1/1

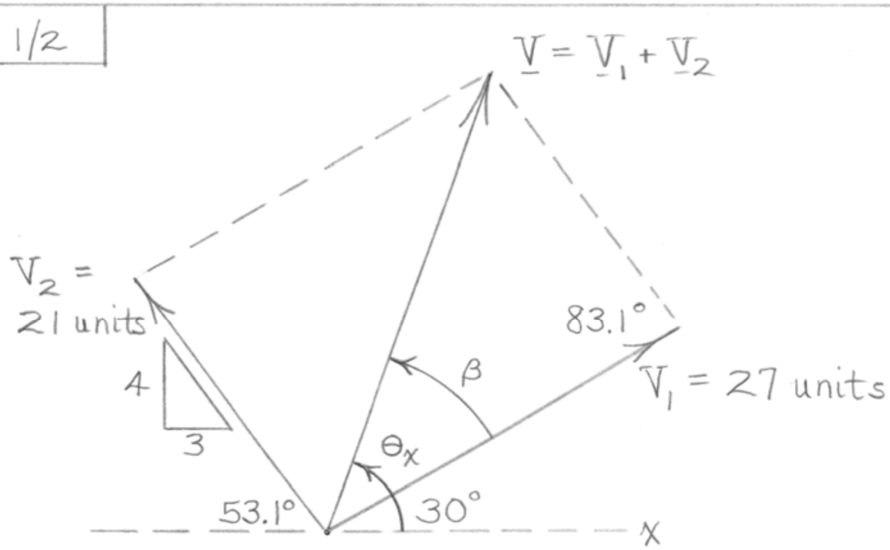
$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{36^2 + 15^2} = 39$$

$$\cos\theta_x = \frac{V_x}{V} = \frac{-36}{39}, \quad \underline{\theta_x = 157.4^\circ}$$

$$\cos\theta_y = \frac{V_y}{V} = \frac{15}{39}, \quad \underline{\theta_y = 67.4^\circ}$$

$$\underline{n} = \frac{\underline{V}}{V} = \frac{-36\underline{i} + 15\underline{j}}{39} = \underline{-0.923\underline{i} + 0.385\underline{j}}$$

1/2



Graphically, $V = \underline{32 \text{ units}}$, $\underline{\theta_x = 70°}$

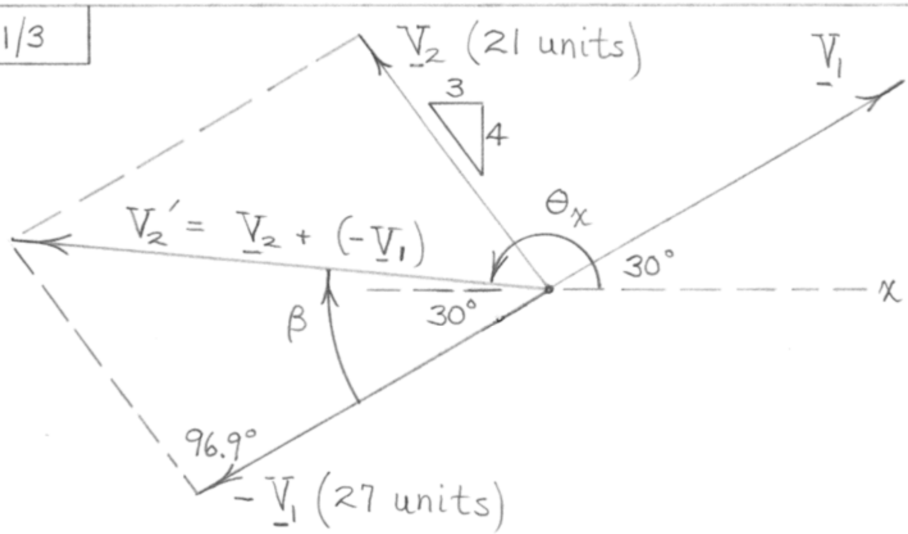
Algebraically, $V^2 = 27^2 + 21^2 - 2(27)(21)\cos 83.1°$

$$V = \underline{32.2 \text{ units}}$$

$$\frac{\sin\beta}{21} = \frac{\sin 83.1°}{32.2}, \quad \beta = 40.4°$$

$$\theta_x = \beta + 30° = 40.4° + 30° = \underline{70.4°}$$

1/3



Graphically, $\underline{V' = 36 \text{ units}}$, $\underline{\theta_x = 175°}$

Algebraically, $V'^2 = 27^2 + 21^2 - 2(27)(21)\cos 96.9°$

$\underline{V' = 36.1 \text{ units}}$

$\dfrac{\sin\beta}{21} = \dfrac{\sin 96.9°}{36.1}$, $\beta = 35.2°$

$\theta_x + \beta = 210°$, $\theta_x = 210 - 35.2° = \underline{174.8°}$

1/4

$$F = \sqrt{160^2 + 80^2 + 120^2} = 215 \text{ N}$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{160}{215} = 0.743, \quad \underline{\theta_x = 42.0°}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{80}{215} = 0.371, \quad \underline{\theta_y = 68.2°}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-120}{215} = -0.557, \quad \underline{\theta_z = 123.9°}$$

1/5

$$m = \frac{W}{g} = \frac{1000}{32.174} = \underline{31.1 \text{ slugs}}$$

$$m = 31.1 \text{ slugs} \left(\frac{14.594 \text{ kg}}{\text{slug}} \right) = \underline{454 \text{ kg}}$$

1/6

$$F = W = \frac{G m_1 m_2}{r^2}$$

where $G = 6.673(10^{-11})\ m^3/(kg \cdot s^2)$

$m_1 = 85\ kg$

$m_2 = 5.976(10^{24})\ kg$

and $r = (6371 + 250)(10^3)\ m$

Substitute these numbers & obtain $\underline{W = 773\ N}$

U.S. units : $W = 773\ N \left(\frac{1\ lb}{4.4482\ N}\right) = \underline{173.8\ lb}$

1/7 $W = (125\text{ lb})\left(\dfrac{4.4482\text{ N}}{\text{lb}}\right) = \underline{556\text{ N}}$

$m = \dfrac{W}{g} = \dfrac{125}{32.2} = \underline{3.88\text{ slugs}}$

$m = \dfrac{W}{g} = \dfrac{556}{9.81} = \underline{56.7\text{ kg}}$

1/8 $\quad A = 8.67, \quad B = 1.429$

$(A + B) = 8.67 + 1.429 = \underline{10.10}$

$(A - B) = 8.67 - 1.429 = \underline{7.24}$

$(AB) = (8.67)(1.429) = \underline{12.39}$

$(A/B) = 8.67/1.429 = \underline{6.07}$

1/9

$$F = \frac{G m_e m_s}{d^2} = \frac{3.439(10^{-8})(1)(333{,}000)(4.095 \cdot 10^{23})^2}{(92.96 \cdot 10^6 \cdot 5280)^2}$$

$$= \underline{7.97(10^{21})\text{ lb}}$$

$$F = 7.97(10^{21})\text{ lb}\left(\frac{4.4482\text{ N}}{\text{lb}}\right) = \underline{3.55(10^{22})\text{ N}}$$

$\boxed{1/10}$ $\underline{F} = F\underline{n} = F\left(\frac{-4\underline{i}-2\underline{j}}{\sqrt{4^2+2^2}}\right),$

where $F = \frac{G m_{cu} m_{st}}{d^2}$

$$= \frac{G\left(\rho_{cu}\,\frac{4}{3}\pi r^3\right)\left(\rho_{st}\,\frac{4}{3}\pi\left(\frac{r}{2}\right)^3\right)}{(4r)^2+(2r)^2}$$

$$= \frac{1}{90}\,G\,\rho_{cu}\,\rho_{st}\,\pi^2 r^4$$

$$= \frac{1}{90}\,(6.673\cdot10^{-11})(8910)(7830)\,\pi^2\,0.050^4$$

$$= 3.19(10^{-9})\ \text{N}$$

Then $\underline{F} = 3.19(10^{-9})\left[\frac{-4\underline{i}-2\underline{j}}{\sqrt{20}}\right]$

$$= \underline{(-2.85\underline{i} - 1.427\underline{j})10^{-9}\ \text{N}}$$

1/11 $E = 3 \sin^2\theta \tan\theta \cos\theta$

Exact: $E = 3 \sin^2 2° \tan 2° \cos 2°$

$= \underline{1.275(10^{-4})}$

Approx: $E_{ap} = 3(\theta^2)(\theta)(1)$

$= 3\theta^3 \quad (\theta \text{ in rad})$

$E_{ap} = 3\left[2\frac{\pi}{180}\right]^3 = \underline{1.276(10^{-4})}$

1/12

SI : $[Q] = (1)(kg)(m^2)/s^2$

$= \underline{kg \cdot m^2/s^2}$

U.S. : $[Q] = (1)(slug)(ft^2)/sec^2$

$= \left(\frac{lb\text{-}sec^2}{ft}\right)(ft)^2/sec^2 = \underline{lb\text{-}ft}$

Note: The SI units reduce to

$(kg \cdot m/s^2)\,m = N \cdot m$, but N is

not a base unit.

2/1

$$\begin{cases} F_x = -800 \sin 35° = \underline{-459 \text{ N}} \\ F_y = 800 \cos 35° = \underline{655 \text{ N}} \end{cases}$$

$$\underline{\mathbf{F} = -459\mathbf{i} + 655\mathbf{j} \text{ N}}$$

2/2

$\underline{F} = 7(-\sin 25^\circ \, \underline{i} + \cos 25^\circ \, \underline{j})$

$\underline{F} = -2.96 \, \underline{i} + 6.34 \, \underline{j}$ kN

• Scalar Components

$F_x = -2.96$ kN

$F_y = 6.34$ kN

2/3

$$\underline{F} = 6.5\left(-\frac{12}{13}\underline{i} - \frac{5}{13}\underline{j}\right)$$

$$= \underline{-6\underline{i} - 2.5\underline{j} \text{ kN}}$$

(Note: Writing 6, rather than 6.00, indicates an exact result.)

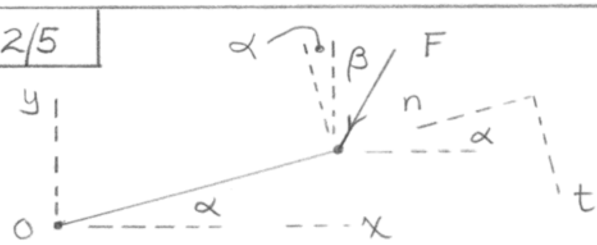
2/4

$$\underline{n} = \frac{13\underline{i} - 15\underline{j}}{\sqrt{13^2 + 15^2}} \longrightarrow \underline{\underline{n} = 0.655\underline{i} - 0.756\underline{j}}$$

• SCALAR COMPONENTS:

$$\begin{cases} F_x = Fn_x = 6(0.655) \longrightarrow \underline{F_x = 3.93 \text{ kN}} \\ F_y = Fn_y = 6(-0.756) \longrightarrow \underline{F_y = -4.53 \text{ kN}} \end{cases}$$

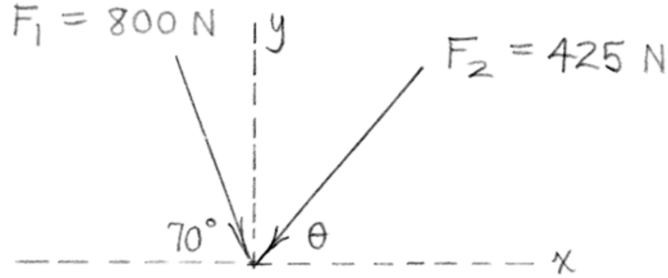
2/5



$$\begin{cases} F_x = -F\sin\beta \\ F_y = -F\cos\beta \end{cases}$$

$$\begin{cases} F_n = F\sin(\alpha+\beta) \\ F_t = F\cos(\alpha+\beta) \end{cases}$$

2/6



$R_x = \Sigma F_x = 800 \cos 70° - 425 \cos\theta = 0$

$\underline{\theta = 49.9°}$

$R_y = \Sigma F_y = -800 \sin 70° - 425 \sin 49.9°$

$= -1077$ N

So $\underline{R = 1077 \text{ N}}$

2/7

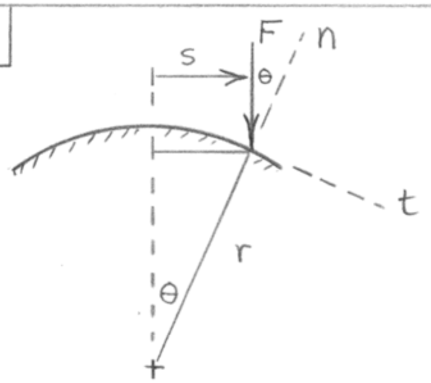
$$\underline{R} = (500 + 350\cos 60^\circ)\,\underline{i} + 350\sin 60^\circ\,\underline{j}$$

$$\underline{\underline{R} = 675\,\underline{i} + 303\,\underline{j}\ \text{N}}$$

$$R = \sqrt{675^2 + 303^2} \longrightarrow \underline{R = 740\ \text{N}}$$

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{675}{740}\right) \longrightarrow \underline{\theta_x = 24.2^\circ\ \text{ABOVE } +x \text{ AXIS}}$$

2/8



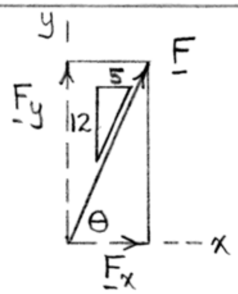
$$\theta = \sin^{-1}\left(\frac{s}{r}\right)$$

$$F_t = F\sin\theta = F\sin\left[\sin^{-1}\left(\frac{s}{r}\right)\right] = \frac{Fs}{r}$$

$$F_n = -F\cos\theta = -F\cos\left[\sin^{-1}\left(\frac{s}{r}\right)\right]$$

$$= -\frac{F\sqrt{r^2-s^2}}{r}$$

2/9



$\cos\theta = \frac{5}{13}$, $\sin\theta = \frac{12}{13}$

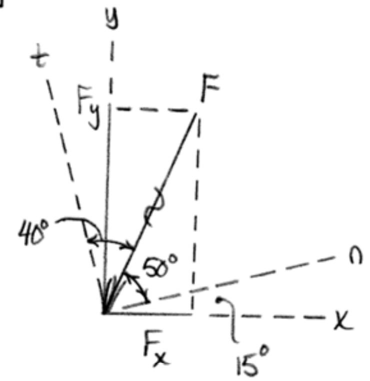
$F_y = F\sin\theta = F\frac{12}{13} = 320 \text{ N}$

$\underline{F = 347 \text{ N}}$

$F_x = F\cos\theta = 347\left(\frac{5}{13}\right) = \underline{133.3 \text{ N}}$

2/10

$F = 65 \text{ kN}$

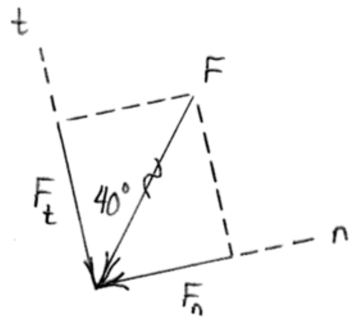


$$F_x = -F\cos 65^\circ = -65\cos 65^\circ$$

$$\underline{F_x = -27.5 \text{ kN}}$$

$$F_y = -F\sin 65^\circ = -65\sin 65^\circ$$

$$\underline{F_y = -58.9 \text{ kN}}$$



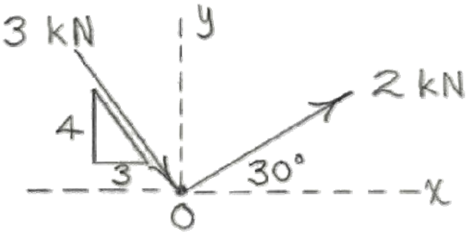
$$F_n = -F\sin 40^\circ = -65\sin 40^\circ$$

$$\underline{F_n = -41.8 \text{ kN}}$$

$$F_t = -F\cos 40^\circ = -65\cos 40^\circ$$

$$\underline{F_t = -49.8 \text{ kN}}$$

2/11



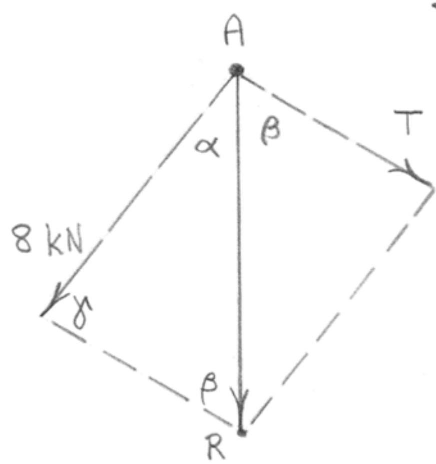
$$\begin{cases} R_x = \Sigma F_x = +3\left(\frac{3}{5}\right) + 2\cos 30° = 3.53 \text{ kN} \\ R_y = \Sigma F_y = -3\left(\frac{4}{5}\right) + 2\sin 30° = -1.4 \text{ kN} \end{cases}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{3.80 \text{ kN}}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-1.4}{3.53}\right) = 338°$$

$\underline{(\text{or } -21.6°)}$

2/12



$$\begin{cases} \alpha = \tan^{-1}\frac{40}{50} = 38.7^\circ \\ \beta = \tan^{-1}\frac{50}{30} = 59.0^\circ \end{cases}$$

$$\gamma = 180^\circ - \alpha - \beta$$

$$= 82.3^\circ$$

$$\frac{\sin\beta}{8} = \frac{\sin\alpha}{T}$$

$$\underline{T = 5.83 \text{ kN}}$$

$$\frac{\sin\beta}{8} = \frac{\sin\gamma}{R}, \qquad \underline{R = 9.25 \text{ kN}}$$

2/113

$$R_x = \Sigma F_x = 400 + 400 \cos 60° = 600 \text{ N}$$

$$R_y = \Sigma F_y = 400 \sin 60° = 346 \text{ N}$$

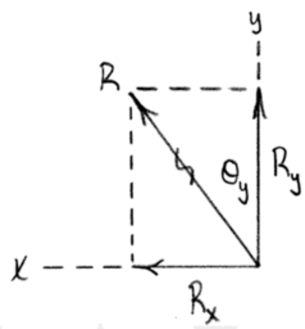
$$\Rightarrow \underline{R} = \underline{600\underline{i} + 346\underline{j} \text{ N}}$$

$$R = \sqrt{600^2 + 346^2} = \underline{693 \text{ N}}$$

2/14

$$\underline{R} = (175\cos 40^\circ + 125\sin 15^\circ)\underline{i} + (175\sin 40^\circ + 125\cos 15^\circ)\underline{j}$$

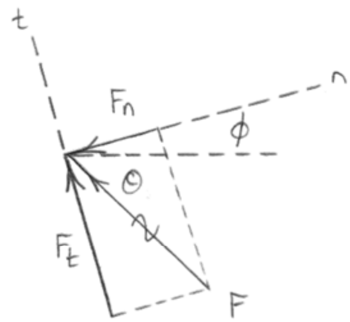
$$\underline{R} = 166.4\,\underline{i} + 233\,\underline{j} \text{ N}$$



$$\theta_y = \tan^{-1}\left(\frac{R_x}{R_y}\right) = \tan^{-1}\left(\frac{166.4}{233}\right)$$

$$\theta_y = 35.5^\circ \text{ CCW OFF } +y\text{-AXIS}$$

2/15



$$\begin{cases} \underline{F_n = -F\cos(\theta+\phi)} \\ \underline{F_t = F\sin(\theta+\phi)} \end{cases}$$

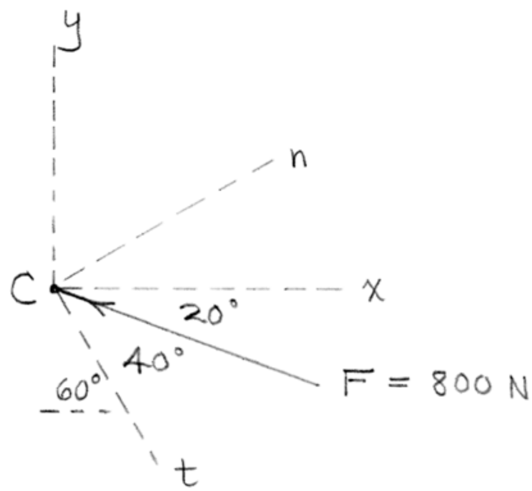
a) $F = 500$ N, $\theta = 60°$, $\phi = 20°$

$$\begin{cases} F_n = -500\cos(60° + 20°) \longrightarrow \underline{F_n = -86.8 \text{ N}} \\ F_t = 500\sin(60° + 20°) \longrightarrow \underline{F_t = 492 \text{ N}} \end{cases}$$

b) $F = 800$ N, $\theta = 45°$, $\phi = 150°$

$$\begin{cases} F_n = -800\cos(45° + 150°) \longrightarrow \underline{F_n = 773 \text{ N}} \\ F_t = 800\sin(45° + 150°) \longrightarrow \underline{F_t = -207 \text{ N}} \end{cases}$$

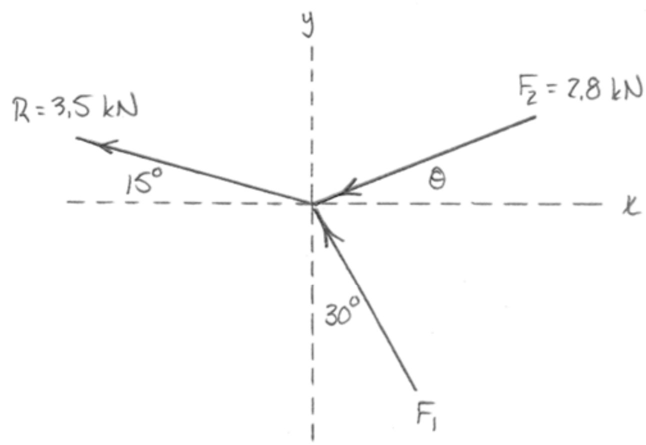
2/16



$$\begin{cases} F_x = -800 \cos 20° = \underline{-752 \text{ N}} \\ F_y = 800 \sin 20° = \underline{274 \text{ N}} \end{cases}$$

$$\begin{cases} F_n = -800 \sin 40° = \underline{-514 \text{ N}} \\ F_t = -800 \cos 40° = \underline{-613 \text{ N}} \end{cases}$$

