Chapter 1

Problems **1-1** through **1-6** are for student research. No standard solutions are provided.

1-7 From Fig. 1-2, cost of grinding to ± 0.0005 in is 270%. Cost of turning to ± 0.003 in is 60%.

Relative cost of grinding vs. turning = 270/60 = 4.5 times *Ans.*

1-8 $C_A = C_B$,

$$10 + 0.8\,P = 60 + 0.8\,P - 0.005\,P^2$$

$P^2 = 50/0.005 \quad \Rightarrow \quad P = 100$ parts *Ans.*

1-9 Max. load = 1.10 P

Min. area = $(0.95)^2 A$

Min. strength = 0.85 S

To offset the absolute uncertainties, the design factor, from Eq. (1-1) should be

$$n_d = \frac{1.10}{0.85\left(0.95\right)^2} = 1.43 \qquad \textit{Ans.}$$

1-10 (**a**) $X_1 + X_2$:

$$x_1 + x_2 = X_1 + e_1 + X_2 + e_2$$
$$\text{error } = e = \left(x_1 + x_2\right) - \left(X_1 + X_2\right)$$
$$= e_1 + e_2 \qquad \textit{Ans.}$$

(**b**) $X_1 - X_2$:

$$x_1 - x_2 = X_1 + e_1 - \left(X_2 + e_2\right)$$
$$e = \left(x_1 - x_2\right) - \left(X_1 - X_2\right) = e_1 - e_2 \qquad \textit{Ans.}$$

(**c**) $X_1\,X_2$:

$$x_1 x_2 = \left(X_1 + e_1\right)\left(X_2 + e_2\right)$$
$$e = x_1 x_2 - X_1 X_2 = X_1 e_2 + X_2 e_1 + e_1 e_2$$
$$\approx X_1 e_2 + X_2 e_1 = X_1 X_2 \left(\frac{e_1}{X_1} + \frac{e_2}{X_2}\right) \qquad \textit{Ans.}$$

(d) X_1/X_2:

$$\frac{x_1}{x_2} = \frac{X_1 + e_1}{X_2 + e_2} = \frac{X_1}{X_2}\left(\frac{1 + e_1/X_1}{1 + e_2/X_2}\right)$$

$$\left(1 + \frac{e_2}{X_2}\right)^{-1} \approx 1 - \frac{e_2}{X_2} \quad \text{then} \quad \left(\frac{1 + e_1/X_1}{1 + e_2/X_2}\right) \approx \left(1 + \frac{e_1}{X_1}\right)\left(1 - \frac{e_2}{X_2}\right) \approx 1 + \frac{e_1}{X_1} - \frac{e_2}{X_2}$$

Thus, $e = \frac{x_1}{x_2} - \frac{X_1}{X_2} \approx \frac{X_1}{X_2}\left(\frac{e_1}{X_1} - \frac{e_2}{X_2}\right)$ *Ans.*

1-11 **(a)** $x_1 = \sqrt{7} = 2.645\ 751\ 311\ 1$

$X_1 = 2.64$ (3 correct digits)

$x_2 = \sqrt{8} = 2.828\ 427\ 124\ 7$

$X_2 = 2.82$ (3 correct digits)

$x_1 + x_2 = 5.474\ 178\ 435\ 8$

$e_1 = x_1 - X_1 = 0.005\ 751\ 311\ 1$

$e_2 = x_2 - X_2 = 0.008\ 427\ 124\ 7$

$e = e_1 + e_2 = 0.014\ 178\ 435\ 8$

Sum $= x_1 + x_2 = X_1 + X_2 + e$

$= 2.64 + 2.82 + 0.014\ 178\ 435\ 8 = 5.474\ 178\ 435\ 8$ Checks

(b) $X_1 = 2.65$, $X_2 = 2.83$ (3 digit significant numbers)

$e_1 = x_1 - X_1 = -\ 0.004\ 248\ 688\ 9$

$e_2 = x_2 - X_2 = -\ 0.001\ 572\ 875\ 3$

$e = e_1 + e_2 = -\ 0.005\ 821\ 564\ 2$

Sum $= x_1 + x_2 = X_1 + X_2 + e$

$= 2.65 + 2.83 - 0.001\ 572\ 875\ 3 = 5.474\ 178\ 435\ 8$ Checks

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1-12 $\sigma = \frac{S}{n_d} \quad \Rightarrow \quad \frac{32(1000)}{\pi d^3} = \frac{25(10^3)}{2.5} \quad \Rightarrow \quad d = 1.006 \text{ in}$ *Ans.*

Table A-17: $d = 1\frac{1}{4}$ in *Ans.*

Factor of safety: $n = \frac{S}{\sigma} = \frac{25(10^3)}{\frac{32(1000)}{\pi(1.25)^3}} = 4.79$ *Ans.*

1-13 **(a)**

x	f	fx	fx^2
60	2	120	7200
70	1	70	4900
80	3	240	19200
90	5	450	40500
100	8	800	80000
110	12	1320	145200
120	6	720	86400
130	10	1300	169000
140	8	1120	156800
150	5	750	112500
160	2	320	51200
170	3	510	86700
180	2	360	64800
190	1	190	36100
200	0	0	0
210	1	210	44100
Σ	69	8480	1 104 600

Eq. (1-6) $\bar{x} = \dfrac{1}{N}\sum_{i=1}^{k} f_i x_i = \dfrac{8480}{69} = 122.9\ \text{kcycles}$

Eq. (1-7)

$$s_x = \sqrt{\frac{\sum_{i=1}^{k} f_i x_i^2 - N\bar{x}^2}{N-1}} = \left[\frac{1\,104\,600 - 69(122.9)^2}{69-1}\right]^{1/2} = 30.3\ \text{kcycles} \quad \textit{Ans.}$$

(b) Eq. (1-5) $z_{115} = \dfrac{x-\mu_x}{\hat{\sigma}_x} = \dfrac{x_{115}-\bar{x}}{s_x} = \dfrac{115-122.9}{30.3} = -0.2607$

Interpolating from Table (A-10)

0.2600	0.3974		
0.2607	x	$\Rightarrow$	$x = 0.3971$
0.2700	0.3936		

$N\Phi(-0.2607) = 69\ (0.3971) = 27.4 \approx 27$ *Ans.*

From the data, the number of instances less than 115 kcycles is

2 + 1 + 3 + 5 + 8 + 12 = 31 (the data is not perfectly normal)

1-14

x	f	fx	fx^2
174	6	1044	181656
182	9	1638	298116
190	44	8360	1588400
198	67	13266	2626668
206	53	10918	2249108
214	12	2568	549552
222	6	1332	295704
Σ	197	39126	7789204

Eq. (1-6)
$$\bar{x} = \frac{1}{N}\sum_{i=1}^{k} f_i x_i = \frac{39\,126}{197} = 198.61 \text{ kpsi}$$

Eq. (1-7)
$$s_x = \sqrt{\frac{\sum_{i=1}^{k} f_i x_i^2 - N\bar{x}^2}{N-1}} = \left[\frac{7\ 789\ 204 - 197(198.61)^2}{197-1}\right]^{1/2} = 9.68 \text{ kpsi} \quad \textit{Ans.}$$

1-15 $\bar{L} = 122.9$ kcycles and $s_L = 30.3$ kcycles

Eq. (1-5)
$$z_{10} = \frac{x-\mu_x}{\hat{\sigma}} = \frac{x_{10}-\bar{L}}{s_L} = \frac{x_{10}-122.9}{30.3}$$

Thus, $x_{10} = 122.9 + 30.3\, z_{10} = L_{10}$

From Table A-10, for 10 percent failure, $z_{10} = -1.282$. Thus,

$$L_{10} = 122.9 + 30.3(-1.282) = 84.1 \text{ kcycles} \qquad \textit{Ans.}$$

1-16

x	f	fx	fx^2
93	19	1767	164331
95	25	2375	225625
97	38	3686	357542
99	17	1683	166617
101	12	1212	122412
103	10	1030	106090
105	5	525	55125
107	4	428	45796
109	4	436	47524
111	2	222	24642
Σ	136	13364	1315704

Eq. (1-6) $$\bar{x} = \frac{1}{N}\sum_{i=1}^{k} f_i x_i = 13\,364 / 136 = 98.26471 = 98.26 \text{ kpsi}$$

Eq. (1-7) $$s_x = \sqrt{\frac{\sum_{i=1}^{k} f_i x_i^2 - N\bar{x}^2}{N-1}} = \left(\frac{1\,315\,704 - 136(98.26471)^2}{136-1}\right)^{1/2} = 4.30 \text{ kpsi}$$

Note, for accuracy in the calculation given above, $\bar{x}$ needs to be of more significant figures than the rounded value.

For a normal distribution, from Eq. (1-5), and a yield strength exceeded by 99 percent ($R = 0.99$, $p_f = 0.01$),

$$z_{0.01} = \frac{x - \mu_x}{\hat{\sigma}_x} = \frac{x_{0.01} - \bar{x}}{s_x} = \frac{x_{0.01} - 98.26}{4.30}$$

Solving for the yield strength gives

$$x_{0.01} = 98.26 + 4.30\, z_{0.01}$$

From Table A-10, $z_{0.01} = -2.326$. Thus

$$x_{0.01} = 98.26 + 4.30(-2.326) = 88.3 \text{ kpsi}$$ *Ans.*

1-17 Eq. (1-9): $$R = \prod_{i=1}^{n} R_i = 0.98(0.96)0.94 = 0.88$$

Overall reliability = 88 percent *Ans.*

1-18 Obtain the coefficients of variance for strength and stress

$$C_S = \frac{\hat{\sigma}_{S_{sy}}}{\bar{S}_{sy}} = \frac{23.5}{312} = 0.07532$$

$$C_\sigma = \frac{\hat{\sigma}_\tau}{\bar{\tau}} = \frac{\hat{\sigma}_T}{\bar{T}} = \frac{145}{1\,500} = 0.09667$$

For $R = 0.99$, from Table A-10, $z = -2.326$.

Eq. (1-12):

$$\bar{n} = \frac{1 + \sqrt{1 - \left(1 - z^2 C_S^2\right)\left(1 - z^2 C_\sigma^2\right)}}{1 - z^2 C_S^2}$$

$$= \frac{1 + \sqrt{1 - \left[1 - (-2.326)^2 (0.07532)^2\right]\left[1 - (-2.326)^2 (0.09667)^2\right]}}{1 - (-2.326)^2 (0.07532)^2} = 1.3229 = 1.32 \qquad \textit{Ans.}$$

From the given equation for stress,

$$\tau_{\max} = \frac{S_{sy}}{\bar{n}} = \frac{16T}{\pi d^3}$$

Solving for d gives

$$d = \left(\frac{16 T \bar{n}}{\pi S_{sy}}\right)^{1/3} = \left[\frac{16(1500)1.3229}{\pi(312)10^6}\right]^{1/3} = 0.0319 \text{ m} = 31.9 \text{ mm} \quad \textit{Ans.}$$

1-19 Obtain the coefficients of variance for stress and strength

$$C_\sigma = \frac{\hat{\sigma}_\sigma}{\mu_\sigma} = \frac{\hat{\sigma}_P}{\bar{P}} = \frac{5}{65} = 0.09231$$

$$C_S = \frac{\hat{\sigma}_S}{\mu_S} = \frac{\hat{\sigma}_{S_y}}{\bar{S}_y} = \frac{6.59}{95.5} = 0.06901$$

(a) $\bar{n} = 1.2$

Eq. (1-11): $$z = -\frac{\bar{n}_d - 1}{\sqrt{\bar{n}_d^2 C_S^2 + C_\sigma^2}} = -\frac{1.2 - 1}{\sqrt{1.2^2\left(0.06901^2\right) + 0.09231^2}} = -1.6127$$

Interpolating Table A-10,

1.61	0.0537		
1.6127	Φ	⇒	$\Phi = 0.0534$
1.62	0.0526		

$R = 1 - 0.0534 = 0.9466$ *Ans.*

$$\bar{n} = \frac{\bar{S}_y}{\bar{\sigma}} = \frac{\bar{S}_y}{\bar{P} / (\pi d^2 / 4)} = \frac{\pi d^2 \bar{S}_y}{4\bar{P}} \Rightarrow d = \sqrt{\frac{4\bar{P}\bar{n}}{\pi \bar{S}_y}} = \sqrt{\frac{4(65)1.2}{\pi(95.5)}} = 1.020 \text{ in} \quad \textit{Ans.}$$

(b) $\bar{n} = 1.5$

$$z = -\frac{1.5 - 1}{\sqrt{1.5^2\left(0.06901^2\right) + 0.09231^2}} = -3.605$$

3.6	0.000159		
3.605	Φ	⇒	$\Phi = 0.00015645$
3.7	0.000108		

$R = 1 - 0.00015645 = 0.9998$ *Ans.*

$$d = \sqrt{\frac{4\bar{P}\bar{n}}{\pi \bar{S}_y}} = \sqrt{\frac{4(65)1.5}{\pi(95.5)}} = 1.140 \text{ in} \quad \textit{Ans.}$$

1-20 $$\mu_{\sigma_{\max}} = \bar{\sigma}_{\max} = \bar{\sigma}_a + \bar{\sigma}_b = 90 + 383 = 473 \text{ MPa}$$

From footnote 9 of text,

$$\hat{\sigma}_{\sigma_{\max}} = \left(\hat{\sigma}_{\sigma_a}^2 + \hat{\sigma}_{\sigma_b}^2\right)^{1/2} = (8.4^2 + 22.3^2)^{1/2} = 23.83 \text{ MPa}$$

$$C_{\sigma_{\max}} = \frac{\hat{\sigma}_{\sigma_{\max}}}{\mu_{\sigma_{\max}}} = \frac{\hat{\sigma}_{\sigma_{\max}}}{\bar{\sigma}_{\max}} = \frac{23.83}{473} = 0.0504$$

$$C_{S_y} = \frac{\hat{\sigma}_{S_y}}{\mu_{S_y}} = \frac{\hat{\sigma}_{S_y}}{\bar{S}_y} = \frac{42.7}{553} = 0.0772$$

$$\bar{n} = \frac{\bar{S}_y}{\bar{\sigma}_{\max}} = \frac{553}{473} = 1.169 = 1.17 \quad Ans.$$

Eq. (1-11): $$z = -\frac{\bar{n}_d - 1}{\sqrt{\bar{n}_d^2 C_S^2 + C_\sigma^2}} = -\frac{1.169 - 1}{\sqrt{1.169^2\left(0.0772^2\right) + 0.0504^2}} = -1.635$$

From Table A-10, $\Phi(-1.635) = 0.05105$

$$R = 1 - 0.05105 = 0.94895 = 94.9 \text{ percent} \quad Ans.$$

1-21 $a = 1.500 \pm 0.001$ in
$b = 2.000 \pm 0.003$ in
$c = 3.000 \pm 0.004$ in
$d = 6.520 \pm 0.010$ in

(**a**) $\bar{w} = \bar{d} - \bar{a} - \bar{b} - \bar{c} = 6.520 - 1.5 - 2 - 3 = 0.020$ in
$t_w = \sum t_{\text{all}} = 0.001 + 0.003 + 0.004 + 0.010 = 0.018$
$w = 0.020 \pm 0.018$ in *Ans.*

(**b**) From part (a), $w_{\min} = 0.002$ in. Thus, must add 0.008 in to $\bar{d}$. Therefore,

$\bar{d} = 6.520 + 0.008 = 6.528$ in *Ans.*

1-22 $V = xyz$, and $x = a \pm \Delta a$, $y = b \pm \Delta b$, $z = c \pm \Delta c$,

$$\bar{V} = abc$$

$$V = (a \pm \Delta a)(b \pm \Delta b)(c \pm \Delta c)$$
$$= abc \pm bc\Delta a \pm ac\Delta b \pm ab\Delta c \pm a\Delta b\Delta c \pm b\Delta c\Delta a \pm c\Delta a\Delta b \pm \Delta a\Delta b\Delta c$$

The higher order terms in Δ are negligible. Thus,

$$\Delta V \approx bc\Delta a + ac\Delta b + ab\Delta c$$

and, $$\frac{\Delta V}{\bar{V}} \approx \frac{bc\Delta a + ac\Delta b + ab\Delta c}{abc} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c} = \frac{\Delta a}{\bar{a}} + \frac{\Delta b}{\bar{b}} + \frac{\Delta c}{\bar{c}} \quad Ans.$$

For the numerical values given, $\bar{V} = 1.500(1.875)3.000 = 8.4375 \text{ in}^3$

$$\frac{\Delta V}{\bar{V}} \approx \frac{0.002}{1.500} + \frac{0.003}{1.875} + \frac{0.004}{3.000} = 0.004267 \quad \Rightarrow \quad \Delta V \approx 0.004267(8.4375) = 0.0360 \text{ in}^3$$

$V = 8.4375 \pm 0.0360 \text{ in}^3$ *Ans.*

This answer yields $V \approx \begin{matrix} 8.4735 \\ 8.4015 \end{matrix}$ in, whereas, exact is $V = \begin{matrix} 8.473551.. \\ 8.401551.. \end{matrix}$ in

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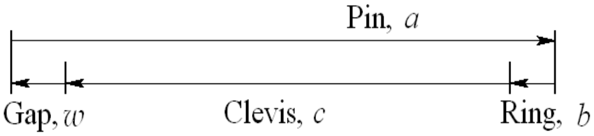
1-23



$w_{\max} = 0.05$ in, $w_{\min} = 0.004$ in

$$\bar{w} = \frac{0.05 + 0.004}{2} = 0.027 \text{ in}$$

Thus, $\Delta w = 0.05 - 0.027 = 0.023$ in, and then, $w = 0.027 \pm 0.023$ in.

$$\bar{w} = \bar{a} - \bar{b} - \bar{c}$$

$$0.027 = \bar{a} - 0.042 - 1.5$$

$$\bar{a} = 1.569 \text{ in}$$

$$t_w = \sum t_{\text{all}} \quad \Rightarrow \quad 0.023 = t_a + 0.002 + 0.005 \quad \Rightarrow \quad t_a = 0.016 \text{ in}$$

Thus, $a = 1.569 \pm 0.016$ in *Ans.*

1-24 $\bar{D}_o = \bar{D}_i + 2\bar{d} = 3.734 + 2(0.139) = 4.012 \text{ in}$

$$t_{D_o} = \sum t_{\text{all}} = 0.028 + 2(0.004) = 0.036 \text{ in}$$

$D_o = 4.012 \pm 0.036$ in *Ans.*

1-25 From O-Rings, Inc. (oringsusa.com), $D_i = 9.19 \pm 0.13$ mm, $d = 2.62 \pm 0.08$ mm

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 9.19 + 2(2.62) = 14.43 \text{ mm}$$

$$t_{D_o} = \sum t_{\text{all}} = 0.13 + 2(0.08) = 0.29 \text{ mm}$$

$D_o = 14.43 \pm 0.29$ mm *Ans.*

1-26 From O-Rings, Inc. (oringsusa.com), D_i = 34.52 ± 0.30 mm, d = 3.53 ± 0.10 mm

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 34.52 + 2(3.53) = 41.58 \quad \text{mm}$$

$$t_{D_o} = \sum t_{\text{all}} = 0.30 + 2(0.10) = 0.50 \quad \text{mm}$$

D_o = 41.58 ± 0.50 mm *Ans.*

1-27 From O-Rings, Inc. (oringsusa.com), D_i = 5.237 ± 0.035 in, d = 0.103 ± 0.003 in

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 5.237 + 2(0.103) = 5.443 \quad \text{in}$$

$$t_{D_o} = \sum t_{\text{all}} = 0.035 + 2(0.003) = 0.041 \quad \text{in}$$

D_o = 5.443 ± 0.041 in *Ans.*

1-28 From O-Rings, Inc. (oringsusa.com), D_i = 1.100 ± 0.012 in, d = 0.210 ± 0.005 in

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 1.100 + 2(0.210) = 1.520 \quad \text{in}$$

$$t_{D_o} = \sum t_{\text{all}} = 0.012 + 2(0.005) = 0.022 \quad \text{in}$$

D_o = 1.520 ± 0.022 in *Ans.*

1-29 From Table A-2,

(a) σ = 150/6.89 = 21.8 kpsi *Ans.*

(b) F = 2 /4.45 = 0.449 kip = 449 lbf *Ans.*

(c) M = 150/0.113 = 1330 lbf · in = 1.33 kip · in *Ans.*

(d) A = 1500/ 25.4^2 = 2.33 in^2 *Ans.*

(e) I = 750/2.54^4 = 18.0 in^4 *Ans.*

(f) E = 145/6.89 = 21.0 Mpsi *Ans.*

(g) v = 75/1.61 = 46.6 mi/h *Ans.*

(h) $V = 1000/946 = 1.06$ qt *Ans.*

1-30 From Table A-2,

(a) $l = 5(0.305) = 1.53$ m *Ans.*

(b) $\sigma = 90(6.89) = 620$ MPa *Ans.*

(c) $p = 25(6.89) = 172$ kPa *Ans.*

(d) $Z = 12(16.4) = 197$ cm^3 *Ans.*

(e) $w = 0.208(175) = 36.4$ N/m *Ans.*

(f) $\delta = 0.001\ 89(25.4) = 0.048\ 0$ mm *Ans.*

(g) $v = 1\ 200(0.0051) = 6.12$ m/s *Ans.*

(h) $\int = 0.002\ 15(1) = 0.002\ 15$ mm/mm *Ans.*

(i) $V = 1830(25.4^3) = 30.0\ (10^6)$ mm^3 *Ans.*

1-31

(a) $\sigma = M/Z = 1770/0.934 = 1895$ psi $= 1.90$ kpsi *Ans.*

(b) $\sigma = F/A = 9440/23.8 = 397$ psi *Ans.*

(c) $y = Fl^3/3EI = 270(31.5)^3/[3(30)10^6(0.154)] = 0.609$ in *Ans.*

(d) $\theta = Tl/GJ = 9\ 740(9.85)/[11.3(10^6)(\pi/32)1.00^4] = 8.648(10^{-2})$ rad $= 4.95°$ *Ans.*

1-32

(a) $\sigma = F/wt = 1000/[25(5)] = 8$ MPa *Ans.*

(b) $I = bh^3/12 = 10(25)^3/12 = 13.0(10^3)$ mm^4 *Ans.*

(c) $I = \pi d^4/64 = \pi(25.4)^4/64 = 20.4(10^3)$ mm^4 *Ans.*

(d) $\tau = 16T/\pi d^3 = 16(25)10^3/[\pi(12.7)^3] = 62.2$ MPa *Ans.*

1-33

(a) $\tau = F/A = 2\,700/[\pi(0.750)^2/4] = 6110 \text{ psi} = 6.11 \text{ kpsi}$ *Ans.*

(b) $\sigma = 32Fa/\pi d^3 = 32(180)31.5/[\pi(1.25)^3] = 29\,570 \text{ psi} = 29.6 \text{ kpsi}$ *Ans.*

(c) $Z = \pi(d_o^4 - d_i^4)/(32\,d_o) = \pi(1.50^4 - 1.00^4)/[32(1.50)] = 0.266 \text{ in}^3$ *Ans.*

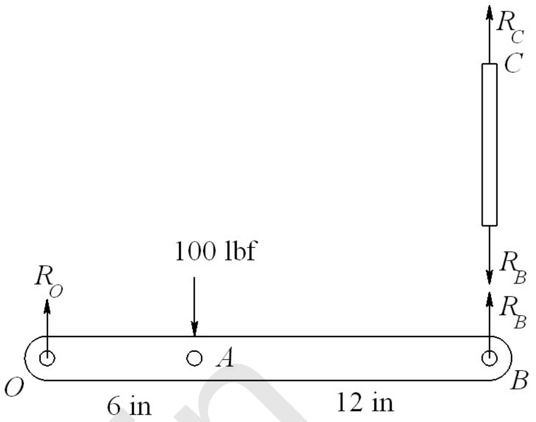
(d) $k = (d^4G)/(8D^3 N) = 0.062\,5^4(11.3)10^6/[8(0.760)^3\,32] = 1.53 \text{ lbf/in}$ *Ans.*

Chapter 3

3-1

$$\Sigma M_O = 0$$
$$18R_B - 6(100) = 0$$
$$R_B = 33.3 \text{ lbf} \quad \textit{Ans.}$$

$$\Sigma F_y = 0$$
$$R_O + R_B - 100 = 0$$
$$R_O = 66.7 \text{ lbf} \quad \textit{Ans.}$$
$$R_C = R_B = 33.3 \text{ lbf} \quad \textit{Ans.}$$



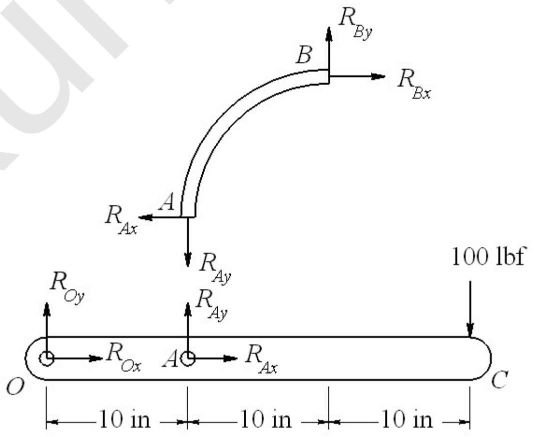
3-2

Body *AB*:

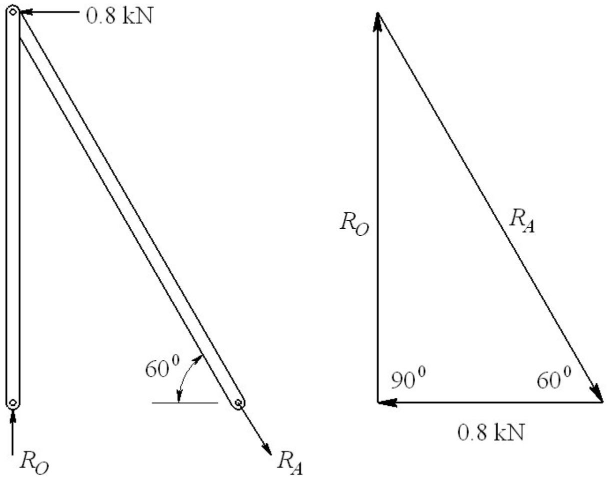
$$\Sigma F_x = 0 \qquad R_{Ax} = R_{Bx}$$
$$\Sigma F_y = 0 \qquad R_{Ay} = R_{By}$$
$$\Sigma M_B = 0 \qquad R_{Ay}(10) - R_{Ax}(10) = 0$$
$$R_{Ax} = R_{Ay}$$

Body *OAC*:

$$\Sigma M_O = 0 \qquad R_{Ay}(10) - 100(30) = 0$$
$$R_{Ay} = 300 \text{ lbf} \quad \textit{Ans.}$$
$$\Sigma F_x = 0 \qquad R_{Ox} = -R_{Ax} = -300 \text{ lbf} \quad \textit{Ans.}$$
$$\Sigma F_y = 0 \qquad R_{Oy} + R_{Ay} - 100 = 0$$
$$R_{Oy} = -200 \text{ lbf} \quad \textit{Ans.}$$



3-3

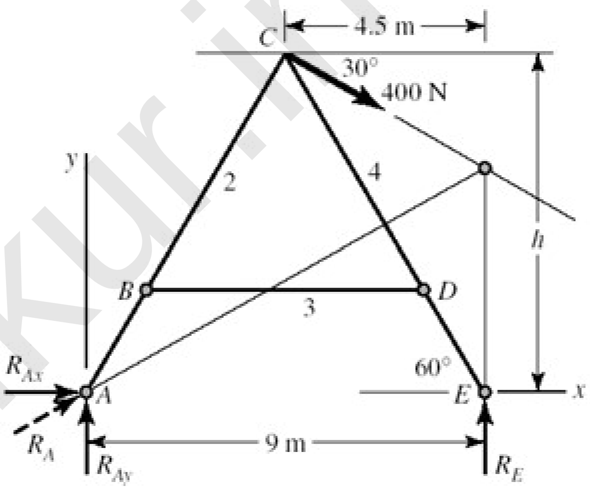


$$R_O = \frac{0.8}{\tan 30^\circ} = 1.39 \text{ kN} \qquad Ans.$$

$$R_A = \frac{0.8}{\sin 30^\circ} = 1.6 \text{ kN} \qquad Ans.$$

3-4

Step 1: Find R_A & R_E



$$h = \frac{4.5}{\tan 30^\circ} = 7.794 \text{ m}$$

$$\Sigma M_A = 0$$

$$9R_E - 7.794(400\cos 30^\circ)$$

$$-4.5(400\sin 30^\circ) = 0$$

$$R_E = 400 \text{ N} \qquad Ans.$$

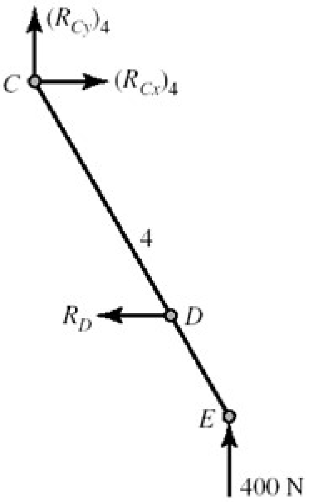
$$\sum F_x = 0 \qquad R_{Ax} + 400\cos 30^\circ = 0$$

$$R_{Ax} = -346.4 \text{ N}$$

$$\sum F_y = 0 \qquad R_{Ay} + 400 - 400\sin 30^\circ = 0$$

$$R_{Ay} = -200 \text{ N}$$

$$R_A = \sqrt{346.4^2 + 200^2} = 400 \text{ N} \qquad Ans.$$



Step 2: Find components of R_C and R_D on link 4

$$\sum M_C = 0$$

$$400(4.5) - (7.794 - 1.9)R_D = 0$$

$$R_D = 305.4 \text{ N} \qquad Ans.$$

$$\sum F_x = 0 \quad \Rightarrow \quad (R_{Cx})_4 = 305.4 \text{ N}$$

$$\sum F_y = 0 \quad \Rightarrow \quad (R_{Cy})_4 = -400 \text{ N}$$

Step 3: Find components of R_C on link 2

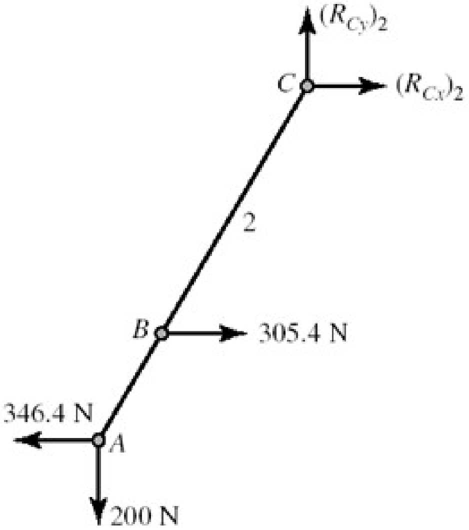
$$\sum F_x = 0$$

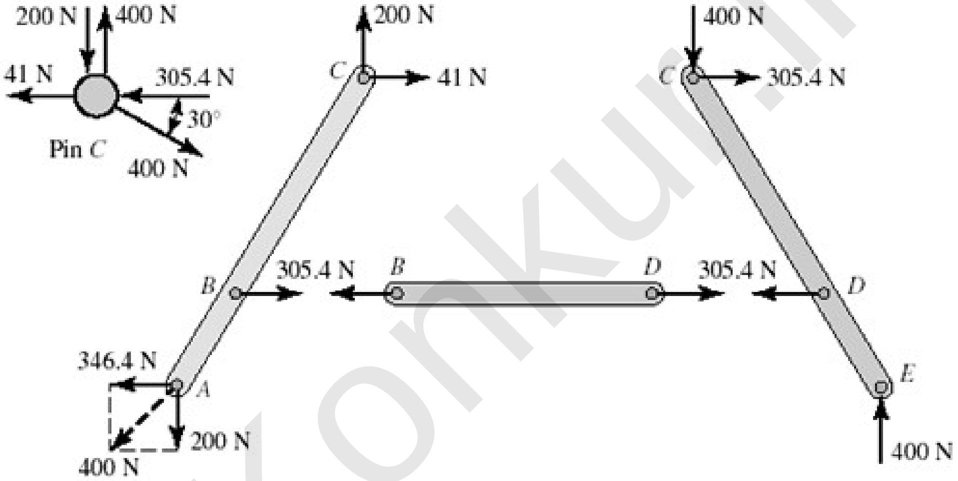
$$\left(R_{Cx}\right)_2 + 305.4 - 346.4 = 0$$

$$\left(R_{Cx}\right)_2 = 41 \text{ N}$$

$$\sum F_y = 0$$

$$\left(R_{Cy}\right)_2 = 200 \text{ N}$$





Ans.

3-5

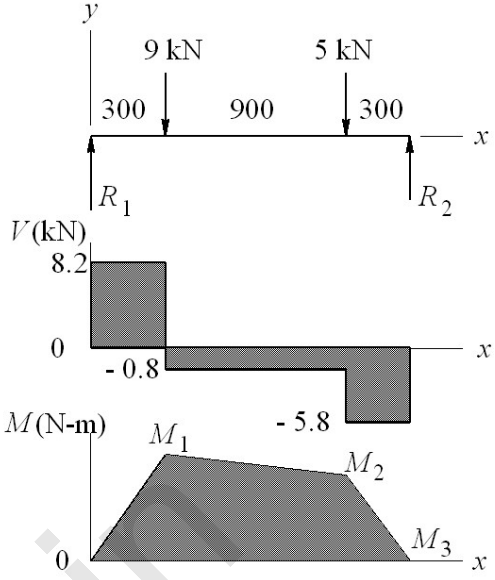
$$\Sigma M_C = 0$$

$$-1500R_1 + 300(5) + 1200(9) = 0$$

$$R_1 = 8.2 \text{ kN} \qquad \textit{Ans.}$$

$$\Sigma F_y = 0$$

$$8.2 - 9 - 5 + R_2 = 0 \qquad R_2 = 5.8 \text{ kN} \qquad \textit{Ans.}$$

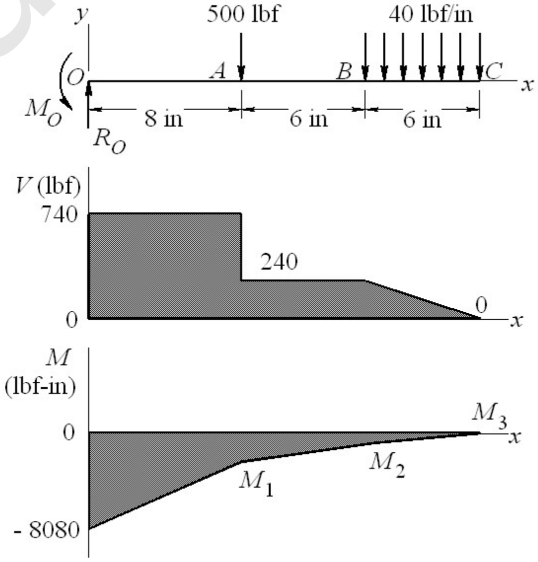


$$M_1 = 8.2(300) = 2460 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

$$M_2 = 2460 - 0.8(900) = 1740 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

$$M_3 = 1740 - 5.8(300) = 0 \quad \text{checks!}$$

3-6



$$\Sigma F_y = 0$$

$$R_O = 500 + 40(6) = 740 \text{ lbf} \qquad \textit{Ans.}$$

$$\Sigma M_O = 0$$

$$M_O = 500(8) + 40(6)(17) = 8080 \text{ lbf}\cdot\text{in} \qquad \textit{Ans.}$$

$$M_1 = -8080 + 740(8) = -2160 \text{ lbf}\cdot\text{in} \qquad \textit{Ans.}$$

$$M_2 = -2160 + 240(6) = -720 \text{ lbf}\cdot\text{in} \qquad \textit{Ans.}$$

$$M_3 = -720 + \frac{1}{2}(240)(6) = 0 \quad \text{checks!}$$

3-7

$\Sigma M_B = 0$

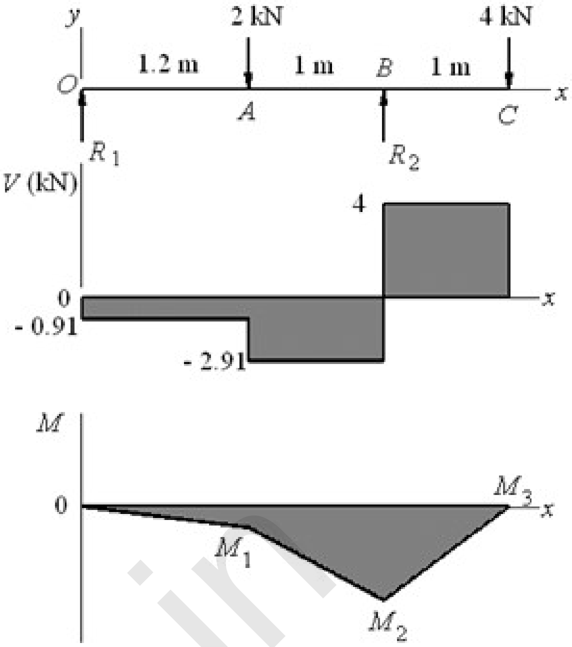
$-2.2R_1 + 1(2) - 1(4) = 0$

$R_1 = -0.91 \text{ kN}$ *Ans.*

$\Sigma F_y = 0$

$-0.91 - 2 + R_2 - 4 = 0$

$R_2 = 6.91 \text{ kN}$ *Ans.*



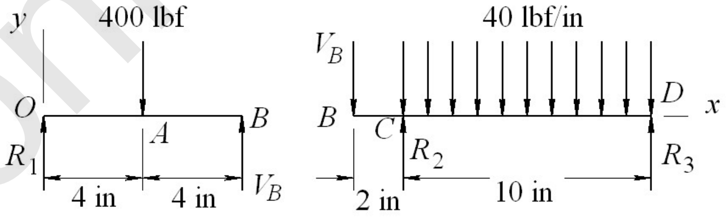
$M_1 = -0.91(1.2) = -1.09 \text{ kN} \cdot \text{m}$ *Ans.*

$M_2 = -1.09 - 2.91(1) = -4 \text{ kN} \cdot \text{m}$ *Ans.*

$M_3 = -4 + 4(1) = 0$ checks!

3-8

Break at the hinge at *B*



Beam *OB*:

From symmetry,

$R_1 = V_B = 200 \text{ lbf}$ *Ans.*

Beam *BD*:

$\Sigma M_D = 0$

$200(12) - R_2(10) + 40(10)(5) = 0$

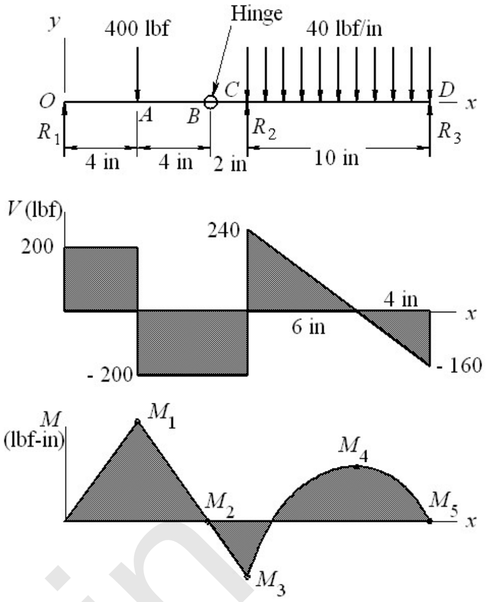
$R_2 = 440 \text{ lbf}$ *Ans.*

$\Sigma F_y = 0$

$-200 + 440 - 40(10) + R_3 = 0$

$R_3 = 160 \text{ lbf}$ *Ans.*

$$M_1 = 200(4) = 800 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$
$$M_2 = 800 - 200(4) = 0 \quad \text{checks at hinge}$$
$$M_3 = 800 - 200(6) = -400 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$
$$M_4 = -400 + \frac{1}{2}(240)(6) = 320 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$
$$M_5 = 320 - \frac{1}{2}(160)(4) = 0 \quad \text{checks!}$$



3-9

$$q = R_1 \langle x \rangle^{-1} - 9\langle x - 300 \rangle^{-1} - 5\langle x - 1200 \rangle^{-1} + R_2 \langle x - 1500 \rangle^{-1}$$
$$V = R_1 - 9\langle x - 300 \rangle^{0} - 5\langle x - 1200 \rangle^{0} + R_2 \langle x - 1500 \rangle^{0} \qquad (1)$$
$$M = R_1 x - 9\langle x - 300 \rangle^{1} - 5\langle x - 1200 \rangle^{1} + R_2 \langle x - 1500 \rangle^{1} \qquad (2)$$

At $x = 1500^+$ $V = M = 0$. Applying Eqs. (1) and (2),

$$R_1 - 9 - 5 + R_2 = 0 \quad \Rightarrow \quad R_1 + R_2 = 14$$

$$1500R_1 - 9(1500 - 300) - 5(1500 - 1200) = 0 \quad \Rightarrow \quad R_1 = 8.2 \text{ kN} \quad \textit{Ans.}$$

$$R_2 = 14 - 8.2 = 5.8 \text{ kN} \quad \textit{Ans.}$$

$0 \le x \le 300$: $V = 8.2$ kN, $M = 8.2x$ N·m
$300 \le x \le 1200$: $V = 8.2 - 9 = -0.8$ kN
$M = 8.2x - 9(x - 300) = -0.8x + 2700$ N·m
$1200 \le x \le 1500$: $V = 8.2 - 9 - 5 = -5.8$ kN
$M = 8.2x - 9(x - 300) - 5(x - 1200) = -5.8x + 8700$ N·m

Plots of V and M are the same as in Prob. 3-5.

3-10

$$q = R_O\langle x\rangle^{-1} - M_O\langle x\rangle^{-2} - 500\langle x-8\rangle^{-1} - 40\langle x-14\rangle^{0} + 40\langle x-20\rangle^{0}$$

$$V = R_O - M_O\langle x\rangle^{-1} - 500\langle x-8\rangle^{0} - 40\langle x-14\rangle^{1} + 40\langle x-20\rangle^{1} \qquad (1)$$

$$M = R_O x - M_O - 500\langle x-8\rangle^{1} - 20\langle x-14\rangle^{2} + 20\langle x-20\rangle^{2} \qquad (2)$$

at $x = 20^+$ in, $V = M = 0$, Eqs. (1) and (2) give

$$R_O - 500 - 40(20-14) = 0 \quad \Rightarrow \quad R_O = 740 \text{ lbf} \qquad \textit{Ans.}$$

$$R_O(20) - M_O - 500(20-8) - 20(20-14)^2 = 0 \quad \Rightarrow \quad M_O = 8080 \text{ lbf}\cdot\text{in} \qquad \textit{Ans.}$$

$0 \le x \le 8$: $V = 740$ lbf, $M = 740x - 8080$ lbf·in

$8 \le x \le 14$: $V = 740 - 500 = 240$ lbf

$M = 740x - 8080 - 500(x-8) = 240x - 4080$ lbf·in

$14 \le x \le 20$: $V = 740 - 500 - 40(x-14) = -40x + 800$ lbf

$M = 740x - 8080 - 500(x-8) - 20(x-14)^2 = -20x^2 + 800x - 8000$ lbf·in

Plots of *V* and *M* are the same as in Prob. 3-6.

3-11

$$q = R_1\langle x\rangle^{-1} - 2\langle x-1.2\rangle^{-1} + R_2\langle x-2.2\rangle^{-1} - 4\langle x-3.2\rangle^{-1}$$

$$V = R_1 - 2\langle x-1.2\rangle^{0} + R_2\langle x-2.2\rangle^{0} - 4\langle x-3.2\rangle^{0} \qquad (1)$$

$$M = R_1 x - 2\langle x-1.2\rangle^{1} + R_2\langle x-2.2\rangle^{1} - 4\langle x-3.2\rangle^{1} \qquad (2)$$

at $x = 3.2^+$, $V = M = 0$. Applying Eqs. (1) and (2),

$$R_1 - 2 + R_2 - 4 = 0 \quad \Rightarrow \quad R_1 + R_2 = 6 \qquad (3)$$

$$3.2R_1 - 2(2) + R_2(1) = 0 \quad \Rightarrow \quad 3.2R_1 + R_2 = 4 \qquad (4)$$

Solving Eqs. (3) and (4) simultaneously,

R_1 = -0.91 kN, R_2 = 6.91 kN *Ans.*

$0 \le x \le 1.2$: $V = -0.91$ kN, $M = -0.91x$ kN·m

$1.2 \le x \le 2.2$: $V = -0.91 - 2 = -2.91$ kN

$M = -0.91x - 2(x-1.2) = -2.91x + 2.4$ kN·m

$2.2 \le x \le 3.2$: $V = -0.91 - 2 + 6.91 = 4$ kN

$M = -0.91x - 2(x-1.2) + 6.91(x-2.2) = 4x - 12.8$ kN·m

Plots of *V* and *M* are the same as in Prob. 3-7.

3-12

$$q = R_1\langle x\rangle^{-1} - 400\langle x-4\rangle^{-1} + R_2\langle x-10\rangle^{-1} - 40\langle x-10\rangle^{0} + 40\langle x-20\rangle^{0} + R_3\langle x-20\rangle^{-1}$$

$$V = R_1 - 400\langle x-4\rangle^{0} + R_2\langle x-10\rangle^{0} - 40\langle x-10\rangle^{1} + 40\langle x-20\rangle^{1} + R_3\langle x-20\rangle^{0} \qquad (1)$$

$$M = R_1 x - 400\langle x-4\rangle^{1} + R_2\langle x-10\rangle^{1} - 20\langle x-10\rangle^{2} + 20\langle x-20\rangle^{2} + R_3\langle x-20\rangle^{1} \qquad (2)$$

$$M = 0 \text{ at } x = 8 \text{ in} \quad \therefore 8R_1 - 400(8-4) = 0 \quad \Rightarrow \quad R_1 = 200 \text{ lbf} \qquad Ans.$$

at $x = 20^+$, $V = M = 0$. Applying Eqs. (1) and (2),

$$200 - 400 + R_2 - 40(10) + R_3 = 0 \quad \Rightarrow \quad R_2 + R_3 = 600$$

$$200(20) - 400(16) + R_2(10) - 20(10)^2 = 0 \quad \Rightarrow \quad R_2 = 440 \text{ lbf} \qquad Ans.$$

$$R_3 = 600 - 440 = 160 \text{ lbf} \qquad Ans.$$

$0 \le x \le 4$: $\quad V = 200$ lbf, $\quad M = 200x$ lbf · in

$4 \le x \le 10$: $\quad V = 200 - 400 = -200$ lbf,

$\qquad M = 200x - 400(x-4) = -200x + 1600$ lbf · in

$10 \le x \le 20$: $\quad V = 200 - 400 + 440 - 40(x-10) = 640 - 40x$ lbf

$\qquad M = 200x - 400(x-4) + 440(x-10) - 20\left(x-10\right)^2 = -20x^2 + 640x - 4800$ lbf · in

Plots of V and M are the same as in Prob. 3-8.

3-13 Solution depends upon the beam selected.

3-14 **(a)** Moment at center,

$$x_c = \frac{(l-2a)}{2}$$

$$M_c = \frac{w}{2}\left[\frac{l}{2}(l-2a) - \left(\frac{l}{2}\right)^2\right] = \frac{wl}{2}\left(\frac{l}{4} - a\right)$$

At reaction, $|M_r| = wa^2/2$

$a = 2.25$, $l = 10$ in, $w = 100$ lbf/in

$$M_c = \frac{100(10)}{2}\left(\frac{10}{4} - 2.25\right) = 125 \text{ lbf} \cdot \text{in}$$

$$|M_r| = \frac{100\left(2.25^2\right)}{2} = 253 \text{ lbf} \cdot \text{in} \qquad Ans.$$

(b) Optimal occurs when $M_c = |M_r|$

$$\frac{wl}{2}\left(\frac{l}{4}-a\right)=\frac{wa^2}{2}\Rightarrow a^2+al-0.25l^2=0$$

Taking the positive root

$$a=\frac{1}{2}\left[-l+\sqrt{l^2+4\left(0.25l^2\right)}\right]=\frac{l}{2}\left(\sqrt{2}-1\right)=0.207\ l \qquad Ans.$$

for $l = 10$ in, $w = 100$ lbf, $a = 0.207(10) = 2.07$ in

$$M_{\min}=\left(100/2\right)2.07^2=214\ \text{lbf}\cdot\text{in}$$

3-15

(a)

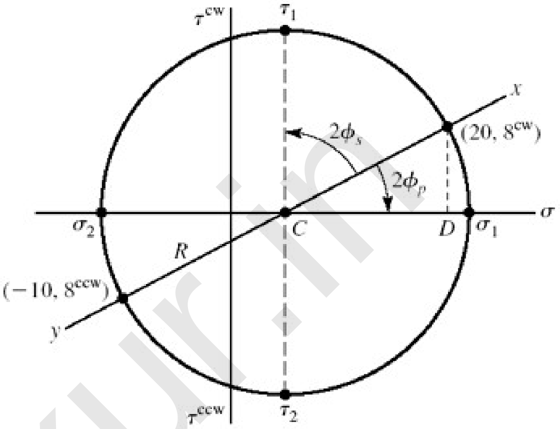
$$C=\frac{20-10}{2}=5\ \text{kpsi}$$

$$CD=\frac{20+10}{2}=15\ \text{kpsi}$$

$$R=\sqrt{15^2+8^2}=17\ \text{kpsi}$$

$$\sigma_1=5+17=22\ \text{kpsi}$$

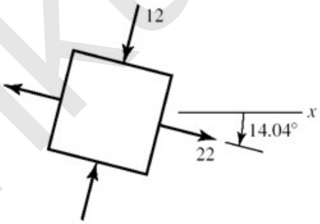
$$\sigma_2=5-17=-12\ \text{kpsi}$$

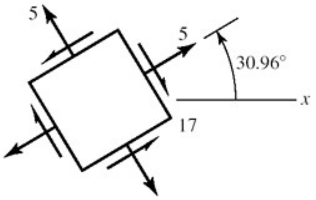


$$\phi_p=\frac{1}{2}\tan^{-1}\left(\frac{8}{15}\right)=14.04^\circ\ \text{cw}$$

$$\tau_1=R=17\ \text{kpsi}$$

$$\phi_s=45^\circ-14.04^\circ=30.96^\circ\ \text{ccw}$$





(b)

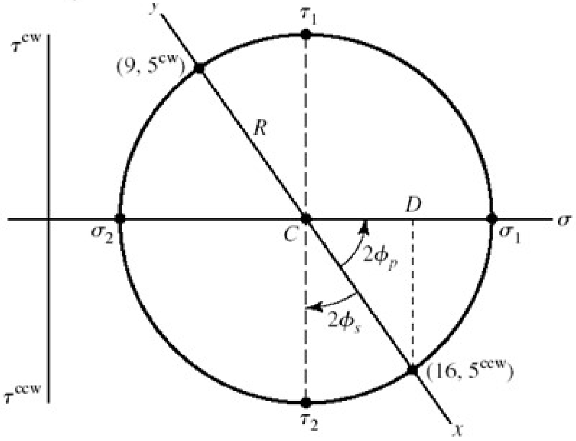
$$C=\frac{9+16}{2}=12.5\ \text{kpsi}$$

$$CD=\frac{16-9}{2}=3.5\ \text{kpsi}$$

$$R=\sqrt{5^2+3.5^2}=6.10\ \text{kpsi}$$

$$\sigma_1=12.5+6.1=18.6\ \text{kpsi}$$

$$\sigma_2=12.5-6.1=6.4\ \text{kpsi}$$

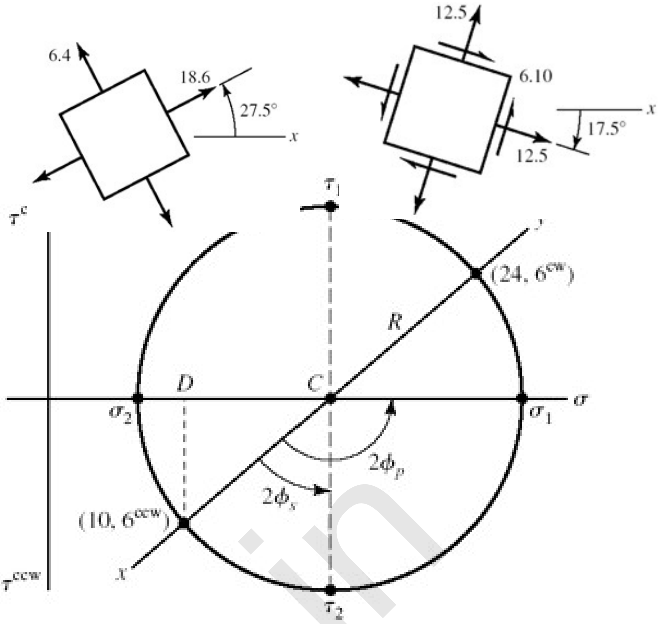


$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{5}{3.5}\right) = 27.5^\circ \text{ ccw}$$

$$\tau_1 = R = 6.10 \text{ kpsi}$$

$$\phi_s = 45^\circ - 27.5^\circ = 17.5^\circ \text{ cw}$$

(c)



$$C = \frac{24+10}{2} = 17 \text{ kpsi}$$

$$CD = \frac{24-10}{2} = 7 \text{ kpsi}$$

$$R = \sqrt{7^2 + 6^2} = 9.22 \text{ kpsi}$$

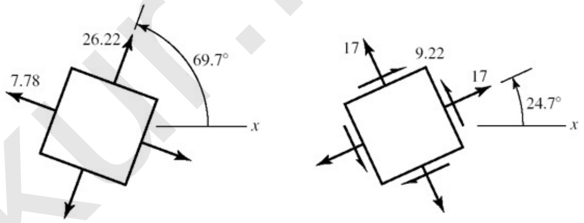
$$\sigma_1 = 17 + 9.22 = 26.22 \text{ kpsi}$$

$$\sigma_2 = 17 - 9.22 = 7.78 \text{ kpsi}$$

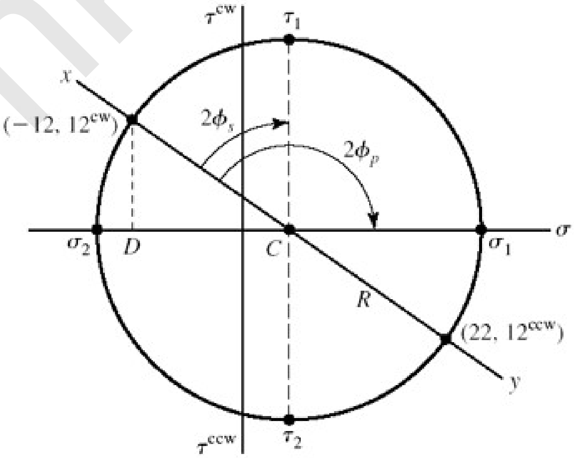
$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{7}{6}\right)\right] = 69.7^\circ \text{ ccw}$$

$$\tau_1 = R = 9.22 \text{ kpsi}$$

$$\phi_s = 69.7^\circ - 45^\circ = 24.7^\circ \text{ ccw}$$



(d)



$$C = \frac{-12+22}{2} = 5 \text{ kpsi}$$

$$CD = \frac{12+22}{2} = 17 \text{ kpsi}$$

$$R = \sqrt{17^2 + 12^2} = 20.81 \text{ kpsi}$$

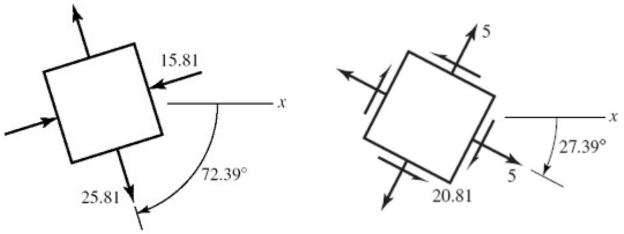
$$\sigma_1 = 5 + 20.81 = 25.81 \text{ kpsi}$$

$$\sigma_2 = 5 - 20.81 = -15.81 \text{ kpsi}$$

$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{17}{12}\right)\right] = 72.39^\circ \text{ cw}$$

$$\tau_1 = R = 20.81 \text{ kpsi}$$

$$\phi_s = 72.39 - 45 = 27.39^\circ \text{ cw}$$



3-16

(a)

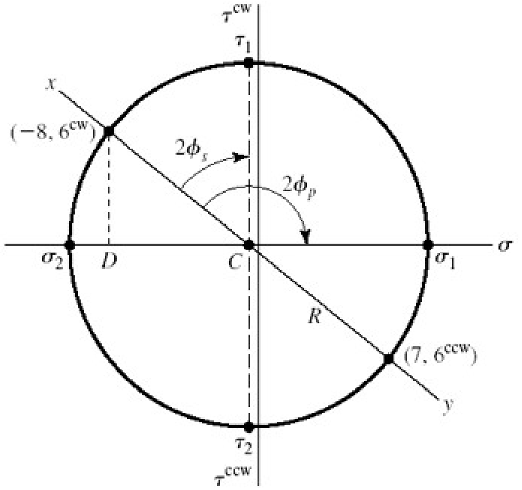
$$C=\frac{-8+7}{2}=-0.5\text{ MPa}$$

$$CD=\frac{8+7}{2}=7.5\text{ MPa}$$

$$R=\sqrt{7.5^2+6^2}=9.60\text{ MPa}$$

$$\sigma_1=9.60-0.5=9.10\text{ MPa}$$

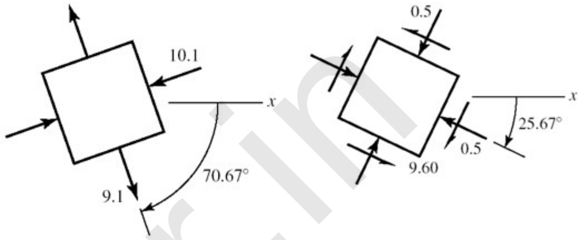
$$\sigma_2=-0.5-9.6=-10.1\text{ Mpa}$$



$$\phi_p=\frac{1}{2}\left[90^\circ+\tan^{-1}\left(\frac{7.5}{6}\right)\right]=70.67^\circ\text{ cw}$$

$$\tau_1=R=9.60\text{ MPa}$$

$$\phi_s=70.67^\circ-45^\circ=25.67^\circ\text{ cw}$$



(b)

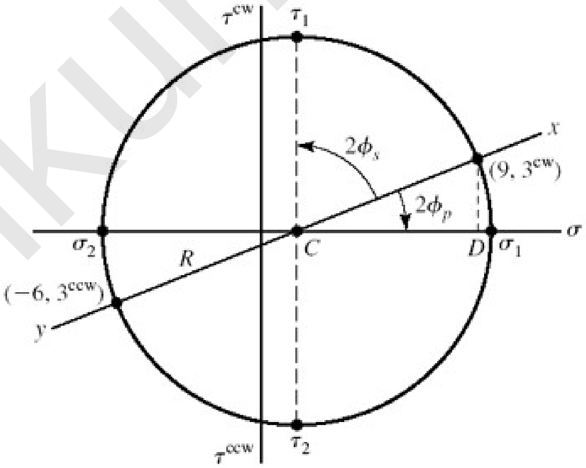
$$C=\frac{9-6}{2}=1.5\text{ MPa}$$

$$CD=\frac{9+6}{2}=7.5\text{ MPa}$$

$$R=\sqrt{7.5^2+3^2}=8.078\text{ MPa}$$

$$\sigma_1=1.5+8.078=9.58\text{ MPa}$$

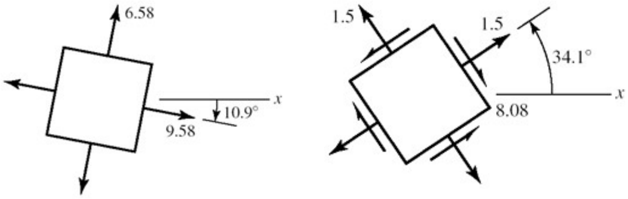
$$\sigma_2=1.5-8.078=-6.58\text{ MPa}$$



$$\phi_p=\frac{1}{2}\tan^{-1}\left(\frac{3}{7.5}\right)=10.9^\circ\text{ cw}$$

$$\tau_1=R=8.078\text{ MPa}$$

$$\phi_s=45^\circ-10.9^\circ=34.1^\circ\text{ ccw}$$



(c)

$$C = \frac{12-4}{2} = 4 \text{ MPa}$$

$$CD = \frac{12+4}{2} = 8 \text{ MPa}$$

$$R = \sqrt{8^2 + 7^2} = 10.63 \text{ MPa}$$

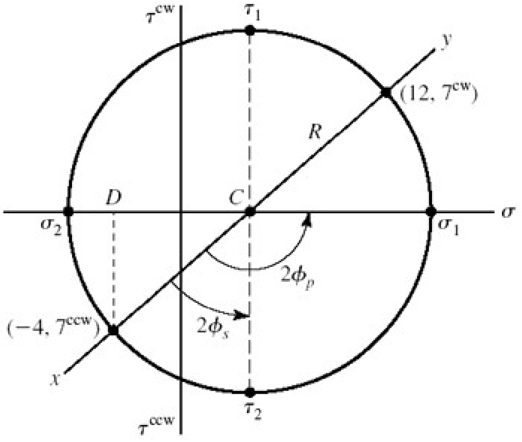
$$\sigma_1 = 4 + 10.63 = 14.63 \text{ MPa}$$

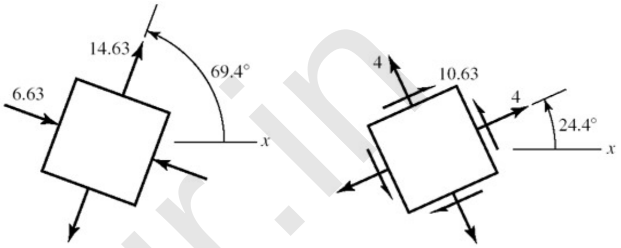
$$\sigma_2 = 4 - 10.63 = -6.63 \text{ MPa}$$

$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{8}{7}\right)\right] = 69.4^\circ \text{ ccw}$$

$$\tau_1 = R = 10.63 \text{ MPa}$$

$$\phi_s = 69.4^\circ - 45^\circ = 24.4^\circ \text{ ccw}$$





(d)

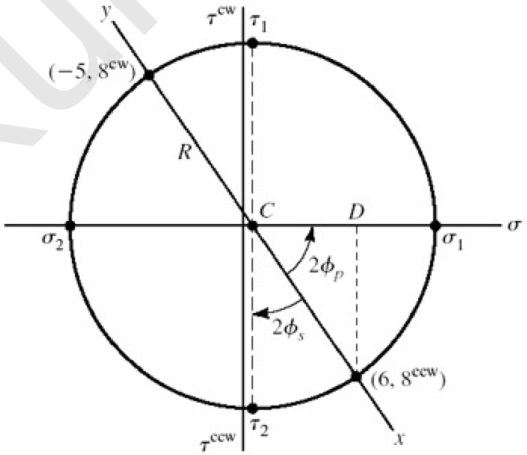
$$C = \frac{6-5}{2} = 0.5 \text{ MPa}$$

$$CD = \frac{6+5}{2} = 5.5 \text{ MPa}$$

$$R = \sqrt{5.5^2 + 8^2} = 9.71 \text{ MPa}$$

$$\sigma_1 = 0.5 + 9.71 = 10.21 \text{ MPa}$$

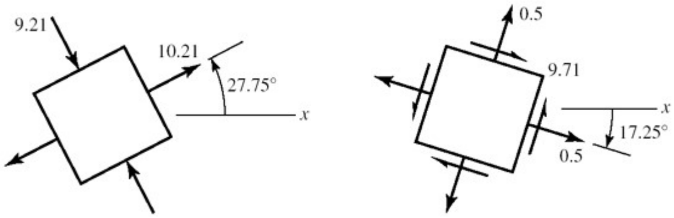
$$\sigma_2 = 0.5 - 9.71 = -9.21 \text{ MPa}$$



$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{8}{5.5}\right) = 27.75^\circ \text{ ccw}$$

$$\tau_1 = R = 9.71 \text{ MPa}$$

$$\phi_s = 45^\circ - 27.75^\circ = 17.25^\circ \text{ cw}$$



3-17

(a)

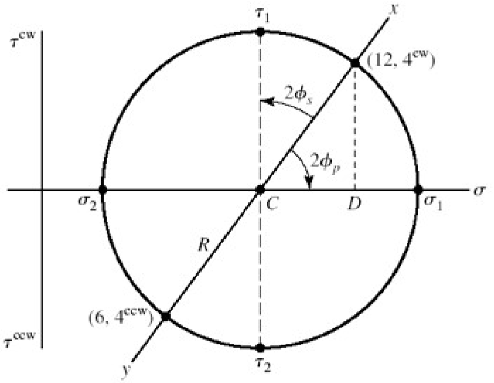
$$C = \frac{12+6}{2} = 9 \text{ kpsi}$$

$$CD = \frac{12-6}{2} = 3 \text{ kpsi}$$

$$R = \sqrt{3^2+4^2} = 5 \text{ kpsi}$$

$$\sigma_1 = 5+9 = 14 \text{ kpsi}$$

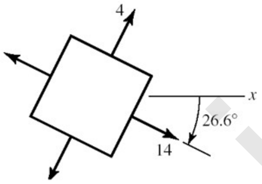
$$\sigma_2 = 9-5 = 4 \text{ kpsi}$$

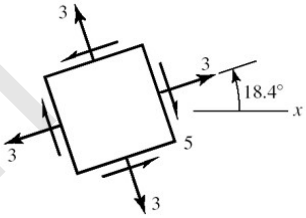


$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right) = 26.6^\circ \text{ ccw}$$

$$\tau_1 = R = 5 \text{ kpsi}$$

$$\phi_s = 45^\circ - 26.6^\circ = 18.4^\circ \text{ ccw}$$





(b)

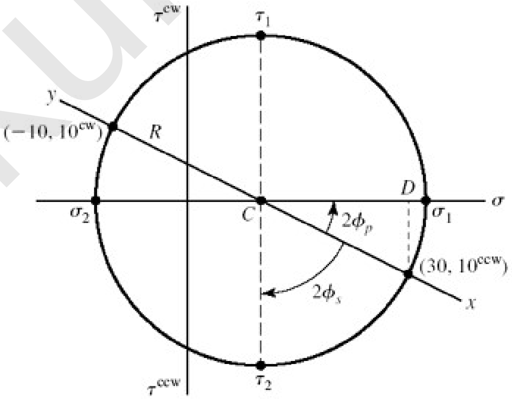
$$C = \frac{30-10}{2} = 10 \text{ kpsi}$$

$$CD = \frac{30+10}{2} = 20 \text{ kpsi}$$

$$R = \sqrt{20^2+10^2} = 22.36 \text{ kpsi}$$

$$\sigma_1 = 10+22.36 = 32.36 \text{ kpsi}$$

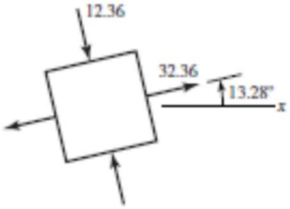
$$\sigma_2 = 10-22.36 = -12.36 \text{ kpsi}$$

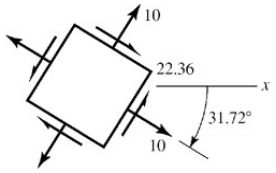


$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{10}{20}\right) = 13.28^\circ \text{ ccw}$$

$$\tau_1 = R = 22.36 \text{ kpsi}$$

$$\phi_s = 45^\circ - 13.28^\circ = 31.72^\circ \text{ cw}$$





(c)

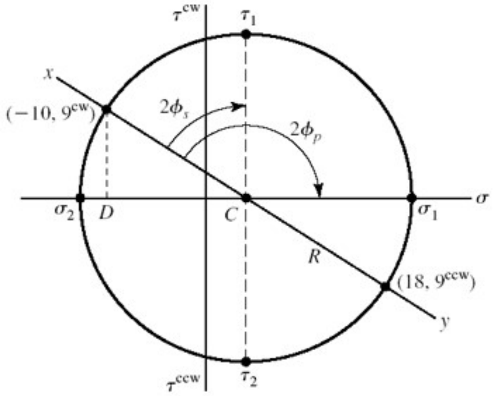
$$C = \frac{-10+18}{2} = 4 \text{ kpsi}$$

$$CD = \frac{10+18}{2} = 14 \text{ kpsi}$$

$$R = \sqrt{14^2 + 9^2} = 16.64 \text{ kpsi}$$

$$\sigma_1 = 4 + 16.64 = 20.64 \text{ kpsi}$$

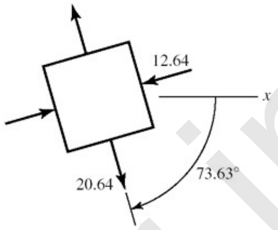
$$\sigma_2 = 4 - 16.64 = -12.64 \text{ kpsi}$$

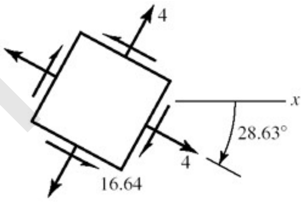


$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{14}{9}\right)\right] = 73.63^\circ \text{ cw}$$

$$\tau_1 = R = 16.64 \text{ kpsi}$$

$$\phi_s = 73.63 - 45 = 28.63^\circ \text{ cw}$$





(d)

$$C = \frac{9+19}{2} = 14 \text{ kpsi}$$

$$CD = \frac{19-9}{2} = 5 \text{ kpsi}$$

$$R = \sqrt{5^2 + 8^2} = 9.434 \text{ kpsi}$$

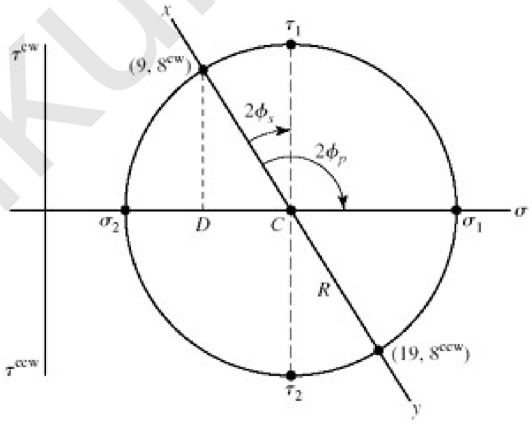
$$\sigma_1 = 14 + 9.43 = 23.43 \text{ kpsi}$$

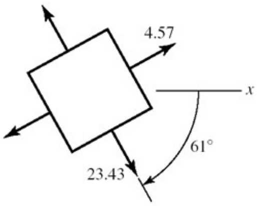
$$\sigma_2 = 14 - 9.43 = 4.57 \text{ kpsi}$$

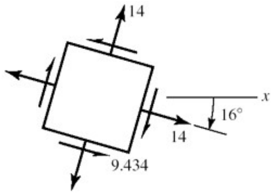
$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{5}{8}\right)\right] = 61.0^\circ \text{ cw}$$

$$\tau_1 = R = 9.34 \text{ kpsi}$$

$$\phi_s = 61^\circ - 45^\circ = 16^\circ \text{ cw}$$







3-18

(a)

$$C = \frac{-80-30}{2} = -55 \text{ MPa}$$

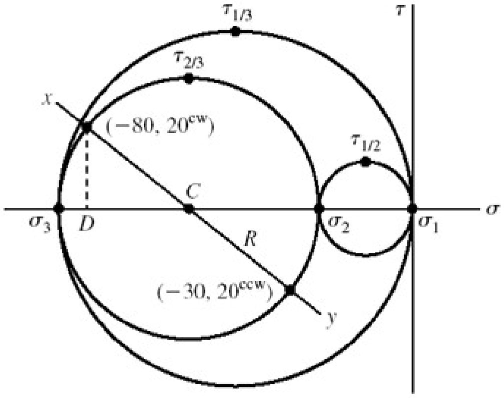
$$CD = \frac{80-30}{2} = 25 \text{ MPa}$$

$$R = \sqrt{25^2 + 20^2} = 32.02 \text{ MPa}$$

$$\sigma_1 = 0 \text{ MPa}$$

$$\sigma_2 = -55 + 32.02 = -22.98 = -23.0 \text{ MPa}$$

$$\sigma_3 = -55 - 32.0 = -87.0 \text{ MPa}$$



$$\tau_{1/2} = \frac{23}{2} = 11.5 \text{ MPa}, \quad \tau_{2/3} = 32.0 \text{ MPa}, \quad \tau_{1/3} = \frac{87}{2} = 43.5 \text{ MPa}$$

(b)

$$C = \frac{30-60}{2} = -15 \text{ MPa}$$

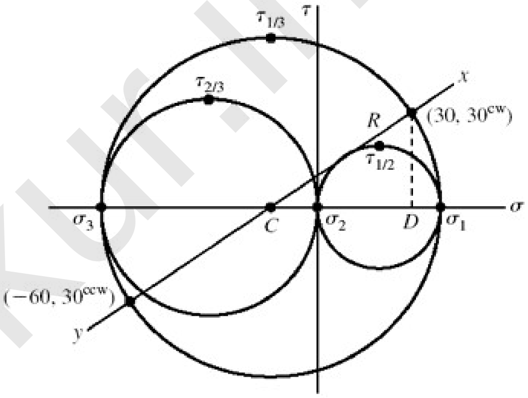
$$CD = \frac{60+30}{2} = 45 \text{ MPa}$$

$$R = \sqrt{45^2 + 30^2} = 54.1 \text{ MPa}$$

$$\sigma_1 = -15 + 54.1 = 39.1 \text{ MPa}$$

$$\sigma_2 = 0 \text{ MPa}$$

$$\sigma_3 = -15 - 54.1 = -69.1 \text{ MPa}$$



$$\tau_{1/3} = \frac{39.1+69.1}{2} = 54.1 \text{ MPa}$$

$$\tau_{1/2} = \frac{39.1}{2} = 19.6 \text{ MPa}$$

$$\tau_{2/3} = \frac{69.1}{2} = 34.6 \text{ MPa}$$

(c)

$$C = \frac{40+0}{2} = 20 \text{ MPa}$$

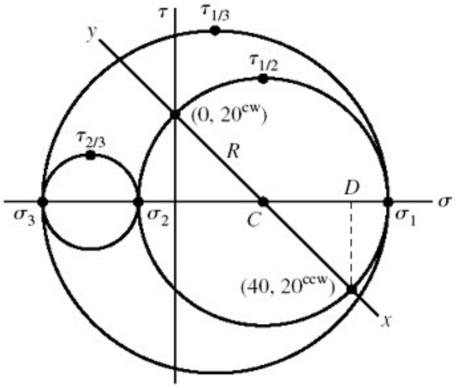
$$CD = \frac{40-0}{2} = 20 \text{ MPa}$$

$$R = \sqrt{20^2 + 20^2} = 28.3 \text{ MPa}$$

$$\sigma_1 = 20 + 28.3 = 48.3 \text{ MPa}$$

$$\sigma_2 = 20 - 28.3 = -8.3 \text{ MPa}$$

$$\sigma_3 = \sigma_z = -30 \text{ MPa}$$



$$\tau_{1/3} = \frac{48.3+30}{2} = 39.1 \text{ MPa}, \qquad \tau_{1/2} = 28.3 \text{ MPa}, \qquad \tau_{2/3} = \frac{30-8.3}{2} = 10.9 \text{ MPa}$$

(d)

$$C = \frac{50}{2} = 25 \text{ MPa}$$

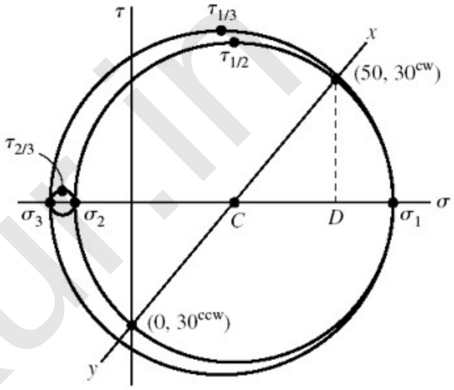
$$CD = \frac{50}{2} = 25 \text{ MPa}$$

$$R = \sqrt{25^2 + 30^2} = 39.1 \text{ MPa}$$

$$\sigma_1 = 25 + 39.1 = 64.1 \text{ MPa}$$

$$\sigma_2 = 25 - 39.1 = -14.1 \text{ MPa}$$

$$\sigma_3 = \sigma_z = -20 \text{ MPa}$$



$$\tau_{1/3} = \frac{64.1+20}{2} = 42.1 \text{ MPa}, \qquad \tau_{1/2} = 39.1 \text{ MPa}, \qquad \tau_{2/3} = \frac{20-14.1}{2} = 2.95 \text{ MPa}$$

3-19

(a)

Since there are no shear stresses on the stress element, the stress element already represents principal stresses.

$$\sigma_1 = \sigma_x = 10 \text{ kpsi}$$

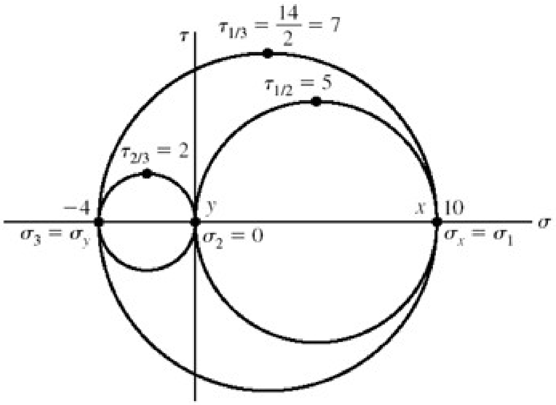
$$\sigma_2 = 0 \text{ kpsi}$$

$$\sigma_3 = \sigma_y = -4 \text{ kpsi}$$

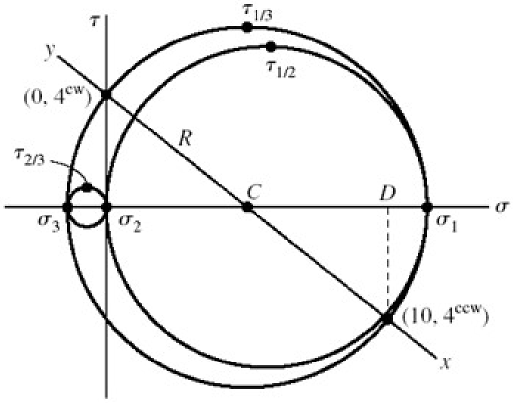
$$\tau_{1/3} = \frac{10-(-4)}{2} = 7 \text{ kpsi}$$

$$\tau_{1/2} = \frac{10}{2} = 5 \text{ kpsi}$$

$$\tau_{2/3} = \frac{0-(-4)}{2} = 2 \text{ kpsi}$$



(b)



$$C = \frac{0+10}{2} = 5 \text{ kpsi}$$

$$CD = \frac{10-0}{2} = 5 \text{ kpsi}$$

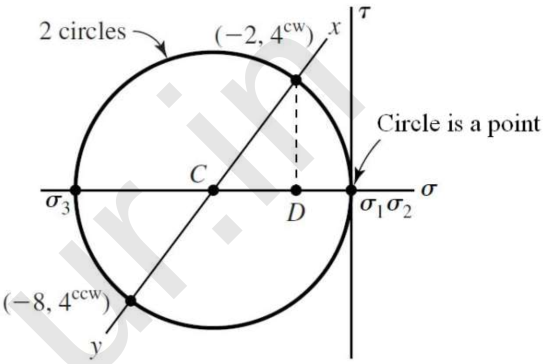
$$R = \sqrt{5^2 + 4^2} = 6.40 \text{ kpsi}$$

$$\sigma_1 = 5 + 6.40 = 11.40 \text{ kpsi}$$

$$\sigma_2 = 0 \text{ kpsi}, \quad \sigma_3 = 5 - 6.40 = -1.40 \text{ kpsi}$$

$$\tau_{1/3} = R = 6.40 \text{ kpsi}, \qquad \tau_{1/2} = \frac{11.40}{2} = 5.70 \text{ kpsi}, \qquad \tau_3 = \frac{1.40}{2} = 0.70 \text{ kpsi}$$

(c)



$$C = \frac{-2-8}{2} = -5 \text{ kpsi}$$

$$CD = \frac{8-2}{2} = 3 \text{ kpsi}$$

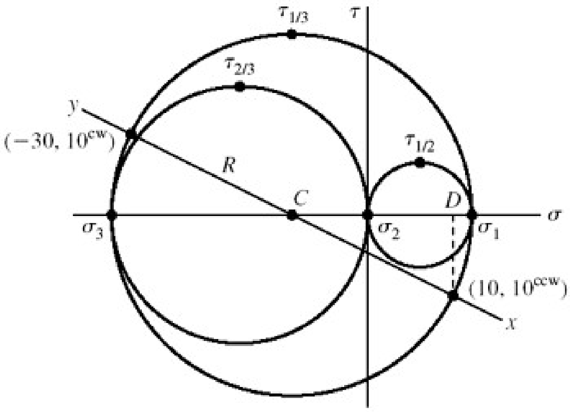
$$R = \sqrt{3^2 + 4^2} = 5 \text{ kpsi}$$

$$\sigma_1 = -5 + 5 = 0 \text{ kpsi}, \quad \sigma_2 = 0 \text{ kpsi}$$

$$\sigma_3 = -5 - 5 = -10 \text{ kpsi}$$

$$\tau_{1/3} = \frac{10}{2} = 5 \text{ kpsi}, \qquad \tau_{1/2} = 0 \text{ kpsi}, \qquad \tau_{2/3} = 5 \text{ kpsi}$$

(d)



$$C = \frac{10-30}{2} = -10 \text{ kpsi}$$

$$CD = \frac{10+30}{2} = 20 \text{ kpsi}$$

$$R = \sqrt{20^2 + 10^2} = 22.36 \text{ kpsi}$$

$$\sigma_1 = -10 + 22.36 = 12.36 \text{ kpsi}$$

$$\sigma_2 = 0 \text{ kpsi}$$

$$\sigma_3 = -10 - 22.36 = -32.36 \text{ kpsi}$$

$$\tau_{1/3} = 22.36 \text{ kpsi}, \qquad \tau_{1/2} = \frac{12.36}{2} = 6.18 \text{ kpsi}, \qquad \tau_{2/3} = \frac{32.36}{2} = 16.18 \text{ kpsi}$$

3-20 From Eq. (3-15),

$$\sigma^3 - (-6+18-12)\sigma^2 + \left[-6(18)+(-6)(-12)+18(-12)-9^2-6^2-(-15)^2\right]\sigma$$

$$-\left[-6(18)(-12)+2(9)(6)(-15)-(-6)(6)^2-18(-15)^2-(-12)(9)^2\right]=0$$

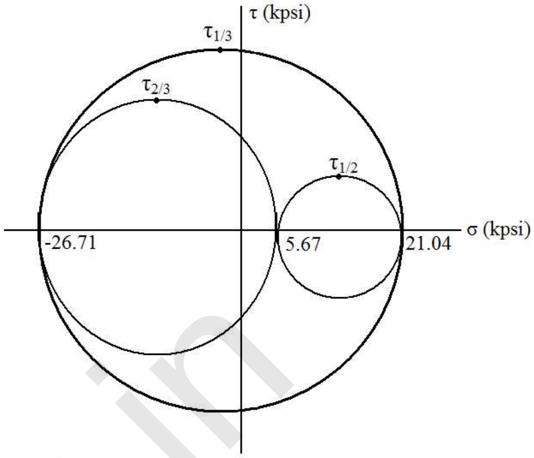
$$\sigma^3 - 594\sigma + 3186 = 0$$

Roots are: 21.04, 5.67, –26.71 kpsi *Ans.*

$$\tau_{1/2} = \frac{21.04-5.67}{2} = 7.69 \text{ kpsi}$$

$$\tau_{2/3} = \frac{5.67+26.71}{2} = 16.19 \text{ kpsi}$$

$$\tau_{\max} = \tau_{1/3} = \frac{21.04+26.71}{2} = 23.88 \text{ kpsi} \qquad \textit{Ans.}$$



3-21

From Eq. (3-15)

$$\sigma^3 - (20+0+20)\sigma^2 + \left[20(0)+20(20)+0(20)-40^2-\left(-20\sqrt{2}\right)^2-0^2\right]\sigma$$

$$-\left[20(0)(20)+2(40)\left(-20\sqrt{2}\right)(0)-20\left(-20\sqrt{2}\right)^2-0(0)^2-20(40)^2\right]=0$$

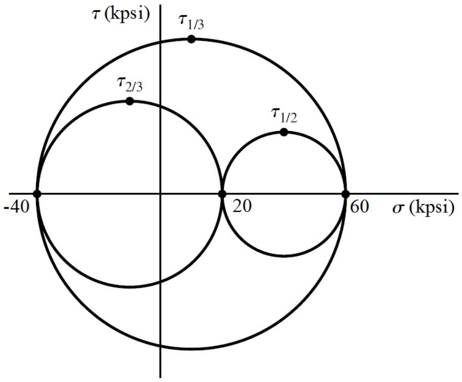
$$\sigma^3 - 40\sigma^2 - 2\ 000\sigma + 48\ 000 = 0$$

Roots are: 60, 20, –40 kpsi *Ans.*

$$\tau_{1/2} = \frac{60-20}{2} = 20 \text{ kpsi}$$

$$\tau_{2/3} = \frac{20+40}{2} = 30 \text{ kpsi}$$

$$\tau_{\max} = \tau_{1/3} = \frac{60+40}{2} = 50 \text{ kpsi} \qquad \textit{Ans.}$$



3-22 From Eq. (3-15)

$$\sigma^3-(10+40+40)\sigma^2+\left[10(40)+10(40)+40(40)-20^2-(-40)^2-(-20)^2\right]\sigma$$

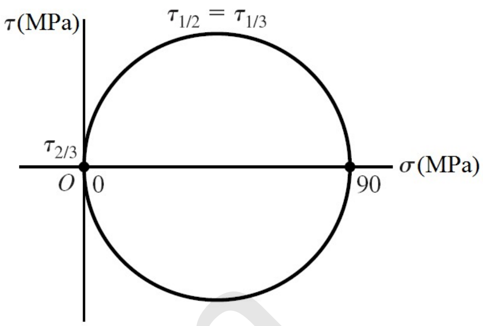
$$-\left[10(40)(40)+2(20)(-40)(-20)-10(-40)^2-40(-20)^2-40(20)^2\right]=0$$

$$\sigma^3-90\sigma^2=0$$

Roots are: 90, 0, 0 MPa *Ans.*

$$\tau_{2/3}=0$$

$$\tau_{1/2}=\tau_{1/3}=\tau_{\max}=\frac{90}{2}=45\text{ MPa}\quad \textit{Ans.}$$



3-23

$$\sigma=\frac{F}{A}=\frac{15000}{(\pi/4)\left(0.75^2\right)}=33\,950\text{ psi}=34.0\text{ kpsi}\quad \textit{Ans.}$$

$$\delta=\frac{FL}{AE}=\sigma\frac{L}{E}=33\,950\frac{60}{30\left(10^6\right)}=0.0679\text{ in}\quad \textit{Ans.}$$

$$\varepsilon_1=\frac{\delta}{L}=\frac{0.0679}{60}=1130\left(10^{-6}\right)=1130\mu\quad \textit{Ans.}$$

From Table A-5, $v=0.292$

$$\varepsilon_2=-v\varepsilon_1=-0.292(1130)=-330\mu\quad \textit{Ans.}$$

$$\Delta d=\varepsilon_2 d=-330\left(10^{-6}\right)(0.75)=-248\left(10^{-6}\right)\text{ in}\quad \textit{Ans.}$$

3-24

$$\sigma=\frac{F}{A}=\frac{3000}{(\pi/4)\left(0.75^2\right)}=6790\text{ psi}=6.79\text{ kpsi}\quad \textit{Ans.}$$

$$\delta=\frac{FL}{AE}=\sigma\frac{L}{E}=6790\frac{60}{10.4\left(10^6\right)}=0.0392\text{ in}\quad \textit{Ans.}$$

$$\varepsilon_1=\frac{\delta}{L}=\frac{0.0392}{60}=653\left(10^{-6}\right)=653\mu\quad \textit{Ans.}$$

From Table A-5, $v=0.333$

$$\varepsilon_2=-v\varepsilon_1=-0.333(653)=-217\mu\quad \textit{Ans.}$$

$$\Delta d=\varepsilon_2 d=-217\left(10^{-6}\right)(0.75)=-163\left(10^{-6}\right)\text{ in}\quad \textit{Ans.}$$

3-25

$$\varepsilon_2 = \frac{\Delta d}{d} = \frac{-0.0001d}{d} = -0.0001$$

From Table A-5, $v = 0.326$, $E = 119$ GPa

$$\varepsilon_1 = \frac{-\varepsilon_2}{v} = \frac{-0.0001}{0.326} = 306.7\left(10^{-6}\right)$$

$$\delta = \frac{FL}{AE} \text{ and } \sigma = \frac{F}{A}, \text{ so}$$

$$\sigma = \frac{\delta E}{L} = \varepsilon_1 E = 306.7\left(10^{-6}\right)(119)\left(10^9\right) = 36.5 \text{ MPa}$$

$$F = \sigma A = 36.5\left(10^6\right)\frac{\pi\left(0.03\right)^2}{4} = 25\ 800 \text{ N} = 25.8 \text{ kN} \qquad Ans.$$

S_y = 70 MPa > σ, so elastic deformation assumption is valid.

3-26

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 20\ 000\frac{8(12)}{10.4\left(10^6\right)} = 0.185 \text{ in} \qquad Ans.$$

3-27

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 140\left(10^6\right)\frac{3}{71.7\left(10^9\right)} = 0.00586 \text{ m} = 5.86 \text{ mm} \qquad Ans.$$

3-28

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 15\ 000\frac{10(12)}{10.4\left(10^6\right)} = 0.173 \text{ in} \qquad Ans.$$

3-29

With $\sigma_z = 0$, solve the first two equations of Eq. (3-19) simulatenously. Place E on the left-hand side of both equations, and using Cramer's rule,

$$\sigma_x = \frac{\begin{vmatrix} E\varepsilon_x & -v \\ E\varepsilon_y & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -v \\ -v & 1 \end{vmatrix}} = \frac{E\varepsilon_x + vE\varepsilon_y}{1-v^2} = \frac{E\left(\varepsilon_x + v\varepsilon_y\right)}{1-v^2}$$

Likewise,

$$\sigma_y = \frac{E\left(\varepsilon_y + \nu\varepsilon_x\right)}{1-\nu^2}$$

From Table A-5, $E = 207$ GPa and $\nu = 0.292$. Thus,

$$\sigma_x = \frac{E\left(\varepsilon_x + \nu\varepsilon_y\right)}{1-\nu^2} = \frac{207\left(10^9\right)\left[0.0019 + 0.292\left(-0.000\ 72\right)\right]}{1-0.292^2}\left(10^{-6}\right) = 382 \text{ MPa} \qquad Ans.$$

$$\sigma_y = \frac{207\left(10^9\right)\left[-0.000\ 72 + 0.292\left(0.0019\right)\right]}{1-0.292^2}\left(10^{-6}\right) = -37.4 \text{ MPa} \qquad Ans.$$

3-30

With $\sigma_z = 0$, solve the first two equations of Eq. (3-19) simulatenously. Place E on the left-hand side of both equations, and using Cramer's rule,

$$\sigma_x = \frac{\begin{vmatrix} E\varepsilon_x & -\nu \\ E\varepsilon_y & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\nu \\ -\nu & 1 \end{vmatrix}} = \frac{E\varepsilon_x + \nu E\varepsilon_y}{1-\nu^2} = \frac{E\left(\varepsilon_x + \nu\varepsilon_y\right)}{1-\nu^2}$$

Likewise,

$$\sigma_y = \frac{E\left(\varepsilon_y + \nu\varepsilon_x\right)}{1-\nu^2}$$

From Table A-5, $E = 71.7$ GPa and $\nu = 0.333$. Thus,

$$\sigma_x = \frac{E\left(\varepsilon_x + \nu\varepsilon_y\right)}{1-\nu^2} = \frac{71.7\left(10^9\right)\left[0.0019 + 0.333\left(-0.000\ 72\right)\right]}{1-0.333^2}\left(10^{-6}\right) = 134 \text{ MPa} \qquad Ans.$$

$$\sigma_y = \frac{71.7\left(10^9\right)\left[-0.000\ 72 + 0.333\left(0.0019\right)\right]}{1-0.333^2}\left(10^{-6}\right) = -7.04 \text{ MPa} \qquad Ans.$$

3-31 For plane strain, $\varepsilon_z = 0$. From the third equation of Eq. (3-19),

$$\sigma_z = \nu\left(\sigma_x + \sigma_y\right) \qquad Ans.$$

First of Eq. (3-19),

$$\begin{aligned}\varepsilon_x &= \frac{1}{E}\left\{\sigma_x - \nu\left[\sigma_y + \nu\left(\sigma_x + \sigma_y\right)\right]\right\} \\ &= \frac{1}{E}\left[\left(1-\nu^2\right)\sigma_x - \nu\left(1+\nu\right)\sigma_y\right] \\ &= \frac{1+\nu}{E}\left[\left(1-\nu\right)\sigma_x - \nu\sigma_y\right] \qquad Ans.\end{aligned}$$

Similarly,

$$\varepsilon_y = \frac{1+\nu}{E}\left[(1-\nu)\sigma_y - \nu\sigma_x\right] \qquad Ans.$$

3-32

(a) $R_1 = \dfrac{c}{l}F \qquad M_{\max} = R_1 a = \dfrac{ac}{l}F$

$$\sigma = \frac{6M}{bh^2} = \frac{6}{bh^2}\frac{ac}{l}F \Rightarrow F = \frac{\sigma b h^2 l}{6ac} \qquad Ans.$$

(b) $$\frac{F_m}{F} = \frac{(\sigma_m/\sigma)(b_m/b)(h_m/h)^2(l_m/l_1)}{(a_m/a)(c_m/c)} = \frac{1(s)(s)^2(s)}{(s)(s)} = s^2 \qquad Ans.$$

For equal stress, the model load varies by the square of the scale factor.

3-33

(a) $$R_1 = \frac{wl}{2}, \qquad M_{\max}\Big|_{x=l/2} = \frac{w}{2}\frac{l}{2}\left(l - \frac{l}{2}\right) = \frac{wl^2}{8}$$

$$\sigma = \frac{6M}{bh^2} = \frac{6}{bh^2}\frac{wl^2}{8} = \frac{3Wl}{4bh^2} \quad \Rightarrow \quad W = \frac{4}{3}\frac{\sigma b h^2}{l} \qquad Ans.$$

(b) $$\frac{W_m}{W} = \frac{(\sigma_m/\sigma)(b_m/b)(h_m/h)^2}{l_m/l} = \frac{1(s)(s)^2}{s} = s^2 \qquad Ans.$$

$$\frac{w_m l_m}{wl} = s^2 \quad \Rightarrow \quad \frac{w_m}{w} = \frac{s^2}{s} = s \qquad Ans.$$

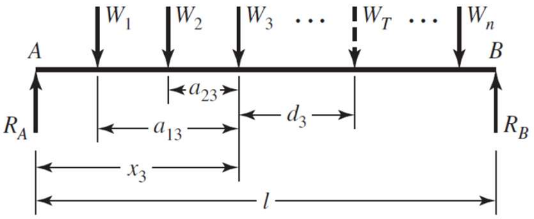
For equal stress, the model load *w* varies linearly with the scale factor.

3-34

(a) Can solve by iteration *or* derive equations for the general case. Find maximum moment under wheel W_3.

$W_T = \Sigma W$ at centroid of W's

$$R_A = \frac{l - x_3 - d_3}{l} W_T$$



Under wheel 3,

$$M_3 = R_A x_3 - W_1 a_{13} - W_2 a_{23} = \frac{(l - x_3 - d_3)}{l} W_T x_3 - W_1 a_{13} - W_2 a_{23}$$

For maximum, $\dfrac{dM_3}{dx_3} = 0 = (l - d_3 - 2x_3)\dfrac{W_T}{l} \quad \Rightarrow \quad x_3 = \dfrac{l - d_3}{2}$

Substitute into $M \Rightarrow M_3 = \dfrac{(l - d_3)^2}{4l} W_T - W_1 a_{13} - W_2 a_{23}$

This means the midpoint of d_3 intersects the midpoint of the beam.

For wheel *i*, $x_i = \dfrac{l-d_i}{2}$, $M_i = \dfrac{(l-d_i)^2}{4l}W_T - \sum_{j=1}^{i-1} W_j a_{ji}$

Note for wheel 1: $\Sigma W_j a_{ji} = 0$

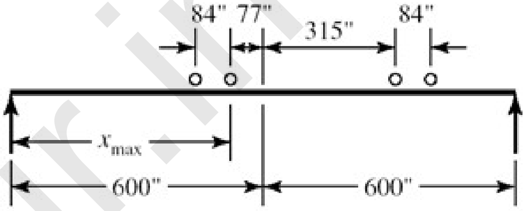
$$W_T = 104.4, \quad W_1 = W_2 = W_3 = W_4 = \frac{104.4}{4} = 26.1 \text{ kips}$$

Wheel 1: $d_1 = \dfrac{476}{2} = 238$ in, $M_1 = \dfrac{(1200-238)^2}{4(1200)}(104.4) = 20\,128$ kip·in

Wheel 2: $d_2 = 238 - 84 = 154$ in

$$M_2 = \frac{(1200-154)^2}{4(1200)}(104.4) - 26.1(84) = 21\,605 \text{ kip}\cdot\text{in} = M_{\max} \qquad \textit{Ans.}$$

Check if all of the wheels are on the rail.



(b) $x_{\max} = 600 - 77 = 523$ in *Ans.*

(c) See above sketch.

(d) Inner axles

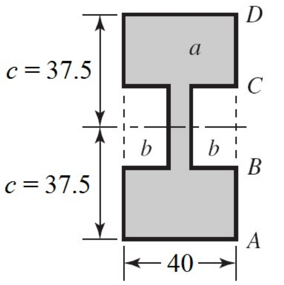
3-35

(a) Let a = total area of entire envelope
Let b = area of side notch

$$A = a - 2b = 40(2)(37.5) - 25(34) = 2150 \text{ mm}^2$$

$$I = I_a - 2I_b = \frac{1}{12}(40)(75)^3 - \frac{1}{12}(34)(25)^3$$

$$I = 1.36\left(10^6\right) \text{ mm}^4 \qquad \textit{Ans.}$$



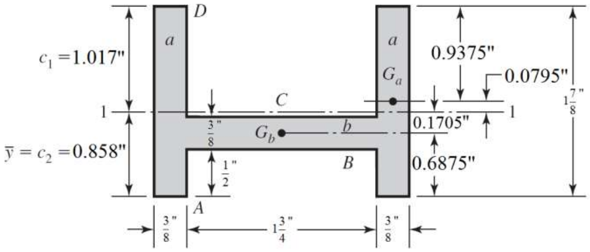
Dimensions in mm.

(b)

$A_a = 0.375(1.875) = 0.703\,125$ in^2

$A_b = 0.375(1.75) = 0.656\,25$ in^2

$A = 2(0.703125) + 0.656\,25 = 2.0625$ in^2



$$\bar{y} = \frac{2(0.703\,125)(0.9375) + 0.656\,25(0.6875)}{2.0625} = 0.858 \text{ in} \quad \textit{Ans.}$$

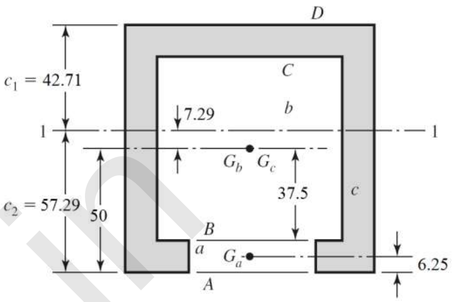
$$I_a = \frac{0.375(1.875)^3}{12} = 0.206 \text{ in}^4$$

$$I_b = \frac{1.75(0.375)^3}{12} = 0.007\,69 \text{ in}^4$$

$$I_1 = 2\left[0.206 + 0.703\,125(0.0795)^2\right] + \left[0.00769 + 0.656\,25(0.1705)^2\right] = 0.448 \text{ in}^4 \quad \textit{Ans.}$$

(c)

Use two negative areas.



$$A_a = 625 \text{ mm}^2,\ A_b = 5625 \text{ mm}^2,\ A_c = 10\,000 \text{ mm}^2$$

$$A = 10\,000 - 5625 - 625 = 3750 \text{ mm}^2;$$

$$\bar{y}_a = 6.25 \text{ mm},\ \bar{y}_b = 50 \text{ mm},\ \bar{y}_c = 50 \text{ mm}$$

$$\bar{y} = \frac{10\,000(50) - 5625(50) - 625(6.25)}{3750} = 57.29 \text{ mm} \quad \textit{Ans.}$$

$$c_1 = 100 - 57.29 = 42.71 \text{ mm} \quad \textit{Ans.}$$

$$I_a = \frac{50(12.5)^3}{12} = 8138 \text{ mm}^4$$

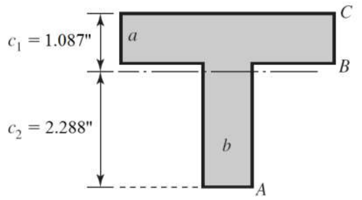
$$I_b = \frac{75(75)^3}{12} = 2.637\left(10^6\right) \text{ mm}^4$$

$$I_c = \frac{100(100)^3}{12} = 8.333\left(10^6\right)\text{in}^4$$

$$I_1 = \left[8.333\left(10^6\right) + 10\,000(7.29)^2\right] - \left[2.637\left(10^6\right) + 5625\left(7.29\right)^2\right] - \left[8138 + 625\left(57.29 - 6.25\right)^2\right]$$

$$I_1 = 4.29\left(10^6\right) \text{ in}^4 \quad \textit{Ans.}$$

(d)



$$A_a = 4(0.875) = 3.5 \text{ in}^2$$

$$A_b = 2.5(0.875) = 2.1875 \text{ in}^2$$

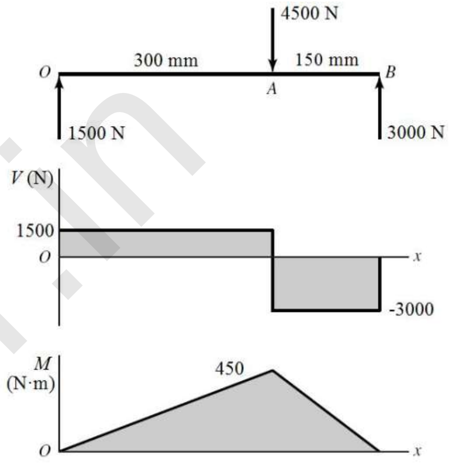
$$A = A_a + A_b = 5.6875 \text{ in}^2$$

$$\bar{y} = \frac{2.9375(3.5) + 1.25(2.1875)}{5.6875} = 2.288 \text{ in} \quad Ans.$$

$$I = \frac{1}{12}(4)(0.875)^3 + 3.5(2.9375 - 2.288)^2 + \frac{1}{12}(0.875)(2.5)^3 + 2.1875(2.288 - 1.25)^2$$

$$I = 5.20 \text{ in}^4 \quad Ans.$$

3-36



$$I = \frac{1}{12}(20)(40)^3 = 1.067\left(10^5\right) \text{ mm}^4$$

$$A = 20(40) = 800 \text{ mm}^2$$

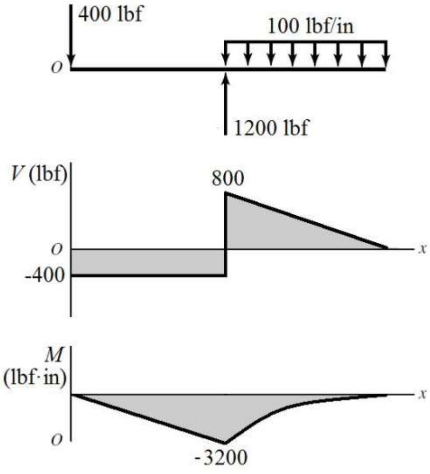
$M_{\max}$ is at A. At the bottom of the section,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{450\ 000(20)}{1.067\left(10^5\right)} = 84.3 \text{ MPa} \quad Ans.$$

Due to V, $\tau_{\max}$ is between A and B at $y = 0$.

$$\tau_{\max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\left(\frac{3000}{800}\right) = 5.63 \text{ MPa} \quad Ans.$$

3-37 $I = \frac{1}{12}(1)(2)^3 = 0.6667 \text{ in}^4$



$$A = 1(2) = 2 \text{ in}^2$$

$$\Sigma M_O = 0$$

$$8R_A - 100(8)(12) = 0$$

$$R_A = 1200 \text{ lbf}$$

$$R_O = 1200 - 100(8) = 400 \text{ lbf}$$

$M_{\max}$ is at A. At the top of the beam,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{3200(0.5)}{0.6667} = 2400 \text{ psi} \quad Ans.$$

Due to V, $\tau_{\max}$ is at A, at $y = 0$.

$$\tau_{\max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\left(\frac{800}{2}\right) = 600 \text{ psi} \quad Ans.$$

3-38 $I = \frac{1}{12}(0.75)(2)^3 = 0.5 \text{ in}^4$

$A = (0.75)(2) = 1.5 \text{ in}^2$

$\Sigma M_A = 0$

$15R_B - 1000(20) = 0$

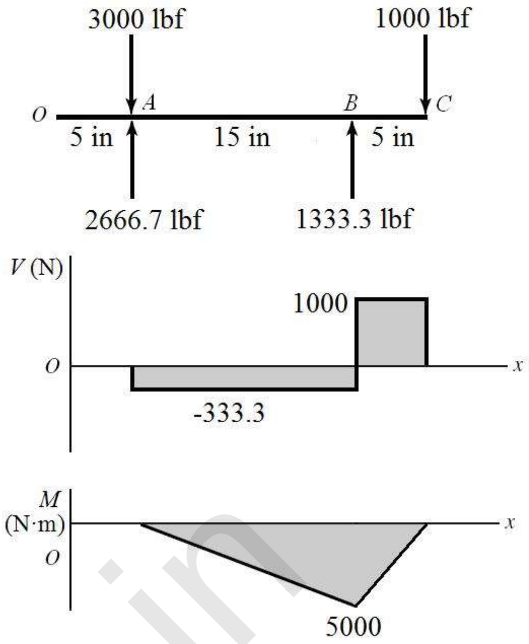
$R_B = 1333.3 \text{ lbf}$

$R_A = 3000 - 1333.3 + 1000 = 2666.7 \text{ lbf}$

$M_{\max}$ is at B. At the top of the beam,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{5000(1)}{0.5} = 10\,000 \text{ psi} \quad \textit{Ans.}$$

Due to V, $\tau_{\max}$ is between B and C at $y = 0$.



$$\tau_{\max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\left(\frac{1000}{1.5}\right) = 1000 \text{ psi} \quad \textit{Ans.}$$

3-39 $I = \frac{\pi d^4}{64} = \frac{\pi(50)^4}{64} = 306.796\left(10^3\right) \text{ mm}^4$

$A = \frac{\pi d^2}{4} = \frac{\pi(50)^2}{4} = 1963 \text{ mm}^2$

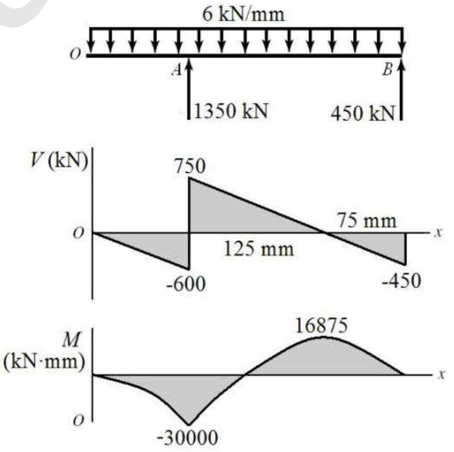
$\Sigma M_B = 0$

$6(300)(150) - 200R_A = 0$

$R_A = 1350 \text{ kN}$

$R_B = 6(300) - 1350 = 450 \text{ kN}$

$M_{\max}$ is at A. At the top,



$$\sigma_{\max} = \frac{Mc}{I} = \frac{30\,000(25)}{306\,796} = 2.44 \text{ kN/mm}^2 = 2.44 \text{ GPa} \quad \textit{Ans.}$$

Due to V, $\tau_{\max}$ is at A, at $y = 0$.

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\left(\frac{750}{1963}\right) = 0.509 \text{ kN/mm}^2 = 509 \text{ MPa} \quad \textit{Ans.}$$

3-40

$$M_{\max} = \frac{wl^2}{8} \Rightarrow \sigma_{\max} = \frac{wl^2c}{8I} \Rightarrow \quad w = \frac{8\sigma_{\max} I}{cl^2}$$

(a) $l = 48$ in; Table A-8, $I = 0.537$ in^4

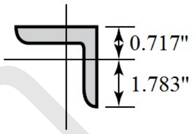
$$w = \frac{8(12)(10^3)(0.537)}{1(48^2)} = 22.38 \text{ lbf/in} \quad \textit{Ans.}$$

(b) $l = 60$ in, $I \approx (1/12)(2)(3^3) - (1/12)(1.625)(2.625^3) = 2.051$ in^4

$$w = \frac{8(12)(10^3)(2.051)}{(1.5)(60^2)} = 36.5 \text{ lbf/in} \quad \textit{Ans.}$$

(c) $l = 60$ in; Table A-6, $I = 2(0.703) = 1.406$ in^4

$y = 0.717$ in, $c_{\max} = 1.783$ in



$$w = \frac{8(12)(10^3)(1.406)}{1.783(60^2)} = 21.0 \text{ lbf/in} \quad \textit{Ans.}$$

(d) $l = 60$ in, Table A-7, $I = 2.07$ in^4

$$w = \frac{8(12)(10^3)(2.07)}{1.5(60^2)} = 36.8 \text{ lbf/in} \quad \textit{Ans.}$$

3-41

$$I = \frac{\pi}{64}(0.5^4) = 3.068(10^{-3}) \text{ in}^4, \ A = \frac{\pi}{4}(0.5^2) = 0.1963 \text{ in}^2$$

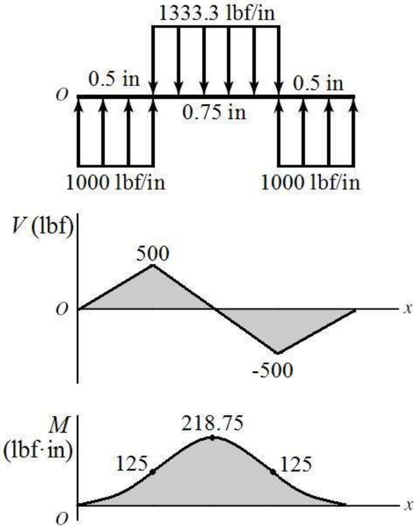
Model (*c*)

$$M = \frac{500(0.5)}{2} + \frac{500(0.75/2)}{2} = 218.75 \text{ lbf} \cdot \text{in}$$

$$\sigma = \frac{Mc}{I} = \frac{218.75(0.25)}{3.068(10^{-3})}$$

$$\sigma = 17\ 825 \text{ psi} = 17.8 \text{ kpsi} \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{500}{0.1963} = 3400 \text{ psi} = 3.4 \text{ kpsi} \quad \textit{Ans.}$$



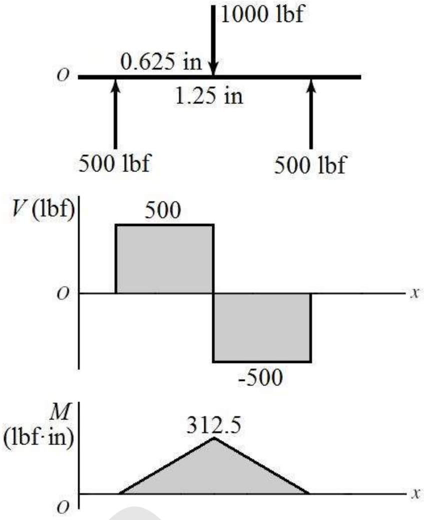
Model (*d*)

$M = 500(0.625) = 312.5 \text{ lbf} \cdot \text{in}$

$$\sigma = \frac{Mc}{I} = \frac{312.5(0.25)}{3.068\left(10^{-3}\right)}$$

$\sigma = 25\ 464 \text{ psi} = 25.5 \text{ kpsi}$ *Ans.*

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{500}{0.1963} = 3400 \text{ psi} = 3.4 \text{ kpsi} \qquad Ans.$$



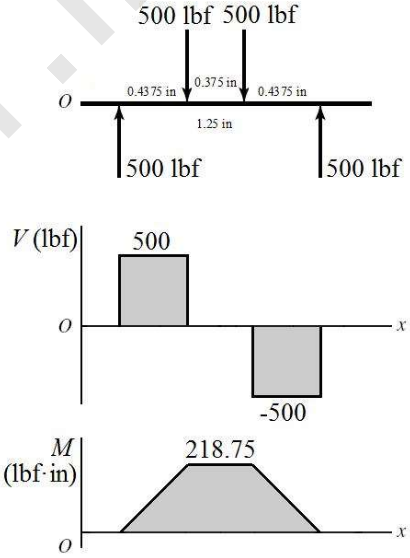
Model (*e*)

$M = 500(0.4375) = 218.75 \text{ lbf} \cdot \text{in}$

$$\sigma = \frac{Mc}{I} = \frac{218.75(0.25)}{3.068\left(10^{-3}\right)}$$

$\sigma = 17\ 825 \text{ psi} = 17.8 \text{ kpsi}$ *Ans.*

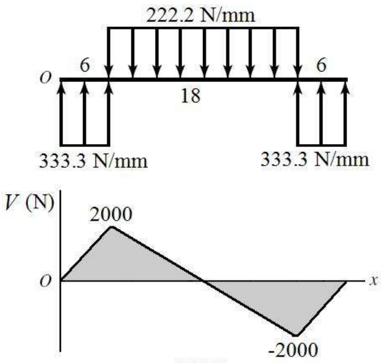
$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{500}{0.1963} = 3400 \text{ psi} = 3.4 \text{ kpsi} \qquad Ans.$$



3-42

$$I=\frac{\pi}{64}\left(12^4\right)=1018\text{ mm}^4,\; A=\frac{\pi}{4}\left(12^2\right)=113.1\text{ mm}^2$$

Model (*c*)



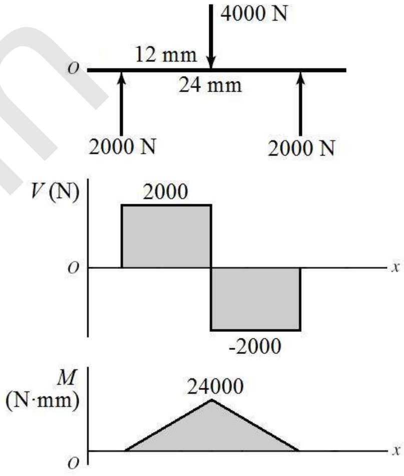
$$M=\frac{2000(6)}{2}+\frac{2000(9)}{2}=15\ 000\text{ N}\cdot\text{mm}$$

$$\sigma=\frac{Mc}{I}=\frac{15\ 000(6)}{1018}$$

$$\sigma=88.4\text{ N/mm}^2=88.4\text{ MPa}\qquad Ans.$$

$$\tau_{\max}=\frac{4}{3}\frac{V}{A}=\frac{4}{3}\left(\frac{2000}{113.1}\right)=23.6\text{ N/mm}^2=23.6\text{ MPa}\qquad Ans.$$

Model (*d*)



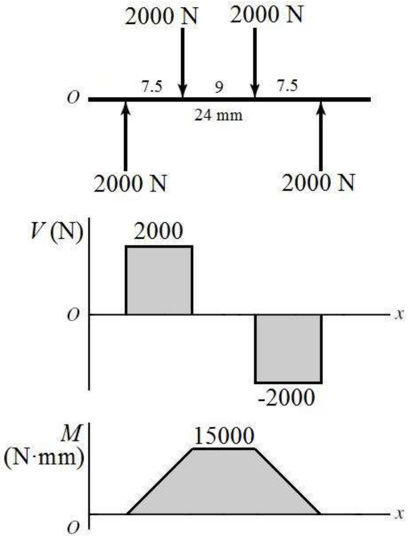
$$M=2000(12)=24\ 000\text{ N}\cdot\text{mm}$$

$$\sigma=\frac{Mc}{I}=\frac{24\ 000(6)}{1018}$$

$$\sigma=141.5\text{ N/mm}^2=141.5\text{ MPa}\qquad Ans.$$

$$\tau_{\max}=\frac{4}{3}\frac{V}{A}=\frac{4}{3}\left(\frac{2000}{113.1}\right)=23.6\text{ N/mm}^2=23.6\text{ MPa}\qquad Ans.$$

Model (e)



$$M=2000(7.5)=15\,000\text{ N}\cdot\text{mm}$$

$$\sigma=\frac{Mc}{I}=\frac{15\,000(6)}{1018}$$

$$\sigma=88.4\text{ N/mm}^2=88.4\text{ MPa}\qquad Ans.$$

$$\tau_{\max}=\frac{4}{3}\frac{V}{A}=\frac{4}{3}\left(\frac{2000}{113.1}\right)=23.6\text{ N/mm}^2=23.6\text{ MPa}\qquad Ans.$$

3-43 **(a)** $\sigma = \dfrac{Mc}{I} = \dfrac{M\left(d/2\right)}{\pi d^4/64} = \dfrac{32M}{\pi d^3}$

$$d = \sqrt[3]{\frac{32M}{\pi\sigma}} = \sqrt[3]{\frac{32(218.75)}{\pi(30\ 000)}} = 0.420\text{ in}\quad Ans.$$

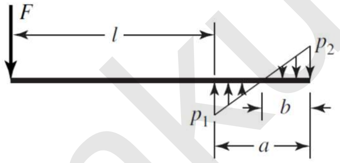
(b) $\tau = \dfrac{V}{A} = \dfrac{V}{\pi d^2/4}$

$$d = \sqrt{\frac{4V}{\pi\tau}} = \sqrt{\frac{4(500)}{\pi(15\,000)}} = 0.206\text{ in}\quad Ans.$$

(c) $\tau = \dfrac{4}{3}\dfrac{V}{A} = \dfrac{4}{3}\dfrac{V}{\left(\pi d^2/4\right)}$

$$d = \sqrt{\frac{4}{3}\frac{4V}{\pi\tau}} = \sqrt{\frac{4}{3}\frac{4(500)}{\pi(15\,000)}} = 0.238\text{ in}\quad Ans.$$

3-44



$$q = -F\langle x\rangle^{-1} + p_1\langle x-l\rangle^0 - \frac{p_1+p_2}{a}\langle x-l\rangle^1 + \text{terms for } x > l+a$$

$$V = -F + p_1\langle x-l\rangle^1 - \frac{p_1+p_2}{2a}\langle x-l\rangle^2 + \text{ terms for } x > l+a$$

$$M = -Fx + \frac{p_1}{2}\langle x-l\rangle^2 - \frac{p_1+p_2}{6a}\langle x-l\rangle^3 + \text{ terms for } x > l+a$$

At $x = (l+a)^+$, $V = M = 0$, terms for $x > l+a = 0$

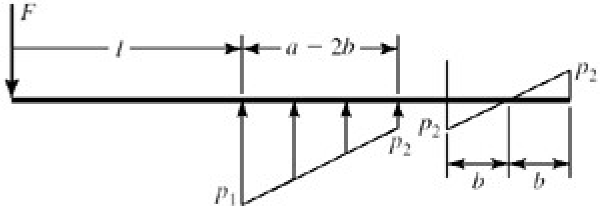
$$-F + p_1 a - \frac{p_1+p_2}{2a}a^2 = 0 \quad\Rightarrow\quad p_1 - p_2 = \frac{2F}{a} \qquad (1)$$

$$-F(l+a) + \frac{p_1 a^2}{2} - \frac{p_1+p_2}{6a}a^3 = 0 \quad\Rightarrow\quad 2p_1 - p_2 = \frac{6F(l+a)}{a^2} \qquad (2)$$

From (1) and (2) $\quad p_1 = \dfrac{2F}{a^2}(3l+2a),\quad p_2 = \dfrac{2F}{a^2}(3l+a) \qquad (3)$

From similar triangles $$\frac{b}{p_2} = \frac{a}{p_1 + p_2} \quad \Rightarrow \quad b = \frac{ap_2}{p_1 + p_2} \qquad (4)$$

$M_{\max}$ occurs where $V = 0$



$$x_{\max} = l + a - 2b$$

$$M_{\max} = -F(l + a - 2b) + \frac{p_1}{2}(a - 2b)^2 - \frac{p_1 + p_2}{6a}(a - 2b)^3$$

$$= -Fl - F(a - 2b) + \frac{p_1}{2}(a - 2b)^2 - \frac{p_1 + p_2}{6a}(a - 2b)^3$$

Normally $M_{\max} = -Fl$

The fractional increase in the magnitude is

$$\Delta = \frac{F(a-2b) - (p_1/2)(a-2b)^2 + \left[(p_1 + p_2)/6a\right](a-2b)^3}{Fl} \qquad (5)$$

For example, consider $F = 1500$ lbf, $a = 1.2$ in, $l = 1.5$ in

From (3) $$p_1 = \frac{2(1500)}{1.2^2}\left[3(1.5) + 2(1.2)\right] = 14\ 375 \text{ lbf/in}$$

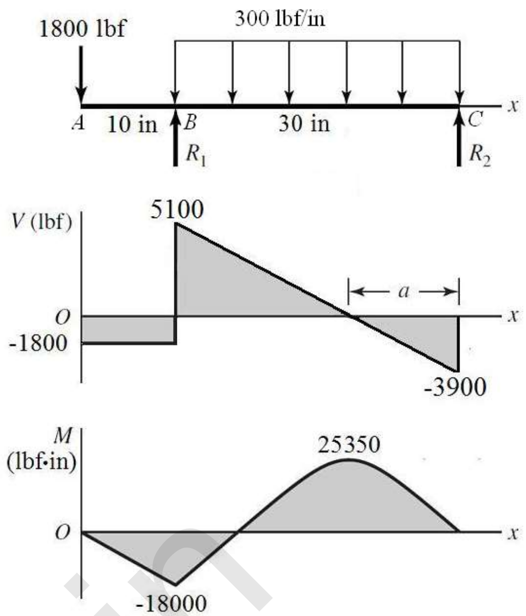
$$p_2 = \frac{2(1500)}{1.2^2}\left[3(1.5) + 1.2\right] = 11\ 875 \text{ lbf/in}$$

From (4) $b = 1.2(11\ 875)/(14\ 375 + 11\ 875) = 0.5429$ in

Substituting into (5) yields

$\Delta = 0.036\ 89$ or 3.7% higher than $-Fl$

3-45



$$R_1 = \frac{300(30)}{2} + \frac{40}{30}1800 = 6900 \text{ lbf}$$

$$R_2 = \frac{300(30)}{2} - \frac{10}{30}1800 = 3900 \text{ lbf}$$

$$a = \frac{3900}{300} = 13 \text{ in}$$

$$M_B = -1800(10) = -18\,000 \text{ lbf·in}$$

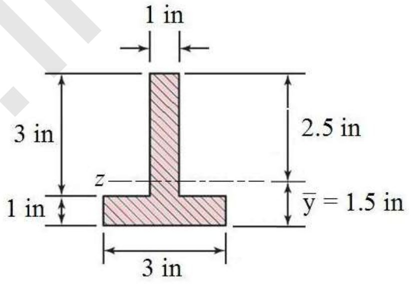
$$M_{x=27\text{ in}} = (1/2)3900(13) = 25\,350 \text{ lbf·in}$$

$$\bar{y} = \frac{0.5(3)+2.5(3)}{6} = 1.5 \text{ in}$$

$$I_1 = \frac{1}{12}(3)(1^3) = 0.25 \text{ in}^4$$

$$I_2 = \frac{1}{12}(1)(3^3) = 2.25 \text{ in}^4$$

Applying the parallel-axis theorem,



$$I_z = \left[0.25 + 3(1.5-0.5)^2\right] + \left[2.25 + 3(2.5-1.5)^2\right] = 8.5 \text{ in}^4$$

(a)

$$\text{At } x = 10 \text{ in}, \quad y = -1.5 \text{ in}, \quad \sigma_x = -\frac{-18000(-1.5)}{8.5} = -3176 \text{ psi}$$

$$\text{At } x = 10 \text{ in}, \quad y = 2.5 \text{ in}, \quad \sigma_x = -\frac{-18000(2.5)}{8.5} = 5294 \text{ psi}$$

$$\text{At } x = 27 \text{ in}, \quad y = -1.5 \text{ in}, \quad \sigma_x = -\frac{25350(-1.5)}{8.5} = 4474 \text{ psi}$$

$$\text{At } x = 27 \text{ in}, \quad y = 2.5 \text{ in}, \quad \sigma_x = -\frac{25350(2.5)}{8.5} = -7456 \text{ psi}$$

Max tension $= 5294$ psi *Ans.*

Max compression $= -7456$ psi *Ans.*

(b) The maximum shear stress due to V is at B, at the neutral axis.

$$V_{\max} = 5100 \text{ lbf}$$

$$Q = \bar{y}'A' = 1.25(2.5)(1) = 3.125 \text{ in}^3$$

$$(\tau_{\max})_V = \frac{VQ}{Ib} = \frac{5100(3.125)}{8.5(1)} = 1875 \text{ psi} \qquad \textit{Ans.}$$

(c) There are three potentially critical locations for the maximum shear stress, all at

$x = 27$ in: (i) at the top where the bending stress is maximum, (ii) at the neutral axis where the transverse shear is maximum, or (iii) in the web just above the flange where bending stress and shear stress are in their largest combination.

For (i):

The maximum bending stress was previously found to be -7456 psi, and the shear stress is zero. From Mohr's circle,

$$\tau_{\max} = \frac{|\sigma_{\max}|}{2} = \frac{7456}{2} = 3728 \text{ psi}$$

For (ii):

The bending stress is zero, and the transverse shear stress was found previously to be 1875 psi. Thus, $\tau_{\max} = 1875$ psi.

For (iii):

The bending stress, at $y = -0.5$ in, is

$$\sigma_x = -\frac{-18000(-0.5)}{8.5} = -1059 \text{ psi}$$

The transverse shear stress is

$$Q = \bar{y}'A' = (1)(3)(1) = 3.0 \text{ in}^3$$

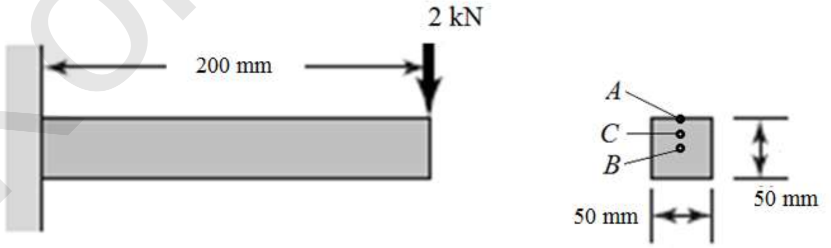
$$\tau = \frac{VQ}{Ib} = \frac{5100(3.0)}{8.5(1)} = 1800 \text{ psi}$$

From Mohr's circle,

$$\tau_{\max} = \sqrt{\left(\frac{-1059}{2}\right)^2 + 1800^2} = 1876 \text{ psi}$$

The critical location is at $x = 27$ in, at the top surface, where $\tau_{\max} = 3728$ psi. *Ans.*

3-46



Cross section at wall

(a) $I = bh^3/12 = 50(50^3)/(12) = 520.83\ (10^3)$ mm^4

Element A:

$$\sigma_A = -\frac{My_A}{I} = -\frac{-2000(200)(50/2)}{520.83(10^3)} = 19.2 \text{ MPa}$$

Ans.

$$\tau_A = \frac{VQ_A}{Ib},\quad Q_A = 0 \Rightarrow \tau_A = 0$$

Element *B*:

$$\sigma_B = -\frac{My_B}{I}, \quad y_B = 0 \;\Rightarrow\; \sigma_B = 0$$

$$Q_B = \bar{y}'_B A' = (25/2)25(50) = 15.625(10^3)\ \text{mm}^3$$

$$\tau_B = \frac{VQ_B}{Ib} = \frac{2000(15.625)10^3}{520.83(10^3)50} = 1.20\ \text{MPa} \qquad \textit{Ans.}$$

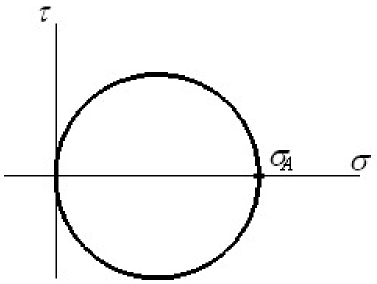
Element *C*:

$$\sigma_C = -\frac{-2\,000(200)(25/2)}{520.83(10^3)} = 9.60\ \text{MPa}$$

$$Q_C = (25/2 + 25/4)(25/2)50 = 11.719(10^3)\ \text{mm}^3$$

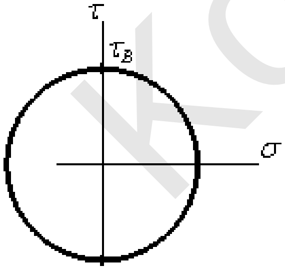
$$\tau_C = \frac{2\,000(11.719)10^3}{520.83(10^3)50} = 0.90\ \text{MPa} \qquad \textit{Ans.}$$

(b) Point *A*:



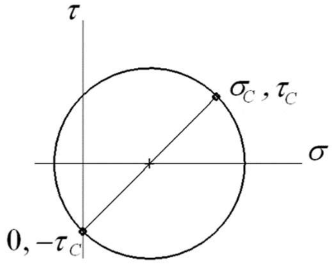
$$\tau_{\max} = \frac{\sigma_A}{2} = \frac{19.2}{2} = 9.6\ \text{MPa} \qquad \textit{Ans.}$$

Point *B*:



$$\tau_{\max} = \tau_B = 1.20\ \text{MPa} \qquad \textit{Ans.}$$

Point *C*:



$$\tau_{\max} = \sqrt{\left(\frac{\sigma_C}{2}\right)^2 + \tau_C^2}$$

$$= \sqrt{\left(\frac{9.60}{2}\right)^2 + 0.9^2} = 4.88\ \text{MPa} \qquad \textit{Ans.}$$

(c) Point *A* is critical. *Ans.*

(d) Transverse shear does not change with respect to the length of the beam.

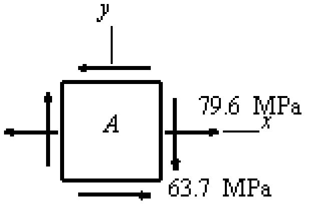
$$(\tau_A)_{\max} = \frac{\sigma_A}{2} = \frac{2\,000L(50/2)}{2(520.83)10^3} = 1.20 \quad \Rightarrow \quad L = 25.0 \text{ mm} \qquad \textit{Ans.}$$

3-47 $I = (\pi/64)40^4 = 125.66(10^3)\ \text{mm}^4,\quad J = 2I$

(a) Point *A*:

$$\sigma_A = \frac{M_A c_A}{I} = \frac{50(10^3)10(40/2)}{125.66} = 79.6 \text{ MPa}$$

$$\tau_A = \frac{T_A r_A}{J} = \frac{800(10^3)(40/2)}{2(125.66)10^3} = 63.7 \text{ MPa}$$

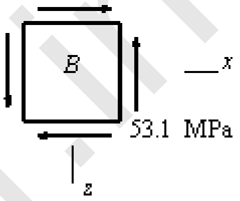


Ans.

Point *B*:

$$\sigma_B = 0$$

$$\tau_B = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{50(10^3)}{\pi(40/2)^2} = 53.1 \text{ MPa}$$

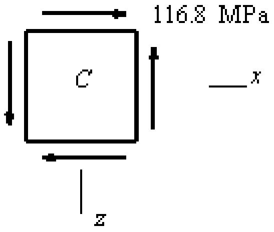


Ans.

Point *C*:

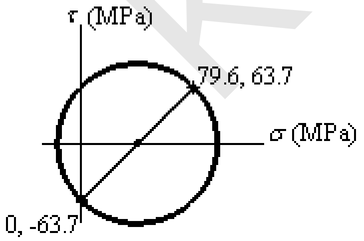
$$\sigma_C = 0$$

$$\tau_C = \tau_B + \frac{Tr}{J} = 53.1 + 63.7 = 116.8 \text{ MPa}$$



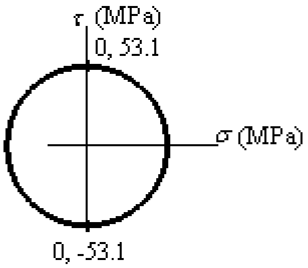
Ans.

(b) Point *A*:



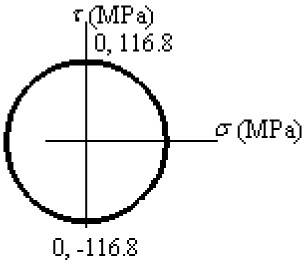
$$(\tau_{\max})_A = \sqrt{\left(\frac{79.6}{2}\right)^2 + 63.7^2} = 75.1 \text{ MPa} \qquad \textit{Ans.}$$

Point *B*:



$(\tau_{\max})_B = 53.1$ MPa *Ans.*

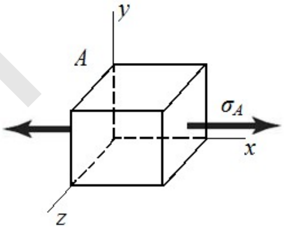
Point C:



$(\tau_{\max})_C = 116.8$ MPa *Ans.*

(c) $\tau_{\max} = 116.8$ MPa at point C *Ans.*

3-48 **(a)** $L = 10$ in. Element A:

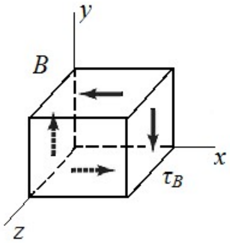


$$\sigma_A = -\frac{My}{I} = -\frac{-(1000)(10)(0.5)}{(\pi/64)(1)^4}\left(10^{-3}\right) = 101.9 \text{ kpsi}$$

$$\tau_A = \frac{VQ}{Ib}, \quad Q = 0 \quad \Rightarrow \quad \tau_A = 0$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau_A^2} = \sqrt{\left(\frac{101.9}{2}\right)^2 + (0)^2} = 50.9 \text{ kpsi} \quad \textit{Ans.}$$

Element B:



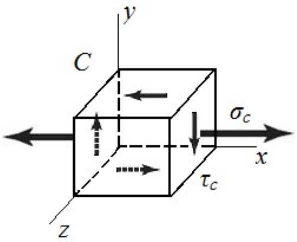
$$\sigma_B = -\frac{My}{I}, \quad y = 0 \quad \Rightarrow \quad \sigma_B = 0$$

$$Q = \bar{y}'A' = \left(\frac{4r}{3\pi}\right)\left(\frac{\pi r^2}{2}\right) = \frac{4r^3}{6} = \frac{4\left(0.5\right)^3}{6} = 1/12 \text{ in}^3$$

$$\tau_B = \frac{VQ}{Ib} = \frac{(1000)(1/12)}{(\pi/64)(1)^4(1)}\left(10^{-3}\right) = 1.698 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\left(\frac{0}{2}\right)^2 + 1.698^2} = 1.698 \text{ kpsi} \quad \textit{Ans.}$$

Element C:

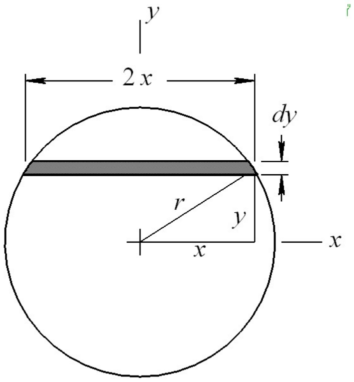


$$\sigma_C = -\frac{My}{I} = -\frac{-(1000)(10)(0.25)}{(\pi/64)(1)^4}\left(10^{-3}\right) = 50.93 \text{ kpsi}$$

$$Q=\int_{y_1}^{r} ydA=\int_{y_1}^{r} y(2x)dy=\int_{y_1}^{r} y\left(2\sqrt{r^2-y^2}\right)dy$$

$$=-\frac{2}{3}\left(r^2-y^2\right)^{3/2}\Big|_{y_1}^{r}=-\frac{2}{3}\left[\left(r^2-r^2\right)^{3/2}-\left(r^2-y_1^2\right)^{3/2}\right]$$

$$=\frac{2}{3}\left(r^2-y_1^2\right)^{3/2}$$



For C, $y_1 = r/2 = 0.25$ in

$$Q=\frac{2}{3}\left(0.5^2-0.25^2\right)^{3/2}=0.05413\ \text{in}^3$$

$$b=2x=2\sqrt{r^2-y_1^2}=2\sqrt{0.5^2-0.25^2}=0.866\ \text{in}$$

$$\tau_C=\frac{VQ}{Ib}=\frac{(1000)(0.05413)}{(\pi/64)(1)^4(0.866)}\left(10^{-3}\right)=1.273\ \text{kpsi}$$

$$\tau_{\max}=\sqrt{\left(\frac{50.93}{2}\right)^2+(1.273)^2}=25.50\ \text{kpsi}\qquad \textit{Ans.}$$

(b) Neglecting transverse shear stress:

Element *A*: Since the transverse shear stress at point *A* is zero, there is no change.

$$\tau_{\max}=50.9\ \text{kpsi}\qquad \textit{Ans.}$$

$$\%\ \text{error}=0\%\qquad \textit{Ans.}$$

Element *B*: Since the only stress at point *B* is transverse shear stress, neglecting the transverse shear stress ignores the entire stress.

$$\tau_{\max}=\sqrt{\left(\frac{0}{2}\right)^2}=0\ \text{psi}\qquad \textit{Ans.}$$

$$\%\ \text{error}=\left(\frac{1.698-0}{1.698}\right)*(100)=100\%\qquad \textit{Ans.}$$

Element *C*:

$$\tau_{\max}=\sqrt{\left(\frac{50.93}{2}\right)^2}=25.47\ \text{kpsi}\qquad \textit{Ans.}$$

$$\%\ \text{error}=\left(\frac{25.50-25.47}{25.50}\right)*(100)=0.12\%\qquad \textit{Ans.}$$

(c) Repeating the process with different beam lengths produces the results in the table.

	Bending stress, σ (kpsi)	Transverse shear stress, τ (kpsi)	Max shear stress, $\tau_{\max}$ (kpsi)	Max shear stress, neglecting τ, $\tau_{\max}$ (kpsi)	% error
L = 10 in					
A	102	0	50.9	50.9	0
B	0	1.70	1.70	0	100
C	50.9	1.27	25.50	25.47	0.12
L = 4 in					
A	40.7	0	20.4	20.4	0
B	0	1.70	1.70	0	100
C	20.4	1.27	10.26	10.19	0.77
L = 1 in					
A	10.2	0	5.09	5.09	0
B	0	1.70	1.70	0	100
C	5.09	1.27	2.85	2.55	10.6
L = 0.1in					
A	1.02	0	0.509	0.509	0
B	0	1.70	1.70	0	100
C	0.509	1.27	1.30	0.255	80.4

Discussion:
The transverse shear stress is only significant in determining the critical stress element as the length of the cantilever beam becomes smaller. As this length decreases, bending stress reduces greatly and transverse shear stress stays the same. This causes the critical element location to go from being at point A, on the surface, to point B, in the center. The maximum shear stress is on the outer surface at point A for all cases except L = 0.1 in, where it is at point B at the center. When the critical stress element is at point A, there is no error from neglecting transverse shear stress, since it is zero at that location. Neglecting the transverse shear stress has extreme significance at the stress element at the center at point B, but that location is probably only of practical significance for very short beam lengths.

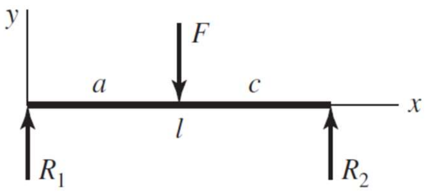
3-49

$$R_1 = \frac{c}{l} F$$

$$M = \frac{c}{l} Fx \qquad 0 \le x \le a$$

$$\sigma = \frac{6M}{bh^2} = \frac{6(c/l)Fx}{bh^2}$$

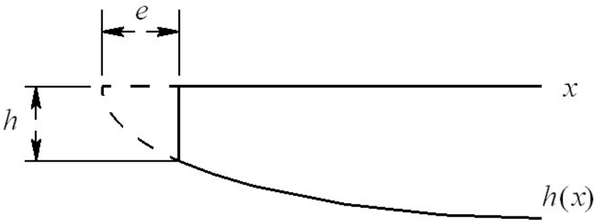
$$h = \sqrt{\frac{6Fcx}{lb\sigma_{\max}}} \qquad 0 \le x \le a \qquad Ans.$$



3-50 From Problem 3-49, $R_1 = \dfrac{c}{l} F = V,\ 0 \le x \le a$

$$\tau_{\max} = \frac{3}{2}\frac{V}{bh} = \frac{3}{2}\frac{(c/l)F}{bh} \qquad \Rightarrow \qquad h = \frac{3}{2}\frac{Fc}{lb\tau_{\max}} \qquad Ans.$$

From Problem 3-49, $h(x) = \sqrt{\dfrac{6Fcx}{lb\sigma_{\max}}}$.



Sub in $x = e$ and equate to h above.

$$\frac{3}{2}\frac{Fc}{lb\tau_{\max}} = \sqrt{\frac{6Fce}{lb\sigma_{\max}}}$$

$$e = \frac{3}{8}\frac{Fc\sigma_{\max}}{lb\tau_{\max}^2} \qquad Ans.$$

3-51 **(a)**

<u>*x-z*</u> plane

$$\Sigma M_O = 0 = 1.5(0.5) + 2(1.5)\sin(30^\circ)(2.25) - R_{2z}(3)$$

$$R_{2z} = 1.375 \text{ kN} \qquad Ans.$$

$$\Sigma F_z = 0 = R_{1z} - 1.5 - 2(1.5)\sin(30^\circ) + 1.375$$

$$R_{1z} = 1.625 \text{ kN} \qquad Ans.$$

x-y plane

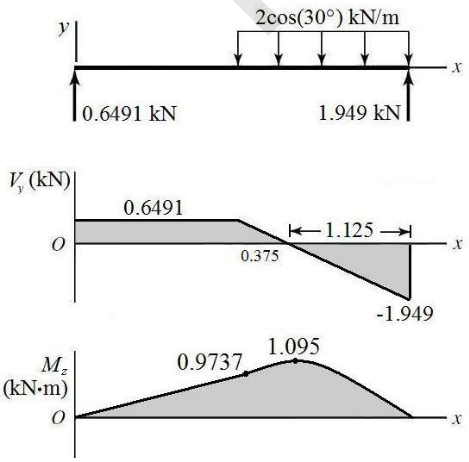
$$\Sigma M_O = 0 = -2(1.5)\cos(30^\circ)(2.25) + R_{2y}(3)$$

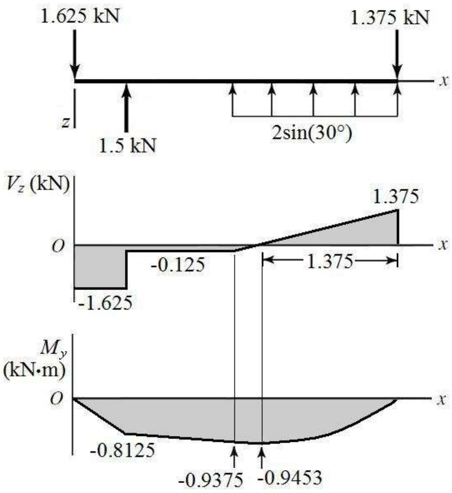
$$R_{2y} = 1.949 \text{ kN} \qquad Ans.$$

$$\Sigma F_y = 0 = R_{1y} - 2(1.5)\cos(30^\circ) + 1.949$$

$$R_{1y} = 0.6491 \text{ kN} \qquad Ans.$$

(b)





(c) The transverse shear and bending moments for most points of interest can readily be taken straight from the diagrams. For 1.5 < x < 3, the bending moment equations are parabolic, and are obtained by integrating the linear expressions for shear. For convenience, use a coordinate shift of $x' = x - 1.5$. Then, for $0 < x' < 1.5$,

$$V_z = x' - 0.125$$

$$M_y = \int V_z dx' = \frac{(x')^2}{2} - 0.125x' + C$$

$$\text{At } x' = 0, M_y = C = -0.9375 \quad \Rightarrow \quad M_y = 0.5\left(x'\right)^2 - 0.125x' + 0.9375$$

$$V_y = -\frac{1.949}{1.125}x' + 0.6491 = -1.732x' + 0.6491$$

$$M_z = \frac{-1.732}{2}\left(x'\right)^2 + 0.6491x' + C$$

$$\text{At } x' = 0, M_z = C = 0.9737 \quad \Rightarrow \quad M_z = -0.8662\left(x'\right)^2 - 0.125x' - 0.9375$$

By programming these bending moment equations, we can find M_y, M_z, and their vector combination at any point along the beam. The maximum combined bending moment is found to be at x = 1.79 m, where M = 1.433 kN·m. The table below shows values at key locations on the shear and bending moment diagrams.

x (m)	V_z (kN)	V_y (kN)	V (kN)	M_y (kN·m)	M_z (kN·m)	M (kN·m)
0	–1.625	0.6491	1.750	0	0	0
0.5^-	–1.625	0.6491	1.750	–0.8125	0.3246	0.8749
1.5	–0.1250	0.6491	0.6610	0.9375	0.9737	1.352
1.625	0	0.4327	0.4327	–0.9453	1.041	1.406
1.875	0.2500	0	0.2500	–0.9141	1.095	1.427
3^-	1.375	–1.949	2.385	0	0	0

(d) The bending stress is obtained from Eq. (3-27),

$$\sigma_x = \frac{-M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

The maximum tensile bending stress will be at point A in the cross section of Prob. 3-35 (a), where distances from the neutral axes for both bending moments will be maximum. At A, for M_z, y_A = –37.5 mm, and for M_y, z_A = –20 mm.

$$I_z = \frac{40(75)^3}{12} - \frac{34(25)^3}{12} = 1.36(10^6)\text{ mm}^4 = 1.36(10^{-6})\text{ m}^4$$

$$I_y = 2\left[\frac{25(40)^3}{12}\right] + \frac{25(6)^3}{12} = 2.67(10^5)\text{ mm}^4 = 2.67(10^{-7})\text{ m}^4$$

It is apparent the maximum bending moment, and thus the maximum stress, will be in the parabolic section of the bending moment diagrams. Programming Eq. (3-27) with the bending moment equations previously derived, the maximum tensile bending stress is found at x = 1.77 m, where M_y = – 0.9408 kN·m, M_z = 1.075 kN·m, and σ_x = 100.1 MPa. *Ans.*

3-52

(a) x-z plane

$$\Sigma M_O = 0 = \frac{3}{5}(1000)(4) - \frac{600}{\sqrt{2}}(10) + M_{Oy}$$

$$M_{Oy} = 1842.6 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$\Sigma F_z = 0 = R_{Oz} - \frac{3}{5}(1000) + \frac{600}{\sqrt{2}}$$

$$R_{Oz} = 175.7 \text{ lbf} \quad \textit{Ans.}$$

x-y plane

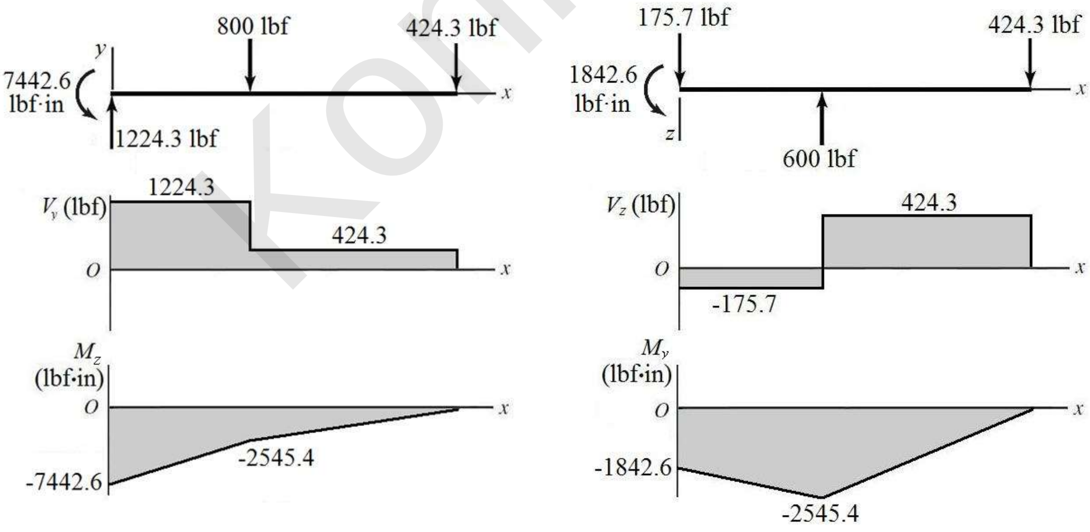
$$\Sigma M_O = 0 = -\frac{4}{5}(1000)(4) - \frac{600}{\sqrt{2}}(10) + M_{Oz}$$

$$M_{Oz} = 7442.5 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$\Sigma F_y = 0 = R_{Oy} - \frac{4}{5}(1000) - \frac{600}{\sqrt{2}}$$

$$R_{Oy} = 1224.3 \text{ lbf} \quad \textit{Ans.}$$

(b)



(c)

$$V(x) = \left[V_y(x)^2 + V_z(x)^2\right]^{1/2}$$

$$M(x) = \left[M_y(x)^2 + M_z(x)^2\right]^{1/2}$$

x (m)	V_z (kN)	V_y (kN)	V (kN)	M_y (kN·m)	M_z (kN·m)	M (kN·m)
0	–175.7	1224.3	1237	–1842.6	–7442.6	7667
4^-	–175.7	1224.3	1237	–2545.4	–2545.4	3600
10^-	424.3	424.3	600	0	0	0

(d) The maximum tensile bending stress will be at the outer corner of the cross section in the positive *y*, negative *z* quadrant, where $y = 1.5$ in and $z = -1$ in.

$$I_z = \frac{2(3)^3}{12} - \frac{(1.625)(2.625)^3}{12} = 2.051 \text{ in}^4$$

$$I_y = \frac{3(2)^3}{12} - \frac{(2.625)(1.625)^3}{12} = 1.601 \text{ in}^4$$

At $x = 0$, using Eq. (3-27),

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

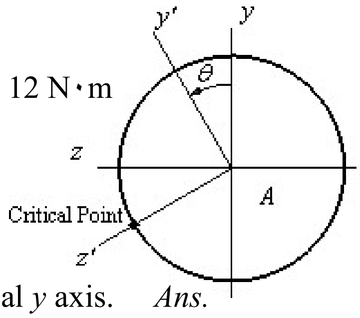
$$\sigma_x = -\frac{(-7442.6)(1.5)}{2.051} + \frac{(-1842.6)(-1)}{1.601} = 6594 \text{ psi}$$

Check at $x = 4$ in,

$$\sigma_x = -\frac{(-2545.4)(1.5)}{2.051} + \frac{(-2545.4)(-1)}{1.601} = 2706 \text{ psi}$$

The critical location is at $x = 0$, where $\sigma_x = 6594$ psi. *Ans.*

3-53 **(a)** Moments at *A*: $M_y = 300(0.050) = 15$ N·m, $M_z = 200(0.055) = 11$ N·m, Torque at *A*: $T_x = 200(0.060) = 12$ N·m



$$|M| = \sqrt{M_y^2 + M_z^2} = \sqrt{15^2 + 11^2} = 18.601 \text{ N}\cdot\text{m}$$

$$\theta = \tan^{-1}\left(\frac{M_z}{M_y}\right) = \tan^{-1}\left(\frac{11}{15}\right) = 36.25^\circ$$

Critical point will be 90° from *θ*, i.e. 126.25° from the vertical *y* axis. *Ans.*

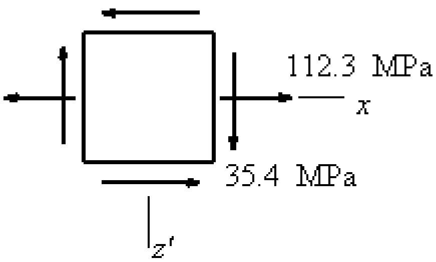
(b)

$$\sigma_{\text{bend}} = \frac{|M|c}{I} = \frac{32|M|}{\pi d^3} = \frac{32(18.601)}{\pi\left(0.012^3\right)} 10^{-6} = 109.6 \text{ MPa}$$

$$\sigma_{\text{axial}} = \frac{F}{A} = \frac{4F}{\pi d^2} = \frac{4(300)}{\pi\left(0.012^2\right)} 10^{-6} = 2.65 \text{ MPa}$$

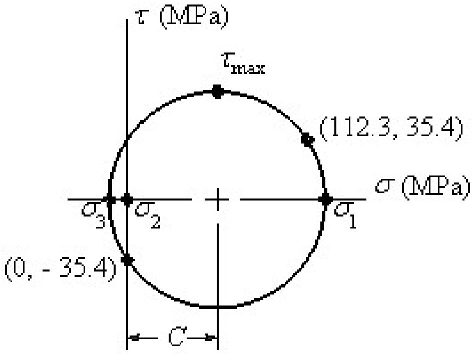
$$\sigma_{\text{total}} = \sigma_{\text{bend}} + \sigma_{\text{axial}} = 109.6 + 2.65 = 112.3 \text{ MPa}$$

$$\tau = \frac{T_x c}{J} = \frac{16T_x}{\pi d^3} = \frac{16(12)}{\pi\left(0.012^3\right)} 10^{-6} = 35.4 \text{ MPa}$$



Ans.

(c)



Ans.

(d) $C = \dfrac{112.3}{2} = 56.2\ MPa, \quad R = \sqrt{\left(\dfrac{112.3}{2}\right)^2 + 35.4^2} = 66.4\ \text{MPa}$

$\sigma_1 = C + R = 56.2 + 66.4 = 122.6\ \text{MPa}, \ \sigma_2 = 0,$

$\sigma_3 = C - R = 56.2 - 66.4 = -10.2\ \text{MPa}$ *Ans.*

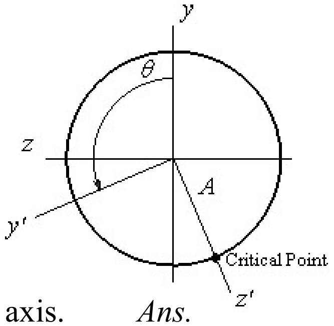
$\tau_{\max} = R = 66.4\ \text{MPa}$

3-54 **(a)** Moments at *A*: $M_y = -200(0.060) = -12$ N·m, $M_z = 300(0.095) = 28.5$ N·m

Torque at *A*: $T_x = -300(0.050) = -15$ N·m

$$\left|M\right| = \sqrt{M_y^2 + M_z^2} = \sqrt{(-12)^2 + 28.5^2} = 30.92\ \text{N} \cdot \text{m}$$

$$\theta = \tan^{-1}\left(\frac{M_z}{M_y}\right) = \tan^{-1}\left(\frac{28.5}{-12}\right) = 112.8^{\circ}$$



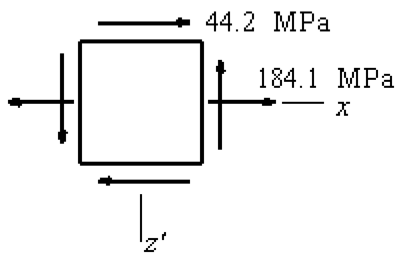
Critical point will be 90° from *θ*, i.e. 202.8° from the vertical *y*. axis. *Ans.*

$$\sigma_{\text{bend}} = \frac{\left|M\right|c}{I} = \frac{32\left|M\right|}{\pi d^3} = \frac{32(30.92)}{\pi\left(0.012^3\right)}10^{-6} = 182.3\ \ \text{MPa}$$

$$\sigma_{\text{axial}} = \frac{F}{A} = \frac{4F}{\pi d^2} = \frac{4(200)}{\pi\left(0.012^2\right)}10^{-6} = 1.77\ \text{MPa}$$

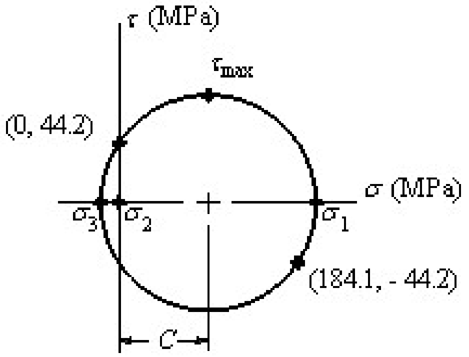
$$\sigma_{\text{total}} = \sigma_{\text{bend}} + \sigma_{\text{axial}} = 182.3 + 1.77 = 184.1\ \text{MPa}$$

$$\tau = \frac{T_x c}{J} = \frac{16T_x}{\pi d^3} = \frac{16(15)}{\pi\left(0.012^3\right)}10^{-6} = 44.2\ \text{MPa}$$



Ans.

(c)



Ans.

(d) $$C=\frac{184.1}{2}=92.1\text{ MPa},\quad R=\sqrt{\left(\frac{184.1}{2}\right)^2+44.2^2}=102.1\text{ MPa}$$

$$\sigma_1=C+R=92.1+102.1=194.2\text{ MPa},\ \sigma_2=0,$$

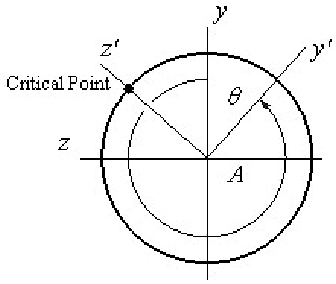
$$\sigma_3=C-R=92.1-102.1=-10\text{ MPa}$$ *Ans.*

$$\tau_{\max}=R=102.1\text{ MPa}$$

3-55 **(a)** Moments at A: M_y = 60(5) + 200(5.5) = 1400 lbf·in,
M_z = – (300 – 75)5.5 = – 1238 lbf·in,
Torque at A: T_x = – 75(5) = – 375 lbf·in

$$|M|=\sqrt{M_y^2+M_z^2}=\sqrt{1400^2+(-1238)^2}=1869\text{ lbf}\cdot\text{in}$$

$$\theta=\tan^{-1}\left(\frac{M_z}{M_y}\right)=\tan^{-1}\left(\frac{-1238}{1400}\right)=318.5=-41.5^{\circ}$$



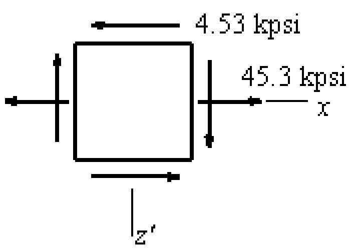
Critical point will be 90° from θ, i.e. 48.5° from the vertical y axis. *Ans.*

(b) $$\sigma_{\text{bend}}=\frac{|M|c}{I}=\frac{32|M|}{\pi d^3}=\frac{32(1869)}{\pi(0.75^3)}10^{-3}=45.13\ \text{kpsi}$$

$$\sigma_{\text{axial}}=\frac{F}{A}=\frac{4F}{\pi d^2}=\frac{4(60)}{\pi(0.75^2)}=136\text{ psi}$$

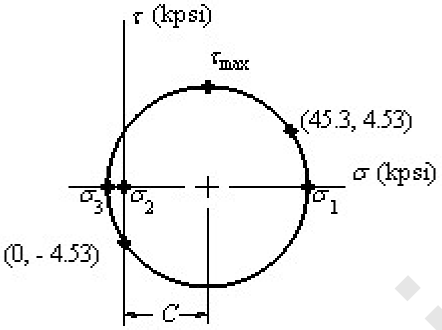
$$\sigma_{\text{total}}=\sigma_{\text{bend}}+\sigma_{\text{axial}}=45.126+0.136=45.3\text{ kpsi}$$

$$\tau=\frac{T_xc}{J}=\frac{16T_x}{\pi d^3}=\frac{16(375)}{\pi(0.75^3)}10^{-3}=4.53\text{ kpsi}$$



Ans.

(c)



Ans.

(d) $$C = \frac{45.3}{2} = 22.7\,\text{kpsi}, \quad R = \sqrt{\left(\frac{45.3}{2}\right)^2 + 4.53^2} = 23.1\text{ kpsi}$$

$$\sigma_1 = C + R = 22.7 + 23.1 = 45.8\,\text{kpsi}, \ \sigma_2 = 0,$$

$$\sigma_3 = C - R = 22.7 - 23.1 = -0.45\text{ kpsi}$$ *Ans.*

$$\tau_{\max} = R = 23.1\text{ kpsi}$$

3-56 Given: $b = 3.6$ in, $c = 2.5$ in, and $l = 40$ in.
From Table A-5, $G = 11.5$ Mpsi. From Table A-20, $S_y = 42$ kpsi.

(a) For the table for Eq. (3-40), $b/c = 3.6/2.5 = 1.44$

α	b/c
0.208	1
α	1.44
0.231	1.5

$$\frac{0.231-\alpha}{0.231-0.208} = \frac{1.5-1.44}{1.5-1} = 0.12 \ \Rightarrow \ \alpha = 0.231 - 0.12(0.231-0.208) = 0.228$$

Equation (3-40):

$$\tau_{\max} = \frac{T}{\alpha bc^2} = \frac{30(10^3)}{0.228(3.6)2.5^2} = 5\,850 \text{ psi} = 5.85 \text{ kpsi}$$ *Ans.*

(b) For the table for Eq. (3-41), $b/c = 3.6/2.5 = 1.44$

β	b/c
0.141	1
β	1.44
0.196	1.5

$$\frac{0.196-\beta}{0.196-0.141}=\frac{1.5-1.44}{1.5-1}=0.12 \;\Rightarrow\; \beta=0.196-0.12\left(0.196-0.141\right)=0.189$$

Equation (3-41):

$$\theta=\frac{Tl}{\beta bc^3G}=\frac{30\left(10^3\right)40}{0.189\left(3.6\right)2.5^3\left(11.5\right)10^6}=9.815\left(10^{-3}\right)\,rad\;=0.562^{\circ}\qquad Ans.$$

(c) S_{sy} = 0.5(42) = 21 kpsi. Yield factor of safety,

$$n_y=\frac{S_{sy}}{\tau_{\max}}=\frac{21}{5.85}=3.59\qquad Ans.$$

3-57 Given: 250 hp at 540 rev/min, τ_{allow} = 15 kpsi.

Eq. (3-42): $T=\frac{63\,025}{n}H=\frac{63\,025}{540}250=29.178\left(10^3\right)$ lbf · in

Eq. (3-41) with table where for square cross-section, b/c =1

$$\tau_{\max}=\frac{T}{0.208bc^2}=\frac{29.178\left(10^3\right)}{0.208b^3}=15\left(10^3\right)\;\Rightarrow\; b=2.107\text{ in}$$

From Table A-17, use b = $2\frac{1}{4}$ in *Ans.*

3-58 Given: T = 50 kN-m. OD = 300 mm, t = 2 mm, and l = 2 m.

J = (π/32)(0.300^4 – 0.296^4) = 4.157(10^{-5}) m^4. From Table A-5, G = 79.3 GPa.

(a) Eq. (3-37):

$$\tau_{\max}=\frac{Tr}{J}=\frac{50\left(10^3\right)0.15}{4.157\left(10^{-5}\right)}10^{-6}=180.4\text{ MPa}\qquad Ans.$$

(b) Eq. (3-45):

$$\tau_{\max}=\frac{T}{2A_mt}=\frac{50\left(10^3\right)}{2\left(\pi/4\right)0.298^2\left(0.002\right)}10^{-6}=179.2\;MPa\qquad Ans.$$

Answer is 0.67 percent lower than part (a).

(c) Eq. (3-35):

$$\theta=\frac{TL}{JG}=\frac{50\left(10^3\right)2}{4.157\left(10^{-5}\right)79.3\left(10^9\right)}=0.0303\text{ rad}=1.74^{\circ}\qquad Ans.$$

(d) Eq. (3-46):

$$\theta=\theta_1 l=\frac{TL_ml}{4GA_m^2t}=\frac{50\left(10^3\right)\pi\left(0.298\right)2}{4\left(79.3\right)10^9\left[\left(\pi/4\right)0.298^2\right]^2 0.002}=0.303\text{ rad}=1.74^{\circ}\qquad Ans.$$

Within the same accuracy, the answer is the same as part (c).

3-59 Given: Rectangular tube with inner dimensions 1.5 in × 2.0 in, $t = \frac{1}{8}$ in, 1035 CD steel, $n = 540$ rev/min, and $S_{sy} = 0.5\ S_y$.

(a) Table A-20, $S_y = 67$ kpsi. Factor of safety against yield, $n_y = 2$. For the table for Eq. (3-40), with $b/c = 2/1.5 = 1.333$,

α	b/c
0.208	1
α	1.333
0.231	1.5

$$\frac{0.231-\alpha}{0.231-0.208} = \frac{1.5-1.333}{1.5-1} = 0.3333 \;\Rightarrow\; \alpha = 0.231 - 0.3333\left(0.231-0.208\right) = 0.2233$$

Eq. (3-40):

$$\tau_{\max} = \frac{T}{\alpha bc^2} = \frac{T}{0.2233\left(2\right)1.5^2} = \frac{0.5S_y}{n_y} = \frac{0.5\left(67\right)10^3}{2} \;\Rightarrow\; T = 16.83\left(10^3\right)\ \text{lbf}\cdot\text{in}$$

Eq. (3-42):

$$H = \frac{Tn}{63\ 025} = \frac{16.83\left(10^3\right)540}{63\ 025} = 144\ \text{hp} \qquad \textit{Ans.}$$

(b) Eq. (3-45):

$$\tau_{\max} = \frac{0.5S_y}{n_y} = \frac{T}{2A_m t}$$

$$\frac{0.5\left(67\right)10^3}{2} = \frac{T}{2\left[\left(1.5+\frac{1}{8}\right)\left(2.0+\frac{1}{8}\right)\right]\frac{1}{8}} \;\Rightarrow\; T = 14.46\left(10^3\right)\ \text{lbf}\cdot\text{in}$$

$$H = \frac{14.46\left(10^3\right)540}{63\ 025} = 124\ \text{hp} \qquad \textit{Ans.}$$

3-60 Outer dimensions 20 × 30 mm, $t = 1$ mm, $l = 1$ m, 1018 CD steel, $S_{sy} = 0.5\ S_y$. Table A-20, $S_y = 370$ MPa. $S_{sy} = 0.5(370) = 185$ MPa. Table A-5, $G = 79.3$ GPa.

(a) $A_m = 19(29) = 551\ \text{mm}^2$.

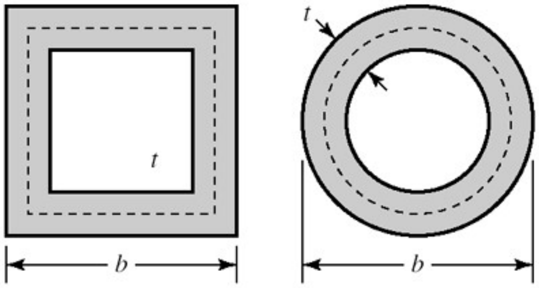
Eq, (3-45):

$$\tau = \frac{T}{2A_m t} = 185\left(10^6\right) \;\Rightarrow\; T = 2(551)10^{-6}\left(0.001\right)185\left(10^6\right) = 204\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

(b) Eq. (3-46):

$$\theta = \theta_1 l = \frac{TL_m l}{4GA_m^2 t} \;\Rightarrow\; 3\left(\frac{\pi}{180}\right) = \frac{T\left(2\right)\left(19+29\right)10^{-3}\left(1\right)}{4\left(79.3\right)10^9\left(551\right)^2 10^{-12}\left(0.001\right)} \;\Rightarrow\; T = 52.5\ \text{N}\cdot\text{m} \quad \textit{Ans.}$$

3-61 **(a)** The area within the wall median line, A_m, is



Square: $A_m = (b-t)^2$. From Eq. (3-45)

$$T_{\text{sq}} = 2A_m t\tau_{\text{all}} = 2(b-t)^2 t\tau_{\text{all}}$$

Round: $A_m = \pi(b-t)^2/4$

$$T_{\text{rd}} = 2\pi(b-t)^2 t\tau_{\text{all}}/4$$

Ratio of Torques

$$\frac{T_{\text{sq}}}{T_{\text{rd}}} = \frac{2(b-t)^2 t\tau_{\text{all}}}{\pi(b-t)^2 t\tau_{\text{all}}/2} = \frac{4}{\pi} = 1.27 \quad \textit{Ans.}$$

(b) Twist per unit length from Eq. (3-46) is

$$\theta_1 = \frac{TL_m}{4GA_m^2 t} = \frac{2A_m t\tau_{\text{all}} L_m}{4GA_m^2 t} = \frac{\tau_{\text{all}}}{2G}\frac{L_m}{A_m} = C\frac{L_m}{A_m}$$

Square:

$$\theta_{\text{sq}} = C\frac{4(b-t)}{(b-t)^2}$$

Round:

$$\theta_{\text{rd}} = C\frac{\pi(b-t)}{\pi(b-t)^2/4} = C\frac{4(b-t)}{(b-t)^2}$$

$\dfrac{\theta_{sq}}{\theta_{rd}} = 1$. Twists are the same. *Ans.*

3-62 **(a)** The area enclosed by the section median line is $A_m = (1 - 0.0625)^2 = 0.8789$ in^2 and the length of the section median line is $L_m = 4(1 - 0.0625) = 3.75$ in. From Eq. (3-45),

$$T = 2A_m t\tau = 2(0.8789)(0.0625)(12\ 000) = 1318 \text{ lbf}\cdot\text{in} \qquad \textit{Ans.}$$

From Eq. (3-46),

$$\theta = \theta_1 l = \frac{TL_m l}{4GA_m^2 t} = \frac{(1318)(3.75)(36)}{4(11.5)\left(10^6\right)(0.8789)^2(0.0625)} = 0.0801 \text{ rad} = 4.59^\circ \quad \textit{Ans.}$$

(b) The radius at the median line is $r_m = 0.125 + (0.5)(0.0625) = 0.15625$ in. The area enclosed by the section median line is $A_m = (1 - 0.0625)^2 - 4(0.15625)^2 + 4(\pi/4)(0.15625)^2 = 0.8579$ in^2. The length of the section median line is $L_m = 4[1 - 0.0625 - 2(0.15625)] + 2\pi(0.15625) = 3.482$ in.

