2400}} - \frac{1}{T_{2000}}) / \overline{R}$$

$$= -12.841 + 453\ 806 (\frac{6\text{-}5}{12000}) / 8.31451 = -12.841 + 4.548$$

$$= -8.293$$

Table A.11 lists –8.280   ($\Delta H°$ not exactly constant)

14.57

A gas mixture of 1 kmol carbon monoxide, 1 kmol nitrogen, and 1 kmol oxygen at 25°C, 150 kPa, is heated in a constant pressure process. The exit mixture can be assumed to be in chemical equilibrium with CO_2, CO, O_2, and N_2 present. The mole fraction of CO_2 at this point is 0.176. Calculate the heat transfer for the process.

initial mix:
1 CO, 1 O_2,
1 N_2

Constant pressure reactor

$\dot{Q}$

Equil. mix:
CO_2, CO, O_2, N_2
$y_{CO2} = 0.176$
P = 150 kPa

reaction	2 CO$_2$	$\Leftrightarrow$	2 CO	+	O$_2$	also,	N$_2$
initial	0		1		1		1
change	+2x		-2x		-x		0
equil.	2x		(1-2x)		(1-x)		1

$$y_{CO2} = 0.176 = \frac{2x}{3-x} \quad \Rightarrow \quad x = 0.242\ 65$$

$$n_{CO2} = 0.4853 \quad n_{O2} = 0.7574 \left.\begin{cases} y_{CO2} = 0.176 \\ y_{CO} = 0.1867 \end{cases}\right. y_{O2} = 0.2747$$
$$n_{CO} = 0.5147 \quad n_{N2} = 1$$

$$K = \frac{y_{CO}^2 y_{O2}}{y_{CO2}^2} \left(\frac{P}{P^0}\right)^1 = \frac{0.1867^2 \times 0.2747}{0.176^2} \left(\frac{150}{100}\right) = 0.4635$$

From A.11, $T_{PROD} = 3213$ K

From A.10, $H_R = -110\ 527$ kJ

$H_P = 0.4853(-393\ 522 + 166\ 134) + 0.5147(-110\ 527 + 101\ 447)$

$\qquad + 0.7574(0 + 106\ 545) + 1(0 + 100\ 617)$

$\quad = +66\ 284$ kJ

$Q_{CV} = H_P - H_R = 66\ 284 - (-110\ 527) = \textbf{+176 811 kJ}$

14.58

A tank contains 0.1 kmol hydrogen and 0.1 kmol of argon gas at 25°C, 200 kPa and the tank keeps constant volume. To what T should it be heated to have a mole fraction of atomic hydrogen, H, of 10%?

For the reaction $H_2 \Leftrightarrow 2H$, $K = \dfrac{y_H^2}{y_{H_2}} \left(\dfrac{P}{P^o}\right)^{2-1}$

Assume the dissociation shifts right with an amount x then we get

reaction	H_2	$\Leftrightarrow$	2 H		also,	Ar	
initial	0.1		0			0.1	
change	-x		2x			0	
equil.	0.1 - x		2x			0.1	Tot: 0.2 + x

$$y_H = \frac{2x}{0.2 + x} = 0.10 \quad \Rightarrow \quad x = 0.010526$$

We need to find T so K will take on the proper value, since K depends on P we need to evaluate P first.

$$P_1 V = n_1 \bar{R} T_1; \quad P_2 V = n_2 \bar{R} T_2 \quad \Rightarrow \quad P_2 = P_1 \frac{n_2 T_2}{n_1 T_1}$$

where we have $n_1 = 0.2$ and $n_2 = 0.2 + x = 0.210526$

$$K = \frac{y_H^2}{y_{H_2}} \left(\frac{P}{P^o}\right)^{2-1} = \frac{(2x)^2}{(0.1 - x) n_2} \frac{200}{100} \frac{n_2 T_2}{0.2 \times 298.15} = 0.0001661 \, T_2$$

Now it is trial and error to get T_2 so the above equation is satisfied with K from A.11 at T_2.

3600 K: ln K = -0.611,   K = 0.5428,   RHS = 0.59796,   error = 0.05516

3800 K: ln K = 0.201,   K = 1.22262,   RHS = 0.63118,   error = -0.59144

Linear interpolation between the two to make zero error

$$T = 3600 + 200 \times \frac{0.05516}{0.05516 + 0.59144} = \textbf{3617 K}$$

14.59

A local stoichiometric combustion of butene C_4H_8 results in only half the C atoms burns to CO_2 and the other half generates CO. This means the products should contain a mixture of H_2O, CO, CO_2, O_2 and N_2 which after heat transfer are at 1000 K.

a) Write the combustion equation assuming no hydrogen in the products

b) Use the water gas reaction to estimate the amount of hydrogen H_2 present.

Select the amount of air for stoichiometric combustion
Combustion:

$$C_4H_8 + 6(O_2 + 3.76N_2) \Rightarrow 2CO_2 + 2\,CO + 4\,H_2O + 1\,O_2 + 22.56\,N_2$$

Since we have 2 CO we did not use 1 O_2 which is then left in the products.

Water-gas reaction: $\quad H_2 \;+\; CO_2 \quad\Leftrightarrow\quad H_2O + CO$

	CO_2	CO	H_2O	H_2	O_2	N_2	
Start	2	2	4	0	1	22.56	
Change	-x	x	x	-x	0	0	
Final	2-x	2 +x	4+x	-x	1	22.56	tot = 31.56

Notice the total number of moles is constant. See Example 14.4 for the equilibrium K

At 1000 K: $\quad \ln K = 0.5(\ln K_1 - \ln K_2) = 0.5[-47.052 - (-46.321)] = -0.3655$

$$K = \exp(-0.3655) = [\,y_{CO}\,y_{H2O} \,/\, y_{CO2}\,y_{H2}]\,1 = \frac{2+x}{2-x}\,\frac{4+x}{-x}$$

Reduce and solve: $\quad (2-x)(-x)\exp(-0.3655) = (2+x)(4+x)$

$\Rightarrow \quad -(2x - x^2)\exp(-0.3655) = 8 + 6x + x^2$

-2 < x < 0 for all pos. concentrations $\qquad \Rightarrow \quad x = -1.136$

$n_{H2} = $ **1.136 mol/mol fuel** or 3.6%

14.60

A liquid fuel can be produced from a lighter fuel in a catalytic reactor according to

$$C_2H_4 + H_2O \quad \Leftrightarrow \quad C_2H_5OH$$

Show the equilibrium constant is ln K = −6.691 at 700 K using $C_P = 63$ kJ/kmol K for ethylene and $C_P = 115$ kJ/kmol K for ethanol at 500 K.

25°C, 5 MPa   1 C_2H_4 IG chem. equil. mixture
C_2H_5OH, C_2H_4, H_2O

300°C, 5 MPa   2 H_2O 700 K, 5 MPa

$1 C_2H_4 + 1 H_2O \Leftrightarrow 1 C_2H_5OH$

$$\Delta H^0_{700\ K} = 1(-235\ 000 + 115(700-298.2)) - 1(+52\ 467 + 63\ (700-298.2))$$

$$- 1(-241\ 826 + 14\ 190)\ = -38\ 935\ kJ$$

$$\Delta S^0_{700\ K} = 1(282.444 + 115 \ln \frac{700}{298.2}) - 1(219.330 + 63 \ln \frac{700}{298.2}) - 1(218.739)$$

$$= -111.253\ kJ/K$$

$$\Delta G^0_{700\ K} = \Delta H^0 - T\Delta S^0 = -38\ 935 - 700(-111.253) = +38\ 942\ kJ$$

$$\ln K = \frac{-\Delta G^0}{\bar{R}T} = -\frac{38\ 942}{8.31451 \times 700} = \mathbf{-6.691}$$

14.61

A rigid container initially contains 2 kmol of carbon monoxide and 2 kmol of oxygen at 25°C, 100 kPa. The content is then heated to 3000 K at which point an equilibrium mixture of CO_2, CO, and O_2 exists. Disregard other possible species and determine the final pressure, the equilibrium composition and the heat transfer for the process.

Equilibrium process: $2\ CO + 2\ O_2 \Leftrightarrow 2\ CO_2 + O_2$

Species: CO O_2 CO_2

Initial   2   2   0

Shift   -2x   -2x+x   2x

Final   2-2x   2-x   2x : $n_{tot} = 2 - 2x + 2 - x + 2x = 4 - x$

$$y_{CO} = \frac{2\text{-}2x}{4\text{-}x}; \quad y_{O_2} = \frac{2\text{-}x}{4\text{-}x}; \quad y_{CO_2} = \frac{2x}{4\text{-}x}$$

Energy equation

$$U_2 - U_1 = {}_1Q_2 = H_2 - H_1 - P_2 v + P_1 v$$

$$= (2 - 2x)\bar{h}_{CO\,2} + (2 - x)\bar{h}_{O_2 2} + 2x\bar{h}_{CO_2} - 2\bar{h}^o_{fCO_2} - 2\bar{h}^o_{fO_2}$$

$$- \bar{R}\,(4 - x)T_2 + 4\bar{R}T_1$$

Notice P_2 is unknown so write it in terms of T_2 and the number of moles. We flipped the reaction relative to the one in A.11 so then $K_{eq} = 1/K_{eq\ A11}$

$$K_{eq} = e^{2.217} = \frac{y^2_{CO_2}}{y_{0_2} y^2_{CO}}\left(\frac{P_2}{P_o}\right)^{-1} = \frac{4x^2}{4(1-x)^2}\,\frac{4-x}{2-x}\,\frac{4T_1}{(4-x)T_2}$$

$$\Rightarrow \left(\frac{x}{1-x}\right)^2 \frac{1}{2-x} = \frac{1}{4}\frac{T_2}{T_1} e^{2.217} = 23.092 \quad \Rightarrow \quad x = 0.8382;$$

$$y_{CO} = \mathbf{0.102}; \quad y_{O_2} = \mathbf{0.368}; \quad y_{CO_2} = \mathbf{0.53}$$

Constant volume and ideal gas approximation $PV = n\bar{R}T$

$$P_2 = P_1(4 - x)\,T_2\,/4T_1 = 100\,\frac{(4 - 0.8382)\times 3000}{4\times 298.15} = \mathbf{795.4\ kPa}$$

$${}_1Q_2 = 0.3236(-110527 + 93504) + 1.1618(98013) + 1.6764(-393522$$

$$+ 152853) - 2(-110527) - 2(\emptyset) + 8.3145[4(298.15) - 3000(3.1618)]$$

$$= \mathbf{-142991\ kJ}$$

Borgnakke and Sonntag

14.62

Use the information in Problem 14.90 to estimate the enthalpy of reaction, ΔH^O, at 700 K using Van't Hoff equation (see problem 14.56) with finite differences for the derivatives.

$$d \ln K = [\Delta H^\circ / \overline{R} T^2] \, dT \quad \text{or} \quad \text{solve for } \Delta H^O$$

$$\Delta H^\circ = \overline{R} T^2 \frac{d \ln K}{dT} = \overline{R} T^2 \frac{\Delta \ln K}{\Delta T}$$

$$= 8.31451 \times 700^2 \times \frac{-0.3362 - (-4.607)}{800 - 600} = 86\,998 \text{ kJ/kmol}$$

[Remark: compare this to A.9 values + A.5, A.10,

$$\Delta H^O = H_C + 2H_{H_2} - H_{CH_4} = 0.61 \times 12 \times (700\text{-}298) + 2 \times 11730$$

$$- 2.254 \times 16.04 \times (700\text{-}298) - (\text{-}74873) = 86\,739 \text{ kJ/kmol} \,]$$

14.63

One kilomole of carbon dioxide, CO_2, and 1 kmol of hydrogen, H_2 at room temperature, 200 kPa is heated to 1200 K at 200 kPa. Use the water gas reaction to determine the mole fraction of CO. Neglect dissociations of H_2 and O_2.

$$1\,CO_2 + 1\,H_2 \Leftrightarrow 1\,CO + 1\,H_2O$$

Initial	1	1	0	0
Shift	-x	-x	+x	+x
Total	1-x	1-x	x	x;

$n_{tot} = 2$

$$y_{H2O} = y_{CO} = x/2, \quad y_{H2} = y_{CO2} = (1-x)/2$$

The water gas reaction, see Example 14.4

At 1200 K, $\ln K_{III} = 0.5\,(-35.736 - (-36.363)) = +0.3135$,

$$K = 1.3682$$

$$\frac{(x/2)(x/2)}{(\frac{1-x}{2})(\frac{1-x}{2})} = K = \frac{x^2}{(1-x)^2} \quad => \quad \frac{x}{1-x} = 1.1697$$

$$x = 1.1697 / 2.1697 = 0.5391$$

$$y_{H2O} = y_{CO} = x/2 = \mathbf{0.27}, \quad y_{H2} = y_{CO2} = (1-x)/2 = \mathbf{0.23}$$

Borgnakke and Sonntag

14.64

A step in the production of a synthetic liquid fuel from organic waste material is the following conversion process at 5 MPa: 1 kmol ethylene gas (converted from the waste) at 25°C and 2 kmol steam at 300°C enter a catalytic reactor. An ideal gas mixture of ethanol, ethylene and water in equilibrium, see previous problem, leaves the reactor at 700 K, 5 MPa. Determine the composition of the mixture.

25°C, 5 MPa	1 C_2H_4			IG chem. equil. mixture
				C_2H_5OH, C_2H_4, H_2O
300°C, 5 MPa	2 H_2O			700 K, 5 MPa

$$1\ C_2H_4 + 1\ H_2O \Leftrightarrow 1\ C_2H_5OH$$

initial	1	2	0	
change	-x	-x	+x	
equil.	(1-x)	(2-x)	x	total = 3 - x

The reaction rate from the previous problem statement is

$$\ln K = \frac{-\Delta G^0}{\bar{R}T} = -6.691 \quad \Rightarrow \quad K = 0.001\ 242 = \frac{y_{C2H5OH}}{y_{C2H4}\ y_{H2O}}\left(\frac{P}{P^0}\right)^{-1}$$

$$\Rightarrow \left(\frac{x}{1-x}\right)\left(\frac{3-x}{2-x}\right) = 0.001242 \times \frac{5.0}{0.1} = 0.0621$$

By trial and error: $x = 0.0404 \Rightarrow$

C_2H_5OH: n = 0.0404, y = 0.01371

C_2H_4: n = 0.9596, y = 0.3242, H_2O: n = 1.9596, y = 0.6621

Borgnakke and Sonntag

14.65

A special coal burner uses a stoichiometric mixture of coal and an oxygen - argon mixture (1:1 mole ratio) with the reactants supplied at Po, To. Consider the dissociation of CO_2 into CO and O_2 and no others. Find the equilibrium composition of the products for T = 4800 K. Is the final temperature, including dissociations, higher or lower than 4800 K?

Combustion: $C + O_2 + Ar \Rightarrow CO_2 + Ar$

Reaction:

$2 CO_2 \Leftrightarrow 2 CO + O_2$		CO_2	CO	O_2	Ar	
	Start	1	0	0	1	
	Change	-2x	2x	x	0	
	Final	1-2x	2x	x	1	tot = 2 + x

At 4800 K: $K = \exp(5.8322) = [\ y_{CO}^2\ y_{O2}\ /\ y_{CO2}^2]\ 1 = [(\dfrac{2x}{2 + x})^2 \dfrac{x}{2 + x} / (\dfrac{1 - 2x}{2+ x})^2]$

Reduce and solve: $x^3 = [\ \exp(5.8322)/4\ ]\ (2 + x)\ (1 – 2x)^2$

$\Rightarrow\ x^3 = 085.277\ (2 + x)\ (1 – 2x)^2\ \Rightarrow\ x = 0.4885$ (close)

$n_{CO} = 0.977,\ n_{O2} = 0.4885,\ n_{CO2} = 0.023,\ n_{Ar} = 1$

Now we can do the energy equation:

$H_{P\ 4800\ K} = 0.023\ (266\ 488 – 393\ 522) + 1 \times 20.773\ (4800 – 298) +$

$0.977\ (161\ 285 – 110\ 527) + 0.4885 \times 172\ 240 = 224\ 325$ kJ/kmol

Since $H_{P\ 4800\ K} > 0$ then **T < 4800 K**

We should have had $H_P = 0$, if we match the energy equation to find the T we have to redo the equilibrium equation as K is different. This is a larger trial and error problem to find the actual T and we even neglected other possible reactions.

14.66

Acetylene gas at 25°C is burned with 140% theoretical air, which enters the burner at 25°C, 100 kPa, 80% relative humidity. The combustion products form a mixture of CO_2, H_2O, N_2, O_2, and NO in chemical equilibrium at 2200 K, 100 kPa. This mixture is then cooled to 1000 K very rapidly, so that the composition does not change. Determine the mole fraction of NO in the products and the heat transfer for the overall process.

$$C_2H_2 + 3.5\,O_2 + 13.16\,N_2 + \text{water} \rightarrow$$
$$2\,CO_2 + 1\,H_2O + 1\,O_2 + 13.16\,N_2 + \text{water}$$

water: $P_V = 0.8 \times 3.169 = 2.535$ kPa

$$n_V = n_A\,P_V/P_A = (3.5 + 13.16)\,2.535/97.465 = 0.433$$

So, total H_2O in products is 1.433.

Reaction: $N_2 + O_2 \Leftrightarrow 2\,NO$

change : $-x$ $-x$ $+2x$

at 2200 K, from A.11: $K = \exp(-6.836) = 0.001\,074$

Equil. products: $n_{CO2} = 2$, $n_{H2O} = 1.433$, $n_{O2} = 1 - x$,

$$n_{N2} = 13.16 - x, \quad n_{NO} = 0 + 2x, \quad n_{TOT} = 17.593$$

$$K = \frac{(2x)^2}{(1 - x)(13.16 - x)} = 0.001\,074 \quad \Rightarrow \quad x = 0.0576$$

$$y_{NO} = \frac{2 \times 0.0576}{17.593} = \mathbf{0.006\,55}$$

Final products (same composition) at 1000 K, reactants at 25°C

$$H_R = 1(226\,731 + 0) + 0.433(-241\,826 + 0) = 122\,020 \text{ kJ}$$

$$H_P = 2(-393\,522 + 33\,397) + 1.433(-241\,826 + 26\,000)$$
$$+ 0.9424(0 + 22\,703) + 13.1024(0 + 21\,463)$$
$$+ 0.1152(90\,291 + 22\,229)$$
$$= -713\,954 \text{ kJ}$$

$$Q_{CV} = H_P - H_R = \mathbf{-835\,974 \text{ kJ}}$$

14.67

Coal is burned with stoichiometric air, with the reactants supplied at the reference conditions Po, To. If no dissociations are considered, the adiabatic flame temperature is found to be 2461 K. What is it if the dissociation of CO_2 is included?

Combustion: $C + 1(O_2 + 3.76\,N_2) \Rightarrow CO_2 + 3.76\,N_2$

The dissociation of CO_2 is: $2CO_2 \Leftrightarrow 2\,CO + O_2$

	CO_2	CO	N_2	O_2
Start	1	0	3.76	0
Change	-2e	2e		e
Final	1-2e	2e	3.76	e; Total = 4.76 + e

Reaction constant: $K = \dfrac{y_{CO}^2 \, y_{O2}}{y_{CO2}^2}\left(\dfrac{P}{P_O}\right)^{3-2} = \dfrac{(2e)^2\,e}{(1-2e)^2(4.76+e)}$ (1)

Products at equilibrium: $(1-2e)\,CO_2 + 2e\,CO + 3.76\,N_2 + e\,O_2$

Energy Eq.: $H_R = H_P = H_P^o + \Delta H_P = 0$

$H_P = (1-2e)\,(\bar{h}_f^0 + \Delta H)_{CO2} + 2e\,(\bar{h}_f^0 + \Delta H)_{CO} + 3.76\,\Delta H_{N2} + e\,\Delta H_{O2}$
$0 = (1-2e)\,(-393\,522 + \Delta H_{CO2}) + 2e\,(-110\,527 + \Delta H_{CO})$
$\quad + 3.76\,\Delta H_{N2} + e\,\Delta H_{O2}$ (2)

We need to satisfy equations (1) and (2), both functions of T and e. We know T must be less than 2461 K as the dissociation requires energy and that e is small.

Guess 2400 K: K = exp(-7.715) = 0.000446, Eq.1 => e = 0.07322
          $H_P = 0.85356(-393522 + 115779) + 0.14644(-110\,527 + 71326)$
               $+ 3.76 \times 70640 + 0.07322 \times 74453 = 28\,247$ kJ/kmol
          This shows that H_P is too high so also then e too high

Guess 2200 K: K = exp(-10.232) = 0.000036, Eq.1 => e = 0.03349
          $H_P = 0.93302(-393522 + 103562) + 0.06698(-110\,527 + 64012)$
               $+ 3.76 \times 63362 + 0.03349 \times 66770 = -33176.8$ kJ/kmol
          Now H_P is too small, so linear interpolation to hit $H_P = 0$

     T = 2200 + 200 [0 – (-33 176.8)]/[28 247 – (-33 176.8)] = 2308 K
To confirm try T = 2300 K: K = exp(-8.9735) = 0.0001267, e = 0.0498
          $H_P = 0.9004(-393522 + 109670.5) + 0.0996(-110\,527 + 67669)$
               $+ 3.76 \times 67001 + 0.03349 \times 70611.5 = -5560$ kJ/kmol
Final linear interpolation: T = 2300 + 100 × 5560/(28 247 + 5560) = **2316 K**

14.68

An important step in the manufacture of chemical fertilizer is the production of ammonia, according to the reaction: $N_2 + 3H_2 \Leftrightarrow 2NH_3$ Show that the equilibrium constant is K = 6.202 at 150°C.

$$1 \ N_2 + 3 \ H_2 \Leftrightarrow 2 \ NH_3 \quad \text{at } 150^o\text{C}$$

Assume ideal gas and constant specific heat for ammonia (we could be more accurate if we used Table B.2 realizing properties depend also on P). We must use the reference h, s values from Table A.10

$$\bar{h}^o_{NH3 \ 150 \ C} = \text{-45 720} + 2.13{\times}17.031(150 - 25) = \text{-41 186 kJ/kmol}$$

$$\bar{s}^o_{NH3 \ 150 \ C} = 192.572 + 2.13{\times}17.031 \ln\frac{423.2}{298.2} = 205.272 \text{ kJ/kmol-K}$$

Use Table A.9 for nitrogen and hydrogen gases

$$\Delta H^o_{150 \ C} = 2(\text{-41 186}) - 1(0{+}3649) - 3(0 + 3636) \ = \text{-96 929 kJ/kmol}$$

$$\Delta S^0_{150 \ C} = 2(205.272) - 1(201.829) - 3(140.860) \ = \text{-213.865 kJ/kmol-K}$$

$$\Delta G^0_{150 \ C} = \text{-96 929} - 423.2(\text{-213.865}) = \text{-6421 kJ/kmol}$$

$$\ln K = \frac{+6421}{8.31451 \times 423.15} = 1.8248, \ \mathbf{K = 6.202}$$

Borgnakke and Sonntag

14.69

Consider the previous reaction in equilibrium at 150°C, 5 MPa. For an initial composition of 25% nitrogen, 75% hydrogen, on a mole basis, calculate the equilibrium composition

Reaction: $\quad 1\,N_2 + 3\,H_2 \Leftrightarrow 2\,NH_3$ at 150^oC progress variable x

Beginning:	1	3	0	
Change:	-x	-3x	2x	
Final:	1-x	3-3x	2x	Total: $n = 4 - 2x$

The reaction constant is $\quad K = 6.202$ so the reaction constant equation is

$$K = \frac{y_{NH3}^2}{y_{N2}\,y_{H2}^3}\left(\frac{P}{P^0}\right)^{-2} = \frac{(2x)^2(4-2x)^2}{3^3(1-x)^4}\left(\frac{P}{P^0}\right)^{-2}$$

$$\text{or}\quad \left(\frac{x}{1-x}\right)^2\left(\frac{2-x}{1-x}\right)^2 = \frac{27}{16}\times 6.202 \times \left(\frac{5}{0.1}\right)^2 = 26165$$

$$\text{or}\quad \left(\frac{x}{1-x}\right)\left(\frac{2-x}{1-x}\right) = 161.755$$

		n	y
$\rightarrow$ Trial & Error:	NH_3	1.843	0.8544
x = 0.9215	N_2	0.0785	0.0364
	H_2	0.2355	0.1092

14.70

At high temperature, NO can form from oxygen and nitrogen. Natural gas (methane) is burned with 150% theoretical air at 100 kPa, and the product temperature is 2000 K. Neglect other reactions and find the equilibrium concentration of NO. Does the formation of the NO change the temperature?

Combustion: $CH_4 + 1.5 \times 2(O_2 + 3.76\ N_2) \rightarrow CO_2 + 2H_2O + 1O_2 + 11.28\ N_2$

Reaction: $N_2 + O_2 \Leftrightarrow 2\ NO$, progress x

	N_2	+	O_2	$\Leftrightarrow$	2 NO	H_2O+CO_2	
initial	11.28		1		0	2+1	
change	-x		-x		+2x	0	
equilibrium	1-x		1-x		2x	3	$n_{TOT} = 5$

$$K = \exp(-7.825) = 0.0003996 = \frac{y_{NO}^2}{y_{N2}\ y_{O2}}\left(\frac{P}{P_o}\right)^{2-2} = \frac{(2x)^2}{(1-x)(1-x)} = \left(\frac{2x}{1-x}\right)^2$$

Solve

$$2x/(1-x) = \sqrt{0.0003996} = 0.01999 \quad \Rightarrow \quad x = 0.009896$$

$$y_{NO} = 2x/5 = \mathbf{0.00396}$$

Borgnakke and Sonntag

14.71

Methane at 25°C, 100 kPa, is burned with 200% theoretical oxygen at 400 K, 100 kPa, in an adiabatic steady state process, and the products of combustion exit at 100 kPa. Assume that the only significant dissociation reaction in the products is that of carbon dioxide going to carbon monoxide and oxygen. Determine the equilibrium composition of the products and also their temperature at the combustor exit.

Combustion: $CH_4 + 4O_2 \rightarrow CO_2 + 2H_2O + 2O_2$

Dissociation:

	$2\,CO_2$	$\Leftrightarrow$	$2\,CO$	+	O_2	,H_2O	inert
initial	1		0		2	2	
change	-2x		+2x		+x	0	
equilibrium	1-2x		2x		2+x	2	$n_{TOT} = 5+x$

Equil. Eq'n: $K = \dfrac{y_{CO}^2 y_{O2}}{y_{CO2}^2}\left(\dfrac{P}{P^0}\right) = \left(\dfrac{x}{0.5 - x}\right)^2\left(\dfrac{2+x}{5+x}\right)\left(\dfrac{P}{P^0}\right)$

or $\left(\dfrac{x}{0.5 - x}\right)^2\left(\dfrac{2+x}{5+x}\right) = \dfrac{K}{(P/P^0)}$ (1)

Energy Eq.: $H_P - H_R = 0$

$(1-2x)(-393\,522 + \Delta\bar{h}_{CO2}) + 2x(-110\,527 + \Delta\bar{h}_{CO})$

$+ 2(-241\,826 + \Delta\bar{h}_{H2O}) + (2+x)\Delta\bar{h}_{O2} - 1(-74\,873) - 4(3027) = 0$

or $(1-2x)\Delta\bar{h}_{CO2} + 2x\Delta\bar{h}_{CO} + 2\Delta\bar{h}_{H2O} + (2+x)\Delta\bar{h}_{O2} + 565\,990x - 814\,409 = 0$

Assume $T_P = 3256$ K. From A.11:   K = 0.6053

Solving (1) by trial & error,   x = 0.2712

Substituting x and the $\Delta\bar{h}$ values from A.9 (at 3256 K) into (2)

$0.4576\times168\,821 + 0.5424\times103\,054 + 2\times140\,914$

$+ 2.2712\times108\,278 + 565\,990\times0.2712 - 814\,409 \approx 0$ OK

$T_P = 3256$ K &   x = 0.2712

$n_{CO2} = 0.4576, n_{CO} = 0.5424, n_{H2O} = 2.0, n_{O2} = 2.2712$

$y_{CO2} = 0.0868, y_{CO} = 0.1029, y_{H2O} = 0.3794, y_{O2} = 0.4309$

14.72

Calculate the irreversibility for the adiabatic combustion process described in the previous problem.

From solution of Prob. 14.71, it is found that the product mixture consists of $0.4576\,CO_2$, $0.5424\,CO$, $2.0\,H_2O$ & $2.2712\,O_2$ at 3256 K, 100 kPa. The reactants include

$$1\,CH_4 \text{ at } 25\,^{\circ}C, 100 \text{ kPa and } 4\,O_2 \text{ at } 400 \text{ K}, 100 \text{ kPa.}$$

Reactants:

$$S_R = 1(186.251) + 4(213.873) = 1041.74 \text{ kJ/K}$$

Products:

	n_i	y_i	$\bar{s}_i^0$	$-\bar{R}\ln\dfrac{y_i P}{P_0}$	$\bar{S}_i^*$
CO_2	0.4576	0.0868	339.278	+20.322	359.600
CO	0.5424	0.1029	276.660	+18.907	295.567
H_2O	2.0	0.3794	291.099	+8.058	299.157
O_2	2.2712	0.4309	287.749	+7.000	294.749

$$S_P = 0.4576(359.600) + 0.5424(295.567) + 2.0(299.157)$$

$$+ 2.2712(294.749) = 1592.62 \text{ kJ/K}$$

$$I = T_0(S_P\text{-}S_R) - Q_{CV} = 298.15(1592.62 - 1041.74) - 0 = \textbf{164 245 kJ}$$

14.73

Consider the stoichiometric combustion of pure carbon with air in a constant-pressure process at 100 kPa. Find the adiabatic flame temperature (no equilibrium reactions). Then find the temperature the mixture should be heated/cooled to so that the concentrations of CO and CO_2 are the same.

Conbustion: $\quad\quad\quad C + 1(O_2 + 3.76\, N_2) \implies \quad CO_2 + 3.76\, N_2$

Energy Eq.: $\quad\quad\quad H_R = H_P = H_P^o + \Delta H_P$

$$\Delta H_P = H_R - H_P^o = -H_{RP}^o = HV = 393\,522 \text{ kJ/kmole}$$
$$= \Delta H_{CO2} + 3.76\, \Delta H_{N2}$$

$\Delta H_{P\,3000\,K} = 152\,853 + 3.76 * 92\,715 = 501\,461 \quad$ too big
$\Delta H_{P\,2400\,K} = 115\,779 + 3.76 * 70\,640 = 381\,385 \quad$ too small
$\Delta H_{P\,2600\,K} = 128\,074 + 3.76 * 77\,963 = 421\,215 \quad$ too big

Interpolate: **T = 2461 K**

Assume now the dissociation of CO_2 as: $\quad\quad\quad 2CO_2 \iff 2\,CO + O_2$

	CO_2	CO	N_2	O_2
Start	1	0	3.76	0
Change	-2e	2e		e
Final	1-2e	2e	3.76	e;	Total = 4.76 + e

When the two concentrations are the same then

$$y_{CO2} = \frac{1 - 2e}{4.76 + e} = y_{CO} = \frac{2e}{4.76 + e} \quad\quad => \quad 1 - 2e = 2e \quad => \quad e = \tfrac14$$

Reaction constant

$$K = \frac{y_{CO}^2\, y_{O2}}{y_{CO2}^2}\left(\frac{P}{P_o}\right)^{3-2} = y_{O2} = \frac{0.25}{4.76 + 0.25} = 0.0499 = \exp(-2.9977)$$

Now look in Table A.11: $\quad \ln K_T = -2.9977 \quad =>$ Interpolate: $\quad T = \textbf{2900 K}$
Check for other reactions possible:

$\quad\quad\quad\quad N_2 \implies 2N \quad\quad\quad\quad K_T = \exp(-23.5) \quad$ no progress to RHS
$\quad\quad\quad\quad O_2 \implies 2O \quad\quad\quad\quad K_T = \exp(-5.09) \quad$ minor progress to RHS
$\quad\quad N_2 + O_2 \implies 2NO \quad\quad\quad K_T = \exp(-4.5) \quad$ minor progress to RHS

To be very accurate we should look at the last two reactions also.

14.74

Hydrides are rare earth metals, M, that have the ability to react with hydrogen to form a different substance MH_x with a release of energy. The hydrogen can then be released, the reaction reversed, by heat addition to the MH_x. In this reaction only the hydrogen is a gas so the formula developed for the chemical equilibrium is inappropriate. Show that the proper expression to be used instead of Eq. 14.14 is

$$\ln (P_{H2}/P^o) = \Delta G^o/RT$$

when the reaction is scaled to 1 kmol of H_2.

$$M + \frac{1}{2} x\, H_2 \quad \Leftrightarrow \quad MH_x$$

At equilibrium $G_P = G_R$, assume g of the solid is a function of T only.

$$\bar{g}_{MHx} = \bar{h}^0_{MHx} - T\bar{s}^0_{MHx} = \bar{g}^0_{MHx}, \quad \bar{g}_M = \bar{h}^0_M - T\bar{s}^0_M = \bar{g}^0_M$$

$$\bar{g}_{H2} = \bar{h}^0_{H2} - T\bar{s}^0_{H2} + \bar{R}T \ln(P_{H2}/P_o) = \bar{g}^0_{H2} + \bar{R}T \ln(P_{H2}/P_o)$$

$$G_P = G_R: \quad \bar{g}_{MHx} = \bar{g}_M + \frac{1}{2} x\, \bar{g}_{H2} = \bar{g}^0_M + \frac{1}{2} x[\bar{g}^0_{H2} + \bar{R}T \ln(P_{H2}/P_o)]$$

$$\Delta\bar{G}^0 = \bar{g}^0_{MHx} - \bar{g}^0_M - x\, \bar{g}^0_{H2}/2 = \bar{g}^0_{MHx} - \bar{g}^0_M$$

Scale to 1 mole of hydrogen

$$\Delta\tilde{G}^0 = (\bar{g}^0_{MHx} - \bar{g}^0_M)/(x/2) = \bar{R}T \ln(P_{H2}/P^o)$$

which is the desired result.

Simultaneous Reactions

14.75

For the process in Problem 14.46 should the dissociation of oxygen also be considered? Verbal answer but supported by numbers.

The dissociation of oxygen

Reaction: $O_2 \leftrightarrow 2\,O$ From A.11: $K_T = \exp(-4.356) = 0.01283$

Equilibrium: $K_T = 0.01283 = \dfrac{y_O^2}{y_{0_2}}\left(\dfrac{P}{P^0}\right) = \dfrac{y_O^2}{y_{0_2}}$

So we need to solve two simultaneous reaction equations. Due to the small equilibrium constant the concentration of O is going to be small, but it will have the effect of requiring a larger heat transfer to reach 3000 K.

From problem 14.46 $y_{O2} = 0.074$ so assume this is nearly the same

$$y_O = \sqrt{y_{O2} K_T} = \sqrt{0.074 \times 0.01283} = 0.03$$

so about half of the oxygen is dissociated and we do need to solve the proper equations.

14.76

Which other reactions should be considered in Problem 14.53 and which components will be present in the final mixture?

$$2H_2 + O_2 \Leftrightarrow 2H_2O \qquad \text{Species} \quad H_2 \quad O_2 \quad H_2O$$

Other possible reactions from table A.11

$$H_2 \leftrightarrow 2\,H \qquad\qquad O_2 \leftrightarrow 2\,O \qquad 2\,H_2O \leftrightarrow H_2 + 2\,OH$$

So the final component list most likely has species as:

$$H_2 \qquad O_2 \qquad H_2O \qquad H \qquad O \qquad OH$$

14.77

Redo Problem 14.46 and include the oxygen dissociation.

Reaction 1: $2\,CO_2 \Leftrightarrow 2\,CO + O_2$ Progress variable x

Reaction 2: $O_2 \Leftrightarrow 2\,O$ Progress variable y

Component	CO_2	CO	O_2	O	Ar
initial	2	1	0	0	1
change	-2x	+2x	x-y	+2y	
equil.	2 - 2x	1 + 2x	x-y	2y	1

The total number of moles is: $n_{TOT} = 4 + x + y$

From Table A.11 at 3000 K:

$$K_1 = \exp(-2.217) = 0.108935 = \frac{y_{CO}^2\, y_{O2}}{y_{CO2}^2}\left(\frac{P}{P_o}\right)^{3-2}$$

$$= \frac{(1+2x)^2}{(4+x+y)^2}\frac{x-y}{4+x+y}\frac{(4+x+y)^2}{(2-2x)^2}(1)^1 = \frac{x-y}{4}\frac{(1+2x)^2}{4+x+y}\frac{1}{(1-x)^2}$$

then

$$0.43574\,(4+x+y)\,(1-x)^2 = (x-y)\,(1+2x)^2 \qquad (1)$$

$$K_2 = \exp(-4.356) = 0.0128296 = \frac{y_O^2}{y_{O2}}\left(\frac{P}{P_o}\right)^{2-1}$$

$$= \frac{(2y)^2}{(4+x+y)^2}\frac{4+x+y}{x-y}(1)^1 = 4\frac{y^2}{4+x+y}\frac{1}{x-y}$$

$$0.0032074\,(4+x+y)\,(x-y) = y^2 \qquad (2)$$

The two eq. (1) and (2) must be solved for the two variables (x,y) with the restrictions $x \geq y \geq 0$ and $0 \leq x \leq 1$. Solution is: $x = 0.347805$, $y = 0.06343$

The outlet has: $1.30439\,CO_2 + 1.69561\,CO + 1\,Ar + 0.28438\,O_2 + 0.12686\,O$

Energy equation gives from Table A.9 and A.5 (for argon $C_p\Delta T$) plus the enthalpy of formation as the composition changes from inlet to exit.

$$\dot{Q} = \sum \dot{n}_{ex}\,\bar{h}_{ex} - \sum \dot{n}_{in}\,\bar{h}_{in}$$

$$= 1.30439\,(152853 - 393522) + 1.69561\,(93504 - 110527)$$
$$+ 1\,(39.948 \times 0.52 \times (3000 - 298)) + 0.28438\,(98013)$$
$$+ 0.12686\,(56574 + 249170) - 2\,(-393522) - 1(-110527) - 1\,(0)$$
$$= \textbf{677 568 kW}$$

14.78

Ethane is burned with 150% theoretical air in a gas turbine combustor. The products exiting consist of a mixture of CO_2, H_2O, O_2, N_2, and NO in chemical equilibrium at 1800 K, 1 MPa. Determine the mole fraction of NO in the products. Is it reasonable to ignore CO in the products?

