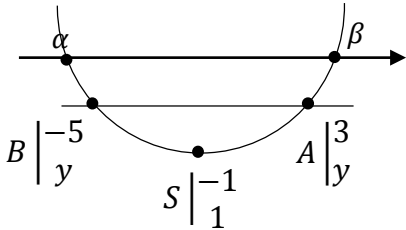


۱- گزینه دوم

$$\div \frac{a}{2}, \frac{ar}{2}, \frac{ar^2}{2}, \Rightarrow ar = \frac{a+ar^2}{2} \Rightarrow r^2 - 2r + 1 = 0 \Rightarrow (r-1)^2 = 0 \Rightarrow r = 1$$

$$\frac{ar}{2} - \frac{a}{2} = d \xrightarrow{r=1} d = 0 \Rightarrow r + d = 1$$

۲- گزینه سوم



$$\frac{\alpha + \beta}{2} = \frac{-5 + 3}{2} = -1$$

$$\boxed{\alpha + \beta = -2} \Rightarrow x_s = -\frac{2}{2} = -1$$

$$\alpha^2 + \beta^2 = 5 \Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 5 \Rightarrow \alpha\beta = -\frac{1}{2}$$

$$y = a \left( x^2 + 2x - \frac{1}{2} \right) \xrightarrow{s \mid 1} 1 = a \left( 1 - 2 - \frac{1}{2} \right) \Rightarrow$$

$$a = -\frac{2}{3} \Rightarrow y = -\frac{2}{3} \left( x^2 + 2x - \frac{1}{2} \right) \xrightarrow{x=0} y = \frac{1}{3}$$

۳- گزینه اول

$$A \times B = B \times A \Rightarrow \sqrt{d} = 6 \Rightarrow \boxed{d = 36}$$

$$2b + 1 = 5 \Rightarrow \boxed{b = 2} \quad \boxed{c = -1}$$

$$a - 2 = 6 \Rightarrow \boxed{a = 8} \Rightarrow a + b + c = 9 \text{ یک حالت}$$

۵- گزینه چهارم

$$ax^2 = ax + b \Rightarrow x^2 = x + \frac{b}{a}, \quad \alpha + \beta = 1 \Rightarrow \boxed{\alpha = 1 - \beta}$$

$$\beta^2 = \beta + \frac{b}{a}, \quad \alpha^2 = \alpha + \frac{b}{a}$$

$$40\beta^2 + 20\alpha^2 - 20\beta = 17 \Rightarrow 40\beta + 40\frac{b}{a} + 20\left(\alpha + \frac{b}{a}\right) - 20\beta = 17$$

$$20(\alpha + \beta) + 60\frac{b}{a} = 17 \Rightarrow 60\frac{b}{a} = -3 \Rightarrow \frac{b}{a} = -\frac{1}{20}$$

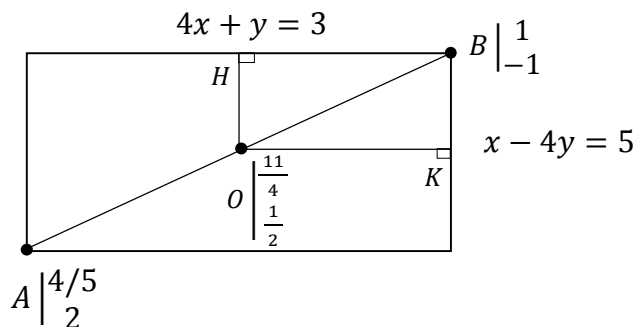
$$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{1}{|a|} \sqrt{a^2 + 4ab} = \sqrt{1 + \frac{4b}{a}} = \sqrt{1 - \frac{4}{20}} = \sqrt{1 - \frac{1}{5}} = \frac{2}{\sqrt{5}}$$