2/17



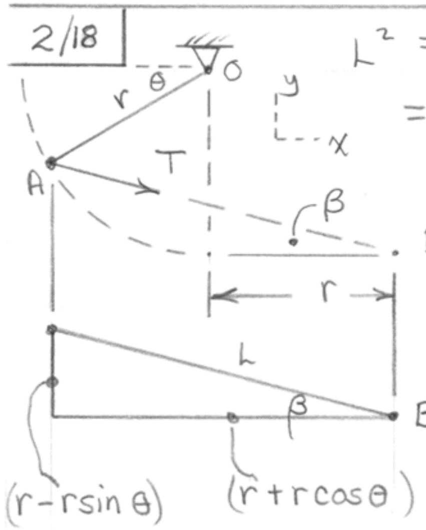
$$R_x = \Sigma F_x: \quad -3.5\cos 15^\circ = -F_1 \sin 30^\circ - 2.8\cos\theta \qquad (1)$$

$$R_y = \Sigma F_y: \quad 3.5 \sin 15^\circ = F_1 \cos 30^\circ - 2.8 \sin\theta \qquad (2)$$

SOLVING (1) AND (2)...

$$\begin{cases} F_1 = 1.165 \text{ kN} \\ \theta = 2.11^\circ \end{cases} \quad \text{OR} \quad \begin{cases} F_1 = 3.78 \text{ kN} \\ \theta = 57.9^\circ \end{cases}$$

2/18



$$L^2 = (r - r\sin\theta)^2 + (r + r\cos\theta)^2$$

$$= r^2 - 2r^2\sin\theta + r^2\sin^2\theta + r^2 + 2r^2\cos\theta + r^2\cos^2\theta$$

$$= r^2(3 + 2\cos\theta - 2\sin\theta)$$

So $\cos\beta = \dfrac{r(1+\cos\theta)}{r\sqrt{3+2\cos\theta-2\sin\theta}}$

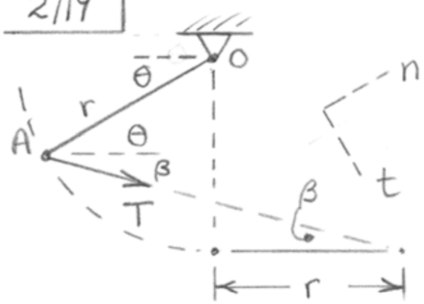
$$= \frac{1+\cos\theta}{\sqrt{3+2\cos\theta-2\sin\theta}}$$

$$\sin\beta = \frac{1-\sin\theta}{\sqrt{3+2\cos\theta-2\sin\theta}}$$

$$T_x = T\cos\beta = \underline{\frac{T(1+\cos\theta)}{\sqrt{3+2\cos\theta-2\sin\theta}}}$$

$$T_y = -T\sin\beta = \underline{\frac{T(\sin\theta-1)}{\sqrt{3+2\cos\theta-2\sin\theta}}}$$

2/19



From solution to previous problem:

$$\beta = \tan^{-1}\left[\frac{1-\sin\theta}{1+\cos\theta}\right]$$

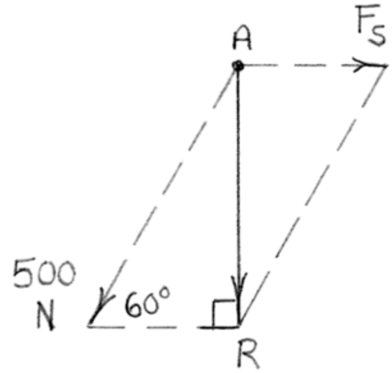
$$\begin{cases} T_n = T\cos(\theta+\beta) \\ T_t = T\sin(\theta+\beta) \end{cases}$$

For $T = 100$ N and $\theta = 35°$:

$\beta = 13.19°$

$$\begin{cases} T_n = 66.7 \text{ N} \\ T_t = 74.5 \text{ N} \end{cases}$$

2/20



$$\cos 60° = \frac{F_S}{500}$$

$$\underline{F_S = 250 \text{ N}}$$

$$\sin 60° = \frac{R}{500}$$

$$\underline{R = 433 \text{ N}}$$

2/21 Using the coordinates of the problem figure:

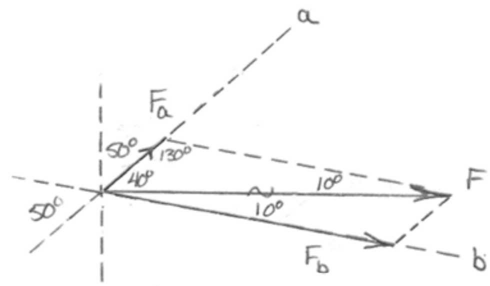
$$R_x = \Sigma F_x = 200 \cos 35° - 150 \sin 30°$$
$$= 88.8 \text{ N}$$

$$R_y = \Sigma F_y = 200 \sin 35° + 150 \cos 30°$$
$$= 245 \text{ N}$$

$$\therefore \underline{R} = 88.8\underline{i} + 245\underline{j} \text{ N}$$

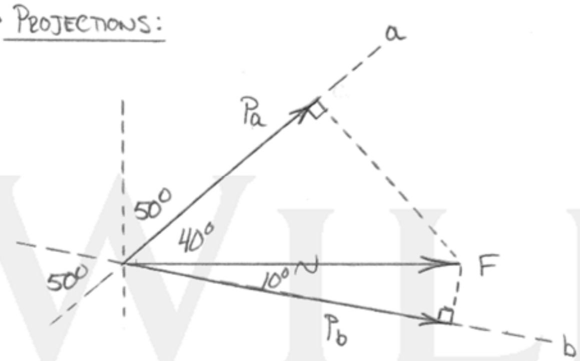
2/22 $F = 2.5$ kN

• COMPONENTS:



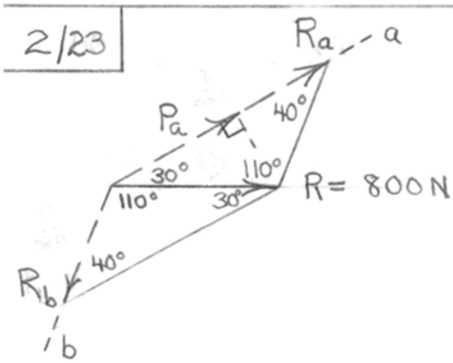
$$\frac{F}{\sin 130°} = \frac{F_a}{\sin 10°} = \frac{F_b}{\sin 40°} \longrightarrow \begin{cases} F_a = 0.567 \text{ kN} \\ F_b = 2.10 \text{ kN} \end{cases}$$

• PROJECTIONS:



$$\begin{cases} P_a = F\cos 40° = 2.5\cos 40° \longrightarrow P_a = 1.915 \text{ kN} \\ P_b = F\cos 10° = 2.5\cos 10° \longrightarrow P_b = 2.46 \text{ kN} \end{cases}$$

2/23



Law of sines:

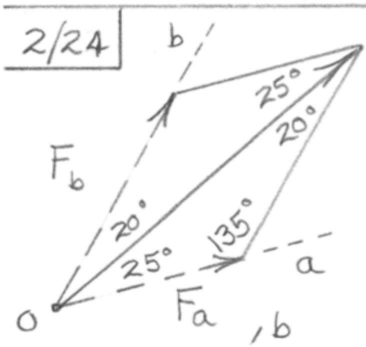
$$\frac{800}{\sin 40^\circ} = \frac{R_a}{\sin 110^\circ} = \frac{R_b}{\sin 30^\circ}$$

$\underline{R_a = 1170\text{ N}}$

$\underline{R_b = 622\text{ N}}$

Projection $P_a = R\cos 30^\circ = 800\cos 30^\circ = \underline{693\text{ N}}$

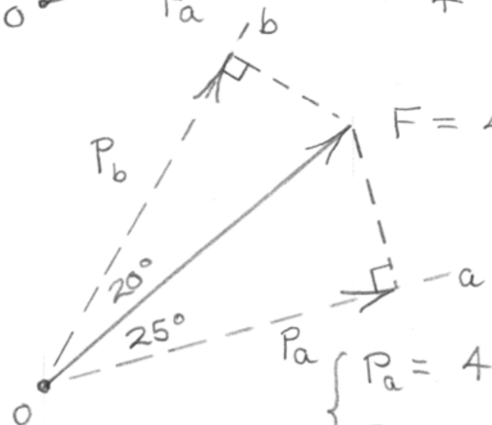
2/24



Components:

$$\frac{\sin 135^\circ}{4} = \frac{\sin 20^\circ}{F_a}, \quad \underline{F_a = 1.935 \text{ kN}}$$

$$\frac{\sin 135^\circ}{4} = \frac{\sin 25^\circ}{F_b}, \quad \underline{F_b = 2.39 \text{ kN}}$$



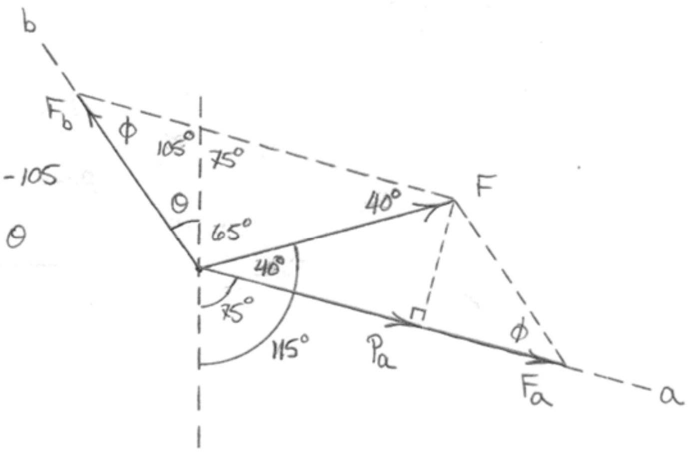
Projections:

$$\begin{cases} P_a = 4 \cos 25^\circ = \underline{3.63 \text{ kN}} \\ P_b = 4 \cos 20^\circ = \underline{3.76 \text{ kN}} \end{cases}$$

2/25

$\phi = 180 - \theta - 105$

$\phi = 75° - \theta$

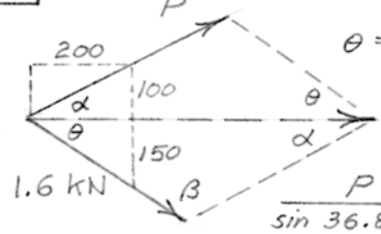


$P_a = F\cos 40° \longrightarrow 325 = F\cos 40° \longrightarrow \underline{F = 424 \text{ N}}$

$\dfrac{F}{\sin\phi} = \dfrac{F_b}{\sin 40°} \longrightarrow \dfrac{424}{\sin(75°-\theta)} = \dfrac{325}{\sin 40°}$

SOLVING THIS YIELDS... $\underline{\theta = 17.95° \text{ OR } -48.0°}$

2/26



$\alpha = \tan^{-1} \frac{100}{200} = 26.57°$

$\theta = \tan^{-1} \frac{150}{200} = 36.87°$

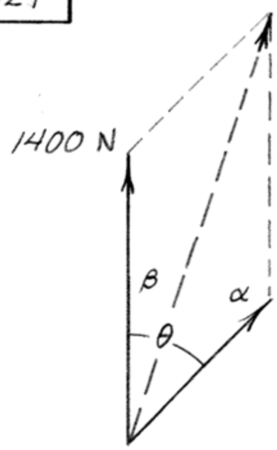
$\beta = 180 - (\alpha + \theta) = 116.57°$

$$\frac{P}{\sin 36.87°} = \frac{1.6}{\sin 26.57}$$

$$P = 1.6 \frac{0.6}{0.4472} = \underline{2.15 \text{ kN}}$$

$$\frac{T}{\sin 116.57°} = \frac{1.6}{\sin 26.57°}, \quad T = 1.6 \frac{0.8944}{0.4472} = \underline{3.20 \text{ kN}}$$

2/27



Law of cosines:

$$2000^2 = 1400^2 + 800^2 - 2(1400)(800)\cos\alpha$$

With $\alpha = 180° - \theta$ and $\cos(180° - \theta) = -\cos\theta$:

$$2000^2 = 1400^2 + 800^2 + 2(1400)(800)\cos\theta$$

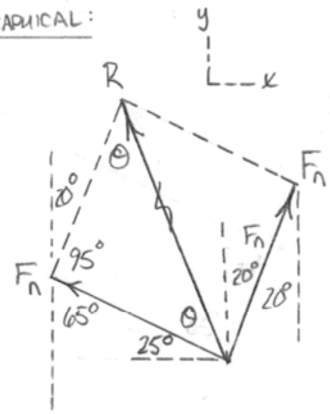
$$\underline{\theta = 51.3°}$$

Law of sines: $\dfrac{800}{\sin\beta} = \dfrac{2000}{\sin(180° - 51.3°)}$

$$\underline{\beta = 18.19°}$$

2/28

• GRAPHICAL:



$$R = \sqrt{F_n^2 + F_n^2 - 2F_n^2 \cos 95^\circ}$$

$$R = \sqrt{2(5500)^2[1 - \cos 95^\circ]}$$

$$\underline{R = 8110 \text{ N}}$$

$$\theta = \frac{180^\circ - 95^\circ}{2} \longrightarrow \theta = 42.5^\circ$$

$$\underline{R = 8110 \text{ N} \text{ @ } 112.5^\circ \text{ CCW FROM } +x \text{ AXIS}}$$

• VECTORS:

$$\underline{R} = (F_n \sin 20^\circ - F_n \sin 65^\circ)\,\underline{i} + (F_n \cos 20^\circ + F_n \cos 65^\circ)\,\underline{j}$$

$$\underline{R} = 5500\left[(\sin 20^\circ - \sin 65^\circ)\,\underline{i} + (\cos 20^\circ + \cos 65^\circ)\,\underline{j}\right]$$

$$\underline{R} = -3100\,\underline{i} + 7490\,\underline{j} \text{ N}$$

$$R = \sqrt{3100^2 + 7490^2} \longrightarrow \underline{R = 8110 \text{ N}}$$

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{-3100}{8110}\right) \longrightarrow \underline{\theta_x = 112.5^\circ \text{ CCW FROM } +x \text{ AXIS}}$$

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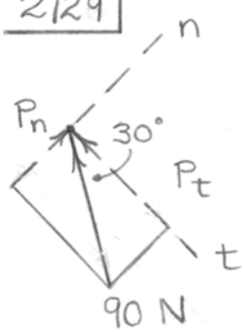
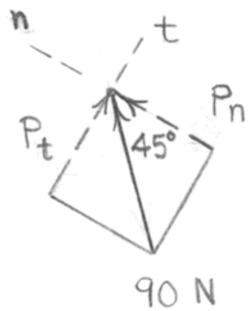


BC

$P_t = -90 \cos 30° = \underline{-77.9 \text{ N}}$

$P_n = 90 \sin 30° = \underline{45.0 \text{ N}}$

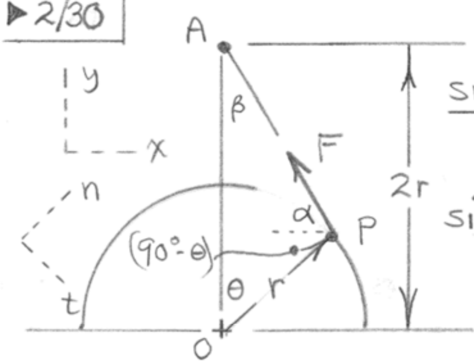


AB

$P_t = 90 \sin 45° = \underline{63.6 \text{ N}}$

$P_n = 90 \cos 45° = \underline{63.6 \text{ N}}$

▶2/30



$$F = k\delta = k(\overline{AP} - r)$$

$$\frac{\sin(\angle OPA)}{2r} = \frac{\sin\theta}{\overline{AP}}$$

$$\sin[\alpha + (90° - \theta)] = \frac{2r\sin\theta}{\overline{AP}}$$

$$\alpha = \theta - 90° + \sin^{-1}\left(\frac{2r\sin\theta}{\overline{AP}}\right)$$

$\overline{AP}$: $\quad \overline{AP}^2 = r^2 + (2r)^2 - 2r(2r)\cos\theta$

$$\overline{AP} = r\sqrt{5 - 4\cos\theta}$$

Then $\quad F_x = -F\cos\alpha, \quad F_y = F\sin\alpha$

For $r = 0.4$ m, $\theta = 40°$, $k = 1400$ N/m :

$$\overline{AP} = 0.4\sqrt{5 - 4\cos 40°} = 0.557 \text{ m}$$

$$\alpha = 40° - 90° + \sin^{-1}\left(\frac{2(0.4)\sin 40°}{0.557}\right) = 62.5°$$

$$F = 1400(0.557 - 0.4) = 219 \text{ N}$$

$$\begin{cases} F_x = -219\cos 62.5° = \underline{-101.2 \text{ N}} \\ F_y = 219\sin 62.5° = \underline{194.4 \text{ N}} \end{cases}$$

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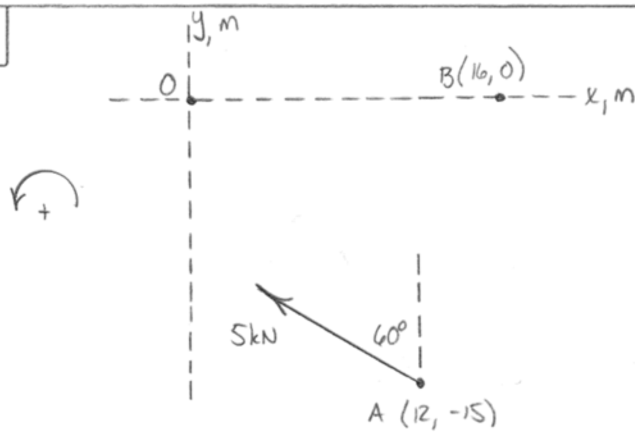


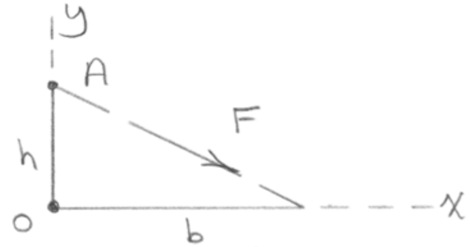
$$M_O = 5\cos 60°(12) - 5\sin 60°(15) = -35.0$$

$\therefore$ $\underline{M_O = 35.0 \text{ kN·m CW}}$

$$M_B = -5\cos 60°(4) - 5\sin 60°(15) = -75.0$$

$\therefore$ $\underline{M_B = 75.0 \text{ kN·m CW}}$

2/32

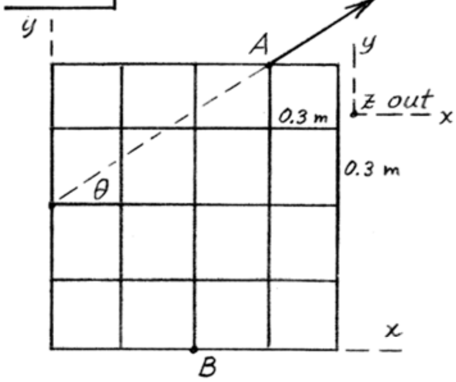


$$\underline{F} = F\left[\frac{b\underline{i} - h\underline{j}}{\sqrt{b^2+h^2}}\right]$$

Acting at A:

$$+\circlearrowright M_O = \frac{Fb}{\sqrt{h^2+b^2}}(h) = \underline{\frac{Fbh}{\sqrt{h^2+b^2}} \quad CW}$$

2/33



$$F_x = F\cos\theta = 150\,\frac{3}{\sqrt{3^2+2^2}} = 450/\sqrt{13}\ \text{N}$$

$$F_y = F\sin\theta = 150\,\frac{2}{\sqrt{3^2+2^2}} = 300/\sqrt{13}\ \text{N}$$

$$\circlearrowright +\ M_B = \frac{450}{\sqrt{13}}(0.6) + \frac{300}{\sqrt{13}}(0.6) = \frac{450}{\sqrt{13}}\ \text{N}\cdot\text{m}$$

or $\underline{M_B = 124.8\ \text{N}\cdot\text{m}\ \ CW}$

With $\theta = \tan^{-1}\frac{2}{3} = 33.7°$, $\underline{F} = 150(\cos 33.7°\,\underline{i} + \sin 33.7°\,\underline{j})$ N

With $\underline{r} = -0.6\underline{i} + 0.6\underline{j}$ m, $\underline{M}_B = \underline{r}\times\underline{F}$ yields

$\underline{M}_B = -124.8\ \underline{k}$ N·m, $\underline{M_B = 124.8\ \text{N}\cdot\text{m}}$, as before.