From Eq. (3-45),

$$T = 2A_m t\tau = 2(0.8579)(0.0625)(12\ 000) = 1287 \text{ lbf}\cdot\text{in} \qquad Ans.$$

From Eq. (3-46),

$$\theta = \theta_1 l = \frac{TL_m l}{4GA_m^2 t} = \frac{(1287)(3.482)(36)}{4(11.5)\left(10^6\right)(0.8579)^2(0.0625)} = 0.0762 \text{ rad} = 4.37^\circ \quad Ans.$$

3-63

$$\theta_1 = \frac{3T_i}{GL_i c_i^3} \qquad \Rightarrow \qquad T_i = \frac{\theta_1 G L_i c_i^3}{3}$$

$$T = T_1 + T_2 + T_3 = \frac{\theta_1 G}{3}\sum_{i=1}^{3} L_i c_i^3 \qquad Ans.$$

From Eq. (3-47), $\tau = G\theta_1 c$

G and θ_1 are constant, therefore the largest shear stress occurs when c is a maximum.

$$\tau_{\max} = G\theta_1 c_{\max} \qquad Ans.$$

3-64

(b) Solve part (b) first since the angle of twist per unit length is needed for part (a).

$$\tau_{\max} = \tau_{\text{allow}} = 12\left(6.89\right) = 82.7 \text{ MPa}$$

$$\theta_1 = \frac{\tau_{\max}}{Gc_{\max}} = \frac{82.7\left(10^6\right)}{79.3\left(10^9\right)(0.003)} = 0.348 \text{ rad/m} \qquad Ans.$$

(a)

$$T_1 = \frac{\theta_1 G L_1 c_1^3}{3} = \frac{0.348(79.3)\left(10^9\right)(0.020)(0.002^3)}{3} = 1.47 \text{ N}\cdot\text{m} \qquad Ans.$$

$$T_2 = \frac{\theta_2 G L_2 c_2^3}{3} = \frac{0.348(79.3)\left(10^9\right)(0.030)(0.003^3)}{3} = 7.45 \text{ N}\cdot\text{m} \qquad Ans.$$

$$T_3 = \frac{\theta_3 G L_3 c_3^3}{3} = \frac{0.348(79.3)\left(10^9\right)(0)(0^3)}{3} = 0 \qquad Ans.$$

$$T = T_1 + T_2 + T_3 = 1.47 + 7.45 + 0 = 8.92 \text{ N}\cdot\text{m} \qquad Ans.$$

3-65

(b) Solve part (b) first since the angle of twist per unit length is needed for part (a).

$$\theta_1 = \frac{\tau_{\max}}{Gc_{\max}} = \frac{12\,000}{11.5\left(10^6\right)(0.125)} = 8.35\left(10^{-3}\right) \text{ rad/in} \quad \textit{Ans.}$$

(a)

$$T_1 = \frac{\theta_1 G L_1 c_1^3}{3} = \frac{(8.35)\left(10^{-3}\right)(11.5)\left(10^6\right)(0.75)\left(0.0625^3\right)}{3} = 5.86 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_2 = \frac{\theta_2 G L_2 c_2^3}{3} = \frac{(8.35)\left(10^{-3}\right)(11.5)\left(10^6\right)(1)\left(0.125^3\right)}{3} = 62.52 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_3 = \frac{\theta_3 G L_3 c_3^3}{3} = \frac{(8.35)\left(10^{-3}\right)(11.5)\left(10^6\right)(0.625)\left(0.0625^3\right)}{3} = 4.88 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T = T_1 + T_2 + T_3 = 5.86 + 62.52 + 4.88 = 73.3 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

3-66

(b) Solve part (b) first since the angle of twist per unit length is needed for part (a).

$$\tau_{\max} = \tau_{\text{allow}} = 12(6.89) = 82.7 \text{ MPa}$$

$$\theta_1 = \frac{\tau_{\max}}{Gc_{\max}} = \frac{82.7\left(10^6\right)}{79.3\left(10^9\right)(0.003)} = 0.348 \text{ rad/m} \quad \textit{Ans.}$$

(a)

$$T_1 = \frac{\theta_1 G L_1 c_1^3}{3} = \frac{0.348(79.3)\left(10^9\right)(0.020)(0.002^3)}{3} = 1.47 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

$$T_2 = \frac{\theta_2 G L_2 c_2^3}{3} = \frac{0.348(79.3)\left(10^9\right)(0.030)(0.003^3)}{3} = 7.45 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

$$T_3 = \frac{\theta_3 G L_3 c_3^3}{3} = \frac{0.348(79.3)\left(10^9\right)(0.025)(0.002^3)}{3} = 1.84 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

$$T = T_1 + T_2 + T_3 = 1.47 + 7.45 + 1.84 = 10.8 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

3-67

(a) From Eq. (3-40), with two 2-mm strips,

$$T = \frac{\tau_{\max} bc^2}{3 + 1.8/(b/c)} = \frac{(80)\left(10^6\right)(0.030)\left(0.002^2\right)}{3 + 1.8/\left(0.030/0.002\right)} = 3.08 \text{ N}\cdot\text{m}$$

$$T_{\max} = 2(3.08) = 6.16 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

From the table for Eqs. (3-40) and (3-41), with $b/c = 30/2 = 15$, $\alpha = \beta$ and has a value between 0.313 and 0.333. From Eq. (3-40),

$$\alpha \approx \frac{1}{3+1.8/(30/2)} = 0.321$$

From Eq. (3-41),

$$\theta = \frac{Tl}{\beta bc^3 G} = \frac{3.08(0.3)}{0.321(0.030)\left(0.002^3\right)(79.3)\left(10^9\right)} = 0.151 \text{ rad} \qquad \textit{Ans.}$$

$$k_t = \frac{T}{\theta} = \frac{6.16}{0.151} = 40.8 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

From Eq. (3-40), with a single 4-mm strip,

$$T_{\max} = \frac{\tau_{\max} bc^2}{3+1.8/(b/c)} = \frac{(80)\left(10^6\right)(0.030)\left(0.004^2\right)}{3+1.8/\left(0.030/0.004\right)} = 11.9 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

Interpolating from the table for Eqs. (3-40) and (3-41), with $b/c = 30/4 = 7.5$,

$$\beta = \frac{7.5-6}{8-6}(0.307-0.299)+0.299 = 0.305$$

From Eq. (3-41)

$$\theta = \frac{Tl}{\beta bc^3 G} = \frac{11.9(0.3)}{0.305(0.030)\left(0.004^3\right)(79.3)\left(10^9\right)} = 0.0769 \text{ rad} \qquad \textit{Ans.}$$

$$k_t = \frac{T}{\theta} = \frac{11.9}{0.0769} = 155 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

(b) From Eq. (3-47), with two 2-mm strips,

$$T = \frac{Lc^2\tau}{3} = \frac{(0.030)\left(0.002^2\right)(80)\left(10^6\right)}{3} = 3.20 \text{ N}\cdot\text{m}$$

$$T_{\max} = 2(3.20) = 6.40 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

$$\theta = \frac{3Tl}{Lc^3 G} = \frac{3(3.20)(0.3)}{(0.030)\left(0.002^3\right)(79.3)\left(10^9\right)} = 0.151 \text{ rad} \qquad \textit{Ans.}$$

$$k_t = T/\theta = 6.40/0.151 = 42.4 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

From Eq. (3-47), with a single 4-mm strip,

$$T_{\max} = \frac{Lc^2\tau}{3} = \frac{(0.030)\left(0.004^2\right)(80)\left(10^6\right)}{3} = 12.8 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

$$\theta = \frac{3Tl}{Lc^3G} = \frac{3(12.8)(0.3)}{(0.030)\left(0.004^3\right)(79.3)\left(10^9\right)} = 0.0757 \text{ rad} \quad Ans.$$

$$k_t = T/\theta = 12.8/0.0757 = 169 \text{ N}\cdot\text{m} \quad Ans.$$

The results for the spring constants when using Eq. (3-47) are slightly larger than when using Eq. (3-40) and Eq. (3-41) because the strips are not infinitesimally thin (i.e. *b*/*c* does not equal infinity). The spring constants when considering one solid strip are significantly larger (almost four times larger) than when considering two thin strips because two thin strips would be able to slip along the center plane.

3-68

(a) Obtain the torque from the given power and speed using Eq. (3-44).

$$T = 9.55\frac{H}{n} = 9.55\frac{(40\,000)}{2500} = 152.8 \text{ N}\cdot\text{m}$$

$$\tau_{\max} = \frac{Tr}{J} = \frac{16T}{\pi d^3}$$

$$d = \left(\frac{16T}{\pi\tau_{\max}}\right)^{1/3} = \left[\frac{16(152.8)}{\pi(70)\left(10^6\right)}\right]^{1/3} = 0.0223 \text{ m} = 22.3 \text{ mm} \quad Ans.$$

(b) $T = 9.55\dfrac{H}{n} = 9.55\dfrac{(40\,000)}{250} = 1528 \text{ N}\cdot\text{m}$

$$d = \left[\frac{16(1528)}{\pi(70)\left(10^6\right)}\right]^{1/3} = 0.0481 \text{ m} = 48.1 \text{ mm} \quad Ans.$$

3-69

(a) Obtain the torque from the given power and speed using Eq. (3-42).

$$T = \frac{63\,025H}{n} = \frac{63\,025(50)}{2500} = 1261 \text{ lbf}\cdot\text{in}$$

$$\tau_{\max} = \frac{Tr}{J} = \frac{16T}{\pi d^3}$$

$$d = \left(\frac{16T}{\pi\tau_{\max}}\right)^{1/3} = \left[\frac{16(1261)}{\pi(20\,000)}\right]^{1/3} = 0.685 \text{ in} \quad Ans.$$

(b) $T = \dfrac{63\,025H}{n} = \dfrac{63\,025(50)}{250} = 12\,610 \text{ lbf}\cdot\text{in}$

$$d = \left[\frac{16(12\,610)}{\pi(20\,000)}\right]^{1/3} = 1.48 \text{ in} \quad Ans.$$

3-70

$$\tau_{\max}=\frac{16T}{\pi d^3} \quad\Rightarrow\quad T=\frac{\tau_{\max}\pi d^3}{16}=\frac{(50)\left(10^6\right)\pi\left(0.03^3\right)}{16}=265\ \text{N}\cdot\text{m}$$

Eq. (3-44), $H=\dfrac{Tn}{9.55}=\dfrac{265(2000)}{9.55}=55.5\left(10^3\right)\ \text{W}=55.5\ \text{kW}$ *Ans.*

3-71

$$\tau=\frac{16T}{\pi d^3} \quad\Rightarrow\quad T=\frac{\pi}{16}\tau d^3=\frac{\pi}{16}(110)\left(10^6\right)\left(0.020^3\right)=173\ \text{N}\cdot\text{m}$$

$$\theta=\frac{Tl}{JG} \quad\Rightarrow\quad l=\frac{\pi d^4 G\theta}{32T}=\frac{\pi\left(0.020^4\right)(79.3)\left(10^9\right)\left(15\frac{\pi}{180}\right)}{32(173)}$$

$l=1.89$ m *Ans.*

3-72

$$\tau=\frac{16T}{\pi d^3} \quad\Rightarrow\quad T=\frac{\pi}{16}\tau d^3=\frac{\pi}{16}(30\,000)\left(0.75^3\right)=2485\ \text{lbf}\cdot\text{in}$$

$$\theta=\frac{Tl}{JG}=\frac{32Tl}{\pi d^4 G}=\frac{32(2485)(24)}{\pi\left(0.75^4\right)(11.5)\left(10^6\right)}=0.167\ \text{rad}=9.57^\circ \quad \textit{Ans.}$$

3-73

(a) $T_{\text{solid}}=\dfrac{J\tau_{\max}}{r}=\dfrac{\pi d_o^4\tau_{\max}}{16d_o}$ $\quad T_{\text{hollow}}=\dfrac{J\tau_{\max}}{r}=\dfrac{\pi(d_o^4-d_i^4)\tau_{\max}}{16d_o}$

$$\%\Delta T=\frac{T_{\text{solid}}-T_{\text{hollow}}}{T_{\text{solid}}}(100\%)=\frac{d_i^4}{d_o^4}(100\%)=\frac{\left(36^4\right)}{\left(40^4\right)}(100\%)=65.6\% \quad \textit{Ans.}$$

(b) $W_{\text{solid}}=kd_o^2,\quad W_{\text{hollow}}=k\left(d_o^{\,2}-d_i^{\,2}\right)$

$$\%\Delta W=\frac{W_{\text{solid}}-W_{\text{hollow}}}{W_{\text{solid}}}(100\%)=\frac{d_i^2}{d_o^2}(100\%)=\frac{\left(36^2\right)}{\left(40^2\right)}(100\%)=81.0\% \quad \textit{Ans.}$$

3-74

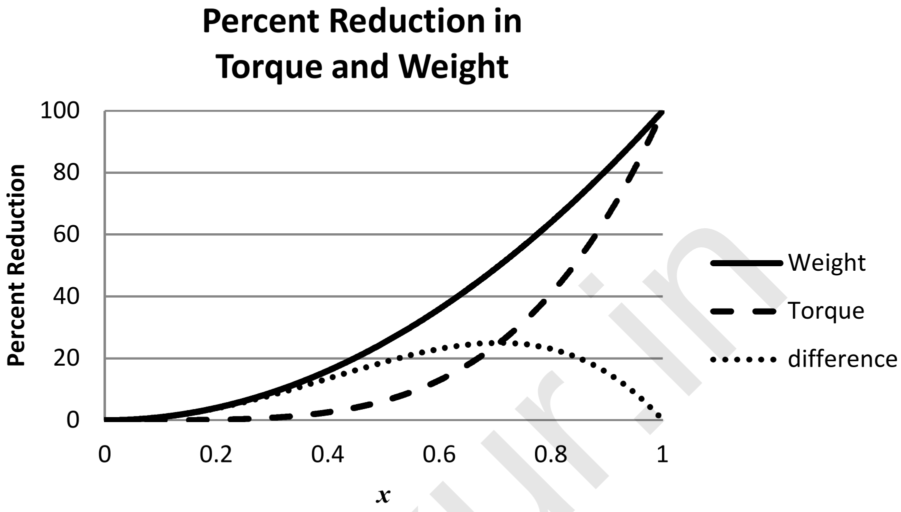
(a) $T_{\text{solid}}=\dfrac{J\tau_{\max}}{r}=\dfrac{\pi d^4\tau_{\max}}{16d}$ $\quad T_{\text{hollow}}=\dfrac{J\tau_{\max}}{r}=\dfrac{\pi\left[d^4-(xd)^4\right]\tau_{\max}}{16d}$

$$\%\Delta T=\frac{T_{\text{solid}}-T_{\text{hollow}}}{T_{\text{solid}}}(100\%)=\frac{(xd)^4}{d^4}(100\%)=x^4(100\%) \quad \textit{Ans.}$$

(b) $W_{\text{solid}} = kd^2 \qquad W_{\text{hollow}} = k\left(d^2 - (xd)^2\right)$

$$\%\Delta W = \frac{W_{\text{solid}} - W_{\text{hollow}}}{W_{\text{solid}}}(100\%) = \frac{(xd)^2}{d^2}(100\%) = x^2(100\%) \qquad \textit{Ans.}$$

Plot $\%\Delta T$ and $\%\Delta W$ versus x.



The value of greatest difference in percent reduction of weight and torque is 25% and occurs at $x = \sqrt{2}/2$.

3-75

(a) $$\tau = \frac{Tc}{J} \quad \Rightarrow \quad 120\left(10^6\right) = \frac{4200(d/2)}{(\pi/32)\left[d^4 - (0.70d)^4\right]} = \frac{2.8149\left(10^4\right)}{d^3}$$

$$d = \left(\frac{2.8149\left(10^4\right)}{120(10^6)}\right)^{1/3} = 6.17\left(10^{-2}\right)\ \text{m} = 61.7\ \text{mm}$$

From Table A-17, the next preferred size is $d = 80$ mm. *Ans.*
$d_i = 0.7d = 56$ mm. The next preferred size smaller is $d_i = 50$ mm *Ans.*

(b)

$$\tau = \frac{Tc}{J} = \frac{4200(d_i/2)}{(\pi/32)\left[d^4 - (d_i)^4\right]} = \frac{4200(0.050/2)}{(\pi/32)\left[(0.080)^4 - (0.050)^4\right]} = 30.8\ \text{MPa} \qquad \textit{Ans.}$$

3-76

$$T = 9.55\frac{H}{n} = 9.55\frac{(1500)}{10} = 1433\ \text{N}\cdot\text{m}$$

$$\tau = \frac{16T}{\pi d_C^3} \quad \Rightarrow \quad d_C = \left(\frac{16T}{\pi\tau}\right)^{1/3} = \left[\frac{16(1433)}{\pi(80)(10^6)}\right]^{1/3} = 0.045\ \text{m} = 45\ \text{mm}$$

From Table A-17, select 50 mm. *Ans.*

(a) $\tau_{\text{start}} = \dfrac{16(2)(1433)}{\pi(0.050^3)} = 117(10^6)\ \text{Pa} = 117\ \text{MPa}$ *Ans.*

(b) Design activity

3-77

$$T = \frac{63\ 025H}{n} = \frac{63\ 025(1)}{8} = 7880\ \text{lbf}\cdot\text{in}$$

$$\tau = \frac{16T}{\pi d_C^3} \quad \Rightarrow \quad d_C = \left(\frac{16T}{\pi\tau}\right)^{1/3} = \left[\frac{16(7880)}{\pi(15\ 000)}\right]^{1/3} = 1.39\ \text{in}$$

From Table A-17, select 1.40 in. *Ans.*

3-78 For a square cross section with side length *b*, and a circular section with diameter *d,*

$$A_{\text{square}} = A_{\text{circular}} \quad \Rightarrow \quad b^2 = \frac{\pi}{4}d^2 \quad \Rightarrow \quad b = \frac{\sqrt{\pi}}{2}d$$

From Eq. (3-40) with $b = c$,

$$(\tau_{\max})_{\text{square}} = \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right) = \frac{T}{b^3}\left(3 + \frac{1.8}{1}\right) = \frac{T}{d^3}\left(\frac{2}{\sqrt{\pi}}\right)^3 (4.8) = 6.896\frac{T}{d^3}$$

For the circular cross section,

$$(\tau_{\max})_{\text{circular}} = \frac{16T}{\pi d^3} = 5.093\frac{T}{d^3}$$

$$\frac{(\tau_{\max})_{\text{square}}}{(\tau_{\max})_{\text{circular}}} = \frac{6.896\dfrac{T}{d^3}}{5.093\dfrac{T}{d^3}} = 1.354$$

The shear stress in the square cross section is 35.4% greater. *Ans.*

(b) For the square cross section, from the table for Eq. (3-41), $\beta = 0.141$. From Eq. (3-41),

$$\theta_{\text{square}} = \frac{Tl}{\beta bc^3 G} = \frac{Tl}{\beta b^4 G} = \frac{Tl}{0.141\left(\dfrac{\sqrt{\pi}}{2}d\right)^4 G} = 11.50\frac{Tl}{d^4 G}$$

For the circular cross section,

$$\theta_{rd} = \frac{Tl}{GJ} = \frac{Tl}{G\left(\pi d^4/32\right)} = 10.19\frac{Tl}{d^4G}$$

$$\frac{\theta_{sq}}{\theta_{rd}} = \frac{11.50\dfrac{Tl}{d^4G}}{10.19\dfrac{Tl}{d^4G}} = 1.129$$

The angle of twist in the square cross section is 12.9% greater. *Ans.*

3-79 **(a)**

$$T_1 = 0.15T_2$$

$$\sum T = 0 = (500-75)(4) - (T_2 - T_1)(5) = 1700 - (T_2 - 0.15T_2)(5)$$

$$1700 - 4.25T_2 = 0 \quad \Rightarrow \quad T_2 = 400 \text{ lbf} \quad \textit{Ans.}$$

$$T_1 = 0.15(400) = 60 \text{ lbf} \quad \textit{Ans.}$$

(b)

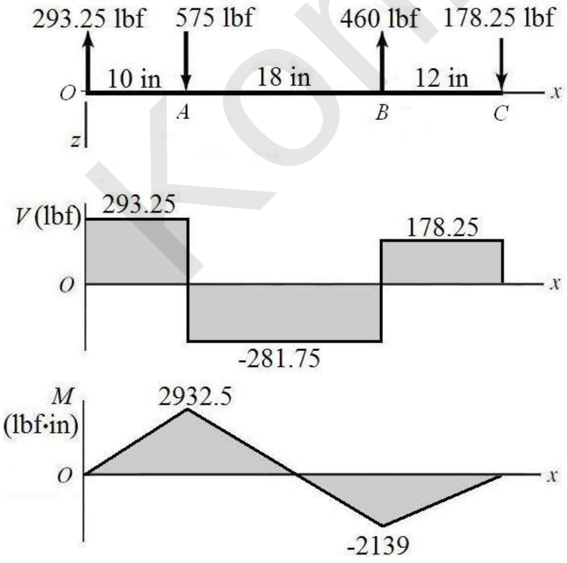
$$\sum M_O = 0 = -575(10) + 460(28) - R_C(40)$$

$$R_C = 178.25 \text{ lbf} \quad \textit{Ans.}$$

$$\sum F = 0 = R_O + 575 - 460 + 178.25$$

$$R_O = -293.25 \text{ lbf} \quad \textit{Ans.}$$

(c)



(d) The maximum bending moment is at $x = 10$ in, and is $M = 2932.5$ lbf·in. Since the shaft rotates, each stress element will experience both positive and negative bending stress as it moves from tension to compression. The torque transmitted through the shaft from *A* to *B* is $T = (500 - 75)(4) = 1700$ lbf·in. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32\left(2932.5\right)}{\pi(1.25)^3} = 15\,294 \text{ psi} = 15.3 \text{ kpsi} \quad Ans.$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(1700)}{\pi(1.25)^3} = 4433 \text{ psi} = 4.43 \text{ kpsi} \qquad Ans.$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \frac{15.3}{2} \pm \sqrt{\left(\frac{15.3}{2}\right)^2 + \left(4.43\right)^2}$$

$$\sigma_1 = 16.5 \text{ kpsi} \qquad Ans.$$

$$\sigma_2 = -1.19 \text{ kpsi} \qquad Ans.$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{15.3}{2}\right)^2 + \left(4.43\right)^2} = 8.84 \text{ kpsi} \qquad Ans.$$

3-80 **(a)**

$$T_2 = 0.15T_1$$

$$\sum T = 0 = \left(1800 - 270\right)\left(200\right) + \left(T_2 - T_1\right)(125) = 306\left(10^3\right) + 125\left(0.15T_1 - T_1\right)$$

$$306\left(10^3\right) - 106.25T_1 = 0 \qquad \Rightarrow \qquad T_1 = 2880 \text{ N} \quad Ans.$$

$$T_2 = 0.15\left(2880\right) = 432 \text{ N} \quad Ans.$$

(b)

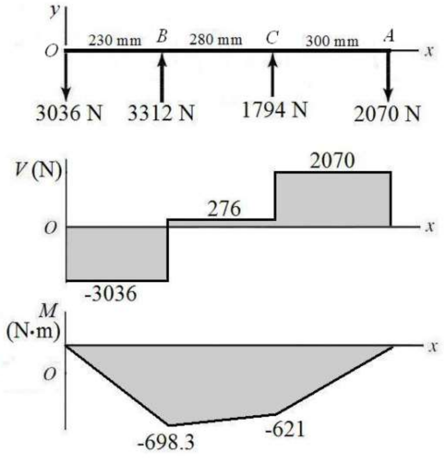
$$\sum M_O = 0 = 3312(230) + R_C(510) - 2070(810)$$

$$R_C = 1794 \text{ N} \quad Ans.$$

$$\sum F_y = 0 = R_O + 3312 + 1794 - 2070$$

$$R_O = -3036 \text{ N} \quad Ans.$$

(c)



(d) The maximum bending moment is at $x = 230$ mm, and is $M = –698.3$ N·m. Since the shaft rotates, each stress element will experience both positive and negative bending stress as it moves from tension to compression. The torque transmitted through the shaft from A to B is $T = (1800 – 270)(0.200) = 306$ N·m. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(698.3)}{\pi(0.030)^3} = 263\left(10^3\right)\text{ Pa} = 263\ \text{MPa} \quad Ans.$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(306)}{\pi(0.030)^3} = 57.7\left(10^6\right)\text{ Pa} = 57.7\text{MPa} \qquad Ans.$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \frac{263}{2} \pm \sqrt{\left(\frac{263}{2}\right)^2 + \left(57.7\right)^2}$$

$$\sigma_1 = 275\text{ MPa} \qquad Ans.$$

$$\sigma_2 = -12.1\text{ MPa} \qquad Ans.$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{263}{2}\right)^2 + \left(57.7\right)^2} = 144\text{ MPa} \qquad Ans.$$

3-81

(a)

$$T_2 = 0.15T_1$$

$$\sum T = 0 = \left(300-50\right)\left(4\right) + \left(T_2 - T_1\right)(3) = 1000 + \left(0.15T_1 - T_1\right)(3)$$

$$1000 - 2.55T_1 = 0 \qquad \Rightarrow \qquad T_1 = 392.16\text{ lbf} \quad Ans.$$

$$T_2 = 0.15\left(392.16\right) = 58.82\text{ lbf} \quad Ans.$$

(b)

$$\sum M_{Oy} = 0 = -450.98(16) - R_{Cz}(22)$$

$$R_{Cz} = -327.99\text{ lbf} \quad Ans.$$

$$\sum F_z = 0 = R_{Oz} + 450.98 - 327.99$$

$$R_{Oz} = -122.99\text{ lbf} \quad Ans.$$

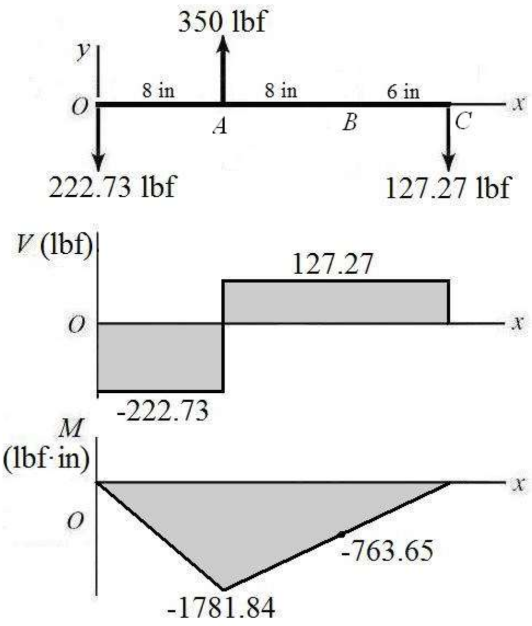
$$\sum M_{Oz} = 0 = 350(8) + R_{Cy}(22)$$

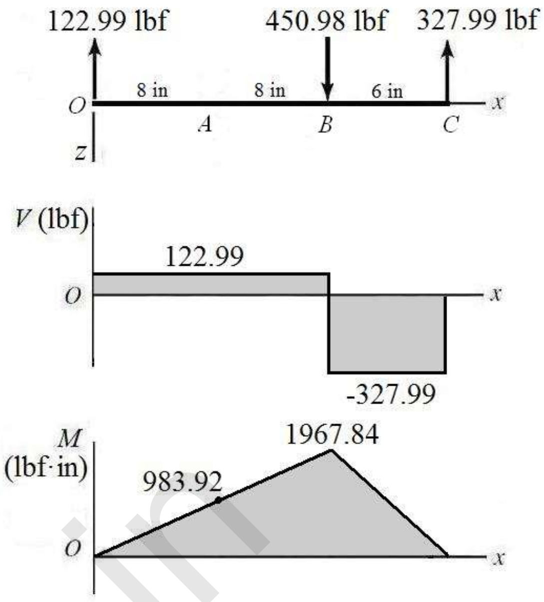
$$R_{Cy} = -127.27\text{ lbf} \quad Ans.$$

$$\sum F_y = 0 = R_{Oy} + 350 - 127.27$$

$$R_{Oy} = -222.73\text{ lbf} \quad Ans.$$

(c)





(d) Combine the bending moments from both planes at *A* and *B* to find the critical location.

$$M_A = \sqrt{(983.92)^2 + (-1781.84)^2} = 2035 \text{ lbf} \cdot \text{in}$$

$$M_B = \sqrt{(1967.84)^2 + (-763.65)^2} = 2111 \text{ lbf} \cdot \text{in}$$

The critical location is at *B*. The torque transmitted through the shaft from *A* to *B* is *T* = (300 – 50)(4) = 1000 lbf·in. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(2111)}{\pi(1)^3} = 21\,502 \text{ psi} = 21.5 \text{ kpsi} \quad Ans.$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi(1)^3} = 5093 \text{ psi} = 5.09 \text{ kpsi} \qquad Ans.$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \frac{21.5}{2} \pm \sqrt{\left(\frac{21.5}{2}\right)^2 + (5.09)^2}$$

$$\sigma_1 = 22.6 \text{ kpsi} \qquad Ans.$$

$$\sigma_2 = -1.14 \text{ kpsi} \qquad Ans.$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{21.5}{2}\right)^2 + (5.09)^2} = 11.9 \text{ kpsi} \qquad Ans.$$

3-82 **(a)**

$$T_2 = 0.15T_1$$

$$\sum T = 0 = (300-45)(125) + (T_2 - T_1)(150) = 31\,875 + (0.15T_1 - T_1)(150)$$

$$31\,875 - 127.5T_1 = 0 \qquad \Rightarrow \qquad T_1 = 250 \text{ N}\cdot\text{mm} \quad \textit{Ans.}$$

$$T_2 = 0.15(250) = 37.5 \text{ N}\cdot\text{mm} \quad \textit{Ans.}$$

(b)

$$\sum M_{Oy} = 0 = 345\sin 45^\circ(300) - 287.5(700) - R_{Cz}(850)$$

$$R_{Cz} = -150.7 \text{ N} \qquad \textit{Ans.}$$

$$\sum F_z = 0 = R_{Oz} - 345\cos 45^\circ + 287.5 - 150.7$$

$$R_{Oz} = 107.2 \text{ N} \qquad \textit{Ans.}$$

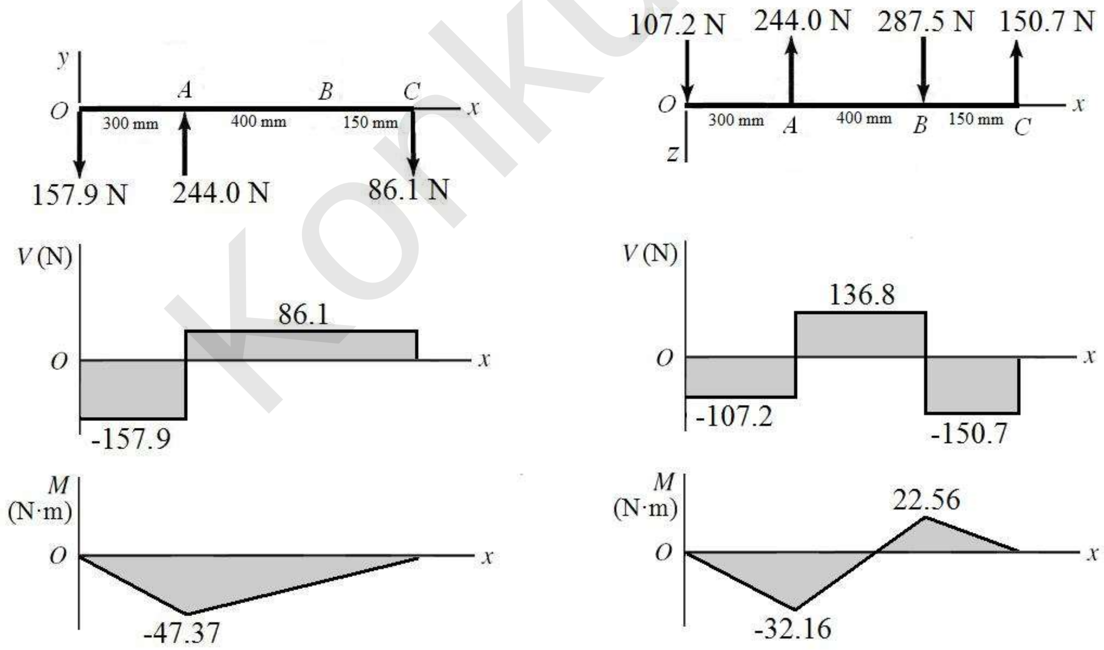
$$\sum M_{Oz} = 0 = 345\sin 45^\circ(300) + R_{Cy}(850)$$

$$R_{Cy} = -86.10 \text{ N} \qquad \textit{Ans.}$$

$$\sum F_y = 0 = R_{Oy} + 345\cos 45^\circ - 86.10$$

$$R_{Oy} = -157.9 \text{ N} \qquad \textit{Ans.}$$

(c)



(d) From the bending moment diagrams, it is clear that the critical location is at *A* where both planes have the maximum bending moment. Combining the bending moments from the two planes,

$$M=\sqrt{(-47.37)^2+(-32.16)^2}=57.26\ \text{N}\cdot\text{m}$$

The torque transmitted through the shaft from A to B is $T = (300 - 45)(0.125) = 31.88$ N·m. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma=\frac{Mc}{I}=\frac{32M}{\pi d^3}=\frac{32(57.26)}{\pi(0.020)^3}=72.9\left(10^6\right)\ \text{Pa}=72.9\ \text{MPa}\qquad Ans.$$

$$\tau=\frac{Tr}{J}=\frac{16T}{\pi d^3}=\frac{16(31.88)}{\pi(0.020)^3}=20.3\left(10^6\right)\ \text{Pa}=20.3\ \text{MPa}\qquad Ans.$$

(e)

$$\sigma_1,\ \sigma_2=\frac{\sigma_x}{2}\pm\sqrt{\left(\frac{\sigma_x}{2}\right)^2+\left(\tau_{xy}\right)^2}=\frac{72.9}{2}\pm\sqrt{\left(\frac{72.9}{2}\right)^2+\left(20.3\right)^2}$$

$$\sigma_1=78.2\ \text{MPa}\qquad Ans.$$

$$\sigma_2=-5.27\ \text{MPa}\qquad Ans.$$

$$\tau_{\max}=\sqrt{\left(\frac{\sigma_x}{2}\right)^2+\left(\tau_{xy}\right)^2}=\sqrt{\left(\frac{72.9}{2}\right)^2+\left(20.3\right)^2}=41.7\ \text{MPa}\qquad Ans.$$

3-83

(a)

$$\sum T=0=-300(\cos 20^\circ)(10)+F_B(\cos 20^\circ)(4)$$

$$F_B=750\ \text{lbf}\quad Ans.$$

(b)

$$\sum M_{Oz}=0=300(\cos 20^\circ)(16)-750(\sin 20^\circ)(39)+R_{Cy}(30)$$

$$R_{Cy}=183.1\ \text{lbf}\quad Ans.$$

$$\sum F_y=0=R_{Oy}+300(\cos 20^\circ)+183.1-750(\sin 20^\circ)$$

$$R_{Oy}=-208.5\ \text{lbf}\quad Ans.$$

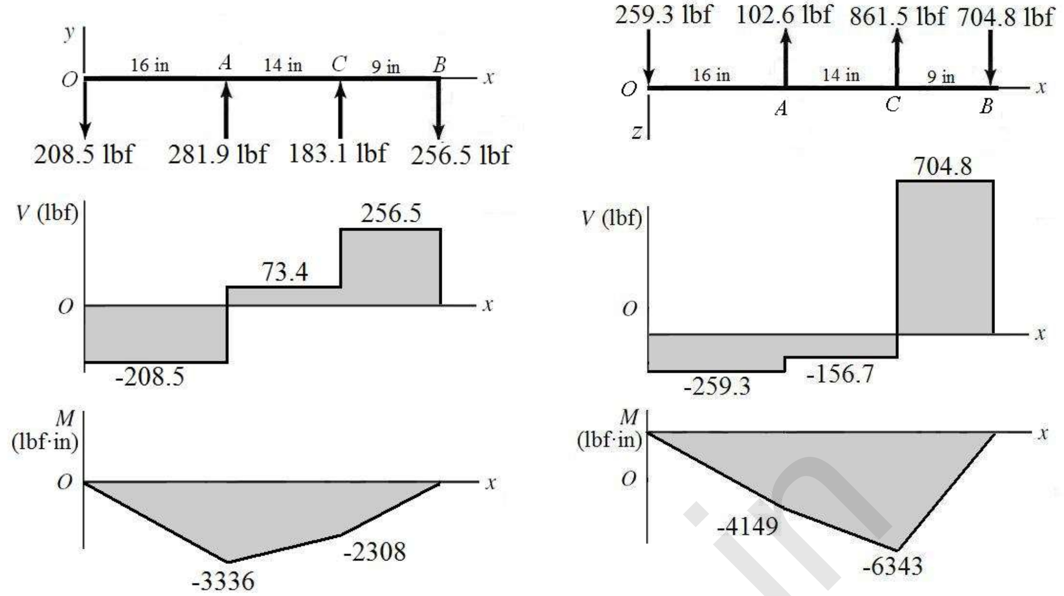
$$\sum M_{Oy}=0=300(\sin 20^\circ)(16)-R_{Cz}(30)-750(\cos 20^\circ)(39)$$

$$R_{Cz}=-861.5\ \text{lbf}\quad Ans.$$

$$\sum F_z=0=R_{Oz}-300(\sin 20^\circ)-861.5+750(\cos 20^\circ)$$

$$R_{Oz}=259.3\ \text{lbf}\quad Ans.$$

(c)



(d) Combine the bending moments from both planes at *A* and C to find the critical location.

$$M_A = \sqrt{(-3336)^2 + (-4149)^2} = 5324 \text{ lbf} \cdot \text{in}$$

$$M_C = \sqrt{(-2308)^2 + (-6343)^2} = 6750 \text{ lbf} \cdot \text{in}$$

The critical location is at *C*. The torque transmitted through the shaft from *A* to *B* is $T = 300\cos(20^\circ)(10) = 2819 \text{ lbf} \cdot \text{in}$. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(6750)}{\pi(1.25)^3} = 35\,203 \text{ psi} = 35.2 \text{ kpsi} \qquad Ans.$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(2819)}{\pi(1.25)^3} = 7351 \text{ psi} = 7.35 \text{ kpsi} \qquad Ans.$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \frac{35.2}{2} \pm \sqrt{\left(\frac{35.2}{2}\right)^2 + (7.35)^2}$$

$$\sigma_1 = 36.7 \text{ kpsi} \qquad Ans.$$

$$\sigma_2 = -1.47 \text{ kpsi} \qquad Ans.$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{35.2}{2}\right)^2 + (7.35)^2} = 19.1 \text{ kpsi} \qquad Ans.$$

3-84

(a)

$$\sum T = 0 = -11\,000(\cos 20^\circ)(300) + F_B(\cos 25^\circ)(150)$$

$$F_B = 22\,810 \text{ N} \qquad \textit{Ans.}$$

(b)

$$\sum M_{Oz} = 0 = -11\,000(\sin 20^\circ)(400) - 22\,810(\sin 25^\circ)(750) + R_{Cy}(1050)$$

$$R_{Cy} = 8319 \text{ N} \qquad \textit{Ans.}$$

$$\sum F_y = 0 = R_{Oy} - 11000(\sin 20^\circ) - 22\,810\sin(25^\circ) + 8319$$

$$R_{Oy} = 5083 \text{ N} \qquad \textit{Ans.}$$

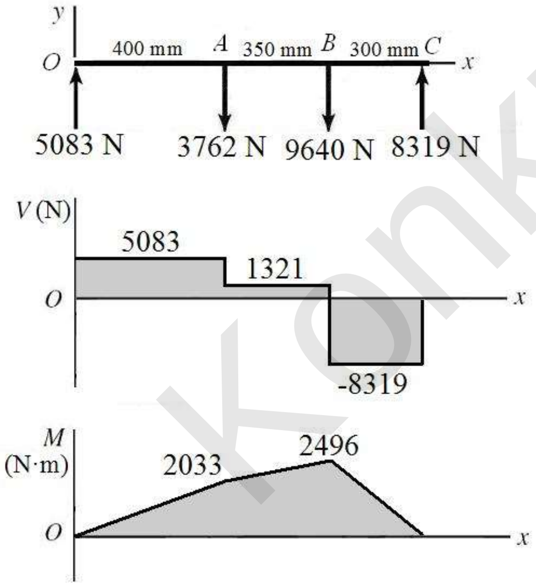
$$\sum M_{Oy} = 0 = 11\,000(\cos 20^\circ)(400) - 22\,810(\cos 25^\circ)(750) - R_{Cz}(1050)$$

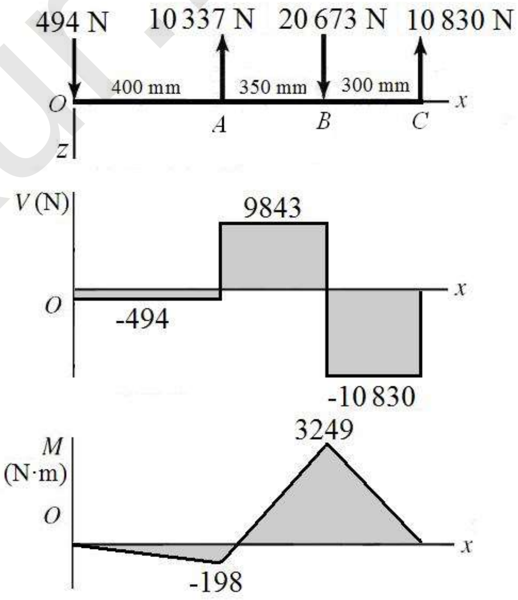
$$R_{Cz} = -10\,830 \text{ N} \quad \textit{Ans.}$$

$$\sum F_z = 0 = R_{Oz} - 11\,000(\cos 20^\circ) + 22\,810(\cos 25^\circ) - 10\,830$$

$$R_{Oz} = 494 \text{ N} \quad \textit{Ans.}$$

(c)





(d) From the bending moment diagrams, it is clear that the critical location is at *B* where both planes have the maximum bending moment. Combining the bending moments from the two planes,

$$M = \sqrt{(2496)^2 + (3249)^2} = 4097 \text{ N}\cdot\text{m}$$

The torque transmitted through the shaft from *A* to *B* is

$$T = 11\,000\cos(20^\circ)(0.3) = 3101 \text{ N}\cdot\text{m}.$$

For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32\left(4097\right)}{\pi (0.050)^3} = 333.9\left(10^6\right) \text{ Pa } = 333.9 \text{ MPa} \qquad Ans.$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(3101)}{\pi (0.050)^3} = 126.3\left(10^6\right) \text{ Pa} = 126.3 \text{ MPa} \qquad Ans.$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \frac{333.9}{2} \pm \sqrt{\left(\frac{333.9}{2}\right)^2 + \left(126.3\right)^2}$$

$$\sigma_1 = 376 \text{ MPa} \qquad Ans.$$

$$\sigma_2 = -42.4 \text{ MPa} \qquad Ans.$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{333.9}{2}\right)^2 + \left(126.3\right)^2} = 209 \text{ MPa} \qquad Ans.$$

3-85

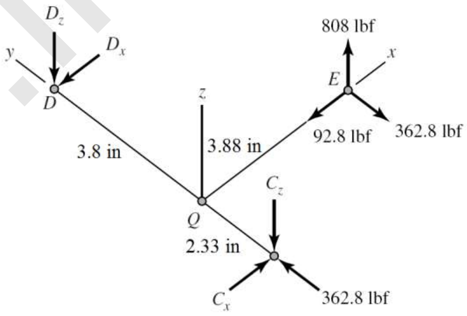
(a)

$$\left(\Sigma M_D\right)_z = 6.13C_x - 3.8(92.8) - 3.88(362.8) = 0$$

$$C_x = 287.2 \text{ lbf} \qquad Ans.$$

$$\left(\Sigma M_C\right)_z = 6.13D_x + 2.33(92.8) - 3.88(362.8) = 0$$

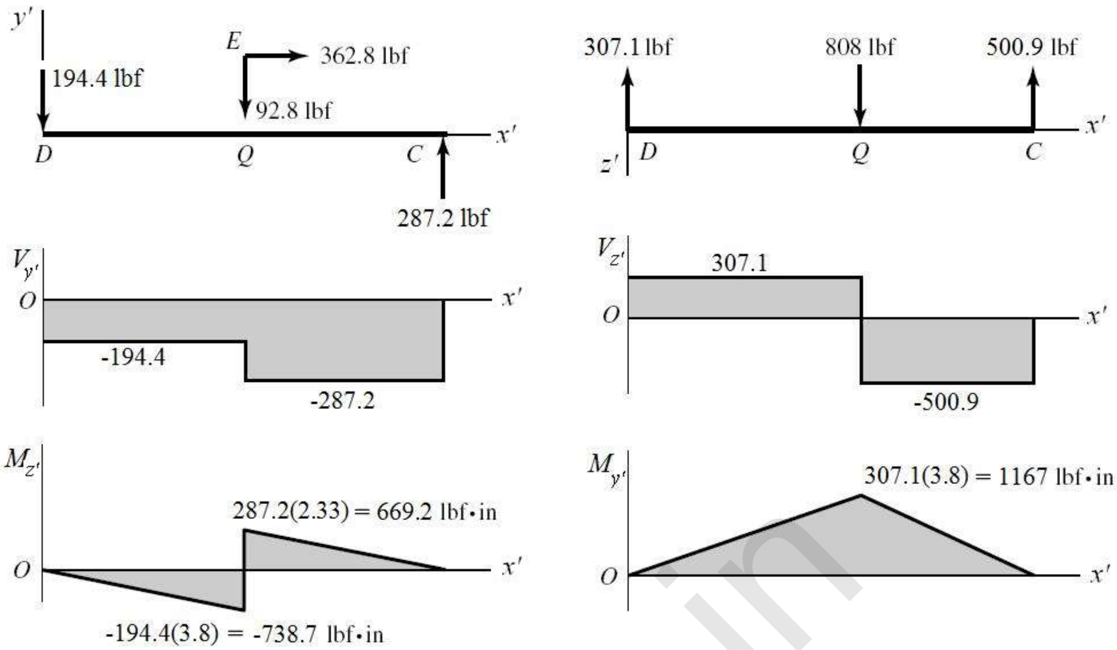
$$D_x = 194.4 \text{ lbf} \qquad Ans.$$



$$\left(\Sigma M_D\right)_x = 0 \quad \Rightarrow \quad C_z = \frac{3.8}{6.13}(808) = 500.9 \text{ lbf} \qquad Ans.$$

$$\left(\Sigma M_C\right)_x = 0 \quad \Rightarrow \quad D_z = \frac{2.33}{6.13}(808) = 307.1 \text{ lbf} \qquad Ans.$$

(b) For *DQC*, let x', y', z' correspond to the original $-y, x, z$ axes.



(c) The critical stress element is just to the right of *Q*, where the bending moment in both planes is a maximum, and where the torsional and axial loads exist.

$$T = 808(3.88) = 3135 \text{ lbf} \cdot \text{in}$$

$$M = \sqrt{669.2^2 + 1167^2} = 1345 \text{ lbf} \cdot \text{in}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(3135)}{\pi\left(1.13^3\right)} = 11\,070 \text{ psi} \qquad \textit{Ans.}$$

$$\sigma_b = \pm\frac{32M}{\pi d^3} = \pm\frac{32(1345)}{\pi\left(1.13^3\right)} = \pm 9495 \text{ psi} \qquad \textit{Ans.}$$

$$\sigma_a = -\frac{F}{A} = -\frac{362.8}{(\pi/4)\left(1.13^2\right)} = -362 \text{ psi} \qquad \textit{Ans.}$$

(d) The critical stress element will be where the bending stress and axial stress are both in compression.

$$\sigma_{\max} = -9495 - 362 = -9857 \text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{-9857}{2}\right)^2 + 11\,070^2} = 12\,118 \text{ psi} = 12.1 \text{ kpsi} \qquad \textit{Ans.}$$

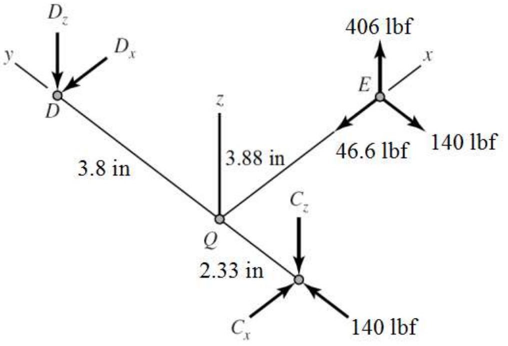
$$\sigma_1, \sigma_2 = \frac{-9857}{2} \pm \sqrt{\left(\frac{-9857}{2}\right)^2 + 11\,070^2}$$

$$\sigma_1 = 7189 \text{ psi} = 7.19 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_2 = -17\,046 \text{ psi} = -17.0 \text{ kpsi} \qquad \textit{Ans.}$$

3-86

(a)



$$\left(\Sigma M_D\right)_z = 0$$

$$6.13C_x - 3.8(46.6) - 3.88(140) = 0$$

$$C_x = 117.5 \text{ lbf} \quad \textit{Ans.}$$

$$\left(\Sigma M_C\right)_z = 0$$

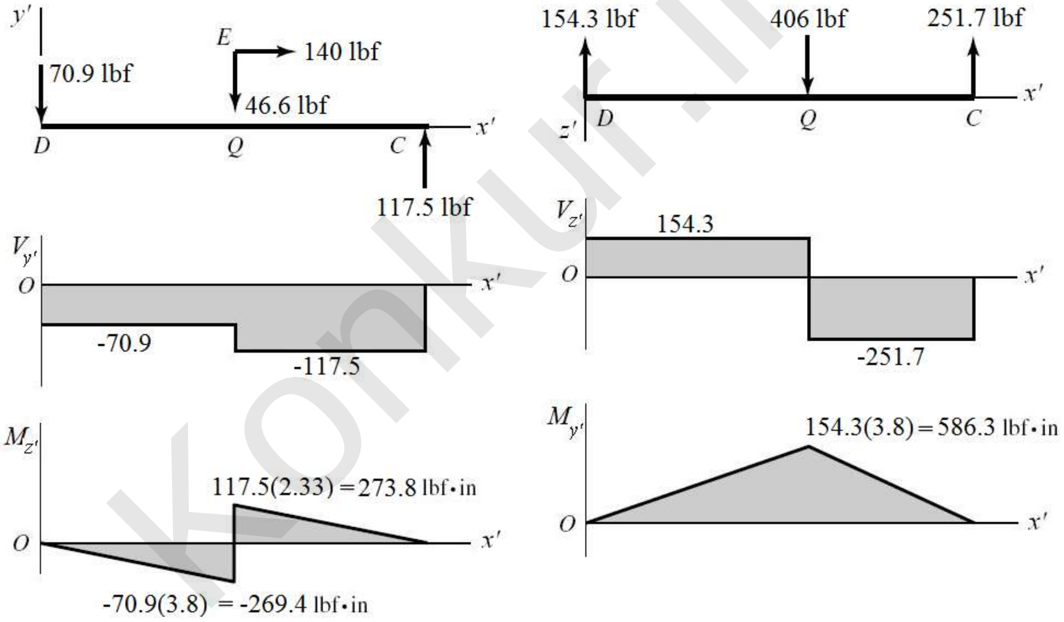
$$-6.13D_x - 2.33(46.6) + 3.88(140) = 0$$

$$D_x = 70.9 \text{ lbf} \quad \textit{Ans.}$$

$$\left(\Sigma M_D\right)_x = 0 \quad \Rightarrow \quad C_z = \frac{3.8}{6.13}(406) = 251.7 \text{ lbf} \quad \textit{Ans.}$$

$$\left(\Sigma M_C\right)_x = 0 \quad \Rightarrow \quad D_z = \frac{2.33}{6.13}(406) = 154.3 \text{ lbf} \quad \textit{Ans.}$$

(b) For *DQC*, let *x′, y′, z′* correspond to the original – *y, x, z* axes.



(c) The critical stress element is just to the right of *Q*, where the bending moment in both planes is a maximum, and where the torsional and axial loads exist.

$$T = 406(3.88) = 1575 \text{ lbf}\cdot\text{in}$$

$$M = \sqrt{273.8^2 + 586.3^2} = 647.1 \text{ lbf}\cdot\text{in}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(1575)}{\pi\left(1^3\right)} = 8021 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_b = \pm\frac{32M}{\pi d^3} = \pm\frac{32(647.1)}{\pi\left(1^3\right)} = \pm 6591 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_a = -\frac{F}{A} = -\frac{140}{(\pi/4)\left(1^2\right)} = -178.3 \text{ psi} \qquad Ans.$$

(d) The critical stress element will be where the bending stress and axial stress are both in compression.

$$\sigma_{\max} = -6591 - 178.3 = -6769 \text{ psi}$$

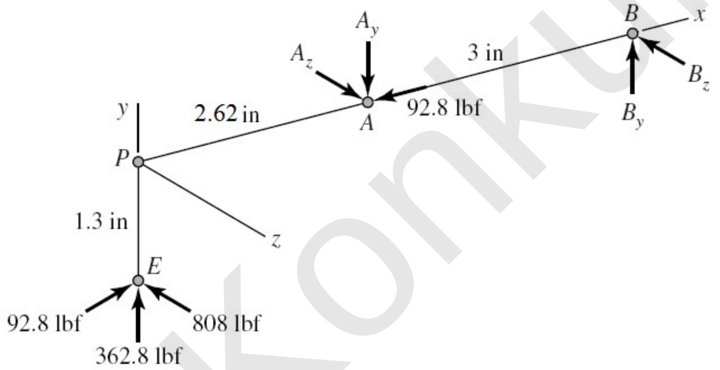
$$\tau_{\max} = \sqrt{\left(\frac{-6769}{2}\right)^2 + 8021^2} = 8706 \text{ psi} = 8.71 \text{ kpsi} \qquad Ans.$$

$$\sigma_1, \sigma_2 = \frac{-6769}{2} \pm \sqrt{\left(\frac{-6769}{2}\right)^2 + 8021^2}$$

$$\sigma_1 = 5321 \text{ psi} = 5.32 \text{ kpsi} \qquad Ans.$$

$$\sigma_2 = -12\,090 \text{ psi} = -12.1 \text{ kpsi} \qquad Ans.$$

3-87



$$\left(\Sigma M_B\right)_z = -5.62(362.8) + 1.3(92.8) + 3A_y = 0$$

$$A_y = 639.4 \text{ lbf} \qquad Ans.$$

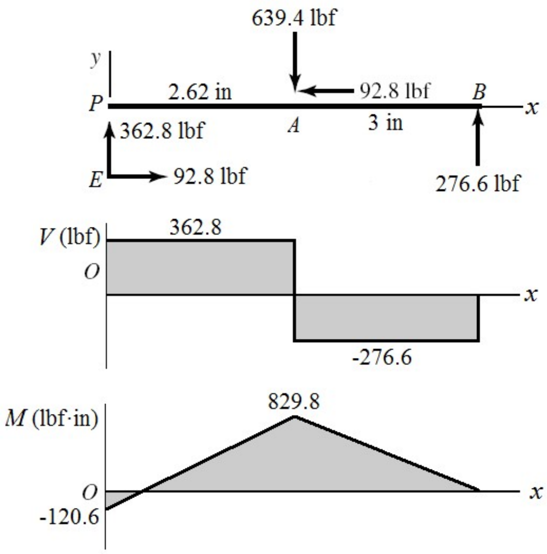
$$\left(\Sigma M_A\right)_z = -2.62(362.8) + 1.3(92.8) + 3B_y = 0$$

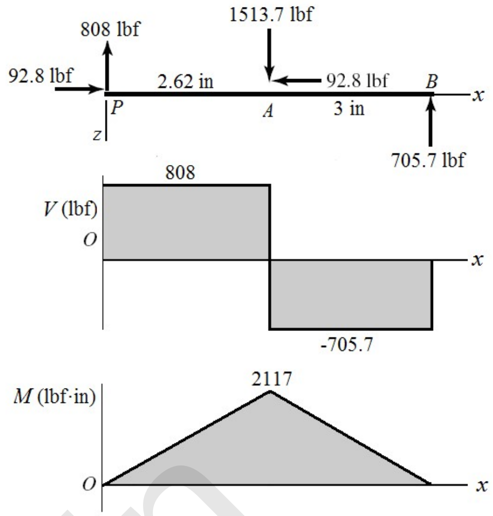
$$B_y = 276.6 \text{ lbf} \qquad Ans.$$

$$\left(\Sigma M_B\right)_y = 0 \quad \Rightarrow \quad A_z = \frac{5.62}{3}(808) = 1513.7 \text{ lbf} \qquad Ans.$$

$$\left(\Sigma M_A\right)_y = 0 \quad \Rightarrow \quad B_z = \frac{2.62}{3}(808) = 705.7 \text{ lbf} \qquad Ans.$$

(b)





(c) The critical stress element is just to the left of *A*, where the bending moment in both planes is a maximum, and where the torsional and axial loads exist.

$$T = 808(1.3) = 1050 \text{ lbf} \cdot \text{in}$$

$$\tau = \frac{16(1050)}{\pi\left(0.88^3\right)} = 7847 \text{ psi} \quad \textit{Ans.}$$

$$M = \sqrt{(829.8)^2 + (2117)^2} = 2274 \text{ lbf} \cdot \text{in}$$

$$\sigma_b = \pm\frac{32M}{\pi d^3} = \pm\frac{32(2274)}{\pi\left(0.88^3\right)} = \pm 33\,990 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_a = -\frac{F}{A} = -\frac{92.8}{(\pi/4)\left(0.88^2\right)} = -153 \text{ psi} \quad \textit{Ans.}$$

(d) The critical stress will occur when the bending stress and axial stress are both in compression.

$$\sigma_{\max} = -33\,990 - 153 = -34\,143 \text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{-34\,143}{2}\right)^2 + 7847^2} = 18\,789 \text{ psi} = 18.8 \text{ kpsi} \quad \textit{Ans.}$$

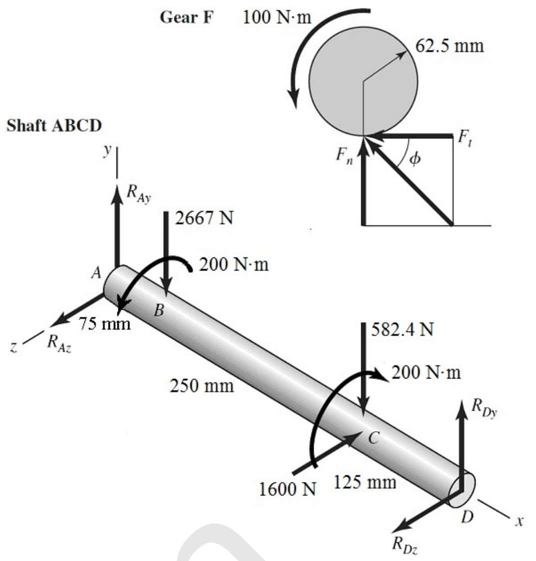
$$\sigma_1, \sigma_2 = \frac{-34143}{2} \pm \sqrt{\left(\frac{-34143}{2}\right)^2 + 7847^2}$$

$$\sigma_1 = 1717 \text{ psi} = 1.72 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_2 = -35\,860 \text{ psi} = -35.9 \text{ kpsi} \quad \textit{Ans.}$$

3-88

$$F_t = \frac{T}{c/2} = \frac{100}{0.125/2} = 1600 \text{ N}$$



$$F_n = 1600 \tan 20 = 582.4 \text{ N} \qquad \sum (M_A)_z = 0$$

$$T_C = F_t(b/2) = 1600(0.250/2) = 200 \text{ N}\cdot\text{m} \quad 450R_{Dy} - 582.4(325) - 2667(75) = 0$$

$$P = \frac{T_C}{(a/2)} = \frac{200}{(0.150/2)} = 2667 \text{ N} \qquad R_{Dy} = 865.1 \text{ N}$$

$$\sum (M_A)_y = 0 = -450R_{Dz} + 1600(325) \quad \Rightarrow \quad R_{Dz} = 1156 \text{ N}$$

$$\sum F_y = 0 = R_{Ay} + 865.1 - 582.4 - 2667 \quad \Rightarrow \quad R_{Ay} = 2384 \text{ N}$$

$$\sum F_z = 0 = R_{Az} + 1156 - 1600 \quad \Rightarrow \quad R_{Az} = 444 \text{ N}$$

AB The maximum bending moment will either be at *B* or *C*. If this is not obvious, sketch the shear and bending moment diagrams. We will directly obtain the combined moments from each plane.

$$M_B = \overline{AB}\sqrt{R_{A_y}^2 + R_{A_z}^2} = 0.075\sqrt{2384^2 + 444^2} = 181.9 \text{ N}\cdot\text{m}$$

$$M_C = \overline{CD}\sqrt{R_{D_y}^2 + R_{D_z}^2} = 0.125\sqrt{865.1^2 + 1156^2} = 180.5 \text{ N}\cdot\text{m}$$

The stresses at *B* and *C* are almost identical, but the maximum stresses occur at *B*. *Ans.*

$$\sigma_B = \frac{32M_B}{\pi d^3} = \frac{32(181.9)}{\pi\left(0.030^3\right)} = 68.6\left(10^6\right) \text{ Pa} = 68.6 \text{ MPa}$$

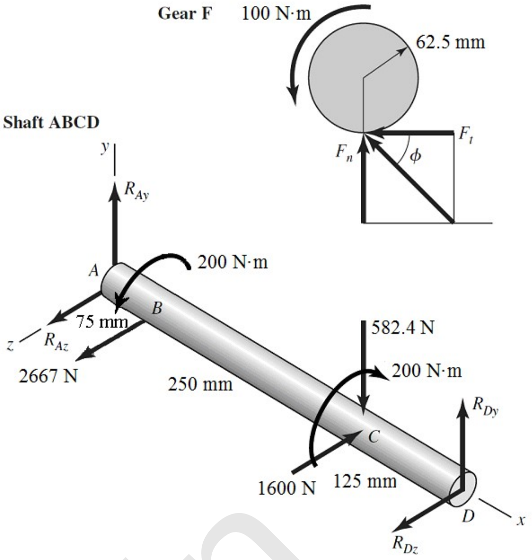
$$\tau_B = \frac{16T_B}{\pi d^3} = \frac{16(200)}{\pi\left(0.030^3\right)} = 37.7\left(10^6\right) \text{ Pa} = 37.7 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_B}{2} + \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \frac{68.6}{2} + \sqrt{\left(\frac{68.6}{2}\right)^2 + 37.7^2} = 85.3 \text{ MPa} \qquad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \sqrt{\left(\frac{68.6}{2}\right)^2 + 37.7^2} = 51.0 \text{ MPa} \qquad \textit{Ans.}$$

3-89

$$F_t = \frac{T}{c/2} = \frac{100}{0.125/2} = 1600 \text{ N}$$



$$F_n = 1600 \tan 20 = 582.4 \text{ N}$$

$$T_C = F_t\left(b/2\right) = 1600\left(0.250/2\right) = 200 \text{ N}\cdot\text{m}$$

$$P = \frac{T_C}{\left(a/2\right)} = \frac{200}{\left(0.150/2\right)} = 2667 \text{ N}$$

$$\sum\left(M_A\right)_z = 0 = 450R_{Dy} - 582.4(325) \quad \Rightarrow \quad R_{Dy} = 420.6 \text{ N}$$

$$\sum\left(M_A\right)_y = 0 = -450R_{Dz} + 1600(325) - 2667(75) \quad \Rightarrow \quad R_{Dz} = 711.1 \text{ N}$$

$$\sum F_y = 0 = R_{Ay} + 420.6 - 582.4 \quad \Rightarrow \quad R_{Ay} = 161.8 \text{ N}$$

$$\sum F_z = 0 = R_{Az} + 711.1 - 1600 + 2667 \quad \Rightarrow \quad R_{Az} = -1778 \text{ N}$$

The maximum bending moment will either be at *B* or *C*. If this is not obvious, sketch shear and bending moment diagrams. We will directly obtain the combined moments from each plane.

$$M_B = \overline{AB}\sqrt{R_{A_y}^2 + R_{A_z}^2} = 0.075\sqrt{161.8^2 + \left(-1778\right)^2} = 133.9 \text{ N}\cdot\text{m}$$

$$M_C = \overline{CD}\sqrt{R_{D_y}^2 + R_{D_z}^2} = 0.125\sqrt{420.6^2 + 711.1^2} = 103.3 \text{ N}\cdot\text{m}$$

The maximum stresses occur at *B*. *Ans.*

$$\sigma_B = \frac{32M_B}{\pi d^3} = \frac{32(133.9)}{\pi\left(0.030^3\right)} = 50.5\left(10^6\right) \text{ Pa} = 50.5 \text{ MPa}$$

$$\tau_B = \frac{16T_B}{\pi d^3} = \frac{16(200)}{\pi\left(0.030^3\right)} = 37.7\left(10^6\right) \text{ Pa} = 37.7 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_B}{2} + \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \frac{50.5}{2} + \sqrt{\left(\frac{50.5}{2}\right)^2 + 37.7^2} = 70.6 \text{ MPa} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \sqrt{\left(\frac{50.5}{2}\right)^2 + 37.7^2} = 45.4 \text{ MPa} \quad \textit{Ans.}$$

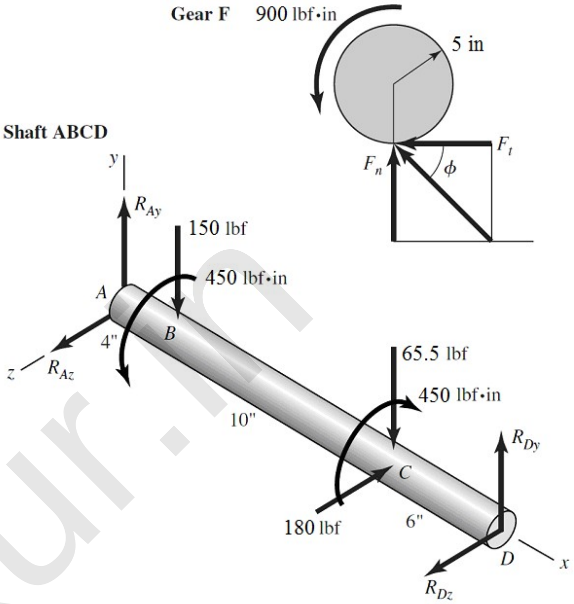
3-90

$$F_t = \frac{T}{c/2} = \frac{900}{10/2} = 180 \text{ lbf}$$

$$F_n = 180 \tan 20 = 65.5 \text{ lbf}$$

$$T_C = F_t\left(b/2\right) = 180\left(5/2\right) = 450 \text{ lbf} \cdot \text{in}$$

$$P = \frac{T_C}{\left(a/2\right)} = \frac{450}{\left(6/2\right)} = 150 \text{ lbf}$$



$$\sum\left(M_A\right)_z = 0 = 20R_{Dy} - 65.5(14) - 150(4) \quad \Rightarrow \quad R_{Dy} = 75.9 \text{ lbf}$$

$$\sum\left(M_A\right)_y = 0 = -20R_{Dz} + 180(14) \quad \Rightarrow \quad R_{Dz} = 126 \text{ lbf}$$

$$\sum F_y = 0 = R_{Ay} + 75.9 - 65.5 - 150 \quad \Rightarrow \quad R_{Ay} = 140 \text{ lbf}$$

$$\sum F_z = 0 = R_{Az} + 126 - 180 \quad \Rightarrow \quad R_{Az} = 54.0 \text{ lbf}$$

The maximum bending moment will either be at *B* or *C*. If this is not obvious, sketch shear and bending moment diagrams. We will directly obtain the combined moments from each plane.

$$M_B = \overline{AB}\sqrt{R_{A_y}^2 + R_{A_z}^2} = 4\sqrt{140^2 + 54^2} = 600 \text{ lbf} \cdot \text{in}$$

$$M_C = \overline{CD}\sqrt{R_{D_y}^2 + R_{D_z}^2} = 6\sqrt{75.9^2 + 126^2} = 883 \text{ lbf} \cdot \text{in}$$

The maximum stresses occur at *C*. *Ans.*

$$\sigma_C = \frac{32M_C}{\pi d^3} = \frac{32(883)}{\pi\left(1.375^3\right)} = 3460 \text{ psi}$$

$$\tau_C = \frac{16T_C}{\pi d^3} = \frac{16(450)}{\pi\left(1.375^3\right)} = 882 \text{ psi}$$

$$\sigma_{\max} = \frac{\sigma_C}{2} + \sqrt{\left(\frac{\sigma_C}{2}\right)^2 + \tau_C^2} = \frac{3460}{2} + \sqrt{\left(\frac{3460}{2}\right)^2 + 882^2} = 3670 \text{ psi} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_C}{2}\right)^2 + \tau_C^2} = \sqrt{\left(\frac{3460}{2}\right)^2 + 882^2} = 1940 \text{ psi} \quad \textit{Ans.}$$

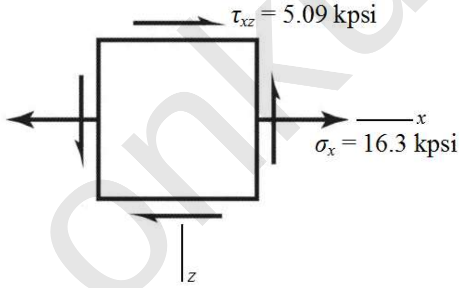
3-91

(a) Rod *AB* experiences constant torsion throughout its length, and maximum bending moment at the wall. Both torsional shear stress and bending stress will be a maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be at the wall, at either the top (compression) or the bottom (tension) on the *y* axis. We will select the bottom element for this analysis.

(b) Transverse shear is zero at the critical stress elements on the top and bottom surfaces.

$$\sigma_x = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(8)(200)}{\pi(1)^3} = 16\,297 \text{ psi} = 16.3 \text{ kpsi}$$

$$\tau_{xz} = \frac{Tr}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3} = \frac{16(5)(200)}{\pi(1)^3} = 5093 \text{ psi} = 5.09 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \frac{16.3}{2} \pm \sqrt{\left(\frac{16.3}{2}\right)^2 + (5.09)^2}$$

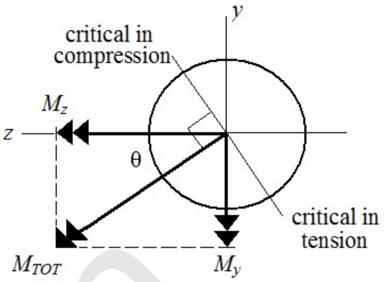
$\sigma_1 = 17.8$ kpsi *Ans.*

$\sigma_2 = -1.46$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \sqrt{\left(\frac{16.3}{2}\right)^2 + (5.09)^2} = 9.61 \text{ kpsi} \quad \textit{Ans.}$$

3-92

(a) Rod *AB* experiences constant torsion throughout its length, and maximum bending moments at the wall in both planes of bending. Both torsional shear stress and bending stress will be a maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface at the wall, with its critical location determined by the plane of the combined bending moments.



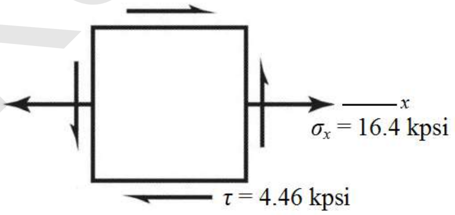
$$M_y = -(100)(8) = -800 \text{ lbf·in}$$
$$M_z = (175)(8) = 1400 \text{ lbf·in}$$
$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$
$$= \sqrt{(-800)^2 + 1400^2} = 1612 \text{ lbf} \cdot \text{in}$$
$$\theta = \tan^{-1}\left(\left|\frac{M_y}{M_z}\right|\right) = \tan^{-1}\left(\frac{800}{1400}\right) = 29.7^{\circ}$$

The combined bending moment vector is at an angle of 29.7º CCW from the *z* axis. The critical bending stress location, and thus the critical stress element, will be ±90º from this vector, as shown. There are two equally critical stress elements, one in tension (119.7º CCW from the *z* axis) and the other in compression (60.3º CW from the *z* axis). We'll continue the analysis with the element in tension.

(b) Transverse shear is zero at the critical stress elements on the outer surfaces.

$$\sigma_x = \frac{M_{\text{tot}} c}{I} = \frac{M_{\text{tot}}(d/2)}{\pi d^4/64} = \frac{32 M_{\text{tot}}}{\pi d^3} = \frac{32(1612)}{\pi(1)^3} = 16\,420 \text{ psi} = 16.4 \text{ kpsi}$$

$$\tau = \frac{Tr}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3} = \frac{16(5)(175)}{\pi(1)^3} = 4456 \text{ psi} = 4.46 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{16.4}{2} \pm \sqrt{\left(\frac{16.4}{2}\right)^2 + (4.46)^2}$$

$\sigma_1 = 17.5$ kpsi *Ans.*

$\sigma_2 = -1.13$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{16.4}{2}\right)^2 + (4.46)^2} = 9.33 \text{ kpsi} \qquad \textit{Ans.}$$

3-93

(a) Rod *AB* experiences constant torsion and constant axial tension throughout its length, and maximum bending moments at the wall from both planes of bending. Both torsional shear stress and bending stress will be maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface at the wall, with its critical location determined by the plane of the combined bending moments.

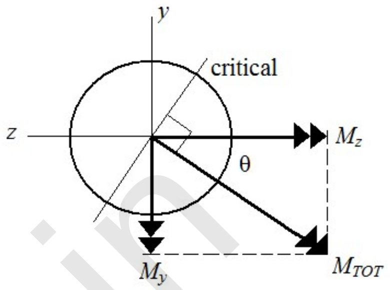
$M_y = -(100)(8) - (75)(5) = -1175$ lbf·in

$M_z = (-200)(8) = -1600$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$

$$= \sqrt{(-1175)^2 + (-1600)^2} = 1985 \text{ lbf} \cdot \text{in}$$

$$\theta = \tan^{-1}\left(\left|\frac{M_y}{M_z}\right|\right) = \tan^{-1}\left(\frac{1175}{1600}\right) = 36.3^{\circ}$$



The combined bending moment vector is at an angle of 36.3° CW from the negative *z* axis. The critical bending stress location will be ±90° from this vector, as shown. Since there is an axial stress in tension, the critical stress element will be where the bending is also in tension. The critical stress element is therefore on the outer surface at the wall, at an angle of 36.3° CW from the *y* axis.

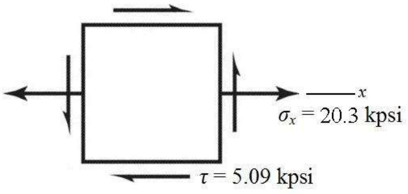
(b) Transverse shear is zero at the critical stress element on the outer surface.

$$\sigma_{x,\text{bend}} = \frac{M_{\text{tot}}c}{I} = \frac{M_{\text{tot}}(d/2)}{\pi d^4/64} = \frac{32M_{\text{tot}}}{\pi d^3} = \frac{32(1985)}{\pi(1)^3} = 20\,220 \text{ psi} = 20.2 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = \frac{F_x}{A} = \frac{F_x}{\pi d^2/4} = \frac{75}{\pi(1)^2/4} = 95.5 \text{ psi} = 0.1 \text{ kpsi}, \text{ which is essentially negligible}$$

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 20\,220 + 95.5 = 20\,316 \text{ psi} = 20.3 \text{ kpsi}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(5)(200)}{\pi(1)^3} = 5093 \text{ psi} = 5.09 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{20.3}{2} \pm \sqrt{\left(\frac{20.3}{2}\right)^2 + (5.09)^2}$$

$\sigma_1 = 21.5$ kpsi *Ans.*

$\sigma_2 = -1.20$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{20.3}{2}\right)^2 + (5.09)^2} = 11.4 \text{ kpsi} \qquad Ans.$$

3-94

$T = (2)(200) = 400$ lbf·in

The maximum shear stress due to torsion occurs in the middle of the longest side of the rectangular cross section. From the table for Eq. (3-40), with $b/c = 1.5/0.25 = 6$, $\alpha = 0.299$. From Eq. (3-40),

$$\tau_{\max} = \frac{T}{\alpha bc^2} = \frac{400}{(0.299)(1.5)(0.25)^2} = 14\,270 \text{ psi} = 14.3 \text{ kpsi} \qquad Ans.$$

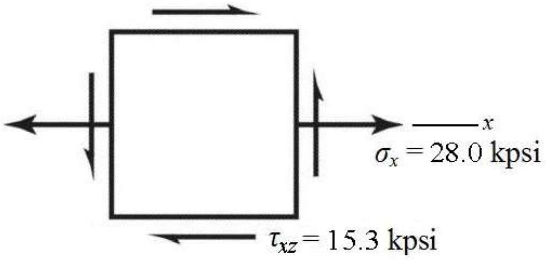
3-95

(a) The cross section at *A* will experience bending, torsion, and transverse shear. Both torsional shear stress and bending stress will be a maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be at either the top (compression) or the bottom (tension) on the *y* axis. We'll select the bottom element for this analysis.

(b) Transverse shear is zero at the critical stress elements on the top and bottom surfaces.

$$\sigma_x = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(11)(250)}{\pi(1)^3} = 28\,011 \text{ psi} = 28.0 \text{ kpsi}$$

$$\tau_{xz} = \frac{Tr}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3} = \frac{16(12)(250)}{\pi(1)^3} = 15\,279 \text{ psi} = 15.3 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2=\frac{\sigma_x}{2}\pm\sqrt{\left(\frac{\sigma_x}{2}\right)^2+\left(\tau_{xz}\right)^2}=\frac{28.0}{2}\pm\sqrt{\left(\frac{28.0}{2}\right)^2+\left(15.3\right)^2}$$

$\sigma_1=34.7$ kpsi *Ans.*

$\sigma_2=-6.7$ kpsi *Ans.*

$$\tau_{\max}=\sqrt{\left(\frac{\sigma_x}{2}\right)^2+\left(\tau_{xz}\right)^2}=\sqrt{\left(\frac{28.0}{2}\right)^2+\left(15.3\right)^2}=20.7\text{ kpsi}\qquad \textit{Ans.}$$

3-96

(a) The cross section at *A* will experience bending, torsion, axial, and transverse shear. Both torsional shear stress and bending stress will be a maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface, with its critical location determined by the plane of the combined bending moments.

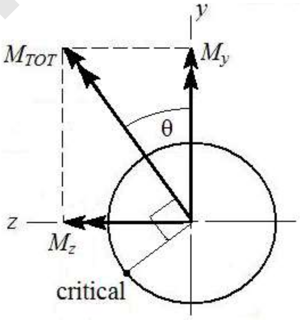
M_y = (300)(12) = 3600 lbf·in

M_z = (250)(11) = 2750 lbf·in

$$M_{\text{tot}}=\sqrt{M_y^2+M_z^2}$$

$$=\sqrt{\left(3600\right)^2+\left(2750\right)^2}=4530\text{ lbf}\cdot\text{in}$$

$$\theta=\tan^{-1}\left(\left|\frac{M_z}{M_y}\right|\right)=\tan^{-1}\left(\frac{2750}{3600}\right)=37.4^{\text{o}}$$



The combined bending moment vector is at an angle of 37.4º CCW from the *y* axis. The critical bending stress location will be 90º CCW from this vector, where the tensile bending stress is additive with the tensile axial stress. The critical stress element is therefore on the outer surface, at an angle of 37.4º CCW from the *z* axis.

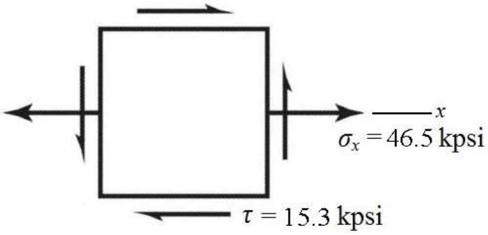
(b)

$$\sigma_{x,\text{bend}}=\frac{M_{\text{tot}}c}{I}=\frac{M_{\text{tot}}\left(d/2\right)}{\pi d^4/64}=\frac{32M_{\text{tot}}}{\pi d^3}=\frac{32\left(4530\right)}{\pi\left(1\right)^3}=46\,142\text{ psi}=46.1\text{ kpsi}$$

$$\sigma_{x,\text{axial}}=\frac{F_x}{A}=\frac{F_x}{\pi d^2/4}=\frac{300}{\pi\left(1\right)^2/4}=382\text{ psi}=0.382\text{ kpsi}$$

$$\sigma_x=\sigma_{x,\text{axial}}+\sigma_{x,\text{bend}}=46\,142+382=46\,524\text{ psi}=46.5\text{ kpsi}$$

$$\tau=\frac{Tr}{J}=\frac{16T}{\pi d^3}=\frac{16\left(12\right)\left(250\right)}{\pi\left(1\right)^3}=15\,279\text{ psi}=15.3\text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{46.5}{2} \pm \sqrt{\left(\frac{46.5}{2}\right)^2 + (15.3)^2}$$

$\sigma_1 = 51.1$ kpsi *Ans.*

$\sigma_2 = -4.58$ kpsi *Ans.*

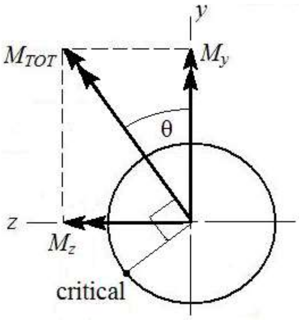
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{46.5}{2}\right)^2 + (15.3)^2} = 27.8 \text{ kpsi} \qquad \textit{Ans.}$$

3-97

(a) The cross section at *A* will experience bending, torsion, axial, and transverse shear. Both torsional shear stress and bending stress will be a maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface, with its critical location determined by the plane of the combined bending moments.

$M_y = (300)(12) - (-100)(11) = 4700$ lbf·in
$M_z = (250)(11) = 2750$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$
$$= \sqrt{(4700)^2 + (2750)^2} = 5445 \text{ lbf} \cdot \text{in}$$
$$\theta = \tan^{-1}\left(\left|\frac{M_z}{M_y}\right|\right) = \tan^{-1}\left(\frac{2750}{4700}\right) = 30.3^\circ$$



The combined bending moment vector is at an angle of 30.3º CCW from the *y* axis. The critical bending stress location will be 90º CCW from this vector, where the tensile bending stress is additive with the tensile axial stress. The critical stress element is therefore on the outer surface, at an angle of 30.3º CCW from the *z* axis.

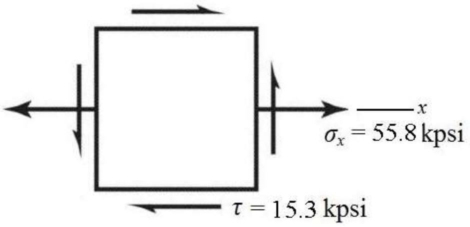
(b)

$$\sigma_{x,\text{bend}} = \frac{M_{\text{tot}}c}{I} = \frac{M_{\text{tot}}(d/2)}{\pi d^4/64} = \frac{32M_{\text{tot}}}{\pi d^3} = \frac{32(5445)}{\pi(1)^3} = 55\,462 \text{ psi} = 55.5 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = \frac{F_x}{A} = \frac{F_x}{\pi d^2/4} = \frac{300}{\pi(1)^2/4} = 382 \text{ psi} = 0.382 \text{ kpsi}$$

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 55\,462 + 382 = 55\,844 \text{ psi} = 55.8 \text{ kpsi}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(12)(250)}{\pi(1)^3} = 15\,279 \text{ psi} = 15.3 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{55.8}{2} \pm \sqrt{\left(\frac{55.8}{2}\right)^2 + (15.3)^2}$$

$\sigma_1 = 59.7$ kpsi *Ans.*

$\sigma_2 = -3.92$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{55.8}{2}\right)^2 + (15.3)^2} = 31.8 \text{ kpsi} \qquad \textit{Ans.}$$

3-98

(a) The cross section at *A* will experience bending, torsion, and transverse shear. Both torsional shear stress and bending stress will be a maximum on the outer surface, where the stress concentration will also be applicable. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be at either the top (compression) or the bottom (tension) on the *y* axis. We'll select the bottom element for this analysis.

(b) Transverse shear is zero at the critical stress elements on the top and bottom surfaces.

$r/d = 0.125/1 = 0.125$

$D/d = 1.5/1 = 1.5$

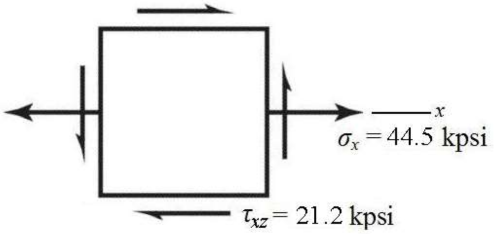
$K_{t,\text{torsion}} = 1.39$ Fig. A-15-8

$K_{t,\text{bend}} = 1.59$ Fig. A-15-9

$$\sigma_x = K_{t,\text{bend}}\frac{Mc}{I} = K_{t,\text{bend}}\frac{32M}{\pi d^3} = (1.59)\frac{32(11)(250)}{\pi(1)^3} = 44\,538 \text{ psi} = 44.5 \text{ kpsi}$$

$$\tau_{xz} = K_{t,\text{torsion}}\frac{Tr}{J} = K_{t,\text{torsion}}\frac{16T}{\pi d^3} = (1.39)\frac{16(12)(250)}{\pi(1)^3} = 21\,238 \text{ psi} = 21.2 \text{ kpsi}$$

(c)



$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xz}\right)^2} = \frac{44.5}{2} \pm \sqrt{\left(\frac{44.5}{2}\right)^2 + (21.2)^2}$$

$\sigma_1 = 53.0$ kpsi *Ans.*

$\sigma_2 = -8.48$ kpsi *Ans.*

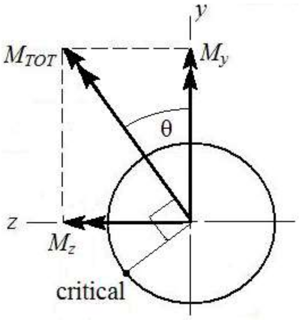
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xz}\right)^2} = \sqrt{\left(\frac{44.5}{2}\right)^2 + (21.2)^2} = 30.7 \text{ kpsi} \qquad \textit{Ans.}$$

3-99

(a) The cross section at *A* will experience bending, torsion, axial, and transverse shear. Both torsional shear stress and bending stress will be a maximum on the outer surface, where the stress concentration will also be applicable. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface, with its critical location determined by the plane of the combined bending moments.

$M_y = (300)(12) = 3600$ lbf·in
$M_z = (250)(11) = 2750$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$
$$= \sqrt{(3600)^2 + (2750)^2} = 4530 \text{ lbf} \cdot \text{in}$$
$$\theta = \tan^{-1}\left(\left|\frac{M_z}{M_y}\right|\right) = \tan^{-1}\left(\frac{2750}{3600}\right) = 37.4^\circ$$



The combined bending moment vector is at an angle of 37.4º CCW from the *y* axis. The critical bending stress location will be 90º CCW from this vector, where the tensile bending stress is additive with the tensile axial stress. The critical stress element is therefore on the outer surface, at an angle of 37.4º CCW from the *z* axis.

(b)

$r / d = 0.125 / 1 = 0.125$

$D / d = 1.5 / 1 = 1.5$

$K_{t,axial} = 1.75$ Fig. A-15-7

$K_{t,\text{torsion}} = 1.39$ Fig. A-15-8

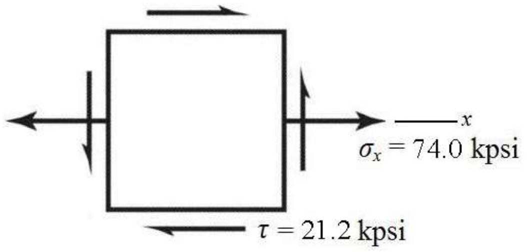
$$K_{t,\text{bend}} = 1.59 \qquad \text{Fig. A-15-9}$$

$$\sigma_{x,\text{bend}} = K_{t,\text{bend}} \frac{Mc}{I} = K_{t,\text{bend}} \frac{32M}{\pi d^3} = (1.59)\frac{32(4530)}{\pi (1)^3} = 73\,366 \text{ psi} = 73.4 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = K_{t,\text{axial}} \frac{F_x}{A} = (1.75)\frac{300}{\pi (1)^2/4} = 668 \text{ psi} = 0.668 \text{ kpsi}$$

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 73\,366 + 668 = 74\,034 \text{ psi} = 74.0 \text{ kpsi}$$

$$\tau = K_{t,\text{torsion}} \frac{Tr}{J} = K_{t,\text{torsion}} \frac{16T}{\pi d^3} = (1.39)\frac{16(12)(250)}{\pi (1)^3} = 21\,238 \text{ psi} = 21.2 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{74.0}{2} \pm \sqrt{\left(\frac{74.0}{2}\right)^2 + (21.2)^2}$$

$\sigma_1 = 79.6$ kpsi *Ans.*

$\sigma_2 = -5.64$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{74.0}{2}\right)^2 + (21.2)^2} = 42.6 \text{ kpsi} \qquad \textit{Ans.}$$

3-100

(a) The cross section at *A* will experience bending, torsion, axial, and transverse shear. Both torsional shear stress and bending stress will be a maximum on the outer surface, where the stress concentration is also applicable. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface, with its critical location determined by the plane of the combined bending moments.

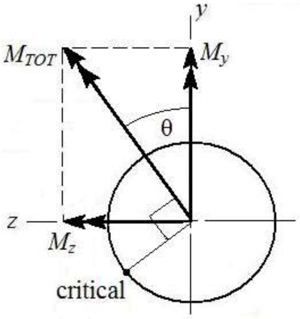
M_y = (300)(12) – (–100)(11) = 4700 lbf·in

M_z = (250)(11) = 2750 lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$

$$= \sqrt{(4700)^2 + (2750)^2} = 5445 \text{ lbf}\cdot\text{in}$$

$$\theta = \tan^{-1}\left(\left|\frac{M_z}{M_y}\right|\right) = \tan^{-1}\left(\frac{2750}{4700}\right) = 30.3^\text{o}$$



The combined bending moment vector is at an angle of 30.3º CCW from the *y* axis. The critical bending stress location will be 90º CCW from this vector, where the tensile bending stress is additive with the tensile axial stress. The critical stress element is therefore on the outer surface, at an angle of 30.3º CCW from the *z* axis.

(b)

$$r/d = 0.125/1 = 0.125$$

$$D/d = 1.5/1 = 1.5$$

$K_{t,axial} = 1.75$ Fig. A-15-7

$K_{t,\text{torsion}} = 1.39$ Fig. A-15-8

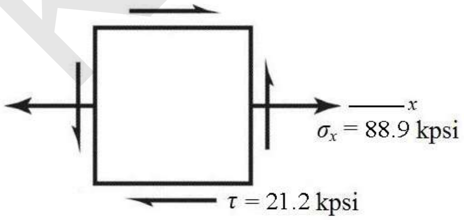
$K_{t,\text{bend}} = 1.59$ Fig. A-15-9

$$\sigma_{x,\text{bend}} = K_{t,\text{bend}}\frac{Mc}{I} = K_{t,\text{bend}}\frac{32M}{\pi d^3} = (1.59)\frac{32(5445)}{\pi(1)^3} = 88\,185 \text{ psi} = 88.2 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = K_{t,\text{axial}}\frac{F_x}{A} = (1.75)\frac{300}{\pi(1)^2/4} = 668 \text{ psi} = 0.668 \text{ kpsi}$$

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 88\,185 + 668 = 88\,853 \text{ psi} = 88.9 \text{ kpsi}$$

$$\tau = K_{t,\text{torsion}}\frac{Tr}{J} = K_{t,\text{torsion}}\frac{16T}{\pi d^3} = (1.39)\frac{16(12)(250)}{\pi(1)^3} = 21\,238 \text{ psi} = 21.2 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{88.9}{2} \pm \sqrt{\left(\frac{88.9}{2}\right)^2 + \left(21.2\right)^2}$$

$\sigma_1 = 93.7$ kpsi *Ans.*

$\sigma_2 = -4.80$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{88.9}{2}\right)^2 + \left(21.2\right)^2} = 49.2 \text{ kpsi} \qquad Ans.$$

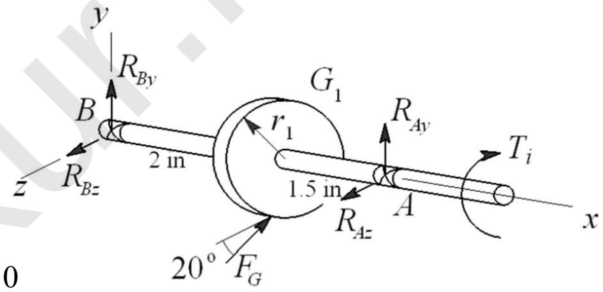
3-101 **(a)** (Eq. 3-42):

$$P = \frac{Tn}{63\ 025} = \frac{200(60)}{63\ 025} = 0.1904 \text{ hp} \qquad Ans.$$

(b) The output torque $T_o = (\omega_i / \omega_o)\ T_i$. But $\omega_i r_1 = \omega_o r_2$. Thus, $T_o = (r_2 / r_1)\ T_i$. So,

$T_o = (2.5/1)\ 200 = 500$ lbf·in *Ans.*

(c)



$\Sigma M_x = 0$, $(F_G \cos 20°)(1) - 200 = 0$

$F_G = 212.84$ lbf

$\Sigma (M_B)_y = 0$, $2(F_G \cos 20°) - 3.5R_{Az} = 0$

$R_{Az} = 2(212.84 \cos 20°)/3.5 = 114.29$ lbf

$\Sigma F_z = 0$, $R_{Bz} + 114.29 - 212.84 \cos 20° = 0$

$R_{Bz} = 85.71$ lbf

$\Sigma (M_B)_z = 0$, $2\ (212.84 \sin 20°) + 3.5\ (F_A)_y = 0$

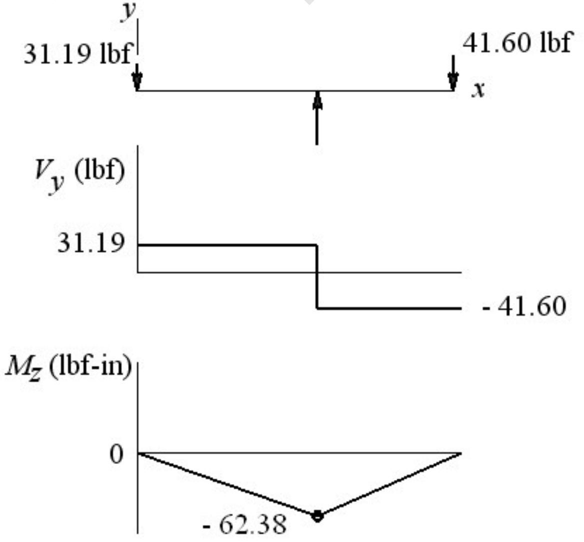
$R_{Ay} = -41.60$ lbf

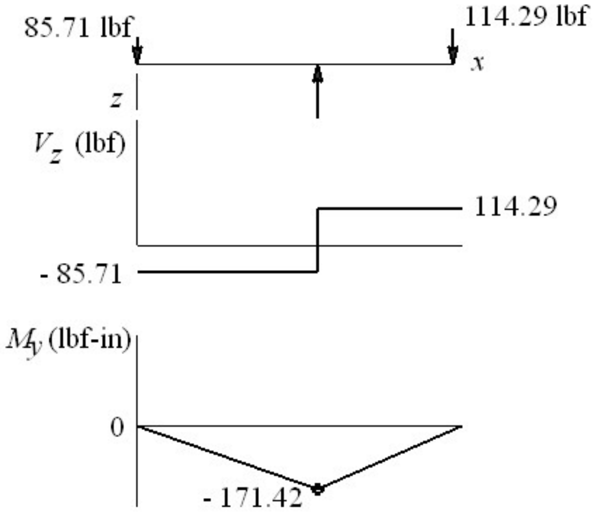
$\Sigma F_y = 0$, $R_{By} - 41.60 + 212.84 \sin 20° = 0 \Rightarrow R_{By} = -31.19$ lbf

$$R_A = \sqrt{R_{Ay}^2 + R_{Az}^2} = \sqrt{\left(-41.60\right)^2 + 114.29^2} = 121.6 \text{ lbf} \qquad Ans.$$

$$R_B = \sqrt{R_{By}^2 + R_{Bz}^2} = \sqrt{\left(-31.19\right)^2 + 85.71^2} = 91.21 \text{ lbf} \qquad Ans.$$

(d)





(e) $(M_y)_{\max} = 2(-85.71) = -171.42$ lbf-in, $(M_z)_{\max} = 2(-31.19) = -62.38$ lbf-in

$$M_{\max} = \sqrt{(-171.42)^2 + (-62.38)^2} = 182.42 \text{ lbf-in}$$

$$\sigma = \frac{32M}{\pi d^3} = \frac{32(182.42)}{\pi(0.5)^3}10^{-3} = 14.87 \text{ kpsi} \qquad Ans.$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(200)}{\pi(0.5)^3}10^{-3} = 8.15 \text{ kpsi} \qquad Ans.$$

(f)

$$\sigma_1, \sigma_2 = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{14.87}{2} \pm \sqrt{\left(\frac{14.87}{2}\right)^2 + 8.15^2}$$
$$= 18.47, -3.60 \quad \text{kpsi} \qquad Ans.$$

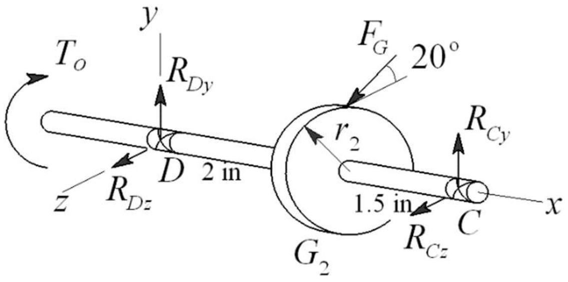
$$\tau_{\max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{14.87}{2}\right)^2 + 8.15^2} = 11.03 \text{ kpsi} \qquad Ans.$$

3-102 **(a)** (Eq. 3-42):

$$P = \frac{Tn}{63\,025} = \frac{200(60)}{63\,025} = 0.1904 \text{ hp} \qquad Ans.$$

(b) The output torque $T_o = (\omega_i / \omega_o)\, T_i$. But $\omega_i r_1 = \omega_o r_2$. Thus, $T_o = (r_2 / r_1)\, T_i$. So,
$T_o = (2.5/1)\, 200 = 500$ lbf·in $\qquad Ans.$

(c)

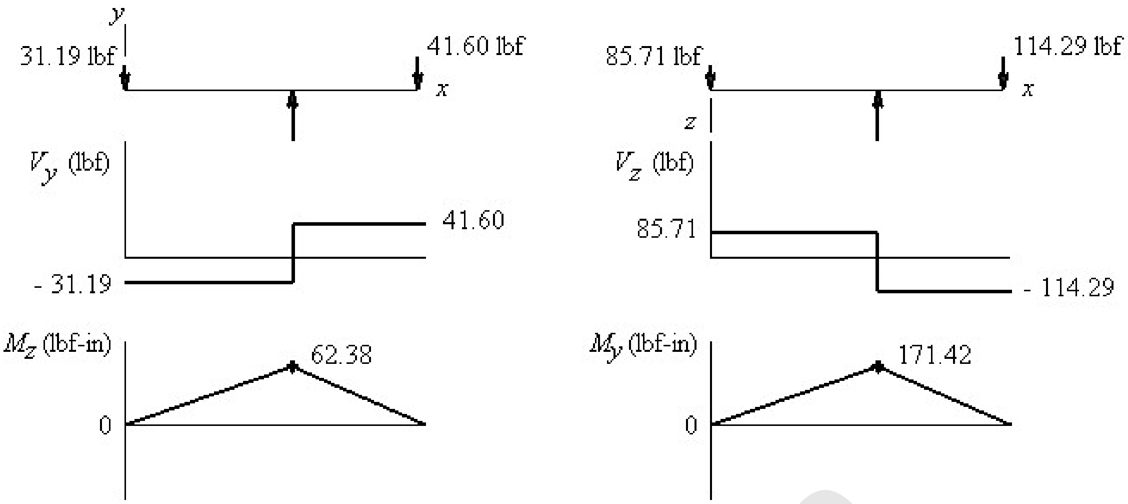


$\Sigma\, M_x = 0 = (F_G \cos 20°)(1) - 200$
$F_G = 212.84$ lbf
$\Sigma\, (M_B)_y = 0 = -2(F_G \cos 20°) - 3.5R_{Cz}$
$R_{Cz} = -2(212.84 \cos 20°)/3.5 = -114.29$ lbf
$\Sigma\, F_z = 0 = R_{Dz} - 114.29 + 212.84 \cos 20°$
$R_{Dz} = -85.71$ lbf
$\Sigma\, (M_B)_z = 0 = -2\,(212.84 \sin 20°) + 3.5\, R_{Cy} \;\Rightarrow\; R_{Cy} = 41.60$ lbf
$\Sigma\, F_y = 0 = R_{Dy} + 41.60 - 212.84 \sin 20° \;\Rightarrow\; R_{Dy} = 31.19$ lbf

$$R_C = \sqrt{R_{Cy}^2 + R_{Cz}^2} = \sqrt{41.60^2 + (-114.29)^2} = 121.6 \text{ lbf} \qquad Ans.$$

$$R_D = \sqrt{R_{Dy}^2 + R_{Dz}^2} = \sqrt{31.19^2 + (-85.71)^2} = 91.21 \text{ lbf} \qquad Ans.$$

(d)



(e) $(M_y)_{\max} = 2(85.71) = 171.42$ lbf·in, $(M_z)_{\max} = 2(31.19) = 62.38$ lbf·in

$$M_{\max} = \sqrt{171.42^2 + 62.38^2} = 182.42 \text{ lbf} \cdot \text{in}$$

$$\sigma = \frac{32M}{\pi d^3} = \frac{32(182.42)}{\pi (0.5)^3} 10^{-3} = 14.87 \text{ kpsi} \qquad Ans.$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(500)}{\pi (0.5)^3} 10^{-3} = 20.37 \text{ kpsi} \qquad Ans.$$

(f)

$$\sigma_1, \sigma_2 = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{14.87}{2} \pm \sqrt{\left(\frac{14.87}{2}\right)^2 + 20.37^2}$$

$$= 29.1, -14.2 \text{ kpsi} \qquad Ans.$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{14.87}{2}\right)^2 + 20.37^2} = 21.7 \text{ kpsi} \qquad Ans.$$

3-103

(a) $M = F(p/4)$, $c = p/4$, $I = bh^3/12$, $b = \pi d_r n_t$, $h = p/2$

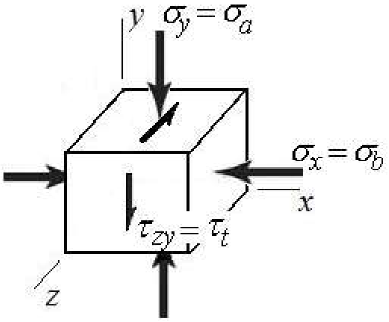
$$\sigma_b = \pm \frac{Mc}{I} = \pm \frac{\left[F(p/4)\right](p/4)}{bh^3/12} = \pm \frac{Fp^2}{16(\pi d_r n_t)(p/2)^3/12}$$

$$\sigma_b = \pm \frac{6F}{\pi d_r n_t p} \qquad Ans.$$

(b) $\sigma_a = -\dfrac{F}{A} = -\dfrac{F}{\pi d_r^2/4} = -\dfrac{4F}{\pi d_r^2} \qquad Ans.$

$$\tau_t = \frac{Tr}{J} = \frac{T(d_r/2)}{\pi d_r^4/32} = \frac{16T}{\pi d_r^3} \qquad Ans.$$

(c) The bending stress causes compression in the *x* direction. The axial stress causes compression in the *y* direction. The torsional stress shears across the *y* face in the negative *z* direction.



(d) Analyze the stress element from part (c) using the equations developed in parts (a) and (b).

$$d_r = d - p = 1.5 - 0.25 = 1.25 \text{ in}$$

$$\sigma_x = \sigma_b = -\frac{6F}{\pi d_r n_t p} = -\frac{6(1500)}{\pi(1.25)(2)(0.25)} = -4584 \text{ psi} = -4.584 \text{ kpsi}$$

$$\sigma_y = \sigma_a = -\frac{4F}{\pi d_r^2} = -\frac{4(1500)}{\pi(1.25^2)} = -1222 \text{ psi} = -1.222 \text{ kpsi}$$

$$\tau_{yz} = -\tau_t = -\frac{16T}{\pi d_r^3} = -\frac{16(235)}{\pi(1.25^3)} = -612.8 \text{ psi} = -0.6128 \text{ kpsi}$$

Use Eq. (3-15) for the three-dimensional stress element.

$$\sigma^3 - (-4.584 - 1.222)\sigma^2 + \left[(-4.584)(-1.222) - (-0.6128)^2\right]\sigma - \left[-(-4.584)(-0.6128)^2\right] = 0$$

$$\sigma^3 + 5.806\sigma^2 + 5.226\sigma - 1.721 = 0$$

The roots are at 0.2543, – 4.584, and –1.476. Thus, the ordered principal stresses are

σ_1 = 0.2543 kpsi, σ_2 = –1.476 kpsi, and σ_3 = – 4.584 kpsi. *Ans.*

From Eq. (3-16), the principal shear stresses are

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2} = \frac{0.2543 - (-1.476)}{2} = 0.8652 \text{ kpsi} \qquad Ans.$$

$$\tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} = \frac{(-1.476) - (-4.584)}{2} = 1.554 \text{ kpsi} \qquad Ans.$$

$$\tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2} = \frac{0.2543 - (-4.584)}{2} = 2.419 \text{ kpsi} \qquad Ans.$$

3-104 As shown in Fig. 3-32, the maximum stresses occur at the inside fiber where $r = r_i$. Therefore, from Eq. (3-50)

$$\sigma_{t,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right)$$

$$= p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) \quad Ans.$$

$$\sigma_{r,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right) = -p_i \quad Ans.$$

3-105 If $p_i = 0$, Eq. (3-49) becomes

$$\sigma_t = \frac{-p_o r_o^2 - r_i^2 r_o^2 p_o / r^2}{r_o^2 - r_i^2}$$

$$= -\frac{p_o r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

The maximum tangential stress occurs at $r = r_i$. So

$$\sigma_{t,\max} = -\frac{2 p_o r_o^2}{r_o^2 - r_i^2} \quad Ans.$$

For σ_r, we have

$$\sigma_r = \frac{-p_o r_o^2 - r_i^2 r_o^2 p_o / r^2}{r_o^2 - r_i^2}$$

$$= \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2} - 1\right)$$

So $\sigma_r = 0$ at $r = r_i$. Thus at $r = r_o$

$$\sigma_{r,\max} = \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2 - r_o^2}{r_o^2}\right) = -p_o \quad Ans.$$

3-106 The force due to the pressure on half of the sphere is resisted by the stress that is distributed around the center plane of the sphere. All planes are the same, so

$$(\sigma_t)_{\text{av}} = \sigma_1 = \sigma_2 = \frac{p(\pi/4)d_i^2}{\pi d_i t} = \frac{p d_i}{4t} \quad Ans.$$

The radial stress on the inner surface of the shell is, $\sigma_3 = -p$ *Ans.*

3-107 $\sigma_t > \sigma_l > \sigma_r$

$\tau_{\max} = (\sigma_t - \sigma_r)/2$ at $r = r_i$

$$\tau_{\max} = \frac{1}{2}\left[\frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) - \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right)\right] = \frac{r_o^2 p_i}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad p_i = \frac{r_o^2 - r_i^2}{r_o^2}\tau_{\max} = \frac{3^2 - 2.75^2}{3^2}(10\ 000) = 1597 \text{ psi} \quad \textit{Ans.}$$

3-108 $\sigma_t > \sigma_l > \sigma_r$

$\tau_{\max} = (\sigma_t - \sigma_r)/2$ at $r = r_i$

$$\tau_{\max} = \frac{1}{2}\left[\frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) - \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right)\right] = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(\frac{r_o^2}{r_i^2}\right) = \frac{r_o^2 p_i}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad r_i = r_o\sqrt{\frac{(\tau_{\max} - p_i)}{\tau_{\max}}} = 100\sqrt{\frac{(25-4)10^6}{25\left(10^6\right)}} = 91.7 \text{ mm}$$

$$t = r_o - r_i = 100 - 91.7 = 8.3 \text{ mm} \quad \textit{Ans.}$$

3-109 $\sigma_t > \sigma_l > \sigma_r$

$\tau_{\max} = (\sigma_t - \sigma_r)/2$ at $r = r_i$

$$\tau_{\max} = \frac{1}{2}\left[\frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) - \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right)\right] = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(\frac{r_o^2}{r_i^2}\right) = \frac{r_o^2 p_i}{r_o^2 - r_i^2}$$

$$= \frac{4^2(500)}{4^2 - 3.75^2} = 4129 \text{ psi} \quad \textit{Ans.}$$

3-110 From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

$$\sigma_r = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right)$$

$\sigma_t > \sigma_l > \sigma_r$, and since σ_t and σ_r are negative,

$\tau_{\max} = (\sigma_r - \sigma_t)/2$ at $r = r_o$

$$\tau_{\max} = \frac{1}{2}\left[-\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r_o^2}\right) + \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r_o^2}\right)\right] = \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r_o^2}\right) = \frac{r_i^2 p_o}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad p_o = \frac{r_o^2 - r_i^2}{r_i^2}\tau_{\max} = \frac{3^2 - 2.75^2}{2.75^2}(10\ 000) = 1900 \text{ psi} \quad \textit{Ans.}$$

3-111 From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

$$\sigma_r = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right)$$

$\sigma_t > \sigma_l > \sigma_r$, and since σ_t and σ_r are negative,
$\tau_{\max} = (\sigma_r - \sigma_t)/2$ at $r = r_o$

$$\tau_{\max} = \frac{1}{2}\left[-\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r_o^2}\right) + \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r_o^2}\right)\right] = \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r_o^2}\right) = \frac{r_i^2 p_o}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad r_i = r_o\sqrt{\frac{\tau_{\max}}{(\tau_{\max} + p_o)}} = 100\sqrt{\frac{25\left(10^6\right)}{(25+4)10^6}} = 92.8 \text{ mm}$$

$$t = r_o - r_i = 100 - 92.8 = 7.2 \text{ mm} \quad \textit{Ans.}$$

3-112 From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

$$\sigma_r = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right)$$

$\sigma_t > \sigma_l > \sigma_r$, and since σ_t and σ_r are negative,
$\tau_{\max} = (\sigma_r - \sigma_t)/2$ at $r = r_o$

$$\tau_{\max} = \frac{1}{2}\left[-\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r_o^2}\right) + \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r_o^2}\right)\right] = \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r_o^2}\right) = \frac{r_i^2 p_o}{r_o^2 - r_i^2}$$

$$= \frac{3.75^2(500)}{4^2 - 3.75^2} = 3629 \text{ psi} \quad \textit{Ans.}$$

3-113 From Table A-20, S_y=490 MPa
From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

Maximum will occur at $r = r_i$

$$\sigma_{t,\max} = -\frac{2r_o^2 p_o}{r_o^2 - r_i^2} \Rightarrow p_o = -\frac{\sigma_{t,\max}(r_o^2 - r_i^2)}{2r_o^2} = -\frac{\left[0.8(-490)\right]\left(25^2 - 19^2\right)}{2(25^2)} = 82.8 \text{ MPa} \quad Ans.$$

3-114 From Table A-20, $S_y = 71$ kpsi
From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

Maximum will occur at $r = r_i$

$$\sigma_{t,\max} = -\frac{2r_o^2 p_o}{r_o^2 - r_i^2} \Rightarrow p_o = -\frac{\sigma_{t,\max}\left(r_o^2 - r_i^2\right)}{2r_o^2} = -\frac{\left[0.8(-71)\right]\left(1^2 - 0.75^2\right)}{2(1^2)} = 12.4 \text{ kpsi} \quad Ans.$$

3-115 From Table A-20, S_y=490 MPa
From Eq. (3-50)

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

Maximum will occur at $r = r_i$

$$\sigma_{t,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) = \frac{p_i\left(r_o^2 + r_i^2\right)}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad p_i = \frac{\sigma_{t,\max}(r_o^2 - r_i^2)}{r_o^2 + r_i^2} = \frac{\left[0.8(490)\right](25^2 - 19^2)}{(25^2 + 19^2)} = 105 \text{ MPa} \quad Ans.$$

3-116 From Table A-20, S_y =71 MPa
From Eq. (3-50)

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

Maximum will occur at $r = r_i$

$$\sigma_{t,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) = \frac{p_i(r_o^2 + r_i^2)}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad p_i = \frac{\sigma_{t,\max}(r_o^2 - r_i^2)}{r_o^2 + r_i^2} = \frac{\left[0.8(71)\right](1^2 - 0.75^2)}{(1^2 + 0.75^2)} = 15.9 \text{ ksi} \quad Ans.$$

3-117 The longitudinal stress will be due to the weight of the vessel above the maximum stress point. From Table A-5, the unit weight of steel is $\gamma_s = 0.282$ lbf/in^3. The area of the wall is

$$A_{\text{wall}} = (\pi/4)(360^2 - 358.5^2) = 846.\ 5\ \text{in}^2$$

The volume of the wall and dome are

$$V_{\text{wall}} = A_{\text{wall}}\, h = 846.5\ (720) = 609.5\ (10^3)\ \text{in}^3$$

$$V_{\text{dome}} = (2\pi/3)(180^3 - 179.25^3) = 152.0\ (10^3)\ \text{in}^3$$

The weight of the structure on the wall area at the tank bottom is

$$W = \gamma_s\, V_{\text{total}} = 0.282(609.5 + 152.0)\ (10^3) = 214.7(10^3)\ \text{lbf}$$

$$\sigma_l = -\frac{W}{A_{\text{wall}}} = -\frac{214.7\left(10^3\right)}{846.5} = -254\ \text{psi}$$

The maximum pressure will occur at the bottom of the tank, $p_i = \gamma_{\text{water}}\, h$. From Eq. (3-50) with $r = r_i$

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) = p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right)$$

$$= \left[62.4(55)\left(\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right)\right]\left(\frac{180^2 + 179.25^2}{180^2 - 179.25^2}\right) = 5708 \ \square\ 5710\ \text{psi} \quad \textit{Ans.}$$

$$\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right) = -p_i = -62.4(55)\left(\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right) = -23.8\ \text{psi} \qquad \textit{Ans.}$$

Note: These stresses are very idealized as the floor of the tank will restrict the values calculated.

Since $\sigma_1 \geq \sigma_2 \geq \sigma_3$, $\sigma_1 = \sigma_t = 5708$ psi, $\sigma_2 = \sigma_r = -24$ psi and$\sigma_3 = \sigma_l = -254$ psi. From Eq. (3-16),

$$\tau_{1/3} = \frac{5708 + 254}{2} = 2981 \approx 2980\ \text{psi}$$

$$\tau_{1/2} = \frac{5708 + 24}{2} = 2866 \approx 2870\ \text{psi} \qquad \textit{Ans.}$$

$$\tau_{2/3} = \frac{-24 + 254}{2} = 115\ \text{psi}$$

3-118 Stresses from additional pressure are,
Eq. (3-52),

$$\left(\sigma_l\right)_{50\text{psi}}=\frac{50\left(179.25^2\right)}{180^2-179.25^2}=5963\text{ psi}$$

$$(\sigma_r)_{50\text{ psi}} = -\ 50\text{ psi}$$

Eq. (3-50)

$$\left(\sigma_t\right)_{50\text{psi}}=50\frac{180^2+179.25^2}{180^2-179.25^2}=11\,975\text{ psi}$$

Adding these to the stresses found in Prob. 3-117 gives

σ_t = 5708 + 11 975 = 17683 psi = 17.7 kpsi *Ans.*
σ_r = – 23.8 – 50 = – 73.8 psi *Ans.*
σ_l = – 254 + 5963 = 5709 psi *Ans.*

Note: These stresses are very idealized as the floor of the tank will restrict the values calculated.

From Eq. (3-16)

$$\tau_{1/3}=\frac{17\,683+73.8}{2}=8879\text{ psi}$$

$$\tau_{1/2}=\frac{17\,683-5709}{2}=5987\text{ psi} \qquad \textit{Ans.}$$

$$\tau_{2/3}=\frac{5709+23.8}{2}=2866\text{ psi}$$

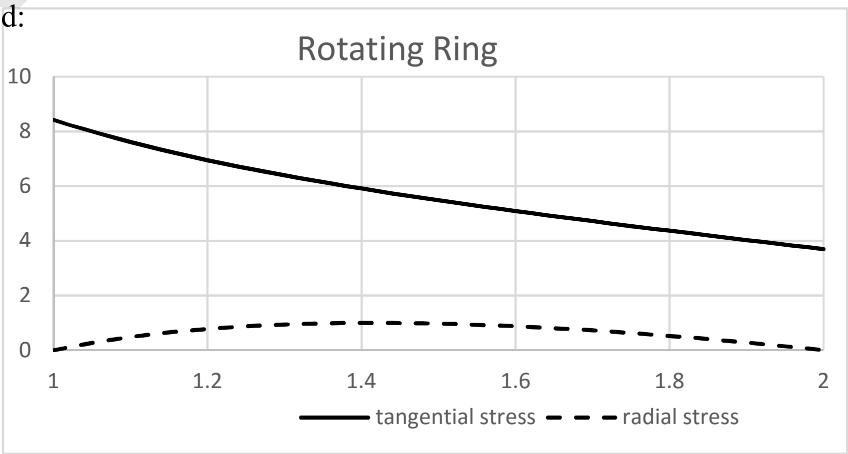
3-119 **(a)** For shapes let $\rho\omega(3+\nu)/8=1$, $\nu=0.3$, $r_o=2$, and $r_i=1$. Then,

$$\frac{1+3\nu}{3+\nu}=\frac{1+3(0.3)}{3+0.3}=0.5758$$

Thus, Eqs. (3-55) are

$$\sigma_t=1+4+\frac{4}{r^2}-0.5758r^2,\qquad \sigma_r=1+4-\frac{4}{r^2}-r^2$$

These equations are plotted:



(b) The tangential stress, σ_t is maximum at $r=r_i$ and is given by

$$(\sigma_t)_{\max} = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r_i^2} - \frac{1+3\nu}{3+\nu}r_i^2\right)$$

$$= \rho\omega^2\left(\frac{3+\nu}{8}\right)\left\{\left[\frac{3+\nu-(1+3\nu)}{3+\nu}\right]r_i^2 + 2r_o^2\right\}$$

$$= \frac{\rho\omega^2}{4}\left[(1-\nu)r_i^2 + (3+\nu)r_o^2\right] \qquad \textit{Ans.}$$

$\sigma_r = 0$ at $r = r_i$ and $r = r_o$. $(\sigma_r)_{\max}$ occurs where $d\sigma_r / dr = 0$.

$$\frac{d\sigma_r}{dr} = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(\frac{2r_i^2 r_o^2}{r^3} - 2r\right) = 0 \;\Rightarrow\; r^4 = r_i^2 r_o^2 \;\Rightarrow\; r = \sqrt{r_i r_o}$$

Thus,

$$(\sigma_r)_{\max} = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r_i r_o} - r_i r_o\right) = \rho\omega^2\left(\frac{3+\nu}{8}\right)(r_o - r_i)^2 \qquad \textit{Ans.}$$

3-120 Since σ_t and σ_r are both positive and $\sigma_t > \sigma_r$

$$\tau_{\max} = (\sigma_t)_{\max}/2$$

From Eq. (3-55), σ_t is maximum at $r = r_i = 0.3125$ in. The term

$$\rho\omega^2\left(\frac{3+\nu}{8}\right) = \frac{0.282}{386}\left[\frac{2\pi(5000)}{60}\right]^2\left(\frac{3+0.292}{8}\right) = 82.42 \text{ lbf/in}$$

$$(\sigma_t)_{\max} = 82.42\left[0.3125^2 + 2.75^2 + \frac{(0.3125^2)(2.75^2)}{0.3125^2} - \frac{1+3(0.292)}{3+0.292}(0.3125^2)\right]$$

$$= 1260 \text{ psi}$$

$$\tau_{\max} = \frac{1260}{2} = 630 \text{ psi} \qquad \textit{Ans.}$$

Radial stress:

$$\sigma_r = k\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)$$

Maxima:

$$\frac{d\sigma_r}{dr} = k\left(2\frac{r_i^2 r_o^2}{r^3} - 2r\right) = 0 \quad\Rightarrow\quad r = \sqrt{r_i r_o} = \sqrt{0.3125(2.75)} = 0.927 \text{ in}$$

$$\left(\sigma_r\right)_{\max} = 82.42\left[0.3125^2 + 2.75^2 - \frac{0.3125^2\left(2.75^2\right)}{0.927^2} - 0.927^2\right]$$
$$= 490 \text{ psi} \quad \textit{Ans.}$$

3-121 $\omega = 2\pi(2000)/60 = 209.4$ rad/s, $\rho = 3320\ 20$ kg/m^3, $\nu = 0.24$, $r_i = 0.01$ m, $r_o = 0.125$ m

Using Eq. (3-55)

$$\sigma_t = 3320(209.4)^2\left(\frac{3+0.24}{8}\right)\left[\left(0.01\right)^2 + (0.125)^2 + (0.125)^2 - \frac{1+3(0.24)}{3+0.24}\left(0.01\right)^2\right](10)^{-6}$$
$$= 1.85 \text{ MPa} \quad \textit{Ans.}$$

3-122 Eq (3-55):

$$\sigma_t = K\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{r^2}{2}\right) \quad (1), \qquad \text{and} \qquad \sigma_r = K\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right) \quad (2)$$

Where $K = \rho\omega^2\,(3 + \nu)/8$ and $(1 + 3\nu)/(3 + \nu) = [1 + 3(0.2)] / (3 + 0.2) = 1/2$.
It can be seen that σ_t is always positive. Check for maxima.

$$\frac{d\sigma_t}{dr} = K\left(-2\frac{r_i^2 r_o^2}{r^3} - r\right) = 0 \;\Rightarrow\; r^4 = -2r_i^2 r_o^2$$ No roots, no maxima. Check at extremes.

At $r = r_i$, $$\left(\sigma_t\right)_i = K\left(r_i^2 + r_o^2 + r_o^2 - \frac{r_i^2}{2}\right) = K\left(2r_o^2 + \frac{r_i^2}{2}\right) \qquad (3)$$

At $r = r_o$, $$\left(\sigma_t\right)_i = K\left(r_i^2 + r_o^2 + r_i^2 - \frac{r_o^2}{2}\right) = K\left(2r_i^2 - \frac{r_o^2}{2}\right) \qquad (4)$$

Eq. (3) > (4). Thus, $(\sigma_t)_{\max} = (\sigma_t)_i$.
For Eq. (2), $\sigma_r = 0$ at $r = r_i$ and $r = r_o$. Check for maxima.

$$\frac{d\sigma_r}{dr} = K\left(2\frac{r_i^2 r_o^2}{r^3} - 2r\right) = 0 \;\Rightarrow\; r = \sqrt{r_i r_o} \qquad (5)$$

Substitute Eq. (5) into (2),

$$\left(\sigma_r\right)_{\max} = K\left(r_i^2 + r_o^2 - r_i r_o - r_i r_o\right) = K\left(r_o^2 - 2r_i r_o + r_i^2\right) = K\left(r_o - r_i\right)^2 \qquad (6)$$

Comparing Eqs. (3) and (6) it is clear that $K\left(2r_o^2 + \dfrac{r_i^2}{2}\right) > K\left(r_o^2 + r_i^2 - 2r_i r_o\right)$. Thus, the maximum stress is given by Eq. (3) simplified as, $\sigma_{\max} = \rho\omega^2\left(\dfrac{3+\nu}{16}\right)\left(4r_o^2 + r_i^2\right)$ *Ans.*

(b) Vol = $(\pi/4)(5^2 - 0.75^2)(1/16) = 1.1996$ in^3. $\omega = 2\pi(12\ 000)/60 = 1256.6$ rad/s.

$$\rho = \frac{5/16}{386\left(1.1996\right)} = 6.749\left(10^{-4}\right) \text{ lbf-s}^2/\text{in}^4$$

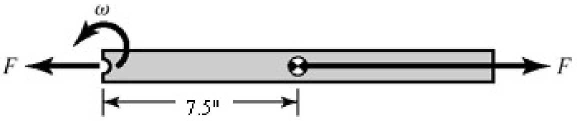
Eq. (3): $$(\sigma_t)_{\max} = 6.749\left(10^{-4}\right)1256.6^2\left(\frac{3+0.2}{8}\right)\left[2\left(2.5\right)^2+\frac{0.375^2}{2}\right] = 5\ 360\ \ \text{psi}\quad Ans.$$

The factor of safety corresponding to fracture is:

$$n = \frac{S_u}{\sigma_{\max}} = \frac{12}{5.36} = 2.24 \qquad Ans.$$

3-123 $\omega = 2\pi(3500)/60 = 366.5$ rad/s,
mass of blade = $m = \rho V = (0.282 / 386)\,[1.25(30)(0.125)] = 3.425(10^{-3})$ lbf·s²/in

$F = (m/2)\,\omega^2 r$
$= [3.425(10^{-3})/2](366.5^2)(7.5)$
$= 1725$ lbf



$A_{\text{nom}} = (1.25 - 0.5)(1/8) = 0.093\ 75\ \text{in}^2$

$\sigma_{\text{nom}} = F/A_{\text{nom}} = 1725/0.093\ 75 = 18\ 400$ psi *Ans.*

Note: Stress concentration Fig. A-15-1 gives $K_t = 2.25$ which increases $\sigma_{\max}$ and fatigue.

3-124 $\nu = 0.292$, $E = 207$ GPa, $r_i = 0$, $R = 25$ mm, $r_o = 50$ mm
Eq. (3-57),

$$p = \frac{207(10^9)\delta}{2(0.025)^3}\left[\frac{(0.05^2-0.025^2)(0.025^2-0)}{(0.05^2-0)}\right]\left(10^{-9}\right) = 3.105(10^3)\delta \qquad (1)$$

where p is in MPa and δ is in mm.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[50.042-50.000] = 0.021\ \text{mm} \qquad Ans.$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[50.026-50.025] = 0.0005\ \text{mm} \qquad Ans.$$

From Eq. (1)

$p_{\max} = 3.105(10^3)(0.021) = 65.2$ MPa *Ans.*

$p_{\min} = 3.105(10^3)(0.0005) = 1.55$ MPa *Ans.*

3-125 $\nu = 0.292$, $E = 30$ Mpsi, $r_i = 0$, $R = 1$ in, $r_o = 2$ in
Eq. (3-57),

$$p = \frac{30(10^6)\delta}{2(1^3)}\left[\frac{(2^2-1^2)(1^2-0)}{(2^2-0)}\right] = 1.125(10^7)\delta \qquad (1)$$

where p is in psi and δ is in inches.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[2.0016 - 2.0000] = 0.0008 \text{ in} \quad \textit{Ans.}$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[2.0010 - 2.0010] = 0 \quad \textit{Ans.}$$

From Eq. (1),

$$p_{\max} = 1.125(10^7)(0.0008) = 9\,000 \text{ psi} \quad \textit{Ans.}$$

$$p_{\min} = 1.125(10^7)(0) = 0 \quad \textit{Ans.}$$

3-126 $\nu = 0.292$, $E = 207$ GPa, $r_i = 0$, $R = 25$ mm, $r_o = 50$ mm

Eq. (3-57),

$$p = \frac{207(10^9)\delta}{2(0.025)^3}\left[\frac{(0.05^2 - 0.025^2)(0.025^2 - 0)}{(0.05^2 - 0)}\right](10^{-9}) = 3.105(10^3)\delta \qquad (1)$$

where p is in MPa and δ is in mm.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[50.059 - 50.000] = 0.0295 \text{ mm} \quad \textit{Ans.}$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[50.043 - 50.025] = 0.009 \text{ mm} \quad \textit{Ans.}$$

From Eq. (1)

$$p_{\max} = 3.105(10^3)(0.0295) = 91.6 \text{ MPa} \quad \textit{Ans.}$$

$$p_{\min} = 3.105(10^3)(0.009) = 27.9 \text{ MPa} \quad \textit{Ans.}$$

3-127 $\nu = 0.292$, $E = 30$ Mpsi, $r_i = 0$, $R = 1$ in, $r_o = 2$ in

Eq. (3-57),

$$p = \frac{30(10^6)\delta}{2(1^3)}\left[\frac{(2^2 - 1^2)(1^2 - 0)}{(2^2 - 0)}\right] = 1.125(10^7)\delta \qquad (1)$$

where p is in psi and δ is in inches.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[2.0023 - 2.0000] = 0.00115 \text{ in} \quad \textit{Ans.}$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[2.0017 - 2.0010] = 0.00035 \qquad Ans.$$

From Eq. (1),

$$p_{\max} = 1.125(10^7)(0.00115) = 12\,940 \text{ psi} \qquad Ans.$$

$$p_{\min} = 1.125(10^7)(0.00035) = 3\,938 \qquad Ans.$$

3-128 $\nu = 0.292$, $E = 207$ GPa, $r_i = 0$, $R = 25$ mm, $r_o = 50$ mm

Eq. (3-57),

$$p = \frac{207(10^9)\delta}{2(0.025)^3}\left[\frac{(0.05^2 - 0.025^2)(0.025^2 - 0)}{(0.05^2 - 0)}\right]\left(10^{-9}\right) = 3.105(10^3)\delta \qquad (1)$$

where p is in MPa and δ is in mm.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[50.086 - 50.000] = 0.043 \text{ mm} \qquad Ans.$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[50.070 - 50.025] = 0.0225 \text{ mm} \qquad Ans.$$

From Eq. (1)

$$p_{\max} = 3.105(10^3)(0.043) = 134 \text{ MPa} \qquad Ans.$$

$$p_{\min} = 3.105(10^3)(0.0225) = 69.9 \text{ MPa} \qquad Ans.$$

3-129 $\nu = 0.292$, $E = 30$ Mpsi, $r_i = 0$, $R = 1$ in, $r_o = 2$ in

Eq. (3-57),

$$p = \frac{30(10^6)\delta}{2(1^3)}\left[\frac{(2^2 - 1^2)(1^2 - 0)}{(2^2 - 0)}\right] = 1.125(10^7)\delta \qquad (1)$$

where p is in psi and δ is in inches.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[2.0034 - 2.0000] = 0.0017 \text{ in} \qquad Ans.$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[2.0028 - 2.0010] = 0.0009 \qquad Ans.$$

From Eq. (1),

$$p_{\max} = 1.125(10^7)(0.0017) = 19\ 130 \text{ psi} \qquad Ans.$$

$$p_{\min} = 1.125(10^7)(0.0009) = 10\ 130 \qquad Ans.$$

3-130 From Table A-5, $E_i = E_o = 30$ Mpsi, $\nu_i = \nu_o = 0.292$. $r_i = 0$, $R = 1$ in, $r_o = 1.5$ in

The radial interference is $\delta = \frac{1}{2}(2.002 - 2.000) = 0.001 \text{ in}$ *Ans.*

Eq. (3-57),

$$p = \frac{E\delta}{2R^3}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2}\right] = \frac{30(10^6)0.001}{2(1^3)}\left[\frac{(1.5^2 - 1^2)(1^2 - 0)}{(1.5^2 - 0)}\right]$$

$$= 8333 \text{ psi} = 8.33 \text{ kpsi} \qquad Ans.$$

The tangential stresses at the interface for the inner and outer members are given by Eqs. (3-58) and (3-59), respectively.

$$(\sigma_t)_i\big|_{r=R} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -(8333)\frac{1^2 + 0^2}{1^2 - 0^2} = -8333 \text{ psi} = -8.33 \text{ kpsi} \qquad Ans.$$

$$(\sigma_t)_o\big|_{r=R} = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = (8333)\frac{1.5^2 + 1^2}{1.5^2 - 1^2} = 21\ 670 \text{ psi } = 21.7 \text{ kpsi} \qquad Ans.$$

3-131 From Table A-5, $E_i = 30$ Mpsi, $E_o = 14.5$ Mpsi, $\nu_i = 0.292$, $\nu_o = 0.211$.
$r_i = 0$, $R = 1$ in, $r_o = 1.5$ in

The radial interference is $\delta = \frac{1}{2}(2.002 - 2.000) = 0.001 \text{ in}$ *Ans.*

Eq. (3-56),

$$p = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \frac{1}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)\right]}$$

$$p = \frac{0.001}{1\left[\frac{1}{14.5(10^6)}\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2} + 0.211\right) + \frac{1}{30(10^6)}\left(\frac{1^2 + 0^2}{1^2 - 0^2} - 0.292\right)\right]} = 4599 \text{ psi} \qquad Ans.$$

The tangential stresses at the interface for the inner and outer members are given by Eqs. (3-58) and (3-59), respectively.

$$(\sigma_t)_i\big|_{r=R} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -(4599)\frac{1^2 + 0^2}{1^2 - 0^2} = -4599 \text{ psi} \qquad Ans.$$

$$(\sigma_t)_o\big|_{r=R} = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = (4599)\frac{1.5^2 + 1^2}{1.5^2 - 1^2} = 11\,960 \text{ psi} \quad \textit{Ans.}$$

3-132 From Table A-5, $E_i = E_o = 30$ Mpsi, $\nu_i = \nu_o = 0.292$. $r_i = 0$, $R = 0.5$ in, $r_o = 1$ in
The minimum and maximum radial interferences are

$$\delta_{\min} = \frac{1}{2}(1.002 - 1.002) = 0.000 \text{ in} \quad \textit{Ans.}$$

$$\delta_{\max} = \frac{1}{2}(1.003 - 1.001) = 0.001 \text{ in} \quad \textit{Ans.}$$

Since the minimum interference is zero, the minimum pressure and tangential stresses are zero. *Ans.*

The maximum pressure is obtained from Eq. (3-57).

$$p = \frac{E\delta}{2R^3}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2}\right]$$

$$p = \frac{30(10^6)0.001}{2(0.5^3)}\left[\frac{(1^2 - 0.5^2)(0.5^2 - 0)}{(1^2 - 0)}\right] = 22\,500 \text{ psi} \qquad \textit{Ans}$$

The maximum tangential stresses at the interface for the inner and outer members are given by Eqs. (3-58) and (3-59), respectively.

$$(\sigma_t)_i\big|_{r=R} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -(22\,500)\frac{0.5^2 + 0^2}{0.5^2 - 0^2} = -22\,500 \text{ psi} \quad \textit{Ans.}$$

$$(\sigma_t)_o\big|_{r=R} = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = (22\,500)\frac{1^2 + 0.5^2}{1^2 - 0.5^2} = 37\,500 \text{ psi} \quad \textit{Ans.}$$

3-133 From Table A-5, $E_i = 10.4$ Mpsi, $E_o = 30$ Mpsi, $\nu_i = 0.333$, $\nu_o = 0.292$.
$r_i = 0$, $R = 1$ in, $r_o = 1.5$ in

The minimum and maximum radial interferences are

$$\delta_{\min} = \frac{1}{2}[2.003 - 2.002] = 0.0005 \text{ in} \quad \textit{Ans.}$$

$$\delta_{\max} = \frac{1}{2}[2.006 - 2.000] = 0.003 \text{ in} \quad \textit{Ans.}$$

Eq. (3-56),

$$p = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2+R^2}{r_o^2-R^2}+\nu_o\right)+\frac{1}{E_i}\left(\frac{R^2+r_i^2}{R^2-r_i^2}-\nu_i\right)\right]}$$

$$p = \frac{\delta}{1\left[\frac{1}{30\left(10^6\right)}\left(\frac{1.5^2+1^2}{1.5^2-1^2}+0.292\right)+\frac{1}{10.4\left(10^6\right)}\left(\frac{1^2+0^2}{1^2-0^2}-0.333\right)\right]}$$

$$p = 6.229\left(10^6\right)\delta \text{ psi} \qquad Ans.$$

$$p_{\min} = 6.229\left(10^6\right)\delta_{\min} = 6.229\left(10^6\right)\left(0.0005\right) = 3114.6 \text{ psi} = 3.11 \text{ kpsi} \qquad Ans.$$

$$p_{\max} = 6.229\left(10^6\right)\delta_{\max} = 6.229\left(10^6\right)\left(0.003\right) = 18\,687 \text{ psi} = 18.7 \text{ kpsi} \qquad Ans.$$

The tangential stresses at the interface for the inner and outer members are given by Eqs. (3-58) and (3-59), respectively.

Minimum interference:

$$(\sigma_t)_i\big|_{\min} = -p_{\min}\frac{R^2+r_i^2}{R^2-r_i^2} = -(3.11)\frac{1^2+0^2}{1^2-0^2} = -3.11 \text{ kpsi} \quad Ans.$$

$$(\sigma_t)_o\big|_{\min} = p_{\min}\frac{r_o^2+R^2}{r_o^2-R^2} = (3.11)\frac{1.5^2+1^2}{1.5^2-1^2} = 8.09 \text{ kpsi} \quad Ans.$$

Maximum interference:

$$(\sigma_t)_i\big|_{\max} = -p_{\max}\frac{R^2+r_i^2}{R^2-r_i^2} = -(18.7)\frac{1^2+0^2}{1^2-0^2} = -18.7 \text{ kpsi} \quad Ans.$$

$$(\sigma_t)_o\big|_{\max} = p_{\max}\frac{r_o^2+R^2}{r_o^2-R^2} = (18.7)\frac{1.5^2+1^2}{1.5^2-1^2} = 48.6 \text{ kpsi} \quad Ans.$$

3-134 $d = 20 \text{ mm}, r_i = 37.5 \text{ mm}, r_o = 57.5 \text{ mm}$

From Table 3-4, for R = 10 mm,

$$r_c = 37.5 + 10 = 47.5 \text{ mm}$$

$$r_n = \frac{10^2}{2\left(47.5-\sqrt{47.5^2-10^2}\right)} = 46.96772 \text{ mm}$$

$$e = r_c - r_n = 47.5 - 46.96772 = 0.53228 \text{ mm}$$

$$c_i = r_n - r_i = 46.9677 - 37.5 = 9.4677 \text{ mm}$$

$$c_o = r_o - r_n = 57.5 - 46.9677 = 10.5323 \text{ mm}$$

$$A = \pi d^2/4 = \pi(20)^2/4 = 314.16 \text{ mm}^2$$

$$M = Fr_c = 4000(47.5) = 190\,000 \text{ N}\cdot\text{mm}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{4000}{314.16} + \frac{190\ 000(9.4677)}{314.16(0.53228)(37.5)} = 300\ \text{MPa} \qquad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{4000}{314.16} - \frac{190\ 000(10.5323)}{314.16(0.53228)(57.5)} = -195\ \text{MPa} \qquad \textit{Ans.}$$

3-135 $d = 0.75$ in, $r_i = 1.25$ in, $r_o = 2.0$ in

From Table 3-4, for $R = 0.375$ in,

$$r_c = 1.25 + 0.375 = 1.625\ \text{in}$$

$$r_n = \frac{0.375^2}{2\left(1.625 - \sqrt{1.625^2 - 0.375^2}\right)} = 1.60307\ \text{in}$$

$$e = r_c - r_n = 1.625 - 1.60307 = 0.02193\ \text{in}$$

$$c_i = r_n - r_i = 1.60307 - 1.25 = 0.35307\ \text{in}$$

$$c_o = r_o - r_n = 2.0 - 1.60307 = 0.39693\ \text{in}$$

$$A = \pi d^2/4 = \pi(0.75)^2/4 = 0.44179\ \text{in}^2$$

$$M = Fr_c = 750(1.625) = 1218.8\ \text{lbf}\cdot\text{in}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{750}{0.44179} + \frac{1218.8(0.35307)}{0.44179(0.02193)(1.25)} = 37\ 230\ \text{psi} = 37.2\ \text{kpsi} \qquad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{750}{0.44179} - \frac{1218.8(0.39693)}{0.44179(0.02193)(2.0)} = -23\ 269\ \text{psi} = -23.3\ \text{kpsi} \qquad \textit{Ans.}$$

3-136 $d = 6$ mm, $r_i = 10$ mm, $r_o = 16$ mm

From Table 3-4, for $R = 3$ mm,

$$r_c = 10 + 3 = 13\ \text{mm}$$

$$r_n = \frac{3^2}{2\left(13 - \sqrt{13^2 - 3^2}\right)} = 12.82456\ \text{mm}$$

$$e = r_c - r_n = 13 - 12.82456 = 0.17544\ \text{mm}$$

$$c_i = r_n - r_i = 12.82456 - 10 = 2.82456\ \text{mm}$$

$$c_o = r_o - r_n = 16 - 12.82456 = 3.17544\ \text{mm}$$

$$A = \pi d^2/4 = \pi(6)^2/4 = 28.2743\ \text{mm}^2$$

$$M = Fr_c = 300(13) = 3900\ \text{N}\cdot\text{mm}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{300}{28.2743} + \frac{3900(2.82456)}{28.2743(0.17544)(10)} = 233 \text{ MPa} \quad \mathit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{300}{28.2743} - \frac{3900(3.17544)}{28.2743(0.17544)(16)} = -145 \text{ MPa} \quad \mathit{Ans.}$$

3-137 $d = 6$ mm, $r_i = 10$ mm, $r_o = 16$ mm

From Table 3-4, for R = 3 mm,

$$r_c = 10 + 3 = 13 \text{ mm}$$

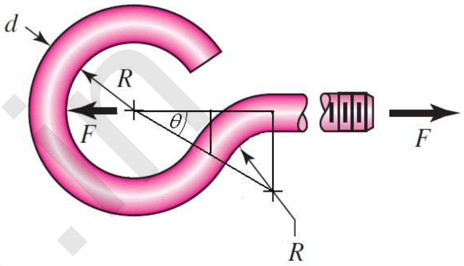
$$r_n = \frac{3^2}{2\left(13 - \sqrt{13^2 - 3^2}\right)} = 12.82456 \text{ mm}$$

$$e = r_c - r_n = 13 - 12.82456 = 0.17544 \text{ mm}$$

$$c_i = r_n - r_i = 12.82456 - 10 = 2.82456 \text{ mm}$$

$$c_o = r_o - r_n = 16 - 12.82456 = 3.17544 \text{ mm}$$

$$A = \pi d^2/4 = \pi(6)^2/4 = 28.2743 \text{ mm}^2$$



The angle θ of the line of radius centers is

$$\theta = \sin^{-1}\left(\frac{R + d/2}{R + d + R}\right) = \sin^{-1}\left(\frac{10 + 6/2}{10 + 6 + 10}\right) = 30^\circ$$

$$M = F\left(R + d/2\right)\sin\theta = 300\left(10 + 6/2\right)\sin 30^\circ = 1950 \text{ N}\cdot\text{mm}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F\sin\theta}{A} + \frac{Mc_i}{Aer_i} = \frac{300\sin 30^\circ}{28.2743} + \frac{1950(2.82456)}{28.2743(0.17544)(10)} = 116 \text{ MPa} \quad \mathit{Ans.}$$

$$\sigma_o = \frac{F\sin\theta}{A} - \frac{Mc_o}{Aer_o} = \frac{300\sin 30^\circ}{28.2743} - \frac{1950(3.17544)}{28.2743(0.17544)(16)} = -72.7 \text{ MPa} \quad \mathit{Ans.}$$

Note that the shear stress due to the shear force is zero at the surface.

3-138 $d = 0.25$ in, $r_i = 0.5$ in, $r_o = 0.75$ in

From Table 3-4, for R = 0.125 in,

$$r_c = 0.5 + 0.125 = 0.625 \text{ in}$$

$$r_n = \frac{0.125^2}{2\left(0.625 - \sqrt{0.625^2 - 0.125^2}\right)} = 0.618686 \text{ in}$$

$$e = r_c - r_n = 0.625 - 0.618686 = 0.006314 \text{ in}$$

$$c_i = r_n - r_i = 0.618686 - 0.5 = 0.118686 \text{ in}$$

$$c_o = r_o - r_n = 0.75 - 0.618686 = 0.131314 \text{ in}$$

$$A = \pi d^2/4 = \pi(0.25)^2/4 = 0.049087 \text{ in}^2$$
$$M = Fr_c = 75(0.625) = 46.875 \text{ lbf}\cdot\text{in}$$

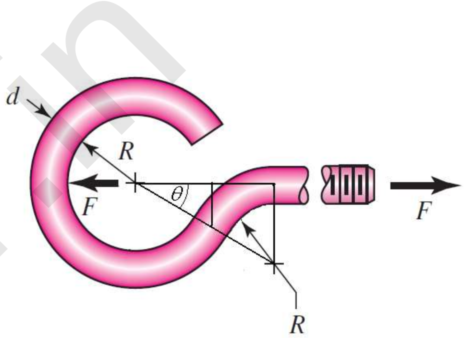
Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{75}{0.049087} + \frac{46.875(0.118686)}{0.049087(0.006314)(0.5)} = 37\,428 \text{ psi} = 37.4 \text{ kpsi} \quad \textit{Ans.}$$
$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{75}{0.049087} - \frac{46.875(0.131314)}{0.049087(0.006314)(0.75)} = -24\,952 \text{ psi} = -25.0 \text{ kpsi} \quad \textit{Ans.}$$

3-139 $d = 0.25$ in, $r_i = 0.5$ in, $r_o = 0.75$ in

From Table 3-4, for $R = 0.125$ in,

$$r_c = 0.5 + 0.125 = 0.625 \text{ in}$$
$$r_n = \frac{0.125^2}{2\left(0.625 - \sqrt{0.625^2 - 0.125^2}\right)} = 0.618686 \text{ in}$$
$$e = r_c - r_n = 0.625 - 0.618686 = 0.006314 \text{ in}$$
$$c_i = r_n - r_i = 0.618686 - 0.5 = 0.118686 \text{ in}$$
$$c_o = r_o - r_n = 0.75 - 0.618686 = 0.131314 \text{ in}$$
$$A = \pi d^2/4 = \pi(0.25)^2/4 = 0.049087 \text{ in}^2$$



The angle θ of the line of radius centers is

$$\theta = \sin^{-1}\left(\frac{R + d/2}{R + d + R}\right) = \sin^{-1}\left(\frac{0.5 + 0.25/2}{0.5 + 0.25 + 0.5}\right) = 30^\circ$$
$$M = F\left(R + d/2\right)\sin\theta = 75\left(0.5 + 0.25/2\right)\sin 30^\circ = 23.44 \text{ lbf}\cdot\text{in}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F\sin\theta}{A} + \frac{Mc_i}{Aer_i} = \frac{75\sin 30^\circ}{0.049087} + \frac{23.44(0.118686)}{0.049087(0.006314)(0.5)} = 18\,716 \text{ psi} = 18.7 \text{ kpsi} \quad \textit{Ans.}$$
$$\sigma_o = \frac{F\sin\theta}{A} - \frac{Mc_o}{Aer_o} = \frac{75\sin 30^\circ}{0.049087} - \frac{23.44(0.131314)}{0.049087(0.006314)(0.75)} = -12\,478 \text{ psi} = -12.5 \text{ kpsi} \quad \textit{Ans.}$$

Note that the shear stress due to the shear force is zero at the surface.

3-140

(a) $\sigma = \pm\dfrac{Mc}{I} = \pm\dfrac{[3(4)][0.5(0.1094)]}{(0.75)\left(0.1094^3\right)/12} = \pm 8021 \text{ psi} = \pm 8.02 \text{ kpsi}$ *Ans.*

(b) $r_i = 0.125$ in, $r_o = r_i + h = 0.125 + 0.1094 = 0.2344$ in

From Table 3-4,

$$r_c = 0.125 + (0.5)(0.1094) = 0.1797 \text{ in}$$

$$r_n = \frac{0.1094}{\ln(0.2344/0.125)} = 0.174006 \text{ in}$$

$$e = r_c - r_n = 0.1797 - 0.174006 = 0.005694 \text{ in}$$

$$c_i = r_n - r_i = 0.174006 - 0.125 = 0.049006 \text{ in}$$

$$c_o = r_o - r_n = 0.2344 - 0.174006 = 0.060394 \text{ in}$$

$$A = bh = 0.75(0.1094) = 0.08205 \text{ in}^2$$

$$M = -3(4) = -12 \text{ lbf} \cdot \text{in}$$

The negative sign on the bending moment is due to the sign convention shown in Fig. 3-35. Using Eq. (3-65),

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-12(0.049006)}{0.08205(0.005694)(0.125)} = -10\ 070 \text{ psi} = -10.1 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_o = -\frac{Mc_o}{Aer_o} = -\frac{-12(0.060394)}{0.08205(0.005694)(0.2344)} = 6618 \text{ psi} = 6.62 \text{ kpsi} \qquad \textit{Ans.}$$

(c) $K_i = \dfrac{\sigma_i}{\sigma} = \dfrac{-10.1}{-8.02} = 1.26 \qquad \textit{Ans.}$

$$K_o = \frac{\sigma_o}{\sigma} = \frac{6.62}{8.02} = 0.825 \qquad \textit{Ans.}$$

3-141

(a) $\sigma = \pm\dfrac{Mc}{I} = \pm\dfrac{[3(4)][0.5(0.1406)]}{(0.75)\left(0.1406^3\right)/12} = \pm 4856 \text{ psi} = \pm 4.86 \text{ kpsi} \qquad \textit{Ans.}$

(b) $r_i = 0.125$ in, $r_o = r_i + h = 0.125 + 0.1406 = 0.2656$ in

From Table 3-4,

$$r_c = 0.125 + (0.5)(0.1406) = 0.1953 \text{ in}$$

$$r_n = \frac{0.1406}{\ln(0.2656/0.125)} = 0.186552 \text{ in}$$

$$e = r_c - r_n = 0.1953 - 0.186552 = 0.008748 \text{ in}$$

$$c_i = r_n - r_i = 0.186552 - 0.125 = 0.061552 \text{ in}$$

$$c_o = r_o - r_n = 0.2656 - 0.186552 = 0.079048 \text{ in}$$

$$A = bh = 0.75(0.1406) = 0.10545 \text{ in}^2$$

$$M = -3(4) = -12 \text{ lbf} \cdot \text{in}$$

The negative sign on the bending moment is due to the sign convention shown in Fig. 3-35. Using Eq. (3-65),

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-12(0.061552)}{0.10545(0.008748)(0.125)} = -6406 \text{ psi} = -6.41 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = -\frac{Mc_o}{Aer_o} = -\frac{-12(0.079048)}{0.10545(0.008748)(0.2656)} = 3872 \text{ psi} = 3.87 \text{ kpsi} \quad \textit{Ans.}$$

(c) $K_i = \frac{\sigma_i}{\sigma} = \frac{-6.41}{-4.86} = 1.32$ *Ans.*

$$K_o = \frac{\sigma_o}{\sigma} = \frac{3.87}{4.86} = 0.80 \quad \textit{Ans.}$$

3-142

(a) $\sigma = \pm\frac{Mc}{I} = \pm\frac{[3(4)][0.5(0.1094)]}{(0.75)\left(0.1094^3\right)/12} = \pm 8021 \text{ psi} = \pm 8.02 \text{ kpsi}$ *Ans.*

(b) $r_i = 0.25$ in, $r_o = r_i + h = 0.25 + 0.1094 = 0.3594$ in

From Table 3-4,

$$r_c = 0.25 + (0.5)(0.1094) = 0.3047 \text{ in}$$

$$r_n = \frac{0.1094}{\ln(0.3594/0.25)} = 0.301398 \text{ in}$$

$$e = r_c - r_n = 0.3047 - 0.301398 = 0.003302 \text{ in}$$

$$c_i = r_n - r_i = 0.301398 - 0.25 = 0.051398 \text{ in}$$

$$c_o = r_o - r_n = 0.3594 - 0.301398 = 0.058002 \text{ in}$$

$$A = bh = 0.75(0.1094) = 0.08205 \text{ in}^2$$

$$M = -3(4) = -12 \text{ lbf} \cdot \text{in}$$

The negative sign on the bending moment is due to the sign convention shown in Fig. 3-35. Using Eq. (3-65),

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-12(0.051398)}{0.08205(0.003302)(0.25)} = -9106 \text{ psi} = -9.11 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = -\frac{Mc_o}{Aer_o} = -\frac{-12(0.058002)}{0.08205(0.003302)(0.3594)} = 7148 \text{ psi} = 7.15 \text{ kpsi} \quad \textit{Ans.}$$

(c) $K_i = \frac{\sigma_i}{\sigma} = \frac{-9.11}{-8.02} = 1.14$ *Ans.*

$$K_o = \frac{\sigma_o}{\sigma} = \frac{7.15}{8.02} = 0.89 \quad \textit{Ans.}$$

3-143 $r_i = 25$ mm, $r_o = r_i + h = 25 + 87 = 112$ mm, $r_c = 25 + 87/2 = 68.5$ mm

The radius of the neutral axis is found from Eq. (3-63), given below.

$$r_n = \frac{A}{\int (dA/r)}$$

For a rectangular area with constant width b, the denominator is

$$\int_{r_i}^{r_o} \left(\frac{bdr}{r} \right) = b \ln \frac{r_o}{r_i}$$

Applying this equation over each of the four rectangular areas,

$$\int \frac{dA}{r} = 9\left(\ln \frac{45}{25} \right) + 31\left(\ln \frac{54.5}{45} \right) + 31\left(\ln \frac{92}{82.5} \right) + 9\left(\ln \frac{112}{92} \right) = 16.3769$$

$$A = 2[20(9) + 31(9.5)] = 949 \text{ mm}^2$$

$$r_n = \frac{A}{\int (dA/r)} = \frac{949}{16.3769} = 57.9475 \text{ mm}$$

$$e = r_c - r_n = 68.5 - 57.9475 = 10.5525 \text{ mm}$$

$$c_i = r_n - r_i = 57.9475 - 25 = 32.9475 \text{ mm}$$

$$c_o = r_o - r_n = 112 - 57.9475 = 54.0525 \text{ mm}$$

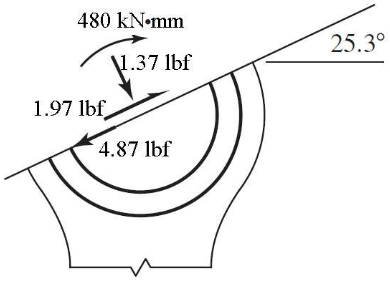
$$M = 150F_2 = 150(3.2) = 480 \text{ kN·mm}$$

We need to find the forces transmitted through the section in order to determine the axial stress. It is not immediately obvious which plane should be used for resolving the axial versus shear directions. It is convenient to use the plane containing the reaction force at the bushing, which assumes its contribution resolves entirely into shear force. To find the angle of this plane, find the resultant of F_1 and F_2.

$$F_x = F_{1x} + F_{2x} = 2.4\cos 60^\circ + 3.2\cos 0^\circ = 4.40 \text{ kN}$$

$$F_y = F_{1y} + F_{2y} = 2.4\sin 60^\circ + 3.2\sin 0^\circ = 2.08 \text{ kN}$$

$$F = \left(4.40^2 + 2.08^2\right)^{1/2} = 4.87 \text{ kN}$$



This is the pin force on the lever which acts in a direction

$$\theta = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{2.08}{4.40} = 25.3^\circ$$

On the surface 25.3° from the horizontal, find the internal forces in the tangential and normal directions. Resolving F_1 into components,

$$F_t = 2.4\cos\left(60^\circ - 25.3^\circ\right) = 1.97 \text{ kN}$$

$$F_n = 2.4\sin\left(60^\circ - 25.3^\circ\right) = 1.37 \text{ kN}$$

The transverse shear stress is zero at the inner and outer surfaces. Using Eq. (3-65) for the bending stress, and combining with the axial stress due to F_n,

$$\sigma_i = \frac{F_n}{A} + \frac{Mc_i}{Aer_i} = \frac{1370}{949} + \frac{\left[(3200)(150)\right](32.9475)}{949(10.5525)(25)} = 64.6 \text{ MPa} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F_n}{A} - \frac{Mc_o}{Aer_o} = \frac{1370}{949} - \frac{\left[(3200)(150)\right](54.0525)}{949(10.5525)(112)} = -21.7 \text{ MPa} \quad \textit{Ans.}$$

3-144 $r_i = 2$ in, $r_o = r_i + h = 2 + 4 = 6$ in, $r_c = 2 + 0.5(4) = 4$ in

$A = (6 - 2 - 0.75)(0.75) = 2.4375 \text{ in}^2$

Similar to Prob. 3-143,

$$\int \frac{dA}{r} = 0.75 \ln \frac{3.625}{2} + 0.75 \ln \frac{6}{4.375} = 0.682\,920 \text{ in}$$

$$r_n = \frac{A}{\int (dA/r)} = \frac{2.4375}{0.682\,920} = 3.56923 \text{ in}$$

$$e = r_c - r_n = 4 - 3.56923 = 0.43077 \text{ in}$$

$$c_i = r_n - r_i = 3.56923 - 2 = 1.56923 \text{ in}$$

$$c_o = r_o - r_n = 6 - 3.56923 = 2.43077 \text{ in}$$

$$M = Fr_c = 6000(4) = 24\,000 \text{ lbf} \cdot \text{in}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{6000}{2.4375} + \frac{24\,000(1.56923)}{2.4375(0.43077)(2)} = 20\,396 \text{ psi} = 20.4 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{6000}{2.4375} - \frac{24\,000(2.43077)}{2.4375(0.43077)(6)} = -6\,799 \text{ psi} = -6.80 \text{ kpsi} \quad \textit{Ans.}$$

3-145 $r_i = 12$ in, $r_o = r_i + h = 12 + 3 = 15$ in, $r_c = 12 + 3/2 = 13.5$ in

$$I = \frac{\pi}{4} a^3 b = \frac{\pi}{4}(1.5^3)(0.75) = 1.988 \text{ in}^4$$

$$A = \pi ab = \pi(1.5)(0.75) = 3.534$$

$$M = 20(3 + 1.5) = 90 \text{ kip} \cdot \text{in}$$

Since the radius is large compared to the cross section, assume Eq. 3-67 is applicable for the bending stress. Combining the bending stress and the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i r_c}{Ir_i} = \frac{20}{3.534} + \frac{90(1.5)(13.5)}{(1.988)(12)} = 82.1 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o r_c}{Ir_o} = \frac{20}{3.534} - \frac{90(1.5)(13.5)}{1.988(15)} = -55.5 \text{ kpsi} \quad \textit{Ans.}$$

3-146 $r_i = 1.25$ in, $r_o = r_i + h = 1.25 + 0.5 + 1 + 0.5 = 3.25$ in

$r_c = (r_i + r_o) / 2 =$ (1.25 + 3.25)/2 = 2.25 in *Ans.*

For outer rectangle, $\left(\int \frac{dA}{r}\right)_{\square} = b \ln \frac{r_o}{r_i}$

For circle, $\left[\frac{A}{\int (dA/r)}\right]_{\text{O}} = \left[\frac{r^2}{2\left(r_c - \sqrt{r_c^2 - r^2}\right)}\right]_{\text{O}}$, $A_{\text{O}} = \pi r^2$

$$\therefore \left[\int \frac{dA}{r}\right]_{\text{O}} = 2\pi (r_c - \sqrt{r_c^2 - r^2})$$

Combine the integrals subtracting the circle from the rectangle

$$\Sigma \int \frac{dA}{r} = 1.25 \ln \frac{3.25}{1.25} - 2\pi \left(2.25 - \sqrt{2.25^2 - 0.5^2}\right) = 0.840\,904 \text{ in}$$

$$A = 1.25(2) - \pi(0.5^2) = 1.714\,60 \text{ in}^2 \quad \textit{Ans.}$$

$$r_n = \frac{A}{\sum \int (dA/r)} = \frac{1.714\,60}{0.840\,904} = 2.0390 \text{ in} \quad \textit{Ans.}$$

$$e = r_c - r_n = 2.25 - 2.0390 = 0.2110 \text{ in} \quad \textit{Ans.}$$

$$c_i = r_n - r_i = 2.0390 - 1.25 = 0.7890 \text{ in}$$

$$c_o = r_o - r_n = 3.25 - 2.0390 = 1.2110 \text{ in}$$

$$M = 2000(4.5 + 1.25 + 0.5 + 0.5) = 13\,500 \text{ lbf} \cdot \text{in}$$

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{2000}{1.7146} + \frac{13\,500(0.7890)}{1.7146(0.2110)(1.25)} = 20\,720 \text{ psi} = 20.7 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{2000}{1.7146} - \frac{13\,500(1.2110)}{1.7146(0.2110)(3.25)} = -12\,738 \text{ psi} = -12.7 \text{ kpsi} \quad \textit{Ans.}$$

3-147 Table A-5: Glass: $E_G = 46.2$ GPa, $\nu_G = 0.245$, Steel: $E_S = 207$ GPa, $\nu_S = 0.292$

Eq. (3-68):

$$a = \sqrt[3]{\frac{3F}{8} \frac{\left(1 - \nu_1^2\right)/E_1 + \left(1 - \nu_2^2\right)/E_2}{1/d_1 + 1/d_2}}$$

$$= \sqrt[3]{\frac{3(5)}{8} \frac{\left(1 - 0.245^2\right)/\left[46.2\left(10^9\right)\right] + \left(1 - 0.292^2\right)/\left[207\left(10^9\right)\right]}{1/0.030 + 1/\infty}}$$

$$= 1.1168\left(10^{-4}\right) \text{ m} = 0.11168 \text{ mm}$$

Eq. (3-69):

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3(5)}{2\pi(0.11168)^2} = 191.4 \text{ MPa} \qquad \textit{Ans.}$$

(b) Eq. (3-70):

$$\sigma_1 = \sigma_2 = -p_{\max}\left\{\left(1-\left|\frac{z}{a}\right|\tan^{-1}\frac{1}{|z/a|}\right)(1+\nu_G)-\frac{1}{2\left[1+(z/a)^2\right]}\right\}$$

$$= -191.4\left\{\left[1-\left(\frac{z}{0.11168}\right)\tan^{-1}\frac{1}{(z/0.11168)}\right](1+0.245)-\frac{1}{2\left[1+(z/0.11168)^2\right]}\right\} \qquad (1)$$

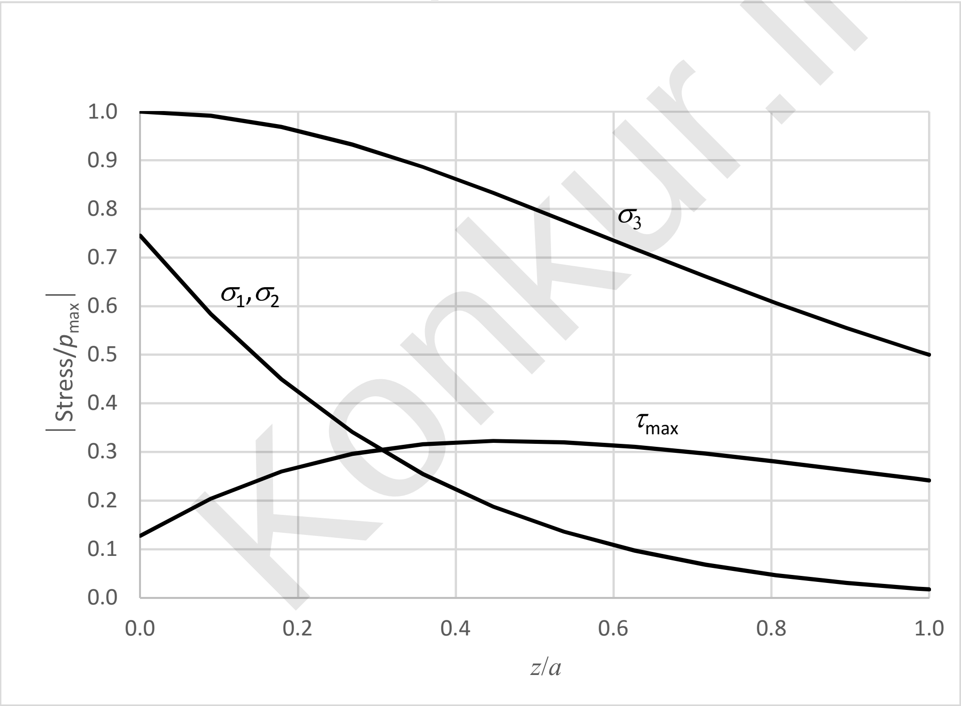
Eq. (3-71):

$$\sigma_3 = \frac{-p_{\max}}{1+(z/a)^2} = \frac{-191.4}{1+(z/0.11168)^2} \qquad (2)$$

The maximum shear stress is given by

$$\tau_{\max} = \left|\frac{\sigma_1-\sigma_3}{2}\right| \qquad (3)$$

(c) Plots of Eqs. (1) – (3) as absolute dimensionless stresses, $|\text{stress}/p_{\max}|$, are given. Note, at $z = 0$, $\sigma_1 = \sigma_2 = -0.745\,p_{\max}$.



Ans.

(d) $\tau_{\max} \approx 0.323\,p_{\max} = 0.323(191.4) = 61.8$ MPa at $z = 0.05$ mm *Ans.*

(e) From Fig. 3-38, $\tau_{\max} = 0.3(191.4) = 57.4$ MPa *Ans.*

3-148 From Eq. (3-68),

$$a = KF^{1/3} = F^{1/3}\left\{\left(\frac{3}{8}\right)\frac{2\left[\left(1-\nu^2\right)/E\right]}{2\left(1/d\right)}\right\}^{1/3}$$

Use $\nu = 0.292$, F in newtons, E in N/mm^2 and d in mm, then

$$K = \left\{\left(\frac{3}{8}\right)\frac{[(1-0.292^2)/207\,000]}{1/30}\right\}^{1/3} = 0.03685$$

From Eq. (3-69),

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3F}{2\pi(KF^{1/3})^2} = \frac{3F^{1/3}}{2\pi K^2} = \frac{3F^{1/3}}{2\pi(0.03685)^2} = 352F^{1/3}\ \text{MPa}$$

From Eq. (3-71), the maximum principal stress occurs on the surface where $z = 0$, and is equal to $-p_{\max}$.

$$\sigma_{\max} = \sigma_z = -p_{\max} = -352F^{1/3}\ \text{MPa} \qquad \textit{Ans.}$$

From Fig. 3-38,

$$\tau_{\max} = 0.3p_{\max} = 106F^{1/3}\ \text{MPa} \qquad \textit{Ans.}$$

3-149 From Eq. (3-68),

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}}$$

$$a = \sqrt[3]{\left(\frac{3(10)}{8}\right)\frac{\left(1-0.292^2\right)/(207\,000) + \left(1-0.333^2\right)/(71\,700)}{1/25 + 1/40}} = 0.0990\ \text{mm}$$

From Eq. (3-69),

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3(10)}{2\pi\left(0.0990^2\right)} = 487.2\ \text{MPa}$$

From Fig. 3-38, the maximum shear stress occurs at a depth of $z = 0.48\,a$.

$$z = 0.48a = 0.48(0.0990) = 0.0475\ \text{mm} \qquad \textit{Ans.}$$

The principal stresses are obtained from Eqs. (3-70) and (3-71) at a depth of $z/a = 0.48$.

$$\sigma_1 = \sigma_2 = -487.2\left\{\left[1-0.48\tan^{-1}(1/0.48)\right](1+0.333) - \frac{1}{2\left(1+0.48^2\right)}\right\} = -101.3\ \text{MPa}$$

$$\sigma_3 = \frac{-487.2}{1+0.48^2} = -396.0\ \text{MPa}$$

From Eq. (3-72),

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{(-101.3)-(-396.0)}{2} = 147.4 \text{ MPa} \qquad \textit{Ans.}$$

Note that if a closer examination of the applicability of the depth assumption from Fig. 3-38 is desired, implementing Eqs. (3-70), (3-71), and (3-72) on a spreadsheet will allow for calculating and plotting the stresses versus the depth for specific values of ν. For ν = 0.333 for aluminum, the maximum shear stress occurs at a depth of $z = 0.492a$ with $\tau_{\max} = 0.3025\, p_{\max}$.

This gives $\tau_{\max} = 0.3025\, p_{\max} = (0.3025)(487.2) = 147.38$ MPa. Even though the depth assumption was a little off, it did not have significant effect on the the maximum shear stress.

3-150 From the solution to Prob. 3-149, $a = 0.0990$ mm and $p_{\max} = 487.2$ MPa. Assuming applicability of Fig. 3-38, the maximum shear stress occurs at a depth of $z = 0.48\, a = 0.0475$ mm. *Ans.*

The principal stresses are obtained from Eqs. (3-70) and (3-71) at a depth of $z/a = 0.48$.

$$\sigma_1 = \sigma_2 = -487.2\left\{\left[1-0.48\tan^{-1}(1/0.48)\right](1+0.292) - \frac{1}{2\left(1+0.48^2\right)}\right\} = -92.09 \text{ MPa}$$

$$\sigma_3 = \frac{-487.2}{1+0.48^2} = -396.0 \text{ MPa}$$

From Eq. (3-72),

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{(-92.09)-(-396.0)}{2} = 152.0 \text{ MPa} \qquad \textit{Ans.}$$

Note that if a closer examination of the applicability of the depth assumption from Fig. 3-38 is desired, implementing Eqs. (3-70), (3-71), and (3-72) on a spreadsheet will allow for calculating and plotting the stresses versus the depth for specific values of ν. For ν = 0.292 for steel, the maximum shear stress occurs at a depth of $z = 0.478a$ with $\tau_{\max} = 0.3119\, p_{\max}$.