Combustion:

$$C_2H_6 + 5.25\,O_2 + 19.74\,N_2 \;\rightarrow\; 2\,CO_2 + 3\,H_2O + 1.75\,O_2 + 19.74\,N_2$$

Products at 1800 K, 1 MPa

Equilibrium mixture: CO_2, H_2O, O_2, N_2, NO

	N_2	+	O_2	$\Leftrightarrow$	2 NO
initial	19.74		1.75		0
change	-x		-x		+2x
equil.	19.74-x		1.75-x		2x

Equil. comp. $n_{CO2} = 2$, $n_{O2} = 1.75-x$, $n_{NO} = 2x$, $n_{H2O} = 3$, $n_{N2} = 19.74-x$

$$K = 1.192\times10^{-4} = \frac{y_{NO}^2}{y_{N2}y_{O2}}\left(\frac{P}{P^0}\right)^0 = \frac{4x^2}{(19.74-x)(1.75-x)}$$

Solving, x = 0.031 75

$$y_{NO} = \frac{2\times0.031\,75}{26.49} = \mathbf{0.0024}$$

b)

	2 CO$_2$	$\Leftrightarrow$	2 CO	+	O_2
initial	2		0		0
change	-2a		+2a		+2x
equil.	2-2a		2a		2x

$$K = 4.194\times10^{-8} = \frac{y_{CO}^2 y_{O2}}{y_{CO2}^2}\left(\frac{P}{P^0}\right)^1 = \left(\frac{2a}{2-2a}\right)^2\left(\frac{1.75-x+a}{26.49+a}\right)\times\frac{1}{0.1}$$

This equation should be solved simultaneously with the equation solved in part a) (modified to include the unknown a). Since x was found to be small and also a will be very small, the two are practically independent. Therefore, use the value   x = 0.031 75 in the equation above, and solve for a.

$$\left(\frac{a}{1-a}\right)^2\left(\frac{1.75-0.031\,75+a}{26.49+a}\right) = \left(\frac{0.1}{1.0}\right) \times 4.194 \times10^{-8}$$

Solving, a = 0.000 254 or $y_{CO} = 1.92\times10^{-5}$ negligible for most applications.

14.79

A mixture of 1 kmol water and 1 kmol oxygen at 400 K is heated to 3000 K, 200 kPa, in a steady flow process. Determine the equilibrium composition at the outlet of the heat exchanger, assuming that the mixture consists of H_2O, H_2, O_2, and OH.

Reactions and equilibrium eq'ns the same as in example 14.7 (but different initial composition).

At equil.: $n_{H2O} = 1 - 2a - 2b$, $n_{H2} = 2a + b$, $n_{O2} = 1 + a$

$n_{OH} = 2b$, $n_{TOT} = 2 + a + b$

Since T = 3000 K is the same, the two equilibrium constants are the same:

From Table A.11: $K_1 = 0.002\ 062$, $K_2 = 0.002\ 893$

The two equilibrium equations are

$$K_1 = \left(\frac{2a + b}{1 - 2a - 2b}\right)^2 \frac{1 + a}{2 + a + b}\left(\frac{P}{P^0}\right); \quad K_2 = \frac{2a + b}{2 + a + b}\left(\frac{2b}{1 - 2a - 2b}\right)^2\left(\frac{P}{P^0}\right)$$

which must be solved simultaneously for a & b. If solving manually, it simplifies the solution to divide the first by the second, which leaves a quadratic equation in a & b - can solve for one in terms of the other using the quadratic formula (with the root that gives all positive moles). This reduces the problem to solving one equation in one unknown, by trial & error.

Solving => b = 0.116, a = -0.038 =>

$n_{H2O} = 0.844$, $n_{H2} = 0.0398$, $n_{O2} = 0.962$, $n_{OH} = 0.232$, $n_{TOT} = 2.0778$

$y_{H2O} = 0.4062$, $y_{H2} = 0.0191$, $y_{O2} = 0.4630$, $y_{OH} = 0.1117$

14.80

Assume dry air (79% N_2 and 21% O_2) is heated to 2000 K in a steady flow process at 200 kPa and only the reactions listed in Table A.11 are possible (and their linear combinations). Find the final composition (anything smaller than 1 ppm, parts per million, is neglected) and the heat transfer needed for 1 kmol of air in.

We could have: $\quad\quad N_2$, N, O_2, O, NO, NO_2

The possible reactions are:

$$N_2 \Leftrightarrow 2\,N \quad\quad\quad\quad K(T) = \exp(-41.655) \approx 0$$
$$O_2 \Leftrightarrow 2\,O \quad\quad\quad\quad K(T) = \exp(-14.619) \approx \text{small}$$
$$N_2 + O_2 \Leftrightarrow 2\,NO \quad\quad\quad\quad K(T) = \exp(-7.825)$$
$$N_2 + 2\,O_2 \Leftrightarrow 2\,NO_2 \quad\quad\quad\quad K(T) = \exp(-19.136) \approx \text{small}$$

So from this we have only NO extra.

	N_2	+	O_2	$\Leftrightarrow$	$2\,NO$
initial	0.79		0.21		0
change	-x		-x		+2x
equil.	0.79 - x		0.21 - x		2x

$$K = \exp(-7.825) = 0.0004 = \frac{y_{NO}^2}{y_{N2}y_{O2}} \left(\frac{P}{P^0}\right)^0 = \frac{4x^2}{(0.79 - x)(0.21 - x)}$$

Solving, x = 0.004

$$y_{N2} = 0.786, \quad y_{O2} = 0.206, \quad y_{NO} = 0.008$$

$$Q = H_{ex} - H_{in} = 0.786\,\Delta\bar{h}_{N2} + 0.206\,\Delta\bar{h}_{O2} + 0.008\,\bar{h}_{NO}$$
$$= 0.786\,(56137) + 0.206\,(59176) + 0.008\,(57859 + 90291)$$
$$= \textbf{57 499 kJ/kmol}$$

If we had solved also for the oxygen O and NO_2 formation we would get approximately (neglect the effect on total moles)

$$(y_O^2 / y_{O2})\,(P/P_o) = \exp(-14.619) \quad\quad \Rightarrow \quad\quad y_O \approx 0.0002$$
$$(y_{NO2}^2 / y_{N2}y_{O2}^2)\,(P/P_o)^{-1} = \exp(-19.136) \quad\quad \Rightarrow \quad\quad y_{NO2} \approx 0.000018$$

and the O energy addition would be: $\quad 0.0002(35713 + 249170) = 57$ kJ/kmol very insignificant.

14.81

One kilomole of water vapor at 100 kPa, 400 K, is heated to 3000 K in a constant pressure steady flow process. Determine the final composition, assuming that H_2O, H_2, H, O_2, and OH are present at equilibrium.

Reactions:

1) $\quad 2\,H_2O \Leftrightarrow 2\,H_2 + O_2 \qquad$ 2) $\quad 2\,H_2O \Leftrightarrow H_2 + 2\,OH$

change -2a $\quad$ +2a $\quad$ +a $\quad$ change $\quad$ -2b $\quad$ +b $\quad$ +2b

3) $\quad H_2 \Leftrightarrow 2\,H$

change -c $\quad$ +2c

At equilibrium (3000 K, 100 kPa)

$n_{H2O} = \quad$ 1-2a-2b $\qquad n_{O2} = \quad$ a $\qquad\qquad n_H = \quad$ 2c

$n_{H2} = \quad$ 2a+b-c $\qquad n_{OH} = \quad$ 2b $\qquad\qquad n_{TOT} = \quad$ 1+a+b+c

$$\frac{K_1}{(P/P^0)} = \frac{2.062\times10^{-3}}{1} = \left(\frac{2a+b-c}{1-2a-2b}\right)^2 \left(\frac{a}{1+a+b+c}\right)$$

$$\frac{K_2}{(P/P^0)} = \frac{2.893\times10^{-3}}{1} = \left(\frac{2a+b-c}{1+a+b+c}\right)\left(\frac{2b}{1-2a-2b}\right)^2$$

$$\frac{K_3}{(P/P^0)} = \frac{2.496\times10^{-2}}{1} = \frac{(2c)^2}{(2a+b-c)(1+a+b+c)}$$

These three equations must be solved simultaneously for a, b & c:

$\quad$ a = 0.0622, $\quad$ b = 0.0570, $\quad$ c = 0.0327

and $\qquad n_{H2O} = 0.7616 \qquad y_{H2O} = 0.6611$

$\qquad\qquad n_{H2} = 0.1487 \qquad y_{H2} = 0.1291$

$\qquad\qquad n_{O2} = 0.0622 \qquad y_{O2} = 0.0540$

$\qquad\qquad n_{OH} = 0.1140 \qquad y_{OH} = 0.0990$

$\qquad\qquad n_{H} = 0.0654 \qquad y_{H} = 0.0568$

14.82

Water from the combustion of hydrogen and pure oxygen is at 3800 K and 50 kPa. Assume we only have H_2O, O_2, OH and H_2 as gases with the two simple water dissociation reactions active find the equilibrium composition.

This problem is very similar to Example 14.7 in the text. The only difference is that we have T = 3800 K and P = 50 kPa. From table A.11 we have

$$\ln K_1 = -1.906 \; ; \quad K_1 = 0.14867; \qquad \ln K_2 = -0.984 \; ; \qquad K_2 = 0.3738$$

$$K_1 = \left(\frac{2a+b}{1-2a-2b}\right)^2 \frac{a}{1+a+b}\left(\frac{P}{P^0}\right); \quad K_2 = \frac{2a+b}{1+a+b}\left(\frac{2b}{1-2a-2b}\right)^2\left(\frac{P}{P^0}\right)$$

So we have two equations as

$$\left(\frac{2a+b}{1-2a-2b}\right)^2 \frac{a}{1+a+b} = K_1 / \left(\frac{P}{P^0}\right) = 0.29734 \tag{1}$$

$$\frac{2a+b}{1+a+b}\left(\frac{2b}{1-2a-2b}\right)^2 = K_2 / \left(\frac{P}{P^0}\right) = 0.7476 \tag{2}$$

Divide the second equation by the first to give

$$\frac{4b^2}{(2a+b)\,a} = \frac{0.7476}{0.29734} = 2.5143$$

or

$$2a^2 + ba - 1.5909\,b^2 = 0$$

$$a = -(b/4) \pm (1/4)\sqrt{b^2 - 4 \times 2 \times (-1.5909\,b^2)} = 0.676256\,b$$

Now we can do trial and error on equation 1 for only one variable, say b:

$$a = 0.14228, \quad b = 0.2104$$

$$n_{H2O} = 1 - 2a - 2b = 0.29464, \quad n_{H2} = 2a + b = 0.49496,$$

$$n_{O2} = a = 0.14228, \quad n_{OH} = 2b = 0.4208$$

14.83

Methane is burned with theoretical oxygen in a steady flow process, and the products exit the combustion chamber at 3200 K, 700 kPa. Calculate the equilibrium composition at this state, assuming that only CO_2, CO, H_2O, H_2, O_2, and OH are present.

Combustion: $\qquad\qquad CH_4 + 2\,O_2 \rightarrow CO_2 + 2\,H_2O$

Dissociation reactions:

$\qquad$ 1) $\quad 2\,H_2O \Leftrightarrow 2\,H_2 + O_2 \qquad\qquad$ 2) $\quad 2\,H_2O \Leftrightarrow H_2 + 2\,OH$

$\qquad$ change -2a $\quad$ +2a +a $\qquad\qquad$ change -2b $\quad$ +b +2b

$\qquad$ 3) $\quad 2\,CO_2 \Leftrightarrow 2\,CO + O_2$

$\qquad$ change -2c $\quad$ +2c +c

$\quad$ At equilibrium:

n_{H2O} =	2-2a-2b	n_{O2} =	a+c	n_{CO2} =	1-2c
n_{H2} =	2a+b	n_{OH} =	2b	n_{CO} =	2c
				n_{TOT} =	3+a+b+c

Products at 3200 K, 700 kPa, from A.11

$$K_1 = \exp(-4.916) = 0.007\ 328 = \left(\frac{2a+b}{2\text{-}2a\text{-}2b}\right)^2 \left(\frac{a+c}{3+a+b+c}\right) \left(\frac{700}{100}\right)$$

$$K_2 = \exp(-4.401) = 0.012\ 265 = \left(\frac{2b}{2\text{-}2a\text{-}2b}\right)^2 \left(\frac{2a+b}{3+a+b+c}\right) \left(\frac{700}{100}\right)$$

$$K_3 = \exp(-0.853) = 0.426\ 135 = \left(\frac{2c}{1\text{-}2c}\right)^2 \left(\frac{a+c}{3+a+b+c}\right) \left(\frac{700}{100}\right)$$

These 3 equations must be solved simultaneously for a, b, & c. If solving by hand divide the first equation by the second, and solve for c = fct(a,b). This reduces the solution to 2 equations in 2 unknowns. Solving,

$$a = 0.024,\ b = 0.1455,\ c = 0.236$$

Substance:	H_2O	H_2	O_2	OH	CO_2	CO
n	1.661	0.1935	0.260	0.291	0.528	0.472
y	0.4877	0.0568	0.0764	0.0855	0.1550	0.1386

14.84

Butane is burned with 200% theoretical air, and the products of combustion, an equilibrium mixture containing only CO_2, H_2O, O_2, N_2, NO, and NO_2, exit from the combustion chamber at 1400 K, 2 MPa. Determine the equilibrium composition at this state.

Combustion:

$$C_4H_{10} + 13\,O_2 + 48.9\,N_2 \rightarrow 4\,CO_2 + 5\,H_2O + 6.5\,O_2 + 48.9\,N_2$$

Dissociation:

1) $\quad N_2 + O_2 \Leftrightarrow 2\,NO$ $\qquad$ 2) $\quad N_2 + 2O_2 \Leftrightarrow 2\,NO_2$

change $\quad$ -a $\quad$ -a $\quad$ +2a $\qquad$ change $\quad$ -b $\quad$ -2b $\quad$ +2b

At equilibrium:

$n_{H2O} = 5$ $\qquad n_{N2} = 48.9\text{-a-b}$ $\qquad\qquad n_{NO} = 2a$

$n_{CO2} = 4$ $\qquad n_{O2} = 6.5\text{-a-2b}$ $\qquad\qquad n_{NO2} = 2b$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad n_{TOT} = 64.4\text{-b}$

At 1400 K, from A.11: $\qquad K_1 = 3.761 \times 10^{-6}$, $\quad K_2 = 9.026 \times 10^{-10}$

$$K_1 = \frac{(2a)^2}{(48.9\text{-a-b})(6.5\text{-a-2b})} \;;\qquad K_2 = \frac{(2b)^2(64.4\text{-b})}{(6.5\text{-a-2b})^2(48.9\text{-a-b})}\left(\frac{P}{P^0}\right)^{-1}$$

As K_1 and K_2 are both very small, with $K_2 << K_1$, the unknowns a & b will both be very small, with b << a. From the equilibrium eq.s, for a first trial

$$a \sim \frac{1}{2}\sqrt{K_1 \times 48.9 \times 6.5} \sim 0.0173\;;\quad b \sim \frac{1}{2} \times 6.5\sqrt{K_2 \times \frac{2}{0.1} \times \frac{48.9}{64.4}} \sim 0.000\,38$$

Then by trial & error,

$$\frac{a^2}{(48.9\text{-a-b})(6.5\text{-a-2b})} = \frac{3.761 \times 10^{-6}}{4} = 0.940\,25 \times 10^{-6}$$

$$\frac{b^2(64.4\text{-b})}{(6.5\text{-a-2b})^2(48.9\text{-a-b})} = \frac{9.026 \times 10^{-10} \times (\frac{2}{0.1})}{4} = 45.13 \times 10^{-10}$$

Solving, $\quad a = 0.017\,27$, $\quad b = 0.000\,379$

$n_{CO2} = 4$, $\qquad\qquad n_{H2O} = 5$, $\qquad\qquad n_{N2} = 48.882$, $\quad n_{O2} = 6.482$,

$y_{CO2} = 0.062\,11$, $\quad y_{H2O} = 0.077\,64$, $\quad y_{N2} = 0.759\,04$, $\quad y_{O2} = 0.100\,65$

$n_{NO} = 0.034\,54$, $\quad n_{NO2} = 0.000\,76$

$y_{NO} = 0.000\,55$, $\quad y_{NO2} = 0.000\,01$

14.85

One kilomole of air (assumed to be 78% nitrogen, 21% oxygen, and 1% argon) at room temperature is heated to 4000 K, 200 kPa. Find the equilibrium composition at this state, assuming that only N_2, O_2, NO, O, and Ar are present.

1 kmol air (0.78 N_2, 0.21 O_2, 0.01 Ar) heated to

4000 K, 200 kPa.

Equil.:

1) $N_2 + O_2 \Leftrightarrow 2\, NO$ $n_{N2} = 0.78\text{-a}$

change -a   -a   +2a $n_{O2} = 0.21\text{-a-b}$

$n_{Ar} = 0.01$

2) $O_2 \Leftrightarrow 2\, O$ $n_O = 2b$

change -b   +2b $n_{NO} = 2a$

$n_{tot} = 1+b$

$$K_1 = 0.0895 = \frac{4a^2}{(0.78\text{-a})(0.21\text{-a-b})}\left(\frac{200}{100}\right)^0$$

$$K_2 = 2.221 = \frac{4b^2}{(1+b)(0.21\text{-a-b})}\left(\frac{200}{100}\right)$$

Divide 1st eq'n by 2nd and solve for a as function(b), using

$$X = \frac{K_1}{K_2}\left(\frac{P}{P^0}\right) = 0.0806$$

Get

$$a = \frac{Xb^2}{2(1+b)}\left[-1+\sqrt{1+\frac{4\times0.78(1+b)}{Xb^2}}\right] \tag{1}$$

Also

$$\frac{b^2}{(1+b)(0.21\text{-a-b})} = \frac{K_2}{4(P/P^0)} = 0.277\,63 \tag{2}$$

Assume   b = 0.1280   From (1), get a = 0.0296

Then, check a & b in (2) $\Rightarrow$ OK

Therefore,

$n_{N2} = 0.7504$ $n_O = 0.2560$ $y_{N2} = 0.6652$ $y_O = 0.2269$

$n_{O2} = 0.0524$ $n_{NO} = 0.0592$ $y_{O2} = 0.0465$ $y_{NO} = 0.0525$

$n_{Ar} = 0.01$ $y_{Ar} = 0.0089$

14.86

Acetylene gas and x times theoretical air ($x > 1$) at room temperature and 500 kPa are burned at constant pressure in an adiabatic steady flow process. The flame temperature is 2600 K, and the combustion products are assumed to consist of N_2, O_2, CO_2, H_2O, CO, and NO. Determine the value of x.

Combustion:

$$C_2H_2 + 2.5x\, O_2 + 9.4x\, N_2 \rightarrow 2\, CO_2 + H_2O + 2.5(x-1)O_2 + 9.4x\, N_2$$

Eq. products 2600 K, 500 kPa: N_2, O_2, CO_2, H_2O, CO & NO

2 Reactions: 1) $2\, CO_2 \Leftrightarrow 2\, CO + O_2$ 2) $N_2 + O_2 \Leftrightarrow 2\, NO$

change -2a   +2a   +a   change -b   -b   +2b

Equil. Comp.: $n_{N2} = 9.4x\text{-}b$, $n_{H2O} = 1$, $n_{CO} = 2a$, $n_{NO} = 2b$

$n_{O2} = 2.5x - 2.5 + a - b$, $n_{CO2} = 2 - 2a$, $n_{TOT} = 11.9x + 0.5 + a$

At 2600 K, from A.11: $K_1 = 3.721 \times 10^{-3}$, $K_2 = 4.913 \times 10^{-3}$

EQ1: $\dfrac{K_1}{(P/P^o)} = \dfrac{3.721 \times 10^{-3}}{5} = \left(\dfrac{a}{1-a}\right)^2 \left(\dfrac{2.5x - 2.5 + a - b}{11.9x + 0.5 + a}\right)$

EQ2: $K_2 = 4.913 \times 10^{-3} = \dfrac{(2b)^2}{(9.4 - b)(2.5x - 2.5 + a - b)}$

Also, from the energy Eq.: $H_P - H_R = 0$ where

$H_R = 1(+226\,731) + 0 + 0 = +226\,731$ kJ

$H_P = (9.4x - b)(0 + 77\,963) + (2.5x - 2.5 + a - b)(0 + 82\,225)$

$\quad + (2 - 2a)(-393\,522 + 128\,074) + 1(-241\,826 + 104\,520)$

$\quad + 2a(-110\,527 + 78\,679) + 2b(90\,291 + 80\,034)$

Substituting H_R and H_P into the energy equation,

EQ3: $988\,415x + 549\,425a + 180\,462b - 1\,100\,496 = 0$

which results in 3 equations in the 3 unknowns x, a, b. Assume x = 1.07 then

EQ1: $7.442 \times 10^{-2} = \left(\dfrac{a}{1\text{-}a}\right)^2 \left(\dfrac{0.175 + a - b}{13.233 + a}\right)$

EQ2: $1.2283 \times 10^{-3} = \dfrac{b^2}{(10.058 - b)(0.175 + a + b)}$

Solving,   a = 0.1595,   b = 0.0585   Then checking in EQ3,

$988\,415 \times 1.07 + 549\,425 \times 0.1595 + 180\,462 \times 0.0585 - 1\,100\,496 \approx 0$

Therefore, x = **1.07**

Gasification

14.87

One approach to using hydrocarbon fuels in a fuel cell is to "reform" the hydrocarbon to obtain hydrogen, which is then fed to the fuel cell. As a part of the analysis of such a procedure, consider the reforming reaction $CH_4 + H_2O \Leftrightarrow 3H_2 + CO$. Determine the equilibrium constant for this reaction at a temperature of 800 K.

$$CH_4 + H_2O \Leftrightarrow 3H_2 + CO$$

The equilibrium constant from Eq.14.15 depends on ΔG^0 and T.

For CH_4, use C_{P0} at average temperature 550 K $\approx \frac{1}{2}$ (298 + 800) K.

Table A.6, $\bar{C}_{P0} = 49.316$ kJ/kmol K

$$\bar{h}^0_{800\ K} = \bar{h}^0_f + \bar{C}_{P0}\Delta T = -74\ 873 + 49.316\ (800 - 298.2)$$

$$= -50\ 126\ \text{kJ/kmol}$$

$$\bar{s}^0_{800\ K} = 186.251 + 49.316 \ln\frac{800}{298.2} = 234.918\ \text{kJ/kmol K}$$

The rest of the properties are from Table A.9 at 800 K.

$$\Delta H^0_{800\ K} = 3(0 + 14\ 681) + 1(-110\ 527 + 15\ 174) - 1(-50\ 126)$$

$$- 1(-241\ 826 + 18\ 002) = +222\ 640\ \text{kJ/kmol}$$

$$\Delta S^0_{800\ K} = 3(159.554) + 1(227.277) - 1(234.918) - 1(223.826)$$

$$= +247.195\ \text{kJ/kmol K}$$

$$\Delta G^0 = \Delta H^0 - T\Delta S^0 = 222\ 640 - 800(247.195) = +24\ 884\ \text{kJ/kmol}$$

$$\ln K = -\frac{\Delta G^0}{\bar{R}T} = \frac{-24\ 884}{8.3145 \times 800} = -3.7411 \quad => \quad K = \mathbf{0.0237}$$

14.88

A coal gasifier produces a mixture of 1 CO and $2H_2$ that is fed to a catalytic converter to produce methane. The reaction is $CO + 3H_2 \Leftrightarrow CH_4 + H_2O$. The equilibrium constant at 600 K is $K = 1.83 \times 10^6$. What is the composition of the exit flow assuming a pressure of 600 kPa?

The reaction equation is:

	CO	+	3 H$_2$	$\Leftrightarrow$	CH$_4$	+	H$_2$O
initial	1		2		0		0
change	−x		−3x		+x		+x
equil.	1 − x		2 − 3x		x		x

$n_{TOTAL} = 3 - 2x$

$$K = \frac{y_{CH_4}\, y_{H_2O}}{y_{H_2}^3\, y_{CO}}\left(\frac{P}{P^o}\right)^{1+1-1-3} = \frac{x^2\,(3-2x)^2}{(1-x)(2-3x)^3}\left(\frac{P}{P^o}\right)^{-2}$$

$$1.83 \times 10^6 \times \left(\frac{600}{100}\right)^2 = 6.588 \times 10^7 = \frac{x^2\,(3-2x)^2}{(1-x)(2-3x)^3}$$

Trial and error to solve for x.

$$x = 0.6654 \quad LHS = 6.719 \times 10^7$$

$$x = 0.66538 \quad LHS = 6.41 \times 10^7$$

$$x = 0.66539 \quad LHS = 6.562 \times 10^7 \qquad \text{close enough}$$

$n_{CH4} = 0.66539, \quad n_{H2O} = 0.66539, \quad n_{CO} = 0.66539, \quad n_{H2} = 0.00383$

so we used up nearly all the hydrogen gas.

14.89

Gasification of char (primarily carbon) with steam following coal pyrolysis yields a gas mixture of 1 kmol CO and 1 kmol H_2. We wish to upgrade the hydrogen content of this syngas fuel mixture, so it is fed to an appropriate catalytic reactor along with 1 kmol of H_2O. Exiting the reactor is a chemical equilibrium gas mixture of CO, H_2, H_2O, and CO_2 at 600 K, 500 kPa. Determine the equilibrium composition. Note: see Example 14.4.

$$1\ CO + 1\ H_2 + 1\ H_2O \longrightarrow \boxed{\text{Constant pressure reactor}} \longrightarrow \text{Chem. Equil. Mix} \quad 600\ \text{K}$$
$$\text{CO, } H_2, H_2O, CO_2 \quad 500\ \text{kPa}$$

(1) $1\ CO + 1\ H_2O \Leftrightarrow 1\ CO_2 + 1\ H_2$ (Water gas reaction)

        -x   -x   +x   +x

(2) $2\ H_2O \Leftrightarrow 2\ H_2 + 1\ O_2$ (3) $2\ CO_2 \Leftrightarrow 2\ CO + 1\ O_2$

Reaction $(1) = \dfrac{1}{2}\,(2) - \dfrac{1}{2}\,(3)$

From Table A.11 at 600 K

$$\ln K_1 = \frac{1}{2}\Big[-85.79 - (-92.49)\Big] = +3.35, \qquad K_1 = 28.503$$

Equilibrium:

$$n_{CO} = 1-x, \quad n_{H2O} = 1-x, \quad n_{CO2} = 0 + x, \quad n_{H2} = 1 + x$$

$\sum n = 3$,   notice the reaction is pressure insensitive.

$$K = \frac{y_{CO2}y_{H2}}{y_{CO}y_{H2O}}\Big(\frac{P}{P^0}\Big)^0 = \frac{y_{CO2}y_{H2}}{y_{CO}y_{H2O}} = \frac{x(1+x)}{(1-x)^2} = 28.503$$

Solve for x: $x = 0.7794$

	n	y	%
CO	0.2206	0.0735	7.35
H_2O	0.2206	0.0735	7.35
CO_2	0.7794	0.2598	26.0
H_2	1.7794	0.5932	59.3

14.90

The equilibrium reaction as: $CH_4 \Leftrightarrow C + 2H_2$. has ln K = -0.3362 at 800 K and ln$K$ = -4.607 at 600 K. By noting the relation of K to temperature show how you would interpolate ln K in $(1/T)$ to find K at 700 K and compare that to a linear interpolation.

A.11:   ln K = − 0.3362 at 800K   ln K = −4.607 at 600K

$$\ln K_{700} = \ln K_{800} + \frac{\dfrac{1}{700} - \dfrac{1}{800}}{\dfrac{1}{600} - \dfrac{1}{800}} \times (-4.607 + 0.3362)$$

$$= -0.3362 + \frac{\dfrac{800}{700} - 1}{\dfrac{800}{600} - 1} \times (-4.2708) = \mathbf{-2.1665}$$

Linear interpolation:

$$\ln K_{700} = \ln K_{600} + \frac{700 - 600}{800 - 600} (\ln K_{800} - \ln K_{600})$$

$$= -4.607 + \frac{1}{2}(-0.3362 + 4.607) = \mathbf{-2.4716}$$

Comment: Look at the van't Hoff equation in Problem 14.56 and integrate it with temperature assuming ΔH^0 is constant. That gives   ln K = C - ΔH^0/T.

14.91

One approach to using hydrocarbon fuels in a fuel cell is to "reform" the hydrocarbon to obtain hydrogen, which is then fed to the fuel cell. As a part of the analysis of such a procedure, consider the reaction $CH_4 + H_2O \Leftrightarrow CO + 3H_2$. One kilomole each of methane and water are fed to a catalytic reformer. A mixture of CH_4, H_2O, H_2, and CO exits in chemical equilibrium at 800 K, 100 kPa; determine the equilibrium composition of this mixture using an equilibrium constant of $K = 0.0237$.

The reaction equation is:

	CH_4	+	H_2O	$\Leftrightarrow$	$3\,H_2$	+	CO
initial	1		1		0		0
change	-x		-x		+3x		+x
equil.	(1-x)		(1-x)		3x		x

$$n_{TOTAL} = 1 - x + 1 - x + 3x + x = 2 + 2x$$

$$K = \frac{y_{H2}^3 y_{CO}}{y_{CH4} y_{H2O}} \left(\frac{P}{P^0}\right)^2 = \frac{(3x)^3 x}{(1-x)(1-x)(2+2x)^2} \left(\frac{100}{100}\right)^2$$

or $\quad \left(\dfrac{x}{1-x}\right)^2 \left(\dfrac{x}{1+x}\right)^2 = \dfrac{4 \times 0.0237}{27 \times 1} = 0.003\ 51$

or $\quad \dfrac{x^2}{1-x^2} = \sqrt{0.003\ 51} = 0.059\ 25 \quad$ Solving, $\quad x = 0.2365$

$$\left.\begin{array}{l} n_{CH4} = 0.7635 \\ n_{H2O} = 0.7635 \\ n_{H2} = 0.7095 \\ n_{CO} = 0.2365 \\ n_{TOT} = 2.473 \end{array}\right\} \left\{\begin{array}{l} y_{CH4} = 0.3087 \\ y_{H2O} = 0.3087 \\ y_{H2} = 0.2870 \\ y_{CO} = 0.0956 \end{array}\right.$$

14.92

Consider a gasifier that receives 4 kmol of carbon-monoxide, 3 kmol hydrogen and 3.76 kmol nitrogen and brings the mixture to equilibrium at 900 K, 1 MPa with the following reaction:

$$2\,CO + 2\,H_2 \iff CH_4 + CO_2$$

which is the sum of Eq.14.32 and 14.33. If the equilibrium constant is K = 2.679 find the exit composition.

The reaction takes place with the nitrogen as a dilutant.

reaction	2 CO	+	2 H$_2$	$\iff$	CH$_4$	+	CO$_2$	also N$_2$
initial	4		3		0		0	3.76
change	-2x		-2x		+x		+x	0
equil.	(4-2x)		(3-2x)		x		x	3.76

$$n_{TOT} = 10.76 - 2x$$

$$K = \frac{y_{CH4}\,y_{CO2}}{y_{CO}^2\,y_{H2}^2}\left(\frac{P}{P^0}\right)^{-2} = \frac{x \times x \times (10.76 - 2x)^2}{(4 - 2x)^2(3 - 2x)^2}\left(\frac{P}{P^0}\right)^{-2}$$

or take the square-root to get

$$\frac{x(10.76 - 2x)}{(4 - 2x)(3 - 2x)} = \frac{P}{P^0}\sqrt{K} = \frac{1}{0.1}\sqrt{2.679} = 16.368$$

By trial & error or solve the quadratic eq. in x,   x = 1.2781

$$n_{CO} = 1.444, \quad n_{H2} = 0.444, \quad n_{CH4} = n_{CO2} = 1.278, \quad n_{N2} = 3.76$$

$$y_{CO} = 0.176, \quad y_{H2} = 0.054, \quad y_{CH4} = y_{CO2} = 0.156, \quad y_{N2} = 0.458$$

14.93

Consider the production of a synthetic fuel (methanol) from coal. A gas mixture of 50% CO and 50% H_2 leaves a coal gasifier at 500 K, 1 MPa, and enters a catalytic converter. A gas mixture of methanol, CO and H_2 in chemical equilibrium with the reaction: $CO + 2H_2 \Leftrightarrow CH_3OH$ leaves the converter at the same temperature and pressure, where it is known that $\ln K = -5.119$.

a. Calculate the equilibrium composition of the mixture leaving the converter.

b. Would it be more desirable to operate the converter at ambient pressure?

1 CO		Equil.	CH_3OH, CO, H_2
1 H_2	⟹ Converter ⟹	Mix	500 K, 1 MPa

Reaction:	CO	+	2 H_2	⟺	CH_3OH
initial	1		1		0
change	-x		-2x		+x
equil.	(1-x)		(1-2x)		x

a) $\quad K = \dfrac{y_{CH3OH}}{y_{CO}y_{H2}^2}\left(\dfrac{P}{P^0}\right)^{-2} = \left(\dfrac{x}{1-x}\right)\left(\dfrac{2-2x}{1-2x}\right)^2\left(\dfrac{P}{P^0}\right)^{-2} \quad => \quad \dfrac{x(1-x)}{(1-2x)^2} = \dfrac{K}{4}\left(\dfrac{P}{P^0}\right)^2$

$\ln K = -5.119, \qquad K = 0.005\,98$

$\dfrac{x(1-x)}{(1-2x)^2} = \dfrac{0.005\,98}{4}\left(\dfrac{1}{0.1}\right)^2 = 0.1495 \quad => \quad x = 0.1045$

$n_{CH3OH} = x = 0.1045, \quad n_{CO} = 1-x = 0.8955, \quad n_{H2} = 1 - 2x = 0.791$

$\qquad y_{CH3OH} = 0.0583, \quad y_{CO} = 0.5000, \quad y_{H2} = \mathbf{0.4417}$

b) For P = 0.1 MPa

$\qquad \dfrac{x(1-x)}{(1-2x)^2} = \dfrac{0.005\,98}{4}\left(\dfrac{0.1}{0.1}\right)^2 = 0.001\,495$

x is much smaller (~ 0.0015) **not good**

Ionization

14.94

At 10 000 K the ionization reaction for Ar is: $\quad$ Ar $\Leftrightarrow$ Ar$^+$ + e$^-$ with equilibrium constant of $\quad K = 4.2 \times 10^{-4}$. What should the pressure be for a mole concentration of argon ions (Ar$^+$) of 10%?

From the reaction (ionization) we recognize that the concentration of electrons must equal that of argon ions so

$$y_{Ar^+} = y_{e^-} = 0.1 \quad \text{and} \quad y_{Ar} = 1 - y_{Ar^+} - y_{e^-} = 0.8$$

Now

$$K = 4.2 \times 10^{-4} = \frac{y_{Ar^+}\, y_{e^-}}{y_{Ar}} \left(\frac{P}{P^o}\right)^{1+1-1} = \frac{0.1 \times 0.1}{0.8}\, \frac{P}{100}$$

$$P = 0.00042 \times \frac{0.8}{0.1 \times 0.1} \times 100 = \textbf{3.36 kPa}$$

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14.95

Repeat the previous problem assuming the argon constitutes 1% of a gas mixture where we neglect any reactions of the other gases and find the pressure that will give a mole-concentration of Ar^+ of 0.1%.

Previous problem:

At 10 000 K the ionization reaction for Ar is: $Ar \Leftrightarrow Ar^+ + e^-$ with equilibrium constant of $K = 4.2 \times 10^{-4}$. What should the pressure be for a mole concentration of argon ions (Ar^+) of 10%?

$$Ar \Leftrightarrow Ar^+ + e^- \quad others$$

	Ar	$\Leftrightarrow$ Ar$^+$	+ e$^-$	others	
Initial:	0.01	0	0	0.99	
Change	-x	x	x	0	
Final	0.01-x	x	x	0.99	Total = 1 + x

From the reaction (ionization) we recognize that the concentration of electrons must equal that of argon ions so

$$y_{Ar+} = y_{e-} = 0.001 \ = x \, / \, (1+x) \quad \Rightarrow \quad x = 0.001001$$

$$y_{Ar} \ = (0.01 - x) \, /(1 +x) = 0.00899$$

Now

$$K = 4.2 \times 10^{-4} = \frac{y_{Ar+} \, y_{e-}}{y_{Ar}} \left(\frac{P}{P^o}\right)^{1+1-1} = \frac{0.001 \times 0.001}{0.00899} \frac{P}{100}$$

$$P = 0.00042 \times 8990 \times 100 = \textbf{377 kPa}$$

14.96

Operation of an MHD converter requires an electrically conducting gas. It is proposed to use helium gas "seeded" with 1.0 mole percent cesium, as shown in Fig. P14.96. The cesium is partly ionized ($Cs \Leftrightarrow Cs^+ + e^-$) by heating the mixture to 1800 K, 1 MPa, in a nuclear reactor to provide free electrons. No helium is ionized in this process, so that the mixture entering the converter consists of He, Cs, Cs^+, and e^-. Determine the mole fraction of electrons in the mixture at 1800 K, where ln $K = 1.402$ for the cesium ionization reaction described.