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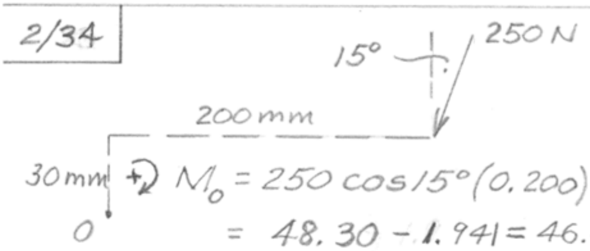
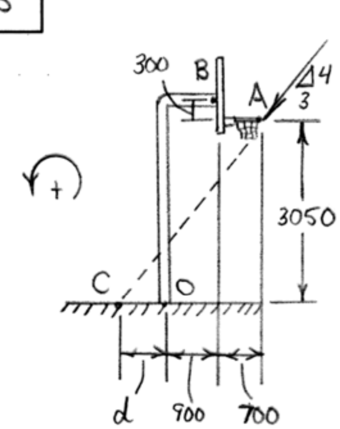


$$\circlearrowleft + \; M_O = 250\cos 15°(0.200) - 250\sin 15°(0.030)$$

$$= 48.30 - 1.941 = \underline{46.4 \text{ N·m}}$$

2/35



$F = 225$ N

DIM. IN mm

$M_O = 3.05\left(\frac{3}{5}F\right) - 1.6\left(\frac{4}{5}F\right) = 123.8 \longrightarrow \underline{M_O = 123.8 \text{ N}\cdot\text{m CCW}}$

$M_B = -0.3\left(\frac{3}{5}F\right) - 0.7\left(\frac{4}{5}F\right) = -166.5 \longrightarrow \underline{M_B = 166.5 \text{ N}\cdot\text{m CW}}$

$M_C = 0 = 3.05\left(\frac{3}{5}F\right) - (1.6+d)\left(\frac{4}{5}F\right) \longrightarrow d = 0.688 \text{ m}$

SO... $\underline{d = 688 \text{ mm LEFT OF O}}$

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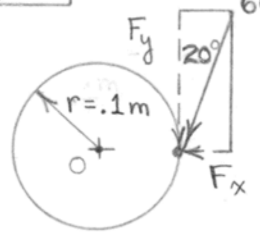
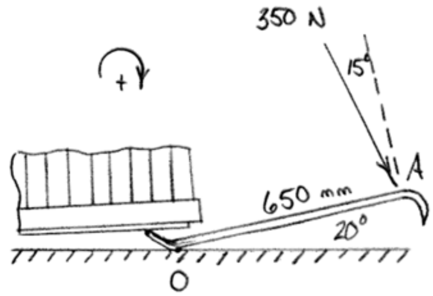


$+\circlearrowleft M_O = r F_y$

$= (0.1)(60 \cos 20°)$

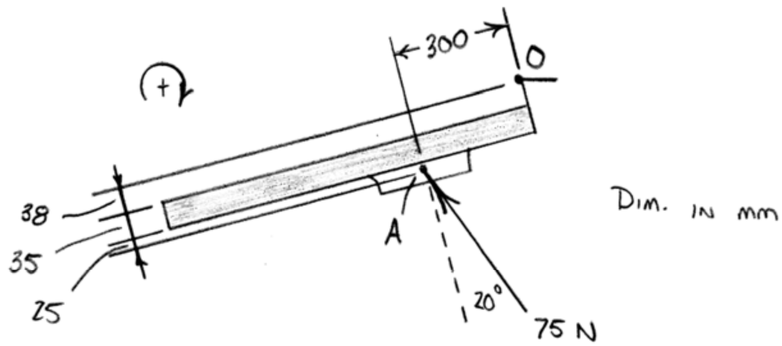
$= \underline{5.64 \text{ N}\cdot\text{m}}$

2/37



$$M_O = 0.650\,(350 \cos 15^\circ) \rightarrow \underline{M_O = 220 \text{ N·m CW}}$$

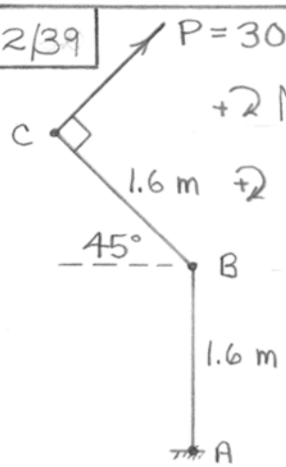
2/38



$$M_O = 0.300(75\cos 20°) + \left(\frac{25+35+38}{1000}\right)(75\sin 20°)$$

$$\underline{M_O = 23.7 \text{ N}\cdot\text{m CW}}$$

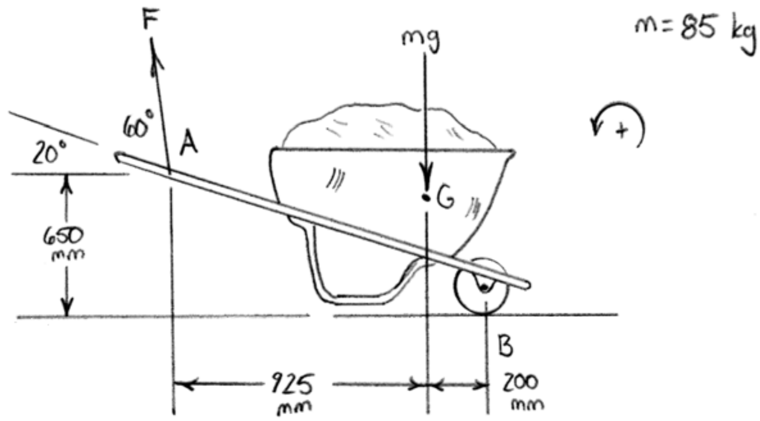
2/39



$$+\circlearrowright M_B = 30(1.6) = \underline{48 \text{ N}\cdot\text{m}}$$

$$+\circlearrowright M_A = 30\cos 45°(1.6 + 1.6\sin 45°) + 30\sin 45°(1.6\cos 45°) = \underline{81.9 \text{ N}\cdot\text{m}}$$

2/40



$$M_B = 0.2(85)(9.81) - F \sin 80^\circ\left(\frac{925+200}{1000}\right) + F \cos 80^\circ\left(\frac{650}{1000}\right) = 0$$

$$\underline{F = 167.6 \text{ N}}$$

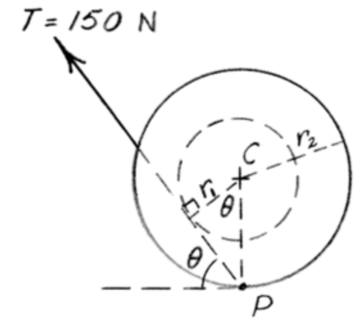
2/41

$M_C = Tr_1 = 150(0.125)$

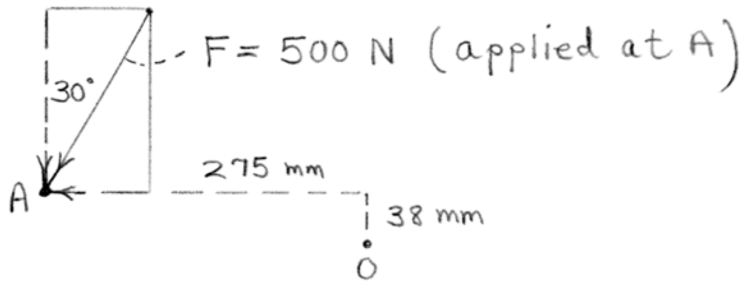
$= \underline{18.75 \text{ N}\cdot\text{m CW}}$

$\cos\theta = \dfrac{r_1}{r_2} = \dfrac{125}{200}$

$\underline{\theta = 51.3°}$



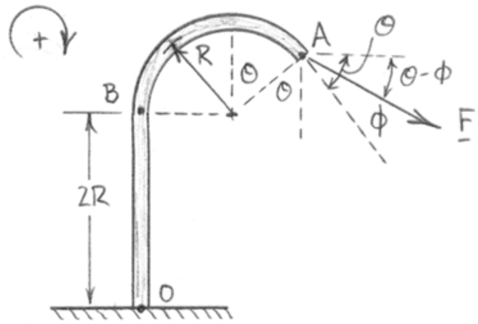
2/42



$$\circlearrowleft_{+} \; M_O = 500 \cos 30° (0.275) + 500 \sin 30° (0.038)$$

$$= \underline{128.6 \text{ N}\cdot\text{m} \quad \text{CCW}}$$

2/43



$M_B = F\sin(\theta-\phi)(R+R\sin\theta) + F\cos(\theta-\phi)(R\cos\theta)$

$M_B = FR[\cos\phi + \sin(\theta-\phi)]$

$M_O = F\sin(\theta-\phi)(R+R\sin\theta) + F\cos(\theta-\phi)(2R+R\cos\theta)$

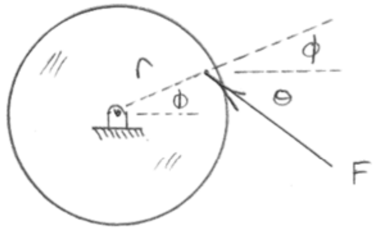
$M_O = FR[2\cos(\theta-\phi) + \cos\phi + \sin(\theta-\phi)]$

IF $F = 750$ N, $R = 2.4$ m, $\theta = 30°$, AND $\phi = 15°$ ...

$M_B = 2200$ N·m CW

$M_O = 5680$ N·m CW

2/44 $r = 0.4\text{ m}$



$M_O = Fr\sin(\theta + \phi)$

$\begin{cases} + \text{ is CCW} \\ - \text{ is CW} \end{cases}$

a) $F = 500\text{ N}$, $\theta = 60°$, $\phi = 20°$:

$$M_O = 500(0.4)\sin(60° + 20°) = 197.0$$

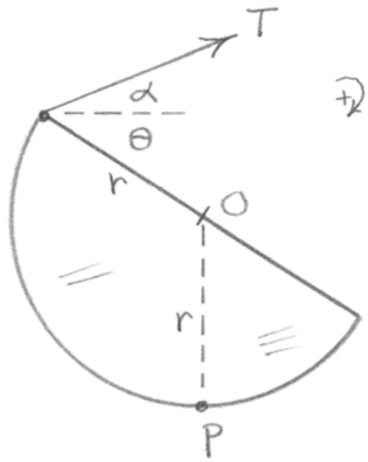
so... $M_O = 197.0\text{ N}\cdot\text{m CCW}$

b) $F = 800\text{ N}$, $\theta = 45°$, $\phi = 150°$:

$$M_O = 800(0.4)\sin(45° + 150°) = -82.8$$

so... $M_O = 82.8\text{ N}\cdot\text{m CW}$

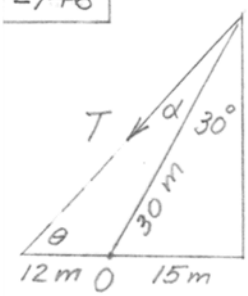
2/45



$$\overset{+}{\curvearrowright} M_O = T\sin(\alpha+\theta)(r)$$
$$= \underline{Tr\sin(\alpha+\theta) \quad CW}$$

$$\overset{+}{\curvearrowright} M_P = T\cos\alpha\,(r + r\sin\theta) + T\sin\alpha\,(r\cos\theta)$$
$$= \underline{Tr\left[\cos\alpha + \sin(\alpha+\theta)\right] \quad CW}$$

2/46



$$\theta = \tan^{-1}\frac{30(0.866)}{12+15} = 43.90°$$

$$\alpha = 90° - (30° + 43.90°) = 16.10°$$

$$M_O = 72 \text{ kN·m}$$

$$= T \sin 16.10° \, (30) = 8.32T$$

$$T = \frac{72}{8.32} = \underline{8.65 \text{ kN}}$$

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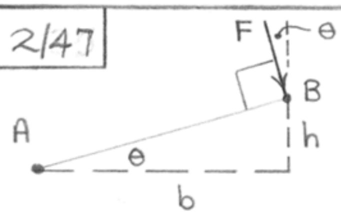
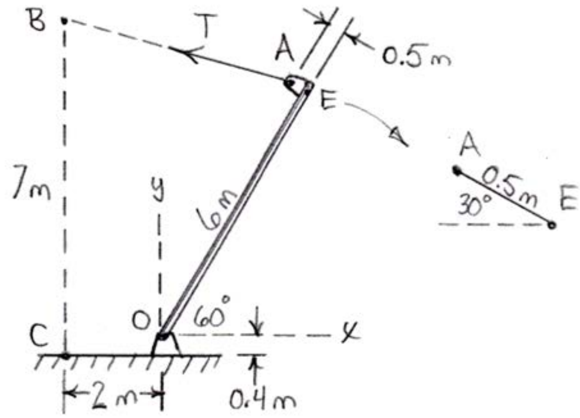


M_A is maximum when $\mathbf{F}$ is perpendicular to AB.

Thus $\underline{\theta = \tan^{-1}(h/b)}$

2/48 $T = 6.75$ kN



$$\underline{r}_{OA} = (6\cos 60° - 0.5\cos 30°)\underline{i} + (6\sin 60° + 0.5\sin 30°)\underline{j}$$

$$= 2.57\underline{i} + 5.45\underline{j} \text{ m}$$

$$\underline{r}_{OB} = -2\underline{i} + (7 - 0.4)\underline{j} = -2\underline{i} + 6.6\underline{j} \text{ m}$$

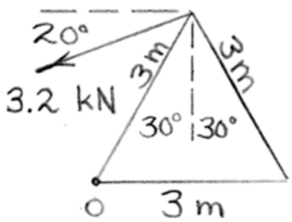
$$\underline{n}_{AB} = \frac{\underline{r}_{OB} - \underline{r}_{OA}}{|\underline{r}_{OB} - \underline{r}_{OA}|} \longrightarrow \underline{n}_{AB} = -0.970\underline{i} + 0.245\underline{j}$$

$$\underline{T} = T\underline{n}_{AB} = 6.75(-0.970\underline{i} + 0.245\underline{j}) = -6.54\underline{i} + 1.653\underline{j} \text{ kN}$$

$$\underline{M}_O = \underline{r}_{OA} \times \underline{T} = (2.57\underline{i} + 5.45\underline{j}) \times (-6.54\underline{i} + 1.653\underline{j})$$

$$\underline{\underline{M}_O = 39.9\,\underline{k} \text{ kN}\cdot\text{m}}$$

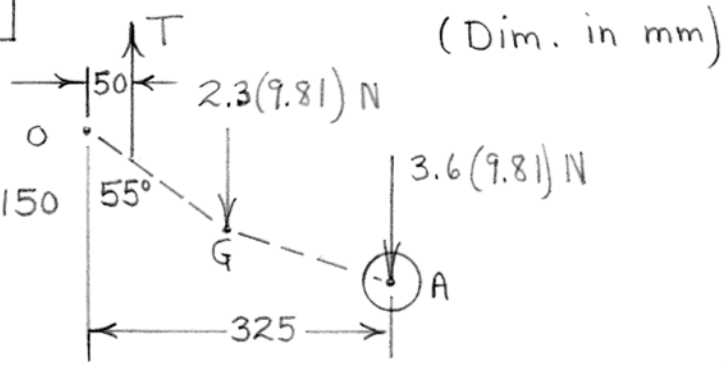
2/49



$$\circlearrowleft + \quad M_O = 3.2 \cos 20° \, (3 \cos 30°) - 3.2 \sin 20° \, (3 \sin 30°)$$

$$= \underline{6.17 \text{ kN}\cdot\text{m} \quad \text{CCW}}$$

2/50

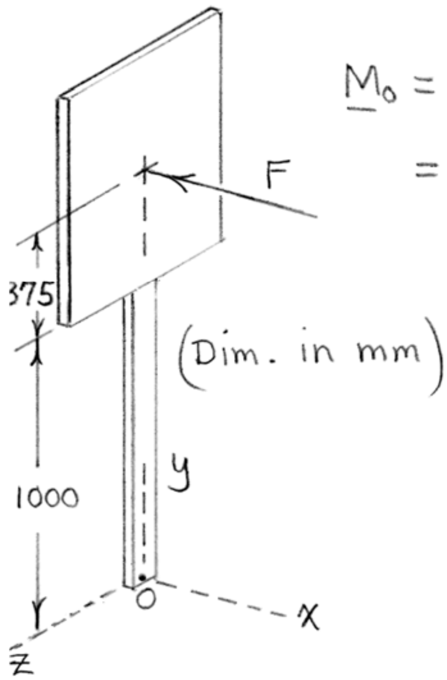


The combined moment about O of the weights of the 2.3-kg and 3.6-kg masses is

$$\circlearrowright + \quad M_O = 2.3(9.81)(0.150 \sin 55°) + 3.6(9.81)(0.325)$$
$$= \underline{14.25 \text{ N·m (CW)}}$$

$$\circlearrowright + \quad \Sigma M_O = 0: \quad -T(0.050) + 14.25 = 0$$
$$\underline{T = 285 \text{ N}}$$

2/51

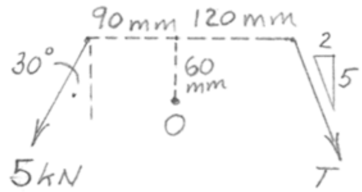


$$F = pA = 175(0.750)(0.6) = 78.8\ \text{N}$$

$$\underline{M}_O = \underline{r} \times \underline{F} = 1.375\underline{j} \times (-78.8\underline{i}) = \underline{108.3\underline{k}\ \text{N}\cdot\text{m}}$$

2/52

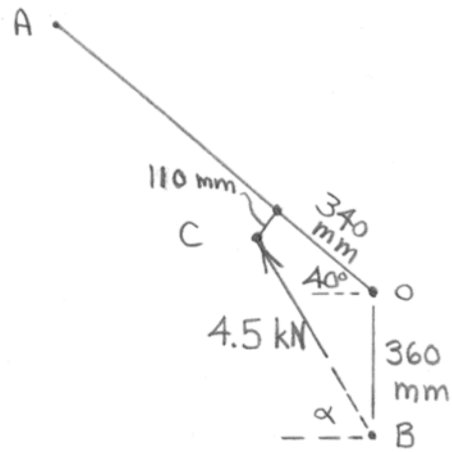
$$M_O = 5[(\cos 30^\circ)90 + (\sin 30^\circ)60] - T\left[\frac{5}{\sqrt{29}}(120) + \frac{2}{\sqrt{29}}(60)\right] = 0$$



$$\sqrt{2^2+5^2} = \sqrt{29}$$

$$539.7 - 133.7T = 0, \quad \underline{T = 4.04 \text{ kN}}$$

2/53



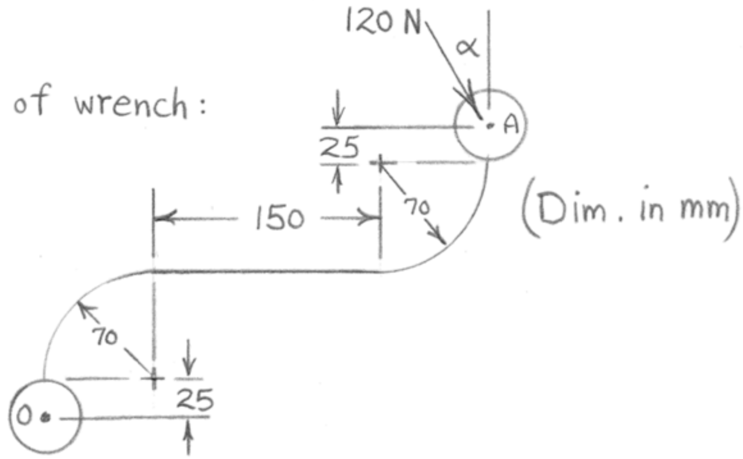
$$\alpha = \tan^{-1}\left[\frac{360 + 340 \sin 40° - 110 \sin 50°}{340 \cos 40° + 110 \cos 50°}\right]$$

$$= 56.2°$$

$$\circlearrowright M_O = 4.5\,(0.360 \cos 56.2°) = \underline{0.902 \text{ kN·m CW}}$$

2/54

Elements of wrench:



$\alpha = 30°$:

$$+\circlearrowright M_O = 120\cos 30°[70 + 150 + 70]$$
$$+ 120 \sin 30°[25 + 70 + 70 + 25] = 41\,500 \text{ N·mm}$$

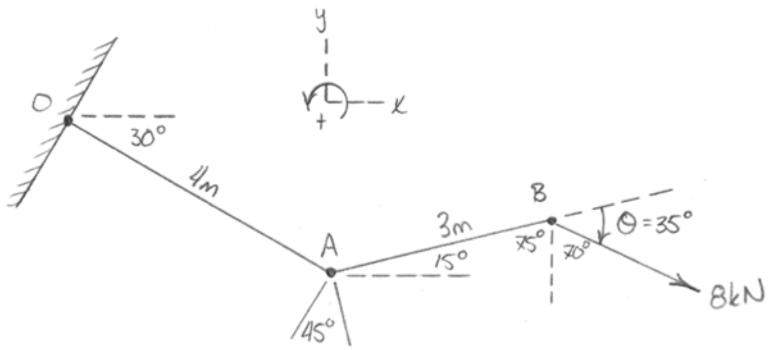
or $\underline{M_O = 41.5 \text{ N·m CW}}$

For maximum M_O:

$$\alpha = \tan^{-1}\left[\frac{25+70+25+70}{70+150+70}\right] = \underline{33.2°}$$

$$(M_O)_{max} = 120\sqrt{(25+70+25+70)^2 + (70+150+70)^2}$$
$$= 41\,600 \text{ N·mm or } \underline{41.6 \text{ N·m CW}}$$

2/55



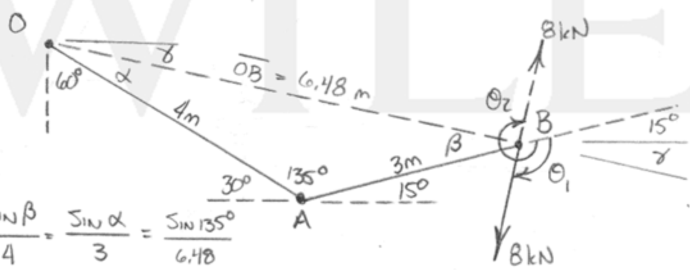
$$\mathbf{M}_O = \left[(4\cos 30^\circ + 3\cos 15^\circ)\mathbf{i} + (-4\sin 30^\circ + 3\sin 15^\circ)\mathbf{j}\right] \times \left(8\sin 70^\circ \mathbf{i} - 8\cos 70^\circ \mathbf{j}\right)$$

$$\therefore \quad \underline{\mathbf{M}_O = -8.21\,\mathbf{k} \text{ kN}\cdot\text{m}}$$

$$\mathbf{M}_A = -8\sin 35^\circ(3)\,\mathbf{k} \longrightarrow \underline{\mathbf{M}_A = -13.77\,\mathbf{k} \text{ kN}\cdot\text{m}}$$

• Angle to find a maximum moment at O: The force must be perpendicular to line $\overline{OB}$.

$$\overline{OB} = \sqrt{4^2 + 3^2 - 2(4)(3)\cos 135^\circ} \longrightarrow \overline{OB} = 6.48\text{m}$$



$$\frac{\sin\beta}{4} = \frac{\sin\alpha}{3} = \frac{\sin 135^\circ}{6.48}$$

$\beta = 25.9^\circ$

$\alpha = 19.11^\circ$

$$\gamma = 90^\circ - 60^\circ - 19.11^\circ = 10.89^\circ$$

$$\theta_1 = 90^\circ + 15^\circ + 10.89^\circ \longrightarrow \underline{\theta_1 = 115.9^\circ}$$

