

## 101. گزینه چهارم

$$x^4 - 7x^2 - 5 = 0$$

$$x' + x'' = s \text{ و } x'.x'' = p$$

$$x^2 = t \geq 0 \Rightarrow t^2 - 7t - 5 = 0 \Rightarrow$$

$$t = t_1 < 0 \text{ غ ق ق } \quad t = t_2 > 0 \Rightarrow x^2 = t \Rightarrow$$

$$x', x'' = \pm\sqrt{t} \Rightarrow s = 0$$

$$x^2 = \frac{7 \pm \sqrt{49 + 20}}{2} \Rightarrow x^2 = \frac{7 + \sqrt{69}}{2}$$

$$2p^2 - 3sp + 2s = 2p^2 = 2 \left( \left( \sqrt{\frac{7 + \sqrt{69}}{2}} \right) \left( -\sqrt{\frac{7 + \sqrt{69}}{2}} \right) \right)^2 =$$

$$\frac{1}{2} \times (7 + \sqrt{69})^2 = \frac{49 + 69 + 14\sqrt{69}}{2} = 59 + 7\sqrt{69}$$

## 102. گزینه سوم

$$(\log^2 5 - \log^2 2) \log_{\frac{5}{2}}(3x - 2) = 1$$

$$\log 10 \times \log \frac{5}{2} \times \frac{\log(3x - 2)}{\log \frac{5}{2}} = 1 \Rightarrow$$

$$\log(3x - 2) = 1 \Rightarrow 3x - 2 = 10 \Rightarrow x = 4$$



## 103. گزینه چهارم

$$\begin{aligned}
 & (\log_{21} 3)^2 + \log_{21} 7^2 \times 3 \times \log_{21} 21^2 \times 3 \\
 &= (\log_{21} 3)^2 + (\log_{21} 21 \times 7) \times (\log_{21} 21^2 + \log_{21} 3) \\
 & \leftarrow 1 + \log_{21} 7 = 1 + \log_{21} \frac{21}{3} = 1 + 1 - \log_{21} 7 \\
 & (\log_{21} 3)^2 + (2 - \log_{21} 3)(2 + \log_{21} 3) = 4
 \end{aligned}$$

## 104. گزینه سوم

به ازای  $x > \frac{3}{2}$   $x$  مخرج همواره مثبت است پس:

$$\begin{aligned}
 & ((m^2 - 1)x^2 - 4mx + 4)(x - 3\sqrt{x} + 2) > 0 \\
 & \quad \quad \quad \underbrace{(\sqrt{x} - 1)}_{+} \underbrace{(\sqrt{x} - 2)}_{-} \\
 & \quad \quad \quad (2, 4) \text{ در } x > \frac{3}{2} \text{ یعنی } \left(\frac{3}{2}, 4\right) \Leftarrow \text{پس باید در } (2, 4)
 \end{aligned}$$

$$(m^2 - 1)x^2 - 4mx + 4 < 0$$

به ازای  $m = 1$  برقرار است:

$$m = 1 \Rightarrow -4x + 4 < 0 \Rightarrow x > 1$$

## 105. گزینه دوم

$$\tan \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 - \tan^2 \frac{\alpha}{2}} = \frac{\frac{1}{2}}{1 - \frac{1}{16}} = \frac{8}{15}$$



$$\sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} = \frac{\frac{1}{2}}{1 + \frac{1}{16}} = \frac{8}{17}$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \frac{15}{17}$$

$$\frac{\tan \alpha - \sin \alpha}{\sin \alpha - \cos \alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{8 \times 17 - 8 \times 15}{15 \times 17}}{\frac{8 - 15}{17}} =$$

$$\frac{\frac{8(17 - 15)}{15}}{8 - 15} = -\frac{16}{105}$$

106. گزینه اول

$$4 \sin \alpha (1 - 2 \sin^2 \alpha) + 2 \sin \alpha = 6 \sin \alpha - 8 \sin^3 \alpha =$$

$$2(3 \sin \alpha - 4 \sin^3 \alpha) = 2 \sin 3\alpha = 2 \sin \frac{41\pi}{3} =$$

$$2 \sin \left( 14\pi - \frac{\pi}{3} \right) = -2 \times \frac{\sqrt{3}}{2} = -\sqrt{3}$$