Reaction: $Cs \Leftrightarrow Cs^+ + e^-$, Also He   ln K = 1.402

initial	0.01	0	0	0.99	=> K = 4.0633
change	-x	+x	+x	0	
Equil	(0.01-x)	x	x	0.99 ;	total: 1 + x

$$K = \frac{y_{e-} \, y_{Cs+}}{y_{Cs}} \left(\frac{P}{P^0}\right) = \left(\frac{x}{0.01-x}\right)\left(\frac{x}{1+x}\right)\left(\frac{P}{P^0}\right)$$

or $\left(\dfrac{x}{0.01-x}\right)\left(\dfrac{x}{1+x}\right) = 4.0633 \, / \, (1/0.1) = 0.40633$

Quadratic equation: x = 0.009767

$$\Rightarrow \quad y_{e-} = \frac{x}{1+x} = \mathbf{0.00967}$$

14.97

One kilomole of argon gas at room temperature is heated to 20 000 K, 100 kPa. Assume that the plasma in this condition consists of an equilibrium mixture of Ar, Ar^+, Ar^{++}, and e^- according to the simultaneous reactions

(1) $Ar \Leftrightarrow Ar^+ + e^-$ (2) $Ar^+ \Leftrightarrow Ar^{++} + e^-$

The ionization equilibrium constants for these reactions at 20 000 K have been calculated from spectroscopic data as $\ln K_1 = 3.11$ and $\ln K_2 = -4.92$. Determine the equilibrium composition of the plasma.

1) $Ar \Leftrightarrow Ar^+ + e^-$ 2) $Ar^+ \Leftrightarrow Ar^{++} + e^-$

ch. -a   +a   +a   ch. -b   +b   +b

Equil. Comp.: $n_{Ar} = 1-a$, $n_{Ar+} = a-b$, $n_{Ar++} = b$, $n_{e-} = a+b$, $n_{TOT} = 1+a+b$

$$K_1 = \frac{y_{Ar+}y_{e-}}{y_{Ar}}\left(\frac{P}{P^0}\right) = \frac{(a-b)(a+b)}{(1-a)(1+a+b)}(1) = 22.421$$

$$K_2 = \frac{y_{Ar++}y_{e-}}{y_{Ar+}}\left(\frac{P}{P^0}\right) = \frac{b(a+b)}{(a-b)(1+a+b)}(1) = 0.0073$$

By trial & error: $a = 0.978\ 57$, $b = 0.014\ 13$

$n_{Ar} = 0.02143$, $n_{Ar+} = 0.96444$, $n_{Ar++} = 0.01413$, $n_{e-} = 0.9927$

$y_{Ar} = 0.0107$, $y_{Ar+} = 0.484$, $y_{Ar++} = 0.0071$, $y_{e-} = 0.4982$

14.98

At 10 000 K the two ionization reactions for N and Ar as

      1:      $Ar \Leftrightarrow Ar^+ + e^-$ 2: $N \Leftrightarrow N^+ + e^-$

have equilibrium constants of $K_1 = 4.2 \times 10^{-4}$ and $K_2 = 6.3 \times 10^{-4}$,
respectively. If we start out with 1 kmol Ar and 0.5 kmol N_2, what is the
equilibrium composition at a pressure of 10 kPa?

At 10 000 K we assume all the nitrogen is dissociated to N.

Assume we shift the argon ionization with a and the nitrogen ionization with b we
get

	Ar	Ar^+	e^-	N	N^+	
Initial	1	0	0	1	0	
Change	-a	a	a + b	-b	b	
Final	1-a	a	a + b	1-b	b	Tot: 2 + a + b

$$K_1 = 4.2 \times 10^{-4} = \frac{y_{Ar+}y_{e-}}{y_{Ar}}\left(\frac{P}{P^0}\right) = \frac{a\,(a + b)}{(1 - a)(2 + a + b)}\left(\frac{10}{100}\right) \qquad (1)$$

$$K_2 = 6.3 \times 10^{-4} = \frac{y_{N+}y_{e-}}{y_{N}}\left(\frac{P}{P^0}\right) = \frac{b\,(a + b)}{(1 - b)(2 + a + b)}\left(\frac{10}{100}\right) \qquad (2)$$

Divide the second equation with the first to get

$$\frac{b}{(1 - b)}\frac{(1 - a)}{a} = \frac{K_2}{K_1} = 1.5 \quad \Rightarrow \quad \frac{b - ab}{a - ab} = 1.5$$

$$b - ab = 1.5\,a - 1.5\,ab \quad \Rightarrow \quad b = 1.5\,a - 0.5\,ab = a(1.5 - 0.5\,b)$$

$$a = \frac{b}{1.5 - 0.5\,b} \quad \text{trial and error on equation (1)}$$

$$a = 0.059 \quad \text{and} \quad b = 0.086$$

$n_{Ar} = 0.941, \quad n_{Ar+} = 0.059, \quad n_N = 0.914, \quad n_{N+} = 0.086, \quad n_{e-} = 0.145$

$y_{Ar} = 0.439, \quad y_{Ar+} = 0.027, \quad y_N = 0.426, \quad y_{N+} = 0.04, \quad y_{e-} = 0.068$

14.99

Plot to scale the equilibrium composition of nitrogen at 10 kPa over the temperature range 5000 K to 15 000 K, assuming that N_2, N, N^+, and e^- are present. For the ionization reaction $N \Leftrightarrow N^+ + e^-$, the ionization equilibrium constant K has been calculated from spectroscopic data as

T [K]	10000	12 000	14 000	16 000
$100K$	0.0626	1.51	15.1	92

1) $N_2 \Leftrightarrow 2N$ 2) $N \Leftrightarrow N^+ + e^-$

change -a   +2a   change -b   +b   +b

Equil. Comp.: $n_{N2} = 1 - a$, $n_N = 2a - b$, $n_{N+} = b$, $n_{e-} = b$

EQ1: $K_1 = \dfrac{y_N^2}{y_{N2}} \left(\dfrac{P}{P^0}\right) = \dfrac{(2a - b)^2}{(1 - a)(1 + a + b)} \left(\dfrac{P}{P^0}\right)$

EQ2: $K_2 = \dfrac{y_{N+} y_{e-}}{y_N} \left(\dfrac{P}{P^0}\right) = \dfrac{b^2}{(2a - b)(1 + a + b)} \left(\dfrac{P}{P^0}\right)$

For T < 10 000 K: b ~ 0 so neglect EQ2: $\Rightarrow$ $K_1 = \dfrac{4a^2}{(1-a^2)} \left(\dfrac{10}{100}\right)$

To extrapolate K_1 above 6000 K: $\ln K_1 \approx 16.845 - \dfrac{118\,260}{T}$

(from values at 5000 K & 6000 K)

T(K)	K_1	a	y_N	y_{N2}
5000	0.0011	0.0524	0.0996	0.9004
6000	0.0570	0.3532	0.5220	0.4780
7000	0.9519	0.8391	0.9125	0.0875
8000	7.866	0.9755	0.9876	0.0124
10000	151.26	0.9987	0.9993	0.0007

For T > 10 000 K: $a \approx 1.0$ => $K_2 = \dfrac{b^2}{(2-b)(2+b)} \left(\dfrac{10}{100}\right) = \dfrac{b^2}{(4-b^2)} 0.1$

T(K)	K_2	b	y_N	y_{N+}
10 000	6.26×10^{-4}	0.1577	0.8538	0.0731
12 000	1.51×10^{-2}	0.7244	0.4862	0.2659
14 000	0.151	1.5512	0.1264	0.4368
16 000	0.92	1.8994	0.0258	0.4871

Note that $b \approx 0$ is not a very good approximation in the vicinity of 10 000 K. In this region, it would be better to solve the original set simultaneously for a & b. The answer would be approximately the same.

Applications

14.100

Are the three reactions in the Zeldovich mechanism pressure sensitive if we look at equilibrium conditions?

No. All three reactions have two moles on the left and right hand sides and the net power to the pressure correction term is zero.

14.101

Assume air is at 3000 K, 1 MPa. Find the time constant for the NO formation. Repeat for 2000 K, 800 kPa.

From the rate in Eq.14.40: $\tau_{NO} = 8 \times 10^{-16}\ T\ (P_o/P)^{1/2}\ \exp(\dfrac{58\ 300}{T})$

Case a: $\tau_{NO} = 8 \times 10^{-16} \times 3000\ (100/1000)^{1/2}\ \exp(\dfrac{58\ 300}{3000})$

$= \mathbf{2.09 \times 10^{-4}\ s} = 0.209\ ms$

Case b: $\tau_{NO} = 8 \times 10^{-16} \times 2000\ (100/800)^{1/2}\ \exp(\dfrac{58\ 300}{2000})$

$= \mathbf{2.58\ s}$

Notice the significant difference in time constants.

14.102

Consider air at 2600 K, 1 MPa. Find the equilibrium concentration of NO neglecting dissociations of oxygen and nitrogen.

1 kmol air (0.78 N_2, 0.21 O_2, 0.01 Ar) at 2600 K, 1 MPa.

N_2	+	O_2 $\Leftrightarrow$	2 NO	Ar		$n_{N2} = 0.78 - x$
0.78		0.21	0	0.01		$n_{O2} = 0.21 - x$
-x		-x	+2x			$n_{NO} = 2x$, $\quad n_{ar} = 0.01$
0.78–x		0.21–x	2x	0.01		$n_{tot} = 1.0$

From A.11 at 2600 K, ln K = -5.316, K = 0.0049124

$$K = \frac{4x^2}{(0.78 - x)(0.21 - x)} \left(\frac{P}{P^0}\right)^0 \qquad \Rightarrow$$

$$\frac{x^2}{(0.78 - x)(0.21 - x)} = \frac{0.0049124}{4} = 0.001228 \quad \text{and} \quad 0 < x < 0.21$$

Solve for x: x = 0.0136 $\Rightarrow$ $y_{NO} = \frac{2x}{1.0} = \mathbf{0.0272}$

Borgnakke and Sonntag

14.103

Redo the previous Problem but include the dissociation of oxygen and nitrogen.

1 kmol air (0.78 N_2, 0.21 O_2, 0.01 Ar) at 2600 K, 1 MPa.

(1) $O_2 \Longleftrightarrow 2\,O$ $\ln K = -7.52$, $K_1 = 0.000542 = (y_O^2/y_{O2})\, P/P_o$

(2) $N_2 \Longleftrightarrow 2\,N$ $\ln K = -28.313$, $K_2 = 5.056 \times 10^{-13} = (y_N^2/y_{N2})\, P/P_o$

(3) $N_2 + O_2 \Longleftrightarrow 2\,NO$ $\ln K = -5.316$, $K_3 = 0.00491 = (y_{NO}^2/y_{N2}\, y_{O2})$

Call the shifts a,b,c respectively so we get

$$n_{O2} = 0.21 - a - c, \quad n_O = 2a, \quad n_{N2} = 0.78 - b - c, \quad n_N = 2b,$$
$$n_{NO} = 2c, \quad n_{Ar} = 0.01, \quad n_{tot} = 1 + a + b$$

From which the mole fractions are formed and substituted into the three equilibrium equations. The result is

$$0.000542 \times 0.1 = y_O^2/y_{O2} = 4a^2 / [(1+a+b)(0.21-a-c)]$$
$$5.056 \times 10^{-14} = y_N^2/y_{N2} = 4b^2 / [(1+a+b)(0.79-b-c)]$$
$$0.00491 = y_{NO}^2/y_{N2}\, y_{O2} = 4c^2 / [(0.79-b-c)(0.21-a-c)]$$

which give 3 eqs. for the unknowns (a,b,c). Trial and error assume b = a = 0 solve for c from the last eq. then for a from the first and finally given the (a,c) solve for b from the second equation. The order chosen according to expected magnitude $K_3 > K_1 > K_2$

$$a = 0.001626, \quad b = 0.99 \times 10^{-7}, \quad c = 0.01355 \ \Rightarrow$$

$$n_{O2} = 0.1948, \quad n_O = 0.00325, \quad n_{N2} = 0.7665, \quad n_N = 1.98 \times 10^{-7},$$
$$n_{NO} = 0.0271, \quad n_{Ar} = 0.01$$
$$y_{NO} = n_{NO} / n_{tot} = 0.0271 / 1.00165 = \mathbf{0.02706}$$

Indeed it is a very small effect to include the additional dissociations.

14.104

Calculate the equilibrium constant for the first reaction in the Zeldovich mechanism at 2600 K, 500 kPa. Notice this is not listed in A.11.

The reaction and equilibrium constant are

$$O + N_2 \Leftrightarrow NO + N$$

$$\ln K = -\frac{\Delta G^0}{\bar{R}T} \, ; \qquad \Delta G^0 = \Delta H^0 - T\Delta S^0$$

$$\Delta H^0 = \bar{h}_{NO} + \bar{h}_N - \bar{h}_O - \bar{h}_{N2} = (80034 + 90291) + (47860 + 472680)$$
$$- \quad (48216 + 249170) - 77963 = 315\ 516 \text{ kJ/kmol}$$

$$\Delta S^0 = \bar{s}_{NO} + \bar{s}_N - \bar{s}_O - \bar{s}_{N2}$$
$$= 282.822 + 198.322 - 206.714 - 261.615 = 12.815 \text{ kJ/kmol-K}$$

$$\Delta G^0 = 315\ 516 - 2600 \times 12.815 = 282\ 197 \text{ kJ/kmol}$$

$$\ln K = -\frac{\Delta G^0}{\bar{R}T} = -\frac{282\ 197}{8.314472 \times 2600} = -13.054, \qquad K = \mathbf{2.14 \times 10^{-6}}$$

14.105

Find the equilibrium constant for the reaction: $2NO + O_2 \Leftrightarrow 2NO_2$ from the elementary reaction in Table A.11 to answer these two questions. Which of the nitrogen oxides NO or NO_2 is the more stable at 25^oC, 100 kPa? At what T do we have an equal amount of each?

The elementary reactions are the last two ones in Table A.11:

$\qquad$ a) $N_2 + O_2 \Leftrightarrow 2NO$ $\qquad\qquad$ b) $N_2 + 2O_2 \Leftrightarrow 2NO_2$

Actual reaction is : $\quad c = b - a \quad \Rightarrow \quad \ln(K_c) = \ln(K_b) - \ln(K_a)$

At 25^oC (approx. 300 K) Table A.11: $\ln(K_a) = -69.868;$ $\qquad \ln(K_b) = -41.355$

so now:

$$\ln(K_c) = -41.355 + 69.868 = 28.5 \quad \Rightarrow \quad K_c = 2.4 \times 10^{12}$$

meaning reaction is pushed completely to the right and **NO_2 is the stable** compound. Assume we start at room T with 1 kmol NO_2: then

	NO	O_2	NO_2	TOT
start	0	0	1	
change	2x	x	-2x	
Final	2x	x	1-2x	1+x

Equal amount of each

$$y(NO) = \frac{2x}{1 + x} = y(NO_2) = \frac{1 - 2x}{1 + x} \quad \Rightarrow \quad x = 0.25$$

$$K(T) = \frac{(1 - 2x)^2}{4x^3} = \frac{0.5^2}{4 \times 0.25^3} = 4 \quad \Rightarrow \quad \ln(K) = 1.386$$

We quickly see

$\qquad\qquad \ln(K)$ at 500 K $= -30.725 + 40.449 = 9.724$

$\qquad\qquad \ln(K)$ at 1000 K $= -23.039 + 18.709 = -4.33$

$\qquad$ Linear interpolation $\quad T = 500 + 0.406 \times 500 = \textbf{703 K}$

14.106

If air at 300 K is brought to 2600 K, 1 MPa, instantly, find the formation rate of NO.

The formation rate of NO is given by Eqs.14.39-40 using the equilibrium concentration in Eq.14.41. If we assume NO formation takes place and that the oxygen and nitrogen have not dissociated their concentrations are (we neglect the argon)

$$N_2 + O_2 \Leftrightarrow 2\ NO$$

$$y_{NOe} = 2x, \qquad y_{O2} = 0.21 - x \quad \text{and} \quad y_{N2} = 0.79 - x$$

At 2600 K from A.11

$$\ln K_4 = -5.316 \qquad \Rightarrow \qquad K_4 = 0.0049124$$

so then we have

$$y_{NOe} = 2x = [K_4\ (0.21 - x)\ (0.79 - x)\ ]^{1/2} \quad \text{solve for } x = 0.0136$$
$$y_{NOe} = 2x = 0.0272$$

From the rate in Eq.14.40: $\tau_{NO} = 8 \times 10^{-16}\ T\ (P_o/P)^{1/2} \exp(\frac{58\ 300}{T})$

$$\tau_{NO} = 8 \times 10^{-16} \times 2600\ (100/1000)^{1/2} \exp(\frac{58\ 300}{2600}) = 0.0036\ s$$

The formation rate is

$$\frac{dy_{NO}}{dt} = \frac{y_{NOe}}{\tau_{NO}} = \frac{0.0272}{0.0036\ s} = \textbf{7.56 s}^{\textbf{-1}}$$

Borgnakke and Sonntag

14.107

Estimate the concentration of oxygen atoms in air at 3000 K, 100 kPa and 0.0001 kPa, Compare this to the result in Fig. 14.11.

For the dissociation of oxygen and nitrogen we get from A.11
$$O_2: \quad \ln K = -4.356, \qquad N_2: \quad \ln K = -22.367$$
from which it is evident that we need only to consider oxygen.

O_2	$\Leftrightarrow$	$2\,O$	N_2	Ar	$n_{N2} = 0.78 - x$
0.21		0	0.78	0.01	$n_{O2} = 0.21 - x$
-x		+2x			$n_{NO} = 2x, \quad n_{ar} = 0.01$
0.21–x		2x	0.78	0.01	$n_{tot} = 1 + x$

$$K(T) = \frac{y_O^2}{y_{O2}}\left(\frac{P}{P_o}\right)^{2-1} = \left(\frac{2x}{1+x}\right)^2 \frac{1+x}{0.21-x}\frac{P}{P_o} = \frac{4x^2}{1+x}\frac{1}{0.21-x}\frac{P}{P_o}$$

$$\Rightarrow \quad \frac{4x^2}{1+x}\frac{1}{0.21-x} = \frac{P_o}{P}K(T) = \frac{P_o}{P}\exp(-4.356)$$

P = 100 kPa: RHS = 0.01283 $\Rightarrow$ x = 0.0247, $y_O = \dfrac{2x}{1+x} = 0.0482$

P = 0.0001 kPa: RHS = 12830 $\Rightarrow$ x = 0.21, $y_O = \dfrac{2x}{1+x} = 0.347$

We notice from Fig. 14.11 that at 3000 K all the oxygen is dissociated so
$$n_O / n_{atoms\ in\ air} = 0.21 \quad so \quad n_O = 0.42, \quad y_O = 0.42/1.21 = 0.347$$
since air is mainly diatomic so $n_{atoms\ in\ air} \approx 2$ (scale is confusing).

14.108

At what temperature range does air become a plasma?

From Fig. 14.11, we note that air becomes predominantly ions and electrons, a plasma, at about 10-12 000 K.

Review Problems

14.109

In a test of a gas-turbine combustor, saturated-liquid methane at 115 K is to be burned with excess air to hold the adiabatic flame temperature to 1600 K. It is assumed that the products consist of a mixture of CO_2, H_2O, N_2, O_2, and NO in chemical equilibrium. Determine the percent excess air used in the combustion, and the percentage of NO in the products.

$$CH_4 + 2x\, O_2 + 7.52x\, N_2 \quad \rightarrow \quad 1\, CO_2 + 2\, H_2O + (2x-2)\, O_2 + 7.52x\, N_2$$

Then	N_2	$+$ O_2	$\Leftrightarrow$	$2\,NO$	Also	CO_2	H_2O
initial	7.52x	2x-2		0		1	2
change	-a	-a		+2a		0	0
final	(7.52x-a)	(2x-2-a)		2a		1	2

$$n_{TOT} = 1 + 9.52x$$

1600 K in A.11: $\quad \ln K = -10.55, \quad K = 2.628\times10^{-5}$

$$2.628\times10^{-5}\, K = \frac{y_{NO}^2}{y_{N2}y_{O2}}\left(\frac{P}{P^0}\right)^0 = \frac{y_{NO}^2}{y_{N2}y_{O2}} = \frac{4a^2}{(7.52x - a)(2x - 2 - a)}$$

From A.9 and B.7,

$$H_R = 1\left[-74\,873 + 16.043(-274.7-624.1)\right] + 0 + 0 = -89\,292\ kJ$$

(Air assumed 25 ^{o}C)

$$H_P = 1(-393\,522 + 67\,569) + 2(-241\,826 + 52\,907)$$
$$+ (7.52x-a)(41\,904) + (2x-2-a)(44\,267) + 2a(90\,291 + 43\,319)$$
$$= -792\,325 + 403\,652\, x + 181\,049\, a$$

Assume a ~ 0, then from $\quad H_P - H_R = 0 \quad \rightarrow \quad x = 1.7417 \quad$ and substitute

$$\frac{a^2}{(13.098 - a)(1.483 - a)} = \frac{2.628\times10^{-5}}{4}, \qquad get \quad a \approx 0.0113$$

Use this a in the energy equation

$$x = \frac{703\,042 - 181\,049\times0.0113}{403\,652} = 1.7366$$

$$\Rightarrow \frac{a^2}{(13.059-a)(1.4732-a)} = \frac{2.628\times10^{-5}}{4}, a = 0.0112 \quad \Rightarrow \quad x = 1.7366$$

$$\% \text{ excess air} = \textbf{73.7 \%}$$

$$\% \text{ NO} = \frac{2\times0.0112\times100}{1+9.52\times1.7366} = \textbf{0.128 \%}$$

14.110

Find the equilibrium constant for the reaction in Problem 14.92.

Reaction: $2\,CO + 2\,H_2 \Leftrightarrow CH_4 + CO_2$

Equilibrium constant Eq. 14.15: $\ln K = -\Delta G^0 / \bar{R}T$

For CH_4 at 600 K (formula in Table A.6), $\bar{C}_{P0} = 52.22$

At 900 K

$\bar{h}^0_{CH4} = -74\,873 + 52.22(900 - 298.2) = $ -43 446 kJ/kmol

$\bar{s}^0_{CH4} = 186.251 + 52.22 \ln(900 / 298.2) = 243.936$ kJ/kmol K

(The integrated-equation values are -43 656 and 240.259)

$\Delta H^0_{900\,K} = 1(-43\,446) + 1(-393\,522 + 28\,030)$

$\qquad - 2(-110\,527 + 18\,397) - 2(0 + 17\,657)\ = $ -259 993 kJ

$\Delta S^0_{900\,K} = 1(243.936) + 1(263.646)$

$\qquad - 2(231.074) - 2(163.060) = -280.687$ kJ/K

$\Delta G^0_{900\,K} = -259\,993 - 900(-280.687) = -7375$ kJ

$\ln K = \dfrac{+7375}{8.3145 \times 900} = 0.9856, \qquad K = \mathbf{2.679}$

Borgnakke and Sonntag

14.111

A space heating unit in Alaska uses propane combustion is the heat supply. Liquid propane comes from an outside tank at -44°C and the air supply is also taken in from the outside at -44°C. The airflow regulator is misadjusted, such that only 90% of the theoretical air enters the combustion chamber resulting in incomplete combustion. The products exit at 1000 K as a chemical equilibrium gas mixture including only CO_2, CO, H_2O, H_2, and N_2. Find the composition of the products. Hint: use the water gas reaction in Example 14.4.

Propane: Liquid, T_1 = -44°C = 229.2 K

Air: T_2 = -44°C = 229.2 K, 90% Theoretical Air

Products: T_3 = 1000 K, CO_2, CO, H_2O, H_2, N_2

Theoretical Air: $C_3H_8 + 5O_2 + 18.8N_2 \implies 3CO_2 + 4H_2O + 18.8N_2$

90% Theoretical Air:

$C_3H_8 + 4.5O_2 + 16.92N_2 \implies aCO_2 + bCO + cH_2O + dH_2 + 16.92N_2$

Carbon: $a + b = 3$;   Hydrogen: $c + d = 4$

Oxygen: $2a + b + c = 9$ Where: $2 \le a \le 3$

Reaction: $CO + H_2O \leftrightarrow CO_2 + H_2$

Initial:	b	c	a	d
Change:	-x	-x	x	x
Equil:	b - x	c - x	a + x	d + x

Chose an Initial guess such as: a = 2, b = 1, c = 4, d = 0

Note: A different initial choice of constants will produce a different value for x, but will result in the same number of moles for each product.

$n_{CO2} = 2 + x$, $n_{CO} = 1 - x$, $n_{H2O} = 4 - x$, $n_{H2} = x$, $n_{N2} = 16.92$

The reaction can be broken down into two known reactions to find K

(1) $2CO_2 \leftrightarrow 2CO + O_2$ @ 1000 K $\ln(K_1)$ = -47.052

(2) $2H_2O \leftrightarrow 2H_2 + O_2$ @ 1000 K $\ln(K_2)$ = -46.321

For the overall reaction: $\ln K = (\ln(K_2) - \ln(K_1))/2 = 0.3655$; K = 1.4412

$$K = \frac{y_{CO2}y_{H2}}{y_{CO}y_{H2O}}\left(\frac{P}{P_o}\right)^{1+1-1-1} = \frac{y_{CO2}y_{H2}}{y_{CO}y_{H2O}} = 1.4412 = \frac{(2+x)x}{(1-4)(4-x)}$$

$\implies$ x = 0.6462

n_{CO2} = 2.6462   **n_{CO} = 0.3538**   **n_{N2} = 16.92**

n_{H2O} = 3.3538   **n_{H2} = 0.6462**

y_{CO2} = 0.111, y_{CO} = 0.015, y_{N2} = 0.707, y_{H2O} = 0.140, y_{H2} = 0.027

14.112

Derive the van't Hoff equation given in problem 14.56, using Eqs.14.12 and 14.15. Note: the $d(\bar{g}/T)$ at constant P for each component can be expressed using the relations in Eqs. 12.18 and 12.19.

Eq. 14.12: $\Delta G^0 = \nu_C\,\bar{g}_C^0 + \nu_D\,\bar{g}_D^0 - \nu_A\,\bar{g}_A^0 - \nu_B\,\bar{g}_B^0$

Eq. 14.15: $\ln K = -\Delta G^0/\,\bar{R}T$ 　　　　Eq. 14.19: 　　$\Delta G^0 = \Delta H^\circ - T\,\Delta S^0$

$$\frac{d\,\ln K}{dT} = -\frac{d}{dT}\left(\frac{\Delta G^0}{\bar{R}T}\right) = -\frac{1}{\bar{R}T}\frac{d\Delta G^0}{dT} + \frac{\Delta G^0}{\bar{R}T^2} = \frac{1}{\bar{R}T^2}\left[\,\Delta G^0 - T\frac{d\Delta G^0}{dT}\,\right]$$

$$= \frac{1}{\bar{R}T^2}[\Delta G^0 + T\,\Delta S^0] \qquad \text{used Eq.12.19} \qquad \frac{d\bar{g}}{dT} = -\,\bar{s}$$

$$= \frac{1}{\bar{R}T^2}\Delta H^0$$

Borgnakke and Sonntag

14.113

Find the equilibrium constant for reaction in Eq. 14.33 at 600 K (see Problem 14.88).

Reaction: $CO + 3\,H_2 \Leftrightarrow CH_4 + H_2O$

Equilibrium constant Eq. 14.15: $\ln K = -\Delta G^0/\bar{R}T$

For CH_4 at 600 K (formula in Table A.6), $\bar{C}_{P0} = 52.22$, A.5: $\bar{C}_{P0} = 36.16$

So use the average for the interval 298 – 600 K, $\bar{C}_{P0} = 44.19$ kJ/kmol-K

At 600 K

$\bar{h}^0_{CH4} = -74\,873 + 44.19(600 - 298.15) = -61\,534$ kJ/kmol

$\bar{s}^0_{CH4} = 186.251 + 44.19 \ln (600 / 298.15) = 217.154$ kJ/kmol K

$\Delta H^0_{600\ K} = 1(-61\,534) + 1(-241\,826 + 10\,499)$

$- (-110\,527 + 8\,942) - 3(0 + 8\,799) = -217\,673$ kJ

$\Delta S^0_{600\ K} = 1(217.154) + 1(213.051) - (218.321) - 3(151.078)$

$= -241.35$ kJ/K

$\Delta G^0_{600\ K} = -217\,673 - 600(-241.35) = -72\,863$ kJ

$$\ln K = -\frac{-72\,863}{8.3145 \times 600} = 14.6, \qquad K = \mathbf{2.2 \times 10^6}$$

Comment: If $\bar{C}_{P0}$ at 298 K for CH_4 was used then $K = 1.83 \times 10^6$

14.114

Combustion of stoichiometric benzene, C_6H_6, and air at 80 kPa with a slight heat loss gives a flame temperature of 2400 K. Consider the dissociation of CO_2 to CO and O_2 as the only equilibrium process possible. Find the fraction of the CO_2 that is dissociated.

Combustion: $C_5H_{12} + 7.5\ O_2 + 7.5 \times 3.76\ N_2 \rightarrow 6\ CO_2 + 3\ H_2O + 28.2\ N_2$

	$2\ CO_2 \Leftrightarrow 2\ CO + 1\ O_2$			also H_2O	N_2
Initial	6	0	0	3	28.2
Change	-2z	+2z	+z	0	0
Equil.	6-2z	2z	z	3	28.2

At 2400 K, $\ln K = -7.715$, $K = 4.461 \times 10^{-4}$ and we have $n_{tot} = 37.2 + z$

Substitute into equilibrium equation,

$$K = \frac{4z^2\ z}{(6 - 2z)^2(37.2 + z)}\left(\frac{80}{100}\right)^1 \Rightarrow \frac{z^3}{(3 - z)^2(37.2 + z)} = = 0.0005576$$

Solve with limit $0 < z < 3$ gives $z = 0.507$ so then $n_{CO_2} = 6 - 2z = 4.986$

Fraction dissociated $= (6 - 4.986)/6 = $ **0.169**

14.115

One kilomole of liquid oxygen, O_2, at 93 K, and x kmol of gaseous hydrogen, H_2, at 25°C, are fed to a combustion chamber. x is greater than 2, such that there is excess hydrogen for the combustion process. There is a heat loss from the chamber of 1000 kJ per kmol of reactants. Products exit the chamber at chemical equilibrium at 3800 K, 400 kPa, and are assumed to include only H_2O, H_2, and O.

a. Determine the equilibrium composition of the products and also x, the amount of H_2 entering the combustion chamber.

b. Should another substance(s) have been included in part (a) as being present in the products? Justify your answer.

$$x\ H_2 + 1\ O_2 \rightarrow 2\ H_2O + (x - 2)\ H_2$$

$$(1)\ \ 1\ H_2O \Leftrightarrow 1\ H_2 + 1\ O$$

shift   -a   +a   +a   and $a > 0$

Equil 2-a   x-2+a   a $a < 2$ and $n_{tot} = x + a$

$$(2)\ 2\ H_2O \Leftrightarrow 2\ H_2 + 1\ O_2 \quad \ln K_2 = -1.906$$

$$(3)\ \ \ 1\ O_2 \Leftrightarrow 2\ O \qquad\qquad \ln K_3 = -0.017$$

$\ln K_1 = 0.5(\ln K_2 + \ln K_3) = -0.9615 \quad => \quad K_1 = 0.3823$

Equil.: $\dfrac{K_1}{(P/P_o)^1} = \dfrac{(x-2+a)a}{(2-a)(x+a)} = \dfrac{0.3823}{4} = 0.95575$

Energy Eq.: $Q + H_R = H_P$, $Q = (1 + x)(-1000)$ kJ

Table A.9: $\Delta\bar{h}_{IG}^{*} = -5980$ kJ/kmol [or $= 0.922 \times 32(93-298.2) = -6054$ kJ/kmol]

Fig. D.2: $T_r = 93/154.6 = 0.601$, $\Delta\bar{h}_f = -5.16 \times \bar{R} \times 154.6 = -6633$

$H_R = x(0 + 0) + 1(0 + \Delta\bar{h}_{IG}^{*} + \Delta\bar{h}_f) = 1(-5980 - 6633) = -12613$ kJ

$H_P = (2-a)(-241\ 826 + 171\ 981) + (x-2+a)(0 + 119077)$

$\qquad + a(249170 + 73424) \ \ = 119077\ x + 511516\ a - 377844$

$\qquad = Q + H_R = -1000 - 1000\ x - 12613$

Rearrange eq. to: $x + 4.2599\ a = 3.03331$

Substitute it into the equilibrium eq.: $\dfrac{(1.03331 + 5.2599\ a)\ a}{(2-a)(3.03331-3.2599\ a)} = 0.095575$

Solve $a = 0.198$, LHS $= 0.09547$,   **$x = 2.1898$**

$y_{H2O} = \dfrac{2-a}{x+a} = 0.755$, $y_{H2} = \dfrac{x-2+a}{x+a} = 0.162$, $y_O = \dfrac{a}{x+a} = 0.083$

Other substances and reactions: $2\ H_2O <=> H_2 + 2\ OH$, $\ln K = -0.984$,

$\qquad H_2 <=> 2\ H$, : $\ln K = 0.201$, $O_2 <=> 2\ O$, : $\ln K = -0.017$

All are significant as K's are of order 1.

Borgnakke and Sonntag

14.116

Dry air is heated from 25°C to 4000 K in a 100-kPa constant-pressure process. List the possible reactions that may take place and determine the equilibrium composition. Find the required heat transfer.

Air assumed to be 21% oxygen and 79% nitrogen by volume.

From the elementary reactions we have at 4000 K (A.11)

$$(1)\ \ O_2 <=> 2\ O \qquad K_1 = 2.221 = y_O^2/y_{O2}$$

$$(2)\ \ N_2 <=> 2\ N \qquad K_2 = 3.141 \times 10^{-6} = y_N^2/y_{N2}$$

$$(3)\ \ N_2 + O_2 <=> 2\ NO \quad K_3 = 0.08955 = y_{NO}^2/y_{N2}\ y_{O2}$$

Call the shifts a,b,c respectively so we get

$$n_{O2} = 0.21 - a - c, \ \ n_O = 2a, \ \ n_{N2} = 0.79 - b - c, \ \ n_N = 2b,$$

$$n_{NO} = 2c, \ \ n_{tot} = 1 + a + b$$

From which the mole fractions are formed and substituted into the three equilibrium equations. The result is

$$K_1 = 2.221 = y_O^2/y_{O2} = 4a^2 / [(1+a+b)(0.21-a-c)]$$

$$K_2 = 3.141 \times 10^{-6} = y_N^2/y_{N2} = 4b^2 / [(1+a+b)(0.79-b-c)]$$

$$K_3 = 0.08955 = y_{NO}^2/y_{N2}\ y_{O2} = 4c^2 / [(0.79-b-c)(0.21-a-c)]$$

which give 3 eqs. for the unknowns (a,b,c). Trial and error assume $b = c = 0$ solve for a from K_1 then for c from K_3 and finally given the (a,c) solve for b from K_2. The order chosen according to expected magnitude $K_1 > K_3 > K_2$

$$a = 0.15, \ \ b = 0.000832, \ \ c = 0.0244 \ \ =>$$

$$n_{O2} = 0.0356, \ \ n_O = 0.3, \ \ n_{N2} = 0.765, \ \ n_N = 0.00167, \ \ n_{NO} = 0.049$$

$$Q = H_{ex} - H_{in} = n_{O2}\Delta\bar{h}_{O2} + n_{N2}\Delta\bar{h}_{N2} + n_O(\bar{h}_{fO} + \Delta\bar{h}_O)$$

$$+ n_N(\bar{h}_{fN} + \Delta\bar{h}_N) + n_{NO}(\bar{h}_{fNO} + \Delta\bar{h}_{NO}) \ - 0$$

$$= 0.0356 \times 138705 + 0.765 \times 130027 + 0.3(249170 + 77675)$$

$$+ 0.00167(472680 + 77532) + 0.049(90291 + 132671)$$

$$= \textbf{214 306 kJ/kmol air}$$

[If no reaction $\ \ Q = n_{O2}\Delta\bar{h}_{O2} + n_{N2}\Delta\bar{h}_{N2} = 131\ 849$ kJ/kmol air]

14.117

Saturated liquid butane (note: use generalized charts) enters an insulated constant pressure combustion chamber at 25°C, and *x* times theoretical oxygen gas enters at the same pressure and temperature. The combustion products exit at 3400 K. Assuming that the products are a chemical equilibrium gas mixture that includes CO, what is *x*?