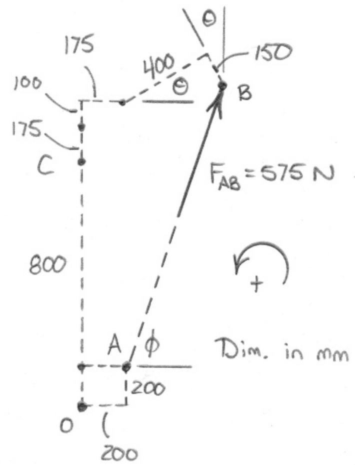
$$\theta_2 = \theta_1 + 180^\circ \longrightarrow \underline{\theta_2 = 296^\circ}$$

$$M_{MAX} = \overline{OB}\,F = 6.48(8) \longrightarrow \underline{M_{MAX} = 51.8 \text{ kN}\cdot\text{m}}$$

2/56

$$\phi = \tan^{-1}\left(\frac{800 + 175 + 100 + 400\sin\theta - 150\cos\theta}{175 + 400\cos\theta + 150\sin\theta - 200}\right) = 70.9^\circ$$

$\theta = 30^\circ$



$$M_O = 575\sin\phi\,(0.200) - 575\cos\phi\,(0.200) \longrightarrow \underline{M_O = 71.1 \text{ N·m CCW}}$$

$$M_C = 575\sin\phi\,(0.200) + 575\cos\phi\,(0.800) \longrightarrow \underline{M_C = 259 \text{ N·m CCW}}$$

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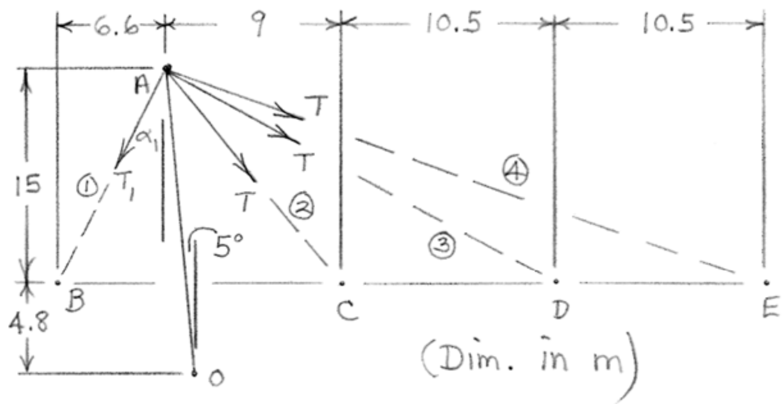


$\alpha_1 = \tan^{-1}\frac{6.6}{15} = 23.7°$; Similarly, $\alpha_2 = 31.0°$,

$\alpha_3 = 52.4°$, $\alpha_4 = 63.4°$ (all relative to vertical)

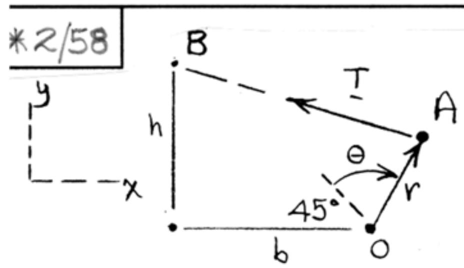
$\circlearrowleft + \Sigma M_O = 0: \left[T_1 \sin(\alpha_1 + 5°) - T\sin(\alpha_2 - 5°) - T\sin(\alpha_3 - 5°) - T\sin(\alpha_4 - 5°)\right] \frac{15 + 4.8}{\cos 5°} = 0$

$T_1 = 4.21T$

$+\downarrow \Sigma F = P = T_1 \cos(\alpha_1 + 5°) + T\left[\cos(\alpha_2 - 5°) + \cos(\alpha_3 - 5°) + \cos(\alpha_4 - 5°)\right]$

or $\underline{P = 5.79T}$ (with $T_1 = 4.21T$)

*2/58



$$\underline{M}_O = \underline{r}_{OA} \times \underline{T}$$

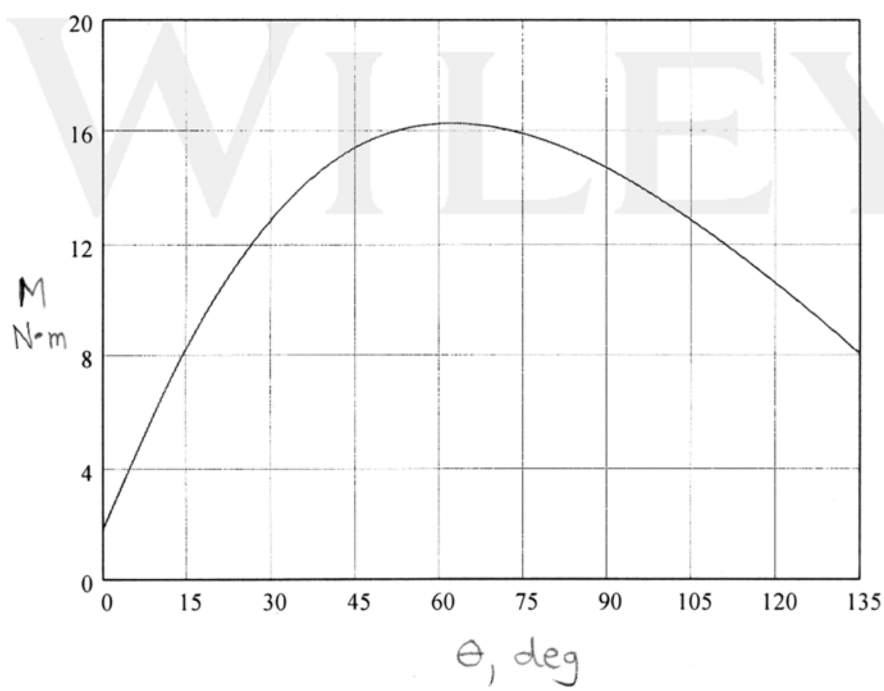
$$\underline{r}_{OA} = r\left[-\cos(\theta+45^\circ)\underline{i} + \sin(\theta+45^\circ)\underline{j}\right]$$

$\underline{T} = mg\,\underline{n}_{AB}$, where

$$\underline{n}_{AB} = \frac{[-b + r\cos(\theta+45^\circ)]\underline{i} + [h - r\sin(\theta+45^\circ)]\underline{j}}{\{[-b+r\cos(\theta+45^\circ)]^2 + [h-\sin(\theta+45^\circ)]^2\}^{1/2}}$$

With $r = 0.325$ m, $h = 0.525$ m, $b = 0.600$ m, $mg = 50$ N, vary θ and take the z-comp. of $\underline{M}_O$ to obtain the following plot.

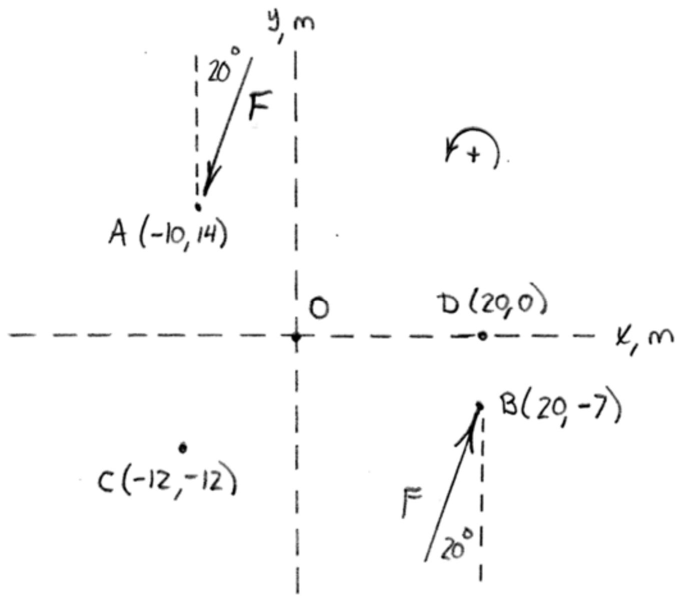
Note that $\underline{M_{max} = 16.25 \text{ N·m at } \theta = 62.1^\circ}$.



$\boxed{2/59}$ $\overset{+}{\curvearrowright}$ $M = Fd = 400(0.035)$

$= \underline{14 \text{ N}\cdot\text{m CW}}$

2/60 $F = 300$ N



a) $M_O = 300 \cos 20° (20 + 10) + 300 \sin 20° (14 + 7)$

$\underline{M_O = 10\ 610 \text{ N·m CCW}}$

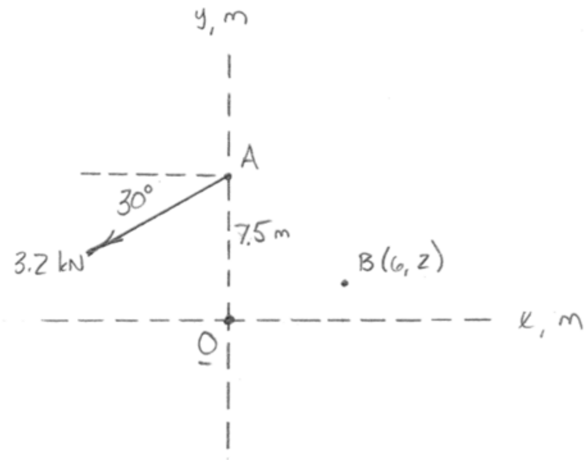
b) By inspection... $\underline{M_C = 10\ 610 \text{ N·m CCW}}$

c) By inspection... $\underline{M_D = 10\ 610 \text{ N·m CCW}}$

2/61 $\underline{\mathbf{R} = 6\mathbf{j} \text{ kN}}$ @ $x = \frac{400}{6000} = 0.0667$ m

or $\underline{x = 66.7 \text{ mm}}$

2/62

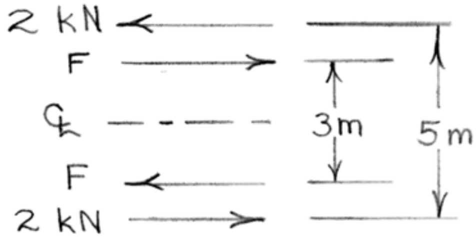


$$\underline{F} = -3.2\cos 30^\circ\,\underline{i} - 3.2\sin 30^\circ\,\underline{j} \longrightarrow \underline{F} = -2.77\,\underline{i} - 1.6\,\underline{j}\ \text{kN}$$

$$\underline{M}_O = 3.2\cos 30^\circ(7.5)\,\underline{k} \longrightarrow \underline{M}_O = 20.8\,\underline{k}\ \text{kN}\cdot\text{m}$$

$$\underline{M}_B = \left[3.2\cos 30^\circ(7.5-2) + 3.2\sin 30^\circ(6)\right]\underline{k} \longrightarrow \underline{M}_B = 24.8\,\underline{k}\ \text{kN}\cdot\text{m}$$

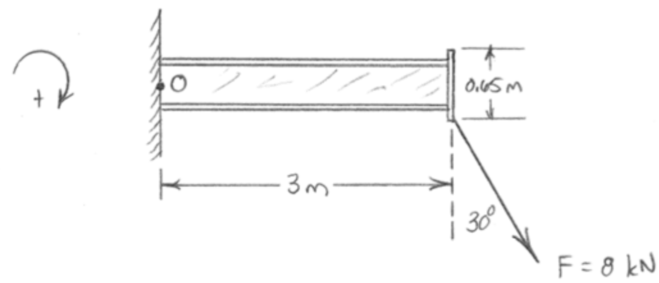
2/63



$\circlearrowleft + \quad \Sigma M = 2(5) - F(3) = 0$

$F = 3.33 \text{ kN}$

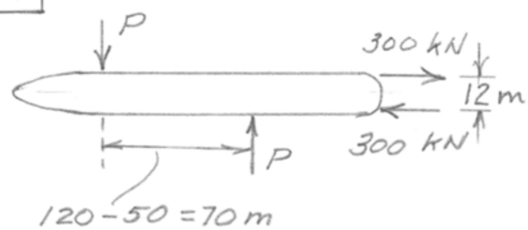
2/64



F = 8 kN @ 60° CW BELOW HORIZONTAL

$$M_O = 8\cos 30^\circ (3) - 8\sin 30^\circ \left(\frac{0.65}{2}\right) \longrightarrow M_O = 19.48 \text{ kN}\cdot\text{m CW}$$

2/65

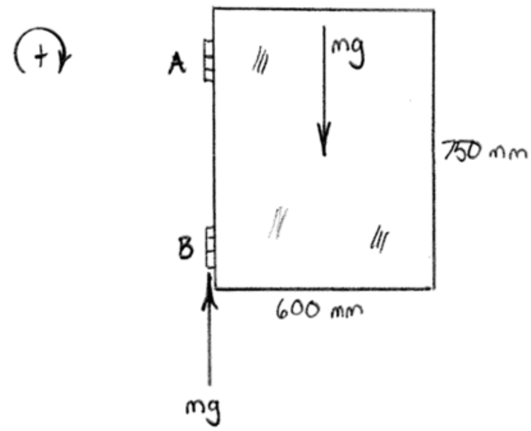


$70P = 300(12)$

$\underline{P = 51.4 \text{ kN}}$

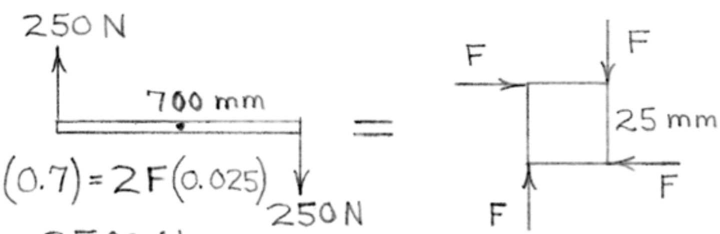
2/66

$m = 5$ kg



$$M = \frac{600/2}{1000}(5)(9.81) \longrightarrow \underline{M = 14.72 \text{ N·m CW}}$$

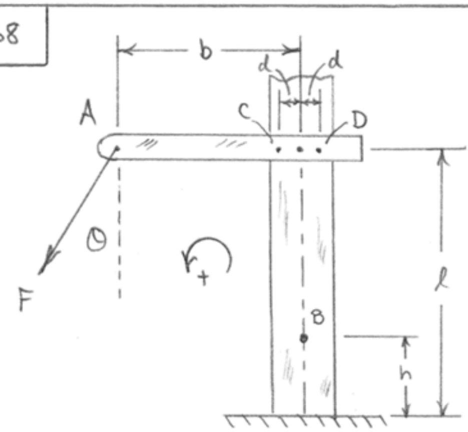
2/67



$$M = 250(0.7) = 2F(0.025)$$

$$\underline{F = 3500 \text{ N}}$$

2/68



$F = 425$ N
$\theta = 30°$
$b = 1.9$ m
$d = 0.2$ m
$h = 0.8$ m
$\ell = 2.75$ m

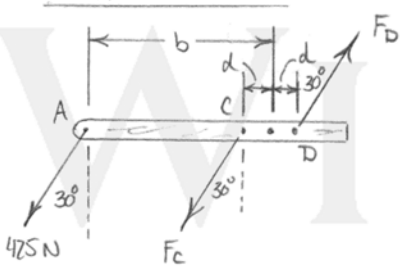
a) Force - Couple At B:

F = 425 N @ 120° CW Below Horizontal

$$M_B = F\cos\theta\, b + F\sin\theta\,(\ell - h) = 425\cos30°(1.9) + 425\sin30°(2.75-0.8)$$

$$\therefore\ M_B = 1114 \text{ N·m CCW}$$

b) Forces At C and D:



$\Sigma F_{\nearrow}$: $F_C - F_D = 425$

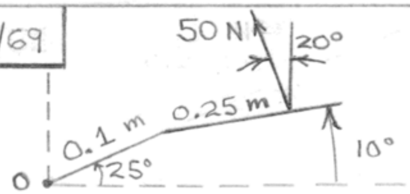
$\Sigma M_A = 0$: $F_d\cos30°(b+d) - F_C\cos30°(b-d) = 0$

Solving...

F_C = 2230 N @ 120° CW Below Horizontal

F_D = 1806 N @ 60° CCW Above Horizontal

2/69



Use principle of moments.

$$\circlearrowleft + \Sigma M_O = 50 \cos 20° [0.1 \cos 25° + 0.25 \cos 10°]$$

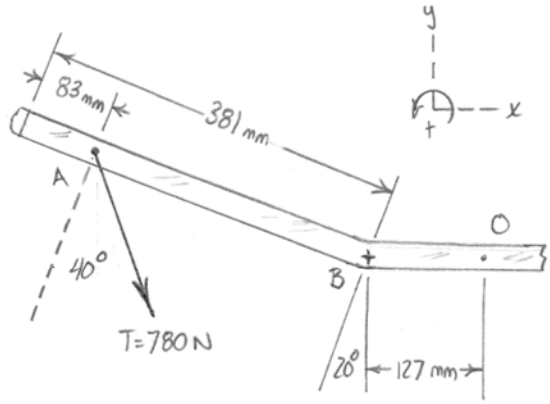
$$+ 50 \sin 20° [0.1 \sin 25° + 0.25 \sin 10°]$$

$$= 17.29 \text{ N}\cdot\text{m}$$

Force - Couple System at O:

$$\begin{cases} R = 50 \text{ N} \ \angle 110° \\ M_O = 17.29 \text{ N}\cdot\text{m} \ \circlearrowleft \end{cases}$$

2/70



$\underline{T} = 780 \sin 20^\circ \underline{i} - 780 \cos 20^\circ \underline{j} \longrightarrow \underline{\underline{T} = 267\underline{i} - 733\underline{j} \text{ N}}$

$\underline{M}_B = 780 \cos 40^\circ \left(\frac{381 - 83}{1000}\right) \underline{k} \longrightarrow \underline{\underline{M}_B = 178.1\, \underline{k} \text{ N}\cdot\text{m}}$

$\underline{M}_O = \underline{M}_B + T\cos 20^\circ (\overline{OB}) = \left(178.1 + 780 \cos 20^\circ \left(\frac{127}{1000}\right)\right) \underline{k}$

$\therefore \quad \underline{\underline{M}_O = 271\, \hat{k} \text{ N}\cdot\text{m}}$

2/71

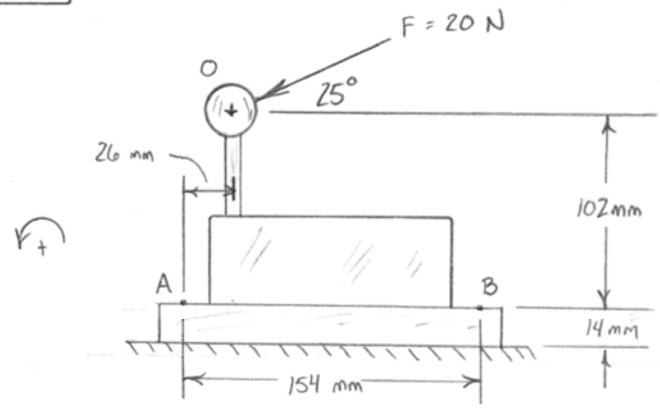
$$\underline{F} = 250(\sin 10° \underline{i} + \cos 10° \underline{j})$$

$$= \underline{43.4\underline{i} + 246\underline{j} \text{ N}}$$

$$\curvearrowright + \; M_O = 250\left[\cos 10°(0.235) + \sin 10°(0.050)\right]$$

$$= \underline{60.0 \text{ N·m CW}}$$

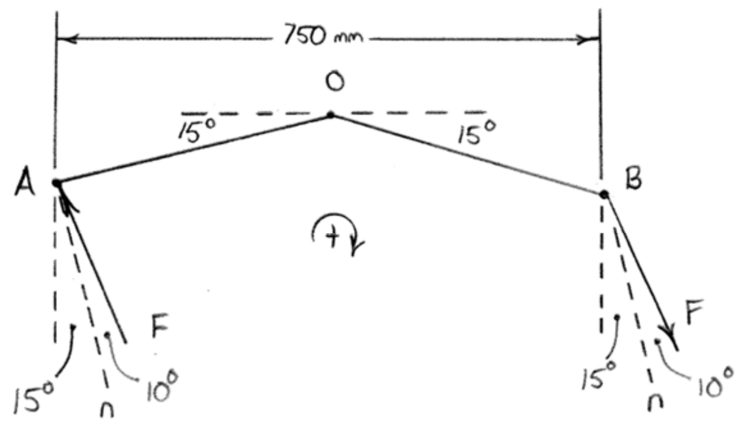
2/72



F = 20 N @ 25° CCW ABOVE HORIZONTAL

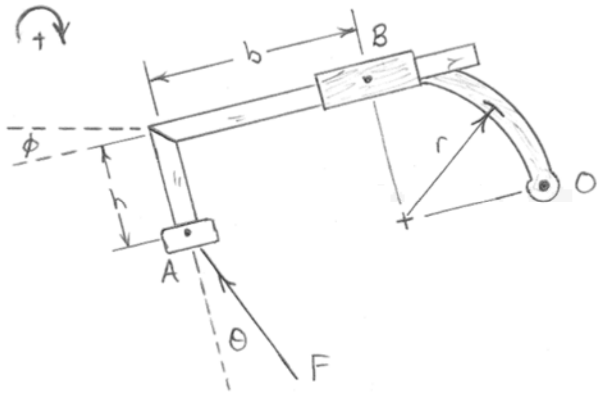
$$M_B = 20\cos 25°\left(\frac{102}{1000}\right) + 20\sin 25°\left(\frac{154-26}{1000}\right) \longrightarrow \underline{M_B = 2.93 \text{ N}\cdot\text{m CCW}}$$

2/73 $F = 150$ N



$$M_O = 0.750\,(150\cos 25^\circ) \longrightarrow \underline{M_O = 102.0 \text{ N}\cdot\text{m CW}}$$

2/74 $b = 450\text{ mm}$, $h = 215\text{ mm}$, $r = 325\text{ mm}$, $F = 520\text{ N}$, $\theta = 15^\circ$, $\phi = 10^\circ$



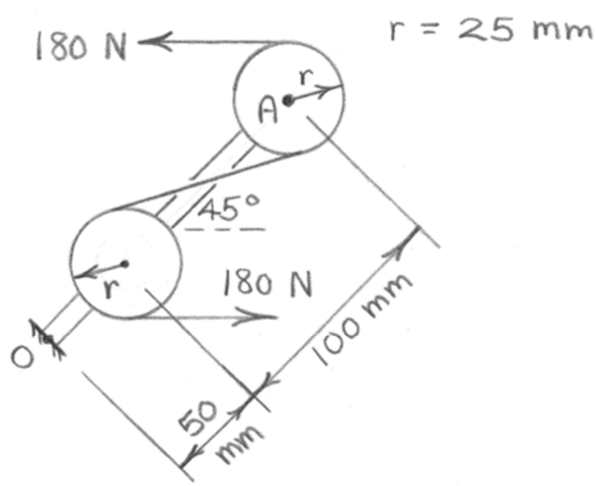
F = 520 N @ 115° CCW ABOVE HORIZONTAL

$$M_O = F\cos\theta(b+r) - F\sin\theta(r-h)$$

$$= 520\cos 15^\circ\left(\frac{450+325}{1000}\right) - 520\sin 15^\circ\left(\frac{325-215}{1000}\right)$$