۶- گزینه اول

$$\frac{1}{x^2} + \frac{1}{(1-x)^2} = \frac{160}{9} = \frac{16+144}{9} = \frac{1}{\left(\frac{3}{4}\right)^2} + \frac{1}{\left(\frac{3}{12}\right)^2}$$

$$x', x'' = \frac{3}{4}, \frac{1}{4} \Rightarrow x' + x'' = 1$$



$$OA = ?$$

$$\begin{cases} 4x + y = 3 \\ x - 4y = 5 \end{cases}$$

$$\boxed{x = 1}, \boxed{y = -1}$$

۷- گزینه اول

$$OH = \frac{|11 + \frac{1}{2} - 3|}{\sqrt{16+1}} = \frac{\sqrt{17}}{2}, \quad OK = \frac{|\frac{11}{4} - 2 - 5|}{\sqrt{17}} = \frac{\sqrt{17}}{4}$$

۸- گزینه چهارم

$$\begin{cases} y = 12 - x \\ y = 10 \end{cases} \Rightarrow x = 2 \Rightarrow \left| \begin{matrix} 2 \\ 10 \end{matrix} \right. \in f^{-1} \Rightarrow \left| \begin{matrix} 10 \\ 2 \end{matrix} \right. \in f$$

$$2 = \sqrt{10 - 2\sqrt{10m - 1}} \Rightarrow m = 1 \Rightarrow f(5) = 1$$

۹- گزینه اول

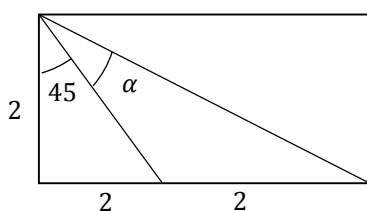
$$A_1 = \frac{8}{9} M \text{ جرم باقی مانده پس از یک ساعت}$$

$$A_2 = \left(\frac{8}{9}\right)^2 M \text{ جرم باقی مانده پس از دو ساعت}$$

$$\left(\frac{8}{9}\right)^n = \frac{1}{6} \Rightarrow n = \log_{\frac{8}{9}} \frac{1}{6} = \frac{-\log_2 6}{\log_2 8 - \log_2 9} = \frac{-1 - \log_2 3}{3 - 2\log_2 3} = \frac{19}{3}$$

$$\log_2 3 = \frac{\log_5 3}{\log_5 2} = \frac{\frac{10}{14}}{\frac{10}{24}} = \frac{12}{7} \quad \text{جواب} = \frac{19}{3} \times 60 = 380$$

۱۰- گزینه دوم



$$\operatorname{tg}(\alpha + 45) = \frac{1 + \operatorname{tg}\alpha}{1 - \operatorname{tg}\alpha} = \frac{4}{2}$$

$$\Rightarrow \operatorname{tg}\alpha = \frac{1}{3} \Rightarrow \operatorname{cotg}\alpha = 3$$



۱۱- گزینه اول

$$4/5 = \frac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha \Rightarrow \sin\alpha = \frac{\sqrt{3}}{2} \Rightarrow \frac{\text{Max}(\alpha)}{\text{Min}(\alpha)} = \frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = 2$$

۱۲- گزینه چهارم

$$f(x) = a + \frac{b}{2} \sin\left(2cx - \frac{3\pi}{2}\right)$$

$$f(x) = a - \frac{b}{2} \cos(2cx)$$

$$T = \frac{2\pi}{2c} = \pi \Rightarrow C = 1$$

$$a + \frac{|b|}{2} = 3$$

$$\Rightarrow \begin{cases} a = 1 \\ b = -4 \end{cases}$$

$$a - \frac{|b|}{2} = -1$$

$$f(x) = 1 - 2\cos 2x = 0$$

$$\cos 2x = \frac{1}{2} \Rightarrow \begin{cases} 2x = \frac{\pi}{3} \Rightarrow x = \frac{\pi}{6} \\ 2x = \frac{5\pi}{3} \Rightarrow x = \frac{5\pi}{6} \end{cases} \quad \text{جواب} = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$$