Butane: $T_1 = 25^O C$, sat. liq., $x_1 = 0$, $T_c = 425.2$ K, $P_c = 3.8$ MPa

   $T_{r1} = 0.7$, Figs. D.1 and D.2, $P_{r1} = 0.10$, $P_1 = P_{r1}P_c = 380$ kPa

   Fig D.2: $\left(\overline{h}_1^* - \overline{h}_1\right)_f = 4.85\ RT_c$

Oxygen: $T_2 = 25^O C$, X * theoretical air   Products: $T_3 = 3400$ K

$$C_4H_{10} + 6.5X\ O_2 \implies 4\ CO_2 + 5\ H_2O + 6.5(X\text{-}1)\ O_2$$

$$2CO_2 \Longleftrightarrow 2CO + O_2$$

Initial	4	0	6.5(X-1)
Change	-2a	2a	a
Equil.	4-2a	2a	6.5(X-1) + a $n_{tot} = 2.5 + a + 6.5X$

$$n_{CO_2} = 4 - 2a, \quad n_{CO} = 2a, \quad n_{O_2} = 6.5(X\text{-}1) + a, \quad n_{H2O} = 5,$$

$$y_{CO} = \frac{2a}{2.5 + a + 6.5X}, \quad y_{CO2} = \frac{4 - 2a}{2.5 + a + 6.5X}, \quad y_{O2} = \frac{6.5(x-1) + a}{2.5 + a + 6.5X}$$

The equilibrium constant is

$$K = \frac{y_{CO}^2 y_{O_2}}{y_{CO_2}^2}\left(\frac{P_1}{P_o}\right)^{2+1-2} = \left(\frac{a}{2\text{-}a}\right)^2\left(\frac{6.5X - 6.5 + a}{6.5X - 2.5 + a}\right)\left(\frac{P_1}{P_o}\right)$$

@ $T_3 = 3400$ K Table A.11, $\ln(K) = 0.346$, $K = 1.4134$

$$1.4134 = \left(\frac{a}{2\text{-}a}\right)^2\left(\frac{6.5X - 6.5 + a}{6.5X - 2.5 + a}\right)(3.76) \qquad \text{Equation 1.}$$

Need a second equation:

Energy eq.: $Q_{cv} + H_R = H_P + W_{cv}$; $Q_{cv} = 0$, $W_{cv} = 0$

$$H_R = \left(\overline{h}_f^o + \Delta\overline{h}\right)_{C_4H_{10}} = (\text{-}126\ 200 - 17\ 146) = \text{-}143\ 346\ \text{kJ}$$

Products @ 3400 K:

$$H_P = n(\overline{h}_f^o + \Delta\overline{h})_{CO2} + n(\overline{h}_f^o + \Delta\overline{h})_{CO} + n(\overline{h}_f^o + \Delta\overline{h})_{O2} + n(\overline{h}_f^o + \Delta\overline{h})_{H2O}$$

$$= (4 - 2a)(-393\ 522 + 177\ 836) + 2a(-110\ 527 + 108\ 440)$$

$$+ [6.5(X - 1) + a](0 + 114101) + 5(-241\ 826 + 149\ 073)$$

$$= -463\ 765 \text{ kJ/kmol}$$

$H_P = H_R$ => $1924820 = 541299a + 741656.5\ X$ Equation 2.

Two equations and two unknowns, solve for X and a.

$$a \cong \mathbf{0.87}, \quad X \cong \mathbf{1.96}$$

ENGLISH UNIT PROBLEMS

Borgnakke Sonntag

Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 14
English Units

8e

UPDATED August 2013

CHAPTER 14

CONTENT CHAPTER 14

Borgnakke and Sonntag

Equilibrium

14.118E

Carbon dioxide at 2200 lbf/in.^{2} is injected into the top of a 3-mi deep well in connection with an enhanced oil recovery process. The fluid column standing in the well is at a uniform temperature of 100 F. What is the pressure at the bottom of the well assuming ideal gas behavior?

Z_1 —

CO_2

Z_2 —

cb

$(Z_1 - Z_2) = 3$ miles $= 15\ 840$ ft

$P_1 = 2200$ lbf/in^2, $\ T = 100\ F = $ const

Equilibrium and ideal gas beahvior

$-w_{REV} = 0 = \Delta g + \Delta PE$

$\qquad = RT \ln (P_2/P_1) + g(Z_2 - Z_1) = 0$

$$\ln \left(\frac{P_2}{P_1}\right) = -\frac{g(Z_2 - Z_1)}{RT}$$

$$= \frac{32.2 \text{ ft/s}^2 \times 15\ 840 \text{ ft}}{32.174 \text{ lbm-ft/s}^2/\text{lbf} \times 35.1 \text{ ft-lbf/lbm-R} \times 559.7 \text{ R}}$$

$$= 0.8063$$

$P_2 = 2200 \text{ lbf/in}^2 \exp(0.8063) = 4927 \text{ lbf/in}^2$

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Chemical equilibrium, Equilibrium Constant

14.119E

Calculate the equilibrium constant for the reaction $O_2 \iff 2O$ at temperatures of 537 R and 10 000 R.

Find the change in Gibbs function at the two T's from Table F.6:

537 R: $\quad \Delta H^0 = 2\bar{h}_O^0 - 1\bar{h}_{O2}^0 = 2 \times 107\ 124 - 0 = 214\ 248$ Btu/lbmol;

$\quad\quad\quad \Delta S^0 = 2\bar{s}_O^0 - 1\bar{s}_{O2}^0 = 2 \times 38.442 - 48.973 = 27.911$ Btu/lbmol R

$\quad\quad\quad \Delta G^0 = \Delta H^0 - T\Delta S^0 = 214\ 248 - 537 \times 27.911 = 199\ 260$ Btu/lbmol

$\quad\quad\quad \ln K = -\dfrac{\Delta G^0}{\bar{R}T} = -\dfrac{+199\ 260}{1.98589 \times 537} = \mathbf{-185.85}$

10 000 R: $\quad \Delta H^0 = 2\bar{h}_O^0 - 1\bar{h}_{O2}^0 = 2\ (107\ 124 + 47\ 897) - 87\ 997$

$\quad\quad\quad\quad\quad = 222\ 045$ Btu/lbmol;

$\quad\quad \Delta S^0 = 2\bar{s}_O^0 - 1\bar{s}_{O2}^0 = 2 \times 53.210 - 74.034 = 32.386$ Btu/lbmol R

$\quad\quad \Delta G^0 = \Delta H^0 - T\Delta S^0 = 222\ 045 - 10\ 000 \times 32.386 = -101\ 815$ Btu/lbmol

$\quad\quad\quad \ln K = -\dfrac{\Delta G^0}{\bar{R}T} = \dfrac{101\ 815}{1.98589 \times 10\ 000} = \mathbf{+5.127}$

14.120E

Consider the dissociation of oxygen, $O_2 \Leftrightarrow 2O$, starting with 1 lbmol oxygen at 77 F and heating it at constant pressure 1 atm. At which temperature will we reach a concentration of monatomic oxygen of 10%?

Look at initially 1 lbmol Oxygen and shift reaction with x

$$O_2 \Leftrightarrow 2\,O$$

Species	O_2	O
Initial	1	0
Change	-x	2x
Equil.	1-x	2x

$n_{tot} = 1 - x + 2x = 1 + x$

$$y_O = \frac{2x}{1 + x} = 0.1 \Rightarrow x = 0.1/(2 - 0.1) = 0.0526, \quad y_{O2} = 0.9$$

$$K = \frac{y_O^2}{y_{0_2}} \left(\frac{P}{P_o}\right)^{2\text{-}1} = \frac{0.1^2}{0.9}\,1.01325 = 0.011258 \Rightarrow \ln K = -4.487$$

Recall $P_o = 100$ kPa for Table A.11, $P/P_o = 101.325 / 100 = 1.01325$

Now look in Table A.11: T = **2982 K = 5368 R**

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14.121E

Redo problem 14.120E for a total pressure of 0.4 atm.

Look at initially 1 lbmol Oxygen and shift reaction with x

$$O_2 \Leftrightarrow 2\,O$$

Species	O_2	O
Initial	1	0
Change	-x	2x
Equil.	1-x	2x

$n_{tot} = 1 - x + 2x = 1 + x$

$$y_O = \frac{2x}{1 + x} = 0.1 \;\;\Rightarrow\;\; x = 0.1/(2 - 0.1) = 0.0526, \;\; y_{O2} = 0.9$$

$$K = \frac{y_O^2}{y_{0_2}} \left(\frac{P}{P_o}\right)^{2\text{-}1} = \frac{0.1^2}{0.9}\,(1.01325 \times 0.4) = 0.004503 \;\;\Rightarrow\;\; \ln K = -5.403$$

Recall $P_o = 100$ kPa for Table A.11, $P/P_o = 101.325 / 100 = 1.01325$

Now look in Table A.11:   T = **2857.6 K = 5143 R**

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14.122E

Redo problem 14.120E, but start with 1 lbmol oxygen and 1 lbmol helium at 77 F, 1 atm.

Look at initially 1 mol Oxygen and shift reaction with x

$$O_2 \Leftrightarrow 2\,O$$

Species	O_2	O	He	
Initial	1	0	1	
Change	-x	2x		
Equil.	1-x	2x	1	$n_{tot} = 1 - x + 2x + 1 = 2 + x$

$$y_O = \frac{2x}{2+x} = 0.1 \;\Rightarrow\; x = 0.2/(2 - 0.1) = 0.10526,$$

$$y_{O2} = (1 - x)/(2 + x) = 0.425$$

$$K = \frac{y_O^2}{y_{0_2}} \left(\frac{P}{P_o}\right)^{2-1} = \frac{0.1^2}{0.425}\,1.01325 = 0.02384 \quad\Rightarrow\quad \ln K = -3.736$$

Recall $P_o = 100$ kPa for Table A.11, $P/P_o = 101.325 / 100 = 1.01325$

Now look in Table A.11: **T = 3096 K = 5573 R**

14.123E

Pure oxygen is heated from 77 F to 5300 F in a steady flow process at a constant pressure of 30 lbf/in.^{2}. Find the exit composition and the heat transfer.

The only reaction will be the dissociation of the oxygen

$$O_2 \Leftrightarrow 2O ; \quad K(5300\ F) = K(3200\ K) = 0.046467$$

Look at initially 1 mol Oxygen and shift the above reaction with x

$$n_{O2} = 1 - x; \quad n_O = 2x; \quad n_{tot} = 1 + x; \quad y_i = n_i/n_{tot}$$

$$K = \frac{y_O^2}{y_{02}}\left(\frac{P}{P_o}\right)^{2-1} = \frac{4x^2}{(1 + x)^2}\frac{1 + x}{1 - x}\ 2 = \frac{8x^2}{1 - x^2}$$

$$x^2 = \frac{K/8}{1 + K/8} \Rightarrow x = 0.07599; \quad y_{02} = \mathbf{0.859}; \quad y_0 = \mathbf{0.141}$$

$$\bar{q} = n_{02ex}\bar{h}_{02ex} + n_{0ex}\bar{h}_{Oex} - \bar{h}_{02in} = (1 + x)(y_{02}\bar{h}_{02} + y_0\bar{h}_O) - 0$$

$$\bar{h}_{02} = 45\ 581; \quad \bar{h}_O = 107124 + 26125 = 133\ 249$$

$$\bar{q} = 1.076(0.859 \times 45581 + 0.141 \times 133249) = 62\ 345\ \text{Btu/lbmol O}_2$$

$$q = \bar{q}/32 = \mathbf{1948\ Btu/lbm}\ \ (= 1424\ \text{if no dissociation})$$

Borgnakke and Sonntag

14.124E

Find the equilibrium constant for $CO_2 \Leftrightarrow CO + \frac{1}{2} O_2$ at 3960 R using Table A.11.

The elementary reaction in A.11 is : $2CO_2 \Leftrightarrow 2CO + O_2$

so the wanted reaction is (1/2) times that and T = 3960 R = 2200 K so

$$K = K_{A.11}^{1/2} = \sqrt{\exp(-10.232)} = \exp(-5.116) = 0.006$$

or

$$\ln K = 0.5 \ln K_{A.11} = 0.5\,(-10.232) = -5.116$$
$$K = \exp(-5.11) = 0.006$$

14.125E

Assume we have air at 4 atm, 3600 R (as 21% O_2 and 79% N_2), and we can neglect dissociations of O_2 and N_2. What is the equilibrium mole fraction of NO? Find the enthalpy difference in the gases due to the formation of NO.

Air initially: $0.79 N_2 + 0.21 O_2$

Final mix: $(0.79 - x) N_2 + (0.21 - x) O_2 + 2x NO$

$$N_2 + O_2 \Leftrightarrow 2 NO \qquad n_{N2} = 0.79 - x$$
$$-x \quad -x \qquad +2x \qquad n_{O2} = 0.21 - x$$
$$n_{NO} = 0 \quad + 2x$$
$$\overline{\quad n = 1.0 \quad}$$

From Table A.11, $T = 3600 R = 2000 K$, $\ln K = -7.825$

$$K = \frac{y_{NO}^2}{y_{N2} y_{O2}} \left(\frac{P}{P^0}\right)^0 = \frac{4x^2}{(0.79 - x)(0.21 - x)} = \exp(-7.825) = 0.0003996$$

So

$$x^2 = 0.000099904 \, (0.79 - x)(0.21 - x) \quad \Rightarrow \quad x = 0.00402$$

$$y_{NO} = 2x = \mathbf{0.00804}$$

$$\Delta H = 2x \, h_{NO} - x h_{N2} - x h_{O2} \qquad \text{all at 3600 R}$$
$$= 0.00402 \, [2 \times (38\,818 + 24\,875) - 24\,135 - 25441] \text{ Btu/lbmol}$$
$$= \mathbf{312.8 \ Btu/lbmol \ air}$$

14.126E

Air (assumed to be 79% nitrogen and 21% oxygen) is heated in a steady flow process at a constant pressure of 14.7 lbf/in.^{2}, and some NO is formed. At what temperature will the mole fraction of NO be 0.001?

$$0.79N_2 + 0.21O_2 \quad \text{heated at 14.7 lbf/in}^2\text{, forms NO}$$

At exit, $y_{NO} = 0.001$

$$N_2 + O_2 \Leftrightarrow 2\,NO \qquad n_{N2} = 0.79 - x$$
$$\text{-x} \quad \text{-x} \qquad \text{+2x} \qquad n_{O2} = 0.21 - x$$
$$\underline{n_{NO} = 0 \quad + 2x}$$
$$n = 1.0$$

$$y_{NO} = 0.001 = \frac{2x}{1.0} \Rightarrow \quad x = 0.0005 \quad \Rightarrow \quad n_{N2} = 0.7895, \quad n_{O2} = 0.2095$$

$$K = \frac{y_{NO}^2}{y_{N2}y_{O2}} \left(\frac{P}{P^0}\right)^0 = \frac{10^{-6}}{0.7895 \times 0.2095} = 6.046 \times 10^{-6} \quad \text{or} \quad \ln K = -12.016$$

From Table A.11, T = 1444 K = **2600 R**

14.127E

The combustion products from burning pentane, C_5H_{12}, with pure oxygen in a stoichiometric ratio exists at 4400 R. Consider the dissociation of only CO_2 and find the equilibrium mole fraction of CO.

$$C_5H_{12} + 8\,O_2 \rightarrow 5\,CO_2 + 6\,H_2O$$

At 4400 R,	$2\,CO_2 \Leftrightarrow 2\,CO + 1\,O_2$		
ln K = -7.226	Initial 5	0	0
$K = 7.272 \times 10^{-4}$	Change -2z	+2z	+z
	Equil. 5-2z	2z	z

Assuming $P = P^o = 0.1$ MPa,

$$K = \frac{y_{CO}^2 y_{O_2}}{y_{CO_2}^2}\left(\frac{P}{P^0}\right) = \left(\frac{2z}{5-2z}\right)^2\left(\frac{z}{5+z}\right)(1) = 7.272 \times 10^{-4};$$

Trial & error on z: z = 0.2673

$n_{CO_2} = 4.4654;$ $n_{CO} = 0.5346;$ $n_{O_2} = 0.2673$

$$y_{CO} = \mathbf{0.1015}$$

14.128E

Pure oxygen is heated from 77 F, 14.7 lbf/in.^{2} to 5300 F in a constant volume container. Find the final pressure, composition, and the heat transfer

As oxygen is heated it dissociates

$$O_2 \Leftrightarrow 2O \qquad \ln K_{eq} = -3.069$$

C. V. Heater: $U_2 - U_1 = {}_1Q_2 = H_2 - H_1 - P_2v + P_1v$

Per mole O_2: ${}_1\bar{q}_2 = \bar{h}_2 - \bar{h}_1 + \bar{R}(T_1 - (n_2/n_1)T_2)$

Shift x in reaction final composition: $(1 - x)O_2 + 2xO$

$$n_1 = 1 \qquad n_2 = 1 - x + 2x = 1 + x$$

$$y_{O2} = (1 - x)/(1 + x) ; \qquad y_O = 2x/(1 + x)$$

Ideal gas and $V_2 = V_1 \Rightarrow P_2 = P_1n_2T_2/n_1T_1 \Rightarrow P_2/P_o = (1 + x)T_2/T_1$

Substitute the molefractions and the pressure into the equilibrium equation

$$K_{eq} = e^{-3.069} = \frac{y_O^2}{y_{02}}(\frac{P_2}{P_o}) = (\frac{2x}{1 + x})^2 (\frac{1 + x}{1 - x}) (\frac{1 + x}{1}) (\frac{T_2}{T_1})$$

$$\Rightarrow \frac{4x^2}{1 - x} = \frac{T_1}{T_2}e^{-3.069} = 0.00433 \Rightarrow x = 0.0324$$

$(n_{O2})_2 = 0.9676, \qquad (n_O)_2 = 0.0648, \qquad n_2 = 1.0324$

$$P_2 = P_o (1 + x)T_2/T_1 = 14.7 (1 + 0.0324) \frac{5760}{460 + 77} = \textbf{162.8 psia}$$

${}_1\bar{q}_2 = 0.9676(45\ 581) + 0.0648(107\ 124 + 26\ 125) - 0$

$\qquad + 1.98589(536.67 - 1.0324 \times 5760) = \textbf{41 996 Btu/lbmol } O_2$

$y_{O2} = 0.9676 / 1.0324 = \textbf{0.937}; \qquad y_O = 0.0648/1.0324 = \textbf{0.0628}$

14.129E

Assume the equilibrium mole fractions of oxygen and nitrogen are close to those in air, find the equilibrium mole fraction for NO at 5400 R, 75 psia disregarding dissocoiations.

Assume the simple reaction to make NO as:

N_2	$+$	$O_2 \Leftrightarrow$	$2\,NO$	Ar	$n_{N2} = 0.78 - x$
0.78		0.21	0	0.01	$n_{O2} = 0.21 - x$
-x		-x	+2x		$n_{NO} = 2x, \quad n_{ar} = 0.01$
0.78–x		0.21–x	2x	0.01	$n_{tot} = 1.0$

From A.11 at 5400 R = 3000 K,   ln K = -4.205,   K = 0.014921

$$K = \frac{4x^2}{(0.78 - x)(0.21 - x)} \left(\frac{P}{P^0}\right)^0 \quad \Rightarrow$$

$$\frac{x^2}{(0.78 - x)(0.21 - x)} = \frac{0.014921}{4} = 0.00373 \quad \text{and} \quad 0 < x < 0.21$$

Solve for x: x = 0.0230 $\Rightarrow$ $y_{NO} = \frac{2x}{1.0} = \mathbf{0.046}$

Comment: Since the reaction is pressure insensitive P does not matter.

14.130E

Acetylene gas, C_2H_2, is burned with stoichiometric air in a torch. The reactants are supplied at the reference conditions P_0, T_0. The products come out from the flame at 5000 R after a small heat loss by radiation. Consider the dissociation of CO_2 into CO and O_2 and no others. Find the equilibrium composition of the products. Are there any other reactions that should be considered?

Combustion: $C_2H_2 + 2.5\ (O_2 + 3.76N_2) \rightarrow 2\ CO_2 + 1\ H_2O + 9.4\ N_2$

At 5000 R = 2778 K

		$2\ CO_2 \Leftrightarrow 2\ CO + 1\ O_2$			H_2O	N_2
ln K = -3.9804	Initial	2	0	0	1	9.4
K = 0.01868	Change	-2z	+2z	+z	0	0
	Equil.	2-2z	2z	z	1	9.4

Assuming $P = P^o = 0.1$ MPa, and $n_{tot} = 2 + z + 1 + 9.4 = 12.4 + z$

$$K = \frac{y_{CO}^2 y_{O_2}}{y_{CO_2}^2} \left(\frac{P}{P^0}\right) = \left(\frac{2z}{2-2z}\right)^2 \left(\frac{z}{12.4+z}\right)(1) = 0.01868;$$

Solve: $z^3 = 0.01868\ (1-z)^2(12.4+z)$ => $z = 0.428$

$$y_{CO} = \frac{2z}{12.4+z} = \mathbf{0.0667};\ \ y_{CO2} = \frac{2-2z}{12.4+z} = \mathbf{0.089};\ \ y_{O2} = \frac{z}{12.4+z} = \mathbf{0.0334}$$

$$y_{H2O} = \frac{1}{12.4+z} = \mathbf{0.0779};\ \ \ y_{N2} = \frac{9.4}{12.4+z} = \mathbf{0.7328}$$

Looking in A.11 there will be no dissociation of nitrogen and only a small dissociation of water and oxygen which could be included.

14.131E

Use the information in problem 14.142E to estimate the enthalpy of reaction,

ΔH^O, at 1260 R using the van't Hoff equation (see problem 14.56) with finite differences for the derivatives.

$$d \ln K = \frac{\Delta H^O}{\bar{R}T^2} \ dT_P^o$$

solve for $\Delta H°$

$$\Delta H° = \bar{R}T^2 \frac{d\ln K}{dT} = \bar{R}T^2 \frac{\Delta \ln K}{\Delta T}$$

$$= 1.98589 \times 1260^2 \times \frac{-0.3362 + 4.607}{1440 - 1080} = 37\ 403 \text{ Btu/lb mol}$$

[Remark: compare this to F.6 values + F.4, F.11,

$$\Delta H° = H_C + 2H_{H_2} - H_{CH_4} \ = \ 0.146 \times 12 \times (1260\text{-}537) + 2 \times 5044$$

$$- 0.538 \times 16.043 \times (1260\text{-}537) \ \text{-}(\text{-}32190) \ = \textbf{37 304} \ ]$$

14.132E

A gas mixture of 1 lbmol carbon monoxide, 1 lbmol nitrogen, and 1 lbmol oxygen at 77 F, 20 lbf/in.^{2}, is heated in a constant pressure steady flow process. The exit mixture can be assumed to be in chemical equilibrium with CO_2, CO, O_2, and N_2 present. The mole fraction of CO_2 at this point is 0.176. Calculate the heat transfer for the process.

initial mix: Equil. mix:

1 CO, 1 O$_2$,   Constant   CO$_2$, CO, O$_2$, N$_2$

1 N$_2$ pressure   y$_{CO2}$ = 0.176

         reactor   P = 20 lbf/in^2

           $\dot{Q}$

reaction	2 CO$_2$	$\Leftrightarrow$	2 CO	+	O$_2$	also,	N$_2$
initial	0		1		1		1
change	+2x		-2x		-x		0
equil.	2x		(1-2x)		(1-x)		1

$$y_{CO2} = 0.176 = \frac{2x}{3-x} \quad \Rightarrow \quad x = 0.242\ 65$$

$$n_{CO2} = 0.4853 \quad n_{O2} = 0.757\ 35 \left.\begin{matrix} \\ \\ \end{matrix}\right\} \left\{\begin{matrix} y_{CO2} = 0.176 \\ y_{CO} = 0.186\ 67 \\ y_{O2} = 0.274\ 67 \end{matrix}\right.$$

$$n_{CO} = 0.5147 \quad n_{N2} = 1$$

$$K = \frac{y_{CO}^2 y_{O2}}{y_{CO2}^2}\left(\frac{P}{P^0}\right)^1 = \frac{0.186\ 67^2 \times 0.274\ 67}{0.176^2}\left(\frac{20}{14.504}\right) = 0.426\ 07$$

Since Table A.11 corresponds to a pressure P_0 of 100 kPa, which is 14.504 lbf/in^2. Then, from A.11, T_{PROD} = 3200 K = 5760 R

$$H_R = -47\ 518 \text{ Btu}$$

$$H_P = 0.4853(-169\ 184 + 71\ 075) + 0.5147(-47\ 518 + 43\ 406)$$

$$+ 0.757\ 35(0 + 45\ 581) + 1(0 + 43\ 050) = +27\ 842 \text{ Btu}$$

$$Q_{CV} = H_P - H_R = 27\ 842 - (-47\ 518) = \textbf{+75 360 Btu}$$

14.133E

Acetylene gas at 77 F is burned with 140% theoretical air, which enters the burner at 77 F, 14.7 lbf/in.^{2}, 80% relative humidity. The combustion products form a mixture of CO_2, H_2O, N_2, O_2, and NO in chemical equilibrium at 3500 F, 14.7 lbf/in.^{2}. This mixture is then cooled to 1340 F very rapidly, so that the composition does not change. Determine the mole fraction of NO in the products and the heat transfer for the overall process.

$$C_2H_2 + 3.5\,O_2 + 13.16\,N_2 + \text{water} \rightarrow$$

$$2\,CO_2 + 1\,H_2O + 1\,O_2 + 13.16\,N_2 + \text{water}$$

Water: $P_V = 0.8{\times}0.46 = 0.368 \text{ lbf/in}^2$

$$n_V = n_A{\times}\frac{P_V}{P_A} = (3.5{+}13.16){\times}\frac{0.368}{14.332} = 0.428$$

So, total H_2O in products is : $1 + n_V = 1.428$.

a) reaction: $N_2 + O_2 \Leftrightarrow 2\,NO$

change : $-x$ $-x$ $+2x$

at 3500 F = 3960 R (=2200 K), from A.11: $K = 0.001\ 074$

Equilibrium products:

$$n_{CO2} = 2, \quad n_{H2O} = 1.428, \quad n_{O2} = 1 - x,$$

$$n_{N2} = 13.16 - x, \quad n_{NO} = 0 + 2x, \quad n_{TOT} = 17.588$$

$$K = \frac{(2x)^2}{(1 - x)(13.16 - x)} = 0.001\ 074$$

By trial and error, $x = 0.0576$

$$y_{NO} = \frac{2 \times 0.0576}{17.588} = \textbf{0.006 55}$$

b) Final products (same composition) at 1340 F = 1800 R

$H_R = 1(97\ 476) + 0.428(\text{-}103\ 966) = 52\ 979 \text{ Btu}$

$H_P = 2(\text{-}169\ 184{+}14\ 358) + 1.428(\text{-}103\ 966{+}11\ 178)$

$+ 0.9424(0{+}9761) + 13.1024(0{+}9227) + 0.1152(38\ 818{+}9557)$

$= \text{-}306\ 486 \text{ Btu}$

$Q_{CV} = H_P - H_R = \textbf{-359 465 Btu}$

Borgnakke and Sonntag

14.134E

A special coal burner uses a stoichiometric mixture of coal and an oxygen - argon mixture (1:1 mole ratio) with the reactants supplied at Po, To. Consider the dissociation of CO_2 into CO and O_2 and no others. Find the equilibrium composition of the products for T = 8600 R. Is the final temperature, including dissociations, higher or lower than 8600 R?

Combustion: $C + O_2 + Ar \Rightarrow CO_2 + Ar$

Reaction:

			CO_2	CO	O_2	Ar
$2 CO_2 \Leftrightarrow 2 CO + O_2$		Start	1	0	0	1
		Change	-2x	2x	x	0
		Final	1-2x	2x	x	1 tot = 2 + x

At 8600 R: $K = \exp(5.7694) = 320.345$

$$K = [\, y_{CO}^2 \, y_{O2} / y_{CO2}^2\,] \, 1 = [(\frac{2x}{2+x})^2 \frac{x}{2+x} / (\frac{1-2x}{2+x})^2]$$

Reduce and solve:

$$x^3 = [\, \exp(5.7694)/4\,] \, (2 + x)(1 - 2x)^2$$
$$\Rightarrow \quad x^3 = 80.09 \,(2 + x)(1 - 2x)^2 \quad \Rightarrow \quad x = 0.48793$$
$$n_{CO} = 0.9759, \; n_{O2} = 0.4879, \; n_{CO2} = 0.0241, \; n_{Ar} = 1$$

Now we can do the energy equation:

$H_{P\,8600\,R} = 0.0241\,(113\,961 - 169\,184) + 1 \times 4.9536\,(8600 - 459.7) +$
$\qquad\qquad 0.9759\,(68\,977 - 47\,518) + 0.4879 \times 73\,648$
$\qquad = 95\,868\ \text{btu/lbmol}$

Since $H_{P\,8600\,R} > 0$ (too high)   then   **T < 8600 R**

We should have had $H_P = 0$, if we match the energy equation to find the T we have to redo the equilibrium equation as K is different. This is a larger trial and error problem to find the actual T and we even neglected other possible reactions.

14.135E

An important step in the manufacture of chemical fertilizer is the production of ammonia, according to the reaction: $N_2 + 3H_2 \Leftrightarrow 2NH_3$. Show that the equilibrium constant is K = 6.826 at 300 F.

$$1N_2 + 3H_2 \Leftrightarrow 2NH_3 \quad \text{at 300 F} = 760 \text{ R}$$

from Table F.4 for NH_3:   M = 17.031, C_{P0} = 0.509 Btu/lbm-R

Assume ideal gas and constant specific heat for ammonia (we could be more accurate if we used Table F.8 realizing properties depend also on P). We must use the reference h, s values from Table F.11

$$\bar{h}^o_{NH3 \ 300 \ F} = -19 \ 656 + 0.509 \times 17.031 \ (300-77) = -17723 \text{ Btu/lbmol}$$

$$\bar{s}^o_{NH3 \ 300 \ F} = 45.969 + 0.509 \times 17.031 \ln \frac{760}{537} = 48.980$$

Use Table F.6 for nitrogen and hydrogen gases

$$\Delta H^o_{300 \ F} = 2(-17723) - 1(0 + \ 1557) - 3(0 + 1552) \ = -41 \ 659 \text{ Btu/lbmol}$$

$$\Delta S^0_{300 \ F} = 2(48.98) - 1(48.164) - 3(33.60) \ = -51.0 \text{ Btu/R-lbmol}$$

$$\Delta G^0_{300 \ F} = -41 \ 659 - 760(-51.0) = -2899 \text{ Btu/lbmol}$$

$$\ln K = \frac{+2899}{1.98589 \times 760} = 1.9208, \quad K = \textbf{6.826}$$

14.136E

Consider the previous reaction in equilibrium at 300 F, 750 psia. For an initial composition of 25% nitrogen, 75% hydrogen, on a mole basis, calculate the equilibrium composition.

$$1N_2 + 3H_2 \Leftrightarrow 2NH_3 \text{ at 300 F}$$

Beginning:	1	3	0	
Change:	-x	-3x	2x	
Final:	1-x	3-3x	2x	Total: $n = 4 - 2x$

$$n_{NH3} = 2x, \quad n_{N2} = 1\text{-}x, \quad n_{H2} = 3\text{-}3x$$

The reaction constant is K = 6.826 so the reaction constant equation is

$$K = \frac{y_{NH3}^2}{y_{N2}y_{H2}^3}\left(\frac{P}{P^0}\right)^{-2} = \frac{(2x)^2 2^2 (2\text{-}x)^2}{3^3 (1\text{-}x)^4}\left(\frac{P}{P^0}\right)^{-2}$$

or $\left(\dfrac{x}{1\text{-}x}\right)^2\left(\dfrac{2\text{-}x}{1\text{-}x}\right)^2 = \dfrac{27}{16} \times 6.826 \times \left(\dfrac{750}{14.7}\right)^2 = 29985$

or $\left(\dfrac{x}{1\text{-}x}\right)\left(\dfrac{2\text{-}x}{1\text{-}x}\right) = 173.16$

$\rightarrow$ Trial & Error:

x = 0.9242

	n	y
NH_3	1.848	0.8591
N_2	0.0758	0.0352
H_2	0.2273	0.1057

Simultaneous Reactions

Borgnakke and Sonntag

14.137E

Ethane is burned with 150% theoretical air in a gas turbine combustor. The products exiting consist of a mixture of CO_2, H_2O, O_2, N_2, and NO in chemical equilibrium at 2800 F, 150 lbf/in.^{2}. Determine the mole fraction of NO in the products. Is it reasonable to ignore CO in the products?

Combustion:

$$C_2H_6 + 5.25\,O_2 + 19.74\,N_2 \rightarrow 2\,CO_2 + 3\,H_2O + 1.75\,O_2 + 19.74\,N_2$$

a) Products at 2800 F, 150 lbf/in^2. Equilibrium mixture: CO_2, H_2O, O_2, N_2, NO

	N_2	+	O_2	$\Leftrightarrow$	$2\,NO$
initial	19.74		1.75		0
change	-x		-x		+2x
equil.	19.74-x		1.75-x		2x

Equil. comp. $n_{CO2} = 2$, $n_{H2O} = 3$, $n_{O2} = 1.75\text{-x}$, $n_{N2} = 19.74\text{-x}$, $n_{NO} = 2x$

$$K = 1.283 \times 10^{-4} = \frac{y_{NO}^2}{y_{N2}y_{O2}}\left(\frac{P}{P^0}\right)^0 = \frac{4x^2}{(19.74\text{-x})(1.75\text{-x})}$$

Solving, $x = 0.032\ 95$

$$y_{NO} = \frac{2 \times 0.032\ 95}{26.49} = \mathbf{0.002\ 49}$$

b)

	$2\,CO_2$	$\Leftrightarrow$	$2\,CO$	+	O_2
initial	2		0		0
change	-2a		+2a		+2x
equil.	2-2a		2a		2x

$$K = 5.259 \times 10^{-8} = \frac{y_{CO}^2 y_{O2}}{y_{CO2}^2}\left(\frac{P}{P^0}\right)^1 = \left(\frac{2a}{2\text{-}2a}\right)^2\left(\frac{1.75\text{-x+a}}{26.49\text{+a}}\right)\left(\frac{150}{14.504}\right)$$

Since Table A.11 corresponds to a pressure P_o of 100 kPa, which is 14.504 lbf/in^2. This equation should be solved simultaneously with the equation solved in part a) (modified to include the unknown a). Since x was found to be small and also a will be very small, the two are practically independent. Therefore, use the value $x = 0.032\ 95$ in the equation above, and solve for a.

$$\left(\frac{a}{1\text{-a}}\right)^2\left(\frac{1.75\text{-0.032\ 95+a}}{26.49\text{+a}}\right) = \left(\frac{14.504}{150}\right) \times 5.259 \times 10^{-8}$$

Solving, $a = 0.000\ 28$ or $y_{CO} = 2.1 \times 10^{-5}$ negligible for most applications.

Borgnakke and Sonntag

14.138E

One pound mole of water vapor at 14.7 $lbf/in.^2$, 720 R, is heated to 5400 R in a constant pressure steady flow process. Determine the final composition, assuming that H_2O, H_2, H, O_2, and OH are present at equilibrium.

Reactions:

1) $2\,H_2O \Leftrightarrow 2\,H_2 + O_2$ 2) $2\,H_2O \Leftrightarrow H_2 + 2\,OH$

change -2a   +2a   +a   change   -2b   +b   +2b

3) $H_2 \Leftrightarrow 2\,H$

change -c   +2c

At equilibrium (5400 R, 14.7 lbf/in^2)

$n_{H2O} =$	1-2a-2b	$n_{OH} =$	2b
$n_{H2} =$	2a+b-c	$n_H =$	2c
$n_{O2} =$	a	$n_{TOT} =$	1+a+b+c

$$\frac{K_1}{(P/P^0)} = \frac{2.062 \times 10^{-3}}{1.03} = \left(\frac{2a+b-c}{1-2a-2b}\right)^2 \left(\frac{a}{1+a+b+c}\right)$$

$$\frac{K_2}{(P/P^0)} = \frac{2.893 \times 10^{-3}}{1.03} = \left(\frac{2a+b-c}{1+a+b+c}\right)\left(\frac{2b}{1-2a-2b}\right)^2$$

$$\frac{K_3}{(P/P^0)} = \frac{2.496 \times 10^{-2}}{1.03} = \frac{(2a)^2}{(2a+b-c)(1+a+b+c)}$$

These three equations must be solved simultaneously for

a, b & c: a = 0.0622, b = 0.0570, c = 0.0327

and $n_{H2O} = 0.7616$ $y_{H2O} = 0.6611$

          $n_{H2} = 0.1487$ $y_{H2} = 0.1291$

          $n_{O2} = 0.0622$ $y_{O2} = 0.0540$

14.139E

Methane is burned with theoretical oxygen in a steady flow process, and the products exit the combustion chamber at 5300 F, 100 lbf/in.^{2}. Calculate the equilibrium composition at this state, assuming that only CO_2, CO, H_2O, H_2, O_2, and OH are present.