$\therefore$ $M_O = 374\text{ N}\cdot\text{m CW}$

2/75



The system at O is a couple.

$$\curvearrowleft + \; M = Fd = 180(100 \sin 45° + 25 + 25)$$

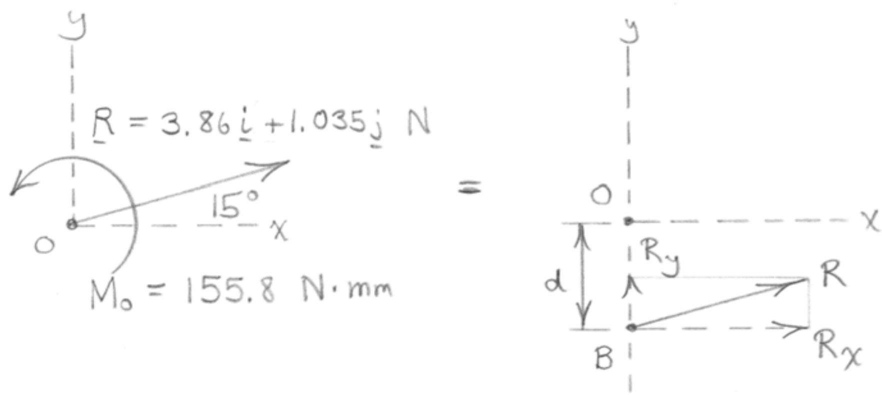
$$= 21\,700 \text{ N}\cdot\text{mm or } \underline{21.7 \text{ N}\cdot\text{m CCW}}$$

2/76 At O:

$\underline{R} = 4(\cos 15° \underline{i} + \sin 15° \underline{j}) = 3.86\underline{i} + 1.035\underline{j}$ N

$\curvearrowleft + M_O = 300 - 4\cos 15°(40) + 4\sin 15°(10)$

$= 155.8$ N·mm CCW

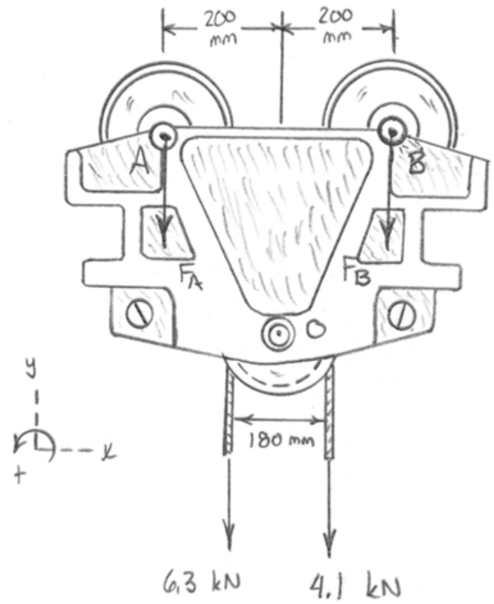


Condition: $R_x d = M_O$

$3.86 d = 155.8$, $d = 40.3$ mm

So $\underline{y = -40.3 \text{ mm}}$

2/77



F_A & F_B ARE DOWN AT A & B.

$$\Sigma F_y: \quad F_A + F_B = 6.3 + 4.1$$

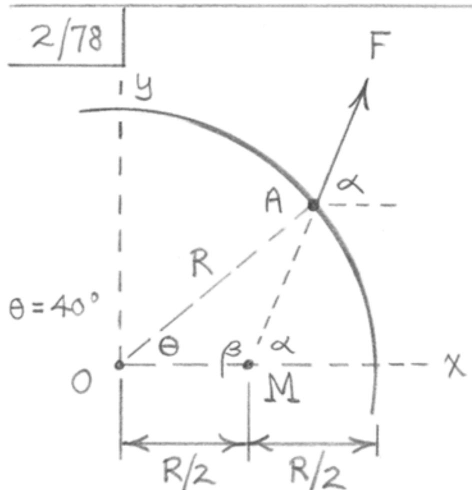
$$\Sigma M_O: \quad 200 F_A - 200 F_B = 90(6.3) - 90(4.1)$$

SOLVING...

$\underline{F_A = 5.70 \text{ kN}}$

$\underline{F_B = 4.70 \text{ kN}}$ (BOTH DOWN AS SHOWN)

2/78



$$\overline{AM}^2 = R^2 + (R/2)^2 - 2(R)\left(\frac{R}{2}\right)\cos 40°$$

$$= 0.484R^2, \quad \overline{AM} = 0.696R$$

$$\frac{\sin\beta}{R} = \frac{\sin 40°}{0.696R}, \quad \beta = 112.5°$$

$$\alpha = 180° - \beta = 180° - 112.5° = 67.5°$$

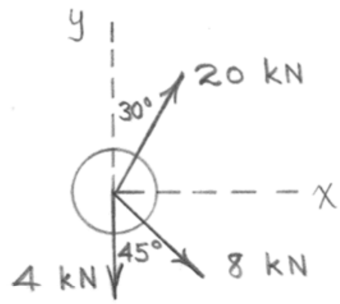
$$M_O = (F\sin\alpha)\frac{R}{2} = (F\sin 67.5°)\frac{R}{2}$$

$$= 0.462\,FR$$

So the force-couple system for $\theta = 40°$ is

$$\begin{cases} F \;\angle 67.5° \\ M_O = 0.462\,FR \quad \text{CCW} \end{cases}$$

2/79



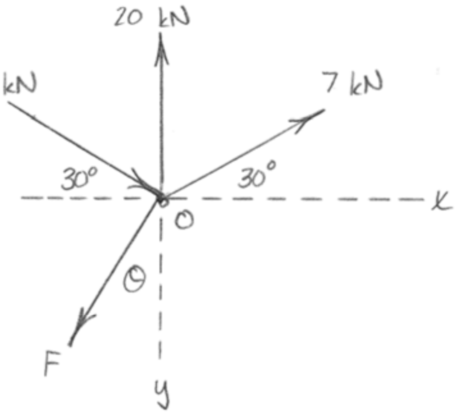
$$R_x = \Sigma F_x = 20 \sin 30° + 8 \sin 45° = 15.66 \text{ kN}$$

$$R_y = \Sigma F_y = 20 \cos 30° - 8 \cos 45° - 4 = 7.66 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{17.43 \text{ kN}}$$

$$\theta_x = \tan^{-1}(R_y / R_x) = \underline{26.1°}$$

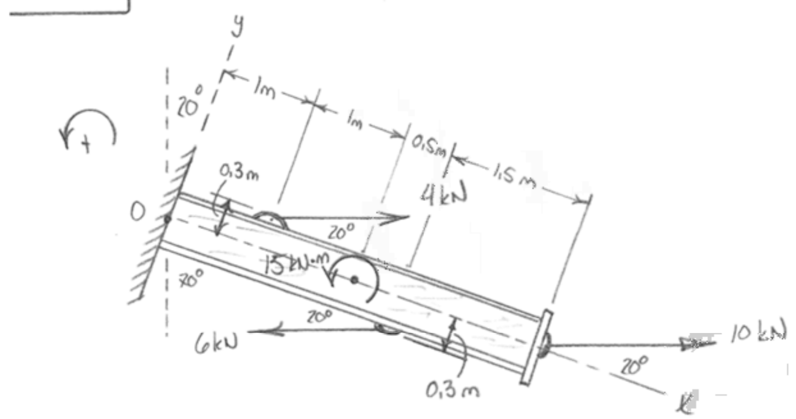
2/80 $R = 9$ kN, RIGHTWARD



$$\begin{cases} R_x = 9 = 11\cos 30^\circ + 7\cos 30^\circ - F\sin\theta \\ R_y = 0 = 11\sin 30^\circ - 7\sin 30^\circ + F\cos\theta - 20 \end{cases}$$

SOLVING... $F = 19.17$ kN AND $\theta = 20.1^\circ$

2/81



$$\begin{cases} R = 10 + 4 - 6 \longrightarrow R = 8\text{ kN} \\ \underline{R} = 8\cos 20°\,\underline{i} + 8\sin 20°\,\underline{j} \longrightarrow \underline{\underline{R} = 7.52\,\underline{i} + 2.74\,\underline{j}\text{ kN}} \end{cases}$$

$$M_O = 15 + 4\sin 20°(1) - 6\sin 20°(2) + 10\sin 20°(4) - 4\cos 20°(0.3) - 6\cos 20°(0.3)$$

$$\therefore\ \underline{M_O = 22.1\text{ kN·m CCW}}$$

• Line-of-Action:

$$\underline{r} \times \underline{R} = \underline{M}_O \longrightarrow (x\,\underline{i} + y\,\underline{j}) \times (7.52\,\underline{i} + 2.74\,\underline{j}) = 22.1\,\underline{k}$$

$$\underline{k}:\quad 2.74x - 7.52y = 22.1$$

$$\therefore\quad \underline{y = 0.364x - 2.94\ (\text{m})}$$

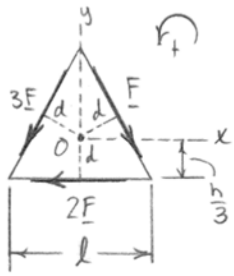
2/82 (a) $\underline{R} = -2F\underline{j}$, $\underline{M}_O = \underline{0}$

(b) $\underline{R} = \underline{0}$, $\underline{M}_O = Fd\underline{k}$ $\quad$ ($+\underline{k}$ is out)

(c) $\underline{R} = -F\underline{i} + F\underline{j}$, $\underline{M}_O = \underline{0}$

2/83 IN EACH CASE, $h = l \sin 60 = \frac{l\sqrt{3}}{2}$ SO.. $\frac{h}{3} = \frac{l\sqrt{3}}{6} = d$

a)



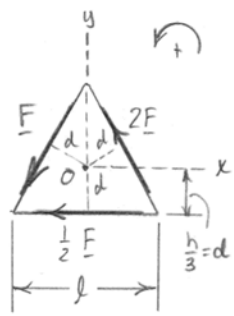
$\underline{R} = (F\cos 60° - 2F - 3F\cos 60°)\underline{i} + (-F\sin 60° - 3F\sin 60°)\underline{j}$

$\underline{R} = -3F\underline{i} - 2\sqrt{3}\,F\underline{j}$

$M_O = 3Fd - Fd - 2Fd \longrightarrow M_O = 0$

$\underline{R}$ ACTS AT $y = 0$.

b)



$\underline{R} = (-2F\cos 60° - F\cos 60° - \frac{1}{2}F)\underline{i} + (2F\sin 60° - F\sin 60°)\underline{j}$

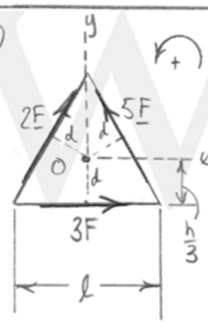
$\underline{R} = -2F\underline{i} + \frac{\sqrt{3}}{2}F\underline{j}$

$M_O = 2Fd + Fd - \frac{1}{2}Fd \longrightarrow M_O = \frac{5\sqrt{3}}{12}Fl$ CCW

TO PRODUCE A CCW MOMENT AT O WITH NEGATIVE R_x, R IS PLACED ABOVE O.

$R_x y = M_O \longrightarrow 2Fy = \frac{5\sqrt{3}}{12}Fl \longrightarrow y = \frac{5\sqrt{3}}{24}l$ ABOVE O

c)



$\underline{R} = (3F - 5F\cos 60° + 2F\cos 60°)\underline{i} + (5F\sin 60° + 2F\sin 60°)\underline{j}$

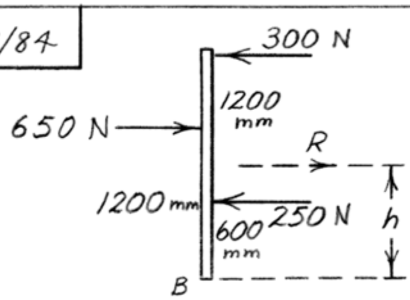
$\underline{R} = \frac{3}{2}F\underline{i} + \frac{7\sqrt{3}}{2}F\underline{j}$

$M_O = 3Fd + 5Fd - 2Fd \longrightarrow M_O = \sqrt{3}\,Fl$ CCW

TO PRODUCE A CCW MOMENT AT O WITH POSITIVE R_x, R IS PLACED BELOW O.

$R_x|y| = M_O \longrightarrow \frac{3}{2}F|y| = \sqrt{3}\,Fl \longrightarrow |y| = \frac{2}{\sqrt{3}}l$ BELOW O

2/84



$R = \Sigma F = 650 - 250 - 300$

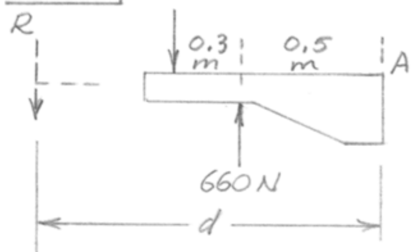
$= 100$ N

$Rh = \Sigma M_B$:

$100h = 650(1.2) - 300(1.8) - 250(0.6)$

$\underline{h = 0.9 \text{ m}}$

2/85



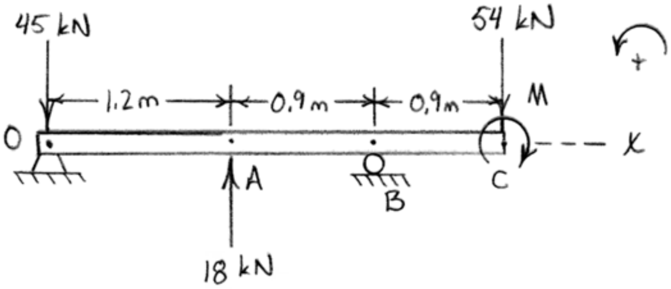
$R = \Sigma F = 680 - 660 = 20$ N

$Rd = \Sigma M_A$

$20d = 680(0.8) - 660(0.5)$

$d = 10.70$ m to the left of A

2/86



$\underline{R = 81 \text{ kN DOWN}}$

$\Sigma M_B = 0: \quad 45(2.1) - 18(0.9) - 0.9(54) - M = 0$

so... $M = 29.7$ kN·m CW

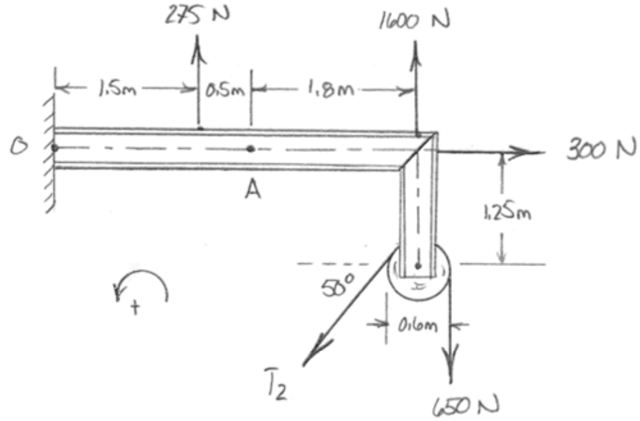
$M_O = 18(1.2) - 54(3) - 29.7 = -170.1$ so... $\underline{M_O = 170.1 \text{ kN·m CW}}$

2/87 $M_O = 0$, so

$$\circlearrowleft + \; M - 400(0.150 \cos 30°) - 320(0.300) = 0$$

$\underline{M = 148.0 \text{ N·m}}$

2/88



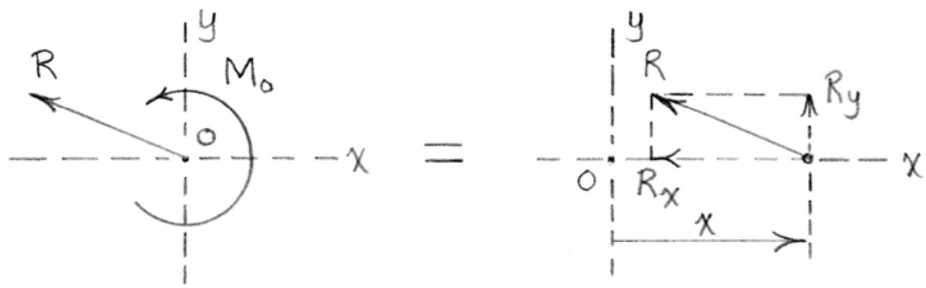
$$\Sigma M_A = 0: \quad -275(0.5) + 1.8(1600) - 650(1.8+0.3) + T_2(0.3) - T_2 \sin 50^\circ (1.8) \ldots$$
$$- T_2 \cos 50^\circ (1.25)$$

$$\underline{T_2 = 732 \text{ N}}$$

2/89

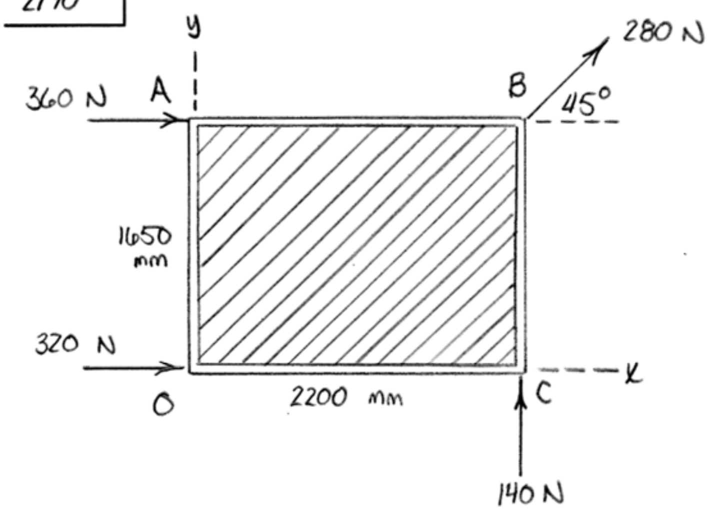
$\underline{\mathbf{R} = -200\mathbf{i} + 80\mathbf{j}\ \mathrm{N}}$

$\curvearrowleft + \ M_O = -160(0.25) + 240(0.50) + 200(0.25) = 130\ \mathrm{N\cdot m}$



$R_y x = M_O, \quad x = \frac{130}{80} = \underline{1.625\ \mathrm{m}\ \ (\text{off pipe})}$

2/90



$\mathbf{R} = (360 + 320 + 280\cos 45°)\,\mathbf{i} + (140 + 280\sin 45°)\,\mathbf{j}$

$\underline{\mathbf{R} = 878\,\mathbf{i} + 338\,\mathbf{j}\ \text{N}}$

$M_O = 2.2(140 + 280\sin 45°) - 1.650(360 + 280\cos 45°) = -177.1\ \text{N}\cdot\text{m}$

$\underline{M_O = 177.1\ \text{N}\cdot\text{m CW}}$

FOR CW MOMENT ABOUT O, POSITIVE R_x IS PLACED ABOVE O.

$R_x y = M_O \longrightarrow 878y = 177.1 \longrightarrow \underline{y = 0.202\text{ m OR } 202\text{ mm ABOVE O}}$

FOR CW MOMENT ABOUT O, POSITIVE R_y IS PLACED LEFT OF O.

$R_y x = M_O \longrightarrow 338x = 177.1 \longrightarrow \underline{x = 0.524\text{ m OR } 524\text{ mm LEFT OF O}}$

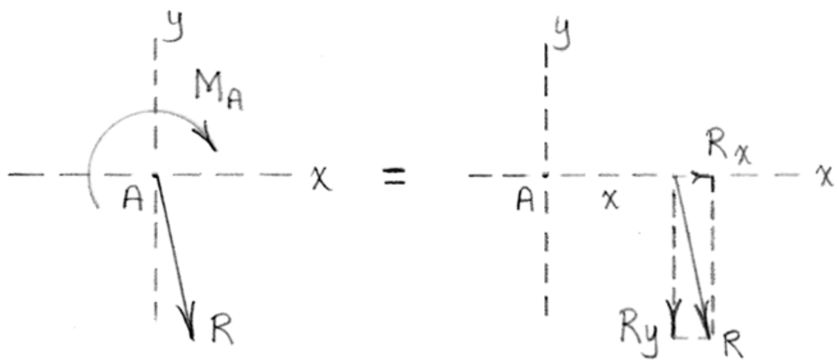
2/91 Equivalent force-couple system at A:

$\underline{R} = -10\underline{j} - 4.8\underline{j} + 3.2(\sin 30^\circ \underline{i} + \cos 30^\circ \underline{j})$

$= \underline{1.6\underline{i} - 12.03\underline{j} \text{ kN}}$

$\circlearrowright M_A = 10(1.2) + 4.8(1.2 + 1.2\cos 30^\circ + 0.9)$
$\quad - 3.2\sin 30^\circ(0.6\sin 30^\circ) - 3.2\cos 30^\circ(1.2 + 0.6\cos 30^\circ)$

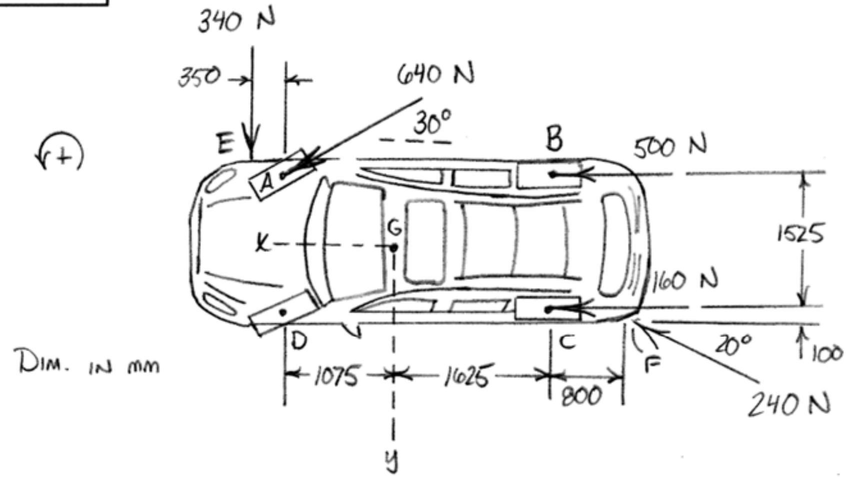
$= \underline{21.8 \text{ kN}\cdot\text{m CW}}$



Condition: $x|R_y| = M_A$

$x = \dfrac{21.8}{12.03} = \underline{1.814 \text{ m}}$

2/92



$$\underline{R} = (500 + 640\cos 30^\circ + 240\cos 20^\circ + 160)\underline{i} + (640\sin 30^\circ + 340 - 240\sin 20^\circ)\underline{j}$$

$$\underline{\underline{R} = 1440\underline{i} + 578\underline{j} \text{ N}}$$

$$\Sigma M_G = \frac{1.525}{2}(500 - 160 + 640\cos 30^\circ) + 340\left(\frac{1075 + 350}{1000}\right) + \frac{1075}{1000}(640\sin 30^\circ)$$
$$+ \frac{1625 + 800}{1000}(240\sin 20^\circ) - \left(\frac{1525/2}{1000} + \frac{100}{1000}\right)(240\cos 20^\circ) = 1515 \text{ N·m CCW}$$

For CCW M_G with positive R_x, R_x is in negative y, above G.