3-151 From Eq. (3-68),

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{2\left(1-\nu^2\right)/E}{1/d_1 + 1/d_2}}$$

$$a = \sqrt[3]{\left(\frac{3(20)}{8}\right)\frac{2\left(1-0.292^2\right)/(207\ 000)}{1/30 + 1/\infty}} = 0.1258 \text{ mm}$$

From Eq. (3-69),

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3(20)}{2\pi\left(0.1258^2\right)} = 603.4 \text{ MPa}$$

From Fig. 3-38, the maximum shear stress occurs at a depth of

$$z = 0.48a = 0.48(0.1258) = 0.0604 \text{ mm} \qquad \textit{Ans.}$$

Also from Fig. 3-38, the maximum shear stress is

$$\tau_{\max} = 0.3 p_{\max} = 0.3(603.4) = 181 \text{ MPa} \qquad \textit{Ans.}$$

3-152 Aluminum **Plate-Ball interface:** From Eq. (3-68),

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}}$$

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{\left(1-0.292^2\right)/\left[(30)\left(10^6\right)\right] + \left(1-0.333^2\right)/\left[(10.4)\left(10^6\right)\right]}{1/1+1/\infty}} = 3.517\left(10^{-3}\right)F^{1/3} \text{ in}$$

From Eq. (3-69),

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3F}{2\pi\left[3.517\left(10^{-3}\right)F^{1/3}\right]^2} = 3.860\left(10^4\right)F^{1/3} \text{ psi}$$

By examination of Eqs. (3-70), (3-71), and (3-72), it can be seen that the only difference in the maximum shear stress for the plate and the ball will be due to Poisson's ratio in Eq. (3-70). The larger Poisson's ratio will create the greater maximum shear stress, so the aluminum plate will be the critical element in this interface. Applying the equations for the aluminum plate,

$$\sigma_1 = -3.86\left(10^4\right)F^{1/3}\left\{\left[1-0.48\tan^{-1}(1/0.48)\right](1+0.333) - \frac{1}{2\left(1+0.48^2\right)}\right\} = -8025F^{1/3} \text{ psi}$$

$$\sigma_3 = \frac{-3.86\left(10^4\right)F^{1/3}}{1+0.48^2} = -3.137\left(10^4\right)F^{1/3} \text{ psi}$$

From Eq. (3-72),

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\left(-8025F^{1/3}\right) - \left(-3.137\left(10^4\right)F^{1/3}\right)}{2} = 1.167\left(10^4\right)F^{1/3} \text{ psi}$$

Comparing this stress to the allowable stress, and solving for *F*,

$$F = \left[\frac{20\,000}{1.167\left(10^4\right)}\right]^3 = 5.03 \text{ lbf}$$

Table-Ball interface: From Eq. (3-68),

$$a=\sqrt[3]{\left(\frac{3F}{8}\right)\frac{\left(1-0.292^2\right)/\left[(30)\left(10^6\right)\right]+\left(1-0.211^2\right)/\left[(14.5)\left(10^6\right)\right]}{1/1+1/\infty}}=3.306\left(10^{-3}\right)F^{1/3}\text{ in}$$

From Eq. (3-69),

$$p_{\max}=\frac{3F}{2\pi a^2}=\frac{3F}{2\pi\left[3.306\left(10^{-3}\right)F^{1/3}\right]^2}=4.369\left(10^4\right)F^{1/3}\text{ psi}$$

The steel ball has a higher Poisson's ratio than the cast iron table, so it will dominate.

$$\sigma_1=-4.369\left(10^4\right)F^{1/3}\left\{\left[1-0.48\tan^{-1}(1/0.48)\right](1+0.292)-\frac{1}{2\left(1+0.48^2\right)}\right\}=-8258F^{1/3}\text{ psi}$$

$$\sigma_3=\frac{-4.369\left(10^4\right)F^{1/3}}{1+0.48^2}=-3.551\left(10^4\right)F^{1/3}\text{ psi}$$

From Eq. (3-72),

$$\tau_{\max}=\frac{\sigma_1-\sigma_3}{2}=\frac{\left(-8258F^{1/3}\right)-\left(-3.551\left(10^4\right)F^{1/3}\right)}{2}=1.363\left(10^4\right)F^{1/3}\text{ psi}$$

Comparing this stress to the allowable stress, and solving for F,

$$F=\left[\frac{20\,000}{1.363\left(10^4\right)}\right]^3=3.16\text{ lbf}$$

The steel ball is critical, with F = 3.16 lbf. *Ans.*

3-153 $v_1 = 0.333$, $E_1 = 10.4$ Mpsi, $l = 2$ in, $d_1 = 1.25$ in, $v_2 = 0.211$, $E_2 = 14.5$ Mpsi, $d_2 = -12$ in.

From Eq. 3-73, with $b = K_cF^{1/2}$

$$K_c=\left(\frac{2}{\pi(2)}\frac{\left(1-0.333^2\right)/\left[10.4\left(10^6\right)\right]+\left(1-0.211^2\right)/\left[14.5\left(10^6\right)\right]}{1/1.25+1/(-12)}\right)^{1/2}$$

$$=2.593\left(10^{-4}\right)$$

By examination of Eqs. (3-75), (3-76), and (3-77), it can be seen that the only difference in the maximum shear stress for the two materials will be due to Poisson's ratio in Eq. (3-75). The larger Poisson's ratio will create the greater maximum shear stress, so the aluminum roller will be the critical element in this interface. Instead of applying these equations, we will assume the Poisson's ratio for aluminum of 0.333 is close enough to 0.3 to make Fig. 3-40 applicable.

$$\tau_{\max} = 0.3 p_{\max}$$

$$p_{\max} = \frac{4000}{0.3} = 13\ 300 \text{ psi}$$

From Eq. (3-74), $p_{\max} = 2F / (\pi bl)$, so we have

$$p_{\max} = \frac{2F}{\pi l K_c F^{1/2}} = \frac{2F^{1/2}}{\pi l K_c}$$

So,

$$F = \left(\frac{\pi l K_c p_{\max}}{2} \right)^2$$

$$= \left(\frac{\pi(2)(2.593)\left(10^{-4}\right)(13\ 300)}{2} \right)^2$$

$$= 117.4 \text{ lbf} \quad \textit{Ans.}$$

3-154

$v = 0.292$, $E = 30$ Mpsi, $l = 0.75$ in, $d_1 = 2(0.47) = 0.94$ in, $d_2 = 2(0.62) = 1.24$ in.
Eq. (3-73):

$$b = \left(\frac{2(40)}{\pi(0.75)} \frac{2\left(1-0.292^2\right) \Big/ \left[30\left(10^6\right)\right]}{1/0.94 + 1/1.24} \right)^{1/2} = 1.052\left(10^{-3}\right) \text{ in}$$

Eq. (3-74):

$$p_{\max} = \frac{2F}{\pi bl} == \frac{2(40)}{\pi(1.052)\left(10^{-3}\right)(0.75)} = 32\ 275 \text{ psi} = 32.3 \text{ kpsi} \qquad \textit{Ans.}$$

From Fig. 3-40,

$$\tau_{\max} = 0.3 p_{\max} = 0.3(32\ 275) = 9682.5 \text{ psi} = 9.68 \text{ kpsi} \qquad \textit{Ans.}$$

3-155 Use Eqs. (3-73) through (3-77).

$$b = \left(\frac{2F}{\pi l} \frac{(1-v_1^2)/E_1 + (1-v_2^2)/E_2}{(1/d_1) + (1/d_2)} \right)^{1/2}$$

$$= \left(\frac{2(600)}{\pi(2)} \frac{(1-0.292^2)/(30(10^6)) + (1-0.292^2)/(30(10^6))}{1/5 + 1/\infty} \right)^{1/2}$$

$$b = 0.007\ 631 \text{ in}$$

$$p_{\max} = \frac{2F}{\pi bl} = \frac{2(600)}{\pi(0.007\ 631)(2)} = 25\ 028 \text{ psi}$$

$$\sigma_x = -2\nu p_{\max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right) = -2(0.292)(25\ 028)\left(\sqrt{1+0.786^2} - 0.786\right)$$

$$= -7102 \text{ psi} = -7.10 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_y = -p_{\max}\left(\frac{1+2\frac{z^2}{b^2}}{\sqrt{1+\frac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right|\right) = -25\ 028\left(\frac{1+2(0.786^2)}{\sqrt{1+(0.786^2)}} - 2(0.786)\right)$$

$$= -4\ 646 \text{ psi} = -4.65 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_z = \frac{-p_{\max}}{\sqrt{1+\frac{z^2}{b^2}}} = \frac{-25\ 028}{\sqrt{1+0.786^2}} = -19\ 677 \text{ psi} = -19.7 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{\max} = \frac{\sigma_y - \sigma_z}{2} = \frac{-4\ 646-(-19\ 677)}{2} = 7\ 516 \text{ psi} = 7.52 \text{ kpsi} \qquad \textit{Ans.}$$

3-156 Use Eqs. (3-73) through (3-77).

$$b = \left(\frac{2F}{\pi l}\frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}\right)^{1/2}$$

$$= \left(\frac{2(2000)}{\pi(40)}\frac{\left(1-0.292^2\right)/\left[207\left(10^3\right)\right] + \left(1-0.211^2\right)/\left[100\left(10^3\right)\right]}{1/150 + 1/\infty}\right)^{1/2}$$

$$b = 0.2583 \text{ mm}$$

$$p_{\max} = \frac{2F}{\pi bl} = \frac{2(2000)}{\pi(0.2583)(40)} = 123.2 \text{ MPa}$$

$$\sigma_x = -2\nu p_{\max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right) = -2(0.292)(123.2)\left(\sqrt{1+0.786^2} - 0.786\right)$$

$$= -35.0 \text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_y = -p_{\max}\left(\frac{1+2\frac{z^2}{b^2}}{\sqrt{1+\frac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right|\right) = -123.2\left(\frac{1+2(0.786^2)}{\sqrt{1+(0.786^2)}} - 2(0.786)\right)$$

$$= -22.9 \text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_z = \frac{-p_{\max}}{\sqrt{1+\dfrac{z^2}{b^2}}} = \frac{-123.2}{\sqrt{1+0.786^2}} = -96.9 \text{ MPa} \qquad \textit{Ans.}$$

$$\tau_{\max} = \frac{\sigma_y - \sigma_z}{2} = \frac{-22.9-(-96.9)}{2} = 37.0 \text{ MPa} \qquad \textit{Ans.}$$

3-157

Use Eqs. (3-73) through (3-77).

$$b = \left(\frac{2F}{\pi l} \frac{\left(1-\nu_1^{\,2}\right)/E_1 + \left(1-\nu_2^{\,2}\right)/E_2}{1/d_1 + 1/d_2} \right)^{1/2}$$

$$= \left(\frac{2(250)}{\pi(1.25)} \frac{\left(1-0.211^2\right)/\left[14.5\left(10^6\right)\right] + \left(1-0.211^2\right)/\left[14.5\left(10^6\right)\right]}{1/3+1/\infty} \right)^{1/2}$$

$$b = 0.007\ 095 \text{ in}$$

$$p_{\max} = \frac{2F}{\pi b l} = \frac{2(250)}{\pi(0.007\ 095)(1.25)} = 17\ 946 \text{ psi}$$

$$\sigma_x = -2\nu p_{\max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right) = -2(0.211)(17\,946)\left(\sqrt{1+0.786^2} - 0.786\right)$$

$$= -3\ 680 \text{ psi} = -3.68 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_y = -p_{\max}\left(\frac{1+2\dfrac{z^2}{b^2}}{\sqrt{1+\dfrac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right| \right) = -17\,946\left(\frac{1+2\left(0.786^2\right)}{\sqrt{1+\left(0.786^2\right)}} - 2(0.786) \right)$$

$$= -3\ 332 \text{ psi} = -3.33 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_z = \frac{-p_{\max}}{\sqrt{1+\dfrac{z^2}{b^2}}} = \frac{-17\,946}{\sqrt{1+0.786^2}} = -14\,109 \text{ psi} = -14.1 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{\max} = \frac{\sigma_y - \sigma_z}{2} = \frac{-3\,332-(-14\,109)}{2} = 5\ 389 \text{ psi} = 5.39 \text{ kpsi} \qquad \textit{Ans.}$$

Chapter 4

4-1 For a torsion bar, $k_T = T/\theta = Fl/\theta$, and so $\theta = Fl/k_T$. For a cantilever, $k_l = F/\delta$, $\delta = F/k_l$. For the assembly, $k = F/y$, or, $y = F/k = l\theta + \delta$

Thus

$$y = \frac{F}{k} = \frac{Fl^2}{k_T} + \frac{F}{k_l}$$

Solving for k

$$k = \frac{1}{\dfrac{l^2}{k_T} + \dfrac{1}{k_l}} = \frac{k_l k_T}{k_l l^2 + k_T} \qquad Ans.$$

4-2 For a torsion bar, $k_T = T/\theta = Fl/\theta$, and so $\theta = Fl/k_T$. For each cantilever, $k_l = F/\delta_l$, $\delta_l = F/k_l$, and, $\delta_L = F/k_L$. For the assembly, $k = F/y$, or, $y = F/k = l\theta + \delta_l + \delta_L$.

Thus

$$y = \frac{F}{k} = \frac{Fl^2}{k_T} + \frac{F}{k_l} + \frac{F}{k_L}$$

Solving for k

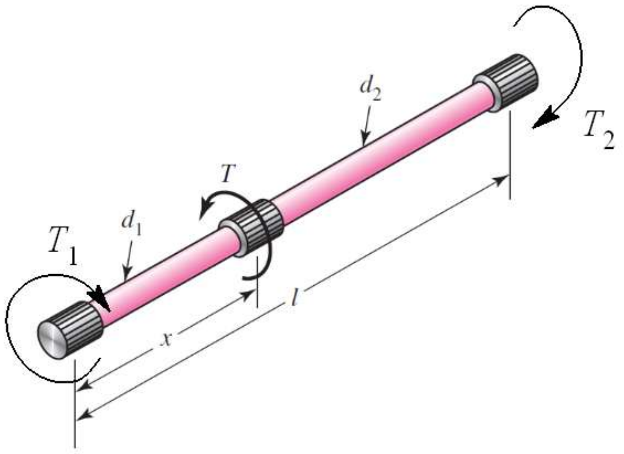
$$k = \frac{1}{\dfrac{l^2}{k_T} + \dfrac{1}{k_l} + \dfrac{1}{k_L}} = \frac{k_L k_l k_T}{k_l k_L l^2 + k_T k_L + k_T k_l} \qquad Ans.$$

4-3 (**a**) For a torsion bar, $k = T/\theta = GJ/l$.

Two springs in parallel, with $J = \pi d_i^{\,4}/32$, and $d_1 = d_2 = d$,

$$k = \frac{J_1 G}{x} + \frac{J_2 G}{l-x} = \frac{\pi}{32} G\left(\frac{d_1^4}{x} + \frac{d_2^4}{l-x}\right)$$

$$= \frac{\pi}{32} G d^4 \left(\frac{1}{x} + \frac{1}{l-x}\right) \qquad Ans. \quad (1)$$



Deflection equation,

$$\theta = \frac{T_1 x}{JG} = \frac{T_2 (l-x)}{JG}$$

results in $\quad T_1 = \dfrac{T_2 (l-x)}{x} \qquad (2)$

From statics, $T_1 + T_2 = T = 1500$. Substitute Eq. (2)

$$T_2\left(\frac{l-x}{x}\right)+T_2=1500 \quad \Rightarrow \quad T_2=1500\frac{x}{l} \qquad Ans. \qquad (3)$$

Substitute into Eq. (2) resulting in $T_1=1500\dfrac{l-x}{l}$ *Ans.* (4)

(**b**) From Eq. (1), $k=\dfrac{\pi}{32}\left(0.5^4\right)11.5\left(10^6\right)\left(\dfrac{1}{5}+\dfrac{1}{10-5}\right)=28.2\left(10^3\right)$ lbf · in/rad *Ans.*

From Eq. (4), $T_1=1500\dfrac{10-5}{10}=750$ lbf · in *Ans.*

From Eq. (3), $T_2=1500\dfrac{5}{10}=750$ lbf · in *Ans.*

From either section, $\tau=\dfrac{16T_i}{\pi d_i^3}=\dfrac{16(1500)}{\pi\left(0.5^3\right)}=30.6\left(10^3\right)$ psi $=30.6$ kpsi *Ans.*

4-4 Deflection to be the same as Prob. 4-3 where $T_1 = 750$ lbf·in, $l_1 = l / 2 = 5$ in, and $d_1 = 0.5$ in

$$\theta_1 = \theta_2 = \theta$$

$$\frac{T_1(4)}{\frac{\pi}{32}d_1^4G}=\frac{T_2(6)}{\frac{\pi}{32}d_2^4G}=\frac{750(5)}{\frac{\pi}{32}\left(0.5^4\right)G} \quad \Rightarrow \quad \frac{4T_1}{d_1^4}=\frac{6T_2}{d_2^4}=60\left(10^3\right) \qquad (1)$$

Or, $T_1=15\left(10^3\right)d_1^4$ (2)

$T_2=10\left(10^3\right)d_2^4$ (3)

Equal stress, $\tau_1=\tau_2 \quad \Rightarrow \quad \dfrac{16T_1}{\pi d_1^3}=\dfrac{16T_2}{\pi d_2^3} \quad \Rightarrow \quad \dfrac{T_1}{d_1^3}=\dfrac{T_2}{d_2^3}$ (4)

Divide Eq. (4) by the first two equations of Eq.(1) results in

$$\frac{\dfrac{T_1}{d_1^3}}{\dfrac{4T_1}{d_1^4}}=\frac{\dfrac{T_2}{d_2^3}}{\dfrac{6T_2}{d_2^4}} \quad \Rightarrow \quad d_2=1.5d_1 \qquad (5)$$

Statics, $T_1 + T_2 = 1500$ (6)

Substitute in Eqs. (2) and (3), with Eq. (5) gives

$$15\left(10^3\right)d_1^4+10\left(10^3\right)\left(1.5d_1\right)^4=1500$$

Solving for d_1 and substituting it back into Eq. (5) gives

$d_1 = 0.388\ 8$ in, $d_2 = 0.583\ 2$ in *Ans.*

From Eqs. (2) and (3),

$T_1 = 15(10^3)(0.388\ 8)^4 = 343$ lbf·in *Ans.*
$T_2 = 10(10^3)(0.583\ 2)^4 = 1\ 157$ lbf·in *Ans.*

Deflection of T is $$\theta_1 = \frac{T_1 l_1}{J_1 G} = \frac{343(4)}{(\pi/32)(0.388\ 8^4)11.5(10^6)} = 0.053\ 18 \text{ rad}$$

Spring constant is $$k = \frac{T}{\theta_1} = \frac{1500}{0.053\ 18} = 28.2(10^3) \text{ lbf}\cdot\text{in} \qquad Ans.$$

The stress in d_1 is $$\tau_1 = \frac{16T_1}{\pi d_1^3} = \frac{16(343)}{\pi(0.388\ 8)^3} = 29.7(10^3) \text{ psi} = 29.7 \text{ kpsi} \qquad Ans.$$

The stress in d_1 is $$\tau_2 = \frac{16T_2}{\pi d_2^3} = \frac{16(1\ 157)}{\pi(0.583\ 2)^3} = 29.7(10^3) \text{ psi} = 29.7 \text{ kpsi} \qquad Ans.$$

4-5 (**a**) Let the radii of the straight sections be $r_1 = d_1/2$ and $r_2 = d_2/2$. Let the angle of the taper be α where $\tan\alpha = (r_2 - r_1)/2$. Thus, the radius in the taper as a function of x is $r = r_1 + x\tan\alpha$, and the area is $A = \pi(r_1 + x\tan\alpha)^2$. The deflection of the tapered portion is

$$\delta = \int_0^l \frac{F}{AE}dx = \frac{F}{\pi E}\int_0^l \frac{dx}{(r_1 + x\tan\alpha)^2} = -\frac{F}{\pi E}\left.\frac{1}{(r_1 + x\tan\alpha)\tan\alpha}\right|_0^l$$
$$= \frac{F}{\pi E}\left[\frac{1}{r_1\tan\alpha} - \frac{1}{\tan\alpha(r_1 + l\tan\alpha)}\right] = \frac{F}{\pi E\tan\alpha}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$
$$= \frac{F}{\pi E\tan\alpha}\frac{r_2 - r_1}{r_1 r_2} = \frac{F}{\pi E\tan\alpha}\frac{l\tan\alpha}{r_1 r_2} = \frac{Fl}{\pi r_1 r_2 E}$$
$$= \frac{4Fl}{\pi d_1 d_2 E} \qquad Ans.$$

(**b**) For section 1,

$$\delta_1 = \frac{Fl}{AE} = \frac{4Fl}{\pi d_1^2 E} = \frac{4(1000)(2)}{\pi(0.5^2)(30)(10^6)} = 3.40(10^{-4}) \text{ in} \qquad Ans.$$

For the tapered section,

$$\delta = \frac{4}{\pi}\frac{Fl}{d_1 d_2 E} = \frac{4}{\pi}\frac{1000(2)}{(0.5)(0.75)(30)(10^6)} = 2.26(10^{-4}) \text{ in} \qquad Ans.$$

For section 2,

$$\delta_2 = \frac{Fl}{AE} = \frac{4Fl}{\pi d_1^2 E} = \frac{4(1000)(2)}{\pi(0.75^2)(30)(10^6)} = 1.51(10^{-4})\ \text{in} \qquad \textit{Ans.}$$

4-6 (**a**) Let the radii of the straight sections be $r_1 = d_1/2$ and $r_2 = d_2/2$. Let the angle of the taper be α where $\tan\alpha = (r_2 - r_1)/2$. Thus, the radius in the taper as a function of x is $r = r_1 + x\tan\alpha$, and the polar second area moment is $J = (\pi/2)(r_1 + x\tan\alpha)^4$. The angular deflection of the tapered portion is

$$\theta = \int_0^l \frac{T}{GJ}dx = \frac{2T}{\pi G}\int_0^l \frac{dx}{(r_1 + x\tan\alpha)^4} = -\frac{1}{3}\frac{2T}{\pi G}\frac{1}{(r_1 + x\tan\alpha)^3\tan\alpha}\Bigg|_0^l$$

$$= \frac{2}{3\pi}\frac{T}{G}\left[\frac{1}{r_1^3\tan\alpha} - \frac{1}{\tan\alpha(r_1 + l\tan\alpha)^3}\right] = \frac{2}{3\pi}\frac{T}{G\tan\alpha}\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right)$$

$$= \frac{2}{3\pi}\frac{T}{G\tan\alpha}\frac{r_2^3 - r_1^3}{r_1^3 r_2^3} = \frac{2}{3\pi}\frac{T}{G}\left(\frac{l}{r_2 - r_1}\right)\frac{r_2^3 - r_1^3}{r_1^3 r_2^3} = \frac{2}{3\pi}\frac{Tl}{G}\frac{(r_1^2 + r_1 r_2 + r_2^2)}{r_1^3 r_2^3}$$

$$= \frac{32}{3\pi}\frac{Tl}{G}\frac{(d_1^2 + d_1 d_2 + d_2^2)}{d_1^3 d_2^3} \qquad \textit{Ans.}$$

(**b**) The deflections, in degrees, are:
Section 1,

$$\theta_1 = \frac{Tl}{GJ}\left(\frac{180}{\pi}\right) = \frac{32Tl}{\pi d_1^4 G}\left(\frac{180}{\pi}\right) = \frac{32(1500)(2)}{\pi(0.5^4)11.5(10^6)}\left(\frac{180}{\pi}\right) = 2.44\ \text{deg} \qquad \textit{Ans.}$$

Tapered section,

$$\theta = \frac{32}{3\pi}\frac{Tl(d_1{}^2 + d_1 d_2 + d_2{}^2)}{Gd_1{}^3 d_2{}^3}\left(\frac{180}{\pi}\right)$$

$$= \frac{32}{3\pi}\frac{(1500)(2)\left[0.5^2 + (0.5)(0.75) + 0.75^2\right]}{11.5(10^6)(0.5^3)(.75^3)}\left(\frac{180}{\pi}\right) = 1.14\ \text{deg} \qquad \textit{Ans.}$$

Section 2,

$$\theta_2 = \frac{Tl}{GJ}\left(\frac{180}{\pi}\right) = \frac{32Tl}{\pi d_2^4 G}\left(\frac{180}{\pi}\right) = \frac{32(1500)(2)}{\pi(0.75^4)11.5(10^6)}\left(\frac{180}{\pi}\right) = 0.481\ \text{deg} \qquad \textit{Ans.}$$

4-7 The area and the elastic modulus remain constant. However, the force changes with respect to x. From Table A-5, the unit weight of steel is $\gamma = 0.282$ lbf/in^3 and the elastic modulus is $E = 30$ Mpsi. Starting from the top of the cable (i.e. $x = 0$, at the top).

$$F = \gamma(A)(l - x)$$

$$\delta_c = \int_o^l \frac{F dx}{AE} = \frac{w}{E}\int_0^l (l-x)dx = \frac{\gamma}{E}\left(lx - \frac{1}{2}x^2\right)\Bigg|_0^l = \frac{\gamma l^2}{2E} = \frac{0.282\left[500(12)\right]^2}{2(30)10^6} = 0.169 \text{ in}$$

From the weight at the bottom of the cable,

$$\delta_W = \frac{Wl}{AE} = \frac{4Wl}{\pi d^2 E} = \frac{4(5000)\left[500(12)\right]}{\pi(0.5^2)30(10^6)} = 5.093 \text{ in}$$

$$\delta = \delta_c + \delta_W = 0.169 + 5.093 = 5.262 \text{ in} \qquad \textit{Ans.}$$

The percentage of total elongation due to the cable's own weight

$$\frac{0.169}{5.262}(100) = 3.21\% \qquad \textit{Ans.}$$

4-8 $\Sigma F_y = 0 = R_1 - F \;\Rightarrow\; R_1 = F$
$\Sigma M_A = 0 = M_1 - Fa \;\Rightarrow\; M_1 = Fa$
$V_{AB} = F,\ M_{AB} = F(x-a),\ V_{BC} = M_{BC} = 0$

Section *AB*:

$$\theta_{AB} = \frac{1}{EI}\int F(x-a)dx = \frac{F}{EI}\left(\frac{x^2}{2} - ax\right) + C_1 \qquad (1)$$

$\theta_{AB} = 0$ at $x = 0 \;\Rightarrow\; C_1 = 0$

$$y_{AB} = \frac{F}{EI}\int\left(\frac{x^2}{2} - ax\right)dx = \frac{F}{EI}\left(\frac{x^3}{6} - a\frac{x^2}{2}\right) + C_2 \qquad (2)$$

$y_{AB} = 0$ at $x = 0 \;\Rightarrow\; C_2 = 0$, and

$$y_{AB} = \frac{Fx^2}{6EI}(x - 3a) \qquad \textit{Ans.}$$

Section *BC*:

$$\theta_{BC} = \frac{1}{EI}\int(0)dx = 0 + C_3$$

From Eq. (1), at $x = a$ (with $C_1 = 0$), $\theta = \dfrac{F}{EI}\left(\dfrac{a^2}{2} - a(a)\right) = -\dfrac{Fa^2}{2EI} = C_3$. Thus,

$$\theta_{BC} = -\frac{Fa^2}{2EI}$$

$$y_{BC} = -\frac{Fa^2}{2EI}\int dx = -\frac{Fa^2}{2EI}x + C_4 \qquad (3)$$

From Eq. (2), at $x = a$ (with $C_2 = 0$), $y = \frac{F}{EI}\left(\frac{a^3}{6} - a\frac{a^2}{2}\right) = -\frac{Fa^3}{3EI}$. Thus, from Eq. (3)

$-\frac{Fa^2}{2EI}a + C_4 = -\frac{Fa^3}{3EI} \quad \Rightarrow \quad C_4 = \frac{Fa^3}{6EI}$ Substitute into Eq. (3), obtaining

$$y_{BC} = -\frac{Fa^2}{2EI}x + \frac{Fa^3}{6EI} = \frac{Fa^2}{6EI}(a - 3x) \qquad \textit{Ans.}$$

The maximum deflection occurs at $x = l$,

$$y_{\max} = \frac{Fa^2}{6EI}(a - 3l) \qquad \textit{Ans.}$$

4-9 $\Sigma M_C = 0 = F(l/2) - R_1 l \quad \Rightarrow \quad R_1 = F/2$

$\Sigma F_y = 0 = F/2 + R_2 - F \quad \Rightarrow \quad R_2 = F/2$

Break at $0 \le x \le l/2$:

$V_{AB} = R_1 = F/2, \qquad M_{AB} = R_1 x = Fx/2$

Break at $l/2 \le x \le l$:

$V_{BC} = R_1 - F = -R_2 = -F/2, \qquad M_{BC} = R_1 x - F(x - l/2) = F(l - x)/2$

Section *AB*:

$$\theta_{AB} = \frac{1}{EI}\int \frac{Fx}{2}dx = \frac{F}{EI}\frac{x^2}{4} + C_1$$

From symmetry, $\theta_{AB} = 0$ at $x = l/2 \Rightarrow \frac{F\left(\frac{l}{2}\right)^2}{4EI} + C_1 = 0 \quad \Rightarrow \quad C_1 = -\frac{Fl^2}{16EI}$. Thus,

$$\theta_{AB} = \frac{F}{EI}\frac{x^2}{4} - \frac{Fl^2}{16EI} = \frac{F}{16EI}\left(4x^2 - l^2\right) \qquad (1)$$

$$y_{AB} = \frac{F}{16EI}\int\left(4x^2 - l^2\right)dx = \frac{F}{16EI}\left(\frac{4x^3}{3} - l^2x\right) + C_2$$

$y_{AB} = 0$ at $x = 0$ $\Rightarrow$ $C_2 = 0$, and,

$$y_{AB} = \frac{Fx}{48EI}\left(4x^2 - 3l^2\right) \qquad (2)$$

y_{BC} is not given, because with symmetry, Eq. (2) can be used in this region. The maximum deflection occurs at $x = l/2$,

$$y_{\max} = \frac{F\left(\frac{l}{2}\right)}{48EI}\left[4\left(\frac{l}{2}\right)^2 - 3l^2\right] = -\frac{Fl^3}{48EI} \qquad Ans.$$

4-10 From Table A-6, for each angle, $I_{1\text{-}1} = 207$ cm^4. Thus, $I = 2(207)\,(10^4) = 4.14(10^6)$ mm^4

From Table A-9, use beam 2 with $F = 2500$ N, $a = 2000$ mm, and $l = 3000$ mm; and beam 3 with $w = 1$ N/mm and $l = 3000$ mm.

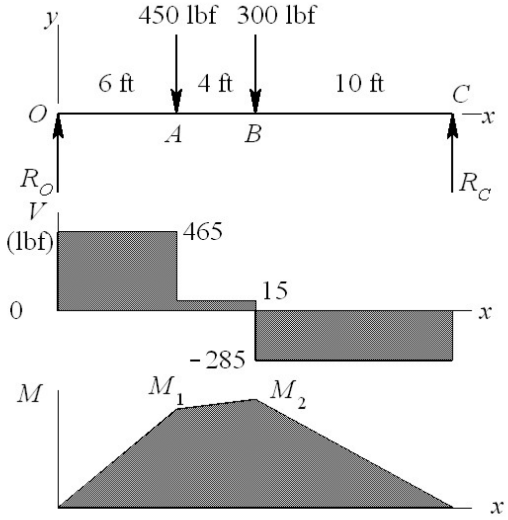
$$\begin{aligned} y_{\max} &= \frac{Fa^2}{6EI}(a - 3l) - \frac{wl^4}{8EI} \\ &= \frac{2500(2000)^2}{6(207)10^3(4.14)10^6}\left[2000 - 3(3000)\right] - \frac{(1)(3000)^4}{8(207)(10^3)(4.14)(10^6)} \\ &= -25.4 \text{ mm} \qquad Ans. \end{aligned}$$

$$\begin{aligned} M_O &= -Fa - (wl^2/2) \\ &= -2500(2000) - [1(3000^2)/2] = -9.5(10^6) \text{ N·mm} \end{aligned}$$

From Table A-6, from centroid to upper surface is $y = 29$ mm. From centroid to bottom surface is $y = 29.0 - 100 = -71$ mm. The maximum stress is compressive at the bottom of the beam at the wall. This stress is

$$\sigma_{\max} = -\frac{My}{I} = -\frac{-9.5(10^6)(-71)}{4.14(10^6)} = -163 \text{ MPa} \qquad Ans.$$

4-11



$$R_O = \frac{14}{20}(450) + \frac{10}{20}(300) = 465 \text{ lbf}$$

$$R_C = \frac{6}{20}(450) + \frac{10}{20}(300) = 285 \text{ lbf}$$

$$M_1 = 465(6)12 = 33.48(10^3) \text{ lbf·in}$$

$$M_2 = 33.48(10^3) + 15(4)12$$

$$= 34.20(10^3) \text{ lbf·in}$$

$$\sigma_{\max} = \frac{M_{\max}}{Z} \quad \Rightarrow \quad 15 = \frac{34.2}{Z} \qquad Z = 2.28 \text{ in}^3$$

For deflections, use beams 5 and 6 of Table A-9

$$y\big|_{x=10\text{ft}} = \frac{F_1 a[l-(l/2)]}{6EIl}\left[\left(\frac{l}{2}\right)^2 + a^2 - 2l\frac{l}{2}\right] - \frac{F_2 l^3}{48EI}$$

$$-0.5 = \frac{450(72)(120)}{6(30)(10^6)I(240)}\left(120^2 + 72^2 - 240^2\right) - \frac{300(240^3)}{48(30)(10^6)I}$$

$$I = 12.60 \text{ in}^4 \ \Rightarrow \ I/2 = 6.30 \text{ in}^4$$

Select two 5 in-6.7 lbf/ft channels from Table A-7, $I = 2(7.49) = 14.98 \text{ in}^4$, $Z = 2(3.00) = 6.00 \text{ in}^3$

$$y_{\text{midspan}} = \frac{12.60}{14.98}\left(-\frac{1}{2}\right) = -0.421 \text{ in}$$

$$\sigma_{\max} = \frac{34.2}{6.00} = 5.70 \text{ kpsi}$$

4-12 $$I = \frac{\pi}{64}(1.5^4) = 0.2485 \text{ in}^4$$

From Table A-9 by superposition of beams 6 and 7, at $x = a = 15$ in, with $b = 24$ in and $l = 39$ in

$$y = \frac{Fba}{6EIl}[a^2 + b^2 - l^2] + \frac{wa}{24EI}(2la^2 - a^3 - l^3)$$

$$y_A = \frac{340(24)15}{6(30)10^6(0.2485)39}\left[15^2 + 24^2 - 39^2\right]$$

$$+ \frac{(150/12)(15)}{24(30)10^6(0.2485)}\left[2(39)(15^2) - 15^3 - 39^3\right] = -0.0978 \text{ in} \qquad \textit{Ans.}$$

At $x = l/2 = 19.5$ in

$$y = \frac{Fa[l-(l/2)]}{6EIl}\left[\left(\frac{l}{2}\right)^2 + a^2 - 2l\frac{l}{2}\right] + \frac{w(l/2)}{24EI}\left[2l\left(\frac{l}{2}\right)^2 - \left(\frac{l}{2}\right)^3 - l^3\right]$$

$$y = \frac{340(15)(19.5)}{6(30)(10^6)(0.2485)(39)}\left[19.5^2 + 15^2 - 39^2\right]$$

$$+\frac{(150/12)(19.5)}{24(30)(10^6)(0.2485)}\left[2(39)(19.5^2) - 19.5^3 - 39^3\right] = -0.1027 \text{ in} \qquad \textit{Ans.}$$

$$\% \text{ difference} = \frac{-0.1027 + 0.0978}{-0.0978}(100) = 5.01\% \qquad \textit{Ans.}$$

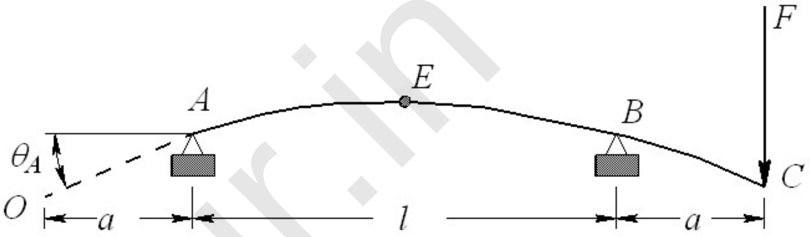
4-13 $I = \dfrac{1}{12}(6)(32^3) = 16.384\left(10^3\right) \text{ mm}^4$

From Table A-9-10, beam 10

$$y_C = -\frac{Fa^2}{3EI}(l+a)$$

$$y_{AB} = \frac{Fax}{6EIl}\left(l^2 - x^2\right)$$

$$\frac{dy_{AB}}{dx} = \frac{Fa}{6EIl}(l^2 - 3x^2)$$



At $x = 0$, $\dfrac{dy_{AB}}{dx} = \theta_A$

$$\theta_A = \frac{Fal^2}{6EIl} = \frac{Fal}{6EI}$$

$$y_O = -\theta_A a = -\frac{Fa^2 l}{6EI}$$

With both loads,

$$y_O = -\frac{Fa^2 l}{6EI} - \frac{Fa^2}{3EI}(l+a)$$

$$= -\frac{Fa^2}{6EI}(3l + 2a) = -\frac{400(300^2)}{6(207)10^3(16.384)10^3}\left[3(500) + 2(300)\right] = -3.72 \text{ mm} \qquad \textit{Ans.}$$

At midspan,

$$y_E = \frac{2Fa(l/2)}{6EIl}\left[l^2 - \left(\frac{l}{2}\right)^2\right] = \frac{3}{24}\frac{Fal^2}{EI} = \frac{3}{24}\frac{400(300)(500^2)}{207\left(10^3\right)16.384\left(10^3\right)} = 1.11 \text{ mm} \qquad \textit{Ans.}$$

4-14 $I = \dfrac{\pi}{64}(2^4 - 1.5^4) = 0.5369 \text{ in}^4$

From Table A-5, $E = 10.4$ Mpsi
From Table A-9, beams 1 and 2, by superposition

$$y_B = -\frac{F_B l^3}{3EI} + \frac{F_A a^2}{6EI}(a - 3l) = \frac{-200\left[4(12)\right]^3}{3(10.4)10^6(0.5369)} + \frac{300\left[2(12)\right]^2}{6(10.4)10^6(0.5369)}\left[2(12) - 3(4)(12)\right]$$

$$y_B = -1.94 \text{ in} \qquad \textit{Ans.}$$

4-15 From Table A-7, $I = 2(1.85) = 3.70$ in^4
From Table A-5, $E = 30.0$ Mpsi
From Table A-9, beams 1 and 3, by superposition

$$y_A = -\frac{Fl^3}{3EI} - \frac{(w + w_c)l^4}{8EI} = -\frac{150(60^3)}{3(30)10^6(3.70)} - \frac{\left[5 + 2(5/12)\right](60^4)}{8(30)10^6(3.70)} = -0.182 \text{ in} \qquad \textit{Ans.}$$

4-16 $I = \dfrac{\pi}{64}d^4$

From Table A-5, $E = 207(10^3)$ MPa
From Table A-9, beams 5 and 9, with $F_C = F_A = F$, by superposition

$$y_B = -\frac{F_B l^3}{48EI} + \frac{Fa}{24EI}(4a^2 - 3l^2) \quad \Rightarrow \quad I = \frac{1}{48Ey_B}\left[-F_B l^3 + 2Fa(4a^2 - 3l^2)\right]$$

$$I = \frac{1}{48(207)10^3(-2)}\left\{-550(1000^3) + 2(375)(250)\left[4(250^2) - 3(1000^2)\right]\right\}$$

$$= 53.624\left(10^3\right) \text{ mm}^4$$

$$d = \sqrt[4]{\frac{64}{\pi}I} = \sqrt[4]{\frac{64}{\pi}(53.624)10^3} = 32.3 \text{ mm} \qquad \textit{Ans.}$$

4-17 From Table A-9, beams 8 (region *BC* for this beam with $a = 0$) and 10 (with $a = a$), by superposition

$$y_{AB} = \frac{M_A}{6EIl}\left(x^3 - 3lx^2 + 2l^2x\right) + \frac{Fax}{6EIl}\left(l^2 - x^2\right)$$

$$= \frac{1}{6EIl}\left[M_A\left(x^3 - 3lx^2 + 2l^2x\right) + Fax\left(l^2 - x^2\right)\right] \qquad \textit{Ans.}$$

$$y_{BC} = \left\{\frac{d}{dx}\left[\frac{M_A}{6EIl}\left(x^3 - 3lx^2 + 2l^2x\right)\right]\right\}_{x=l}(x - l) + \frac{F(x - l)}{6EI}[(x - l)^2 - a(3x - l)]$$

$$= -\frac{M_A l}{6EI}(x - l) + \frac{F(x - l)}{6EI}[(x - l)^2 - a(3x - l)]$$

$$= \frac{(x - l)}{6EI}\left\{-M_A l + F\left[(x - l)^2 - a(3x - l)\right]\right\} \qquad \textit{Ans.}$$

4-18 **Note to the instructor:** Beams with discontinuous loading are better solved using singularity functions. This eliminates matching the slopes and displacements at the discontinuity as is done in this solution.

$$\sum M_C = 0 = R_1 l - wa\left(l - a + \frac{a}{2}\right) \quad \Rightarrow \quad R_1 = \frac{wa}{2l}(2l - a) \quad Ans.$$

$$\sum F_y = 0 = \frac{wa}{2l}(2l - a) + R_2 - wa \quad \Rightarrow \quad R_2 = \frac{wa^2}{2l} \quad Ans.$$

$$V_{AB} = R_1 - wx = \frac{wa}{2l}(2l - a) - wx = \frac{w}{2l}\left[2l(a - x) - a^2\right] \quad Ans.$$

$$V_{BC} = -R_2 = -\frac{wa^2}{2l} \quad Ans.$$

$$M_{AB} = \int V_{AB}dx = \frac{w}{2l}\left[2l\left(ax - \frac{x^2}{2}\right) - a^2 x\right] + C_1$$

$$M_{AB} = 0 \text{ at } x = 0 \ \therefore C_1 = 0 \quad \Rightarrow \quad M_{AB} = \frac{wx}{2l}\left[2al - a^2 - lx\right] \quad Ans.$$

$$M_{BC} = \int V_{BC}dx = \int -\frac{wa^2}{2l}dx = -\frac{wa^2}{2l}x + C_2$$

$$M_{BC} = 0 \text{ at } x = l \ \therefore \ C_2 = \frac{wa^2}{2} \quad \Rightarrow \quad M_{BC} = \frac{wa^2}{2l}(l - x) \quad Ans.$$

$$\theta_{AB} = \int \frac{M_{AB}}{EI}dx = \frac{1}{EI}\int \frac{wx}{2l}\left(2al - a^2 - lx\right)dx = \frac{1}{EI}\left[\frac{w}{2l}\left(alx^2 - \frac{1}{2}a^2x^2 - \frac{1}{3}lx^3\right) + C_3\right]$$

$$y_{AB} = \int \theta_{AB}dx = \frac{1}{EI}\int\left[\frac{w}{2l}\left(alx^2 - \frac{1}{2}a^2x^2 - \frac{1}{3}lx^3\right) + C_3\right]dx$$

$$= \frac{1}{EI}\left[\frac{w}{2l}\left(\frac{1}{3}alx^3 - \frac{1}{6}a^2x^3 - \frac{1}{12}lx^4\right) + C_3x + C_4\right]$$

$$y_{AB} = 0 \text{ at } x = 0 \ \therefore \ C_4 = 0$$

$$\theta_{BC} = \int \frac{M_{BC}}{EI}dx = \frac{1}{EI}\int \frac{wa^2}{2l}(l - x)\,dx = \frac{1}{EI}\left[\frac{wa^2}{2l}\left(lx - \frac{1}{2}x^2\right) + C_5\right]$$

$$\theta_{AB} = \theta_{BC} \text{ at } x = a \therefore$$

$$\frac{1}{EI}\left[\frac{w}{2l}\left(ala^2 - \frac{1}{2}a^4 - \frac{1}{3}la^3\right) + C_3\right] = \frac{1}{EI}\left[\frac{wa^2}{2l}\left(la - \frac{1}{2}a^2\right) + C_5\right] \Rightarrow C_3 = \frac{wa^3}{6} + C_5 \qquad (1)$$

$$y_{BC}=\int \theta_{BC}\,dx=\frac{1}{EI}\int\left[\frac{wa^2}{2l}\left(lx-\frac{1}{2}x^2\right)+C_5\right]dx=\frac{1}{EI}\left[\frac{wa^2}{2l}\left(\frac{1}{2}lx^2-\frac{1}{6}x^3\right)+C_5x+C_6\right]$$

$y_{BC}=0$ at $x=l \;\therefore\; C_6=-\dfrac{wa^2l^2}{6}-C_5l$

$$y_{BC}=\frac{1}{EI}\left[\frac{wa^2}{2l}\left(\frac{1}{2}lx^2-\frac{1}{6}x^3-\frac{1}{3}l^3\right)+C_5(x-l)\right]$$

$y_{AB}=y_{BC}$ at $x=a \;\therefore$

$$\frac{w}{2l}\left(\frac{1}{3}ala^3-\frac{1}{6}a^5-\frac{1}{12}la^4\right)+C_3a=\frac{wa^2}{2l}\left(\frac{1}{2}la^2-\frac{1}{6}a^3-\frac{1}{3}l^3\right)+C_5(a-l)$$

$$C_3a=\frac{wa^2}{24l}\left(3la^2-4l^3\right)+C_5(a-l) \qquad (2)$$

Substituting (1) into (2) yields $C_5=\dfrac{wa^2}{24l}\left(-a^2-4l^2\right)$. Substituting this back into (2) gives

$C_3=\dfrac{wa^2}{24l}\left(4al-a^2-4l^2\right)$. Thus,

$$y_{AB}=\frac{w}{24EIl}\left(4alx^3-2a^2x^3-lx^4+4a^3lx-a^4x-4a^2l^2x\right)$$

$$\Rightarrow \quad y_{AB}=\frac{wx}{24EIl}\left[2ax^2(2l-a)-lx^3-a^2(2l-a)^2\right] \qquad Ans.$$

$$y_{BC}=\frac{w}{24EIl}\left(6a^2lx^2-2a^2x^3-a^4x-4a^2l^2x+a^4l\right) \qquad Ans.$$

This result is sufficient for y_{BC}. However, this can be shown to be equivalent to

$$y_{BC}=\frac{w}{24EIl}\left(4alx^3-2a^2x^3-lx^4-4a^2l^2x+4a^3lx-a^4x\right)+\frac{w}{24EI}(x-a)^4$$

$$y_{BC}=y_{AB}+\frac{w}{24EI}(x-a)^4 \qquad Ans.$$

by expanding this or by solving the problem using singularity functions.

4-19 The beam can be broken up into a uniform load w downward from points A to C and a uniform load w upward from points A to B. Using the results of Prob. 4-18, with $b = a$ for A to C and $a = a$ for A to B, results in

$$y_{AB}=\frac{wx}{24EIl}\left[2bx^2(2l-b)-lx^3-b^2(2l-b)^2\right]-\frac{wx}{24EIl}\left[2ax^2(2l-a)-lx^3-a^2(2l-a)^2\right]$$

$$=\frac{wx}{24EIl}\left[2bx^2(2l-b)-b^2(2l-b)^2-2ax^2(2l-a)+a^2(2l-a)^2\right] \qquad Ans.$$

$$y_{BC}=\frac{w}{24EIl}\Big[2bx^3(2l-b)-lx^4-b^2x(2l-b)^2$$
$$-\left(4alx^3-2a^2x^3-lx^4-4a^2l^2x+4a^3lx-a^4x\right)-l(x-a)^4\Big] \qquad Ans.$$

$$\begin{aligned} y_{CD} &= \frac{w}{24EIl}\left[4blx^3 - 2b^2x^3 - lx^4 - 4b^2l^2x + 4b^3lx - b^4x + l(x-b)^4\right] \\ &\quad - \frac{w}{24EIl}\left[4alx^3 - 2a^2x^3 - lx^4 - 4a^2l^2x + 4a^3lx - a^4x + l(x-a)^4\right] \\ &= \frac{w}{24EI}\left[(x-b)^4 - (x-a)^4\right] + y_{AB} \qquad Ans. \end{aligned}$$

4-20 **Note to the instructor:** See the note in the solution for Problem 4-18.

$$\sum F_y = 0 = R_B - \frac{wa^2}{2l} - wa \quad \Rightarrow \quad R_B = \frac{wa}{2l}(2l + a) \qquad Ans.$$

For region BC, isolate right-hand element of length $(l + a - x)$

$$V_{AB} = -R_A = -\frac{wa^2}{2l}, \qquad V_{BC} = w(l + a - x) \qquad Ans.$$

$$M_{AB} = -R_A x = -\frac{wa^2}{2l}x, \qquad M_{BC} = -\frac{w}{2}(l + a - x)^2 \qquad Ans.$$

$$EI\theta_{AB} = \int M_{AB}\,dx = -\frac{wa^2}{4l}x^2 + C_1$$

$$EIy_{AB} = -\frac{wa^2}{12l}x^3 + C_1x + C_2$$

$y_{AB} = 0$ at $x = 0$ $\quad \Rightarrow \quad C_2 = 0 \quad \therefore \quad EIy_{AB} = -\dfrac{wa^2}{12l}x^3 + C_1x$

$y_{AB} = 0$ at $x = l$ $\quad \Rightarrow \quad C_1 = \dfrac{wa^2l}{12} \quad \therefore$

$$EIy_{AB} = -\frac{wa^2}{12l}x^3 + \frac{wa^2l}{12}x = \frac{wa^2x}{12l}\left(l^2 - x^2\right) \quad \Rightarrow \quad y_{AB} = \frac{wa^2x}{12EIl}\left(l^2 - x^2\right) \qquad Ans.$$

$$EI\theta_{BC} = \int M_{BC}\,dx = -\frac{w}{6}(l + a - x)^3 + C_3$$

$$EIy_{BC} = -\frac{w}{24}(l + a - x)^4 + C_3x + C_4$$

$y_{BC} = 0$ at $x = l$ $\quad \Rightarrow \quad -\dfrac{wa^4}{24} + C_3l + C_4 = 0 \quad \Rightarrow \quad C_4 = \dfrac{wa^4}{24} - C_3l \qquad (1)$

$\theta_{AB} = \theta_{BC}$ at $x = l$ $\quad \Rightarrow \quad -\dfrac{wa^2l}{4} + \dfrac{wa^2l}{12} = \dfrac{wa^3}{6} + C_3 \quad \Rightarrow \quad C_3 = -\dfrac{wa^2}{6}(l + a)$

Substitute C_3 into Eq. (1) gives $\quad C_4 = \dfrac{wa^2}{24}\left[a^2 + 4l(l + a)\right]$. Substitute back into y_{BC}

$$\begin{aligned} y_{BC} &= \frac{1}{EI}\left[-\frac{w}{24}(l + a - x)^4 - \frac{wa^2}{6}x(l + a) + \frac{wa^4}{24} + \frac{wa^2l}{6}(l + a)\right] \\ &= -\frac{w}{24EI}\left[(l + a - x)^4 - 4a^2(l - x)(l + a) - a^4\right] \qquad Ans. \end{aligned}$$

4-21 Table A-9, beam 7,

$$R_1 = R_2 = \frac{wl}{2} = \frac{100(10)}{2} = 500 \text{ lbf} \uparrow$$

$$y_{AB} = \frac{wx}{24EI}\left(2lx^2 - x^3 - l^3\right) = \frac{100x}{24(30)10^6(0.05)}\left[2(10)x^2 - x^3 - 10^3\right]$$

$$= 2.7778\left(10^{-6}\right)x\left(20x^2 - x^3 - 1000\right)$$

Slope: $\theta_{AB} = \dfrac{d\,y_{AB}}{d\,x} = \dfrac{w}{24EI}\left(6lx^2 - 4x^3 - l^3\right)$

At $x = l$, $\theta_{AB}\big|_{x=l} = \dfrac{w}{24EI}\left(6l\,l^2 - 4l^3 - l^3\right) = \dfrac{wl^3}{24EI}$

$$y_{BC} = \theta_{AB}\big|_{x=l}(x-l) = \frac{wl^3}{24EI}(x-l) = \frac{100\left(10^3\right)}{24(30)10^6(0.05)}(x-10) = 2.7778\left(10^{-3}\right)(x-10)$$

From Prob. 4-20,

$$R_A = \frac{wa^2}{2l} = \frac{100\left(4^2\right)}{2(10)} = 80 \text{ lbf} \downarrow \qquad R_B = \frac{wa}{2l}(2l+a) = \frac{100(4)}{2(10)}\left[2(10)+4\right] = 480 \text{ lbf} \uparrow$$

$$y_{AB} = \frac{wa^2x}{12EIl}\left(l^2 - x^2\right) = \frac{100\left(4^2\right)x}{12(30)10^6(0.05)}\left(10^2 - x^2\right) = 8.8889\left(10^{-6}\right)x\left(100 - x^2\right)$$

$$y_{BC} = -\frac{w}{24EI}\left[(l+a-x)^4 - 4a^2(l-x)(l+a) - a^4\right]$$

$$= -\frac{100}{24(30)10^6(0.05)}\left[(10+4-x)^4 - 4\left(4^2\right)(10-x)(10+4) - 4^4\right]$$

$$= -2.7778\left(10^{-6}\right)\left[(14-x)^4 + 896x - 9216\right]$$

Superposition,

$R_A = 500 - 80 = 420 \text{ lbf} \uparrow \qquad R_B = 500 + 480 = 980 \text{ lbf} \uparrow$ *Ans.*

$y_{AB} = 2.7778\left(10^{-6}\right)x\left(20x^2 - x^3 - 1000\right) + 8.8889\left(10^{-6}\right)x\left(100 - x^2\right)$ *Ans.*

$y_{BC} = 2.7778\left(10^{-3}\right)(x-10) - 2.7778\left(10^{-6}\right)\left[(14-x)^4 + 896x - 9216\right]$ *Ans.*

The deflection equations can be simplified further. However, they are sufficient for plotting.

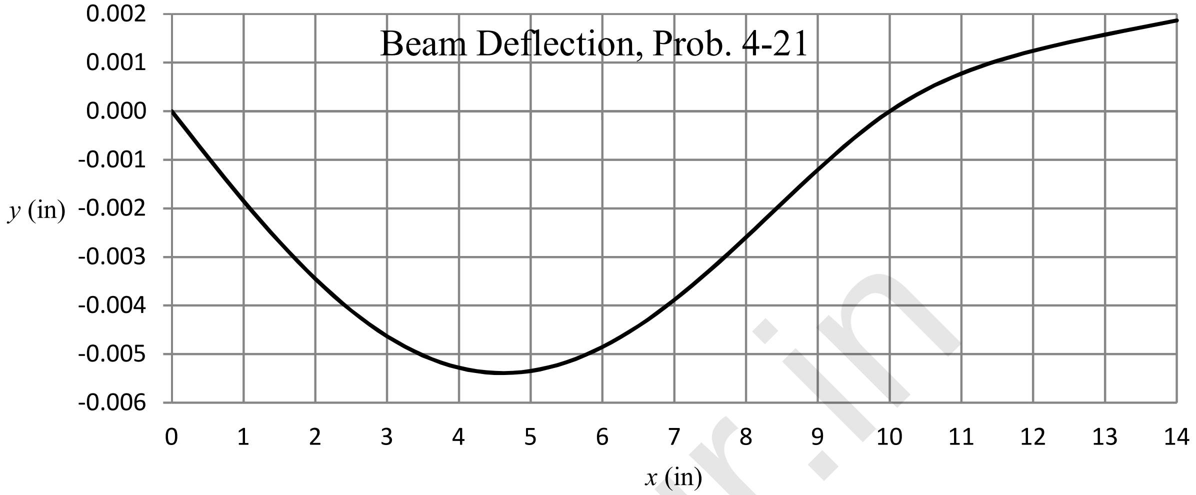
Using a spreadsheet,

x	0	0.5	1	1.5	2	2.5	3	3.5
y	0.000000	-0.000939	-0.001845	-0.002690	-0.003449	-0.004102	-0.004632	-0.005027

x	4	4.5	5	5.5	6	6.5	7	7.5
y	-0.005280	-0.005387	-0.005347	-0.005167	-0.004853	-0.004421	-0.003885	-0.003268

x	8	8.5	9	9.5	10	10.5	11	11.5
y	-0.002596	-0.001897	-0.001205	-0.000559	0.000000	0.000439	0.000775	0.001036

x	12	12.5	13	13.5	14
y	0.001244	0.001419	0.001575	0.001722	0.001867



4-22 (**a**) Useful relations

$$k = \frac{F}{y} = \frac{48EI}{l^3}$$

$$I = \frac{kl^3}{48E} = \frac{1800\left(36^3\right)}{48(30)10^6} = 0.05832 \text{ in}^4$$

From $I = bh^3/12$, and $b = 10\,h$, then $I = 5\,h^4/6$, or,

$$h = \sqrt[4]{\frac{6I}{5}} = \sqrt[4]{\frac{6(0.05832)}{5}} = 0.514 \text{ in}$$

h is close to 1/2 in and 9/16 in, while b is close to 5.14 in. Changing the height drastically changes the spring rate, so changing the base will make finding a close solution easier. Trial and error was applied to find the combination of values from Table A-17 that yielded the closet desired spring rate.

h (in)	b (in)	b/h	k (lbf/in)
1/2	5	10	1608
1/2	5½	11	1768
1/2	5¾	11.5	1849
9/16	5	8.89	2289
9/16	4	7.11	1831

h = ½ in, b = 5 ½ in should be selected because it results in a close spring rate and b/h is still reasonably close to 10.

(b) $I = 5.5(0.5)^3 / 12 = 0.05729\ \text{in}^4$

$$\sigma = \frac{Mc}{I} = \frac{(Fl/4)c}{I} \quad \Rightarrow \quad F = \frac{4\sigma I}{lc} = \frac{4(60)10^3(0.05729)}{(36)(0.25)} = 1528\ \text{lbf}$$

$$y = \frac{Fl^3}{48EI} = \frac{(1528)\left(36^3\right)}{48(30)10^6(0.05729)} = 0.864\ \text{in} \qquad Ans.$$

4-23 From the solutions to Prob. 3-79, $T_1 = 60$ lbf and $T_2 = 400$ lbf

$$I = \frac{\pi d^4}{64} = \frac{\pi(1.25)^4}{64} = 0.1198\ \text{in}^4$$

From Table A-9, beam 6,

$$z_A = \left[\frac{F_1 b_1 x}{6EIl}(x^2 + b_1^2 - l^2) + \frac{F_2 b_2 x}{6EIl}(x^2 + b_2^2 - l^2)\right]_{x=10\text{in}}$$

$$= \frac{(-575)(30)(10)}{6(30)10^6(0.1198)(40)}\left(10^2 + 30^2 - 40^2\right)$$

$$+ \frac{460(12)(10)}{6(30)10^6(0.1198)(40)}\left(10^2 + 12^2 - 40^2\right) = 0.0332\ \text{in} \qquad Ans.$$

$$(\theta_A)_y = -\left(\frac{dz}{dx}\right)_{x=10\text{in}} = -\left\{\frac{d}{dx}\left[\frac{F_1 b_1 x}{6EIl}(x^2 + b_1^2 - l^2) + \frac{F_2 b_2 x}{6EIl}(x^2 + b_2^2 - l^2)\right]\right\}_{x=10\text{in}}$$

$$= -\left\{\frac{F_1 b_1}{6EIl}(3x^2 + b_1^2 - l^2) + \frac{F_2 b_2}{6EIl}(3x^2 + b_2^2 - l^2)\right\}_{x=10\text{in}}$$

$$= -\frac{(575)(30)}{6(30)10^6(0.1198)(40)}\left[3\left(10^2\right) + 30^2 - 40^2\right]$$

$$- \frac{-460(12)}{6(30)10^6(0.1198)(40)}\left[3\left(10^2\right) + 12^2 - 40^2\right]$$

$$= 6.02(10^{-4})\ \text{rad} \qquad Ans.$$

4-24 From the solutions to Prob. 3-80, $T_1 = 2880$ N and $T_2 = 432$ N

$$I = \frac{\pi d^4}{64} = \frac{\pi(30)^4}{64} = 39.76\left(10^3\right)\ \text{mm}^4$$

The load in between the supports supplies an angle to the overhanging end of the beam. That angle is found by taking the derivative of the deflection from that load. From Table A-9, beams 6 (subscript 1) and 10 (subscript 2),

$$y_A = \left[\theta_{BC}\big|_C \left(a_2\right)\right]_{\text{beam}\,6} + \left(y_A\right)_{\text{beam}\,10} \qquad (1)$$

$$\theta_{BC}\big|_C = \left\{\frac{d}{dx}\left[\frac{F_1 a_1 (l-x)}{6EIl}\left(x^2 + a_1{}^2 - 2lx\right)\right]\right\}_{x=l} = \left[\frac{F_1 a_1}{6EIl}\left(6lx - 3x^2 - a_1{}^2 - 2l^2\right)\right]_{x=l}$$

$$= \frac{F_1 a_1}{6EIl}\left(l^2 - a_1{}^2\right)$$

Equation (1) is thus

$$y_A = \frac{F_1 a_1}{6EIl}\left(l^2 - a_1{}^2\right)a_2 - \frac{F_2 a_2{}^2}{3EI}(l + a_2)$$

$$= \frac{-3312(230)}{6(207)10^3(39.76)10^3(510)}\left(510^2 - 230^2\right)(300) - \frac{2070(300^2)}{3(207)10^3(39.76)10^3}\left(510 + 300\right)$$

$$= -7.99 \text{ mm} \qquad \textit{Ans.}$$

The slope at A, relative to the z axis is

$$\left(\theta_A\right)_z = \frac{F_1 a_1}{6EIl}(l^2 - a_1{}^2) + \left\{\frac{d}{dx}\left[\frac{F_2 (x-l)}{6EI}\left[(x-l)^2 - a_2(3x-l)\right]\right]\right\}_{x=l+a_2}$$

$$= \frac{F_1 a_1}{6EIl}\left(l^2 - a_1{}^2\right) + \frac{F_2}{6EI}\left[3(x-l)^2 - 3a_2(x-l) - a_2(3x-l)\right]_{x=l+a_2}$$

$$= \frac{F_1 a_1}{6EIl}(l^2 - a_1{}^2) - \frac{F_2}{6EI}\left(3a_2{}^2 + 2la_2\right)$$

$$= \frac{-3312(230)}{6(207)10^3(39.76)10^3(510)}\left(510^2 - 230^2\right)$$

$$- \frac{2070}{6(207)10^3(39.76)10^3}\left[3(300^2) + 2(510)(300)\right]$$

$$= -0.0304 \text{ rad} \qquad \textit{Ans.}$$

4-25 From the solutions to Prob. 3-81, $T_1 = 392.16$ lbf and $T_2 = 58.82$ lbf

$$I = \frac{\pi d^4}{64} = \frac{\pi (1)^4}{64} = 0.049\ 09 \text{ in}^4$$

From Table A-9, beam 6,

$$y_A = \left[\frac{F_1 b_1 x}{6EIl}\left(x^2 + b_1^{\,2} - l^2\right)\right]_{x=8\text{in}} = \frac{(-350)(14)(8)}{6(30)10^6(0.049\ 09)(22)}\left(8^2 + 14^2 - 22^2\right) = 0.0452 \text{ in} \quad \textit{Ans.}$$

$$z_A = \left[\frac{F_2 b_2 x}{6EIl}(x^2 + b_2^{\,2} - l^2)\right]_{x=8\text{in}} = \frac{(-450.98)(6)(8)}{6(30)10^6(0.049\ 09)(22)}\left(8^2 + 6^2 - 22^2\right) = 0.0428 \text{ in} \quad \textit{Ans.}$$

The displacement magnitude is $\delta = \sqrt{y_A^2 + z_A^2} = \sqrt{0.0452^2 + 0.0428^2} = 0.0622$ in *Ans.*

$$(\theta_A)_z = \left(\frac{d\,y}{d\,x}\right)_{x=a_1} = \left\{\frac{d}{dx}\left[\frac{F_1 b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right)\right]\right\}_{x=a_1} = \frac{F_1 b_1}{6EIl}(3a_1^2 + b_1^2 - l^2)$$

$$= \frac{(-350)(14)}{6(30)10^6(0.04909)(22)}\left[3\left(8^2\right) + 14^2 - 22^2\right] = 0.00242 \text{ rad} \quad \textit{Ans.}$$

$$(\theta_A)_y = \left(-\frac{d\,z}{d\,x}\right)_{x=a_1} = -\left\{\frac{d}{dx}\left[\frac{-F_2 b_2 x}{6EIl}(x^2 + b_2^2 - l^2)\right]\right\}_{x=a_1} = \frac{F_2 b_2}{6EIl}\left(3a_1^2 + b_2^2 - l^2\right)$$

$$= \frac{(450.98)(6)}{6(30)10^6(0.04909)(22)}\left[3\left(8^2\right) + 6^2 - 22^2\right] = -0.00356 \text{ rad} \quad \textit{Ans.}$$

The slope magnitude is $\Theta_A = \sqrt{0.00242^2 + \left(-0.00356\right)^2} = 0.00430$ rad *Ans.*

4-26 From the solutions to Prob. 3-82, $T_1 = 250$ N and $T_2 = 37.5$ N

$$I = \frac{\pi d^4}{64} = \frac{\pi(20)^4}{64} = 7\ 854 \text{ mm}^4$$

$$y_A = \left[\frac{F_{1y} b_1 x}{6EIl}(x^2 + b_1^{\,2} - l^2)\right]_{x=300\text{mm}} = \frac{\left(-345 \sin 45^\circ\right)(550)(300)}{6(207)10^3(7\ 854)(850)}\left(300^2 + 550^2 - 850^2\right)$$

$$= 1.60 \text{ mm} \quad \textit{Ans.}$$

$$z_A = \left[\frac{F_{1z} b_1 x}{6EIl}(x^2 + b_1^{\,2} - l^2) + \frac{F_2 b_2 x}{6EIl}(x^2 + b_2^{\,2} - l^2)\right]_{x=300\text{mm}}$$

$$= \frac{\left(345 \cos 45^\circ\right)(550)(300)}{6(207)10^3(7\ 854)(850)}\left(300^2 + 550^2 - 850^2\right)$$

$$+ \frac{-287.5(150)(300)}{6(207)10^3(7\ 854)(850)}\left(300^2 + 150^2 - 850^2\right) = -0.650 \text{ mm} \quad \textit{Ans.}$$

The displacement magnitude is $\delta = \sqrt{y_A^2 + z_A^2} = \sqrt{1.60^2 + \left(-0.650\right)^2} = 1.73$ mm *Ans.*

$$(\theta_A)_z = \left(\frac{d\,y}{d\,x}\right)_{x=a_1} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)\right]\right\}_{x=a_1} = \frac{F_{1y}b_1}{6EIl}(3a_1^2+b_1^2-l^2)$$

$$= \frac{-\left(345\sin 45^\circ\right)(550)}{6(207)10^3(7\ 854)(850)}\left[3\left(300^2\right)+550^2-850^2\right] = 0.00243\text{ rad} \qquad \textit{Ans.}$$

$$(\theta_A)_y = -\left(\frac{d\,z}{d\,x}\right)_{x=a_1} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1{}^2-l^2\right)+\frac{F_2b_2x}{6EIl}\left(x^2+b_2{}^2-l^2\right)\right]\right\}_{x=a_1}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(3a_1^2+b_1^2-l^2\right)-\frac{F_2b_2}{6EIl}\left(3a_1^2+b_2^2-l^2\right)$$

$$= -\frac{\left(345\cos 45^\circ\right)(550)}{6(207)10^3(7\ 854)(850)}\left[3\left(300^2\right)+550^2-850^2\right]$$

$$-\frac{-287.5(150)}{6(207)10^3(7\ 854)(850)}\left[3\left(300^2\right)+150^2-850^2\right] = 1.91\cdot 10^{-4}\text{rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_A = \sqrt{0.00243^2+0.000191^2} = 0.00244$ rad *Ans.*

4-27 From the solutions to Prob. 3-83, $F_B = 750$ lbf

$$I = \frac{\pi d^4}{64} = \frac{\pi(1.25)^4}{64} = 0.1198\text{ in}^4$$

From Table A-9, beams 6 (subscript 1) and 10 (subscript 2)

$$y_A = \left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2y}a_2x}{6EIl}\left(l^2-x^2\right)\right]_{x=16\text{in}}$$

$$= \frac{\left(-300\cos 20^\circ\right)(14)(16)}{6(30)10^6(0.119\ 8)(30)}\left(16^2+14^2-30^2\right)+\frac{\left(750\sin 20^\circ\right)(9)(16)}{6(30)10^6(0.119\ 8)(30)}\left(30^2-16^2\right)$$

$$= 0.0805\text{ in} \qquad \textit{Ans.}$$

$$z_A = \left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1{}^2-l^2\right)+\frac{F_{2z}a_2x}{6EIl}\left(l^2-x^2\right)\right]_{x=16\text{in}}$$

$$= \frac{\left(300\sin 20^\circ\right)(14)(16)}{6(30)10^6(0.119\ 8)(30)}\left(16^2+14^2-30^2\right)+\frac{\left(-750\cos 20^\circ\right)(9)(16)}{6(30)10^6(0.119\ 8)(30)}\left(30^2-16^2\right)$$

$$= -0.1169\text{ in} \qquad \textit{Ans.}$$

The displacement magnitude is $\delta = \sqrt{y_A^2+z_A^2} = \sqrt{0.0805^2+\left(-0.1169\right)^2} = 0.142$ in *Ans.*

$$(\theta_A)_z = \left(\frac{dy}{dx}\right)_{x=a_1} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2y}a_2x}{6EIl}\left(l^2-x^2\right)\right]\right\}_{x=a_1}$$

$$=\frac{F_{1y}b_1}{6EIl}\left(3a_1^2+b_1^2-l^2\right)+\frac{F_{2y}a_2}{6EIl}\left(l^2-3a_1^2\right)$$

$$=\frac{\left(-300\cos 20^\circ\right)(14)}{6(30)10^6(0.119\ 8)(30)}\left[3\left(16^2\right)+14^2-30^2\right]$$

$$+\frac{\left(750\sin 20^\circ\right)(9)}{6(30)10^6(0.119\ 8)(30)}\left[30^2-3\left(16^2\right)\right]=8.06\left(10^{-5}\right)\text{rad} \qquad \textit{Ans.}$$

$$(\theta_A)_y = -\left(\frac{dz}{dx}\right)_{x=a_1} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2z}a_2x}{6EIl}\left(l^2-x^2\right)\right]\right\}_{x=a_1}$$

$$=-\frac{F_{1z}b_1}{6EIl}\left(3a_1^2+b_1^2-l^2\right)-\frac{F_{2z}a_2}{6EIl}\left(l^2-3a_1^2\right)$$

$$=-\frac{\left(300\sin 20^\circ\right)(14)}{6(30)10^6(0.119\ 8)(30)}\left[3\left(16^2\right)+14^2-30^2\right]-\frac{\left(-750\cos 20^\circ\right)(9)}{6(30)10^6(0.119\ 8)(30)}\left[30^2-3\left(16^2\right)\right]$$

$$=0.00115 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_A=\sqrt{\left[8.06\left(10^{-5}\right)\right]^2+0.00115^2}=0.00115 \text{ rad} \quad \textit{Ans.}$

4-28 From the solutions to Prob. 3-84, $F_B = 22.8\ (10^3)$ N

$$I=\frac{\pi d^4}{64}=\frac{\pi\left(50^4\right)}{64}=306.8\left(10^3\right)\ \text{mm}^4$$

From Table A-9, beam 6,

$$y_A=\left[\frac{F_{1y}b_1x}{6EIl}(x^2+b_1^2-l^2)+\frac{F_{2y}b_2x}{6EIl}(x^2+b_2^2-l^2)\right]_{x=400\text{mm}}$$

$$=\frac{\left[11\left(10^3\right)\sin 20^\circ\right](650)(400)}{6(207)10^3(306.8)10^3(1050)}\left(400^2+650^2-1050^2\right)$$

$$+\frac{\left[22.8\left(10^3\right)\sin 25^\circ\right](300)(400)}{6(207)10^3(306.8)10^3(1050)}\left(400^2+300^2-1050^2\right)$$

$$=-3.735 \text{ mm} \qquad \textit{Ans.}$$

$$z_A=\left[\frac{F_{1z}b_1x}{6EIl}(x^2+b_1{}^2-l^2)+\frac{F_{2z}b_2x}{6EIl}(x^2+b_2{}^2-l^2)\right]_{x=400\text{mm}}$$

$$=\frac{\left[11\left(10^3\right)\cos 20^\circ\right](650)(400)}{6(207)10^3(306.8)10^3(1050)}\left(400^2+650^2-1050^2\right)$$

$$+\frac{\left[-22.8\left(10^3\right)\cos 25^\circ\right](300)(400)}{6(207)10^3(306.8)10^3(1050)}\left(400^2+300^2-1050^2\right)=1.791\text{ mm}\qquad \textit{Ans.}$$

The displacement magnitude is $\delta=\sqrt{y_A^2+z_A^2}=\sqrt{\left(-3.735\right)^2+1.791^2}=4.14\text{ mm}$ *Ans.*

$$\left(\theta_A\right)_z=\left(\frac{d\,y}{d\,x}\right)_{x=a_1}=\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1{}^2-l^2\right)+\frac{F_{2z}b_2x}{6EIl}\left(x^2+b_2{}^2-l^2\right)\right]\right\}_{x=a_1}$$

$$=\frac{F_{1y}b_1}{6EIl}\left(3a_1^2+b_1^2-l^2\right)+\frac{F_{2y}b_2}{6EIl}\left(3a_1^2+b_2^2-l^2\right)$$

$$=\frac{\left[11\left(10^3\right)\sin 20^\circ\right](650)}{6(207)10^3(306.8)10^3(1050)}\left[3\left(400^2\right)+650^2-1050^2\right]$$

$$+\frac{\left[22.8\left(10^3\right)\sin 25^\circ\right](300)}{6(207)10^3(306.8)10^3(1050)}\left[3\left(400^2\right)+300^2-1050^2\right]$$

$$=-0.00507\text{ rad}\qquad \textit{Ans.}$$

$$\left(\theta_A\right)_y=-\left(\frac{d\,z}{d\,x}\right)_{x=a_1}=-\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1{}^2-l^2\right)+\frac{F_{2z}b_2x}{6EIl}\left(x^2+b_2{}^2-l^2\right)\right]\right\}_{x=a_1}$$

$$=-\frac{F_{1z}b_1}{6EIl}\left(3a_1^2+b_1^2-l^2\right)-\frac{F_{2z}b_2}{6EIl}\left(3a_1^2+b_2^2-l^2\right)$$

$$=-\frac{\left[11\left(10^3\right)\cos 20^\circ\right](650)}{6(207)10^3(306.8)10^3(1050)}\left[3\left(400^2\right)+650^2-1050^2\right]$$

$$-\frac{\left[-22.8\left(10^3\right)\cos 25^\circ\right](300)}{6(207)10^3(306.8)10^3(1050)}\left[3\left(400^2\right)+300^2-1050^2\right]$$

$$=-0.00489\text{ rad}\qquad \textit{Ans.}$$

The slope magnitude is $\Theta_A=\sqrt{\left(-0.00507\right)^2+\left(-0.00489\right)^2}=0.00704\text{ rad}$ *Ans.*

4-29 From the solutions to Prob. 3-79, $T_1 = 60$ lbf and $T_2 = 400$ lbf , and Prob. 4-23, $I = 0.119\,8$ in^4. From Table A-9, beam 6,

$$(\theta_O)_y = -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2z}b_2x}{6EIl}\left(x^2+b_2^2-l^2\right)\right]\right\}_{x=0}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(b_1^{\,2}-l^2\right)-\frac{F_{2z}b_2}{6EIl}\left(b_2^{\,2}-l^2\right) = -\frac{-575(30)}{6(30)10^6(0.119\ 8)(40)}\left(30^2-40^2\right)$$

$$-\frac{460(12)}{6(30)10^6(0.119\ 8)(40)}\left(12^2-40^2\right) = -0.00468 \text{ rad} \qquad Ans.$$

$$(\theta_C)_y = -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1\left(l-x\right)}{6EIl}\left(x^2+a_1^2-2lx\right)+\frac{F_{2z}a_2\left(l-x\right)}{6EIl}\left(x^2+a_2^2-2lx\right)\right]\right\}_{x=l}$$

$$= -\left[\frac{F_{1z}a_1}{6EIl}\left(6lx-2l^2-3x^2-a_1^2\right)+\frac{F_{2z}a_2}{6EIl}\left(6lx-2l^2-3x^2-a_2^2\right)\right]_{x=l}$$

$$= -\frac{F_{1z}a_1}{6EIl}\left(l^2-a_1^2\right)-\frac{F_{2z}a_2}{6EIl}\left(l^2-a_2^2\right)$$

$$= -\frac{-575(10)\left(40^2-10^2\right)}{6(30)10^6(0.119\ 8)(40)}-\frac{460(28)\left(40^2-28^2\right)}{6(30)10^6(0.119\ 8)(40)} = -0.00219 \text{ rad} \qquad Ans.$$

4-30 From the solutions to Prob. 3-80, $T_1 = 2\ 880$ N and $T_2 = 432$ N, and Prob. 4-24, $I = 39.76$ (10^3) mm^4. From Table A-9, beams 6 and 10

$$(\theta_O)_z = \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_1b_1x}{6EIl}(x^2+b_1^2-l^2)+\frac{F_2a_2x}{6EIl}(l^2-x^2)\right]\right\}_{x=0}$$

$$= \left[\frac{F_1b_1}{6EIl}(3x^2+b_1^2-l^2)+\frac{F_2a_2}{6EIl}(l^2-3x^2)\right]_{x=0} = \frac{F_1b_1}{6EIl}(b_1^2-l^2)+\frac{F_2a_2l}{6EI}$$

$$= \frac{-3\ 312(280)}{6(207)10^3(39.76)10^3(510)}\left(280^2-510^2\right)+\frac{2\ 070(300)(510)}{6(207)10^3(39.76)10^3}$$

$$= 0.0131 \text{ rad} \qquad Ans.$$

$$(\theta_C)_z = \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_1a_1(l-x)}{6EIl}(x^2+a_1^2-2lx)+\frac{F_2a_2x}{6EIl}(l^2-x^2)\right]\right\}_{x=l}$$

$$= \left[\frac{F_1a_1}{6EIl}(6lx-2l^2-3x^2-a_1^2)+\frac{F_2a_2}{6EIl}(l^2-3x^2)\right]_{x=l} = \frac{F_1a_1}{6EIl}(l^2-a_1^{\,2})-\frac{F_2a_2l}{3EI}$$

$$= \frac{-3\ 312(230)}{6(207)10^3(39.76)10^3(510)}(510^2-230^2)-\frac{2\ 070(300)(510)}{3(207)10^3(39.76)10^3}$$

$$= -0.0191 \text{ rad} \qquad Ans.$$

4-31 From the solutions to Prob. 3-81, $T_1 = 392.19$ lbf and $T_2 = 58.82$ lbf, and Prob. 4-25, $I = 0.0490\ 9$ in^4. From Table A-9, beam 6

$$(\theta_O)_z = \left(\frac{d\,y}{d\,x}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)\right]\right\}_{x=0} = \frac{F_{1y}b_1}{6EIl}(b_1^{\,2}-l^2)$$

$$= \frac{-350(14)}{6(30)10^6(0.04909)(22)}\left(14^2-22^2\right) = 0.00726 \text{ rad} \qquad Ans.$$

$$(\theta_O)_y = -\left(\frac{d\,z}{d\,x}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{2z}b_2x}{6EIl}\left(x^2+b_2^2-l^2\right)\right]\right\}_{x=0} = -\frac{F_{2z}b_2}{6EIl}\left(b_2^2-l^2\right)$$

$$= -\frac{-450.98(6)}{6(30)10^6(0.04909)(22)}\left(6^2-22^2\right)$$

$$= -0.00624 \text{ rad} \qquad Ans.$$

The slope magnitude is $\Theta_O = \sqrt{0.00726^2+\left(-0.00624\right)^2} = 0.00957$ rad *Ans.*

$$(\theta_C)_z = \left(\frac{d\,y}{d\,x}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)\right]\right\}_{x=l}$$

$$= \left[\frac{F_{1y}a_1}{6EIl}\left(6lx-2l^2-3x^2-a_1^2\right)\right]_{x=l} = \frac{F_{1y}a_1}{6EIl}(l^2-a_1^{\,2})$$

$$= \frac{-350(8)}{6(30)10^6(0.0491)(22)}\left(22^2-8^2\right) = -0.00605 \text{ rad} \qquad Ans.$$

$$(\theta_C)_y = -\left(\frac{d\,z}{d\,x}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{2z}a_2(l-x)}{6EIl}\left(x^2+a_2^2-2lx\right)\right]\right\}_{x=l}$$

$$= -\left[\frac{F_{2z}a_2}{6EIl}\left(6lx-2l^2-3x^2-a_2^2\right)\right]_{x=l} = -\frac{F_{2z}a_2}{6EIl}\left(l^2-a_2^2\right)$$

$$= -\frac{-450.98(16)}{6(30)10^6(0.04909)(22)}\left(22^2-16^2\right) = 0.00846 \text{ rad} \qquad Ans.$$

The slope magnitude is $\Theta_C = \sqrt{\left(-0.00605\right)^2+0.00846^2} = 0.0104$ rad *Ans.*

4-32 From the solutions to Prob. 3-82, T_1 =250 N and T_1 =37.5 N, and Prob. 4-26, I = 7 854 mm^4. From Table A-9, beam 6

$$(\theta_O)_z = \left(\frac{d\,y}{d\,x}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)\right]\right\}_{x=0} = \frac{F_{1y}b_1}{6EIl}(b_1^{\,2}-l^2)$$

$$= \frac{\left[-345\sin 45^\circ\right](550)}{6(207)10^3(7\ 854)(850)}\left(550^2-850^2\right) = 0.00680 \text{ rad} \qquad Ans.$$

$$(\theta_O)_y = -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2z}b_2x}{6EIl}\left(x^2+b_2^2-l^2\right)\right]\right\}_{x=0}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(b_1^2-l^2\right)-\frac{F_{2z}b_2}{6EIl}\left(b_2^2-l^2\right) = -\frac{\left[345\cos 45^\circ\right](550)}{6(207)10^3(7\ 854)(850)}\left(550^2-850^2\right)$$

$$-\frac{-287.5(150)}{6(207)10^3(7\ 854)(850)}\left(150^2-850^2\right) = 0.00316 \text{ rad} \qquad Ans.$$

The slope magnitude is $\Theta_O = \sqrt{0.00680^2+0.00316^2} = 0.00750$ rad *Ans.*

$$(\theta_C)_z = \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)\right]\right\}_{x=l} = \left[\frac{F_{1y}a_1}{6EIl}\left(6lx-2l^2-3x^2-a_1^2\right)\right]_{x=l}$$

$$= \frac{F_{1y}a_1}{6EIl}(l^2-a_1^2) = \frac{\left[-345\sin 45^\circ\right](300)}{6(207)10^3(7\ 854)(850)}\left(850^2-300^2\right) = -0.00558 \text{ rad} \qquad Ans.$$

$$(\theta_C)_y = -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)+\frac{F_{2z}a_2(l-x)}{6EIl}\left(x^2+a_2^2-2lx\right)\right]\right\}_{x=l}$$

$$= -\frac{F_{1z}a_1}{6EIl}\left(l^2-a_1^2\right)-\frac{F_{2z}a_2}{6EIl}\left(l^2-a_2^2\right) = -\frac{\left[345\cos 45^\circ\right](300)}{6(207)10^3(7\ 854)(850)}\left(850^2-300^2\right)$$

$$-\frac{-287.5(700)}{6(207)10^3(7\ 854)(850)}\left(850^2-700^2\right) = 6.04\left(10^{-5}\right) \text{ rad} \qquad Ans.$$

The slope magnitude is $\Theta_C = \sqrt{(-0.00558)^2+\left[6.04\left(10^{-5}\right)\right]^2} = 0.00558$ rad *Ans.*

4-33 From the solutions to Prob. 3-83, $F_B = 750$ lbf, and Prob. 4-27, $I = 0.119\ 8$ in^4. From Table A-9, beams 6 and 10

$$(\theta_O)_z = \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2y}a_2x}{6EIl}\left(l^2-x^2\right)\right]\right\}_{x=0}$$

$$= \left[\frac{F_{1y}b_1}{6EIl}\left(3x^2+b_1^2-l^2\right)+\frac{F_{2y}a_2}{6EIl}\left(l^2-3x^2\right)\right]_{x=0} = \frac{F_{1y}b_1}{6EIl}\left(b_1^2-l^2\right)+\frac{F_{2y}a_2l}{6EI}$$

$$= \frac{\left[-300\cos 20^\circ\right](14)}{6(30)10^6(0.119\ 8)(30)}\left(14^2-30^2\right)+\frac{\left[750\sin 20^\circ\right](9)(30)}{6(30)10^6(0.119\ 8)} = 0.00751 \text{ rad} \qquad Ans.$$

$$(\theta_O)_y = -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2z}a_2x}{6EIl}\left(l^2-x^2\right)\right]\right\}_{x=0}$$

$$= -\left[\frac{F_{1z}b_1}{6EIl}\left(3x^2+b_1^2-l^2\right)+\frac{F_{2z}a_2}{6EIl}\left(l^2-3x^2\right)\right]_{x=0} = -\frac{F_{1z}b_1}{6EIl}\left(b_1^2-l^2\right)-\frac{F_{2z}a_2l}{6EI}$$

$$= -\frac{\left[300\sin 20^\circ\right](14)}{6(30)10^6(0.119\,8)(30)}\left(14^2-30^2\right)-\frac{\left[-750\cos 20^\circ\right](9)(30)}{6(30)10^6(0.119\,8)} = 0.0104 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_O = \sqrt{0.00751^2+0.0104^2} = 0.0128$ rad *Ans.*

$$(\theta_C)_z = \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)+\frac{F_{2y}a_2x}{6EIl}\left(l^2-x^2\right)\right]\right\}_{x=l}$$

$$= \left[\frac{F_{1y}a_1}{6EIl}\left(6lx-2l^2-3x^2-a_1^2\right)+\frac{F_{2y}a_2}{6EIl}\left(l^2-3x^2\right)\right]_{x=l} = \frac{F_{1y}a_1}{6EIl}(l^2-a_1{}^2)-\frac{F_{2y}a_2l}{3EI}$$

$$= \frac{\left[-300\cos 20^\circ\right](16)}{6(30)10^6(0.119\,8)(30)}\left(30^2-16^2\right)-\frac{\left[750\sin 20^\circ\right](9)(30)}{3(30)10^6(0.119\,8)} = -0.0109 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_C)_y = -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)+\frac{F_{2z}a_2x}{6EIl}\left(l^2-x^2\right)\right]\right\}_{x=l}$$

$$= -\left[\frac{F_{1z}a_1}{6EIl}\left(6lx-2l^2-3x^2-a_1^2\right)+\frac{F_{2z}a_2}{6EIl}\left(l^2-3x^2\right)\right]_{x=l} = -\frac{F_{1z}a_1}{6EIl}\left(l^2-a_1^2\right)+\frac{F_{2z}a_2l}{3EI}$$

$$= -\frac{\left[300\sin 20^\circ\right](16)}{6(30)10^6(0.119\,8)(30)}\left(30^2-16^2\right)+\frac{\left[-750\cos 20^\circ\right](9)(30)}{3(30)10^6(0.119\,8)} = -0.0193 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_C = \sqrt{(-0.0109)^2+(-0.0193)^2} = 0.0222$ rad *Ans.*

4-34 From the solutions to Prob. 3-84, $F_B = 22.8$ kN, and Prob. 4-28, $I = 306.8\ (10^3)$ mm^4. From Table A-9, beam 6

$$(\theta_O)_z = \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2y}b_2x}{6EIl}\left(x^2+b_2^2-l^2\right)\right]\right\}_{x=0}$$

$$= \frac{F_{1y}b_1}{6EIl}\left(b_1^2-l^2\right)+\frac{F_{2y}b_2}{6EIl}\left(b_2^2-l^2\right) = \frac{\left[11\left(10^3\right)\sin 20^\circ\right](650)}{6(207)10^3(306.8)10^3(1050)}\left(650^2-1050^2\right)$$

$$+\frac{\left[22.8\left(10^3\right)\sin 25^\circ\right](300)}{6(207)10^3(306.8)10^3(1050)}\left(300^2-1050^2\right) = -0.0115 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_O)_y = -\left(\frac{d\,z}{d\,x}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2z}b_2x}{6EIl}\left(x^2+b_2^2-l^2\right)\right]\right\}_{x=0}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(b_1^2-l^2\right)-\frac{F_{2z}b_2}{6EIl}\left(b_2^2-l^2\right)$$

$$= -\frac{\left[11\left(10^3\right)\cos 20^\circ\right](650)}{6(207)10^3(306.8)10^3(1050)}\left(650^2-1050^2\right)$$

$$-\frac{\left[-22.8\left(10^3\right)\cos 25^\circ\right](300)}{6(207)10^3(306.8)10^3(1050)}\left(300^2-1050^2\right) = -0.00427 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_O = \sqrt{(-0.0115)^2+(-0.00427)^2} = 0.0123$ rad *Ans.*

$$(\theta_C)_z = \left(\frac{d\,y}{d\,x}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)+\frac{F_{2y}a_2(l-x)}{6EIl}\left(x^2+a_2^2-2lx\right)\right]\right\}_{x=l}$$

$$= \left[\frac{F_{1y}a_1}{6EIl}(6lx-2l^2-3x^2-a_1^2)+\frac{F_{2y}a_2}{6EIl}\left(6lx-2l^2-3x^2-a_2^2\right)\right]_{x=l}$$

$$= \frac{F_{1y}a_1}{6EIl}\left(l^2-a_1^2\right)+\frac{F_{2y}a_2}{6EIl}\left(l^2-a_2^2\right) = \frac{\left[11\left(10^3\right)\sin 20^\circ\right](400)}{6(207)10^3(306.8)10^3(1050)}\left(1050^2-400^2\right)$$

$$+\frac{\left[22.8\left(10^3\right)\sin 25^\circ\right](750)}{6(207)10^3(306.8)10^3(1050)}\left(1050^2-750^2\right) = 0.0133 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_C)_y = -\left(\frac{d\,z}{d\,x}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)+\frac{F_{2z}a_2(l-x)}{6EIl}\left(x^2+a_2^2-2lx\right)\right]\right\}_{x=l}$$

$$= -\left[\frac{F_{1z}a_1}{6EIl}\left(6lx-2l^2-3x^2-a_1^2\right)+\frac{F_{2z}a_2}{6EIl}\left(6lx-2l^2-3x^2-a_2^2\right)\right]_{x=l}$$

$$= -\frac{F_{1z}a_1}{6EIl}\left(l^2-a_1^2\right)-\frac{F_{2z}a_2}{6EIl}\left(l^2-a_2^2\right) = -\frac{\left[11\left(10^3\right)\cos 20^\circ\right](400)}{6(207)10^3(306.8)10^3(1050)}\left(1050^2-400^2\right)$$

$$-\frac{\left[-22.8\left(10^3\right)\cos 25^\circ\right](750)}{6(207)10^3(306.8)10^3(1050)}\left(1050^2-750^2\right) = 0.0112 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_C = \sqrt{0.0133^2+0.0112^2} = 0.0174$ rad *Ans.*

4-35 The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105$ rad.
In Prob. 4-29, $I = 0.119\,8$ in^4, and it was found that the greater angle occurs at the bearing at O where $(\theta_O)_y = -\,0.00468$ rad.

Since θ is inversely proportional to I,

$$\theta_{\text{new}}\, I_{\text{new}} = \theta_{\text{old}}\, I_{\text{old}} \quad \Rightarrow \quad I_{\text{new}} = \pi\, d_{\text{new}}^4/64 = \theta_{\text{old}}\, I_{\text{old}}/\theta_{\text{new}}$$

or,
$$d_{\text{new}} = \left(\frac{64}{\pi}\left|\frac{\theta_{\text{old}}}{\theta_{\text{new}}}\right| I_{\text{old}}\right)^{1/4}$$

The absolute sign is used as the old slope may be negative.

$$d_{\text{new}} = \left(\frac{64}{\pi}\left|\frac{-0.00468}{0.00105}\right| 0.119\,8\right)^{1/4} = 1.82 \text{ in} \qquad \textit{Ans.}$$

4-36 The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105$ rad. In Prob. 4-30, $I = 39.76\ (10^3)\ \text{mm}^4$, and it was found that the greater angle occurs at the bearing at C where $(\theta_C)_y = -\ 0.0191$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left(\frac{64}{\pi}\left|\frac{\theta_{\text{old}}}{\theta_{\text{new}}}\right| I_{\text{old}}\right)^{1/4}$$

$$d_{\text{new}} = \left(\frac{64}{\pi}\left|\frac{-0.0191}{0.00105}\right| 39.76\left(10^3\right)\right)^{1/4} = 62.0 \text{ mm} \qquad \textit{Ans.}$$

4-37 The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105$ rad. In Prob. 4-31, $I = 0.0491\ \text{in}^4$, and the maximum slope is $\Theta_C = 0.0104$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left(\frac{64}{\pi}\left|\frac{\theta_{\text{old}}}{\theta_{\text{new}}}\right| I_{\text{old}}\right)^{1/4}$$

$$d_{\text{new}} = \left(\frac{64}{\pi}\left|\frac{0.0104}{0.00105}\right| 0.0491\right)^{1/4} = 1.77 \text{ in} \qquad \textit{Ans.}$$

4-38 The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105$ rad. In Prob. 4-32, $I = 7\,854\ \text{mm}^4$, and the maximum slope is $\Theta_O = 0.00750$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{0.00750}{0.00105} \right| 7\,854 \right)^{1/4} = 32.7 \text{ mm} \qquad \textit{Ans.}$$

4-39 The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105$ rad. In Prob. 4-33, $I = 0.119\,8$ in^4, and the maximum slope $\Theta = 0.0222$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{0.0222}{0.00105} \right| 0.119\,8 \right)^{1/4} = 2.68 \text{ in} \qquad \textit{Ans.}$$

4-40 The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105$ rad. In Prob. 4-34, $I = 306.8\ (10^3)$ mm^4, and the maximum slope is $\Theta_C = 0.0174$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{0.0174}{0.00105} \right| 306.8\left(10^3\right) \right)^{1/4} = 100.9 \text{ mm} \qquad \textit{Ans.}$$

4-41 $I_{AB} = \pi\, 1^4/64 = 0.04909$ in^4, $J_{AB} = 2\, I_{AB} = 0.09818$ in^4, $I_{BC} = (0.25)(1.5)^3/12 = 0.07031$ in^4, $I_{CD} = \pi(3/4)^4/64 = 0.01553$ in^4. For Eq. (3-41), $b/c = 1.5/0.25 = 6 \Rightarrow \beta = 0.299$.

The deflection can be broken down into several parts

1. The vertical deflection of B due to force and moment acting on B (y_1).
2. The vertical deflection due to the slope at B, θ_{B1}, due to the force and moment acting on B ($y_2 = \overline{CD}\, \theta_{B1} = 2\theta_{B1}$).

3. The vertical deflection due to the rotation at B, θ_{B2}, due to the torsion acting at B ($y_3 = \overline{BC}\,\theta_{B1} = 5\theta_{B1}$).
4. The vertical deflection of C due to the force acting on C (y_4).
5. The rotation at C, θ_C, due to the torsion acting at C ($y_3 = \overline{CD}\,\theta_C = 2\theta_C$).
6. The vertical deflection of D due to the force acting on D (y_5).

1. From Table A-9, beams 1 and 4 with $F = -200$ lbf and $M_B = 2(200) = 400$ lbf·in

$$y_1 = -\frac{-200\left(6^3\right)}{3(30)10^6(0.04909)} + \frac{400\left(6^2\right)}{2(30)10^6(0.04909)} = 0.01467 \text{ in}$$

2. From Table A-9, beams 1 and 4

$$\theta_{B1} = \left\{\frac{d}{dx}\left[\frac{Fx^2}{6EI}(x-3l) + \frac{M_B x^2}{2EI}\right]\right\}_{x=l} = \left[\frac{Fx}{6EI}(3x-6l) + \frac{M_B x}{EI}\right]_{x=l}$$

$$= \left\{\frac{l}{2EI}\left[-Fl + 2M_B\right]\right\} = \frac{6}{2(30)10^6(0.04909)}\left[-(-200)(6) + 2(400)\right] = 0.004074 \text{ rad}$$

$$y_2 = 2(0.004072) = 0.00815 \text{ in}$$

3. The torsion at B is $T_B = 5(200) = 1000$ lbf·in. From Eq. (4-5)

$$\theta_{B2} = \left(\frac{TL}{JG}\right)_{AB} = \frac{1000(6)}{0.09818(11.5)10^6} = 0.005314 \text{ rad}$$

$$y_3 = 5(0.005314) = 0.02657 \text{ in}$$

4. For bending of BC, from Table A-9, beam 1

$$y_4 = -\frac{-200\left(5^3\right)}{3(30)10^6(0.07031)} = 0.00395 \text{ in}$$

5. For twist of BC, from Eq. (3-41), with $T = 2(200) = 400$ lbf·in

$$\theta_C = \frac{400(5)}{0.299(1.5)0.25^3(11.5)10^6} = 0.02482 \text{ rad}$$

$$y_5 = 2(0.02482) = 0.04964 \text{ in}$$

6. For bending of CD, from Table A-9, beam 1

$$y_6 = -\frac{-200\left(2^3\right)}{3(30)10^6(0.01553)} = 0.00114 \text{ in}$$

Summing the deflections results in

$$y_D = \sum_{i=1}^{6} y_i = 0.01467 + 0.00815 + 0.02657 + 0.00395 + 0.04964 + 0.00114 = 0.1041 \text{ in} \quad Ans.$$

This problem is solved more easily using Castigliano's theorem. See Prob. 4-78.

4-42 The deflection of *D* in the *x* direction due to F_z is from:

1. The deflection due to the slope at *B*, θ_{B1}, due to the force and moment acting on *B* ($x_1 = \overline{BC}\,\theta_{B1} = 5\theta_{B1}$).
2. The deflection due to the moment acting on *C* (x_2).

1. For *AB*, $I_{AB} = \pi\, 1^4/64 = 0.04909$ in^4. From Table A-9, beams 1 and 4

$$\theta_{B1} = \left\{\frac{d}{dx}\left[\frac{Fx^2}{6EI}(x-3l) + \frac{M_B x^2}{2EI}\right]\right\}_{x=l} = \left[\frac{Fx}{6EI}(3x-6l) + \frac{M_B x}{EI}\right]_{x=l}$$

$$= \left\{\frac{l}{2EI}\left[-Fl + 2M_B\right]\right\} = \frac{6}{2(30)10^6(0.04909)}\left[-(100)(6) + 2(-200)\right] = -0.002037 \text{ rad}$$

$x_1 = 5(-0.002037) = -0.01019$ in

2. For *BC*, $I_{BC} = (1.5)(0.25)^3/12 = 0.001953$ in^4. From Table A-9, beam 4

$$x_2 = \frac{M_C l^2}{2EI} = \frac{2(-100)5}{2(30)10^6(0.001953)} = -0.04267 \text{ in}$$

The deflection of *D* in the *x* direction due to F_x is from:

3. The elongation of *AB* due to the tension. For *AB*, the area is $A = \pi\, 1^2/4 = 0.7854$ in^2

$$x_3 = \left(\frac{Fl}{AE}\right)_{AB} = \frac{-150(6)}{0.7854(30)10^6} = -3.82\left(10^{-5}\right) \text{ in}$$

4. The deflection due to the slope at *B*, θ_{B2}, due to the moment acting on *B* ($x_1 = \overline{BC}\,\theta_{B2} = 5\theta_{B2}$). With $I_{AB} = 0.04907$ in^4,

$$\theta_{B2} = \frac{M_B l}{EI} = \frac{5(-150)6}{30\left(10^6\right)0.04909} = -0.003056 \text{ rad}$$

$x_4 = 5(-0.003056) = -0.01528$ in

5. The deflection at C due to the bending force acting on C. With $I_{BC} = 0.001953$ in^4

$$x_5 = \left(-\frac{Fl^3}{3EI}\right)_{BC} = -\frac{150\left(5^3\right)}{3(30)10^6\left(0.001953\right)} = -0.10667 \text{ in}$$

6. The elongation of CD due to the tension. For CD, the area is $A = \pi(0.75^2)/4 = 0.4418$ in^2

$$x_6 = \left(\frac{Fl}{AE}\right)_{CD} = \frac{-150(2)}{0.4418(30)10^6} = -2.26\left(10^{-5}\right) \text{ in}$$

Summing the deflections results in

$$x_D = \sum_{i=1}^{6} x_i = -0.01019 - 0.04267 - 3.82\left(10^{-5}\right)$$

$$-0.01528 - 0.10667 - 2.26\left(10^{-5}\right) = -0.1749 \text{ in} \quad \textit{Ans.}$$

4-43 $J_{OA} = J_{BC} = \pi(1.5^4)/32 = 0.4970$ in^4, $J_{AB} = \pi(1^4)/32 = 0.09817$ in^4, $I_{AB} = \pi(1^4)/64 = 0.04909$ in^4, and $I_{CD} = \pi(0.75^4)/64 = 0.01553$ in^4.

$$\theta = \left(\frac{Tl}{GJ}\right)_{OA} + \left(\frac{Tl}{GJ}\right)_{AB} + \left(\frac{Tl}{GJ}\right)_{BC} = \frac{T}{G}\left(\frac{l_{OA}}{J_{OA}} + \frac{l_{AB}}{J_{AB}} + \frac{l_{BC}}{J_{BC}}\right)$$

$$= \frac{250(12)}{11.5(10^6)}\left(\frac{2}{0.4970} + \frac{9}{0.09817} + \frac{2}{0.4970}\right) = 0.0260 \text{ rad} \qquad \textit{Ans.}$$

Simplified

$$\theta_s = \frac{Tl}{GJ} = \frac{250(12)(13)}{11.5\left(10^6\right)\left(0.09817\right)}$$

$$\theta_s = 0.0345 \text{ rad} \qquad \textit{Ans.}$$

Simplified is 0.0345/0.0260 = 1.33 times greater *Ans.*

$$y_D = \frac{F_y l_{OC}{}^3}{3EI_{AB}} + \theta_s\left(l_{CD}\right) + \frac{F_y l_{CD}{}^3}{3EI_{CD}} = \frac{250\left(13^3\right)}{3(30)10^6\left(0.04909\right)} + 0.0345(12) + \frac{250\left(12^3\right)}{3(30)10^6\left(0.01553\right)}$$

$$y_D = 0.847 \text{ in} \qquad \textit{Ans.}$$

4-44 Reverse the deflection equation of beam 7 of Table A-9. Using units in lbf, inches

$$y=-\frac{wx}{24EI}\left(2lx^2-x^3-l^3\right)=-\frac{(3000/12)x}{24(30)10^6(485)}\left\{2(25)x^2-x^3-[25(12)]^3\right\}$$

$$=7.159\left(10^{-10}\right)x\left[27\left(10^6\right)-600x^2+x^3\right] \qquad Ans.$$

The maximum height occurs at $x = 25(12)/2 = 150$ in

$$y_{\max}=7.159\left(10^{-10}\right)150\left[27\left(10^6\right)-600\left(150^2\right)+150^3\right]=1.812 \text{ in} \qquad Ans.$$

4-45 From Table A-9, beam 6,

$$y_L=\frac{Fbx}{6EIl}\left(x^2+b^2-l^2\right)$$

$$y_L=\frac{Fb}{6EIl}\left(x^3+b^2x-l^2x\right)$$

$$\frac{dy_L}{dx}=\frac{Fb}{6EIl}\left(3x^2+b^2-l^2\right)$$

$$\left.\frac{dy_L}{dx}\right|_{x=0}=\frac{Fb\left(b^2-l^2\right)}{6EIl}$$

Let $\xi=\left.\frac{dy_L}{dx}\right|_{x=0}$ and set $I=\frac{\pi d_L^4}{64}$. Thus,

$$d_L=\left|\frac{32Fb\left(b^2-l^2\right)}{3\pi El\xi}\right|^{1/4} \qquad Ans.$$

For the other end view, observe beam 6 of Table A-9 from the back of the page, noting that *a* and *b* interchange as do *x* and –*x*

$$d_R=\left|\frac{32Fa\left(l^2-a^2\right)}{3\pi El\xi}\right|^{1/4} \qquad Ans.$$

For a uniform diameter shaft the necessary diameter is the larger of d_L and d_R.

4-46 The maximum slope will occur at the left bearing. Incorporating a design factor into the solution for d_L of Prob. 4-45,

$$d=\left[\frac{32nFb\left(l^2-b^2\right)}{3\pi El\xi}\right]^{1/4}$$

$$d=\sqrt[4]{\frac{32(1.28)(3000)(200)\left(300^2-200^2\right)}{3\pi(207)10^3(300)(0.001)}}$$

$$d=38.1\text{ mm}\qquad Ans.$$

$$I=\frac{\pi\left(38.1^4\right)}{64}=103.4\left(10^3\right)\text{ mm}^4$$

From Table A-9, beam 6, the maximum deflection will occur in *BC* where $dy_{BC}/dx=0$

$$\frac{d}{dx}\left[\frac{Fa(l-x)}{6EIl}\left(x^2+a^2-2lx\right)\right]=0\quad\Rightarrow\quad 3x^2-6lx+\left(a^2+2l^2\right)=0$$

$$3x^2-6(300)x+\left[100^2+2\left(300^2\right)\right]=0\quad\Rightarrow\quad x^2-600x+63333=0$$

$$x=\frac{1}{2}\left[600\pm\sqrt{600^2-4(1)63\,333}\right]=463.3,\,136.7\text{ mm}$$

$x = 136.7$ mm is acceptable.

$$y_{\max}=\left[\frac{Fa(l-x)}{6EIl}\left(x^2+a^2-2lx\right)\right]_{x=136.7\text{ mm}}$$

$$=\frac{3\left(10^3\right)100(300-136.7)}{6(207)10^3(103.4)10^3(300)}\left[136.7^2+100^2-2(300)136.7\right]=-0.0678\text{ mm}\qquad Ans.$$

4-47 $I=\pi(1.25^4)/64=0.1198\text{ in}^4$. From Table A-9, beam 6

$$\delta=\sqrt{\left[\frac{F_1a_1(l-x)}{6EIl}(x^2+a_1{}^2-2lx)\right]^2+\left[\frac{F_2b_2x}{6EIl}(x^2+b_2{}^2-l^2)\right]^2}$$

$$=\left\{\left[\frac{150(5)(20-8)}{6(30)10^6(0.1198)(20)}\left(8^2+5^2-2(20)(8)\right)\right]^2\right.$$

$$\left.+\left[\frac{250(10)(8)}{6(30)10^6(0.1198)(20)}\left(8^2+10^2-20^2\right)\right]^2\right\}^{1/2}$$

$$=0.0120\text{ in}\qquad Ans.$$

4-48 $I = \pi(1.25^4)/64 = 0.1198$ in^4. For both forces use beam 6 of Table A-9.

For $F_1 = 150$ lbf:

$\underline{0 \le x \le 5}$

$$y = \frac{F_1 b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) = \frac{150(15)x}{6(30)10^6(0.1198)(20)}\left(x^2 + 15^2 - 20^2\right)$$

$$= 5.217\left(10^{-6}\right)x\left(x^2 - 175\right) \qquad (1)$$

$\underline{5 \le x \le 20}$

$$y = \frac{F_1 a_1 (l-x)}{6EIl}\left(x^2 + a_1^2 - 2lx\right) = \frac{150(5)(20-x)}{6(30)10^6(0.1198)(20)}\left[x^2 + 5^2 - 2(20)x\right]$$

$$= 1.739\left(10^{-6}\right)(20-x)\left(x^2 - 40x + 25\right) \qquad (2)$$

For $F_2 = 250$ lbf:

$\underline{0 \le x \le 10}$

$$z = \frac{F_2 b_2 x}{6EIl}\left(x^2 + b_2^2 - l^2\right) = \frac{250(10)x}{6(30)10^6(0.1198)(20)}\left(x^2 + 10^2 - 20^2\right)$$

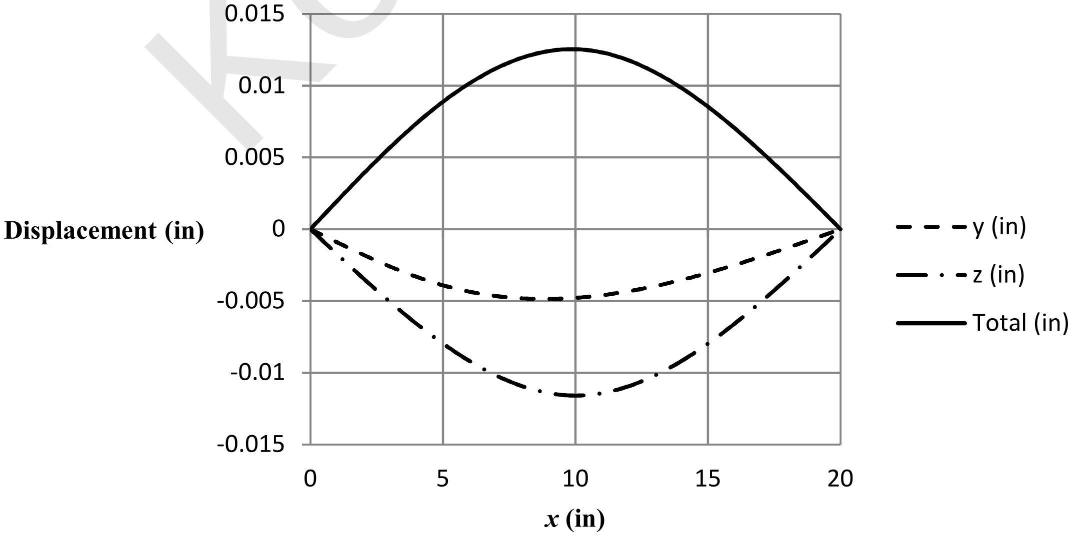
$$= 5.797\left(10^{-6}\right)x\left(x^2 - 300\right) \qquad (3)$$

$\underline{10 \le x \le 20}$

$$z = \frac{F_2 a_2 (l-x)}{6EIl}\left(x^2 + a_2^2 - 2lx\right) = \frac{250(10)(20-x)}{6(30)10^6(0.1198)(20)}\left[x^2 + 10^2 - 2(20)x\right]$$

$$= 5.797\left(10^{-6}\right)(20-x)\left(x^2 - 40x + 100\right) \qquad (4)$$

Plot Eqs. (1) to (4) for each 0.1 in using a spreadsheet. There are 201 data points, too numerous to tabulate here but the plot is shown below, where the maximum deflection of $\delta = 0.01255$ in occurs at $x = 9.9$ in. *Ans.*



4-49 The larger slope will occur at the left end.
From Table A-9, beam 8

$$y_{AB} = \frac{M_B x}{6EIl}(x^2 + 3a^2 - 6al + 2l^2)$$

$$\frac{dy_{AB}}{dx} = \frac{M_B}{6EIl}(3x^2 + 3a^2 - 6al + 2l^2)$$

With $I = \pi d^4/64$, the slope at the left bearing is

$$\left.\frac{dy_{AB}}{dx}\right|_{x=0} = \theta_A = \frac{M_B}{6E\left(\pi d^4/64\right)l}(3a^2 - 6al + 2l^2)$$

Solving for d

$$d = \sqrt[4]{\frac{32M_B}{3\pi E\theta_A l}\left(3a^2 - 6al + 2l^2\right)} = \sqrt[4]{\frac{32(1000)}{3\pi(30)10^6(0.002)(10)}\left[3(4^2) - 6(4)(10) + 2\left(10^2\right)\right]}$$

$$= 0.461 \text{ in} \qquad \textit{Ans.}$$

4-50 From Table A-5, $E = 10.4$ Mpsi
$\Sigma M_O = 0 = 18\,F_{BC} - 6(100) \quad \Rightarrow \quad F_{BC} = 33.33$ lbf
The cross sectional area of rod BC is $A = \pi(0.5^2)/4 = 0.1963$ in^2.

The deflection at point B will be equal to the elongation of the rod BC.

$$y_B = \left(\frac{FL}{AE}\right)_{BC} = \frac{33.33(12)}{(0.1963)30\left(10^6\right)} = 6.79\left(10^{-5}\right) \text{ in} \qquad \textit{Ans.}$$

4-51 $\Sigma M_O = 0 = 6\,F_{AC} - 11(100) \quad \Rightarrow \quad F_{AC} = 183.3$ lbf

The deflection at point A in the negative y direction is equal to the elongation of the rod AC. From Table A-5, $E_s = 30$ Mpsi.

$$y_A = -\left(\frac{FL}{AE}\right)_{AC} = -\frac{183.3(12)}{\left[\pi\left(0.5^2\right)/4\right]30\left(10^6\right)} = -3.735\left(10^{-4}\right) \text{ in}$$

By similar triangles the deflection at B due to the elongation of the rod AC is

$$\frac{y_A}{6} = \frac{y_{B1}}{18} \quad \Rightarrow \quad y_{B1} = 3y_A = 3(-3.735)10^{-4} = -0.00112 \text{ in}$$

From Table A-5, $E_a = 10.4$ Mpsi

The bar can then be treated as a simply supported beam with an overhang AB. From Table A-9, beam 10,

$$y_{B2} = \left(\overline{BD}\right)\left(\left.\frac{dy_{BC}}{dx}\right|_{x=l+a}\right) - \frac{Fa^2}{3EI}(l+a) = 7\left\{\frac{d}{dx}\left(\frac{F(x-l)}{6EI}\left[(x-l)^2 - a(3x-l)\right]\right)\right\}_{x=l+a} - \frac{Fa^2}{3EI}(l+a)$$

$$= 7\frac{F}{6EI}\left[3(x-l)^2 - 3a(x-l) - a(3x-l)\right]\Big|_{x=l+a} - \frac{Fa^2}{3EI}(l+a) = -\frac{7Fa}{6EI}(2l+3a) - \frac{Fa^2}{3EI}(l+a)$$

$$= -\frac{7(100)5}{6(10.4)10^6\left(0.25(2^3)/12\right)}\left[2(6)+3(5)\right] - \frac{100\left(5^2\right)}{3(10.4)10^6\left(0.25(2^3)/12\right)}(6+5)$$

$$= -0.01438 \text{ in}$$

$y_B = y_{B1} + y_{B2} = -0.00112 - 0.01438 = -0.0155$ in *Ans.*

4-52 From Table A-5, $E_A = 71.7$ GPa, $E_S = 207$ GPa.

$\Sigma M_O = 0 = 450\,F_{CD} - 650(4000) \quad \Rightarrow \quad F_{CD} = 5777.8$ N

$$y_D = -\left(\frac{FL}{AE}\right)_{CD} = -\frac{5777.8(220)}{(\pi/4)0.006^2(207)10^9} = -0.2172 \text{ mm}$$

The deflection of B due to y_D is

$(y_B)_1 = (650/450)(-0.2172) = -0.3137$ mm

Treat beam *OADB* as simply-supported at *O* and *D*. Use beam 10 of Table A-9 and use the equation for y_C,

$$\left(y_B\right)_2 = -\frac{Fa^2(l+a)}{3EI} = -\frac{4000(0.2)^2(0.450+0.2)}{3(71.7)10^9(0.012)0.050^3/(12)}$$

$$= -3.868\left(10^{-3}\right) \text{ m} = -3.868 \text{ mm}$$

Superposition:

$y_B = (y_B)_1 + (y_B)_2 = -0.3137 - 3.868 = -4.18$ mm *Ans.*

4-53 From Table A-5, $E_A = 71.7$ MPa, $E_S = 207$ MPa

$\Sigma M_O = 0 = 450\,F_{CD} - 150(4000) \quad \Rightarrow \quad F_{CD} = 1333$ N

$$y_D = -\left(\frac{FL}{AE}\right)_{CD} = -\frac{1333(220)}{(\pi/4)0.006^2(207)10^9} = -0.0501 \text{ mm}$$

The deflection of B due to y_D is

$(y_B)_1 = (650/450)(-0.0501) = -0.0724$ mm

Treat beam *OADB* as simply-supported at *O* and *D*. Find slope at *B* for beam 6 of Table A-9,

$$y_{BC} = \frac{Fa}{6EIl}\left[-x^3 + 3lx^2 - \left(2l^2 + a^2\right)x + la^2\right]$$

$$\theta_{BC} = \frac{dy_{BC}}{dx} = \frac{Fa}{6EIl}\left[-3x^2 + 6lx - 2l^2 - a^2\right]$$

For *OADB*,

$$\theta_D = \frac{4000(0.15)\left[-3(0.450)^2 + 6(0.450)0.450 - 2(0.450)^2 - 0.15^2\right]}{6(71.7)10^9(0.012)\left(0.050^3/12\right)0.450}$$

$$= 0.004\ 463 \text{ rad}$$

Deflection of *B* due to slope at *D* is

$$(y_B)_2 = 0.004\ 463(200) = 0.8926 \text{ mm}$$

Superposition:

$$y_B = (y_B)_1 + (y_B)_2 = -0.0725 + 0.8926 = 0.820 \text{ mm}$$ *Ans.*

4-54 From Table A-5, $E_A = 10.4$ Mpsi, $E_S = 30$ Mpsi

$\Sigma M_O = 0 = 18\,F_{CD} - 6(100) \quad \Rightarrow \quad F_{CD} = 33.33$ lbf

$$y_B = -\left(\frac{FL}{AE}\right)_{BC} = -\frac{33.33(12)}{(\pi/4)0.5^2(30)10^6} = -6.792\left(10^{-5}\right) \text{ in}$$

The deflection of *A* due to y_B is

$(y_A)_1 = (6/18)(-6.792)10^{-5} = -2.264\ (10^{-5})$ in

Treat beam *OAB* as simply-supported at *O* and *D*. Use beam 6 of Table A-9

$$(y_A)_2 = -\frac{Fbx}{6EIl}\left(x^2 + b^2 - l^2\right) = -\frac{100(12)6\left(6^2 + 12^2 - 18^2\right)}{6(10.4)10^6\left[(0.25)2^3/(12)\right]18} = -5.538\left(10^{-3}\right) \text{ in}$$

Superposition:

$$y_A = (y_A)_1 + (y_A)_2 = -2.264\ (10^{-5}) - 5.538\ (10^{-3}) = -5.56\ (10^{-3}) \text{ in}$$ *Ans.*

4-55 From Table A-5, $E_S = 30$ Mpsi

$\Sigma M_O = 0 = 18\,F_{AC} - 11(100) \quad \Rightarrow \quad F_{AC} = 183.3$ lbf

$$y_A = -\left(\frac{FL}{AE}\right)_{AC} = -\frac{183.3(12)}{(\pi/4)0.5^2(30)10^6} = -3.734\left(10^{-4}\right) \text{ in}$$ *Ans.*

4-56 From Table A-5, $E_A = 71.7$ MPa, $E_S = 207$ MPa

$\Sigma M_O = 0 = 450\,F_{CD} - 650(4000) \quad \Rightarrow \quad F_{CD} = 5777.8$ N

$$y_D = -\left(\frac{FL}{AE}\right)_{CD} = -\frac{5777.8(220)}{(\pi/4)0.006^2(207)10^9} = -0.2172 \text{ mm}$$

The deflection of *A* due to y_D is

$(y_A)_1 = (150/300)(-0.2172) = -0.1086$ mm

Treat beam *OADB* as simply-supported at *O* and *D*. Use beam 10 of Table A-9

$$(y_A)_2 = \frac{Fax}{6EIl}\left(l^2 - x^2\right) = \frac{4000(0.2)0.15\left(0.450^2 - 0.150^2\right)}{6(71.7)10^9\left[(0.012)0.050^3/(12)\right]0.450}$$

$$= 0.8926\left(10^{-3}\right) \text{ m} = 0.8926 \text{ mm}$$

Superposition:

$$y_A = (y_A)_1 + (y_A)_2 = -0.1086 + 0.8926 = 0.784 \text{ mm}$$ *Ans.*

4-57 From Table A-5, $E_A = 71.7$ MPa, $E_S = 207$ MPa

$\Sigma M_O = 0 = 450\, F_{CD} - 150(4000) \quad \Rightarrow \quad F_{CD} = 1333$ N

$$y_D = -\left(\frac{FL}{AE}\right)_{CD} = -\frac{1333(220)}{(\pi/4)0.006^2(207)10^9} = -0.0501 \text{ mm}$$

The deflection of A due to y_D is

$(y_A)_1 = (150/300)\,(-\,0.0501) = -\,0.02505$ mm

Treat beam $OADB$ as simply-supported at O and D. Use beam 6 of Table A-9

$$(y_A)_2 = \frac{Fbx}{6EIl}\left(x^2 + b^2 - l^2\right) = \frac{4000(0.3)0.15\left(0.15^2 + 0.3^2 - 0.45^2\right)}{6(71.7)10^9\left[(0.012)0.050^3/(12)\right]0.450}$$

$$= -0.6695\left(10^{-3}\right) \text{ m} = -0.6695 \text{ mm}$$

Superposition:

$y_A = (y_A)_1 + (y_A)_2 = -\,0.02505 -\, 0.6695 = -\,0.695$ mm *Ans.*

4-58 From Table A-5, $E = 207$ GPa, and $G = 79.3$ GPa.

$$|y_B| = \left(\frac{Tl}{GJ}\right)_{OC} l_{AB} + \left(\frac{Tl}{GJ}\right)_{AC} l_{AB} + \frac{Fl_{AB}{}^3}{3EI_{AB}} = \frac{Fl_{OC}l_{AB}{}^2}{G\left(\pi d_{OC}{}^4/32\right)} + \frac{Fl_{AC}l_{AB}{}^2}{G\left(\pi d_{AC}{}^4/32\right)} + \frac{Fl_{AB}{}^3}{3E\left(\pi d_3{}^4/64\right)}$$

$$= \frac{32Fl_{AB}{}^2}{\pi}\left[\frac{l_{OC}}{Gd_{OC}{}^4} + \frac{l_{AC}}{Gd_{AC}{}^4} + \frac{2l_{AB}}{3Ed_{AB}{}^4}\right]$$

The spring rate is $k = F/\,|y_B|$. Thus

$$k = \left\{\frac{32l_{AB}{}^2}{\pi}\left[\frac{l_{OC}}{Gd_{OC}{}^4} + \frac{l_{AC}}{Gd_{AC}{}^4} + \frac{2l_{AB}}{3Ed_{AB}{}^4}\right]\right\}^{-1}$$

$$= \left\{\frac{32\left(200^2\right)}{\pi}\left[\frac{200}{79.3\left(10^3\right)18^4} + \frac{200}{79.3\left(10^3\right)12^4} + \frac{2(200)}{3(207)10^3\left(8^4\right)}\right]\right\}^{-1}$$

$= 8.10$ N/mm *Ans.*

4-59 For the beam deflection, use beam 5 of Table A-9.

$$R_1 = R_2 = \frac{F}{2}$$

$$\delta_1 = \frac{F}{2k_1}, \text{ and } \delta_2 = \frac{F}{2k_2}$$

$$y_{AB} = -\delta_1 + \frac{\delta_1 - \delta_2}{l}x + \frac{Fx}{48EI}(4x^2 - 3l^3)$$

$$y_{AB} = F\left[-\frac{1}{2k_1} + \frac{k_2 - k_1}{2k_1k_2l}x + \frac{x}{48EI}(4x^2 - 3l^3)\right] \qquad \textit{Ans.}$$

For *BC*, since Table A-9 does not have an equation (because of symmetry) an equation will need to be developed as the problem is no longer symmetric. This can be done easily using beam 6 of Table A-9 with $a = l/2$

$$y_{BC} = \frac{-F}{2k_1} + \frac{Fk_2 - Fk_1}{2k_1k_2l}x + \frac{F(l/2)(l-x)}{EIl}\left(x^2 + \frac{l^2}{4} - 2lx\right)$$

$$= F\left[-\frac{1}{2k_1} + \frac{k_2 - k_1}{2k_1k_2l}x + \frac{(l-x)}{48EI}\left(4x^2 + l^2 - 8lx\right)\right] \qquad Ans.$$

4-60

$$R_1 = \frac{Fa}{l}, \text{ and } R_2 = \frac{F}{l}(l+a)$$

$$\delta_1 = \frac{Fa}{lk_1}, \text{ and } \delta_2 = \frac{F}{lk_2}(l+a)$$

$$y_{AB} = -\delta_1 + \frac{\delta_1 - \delta_2}{l}x + \frac{Fax}{6EIl}(l^2 - x^2)$$

$$y_{AB} = F\left\{-\frac{a}{k_1l} + \frac{x}{k_1k_2l^2}\left[k_2a - k_1(l+a)\right] + \frac{ax}{6EIl}(l^2 - x^2)\right\} \qquad Ans.$$

$$y_{BC} = -\delta_1 + \frac{\delta_1 - \delta_2}{l}x + \frac{F(x-l)}{6EI}\left[(x-l)^2 - a(3x-l)\right]$$

$$y_{BC} = F\left\{-\frac{a}{k_1l} + \frac{x}{k_1k_2l^2}\left[k_2a - k_1(l+a)\right] + \frac{(x-l)}{6EI}\left[(x-l)^2 - a(3x-l)\right]\right\} \qquad Ans.$$

4-61 Let the load be at $x \geq l/2$. The maximum deflection will be in Section *AB* (Table A-9, beam 6)

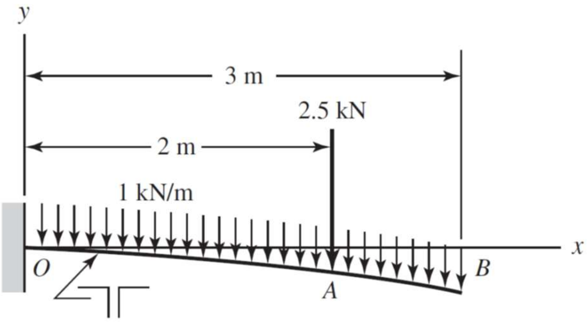
$$y_{AB} = \frac{Fbx}{6EIl}\left(x^2 + b^2 - l^2\right)$$

$$\frac{dy_{AB}}{dx} = \frac{Fb}{6EIl}\left(3x^2 + b^2 - l^2\right) = 0 \qquad \Rightarrow \qquad 3x^2 + b^2 - l^2 = 0$$

$$x = \sqrt{\frac{l^2 - b^2}{3}}, \quad x_{\max} = \sqrt{\frac{l^2}{3}} = 0.577l \qquad Ans.$$

For $x \leq l/2$, $x_{\min} = l - 0.577l = 0.423l \qquad Ans.$

4-62



$$M_O = 1(3000)(1500) + 2500(2000)$$
$$= 9.5\left(10^6\right) \text{ N·mm}$$
$$R_O = 1(3000) + 2500 = 5\ 500 \text{ N}$$

From Prob. 4-10, $I = 4.14(10^6)\text{ mm}^4$

$$M = -9.5\left(10^6\right) + 5500x - \frac{x^2}{2} - 2\,500\langle x\text{-}2\,000\rangle^1$$

$$EI\frac{dy}{dx} = -9.5\left(10^6\right)x + 2\,750x^2 - \frac{x^3}{6} - 1250\langle x-2000\rangle^2 + C_1$$

$$\frac{dy}{dx} = 0 \text{ at } x = 0 \quad \therefore C_1 = 0$$

$$EI\frac{dy}{dx} = -9.5\left(10^6\right)x + 2\,750x^2 - \frac{x^3}{6} - 1250\langle x-2000\rangle^2$$

$$EIy = -4.75\left(10^6\right)x^2 + 916.67x^3 - \frac{x^4}{24} - 416.67\langle x-2000\rangle^3 + C_2$$

$y = 0$ at $x = 0 \quad \therefore C_2 = 0$, and therefore

$$y = -\frac{1}{24EI}\left[114\left(10^6\right)x^2 - 22\left(10^3\right)x^3 + x^4 + 10\left(10^3\right)\langle x-2000\rangle^3\right]$$

$$y_B = -\frac{1}{24\left(207\right)10^3\left(4.14\right)10^6}\left[114\left(10^6\right)3000^2 - 22\left(10^3\right)3000^3 + 3000^4 + 10\left(10^3\right)\left(3000-2000\right)^3\right]$$

$$= -25.4 \text{ mm} \qquad Ans.$$

$M_O = 9.5\ (10^6)$ N·m. The maximum stress is compressive at the bottom of the beam where $y = 29.0 - 100 = -71$ mm

$$\sigma_{\max} = -\frac{My}{I} = -\frac{-9.5\left(10^6\right)(-71)}{4.14(10^6)} = -163\left(10^6\right)\text{Pa} = -163\text{MPa} \qquad Ans.$$

The solutions are the same as Prob. 4-10.

4-63 See Prob. 4-11 for reactions: $R_O = 465$ lbf and $R_C = 285$ lbf. Using lbf and inch units

$$M = 465\,x - 450\,\langle x - 72\rangle^1 - 300\,\langle x - 120\rangle^1$$

$$EI\frac{dy}{dx} = 232.5x^2 - 225\langle x-72\rangle^2 - 150\langle x-120\rangle^2 + C_1$$

$$EIy = 77.5\,x^3 - 75\,\langle x - 72\rangle^3 - 50\,\langle x - 120\rangle^3 - C_1x$$

$y = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$

$y = 0$ at $x = 240$ in

$0 = 77.5(240^3) - 75(240 - 72)^3 - 50(240 - 120)^3 + C_1\,x \quad \Rightarrow \quad C_1 = -\,2.622(10^6)$ lbf·in^2

and,

$$EIy = 77.5\,x^3 - 75\,\langle x - 72\rangle^3 - 50\,\langle x - 120\rangle^3 - 2.622(10^6)\,x$$

Substituting $y = -\,0.5$ in at $x = 120$ in gives

$$30(10^6)\,I\,(-\,0.5) = 77.5\,(120^3) - 75(120 - 72)^3 - 50(120 - 120)^3 - 2.622(10^6)(120)$$

$$I = 12.60 \text{ in}^4$$

Select two 5 in × 6.7 lbf/ft channels; from Table A-7, $I = 2(7.49) = 14.98$ in^4

$$y_{\text{midspan}} = \frac{12.60}{14.98}\left(-\frac{1}{2}\right) = -0.421 \text{ in} \qquad \textit{Ans.}$$

The maximum moment occurs at $x = 120$ in where $M_{\max} = 34.2(10^3)$ lbf·in

$$\sigma_{\max} = \frac{Mc}{I} = \frac{34.2(10^3)(2.5)}{14.98} = 5\,710 \text{ psi} \qquad \text{O.K.}$$

The solutions are the same as Prob. 4-11.

4-64 $I = \pi(1.5^4)/64 = 0.2485$ in^4, and $w = 150/12 = 12.5$ lbf/in.

$$R_O = \frac{1}{2}(12.5)39 + \frac{24}{39}(340) = 453.0 \text{ lbf}$$

$$M = 453.0x - \frac{12.5}{2}x^2 - 340\langle x-15\rangle^1$$

$$EI\frac{dy}{dx} = 226.5x^2 - \frac{12.5}{6}x^3 - 170\langle x-15\rangle^2 + C_1$$

$$EIy = 75.5x^3 - 0.5208x^4 - 56.67\langle x-15\rangle^3 + C_1x + C_2$$

$y = 0 \text{ at } x = 0 \quad \Rightarrow \quad C_2 = 0$

$y = 0 \text{ at } x = 39 \text{ in} \quad \Rightarrow \quad C_1 = -6.385(10^4) \text{ lbf}\cdot\text{in}^2$ Thus,

$$y = \frac{1}{EI}\left[75.5x^3 - 0.5208x^4 - 56.67\langle x-15\rangle^3 - 6.385\left(10^4\right)x\right]$$

Evaluating at $x = 15$ in,

$$y_A = \frac{1}{30(10^6)(0.2485)}\left[75.5\left(15^3\right)-0.5208\left(15^4\right)-56.67\left(15-15\right)^3-6.385\left(10^4\right)(15)\right]$$
$$= -0.0978 \text{ in} \quad Ans.$$

$$y_{\text{midspan}} = \frac{1}{30(10^6)(0.2485)}\left[75.5\left(19.5^3\right)-0.5208\left(19.5^4\right)-56.67\left(19.5-15\right)^3-6.385\left(10^4\right)(19.5)\right]$$
$$= \ -0.1027 \text{ in} \quad Ans.$$

5 % difference *Ans.*

The solutions are the same as Prob. 4-12.

4-65 $I = 0.05 \text{ in}^4$, $R_A = \dfrac{3(14)100}{10} = 420 \text{ lbf}\uparrow$ and $R_B = \dfrac{7(14)100}{10} = 980 \text{ lbf}\uparrow$

$M = 420\,x - 50\,x^2 + 980\,\langle x - 10\rangle^1$

$$EI\frac{dy}{dx} = 210x^2 - 16.667x^3 + 490\langle x-10\rangle^2 + C_1$$

$$EIy = 70x^3 - 4.167x^4 + 163.3\langle x-10\rangle^3 + C_1x + C_2$$

$y = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$

$y = 0$ at $x = 10$ in $\quad \Rightarrow \quad C_1 = -2\,833 \text{ lbf·in}^2$. Thus,

$$y = \frac{1}{30\left(10^6\right)0.05}\left[70x^3 - 4.167x^4 + 163.3\langle x-10\rangle^3 - 2833x\right]$$

$$= 6.667\left(10^{-7}\right)\left[70x^3 - 4.167x^4 + 163.3\langle x-10\rangle^3 - 2833x\right] \qquad Ans.$$

The tabular results and plot are exactly the same as Prob. 4-21.

4-66 $R_A = R_B = 400$ N, and $I = 6(32^3)/12 = 16\,384 \text{ mm}^4$.

First half of beam,

$M = -400\,x + 400\,\langle x - 300\rangle^1$

$$EI\frac{dy}{dx} = -200x^2 + 200\langle x-300\rangle^2 + C_1$$

From symmetry, $dy/dx = 0$ at $x = 550$ mm $\quad \Rightarrow \quad 0 = -200(550^2) + 200(550 - 300)^2 + C_1$

$\Rightarrow \quad C_1 = 48(10^6) \text{ N·mm}^2$

$EIy = -66.67\,x^3 + 66.67\,\langle x - 300\rangle^3 + 48(10^6)\,x + C_2$

$y = 0$ at $x = 300$ mm $\quad\Rightarrow\quad C_2 = -\,12.60(10^9)\ \text{N}\cdot\text{mm}^3$.

The term $(EI)^{-1} = [207(10^3)16\,384]^{-1} = 2.949\,(10^{-10})$ Thus

$$y = 2.949\,(10^{-10})\,[-\,66.67\,x^3 + 66.67\,\langle x - 300\rangle^3 + 48(10^6)\,x - 12.60(10^9)]$$

$y_O = -\,3.72$ mm *Ans.*

$$y|_{x\,=\,550\text{ mm}} = 2.949\,(10^{-10})\,[-\,66.67\,(550^3) + 66.67\,(550 - 300)^3 + 48(10^6)\,550 - 12.60(10^9)] = 1.11\text{ mm} \quad \textit{Ans.}$$

The solutions are the same as Prob. 4-13.

4-67

$$\sum M_B = 0 = R_1 l + Fa - M_A \quad\Rightarrow\quad R_1 = \frac{1}{l}\left(M_A - Fa\right)$$

$$\sum M_A = 0 = M_A + R_2 l - F(l + a) \quad\Rightarrow\quad R_2 = \frac{1}{l}\left(Fl + Fa - M_A\right)$$

$$M = R_1 x - M_A + R_2\langle x - l\rangle^1$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_1 x^2 - M_A x + \frac{1}{2}R_2\langle x - l\rangle^2 + C_1$$

$$EIy = \frac{1}{6}R_1 x^3 - \frac{1}{2}M_A x^2 + \frac{1}{6}R_2\langle x - l\rangle^3 + C_1 x + C_2$$

$y = 0$ at $x = 0 \quad\Rightarrow\quad C_2 = 0$

$y = 0$ at $x = l \quad\Rightarrow\quad C_1 = -\frac{1}{6}R_1 l^2 + \frac{1}{2}M_A l$. Thus,

$$EIy = \frac{1}{6}R_1 x^3 - \frac{1}{2}M_A x^2 + \frac{1}{6}R_2\langle x - l\rangle^3 + \left(-\frac{1}{6}R_1 l^2 + \frac{1}{2}M_A l\right)x$$

$$y = \frac{1}{6EIl}\left[\left(M_A - Fa\right)x^3 - 3M_A x^2 l + \left(Fl + Fa - M_A\right)\langle x - l\rangle^3 + \left(Fal^2 + 2M_A l^2\right)x\right] \qquad \textit{Ans.}$$

In regions,

$$y_{AB} = \frac{1}{6EIl}\left[\left(M_A - Fa\right)x^3 - 3M_A x^2 l + \left(Fal^2 + 2M_A l^2\right)x\right]$$

$$= \frac{x}{6EIl}\left[M_A\left(x^2 - 3lx + 2l^2\right) + Fa\left(l^2 - x^2\right)\right] \qquad \textit{Ans.}$$

$$y_{BC}=\frac{1}{6EIl}\left[\left(M_A-Fa\right)x^3-3M_Ax^2l+\left(Fl+Fa-M_A\right)\left(x-l\right)^3+\left(Fal^2+2M_Al^2\right)x\right]$$

$$=\frac{1}{6EIl}\left\{M_A\left[x^3-3x^2l-\left(x-l\right)^3+2xl^2\right]+F\left[-ax^3+\left(l+a\right)\left(x-l\right)^3+axl^2\right]\right\}$$

$$=\frac{1}{6EIl}\left\{-M_A\left(x-l\right)l^2+Fl\left(x-l\right)\left[\left(x-l\right)^2-a\left(3x-l\right)\right]\right\}$$

$$=\frac{\left(x-l\right)}{6EI}\left\{-M_Al+F\left[\left(x-l\right)^2-a\left(3x-l\right)\right]\right\} \qquad Ans.$$

The solutions reduce to the same as Prob. 4-17.

4-68 $\sum M_D=0=R_1l-w\left(b-a\right)\left[l-b+\frac{1}{2}\left(b-a\right)\right] \quad \Rightarrow \quad R_1=\frac{w\left(b-a\right)}{2l}\left(2l-b-a\right)$

$$M=R_1x-\frac{w}{2}\left\langle x-a\right\rangle^2+\frac{w}{2}\left\langle x-b\right\rangle^2$$

$$EI\frac{dy}{dx}=\frac{1}{2}R_1x^2-\frac{w}{6}\left\langle x-a\right\rangle^3+\frac{w}{6}\left\langle x-b\right\rangle^3+C_1$$

$$EIy=\frac{1}{6}R_1x^3-\frac{w}{24}\left\langle x-a\right\rangle^4+\frac{w}{24}\left\langle x-b\right\rangle^4+C_1x+C_2$$

$y=0$ at $x=0 \quad \Rightarrow \quad C_2=0$
$y=0$ at $x=l$

$$C_1=-\frac{1}{l}\left[\frac{1}{6}R_1l^3-\frac{w}{24}\left(l-a\right)^4+\frac{w}{24}\left(l-b\right)^4\right]$$

$$y=\frac{1}{EI}\left\{\frac{1}{6}\frac{w\left(b-a\right)}{2l}\left(2l-b-a\right)x^3-\frac{w}{24}\left\langle x-a\right\rangle^4+\frac{w}{24}\left\langle x-b\right\rangle^4\right.$$
$$\left.-x\frac{1}{l}\left[\frac{1}{6}\frac{w\left(b-a\right)}{2l}\left(2l-b-a\right)l^3-\frac{w}{24}\left(l-a\right)^4+\frac{w}{24}\left(l-b\right)^4\right]\right\}$$

$$=\frac{w}{24EIl}\left\{2\left(b-a\right)\left(2l-b-a\right)x^3-l\left\langle x-a\right\rangle^4+l\left\langle x-b\right\rangle^4\right.$$
$$\left.-x\left[2\left(b-a\right)\left(2l-b-a\right)l^2-\left(l-a\right)^4+\left(l-b\right)^4\right]\right\} \qquad Ans.$$

The above answer is sufficient. In regions,

$$y_{AB}=\frac{w}{24EIl}\left\{2\left(b-a\right)\left(2l-b-a\right)x^3-x\left[2\left(b-a\right)\left(2l-b-a\right)l^2-\left(l-a\right)^4+\left(l-b\right)^4\right]\right\}$$

$$=\frac{wx}{24EIl}\left[2\left(b-a\right)\left(2l-b-a\right)x^2-2\left(b-a\right)\left(2l-b-a\right)l^2+\left(l-a\right)^4-\left(l-b\right)^4\right]$$

$$y_{BC}=\frac{w}{24EIl}\left\{2(b-a)(2l-b-a)x^3-l(x-a)^4\right.$$
$$\left.-x\left[2(b-a)(2l-b-a)l^2-(l-a)^4+(l-b)^4\right]\right\}$$

$$y_{CD}=\frac{w}{24EIl}\left\{2(b-a)(2l-b-a)x^3-l(x-a)^4+l(x-b)^4\right.$$
$$\left.-x\left[2(b-a)(2l-b-a)l^2-(l-a)^4+(l-b)^4\right]\right\}$$

These equations can be shown to be equivalent to the results found in Prob. 4-19.

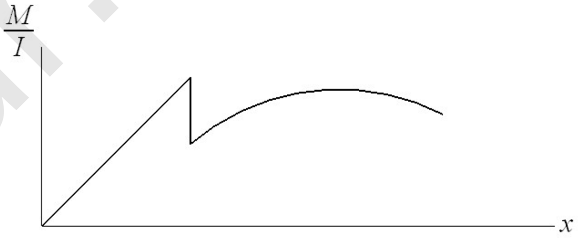
4-69 $I_1=\pi(1.375^4)/64=0.1755\text{ in}^4$, $I_2=\pi(1.75^4)/64=0.4604\text{ in}^4$,

$R_1=0.5(180)(10)=900$ lbf

Since the loading and geometry are symmetric, we will only write the equations for the first half of the beam

For <u>$0\le x\le 8$ in</u> $\quad M=900x-90\langle x-3\rangle^2$

At $x=3$, $M=2700$ lbf·in



Writing an equation for M/I, as seen in the figure, the magnitude and slope reduce since $I_2>I_1$. To reduce the magnitude at $x=3$ in, we add the term, $-2700(1/I_1-1/I_2)\langle x-3\rangle^0$. The slope of 900 at $x=3$ in is also reduced. We account for this with a ramp function, $\langle x-3\rangle^1$. Thus,

$$\frac{M}{I}=\frac{900x}{I_1}-2700\left(\frac{1}{I_1}-\frac{1}{I_2}\right)\langle x-3\rangle^0-900\left(\frac{1}{I_1}-\frac{1}{I_2}\right)\langle x-3\rangle^1-\frac{90}{I_2}\langle x-3\rangle^2$$
$$=5128x-9520\langle x-3\rangle^0-3173\langle x-3\rangle^1-195.5\langle x-3\rangle^2$$

$$E\frac{dy}{dx}=2564x^2-9520\langle x-3\rangle^1-1587\langle x-3\rangle^2-65.17\langle x-3\rangle^3+C_1$$

Boundary Condition: $\frac{dy}{dx}=0$ at $x=8$ in

$0=2564(8)^2-9520(8-3)-1587(8-3)^2-65.17(8-3)^3+C_1\ \Rightarrow$
$C_1=-68.67\ (10^3)\ \text{lbf/in}^2$

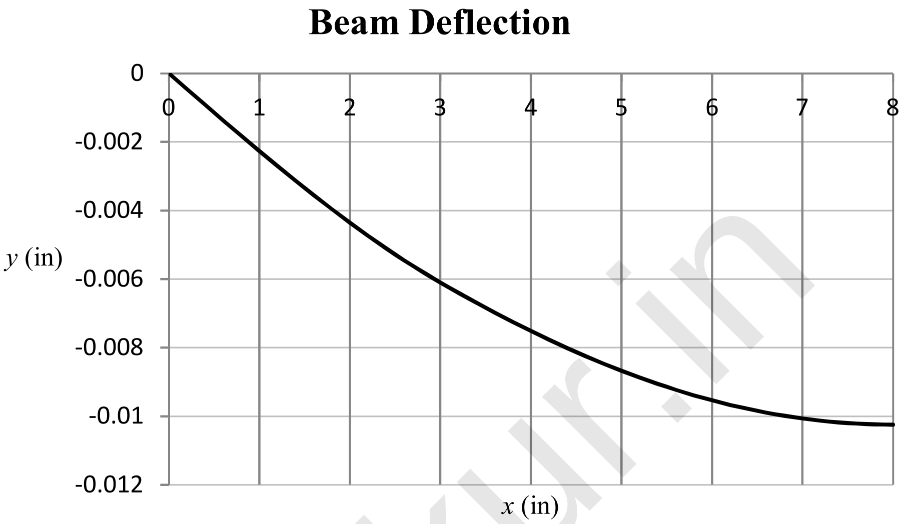
$$Ey=854.7x^3-4760\langle x-3\rangle^2-529\langle x-3\rangle^3-16.29\langle x-3\rangle^4-68.67(10^3)x+C_2$$

$y = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$

Thus, for $\underline{0 \le x \le 8 \text{ in}}$

$$y = \frac{1}{30(10^6)}\left[854.7x^3 - 4760\langle x-3\rangle^2 - 529\langle x-3\rangle^3 - 16.29\langle x-3\rangle^4 - 68.7(10^3)x\right] \quad \textit{Ans.}$$

Using a spreadsheet, the following graph represents the deflection equation found above

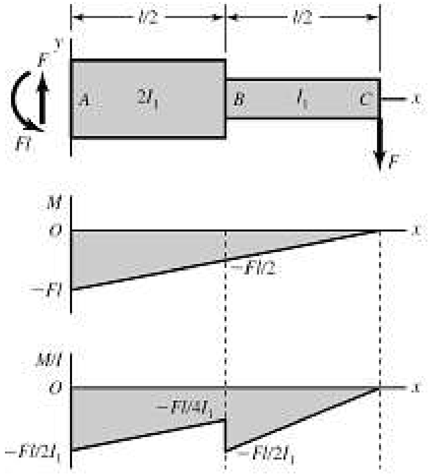


The maximum is $y_{\max} = -0.0102$ in at $x = 8$ in *Ans.*

4-70 The force and moment reactions at the left support are F and Fl respectively. The bending moment equation is

$$M = Fx - Fl$$

Plots for M and M/I are shown.



M/I can be expressed using singularity functions

$$\frac{M}{I} = \frac{F}{2I_1}x - \frac{Fl}{2I_1} - \frac{Fl}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^0 + \frac{F}{2I_1}\left\langle x - \frac{l}{2}\right\rangle^1$$

where the step down and increase in slope at $x = l/2$ are given by the last two terms. Integrate

$$E\frac{dy}{dx} = \frac{F}{4I_1}x^2 - \frac{Fl}{2I_1}x - \frac{Fl}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^1 + \frac{F}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^2 + C_1$$

$dy/dx = 0$ at $x = 0 \quad \Rightarrow \quad C_1 = 0$

$$Ey = \frac{F}{12I_1}x^3 - \frac{Fl}{4I_1}x^2 - \frac{Fl}{8I_1}\left\langle x - \frac{l}{2}\right\rangle^2 + \frac{F}{12I_1}\left\langle x - \frac{l}{2}\right\rangle^3 + C_2$$

$y = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$

$$y = \frac{F}{24EI_1}\left(2x^3 - 6lx^2 - 3l\left\langle x - \frac{l}{2}\right\rangle^2 + 2\left\langle x - \frac{l}{2}\right\rangle^3\right)$$

$$y\Big|_{x=l/2} = \frac{F}{24EI_1}\left[2\left(\frac{l}{2}\right)^3 - 6l\left(\frac{l}{2}\right)^2 - 3l(0) + 2(0)\right] = -\frac{5Fl^3}{96EI_1} \qquad Ans.$$

$$y\Big|_{x=l} = \frac{F}{24EI_1}\left[2(l)^3 - 6l(l^2) - 3l\left(l - \frac{l}{2}\right)^2 + 2\left(x - \frac{l}{2}\right)^3\right] = -\frac{3Fl^3}{16EI_1} \qquad Ans.$$

The answers are identical to Ex. 4-10.

4-71 Place a fictitious force, Q, at the center. The reaction, $R_1 = wl/2 + Q/2$

$$M = \left(\frac{wl}{2} + \frac{Q}{2}\right)x - \frac{wx^2}{2} \qquad \frac{\partial M}{\partial Q} = \frac{x}{2}$$

Integrating for half the beam and doubling the results

$$y_{\max} = \left(2\frac{1}{EI}\int_0^{l/2} M\left(\frac{\partial M}{\partial Q}\right)dx\right)_{Q=0} = \frac{2}{EI}\int_0^{l/2}\left[\left(\frac{wl}{2}\right)x - \frac{wx^2}{2}\right]\left(\frac{x}{2}\right)dx$$

Note, after differentiating with respect to Q, it can be set to zero

$$y_{\max} = \frac{w}{2EI}\int_0^{l/2} x^2(l - x)dx = \frac{w}{2EI}\left(\frac{x^3 l}{3} - \frac{x^4}{4}\right)\Bigg|_0^{l/2} = \frac{5w}{384EI} \qquad Ans.$$

4-72 Place a fictitious force Q pointing downwards at the end. Use the variable $\bar{x}$ originating at the free end and positive to the left

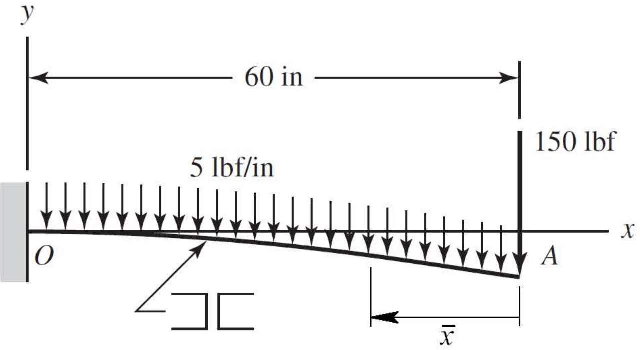
$$M = -Qx - \frac{wx^2}{2} \qquad \frac{\partial M}{\partial Q} = -x$$

$$y_{\max} = \left[\frac{1}{EI}\int_0^l M\left(\frac{\partial M}{\delta Q}\right)dx\right]_{Q=0} = \frac{1}{EI}\int_0^l\left(-\frac{wx^2}{2}\right)(-x)\,dx = \frac{w}{2EI}\int_0^l x^3 dx$$

$$= \frac{wl^4}{8EI} \qquad Ans.$$

4-73 From Table A-7, $I_{1\text{-}1} = 1.85$ in^4. Thus, $I = 2(1.85) = 3.70$ in^4

First treat the end force as a variable, F. Adding weight of channels of 2(5)/12 = 0.833 lbf/in. Using the variable $\bar{x}$ as shown in the figure



$$M = -F\,\bar{x} - \frac{5.833}{2}\bar{x}^2 = -F\,\bar{x} - 2.917\bar{x}^2$$

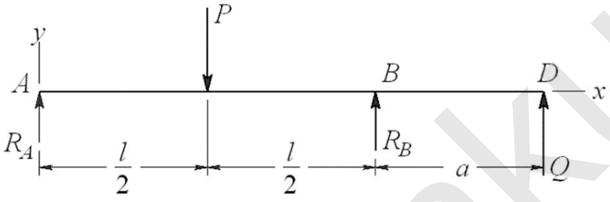
$$\frac{\partial M}{\partial F} = -\bar{x}$$

$$\delta_A = \frac{1}{EI}\int_0^{60} M\frac{\partial M}{\partial F}\,d\,\bar{x} = \frac{1}{EI}\int_0^{60}(F\,\bar{x} + 2.917\bar{x}^2)(\bar{x})\;d\,\bar{x} = \frac{1}{EI}(F\bar{x}^3/3 + 2.917\bar{x}^4/4)\Big|_0^{60}$$

$$= \frac{(150)(60^3)/3 + (2.917)(60^4)/4}{30(10^6)(3.70)} = 0.182 \text{ in} \quad \text{in the direction of the 150 lbf force}$$

$$\therefore y_A = -0.182 \text{ in} \qquad \textit{Ans.}$$

4-74



$$R_A = \frac{P}{2} + \frac{a}{l}Q$$

$$R_B = \frac{P}{2} - \frac{(l+a)}{l}Q$$

$$0 \le x \le \frac{l}{2} \qquad M = \left(\frac{P}{2} + \frac{a}{l}Q\right)x \qquad \frac{\partial M}{\partial Q} = \frac{a}{l}x$$

$$\frac{l}{2} \le x \le l \qquad M = \left(\frac{P}{2} + \frac{a}{l}Q\right)x - P\left(x - \frac{l}{2}\right) \qquad \frac{\partial M}{\partial Q} = \frac{a}{l}x$$

$$l \le x \le (l+a) \qquad M = Q(l+a-x) \qquad \frac{\partial M}{\partial Q} = (l+a-x)$$

We observe that for section BD, the only force is Q, which is zero, so there is no contribution to the deflection at D from the strain energy in section BD.

$$y_D = \left(\frac{1}{EI}\int_0^{l+a} M\frac{\partial M}{\partial Q}dx\right)_{Q=0} = \frac{1}{EI}\left[\int_0^{l/2}\frac{Pa}{2l}x^2dx + \int_{l/2}^{l}\left(\frac{Pa}{2l}x^2 - \frac{Pa}{l}x^2 + \frac{Pa}{2}x\right)dx\right]$$

The first two integrals can be combined from 0 to l,

$$y_D = \frac{1}{EI}\left\{\frac{Pal^3}{6l} - \frac{Pa}{l}\left[l^3 - \left(\frac{l}{2}\right)^3\right] + \frac{Pa}{4}\left[l^2 - \left(\frac{l}{2}\right)^2\right]\right\} = \frac{Pal^2}{16EI} \qquad \textit{Ans.}$$

(b) Table A-9, beam 5 with $F = P$,

$$\text{Slope:} \quad \theta = \frac{dy}{dx} = \frac{P}{48EI}\left(12x^2 - 3l^2\right) = \frac{P}{16EI}\left(4x^2 - l^2\right)$$

At $x = 0$, $\theta_A = -\dfrac{Pl^2}{16EI}$. By symmetry, $\theta_C = \dfrac{Pl^2}{16EI}$

$$y_C = a\theta_C = \frac{Pal^2}{16EI} \qquad \textit{Ans.} \qquad \text{This agrees with part (a)}$$

4-75 The energy includes torsion in AC, torsion in CO, and bending in AB.

Neglecting transverse shear in AB

$$M = Fx, \ \frac{\partial M}{\partial F} = x$$

In AC and CO,

$$T = Fl_{AB}, \ \frac{\partial T}{\partial F} = l_{AB}$$

The total energy is

$$U = \left(\frac{T^2 l}{2GJ}\right)_{AC} + \left(\frac{T^2 l}{2GJ}\right)_{CO} + \int_0^{l_{AB}} \frac{M^2}{2EI_{AB}} dx$$

The deflection at the tip is

$$\delta = \frac{\partial U}{\partial F} = \frac{Tl_{AC}}{GJ_{AC}}\frac{\partial T}{\partial F} + \frac{Tl_{CO}}{GJ_{CO}}\frac{\partial T}{\partial F} + \int_0^{l_{AB}} \frac{M}{EI_3}\frac{\partial M}{\partial F}dx = \frac{Tl_{AC}l_{AB}}{GJ_{AC}} + \frac{Tl_{CO}l_{AB}}{GJ_{CO}} + \frac{1}{EI_{AB}}\int_0^{l_{AB}} Fx^2 dx$$

$$\delta = \frac{Tl_{AC}l_{AB}}{GJ_{AC}} + \frac{Tl_{CO}l_{AB}}{GJ_{CO}} + \frac{Fl_{AB}^3}{3EI_{AB}} = \frac{Fl_{AC}l_{AB}^2}{G\left(\pi d_{AC}^4/32\right)} + \frac{Fl_{CO}l_{AB}^2}{G\left(\pi d_{CO}^4/32\right)} + \frac{Fl_{AB}^3}{3E\left(\pi d_{AB}^4/64\right)}$$

$$= \frac{32Fl_{AB}^2}{\pi}\left(\frac{l_{AC}}{Gd_{AC}^4} + \frac{l_{CO}}{Gd_{CO}^4} + \frac{2l_{AB}}{3Ed_{AB}^4}\right)$$

$$k = \frac{F}{\delta} = \frac{\pi}{32l_{AB}^2}\left(\frac{l_{AC}}{Gd_{AC}^4} + \frac{l_{CO}}{Gd_{CO}^4} + \frac{2l_{AB}}{3Ed_{AB}^4}\right)^{-1}$$

$$= \frac{\pi}{32\left(200^2\right)}\left(\frac{200}{79.3\left(10^3\right)18^4} + \frac{200}{79.3\left(10^3\right)12^4} + \frac{2\left(200\right)}{3\left(207\right)10^3\left(8^4\right)}\right)^{-1} = 8.10 \text{ N/mm} \qquad \textit{Ans.}$$

4-76 $I_1 = \pi(1.375^4)/64 = 0.1755 \text{ in}^4$, $I_2 = \pi(1.75^4)/64 = 0.4604 \text{ in}^4$

Place a fictitious force Q pointing downwards at the midspan of the beam, $x = 8$ in

$$R_1 = \frac{1}{2}(10)180 + \frac{1}{2}Q = 900 + 0.5Q$$

For $\underline{0 \le x \le 3 \text{ in}}$ $\quad M = (900 + 0.5Q)x \qquad \dfrac{\partial M}{\partial Q} = 0.5x$

For $\underline{3 \le x \le 13 \text{ in}}$ $\quad M = (900 + 0.5Q)x - 90(x-3)^2 \qquad \dfrac{\partial M}{\partial Q} = 0.5x$

By symmetry it is equivalent to use twice the integral from 0 to 8

$$\delta = \left(2\int_0^8 \frac{M}{EI}\frac{\partial M}{\partial Q} dx \right)_{Q=0} = \frac{1}{EI_1}\int_0^3 900x^2 dx + \frac{1}{EI_2}\int_3^8 \left[900x - 90(x-3)^2 \right] x\,dx$$

$$= \frac{300x^3}{EI_1}\Bigg|_0^3 + \frac{1}{EI_2}\left[300x^3 - 90(\frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2) \right]\Bigg|_3^8$$

$$= \frac{8100}{EI_1} + \frac{1}{EI_2}\left[145.5(10^3) - 25.31(10^3) \right] = \frac{8100}{30(10^6)0.1755} + \frac{120.2(10^3)}{30(10^6)0.4604}$$

$$= 0.0102 \text{ in} \qquad \textit{Ans.}$$

4-77 $I = \pi(0.5^4)/64 = 3.068\ (10^{-3})\ \text{in}^4$, $J = 2I = 6.136\ (10^{-3})\ \text{in}^4$, $A = \pi(0.5^2)/4 = 0.1963\ \text{in}^2$.

Consider x to be in the direction of OA, y vertically upward, and z in the direction of AB. Resolve the force F into components in the x and y directions obtaining 0.6 F in the horizontal direction and 0.8 F in the negative vertical direction. The 0.6 F force creates strain energy in the form of bending in AB and OA, and tension in OA. The 0.8 F force creates strain energy in the form of bending in AB and OA, and torsion in OA. Use the dummy variable $\bar{x}$ to originate at the end where the loads are applied on each segment,

$\underline{0.6\ F}$: $\quad AB \quad M = 0.6F\bar{x} \qquad \dfrac{\partial M}{\partial F} = 0.6\bar{x}$

$\quad OA \quad M = 4.2F \qquad \dfrac{\partial M}{\partial F} = 4.2$

$\quad F_a = 0.6F \qquad \dfrac{\partial F_a}{\partial F} = 0.6$

$\underline{0.8\ F}$: $\quad AB \quad M = 0.8F\bar{x} \qquad \dfrac{\partial M}{\partial F} = 0.8\bar{x}$

$\quad OA \quad M = 0.8F\bar{x} \qquad \dfrac{\partial M}{\partial F} = 0.8\bar{x}$

$$T = 5.6F \qquad \frac{\partial T}{\partial F} = 5.6$$

Once the derivatives are taken the value of $F = 15$ lbf can be substituted in. The deflection of B in the direction of F is*

$$\begin{aligned}
(\delta_B)_F &= \frac{\partial U}{\partial F} = \left(\frac{F_a L}{AE}\right)_{OA}\frac{\partial F_a}{\partial F} + \left(\frac{TL}{JG}\right)_{OA}\frac{\partial T}{\partial F} + \frac{1}{EI}\sum\int M\frac{\partial M}{\partial F}\,d\bar{x} \\
&= \frac{0.6(15)15}{0.1963(30)10^6}(0.6) + \frac{5.6(15)15}{6.136\left(10^{-3}\right)11.5\left(10^6\right)}(5.6) \\
&\quad + \frac{15}{30\left(10^6\right)3.068\left(10^{-3}\right)}\int_0^7 (0.6\bar{x})^2\,d\bar{x} + \frac{15\left(4.2^2\right)}{30\left(10^6\right)3.068\left(10^{-3}\right)}\int_0^{15} d\bar{x} + \\
&\quad + \frac{15}{30\left(10^6\right)3.068\left(10^{-3}\right)}\int_0^7 (0.8\bar{x})^2\,d\bar{x} + \frac{15}{30\left(10^6\right)3.068\left(10^{-3}\right)}\int_0^{15} (0.8\bar{x})^2\,d\bar{x} \\
&= 1.38\left(10^{-5}\right) + 0.1000 + 6.71\left(10^{-3}\right) + 0.0431 + 0.0119 + 0.1173 \\
&= 0.279 \text{ in} \qquad \textit{Ans.}
\end{aligned}$$

*Note. Be careful, this is not the actual deflection of point B. For this, fictitious forces must be placed on B in the x, y, and z directions. Determine the energy due to each, take derivatives, and then substitute the values of $F_x = 9$ lbf, $F_y = -12$ lbf, and $F_z = 0$. This can be done separately and then added by vector addition. The actual deflections of B are found to be

$$\boldsymbol{\delta_B} = 0.0831\,\mathbf{i} - 0.2862\,\mathbf{j} - 0.00770\,\mathbf{k} \text{ in}$$

From this, the deflection of B <u>in the direction of F</u> is

$$(\delta_B)_F = 0.6(0.0831) + 0.8(0.2862) = 0.279 \text{ in}$$

which agrees with our result.

4-78 Strain energy. *AB*: Bending and torsion, *BC*: Bending and torsion, *CD*: Bending.
$I_{AB} = \pi(1^4)/64 = 0.04909$ in^4, $J_{AB} = 2\,I_{AB} = 0.09818$ in^4, $I_{BC} = 0.25(1.5^3)/12 = 0.07031$ in^4, $I_{CD} = \pi(0.75^4)/64 = 0.01553$ in^4.
For the torsion of bar BC, Eq. (3-41) is in the form of $\theta = TL/(JG)$, where the equivalent of J is $J_{\text{eq}} = \beta bc^3$. With $b/c = 1.5/0.25 = 6$, $J_{BC} = \beta bc^3 = 0.299(1.5)0.25^3 = 7.008\,(10^{-3})$ in^4.

Use the dummy variable $\bar{x}$ to originate at the end where the loads are applied on each segment,

$$AB\text{: Bending} \quad M = F\bar{x} + 2F \qquad \frac{\partial M}{\partial F} = \bar{x} + 2$$

$$\text{Torsion} \quad T = 5F \qquad \frac{\partial T}{\partial F} = 5$$

$$BC\text{: Bending} \quad M = F\bar{x} \qquad \frac{\partial M}{\partial F} = \bar{x}$$

$$\text{Torsion} \quad T = 2F \qquad \frac{\partial T}{\partial F} = 2$$

$$CD\text{: Bending} \quad M = F\bar{x} \qquad \frac{\partial M}{\partial F} = \bar{x}$$

$$\delta_D = \frac{\partial U}{\partial F} = \sum \frac{Tl}{JG}\frac{\partial T}{\partial F} + \sum \frac{1}{EI}\int M\frac{\partial M}{\partial F}d\bar{x}$$

$$= \frac{5F(6)}{0.09818(11.5)10^6}(5) + \frac{2F(5)}{7.008(10^{-3})11.5(10^6)}2 + \frac{1}{30(10^6)0.04909}\int_0^6 F(\bar{x}+2)^2 d\bar{x}$$

$$+ \frac{1}{30(10^6)0.07031}\int_0^5 F\bar{x}^2 d\bar{x} + \frac{1}{30(10^6)0.01553}\int_0^2 F\bar{x}^2 d\bar{x}$$

$$= 1.329(10^{-4})F + 2.482(10^{-4})F + 1.141(10^{-4})F + 1.98(10^{-5})F + 5.72(10^{-6})F$$

$$= 5.207(10^{-4})F = 5.207(10^{-4})200 = 0.104 \text{ in} \qquad Ans.$$

4-79 $A_{AB} = \pi(1^2)/4 = 0.7854$ in², $I_{AB} = \pi(1^4)/64 = 0.04909$ in⁴, $I_{BC} = 1.5\ (0.25^3)/12 = 1.953\ (10^{-3})$ in⁴, $A_{CD} = \pi(0.75^2)/4 = 0.4418$ in², $I_{AB} = \pi(0.75^4)/64 = 0.01553$ in⁴. For $(\delta_D)_x$ let $F = F_x = -150$ lbf and $F_z = -100$ lbf. Use the dummy variable $\bar{x}$ to originate at the end where the loads are applied on each segment,

$$CD\text{:} \quad M_y = F_z\bar{x} \qquad \frac{\partial M_y}{\partial F} = 0$$

$$F_a = F \qquad \frac{\partial F_a}{\partial F} = 1$$

$$BC\text{:} \quad M_y = F\bar{x} + 2F_z \qquad \frac{\partial M_y}{\partial F} = \bar{x}$$

$$F_a = F_z \qquad \frac{\partial F_a}{\partial F} = 0$$

$$AB\text{:} \quad M_y = 5F + 2F_z + F_z\bar{x} \qquad \frac{\partial M_y}{\partial F} = 5$$

$$F_a = F \qquad \frac{\partial F_a}{\partial F} = 1$$

$$(\delta_D)_x = \frac{\partial U}{\partial F} = \left(\frac{FL}{AE}\right)_{CD}\frac{\partial F_a}{\partial F} + \frac{1}{EI_{BC}}\int_0^5 \left(F\,\bar{x} + 2F_z\right)\bar{x}\,d\,\bar{x}$$

$$+\frac{1}{EI_{AB}}\int_0^6 \left(5F + 2F_z + F_z\bar{x}\right)(5)\,d\,\bar{x} + \left(\frac{FL}{AE}\right)_{AB}\frac{\partial F_a}{\partial F}$$

$$= \frac{F(2)}{0.4418(30)10^6}(1) + \frac{1}{30(10^6)1.953(10^{-3})}\left[\frac{F}{3}(5)^3 + F_z(5^2)\right]$$

$$+\frac{1}{30(10^6)0.04909}\left[25F(6) + 10F_z(6) + \frac{F_z}{2}(6^2)5\right] + \frac{F(6)}{0.7854(30)10^6}(1)$$

$$= 1.509(10^{-7})F + 7.112(10^{-4})F + 4.267(10^{-4})F_z + 1.019(10^{-4})F$$

$$+1.019(10^{-4})F_z + 2.546(10^{-7})F = 8.135(10^{-4})F + 5.286(10^{-4})F_z$$

Substituting $F = F_x = -\ 150$ lbf and $F_z = -\ 100$ lbf gives

$$(\delta_D)_x = 8.135(10^{-4})(-150) + 5.286(10^{-4})(-100) = -0.1749 \text{ in} \qquad Ans.$$

4-80 $I_{OA} = I_{BC} = \pi(1.5^4)/64 = 0.2485$ in^4, $J_{OA} = J_{BC} = 2\,I_{OA} = 0.4970$ in^4, $I_{AB} = \pi(1^4)/64 = 0.04909$ in^4, $J_{AB} = 2\,I_{AB} = 0.09818$ in^4, $I_{CD} = \pi(0.75^4)/64 = 0.01553$ in^4

Let $F_y = F$, and use the dummy variable $\bar{x}$ to originate at the end where the loads are applied on each segment,

OC: $\quad M = F\,\bar{x} \quad \frac{\partial M}{\partial F} = \bar{x}, \quad T = 12F \quad \frac{\partial T}{\partial F} = 12$

DC: $\quad M = F\,\bar{x} \quad \frac{\partial M}{\partial F} = \bar{x}$

$$(\delta_D)_y = \frac{\partial U}{\partial F} = \sum\left(\frac{TL}{JG}\right)_{OC}\frac{\partial T}{\partial F} + \sum\frac{1}{EI}\int M\frac{\partial M}{\partial F}\,d\,\bar{x}$$

The terms involving the torsion and bending moments in OC must be split up because of the changing second-area moments.

$$(\delta_D)_y = \frac{12F(4)}{0.4970(11.5)10^6}(12) + \frac{12F(9)}{0.09818(11.5)10^6}(12) + \frac{1}{30(10^6)0.2485}\int_0^2 F\bar{x}^2 d\bar{x}$$
$$+\frac{1}{30(10^6)0.04909}\int_2^{11} F\bar{x}^2 d\bar{x} + \frac{1}{30(10^6)0.2485}\int_{11}^{13} F\bar{x}^2 d\bar{x} + \frac{1}{30(10^6)0.01553}\int_0^{12} F\bar{x}^2 d\bar{x}$$
$$= 1.008(10^{-4})F + 1.148(10^{-3})F + 3.58(10^{-7})F$$
$$+2.994(10^{-4})F + 3.872(10^{-5})F + 1.2363(10^{-3})F$$
$$= 2.824(10^{-3})F = 2.824(10^{-3})250 = 0.706 \text{ in} \qquad \textit{Ans.}$$

For the simplified shaft *OC*,

$$(\delta_B)_y = \frac{12F(13)}{0.09818(11.5)10^6}(12) + \frac{1}{30(10^6)0.04909}\int_0^{13} F\bar{x}^2 d\bar{x} + \frac{1}{30(10^6)0.01553}\int_0^{12} F\bar{x}^2 d\bar{x}$$
$$= 1.6580(10^{-3})F + 4.973(10^{-4})F + 1.2363(10^{-3})F = 3.392(10^{-3})F = 3.392(10^{-3})250$$
$$= 0.848 \text{ in} \qquad \textit{Ans.}$$

Simplified is 0.848/0.706 = 1.20 times greater *Ans.*

4-81 Place a fictitious force *Q* pointing downwards at point *B*. The reaction at *C* is

$R_C = Q + (6/18)100 = Q + 33.33$

This is the axial force in member *BC*. Isolating the beam, we find that the moment is not a function of *Q*, and thus does not contribute to the strain energy. Thus, only energy in the member *BC* needs to be considered. Let the axial force in *BC* be *F*, where

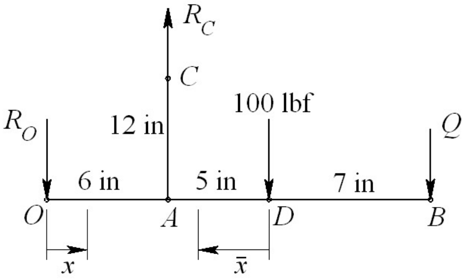
$$F = Q + 33.33 \qquad \frac{\partial F}{\partial Q} = 1$$

$$\delta_B = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} = \left[\left(\frac{FL}{AE}\right)_{BC}\frac{\partial F}{\partial Q}\right]_{Q=0} = \frac{(0+33.33)12}{\left[\pi(0.5^2)/4\right]30(10^6)}(1) = 6.79(10^{-5}) \text{ in} \qquad \textit{Ans.}$$

4-82 $I_{OB} = 0.25(2^3)/12 = 0.1667 \text{ in}^4$

$A_{AC} = \pi(0.5^2)/4 = 0.1963 \text{ in}^2$

$\Sigma M_O = 0 = 6R_C - 11(100) - 18Q$



$R_C = 3Q + 183.3$

$\Sigma M_A = 0 = 6\,R_O - 5(100) - 12\,Q \quad \Rightarrow \quad R_O = 2Q + 83.33$

Bending in *OB*.