Combustion: $CH_4 + 2\,O_2 \rightarrow CO_2 + 2\,H_2O$

Dissociation reactions: At equilibrium:

1) $2\,H_2O \Leftrightarrow 2\,H_2 + O_2$ $n_{H2O} =$ 2-2a-2b

change -2a   +2a   +a $n_{H2} =$ 2a+b

2) $2\,H_2O \Leftrightarrow H_2 + 2\,OH$ $n_{O2} =$ a+c

change -2b   +b   +2b $n_{OH} =$ 2b

3) $2\,CO_2 \Leftrightarrow 2\,CO + O_2$ $n_{CO2} =$ 1-2c

change -2c   +2c   +c $n_{CO} =$ 2c
_____

$n_{TOT} =$ 3+a+b+c

Products at 5300F, 100 lbf/in^2

$$K_1 = 0.007\,328 = \left(\frac{2a+b}{2\text{-}2a\text{-}2b}\right)^2 \left(\frac{a+c}{3+a+b+c}\right)\left(\frac{100}{14.504}\right)$$

$$K_2 = 0.012\,265 = \left(\frac{2b}{2\text{-}2a\text{-}2b}\right)^2 \left(\frac{2a+b}{3+a+b+c}\right)\left(\frac{100}{14.504}\right)$$

$$K_3 = 0.426\,135 = \left(\frac{2c}{1\text{-}2c}\right)^2 \left(\frac{a+c}{3+a+b+c}\right)\left(\frac{100}{14.504}\right)$$

These 3 equations must be solved simultaneously for a, b, & c. If solving by hand divide the first equation by the second, and solve for c = fn(a,b). This reduces the solution to 2 equations in 2 unknowns. Solving,

a = 0.0245, b = 0.1460, c = 0.2365

Substance:	H2O	H2	O2	OH	CO2	CO
n	1.659	0.195	0.260	0.292	0.527	0.473
y	0.4871	0.0573	0.0763	0.0857	0.1547	0.1389

14.140E

One pound mole of air (assumed to be 78% nitrogen, 21% oxygen, and 1% argon) at room temperature is heated to 7200 R, 30 lbf/in.^{2}. Find the equilibrium composition at this state, assuming that only N_2, O_2, NO, O, and Ar are present.

1 lbmol air (0.78 N_2, 0.21 O_2, 0.01 Ar) heated to 7200 R, 30 lbf/in^2.

    1) $N_2 + O_2 \Leftrightarrow 2\,NO$ 2) $O_2 \Leftrightarrow 2\,O$

change -a   -a   +2a   change   -b   +2b

Equil.:

$n_{N2} =$	0.78-a	$n_{Ar} =$	0.01	$n_{NO} =$	2a
$n_{O2} =$	0.21-a-b	$n_O =$	2b	$n =$	1+b

$$K_1 = 0.0895 = \frac{4a^2}{(0.78\text{-}a)(0.21\text{-}a\text{-}b)} \left(\frac{30}{14.504}\right)^0$$

$$K_2 = 2.221 = \frac{4b^2}{(1\text{+}b)(0.21\text{-}a\text{-}b)} \left(\frac{30}{14.504}\right)$$

Divide 1st eq'n by 2nd and solve for a as function(b), using

$$X = \frac{K_1}{K_2} \left(\frac{P}{P^0}\right) = 0.083\ 35$$

Get

$$a = \frac{Xb^2}{2(1\text{+}b)} \left[-1 + \sqrt{1 + \frac{4\times0.78(1\text{+}b)}{Xb^2}} \right] \tag{1}$$

Also

$$\frac{b^2}{(1\text{+}b)(0.21\text{-}a\text{-}b)} = \frac{K_2}{4(P/P^0)} = 0.268\ 44 \tag{2}$$

Assume b = 0.1269   From (1), get a = 0.0299

Then, check a & b in (2) $\Rightarrow$ OK

Therefore,

Subst.	N2	O2	Ar	O	NO
n	0.7501	0.0532	0.01	0.2538	0.0598
y	0.6656	0.0472	0.0089	0.2252	0.0531

14.141E

Acetylene gas and x times theoretical air ($x > 1$) at room temperature and 75 lbf/in.^{2} are burned at constant pressure in an adiabatic steady flow process. The flame temperature is 4600 R, and the combustion products are assumed to consist of N_2, O_2, CO_2, H_2O, CO, and NO. Determine the value of x.

Combustion:

$$C_2H_2 + 2.5x\, O_2 + 9.4x\, N_2 \rightarrow 2\, CO_2 + H_2O + 2.5(x-1)O_2 + 9.4x\, N_2$$

Eq. products 4600 R, 75 lbf/in^2: N_2, O_2, CO_2, H_2O, CO, NO

2 Reactions:

1) $2\,CO_2 \Leftrightarrow 2\,CO + O_2$ 2) $N_2 + O_2 \Leftrightarrow 2\,NO$

change -2a   +2a   +a   change   -b   -b   +2b

Equil. Comp.:

$n_{N2} =$	9.4x-b	$n_{CO2} =$	2-2a
$n_{O2} =$	2.5x-2.5+a-b	$n_{H2O} =$	1
$n_{CO} =$	2a	$n_{NO} =$	2b
$n_{TOT} =$	11.9x+0.5+a		

At 4600 R, from A.11: $K_1 = 2.359 \times 10^{-3}$, $K_2 = 4.249 \times 10^{-3}$

$$\frac{K_1}{(P/P^0)} = \frac{2.359 \times 10^{-3}}{5.103} = 4.622 \times 10^{-4} = \left(\frac{a}{1-a}\right)^2 \left(\frac{2.5x-2.5+a-b}{11.9x+0.5+a}\right)$$

$$K_2 = 4.249 \times 10^{-3} = \frac{(2b)^2}{(9.4-b)(2.5x-2.5+a-b)}$$

Also, from the energy Eq.: $H_P - H_R = 0$ where

$H_R = 1(+97\ 477) + 0 + 0 = +97\ 497$ Btu

$H_P = (9.4x-b)(0+32\ 817) + (2.5x-2.5+a-b)(0+34\ 605)$

        $+ (2-2a)(-169\ 184+53\ 885) + 1(-103\ 966+43\ 899)$

        $+ 2a(-47\ 518+33\ 122) + 2b(38\ 818+31\ 161)$

Substituting,

$394\ 992 \times x + 236\ 411 \times a + 72\ 536 \times b - 377\ 178 = 97477$

which results in a set of 3 equations in the 3 unknowns x,a,b. Trial and error solution from the last eq. and the ones for K_1 and K_2. The result is

x = 1.12 ,   a = 0.1182,   b = 0.05963

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Review problems

14.142E

The equilibrium reaction with methane as $CH_4 \Leftrightarrow C + 2H_2$ has ln K = -0.3362 at 1440 R and ln K = -4.607 at 1080 R. By noting the relation of K to temperature, show how you would interpolate lnK in (1/T) to find K at 1260 R and compare that to a linear interpolation.

ln K = -0.3362 at 1440 R   ln K = -4.607 at 1080 R

$$\ln K_{1260} = \ln K_{1440} + \frac{\dfrac{1}{1260} - \dfrac{1}{1440}}{\dfrac{1}{1080} - \dfrac{1}{1440}} \times (-4.607 + 0.3362)$$

$$= -0.3362 + \frac{\dfrac{1440}{1260} - 1}{\dfrac{1440}{1080} - 1} \times (-4.2708) = -2.1665$$

Linear interpolation:

$$\ln K_{1260} = \ln K_{1080} + \frac{1260 - 1080}{1440 - 1080}(\ln K_{1440} - \ln K_{1080})$$

$$= -4.607 + \frac{1}{2}(-0.3362 + 4.607) = \textbf{-2.4716}$$

14.143E

In a test of a gas-turbine combustor, saturated-liquid methane at 210 R is to be burned with excess air to hold the adiabatic flame temperature to 2880 R. It is assumed that the products consist of a mixture of CO_2, H_2O, N_2, O_2, and NO in chemical equilibrium. Determine the percent excess air used in the combustion, and the percentage of NO in the products.

$$CH_4 + 2x\,O_2 + 7.52x\,N_2$$

$$\rightarrow 1\,CO_2 + 2\,H_2O + (2x-2)\,O_2 + 7.52x\,N_2$$

							Also	CO_2	H_2O
Then		N_2	+	O_2	$\Leftrightarrow$	2 NO			
init		7.52x		2x-2		0		1	2
ch.		-a		-a		+2a		0	0
equil.		(7.52x-a)		(2x-2-a)		2a		1	2

$$n_{TOT} = 1 + 9.52x$$

2880 R: $\ln K = -10.55$, $K = 2.628 \times 10^{-5}$

$$2.628 \times 10^{-5}\ K = \frac{y_{NO}^2}{y_{N2}y_{O2}}\left(\frac{P}{P^0}\right)^0 = \frac{y_{NO}^2}{y_{N2}y_{O2}} = \frac{4a^2}{(7.52x-a)(2x-2-a)}$$

$$H_R = 1\left[-32\ 190 + (-1854-4300)\right] + 0 + 0 \ (\text{assume } 77\ F) = -38\ 344\ Btu$$

$$H_P = 1(-169\ 184 + 29\ 049) + 2(-103\ 966 + 22\ 746)$$

$$+ (7.52x-a)(18\ 015) + (2x-2-a)(19\ 031) + 2a(38\ 818 + 18\ 624)$$

$$= -340\ 639 + 173\ 535\ x + 77\ 838\ a$$

Assume $a \sim 0$, then from $H_P - H_R = 0$ $\rightarrow$ $x = 1.742$

Subst.

$$\frac{a^2}{(13.1-a)(1.484-a)} = \frac{2.628 \times 10^{-5}}{4},\qquad \text{get}\quad a \approx 0.01125$$

Use this a in energy equation

$$x = \frac{302\ 295 - 77\ 838 \times 0.01125}{173\ 535} = 1.737$$

$$\Rightarrow \qquad \frac{a^2}{(13.062-a)(1.474-a)} = \frac{2.628 \times 10^{-5}}{4},\ a = 0.0112$$

$$\Rightarrow \qquad x = 1.737 \qquad\qquad \% \text{ excess air} = \textbf{73.7 \%}$$

$$\% \text{ NO} = \frac{2 \times 0.0112 \times 100}{1 + 9.52 \times 1.737} = \textbf{0.128 \%}$$

14.144E

Dry air is heated from 77 F to 7200 R in a 14.7 lbf/in.^{2} constant-pressure process. List the possible reactions that may take place and determine the equilibrium composition. Find the required heat transfer.

Air assumed to be 21% oxygen and 79% nitrogen by volume.

From the elementary reactions at 4000 K = 7200 R (A.11):

(1) $O_2 <=> 2\ O$ $K_1 = 2.221 = y_O^2/y_{O2}$

(2) $N_2 <=> 2\ N$ $K_2 = 3.141 \times 10^{-6} = y_N^2/y_{N2}$

(3) $N_2 + O_2 <=> 2\ NO$ $K_3 = 0.08955 = y_{NO}^2/y_{N2}\ y_{O2}$

Call the shifts a,b,c respectively so we get

$n_{O2} = 0.21\text{-}a\text{-}c, \quad n_O = 2a, \quad n_{N2} = 0.79\text{-}b\text{-}c, \quad n_N = 2b,$

$n_{NO} = 2c, \quad n_{tot} = 1+a+b$

From which the molefractions are formed and substituted into the three equilibrium equations. The result is corrected for 1 atm = 14.7 lbf/in^2 = 101.325 kPa versus the tables 100 kPa

$K_1 = 2.1511 = y_O^2/y_{O2} = 4a^2/[(1+a+b)(0.21\text{-}a\text{-}c)]$

$K_2 = 3.042 \times 10^{-6} = y_N^2/y_{N2} = 4b^2/[(1+a+b)(0.79\text{-}b\text{-}c)]$

$K_3 = 0.08955 = y_{NO}^2/y_{N2}\ y_{O2} = 4c^2/[(0.79\text{-}b\text{-}c)(0.21\text{-}a\text{-}c)]$

which give 3 eqs. for the unknowns (a,b,c). Trial and error assume b = c = 0 solve for a from K_1 then for c from K_3 and finally given the (a,c) solve for b from K_2. The order chosen according to expected magnitude $K_1 > K_3 > K_2$

$a = 0.15, \quad b = 0.000832, \quad c = 0.0244 \quad =>$

$n_{O2} = 0.0356, \quad n_O = 0.3, \quad n_{N2} = 0.765, \quad n_N = 0.00167, \quad n_{NO} = 0.049$

$Q = H_{ex} - H_{in} = n_{O2}\Delta\bar{h}_{O2} + n_{N2}\Delta\bar{h}_{N2} + n_O(\bar{h}_{fO} + \Delta\bar{h}_O)$

$+ n_N(\bar{h}_{fN} + \Delta\bar{h}_N) + n_{NO}(\bar{h}_{fNO} + \Delta\bar{h}_{NO}) - 0$

$= 0.0356 \times 59\ 632 + 0.765 \times 55\ 902 + 0.3(107\ 124 + 33\ 394)$

$+ 0.00167(203\ 216 + 33\ 333) + 0.049(38\ 818 + 57\ 038)$

$= \textbf{92 135 Btu/lbmol air}$

[If no reaction: $Q = n_{O2}\Delta\bar{h}_{O2} + n_{N2}\Delta\bar{h}_{N2} = 56\ 685$ Btu/lbmol air]

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Fundamentals of Thermodynamics

SOLUTION MANUAL
CHAPTER 15

8e

Updated June 2013

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Fundamentals of Thermodynamics 8th Edition
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CONTENT CHAPTER 15

In-Text Concept Questions

15.a

Is stagnation temperature always higher than free stream temperature? Why?

Yes. Since kinetic energy can only be positive we have

$$h_0 = h_1 + \mathbf{V}_1^2/2 \; > \; h_1$$

If it is a gas with constant heat capacity we get

$$T_0 = T_1 + \mathbf{V}_1^2/2C_p$$

15.b

By looking at Eq. 15.25, rank the speed of sound for a solid, a liquid, and a gas.

Speed of sound: $\left(\dfrac{\partial P}{\partial \rho}\right)_s = c^2$

For a solid and liquid phase the density varies only slightly with temperature and constant s is also nearly constant T. We thus expect the derivative to be very high that is we need very large changes in P to give small changes in density.

A gas is highly compressible so the formula reduces to Eq.15.28 which gives modest values for the speed of sound.

15.c

Does speed of sound in an ideal gas depend on pressure? What about a real gas?

No. For an ideal gas the speed of sound is given by Eq.15.28
$$c = \sqrt{kRT}$$
and is only a function of temperature T.

For a real gas we do not recover the simple expression above and there is a dependency on P particularly in the dense gas region above the critical point.

15.d

Can a convergent adiabatic nozzle produce a supersonic flow?

No. From Eq.15.33 and a nozzle so $dP < 0$ it is required to have $dA > 0$ to reach $M > 1$. A convergent nozzle will have $M = 1$ at the exit, which is the smallest area. For lower back pressures there may be a shock standing outside the exit plane.

15.e

To maximize the mass flow rate of air through a given nozzle, which properties should I try to change and in which direction, higher or lower?

The mass flow rate is given by Eq.15.41 and if we have $M = 1$ at the throat then Eq.15.42 gives the maximum mass flow rate possible.

Max flow for:
> Higher upstream stagnation pressure
> Lower upstream stagnation temperature

15.f

How do the stagnation temperature and pressure change in an isentropic flow?

The stagnation temperature and stagnation pressure are constant.

15.g

Which of the cases in Fig. 15.17 (a-h) have entropy generation and which do not?

a. There is no flow so $s_{gen} = 0$.

b. Subsonic flow, reversible, so $s_{gen} = 0$.

c. Limit for subsonic flow, reversible, so $s_{gen} = 0$.

d. The only supersonic reversible flow solution, so $s_{gen} = 0$.

e. Supersonic reversible in nozzle $s_{gen} = 0$, irreversible outside.

f. Supersonic reversible in nozzle $s_{gen} = 0$, compression outside.

g. Shock stands at exit plane, $s_{gen} > 0$ across shock.

h. Shock is located inside nozzle, $s_{gen} > 0$ across shock.

15.h

How does the stagnation temperature and pressure change in an adiabatic nozzle flow with an efficiency of less than 100%?

The stagnation temperature stays constant (energy eq.)
The stagnation pressure drops (s is generated, less kinetic energy).

15.i

Table A.13 has a column for P_{oy}/P_{ox} why is there not one for T_{oy}/T_{ox}?

The stagnation pressure drops across the shock (irreversible flow) whereas the stagnation temperature is constant (energy equation).

15.j

How high can a gas velocity (Mach number) be and still treat it as incompressible flow within 2% error?

The relative error in the ΔP versus kinetic energy, Eq.15.66, becomes

$$e = \frac{1}{4}\left(\frac{\mathbf{V}}{c_o}\right)^2 = 0.02 \quad \Rightarrow \quad M = \frac{\mathbf{V}}{c_o} = \sqrt{4 \times 0.02} = \mathbf{0.283}$$

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Concept-Study Guide Problems

Borgnakke and Sonntag

15.1

Which temperature does a thermometer or thermocouple measure? Would you ever need to make a correction to that?

Since the probe with the thermocouple in its tip is stationary relative to the moving fluid it will measure something close to the stagnation temperature. If that is high relative to the free stream temperature there will be significant heat transfer (convection and radiation) from the probe and it will measure a little less. For very high accuracy temperature measurements you must make some corrections for these effects.

15.2

The jet engine thrust is found from the overall momentum equation. Where is the actual force acting (it is not a long-range force in the flow)?

The compressor is generating the high pressure flow so the blades pushes hard on the flow and thus a force acts in the forward direction on the shaft holding the rotating blades. The high pressure in the chamber with combustion also has a net force in the forward direction as the flow leaves in the backwards direction so less wall area there and higher velocity. The pressure drop in the turbine means its blades pushes in the other direction but as the turbine exit pressure is higher than the ambient pressure the axial force is less than that of the compressor.

In front of the nozzle section there is a higher upstream pressure and a lower downstream pressure which is the pressure difference that generates the large exit velocity. This higher pressure plus the pressure difference across the turbine equals the pressure in the combustion chamber which is the compressor exit pressure. If the nozzle was not there this pressure would be lower and the pressure on all the surfaces including the compressor blades would provide less thrust.

15.3

Most compressors have a small diffuser at the exit to reduce the high gas velocity near the rotating blades and increase the pressure in the exit flow. What does this do to the stagnation pressure?

For a reversible flow (ideal case) the stagnation pressure is constant. However, the reason it is done is to raise the pressure in a near reversible flow (diffuser) rather than let the flow reduce the peak velocities in a less reversible fashion which would lower the stagnation pressure.

15.4

A diffuser is a divergent nozzle used to reduce a flow velocity. Is there a limit for the Mach number for it to work like this?

Yes, the flow must be subsonic. If the flow was supersonic then increasing the flow area would increase the velocity.

15.5

Sketch the variation in **V**, T, P, ρ and *M* for a subsonic flow into a convergent nozzle with *M* = 1 at the exit plane?

$$\mathbf{V} = M\,c = M\sqrt{kRT} = \sqrt{2C_p(T_o - T)}$$

Since we do not know the area versus length, we plot it versus mach number *M*. T, P and ρ relative to the stagnation state is listed in Table A.12 and given in eqs.15.34-36. A small spread sheet (M step 0.1) did the calculations.

Only the first part   0 < M < 1 is the answer to this question.

The curves are plotted as the variables:

$$T/T_o$$

$$\rho/\rho_o$$

$$P/P_o$$

$$V/\sqrt{2C_p\,T_o}$$

and for k = 1.4

15.6

Sketch the variation in **V**, T, P, ρ and *M* for a sonic (*M* = 1) flow into a divergent nozzle with *M* = 2 at the exit plane?

$$\mathbf{V} = M\,c = M\sqrt{kRT} = \sqrt{2C_p(T_o - T)}$$

Since we do not know the area versus length, we plot it versus mach number *M*. T, P and ρ relative to the stagnation state is listed in Table A.12 and given in eqs.15.34-36.

Only the last part $1 < M < 2$ is the answer to this question.

The curves are plotted as the variables:

$$T\,/\,T_o$$
$$\rho\,/\,\rho_o$$
$$P\,/\,P_o$$
$$V\,/\,\sqrt{2C_p\,T_o}$$

and for k = 1.4

15.7

Can any low enough backup pressure generate an isentropic supersonic flow?

No. Only one back pressure corresponds to a supersonic flow, which is the exit pressure at state d in Figure 15.13. However a pressure lower than that can give an isentropic flow in the nozzle, case e, with a drop in pressure outside the nozzle. This is irreversible leading to an increase in s and therefore not isentropic.

15.8

Is there any benefit to operate a nozzle choked?

Yes. Since the mass flow rate is constant (max value) between points c and d in Fig. 15.12 a small variation in the back pressure will not have any influence. The nozzle then provides a constant mass flow rate free of surges up or down which is very useful for flow calibrations or other measurements where a constant mass flow rate is essential.

15.9

Can a shock be located upstream from the throat?

No. The flow adjust so M = 1 at the throat.

15.10

The high velocity exit flow in Example 15.7 is at 183 K. Can that flow be used to cool a room?

Being that cold it sounds like it could. However when the flow enters a room it eventually would have to slow down and then it has the stagnation temperature. If you let the flow run over a surface there will be a boundary layer with zero velocity at the surface and again there the temperature is close to the stagnation temperature.

15.11

A convergent-divergent nozzle is presented for an application that requires a supersonic exit flow. What features of the nozzle do you look at first?

You look at the cross section area change through the nozzle. At the throat M = 1 so in the divergent section the velocity increases and the ratio A/A* determines how the flow changes. The exit area can then tell you what the exit mach number will be and if you can have a reversible flow or not.

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15.12

To increase the flow through a choked nozzle, the flow can be heated/cooled or compressed/expanded (four processes) before or after the nozzle. Explain which of these eight possibilities will help and which will not.

The mass flow rate through a choked nozzle is given by Eq.15.42. Since k and R are constant it varies with the upstream stagnation properties P_o and T_o.

After nozzle: Any downstream changes have **no effects**.

Before nozzle: Upstream changes in P_o and T_o has an influence.

a. Heat   This lowers mass flow rate (T_o increases)

b. Cool   This raises mass flow rate (T_o decreases)

c. Compress. Raises P_o and T_o opposite effects.

Isentropic: $P_{o\,new} = P_o\,r_p$ and $T_{o\,new} = T_o\,(r_p)^{\frac{k-1}{k}}$

$P_{o\,new} / \sqrt{T_{o\,new}} = (r_p)^{\frac{k+1}{2k}} \left[P_o / \sqrt{T_o}\right] \; > \; \left[P_o / \sqrt{T_o}\right]$

So the mass flow rate increases

d. Expand. Lowers P_o and T_o opposite effects. Assume isentropic, then mass flow rate decreases.

15.13

Suppose a convergent-divergent nozzle is operated as case h in Fig. 15.17. What kind of nozzle could have the same exit pressure but with a reversible flow?

A convergent nozzle, having subsonic flow everywhere assuming the pressure ratio is higher than the critical.

Stagnation Properties

15.14

A stationary thermometer measures 80°C in an air flow that has a velocity of 200 m/s. What is the actual flow temperature?

We assume that the thermometer measures the stagnation temperature, that is the probe (bulb, thermistor or thermo-couple junction sits stationary).

$$h_0 = h_1 + \mathbf{V}_1^2/2 \qquad \Rightarrow \qquad T_1 = T_0 - \mathbf{V}_1^2/2C_p$$

$$T_1 = 80°C \ - \frac{200^2}{2000 \times 1.004} \frac{(m/s)^2}{J/kg\text{-}K} = \mathbf{60°C}$$

15.15

Steam leaves a nozzle with a pressure of 500 kPa, a temperature of 350°C, and a velocity of 250 m/s. What is the isentropic stagnation pressure and temperature?

Stagnation enthalpy from energy equation and values from steam tables B.1.3

$$h_0 = h_1 + \mathbf{V}_1^2/2 = 3167.7 \text{ kJ/kg} + \frac{250^2}{2000} \frac{(\text{m/s})^2}{\text{J/kJ}} = 3198.4 \text{ kJ/kg}$$

$$s_0 = s_1 = 7.6329 \text{ kJ/kg K}$$

It can be linearly interpolated from the printed tables

Computer software: $(h_o, s_o) \Rightarrow T_o = \mathbf{365°C}, \ P_o = \mathbf{556 \ kPa}$

15.16

Steam at 1600 kPa, 300°C flows so it has a stagnation (total) pressure of 1800 kPa. Find the velocity and the stagnation temperature.

The stagnation state has the same entropy as the inlet state so:

1: 1600 kPa, 300°C   has   h = 3034.83 kJ/kg, s = 6.8844 kJ/kg-K
Stagnation: 1800 kPa, s_0 = 6.8844 kJ/kg-K   has

Table B.1.3: T_{01} = **316.5°C**, h_{01} = 3066.155 kJ/kg

$$V_1^2/2 = h_{01} - h_1 = 3066.155 - 3034.83 = 31.325 \text{ kJ/kg}$$
$$V_1 = \sqrt{2 \times (h_{o1} - h_1)} = \sqrt{2 \times 31.325 \times 1000 \text{ J/kg}} = \textbf{250.3 m/s}$$

15.17

An object from space enters the earth's upper atmosphere at 5 kPa, 100 K, with a relative velocity of 2500 m/s or more . Estimate the object`s surface temperature.

$$h_{o1} - h_1 = \mathbf{V}_1^2/2 = 2500^2/2000 = 3125 \text{ kJ/kg}$$
$$h_{o1} = h_1 + 2000 = 100 + 3125 = 3225 \text{ kJ/kg} \quad => \quad T = \mathbf{2767 \text{ K}}$$

The value for h_1 from ideal gas table A.7 was estimated since the lowest T in the table is 200 K.

15.18

The products of combustion of a jet engine leave the engine with a velocity relative to the plane of 500 m/s, a temperature of 525°C, and a pressure of 75 kPa. Assuming that $k = 1.32$, $C_p = 1.15$ kJ/kg K for the products, determine the stagnation pressure and temperature of the products relative to the airplane.

Energy Eq.: $\quad h_{o1} - h_1 = \mathbf{V}_1^2/2 = 500^2/2000 = 125$ kJ/kg

$\qquad T_{o1} - T_1 = (h_{o1} - h_1)/C_p = 125/1.15 = 108.7$ K

$\qquad T_{o1} = 525 + 273.15 + 108.7 = \mathbf{906.8\ K}$

Isentropic process relates to the stagnation pressure

$\qquad P_{o1} = P_1(T_{o1}/T_1)^{k/(k-1)} = 75(906.8/798.15)^{4.125} = \mathbf{127\ kPa}$

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15.19

Steam is flowing to a nozzle with a pressure of 400 kPa. The stagnation pressure and temperature are measured to be 600 kPa and 350°C, respectively. What are the flow velocity and temperature?

Stagnation state Table B.1.3: h_{o1} = 3165.66 kJ/kg, s_{o1} = 7.5463 kJ/kg K

State 1: 400 kPa, $s_1 = s_{o1}$ = 7.5463 kJ/kg K

$$T_1 = 250 + (300 - 250)\,\frac{7.5463 - 7.3788}{7.5661 - 7.3788} = \mathbf{294.7^oC}$$

$$h_1 = 2964.16 + \frac{7.5463 - 7.3788}{7.5661 - 7.3788}\,(3066.75 - 2964.16) = 3055.9 \text{ kJ/kg}$$

Energy equation gives

$$\mathbf{V}_1^2/2 = h_{o1} - h_1 = 3165.66 - 3055.9 = 109.76 \text{ kJ/kg}$$

$$\mathbf{V}_1 = \sqrt{2 \times (h_{o1} - h_1)} = \sqrt{(2 \times 109.76 \times 1000) \text{ J/kg}} = \mathbf{468.5 \text{ m/s}}$$

15.20

A meteorite melts and burn up at temperatures of 3000 K. If it hits air at 5 kPa, 50 K how high a velocity should it have to experience such a temperature?

Assume we have a stagnation T = 3000 K

$h_1 + V_1^2/2 = h_{stagn.}$

Use table A.7, $h_{stagn.} = 3525.36$ kJ/kg, $h_1 = 50$ kJ/kg

$V_1^2/2 = 3525.36 - 50 = 3475.4$ kJ/kg (remember convert to J/kg = m^2/s^2)

$V_1 = \sqrt{2 \times 3475.4 \times 1000 \text{ J/kg}} = $ **2636 m/s**

15.21

Air leaves a compressor in a pipe with a stagnation temperature and pressure of 150°C, 300 kPa, and a velocity of 125 m/s. The pipe has a cross-sectional area of 0.02 m^2. Determine the static temperature and pressure and the mass flow rate.

$$h_{o1} - h_1 = \mathbf{V}_1^2/2 = 125^2/2000 = 7.8125 \text{ kJ/kg}$$

$$T_{o1} - T_1 = (h_{o1} - h_1)/C_p = 7.8125/1.004 = 7.8 \text{ K}$$

$$T_1 = T_{o1} - \Delta T = 150 - 7.8 = \textbf{142.2 °C = 415.4 K}$$

$$P_1 = P_{o1}(T_1/T_{o1})^{k/(k-1)} = 300 \text{ kPa } (415.4/423.15)^{3.5} = \textbf{281 kPa}$$

$$\dot{m} = \rho A \mathbf{V} = \frac{A\mathbf{V}}{v} = \frac{P_1 A \mathbf{V}_1}{RT_1} = \frac{281.2(0.02)(125)}{0.287(415.4)} = \textbf{5.9 kg/s}$$

15.22

I drive down the highway at 110 km/h on a day with 25°C, 101.3 kPa. I put my hand, cross sectional area 0.01 m², flat out the window. What is the force on my hand and what temperature do I feel?

The air stagnates on the hand surface : $\quad h_1 + \mathbf{V}_1^2/2 = h_{stagn.}$

Use constant heat capacity

$$T_{stagn.} = T_1 + \frac{\mathbf{V}_1^2/2}{C_p} = 25 + \frac{0.5 \times 110^2 \times (1000/3600)^2}{1004} = \mathbf{25.465°C}$$

Assume a reversible adiabatic compression

$$P_{stagn.} = P_1 \, (T_{stagn.}/T_1)^{k/(k-1)} = 101.3 \text{ kPa } (298.615/298.15)^{3.5}$$
$$= \mathbf{101.85 \text{ kPa}}$$

15.23

A stagnation pressure of 110 kPa is measured for an airflow where the pressure is 100 kPa and 20°C in the approach flow. What is the incomming velocity?

Assume a reversible adiabatic compression

$$T_{o1} = T_1 \times (P_{o1}/P_1)^{(k-1)/k} = 293.15 \text{ K} \times \left(\frac{110}{100}\right)^{0.2857} = 301.24 \text{ K}$$

$$V_1^2/2 = h_{o1} - h_1 = C_p (T_{o1} - T_1) = 1.004 (301.24 - 293.15) = 8.1246 \text{ kJ/kg}$$

$$V_1 = \sqrt{2 \times 8.1246 \times 1000} = \textbf{127.5 m/s}$$

To the left a Pitot tube, blue inner tube measures stagnation pressure and yellow outer tube with holes in it measures static pressure. To the right is a stagnation point on a wall relative to the free stream flow at state 1.

15.24

A 4-cm inner diameter pipe has an inlet flow of 10 kg/s water at 20°C, 200 kPa. After a 90 degree bend as shown in Fig. P15.24, the exit flow is at 20°C, 190 kPa. Neglect gravitational effects and find the anchoring forces F_x and F_y.

$$D = 0.04 \text{ m} \quad \Rightarrow \quad A = \frac{\pi}{4} D^2 = 0.001257 \text{ m}^2$$

$$\mathbf{V}_{avg} = \frac{\dot{m}}{\rho A} = \frac{10 \times 0.001002}{0.001257} = 7.971 \text{ m/s}$$

Now we can do the x and y direction momentum equations for steady flow and the same magnitude of the velocity, but different directions

X-dir: $0 = \dot{m} \, \mathbf{V}_{avg\,1} + F_x - \dot{m} \times 0 + (P_1 - P_o) \, A$

Y-dir: $0 = \dot{m} \times 0 + F_y - \dot{m} \times (-\mathbf{V}_{avg\,2}) + (P_2 - P_o) \, A$

$$F_x = -\dot{m} \, \mathbf{V}_{avg\,1} - (P_1 - P_o) \, A$$
$$= -10 \times 7.97 - 100 \times 0.001257 \times 1000 = \mathbf{-205 \ N}$$

$$F_y = -\dot{m} \, \mathbf{V}_{avg\,2} - (P_2 - P_o) \, A$$
$$= -10 \times 7.97 - 90 \times 0.001257 \times 1000 = \mathbf{-193 \ N}$$

15.25

A jet engine receives a flow of 150 m/s air at 75 kPa, 5°C across an area of 0.6 m^2 with an exit flow at 450 m/s, 75 kPa, 800 K. Find the mass flow rate and thrust.

$$\dot{m} = \rho A \mathbf{V}; \quad \text{ideal gas} \quad \rho = P/RT$$

$$\dot{m} = (P/RT)A\mathbf{V} \;=\; (\frac{75}{0.287 \times 278.15}) \times 0.6 \times 150 = 0.9395 \times 0.6 \times 150$$

$$= \mathbf{84.555 \ kg/s}$$

$$F_{net} = \dot{m} \, (\mathbf{V}_{ex} - \mathbf{V}_{in}) \;=\; 84.555 \times (450 - 150) = \mathbf{25\ 367\ N}$$

Inlet   High P   Low P exit

$\leftarrow \; F_{net}$

The shaft must have axial load bearings to transmit thrust to aircraft.

15.26

How large a force must be applied to a squirt gun to have 0.1 kg/s water flow out at 20 m/s? What pressure inside the chamber is needed?

$$F = \frac{d\,m\mathbf{V}}{dt} = \dot{m}\,\mathbf{V} = 0.1 \times 20 \text{ kg m/s}^2 = \mathbf{2\ N}$$

Eq.15.21: $\quad v\Delta P = 0.5\,\mathbf{V}^2$

$$\Delta P = 0.5\,\mathbf{V}^2/v = 0.5 \times 20^2 / 0.001$$
$$= 200\,000 \text{ Pa} = \mathbf{200\ kPa}$$

15.27

A jet engine at takeoff has air at 20°C, 100 kPa coming at 25 m/s through the 1.0 m diameter inlet. The exit flow is at 1200 K, 100 kPa, through the exit nozzle of 0.4 m diameter. Neglect the fuel flow rate and find the net force (thrust) on the engine.

$$A_1 = \frac{\pi}{4} D^2 = 0.7854 \text{ m}^2; \qquad A_2 = \frac{\pi}{4} D^2 = 0.1257 \text{ m}^2$$

$$v_1 = \frac{RT}{P} = \frac{0.287 \times 293.15}{100} = 0.8409 \text{ m}^3/\text{kg}; \quad v_2 = 3.444 \text{ m}^3/\text{kg}$$

$$\dot{m} = AV/v = A_1 V_1 / v_1 = \frac{0.7854 \times 25}{0.8409} = 48.0 \text{ kg/s}$$

$$\mathbf{V}_2 = \frac{\dot{m} v_2}{A_2} = \frac{48.0 \times 3.444}{0.1257} = 1315 \text{ m/s}$$

Now we can do the x direction momentum equation for steady flow and the same mass flow rate in and out

X-dir: $\qquad 0 = \dot{m} \, \mathbf{V}_1 + F_x + (P_1 - P_o) \, A_1 - \dot{m} \, \mathbf{V}_2 - (P_2 - P_o) \, A_2$

$$F_x = -\dot{m} \, \mathbf{V}_1 - (P_1 - P_o) \, A_1 + \dot{m} \, \mathbf{V}_2 + (P_2 - P_o) \, A_2$$

$$= \dot{m} \, (\mathbf{V}_2 - \mathbf{V}_1) - 0 + 0 = 48 \, (1315 - 25) = \textbf{61 920 N}$$

Inlet High P Low P exit

$\leftarrow \ F_{net}$

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15.28

A water turbine using nozzles is located at the bottom of Hoover Dam 175 m below the surface of Lake Mead. The water enters the nozzles at a stagnation pressure corresponding to the column of water above it minus 20% due to losses. The temperature is 15°C and the water leaves at standard atmospheric pressure. If the flow through the nozzle is reversible and adiabatic, determine the velocity and kinetic energy per kilogram of water leaving the nozzle.

The static pressure at the 175 m depth is:

$$\Delta P = \rho g \Delta Z = \frac{g \Delta Z}{v} = \frac{9.807 \times 175}{0.001001 \times 1000} = 1714.5 \text{ kPa}$$

$$\Delta P_{ac} = 0.8 \ \Delta P = 1371.6 \text{ kPa}$$

Use Bernoulli's equation: $\quad v\Delta P = \mathbf{V}_{ex}^2/2$

$$\Rightarrow \mathbf{V}_{ex} = \sqrt{2v\Delta P}$$
$$\mathbf{V}_{ex} = \sqrt{2 \times 0.001001 \times 1000 \times 1371.6} = \textbf{62.4 m/s}$$
$$\mathbf{V}_{ex}^2/2 = v\Delta P = \textbf{1.373 kJ/kg}$$

15.29

A water cannon sprays 1 kg/s liquid water at a velocity of 100 m/s horizontally out from a nozzle. It is driven by a pump that receives the water from a tank at 15°C, 100 kPa. Neglect elevation differences and the kinetic energy of the water flow in the pump and hose to the nozzle. Find the nozzle exit area, the required pressure out of the pump and the horizontal force needed to hold the cannon.

$$\dot{m} = \rho A \mathbf{V} = A\mathbf{V}/v \implies A = \dot{m}v/\mathbf{V} = 1\,\frac{0.001001}{100} = \mathbf{1.0 \times 10^{-5}\ m^2}$$

$$\dot{W}_p = \dot{m}w_p = \dot{m}v(P_{ex} - P_{in}) = \dot{m}\mathbf{V}_{ex}^2/2$$

$$P_{ex} = P_{in} + \mathbf{V}_{ex}^2/2v = 100 + 100^2/2 \times 1000 \times 0.001 = \mathbf{150\ kPa}$$

$$F = \dot{m}\mathbf{V}_{ex} = 1 \times 100 = \mathbf{100\ N}$$

15.30

An irrigation pump takes water from a lake and discharges it through a nozzle as shown in Fig. P15.30. At the pump exit the pressure is 900 kPa, and the temperature is 20°C. The nozzle is located 15 m above the pump and the atmospheric pressure is 100 kPa. Assuming reversible flow through the system determine the velocity of the water leaving the nozzle.