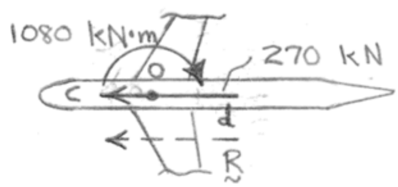
$R_x|y| = M_G \longrightarrow 1440|y| = 1515 \longrightarrow |y| = 1.052$ m so... $\underline{(0, -1.052) \text{ m}}$

For CCW M_G with positive R_y, R_y is in positive x, left of G.

$R_y x = M_G \longrightarrow 578x = 1515 \longrightarrow x = 2.62$ m so... $\underline{(2.62, 0) \text{ m}}$

2/93 Force – Couple system at point O:

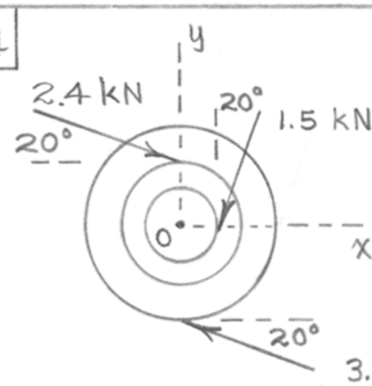
$$\begin{cases} R = 3(90) = \underline{270 \text{ kN} \ (\leftarrow)} \\ +\circlearrowleft M_O = 12(90) = 1080 \text{ kN}\cdot\text{m} \end{cases}$$



$$d = \frac{M_O}{R} = \frac{1080}{270}$$

$$= \underline{4 \text{ m}}$$

2/94



$$\begin{cases} r_1 = 0.120 \text{ m} \\ r_2 = 0.200 \text{ m} \\ r_3 = 0.300 \text{ m} \end{cases}$$

$$\underline{R} = \Sigma \underline{F} = 2.4(c20°\underline{i} - s20°\underline{j}) + 1.5(-s20°\underline{i} - c20°\underline{j})$$
$$+ 3.6(-c20°\underline{i} + s20°\underline{j}) = -1.641\underline{i} - 0.999\underline{j} \text{ kN}$$

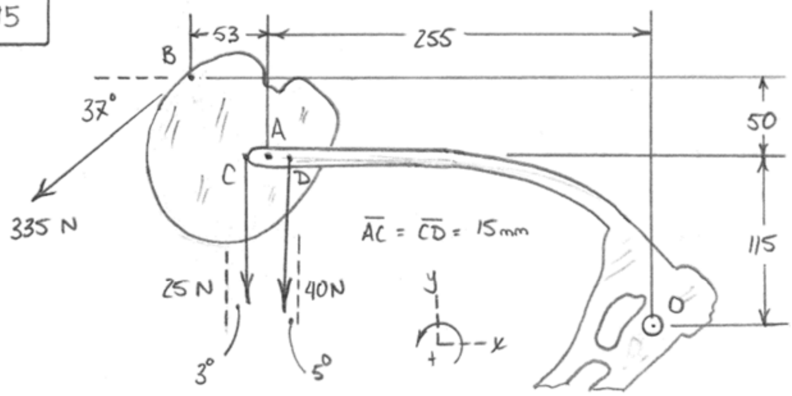
$$\circlearrowright M_O = (2.4(0.2) + 1.5(0.12) + 3.6(0.3))\cos 20° = 1.635 \text{ kN·m}$$

$$\underline{r} \times \underline{R} = \underline{M}_O: (x\underline{i} + y\underline{j}) \times (-1.641\underline{i} - 0.999\underline{j}) = -1.635$$

$$\Rightarrow -0.999x + 1.641y = -1.635$$

Axis intercepts : $\underline{x = 1.637 \text{ m}, \; y = -0.997 \text{ m}}$

2/95



DIMENSIONS IN mm

$$\underline{R} = (-335\cos 37^\circ + 25\sin 3^\circ - 40\sin 5^\circ)\underline{i} - (25\cos 3^\circ + 40\cos 5^\circ + 335\sin 37^\circ)\underline{j}$$

$$\therefore \ \underline{R} = -270\underline{i} - 266\underline{j} \text{ N}$$

$$\Sigma M_o = 335\cos 37^\circ\left(\frac{50+115}{1000}\right) + 335\sin 37^\circ\left(\frac{53+255}{1000}\right) + 25\cos 3^\circ\left(\frac{15+255}{1000}\right) - 25\sin 3^\circ\left(\frac{115}{1000}\right)\ldots$$

$$+ 40\cos 5^\circ\left(\frac{255-15}{1000}\right) + 40\sin 5^\circ\left(\frac{115}{1000}\right)$$

$$\therefore \ M_o = 122.8 \text{ N}\cdot\text{m CCW}$$

2/96 Equivalent force - couple system at point O:

$$\underline{R} = \Sigma \underline{F} = (-25 + 20 \sin 30°)\,\underline{i} + (-30 - 20 \cos 30°)\,\underline{j} = \underline{-15\underline{i} - 47.3\underline{j}\ \text{kN}}$$

$$\curvearrowleft + M_O = 25(5) - 30(9) - (20 \cos 30°)9 - (20 \sin 30°)5 = -351\ \text{kN·m}$$

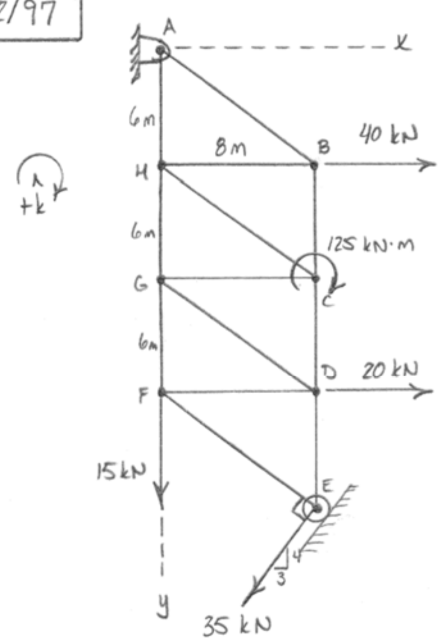
For final location of $\underline{R}$:

$$\underline{r} \times \underline{R} = \underline{M}_O\,,\quad (x\underline{i} + y\underline{j}) \times (-15\underline{i} - 47.3\,\underline{j}) = -351\,\underline{k}$$

$$-47.3x + 15y = -351$$

Axis intersections: $\underline{x = 7.42\ \text{m},\ y = -23.4\ \text{m}}$

2/97



$$\mathbf{R} = \left(40 + 20 - \tfrac{3}{5}35\right)\mathbf{i} + \left(15 + \tfrac{4}{5}35\right)\mathbf{j}$$

$$\mathbf{R} = 39\mathbf{i} + 43\mathbf{j} \text{ kN}$$

$$\mathbf{M}_A = \left[-40(6) - 20(18) + 125 + 35(10) + \tfrac{3}{5}35(18)\right]\mathbf{k} \longrightarrow \mathbf{M}_A = 253\,\mathbf{k} \text{ kN·m}$$

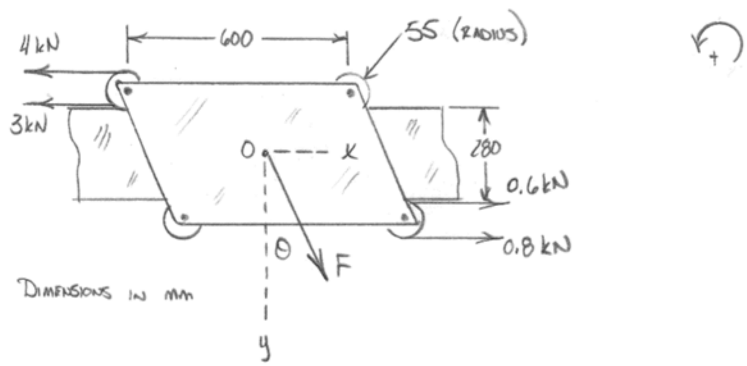
$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_A \longrightarrow (x\mathbf{i} + y\mathbf{j}) \times (39\mathbf{i} - 43\mathbf{j}) = 253\mathbf{k}$$

$$\mathbf{k}: \; 43x - 39y = 253 \longrightarrow \underline{y = 1.103x - 6.49 \text{ (m)}}$$

$\underline{x\text{-Axis:}}$ $y = 0 = 1.103x - 6.49 \longrightarrow \underline{x = 5.88 \text{ m} \quad \text{so} \quad (5.88, 0)\text{ m}}$

$\underline{y\text{-Axis:}}$ $x = 0 \longrightarrow \underline{y = -6.49 \text{ m} \quad \text{so} \quad (0, -6.49)\text{ m}}$

2/98 $F = 5\text{ kN}$, $\theta = 30°$



$$\underline{R} = (0.8 + 0.6 + 5\sin 30° - 4 - 3)\,\underline{i} + 5\cos 30°\,\underline{j} \longrightarrow \underline{R} = -3.10\,\underline{i} + 4.33\,\underline{j}\ \text{kN}$$

$$\Sigma M_O = 0.6\left(\frac{140}{1000}\right) + 0.8\left(\frac{140+110}{1000}\right) + 3\left(\frac{140}{1000}\right) + 4\left(\frac{140+110}{1000}\right) =$$

$$\therefore\ \Sigma M_O = 1.704\ \text{kN·m CCW}$$

FOR A CCW M_O WITH NEGATIVE R_x, R IS PLACED ABOVE O IN MINUS y.

$$R_x y = M_O \longrightarrow 3.10|y| = 1.704 \longrightarrow |y| = 0.550$$

$$\therefore\ \underline{|y| = 550\text{ mm ABOVE O OR } (0, -550)\ (\text{mm})}$$

2/99

$$\underline{R} = \Sigma \underline{F} = 400(\cos 45^\circ \underline{i} - \sin 45^\circ \underline{j}) + 500(\sin 15^\circ \underline{i} - \cos 15^\circ \underline{j})$$

$$= 412\underline{i} - 766\underline{j} \text{ N}$$

$$\overset{+}{\circlearrowleft} M_O = (500-400)(0.060) = 6 \text{ N}\cdot\text{m}$$

For the line of action of the standalone force:

$$\underline{r} \times \underline{R} = \underline{M}_O$$

$$(x\underline{i} + y\underline{j}) \times (412\underline{i} - 766\underline{j}) = -6\underline{k}$$

$$-766x - 412y = -6$$

$$\begin{cases} \text{For } x = 0: & y = 0.01455 \text{ m or } \underline{y = 14.55 \text{ mm}} \\ \text{For } y = 0: & x = 0.00783 \text{ m or } \underline{x = 7.83 \text{ mm}} \end{cases}$$

2/100 For a zero force-couple system at point O:

$$\underline{R} = \Sigma \underline{F} = (-F_C \sin 30° + F_D \sin 30°)\,\underline{i}$$
$$+ (50 - 10 - 100 - 50 + F_B + F_C \cos 30° + F_D \cos 30°)\,\underline{j} = \underline{0}$$

$$\Rightarrow F_C = F_D = F$$

$$\circlearrowleft M_O = -10(0.5) + 50(0.7) - 100(1.35) + F_B(2) - 50(2.5) + 2F \cos 30° (2.9) = 0$$

$$\underline{F = F_C = F_D = 6.42 \text{ N}}\,, \quad \underline{F_B = 98.9 \text{ N}}$$

2/101

$$\underline{F} = F\underline{n}$$

$$= 60\left[\frac{40\underline{i} - 50\underline{j} + 110\underline{k}}{\sqrt{40^2 + 50^2 + 110^2}}\right]$$

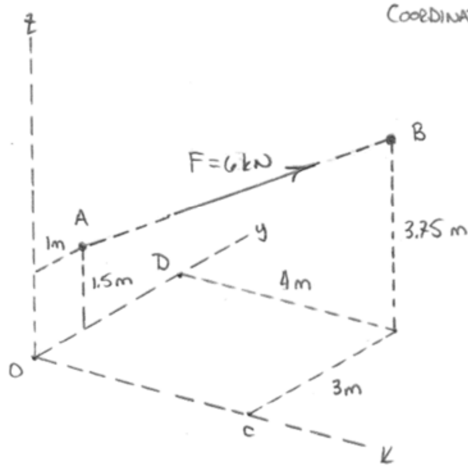
$$= \underline{18.86\underline{i} - 23.6\underline{j} + 51.9\underline{k} \text{ N}}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-23.6}{60}, \quad \underline{\theta_y = 113.1^\circ}$$

2/102

COORDINATES OF $A = (0, 1, 1.5)$ m

COORDINATES OF $B = (4, 3, 3.75)$ m



$$\underline{n}_{AB} = \frac{(4-0)\underline{i} + (3-1)\underline{j} + (3.75-1.5)\underline{k}}{\left[(4-0)^2 + (3-1)^2 + (3.75-1.5)^2\right]^{1/2}} \longrightarrow \underline{n}_{AB} = 0.799\underline{i} + 0.400\underline{j} + 0.449\underline{k}$$

$$\underline{F} = F\underline{n}_{AB} = 6(0.799\underline{i} + 0.400\underline{j} + 0.449\underline{k}) \longrightarrow \underline{\underline{F} = 4.79\underline{i} + 2.40\underline{j} + 2.70\underline{k} \text{ kN}}$$

$$\theta_x = \cos^{-1}\left(\frac{\underline{F}\cdot\underline{i}}{F}\right) = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{4.79}{6}\right) \longrightarrow \underline{\theta_x = 37.0°}$$

2/103 $F_z = 5 \sin 35° = 2.87$ kN

$F_{xy} = 5 \cos 35° = 4.10$ kN

$F_x = 4.10 \cos 60° = 2.05$ kN

$F_y = 4.10 \sin 60° = 3.55$ kN

So $\underline{F} = 2.05\underline{i} + 3.55\underline{j} + 2.87\underline{k}$ kN

The projection of $\underline{F}$ onto the x-axis is

$F_x = 2.05$ kN

The projection of $\underline{F}$ onto line OA is

$F_{OA} = \underline{F} \cdot \underline{n}_{OA}$

$= (2.05\underline{i} + 3.55\underline{j} + 2.87\underline{k}) \cdot (\cos 30°\underline{i} + \sin 30°\underline{j})$

$= 3.55$ kN

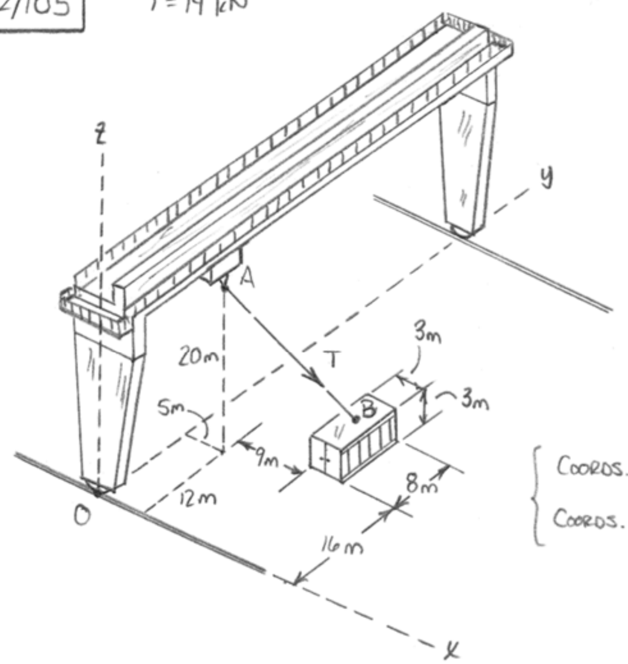
2/104

$$\underline{F} = F\underline{n} = 900\frac{2\underline{i} - 4\underline{j} - 4\underline{k}}{\sqrt{2^2 + 4^2 + 4^2}}$$

$$= 900\left(\frac{1}{3}\underline{i} - \frac{2}{3}\underline{j} - \frac{2}{3}\underline{k}\right) \text{ N}$$

$\underline{F_x = 300 \text{ N}}$, $\underline{F_y = -600 \text{ N}}$, $\underline{F_z = -600 \text{ N}}$

2/105 $T = 14$ kN



COORDS. OF A ARE $(5, 12, 20)$ m

COORDS. OF B ARE $(15.5, 20, 3)$ m

$$\mathbf{n}_{AB} = \frac{(15.5-5)\mathbf{i} + (20-12)\mathbf{j} + (3-20)\mathbf{k}}{\left[(15.5-5)^2 + (20-12)^2 + (3-20)^2\right]^{1/2}} \longrightarrow \underline{\hat{\mathbf{n}}_{AB} = 0.488\mathbf{i} + 0.372\mathbf{j} - 0.790\mathbf{k}}$$

$$T_x = T n_x = 14(0.488) \longrightarrow \underline{T_x = 6.83 \text{ kN}}$$

$$T_y = T n_y = 14(0.372) \longrightarrow \underline{T_y = 5.20 \text{ kN}}$$

$$T_z = T n_z = 14(-0.790) \longrightarrow \underline{T_z = -11.06 \text{ kN}}$$

2/106 $\underline{T} = T\underline{n}_{AB} = 2.4\left(\dfrac{2\underline{i} + \underline{j} - 5\underline{k}}{\sqrt{2^2 + 1^2 + 5^2}}\right)$

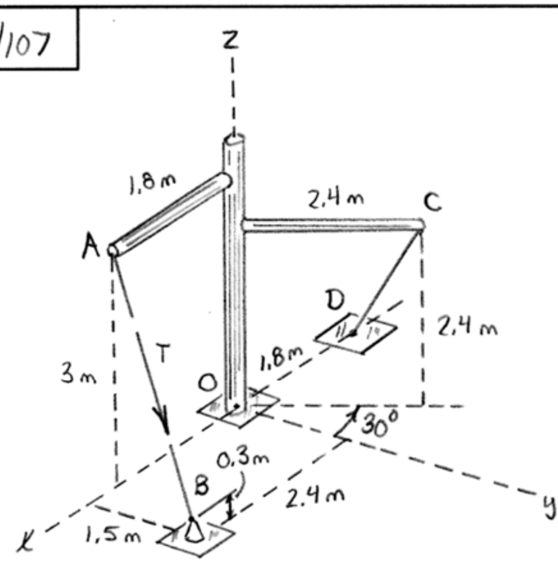
$= 0.876\underline{i} + 0.438\underline{j} - 2.19\underline{k}$ kN

Projection $T_{AC} = \underline{T} \cdot \underline{n}_{AC}$

$= (0.876\underline{i} + 0.438\underline{j} - 2.19\underline{k}) \cdot \left(\dfrac{2\underline{i} - 2\underline{j} - 5\underline{k}}{\sqrt{2^2 + 2^2 + 5^2}}\right)$

$=$ 2.06 kN

2/107



$T = 8$ kN

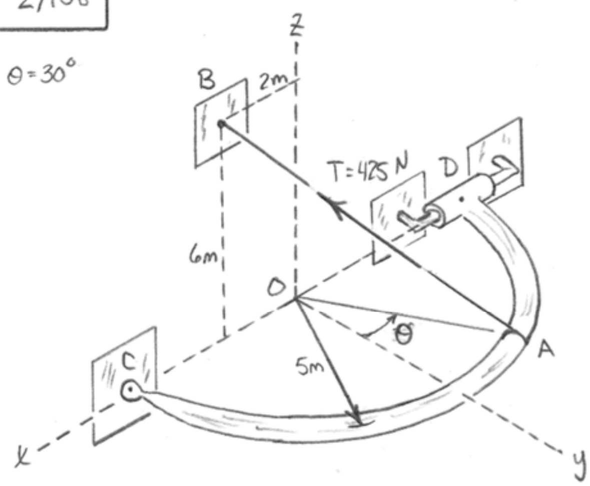
$$\begin{cases} \underline{r}_{OA} = (1.8,\ 0,\ 3)\ \text{m} \\ \underline{r}_{OB} = (2.4,\ 1.5,\ 0.3)\ \text{m} \end{cases}$$

$$\bar{n}_{AB} = \frac{(2.4-1.8)\underline{i} + (1.5-0)\underline{j} + (0.3-3)\underline{k}}{\sqrt{(2.4-1.8)^2 + (1.5-0)^2 + (0.3-3)^2}} \rightarrow \underline{n}_{AB} = 0.1907\,\underline{i} + 0.477\,\underline{j} - 0.858\,\underline{k}$$

$$\begin{cases} \theta_x = \cos^{-1}(n_x) = \cos^{-1}(0.1907) \longrightarrow \underline{\theta_x = 79.0^\circ} \\ \theta_y = \cos^{-1}(n_y) = \cos^{-1}(0.477) \longrightarrow \underline{\theta_y = 61.5^\circ} \\ \theta_z = \cos^{-1}(n_z) = \cos^{-1}(-0.858) \longrightarrow \underline{\theta_z = 149.1^\circ} \end{cases}$$

2/108

$\theta = 30°$



$$\begin{cases} A(-5\sin 30,\ 5\cos 30,\ 0)\ \text{m} \\ B(2, 0, 6)\ \text{m} \end{cases}$$