BD: Bending in *BD* is only due to *Q* which when set to zero after differentiation gives no contribution.

AD: Using the variable $\bar{x}$ as shown in the figure above

$$M = -100\bar{x} - Q\left(7 + \bar{x}\right) \qquad \frac{\partial M}{\partial Q} = -\left(7 + \bar{x}\right)$$

OA: Using the variable *x* as shown in the figure above

$$M = -\left(2Q + 83.33\right)x \qquad \frac{\partial M}{\partial Q} = -2x$$

Axial in *AC*:

$$F = 3Q + 183.3 \qquad \frac{\partial F}{\partial Q} = 3$$

$$\delta_B = \left(\frac{\partial U}{\partial Q}\right)_{Q=0} = \left[\left(\frac{FL}{AE}\right)\frac{\partial F}{\partial Q}\right]_{Q=0} + \left(\frac{1}{EI}\sum M\frac{\partial M}{\partial Q}dx\right)_{Q=0}$$

$$= \frac{183.3\left(12\right)}{0.1963\left(30\right)10^6}\left(3\right) + \frac{1}{EI}\int_0^5 \left(100\,\bar{x}\right)\left(7 + \bar{x}\right)d\,\bar{x} + \int_0^6 2\left(83.33\right)x^2 dx$$

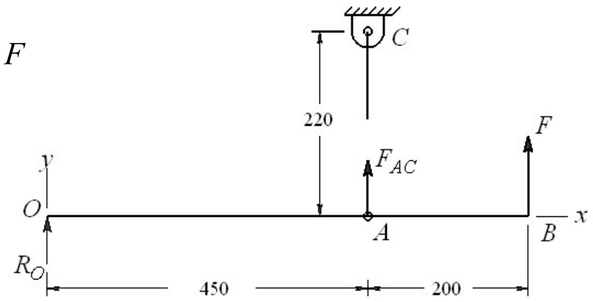
$$= 1.121\left(10^{-3}\right) + \frac{1}{10.4\left(10^6\right)0.1667}\left[100\int_0^5 \bar{x}\left(7 + \bar{x}\right)d\,\bar{x} + 166.7\int_0^6 x^2 dx\right]$$

$$= 1.121\left(10^{-3}\right) + 5.768\left(10^{-7}\right)\left[100\left(129.2\right) + 166.7\left(72\right)\right] = 0.0155 \text{ in} \qquad \textit{Ans.}$$

4-83 Table A-5, $E_A = 71.7$ GPa, $E_S = 207$ GPa, $EI = 71.7(10^9)0.012(0.05^3)/12 = 8962.5\ \text{N}\cdot\text{m}^2$

$F = -\,4000$ lbf

$\Sigma M_O = 0 = 450\,F_{AC} + 650F \quad \Rightarrow \quad F_{AC} = -\,1.444\,F$



$$\left(y_B\right)_1 = \frac{F_{AC}\dfrac{\partial F_{AC}}{\partial F}L_{AC}}{EA}$$

$$= \frac{-1.444\left(-4000\right)\left(-1.444\right)220}{207\left(10^9\right)\left(\pi/4\right)0.006^2}$$

$$= -0.3135 \text{ mm}$$

$\Sigma M_A = 0 = -\,450\,R_O + 200F \quad \Rightarrow \quad R_O = 0.4444\,F$

$0 \le x \le 0.450\,\text{m} \qquad M = 0.4444Fx \qquad \partial M / \partial F = 0.4444x$

$450 \le x \le 0.650\,\text{m} \qquad M = F(0.650 - x) \qquad \partial M / \partial F = (0.650 - x)$

$$(y_B)_2 = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial F}dx = \frac{1}{EI}\left[\int_0^{0.45}(0.4444)Fx^2dx + \int_{0.45}^{0.65}F(0.65-x)^2\,dx\right]$$

$$= \frac{-4000}{8962.5}\left[\frac{(0.4444)^2}{3}0.45^3 - \frac{1}{3}(0.65-x)^3\Big|_{0.45}^{0.65}\right] = -3.867\ \ \text{mm}$$

$y_B = (y_B)_1 + (y_B)_2 = -\,0.3135 - 3.867 = -\,4.18$ mm Ans.

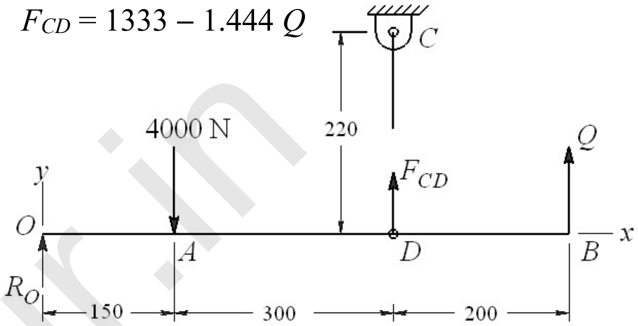
4-84 Table A-5, $E_A = 71.7$ GPa, $E_S = 207$ GPa, $EI = 71.7(10^9)0.012(0.05^3)/12 = 8962.5\ \text{N}\cdot\text{m}^2$

$\Sigma M_O = 0 = 450\,F_{CD} + 650Q - 150(4000) \quad \Rightarrow \quad F_{CD} = 1333 - 1.444\,Q$

$$(y_B)_1 = \frac{F_{CD}\dfrac{\partial F_{CD}}{\partial Q}L_{CD}\Big|_{Q=0}}{EA}$$

$$= \frac{1333(-1.444)220}{207(10^9)(\pi/4)0.006^2}$$

$$= -0.0724\ \ \text{mm}$$



$\Sigma F_y = 0 = R_O - 4000 + 1333 - 1.444\,Q + Q \quad \Rightarrow \quad R_O = 2667 + 0.444\,Q$

$0 \le x \le 0.150\,\text{m} \qquad M = (2667 + 0.444Q)x \qquad \partial M / \partial Q = 0.444x$

$150 \le x \le 0.450\,\text{m} \qquad M = (2667 + 0.444Q)x - 4000(x - 0.150) = (-1333 + 0.444Q)x + 600$

$\partial M / \partial Q = 0.444x$

$0.450 \le x \le 0.650\,\text{m} \qquad M = (0.65 - x)Q \qquad \partial M / \partial Q = -x$

$$(y_B)_2 = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial Q}dx\Big|_{Q=0} = \frac{1}{EI}\left[\int_0^{0.15}2667x(0.444x)dx + \int_{0.15}^{0.45}(-1333x + 600)(0.444x)dx\right]$$

$$= \frac{1}{EI}\left\{\frac{2667(0.444)0.15^3}{3} + \left[\frac{-1333(0.444)}{3}(0.45^3 - 0.15^3) + \frac{600(0.444)}{2}(0.45^2 - 0.15^2)\right]\right\}$$

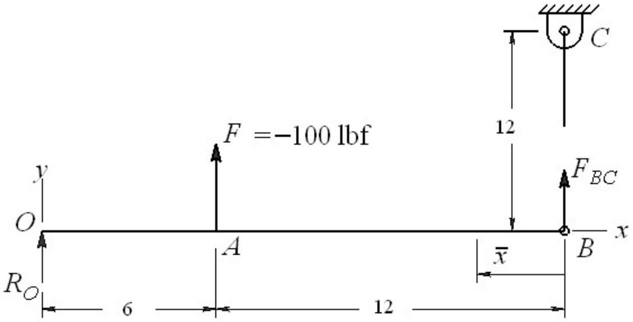
$$= \frac{1}{8962.5}(7.996) = 8.922(10^{-4})\ \ \text{m} \ = 0.8922\ \ \text{mm}$$

$y_B = (y_B)_1 + (y_B)_2 = -\,0.0724 + 0.8922 = 0.820$ mm *Ans.*

4-85 Table A-5, $E_A = 10.4$ Mpsi, $E_S = 30$ Mpsi, $EI = 10.4(10^6)0.25(2^3)/12 = 1.733\,(10^6)\ \text{lbf}\cdot\text{in}^3$

$\Sigma M_O = 0 = 18\,F_{BC} + 6F \quad \Rightarrow \quad F_{BC} = -\,F/3$

$$(y_A)_1 = \frac{F_{BC}\dfrac{\partial F_{BC}}{\partial F}L_{BC}}{AE} = \frac{(-1/3)^2(-100)12}{(\pi/4)0.5^2(30)10^6} = -2.264\left(10^{-5}\right)\text{ in}$$



$$\Sigma F_y = 0 = R_O + F + F_{BC} \qquad \Rightarrow \qquad R_O = -\,2\,F/3$$

$$0 \le x \le 6\text{ in} \qquad M = -\frac{2}{3}Fx \qquad \partial M/\partial F = -\frac{2}{3}x$$

$$0 \le \bar{x} \le 12\text{ in} \qquad M = F_{BC}\bar{x} = -\frac{1}{3}F\,\bar{x} \qquad \partial M/\partial F = -\frac{1}{3}\bar{x}$$

$$(y_A)_2 = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial F}dx = \frac{1}{EI}\left[\int_0^6 (-2/3)^2 Fx^2dx + \int_0^{12}(-1/3)^2 F\,\bar{x}^2d\,\bar{x}\right]$$

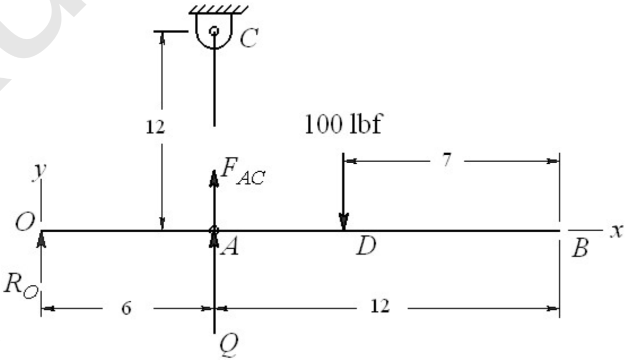
$$= \frac{F}{EI}\left(\frac{4}{27}6^3 + \frac{1}{27}12^3\right) = \frac{-100}{1.733\left(10^6\right)}(96) = -5.540\left(10^{-3}\right)\text{ in}$$

$y_A = (y_A)_1 + (y_A)_2 = -\,2.264\,(10^{-5}) - 5.540\,(10^{-3}) = -\,5.56\,(10^{-3})$ in *Ans.*

4-86 Table A-5, $E_A = 10.4$ Mpsi, $E_S = 30$ Mpsi

$\Sigma M_O = 0 = 6(F_{AC} + Q) - 11(100)$

$F_{AC} = 183.3 - Q \qquad \dfrac{\partial F_{AC}}{\partial Q} = -1$



$$(y_A)_1 = \frac{F_{AC}\dfrac{\partial F_{AC}}{\partial Q}L_{AC}\Big|_{Q=0}}{AE} = \frac{183.3(-1)12}{(\pi/4)0.5^2(30)10^6} = -3.734\left(10^{-4}\right)\text{ in}$$

Treating beam *OADB* as a simply-supported beam pinned at *O* and *A* we see that the force *Q* does not induce any bending. Thus,

$y_A = (y_A)_1 = -\,3.734\,(10^{-4})$ in *Ans.*

4-87 Table A-5, $E_A = 71.7$ GPa,

$k = P/\delta = 10\,(10^3)/(10^{-3})$

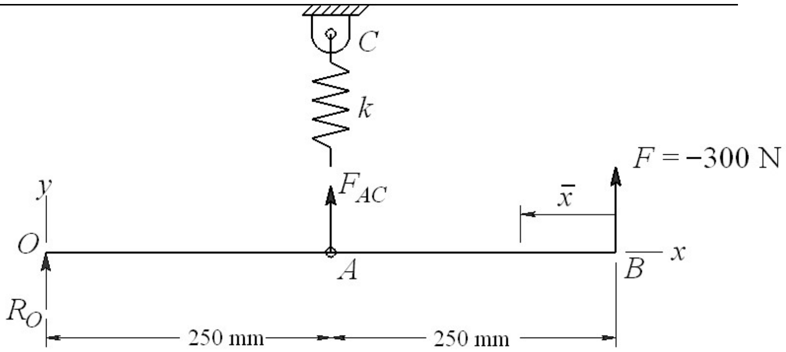
$= 10\,(10^6)$ N/m,

$EI = 71.7\,(10^9)0.005(0.03^3)/12$

$= 806.6\ \text{N}\cdot\text{m}^2$

(a) OA: $R_O = F$

$$M = R_O x = Fx \qquad \frac{\partial M}{\partial x} = x$$



$$\left(y_B\right)_1 = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial F}dx = \frac{1}{EI}\int_0^{0.250} Fx^2 dx = \frac{-300}{3\left(806.6\right)}\left(0.250^3\right)10^3 = -1.937 \text{ mm} \qquad \textit{Ans.}$$

(b) *AB*: $M = F\,\bar{x}$ $\qquad \dfrac{\partial M}{\partial F} = \bar{x}$

Since *AB* is the same length as *OA*, the integration is identical to part (a). Thus

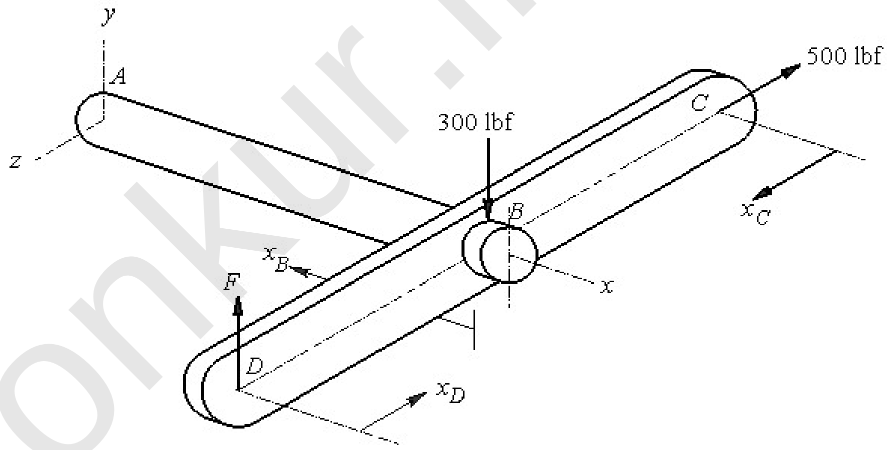
$(y_B)_2 = -1.937$ mm *Ans.*

(c) *AC*: Eq. (4-15): $\Sigma M_O = 0 = 250\,F_{AC} + 500\,F \;\Rightarrow\; F_{AC} = -2F$

$$U = \frac{F_{AC}^2}{2k} \quad\Rightarrow\quad \left(y_B\right)_3 = \frac{\partial U}{\partial F} = \frac{F_{AC}\dfrac{\partial F_{AC}}{\partial F}}{k} = \frac{2F\left(2\right)}{k} = \frac{4\left(-300\right)}{10\left(10^6\right)} = -0.12 \text{ mm} \qquad \textit{Ans.}$$

(d) $y_B = (y_B)_1 + (y_B)_2 + (y_B)_3 = 2(-1.937) - 0.12 = -3.994$ mm *Ans.*

4-88 Table A-5, $E = 30$ Mpsi, $G = 11.5$ Mpsi. $(EI)_{AB} = 30(10^6)\,(\pi/64)1^4 = 1.473\,(10^6)$ lbf·in^2, $(JG)_{AB} = (\pi/32)1^4\,(11.5)10^6 = 1.129\,(10^6)$ lbf·in^2, $(EI)_{BD} = 30(10^6)\,0.25\,(1.25^3)/12 = 1.221\,(10^6)$ lbf·in^2. $F = 200$ lbf.



(a) *AB* at x_B: $M_y = -500x_B$ $\qquad \dfrac{\partial M_y}{\partial F} = 0$ $\qquad$ No contribution from *F*

$$M_z = \left(F - 300\right)x_B \qquad \frac{\partial M_z}{\partial F} = x_B$$

$$\left(y_D\right)_i = \frac{1}{\left(EI\right)_{AB}}\int_0^{l_{AB}} M_z\frac{\partial M_z}{\partial F}dx_B = \frac{1}{1.473\,\left(10^6\right)}\int_0^6\left(F - 300\right)x_B^2 dx_B = \frac{\left(200 - 300\right)6^3}{3\left(1.473\right)10^6}$$

$$= -4.888\left(10^{-3}\right) \text{ in}$$

$$T_x = 4F \qquad \frac{\partial T_x}{\partial F} = 4 \qquad \left(y_D\right)_{ii} = \frac{T_x\dfrac{\partial T_x}{\partial F}L_{AB}}{\left(JG\right)_{AB}} = \frac{4\left(200\right)4\left(6\right)}{1.129\left(10^6\right)} = 0.01701 \text{ in}$$

$(y_D)_1 = (y_D)_i + (y_D)_{ii} = -4.888\ (10^{-3}) + 0.01701 = 0.0121$ in

(b) *BC* at x_C has no *F*. Therefore, $(y_D)_2 = 0$ *Ans.*

(c) *BD* at x_D:

$$M_x = Fx_D \qquad \frac{\partial M_x}{\partial F} = x_D \qquad (y_D)_3 = \frac{1}{(EI)_{BD}}\int_0^4 Fx_D^2 dx_D = \frac{200(4^3)}{3(1.221)10^6}$$

$$= 3.494(10^{-3}) \text{ in} \qquad Ans.$$

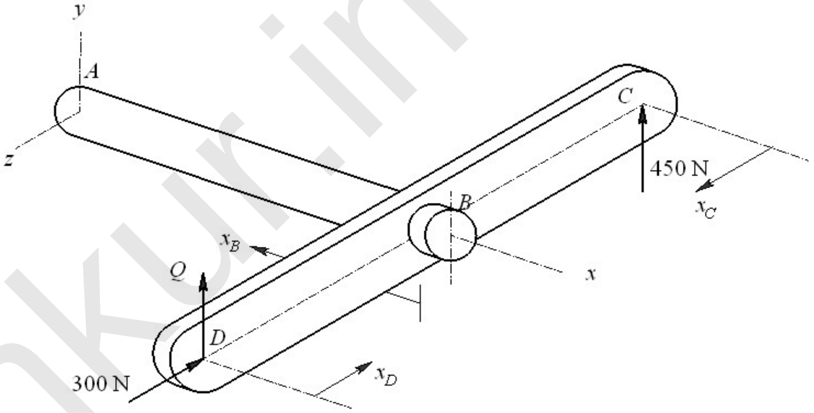
(d) $y_D = (y_D)_1 + (y_D)_2 + (y_D)_3 = 0.0121 + 0 + 0.0035 = 0.0156$ in *Ans.*

4-89 Table A-5, $E = 207$ GPa, $G = 79.3$ GPa, $(EI)_{AB} = 207\ (10^9)(\pi/64)\ 0.025^4 = 3.969\ (10^3)$ m^4, $(JG)_{AB} = (\pi/32)\ 0.025^4\ (79.3)10^9 = 3.041\ (10^3)$ m^4

(a) *AB*: Bending:

$$M_y = -300x_B \qquad \frac{\partial M_y}{\partial Q} = 0$$

No contribution from *Q*.



$$M_z = (450 + Q)x_B \qquad \frac{\partial M_z}{\partial Q} = x_B$$

$$(y_D)_i = \frac{1}{EI}\int_0^l M_z \frac{\partial M_z}{\partial Q} dx_B \Bigg|_{Q=0}$$

$$= \frac{1}{(EI)_{AB}}\int_0^{0.150} 450\,x_B^2\,dx_B$$

$$= \frac{450(0.150^3)}{3(3.969)\ 10^3} = 1.276(10^{-4}) \text{ m} = 0.1275 \text{ mm}$$

Torsion: $T_x = Q(0.1) - 450(0.125) = 0.1Q - 56.25 \qquad \dfrac{\partial T_x}{\partial Q} = 0.1$

$$(y_D)_{ii} = \frac{T_x \dfrac{\partial T_x}{\partial Q} L_{AB}\Big|_{Q=0}}{(JG)_{AB}} = \frac{-56.25(0.1)0.150}{3.041\ (10^3)} = -2.775(10^{-4}) \text{ m} = -0.2775 \text{ mm}$$

$(y_D)_1 = (y_D)_i + (y_D)_{ii} = 0.1275 - 0.2775 = -0.150$ mm *Ans.*

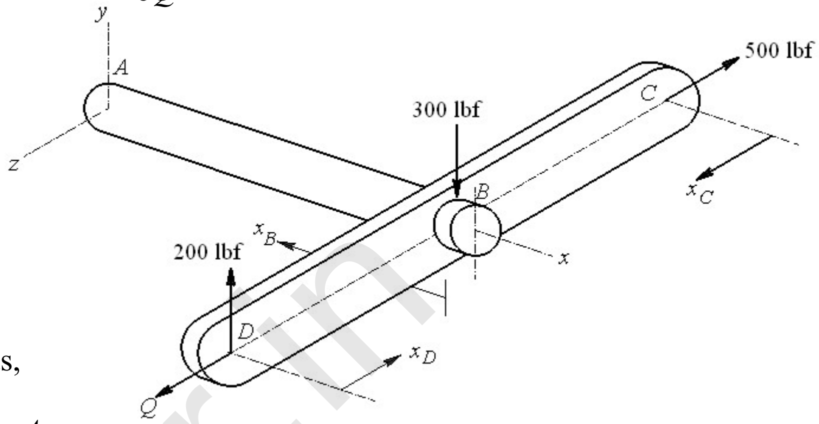
(b) *BC*: Break at x_C has no *Q* in it. Thus, $(y_D)_2 = 0$ *Ans.*

(c) *BD*: Axial has no *Q*. Thus no contribution. Bending only has *Q* and since $Q = 0$, no contribution. Therefore, $(y_D)_3 = 0$ *Ans.*

(d) $y_D = (y_D)_1 + (y_D)_2 + (y_D)_3 = -\ 0.150 + 0 + 0 = -\ 0.150$ mm *Ans.*

4-90 Table A-5, $E = 30$ Mpsi, $G = 11.5$ Mpsi. $(EI)_{AB} = 30\ (10^6)\ (\pi/64)\ 1^4 = 1.473\ (10^6)$ lbf·in^2.

(a) *AB*: Break at x_B: $M_y = (Q - 500)\,x_B \qquad \dfrac{\partial M_y}{\partial Q} = x_B$



$$(z_D)_i = \frac{1}{(EI)_{AB}} \int_0^l M_y \frac{\partial M_y}{\partial Q} dx_B \bigg|_{Q=0}$$

$$= \frac{-500(6^3)}{3(1.473)10^6}$$

$$= -0.0244 \text{ in}$$

No contribution of Q to T_x, M$_z$. Thus,

$(z_D)_1 = (z_D)_i = -\ 0.0244$ in *Ans.*

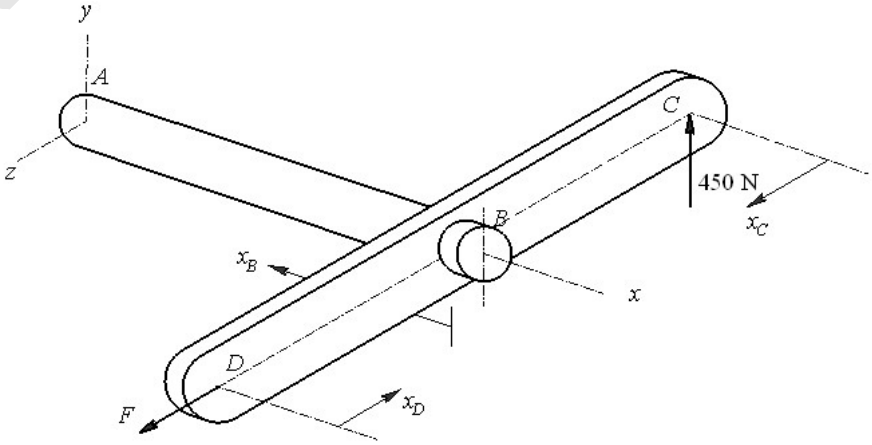
(b) *BC*: Break x_C shows no contributions from Q. Thus, $(z_D)_2 = 0$ *Ans.*

(c) *BD*: Break x_D shows no contributions from Q for M_x, M_y, or T_z. For axial,

$F = Q \qquad \dfrac{\partial F}{\partial Q} = 1$ but setting $Q = 0$ gives nothing. Thus, $(z_D)_3 = 0$ *Ans.*

(d) $z_D = (z_D)_1 = -\ 0.0244$ in *Ans.*

4-91 Table A-5, $E = 207$ GPa, $(EI)_{AB} = 207\ (10^9)(\pi/64)\ 0.025^4 = 3.969\ (10^3)$ m^4,



$(EA)_{BD} = 207(10^9)\ (\pi/4)\ 0.025^2 = 101.6\ (10^6)$ m^2. $F = -\ 300$ N.

(a) *AB*: Break at x_B shows only contribution to M_y from *F*

$$M_y = Fx_B \qquad \frac{\partial M_y}{\partial F} = x_B$$

$$\left(z_D\right)_1 = \frac{1}{EI}\int_0^l M_y \frac{\partial M_y}{\partial F} dx = \frac{1}{\left(EI\right)_{AB}}\int_0^{0.150} Fx_B^2 dx_B = \frac{-300\left(0.150^3\right)}{3\left(3.969\right)10^3}$$

$$= -8.50\left(10^{-5}\right) \text{ m} = -0.0850 \text{ mm} \qquad Ans.$$

(b) *BC*: Break at x_B shows no contribution from *F*. Thus, $(z_D)_2 = 0$ *Ans*.

(c) *BD*: Break at x_D shows only contribution to axial force from *F*

$$F_z = F \qquad \frac{\partial F_z}{\partial F} = 1$$

$$\left(z_D\right)_3 = \frac{F_z \dfrac{\partial F_z}{\partial F} L_{BD}}{\left(EA\right)_{BD}} = \frac{-300\left(1\right)0.100}{101.6\ \left(10^6\right)} = -2.95\left(10^{-7}\right) \text{ m} = -2.95\left(10^{-4}\right) \text{ mm} \quad Ans.$$

(d) $z_D = -\ 0.0850 + 0 - 2.95\ (10^{-4}) = -\ 0.0853$ mm *Ans*.

4-92 There is no bending in *AB*. Using the variable θ, rotating counterclockwise from *B*

$$M = PR\sin\theta \qquad \frac{\partial M}{\partial P} = R\sin\theta$$

$$F_r = P\cos\theta \qquad \frac{\partial F_r}{\partial P} = \cos\theta$$

$$F_\theta = P\sin\theta \qquad \frac{\partial F_\theta}{\partial P} = \sin\theta$$

$$\frac{\partial MF_\theta}{\partial P} = 2PR\sin^2\theta$$

$A = 6(4) = 24 \text{ mm}^2, \ r_o = 40 + \tfrac{1}{2}(6) = 43 \text{ mm}, \ r_i = 40 - \tfrac{1}{2}(6) = 37 \text{ mm},$

From Table 3-4, for a rectangular cross section

$$r_n = \frac{6}{\ln(43/37)} = 39.92489 \text{ mm}$$

From Eq. (4-33), the eccentricity is $e = R - r_n = 40 - 39.92489 = 0.07511$ mm
From Table A-5, $E = 207(10^3)$ MPa, $G = 79.3(10^3)$ MPa
From Table 4-1, $C = 1.2$

From Eq. (4-38)

$$\delta = \int_0^{\frac{\pi}{2}} \frac{M}{AeE}\left(\frac{\partial M}{\partial P}\right) d\theta + \int_0^{\frac{\pi}{2}} \frac{F_\theta R}{AE}\left(\frac{\partial F_\theta}{\partial P}\right) d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{AE}\frac{\partial\left(MF_\theta\right)}{\partial P} d\theta + \int_0^{\frac{\pi}{2}} \frac{CF_r R}{AG}\left(\frac{\partial F_r}{\partial P}\right) d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{P\left(R\sin\theta\right)^2}{AeE} d\theta + \int_0^{\frac{\pi}{2}} \frac{PR\left(\sin\theta\right)^2}{AE} d\theta - \int_0^{\frac{\pi}{2}} \frac{2PR\sin^2\theta}{AE} d\theta + \int_0^{\frac{\pi}{2}} \frac{CPR\left(\cos\theta\right)^2}{AG} d\theta$$

$$= \frac{\pi PR}{4AE}\left(\frac{R}{e} + 1 - 2 + \frac{EC}{G}\right) = \frac{\pi(10)(40)}{4(24)(207\cdot 10^3)}\left(\frac{40}{0.07511} + 1 - 2 + \frac{(207\cdot 10^3)(1.2)}{79.3\cdot 10^3}\right)$$

$$\delta = 0.0338 \text{ mm} \qquad \textit{Ans.}$$

4-93 Place a fictitious force Q pointing downwards at point A. Bending in AB is only due to Q which when set to zero after differentiation gives no contribution. For section BC use the variable θ, rotating counterclockwise from B

$$M = PR\sin\theta + Q\left(R + R\sin\theta\right) \qquad \frac{\partial M}{\partial Q} = R\left(1+\sin\theta\right)$$

$$F_r = \left(P+Q\right)\cos\theta \qquad \frac{\partial F_r}{\partial Q} = \cos\theta$$

$$F_\theta = \left(P+Q\right)\sin\theta \qquad \frac{\partial F_\theta}{\partial Q} = \sin\theta$$

$$MF_\theta = \left[PR\sin\theta + QR\left(1+\sin\theta\right)\right]\left(P+Q\right)\sin\theta$$

$$\frac{\partial MF_\theta}{\partial Q} = PR\sin^2\theta + +PR\sin\theta\left(1+\sin\theta\right) + 2QR\sin\theta\left(1+\sin\theta\right)$$

But after differentiation, we can set $Q = 0$. Thus,

$$\frac{\partial MF_\theta}{\partial Q} = PR\sin\theta\left(1+2\sin\theta\right)$$

$$A = 6(4) = 24 \text{ mm}^2, \quad r_o = 40 + \tfrac{1}{2}(6) = 43 \text{ mm}, \quad r_i = 40 - \tfrac{1}{2}(6) = 37 \text{ mm},$$

From Table 3-4, for a rectangular cross section

$$r_n = \frac{6}{\ln(43/37)} = 39.92489 \text{ mm}$$

From Eq. (4-33), the eccentricity is $e = R - r_n = 40 - 39.92489 = 0.07511$ mm

From Table A-5, $E = 207(10^3)$ MPa, $G = 79.3(10^3)$ MPa

From Table 4-1, $C = 1.2$

From Eq. (4-38),

$$\delta = \int_0^{\frac{\pi}{2}} \frac{M}{AeE}\left(\frac{\partial M}{\partial Q}\right)d\theta + \int_0^{\frac{\pi}{2}} \frac{F_\theta R}{AE}\left(\frac{\partial F_\theta}{\partial Q}\right)d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{AE}\frac{\partial\left(MF_\theta\right)}{\partial Q}d\theta + \int_0^{\frac{\pi}{2}} \frac{CF_r R}{AG}\left(\frac{\partial F_r}{\partial Q}\right)d\theta$$

$$= \frac{PR^2}{AeE}\int_0^{\frac{\pi}{2}} \sin\theta\left(1+\sin\theta\right)d\theta + \frac{PR}{AE}\int_0^{\frac{\pi}{2}} \sin^2\theta d\theta - \frac{PR}{AE}\int_0^{\frac{\pi}{2}} \sin\theta\left(1+2\sin\theta\right)d\theta$$

$$+\frac{CPR}{AG}\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta$$

$$= \left(\frac{\pi}{4}+1\right)\frac{PR^2}{AeE} + \frac{\pi}{4}\frac{PR}{AE} - \left(\frac{\pi}{4}+2\right)\frac{PR}{AE} + \frac{\pi}{4}\frac{CPR}{AG} = \frac{PR}{AE}\left[\left(\frac{\pi}{4}+1\right)\frac{R}{e} - 2 + \frac{\pi}{4}\frac{CE}{G}\right]$$

$$= \frac{10\left(40\right)}{24\left(207\right)10^3}\left[\left(\frac{\pi}{4}+1\right)\frac{40}{0.07511} - 2 + \frac{\pi}{4}\frac{1.2\left(207\right)10^3}{79.3\left(10^3\right)}\right]$$

$= 0.0766$ mm *Ans.*

4-94 $A = 3(2.25) - 2.25(1.5) = 3.375\text{ in}^2$

$$R = \frac{(1+1.5)(3)(2.25) - (1+0.75+1.125)(1.5)(2.25)}{3.375} = 2.125\text{ in}$$

Section is equivalent to the “T” section of Table 3-4,

$$r_n = \frac{2.25(0.75) + 0.75(2.25)}{2.25\ln[(1+0.75)/1] + 0.75\ln[(1+3)/(1+0.75)]} = 1.7960\text{ in}$$

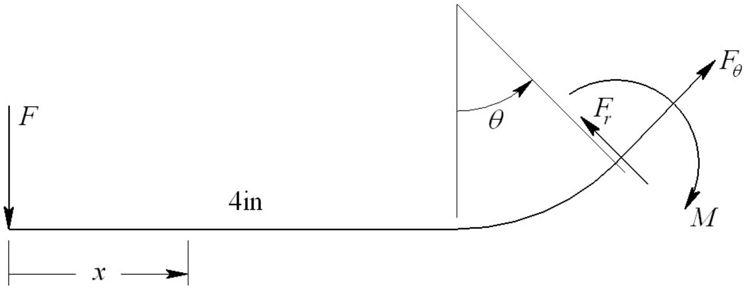
$$e = R - r_n = 2.125 - 1.7960 = 0.329\text{ in}$$

For the straight section

$$I_z = \frac{1}{12}(2.25)\left(3^3\right) + 2.25(3)(1.5-1.125)^2$$

$$-\left[\frac{1}{12}(1.5)\left(2.25^3\right) + 1.5(2.25)\left(0.75 + \frac{2.25}{2} - 1.125\right)^2\right]$$

$$= 2.689\text{ in}^4$$



For $\underline{0 \le x \le 4\text{ in}}$

$$M = -Fx \quad \frac{\partial M}{\partial F} = -x, \qquad V = F \quad \frac{\partial V}{\partial F} = 1$$

For $\underline{\theta \le \pi/2}$

$$F_r = F\cos\theta \quad \frac{\partial F_r}{\partial F} = \cos\theta, \qquad F_\theta = F\sin\theta \quad \frac{\partial F_\theta}{\partial F} = \sin\theta$$

$$M = F(4 + 2.125\sin\theta) \quad \frac{\partial M}{\partial F} = (4 + 2.125\sin\theta)$$

$$MF_\theta = F(4 + 2.125\sin\theta)F\sin\theta \qquad \frac{\partial MF_\theta}{\partial F} = 2F(4 + 2.365\sin\theta)\sin\theta$$

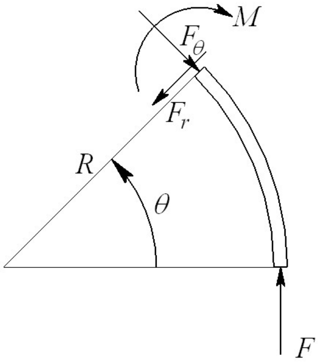
Use Eqs. (4-31) and (4-24) (with $C = 1$) for the straight part, and Eq. (4-38) for the curved part, integrating from 0 to π/2, and double the results

$$\delta = \frac{2}{E}\left\{\frac{1}{I}\int_0^4 Fx^2 dx + \frac{F(4)(1)}{3.375(G/E)} + \int_0^{\pi/2} F\frac{(4+2.125\sin\theta)^2}{3.375(0.329)}d\theta\right.$$
$$+ \int_0^{\pi/2}\frac{F\sin^2\theta(2.125)}{3.375}d\theta - \int_0^{\pi/2}\frac{2F(4+2.125\sin\theta)\sin\theta}{3.375}d\theta$$
$$\left. + \int_0^{\pi/2}\frac{(1)F\cos^2\theta(2.125)}{3.375(G/E)}d\theta\right\}$$

Substitute $I = 2.689$ in^4, $F = 6700$ lbf, $E = 30$ (10^6) psi, $G = 11.5$ (10^6) psi

$$\delta = \frac{2(6700)}{30(10^6)}\left\{\frac{4^3}{3(2.689)} + \frac{4}{3.375(11.5/30)} + \frac{1}{3.375(0.329)}\left[16\left(\frac{\pi}{2}\right) + 17(1) + 4.516\left(\frac{\pi}{4}\right)\right]\right.$$
$$\left. + \frac{2.125}{3.375}\left(\frac{\pi}{4}\right) - \frac{2}{3.375}\left[4(1) + 2.125\left(\frac{\pi}{4}\right)\right] + \frac{2.125}{3.375(11.5/30)}\left(\frac{\pi}{4}\right)\right\}$$
$$= 0.0226 \text{ in} \qquad Ans.$$

4-95 Since $R/h = 35/4.5 = 7.78$ use Eq. (4-38), integrate from 0 to π, and double the results



$$M = FR(1 - \cos\theta) \qquad \frac{\partial M}{\partial F} = R(1 - \cos\theta)$$

$$F_r = F\sin\theta \qquad \frac{\partial F_r}{\partial F} = \sin\theta$$

$$F_\theta = F\cos\theta \qquad \frac{\partial F_\theta}{\partial F} = \cos\theta$$

$$MF_\theta = F^2R\cos\theta(1 - \cos\theta)$$

$$\frac{\partial(MF_\theta)}{\partial F} = 2FR\cos\theta(1 - \cos\theta)$$

From Eq. (4-38),

$$\delta = 2\left[\frac{FR^2}{AeE}\int_0^\pi (1-\cos\theta)^2 d\theta + \frac{FR}{AE}\int_0^\pi \cos^2\theta d\theta \right.$$
$$\left. -\frac{2FR}{AE}\int_0^\pi \cos\theta\left(1-\cos\theta\right)d\theta + \frac{1.2FR}{AG}\int_0^\pi \sin^2\theta d\theta\right]$$
$$= \frac{2FR}{AE}\left(\frac{3\pi}{2}\frac{R}{e} + \frac{3\pi}{2} + 0.6\pi\frac{E}{G}\right)$$

$A = 4.5(3) = 13.5\ \text{mm}^2$, $E = 207\ (10^3)\ \text{N/mm}^2$, $G = 79.3\ (10^3)\ \text{N/mm}^2$, and from Table 3-4,

$$r_n = \frac{h}{\ln\dfrac{r_o}{r_i}} = \frac{4.5}{\ln\dfrac{37.25}{32.75}} = 34.95173\ \text{mm}$$

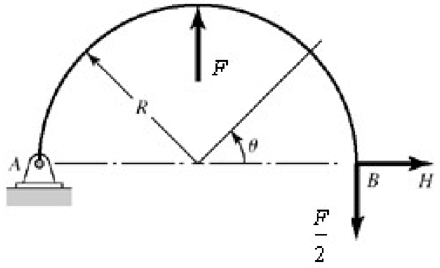
and $e = R - r_n = 35 - 34.95173 = 0.04827$ mm. Thus,

$$\delta = \frac{2F\left(35\right)}{13.5\left(207\right)10^3}\left(\frac{3\pi}{2}\frac{35}{0.04827} + \frac{3\pi}{2} + 0.6\pi\frac{207}{79.3}\right) = 0.08583F$$

where F is in N. For $\delta = 1$ mm, $F = \dfrac{1}{0.08583} = 11.65\ \text{N}$ *Ans.*

Note: The first term in the equation for δ dominates and this is from the bending moment. Try Eq. (4-41), and compare the results.

4-96 $R/h = 20 > 10$ so Eq. (4-41) can be used to determine deflections. Consider the horizontal reaction to be applied at B and subject to the constraint $(\delta_B)_H = 0$.



$$M = \frac{FR}{2}(1-\cos\theta) - HR\sin\theta \qquad \frac{\partial M}{\partial H} = -R\sin\theta \qquad 0 < \theta < \frac{\pi}{2}$$

By symmetry, we may consider only half of the wire form and use twice the strain energy Eq. (4-41) then becomes,

$$(\delta_B)_H = \frac{\partial U}{\partial H} \approx \frac{2}{EI}\int_0^{\pi/2}\left(M\frac{\partial M}{\partial H}\right)Rd\theta = 0$$

$$\int_0^{\pi/2}\left[\frac{FR}{2}(1-\cos\theta) - HR\sin\theta\right](-R\sin\theta)R\ d\theta = 0$$

$$-\frac{F}{2} + \frac{F}{4} + H\frac{\pi}{4} = 0 \Rightarrow H = \frac{F}{\pi} = \frac{30}{\pi} = 9.55\ \text{N} \quad \textit{Ans.}$$

The reaction at *A* is the same where *H* goes to the left. Substituting *H* into the moment equation we get,

$$M=\frac{FR}{2\pi}\left[\pi(1-\cos\theta)-2\sin\theta\right]\qquad\frac{\partial M}{\partial F}=\frac{R}{2\pi}[\pi(1-\cos\theta)-2\sin\theta]\qquad 0<\theta<\frac{\pi}{2}$$

$$\delta_P=\frac{\partial U}{\partial P}\approx\int\frac{2}{EI}\left(M\frac{\partial M}{\partial F}\right)Rd\theta=\frac{2}{EI}\int_0^{\pi/2}\frac{FR^2}{4\pi^2}[\pi(1-\cos\theta)-2\sin\theta]^2 R\,d\theta$$

$$=\frac{FR^3}{2\pi^2 EI}\int_0^{\pi/2}(\pi^2+\pi^2\cos^2\theta+4\sin^2\theta-2\pi^2\cos\theta-4\pi\sin\theta+4\pi\sin\theta\cos\theta)\,d\theta$$

$$=\frac{FR^3}{2\pi^2 EI}\left[\pi^2\left(\frac{\pi}{2}\right)+\pi^2\left(\frac{\pi}{4}\right)+4\left(\frac{\pi}{4}\right)-2\pi^2-4\pi+2\pi\right]$$

$$=\frac{(3\pi^2-8\pi-4)}{8\pi}\frac{FR^3}{EI}=\frac{(3\pi^2-8\pi-4)}{8\pi}\frac{(30)(40^3)}{207\left(10^3\right)\left[\pi\left(2^4\right)/64\right]}=0.224\text{ mm}\qquad \textit{Ans.}$$

4-97 The radius is sufficiently large compared to the wire diameter to use Eq. (4-41) for the curved beam portion. The shear and axial components will be negligible compared to bending.

Place a fictitious force *Q* pointing to the left at point *A*.

$$M=PR\sin\theta+Q(R\sin\theta+l)\qquad\frac{\partial M}{\partial Q}=R\sin\theta+l$$

Note that the strain energy in the straight portion is zero since there is no real force in that section.

From Eq. (4-41),

$$\delta_A=\left[\int_0^{\pi/2}\frac{1}{EI}\left(M\frac{\partial M}{\partial Q}\right)Rd\theta\right]_{Q=0}=\frac{1}{EI}\int_0^{\pi/2}PR\sin\theta\left(R\sin\theta+l\right)Rd\theta$$

$$=\frac{PR^2}{EI}\int_0^{\pi/2}\left(R\sin^2\theta+l\sin\theta\right)d\theta=\frac{PR^2}{EI}\left(\frac{\pi}{4}R+l\right)=\frac{1(2.5^2)}{30\left(10^6\right)\left[\pi\left(0.125^4\right)/64\right]}\left(\frac{\pi}{4}(2.5)+2\right)$$

$$=0.0689\text{ in}\qquad \textit{Ans.}$$

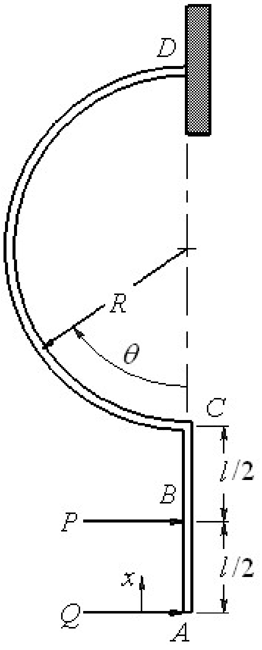
4-98 Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

Straight portion: $M_{AB}=Px\qquad\dfrac{\partial M_{AB}}{\partial P}=x$

Curved portion: $M_{BC}=P\left[R(1-\cos\theta)+l\right]\qquad\dfrac{\partial M_{BC}}{\partial P}=\left[R(1-\cos\theta)+l\right]$

From Eq. (4-41) with the addition of the bending strain energy in the straight portion of the wire,

$$
\begin{aligned}
\delta_A &= \int_0^l \frac{1}{EI}\left(M_{AB}\frac{\partial M_{AB}}{\partial P}\right)dx + \int_0^{\pi/2}\frac{1}{EI}\left(M_{BC}\frac{\partial M_{BC}}{\partial P}\right)Rd\theta \\
&= \frac{P}{EI}\int_0^l x^2dx + \frac{PR}{EI}\int_0^{\pi/2}\left[R(1-\cos\theta)+l\right]^2 d\theta \\
&= \frac{Pl^3}{3EI} + \frac{PR}{EI}\int_0^{\pi/2}\left[R^2(1-2\cos\theta+\cos^2\theta)+2Rl(1-\cos\theta)+l^2\right]d\theta \\
&= \frac{Pl^3}{3EI} + \frac{PR}{EI}\int_0^{\pi/2}\left[R^2\cos^2\theta-\left(2R^2+2Rl\right)\cos\theta+(R+l)^2\right]d\theta \\
&= \frac{Pl^3}{3EI} + \frac{PR}{EI}\left[\frac{\pi}{4}R^2-\left(2R^2+2Rl\right)+\frac{\pi}{2}(R+l)^2\right] \\
&= \frac{P}{EI}\left[\frac{l^3}{3}+\frac{\pi}{4}R^3-R\left(2R^2+2Rl\right)+\frac{\pi}{2}R(R+l)^2\right] \\
&= \frac{1}{30\left(10^6\right)\pi\left(0.125^4\right)/64}\left[\frac{2^3}{3}+\frac{\pi}{4}(2.5^3)-2.5\left[2(2.5^2)+2(2.5)(2)\right]+\frac{\pi}{2}(2.5)(2.5+2)^2\right] \\
&= 0.106 \text{ in} \quad \textit{Ans.}
\end{aligned}
$$



4-99 $R = 2.5$ in, $d = 0.125$ in, $l = 2$ in, $P = 1$ lbf, $E = 30$ Mpsi.

$EI = 30\,(10^6)\,(\pi/64)\,0.125^4 = 359.53$ lbf·in^2.

Member *ABC*: $0 \le x \le l/2 \qquad M = Qx \qquad \dfrac{\partial M}{\partial Q} = x$

Since $Q = 0$, there is no contribution.

$l/2 \le x \le l \qquad M = Qx + P\left(x - l/2\right) \qquad \dfrac{\partial M}{\partial Q} = x$

$$
\begin{aligned}
\left(\delta_A\right)_1 &= \left[\frac{1}{EI}\int_0^l M\frac{\partial M}{\partial Q}dx\right]_{Q=0} = \frac{1}{EI}\left[0+\int_{l/2}^{l}P\left(x-l/2\right)xdx\right] \\
&= \frac{P}{EI}\left(\frac{x^3}{3}-\frac{x^2 l}{4}\right)\Bigg|_{l/2}^{l} = \frac{P}{EI}\left\{\frac{l^3}{3}-\left[\frac{(l/2)^3}{3}\right]-\frac{l^3}{4}+\frac{(l/2)^2\,l}{4}\right\} \\
&= \frac{5Pl^3}{48EI} = \frac{5(1)2^3}{48(359.53)} = 2.32\left(10^{-3}\right) \text{ in}
\end{aligned}
$$

Member *CD*:

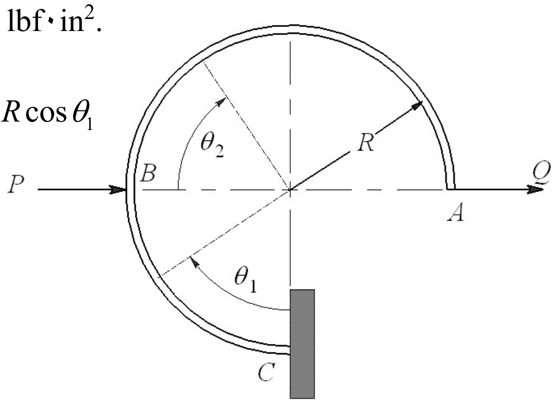
$$0 \le \theta \le \pi \qquad M = Q\left[l + R(1-\cos\theta)\right] + P\left[l/2 + R(1-\cos\theta)\right]$$

$$\frac{\partial M}{\partial Q} = \left[l + R(1-\cos\theta)\right]$$

$$\begin{aligned}
(\delta_A)_2 &= \frac{1}{EI}\int_0^{\pi} P\left[l/2 + R(1-\cos\theta)\right]\left[l + R(1-\cos\theta)\right]Rd\theta \\
&= \frac{PR}{EI}\int_0^{\pi}\left[l^2/2 + (3l/2)R(1-\cos\theta) + R^2(1-\cos\theta)^2\right]d\theta \\
&= \frac{PR}{EI}\int_0^{\pi}\left[l^2/2 + (3l/2)R - (3l/2)R\cos\theta + R^2 - 2R^2\cos\theta + R^2\cos^2\theta\right]d\theta \\
&= \frac{PR}{EI}\int_0^{\pi}\left[l^2/2 + (3l/2)R + R^2 - (3l/2)R\cos\theta - 2R^2\cos\theta + R^2\left(\frac{1+\cos 2\theta}{2}\right)\right]d\theta \\
&= \frac{PR}{EI}\int_0^{\pi}\left\{\left[l^2/2 + (3l/2)R + (3/2)R^2\right] - \left[(3l/2)R + 2R^2\right]\cos\theta + (1/2)R^2\cos 2\theta\right\}d\theta \\
&= \frac{\pi PR}{EI}\left[l^2/2 + (3l/2)R + (3/2)R^2\right] = \frac{\pi(1)2.5}{(359.53)}\left\{(2^2/2) + \left[3(2)/2\right](2.5) + (3/2)2.5^2\right\} \\
&= 0.4123 \text{ in}
\end{aligned}$$

$\delta_A = (\delta_A)_1 + (\delta_A)_2 = 2.32\ (10^{-3}) + 0.4123 = 0.4146$ in *Ans.*

4-100 $E = 30$ Mpsi, $EI = 30\ (10^6)\ (\pi/64)\ (0.125)^4 = 359.53$ lbf·in^2.



$$0 \le \theta_1 \le \pi/2 \quad M = (P+Q)R\cos\theta_1 \qquad \frac{\partial M}{\partial Q} = R\cos\theta_1$$

$$0 \le \theta_2 \le \pi \qquad M = QR\cos\theta_2 \qquad \frac{\partial M}{\partial Q} = R\cos\theta_2$$

No contribution in θ_2 range since $Q = 0$. Thus

$$\delta_A = \frac{1}{EI}\int_0^{\pi/2} M\frac{\partial M}{\partial Q}Rd\theta_1\Big|_{Q=0} = \frac{1}{EI}\int_0^{\pi/2} PR^2\cos^2\theta_1 Rd\theta_1 = \frac{PR^3}{2EI}\int_0^{\pi/2}(1+\cos 2\theta_1)d\theta_1$$

$$= \frac{PR^3}{2EI}\left(\frac{\pi}{2} + \frac{\sin 2\theta_1}{2}\Big|_0^{\pi/2}\right) = \frac{\pi PR^3}{4EI} = \frac{\pi(1)2.5^3}{4(359.53)} = 0.0341 \text{ in} \qquad \textit{Ans.}$$

4-101 Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

Place a fictitious force, Q, at A vertically downward. The only load in the straight section is the axial force, Q. Since this will be zero, there is no contribution.

In the curved section

$$M = PR\sin\theta + QR(1-\cos\theta) \qquad \frac{\partial M}{\partial Q} = R(1-\cos\theta)$$

From Eq. (4-41)

$$(\delta_A)_V = \left[\int_0^{\pi/2} \frac{1}{EI}\left(M\frac{\partial M}{\partial Q}\right)R\,d\theta\right]_{Q=0} = \frac{1}{EI}\int_0^{\pi/2} PR\sin\theta\left[R(1-\cos\theta)\right]R\,d\theta$$

$$= \frac{PR^3}{EI}\int_0^{\pi/2}(\sin\theta - \sin\theta\cos\theta)\,d\theta = \frac{PR^3}{EI}\left(1-\frac{1}{2}\right) = \frac{PR^3}{2EI}$$

$$= \frac{1(2.5^3)}{2(30)10^6\left[\pi(0.125^4)/64\right]} = 0.0217 \text{ in} \downarrow \quad \textit{Ans.}$$

4-102 Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

Place a fictitious force, Q, at A vertically downward. The load in the straight section is the axial force, Q, whereas the bending moment is only a function of P and is not a function of Q. When setting $Q = 0$, there is no axial or bending contribution.

In the curved section

$$M = P\left[R(1-\cos\theta)+l\right] - QR\sin\theta \qquad \frac{\partial M}{\partial Q} = -R\sin\theta$$

From Eq. (4-41)

$$(\delta_A)_V = \left[\int_0^{\pi/2} \frac{1}{EI}\left(M\frac{\partial M}{\partial Q}\right)R\,d\theta\right]_{Q=0} = \frac{1}{EI}\int_0^{\pi/2} P\left[R(1-\cos\theta)+l\right](-R\sin\theta)\,R\,d\theta$$

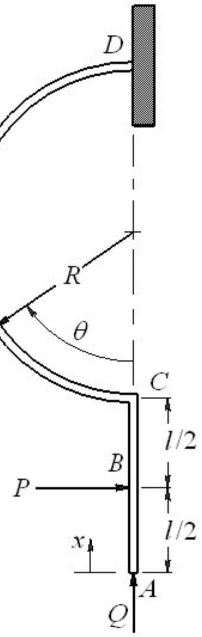
$$= -\frac{PR^2}{EI}\int_0^{\pi/2}(R\sin\theta - R\sin\theta\cos\theta + l\sin\theta)\,d\theta = -\frac{PR^2}{EI}\left(R + l - \frac{1}{2}R\right) = -\frac{PR^2}{2EI}(R+2l)$$

$$= -\frac{1(2.5^2)}{2(30)10^6\left[\pi(0.125^4)/64\right]}\left[2.5+2(2)\right] = -0.0565 \text{ in}$$

Since the deflection is negative, δ is in the opposite direction of Q. Thus the deflection is

$$\delta = 0.0565 \text{ in} \uparrow \qquad \textit{Ans.}$$

4-103 $EI = 30(10^6)(\pi/64)(0.125)^4 = 359.53$ lbf·in^2.



AB: Only *Q* and since $Q = 0$, no contribution to $(\delta_A)_V$.

CD:

$$M = QR\sin\theta + P\left[(l/2) + R(1-\cos\theta)\right] \qquad \frac{\partial M}{\partial Q} = R\sin\theta$$

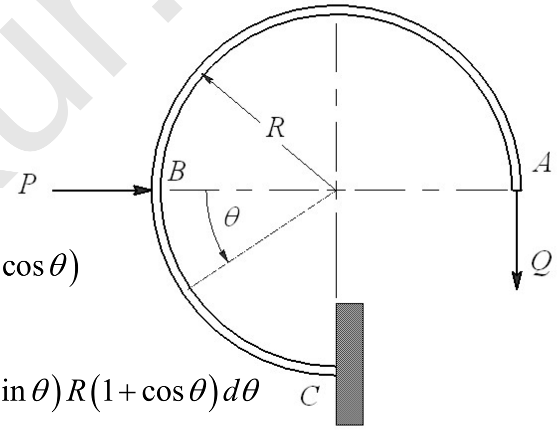
$$(\delta_A)_V = \frac{1}{EI}\int_0^{\pi} M\frac{\partial M}{\partial Q}Rd\theta\bigg|_{Q=0}$$

$$= \frac{1}{EI}\int_0^{\pi} P\left[(l/2) + R(1-\cos\theta)\right]R\sin\theta Rd\theta$$

$$= \frac{PR^2}{EI}\left[-(l/2)\cos\theta - R\cos\theta - (R/2)\sin^2\theta\right]\Big|_0^{\pi} = \frac{PR^2}{EI}(l+2R)$$

$$= \frac{(1)2.5^2}{359.53}\left[2+2(2.5)\right] = 0.1217 \text{ in } \uparrow \qquad \textit{Ans.}$$

4-104 $EI = 30(10^6)\,(\pi/64)(0.125)^4 = 359.53$ lbf·in^2.



AB: Only *Q*. Since $Q = 0$, no contribution.

BC:

$$M = QR(1+\cos\theta) + PR\sin\theta \qquad \frac{\partial M}{\partial Q} = R(1+\cos\theta)$$

$$(\delta_A)_V = \frac{1}{EI}\int_0^{\pi/2} M\frac{\partial M}{\partial Q}Rd\theta\bigg|_{Q=0} = \frac{1}{EI}\int_0^{\pi/2}(PR\sin\theta)R(1+\cos\theta)d\theta$$

$$= \frac{PR^3}{EI}\left(-\cos\theta + \frac{1}{2}\sin^2\theta\right)_0^{\pi/2} = \frac{PR^3}{EI}\left(1+\frac{1}{2}\right)$$

$$= \frac{3PR^3}{2EI} = \frac{3(2.5^3)}{2(359.53)} = 0.0652 \text{ in } \downarrow \quad \textit{Ans.}$$

4-105 Consider the force of the mass to be F, where $F = 9.81(1) = 9.81$ N. The load in AB is tension

$$F_{AB} = F \qquad \frac{\partial F_{AB}}{\partial F} = 1$$

For the curved section, the radius is sufficiently large to use Eq. (4-41). There is no bending in section DE. For section BCD, let θ be counterclockwise originating at D

$$M = FR\sin\theta \qquad \frac{\partial M}{\partial F} = R\sin\theta \qquad 0 \le \theta \le \pi$$

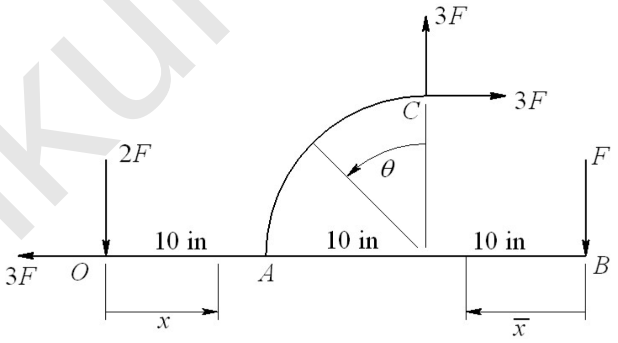
Using Eqs. (4-29) and (4-41)

$$\delta = \left(\frac{Fl}{AE}\right)_{AB}\frac{\partial F_{AB}}{\partial F} + \int_0^{\pi}\frac{1}{EI}\left(M\frac{\partial M}{\partial F}\right)R\,d\theta = \frac{Fl}{AE}(1) + \int_0^{\pi}\frac{FR^3}{EI}\sin^2\theta\,d\theta$$

$$= \frac{Fl}{AE} + \frac{\pi FR^3}{2EI} = \frac{F}{E}\left(\frac{l}{A} + \frac{\pi R^3}{2I}\right) = \frac{9.81}{207(10^3)}\left[\frac{80}{\left[\pi(2^2)/4\right]} + \frac{\pi(40^3)}{2\left[\pi(2^4)/64\right]}\right]$$

$$= 6.067 \text{ mm} \qquad \textit{Ans.}$$

4-106 $A_{OA} = 2(0.25) = 0.5$ in^2,
$I_{OAB} = 0.25(2^3)/12 = 0.1667$ in^4,
$I_{AC} = \pi(0.5^4)/64 = 3.068\ (10^{-3})$ in^4



Applying a force F at point B, using statics (AC is a two-force member), the reaction forces at O and C are as shown.

OA: Axial $\quad F_{OA} = 3F \qquad \dfrac{\partial F_{OA}}{\partial F} = 3$

Bending $\quad M_{OA} = -2Fx \qquad \dfrac{\partial M_{OA}}{\partial F} = -2x$

AB: Bending $\quad M_{AB} = -F\bar{x} \qquad \dfrac{\partial M_{AB}}{\partial F} = -\bar{x}$

AC: Isolating the upper curved section

$$M_{AC} = 3FR(\sin\theta + \cos\theta - 1) \qquad \frac{\partial M_{AC}}{\partial F} = 3R(\sin\theta + \cos\theta - 1)$$

$$\delta = \left(\frac{Fl}{AE}\right)_{OA}\frac{\partial F_{OA}}{\partial F} + \frac{1}{(EI)_{OAB}}\int_0^{10} 4Fx^2 dx + \frac{1}{(EI)_{OAB}}\int_0^{20} F\,\bar{x}^2 d\,\bar{x}$$

$$+\frac{9FR^3}{(EI)_{AC}}\int_0^{\pi/2}(\sin\theta+\cos\theta-1)^2\,d\theta$$

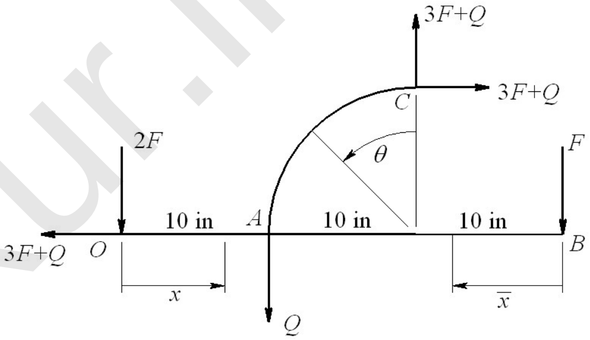
$$=\frac{3F(10)}{0.5(10.4)10^6}(3)+\frac{4F\left(10^3\right)}{3(10.4)10^6(0.1667)}+\frac{F\left(20^3\right)}{3(10.4)10^6(0.1667)}$$

$$+\frac{9F\left(10^3\right)}{30\left(10^6\right)3.068\left(10^{-3}\right)}\int_0^{\pi/2}\left(\sin^2\theta+2\sin\theta\cos\theta-2\sin\theta+\cos^2\theta-2\cos\theta+1\right)d\theta$$

$$=1.731\left(10^{-5}\right)F+7.691\left(10^{-4}\right)F+1.538\left(10^{-3}\right)F+0.09778F\left(\frac{\pi}{4}+1-2+\frac{\pi}{4}-2+\frac{\pi}{2}\right)$$

$$=0.0162F=0.0162(100)=1.62 \text{ in} \qquad \textit{Ans.}$$

4-107 $A_{OA} = 2(0.25) = 0.5 \text{ in}^2$,
$I_{OAB} = 0.25(2^3)/12 = 0.1667 \text{ in}^4$,
$I_{AC} = \pi(0.5^4)/64 = 3.068\ (10^{-3}) \text{ in}^4$
Applying a vertical fictitious force, Q, at A, from statics the reactions are as shown. The fictitious force is transmitted through section OA and member AC.



$$OA:\quad F_{OA}=3F+Q \qquad \frac{\partial F_{OA}}{\partial Q}=1$$

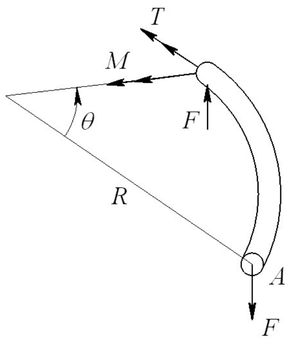
$$AC:\quad M_{AC}=(3F+Q)R\sin\theta-(3F+Q)R(1-\cos\theta) \qquad \frac{\partial M_{AC}}{\partial Q}=R(\sin\theta+\cos\theta-1)$$

$$\delta=\left[\left(\frac{Fl}{AE}\right)_{OA}\left(\frac{\partial F_{OA}}{\partial Q}\right)+\left(\frac{1}{EI}\right)_{AC}\int_0^{\pi/2}M_{AC}\frac{\partial M_{AC}}{\partial Q}R\,d\theta\right]_{Q=0}$$

$$=\frac{3Fl_{OA}}{(AE)_{OA}}+\frac{3FR^3}{(EI)_{AC}}\int_0^{\pi/2}(\sin\theta+\cos\theta-1)^2\,d\theta$$

$$=\frac{3(100)10}{10.4\left(10^6\right)0.5}+\frac{3(100)10^3}{30\left(10^6\right)3.068\left(10^{-3}\right)}\left(\frac{\pi}{4}+1-2+\frac{\pi}{4}-2+\frac{\pi}{2}\right)=0.462 \text{ in} \qquad \textit{Ans.}$$

4-108 $I = \pi(6^4)/64 = 63.62 \text{ mm}^4$



$\underline{0 \le \theta \le \pi/2}$

$$M = FR\sin\theta \qquad \frac{\partial M}{\partial F} = R\sin\theta$$

$$T = FR(1-\cos\theta) \qquad \frac{\partial T}{\partial F} = R(1-\cos\theta)$$

According to Castigliano's theorem, a positive ∂ *U*/∂ *F* will yield a deflection of *A* in the negative *y* direction. Thus the deflection in the positive *y* direction is

$$(\delta_A)_y = -\frac{\partial U}{\partial F} = -\left\{\frac{1}{EI}\int_0^{\pi/2} F(R\sin\theta)^2 R\,d\theta + \frac{1}{GJ}\int_0^{\pi/2} F[R(1-\cos\theta)]^2 R\,d\theta\right\}$$

Integrating and substituting $J = 2I$ and $G = E/\left[2(1+\nu)\right]$

$$(\delta_A)_y = -\frac{FR^3}{EI}\left[\frac{\pi}{4} + (1+\nu)\left(\frac{3\pi}{4} - 2\right)\right] = -\left[4\pi - 8 + (3\pi - 8)\nu\right]\frac{FR^3}{4EI}$$

$$= -[4\pi - 8 + (3\pi - 8)(0.29)]\frac{(250)(80)^3}{4(200)10^3(63.62)} = -12.5 \text{ mm} \qquad Ans.$$

4-109 The force applied to the copper and steel wire assembly is

$$F_c + F_s = 400 \text{ lbf} \qquad (1)$$

Since the deflections are equal, $\delta_c = \delta_s$

$$\left(\frac{Fl}{AE}\right)_c = \left(\frac{Fl}{AE}\right)_s$$

$$\frac{F_c l}{3(\pi/4)(0.1019)^2(17.2)10^6} = \frac{F_s l}{(\pi/4)(0.1055)^2(30)10^6}$$

Yields, $F_c = 1.6046\ F_s$. Substituting this into Eq. (1) gives

$$1.6046F_s + F_s = 2.6046F_s = 400 \quad \Rightarrow \quad F_s = 153.6 \text{ lbf}$$

$$F_c = 1.6046F_s = 246.5 \text{ lbf}$$

$$\sigma_c = \frac{F_c}{A_c} = \frac{246.5}{3(\pi/4)(0.1019)^2} = 10\ 075 \text{ psi} = 10.1 \text{ kpsi} \qquad Ans.$$

$$\sigma_s = \frac{F_s}{A_s} = \frac{153.6}{(\pi/4)(0.1055^2)} = 17\ 571 \text{ psi} = 17.6 \text{ kpsi} \qquad Ans.$$

$$\delta = \left(\frac{Fl}{AE}\right)_s = \frac{153.6(100)(12)}{(\pi/4)(0.1055)^2(30)10^6} = 0.703 \text{ in} \qquad Ans.$$

4-110 **(a)** Bolt stress $\sigma_b = 0.75(65) = 48.8 \text{ kpsi}$ *Ans.*

Total bolt force $$F_b = 6\sigma_b A_b = 6(48.8)\left(\frac{\pi}{4}\right)(0.5^2) = 57.5 \text{ kips}$$

Cylinder stress $$\sigma_c = -\frac{F_b}{A_c} = \frac{57.43}{(\pi/4)(5.5^2 - 5^2)} = -13.9 \text{ kpsi} \quad \textit{Ans.}$$

(b) Force from pressure

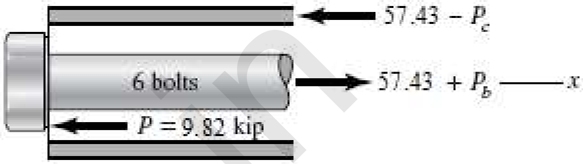
$$P = \frac{\pi D^2}{4} p = \frac{\pi(5^2)}{4}(500) = 9817 \text{ lbf} = 9.82 \text{ kip}$$

$\Sigma F_x = 0$

$P_b + P_c = 9.82$ (1)

Since $\delta_c = \delta_b$,

$$\frac{P_c l}{(\pi/4)(5.5^2 - 5^2)E} = \frac{P_b l}{6(\pi/4)(0.5^2)E}$$



$P_c = 3.5\ P_b$ (2)

Substituting this into Eq. (1)

$P_b + 3.5\ P_b = 4.5\ P_b = 9.82 \Rightarrow P_b = 2.182$ kip. From Eq. (2), $P_c = 7.638$ kip

Using the results of (**a**) above, the total bolt and cylinder stresses are

$$\sigma_b = 48.8 + \frac{2.182}{6(\pi/4)(0.5^2)} = 50.7 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_c = -13.9 + \frac{7.638}{(\pi/4)(5.5^2 - 5^2)} = -12.0 \text{ kpsi} \quad \textit{Ans.}$$

4-111 $T_c + T_s = T$ (1)

$$\theta_c = \theta_s \quad \Rightarrow \quad \frac{T_c l}{(JG)_c} = \frac{T_s l}{(JG)_s} \quad \Rightarrow \quad T_c = \frac{(JG)_c}{(JG)_s} T_s \qquad (2)$$

Substitute this into Eq. (1)

$$\frac{(JG)_c}{(JG)_s} T_s + T_s = T \quad \Rightarrow \quad T_s = \frac{(JG)_s}{(JG)_s + (JG)_c} T$$

The percentage of the total torque carried by the shell is

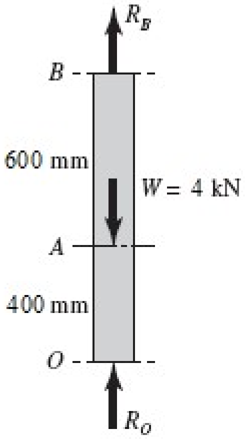
$$\% \text{ Torque} = \frac{100(JG)_s}{(JG)_s + (JG)_c} \qquad \textit{Ans.}$$

4-112 $R_O + R_B = W$ (1)

$\delta_{OA} = \delta_{AB}$

$$\left(\frac{Fl}{AE}\right)_{OA} = \left(\frac{Fl}{AE}\right)_{AB}$$

$$\frac{400R_O}{AE} = \frac{600R_B}{AE} \quad \Rightarrow \quad R_O = \frac{3}{2}R_B \qquad (2)$$



Substitute this unto Eq. (1)

$$\frac{3}{2}R_B + R_B = 4 \quad \Rightarrow \quad R_B = 1.6 \text{ kN} \quad Ans.$$

From Eq. (2) $R_O = \frac{3}{2}1.6 = 2.4 \text{ kN}$ *Ans.*

$$\delta_A = \left(\frac{Fl}{AE}\right)_{OA} = \frac{2\,400(400)}{10(60)(71.7)(10^3)} = 0.0223 \text{ mm} \quad Ans.$$

4-113 See figure in Prob. 4-112 solution.

Procedure 1:

1. Let R_B be the redundant reaction.

2. Statics. $R_O + R_B = 4\,000$ N $\Rightarrow$ $R_O = 4\,000 - R_B$ (1)

3. Deflection of point *B*. $\delta_B = \frac{R_B(600)}{AE} + \frac{(R_B - 4000)(400)}{AE} = 0$ (2)

4. From Eq. (2), *AE* cancels and $R_B = 1\,600$ N *Ans.*
 and from Eq. (1), $R_O = 4\,000 - 1\,600 = 2\,400$ N *Ans.*

$$\delta_A = \left(\frac{Fl}{AE}\right)_{OA} = \frac{2\,400(400)}{10(60)(71.7)(10^3)} = 0.0223 \text{ mm} \quad Ans.$$

4-114 (**a**) Without the right-hand wall, the deflection of point *C* would be

$$\delta_C = \sum\frac{Fl}{AE} = \frac{5(10^3)8}{(\pi/4)0.75^2(10.4)10^6} + \frac{2(10^3)5}{(\pi/4)0.5^2(10.4)10^6}$$
$$= 0.01360 \text{ in} > 0.005 \text{ in} \quad \therefore \text{ Hits wall} \quad Ans.$$

(**b**) Let R_C be the reaction of the wall at C acting to the left (←). Thus, the deflection of point C is now

$$\delta_C = \frac{\left[5\left(10^3\right) - R_C\right]8}{(\pi/4)0.75^2(10.4)10^6} + \frac{\left[2\left(10^3\right) - R_C\right]5}{(\pi/4)0.5^2(10.4)10^6}$$

$$= 0.01360 - \frac{4R_C}{\pi(10.4)10^6}\left(\frac{8}{0.75^2} + \frac{5}{0.5^2}\right) = 0.005$$

or,

$$0.01360 - 4.190\left(10^{-6}\right)R_C = 0.005 \quad \Rightarrow \quad R_C = 2\,053 \text{ lbf} = 2.05 \text{ kip} \leftarrow \quad \textit{Ans.}$$

$\Sigma F_x = 5\,000 + R_A - 2\,053 = 0 \;\; \Rightarrow \;\; R_A = -2\,947$ lbf, $\Rightarrow \;\; R_A = 2.95$ kip ← *Ans.*

Deflection. AB is 2 947 lbf in tension. Thus

$$\delta_B = \delta_{AB} = \frac{R_A(8)}{A_{AB}E} = \frac{2\,947(8)}{(\pi/4)0.75^2(10.4)10^6} = 5.13\left(10^{-3}\right) \text{ in} \;\rightarrow \quad \textit{Ans.}$$

4-115 Since $\theta_{OA} = \theta_{AB}$,

$$\frac{T_{OA}(4)}{JG} = \frac{T_{AB}(6)}{JG} \quad \Rightarrow \quad T_{OA} = \frac{3}{2}T_{AB} \qquad (1)$$

Statics. $T_{OA} + T_{AB} = 200$ (2)

Substitute Eq. (1) into Eq. (2),

$$\frac{3}{2}T_{AB} + T_{AB} = \frac{5}{2}T_{AB} = 200 \qquad \Rightarrow \qquad T_{AB} = 80 \text{ lbf}\cdot\text{in} \qquad \textit{Ans.}$$

From Eq. (1) $\quad T_{OA} = \frac{3}{2}T_{AB} = \frac{3}{2}80 = 120 \text{ lbf}\cdot\text{in} \qquad$ *Ans.*

$$\theta_A = \left(\frac{TL}{JG}\right)_{AB} = \frac{80(6)}{(\pi/32)0.5^4(11.5)10^6}\frac{180}{\pi} = 0.390^0 \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{16T}{\pi d^3} \qquad \Rightarrow \qquad \tau_{OA} = \frac{16(120)}{\pi\left(0.5^3\right)} = 4890 \text{ psi} = 4.89 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{AB} = \frac{16(80)}{\pi\left(0.5^3\right)} = 3260 \text{ psi} = 3.26 \text{ kpsi} \qquad \textit{Ans.}$$

4-116 Since $\theta_{OA} = \theta_{AB}$,

$$\frac{T_{OA}(4)}{(\pi/32)0.5^4 G} = \frac{T_{AB}(6)}{(\pi/32)0.75^4 G} \quad \Rightarrow \quad T_{OA} = 0.2963\,T_{AB} \qquad (1)$$

Statics. $T_{OA} + T_{AB} = 200$ (2)

Substitute Eq. (1) into Eq. (2),

$$0.2963T_{AB} + T_{AB} = 1.2963T_{AB} = 200 \quad \Rightarrow \quad T_{AB} = 154.3 \text{ lbf}\cdot\text{in} \qquad Ans.$$

From Eq. (1) $T_{OA} = 0.2963T_{AB} = 0.2963(154.3) = 45.7 \text{ lbf}\cdot\text{in}$ *Ans.*

$$\theta_A = \frac{154.3(6)}{(\pi/32)0.75^4(11.5)10^6}\frac{180}{\pi} = 0.148^0 \quad Ans.$$

$$\tau_{\max} = \frac{16T}{\pi d^3} \quad \Rightarrow \quad \tau_{OA} = \frac{16(45.7)}{\pi(0.5^3)} = 1862 \text{ psi} = 1.86 \text{ kpsi} \qquad Ans.$$

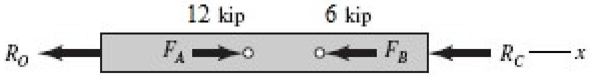
$$\tau_{AB} = \frac{16(154.3)}{\pi(0.75^3)} = 1862 \text{ psi} = 1.86 \text{ kpsi} \qquad Ans.$$

4-117 Procedure 1

1. Arbitrarily, choose R_C as a redundant reaction.
2. Statics. $\Sigma F_x = 0$,

$12(10^3) - 6(10^3) - R_O - R_C = 0$

$R_O = 6(10^3) - R_C$ (1)



3. The deflection of point C.

$$\delta_C = \frac{[12(10^3) - 6(10^3) - R_C](20)}{AE} - \frac{[6(10^3) + R_C](10)}{AE} - \frac{R_C(15)}{AE} = 0$$

4. The deflection equation simplifies to

$$-45R_C + 60(10^3) = 0 \quad \Rightarrow \quad R_C = 1\,333 \text{ lbf} = 1.33 \text{ kip} \quad Ans.$$

From Eq. (1), $R_O = 6(10^3) - 1\,333 = 4\,667 \text{ lbf} = 4.67 \text{ kip}$ *Ans.*

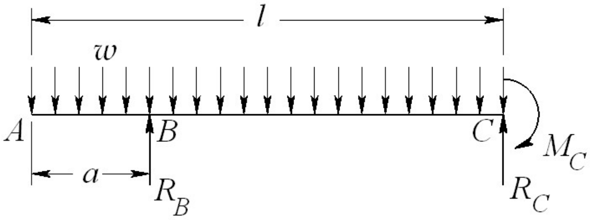
$F_{AB} = F_B + R_C = 6 + 1.333 = 7.333$ kips compression

$$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{-7.333}{(0.5)(1)} = -14.7 \text{ kpsi} \qquad Ans.$$

Deflection of *A*. Since *OA* is in tension,

$$\delta_A = \delta_{OA} = \frac{R_O l_{OA}}{AE} = \frac{4\,667(20)}{(0.5)(1)(30)10^6} = 0.00622 \text{ in} \qquad Ans.$$

4-118 Procedure 1



1. Choose R_B as redundant reaction.

2. Statics. $R_C = wl - R_B$ (1)

$$M_C = \frac{1}{2} w l^2 - R_B (l - a) \qquad (2)$$

3. Deflection equation for point *B*. Superposition of beams 2 and 3 of Table A-9,

$$y_B = \frac{R_B (l-a)^3}{3EI} + \frac{w(l-a)^2}{24EI}\left[4l(l-a) - (l-a)^2 - 6l^2\right] = 0$$

4. Solving for R_B.

$$R_B = \frac{w}{8(l-a)}\left[6l^2 - 4l(l-a) + (l-a)^2\right]$$

$$= \frac{w}{8(l-a)}\left(3l^2 + 2al + a^2\right) \qquad Ans.$$

Substituting this into Eqs. (1) and (2) gives

$$R_C = wl - R_B = \frac{w}{8(l-a)}\left(5l^2 - 10al - a^2\right) \qquad Ans.$$

$$M_C = \frac{1}{2} wl^2 - R_B (l-a) = \frac{w}{8}\left(l^2 - 2al - a^2\right) \qquad Ans.$$

4-119 See figure in Prob. 4-118 solution.

Procedure 1

1. Choose R_B as redundant reaction.

2. Statics. $R_C = wl - R_B$ (1)

$$M_C = \frac{1}{2} wl^2 - R_B (l-a) \qquad (2)$$

3. Deflection equation for point B. Let the variable x start at point A and to the right. Using singularity functions, the bending moment as a function of x is

$$M = -\frac{1}{2}wx^2 + R_B\langle x-a\rangle^1 \qquad \frac{\partial M}{\partial R_B} = \langle x-a\rangle^1$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial R_B}dx$$

$$= \frac{1}{EI}\int_0^l -\frac{1}{2}wx^2(0)dx + \frac{1}{EI}\int_a^l\left[-\frac{1}{2}wx^2 + R_B(x-a)\right](x-a)dx = 0$$

or,

$$-\frac{1}{2}w\left[\frac{1}{4}\left(l^4-a^4\right)-\frac{a}{3}\left(l^3-a^3\right)\right]+\frac{R_B}{3}\left[(l-a)^3-(a-a)^3\right]=0$$

Solving for R_B gives

$$R_B = \frac{w}{8(l-a)^3}\left[3\left(l^4-a^4\right)-4a\left(l^3-a^3\right)\right] = \frac{w}{8(l-a)}\left(3l^2+2al+a^2\right) \qquad Ans.$$

From Eqs. (1) and (2)

$$R_C = wl - R_B = \frac{w}{8(l-a)}\left(5l^2-10al-a^2\right) \qquad Ans.$$

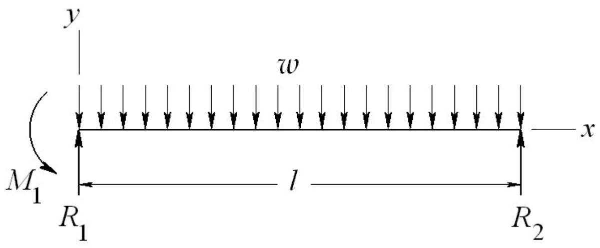
$$M_C = \frac{1}{2}wl^2 - R_B(l-a) = \frac{w}{8}\left(l^2-2al-a^2\right) \qquad Ans.$$

4-120 **Note:** When setting up the equations for this problem, no rounding of numbers was made in the calculations. It turns out that the deflection equation is very sensitive to rounding.

Procedure 2

1. Statics. $R_1 + R_2 = wl$ (1)

$$R_2 l + M_1 = \frac{1}{2}wl^2 \qquad (2)$$



2. Bending moment equation.

$$M = R_1 x - \frac{1}{2} w x^2 - M_1$$

$$EI\frac{dy}{dx} = \frac{1}{2} R_1 x^2 - \frac{1}{6} w x^3 - M_1 x + C_1 \qquad (3)$$

$$EIy = \frac{1}{6} R_1 x^3 - \frac{1}{24} w x^4 - \frac{1}{2} M_1 x^2 + C_1 x + C_2 \qquad (4)$$

$EI = 30(10^6)(0.85) = 25.5(10^6)$ lbf·in^2.

3. Boundary condition 1. At $x = 0$, $y = -R_1/k_1 = -R_1/[1.5(10^6)]$. Substitute into Eq. (4) with value of EI yields $C_2 = -17R_1$.

Boundary condition 2. At $x = 0$, $dy/dx = -M_1/k_2 = -M_1/[2.5(10^6)]$. Substitute into Eq. (3) with value of EI yields $C_1 = -10.2M_1$.

Boundary condition 3. At $x = l$, $y = -R_2/k_3 = -R_1/[2.0(10^6)]$. Substitute into Eq. (4) with value of EI yields

$$-12.75R_2 = \frac{1}{6} R_1 l^3 - \frac{1}{24} w l^4 - \frac{1}{2} M_1 l^2 - 10.2M_1 l - 17R_1 \qquad (5)$$

Equations (1), (2), and (5), written in matrix form with $w = 500/12$ lbf/in and $l = 24$ in, are

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 24 & 1 \\ 2287 & 12.75 & -532.8 \end{pmatrix} \begin{Bmatrix} R_1 \\ R_2 \\ M_1 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 12 \\ 576 \end{Bmatrix} (10^3)$$

Solving, the simultaneous equations yields

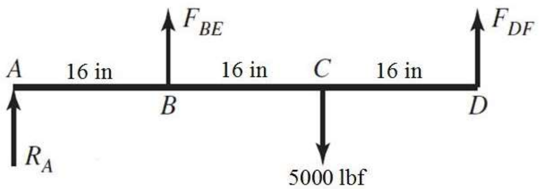
$R_1 = 554.59$ lbf, $R_2 = 445.41.59$ lbf, $M_1 = 1310.1$ lbf·in *Ans.*

For the deflection at $x = l/2 = 12$ in, Eq. (4) gives

$$y\big|_{x=12\text{in}} = \frac{1}{25.5(10^6)}\left[\frac{1}{6}(554.59)12^3 - \frac{1}{24}\frac{500}{12}12^4 - \frac{1}{2}(1310.1)12^2 \right.$$
$$\left. -10.2(1310.1)12 - 17(554.59)\right]$$
$$= -5.51(10^{-3}) \text{ in} \qquad \textit{Ans.}$$

4-121 Cable area, $A = \frac{\pi}{4}(0.5^2) = 0.1963$ in^2

Procedure 2



1. Statics. $R_A + F_{BE} + F_{DF} = 5(10^3)$ (1)

$3\,F_{DF} + F_{BE} = 10(10^3)$ (2)

2. Bending moment equation.

$$M = R_A x + F_{BE}\langle x-16\rangle^1 - 5000\langle x-32\rangle^1$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + \frac{1}{2}F_{BE}\langle x-16\rangle^2 - 2500\langle x-32\rangle^2 + C_1 \qquad (3)$$

$$EIy = \frac{1}{6}R_A x^3 + \frac{1}{6}F_{BE}\langle x-16\rangle^3 - \frac{2500}{3}\langle x-32\rangle^3 + C_1 x + C_2 \qquad (4)$$

3. B.C. 1: At $x = 0$, $y = 0 \quad \Rightarrow \quad C_2 = 0$

B.C. 2: At $x = 16$ in,

$$y_B = -\left(\frac{Fl}{AE}\right)_{BE} = -\frac{F_{BE}(38)}{0.1963(30)10^6} = -6.453(10^{-6})F_{BE}$$

Substituting into Eq. (4) and evaluating at $x = 16$ in

$$EIy_B = 30(10^6)(1.2)(-6.453)(10^{-6})F_{BE} = \frac{1}{6}R_A\left(16^3\right) + C_1(16)$$

Simplifying gives $682.7\,R_A + 232.3\,F_{BE} + 16\,C_1 = 0$ (5)

B.C. 2: At $x = 48$ in,

$$y_D = -\left(\frac{Fl}{AE}\right)_{DF} = -\frac{F_{DF}(38)}{0.1963(30)10^6} = -6.453(10^{-6})F_{DF}$$

Substituting into Eq. (4) and evaluating at $x = 48$ in,

$$EIy_D = -232.3F_{DF} = \frac{1}{6}R_A\left(48^3\right) + \frac{1}{6}F_{BE}(48-16)^3 - \frac{2500}{3}(48-32)^3 + 48C_1$$

Simplifying gives $18\,432\,R_A + 5\,461\,F_{BE} + 232.3\,F_{DF} + 48\,C_1 = 3.413(10^6)$ (6)

Equations (1), (2), (5) and (6) in matrix form are

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 682.7 & 232.3 & 0 & 16 \\ 18432 & 5461 & 232.3 & 48 \end{pmatrix} \begin{Bmatrix} R_A \\ F_{BE} \\ F_{DF} \\ C_1 \end{Bmatrix} = \begin{Bmatrix} 5000 \\ 10000 \\ 0 \\ 3.413\left(10^6\right) \end{Bmatrix}$$

Solve simultaneously or use software. The results are

$R_A = -\,970.5$ lbf, $F_{BE} = 3956$ lbf, $F_{DF} = 2015$ lbf, and $C_1 = -\,16\,020$ lbf·in^2.

$$\sigma_{BE} = \frac{3956}{0.1963} = 20.2 \text{ kpsi}, \qquad \sigma_{DF} = \frac{2015}{0.1963} = 10.3 \text{ kpsi} \qquad Ans.$$

$EI = 30(10^6)(1.2) = 36(10^6)$ lbf·in^2

$$y = \frac{1}{36(10^6)}\left(-\frac{970.5}{6}x^3 + \frac{3956}{6}\langle x-16\rangle^3 - \frac{2500}{3}\langle x-32\rangle^3 - 16\ 020x\right)$$

$$= \frac{1}{36(10^6)}\left(-161.8x^3 + 659.3\langle x-16\rangle^3 - 833.3\langle x-32\rangle^3 - 16\ 020x\right)$$

B: $x = 16$ in, $\quad y_B = \dfrac{1}{36(10^6)}\left[-161.8(16^3) - 16\ 020(16)\right] = -0.0255$ in $\qquad$ *Ans.*

C: $x = 32$ in,

$$y_C = \frac{1}{36(10^6)}\left[-161.8(32^3) + 659.3(32-16)^3 - 16\ 020(32)\right]$$

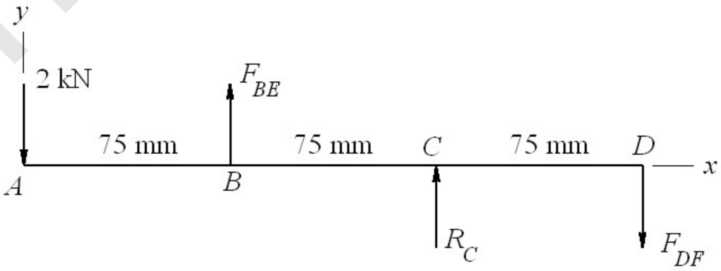
$$= -0.0865 \text{ in} \qquad Ans.$$

D: $x = 48$ in,

$$y_D = \frac{1}{36(10^6)}\left[-161.8(48^3) + 659.3(48-16)^3 - 833.3(48-32)^3 - 16\ 020(48)\right]$$

$$= -0.0131 \text{ in} \qquad Ans.$$

4-122 Beam: $EI = 207(10^3)21(10^3)$
$= 4.347(10^9)$ N·mm^2.
Rods: $A = (\pi/4)8^2 = 50.27$ mm^2.
Procedure 2



1. Statics.

$R_C + F_{BE} - F_{DF} = 2\ 000 \qquad (1)$

$R_C + 2F_{BE} = 6\ 000 \qquad (2)$

2. Bending moment equation.

$M = -2\ 000\,x + F_{BE}\langle x - 75\rangle^1 + R_C\langle x - 150\rangle^1$

$$EI\frac{dy}{dx} = -1000x^2 + \frac{1}{2}F_{BE}\langle x-75\rangle^2 + \frac{1}{2}R_C\langle x-150\rangle^2 + C_1 \qquad (3)$$

$$EIy = -\frac{1000}{3}x^3 + \frac{1}{6}F_{BE}\langle x-75\rangle^3 + \frac{1}{6}R_C\langle x-150\rangle^3 + C_1x + C_2 \qquad (4)$$

3. <u>B.C 1</u>. At x = 75 mm,

$$y_B = -\left(\frac{Fl}{AE}\right)_{BE} = -\frac{F_{BE}(50)}{50.27(207)10^3} = -4.805(10^{-6})F_{BE}$$

Substituting into Eq. (4) at x = 75 mm,

$$4.347(10^9)\left[-4.805(10^{-6})F_{BE}\right] = -\frac{1000}{3}(75^3) + C_1(75) + C_2$$

Simplifying gives

$$20.89(10^3)F_{BE} + 75C_1 + C_2 = 140.6(10^6) \qquad (5)$$

<u>B.C 2</u>. At x = 150 mm, y = 0. From Eq. (4),

$$-\frac{1000}{3}(150^3) + \frac{1}{6}F_{BE}(150-75)^3 + C_1(150) + C_2 = 0$$

or,

$$70.31(10^3)F_{BE} + 150C_1 + C_2 = 1.125(10^9) \qquad (6)$$

<u>B.C 3</u>. At x = 225 mm,

$$y_D = \left(\frac{Fl}{AE}\right)_{DF} = \frac{F_{DF}(65)}{50.27(207)10^3} = 6.246(10^{-6})F_{DF}$$

Substituting into Eq. (4) at x = 225 mm,

$$4.347(10^9)\left[6.246(10^{-6})F_{DF}\right] = -\frac{1000}{3}(225^3) + \frac{1}{6}F_{BE}(225-75)^3 + \frac{1}{6}R_C(225-150)^3 + C_1(225) + C_2$$

Simplifying gives

$$70.31(10^3)R_C + 562.5(10^3)F_{BE} - 27.15(10^3)F_{DF} + 225C_1 + C_2 = 3.797(10^9) \qquad (7)$$

Equations (1), (2), (5), (6), and (7) in matrix form are

$$\begin{pmatrix} 1 & 1 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 20.89\left(10^3\right) & 0 & 75 & 1 \\ 0 & 70.31\left(10^3\right) & 0 & 150 & 1 \\ 70.31\left(10^3\right) & 562.5\left(10^3\right) & -27.15\left(10^3\right) & 225 & 1 \end{pmatrix} \begin{Bmatrix} R_C \\ F_{BE} \\ F_{DF} \\ C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 2\left(10^3\right) \\ 6\left(10^3\right) \\ 140.6\left(10^6\right) \\ 1.125\left(10^9\right) \\ 3.797\left(10^9\right) \end{Bmatrix}$$

Solve simultaneously or use software. The results are

$R_C = -2378$ N, $F_{BE} = 4189$ N, $F_{DF} = -189.2$ N *Ans.*
and $C_1 = 1.036\ (10^7)$ N·mm^2, $C_2 = -7.243\ (10^8)$ N·mm^3.

The bolt stresses are $\sigma_{BE} = 4189/50.27 = 83.3$ MPa, $\sigma_{DF} = -189/50.27 = -3.8$ MPa *Ans.*

The deflections are

From Eq. (4) $y_A = \dfrac{1}{4.347\left(10^9\right)}\left[-7.243\left(10^8\right)\right] = -0.167$ mm *Ans.*

For points *B* and *D* use the axial deflection equations*.

$$y_B = -\left(\frac{Fl}{AE}\right)_{BE} = -\frac{4189(50)}{50.27(207)10^3} = -0.0201 \text{ mm} \qquad Ans.$$

$$y_D = \left(\frac{Fl}{AE}\right)_{DF} = \frac{-189(65)}{50.27(207)10^3} = -1.18\left(10^{-3}\right) \text{ mm} \qquad Ans.$$

*Note. The terms in Eq. (4) are quite large, and due to rounding are not very accurate for calculating the very small deflections, especially for point *D*.

4-123 Everything in Ex. 4-15 is the same except $k_B = 40$ MN/m.

$y_B = -F_B / k_B = -F_B / 40\ (10^6) = -2.5\ (10^{-8})\ F_B$

Equation (7) is replaced by

$$EI\left(-2.5\right)10^{-8}F_B = -\frac{F_A}{6}(0.2)^3 + \frac{F_B}{6}(0)^3 + 0.2C_1 + C_2$$

With $EI = 1.25\ (10^4)$ N·m^2, the equation reduces to

$-1.3333\ (10^{-3})\ F_A + 3.125\ (10^{-4})\ F_B + 0.2\ C_1 + C_2$

The remaining equations come from Ex. 4-15

$$\begin{bmatrix} -1 & 1 & -1 & 0 & 0 \\ 4 & 0 & -3 & 0 & 0 \\ -4.7746\left(10^{-4}\right) & 0 & 0 & 0 & 1 \\ -1.3333\left(10^{-3}\right) & 3.125\left(10^{-4}\right) & 0 & 0.2 & 1 \\ -7.1458\left(10^{-3}\right) & 5.625\left(10^{-4}\right) & -6.3662\left(10^{-4}\right) & 0.35 & 1 \end{bmatrix} \begin{Bmatrix} F_A \\ F_B \\ F_C \\ C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -18.75 \\ 0 \\ 0 \end{Bmatrix}$$

Solving for the unknowns gives

$$\begin{Bmatrix} F_A \\ F_B \\ F_C \\ C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 2350 \text{ N} \\ 5484 \text{ N} \\ 3134 \text{ N} \\ 95.239 \text{ N}\cdot\text{m}^2 \\ -17.628 \text{ N}\cdot\text{m}^3 \end{Bmatrix} \qquad \textit{Ans.}$$

Equation (3): $1.25\left(10^4\right)y = -\dfrac{2350}{6}x^3 + \dfrac{5484}{6}\langle x-0.2\rangle^3 + 95.239x - 17.628$

Which reduces to $y = -0.03133\, x^3 + 0.07312\, \langle x - 0.2\rangle^3 + 7.619\, (10^{-3})\, x - 1.410\, (10^{-3})$

At $x = 0$, $y = y_A = -1.410\, (10^{-3})$ m $= -1.41$ mm *Ans.*

At $x = 0.2$ m, $y = y_B = -0.03133(0.2)^3 + 7.619(10^{-3})0.2 - 1.410(10^{-3})$
$= -1.37\, (10^{-4})$ m $= -0.137$ mm *Ans.*

At $x = 0.35$ m,

$$y = y_C = -0.03133(0.35)^3 + 0.07312\, (0.35 - 0.2)^3 + 7.619(10^{-3})0.35 - 1.410(10^{-3})$$

$$= 1.60\, (10^{-4}) \text{ m} = -0.160 \text{ mm} \qquad \textit{Ans.}$$

4-124 (**a**) The cross section at *A* does not rotate. Thus, for a single quadrant we have

$$\frac{\partial U}{\partial M_A} = 0$$

The bending moment at an angle θ to the x axis is

$$M = M_A - \frac{FR}{2}(1-\cos\theta) \qquad \frac{\partial M}{\partial M_A} = 1$$

The rotation at *A* is

$$\theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI}\int_0^{\pi/2} M \frac{\partial M}{\partial M_A} R\, d\theta = 0$$

Thus, $\frac{1}{EI}\int_0^{\pi/2}\left[M_A-\frac{FR}{2}(1-\cos\theta)\right](1)Rd\theta=0 \quad\Rightarrow\quad \left(M_A-\frac{FR}{2}\right)\frac{\pi}{2}+\frac{FR}{2}=0$

or,

$$M_A=\frac{FR}{2}\left(1-\frac{2}{\pi}\right)$$

Substituting this into the equation for M gives

$$M=\frac{FR}{2}\left(\cos\theta-\frac{2}{\pi}\right) \qquad (1)$$

The maximum occurs at B where $\theta=\pi/2$

$$M_{\max}=M_B=-\frac{FR}{\pi} \qquad Ans.$$

(**b**) Assume B is supported on a knife edge. The deflection of point D is $\partial U/\partial F$. We will deal with the quarter-ring segment and multiply the results by 4. From Eq. (1)

$$\frac{\partial M}{\partial F}=\frac{R}{2}\left(\cos\theta-\frac{2}{\pi}\right)$$

Thus,

$$\delta_D=\frac{\partial U}{\partial F}=\frac{4}{EI}\int_0^{\pi/2}M\frac{\partial M}{\partial F}Rd\theta=\frac{FR^3}{EI}\int_0^{\pi/2}\left(\cos\theta-\frac{2}{\pi}\right)^2d\theta=\frac{FR^3}{EI}\left(\frac{\pi}{4}-\frac{2}{\pi}\right)$$

$$=\frac{FR^3}{4\pi EI}\left(\pi^2-8\right) \quad Ans.$$

4-125

$$P_{\text{cr}}=\frac{C\pi^2EI}{l^2}$$

$$I=\frac{\pi}{64}\left(D^4-d^4\right)=\frac{\pi D^4}{64}\left(1-K^4\right) \qquad \text{where } K=\frac{d}{D}$$

$$P_{\text{cr}}=\frac{C\pi^2E}{l^2}\left[\frac{\pi D^4}{64}\left(1-K^4\right)\right]$$

$$D=\left[\frac{64P_{\text{cr}}l^2}{\pi^3CE\left(1-K^4\right)}\right]^{1/4} \qquad Ans.$$

4-126 $A=\frac{\pi}{4}D^2\left(1-K^2\right), \quad I=\frac{\pi}{64}D^4\left(1-K^4\right)=\frac{\pi}{64}D^4\left(1-K^2\right)\left(1+K^2\right)$, where $K=d/D$.

The radius of gyration, k, is given by

$$k^2 = \frac{I}{A} = \frac{D^2}{16}\left(1+K^2\right)$$

From Eq. (4-46)

$$\frac{P_{\text{cr}}}{(\pi/4)D^2\left(1-K^2\right)} = S_y - \frac{S_y^2 l^2}{4\pi^2 k^2 CE} = S_y - \frac{S_y^2 l^2}{4\pi^2\left(D^2/16\right)\left(1+K^2\right)CE}$$

$$4P_{\text{cr}} = \pi D^2\left(1-K^2\right)S_y - \frac{4S_y^2 l^2 \pi D^2\left(1-K^2\right)}{\pi^2 D^2\left(1+K^2\right)CE}$$

$$\pi D^2\left(1-K^2\right)S_y = 4P_{\text{cr}} + \frac{4S_y^2 l^2\left(1-K^2\right)}{\pi\left(1+K^2\right)CE}$$

$$D = \left[\frac{4P_{\text{cr}}}{\pi S_y\left(1-K^2\right)} + \frac{4S_y^2 l^2\left(1-K^2\right)}{\pi\left(1+K^2\right)CE\pi\left(1-K^2\right)S_y}\right]^{1/2}$$

$$= 2\left[\frac{P_{\text{cr}}}{\pi S_y\left(1-K^2\right)} + \frac{S_y l^2}{\pi^2 CE\left(1+K^2\right)}\right]^{1/2} \qquad Ans.$$

4-127 **(a)** $\Sigma M_A = 0,\quad (0.75)(800) - \frac{0.9}{\sqrt{0.9^2+0.5^2}}F_{BO}(0.5) = 0 \quad\Rightarrow\quad F_{BO} = 1373\text{ N}$

Using $n_d = 4$, design for $F_{\text{cr}} = n_d F_{BO} = 4(1373) = 5492$ N

$$l = \sqrt{0.9^2+0.5^2} = 1.03\text{ m},\quad S_y = 165\text{ MPa}$$

In-plane:

$$k = \left(\frac{I}{A}\right)^{1/2} = \left(\frac{bh^3/12}{bh}\right)^{1/2} = 0.2887h = 0.2887(0.025) = 0.007\,218\text{ m},\qquad C = 1.0$$

$$\frac{l}{k} = \frac{1.03}{0.007218} = 142.7$$

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2(207)(10^9)}{165(10^6)}\right)^{1/2} = 157.4$$

Since $(l/k)_1 > (l/k)$ use Johnson formula.

Try 25 mm x 12 mm,

$$P_{\text{cr}} = 0.025(0.012)\left\{165\left(10^6\right) - \left[\frac{165\left(10^6\right)}{2\pi}(142.7)\right]^2 \frac{1}{1(207)10^9}\right\} = 29.1\text{ kN}$$

This is significantly greater than the design load of 5492 N found earlier. Check out-of-plane.

Out-of-plane: $k = 0.2887(0.012) = 0.003\ 464$ in, $C = 1.2$

$$\frac{l}{k} = \frac{1.03}{0.003\ 464} = 297.3$$

Since $(l/k)_1 < (l/k)$ use Euler equation.

$$P_{\text{cr}} = 0.025(0.012)\frac{1.2\pi^2(207)10^9}{297.3^2} = 8321 \text{ N}$$

This is greater than the design load of 5492 N found earlier. It is also significantly less than the in-plane P_{cr} found earlier, so the out-of-plane condition will dominate. Iterate the process to find the minimum h that gives P_{cr} greater than the design load.

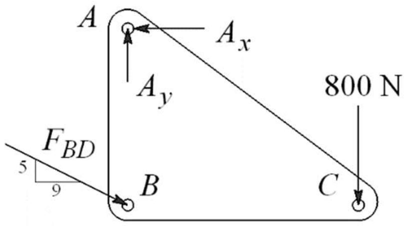
With $h = 0.010$, $P_{\text{cr}} = 4815$ N (too small)
$h = 0.011$, $P_{\text{cr}} = 6409$ N (acceptable)

Use 25 mm x 11 mm. If standard size is preferred, use 25 mm x 12 mm. *Ans.*

(b) $\sigma_b = -\dfrac{P}{dh} = -\dfrac{1373}{0.012(0.011)} = -10.4\left(10^6\right) \text{ Pa} = -10.4 \text{ MPa}$

No, bearing stress is not significant. *Ans.*

4-128 **(a)** $\Sigma M_A = 0 = 800(750) - \dfrac{9}{\sqrt{9^2+5^2}} F_{BD}(500)$



$F_{BD} = 1373$ N *Ans.*

(b) $\sigma = \dfrac{F}{A} = \dfrac{1373}{25(11)} = 4.99$ MPa *Ans.*

(c) $l_{BD} = \sqrt{0.9^2 + 0.5^2} = 1.030$ m

In plane, Table 4-2 $\Rightarrow$ $C = 1$

$$k = \sqrt{I/A} = \left(\frac{bh^3/12}{bh}\right)^{1/2} = h/\sqrt{12} = 0.025/\sqrt{12} = 0.007\ 217 \text{ m}$$

$l/k = 1.030/0.007\ 217 = 142.7.$

Eq. (4-45): $\left(\dfrac{l}{k}\right)_1 = \left(\dfrac{2\pi^2 CE}{S_y}\right)^{1/2} = \left[\dfrac{2\pi^2(1)207\left(10^9\right)}{165\left(10^6\right)}\right]^{1/2} = 157.4$

Since $(l/k)_1 > l/k$, use Johnson formula, Eq. (4-48)

$$P_{cr} = A\left[S_y - \left(\frac{S_y}{2\pi}\frac{l}{k}\right)^2 \frac{1}{CE}\right] = 0.25(0.011)\left\{165(10^6) - \left[\frac{165(10^6)}{2\pi}(142.7)\right]^2 \frac{1}{(1)207(10^9)}\right\}$$

$$= 26\ 720\ \text{ N} = 26.72\ \text{ kN}$$

$$n = \frac{P_{cr}}{F_{BD}} = \frac{26.72}{1.373} = 19.5 \qquad \textit{Ans.}$$

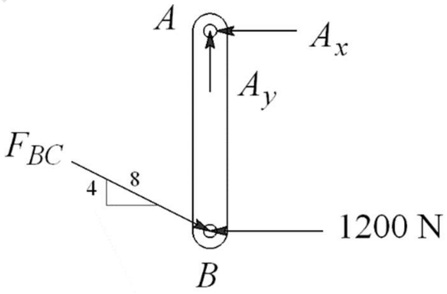
(d) Out of plane, Table 4-2, $C = 1.2$, $k = 0.011/\sqrt{12} = 3.175\ (10^{-3})$ m,

$l\,/\,k = 1.030/3.175\ (10^{-3}) = 324.4$. Since $l\,/\,k > (l\,/\,k)_1$, use the Euler formula, Eq. (4-44)

$$P_{cr} = A\left[\frac{C\pi^2 E}{(l/k)^2}\right] = 0.025(0.011)\left[\frac{1.2\pi^2(207)10^9}{324.4^2}\right] = 6\ 407\ \text{ N}$$

$$n = \frac{P_{cr}}{F_{BD}} = \frac{6\ 407}{1373} = 4.67 \qquad \textit{Ans.}$$

4-129 Out of plane bending, C =1.2.



$$\Sigma M_A = 0 = 400\left(1200 - \frac{\sqrt{8^2 - 4^2}}{8}F_{BC}\right)$$

$$F_{BC} = 1386\ \text{N} \qquad \textit{Ans.}$$

$$k = \sqrt{I/A} = \sqrt{(bh^3/12)/(bh)} = h/\sqrt{12} = 0.010/\sqrt{12} = 2.887\left(10^{-3}\right)\ \text{ m}$$

$l\,/\,k = 0.8/[2.887(10^{-3})] = 277.1$

Eq. (4-45): $\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2 CE}{S_y}\right)^{1/2} = \left[\frac{2\pi^2(1.2)207(10^9)}{200(10^6)}\right]^{1/2} = 156.6$

Since $l\,/\,k > (l\,/\,k)_1$, use the Euler formula, Eq. (4-44)

$$P_{cr} = A\left[\frac{C\pi^2 E}{(l/k)^2}\right] = 0.02(0.01)\left[\frac{1.2\pi^2(207)10^9}{277.1^2}\right] = 6\ 386\ \text{ N}$$

$$n = \frac{P_{cr}}{F_{BD}} = \frac{6\ 386}{1386} = 4.6\ \textit{Ans.}$$

4-130 This is an open-ended design problem with no one distinct solution.

4-131 $F = 1500(\pi/4)2^2 = 4712$ lbf. From Table A-20, $S_y = 37.5$ kpsi
$P_{\text{cr}} = n_d F = 2.5(4712) = 11\ 780$ lbf

(**a**) Assume Euler with $C = 1$

$$I = \frac{\pi}{64}d^4 = \frac{P_{\text{cr}}l^2}{C\pi^2 E} \quad \Rightarrow \quad d = \left(\frac{64P_{\text{cr}}l^2}{\pi^3 CE}\right)^{1/4} = \left[\frac{64(11790)50^2}{\pi^3(1)30(10^6)}\right]^{1/4} = 1.193 \text{ in}$$

Use $d = 1.25$ in. The radius of gyration, $k = (I/A)^{1/2} = d/4 = 0.3125$ in

$$\frac{l}{k} = \frac{50}{0.3125} = 160$$

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2 CE}{S_y}\right)^{1/2} = \left(\frac{2\pi^2(1)30(10^6)}{37.5(10^3)}\right)^{1/2} = 126 \qquad \therefore \text{ use Euler}$$

$$P_{\text{cr}} = \frac{\pi^2(30)10^6(\pi/64)1.25^4}{50^2} = 14194 \text{ lbf}$$

Since 14 194 lbf > 11 780 lbf, $d = 1.25$ in is satisfactory. *Ans.*

(**b**) $$d = \left[\frac{64(11780)16^2}{\pi^3(1)30(10^6)}\right]^{1/4} = 0.675 \text{ in, so use } d = 0.750 \text{ in}$$

$k = 0.750/4 = 0.1875$ in

$$\frac{l}{k} = \frac{16}{0.1875} = 85.33 \qquad \text{use Johnson}$$

$$P_{\text{cr}} = \frac{\pi}{4}(0.750^2)\left\{37.5(10^3) - \left[\frac{37.5(10^3)}{2\pi}85.33\right]^2 \frac{1}{1(30)10^6}\right\} = 12748 \text{ lbf}$$

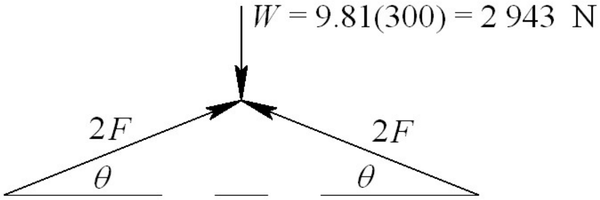
Use $d = 0.75$ in.

(**c**)

$$n_{(a)} = \frac{14194}{4712} = 3.01 \qquad Ans.$$

$$n_{(b)} = \frac{12748}{4712} = 2.71 \qquad Ans.$$

4-132 From Table A-20, $S_y = 180$ MPa



$$4F \sin\theta = 2\,943$$

$$F = \frac{735.8}{\sin\theta}$$

In range of operation, F is maximum when $\theta = 15°$

$$F_{\max} = \frac{735.8}{\sin 15°} = 2843 \text{ N per bar}$$

$$P_{cr} = n_d F_{\max} = 3.50\ (2\,843) = 9\,951 \text{ N}$$

$l = 350$ mm, $h = 30$ mm

Try $b = 5$ mm. Out of plane, $k = b / \sqrt{12} = 5/\sqrt{12} = 1.443$ mm

$$\frac{l}{k} = \frac{350}{1.443} = 242.6$$

$$\left(\frac{l}{k}\right)_1 = \left[\frac{2\pi^2(1.4)207\left(10^9\right)}{180\left(10^6\right)}\right]^{1/2} = 178.3 \qquad \therefore \text{ use Euler}$$

$$P_{cr} = A\frac{C\pi^2 E}{(l/k)^2} = 5(30)\frac{1.4\pi^2(207)10^3}{(242.6)^2} = 7\,290 \text{ N}$$

Too low. Try $b = 6$ mm. $k = 6/\sqrt{12} = 1.732$ mm

$$\frac{l}{k} = \frac{350}{1.732} = 202.1$$

$$P_{cr} = A\frac{C\pi^2 E}{(l/k)^2} = 6(30)\frac{1.4\pi^2(207)10^3}{(202.1)^2} = 12\,605 \text{ N}$$

O.K. Use 25 × 6 mm bars *Ans.* The factor of safety is

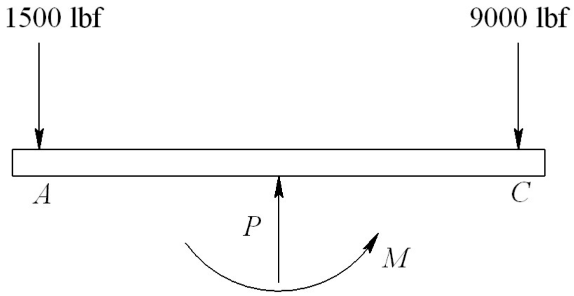
$$n = \frac{12\,605}{2843} = 4.43 \qquad Ans.$$

4-133 $P = 1\,500 + 9\,000 = 10\,500$ lbf *Ans.*

ΣM_A = 10 500 (4.5/2) – 9 000 (4.5) +M = 0

M = 16 874 lbf·in

$e = M / P$ = 16 874/10 500 = 1.607 in *Ans.*



From Table A-8, A = 2.160 in^2, and I = 2.059 in^4. The stresses are determined using Eq. (4-55)

$$k^2 = \frac{I}{A} = \frac{2.059}{2.160} = 0.953 \text{ in}^2$$

$$\sigma_c = -\frac{P}{A}\left(1+\frac{ec}{k^2}\right) = -\frac{10\,500}{2.160}\left[1+\frac{1.607(3/2)}{0.953}\right] = -17157 \text{ psi} = -17.16 \text{ kpsi} \quad Ans.$$

4-134 This is a design problem which has no single distinct solution.

4-135 Loss of potential energy of weight = $W(h + \delta)$

Increase in potential energy of spring = $\frac{1}{2}k\delta^2$

$$W(h+\delta) = \frac{1}{2}k\delta^2$$

or, $\delta^2 - \frac{2W}{k}\delta - \frac{2W}{k}h = 0$. W = 30 lbf, k = 100 lbf/in, h = 2 in yields

$$\delta^2 - 0.6\,\delta - 1.2 = 0$$

Taking the positive root [see discussion after Eq. (*b*), Sec. 4-17]

$$\delta_{\max} = \frac{1}{2}\left[0.6+\sqrt{(-0.6)^2+4(1.2)}\right] = 1.436 \text{ in} \quad Ans.$$

$$F_{\max} = k\,\delta_{\max} = 100\,(1.436) = 143.6 \text{ lbf} \quad Ans.$$

4-136 The drop of weight W_1 converts potential energy, $W_1\,h$, to kinetic energy $\frac{1}{2}\frac{W_1}{g}v_1^2$.

Equating these provides the velocity of W_1 at impact with W_2.

$$W_1 h = \frac{1}{2}\frac{W_1}{g}v_1^2 \quad \Rightarrow \quad v_1 = \sqrt{2gh} \qquad (1)$$

Since the collision is inelastic, momentum is conserved. That is, $(m_1 + m_2)\, v_2 = m_1\, v_1$, where v_2 is the velocity of $W_1 + W_2$ after impact. Thus

$$\frac{W_1 + W_2}{g} v_2 = \frac{W_1}{g} v_1 \quad \Rightarrow \quad v_2 = \frac{W_1}{W_1 + W_2} v_1 = \frac{W_1}{W_1 + W_2}\sqrt{2gh} \qquad (2)$$

The kinetic and potential energies of $W_1 + W_2$ are then converted to potential energy of the spring. Thus,

$$\frac{1}{2}\frac{W_1 + W_2}{g} v_2^2 + \left(W_1 + W_2\right)\delta = \frac{1}{2}k\delta^2$$

Substituting in Eq. (1) and rearranging results in

$$\delta^2 - 2\frac{W_1 + W_2}{k}\delta - 2\frac{W_1^2}{W_1 + W_2}\frac{h}{k} = 0 \qquad (3)$$

Solving for the positive root [see discussion after Eq. (*b*), Sec. 4-17]

$$\delta = \frac{1}{2}\left[2\frac{W_1 + W_2}{k} + \sqrt{4\left(\frac{W_1 + W_2}{k}\right)^2 + 8\frac{W_1^2}{W_1 + W_2}\frac{h}{k}}\right] \qquad (4)$$

$W_1 = 40$ N, $W_2 = 400$ N, $h = 200$ mm, $k = 32$ kN/m $= 32$ N/mm.

$$\delta = \frac{1}{2}\left[2\left(\frac{40 + 400}{32}\right) + \sqrt{4\left(\frac{40 + 400}{32}\right)^2 + 8\frac{40^2}{40 + 400}\frac{200}{32}}\right] = 29.06 \text{ mm} \qquad \textit{Ans.}$$

$F_{\max} = k\delta = 32(29.06) = 930$ N *Ans.*

4-137 The initial potential energy of the k_1 spring is $V_i = \frac{1}{2}k_1 a^2$. The movement of the weight W the distance y gives a final potential of $V_f = \frac{1}{2}k_1\left(a - y\right)^2 + \frac{1}{2}k_2 y^2$. Equating the two energies give

$$\frac{1}{2}k_1 a^2 = \frac{1}{2}k_1\left(a - y\right)^2 + \frac{1}{2}k_2 y^2$$

Simplifying gives

$$\left(k_1 + k_2\right) y^2 - 2ak_1 y = 0$$

This has two roots, $y = 0, \dfrac{2k_1 a}{k_1 + k_2}$. Without damping the weight will vibrate between these two limits. The maximum displacement is thus $y_{\max} = \dfrac{2k_1 a}{k_1 + k_2}$ *Ans*.

With $W = 5$ lbf, $k_1 = 10$ lbf/in, $k_2 = 20$ lbf/in, and $a = 0.25$ in

$$y_{\max} = \frac{2(0.25)10}{10+20} = 0.1667 \text{ in} \qquad \textit{Ans.}$$

Chapter 5

5-1 $S_y = 350$ MPa.

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$

$$n = \frac{S_y}{\sigma'}$$

(a) MSS: $\sigma_1 = 100$ MPa, $\sigma_2 = 100$ MPa, $\sigma_3 = 0$

$$n = \frac{350}{100-0} = 3.5 \quad \textit{Ans.}$$

DE: $\sigma' = (100^2 - 100(100) + 100^2)^{1/2} = 100 \text{ MPa}, \quad n = \dfrac{350}{100} = 3.5 \quad \textit{Ans.}$

(b) MSS: $\sigma_1 = 100$ MPa, $\sigma_2 = 50$ MPa, $\sigma_3 = 0$

$$n = \frac{350}{100-0} = 3.5 \quad \textit{Ans.}$$

DE: $\sigma' = (100^2 - 100(50) + 50^2)^{1/2} = 86.6 \text{ MPa}, \quad n = \dfrac{350}{86.6} = 4.04 \quad \textit{Ans.}$

(c) $\sigma_A,\ \sigma_B = \dfrac{100}{2} \pm \sqrt{\left(\dfrac{100}{2}\right)^2 + (-75)^2} = 140,\ -40 \text{ MPa}$

$$\sigma_1 = 140,\ \sigma_2 = 0,\ \sigma_3 = -40 \text{ MPa}$$

MSS: $n = \dfrac{350}{140-(-40)} = 1.94 \quad \textit{Ans.}$

DE: $\sigma' = \left[100^2 + 3\left(-75^2\right)\right]^{1/2} = 164 \text{ MPa}, \quad n = \dfrac{350}{164} = 2.13 \quad \textit{Ans.}$

(d) $\sigma_A, \sigma_B = \dfrac{-50-75}{2} \pm \sqrt{\left(\dfrac{-50+75}{2}\right)^2 + (-50)^2} = -11.0, -114.0 \text{ MPa}$

$$\sigma_1 = 0,\ \sigma_2 = -11.0,\ \sigma_3 = -114.0 \text{ MPa}$$

MSS: $n = \dfrac{350}{0-(-114.0)} = 3.07 \quad \textit{Ans.}$

DE: $\sigma' = [(-50)^2 - (-50)(-75) + (-75)^2 + 3(-50)^2]^{1/2} = 109.0 \text{ MPa}$

$$n = \frac{350}{109.0} = 3.21 \quad \textit{Ans.}$$

(e) $\sigma_A, \sigma_B = \dfrac{100+20}{2} \pm \sqrt{\left(\dfrac{100-20}{2}\right)^2 + \left(-20\right)^2} = 104.7,\ 15.3 \text{ MPa}$

$\sigma_1 = 104.7,\ \sigma_2 = 15.3,\ \sigma_3 = 0$ MPa

MSS: $n = \dfrac{350}{104.7 - 0} = 3.34$ *Ans.*

DE: $\sigma' = \left[100^2 - 100(20) + 20^2 + 3(-20)^2\right]^{1/2} = 98.0$ MPa

$n = \dfrac{350}{98.0} = 3.57$ *Ans.*

5-2 $S_y = 350$ MPa.

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \dfrac{S_y}{n} \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma'}$

(a) MSS: $\sigma_1 = 100$ MPa, $\sigma_3 = 0 \ \Rightarrow \ n = \dfrac{350}{100 - 0} = 3.5$ *Ans.*

DE: $n = \dfrac{350}{[100^2 - (100)(100) + 100^2]^{1/2}} = 3.5$ *Ans.*

(b) MSS: $\sigma_1 = 100$, $\sigma_3 = -100$ MPa $\Rightarrow \ n = \dfrac{350}{100 - (-100)} = 1.75$ *Ans.*

DE: $n = \dfrac{350}{\left[100^2 - (100)(-100) + (-100)^2\right]^{1/2}} = 2.02$ *Ans.*

(c) MSS: $\sigma_1 = 100$ MPa, $\sigma_3 = 0 \ \Rightarrow \ n = \dfrac{350}{100 - 0} = 3.5$ *Ans.*

DE: $n = \dfrac{350}{\left[100^2 - (100)(50) + 50^2\right]^{1/2}} = 4.04$ *Ans.*

(d) MSS: $\sigma_1 = 100,\ \sigma_3 = -50$ MPa $\Rightarrow \ n = \dfrac{350}{100 - (-50)} = 2.33$ *Ans.*

DE: $n = \dfrac{350}{\left[100^2 - (100)(-50) + (-50)^2\right]^{1/2}} = 2.65$ *Ans.*

(e) MSS: $\sigma_1 = 0,\ \sigma_3 = -100$ MPa $\Rightarrow \ n = \dfrac{350}{0 - (-100)} = 3.5$ *Ans.*

DE: $n = \dfrac{350}{\left[(-50)^2 - (-50)(-100) + (-100)^2\right]^{1/2}} = 4.04$ *Ans.*

5-3 From Table A-20, $S_y = 37.5$ kpsi

MSS: $\sigma_1 - \sigma_3 = S_y / n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$

$$n = \frac{S_y}{\sigma'}$$

(a) MSS: $\sigma_1 = 25\text{ kpsi},\ \sigma_3 = 0 \quad \Rightarrow \quad n = \dfrac{37.5}{25-0} = 1.5$ *Ans.*

DE: $n = \dfrac{37.5}{\left[25^2 - (25)(15) + 15^2\right]^{1/2}} = 1.72$ *Ans.*

(b) MSS: $\sigma_1 = 15\text{ kpsi},\ \sigma_3 = -15 \quad \Rightarrow \quad n = \dfrac{37.5}{15-(-15)} = 1.25$ *Ans.*

DE: $n = \dfrac{37.5}{\left[15^2 - (15)(-15) + (-15)^2\right]^{1/2}} = 1.44$ *Ans.*

(c) $\sigma_A,\ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2}\right)^2 + (-10)^2} = 24.1,\ -4.1\text{ kpsi}$

$\sigma_1 = 24.1,\ \sigma_2 = 0,\ \sigma_3 = -4.1\text{ kpsi}$

MSS: $n = \dfrac{37.5}{24.1-(-4.1)} = 1.33$ *Ans.*

DE: $\sigma' = \left[20^2 + 3\left(-10^2\right)\right]^{1/2} = 26.5\text{ kpsi} \quad \Rightarrow \quad n = \dfrac{37.5}{26.5} = 1.42$ *Ans.*

(d) $\sigma_A, \sigma_B = \dfrac{-12+15}{2} \pm \sqrt{\left(\dfrac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7\text{ kpsi}$

$\sigma_1 = 17.7,\ \sigma_2 = 0,\ \sigma_3 = -14.7\text{ kpsi}$

MSS: $n = \dfrac{37.5}{17.7-(-14.7)} = 1.16$ *Ans.*

DE: $\sigma' = \left[(-12)^2 - (-12)(15) + 15^2 + 3(-9)^2\right]^{1/2} = 28.1\text{ kpsi}$

$n = \dfrac{37.5}{28.1} = 1.33$ *Ans.*

(e) $\sigma_A, \sigma_B = \dfrac{-24-24}{2} \pm \sqrt{\left(\dfrac{-24+24}{2}\right)^2 + (-15)^2} = -9,\ -39\text{ kpsi}$

$\sigma_1 = 0,\ \sigma_2 = -9,\ \sigma_3 = -39\text{ kpsi}$

MSS: $n = \dfrac{37.5}{0-(-39)} = 0.96$ *Ans.*

DE: $$\sigma' = \left[\left(-24\right)^2 - \left(-24\right)\left(-24\right) + \left(-24\right)^2 + 3\left(-15\right)^2\right]^{1/2} = 35.4 \text{ kpsi}$$

$$n = \frac{37.5}{35.4} = 1.06 \quad \textit{Ans.}$$

5-4 From Table A-20, $S_y = 47$ kpsi.

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \dfrac{S_y}{n} \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma'}$

(a) MSS: $\sigma_1 = 30 \text{ kpsi}, \ \sigma_3 = 0 \quad \Rightarrow \quad n = \dfrac{47}{30-0} = 1.57 \quad \textit{Ans.}$

DE: $$n = \frac{47}{[30^2 - (30)(30) + 30^2]^{1/2}} = 1.57 \quad \textit{Ans.}$$

(b) MSS: $\sigma_1 = 30\ , \ \sigma_3 = -30 \text{ kpsi} \quad \Rightarrow \quad n = \dfrac{47}{30-(-30)} = 0.78 \quad \textit{Ans.}$

DE: $$n = \frac{47}{\left[30^2 - (30)(-30) + \left(-30\right)^2\right]^{1/2}} = 0.90 \quad \textit{Ans.}$$

(c) MSS: $\sigma_1 = 30 \text{ kpsi}, \ \sigma_3 = 0 \quad \Rightarrow \quad n = \dfrac{47}{30-0} = 1.57 \quad \textit{Ans.}$

DE: $$n = \frac{47}{\left[30^2 - (30)(15) + 15^2\right]^{1/2}} = 1.81 \quad \textit{Ans.}$$

(d) MSS: $\sigma_1 = 0, \ \sigma_3 = -30 \text{ kpsi} \quad \Rightarrow \quad n = \dfrac{47}{0-(-30)} = 1.57 \quad \textit{Ans.}$

DE: $$n = \frac{47}{\left[\left(-30\right)^2 - (-30)(-15) + \left(-15\right)^2\right]^{1/2}} = 1.81 \quad \textit{Ans.}$$

(e) MSS: $\sigma_1 = 10, \ \sigma_3 = -50 \text{ kpsi} \quad \Rightarrow \quad n = \dfrac{47}{10-(-50)} = 0.78 \quad \textit{Ans.}$

DE: $$n = \frac{47}{\left[\left(-50\right)^2 - (-50)(10) + 10^2\right]^{1/2}} = 0.84 \quad \textit{Ans.}$$

5-5 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

(**a**) MSS and DE:

$$n = \frac{OB}{OA} = \frac{4.95''}{1.41''} = 3.51 \quad Ans.$$

(**b**) MSS:

$$n = \frac{OD}{OC} = \frac{3.91''}{1.12''} = 3.49 \quad Ans.$$

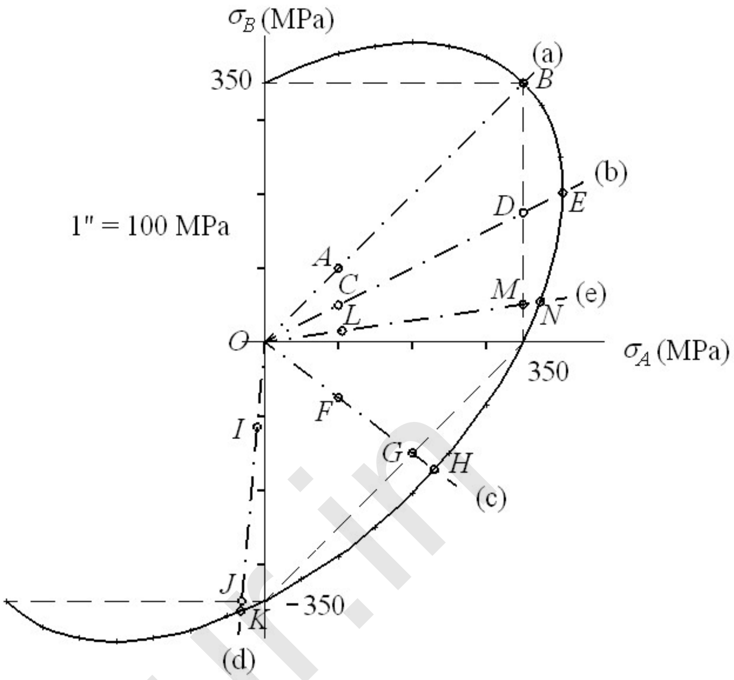
DE:

$$n = \frac{OE}{OC} = \frac{4.51''}{1.12''} = 4.03 \quad Ans.$$

(**c**) MSS:

$$n = \frac{OG}{OF} = \frac{2.50''}{1.25''} = 2.00 \quad Ans.$$

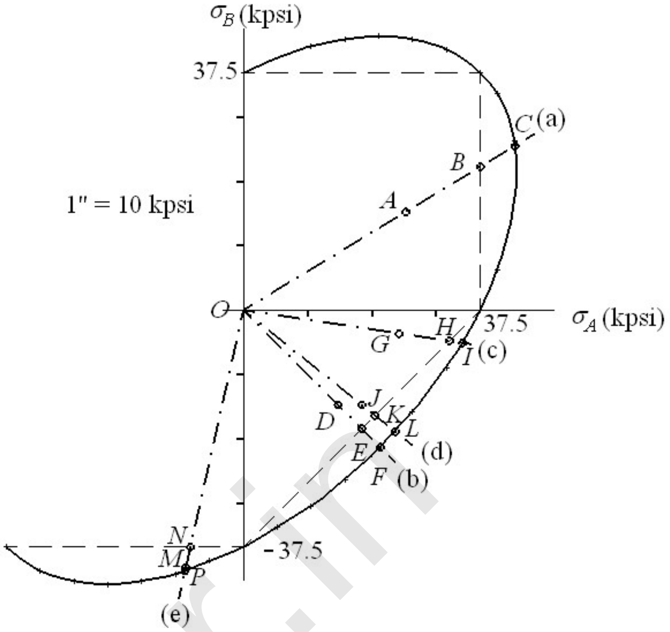
DE: $n = \dfrac{OH}{OF} = \dfrac{2.86''}{1.25''} = 2.29 \quad Ans.$



(**d**) MSS: $n = \dfrac{OJ}{OI} = \dfrac{3.51''}{1.15''} = 3.05 \quad Ans.$, DE: $n = \dfrac{OK}{OI} = \dfrac{3.65''}{1.15''} = 3.17 \quad Ans.$

(**e**) MSS: $n = \dfrac{OM}{OL} = \dfrac{3.54''}{1.06''} = 3.34 \quad Ans.$, DE: $n = \dfrac{ON}{OL} = \dfrac{3.77''}{1.06''} = 3.56 \quad Ans.$

5-6 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.



(**a**) $\sigma_A = 25$ kpsi, $\sigma_B = 15$ kpsi

MSS:

$$n = \frac{OB}{OA} = \frac{4.37''}{2.92''} = 1.50 \quad Ans.$$

DE:

$$n = \frac{OC}{OA} = \frac{5.02''}{2.92''} = 1.72 \quad Ans.$$

(**b**) $\sigma_A = 15$ kpsi, $\sigma_B = -15$ kpsi

MSS:

$$n = \frac{OE}{OD} = \frac{2.66''}{2.12''} = 1.25 \quad Ans.$$

DE:

$$n = \frac{OF}{OD} = \frac{3.05''}{2.12''} = 1.44 \quad Ans.$$

(**c**) $$\sigma_A,\ \sigma_B = \frac{20}{2} \pm \sqrt{\left(\frac{20}{2}\right)^2 + (-10)^2} = 24.1,\ -4.1 \text{ kpsi}$$

MSS: $n = \dfrac{OH}{OG} = \dfrac{3.25''}{2.43''} = 1.34$ *Ans.* DE: $n = \dfrac{OI}{OG} = \dfrac{3.46''}{2.43''} = 1.42$ *Ans.*

(**d**) $$\sigma_A, \sigma_B = \frac{-12+15}{2} \pm \sqrt{\left(\frac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7 \text{ MPa}$$

MSS: $n = \dfrac{OK}{OJ} = \dfrac{2.67''}{2.30''} = 1.16$ *Ans.* DE: $n = \dfrac{OL}{OJ} = \dfrac{3.06''}{2.30''} = 1.33$ *Ans.*

(**e**) $$\sigma_A, \sigma_B = \frac{-24-24}{2} \pm \sqrt{\left(\frac{-24+24}{2}\right)^2 + \left(-15\right)^2} = -9,\ -39 \text{ kpsi}$$

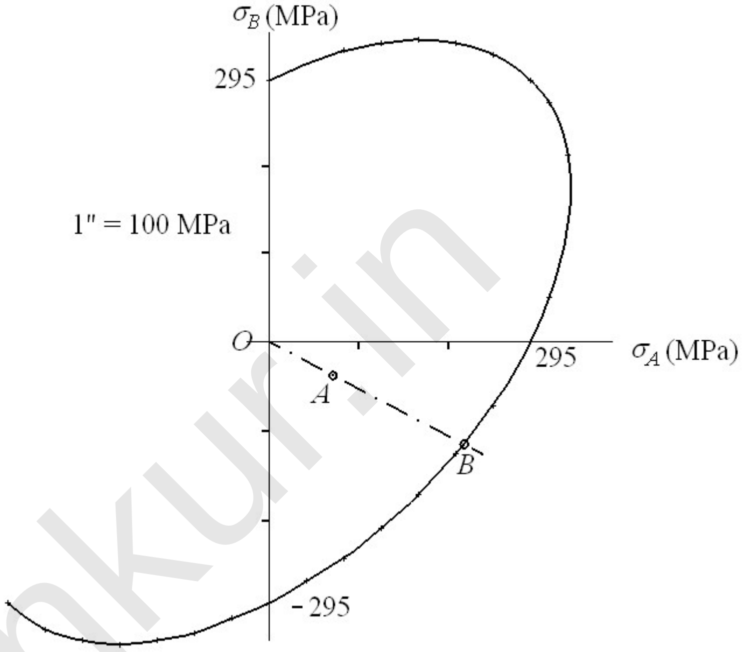
MSS: $n = \dfrac{ON}{OM} = \dfrac{3.85''}{4.00''} = 0.96$ *Ans.* DE: $n = \dfrac{OP}{OM} = \dfrac{4.23''}{4.00''} = 1.06$ *Ans.*

5-7 $S_y = 295$ MPa, $\sigma_A = 75$ MPa, $\sigma_B = -35$ MPa,

(a) $$n = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{295}{\left[75^2 - 75(-35) + (-35)^2\right]^{1/2}} = 3.03 \quad Ans.$$

(b) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.50''}{0.83''} = 3.01 \quad Ans.$$

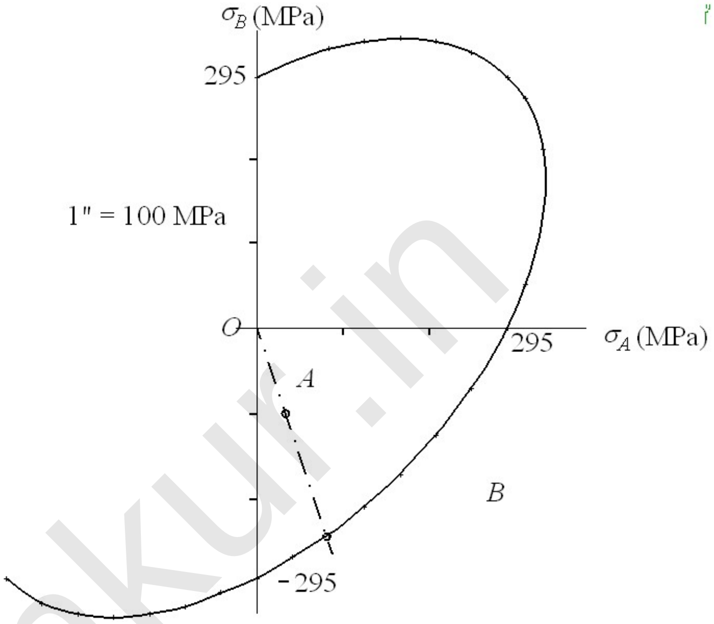


5-8 $S_y = 295$ MPa, $\sigma_A = 30$ MPa, $\sigma_B = -100$ MPa,

(a) $$n = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{295}{\left[30^2 - 30(-100) + (-100)^2\right]^{1/2}} = 2.50 \quad Ans.$$

(b) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.50''}{0.83''} = 3.01 \quad Ans.$$

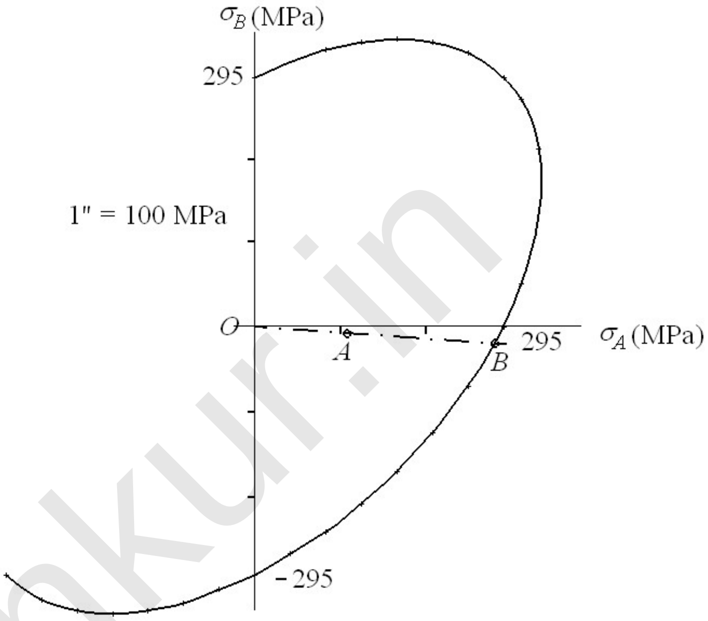


5-9 $S_y = 295\text{ MPa},\ \sigma_A,\ \sigma_B = \dfrac{100}{2} \pm \sqrt{\left(\dfrac{100}{2}\right)^2 + (-25)^2} = 105.9,\ -5.9\text{ MPa}$

(a) $$n = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{295}{\left[105.9^2 - 105.9(-5.9) + (-5.9)^2\right]^{1/2}} = 2.71 \quad Ans.$$

(b) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.87''}{1.06''} = 2.71 \quad Ans.$$

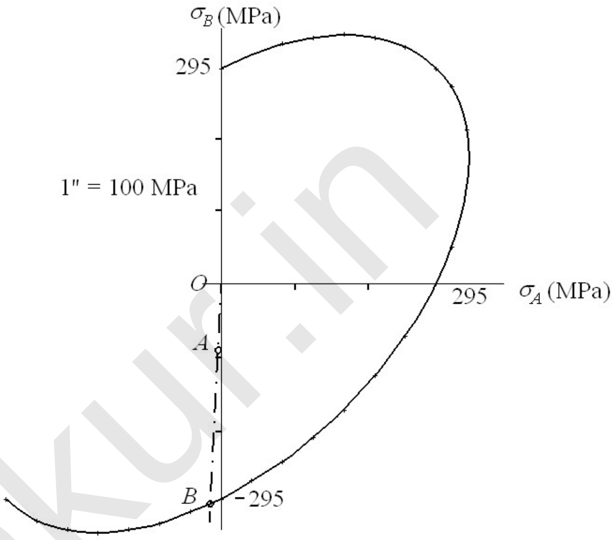


5-10 $S_y = 295$ MPa, $\sigma_A, \sigma_B = \dfrac{-30-65}{2} \pm \sqrt{\left(\dfrac{-30+65}{2}\right)^2 + 40^2} = -3.8, -91.2$ MPa

(a) $$n = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{295}{\left[(-3.8)^2 - (-3.8)(-91.2) + (-91.2)^2\right]^{1/2}} = 3.30 \quad Ans.$$

(**b**) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{3.00''}{0.90''} = 3.33 \quad Ans.$$

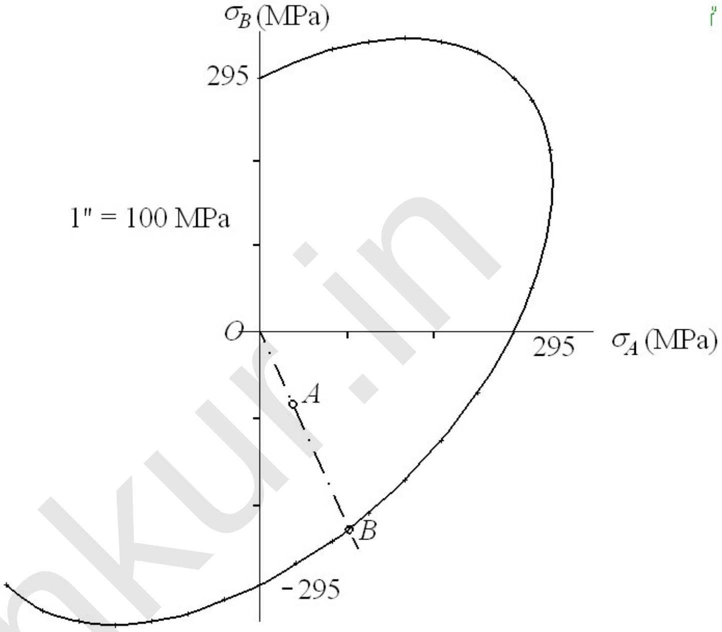


5-11 $S_y = 295$ MPa, $\sigma_A, \sigma_B = \dfrac{-80+30}{2} \pm \sqrt{\left(\dfrac{-80-30}{2}\right)^2 + (-10)^2} = 30.9, -80.9$ MPa

(a) $$n = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{295}{\left[30.9^2 - 30.9(-80.9) + (-80.9)^2\right]^{1/2}} = 2.95 \quad Ans.$$

(b) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.55''}{0.87''} = 2.93 \quad Ans.$$



5-12 $S_{yt} = 60$ kpsi, $S_{yc} = 75$ kpsi. Eq. (5-26) for yield is

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1}$$

(a) $\sigma_1 = 25$ kpsi, $\sigma_3 = 0 \quad \Rightarrow \quad n = \left(\dfrac{25}{60} - \dfrac{0}{75}\right)^{-1} = 2.40 \quad Ans.$

(b) $\sigma_1 = 15,\ \sigma_3 = -15$ kpsi $\quad \Rightarrow \quad n = \left(\dfrac{15}{60} - \dfrac{-15}{75}\right)^{-1} = 2.22 \quad Ans.$

(c) $\sigma_A,\ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2}\right)^2 + (-10)^2} = 24.1,\ -4.1 \text{ kpsi}$,

$\sigma_1 = 24.1,\ \sigma_2 = 0,\ \sigma_3 = -4.1 \text{ kpsi} \quad \Rightarrow \quad n = \left(\dfrac{24.1}{60} - \dfrac{-4.1}{75}\right)^{-1} = 2.19 \quad$ *Ans.*

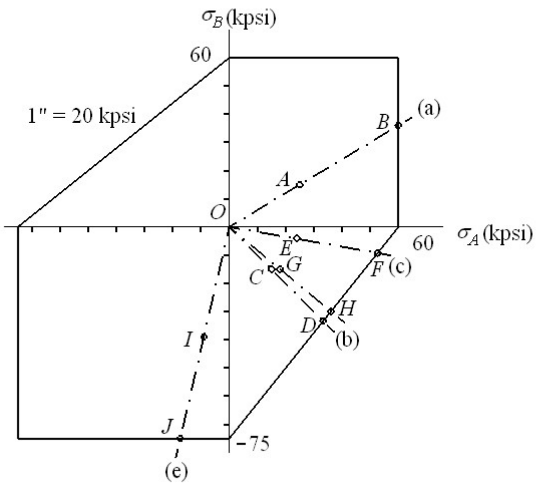
(d) $\sigma_A, \sigma_B = \dfrac{-12+15}{2} \pm \sqrt{\left(\dfrac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7 \text{ kpsi}$

$\sigma_1 = 17.7,\ \sigma_2 = 0,\ \sigma_3 = -14.7 \text{ kpsi} \quad \Rightarrow \quad n = \left(\dfrac{17.7}{60} - \dfrac{-14.7}{75}\right)^{-1} = 2.04 \quad$ *Ans.*

(e) $\sigma_A, \sigma_B = \dfrac{-24-24}{2} \pm \sqrt{\left(\dfrac{-24+24}{2}\right)^2 + \left(-15\right)^2} = -9,\ -39 \text{ kpsi}$

$\sigma_1 = 0,\ \sigma_2 = -9,\ \sigma_3 = -39 \text{ kpsi} \quad \Rightarrow \quad n = \left(\dfrac{0}{60} - \dfrac{-39}{75}\right)^{-1} = 1.92 \quad$ *Ans.*

5-13 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.



(a) $\sigma_A = 25,\ \sigma_B = 15 \text{ kpsi}$

$n = \dfrac{OB}{OA} = \dfrac{3.49"}{1.46"} = 2.39 \quad$ *Ans.*

(b) $\sigma_A = 15,\ \sigma_B = -15 \text{ kpsi}$

$n = \dfrac{OD}{OC} = \dfrac{2.36"}{1.06"} = 2.23 \quad$ *Ans.*

(c)

$\sigma_A,\ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2}\right)^2 + (-10)^2} = 24.1,\ -4.1 \text{ kpsi}$

$n = \dfrac{OF}{OE} = \dfrac{2.67"}{1.22"} = 2.19 \quad$ *Ans.*

(d)

$\sigma_A, \sigma_B = \dfrac{-12+15}{2} \pm \sqrt{\left(\dfrac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7 \text{ kpsi}$

$$n = \frac{OH}{OG} = \frac{2.34"}{1.15"} = 2.03 \quad Ans.$$

(e)

$$\sigma_A, \sigma_B = \frac{-24-24}{2} \pm \sqrt{\left(\frac{-24+24}{2}\right)^2 + (-15)^2} = -9,\ -39\ \text{kpsi}$$

$$n = \frac{OJ}{OI} = \frac{3.85"}{2.00"} = 1.93 \quad Ans.$$

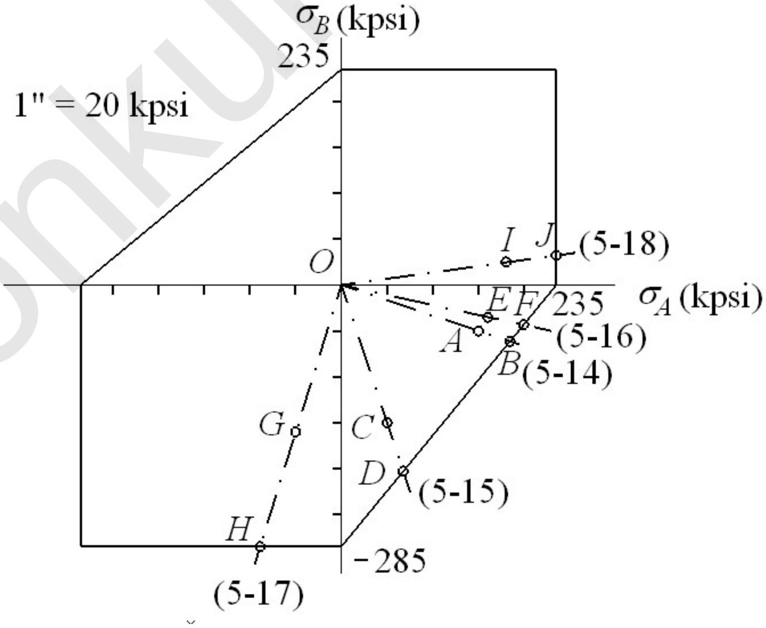
5-14 Since $\varepsilon_f > 0.05$, and $S_{yt} \neq S_{yc}$, the Coulomb-Mohr theory for ductile materials will be used.

(**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1} = \left(\frac{150}{235} - \frac{-50}{285}\right)^{-1} = 1.23 \quad Ans.$$

(**b**) Plots for Problems 5-14 to 5-18 are found here. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{1.94"}{1.58"} = 1.23 \quad Ans.$$



5-15 (**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1} = \left(\frac{50}{235} - \frac{-150}{285}\right)^{-1} = 1.35 \quad Ans.$$

(**b**) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OD}{OC} = \frac{2.14"}{1.58"} = 1.35 \quad Ans.$$

5-16 $$\sigma_A,\ \sigma_B = \frac{125}{2} \pm \sqrt{\left(\frac{125}{2}\right)^2 + (-75)^2} = 160,\ -35 \text{ kpsi}$$

(**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1} = \left(\frac{160}{235} - \frac{-35}{285}\right)^{-1} = 1.24 \quad Ans.$$

(**b**) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OF}{OE} = \frac{2.04"}{1.64"} = 1.24 \quad Ans.$$

5-17 $$\sigma_A,\ \sigma_B = \frac{-80-125}{2} \pm \sqrt{\left(\frac{-80+125}{2}\right)^2 + 50^2} = -47.7,\ -157.3 \text{ kpsi}$$

(**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1} = \left(\frac{0}{235} - \frac{-157.3}{285}\right)^{-1} = 1.81 \quad Ans.$$

(**b**) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OH}{OG} = \frac{2.99"}{1.64"} = 1.82 \quad Ans.$$

5-18 $$\sigma_A,\ \sigma_B = \frac{125+80}{2} \pm \sqrt{\left(\frac{125-80}{2}\right)^2 + (-75)^2} = 180.8,\ 24.2 \text{ kpsi}$$

(**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} \right)^{-1} = \left(\frac{180.8}{235} - \frac{0}{285} \right)^{-1} = 1.30 \quad Ans.$$

(**b**) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OJ}{OI} = \frac{2.37"}{1.83"} = 1.30 \quad Ans.$$

5-19 S_{ut} = 30 kpsi, S_{uc} = 90 kpsi

BCM: Eqs. (5-31), MM: Eqs. (5-32)

(**a**) σ_A = 25 kpsi, σ_B = 15 kpsi

$$\text{BCM : Eq. (5-31}a\text{)}, \; n = \frac{S_{ut}}{\sigma_A} = \frac{30}{25} = 1.2 \quad Ans.$$

$$\text{MM: Eq. (5-32}a\text{)}, \; n = \frac{S_{ut}}{\sigma_A} = \frac{30}{25} = 1.2 \quad Ans.$$

(**b**) σ_A = 15 kpsi, σ_B = −15 kpsi,

$$\text{BCM: Eq. (5-31}a\text{)}, \; n = \left(\frac{15}{30} - \frac{-15}{90} \right)^{-1} = 1.5 \quad Ans.$$

MM: $\sigma_A \geq 0 \geq \sigma_B$, and $|\sigma_B / \sigma_A| \leq 1$, Eq. (5-32a), $n = \dfrac{S_{ut}}{\sigma_A} = \dfrac{30}{15} = 2.0 \quad Ans.$

(**c**) $\sigma_A, \; \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2} \right)^2 + (-10)^2} = 24.14, \; -4.14$ kpsi

$$\text{BCM: Eq. (5-31}b\text{)}, \; n = \left(\frac{24.14}{30} - \frac{-4.14}{90} \right)^{-1} = 1.18 \quad Ans.$$

MM: $\sigma_A \geq 0 \geq \sigma_B$, and $|\sigma_B / \sigma_A| \leq 1$, Eq. (5-32$a$), $n = \dfrac{S_{ut}}{\sigma_A} = \dfrac{30}{24.14} = 1.24 \quad Ans.$

(**d**) $\sigma_A, \; \sigma_B = \dfrac{-15+10}{2} \pm \sqrt{\left(\dfrac{-15-10}{2} \right)^2 + (-15)^2} = 17.03, \; -22.03$ kpsi

BCM: Eq. (5-31*b*), $n = \left(\dfrac{17.03}{30} - \dfrac{-22.03}{90} \right)^{-1} = 1.23$ *Ans.*

MM: $\sigma_A \geq 0 \geq \sigma_B$, and $|\sigma_B / \sigma_A| \geq 1$, Eq. (5-32*b*),

$$n = \left[\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc} S_{ut}} - \frac{\sigma_B}{S_{uc}} \right]^{-1} = \left[\frac{(90-30)17.03}{90(30)} - \frac{-22.03}{90} \right]^{-1} = 1.60 \quad Ans.$$

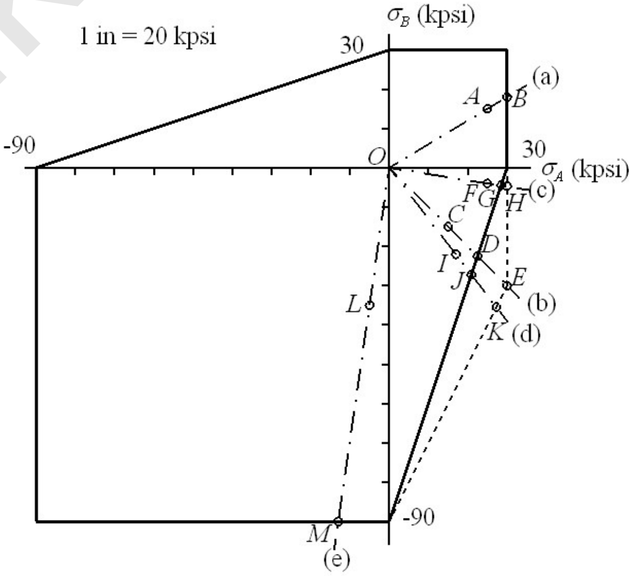
(e) $\sigma_A,\ \sigma_B = \dfrac{-20-20}{2} \pm \sqrt{\left(\dfrac{-20+20}{2} \right)^2 + (-15)^2} = -5,\ -35$ kpsi

BCM: Eq. (5-31*c*), $n = -\dfrac{S_{uc}}{\sigma_B} = -\dfrac{90}{-35} = 2.57$ *Ans.*

MM: Eq. (5-32*c*), $n = -\dfrac{S_{uc}}{\sigma_B} = -\dfrac{90}{-35} = 2.57$ *Ans.*

5-20 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

(a) $\sigma_A = 25$, $\sigma_B = 15$ kpsi



BCM & MM:

$$n = \frac{OB}{OA} = \frac{1.74''}{1.46''} = 1.19 \quad Ans.$$

(b) $\sigma_A = 15$, $\sigma_B = -15$ kpsi

BCM: $n = \dfrac{OC}{OD} = \dfrac{1.59''}{1.06''} = 1.5$ *Ans.*

MM: $n = \dfrac{OE}{OC} = \dfrac{2.12''}{1.06''} = 2.0$ *Ans.*

(c) $\sigma_A,\ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2} \right)^2 + (-10)^2}$

$= 24.14,\ -4.14$ kpsi

BCM: $n = \dfrac{OG}{OF} = \dfrac{1.44''}{1.22''} = 1.18$ *Ans.*

MM: $n = \dfrac{OH}{OF} = \dfrac{1.52''}{1.22''} = 1.25$ *Ans.*

(d) $$\sigma_A,\ \sigma_B = \frac{-15+10}{2} \pm \sqrt{\left(\frac{-15-10}{2}\right)^2 + (-15)^2} = 17.03,\ -22.03 \text{ kpsi}$$

BCM: $n = \dfrac{OJ}{OI} = \dfrac{1.72"}{1.39"} = 1.24$ *Ans.*

MM: $n = \dfrac{OK}{OI} = \dfrac{2.24"}{1.39"} = 1.61$ *Ans.*

(e) $$\sigma_A,\ \sigma_B = \frac{-20-20}{2} \pm \sqrt{\left(\frac{-20+20}{2}\right)^2 + (-15)^2} = -5,\ -35 \text{ kpsi}$$

BCM and MM: $n = \dfrac{OM}{OL} = \dfrac{4.55"}{1.77"} = 2.57$ *Ans.*

5-21 From Table A-24, S_{ut} = 31 kpsi, S_{uc} = 109 kpsi

BCM: Eqs. (5-31), MM: Eqs. (5-32)

(a) σ_A = 15, σ_B = 10 kpsi.

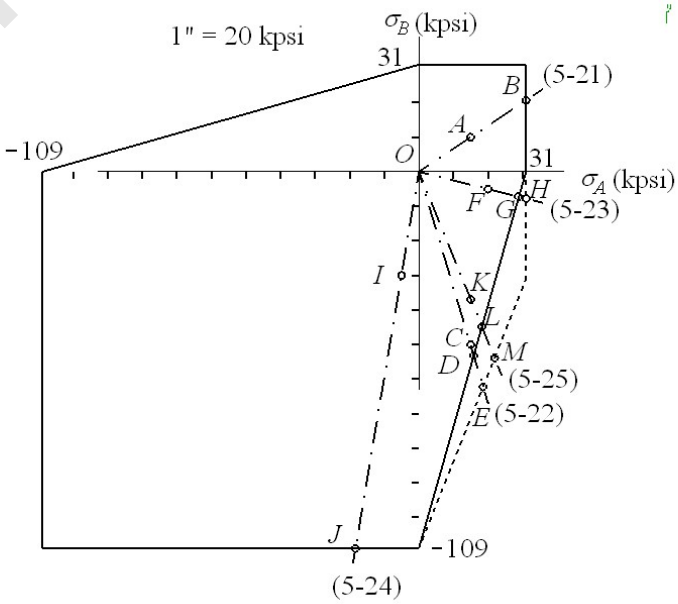
BCM: Eq. (5-31*a*), $n = \dfrac{S_{ut}}{\sigma_A} = \dfrac{31}{15} = 2.07$ *Ans.*

MM: Eq. (5-32*a*), $n = \dfrac{S_{ut}}{\sigma_A} = \dfrac{31}{15} = 2.07$ *Ans.*

(b), (c) The plot is shown below is for Probs. 5-21 to 5-25. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

BCM and MM:

$n = \dfrac{OB}{OA} = \dfrac{1.86"}{0.90"} = 2.07$ *Ans.*



5-22 $S_{ut} = 31$ kpsi, $S_{uc} = 109$ kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

(**a**) $\sigma_A = 15$, $\sigma_B = -50$ kpsi, $|\sigma_B / \sigma_A| > 1$

$$\text{BCM: Eq. (5-31}b\text{)},\ n = \left(\frac{15}{31} - \frac{-50}{109}\right)^{-1} = 1.06 \quad Ans.$$

$$\text{MM: Eq. (5-32}b\text{)},\ n = \left[\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}}\right]^{-1} = \left[\frac{(109-31)15}{109(31)} - \frac{-50}{109}\right]^{-1} = 1.24 \quad Ans.$$

(**b**), (**c**) The plot is shown in the solution to Prob. 5-21.

$$\text{BCM: } n = \frac{OD}{OC} = \frac{2.78"}{2.61"} = 1.07 \quad Ans.$$

$$\text{MM: } n = \frac{OE}{OC} = \frac{3.25"}{2.61"} = 1.25 \quad Ans.$$

5-23 From Table A-24, $S_{ut} = 31$ kpsi, $S_{uc} = 109$ kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

$$\sigma_A,\ \sigma_B = \frac{15}{2} \pm \sqrt{\left(\frac{15}{2}\right)^2 + (-10)^2} = 20,\ -5 \text{ kpsi}$$

$$(\mathbf{a})\ \text{BCM: Eq. (5-32}b\text{)},\ n = \left(\frac{20}{31} - \frac{-5}{109}\right)^{-1} = 1.45 \quad Ans.$$

$$\text{MM: Eq. (5-32}a\text{)},\quad n = \frac{S_{ut}}{\sigma_A} = \frac{31}{20} = 1.55 \quad Ans.$$

(**b**), (**c**) The plot is shown in the solution to Prob. 5-21.

$$\text{BCM: } n = \frac{OG}{OF} = \frac{1.48"}{1.03"} = 1.44 \quad Ans.$$

$$\text{MM: } n = \frac{OH}{OF} = \frac{1.60"}{1.03"} = 1.55 \quad Ans.$$

5-24 From Table A-24, $S_{ut} = 31$ kpsi, $S_{uc} = 109$ kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

$$\sigma_A,\ \sigma_B = \frac{-10-25}{2} \pm \sqrt{\left(\frac{-10+25}{2}\right)^2 + (-10)^2} = -5,\ -30 \text{ kpsi}$$

(**a**) BCM: Eq. (5-31*c*), $n = -\frac{S_{uc}}{\sigma_B} - \frac{109}{-30} = 3.63$ *Ans.*

MM: Eq. (5-32*c*), $n = -\frac{S_{uc}}{\sigma_B} = -\frac{109}{-30} = 3.63$ *Ans.*

(**b**), (**c**) The plot is shown in the solution to Prob. 5-21.

BCM and MM: $n = \frac{OJ}{OI} = \frac{5.53"}{1.52"} = 3.64$ *Ans.*

5-25 From Table A-24, S_{ut} = 31 kpsi, S_{uc} = 109 kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

$$\sigma_A,\ \sigma_B = \frac{-35+13}{2} \pm \sqrt{\left(\frac{-35-13}{2}\right)^2 + (-10)^2} = 15,\ -37 \text{ kpsi}$$

(**a**) BCM: Eq. (5-31*b*), $n = \left(\frac{15}{31} - \frac{-37}{109}\right)^{-1} = 1.21$ *Ans.*

MM: Eq. (5-32*b*), $n = \left[\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}}\right]^{-1} = \left[\frac{(109-31)15}{109(31)} - \frac{-37}{109}\right]^{-1} = 1.46$ *Ans.*

(**b**), (**c**) The plot is shown in the solution to Prob. 5-21.

BCM: $n = \frac{OL}{OK} = \frac{2.42"}{2.00"} = 1.21$ *Ans.*

MM: $n = \frac{OM}{OK} = \frac{2.91"}{2.00"} = 1.46$ *Ans.*

5-26 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

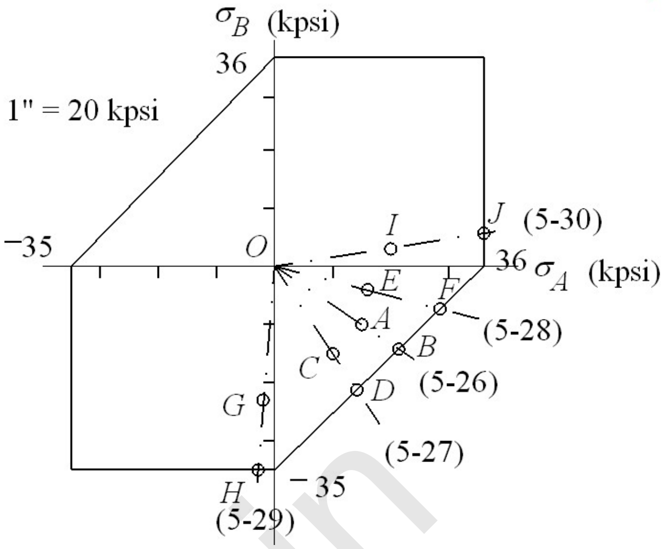
BCM: Eq. (5-31),

(**a**) σ_A = 15, σ_B = – 10 kpsi.

BCM: Eq. (5-31*b*), $n = \left(\frac{15}{36} - \frac{-10}{35}\right)^{-1} = 1.42$ *Ans.*

(**b**) The plot is shown below is for Probs. 5-26 to 5-30. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{1.28"}{0.90"} = 1.42 \quad Ans.$$



5-27 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

BCM: Eq. (5-31),

(**a**) σ_A = 15, σ_B = − 15 kpsi.

BCM: Eq. (5-31*b*), $n = \left(\frac{10}{36} - \frac{-15}{35}\right)^{-1} = 1.42 \quad Ans.$

(**b**) The plot is shown in the solution to Prob. 5-26.

$$n = \frac{OD}{OC} = \frac{1.28"}{0.90"} = 1.42 \quad Ans.$$

5-28 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

BCM: Eq. (5-31),

(**a**) $\sigma_A,\ \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + (-8)^2} = 16,\ -4$ kpsi

BCM: Eq. (5-31*b*), $n = \left(\frac{16}{36} - \frac{-4}{35}\right)^{-1} = 1.79 \quad Ans.$

(**b**) The plot is shown in the solution to Prob. 5-26.

$$n = \frac{OF}{OE} = \frac{1.47"}{0.82"} = 1.79 \quad Ans.$$

5-29 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

BCM: Eq. (5-31),

(a) $$\sigma_A,\ \sigma_B = \frac{-10-15}{2} \pm \sqrt{\left(\frac{-10+15}{2}\right)^2 + 10^2} = -2.2,\ -22.8 \text{ kpsi}$$

BCM: Eq. (5-31*c*), $n = -\dfrac{35}{-22.8} = 1.54 \quad Ans.$

(**b**) The plot is shown in the solution to Prob. 5-26.

$$n = \frac{OH}{OG} = \frac{1.76"}{1.15"} = 1.53 \quad Ans.$$

5-30 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

BCM: Eq. (5-31),

(a) $$\sigma_A,\ \sigma_B = \frac{15+8}{2} \pm \sqrt{\left(\frac{15-8}{2}\right)^2 + (-8)^2} = 20.2,\ 2.8 \text{ kpsi}$$

BCM: Eq. (5-31*a*), $n = \dfrac{36}{20.2} = 1.78 \quad Ans.$

(**b**) The plot is shown in the solution to Prob. 5-26.

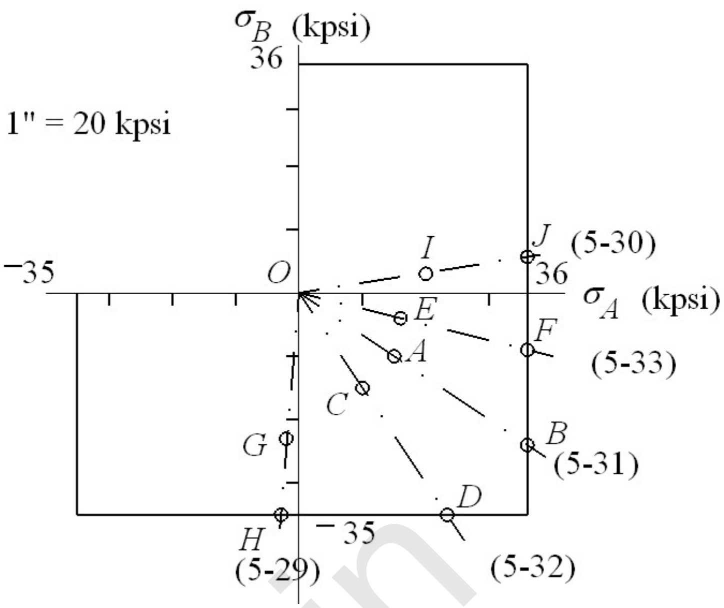
$$n = \frac{OJ}{OI} = \frac{1.82"}{1.02"} = 1.78 \quad Ans.$$

5-31 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32). For this problem, MM reduces to the MNS theory.

(**a**) σ_A = 15, σ_B = – 10 kpsi. Eq. (5-32*a*), $n = \dfrac{S_{ut}}{\sigma_A} = \dfrac{36}{15} = 2.4 \quad Ans.$

(b) The plot on the next page is for Probs. 5-31 to 5-35. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.16"}{0.90"} = 2.43 \quad Ans.$$



5-32 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32).For this problem, MM reduces to the MNS theory.

(a) σ_A = 10, σ_B = – 15 kpsi. Eq. (5-32b) is not valid and must use Eq, (5-32c),

$$n = -\frac{S_{uc}}{\sigma_B} = -\frac{35}{-15} = 2.33 \quad Ans.$$

(b) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OD}{OC} = \frac{2.10"}{0.90"} = 2.33 \quad Ans.$$

5-33 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32).For this problem, MM reduces to the MNS theory.

(a) $$\sigma_A,\ \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + (-8)^2} = 16,\ -4 \text{ kpsi}$$

$$n = \frac{S_{ut}}{\sigma_A} = \frac{36}{16} = 2.25 \quad Ans.$$

(b) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OF}{OE} = \frac{1.86"}{0.82"} = 2.27 \quad Ans.$$

5-34 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32).For this problem, MM reduces to the MNS theory.

(a) $\sigma_A,\ \sigma_B = \dfrac{-10-15}{2} \pm \sqrt{\left(\dfrac{-10+15}{2}\right)^2 + 10^2} = -2.2,\ \ -22.8 \text{ kpsi}$

$$n = -\frac{S_{uc}}{\sigma_B} = -\frac{35}{-22.8} = 1.54 \quad Ans.$$

(b) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OH}{OG} = \frac{1.76"}{1.15"} = 1.53 \quad Ans.$$

5-35 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32).For this problem, MM reduces to the MNS theory.