Assume we can neglect kinetic energy in the pipe in and out of the pump. Incompressible flow so Bernoulli's equation applies ($V_1 \cong V_2 \cong V_3 \cong 0$)

Bernoulli: $\quad v(P_3 - P_2) + (V_3^2 - V_2^2)/2 + g(Z_3 - Z_2) = 0$

$$P_3 = P_2 - \frac{g(Z_3 - Z_2)}{v} = 900 - \frac{9.807 \times 15}{1000 \times 0.001002} = 753 \text{ kPa}$$

Bernoulli: $\quad V_4^2/2 = v(P_3 - P_4)$

$$\Rightarrow V_4 = \sqrt{2v(P_3 - P_4)} = \sqrt{2 \times 0.001002 \times 653 \times 1000} = \textbf{36.2 m/s}$$

15.31

A water tower on a farm holds 1 m^3 liquid water at 20°C, 100 kPa in a tank on top of a 5 m tall tower. A pipe leads to the ground level with a tap that can open a 1.5 cm diameter hole. Neglect friction and pipe losses, and estimate the time it will take to empty the tank for water.

Incompressible flow so we can use Bernoulli Equation.

$$P_e = P_i; \quad \mathbf{V}_i = 0; \quad Z_e = 0; \quad Z_i = H$$

$$\mathbf{V}_e^2/2 = gZ_i \quad \Rightarrow \quad \mathbf{V}_e = \sqrt{2gZ} = \sqrt{2 \times 9.807 \times 5} = 9.9 \text{ m/s}$$

$$\dot{m} = \rho A \mathbf{V}_e = A\mathbf{V}_e/v = \Delta m/\Delta t$$

$$\Delta m = V/v; \quad A = \pi D^2/4 = \pi \times 0.015^2 / 4 = 1.77 \times 10^{-4} \text{ m}^2$$

$$\Rightarrow \quad \Delta t = \Delta mv/A\mathbf{V}_e = V/A\mathbf{V}_e$$

$$\Delta t = \frac{1}{1.77 \times 10^{-4} \times 9.9} = \textbf{571.6 sec} = \textbf{9.53 min}$$

15.32

Find the speed of sound for air at 100 kPa at the two temperatures 0°C and 30°C. Repeat the answer for carbon dioxide and argon gases.

From eq. 15.28 we have

$$c_0 = \sqrt{kRT} = \sqrt{1.4 \times 0.287 \times 273.15 \times 1000} = \textbf{331 m/s}$$

$$c_{30} = \sqrt{1.4 \times 0.287 \times 303.15 \times 1000} = \textbf{349 m/s}$$

For Carbon Dioxide: R = 0.1889 kJ/kg K, k = 1.289

$$c_0 = \sqrt{1.289 \times 0.1889 \times 273.15 \times 1000} = \textbf{257.9 m/s}$$

$$c_{30} = \sqrt{1.289 \times 0.1889 \times 303.15 \times 1000} = \textbf{271.7 m/s}$$

For Argon: R = 0.2081 kJ/kg K, k = 1.667

$$c_0 = \sqrt{1.667 \times 0.2081 \times 273.15 \times 1000} = \textbf{307.8 m/s}$$

$$c_{30} = \sqrt{1.667 \times 0.2081 \times 303.15 \times 1000} = \textbf{324.3 m/s}$$

15.33

Find the expression for the anchoring force R_x for an incompressible flow like in Figure 15.6. Show that it can be written as

$$R_x = \frac{\mathbf{V}_i - \mathbf{V}_e}{\mathbf{V}_i + \mathbf{V}_e} \left[(P_i - P_o)A_i + (P_e - P_o)A_e \right]$$

Apply the X-dir momentum equation for a steady flow

$$0 = R_x + (P_i - P_o)A_i - (P_e - P_o)A_e + \dot{m}\mathbf{V}_i - \dot{m}\mathbf{V}_e$$

Bernoulli equation for the flow is

$$0.5(\mathbf{V}_e^2 - \mathbf{V}_i^2) + v(P_e - P_i) = 0 \qquad \Rightarrow \qquad \mathbf{V}_e - \mathbf{V}_i = \frac{2v(P_i - P_e)}{\mathbf{V}_i + \mathbf{V}_e}$$

Continuity equation gives

$$\dot{m} = A_i\mathbf{V}_i /v = A_e\mathbf{V}_e /v$$

Solve for R_x from the momentum equation

$$R_x = \dot{m}(\mathbf{V}_e - \mathbf{V}_i) + (P_e - P_o)A_e - (P_i - P_o)A_i$$

$$= \frac{A_i\mathbf{V}_i}{v} \frac{2v(P_i - P_e)}{\mathbf{V}_i + \mathbf{V}_e} + (P_e - P_o)A_e - (P_i - P_o)A_i$$

Multiply in and use continuity equation for second term

$$R_x = \frac{2}{\mathbf{V}_i + \mathbf{V}_e} \left[P_i A_i\mathbf{V}_i - P_e A_e\mathbf{V}_e \right] + (P_e - P_o)A_e - (P_i - P_o)A_i$$

$$= \frac{2}{\mathbf{V}_i + \mathbf{V}_e} \left[P_i A_i\mathbf{V}_i - P_e A_e\mathbf{V}_e + \tfrac{1}{2}(P_e - P_o)A_e\mathbf{V}_e - \tfrac{1}{2}(P_i - P_o)A_i\mathbf{V}_i \right.$$

$$\left. + \tfrac{1}{2}(P_e - P_o)A_e\mathbf{V}_i - \tfrac{1}{2}(P_i - P_o)A_i\mathbf{V}_e \right]$$

Now put the first four terms together

$$R_x = \frac{2}{\mathbf{V}_i + \mathbf{V}_e} \left[\tfrac{1}{2}(P_i - P_o)A_i\mathbf{V}_i - \tfrac{1}{2}(P_e - P_o)A_e\mathbf{V}_e \right.$$

$$\left. + \tfrac{1}{2}(P_e - P_o)A_e\mathbf{V}_i - \tfrac{1}{2}(P_i - P_o)A_i\mathbf{V}_e \right]$$

$$= \frac{2}{\mathbf{V}_i + \mathbf{V}_e} \left[\tfrac{1}{2}(P_i - P_o)A_i(\mathbf{V}_i - \mathbf{V}_e) + \tfrac{1}{2}(P_e - P_o)A_e(\mathbf{V}_i - \mathbf{V}_e) \right]$$

$$= \frac{\mathbf{V}_i - \mathbf{V}_e}{\mathbf{V}_i + \mathbf{V}_e} \left[(P_i - P_o)A_i + (P_e - P_o)A_e \right]$$

15.34

Estimate the speed of sound for steam directly from Eq. 15.25 and the steam tables for a state of 6 MPa, 400°C. Use table values at 5 and 7 MPa at the same entropy as the wanted state. Eq. 15.25 is then done by finite difference. Find also the answer for the speed of sound assuming steam is an ideal gas.

Eq.15.25: $\quad c^2 = \left(\dfrac{\partial P}{\partial \rho}\right)_s = \left(\dfrac{\Delta P}{\Delta \rho}\right)_s$

State 6 MPa, 400°C $\Rightarrow \quad s = 6.5407$ kJ/kg K

$\quad$ 7 MPa, s $\quad\Rightarrow\quad$ v = 0.04205 m^3/kg; $\quad \rho = 1/v = 23.777$ kg/m^3

$\quad$ 5 MPa, s $\quad\Rightarrow\quad$ v = 0.05467 m^3/kg; $\quad \rho = 1/v = 18.2909$ kg/m^3

$c^2 = \dfrac{7000 - 5000}{23.777 - 18.2909} = 364.56 \times 1000 \quad \Rightarrow \quad$ c = **603.8 m/s**

From Table A.8: $\quad C_p = \dfrac{1338.56 - 1235.3}{50} = 2.0652$ kJ/kg K

$C_v = C_p - R = 2.0652 - 0.4615 = 1.6037$ kJ/kg K

$k = C_p/C_v = 1.288; \quad R = 0.4615$ kJ/kg K (from A.5)

Now do the speed of sound from Eq.15.28

$c = \sqrt{kRT} = \sqrt{1.288 \times 0.4615 \times 673.15 \times 1000} =$ **632.6 m/s**

15.35

Use the CATT3 software to solve the previous problem.

From Eq. 15.25: $\quad c^2 = \left(\dfrac{\partial P}{\partial \rho}\right)_s = -v^2\left(\dfrac{\partial P}{\partial v}\right)_s$

Superheated vapor water at 400°C, 6000 kPa

CATT3: $\quad v = 0.04739$ m^3/kg, $\quad s = 6.541$ kJ/kg K

At P = 6200 kPa & s = 6.541 kJ/kg K: $\quad$ T = 405.1°C, $\quad$ v = 0.0462 m^3/kg

At P = 5800 kPa & s = 6.541 kJ/kg K: $\quad$ T = 394.8°C, $\quad$ v = 0.04866 m^3/kg

$$c^2 = -(0.04739)^2 \left(\frac{6.2 - 5.8}{0.0462 - 0.04866}\right) \frac{\text{MJ}}{\text{kg}} = 0.36517 \times 10^6 \text{ m}^2/\text{s}^2$$

$$\Rightarrow \quad c = \textbf{604 m/s}$$

From Table A.8: $\quad C_p = \dfrac{1338.56 - 1235.3}{50} = 2.0652$ kJ/kg K

$\quad C_v = C_p - R = 2.0652 - 0.4615 = 1.6037$ kJ/kg K

$\quad k = C_p/C_v = 1.288; \quad R = 0.4615$ kJ/kg K (from A.5)

Now do the speed of sound from Eq.15.28

$\quad c = \sqrt{kRT} = \sqrt{1.288 \times 0.4615 \times 673.15 \times 1000} = \textbf{632.6 m/s}$

15.36

If the sound of thunder is heard 5 seconds after the lightning is seen and the weather is 20°C. How far away is the lightning taking place?

The sound travels with the speed of sound in air (ideal gas). Use the formula in Eq.15.28

$$L = c \times t = \sqrt{kRT} \times t = \sqrt{1.4 \times 0.287 \times 293.15 \times 1000} \times 5 = \mathbf{1716\ m}$$

For every 3 seconds after the lightning the sound travels about 1 km.

15.37

Find the speed of sound for carbon dioxide at 2500 kPa, 60°C using either the tables or the CATT3 software (same procedure as in Problem 15.34) and compare that with Eq.15.28.

From Eq. 15.25: $c^2 = \left(\dfrac{\partial P}{\partial \rho}\right)_s = -v^2\left(\dfrac{\partial P}{\partial v}\right)_s$

Superheated carbon dioxide at 60°C, 2500 kPa

CATT3: $v = 0.02291$ m^3/kg, $s = 1.521$ kJ/kg K

At P = 2600 kPa & s = 1.521 kJ/kg K: T = 63.02°C, $v = 0.02221$ m^3/kg

At P = 2400 kPa & s = 1.521 kJ/kg K: T = 56.9°C, $v = 0.02365$ m^3/kg

$$c^2 = -(0.02291)^2 \left(\frac{2.6 - 2.4}{0.02221 - 0.02365}\right) \frac{MJ}{kg} = 7.8983 \times 10^4 \text{ m}^2/\text{s}^2$$

$\Rightarrow$ c = **270 m/s**

From Table A.5: $k = C_p/C_v = 1.289$;   R = 0.1889 kJ/kg K

Now do the speed of sound from Eq.15.28

$$c = \sqrt{kRT} = \sqrt{1.289 \times 0.1889 \times 333.15 \times 1000} = \textbf{284.8 m/s}$$

Borgnakke and Sonntag

15.38

A jet flies at an altitude of 12 km where the air is at -40°C, 45 kPa with a velocity of 1000 km/h. Find the Mach number and the stagnation temperature on the nose.

From Table A.5:   k = C$_p$/C$_v$ = 1.4;   R = 0.287 kJ/kg K

Now do the speed of sound from Eq.15.28

$$c = \sqrt{kRT} = \sqrt{1.4 \times 0.287 \times 233.15 \times 1000} = 306 \text{ m/s}$$

$$\mathbf{V} = 1000 \text{ km/h} = 1000 \text{ (km/h)} \times 1000 \text{ (m/km)} /3600 \text{ (s/h)} = 278 \text{ m/s}$$

$$M = \mathbf{V} / c = 278/306 = \mathbf{0.908}$$

$$h_0 = h_1 + \mathbf{V}_1^2/2 \quad \Rightarrow \quad T_0 = T_1 + \mathbf{V}_1^2 / 2C_p = -40 + \frac{278^2}{2 \times 1004} = \mathbf{-1.5^oC}$$

15.39

The speed of sound in liquid water at 25$^{\text{o}}$C is about 1500 m/s. Find the stagnation pressure and temperature for a $M = 0.1$ flow at 25$^{\text{o}}$C, 100 kPa. Is it possible to get a significant mach number flow of liquid water?

$$\mathbf{V} = M\,c = 0.1 \times 1500 = 150 \text{ m/s}$$

$$h_0 = h_1 + \mathbf{V}_1^2/2$$

Bernoulli Eq.: $\quad \Delta P = \mathbf{V}_1^2/2v = \dfrac{150^2}{2 \times 0.001} = 11.25 \times 10^6 \text{ Pa} = 11.25 \text{ MPa}$

$$P_0 = P_1 + \Delta P = 100 + 11\,250 = \mathbf{11\,350\ kPa}$$

$$T_0 = T_1 + \mathbf{V}_1^2 / 2C_p = 25 + \dfrac{150^2}{2 \times 4180} = \mathbf{27.7^oC}$$

Remark: Notice the very high pressure. To get a higher velocity you need a higher pressure to accelerate the fluid, that is not feasible for any large flow rate.

15.40

Steam flowing at 15 m/s 1800 kPa, 300°C expands to 1600 kPa in a converging nozzle. Find the exit velocity and area ratio A_e / A_i.

Solve the problem with the steam tables.

Inlet state: v_i = 0.14021 m^3/kg, h_i = 3029.21 kJ/kg, s_i = 6.8226 kJ/kg-K

Exit state: $(P_e, s_e = s_i)$ v_e = 0.15371 m^3/kg, h_e = 3000.995 kJ/kg

Energy Eq.: $\mathbf{V}_i^2 / 2 + h_i = \mathbf{V}_e^2 / 2 + h_e$; $\mathbf{V}_e^2 = \mathbf{V}_i^2 + 2(h_i - h_e)$

$$\mathbf{V}_e = \sqrt{15 \times 15 + 2000(3029.21 - 3000.995)} = \textbf{238 m/s}$$

Same mass flow rate so

$$A_e/A_i = (v_e/v_i)(V_i/V_e) = \frac{0.15371}{0.14021} \times \frac{15}{238} = \textbf{0.06909}$$

If we solved as ideal gas with constant specific heat we get (k = 1.327)

$$T_e = T_i \, (P_e/P_i)^{(k-1)/k} = 573.15 \, (1600/1800)^{0.2464} = 556.76 \text{ K}$$

$$\mathbf{V}_e = \sqrt{\mathbf{V}_i^2 + 2C_p(T_i - T_e)} = \sqrt{15 \times 15 + 2 \times 1872(573.15 - 556.76)}$$
$$= \textbf{248 m/s}$$

$$A_e/A_i = (v_e/v_i)(V_i/V_e) = (P_i/P_e)^{1/k}(V_i/V_e) = \left(\frac{1800}{1600}\right)^{0.7536} \times \frac{15}{248} = \textbf{0.0661}$$

15.41

A convergent nozzle has a minimum area of 0.1 m^2 and receives air at 175 kPa, 1000 K flowing with 100 m/s. What is the back pressure that will produce the maximum flow rate and find that flow rate?

$$\frac{P^*}{P_o} = (\frac{2}{k+1})^{\frac{k}{k-1}} = 0.528 \qquad \text{Critical Pressure Ratio}$$

Find P_o:

$$h_0 = h_1 + \mathbf{V}_1^2/2 = 1046.22 + 100^2/2000 = 1051.22 \text{ kJ/kg}$$

$$T_0 = T_i + 4.4 = 1004.4 \text{ K} \quad \text{from table A.7}$$

$$P_0 = P_i \, (T_0/T_i)^{k/(k-1)} = 175 \times \ (1004.4/1000)^{3.5} = 177.71 \text{ kPa}$$

The mass flow rate comes from the throat properties

$$P^* = 0.528 \, P_o = 0.528 \times 177.71 = \textbf{93.83 kPa}$$

$$T^* = 0.8333 \, T_o = 836.97 \text{ K}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{93.83}{0.287 \times 836.97} = 0.3906 \text{ kg/m}^3$$

$$\mathbf{V} = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 836.97} = \textbf{579.9 m/s}$$

$$\dot{m} = \rho A \mathbf{V} = 0.3906 \times 0.1 \times 579.9 = \textbf{22.65 kg/s}$$

15.42

A convergent-divergent nozzle has a throat area of 100 mm^2 and an exit area of 175 mm^2. The inlet flow is helium at a stagnation pressure of 1 MPa, stagnation temperature of 375 K. What is the back pressure that will give sonic condition at the throat, but subsonic everywhere else?

For this flow we have helium with $k_{He} = 1.667$, so we cannot use the tables for air. We need the solution to the curve labeled c in Fig. 15.13. For critical flow at the throat we have from Table 15.1 last column

$$P^* = 0.4867\ P_o = 486.7\ kPa$$

Now we need to find the conditions where the area ratio is

$$A_E/A^* = 175/100 = 1.75$$

that is solve for M in Eq. 15.43 given the area ratio. This is nonlinear so we have to iterate on it. Here $(k+1)/2(k-1) = 2$ so look also at Fig. 15.10 for the general shape.

$$M = 0.4 \implies A/A^* = (1/0.4)\ [\ 0.75(1 + 0.3333*0.4^2)\ ]^2 = 1.5602$$

$$M = 0.3 \implies A/A^* = (1/0.3)\ [\ 0.75(1 + 0.3333*0.3^2)\ ]^2 = 1.9892$$

$$M = 0.35 \implies A/A^* = (1/0.35)\ [\ 0.75(1 + 0.3333*0.35^2)\ ]^2 = 1.7410$$

$$M = 0.34 \implies A/A^* = (1/0.34)\ [\ 0.75(1 + 0.3333*0.34^2)\ ]^2 = 1.7844$$

Now do a linear interpolation for the rest to get $M_E = 0.348$;

Eq. 15.35 $P_E/P_o = [1 + 0.3333*0.348^2\ ]^{-2.5} = 0.9058$

$$P_E = 0.9058 \times 1000 = \textbf{906 kPa}$$

15.43

To what pressure should the steam in problem 15.40 expand to reach Mach one? Use constant specific heats to solve.

Find stagnation properties

$$T_0 = T_1 + \mathbf{V}_1^2/2C_p = 573.15 \text{ K} + 15^2 \text{ (m/s)}^2/(2 \times 1872 \text{ J/kgK}) = 573.21 \text{ K}$$

$$P_0 = P_1 (T_0/T_1)^{k/(k-1)} = 1800 \text{ kPa } (573.21/573.15)^{4.058} = 1800.765 \text{ kPa}$$

From Eq.15.35 we get (k = 1.327)

$$P = P_0 [1 + \frac{k-1}{2} M^2]^{-k/(k-1)} = 1800.765 \text{ kPa } [1 + 0.1635]^{-4.058} = \mathbf{974 \text{ kPa}}$$

15.44

A jet plane travels through the air with a speed of 1000 km/h at an altitude of 6 km, where the pressure is 40 kPa and the temperature is −12°C. Consider the inlet diffuser of the engine where air leaves with a velocity of 100 m/s. Determine the pressure and temperature leaving the diffuser, and the ratio of inlet to exit area of the diffuser, assuming the flow to be reversible and adiabatic.

$\mathbf{V}$ = 1000 km/h = 277.8 m/s, v_1 = RT/P = 0.287 × 261.15/40 = 1.874 m^3/kg

h_1 = 261.48 kJ/kg,

h_{o1} = 261.48 + 277.8^2/2000 = 300.07 kJ/kg

$\Rightarrow$ T_{o1} = 299.7 K,

P_{o1} = P_1 $(T_{o1}/T_1)^{k/(k-1)}$ = 40 × $(299.7/261.15)^{3.5}$ = 64.766 kPa

h_2 = 300.07 - 100^2/2000 = 295.07 kJ/kg $\Rightarrow$ T_2 = **294.7 K**,

P_2 = P_{o1} $(T_2/T_{o1})^{k/(k-1)}$ = 64.766 × $(294.7/299.7)^{3.5}$ = **61 kPa**

v_2 = RT_2/P_2 = 0.287 kJ/kg-K × 294.7 K/61 kPa = 1.386 m^3/kg

A_1/A_2 = $(v_1/v_2)(V_2/V_1)$ = (1.874/1.386)(100/277.8) = **0.487**

15.45

Air flows into a convergent-divergent nozzle with an exit area of 1.59 times the throat area of 0.005 m^2. The inlet stagnation state is 1 MPa, 600 K. Find the backpressure that will cause subsonic flow throughout the entire nozzle with M = 1 at the throat. What is the mass flow rate?

This corresponds to case c and is a reversible flow.

$$A_E/A^* = 1.59 \qquad \text{Look at top in Table A.12 } (M < 1)$$

$$M_E = 0.4 \quad \text{and} \quad P_E/P_o = 0.8956$$

$$P_E = 0.8956 \; P_o = 0.8956 \times 1000 \text{ kPa} = \textbf{896 kPa}$$

To find the mass flow rate we have the maximum flow rate from Eq. 15.42,

$$\dot{m} = A^* \, P_o \, \sqrt{k/RT_o} \times [\frac{k+1}{2}]^{-(k+1)/2(k-1)}$$

$$= 0.005 \times 1000 \times 1000 \text{ N} \sqrt{1.4/(0.287 \times 600 \times 1000)} \text{ s/m} \times 1.2^{-2.4/0.8}$$
$$= \textbf{8.25 kg/s}$$

Notice kPa must be converted to Pa and also units for RT [kJ/kg = 1000 $(m/s)^2$]

15.46

A nozzle is designed assuming reversible adiabatic flow with an exit Mach number of 2.8 while flowing air with a stagnation pressure and temperature of 2 MPa and 150°C, respectively. The mass flow rate is 5 kg/s, and *k* may be assumed to be 1.40 and constant. Determine the exit pressure, temperature, exit area, and the throat area.

Since $M_E = 2.8$ we must have choked flow with $M^* = 1$, then the mass flow rate is maximum and follows Eq.15.42

$$\dot{m}/A^* = P_o \sqrt{k/RT_o} \times [\frac{k+1}{2}]^{-(k+1)/2(k-1)}$$

$$= 2000 \times 1000 \text{ N/m}^2 \sqrt{1.4/(0.287 \times 423.15 \times 1000)} \text{ s/m} \times 1.2^{-2.4/0.8}$$
$$= 3929.72 \text{ kg/s-m}^2$$

$$A^* = \dot{m} / (3929.72 \text{ kg/sm}^2) = (5 / 3929.72) \text{ m}^2 = \mathbf{1.272 \times 10^{-3} \, m^2}$$

From Table A.12: $M_E = 2.8$: $T_E / T_o = 0.38941$, $P_E / P_o = 0.036848$

$$A_E = A^* (A_E/A^*) = 1.272 \times 10^{-3} \times 3.5001 = \mathbf{4.452 \times 10^{-3} \, m^2}$$

$$T_E = T_o (T_E / T_o) = 423.15 \text{ K} \times 0.38941 = \mathbf{164.8 \, K}$$

$$P_E = P_o (P_E / P_o) = 2000 \text{ kPa} \times 0.036848 = \mathbf{73.7 \, kPa}$$

15.47

An air flow at 600 kPa, 600 K, $M = 0.3$ flows into a convergent-divergent nozzle with $M = 1$ at the throat. Assume a reversible flow with an exit area twice the throat area and find the exit pressure and temperature for subsonic exit flow to exist.

To find these properties we need the stagnation properties from the inlet state

From Table A.12: $M_i = 0.3$: $P_i/P_o = 0.93947$, $T_i/T_o = 0.98232$

$P_o = 600 \text{ kPa}/ 0.93947 = 638.66 \text{ kPa}$, $T_o = 600 \text{ K}/ 0.98232 = 610.8 \text{ K}$

This flow is case c in Figure 15.13. From Table A.12: $A_E/A^* = 2$

$P_E/P_o = 0.9360$, $T_E/T_o = 0.98127$

$P_E = 0.9360 \ P_o = 0.936 \times 638.66 \text{ kPa} = \textbf{597.8 kPa}$

$T_E = 0.98127 \ T_o = 0.98127 \times 610.8 \text{ K} = \textbf{599.4 K}$

15.48

Air at 150 kPa, 290 K expands to the atmosphere at 100 kPa through a convergent nozzle with exit area of 0.01 m^2. Assume an ideal nozzle. What is the percent error in mass flow rate if the flow is assumed incompressible?

$$T_e = T_i \left(\frac{P_e}{P_i}\right)^{\frac{k-1}{k}} = 258.28 \text{ K}$$

$$\mathbf{V}_e^2/2 = h_i - h_e = C_p (T_i - T_e) = 1.004 (290 - 258.28) = 31.83 \text{ kJ/kg}$$

$$\mathbf{V}_e = 252.3 \text{ m/s}; \quad v_e = \frac{RT_e}{P_e} = \frac{0.287 \times 258.28}{100} = 0.7412 \text{ m}^3/\text{kg}$$

$$\dot{m} = A\mathbf{V}_e / v_e = \frac{0.01 \times 252.3}{0.7413} = 3.4 \text{ kg/s}$$

Incompressible Flow: $v_i = RT/P = 0.287 \times 290/150 = 0.55487 \text{ m}^3/\text{kg}$

$$\mathbf{V}_e^2/2 = v \Delta P = v_i (P_i - P_e) = 0.55487 (150 - 100) = 27.74 \text{ kJ/kg}$$

$\Rightarrow$ $\mathbf{V}_e = 235 \text{ m/s}$ $\Rightarrow$ $\dot{m} = A\mathbf{V}_e / v_i = 0.01 \times 235 / 0.55487 = 4.23 \text{ kg/s}$

$$\frac{\dot{m}_{\text{incompressible}}}{\dot{m}_{\text{compressible}}} = \frac{4.23}{3.4} = 1.25 \qquad \text{about } \textbf{25\% overestimation.}$$

15.49

Find the exit pressure and temperature for supersonic exit flow to exist in the nozzle flow of Problem 15.47.

We assume a reversible as the possibility which is case d in Figure 15.13.

To find these properties we need the stagnation properties from the inlet state

From Table A.12: $M_i = 0.3$: $P_i/P_o = 0.93947$, $T_i/T_o = 0.98232$

$P_o = 600$ kPa/ $0.93947 = 638.66$ kPa, $T_o = 600$ K/ $0.98232 = 610.8$ K

From Table A.12: $A_E/A^* = 2$, $P_E/P_o = 0.09352$, $T_E/T_o = 0.50813$

$P_E = 0.09352\ P_o = 0.09352 \times 638.66 =$ **59.7 kPa**

$T_E = 0.50813\ T_o = 0.50813 \times 610.8 =$ **310.4 K**

This is significant lower P and T, but then we also have M = 2.2

15.50

Air is expanded in a nozzle from a stagnation state of 2 MPa, 600 K to a backpressure of 1.9 MPa. If the exit cross-sectional area is 0.003 m^2, find the mass flow rate.

This corresponds to case c and is a reversible flow.

$$P_E/P_{ox} = 1.9/2.0 = 0.95 \quad\quad \Rightarrow \text{ Table A.12:} \quad\quad M_E = 0.268$$

$$T_E = (T/T_o)_E \, T_o = 0.9854 \times 600 = 591.2 \text{ K}$$

$$c_E = \sqrt{kRT_E} = \sqrt{1.4 \times 1000 \times 0.287 \times 591.2} = 487.4 \text{ m/s}$$

$$\mathbf{V}_E = M_E c_E = 0.268 \times 487.4 = 130.6 \text{ m/s}$$

$$v_E = RT/P = 0.287 \text{ kJ/kg-K} \times 591.2 \text{ K}/1900 \text{ kPa} = 0.0893 \text{ m}^3/\text{kg}$$

$$\dot{m} = A_E \mathbf{V}_E / v_E = 0.003 \text{ m}^2 \times 130.6 \text{ (m/s)}/0.0893 \text{ m}^3/\text{kg} = \textbf{4.387 kg/s}$$

15.51

A 1-m^3 insulated tank contains air at 1 MPa, 560 K. The tank is now discharged through a small convergent nozzle to the atmosphere at 100 kPa. The nozzle has an exit area of 2×10^{-5} m^2.

    a. Find the initial mass flow rate out of the tank.

    b. Find the mass flow rate when half the mass has been discharged.

    a. The back pressure ratio:

$$P_B/P_{o1} = 100/1000 = 0.1 < (P^*/P_o)_{crit} = 0.5283$$

so the initial flow is choked with the maximum possible flow rate.

$$M_E = 1 \; ; \; P_E = 0.5283 \times 1000 = 528.3 \text{ kPa}$$

$$T_E = T^* = 0.8333 \times 560 \text{ K} = 466.7 \text{ K}$$

$$\mathbf{V}_E = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 466.7} = 433 \text{ m/s}$$

$$v_E = RT^*/P_E = 0.287 \text{ kJ/kg-K} \times 466.7 \text{ K}/528.3 \text{ kPa} = 0.2535 \text{ m}^3/\text{kg}$$

$$\dot{m}_1 = AV_E/v_E = 2 \times 10^{-5} \text{ m}^2 \times 433 \text{ (m/s)}/0.2535 \text{ m}^3/\text{kg} = \textbf{0.0342 kg/s}$$

    b. The initial mass is

$$m_1 = P_1 V/RT_1 = 1000 \times 1/(0.287 \times 560) = 6.222 \text{ kg}$$

with a mass at state 2 as $m_2 = m_1/2 = 3.111$ kg.

Assume an adiabatic reversible expansion of the mass that remains in the tank.

$$P_2 = P_1(v_1/v_2)^k = 100 \times 0.5^{1.4} = 378.9 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 560 \times 0.5^{0.4} = 424 \text{ K}$$

The pressure ratio is still less than critical and the flow thus choked.

$$P_B/P_{o2} = 100/378.9 = 0.264 < (P^*/P_o)_{crit}$$

$$M_E = 1 \; ; \; P_E = 0.5283 \times 378.9 = 200.2 \text{ kPa}$$

$$T_E = T^* = 0.8333 \times 424 = 353.7 \text{ K}$$

$$\mathbf{V}_E = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 353.7} = 377 \text{ m/s}$$

$$\dot{m}_2 = AV_E P_E/RT_E = \frac{2 \times 10^{-5}(377)(200.2)}{0.287(353.7)} = \textbf{0.0149 kg/s}$$

We could also have found the two mass flow rates from Eq. 15.42 without finding the exit plane conditions.

15.52

A convergent-divergent nozzle has a throat diameter of 0.05 m and an exit diameter of 0.1 m. The inlet stagnation state is 500 kPa, 500 K. Find the back pressure that will lead to the maximum possible flow rate and the mass flow rate for three different gases as: air; hydrogen or carbon dioxide.

There is a maximum possible flow when M = 1 at the throat,

$$T^* = \frac{2}{k+1} T_o; \quad P^* = P_o (\frac{2}{k+1})^{\frac{k}{k-1}}; \quad \rho^* = \rho_o (\frac{2}{k+1})^{\frac{1}{k-1}}$$

$$\dot{m} = \rho^* A^* V = \rho^* A^* c = P^* A^* \sqrt{k/RT^*}$$

$$A^* = \pi D^2/4 = 0.001963 \text{ m}^2$$

	k	T^*	P^*	c	ρ^*	$\dot{m}$
a)	1.400	416.7	264.1	448.2	2.209	1.944
b)	1.409	415.1	263.4	1704.5	0.154	0.515
c)	1.289	436.9	273.9	348.9	3.318	2.273

$A_E/A^* = (D_E/D^*)^2 = 4$. There are 2 possible solutions corresponding to points c and d in Fig. 15.13 and Fig. 15.17. For these we have

	Subsonic solution		Supersonic solution	
	M_E	P_E/P_o	M_E	P_E/P_o
a)	0.1466	0.985	2.940	0.0298
b)	0.1464	0.985	2.956	0.0293
c)	0.1483	0.986	2.757	0.0367

$P_B = P_E \cong 0.985 \times 500 = 492.5$ kPa all cases point c

a) $\quad P_B = P_E = 0.0298 \times 500 = 14.9$ kPa, point d

b) $\quad P_B = P_E = 0.0293 \times 500 = 14.65$ kPa, point d

c) $\quad P_B = P_E = 0.0367 \times 500 = 18.35$ kPa, point d

15.53

Air is expanded in a nozzle from a stagnation state of 2 MPa, 600 K, to a static pressure of 200 kPa. The mass flow rate through the nozzle is 5 kg/s. Assume the flow is reversible and adiabatic and determine the throat and exit areas for the nozzle.

$$P^* = P_o \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$
$$= 2 \times 0.5283 = 1.056 \text{ MPa}$$
$$T^* = T_o \times \frac{2}{k+1} = 600 \times 0.8333 = 500 \text{ K}$$
$$v^* = RT^*/P^* = 0.287 \times 500/1056$$
$$= 0.1359 \text{ m}^3/\text{kg}$$

The critical speed of sound is

$$c^* = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 500} = 448.2 \text{ m/s}$$

$$A^* = \dot{m}v^*/c^* = 5 \times 0.1359/448.2 = \mathbf{0.00152 \ m^2}$$

$$P_2/P_o = 200/2000 = 0.1 \qquad \Rightarrow \qquad M_2^* = 1.701 = \mathbf{V}_2/c^*$$

We used the column in Table A.12 with mach no. based on throat speed of sound.

$$\mathbf{V}_2 = 1.701 \times 448.2 = 762.4 \text{ m/s}$$

$$T_2 = T_o \ (T_2/T_o) = 600 \times 0.5176 = 310.56 \text{ K}$$

$$v_2 = RT_2/P_2 = 0.287 \text{ kJ/kg-K} \times 310.56 \text{ K}/200 \text{ kPa} = 0.4456 \text{ m}^3/\text{kg}$$

$$A_2 = \dot{m}v_2/\mathbf{V}_2 = 5 \text{ kg/s} \times 0.4456 \text{ (m}^3/\text{kg)} / 762.4 \text{ m/s} = \mathbf{0.00292 \ m^2}$$

15.54

Air flows into a convergent-divergent nozzle with an exit area of 2.0 times the throat area of 0.005 m^2. The inlet stagnation state is 1.2 MPa, 600 K. Find the backpressure that will cause a reversible supersonic exit flow with M = 1 at the throat. What is the mass flow rate?

This flow is case d in Fig.15.17 the only reversible supersonic flow.

$$A_E/A^* = 2 \quad \text{see Table A.12} \quad (M > 1)$$

$$\Rightarrow \quad M_E = 2.2 \quad \text{and} \quad P_E/P_o = 0.09399$$

$$P_E = 0.09399 \times 1200 = \textbf{112.8 kPa}$$

Using Eq.15.42 for the chocked flow rate

$$\dot{m} = A^* P_o \sqrt{k/RT_o} \times [\frac{k+1}{2}]^{-(k+1)/2(k-1)}$$

$$= 0.005 \times 1200 \times 1000 \sqrt{1.4/(0.287 \times 600 \times 1000)} \times 1.2^{-2.4/0.8}$$

$$= \textbf{9.9 kg/s}$$

Notice kPa must be converted to Pa and also units for RT [kJ/kg = 1000 (m/s)^2]

15.55

What is the exit pressure that will allow a reversible subsonic exit flow in the previous problem?

This flow is case c in Fig.15.17 (and c in Fig. 15.13) the only reversible subsonic flow with $M = 1$ at the throat.

$$A_E/A^* = 2 \quad \text{see Table A.12} \quad (M < 1)$$

$$P_E/P_o = 0.9360, \quad T_E/T_o = 0.98127$$

$$P_E = 0.9360 \, P_o = 0.936 \times 1200 = \mathbf{1123 \ kPa}$$

$$(\quad T_E = 0.98127 \, T_o = 0.98127 \times 600 = 588.8 \text{ K} \,)$$

15.56

A flow of helium flows at 500 kPa, 500 K with 100 m/s into a convergent-divergent nozzle. Find the throat pressure and temperature for reversible flow and $M = 1$ at the throat.

We need to find the stagnation properties first (k = 1.667)

$$T_o = T_1 + \mathbf{V}_1^2/2C_p = 500 + 100^2/(2 \times 5193) = 500.963 \text{ K}$$

$$P_o = P_1 \, (T_0/T_1)^{k/(k-1)} = 500 \, (500.963/500)^{2.5} = 502.41 \text{ kPa}$$

From the analysis we get Eqs.15.37-38

$$P^* = P_o \left(\frac{2}{k+1}\right)^{k/(k-1)} = 502.41 \left(\frac{2}{1.667+1}\right)^{2.5} = \mathbf{244.7 \ kPa}$$

$$T^* = T_o \frac{2}{k+1} = 500.963 \times \frac{2}{1.667+1} = \mathbf{375.7 \ K}$$

15.57

Assume the same tank and conditions as in Problem 15.51. After some flow out the nozzle flow changes to become subsonic. Find the mass in the tank and the mass flow rate out at that instant.