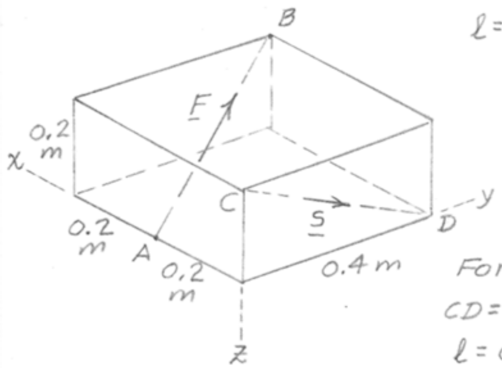
$$\underline{n}_{AB} = \frac{(2+5\sin 30)\underline{i} + (0-5\cos 30)\underline{j} + (6-0)\underline{k}}{\left[(2+5\sin 30)^2 + (5\cos 30)^2 + 6^2\right]^{1/2}}$$

$$\therefore \quad \underline{n}_{AB} = 0.520\underline{i} - 0.5\underline{j} + 0.693\underline{k}$$

$$\underline{T}_A = T\underline{n}_{AB} = 425(0.520\hat{i} - 0.5\hat{j} + 0.693\hat{k}) \longrightarrow \underline{\underline{T}_A = 221\underline{i} - 212\underline{j} + 294\underline{k}\ \text{N}}$$

$$\underline{T}_B = -\underline{T}_A \longrightarrow \underline{\underline{T}_B = -221\underline{i} + 212\underline{j} - 294\underline{k}\ \text{N}}$$

2/109 $F = 2$ kN; For $\underline{F}$, $\overline{AB} = \sqrt{\overline{0.2}^2 + \overline{0.4}^2 + \overline{0.2}^2} = \sqrt{0.24}$ m



$$l = \frac{0.2}{\sqrt{0.24}} = \frac{1}{\sqrt{6}}, \quad m = \frac{0.4}{\sqrt{0.24}} = \frac{2}{\sqrt{6}}$$

$$n = -\frac{1}{\sqrt{6}}$$

$$\underline{F} = \frac{2}{\sqrt{6}}(\underline{i} + 2\underline{j} - \underline{k}) \text{ kN}$$

For unit vector $\underline{s}$

$$CD = \sqrt{\overline{0.2}^2 + \overline{0.4}^2} = \sqrt{0.20} \text{ m}$$

$$l = 0, \quad m = \frac{0.4}{\sqrt{0.2}} = \frac{2}{\sqrt{5}}, \quad n = \frac{1}{\sqrt{5}}$$

$$\underline{s} = \frac{1}{\sqrt{5}}(2\underline{j} + \underline{k})$$

$$F_{CD} = \underline{F} \cdot \underline{s} = \frac{2}{\sqrt{6}}(\underline{i} + 2\underline{j} - \underline{k}) \cdot \frac{1}{\sqrt{5}}(2\underline{j} + \underline{k})$$

$$= \frac{2}{\sqrt{30}}(4 - 1) = \frac{6}{\sqrt{30}} = \underline{\sqrt{6/5} \text{ kN}}$$

$$\cos\theta = \frac{\underline{F} \cdot \underline{s}}{F} = \frac{\sqrt{6/5}}{2}, \quad \underline{\theta = 56.8°}$$

2/110

$$\underline{T} = T\underline{n}_{AB} = 10\left[\frac{4\underline{i} - 7.5\underline{j} + 5\underline{k}}{(4^2 + (-7.5)^2 + 5^2)^{1/2}}\right]$$

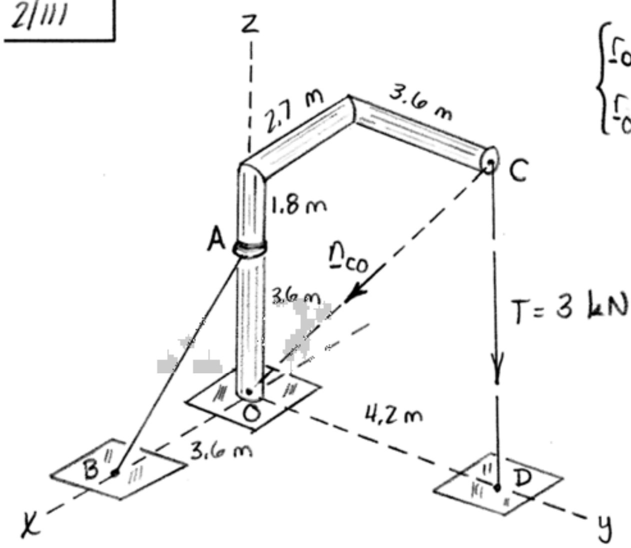
$$= \underline{10\,(0.406\,\underline{i} - 0.761\,\underline{j} + 0.507\,\underline{k})\text{ kN}}$$

$\cos\theta_x = 0.406$, $\theta_x = 66.1°$

$\cos\theta_y = -0.761$, $\theta_y = 139.5°$

$\cos\theta_z = 0.507$, $\underline{\theta_z = 59.5°}$

2/111



$$\begin{cases} \underline{r}_{OC} = (-2.7, 3.6, 5.4) \text{ m} \\ \underline{r}_{OD} = (0, 4.2, 0) \text{ m} \end{cases}$$

$$\underline{n}_{CD} = \frac{(0+2.7)\underline{i} + (4.2-3.6)\underline{j} + (0-5.4)\underline{k}}{\sqrt{2.7^2 + (4.2-3.6)^2 + 5.4^2}} \longrightarrow \underline{n}_{CD} = 0.445\underline{i} + 0.989\underline{j} - 0.890\underline{k}$$

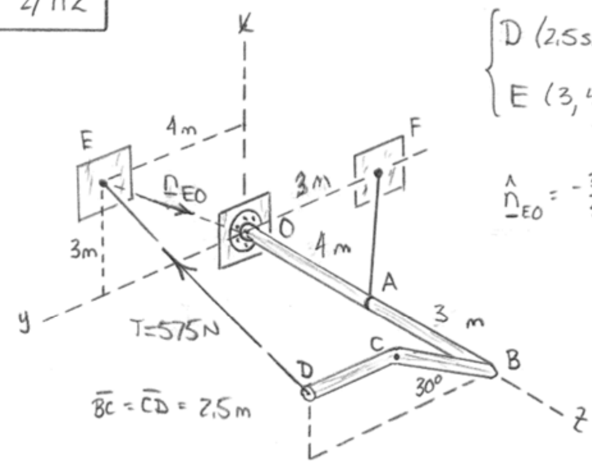
$$\underline{T} = T\underline{n}_{CD} = 3(0.445\underline{i} + 0.989\underline{j} - 0.890\underline{k}) \longrightarrow \underline{T} = 1.335\underline{i} + 0.297\underline{j} - 2.67\underline{k} \text{ kN}$$

$$\underline{n}_{CO} = \frac{2.7\underline{i} - 3.6\underline{j} - 5.4\underline{k}}{\sqrt{2.7^2 + 3.6^2 + 5.4^2}} \longrightarrow \underline{n}_{CO} = 0.384\underline{i} - 0.512\underline{j} - 0.768\underline{k}$$

$$T_{CO} = \underline{T} \cdot \underline{n}_{CO} = (1.335\underline{i} + 0.297\underline{j} - 2.67\underline{k}) \cdot (0.384\underline{i} - 0.512\underline{j} - 0.768\underline{k})$$

$$\underline{T_{CO} = 2.41 \text{ kN}}$$

2/112



$$\begin{cases} D\ (2.5\sin 30^\circ,\ 2.5+2.5\cos 30^\circ,\ 7) \\ E\ (3, 4, 0) \end{cases}$$

$$\hat{n}_{EO} = -\frac{3}{5}\underline{i} - \frac{4}{5}\underline{j}$$

$$n_{DE} = \frac{(3-2.5\sin 30^\circ)\underline{i} + (4-(2.5+2.5\cos 30^\circ))\underline{j} + (0-7)\underline{k}}{\left[(3-2.5\sin 30^\circ)^2 + (4-(2.5+2.5\cos 30^\circ))^2 + 7^2\right]^{1/2}}$$

$$n_{DE} = 0.242\underline{i} - 0.0918\underline{j} - 0.966\underline{k}$$

$$\underline{T} = T n_{DE} = 575(0.242\underline{i} - 0.0918\underline{j} - 0.966\underline{k}) \longrightarrow \underline{T} = 138.9\underline{i} - 52.8\underline{j} - 555\underline{k}\ \text{N}$$

$$T_{EO} = \underline{T} \cdot n_{EO} = (138.9\underline{i} - 52.8\underline{j} - 555\underline{k}) \cdot \left(-\frac{3}{5}\underline{i} - \frac{4}{5}\underline{j}\right) \longrightarrow \underline{T_{EO} = -41.1\ \text{N}}$$

$$\underline{T}_{EO} = T_{EO}\, n_{EO} = -41.1\left(-\frac{3}{5}\underline{i} - \frac{4}{5}\underline{j}\right) \longrightarrow \underline{\underline{T}_{EO} = 24.7\underline{i} + 32.9\underline{j}\ \text{N}}$$

2/113

$$\mathbf{F} = F\mathbf{n}_{AB} = 200\left[\frac{-120\mathbf{i} + 240\mathbf{j} + 80\mathbf{k}}{\sqrt{120^2 + 240^2 + 80^2}}\right]$$

$$= -85.7\mathbf{i} + 171.4\mathbf{j} + 57.1\mathbf{k} \text{ N}$$

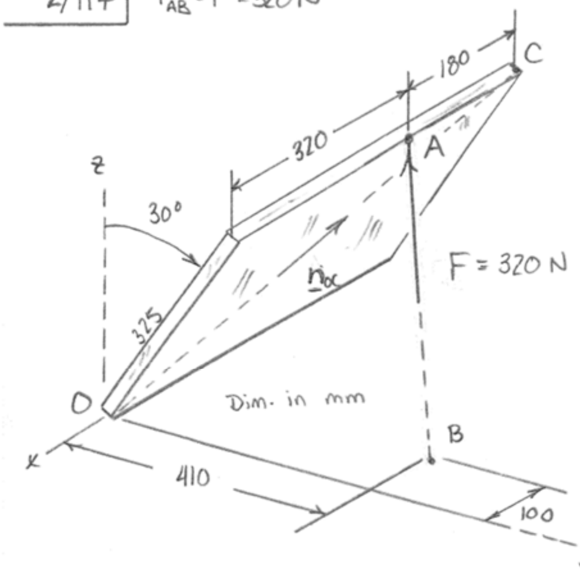
$\mathbf{OC} = 120\mathbf{i} + 240\mathbf{j}$ mm

The angle θ between $\mathbf{F}$ and $\mathbf{OC}$ is

$$\theta = \cos^{-1}\frac{\mathbf{F}\cdot\mathbf{OC}}{F(\overline{OC})} = \cos^{-1}\left[\frac{-85.7(120) + 171.4(240)}{200\sqrt{120^2 + 240^2}}\right]$$

$$= \underline{54.9°}$$

2/114 $F_{AB} = F = 320$ N



$$\begin{cases} A\,(-320,\ 325\sin 30^\circ,\ 325\cos 30^\circ) \\ B\,(-100,\ 410,\ 0) \\ C\,(-500,\ 325\sin 30^\circ,\ 325\cos 30^\circ) \end{cases}$$

$$\underline{n}_{BA} = \frac{(-320+100)\underline{i} + (325\sin 30^\circ - 410)\underline{j} + 325\cos 30^\circ \underline{k}}{\left[(-320+100)^2 + (325\sin 30^\circ - 410)^2 + (325\cos 30^\circ)^2\right]^{1/2}}$$

$$\therefore \quad \underline{n}_{BA} = -0.506\underline{i} - 0.569\underline{j} + 0.648\underline{k}$$

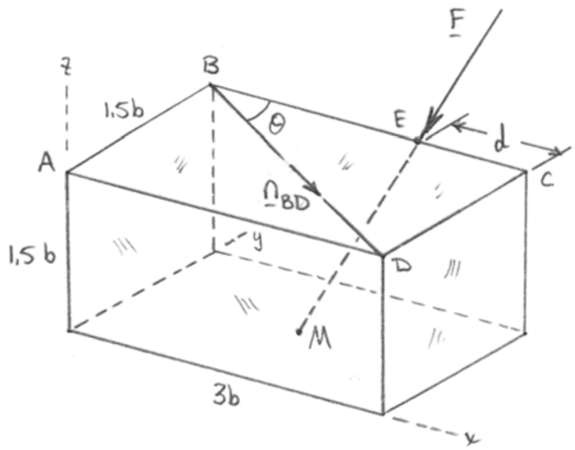
$$\underline{F} = F\underline{n}_{BA} = 320(-0.506\underline{i} - 0.569\underline{j} + 0.648\underline{k}) \longrightarrow \underline{F} = -162.0\underline{i} - 182.2\underline{j} + 207\underline{k} \text{ N}$$

$$\underline{n}_{OC} = \frac{-500\underline{i} + 325\sin 30^\circ\underline{j} + 325\cos 30^\circ\underline{k}}{\sqrt{500^2 + 325^2}} \longrightarrow \underline{n}_{OC} = -0.838\underline{i} + 0.272\underline{j} + 0.472\underline{k}$$

$$F_{OC} = \underline{F}\cdot\underline{n}_{OC} = (-162.0\underline{i} - 182.2\underline{j} + 207\underline{k})\cdot(-0.838\underline{i} + 0.272\underline{j} + 0.472\underline{k})$$

$$\therefore \quad F_{OC} = 184.0 \text{ N}$$

2/115



$$\begin{cases} B\ (0,\ 1.5b,\ 1.5b) \\ D\ (3b,\ 0,\ 1.5b) \\ E\ (3b-d,\ 1.5b,\ 1.5b) \\ M\ \left(\frac{3b}{2},\ \frac{3b}{4},\ 0\right) \end{cases}$$

$$\theta = \tan^{-1}\left(\frac{1.5b}{3b}\right) = 26.6^\circ$$

$$\underline{n}_{BD} = \cos\theta\,\underline{i} - \sin\theta\,\underline{j} \longrightarrow \underline{n}_{BD} = 0.894\,\underline{i} - 0.447\,\underline{j}$$

$$\underline{n}_{EM} = \frac{\left(\frac{3b}{2} - 3b + d\right)\underline{i} + \left(\frac{3b}{4} - 1.5b\right)\underline{j} + (0 - 1.5b)\underline{k}}{\left[\left(\frac{3b}{2} - 3b + d\right)^2 + \left(\frac{3b}{4} - 1.5b\right)^2 + (-1.5b)^2\right]^{1/2}}$$

$$\therefore\ \underline{n}_{EM} = \frac{(4d - 6b)\underline{i} - 3b\,\underline{j} - 6b\,\underline{k}}{\sqrt{81b^2 - 48bd + 16d^2}} \qquad \text{AND} \qquad \underline{F} = F\hat{n}_{EM}$$

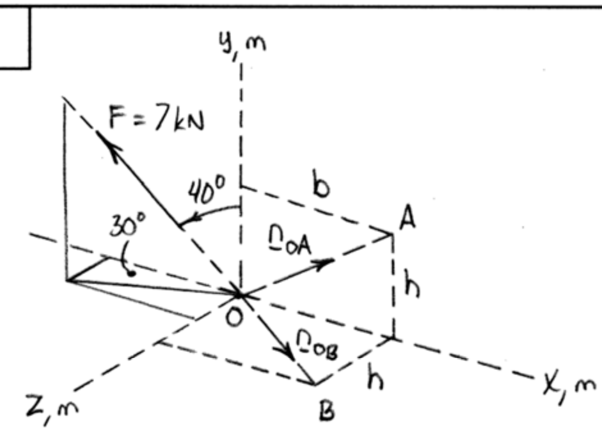
$$F_{BD} = \underline{F} \cdot \underline{n}_{BD} = F\left(\frac{(4d-6b)\underline{i} - 3b\underline{j} - 6b\underline{k}}{\sqrt{81b^2 - 48bd + 16d^2}}\right) \cdot \left(0.894\,\underline{i} - 0.447\,\underline{j}\right)$$

$$\therefore\ F_{BD} = \frac{(8d - 9b)F}{\sqrt{5}\sqrt{81b^2 - 48bd + 16d^2}}$$

$$\begin{cases} \text{IF } d = \frac{b}{2} \ldots & \underline{F_{BD} = -0.286\,F} \\ \text{IF } d = \frac{5b}{2} \ldots & \underline{F_{BD} = 0.630\,F} \end{cases}$$

2/116

$$\begin{cases} b = 2\,m \\ F_{OA} = 0 \end{cases}$$



$$\underline{F} = -7\sin 40°\cos 30°\,\underline{i} + 7\cos 40°\,\underline{j} + 7\sin 40°\sin 30° = -3.90\,\underline{i} + 5.36\,\underline{j} + 2.25\,\underline{k}$$

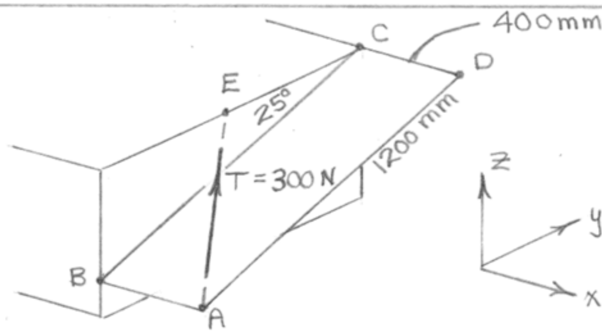
$$\underline{n}_{OA} = \frac{b\underline{i} + h\underline{j}}{\sqrt{b^2 + h^2}} \quad \text{so...} \quad \underline{F}\cdot\underline{n}_{OA} = F_{OA} = 0 = (-3.90\,\underline{i} + 5.36\,\underline{j} + 2.25\,\underline{k}) \cdot \frac{2\underline{i} + h\underline{j}}{\sqrt{2^2 + h^2}}$$

$$\therefore\ 0 = \frac{5.36\,h - 7.79}{\sqrt{4 + h^2}} \longrightarrow h = 1.453\ m$$

$$\underline{n}_{OB} = \frac{b\underline{i} + h\underline{k}}{\sqrt{b^2 + h^2}} = 0.809\,\underline{i} + 0.588\,\underline{k}$$

$$F_{OB} = \underline{F}\cdot\underline{n}_{OB} = (-3.90\,\underline{i} + 5.36\,\underline{j} + 2.25\,\underline{k})\cdot(0.809\,\underline{i} + 0.588\,\underline{k}) \longrightarrow \underline{F_{OB} = -1.830\ kN}$$

2/117



$$\underline{T} = T\underline{n}_{AE} = 300\left[\frac{-400\underline{i} + 544\underline{j} + 507\underline{k}}{\sqrt{400^2 + 544^2 + 507^2}}\right]$$

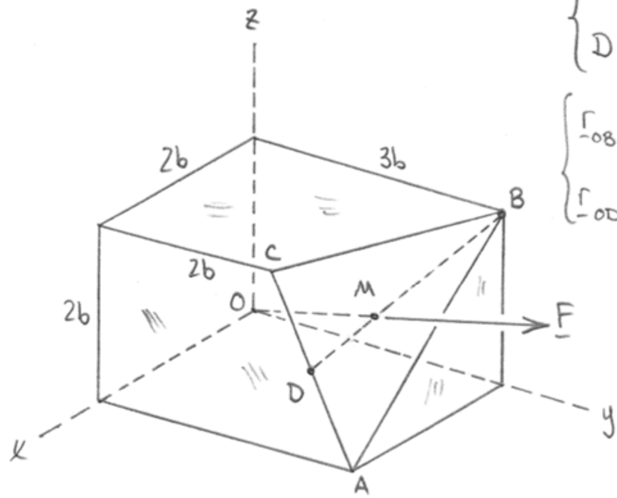
$$= 300\left[-0.474\underline{i} + 0.644\underline{j} + 0.601\underline{k}\right] \text{ N}$$

$$\underline{n}_{BC} = \cos 25°\underline{j} + \sin 25°\underline{k}$$

Carry out $T_{BC} = \underline{T}\cdot\underline{n}_{BC}$ to obtain

$$\underline{T_{BC} = 251 \text{ N}}$$

2/118



$$\begin{cases} B\ (0, 3b, 2b) \\ D\ (2b, 2b+\frac{b}{2}, b) \end{cases}$$

$$\begin{cases} \underline{r}_{OB} = 3b\,\underline{j} + 2b\,\underline{k} \\ \underline{r}_{OD} = 2b\,\underline{i} + \frac{5b}{2}\underline{j} + b\,\underline{k} \end{cases}$$

$$\underline{r}_{OM} = \underline{r}_{OD} + \frac{1}{3}(\underline{r}_{OB} - \underline{r}_{OD}) = \frac{4b}{3}\underline{i} + \frac{8b}{3}\underline{j} + \frac{4b}{3}\underline{k}$$

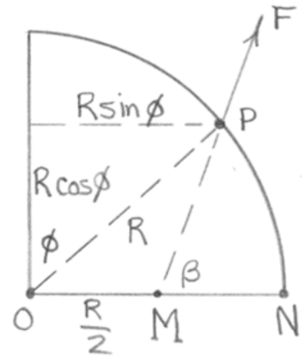
$$\underline{n}_{OM} = \frac{\underline{r}_{OM}}{|\underline{r}_{OM}|} = \frac{\frac{4b}{3}\underline{i} + \frac{8b}{3}\underline{j} + \frac{4b}{3}\underline{k}}{\sqrt{\left(\frac{4b}{3}\right)^2 + \left(\frac{8b}{3}\right)^2 + \left(\frac{4b}{3}\right)^2}} \longrightarrow \hat{n}_{OM} = \frac{1}{\sqrt{6}}\underline{i} + \sqrt{\frac{2}{3}}\underline{j} + \frac{1}{\sqrt{6}}\underline{k}$$

$$\underline{F} = F\underline{n}_{OM} \longrightarrow \underline{F} = \frac{F}{\sqrt{6}}\underline{i} + \sqrt{\frac{2}{3}}F\underline{j} + \frac{F}{\sqrt{6}}\underline{k}$$