107. گزینه چهارم

$$(1 + \cos 2\alpha)(1 + \cos 4\alpha)(1 + \cos 8\alpha) = \frac{1}{8}$$

$$2 \cos^2 \alpha \times 2 \cos^2 2\alpha \times 2 \cos^2 4\alpha = \frac{1}{8}$$

$$\cos \alpha \cos 2\alpha \cos 4\alpha = \frac{1}{8}$$

$$\frac{\sin \alpha \cos \alpha \cos 2\alpha \cos 4\alpha}{\sin \alpha} = \frac{1}{8} \Rightarrow \frac{\frac{1}{8} \sin 8\alpha}{\sin \alpha} = \frac{1}{8} \Rightarrow \sin 8\alpha = \sin \alpha$$

$$\begin{cases} 8\alpha = 2k\pi + \alpha \\ 8\alpha = 2k\pi + \pi - \alpha \end{cases} \Rightarrow \begin{cases} \alpha = \frac{2k\pi}{7} \Rightarrow \max = \frac{6\pi}{7} \\ \alpha = \frac{(2k+1)\pi}{9} \Rightarrow \max = \frac{8\pi}{9} \end{cases} \Rightarrow \max = \frac{8\pi}{9}$$

108. گزینه سوم

$$ax^2 + bx + c = (2ax + b)\left(\frac{x}{2} + 1\right) - 2 \Rightarrow$$

$$b = 2a + \frac{b}{2} \Rightarrow b = 4a$$

$$b - 2 = c \Rightarrow c = 4a - 2$$

$$a = 1 \Rightarrow b = 4 \Rightarrow c = 2 \quad \min(a + b + c) = 7$$

109. گزینه اول

$$a_{100} = \frac{1}{a_{99}} + 1 \Rightarrow a_{99} = \frac{1}{a_{100} - 1}$$

$$a_{99} = \frac{1}{a_{98}} + 1 \Rightarrow a_{98} = \frac{1}{a_{99} - 1} = \frac{1}{\frac{1}{\frac{k}{m} - 1} - 1} = \frac{\frac{k}{m} - 1}{2 - \frac{k}{m}} = \frac{k - m}{2m - k}$$

110. گزینه اول

$$a_0 = 1, \quad a_1 = 4, \quad a_2 = 1 + a, \quad a_3 = 2, \quad a_4 = 2$$

$$a_5 = 1 + 5, \quad a_6 = 4, \quad a_7 = 0, \quad a_8 = 2 + a, \quad a_9 = 8$$

$$\text{جمع} = 25 + 3a = 19 \Rightarrow a = -2$$

$$a_2 + a_5 + \dots + a_{29} =$$

$$\begin{array}{c} \downarrow \quad \downarrow \\ -1 - 1 + 0 + 0 + \dots + 0 = -2 \end{array}$$



## 111. گزینه چهارم

تابع  $2^x$  صعودی و  $2^{-x}$  نزولی است و  $-2^{-x}$  صعودی و در نتیجه  $y = 2^x - 2^{-x}$  صعودی است.

$$\cos^2 x = 0 \Rightarrow y = -\frac{3}{2}$$

$$\cos^2 x = 1 \Rightarrow y = 2^2 - 2^{-2} = \frac{15}{4}$$

$$b - a = \frac{15}{4} + \frac{6}{4} = \frac{21}{4}$$

## 112. گزینه اول

$$x = 0 \Rightarrow \text{ص} = \frac{1}{6} \text{ لگاریتم} \Rightarrow \textcircled{3}$$

$$x = -4 \Rightarrow \frac{1}{6+2-4} > 0 \Rightarrow \text{گزینه ۱}$$

## 113. گزینه سوم

$$y = \sqrt{4-x} + k$$

$$\left| \frac{1}{1} \right| \text{ باید در } y = \sqrt{4 - (x - k + 2)} + k \text{ صدق کند}$$

$$k = 0$$

$$y = \sqrt{2-x} - 1 = 0 \Rightarrow x = 1$$

## 114. گزینه سوم

$f(x)$  در  $x = \pm 1$  مشتق ناپذیر است

و  $f \circ g$  وقتی  $g(x) = \pm 1$  مشتق ناپذیر است

$$g(x) = -1 \Rightarrow 1 - x^2 = -1 \Rightarrow x = \pm\sqrt{2}$$

115. گزینه دوم

$$9^{\log_3 x} = x^{\log_3 9} = x^2 \Rightarrow x > 0$$

116. گزینه چهارم

$$\lim_{x \rightarrow 0^+} \frac{\tan^2\left(\frac{1}{\sqrt{1-x^2}} - 1\right)}{(1 - \cos\sqrt{2x})^n} = a$$

$$\lim_{x \rightarrow 0^+} \frac{\left(\frac{1 - \sqrt{1-x^2}}{\sqrt{1-x^2}}\right)^2}{\left(\frac{(\sqrt{2x})^2}{2}\right)^n} = \lim_{x \rightarrow 0^+} \frac{\left(1 - \left(1 - \frac{1}{2}x^2\right)\right)^2}{x^n} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{4}x^4}{x^n} = \frac{1}{4}$$

$$n = 4, \quad a = \frac{1}{4} \Rightarrow a + \frac{1}{4} = \frac{17}{4}$$

117. گزینه اول

$$\text{حد} = \frac{-10 - \varepsilon + \left[\frac{3}{\frac{1}{4} + \varepsilon + \varepsilon^2}\right]}{16\left(-\frac{1}{2} - \varepsilon\right) - \left[\frac{-2}{\frac{1}{4} + \varepsilon + \varepsilon^2}\right]} = \frac{-10 + 11}{-8 - \varepsilon + 8} = -\infty$$