The initial mass is
$$m_1 = P_1 V/RT_1 = 1000 \times 1/(0.287 \times 560) = 6.222 \text{ kg}$$
The flow changes to subsonic when the pressure ratio reaches critical.
$$P_B/P_{o3} = 0.5283 \implies P_{o3} = 189.3 \text{ kPa}$$
$$v_1/v_3 = (P_{o3}/P_1)^{1/k} = (189.3/1000)^{0.7143} = 0.3046$$
$$m_3 = m_1 v_1/v_3 = 6.222 \times 0.3046 = \textbf{1.895 kg}$$
At this point the tank temperatures is (assuming isentropic expansion)
$$T_3 = T_1 (v_1/v_3)^{k-1} = 560 \times 0.3046^{0.4} = 348 \text{ K}$$
$$P_E = P_B = 100 \text{ kPa} ; \quad M_E = 1$$
$$T_E = 0.8333 \times 348 = 290 \text{ K} ; \quad V_E = \sqrt{kRT_E} = 341.4 \text{ m/s}$$
$$\dot{m}_3 = A V_E P_E/RT_E = \frac{2 \times 10^{-5}(341.4)(100)}{0.287(290)} = \textbf{0.0082 kg/s}$$
We could have used Eq.15.42 with $P_o = P_{o3} = 189.3$ kPa, $T_o = T_3 = 348$ K

15.58

A given convergent nozzle operates so it is choked with stagnation inlet flow properties of 400 kPa, 400 K. To increase the flow, a reversible adiabatic compressor is added before the nozzle to increase the stagnation flow pressure to 500 kPa. What happens to the flow rate?

Since the nozzle is choked the mass flow rate is given by Eq.15.42. The compressor changes the stagnation pressure and temperature.

Isentropic: $P_{o\ new} = P_o\ r_p$ and $T_{o\ new} = T_o\ (r_p)^{\frac{k-1}{k}}$

$$P_{o\ new} / \sqrt{T_{o\ new}} = (r_p)^{\frac{k+1}{2k}} \left[P_o / \sqrt{T_o} \right]$$

so the mass flow rate is multiplied with the factor

$$(r_p)^{\frac{k+1}{2k}} = \left(\frac{500}{400}\right)^{\frac{2.4}{2.8}} = 1.21$$

15.59

A 1-m^3 uninsulated tank contains air at 1 MPa, 560 K. The tank is now discharged through a small convergent nozzle to the atmosphere at 100 kPa while heat transfer from some source keeps the air temperature in the tank at 560 K. The nozzle has an exit area of 2×10^{-5} m^2.

  a. Find the initial mass flow rate out of the tank.

  b. Find the mass flow rate when half the mass has been discharged.

    a. The back pressure ratio:

$$P_B/P_{o1} = 100/1000 = 0.1 < (P^*/P_o)_{crit} = 0.5283$$

so the initial flow is choked with the maximum possible flow rate given in Eq.15.42

$$\dot{m} = A^* P_o \sqrt{k/RT_o} \times [\frac{k+1}{2}]^{-(k+1)/2(k-1)}$$

$$= 2 \times 10^{-5} \times 1 \times 10^6 \sqrt{1.4/(0.287 \times 560 \times 1000)} \times 1.2^{-2.4/0.8}$$

$$= \textbf{0.034 kg/s}$$

    b. The initial mass is

$$m_1 = P_1 V/RT_1 = 1000 \times 1/(0.287 \times 560) = 6.222 \text{ kg}$$

with a mass at state 2 as $m_2 = m_1/2 = 3.111$ kg.

$$P_2 = P_1/2 = 500 \text{ kPa} ; \ T_2 = T_1 ; \qquad P_B/P_2 = 100/500 = 0.2 < (P^*/P_o)_{crit}$$

The flow is choked and the mass flow rate is similar to the previously stated one but with half the stagnation pressure (same stagnation temperature):

$$\dot{m}_2 = A^* P_o \sqrt{k/RT_o} \times [\frac{k+1}{2}]^{-(k+1)/2(k-1)} = \dot{m}_1 / 2 = \textbf{0.017 kg/s}$$

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15.60

Assume the same tank and conditions as in Problem 15.59. After some flow out the nozzle flow changes to become subsonic. Find the mass in the tank and the mass flow rate out at that instant.

The initial mass is
$$m_1 = P_1 V/RT_1 = 1000 \times 1/(0.287 \times 560) = 6.222 \text{ kg}$$
Flow changes to subsonic when the pressure ratio reaches critical.
$$P_B/P_{o3} = 0.5283 ; \quad P_3 = P_{o3} = P_B/0.5283 = 100/0.5283 = 189.3 \text{ kPa}$$
$$m_3 = m_1 P_3/P_1 = \textbf{1.178 kg} ;$$
$$T_3 = T_1$$
$$T_E = T^* = 0.8333 \times 560 = 466.7 \text{ K}, \ P_E = P_B = 100 \text{ kPa}$$
$$\mathbf{V}_E = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 466.7} = 433 \text{ m/s}$$
$$\dot{m}_3 = A\mathbf{V}_E P_E/RT_E = \frac{2\times10^{-5}(433)(100)}{0.287(466.7)} = \textbf{0.00646 kg/s}$$
Flow is critical so we could have used Eq.15.42 with $P_o = 189.3$ kPa, $T_o = 560$ K

Normal Shocks

Borgnakke and Sonntag

15.61

The products of combustion, use air, enter a convergent nozzle of a jet engine at a total pressure of 125 kPa, and a total temperature of 650°C. The atmospheric pressure is 45 kPa and the flow is adiabatic, with a rate of 25 kg/s. Determine the exit area of the nozzle.

The critical pressure Table 15.1: $P_{crit} = P_2 = 125 \times 0.5283 = 66$ kPa $> P_{amb}$

The flow is then choked with M = 1 at the exit and a pressure drop outside of the nozzle.

$T_2 = 923.15 \times 0.8333 = 769.3$ K

$\mathbf{V}_2 = c_2 = \sqrt{1.4 \times 1000 \times 0.287 \times 769.3} = 556$ m/s

$v_2 = RT_2/P_2 = 0.287 \times 769.3/66 = 3.3453$ m^3/kg

$A_2 = \dot{m}v_2 / \mathbf{V}_2 = 25 \times 3.3453/556 = \mathbf{0.1504 \ m^2}$

15.62

Redo the previous problem for a mixture with k = 1.3 and molecular mass of 31.

The critical pressure Table 15.1: $P_{crit} = P_2 = 125 \times 0.5457 = 68.2$ kPa $> P_{amb}$

The flow is then choked. $T_2 = 923.15 \times 0.8696 = 802.8$ K

The gas constant is R = 8.31451 / 31 = 0.2682 kJ/kg-K

$$\mathbf{V}_2 = c_2 = \sqrt{1.3 \times 1000 \times 0.2682 \times 802.8} = 529 \text{ m/s}$$

$$v_2 = RT_2/P_2 = 0.2682 \times 802.8/68.2 = 3.157 \text{ m}^3/\text{kg}$$

$$A_2 = \dot{m}v_2 / \mathbf{V}_2 = 25 \times 3.157/529 = \mathbf{0.149 \ m^2}$$

15.63

At what Mach number will the normal shock occur in the nozzle of Problem 15.52 flowing with air if the back pressure is halfway between the pressures at *c* and *d* in Fig. 15.17?

First find the two pressures that will give exit at c and d. See solution to 15.52

a)

$$A_E/A^* = (D_E/D^*)^2 = 4 \quad \Rightarrow$$

For case c: $P_E = 492.5$ kPa   For case d: $P_E = 14.9$ kPa

Actual case: $P_E = (492.5 + 14.9)/2 = 253.7$ kPa

Assume $M_x = 2.4 \Rightarrow$ $M_y = 0.5231$; $P_{oy}/P_{ox} = 0.54015$

$$A_x/A_x^* = 2.4031 ; \quad A_x/A_y^* = 1.298$$

Work backwards to get P_E:

$$A_E/A_y^* = (A_E/A_x^*)(A_x/A_y^*)/(A_x/A_x^*) = 4 \times 1.298/2.4031 = 2.1605$$

$$\Rightarrow \quad M_E = 0.2807 ; \quad P_E/P_{oy} = 0.94675$$

$$P_E = (P_E/P_{oy})(P_{oy}/P_{ox}) P_{ox} = 0.94675 \times 0.54015 \times 500 = 255.7 \text{ kPa}$$

Repeat if $M_x = 2.5 \quad \Rightarrow \quad P_E = 233.8$ kPa

Interpolate to match the desired pressure => $M_x = $ **2.41**

15.64

Consider the nozzle of Problem 15.53 and determine what back pressure will cause a normal shock to stand in the exit plane of the nozzle. This is case g in Fig. 15.17. What is the mass flow rate under these conditions?

We assume reversible flow up to the shock

Table A.12: $P_E/P_o = 200/2000 = 0.1$; $M_E = 2.1591 = M_x$

Shock functions Table A.13: $M_y = 0.5529$; $P_y/P_x = 5.275$

$P_B = P_y = 5.275 \times P_x = 5.275 \times 200 = $ **1055 kPa**

$\dot{m} = $ **5 kg/s** same as in Problem 15.53 since M = 1 at throat.

15.65

A normal shock in air has upstream total pressure of 500 kPa, stagnation temperature of 500 K and $M_x = 1.4$. Find the downstream stagnation pressure.

From the normal shock relations in Section 15.8 found in Table A.13 we get

$$M_x = 1.4: \qquad P_{o\,y}/P_{o\,x} = 0.95819$$

$$P_{o\,y} = 0.95819\ P_{o\,x} = 0.95819 \times 500 = \textbf{479.1 kPa}$$

Remark: The stagnation temperature would be unchanged (energy equation).

15.66

How much entropy per kg flow is generated in the shock in Example 15.9?

The change in entropy is

$$s_{gen} = s_y - s_x = C_p \ln \frac{T_y}{T_x} - R \ln \frac{P_y}{P_x}$$

$$= 1.004 \ln 1.32 - 0.287 \ln 2.4583$$

$$= 0.27874 - 0.25815 = \mathbf{0.0206 \ kJ/kg \ K}$$

Notice that this function could have been tabulated also as

$$s_{gen}/R = \frac{k}{k-1} \ln \frac{T_y}{T_x} - \ln \frac{P_y}{P_x}$$

15.67

Consider the diffuser of a supersonic aircraft flying at $M = 1.4$ at such an altitude that the temperature is $-20°C$, and the atmospheric pressure is 50 kPa. Consider two possible ways in which the diffuser might operate, and for each case calculate the throat area required for a flow of 50 kg/s.

a. The diffuser operates as reversible adiabatic with subsonic exit velocity.

b. A normal shock stands at the entrance to the diffuser. Except for the normal shock the flow is reversible and adiabatic, and the exit velocity is subsonic. This is shown in Fig. P15.67.

a. Assume a convergent-divergent diffuser with $M = 1$ at the throat. Remember that for the supersonic flow the convergent front will reduce the velocity.

Relate the inlet state to the stagnation state state table A.12 with $M = 1.4$:

$P/P_O = 0.31424 \quad => \quad P_O = 50 \text{ kPa} / 0.31424 = 159.11 \text{ kPa}$

$T/T_O = 0.71839 \quad => \quad T_O = 253.15 \text{ K} / 0.71839 = 352.385 \text{ K}$

The maximum flow rate for chocked flow from Eq.15.42

$$\dot{m}/A^* = P_o \sqrt{k/RT_o} \times [\frac{k+1}{2}]^{-(k+1)/2(k-1)}$$

$$= 159.11 \times 10^3 \sqrt{1.4/(0.287 \times 253.15 \times 1000)} \times 1.2^{-2.4/0.8}$$

$$= 342.59 \text{ kg/sm}^2$$

$$A^* = \dot{m}/342.59 \text{ kg/sm}^2 = \textbf{0.1459 m}^2$$

b. Across the shock we have, $M_x = 1.4$ so from Table A.13

$M_y = 0.7397 ; \quad P_{oy}/P_{ox} = 0.95819 ;$

The stagnation properties after the shock are:

$T_{oy} = T_{ox} = 253.15 / 0.71839 = 352.385 \text{ K (as before)}$

$P_{oy} = P_{ox} \times 0.95819 = (50 \text{ kPa} / 0.31424) \times 0.95819 = 152.46 \text{ kPa}$

Only the stagnation pressure changed due to the shock so Eq.15.42 gives

$$\dot{m}/A^* = 0.95819 \times 342.59 \text{ kg/sm}^2 = 328.266 \text{ kg/sm}^2$$

$$A^* = \dot{m}/328.266 \text{ kg/sm}^2 = \textbf{0.1523 m}^2$$

15.68

A flow into a normal shock in air has a total pressure 400 kPa, stagnation temperature of 600 K and $M_x = 1.2$. Find the upstream temperature T_x, the specific entropy generation in the shock and the downstream velocity.

From Table A.12: $M_x = 1.2$ has $T_x/T_o = 0.7764$, $P_x/P_o = 0.41238$

$$T_x = 0.7764\ T_o = 0.7764 \times 600\ K = \textbf{465.84 K}$$

From Table A.13: $T_y/T_x = 1.128$, $P_y/P_x = 1.5133$, $M_y = 0.84217$

The change in entropy is

$$s_{gen} = s_y - s_x = C_p \ln \frac{T_y}{T_x} - R \ln \frac{P_y}{P_x}$$

$$= 1.004 \ln 1.128\ - 0.287 \ln 1.5133$$

$$= 0.12093 - 0.11890 = \textbf{0.00203 kJ/kg K}$$

From the shock relations we had

$$T_y = 1.128\ T_x = 1.128 \times 465.84\ K = 525.47\ K$$

$$\textbf{V}_y = M_y\ c_y = M_y \sqrt{kRT_y} = 0.84217 \sqrt{1.4 \times 0.287 \times 525.47 \times 1000}$$

$$= 0.84217 \times 459.49 = \textbf{387 m/s}$$

15.69

Consider the nozzle in problem 15.42 flowing helium. What should the backpressure be for a normal shock to stand at the exit plane? This is case g in Fig.15.17. What is the exit velocity after the shock?

Reversible flow up to the shock with M = 1 at the throat.

$$P_{x\,o} = P_o, \quad T_{x\,o} = T_o, \qquad A_E/A^* = 175 / 100 = 1.75$$

As we do not have a Table 12 made for helium (k = 5/3 = 1.667) we need to use the formulas. The area to M relation is given in Eq.15.43 so

$$A/A^* = 1.75 = (1/M_x)\left[\frac{2}{k+1}(1 + (1/3)M_x^2)\right]^2 = [3 + M_x^2]^2 / 16\,M_x$$

There are two solutions and we need the one with $M_x > 1$, see fig 15.10 we should expect a solution around $M_x = 2$. With trial and error on

$$3 + M_x^2 = \sqrt{16 \times 1.75\,M_x} \quad \text{which gives } M_x = 2.2033$$

Now we can do the normal shock from Eq.15.53

$$M_x = 2.2033 \quad \Rightarrow \quad M_y^2 = \frac{2.2033^2 + 3}{5 \times 2.2033^2 - 1} = 0.3375,$$

The temperature ratio from Eq. 15.49 and the T_x from Eq. 15.34:

$$T_y = T_x \frac{1 + 0.5(k-1)\,M_x^2}{1 + 0.5(k-1)\,M_y^2} = \frac{T_{x\,o}}{1 + 0.5(k-1)\,M_y^2} = \frac{375}{1 + 0.333 \times 0.3375} = 337.1$$

K

$$V_y = M_y\,c_y = M_y\sqrt{kRT_y} = \sqrt{0.3375} \times \sqrt{1.667 \times 2.0771 \times 337.1 \times 1000}$$
$$= 0.58095 \times 1080.4 = \textbf{627.6 m/s}$$

15.70

Find the specific entropy generation in the shock of the previous Problem.

Reversible flow up to the shock with $M = 1$ at the throat.

$$P_{x\,o} = P_o, \quad T_{x\,o} = T_o, \quad A_E/A^* = 175 \,/\, 100 = 1.75$$

Table A.12: $\quad M_E = M_x = 2.042, \quad P_x/P_{o\,x} = 0.12, \; T_x/T_{o\,x} = 0.5454$

Now we can do the normal shock from Table A.13

$$M_x = 2.042 \;\Rightarrow\; M_y = 0.5704, \; P_y/P_x = 4.6984, \; T_y/T_x = 1.7219$$

The change in entropy is

$$s_{gen} = s_y - s_x = C_p \ln \frac{T_y}{T_x} - R \ln \frac{P_y}{P_x}$$

$$= 1.004 \ln 1.7219 \; - 0.287 \ln 4.6984$$

$$= 0.5456 - 0.44405 = \mathbf{0.1015 \; kJ/kg \; K}$$

Nozzles, Diffusers, and Orifices

15.71

Steam at 600 kPa, 300°C is fed to a set of convergent nozzles in a steam turbine. The total nozzle exit area is 0.005 m^2 and they have a discharge coefficient of 0.94. The mass flow rate should be estimated from the measurement of the pressure drop across the nozzles, which is measured to be 200 kPa. Determine the mass flow rate.

Inlet B.1.3 $h_i = 3061.6$ kJ/kg, $s_i = 7.3724$ kJ/kg K

Exit: $(P_e, s_{e,s})$ $P_e = P_i - 200 = 400$ kPa, $s_{e,s} = s_i = 7.3724$ kJ/kg K

$\Rightarrow h_{e,s} = 2961$ kJ/kg and $v_{e,s} = 0.5932$ m^3/kg,

$\mathbf{V}_{e,s} = \sqrt{2 \times 1000(3061.6 - 2961)} = 448.55$ m/s

$\dot{m}_s = A\mathbf{V}_{e,s}/v_{e,s} = 0.005 \times 448.55/0.5932 = 3.781$ kg/s

$\dot{m}_a = C_D\dot{m}_s = 0.94 \times 3.781 = \mathbf{3.554\ kg/s}$

15.72

Air enters a diffuser with a velocity of 200 m/s, a static pressure of 70 kPa, and a temperature of −6°C. The velocity leaving the diffuser is 60 m/s and the static pressure at the diffuser exit is 80 kPa. Determine the static temperature at the diffuser exit and the diffuser efficiency. Compare the stagnation pressures at the inlet and the exit.

Stagnation T at the inlet

$$T_{o1} = T_1 + \mathbf{V}_1^2/2C_p = 267.15 + 200^2/(2000 \times 1.004) = 287.1 \text{ K}$$

Energy Eq. gives the same stagnation T at exit

$$T_{o2} = T_{o1} \Rightarrow T_2 = T_{o2} - \mathbf{V}_2^2/2C_p = 287.1 - 60^2/(2000 \times 1.004) = 285.3 \text{ K}$$

$$\frac{T_{o1} - T_1}{T_1} = \frac{k-1}{k}\frac{P_{o1} - P_1}{P_1} \qquad \Rightarrow \quad P_{o1} - P_1 = 18.25 \Rightarrow P_{o1} = 88.3 \text{ kPa}$$

$$\frac{T_{o2} - T_2}{T_2} = \frac{k-1}{k}\frac{P_{o2} - P_2}{P_2} \qquad \Rightarrow \quad P_{o2} - P_2 = 1.77 \Rightarrow P_{o2} = 81.8 \text{ kPa}$$

$$T_s^{ex} = T_1 (P_{o2}/P_1)^{k-1/k} = 267.15 \times 1.0454 = \mathbf{279.3 \text{ K}}$$

$$\eta_D = \frac{T_s^{ex} - T_1}{T_{o1} - T_1} = \frac{279.3 - 267.15}{287.1 - 267.15} = \mathbf{0.608}$$

15.73

Repeat Problem 15.44 assuming a diffuser efficiency of 80%.

$V = 1000$ km/h $= 277.8$ m/s, $\quad v_1 = RT/P = 0.287 \times 261.15/40 = 1.874$ m^3/kg

$h_1 = 261.48$ kJ/kg,

$h_{o1} = 261.48 + 277.8^2/2000 = 300.07$ kJ/kg

$\Rightarrow T_{o1} = 299.7$ K,

$P_{o1} = P_1 (T_{o1}/T_1)^{k/(k-1)} = 40 \times (299.7/261.15)^{3.5} = 64.766$ kPa

Same as problem 15.44, except

$\eta_D = 0.80$. We thus have from 15.44

$$\frac{h_3 - h_1}{h_{o1} - h_1} = \frac{h_3 - 261.48}{300.07 - 261.48} = 0.8$$

$\Rightarrow h_3 = 292.35$ kJ/kg, $T_3 = 291.9$ K

$P_{o2} = P_3 = P_1 (T_3/T_1)^{k/(k-1)}$

$= 40 (291.9/261.15)^{3.5} = 59.06$ kPa

$T_{o2} = T_{o1} = 299.7$ K

$h_2 = 300.07 - 100^2/2000 = 295.07$ kJ/kg $\quad \Rightarrow T_2 = $ **294.7 K**,

$P_2 = P_{o2} (T_2/T_{o1})^{k/(k-1)} = 59.06 \times (294.7/299.7)^{3.5} = $ **55.68 kPa**

$v_2 = RT_2/P_2 = 0.287 \times 294.7/55.68 = 1.519$ m^3/kg

$A_1/A_2 = (v_1/v_2)(V_2/V_1) = (1.874/1.519) (100/277.8) = $ **0.444**

15.74

A sharp-edged orifice is used to measure the flow of air in a pipe. The pipe diameter is 100 mm and the diameter of the orifice is 25 mm. Upstream of the orifice, the absolute pressure is 150 kPa and the temperature is 35°C. The pressure drop across the orifice is 15 kPa, and the coefficient of discharge is 0.62. Determine the mass flow rate in the pipeline.

$$\Delta T = T_i\left(\frac{k-1}{k}\right)\frac{\Delta P}{P_i} = 308.15 \times \frac{0.4}{1.4} \times \frac{15}{150} = 8.8 \text{ K}$$

$$v_i = RT_i/P_i = 0.5896 \text{ m}^3/\text{kg}$$

$$P_e = 135 \text{ kPa},\ \ T_e = 299.35 \text{ K},\ \ v_e = 0.6364 \text{ m}^3/\text{kg}$$

$$\dot{m}_i = \dot{m}_e \implies \quad \mathbf{V}_i / \mathbf{V}_e = (D_e/D_i)^2\, v_i/v_e = 0.0579$$

$$h_i - h_e = \mathbf{V}_e^2(1 - 0.0579^2)/2 = C_p(T_i - T_e)$$

$$\mathbf{V}_{e\,s} = \sqrt{2 \times 1000 \times 1.004 \times 8.8/(1 - 0.0579)^2} = 133.1 \text{ m/s}$$

$$\dot{m} = C_D\, A\mathbf{V}/v = 0.62\ (\pi/4)\ (0.025)^2\ 133.1\ /\ 0.6364 = \mathbf{0.06365 \text{ kg/s}}$$

15.75

A critical nozzle is used for the accurate measurement of the flow rate of air. Exhaust from a car engine is diluted with air so its temperature is 50°C at a total pressure of 100 kPa. It flows through the nozzle with throat area of 700 mm^2 by suction from a blower. Find the needed suction pressure that will lead to critical flow in the nozzle and the mass flow rate.

$$P^* = 0.5283\ P_o = \mathbf{52.83\ kPa},\ \ T^* = 0.8333\ T_o = 269.3\ K$$

$$v^* = RT^*/P^* = 0.287 \times 269.3/52.83 = 1.463\ m^3/kg$$

$$c^* = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 269.3} = 328.9\ m/s$$

$$\dot{m} = Ac^*/v^* = 700 \times 10^{-6} \times 328.9/1.463 = \mathbf{0.157\ kg/s}$$

15.76

Air is expanded in a nozzle from 700 kPa, 200°C, to 150 kPa in a nozzle having an efficiency of 90%. The mass flow rate is 4 kg/s. Determine the exit area of the nozzle, the exit velocity, and the increase of entropy per kilogram of air. Compare these results with those of a reversible adiabatic nozzle.

$$T_{2s} = T_1(P_2/P_1)^{(k-1)/k} = 473.2 \ (150/700)^{0.286} = 304.6 \text{ K}$$

$$V_{2s}^2 = 2 \times 1000 \times 1.004(473.2 - 304.6) = 338 \ 400 \text{ J/kg}$$

$$V_2^2 = 0.9 \times 338 \ 400 \text{ m}^2/\text{s}^2 \ \Rightarrow \ V_2 = \textbf{552 m/s}$$

$$h_2 + V_2^2/2 = h_1 \ \Rightarrow \ T_2 = T_1 - V_2^2/2C_p$$

$$T_2 = 473.2 - 552^2/(2 \times 1000 \times 1.004) = 321.4 \text{ K} \ ;$$

$$v_2 = RT/P = 0.287 \text{ kJ/kg} \times 321.4 \text{ K}/150 \text{ kPa} = 0.6149 \text{ m}^3/\text{kg}$$

$$A_2 = \dot{m} \ v_2/V_2 = 4 \text{ kg/s} \times 0.6149 \text{ m}^3/\text{kg} \ /(552 \text{ m/s}) = 0.00446 \text{ m}^2 = \textbf{4460 mm}^2$$

$$s_2 - s_1 = 1.0035 \ \ln\!\left(\frac{321.4}{473.2}\right) - 0.287 \ \ln\!\left(\frac{150}{700}\right) = \textbf{0.0539 kJ/kg K}$$

15.77

Steam at a pressure of 1 MPa and temperature of 400°C expands in a nozzle to a pressure of 200 kPa. The nozzle efficiency is 90% and the mass flow rate is 10 kg/s. Determine the nozzle exit area and the exit velocity.

First do the ideal reversible adiabatic nozzle

$$s_{2s} = s_1 = 7.4651 \text{ kJ/kg K}, \quad h_1 = 3263.9 \text{ kJ/kg}$$

$$\Rightarrow \ T_{2s} = 190.4°C \ ; \quad h_{2s} = 2851 \text{ kJ/kg}$$

Now the actual nozzle can be calculated

$$h_1 - h_{2ac} = \eta_D(h_1 - h_{2s}) = 0.9(3263.9 - 2851) = 371.6 \text{ kJ/kg}$$

$$h_{2ac} = 2892.3 \text{ kJ/kg}, \quad T_2 = 210.9°C, \quad v_2 = 1.1062 \text{ m}^3\text{/kg}$$

$$\mathbf{V_2} = \sqrt{2000(3263.9 - 2892.3)} = \mathbf{862 \text{ m/s}}$$

$$A_2 = \dot{m}v_2/\mathbf{V_2} = 10 \text{ kg/s} \times 1.1062 \text{ m}^3\text{/kg} /(862 \text{ m/s}) = \mathbf{0.01283 \text{ m}^2}$$

15.78

Steam at 800 kPa, 350°C flows through a convergent-divergent nozzle that has a throat area of 350 mm^2. The pressure at the exit plane is 150 kPa and the exit velocity is 800 m/s. The flow from the nozzle entrance to the throat is reversible and adiabatic. Determine the exit area of the nozzle, the overall nozzle efficiency, and the entropy generation in the process.

$h_{o1} = 3161.7$ kJ/kg, $s_{o1} = 7.4089$ kJ/kg K

$P^*/P_{o1} = (2/(k+1))^{k/(k-1)} = 0.54099 \Rightarrow P^* = 432.7$ kPa

At *: $(P^*, s^* = s_{o1}) \Rightarrow h^* = 2999.3$ kJ/kg, $v^* = 0.5687$ m^3/kg

$\Delta h = \mathbf{V}^2/2 \Rightarrow \mathbf{V}^* = \sqrt{2000(3161.7-2999.3)} = 569.9$ m/s

$\dot{m} = A\mathbf{V}^*/v^* = 350 \times 10^{-6} \times 569.9/0.5687 = 0.3507$ kg/s

$h_e = h_{o1} - \mathbf{V}_e^2/2 = 3161.7 - 800^2/2 \times 1000 = 2841.7$ kJ/kg

Exit: P_e, h_e: $v_e = 1.395$ m^3/kg, $s_e = 7.576$ kJ/kg K

$A_e = \dot{m}v_e/\mathbf{V}_e = 0.3507 \times 1.395/800 = \mathbf{6.115 \times 10^{-4}\ m^2}$

$s_{gen} = s_e - s_{o1} = 7.576 - 7.4089 = \mathbf{0.167\ kJ/kg\ K}$

15.79

A convergent nozzle with exit diameter of 2 cm has an air inlet flow of 20°C, 101 kPa (stagnation conditions). The nozzle has an isentropic efficiency of 95% and the pressure drop is measured to 50 cm water column. Find the mass flow rate assuming compressible adiabatic flow. Repeat calculation for incompressible flow.

Convert ΔP to kPa:

$\Delta P = 50$ cm $H_2O = 0.5 \times 9.8064 = 4.903$ kPa

$T_0 = 20°C = 293.15$ K $\qquad P_0 = 101$ kPa

Assume inlet $V_i = 0$ $\qquad P_e = P_0 - \Delta P = 101 - 4.903 = 96.097$ kPa

$T_e = T_0 \left(\dfrac{P_e}{P_0}\right)^{\frac{k-1}{k}} = 293.15 \times \left(\dfrac{96.097}{101}\right)^{0.2857} = 289.01$

$V_e^2/2 = h_i - h_e = C_p (T_i - T_e) = 1.004 \times (293.15 - 289.01)$

$\qquad = 4.1545$ kJ/kg $= 4254.5$ J/kg $\qquad\qquad \Rightarrow \; V_e = 91.15$ m/s

$V_{e\,ac}^2/2 = \eta\, V_{e\,s}^2/2 = 0.95 \times 4154.5 = 3946.78 \qquad \Rightarrow \; V_{e\,ac} = 88.85$ m/s

$T_{e\,ac} = T_i - \dfrac{V_{e\,ac}^2/2}{C_p} = 293.15 - \dfrac{3.9468}{1.0035} = 289.2$ K

$\rho_{e\,ac} = \dfrac{P_e}{RT_p} = \dfrac{96.097}{0.287 \times 289.2} = 1.158$ kg/m^3

$\dot{m} = \rho A V = 1.158$ kg/m$^3 \times \dfrac{\pi}{4} \times 0.02^2$ m$^2 \times 88.85$ m/s $= \mathbf{0.0323}$ **kg/s**

For incompressible flow:

$V_e^2/2 = (P_i - P_e)v = (P_i - P_e)\dfrac{RT_i}{P_i} = 4.903$ kPa $\times \dfrac{0.287 \text{ kJ/kg-K } 293.15 \text{ K}}{101 \text{ kPa}}$

$\qquad = 4.903 \times 0.833$ kJ/kg $= 4.084$ kJ/kg $= 4084$ J/kg

$V_{e\,ac}^2/2 = \eta\, V_{e\,s}^2/2 = 0.95 \times 4084 = 3880$ J/kg $\qquad \Rightarrow \; V_{e\,ac} = 88.09$ m/s

$\dot{m} = \rho A V = (1/0.833)$ kg/m$^3 \times \dfrac{\pi}{4} \times 0.02^2$ m$^2 \times 88.09$ m/s $= \mathbf{0.0332}$ **kg/s**

15.80

The coefficient of discharge of a sharp-edged orifice is determined at one set of conditions by use of an accurately calibrated gasometer. The orifice has a diameter of 20 mm and the pipe diameter is 50 mm. The absolute upstream pressure is 200 kPa and the pressure drop across the orifice is 82 mm of mercury. The temperature of the air entering the orifice is 25°C and the mass flow rate measured with the gasometer is 2.4 kg/min. What is the coefficient of discharge of the orifice at these conditions?

$$\Delta P = 82 \times 101.325/760 = 10.93 \text{ kPa}$$

$$\Delta T = T_i \left(\frac{k\text{-}1}{k}\right) \Delta P/P_i = 298.15 \times \frac{0.4}{1.4} \times 10.93/200 = 4.66$$

$$v_i = RT_i/P_i = 0.4278 \text{ m}^3/\text{kg}, \quad v_e = RT_e/P_e = 0.4455 \text{ m}^3/\text{kg}$$

$$\mathbf{V}_i = \mathbf{V}_e A_e v_i / A_i v_e = 0.1536 \, \mathbf{V}_e$$

$$(\mathbf{V}_e^2 - \mathbf{V}_i^2)/2 = \mathbf{V}_e^2 (1 - 0.1536^2)/2 = h_i - h_e = C_p \Delta T$$

$$\mathbf{V}_e = \sqrt{2 \times 1000 \times 1.004 \times 4.66/(1 - 0.1536^2)} = 97.9 \text{ m/s}$$

$$\dot{m}_s = A_e \mathbf{V}_e / v_e = \frac{\pi}{4} \times 0.02^2 \times 97.9/0.4455 = 0.069 \text{ kg/s}$$

$$C_D = \dot{m}_a / \dot{m}_s = 2.4/(60 \times 0.069) = \mathbf{0.58}$$

15.81

A convergent nozzle is used to measure the flow of air to an engine. The atmosphere is at 100 kPa, 25°C. The nozzle used has a minimum area of 2000 mm^2 and the coefficient of discharge is 0.95. A pressure difference across the nozzle is measured to 2.5 kPa. Find the mass flow rate assuming incompressible flow. Also find the mass flow rate assuming compressible adiabatic flow.

Assume $\mathbf{V}_i \cong 0$, $\quad v_i = RT_i/P_i = 0.287 \times 298.15/100 = 0.8557 \text{ m}^3/\text{kg}$

Incompressible flow:

$\mathbf{V}_{e,s}^{\ 2}/2 = h_i - h_{e,s} = v_i(P_i - P_e) = 2.1393 \text{ kJ/kg}$

$\mathbf{V}_{e,s} = \sqrt{2 \times 1000 \times 2.1393} = 65.41 \text{ m/s}$

$\dot{m}_s = A\mathbf{V}_{e,s}/v_i = 2000 \times 10^{-6} \times 65.41/0.8557 = 0.153 \text{ kg/s}$

$\dot{m}_a = C_D\dot{m}_s = \mathbf{0.1454 \text{ kg/s}}$

Compressible flow:

$T_{e,s} = T_i\ (P_e/P_i)^{(k-1)/k} = 298.15(97.5/100)^{0.2857} = 296 \text{ K}$

$\Delta h = C_p\Delta T = 1.0035 \times 2.15 = 2.1575 = \mathbf{V}_{e,s}^{\ 2}/2$

$\mathbf{V}_{e,s} = \sqrt{2 \times 1000 \times 2.1575} = 65.69 \text{ m/s}$

$v_{e,s} = 0.287 \times 296/97.5 = 0.8713 \text{ m}^3/\text{kg}$

$\dot{m}_s = A\mathbf{V}_{e,s}/v_{e,s} = 2000 \times 10^{-6} \times 65.69/0.8713 = 0.1508 \text{ kg/s}$

$\dot{m}_a = C_D\dot{m}_s = \mathbf{0.1433 \text{ kg/s}}$

Review Problems

15.82

Atmospheric air is at 20°C, 100 kPa with zero velocity. An adiabatic reversible compressor takes atmospheric air in through a pipe with cross-sectional area of 0.1 m^2 at a rate of 1 kg/s. It is compressed up to a measured stagnation pressure of 500 kPa and leaves through a pipe with cross-sectional area of 0.01 m^2. What are the required compressor work and the air velocity, static pressure, and temperature in the exit pipeline?

C.V. compressor out to standing air and exit to stagnation point.

$$\dot{m}\,h_{o1} + \dot{W}_c = \dot{m}(h + \mathbf{V}^2/2)_{ex} = \dot{m}h_{o,ex}$$

$$\dot{m}s_{o1} = \dot{m}s_{o,ex} \Rightarrow P_{r,o,ex} = P_{r,o1}$$

$$(P_{st,ex}/P_{o1}) = 1.028(500/100) = 5.14$$

$$\Rightarrow T_{o,ex} = 463 \text{ K}, \quad h_{o,ex} = 465.38 \text{ kJ/kg}, \quad h_{o1} = 209.45 \text{ kJ/kg}$$

$$\dot{W}_c = \dot{m}(h_{o,ex} - h_{o1}) = 1(465.38 - 209.45) = \textbf{255.9 kW}$$

$$P_{ex} = P_{o,ex}(T_{ex}/T_{o,ex})^{k/(k-1)} \qquad T_{ex} = T_{o,ex} - \mathbf{V}^2_{ex}/2C_p$$

$$\dot{m} = 1 \text{ kg/s} = (\rho A\mathbf{V})_{ex} = P_{ex}A\mathbf{V}_{ex}/RT_{ex}$$

Now select 1 unknown amongst P_{ex}, T_{ex}, $\mathbf{V}_{ex}$ and write the continuity eq. $\dot{m}$ and solve the nonlinear equation. Say, use T_{ex} then

$$\mathbf{V}_{ex} = \sqrt{2C_p(T_{o,ex} - T_{ex})}$$

$$\dot{m} = 1 \text{ kg/s} = P_{o,ex}(T_{ex}/T_{o,ex})^{k/k-1}A\sqrt{2C_p(T_{o,ex} - T_{ex})}/RT_{ex}$$

solve for $T_{ex}/T_{o,ex}$ (close to 1)

$$T_{ex} = \textbf{462.6 K} \Rightarrow \mathbf{V}_{ex} = \textbf{28.3 m/s}, \quad P_{ex} = \textbf{498.6 kPa}$$

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15.83

The nozzle in Problem 15.46 will have a throat area of 0.001272 m^2 and an exit area 3.50 times as large. Suppose the back pressure is raised to 1.4 MPa and that the flow remains isentropic except for a normal shock wave. Verify that the shock mach number (M_x) is close to 2 and find the exit mach number, the temperature and the mass flow rate through the nozzle.