(a) $\sigma_A,\ \sigma_B = \dfrac{15+8}{2} \pm \sqrt{\left(\dfrac{15-8}{2}\right)^2 + (-8)^2} = 20.2,\ \ 2.8 \text{ kpsi}$

$$n = \frac{S_{ut}}{\sigma_A} = \frac{36}{20.2} = 1.78 \quad Ans.$$

(b) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OJ}{OI} = \frac{1.82"}{1.02"} = 1.78 \quad Ans.$$

5-36 Given: AISI 1006 CD steel, F = 0.55 kN, P = 4.0 kN, and T = 25 N·m. From Table A-20, S_y =280 MPa. Apply the DE theory to stress elements A and B

A: $$\sigma_x = \frac{4P}{\pi d^2} = \frac{4(4)10^3}{\pi(0.015^2)} = 22.6(10^6) \text{ Pa} = 22.6 \text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} - \frac{4}{3}\frac{V}{A} = \frac{16(25)}{\pi(0.015^3)} - \frac{4}{3}\left[\frac{0.55(10^3)}{(\pi/4)0.015^2}\right] = 33.6(10^6) \text{ Pa} = 33.6 \text{ MPa}$$

$$\sigma' = \left[22.6^2 + 3(33.6^2)\right]^{1/2} = 62.4 \text{ MPa}$$

$$n = \frac{280}{62.4} = 4.49 \qquad Ans.$$

B: $$\sigma_x = \frac{32Fl}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{32(0.55)10^3(0.1)}{\pi(0.015^3)} + \frac{4(4)10^3}{\pi(0.015^2)} = 189(10^6) \text{ Pa} = 189 \text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(25)}{\pi(0.015^3)} = 37.7(10^6) \text{ Pa} = 37.7 \text{ MPa}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = \left[189^2 + 3\left(37.7^2\right)\right]^{1/2} = 200\text{ MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{280}{200} = 1.4 \qquad \textit{Ans.}$$

5-37 From Prob. 3-45, the critical location is at the top of the beam at $x = 27$ in from the left end, where there is only a bending stress of $\sigma = -\ 7\,456$ psi. Thus, $\sigma' = 7\,456$ psi and

$(S_y)_{\min} = n\sigma' = 2(7\,456) = 14\,912$ psi

Choose $(S_y)_{\min} = 15$ kpsi *Ans.*

5-38 From Table A-20 for 1020 CD steel, $S_y = 57$ kpsi. From Eq. (3-42)

$$T = \frac{63\,025H}{n} \qquad (1)$$

where n is the shaft speed in rev/min. From Eq. (5-3), for the MSS theory,

$$\tau_{\max} = \frac{S_y}{2n_d} = \frac{16T}{\pi d^3} \qquad (2)$$

where n_d is the design factor. Substituting Eq. (1) into Eq. (2) and solving for d gives

$$d = \left[\frac{32(63\,025)Hn_d}{n\pi S_y}\right]^{1/3} \qquad (3)$$

Substituting $H = 20$ hp, $n_d = 3$, $n = 1750$ rev/min, and $S_y = 57(10^3)$ psi results in

$$d_{\min} = \left[\frac{32(63\,025)20(3)}{1750\pi(57)10^3}\right]^{1/3} = 0.728\text{ in} \qquad \textit{Ans.}$$

5-39 Given: $d = 30$ mm, AISI 1018 steel, $H = 10$ kW, $n = 200$ rev/min.
Table A-20, $S_y = 220$ MPa
Eq. (3-44): $T = 9.55\,H/n = 9.55(10)10^3/200 = 477.5$ N·m

$$\tau_{\max} = \frac{16T}{\pi d^3} = \frac{16(477.5)}{\pi(0.030)^3}10^{-6} = 90.07\text{ MPa}$$

(a) Eq. (5-3): $n = \dfrac{S_y}{2\tau_{\max}} = \dfrac{220}{2(90.07)} = 1.22$ *Ans.*

(b) From Eq. (5-13), $\sigma'_{\max} = \sqrt{3\tau^2_{\max}} = \sqrt{3}\left(90.07\right) = 156.0 \text{ MPa}$

Eq. (5-19): $n = \dfrac{S_y}{\sigma'_{\max}} = \dfrac{220}{156} = 1.41$ *Ans.*

5-40 Given: $d = 20$ mm, AISC 1035 HR steel, $n_s = 400$ rev/min, $n_y = 1.5$.
Table A-20, $S_y = 270$ MPa

(a) Eq. (5-30): $\tau_{\max} = \dfrac{S_y}{2n_y} = \dfrac{270}{2\left(1.5\right)} = 90 \text{ MPa}$

Substituting Eq. (3-44) in the equation for $\tau_{\max}$ gives

$$\tau_{\max} = \frac{16T}{\pi d^3} = \frac{16}{\pi d^3}\left(9.55\frac{H}{n_s}\right) \quad \Rightarrow \quad H = \frac{\pi d^3 n_s \tau_{\max}}{16\left(9.55\right)} = \frac{\pi\left(0.02^3\right)400\left(90\right)10^6}{16\left(9.55\right)}$$

$$= 5.92\left(10^3\right) \text{ W} = 5.92 \text{ kW} \qquad \textit{Ans.}$$

(b) From Eq. (5-13): $\sigma'_{\max} = \sqrt{3}\ \tau_{\max} = \dfrac{S_y}{n_y} = \dfrac{270}{1.5} \quad \Rightarrow \quad \tau_{\max} = \dfrac{270}{1.5\sqrt{3}} = 103.9 \text{ MPa}$

Thus,

$$H = \frac{\pi d^3 n_s \tau_{\max}}{16\left(9.55\right)} = \frac{\pi\left(0.02^3\right)400\left(103.9\right)10^6}{16\left(9.55\right)} = 6.84\left(10^3\right) \text{ W} = 6.84 \text{ kW} \qquad \textit{Ans}$$

5-41 Table A-20 for AISI 1040 CD steel, $S_y = 490$ MPa.
From Prob. 3-47,

A: $\sigma_x = 79.6$ MPa, $\tau_{xy} = 63.7$ MPa. $\tau_{\max} = \sqrt{\left(\dfrac{\sigma}{2}\right)^2 + \tau^2_{xy}} = \sqrt{\left(\dfrac{79.6}{2}\right)^2 + 63.7^2} = 75.1 \text{ MPa.}$

B: $\tau_{\max} = \tau_{zx} = 53.1$ MPa. *C:* $\tau_{\max} = \tau_{zx} = 116.8$ MPa.
Critical case is at point *C*.

(a) MSS Theory: $n_y = \dfrac{S_y}{2\tau_{\max}} = \dfrac{490}{2\left(116.8\right)} = 2.1$ *Ans.*

(b) DE Theory: $n = \dfrac{S_y}{\sqrt{3}\tau_{\max}} = \dfrac{490}{\sqrt{3}\left(116.8\right)} = 2.4$ *Ans.*

5-42 Table A-20 for AISI 1040 CD steel, $S_y = 490$ MPa
From Prob. 3-53, $\sigma_1 = 122.6$ MPa, $\sigma_2 = 0$, $\sigma_3 = -10.2$ MPa, $\tau_{\max} = R = 66.4$ MPa

(a) MSS Theory: $n_y = \dfrac{S_y}{2\tau_{\max}} = \dfrac{490}{2\left(66.4\right)} = 3.69$ *Ans.*

(b) DE Theory, Eq. (5-13): $\sigma' = \left[122.6^2 - 122.6\left(-10.2\right) + \left(-10.2\right)^2\right]^{1/2} = 128 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'} = \frac{490}{128} = 3.83 \qquad Ans.$$

5-43 Table A-20 for AISI 1020 CD steel, S_y = 390 MPa
From Prob. 3-54, $\sigma_1 = 194.2$ MPa, $\sigma_2 = 0$, $\sigma_3 = -10$ MPa, $\tau_{\max} = 102.1$ MPa

(a) MSS Theory: $n_y = \frac{S_y}{2\tau_{\max}} = \frac{490}{2(102.1)} = 1.91 \qquad Ans.$

(b) DE Theory, Eq. (5-13): $\sigma' = \left[194.2^2 - 194.2(-10) + (-10)^2\right]^{1/2} = 199.4 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'} = \frac{490}{194.2} = 1.96 \qquad Ans.$$

5-44 Table A-20 for AISI 1035 CD steel, S_y = 67 kpsi
From Prob. 3-55, $\sigma_1 = 45.8$ kpsi, $\sigma_2 = 0$, $\sigma_3 = -0.45$ kpsi, $\tau_{\max} = 23.1$ kpsi

(a) MSS Theory: $n_y = \frac{S_y}{2\tau_{\max}} = \frac{67}{2(23.1)} = 1.45 \qquad Ans.$

(b) DE Theory, Eq. (5-13): $\sigma' = \left[45.8^2 - 45.8(-0.45) + (-0.45)^2\right]^{1/2} = 46.0 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'} = \frac{67}{46} = 1.46 \qquad Ans.$$

5-45 Table A-20 for AISI 1040 CD steel, S_y = 71 kpsi
From Prob. 3-101, $\sigma_1 = 18.47$ kpsi, $\sigma_2 = -3.60$ kpsi, $\tau_{\max} = 11.03$ kpsi

(a) MSS Theory: $n_y = \frac{S_y}{2\tau_{\max}} = \frac{71}{2(11.03)} = 3.22 \qquad Ans.$

(b) DE Theory, Eq. (5-13): $\sigma' = \left[18.47^2 - 18.47(-3.60) + (-3.60)^2\right]^{1/2} = 20.51 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'} = \frac{71}{20.51} = 3.46 \qquad Ans.$$

5-46 Table A-20 for AISI 1040 CD steel, S_y = 71 kpsi
From Prob. 3-102, $\sigma_1 = 29.1$ kpsi, $\sigma_2 = -14.2$ kpsi, $\tau_{\max} = 21.7$ kpsi

(a) MSS Theory: $n_y = \frac{S_y}{2\tau_{\max}} = \frac{71}{2(21.7)} = 1.64 \qquad Ans.$

(b) DE Theory, Eq. (5-13): $\sigma' = \left[29.1^2 - 29.1(-14.2) + (-14.2)^2\right]^{1/2} = 38.2 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'} = \frac{71}{38.2} = 1.86 \qquad Ans.$$

5-47 Table A-21 for AISI 4140 steel Q & T 400° F , $S_y = 238$ kpsi, $F = 15$ kip.
$\Sigma M_A = 0 = 3\,R_D - 2\,F \quad \Rightarrow \quad R_D = 2\,(15)/3 = 10$ kip,
$\Sigma F_y = 0 = R_A + R_D - F \quad \Rightarrow \quad R_A = 15 - 10 = 5$ kip
Critical sections are at points B and C where the areas are minimal.
B: $d_B = 1.1$ in, $M_B = R_A\,(1) = 5$ kip·in, $V_B = R_A = 5$ kip, $T_B = 7$ kip·in
$A_B = (\pi/4)\,1.1^2 = 0.9503$ in^2,
C: $d_C = 1.3$ in, $M_C = R_D\,(1) = 10$ kip·in, $V_C = R_D = 10$ kip, $T_C = 7$ kip·in
$A_C = (\pi/4)\,1.3^2 = 1.327$ in^2,
Critical locations are at the outer surfaces where bending stresses are maximum, and at the center planes where the transverse shear stresses are maximum. In both cases, there exists the torsional shear stresses.
B: Outer surface:

$$\sigma_B = \frac{32M_B}{\pi d^3} = \frac{32(5)}{\pi(1.1)^3} = 38.26 \text{ kpsi}, \quad \tau_B = \frac{16T_B}{\pi d^3} = \frac{16(7)}{\pi(1.1)^3} = 26.78 \text{ kpsi}$$

$$(\tau_B)_{\max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \sqrt{\left(\frac{38.26}{2}\right)^2 + 26.78^2} = 32.9 \text{ kpsi}$$

$$\sigma'_B = \sqrt{\sigma_B^2 + 3\tau_B^2} = \sqrt{38.26^2 + 3(26.78)^2} = 60.1 \text{ kpsi}$$

Center plane:

$$(\tau_B)_V = \frac{4V_B}{3A_B} = \frac{4(5)}{3(0.9503)} = 7.02 \text{ kpsi}$$

$$(\tau_B)_{\max} = \tau_B + (\tau_B)_V = 26.78 + 7.02 = 33.8 \text{ kpsi}$$

$$\sigma'_B = \sqrt{3}(\tau_B)_{\max} = \sqrt{3}(33.8) = 58.5 \text{ kpsi}$$

C: Outer surface:

$$\sigma_C = \frac{32M_C}{\pi d^3} = \frac{32(10)}{\pi(1.3)^3} = 46.36 \text{ kpsi}, \quad \tau_C = \frac{16T_C}{\pi d^3} = \frac{16(7)}{\pi(1.3)^3} = 16.23 \text{ kpsi}$$

$$(\tau_C)_{\max} = \sqrt{\left(\frac{\sigma_C}{2}\right)^2 + \tau_C^2} = \sqrt{\left(\frac{46.36}{2}\right)^2 + 16.23^2} = 28.30 \text{ kpsi}$$

$$\sigma'_C = \sqrt{\sigma_C^2 + 3\tau_C^2} = \sqrt{46.36^2 + 3(16.23)^2} = 54.2 \text{ kpsi}$$

Center plane:

$$(\tau_C)_V = \frac{4V_C}{3A_C} = \frac{4(10)}{3(1.327)} = 10.05 \text{ kpsi}$$

$$(\tau_C)_{\max} = \tau_C + (\tau_C)_V = 16.23 + 10.05 = 26.28 \text{ kpsi}$$

$$\sigma'_C = \sqrt{3}(\tau_C)_{\max} = \sqrt{3}(26.28) = 45.5 \text{ kpsi}$$

(a) MSS Theory: Critical location is at point B at the center plane:

$$n_y = \frac{S_y}{2\tau_{\max}} = \frac{238}{2(33.8)} = 3.52 \qquad Ans.$$

(b) DE Theory: Critical location is at point B at the outer surface:

$$n_y = \frac{S_y}{\sigma'_B} = \frac{238}{60.1} = 3.96 \qquad Ans.$$

5-48 $\Sigma M_O = 0 = 40\,R_C - 30(575) + 12(460)$

$R_C = 293.25$ lbf

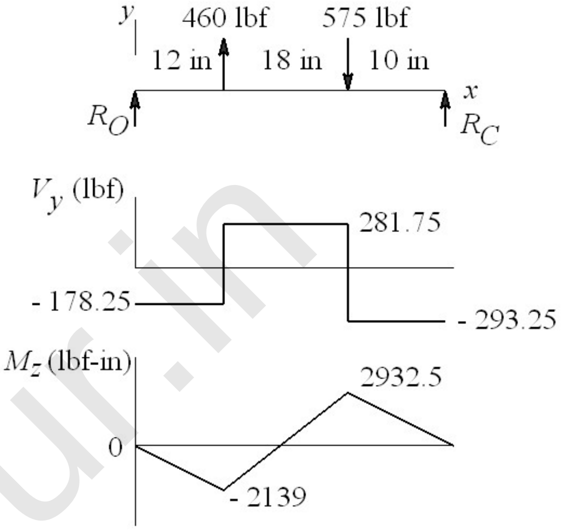
$\Sigma F_y = 0 = R_O + 293.25 + 460 - 575$

$R_O = -\,178.25$ lbf

$M_{\max} = 2.9325$ kip·in

$$\sigma = \frac{32M_{\max}}{\pi d^3} = \frac{32(2.9325)}{\pi(1^3)} = 29.87 \text{ kpsi}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(1.5)}{\pi(1^3)} = 7.64 \text{ kpsi}$$



(a) $$\tau_{\max} = \sqrt{\left(\frac{29.87}{2}\right)^2 + 7.64^2} = 16.78 \text{ kpsi}$$

$$n_y = \frac{S_y}{2\tau_{\max}} = \frac{50}{2(16.78)} = 1.49 \qquad Ans.$$

(b) Eq. (5-15): $\sigma' = [29.87^2 + 3\,(7.64)^2]^{1/2} = 32.67$ kpsi

$$n_y = \frac{S_y}{\sigma'} = \frac{50}{32.67} = 1.53 \qquad Ans.$$

5-49 Given: AISI 1010 HR, $n_y = 2$, $L = 0.5$ m, $F = 150$ N, $T = 25$ N·m
Table A-20, $S_y = 180$ MPa.

$M_z = FL = 150\,(0.5) = 75$ N·m

$$\sigma = \frac{32M_z}{\pi d^3} = \frac{32(75)}{\pi d^3} = \frac{2400}{\pi d^3}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(25)}{\pi d^3} = \frac{400}{\pi d^3}$$

(a) $$\tau_{\max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{1200}{\pi d^3}\right)^2 + \left(\frac{400}{\pi d^3}\right)^2} = \frac{402.63}{d^3} = \frac{S_y}{2n} = \frac{180(10^6)}{2(2)}$$

$$d = \left[\frac{4(402.63)}{180(10^6)}\right]^{1/3} = 0.0208 \text{ m} = 20.8 \text{ mm} \qquad Ans.$$

(b) $\sigma' = \left(\sigma^2 + 3\tau^2\right)^{1/2} = \left[\left(\dfrac{2400}{\pi d^3}\right)^2 + 3\left(\dfrac{400}{\pi d^3}\right)^2\right]^{1/2} = \dfrac{795.14}{d^3} = \dfrac{S_y}{n_y} = \dfrac{180\left(10^6\right)}{2}$

$$d = \left[\frac{2(795.14)}{180\left(10^6\right)}\right]^{1/3} = 0.0207 \ \text{ m} = 20.7 \ \text{ mm} \qquad Ans.$$

5-50 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-79, in the plane of analysis

σ_1 = 16.5 kpsi, σ_2 = – 1.19 kpsi, and $\tau_{\max}$ = 8.84 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 16.5 kpsi, σ_2 = 0, and σ_3 = – 1.19 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{16.5 - (-1.19)} = 3.05 \quad Ans.$

Note: Whenever the two principal stresses of a plane stress state are of opposite sign, the maximum shear stress found in the analysis is the *true* maximum shear stress. Thus, the factor of safety could have been found from

$$n = \frac{S_y}{2\tau_{\max}} = \frac{54}{2(8.84)} = 3.05 \quad Ans.$$

DE: The von Mises stress can be found from the principal stresses or from the stresses found in part (d) of Prob. 3-79. That is,

Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[16.5^2 - 16.5(-1.19) + (-1.19)^2\right]^{1/2}}$$
$$= 3.15 \quad Ans.$$

or, Eqs. (5-15) and (5-19) using the results of part (d) of Prob. 3-79

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma^2 + 3\tau^2\right)^{1/2}} = \frac{54}{\left[15.3^2 + 3\left(4.43^2\right)\right]^{1/2}}$$
$$= 3.15 \quad Ans.$$

5-51 From Table A-20, S_y = 370 MPa. From the solution of Prob. 3-80, in the plane of analysis

σ_1 = 275 MPa, σ_2 = – 12.1 MPa, and $\tau_{\max}$ = 144 MPa

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 275 MPa, σ_2 = 0, and σ_3 = – 12.1 MPa

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{370}{275-(-12.1)} = 1.29$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{370}{\left[275^2 - 275(-12.1) + (-12.1)^2\right]^{1/2}}$$
$$= 1.32 \quad \textit{Ans.}$$

5-52 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-81, in the plane of analysis

σ_1 = 22.6 kpsi, σ_2 = – 1.14 kpsi, and $\tau_{\max}$ = 11.9 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 22.6 kpsi, σ_2 = 0, and σ_3 = – 1.14 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{22.6-(-1.14)} = 2.27$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[22.6^2 - 22.6(-1.14) + (-1.14)^2\right]^{1/2}}$$
$$= 2.33 \quad \textit{Ans.}$$

5-53 From Table A-20, S_y = 370 MPa. From the solution of Prob. 3-82, in the plane of analysis

σ_1 = 78.2 MPa, σ_2 = – 5.27 MPa, and $\tau_{\max}$ = 41.7 MPa

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 78.2 MPa, σ_2 = 0, and σ_3 = – 5.27 MPa

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{370}{78.2 - (-5.27)} = 4.43$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{370}{\left[78.2^2 - 78.2(-5.27) + (-5.27)^2\right]^{1/2}}$$
$$= 4.57 \quad Ans.$$

5-54 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-83, in the plane of analysis

σ_1 = 36.7 kpsi, σ_2 = – 1.47 kpsi, and $\tau_{\max}$ = 19.1 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 36.7 kpsi, σ_2 = 0, and σ_3 = – 1.47 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{36.7 - (-1.47)} = 1.41$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[36.7^2 - 36.7(-1.47) + (-1.47)^2\right]^{1/2}}$$
$$= 1.44 \quad Ans.$$

5-55 From Table A-20, S_y = 370 MPa. From the solution of Prob. 3-84, in the plane of analysis

σ_1 = 376 MPa, σ_2 = – 42.4 MPa, and $\tau_{\max}$ = 209 MPa

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 376 MPa, σ_2 = 0, and σ_3 = – 42.4 MPa

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{370}{376 - (-42.4)} = 0.88$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n=\frac{S_y}{\sigma'}=\frac{S_y}{\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_B^2\right)^{1/2}}=\frac{370}{\left[376^2-376(-42.4)+(-42.4)^2\right]^{1/2}}$$

$$=0.93 \qquad Ans.$$

5-56 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-85, in the plane of analysis

σ_1 = 7.19 kpsi, σ_2 = – 17.0 kpsi, and $\tau_{\max}$ = 12.1 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 7.19 kpsi, σ_2 = 0, and σ_3 = – 17.0 kpsi

MSS: From Eq. (5-3), $n=\dfrac{S_y}{\sigma_1-\sigma_3}=\dfrac{54}{7.19-(-17.0)}=2.23 \qquad Ans.$

DE: From Eqs. (5-13) and (5-19)

$$n=\frac{S_y}{\sigma'}=\frac{S_y}{\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_B^2\right)^{1/2}}=\frac{54}{\left[7.19^2-7.19(-17.0)+(-17.0)^2\right]^{1/2}}$$

$$=2.51 \qquad Ans.$$

5-57 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-87, in the plane of analysis

σ_1 = 1.72 kpsi, σ_2 = – 35.9 kpsi, and $\tau_{\max}$ = 18.8 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 1.72 kpsi, σ_2 = 0, and σ_3 = – 35.9 kpsi

MSS: From Eq. (5-3), $n=\dfrac{S_y}{\sigma_1-\sigma_3}=\dfrac{54}{1.72-(-35.9)}=1.44 \qquad Ans.$

DE: From Eqs. (5-13) and (5-19)

$$n=\frac{S_y}{\sigma'}=\frac{S_y}{\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_B^2\right)^{1/2}}=\frac{54}{\left[1.72^2-1.72(-35.9)+(-35.9)^2\right]^{1/2}}$$

$$=1.47 \qquad Ans.$$

5-58 From Table A-20, S_y = 370 MPa. From the solution of Prob. 3-88,

Bending: σ_B = 68.6 MPa, Torsion: τ_B = 37.7 MPa

For a plane stress analysis it was found that $\tau_{\max} = 51.0$ MPa. With combined bending and torsion, the plane stress analysis yields the true $\tau_{\max}$.

MSS: From Eq. (5-3), $n = \dfrac{S_y}{2\tau_{\max}} = \dfrac{370}{2(51.0)} = 3.63$ *Ans.*

DE: From Eqs. (5-15) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_B^2 + 3\tau_B^2\right)^{1/2}} = \frac{370}{\left[68.6^2 + 3\left(37.7^2\right)\right]^{1/2}}$$

$$= 3.91 \quad \textit{Ans.}$$

5-59 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-90,

Bending: $\sigma_C = 3460$ psi, Torsion: $\tau_C = 882$ kpsi

For a plane stress analysis it was found that $\tau_{\max} = 1940$ psi. With combined bending and torsion, the plane stress analysis yields the true $\tau_{\max}$.

MSS: From Eq. (5-3), $n = \dfrac{S_y}{2\tau_{\max}} = \dfrac{54\left(10^3\right)}{2(1940)} = 13.9$ *Ans.*

DE: From Eqs. (5-15) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_C^2 + 3\tau_C^2\right)^{1/2}} = \frac{54\left(10^3\right)}{\left[3460^2 + 3\left(882^2\right)\right]^{1/2}}$$

$$= 14.3 \quad \textit{Ans.}$$

5-60 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-91, in the plane of analysis

$\sigma_1 = 17.8$ kpsi, $\sigma_2 = -\,1.46$ kpsi, and $\tau_{\max} = 9.61$ kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$\sigma_1 = 17.8$ kpsi, $\sigma_2 = 0$, and $\sigma_3 = -\,1.46$ kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{17.8 - (-1.46)} = 2.80$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[17.8^2 - 17.8(-1.46) + (-1.46)^2\right]^{1/2}}$$

$$= 2.91 \qquad Ans.$$

5-61 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-92, in the plane of analysis

σ_1 = 17.5 kpsi, σ_2 = – 1.13 kpsi, and $\tau_{\max}$ = 9.33 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 17.5 kpsi, σ_2 = 0, and σ_3 = – 1.13 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{17.5 - (-1.13)} = 2.90 \qquad Ans.$

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[17.5^2 - 17.5(-1.13) + (-1.13)^2\right]^{1/2}}$$

$$= 2.98 \qquad Ans.$$

5-62 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-93, in the plane of analysis

σ_1 = 21.5 kpsi, σ_2 = – 1.20 kpsi, and $\tau_{\max}$ = 11.4 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 21.5 kpsi, σ_2 = 0, and σ_3 = – 1.20 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{21.5 - (-1.20)} = 2.38 \qquad Ans.$

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[21.5^2 - 21.5(-1.20) + (-1.20)^2\right]^{1/2}}$$

$$= 2.44 \qquad Ans.$$

5-63 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-94, the concern was failure due to twisting of the flat bar where it was found that $\tau_{\max}$ = 14.3 kpsi in the middle of the longest side of the rectangular cross section. The bar is also in bending, but the bending stress is zero where $\tau_{\max}$ exists.

MSS: From Eq. (5-3), $n = \dfrac{S_y}{2\tau_{\max}} = \dfrac{54}{2(14.3)} = 1.89$ *Ans.*

DE: From Eqs. (5-15) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(3\tau_{\max}^2\right)^{1/2}} = \frac{54}{\left[3\left(14.3^2\right)\right]^{1/2}} = 2.18 \qquad Ans.$$

5-64 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-95, in the plane of analysis

σ_1 = 34.7 kpsi, σ_2 = – 6.7 kpsi, and $\tau_{\max}$ = 20.7 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 34.7 kpsi, σ_2 = 0, and σ_3 = – 6.7 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{34.7-(-6.7)} = 1.30$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[34.7^2 - 34.7(-6.7) + (-6.7)^2\right]^{1/2}}$$

$$= 1.40 \qquad Ans.$$

5-65 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-96, in the plane of analysis

σ_1 = 51.1 kpsi, σ_2 = – 4.58 kpsi, and $\tau_{\max}$ = 27.8 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 51.1 kpsi, σ_2 = 0, and σ_3 = – 4.58 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{51.1-(-4.58)} = 0.97$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n=\frac{S_y}{\sigma'}=\frac{S_y}{\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_B^2\right)^{1/2}}=\frac{54}{\left[51.1^2-51.1(-4.58)+(-4.58)^2\right]^{1/2}}$$

$$=1.01 \qquad Ans.$$

5-66 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-97, in the plane of analysis

σ_1 = 59.7 kpsi, σ_2 = – 3.92 kpsi, and $\tau_{\max}$ = 31.8 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 59.7 kpsi, σ_2 = 0, and σ_3 = – 3.92 kpsi

MSS: From Eq. (5-3), $n=\dfrac{S_y}{\sigma_1-\sigma_3}=\dfrac{54}{59.7-(-3.92)}=0.85 \qquad Ans.$

DE: From Eqs. (5-13) and (5-19)

$$n=\frac{S_y}{\sigma'}=\frac{S_y}{\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_B^2\right)^{1/2}}=\frac{54}{\left[59.7^2-59.7(-3.92)+(-3.92)^2\right]^{1/2}}$$

$$=0.87 \qquad Ans.$$

5-67 For Prob. 3-95, from Prob. 5-64 solution, with 1018 CD, DE theory yields, n = 1.40.

From Table A-21, for 4140 Q&T @400°F, S_y = 238 kpsi. From Prob. 3-98 solution which considered stress concentrations for Prob. 3-95

σ_1 = 53.0 kpsi, σ_2 = – 8.48 kpsi, and $\tau_{\max}$ = 30.7 kpsi

DE: From Eqs. (5-13) and (5-19)

$$n=\frac{S_y}{\sigma'}=\frac{S_y}{\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_B^2\right)^{1/2}}=\frac{238}{\left[53.0^2-53.0(-8.48)+(-8.48)^2\right]^{1/2}}$$

$$=4.12 \qquad Ans.$$

Using the 4140 versus the 1018 CD, the factor of safety increases by a factor of 4.12/1.40 = 2.94. *Ans.*

5-68 Design Decisions Required:

- Material and condition
- Design factor
- Failure model
- Diameter of pin

Using $F = 416$ lbf from Ex. 5-3,

$$\sigma_{\max} = \frac{32M}{\pi d^3}$$

$$d = \left(\frac{32M}{\pi \sigma_{\max}}\right)^{\frac{1}{3}}$$

Decision 1: Select the same material and condition of Ex. 5-3 (AISI 1035 steel, $S_y = 81$ kpsi)

Decision 2: Since we prefer the pin to yield, set n_d a little larger than 1. Further explanation will follow.

Decision 3: Use the Distortion Energy static failure theory.

Decision 4: Initially set $n_d = 1$

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{S_y}{1} = 81\,000 \text{ psi}$$

$$d = \left(\frac{32(416)(15)}{\pi(81\,000)}\right)^{\frac{1}{3}} = 0.922 \text{ in}$$

Choose preferred size of $d = 1.000$ in

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.27$$

Set design factor to $n_d = 1.27$

Adequacy Assessment:

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{81\,000}{1.27} = 63\,800 \text{ psi}$$

$$d = \left(\frac{32(416)(15)}{\pi(63\,800)}\right)^{\frac{1}{3}} = 1.00 \text{ in (OK)}$$

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.27 \text{ (OK)}$$

5-69 From Table A-20, for a thin walled cylinder made of AISI 1020 CD steel, S_{yt} = 57 kpsi, S_{ut} = 68 kpsi.

Since r/t = 7.5/0.0625 = 120 > 10, the shell can be considered thin-wall. From the solution of Prob. 3-106 the principal stresses are

$$\sigma_1 = \sigma_2 = \frac{pd}{4t} = \frac{p(15)}{4(0.0625)} = 60p, \qquad \sigma_3 = -p$$

From Eq. (5-12)

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]^{1/2}$$

$$= \frac{1}{\sqrt{2}}\left[(60p-60p)^2 + (60p+p)^2 + (-p-60p)^2\right]^{1/2} = 61p$$

For yield, $\sigma' = S_y \;\Rightarrow\; 61p = 57\,(10^3) \;\Rightarrow\; p = 934$ psi *Ans.*
For rupture, $61p = 68 \;\Rightarrow\; p = 1.11$ kpsi *Ans.*

5-70 Given: AISI CD 1040 steel, n_y = 2, OD = 50 mm, ID = 42 mm, L = 150 mm.
Table A-20, S_y = 490 MPa
At $r = r_i$ = 21 mm, Eq. (3-51) gives

$$(\sigma_t)_{\max} = p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} = p_i \frac{25^2+21^2}{25^2-21^2} = 5.793\,p_i = \sigma_1$$

$$(\sigma_r)_{\max} = -p_i = \sigma_3$$

Closed end, Eq. (3-52) gives

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2} = \frac{p_i(21)^2}{25^2-21^2} = 2.397\,p_i = \sigma_2$$

(a) $\tau_{\max} = \dfrac{\sigma_1-\sigma_3}{2} = \dfrac{5.793p_i - (-p_i)}{2} = 3.397p_i$

$$\tau_{\max} = \frac{S_y}{2n_y} \quad\Rightarrow\quad 3.397p_i = \frac{490}{2(2)} \quad\Rightarrow\quad p_i = 36.1 \text{ MPa} \qquad \textit{Ans.}$$

(b) Eq. (5-12):

$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2}$$

$$= \left[\frac{(5.793-2.397)^2+(2.397+1)^2+(-1-5.793)^2}{2}\right]^{1/2} p_i = 5.883 p_i$$

$$\sigma' = \frac{S_y}{n_y} \quad \Rightarrow \quad 5.883 p_i = \frac{490}{2} \quad \Rightarrow \quad p_i = 41.6 \;\; \text{MPa} \qquad \textit{Ans.}$$

5-71 Given: AISI 1040 CD steel, OD = 50 mm, ID = 42 mm, L = 150 mm, p_i = 40 MPa
Table A-20, S_y = 490 MPa
At $r = r_i$ = 21 mm, Eq. (3-51) gives

$$(\sigma_t)_{\max} = p_i \frac{r_o^2+r_i^2}{r_o^2-r_i^2} = 40\frac{25^2+21^2}{25^2-21^2} = 231.74 \;\; \text{MPa} \;\; = \sigma_1$$

$$(\sigma_r)_{\max} = -p_i = -40 \;\; \text{MPa} \;\; = \sigma_3$$

Closed end, Eq. (3-52) gives

$$\sigma_l = \frac{p_i r_i^2}{r_o^2-r_i^2} = \frac{40(21)^2}{25^2-21^2} = 95.87 \;\; \text{MPa} \;\; = \sigma_2$$

(a) $$\tau_{\max} = \frac{\sigma_1-\sigma_3}{2} = \frac{231.74-(-40)}{2} = 135.87 \;\; \text{MPa}$$

$$n_y = \frac{S_y}{2\tau_{\max}} = \frac{490}{2(135.87)} = 1.80 \qquad \textit{Ans.}$$

(b) Eq. (5-12):

$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2}$$

$$= \left[\frac{(231.74-95.87)^2+(95.87+40)^2+(-40-231.74)^2}{2}\right]^{1/2} = 235.3 \;\; \text{MPa}$$

$$n_y = \frac{S_y}{\sigma'} = \frac{490}{235.3} = 2.08 \qquad \textit{Ans.}$$

5-72 For AISI 1020 HR steel, from Tables A-5 and A-20, w = 0.282 lbf/in^3, S_y = 30 kpsi, and ν = 0.292. Then, $\rho = w/g$ = 0.282/386 lbf·s^2/in. For the problem, r_i = 3 in, and r_o = 5 in. Substituting into Eqs. (3-55), gives

$$\sigma_t = \frac{0.282}{386}\omega^2\left(\frac{3+0.292}{8}\right)\left[9+25+\frac{9(25)}{r^2}-\frac{1+3(0.292)}{3+0.292}r^2\right]$$

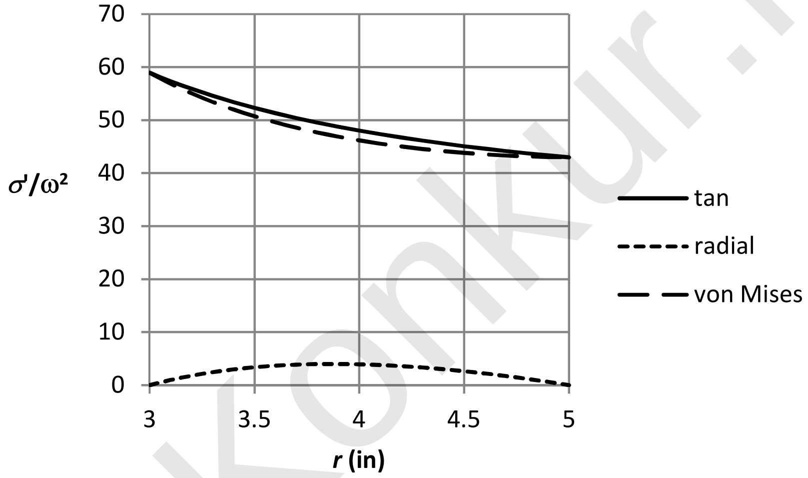
$$= 3.006\left(10^{-4}\right)\omega^2\left(34+\frac{225}{r^2}-0.5699r^2\right) = F(r)\omega^2 \qquad (1)$$

$$\sigma_r = 3.006\left(10^{-4}\right)\omega^2\left(34-\frac{225}{r^2}-r^2\right) = G(r)\omega^2 \qquad (2)$$

For the distortion-energy theory, the von Mises stress will be

$$\sigma' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_2^2\right)^{1/2} = \omega^2\left[F^2(r) - F(r)G(r) + G^2(r)\right]^{1/2} \qquad (3)$$

Although it was noted that the maximum radial stress occurs at $r = (r_o r_i)^{1/2}$ we are more interested as to where the von Mises stress is a maximum. One could take the derivative of Eq. (3) and set it to zero to find where the maximum occurs. However, it is much easier to plot σ'/ω^2 for $3 \le r \le 5$ in. Plotting Eqs. (1) through (3) results in



It can be seen that there is no maxima, and the greatest value of the von Mises stress is the tangential stress at $r = r_i$. Substituting $r = 3$ in into Eq. (1) and setting $\sigma' = S_y$ gives

$$\omega = \left[\frac{30\left(10^3\right)}{3.006\left(10^{-4}\right)\left(34+\frac{225}{3^2}-0.5699\left(3^2\right)\right)}\right]^{1/2} = 1361 \text{ rad/s}$$

$$n = \frac{60\omega}{2\pi} = \frac{60(1361)}{2\pi} = 13\,000 \text{ rev/min} \qquad \textit{Ans.}$$

5-73 Since $r/t = 1.75/0.065 = 26.9 > 10$, we can use thin-walled equations. From Eqs. (3-53) and (3-54),

$$d_i = 3.5 - 2(0.065) = 3.37 \text{ in}$$

$$\sigma_t = \frac{p(d_i + t)}{2t}$$

$$\sigma_t = \frac{500(3.37 + 0.065)}{2(0.065)} = 13\,212 \text{ psi} = 13.2 \text{ kpsi}$$

$$\sigma_l = \frac{pd_i}{4t} = \frac{500(3.37)}{4(0.065)} = 6481 \text{ psi} = 6.48 \text{ kpsi}$$

$$\sigma_r = -p_i = -500 \text{ psi} = -0.5 \text{ kpsi}$$

These are all principal stresses, thus, from Eq. (5-12),

$$\sigma' = \frac{1}{\sqrt{2}}\left\{(13.2 - 6.48)^2 + [6.48 - (-0.5)]^2 + (-0.5 - 13.2)^2\right\}^{1/2}$$

$$= 11.87 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{46}{11.87}$$

$$n = 3.88 \quad \textit{Ans.}$$

5-74 From Table A-20, $S_y = 320$ MPa

With $p_i = 0$, Eqs. (3-49) are

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right) = c\left(1 + \frac{b^2}{r^2}\right) \qquad (1)$$

$$\sigma_r = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right) = c\left(1 - \frac{b^2}{r^2}\right)$$

For the distortion-energy theory, the von Mises stress is

$$\sigma' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = c\left[\left(1 + \frac{b^2}{r^2}\right)^2 - \left(1 + \frac{b^2}{r^2}\right)\left(1 - \frac{b^2}{r^2}\right) + \left(1 - \frac{b^2}{r^2}\right)^2\right]^{1/2}$$

$$= c\left(1 + 3\frac{b^4}{r^4}\right)^{1/2}$$

We see that the maximum von Mises stress occurs where r is a minimum at $r = r_i$. Here, $\sigma_r = 0$ and thus $\sigma' = -\sigma_t$. Setting $-\sigma_t = S_y = 320$ MPa at $r = 0.1$ m in Eq. (1) results in

$$-\sigma_t\Big|_{r=r_i} = \frac{2r_o^2 p_o}{r_o^2 - r_i^2} = \frac{2\left(0.15^2\right)p_o}{0.15^2 - 0.1^2} = 3.6p_o = 320 \;\Rightarrow\; p_o = 88.9 \text{ MPa} \qquad \textit{Ans.}$$

5-75 From Table A-24, S_{ut} = 31 kpsi for grade 30 cast iron. From Table A-5, ν = 0.211 and w = 0.260 lbf/in^3. In Prob. 5-72, it was determined that the maximum stress was the tangential stress at the inner radius, where the radial stress is zero. Thus at the inner radius, Eq. (3-55) gives

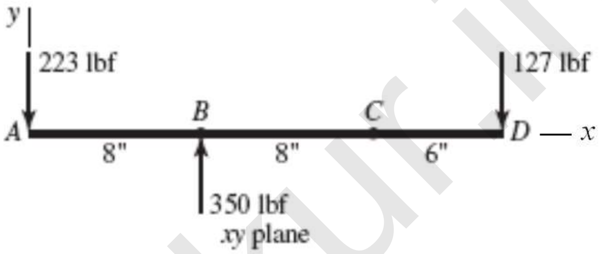
$$\sigma_t = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(2r_o^2 + r_i^2 - \frac{1+3\nu}{3+\nu}r_i^2\right) = \frac{0.260}{386}\omega^2\left(\frac{3.211}{8}\right)\left[2\left(5^2\right)+3^2-\frac{1+3(0.211)}{3.211}3^2\right]$$

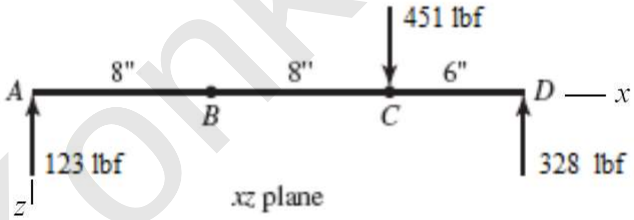
$$= 0.01471\omega^2 = 31\left(10^3\right) \qquad \Rightarrow \qquad 1452 \text{ rad/ sec}$$

$$n = 60(1452)/(2\pi) = 13\ 870 \text{ rev/min} \qquad \textit{Ans.}$$

5-76 From Table A-20, for AISI 1035 CD, S_y = 67 kpsi.

From force and bending-moment equations, the ground reaction forces are found in two planes as shown.





The maximum bending moment will be at B or C. Check which is larger. In the xy plane,

$M_B = 223(8) = 1784$ lbf · in and $M_C = 127(6) = 762$ lbf · in.

In the xz plane, $M_B = 123(8) = 984$ lbf · in and $M_C = 328(6) = 1968$ lbf · in.

$$M_B = [(1784)^2 + (984)^2]^{\frac{1}{2}} = 2037 \text{ lbf} \cdot \text{in}$$

$$M_C = [(762)^2 + (1968)^2]^{\frac{1}{2}} = 2110 \text{ lbf} \cdot \text{in}$$

So point C governs. The torque transmitted between B and C is T = (300 − 50)(4) = 1000 lbf·in. The stresses are

$$\tau_{xz} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi d^3} = \frac{5093}{d^3} \text{ psi}$$

$$\sigma_x = \frac{32M_C}{\pi d^3} = \frac{32(2110)}{\pi d^3} = \frac{21\,492}{d^3} \text{ psi}$$

For combined bending and torsion, the maximum shear stress is found from

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xz}^2\right]^{1/2} = \left[\left(\frac{21.49}{2d^3}\right)^2 + \left(\frac{5.09}{d^3}\right)^2\right]^{1/2} = \frac{11.89}{d^3} \text{ kpsi}$$

Max Shear Stress theory is chosen as a conservative failure theory. From Eq. (5-3)

$$\tau_{\max} = \frac{S_y}{2n} = \frac{11.89}{d^3} = \frac{67}{2(2)} \quad \Rightarrow \quad d = 0.892 \text{ in} \quad \textit{Ans.}$$

5-77 As in Prob. 5-76, we will assume this to be a statics problem. Since the proportions are unchanged, the bearing reactions will be the same as in Prob. 5-76 and the bending moment will still be a maximum at point *C*. Thus

xy plane: $M_C = 127(3) = 381 \text{ lbf}\cdot\text{in}$
xz plane: $M_C = 328(3) = 984 \text{ lbf}\cdot\text{in}$

So

$$M_{\max} = \left[(381)^2 + (984)^2\right]^{1/2} = 1055 \text{ lbf}\cdot\text{in}$$

$$\sigma_x = \frac{32M_C}{\pi d^3} = \frac{32(1055)}{\pi d^3} = \frac{10\,746}{d^3} \text{ psi} = \frac{10.75}{d^3} \text{ kpsi}$$

Since the torsional stress is unchanged,

$$\tau_{xz} = \frac{5.09}{d^3} \text{ kpsi}$$

For combined bending and torsion, the maximum shear stress is found from

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xz}^2\right]^{1/2} = \left[\left(\frac{10.75}{2d^3}\right)^2 + \left(\frac{5.09}{d^3}\right)^2\right]^{1/2} = \frac{7.40}{d^3} \text{ kpsi}$$

Using the MSS theory, as was used in Prob. 5-76, gives

$$\tau_{\max} = \frac{S_y}{2n} = \frac{7.40}{d^3} = \frac{67}{2(2)} \quad \Rightarrow \quad d = 0.762 \text{ in} \quad \textit{Ans.}$$

5-78 For AISI 1018 HR, Table A-20 gives $S_y = 32$ kpsi. Transverse shear stress is a maximum at the neutral axis, and zero at the outer radius. Bending stress is a maximum at the outer radius, and zero at the neutral axis.

Model (*c*): From Prob. 3-41, at outer radius,

$$\sigma' = \sigma = 17.8 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{17.8} = 1.80$$

At neutral axis,

$$\sigma' = \sqrt{3\tau^2} = \sqrt{3(3.4)^2} = 5.89 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{5.89} = 5.43$$

The bending stress at the outer radius dominates. $n = 1.80$ *Ans.*

Model (*d*): Assume the bending stress at the outer radius will dominate, as in model (*c*). From Prob. 3-41,

$$\sigma' = \sigma = 25.5 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{25.5} = 1.25 \qquad \textit{Ans.}$$

Model (*e*): From Prob. 3-41,

$$\sigma' = \sigma = 17.8 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{17.8} = 1.80 \qquad \textit{Ans.}$$

Model (*d*) is the most conservative, thus safest, and requires the least modeling time. Model (*c*) is probably the most accurate, but model (*e*) yields the same results with less modeling effort.

5-79 For AISI 1018 HR, from Table A-20, $S_y = 32$ kpsi. Model (*d*) yields the largest bending moment, so designing to it is the most conservative approach. The bending moment is $M = 312.5$ lbf·in. For this case, the principal stresses are

$$\sigma_1 = \frac{32M}{\pi d^3}, \ \sigma_2 = \sigma_3 = 0$$

Using a conservative yielding failure theory use the MSS theory and Eq. (5-3)

$$\sigma_1 - \sigma_3 = \frac{S_y}{n} \quad \Rightarrow \quad \frac{32M}{\pi d^3} = \frac{S_y}{n} \quad \Rightarrow \quad d = \left(\frac{32Mn}{\pi S_y}\right)^{1/3}$$

Thus, $d = \left[\dfrac{32(312.5)2.5}{\pi(32)10^3}\right]^{1/3} = 0.629 \text{ in} \ \therefore$ Use $d = \dfrac{11}{16}$ in *Ans.*

5-80 When the ring is set, the hoop tension in the ring is equal to the screw tension.

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

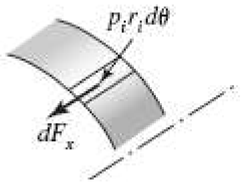
The differential hoop tension dF at r for the ring of width w, is $dF = w\sigma_t dr$. Integration yields

$$F = \int_{r_i}^{r_o} w\sigma_t dr = \frac{w r_i^2 p_i}{r_o^2 - r_i^2}\int_{r_i}^{r_o}\left(1 + \frac{r_o^2}{r^2}\right)dr = \frac{w r_i^2 p_i}{r_o^2 - r_i^2}\left(r - \frac{r_o^2}{r}\right)\Bigg|_{r_i}^{r_o} = w r_i p_i \quad (1)$$

The screw equation is

$$F_i = \frac{T}{0.2d} \qquad (2)$$

From Eqs. (1) and (2)



$$p_i = \frac{F}{w r_i} = \frac{T}{0.2 d w r_i}$$

$$dF_x = f p_i r_i d\theta$$

$$F_x = \int_0^{2\pi} f p_i w r_i d\theta = \frac{f T w}{0.2 d w r_i} r_i \int_0^{2\pi} d\theta$$

$$= \frac{2\pi f T}{0.2d} \qquad Ans.$$

5-81 $T = 20$ N·m, $S_y = 450$ MPa

(a) From Prob. 5-80, $T = 0.2\, F_i\, d$

$$F_i = \frac{T}{0.2d} = \frac{20}{0.2\left[6\left(10^{-3}\right)\right]} = 16.7\left(10^3\right)\text{ N} = 16.7\text{ kN} \qquad Ans.$$

(b) From Prob. 5-80, $F = w r_i p_i$

$$p_i = \frac{F}{w r_i} = \frac{F_i}{w r_i} = \frac{16.7\left(10^3\right)}{\left[12\left(10^{-3}\right)\right]\left[(25/2)\left(10^{-3}\right)\right]} = 111.3\left(10^6\right)\text{ Pa} = 111.3\text{ MPa} \qquad Ans.$$

(c) $$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1+\frac{r_o^2}{r}\right)_{r=r_i} = \frac{p_i\left(r_i^2+r_o^2\right)}{r_o^2-r_i^2}$$

$$= \frac{111.3\left(0.0125^2+0.025^2\right)}{0.025^2-0.0125^2} = 185.5 \text{ MPa} \qquad Ans.$$

$$\sigma_r = -p_i = -111.3 \text{ MPa}$$

(d)

$$\tau_{\max} = \frac{\sigma_1-\sigma_3}{2} = \frac{\sigma_t-\sigma_r}{2}$$

$$= \frac{185.5-(-111.3)}{2} = 148.4 \text{ MPa} \qquad Ans.$$

$$\sigma' = (\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2)^{1/2}$$

$$= \left[185.5^2 - (185.5)(-111.3) + (-111.3)^2\right]^{1/2}$$

$$= 259.7 \text{ MPa} \qquad Ans.$$

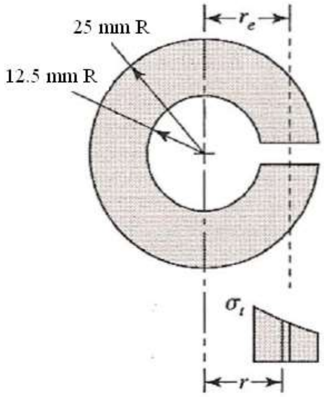
(e) Maximum Shear Stress Theory

$$n = \frac{S_y}{2\tau_{\max}} = \frac{450}{2(148.4)} = 1.52 \qquad Ans.$$

Distortion Energy theory

$$n = \frac{S_y}{\sigma'} = \frac{450}{259.7} = 1.73 \qquad Ans.$$

5-82 The moment about the center caused by the force F is $F r_e$ where r_e is the effective radius. This is balanced by the moment about the center caused by the tangential (hoop) stress. For the ring of width w



$$Fr_e = \int_{r_i}^{r_o} r\sigma_t w\,dr$$

$$= \frac{wp_i r_i^2}{r_o^2 - r_i^2}\int_{r_i}^{r_o}\left(r + \frac{r_o^2}{r}\right)dr$$

$$r_e = \frac{wp_i r_i^2}{F\left(r_o^2 - r_i^2\right)}\left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln\frac{r_o}{r_i}\right)$$

From Prob. 5-80, $F = wr_i p_i$. Therefore,

$$r_e = \frac{r_i}{r_o^2 - r_i^2}\left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln\frac{r_o}{r_i}\right)$$

For the conditions of Prob. 5-80, $r_i = 12.5$ mm and $r_o = 25$ mm

$$r_e = \frac{12.5}{25^2 - 12.5^2}\left(\frac{25^2 - 12.5^2}{2} + 25^2 \ln\frac{25}{12.5}\right) = 17.8 \text{ mm} \qquad \textit{Ans.}$$

5-83 (**a**) The nominal radial interference is $\delta_{\text{nom}} = (2.002 - 2.001)/2 = 0.0005$ in.

From Eq. (3-57),

$$p = \frac{E\delta}{2R^3}\left[\frac{\left(r_o^2 - R^2\right)\left(R^2 - r_i^2\right)}{r_o^2 - r_i^2}\right]$$

$$= \frac{30\left(10^6\right)0.0005}{2\left(1^3\right)}\left[\frac{\left(1.5^2 - 1^2\right)\left(1^2 - 0.625^2\right)}{1.5^2 - 0.625^2}\right] = 3072 \text{ psi} \quad \textit{Ans.}$$

Inner member: $p_i = 0$, $p_o = p = 3072$ psi. At fit surface $r = R = 1$ in,

Eq. (3-49): $$\sigma_t = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -3072\left(\frac{1^2 + 0.625^2}{1^2 - 0.625^2}\right) = -7010 \text{ psi}$$

$$\sigma_r = -p = -3072 \text{ psi}$$

Eq. (5-13):

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2\right)^{1/2}$$

$$= \left[(-7010)^2 - (-7010)(-3072) + (-3072)\right]^{1/2} = 6086 \text{ psi} \quad \textit{Ans.}$$

Outer member: $p_i = p = 3072$ psi, $p_o = 0$. At fit surface $r = R = 1$ in,

Eq. (3-49): $$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 3072\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 7987 \text{ psi}$$

$$\sigma_r = -p = -3072 \text{ psi}$$

Eq. (5-13):

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2\right)^{1/2}$$

$$= \left[7987^2 - 7987(-3072) + (-3072)\right]^{1/2} = 9888 \text{ psi} \quad \textit{Ans.}$$

(**b**) For a solid inner tube,

$$p = \frac{30(10^6)0.0005}{2(1^3)}\left[\frac{(1.5^2 - 1^2)(1^2)}{1.5^2}\right] = 4167 \text{ psi} \quad \textit{Ans.}$$

Inner member: $\sigma_t = \sigma_r = -p = -4167$ psi

$$\sigma' = \left[(-4167)^2 - (-4167)(-4167) + (-4167)^2\right]^{1/2} = 4167 \text{ psi} \quad \textit{Ans.}$$

Outer member: $p_i = p = 4167$ psi, $p_o = 0$. At fit surface $r = R = 1$ in,

Eq. (3-49): $$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 4167\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 10834 \text{ psi}$$

$$\sigma_r = -p = -4167 \text{ psi}$$

Eq. (5-13):

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2\right)^{1/2}$$

$$= \left[10\,834^2 - 10\,834(-4167) + (-4167)\right]^{1/2} = 13\,410 \text{ psi} \quad \textit{Ans.}$$

5-84 From Table A-5, E = 207 (10^3) MPa. The nominal radial interference is δ_{nom} = (40 – 39.98) /2 = 0.01 mm.

From Eq. (3-57),

$$p=\frac{E\delta}{2R^3}\left[\frac{\left(r_o^2-R^2\right)\left(R^2-r_i^2\right)}{r_o^2-r_i^2}\right]$$

$$=\frac{207\left(10^3\right)0.01}{2\left(20^3\right)}\left[\frac{\left(32.5^2-20^2\right)\left(20^2-10^2\right)}{32.5^2-10^2}\right]=26.64\text{ MPa}\quad \textit{Ans.}$$

Inner member: $p_i = 0$, $p_o = p = 26.64$ MPa. At fit surface $r = R = 20$ mm,

Eq. (3-49): $$\sigma_t=-p\frac{R^2+r_i^2}{R^2-r_i^2}=-26.64\left(\frac{20^2+10^2}{20^2-10^2}\right)=-44.40\text{ MPa}$$

$$\sigma_r = -\,p = -\,26.64\text{ MPa}$$

Eq. (5-13):

$$\sigma'=\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_A^2\right)^{1/2}$$

$$=\left[(-44.40)^2-(-44.40)(-26.64)+(-26.64)\right]^{1/2}=38.71\text{ MPa}\quad \textit{Ans.}$$

Outer member: $p_i = p = 26.64$ MPa, $p_o = 0$. At fit surface $r = R = 20$ mm,

Eq. (3-49): $$\sigma_t=p\frac{r_o^2+R^2}{r_o^2-R^2}=26.64\left(\frac{32.5^2+20^2}{32.5^2-20^2}\right)=59.12\text{ MPa}$$

$$\sigma_r = -\,p = -\,26.64\text{ MPa}$$

Eq. (5-13):

$$\sigma'=\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_A^2\right)^{1/2}$$

$$=\left[59.12^2-59.12(-26.64)+(-26.64)\right]^{1/2}=76.03\text{ MPa}\quad \textit{Ans.}$$

5-85 From Table A-5, E = 207 (10^3) MPa. The nominal radial interference is δ_{nom} = (40.008 – 39.972) /2 = 0.018 mm.

From Eq. (3-57),

$$p=\frac{E\delta}{2R^3}\left[\frac{\left(r_o^2-R^2\right)\left(R^2-r_i^2\right)}{r_o^2-r_i^2}\right]$$

$$=\frac{207\left(10^3\right)0.018}{2\left(20^3\right)}\left[\frac{\left(32.5^2-20^2\right)\left(20^2-10^2\right)}{32.5^2-10^2}\right]=47.94\text{ MPa}\quad Ans.$$

Inner member: $p_i = 0$, $p_o = p = 47.94$ MPa. At fit surface $r = R = 20$ mm,

Eq. (3-49): $$\sigma_t=-p\frac{R^2+r_i^2}{R^2-r_i^2}=-47.94\left(\frac{20^2+10^2}{20^2-10^2}\right)=-79.90\text{ MPa}$$

$$\sigma_r = -p = -47.94\text{ MPa}$$

Eq. (5-13):
$$\sigma'=\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_A^2\right)^{1/2}$$
$$=\left[\left(-79.90\right)^2-\left(-79.90\right)\left(-47.94\right)+\left(-47.94\right)\right]^{1/2}=69.66\text{ MPa}\quad Ans.$$

Outer member: $p_i = p = 47.94$ MPa, $p_o = 0$. At fit surface $r = R = 20$ mm,

Eq. (3-49): $$\sigma_t=p\frac{r_o^2+R^2}{r_o^2-R^2}=47.94\left(\frac{32.5^2+20^2}{32.5^2-20^2}\right)=106.4\text{ MPa}$$

$$\sigma_r = -p = -47.94\text{ MPa}$$

Eq. (5-13):
$$\sigma'=\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_A^2\right)^{1/2}$$
$$=\left[106.4^2-106.4\left(-47.94\right)+\left(-47.94\right)\right]^{1/2}=136.8\text{ MPa}\quad Ans.$$

5-86 From Table A-5, for carbon steel, $E_s = 30$ kpsi, and $\nu_s = 0.292$. While for $E_{ci} = 14.5$ Mpsi, and $\nu_{ci} = 0.211$. For ASTM grade 20 cast iron, from Table A-24, $S_{ut} = 22$ kpsi.

For midrange values, $\delta = (2.001 - 2.0002)/2 = 0.0004$ in.

Eq. (3-50):

$$p = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2+R^2}{r_o^2-R^2}+\nu_o\right)+\frac{1}{E_i}\left(\frac{R^2+r_i^2}{R^2-r_i^2}-\nu_i\right)\right]}$$

$$= \frac{0.0004}{1\left[\frac{1}{14.5\left(10^6\right)}\left(\frac{2^2+1^2}{2^2-1^2}+0.211\right)+\frac{1}{30\left(10^6\right)}\left(\frac{1^2}{1^2}-0.292\right)\right]} = 2613 \text{ psi}$$

At fit surface, with $p_i = p$ =2613 psi, and p_o = 0, from Eq. (3-50)

$$\sigma_t = p\frac{r_o^2+R^2}{r_o^2-R^2} = 2613\left(\frac{2^2+1^2}{2^2-1^2}\right) = 4355 \text{ psi}$$

$$\sigma_r = -p = -2613 \text{ psi}$$

From Modified-Mohr theory, Eq. (5-32a), since $\sigma_A > 0 > \sigma_B$ and $|\sigma_B/\sigma_A| < 1$,

$$n = \frac{S_{ut}}{\sigma_A} = \frac{22}{4.355} = 5.05 \qquad Ans.$$

5-87 $E = 207$ GPa

Eq. (3-57) can be written in terms of diameters,

$$p = \frac{E\delta_d}{2D^3}\left[\frac{(d_o^2-D^2)(D^2-d_i^2)}{(d_o^2-d_i^2)}\right] = \frac{207\left(10^3\right)(0.062)}{2\left(45\right)^3}\left[\frac{(50^2-45^2)(45^2-40^2)}{(50^2-40^2)}\right]$$

$$p = 15.80 \text{ MPa}$$

Outer member: From Eq. (3-50),

Outer radius: $$\left(\sigma_t\right)_o = \frac{45^2(15.80)}{50^2-45^2}(2) = 134.7 \text{ MPa}$$

$$\left(\sigma_r\right)_o = 0$$

Inner radius: $$\left(\sigma_t\right)_i = \frac{45^2(15.80)}{50^2-45^2}\left(1+\frac{50^2}{45^2}\right) = 150.5 \text{ MPa}$$

$$\left(\sigma_r\right)_i = -15.80 \text{ MPa}$$

Bending (no slipping): $I = (\pi/64)(50^4 - 40^4) = 181.1\ (10^3)\ \text{mm}^4$

$$\text{At } r_o: \qquad (\sigma_x)_o = \pm\frac{Mc}{I} = \pm\frac{675(0.05/2)}{181.1\left(10^{-9}\right)} = \pm 93.2(10^6)\ \text{Pa} = \pm 93.2\ \text{MPa}$$

$$\text{At } r_i: \qquad (\sigma_x)_i = \pm\frac{675(0.045/2)}{181.1\left(10^{-9}\right)} = \pm 83.9\left(10^6\right)\ \text{Pa} = \pm 83.9\ \text{MPa}$$

Torsion: $J = 2I = 362.2\ (10^3)\ \text{mm}^4$

$$\text{At } r_o: \qquad \left(\tau_{xy}\right)_o = \frac{Tc}{J} = \frac{900(0.05/2)}{362.2\left(10^{-9}\right)} = 62.1\left(10^6\right)\ \text{Pa} = 62.1\ \text{MPa}$$

$$\text{At } r_i: \qquad \left(\tau_{xy}\right)_i = \frac{900(0.045/2)}{362.2\left(10^{-9}\right)} = 55.9\left(10^6\right)\ \text{Pa} = 55.9\ \text{MPa}$$

Outer radius, is plane stress. Since the tangential stress is positive the von Mises stress will be a maximum with a negative bending stress. That is,

$$\sigma_x = -93.2\ \text{MPa},\ \sigma_y = 134.7\ \text{MPa},\ \tau_{xy} = 62.1\ \text{MPa}$$

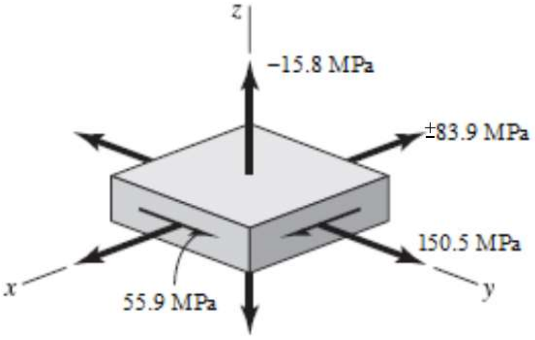
Eq. (5-15)

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$

$$= \left[(-93.2)^2 - (-93.2)(134.7) + 134.7^2 + 3(62.1)^2\right]^{1/2} = 226\ \text{MPa}$$

$$n_o = \frac{S_y}{\sigma'} = \frac{415}{226} = 1.84 \qquad \textit{Ans.}$$

Inner radius, 3D state of stress



From Eq. (5-14) with $\tau_{yz} = \tau_{zx} = 0$ and $\sigma_x = +\ 83.9$ MPa

$$\sigma' = \frac{1}{\sqrt{2}}\left[(83.9-150.5)^2 + (150.5+15.8)^2 + (-15.8-83.9)^2 + 6(55.9)^2\right]^{1/2} = 174\ \text{MPa}$$

With $\sigma_x = -\ 83.9$ MPa

$$\sigma' = \frac{1}{\sqrt{2}}\left[(-83.9-150.5)^2 + (150.5+15.8)^2 + (-15.8+83.9)^2 + 6(55.9)^2\right]^{1/2} = 230\ \text{MPa}$$

$$n_i = \frac{S_y}{\sigma'} = \frac{415}{230} = 1.80 \qquad Ans.$$

5-88 From the solution of Prob. 5-87, p = 15.80 MPa

Inner member: From Eq. (3-50),

Outer radius: $$(\sigma_t)_o = -\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} p_o = -\frac{45^2 + 40^2}{45^2 - 40^2}(15.80) = -134.8 \text{ MPa}$$

$$(\sigma_r)_o = -p = -15.80 \text{ MPa}$$

Inner radius: $$(\sigma_t)_i = -\frac{2r_o^2}{r_o^2 - r_i^2} p_o = -\frac{2\left(45^2\right)}{45^2 - 40^2}(15.80) = -150.6 \text{ MPa}$$

$$(\sigma_r)_i = 0$$

Bending (no slipping): $I = (\pi/64)(50^4 - 40^4) = 181.1\ (10^3)\ \text{mm}^4$

At r_o: $$(\sigma_x)_o = \pm\frac{Mc}{I} = \pm\frac{675(0.045/2)}{181.1\left(10^{-9}\right)} = \pm 83.9(10^6)\text{ Pa} = \pm 83.9 \text{ MPa}$$

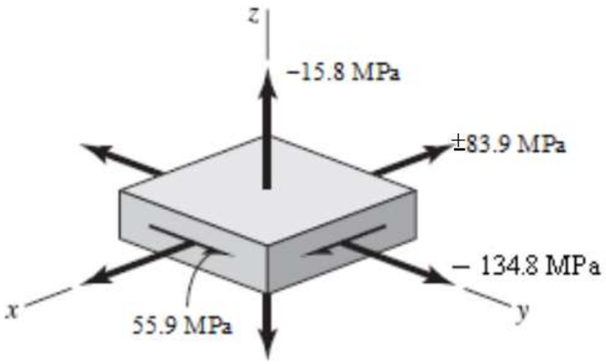
At r_i: $$(\sigma_x)_i = \pm\frac{675(0.040/2)}{181.1\left(10^{-9}\right)} = \pm 74.5\left(10^6\right)\text{ Pa} = \pm 74.5 \text{ MPa}$$

Torsion: $J = 2I = 362.2\ (10^3)\ \text{mm}^4$

At r_o: $$(\tau_{xy})_o = \frac{Tc}{J} = \frac{900(0.045/2)}{362.2\left(10^{-9}\right)} = 55.9\left(10^6\right)\text{ Pa} = 55.9 \text{ MPa}$$

At r_i: $$(\tau_{xy})_i = \frac{900(0.040/2)}{362.2\left(10^{-9}\right)} = 49.7\left(10^6\right)\text{ Pa} = 49.7 \text{ MPa}$$

Outer radius, 3D state of stress



From Eq. (5-14) with $\tau_{yz} = \tau_{zx} = 0$ and $\sigma_x = +\,83.9$ MPa

$$\sigma' = \frac{1}{\sqrt{2}}\left[(83.9+134.8)^2+(-134.8+15.8)^2+(-15.8-83.9)^2+6(55.9)^2\right]^{1/2} = 213\text{ MPa}$$

With $\sigma_x = -\,83.9$ MPa

$$\sigma' = \frac{1}{\sqrt{2}}\left[(-83.9+134.8)^2+(-134.8+15.8)^2+(-15.8+83.9)^2+6(55.9)^2\right]^{1/2} = 142\text{ MPa}$$

$$n_o = \frac{S_y}{\sigma'} = \frac{415}{213} = 1.95 \qquad Ans.$$

Inner radius, plane stress. Worst case is when σ_x is positive

$$\sigma_x = 74.5\text{ MPa},\ \sigma_y = -150.6\text{ MPa},\ \tau_{xy} = 49.7\text{ MPa}$$

Eq. (5-15)

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$

$$= \left[74.5^2 - 74.5(-150.6) + (-150.6)^2 + 3(49.7)^2\right]^{1/2} = 216\text{ MPa}$$

$$n_i = \frac{S_y}{\sigma'} = \frac{415}{216} = 1.92 \qquad Ans.$$

5-89 For AISI 1040 HR, from Table A-20, $S_y = 290$ MPa.

From Prob. 3-124, $p_{\max} = 65.2$ MPa. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 65.2\frac{50^2+25^2}{50^2-25^2} = 108.7\text{ MPa}$$

$$\sigma_r = -p = -65.2\text{ MPa}$$

These are principal stresses. From Eq. (5-13)

$$\sigma_o' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[108.7^2 - 108.7(-65.2) + (-65.2)^2\right]^{1/2} = 152.2\text{ MPa}$$

$$n = \frac{S_y}{\sigma_o'} = \frac{290}{152.2} = 1.91 \qquad Ans.$$

5-90 For AISI 1040 HR, from Table A-20, $S_y = 42$ kpsi.

From Prob. 3-125, $p_{\max} = 9$ kpsi. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 9\frac{2^2 + 1^2}{2^2 - 1^2} = 15 \text{ kpsi}$$

$$\sigma_r = -p = -9 \text{ kpsi}$$

These are principal stresses. From Eq. (5-13)

$$\sigma_o' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[15^2 - 15(-9) + (-9)^2\right]^{1/2} = 21 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma_o'} = \frac{42}{21} = 2 \qquad Ans.$$

5-91 For AISI 1040 HR, from Table A-20, $S_y = 290$ MPa.

From Prob. 3-126, $p_{\max} = 91.6$ MPa. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 91.6\frac{50^2 + 25^2}{50^2 - 25^2} = 152.7 \text{ MPa}$$

$$\sigma_r = -p = -91.6 \text{ MPa}$$

These are principal stresses. From Eq. (5-13)

$$\sigma_o' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[152.7^2 - 152.7(-91.6) + (-91.6)^2\right]^{1/2} = 213.8 \text{ MPa}$$

$$n = \frac{S_y}{\sigma_o'} = \frac{290}{213.8} = 1.36 \qquad Ans.$$

5-92 For AISI 1040 HR, from Table A-20, $S_y = 42$ kpsi.

From Prob. 3-127, $p_{\max} = 12.94$ kpsi. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 12.94\frac{2^2 + 1^2}{2^2 - 1^2} = 21.57 \text{ kpsi}$$

$$\sigma_r = -p = -12.94 \text{ kpsi}$$

These are principal stresses. From Eq. (5-13)

$$\sigma_o' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[21.57^2 - 21.57(-12.94) + (-12.94)^2\right]^{1/2} = 30.20 \text{ kpsi}$$

$$n=\frac{S_y}{\sigma'_o}=\frac{42}{30.2}=1.39 \qquad Ans.$$

5-93 For AISI 1040 HR, from Table A-20, S_y = 290 MPa.

From Prob. 3-128, $p_{\max}$ = 134 MPa. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t=p\frac{r_o^2+R^2}{r_o^2-R^2}=134\frac{50^2+25^2}{50^2-25^2}=223.3\text{ MPa}$$

$$\sigma_r=-p=-134\text{ MPa}$$

These are principal stresses. From Eq. (5-13)

$$\sigma'_o=\left(\sigma_t^2-\sigma_t\sigma_r+\sigma_r^2\right)^{1/2}=\left[223.3^2-223.3(-134)+(-134)^2\right]^{1/2}=312.6\text{ MPa}$$

$$n=\frac{S_y}{\sigma'_o}=\frac{290}{312.6}=0.93 \qquad Ans.$$

5-94 For AISI 1040 HR, from Table A-20, S_y = 42 kpsi.

From Prob. 3-129, $p_{\max}$ = 19.13 kpsi. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t=p\frac{r_o^2+R^2}{r_o^2-R^2}=19.13\frac{2^2+1^2}{2^2-1^2}=31.88\text{ kpsi}$$

$$\sigma_r=-p=-19.13\text{ kpsi}$$

These are principal stresses. From Eq. (5-13)

$$\sigma'_o=\left(\sigma_t^2-\sigma_t\sigma_r+\sigma_r^2\right)^{1/2}=\left[31.88^2-31.88(-19.13)+(-19.13)^2\right]^{1/2}=44.63\text{ kpsi}$$

$$n=\frac{S_y}{\sigma'_o}=\frac{42}{44.63}=0.94 \qquad Ans.$$

5-95

$$\sigma_p=\frac{1}{2}\left(2\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\right)\pm\left[\left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\sin\frac{3\theta}{2}\right)^2+\left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)^2\right]^{1/2}$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left[\cos\frac{\theta}{2} \pm \left(\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\sin^2\frac{3\theta}{2} + \sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\cos^2\frac{3\theta}{2}\right)^{1/2}\right]$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left(\cos\frac{\theta}{2} \pm \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 \pm \sin\frac{\theta}{2}\right)$$

Plane stress: The third principal stress is zero and

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\right), \ \sigma_2 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right), \ \sigma_3 = 0 \quad \textit{Ans.}$$

Plane strain: Equations for σ_1 andσ_2 are still valid,. However,

$$\sigma_3 = \nu(\sigma_1 + \sigma_2) = 2\nu\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2} \qquad \textit{Ans.}$$

5-96 For $\theta = 0$ and plane strain, the principal stress equations of Prob. 5-95 give

$$\sigma_1 = \sigma_2 = \frac{K_I}{\sqrt{2\pi r}}, \quad \sigma_3 = 2\nu\frac{K_I}{\sqrt{2\pi r}} = 2\nu\sigma_1$$

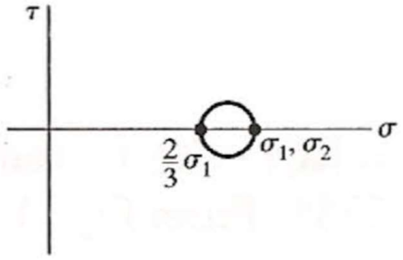
(**a**) DE: Eq. (5-18) $\frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_1)^2 + (\sigma_1 - 2\nu\sigma_1)^2 + (2\nu\sigma_1 - \sigma_1)^2\right]^{1/2} = S_y$

or, $\sigma_1 - 2\nu\sigma_1 = S_y$

For $\nu = \frac{1}{3}$, $\left[1-2\left(\frac{1}{3}\right)\right]\sigma_1 = S_y \quad \Rightarrow \quad \sigma_1 = 3S_y \quad$ *Ans.*

(**a**) MSS: Eq. (5-3) , with n =1 $\quad \sigma_1 - \sigma_3 = S_y \quad \Rightarrow \quad \sigma_1 - 2\nu\sigma_1 = S_y$

$$\nu = \frac{1}{3} \quad \Rightarrow \quad \sigma_1 = 3S_y \qquad \textit{Ans.}$$



$$\sigma_3 = \sigma_1 - S_y = 2S_y \quad \Rightarrow \quad \sigma_3 = \frac{2}{3}\sigma_1$$

Radius of largest circle

$$R = \frac{1}{2}\left(\sigma_1 - \frac{2}{3}\sigma_1\right) = \frac{\sigma_1}{6}$$

5-97 Given: $a = 16$ mm, $K_{Ic} = 80\ \text{MPa}\cdot\sqrt{\text{m}}$ and $S_y = 950$ MPa

(a) Ignoring stress concentration

$F = S_yA = 950(100 - 16)(12) = 958(10^3)\ \text{N} = 958\ \text{kN}$ *Ans.*

(b) From Fig. 5-26: $h/b = 1$, $a/b = 16/100 = 0.16$, $\beta = 1.3$

Eq. (5-37) $K_I = \beta\sigma\sqrt{\pi a}$

$$80 = 1.3\frac{F}{100(12)}\sqrt{\pi(16)10^{-3}}$$

$F = 329.4(10^3)\ \text{N} = 329.4\ \text{kN}$ *Ans.*

5-98 Given: $a = 0.5$ in, $K_{Ic} = 72\ \text{kpsi}\cdot\sqrt{\text{in}}$ and $S_y = 170$ kpsi, $S_{ut} = 192$ kpsi

$r_o = 14/2 = 7$ in, $r_i = (14 - 2)/2 = 6$ in

$$\frac{a}{r_o - r_i} = \frac{0.5}{7-6} = 0.5, \qquad \frac{r_i}{r_o} = \frac{6}{7} = 0.857$$

Fig. 5-30: β B 2.4

Eq. (5-37): $K_{Ic} = \beta\sigma\sqrt{\pi a} \quad \Rightarrow \quad 72 = 2.4\sigma\sqrt{\pi(0.5)} \quad \Rightarrow \quad \sigma = 23.9$ kpsi

Eq. (3-50) at $r = r_o = 7$ in:

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}(2) \quad \Rightarrow \quad 23.9 = \frac{6^2 p_i}{7^2 - 6^2}(2) \quad \Rightarrow \quad p_i = 4.315\ \text{kpsi} \qquad \textit{Ans.}$$

Chapter 6

6-1 Eq. (2-36): $S_{ut} = 3.4H_B = 3.4(300) = 1020$ MPa

Eq. (6-10): $S'_e = 0.5S_{ut} = 0.5(1020) = 510$ MPa

Table 6-2: $a = 1.38,\ b = -0.067$

Eq. (6-18): $k_a = aS_{ut}^b = 1.38(1020)^{-0.067} = 0.868$

Eq. (6-19): $k_b = 1.24d^{-0.107} = 1.24(10)^{-0.107} = 0.969$

Eq. (6-17): $S_e = k_a k_b S'_e = (0.868)(0.969)(510) = 429$ MPa *Ans.*

6-2 **(a)** Table A-20: $S_{ut} = 80$ kpsi

Eq. (6-10): $S'_e = 0.5(80) = 40$ kpsi *Ans.*

(b) Table A-20: $S_{ut} = 90$ kpsi

Eq. (6-10): $S'_e = 0.5(90) = 45$ kpsi *Ans.*

(c) Aluminum has no endurance limit. *Ans.*

(d) Eq. (6-10): $S_{ut} > 200$ kpsi, $S'_e = 100$ kpsi *Ans.*

6-3 $S_{ut} = 120$ kpsi, $\sigma_{ar} = 70$ kpsi

Fig. 6-23: $f = 0.82$

Eq. (6-10): $S'_e = S_e = 0.5(120) = 60$ kpsi

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.82(120)\right]^2}{60} = 161.4 \text{ kpsi}$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.82(120)}{60}\right) = -0.0716$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{70}{161.4}\right)^{\frac{1}{-0.0716}} = 117\,000 \text{ cycles} \quad \textit{Ans.}$$

6-4 $S_{ut} = 1600$ MPa, $\sigma_{ar} = 900$ MPa

Fig. 6-23: S_{ut} = 1600 MPa. Off the graph, so estimate f = 0.77.

Eq. (6-10): $S_{ut} > 1400$ MPa, so $S_e = 700$ MPa

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.77(1600)\right]^2}{700} = 2168.3 \text{ MPa}$$

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.77(1600)}{700}\right) = -0.081838$

Eq. (6-15): $N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{900}{2168.3}\right)^{\frac{1}{-0.081838}} = 46\,400 \text{ cycles}$ *Ans.*

6-5 $S_{ut} = 230 \text{ kpsi}, N = 150\,000 \text{ cycles}$

Fig. 6-23, point is off the graph, so estimate: $f = 0.77$

Eq. (6-10): $S_{ut} > 200$ kpsi, so $S'_e = S_e = 100$ kpsi

Eq. (6-13): $a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.77(230)]^2}{100} = 313.6 \text{ kpsi}$

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.77(230)}{100}\right) = -0.08274$

Eq. (6-12): $S_f = aN^b = 313.6(150\,000)^{-0.08274} = 117.0 \text{ kpsi}$ *Ans.*

6-6 $S_{ut} = 1100 \text{ MPa} = 160 \text{ kpsi}$

Fig. 6-23: $f = 0.79$

Eq. (6-10): $S'_e = S_e = 0.5(1100) = 550 \text{ MPa}$

Eq. (6-13): $a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.79(1100)]^2}{550} = 1373 \text{ MPa}$

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.79(1100)}{550}\right) = -0.06622$

Eq. (6-12): $S_f = aN^b = 1373(150\,000)^{-0.06622} = 624 \text{ MPa}$ *Ans.*

6-7 $S_{ut} = 150 \text{ kpsi}, S_{yt} = 135 \text{ kpsi}, N = 500 \text{ cycles}$

Fig. 6-23: $f = 0.80$

From Fig. 6-21, we note that below 10^3 cycles on the *S-N* diagram constitutes the low-cycle region. The stress-life approach is not very reliable in this region, but for a rough response to this question, we can write an equation in log-log scale for the line between (10^0, S_{ut}) and (10^3, fS_{ut}) as

$$S_f = S_{ut} N^{(\log f)/3} = 150(500)^{[\log(0.80)]/3} = 123 \text{ kpsi} \qquad Ans.$$

The testing should be done at a completely reversed stress of 123 kpsi, which is below the yield strength, so it is possible. *Ans.*

6-8 $d = 1.5$ in, $S_{ut} = 110$ kpsi

Eq. (6-10): $S'_e = 0.5(110) = 55$ kpsi

Table 6-2: $a = 2.00$, $b = -0.217$

Eq. (6-18): $k_a = aS_{ut}{}^{b} = 2.00(110)^{-0.217} = 0.721$

Eq. (6-19): $k_b = 0.879\, d^{-0.107} = 0.879(1.5)^{-0.107} = 0.842$

Eq. (6-17): $S_e = k_a k_b S'_e = 0.721(0.842)(55) = 33.4$ kpsi *Ans.*

6-9 For AISI 4340 as-forged steel,

Eq. (6-10): $S_e = 100$ kpsi

Table 6-2: $a = 12.7$, $b = -0.758$

Eq. (6-18): $k_a = 12.7(260)^{-0.758} = 0.188$

Eq. (6-19): $k_b = \left(\dfrac{0.75}{0.30}\right)^{-0.107} = 0.907$

Each of the other modifying factors is unity.

$$S_e = 0.188(0.907)(100) = 17.1 \text{ kpsi} \qquad Ans.$$

For AISI 1040:

$$S'_e = 0.5(113) = 56.5 \text{ kpsi}$$

$$k_a = 12.7(113)^{-0.758} = 0.353$$

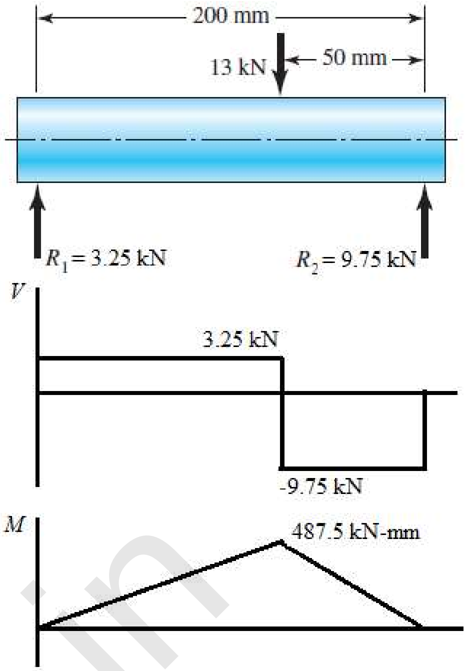
$$k_b = 0.907 \text{ (same as 4340)}$$

Each of the other modifying factors is unity

$$S_e = 0.353(0.907)(56.5) = 18.1 \text{ kpsi} \qquad Ans.$$

Not only is AISI 1040 steel a contender, it has a superior endurance strength. *Ans.*

6-10 From Table A-20, $S_{ut} = 570$ MPa, $S_y = 310$ MPa. From a free-body diagram analysis, the bearing reaction forces are found to be $R_1 = 3.25$ kN and $R_2 = 9.75$ kN. The shear-force and bending-moment diagrams are shown. The critical location is at the section where the bending moment is maximum, on the outer surface where the bending stress is maximum. With a rotating shaft, the bending stress will be completely reversed.



$$\sigma_{\max} = \sigma_{ar} = \frac{Mc}{I} = \frac{487\,500(25/2)}{(\pi/64)(25)^4} = 317.8 \text{ MPa}$$

(a) $$n_y = \frac{S_y}{\sigma_{\max}} = \frac{310}{317.8} = 0.98 \qquad \textit{Ans.}$$

Yielding is predicted, on the outer surface. For some applications, this might not prevent the part from being used, so we will continue checking for fatigue.

(b) Eq. (6-10): $S_e' = 0.5S_{ut} = 0.5(570) = 285$ MPa

Eq. (6-18): $k_a = aS_{ut}^b = 3.04(570)^{-0.217} = 0.767$

Eq. (6-19): $k_b = 1.24d^{-0.107} = 1.24(25)^{-0.107} = 0.879$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.767)(0.879)(1)(285) = 192$ MPa *Ans.*

(c) For completely reversed stress, the fatigue factor of safety can be assessed as the ratio of the endurance limit to the completely reversed stress.

$$n_f = \frac{S_e}{\sigma_{ar}} = \frac{192}{317.8} = 0.60 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S-N* diagram to estimate the life.

(d) Fig. 6-23, or Eq. (6-11): $f = 0.87$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.87(570)]^2}{192} = 1280.8$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(570)}{192}\right) = -0.1374$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{\frac{1}{b}} = \left(\frac{317.8}{1280.8}\right)^{\frac{1}{-0.1374}} = 25444$$

$N = 25000$ cycles *Ans.*

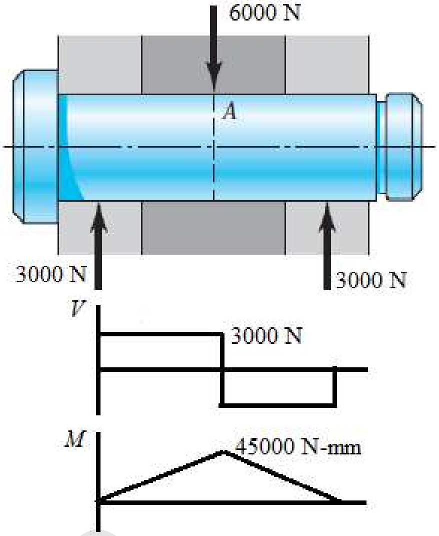
6-11 From Table A-20, $S_{ut} = 400$ MPa, $S_y = 220$ MPa. Free-body, shear-force, and bending-moment diagrams are shown.

$$\sigma_{\max} = \frac{32M}{\pi d^3} = \frac{32(45000)}{\pi d^3} = 458366 / d^3$$

The load is repeatedly applied and released, so from Eqs. (6-8) and (6-9),

$$\sigma_m = \sigma_a = \sigma_{\max} / 2 = 229183 / d^3$$

Be sure to confirm that the units are legitimate for stress in MPa and d in mm.



For yielding,

$$n_y = 1.5 = \frac{S_y}{\sigma_{\max}} = \frac{220}{458366 / d^3}$$

$$d = 14.62 \text{ mm}$$

Now check fatigue, opting for the linear Goodman criterion for simplicity of solving for the diameter. First, determine the adjusted endurance limit.

Eq. (6-10): $S_e^{'} = 0.5 S_{ut} = 0.5(400) = 200$ MPa

Eq. (6-18): $k_a = a S_{ut}^b = 3.04(400)^{-0.217} = 0.828$

Estimate the size factor from the diameter determined for yielding. It can be adjusted later.

Eq. (6-19): $k_b = 1.24 d^{-0.107} = 1.24(15)^{-0.107} = 0.93$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e^{'} = (0.767)(0.879)(1)(285) = 192$ MPa *Ans.*

(c) For completely reversed stress, the fatigue factor of safety can be assessed as the ratio of the endurance limit to the completely reversed stress.

$$n_f = \frac{S_e}{\sigma_{ar}} = \frac{192}{317.8} = 0.60 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S-N* diagram to estimate the life.

(d) Fig. 6-23, or Eq. (6-11): $f = 0.87$

Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.87(570)]^2}{192} = 1280.8$

Eq. (6-14): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.87(570)}{192}\right) = -0.1374$

Eq. (6-15): $N = \left(\dfrac{\sigma_{ar}}{a}\right)^{\frac{1}{b}} = \left(\dfrac{317.8}{1280.8}\right)^{\frac{1}{-0.1374}} = 25444$

$N = 25000$ cycles *Ans.*

6-12 $D = 1$ in, $d = 0.8$ in, $T = 1800$ lbf·in, and from Table A-20 for AISI 1020 CD, $S_{ut} = 68$ kpsi, and $S_y = 57$ kpsi.

(a) Fig. A-15-15: $\dfrac{r}{d} = \dfrac{0.1}{0.8} = 0.125, \dfrac{D}{d} = \dfrac{1}{0.8} = 1.25, K_{ts} = 1.40$

Get the notch sensitivity either from Fig. 6-27, or from the curve-fit Eqs. (6-33) and (6-36). Using the equations,

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)(68) + 1.35\left(10^{-5}\right)(68)^2 - 2.67\left(10^{-8}\right)\left(68^3\right) = 0.07335$$

$$q_s = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07335}{\sqrt{0.1}}} = 0.812$$

Eq. (6-32): $K_{fs} = 1 + q_s\,(K_{ts} - 1) = 1 + 0.812(1.40 - 1) = 1.32$

For a purely reversing torque of $T = 1800$ lbf·in,

$$\tau_a = K_{fs}\frac{Tr}{J} = \frac{K_{fs}16T}{\pi d^3} = \frac{1.32(16)(1800)}{\pi(0.8)^3} = 23\,635 \text{ psi} = 23.6 \text{ kpsi}$$

Eq. (6-10): $S'_e = 0.5(68) = 34$ kpsi

Eq. (6-18): $k_a = 2.00(68)^{-0.217} = 0.80$
Eq. (6-19): $k_b = 0.879(0.8)^{-0.107} = 0.90$
Eq. (6-25): $k_c = 0.59$
Eq. (6-17) (labeling for shear): $S_{se} = 0.80(0.90)(0.59)(34) = 14.4$ kpsi

For purely reversing torsion, use Eq. (6-58) for the ultimate strength in shear.
Eq. (6-58): $S_{su} = 0.67\,S_{ut} = 0.67(68) = 45.6$ kpsi
Fig. 6-23: $f = 0.9$

Adjusting the fatigue strength equations for shear,

Eq. (6-13): $$a = \frac{(f\,S_{su})^2}{S_{se}} = \frac{[0.9(45.6)]^2}{14.4} = 117.0 \text{ kpsi}$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{su}}{S_{se}}\right) = -\frac{1}{3}\log\left(\frac{0.9(45.6)}{14.4}\right) = -0.151\,61$$

Eq. (6-15): $$N = \left(\frac{\tau_a}{a}\right)^{\frac{1}{b}} = \left(\frac{23.6}{117.0}\right)^{\frac{1}{-0.151\,61}} = 38.5\left(10^3\right) \text{ cycles} \quad \textit{Ans.}$$

(b) Estimate the ultimate strength at the operating temperature.

Eq. (6-26): $(S_T/S_{RT})_{750^\circ} = 0.98 + 3.5(10^{-4})(750) - 6.3(10^{-7})750^2 = 0.89$

Thus, $(S_{ut})_{750^\circ} = (S_T/S_{RT})_{750^\circ}\,(S_{ut})_{70^\circ} = 0.89(68) = 60.5$ kpsi

Eq. (6-10): $(S_e')_{750^\circ} = 0.5(S_{ut})_{750^\circ} = 0.5(60.5) = 30.3$ kpsi

Eq. (6-17): $S_{se} = 0.80(0.90)(0.59)(30.3) = 12.9$ kpsi

Note that we use $k_d = 1$ since the ultimate strength has been adjusted for the operating temperature.

Eq. (6-58): $S_{su} = 0.67(S_{ut})_{750^\circ} = 0.67(60.5) = 40.5$ kpsi

$$a = \frac{(f\,S_{su})^2}{S_{se}} = \frac{[0.9(40.5)]^2}{12.9} = 103.0 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{su}}{S_{se}}\right) = -\frac{1}{3}\log\left(\frac{0.9(40.5)}{12.9}\right) = -0.150\,37$$

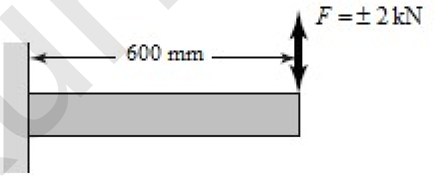
$$N = \left(\frac{\tau_a}{a}\right)^{\frac{1}{b}} = \left(\frac{23.6}{103.0}\right)^{\frac{1}{-0.150\,37}} = 18.0(10^3) \text{ cycles} \qquad \textit{Ans.}$$

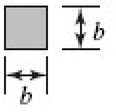
6-13 $L = 0.6$ m, $F_a = 2$ kN, $n = 1.5$, $N = 10^4$ cycles, $S_{ut} = 770$ MPa, $S_y = 420$ MPa (Table A-20)

First evaluate the fatigue strength.

$$S_e' = 0.5(770) = 385 \text{ MPa}$$

$$k_a = 38.6(770)^{-0.650} = 0.51$$





Since the size is not yet known, assume a typical value of $k_b = 0.85$ and check later. All other modifiers are equal to one.

Eq. (6-17): $S_e = 0.51(0.85)(385) = 167$ MPa

Fig. 6-23: $f = 0.83$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.83(770)]^2}{167} = 2446 \text{ MPa}$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.83(770)}{167}\right) = -0.1943$$

Eq. (6-12): $S_f = aN^b = 2446(10^4)^{-0.1943} = 409$ MPa

Now evaluate the stress.

$$M_{\max} = (2000 \text{ N})(0.6 \text{ m}) = 1200 \text{ N}\cdot\text{m}$$

$$\sigma_a = \sigma_{\max} = \frac{Mc}{I} = \frac{M(b/2)}{b(b^3)/12} = \frac{6M}{b^3} = \frac{6(1200)}{b^3} = \frac{7200}{b^3} \text{ Pa, with } b \text{ in m.}$$

Compare strength to stress and solve for the necessary b.

$$n = \frac{S_f}{\sigma_a} = \frac{409\left(10^6\right)}{7200/b^3} = 1.5$$

$$b = 0.0298 \text{ m} \quad \text{Select } b = 30 \text{ mm.}$$

Since the size factor was guessed, go back and check it now.

Eq. (6-24): $d_e = 0.808\left(hb\right)^{1/2} = 0.808b = 0.808\left(30\right) = 24.2 \text{ mm}$

Eq. (6-19): $k_b = \left(\dfrac{24.2}{7.62}\right)^{-0.107} = 0.88$

Our guess of 0.85 was slightly conservative, so we will accept the result of

$$b = 30 \text{ mm.} \qquad \textit{Ans.}$$

Checking yield,

$$\sigma_{\max} = \frac{7200}{0.030^3}\left(10^{-6}\right) = 267 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{420}{267} = 1.57$$

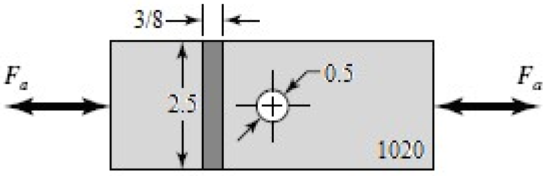
6-14 Given: w =2.5 in, t = 3/8 in, d = 0.5 in, n_d = 2. From Table A-20, for AISI 1020 CD, S_{ut} = 68 kpsi and S_y = 57 kpsi.

Eq. (6-10): $S'_e = 0.5(68) = 34$ kpsi

Table 6-2: $k_a = 2.00(68)^{-0.217} = 0.80$

Eq. (6-20): k_b = 1 (axial loading)

Eq. (6-25): k_c = 0.85



Eq. (6-17): S_e = 0.80(1)(0.85)(34) = 23.1 kpsi

Table A-15-1: $d/w = 0.5/2.5 = 0.2,\ K_t = 2.5$

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). The relatively large radius is off the graph of Fig. 6-26, so we will assume the curves continue according to the same trend and use the equations to estimate the notch sensitivity.

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)\left(68\right) + 1.51\left(10^{-5}\right)\left(68\right)^2 - 2.67\left(10^{-8}\right)\left(68^3\right) = 0.09799$$

$$q = \frac{1}{1+\dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\dfrac{0.09799}{\sqrt{0.25}}} = 0.836$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.836(2.5 - 1) = 2.25$

$$\sigma_a = K_f \frac{F_a}{A} = \frac{2.25F_a}{(3/8)(2.5-0.5)} = 3F_a$$

Since a finite life was not mentioned, we'll assume infinite life is desired, so the completely reversed stress must stay below the endurance limit.

$$n_f = \frac{S_e}{\sigma_a} = \frac{23.1}{3F_a} = 2$$

$$F_a = 3.85 \text{ kips} \quad \textit{Ans.}$$

6-15 Given: $D = 2$ in, $d = 1.8$ in, $r = 0.1$ in, $M_{\max} = 25\,000$ lbf $\cdot$ in, $M_{\min} = 0$.
From Table A-20, for AISI 1095 HR, S_{ut} = 120 kpsi and S_y = 66 kpsi.

Eq. (6-10): $S_e' = 0.5S_{ut} = 0.5(120) = 60$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(120)^{-0.217} = 0.71$

Eq. (6-23): $d_e = 0.370d = 0.370(1.8) = 0.666$ in

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.666)^{-0.107} = 0.92$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.71)(0.92)(1)(60) = 39.2$ kpsi

Fig. A-15-14: $D/d = 2/1.8 = 1.11, \quad r/d = 0.1/1.8 = 0.056 \quad \Rightarrow K_t = 2.1$

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). Using the equations,

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(120) + 1.51\left(10^{-5}\right)(120)^2 - 2.67\left(10^{-8}\right)\left(120^3\right) = 0.04770$$

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\frac{0.04770}{\sqrt{0.1}}} = 0.87$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.87(2.1-1) = 1.96$

$$I = (\pi/64)d^4 = (\pi/64)(1.8)^4 = 0.5153 \text{ in}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{25\,000(1.8/2)}{0.5153} = 43\,664 \text{ psi} = 43.7 \text{ kpsi}$$

$$\sigma_{\min} = 0$$

Eqs. (6-8) and (6-9): $\sigma_m = K_f \dfrac{\sigma_{\max}+\sigma_{\min}}{2} = (1.96)\dfrac{(43.7+0)}{2} = 42.8 \text{ kpsi}$

$$\sigma_a = K_f \left|\frac{\sigma_{\max}-\sigma_{\min}}{2}\right| = (1.96)\left|\frac{(43.7-0)}{2}\right| = 42.8 \text{ kpsi}$$

Eq. (6-41): $n_f = \left(\dfrac{\sigma_a}{S_e}+\dfrac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\dfrac{42.8}{39.2}+\dfrac{42.8}{120}\right)^{-1}$

$$n_f = 0.69 \qquad Ans.$$

A factor of safety less than unity indicates a finite life.

Check for yielding. It is not necessary to include the stress concentration for static yielding of a ductile material.

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{66}{43.7} = 1.51 \qquad Ans.$$

6-16 From a free-body diagram analysis, the bearing reaction forces are found to be 2.1 kN at the left bearing and 3.9 kN at the right bearing. The critical location will be at the shoulder fillet between the 35 mm and the 50 mm diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists. The bending moment at this point is M = 2.1(200) = 420 kN·mm. With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{ar} = \frac{Mc}{I} = \frac{420\,(35/2)}{(\pi/64)(35)^4} = 0.09978 \text{ kN/mm}^2 = 99.8 \text{ MPa}$$

This stress is far below the yield strength of 390 MPa, so yielding is not predicted. Find the stress concentration factor for the fatigue analysis.

Fig. A-15-9: r/d = 3/35 = 0.086, D/d = 50/35 = 1.43, K_t =1.7

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). Using the equations, with S_{ut} = 470 MPa and r = 3 mm,

$$\sqrt{a} = 1.24 - 2.25\left(10^{-3}\right)(470) + 1.60\left(10^{-6}\right)(470)^2 - 4.11\left(10^{-10}\right)(470)^3 = 0.4933$$

$$q = \frac{1}{1+\dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\dfrac{0.4933}{\sqrt{3}}} = 0.78$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.78(1.7 - 1) = 1.55$

Eq. (6-10): $S_e' = 0.5 S_{ut} = 0.5(470) = 235 \text{ MPa}$

Eq. (6-18): $k_a = a S_{ut}^b = 3.04(470)^{-0.217} = 0.80$

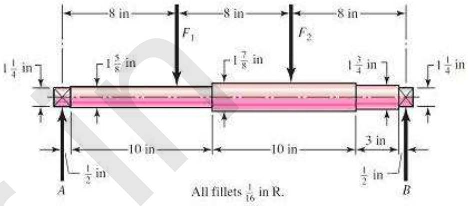
Eq. (6-19): $k_b = 1.24 d^{-0.107} = 1.24(35)^{-0.107} = 0.85$

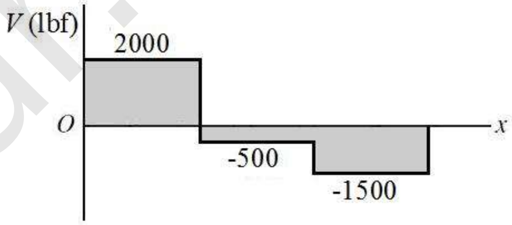
Eq. (6-25): $k_c = 1$

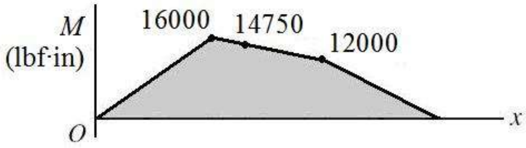
Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.80)(0.85)(1)(235) = 160 \text{ MPa}$

$$n_f = \frac{S_e}{K_f \sigma_{ar}} = \frac{160}{1.55(99.8)} = 1.03$$ Infinite life is predicted. *Ans.*

6-17 From a free-body diagram analysis, the bearing reaction forces are found to be R_A = 2000 lbf and R_B = 1500 lbf. The shear-force and bending-moment diagrams are shown. The critical location will be at the shoulder fillet between the 1-5/8 in and the 1-7/8 in diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists.







M = 16 000 – 500 (2.5) = 14 750 lbf · in

With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{ar} = \frac{Mc}{I} = \frac{14\,750(1.625/2)}{(\pi/64)(1.625)^4} = 35.0 \text{ kpsi}$$

This stress is far below the yield strength of 71 kpsi, so yielding is not predicted.

Fig. A-15-9: $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_t = 1.95$

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). Using the equations,

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(85) + 1.51\left(10^{-5}\right)(85)^2 - 2.67\left(10^{-8}\right)(85)^3 = 0.07690$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07690}{\sqrt{0.0625}}} = 0.76.$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.76(1.95 - 1) = 1.72$

Eq. (6-10): $S_e' = 0.5S_{ut} = 0.5(85) = 42.5$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(85)^{-0.217} = 0.76$

Eq. (6-19): $k_b = 0.879d^{-0.107} = 0.879(1.625)^{-0.107} = 0.835$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.76)(0.835)(1)(42.5) = 27.0$ kpsi

$$n_f = \frac{S_e}{K_f \sigma_{\text{ar}}} = \frac{27.0}{1.72\left(35.0\right)} = 0.45 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S-N* diagram to estimate the life.

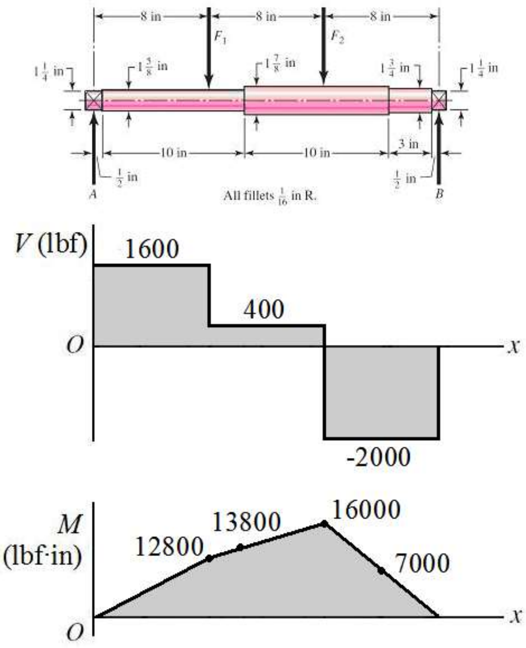
Fig. 6-23: $f = 0.87$

Eq. (6-13): $$a = \frac{\left(f\,S_{ut}\right)^2}{S_e} = \frac{\left[0.87(85)\right]^2}{27.0} = 202.5$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(85)}{27.0}\right) = -0.1459$$

Eq. (6-15): $$N = \left(\frac{K_f \sigma_{ar}}{a}\right)^{\frac{1}{b}} = \left(\frac{(1.72)(35.0)}{202.5}\right)^{\frac{1}{-0.1459}} = 4082 \text{ cycles}$$

$N = 4100$ cycles *Ans.*



6-18 From a free-body diagram analysis, the bearing reaction forces are found to be R_A = 1600 lbf and R_B = 2000 lbf. The shear-force and bending-moment diagrams are shown. The critical location will be at the shoulder fillet between the 1-5/8 in and the 1-7/8 in diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists.

M = 12 800 + 400 (2.5) = 13 800 lbf · in

With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{ar} = \frac{Mc}{I} = \frac{13\,800(1.625/2)}{(\pi/64)(1.625)^4} = 32.8 \text{ kpsi}$$

This stress is far below the yield strength of 71 kpsi, so yielding is not predicted.

Fig. A-15-9: $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_t = 1.95$

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). Using the equations,

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)\left(85\right) + 1.51\left(10^{-5}\right)\left(85\right)^2 - 2.67\left(10^{-8}\right)\left(85\right)^3 = 0.07690$$

$$q = \frac{1}{1+\dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\dfrac{0.07690}{\sqrt{0.0625}}} = 0.76$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.76(1.95 - 1) = 1.72$

Eq. (6-10): $S_e^{'} = 0.5S_{ut} = 0.5(85) = 42.5$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(85)^{-0.217} = 0.76$

Eq. (6-19): $k_b = 0.879d^{-0.107} = 0.879(1.625)^{-0.107} = 0.835$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e^{'} = (0.76)(0.835)(1)(42.5) = 27.0$ kpsi

$$n_f = \frac{S_e}{K_f \sigma_{ar}} = \frac{27.0}{1.72\left(32.8\right)} = 0.48 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S-N* diagram to estimate the life.

Fig. 6-23: $f = 0.87$

Eq. (6-13): $$a = \frac{\left(f\,S_{ut}\right)^2}{S_e} = \frac{\left[0.87(85)\right]^2}{27.0} = 202.5$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(85)}{27.0}\right) = -0.1459$$

Eq. (6-15): $$N = \left(\frac{K_f \sigma_{ar}}{a}\right)^{\frac{1}{b}} = \left(\frac{(1.72)(32.8)}{202.5}\right)^{\frac{1}{-0.1459}} = 6370 \text{ cycles}$$

$N = 6400$ cycles *Ans.*

6-19 Table A-20: $S_{ut} = 120$ kpsi, $S_y = 66$ kpsi

N = (950 rev/min)(10 hr)(60 min/hr) = 570 000 cycles

One approach is to guess a diameter and solve the problem as an iterative analysis problem. Alternatively, we can estimate the few modifying parameters that are dependent on the diameter and solve the stress equation for the diameter, then iterate to check the estimates. We'll use the second approach since it should require only one iteration, since the estimates on the modifying parameters should be pretty close.

First, we will evaluate the stress. From a free-body diagram analysis, the reaction forces at the bearings are $R_1 = 2$ kips and $R_2 = 6$ kips. The critical stress location is in the middle of the span at the shoulder, where the bending moment is high, the shaft diameter is smaller, and a stress concentration factor exists. If the critical location is not obvious, prepare a complete bending moment diagram and evaluate at any potentially critical locations. Evaluating at the critical shoulder,

$$M = 2\text{ kip}(10\text{ in}) = 20\text{ kip}\cdot\text{in}$$

$$\sigma_{ar} = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(20)}{\pi d^3} = \frac{203.7}{d^3}\text{ kpsi}$$

Now we will get the notch sensitivity and stress concentration factor. The notch sensitivity depends on the fillet radius, which depends on the unknown diameter. For now, let us estimate a value of $q = 0.85$ from observation of Fig. 6-26, and check it later.

Fig. A-15-9: $D/d = 1.4d/d = 1.4, \quad r/d = 0.1d/d = 0.1, \quad K_t = 1.65$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.85(1.65 - 1) = 1.55$

Now, evaluate the fatigue strength.

$$S_e' = 0.5(120) = 60\text{ kpsi}$$
$$k_a = 2.00(120)^{-0.217} = 0.71$$

Since the diameter is not yet known, assume a typical value of $k_b = 0.85$ and check later. All other modifiers are equal to one.

$$S_e = (0.71)(0.85)(60) = 36.2\text{ kpsi}$$

Determine the desired fatigue strength from the *S-N* diagram.

Fig. 6-23: $f = 0.82$

Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.82(120)]^2}{36.2} = 267.5$

Eq. (6-14): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.82(120)}{36.2}\right) = -0.1448$

Eq. (6-12): $S_f = aN^b = 267.5(570\,000)^{-0.1448} = 39.3\text{ kpsi}$

Compare strength to stress and solve for the necessary *d*.

$$n_f = \frac{S_f}{K_f \sigma_{ar}} = \frac{39.3}{(1.55)(203.7/d^3)} = 1.6$$

$$d = 2.34 \text{ in}$$

Since the size factor and notch sensitivity were guessed, go back and check them now.

Eq. (6-19): $k_b = 0.91 d^{-0.157} = 0.91(2.34)^{-0.157} = 0.80$

This is a little lower than our initial guess.
From Fig. 6-26 with $r = d/10 = 0.234$ in, we are off the graph, but it appears our guess for q of 0.85 is low. Assuming the trend of the graph continues, we'll choose $q = 0.91$ and iterate the problem with the new values of k_b and q.
Intermediate results are $S_e = 34.1$ kpsi, $S_f = 37.2$ kpsi, and $K_f = 1.59$. This gives

$$n_f = \frac{S_f}{K_f \sigma_{ar}} = \frac{37.2}{(1.59)(203.7/d^3)} = 1.6$$

$d = 2.41$ in *Ans.*

A quick check of k_b and q show that our estimates are still reasonable for this diameter.

6-20 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_m = 15$ kpsi, $\sigma_a = 25$ kpsi, $\sigma_m = \tau_a = 0$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[25^2 + 3(0)^2\right]^{1/2} = 25.00 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(15)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2\right]^{1/2}$$

$$= \left[25^2 + 3(15^2)\right]^{1/2} = 36.06 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{36.06} = 1.66 \quad \textit{Ans.}$$

(a) Goodman, Equation (6-41)

$$n_f = \frac{1}{(25.00/40) + (25.98/80)} = 1.05 \quad \textit{Ans.}$$

(b) Gerber, Equation (6-48)

$$n_f = \frac{1}{2}\left(\frac{80}{25.98}\right)^2 \left(\frac{25.00}{40}\right)\left[-1 + \sqrt{1 + \left(\frac{2(25.98)(40)}{80(25.00)}\right)^2}\right] = 1.31 \quad \textit{Ans.}$$

(c) Morrow

Estimate the fatigue strength coefficient.

Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 80 + 50 = 130$ kpsi

Eq. (6-46): $$n_f = \frac{1}{(25.00/40) + (25.98/130)} = 1.21 \qquad \textit{Ans.}$$

6-21 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_m = 20$ kpsi, $\sigma_a = 10$ kpsi, $\sigma_m = \tau_a = 0$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[10^2 + 3\left(0\right)^2\right]^{1/2} = 10.00 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(20\right)^2\right]^{1/2} = 34.64 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[10^2 + 3\left(20^2\right)\right]^{1/2} = 36.06 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{36.06} = 1.66 \qquad \textit{Ans.}$$

(a) Goodman, Equation (6-41)

$$n_f = \frac{1}{(10.00/40) + (34.64/80)} = 1.46 \qquad \textit{Ans.}$$

(b) Gerber, Equation (6-48)

$$n_f = \frac{1}{2}\left(\frac{80}{34.64}\right)^2\left(\frac{10.00}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(34.64)(40)}{80(10.00)}\right)^2}\right\} = 1.74 \qquad \textit{Ans.}$$

(c) Morrow

Estimate the fatigue strength coefficient.

Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 80 + 50 = 130$ kpsi

Eq. (6-46): $$n_f = \frac{1}{(10.00/40) + (34.64/130)} = 1.94 \qquad \textit{Ans.}$$

6-22 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 10$ kpsi, $\tau_m = 15$ kpsi, $\sigma_a = 12$ kpsi, $\sigma_m = 0$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[12^2 + 3\left(10\right)^2\right]^{1/2} = 21.07 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(15\right)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[\left(12+0\right)^2 + 3\left(10+15\right)^2\right]^{1/2} = 44.93 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{44.93} = 1.34 \quad \textit{Ans.}$$

(a) Goodman, Equation (6-41)

$$n_f = \frac{1}{(21.07/40)+(25.98/80)} = 1.17 \quad \textit{Ans.}$$

(b) Gerber, Equation (6-48)

$$n_f = \frac{1}{2}\left(\frac{80}{25.98}\right)^2\left(\frac{21.07}{40}\right)\left\{-1+\sqrt{1+\left(\frac{2(25.98)(40)}{80(21.07)}\right)^2}\right\} = 1.47 \quad \textit{Ans.}$$

(c) Morrow
Estimate the fatigue strength coefficient.
Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 80 + 50 = 130$ kpsi

Eq. (6-46): $n_f = \dfrac{1}{(21.07/40)+(25.98/130)} = 1.38 \quad \textit{Ans.}$

6-23 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 30$ kpsi, $\sigma_m = \sigma_a = \tau_a = 0$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[0^2 + 3\left(30\right)^2\right]^{1/2} = 51.96 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = 0 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[3\left(30\right)^2\right]^{1/2} = 51.96 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{51.96} = 1.15 \quad \textit{Ans.}$$

(a) through (c)
With a mean stress of zero, the Goodman, Gerber, and Morrow criteria all simplify to the same simple comparison of the alternating stress to the endurance limit,

$$n_f = \frac{S_e}{\sigma'_a} = \frac{40}{51.96} = 0.77 \quad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. Since $\sigma'_m = 0$, the stress state is completely reversed and the *S-N* diagram is applicable for σ'_a.

Fig. 6-23: $f = 0.875$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.875(80)]^2}{40} = 122.5$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.875(80)}{40}\right) = -0.08101$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{51.96}{122.5}\right)^{\frac{1}{-0.08101}} = 39\ 600 \text{ cycles} \qquad Ans.$$

6-24 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 15$ kpsi, $\sigma_m = 15$ kpsi, $\tau_m = \sigma_a = 0$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[0^2 + 3\left(15\right)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[15^2 + 3\left(0\right)^2\right]^{1/2} = 15.00 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[\left(15\right)^2 + 3\left(15\right)^2\right]^{1/2} = 30.00 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{30} = 2.00 \qquad Ans.$$

(a) Goodman, Eq. (6-41)

$$n_f = \frac{1}{(25.98/40) + (15.00/80)} = 1.19 \qquad Ans.$$

(b) Gerber, Eq. (6-48)

$$n_f = \frac{1}{2}\left(\frac{80}{15.00}\right)^2\left(\frac{25.98}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(15.00)(40)}{80(25.98)}\right)^2}\right\} = 1.43 \qquad Ans.$$

(c) Morrow

Estimate the fatigue strength coefficient.

Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 80 + 50 = 130$ kpsi

Eq. (6-46): $$n_f = \frac{1}{(25.98/40) + (15.00/130)} = 1.31 \qquad Ans.$$

6-25 Given: $F_{\max} = 28\text{ kN}, F_{\min} = -28\text{ kN}$. From Table A-20, for AISI 1040 CD, $S_{ut} = 590\text{ MPa}, S_y = 490\text{ MPa},$

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4\text{ N/mm}^2 = 147.4\text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \qquad \textit{Ans.}$$

Determine the fatigue factor of safety based on infinite life

Eq. (6-10): $S_e' = 0.5(590) = 295\text{ MPa}$

Eq. (6-18): $k_a = aS_{ut}^b = 3.04(590)^{-0.217} = 0.76$

Eq. (6-20): $k_b = 1 \quad \text{(axial)}$

Eq. (6-25): $k_c = 0.85$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.76)(1)(0.85)(295) = 190.6\text{ MPa}$

Fig. 6-26: $q = 0.83$

Fig. A-15-1: $d/w = 0.24, K_t = 2.44$

$$K_f = 1 + q(K_t - 1) = 1 + 0.83(2.44 - 1) = 2.20$$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - (-28\,000)}{2(10)(25-6)} \right| = 324.2\text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 0$$

Note, since $\sigma_m = 0$, the stress is completely reversing, and

$$n_f = \frac{S_e}{\sigma_a} = \frac{190.6}{324.2} = 0.59 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate the life from the *S-N* diagram. With $\sigma_m = 0$, the stress state is completely reversed, and the *S-N* diagram is applicable for σ_a.

Fig. 6-23: $f = 0.87$

Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.87(590)]^2}{190.6} = 1382$

Eq. (6-14): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.87(590)}{190.6}\right) = -0.1434$

Eq. (6-15): $N = \left(\dfrac{\sigma_{ar}}{a}\right)^{1/b} = \left(\dfrac{324.2}{1382}\right)^{\frac{1}{-0.1434}} = 24\,613\text{ cycles}$

$$N = 25\,000 \text{ cycles} \qquad Ans.$$

6-26 $S_{ut} = 590$ MPa, $S_y = 490$ MPa, $F_{\max} = 28$ kN, $F_{\min} = 12$ kN

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \qquad Ans.$$

Determine the fatigue factor of safety based on infinite life.
From Prob. 6-25: $S_e = 190.6$ MPa, $K_f = 2.2$

$$\sigma_a = K_f \left|\frac{F_{\max} - F_{\min}}{2A}\right| = 2.2\left|\frac{28\,000 - (12\,000)}{2(10)(25-6)}\right| = 92.63 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2\left[\frac{28\,000 + 12\,000}{2(10)(25-6)}\right] = 231.6 \text{ MPa}$$

Goodman criteria, Equation (6-41):

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\frac{92.63}{190.6} + \frac{231.6}{590}\right)^{-1}$$

$$n_f = 1.14 \qquad Ans.$$

Gerber criteria, Equation (6-48):

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right]$$

$$= \frac{1}{2}\left(\frac{590}{231.6}\right)^2 \frac{92.63}{190.6}\left[-1 + \sqrt{1 + \left(\frac{2(231.6)(190.6)}{590(92.63)}\right)^2}\,\right]$$

$$n_f = 1.42 \qquad Ans.$$

Morrow criteria:

Estimate the fatigue strength coefficient.

Eq. (6-44): $\sigma'_f = S_{ut} + 345 = 590 + 345 = 935$ MPa

Eq. (6-46): $$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{\sigma'_f}\right)^{-1} = \left(\frac{92.63}{190.6} + \frac{231.6}{935}\right)^{-1}$$

$$n_f = 1.36 \qquad Ans.$$

The results are consistent with Fig. 6-36, where for a mean stress that is about half of the yield strength, the Goodman line should predict failure significantly before the other two.

6-27 $S_{ut} = 590$ MPa, $S_y = 490$ MPa

From Prob. 6-25: $S_e = 190.6$ MPa, $K_f = 2.2$

(a) $F_{\max} = 28$ kN, $F_{\min} = 0$ kN

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \qquad Ans.$$

$$\sigma_a = K_f \left|\frac{F_{\max} - F_{\min}}{2A}\right| = 2.2\left|\frac{28\,000-0}{2(10)(25-6)}\right| = 162.1 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2\left[\frac{28\,000+0}{2(10)(25-6)}\right] = 162.1 \text{ MPa}$$

For the Goodman criteria, Eq. (6-41):

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\frac{162.1}{190.6} + \frac{162.1}{590}\right)^{-1} = 0.89 \qquad Ans.$$

Since infinite life is not predicted, estimate a life from the *S*-*N* diagram. First, find an equivalent completely reversed stress. Using the Goodman criterion,

Eq. (6-58): $$\sigma_{ar} = \frac{\sigma_a}{1-(\sigma_m/S_{ut})} = \frac{162.1}{1-(162.1/590)} = 223.5 \text{ MPa}$$

Fig. 6-23: $f = 0.87$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.87(590)]^2}{190.6} = 1382$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(590)}{190.6}\right) = -0.1434$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{223.5}{1382}\right)^{\frac{1}{-0.1434}} = 329\,000 \text{ cycles} \qquad Ans.$$

(b) $F_{\max} = 28$ kN, $F_{\min} = 12$ kN

The maximum load is the same as in part (a), so

$$\sigma_{\max} = 147.4 \text{ MPa}$$

$$n_y = 3.32 \qquad \textit{Ans.}$$

Factor of safety based on infinite life:

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - 12\,000}{2(10)(25-6)} \right| = 92.63 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{28\,000 + 12\,000}{2(10)(25-6)} \right] = 231.6 \text{ MPa}$$

Eq. (6-41): $$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} = \left(\frac{92.63}{190.6} + \frac{231.6}{590} \right)^{-1} = 1.14 \qquad \textit{Ans.}$$

(c) $F_{\max} = 12$ kN, $F_{\min} = -28$ kN

The compressive load is the largest, so check it for yielding.

$$\sigma_{\min} = \frac{F_{\min}}{A} = \frac{-28\,000}{10(25-6)} = -147.4 \text{ MPa}$$

$$n_y = \frac{S_{yc}}{\sigma_{\min}} = \frac{-490}{-147.4} = 3.32 \qquad \textit{Ans.}$$

Factor of safety based on infinite life:

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{12\,000 - (-28\,000)}{2(10)(25-6)} \right| = 231.6 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{12\,000 + (-28\,000)}{2(10)(25-6)} \right] = -92.63 \text{ MPa}$$

For $\sigma_m < 0$, Eq. (6-42): $$n_f = \frac{S_e}{\sigma_a} = \frac{190.6}{231.6} = 0.82 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. For a negative mean stress, we shall assume the equivalent completely reversed stress is the same as the actual alternating stress, consistent with the horizontal fatigue line in Fig. 6-34. Get *a* and *b* from part (a).

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a} \right)^{1/b} = \left(\frac{231.6}{1382} \right)^{\frac{1}{-0.1434}} = 257\,000 \text{ cycles} \qquad \textit{Ans.}$$

6-28 Eq. (2-36): $S_{ut} = 0.5(400) = 200$ kpsi

Eq. (6-10): $S_e' = 0.5(200) = 100 \text{ kpsi}$

Eq. (6-18): $k_a = aS_{ut}^b = 11.0(200)^{-0.650} = 0.35$

Eq. (6-24): $d_e = 0.37d = 0.37(0.375) = 0.1388 \text{ in}$

Eq. (6-19): $k_b = 0.879 d_e^{-0.107} = 0.879(0.1388)^{-0.107} = 1.09$

Since we have used the equivalent diameter method to get the size factor, and in doing so introduced greater uncertainties, we will choose not to use a size factor greater than one. Let $k_b = 1$.

Eq. (6-17): $S_e = (0.35)(1)(100) = 35.0 \text{ kpsi}$

$$F_a = \frac{40-20}{2} = 10 \text{ lb} \qquad F_m = \frac{40+20}{2} = 30 \text{ lb}$$

$$\sigma_a = \frac{32M_a}{\pi d^3} = \frac{32(10)(12)}{\pi(0.375)^3} = 23.18 \text{ kpsi}$$

$$\sigma_m = \frac{32M_m}{\pi d^3} = \frac{32(30)(12)}{\pi(0.375)^3} = 69.54 \text{ kpsi}$$

(a) Goodman criterion, Eq. (6-41):

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{23.18}{35.0} + \frac{69.54}{200}$$

$$n_f = 0.99 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress. Using the Goodman criterion,

Eq. (6-58): $\sigma_{ar} = \dfrac{\sigma_a}{1-(\sigma_m / S_{ut})} = \dfrac{23.18}{1-(69.54/200)} = 35.54 \text{ kpsi}$

Fig. 6-23: $f = 0.78$

Eq. (6-13): $a = \dfrac{(f\, S_{ut})^2}{S_e} = \dfrac{\left[0.78(200)\right]^2}{35} = 695.3$

Eq. (6-14): $b = -\dfrac{1}{3}\log\left(\dfrac{f\, S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.78(200)}{35.0}\right) = -0.2164$

Eq. (6-15): $N = \left(\dfrac{\sigma_{ar}}{a}\right)^{1/b} = \left(\dfrac{35.54}{695.3}\right)^{\frac{1}{-0.2164}} = 929\,000 \text{ cycles} \qquad \textit{Ans.}$

(b) Gerber criterion, Eq. (6-48):

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1+\sqrt{1+\left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{200}{69.54}\right)^2 \frac{23.18}{35.0}\left[-1+\sqrt{1+\left(\frac{2(69.54)(35.0)}{200(23.18)}\right)^2}\right]$$

$$= 1.23 \quad \text{Infinite life is predicted} \qquad \textit{Ans.}$$

6-29 $E = 207.0$ GPa

(a) $I = \dfrac{1}{12}(20)(4^3) = 106.7\ \text{mm}^4$

$$y = \frac{Fl^3}{3EI} \quad \Rightarrow \quad F = \frac{3EIy}{l^3}$$

$$F_{\min} = \frac{3(207)(10^9)(106.7)(10^{-12})(2)(10^{-3})}{140^3(10^{-9})} = 48.3\ \text{N} \qquad \textit{Ans.}$$

$$F_{\max} = \frac{3(207)(10^9)(106.7)(10^{-12})(6)(10^{-3})}{140^3(10^{-9})} = 144.9\ \text{N} \qquad \textit{Ans.}$$

(b) Get the fatigue strength information.

Eq. (2-36): $S_{ut} = =3.4H_B = 3.4(490) = 1666$ MPa

From problem statement: $S_y = 0.9S_{ut} = 0.9(1666) = 1499$ MPa

Eq. (6-10): $S'_e = 700$ MPa

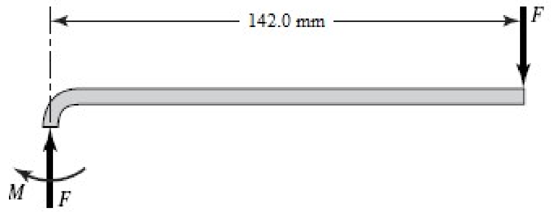
Eq. (6-18): $k_a = 1.38(1666)^{-0.067} = 0.84$

Eq. (6-24): $d_e = 0.808[20(4)]^{1/2} = 7.23$ mm

Eq. (6-19): $k_b = 1.24(7.23)^{-0.107} = 1.00$

Eq. (6-17): $S_e = 0.84(1)(700) = 588$ MPa

This is a relatively thick curved beam, so use the method in Sect. 3-18 to find the stresses. The maximum bending moment will be to the centroid of the section as shown.



$M = 142F$ N·mm, $A = 4(20) = 80\ \text{mm}^2$,
$h = 4$ mm, $r_i = 4$ mm, $r_o = r_i + h = 8$ mm,
$r_c = r_i + h/2 = 6$ mm

Table 3-4: $r_n = \dfrac{h}{\ln(r_o/r_i)} = \dfrac{4}{\ln(8/4)} = 5.7708$ mm

$$e = r_c - r_n = 6 - 5.7708 = 0.2292 \text{ mm}$$
$$c_i = r_n - r_i = 5.7708 - 4 = 1.7708 \text{ mm}$$
$$c_o = r_o - r_n = 8 - 5.7708 = 2.2292 \text{ mm}$$

Get the stresses at the inner and outer surfaces from Eq. (3-76) with the axial stresses added. The signs have been set to account for tension and compression as appropriate.

$$\sigma_i = -\frac{Mc_i}{Aer_i} - \frac{F}{A} = -\frac{(142F)(1.7708)}{80(0.2292)(4)} - \frac{F}{80} = -3.441F \text{ MPa}$$
$$\sigma_o = \frac{Mc_o}{Aer_o} - \frac{F}{A} = \frac{(142F)(2.2292)}{80(0.2292)(8)} - \frac{F}{80} = 2.145F \text{ MPa}$$
$$(\sigma_i)_{\min} = -3.441(144.9) = -498.6 \text{ MPa}$$
$$(\sigma_i)_{\max} = -3.441(48.3) = -166.2 \text{ MPa}$$
$$(\sigma_o)_{\min} = 2.145(48.3) = 103.6 \text{ MPa}$$
$$(\sigma_o)_{\max} = 2.145(144.9) = 310.8 \text{ MPa}$$
$$(\sigma_i)_a = \left|\frac{-166.2 - (-498.6)}{2}\right| = 166.2 \text{ MPa}$$
$$(\sigma_i)_m = \frac{-166.2 + (-498.6)}{2} = -332.4 \text{ MPa}$$
$$(\sigma_o)_a = \left|\frac{310.8 - 103.6}{2}\right| = 103.6 \text{ MPa}$$
$$(\sigma_o)_m = \frac{310.8 + 103.6}{2} = 207.2 \text{ MPa}$$

To check for yielding, we note that the largest stress is –498.6 MPa (compression) on the inner radius. This is considerably less than the estimated yield strength of 1499 MPa, so yielding is not predicted.

Check for fatigue on both inner and outer radii since one has a compressive mean stress and the other has a tensile mean stress.

Inner radius:

Since $\sigma_m < 0$, Eq. (6-42): $n_f = \dfrac{S_e}{\sigma_a} = \dfrac{588}{166.2} = 3.54$

Outer radius:

Since $\sigma_m > 0$, using the Goodman line, Eq. (6-41),

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\frac{103.6}{588} + \frac{207.2}{1666}\right)^{-1}$$

$$n_f = 3.33$$

Infinite life is predicted at both inner and outer radii. The outer radius is critical, with a fatigue factor of safety of $n_f = 3.33$. *Ans.*

6-30 From Table A-20, for AISI 1018 CD, $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

Eq. (6-10): $S_e' = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 1$ (axial)

Eq. (6-25): $k_c = 0.85$

Eq. (6-17): $S_e = (0.81)(1)(0.85)(32) = 22.0$ kpsi

Fillet:

Fig. A-15-5: $D/d = 3.5/3 = 1.17, \quad r/d = 0.25/3 = 0.083, \quad K_t = 1.85$

Use Fig. 6-26 or Eqs. (6-33) and (6-35) for q. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + q(K_t - 1) = 1 + 0.85(1.85 - 1) = 1.72$$

$$\sigma_{\max} = \frac{F_{\max}}{w_2 h} = \frac{5}{3.0(0.5)} = 3.33 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-16}{3.0(0.5)} = -10.67 \text{ kpsi}$$

$$\sigma_a = K_f \left|\frac{\sigma_{\max} - \sigma_{\min}}{2}\right| = 1.72\left|\frac{3.33 - (-10.67)}{2}\right| = 12.0 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2}\right) = 1.72\left(\frac{3.33 + (-10.67)}{2}\right) = -6.31 \text{ kpsi}$$

$$n_y = \left|\frac{S_y}{\sigma_{\min}}\right| = \left|\frac{54}{-10.67}\right| = 5.06 \quad \therefore \text{Does not yield.}$$

Since the mean stress is negative, use Eq. (6-42).

$$n_f = \frac{S_e}{\sigma_a} = \frac{22.0}{12.0} = 1.83$$

Hole:

Fig. A-15-1: $d/w_1 = 0.4/3.5 = 0.11 \quad \therefore K_t = 2.68$

Use Fig. 6-26 or Eqs. (6-33) and (6-35) for q. Estimate a little high since it is off the graph, $q = 0.85$

$$K_f = 1 + 0.85(2.68 - 1) = 2.43$$

$$\sigma_{\max} = \frac{F_{\max}}{h(w_1 - d)} = \frac{5}{0.5(3.5 - 0.4)} = 3.226 \text{ kpsi}$$

$$\sigma_{\min} = \frac{F_{\min}}{h(w_1 - d)} = \frac{-16}{0.5(3.5 - 0.4)} = -10.32 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 2.43 \left| \frac{3.226 - (-10.32)}{2} \right| = 16.5 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 2.43 \left(\frac{3.226 + (-10.32)}{2} \right) = -8.62 \text{ kpsi}$$

$$n_y = \left| \frac{S_y}{\sigma_{\min}} \right| = \left| \frac{54}{-10.32} \right| = 5.23 \quad \therefore \text{does not yield}$$

Since the mean stress is negative, use Eq. (6-42).

$$n_f = \frac{S_e}{\sigma_a} = \frac{22.0}{16.5} = 1.33$$

Thus the design is controlled by the threat of fatigue at the hole with a minimum factor of safety of $n_f = 1.33$. *Ans.*

6-31 $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

Eq. (6-10): $S_e' = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 1$ (axial)

Eq. (6-25): $k_c = 0.85$

Eq. (6-17): $S_e = (0.81)(1)(0.85)(32) = 22.0$ kpsi

Fillet:

Fig. A-15-5: $D/d = 2.5/1.5 = 1.67, \quad r/d = 0.25/1.5 = 0.17, \quad K_t \approx 2.1$

Use Fig. 6-26 or Eqs. (6-33) and (6-35) for q. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + q(K_t - 1) = 1 + 0.85(2.1 - 1) = 1.94$$

$$\sigma_{\max} = \frac{F_{\max}}{w_2 h} = \frac{16}{1.5(0.5)} = 21.3 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-4}{1.5(0.5)} = -5.33 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 1.94 \left| \frac{21.3 - (-5.33)}{2} \right| = 25.8 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 1.94 \left(\frac{21.3 + (-5.33)}{2} \right) = 15.5 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{54}{21.3} = 2.54 \qquad \therefore \text{Does not yield.}$$

Using Goodman criteria, Eq. (6-41),

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} = \left(\frac{25.8}{22.0} + \frac{15.5}{64} \right)^{-1} = 0.71$$

Hole:
Fig. A-15-1: $d / w_1 = 0.4 / 2.5 = 0.16 \qquad \therefore K_t = 2.55$
Use Fig. 6-26 or Eqs. (6-33) and (6-35) for q. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + 0.85(2.55 - 1) = 2.32$$

$$\sigma_{\max} = \frac{F_{\max}}{h(w_1 - d)} = \frac{16}{0.5(2.5 - 0.4)} = 15.2 \text{ kpsi}$$

$$\sigma_{\min} = \frac{F_{\min}}{h(w_1 - d)} = \frac{-4}{0.5(2.5 - 0.4)} = -3.81 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 2.32 \left(\frac{15.2 - (-3.81)}{2} \right) = 22.1 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 2.32 \left(\frac{15.2 + (-3.81)}{2} \right) = 13.2 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{54}{15.2} = 3.55 \qquad \therefore \text{Does not yield.}$$

Using Goodman criteria, Eq. (6-41),

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} = \left(\frac{22.1}{22.0} + \frac{13.2}{64} \right)^{-1} = 0.83$$

Thus the design is controlled by the threat of fatigue at the fillet with a minimum factor of safety of $n_f = 0.71$ *Ans.*

6-32 $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 6-30, the fatigue factor of safety at the hole is $n_f = 1.33$. To match this at the fillet,

$$n_f = \frac{S_e}{\sigma_a} \quad \Rightarrow \quad \sigma_a = \frac{S_e}{n_f} = \frac{22.0}{1.33} = 16.5 \text{ kpsi}$$

where S_e is unchanged from Prob. 6-30. The only aspect of σ_a that is affected by the fillet radius is the fatigue stress concentration factor. Obtaining σ_a in terms of K_f,

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = K_f \left| \frac{3.33 - (-10.67)}{2} \right| = 7.00 K_f$$

Equating to the desired stress, and solving for K_f,

$$\sigma_a = 7.00 K_f = 16.5 \qquad \Rightarrow \qquad K_f = 2.36$$

Assume since we are expecting to get a smaller fillet radius than the original, that q will be back on the graph of Fig. 6-26, so we will estimate $q = 0.8$.

$$K_f = 1 + 0.80(K_t - 1) = 2.36 \quad \Rightarrow \qquad K_t = 2.7$$

From Fig. A-15-5, with $D / d = 3.5/3 = 1.17$ and $K_t = 2.6$, find r / d. Choosing $r / d = 0.03$, and with $d = w_2 = 3.0$,

$$r = 0.03 w_2 = 0.03(3.0) = 0.09 \text{ in}$$

At this small radius, our estimate for q is too high. From Fig. 6-26, with $r = 0.09$, q should be about 0.75. Iterating, we get $K_t = 2.8$. This is at a difficult range on Fig. A-15-5 to read the graph with any confidence, but we'll estimate $r / d = 0.02$, giving $r = 0.06$ in. This is a very rough estimate, but it clearly demonstrates that the fillet radius can be relatively sharp to match the fatigue factor of safety of the hole. *Ans.*

6-33 $S_y = 60$ kpsi, $S_{ut} = 110$ kpsi

Inner fiber where $r_c = 3/4$ in

$$r_o = \frac{3}{4} + \frac{3}{16(2)} = 0.84375$$

$$r_i = \frac{3}{4} - \frac{3}{32} = 0.65625$$

Table 3-4,

$$r_n = \frac{h}{\ln \frac{r_o}{r_i}} = \frac{3/16}{\ln \frac{0.84375}{0.65625}} = 0.74608 \text{ in}$$

$$e = r_c - r_n = 0.75 - 0.74608 = 0.00392 \text{ in}$$
$$c_i = r_n - r_i = 0.74608 - 0.65625 = 0.08983$$
$$A = \left(\frac{3}{16}\right)\left(\frac{3}{16}\right) = 0.035156 \text{ in}^2$$

Eq. (3-65),

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-T(0.08983)}{(0.035156)(0.00392)(0.65625)} = -993.3T$$

where T is in lbf·in and σ_i is in psi.

$$\sigma_m = \frac{1}{2}(-993.3)T = -496.7T$$
$$\sigma_a = 496.7T$$

Eq. (6-10): $S_e' = 0.5(110) = 55$ kpsi

Eq. (6-18): $k_a = 2.00(110)^{-0.217} = 0.72$

Eq. (6-24): $d_e = 0.808\left[(3/16)(3/16)\right]^{1/2} = 0.1515$ in

Eq. (6-19): $k_b = 0.879(0.1515)^{-0.107} = 1.08$ (round to 1)

Eq. (6-18): $S_e = (0.72)(1)(55) = 39.6$ kpsi

For a compressive mean component, from Eq. (6-42), $\sigma_a = S_e / n_f$. Thus,

$$0.4967T = \frac{39.6}{3}$$
$$T = 26.6 \text{ lbf} \cdot \text{in}$$

Outer fiber where $r_c = 2.5$ in

$$r_o = 2.5 + \frac{3}{32} = 2.59375$$
$$r_i = 2.5 - \frac{3}{32} = 2.40625$$
$$r_n = \frac{3/16}{\ln \frac{2.59375}{2.40625}} = 2.49883$$
$$e = 2.5 - 2.49883 = 0.00117 \text{ in}$$
$$c_o = 2.59375 - 2.49883 = 0.09492 \text{ in}$$

$$\sigma_o = \frac{Mc_o}{Aer_o} = \frac{T(0.09492)}{(0.035156)(0.00117)(2.59375)} = 889.7T \text{ psi}$$

$$\sigma_m = \sigma_a = \frac{1}{2}(889.7T) = 444.9T \text{ psi}$$

(a) Using Eq. (6-41), for Goodman, we have

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\frac{0.4449T}{39.6} + \frac{0.4449T}{110}\right)^{-1} = 3$$

$$T = 21.8 \text{ lbf} \cdot \text{in} \qquad Ans.$$

(b) For Morrow, estimate the fatigue strength coefficient from Eq. (6-44),

$$\sigma'_f = S_{ut} + 50 = 110 + 50 = 160 \text{ kpsi}$$

Eq. (6-46):

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{\sigma'_f}\right)^{-1} = \left(\frac{0.4449T}{39.6} + \frac{0.4449T}{160}\right)^{-1} = 3$$

$$T = 23.8 \text{ lbf} \cdot \text{in} \qquad Ans.$$

(c) To guard against yield, use T of part (b) and the inner stress.

$$n_y = \frac{S_y}{\sigma_i} = \frac{60}{0.9933(23.8)} = 2.54 \qquad Ans.$$

6-34 From Prob. 6-33, $S_e = 39.6$ kpsi, $S_y = 60$ kpsi, and $S_{ut} = 110$ kpsi

(a) Assuming the beam is straight,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M(h/2)}{bh^3/12} = \frac{6M}{bh^2} = \frac{6T}{(3/16)^3} = 910.2T$$

Using Eq. (6-41), for Goodman, we have

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\frac{0.4551T}{39.6} + \frac{0.4551T}{110}\right)^{-1} = 3$$

$$T = 21.3 \text{ lbf} \cdot \text{in} \qquad Ans.$$

(b)) For Morrow, estimate the fatigue strength coefficient from Eq. (6-44),

$$\sigma'_f = S_{ut} + 50 = 110 + 50 = 160 \text{ kpsi}$$

Eq. (6-46):

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{\sigma'_f}\right)^{-1} = \left(\frac{0.4551T}{39.6} + \frac{0.4551T}{160}\right)^{-1} = 3$$

$$T = 23.3 \text{ lbf} \cdot \text{in} \qquad Ans.$$

(c) $$n_y = \frac{S_y}{\sigma_{\max}} = \frac{60}{0.9102(23.3)} = 2.83 \quad \textit{Ans.}$$

6-35 $K_{f,\text{bend}} = 1.4,\ K_{f,\text{axial}} = 1.1,\ K_{f,\text{tors}} = 2.0,\ S_y = 300\text{ MPa},\ S_{ut} = 400\text{ MPa},\ S_e = 160\text{ MPa}$

Bending: $\sigma_m = 0,\ \sigma_a = 60\text{ MPa}$

Axial: $\sigma_m = 20\text{ MPa},\ \sigma_a = 0$

Torsion: $\tau_m = 35\text{ MPa},\ \tau_a = 35\text{ MPa}$

Eqs. (6-66) and (6-67):

$$\sigma'_a = \sqrt{[1.4(60)+0]^2 + 3[2.0(35)]^2} = 147.5\text{ MPa}$$

$$\sigma'_m = \sqrt{[0+1.1(20)]^2 + 3[2.0(35)]^2} = 123.2\text{ MPa}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{300}{147.5 + 123.2} = 1.11 \quad \text{Yielding is not predicted.} \quad \textit{Ans.}$$

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{147.5}{160} + \frac{123.2}{400}\right)^{-1}$$

$$n_f = 0.81 \quad \textit{Ans.}$$

Finite life is predicted. To use the Walker criterion for estimating an equivalent completely reversed stress, estimate the material fitting parameter for steels with Eq. (6-57).

$$\gamma = -0.0002S_{ut} + 0.8818 = -0.0002(400) + 0.8818 = 0.8018$$

Eq. (6-61): $\sigma_{ar} = (\sigma_m + \sigma_a)^{1-\gamma}\,\sigma_a^{\gamma} = (123.2 + 147.5)^{1-0.8018}\,147.5^{0.8018} = 166.4\text{ MPa}$

Fig. 6-23: Off the chart, so use *f* = 0.9

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.9(400)]^2}{160} = 810$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.9(400)}{160}\right) = -0.1174$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{166.4}{810}\right)^{\frac{1}{-0.1174}} = 716\,000\text{ cycles} \qquad \textit{Ans.}$$

6-36 $K_{f,\text{bend}} = 1.4$, $K_{f,\text{tors}} = 2.0$, $S_y = 300$ MPa, $S_{ut} = 400$ MPa, $S_e = 160$ MPa

Bending: $\sigma_{\max} = 150$ MPa, $\sigma_{\min} = -40$ MPa, $\sigma_m = 55$ MPa, $\sigma_a = 95$ MPa

Torsion: $\tau_m = 90$ MPa, $\tau_a = 9$ MPa

Eqs. (6-66) and (6-67):

$$\sigma'_a = \sqrt{[1.4(95)]^2 + 3[2.0(9)]^2} = 136.6 \text{ MPa}$$

$$\sigma'_m = \sqrt{[1.4(55)]^2 + 3[2.0(90)]^2} = 321.1 \text{ MPa}$$

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{136.6}{160} + \frac{321.1}{400}\right)^{-1} = 0.60 \qquad Ans.$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{300}{136.6 + 321.1} = 0.66 \qquad Ans.$$

Since the conservative yield check indicates yielding, we will check more carefully with $\sigma'_{\max}$ obtained directly from the maximum stresses, using the distortion energy failure theory, without stress concentrations. Note that this is exactly the method used for static failure in Ch. 5.

$$\sigma'_{\max} = \sqrt{(\sigma_{\max})^2 + 3(\tau_{\max})^2} = \sqrt{(150)^2 + 3(90+9)^2} = 227.8 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{300}{227.8} = 1.32 \qquad Ans.$$

Since yielding is not predicted, and infinite life is not predicted, we would like to estimate a life from the *S-N* diagram.

To use the Walker criterion for estimating an equivalent completely reversed stress, estimate the material fitting parameter for steels with Eq. (6-57).

$$\gamma = -0.0002 S_{ut} + 0.8818 = -0.0002(400) + 0.8818 = 0.8018$$

Eq. (6-61): $\sigma_{ar} = (\sigma_m + \sigma_a)^{1-\gamma}\,\sigma_a^{\gamma} = (321.1 + 136.6)^{1-0.8018}\,136.6^{0.8018} = 173.6$ MPa

Fig. 6-23: Off the chart, so use $f = 0.9$

Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.9(400)]^2}{160} = 810$

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.9(400)}{160}\right) = -0.1174$

Eq. (6-15): $N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{173.6}{810}\right)^{\frac{1}{-0.1174}} = 499\ 000 \text{ cycles}$ *Ans.*

6-37 Table A-20: $S_{\text{ut}} = 64$ kpsi, $S_{\text{y}} = 54$ kpsi

From Prob. 3-79, the critical stress element experiences $\sigma = 15.3$ kpsi and $\tau = 4.43$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 15.3$ kpsi, $\sigma_m = 0$ kpsi, $\tau_a = 0$ kpsi, $\tau_m = 4.43$ kpsi. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[15.3^2 + 3\left(0\right)^2\right]^{1/2} = 15.3 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(4.43\right)^2\right]^{1/2} = 7.67 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[15.3^2 + 3\left(4.43\right)^2\right]^{1/2} = 17.11 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{17.11} = 3.16$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5\left(64\right) = 32$ kpsi

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 0.879(1.25)^{-0.107} = 0.86$

Eq. (6-17): $S_e = 0.81(0.86)(32) = 22.3$ kpsi

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{15.3}{22.3} + \frac{7.67}{64}\right)^{-1} = 1.24 \qquad \textit{Ans.}$$

6-38 Table A-20: $S_{\text{ut}} = 440$ MPa, $S_{\text{y}} = 370$ MPa

From Prob. 3-80, the critical stress element experiences $\sigma = 263$ MPa and $\tau = 57.7$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 263$ MPa, $\sigma_m = 0$, $\tau_a = 0$ MPa, $\tau_m = 57.7$ MPa. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[263^2 + 3\left(0\right)^2\right]^{1/2} = 263 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(57.7\right)^2\right]^{1/2} = 99.9 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[263^2 + 3\left(57.7\right)^2\right]^{1/2} = 281 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{281} = 1.32$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5\left(440\right) = 220 \text{ MPa}$

Eq. (6-18): $k_a = 3.04(440)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 1.24(30)^{-0.107} = 0.86$

Eq. (6-17): $S_e = 0.81(0.86)(220) = 153 \text{ MPa}$

Using Goodman,

Eq. (6-41): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{263}{153} + \frac{99.9}{440}\right)^{-1}$$

$n_f = 0.51$ Infinite life is not predicted. *Ans.*

6-39 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-81, the critical stress element experiences $\sigma = 21.5$ kpsi and $\tau = 5.09$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 21.5$ kpsi, $\sigma_m = 0$ kpsi, $\tau_a = 0$ kpsi, $\tau_m = 5.09$ kpsi. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[21.5^2 + 3\left(0\right)^2\right]^{1/2} = 21.5 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(5.09\right)^2\right]^{1/2} = 8.82 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[21.5^2 + 3\left(5.09\right)^2\right]^{1/2} = 23.24 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{23.24} = 2.32$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 0.879(1)^{-0.107} = 0.88$

Eq. (6-17): $S_e = 0.81(0.88)(32) = 22.8$ kpsi

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{21.5}{22.8} + \frac{8.82}{64}\right)^{-1} = 0.93 \qquad Ans.$$

6-40 Table A-20: $S_{\mathrm{ut}} = 440$ MPa, $S_{\mathrm{y}} = 370$ MPa

From Prob. 3-82, the critical stress element experiences $\sigma = 72.9$ MPa and $\tau = 20.3$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 72.9$ MPa, $\sigma_m = 0$ MPa, $\tau_a = 0$ MPa, $\tau_m = 20.3$ MPa. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[72.9^2 + 3(0)^2\right]^{1/2} = 72.9 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(20.3)^2\right]^{1/2} = 35.2 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[72.9^2 + 3(20.3)^2\right]^{1/2} = 80.9 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{80.9} = 4.57$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(440) = 220$ MPa

Eq. (6-18): $k_a = 3.04(440)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 1.24(20)^{-0.107} = 0.90$

Eq. (6-17): $S_e = 0.81(0.90)(220) = 160.4$ MPa

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{72.9}{160.4} + \frac{35.2}{440}\right)^{-1} = 1.87 \qquad Ans.$$

6-41 Table A-20: $S_{\mathrm{ut}} = 64$ kpsi, $S_{\mathrm{y}} = 54$ kpsi

From Prob. 3-83, the critical stress element experiences $\sigma = 35.2$ kpsi and $\tau = 7.35$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 35.2$ kpsi, $\sigma_m = 0$ kpsi, $\tau_a = 0$ kpsi, $\tau_m = 7.35$ kpsi. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[35.2^2 + 3\left(0\right)^2\right]^{1/2} = 35.2 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(7.35\right)^2\right]^{1/2} = 12.7 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[35.2^2 + 3\left(7.35\right)^2\right]^{1/2} = 37.4 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{37.4} = 1.44$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 0.879(1.25)^{-0.107} = 0.86$

Eq. (6-17): $S_e = 0.81(0.86)(32) = 22.3$ kpsi

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{35.2}{22.3} + \frac{12.7}{64}\right)^{-1} = 0.56 \qquad \textit{Ans.}$$

6-42 Table A-20: $S_{\text{ut}} = 440$ MPa, $S_{\text{y}} = 370$ MPa

From Prob. 3-84, the critical stress element experiences $\sigma = 333.9$ MPa and $\tau = 126.3$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 333.9$ MPa, $\sigma_m = 0$ MPa, $\tau_a = 0$ MPa, $\tau_m = 126.3$ MPa. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[333.9^2 + 3\left(0\right)^2\right]^{1/2} = 333.9 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(126.3\right)^2\right]^{1/2} = 218.8 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[333.9^2 + 3\left(126.3\right)^2\right]^{1/2} = 399.2 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{399.2} = 0.93$$

The sample fails by yielding, infinite life is not predicted. *Ans.*

The fatigue analysis will be continued only to obtain the requested fatigue factor of safety, though the yielding failure will dictate the life.

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(440) = 220 \text{ MPa}$

Eq. (6-18): $k_a = 3.04(440)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 1.24(50)^{-0.107} = 0.82$

Eq. (6-17): $S_e = 0.81(0.82)(220) = 146.1 \text{ MPa}$

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{333.9}{146.1} + \frac{218.8}{440}\right)^{-1} = 0.36 \qquad \textit{Ans.}$$

6-43 Table A-20: $S_{ut} = 64 \text{ kpsi}$, $S_y = 54 \text{ kpsi}$

From Prob. 3-85, the critical stress element experiences completely reversed bending stress due to the rotation, and steady torsional and axial stresses.

$$\sigma_{a,\text{bend}} = 9.495 \text{ kpsi}, \qquad \sigma_{m,\text{bend}} = 0 \text{ kpsi}$$
$$\sigma_{a,\text{axial}} = 0 \text{ kpsi}, \qquad \sigma_{m,\text{axial}} = -0.362 \text{ kpsi}$$
$$\tau_a = 0 \text{ kpsi}, \qquad \tau_m = 11.07 \text{ kpsi}$$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[(9.495)^2 + 3(0)^2\right]^{1/2} = 9.495 \text{ kpsi}$$
$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[(-0.362)^2 + 3(11.07)^2\right]^{1/2} = 19.18 \text{ kpsi}$$
$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[(-9.495 - 0.362)^2 + 3(11.07)^2\right]^{1/2} = 21.56 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{21.56} = 2.50$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S_e' = 0.5(64) = 32\ \text{kpsi}$

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 0.879(1.13)^{-0.107} = 0.87$

Eq. (6-17): $S_e = 0.81(0.87)(32) = 22.6\ \text{kpsi}$

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}\right)^{-1} = \left(\frac{9.495}{22.6} + \frac{19.18}{64}\right)^{-1} = 1.39 \qquad \textit{Ans.}$$

6-44 Table A-20: $S_{\text{ut}} = 64\ \text{kpsi},\ S_{\text{y}} = 54\ \text{kpsi}$

From Prob. 3-87, the critical stress element experiences completely reversed bending stress due to the rotation, and steady torsional and axial stresses.

$$\sigma_{a,\text{bend}} = 33.99\ \text{kpsi}, \qquad \sigma_{m,\text{bend}} = 0\ \text{kpsi}$$
$$\sigma_{a,\text{axial}} = 0\ \text{kpsi}, \qquad \sigma_{m,\text{axial}} = -0.153\ \text{kpsi}$$
$$\tau_a = 0\ \text{kpsi}, \qquad \tau_m = 7.847\ \text{kpsi}$$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma_a' = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[(33.99)^2 + 3(0)^2\right]^{1/2} = 33.99\ \text{kpsi}$$
$$\sigma_m' = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[(-0.153)^2 + 3(7.847)^2\right]^{1/2} = 13.59\ \text{kpsi}$$
$$\sigma_{\max}' = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[(-33.99 - 0.153)^2 + 3(7.847)^2\right]^{1/2} = 36.75\ \text{kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma_{\max}'} = \frac{54}{36.75} = 1.47$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S_e' = 0.5(64) = 32\ \text{kpsi}$

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 0.879(0.88)^{-0.107} = 0.89$

Eq. (6-17): $S_e = 0.81(0.89)(32) = 23.1\ \text{kpsi}$

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}\right)^{-1} = \left(\frac{33.99}{23.1} + \frac{13.59}{64}\right)^{-1} = 0.59 \qquad \textit{Ans.}$$

6-45 Table A-20: $S_{\mathrm{ut}} = 440$ MPa, $S_{\mathrm{y}} = 370$ MPa

From Prob. 3-88, the critical stress element experiences $\sigma = 68.6$ MPa and $\tau = 37.7$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 68.6$ MPa, $\sigma_m = 0$ MPa, $\tau_a = 0$ MPa, $\tau_m = 37.7$ MPa. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[68.6^2 + 3\left(0\right)^2\right]^{1/2} = 68.6 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(37.7\right)^2\right]^{1/2} = 65.3 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[68.6^2 + 3\left(37.7\right)^2\right]^{1/2} = 94.7 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{94.7} = 3.91$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(440) = 220$ MPa

Eq. (6-18): $k_a = 3.04(440)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 1.24(30)^{-0.107} = 0.86$

Eq. (6-17): $S_e = 0.81(0.86)(220) = 153$ MPa

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{68.6}{153} + \frac{65.3}{440}\right)^{-1} = 1.68 \qquad \textit{Ans.}$$

6-46 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-90, the critical stress element experiences $\sigma = 3.46$ kpsi and $\tau = 0.882$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 3.46$ kpsi, $\sigma_m = 0$, $\tau_a = 0$ kpsi, $\tau_m = 0.882$ kpsi. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[3.46^2 + 3\left(0\right)^2\right]^{1/2} = 3.46 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(0.882\right)^2\right]^{1/2} = 1.53 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[3.46^2 + 3\left(0.882\right)^2\right]^{1/2} = 3.78 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{3.78} = 14.3$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(64) = 32\ \text{kpsi}$

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 0.879(1.375)^{-0.107} = 0.85$

Eq. (6-17): $S_e = 0.81(0.85)(32) = 22.0\ \text{kpsi}$

Using Goodman,

Eq. (6-41): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{3.46}{22.0} + \frac{1.53}{64}\right)^{-1}$$

$n_f = 5.5$ *Ans.*

6-47 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-91, the critical stress element experiences $\sigma = 16.3$ kpsi and $\tau = 5.09$ kpsi. Since the load is applied and released repeatedly, this gives $\sigma_{\max} = 16.3$ kpsi, $\sigma_{\min} = 0$ kpsi, $\tau_{\max} = 5.09$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 8.15$ kpsi, $\tau_m = \tau_a = 2.55$ kpsi.

For bending, from Eqs. (6-33) and (6-35),

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(64) + 1.51\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.10373$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.10373}{\sqrt{0.1}}} = 0.75$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$

For torsion, from Eqs. (6-33) and (6-36),

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)(64) + 1.35\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.07800$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07800}{\sqrt{0.1}}} = 0.80$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma_a' = \left\{\left[(1.38)(8.15)\right]^2 + 3\left[(1.88)(2.55)\right]^2\right\}^{1/2} = 13.98 \text{ kpsi}$$

$$\sigma_m' = \sigma_a' = 13.98 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma_{\max}' = \sigma_a' + \sigma_m'$,

$$n_y = \frac{S_y}{\sigma_a' + \sigma_m'} = \frac{54}{13.98 + 13.98} = 1.93$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S_e' = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

Eq. (6-23): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-17): $S_e = (0.81)(0.98)(32) = 25.4$ kpsi

Using Goodman,

Eq. (6-41): $$n_f = \left(\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}\right)^{-1} = \left(\frac{13.98}{25.4} + \frac{13.98}{64}\right)^{-1}$$

$n_f = 1.3$ *Ans.*

6-48 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-92, the critical stress element experiences $\sigma = 16.4$ kpsi and $\tau = 4.46$ kpsi. Since the load is applied and released repeatedly, this gives $\sigma_{\max} = 16.4$ kpsi, $\sigma_{\min} = 0$ kpsi, $\tau_{\max} = 4.46$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 8.20$ kpsi, $\tau_m = \tau_a = 2.23$ kpsi.

For bending, from Eqs. (6-33) and (6-35),

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(64) + 1.51\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.10373$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.10373}{\sqrt{0.1}}} = 0.75$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$

For torsion, from Eqs. (6-33) and (6-36),

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)(64) + 1.35\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.07800$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07800}{\sqrt{0.1}}} = 0.80$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma_a' = \left\{\left[(1.38)(8.20)\right]^2 + 3\left[(1.88)(2.23)\right]^2\right\}^{1/2} = 13.45 \text{ kpsi}$$

$$\sigma_m' = \sigma_a' = 13.45 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma_{\max}' = \sigma_a' + \sigma_m'$,

$$n_y = \frac{S_y}{\sigma_a' + \sigma_m'} = \frac{54}{13.45 + 13.45} = 2.01$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S_e' = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

Eq. (6-23): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-17): $S_e = (0.81)(0.98)(32) = 25.4$ kpsi

Using Goodman,

Eq. (6-41): $$n_f = \left(\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}\right)^{-1} = \left(\frac{13.45}{25.4} + \frac{13.45}{64}\right)^{-1}$$

$$n_f = 1.35 \quad \textit{Ans.}$$

6-49 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-93, the critical stress element experiences repeatedly applied bending, axial, and torsional stresses of $\sigma_{x,\text{bend}} = 20.2$ kpsi, $\sigma_{x,\text{axial}} = 0.1$ kpsi, and $\tau = 5.09$ kpsi.. Since the axial stress is practically negligible compared to the bending stress, we will simply combine the two and not treat the axial stress separately for stress concentration factor and load factor. This gives $\sigma_{\max} = 20.3$ kpsi, $\sigma_{\min} = 0$ kpsi, $\tau_{\max} = 5.09$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 10.15$ kpsi, $\tau_m = \tau_a = 2.55$ kpsi.

For bending, from Eqs. (6-33) and (6-35),

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)\left(64\right) + 1.51\left(10^{-5}\right)\left(64\right)^2 - 2.67\left(10^{-8}\right)\left(64\right)^3 = 0.10373$$

$$q = \frac{1}{1+\dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\dfrac{0.10373}{\sqrt{0.1}}} = 0.75$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$

For torsion, from Eqs. (6-33) and (6-36),

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)\left(64\right) + 1.35\left(10^{-5}\right)\left(64\right)^2 - 2.67\left(10^{-8}\right)\left(64\right)^3 = 0.07800$$

$$q = \frac{1}{1+\dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\dfrac{0.07800}{\sqrt{0.1}}} = 0.80$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma'_a = \left\{\left[\left(1.38\right)\left(10.15\right)\right]^2 + 3\left[\left(1.88\right)\left(2.55\right)\right]^2\right\}^{1/2} = 16.28 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 16.28 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{16.28 + 16.28} = 1.66$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

Eq. (6-23): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-17): $S_e = (0.81)(0.98)(32) = 25.4$ kpsi

Using Goodman,

Eq. (6-41): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{16.28}{25.4} + \frac{16.28}{64}\right)^{-1}$$

$$n_f = 1.12 \quad \textit{Ans.}$$

6-50 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-94, the critical stress element on the neutral axis in the middle of the longest side of the rectangular cross section experiences a repeatedly applied shear stress of $\tau_{\max} = 14.3$ kpsi, $\tau_{\min} = 0$ kpsi. Thus, $\tau_m = \tau_a = 7.15$ kpsi. Since the stress is entirely shear, it is convenient to check for yielding using the standard Maximum Shear Stress theory.

$$n_y = \frac{S_y / 2}{\tau_{\max}} = \frac{54/2}{14.3} = 1.89$$

Find the modifiers and endurance limit.

Eq. (6-10): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

The size factor for a rectangular cross section loaded in torsion is not readily available. An equivalent diameter based on the 95 percent stress area is not readily obtained, since the stress situation in this case is nonlinear, as described in Section 3-12. Noting that the maximum stress occurs at the middle of the longest side, or with a radius from the center of the cross section equal to half of the shortest side, we will simply choose an equivalent diameter equal to the length of the shortest side.

$d_e = 0.25$ in

Eq. (6-19): $k_b = 0.879 d_e^{-0.107} = 0.879(0.25)^{-0.107} = 1.02$

We will round down to $k_b = 1$.

Eq. (6-25): $k_c = 0.59$

Eq. (6-17): $S_{se} = 0.81(1)(0.59)(32) = 15.3$ kpsi

Since the stress is entirely shear, we choose to use a load factor $k_c = 0.59$, and convert the ultimate strength to a shear value rather than using the combination loading method of Sec. 6-16. From Eq. (6-58), $S_{su} = 0.67 S_u = 0.67\ (64) = 42.9$ kpsi.

Using Goodman,

Eq. (6-41): $$n_f = \left(\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}\right)^{-1} = \left(\frac{7.15}{15.3} + \frac{7.15}{42.9}\right)^{-1} = 1.58 \qquad \textit{Ans.}$$

6-51 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-95, the critical stress element experiences $\sigma = 28.0$ kpsi and $\tau = 15.3$ kpsi. Since the load is applied and released repeatedly, this gives $\sigma_{\max} = 28.0$ kpsi, $\sigma_{\min} = 0$

kpsi, $\tau_{\max} = 15.3$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 14.0$ kpsi, $\tau_m = \tau_a = 7.65$ kpsi. From Table A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \qquad r/d = 0.125/1 = 0.125$$
$$K_{t,\text{bend}} = 1.60, \qquad K_{t,\text{tors}} = 1.39$$

Figs. 6-26 and 6-27: $q_{\text{bend}} = 0.78$, $q_{\text{tors}} = 0.82$

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.78\left(1.60 - 1\right) = 1.47$$
$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.82\left(1.39 - 1\right) = 1.32$$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma_a' = \left\{\left[\left(1.47\right)\left(14.0\right)\right]^2 + 3\left[\left(1.32\right)\left(7.65\right)\right]^2\right\}^{1/2} = 27.0 \text{ kpsi}$$
$$\sigma_m' = \sigma_a' = 27.0 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma_{\max}' = \sigma_a' + \sigma_m'$,

$$n_y = \frac{S_y}{\sigma_a' + \sigma_m'} = \frac{54}{27.0 + 27.0} = 1.00$$

Since stress concentrations are included in this quick yield check, the low factor of safety is acceptable.

Eq. (6-10): $S_e' = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

Eq. (6-23): $d_e = 0.370d = 0.370\left(1\right) = 0.370$ in

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-17): $S_e = (0.81)(0.98)(0.5)(64) = 25.4$ kpsi

For the Morrow criterion, estimate the fatigue strength coefficient for steel.

Eq. (6-44): $\sigma_f' = S_{ut} + 50 = 64 + 50 = 114$ kpsi

Eq. (6-46):
$$n_f = \left(\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{\sigma_f'}\right)^{-1} = \left(\frac{27.0}{25.4} + \frac{27.0}{114}\right)^{-1}$$
$$n_f = 0.77 \qquad Ans.$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress, again using Morrow.

Eq. (6-59): $$\sigma_{ar} = \frac{\sigma'_a}{1-(\sigma'_m/\sigma'_f)} = \frac{27.0}{1-(27.0/114)} = 35.4 \text{ kpsi}$$

Fig. 6-23: Off the chart, so use $f = 0.9$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.9(64)]^2}{25.4} = 130.6$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.9(64)}{25.4}\right) = -0.1185$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{35.4}{130.6}\right)^{\frac{1}{-0.1185}} = 60856 \text{ cycles} \approx 61\,000 \text{ cycles} \qquad Ans.$$

6-52 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-96, the critical stress element experiences $\sigma_{x,\text{bend}} = 46.1$ kpsi, $\sigma_{x,\text{axial}} = 0.382$ kpsi and $\tau = 15.3$ kpsi. The axial load is practically negligible, but we'll include it to demonstrate the process. Since the load is applied and released repeatedly, this gives $\sigma_{\max,\text{bend}} = 46.1$ kpsi, $\sigma_{\min,\text{bend}} = 0$ kpsi, $\sigma_{\max,\text{axial}} = 0.382$ kpsi, $\sigma_{\min,\text{axial}} = 0$ kpsi, $\tau_{\max} = 15.3$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_{m,\text{bend}} = \sigma_{a,\text{bend}} = 23.05$ kpsi, $\sigma_{m,\text{axial}} = \sigma_{a,\text{axial}} = 0.191$ kpsi, $\tau_m = \tau_a = 7.65$ kpsi. From Table A-15-7, A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \qquad r/d = 0.125/1 = 0.125$$
$$K_{t,\text{bend}} = 1.60, \qquad K_{t,\text{tors}} = 1.39, \qquad K_{t,\text{axial}} = 1.75$$

Eqs. (6-33), (6-35), and (6-36), or Figs. 6-26 and 6-27: $q_{\text{bend}} = q_{\text{axial}} = 0.78$, $q_{\text{tors}} = 0.82$
Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.78(1.60-1) = 1.47$$
$$K_{f,\text{axial}} = 1 + q_{\text{axial}}\left(K_{t,\text{axial}} - 1\right) = 1 + 0.78(1.75-1) = 1.59$$
$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.82(1.39-1) = 1.32$$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma'_a = \left\{\left[(1.47)(23.05) + (1.59)(0.191)\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 38.4 \text{ kpsi}$$
$$\sigma'_m = \left\{\left[(1.47)(23.05) + (1.59)(0.191)\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 38.4 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{38.4 + 38.4} = 0.70$$

Since the conservative yield check indicates yielding, we will check more carefully with $\sigma'_{\max}$ obtained directly from the maximum stresses, using the distortion energy failure theory, without stress concentrations. Note that this is exactly the method used for static failure in Ch. 5.

$$\sigma'_{\max} = \sqrt{\left(\sigma_{\max,\text{bend}} + \sigma_{\max,\text{axial}}\right)^2 + 3\left(\tau_{\max}\right)^2} = \sqrt{\left(46.1 + 0.382\right)^2 + 3\left(15.3\right)^2} = 53.5 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{53.5} = 1.01 \qquad Ans.$$

This shows that yielding is imminent, and further analysis of fatigue life should not be interpreted as a guarantee of more than one cycle of life.

Eq. (6-10): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

Eq. (6-23): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-17): $S_e = (0.81)(0.98)(0.5)(64) = 25.4$ kpsi

For the Morrow criterion, estimate the fatigue strength coefficient for steel.

Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 64 + 50 = 114$ kpsi

Eq. (6-46): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{\sigma'_f}\right)^{-1} = \left(\frac{38.4}{25.4} + \frac{38.4}{114}\right)^{-1}$$

$$n_f = 0.54 \qquad Ans.$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress, again using Morrow.

Eq. (6-59): $$\sigma_{ar} = \frac{\sigma'_a}{1 - (\sigma'_m / \sigma'_f)} = \frac{38.4}{1 - (38.4/114)} = 57.9 \text{ kpsi}$$

Fig. 6-23: Off the chart, so use $f = 0.9$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.9(64)\right]^2}{25.4} = 130.6$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.9(64)}{25.4}\right) = -0.1185$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{57.9}{130.6}\right)^{\frac{1}{-0.1185}} = 960 \text{ cycles} \qquad Ans.$$

6-53 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-97, the critical stress element experiences $\sigma_{x,\text{bend}} = 55.5$ kpsi, $\sigma_{x,\text{axial}} = 0.382$ kpsi and $\tau = 15.3$ kpsi. The axial load is practically negligible, but we'll include it to demonstrate the process. Since the load is applied and released repeatedly, this gives $\sigma_{\text{max,bend}} = 55.5$ kpsi, $\sigma_{\text{min,bend}} = 0$ kpsi, $\sigma_{\text{max,axial}} = 0.382$ kpsi, $\sigma_{\text{min,axial}} = 0$ kpsi, $\tau_{\text{max}} = 15.3$ kpsi, $\tau_{\text{min}} = 0$ kpsi. Consequently, $\sigma_{m,\text{bend}} = \sigma_{a,\text{bend}} = 27.75$ kpsi, $\sigma_{m,\text{axial}} = \sigma_{a,\text{axial}} = 0.191$ kpsi, $\tau_m = \tau_a = 7.65$ kpsi. From Table A-15-7, A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \qquad r/d = 0.125/1 = 0.125$$

$$K_{t,\text{bend}} = 1.60, \qquad K_{t,\text{tors}} = 1.39, \qquad K_{t,\text{axial}} = 1.75$$

Eqs. (6-33), (6-35), and (6-36), or Figs. 6-26 and 6-27: $q_{\text{bend}} = q_{\text{axial}} = 0.78$, $q_{\text{tors}} = 0.82$

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.78\left(1.60 - 1\right) = 1.47$$

$$K_{f,\text{axial}} = 1 + q_{\text{axial}}\left(K_{t,\text{axial}} - 1\right) = 1 + 0.78\left(1.75 - 1\right) = 1.59$$

$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.82\left(1.39 - 1\right) = 1.32$$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma'_a = \left\{\left[(1.47)(27.75) + (1.59)(0.191)\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 44.66 \text{ kpsi}$$

$$\sigma'_m = \left\{\left[(1.47)(27.75) + (1.59)(0.191)\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 44.66 \text{ kpsi}$$

Since these stresses are relatively high compared to the yield strength, we will go ahead and check for yielding using the distortion energy failure theory.

$$\sigma'_{\text{max}} = \sqrt{\left(\sigma_{\text{max,bend}} + \sigma_{\text{max,axial}}\right)^2 + 3\left(\tau_{\text{max}}\right)^2} = \sqrt{\left(55.5 + 0.382\right)^2 + 3\left(15.3\right)^2} = 61.8 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\text{max}}} = \frac{54}{61.8} = 0.87 \qquad \textit{Ans.}$$

This shows that yielding is predicted. Further analysis of fatigue life is just to be able to report the fatigue factor of safety, though the life will be dictated by the static yielding failure, i.e. $N = 1/2$ cycle. *Ans.*

Eq. (6-10): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

Eq. (6-23): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-17): $S_e = (0.81)(0.98)(0.5)(64) = 25.4$ kpsi

For the Morrow criterion, estimate the fatigue strength coefficient for steel.

Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 64 + 50 = 114$ kpsi

Eq. (6-46): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{\sigma'_f}\right)^{-1} = \left(\frac{44.66}{25.4} + \frac{44.66}{114}\right)^{-1}$$

$$n_f = 0.47 \qquad \textit{Ans.}$$

6-54 From Table A-20, for AISI 1040 CD, S_{ut} = 85 kpsi and S_y = 71 kpsi. From the solution to Prob. 6-17 we find the completely reversed stress at the critical shoulder fillet to be σ_{ar} = 35.0 kpsi, producing σ_a = 35.0 kpsi and σ_m = 0 kpsi. This problem adds a steady torque which creates torsional stresses of

$$\tau_m = \frac{Tr}{J} = \frac{2500(1.625/2)}{\pi(1.625^4)/32} = 2967 \text{ psi} = 2.97 \text{ kpsi}, \qquad \tau_a = 0 \text{ kpsi}$$

From Table A-15-8 and A-15-9, $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_{t,\text{bend}} = 1.95$, $K_{t,\text{tors}} = 1.60$
Eqs. (6-33), (6-35) and (6-36), or Figs. 6-26 and 6-27: $q_{\text{bend}} = 0.76$, $q_{\text{tors}} = 0.81$
Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.76(1.95 - 1) = 1.72$$

$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.81(1.60 - 1) = 1.49$$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma'_a = \left\{\left[(1.72)(35.0)\right]^2 + 3\left[(1.49)(0)\right]^2\right\}^{1/2} = 60.2 \text{ kpsi}$$

$$\sigma'_m = \left\{\left[(1.72)(0)\right]^2 + 3\left[(1.49)(2.97)\right]^2\right\}^{1/2} = 7.66 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{71}{60.2 + 7.66} = 1.05$$

From the solution to Prob. 6-17, S_e = 27.0 kpsi. Using Goodman,

Eq. (6-41): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{60.2}{27.0} + \frac{7.66}{85}\right)^{-1}$$

$$n_f = 0.43 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress. Choosing the Goodman criterion, Eq. (6-58): $\sigma_{ar} = \dfrac{\sigma'_a}{1-(\sigma'_m / S_{ut})} = \dfrac{60.2}{1-(7.66/85)} = 66.2$ kpsi

Fig. 6-23: $f = 0.867$

Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.867(85)]^2}{27.0} = 201.1$

Eq. (6-14): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.867(85)}{27.0}\right) = -0.1454$

Eq. (6-15): $N = \left(\dfrac{\sigma_{ar}}{a}\right)^{1/b} = \left(\dfrac{66.2}{201.1}\right)^{\frac{1}{-0.1454}} = 2084$ cycles

$N = 2100$ cycles *Ans.*

6-55 From the solution to Prob. 6-18 we find the completely reversed stress at the critical shoulder fillet to be $\sigma_{\text{rev}} = 32.8$ kpsi, producing $\sigma_a = 32.8$ kpsi and $\sigma_m = 0$ kpsi. This problem adds a steady torque which creates torsional stresses of

$$\tau_m = \frac{Tr}{J} = \frac{2200(1.625/2)}{\pi\left(1.625^4\right)/32} = 2611 \text{ psi} = 2.61 \text{ kpsi}, \quad \tau_a = 0 \text{ kpsi}$$

From Table A-15-8 and A-15-9, $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_{t,\text{bend}} = 1.95$, $K_{t,\text{tors}} = 1.60$

Eqs. (6-33), (6-35) and (6-36), or Figs. 6-26 and 6-27: $q_{\text{bend}} = 0.76$, $q_{\text{tors}} = 0.81$
Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.76(1.95 - 1) = 1.72$$
$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.81(1.60 - 1) = 1.49$$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma'_a = \left\{\left[(1.72)(32.8)\right]^2 + 3\left[(1.49)(0)\right]^2\right\}^{1/2} = 56.4 \text{ kpsi}$$
$$\sigma'_m = \left\{\left[(1.72)(0)\right]^2 + 3\left[(1.49)(2.61)\right]^2\right\}^{1/2} = 6.74 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{71}{56.4 + 6.74} = 1.12$$

From the solution to Prob. 6-18, $S_e = 27.0$ kpsi. Using Goodman, Eq. (6-41): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{56.4}{27.0} + \frac{6.74}{85}\right)^{-1}$$

$$n_f = 0.46 \qquad Ans.$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress. Choosing the Goodman criterion,

Eq. (6-58): $$\sigma_{ar} = \frac{\sigma'_a}{1-(\sigma'_m / S_{ut})} = \frac{56.4}{1-(6.74/85)} = 61.3 \text{ kpsi}$$

Fig. 6-23: $f = 0.867$

Eq. (6-13): $$a = \frac{(f\, S_{ut})^2}{S_e} = \frac{[0.867(85)]^2}{27.0} = 201.1$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\, S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.867(85)}{27.0}\right) = -0.1454$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{61.3}{201.1}\right)^{\frac{1}{-0.1454}} = 3536 \text{ cycles}$$

$$N = 3500 \text{ cycles} \qquad Ans.$$

6-56 $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi, $K_{ts} = 1.6$, $L = 2$ ft, $F_{\min} = 150$ lbf, $F_{\max} = 500$ lbf

Eqs. (6-33) and (6-36), or Fig. 6-27: $q_s = 0.80$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(1.6-1) = 1.48$

$$T_{\max} = 500(2) = 1000 \text{ lbf}\cdot\text{in}, \quad T_{\min} = 150(2) = 300 \text{ lbf}\cdot\text{in}$$

$$\tau_{\max} = \frac{16K_{fs}T_{\max}}{\pi d^3} = \frac{16(1.48)(1000)}{\pi(0.875)^3} = 11\,251 \text{ psi} = 11.25 \text{ kpsi}$$

$$\tau_{\min} = \frac{16K_{fs}T_{\min}}{\pi d^3} = \frac{16(1.48)(300)}{\pi(0.875)^3} = 3375 \text{ psi} = 3.38 \text{ kpsi}$$

$$\tau_m = \frac{\tau_{\max} + \tau_{\min}}{2} = \frac{11.25 + 3.38}{2} = 7.32 \text{ kpsi}$$

$$\tau_a = \frac{\tau_{\max} - \tau_{\min}}{2} = \frac{11.25 - 3.38}{2} = 3.94 \text{ kpsi}$$

Since the stress is entirely shear, it is convenient to check for yielding using the standard Maximum Shear Stress theory.

$$n_y = \frac{S_y / 2}{\tau_{\max}} = \frac{30/2}{11.25} = 1.33$$

Find the modifiers and endurance limit.

Eq. (6-10): $S'_e = 0.5(55) = 27.5$ kpsi

Eq. (6-18): $k_a = 11.0(55)^{-0.650} = 0.81$

Eq. (6-23): $d_e = 0.370(0.875) = 0.324$ in

Eq. (6-19): $k_b = 0.879(0.324)^{-0.107} = 0.99$

Eq. (6-25): $k_c = 0.59$

Eq. (6-17): $S_{se} = 0.81(0.99)(0.59)(27.5) = 13.0$ kpsi

Since the stress is entirely shear, we will use a load factor $k_c = 0.59$, and convert the ultimate strength to a shear value rather than using the combination loading method of Sec. 6-16. From Eq. (6-58), $S_{su} = 0.67S_u = 0.67\ (55) = 36.9$ kpsi.

(a) Goodman,

Eq. (6-41): $$n_f = \left(\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}\right)^{-1} = \left(\frac{3.94}{13.0} + \frac{7.32}{36.9}\right)^{-1} = 1.99 \qquad Ans.$$

(b) Gerber

Eq. (6-48): $$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)^2}\right]$$

$$n_f = \frac{1}{2}\left(\frac{36.9}{7.32}\right)^2 \left(\frac{3.94}{13.0}\right)\left[-1 + \sqrt{1 + \left(\frac{2(7.32)(13.0)}{36.9(3.94)}\right)^2}\right]$$

$$n_f = 2.49 \qquad Ans.$$

6-57 $S_{ut} = 145$ kpsi, $S_y = 120$ kpsi

From Eqs. (6-33) and (6-35), or Fig. 6-26, with a notch radius of 0.1 in, $q = 0.9$. Thus, with $K_t = 3$ from the problem statement,

$$K_f = 1 + q(K_t - 1) = 1 + 0.9(3-1) = 2.80$$

$$\sigma_{\max} = -K_f \frac{4P}{\pi d^2} = \frac{-2.80(4)(P)}{\pi(1.2)^2} = -2.476P$$

$$\sigma_m = -\sigma_a = \frac{1}{2}(-2.476P) = -1.238P$$

$$T_{\max} = \frac{f\,P\left(D+d\right)}{4} = \frac{0.3P\left(6+1.2\right)}{4} = 0.54P$$

From Eqs. (6-33) and (6-36), or Fig. 6-27, with a notch radius of 0.1 in, $q_s = 0.92$. Thus, with K_{ts} = 1.8 from the problem statement,

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.92(1.8 - 1) = 1.74$$

$$\tau_{\max} = \frac{16K_{fs}T}{\pi d^3} = \frac{16(1.74)(0.54P)}{\pi(1.2)^3} = 2.769P$$

$$\tau_a = \tau_m = \frac{\tau_{\max}}{2} = \frac{2.769P}{2} = 1.385P$$

Eqs. (6-66) and (6-67):

$$\sigma'_a = [\sigma_a{}^2 + 3\tau_a^2]^{1/2} = [(1.238P)^2 + 3(1.385P)^2]^{1/2} = 2.70P$$

$$\sigma'_m = [\sigma_m{}^2 + 3\tau_m^2]^{1/2} = [(-1.238P)^2 + 3(1.385P)^2]^{1/2} = 2.70P$$

Eq. (6-10): $S'_e = 0.5(145) = 72.5$ kpsi

Eq. (6-18): $k_a = 2.00(145)^{-0.217} = 0.68$

Eq. (6-19): $k_b = 0.879(1.2)^{-0.107} = 0.862$

Eq. (6-17): $S_e = (0.68)(0.862)(72.5) = 42.5$ kpsi

Eq. (6-41): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{2.70P}{42.5} + \frac{2.70P}{145}\right)^{-1} = 3$$

$P = 4.1$ kips *Ans.*

Yield (conservative):

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{120}{(2.70)(4.1) + (2.70)(4.1)} = 5.4$$ Yielding is not predicted. *Ans.*

6-58 From Prob. 6-57, $K_f = 2.80, K_{fs} = 1.74, S_e = 42.5$ kpsi

$$\sigma_{\max} = -K_f \frac{4P_{\max}}{\pi d^2} = -2.80\frac{4(18)}{\pi(1.2^2)} = -44.56 \text{ kpsi}$$

$$\sigma_{\min} = -K_f \frac{4P_{\min}}{\pi d^2} = -2.80\frac{4(4.5)}{\pi(1.2)^2} = -11.14 \text{ kpsi}$$

$$T_{\max} = f\,P_{\max}\left(\frac{D+d}{4}\right) = 0.3(18)\left(\frac{6+1.2}{4}\right) = 9.72 \text{ kip}\cdot\text{in}$$

$$T_{\min} = f\,P_{\min}\left(\frac{D+d}{4}\right) = 0.3(4.5)\left(\frac{6+1.2}{4}\right) = 2.43 \text{ kip}\cdot\text{in}$$

$$\tau_{\max} = K_{fs}\frac{16T_{\max}}{\pi d^3} = 1.74\frac{16(9.72)}{\pi(1.2)^3} = 49.85 \text{ kpsi}$$

$$\tau_{\min} = K_{fs}\frac{16T_{\min}}{\pi d^3} = 1.74\frac{16(2.43)}{\pi(1.2)^3} = 12.46 \text{ kpsi}$$

$$\sigma_a = \left|\frac{-44.56-(-11.14)}{2}\right| = 16.71 \text{ kpsi}$$
$$\sigma_m = \frac{-44.56+(-11.14)}{2} = -27.85 \text{ kpsi}$$
$$\tau_a = \frac{49.85-12.46}{2} = 18.70 \text{ kpsi}$$
$$\tau_m = \frac{49.85+12.46}{2} = 31.16 \text{ kpsi}$$

Eqs. (6-66) and (6-67):

$$\sigma'_a = [(\sigma_a/0.85)^2 + 3\tau_a^2]^{1/2} = [(16.71/0.85)^2 + 3(18.70)^2]^{1/2} = 37.89 \text{ kpsi}$$
$$\sigma'_m = [\sigma_m{}^2 + 3\tau_m^2]^{1/2} = [(-27.85)^2 + 3(31.16)^2]^{1/2} = 60.73 \text{ kpsi}$$

Goodman:

Eq. (6-41): $n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{37.89}{42.5} + \frac{60.73}{145}\right)^{-1}$

$n_f = 0.76$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).
Choosing the Goodman criterion,

Eq. (6-58): $\sigma_{ar} = \frac{\sigma'_a}{1-(\sigma'_m/S_{ut})} = \frac{37.89}{1-(60.73/145)} = 65.2 \text{ kpsi}$

Fig. 6-23: $f = 0.8$

Eq. (6-13): $a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.8(145)]^2}{42.5} = 316.6$

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.8(145)}{42.5}\right) = -0.1454$

Eq. (6-15): $N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{65.2}{316.6}\right)^{\frac{1}{-0.1454}} = 52\,460 \text{ cycles}$

$N = 52\ 500$ cycles *Ans.*

6-59 For AISI 1020 CD, From Table A-20, $S_y = 390$ MPa, $S_{ut} = 470$ MPa. Given: $S_e = 175$ MPa.

First Loading: $(\sigma_m)_1 = \dfrac{360+160}{2} = 260 \text{ MPa}, \quad (\sigma_a)_1 = \dfrac{360-160}{2} = 100 \text{ MPa}$

Goodman, Eq. (6-58):

$$(\sigma_a)_{e1} = \frac{(\sigma_a)_1}{1-(\sigma_m)_1/S_{ut}} = \frac{100}{1-260/470} = 223.8 \text{ MPa} > S_e \;\therefore \text{ finite life}$$

Fig. 6-23: Off the graph, so let $f = 0.9$.

$$a = \frac{[0.9(470)]^2}{175} = 1022.5 \text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{0.9(470)}{175} = -0.127\,767$$

$$N = \left(\frac{223.8}{1022.5}\right)^{-1/0.127767} = 145\ 920 \text{ cycles}$$

Second loading: $(\sigma_m)_2 = \dfrac{320+(-200)}{2} = 60 \text{ MPa}, \quad (\sigma_a)_2 = \dfrac{320-(-200)}{2} = 260 \text{ MPa}$

$$(\sigma_a)_{e2} = \frac{260}{1-60/470} = 298.0 \text{ MPa}$$

(**a**) Miner's method: $N_2 = \left(\dfrac{298.0}{1022.5}\right)^{-1/0.127767} = 15\ 520 \text{ cycles}$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \quad \Rightarrow \quad \frac{80\ 000}{145\ 920} + \frac{n_2}{15\ 520} = 1 \quad \Rightarrow \quad n_2 = 7000 \text{ cycles} \quad \textit{Ans.}$$

(**b**) Manson's method: The number of cycles remaining after the first loading

$N_{\text{remaining}} = 145\ 920 - 80\ 000 = 65\ 920$ cycles

Two data points: 0.9(470) MPa, 10^3 cycles
223.8 MPa, 65 920 cycles

$$\frac{0.9(470)}{223.8} = \frac{a_2\left(10^3\right)^{b_2}}{a_2\left(65\ 920\right)^{b_2}}$$

$$1.8901 = \left(0.015170\right)^{b_2}$$

$$b_2 = \frac{\log 1.8901}{\log 0.015170} = -0.151\ 997$$

$$a_2 = \frac{223.8}{\left(65\ 920\right)^{-0.151\ 997}} = 1208.7 \text{ MPa}$$

$$n_2 = \left(\frac{298.0}{1208.7}\right)^{1/-0.151\ 997} = 10\ 000 \text{ cycles} \qquad Ans.$$

6-60 Given: S_e = 50 kpsi, S_{ut} = 140 kpsi, f =0.8. Using Miner's method,

$$a = \frac{\left[0.8(140)\right]^2}{50} = 250.88 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\frac{0.8(140)}{50} = -0.116\ 749$$

$$\sigma_1 = 95 \text{ kpsi}, \qquad N_1 = \left(\frac{95}{250.88}\right)^{1/-0.116\ 749} = 4100 \text{ cycles}$$

$$\sigma_2 = 80 \text{ kpsi}, \qquad N_2 = \left(\frac{80}{250.88}\right)^{1/-0.116\ 749} = 17\ 850 \text{ cycles}$$

$$\sigma_3 = 65 \text{ kpsi}, \qquad N_3 = \left(\frac{65}{250.88}\right)^{1/-0.116\ 749} = 105\ 700 \text{ cycles}$$

$$\frac{0.2N}{4100} + \frac{0.5N}{17\ 850} + \frac{0.3N}{105\ 700} = 1 \quad \Rightarrow \quad N = 12\ 600 \text{ cycles} \quad Ans.$$

6-61 Given: S_{ut} = 530 MPa, S_e = 210 MPa, and f = 0.9.

(a) Miner's method

$$a = \frac{\left[0.9(530)\right]^2}{210} = 1083.47 \text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{0.9(530)}{210} = -0.118\ 766$$

$$\sigma_1 = 350 \text{ MPa}, \quad N_1 = \left(\frac{350}{1083.47}\right)^{1/-0.118\ 766} = 13\ 550 \text{ cycles}$$

$$\sigma_2 = 260\ \text{MPa},\quad N_2 = \left(\frac{260}{1083.47}\right)^{1/-0.118\,766} = 165\ 600\ \text{cycles}$$

$$\sigma_3 = 225\ \text{MPa},\quad N_3 = \left(\frac{225}{1083.47}\right)^{1/-0.118\,766} = 559\ 400\ \text{cycles}$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1$$

$$\frac{5000}{13\ 550} + \frac{50\ 000}{165\ 600} + \frac{n_3}{559\ 400} = 184\ 100\ \text{cycles} \qquad \textit{Ans.}$$

(**b**) Manson's method:

The life remaining after the first series of cycling is N_{R1} = 13 550 − 5000 = 8550 cycles. The two data points required to define $S'_{e,1}$ are [0.9(530), 10^3] and (350, 8550).

$$\frac{0.9(530)}{350} = \frac{a_2\left(10^3\right)^{b_2}}{a_2\left(8550\right)^{b_2}} \quad\Rightarrow\quad 1.3629 = \left(0.11696\right)^{b_2}$$

$$b_2 = \frac{\log\left(1.362\ 9\right)}{\log\left(0.116\ 96\right)} = -0.144\,280$$

$$a_2 = \frac{350}{\left(8550\right)^{-0.144\ 280}} = 1292.3\ \text{MPa}$$

$$N_2 = \left(\frac{260}{1292.3}\right)^{-1/0.144\ 280} = 67\ 090\ \text{cycles}$$

$$N_{R2} = 67\ 090 - 50\ 000 = 17\ 090\ \text{cycles}$$

$$\frac{0.9(530)}{260} = \frac{a_3\left(10^3\right)^{b_3}}{a_3\left(17\ 090\right)^{b_3}} \quad\Rightarrow\quad 1.834\ 6 = \left(0.058\ 514\right)^{b_2}$$

$$b_3 = \frac{\log\left(1.834\ 6\right)}{\log\left(0.058\ 514\right)} = -0.213\ 785,\quad a_3 = \frac{260}{\left(17\ 090\right)^{-0.213\ 785}} = 2088.7\ \text{MPa}$$

$$N_3 = \left(\frac{225}{2088.7}\right)^{-1/0.213\ 785} = 33\ 610\ \text{cycles} \qquad \textit{Ans.}$$

6-62 Given: S_e = 45 kpsi, S_{ut} = 85 kpsi, f = 0.86, and σ_a = 35 kpsi and σ_m = 30 kpsi for 12 (10^3) cycles.

Goodman equivalent reversing stress, Eq. (6-58):

$$\sigma_{ar} = \frac{\sigma_a}{1-(\sigma_m / S_{ut})} = \frac{35}{1-(30/85)} = 54.09 \text{ kpsi}$$

Initial cycling

$$a = \frac{[0.86(85)]^2}{45} = 116.00 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\frac{0.86(85)}{45} = -0.070\ 235$$

$$\sigma_1 = 54.09 \text{ kpsi}, \qquad N_1 = \left(\frac{54.09}{116.00}\right)^{1/-0.070\ 235} = 52\ 190 \text{ cycles}$$

(**a**) Miner's method: The number of remaining cycles at 54.09 kpsi is

$N_{\text{remaining}} = 52\ 190 - 12\ 000 = 40\ 190$ cycles.

The new coefficients are $b' = b$, and $a' = S_f / N^b = 54.09/(40\ 190)^{-0.070\ 235} = 113.89$ kpsi.

The new endurance limit is

$$S'_{e,1} = a'N_e^{b'} = 113.89\left(10^6\right)^{-0.070\ 235} = 43.2 \text{ kpsi} \qquad \textit{Ans.}$$

(**b**) Manson's method: The number of remaining cycles at 54.09 kpsi is

$N_{\text{remaining}} = 52\ 190 - 12\ 000 = 40\ 190$ cycles.

At 10^3 cycles,

$S_f = 0.86(85) = 73.1$ kpsi.

The new coefficients are

$b' = [\log(73.1/54.09)]/\log(10^3/40\ 190) = -0.081\ 540$

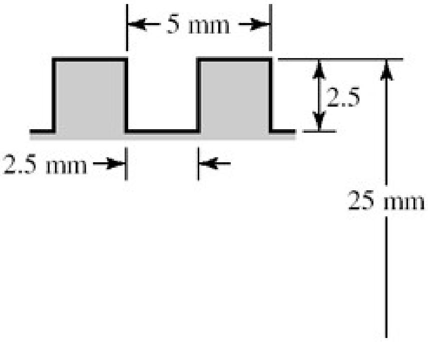
and $\quad a' = \sigma_1 / (N_{\text{remaining}})^{b'} = 54.09/(40\ 190)^{-0.081\ 540} = 128.39$ kpsi.

The new endurance limit is

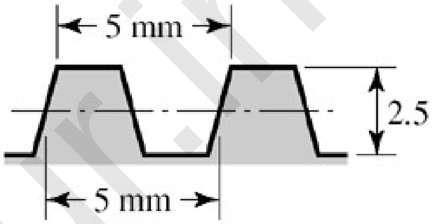
$$S'_{e,1} = a'N_e^{b'} = 128.39\left(10^6\right)^{-0.081\ 540} = 41.6 \text{ kpsi} \qquad \textit{Ans.}$$

Chapter 8

8-1 **(a)** Thread depth= 2.5 mm *Ans.*
Width = 2.5 mm *Ans.*
d_m = 25 - 1.25 - 1.25 = 22.5 mm
d_r = 25 - 5 = 20 mm
$l = p$ = 5 mm *Ans.*



(b) Thread depth = 2.5 mm *Ans.*
Width at pitch line = 2.5 mm *Ans.*
d_m = 22.5 mm
d_r = 20 mm
$l = p$ = 5 mm *Ans.*



8-2 From Table 8-1,

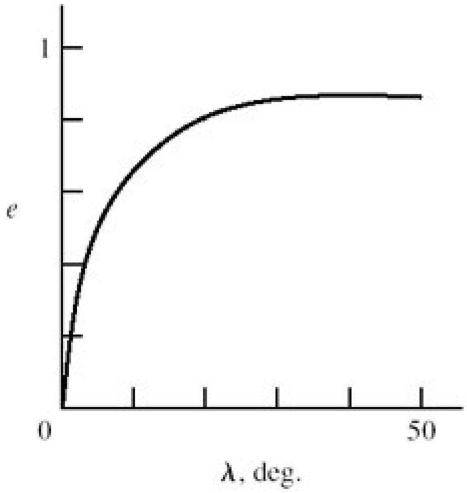
$$d_r = d - 1.226\,869p$$
$$d_m = d - 0.649\,519p$$
$$\bar{d} = \frac{d - 1.226\,869p + d - 0.649\,519p}{2} = d - 0.938\,194p$$
$$A_t = \frac{\pi\bar{d}^2}{4} = \frac{\pi}{4}(d - 0.938\,194p)^2 \qquad Ans.$$

8-3 From Eq. (*c*) of Sec. 8-2,

$$P_R = F\frac{\tan\lambda + f}{1 - f\tan\lambda}$$
$$T_R = \frac{P_R d_m}{2} = \frac{Fd_m}{2}\frac{\tan\lambda + f}{1 - f\tan\lambda}$$
$$e = \frac{T_0}{T_R} = \frac{Fl/(2\pi)}{Fd_m/2}\frac{1 - f\tan\lambda}{\tan\lambda + f} = \tan\lambda\frac{1 - f\tan\lambda}{\tan\lambda + f} \qquad Ans.$$

Using $f = 0.08$, form a table and plot the efficiency curve.

λ, deg.	e
0	0
0	0.678
20	0.796
30	0.838
40	0.8517
45	0.8519



8-4 Given $F = 5$ kN, $l = 5$ mm, and $d_m = d - p/2 = 25 - 5/2 = 22.5$ mm, the torque required to raise the load is found using Eqs. (8-1) and (8-6)

$$T_R = \frac{5(22.5)}{2}\left[\frac{5+\pi(0.09)22.5}{\pi(22.5)-0.09(5)}\right] + \frac{5(0.06)45}{2} = 15.85\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

The torque required to lower the load, from Eqs. (8-2) and (8-6) is

$$T_L = \frac{5(22.5)}{2}\left[\frac{\pi(0.09)22.5-5}{\pi(22.5)+0.09(5)}\right] + \frac{5(0.06)45}{2} = 7.83\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

Since T_L is positive, the thread is self-locking. From Eq.(8-4) the efficiency is

$$e = \frac{5(5)}{2\pi(15.85)} = 0.251 \qquad \textit{Ans.}$$

8-5 Collar (thrust) bearings, at the bottom of the screws, must bear on the collars. The bottom segment of the screws must be in compression. Whereas, tension specimens and their grips must be in tension. Both screws must be of the same-hand threads.

8-6 Screws rotate at an angular rate of

$$n = \frac{1720}{60} = 28.67 \text{ rev/min}$$

(a) The lead is 0.25 in, so the linear speed of the press head is

$$V = 28.67(0.25) = 7.17 \text{ in/min} \quad \textit{Ans.}$$

(b) $F = 2500$ lbf/screw

$$d_m = 2 - 0.25 / 2 = 1.875 \text{ in}$$
$$\sec \alpha = 1 / \cos(29^\circ / 2) = 1.033$$

Eq. (8-5):

$$T_R = \frac{2500(1.875)}{2}\left(\frac{0.25 + \pi(0.05)(1.875)(1.033)}{\pi(1.875) - 0.05(0.25)(1.033)}\right) = 221.0 \text{ lbf} \cdot \text{in}$$

Eq. (8-6):

$$T_c = 2500(0.08)(3.5 / 2) = 350 \text{ lbf} \cdot \text{in}$$
$$T_{total} = 350 + 221.0 = 571 \text{ lbf} \cdot \text{in/screw}$$
$$T_{motor} = \frac{571(2)}{60(0.95)} = 20.04 \text{ lbf} \cdot \text{in}$$
$$H = \frac{Tn}{63\ 025} = \frac{20.04(1720)}{63\ 025} = 0.547 \text{ hp} \quad \textit{Ans.}$$

8-7 AISI 1006 CD steel. Table A-20: $S_y = 41$ kpsi.

(a) The handle has maximum bending moment where it enters the screw body.

$$M = (3.5 - 0.375)\ F = 3.125\ F$$
$$S_y = \sigma = \frac{32M}{\pi d^3} = \frac{32(3.125)F}{\pi(0.375)^3} = 41\ 000$$
$$F = 67.9 \text{ lbf} \quad \textit{Ans.}$$

(b) Using Fig. 8-3 for the Acme thread, $p = l = 1/6 = 0.1667$ in

$$d_m = d - p/2 = 0.75 - 1/12 = 0.6667 \text{ in}, d_r = d - p = 0.75 - 1/6 = 0.5833 \text{ in}$$
$$\alpha = 29^\circ/2 = 14.5^\circ, \quad \sec 14.5^\circ = 1.033$$

From Eqs. (8-5) and (8-6),

$$T_{\text{total}} = \frac{Fd_m}{2}\left(\frac{l + \pi f d_m \sec\alpha}{\pi d_m - fl \sec\alpha}\right) + \frac{Ff_c d_c}{2}$$
$$= \frac{F(0.6667)}{2}\left[\frac{0.1667 + \pi(0.15)(0.6667)(1.033)}{\pi(0.6667) - 0.15(0.1667)(1.033)}\right] + \frac{F(0.15)(1)}{2} = 0.1542F$$

From part (a), $T_{\text{total}} = 3.5\ F = 3.5(67.9) = 237.7$ lbf·in

$$F = \frac{237.7}{0.1542} = 1542 \text{ lbf} \quad \textit{Ans.}$$

(c) Using Eqs. (8-11), (8-8), (8-7), and (8-12):

Bending stress in first thread, with the force on the first thread being $0.38F$ and $n_t = 1$,

$$\sigma_x = \frac{6(0.38F)}{\pi d_r n_t p} = \frac{6(0.38)(1542)}{\pi(0.5833)(1)(0.1667)} = 11\ 510 \text{ psi} = 11.5 \text{ kpsi}$$

Axial stress in body of screw,

$$\sigma_y = -\frac{4F}{\pi d_r^{\,2}} = -\frac{4(1542)}{\pi(0.5833)^2} = -5770 \text{ psi} = -5.77 \text{ kpsi}$$

Torsion in body of screw:

$$\tau_{yz} = \frac{16T}{\pi d_r^{\,3}} = \frac{16(237.7)}{\pi(0.5833)^3} = 6100 \text{ psi} = 6.10 \text{ kpsi}$$

The tangential shear stress given by Eq. (8-12) with one thread carrying 0.38 T:

$$\tau_{zx} = -\frac{4(0.38T)}{\pi d_r^2(1)p} = -\frac{4(0.38)237.7}{\pi(0.5833)^2(1)0.1667} = -2030 \text{ psi} = -2.03 \text{ kpsi}$$

From Eq. (5-14),

$$\sigma' = \frac{1}{\sqrt{2}}\left\{[11.5-(-5.77)]^2 + (-5.77-0)^2 + (0-11.5)^2 + 6(6.10)^2 + 6(-2.03)\right\}^{1/2}$$

$$= 18.9 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'} = \frac{41}{18.9} = 2.2 \quad \textit{Ans.}$$

(d) The column has one end fixed and the other end pivoted in the swivel joint of the anvil striker, so from Table 4-2, $C = 1.2$. We will use the root diameter of the screw body to check buckling, neglecting the effect of the threads.

$A = \pi(0.5833^2)/4 = 0.267 \text{ in}^2$, $S_y = 41$ kpsi, $E = 30(10^6)$ psi, $L = 8$ in,

$$k = \sqrt{I/A} = \sqrt{\frac{\pi d^4/64}{\pi d^2/4}} = d/4 = 0.5833/4 = 0.1458 \text{ in}, \; L/k = 8/0.1458 = 54.9$$

From Eq. (4-45),

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2 CE}{S_y}\right)^{1/2} = \left[\frac{2\pi^2(1.2)30(10^6)}{41\ 000}\right]^{1/2} = 131.7$$

Since 54.9 < 131.7, the J.B. Johnson formula is applicable. From Eq. (4-48), the critical clamping force for buckling is

$$P_{\text{cr}} = A\left[S_y - \left(\frac{S_y}{2\pi}\frac{l}{k}\right)^2 \frac{1}{CE}\right]$$

$$= 0.267\left\{41(10^3) - \left[\frac{41(10^3)}{2\pi}(54.9)\right]^2 \frac{1}{1.2(30)10^6}\right\} = 9995 \text{ lbf} \qquad \textit{Ans}$$

$$n = \frac{P_{cr}}{F} = \frac{9995}{1542} = 6.5 \quad \textit{Ans.}$$

It is confirmed that the weak link is the yielding of the handle.

8-8 $T = 8(3.5) = 28 \text{ lbf} \cdot \text{in}$

$$d_m = \frac{3}{4} - \frac{1}{12} = 0.6667 \text{ in}$$

$$l = \frac{1}{6} = 0.1667 \text{ in}, \quad \alpha = \frac{29^0}{2} = 14.5^0, \quad \sec 14.5^0 = 1.033$$

From Eqs. (8-5) and (8-6)

$$T_{\text{total}} = \frac{0.6667F}{2}\left[\frac{0.1667+\pi(0.15)(0.6667)(1.033)}{\pi(0.6667)-0.15(0.1667)(1.033)}\right]+\frac{0.15(1)F}{2} = 0.1542F$$

$$F = \frac{28}{0.1542} = 182 \text{ lbf} \qquad \textit{Ans.}$$

8-9 $d_m = 1.5 - 0.25/2 = 1.375$ in, $l = 2(0.25) = 0.5$ in

From Eq. (8-1) and Eq. (8-6),

$$T_R = \frac{2.2(10^3)(1.375)}{2}\left[\frac{0.5 + \pi(0.10)(1.375)}{\pi(1.375) - 0.10(0.5)}\right] + \frac{2.2(10^3)(0.15)(2.25)}{2}$$
$$= 330 + 371 = 701 \text{ lbf} \cdot \text{in}$$

Since $n = V/l = 2/0.5 = 4$ rev/s = 240 rev/min

so the power is

$$H = \frac{Tn}{63\ 025} = \frac{701(240)}{63\ 025} = 2.67 \text{ hp} \qquad \textit{Ans.}$$

8-10 $d_m = 40 - 4 = 36$ mm, $l = p = 8$ mm

From Eqs. (8-1) and (8-6)

$$T = \frac{36F}{2}\left[\frac{8 + \pi(0.14)(36)}{\pi(36) - 0.14(8)}\right] + \frac{0.09(100)F}{2}$$
$$= (3.831 + 4.5)F = 8.33F \quad \text{N} \cdot \text{m} \quad (F \text{ in kN})$$
$$\omega = 2\pi n = 2\pi(1) = 2\pi \text{ rad/s}$$
$$H = T\omega$$
$$T = \frac{H}{\omega} = \frac{3000}{2\pi} = 477 \text{ N} \cdot \text{m}$$
$$F = \frac{477}{8.33} = 57.3 \text{ kN} \qquad \textit{Ans.}$$

$$e = \frac{Fl}{2\pi T} = \frac{57.3(8)}{2\pi(477)} = 0.153 \quad Ans.$$

8-11 (**a**) Table A-31, nut height $H = 12.8$ mm. $L \geq l + H = 2(15) + 12.8 = 42.8$ mm. Rounding up,

$L = 45$ mm *Ans.*

(**b**) From Eq. (8-14), $L_T = 2d + 6 = 2(14) + 6 = 34$ mm
From Table 8-7, $l_d = L - L_T = 45 - 34 = 11$ mm, $l_t = l - l_d = 2(15) - 11 = 19$ mm,

$A_d = \pi(14^2)/4 = 153.9$ mm^2. From Table 8-1, $A_t = 115$ mm^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{153.9(115)207}{153.9(19) + 115(11)} = 874.6 \text{ MN/m} \quad Ans.$$

(**c**) From Eq. (8-22), with $l = 2(15) = 30$ mm

$$k_m = \frac{0.5774\pi E d}{2\ln\left(5\dfrac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)} = \frac{0.5774\pi(207)14}{2\ln\left[5\dfrac{0.5774(30) + 0.5(14)}{0.5774(30) + 2.5(14)}\right]} = 3\,116.5 \text{ MN/m} \quad Ans.$$

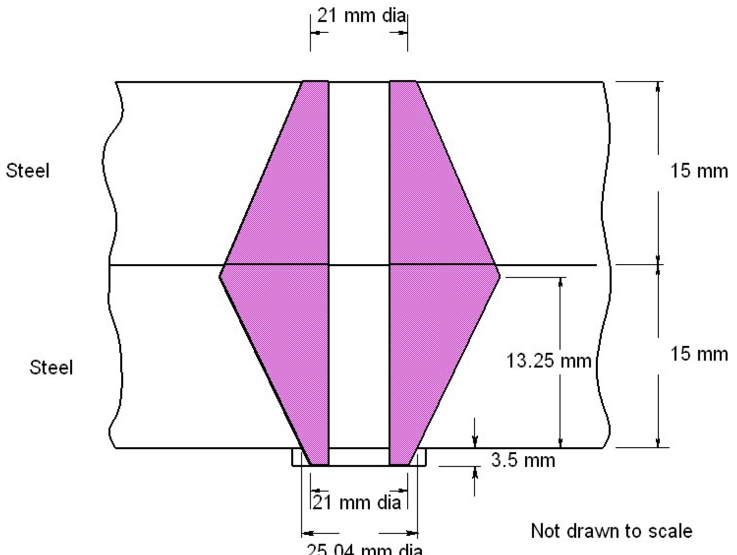
8-12 (**a**) Table A-31, nut height $H = 12.8$ mm. Table A-33, washer thickness $t = 3.5$ mm. Thus, the grip is $l = 2(15) + 3.5 = 33.5$ mm. $L \geq l + H = 33.5 + 12.8 = 46.3$ mm. Rounding up

$L = 50$ mm *Ans.*

(**b**) From Eq. (8-14), $L_T = 2d + 6 = 2(14) + 6 = 34$ mm
From Table 8-7, $l_d = L - L_T = 50 - 34 = 16$ mm, $l_t = l - l_d = 33.5 - 16 = 17.5$ mm,
$A_d = \pi(14^2)/4 = 153.9$ mm^2. From Table 8-1, $A_t = 115$ mm^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{153.9(115)207}{153.9(17.5) + 115(16)} = 808.2 \text{ MN/m} \quad Ans.$$

(**c**)



From Eq. (8-22)

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\dfrac{0.5774l+0.5d}{0.5774l+2.5d}\right)} = \frac{0.5774\pi(207)14}{2\ln\left[5\dfrac{0.5774(33.5)+0.5(14)}{0.5774(33.5)+2.5(14)}\right]} = 2\ 969\ \text{MN/m} \qquad \textit{Ans.}$$

8-13 (**a**) Table 8-7, $l = h + d/2 = 15 + 14/2 = 22$ mm. $L \geq h + 1.5d = 36$ mm. Rounding up $L = 40$ mm *Ans.*

(**b**) From Eq. (8-14), $L_T = 2d + 6 = 2(14) + 6 = 34$ mm
From Table 8-7, $l_d = L - L_T = 40 - 34 = 6$ mm, $l_t = l - l_d = 22 - 6 = 16$ mm

$A_d = \pi(14^2)/4 = 153.9$ mm^2. From Table 8-1, $A_t = 115$ mm^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{153.9(115)207}{153.9(16)+115(6)} = 1\ 162.2\ \text{MN/m} \qquad \textit{Ans.}$$

(**c**) From Eq. (8-22), with $l = 22$ mm

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\dfrac{0.5774l+0.5d}{0.5774l+2.5d}\right)} = \frac{0.5774\pi(207)14}{2\ln\left[5\dfrac{0.5774(22)+0.5(14)}{0.5774(22)+2.5(14)}\right]} = 3\ 624.4\ \text{MN/m} \qquad \textit{Ans.}$$

8-14 (**a**) From Table A-31, the nut height is $H = 7/16$ in. $L \geq l + H = 2 + 1 + 7/16 = 3\ 7/16$ in. Rounding up, $L = 3.5$ in *Ans.*

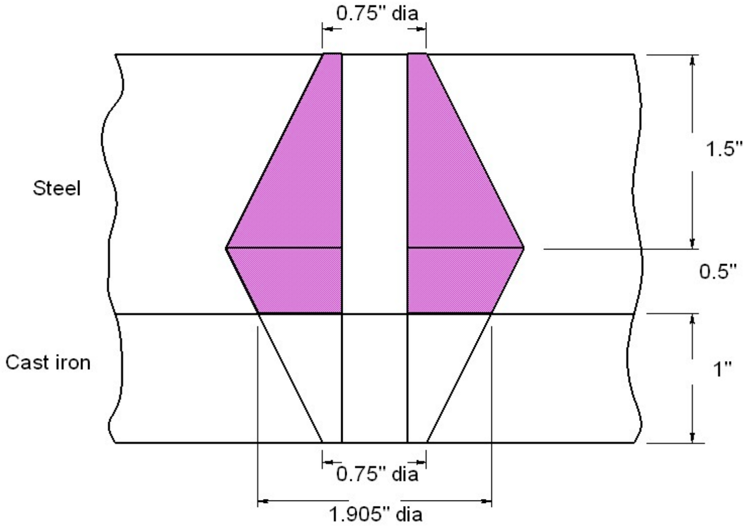
(**b**) From Eq. (8-13), $L_T = 2d + 1/4 = 2(0.5) + 0.25 = 1.25$ in

From Table 8-7, $l_d = L - L_T = 3.5 - 1.25 = 2.25$ in, $l_t = l - l_d = 3 - 2.25 = 0.75$ in

$A_d = \pi(0.5^2)/4 = 0.1963$ in^2. From Table 8-2, $A_t = 0.1419$ in^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.1963(0.1419)30}{0.1963(0.75)+0.1419(2.25)} = 1.79\ \text{Mlbf/in} \qquad \textit{Ans.}$$

(**c**)



Top steel frustum: $t = 1.5$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 30$ Mpsi. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(30)0.5}{\ln\dfrac{\left[1.155(1.5)+0.75-0.5\right](0.75+0.5)}{\left[1.155(1.5)+0.75+0.5\right](0.75-0.5)}} = 22.65\ \text{Mlbf/in}$$

Lower steel frustum: $t = 0.5$ in, $d = 0.5$ in, $D = 0.75 + 2(1)\tan 30° = 1.905$ in, $E = 30$ Mpsi. Eq. (8-20) $\Rightarrow$ $k_2 = 210.7$ Mlbf/in

Cast iron: $t = 1$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 14.5$ Mpsi (Table 8-8). Eq. (8-20) $\Rightarrow$ $k_3 = 12.27$ Mlbf/in

From Eq. (8-18),
$k_m = (1/k_1 + 1/k_2 + 1/k_3)^{-1} = (1/22.65 + 1/210.7 + 1/12.27)^{-1} = 7.67$ Mlbf/in *Ans.*

8-15 (**a**) From Table A-32, the washer thickness is 0.095 in. Thus, $l = 2 + 1 + 2(0.095) = 3.19$ in. From Table A-31, the nut height is $H = 7/16$ in. $L \geq l + H = 3.19 + 7/16 = 3.63$ in. Rounding up, $L = 3.75$ in *Ans.*

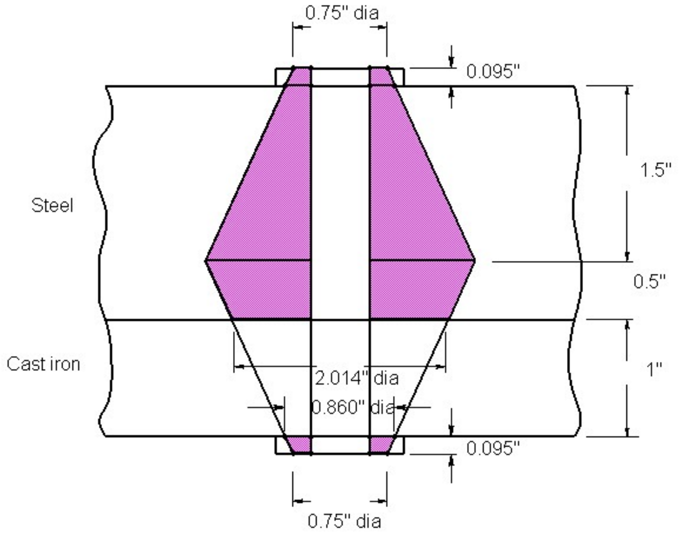
(**b**) From Eq. (8-13), $L_T = 2d + 1/4 = 2(0.5) + 0.25 = 1.25$ in

From Table 8-7, $l_d = L - L_T = 3.75 - 1.25 = 2.5$ in, $l_t = l - l_d = 3.19 - 2.5 = 0.69$ in

$A_d = \pi(0.5^2)/4 = 0.1963$ in^2. From Table 8-2, $A_t = 0.1419$ in^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.1963(0.1419)30}{0.1963(0.69)+0.1419(2.5)} = 1.705\ \text{Mlbf/in} \qquad \textit{Ans.}$$

(c)



Each steel washer frustum: $t = 0.095$ in, $d = 0.531$ in (Table A-32), $D = 0.75$ in, $E = 30$ Mpsi. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(30)0.531}{\ln\dfrac{[1.155(0.095)+0.75-0.531](0.75+0.531)}{[1.155(0.095)+0.75+0.531](0.75-0.531)}} = 89.20 \text{ Mlbf/in}$$

Top plate, top steel frustum: $t = 1.5$ in, $d = 0.5$ in, $D = 0.75 + 2(0.095) \tan 30° = 0.860$ in, $E = 30$ Mpsi. Eq. (8-20) $\Rightarrow k_2 = 28.99$ Mlbf/in

Top plate, lower steel frustum: $t = 0.5$ in, $d = 0.5$ in, $D = 0.860 + 2(1) \tan 30° = 2.015$ in, $E = 30$ Mpsi. Eq. (8-20) $\Rightarrow k_3 = 234.08$ Mlbf/in

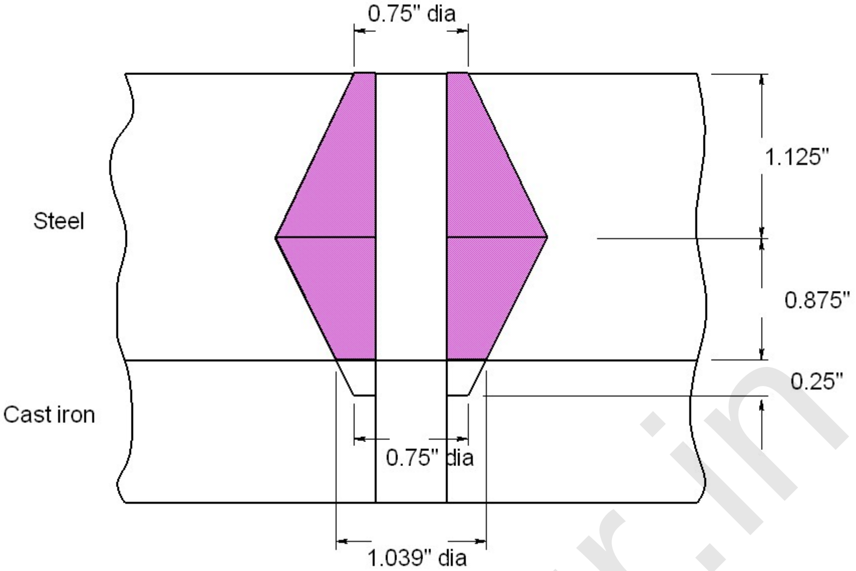
Cast iron: $t = 1$ in, $d = 0.5$ in, $D = 0.75 + 2(0.095) \tan 30° = 0.860$ in, $E = 14.5$ Mpsi (Table 8-8). Eq. (8-20) $\Rightarrow k_4 = 15.99$ Mlbf/in
From Eq. (8-18)
$k_m = (2/k_1 + 1/k_2 + 1/k_3 + 1/k_4)^{-1} = (2/89.20 + 1/28.99 + 1/234.08 + 1/15.99)^{-1}$
$= 8.08$ Mlbf/in *Ans.*

8-16 (**a**) From Table 8-7, $l = h + d/2 = 2 + 0.5/2 = 2.25$ in.
$L \geq h + 1.5\,d = 2 + 1.5(0.5) = 2.75$ in *Ans.*

(**b**) From Table 8-7, $L_T = 2d + 1/4 = 2(0.5) + 0.25 = 1.25$ in
$l_d = L - L_T = 2.75 - 1.25 = 1.5$ in, $l_t = l - l_d = 2.25 - 1.5 = 0.75$ in
$A_d = \pi(0.5^2)/4 = 0.1963$ in^2. From Table 8-2, $A_t = 0.1419$ in^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.1963(0.1419)30}{0.1963(0.75)+0.1419(1.5)} = 2.321 \text{ Mlbf/in} \qquad \textit{Ans.}$$

(c)



Top steel frustum: $t = 1.125$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 30$ Mpsi. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(30)0.5}{\ln\dfrac{[1.155(1.125)+0.75-0.5](0.75+0.5)}{[1.155(1.125)+0.75+0.5](0.75-0.5)}} = 24.48 \text{ Mlbf/in}$$

Lower steel frustum: $t = 0.875$ in, $d = 0.5$ in, $D = 0.75 + 2(0.25)\tan 30° = 1.039$ in, $E = 30$ Mpsi. Eq. (8-20) $\Rightarrow$ $k_2 = 49.36$ Mlbf/in

Cast iron: $t = 0.25$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 14.5$ Mpsi (Table 8-8). Eq. (8-20) $\Rightarrow$ $k_3 = 23.49$ Mlbf/in

From Eq. (8-18)

$$k_m = (1/k_1 + 1/k_2 + 1/k_3)^{-1} = (1/24.48 + 1/49.36 + 1/23.49)^{-1} = 9.645 \text{ Mlbf/in} \qquad \textit{Ans.}$$

8-17 **a**) Grip, $l = 2(2 + 0.095) = 4.19$ in. $L \geq 4.19 + 7/16 = 4.628$ in.
Rounding up, $L = 4.75$ in *Ans.*

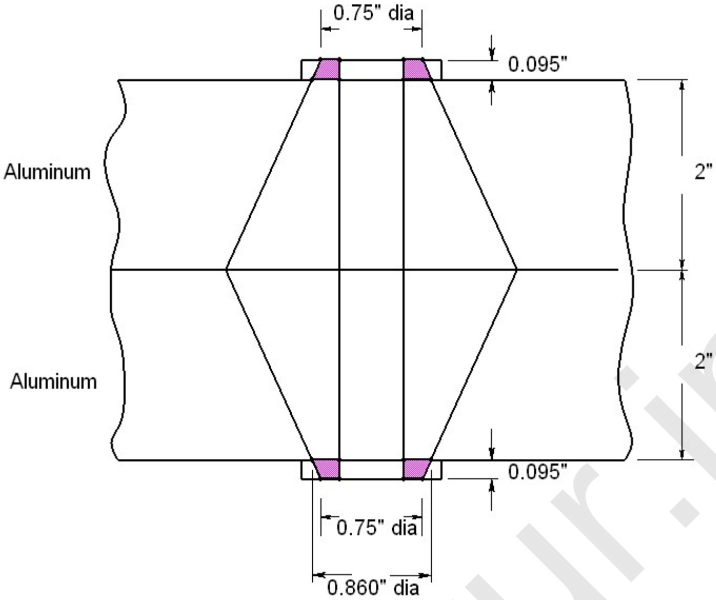
(**b**) From Eq. (8-13), $L_T = 2d + 1/4 = 2(0.5) + 0.25 = 1.25$ in

From Table 8-7, $l_d = L - L_T = 4.75 - 1.25 = 3.5$ in, $l_t = l - l_d = 4.19 - 3.5 = 0.69$ in

$A_d = \pi(0.5^2)/4 = 0.1963$ in^2. From Table 8-2, $A_t = 0.1419$ in^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.1963(0.1419)30}{0.1963(0.69) + 0.1419(3.5)} = 1.322 \text{ Mlbf/in} \qquad \textit{Ans.}$$

(c)



Upper and lower halves are the same. For the upper half,
Steel frustum: $t = 0.095$ in, $d = 0.531$ in, $D = 0.75$ in, and $E = 30$ Mpsi. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(30)0.531}{\ln\dfrac{[1.155(0.095)+0.75-0.531](0.75+0.531)}{[1.155(0.095)+0.75+0.531](0.75-0.531)}} = 89.20 \text{ Mlbf/in}$$

Aluminum: $t = 2$ in, $d = 0.5$ in, $D = 0.75 + 2(0.095)\tan 30° = 0.860$ in, and $E = 10.3$ Mpsi. Eq. (8-20) $\Rightarrow$ $k_2 = 9.24$ Mlbf/in

For the top half, $k'_m = (1/k_1 + 1/k_2)^{-1} = (1/89.20 + 1/9.24)^{-1} = 8.373$ Mlbf/in

Since the bottom half is the same, the overall stiffness is given by

$$k_m = (1/k'_m + 1/k'_m)^{-1} = k'_m/2 = 8.373/2 = 4.19 \text{ Mlbf/in} \qquad \textit{Ans}$$

8-18 (**a**) Grip, $l = 2(2 + 0.095) = 4.19$ in. $L \geq 4.19 + 7/16 = 4.628$ in.
Rounding up, $L = 4.75$ in *Ans.*

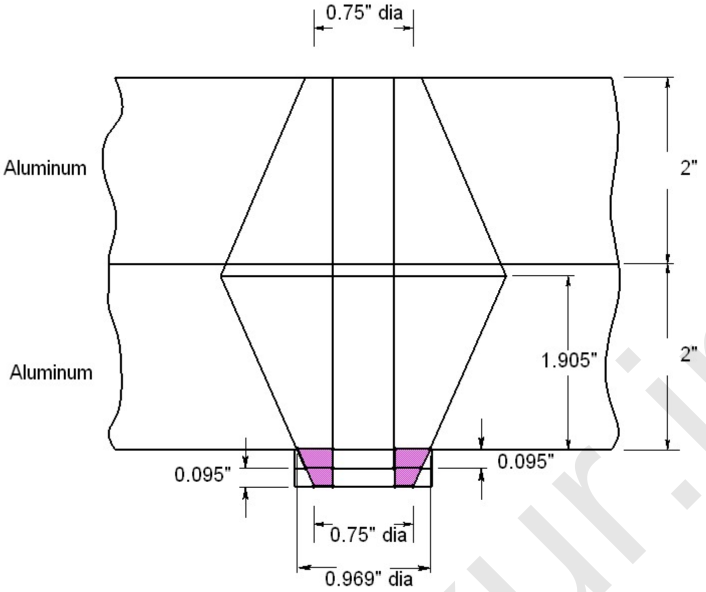
(**b**) From Eq. (8-13), $L_T = 2d + 1/4 = 2(0.5) + 0.25 = 1.25$ in

From Table 8-7, $l_d = L - L_T = 4.75 - 1.25 = 3.5$ in, $l_t = l - l_d = 4.19 - 3.5 = 0.69$ in

$A_d = \pi(0.5^2)/4 = 0.1963$ in^2. From Table 8-2, $A_t = 0.1419$ in^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.1963(0.1419)30}{0.1963(0.69)+0.1419(3.5)} = 1.322 \text{ Mlbf/in} \qquad \textit{Ans.}$$

(**c**)



Upper aluminum frustum: $t = [4 + 2(0.095)]\ /2 = 2.095$ in, $d = 0.5$ in, $D = 0.75$ in, and $E = 10.3$ Mpsi. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(10.3)0.5}{\ln\dfrac{\left[1.155(2.095)+0.75-0.5\right](0.75+0.5)}{\left[1.155(2.095)+0.75+0.5\right](0.75-0.5)}} = 7.23 \text{ Mlbf/in}$$

Lower aluminum frustum: $t = 4 - 2.095 = 1.905$ in, $d = 0.5$ in, $D = 0.75 + 4(0.095)\tan 30° = 0.969$ in, and $E = 10.3$ Mpsi. Eq. (8-20) $\Rightarrow k_2 = 11.34$ Mlbf/in

Steel washers frustum: $t = 2(0.095) = 0.190$ in, $d = 0.531$ in, $D = 0.75$ in, and $E = 30$ Mpsi. Eq. (8-20) $\Rightarrow k_3 = 53.91$ Mlbf/in

From Eq. (8-18)

$k_m = (1/k_1 + 1/k_2 + 1/k_3)^{-1} = (1/7.23 + 1/11.34 + 1/53.91)^{-1} = 4.08$ Mlbf/in *Ans.*

8-19 (**a**) From Table A-31, the nut height is $H = 8.4$ mm. $L > l + H = 50 + 8.4 = 58.4$ mm.

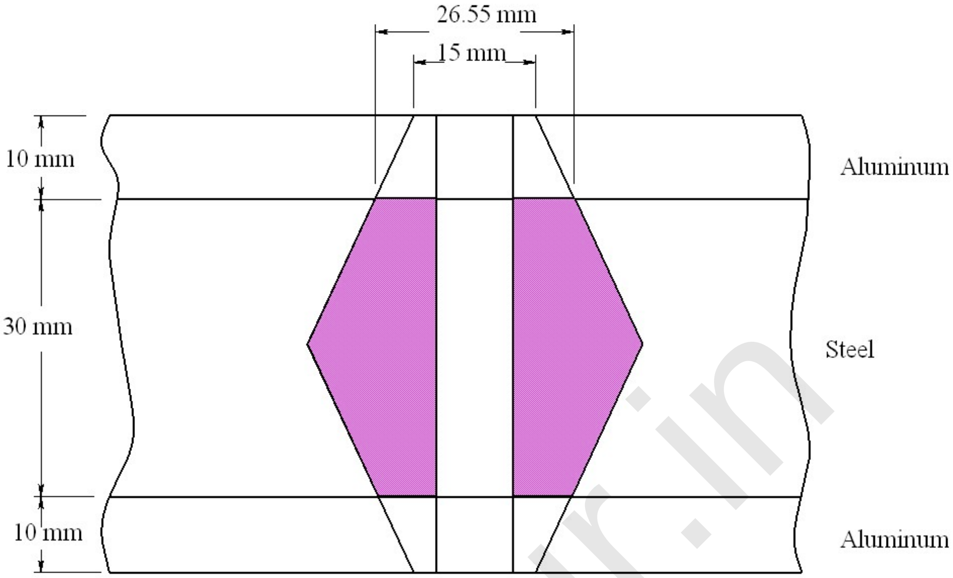
Rounding up, $L = 60$ mm *Ans.*

(**b**) From Eq. (8-14), $L_T = 2d + 6 = 2(10) + 6 = 26$ mm, $l_d = L - L_T = 60 - 26 = 34$ mm, $l_t = l - l = 50 - 34 = 16$ mm. $A_d = \pi(10^2)\ /\ 4 = 78.54$ mm^2. From Table 8-1,

$A_t = 58\ \text{mm}^2$. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{78.54(58.0)207}{78.54(16) + 58.0(34)} = 292.1\ \text{MN/m} \qquad \textit{Ans.}$$

(c)



Upper and lower frustums are the same. For the upper half,

Aluminum: $t = 10$ mm, $d = 10$ mm, $D = 15$ mm, and from Table 8-8, $E = 71$ GPa. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(71)10}{\ln\dfrac{[1.155(10)+15-10](15+10)}{[1.155(10)+15+10](15-10)}} = 1576\ \text{MN/m}$$

Steel: $t = 15$ mm, $d = 10$ mm, $D = 15 + 2(10)\tan 30° = 26.55$ mm, and $E = 207$ GPa. From Eq. (8-20)

$$k_2 = \frac{0.5774\pi(207)10}{\ln\dfrac{[1.155(15)+26.55-10](26.55+10)}{[1.155(15)+26.55+10](26.55-10)}} = 11\ 440\ \text{MN/m}$$

For the top half, $k'_m = (1/k_1 + 1/k_2)^{-1} = (1/1576 + 1/11\ 440)^{-1} = 1385$ MN/m

Since the bottom half is the same, the overall stiffness is given by

$k_m = (1/k'_m + 1/k'_m)^{-1} = k'_m/2 = 1385/2 = 692.5$ MN/m *Ans.*

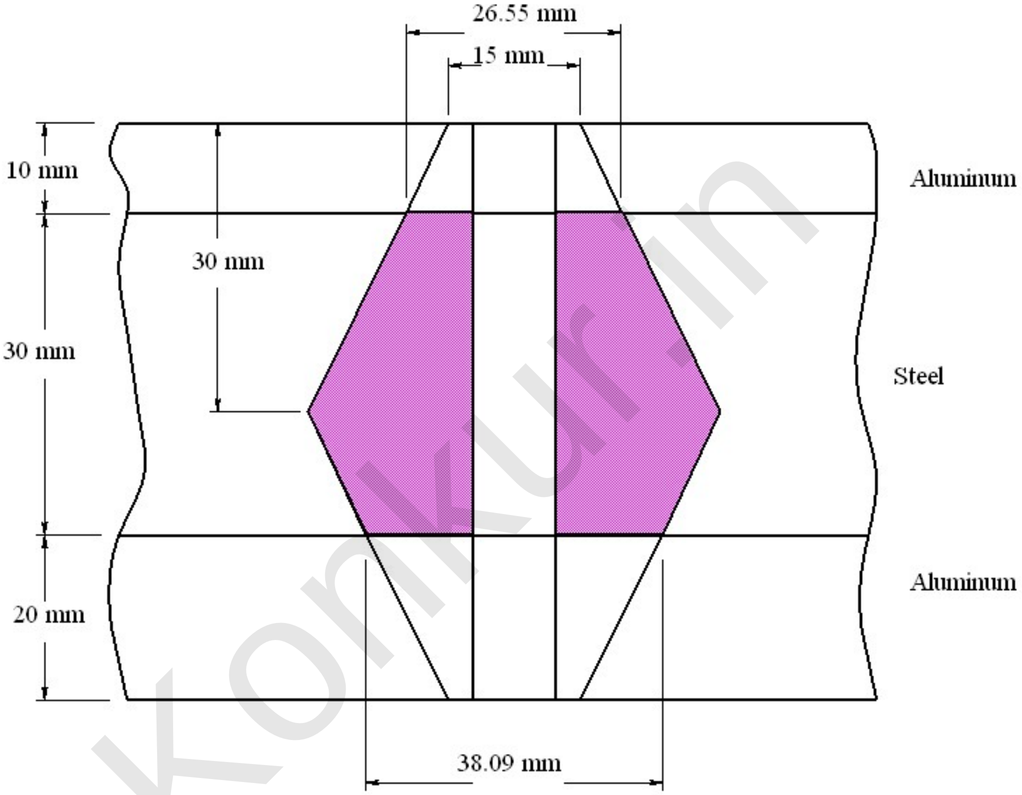
8-20 (**a**) From Table A-31, the nut height is $H = 8.4$ mm. $L > l + H = 60 + 8.4 = 68.4$ mm.

Rounding up, $L = 70$ mm *Ans.*

(**b**) From Eq. (8-14), $L_T = 2d + 6 = 2(10) + 6 = 26$ mm, $l_d = L - L_T = 70 - 26 =$ 44 mm, $l_t = l - l_d = 60 - 44 = 16$ mm. $A_d = \pi(10^2) / 4 = 78.54$ mm^2. From Table 8-1, $A_t = 58$ mm^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{78.54(58.0)207}{78.54(16)+58.0(44)} = 247.6 \text{ MN/m} \qquad \textit{Ans.}$$

(**c**)



Upper aluminum frustum: $t = 10$ mm, $d = 10$ mm, $D = 15$ mm, and $E = 71$ GPa. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(10.3)71}{\ln\dfrac{[1.155(2.095)+15-10](15+10)}{[1.155(2.095)+15+10](15-10)}} = 1576 \text{ MN/m}$$

Lower aluminum frustum: $t = 20$ mm, $d = 10$ mm, $D = 15$ mm, and $E = 71$ GPa. Eq. (8-20) $\Rightarrow k_2 = 1\ 201$ MN/m

Top steel frustum: $t = 20$ mm, $d = 10$ mm, $D = 15 + 2(10)\tan 30° = 26.55$ mm, and $E =$ 207 GPa. Eq. (8-20) $\Rightarrow k_3 = 9\ 781$ MN/m

Lower steel frustum: $t = 10$ mm, $d = 10$ mm, $D = 15 + 2(20)\tan 30° = 38.09$ mm, and $E =$ 207 GPa. Eq. (8-20) $\Rightarrow k_4 = 29\ 070$ MN/m

From Eq. (8-18)

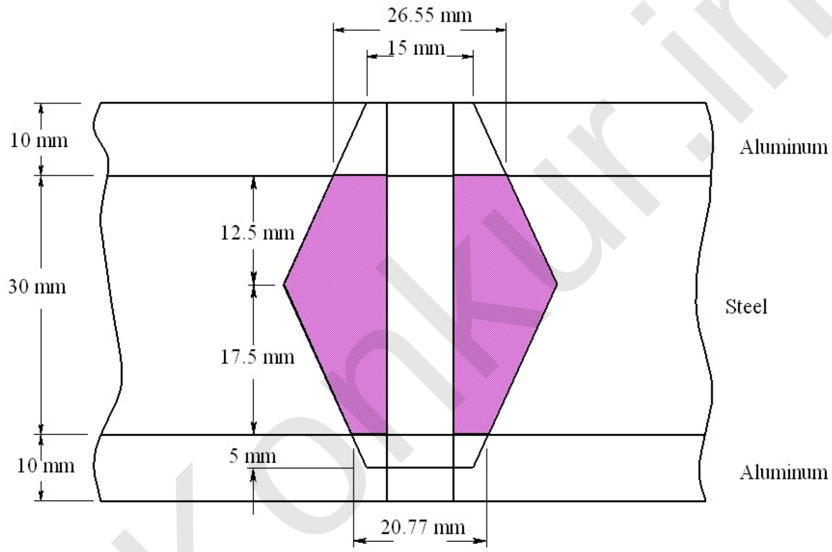
$k_m = (1/k_1 + 1/k_2 + 1/k_3 + 1/k_4)^{-1} = (1/1\ 576 + 1/1\ 201 + 1/9\ 781 + 1/29\ 070)^{-1}$
$= 623.5$ MN/m *Ans.*

8-21 (**a**) From Table 8-7, $l = h + d/2 = 10 + 30 + 10/2 = 45$ mm. $L \geq h + 1.5\,d = 10 + 30 + 1.5(10) = 55$ mm *Ans.*

(**b**) From Eq. (8-14), $L_T = 2d + 6 = 2(10) + 6 = 26$ mm, $l_d = L - L_T = 55 - 26 = 29$ mm, $l_t = l - l_d = 45 - 29 = 16$ mm. $A_d = \pi(10^2)/4 = 78.54$ mm^2. From Table 8-1, $A_t = 58$ mm^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{78.54(58.0)207}{78.54(16) + 58.0(29)} = 320.9 \text{ MN/m} \qquad \textit{Ans.}$$

(**c**)



Upper aluminum frustum: $t = 10$ mm, $d = 10$ mm, $D = 15$ mm, and $E = 71$ GPa. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(10.3)71}{\ln\dfrac{[1.155(2.095)+15-10](15+10)}{[1.155(2.095)+15+10](15-10)}} = 1576 \text{ MN/m}$$

Lower aluminum frustum: $t = 5$ mm, $d = 10$ mm, $D = 15$ mm, and $E = 71$ GPa. Eq. (8-20) $\Rightarrow k_2 = 2\ 300$ MN/m
Top steel frustum: $t = 12.5$ mm, $d = 10$ mm, $D = 15 + 2(10)\tan 30° = 26.55$ mm, and $E = 207$ GPa. Eq. (8-20) $\Rightarrow k_3 = 12\ 759$ MN/m
Lower steel frustum: $t = 17.5$ mm, $d = 10$ mm, $D = 15 + 2(5)\tan 30° = 20.77$ mm, and $E = 207$ GPa. Eq. (8-20) $\Rightarrow k_4 = 6\ 806$ MN/m

From Eq. (8-18)

$k_m = (1/k_1 + 1/k_2 + 1/k_3 + 1/k_4)^{-1} = (1/1\,576 + 1/2\,300 + 1/12\,759 + 1/6\,806)^{-1}$
$= 772.4$ MN/m *Ans.*

8-22 Equation (*f*), Sec. 8-7: $C = \dfrac{k_b}{k_b + k_m}$

Eq. (8-17): $k_b = \dfrac{A_d A_t E}{A_d l_t + A_t l_d}$

Eq. (8-22): $$k_m = \frac{0.5774\pi(207)d}{2\ln\left[5\dfrac{0.5774(40)+0.5d}{0.5774(40)+2.5d}\right]}$$

See Table 8-7 for other terms used.
Using a spreadsheet, with coarse-pitch bolts (units are mm, mm^2, MN/m):

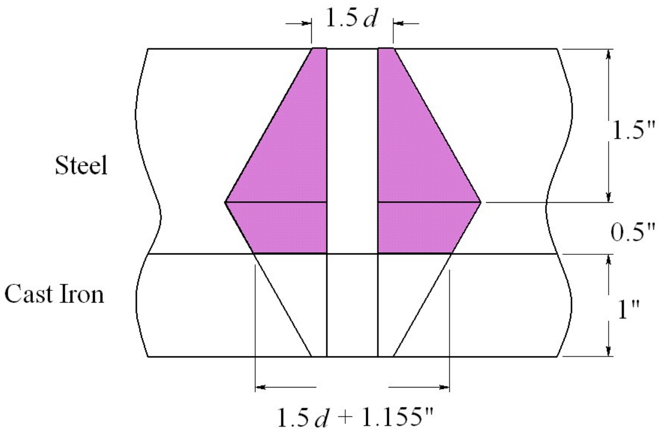
d	A_t	A_d	H	$L >$	L	L_T
10	58	78.53982	8.4	48.4	50	26
12	84.3	113.0973	10.8	50.8	55	30
14	115	153.938	12.8	52.8	55	34
16	157	201.0619	14.8	54.8	55	38
20	245	314.1593	18	58	60	46
24	353	452.3893	21.5	61.5	65	54
30	561	706.8583	25.6	65.6	70	66

d	l	l_d	l_t	k_b	k_m	C
10	40	24	16	356.0129	1751.566	0.16892
12	40	25	15	518.8172	2235.192	0.188386
14	40	21	19	686.2578	2761.721	**0.199032**
16	40	17	23	895.9182	3330.796	0.211966
20	40	14	26	1373.719	4595.515	0.230133
24	40	11	29	1944.24	6027.684	0.243886
30	40	4	36	2964.343	8487.533	0.258852

Use a M14 × 2 bolt, with length 55 mm. *Ans.*

8-23 Equation (*f*), Sec. 8-7: $C = \dfrac{k_b}{k_b + k_m}$

Eq. (8-17): $k_b = \dfrac{A_d A_t E}{A_d l_t + A_t l_d}$



For upper frustum, Eq. (8-20), with *D* = 1.5 *d* and *t* = 1.5 in:

$$k_1 = \frac{0.5774\pi(30)d}{\ln\left[\frac{[1.155(1.5)+0.5d](2.5d)}{[1.155(1.5)+2.5d](0.5d)}\right]} = \frac{0.5774\pi(30)d}{\ln\left[5\frac{(1.733+0.5d)}{(1.733+2.5d)}\right]}$$

Lower steel frustum, with *D* = 1.5*d* + 2(1) tan 30° = 1.5*d* + 1.155, and *t* = 0.5 in:

$$k_2 = \frac{0.5774\pi(30)d}{\ln\left[\frac{(1.733+0.5d)(2.5d+1.155)}{(1.733+2.5d)(0.5d+1.155)}\right]}$$

For cast iron frustum, let *E* = 14. 5 Mpsi, and *D* = 1.5 *d*, and *t* = 1 in:

$$k_3 = \frac{0.5774\pi(14.5)d}{\ln\left[5\frac{(1.155+0.5d)}{(1.155+2.5d)}\right]}$$

Overall, $k_m = (1/k_1 + 1/k_2 + 1/k_3)^{-1}$

See Table 8-7 for other terms used.
Using a spreadsheet, with coarse-pitch bolts (units are in, in^2, Mlbf/in):

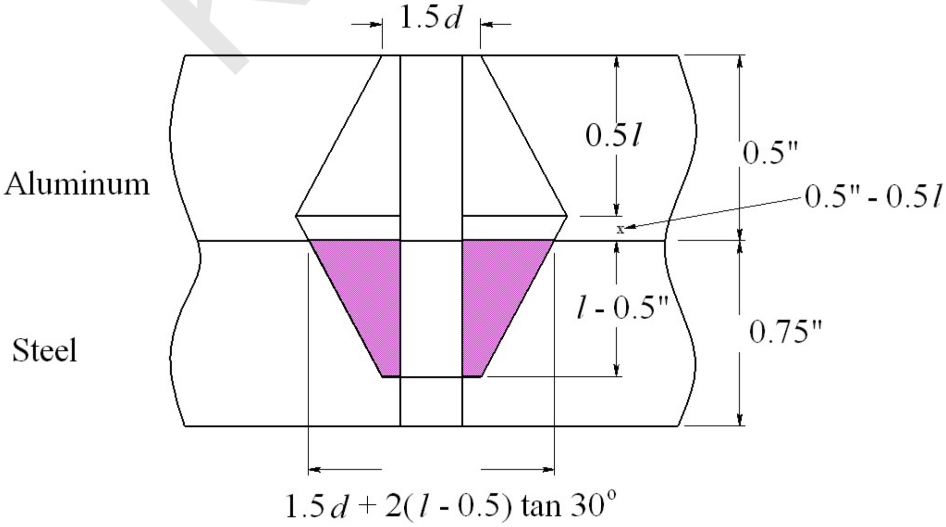
d	A_t	A_d	H	$L >$	L	L_T	l
0.375	0.0775	0.110447	0.328125	3.328125	3.5	1	3
0.4375	0.1063	0.15033	0.375	3.375	3.5	1.125	3
0.5	0.1419	0.19635	0.4375	3.4375	3.5	1.25	3
0.5625	0.182	0.248505	0.484375	3.484375	3.5	1.375	3
0.625	0.226	0.306796	0.546875	3.546875	3.75	1.5	3
0.75	0.334	0.441786	0.640625	3.640625	3.75	1.75	3
0.875	0.462	0.60132	0.75	3.75	3.75	2	3

d	l_d	l_t	k_b	k_1	k_2	k_3	k_m	C
0.375	2.5	0.5	1.031389	15.94599	178.7801	8.461979	5.362481	0.161309
0.4375	2.375	0.625	1.383882	19.21506	194.465	10.30557	6.484256	0.175884
0.5	2.25	0.75	1.791626	22.65332	210.6084	12.26874	7.668728	0.189383
0.5625	2.125	0.875	2.245705	26.25931	227.2109	14.35052	8.915294	**0.20121**
0.625	2.25	0.75	2.816255	30.03179	244.2728	16.55009	10.22344	0.215976
0.75	2	1	3.988786	38.07191	279.7762	21.29991	13.02271	0.234476
0.875	1.75	1.25	5.341985	46.7663	317.1203	26.51374	16.06359	0.24956

Use a $\frac{9}{16}$−12 UNC × 3.5 in long bolt *Ans.*

8-24 Equation (f), Sec. 8-7: $C = \dfrac{k_b}{k_b + k_m}$

Eq. (8-17): $k_b = \dfrac{A_d A_t E}{A_d l_t + A_t l_d}$



Top frustum, Eq. (8-20), with $E = 10.3\text{Mpsi}$, $D = 1.5\,d$, and $t = l/2$:

$$k_1 = \frac{0.5774\pi(10.3)d}{\ln\left[5\dfrac{1.155\,l/2+0.5d}{1.155\,l/2+2.5d}\right]}$$

Middle frustum, with $E = 10.3$ Mpsi, $D = 1.5d + 2(l - 0.5)\tan 30°$, and $t = 0.5 - l/2$

$$k_2 = \frac{0.5774\pi(10.3)d}{\ln\dfrac{\left\{\left[1.155(0.5-0.5l)+0.5d+2(l-0.5)\tan 30^0\right]\left[2.5d+2(l-0.5)\tan 30^0\right]\right\}}{\left\{\left[1.155(0.5-0.5l)+2.5d+2(l-0.5)\tan 30^0\right]\left[0.5d+2(l-0.5)\tan 30^0\right]\right\}}}$$

Lower frustum, with $E = 30\text{Mpsi}$, $D = 1.5\,d$, $t = l - 0.5$

$$k_3 = \frac{0.5774\pi(30)d}{\ln 5\left\{\dfrac{\left[1.155(l-0.5)+0.5d\right]}{\left[1.155(l-0.5)+2.5d\right]}\right\}}$$

See Table 8-7 for other terms used.
Using a spreadsheet, with coarse-pitch bolts (units are in, in², Mlbf/in)

Size	d	A_t	A_d	$L >$	L	L_T	l	l_d
1	0.073	0.00263	0.004185	0.6095	0.75	0.396	0.5365	0.354
2	0.086	0.0037	0.005809	0.629	0.75	0.422	0.543	0.328
3	0.099	0.00487	0.007698	0.6485	0.75	0.448	0.5495	0.302
4	0.112	0.00604	0.009852	0.668	0.75	0.474	0.556	0.276
5	0.125	0.00796	0.012272	0.6875	0.75	0.5	0.5625	0.25
6	0.138	0.00909	0.014957	0.707	0.75	0.526	0.569	0.224
8	0.164	0.014	0.021124	0.746	0.75	0.578	0.582	0.172
10	0.19	0.0175	0.028353	0.785	1	0.63	0.595	0.37
Size	d	l_t	k_b	k_1	k_2	k_3	k_m	C
1	0.073	0.1825	0.194841	1.084468	1.954599	7.09432	0.635049	0.23478
2	0.086	0.215	0.261839	1.321595	2.449694	8.357692	0.778497	**0.251687**
3	0.099	0.2475	0.333134	1.570439	2.993366	9.621064	0.930427	0.263647
4	0.112	0.28	0.403377	1.830494	3.587564	10.88444	1.090613	0.27
5	0.125	0.3125	0.503097	2.101297	4.234381	12.14781	1.258846	0.285535
6	0.138	0.345	0.566787	2.382414	4.936066	13.41118	1.434931	0.28315
8	0.164	0.41	0.801537	2.974009	6.513824	15.93792	1.809923	0.306931
10	0.19	0.225	1.15799	3.602349	8.342138	18.46467	2.214214	0.343393

Use a 2−56 UNC × 0.75 in long bolt. *Ans.*

8-25 For half of joint, Eq. (8-20): $t = 20$ mm, $d = 14$ mm, $D = 21$ mm, and $E = 207$ GPa

$$k_1 = \frac{0.5774\pi(207)14}{\ln\dfrac{[1.155(20)+21-14](21+14)}{[1.155(20)+21+14](21-14)}} = 5523 \text{ MN/m}$$

$k_m = (1/k_1 + 1/k_1)^{-1} = k_1/2 = 5523/2 = 2762$ MN/m *Ans.*

From Eq. (8-22) with $l = 40$ mm

$$k_m = \frac{0.5774\pi(207)14}{2\ln\left[5\dfrac{0.5774(40)+0.5(14)}{0.5774(40)+2.5(14)}\right]} = 2762 \text{ MN/m} \qquad Ans.$$

which agrees with the earlier calculation.

For Eq. (8-23), from Table 8-8, $A = 0.787\ 15$, $B = 0.628\ 73$

$k_m = 207(14)(0.78\ 715) \exp[0.628\ 73(14)/40] = 2843$ MN/m *Ans.*

This is 2.9% higher than the earlier calculations.

8-26 (**a**) Grip, $l = 10$ in. Nut height, $H = 41/64$ in (Table A-31).
$L \geq l + H = 10 + 41/64 = 10.641$ in. Let $L = 10.75$ in.
Table 8-7, $L_T = 2d + 0.5 = 2(0.75) + 0.5 = 2$ in, $l_d = L - L_T = 10.75 - 2 = 8.75$ in,
$l_t = l - l_d = 10 - 8.75 = 1.25$ in
$A_d = \pi(0.75^2)/4 = 0.4418$ in^2, $A_t = 0.373$ in^2 (Table 8-2)
Eq. (8-17),

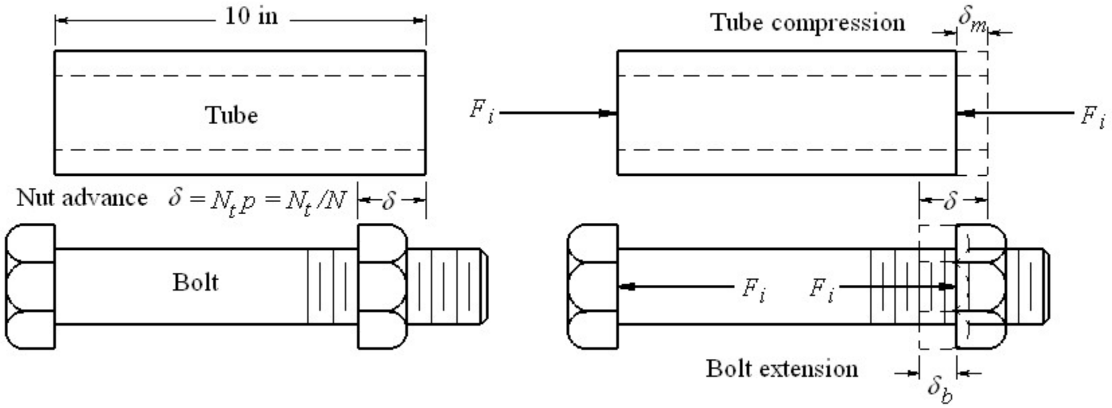
$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.4418(0.373)30}{0.4418(1.25)+0.373(8.75)} = 1.296 \quad \text{Mlbf/in} \qquad Ans.$$

Eq. (4-4),

$$k_m = \frac{A_m E_m}{l} = \frac{(\pi/4)(1.125^2 - 0.75^2)30}{10} = 1.657 \text{ Mlbf/in} \qquad Ans.$$

Eq. (*f*), Sec. 8-7, $C = k_b/(k_b + k_m) = 1.296/(1.296 + 1.657) = 0.439$ *Ans.*

(b)



Let: N_t = no. of turns, p = pitch of thread (in), N = no. of threads per in = $1/p$. Then,

$$\delta = \delta_b + \delta_m = N_t\, p = N_t / N \qquad (1)$$

But, $\delta_b = F_i / k_b$, and, $\delta_m = F_i / k_m$. Substituting these into Eq. (1) and solving for F_i gives

$$F_i = \frac{k_b k_m}{k_b + k_m} \frac{N_t}{N} \qquad (2)$$

$$= \frac{1.296(1.657)10^6}{1.296 + 1.657} \frac{1/3}{16} = 15\,150 \quad \text{lbf} \qquad \textit{Ans.}$$

8-27 Proof for the turn-of-nut equation is given in the solution of Prob. 8-26, Eq. (2), where $N_t = \theta / 360°$.

The relationship between the turn-of-nut method and the torque-wrench method is as follows.

$$N_t = \left(\frac{k_b + k_m}{k_b k_m}\right) F_i N \qquad \text{(turn-of-nut)}$$

$$T = K F_i d \qquad \text{(torque-wrench)}$$

Eliminate F_i

$$N_t = \left(\frac{k_b + k_m}{k_b k_m}\right) \frac{NT}{Kd} = \frac{\theta}{360^\circ} \qquad \textit{Ans.}$$

8-28 **(a)** From Ex. 8-4, F_i = 14.4 kip, $k_b = 5.21(10^6)$ lbf/in, $k_m = 8.95(10^6)$ lbf/in
Eq. (8-27): $T = kF_i d = 0.2(14.4)(10^3)(5/8) = 1800$ lbf · in *Ans.*
From Prob. 8-27,

$$N_t = \left(\frac{k_b + k_m}{k_b k_m}\right) F_i N = \left[\frac{5.21 + 8.95}{5.21(8.95)10^6}\right](14.4)(10^3)11$$
$$= 0.0481 \text{ turns } = 17.3^\circ \qquad Ans.$$

Bolt group is (1.5) / (5/8) = 2.4 diameters. Answer is much lower than RB&W recommendations.

8-29 $C = k_b / (k_b + k_m) = 3/(3+12) = 0.2$, $P = P_{\text{total}}/ N = 80/6 = 13.33$ kips/bolt
Table 8-2, $A_t = 0.141\,9$ in^2; Table 8-9, $S_p = 120$ kpsi; Eqs. (8-31) and (8-32),
$F_i = 0.75\, A_t\, S_p = 0.75(0.141\,9)(120) = 12.77$ kips
(**a**) From Eq. (8-28), the factor of safety for yielding is

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{120(0.141\,9)}{0.2(13.33)+12.77} = 1.10 \qquad Ans.$$

(**b**) From Eq. (8-29), the overload factor is

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{120(0.141\,9)-12.77}{0.2(13.33)} = 1.60 \qquad Ans.$$

(**c**) From Eq. (8-30), the joint separation factor of safety is

$$n_0 = \frac{F_i}{P(1-C)} = \frac{12.77}{13.33(1-0.2)} = 1.20 \qquad Ans.$$

8-30 1/2 – 13 UNC Grade 8 bolt, $K = 0.20$
(**a**) Proof strength, Table 8-9, $S_p = 120$ kpsi
Table 8-2, $A_t = 0.141\,9$ in^2
Maximum, $F_i = S_p\, A_t = 120(0.141\,9) = 17.0$ kips *Ans.*
(**b**) From Prob. 8-29, $C = 0.2$, $P = 13.33$ kips
Joint separation, Eq. (8-30) with $n_0 = 1$
Minimum $F_i = P\,(1 - C) = 13.33(1 - 0.2) = 10.66$ kips *Ans.*
(**c**) $\bar{F}_i = (17.0 + 10.66)/2 = 13.8$ kips
Eq. (8-27), $T = KF_i\, d = 0.2(13.8)10^3(0.5)/12 = 115$ lbf · ft *Ans.*

8-31 (**a**) Table 8-1, $A_t = 20.1$ mm^2. Table 8-11, $S_p = 380$ MPa.

Eq. (8-31), $F_i = 0.75\, F_p = 0.75\, A_t\, S_p = 0.75(20.1)380(10^{-3}) = 5.73$ kN

Eq. (f), Sec. 8-7, $$C = \frac{k_b}{k_b + k_m} = \frac{1}{1+2.6} = 0.278$$

Eq. (8-28) with $n_p = 1$,

$$P = \frac{S_p A_t - F_i}{C} = \frac{0.25 S_p A_t}{C} = \frac{0.25(20.1)380(10^{-3})}{0.278} = 6.869 \text{ kN}$$

$P_{\text{total}} = NP = 8(6.869) = 55.0$ kN *Ans.*

(**b**) Eq. (8-30) with $n_0 = 1$,

$$P = \frac{F_i}{1-C} = \frac{5.73}{1-0.278} = 7.94 \text{ kN}$$

$P_{\text{total}} = NP = 8(7.94) = 63.5$ kN *Ans.* Bolt stress would exceed proof strength

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8-32 (**a**) Table 8-2, $A_t = 0.141\,9$ in^2. Table 8-9, $S_p = 120$ kpsi.

Eq. (8-31), $F_i = 0.75\,F_p = 0.75\,A_t\,S_p = 0.75(0.141\,9)120 = 12.77$ kips

Eq. (f), Sec. 8-7, $$C = \frac{k_b}{k_b + k_m} = \frac{4}{4+12} = 0.25$$

Eq. (8-28) with $n_p = 1$,

$$P_{\text{total}} = N\left(\frac{S_p A_t - F_i}{C}\right) = \frac{0.25 N S_p A_t}{C}$$

$$N = \frac{P_{\text{total}} C}{0.25 S_p A_t} = \frac{80(0.25)}{0.25(120)0.141\,9} = 4.70$$

Round to $N = 5$ bolts *Ans.*

(**b**) Eq. (8-30) with $n_0 = 1$,

$$P_{\text{total}} = N\left(\frac{F_i}{1-C}\right)$$

$$N = \frac{P_{\text{total}}(1-C)}{F_i} = \frac{80(1-0.25)}{12.77} = 4.70$$

Round to $N = 5$ bolts *Ans.*

8-33 Bolts: From Table A-31, the nut height is $H = 10.8$ mm. $L \geq l + H = 40 + 10.8 = 50.8$ mm.

Round up to $L = 55$ mm *Ans.*

Eq. (8-14): $L_T = 2d + 6 = 2(12) + 6 = 30$ mm

Table 8-7: $l_d = L - L_T = 55 - 30 = 25$ mm, $l_t = l - l_d = 40 - 25 = 15$ mm

$A_d = \pi(12^2)/4 = 113.1$ mm^2, Table 8-1: $A_t = 84.3$ mm^2

Eq. (8-17):

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{113.1(84.3)207}{113.1(15)+84.3(25)} = 518.8 \text{ MN/m}$$

Members: Steel cyl. head: $t = 20$ mm, $d = 12$ mm, $D = 18$ mm, $E = 207$ GPa. Eq. (8-20),

$$k_1 = \frac{0.5774\pi(207)12}{\ln \frac{[1.155(20)+18-12](18+12)}{[1.155(20)+18+12](18-12)}} = 4470 \text{ MN/m}$$

Cast iron: $t = 20$ mm, $d = 12$ mm, $D = 18$ mm, $E = 100$ GPa (from Table 8-8). The only difference from k_1 is the material

$$k_2 = (100/207)(4470) = 2159 \text{ MN/m}$$

Eq. (8-18): $k_m = (1/4470 + 1/2159)^{-1} = 1456$ MN/m

$C = k_b / (k_b + k_m) = 518.8/(518.8+1456) = 0.263$

Table 8-11: $S_p = 650$ MPa

For a non-permanent connection, using Eqs. (8-31) and (8-32)

$$F_i = 0.75\, A_t\, S_p = 0.75(84.3)(650)10^{-3} = 41.1 \text{ kN}$$

The total external load is $P_{\text{total}} = p_g A_g$, where A_g is the effective area of the cylinder, based on the effective sealing diameter of 100 mm. The external load per bolt is $P = P_{\text{total}} / N$. Thus

$$P = [6\pi(100^2)/4](10^{-3})/10 = 4.712 \text{ kN/bolt}$$

Yielding factor of safety, Eq. (8-28):

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{650(84.3)10^{-3}}{0.263(4.712)+41.10} = 1.29 \qquad \textit{Ans.}$$

Overload factor of safety, Eq. (8-29):

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{650(84.3)10^{-3} - 41.10}{0.263(4.712)} = 11.1 \qquad \textit{Ans.}$$

Separation factor of safety, Eq. (8-30):

$$n_0 = \frac{F_i}{P(1-C)} = \frac{41.10}{4.712(1-0.263)} = 11.8 \qquad Ans.$$

8-34 Bolts: Grip, $l = 1/2 + 5/8 = 1.125$ in. From Table A-31, the nut height is $H = 7/16$ in. $L \geq l + H = 1.125 + 7/16 = 1.563$ in.

Round up to $L = 1.75$ in *Ans.*
Eq. (8-13): $L_T = 2d + 0.25 = 2(0.5) + 0.25 = 1.25$ in
Table 8-7: $l_d = L - L_T = 1.75 - 1.25 = 0.5$ in, $l_t = l - l_d = 1.125 - 0.5 = 0.625$ in
$A_d = \pi(0.5^2)/4 = 0.196\ 3$ in^2, Table 8-2: $A_t = 0.141\ 9$ in^2
Eq. (8-17):

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.196\ 3(0.141\ 9)30}{0.196\ 3(0.625) + 0.141\ 9(0.5)} = 4.316 \text{ Mlbf/in}$$

Members: Steel cyl. head: $t = 0.5$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 30$ Mpsi. Eq. (8-20),

$$k_1 = \frac{0.5774\pi(30)0.5}{\ln\dfrac{[1.155(0.5)+0.75-0.5](0.75+0.5)}{[1.155(0.5)+0.75+0.5](0.75-0.5)}} = 33.30 \text{ Mlbf/in}$$

Cast iron: Has two frusta. Midpoint of complete joint is at $(1/2 + 5/8)/2 = 0.5625$ in.
Upper frustum, $t = 0.5625 - 0.5 = 0.0625$ in, $d = 0.5$ in,
$D = 0.75 + 2(0.5) \tan 30° = 1.327$ in, $E = 14.5$ Mpsi (from Table 8-8)
Eq. (8-20) $\Rightarrow$ $k_2 = 292.7$ Mlbf/in
Lower frustum, $t = 0.5625$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 14.5$ Mpsi
Eq. (8-20) $\Rightarrow$ $k_3 = 15.26$ Mlbf/in
Eq. (8-18): $k_m = (1/33.30 + 1/292.7 + 1/15.26)^{-1} = 10.10$ Mlbf/in
$C = k_b / (k_b + k_m) = 4.316/(4.316+10.10) = 0.299$
Table 8-9: $S_p = 85$ kpsi
For a non-permanent connection, using Eqs. (8-31) and (8-32)

$$F_i = 0.75\ A_t\ S_p = 0.75(0.141\ 9)(85) = 9.05 \text{ kips}$$

The total external load is $P_{\text{total}} = p_g A_g$, where A_g is the effective area of the cylinder, based on the effective sealing diameter of 3.5 in. The external load per bolt is $P = P_{\text{total}} / N$. Thus,

$$P = [1\ 500\pi(3.5^2)/4](10^{-3})/10 = 1.443 \text{ kips/bolt}$$

Yielding factor of safety, Eq. (8-28):

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{85(0.141\ 9)}{0.299(1.443) + 9.05} = 1.27 \qquad Ans.$$

Overload factor of safety, Eq. (8-29):

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{85(0.141\,9) - 9.05}{0.299(1.443)} = 6.98 \qquad Ans.$$

Separation factor of safety, Eq. (8-30):

$$n_0 = \frac{F_i}{P(1-C)} = \frac{9.05}{1.443(1-0.299)} = 8.95 \qquad Ans.$$

8-35 Bolts: Grip: $l = 20 + 25 = 45$ mm. From Table A-31, the nut height is $H = 8.4$ mm.
$L \geq l + H = 45 + 8.4 = 53.4$ mm.

Round up to $L = 55$ mm *Ans.*

Eq. (8-14): $L_T = 2d + 6 = 2(10) + 6 = 26$ mm

Table 8-7: $l_d = L - L_T = 55 - 26 = 29$ mm, $l_t = l - l_d = 45 - 29 = 16$ mm

$A_d = \pi(10^2)/4 = 78.5$ mm^2, Table 8-1: $A_t = 58.0$ mm^2

Eq. (8-17):

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{78.5(58.0)207}{78.5(16) + 58.0(29)} = 320.8 \text{ MN/m}$$

Members: Steel cyl. head: $t = 20$ mm, $d = 10$ mm, $D = 15$ mm, $E = 207$ GPa. Eq. (8-20),

$$k_1 = \frac{0.5774\pi(207)10}{\ln \dfrac{[1.155(20) + 15 - 10](15+10)}{[1.155(20) + 15 + 10](15-10)}} = 3503 \text{ MN/m}$$

Cast iron: Has two frusta. Midpoint of complete joint is at $(20 + 25)/2 = 22.5$ mm

Upper frustum, $t = 22.5 - 20 = 2.5$ mm, $d = 10$ mm,
$D = 15 + 2(20) \tan 30° = 38.09$ mm, $E = 100$ GPa (from Table 8-8),
Eq. (8-20) $\Rightarrow k_2 = 45\,880$ MN/m

Lower frustum, $t = 22.5$ mm, $d = 10$ mm, $D = 15$ mm, $E = 100$ GPa
Eq. (8-20) $\Rightarrow k_3 = 1632$ MN/m

Eq. (8-18): $k_m = (1/3503 + 1/45\,880 + 1/1632)^{-1} = 1087$ MN/m

$C = k_b / (k_b + k_m) = 320.8/(320.8+1087) = 0.228$

Table 8-11: $S_p = 830$ MPa

For a non-permanent connection, using Eqs. (8-31) and (8-32)

$$F_i = 0.75\, A_t\, S_p = 0.75(58.0)(830)10^{-3} = 36.1 \text{ kN}$$

The total external load is $P_{\text{total}} = p_g A_g$, where A_g is the effective area of the cylinder, based on the effective sealing diameter of 0.8 m. The external load per bolt is $P = P_{\text{total}} / N$. Thus,

$$P = [550\pi(0.8^2)/4]/36 = 7.679 \text{ kN/bolt}$$

Yielding factor of safety, Eq. (8-28):

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{830(58.0)10^{-3}}{0.228(7.679) + 36.1} = 1.27 \qquad Ans.$$

Overload factor of safety, Eq. (8-29):

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{830(58.0)10^{-3} - 36.1}{0.228(7.679)} = 6.88 \; Ans.$$

Separation factor of safety, Eq. (8-30):

$$n_0 = \frac{F_i}{P(1-C)} = \frac{36.1}{7.679(1-0.228)} = 6.09 \qquad Ans.$$

8-36

Bolts: Grip, $l = 3/8 + 1/2 = 0.875$ in. From Table A-31, the nut height is $H = 3/8$ in. $L \geq l + H = 0.875 + 3/8 = 1.25$ in.

Let $L = 1.25$ in *Ans.*

Eq. (8-13): $L_T = 2d + 0.25 = 2(7/16) + 0.25 = 1.125$ in

Table 8-7: $l_d = L - L_T = 1.25 - 1.125 = 0.125$ in, $l_t = l - l_d = 0.875 - 0.125 = 0.75$ in

$A_d = \pi(7/16)^2/4 = 0.150\ 3$ in^2, Table 8-2: $A_t = 0.106\ 3$ in^2

Eq. (8-17),

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.150\ 3(0.106\ 3)30}{0.150\ 3(0.75) + 0.106\ 3(0.125)} = 3.804 \text{ Mlbf/in}$$

Members: Steel cyl. head: $t = 0.375$ in, $d = 0.4375$ in, $D = 0.65625$ in, $E = 30$ Mpsi. Eq. (8-20),

$$k_1 = \frac{0.5774\pi(30)0.4375}{\ln\dfrac{[1.155(0.375) + 0.65625 - 0.4375](0.65625 + 0.4375)}{[1.155(0.375) + 0.65625 + 0.4375](0.65625 - 0.4375)}} = 31.40 \text{ Mlbf/in}$$

Cast iron: Has two frusta. Midpoint of complete joint is at $(3/8 + 1/2)/2 = 0.4375$ in.

Upper frustum, $t = 0.4375 - 0.375 = 0.0625$ in, $d = 0.4375$ in,
$D = 0.65625 + 2(0.375)\tan 30° = 1.089$ in, $E = 14.5$ Mpsi (from Table 8-8)

Eq. (8-20) $\Rightarrow$ $k_2 = 195.5$ Mlbf/in

Lower frustum, $t = 0.4375$ in, $d = 0.4375$ in, $D = 0.65625$ in, $E = 14.5$ Mpsi

Eq. (8-20) $\Rightarrow$ $k_3 = 14.08$ Mlbf/in

Eq. (8-18): $k_m = (1/31.40 + 1/195.5 + 1/14.08)^{-1} = 9.261$ Mlbf/in

$C = k_b / (k_b + k_m) = 3.804/(3.804 + 9.261) = 0.291$

Table 8-9: $S_p = 120$ kpsi

For a non-permanent connection, using Eqs. (8-31) and (8-32)

$$F_i = 0.75\, A_t\, S_p = 0.75(0.106\ 3)(120) \ = 9.57 \text{ kips}$$

The total external load is $P_{\text{total}} = p_g\, A_g$, where A_g is the effective area of the cylinder, based on the effective sealing diameter of 3.25 in. The external load per bolt is $P = P_{\text{total}}/N$. Thus,

$$P = [1\ 200\pi(3.25^2)/4](10^{-3})/8 \ = 1.244 \ \text{kips/bolt}$$

Yielding factor of safety, Eq. (8-28):

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{120(0.106\ 3)}{0.291(1.244) + 9.57} = 1.28 \qquad \textit{Ans.}$$

Overload factor of safety, Eq. (8-29):

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{120(0.106\ 3) - 9.57}{0.291(1.244)} = 8.80 \quad \textit{Ans.}$$

Separation factor of safety, Eq. (8-30):

$$n_0 = \frac{F_i}{P(1-C)} = \frac{9.57}{1.244(1-0.291)} = 10.9 \qquad \textit{Ans.}$$

8-37

From Table 8-7, $h = t_1 = 20$ mm
For $t_2 > d$, $l = h + d/2 \ = 20 + 12/2 \ = 26$ mm
$L \ge h + 1.5\, d = 20 + 1.5(12) = 38$ mm. Round up to $L = 40$ mm
$L_T = 2d + 6 = 2(12) + 6 = 30$ mm
$l_d = L - L_T \ = 40 - 20 = 10$ mm
$l_t = l - l_d \ = 26 - 10 = 16$ mm
From Table 8-1, $A_t = 84.3$ mm^2. $A_d = \pi(12^2)/4 = 113.1$ mm^2
Eq. (8-17),

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{113.1(84.3)207}{113.1(16) + 84.3(10)} = 744.0 \text{ MN/m}$$

Similar to Fig. 8-23, we have three frusta.
Top frusta, steel: $t = l/2 = 13$ mm, $d = 12$ mm, $D = 18$ mm, $E \ = 207$ GPa. Eq. (8-20)

$$k_1 = \frac{0.5774\pi(207)12}{\ln \dfrac{[1.155(13) + 18 - 12](18 + 12)}{[1.155(13) + 18 + 12](18 - 12)}} = 5\ 316 \text{ MN/m}$$

Middle frusta, steel: $t = 20 - 13 = 7$ mm, $d = 12$ mm, $D = 18 + 2(13 - 7)\tan 30° \ = 24.93$ mm, $E \ = 207$ GPa. Eq. (8-20) $\Rightarrow$ $k_2 = 15\ 660$ MN/m
Lower frusta, cast iron: $t = 26 - 20 = 6$ mm, $d = 12$ mm, $D = 18$ mm, $E \ = 100$ GPa (see Table 8-8). Eq. (8-20) $\Rightarrow$ $k_3 = 3\ 887$ MN/m
Eq. (8-18), $k_m = (1/5\ 316 + 1/15\ 660 + 1/3\ 887)^{-1} \ = 1\ 964$ MN/m

$$C = k_b / (k_b + k_m) = 744.0/(744.0 + 1\,964) = 0.275$$

Table 8-11: $S_p = 650$ MPa. For a non-permanent connection, using Eqs. (8-31) and (8-32)

$$F_i = 0.75\, A_t\, S_p = 0.75(84.3)(650)10^{-3} = 41.1 \text{ kN}$$

The total external load is $P_{\text{total}} = p_g\, A_g$, where A_g is the effective area of the cylinder, based on the effective sealing diameter of 100 mm. The external load per bolt is $P = P_{\text{total}} / N$. Thus

$$P = [6\pi(100^2)/4](10^{-3})/10 = 4.712 \text{ kN/bolt}$$

Yielding factor of safety, Eq. (8-28)

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{650(84.3)10^{-3}}{0.275(4.712) + 41.1} = 1.29 \qquad Ans.$$

Overload factor of safety, Eq. (8-29)

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{650(84.3)10^{-3} - 41.1}{0.275(4.712)} = 10.7 \qquad Ans.$$

Separation factor of safety, Eq. (8-30)

$$n_0 = \frac{F_i}{P(1-C)} = \frac{41.1}{4.712(1-0.275)} = 12.0 \qquad Ans.$$

8-38

From Table 8-7, $h = t_1 = 0.5$ in

For $t_2 > d$, $l = h + d/2 = 0.5 + 0.5/2 = 0.75$ in

$L \geq h + 1.5\, d = 0.5 + 1.5(0.5) = 1.25$ in. Let $L = 1.25$ in

$L_T = 2d + 0.25 = 2(0.5) + 0.25 = 1.25$ in. All threaded.

From Table 8-1, $A_t = 0.141\,9$ in^2. The bolt stiffness is $k_b = A_t\, E / l = 0.141\,9(30)/0.75 = 5.676$ Mlbf/in

Similar to Fig. 8-23, we have three frusta.

Top frusta, steel: $t = l/2 = 0.375$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 30$ Mpsi

$$k_1 = \frac{0.5774\pi(30)0.5}{\ln \dfrac{[1.155(0.375) + 0.75 - 0.5](0.75 + 0.5)}{[1.155(0.375) + 0.75 + 0.5](0.75 - 0.5)}} = 38.45 \text{ Mlbf/in}$$

Middle frusta, steel: $t = 0.5 - 0.375 = 0.125$ in, $d = 0.5$ in,

$D = 0.75 + 2(0.75 - 0.5) \tan 30° = 1.039$ in, $E = 30$ Mpsi.

Eq. (8-20) $\Rightarrow$ $k_2 = 184.3$ Mlbf/in

Lower frusta, cast iron: $t = 0.75 - 0.5 = 0.25$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 14.5$ Mpsi.

Eq. (8-20) $\Rightarrow$ $k_3 = 23.49$ Mlbf/in

Eq. (8-18), $k_m = (1/38.45 + 1/184.3 + 1/23.49)^{-1} = 13.51$ Mlbf/in

$$C = k_b / (k_b + k_m) = 5.676 / (5.676 + 13.51) = 0.296$$

Table 8-9, $S_p = 85$ kpsi. For a non-permanent connection, using Eqs. (8-31) and (8-32)

$$F_i = 0.75\, A_t\, S_p = 0.75(0.141\,9)(85) = 9.05 \text{ kips}$$

The total external load is $P_{\text{total}} = p_g A_g$, where A_g is the effective area of the cylinder, based on the effective sealing diameter of 3.5 in. The external load per bolt is $P = P_{\text{total}}/N$. Thus

$$P = [1\ 500\pi(3.5^2)/4](10^{-3})/10 = 1.443 \text{ kips/bolt}$$

Yielding factor of safety, Eq. (8-28)

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{85(0.141\,9)}{0.296(1.443)+9.05} = 1.27 \qquad Ans.$$

Overload factor of safety, Eq. (8-29)

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{85(0.141\,9)-9.05}{0.296(1.443)} = 7.05 \qquad Ans.$$

Separation factor of safety, Eq. (8-30)

$$n_0 = \frac{F_i}{P(1-C)} = \frac{9.05}{1.443(1-0.296)} = 8.91 \qquad Ans.$$

8-39

From Table 8-7, $h = t_1 = 20$ mm
For $t_2 > d$, $l = h + d/2 = 20 + 10/2 = 25$ mm
$L \geq h + 1.5\,d = 20 + 1.5(10) = 35$ mm. Let $L = 35$ mm
$L_T = 2d + 6 = 2(10) + 6 = 26$ mm
$l_d = L - L_T = 35 - 26 = 9$ mm
$l_t = l - l_d = 25 - 9 = 16$ mm

From Table 8-1, $A_t = 58.0$ mm^2. $A_d = \pi(10^2)/4 = 78.5$ mm^2
Eq. (8-17),

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{78.5(58.0)207}{78.5(16)+58.0(9)} = 530.1 \text{ MN/m}$$

Similar to Fig. 8-23, we have three frusta.
Top frusta, steel: $t = l/2 = 12.5$ mm, $d = 10$ mm, $D = 15$ mm, $E = 207$ GPa. Eq. (8-20)

$$k_1 = \frac{0.5774\pi(207)10}{\ln\dfrac{[1.155(12.5)+15-10](15+10)}{[1.155(12.5)+15+10](15-10)}} = 4\ 163 \text{ MN/m}$$

Middle frusta, steel: $t = 20 - 12.5 = 7.5$ mm, $d = 10$ mm, $D = 15 + 2(12.5 - 7.5)\tan 30° = 20.77$ mm, $E = 207$ GPa. Eq. (8-20) $\Rightarrow$ $k_2 = 10\ 975$ MN/m
Lower frusta, cast iron: $t = 25 - 20 = 5$ mm, $d = 10$ mm, $D = 15$ mm, $E = 100$ GPa (see Table 8-8). Eq. (8-20) $\Rightarrow$ $k_3 = 3\ 239$ MN/m
Eq. (8-18), $k_m = (1/4\ 163 + 1/10\ 975 + 1/3\ 239)^{-1} = 1\ 562$ MN/m
$C = k_b/(k_b + k_m) = 530.1/(530.1 + 1\ 562) = 0.253$
Table 8-11: $S_p = 830$ MPa. For a non-permanent connection, using Eqs. (8-31) and (8-32)
$F_i = 0.75\,A_t S_p = 0.75(58.0)(830)10^{-3} = 36.1$ kN
The total external load is $P_{\text{total}} = p_g A_g$, where A_g is the effective area of the cylinder, based on the effective sealing diameter of 0.8 m. The external load per bolt is $P = P_{\text{total}}/N$. Thus

$$P = [550\pi(0.8^2)/4]/36 = 7.679 \text{ kN/bolt}$$

Yielding factor of safety, Eq. (8-28)

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{830(58.0)10^{-3}}{0.253(7.679)+36.1} = 1.27 \qquad Ans.$$

Overload factor of safety, Eq. (8-29)

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{830(58.0)10^{-3} - 36.1}{0.253(7.679)} = 6.20 \qquad Ans.$$

Separation factor of safety, Eq. (8-30)

$$n_0 = \frac{F_i}{P(1-C)} = \frac{36.1}{7.679(1-0.253)} = 6.29 \qquad Ans.$$

8-40

From Table 8-7, $h = t_1 = 0.375$ in

For $t_2 > d$, $l = h + d/2 = 0.375 + 0.4375/2 = 0.59375$ in

$L \geq h + 1.5\,d = 0.375 + 1.5(0.4375) = 1.031$ in. Round up to $L = 1.25$ in

$L_T = 2d + 0.25 = 2(0.4375) + 0.25 = 1.125$ in

$l_d = L - L_T = 1.25 - 1.125 = 0.125$

$l_t = l - l_d = 0.59375 - 0.125 = 0.46875$ in

$A_d = \pi(7/16)^2/4 = 0.150\ 3$ in^2, Table 8-2: $A_t = 0.106\ 3$ in^2

Eq. (8-17),

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.150\ 3(0.106\ 3)30}{0.150\ 3(0.46875) + 0.106\ 3(0.125)} = 5.724 \text{ Mlbf/in}$$

Similar to Fig. 8-23, we have three frusta.

Top frusta, steel: $t = l/2 = 0.296875$ in, $d = 0.4375$ in, $D = 0.65625$ in, $E = 30$ Mpsi

$$k_1 = \frac{0.5774\pi(30)0.4375}{\ln\dfrac{\left[1.155(0.296875)+0.656255-0.4375\right](0.75+0.656255)}{\left[1.155(0.296875)+0.75+0.656255\right](0.75-0.656255)}} = 35.52 \text{ Mlbf/in}$$

Middle frusta, steel: $t = 0.375 - 0.296875 = 0.078125$ in, $d = 0.4375$ in,
$D = 0.65625 + 2(0.59375 - 0.375) \tan 30° = 0.9088$ in, $E = 30$ Mpsi.

Eq. (8-20) $\Rightarrow$ $k_2 = 215.8$ Mlbf/in

Lower frusta, cast iron: $t = 0.59375 - 0.375 = 0.21875$ in, $d = 0.4375$ in, $D = 0.65625$ in,
$E = 14.5$ Mpsi. Eq. (8-20) $\Rightarrow$ $k_3 = 20.55$ Mlbf/in

Eq. (8-18), $k_m = (1/35.52 + 1/215.8 + 1/20.55)^{-1} = 12.28$ Mlbf/in

$$C = k_b/(k_b + k_m) = 5.724/(5.724 + 12.28) = 0.318$$

Table 8-9, $S_p = 120$ kpsi. For a non-permanent connection, using Eqs. (8-31) and (8-32)

$$F_i = 0.75\,A_t S_p = 0.75(0.106\ 3)(120) = 9.57 \text{ kips}$$

The total external load is $P_{\text{total}} = p_g A_g$, where A_g is the effective area of the cylinder, based on the effective sealing diameter of 3.25 in. The external load per bolt is $P = P_{\text{total}} /N$. Thus

$$P = [1\,200\pi(3.25^2)/4](10^{-3})/8 \;= 1.244 \text{ kips/bolt}$$

Yielding factor of safety, Eq. (8-28)

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{120(0.106\,3)}{0.318(1.244)+9.57} = 1.28 \qquad \textit{Ans.}$$

Overload factor of safety, Eq. (8-29)

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{120(0.106\,3)-9.57}{0.318(1.244)} = 8.05 \qquad \textit{Ans.}$$

Separation factor of safety, Eq. (8-30)

$$n_0 = \frac{F_i}{P(1-C)} = \frac{9.57}{1.244(1-0.318)} = 11.3 \qquad \textit{Ans.}$$

8-41

This is a design problem and there is no closed-form solution path or a unique solution. What is presented here is one possible iterative approach. We will demonstrate this with an example.

1. Select the diameter, d. For this example, let $d = 10$ mm. Using Eq. (8-20) on members, and combining using Eq. (8-18), yields $k_m = 1\,141$ MN/m (see Prob. 8-33 for method of calculation.

2. Look up the nut height in Table A-31. For the example, $H = 8.4$ mm. From this, L is rounded up from the calculation of $l + H = 40 + 8.4 = 48.4$ mm to 50 mm. Next, calculations are made for $L_T = 2(10) + 6 = 26$ mm, $l_d = 50 - 26 = 24$ mm, $l_t = 40 - 24 = 16$ mm. From step 1, $A_d = \pi(10^2)/4 = 78.54$ mm^2. Next, from Table 8-1, $A_t = 78.54$ mm^2. From Eq. (8-17), $k_b = 356$ MN/m. Finally, from Eq. (f), Sec. 8-7, $C = 0.238$.

3. From Prob. 8-33, the bolt circle diameter is $E = 200$ mm. Substituting this for D_b in Eq. (8-34), the number of bolts are

$$N = \frac{\pi D_b}{4d} = \frac{\pi(200)}{4(10)} = 15.7$$

Rounding this up gives $N = 16$.

4. Next, select a grade bolt. Based on the solution to Prob. 8-33, the strength of ISO 9.8 was so high to give very large factors of safety for overload and separation. Try ISO 4.6 with $S_p = 225$ MPa. From Eqs. (8-31) and (8-32) for a non-permanent connection, $F_i = 9.79$ kN.

5. The external load requirement per bolt is $P = 1.15\, p_g\, A_g/N$, where from Prob 8-33, $p_g =$ 6 MPa, and $A_g = \pi(100^2)/4$. This gives $P = 3.39$ kN/bolt.

6. Using Eqs. (8-28) to (8-30) yield $n_p = 1.23$, $n_L = 4.05$, and $n_0 = 3.79$.

Steps 1 - 6 can be easily implemented on a spreadsheet with lookup tables for the tables used from the text. The results for four bolt sizes are shown below. The dimension of each term is consistent with the example given above.

d	k_m	H	L	L_T	l_d	l_t	A_d	A_t	k_b
8	854	6.8	50	22	28	12	50.26	36.6	233.9
10	1141	8.4	50	26	24	16	78.54	58	356
12	1456	10.8	55	30	25	15	113.1	84.3	518.8
14	1950	12.8	55	34	21	19	153.9	115	686.3

d	C	N	S_p	F_i	P	n_p	n_L	n_0
8	0.215	20	225	6.18	2.71	1.22	3.53	2.90
10	0.238	16	225	9.79	3.39	1.23	4.05	3.79
12	0.263	13*	225	14.23	4.17	1.24	4.33	4.63
14	0.276	12	225	19.41	4.52	1.25	5.19	5.94

*Rounded down from13.08997, so spacing is slightly greater than four diameters.

Any one of the solutions is acceptable. A decision-maker might be cost such as N × cost/bolt, and/or N × cost per hole, etc.

8-42 This is a design problem and there is no closed-form solution path or a unique solution. What is presented here is one possible iterative approach. We will demonstrate this with an example.

1. Select the diameter, d. For this example, let $d = 0.5$ in. Using Eq. (8-20) on three frusta (see Prob. 8-34 solution), and combining using Eq. (8-19), yields $k_m = 10.10$ Mlbf/in.

2. Look up the nut height in Table A-31. For the example, $H = 0.4375$ in. From this, L is rounded up from the calculation of $l + H = 1.125 + 0.4375 = 1.5625$ in to 1.75 in. Next, calculations are made for $L_T = 2(0.5) + 0.25 = 1.25$ in, $l_d = 1.75 - 1.25 = 0.5$ in, $l_t = 1.125 - 0.5 = 0.625$ in. From step 1, $A_d = \pi(0.5^2)/4 = 0.1963$ in^2. Next, from Table 8-1, $A_t = 0.141\,9$ in^2. From Eq. (8-17), $k_b = 4.316$ Mlbf/in. Finally, from Eq. (*f*), Sec. 8-7, $C = 0.299$.

3. From Prob. 8-34, the bolt circle diameter is $E = 6$ in. Substituting this for D_b in Eq. (8-34), for the number of bolts

$$N = \frac{\pi D_b}{4d} = \frac{\pi(6)}{4(0.5)} = 9.425$$

Rounding this up gives $N = 10$.

4. Next, select a grade bolt. Based on the solution to Prob. 8-34, the strength of SAE grade 5 was adequate. Use this with $S_p = 85$ kpsi. From Eqs. (8-31) and (8-32) for a non-permanent connection, $F_i = 9.046$ kips.

5. The external load requirement per bolt is $P = 1.15\, p_g\, A_g/N$, where from Prob 8-34, $p_g = 1\,500$ psi, and $A_g = \pi(3.5^2)/4$. This gives $P = 1.660$ kips/bolt.
6. Using Eqs. (8-28) to (8-30) yield $n_p = 1.26$, $n_L = 6.07$, and $n_0 = 7.78$.

d	k_m	H	L	L_T	l_d	l_t	A_d	A_t	k_b
0.375	6.75	0.3281	1.5	1	0.5	0.625	0.1104	0.0775	2.383
0.4375	9.17	0.375	1.5	1.125	0.375	0.75	0.1503	0.1063	3.141
0.5	10.10	0.4375	1.75	1.25	0.5	0.625	0.1963	0.1419	4.316
0.5625	11.98	0.4844	1.75	1.375	0.375	0.75	0.2485	0.182	5.329

d	C	N	S_p	F_i	P	n_p	n_L	n_0
0.375	0.261	13	85	4.941	1.277	1.25	4.95	5.24
0.4375	0.273	11	85	6.777	1.509	1.26	5.48	6.18
0.5	0.299	10	85	9.046	1.660	1.26	6.07	7.78
0.5625	0.308	9	85	11.6	1.844	1.27	6.81	9.09

Any one of the solutions is acceptable. A decision-maker might be cost such as N × cost/bolt, and/or N × cost per hole, etc.

8-43 This is a design problem and there is no closed-form solution path or a unique solution. What is presented here is one possible iterative approach. We will demonstrate this with an example.

1. Select the diameter, d. For this example, let $d = 10$ mm. Using Eq. (8-20) on three frusta (see Prob. 8-35 solution), and combining using Eq. (8-19), yields $k_m = 1\ 087$ MN/m.

2. Look up the nut height in Table A-31. For the example, $H = 8.4$ mm. From this, L is rounded up from the calculation of $l + H = 45 + 8.4 = 53.4$ mm to 55 mm. Next, calculations are made for $L_T = 2(10) + 6 = 26$ mm, $l_d = 55 - 26 = 29$ mm, $l_t = 45 - 29 = 16$ mm. From step 1, $A_d = \pi(10^2)/4 = 78.54$ mm^2. Next, from Table 8-1, $A_t = 58.0$ mm^2. From Eq. (8-17), $k_b = 320.9$ MN/m. Finally, from Eq. (f), Sec. 8-7, $C = 0.228$.

3. From Prob. 8-35, the bolt circle diameter is $E = 1000$ mm. Substituting this for D_b in Eq. (8-34), for the number of bolts

$$N = \frac{\pi D_b}{4d} = \frac{\pi(1000)}{4(10)} = 78.5$$

Rounding this up gives $N = 79$. A rather large number, since the bolt circle diameter, E is so large. Try larger bolts.

4. Next, select a grade bolt. Based on the solution to Prob. 8-35, the strength of ISO 9.8 was so high to give very large factors of safety for overload and separation. Try ISO 5.8 with $S_p = 380$ MPa. From Eqs. (8-31) and (8-32) for a non-permanent connection, $F_i =$ 16.53 kN.

5. The external load requirement per bolt is $P = 1.15\ p_g\ A_g/N$, where from Prob 8-35, p_g = 0.550 MPa, and $A_g = \pi(800^2)/4$. This gives $P = 4.024$ kN/bolt.

6. Using Eqs. (8-28) to (8-30) yield $n_p = 1.26$, $n_L = 6.01$, and $n_0 = 5.32$.

Steps 1 - 6 can be easily implemented on a spreadsheet with lookup tables for the tables used from the text. The results for three bolt sizes are shown below. The dimension of each term is consistent with the example given above.

d	k_m	H	L	L_T	l_d	l_t	A_d	A_t	k_b
10	1087	8.4	55	26	29	16	78.54	58	320.9
20	3055	18	65	46	19	26	314.2	245	1242
36	6725	31	80	78	2	43	1018	817	3791

d	C	N	S_p	F_i	P	n_p	n_L	n_0
10	0.228	79	380	16.53	4.024	1.26	6.01	5.32
20	0.308	40	380	69.83	7.948	1.29	9.5	12.7
36	0.361	22	380	232.8	14.45	1.3	14.9	25.2

A large range is presented here. Any one of the solutions is acceptable. A decision-maker might be cost such as N × cost/bolt, and/or N × cost per hole, etc.

8-44 This is a design problem and there is no closed-form solution path or a unique solution. What is presented here is one possible iterative approach. We will demonstrate this with an example.

1. Select the diameter, d. For this example, let $d = 0.375$ in. Using Eq. (8-20) on three frusta (see Prob. 8-36 solution), and combining using Eq. (8-19), yields $k_m = 7.42$ Mlbf/in.

2. Look up the nut height in Table A-31. For the example, $H = 0.3281$ in. From this, $L \geq l + H = 0.875 + 0.3281 = 1.2031$ in. Rounding up, $L = 1.25$. Next, calculations are made for $L_T = 2(0.375) + 0.25 = 1$ in, $l_d = 1.25 - 1 = 0.25$ in, $l_t = 0.875 - 0.25 = 0.625$ in. From step 1, $A_d = \pi(0.375^2)/4 = 0.1104$ in^2. Next, from Table 8-1, $A_t = 0.0775$ in^2. From Eq. (8-17), $k_b = 2.905$ Mlbf/in. Finally, from Eq. (f), Sec. 8-7, $C = 0.263$.

3. From Prob. 8-36, the bolt circle diameter is $E = 6$ in. Substituting this for D_b in Eq. (8-34), for the number of bolts

$$N = \frac{\pi D_b}{4d} = \frac{\pi(6)}{4(0.375)} = 12.6$$

Rounding this up gives $N = 13$.

4. Next, select a grade bolt. Based on the solution to Prob. 8-36, the strength of SAE grade 8 seemed high for overload and separation. Try SAE grade 5 with $S_p = 85$ kpsi. From Eqs. (8-31) and (8-32) for a non-permanent connection, $F_i = 4.941$ kips.

5. The external load requirement per bolt is $P = 1.15\, p_g\, A_g/N$, where from Prob 8-36, $p_g = 1\,200$ psi, and $A_g = \pi(3.25^2)/4$. This gives $P = 0.881$ kips/bolt.

6. Using Eqs. (8-28) to (8-30) yield $n_p = 1.27$, $n_L = 6.65$, and $n_0 = 7.81$.

Steps 1 - 6 can be easily implemented on a spreadsheet with lookup tables for the tables used from the text. For this solution we only looked at one bolt size, $\frac{3}{8}-16$, but evaluated changing the bolt grade. The results for four bolt grades are shown below. The dimension of each term is consistent with the example given above.

d	k_m	H	L	L_T	l_d	l_t	A_d	A_t	k_b
0.375	7.42	0.3281	1.25	1	0.25	0.625	0.1104	0.0775	2.905

d	C	N	SAE grade	S_p	F_i	P	n_p	n_L	n_0
0.375	0.281	13	1	33	1.918	0.881	1.18	2.58	3.03
0.375	0.281	13	2	55	3.197	0.881	1.24	4.30	5.05
0.375	0.281	13	4	65	3.778	0.881	1.25	5.08	5.97
0.375	0.281	13	5	85	4.941	0.881	1.27	6.65	7.81

Note that changing the bolt grade only affects S_p, F_i, n_p, n_L, and n_0. Any one of the solutions is acceptable, especially the lowest grade bolt.

8-45 **(a)** $F_b' = RF_{b,\max}' \sin\theta$

Half of the external moment is contributed by the line load in the interval $0 \le \theta \le \pi$

$$\frac{M}{2} = \int_0^\pi F_b' R^2 \sin\theta \, d\theta = \int_0^\pi F_{b,\max}' R^2 \sin^2\theta \, d\theta$$
$$\frac{M}{2} = \frac{\pi}{2} F_{b,\max}' R^2$$

from which $F_{b,\max}' = \dfrac{M}{\pi R^2}$

$$F_{\max} = \int_{\phi_1}^{\phi_2} F_b' R \sin\theta \, d\theta = \frac{M}{\pi R^2} \int_{\phi_1}^{\phi_2} R \sin\theta \, d\theta = \frac{M}{\pi R}(\cos\phi_1 - \cos\phi_2)$$

Noting $\phi_1 = 75°$, $\phi_2 = 105°$,

$$F_{\max} = \frac{12\ 000}{\pi(8/2)}(\cos 75^\circ - \cos 105^\circ) = 494 \text{ lbf} \quad \textit{Ans.}$$

(b)

$$F_{\max} = F_{b,\max}' R\Delta\phi = \frac{M}{\pi R^2}(R)\left(\frac{2\pi}{N}\right) = \frac{2M}{RN}$$
$$F_{\max} = \frac{2(12\ 000)}{(8/2)(12)} = 500 \text{ lbf} \quad \textit{Ans.}$$

(c) $F = F_{\max} \sin\theta$

$$M = 2\, F_{\max} R\, [(1)\sin^2 90° + 2\sin^2 60° + 2\sin^2 30° + (1)\sin^2(0)] = 6F_{\max}R$$

from which,

$$F_{\max} = \frac{M}{6R} = \frac{12\ 000}{6(8/2)} = 500 \text{ lbf} \quad \textit{Ans.}$$

The simple general equation resulted from part (*b*)

$$F_{\max} = \frac{2M}{RN}$$

8-46

(**a**) From Table 8-11, $S_p = 600$ MPa. From Table 8-1, $A_t = 353\ \text{mm}^2$.

Eq. (8-31): $$F_i = 0.9A_t S_p = 0.9(353)(600)\left(10^{-3}\right) = 190.6\ \text{kN}$$

Table 8-15: $K = 0.18$

Eq. (8-27): $T = K F_i d = 0.18(190.6)(24) = 823$ N·m *Ans.*

(**b**) Washers: $t = 4.6$ mm, $d = 24$ mm, $D = 1.5(24) = 36$ mm, $E = 207$ GPa.

Eq. (8-20),

$$k_1 = \frac{0.5774\pi(207)24}{\ln\dfrac{\left[1.155(4.6)+36-24\right](36+24)}{\left[1.155(4.6)+36+24\right](36-24)}} = 31\,990\ \text{MN/m}$$

Cast iron: $t = 20$ mm, $d = 24$ mm, $D = 36 + 2(4.6)\tan 30° = 41.31$ mm, $E = 135$ GPa. Eq. (8-20) $\Rightarrow$ $k_2 = 10\,785$ MN/m

Steel joist: $t = 20$ mm, $d = 24$ mm, $D = 41.31$ mm, $E = 207$ GPa. Eq. (8-20) $\Rightarrow$ $k_3 = 16\,537$ MN/m

Eq. (8-18): $k_m = (2 / 31\,990 + 1 / 10\,785 + 1 / 16\,537)^{-1} = 4\,636$ MN/m

Bolt: $l = 2(4.6) + 2(20) = 49.2$ mm. Nut, Table A-31, $H = 21.5$ mm. $L > 49.2 + 21.5 = 70.7$ mm. From Table A-17, use $L = 80$ mm. From Eq. (8-14)

$L_T = 2(24) + 6 = 54$ mm, $l_d = 80 - 54 = 26$ mm, $l_t = 49.2 - 26 = 23.2$ mm

From Table (8-1), $A_t = 353\ \text{mm}^2$, $A_d = \pi(24^2) / 4 = 452.4\ \text{mm}^2$

Eq. (8-17):

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{452.4(353)207}{452.4(23.2)+353(26)} = 1680\ \text{MN/m}$$

$C = k_b / (k_b + k_m) = 1680 / (1680 + 4636) = 0.266$, $S_p = 600$ MPa, $F_i = 190.6$ kN,
$P = P_{\text{total}} / \text{N} = 18/4 = 4.5$ kN

Yield: From Eq. (8-28)

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{600(353)10^{-3}}{0.266(4.5) + 190.6} = 1.10 \quad Ans.$$

Load factor: From Eq. (8-29)

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{600(353)10^{-3} - 190.6}{0.266(4.5)} = 17.7 \qquad Ans.$$

Separation: From Eq. (8-30)

$$n_0 = \frac{F_i}{P(1-C)} = \frac{190.6}{4.5(1-0.266)} = 57.7 \qquad Ans.$$

As was stated in the text, bolts are typically preloaded such that the yielding factor of safety is not much greater than unity which is the case for this problem. However, the other load factors indicate that the bolts are oversized for the external load.

8-47 (**a**) ISO M 20 × 2.5 grade 8.8 coarse pitch bolts, lubricated.

Table 8-2, $A_t = 245\ \text{mm}^2$

Table 8-11, $S_p = 600$ MPa

$F_i = 0.90\, A_t\, S_p = 0.90(245)600(10^{-3}) = 132.3$ kN

Table 8-15, $K = 0.18$

Eq. (8-27), $T = KF_i\, d = 0.18(132.3)20 = 476\ \text{N} \cdot \text{m}$ *Ans.*

(**b**) Table A-31, $H = 18$ mm, $L \geq L_G + H = 48 + 18 = 66$ mm. Round up to $L = 80$ mm per Table A-17.

$$L_T = 2d + 6 = 2(20) + 6 = 46 \text{ mm}$$
$$l_d = L - L_T = 80 - 46 = 34 \text{ mm}$$
$$l_t = l - l_d = 48 - 34 = 14 \text{ mm}$$

$A_d = \pi(20^2)/4 = 314.2\ \text{mm}^2$,

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{314.2(245)(207)}{314.2(14) + 245(34)} = 1251.9 \text{ MN/m}$$

Members: Since all members are steel use Eq. (8-22) with $E = 207$ MPa, $l = 48$ mm, $d =$ 20mm

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\dfrac{0.5774l+0.5d}{0.5774l+2.5d}\right)} = \frac{0.5774\pi(207)20}{2\ln\left[5\dfrac{0.5774(48)+0.5(20)}{0.5774(48)+2.5(20)}\right]} = 4236 \text{ MN/m}$$

$$C = \frac{k_b}{k_b+k_m} = \frac{1251.9}{1251.9+4236} = 0.228$$

$P = P_{\text{total}} / N = 40/2 = 20$ kN,

Yield: From Eq. (8-28)

$$n_p = \frac{S_pA_t}{CP+F_i} = \frac{600(245)10^{-3}}{0.228(20)+132.3} = 1.07 \quad \textit{Ans.}$$

Load factor: From Eq. (8-29)

$$n_L = \frac{S_pA_t - F_i}{CP} = \frac{600(245)10^{-3}-132.3}{0.228(20)} = 3.22 \quad \textit{Ans.}$$

Separation: From Eq. (8-30)

$$n_0 = \frac{F_i}{P(1-C)} = \frac{132.3}{20(1-0.228)} = 8.57 \quad \textit{Ans.}$$

8-48 From Prob. 8-29 solution, $P_{\max}$ =13.33 kips, $C = 0.2$, $F_i = 12.77$ kips, $A_t = 0.141\,9$ in^2

$$\sigma_i = \frac{F_i}{A_t} = \frac{12.77}{0.141\,9} = 90.0 \text{ kpsi}$$

Eq. (8-39), $$\sigma_a = \frac{CP}{2A_t} = \frac{0.2(13.33)}{2(0.141\,9)} = 9.39 \text{ kpsi}$$

Eq. (8-41), $$\sigma_m = \sigma_a + \sigma_i = 9.39 + 90.0 = 99.39 \text{ kpsi}$$

(**a**) Goodman Eq. (8-45) for grade 8 bolts, $S_e = 23.2$ kpsi (Table 8-17), $S_{ut} = 150$ kpsi (Table 8-9)

$$n_f = \frac{S_e(S_{ut}-\sigma_i)}{\sigma_a(S_{ut}+S_e)} = \frac{23.2(150-90.0)}{9.39(150+23.2)} = 0.856 \quad \textit{Ans.}$$

(**b**) Gerber Eq. (8-46)

$$n_f = \frac{1}{2\sigma_aS_e}\left[S_{ut}\sqrt{S_{ut}^2+4S_e(S_e+\sigma_i)} - S_{ut}^2 - 2\sigma_iS_e\right]$$

$$= \frac{1}{2(9.39)23.2}\left[150\sqrt{150^2+4(23.2)(23.2+90.0)} - 150^2 - 2(90.0)23.2\right] = 1.32 \quad \textit{Ans.}$$

(**c**) ASME-elliptic Eq. (8-47) with $S_p = 120$ kpsi (Table 8-9)

$$n_f = \frac{S_e}{\sigma_a\left(S_p^2 + S_e^2\right)}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right)$$

$$= \frac{23.2}{9.39\left(120^2 + 23.2^2\right)}\left[120\sqrt{120^2 + 23.2^2 - 90^2} - 90\left(23.2\right)\right] = 1.30 \quad \textit{Ans.}$$

8-49 **Attention to the Instructor**. Part (**d**) requires the determination of the endurance strength, S_e, of a class 5.8 bolt. Table 8-17 does not provide this and the student will be required to estimate it by other means [see the solution of part (**d**)].

Per bolt, $P_{b\max} = 60/8 = 7.5$ kN, $P_{b\min} = 20/8 = 2.5$ kN

$$C = \frac{k_b}{k_b + k_m} = \frac{1}{1 + 2.6} = 0.278$$

(**a**) Table 8-1, $A_t = 20.1$ mm^2; Table 8-11, $S_p = 380$ MPa

Eqs. (8-31) and (8-32), $F_i = 0.75\, A_t\, S_p = 0.75(20.1)380(10^{-3}) = 5.73$ kN

Yield, Eq. (8-28), $$n_p = \frac{S_p A_t}{CP + F_i} = \frac{380\left(20.1\right)10^{-3}}{0.278\left(7.5\right) + 5.73} = 0.98 \qquad \textit{Ans.}$$

(**b**) Overload, Eq. (8-29), $$n_L = \frac{S_p A_t - F_i}{CP} = \frac{380\left(20.1\right)10^{-3} - 5.73}{0.278\left(7.5\right)} = 0.915 \qquad \textit{Ans.}$$

(**c**) Separation, Eq. (8-30), $$n_0 = \frac{F_i}{P\left(1 - C\right)} = \frac{5.73}{7.5\left(1 - 0.278\right)} = 1.06 \qquad \textit{Ans.}$$

(**d**) Goodman, Eq. (8-35), $$\sigma_a = \frac{C\left(P_{b\max} - P_{b\min}\right)}{2A_t} = \frac{0.278\left(7.5 - 2.5\right)10^3}{2\left(20.1\right)} = 34.6 \text{ MPa}$$

Eq. (8-36), $$\sigma_m = \frac{C\left(P_{b\max} + P_{b\min}\right)}{2A_t} + \frac{F_i}{A_t} = \frac{0.278\left(7.5 + 2.5\right)10^3}{2\left(20.1\right)} + \frac{5.73\left(10^3\right)}{20.1} = 354.2 \text{ MPa}$$

Table 8-11, $S_{ut} = 520$ MPa, $\sigma_i = F_i / A_t = 5.73(10^3)/20.1 = 285$ MPa

We have a problem for S_e. Table 8-17 does not list S_e for class 5.8 bolts. Here, we will estimate S_e using the methods of Chapter 6. Estimate S_e' from the,

Eq. (6-10): $S_e' = 0.5 S_{ut} = 0.5\left(520\right) = 260$ MPa .

Table 6-2: $a = 3.04$, $b = -\,0.217$

Eq. (6-18): $k_a = a S_{ut}^b = 3.04\left(520^{-0.217}\right) = 0.783$

Eq. (6-19): $k_b = 1$

Eq. (6-25): $k_c = 0.85$

The fatigue stress-concentration factor, from Table 8-16, is $K_f = 2.2$. For simple axial loading and infinite-life it is acceptable to reduce the endurance limit by K_f and use the nominal stresses in the stress/strength/design factor equations. Thus, from

Eq. (6-17), $S_e = k_a\, k_b\, k_c\, S_e' \,/\, K_f = 0.783(1)0.85(260) / 2.2 = 78.7$ MPa

Eq. (8-38),

$$n_f = \frac{S_e\left(S_{ut}-\sigma_i\right)}{S_{ut}\sigma_a + S_e\left(\sigma_m-\sigma_i\right)} = \frac{78.7\left(520-285\right)}{520\left(34.6\right)+78.7\left(354.2-285\right)} = 0.789 \qquad Ans.$$

It is obvious from the various answers obtained, the bolted assembly is undersized. This can be rectified by a one or more of the following: more bolts, larger bolts, higher class bolts.

8-50 Per bolt, $P_{b\max} = P_{\max}/N = 80/10 = 8$ kips, $P_{b\min} = P_{\min}/N = 20/10 = 2$ kips
$C = k_b/(k_b+k_m) = 4/(4+12) = 0.25$
(**a**) Table 8-2, $A_t = 0.141\,9$ in^2, Table 8-9, $S_p = 120$ kpsi and $S_{ut} = 150$ kpsi
Table 8-17, $S_e = 23.2$ kpsi
Eqs. (8-31) and (8-32), $F_i = 0.75\,A_t S_p \;\Rightarrow\; \sigma_i = F_i/A_t = 0.75\,S_p = 0.75(120) = 90$ kpsi

Eq. (8-35), $$\sigma_a = \frac{C\left(P_{b\max}-P_{b\min}\right)}{2A_t} = \frac{0.25\left(8-2\right)}{2\left(0.141\,9\right)} = 5.29 \text{ kpsi}$$

Eq. (8-36), $$\sigma_m = \frac{C\left(P_{b\max}+P_{b\min}\right)}{2A_t} + \sigma_i = \frac{0.25\left(8+2\right)}{2\left(0.141\,9\right)} + 90 = 98.81 \text{ kpsi}$$

Eq. (8-38),

$$n_f = \frac{S_e\left(S_{ut}-\sigma_i\right)}{S_{ut}\sigma_a + S_e\left(\sigma_m-\sigma_i\right)} = \frac{23.2\left(150-90\right)}{150\left(5.29\right)+23.2\left(98.81-90\right)} = 1.39 \qquad Ans.$$

8-51 Given: (8) M8 × 1.5 class 9.8 bolts, $k_b = 1.5$ MN/mm, $k_m = 3.9$ MN/mm, reversing load with $P_{\max} = 50$ kN.
Per bolt, $P = P_{\max}/N = 50/8 = 6.25$ kN
Table 8-1, $A_t = 36.6$ mm^2, Table 8-11, $S_{ut} = 900$ MPa, $S_p = 650$ MPa,
Table 8-17, $S_e = 140$ MPa
Eq. (*f*), Sec. 8-7: $C = k_b/(k_b + k_m) = 1.5/(1.5+3.9) = 0.278$
Eqs. (8-31) and (8-32): $F_i = 0.75\,A_t S_p = 0.75(36.6)650(10^{-3}) = 17.84$ kN
$\sigma_i = F_i/A_t = 17.84(10^3)/36.6 = 487.5$ MPa
Eq. (8-39): $\sigma_a = CP/(2A_t) = 0.278(6.25)10^3/[2(36.6)] = 23.74$ MPa
(**a**) Goodman: Eq. (8-45):

$$n_f = \frac{S_e\left(S_{ut}-\sigma_i\right)}{\sigma_a\left(S_{ut}+S_e\right)} = \frac{140\left(900-487.5\right)}{23.74\left(900+140\right)} = 2.34 \qquad Ans.$$

(**b**) Gerber: Eq. (8-46):

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e\left(S_e+\sigma_i\right)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2\left(23.74\right)\left(140\right)}\left[900\sqrt{900^2+4\left(140\right)\left(140+487.5\right)} - 900^2 - 2\left(487.5\right)\right] = 3.52 \quad Ans.$$

(**c**) Morrow: Use Goodman Eq. (8-45) replacing S_{ut} with σ_f'. Equation (6-44) gives

$\sigma'_f = S_{ut} + 345 = 900 + 345 = 1\ 245\text{MPa}$

$$n_f = \frac{S_e\left(\sigma'_f - \sigma_i\right)}{\sigma_a\left(\sigma'_f + S_e\right)} = \frac{140(1245 - 487.5)}{23.74(1245 + 140)} = 3.23 \qquad Ans.$$

8-52 From Prob. 8-33, $C = 0.263$, $P_{\max} =$ 4.712 kN / bolt, F_i = 41.1 kN, $S_p = 650$ MPa, and A_t = 84.3 mm^2

$$\sigma_i = 0.75\ S_p = 0.75(650) = 487.5 \text{ MPa}$$

Eq. (8-39): $$\sigma_a = \frac{CP}{2A_t} = \frac{0.263(4.712)10^3}{2(84.3)} = 7.350 \text{ MPa}$$

Eq. (8-40) $$\sigma_m = \frac{CP}{2A_t} + \frac{F_i}{A_t} = 7.350 + 487.5 = 494.9 \text{ MPa}$$

(**a**) Goodman: From Table 8-11, $S_{ut} = 900$ MPa, and from Table 8-17, S_e = 140 MPa

Eq. (8-45): $$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{\sigma_a\left(S_{ut} + S_e\right)} = \frac{140(900 - 487.5)}{7.350(900 + 140)} = 7.55 \qquad Ans.$$

(**b**) Gerber:
Eq. (8-46):

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e\left(S_e + \sigma_i\right)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(7.350)140}\left[900\sqrt{900^2 + 4(140)(140 + 487.5)} - 900^2 - 2(487.5)(140)\right]$$

$$= 11.4 \qquad Ans.$$

(**c**) ASME-elliptic:

Eq. (8-47):

$$n_f = \frac{S_e}{\sigma_a\left(S_p^2 + S_e^2\right)}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right)$$

$$= \frac{140}{7.350\left(650^2 + 140^2\right)}\left[650\sqrt{650^2 + 140^2 - 487.5^2} - 487.5(140)\right] = 9.73 \qquad Ans.$$

8-53 From Prob. 8-34, $C = 0.299$, $P_{\max} = 1.443$ kips/bolt, F_i = 9.05 kips, $S_p = 85$ kpsi, and $A_t = 0.141\ 9$ in^2

$$\sigma_i = 0.75 S_p = 0.75(85) = 63.75 \text{ kpsi}$$

Eq. (8-37): $$\sigma_a = \frac{CP}{2A_t} = \frac{0.299(1.443)}{2(0.141\,9)} = 1.520 \text{ kpsi}$$

Eq. (8-38) $$\sigma_m = \frac{CP}{2A_t} + \sigma_i = 1.520 + 63.75 = 65.27 \text{ kpsi}$$

(**a**) Goodman: From Table 8-9, S_{ut} = 120 kpsi, and from Table 8-17, S_e = 18.8 kpsi

Eq. (8-45): $$n_f = \frac{S_e(S_{ut} - \sigma_i)}{\sigma_a(S_{ut} + S_e)} = \frac{18.8(120 - 63.75)}{1.520(120 + 18.8)} = 5.01 \qquad Ans.$$

(**b**) Gerber:

Eq. (8-46):

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(1.520)18.6}\left[120\sqrt{120^2 + 4(18.6)(18.6 + 63.75)} - 120^2 - 2(63.75)(18.6)\right]$$

$$= 7.45 \qquad Ans.$$

(**c**) ASME-elliptic:

Eq. (8-47):

$$n_f = \frac{S_e}{\sigma_a(S_p^2 + S_e^2)}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right)$$

$$= \frac{18.6}{1.520(85^2 + 18.6^2)}\left[85\sqrt{85^2 + 18.6^2 - 63.75^2} - 63.75(18.6)\right] = 6.22 \qquad Ans.$$

8-54 From Prob. 8-35, $C = 0.228$, $P_{\max}$ = 7.679 kN/bolt, F_i = 36.1 kN, S_p = 830 MPa, and A_t = 58.0 mm^2

$$\sigma_i = 0.75\, S_p = 0.75(830) = 622.5 \text{ MPa}$$

Eq. (8-37): $$\sigma_a = \frac{CP}{2A_t} = \frac{0.228(7.679)10^3}{2(58.0)} = 15.09 \text{ MPa}$$

Eq. (8-38) $$\sigma_m = \frac{CP}{2A_t} + \sigma_i = 15.09 + 622.5 = 637.6 \text{ MPa}$$

(**a**) Goodman: From Table 8-11, S_{ut} = 1040 MPa, and from Table 8-17, S_e = 162 MPa

Eq. (8-45): $$n_f = \frac{S_e\left(S_{ut}-\sigma_i\right)}{\sigma_a\left(S_{ut}+S_e\right)} = \frac{162(1040-622.5)}{15.09(1040+162)} = 3.73 \qquad Ans.$$

(**b**) Gerber:
Eq. (8-46):

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2+4S_e\left(S_e+\sigma_i\right)}-S_{ut}^2-2\sigma_i S_e\right]$$

$$= \frac{1}{2(15.09)162}\left[1040\sqrt{1040^2+4(162)(162+622.5)}-1040^2-2(622.5)(162)\right]$$

$$= 5.74 \qquad Ans.$$

(**c**) ASME-elliptic:

Eq. (8-47):

$$n_f = \frac{S_e}{\sigma_a\left(S_p^2+S_e^2\right)}\left(S_p\sqrt{S_p^2+S_e^2-\sigma_i^2}-\sigma_i S_e\right)$$

$$= \frac{162}{15.09\left(830^2+162^2\right)}\left[830\sqrt{830^2+162^2-622.5^2}-622.5(162)\right] = 5.62 \qquad Ans.$$

8-55 From Prob. 8-36, $C = 0.291$, $P_{\max} = 1.244$ kips/bolt, $F_i = 9.57$ kips, $S_p = 120$ kpsi, and $A_t = 0.106\ 3$ in^2

$$\sigma_i = 0.75S_p = 0.75(120) = 90 \text{ kpsi}$$

Eq. (8-37): $$\sigma_a = \frac{CP}{2A_t} = \frac{0.291(1.244)}{2(0.106\ 3)} = 1.703 \text{ kpsi}$$

Eq. (8-38) $$\sigma_m = \frac{CP}{2A_t} + \sigma_i = 1.703 + 90 = 91.70 \text{ kpsi}$$

(**a**) Goodman: From Table 8-9, S_{ut} = 150 kpsi, and from Table 8-17, S_e = 23.2 kpsi

Eq. (8-45): $$n_f = \frac{S_e\left(S_{ut}-\sigma_i\right)}{\sigma_a\left(S_{ut}+S_e\right)} = \frac{23.2(150-90)}{1.703(150+23.2)} = 4.72 \qquad Ans.$$

(**b**) Gerber:
Eq. (8-46):

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e\left(S_e + \sigma_i\right)} - S_{ut}^2 - 2\sigma_i S_e \right]$$

$$= \frac{1}{2\left(1.703\right)23.2}\left[150\sqrt{150^2 + 4\left(23.2\right)\left(23.2+90\right)} - 150^2 - 2\left(90\right)\left(23.2\right)\right]$$

$$= 7.28 \qquad Ans.$$

(**c**) ASME-elliptic:

Eq. (8-47):

$$n_f = \frac{S_e}{\sigma_a\left(S_p^2 + S_e^2\right)}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right)$$

$$= \frac{23.2}{1.703\left(120^2 + 18.6^2\right)}\left[120\sqrt{120^2 + 23.2^2 - 90^2} - 90\left(23.2\right)\right] = 7.24 \qquad Ans.$$

8-56 From Prob. 8-52, $C = 0.263$, $S_e = 140$ MPa, $S_{ut} = 900$ MPa, $A_t = 84.4\ \text{mm}^2$, $\sigma_i = 487.5$ MPa, and $P_{\max} = 4.712$ kN.

$$P_{\min} = P_{\max}/2 = 4.712/2 = 2.356 \text{ kN}$$

Eq. (8-35):
$$\sigma_a = \frac{C\left(P_{\max} - P_{\min}\right)}{2A_t} = \frac{0.263\left(4.712 - 2.356\right)10^3}{2\left(84.3\right)} = 3.675 \text{ MPa}$$

Eq. (8-36):

$$\sigma_m = \frac{C\left(P_{\max} + P_{\min}\right)}{2A_t} + \sigma_i$$

$$= \frac{0.263\left(4.712 + 2.356\right)10^3}{2\left(84.3\right)} + 487.5 = 498.5 \text{ MPa}$$

Eq. (8-38):

$$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut}\sigma_a + S_e\left(\sigma_m - \sigma_i\right)} = \frac{140\left(900 - 487.5\right)}{900\left(3.675\right) + 140\left(498.5 - 487.5\right)} = 11.9 \qquad Ans.$$

8-57 From Prob. 8-53, $C = 0.299$, $S_e = 18.8$ kpsi, $S_{ut} = 120$ kpsi, $A_t = 0.141\,9$ in^2, $\sigma_i = 63.75$ kpsi, and $P_{\max} = 1.443$ kips

$$P_{\min} = P_{\max} / 2 = 1.443/2 = 0.722 \text{ kips}$$

Eq. (8-35): $$\sigma_a = \frac{C\left(P_{\max} - P_{\min}\right)}{2A_t} = \frac{0.299\left(1.443 - 0.722\right)}{2\left(0.141\,9\right)} = 0.760 \text{ kpsi}$$

Eq. (8-36):

$$\sigma_m = \frac{C\left(P_{\max} + P_{\min}\right)}{2A_t} + \sigma_i$$

$$= \frac{0.299\left(1.443 + 0.722\right)}{2\left(0.141\,9\right)} + 63.75 = 66.03 \text{ kpsi}$$

Eq. (8-38):

$$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut}\sigma_a + S_e\left(\sigma_m - \sigma_i\right)} = \frac{18.8\left(120 - 63.75\right)}{120\left(0.760\right) + 18.8\left(66.03 - 63.75\right)} = 7.89 \qquad \textit{Ans.}$$

8-58 From Prob. 8-54, $C = 0.228$, $S_e = 162$ MPa, $S_{ut} = 1040$ MPa, $A_t = 58.0$ mm^2, $\sigma_i = 622.5$ MPa, and $P_{\max} = 7.679$ kN.

$$P_{\min} = P_{\max} / 2 = 7.679/2 = 3.840 \text{ kN}$$

Eq. (8-35): $$\sigma_a = \frac{C\left(P_{\max} - P_{\min}\right)}{2A_t} = \frac{0.228\left(7.679 - 3.840\right)10^3}{2\left(58.0\right)} = 7.546 \text{ MPa}$$

Eq. (8-36):

$$\sigma_m = \frac{C\left(P_{\max} + P_{\min}\right)}{2A_t} + \sigma_i$$

$$= \frac{0.228\left(7.679 + 3.840\right)10^3}{2\left(58.0\right)} + 622.5 = 645.1 \text{ MPa}$$

Eq. (8-38):

$$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut}\sigma_a + S_e\left(\sigma_m - \sigma_i\right)} = \frac{162\left(1040 - 622.5\right)}{1040\left(7.546\right) + 162\left(645.1 - 622.5\right)} = 5.88 \qquad Ans.$$

8-59 From Prob. 8-55, $C = 0.291$, $S_e = 23.2$ kpsi, $S_{ut} = 150$ kpsi, $A_t = 0.106\ 3$ in^2, $\sigma_i = 90$ kpsi, and $P_{\max} = 1.244$ kips

$$P_{\min} = P_{\max} / 2 = 1.244/2 = 0.622 \text{ kips}$$

Eq. (8-35):
$$\sigma_a = \frac{C\left(P_{\max} - P_{\min}\right)}{2A_t} = \frac{0.291\left(1.244 - 0.622\right)}{2\left(0.106\ 3\right)} = 0.851 \text{ kpsi}$$

Eq. (8-36):
$$\sigma_m = \frac{C\left(P_{\max} + P_{\min}\right)}{2A_t} + \sigma_i$$

$$= \frac{0.291\left(1.244 + 0.622\right)}{2\left(0.106\ 3\right)} + 90 = 92.55 \text{ kpsi}$$

Eq. (8-38):
$$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut}\sigma_a + S_e\left(\sigma_m - \sigma_i\right)} = \frac{23.2\left(150 - 90\right)}{150\left(0.851\right) + 23.2\left(92.55 - 90\right)} = 7.45 \qquad Ans.$$

8-60 Let the repeatedly-applied load be designated as P. From Table A-22, $S_{ut} = 93.7$ kpsi. Referring to the Figure of Prob. 3-136, the following notation will be used for the radii of Section AA.

$$r_i = 1.5 \text{ in}, \quad r_o = 2.5 \text{ in}, \quad r_c = 2.0 \text{ in}$$

From Table 3-4, with $R = 0.5$ in,

$$r_n = \frac{R^2}{2\left(r_c - \sqrt{r_c^2 - R^2}\right)} = \frac{0.5^2}{2\left(2 - \sqrt{2^2 - 0.5^2}\right)} = 1.968\ 246 \text{ in}$$

$$e = r_c - r_n = 2.0 - 1.968\ 246 = 0.031\ 754 \text{ in}$$

$$c_o = r_o - r_n = 2.5 - 1.968\ 246 = 0.531\ 754 \text{ in}$$

$$c_i = r_n - r_i = 1.968\ 246 - 1.5 = 0.468\ 246 \text{ in}$$

$$A = \pi(1^2) / 4 = 0.7854 \text{ in}^2$$

If P is the maximum load

$$M = Pr_c = 2P$$

$$\sigma_i = \frac{P}{A}\left(1 + \frac{r_c c_i}{e r_i}\right) = \frac{P}{0.785\ 4}\left(1 + \frac{2(0.468)}{0.031\ 754(1.5)}\right) = 26.29P$$

$$\sigma_a = \sigma_m = \frac{\sigma_i}{2} = \frac{26.294P}{2} = 13.15P$$

(**a**) *Eye:* Section AA,

Table 6-2: $a = 11.0$ kpsi, $b = -0.650$

Eq. (6-18): $k_a = 11.0(93.7)^{-0.650} = 0.575$

Eq. (6-23): $d_e = 0.370\, d$

Eq. (6-19): $k_b = \left(\frac{0.37}{0.30}\right)^{-0.107} = 0.978$

Eq. (6-25): $k_c = 0.85$

Eq. (6-10): $S'_e = 0.5S_{ut} = 0.5(93.7) = 46.85$ kpsi

Eq. (6-18): $S_e = 0.575(0.978)0.85(46.85) = 22.4$ kpsi

From Eq. 6-48, for Gerber,

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1+\sqrt{1+\left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

With $\sigma_m = \sigma_a$,

$$n_f = \frac{1}{2}\frac{S_{ut}^2}{\sigma_a S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{S_{ut}}\right)^2}\right] = \frac{1}{2}\frac{93.7^2}{13.15P(22.4)}\left[-1 + \sqrt{1 + \left(\frac{2(22.4)}{93.7}\right)^2}\right] = \frac{1.616}{P}$$

where P is in kips.

Thread: Die cut. Table 8-17 gives $S_e = 18.6$ kpsi for rolled threads. Use Table 8-16 to find S_e for die cut threads

$$S_e = 18.6(3.0/3.8) = 14.7 \text{ kpsi}$$

Table 8-2, $A_t = 0.663$ in^2, $\sigma = P/A_t = P/0.663 = 1.51\,P$, $\sigma_a = \sigma_m = \sigma/2 = 0.755\,P$

From Eq. 6-48, Gerber

$$n_f = \frac{1}{2}\frac{S_{ut}^2}{\sigma_a S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{S_{ut}}\right)^2}\right] = \frac{1}{2}\frac{93.7^2}{0.755P(14.7)}\left[-1 + \sqrt{1 + \left(\frac{2(14.7)}{93.7}\right)^2}\right] = \frac{19.01}{P}$$

Comparing 19.01/*P* with 1.616/*P*, we conclude that the *eye* is weaker in fatigue. *Ans.*

(**b**) Strengthening steps can include heat treatment, cold forming, cross section change (a round section is a poor cross section for a curved bar in bending because the bulk of the material is located where the stress is small). *Ans.*

(**c**) For $n_f = 2$

$$P = \frac{1.616\left(10^3\right)}{2} = 808 \text{ lbf, max. load} \quad \textit{Ans.}$$

8-61 Member, Eq. (8-22) with E =16 Mpsi, d = 0.75 in, and l = 1.5 in

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\dfrac{0.5774l+0.5d}{0.5774l+2.5d}\right)} = \frac{0.5774\pi(16)0.75}{2\ln\left[5\dfrac{0.5774(1.5)+0.5(0.75)}{0.5774(1.5)+2.5(0.75)}\right]} = 13.32 \text{ Mlbf/in}$$

Bolt, Eq. (8-13),

$$L_T = 2d + 0.25 = 2(0.75) + 0.25 = 1.75 \text{ in}$$

$$l = 1.5 \text{ in}$$

$$l_d = L - L_T = 2.5 - 1.75 = 0.75 \text{ in}$$

$$l_t = l - l_d = 1.5 - 0.75 = 0.75 \text{ in}$$

Table 8-2,

$$A_t = 0.373 \text{ in}^2$$

$$A_d = \pi(0.75^2)/4 = 0.442 \text{ in}^2$$

Eq. (8-17),

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.442(0.373)30}{0.442(0.75)+0.373(0.75)} = 8.09 \text{ Mlbf/in}$$

$$C = \frac{k_b}{k_b + k_m} = \frac{8.09}{8.09+13.32} = 0.378$$

Eq. (8-35),

$$\sigma_a = \frac{C(P_{\max} - P_{\min})}{2A_t} = \frac{0.378(6-4)}{2(0.373)} = 1.013 \text{ kpsi}$$

Eq.(8-36),

$$\sigma_m = \frac{C(P_{\max} + P_{\min})}{2A_t} + \frac{F_i}{A_t} = \frac{0.378(6+4)}{2(0.373)} + \frac{25}{0.373} = 72.09 \text{ kpsi}$$

(**a**) From Table 8-9, S_p = 85 kpsi, and Eq. (8-51), the yielding factor of safety is

$$n_p = \frac{S_p}{\sigma_m + \sigma_a} = \frac{85}{72.09+1.013} = 1.16 \qquad \textit{Ans.}$$

(**b**) From Eq. (8-29), the overload factor of safety is

$$n_L = \frac{S_p A_t - F_i}{CP_{\max}} = \frac{85(0.373) - 25}{0.378(6)} = 2.96 \quad Ans.$$

(**c**) From Eq. (8-30), the factor of safety based on joint separation is

$$n_0 = \frac{F_i}{P_{\max}(1 - C)} = \frac{25}{6(1 - 0.378)} = 6.70 \quad Ans.$$

(**d**) From Table 8-17, S_e = 18.6 kpsi; Table 8-9, S_{ut} = 120 kps; the preload stress is $\sigma_i = F_i / A_t$ = 25/0.373 = 67.0 kpsi; and from Eq. (8-38)

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)} = \frac{18.6(120 - 67.0)}{120(1.013) + 18.6(72.09 - 67.0)} = 4.56 \quad Ans.$$

8-62 (**a**) Table 8-2, $A_t = 0.1419\ \text{in}^2$
Table 8-9, S_p = 120 kpsi, S_{ut} = 150 kpsi
Table 8-17, S_e = 23.2 kpsi
Eqs. (8-31) and (8-32), $\sigma_i = 0.75\ S_p = 0.75(120) = 90$ kpsi

$$C = \frac{k_b}{k_b + k_m} = \frac{4}{4 + 16} = 0.2$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.2P}{2(0.141\,9)} = 0.705P \text{ kpsi}$$

Eq. (8-45) for the Goodman criterion,

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{\sigma_a(S_{ut} + S_e)} = \frac{23.2(150 - 90)}{0.705P(150 + 23.2)} = \frac{11.4}{P} = 2 \quad \Rightarrow \quad P = 5.70 \text{ kips} \quad Ans.$$

(**b**) $F_i = 0.75 A_t S_p = 0.75(0.141\,9)120 = 12.77$ kips
Yield, Eq. (8-28),

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{120(0.141\,9)}{0.2(5.70) + 12.77} = 1.22 \quad Ans.$$

Load factor, Eq. (8-29),

$$n_L = \frac{S_p A_t \text{ - } F_i}{CP} = \frac{120(0.141\,9) - 12.77}{0.2(5.70)} = 3.74 \quad Ans.$$

Separation load factor, Eq. (8-30)

$$n_0 = \frac{F_i}{P(1 - C)} = \frac{12.77}{5.70(1-0.2)} = 2.80 \quad \textit{Ans.}$$

8-63 Table 8-2, $A_t = 0.969 \text{ in}^2$ (coarse), $A_t = 1.073 \text{ in}^2$ (fine)
Table 8-9, $S_p = 74$ kpsi, $S_{ut} = 105$ kpsi
Table 8-17, $S_e = 16.3$ kpsi

Coarse thread,

$$F_i = 0.75\, A_t\, S_p = 0.75(0.969)74 = 53.78 \text{ kips}$$
$$\sigma_i = 0.75\, S_p = 0.75(74) = 55.5 \text{ kpsi}$$
$$\sigma_a = \frac{CP}{2A_t} = \frac{0.30P}{2(0.969)} = 0.155P \text{ kpsi}$$

Gerber, Eq. (8-46),

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e\left(S_e + \sigma_i\right)} - S_{ut}^2 - 2\sigma_i S_e\right]$$
$$= \frac{1}{2\left(0.155P\right)16.3}\left[105\sqrt{105^2 + 4\left(16.3\right)\left(16.3 + 55.5\right)} - 105^2 - 2\left(55.5\right)16.3\right] = \frac{64.28}{P}$$

With $n_f = 2$,

$$P = \frac{64.28}{2} = 32.14 \text{ kip} \quad \textit{Ans.}$$

Fine thread,

$$F_i = 0.75\, A_t\, S_p = 0.75(1.073)74 = 59.55 \text{kips}$$
$$\sigma_i = 0.75\, S_p = 0.75(74) = 55.5 \text{ kpsi}$$
$$\sigma_a = \frac{CP}{2A_t} = \frac{0.32P}{2(1.073)} = 0.149P \text{ kpsi}$$

The only thing that changes in Eq. (8-46) is σ_a. Thus,

$$n_f = \frac{0.155}{0.149}\frac{64.28}{P} = \frac{66.87}{P} = 2 \quad \Rightarrow \quad P = 33.43 \text{ kips} \qquad \textit{Ans.}$$

Percent improvement,

$$\frac{33.43 - 32.14}{32.14}(100) \approx 4\% \quad \textit{Ans.}$$

8-64 For an M 30 × 3.5 ISO 8.8 bolt with $P = 65$ kN/bolt and $C = 0.28$

Table 8-1, $A_t = 561 \text{ mm}^2$
Table 8-11, $S_p = 600$ MPa, $S_{ut} = 830$ MPa
Table 8-17, $S_e = 129$ MPa

Eq. (8-31), $F_i = 0.75F_p = 0.75\,A_t\,S_p$

$= 0.75(5610600(10^{-3}) = 252.45$ kN

$\sigma_i = 0.75\,S_p = 0.75(600) = 450$ MPa

Eq. (8-39), $\sigma_a = \dfrac{CP}{2A_t} = \dfrac{0.28(65)10^3}{2(561)} = 16.22$ MPa

Gerber, Eq. (8-46),

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e\left(S_e + \sigma_i\right)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(16.22)129}\left[830\sqrt{830^2 + 4(129)(129+450)} - 830^2 - 2(450)129\right]$$

$$= 4.75 \qquad Ans.$$

The yielding factor of safety, from Eq. (8-28) is

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{600(561)10^{-3}}{0.28(65) + 252.45} = 1.24 \qquad Ans.$$

From Eq. (8-29), the load factor is

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{600(561)10^{-3} - 252.45}{0.28(65)} = 4.62 \qquad Ans.$$

The separation factor, from Eq. (8-30) is

$$n_0 = \frac{F_i}{P(1-C)} = \frac{252.45}{65(1-0.28)} = 5.39 \qquad Ans.$$

8-65 (**a**) Table 8-2, $A_t = 0.077\,5$ in^2

Table 8-9, $S_p = 85$ kpsi, $S_{ut} = 120$ kpsi

Table 8-17, $S_e = 18.6$ kpsi

Unthreaded grip,

$$k_b = \frac{A_d E}{l} = \frac{\pi(0.375)^2(30)}{4(13.5)} = 0.245 \text{ Mlbf/in per bolt} \qquad Ans.$$

$$A_m = \frac{\pi}{4}[(D + 2t)^2 - D^2] = \frac{\pi}{4}(4.75^2 - 4^2) = 5.154 \text{ in}^2$$

$$k_m = \frac{A_m E}{l} = \frac{5.154(30)}{12}\left(\frac{1}{6}\right) = 2.148 \text{ Mlbf/in/bolt.} \qquad Ans.$$

(b) $F_i = 0.75\, A_t\, S_p = 0.75(0.0775)\,(85) = 4.94$ kip

$$\sigma_i = 0.75 S_p = 0.75(85) = 63.75 \text{ kpsi}$$

$$P = pA = \frac{2000}{6}\left[\frac{\pi}{4}(4)^2\right] = 4189 \text{ lbf/bolt}$$

$$C = \frac{k_b}{k_b + k_m} = \frac{0.245}{0.245 + 2.148} = 0.102$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.102(4.189)}{2(0.0775)} = 2.77 \text{ kpsi}$$

From Eq. (8-46) for Gerber fatigue criterion,

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e\left(S_e + \sigma_i\right)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2\left(2.77\right)18.6}\left[120\sqrt{120^2 + 4\left(18.6\right)\left(18.6 + 63.75\right)} - 120^2 - 2\left(63.75\right)18.6\right] = 4.09 \quad \textit{Ans.}$$

(**c**) Pressure causing joint separation from Eq. (8-30)

$$n_0 = \frac{F_i}{P(1 - C)} = 1$$

$$P = \frac{F_i}{1 - C} = \frac{4.94}{1 - 0.102} = 5.50 \text{ kip}$$

$$p = \frac{P}{A} = \frac{5.50}{\pi(4^2)\,/\,4}6 = 2.63 \text{ kpsi} \quad \textit{Ans.}$$

8-66 From the solution of Prob. 8-64, $A_t = 0.077\,5$ in^2, $S_{ut} = 120$ kpsi, $S_e = 18.6$ kpsi, $C = 0.102$, $\sigma_i = 63.75$ kpsi

$P_{\max} = p_{\max}A = 2\ \pi\,(4^2)/4 = 25.13$ kpsi, $P_{\min} = p_{\min}A = 1.2\ \pi\,(4^2)/4 = 15.08$ kpsi,

Eq. (8-35), $$\sigma_a = \frac{C\left(P_{\max} - P_{\min}\right)}{2A_t} = \frac{0.102\left(25.13 - 15.08\right)}{2\left(0.077\,5\right)} = 6.61 \text{ kpsi}$$

Eq. (8-36), $$\sigma_m = \frac{C\left(P_{\max} + P_{\min}\right)}{2A_t} + \sigma_i = \frac{0.102\left(25.13 + 15.08\right)}{2\left(0.077\,5\right)} + 63.75 = 90.21 \text{ kpsi}$$

Eq. (8-38),

$$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut}\sigma_a + S_e\left(\sigma_m - \sigma_i\right)} = \frac{18.6\left(120 - 63.75\right)}{120\left(6.61\right) + 18.6\left(90.21 - 63.75\right)} = 0.814 \qquad \textit{Ans.}$$

This predicts a fatigue failure.

8-67 Members: $S_y = 57$ kpsi, $S_{sy} = 0.577(57) = 32.89$ kpsi.

Bolts: SAE grade 5, $S_y = 92$ kpsi, $S_{sy} = 0.577(92) = 53.08$ kpsi

Shear in bolts,

$$A_s = 2\left[\frac{\pi(0.25^2)}{4}\right] = 0.0982\ \text{in}^2$$

$$F_s = \frac{A_s S_{sy}}{n} = \frac{0.0982(53.08)}{2} = 2.61\ \text{kips}$$

Bearing on bolts,

$$A_b = 2(0.25)0.25 = 0.125\ \text{in}^2$$

$$F_b = \frac{A_b S_{yc}}{n} = \frac{0.125(92)}{2} = 5.75\ \text{kips}$$

Bearing on member,

$$F_b = \frac{0.125(57)}{2} = 3.56\ \text{kips}$$

Tension of members,

$$A_t = (1.25 - 0.25)\ (0.25) = 0.25\ \text{in}^2$$

$$F_t = \frac{0.25(57)}{2} = 7.13\ \text{kip}$$

$$F = \min(2.61,\ 5.75,\ 3.56,\ 7.13) = 2.61\ \text{kip} \qquad \textit{Ans.}$$

The shear in the bolts controls the design.

8-68 Members, Table A-20, $S_y = 42$ kpsi
Bolts, Table 8-9, $S_y = 130$ kpsi, $S_{sy} = 0.577(130) = 75.01$ kpsi

Shear of bolts,

$$A_s = 2\left[\frac{\pi(5/16)^2}{4}\right] = 0.1534\ \text{in}^2$$

$$\tau = \frac{F_s}{A_s} = \frac{5}{0.1534} = 32.6\ \text{kpsi}$$

$$n = \frac{S_{sy}}{\tau} = \frac{75.01}{32.6} = 2.30 \qquad \textit{Ans.}$$

Bearing on bolts,

$$A_b = 2(0.25)\ (5/16) = 0.1563\ \text{in}^2$$

$$\sigma_b = -\frac{5}{0.1563} = -32.0\ \text{kpsi}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{130}{32.0} = 4.06 \qquad Ans.$$

Bearing on members,

$$n = \frac{S_y}{|\sigma_b|} = \frac{42}{32} = 1.31 \qquad Ans.$$

Tension of members,

$$A_t = [2.375 - 2(5/16)]\ (1/4) = 0.4375\ \text{in}^2$$

$$\sigma_t = \frac{5}{0.4375} = 11.4\ \text{kpsi}$$

$$n = \frac{S_y}{\sigma_t} = \frac{42}{11.4} = 3.68 \qquad Ans.$$

8-69 Members: Table A-20, $S_y = 490$ MPa, $S_{sy} = 0.577(490) = 282.7$ MPa
Bolts: Table 8-11, ISO class 5.8, $S_y = 420$ MPa, $S_{sy} = 0.577(420) = 242.3$ MPa

Shear in bolts,

$$A_s = 2\left[\frac{\pi(20^2)}{4}\right] = 628.3\ \text{mm}^2$$

$$F_s = \frac{A_s S_{sy}}{n} = \frac{628.3(242.3)10^{-3}}{2.5} = 60.9\ \text{kN}$$

Bearing on bolts,

$$A_b = 2(20)20 = 800\ \text{mm}^2$$

$$F_b = \frac{A_b S_{yc}}{n} = \frac{800(420)10^{-3}}{2.5} = 134\ \text{kN}$$

Bearing on member,

$$F_b = \frac{800(490)10^{-3}}{2.5} = 157\ \text{kN}$$

Tension of members,

$$A_t = (80 - 20)\ (20) = 1\ 200\ \text{mm}^2$$

$$F_t = \frac{1\ 200(490)10^{-3}}{2.5} = 235\ \text{kN}$$

$$F = \min(60.9,\ 134,\ 157,\ 235) = 60.9\ \text{kN} \qquad Ans.$$

The shear in the bolts controls the design.