118. گزینه چهارم

$$\frac{-8x^3 + 6x^2 + 2}{-8x^3 + 6x + 2} = \frac{(x-1)(-8x^2 - 2x - 2)}{-2(x-1)(2x+1)^2}$$

119. گزینه دوم

$$\lim_{x \rightarrow -\infty} \frac{50\sqrt{a^{2 \times \frac{50 \times 51}{2}} \times x^{51 \times 50}}}{a^{49} x^k} = -1$$



$$\lim_{x \rightarrow -\infty} \frac{|a^{51} x^{51}|}{a^{49} x^k} = -1$$

$$a = 1, k = 51$$

120. گزینه سوم

باید به ازای  $x = 0$  صورت صفر شود:

$$1 + a(0) + b = 0 \Rightarrow b = -1$$

$$\lim_{x \rightarrow 0^-} \frac{f'(x)}{x} = 2 \Rightarrow \lim_{x \rightarrow 0^-} \frac{-6\cos^3 2x \sin 2x + 2ax}{x} = 2$$

$$\lim_{x \rightarrow 0^-} \frac{-12x + 2ax}{x} = 2 \Rightarrow a = 7 \quad a + b = 6$$

121. گزینه سوم

$$f(x) = |\sin 2x| + 1 \quad A' \begin{vmatrix} 0 \\ 1 \end{vmatrix}$$

$$x \rightarrow 0^+ \Rightarrow f'(x) = 2\cos 2x \Rightarrow m = 2 \Rightarrow y - 1 = 2(x - 0)$$

$$x \rightarrow 0^- \Rightarrow f'(x) = -2\cos 2x \Rightarrow m' = -2 \Rightarrow y - 1 = -2(x - 0)$$

$$y = 2x + 1 = -x \Rightarrow x = -\frac{1}{3} \Rightarrow y = \frac{1}{3} \quad A \begin{vmatrix} -\frac{1}{3} \\ \frac{1}{3} \end{vmatrix}$$

$$y = -2x + 1 = -x \Rightarrow x = 1 \Rightarrow y = -1 \quad B \begin{vmatrix} 1 \\ -1 \end{vmatrix}$$

$$AB = \sqrt{\frac{16}{9} + \frac{16}{9}} = \frac{4\sqrt{2}}{3}$$



## 122. گزینه دوم

$$f(0) = \frac{3}{2}, \quad f(1^-) = +\infty$$

$$f(1^+) = -\infty, \quad f(+\infty) = +\infty$$

تابع در  $(0, 1)$  و  $(1, \infty)$  صعودی است.

## 123. گزینه چهارم

$$f'(x) = \frac{4x^3(x^3 - 8) - 3x^2(x^4)}{(x^3 - 8)^2} = 0$$

$$x^3(4x^3 - 32 - 3x^3) = 0 \Rightarrow x = 0, \sqrt[3]{32}$$

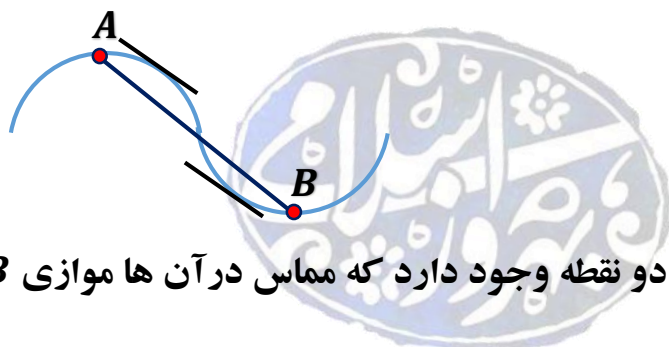
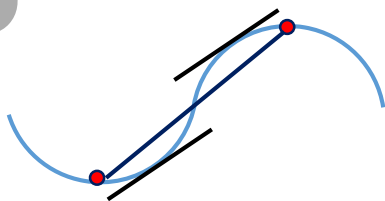
$x$	$-\infty$	$0$	$2$	$\sqrt[3]{32}$	$+\infty$
$y'$	$+$		$-$	$-$	$+$
$y$	$\nearrow$		$\searrow$	$\searrow$	$\nearrow$

در  $(0, 2) \Leftarrow$  طول = 2

در  $(2, 2\sqrt[3]{4}) \Leftarrow$  طول =  $2(\sqrt[3]{4} - 1)$

## 124. گزینه سوم

اگر نمودار درجه 3 دو اکسترمم داشته باشد:



دو نقطه وجود دارد که مماس در آن ها موازی AB است.