۱۳- گزینه سوم

$$-m(\sin x - \cos x) - 3\sqrt{6}(\sin 2x - 1) = 4\sqrt{6}$$

$$+m\sqrt{2} \cos\left(x + \frac{\pi}{4}\right) - 3\sqrt{6} \left(-\sqrt{2} \cos\left(x + \frac{\pi}{4}\right)\right)^2 = 4\sqrt{6}$$

$$m \times \sqrt{2} \times \frac{1}{\sqrt{3}} - 3\sqrt{6} \times 2 \times \frac{1}{3} = 4\sqrt{6} \Rightarrow m = 18$$



۱۴- گزینه اول

$$m^2 - m - 5 > -3 + 2m - m^2$$

$$2m^2 - 3m - 2 > 0 \Rightarrow m = \frac{3 \pm 5}{4} = \begin{matrix} \rightarrow 2 \\ \rightarrow -\frac{1}{2} \end{matrix}$$

$$m < -\frac{1}{2} \text{ یا } m > 2 \quad \textcircled{1}$$

$$m^2 - m - 5 < 0 \Rightarrow \frac{1-\sqrt{22}}{2} < m < \frac{1+\sqrt{22}}{2} \quad \textcircled{2}$$

$$-3 + 2m - m^2 < 0 \Rightarrow \Delta < 0$$

$$\textcircled{1} \cap \textcircled{2} \quad \frac{1-\sqrt{22}}{2} < m < -\frac{1}{2} \Rightarrow -\frac{1}{8} < m < -\frac{1}{2} \Rightarrow m = -1 \quad \text{مقدار صحیح}$$

۱۵- گزینه سوم

$$f(x) = \frac{ax+b}{cx+d} \Rightarrow g(x) = \frac{cx+d}{ax+b}$$

$$\lim_{x \rightarrow -\infty} \frac{\frac{ax+b}{cx+d}}{\frac{-bx+d}{ax-c}} = \lim_{x \rightarrow +\infty} \frac{\frac{-bx+d}{ax-c}}{\frac{cx+d}{ax+b}} \Rightarrow \frac{\frac{a}{c}}{\frac{-b}{-a}} = \frac{-\frac{b}{a}}{\frac{c}{a}} \Rightarrow \frac{b}{a} = \pm 1$$

$$\lim_{x \rightarrow 0^+} f^{-1}(x) = \lim_{x \rightarrow 0^+} \frac{-dx+b}{cx-a} = -\frac{b}{a} = \mp 1$$

۱۶- گزینه دوم

$$\begin{cases} \lim_{x \rightarrow 2^+} |x - [-x]| = \lim_{x \rightarrow 2^+} |x + 1 + [x]| = 5 \\ \lim_{x \rightarrow 2^-} (x - [x] + k) = 2 - 1 + k = k + 1 \end{cases} \Rightarrow k + 1 = 5 \Rightarrow \begin{cases} k = 4 \\ n = 2 \end{cases}$$

$$\begin{cases} \lim_{x \rightarrow -2^+} |x - [-x]| = \lim_{x \rightarrow -2^+} |x + 1 + [x]| = -2 + 1 - 2 = -3 \\ \lim_{x \rightarrow -2^-} (x - [x] + k) = -2 + 3 + k \end{cases} \Rightarrow \begin{cases} k + 1 = 3 \\ k = -4 \end{cases} \quad \times$$

$$\begin{cases} \lim_{x \rightarrow 3^+} (x - [x] + k) = 3 - 3 + k = k \\ \lim_{x \rightarrow 3^-} |x - [-x]| = \lim_{x \rightarrow 3^-} |x + 1 + [x]| = 3 + 1 + 2 = 6 \end{cases} \Rightarrow k = 6$$

$$\begin{cases} \lim_{x \rightarrow -3^+} (x - [x] + k) = -3 + 3 + k = k \\ \lim_{x \rightarrow -3^-} |x - [-x]| = \lim_{x \rightarrow -3^-} |x + 1 + [x]| = -3 + 1 - 4 = 6 \end{cases} \Rightarrow k = 6 \quad \checkmark$$