8-70 Members: Table A-20, $S_y = 320$ MPa
Bolts: Table 8-11, ISO class 5.8, $S_y = 420$ MPa, $S_{sy} = 0.577(420) = 242.3$ MPa

Shear of bolts,

$$A_s = \pi(20^2)/4 = 314.2\ \text{mm}^2$$

$$\tau_s = \frac{90(10^3)}{3(314.2)} = 95.48\ \text{MPa}$$

$$n = \frac{S_{sy}}{\tau_s} = \frac{242.3}{95.48} = 2.54 \quad \textit{Ans.}$$

Bearing on bolt,

$$A_b = 3(20)15 = 900\ \text{mm}^2$$

$$\sigma_b = -\frac{90(10^3)}{900} = -100\ \text{MPa}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{420}{100} = 4.2 \quad \textit{Ans.}$$

Bearing on members,

$$n = \frac{S_y}{|\sigma_b|} = \frac{320}{100} = 3.2 \quad \textit{Ans.}$$

Tension on members,

$$\sigma_t = \frac{F}{A} = \frac{90(10^3)}{15[190 - 3(20)]} = 46.15\ \text{MPa}$$

$$n = \frac{S_y}{\sigma_t} = \frac{320}{46.15} = 6.93 \quad \textit{Ans.}$$

8-71 Members: $S_y = 57$ kpsi
Bolts: $S_y = 100$ kpsi, $S_{sy} = 0.577(100) = 57.7$ kpsi
Shear of bolts,

$$A = 3\left[\frac{\pi(1/4)^2}{4}\right] = 0.1473\ \text{in}^2$$

$$\tau_s = \frac{F}{A_s} = \frac{5}{0.1473} = 33.94\ \text{kpsi}$$

$$n = \frac{S_{sy}}{\tau_s} = \frac{57.7}{33.94} = 1.70 \qquad \textit{Ans.}$$

Bearing on bolts,

$$A_b = 3(1/4)(5/16) = 0.2344\ \text{in}^2$$

$$\sigma_b = -\frac{F}{A_b} = -\frac{5}{0.2344} = -21.3 \text{ kpsi}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{100}{21.3} = 4.69 \qquad Ans.$$

Bearing on members,

$A_b = 0.2344$ in^2 (From bearing on bolts calculation)

$\sigma_b = -21.3$ kpsi (From bearing on bolts calculation)

$$n = \frac{S_y}{|\sigma_b|} = \frac{57}{21.3} = 2.68 \qquad Ans.$$

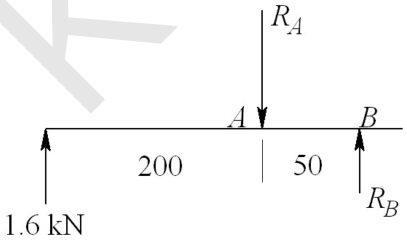
Tension in members, failure across two bolts,

$$A_t = \frac{5}{16}\left[2.375 - 2(1/4)\right] = 0.5859 \text{ in}^2$$

$$\sigma_t = \frac{F}{A_t} = \frac{5}{0.5859} = 8.534 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma_t} = \frac{57}{8.534} = 6.68 \qquad Ans.$$

8-72 By symmetry, the reactions at each support is 1.6 kN. The free-body diagram for the left member is



$$\sum M_B = 0 \quad 1.6(250) - 50R_A = 0 \quad \Rightarrow \quad R_A = 8 \text{ kN}$$
$$\sum M_A = 0 \quad 200(1.6) - 50R_B = 0 \quad \Rightarrow \quad R_B = 6.4 \text{ kN}$$

Members: Table A-20, $S_y = 370$ MPa
Bolts: Table 8-11, $S_y = 420$ MPa, $S_{sy} = 0.577(420) = 242.3$ MPa

Bolt shear, $A_s = \frac{\pi}{4}(12^2) = 113.1 \text{ mm}^2$

$$\tau = \frac{F_{\max}}{A_s} = \frac{8(10^3)}{113.1} = 70.73 \text{ MPa}$$

$$n = \frac{S_{sy}}{\tau} = \frac{242.3}{70.73} = 3.43$$

Bearing on member, $A_b = td = 10(12) = 120 \text{ mm}^2$

$$\sigma_b = -\frac{8(10^3)}{120} = -66.67 \text{ MPa}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{370}{66.67} = 5.55$$

Strength of member. The bending moments at the hole locations are:
In the left member at A, $M_A = 1.6(200) = 320$ N · m. In the right member at B, $M_B = 8(50) = 400$ N · m. The bending moment is greater at B

$$I_B = \frac{1}{12}[10(50^3) - 10(12^3)] = 102.7(10^3) \text{ mm}^4$$

$$\sigma_B = \frac{M_A c}{I_A} = \frac{400(25)}{102.7(10^3)}(10^3) = 97.37 \text{ MPa}$$

$$n = \frac{S_y}{\sigma_A} = \frac{370}{97.37} = 3.80$$

At the center, call it point C,

$$M_C = 1.6(350) = 560 \text{ N} \cdot \text{m}$$

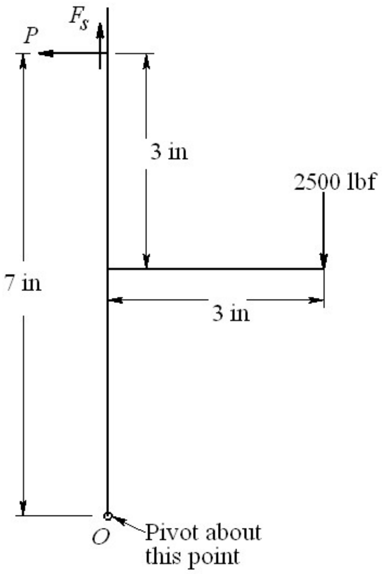
$$I_C = \frac{1}{12}(10)(50^3) = 104.2(10^3) \text{ mm}^4$$

$$\sigma_C = \frac{M_C c}{I_C} = \frac{560(25)}{104.2(10^3)}(10^3) = 134.4 \text{ MPa}$$

$$n = \frac{S_y}{\sigma_C} = \frac{370}{134.4} = 2.75 < 3.80 \quad \text{more critical at } C$$

$$n = \min(3.04,\ 3.80,\ 2.75) = 2.72 \qquad \textit{Ans.}$$

8-73 The free-body diagram of the bracket, assuming the upper bolt takes all the shear and tensile load is



$$F_s = 2500 \text{ lbf}$$

$$P = \frac{2500(3)}{7} = 1071 \text{ lbf}$$

Table A-31, $H = 7/16 = 0.4375$ in. Grip, $l = 2(1/2) = 1$ in. $L \geq l + H = 1.4375$ in. Use 1.5 in bolts.

Eq. (8-13), $L_T = 2d + 0.25 = 2(0.5) + 0.25 = 1.25$ in

Table 8-7, $l_d = L - L_T = 1.5 - 1.25 = 0.25$ in

$l_t = l - l_d = 1 - 0.25 = 0.75$ in

Table 8-2, $A_t = 0.141\ 9 \text{ in}^2$

$A_d = \pi(0.5^2)/4 = 0.196\ 3 \text{ in}^2$

Eq. (8-17), $$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.196\ 3(0.141\ 9)30}{0.196\ 3(0.75) + 0.141\ 9(0.25)} = 4.574 \text{ Mlbf/in}$$

Eq. (8-22),

$$k_m = \frac{0.5774\pi E d}{2\ln\left(5\dfrac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)} = \frac{0.5774\pi(30)0.5}{2\ln\left(5\dfrac{0.5774(1) + 0.5(0.5)}{0.5774(1) + 2.5(0.5)}\right)} = 16.65 \text{ Mlbf/in}$$

$$C = \frac{k_b}{k_b + k_m} = \frac{4.574}{4.574 + 16.65} = 0.216$$

Table 8-9, $S_p = 65$ kpsi

Eqs. (8-31) and (8-32), $F_i = 0.75\ A_t\ S_p = 0.75(0.141\ 9)65 = 6.918$ kips

$\sigma_i = 0.75\ S_p = 0.75(65) = 48.75$ kips

Eq. (*a*): $$\sigma_b = \frac{CP + F_i}{A_t} = \frac{0.216(1.071) + 6.918}{0.141\ 9} = 50.38 \text{ kpsi}$$

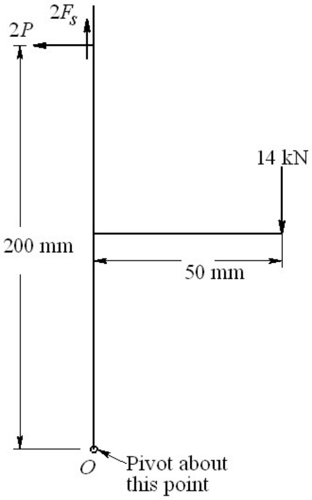
Direct shear, $$\tau_s \approx \frac{F_s}{A_t} = \frac{3}{0.141\ 9} = 21.14 \text{ kpsi}$$

von Mises stress, Eq. (5-15),

$$\sigma' = \left(\sigma_b^2 + 3\tau_s^2\right)^{1/2} = \left[50.38^2 + 3\left(21.14^2\right)\right]^{1/2} = 62.3 \text{ kpsi}$$

$$n = \frac{S_p}{\sigma'} = \frac{65}{62.3} = 1.04 \qquad \textit{Ans.}$$

8-74



$$\begin{aligned} 2P(200) &= 14(50) \\ P &= \frac{14(50)}{2(200)} = 1.75 \text{ kN per bolt} \\ F_s &= 7 \text{ kN/bolt} \\ S_p &= 380 \text{ MPa} \\ A_t &= 245 \text{ mm}^2,\ A_d = \frac{\pi}{4}(20^2) = 314.2 \text{ mm}^2 \\ F_i &= 0.75(245)(380)(10^{-3}) = 69.83 \text{ kN} \\ \sigma_i &= 0.75\left(380\right) = 285 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \sigma_b &= \frac{CP + F_i}{A_t} = \left(\frac{0.25(1.75) + 69.83}{245}\right)(10^3) = 287 \text{ MPa} \\ \tau &= \frac{F_s}{A_d} = \frac{7(10^3)}{314.2} = 22.3 \text{ MPa} \\ \sigma' &= [287^2 + 3(22.3^2)]^{1/2} = 290 \text{ MPa} \end{aligned}$$

$$n = \frac{S_p}{\sigma'} = \frac{380}{290} = 1.31 \qquad \textit{Ans.}$$

8-75 Using the result of Prob. 5-80 for lubricated assembly (replace 0.2 with 0.18 per Table 8-15)

$$F_x = \frac{2\pi f\, T}{0.18d}$$

With a design factor of n_d gives

$$T = \frac{0.18 n_d\, F_x d}{2\pi f} = \frac{0.18(3)(1000)d}{2\pi(0.12)} = 716d$$

or $T/d = 716$. Also,

$$\begin{aligned}\frac{T}{d} &= K(0.75S_pA_t)\\ &= 0.18(0.75)(85\ 000)A_t\\ &= 11\ 475A_t\end{aligned}$$

Form a table

Size	A_t	$T/d = 11\ 475A_t$	n
$\frac{1}{4}$ - 28	0.0364	417.70	1.75
$\frac{5}{16}$ - 24	0.058	665.55	2.8
$\frac{3}{8}$ – 24	0.0878	1007.50	4.23

where the factor of safety in the last column of the table comes from

$$n = \frac{2\pi f(T/d)}{0.18F_x} = \frac{2\pi(0.12)(T/d)}{0.18(1000)} = 0.0042(T/d)$$

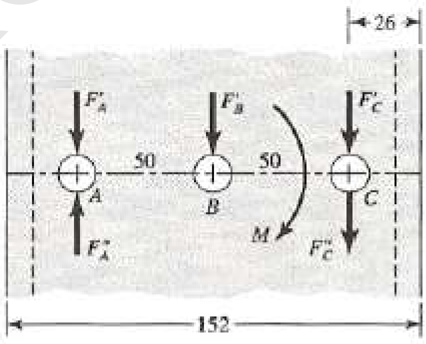
Select a $\frac{3}{8}''$ - 24 UNF cap screw. The setting is given by

$$T = (11\ 475A_t)d = 1007.5(0.375) = 378 \text{ lbf} \cdot \text{in}$$

Given the coarse scale on a torque wrench, specify a torque wrench setting of 400 lbf · in. Check the factor of safety

$$n = \frac{2\pi f\ T}{0.18F_xd} = \frac{2\pi(0.12)(400)}{0.18(1000)(0.375)} = 4.47$$

8-76



Bolts, from Table 8-11, $S_y = 420$ MPa
Channel, From Table A-20, $S_y = 170$ MPa. From Table A-7, $t = 6.4$ mm
Cantilever, from Table A-20, $S_y = 190$ MPa

$$F'_A = F'_B = F'_C = F/3$$

$$M = (50 + 26 + 125)\ F\ =\ 201\ F$$

$$F''_A = F''_C = \frac{201F}{2(50)} = 2.01F$$

Max. force, $$F_C = F'_C + F''_C = \left(\frac{1}{3} + 2.01\right)F = 2.343F \qquad (1)$$

Shear on Bolts: The shoulder bolt shear area, $A_s = \pi(10^2) / 4 = 78.54$ mm^2

$$S_{sy} = 0.577(420)\ =\ 242.3\ \text{KPa}$$

$$\tau_{\max} = \frac{F_C}{A_s} = \frac{S_{sy}}{n}$$

From Eq. (1), $F_C = 2.343\ F$. Thus

$$F = \frac{S_{sy}}{n}\left(\frac{A_s}{2.343}\right) = \frac{242.3}{2.0}\left(\frac{78.54}{2.343}\right)10^{-3} = 4.06\ \text{kN}$$

Bearing on bolt: The bearing area is $A_b = td = 6.4(10) = 64$ mm^2. Similar to shear

$$F = \frac{S_y}{n}\left(\frac{A_b}{2.343}\right) = \frac{420}{2.0}\left(\frac{64}{2.343}\right)10^{-3} = 5.74\ \text{kN}$$

Bearing on channel: $A_b = 64$ mm^2, $S_y = 170$ MPa.

$$F = \frac{S_y}{n}\left(\frac{A_b}{2.343}\right) = \frac{170}{2.0}\left(\frac{64}{2.343}\right)10^{-3} = 2.32\ \text{kN}$$

Bearing on cantilever: $A_b = 12(10) = 120$ mm^2, $S_y = 190$ MPa.

$$F = \frac{S_y}{n}\left(\frac{A_b}{2.343}\right) = \frac{190}{2.0}\left(\frac{120}{2.343}\right)10^{-3} = 4.87\ \text{kN}$$

Bending of cantilever: At C

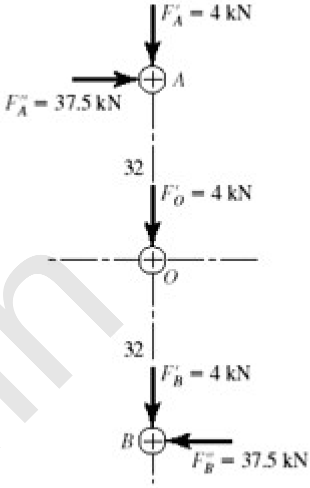
$$I = \frac{1}{12}(12)\left(50^3 - 10^3\right) = 1.24\left(10^5\right)\ \text{mm}^4$$

$$\sigma_{\max} = \frac{S_y}{n} = \frac{Mc}{I} = \frac{151Fc}{I} \quad \Rightarrow \quad F = \frac{S_y}{n}\left(\frac{I}{151c}\right)$$

$$F = \frac{190}{2.0}\left[\frac{1.24\left(10^5\right)}{151\left(25\right)}\right]10^{-3} = 3.12 \text{ kN}$$

So $F = 2.32$ kN based on bearing on channel. *Ans.*

8-77 Bolts, from Table 8-11, $S_y = 420$ MPa
Bracket, from Table A-20, $S_y = 210$ MPa



$$\begin{aligned} F' &= \frac{12}{3} = 4 \text{ kN; } M = 12(200) = 2400 \text{ N} \cdot \text{m} \\ F''_A &= F''_B = \frac{2400}{64} = 37.5 \text{ kN} \\ F_A &= F_B = \sqrt{(4)^2 + (37.5)^2} = 37.7 \text{ kN} \\ F_O &= 4 \text{ kN} \end{aligned}$$

Bolt shear:
The shoulder bolt shear area, $A_s = \pi(12^2) / 4 = 113.1$ mm^2

$$S_{sy} = 0.577(420) = 242.3 \text{ KPa}$$

$$\begin{aligned} \tau &= \frac{37.7(10)^3}{113} = 333 \text{ MPa} \\ n &= \frac{S_{sy}}{\tau} = \frac{242.3}{333} = 0.728 \qquad \textit{Ans.} \end{aligned}$$

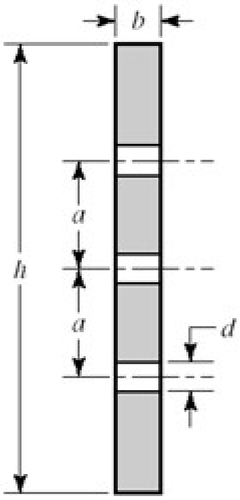
Bearing on bolts:

$$\begin{aligned} A_b &= 12(8) = 96 \text{ mm}^2 \\ \sigma_b &= -\frac{37.7(10)^3}{96} = -393 \text{ MPa} \\ n &= \frac{S_{yc}}{|\sigma_b|} = \frac{420}{393} = 1.07 \qquad \textit{Ans.} \end{aligned}$$

Bearing on member:

$$\begin{aligned} \sigma_b &= -393 \text{ MPa} \\ n &= \frac{S_{yc}}{|\sigma_b|} = \frac{210}{393} = 0.534 \qquad \textit{Ans.} \end{aligned}$$

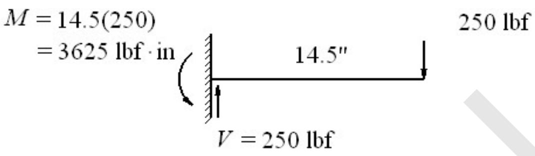
Bending stress in plate:

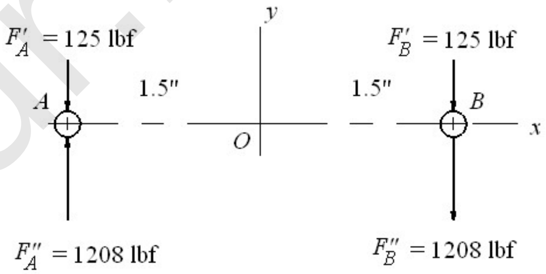


$$I = \frac{bh^3}{12} - \frac{bd^3}{12} - 2\left(\frac{bd^3}{12} + a^2bd\right)$$
$$= \frac{8(136)^3}{12} - \frac{8(12)^3}{12} - 2\left[\frac{8(12)^3}{12} + (32)^2(8)(12)\right]$$
$$= 1.48(10)^6 \text{ mm}^4 \quad \textit{Ans.}$$
$$\sigma = \frac{Mc}{I} = \frac{2400(68)}{1.48(10)^6}(10)^3 = 110 \text{ MPa}$$
$$n = \frac{S_y}{\sigma} = \frac{210}{110} = 1.91 \qquad \textit{Ans.}$$

Failure is predicted for bolt shear and bearing on member.

8-78





$$F_A'' = F_B'' = \left(\frac{3625}{3}\right) = 1208 \text{ lbf}$$

$$F_A = 1208 - 125 = 1083 \text{ lbf}, \qquad F_B = 1208 + 125 = 1333 \text{ lbf}$$

Bolt shear:

$$A_s = (\pi/4)(0.375^2) = 0.1104 \text{ in}^2$$

$$\tau_{\max} = \frac{F_{\max}}{A_s} = \frac{1333}{0.1104} = 12\ 070 \text{ psi}$$

From Table 8-10, $S_y = 100$ kpsi, $S_{sy} = 0.577(100) = 57.7$ kpsi

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{57.7}{12.07} = 4.78 \qquad \textit{Ans.}$$

Bearing on bolt: Bearing area is $A_b = td = 0.375\ (0.375) = 0.1406$ in^2.

$$\sigma_b = -\frac{F}{A_b} = -\frac{1333}{0.1406} = -9\ 481 \text{ psi}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{100}{9.481} = 10.55 \qquad Ans.$$

Bearing on member: From Table A-20, $S_y = 54$ kpsi. Bearing stress same as bolt

$$n = \frac{S_y}{|\sigma_b|} = \frac{54}{9.481} = 5.70 \qquad Ans.$$

Bending of member: At B, $M = 250(13) = 3250$ lbf·in

$$I = \frac{1}{12}\left(\frac{3}{8}\right)\left[2^3 - \left(\frac{3}{8}\right)^3\right] = 0.2484 \text{ in}^4$$

$$\sigma = \frac{Mc}{I} = \frac{3250(1)}{0.2484} = 13\ 080 \text{ psi}$$

$$n = \frac{S_y}{\sigma} = \frac{54}{13.08} = 4.13 \qquad Ans.$$

8-79 The direct shear load per bolt is $F' = 2000/6 = 333.3$ lbf. The moment is taken only by the four outside bolts. This moment is $M = 2000(5) = 10\ 000$ lbf · in.

Thus $F'' = \dfrac{10\ 000}{2(5)} = 1000$ lbf and the resultant bolt load is

$$F = \sqrt{(333.3)^2 + (1000)^2} = 1054 \text{ lbf}$$

Bolt strength, Table 8-9, $S_y = 100$ kpsi; Channel and Plate strength, $S_y = 42$ kpsi

Shear of bolt: $A_s = \pi(0.5)^2/4 = 0.1963$ in^2

$$n = \frac{S_{sy}}{\tau} = \frac{(0.577)(100)}{1.054\ /\ 0.1963} = 10.7 \qquad Ans.$$

Bearing on bolt: Channel thickness is $t = 3/16$ in, $A_b = 0.5(3/16) = 0.09375$ in^2

$$n = \frac{100}{1.054\ /\ 0.09375} = 8.89 \qquad Ans.$$

Bearing on channel: $n = \dfrac{42}{1.054\ /\ 0.09375} = 3.74$ *Ans.*

Bearing on plate: $A_b = 0.5(0.25) = 0.125\ \text{in}^2$

$$n = \frac{42}{1.054\ /\ 0.125} = 4.98 \quad \textit{Ans.}$$

Strength of plate:

$$I = \frac{0.25(7.5)^3}{12} - \frac{0.25(0.5)^3}{12} - 2\left[\frac{0.25(0.5)^3}{12} + (0.25)(0.5)(2.5)^2\right] = 7.219\ \text{in}^4$$

$$M = 5000\ \text{lbf} \cdot \text{in per plate}$$

$$\sigma = \frac{Mc}{I} = \frac{5000(3.75)}{7.219} = 2597\ \text{psi}$$

$$n = \frac{42}{2.597} = 16.2 \quad \textit{Ans.}$$

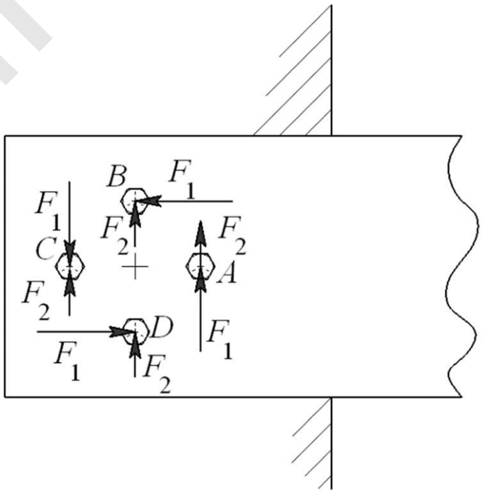
8-80 $A = 58\ \text{mm}^2$. From Table A-9, beam 14,
$R_1 = R_2 = F/2 = 10/2 = 5\ \text{kN}$,
$M_1 = M_2 = Fl/8 = 10(0.4)10^3/8 = 500\ \text{N}\cdot\text{m}$.

$$M_1 = 500 = 4F_1\left(\frac{0.050}{2}\right) \quad \Rightarrow \quad F_1 = 5000\ \text{N}$$

$$F_2 = \frac{R_1}{4} = \frac{5000}{4} = 1250\ \text{N}$$

Bolt A has the maximum force of
$F_A = F_1 + F_2 = 5000 + 1250 = 6250\ \text{N}$

$$\tau_{\max} = \tau_A = \frac{F_A}{A} = \frac{6250}{58} = 108\ \text{MPa} \qquad \textit{Ans.}$$



8-81 Given: 250 hp at 600 rev/min, $r_1 = 2.5$ in, $r_2 = 5$ in.

Eq. (3-42): $T = \dfrac{63\,025(250)}{600} = 26.26(10^3)\ \text{lbf}\cdot\text{in}$

(a) $\dfrac{F_1}{r_1} = \dfrac{F_2}{r_2} \quad \Rightarrow \quad F_2 = \dfrac{r_2}{r_1}F_1 = \dfrac{5}{2.5}F_1 = 2F_1$

$4F_1r_1 + 4F_2r_2 = T \quad \Rightarrow \quad 4F_1(2.5) + 4(2F_1)5 = 26.26(10^3)$

Yields $F_1 = 525.2$ lbf and $F_2 = 1050.4$ lbf

$$\tau_{\max} = \frac{4F_2}{\pi d^2} \quad \Rightarrow \quad d = \sqrt{\frac{4F_2}{\pi \tau_{\max}}} = \sqrt{\frac{4(1050.4)}{\pi(20)10^3}} = 0.258\ \text{in}$$

From Table A-17 select $d = \frac{5}{16}$ in *Ans.*

(b) $F = \dfrac{\tau_{\max} \pi d^2}{4} = \dfrac{20\left(10^3\right)\pi\left(\frac{5}{16}\right)^2}{4} = 1534 \text{ lbf}$

$nFr_2 = T \quad \Rightarrow \quad n\,(1534)\,5 = 26.26\,(10^3) \quad \Rightarrow \quad n = 3.4$ bolts

Use four bolts without need of the inner bolts *Ans.*

8-82 to 8-84 Specifying bolts, screws, dowels and rivets is the way a student learns about such components. However, choosing an array a priori is based on experience. Here is a chance for students to build some experience.

8-85 Assume tensile forces to be proportional to deflections. Thus,

$$\frac{F_1}{1} = \frac{F_2}{9} \quad \Rightarrow \quad F_2 = 9F_1 \qquad (1)$$

$\Sigma M_A = 0 = 2\,F_1\,(1) + 2\,F_2\,(9) - 1000\,(10)$

Substitute Eq. (1) in,

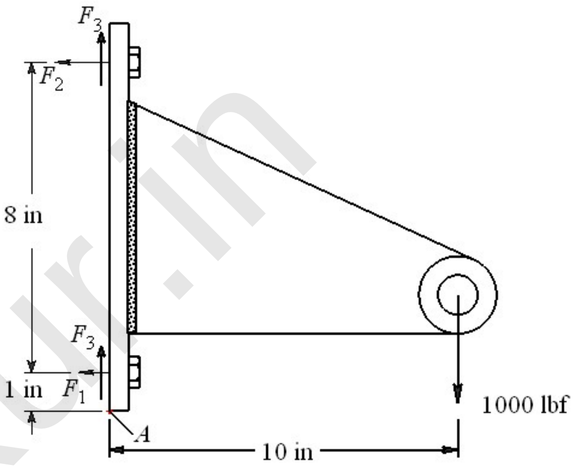
$2\,F_1\,(1) + 2\,(9\,F_1\,)\,(9) - 1000\,(10) = 0$

Yields $F_1 = 60.98$ lbf, $F_2 = 548.8$ lbf.

Combining the preload,

$$\sigma = \frac{5000 + 548}{0.2} = 27.7\left(10^3\right) \text{ psi} = 27.7 \text{ kpsi}$$

The shear forces are $F_3 = 1000/4 = 250$ lbf.



The shear stress is,

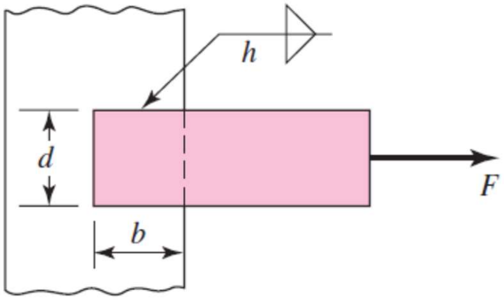
$$\tau = \frac{F_3}{A} = \frac{250}{0.2} = 1250 \text{ psi} = 1.25 \text{ kpsi}$$

The maximum tensile stress is,

$$\sigma_{\max} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{27.7}{2} + \sqrt{\left(\frac{27.7}{2}\right)^2 + 1.25^2} = 27.8 \text{ kpsi} \qquad Ans.$$

Chapter 9

Figure for Probs. 9-1 to 9-4



9-1 Given, $b = 50$ mm, $d = 50$ mm, $h = 5$ mm, $\tau_{\text{allow}} = 140$ MPa.

$$F = 0.707\, hl\tau_{\text{allow}} = 0.707(5)[2(50)](140)(10^{-3}) = 49.5 \text{ kN} \qquad \textit{Ans.}$$

9-2 Given, $b = 2$ in, $d = 2$ in, $h = 5/16$ in, $\tau_{\text{allow}} = 25$ kpsi.

$$F = 0.707\, hl\tau_{\text{allow}} = 0.707(5/16)[2(2)](25) = 22.1 \text{ kip} \qquad \textit{Ans.}$$

9-3 Given, $b = 50$ mm, $d = 30$ mm, $h = 5$ mm, $\tau_{\text{allow}} = 140$ MPa.

$$F = 0.707\, hl\tau_{\text{allow}} = 0.707(5)[2(50)](140)(10^{-3}) = 49.5 \text{ kN} \qquad \textit{Ans.}$$

9-4 Given, $b = 4$ in, $d = 2$ in, $h = 5/16$ in, $\tau_{\text{allow}} = 25$ kpsi.

$$F = 0.707\, hl\tau_{\text{allow}} = 0.707(5/16)[2(4)](25) = 44.2 \text{ kip} \qquad \textit{Ans.}$$

9-5 Prob. 9-1 with E7010 Electrode.

Table 9-6: $f = 14.85\, h$ kip/in $= 14.85\, [5 \text{ mm}/(25.4 \text{ mm/in})] = 2.923$ kip/in

$= 2.923(4.45/25.4) = 0.512$ kN/mm

$F = fl = 0.512[2(50)] = 51.2$ kN *Ans.*

9-6 Prob. 9-2 with E6010 Electrode.

Table 9-6: $f = 14.85\, h$ kip/in $= 14.85(5/16) = 4.64$ kip/in

$F = f\,l = 4.64[2(2)] = 18.6$ kip *Ans.*

9-7 Prob. 9-3 with E7010 Electrode.

Table 9-6: $f = 14.85\,h$ kip/in $= 14.85$ [5 mm/(25.4 mm/in)] $= 2.923$ kip/in

$= 2.923(4.45/25.4) = 0.512$ kN/mm

$F = f\,l = 0.512[2(50)] = 51.2$ kN *Ans.*

9-8 Prob. 9-4 with E6010 Electrode.

Table 9-6: $f = 14.85\,h$ kip/in $= 14.85(5/16) = 4.64$ kip/in

$F = f\,l = 4.64[2(4)] = 37.1$ kip *Ans.*

9-9 Table A-20:

1018 CD: $S_{ut} = 440$ MPa, $S_y = 370$ MPa
1018 HR: $S_{ut} = 400$ MPa, $S_y = 220$ MPa

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.
Table 9-4:

$$\begin{aligned}\tau_{\text{all}} &= \min(0.30S_{ut},\ 0.40S_y)\\ &= \min[0.30(400),\ 0.40(220)]\\ &= \min(120,\ 88) = 88\text{ MPa}\end{aligned}$$

for both materials.
Eq. (9-3): $F = 0.707hl\tau_{\text{all}} = 0.707(5)[2(50)](88)(10^{-3}) = 31.1$ kN *Ans.*

9-10 Table A-20:

1020 CD: $S_{ut} = 68$ kpsi, $S_y = 57$ kpsi
1020 HR: $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.
Table 9-4:

$$\begin{aligned}\tau_{\text{all}} &= \min(0.30S_{ut},\ 0.40S_y)\\ &= \min[0.30(55),\ 0.40(30)]\\ &= \min(16.5,\ 12.0) = 12.0\text{ kpsi}\end{aligned}$$

for both materials.
Eq. (9-3): $F = 0.707hl\tau_{\text{all}} = 0.707(5/16)[2(2)](12.0) = 10.6$ kip *Ans.*

9-11 Table A-20:

1035 HR: S_{ut} = 500 MPa, S_y = 270 MPa

1035 CD: S_{ut} = 550 MPa, S_y = 460 MPa

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned}\tau_{\text{all}} &= \min(0.30S_{ut},\ 0.40S_y)\\ &= \min[0.30(500),\ 0.40(270)]\\ &= \min(150,\ 108) = 108 \text{ MPa}\end{aligned}$$

for both materials.

Eq. (9-3): $F = 0.707hl\tau_{\text{all}} = 0.707(5)[2(50)](108)(10^{-3}) = 38.2$ kN *Ans.*

9-12 Table A-20:

1035 HR: S_{ut} = 72 kpsi, S_y = 39.5 kpsi

1020 CD: S_{ut} = 68 kpsi, S_y = 57 kpsi, 1020 HR: S_{ut} = 55 kpsi, S_y = 30 kpsi

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned}\tau_{\text{all}} &= \min(0.30S_{ut},\ 0.40S_y)\\ &= \min[0.30(55),\ 0.40(30)]\\ &= \min(16.5,\ 12.0) = 12.0 \text{ kpsi}\end{aligned}$$

for both materials.

Eq. (9-3): $F = 0.707hl\tau_{\text{all}} = 0.707(5/16)[2(4)](12.0) = 21.2$ kip *Ans.*

9-13

Eq. (9-3): $$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(100)(10^3)}{5[2(50+50)]} = 141 \text{ MPa} \qquad \textit{Ans.}$$

9-14

Eq. (9-3): $$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(40)}{(5/16)[2(2+2)]} = 22.6 \text{ kpsi} \qquad \textit{Ans.}$$

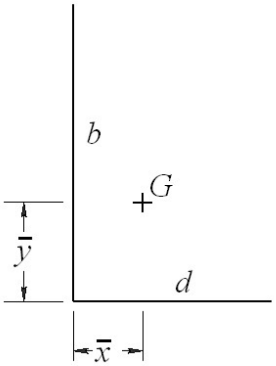
9-15 Eq. (9-3): $$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(100)(10^3)}{5[2(50+30)]} = 177 \text{ MPa} \qquad \textit{Ans.}$$

9-16 Eq. (9-3): $$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(40)}{(5/16)[2(4+2)]} = 15.1 \text{ kpsi} \qquad \textit{Ans.}$$

9-17 A = (throat area)(length) = $0.707h\ (b + d)$ *Ans.*

$$\bar{x}(b+d)=0(d)+\frac{b}{2}(b) \quad\Rightarrow\quad \bar{x}=\frac{b^2}{2(b+d)} \qquad Ans.$$

$$\bar{y}(b+d)=0(b)+\frac{d}{2}(d) \quad\Rightarrow\quad \bar{x}=\frac{d^2}{2(b+d)} \qquad Ans.$$



For line b:

$$(J_u)_b=\frac{b^3}{12}+b\left[\left(\frac{b}{2}-\bar{x}\right)^2+\bar{y}^2\right]=\frac{b^3}{12}+b\left(\frac{b^2}{4}-b\bar{x}+\bar{x}^2+\bar{y}^2\right)$$

$$=\frac{b^3}{12}+b\left[\frac{b^2}{4}-\frac{b^3}{2(b+d)}+\frac{b^4}{4(b+d)^2}+\frac{d^4}{4(b+d)^2}\right]=\frac{4b^3(b+d)^2-6b^4(b+d)+3b^5+3bd^4}{12(b+d)^2}$$

Similarly,

$$(J_u)_d=\frac{4d^3(b+d)^2-6d^4(b+d)+3d^5+3b^4d}{12(b+d)^2}$$

$$J_u=(J_u)_b+(J_u)_d$$

$$=\frac{4b^3(b+d)^2-6b^4(b+d)+3b^5+3bd^4+4d^3(b+d)^2-6d^4(b+d)+3d^5+3b^4d}{12(b+d)^2}$$

$$=\frac{4b^3(b+d)^2-6b^4(b+d)+3b^4(b+d)+3d^4(b+d)+4d^3(b+d)^2-6d^4(b+d)}{12(b+d)^2}$$

This reduces to

$$J_u=\frac{b^4+4b^3d+d^4+4bd^3}{12(b+d)} \qquad (1)$$

Add and subtract $6b^2d^2$ to Eq. (1) giving

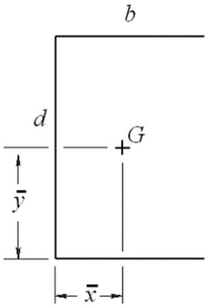
$$J_u=\frac{\left(b^4+4b^3d+6b^2d^2+4bd^3+d^4\right)-6b^2d^2}{12(b+d)}$$

$$=\frac{(b+d)^4-6b^2d^2}{12(b+d)} \qquad \text{which is the same as Table 9}-1 \qquad Ans.$$

9-18 A = (throat area) (length) = $0.707h\ (2b + d)$ *Ans.*

$$\bar{x}(2b+d)=0(d)+\frac{d}{2}(b) \quad\Rightarrow\quad \bar{x}=\frac{b^2}{2b+d} \qquad Ans.$$

$$\bar{y}(2b+d)=0(b)+d\left(\frac{d}{2}\right)+d(b) \quad\Rightarrow\quad \bar{y}=\frac{d\left(b+\dfrac{d}{2}\right)}{2b+d}=\frac{d}{2} \qquad Ans.$$



For lines b:

$$\left(J_u\right)_b = 2\left\{\frac{b^3}{12} + b\left[\left(\frac{b}{2} - \bar{x}\right)^2 + \bar{y}^2\right]\right\} = \frac{b^3}{6} + 2b\left(\frac{b^2}{4} - b\bar{x} + \bar{x}^2 + \bar{y}^2\right)$$

$$= \frac{b^3}{6} + \frac{b^3}{2} - \frac{2b^4}{2b+d} + \frac{2b^5}{\left(2b+d\right)^2} + \frac{bd^2}{2} = \frac{2b^3}{3} - \frac{2b^4}{2b+d} + \frac{2b^5}{\left(2b+d\right)^2} + \frac{bd^2}{2}$$

Line d:

$$\left(J_u\right)_d = \frac{d^3}{12} + d\,\bar{x}^2 = \frac{d^3}{12} + \frac{b^4 d}{\left(2b+d\right)^2}$$

$J_u = (J_u)_b + (J_u)_d$

$$J_u = \frac{2b^3}{3} - \frac{2b^4}{2b+d} + \frac{2b^5}{\left(2b+d\right)^2} + \frac{bd^2}{2} + \frac{d^3}{12} + \frac{b^4 d}{\left(2b+d\right)^2}$$

$$= \frac{8b^3 + 6bd^2 + d^3}{12} + \frac{-2b^4\left(2b+d\right) + 2b^5 + b^4 d}{\left(2b+d\right)^2}$$

$$= \frac{8b^3 + 6bd^2 + d^3}{12} + \frac{-2b^4\left(2b+d\right) + b^4\left(2b+d\right)}{\left(2b+d\right)^2}$$

$$= \frac{8b^3 + 6bd^2 + d^3}{12} - \frac{b^4}{2b+d} \qquad \textit{Ans.}$$

9-19 $b = d = 50$ mm, $c = 150$ mm, $h = 5$ mm, and $\tau_{\text{allow}} = 140$ MPa.

(a) *Primary shear*, Table 9-1, Case 2 (Note: b and d are interchanged between problem figure and table figure. Note, also, F in kN and τ in MPa):

$$\tau'_y = \frac{V}{A} = \frac{F\left(10^3\right)}{1.414\left(5\right)\left(50\right)} = 2.829F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d\left(3b^2 + d^2\right)}{6} = \frac{50\left[3\left(50^2\right) + 50^2\right]}{6} = 83.33\left(10^3\right)\text{ mm}^3$$

$$J = 0.707\,h\,J_u = 0.707(5)(83.33)(10^3) = 294.6(10^3)\text{ mm}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{175F\left(10^3\right)\left(25\right)}{294.6\left(10^3\right)} = 14.85F$$

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{14.85^2 + \left(2.829 + 14.85\right)^2} = 23.1F \qquad (1)$$

$$F = \frac{\tau_{\text{allow}}}{23.1} = \frac{140}{23.1} = 6.06 \text{ kN} \quad \textit{Ans.}$$

(**b**) For E7010 from Table 9-6, $\tau_{\text{allow}} = 21$ kpsi = 21(6.89) = 145 MPa

1020 HR bar: $S_{ut} = 380$ MPa, $S_y = 210$ MPa

1015 HR support: $S_{ut} = 340$ MPa, $S_y = 190$ MPa

Table 9-3, E7010 Electrode: $S_{ut} = 482$ MPa, $S_y = 393$ MPa

The support controls the design.

Table 9-4: $\tau_{\text{allow}} = \min(0.30S_{ut}, 0.40S_y) = \min[0.30(340), 0.40(190) = \min(102, 76)$

$= 76$ MPa

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{23.1} = \frac{76}{23.1} = 3.29 \text{ kN} \qquad \textit{Ans.}$$

9-20 $b = d = 2$ in, $c = 6$ in, $h = 5/16$ in, and $\tau_{\text{allow}} = 25$ kpsi.
(**a**) *Primary shear*, Table 9-1(Note: b and d are interchanged between problem figure and table figure. Note, also, F in kip and τ in kpsi):

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(2)} = 1.132F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d\left(3b^2 + d^2\right)}{6} = \frac{2\left[3\left(2^2\right) + 2^2\right]}{6} = 5.333 \text{ in}^3$$

$$J = 0.707\, h\, J_u = 0.707(5/16)(5.333) = 1.178 \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{7F(1)}{1.178} = 5.942F$$

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{5.942^2 + \left(1.132 + 5.942\right)^2} = 9.24F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{9.24} = \frac{25}{9.24} = 2.71 \text{ kip} \quad \textit{Ans.}$$

(**b**) For E7010 from Table 9-6, $\tau_{\text{allow}} = 21$ kpsi

1020 HR bar: $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi

1015 HR support: $S_{ut} = 50$ kpsi, $S_y = 27.5$ kpsi

Table 9-3, E7010 Electrode: $S_{ut} = 70$ kpsi, $S_y = 57$ kpsi

The support controls the design.

Table 9-4: $\tau_{\text{allow}} = \min(0.30S_{ut},\ 0.40S_y) = \min[0.30(50),\ 0.40(27.5) = \min(15,\ 11)$
$= 11$ kpsi

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{9.24} = \frac{11}{9.24} = 1.19 \text{ kip} \qquad \textit{Ans.}$$

9-21 $b = 50$ mm, $c = 150$ mm, $d = 30$ mm, $h = 5$ mm, and $\tau_{\text{allow}} = 140$ MPa.
(**a**) *Primary shear*, Table 9-1, Case 2 (Note: b and d are interchanged between problem figure and table figure. Note, also, F in kN and τ in MPa):

$$\tau'_y = \frac{V}{A} = \frac{F\left(10^3\right)}{1.414(5)(50)} = 2.829F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d\left(3b^2 + d^2\right)}{6} = \frac{50\left[3\left(30^2\right) + 50^2\right]}{6} = 43.33\left(10^3\right) \text{ mm}^3$$

$$J = 0.707\, h\, J_u = 0.707(5)(43.33)(10^3) = 153.2(10^3) \text{ mm}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{175F\left(10^3\right)(15)}{153.2\left(10^3\right)} = 17.13F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{175F\left(10^3\right)(25)}{153.2\left(10^3\right)} = 28.55F$$

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{17.13^2 + \left(2.829 + 28.55\right)^2} = 35.8F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{35.8} = \frac{140}{35.8} = 3.91 \text{ kN} \quad \textit{Ans.}$$

(**b**) For E7010 from Table 9-6, $\tau_{\text{allow}} = 21$ kpsi = 21(6.89) = 145 MPa

1020 HR bar: $S_{ut} = 380$ MPa, $S_y = 210$ MPa

1015 HR support: $S_{ut} = 340$ MPa, $S_y = 190$ MPa

Table 9-3, E7010 Electrode: $S_{ut} = 482$ MPa, $S_y = 393$ MPa

The support controls the design.

Table 9-4: $\tau_{\text{allow}} = \min(0.30S_{ut}, 0.40S_y) = \min[0.30(340), 0.40(190) = \min(102, 76)$

$= 76$ MPa

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{35.8} = \frac{76}{35.8} = 2.12 \text{ kN} \qquad \textit{Ans.}$$

9-22 $b = 4$ in, $c = 6$ in, $d = 2$ in, $h = 5/16$ in, and $\tau_{\text{allow}} = 25$ kpsi.
(**a**) *Primary shear*, Table 9-1(Note: b and d are interchanged between problem figure and table figure. Note, also, F in kip and τ in kpsi):

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(4)} = 0.5658F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d\left(3b^2 + d^2\right)}{6} = \frac{4\left[3\left(2^2\right) + 4^2\right]}{6} = 18.67 \text{ in}^3$$

$$J = 0.707\, h\, J_u = 0.707(5/16)(18.67) = 4.125 \text{ in}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{8F(1)}{4.125} = 1.939F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{8F(2)}{4.125} = 3.879F$$

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{1.939^2 + \left(0.5658 + 3.879\right)^2} = 4.85F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{4.85} = \frac{25}{4.85} = 5.15 \text{ kip} \quad Ans.$$

(**b**) For E7010 from Table 9-6, $\tau_{\text{allow}} = 21$ kpsi

1020 HR bar: $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi

1015 HR support: $S_{ut} = 50$ kpsi, $S_y = 27.5$ kpsi

Table 9-3, E7010 Electrode: $S_{ut} = 70$ kpsi, $S_y = 57$ kpsi

The support controls the design.

Table 9-4: $\tau_{\text{allow}} = \min(0.30S_{ut}, 0.40S_y\ ) = \min[0.30(50), 0.40(27.5) = \min(15, 11)$

$= 11$ kpsi

The allowable load, from Eq. (1) is

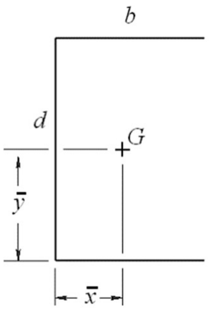
$$F = \frac{\tau_{\text{allow}}}{4.85} = \frac{11}{4.85} = 2.27 \text{ kip} \qquad Ans.$$

9-23 A = (throat area)(length) = $0.707h\ (2b + d)$ *Ans.*

$$\bar{x}\left(2b+d\right) = 0\left(d\right) + \frac{d}{2}\left(b\right) \quad \Rightarrow \quad \bar{x} = \frac{b^2}{2b+d} \qquad Ans.$$

$$\bar{y}\left(2b+d\right) = 0\left(b\right) + d\left(\frac{d}{2}\right) + d\left(b\right) \quad \Rightarrow \quad \bar{y} = \frac{d\left(b+\frac{d}{2}\right)}{2b+d} = \frac{d}{2} \qquad Ans.$$

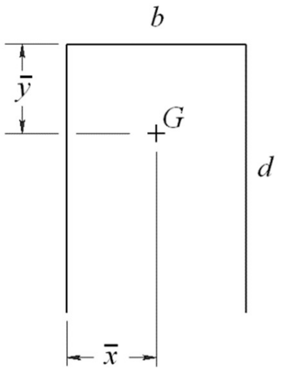
$$I_u = \frac{d^3}{12} + 2b\left(\frac{d}{2}\right)^2 = \frac{d^2}{12}\left(6b+d\right) \qquad Ans.$$



9-24 A = (throat area)(length) = $0.707h\ (b + 2d)$ *Ans.*

$$\bar{x}\left(b+2d\right) = 0\left(d\right) + \frac{b}{2}\left(b\right) + b\left(d\right) \quad \Rightarrow \quad \bar{x} = \frac{b\left(\frac{b}{2}+d\right)}{b+2d} = \frac{b}{2} \qquad Ans.$$

$$\bar{y}\left(b+2d\right) = 0\left(b\right) + \frac{d}{2}\left(2d\right) \quad \Rightarrow \quad \bar{y} = \frac{d^2}{b+2d} \qquad Ans.$$



$$I_u = b\overline{y}^2 + \frac{2d^3}{12} + 2d\left(\frac{d}{2} - \overline{y}\right)^2 = \frac{d^3}{6} + \frac{d^3}{2} + b\,\overline{y}^2 - 2d^2\overline{y} + 2d\,\overline{y}^2$$

$$= \frac{2}{3}d^3 - 2d^2\overline{y} + (b + 2d)\overline{y}^2 \qquad \textit{Ans.}$$

9-25 Given, $b = 50$ mm, $c = 150$ mm, $d = 50$ mm, $h = 5$ mm, $\tau_{\text{allow}} = 140$ MPa.

Primary shear (F in kN, τ in MPa, A in mm^2):

$$\tau_y' = \frac{V}{A} = \frac{F(10^3)}{1.414(5)(50+50)} = 1.414F$$

Secondary shear:

Table 9-1: $$J_u = \frac{(b+d)^3}{6} = \frac{(50+50)^3}{6} = 166.7(10^3)\ \text{mm}^3$$

$$J = 0.707\, h\, J_u = 0.707(5)166.7(10^3) = 589.2(10^3)\ \text{mm}^4$$

$$\tau_x'' = \tau_y'' = \frac{Mr_y}{J} = \frac{175F(10^3)(25)}{589.2(10^3)} = 7.425F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau_x''^2 + (\tau_y' + \tau_y'')^2} = F\sqrt{7.425^2 + (1.414 + 7.425)^2} = 11.54F$$

$$F = \frac{\tau_{allow}}{11.54} = \frac{140}{11.54} = 12.1\ \text{kN} \qquad \textit{Ans.}$$

9-26 Given, $b = 2$ in, $c = 6$ in, $d = 2$ in, $h = 5/16$ in, $\tau_{\text{allow}} = 25$ kpsi.

Primary shear:

$$\tau_y' = \frac{V}{A} = \frac{F}{1.414(5/16)(2+2)} = 0.5658F$$

Secondary shear:

Table 9-1: $$J_u = \frac{(b+d)^3}{6} = \frac{(2+2)^3}{6} = 10.67\ \text{in}^3$$

$$J = 0.707\, h\, J_u = 0.707(5/16)10.67 = 2.357\ \text{in}^4$$

$$\tau_x'' = \tau_y'' = \frac{Mr_y}{J} = \frac{7F(1)}{2.357} = 2.970F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau_x''^2 + \left(\tau_y' + \tau_y''\right)^2} = F\sqrt{2.970^2 + \left(0.566 + 2.970\right)^2} = 4.618F$$

$$F = \frac{\tau_{allow}}{4.618} = \frac{25}{4.618} = 5.41 \text{ kip} \quad \textit{Ans.}$$

9-27 Given, $b = 50$ mm, $c = 150$ mm, $d = 30$ mm, $h = 5$ mm, $\tau_{\text{allow}} = 140$ MPa.

Primary shear (F in kN, τ in MPa, A in mm^2):

$$\tau_y' = \frac{V}{A} = \frac{F\left(10^3\right)}{1.414(5)(50+30)} = 1.768F$$

Secondary shear:

Table 9-1: $$J_u = \frac{(b+d)^3}{6} = \frac{(50+30)^3}{6} = 85.33\left(10^3\right) \text{ mm}^3$$

$$J = 0.707\, h\, J_u = 0.707(5)85.33(10^3) = 301.6(10^3) \text{ mm}^4$$

$$\tau_x'' = \frac{Mr_y}{J} = \frac{175F\left(10^3\right)(15)}{301.6\left(10^3\right)} = 8.704F$$

$$\tau_y'' = \frac{Mr_x}{J} = \frac{175F\left(10^3\right)(25)}{301.6\left(10^3\right)} = 14.51F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau_x''^2 + \left(\tau_y' + \tau_y''\right)^2} = F\sqrt{8.704^2 + \left(1.768 + 14.51\right)^2} = 18.46F$$

$$F = \frac{\tau_{allow}}{18.46} = \frac{140}{18.46} = 7.58 \text{ kN} \quad \textit{Ans.}$$

9-28 Given, $b = 4$ in, $c = 6$ in, $d = 2$ in, $h = 5/16$ in, $\tau_{\text{allow}} = 25$ kpsi.

Primary shear:

$$\tau_y' = \frac{V}{A} = \frac{F}{1.414(5/16)(4+2)} = 0.3772F$$

Secondary shear:

Table 9-1: $$J_u = \frac{(b+d)^3}{6} = \frac{(4+2)^3}{6} = 36 \text{ in}^3$$

$$J = 0.707\, h\, J_u = 0.707(5/16)36 = 7.954 \text{ in}^4$$

$$\tau_x'' = \frac{Mr_y}{J} = \frac{8F(1)}{7.954} = 1.006F$$

$$\tau_y'' = \frac{Mr_x}{J} = \frac{8F(2)}{7.954} = 2.012F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau_x''^2 + \left(\tau_y' + \tau_y''\right)^2} = F\sqrt{1.006^2 + \left(0.3772 + 2.012\right)^2} = 2.592F$$

$$F = \frac{\tau_{allow}}{2.592} = \frac{25}{2.592} = 9.65\,\text{kip} \qquad \textit{Ans.}$$

9-29 Given: $b = 50$ mm, $c = 150$ mm, $d = 50$ mm, $h = 5$ mm, $\tau_{\text{allow}} = 140$ MPa.

Primary shear (F in kN): $\tau_x' = \tau_y' = \dfrac{V}{A} = \dfrac{F\left(10^3\right)\sin 45^{\circ}}{1.414(5)(50+50)} = F$

Secondary shear: $M = 0.707\,F\,(10^3)(175 - 25) = 106.05(10^3)\,F$

Table 9-1: $J_u = (b+d)^3/6 = (50+50)^3/6 = 166.7\left(10^3\right)$ mm^3

$$J = 0.707hJ_u = 0.707(5)166.7\left(10^3\right) = 589.3\left(10^3\right)\ \text{mm}^4$$

$$\tau_x'' = \tau_y'' = \frac{Mr_y}{J} = \frac{106.5\left(10^3\right)F(25)}{589.3\left(10^3\right)} = 4.50F$$

Upper right end of weld:

$$\tau_{\max} = \sqrt{\left(\tau_x' + \tau_x''\right)^2 + \left(\tau_y' + \tau_y''\right)^2} = \sqrt{2\left(F + 4.5F\right)^2} = 7.778F$$

$$\tau_{\max} = \tau_{\text{allow}} \quad \Rightarrow \quad F = \frac{140}{7.778} = 18.0\ \text{kN} \qquad \textit{Ans.}$$

9-30 Given: $b = 2$ in, $c = 6$ in, $d = 2$ in, $h = \frac{5}{16}$ in, $\tau_{\text{allow}} = 25$ kpsi.

Primary shear (F in kip) : $\tau_x' = \tau_y' = \dfrac{V}{A} = \dfrac{0.707F}{1.414\left(\frac{5}{16}\right)(2+2)} = 0.4F$

Secondary shear: $M = 0.707\,F\,(7 - 1) = 4.242\,F$

Table 9-1: $J_u = (b+d)^3/6 = (2+2)^3/6 = 10.667$ in^3

$$J = 0.707hJ_u = 0.707\left(\tfrac{5}{16}\right)10.667 = 2.357\ \text{in}^4$$

$$\tau_x'' = \tau_y'' = \frac{Mr_y}{J} = \frac{4.242F(1)}{2.357} = 1.80F$$

Upper right end of weld:

$$\tau_{\max} = \sqrt{\left(\tau_x' + \tau_x''\right)^2 + \left(\tau_y' + \tau_y''\right)^2} = \sqrt{2\left(0.4F + 1.80F\right)^2} = 3.111F$$

$$\tau_{\max} = \tau_{\text{allow}} \quad \Rightarrow \quad F = \frac{25}{3.111} = 8.04\ \text{kip} \qquad \textit{Ans.}$$

9-31 Given: $b = 50$ mm, $c = 150$ mm, $d = 30$ mm, $h = 5$ mm, $\tau_{\text{allow}} = 140$ MPa.

Primary shear (F in kN): $\tau'_x = \tau'_y = \dfrac{V}{A} = \dfrac{F\left(10^3\right)\sin 45^\circ}{1.414(5)(50+30)} = 1.25F$

Secondary shear: $M = 0.707\ F\ (10^3)(175 - 15) = 113.12\ (10^3)\ F$

Table 9-1: $J_u = (b+d)^3/6 = (50+30)^3/6 = 85.33\left(10^3\right)\ \text{mm}^3$

$$J = 0.707 h J_u = 0.707(5)85.33\left(10^3\right) = 301.65\left(10^3\right)\ \text{mm}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{113.12\left(10^3\right)F(15)}{301.65\left(10^3\right)} = 5.625F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{113.12\left(10^3\right)F(25)}{301.65\left(10^3\right)} = 9.375F$$

Upper right end of weld:

$$\tau_{\max} = \sqrt{\left(\tau'_x+\tau''_x\right)^2 + \left(\tau'_y+\tau''_y\right)^2} = F\sqrt{\left[(1.25+5.625)^2 + (1.25+9.375)^2\right]} = 12.66F$$

$$\tau_{\max} = \tau_{\text{allow}} \quad \Rightarrow \quad F = \frac{140}{12.66} = 11.1\ \text{kN} \qquad \textit{Ans.}$$

9-32 Given: $b = 4$ in, $c = 6$ in, $d = 2$ in, $h = \frac{5}{16}$ in, $\tau_{\text{allow}} = 25$ kpsi.

Primary shear (F in kip) : $\tau'_x = \tau'_y = \dfrac{V}{A} = \dfrac{0.707F}{1.414\left(\frac{5}{16}\right)(4+2)} = 0.2667F$

Secondary shear: $M = 0.707\ F\ (8 - 1) = 4.949\ F$

Table 9-1: $J_u = (b+d)^3/6 = (4+2)^3/6 = 36\ \text{in}^3$

$$J = 0.707 h J_u = 0.707\left(\tfrac{5}{16}\right)36 = 7.954\ \text{in}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{4.949F(1)}{7.954} = 0.6222F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{4.949F(2)}{7.954} = 1.244F$$

Upper right end of weld:

$$\tau_{\max} = \sqrt{\left(\tau'_x+\tau''_x\right)^2 + \left(\tau'_y+\tau''_y\right)^2} = F\sqrt{\left[(0.2667+0.6222)^2 + (0.2667+1.244)^2\right]} = 1.753F$$

$$\tau_{\max} = \tau_{\text{allow}} \quad \Rightarrow \quad F = \frac{25}{1.753} = 14.3\ \text{kip} \qquad \textit{Ans.}$$

9-33 Given, $b = 50$ mm, $d = 50$ mm, $h = 5$ mm, E6010 electrode.

$$A = 0.707(5)(50 + 50 + 50) = 530.3 \text{ mm}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, $S_{ut} = 320$ MPa.

Eq. (6-18) and Table 6-2: $k_a = 54.9(320)^{-0.758} = 0.693$

$k_b = 1$ (uniform shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

Eqs. (6-10) and (6-17): $S_e = 0.693(1)(0.59)(1)(0.5)(320) = 65.4$ MPa

Electrode endurance: E6010, Table 9-3, $S_{ut} = 427$ MPa

Eq. (6-18) and Table 6-2: $k_a = 54.9(427)^{-0.758} = 0.557$

As before, $k_b = 1$ (direct shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

$$S_e = 0.557(1)(0.59)(1)(0.5)(427) = 70.2 \text{ MPa}$$

The members and electrode are basically of equal strength. We will use $S_e = 65.4$ MPa.
For a factor of safety of 1, and with $K_{fs} = 2.7$ (Table 9-5)

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{65.4(530.3)}{2.7} = 12.8\left(10^3\right) \text{ N} = 12.8 \text{ kN} \qquad \textit{Ans.}$$

9-34 Given, $b = 2$ in, $d = 2$ in, $h = 5/16$ in, E6010 electrode.

$$A = 0.707(5/16)(2 + 2 + 2) = 1.326 \text{ in}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, $S_{ut} = 47$ kpsi.

Eq. (6-18) and Table 6-2: $k_a = 12.7(47)^{-0.758} = 0.686$

$k_b = 1$ (uniform shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

Eqs. (6-10) and (6-17): $S_e = 0.686(1)(0.59)(1)(0.5)(47) = 9.51$ kpsi

Electrode endurance: E6010, Table 9-3, $S_{ut} = 62$ kpsi

Eq. (6-18) and Table 6-2: $k_a = 12.7(62)^{-0.758} = 0.556$
As before, $k_b = 1$ (uniform shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

$$S_e = 0.556(1)(0.59)(1)(0.5)(62) = 10.2 \text{ kpsi}$$

For a factor of safety of 1, with $K_{fs} = 2.7$ (Table 9-5), using $S_e = 9.51$ kpsi

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{9.51(1.326)}{2.7} = 4.67 \text{ kip} \qquad \textit{Ans.}$$

9-35 Given, b = 50 mm, d = 30 mm, h = 5 mm, E7010 electrode.

$$A = 0.707(5)(50 + 50 + 30) = 459.6 \text{ mm}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, S_{ut} = 320 MPa.

Eq. (6-18) and Table 6-2: $k_a = 54.9(320)^{-0.758} = 0.693$

k_b = 1 (direct shear), k_c = 0.59 (torsion, shear), k_d = 1

Eqs. (6-10) and (6-17): $S_e = 0.693(1)(0.59)(1)(0.5)(320) = 65.4$ MPa

Electrode endurance: E6010, Table 9-3, S_{ut} = 482 MPa

Eq. (6-18) and Table 6-2: $k_a = 54.9(482)^{-0.758} = 0.508$

As before, k_b = 1 (direct shear), k_c = 0.59 (torsion, shear), k_d = 1

$$S_e = 0.508(1)(0.59)(1)(0.5)(482) = 72.2 \text{ MPa}$$

For a factor of safety of 1,with K_{fs} = 2.7 (Table 9-5), and using S_e =65.4 MPa

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{65.4(459.6)}{2.7} = 11.1(10^3) \text{ N} = 11.1 \text{ kN} \qquad \textit{Ans.}$$

9-36 Given, b = 4 in, d = 2 in, h = 5/16 in, E7010 electrode.

$$A = 0.707(5/16)(4 + 4 + 2) = 2.209 \text{ in}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, S_{ut} = 47 kpsi.

Eq. (6-18) and Table 6-2: $k_a = 12.7(47)^{-0.758} = 0.686$

k_b = 1 (direct shear), k_c = 0.59 (torsion, shear), k_d = 1

Eqs. (6-10) and (6-17): $S_e = 0.686(1)(0.59)(1)(0.5)(47) = 9.51$ kpsi

Electrode endurance: E7010, Table 9-3, S_{ut} = 70 kpsi

Eq. (6-18) and Table 6-2: $k_a = 12.7(70)^{-0.758} = 0.507$

As before, k_b = 1 (direct shear), k_c = 0.59 (torsion, shear), k_d = 1

$$S_e = 0.507(1)(0.59)(1)(0.5)(70) = 10.5 \text{ kpsi}$$

For a factor of safety of 1, with K_{fs} = 2.7 (Table 9-5), and using S_e = 9.51 kpsi

$$F=\frac{\tau_{\text{allow}}A}{K_{fs}}=\frac{9.51\left(2.209\right)}{2.7}=7.78\text{ kip}\qquad \textit{Ans.}$$

9-37 *Primary shear*: $\tau'=0$ (why?)
Secondary shear:

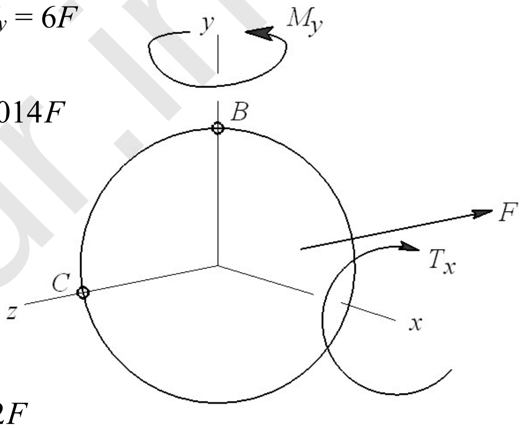
Table 9-1: $J_u=2\pi r^3=2\pi(1.5)^3=21.21\text{ in}^3$

$$J=0.707\,h\,J_u=0.707(1/4)(21.21)=3.749\text{ in}^4$$

2 welds: $$\tau''=\frac{Mr}{2J}=\frac{8F\left(1.5\right)}{2\left(3.749\right)}=1.600F$$

$$\tau''=\tau_{\text{allow}}\quad\Rightarrow\quad 1.600F=20\quad\Rightarrow\quad F=12.5\text{ kip}\qquad \textit{Ans.}$$

9-38 Direct shear $V_z=F$, Torsion $T_x=8F$, Bending $M_y=6F$



B: Direct shear:

$$\left(\tau\right)_V=\tau'_z=\frac{V_z}{A}=\frac{F}{1.414\pi hr}=\frac{F}{1.414\pi\left(\frac{1}{4}\right)1.5}=0.6014F$$

Torsion, Table 9-1:
$J_u=2\pi r^3=2\pi\,(1.5)^3=21.21\text{ in}^3$
$J=0.707h\,J_u=0.707\,\left(\frac{1}{4}\right)21.21=3.749\text{ in}^4$

$$\left(\tau\right)_T=\tau''_z=\frac{T_x r}{J}=\frac{8F\left(1.5\right)}{3.749}=3.201F$$

$$\tau_{\max}=\tau'_z+\tau''_z=0.6014F+3.201F=3.802F$$

$$F=\frac{\tau_{\text{allow}}}{3.802}=\frac{20}{3.802}=5.26\text{ kip}\qquad \textit{Ans.}$$

C: Direct shear, torsion, and bending.
Bending, Table 9-2:

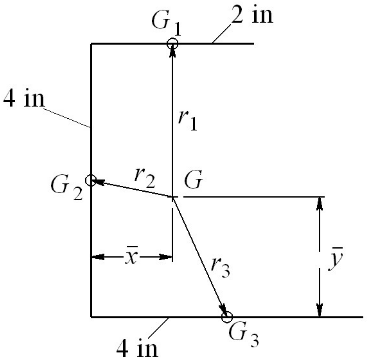
$I_u=\pi r^3=\pi\,(1.5)^3=10.603\text{ in}^3$, $I=0.707h\,I_u=0.707\left(\frac{1}{4}\right)10.603=1.874\text{ in}^4$.

$$\left(\tau\right)_M=\frac{Mc}{I}=\frac{6F\left(1.5\right)}{1.874}=4.803F$$

$$\tau_{\max}=\sqrt{\left(\tau\right)_V^2+\left(\tau\right)_T^2+\left(\tau\right)_M^2}=\sqrt{\left(0.6014F\right)^2+\left(3.201F\right)^2+\left(4.803F\right)^2}=5.803F$$

$$F=\frac{\tau_{\text{allow}}}{5.803}=\frac{20}{5.803}=3.45\text{ kip}\qquad \textit{Ans.}$$

9-39 $l = 2 + 4 + 4 = 10$ in



$$\bar{x} = \frac{2(1)+4(0)+4(2)}{10} = 1 \text{ in}$$

$$\bar{y} = \frac{2(4)+4(2)+4(0)}{10} = 1.6 \text{ in}$$

$$M = FR = F\,(10 - 1) = 9\,F$$

$$r_1 = \sqrt{(1-1)^2 + (4-1.6)^2} = 2.4 \text{ in}$$

$$r_2 = \sqrt{1^2 + (2-1.6)^2} = 1.077 \text{ in}$$

$$r_3 = \sqrt{(2-1)^2 + 1.6^2} = 1.887 \text{ in}$$

$$J_{G_1} = \frac{1}{12}(0.707)(5/16)(2^3) = 0.1473 \text{ in}^4$$

$$J_{G_2} = J_{G_3} = \frac{1}{12}(0.707)(5/16)(4^3) = 1.178 \text{ in}^4$$

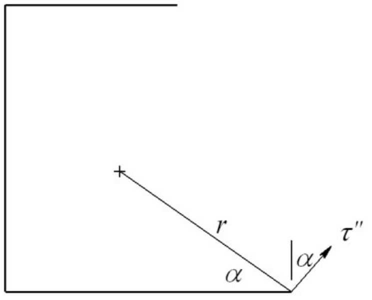
$$J = \sum_{i=1}^{3}\left(J_i + A_i r_{G_i}^2\right)$$

$$= 0.1473 + 0.707(5/16)(2)(2.4^2) + 1.178 + 0.707(5/16)(4)(1.077^2)$$

$$+ 1.178 + 0.707(5/16)(4)(1.887^2) = 9.220 \text{ in}^4$$

$$\alpha = \tan^{-1}\left(\frac{1.6}{4-1}\right) = 28.07^\circ$$

$$r = \sqrt{1.6^2 + (4-1)^2} = 3.4 \text{ in}$$



Primary shear (τ in kpsi, F in kip) :

$$\tau' = \frac{V}{A} = \frac{F}{0.707(5/16)(10)} = 0.4526F$$

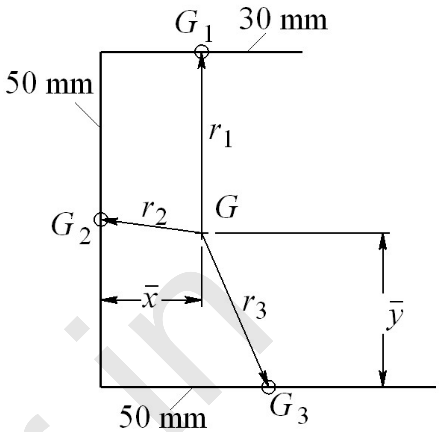
Secondary shear:

$$\tau'' = \frac{Mr}{J} = \frac{9F(3.4)}{9.220} = 3.319F$$

$$\tau_{\max} = \sqrt{(3.319F\sin 28.07^\circ)^2 + (3.319F\cos 28.07^\circ + 0.4526F)^2}$$
$$= 3.724F$$

$\tau_{\max} = \tau_{\text{allow}} \Rightarrow 3.724\,F = 25 \Rightarrow F = 6.71$ kip *Ans.*

9-40 $l = 30 + 50 + 50 = 130$ mm



$$\bar{x} = \frac{30(15)+50(0)+50(25)}{130} = 13.08 \text{ mm}$$
$$\bar{y} = \frac{30(50)+50(25)+50(0)}{130} = 21.15 \text{ mm}$$

$M = FR = F(200 - 13.08)$
$= 186.92\,F$ (M in N·m, F in kN)

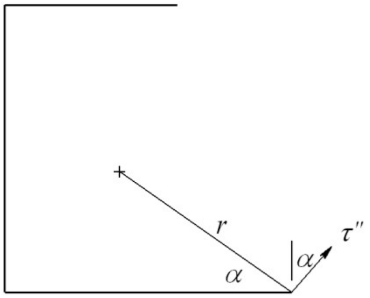
$$r_1 = \sqrt{(15-13.08)^2 + (50-21.15)^2} = 28.92 \text{ mm}$$
$$r_2 = \sqrt{13.08^2 + (25-21.15)^2} = 13.63 \text{ mm}$$
$$r_3 = \sqrt{(25-13.08)^2 + 21.15^2} = 24.28 \text{ mm}$$

$$J_{G_1} = \frac{1}{12}(0.707)(5)(30^3) = 7.954(10^3) \text{ mm}^4$$
$$J_{G_2} = J_{G_3} = \frac{1}{12}(0.707)(5)(50^3) = 36.82(10^3) \text{ mm}^4$$

$$J = \sum_{i=1}^{3}\left(J_i + A_i r_{G_i}^2\right)$$
$$= 7.954(10^3) + 0.707(5)(30)(28.92^2) + 36.82(10^3) + 0.707(5)(50)(13.63^2)$$
$$+ 36.82(10^3) + 0.707(5)(50)(24.28^2) = 307.3(10^3) \text{ mm}^4$$

$$\alpha = \tan^{-1}\left(\frac{21.15}{50-13.08}\right) = 29.81^\circ$$

$$r = \sqrt{21.15^2 + (50-13.08)^2} = 42.55 \text{ mm}$$



Primary shear (τ in MPa, F in kN) :

$$\tau' = \frac{V}{A} = \frac{F\left(10^3\right)}{0.707(5)(130)} = 2.176F$$

Secondary shear:

$$\tau'' = \frac{Mr}{J} = \frac{186.92F\left(10^3\right)(42.55)}{307.3\left(10^3\right)} = 25.88F$$

$$\tau_{\max} = \sqrt{\left(25.88F\sin 29.81^\circ\right)^2 + \left(25.88F\cos 29.81^\circ + 2.176F\right)^2}$$
$$= 27.79F$$

$$\tau_{\max} = \tau_{\text{allow}} \quad \Rightarrow \quad 27.79\,F = 140 \quad \Rightarrow \quad F = 5.04\text{ kN} \qquad \textit{Ans.}$$

9-41

Weld Pattern	Figure of merit	Rank
1.	$\text{fom}' = \frac{J_u}{lh} = \frac{a^3/12}{ah} = \frac{a^2}{12h} = 0.0833\left(\frac{a^2}{h}\right)$	**5**
2.	$\text{fom}' = \frac{a\left(3a^2+a^2\right)}{6(2a)h} = \frac{a^2}{3h} = 0.3333\left(\frac{a^2}{h}\right)$	**1**
3.	$\text{fom}' = \frac{(2a)^4 - 6a^2a^2}{12(a+a)2ah} = \frac{5a^2}{24h} = 0.2083\left(\frac{a^2}{h}\right)$	**4**
4.	$\text{fom}' = \frac{1}{3ah}\left(\frac{8a^3+6a^3+a^3}{12} - \frac{a^4}{2a+a}\right) = 0.3056\left(\frac{a^2}{h}\right)$	**2**
5.	$\text{fom}' = \frac{(2a)^3}{6h}\frac{1}{4a} = \frac{8a^3}{24ah} = 0.3333\left(\frac{a^2}{h}\right)$	**1**
6.	$\text{fom}' = \frac{2\pi(a/2)^3}{\pi ah} = \frac{a^3}{4ah} = 0.25\left(\frac{a^2}{h}\right)$	**3**

9-42

Weld Pattern	Figure of merit	Rank
1.	$\text{fom}' = \frac{I_u}{lh} = \frac{\left(a^3/12\right)}{ah} = 0.0833\left(\frac{a^2}{h}\right)$	**6**
2.	$\text{fom}' = \frac{\left(a^3/6\right)}{2ah} = 0.0833\left(\frac{a^2}{h}\right)$	**6**
3.	$\text{fom}' = \frac{\left(aa^2/2\right)}{2ah} = 0.25\left(\frac{a^2}{h}\right)$	**1**
4.*	$\text{fom}' = \frac{\left(a^2/12\right)\left(6a+a\right)}{3ah} = \frac{7a^2}{36h} = 0.1944\left(\frac{a^2}{h}\right)$	**2**
5. & 7.	$\bar{x} = \frac{a}{2}, \quad \bar{y} = \frac{a^2}{a+2a} = \frac{a}{3}$ $I_u = \frac{2a^3}{3} - 2a^2\frac{a}{3} + \left(a+2a\right)\left(\frac{a}{3}\right)^2 = \frac{a^3}{3}$ $\text{fom}' = \frac{I_u}{lh} = \frac{\left(a^3/3\right)}{3ah} = \frac{1}{9}\left(\frac{a^2}{h}\right) = 0.1111\left(\frac{a^2}{h}\right)$	**5**
6. & 8.	$\text{fom}' = \frac{\left(a^2/6\right)\left(3a+a\right)}{4ah} = \frac{1}{6}\left(\frac{a^2}{h}\right) = 0.1667\left(\frac{a^2}{h}\right)$	**3**
9.	$\text{fom}' = \frac{\pi\left(a/2\right)^3}{\pi ah} = \frac{a^2}{8h} = 0.125\left(\frac{a^2}{h}\right)$	**4**

*Note. Because this section is not symmetric with the vertical axis, out-of-plane deflection may occur unless special precautions are taken. See the topic of “shear center” in books with more advanced treatments of mechanics of materials.

9-43 Attachment and member (1018 HR), $S_y = 220$ MPa and $S_{ut} = 400$ MPa.

The member and attachment are weak compared to the properties of the lowest electrode.

Decision Specify the E6010 electrode

Controlling property, Table 9-4: $\tau_{\text{all}} = \min[0.3(400), 0.4(220)] = \min(120, 88) = 88$ MPa

For a static load, the parallel and transverse fillets are the same. Let the length of a bead be $l = 75$ mm, and n be the number of beads.

$$\tau = \frac{F}{n(0.707)hl} = \tau_{all}$$

$$nh = \frac{F}{0.707l\tau_{\text{all}}} = \frac{100(10^3)}{0.707(75)(88)} = 21.43$$

where h is in millimeters. Make a table

Number of beads, n	Leg size, h (mm)
1	21.43
2	10.71
3	7.14
4	5.36 → 6 mm

Decision Specify h = 6 mm on all four sides.

Weldment specification:

Pattern: All-around square, four beads each side, 75 mm long

Electrode: E6010

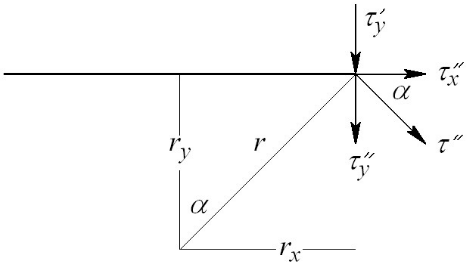
Leg size: h = 6 mm

9-44 *Decision*: Choose a parallel fillet weldment pattern. By so-doing, we've chosen an optimal pattern (see Prob. 9-41) and have thus reduced a synthesis problem to an analysis problem:

Table 9-1, case 2, rotated 90°: $A = 1.414hd = 1.414(h)(75) = 106.05h\ \text{mm}^2$

Primary shear

$$\tau'_y = \frac{V}{A} = \frac{12(10^3)}{106.05h} = \frac{113.2}{h}$$



Secondary shear:

$$J_u = \frac{d(3b^2 + d^2)}{6}$$
$$= \frac{75[3(75^2) + 75^2]}{6} = 281.3(10^3)\ \text{mm}^3$$
$$J = 0.707(h)(281.3)(10^3) = 198.8(10^3)h\ \text{mm}^4$$

With $\alpha = 45°$,

$$\tau''_x = \frac{Mr\cos 45^\circ}{J} = \frac{Mr_y}{J} = \frac{12\left(10^3\right)(187.5)(37.5)}{198.8\left(10^3\right)h} = \frac{424.4}{h} = \tau''_y$$

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = \frac{1}{h}\sqrt{424.4^2 + (113.2 + 424.4)^2} = \frac{684.9}{h}$$

Attachment and member (1018 HR): $S_y = 220$ MPa, $S_{ut} = 400$ MPa
Decision: Use E60XX electrode which is stronger

$$\tau_{\text{all}} = \min[0.3(400),\ 0.4(220)] = 88 \text{ MPa}$$
$$\tau_{\max} = \tau_{\text{all}} = \frac{684.9}{h} = 88 \text{ MPa}$$
$$h = \frac{684.9}{88} = 7.78 \text{ mm}$$

Decision: Specify 8 mm leg size
Weldment Specifications:
Pattern: Parallel horizontal fillet welds
Electrode: E6010
Type: Fillet
Length of each bead: 75 mm
Leg size: 8 mm

9-45 Problem 9-44 solves the problem using parallel horizontal fillet welds, each 75 mm long obtaining a leg size rounded up to 8 mm.
For this problem, since the width of the plate is fixed and the length has not been determined, we will explore reducing the leg size by using two vertical beads 75 mm long and two horizontal beads such that the beads have a leg size of 6 mm.

Decision: Use a rectangular weld bead pattern with a leg size of 6 mm (case 5 of Table 9-1 with b unknown and $d = 75$ mm).
Materials:
Attachment and member (1018 HR): $S_y = 220$ MPa, $S_{ut} = 400$ MPa
From Table 9-4, AISC welding code,

$$\tau_{\text{all}} = \min[0.3(400), 0.4(220)] = \min(120, 88) = 88 \text{ MPa}$$

Select a stronger electrode material from Table 9-3.
Decision: Specify E6010

Solving for b: In Prob. 9-44, every term was linear in the unknown h. This made solving for h relatively easy. In this problem, the terms will not be linear in b, and so we will use an iterative solution with a spreadsheet.

Throat area and other properties from Table 9-1:

$$A = 1.414(6)(b + 75) = 8.484(b + 75) \qquad (1)$$

$$J_u = \frac{(b+75)^3}{6}, \ J = 0.707\ (6)\ J_u \ = 0.707(b+75)^3 \qquad (2)$$

Primary shear (τ in MPa, h in mm):

$$\tau'_y = \frac{V}{A} = \frac{12(10^3)}{A} \qquad (3)$$

Secondary shear (See Prob. 9-44 solution for the definition of α) :

$$\tau'' = \frac{Mr}{J}$$

$$\tau''_x = \tau'' \cos\alpha = \frac{Mr}{J}\cos\alpha = \frac{Mr_y}{J} = \frac{12(10^3)(150 + b/2)(37.5)}{0.707(b+75)^3} \qquad (4)$$

$$\tau''_y = \tau'' \sin\alpha = \frac{Mr}{J}\sin\alpha = \frac{Mr_x}{J} = \frac{12(10^3)(150 + b/2)(b/2)}{0.707(b+75)^3} \qquad (5)$$

$$\tau_{\max} = \sqrt{\tau'^{\,2}_y + (\tau''_x + \tau''_y)^2} \qquad (6)$$

Enter Eqs. (1) to (6) into a spreadsheet and iterate for various values of b. A portion of the spreadsheet is shown below.

b (mm)	A (mm^2)	J (mm^4)	τ'_y (Mpa)	τ''_y (Mpa)	τ''_x (Mpa)	$\tau_{\max}$ (Mpa)	
41	984.144	1103553.5	12.19334	69.5254	38.00722	90.12492	
42	992.628	1132340.4	12.08912	67.9566	38.05569	88.63156	
43	1001.112	1161623.6	11.98667	66.43718	38.09065	87.18485	< 88 Mpa
44	1009.596	1191407.4	11.88594	64.96518	38.11291	85.7828	

We see that $b \geq 43$ mm meets the strength goal.

Weldment Specifications:

Pattern: Horizontal parallel weld tracks 43 mm long, vertical parallel weld tracks 75 mm long

Electrode: E6010

Leg size: 6 mm

9-46 *Materials:*

Member and attachment (1018 HR): $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

Table 9-4: $\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = 12.8$ kpsi

Decision: Use E6010 electrode. From Table 9-3: $S_y = 50$ kpsi, $S_{ut} = 62$ kpsi,

$$\tau_{\text{all}} = \min[0.3(62),\ 0.4(50)] = 20 \text{ kpsi}$$

Decision: Since 1018 HR is weaker than the E6010 electrode, use $\tau_{\text{all}} = 12.8$ kpsi

Decision: Use an all-around square weld bead track.

$$l_1 = 6 + a = 6 + 6.25 = 12.25 \text{ in}$$

Throat area and other properties from Table 9-1:

$$A = 1.414h(b+d) = 1.414(h)(6+6) = 16.97h$$

Primary shear

$$\tau_y' = \frac{V}{A} = \frac{F}{A} = \frac{20\left(10^3\right)}{16.97h} = \frac{1179}{h} \text{ psi}$$

Secondary shear

$$J_u = \frac{(b+d)^3}{6} = \frac{(6+6)^3}{6} = 288 \text{ in}^3$$

$$J = 0.707h(288) = 203.6h \text{ in}^4$$

$$\tau_x'' = \tau_y'' = \frac{Mr_y}{J} = \frac{20\left(10^3\right)(6.25+3)(3)}{203.6h} = \frac{2726}{h} \text{ psi}$$

$$\tau_{\max} = \sqrt{\tau_x''^2 + (\tau_y' + \tau_y'')^2} = \frac{1}{h}\sqrt{2726^2 + (1179+2726)^2} = \frac{4762}{h} \text{ psi}$$

Relate stress to strength

$$\tau_{\max} = \tau_{\text{all}} \quad \Rightarrow \quad \frac{4762}{h} = 12.8\left(10^3\right) \quad \Rightarrow \quad h = \frac{4762}{12.8\left(10^3\right)} = 0.372 \text{ in}$$

Decision:

Specify 3/8 in leg size

Specifications:

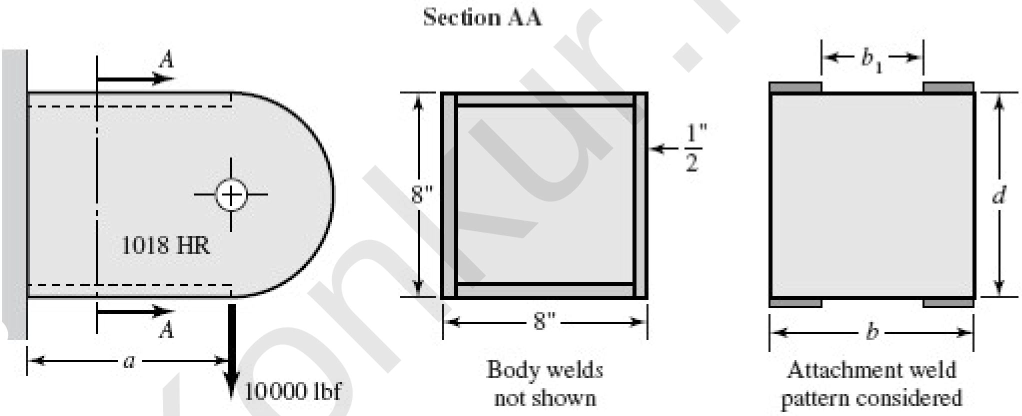
Pattern: All-around square weld bead track
Electrode: E6010
Type of weld: Fillet
Weld bead length: 24 in
Leg size: 3/8 in
Attachment length: 12.25 in

9-47 This is a good analysis task to test a student's understanding.

(1) Solicit information related to a priori decisions.
(2) Solicit design variables b and d.
(3) Find h and round and output all parameters on a single screen. Allow return to Step 1 or Step 2.
(4) When the iteration is complete, the final display can be the bulk of your adequacy assessment.

Such a program can teach too.

9-48 The objective of this design task is to have the students teach themselves that the weld patterns of Table 9-2 can be added or subtracted to obtain the properties of a contemplated weld pattern. The instructor can control the level of complication. We have left the presentation of the drawing to you. Here is *one* possibility. Study the problem's opportunities, and then present this (or your sketch) with the problem assignment.



Use b_1 as the design variable. Express properties as a function of b_1. From Table 9-3, case 3:

$$A = 1.414h(b - b_1)$$

$$I_u = \frac{bd^2}{2} - \frac{b_1 d^2}{2} = \frac{(b - b_1)d^2}{2}$$

$$I = 0.707hI_u$$

$$\tau' = \frac{V}{A} = \frac{F}{1.414h(b - b_1)}$$

$$\tau'' = \frac{Mc}{I} = \frac{Fa(d/2)}{0.707hI_u}$$

Parametric study

Let $a = 10$ in, $b = 8$ in, $d = 8$ in, $b_1 = 2$ in, $\tau_{\text{all}} = 12.8$ kpsi, $l = 2(8 - 2) = 12$ in

$$A = 1.414h(8-2) = 8.48h \text{ in}^2$$
$$I_u = (8-2)(8^2/2) = 192 \text{ in}^3$$
$$I = 0.707(h)(192) = 135.7h \text{ in}^4$$
$$\tau' = \frac{10\,000}{8.48h} = \frac{1179}{h} \text{ psi}$$
$$\tau'' = \frac{10\,000(10)(8/2)}{135.7h} = \frac{2948}{h} \text{ psi}$$
$$\tau_{\max} = \frac{1}{h}\sqrt{1179^2 + 2948^2} = \frac{3175}{h} = 12\,800 \text{ psi}$$

from which $h = 0.248$ in. Do not round off the leg size – something to learn.

$$\text{fom}' = \frac{I_u}{hl} = \frac{192}{0.248(12)} = 64.5 \text{ in}$$
$$A = 8.48(0.248) = 2.10 \text{ in}^2$$
$$I = 135.7(0.248) = 33.65 \text{ in}^4$$
$$\text{vol} = \frac{h^2}{2}l = \frac{0.248^2}{2}12 = 0.369 \text{ in}^3$$
$$\text{eff} = \frac{I}{\text{vol}} = \frac{33.65}{0.369} = 91.2 \text{ in}$$
$$\tau' = \frac{1179}{0.248} = 4754 \text{ psi}$$
$$\tau'' = \frac{2948}{0.248} = 11\,887 \text{ psi}$$
$$\tau_{\max} = \frac{3175}{0.248} = 12\,800 \text{ psi}$$

Now consider the case of uninterrupted welds,

$$b_1 = 0$$
$$A = 1.414(h)(8-0) = 11.31h$$
$$I_u = (8-0)(8^2/2) = 256 \text{ in}^3$$
$$I = 0.707(256)h = 181h \text{ in}^4$$
$$\tau' = \frac{10\,000}{11.31h} = \frac{884}{h}$$
$$\tau'' = \frac{10\,000(10)(8/2)}{181h} = \frac{2210}{h}$$
$$\tau_{\max} = \frac{1}{h}\sqrt{884^2 + 2210^2} = \frac{2380}{h} = \tau_{\text{all}}$$
$$h = \frac{\tau_{\max}}{\tau_{\text{all}}} = \frac{2380}{12\,800} = 0.186 \text{ in}$$

Do not round off h.

$$A = 11.31(0.186) = 2.10 \text{ in}^2$$

$$I = 181(0.186) = 33.67 \text{ in}^4$$

$$\tau' = \frac{884}{0.186} = 4753 \text{ psi}, \quad \text{vol} = \frac{0.186^2}{2}16 = 0.277 \text{ in}^3$$

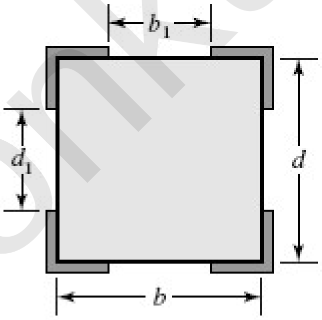
$$\tau'' = \frac{2210}{0.186} = 11882 \text{ psi}$$

$$\text{fom}' = \frac{I_u}{hl} = \frac{256}{0.186(16)} = 86.0 \text{ in}$$

$$\text{eff} = \frac{I}{(h^2/2)l} = \frac{33.67}{(0.186^2/2)16} = 121.7 \text{ in}$$

Conclusions: To meet allowable stress limitations, I and A do not change, nor do τ and σ. To meet the shortened bead length, h is increased proportionately. However, volume of bead laid down increases as h^2. The uninterrupted bead is superior. In this example, we did not round h and as a result we learned something. Our measures of merit are also sensitive to rounding. When the design decision is made, rounding to the next larger standard weld fillet size will decrease the merit.

Had the weld bead gone around the corners, the situation would change. Here is a follow up task analyzing an alternative weld pattern.



9-49 From Table 9-2

For the box $\quad A = 1.414h(b+d)$

Subtracting b_1 from b and d_1 from d

$$A = 1.414h\left(b - b_1 + d - d_1\right)$$

$$I_u = \frac{d^2}{6}(3b+d) - \frac{d_1^3}{6} - \frac{b_1 d^2}{2} = \frac{1}{2}\left(b-b_1\right)d^2 + \frac{1}{6}\left(d^3 - d_1^3\right)$$

Length of bead $\quad l = 2(b - b_1 + d - d_1)$

$$\text{fom} = I_u / hl$$

9-50 Computer programs will vary.

9-51 τ_{all} = 12 kpsi. Use Fig. 9-17(a) for general geometry, but employ = beads and then | | beads.

Horizontal parallel weld bead pattern

$b = 3$ in, $d = 6$ in

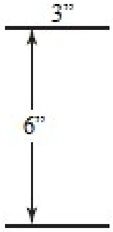


Table 9-2: $A = 1.414hb = 1.414(h)(3) = 4.24h \text{ in}^2$

$$I_u = \frac{bd^2}{2} = \frac{3(6)^2}{2} = 54 \text{ in}^3$$

$$I = 0.707hI_u = 0.707(h)(54) = 38.2h \text{ in}^4$$

$$\tau' = \frac{10}{4.24h} = \frac{2.358}{h} \text{ kpsi}$$

$$\tau'' = \frac{Mc}{I} = \frac{10(10)(6/2)}{38.2h} = \frac{7.853}{h} \text{ kpsi}$$

$$\tau_{\text{max}} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h}\sqrt{2.358^2 + 7.853^2} = \frac{8.199}{h} \text{ kpsi}$$

Equate the maximum and allowable shear stresses.

$$\tau_{\text{max}} = \tau_{\text{all}} = \frac{8.199}{h} = 12$$

from which $h = 0.683$ in. It follows that

$$I = 38.2(0.683) = 26.1 \text{ in}^4$$

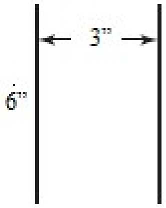
The volume of the weld metal is

$$\text{vol} = \frac{h^2 l}{2} = \frac{(0.683)^2(3+3)}{2} = 1.40 \text{ in}^3$$

The effectiveness, $(\text{eff})_{\text{H}}$, is

$$(\text{eff})_{\text{H}} = \frac{I}{\text{vol}} = \frac{26.1}{1.4} = 18.6 \text{ in} \qquad \textit{Ans.}$$

Vertical parallel weld beads



$b = 3$ in
$d = 6$ in

From Table 9-2, case 2

$$A = 1.414hd = 1.414(h)(6) = 8.48h \text{ in}^2$$

$$I_u = \frac{d^3}{6} = \frac{6^3}{6} = 72 \text{ in}^3$$

$$I = 0.707hI_u = 0.707(h)(72) = 50.9h$$

$$\tau' = \frac{10}{8.48h} = \frac{1.179}{h} \text{ psi}$$

$$\tau'' = \frac{Mc}{I} = \frac{10(10)(6/2)}{50.9h} = \frac{5.894}{h} \text{ psi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h}\sqrt{1.179^2 + 5.894^2} = \frac{6.011}{h} \text{ kpsi}$$

Equating $\tau_{\max}$ to τ_{all} gives $h = 0.501$ in. It follows that

$$I = 50.9(0.501) = 25.5 \text{ in}^4$$

$$\text{vol} = \frac{h^2 l}{2} = \frac{0.501^2}{2}(6+6) = 1.51 \text{ in}^3$$

$$(\text{eff})_V = \frac{I}{\text{vol}} = \frac{25.5}{1.51} = 16.7 \text{ in} \qquad \textit{Ans.}$$

9-52 $F = 0$, $T = 15$ kip·in.

Table 9-1: $J_u = 2\pi r^3 = 2\pi(1)^3 = 6.283 \text{ in}^3$, $J = 0.707(1/4)\,6.283 = 1.111 \text{ in}^4$

$$\tau_{\max} = \frac{Tr}{J} = \frac{15(1)}{1.111} = 13.5 \text{ kpsi} \qquad \textit{Ans.}$$

9-53 $F = 2$ kip, $T = 0$.

Table 9-2: $A = 1.414\,\pi h r = 1.414\,\pi(1/4)(1) = 1.111 \text{ in}^2$
$I_u = \pi r^3 = \pi(1)^3 = 3.142 \text{ in}^3$, $I = 0.707(1/4)\,3.142 = 0.5553 \text{ in}^4$

$$\tau' = \frac{V}{A} = \frac{2}{1.111} = 1.80 \text{ kpsi}$$

$$\tau'' = \frac{Mr}{I} = \frac{2(6)(1)}{0.5553} = 21.6 \text{ kpsi}$$

$$\tau_{\max} = (\tau'^2 + \tau''^2)^{1/2} = (1.80^2 + 21.6^2)^{1/2} = 21.7 \text{ kpsi} \quad \textit{Ans.}$$

9-54 $F = 2$ kip, $T = 15$ kip·in.

Bending:

Table 9-2: $A = 1.414\,\pi h\, r = 1.414\,\pi(1/4)(1) = 1.111 \text{ in}^2$

$I_u = \pi r^3 = \pi(1)^3 = 3.142 \text{ in}^3, \quad I = 0.707(1/4)\,3.142 = 0.5553 \text{ in}^4$

$$\tau' = \frac{V}{A} = \frac{2}{1.111} = 1.80 \text{ kpsi}$$

$$(\tau'')_M = \frac{Mr}{I} = \frac{2(6)(1)}{0.5553} = 21.6 \text{ kpsi}$$

Torsion:

Table 9-1: $J_u = 2\pi r^3 = 2\pi(1)^3 = 6.283 \text{ in}^3, \quad J = 0.707(1/4)\,6.283 = 1.111 \text{ in}^4$

$$(\tau'')_T = \frac{Tr}{J} = \frac{15(1)}{1.111} = 13.5 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + (\tau'')_M^2 + (\tau'')_T^2} = \sqrt{1.80^2 + 21.6^2 + 13.5^2} = 25.5 \text{ kpsi} \quad \textit{Ans.}$$

9-55 $F = 2$ kip, $T = 15$ kip·in.

Bending:

Table 9-2: $A = 1.414\,\pi h\, r = 1.414\,\pi h\,(1) = 4.442h \text{ in}^2$

$I_u = \pi r^3 = \pi(1)^3 = 3.142 \text{ in}^3, \quad I = 0.707\, h\,(3.142) = 2.221h \text{ in}^4$

$$\tau' = \frac{V}{A} = \frac{2}{4.442h} = \frac{0.4502}{h} \text{ kpsi}$$

$$(\tau'')_M = \frac{Mr}{I} = \frac{2(6)(1)}{2.221h} = \frac{5.403}{h} \text{ kpsi}$$

Torsion:

Table 9-1: $J_u = 2\pi r^3 = 2\pi(1)^3 = 6.283 \text{ in}^3$, $J = 0.707\,h\,(6.283) = 4.442 \text{ in}^4$

$$(\tau'')_T = \frac{Tr}{J} = \frac{15(1)}{4.442h} = \frac{3.377}{h} \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + (\tau'')_M^2 + (\tau'')_T^2} = \sqrt{\left(\frac{0.4502}{h}\right)^2 + \left(\frac{5.403}{h}\right)^2 + \left(\frac{3.377}{h}\right)^2} = \frac{6.387}{h} \text{ kpsi}$$

$$\tau_{\max} = \tau_{\text{all}} \quad \Rightarrow \quad \frac{6.387}{h} = 20 \quad \Rightarrow \quad h = 0.319 \text{ in} \quad \textit{Ans.}$$

Should specify a $\frac{3}{8}$ in weld. *Ans.*

9-56 $h = 9$ mm, $d = 200$ mm, $b = 25$ mm

From Table 9-2, case 2:

$$A = 1.414(9)(200) = 2.545(10^3) \text{ mm}^2$$

$$I_u = \frac{d^3}{6} = \frac{200^3}{6} = 1.333\left(10^6\right) \text{ mm}^3$$

$$I = 0.707h\, I_u = 0.707(9)(1.333)(10^6) = 8.484(10^6) \text{ mm}^4$$

$$\tau' = \frac{F}{A} = \frac{25\left(10^3\right)}{2.545(10^3)} = 9.82 \text{ MPa}$$

$$M = 25(150) = 3750 \text{ N}\cdot\text{m}$$

$$\tau'' = \frac{Mc}{I} = \frac{3750(100)}{8.484(10^6)}\left(10^3\right) = 44.20 \text{ MPa}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{9.82^2 + 44.20^2} = 45.3 \text{ MPa} \quad \textit{Ans.}$$

9-57 $h = 0.25$ in, $b = 2.5$ in, $d = 5$ in.

Table 9-2, case 5: $A = 0.707h\,(b + 2d) = 0.707(0.25)[2.5 + 2(5)] = 2.209 \text{ in}^2$

$$\bar{y} = \frac{d^2}{b+2d} = \frac{5^2}{2.5+2(5)} = 2 \text{ in}$$

$$I_u = \frac{2d^3}{3} - 2d^2\bar{y} + (b+2d)\bar{y}^2$$

$$= \frac{2(5^3)}{3} - 2(5^2)(2) + [2.5+2(5)](2^2) = 33.33 \text{ in}^3$$

$$I = 0.707\, h\, I_u = 0.707(1/4)(33.33) = 5.891 \text{ in}^4$$

Primary shear:

$$\tau' = \frac{F}{A} = \frac{2}{2.209} = 0.905 \text{ kpsi}$$

Secondary shear (the critical location is at the bottom of the bracket):

$$y = 5 - 2 = 3 \text{ in}$$

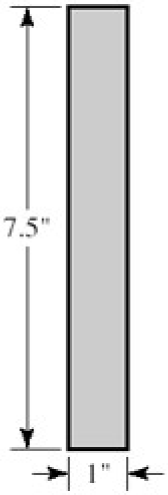
$$\tau'' = \frac{My}{I} = \frac{2(5)(3)}{5.891} = 5.093 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{0.905^2 + 5.093^2} = 5.173 \text{ kpsi}$$

$$n = \frac{\tau_{\text{all}}}{\tau_{\max}} = \frac{18}{5.173} = 3.48 \qquad \textit{Ans.}$$

9-58 The largest possible weld size is 1/16 in. This is a small weld and thus difficult to accomplish. The bracket's load-carrying capability is not known. There are geometry problems associated with sheet metal folding, load-placement and location of the center of twist. This is not available to us. We will identify the strongest possible weldment. Use a rectangular, weld-all-around pattern – Table 9-2, case 6:

$$\begin{aligned} A &= 1.414\,h(b+d) = 1.414(1/16)(1+7.5) \\ &= 0.7512 \text{ in}^2 \\ \bar{x} &= b/2 = 0.5 \text{ in} \\ \bar{y} &= d/2 = 7.5/2 = 3.75 \text{ in} \\ I_u &= \frac{d^2}{6}(3b+d) = \frac{7.5^2}{6}[3(1)+7.5] = 98.44 \text{ in}^3 \\ I &= 0.707hI_u = 0.707(1/16)(98.44) = 4.350 \text{ in}^4 \\ M &= (3.75+0.5)W = 4.25W \\ \tau' &= \frac{V}{A} = \frac{W}{0.7512} = 1.331W \\ \tau'' &= \frac{Mc}{I} = \frac{4.25W(7.5/2)}{4.350} = 3.664W \\ \tau_{\max} &= \sqrt{\tau'^2+\tau''^2} = W\sqrt{1.331^2+3.664^2} = 3.90W \end{aligned}$$



Material properties: The allowable stress given is low. Let's demonstrate that. For the 1020 CD bracket, use HR properties of $S_y = 30$ kpsi and $S_{ut} = 55$. The 1030 HR support, $S_y = 37.5$ kpsi and $S_{ut} = 68$. The E6010 electrode has strengths of $S_y = 50$ and $S_{ut} = 62$ kpsi.

Allowable stresses:

1020 HR: $\tau_{\text{all}} = \min[0.3(55), 0.4(30)] = \min(16.5, 12) = 12$ kpsi

1020 HR: $\tau_{\text{all}} = \min[0.3(68), 0.4(37.5)] = \min(20.4, 15) = 15$ kpsi

E6010: $\tau_{\text{all}} = \min[0.3(62), 0.4(50)] = \min(18.6, 20) = 18.6$ kpsi

Since Table 9-6 gives 18.0 kpsi as the allowable shear stress, use this lower value. Therefore, the allowable shear stress is

$$\tau_{\text{all}} = \min(14.4, 12, 18.0) = 12 \text{ kpsi}$$

However, the allowable stress in the problem statement is 1.5 kpsi which is low from the weldment perspective. The load associated with this strength is

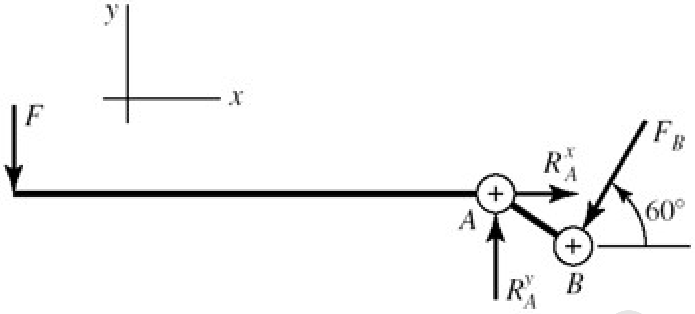
$$\begin{aligned} \tau_{\max} &= \tau_{\text{all}} = 3.90W = 1500 \\ W &= \frac{1500}{3.90} = 385 \text{ lbf} \end{aligned}$$

If the welding can be accomplished (1/16 leg size is a small weld), the weld strength is 12 000 psi and the load associated with this strength is $W = 12\,000/3.90 = 3077$ lbf. Can the bracket carry such a load?

There are geometry problems associated with sheet metal folding. Load placement is important and the center of twist has not been identified. Also, the load-carrying capability of the top bend is unknown.

These uncertainties may require the use of a different weld pattern. Our solution provides the best weldment and thus insight for comparing a welded joint to one which employs screw fasteners.

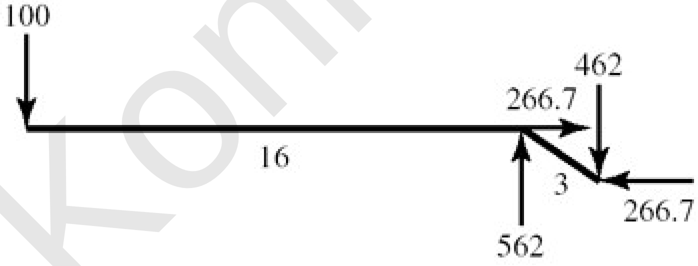
9-59



$$F = 100 \text{ lbf}, \quad \tau_{\text{all}} = 3 \text{ kpsi}$$
$$F_B = 100(16/3) = 533.3 \text{ lbf}$$
$$F_B^x = -533.3\cos 60^\circ = -266.7 \text{ lbf}$$
$$F_B^y = -533.3\cos 30^\circ = -462 \text{ lbf}$$

It follows that $R_A^y = 562$ lbf and $R_A^x = 266.7$ lbf, $R_A = 622$ lbf

$$M = 100(16) = 1600 \text{ lbf} \cdot \text{in}$$



The OD of the tubes is 1 in. From Table 9-1, case 6:

$$A = 2\left[1.414(\pi hr)\right] = 2(1.414)(\pi h)(1/2) = 4.442h \text{ in}^2$$
$$J_u = 2\pi r^3 = 2\pi(1/2)^3 = 0.7854 \text{ in}^3$$
$$J = 2(0.707)hJ_u = 1.414(0.7854)h = 1.111h \text{ in}^4$$

The weld only carries the torsional load between the handle and tube *A*. Consequently, the primary shear in the weld is zero, and the maximum shear stress is comprised entirely of the secondary shear.

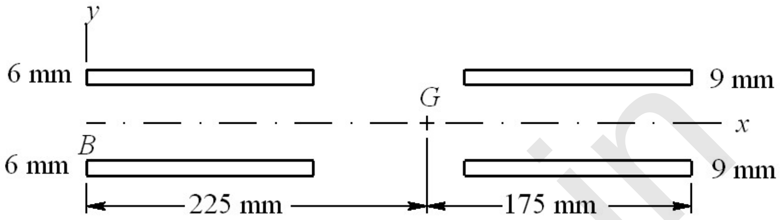
$$\tau_{\max} = \tau'' = \frac{Tc}{J} = \frac{Mc}{J} = \frac{1600(0.5)}{1.111h} = \frac{720.1}{h}\text{psi}$$

$$\tau_{\max} = \tau_{\text{all}} = \frac{720.1}{h} = 3000$$

$$h = \frac{720.1}{3000} = 0.240 \rightarrow 1/4 \text{ in}$$

Decision: Use 1/4 in fillet welds *Ans.*

9-60



For the pattern in bending shown, find the centroid *G* of the weld group.

$$\bar{x} = \frac{75(6)(150) + 325(9)(150)}{(6)(150) + (9)(150)} = 225 \text{ mm}$$

$$I_{6\text{mm}} = 2\left(I_G + A\bar{x}^2\right)_{6\text{mm}}$$

$$= 2\left[\frac{0.707(6)(150^3)}{12} + 0.707(6)(150)(225-75)^2\right] = 31.02(10^6) \text{ mm}^4$$

$$I_{9\text{mm}} = 2\left[\frac{0.707(9)(150^3)}{12} + 0.707(9)(150)(175-75)^2\right] = 22.67(10^6) \text{ mm}^4$$

$$I = I_{6\text{ mm}} + I_{9\text{ mm}} = (31.02 + 22.67)(10^6) = 53.69(10^6) \text{ mm}^4$$

The critical location is at *B*. With τ in MPa, and *F* in kN

$$\tau' = \frac{V}{A} = \frac{F(10^3)}{2[0.707(6+9)(150)]} = 0.3143F$$

$$\tau'' = \frac{Mc}{I} = \frac{200F(10^3)(225)}{53.69(10^6)} = 0.8381F$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = F\sqrt{0.3143^2 + 0.8381^2} = 0.8951F$$

Materials:

1015 HR (Table A-20): $S_y = 190$ MPa, E6010 Electrode(Table 9-3): $S_y = 345$ MPa

Eq. (5-21): $\tau_{\text{all}} = 0.577(190) = 109.6$ MPa

$$F = \frac{\tau_{\text{all}}/n}{0.8951} = \frac{109.6/2}{0.8951} = 61.2 \text{ kN} \qquad \textit{Ans.}$$

9-61 In the textbook, Fig. Problem 9-61*b* is a free-body diagram of the bracket. Forces and moments that act on the welds are equal, but of opposite sense.

(a) $M = 1200(0.366) = 439$ lbf · in *Ans.*

(b) $F_y = 1200 \sin 30° = 600$ lbf *Ans.*

(c) $F_x = 1200 \cos 30° = 1039$ lbf *Ans.*

(d) From Table 9-2, case 6:

$$A = 1.414(0.25)(0.25 + 2.5) = 0.972 \text{ in}^2$$

$$I_u = \frac{d^2}{6}(3b + d) = \frac{2.5^2}{6}[3(0.25) + 2.5] = 3.39 \text{ in}^3$$

The second area moment about an axis through G and parallel to z is

$$I = 0.707hI_u = 0.707(0.25)(3.39) = 0.599 \text{ in}^4 \qquad \textit{Ans.}$$

(e) Refer to Fig. Problem 9-61*b*. The shear stress due to F_y is

$$\tau_1 = \frac{F_y}{A} = \frac{600}{0.972} = 617 \text{ psi}$$

The shear stress along the throat due to F_x is

$$\tau_2 = \frac{F_x}{A} = \frac{1039}{0.972} = 1069 \text{ psi}$$

The resultant of τ_1 and τ_2 is in the throat plane

$$\tau' = \sqrt{\tau_1^2 + \tau_2^2} = \sqrt{617^2 + 1069^2} = 1234 \text{ psi}$$

The bending of the throat gives

$$\tau'' = \frac{Mc}{I} = \frac{439(1.25)}{0.599} = 916 \text{ psi}$$

The maximum shear stress is

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{1234^2 + 916^2} = 1537 \text{ psi} \quad \textit{Ans.}$$

(f) *Materials:*

1018 HR Member: $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi (Table A-20)
E6010 Electrode: $S_y = 50$ kpsi (Table 9-3)

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{0.577 S_y}{\tau_{\max}} = \frac{0.577(32)}{1.537} = 12.0 \quad \textit{Ans.}$$

(g) Bending in the attachment near the base. The cross-sectional area is approximately equal to *bh*.

$$A_1 \approx bh = 0.25(2.5) = 0.625 \text{ in}^2$$

$$\tau_{xy} = \frac{F_x}{A_1} = \frac{1039}{0.625} = 1662 \text{ psi}$$

$$\frac{I}{c} = \frac{bd^2}{6} = \frac{0.25(2.5)^2}{6} = 0.260 \text{ in}^3$$

At location *A*,

$$\sigma_y = \frac{F_y}{A_1} + \frac{M}{I/c}$$

$$\sigma_y = \frac{600}{0.625} + \frac{439}{0.260} = 2648 \text{ psi}$$

The von Mises stress σ' is

$$\sigma' = \sqrt{\sigma_y^2 + 3\tau_{xy}^2} = \sqrt{2648^2 + 3(1662)^2} = 3912 \text{ psi}$$

Thus, the factor of safety is,

$$n = \frac{S_y}{\sigma'} = \frac{32}{3.912} = 8.18 \quad \textit{Ans.}$$

The clip on the mooring line bears against the side of the 1/2-in hole. If the clip fills the hole

$$\sigma = \frac{F}{td} = \frac{-1200}{0.25(0.50)} = -9600 \text{ psi}$$

$$n = -\frac{S_y}{\sigma'} = -\frac{32(10^3)}{-9600} = 3.33 \quad \textit{Ans.}$$

Further investigation of this situation requires more detail than is included in the task statement.

(h) In shear fatigue, the weakest constituent of the weld melt is 1018 HR with $S_{ut} = 58$ kpsi, Eq. (6-10), gives

$$S'_e = 0.5S_{ut} = 0.5(58) = 29.0 \text{ kpsi}$$

Eq. (6-18): $k_a = 12.7(58)^{-0.758} = 0.585$

For uniform shear stress on the throat, assume $k_b = 1$.

Eq.(6-25): $k_c = 0.59$

From Eq. (6-17), the endurance strength in shear is

$$S_{se} = 0.585(1)(0.59)(29.0) = 10.0 \text{ kpsi}$$

From Table 9-5, the shear stress-concentration factor is $K_{fs} = 2.7$. The loading is repeatedly-applied

$$\tau_a = \tau_m = K_{fs}\frac{\tau_{\max}}{2} = 2.7\frac{1.537}{2} = 2.07 \text{ kpsi}$$

Eq. (6-48): Gerber factor of safety n_f, adjusted for shear, with $S_{su} = 0.67S_{ut}$

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)}\right]$$

$$= \frac{1}{2}\left[\frac{0.67(58)}{2.07}\right]^2 \left(\frac{2.07}{10.0}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(2.07)(10.0)}{0.67(58)(2.07)}\right]^2}\right\} = 4.55 \quad Ans.$$

Attachment metal should be checked for bending fatigue.

9-62 (**a**) Use $b = d = 4$ in. Since $h = 5/8$ in, the primary shear is

$$\tau' = \frac{F}{1.414(5/8)(4)} = 0.2829F$$

The secondary shear calculations, for a moment arm of 14 in give

$$J_u = \frac{4[3(4^2) + 4^2]}{6} = 42.67 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(5/8)42.67 = 18.85 \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{14F(2)}{18.85} = 1.485F$$

Thus, the maximum shear and allowable load are:

$$\tau_{\max} = F\sqrt{1.485^2 + (0.2829 + 1.485)^2} = 2.309F$$

$$F = \frac{\tau_{\text{all}}}{2.309} = \frac{25}{2.309} = 10.8 \text{ kip} \quad \textit{Ans.}$$

The load for part (a) has increased by a factor of 10.8/2.71 = 3.99 *Ans*.

(**b**) From Prob. 9-20*b*, $\tau_{\text{all}} = 11$ kpsi

$$F_{\text{all}} = \frac{\tau_{\text{all}}}{2.309} = \frac{11}{2.309} = 4.76 \text{ kip}$$

The allowable load in part (b) has increased by a factor of 4.76/1.19 = 4 *Ans*.

9-63 Purchase the hook having the design shown in Fig. Problem 9-63*b*. Referring to text Fig. 9-29*a*, this design reduces peel stresses.

9-64 **(a)**

$$\bar{\tau} = \frac{1}{l}\int_{-l/2}^{l/2} \frac{P\omega\cosh(\omega x)}{4b\sinh(\omega l\ /\ 2)}dx = A_1\int_{-l/2}^{l/2}\cosh(\omega x)\,dx = \frac{A_1}{\omega}\sinh(\omega x)\Big|_{-l/2}^{l/2}$$

$$= \frac{A_1}{\omega}[\sinh(\omega l\ /\ 2) - \sinh(-\omega l\ /\ 2)] = \frac{A_1}{\omega}[\sinh(\omega l\ /\ 2) - (-\sinh(\omega l\ /\ 2))]$$

$$= \frac{2A_1\sinh(\omega l\ /\ 2)}{\omega} = \frac{P\omega}{4bl\sinh(\omega l\ /\ 2)}[2\sinh(\omega l\ /\ 2)] = \frac{P}{2bl} \quad \textit{Ans.}$$

(b) $$\tau(l/2) = \frac{P\omega\cosh(\omega l/2)}{4b\sinh(\omega l/2)} = \frac{P\omega}{4b\tanh(\omega l/2)} \quad \textit{Ans.}$$

(c) $$K = \frac{\tau(l\ /\ 2)}{\bar{\tau}} = \frac{P\omega}{4b\tanh(\omega l\ /\ 2)}\left(\frac{2bl}{P}\right) = \frac{\omega l\ /\ 2}{\tanh(\omega l\ /\ 2)} \quad \textit{Ans.}$$

For computer programming, it can be useful to express the hyperbolic tangent in terms of exponentials:

$$K = \frac{\omega l}{2}\,\frac{\exp(\omega l\ /\ 2) - \exp(-\omega l\ /\ 2)}{\exp(\omega l\ /\ 2) + \exp(-\omega l\ /\ 2)} \quad \textit{Ans.}$$

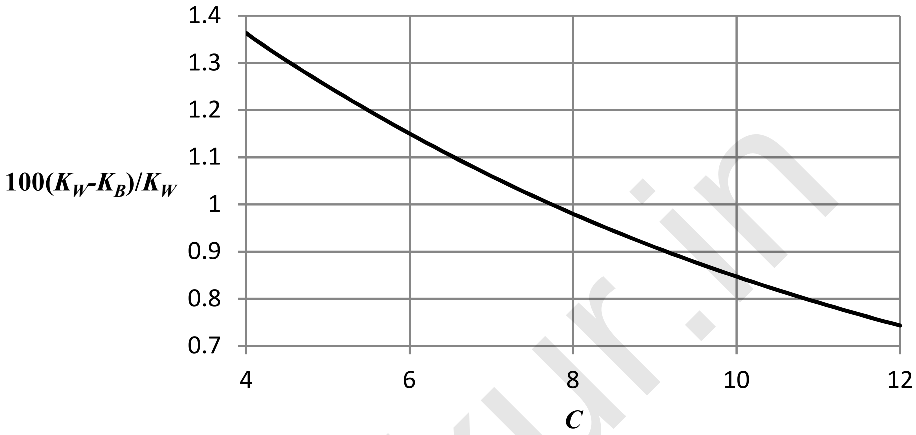
9-65 This is a computer programming exercise. All programs will vary.

Chapter 10

10-1 From Eqs. (10-4) and (10-5)

$$K_W - K_B = \frac{4C-1}{4C-4} + \frac{0.615}{C} - \frac{4C+2}{4C-3}$$

Plot $100(K_W - K_B)/K_W$ vs. C for $4 \le C \le 12$ obtaining



We see the maximum and minimum occur at C = 4 and 12 respectively where

Maximum = 1.36 % *Ans*., and Minimum = 0.743 % *Ans*.

10-2 $A = Sd^m$

$\dim(A_{\text{uscu}}) = [\dim(S)\dim(d^m)]_{\text{uscu}} = \text{kpsi}\cdot\text{in}^m$

$\dim(A_{\text{SI}}) = [\dim(S)\dim(d^m)]_{\text{SI}} = \text{MPa}\cdot\text{mm}^m$

$$A_{\text{SI}} = \frac{\text{MPa}}{\text{kpsi}}\cdot\frac{\text{mm}^m}{\text{in}^m}A_{\text{uscu}} = 6.894\,757(25.4)^m A_{\text{uscu}} \doteq 6.895(25.4)^m A_{\text{uscu}} \quad \textit{Ans.}$$

For music wire, from Table 10-4:

$A_{\text{uscu}} = 201$ kpsi·in^m, $m = 0.145$; what is A_{SI}?

$A_{\text{SI}} = 6.895(25.4)^{0.145}(201) = 2215$ MPa·mm^m *Ans*.

10-3 Given: Music wire, d = 2.5 mm, OD = 31 mm, plain ground ends, N_t = 14 coils.

(a) Table 10-1: $N_a = N_t - 1 = 14 - 1 = 13$ coils

$D = \text{OD} - d = 31 - 2.5 = 28.5$ mm

$C = D/d = 28.5/2.5 = 11.4$

Table 10-5: $d = 2.5/25.4 = 0.098$ in $\Rightarrow$ $G = 81.0(10^3)$ MPa

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{2.5^4(81)10^3}{8(28.5^3)13} = 1.314 \text{ N/mm}$$ *Ans.*

(b) Table 10-1: $L_s = d\,N_t = 2.5(14) = 35$ mm

Table 10-4: $m = 0.145$, $A = 2211$ MPa·mm^m

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{2211}{2.5^{0.145}} = 1936 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.45(1936) = 871.2$ MPa

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(11.4)+2}{4(11.4)-3} = 1.117$$

Eq. (10-7): $$F_s = \frac{\pi d^3 S_{sy}}{8K_B D} = \frac{\pi(2.5^3)871.2}{8(1.117)28.5} = 167.9 \text{ N}$$ *Ans.*

(c) $$L_0 = \frac{F_s}{k} + L_s = \frac{167.9}{1.314} + 35 = 162.8 \text{ mm}$$ *Ans.*

(d) $$(L_0)_{\text{cr}} = \frac{2.63(28.5)}{0.5} = 149.9 \text{ mm}$$. Spring needs to be supported. *Ans.*

10-4 Given: Design load, $F_1 = 130$ N.

Referring to Prob. 10-3 solution, $C = 11.4$, $N_a = 13$ coils, $S_{sy} = 871.2$ MPa, $F_s = 167.9$ N, $L_0 = 162.8$ mm and $(L_0)_{\text{cr}} = 149.9$ mm.
Eq. (10-18): $4 \le C \le 12$ $C = 11.4$ *O.K.*
Eq. (10-19): $3 \le N_a \le 15$ $N_a = 13$ *O.K.*
Eq. (10-17): $$\xi = \frac{F_s}{F_1} - 1 = \frac{167.9}{130} - 1 = 0.29$$

Eq. (10-20): $\xi \geq 0.15$, $\xi = 0.29$ *O.K.*

From Eq. (10-7) for static service

$$\tau_1 = K_B\left(\frac{8F_1D}{\pi d^3}\right) = 1.117\frac{8(130)(28.5)}{\pi(2.5)^3} = 674 \text{ MPa}$$

$$n = \frac{S_{sy}}{\tau_1} = \frac{871.2}{674} = 1.29$$

Eq. (10-21): $n_s \geq 1.2$, $n = 1.29$ *O.K.*

$$\tau_s = \tau_1\left(\frac{167.9}{130}\right) = 674\left(\frac{167.9}{130}\right) = 870.5 \text{ MPa}$$

$$S_{sy} / \tau_s = 871.2 / 870.5 \square 1$$

$S_{sy}/\tau_s \geq (n_s)_d$: Not solid-safe (but was the basis of the design). *Not O.K.*

$L_0 \leq (L_0)_{\text{cr}}$: 162.8 ≥ 149.9 *Not O.K.*

Design is unsatisfactory. Operate over a rod? *Ans.*

10-5 Given: Oil-tempered wire, $d = 0.2$ in, $D = 2$ in, $N_t = 12$ coils, $L_0 = 5$ in, squared ends.

(**a**) Table 10-1: $L_s = d\,(N_t + 1) = 0.2(12 + 1) = 2.6$ in *Ans.*

(**b**) Table 10-1: $N_a = N_t - 2 = 12 - 2 = 10$ coils

Table 10-5: $G = 11.2$ Mpsi

Eq. (10-9):
$$k = \frac{d^4G}{8D^3N_a} = \frac{0.2^4\left(11.2\right)10^6}{8\left(2^3\right)10} = 28 \text{ lbf/in}$$

$F_s = k\,y_s = k\,(L_0 - L_s) = 28(5 - 2.6) = 67.2$ lbf *Ans.*

(**c**) Eq. (10-1): $C = D/d = 2/0.2 = 10$

Eq. (10-5):
$$K_B = \frac{4C+2}{4C-3} = \frac{4\left(10\right)+2}{4\left(10\right)-3} = 1.135$$

Eq. (10-7):
$$\tau_s = K_B\frac{8F_sD}{\pi d^3} = 1.135\frac{8\left(67.2\right)2}{\pi\left(0.2^3\right)} = 48.56\left(10^3\right) \text{ psi}$$

Table 10-4: $m = 0.187$, $A = 147 \text{ kpsi}\cdot\text{in}^m$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{147}{0.2^{0.187}} = 198.6 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.50\ S_{ut} = 0.50(198.6) = 99.3$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{99.3}{48.56} = 2.04 \qquad \textit{Ans.}$$

10-6 Given: Oil-tempered wire, $d = 4$ mm, $C = 10$, plain ends, $L_0 = 80$ mm, and at $F = 50$ N, $y = 15$ mm.

(a) $k = F/y = 50/15 = 3.333$ N/mm *Ans.*

(b) $D = Cd = 10(4) = 40$ mm

OD $= D + d = 40 + 4 = 44$ mm *Ans.*

(c) From Table 10-5, $G = 77.2$ GPa

Eq. (10-9): $$N_a = \frac{d^4G}{8kD^3} = \frac{4^4(77.2)10^3}{8(3.333)40^3} = 11.6 \text{ coils}$$

Table 10-1: $N_t = N_a = 11.6$ coils *Ans.*

(d) Table 10-1: $L_s = d\,(N_t + 1) = 4(11.6 + 1) = 50.4$ mm *Ans.*

(e) Table 10-4: $m = 0.187$, $A = 1855$ MPa·mm^m

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{1855}{4^{0.187}} = 1431 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.50\ S_{ut} = 0.50(1431) = 715.5$ MPa

$y_s = L_0 - L_s = 80 - 50.4 = 29.6$ mm

$F_s = k\,y_s = 3.333(29.6) = 98.66$ N

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$$

Eq. (10-7): $$\tau_s = K_B\frac{8F_sD}{\pi d^3} = 1.135\frac{8(98.66)40}{\pi(4^3)} = 178.2 \text{ MPa}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{715.5}{178.2} = 4.02 \qquad Ans.$$

10-7 Static service spring with: HD steel wire, $d = 0.080$ in, OD = 0.880 in, $N_t = 8$ coils, plain and ground ends.

Preliminaries

Table 10-5: $A = 140 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.190$

Eq. (10-14): $S_{ut} = \frac{A}{d^m} = \frac{140}{0.080^{0.190}} = 226.2 \text{ kpsi}$

Table 10-6: $S_{sy} = 0.45(226.2) = 101.8$ kpsi

Then,

$D = \text{OD} - d = 0.880 - 0.080 = 0.8$ in

Eq. (10-1): $C = D/d = 0.8/0.08 = 10$

Eq. (10-5): $K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$

Table 10-1: $N_a = N_t - 1 = 8 - 1 = 7$ coils

$L_s = dN_t = 0.08(8) = 0.64$ in

Eq. (10-7) For solid-safe, $n_s = 1.2$:

$$F_s = \frac{\pi d^3 S_{sy} / n_s}{8K_B D} = \frac{\pi\left(0.08^3\right)\left[101.8\left(10^3\right)/1.2\right]}{8(1.135)(0.8)} = 18.78 \text{ lbf}$$

Eq. (10-9): $k = \frac{d^4 G}{8D^3 N_a} = \frac{0.08^4\left(11.5\right)10^6}{8\left(0.8^3\right)7} = 16.43 \text{ lbf/in}$

$$y_s = \frac{F_s}{k} = \frac{18.78}{16.43} = 1.14 \text{ in}$$

(a) $L_0 = y_s + L_s = 1.14 + 0.64 = 1.78$ in *Ans.*

(b) Table 10-1: $p = \frac{L_0}{N_t} = \frac{1.78}{8} = 0.223$ in *Ans.*

(c) From above: $F_s = 18.78$ lbf *Ans.*

(d) From above: $k = 16.43$ lbf/in *Ans.*

(e) Table 10-2 and Eq. (10-13): $(L_0)_{\text{cr}} = \frac{2.63D}{\alpha} = \frac{2.63(0.8)}{0.5} = 4.21$ in

Since $L_0 < (L_0)_{\text{cr}}$, buckling is unlikely *Ans.*

10-8 Given: Design load, $F_1 = 16.5$ lbf.

Referring to Prob. 10-7 solution, $C = 10$, $N_a = 7$ coils, $S_{sy} = 101.8$ kpsi, $F_s = 18.78$ lbf, $y_s = 1.14$ in, $L_0 = 1.78$ in, and $(L_0)_{\text{cr}} = 4.21$ in.

Eq. (10-18): $4 \le C \le 12 \qquad C = 10 \qquad$ *O.K.*

Eq. (10-19): $3 \le N_a \le 15$ $N_a = 7$ *O.K.*

Eq. (10-17): $\xi = \dfrac{F_s}{F_1} - 1 = \dfrac{18.78}{16.5} - 1 = 0.14$

Eq. (10-20): $\xi \ge 0.15$, $\xi = 0.14$ *not O.K.*, but probably acceptable.

From Eq. (10-7) for static service

$$\tau_1 = K_B\left(\frac{8F_1D}{\pi d^3}\right) = 1.135\frac{8(16.5)(0.8)}{\pi(0.080)^3} = 74.5\left(10^3\right) \text{ psi} = 74.5 \text{ kpsi}$$

$$n = \frac{S_{sy}}{\tau_1} = \frac{101.8}{74.5} = 1.37$$

Eq. (10-21): $n_s \ge 1.2$, $n = 1.37$ *O.K.*

$$\tau_s = \tau_1\left(\frac{18.78}{16.5}\right) = 74.5\left(\frac{18.78}{16.5}\right) = 84.8 \text{ kpsi}$$

$$n_s = S_{sy} / \tau_s = 101.8 / 84.8 = 1.20$$

Eq. (10-21): $n_s \ge 1.2$, $n_s = 1.2$ It is solid-safe (basis of design). *O.K.*

Eq. (10-13) and Table 10-2: $L_0 \le (L_0)_{cr}$ 1.78 in ≤ 4.21 in *O.K.*

10-9 Given: A228 music wire, squared and ground ends, $d = 0.007$ in, OD = 0.038 in, $L_0 = 0.58$ in, $N_t = 38$ coils.

$D = \text{OD} - d = 0.038 - 0.007 = 0.031$ in

Eq. (10-1): $C = D/d = 0.031/0.007 = 4.429$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(4.429)+2}{4(4.429)-3} = 1.340$

Table 10-1: $N_a = N_t - 2 = 38 - 2 = 36$ coils (high)

Table 10-5: $G = 12.0$ Mpsi

Eq. (10-9): $k = \dfrac{d^4G}{8D^3N_a} = \dfrac{0.007^4(12.0)10^6}{8\left(0.031^3\right)36} = 3.358$ lbf/in

Table 10-1: $L_s = dN_t = 0.007(38) = 0.266$ in

$y_s = L_0 - L_s = 0.58 - 0.266 = 0.314$ in

$F_s = ky_s = 3.358(0.314) = 1.054$ lbf

Eq. (10-7): $\tau_s = K_B\dfrac{8F_sD}{\pi d^3} = 1.340\dfrac{8(1.054)0.031}{\pi\left(0.007^3\right)} = 325.1\left(10^3\right)$ psi (1)

Table 10-4: $A = 201$ kpsi·in^m, $m = 0.145$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{201}{0.007^{0.145}} = 412.7\ \text{kpsi}$

Table 10-6: $S_{sy} = 0.45\ S_{ut} = 0.45(412.7) = 185.7$ kpsi

$\tau_s > S_{sy}$, that is, 325.1 > 185.7 kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_B kD} = \frac{\left[185.7\left(10^3\right)/1.2\right]\pi\left(0.007^3\right)}{8\left(1.340\right)3.358\left(0.031\right)} = 0.149\ \text{in}$$

The free length should be wound to

$$L_0 = L_s + y_s = 0.266 + 0.149 = 0.415\ \text{in} \qquad Ans.$$

This only addresses the solid-safe criteria. There are additional problems.

10-10 Given: B159 phosphor-bronze, squared and ground. ends, $d = 0.014$ in, OD = 0.128 in, $L_0 = 0.50$ in, $N_t = 16$ coils.

$D = \text{OD} - d = 0.128 - 0.014 = 0.114$ in

Eq. (10-1): $C = D/d = 0.114/0.014 = 8.143$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4\left(8.143\right)+2}{4\left(8.143\right)-3} = 1.169$

Table 10-1: $N_a = N_t - 2 = 16 - 2 = 14$ coils

Table 10-5: $G = 6$ Mpsi

Eq. (10-9): $k = \dfrac{d^4G}{8D^3N_a} = \dfrac{0.014^4\left(6\right)10^6}{8\left(0.114^3\right)14} = 1.389\ \text{lbf/in}$

Table 10-1: $L_s = dN_t = 0.014(16) = 0.224$ in

$y_s = L_0 - L_s = 0.50 - 0.224 = 0.276$ in

$F_s = ky_s = 1.389(0.276) = 0.3834$ lbf

Eq. (10-7): $\tau_s = K_B \dfrac{8F_sD}{\pi d^3} = 1.169\dfrac{8\left(0.3834\right)0.114}{\pi\left(0.014^3\right)} = 47.42\left(10^3\right)\ \text{psi} \qquad (1)$

Table 10-4: $A = 145\ \text{kpsi}\cdot\text{in}^m$, $m = 0$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{145}{0.014^0} = 145\ \text{kpsi}$

Table 10-6: $S_{sy} = 0.35\ S_{ut} = 0.35(135) = 47.25$ kpsi

$\tau_s > S_{sy}$, that is, 47.42 > 47.25 kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_B kD} = \frac{\left[47.25\left(10^3\right)/1.2\right]\pi\left(0.014^3\right)}{8\left(1.169\right)1.389\left(0.114\right)} = 0.229\ \text{in}$$

The free length should be wound to

$L_0 = L_s + y_s = 0.224 + 0.229 = 0.453$ in *Ans.*

10-11 Given: A313 stainless steel, squared and ground ends, $d = 0.050$ in, OD = 0.250 in, $L_0 = 0.68$ in, $N_t = 11.2$ coils.

$D = \text{OD} - d = 0.250 - 0.050 = 0.200$ in

Eq. (10-1): $C = D/d = 0.200/0.050 = 4$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(4)+2}{4(4)-3} = 1.385$$

Table 10-1: $N_a = N_t - 2 = 11.2 - 2 = 9.2$ coils

Table 10-5: $G = 10$ Mpsi

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.050^4 (10)10^6}{8(0.2^3)9.2} = 106.1 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.050(11.2) = 0.56$ in

$y_s = L_0 - L_s = 0.68 - 0.56 = 0.12$ in

$F_s = ky_s = 106.1(0.12) = 12.73$ lbf

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.385 \frac{8(12.73)0.2}{\pi(0.050^3)} = 71.8(10^3) \text{ psi}$$

Table 10-4: $A = 169$ kpsi·in^m, $m = 0.146$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{169}{0.050^{0.146}} = 261.7 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.35(261.7) = 91.6$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{91.6}{71.8} = 1.28$$ Spring is solid-safe ($n_s > 1.2$) *Ans.*

10-12 Given: A227 hard-drawn wire, squared and ground ends, $d = 0.148$ in, OD = 2.12 in, $L_0 = 2.5$ in, $N_t = 5.75$ coils.

$D = \text{OD} - d = 2.12 - 0.148 = 1.972$ in

Eq. (10-1): $C = D/d = 1.972/0.148 = 13.32$ (high)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(13.32)+2}{4(13.32)-3} = 1.099$$

Table 10-1: $N_a = N_t - 2 = 5.75 - 2 = 3.75$ coils

Table 10-5: $G = 11.4$ Mpsi

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.148^4 (11.4)10^6}{8(1.972^3)3.75} = 23.77 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.148(5.75) = 0.851$ in

$y_s = L_0 - L_s = 2.5 - 0.851 = 1.649$ in

$F_s = ky_s = 23.77(1.649) = 39.20$ lbf

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.099\dfrac{8(39.20)1.972}{\pi(0.148^3)} = 66.7(10^3)$ psi

Table 10-4: $A = 140$ kpsi·in^m, $m = 0.190$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{140}{0.148^{0.190}} = 201.3$ kpsi

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.45(201.3) = 90.6$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{90.6}{66.7} = 1.36$$ Spring is solid-safe ($n_s > 1.2$) *Ans.*

10-13 Given: A229 OQ&T steel, squared and ground ends, $d = 0.138$ in, OD = 0.92 in, $L_0 = 2.86$ in, $N_t = 12$ coils.

$D = \text{OD} - d = 0.92 - 0.138 = 0.782$ in

Eq. (10-1): $C = D/d = 0.782/0.138 = 5.667$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(5.667)+2}{4(5.667)-3} = 1.254$

Table 10-1: $N_a = N_t - 2 = 12 - 2 = 10$ coils

A229 OQ&T steel is not given in Table 10-5. From Table A-5, for carbon steels, $G = 11.5$ Mpsi.

Eq. (10-9): $k = \dfrac{d^4 G}{8D^3 N_a} = \dfrac{0.138^4(11.5)10^6}{8(0.782^3)10} = 109.0$ lbf/in

Table 10-1: $L_s = dN_t = 0.138(12) = 1.656$ in
$y_s = L_0 - L_s = 2.86 - 1.656 = 1.204$ in
$F_s = ky_s = 109.0(1.204) = 131.2$ lbf

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.254\dfrac{8(131.2)0.782}{\pi(0.138^3)} = 124.7(10^3)$ psi (1)

Table 10-4: $A = 147$ kpsi·in^m, $m = 0.187$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{147}{0.138^{0.187}} = 212.9$ kpsi

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(212.9) = 106.5$ kpsi

$\tau_s > S_{sy}$, that is, 124.7 > 106.5 kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B kD} = \frac{\left[106.5(10^3)/1.2\right]\pi(0.138^3)}{8(1.254)109.0(0.782)} = 0.857 \text{ in}$$

The free length should be wound to

$L_0 = L_s + y_s = 1.656 + 0.857 = 2.51$ in *Ans.*

10-14 Given: A232 chrome-vanadium steel, squared and ground ends, $d = 0.185$ in, OD = 2.75 in, $L_0 = 7.5$ in, $N_t = 8$ coils.

$D = \text{OD} - d = 2.75 - 0.185 = 2.565$ in

Eq. (10-1): $C = D/d = 2.565/0.185 = 13.86$ (high)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(13.86)+2}{4(13.86)-3} = 1.095$$

Table 10-1: $N_a = N_t - 2 = 8 - 2 = 6$ coils

Table 10-5: $G = 11.2$ Mpsi.

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.185^4(11.2)10^6}{8(2.565^3)6} = 16.20 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.185(8) = 1.48$ in

$y_s = L_0 - L_s = 7.5 - 1.48 = 6.02$ in

$F_s = ky_s = 16.20(6.02) = 97.5$ lbf

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.095 \frac{8(97.5)2.565}{\pi(0.185^3)} = 110.1(10^3) \text{ psi} \qquad (1)$$

Table 10-4: $A = 169$ kpsi·in^m, $m = 0.168$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{169}{0.185^{0.168}} = 224.4 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(224.4) = 112.2$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{112.2}{110.1} = 1.02$$ Spring is not solid-safe ($n_s < 1.2$)

Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B kD} = \frac{\left[112.2(10^3)/1.2\right]\pi(0.185^3)}{8(1.095)16.20(2.565)} = 5.109 \text{ in}$$

The free length should be wound to

$L_0 = L_s + y_s = 1.48 + 5.109 = 6.59$ in *Ans.*

10-15 Given: A313 stainless steel, squared and ground ends, $d = 0.25$ mm, OD = 0.95 mm, $L_0 = 12.1$ mm, $N_t = 38$ coils.

$D = \text{OD} - d = 0.95 - 0.25 = 0.7$ mm

Eq. (10-1): $C = D/d = 0.7/0.25 = 2.8$ (low)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(2.8)+2}{4(2.8)-3} = 1.610$$

Table 10-1: $N_a = N_t - 2 = 38 - 2 = 36$ coils (high)

Table 10-5: $G = 69.0(10^3)$ MPa.

Eq. (10-9): $$k = \frac{d^4G}{8D^3N_a} = \frac{0.25^4(69.0)10^3}{8(0.7^3)36} = 2.728 \text{ N/mm}$$

Table 10-1: $L_s = dN_t = 0.25(38) = 9.5$ mm

$y_s = L_0 - L_s = 12.1 - 9.5 = 2.6$ mm

$F_s = ky_s = 2.728(2.6) = 7.093$ N

Eq. (10-7): $$\tau_s = K_B\frac{8F_sD}{\pi d^3} = 1.610\frac{8(7.093)0.7}{\pi(0.25^3)} = 1303 \text{ MPa} \qquad (1)$$

Table 10-4 (dia. less than table): $A = 1867$ MPa·mm^m, $m = 0.146$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{1867}{0.25^{0.146}} = 2286 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.35(2286) = 734$ MPa

$\tau_s > S_{sy}$, that is, 1303 > 734 MPa, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_BkD} = \frac{(734/1.2)\pi(0.25^3)}{8(1.610)2.728(0.7)} = 1.22 \text{ mm}$$

The free length should be wound to

$$L_0 = L_s + y_s = 9.5 + 1.22 = 10.72 \text{ mm} \qquad \textit{Ans.}$$

This only addresses the solid-safe criteria. There are additional problems.

10-16 Given: A228 music wire, squared and ground ends, $d = 1.2$ mm, OD = 6.5 mm, $L_0 = 15.7$ mm, $N_t = 10.2$ coils.

$D = \text{OD} - d = 6.5 - 1.2 = 5.3$ mm

Eq. (10-1): $C = D/d = 5.3/1.2 = 4.417$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(4.417)+2}{4(4.417)-3} = 1.368$$

Table (10-1): $N_a = N_t - 2 = 10.2 - 2 = 8.2$ coils

Table 10-5 ($d = 1.2/25.4 = 0.0472$ in): $G = 81.7(10^3)$ MPa.

Eq. (10-9): $$k = \frac{d^4G}{8D^3N_a} = \frac{1.2^4(81.7)10^3}{8(5.3^3)8.2} = 17.35 \text{ N/mm}$$

Table 10-1: $L_s = dN_t = 1.2(10.2) = 12.24$ mm

$y_s = L_0 - L_s = 15.7 - 12.24 = 3.46$ mm

$F_s = ky_s = 17.35(3.46) = 60.03$ N

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.368 \dfrac{8(60.03)5.3}{\pi(1.2^3)} = 641.4 \text{ MPa}$ (1)

Table 10-4: $A = 2211 \text{ MPa}\cdot\text{mm}^m,\ m = 0.145$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{2211}{1.2^{0.145}} = 2153 \text{ MPa}$

Table 10-6: $S_{sy} = 0.45\ S_{ut} = 0.45(2153) = 969$ MPa

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{969}{641.4} = 1.51$$ Spring is solid-safe ($n_s > 1.2$) *Ans.*

10-17 Given: A229 OQ&T steel, squared and ground ends, $d = 3.5$ mm, OD = 50.6 mm, $L_0 = 75.5$ mm, $N_t = 5.5$ coils.

$D = \text{OD} - d = 50.6 - 3.5 = 47.1$ mm

Eq. (10-1): $C = D/d = 47.1/3.5 = 13.46$ (high)

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(13.46)+2}{4(13.46)-3} = 1.098$

Table 10-1: $N_a = N_t - 2 = 5.5 - 2 = 3.5$ coils

A229 OQ&T steel is not given in Table 10-5. From Table A-5, for carbon steels, $G = 79.3(10^3)$ MPa.

Eq. (10-9): $k = \dfrac{d^4 G}{8D^3 N_a} = \dfrac{3.5^4(79.3)10^3}{8(47.1^3)3.5} = 4.067 \text{ N/mm}$

Table 10-1: $L_s = dN_t = 3.5(5.5) = 19.25$ mm

$y_s = L_0 - L_s = 75.5 - 19.25 = 56.25$ mm

$F_s = ky_s = 4.067(56.25) = 228.8$ N

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.098 \dfrac{8(228.8)47.1}{\pi(3.5^3)} = 702.8 \text{ MPa}$ (1)

Table 10-4: $A = 1855 \text{ MPa}\cdot\text{mm}^m,\ m = 0.187$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{1855}{3.5^{0.187}} = 1468 \text{ MPa}$

Table 10-6: $S_{sy} = 0.50\ S_{ut} = 0.50(1468) = 734$ MPa

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{734}{702.8} = 1.04$$ Spring is not solid-safe ($n_s < 1.2$)

Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B kD} = \frac{(734/1.2)\pi(3.5^3)}{8(1.098)4.067(47.1)} = 48.96 \text{ mm}$$

The free length should be wound to

$L_0 = L_s + y_s = 19.25 + 48.96 = 68.2$ mm *Ans.*

10-18 Given: B159 phosphor-bronze, squared and ground ends, $d = 3.8$ mm, OD = 31.4 mm, $L_0 = 71.4$ mm, $N_t = 12.8$ coils.

$D = \text{OD} - d = 31.4 - 3.8 = 27.6$ mm

Eq. (10-1): $C = D/d = 27.6/3.8 = 7.263$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(7.263)+2}{4(7.263)-3} = 1.192$

Table 10-1: $N_a = N_t - 2 = 12.8 - 2 = 10.8$ coils

Table 10-5: $G = 41.4(10^3)$ MPa.

Eq. (10-9): $k = \dfrac{d^4G}{8D^3N_a} = \dfrac{3.8^4(41.4)10^3}{8(27.6^3)10.8} = 4.752\ \text{N/mm}$

Table 10-1: $L_s = dN_t = 3.8(12.8) = 48.64$ mm

$y_s = L_0 - L_s = 71.4 - 48.64 = 22.76$ mm

$F_s = ky_s = 4.752(22.76) = 108.2$ N

Eq. (10-7): $\tau_s = K_B\dfrac{8F_sD}{\pi d^3} = 1.192\dfrac{8(108.2)27.6}{\pi(3.8^3)} = 165.2\ \text{MPa}$ (1)

Table 10-4 ($d = 3.8/25.4 = 0.150$ in): $A = 932\ \text{MPa}\cdot\text{mm}^m$, $m = 0.064$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{932}{3.8^{0.064}} = 855.7\ \text{MPa}$

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.35(855.7) = 299.5$ MPa

$n_s = \dfrac{S_{sy}}{\tau_s} = \dfrac{299.5}{165.2} = 1.81$ Spring is solid-safe ($n_s > 1.2$) *Ans.*

10-19 Given: A232 chrome-vanadium steel, squared and ground ends, $d = 4.5$ mm, OD = 69.2 mm, $L_0 = 215.6$ mm, $N_t = 8.2$ coils.

$D = \text{OD} - d = 69.2 - 4.5 = 64.7$ mm

Eq. (10-1): $C = D/d = 64.7/4.5 = 14.38$ (high)

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(14.38)+2}{4(14.38)-3} = 1.092$

Table 10-1: $N_a = N_t - 2 = 8.2 - 2 = 6.2$ coils

Table 10-5: $G = 77.2(10^3)$ MPa.

Eq. (10-9): $k = \dfrac{d^4G}{8D^3N_a} = \dfrac{4.5^4(77.2)10^3}{8(64.7^3)6.2} = 2.357\ \text{N/mm}$

Table 10-1: $L_s = dN_t = 4.5(8.2) = 36.9$ mm

$y_s = L_0 - L_s = 215.6 - 36.9 = 178.7$ mm
$F_s = ky_s = 2.357(178.7) = 421.2$ N

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.092\frac{8(421.2)64.7}{\pi(4.5^3)} = 832 \text{ MPa} \qquad (1)$$

Table 10-4: $A = 2005 \text{ MPa}\cdot\text{mm}^m,\ m = 0.168$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{2005}{4.5^{0.168}} = 1557 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(1557) = 779$ MPa

$\tau_s > S_{sy}$, that is, $832 > 779$ MPa, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B kD} = \frac{(779/1.2)\pi(4.5^3)}{8(1.092)2.357(64.7)} = 139.5 \text{ mm}$$

The free length should be wound to

$$L_0 = L_s + y_s = 36.9 + 139.5 = 176.4 \text{ mm} \qquad \textit{Ans.}$$

This only addresses the solid-safe criteria. There are additional problems.

10-20 Given: A227 HD steel.

From the figure: $L_0 = 4.75$ in, OD = 2 in, and $d = 0.135$ in. Thus

$$D = \text{OD} - d = 2 - 0.135 = 1.865 \text{ in}$$

(a) By counting, $N_t = 12.5$ coils. Since the ends are squared along 1/4 turn on each end,

$$N_a = 12.5 - 0.5 = 12 \text{ turns} \qquad \textit{Ans.}$$
$$p = 4.75 / 12 = 0.396 \text{ in} \qquad \textit{Ans.}$$

The solid stack is 13 wire diameters

$$L_s = 13(0.135) = 1.755 \text{ in} \qquad \textit{Ans.}$$

(b) From Table 10-5, $G = 11.4$ Mpsi

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.135^4(11.4)(10^6)}{8(1.865^3)(12)} = 6.08 \text{ lbf/in} \qquad \textit{Ans.}$$

(c) $F_s = k(L_0 - L_s) = 6.08(4.75 - 1.755) = 18.2$ lbf *Ans.*

(d) $C = D/d = 1.865/0.135 = 13.81$

$$K_B = \frac{4(13.81) + 2}{4(13.81) - 3} = 1.096$$

$$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.096 \frac{8(18.2)(1.865)}{\pi\left(0.135^3\right)} = 38.5\left(10^3\right) \text{ psi} = 38.5 \text{ kpsi} \quad Ans.$$

10-21 Given: Plain end, hard drawn steel, 12 gauge W & M wire, OD = 0.75 in, $N_t = 20$ coils, $L_0 = 3.75$ in.

Table A-28: $d = 0.1055$ in

(a) $D = \text{OD} - d = 0.75 - 0.1055 = 0.6445$ in. $C = D / d = 0.6445/0.1055 = 6.109$ *Ans.*

(b) Table 10-1: $N_a = N_t = 20$ coils,

$p = (L_0 - d)/ N_a = (3.75 - 0.1055)/20 = 0.1822$ in/coil *Ans.*

(c) Table 10-1: $L_s = d\,(N_t + 1) = 0.1055\ (20 + 1) = 2.2155$ in,

$y_s = L_0 - L_s = 3.75 - 2.2155 = 1.5345$ in *Ans.*

(d) Eq. (10-8):

$$F_s = \frac{d^4 G y_s}{8D^3 N\left(1+\dfrac{1}{2C^2}\right)} = \frac{0.1055^4\left(11.5\right)10^6\left(1.5345\right)}{8\left(0.6445\right)^3\left(20\right)\left[1+\dfrac{1}{2\left(6.109\right)^2}\right]} = 50.36 \text{ lbf} \quad Ans.$$

(e) Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4\left(6.109\right)+2}{4\left(6.109\right)-3} = 1.233$

Eq. (10-7):

$$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.233 \frac{8\left(50.36\right)0.6445}{\pi\left(0.1055\right)^3} = 86.8\left(10^3\right) \text{ psi} = 86.8 \text{ kpsi} \quad Ans.$$

(f) Table 10-4 and Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{140}{\left(0.1055\right)^{0.190}} = 214.6 \text{ kpsi}$

Table 10-6: $S_{sy} = 0.45\ S_{ut} = 0.45(214.6) = 96.57$ kpsi.

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{96.57}{86.8} = 1.11 \quad Ans.$$

(g) Exact, $k = F_s / y_s = 50.36/1.5345 = 32.82$ lbf/in *Ans.*

Approximate using Eq. (10-9): $k \approx \dfrac{d^4 G}{8D^3 N} = \dfrac{0.1055^4\left(11.5\right)10^6}{8\left(0.6445\right)^3 20} = 33.26 \text{ lbf/in}$ *Ans.*

Approximate is 1.34 percent higher than the exact. *Ans.*

10-22 Given: Squared and ground, oil tempered steel, $d = 3$ mm, OD = 30 mm, $N_t = 32$ coils, $L_0 = 240$ mm.

Table A-28: $d = 0.1055$ in

(a) $D = \text{OD} - d = 30 - 3 = 27$ mm. $C = D / d = 27/3 = 9$ *Ans.*

(b) Table 10-1: $N_a = N_t - 2 = 32 - 2 = 30$ coils,

$p = (L_0 - 2d)/ N_a = [240 - 2(3)] / 30 = 7.8$ mm/coil *Ans.*

(c) Table 10-1: $L_s = d\,N_t = 3\ (32) = 96$ mm,

$y_s = L_0 - L_s = 240 - 96 = 144$ mm *Ans.*

(d) Eq. (10-8):

$$F_s = \frac{d^4 G y_s}{8D^3 N\left(1+\dfrac{1}{2C^2}\right)} = \frac{(0.003)^4\, 77.2\left(10^9\right)0.144}{8(0.027)^3(30)\left[1+\dfrac{1}{2(9)^2}\right]} = 189.45 \text{ N} \qquad Ans.$$

(e) Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(9)+2}{4(9)-3} = 1.152$

Eq. (10-7):

$$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.152 \frac{8(189.45)0.027}{\pi(0.003)^3}\left(10^{-6}\right) = 555.8 \text{ MPa} \qquad Ans.$$

(f) Table 10-4 and Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{1855}{(3)^{0.187}} = 1510.5 \text{ MPa}$

Table 10-5: $S_{sy} = 0.45\, S_{ut} = 0.45(1510.5) = 679.7$ MPa.

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{679.7}{555.8} = 1.22 \qquad Ans.$$

(g) Exact, $k = F_s / y_s = 189.45/144 = 1.316$ N/mm *Ans.*
Approximate using Eq. (10-9):

$$k \approx \frac{d^4 G}{8D^3 N} = \frac{(0.003)^4\, 77.2\left(10^9\right)}{8(0.027)^3\, 30}\left(10^{-3}\right) = 1.324 \text{ N/mm} \qquad Ans.$$

Approximate is 0.61 percent higher than the exact. *Ans.*

10-23 $y = 50$ mm, $F = 90$ N, $k = F / y = 90/50 = 1.8$ N/mm *Ans.*
$y_s = 60$ mm, $F_s = k\, y_s = 1.8(60) = 108$ N.

Eq. (10-14), Table 10-4, assume $2.5 \le d \le 5$ mm: $S_{ut} = \dfrac{A}{d^m} = \dfrac{2065}{d^{0.263}}$

Table 10-6 (includes K_B): $S_{ys} = 0.35 S_{ut} = 0.35 \dfrac{2065}{d^{0.263}} = \dfrac{722.75}{d^{0.263}}$ (1)

Eq. (10-7) (with $n_s = 1.2$ but without K_B):

$$\tau_{\max} = \frac{8 n_s F_s D}{\pi d^3} = \frac{8 n_s F_s C}{\pi d^2} = \frac{8(1.2)108(10)}{\pi d^2} = \frac{3300}{d^2} \text{ MPa} \qquad (2)$$

Equate Eqs. (1) and (2), $\dfrac{722.75}{d^{0.263}} = \dfrac{3300}{d^2} \Rightarrow d^{1.737} = \dfrac{3300}{722.75} \Rightarrow d = 2.40$ mm

Since this is less than 2.5 mm, return to Eq. (10-14), Table 10-4, for $0.3 \le d \le 2.5$ mm:

$$S_{ut} = \frac{A}{d^m} = \frac{1867}{d^{0.146}} \Rightarrow S_{ys} = \frac{653.45}{d^{0.146}} \qquad (1)$$

Again, equate Eqs. (1) and (2),

$$\frac{653.45}{d^{0.146}} = \frac{3300}{d^2} \Rightarrow d^{1.854} = \frac{3300}{653.45} \Rightarrow d = 2.40 \text{ mm} \qquad Ans.$$

The final factor of safety is

$$n_s = \frac{S_{ys}}{\tau_s} = \frac{653.45 / d^{0.146}}{\left(\frac{8F_sC}{\pi d^2}\right)} = \frac{653.45\pi\left(2.40\right)^{1.854}}{8\left(108\right)10} = 1.20 \qquad \textit{Ans.}$$

OD $= Cd + d = (C + 1)\, d = 11(2.40) = 26.4$ mm *Ans.*
ID $= (C - 1)\, d = 9(2.40) = 21.6$ mm *Ans.*
$k = 1.8$ N/mm *Ans.* (found earlier)
Table 10-5, $G = 69.0$ GPa, Eq. (10-9):

$$N_a = \frac{d^4 G}{8D^3 k} = \frac{dG}{8C^3 k} = \frac{2.4\left(10^{-3}\right)69\left(10^9\right)}{8\left(10^3\right)1.8\left(10^3\right)} = 11.5 \text{ coils}$$

Table 10-1: $N_t = N_a + 2 = 13.5$ coils *Ans.*
$L_s = d\,(N_t + 1) = 2.4(13.5 + 1) = 34.8$ mm *Ans.*
$L_0 = L_s + y_s = 34.8 + 60 = 94.8$ mm *Ans.*

Eq. (10-13): $\alpha = 2.63\dfrac{D}{L_0} = 2.63\dfrac{10\left(2.4\right)}{94.8} = 0.666$

Stable if supported between fixed-fixed ends. Otherwise would need to be supported by hole or rod.

10-24 Phosphor-bronze, closed ends, $C = 10$, at $y = 2$ in $F = 15$ lbf, $y_s = 3$ in, $n_s = 1.2$.
$k = F / y = 15/2 = 7.5$ lbf/in, $F_s = k\, y_s = 7.5\,(3) = 22.5$ lbf.
Eq. (10-14) with Table 10-4 assuming $0.022 \le d \le 0.075$ in, $A = 121$ kpsi·in$^{\text{m}}$ and $m = 0.028$. Then, from Table 10-5, $S_{ys} = 0.45\, S_{ut}$:

$$S_{ys} = 0.45\frac{121}{d^{0.028}} = \frac{54.45}{d^{0.028}} \qquad (1)$$

Eq. (10-5): $K_B = \dfrac{4C + 2}{4C - 3} = \dfrac{4\left(10\right) + 2}{4\left(10\right) - 3} = 1.135$

Eq. (10-7) with $n_s = 1.2$:

$$\tau_{\max} = n_s K_B \frac{8F_sC}{\pi d^2} = 1.2\left(1.135\right)\frac{8\left(22.5\right)10}{\pi d^2}\left(10^{-3}\right) = \frac{0.7804}{d^2} \qquad (2)$$

Where $\tau_{\max}$ is in kpsi. Equating (1) and (2) gives

$$\frac{54.45}{d^{0.028}} = \frac{0.7804}{d^2} \quad \Rightarrow \quad d^{1.972} = \frac{0.7804}{54.45} \quad \Rightarrow \quad d = 0.116 \text{ in}$$

Returning to Table 10-4, use $A = 110$ kpsi·in$^{\text{m}}$ and $m = 0.028$ for $0.075 \le d \le 0.3$ in,

$$S_{ys} = 0.45\frac{110}{d^{0.064}} = \frac{49.5}{d^{0.064}} = \frac{0.7804}{d^2} \quad \Rightarrow \quad d^{1.936} = \frac{0.7804}{49.5} \quad \Rightarrow \quad d = 0.117 \text{ in}$$

Table A-17, select the preferred size of: $d = 0.12$ in *Ans.*

Check n_s, $n_s = \dfrac{S_{ys}}{\tau_s} = \dfrac{49.5/\left(d^{0.064}\right)}{K_B\left(8\dfrac{F_sC}{\pi d^2}10^{-3}\right)} = \dfrac{49.5\left(10^3\right)\pi d^{1.936}}{8K_BF_sC}$ (3)

$$n_s = \frac{49.5\left(10^3\right)\pi\left(0.12\right)^{1.936}}{8\left(1.135\right)22.5\left(10\right)} = 1.26 \qquad Ans.$$

OD = $Cd + d = (C + 1)\,d = (10 + 1)0.12 = 1.32$ in *Ans.*

ID = $(C - 1)\,d = (10 - 1)0.12 = 1.08$ in

Found earlier, $k = 7.5$ lbf/in *Ans.*

Table 10-5, $G = 6$ Mpsi. Eq. (10-9): $N_a = \dfrac{dG}{8C^3k} = \dfrac{0.12(6)10^6}{8\left(10^3\right)7.5} = 12$ coils

Table 10-1: $N_t = N_a + 2 = 14$ coils *Ans.*

$L_s = d\,(N_t + 1) = 0.12(14 + 1) = 1.8$ in *Ans.*

$L_0 = L_s + y_s = 1.8 + 3 = 4.8$ in *Ans.*

Eq. (10-13): $\alpha = 2.63\, D\, /\, L_0 = 2.63\ (10)0.12/4.8 = 0.658$
Table 10-2: Stable if supported between fixed-fixed ends. Otherwise would need to be supported by hole or rod.

10-25 From Prob. 10-24, $d = 0.12$ in and from Eq. (3),

$$C = \frac{49.5\left(10^3\right)\pi d^{1.936}}{8n_sK_BF_s} = \frac{49.5\left(10^3\right)\pi\left(0.12\right)^{1.936}}{8\left(1.2\right)1.135\left(22.5\right)} = 10.46 \qquad Ans.$$

OD = $(C + 1)\,d = 11.46(0.12) = 1.375$ in *Ans.*
ID = $(C - 1)\,d = 9.46(0.12) = 1.135$ in *Ans.*

Table 10-5, $G = 6$ Mpsi, Eq. (10-9): $N_a = \dfrac{dG}{8C^3k} = \dfrac{0.12(6)10^6}{8\left(10.46^3\right)7.5} = 10.5$ coils

Table 10-1: $N_t = N_a + 2 = 12.5$ coils *Ans.*

$L_s = d\,(N_t + 1) = 0.12(12.5 + 1) = 1.62$ in *Ans.*

$L_0 = L_s + y_s = 1.62 + 3 = 4.62$ in *Ans.*

Eq. (10-13): $\alpha = 2.63\, D\, /\, L_0 = 2.63\ (10.46)0.12/4.62 = 0.715$

Table 10-2: Stable if supported between fixed-fixed ends, or one end on flat surface and other end hinged. Otherwise would need to be supported by hole or rod.

10-26 For the wire diameter analyzed, $G = 11.75$ Mpsi per Table 10-5. Use squared and ground ends. The following is a spread-sheet study using Fig. 10-3 for parts (*a*) and (*b*). For N_a, $k = F_{\max}/y = 20/2 = 10$ lbf/in. For τ_s, $F = F_s = 20(1 + \xi) = 20(1 + 0.15) = 23$ lbf.

(a) Spring over a Rod					(b) Spring in a Hole				
Source	Parameter	Values			Source	Parameter	Values		
	d	0.075	0.080	0.085		d	0.075	0.080	0.085
	ID	0.800	0.800	0.800		OD	0.950	0.950	0.950
	D	0.875	0.880	0.885		D	0.875	0.870	0.865
Eq. (10-1)	C	11.667	11.000	10.412	Eq. (10-1)	C	11.667	10.875	10.176
Eq. (10-9)	N_a	6.937	8.828	11.061	Eq. (10-9)	N_a	6.937	9.136	11.846
Table 10-1	N_t	8.937	10.828	13.061	Table 10-1	N_t	8.937	11.136	13.846
Table 10-1	L_s	0.670	0.866	1.110	Table 10-1	L_s	0.670	0.891	1.177
$1.15y + L_s$	L_0	2.970	3.166	3.410	$1.15y + L_s$	L_0	2.970	3.191	3.477
Eq. (10-13)	$(L_0)_{\mathrm{cr}}$	4.603	4.629	4.655	Eq. (10-13)	$(L_0)_{\mathrm{cr}}$	4.603	4.576	4.550
Table 10-4	A	201.000	201.000	201.000	Table 10-4	A	201.000	201.000	201.000
Table 10-4	m	0.145	0.145	0.145	Table 10-4	m	0.145	0.145	0.145
Eq. (10-14)	S_{ut}	292.626	289.900	287.363	Eq. (10-14)	S_{ut}	292.626	289.900	287.363
Table 10-6	S_{sy}	131.681	130.455	129.313	Table 10-6	S_{sy}	131.681	130.455	129.313
Eq. (10-5)	K_B	1.115	1.122	1.129	Eq. (10-5)	K_B	1.115	1.123	1.133
Eq. (10-7)	τ_s	135.335	112.948	95.293	Eq. (10-7)	τ_s	135.335	111.787	93.434
Eq. (10-3)	n_s	0.973	1.155	1.357	Eq. (10-3)	n_s	0.973	1.167	1.384
Eq. (10-22)	fom	−0.282	−0.391	−0.536	Eq. (10-22)	fom	−0.282	−0.398	−0.555

For $n_s \geq 1.2$, the optimal size is $d = 0.085$ in for both cases.

10-27 In Prob. 10-26, there is an advantage of first selecting d as one can select from the available sizes (Table A-28). Selecting C first requires a calculation of d where then a size must be selected from Table A-28.
Consider part (a) of the problem. It is required that

$$\text{ID} = D - d = 0.800 \text{ in.} \qquad (1)$$

From Eq. (10-1), $D = Cd$. Substituting this into the first equation yields

$$d = \frac{0.800}{C-1} \qquad (2)$$

Starting with $C = 10$, from Eq. (2) we find that $d = 0.089$ in. From Table A-28, the closest diameter is $d = 0.090$ in. Substituting this back into Eq. (1) gives $D = 0.890$ in, with $C = 0.890/0.090 = 9.889$, which are acceptable. From this point the solution is the same as Prob. 10-26. For part (b), use

$$\text{OD} = D + d = 0.950 \text{ in.} \qquad (3)$$

and, $$d = \frac{0.800}{C-1} \qquad (4)$$

(a) Spring over a rod				(b) Spring in a Hole		
Source		Parameter	Values	Source	Parameter	Values
	C	10.000	10.5		C	10.000
Eq. (2)	d	0.089	0.084	Eq. (4)	d	0.086
Table A-28	d	0.090	0.085	Table A-28	d	0.085
Eq. (1)	D	0.890	0.885	Eq. (3)	D	0.865
Eq. (10-1)	C	9.889	10.412	Eq. (10-1)	C	10.176
Eq. (10-9)	N_a	13.669	11.061	Eq. (10-9)	N_a	11.846
Table 10-1	N_t	15.669	13.061	Table 10-1	N_t	13.846
Table 10-1	L_s	1.410	1.110	Table 10-1	L_s	1.177
$1.15y + L_s$	L_0	3.710	3.410	$1.15y + L_s$	L_0	3.477
Eq. (10-13)	$(L_0)_{\mathrm{cr}}$	4.681	4.655	Eq. (10-13)	$(L_0)_{\mathrm{cr}}$	4.550
Table 10-4	A	201.000	201.000	Table 10-4	A	201.000
Table 10-4	m	0.145	0.145	Table 10-4	m	0.145
Eq. (10-14)	S_{ut}	284.991	287.363	Eq. (10-14)	S_{ut}	287.363
Table 10-6	S_{sy}	128.246	129.313	Table 10-6	S_{sy}	129.313
Eq. (10-5)	K_B	1.135	1.128	Eq. (10-5)	K_B	1.135
Eq. (10-7)	τ_s	81.167	95.223	Eq. (10-7)	τ_s	93.643
$n_s = S_{sy}/\tau_s$	n_s	1.580	1.358	$n_s = S_{sy}/\tau_s$	n_s	1.381
Eq. (10-22)	fom	-0.725	-0.536	Eq. (10-22)	fom	-0.555

Again, for $n_s \geq 1.2$, the optimal size is = 0.085 in.

Although this approach used less iterations than in Prob. 10-26, this was due to the initial values picked and not the approach.

10-28 One approach is to select A227 HD steel for its low cost. Try $L_0 = 48$ mm, then for $y = 48 - 37.5 = 10.5$ mm when $F = 45$ N. The spring rate is $k = F/y = 45/10.5 = 4.286$ N/mm.

For a clearance of 1.25 mm with screw, ID = 10 + 1.25 = 11.25 mm. Starting with $d = 2$ mm,

$$D = \mathrm{ID} + d = 11.25 + 2 = 13.25 \text{ mm}$$

$$C = D/d = 13.25/2 = 6.625 \qquad \text{(acceptable)}$$

Table 10-5 ($d = 2/25.4 = 0.0787$ in): $G = 79.3$ GPa

Eq. (10-9): $$N_a = \frac{d^4 G}{8kD^3} = \frac{2^4(79.3)10^3}{8(4.286)13.25^3} = 15.9 \text{ coils}$$

Assume squared and closed.

Table 10-1: $N_t = N_a + 2 = 15.9 + 2 = 17.9$ coils

$L_s = dN_t = 2(17.9) = 35.8$ mm

$$y_s = L_0 - L_s = 48 - 35.8 = 12.2 \text{ mm}$$
$$F_s = ky_s = 4.286(12.2) = 52.29 \text{ N}$$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(6.625)+2}{4(6.625)-3} = 1.213$$

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.213\left[\frac{8(52.29)13.25}{\pi(2^3)}\right] = 267.5 \text{ MPa}$$

Table 10-4: $A = 1783 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.190$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{1783}{2^{0.190}} = 1563 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.45 S_{ut} = 0.45(1563) = 703.3 \text{ MPa}$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{703.3}{267.5} = 2.63 > 1.2 \qquad O.K.$$

No other diameters in the given range work. So specify

A227-47 HD steel, $d = 2$ mm, $D = 13.25$ mm, ID = 11.25 mm, OD = 15.25 mm, squared and closed, $N_t = 17.9$ coils, $N_a = 15.9$ coils, $k = 4.286$ N/mm, $L_s = 35.8$ mm, and $L_0 = 48$ mm. *Ans.*

10-29 Select A227 HD steel for its low cost. Try $L_0 = 48$ mm, then for $y = 48 - 37.5 = 10.5$ mm when $F = 45$ N. The spring rate is $k = F/y = 45/10.5 = 4.286$ N/mm.

For a clearance of 1.25 mm with screw, ID = 10 + 1.25 = 11.25 mm.

$$D - d = 11.25 \qquad (1)$$

and, $$D = Cd \qquad (2)$$

Starting with $C = 8$, gives $D = 8d$. Substitute into Eq. (1) resulting in $d = 1.607$ mm. Selecting the nearest diameter in the given range, $d = 1.6$ mm. From this point, the calculations are shown in the third column of the spreadsheet output shown. We see that for $d = 1.6$ mm, the spring is not solid safe. Iterating on C we find that $C = 6.5$ provides acceptable results with the specifications

A227-47 HD steel, $d = 2$ mm, $D = 13.25$ mm, ID = 11.25 mm, OD = 15.25 mm, squared and closed, $N_t = 17.9$ coils, $N_a = 15.9$ coils, $k = 4.286$ N/mm, $L_s = 35.8$ mm, and $L_0 = 48$ mm. *Ans.*

Source		Parameter Values		
	C	8.000	7	6.500
Eq. (2)	d	1.607	1.875	2.045
Table A-28	d	1.600	1.800	2.000
Eq. (1)	D	12.850	13.050	13.250
Eq. (10-1)	C	8.031	7.250	6.625
Eq. (10-9)	N_a	7.206	10.924	15.908
Table 10-1	N_t	9.206	12.924	17.908
Table 10-1	L_s	14.730	23.264	35.815
$L_0 - L_s$	y_s	33.270	24.736	12.185
$F_s = ky_s$	F_s	142.594	106.020	52.224
Table 10-4	A	1783.000	1783.000	1783.000
Table 10-4	m	0.190	0.190	0.190
Eq. (10-14)	S_{ut}	1630.679	1594.592	1562.988
Table 10-6	S_{sy}	733.806	717.566	703.345
Eq. (10-5)	K_B	1.172	1.200	1.217
Eq. (10-7)	τ_s	1335.568	724.943	268.145
$n_s = S_{sy}/\tau_s$	n_s	0.549	0.990	2.623

The only difference between selecting C first rather than d as was done in Prob. 10-28, is that once d is calculated, the closest wire size must be selected. Iterating on d uses available wire sizes from the beginning.