Since a shock is present we must have choked flow with $M^* = 1$, then the mass flow rate is maximum and follows Eq.15.42

$$\dot{m}/A^* = P_o \sqrt{k/RT_o} \times [\frac{k+1}{2}]^{-(k+1)/2(k-1)}$$

$$= 2000 \text{ kPa} \times 1000\sqrt{1.4/(0.287 \times 423.15 \times 1000)} \text{ m/s} \times 1.2^{-2.4/0.8}$$

$$= 3929.72 \text{ kg/sm}^2$$

$$\dot{m} = A^* \times 3929.72 \text{ kg/sm}^2 = \textbf{5 kg/s}$$

Assume $M_x = 2$ then

Table A.13: $M_y = 0.57735$, $P_{oy}/P_{ox} = 0.72087$, $A_E/A_x^* = 3.50$

From A.12, $M_x = 2$: $A_x/A_x^* = 1.6875$,   (upstream),

$M_y = 0.57735$: $A_y/A_y^* = 1.2225$, (downstream)

Now since $A_y = A_x$ then (see Ex. 15.9 page 734)

$$A_E/A_y^* = [A_E/A_x^*][A_y/A_y^*][A_x^*/A_x] = 3.5 \times 1.2225/1.6875 = 2.5356$$

From A.12 with A/A* then: $M_E = \textbf{0.246}$, $P_E/P_{oy} = 0.95728$

$P_E = 0.95728 \times 0.72087 \times 2.0 = 1.38$ MPa, OK close to the 1.4 MPa

$T_E = 0.9876 \times 423.15 = \textbf{418 K}$,

15.84

At what Mach number will the normal shock occur in the nozzle of Problem 15.53 if the back pressure is 1.4 MPa? (trial and error on M_x)

Relate the inlet and exit conditions to the shock conditions with reversible flow before and after the shock. It becomes trial and error.

Assume $M_x = 1.8 \Rightarrow M_y = 0.6165$; $P_{oy}/P_{ox} = 0.8127$

$A_E/A_x^* = A_2/A^* = 0.002435/0.001516 = 1.6062$

$A_x/A_x^* = 1.439$; $A_x/A_y^* = 1.1694$

$A_E/A_y^* = (A_E/A_x^*)(A_x/A_y^*)/(A_x/A_x^*) = \dfrac{1.6062(1.1694)}{1.439} = 1.3053$

$\Rightarrow M_E = 0.5189$; $P_E/P_{oy} = 0.8323$

$P_E = (P_E/P_{oy})(P_{oy}/P_{ox})P_{ox} = 0.8323 \times 0.8127 \times 2000 = 1353$ kPa < 1.4 MPa

So select the mach number a little less

$M_x = 1.7 \Rightarrow M_y = 0.64055$; $P_{oy}/P_{ox} = 0.85573$

$A_x/A_x^* = 1.3376$; $A_x/A_y^* = 1.1446$

$A_E/A_y^* = (A_E/A_x^*)(A_x/A_y^*)/(A_x/A_x^*) = \dfrac{1.6062(1.1446)}{1.3376} = 1.3744$

$\Rightarrow M_E = 0.482$; $P_E/P_{oy} = 0.853$

$P_E = (P_E/P_{oy})(P_{oy}/P_{ox})P_{ox} = 0.853 \times 0.85573 \times 2000 = 1459.9$ kPa

Now interpolate between the two

$M_x = \mathbf{1.756}$ and we check $\Rightarrow M_y = 0.6266$; $P_{oy}/P_{ox} = 0.832$

$A_x/A_x^* = 1.3926$; $A_x/A_y^* = 1.1586$

$A_E/A_y^* = 1.6062 \times 1.1586/1.3926 = 1.3363$

$\Rightarrow M_E = 0.5$; $P_E/P_{oy} = 0.843$

$P_E = 0.843 \times 0.832 \times 2000 = 1402.7$ kPa **OK**

Solution using the Pr or vr functions

15.44

A jet plane travels through the air with a speed of 1000 km/h at an altitude of 6 km, where the pressure is 40 kPa and the temperature is −12°C. Consider the inlet diffuser of the engine where air leaves with a velocity of 100 m/s. Determine the pressure and temperature leaving the diffuser, and the ratio of inlet to exit area of the diffuser, assuming the flow to be reversible and adiabatic.

$$\mathbf{V} = 1000 \text{ km/h} = 277.8 \text{ m/s}, \quad v_1 = \frac{RT}{P} = \frac{0.287 \times 261.15}{40} = 1.874 \text{ m}^3/\text{kg}$$

$$h_1 = 261.48 \text{ kJ/kg}, \quad P_{r1} = 0.6862$$

$$h_{o1} = 261.48 + 277.8^2/2000 = 300.07 \text{ kJ/kg}$$

$$\Rightarrow T_{o1} = 299.7 \text{ K}, \quad P_{ro1} = 1.1107$$

The ratio of the pressures equals the ratio of the Pr functions when s = constant

$$P_{o1} = P \, P_{ro1}/ P_{r1} = 40 \times 1.1107/0.6862 = 64.74 \text{ kPa}$$

$$h_2 = 300.07 - 100^2/2000 = 295.07 \quad \Rightarrow \quad T_2 = \mathbf{294.7 \text{ K}}, \quad P_{r2} = 1.0462$$

$$P_2 = 64.74 \times 1.0462/1.1107 = \mathbf{61 \text{ kPa}}$$

$$v_2 = RT_2/P_2 = 0.287 \times 294.7/61 = 1.386 \text{ m}^3/\text{kg}$$

$$A_1/A_2 = (v_1/v_2)(V_2/V_1) = (1.874/1.386)(100/277.8) = \mathbf{0.487}$$

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15.73

Repeat Problem 15.44 assuming a diffuser efficiency of 80%.

$$V = 1000 \text{ km/h} = 277.8 \text{ m/s}, \quad v_1 = \frac{RT}{P} = \frac{0.287 \times 261.15}{40} = 1.874 \text{ m}^3/\text{kg}$$

$$h_1 = 261.48 \text{ kJ/kg}, \quad P_{r1} = 0.6862$$

$$h_{o1} = 261.48 + 277.8^2/2000 = 300.07 \text{ kJ/kg}$$

$$\Rightarrow T_{o1} = 299.7 \text{ K}, \quad P_{ro1} = 1.1107$$

Same as problem 15.44, except
$\eta_D = 0.80$. We thus have from 15.44

$$\frac{h_3 - h_1}{h_{o1} - h_1} = \frac{h_3 - 261.48}{300.07 - 261.48} = 0.8$$

$$\Rightarrow h_3 = 292.35 \text{ kJ/kg}, \quad P_{r3} = 1.0129$$

$$P_{o2} = P_3 = 40 \times 1.0129/0.6862 = 59.04 \text{ kPa}$$

$$P_{ro2} = P_{ro1} = 1.1107$$

$$h_2 = 300.07 - 100^2/2000 = 295.07 \text{ kJ/kg} \Rightarrow T_2 = \textbf{294.7 K}, \quad P_{r2} = 1.0462$$

$$P_2 = P_{o2} \, P_{r2} / P_{ro2} = 59.04 \times 1.0462/1.1107 = \textbf{55.6 kPa}$$

$$v_2 = RT_2/P_2 = 0.287 \times 294.7/55.6 = 1.521 \text{ m}^3/\text{kg}$$

$$A_1/A_2 = (v_1/v_2)(V_2/V_1) = (1.874/1.521)(100/277.8) = \textbf{0.444}$$

ENGLISH UNIT PROBLEMS

Borgnakke Sonntag

Fundamentals of
Thermodynamics

SOLUTION MANUAL
CHAPTER 15
English Units

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CHAPTER 15

CONTENT CHAPTER 15

Stagnation properties

15.85E

Steam leaves a nozzle with a velocity of 800 ft/s. The stagnation pressure is 100 lbf/in^2, and the stagnation temperature is 500 F. What is the static pressure and temperature?

$$h_1 = h_{o1} - \mathbf{V}_1^2/2 = 1279.1 - \frac{800^2}{2 \times 32.174 \times 778} = 1266.3 \frac{Btu}{lbm}$$

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 (= 32.174 × 778.1693)

Assume isentropic flow

$s_1 = s_0 = 1.7085$ Btu/lbm R

(h, s) Computer table $\Rightarrow$ P$_1$ = **88 lbf/in.^{2}**, T = **466 F**

15.86E

Air leaves the compressor of a jet engine at a temperature of 300 F, a pressure of 45 lbf/in^2, and a velocity of 400 ft/s. Determine the isentropic stagnation temperature and pressure.

$$h_{o1} - h_1 = \mathbf{V}_1^2/2 = 400^2/(2 \times 32.174 \times 778) = 3.2 \; \text{Btu/lbm}$$

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 (= 32.174 × 778.1693)

$$T_{o1} - T - 1 = (h_{o1} - h_1)/C_p = 3.2/0.24 = 13.3$$
$$T_{o1} = T + \Delta T = 300 + 13.3 = 313.3 \; F = \mathbf{773 \; R}$$
$$P_{o1} = P_1 \left(T_{o1}/T_1\right)^{\frac{k}{k-1}} = 45(773/759.67)^{3.5} = \mathbf{47.82 \; lbf/in^2}$$

15.87E

A meteorite melts and burn up at temperatures of 5400 R. If it hits air at 0.75 lbf/in.^{2}, 90 R how high a velocity should it have to reach such temperature?

Assume we have a stagnation T = 5400 R

$h_1 + \mathbf{V}_1^2/2 = h_{stagn.}$

Extrapolating from table F.5, $h_{stagn.} = 1515.6$, $h_1 = 21.4$ Btu/lbm

$\mathbf{V}_1^2/2 = 1515.6 - 21.4 = 1494.2$ Btu/lbm

$\mathbf{V}_1 = \sqrt{2 \times 32.174 \times 778 \times 1494.2} = $ **8649 ft/s**

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 (= 32.174 × 778.1693)
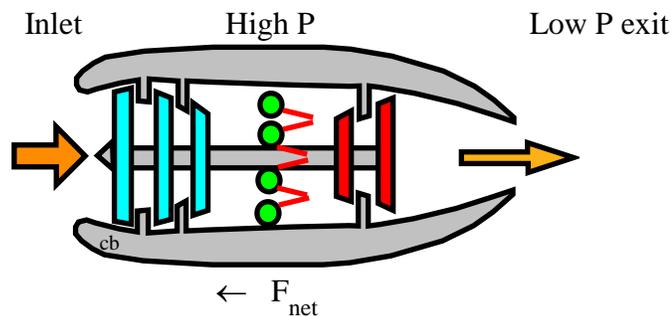
Momentum Equation and Forces

15.88E

A jet engine receives a flow of 500 ft/s air at 10 lbf/in.^{2}, 40 F inlet area of 7 ft^2 with an exit at 1500 ft/s, 10 lbf/in.^{2}, 1400 R. Find the mass flow rate and thrust.

$$\dot{m} = \rho A\mathbf{V}; \quad \text{ideal gas} \quad \rho = P/RT$$

$$\dot{m} = (P/RT)A\,\mathbf{V} = \frac{10 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2}{53.34 \text{ lbf-ft/lbm-R} \times 499.7 \text{ R}} \times 7 \text{ ft}^2 \times 500 \text{ ft/s}$$

$$= 189.1 \text{ lbm/s}$$

$$F_{net} = \dot{m}\,(\mathbf{V}_{ex} - \mathbf{V}_{in}) = \frac{189.1 \text{ lbm/s} \times (1500 - 500) \text{ ft/s}}{32.174 \text{ lbm-ft/s}^2\text{-lbf}} = \mathbf{5877\ lbf}$$

Inlet　　　　High P　　　　Low P exit

cb

$\leftarrow\ F_{net}$

The shaft must have axial load bearings to transmit thrust to aircraft.

15.89E

A water turbine using nozzles is located at the bottom of Hoover Dam 575 ft below the surface of Lake Mead. The water enters the nozzles at a stagnation pressure corresponding to the column of water above it minus 20% due to friction. The temperature is 60 F and the water leaves at standard atmospheric pressure. If the flow through the nozzle is reversible and adiabatic, determine the velocity and kinetic energy per kilogram of water leaving the nozzle.

$$\Delta P = \rho \, g \Delta Z = g(\Delta Z / v) = \frac{32.174 \times 575}{0.016035 \times 144 \times 32.174} = 249 \text{ lbf/in.}^2$$

$$\Delta P_{ac} = 0.8\Delta P = 199.2 \text{ lbf/in.}^2 \quad \text{and Bernoulli} \quad v\Delta P = V_{ex}{}^2/2$$

$$V_{ex} = \sqrt{2v\Delta P} = \sqrt{2g\Delta Z} = \sqrt{2 \times 32.174 \text{ ft/s}^2 \times 575 \text{ ft}} = \mathbf{192.4 \text{ ft/s}}$$

$$V_{ex}^2/2 = v\Delta P = g\Delta Z = 575/778 = \mathbf{0.739 \text{ Btu/lbm}}$$

Borgnakke and Sonntag

Velocity of Sound

15.90E

Find the speed of sound for air at 15 lbf/in.^{2}, at the two temperatures of 32 F and 90 F. Repeat the answer for carbon dioxide and argon gases.

We assume all gases can be treated as ideal gas behavior

From eq. 15.28 we have

$$c_{32} = \sqrt{kRT} = \sqrt{1.4 \times 32.174 \text{ lbm-ft/s}^2\text{-lbf} \times 53.34 \text{ lbf-ft/lbm-R} \times 491.7 \text{ R}}$$

$$= \textbf{1087 ft/s}$$

$$c_{90} = \sqrt{1.4 \times 32.174 \times 53.34 \times 549.7} = \textbf{1149 ft/s}$$

For Carbon Dioxide: $R = 35.1$ lbf-ft/lbm-R, $\quad k = 1.289$

$$c_{32} = \sqrt{1.289 \times 32.174 \times 35.1 \times 491.7} = \textbf{846 ft/s}$$

$$c_{90} = \sqrt{1.289 \times 32.174 \times 35.1 \times 549.7} = \textbf{894.5 ft/s}$$

For Argon: $R = 38.68$ lbf-ft/lbm-R, $\quad k = 1.667$

$$c_{32} = \sqrt{1.667 \times 32.174 \times 38.68 \times 491.7} = \textbf{1010 ft/s}$$

$$c_{90} = \sqrt{1.667 \times 32.174 \times 38.68 \times 549.7} = \textbf{1068 ft/s}$$

15.91E

A jet flies at an altitude of 40 000 ft where the air is at -40 F, 6.5 psia with a velocity of 625 mi/h. Find the Mach number and the stagnation temperature on the nose.

From Table F.4: $k = C_p/C_v = 1.4$; $R = 53.34$ ft-lbf/lbm-R

Now do the speed of sound from Eq.15.28

$$c = \sqrt{kRT} = \sqrt{1.4 \times 32.174 \text{ lbm-ft/s}^2\text{-lbf} \times 53.34 \text{ lbf-ft/lbm-R} \times 419.7 \text{ R}}$$
$$= 1004 \text{ ft/s}$$

$\mathbf{V} = 625$ mi/h $= 625$ (mi/h) $\times 1.46667$ (ft/s) / (mi/h) $= 917$ ft/s

$M = \mathbf{V} / c = 917/1004 = \mathbf{0.913}$

$h_0 = h_1 + \mathbf{V}_1^2/2$ ⇨

$T_0 = T_1 + \mathbf{V}_1^2 / 2C_p = -40 + \dfrac{917^2}{2 \times 0.24 \times 25037} = \mathbf{29.97 \ F}$

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 (= 32.174 × 778.1693)

Borgnakke and Sonntag

Flow Through Nozzles, Shocks

Borgnakke and Sonntag

15.92E

Steam flowing at 50 ft/s 200 psia, 600 F expands to 150 psia in a converging nozzle. Find the exit velocity and area ratio A_e / A_i.

Solve the problem with the steam tables.

Inlet state: $v_i = 3.058$ ft^3/lbm, $h_i = 1322.05$ Btu/lbm, $s_i = 1.6767$ Btu/lbm-R

Exit state: $(P_e, s_e = s_i)$ $v_e = 3.8185$ ft^3/lbm, $h_e = 1290.69$ Btu/lbm

Energy Eq.: $\mathbf{V}_i^2 / 2 + h_i = \mathbf{V}_e^2 / 2 + h_e$; $\mathbf{V}_e^2 = \mathbf{V}_i^2 + 2(h_i - h_e)$

$$\mathbf{V}_e = \sqrt{50 \times 50 + 2 \times 25037 \times (1322.05 - 1290.69)} = \mathbf{1254\ ft/s}$$

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 (= 32.174 × 778.1693)

Same mass flow rate so

$$A_e/A_i = (v_e/v_i)(V_i/V_e) = \frac{3.8185}{3.058} \times \frac{50}{1254} = \mathbf{0.0498}$$

If we solved as ideal gas with constant specific heat we get (k = 1.327)

$$T_e = T_i\, (P_e/P_i)^{(k-1)/k} = 1059.7\, (150/200)^{0.2464} = 987.2\ R$$

$$\mathbf{V}_e = \sqrt{\mathbf{V}_i^2 + 2C_p(T_i - T_e)} = \sqrt{50 \times 50 + 2 \times 0.447 \times 25037(1059.7 - 987.2)}$$
$$= 1274.9\ ft/s$$

$$A_e/A_i = (v_e/v_i)(V_i/V_e) = (P_i/P_e)^{1/k}(V_i/V_e) = \left(\frac{200}{150}\right)^{0.7536} \times \frac{50}{1274.9}$$
$$= \mathbf{0.0487}$$

Borgnakke and Sonntag

15.93E

A convergent nozzle has a minimum area of 1 ft^2 and receives air at 25 lbf/in.^{2}, 1800 R flowing with 330 ft/s. What is the back pressure that will produce the maximum flow rate and find that flow rate?

$$\frac{P^*}{P_o} = (\frac{2}{k+1})^{\frac{k}{k-1}} = 0.528 \qquad \text{Critical Pressure Ratio}$$

Find P_o: $C_p = (463.445 - 449.794)/50 = 0.273$ Btu/lbm-R from table F.5

$$h_0 = h_1 + \mathbf{V}_1^2/2 \quad \Rightarrow \quad T_0 = T_i + \mathbf{V}^2/2C_p$$

$$T_0 = 1800 + \frac{330^2/2}{25\ 037 \times 0.273} = 1807.97 \ \Rightarrow\ T^* = 0.8333\ T_o = 1506.6\ R$$

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 (= 32.174 × 778.1693)

$$P_0 = P_i\ (T_0/T_i)^{k/(k-1)} = 25 \times (1807.97/1800)^{3.5} = 25.39\ \text{lbf/in.}^2$$

$$P^* = 0.528\ P_o = 0.528 \times 25.39 = \mathbf{13.406\ lbf/in^2}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{13.406 \times 144}{53.34 \times 1506.6} = 0.024\ \text{lbm/ft}^3$$

$$\mathbf{V} = c = \sqrt{kRT^*} = \sqrt{1.4 \times 53.34 \times 1506.6 \times 32.174} = \mathbf{1902.6\ ft/s}$$

$$\dot{m} = \rho A \mathbf{V} = 0.024 \times 1 \times 1902.6 = \mathbf{45.66\ lbm/s}$$

15.94E

A jet plane travels through the air with a speed of 600 mi/h at an altitude of 20000 ft, where the pressure is 5.75 lbf/in.^{2} and the temperature is 25 F. Consider the diffuser of the engine where air leaves at with a velocity of 300 ft/s. Determine the pressure and temperature leaving the diffuser, and the ratio of inlet to exit area of the diffuser, assuming the flow to be reversible and adiabatic.

$\text{V} = 600 \text{ mi/h} = 880 \text{ ft/s}$

$v_1 = 53.34 \times 484.67/(5.75 \times 144) = 31.223 \text{ ft}^3/\text{lbm},$

$h_1 = 115.91 \text{ Btu/lbm},$

$h_{o1} = 115.91 + 880^2/(2 \times 25\ 037) = 131.38 \text{ Btu/lbm}$

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 (= 32.174 × 778.1693)

Table F.5 $\Rightarrow$ $T_{o1} = 549.2 \text{ R},$

$\quad P_{o1} = P_1\ (T_{o1}/T_1)^{k/(k-1)} = 5.75 \times (549.2/484.67)^{3.5} = 8.9 \text{ lbf/in.}^2$

$h_2 = 131.38 - 300^2/(2 \times 32.174 \times 778) = 129.58 \text{ Btu/lbm}$

$T_2 = \textbf{542 R}, \quad =>$

$P_2 = P_{o1}\ (T_2/T_{o1})^{k/(k-1)} = 8.9 \times (542/549.2)^{3.5} = \textbf{8.5 lbf/in.}^2$

$v_2 = 53.34 \times 542/(8.5 \times 144) = 23.62 \text{ ft}^3/\text{lbm}$

$A_1/A_2 = (v_1/v_2)(V_2/V_1) = (31.223/23.62)(300/880) = \textbf{0.45}$

15.95E

An air flow at 90 psia, 1100 R, $M = 0.3$ flows into a convergent-divergent nozzle with $M = 1$ at the throat. Assume a reversible flow with an exit area twice the throat area and find the exit pressure and temperature for subsonic exit flow to exist.

To find these properties we need the stagnation properties from the inlet state

From Table A.12: $M_i = 0.3$: $P_i/P_o = 0.93947$, $T_i/T_o = 0.98232$

$P_o = 90 / 0.93947 = 95.8$ psia, $T_o = 1100 / 0.98232 = 1119.8$ R

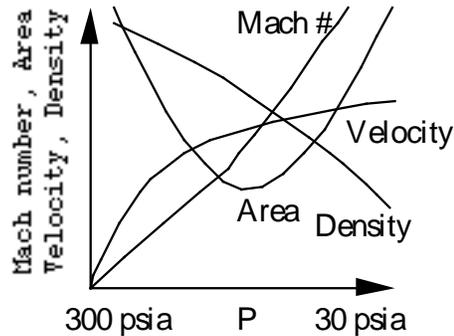This flow is case c in Figure 15.13. From Table A.12: $A_E/A^* = 2$

$P_E/P_o = 0.9360$, $T_E/T_o = 0.98127$

$P_E = 0.9360 \ P_o = 0.936 \times 95.8 = $ **89.67 psia**

$T_E = 0.98127 \ T_o = 0.98127 \times 1119.8 = $ **1099 R**

15.96E

Air is expanded in a nozzle from 300 lbf/in.2, 1100 R to 30 lbf/in.^{2}. The mass flow rate through the nozzle is 10 lbm/s. Assume the flow is reversible and adiabatic and determine the throat and exit areas for the nozzle.

$$P^* = P_o \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$
$$= 300 \times 0.5283 = 158.5 \text{ lbf/in.}^2.$$
$$T^* = T_o \times 2/(k+1) = 1100 \times 0.8333 = 916.6 \text{ R}$$
$$v^* = RT^*/P^* = 53.34 \times 916.6/(158.5 \times 144)$$
$$= 2.1421 \text{ ft}^3/\text{lbm}$$

The critical speed of sound is

$$c^* = \sqrt{kRT^*} = \sqrt{1.4 \times 32.174 \times 53.34 \times 916.6} = 1484 \text{ ft/s}$$

$$A^* = \dot{m}v^*/c^* = 10 \times 2.1421/1484 = \mathbf{0.0144 \text{ ft}^2}$$

$$P_2/P_o = 30/300 = 0.1 \quad \text{Table A.11} \implies M_2^* = 1.701 = V_2/c^*$$

We used the column in Table A.12 with mach no. based on throat speed of sound.

$$V_2 = 1.701 \times 1484 = 2524 \text{ ft/s}$$
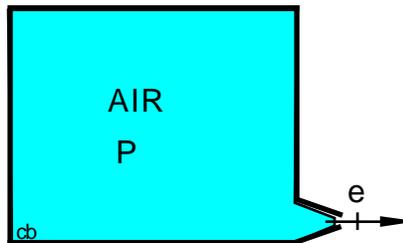$$T_2 = 916.6 \times 0.5176 = 474.4 \text{ R}$$
$$v_2 = RT_2/P_2 = 53.34 \times 474.4/(30 \times 144) = 5.8579 \text{ ft}^3/\text{lbm}$$
$$A_2 = \dot{m}v_2/V_2 = 10 \times 5.8579 / 2524 = \mathbf{0.0232 \text{ ft}^2}$$

15.97E

A 50-ft^3 uninsulated tank contains air at 150 lbf/in.^{2}, 1000 R. The tank is now discharged through a small convergent nozzle to the atmosphere at 14.7 lbf/in.^{2} while heat transfer from some source keeps the air temperature in the tank at 1000 R. The nozzle has an exit area of 2×10^{-4} ft^2.
a. Find the initial mass flow rate out of the tank.
b. Find the mass flow rate when half the mass has been discharged.
c. Find the mass of air in the tank and the mass flow rate out of the tank
   when the nozzle flow changes to become subsonic.

$$P_B/P_o = 14.7/150 = 0.098 \; < \; (P^*/P_o)_{crit} = 0.5283$$

a. The flow is choked, max possible flow rate
$$M_E = 1 \; ; \quad P_E = 0.5283 \times 150 = 79.245 \text{ lbf/in.}^2$$
$$T_E = T^* = 0.8333 \times 1000 = 833.3 \text{ R}$$
$$\mathbf{V}_E = c = \sqrt{kRT^*} = \sqrt{1.4 \times 53.34 \times 833.3 \times 32.174} = 1415 \text{ ft/s}$$
$$v_E = RT^*/P_E = 53.34 \times 833.3/(79.245 \times 144) = 3.895 \text{ ft}^3/\text{lbm}$$

Mass flow rate is : $\dot{m}_1 = A\mathbf{V}_E/v_E = 2 \times 10^{-4} \times 1415/3.895 = \mathbf{0.0727 \ lbm/s}$

b. $m_1 = P_1V/RT_1 = 150 \times 50 \times 144/53.34 \times 1000 = 20.247$ lbm
$$m_2 = m_1/2 = 10.124 \text{ lbm}, \quad P_2 = P_1/2 = 75 \text{ lbf/in.}^2 \; ; \quad T_2 = T_1$$
$$P_B/P_2 = 14.7/75 = 0.196 \; < \; (P^*/P_o)_{crit}$$
The flow is choked and the velocity is the same as in a)
$$P_E = 0.5283 \times 75 = 39.623 \text{ lbf/in.}^2 \; ; \quad M_E = 1$$
$$\dot{m}_2 = A\mathbf{V}_EP_E/RT_E = \frac{2 \times 10^{-4} \times 1415 \times 39.623 \times 144}{53.34 \times 1000} = \mathbf{0.0303 \ lbm/s}$$

c. Flow changes to subsonic when the pressure ratio reaches critical.

$P_B/P_o = 0.5283$ $P_3 = 27.825$ lbf/in.^{2}

$m_3 = m_1P_3/P_1 = $ **3.756 lbm** ; $T_3 = T_1$ $\Rightarrow$ $V_E = 1415$ ft/s

$\dot{m}_3 = AV_EP_E/RT_E = \dfrac{2 \times 10^{-4} \times 1415 \times 27.825 \times 144}{53.34 \times 1000} = $ **0.02125 lbm/s**

15.98E

A flow of helium flows at 75 psia, 900 R with 330 ft/s into a convergent-divergent nozzle. Find the throat pressure and temperature for reversible flow and $M = 1$ at the throat.

We need to find the stagnation properties first (k = 1.667)
$$T_0 = T_1 + \mathbf{V}_1^2/2C_p = 900 + 330^2/(2 \times 25037 \times 1.24) = 901.754 \text{ R}$$
Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 (= 32.174 $\times$ 778.1693)
$$P_0 = P_1 (T_0/T_1)^{k/(k-1)} = 75 (901.754/900)^{2.5} = 75.366 \text{ psia}$$

From the analysis we get Eqs.15.37-38

$$P^* = P_0 \left(\frac{2}{k+1}\right)^{k/(k-1)} = 75.366 \left(\frac{2}{1.667+1}\right)^{2.5} = \mathbf{36.7 \text{ psia}}$$

$$T^* = T_0 \frac{2}{k+1} = 901.754 \times \frac{2}{1.667+1} = \mathbf{676.2 \text{ R}}$$

Borgnakke and Sonntag

15.99E

The products of combustion enter a nozzle of a jet engine at a total pressure of 18 lbf/in.^{2}, and a total temperature of 1200 F. The atmospheric pressure is 6.75 lbf/in.^{2}. The nozzle is convergent, and the mass flow rate is 50 lbm/s. Assume the flow is adiabatic. Determine the exit area of the nozzle.

$$P_{crit} = P_2 = 18 \times 0.5283 = 9.5 \ \text{lbf/in.}^2 > P_{amb}$$

The flow is then choked.

$$T_2 = 1660 \times 0.8333 = 1382 \ R$$

$$V_2 = c_2 = \sqrt{kRT} = \sqrt{1.4 \times 32.174 \times 53.34 \times 1382} = 1822 \ \text{ft/s}$$

$$v_2 = 53.34 \times 1382/9.5 \times 144 = 53.9 \ \text{ft}^3/\text{lbm}$$

$$A_2 = \dot{m} \ v_2 / V_2 = 50 \times 53.9/1822 = \mathbf{1.479 \ ft^2}$$

Borgnakke and Sonntag

15.100E

A normal shock in air has upstream total pressure of 75 psia, stagnation temperature of 900 R and $M_x = 1.4$. Find the downstream stagnation pressure.

From the normal shock relations in Section 15.8 found in Table A.13 we get

$M_x = 1.4$: $P_{oy}/P_{ox} = 0.95819$

$P_{oy} = 0.95819\, P_{ox} = 0.95819 \times 75 = \textbf{71.86 psi}$

Remark: The stagnation temperature would be unchanged (energy equation).

Borgnakke and Sonntag

Nozzles, Diffusers and Orifices

15.101E

Air enters a diffuser with a velocity of 600 ft/s, a static pressure of 10 lbf/in.^{2}, and a temperature of 20 F. The velocity leaving the diffuser is 200 ft/s and the static pressure at the diffuser exit is 11.7 lbf/in.^{2}. Determine the static temperature at the diffuser exit and the diffuser efficiency. Compare the stagnation pressures at the inlet and the exit.

$$T_{o1} = T_1 + \frac{V_1^2}{2g_c C_p} = 480 + 600^2/(2 \times 25\ 037 \times 0.24) = 510 \text{ R}$$

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2 (= 32.174 × 778.1693)

$$\frac{T_{o1} - T_1}{T_1} = \frac{k-1}{k}\frac{P_{o1} - P_1}{P_1} \quad\Rightarrow\quad P_{o1} - P_1 = 2.1875 \Rightarrow P_{o1} = 12.2 \text{ lbf/in.}^2$$

$$T_{o2} = T_{o1} \quad\Rightarrow$$

$$T_2 = T_{o2} - V_2^2/2C_p = 510 - 200^2/(2 \times 25\ 037 \times 0.24) = 506.7 \text{ R}$$

$$\frac{T_{o1} - T_1}{T_1} = \frac{k-1}{k}\frac{P_{o1} - P_1}{P_1} \quad\Rightarrow\quad P_{o1} - P_1 = 2.1875 \Rightarrow P_{o1} = 12.2 \text{ lbf/in.}^2$$

$$\frac{T_{o2} - T_2}{T_2} = \frac{k-1}{k}\frac{P_{o2} - P_2}{P_2} \quad\Rightarrow\quad P_{o2} - P_2 = 0.267 \Rightarrow P_{o2} = 11.97 \text{ lbf/in.}^2$$

$$T_{ex,s} = T_1 (P_{o2}/P_1)^{(k-1)/k} = 480 \text{ R} \times 1.0528 = \textbf{505.3 R}$$

$$\eta_D = \frac{T_{ex,s} - T_1}{T_{o1} - T_1} = \frac{505.3 - 480}{51 - 480} = \textbf{0.844}$$

15.102E

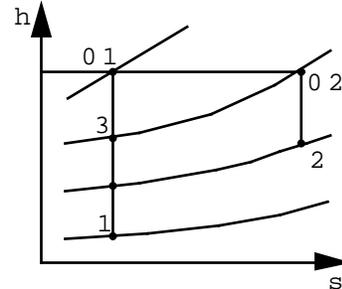Repeat Problem 15.94 assuming a diffuser efficiency of 80%.

From solution to 15.94

$h_1 = 115.91$ Btu/lbm, $v_1 = 31.223$ ft^3/lbm

$h_{o1} = 115.91 + 880^2/(2 \times 25\ 037)$

$\quad = 131.38$ Btu/lbm

Recall conversion 1 Btu/lbm = 25 037 ft^2/s^2

$(= 32.174 \times 778.1693)$

Table F.5 $\quad \Rightarrow T_{o1} = 549.2$ R,

$\eta_D = (h_3 - h_1)/(h_{o1} - h_1) = 0.8 \quad \Rightarrow \quad h_3 = 128.29$ Btu/lbm, $T_3 = 536.29$ R

$P_{o2} = P_3 = P_1 (T_3/T_1)^{k/(k-1)} = 5.75 \times (536.29/484.67)^{3.5} = 8.194$ lbf/in.^{2}

$T_{o2} = T_{o1} = 549.2$ R

$h_2 = 131.38 - 300^2/(2 \times 25\ 037) = 129.58$ Btu/lbm

$\quad\quad T_2 = \textbf{542 R}, \quad =>$

$P_2 = P_{o2} (T_2/T_{o1})^{k/(k-1)} = 8.194 \times (542/549.2)^{3.5} = \textbf{7.824 lbf/in.}^2$

$\quad \Rightarrow \quad v_2 = \dfrac{53.34 \times 542}{7.824 \times 144} = 25.66$ ft^3/lbm

$A_1/A_2 = v_1 V_2/v_2 V_1 = 31.223 \times 300/(25.66 \times 880) = \textbf{0.415}$

Borgnakke and Sonntag

15.103E

A convergent nozzle with exit diameter of 1 in. has an air inlet flow of 68 F, 14.7 lbf/in.^{2} (stagnation conditions). The nozzle has an isentropic efficiency of 95% and the pressure drop is measured to 20 in. water column. Find the mass flow rate assuming compressible adiabatic flow. Repeat calculation for incompressible flow.

Convert ΔP to lbf/in^2

$\Delta P = 20$ in $H_2O = 20 \times 0.03613 = 0.7226$ lbf/in^2

$T_0 = 68$ F $= 527.7$ R $\qquad\qquad P_0 = 14.7$ lbf/in^2

Assume inlet $\mathbf{V}_i = 0 \qquad\qquad P_e = P_0 - \Delta P = 14.7 - 0.7226 = 13.977$ lbf/in^2

$$T_e = T_0 \left(\frac{P_e}{P_0}\right)^{\frac{k-1}{k}} = 527.7 \times \left(\frac{13.977}{14.7}\right)^{0.2857} = 520.15 \text{ R}$$

$$\mathbf{V}_e^2/2 = h_i - h_e = C_p (T_i - T_e) = 0.24 \times (527.7 - 520.15) = 1.812 \text{ Btu/lbm}$$

$$\mathbf{V}_{e\,ac}^2/2 = \eta\, \mathbf{V}_e^2/2 = 0.95 \times 1.812 = 1.7214 \text{ Btu/lbm}$$

$$\Rightarrow \mathbf{V}_{e\,ac} = \sqrt{2 \times 25\,037 \times 1.7214} = 293.6 \text{ ft/s}$$

Recall conversion 1 Btu/lbm $= 25\,037$ ft^2/s^2 ($= 32.174 \times 778.1693$)

$$T_{e\,ac} = T_i - \frac{\mathbf{V}_{e\,ac}^2/2}{C_p} = 527.7 - \frac{1.7214}{0.24} = 520.53 \text{ R}$$

$$\rho_{e\,ac} = \frac{P_e}{RT_{e\,ac}} = \frac{13.977 \times 144}{53.34 \times 520.53} = 0.07249 \text{ lbm/ft}^3$$

$$\dot{m} = \rho A \mathbf{V} = 0.07249 \times \frac{\pi}{4} \times \left(\frac{1}{12}\right)^2 \times 293.6 = \textbf{0.116 lbm/s}$$

Incompressible: $\qquad \rho_i = \frac{P_0}{RT_0} = \frac{14.7 \times 144}{53.34 \times 527.7} = 0.0752 \text{ lbm/ft}^3$

$$\mathbf{V}_e^2/2 = v_i (P_i - P_e) = \frac{\Delta P}{\rho_i} = \frac{0.7226 \times 144}{0.0752 \times 778} = 1.7785 \text{ Btu/lbm}$$

$$\mathbf{V}_{e\,ac}^2/2 = \eta\, \mathbf{V}_e^2/2 = 0.95 \times 1.7785 = 1.6896 \text{ Btu/lbm}$$

$$\Rightarrow \mathbf{V}_{e\,ac} = \sqrt{2 \times 25\,037 \times 1.6896} = 290.84 \text{ ft/s}$$

$$\dot{m} = \rho A \mathbf{V} = 0.0752 \times \frac{\pi}{4} \times \left(\frac{1}{12}\right)^2 \times 290.84 = \textbf{0.119 lbm/s}$$