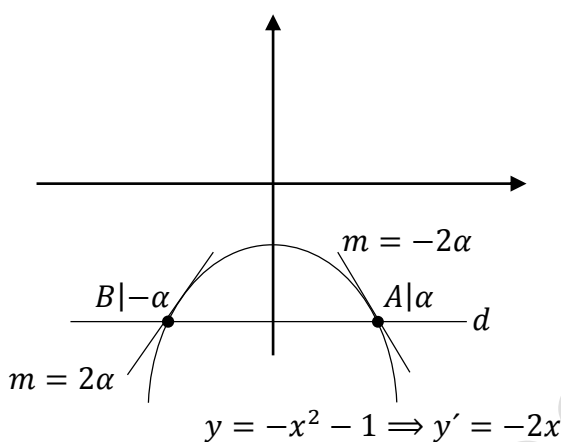
۱۷- گزینه سوم

$$f(x) = \left( \frac{1-\sin x}{1+\sin x} \right)^2 = \left( \frac{\sin \frac{x}{2} - \cos \frac{x}{2}}{\sin \frac{x}{2} + \cos \frac{x}{2}} \right)^4 = \left( \frac{tg \frac{x}{2} - 1}{tg \frac{x}{2} + 1} \right)^4 = tg^4 \left( \frac{x}{2} - \frac{\pi}{4} \right)$$

$$g(x) = \frac{f(x)-1}{x} \Rightarrow \lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} \frac{tg^4 \left( \frac{x}{2} - \frac{\pi}{4} \right) - 1}{x} \xrightarrow{\text{hopital}}$$

$$\lim_{x \rightarrow 0} 4 \times \frac{1}{2} \left( 1 + tg^2 \left( \frac{x}{2} - \frac{\pi}{4} \right) \right) tg^3 \left( \frac{x}{2} - \frac{\pi}{4} \right) = -4$$

۱۸- گزینه یک



$$(-2\alpha)(2\alpha) = -1 \Rightarrow \alpha = \pm \frac{1}{2}$$

$$A \begin{vmatrix} \frac{1}{2} \\ -\frac{5}{4} \end{vmatrix} \quad B \begin{vmatrix} -\frac{1}{2} \\ -\frac{5}{4} \end{vmatrix}$$

$$\text{جواب} = \frac{5}{4} = 1/25$$

۱۹- گزینه چهارم

$$\text{عطف } x = \frac{-(k+1)}{3k} < 0 \Rightarrow k < -1 \text{ یا } k > 0 \xrightarrow[k < 0]{\textcircled{1}} \boxed{k < -1}$$

$$\text{عطف } y = k \left( \frac{-(k+1)^3}{27k^3} \right) + \frac{(k+1)^3}{9k^2} > 0$$

$$\frac{(k+1)^3}{9k^2} \left( -\frac{k}{3} + 1 \right) > 0 \Rightarrow (k+1)(3-k) > 0 \Rightarrow \boxed{-1 < k < 3} \textcircled{2}$$

$$\textcircled{1} \cap \textcircled{2} \Rightarrow \emptyset$$



۲۰- گزینه دوم

$$x - [x^2] \geq 0 \Rightarrow 0 \leq x \leq \sqrt{2}$$

$$0 \leq x < 1 \Rightarrow y = \sqrt{x-0} \Rightarrow y = \sqrt{x} \xrightarrow{|\sqrt{\alpha}|} d = \frac{|2\alpha - \sqrt{\alpha} + 2|}{\sqrt{5}}$$

$$d' = 0 \Rightarrow \alpha = \frac{1}{16} \Rightarrow d = \frac{3\sqrt{5}}{8}$$

$$1 \leq x < \sqrt{2} \Rightarrow y = \sqrt{x-1} \xrightarrow{|\sqrt{\alpha-1}|} d = \frac{|2\alpha - \sqrt{\alpha-1} + 2|}{\sqrt{5}}$$

$$d' = 0 \Rightarrow \alpha = \frac{17}{16} \Rightarrow d = \frac{31}{8\sqrt{5}}$$

## مهندس بهروز اسلامی



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