10-30 A stock spring catalog may have over two hundred pages of compression springs with up to 80 springs per page listed.

- Students should be made aware that such catalogs exist.
- Many springs are selected from catalogs rather than designed.
- The wire size you want may not be listed.
- Catalogs may also be available on disk or the web through search routines.
- It is better to familiarize yourself with vendor resources rather than invent them yourself.
- Sample catalog pages can be given to students for study.

10-31 Given: ID = 0.6 in, $C = 10$, $L_0 = 5$ in, $L_s = 5 - 3 = 2$ in, sq. & grd ends, unpeened, HD A227 wire.

(**a**) With ID $= D - d = 0.6$ in and $C = D/d = 10 \Rightarrow 10\,d - d = 0.6 \;\Rightarrow d = 0.0667$ in *Ans.*, and $D = 0.667$ in.

(**b**) Table 10-1: $L_s = dN_t = 2$ in $\Rightarrow N_t = 2/0.0667 = 30$ coils *Ans.*

(**c**) Table 10-1: $N_a = N_t - 2 = 30 - 2 = 28$ coils

Table 10-5: $G = 11.5$ Mpsi

Eq. (10-9):
$$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.0667^4\left(11.5\right)10^6}{8\left(0.667^3\right)28} = 3.424 \text{ lbf/in} \qquad Ans.$$

(**d**) Table 10-4: $A = 140$ kpsi·in^m, $m = 0.190$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{140}{0.0667^{0.190}} = 234.2 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45\, S_{ut} = 0.45\,(234.2) = 105.4$ kpsi

$F_s = ky_s = 3.424(3) = 10.27$ lbf

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$$

Eq. (10-7):

$$\tau_s = K_B \frac{8F_sD}{\pi d^3} = 1.135 \frac{8(10.27)0.667}{\pi\left(0.0667^3\right)}$$

$$= 66.72\left(10^3\right) \text{ psi} = 66.72 \text{ kpsi}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{105.4}{66.72} = 1.58 \qquad Ans.$$

(**e**) $\tau_a = \tau_m = 0.5\,\tau_s = 0.5(66.72) = 33.36$ kpsi, $r = \tau_a / \tau_m = 1$. Using the Gerber fatigue failure criterion with Zimmerli data,

Eq. (10-30): $S_{su} = 0.67\, S_{ut} = 0.67(234.2) = 156.9$ kpsi

The Gerber ordinate intercept for the Zimmerli data is obtained using Eqs. (10-28) and (10-29b).

$$S_{se} = \frac{S_{sa}}{1-\left(S_{sm}/S_{su}\right)^2} = \frac{35}{1-\left(55/156.9\right)^2} = 39.9 \text{ kpsi}$$

The Gerber fatigue criterion from Eq. (6-48), adapted for shear,

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)^2}\,\right]$$

$$= \frac{1}{2}\left(\frac{156.9}{33.36}\right)^2 \frac{33.36}{39.9}\left[-1+\sqrt{1+\left(\frac{2(33.36)39.9}{156.9(33.36)}\right)^2}\,\right]$$

$$= 1.13 \qquad Ans.$$

10-32 Given: OD ≤ 0.9 in, $C = 8$, $L_0 = 3$ in, $L_s = 1$ in, $y_s = 3 - 1 = 2$ in, sq. ends, unpeened, music wire.

(**a**) Try OD $= D + d = 0.9$ in, $C = D/d = 8 \Rightarrow D = 8d \Rightarrow 9d = 0.9 \Rightarrow d = 0.1$ *Ans.*
$D = 8(0.1) = 0.8$ in

(**b**) Table 10-1: $L_s = d\,(N_t + 1) \Rightarrow N_t = L_s / d - 1 = 1/0.1 - 1 = 9$ coils *Ans.*

Table 10-1: $N_a = N_t - 2 = 9 - 2 = 7$ coils (**c**) Table 10-5: $G = 11.75$ Mpsi

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.1^4\left(11.75\right)10^6}{8\left(0.8^3\right)7} = 40.98 \text{ lbf/in} \qquad \textit{Ans.}$$

(**d**) $$F_s = ky_s = 40.98(2) = 81.96 \text{ lbf}$$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4\left(8\right)+2}{4\left(8\right)-3} = 1.172$$

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.172 \frac{8\left(81.96\right)0.8}{\pi\left(0.1^3\right)} = 195.7\left(10^3\right) \text{ psi} = 195.7 \text{ kpsi}$$

Table 10-4: $A = 201$ kpsi·in^m, $m = 0.145$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{201}{0.1^{0.145}} = 280.7 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45\, S_{ut} = 0.45(280.7) = 126.3$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{126.3}{195.7} = 0.645 \qquad \textit{Ans.}$$

(**e**) $\tau_a = \tau_m = \tau_s/2 = 195.7/2 = 97.85$ kpsi. Using the Gerber fatigue failure criterion with Zimmerli data,

Eq. (10-30): $S_{su} = 0.67\, S_{ut} = 0.67(280.7) = 188.1$ kpsi

The Gerber ordinate intercept for the Zimmerli data is obtained using Eqs. (10-28) and (10-29b).

$$S_{se} = \frac{S_{sa}}{1-\left(S_{sm}/S_{su}\right)^2} = \frac{35}{1-\left(55/188.1\right)^2} = 38.3 \text{ kpsi}$$

The Gerber fatigue criterion from Eq. (6-48), adapted for shear,

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{188.1}{97.85}\right)^2 \frac{97.85}{38.3}\left[-1+\sqrt{1+\left(\frac{2\left(97.85\right)38.3}{188.1\left(97.85\right)}\right)^2}\right]$$

$$= 0.38 \qquad \textit{Ans.}$$

Obviously, the spring is severely under designed and will fail statically and in fatigue. Increasing C would improve matters. Try $C = 12$. This yields $n_s = 1.83$ and $n_f = 1.00$.

10-33 Given: $F_{\max} = 300$ lbf, $F_{\min} = 150$ lbf, $\Delta y = 1$ in, OD = 2.1 − 0.2 = 1.9 in, $C = 7$, unpeened, squared & ground, oil-tempered wire.

(a) $D = \text{OD} - d = 1.9 - d$ (1)

$C = D/d = 7 \Rightarrow D = 7d$ (2)

Substitute Eq. (2) into (1)

$7d = 1.9 - d \Rightarrow d = 1.9/8 = 0.2375$ in *Ans.*

(b) From Eq. (2): $D = 7d = 7(0.2375) = 1.663$ in *Ans.*

(c) $$k = \frac{\Delta F}{\Delta y} = \frac{300-150}{1} = 150 \text{ lbf/in}$$ *Ans.*

(d) Table 10-5: $G = 11.6$ Mpsi

Eq. (10-9): $$N_a = \frac{d^4 G}{8D^3 k} = \frac{0.2375^4\left(11.6\right)10^6}{8\left(1.663^3\right)150} = 6.69 \text{ coils}$$

Table 10-1: $N_t = N_a + 2 = 8.69$ coils *Ans.*

(e) Table 10-4: $A = 147$ kpsi·in^m, $m = 0.187$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{147}{0.2375^{0.187}} = 192.3 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.5\,S_{ut} = 0.5(192.3) = 96.15$ kpsi

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4\left(7\right)+2}{4\left(7\right)-3} = 1.2$$

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = S_{sy}$$

$$F_s = \frac{\pi d^3 S_{sy}}{8K_B D} = \frac{\pi\left(0.2375^3\right)96.15\left(10^3\right)}{8\left(1.2\right)1.663} = 253.5 \text{ lbf}$$

$y_s = F_s / k = 253.5/150 = 1.69$ in

Table 10-1: $L_s = N_t\, d = 8.46(0.2375) = 2.01$ in

$$L_0 = L_s + y_s = 2.01 + 1.69 = 3.70 \text{ in} \qquad Ans.$$

10-34 For a coil radius given by:

$$R = R_1 + \frac{R_2 - R_1}{2\pi N}\theta$$

The torsion of a section is $T = PR$ where $dL = R\,d\theta$

$$\begin{aligned}
\delta_P &= \frac{\partial U}{\partial P} = \frac{1}{GJ}\int T\frac{\partial T}{\partial P}dL = \frac{1}{GJ}\int_0^{2\pi N} PR^3\,d\theta \\
&= \frac{P}{GJ}\int_0^{2\pi N}\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^3 d\theta \\
&= \frac{P}{GJ}\left(\frac{1}{4}\right)\left(\frac{2\pi N}{R_2 - R_1}\right)\left[\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^4\right]_0^{2\pi N} \\
&= \frac{\pi PN}{2GJ(R_2 - R_1)}\left(R_2^4 - R_1^4\right) = \frac{\pi PN}{2GJ}(R_1 + R_2)\left(R_1^2 + R_2^2\right)
\end{aligned}$$

$$J = \frac{\pi}{32}d^4 \quad \therefore\ \delta_p = \frac{16PN}{Gd^4}(R_1 + R_2)\left(R_1^2 + R_2^2\right)$$

$$k = \frac{P}{\delta_P} = \frac{d^4G}{16N(R_1 + R_2)\left(R_1^2 + R_2^2\right)} \qquad Ans.$$

10-35 Given: $F_{\min} = 4$ lbf, $F_{\max} = 18$ lbf, $k = 9.5$ lbf/in, OD ≤ 2.5 in, $n_f = 1.5$.

For a food service machinery application select A313 Stainless wire.

Table 10-5: $G = 10(10^6)$ psi

Note that for $0.013 \le d \le 0.10$ in $\quad A = 169, \quad m = 0.146$

$0.10 < d \le 0.20$ in $\quad A = 128, \quad m = 0.263$

$$F_a = \frac{18 - 4}{2} = 7 \text{ lbf}, \quad F_m = \frac{18 + 4}{2} = 11 \text{ lbf}, \quad r = 7/11$$

Try, $d = 0.080$ in, $S_{ut} = \dfrac{169}{(0.08)^{0.146}} = 244.4$ kpsi

$S_{su} = 0.67S_{ut} = 163.7$ kpsi, $\quad S_{sy} = 0.35S_{ut} = 85.5$ kpsi

The Gerber ordinate intercept for the Zimmerli data is obtained using Eqs. (10-28) and (10-29b).

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})^2} = \frac{35}{1 - (55/163.7)^2} = 39.5 \text{ kpsi}$$

Let $r = \tau_a / \tau_m$ = 7/11. The Gerber fatigue criterion from Eq. (6-48), adapted for shear,

$$n_f = \frac{1}{2}\frac{S_{su}^2}{\tau_m}\frac{r}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2S_{se}}{S_{su}r}\right)^2}\right]$$

Solving for τ_m gives,

$$\tau_m = \frac{1}{2}\frac{S_{su}^2}{n_f}\frac{r}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2S_{se}}{S_{su}r}\right)^2}\right] = \frac{1}{2}\frac{(163.7)^2}{(1.5)}\frac{(7/11)}{39.5}\left[-1+\sqrt{1+\left(\frac{2(39.5)}{163.7(7/11)}\right)^2}\right]$$

$$= 36.70 \text{ kpsi}$$

But,

$$\tau_m = K_B\frac{8F_mC}{\pi d^2} = \frac{4C+2}{4C-3}\left(\frac{8F_mC}{\pi d^2}\right)$$

Let $\alpha = \tau_m$ = 36.70 kpsi, and $\beta = \dfrac{8F_m}{\pi d^2} = \dfrac{8(11)10^{-3}}{\pi(0.08)^2} = 4.377$ kpsi From Eq. (10-23),

$$C = \frac{2\alpha-\beta}{4\beta}+\sqrt{\left(\frac{2\alpha-\beta}{4\beta}\right)^2-\frac{3\alpha}{4\beta}}$$

$$= \frac{2(36.70)-4.377}{4(4.377)}+\sqrt{\left(\frac{2(36.70)-4.377}{4(4.377)}\right)^2-\frac{3(36.70)}{4(4.377)}} = 6.98$$

$$D = Cd = 6.98(0.08) = 0.558 \text{ in}$$

$$K_B = \frac{4C+2}{4C-3} = \frac{4(6.98)+2}{4(6.98)-3} = 1.201$$

$$\tau_a = K_B\left(\frac{8F_aD}{\pi d^3}\right) = 1.201\left[\frac{8(7)(0.558)}{\pi(0.08^3)}(10^{-3})\right] = 23.3 \text{ kpsi}$$

The Gerber fatigue criterion from Eq. (6-48), adapted for shear,

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2\frac{\tau_a}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2\tau_mS_{se}}{S_{su}\tau_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{163.7}{36.6}\right)^2\frac{23.3}{39.5}\left\{-1+\sqrt{1+\left[2\left(\frac{11}{7}\right)\frac{39.5}{163.7}\right]^2}\right\}$$

$$= 1.50 \qquad \text{checks}$$

$$N_a = \frac{Gd^4}{8kD^3} = \frac{10(10^6)(0.08)^4}{8(9.5)(0.558)^3} = 31.02 \text{ coils}$$

$N_t = 31.02 + 2 = 33$ coils

$L_s = dN_t = 0.08(33) = 2.64$ in

$y_{\max} = F_{\max} / k = 18 / 9.5 = 1.895$ in

$y_s = (1 + \xi)\, y_{\max} = (1 + 0.15)(1.895) = 2.179$ in

$L_0 = L_s + y_{\max} = 2.64 + 2.179 = 4.819$ in

$(L_0)_{\text{cr}} = 2.63\, D / \alpha = 2.63(0.558) / 0.5 = 2.935$ in

$\tau_s = (1 + \xi)(\, F_{\max} / F_a)\ \tau_a\ = 1.15(18/7)23.3 = 68.8$ kpsi

$n_s = S_{sy} / \tau_s = 85.5/68.9 = 1.24$

$$f = \sqrt{\frac{kg}{\pi^2 d^2 D N_a \gamma}} = \sqrt{\frac{9.5(386)}{\pi^2(0.08^2)(0.558)(31.02)(0.283)}} = 109 \text{ Hz}$$

These steps are easily implemented on a spreadsheet, as shown below, for different diameters.

	d_1	d_2	d_3	d_4
d	0.080	0.092	0.106	0.121
m	0.146	0.146	0.263	0.263
A	169.000	169.000	128.000	128.000
S_{ut}	244.363	239.618	231.257	223.311
S_{su}	163.723	160.544	154.942	149.618
S_{sy}	85.527	83.866	80.940	78.159
S_{sa}	35.000	35.000	35.000	35.000
S_{se}	39.452	39.654	40.046	40.469
τ_m	36.667	36.667	36.667	36.667
α	36.667	36.667	36.667	36.667
β	4.377	3.346	2.517	1.929
C	6.977	9.603	13.244	17.702
D	0.558	0.879	1.397	2.133
K_B	1.201	1.141	1.100	1.074
τ_a	23.333	23.333	23.333	23.333
n_f	1.500	1.500	1.500	1.500
N_a	30.993	13.594	5.975	2.858
N_t	32.993	15.594	7.975	4.858
L_s	2.639	1.427	0.841	0.585
y_s	2.179	2.179	2.179	2.179

L_0	4.818	3.606	3.020	2.764
(L_0)cr	2.936	4.622	7.350	11.220
τ_s	69.000	69.000	69.000	69.000
n_s	1.240	1.215	1.173	1.133
f, (Hz)	108.895	114.578	118.863	121.775

The shaded areas depict conditions outside the recommended design conditions. Thus, one spring is satisfactory. The specifications are: A313 stainless wire, unpeened, squared and ground, $d = 0.0915$ in, OD = 0.879 + 0.092 = 0.971 in, $L_0 = 3.606$ in, and $N_t = 15.59$ turns *Ans.*

10-36 The steps are the same as in Prob. 10-35 except that the Gerber-Zimmerli criterion is replaced with the Goodman-Zimmerli relationship of Eq. (10-29*a*) :

$$S_{se} = \frac{S_{sa}}{1 - \left(S_{sm}/S_{su}\right)}$$

The problem then proceeds as in Prob. 10-30. The results for the wire sizes are shown below (see solution to Prob. 10-35 for additional details).

Iteration of d for the first trial									
	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.151	1.108	1.078	1.058
A	169.000	169.000	128.000	128.000	τ_a	29.008	29.040	29.090	29.127
S_{ut}	244.363	239.618	231.257	223.311	n_f	1.500	1.500	1.500	1.500
S_{su}	163.723	160.544	154.942	149.618	N_a	14.191	6.456	2.899	1.404
S_{sy}	85.527	83.866	80.940	78.159	N_t	16.191	8.456	4.899	3.404
S_{sa}	35.000	35.000	35.000	35.000	L_s	1.295	0.774	0.517	0.410
S_{se}	52.706	53.239	54.261	55.345	y_s	2.179	2.179	2.179	2.179
τ_m	45.585	45.635	45.712	45.771	L_0	3.474	2.953	2.696	2.589
α	45.585	45.635	45.712	45.771	$(L_0)_{cr}$	3.809	5.924	9.354	14.219
β	4.377	3.346	2.517	1.929	τ_s	85.782	85.876	86.022	86.133
C	9.052	12.309	16.856	22.433	n_s	0.997	0.977	0.941	0.907
D	0.724	1.126	1.778	2.703	f, (Hz)	141.284	146.853	151.271	154.326

Without checking all of the design conditions, it is obvious that none of the wire sizes satisfy $n_s \geq 1.2$. Also, the Gerber line is closer to the yield line than the Goodman. Setting $n_f = 1.5$ for Goodman makes it impossible to reach the yield line ($n_s < 1$) . The table below uses $n_f = 2$.

Iteration of d for the second trial									
	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.221	1.154	1.108	1.079
A	169.000	169.000	128.000	128.000	τ_a	21.756	21.780	21.817	21.845
S_{ut}	244.363	239.618	231.257	223.311	n_f	2.000	2.000	2.000	2.000
S_{su}	163.723	160.544	154.942	149.618	N_a	40.243	17.286	7.475	3.539
S_{sy}	85.527	83.866	80.940	78.159	N_t	42.243	19.286	9.475	5.539
S_{sa}	35.000	35.000	35.000	35.000	L_s	3.379	1.765	1.000	0.667
S_{se}	52.706	53.239	54.261	55.345	y_s	2.179	2.179	2.179	2.179
τ_m	34.188	34.226	34.284	34.329	L_0	5.558	3.944	3.179	2.846
α	34.188	34.226	34.284	34.329	$(L_0)_{cr}$	2.691	4.266	6.821	10.449
β	4.377	3.346	2.517	1.929	τ_s	64.336	64.407	64.517	64.600
C	6.395	8.864	12.292	16.485	n_s	1.329	1.302	1.255	1.210
D	0.512	0.811	1.297	1.986	f, (Hz)	99.816	105.759	110.312	113.408

The satisfactory spring has design specifications of: A313 stainless wire, unpeened, squared and ground, $d = 0.0915$ in, OD = 0.811 + 0.092 = 0.903 in, $L_0 = 3.944$ in, and .$N_t = 19.3$ turns. *Ans.*

10-37 This is the same as Prob. 10-35 since $S_{sa} = 35$ kpsi. Therefore, the specifications are: A313 stainless wire, unpeened, squared and ground, $d = 0.0915$ in, OD = 0.879 + 0.092 = 0.971 in, $L_0 = 3.606$ in, and $N_t = 15.59$ turns *Ans.*

10-38 For the Gerber-Zimmerli fatigue-failure criterion, $S_{su} = 0.67S_{ut}$,

$$S_{se} = \frac{S_{sa}}{1-(S_{sm}/S_{su})^2}\ , \qquad \tau_m = \frac{1}{2}\frac{S_{su}^2}{n_f}\frac{r}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2S_{se}}{S_{su}r}\right)^2}\right]$$

See the process used in Prob. 10-36. The last 2 columns of diameters of Ex. 10-5 are presented below with additional calculations.

d	0.105	0.112	d	0.105	0.112
S_{ut}	278.691	276.096	N_t	10.915	8.190
S_{su}	186.723	184.984	L_s	1.146	0.917
S_{se}	38.325	38.394	L_0	3.446	3.217
τ_m	38.508	38.502	$(L_0)_{cr}$	6.630	8.160
α	38.508	38.502	K_B	1.111	1.095
β	2.887	2.538	τ_a	23.105	23.101
C	12.004	13.851	n_f	1.500	1.500
D	1.260	1.551	τ_s	70.855	70.844
ID	1.155	1.439	n_s	1.770	1.754
OD	1.365	1.663	f_n	105.433	106.922
N_a	8.915	6.190	fom	-0.973	-1.022

There are only slight changes in the results.

10-39 As in Prob. 10-38, the basic change is S_{sa}.

For Goodman, using Eq. (10-29*a*): $S_{se} = \dfrac{S_{sa}}{1 - (S_{sm} / S_{su})}$

Recalculate τ_m using Eq. (6-41) for shear. That is,

$$n_f = \left(\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}} \right)^{-1} = \frac{S_{se} S_{su}}{\tau_a S_{su} + \tau_m S_{se}} = \frac{S_{se} S_{su}}{\tau_m \left(r S_{su} + S_{se} \right)}$$

Where $r = \tau_a / \tau_m$. Thus,

$$\tau_m = \frac{S_{se} S_{su}}{n_f \left(r S_{su} + S_{se} \right)}$$

See the process used in Prob. 10-36. Calculations for the last 2 diameters of Ex. 10-5 are given below.

d	0.105	0.112	d	0.105	0.112
S_{ut}	278.691	276.096	N_t	11.153	8.353
S_{su}	186.723	184.984	L_s	1.171	0.936
S_{se}	49.614	49.810	L_0	3.471	3.236
τ_m	38.207	38.201	$(L_0)_{cr}$	6.572	8.090
α	38.207	38.201	K_B	1.112	1.096
β	2.887	2.538	τ_a	22.924	22.920
C	11.899	13.732	n_f	1.500	1.500
D	1.249	1.538	τ_s	70.301	70.289
ID	1.144	1.426	n_s	1.784	1.768
OD	1.354	1.650	f_n	104.509	106.000
N_a	9.153	6.353	fom	-0.986	-1.034

There are only slight differences in the results.

10-40 Use: $E = 28.6$ Mpsi, $G = 11.5$ Mpsi, $A = 140$ kpsi · in^m, $m = 0.190$, rel cost = 1.

Try $$d = 0.067\ in, \quad S_{ut} = \frac{140}{(0.067)^{0.190}} = 234.0 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45S_{ut} = 105.3$ kpsi
Table 10-7: $S_y = 0.75S_{ut} = 175.5$ kpsi
Eq. (10-34) with $D/d = C$ and $C_1 = C$

$$\sigma_A = \frac{F_{\max}}{\pi d^2}[(K)_A(16C) + 4] = \frac{S_y}{n_y}$$

$$\frac{4C^2 - C - 1}{4C(C-1)}(16C) + 4 = \frac{\pi d^2 S_y}{n_y F_{\max}}$$

$$4C^2 - C - 1 = (C-1)\left(\frac{\pi d^2 S_y}{4n_y F_{\max}} - 1\right)$$

$$C^2 - \frac{1}{4}\left(1 + \frac{\pi d^2 S_y}{4n_y F_{\max}} - 1\right)C + \frac{1}{4}\left(\frac{\pi d^2 S_y}{4n_y F_{\max}} - 2\right) = 0$$

$$C = \frac{1}{2}\left[\frac{\pi d^2 S_y}{16 n_y F_{\max}} \pm \sqrt{\left(\frac{\pi d^2 S_y}{16 n_y F_{\max}}\right)^2 - \frac{\pi d^2 S_y}{4 n_y F_{\max}} + 2}\,\right] \text{ take positive root}$$

$$= \frac{1}{2}\left\{\frac{\pi(0.067^2)(175.5)(10^3)}{16(1.5)(18)} + \sqrt{\left[\frac{\pi(0.067)^2(175.5)(10^3)}{16(1.5)(18)}\right]^2 - \frac{\pi(0.067)^2(175.5)(10^3)}{4(1.5)(18)} + 2}\right\} = 4.590$$

$$D = Cd = 4.59\left(0.067\right) = 0.3075 \text{ in}$$

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\ 500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range. This results in the best fom.

$$F_i = \frac{\pi(0.067)^3}{8(0.3075)}\left\{\frac{33\ 500}{\exp[0.105(4.590)]} - 1000\left(4 - \frac{4.590-3}{6.5}\right)\right\} = 6.505 \text{ lbf}$$

For simplicity, we will round up to the next integer or half integer. Therefore, use $F_i = 7$ lbf

$$k = \frac{18-7}{0.5} = 22 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.067)^4(11.5)(10^6)}{8(22)(0.3075)^3} = 45.28 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 45.28 - \frac{11.5}{28.6} = 44.88 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.590) - 1 + 44.88](0.067) = 3.555 \text{ in}$$

$$L_{18 \text{ lbf}} = 3.555 + 0.5 = 4.055 \text{ in}$$

Body: $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(4.590)+2}{4(4.590)-3} = 1.326$

$$\tau_{\max} = \frac{8K_B F_{\max} D}{\pi d^3} = \frac{8(1.326)(18)(0.3075)}{\pi(0.067)^3}(10^{-3}) = 62.1 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{S_{sy}}{\tau_{\max}} = \frac{105.3}{62.1} = 1.70$$

$$r_2 = 2d = 2(0.067) = 0.134 \text{ in}, \quad C_2 = \frac{2r_2}{d} = \frac{2(0.134)}{0.067} = 4$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4)-1}{4(4)-4} = 1.25$$

$$\tau_B = (K)_B\left[\frac{8F_{\max}D}{\pi d^3}\right] = 1.25\left[\frac{8(18)(0.3075)}{\pi(0.067)^3}\right](10^{-3}) = 58.58\text{ kpsi}$$

$$(n_y)_B = \frac{S_{sy}}{\tau_B} = \frac{105.3}{58.58} = 1.80$$

$$\text{fom} = -(1)\frac{\pi^2 d^2(N_b+2)D}{4} = -\frac{\pi^2(0.067)^2(44.88+2)(0.3075)}{4} = -0.160$$

Several diameters, evaluated using a spreadsheet, are shown below.

d	0.067	0.072	0.076	0.081	0.085	0.09	0.095	0.104
S_{ut}	233.977	230.799	228.441	225.692	223.634	221.219	218.958	215.224
S_{sy}	105.290	103.860	102.798	101.561	100.635	99.548	98.531	96.851
S_y	175.483	173.100	171.331	169.269	167.726	165.914	164.218	161.418
C	4.589	5.412	6.099	6.993	7.738	8.708	9.721	11.650
D	0.307	0.390	0.463	0.566	0.658	0.784	0.923	1.212
F_i (calc)	6.505	5.773	5.257	4.675	4.251	3.764	3.320	2.621
F_i (rd)	7.0	6.0	5.5	5.0	4.5	4.0	3.5	3.0
k	22.000	24.000	25.000	26.000	27.000	28.000	29.000	30.000
N_a	45.29	27.20	19.27	13.10	9.77	7.00	5.13	3.15
N_b	44.89	26.80	18.86	12.69	9.36	6.59	4.72	2.75
L_0	3.556	2.637	2.285	2.080	2.026	2.071	2.201	2.605
$L_{18\text{ lbf}}$	4.056	3.137	2.785	2.580	2.526	2.571	2.701	3.105
K_B	1.326	1.268	1.234	1.200	1.179	1.157	1.139	1.115
$\tau_{\max}$	62.118	60.686	59.707	58.636	57.875	57.019	56.249	55.031
$(n_y)_{\text{body}}$	1.695	1.711	1.722	1.732	1.739	1.746	1.752	1.760
τ_B	58.576	59.820	60.495	61.067	61.367	61.598	61.712	61.712
$(n_y)_B$	1.797	1.736	1.699	1.663	1.640	1.616	1.597	1.569
$(n_y)_A$	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500
fom	–0.160	–0.144	-0.138	–0.135	–0.133	–0.135	–0.138	–0.154

Except for the 0.067 in wire, all springs satisfy the requirements of length and number of coils. The 0.085 in wire has the highest fom.

10-41 Given: $N_b = 84$ coils, $F_i = 16$ lbf, OQ&T steel, OD = 1.5 in, $d = 0.162$ in.

$$D = \text{OD} - d = 1.5 - 0.162 = 1.338\text{ in}$$

(a) Eq. (10-39):

$$L_0 = 2(D-d) + (N_b+1)d$$
$$= 2(1.338 - 0.162) + (84+1)(0.162) = 16.12\text{ in} \quad \textit{Ans.}$$

or $$2d + L_0 = 2(0.162) + 16.12 = 16.45\text{ in overall}$$

(b)

$$C = \frac{D}{d} = \frac{1.338}{0.162} = 8.26$$

$$K_B = \frac{4C+2}{4C-3} = \frac{4(8.26)+2}{4(8.26)-3} = 1.166$$

$$\tau_i = K_B\left[\frac{8F_iD}{\pi d^3}\right] = 1.166\frac{8(16)(1.338)}{\pi(0.162)^3} = 14\ 950 \text{ psi} \qquad \textit{Ans.}$$

(c) From Table 10-5 use: $G = 11.4(10^6)$ psi and $E = 28.5(10^6)$ psi

$$N_a = N_b + \frac{G}{E} = 84 + \frac{11.4}{28.5} = 84.4 \text{ turns}$$

$$k = \frac{d^4G}{8D^3N_a} = \frac{(0.162)^4(11.4)(10^6)}{8(1.338)^3(84.4)} = 4.855 \text{ lbf/in} \qquad \textit{Ans.}$$

(d) Table 10-4: $A = 147 \text{ psi} \cdot \text{in}^m$, $m = 0.187$

$$S_{ut} = \frac{147}{(0.162)^{0.187}} = 207.1 \text{ kpsi}$$

$$S_y = 0.75(207.1) = 155.3 \text{ kpsi}$$

$$S_{sy} = 0.50(207.1) = 103.5 \text{ kpsi}$$

Body

$$F = \frac{\pi d^3 S_{sy}}{\pi K_B D}$$

$$= \frac{\pi(0.162)^3(103.5)(10^3)}{8(1.166)(1.338)} = 110.8 \text{ lbf}$$

Torsional stress on hook point B

$$C_2 = \frac{2r_2}{d} = \frac{2(0.25 + 0.162\ /\ 2)}{0.162} = 4.086$$

$$(K)_B = \frac{4C_2-1}{4C_2-4} = \frac{4(4.086)-1}{4(4.086)-4} = 1.243$$

$$F = \frac{\pi(0.162)^3(103.5)(10^3)}{8(1.243)(1.338)} = 103.9 \text{ lbf}$$

Normal stress on hook point A

$$C_1 = \frac{2r_1}{d} = \frac{1.338}{0.162} = 8.26$$

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1-1)} = \frac{4(8.26)^2 - 8.26 - 1}{4(8.26)(8.26-1)} = 1.099$$

$$S_{yt} = \sigma = F\left[\frac{16(K)_A D}{\pi d^3} + \frac{4}{\pi d^2}\right]$$

$$F = \frac{155.3(10^3)}{\left[16(1.099)(1.338)\right] / \left[\pi(0.162)^3\right] + \left\{4 / \left[\pi(0.162)^2\right]\right\}} = 85.8 \text{ lbf}$$

$$= \min(110.8,\ 103.9,\ 85.8) = 85.8 \text{ lbf} \quad \textit{Ans.}$$

(e) Eq. (10-48):

$$y = \frac{F - F_i}{k} = \frac{85.8 - 16}{4.855} = 14.4 \text{ in} \quad \textit{Ans.}$$

10-42 $F_{\min} = 9$ lbf, $F_{\max} = 18$ lbf

$$F_a = \frac{18-9}{2} = 4.5 \text{ lbf}, \quad F_m = \frac{18+9}{2} = 13.5 \text{ lbf}$$

A313 stainless: $0.013 \le d \le 0.1$ $A = 169 \text{ kpsi} \cdot \text{in}^m$, $m = 0.146$

$0.1 \le d \le 0.2$ $A = 128 \text{ kpsi} \cdot \text{in}^m$, $m = 0.263$

$E = 28$ Mpsi, $G = 10$ Gpsi

Try $d = 0.081$ in and refer to the discussion following Ex. 10-7

$$S_{ut} = \frac{169}{(0.081)^{0.146}} = 243.9 \text{ kpsi}$$

$$S_{su} = 0.67 S_{ut} = 163.4 \text{ kpsi}$$

$$S_{sy} = 0.35 S_{ut} = 85.4 \text{ kpsi}$$

$$S_y = 0.55 S_{ut} = 134.2 \text{ kpsi}$$

Table 10-8: $S_r = 0.45 S_{ut} = 109.8$ kpsi

$$S_e = \frac{S_r / 2}{1 - [S_r / (2S_{ut})]^2} = \frac{109.8 / 2}{1 - [(109.8 / 2) / 243.9]^2} = 57.8 \text{ kpsi}$$

$$r = \tau_a / \tau_m = F_a / F_m = 4.5 / 13.5$$

For Gerber, Eq. (6-48), solving for $(\sigma_m)_A$ gives

$$(\sigma_m)_A = \frac{1}{2}\frac{S_{ut}^2 r}{n_f S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{S_{ut} r}\right)^2}\right]$$

$$= \frac{1}{2}\frac{243.9^2(4.5/13.5)}{2(57.8)}\left\{-1 + \sqrt{1 + \left[\frac{2(57.8)}{243.9(4.5/13.5)}\right]^2}\right\} = 63.3 \text{ kpsi}$$

Hook bending

$$(\sigma_m)_A = F_m\left[(K)_A \frac{16C}{\pi d^2} + \frac{4}{\pi d^2}\right] = \frac{13.5}{\pi d^2}\left[\frac{(4C^2 - C - 1)16C}{4C(C-1)} + 4\right] \qquad (1)$$

Let $\alpha = \dfrac{(\sigma_a)_A 10^3 \pi d^2}{13.5} = \dfrac{63.3(10^3)\pi(0.081)^2}{13.5} = 96.65$

Equation (1) reduces to

$$C^2 - \frac{\alpha}{16}C + \frac{\alpha - 8}{16} = 0$$

The useable root for C is

$$C = \frac{1}{4}\left(\frac{\alpha}{8} + \sqrt{\left(\frac{\alpha}{8}\right)^2 - \alpha + 8}\right) = \frac{1}{4}\left(\frac{96.647}{8} + \sqrt{\left(\frac{96.647}{8}\right)^2 - 96.647 + 8}\right)$$

$$= 4.91$$

$$(\sigma_a)_A = \frac{F_a}{F_m}(\sigma_m)_A = \frac{4.5}{13.5}63.3 = 21.1 \text{ kpsi}$$

$$(n_f)_A = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2\left(\frac{\sigma_a}{S_e}\right)\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{243.9}{63.3}\right)^2\left(\frac{21.1}{57.8}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(63.3)(57.8)}{243.9(21.1)}\right]^2}\right\} = 2.00 \qquad \text{checks}$$

$$D = Cd = 0.398 \text{ in}$$

Using Eq. (10-4) for τ_i

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\,500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range.

$$F_i = \frac{\pi(0.081)^3}{8(0.398)}\left[\frac{33\,500}{\exp[0.105(4.91)]} - 1000\left(4 - \frac{4.91-3}{6.5}\right)\right]$$
$$= 8.55 \text{ lbf}$$

For simplicity we will round F_i up to next 1/4 integer. Let $F_i = 8.75$ lbf.

$$k = \frac{18-9}{0.25} = 36 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.081)^4(10)(10^6)}{8(36)(0.398)^3} = 23.7 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 23.7 - \frac{10}{28} = 23.3 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.91) - 1 + 23.3](0.081) = 2.602 \text{ in}$$

$$L_{\max} = L_0 + (F_{\max} - F_i)/k = 2.602 + (18 - 8.75)/36 = 2.859 \text{ in}$$

$$\left(\sigma_a\right)_A = \frac{F_a}{F_m}\left(\sigma_m\right)_A = \frac{4.5}{13.5}63.3 = 21.1 \text{ kpsi}$$

Body: $$K_B = \frac{4C+2}{4C-3} = \frac{4(4.91)+2}{4(4.91)-3} = 1.300$$

$$\left(\tau_a\right)_{\text{body}} = \frac{8(1.300)(4.5)(0.398)}{\pi(0.081)^3}(10^{-3}) = 11.16 \text{ kpsi}$$

$$\left(\tau_m\right)_{\text{body}} = \frac{F_m}{F_a}\left(\tau_a\right)_{\text{body}} = \frac{13.5}{4.5}(11.16) = 33.48 \text{ kpsi}$$

The repeating allowable stress from Table 10-8 is

$$S_{sr} = 0.30S_{ut} = 0.30(243.9) = 73.17 \text{ kpsi}$$

The Gerber intercept is given by Eq. (10-42) as

$$S_{se} = \frac{73.17\ /\ 2}{1-[(73.17\ /\ 2)\ /\ 163.4]^2} = 38.5 \text{ kpsi}$$

From Eq. (6-48),

$$(n_f)_{\text{body}} = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2\left(\frac{\tau_a}{S_{se}}\right)\left[-1+\sqrt{1+\left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{163.4}{33.47}\right)^2\left(\frac{11.16}{38.5}\right)\left\{-1+\sqrt{1+\left[\frac{2(33.47)(38.5)}{163.4(11.16)}\right]^2}\right\} = 2.53$$

Let $r_2 = 2d = 2(0.081) = 0.162$

$$C_2 = \frac{2r_2}{d} = 4, \qquad (K)_B = \frac{4(4)-1}{4(4)-4} = 1.25$$

$$(\tau_a)_B = \frac{(K)_B}{K_B}\tau_a = \frac{1.25}{1.30}(11.16) = 10.73 \text{ kpsi}$$

$$(\tau_m)_B = \frac{(K)_B}{K_B}\tau_m = \frac{1.25}{1.30}(33.48) = 32.19 \text{ kpsi}$$

Table 10-8: $(S_{sr})_B = 0.28S_{ut} = 0.28(243.9) = 68.3$ kpsi

$$(S_{se})_B = \frac{68.3\ /\ 2}{1-[(68.3\ /\ 2)\ /\ 163.4]^2} = 35.7 \text{ kpsi}$$

$$(n_f)_B = \frac{1}{2}\left(\frac{163.4}{32.18}\right)^2\left(\frac{10.73}{35.7}\right)\left\{-1+\sqrt{1+\left[\frac{2(32.18)(35.7)}{163.4(10.73)}\right]^2}\right\} = 2.51$$

Yield
Bending:

$$(\sigma_A)_{\max} = \frac{4F_{\max}}{\pi d^2}\left[\frac{(4C^2 - C - 1)}{C - 1} + 1\right]$$

$$= \frac{4(18)}{\pi(0.081^2)}\left[\frac{4(4.91)^2 - 4.91 - 1}{4.91 - 1} + 1\right](10^{-3}) = 84.4 \text{ kpsi}$$

$$(n_y)_A = \frac{134.2}{84.4} = 1.59$$

Body:

$$\tau_i = (F_i / F_a)\tau_a = (8.75 / 4.5)(11.16) = 21.7 \text{ kpsi}$$

$$r = \tau_a/(\tau_m - \tau_i) = 11.16 / (33.47 - 21.7) = 0.948$$

$$(S_{sa})_y = \frac{r}{r+1}(S_{sy} - \tau_i) = \frac{0.948}{0.948+1}(85.4 - 21.7) = 31.0 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{(S_{sa})_y}{\tau_a} = \frac{31.0}{11.16} = 2.78$$

Hook shear:

$$S_{sy} = 0.3S_{ut} = 0.3(243.9) = 73.2 \text{ kpsi}$$

$$\tau_{\max} = (\tau_a)_B + (\tau_m)_B = 10.73 + 32.18 = 42.9 \text{ kpsi}$$

$$(n_y)_B = \frac{73.2}{42.9} = 1.71$$

$$\text{fom} = -\frac{7.6\pi^2 d^2 (N_b + 2)D}{4} = -\frac{7.6\pi^2(0.081)^2(23.3 + 2)(0.398)}{4} = -1.239$$

A tabulation of several wire sizes follow

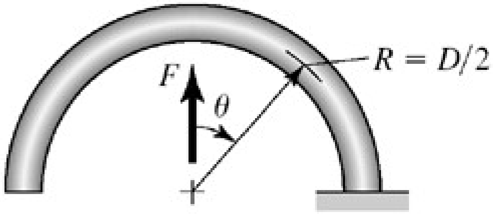
d	0.081	0.085	0.092	0.098	0.105	0.120
S_{ut}	243.920	242.210	239.427	237.229	234.851	230.317
S_{su}	163.427	162.281	160.416	158.943	157.350	154.312
S_{sy}	85.372	84.773	83.800	83.030	82.198	80.611
S_r	109.764	108.994	107.742	106.753	105.683	103.643
S_e	57.809	57.403	56.744	56.223	55.659	54.585
$(\sigma_m)_A$	63.331	62.887	62.164	61.594	60.976	59.799
α	96.695	105.734	122.443	137.659	156.443	200.389
C	4.916	5.497	6.563	7.527	8.713	11.477
$(\sigma_a)_A$	21.110	20.962	20.721	20.531	20.325	19.933
$(n_f)_A$	2.000	2.000	2.000	2.000	2.000	2.000
D	0.398	0.467	0.604	0.738	0.915	1.377
OD	0.479	0.552	0.696	0.836	1.020	1.497
F_i (calc)	8.537	7.842	6.769	5.960	5.117	3.618
F_i (rd)	8.750	8.750	8.750	8.750	8.750	8.750

k	36.000	36.000	36.000	36.000	36.000	36.000
N_a	23.677	17.767	11.301	7.979	5.512	2.756
N_b	23.320	17.410	10.944	7.622	5.155	2.399
L_0	2.604	2.329	2.122	2.124	2.266	2.922
$L_{\max}$	2.861	2.586	2.379	2.381	2.523	3.179
K_B	1.300	1.263	1.215	1.184	1.157	1.117
$(\tau_a)_{\text{body}}$	11.162	11.015	10.796	10.638	10.478	10.197
$(\tau_m)_{\text{body}}$	33.486	33.044	32.388	31.913	31.433	30.591
S_{sr}	73.176	72.663	71.828	71.169	70.455	69.095
S_{se}	38.519	38.249	37.809	37.462	37.087	36.371
$(n_f)_{\text{body}}$	2.526	2.542	2.564	2.578	2.591	2.611
$(K_B)_B$	4.000	4.000	4.000	4.000	4.000	4.000
$(\tau_a)_B$	10.732	10.898	11.106	11.226	11.320	11.416
$(\tau_m)_B$	32.196	32.695	33.319	33.679	33.960	34.248
$(S_{sr})_B$	68.298	67.819	67.040	66.424	65.758	64.489
$(S_{se})_B$	35.708	35.458	35.050	34.728	34.380	33.717
$(n_f)_B$	2.512	2.457	2.383	2.336	2.293	2.230
S_y	134.156	133.215	131.685	130.476	129.168	126.674
$(\sigma_A)_{\max}$	84.441	83.849	82.886	82.125	81.302	79.732
$(n_y)_A$	1.589	1.589	1.589	1.589	1.589	1.589
τ_i	21.704	21.417	20.992	20.684	20.373	19.828
r	0.947	0.947	0.947	0.947	0.947	0.947
$(S_{sa})_y$	30.974	30.822	30.555	30.330	30.077	29.570
$(n_y)_{\text{body}}$	2.775	2.798	2.830	2.851	2.871	2.900
$(S_{sy})_B$	73.176	72.663	71.828	71.169	70.455	69.095
$(\tau_B)_{\max}$	42.928	43.594	44.426	44.905	45.280	45.664
$(n_y)_B$	1.705	1.667	1.617	1.585	1.556	1.513
fom	-1.240	-1.229	-1.240	-1.278	-1.353	-1.636

↑ optimal fom

The shaded areas show the conditions not satisfied.

10-43 For the hook,



$$M = FR\sin\theta, \quad \partial M/\partial F = R\sin\theta$$

$$\delta_F = \frac{1}{EI}\int_0^{\pi/2} F\left(R\sin\theta\right)^2 R\,d\theta = \frac{\pi}{2}\frac{FR^3}{EI}$$

The total deflection of the body and the two hooks

$$\delta = \frac{8FD^3N_b}{d^4G} + 2\left(\frac{\pi}{2}\frac{FR^3}{EI}\right) = \frac{8FD^3N_b}{d^4G} + \frac{\pi F(D/2)^3}{E(\pi/64)(d^4)}$$

$$= \frac{8FD^3}{d^4G}\left(N_b + \frac{G}{E}\right) = \frac{8FD^3N_a}{d^4G}$$

$$\therefore\ N_a = N_b + \frac{G}{E} \qquad \text{Q.E.D.}$$

10-44 Table 10-5 (d = 4 mm = 0.1575 in): E = 196.5 GPa

Table 10-4 for A227:

$$A = 1783\ \text{MPa}\cdot\text{mm}^m, \qquad m = 0.190$$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{1783}{4^{0.190}} = 1370\ \text{MPa}$

Eq. (10-57): $S_y = \sigma_{\text{all}} = 0.78\,S_{ut} = 0.78(1370) = 1069\ \text{MPa}$

$D = \text{OD} - d = 32 - 4 = 28\ \text{mm}$

$C = D/d = 28/4 = 7$

Eq. (10-43): $K_i = \dfrac{4C^2 - C - 1}{4C(C-1)} = \dfrac{4\left(7^2\right) - 7 - 1}{4(7)(7-1)} = 1.119$

Eq. (10-44): $\sigma = K_i\dfrac{32Fr}{\pi d^3}$

At yield, $Fr = M_y$, $\sigma = S_y$. Thus,

$$M_y = \frac{\pi d^3 S_y}{32K_i} = \frac{\pi\left(4^3\right)1069\left(10^{-3}\right)}{32(1.119)} = 6.00\ \text{N}\cdot\text{m}$$

Count the turns when $M = 0$

$$N = 2.5 - \frac{M_y}{k}$$

where from Eq. (10-51): $k = \dfrac{d^4E}{10.8DN}$

Thus,

$$N = 2.5 - \frac{M_y}{d^4E / (10.8DN)}$$

Solving for N gives

$$N = \frac{2.5}{1 + [10.8DM_y / (d^4E)]}$$
$$= \frac{2.5}{1 + \{[10.8(28)(6.00)] / [4^4(196.5)]\}} = 2.413 \text{ turns}$$

This means (2.5 − 2.413)(360°) or 31.3° from closed. *Ans.*

Treating the hand force as in the middle of the grip,

$$r = 112.5 - 87.5 + \frac{87.5}{2} = 68.75 \text{ mm}$$
$$F_{\max} = \frac{M_y}{r} = \frac{6.00\left(10^3\right)}{68.75} = 87.3 \text{ N} \quad \textit{Ans.}$$

10-45 The spring material and condition are unknown. Given $d = 0.081$ in and OD = 0.500,
(a) $D = 0.500 - 0.081 = 0.419$ in
Using $E = 28.6$ Mpsi for an estimate

$$k' = \frac{d^4E}{10.8DN} = \frac{(0.081)^4(28.6)(10^6)}{10.8(0.419)(11)} = 24.7 \text{ lbf} \cdot \text{in/turn}$$

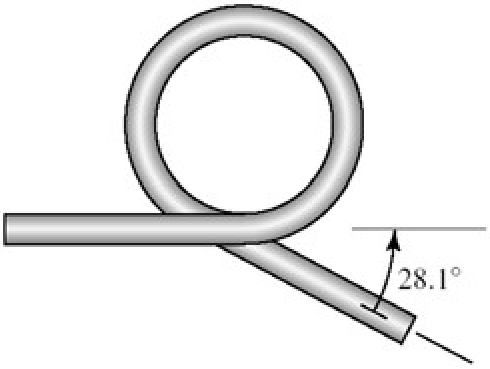
for each spring. The moment corresponding to a force of 8 lbf

$$Fr = (8/2)(3.3125) = 13.25 \text{ lbf} \cdot \text{in/spring}$$

The fraction windup turn is

$$n = \frac{Fr}{k'} = \frac{13.25}{24.7} = 0.536 \text{ turns}$$

The arm swings through an arc of slightly less than 180°, say 165°. This uses up 165/360 or 0.458 turns. So $n = 0.536 - 0.458 = 0.078$ turns are left (or $0.078(360°) = 28.1°$). The original configuration of the spring was



Ans.

(b)

$$C = \frac{D}{d} = \frac{0.419}{0.081} = 5.17$$

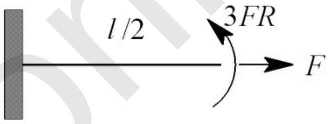
$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4(5.17)^2 - 5.17 - 1}{4(5.17)(5.17 - 1)} = 1.168$$

$$\sigma = K_i \frac{32M}{\pi d^3} = 1.168\left[\frac{32(13.25)}{\pi(0.081)^3}\right] = 297\left(10^3\right) \text{ psi} = 297 \text{ kpsi} \qquad \textit{Ans.}$$

To achieve this stress level, the spring had to have set removed.

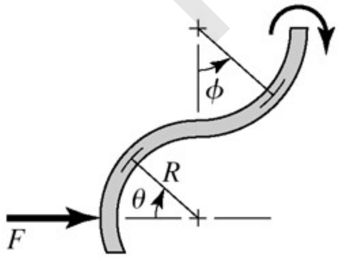
10-46 (**a**) Consider half and double results

Straight section:



$M = 3FR, \quad \dfrac{\partial M}{\partial F} = 3R$

Upper 180° section:



$$\begin{aligned} M &= F[R + R(1 - \cos\phi)] \\ &= FR(2 - \cos\phi), \quad \frac{\partial M}{\partial F} = R(2 - \cos\phi) \end{aligned}$$

Lower section: $M = FR\sin\theta, \quad \dfrac{\partial M}{\partial F} = R\sin\theta$

Considering bending only:

$$\delta = \frac{\partial U}{\partial F} = \frac{2}{EI}\left[\int_0^{l/2} 9FR^2\,dx + \int_0^{\pi} FR^2(2-\cos\phi)^2 R\,d\phi + \int_0^{\pi/2} F(R\sin\theta)^2 R\,d\theta\right]$$

$$= \frac{2F}{EI}\left[\frac{9}{2}R^2 l + R^3\left(4\pi - 4\sin\phi\Big|_0^{\pi} + \frac{\pi}{2}\right) + R^3\left(\frac{\pi}{4}\right)\right]$$

$$= \frac{2FR^2}{EI}\left(\frac{19\pi}{4}R + \frac{9}{2}l\right) = \frac{FR^2}{2EI}(19\pi R + 18l)$$

The spring rate is

$$k = \frac{F}{\delta} = \frac{2EI}{R^2(19\pi R + 18l)} \qquad Ans.$$

(**b**) Given: A227 HD wire, $d = 2$ mm, $R = 6$ mm, and $l = 25$ mm.

Table 10-5 ($d = 2$ mm $= 0.0787$ in): $E = 197.2$ GPa

$$k = \frac{2(197.2)10^9\pi\left(0.002^4\right)/(64)}{0.006^2\left[19\pi(0.006)+18(0.025)\right]} = 10.65\left(10^3\right)\text{ N/m} = 10.65\text{ N/mm} \qquad Ans.$$

(**c**) The maximum stress will occur at the bottom of the top hook where the bending-moment is $3FR$ and the axial fore is F. Using curved beam theory for bending,

Eq. (3-65): $$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{3FRc_i}{\left(\pi d^2/4\right)e\left(R-d/2\right)}$$

Axial: $$\sigma_a = \frac{F}{A} = \frac{F}{\pi d^2/4}$$

Combining, $$\sigma_{\max} = \sigma_i + \sigma_a = \frac{4F}{\pi d^2}\left[\frac{3Rc_i}{e\left(R-d/2\right)} + 1\right] = S_y$$

$$F = \frac{\pi d^2 S_y}{4\left[\dfrac{3Rc_i}{e\left(R-d/2\right)} + 1\right]} \qquad (1) \qquad Ans.$$

For the clip in part (b),

Eq. (10-14) and Table 10-4: $S_{ut} = A/d^m = 1783/2^{0.190} = 1563$ MPa

Eq. (10-57): $S_y = 0.78\,S_{ut} = 0.78(1563) = 1219$ MPa

Table 3-4:

$$r_n = \frac{1^2}{2\left(6-\sqrt{6^2-1^2}\right)} = 5.95804 \text{ mm}$$

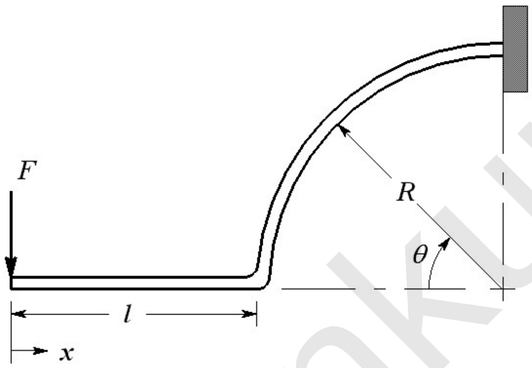
$$e = r_c - r_n = 6 - 5.95804 = 0.04196 \text{ mm}$$

$$c_i = r_n - (R - d/2) = 5.95804 - (6 - 2/2) = 0.95804 \text{ mm}$$

Eq. (1):

$$F = \frac{\pi\left(0.002^2\right)1219\left(10^6\right)}{4\left[\dfrac{3(6)0.95804}{0.04196(6-1)}+1\right]} = 46.0 \text{ N} \qquad Ans.$$

10-47 **(a)**



$$M = -Fx, \quad \frac{\partial M}{\partial F} = -x \quad 0 \le x \le l$$

$$M = Fl + FR(1-\cos\theta), \quad \frac{\partial M}{\partial F} = l + R(1-\cos\theta) \quad 0 \le \theta \le \pi/2$$

$$\delta_F = \frac{1}{EI}\left\{\int_0^l -Fx(-x)dx + \int_0^{\pi/2} F\left[l+R(1-\cos\theta)\right]^2 Rd\theta\right\}$$

$$= \frac{F}{12EI}\left\{4l^3 + 3R\left[2\pi l^2 + 4(\pi-2)lR + (3\pi-8)R^2\right]\right\}$$

The spring rate is

$$k = \frac{F}{\delta_F} = \frac{12EI}{4l^3 + 3R\left[2\pi l^2 + 4(\pi-2)lR + (3\pi-8)R^2\right]} \qquad Ans.$$

(**b**) Given: A313 stainless wire, $d = 0.063$ in, $R = 0.625$ in, and $l = 0.5$ in.

Table 10-5: $E = 28$ Mpsi

$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64}\left(0.063^4\right) = 7.733\left(10^{-7}\right)\text{ in}^4$$

$$k = \frac{12(28)10^6(7.733)10^{-7}}{4\left(0.5^3\right)+3(0.625)\left[2\pi\left(0.5^2\right)+4(\pi-2)0.5(0.625)+(3\pi-8)\left(0.625^2\right)\right]}$$
$$= 36.3\text{ lbf/in} \qquad Ans.$$

(**c**) Table 10-4: $A = 169$ kpsi·in^m, $m = 0.146$

Eq. (10-14): $S_{ut} = A/d^m = 169/0.063^{0.146} = 253.0$ kpsi

Eq. (10-57): $S_y = 0.61\ S_{ut} = 0.61(253.0) = 154.4$ kpsi

One can use curved beam theory as in the solution for Prob. 10-41. However, the equations developed in Sec. 10-12 are equally valid.

$$C = D/d = 2(0.625 + 0.063/2)/0.063 = 20.8$$

Eq. (10-43): $$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4\left(20.8^2\right) - 20.8 - 1}{4(20.8)(20.8-1)} = 1.037$$

Eq. (10-44), setting $\sigma = S_y$:

$$K_i\frac{32Fr}{\pi d^3} = S_y \quad \Rightarrow \quad 1.037\frac{32F(0.5+0.625)}{\pi\left(0.063^3\right)} = 154.4\left(10^3\right)$$

Solving for F yields $F = 3.25$ lbf *Ans.*

Try solving part (c) of this problem using curved beam theory. You should obtain the same answer.

10-48 (**a**) $M = -Fx$

$$\sigma = \left|\frac{M}{I/c}\right| = \frac{Fx}{I/c} = \frac{Fx}{bh^2/6}$$

Constant stress,

$$\frac{bh^2}{6} = \frac{Fx}{\sigma} \quad \Rightarrow \quad h = \sqrt{\frac{6Fx}{b\sigma}} \qquad (1) \qquad Ans.$$

At $x = l$,

$$h_o = \sqrt{\frac{6Fl}{b\sigma}} \qquad \Rightarrow \qquad h = h_o\sqrt{x/l} \qquad Ans.$$

(b) $M = -Fx, \quad \partial M/\partial F = -x$

$$y = \int_0^l \frac{M\left(\partial M/\partial F\right)}{EI}dx = \frac{1}{E}\int_0^l \frac{-Fx\left(-x\right)}{\frac{1}{12}bh_o^3\left(x/l\right)^{3/2}}dx = \frac{12Fl^{3/2}}{bh_o^3E}\int_0^l x^{1/2}dx$$

$$= \frac{2}{3}\frac{12Fl^{3/2}}{bh_o^3E}l^{3/2} = \frac{8Fl^3}{bh_o^3E}$$

$$k = \frac{F}{y} = \frac{bh_o^3E}{8l^3} \qquad Ans.$$

10-49 Computer programs will vary.

10-50 Computer programs will vary.

Chapter 13

13-1

$$d_P = 17/8 = 2.125 \text{ in}$$

$$d_G = \frac{N_2}{N_3} d_P = \frac{1120}{544}(2.125) = 4.375 \text{ in}$$

$$N_G = Pd_G = 8(4.375) = 35 \text{ teeth} \qquad \textit{Ans.}$$

$$C = (2.125 + 4.375)/2 = 3.25 \text{ in} \qquad \textit{Ans.}$$

13-2

$$n_G = 1600(15/60) = 400 \text{ rev/min} \qquad \textit{Ans.}$$

$$p = \pi m = 3\pi \text{ mm} \qquad \textit{Ans.}$$

$$C = \left[3(15+60)\right]/2 = 112.5 \text{ mm} \qquad \textit{Ans.}$$

13-3

$$N_G = 16(4) = 64 \text{ teeth} \qquad \textit{Ans.}$$

$$d_G = N_G m = 64(6) = 384 \text{ mm} \qquad \textit{Ans.}$$

$$d_P = N_P m = 16(6) = 96 \text{ mm} \qquad \textit{Ans.}$$

$$C = (384 + 96)/2 = 240 \text{ mm} \qquad \textit{Ans.}$$

13-4 *Mesh:*

$$a = 1/P = 1/3 = 0.3333 \text{ in} \qquad \textit{Ans.}$$
$$b = 1.25/P = 1.25/3 = 0.4167 \text{ in} \qquad \textit{Ans.}$$
$$c = b - a = 0.0834 \text{ in} \qquad \textit{Ans.}$$
$$p = \pi/P = \pi/3 = 1.047 \text{ in} \qquad \textit{Ans.}$$
$$t = p/2 = 1.047/2 = 0.523 \text{ in} \qquad \textit{Ans.}$$

Pinion Base-Circle:

$$d_1 = N_1/P = 21/3 = 7 \text{ in}$$

$$d_{1b} = 7\cos 20^\circ = 6.578 \text{ in} \qquad \textit{Ans.}$$

Gear Base-Circle:

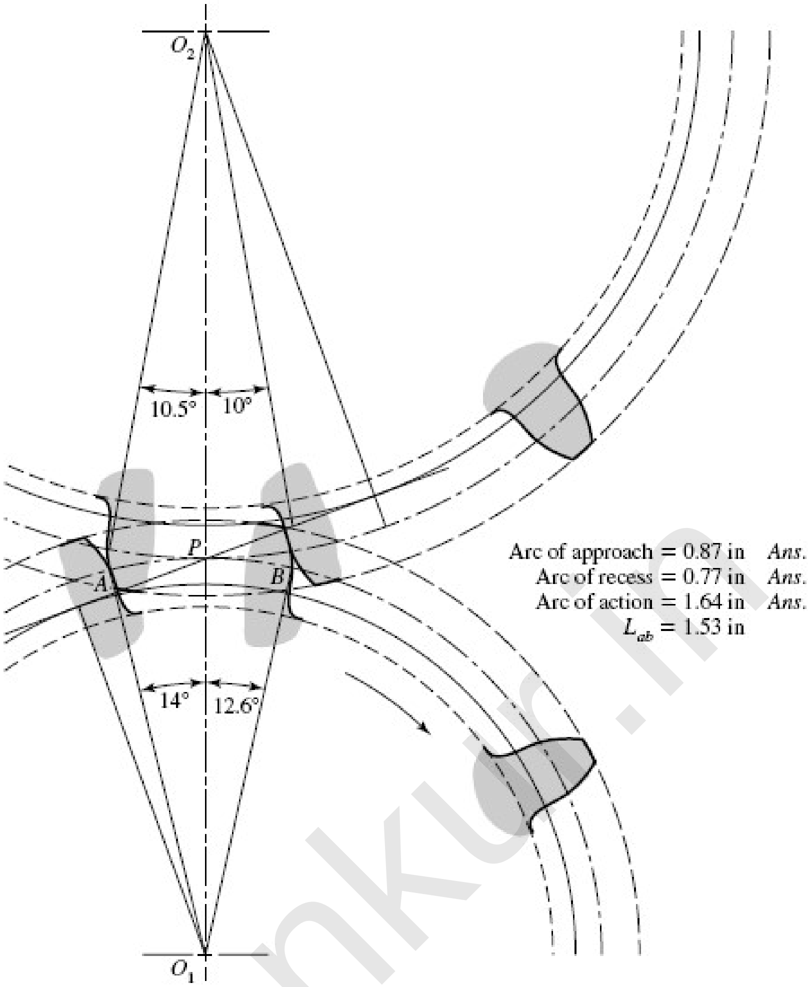
$$d_2 = N_2/P = 28/3 = 9.333 \text{ in}$$

$$d_{2b} = 9.333\cos 20^\circ = 8.770 \text{ in} \qquad \textit{Ans.}$$

Base pitch: $p_b = p_c \cos\phi = (\pi/3)\cos 20^\circ = 0.984 \text{ in} \qquad \textit{Ans.}$

Contact Ratio: $m_c = L_{ab}/p_b = 1.53/0.984 = 1.55 \qquad \textit{Ans.}$

See the following figure for a drawing of the gears and the arc lengths.



Arc of approach = 0.87 in *Ans.*
Arc of recess = 0.77 in *Ans.*
Arc of action = 1.64 in *Ans.*
L_{ab} = 1.53 in

13-5

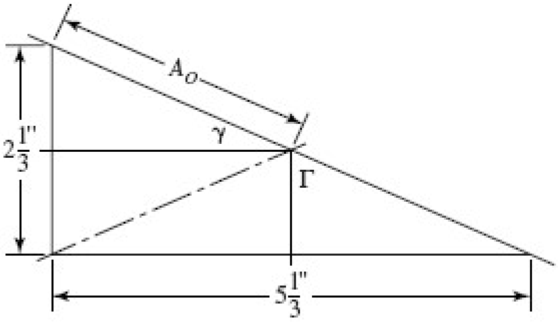
(a)
$$A_O = \left[\left(\frac{14/6}{2}\right)^2 + \left(\frac{32/6}{2}\right)^2\right]^{1/2} = 2.910 \text{ in} \quad \textit{Ans.}$$

(b)
$$\gamma = \tan^{-1}(14/32) = 23.63^\circ \quad \textit{Ans.}$$
$$\Gamma = \tan^{-1}(32/14) = 66.37^\circ \quad \textit{Ans.}$$

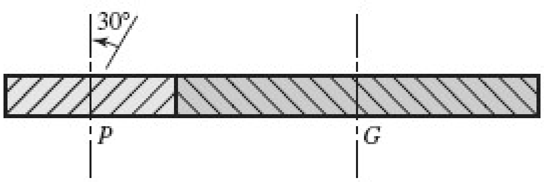
(c)
$$d_P = 14/6 = 2.333 \text{ in} \quad \textit{Ans.}$$
$$d_G = 32/6 = 5.333 \text{ in} \quad \textit{Ans.}$$

(d) From Table 13-3, $0.3A_O = 0.3(2.910)$ $= 0.873$ in and $10/P = 10/6 = 1.67$

$$0.873 < 1.67 \quad \therefore\ F = 0.873 \text{ in} \quad \textit{Ans.}$$



13-6



(a) $p_n = \pi / P_n = \pi / 4 = 0.7854$ in

$p_t = p_n / \cos\psi = 0.7854 / \cos 30^\circ = 0.9069$ in

$p_x = p_t / \tan\psi = 0.9069 / \tan 30^\circ = 1.571$ in

(b) Eq. (13-7): $p_{nb} = p_n \cos\phi_n = 0.7854 \cos 25^\circ = 0.7380$ in *Ans.*

(c) $P_t = P_n \cos\psi = 4 \cos 30^\circ = 3.464$ teeth/in

$\phi_t = \tan^{-1}\left(\tan\phi_n / \cos\psi\right) = \tan^{-1}(\tan 25^\circ / \cos 30^\circ) = 28.3^\circ$ *Ans.*

(d) Table 13-4:

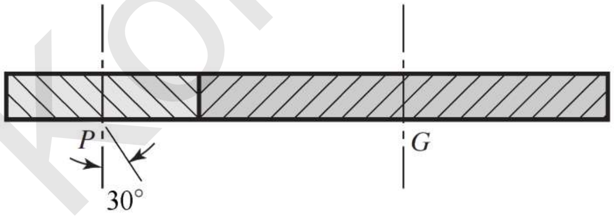
$a = 1/4 = 0.250$ in *Ans.*

$b = 1.25/4 = 0.3125$ in *Ans.*

$$d_P = \frac{20}{4\cos 30^\circ} = 5.774 \text{ in} \quad \textit{Ans.}$$

$$d_G = \frac{36}{4\cos 30^\circ} = 10.39 \text{ in} \quad \textit{Ans.}$$

13-7



$N_P = 19$ teeth, $N_G = 57$ teeth, $\phi_n = 20^\circ, m_n = 2.5$ mm

(a) $p_n = \pi m_n = \pi(2.5) = 7.854$ mm *Ans.*

$$p_t = \frac{p_n}{\cos\psi} = \frac{7.854}{\cos 30^\circ} = 9.069 \text{ mm} \quad \textit{Ans.}$$

$$p_x = \frac{p_t}{\tan\psi} = \frac{9.069}{\tan 30^\circ} = 15.71 \text{ mm} \quad \textit{Ans.}$$

(b) $$m_t = \frac{m_n}{\cos\psi} = \frac{2.5}{\cos 30^\circ} = 2.887 \text{ mm} \quad \textit{Ans.}$$

$$\phi_t = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.80^\circ \quad \textit{Ans.}$$

(c) $a = m_n = 2.5\,\text{mm} \quad \textit{Ans.}$

$$b = 1.25m_n = 1.25(2.5) = 3.125 \text{ mm} \quad \textit{Ans.}$$

$$d_P = \frac{N}{P_t} = Nm_t = 19(2.887) = 54.85 \text{ mm} \quad \textit{Ans.}$$

$$d_G = 57(2.887) = 164.6 \text{ mm} \quad \textit{Ans.}$$

13-8 **(a)** Using Eq. (13-11) with $k = 1$, $\phi = 20^\circ$, and $m = 2$,

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{[1+2(2)]\sin^2(20^\circ)}\left\{(2) + \sqrt{(2)^2 + [1+2(2)]\sin^2(20^\circ)}\right\} = 14.16 \text{ teeth}$$

Round up for the minimum integer number of teeth.

$N_P = 15$ teeth *Ans.*

(b) Repeating (a) with $m = 3$, $N_P = 14.98$ teeth. Rounding up, $N_P = 15$ teeth. *Ans.*
(c) Repeating (a) with $m = 4$, $N_P = 15.44$ teeth. Rounding up, $N_P = 16$ teeth. *Ans.*
(d) Repeating (a) with $m = 5$, $N_P = 15.74$ teeth. Rounding up, $N_P = 16$ teeth. *Ans.*

Alternatively, a useful table can be generated to determine the largest gear that can mesh with a specified pinion, and thus also the maximum gear ratio with a specified pinion. The Max N_G column was generated using Eq. (13-12) with $k = 1$, $\phi = 20^\circ$, and rounding up to the next integer.

Min N_P	**Max N_G**	**Max m = Max N_G / Min N_P**
13	16	1.23
14	26	1.86
15	45	3.00
16	101	6.31
17	1309	77.00
18	unlimited	unlimited

With this table, we can readily see that gear ratios up to 3 can be obtained with a minimum N_P of 15 teeth, and gear ratios up to 6.31 can be obtained with a minimum N_P of 16 teeth. This is consistent with the results previously obtained.

13-9 Repeating the process shown in the solution to Prob. 13-8, except with $\phi = 25^\circ$, we obtain the following results.

(a) For $m = 2$, $N_P = 9.43$ teeth. Rounding up, $N_P = 10$ teeth. *Ans.*
(b) For $m = 3$, $N_P = 9.92$ teeth. Rounding up, $N_P = 10$ teeth. *Ans.*
(c) For $m = 4$, $N_P = 10.20$ teeth. Rounding up, $N_P = 11$ teeth. *Ans.*
(d) For $m = 5$, $N_P = 10.38$ teeth. Rounding up, $N_P = 11$ teeth. *Ans.*

For convenient reference, we will also generate the table from Eq. (13-12) for $\phi = 25^\circ$.

Min N_P	**Max N_G**	**Max m = Max N_G / Min N_P**
9	13	1.44
10	32	3.20
11	249	22.64
12	unlimited	unlimited

13-10 **(a)** The smallest pinion tooth count that will run with itself is found from Eq. (13-10).

$$\begin{aligned} N_P &\ge \frac{2k}{3\sin^2\phi}\left(1+\sqrt{1+3\sin^2\phi}\right) \\ &\ge \frac{2(1)}{3\sin^2 20^\circ}\left(1+\sqrt{1+3\sin^2 20^\circ}\right) \\ &\ge 12.32 \quad \rightarrow \quad 13 \text{ teeth} \quad \textit{Ans.} \end{aligned}$$

(b) The smallest pinion that will mesh with a gear ratio of $m_G = 2.5$, from Eq. (13-11) is

$$\begin{aligned} N_P &\ge \frac{2k}{(1+2m)\sin^2\phi}\left(m+\sqrt{m^2+(1+2m)\sin^2\phi}\right) \\ &\ge \frac{2(1)}{\left[1+2(2.5)\right]\sin^2 20^\circ}\left\{2.5+\sqrt{2.5^2+\left[1+2(2.5)\right]\sin^2 20^\circ}\right\} \\ &\ge 14.64 \quad \rightarrow \quad 15 \text{ teeth} \quad \textit{Ans.} \end{aligned}$$

The largest gear-tooth count possible to mesh with this pinion, from Eq. (13-12) is

$$\begin{aligned} N_G &\le \frac{N_P^2\sin^2\phi-4k^2}{4k-2N_P\sin^2\phi} \\ &\le \frac{15^2\sin^2 20^\circ-4(1)^2}{4(1)-2(15)\sin^2 20^\circ} \\ &\le 45.49 \quad \rightarrow \quad 45 \text{ teeth} \quad \textit{Ans.} \end{aligned}$$

(c) The smallest pinion that will mesh with a rack, from Eq. (13-13),

$$N_P \geq \frac{2k}{\sin^2 \phi} = \frac{2(1)}{\sin^2 20^\circ}$$
$$\geq 17.097 \quad \rightarrow \quad 18 \text{ teeth} \qquad Ans.$$

13-11 $\phi_n = 20^\circ, \psi = 30^\circ$

From Eq. (13-19), $\phi_t = \tan^{-1}\left(\tan 20^\circ / \cos 30^\circ\right) = 22.80^\circ$

(a) The smallest pinion tooth count that will run with itself, from Eq. (13-21) is

$$N_P \geq \frac{2k\cos\psi}{3\sin^2\phi_t}\left(1+\sqrt{1+3\sin^2\phi_t}\right)$$
$$\geq \frac{2(1)\cos 30^\circ}{3\sin^2 22.80^\circ}\left(1+\sqrt{1+3\sin^2 22.80^\circ}\right)$$
$$\geq 8.48 \quad \rightarrow \quad 9 \text{ teeth} \qquad Ans.$$

(b) The smallest pinion that will mesh with a gear ratio of $m = 2.5$, from Eq. (13-22) is

$$N_P \geq \frac{2(1)\cos 30^\circ}{\left[1+2(2.5)\right]\sin^2 22.80^\circ}\left\{2.5+\sqrt{2.5^2+\left[1+2(2.5)\right]\sin^2 22.80^\circ}\right\}$$
$$\geq 9.95 \quad \rightarrow \quad 10 \text{ teeth} \qquad Ans.$$

The largest gear-tooth count possible to mesh with this pinion, from Eq. (13-23) is

$$N_G \leq \frac{N_P^2\sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_P\sin^2\phi_t}$$
$$\leq \frac{10^2\sin^2 22.80^\circ - 4(1)\cos^2 30^\circ}{4(1)\cos^2 30^\circ - 2(20)\sin^2 22.80^\circ}$$
$$\leq 26.08 \quad \rightarrow \quad 26 \text{ teeth} \qquad Ans.$$

(c) The smallest pinion that will mesh with a rack, from Eq. (13-24) is

$$N_P \geq \frac{2k\cos\psi}{\sin^2\phi_t} = \frac{2(1)\cos 30^\circ}{\sin^2 22.80^\circ}$$
$$\geq 11.53 \quad \rightarrow \quad 12 \text{ teeth} \qquad Ans.$$

13-12 From Eq. (13-19), $\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.796^\circ$

Program Eq. (13-23) on a computer using a spreadsheet or code, and increment N_P. The first value of N_P that can be doubled is $N_P = 10$ teeth, where $N_G \le 26.01$ teeth. So $N_G = 20$ teeth will work. Higher tooth counts will work also, for example 11:22, 12:24, etc.

Use $N_P = 10$ teeth, $N_G = 20$ teeth *Ans.*

Note that the given diametral pitch (tooth size) is not relevant to the interference problem.

13-13 From Eq. (13-19), $\phi_t = \tan^{-1}\left(\dfrac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\dfrac{\tan 20^\circ}{\cos 45^\circ}\right) = 27.236^\circ$

Program Eq. (13-23) on a computer using a spreadsheet or code, and increment N_P. The first value of N_P that can be doubled is $N_P = 6$ teeth, where $N_G \le 17.6$ teeth. So $N_G = 12$ teeth will work. Higher tooth counts will work also, for example 7:14, 8:16, etc.

Use $N_P = 6$ teeth, $N_G = 12$ teeth *Ans.*

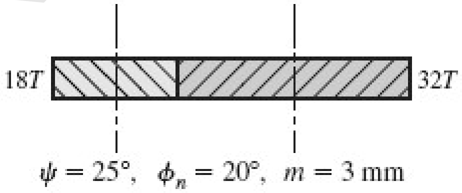
13-14 The smallest pinion that will operate with a rack without interference is given by Eq. (13-13).

$$N_P = \frac{2k}{\sin^2\phi}$$

Setting $k = 1$ for full depth teeth, $N_P = 9$ teeth, and solving for ϕ,

$$\phi = \sin^{-1}\sqrt{\frac{2k}{N_P}} = \sin^{-1}\sqrt{\frac{2(1)}{9}} = 28.126^\circ \qquad Ans.$$

13-15



(a) Eq. (13-3): $p_n = \pi m_n = 3\pi$ mm *Ans.*

Eq. (13-16): $p_t = p_n / \cos\psi = 3\pi / \cos 25^\circ = 10.40$ mm *Ans.*

Eq. (13-17): $p_x = p_t / \tan\psi = 10.40 / \tan 25^\circ = 22.30$ mm *Ans.*

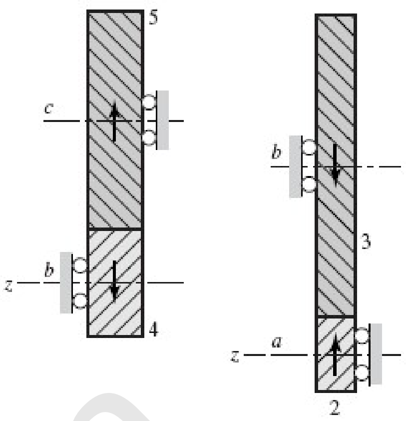
(b) Eq. (13-3): $m_t = p_t / \pi = 10.40 / \pi = 3.310$ mm *Ans.*

Eq. (13-19): $\phi_t = \tan^{-1}\dfrac{\tan\phi_n}{\cos\psi} = \tan^{-1}\dfrac{\tan 20^\circ}{\cos 25^\circ} = 21.88^\circ$ *Ans.*

(c) Eq. (13-2): $d_P = m_t N_P = 3.310\ (18) = 59.58$ mm *Ans.*

Eq. (13-2): $d_G = m_t N_G = 3.310\ (32) = 105.92$ mm *Ans.*

13-16 **(a)** Sketches of the figures are shown to determine the axial forces by inspection.



The axial force of gear 2 on shaft *a* is in the negative *z*-direction. The axial force of gear 3 on shaft *b* is in the positive *z*-direction. *Ans.*

The axial force of gear 4 on shaft *b* is in the positive *z*-direction. The axial force of gear 5 on shaft *c* is in the negative *z*-direction. *Ans.*

(b) $$n_c = n_5 = \frac{12}{48}\left(\frac{16}{36}\right)(700) = +77.78 \text{ rev/min ccw} \quad \textit{Ans.}$$

(c) $$d_{P2} = 12/\left(12\cos 30^\circ\right) = 1.155 \text{ in}$$

$$d_{G3} = 48/\left(12\cos 30^\circ\right) = 4.619 \text{ in}$$

$$C_{ab} = \frac{1.155 + 4.619}{2} = 2.887 \text{ in} \quad \textit{Ans.}$$

$$d_{P4} = 16/\left(8\cos 25^\circ\right) = 2.207 \text{ in}$$

$$d_{G5} = 36/\left(8\cos 25^\circ\right) = 4.965 \text{ in}$$

$$C_{bc} = 3.586 \text{ in} \quad \textit{Ans.}$$

13-17

$$e = \frac{20}{40}\left(\frac{8}{17}\right)\left(\frac{20}{60}\right) = \frac{4}{51}$$

$$n_d = \frac{4}{51}(600) = 47.06 \text{ rev/min cw} \quad \textit{Ans.}$$

13-18

$$e = \frac{6}{10}\left(\frac{18}{38}\right)\left(\frac{20}{48}\right)\left(\frac{3}{36}\right) = \frac{3}{304}$$

$$n_9 = \frac{3}{304}(1200) = 11.84 \text{ rev/min cw} \quad \textit{Ans.}$$

13-19 **(a)**

$$\left.\begin{array}{r} n_c = \dfrac{12}{40}\cdot\dfrac{1}{1}(540) = 162 \text{ rev/min} \\ \\ \text{cw about } x,\ \text{as viewed from the positive } x \text{ axis} \end{array}\right\} \quad Ans.$$

(b)

$$d_P = 12/\left(8\cos 23^\circ\right) = 1.630 \text{ in}$$

$$d_G = 40/\left(8\cos 23^\circ\right) = 5.432 \text{ in}$$

$$\frac{d_P + d_G}{2} = 3.531 \text{ in} \qquad Ans.$$

(c)

$$d = \frac{N}{P} = \frac{32}{4} = 8 \text{ in} \qquad Ans.$$

13-20 Applying Eq. (13-30), $e = (N_2 / N_3)\,(N_4 / N_5) = 45$. For an exact ratio, we will choose to factor the train value into integers, such that

$N_2 / N_3 = 9$ (1)
$N_4 / N_5 = 5$ (2)

Assuming a constant diametral pitch in both stages, the geometry condition to satisfy the in-line requirement of the compound reverted configuration is

$N_2 + N_3 = N_4 + N_5$ (3)

With three equations and four unknowns, one free choice is available. It is necessary that all of the unknowns be integers. We will use a normalized approach to find the minimum free choice to guarantee integers; that is, set the smallest gear of the largest stage to unity, thus $N_3 = 1$. From (1), $N_2 = 9$. From (3),

$$N_2 + N_3 = 9 + 1 = 10 = N_4 + N_5$$

Substituting $N_4 = 5\,N_5$ from (2) gives

$$10 = 5\,N_5 + N_5 = 6\,N_5$$
$$N_5 = 10 / 6 = 5 / 3$$

To eliminate this fraction, we need to multiply the original free choice by a multiple of 3. In addition, the smallest gear needs to have sufficient teeth to avoid interference. From Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = 9$, the minimum number of teeth on the pinion to avoid interference is 17. Therefore, the smallest multiple of 3 greater than 17 is 18. Setting $N_3 = 18$ and repeating the solution of equations (1), (2), and (3) yields

N_2 = 162 teeth
N_3 = 18 teeth
N_4 = 150 teeth
N_5 = 30 teeth *Ans.*

13-21 The solution to Prob. 13-20 applies up to the point of determining the minimum number of teeth to avoid interference. From Eq. (13-11), with $k = 1$, $\phi = 25°$, and $m = 9$, the minimum number of teeth on the pinion to avoid interference is 11. Therefore, the smallest multiple of 3 greater than 11 is 12. Setting $N_3 = 12$ and repeating the solution of equations (1), (2), and (3) of Prob. 13-20 yields

N_2 = 108 teeth
N_3 = 12 teeth
N_4 = 100 teeth
N_5 = 20 teeth *Ans.*

13-22 Applying Eq. (13-30), $e = (N_2 / N_3)\,(N_4 / N_5) = 30$. For an exact ratio, we will choose to factor the train value into integers, such that

$$N_2 / N_3 = 6 \qquad (1)$$
$$N_4 / N_5 = 5 \qquad (2)$$

Assuming a constant diametral pitch in both stages, the geometry condition to satisfy the in-line requirement of the compound reverted configuration is

$$N_2 + N_3 = N_4 + N_5 \qquad (3)$$

With three equations and four unknowns, one free choice is available. It is necessary that all of the unknowns be integers. We will use a normalized approach to find the minimum free choice to guarantee integers; that is, set the smallest gear of the largest stage to unity, thus $N_3 = 1$. From (1), $N_2 = 6$. From (3),

$$N_2 + N_3 = 6 + 1 = 7 = N_4 + N_5$$

Substituting $N_4 = 5\,N_5$ from (2) gives

$$7 = 5\,N_5 + N_5 = 6\,N_5$$
$$N_5 = 7 / 6$$

To eliminate this fraction, we need to multiply the original free choice by a multiple of 6. In addition, the smallest gear needs to have sufficient teeth to avoid interference. From Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = 6$, the minimum number of teeth on the pinion to avoid interference is 16. Therefore, the smallest multiple of 6 greater than 16 is 18. Setting $N_3 = 18$ and repeating the solution of equations (1), (2), and (3) yields

$N_2 = 108$ teeth
$N_3 = 18$ teeth
$N_4 = 105$ teeth
$N_5 = 21$ teeth *Ans.*

13-23 Applying Eq. (13-30), $e = (N_2 / N_3)(N_4 / N_5) = 45$. For an approximate ratio, we will choose to factor the train value into two equal stages, such that

$$N_2 / N_3 = N_4 / N_5 = \sqrt{45}$$

If we choose identical pinions such that interference is avoided, both stages will be identical and the in-line geometry condition will automatically be satisfied. From Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = \sqrt{45}$, the minimum number of teeth on the pinions to avoid interference is 17. Setting $N_3 = N_5 = 17$, we get

$$N_2 = N_4 = 17\sqrt{45} = 114.04 \text{ teeth}$$

Rounding to the nearest integer, we obtain

$N_2 = N_4 = 114$ teeth
$N_3 = N_5 = 17$ teeth *Ans.*

Checking, the overall train value is $e = (114 / 17)(114 / 17) = 44.97$.

13-24 $H = 25$ hp, $\omega_i = 2500$ rev/min
Let $\omega_o = 300$ rev/min for minimal gear ratio to minimize gear size.

$$\frac{\omega_o}{\omega_i} = \frac{300}{2500} = \frac{1}{8.333} = \frac{N_2}{N_3}\frac{N_4}{N_5}$$

Let $$\frac{N_2}{N_3} = \frac{N_4}{N_5} = \sqrt{\frac{1}{8.333}} = \frac{1}{2.887}$$

From Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = 2.887$, the minimum number of teeth on the pinions to avoid interference is 15.

Let $N_2 = N_4 = 15$ teeth
$N_3 = N_5 = 2.887(15) = 43.31$ teeth

Try $N_3 = N_5 = 43$ teeth.

$$\omega_o = \left(\frac{15}{43}\right)\left(\frac{15}{43}\right)(2500) = 304.2$$

Too big. Try $N_3 = N_5 = 44$.

$$\omega_o = \left(\frac{15}{44}\right)\left(\frac{15}{44}\right)(2500) = 290.55 \text{ rev/min}$$

$N_2 = N_4 = 15$ teeth, $N_3 = N_5 = 44$ teeth *Ans.*

13-25 **(a)** The planet gears act as keys and the wheel speeds are the same as that of the ring gear. Thus,

$$n_A = n_3 = 900(16/48) = 300 \text{ rev/min} \quad \textit{Ans.}$$

(b)

$$n_F = n_5 = 0, \quad n_L = n_6, \quad e = -1$$

$$-1 = \frac{n_6 - 300}{0 - 300}$$

$$300 = n_6 - 300$$

$$n_6 = 600 \text{ rev/min} \quad \textit{Ans.}$$

(c) The wheel spins freely on icy surfaces, leaving no traction for the other wheel. The car is stalled. *Ans.*

13-26 **(a)** The motive power is divided equally among four wheels instead of two.

(b) Locking the center differential causes 50 percent of the power to be applied to the rear wheels and 50 percent to the front wheels. If one of the rear wheels rests on a slippery surface such as ice, the other rear wheel has no traction. But the front wheels still provide traction, and so you have two-wheel drive. However, if the rear differential is locked, you have 3-wheel drive because the rear-wheel power is now distributed 50-50.

13-27 Let gear 2 be first, then $n_F = n_2 = 0$. Let gear 6 be last, then $n_L = n_6 = -12$ rev/min.

$$e = \frac{20}{30}\left(\frac{16}{34}\right) = \frac{16}{51} = \frac{n_L - n_A}{n_F - n_A}$$

$$(0 - n_A)\frac{16}{51} = -12 - n_A$$

$$\left.\begin{array}{l} n_A = \dfrac{-12}{35/51} = -17.49 \text{ rev/min} \\ \\ \text{(negative indicates cw as viewed from the bottom of the figure)} \end{array}\right\} \textit{Ans.}$$

13-28 Let gear 2 be first, then $n_F = n_2 = 0$ rev/min. Let gear 6 be last, then $n_L = n_6 = 85$ rev/min.

$$e = \frac{20}{30}\left(\frac{16}{34}\right) = \frac{16}{51} = \frac{n_L - n_A}{n_F - n_A}$$

$$(0 - n_A)\frac{16}{51} = (85 - n_A)$$

$$-n_A\left(\frac{16}{51}\right) + n_A = 85$$

$$n_A\left(1 - \frac{16}{51}\right) = 85$$

$$n_A = \frac{85}{1 - \dfrac{16}{51}} = 123.9 \text{ rev/min}$$

The positive sign indicates the same direction as n_6. $\therefore n_A = 123.9$ rev/min ccw *Ans.*

13-29 $\phi = 20^\circ$, $P = 6$ teeth/in. Since N α d, then,

$$N_2 + N_3 = N_4 + N_5 \qquad (1)$$

Hour hand moves 1/12 of minute hand. Thus, $\omega_5 / \omega_2 = \frac{1}{12}$. Now,

$\omega_5 / \omega_4 = N_4 / N_5$, $\omega_3 / \omega_2 = N_2 / N_3$, and $\omega_4 = \omega_3$. Thus,

$$\frac{\omega_2}{\omega_2} = \frac{N_3}{N_2}\frac{N_5}{N_4} = 12 = 4 \times 3$$

So, try $N_3 = 4\ N_2$, and $N_5 = 3\ N_4$. Substituting $N_5 = 3\ N_4$ into Eq. (1) gives

$N_2 + N_3 = 4\ N_4 \quad \Rightarrow \quad N_4 = (N_2 + N_3)/4$. Let $N_2 = 1$. Then, $N_3 = 4N_2 = 4(1) = 4$,

$N_4 = (N_2 + N_3)/4 = (1 + 4)/4 = 5/4$, $N_5 = 3\ N_4 = 3(5/4) = 15/4$. Teeth must be a multiple of 4. To avoid interference for the smaller pinion use Eq. (13-11) with $m = N_3 / N_2 = 4$:

$$N_2 = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{\left[1+2(4)\right]\sin^2 20^\circ}\left(4 + \sqrt{4^2 + \left[1+2(4)\right]\sin^2 20^\circ}\right) = 15.4 \text{ teeth}$$

Use $N_2 = 16$ teeth which is a multiple of 4. Then, $N_3 = 4(16) = 64$ teeth, $N_4 = (5/4)16 = 20$ teeth, and $N_5 = (15/4)\ 16 = 60$ teeth. Thus,

$N_2 = 16$ teeth, $N_3 = 64$ teeth, $N_4 = 20$ teeth, and $N_5 = 60$ teeth *Ans.*

13-30 The geometry condition is $d_5/2 = d_2/2 + d_3 + d_4$. Since all the gears are meshed, they will all have the same diametral pitch. Applying $d = N/P$,

$$N_5/(2P) = N_2/(2P) + N_3/P + N_4/P$$
$$N_5 = N_2 + 2N_3 + 2N_4 = 12 + 2(16) + 2(12) = 68 \text{ teeth} \quad \textit{Ans.}$$

Let gear 2 be first, $n_F = n_2 = 320$ rev/min. Let gear 5 be last, $n_L = n_5 = 0$ rev/min.

$$e = \frac{12}{16}\left(\frac{16}{12}\right)\left(\frac{12}{68}\right) = \frac{3}{17} = \frac{n_L - n_A}{n_F - n_A}$$

$$320 - n_A = \frac{17}{3}(0 - n_A)$$
$$n_A = -\frac{3}{14}(320) = -68.57 \text{ rev/min}$$

The negative sign indicates opposite of n_2. $\therefore n_A = 68.57$ rev/min cw *Ans.*

13-31 Let $n_F = n_2$, then $n_L = n_7 = 0$.

$$e = -\frac{20}{16}\left(\frac{16}{30}\right)\left(\frac{36}{46}\right) = -0.5217 = \frac{n_L - n_5}{n_F - n_5}$$

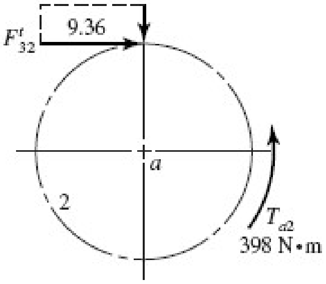
$$\frac{0 - n_5}{10 - n_5} = -0.5217$$
$$-0.5217(10 - n_5) = -n_5$$
$$-5.217 + 0.5217 n_5 + n_5 = 0$$
$$n_5(1 + 0.5217) = 5.217$$
$$n_5 = \frac{5.217}{1.5217}$$
$$n_5 = n_b = 3.428 \text{ turns in same direction} \quad \textit{Ans.}$$

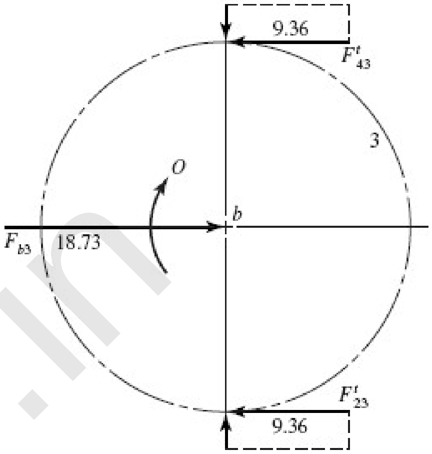
13-32 **(a)**

$$\omega = 2\pi n/60$$
$$H = T\omega = 2\pi T n/60 \quad (T \text{ in N}\cdot\text{m}, H \text{ in W})$$

So
$$T = \frac{60H\left(10^3\right)}{2\pi n}$$
$$= 9550\ H/n \qquad (H \text{ in kW, } n \text{ in rev/min})$$
$$T_a = \frac{9550(75)}{1800} = 398\ \text{N}\cdot\text{m}$$
$$r_2 = \frac{mN_2}{2} = \frac{5(17)}{2} = 42.5\ \text{mm}$$

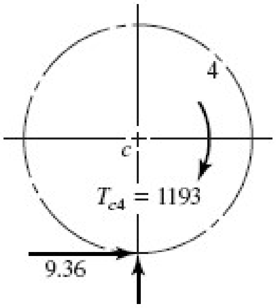
So
$$F_{32}^t = \frac{T_a}{r_2} = \frac{398}{42.5} = 9.36\ \text{kN}$$





$$F_{3b} = -F_{b3} = 2(9.36) = 18.73\ \text{kN in the positive } x\text{-direction.} \qquad \textit{Ans.}$$

(b)
$$r_4 = \frac{mN_4}{2} = \frac{5(51)}{2} = 127.5\ \text{mm}$$
$$T_{c4} = 9.36(127.5) = 1193\ \text{N}\cdot\text{m ccw}$$
$$\therefore T_{4c} = 1193\ \text{N}\cdot\text{m cw} \qquad \textit{Ans.}$$



Note: The solution is independent of the pressure angle.

13-33
$$d = \frac{N}{P} = \frac{N}{6}$$
$$d_2 = 4\ \text{in},\ \ d_4 = 4\ \text{in},\ \ d_5 = 6\ \text{in},\ \ d_6 = 24\ \text{in}$$
$$e = \left(-\frac{24}{24}\right)\left(-\frac{24}{36}\right)\left(+\frac{36}{144}\right) = 1/6$$
$$n_F = n_2 = 1000\ \text{rev/min}$$
$$n_L = n_6 = 0$$
$$e = \frac{n_L - n_A}{n_F - n_A} = \frac{0 - n_A}{1000 - n_A} = \frac{1}{6}$$

$$n_A = -200 \text{ rev/min}$$

Noting that power equals torque times angular velocity, the input torque is

$$T_2 = \frac{H}{n_2} = \frac{25 \text{ hp}}{1000 \text{ rev/min}}\left(\frac{550 \text{ lbf}\cdot\text{ft/s}}{\text{hp}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{12 \text{ in}}{\text{ft}}\right) = 1576 \text{ lbf}\cdot\text{in}$$

For 100 percent gear efficiency, the output power equals the input power, so

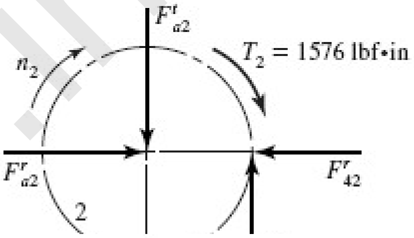
$$T_{arm} = \frac{H}{n_A} = \frac{25 \text{ hp}}{200 \text{ rev/min}}\left(\frac{550 \text{ lbf}\cdot\text{ft/s}}{\text{hp}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{12 \text{ in}}{\text{ft}}\right) = 7878 \text{ lbf}\cdot\text{in}$$

Next, we'll confirm the output torque as we work through the force analysis and complete the free body diagrams.

Gear 2

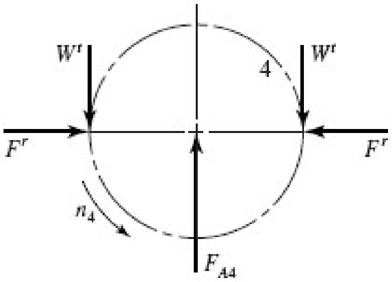
$$W^t = \frac{1576}{2} = 788 \text{ lbf}$$

$$F_{32}^r = 788 \tan 20^\circ = 287 \text{ lbf}$$

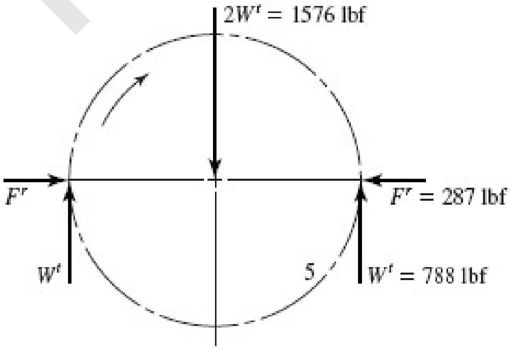


Gear 4

$$F_{A4} = 2W^t = 2(788) = 1576 \text{ lbf}$$

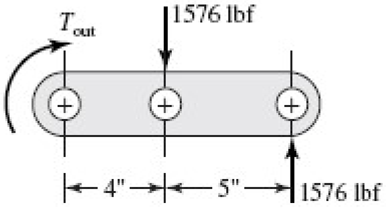


Gear 5



Arm

$$T_{\text{out}} = 1576(9) - 1576(4) = 7880 \text{ lbf} \cdot \text{in} \qquad \textit{Ans.}$$



13-34 **(a)** $\phi = 20°$, $P = 6$ teeth/in. Since $N \alpha d$, then,

$$N_2 + N_3 = N_4 + N_5 \qquad (1)$$

$e = 40 = 8 \times 5$. Then, let $N_2 / N_3 = 8$ and $N_4 / N_5 = 5$. Thus, $N_2 = 8 N_3$ and $N_4 = 5 N_5$. Substituting $N_4 = 5 N_5$ into Eq. (1) gives $N_2 + N_3 = 6N_5 \;\Rightarrow\; N_5 = (N_2 + N_3)/6$. To avoid interference use Eq. (13-11) with $m = N_2 / N_3 = 8$:

$$N_2 = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{\left[1+2(8)\right]\sin^2 20°}\left(8 + \sqrt{8^2 + \left[1+2(8)\right]\sin^2 20°}\right) = 16.2 \text{ teeth}$$

Try $N_3 = 17$ teeth $\Rightarrow$ $N_2 = 8 N_3 = 8(17) = 136$ teeth, $N_5 = (136 + 17)/6 = 25.5$ teeth which is unacceptable. Next, try $N_3 = 18$ teeth $\Rightarrow$ $N_2 = 8 N_3 = 8(18) = 144$ teeth, $N_5 = (144 + 18)/6 = 27$ teeth, and $N_4 = 5 N_5 = 5\,(27) = 135$ teeth. Thus,

$N_2 = 144$ teeth, $N_3 = 18$ teeth, $N_4 = 135$ teeth, and $N_5 = 27$ teeth *Ans.*

(b) Gear diameter is $d = N / P$ (with $P = 6$ teeth/in), 0.5 in wall clearance on two sides, addendum of each gear is $1/P = 1/6$ in, and each wall thickness is 0.75 in. Thus,

$$Y = \frac{N_2}{P} + \frac{N_3/2}{P} + \frac{N_4/2}{P} + 2(0.5) + 2\left(\frac{1}{P}\right) + 2(0.75)$$

$$= \frac{144}{6} + \frac{18/2}{6} + \frac{135/2}{6} + 2(0.5) + 2\left(\frac{1}{6}\right) + 2(0.75) = 39.6 \text{ in} \qquad \textit{Ans.}$$

13-35 **(a)** $\phi = 25°$, $P = 6$ teeth/in. Since $N \alpha d$, then,

$$N_2 + N_3 = N_4 + N_5 \qquad (1)$$

$e = 40 = 8 \times 5$. Let $N_2 / N_3 = 8$ and $N_4 / N_5 = 5$. Thus, $N_2 = 8 N_3$ and $N_4 = 5 N_5$. Substituting $N_4 = 5 N_5$ into Eq. (1) gives $N_2 + N_3 = 6N_5 \;\Rightarrow\; N_5 = (N_2 + N_3)/6$. To avoid interference use Eq. (13-11) with $m = N_2 / N_3 = 8$:

$$N_2 = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{\left[1+2(8)\right]\sin^2 25°}\left(8 + \sqrt{8^2 + \left[1+2(8)\right]\sin^2 25°}\right) = 10.7 \text{ teeth}$$

Try $N_3 = 11$ teeth $\Rightarrow$ $N_2 = 8 N_3 = 8(11) = 88$ teeth, $N_5 = (88 + 11)/6 = 16.5$ teeth which is unacceptable. Next, try $N_3 = 12$ teeth $\Rightarrow$ $N_2 = 8 N_3 = 8(12) = 96$ teeth, $N_5 = (96 + 12)/6 = 18$ teeth, and $N_4 = 5 N_5 = 5\,(18) = 90$ teeth. Thus,

$N_2 = 96$ teeth, $N_3 = 12$ teeth, $N_4 = 90$ teeth, and $N_5 = 18$ teeth *Ans.*

(b) Gear diameter is $d = N / P$ (with $P = 6$ teeth/in), 0.5 in wall clearance on two sides, addendum of each gear is $1/P = 1/6$ in, and each wall thickness is 0.75 in. Thus,

$$Y = \frac{N_2}{P} + \frac{N_3/2}{P} + \frac{N_4/2}{P} + 2(0.5) + 2\left(\frac{1}{P}\right) + 2(0.75)$$

$$= \frac{96}{6} + \frac{12/2}{6} + \frac{90/2}{6} + 2(0.5) + 2\left(\frac{1}{6}\right) + 2(0.75) = 27.3 \text{ in } \quad Ans.$$

13-36 **(a)** $\phi_n = 20^\circ$, $\psi = 45^\circ$, $P = 6$ teeth/in. Since $N \propto d$, then,

$$N_2 + N_3 = N_4 + N_5 \qquad (1)$$

$e = 40 = 8 \times 5$. Let $N_2 / N_3 = 8$ and $N_4 / N_5 = 5$. Thus $N_2 = 8\,N_3$ and $N_4 = 5\,N_5$.
Substituting $N_4 = 5\,N_5$ into Eq. (1) gives $N_2 + N_3 = 6N_5 \;\Rightarrow\; N_5 = (N_2 + N_3)/6$.
From Eq. (13-19),

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\tan\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\tan 45^\circ}\right) = 20^\circ$$

Eq. (13-22):

$$N_p = \frac{2k\cos\psi}{(1+2m)\sin^2\phi_t}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi_t}\right)$$

$$= \frac{2(1)\cos 45^\circ}{[1+2(8)]\sin^2 20^\circ}\left(8 + \sqrt{8^2 + [1+2(8)]\sin^2 20^\circ}\right) = 11.5 \text{ teeth}$$

Try $N_3 = 12$ teeth $\Rightarrow$ $N_2 = 8\,N_3 = 8(12) = 96$ teeth, $N_5 = (96 + 12)/6 = 18$ teeth, and $N_4 = 5\,N_5 = 5\,(18) = 90$ teeth. Thus,

$N_2 = 96$ teeth, $N_3 = 12$ teeth, $N_4 = 90$ teeth, and $N_5 = 18$ teeth *Ans.*

(b) Gear diameter is $d = N / P$ (with $P = 6$ teeth/in), 0.5 in wall clearance on two sides, addendum of each gear is $1/P = 1/6$ in, and each wall thickness is 0.75 in. Thus,

$$Y = \frac{N_2}{P} + \frac{N_3/2}{P} + \frac{N_4/2}{P} + 2(0.5) + 2\left(\frac{1}{P}\right) + 2(0.75)$$

$$= \frac{96}{6} + \frac{12/2}{6} + \frac{90/2}{6} + 2(0.5) + 2\left(\frac{1}{6}\right) + 2(0.75) = 27.3 \text{ in } \quad Ans.$$

13-37 $e \approx 40$, $P = 6$ teeth/in, $\phi = 20^\circ$.

(a) Minimum size is $N_2 / N_3 = N_4 / N_5 = \sqrt{40} = 6.325$. To avoid interference use Eq. (13-11) with $m = 6.325$:

$$N_2 = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{[1+2(6.325)]\sin^2 20^\circ}\left(6.325 + \sqrt{6.235^2 + [1+2(6.325)]\sin^2 20^\circ}\right) = 16.0 \text{ teeth}$$

Let $N_3 = 16$ teeth. $N_2 = 16\sqrt{40} = 101.2$. Use $N_2 = 101$ teeth, $e = (101/16)^2 = 39.85$ which is ok since it is between 38 and 42. Thus,

$$N_2 = N_4 = 101 \text{ teeth, and } N_3 = N_5 = 16 \text{ teeth} \qquad Ans.$$

(b) Gear diameter is $d = N / P$ (with $P = 6$ teeth/in), 0.5 in wall clearance on two sides, addendum of each gear is $1/P = 1/6$ in, and each wall thickness is 0.75 in. Thus,

$$Y = \frac{N_2}{P} + \frac{N_3/2}{P} + \frac{N_4/2}{P} + 2(0.5) + 2\left(\frac{1}{P}\right) + 2(0.75)$$

$$= \frac{101}{6} + \frac{116/2}{6} + \frac{101/2}{6} + 2(0.5) + 2\left(\frac{1}{6}\right) + 2(0.75) = 29.4 \text{ in} \qquad Ans.$$

Comparing this to Prob. 13-34 where $Y = 39.6$ in, we see a large reduction in gearbox size.

13-38 **(a)** $P = 6$ teeth/in, $\phi = 20^\circ$. Since $N \alpha d$, then,

$$\frac{N_2}{2} + N_3 + \frac{N_4}{2} = \frac{N_5}{2} + N_6 + \frac{N_7}{2} \quad \Rightarrow \quad \frac{30}{2} + 20 + \frac{60}{2} = \frac{20}{2} + N_6 + \frac{80}{2}$$

Solving for N_6 yields $\qquad N_6 = 15$ teeth $\qquad Ans.$

(b) $$Y = \frac{N_4}{P} + \frac{N_3}{P} + \frac{N_2/2}{P} + \frac{N_7/2}{P} + \frac{2}{P} = \frac{60}{6} + \frac{20}{6} + \frac{30/2}{6} + \frac{80/2}{6} + \frac{2}{6} = 22.83 \text{ in} \qquad Ans.$$

(c) Gear 2, $n_2 = 300$ rev/min,

Eq, (13-34): $$V_2 = \frac{\pi d_2 n_2}{12} = \frac{\pi (N_2/P) n_2}{12} = \frac{\pi (30/6) 300}{12} = 392.7 \text{ ft/min} \qquad Ans.$$

(d) Eq. (13-35): $$W_t = 33\,000 \frac{H}{V} = 33\,000 \frac{4}{392.7} = 336.1 \text{ lbf} \qquad Ans.$$

(e) $\quad F_r = W_t \tan \phi = 336.1 \tan 20^\circ = 122.3$ lbf $\qquad Ans,$

(f) $$T_i = \frac{63\,025 H}{12 n} = \frac{63\,025(4)}{12(300)} = 70.0 \text{ lbf} \cdot \text{in} \qquad Ans.$$

(g) $$e = \frac{N_2}{\cancel{N_3}} \frac{\cancel{N_3}}{N_4} \frac{N_5}{\cancel{N_6}} \frac{\cancel{N_6}}{N_7} = \frac{N_2 N_5}{N_4 N_7} = \frac{30(20)}{60(80)} = 0.125$$

$$T_o = T_i \left(\frac{1}{e}\right) = 70.0 \left(\frac{1}{0.125}\right) = 560 \text{ lbf} \cdot \text{ft} \qquad Ans.$$

(h) $\quad \omega_o = e\omega_i = 0.125\ (300) = 37.5$ rev/min $\qquad Ans.$

(i) Assuming no losses, $\qquad P_o = P_i = 4$ hp $\qquad Ans.$

13-39 Given: $m = 12$ mm, $n_P = 1800$ rev/min cw,
$N_2 = 18T$, $N_3 = 32T$, $N_4 = 18T$, $N_5 = 48T$

Pitch Diameters: $\quad d_2 = 18(12) = 216$ mm, $d_3 = 32(12) = 384$ mm,
$d_4 = 18(12) = 216$ mm, $d_5 = 48(12) = 576$ mm

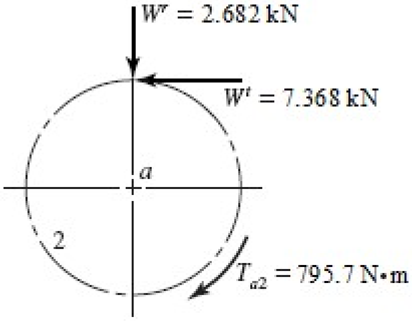
Gear 2

From Eq. (13-36),

$$W_t = \frac{60\,000H}{\pi dn} = \frac{60\,000(150)}{\pi(216)(1800)} = 7.368 \text{ kN}$$

$$T_{a2} = W_t\left(\frac{d_2}{2}\right) = 7.368\left(\frac{216}{2}\right) = 795.7 \text{ N}\cdot\text{m}$$

$$W^r = 7.368 \tan 20^\circ = 2.682 \text{ kN}$$



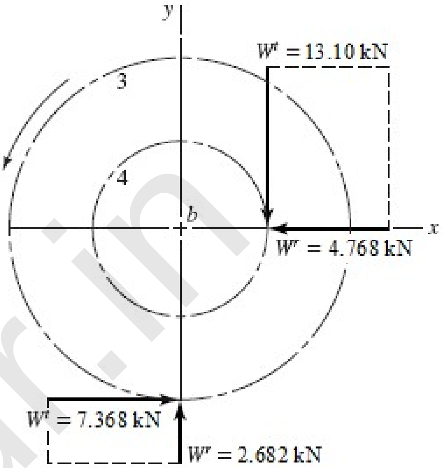
Gears 3 *and* 4

$$W^t\left(\frac{216}{2}\right) = 7.368\frac{(384)}{2}$$

$$W^t = 13.10 \text{ kN}$$

$$W^r = 13.10 \tan 20^\circ = 4.768 \text{ kN}$$

Ans.



13-40 Given: $P = 5$ teeth/in, $N_2 = 18T$, $N_3 = 45T$,
$\phi_n = 20^\circ$, $H = 32$ hp, $n_2 = 1800$ rev/min

Gear 2

$$T_{\text{in}} = \frac{63\,025(32)}{1800} = 1120 \text{ lbf}\cdot\text{in}$$

$$d_P = \frac{18}{5} = 3.600 \text{ in}$$

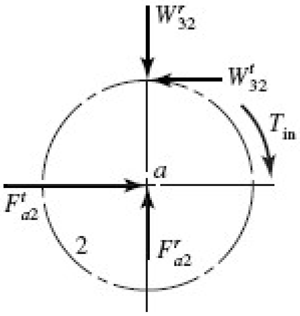
$$d_G = \frac{45}{5} = 9.000 \text{ in}$$

$$W_{32}^t = \frac{1120}{3.6/2} = 622 \text{ lbf}$$

$$W_{32}^r = 622 \tan 20^\circ = 226 \text{ lbf}$$

$$F_{a2}^t = W_{32}^t = 622 \text{ lbf}, \quad F_{a2}^r = W_{32}^r = 226 \text{ lbf}$$

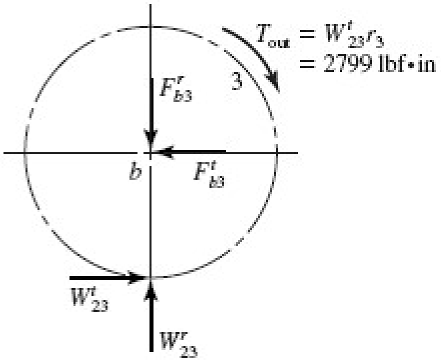
$$F_{a2} = \left(622^2 + 226^2\right)^{1/2} = 662 \text{ lbf}$$



Each bearing on shaft *a* has the same radial load of $R_A = R_B = 662/2 = 331$ lbf.

Gear 3

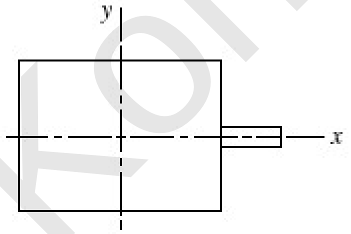
$$W_{23}^t = W_{32}^t = 622 \text{ lbf}$$
$$W_{23}^r = W_{32}^r = 226 \text{ lbf}$$
$$F_{b3} = F_{b2} = 662 \text{ lbf}$$
$$R_C = R_D = 662/2 = 331 \text{ lbf}$$

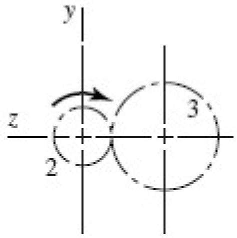


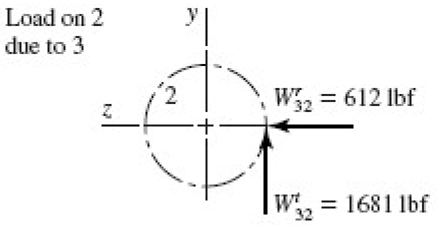
Each bearing on shaft *b* has the same radial load which is equal to the radial load of bearings *A* and *B*. Thus, all four bearings have the same radial load of 331 lbf. *Ans.*

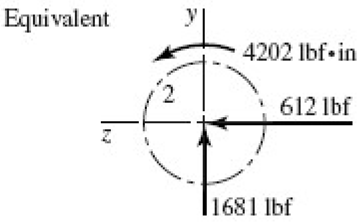
13-41 Given: $P = 4$ teeth/in, $\phi_n = 20^\circ$, $N_P = 20T$, $n_2 = 900$ rev/min

$$d_2 = \frac{N_P}{P} = \frac{20}{4} = 5.000 \text{ in}$$
$$T_{\text{in}} = \frac{63\,025(30)(2)}{900} = 4202 \text{ lbf}\cdot\text{in}$$
$$W_{32}^t = T_{\text{in}}/(d_2/2) = 4202/(5/2) = 1681 \text{ lbf}$$
$$W_{32}^r = 1681 \ \tan 20^\circ = 612 \text{ lbf}$$





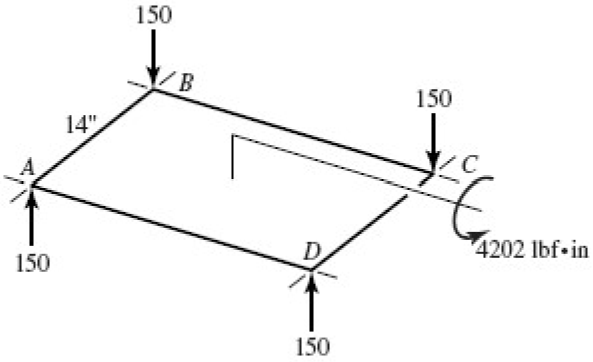




The motor mount resists the equivalent forces and torque.
The radial force due to torque is

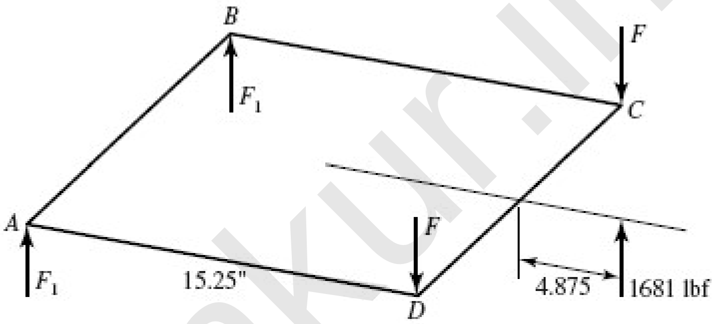
$$F^r = \frac{4202}{14(2)} = 150 \text{ lbf}$$

Forces reverse with rotational sense as torque reverses.



The compressive loads at *A* and *D* are absorbed by the base plate, not the bolts. For W^t_{32}, the tensions in *C* and *D* are

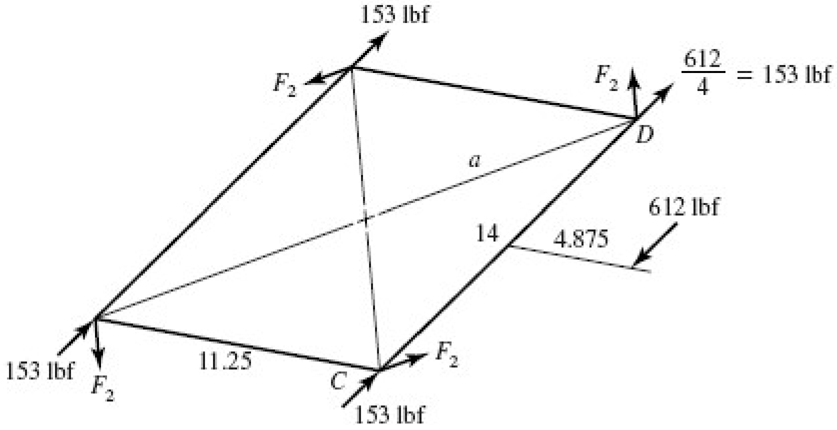
$$\Sigma M_{AB} = 0 \qquad 1681(4.875+15.25) - 2F(15.25) = 0 \qquad F = 1109 \text{ lbf}$$



If W^t_{32} reverses, 15.25 in changes to 13.25 in, 4.815 in changes to 2.875 in, and the forces change direction. For *A* and *B*,

$$1681(2.875) - 2F_1(13.25) = 0 \quad \Rightarrow \quad F_1 = 182.4 \text{ lbf}$$

For W^r_{32},



$$M = 612\left(4.875 + 11.25/2\right) = 6426 \text{ lbf}\cdot\text{in}$$

$$a = \sqrt{\left(14/2\right)^2 + \left(11.25/2\right)^2} = 8.98 \text{ in}$$

$$F_2 = \frac{6426}{4\left(8.98\right)} = 179 \text{ lbf}$$

At *C* and *D*, the shear forces are:

$$F_{S1} = \sqrt{\left[153 + 179\left(5.625/8.98\right)\right]^2 + \left[179\left(7/8.98\right)\right]^2}$$

At *A* and *B*, the shear forces are:

$$F_{S2} = \sqrt{\left[153 - 179\left(5.625/8.98\right)\right]^2 + \left[179\left(7/8.98\right)\right]^2}$$

$$= 145 \text{ lbf}$$

The shear forces are independent of the rotational sense.
The bolt tensions and the shear forces for cw rotation are,

	Tension (lbf)	Shear (lbf)
A	0	145
B	0	145
C	1109	300
D	1109	300

For ccw rotation,

	Tension (lbf)	Shear (lbf)
A	182	145
B	182	145
C	0	300
D	0	300

13-42 **(a)** $N_2 = N_4 = 15$ teeth, $N_3 = N_5 = 44$ teeth

$$P = \frac{N}{d} \quad \Rightarrow \quad d = \frac{N}{P}$$

$$d_2 = d_4 = \frac{15}{6} = 2.5 \text{ in} \qquad \textit{Ans.}$$

$$d_3 = d_5 = \frac{44}{6} = 7.33 \text{ in} \qquad \textit{Ans.}$$

(b) $$V_i = V_2 = V_3 = \frac{\pi d_2 n_2}{12} = \frac{\pi(2.5)(2500)}{12} = 1636 \text{ ft/min} \quad \textit{Ans.}$$

$$V_o = V_4 = V_5 = \frac{\pi d_4 n_4}{12} = \frac{\pi(2.5)\left[(2500)(15/44)\right]}{12} = 558 \text{ ft/min} \quad \textit{Ans.}$$

(c) Input gears:

$$W_{ti} = 33\,000\frac{H}{V_i} = \frac{33\,000(25)}{1636} = 504.3 \text{ lbf} = 504 \text{ lbf} \quad \textit{Ans.}$$

$$W_{ri} = W_{ti}\tan\phi = 504.3\tan 20^\circ = 184 \text{ lbf} \quad \textit{Ans.}$$

$$W_i = \frac{W_{ti}}{\cos\phi} = \frac{504.3}{\cos 20^\circ} = 537 \text{ lbf} \quad \textit{Ans.}$$

Output gears:

$$W_{to} = 33\,000\frac{H}{V_o} = \frac{33\,000(25)}{558} = 1478 \text{ lbf} \quad \textit{Ans.}$$

$$W_{ro} = W_{to}\tan\phi = 1478\tan 20^\circ = 538 \text{ lbf} \quad \textit{Ans.}$$

$$W_o = \frac{W_{to}}{\cos 20^\circ} = \frac{1478}{\cos 20^\circ} = 1573 \text{ lbf} \quad \textit{Ans.}$$

(d) $$T_i = W_{ti}\left(\frac{d_2}{2}\right) = 504.3\left(\frac{2.5}{2}\right) = 630 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

(e) $$T_o = T_i\left(\frac{44}{15}\right)^2 = 630\left(\frac{44}{15}\right)^2 = 5420 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

13-43 $H = 35$ hp, $n_i = 1200$ rev/min, $\phi = 20^\circ$

$N_2 = N_4 = 16$ teeth, $N_3 = N_5 = 48$ teeth, $P = 10$ teeth/in

(a) $$n_{\text{intermediate}} = n_3 = n_4 = \frac{N_2}{N_3}n_i = \frac{16}{48}(1200) = 400 \text{ rev/min} \quad \textit{Ans.}$$

$$n_o = \frac{N_2}{N_3}\frac{N_4}{N_5}n_i = \frac{16}{48}\left(\frac{16}{48}\right)(1200) = 133.3 \text{ rev/min} \quad \textit{Ans.}$$

(b) $$P = \frac{N}{d} \quad\Rightarrow\quad d = \frac{N}{P}$$

$$d_2 = d_4 = \frac{16}{10} = 1.6 \text{ in} \quad \textit{Ans.}$$

$$d_3 = d_5 = \frac{48}{10} = 4.8 \text{ in} \quad \textit{Ans.}$$

$$V_i = V_2 = V_3 = \frac{\pi d_2 n_2}{12} = \frac{\pi(1.6)(1200)}{12} = 502.7 \text{ ft/min} \quad Ans.$$

$$V_o = V_4 = V_5 = \frac{\pi d_4 n_4}{12} = \frac{\pi(1.6)(400)}{12} = 167.6 \text{ ft/min} \quad Ans.$$

(c) $$W_{ti} = 33\,000\frac{H}{V_i} = \frac{33\,000(35)}{502.7} = 2298 \text{ lbf lbf} \quad Ans.$$

$$W_{ri} = W_{ti}\tan\phi = 2298\tan 20^\circ = 836.4 \text{ lbf} \quad Ans.$$

$$W_i = \frac{W_{ti}}{\cos\phi} = \frac{2298}{\cos 20^\circ} = 2445 \text{ lbf} \quad Ans.$$

$$W_{to} = 33\,000\frac{H}{V_o} = \frac{33\,000(35)}{167.6} = 6891 \text{ lbf} \quad Ans.$$

$$W_{ro} = W_{to}\tan\phi = 6891\tan 20^\circ = 2508 \text{ lbf} \quad Ans.$$

$$W_o = \frac{W_{to}}{\cos 20^\circ} = \frac{6891}{\cos 20^\circ} = 7333 \text{ lbf} \quad Ans.$$

(d) $$T_i = W_{ti}\left(\frac{d_2}{2}\right) = 2298\left(\frac{1.6}{2}\right) = 1838 \text{ lbf}\cdot\text{in} \quad Ans.$$

(e) $$T_o = T_i\left(\frac{48}{16}\right)^2 = 1838\left(\frac{48}{16}\right)^2 = 16\,540 \text{ lbf}\cdot\text{in} \quad Ans.$$

13-44 **(a)** For $\frac{\omega_o}{\omega_i} = \frac{2}{1}$, from Eq. (13-11), with $m = 2$, $k = 1$, $\phi = 20^\circ$

$$N_P = \frac{2(1)}{[1+2(2)]\sin^2 20^\circ}\left\{2+\sqrt{2^2+[1+2(2)]\sin^2 20^\circ}\right\} = 14.16$$

So $$N_{P\min} = 15 \quad Ans.$$

(b) $$P = \frac{N}{d} = \frac{15}{8} = 1.875 \text{ teeth/in} \quad Ans.$$

(c) To transmit the same power with no change in pitch diameters, the speed and transmitted force must remain the same.

For A, with $\phi = 20°$,

$$W_{tA} = F_A \cos 20° = 300 \cos 20° = 281.9 \text{ lbf}$$

For A, with $\phi = 25°$, same transmitted load,

$$F_A = W_{tA}/\cos 25° = 281.9/\cos 25° = 311.0 \text{ lbf} \quad \textit{Ans.}$$

Summing the torque about the shaft axis,

$$W_{tA}\left(\frac{d_A}{2}\right) = W_{tB}\left(\frac{d_B}{2}\right)$$

$$W_{tB} = W_{tA}\frac{(d_A/2)}{(d_B/2)} = W_{tA}\left(\frac{d_A}{d_B}\right) = (281.9)\left(\frac{20}{8}\right) = 704.75 \text{ lbf}$$

$$F_B = \frac{W_{tB}}{\cos 25^\circ} = \frac{704.75}{\cos 25^\circ} = 777.6 \text{ lbf} \quad \textit{Ans.}$$

13-45 **(a)** For $\frac{\omega_o}{\omega_i} = \frac{5}{1}$, from Eq. (13-11), with $m = 5$, $k = 1$, $\phi = 20^\circ$

$$N_P = \frac{2(1)}{[1+2(5)]\sin^2 25^\circ}\left\{5+\sqrt{5^2+[1+2(5)]\sin^2 25^\circ}\right\} = 10.4$$

So $N_{P\min} = 11 \quad \textit{Ans.}$

(b) $m = \frac{d}{N} = \frac{300}{11} = 27.3 \text{ mm/tooth} \quad \textit{Ans.}$

(c) To transmit the same power with no change in pitch diameters, the speed and transmitted force must remain the same.
For A, with $\phi = 20°$,

$$W_{tA} = F_A \cos 20° = 11 \cos 20° = 10.33 \text{ kN}$$

For A, with $\phi = 25°$, same transmitted load,

$$F_A = W_{tA}/\cos 25° = 10.33 / \cos 25° = 11.40 \text{ kN} \quad \textit{Ans.}$$

Summing the torque about the shaft axis,

$$W_{tA}\left(\frac{d_A}{2}\right) = W_{tB}\left(\frac{d_B}{2}\right)$$

$$W_{tB} = W_{tA}\frac{(d_A/2)}{(d_B/2)} = W_{tA}\left(\frac{d_A}{d_B}\right) = (11.40)\left(\frac{600}{300}\right) = 22.80 \text{ kN}$$

$$F_B = \frac{W_{tB}}{\cos 25^\circ} = \frac{22.80}{\cos 25^\circ} = 25.16 \text{ kN} \quad \textit{Ans.}$$

13-46 **(a)** Using Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = 2$,

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m+\sqrt{m^2+(1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{\left[1+2(2)\right]\sin^2(20^\circ)}\left\{(2)+\sqrt{(2)^2+\left[1+2(2)\right]\sin^2(20^\circ)}\right\} = 14.16 \text{ teeth}$$

Round up for the minimum integer number of teeth.

N_F = 15 teeth, N_C = 30 teeth *Ans.*

(b) $m = \frac{d}{N} = \frac{125}{15} = 8.33$ mm/tooth *Ans.*

(c) $T = \frac{H}{\omega} = \frac{2 \text{ kW}}{191 \text{ rev/min}}\left(\frac{1000 \text{ W}}{\text{kW}}\right)\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 100 \text{ N}\cdot\text{m}$

(d) From Eq. (13-36),

$$W_t = \frac{60\,000H}{\pi dn} = \frac{60\,000(2)}{\pi(125)(191)} = 1.60 \text{ kN} = 1600 \text{ N} \qquad \textit{Ans.}$$

Or, we could have obtained W_t directly from the torque and radius,

$$W_t = \frac{T}{d/2} = \frac{100}{0.125/2} = 1600 \text{ N}$$

$$W_r = W_t \tan\phi = 1600 \tan 20^\circ = 583 \text{ N} \qquad \textit{Ans.}$$

$$W = \frac{W_t}{\cos\phi} = \frac{1600}{\cos 20^\circ} = 1700 \text{ N} \qquad \textit{Ans.}$$

13-47 **(a)** Using Eq. (13-11) with $k = 1$, $\phi = 20^\circ$, and $m = 2$,

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m+\sqrt{m^2+(1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{\left[1+2(2)\right]\sin^2(20^\circ)}\left\{(2)+\sqrt{(2)^2+\left[1+2(2)\right]\sin^2(20^\circ)}\right\} = 14.16 \text{ teeth}$$

Round up for the minimum integer number of teeth.

N_C = 15 teeth, N_F = 30 teeth *Ans.*

(b) $$P = \frac{N}{d} = \frac{30}{10} = 3 \text{ teeth/in} \quad Ans.$$

(c) $$T = \frac{H}{\omega} = \frac{1 \text{ hp}}{70 \text{ rev/min}}\left(\frac{550 \text{ lbf}\cdot\text{ft/s}}{\text{hp}}\right)\left(\frac{12 \text{ in}}{\text{ft}}\right)\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right)$$

$$T = 900 \text{ lbf}\cdot\text{in} \quad Ans.$$

(d) From Eqs. (13-34) and (13-35),

$$V = \frac{\pi dn}{12} = \frac{\pi(10)(70)}{12} = 183.3 \text{ ft/min}$$

$$W_t = 33\,000\frac{H}{V} = \frac{33\,000(1)}{183.3} = 180 \text{ lbf} \quad Ans.$$

$$W_r = W_t \tan\phi = 180 \tan 20^\circ = 65.5 \text{ lbf} \quad Ans.$$

$$W = \frac{W_t}{\cos\phi} = \frac{180}{\cos 20^\circ} = 192 \text{ lbf} \quad Ans.$$

13-48 **(a)** Eq. (13-14): $\gamma = \tan^{-1}\left(\frac{N_P}{N_G}\right) = \tan^{-1}\left(\frac{d_P}{d_G}\right) = \tan^{-1}\left(\frac{1.30}{3.88}\right) = 18.5^\circ$ *Ans.*

(b) Eq. (13-34): $V = \frac{\pi dn}{12} = \frac{\pi(2)(1.30)(600)}{12} = 408.4 \text{ ft/min}$ *Ans.*

(c) Eq. (13-35): $W_t = 33\,000\frac{H}{V} = 33\,000\left(\frac{10}{408.4}\right) = 808 \text{ lbf}$ *Ans.*

Eq. (13-38): $W_r = W_t \tan\phi \cos\gamma = 808 \tan 20^\circ \cos 18.5^\circ = 279 \text{ lbf}$ *Ans.*

Eq. (13-38): $W_a = W_t \tan\phi \sin\gamma = 808 \tan 20^\circ \sin 18.5^\circ = 93.3 \text{ lbf}$ *Ans.*

The tangential and axial forces agree with Prob. 3-85, but the radial force given in Prob. 3-85 is shown here to be incorrect. *Ans.*

13-49

$$\gamma = \tan^{-1}(2/4) = 26.565^\circ$$

$$\Gamma = \tan^{-1}(4/2) = 63.435^\circ$$

$$r_{\text{av}} = 2 - (1.5 \sin 26.565^0)/2 = 1.665 \text{ in}$$

$$T_{\text{in}} = 63\,025H/n = 63025(2.5)/240 = 656.5 \text{ lbf}\cdot\text{in}$$

$$W^t = T/r_{\text{av}} = 656.5/1.665 = 394.3 \text{ lbf}$$

$$a = 2 + (1.5 \cos 26.565^\circ)/2 = 2.671 \text{ in}$$

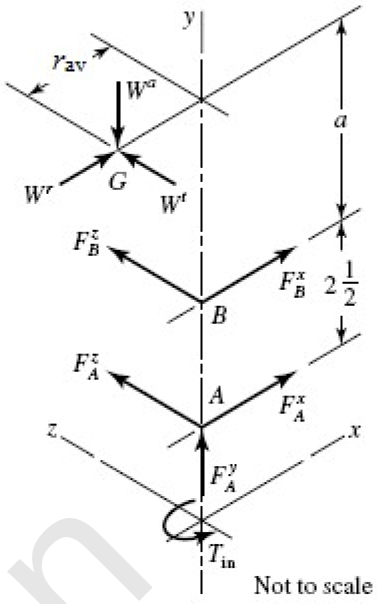
$$W^r = 394.3 \tan 20^\circ \cos 26.565^\circ = 128.4 \text{ lbf}$$

$$W^a = 394.3\tan 20^\circ \sin 26.565^\circ = 64.2 \text{ lbf}$$

$$\mathbf{W} = 128.4\mathbf{i} - 64.2\mathbf{j} + 394.3\mathbf{k} \text{ lbf}$$

$$\mathbf{R}_{AG} = -1.665\mathbf{i} + 5.171\mathbf{j}, \quad \mathbf{R}_{AB} = 2.5\mathbf{j}$$

$$\Sigma\mathbf{M}_4 = \mathbf{R}_{AG}\times\mathbf{W} + \mathbf{R}_{AB}\times\mathbf{F}_B + \mathbf{T} = \mathbf{0}$$



Solving gives

$$\mathbf{R}_{AB}\times\mathbf{F}_B = 2.5F_B^z\mathbf{i} - 2.5F_B^x\mathbf{k}$$
$$\mathbf{R}_{AG}\times\mathbf{W} = 2039\mathbf{i} + 656.5\mathbf{j} - 557.1\mathbf{k}$$

So,

$$(2039\mathbf{i} + 656.5\mathbf{j} - 557.1\mathbf{k}) + (2.5F_B^z\mathbf{i} - 2.5F_B^x\mathbf{k} + T\mathbf{j}) = \mathbf{0}$$
$$F_B^z = -2039/2.5 = -815.6 \text{ lbf}$$
$$T = -656.5 \text{ lbf}\cdot\text{in}$$
$$F_B^x = -557.1/2.5 = -222.8 \text{ lbf}$$

So, $\mathbf{F}_B = -222.8\mathbf{i} - 815.6\mathbf{k}$ lbf *Ans.*

$$F_B = \left[(-222.88)^2 + (-815.6)^2\right]^{1/2} = 845.5 \text{ lbf}$$

$$\mathbf{F}_A = -(\mathbf{F}_B + \mathbf{W})$$
$$= -(-222.8\mathbf{i} - 815.6\mathbf{k} + 128.4\mathbf{i} - 64.2\mathbf{j} + 394.3\mathbf{k})$$
$$= 94.4\mathbf{i} + 64.2\mathbf{j} + 421.3\mathbf{k} \quad \textit{Ans.}$$

$$F_A(\text{radial}) = (94.4^2 + 421.3^2)^{1/2} = 431.7 \text{ lbf}$$

$$F_A(\text{thrust}) = 64.2 \text{ lbf}$$

13-50 $d_2 = 18/10 = 1.8$ in, $d_3 = 30/10 = 3.0$ in

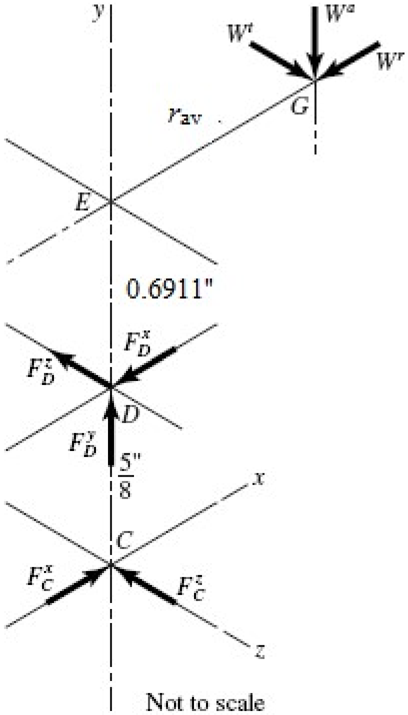
$$\gamma = \tan^{-1}\left(\frac{d_2/2}{d_3/2}\right) = \tan^{-1}\left(\frac{0.9}{1.5}\right) = 30.96^\circ$$

$$\Gamma = 90^\circ - \gamma = 59.04^\circ$$

$$r_{av} = 3.0/2 - (0.5\sin 59.04^\circ)/2 = 1.286 \text{ in}$$

$$DE = \frac{9}{16} + (0.5\cos 59.04^\circ)/2 = 0.6911 \text{ in}$$

$$W^t = 25 \text{ lbf}$$
$$W^r = 25\tan 20^\circ \cos 59.04^\circ = 4.681 \text{ lbf}$$
$$W^a = 25\tan 20^\circ \sin 59.04^\circ = 7.803 \text{ lbf}$$
$$\mathbf{W} = -4.681\mathbf{i} - 7.803\mathbf{j} + 25\mathbf{k}$$



$$\mathbf{R}_{DG} = 1.286\mathbf{i} + 0.6911\mathbf{j}$$

$$\mathbf{R}_{DC} = -0.625\mathbf{j}$$

$$\Sigma\mathbf{M}_D = \mathbf{R}_{DG} \times \mathbf{W} + \mathbf{R}_{DC} \times \mathbf{F}_C + \mathbf{T} = \mathbf{0}$$

$$\mathbf{R}_{DG} \times \mathbf{W} = 17.28\mathbf{i} - 32.15\mathbf{j} - 6.800\mathbf{k}$$

$$\mathbf{R}_{DC} \times \mathbf{F}_C = -0.625 F_C^z \mathbf{i} + 0.625 F_C^x \mathbf{k}$$

$$\left(17.28\mathbf{i} - 32.15\mathbf{j} - 6.800\mathbf{k}\right) + \left(-0.625 F_C^z \mathbf{i} + 0.625 F_C^x \mathbf{k}\right) + T\mathbf{j} = \mathbf{0}$$

$$T = 32.15 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$\mathbf{F}_C = 10.88\mathbf{i} + 27.65\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

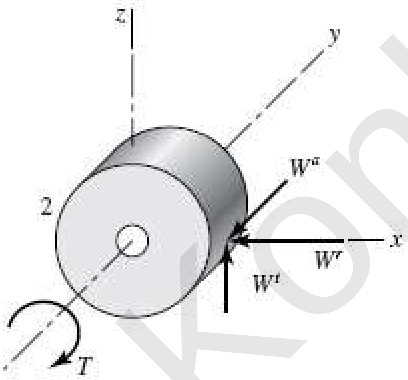
$$F_C = \left(10.88^2 + 27.65^2\right)^{1/2} = 29.7 \text{ lbf} \quad \textit{Ans.}$$

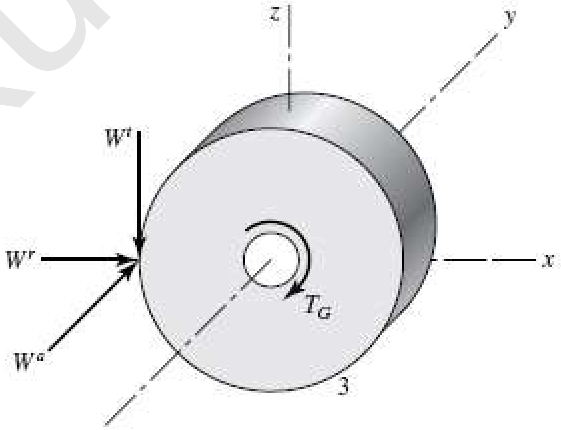
$$\Sigma\mathbf{F} = 0 \qquad \mathbf{F}_D = -6.20\mathbf{i} + 7.80\mathbf{j} - 52.65\mathbf{k} \text{ lbf}$$

$$F_D(\text{radial}) = \left[\left(-6.20\right)^2 + \left(-52.65\right)^2\right]^{1/2} = 53.0 \text{ lbf} \quad \textit{Ans.}$$

$$F_D(\text{thrust}) = W^a = 7.80 \text{ lbf} \quad \textit{Ans.}$$

13-51





NOTE: The shaft forces exerted on the gears are not shown in the figures above.

$$P_t = P_n \cos\psi = 5\cos 30^\circ = 4.330 \text{ teeth/in}$$

$$\phi_t = \tan^{-1}\frac{\tan\phi_n}{\cos\psi} = \tan^{-1}\frac{\tan 20^\circ}{\cos 30^\circ} = 22.80^\circ$$

$$d_P = \frac{18}{4.330} = 4.157 \text{ in}$$

The forces on the shafts will be equal to the forces transmitted to the gears through the meshing teeth.

Pinion (*Gear* 2)

$$W^r = W^t \tan\phi_t = 800\tan 22.80^\circ = 336 \text{ lbf}$$

$$W^a = W^t \tan\psi = 800\tan 30^\circ = 462 \text{ lbf}$$

$$\mathbf{W} = -336\mathbf{i} - 462\mathbf{j} + 800\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

$$W = \left[(-336)^2 + (-462)^2 + 800^2\right]^{1/2} = 983 \text{ lbf} \quad \textit{Ans.}$$

Gear 3

$$\mathbf{W} = 336\mathbf{i} + 462\mathbf{j} - 800\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

$$W = 983 \text{ lbf} \quad \textit{Ans.}$$

$$d_G = \frac{32}{4.330} = 7.390 \text{ in}$$

$$T_G = W^t r = 800(7.390) = 5912 \text{ lbf} \cdot \text{in}$$

13-52 $$\phi_t = \tan^{-1}\frac{\tan\phi_n}{\cos\psi} = \tan^{-1}\frac{\tan 20^\circ}{\cos 30^\circ} = 22.80^\circ$$

Pinion (*Gear* 2)

$$W^r = W^t \tan\phi_t = 800\tan 22.80^\circ = 336 \text{ lbf}$$

$$W^a = W^t \tan\psi = 800\tan 30^\circ = 462 \text{ lbf}$$

$$\mathbf{W} = -336\mathbf{i} - 462\mathbf{j} - 800\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

$$W = \left[(-336)^2 + (-462)^2 + (-800)^2\right]^{1/2} = 983 \text{ lbf} \quad \textit{Ans.}$$

Idler (*Gear* 3)

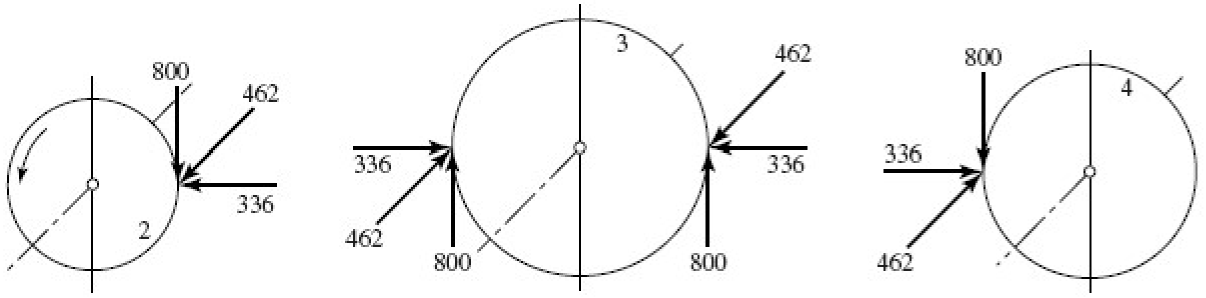
From the diagram for the idler, noting that the radial and axial forces from gears 2 and 4 cancel each other, the force acting on the shaft is

$$\mathbf{W} = +1600\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

Output gear (*Gear* 4)

$$\mathbf{W} = 336\mathbf{i} - 462\mathbf{j} - 800\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

$$W = \left[(-336)^2 + (-462)^2 + (-800)^2\right]^{1/2} = 983 \text{ lbf} \quad \textit{Ans.}$$



NOTE: For simplicity, the above figures only show the gear contact forces.

Also, notice that the idler shaft reaction contains a couple tending to turn the shaft end-over-end. Also the idler teeth are bent both ways. Idlers are more severely loaded than other gears, belying their name. Thus, be cautious.

13-53 *Gear* 3:

$$P_t = P_n \cos\psi = 7\cos 30^\circ = 6.062 \text{ teeth/in}$$

$$\tan\phi_t = \frac{\tan 20^\circ}{\cos 30^\circ} = 0.4203, \quad \phi_t = 22.8^\circ$$

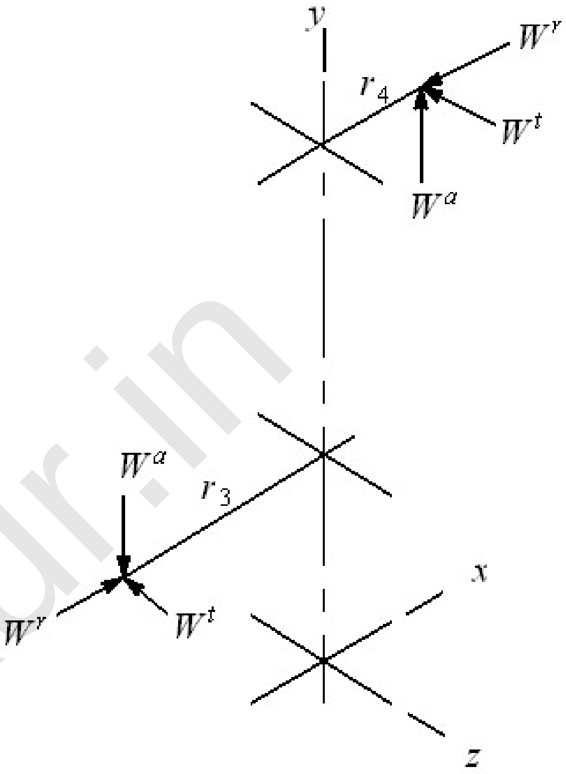
$$d_3 = \frac{54}{6.062} = 8.908 \text{ in}$$

$$W^t = 500 \text{ lbf}$$

$$W^a = 500\tan 30^\circ = 288.7 \text{ lbf}$$

$$W^r = 500\tan 22.8^\circ = 210.2 \text{ lbf}$$

$$\mathbf{W}_3 = 210.2\mathbf{i} - 288.7\mathbf{j} - 500\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$



Gear 4:

$$d_4 = \frac{14}{6.062} = 2.309 \text{ in}$$

$$W^t = 500\frac{8.908}{2.309} = 1929 \text{ lbf}$$

$$W^a = 1929\tan 30^\circ = 1114 \text{ lbf}$$

$$W^r = 1929\tan 22.8^\circ = 811 \text{ lbf}$$

$$\mathbf{W}_4 = -811\mathbf{i} + 1114\mathbf{j} - 1929\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

13-54

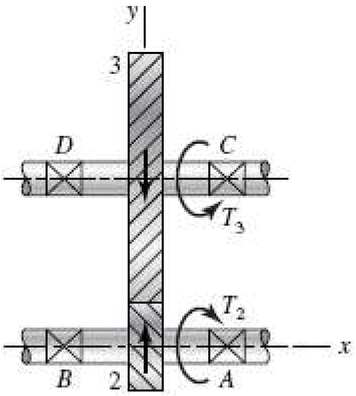
$$P_t = 6\cos 30^\circ = 5.196 \text{ teeth/in}$$

$$d_3 = \frac{42}{5.196} = 8.083 \text{ in}$$

$$\phi_t = 22.8^\circ$$

$$d_2 = \frac{16}{5.196} = 3.079 \text{ in}$$

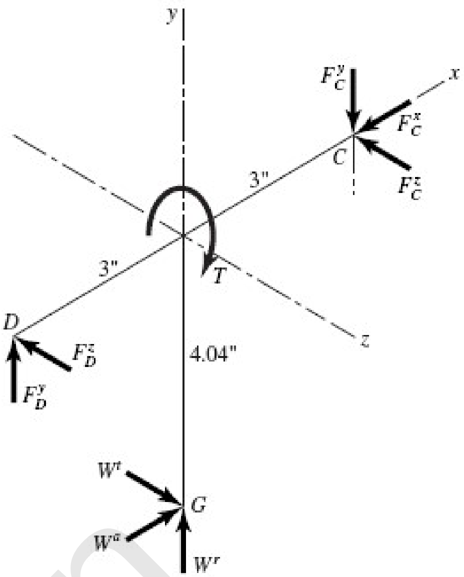
$$T_2 = \frac{63\,025(25)}{1720} = 916 \text{ lbf}\cdot\text{in}$$



$$W^t = \frac{T}{r} = \frac{916}{3.079/2} = 595 \text{ lbf}$$
$$W^a = 595 \tan 30^\circ = 344 \text{ lbf}$$
$$W^r = 595 \tan 22.8^\circ = 250 \text{ lbf}$$

$$\mathbf{W} = 344\mathbf{i} + 250\mathbf{j} + 595\mathbf{k} \text{ lbf}$$

$$\mathbf{R}_{DC} = 6\mathbf{i}, \quad \mathbf{R}_{DG} = 3\mathbf{i} - 4.04\mathbf{j}$$



$$\Sigma\mathbf{M}_D = \mathbf{R}_{DC} \times \mathbf{F}_C + \mathbf{R}_{DG} \times \mathbf{W} + \mathbf{T} = \mathbf{0} \tag{1}$$

$$\mathbf{R}_{DG} \times \mathbf{W} = -2404\mathbf{i} - 1785\mathbf{j} + 2140\mathbf{k}$$
$$\mathbf{R}_{DC} \times \mathbf{F}_C = -6F_C^z\mathbf{j} + 6F_C^y\mathbf{k}$$

Substituting and solving Eq. (1) gives

$$\mathbf{T} = 2404\mathbf{i} \text{ lbf} \cdot \text{in}$$
$$F_C^z = -297.5 \text{ lbf}$$
$$F_C^y = -365.7 \text{ lbf}$$

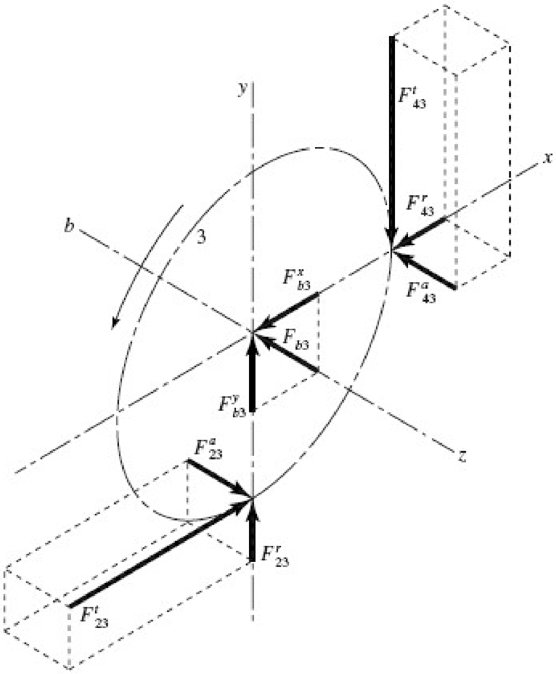
$$\Sigma\mathbf{F} = \mathbf{F}_D + \mathbf{F}_C + \mathbf{W} = \mathbf{0}$$

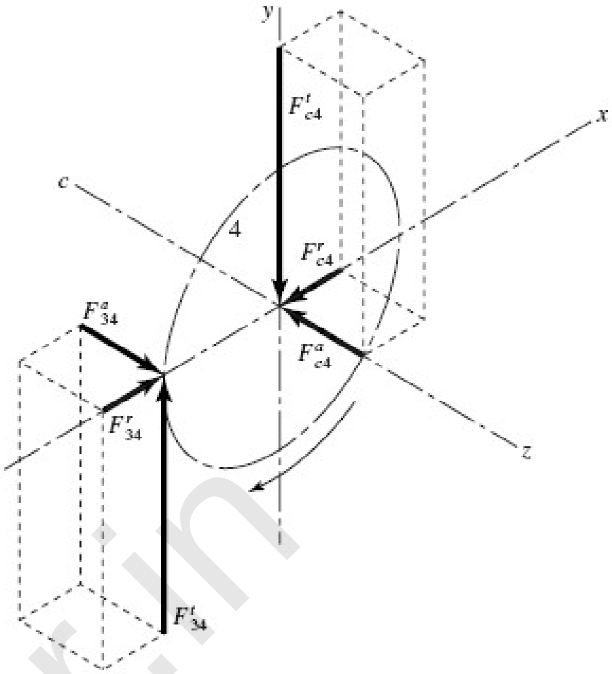
Substituting and solving gives

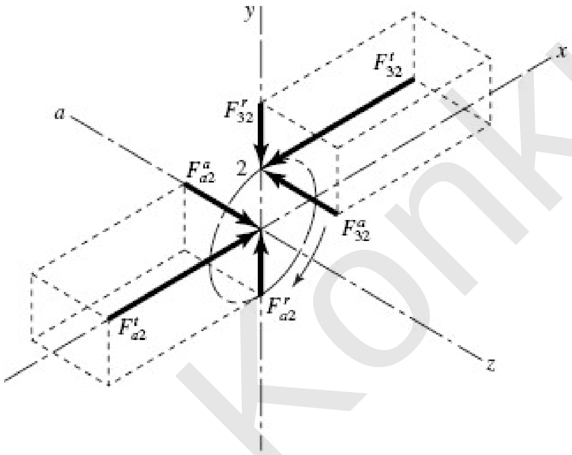
$$F_C^x = -344 \text{ lbf}$$
$$F_D^y = 106.7 \text{ lbf}$$
$$F_D^z = -297.5 \text{ lbf}$$

$$\mathbf{F}_C = -344\mathbf{i} - 356.7\mathbf{j} - 297.5\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$
$$\mathbf{F}_D = 106.7\mathbf{j} - 297.5\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

13-55







Since the transverse pressure angle is specified, we will assume the given module is also in terms of the transverse orientation.

$$d_2 = mN_2 = 4(16) = 64 \text{ mm}$$

$$d_3 = mN_3 = 4(36) = 144 \text{ mm}$$

$$d_4 = mN_4 = 4(28) = 112 \text{ mm}$$

$$T = \frac{H}{\omega} = \frac{6 \text{ kW}}{1600 \text{ rev/min}}\left(\frac{1000 \text{ W}}{\text{kW}}\right)\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 35.81 \text{ N}\cdot\text{m}$$

$$W^t = \frac{T}{d_2/2} = \frac{35.81}{0.064/2} = 1119 \text{ N}$$

$$W^r = W^t \tan\phi_t = 1119 \tan 20^\circ = 407.3 \text{ N}$$

$$W^a = W^t \tan\psi = 1119 \tan 15^\circ = 299.8 \text{ N}$$

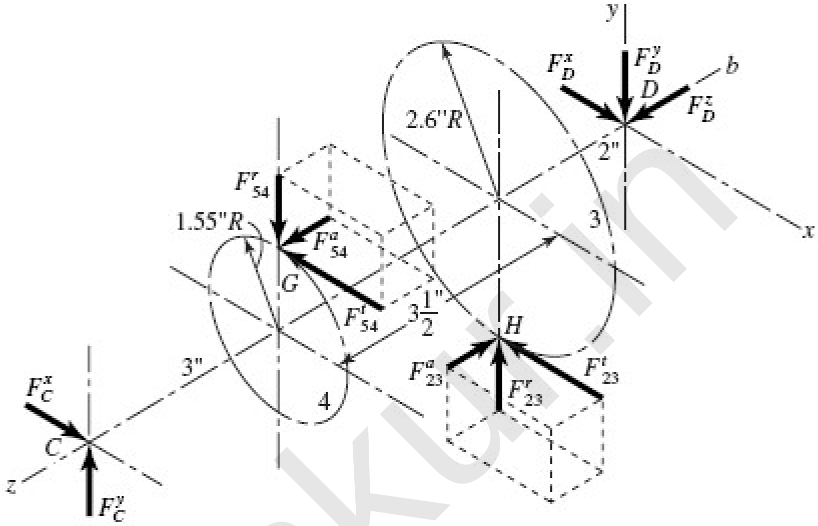
$$\mathbf{F}_{2a} = -1119\mathbf{i} - 407.3\mathbf{j} - 299.8\mathbf{k} \text{ N} \quad \textit{Ans.}$$

$$\mathbf{F}_{3b} = (1119 - 407.3)\mathbf{i} - (1119 - 407.3)\mathbf{j}$$

$$= 711.7\mathbf{i} - 711.7\mathbf{j} \text{ N} \quad \textit{Ans.}$$

$$\mathbf{F}_{4c} = 407.3\mathbf{i} + 1119\mathbf{j} + 299.8\mathbf{k} \text{ N} \quad \textit{Ans.}$$

13-56



$$d_2 = \frac{N}{P_n \cos\psi} = \frac{14}{8\cos 30^\circ} = 2.021 \text{ in}, \quad d_3 = \frac{36}{8\cos 30^\circ} = 5.196 \text{ in}$$

$$d_4 = \frac{15}{5\cos 15^\circ} = 3.106 \text{ in}, \quad d_5 = \frac{45}{5\cos 15^\circ} = 9.317 \text{ in}$$

For gears 2 and 3: $\phi_t = \tan^{-1}(\tan\phi_n / \cos\psi) = \tan^{-1}(\tan 20^\circ / \cos 30^\circ) = 22.8^\circ$

For gears 4 and 5: $\phi_t = \tan^{-1}(\tan 20^\circ / \cos 15^\circ) = 20.6^\circ$

$$F_{23}^t = T_2 / r_2 = 1200 / (2.021/2) = 1188 \text{ lbf}$$

$$F_{54}^t = 1188 \frac{5.196}{3.106} = 1987 \text{ lbf}$$

$$F_{23}^r = F_{23}^t \tan\phi_t = 1188 \tan 22.8^\circ = 499 \text{ lbf}$$

$$F_{54}^r = 1986 \tan 20.6^\circ = 746 \text{ lbf}$$

$$F_{23}^a = F_{23}^t \tan\psi = 1188 \tan 30^\circ = 686 \text{ lbf}$$

$$F_{54}^a = 1986 \tan 15^\circ = 532 \text{ lbf}$$

Next, designate the points of action on gears 4 and 3, respectively, as points *G* and *H*, as shown. Position vectors are

$$\mathbf{R}_{CG} = 1.553\mathbf{j} - 3\mathbf{k}$$
$$\mathbf{R}_{CH} = -2.598\mathbf{j} - 6.5\mathbf{k}$$
$$\mathbf{R}_{CD} = -8.5\mathbf{k}$$

Force vectors are

$$\mathbf{F}_{54} = -1986\mathbf{i} - 748\mathbf{j} + 532\mathbf{k}$$
$$\mathbf{F}_{23} = -1188\mathbf{i} + 500\mathbf{j} - 686\mathbf{k}$$
$$\mathbf{F}_{C} = F_C^x\mathbf{i} + F_C^y\mathbf{j}$$
$$\mathbf{F}_{D} = F_D^x\mathbf{i} + F_D^y\mathbf{j} + F_D^z\mathbf{k}$$

Now, a summation of moments about bearing *C* gives

$$\Sigma\mathbf{M}_C = \mathbf{R}_{CG} \times \mathbf{F}_{54} + \mathbf{R}_{CH} \times \mathbf{F}_{23} + \mathbf{R}_{CD} \times \mathbf{F}_D = \mathbf{0}$$

The terms for this equation are found to be

$$\mathbf{R}_{CG} \times \mathbf{F}_{54} = -1412\mathbf{i} + 5961\mathbf{j} + 3086\mathbf{k}$$
$$\mathbf{R}_{CH} \times \mathbf{F}_{23} = 5026\mathbf{i} + 7722\mathbf{j} - 3086\mathbf{k}$$
$$\mathbf{R}_{CD} \times \mathbf{F}_{D} = 8.5F_D^y\mathbf{i} - 8.5F_D^x\mathbf{j}$$

When these terms are placed back into the moment equation, the **k** terms, representing the shaft torque, cancel. The **i** and **j** terms give

$$F_D^y = -\frac{3614}{8.5} = -425 \text{ lbf}$$
$$F_D^x = \frac{13683}{8.5} = 1610 \text{ lbf}$$

Next, we sum the forces to zero.

$$\Sigma\mathbf{F} = \mathbf{F}_C + \mathbf{F}_{54} + \mathbf{F}_{23} + \mathbf{F}_D = \mathbf{0}$$

Substituting, gives

$$\left(F_C^x\mathbf{i} + F_C^y\mathbf{j}\right) + \left(-1987\mathbf{i} - 746\mathbf{j} + 532\mathbf{k}\right) + \left(-1188\mathbf{i} + 499\mathbf{j} - 686\mathbf{k}\right) + \left(1610\mathbf{i} - 425\mathbf{j} + F_D^z\mathbf{k}\right) = \mathbf{0}$$

Solving gives

$$F_C^x = 1987 + 1188 - 1610 = 1565 \text{ lbf}$$
$$F_C^y = 746 - 499 + 425 = 672 \text{ lbf}$$
$$F_D^z = -532 + 686 = 154 \text{ lbf}$$

So, $\mathbf{F}_C = 1565\mathbf{i} + 672\mathbf{j}$ lbf *Ans.*

$\mathbf{F}_D = 1610\mathbf{i} - 425\mathbf{j} + 154\mathbf{k}$ lbf *Ans.*

13-57

$$V_W = \frac{\pi d_W n_W}{60} = \frac{\pi(0.100)(600)}{60} = \pi \text{ m/s}$$

$$W_{Wt} = \frac{H}{V_W} = \frac{2000}{\pi} = 637 \text{ N}$$

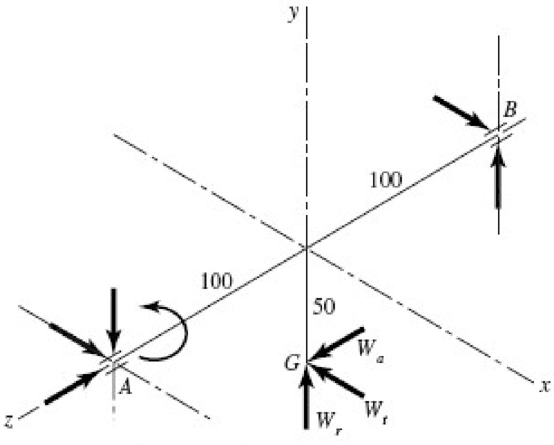
$$L = p_x N_W = 25(1) = 25 \text{ mm}$$

$$\lambda = \tan^{-1}\frac{L}{\pi d_W}$$

$$= \tan^{-1}\frac{25}{\pi(100)} = 4.550^\circ \text{ lead angle}$$

$$W = \frac{W_{Wt}}{\cos\phi_n \sin\lambda + f\cos\lambda}$$

$$V_S = \frac{V_W}{\cos\lambda} = \frac{\pi}{\cos 4.550^\circ} = 3.152 \text{ m/s}$$



Worm shaft diagram

In ft/min: $V_S = 3.28(3.152) = 10.33$ ft/s = 620 ft/min
Use $f = 0.043$ from curve A of Fig. 13-38. Then, from the first of Eq. (13-43)

$$W = \frac{637}{\cos 14.5^\circ(\sin 4.55^\circ) + 0.043\cos 4.55^\circ} = 5323 \text{ N}$$

$$W^y = W\sin\phi_n = 5323\sin 14.5^\circ = 1333 \text{ N}$$

$$W^z = 5323\left[\cos 14.5^\circ(\cos 4.55^\circ) - 0.043\sin 4.55^\circ\right] = 5119 \text{ N}$$

The force acting against the worm is

$$\mathbf{W} = -637\mathbf{i} + 1333\mathbf{j} + 5119\mathbf{k} \text{ N}$$

Thus, A is the thrust bearing. *Ans.*

$$\mathbf{R}_{AG} = -0.05\mathbf{j} - 0.10\mathbf{k} \text{ m}, \quad \mathbf{R}_{AB} = -0.20\mathbf{k} \text{ m}$$

$$\Sigma\mathbf{M}_A = \mathbf{R}_{AG}\times\mathbf{W} + \mathbf{R}_{AB}\times\mathbf{F}_B + \mathbf{T} = \mathbf{0}$$

$$\mathbf{R}_{AG}\times\mathbf{W} = -122.6\mathbf{i} + 63.7\mathbf{j} - 31.85\mathbf{k} \quad \text{N}\cdot\text{m}$$

$$\mathbf{R}_{AB}\times\mathbf{F}_B = 0.2F_B^y\mathbf{i} - 0.2F_B^x\mathbf{j}$$

Substituting and solving gives

$T = 31.85$ N·m *Ans.*

$F_B^x = 318.5$ N, $F_B^y = 613$ N

So $\mathbf{F}_B = 318.5\mathbf{i} + 613\mathbf{j}$ N *Ans.*

Or $F_B = \left[(613)^2 + (318.5)^2\right]^{1/2} = 691 \text{ N radial}$

$$\Sigma \mathbf{F} = \mathbf{F}_A + \mathbf{W} + \mathbf{R}_B = \mathbf{0}$$

$$\mathbf{F}_A = -(\mathbf{W} + \mathbf{F}_B) = -(-637\mathbf{i} + 1333\mathbf{j} + 5119\mathbf{k} + 318.5\mathbf{i} + 613\mathbf{j})$$

$$= 318.5\mathbf{i} - 1946\mathbf{j} - 5119\mathbf{k} \quad \textit{Ans.}$$

Radial $\mathbf{F}_A^r = 318.5\mathbf{i} - 1946\mathbf{j} \text{ N}$

$$F_A^r = \left[(318.5)^2 + (-1946)^2\right]^{1/2} = 1972 \text{ N}$$

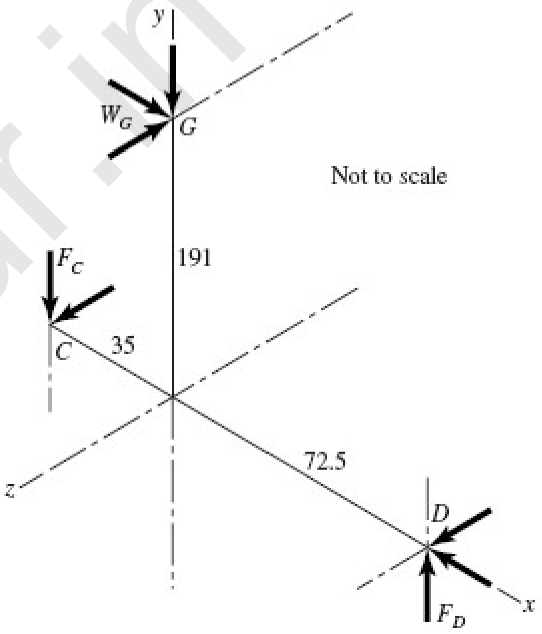
Thrust $F_A^a = -5119 \text{ N}$

13-58 From Prob. 13-57,

$$\mathbf{W}_G = 637\mathbf{i} - 1333\mathbf{j} - 5119\mathbf{k} \text{ N}$$

$$p_t = p_x$$

So $d_G = \dfrac{N_G p_x}{\pi} = \dfrac{48(25)}{\pi} = 382 \text{ mm}$



Bearing D takes the thrust load.

$$\Sigma \mathbf{M}_D = \mathbf{R}_{DG} \times \mathbf{W}_G + \mathbf{R}_{DC} \times \mathbf{F}_C + \mathbf{T} = \mathbf{0}$$

$$\mathbf{R}_{DG} = -0.0725\mathbf{i} + 0.191\mathbf{j} \text{ m}$$

$$\mathbf{R}_{DC} = -0.1075\mathbf{i} \text{ m}$$

$$\mathbf{R}_{DG} \times \mathbf{W}_G = -977.7\mathbf{i} - 371.1\mathbf{j} - 25.02\mathbf{k} \text{ N} \cdot \text{m}$$

$$\mathbf{R}_{DC} \times \mathbf{F}_C = 0.1075 F_C^z \mathbf{j} - 0.1075 F_C^y \mathbf{k} \text{ N} \cdot \text{m}$$

Putting it together and solving,

$$T = 977.7 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

$$\mathbf{F}_C = -233\mathbf{j} + 3450\mathbf{k} \text{ N}, \quad F_C = 3460 \text{ N} \quad \textit{Ans.}$$

$$\Sigma \mathbf{F} = \mathbf{F}_C + \mathbf{W}_G + \mathbf{F}_D = \mathbf{0}$$

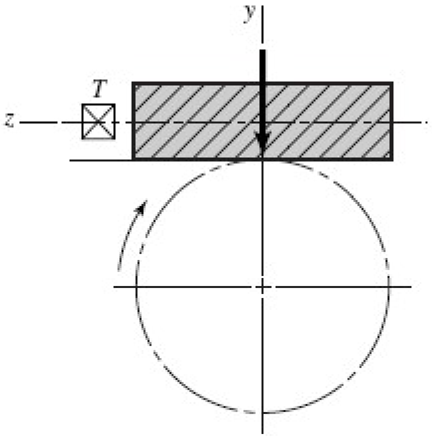
$$\mathbf{F}_D = -(\mathbf{F}_C + \mathbf{W}_G) = -637\mathbf{i} + 1566\mathbf{j} + 1669\mathbf{k} \text{ N} \quad \textit{Ans.}$$

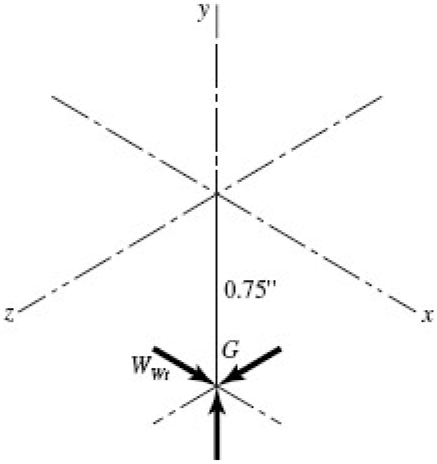
Radial $\mathbf{F}_D^r = 1566\mathbf{j} + 1669\mathbf{k} \text{ N}$

Or $F_D^r = (1566^2 + 1669^2)^{1/2} = 2289 \text{ N (total radial)}$

$\mathbf{F}_D^t = -637\mathbf{i} \text{ N} \quad \text{(thrust)}$

13-59





$$V_W = \frac{\pi(1.5)(600)}{12} = 235.7 \text{ ft/min}$$

$$W^x = W_{Wt} = \frac{33000(0.75)}{235.7} = 105.0 \text{ lbf}$$

$$p_t = p_x = \frac{\pi}{8} = 0.3927 \text{ in}$$

$$L = 0.3927(2) = 0.7854 \text{ in}$$

$$\lambda = \tan^{-1}\frac{0.7854}{\pi(1.5)} = 9.46^\circ$$

$$W = \frac{105.0}{\cos 20^\circ \sin 9.46^\circ + 0.05\cos 9.46^\circ} = 515.3 \text{ lbf}$$

$$W^y = 515.3\sin 20^\circ = 176.2 \text{ lbf}$$

$$W^z = 515.3\left[\cos 20^\circ\left(\cos 9.46^\circ\right) - 0.05\sin 9.46^\circ\right] = 473.4 \text{ lbf}$$

So $\mathbf{W} = 105\mathbf{i} + 176\mathbf{j} + 473\mathbf{k} \text{ lbf}$ *Ans.*

$T = 105(0.75) = 78.8 \text{ lbf}\cdot\text{in}$ *Ans.*

13-60 Computer programs will vary.