▶2/119

$F_x = F_{xy}\cos\theta$, $F_y = F_{xy}\sin\theta$

$F_z = F\sin\beta$, $F_{xy} = F\cos\beta$



$$\tan\beta = \frac{R\cos\phi}{R\sin\phi - \frac{R}{2}} = \frac{2\cos\phi}{2\sin\phi - 1}$$

$$\text{So } \sin\beta = \frac{2\cos\phi}{\sqrt{(2\sin\phi-1)^2+(2\cos\phi)^2}}$$

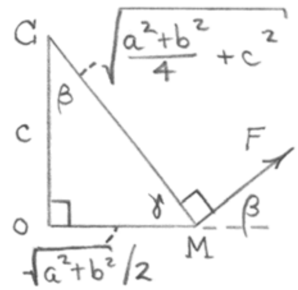
$$\cos\beta = \frac{2\sin\phi - 1}{\sqrt{(2\sin\phi-1)^2+(2\cos\phi)^2}}$$

Note that $\sqrt{(2\sin\phi-1)^2+(2\cos\phi)^2} = \sqrt{5-4\sin\phi}$

So $\underline{F} = F\left[\cos\theta\cos\beta\,\underline{i} + \sin\theta\cos\beta\,\underline{j} + \sin\beta\,\underline{k}\right]$

$$= \frac{F}{\sqrt{5-4\sin\phi}}\left[(2\sin\phi-1)(\cos\theta\,\underline{i}+\sin\theta\,\underline{j}) + 2\cos\phi\,\underline{k}\right]$$

▶2/120



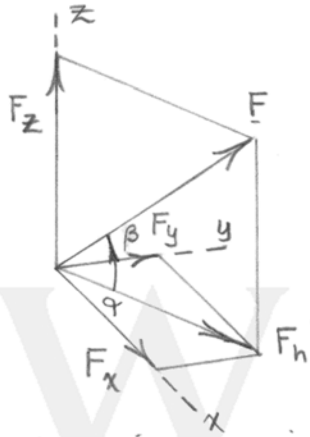
$$\tan\gamma = \frac{c}{\sqrt{a^2+b^2}/2} = \frac{2c}{\sqrt{a^2+b^2}}$$

$$\gamma + 90° + \beta = 180°$$

$$\beta = 90° - \gamma = 90° - \tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}$$

$$\tan\alpha = \frac{b}{a}$$

$$\begin{cases} \cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}} \\ \sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}} \end{cases}$$



$$\begin{cases} F_z = F\sin\beta \\ F_h = F\cos\beta \\ F_x = F_h\cos\alpha = F\cos\beta\cos\alpha \\ F_y = F_h\sin\alpha = F\cos\beta\sin\alpha \end{cases}$$

Now simplify $\sin\beta$ & $\cos\beta$ expressions:

$$\sin\beta = \sin\left[90° - \tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right]$$

$$= \sin 90°\cos\left[\tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right] - \cancelto{0}{\cos 90°}\sin\left[\tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right]$$

$$= \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2+4c^2}}$$ (Also see that $\sin\beta = \cos\gamma$!)

$$\cos\beta = \sin\gamma = \frac{c}{\sqrt{\frac{a^2+b^2}{4}+c^2}} = \frac{2c}{\sqrt{a^2+b^2+4c^2}}$$

Finally,

$$F_x = F\frac{2c}{\sqrt{a^2+b^2+4c^2}}\frac{a}{\sqrt{a^2+b^2}} = \frac{2acF}{\sqrt{a^2+b^2}\sqrt{a^2+b^2+4c^2}}$$

$$F_y = F\frac{2c}{\sqrt{a^2+b^2+4c^2}}\frac{b}{\sqrt{a^2+b^2}} = \frac{2bcF}{\sqrt{a^2+b^2}\sqrt{a^2+b^2+4c^2}}$$

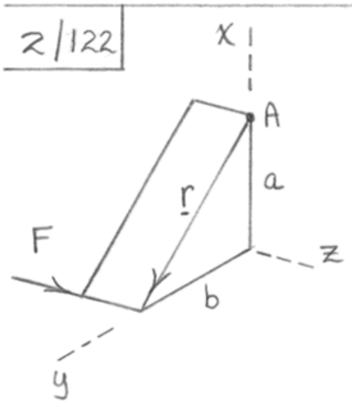
$$F_z = F\sqrt{\frac{a^2+b^2}{a^2+b^2+4c^2}}$$

2/121 By inspection, $\underline{\mathbf{M}_O = F(c\mathbf{j} - b\mathbf{k})}$

Or, $\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = (b\mathbf{j} + c\mathbf{k}) \times F\mathbf{i}$

$= \underline{F(c\mathbf{j} - b\mathbf{k})}$ ✓

2/122



$$\underline{M}_A = \underline{r} \times \underline{F}$$

$$= (-a\underline{i} + b\underline{j}) \times F\underline{k}$$

$$= \underline{F(b\underline{i} + a\underline{j})}$$

2/123 (a) $\underline{M}_O = \underline{FL\underline{i}}$

(b) $\underline{M}_O = FL\underline{i} + FD\underline{k} = \underline{F(L\underline{i} + D\underline{k})}$

2/124

$$\underline{M}_O = \underline{r}_{OA} \times \underline{F}$$

$$= (36\underline{j} + 18\underline{k}) \times 24(-\cos 30°\,\underline{i} + \sin 30°\,\underline{j})$$

$$= \underline{-216\underline{i} - 374\underline{j} + 748\underline{k} \quad \text{N}\cdot\text{mm}}$$

2/125 $R = \Sigma F = \underline{1000 \text{ kN}}$

$\underline{M} = -600(100)\underline{i} + (600-400)(200)\underline{j}$ N·m

$= \underline{(-60\underline{i} + 40\underline{j})10^3 \text{ N·m}}$

$\boxed{2/126}$ $\underline{M} = \underline{r} \times \underline{F}$

$= -0.5\underline{i} \times 400(\cos 15° \underline{j} + \sin 15° \underline{k})$

$= \underline{51.8\underline{j} - 193.2\underline{k} \text{ N}\cdot\text{m}}$

2/127 $\overline{AB} = \sqrt{0.8^2 + 1.5^2 + 2^2} = 2.62 \text{ m}$

$\underline{T} = \frac{1.2}{2.62}(0.8\underline{i} + 1.5\underline{j} - 2\underline{k}) \text{ kN}$

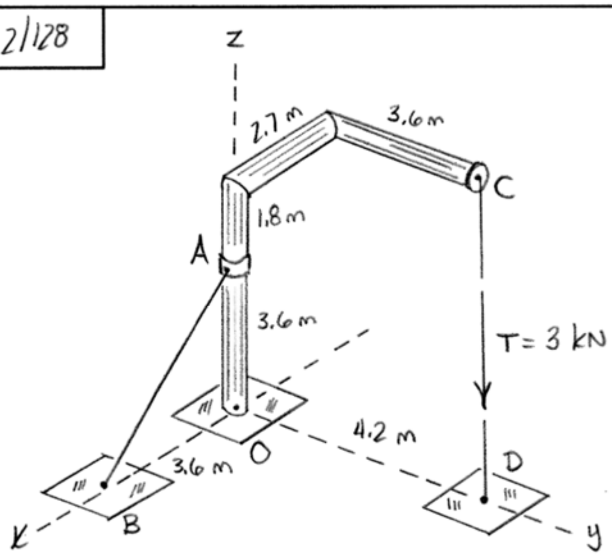
Take $\underline{r} = \overrightarrow{OA} = 1.6\underline{i} + 2\underline{k} \text{ m}$

$$\underline{M}_O = \underline{r} \times \underline{T} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1.6 & 0 & 2 \\ 0.8 & 1.5 & -2 \end{vmatrix} \frac{1.2}{2.62}$$

$$\underline{M}_O = 0.457(-3\underline{i} + 4.8\underline{j} + 2.40\underline{k}) \text{ kN}\cdot\text{m}$$

$$M_O = |\underline{M}_O| = 0.457\sqrt{3^2 + 4.8^2 + 2.40^2} = \underline{2.81 \text{ kN}\cdot\text{m}}$$

2/128



FROM 2/111 ... $\underline{T} = 1.335\underline{i} + 0.297\underline{j} - 2.67\underline{k}$ kN

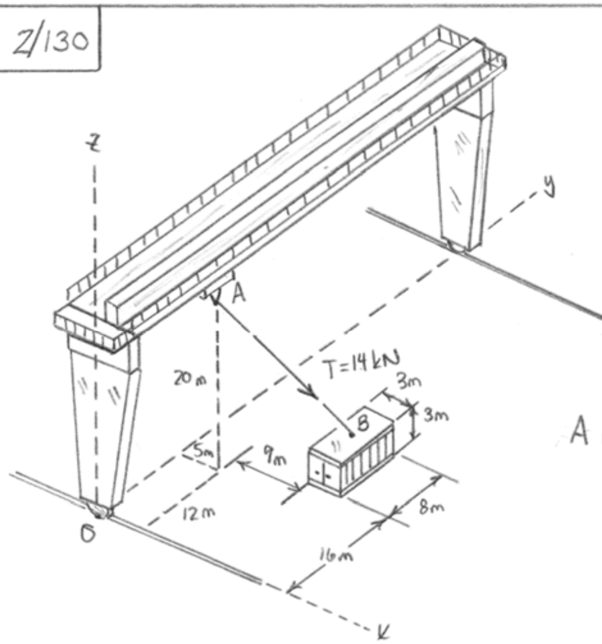
$\underline{M}_O = \underline{r}_{OD} \times \underline{T} = 4.2\underline{j} \times (1.335\underline{i} + 0.297\underline{j} - 2.67\underline{k})$

$\therefore$ $\underline{M}_O = -11.21\underline{i} - 5.61\underline{k}$ kN·m

2/129 $\underline{M} = -150(0.250+0.250)\underline{i} + 150(0.150)\underline{j}$

$= \underline{-75\underline{i} + 22.5\underline{j} \ \text{N}\cdot\text{m}}$

2/130



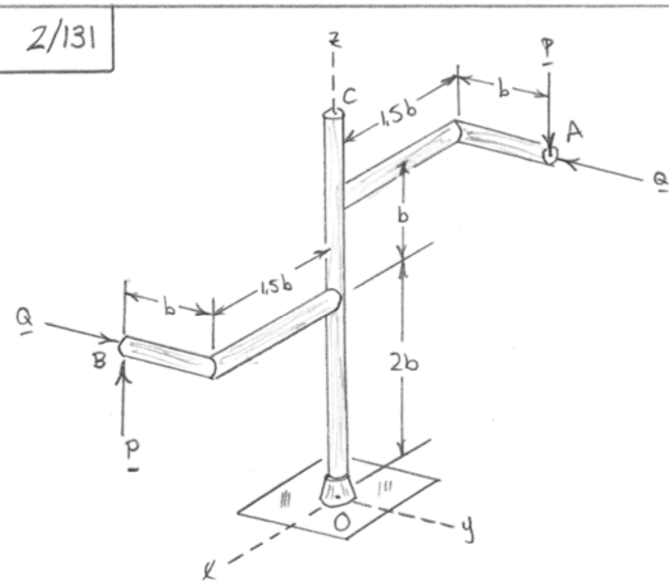
A (5, 12, 20) m

From 2/105 ... $\underline{T} = \underline{R} = 6.83\underline{i} + 5.20\underline{j} - 11.06\underline{k}$ kN

$\underline{M}_O = \underline{r}_{OA} \times \underline{T} = (5\underline{i} + 12\underline{j} + 20\underline{k}) \times (6.83\underline{i} + 5.20\underline{j} - 11.06\underline{k})$

$\underline{M}_O = -237\underline{i} + 191.9\underline{j} - 55.9\underline{k}$ kN·m

2/131



$b = 3\text{ m}$
$P = 4\text{ kN}$
$Q = 7.5\text{ kN}$

$$\underline{M}_O = \underline{M}_C = (bQ - 2bP)\underline{i} - 3bP\underline{j} + 3bQ\underline{k}$$

$$= \big(3(7.5) - 2(3)(4)\big)\underline{i} - 3(3)(4)\underline{j} + 3(3)(7.5)\underline{k}$$

$$\therefore \quad \underline{M}_O = \underline{M}_C = -1.5\underline{i} - 36\underline{j} + 67.5\underline{k} \text{ kN}\cdot\text{m}$$

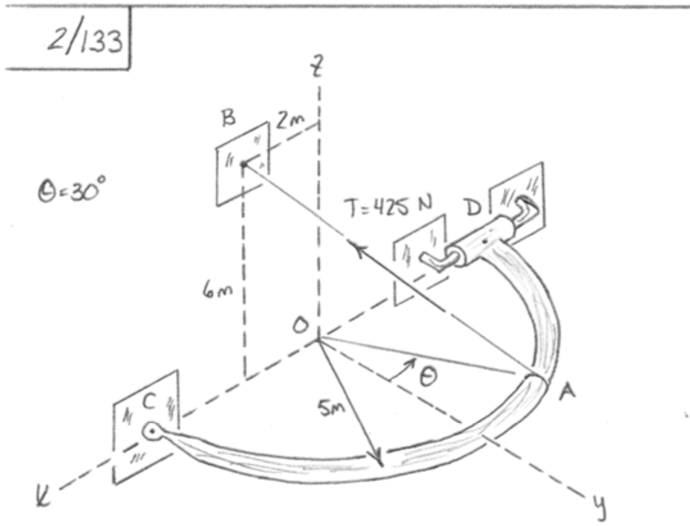
2/132

$$\underline{M}_O = \underline{r} \times \underline{F}$$

$$= (-6\underline{i} + 0.8\underline{j} + 1.2\underline{k}) \times (-400\underline{j})$$

$$= \underline{480\underline{i} + 2400\underline{k} \ \text{N}\cdot\text{m}}$$

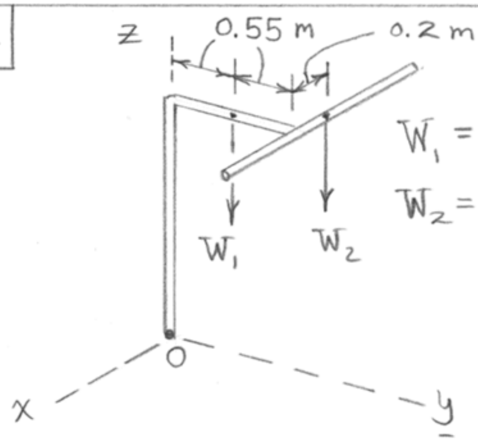
2/133



FROM 2/108 ... $\underline{T} = 221\underline{i} - 212\underline{j} + 294\underline{k}$ N

$$M_{O_x} = r\cos\theta \, T_z = 5\cos 30^\circ(294) \longrightarrow \underline{M_{O_x} = 1275 \text{ N}\cdot\text{m}}$$

2/134



$W_1 = 1.1(7)(9.81) = 75.5$ N

$W_2 = 2(7)(9.81) = 137.3$ N

$$\begin{cases} M_{O_x} = -75.5(0.55) - 137.3(1.1) = -192.6 \text{ N·m} \\ M_{O_y} = -137.3(0.2) = -27.5 \text{ N·m} \\ M_{O_z} = 0 \end{cases}$$

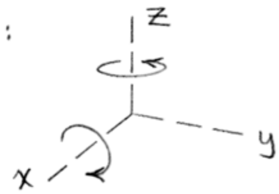
$$\therefore \underline{\mathbf{M}_O = -192.6\mathbf{i} - 27.5\mathbf{j} \text{ N·m}}$$

$$\underline{M_O = 194.6 \text{ N·m}}$$

2/135 $\underline{M} = 4N(1m)\,\underline{k} - 4N(1.25m)\,\underline{i}$

$= \underline{-5\underline{i} + 4\underline{k}\ \text{N·m}}$

Spacecraft will begin to rotate about its x- and z-axes:



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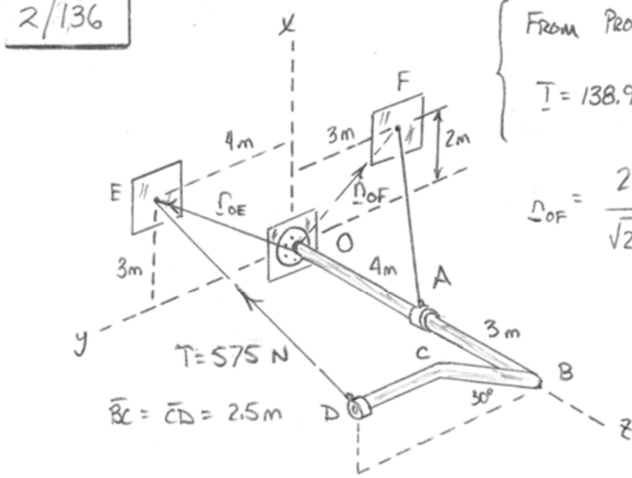


FROM PROBLEM 2/112 ...

$\mathbf{T} = 138.9\mathbf{i} - 52.8\mathbf{j} - 555\mathbf{k}$ N

$$n_{OF} = \frac{2\mathbf{i} - 3\mathbf{j}}{\sqrt{2^2 + 3^2}} = \frac{2}{\sqrt{13}}\mathbf{i} - \frac{3}{\sqrt{13}}\mathbf{j}$$

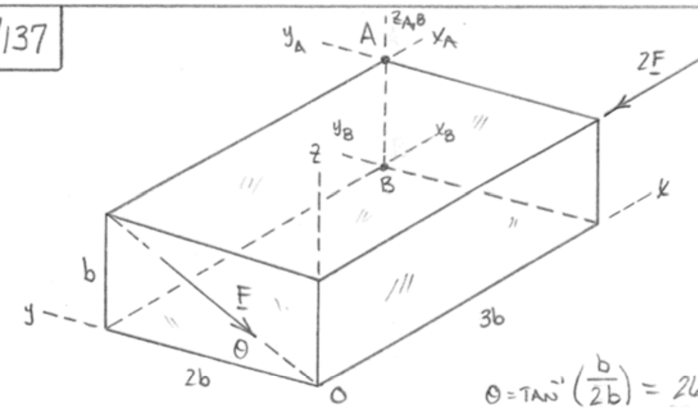
$$\mathbf{M}_O = \mathbf{r}_{OE} \times \mathbf{T} = (3\mathbf{i} + 4\mathbf{j}) \times (138.9\mathbf{i} - 52.8\mathbf{j} - 555\mathbf{k})$$

$$\mathbf{M}_O = -2220\mathbf{i} + 1666\mathbf{j} - 714\mathbf{k} \text{ N·m}$$

$$\mathbf{M}_{OF} = (\mathbf{M}_O \cdot \mathbf{n}_{OF})\,\mathbf{n}_{OF} = \left[(-2220\mathbf{i} + 1666\mathbf{j} - 714\mathbf{k}) \cdot \left(\frac{2}{\sqrt{13}}\mathbf{i} - \frac{3}{\sqrt{13}}\mathbf{j}\right)\right]\left(\frac{2}{\sqrt{13}}\mathbf{i} - \frac{3}{\sqrt{13}}\mathbf{j}\right)$$

$$\mathbf{M}_{OF} = -1453\mathbf{i} + 2180\mathbf{j} \text{ N·m}$$

2/137



$\theta = \tan^{-1}\left(\frac{b}{2b}\right) = 26.6°$

· Force $\underline{F}$

$$\underline{M}_A = F\cos\theta\,(3b)\underline{k} - F\sin\theta\,(3b)\underline{j} \longrightarrow \underline{M}_A = \frac{Fb}{\sqrt{5}}(-3\underline{j} + 6\underline{k})$$

$$\underline{M}_B = F\cos\theta\,(3b)\underline{k} - F\sin\theta\,(3b)\underline{j} + F\sin\theta\,(2b)\underline{i}$$

$$\therefore\ \underline{M}_B = \frac{Fb}{\sqrt{5}}(2\underline{i} - 3\underline{j} + 6\underline{k})$$

· Force $2\underline{F}$

$$\underline{M}_A = -2F(2b)\underline{k} \longrightarrow \underline{M}_A = -4Fb\,\underline{k}$$

$$\underline{M}_B = -2F(2b)\underline{k} - 2F(b)\underline{j} \longrightarrow \underline{M}_B = -2Fb(\underline{j} + 2\underline{k})$$

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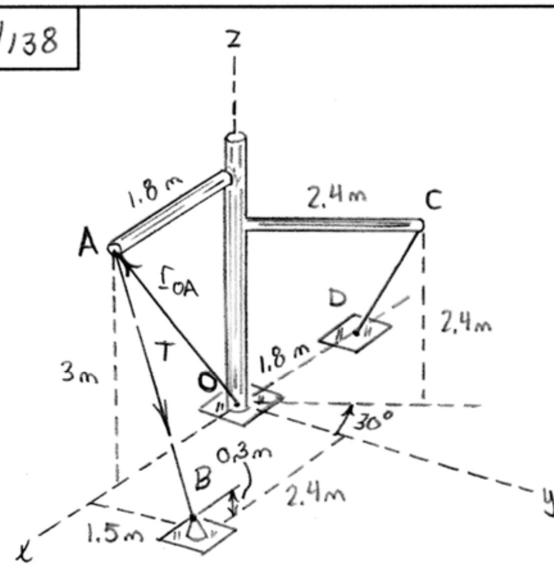


T = 8 kN

From 2/107... $\underline{n}_{AB} = 0.1907\,\underline{i} + 0.477\,\underline{j} - 0.858\,\underline{k}$

$$\underline{T} = T\underline{n}_{AB} = 8(0.1907\,\underline{i} + 0.477\,\underline{j} - 0.858\,\underline{k}) = 1.526\,\underline{i} + 3.81\,\underline{j} - 6.86\,\underline{k} \text{ kN}$$

$$\therefore \quad \underline{R} = 1.526\,\underline{i} + 3.81\,\underline{j} - 6.86\,\underline{k} \text{ kN}$$

$$\underline{M}_O = \underline{r}_{OA} \times \underline{T} = (1.8\,\underline{i} + 3\,\underline{k}) \times (1.526\,\underline{i} + 3.81\,\underline{j} - 6.86\,\underline{k})$$

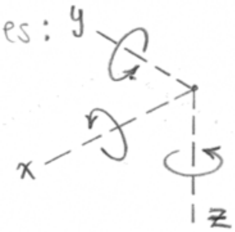
$$\therefore \quad \underline{M}_O = -11.44\,\underline{i} + 16.93\,\underline{j} + 6.86\,\underline{k} \text{ kN·m}$$

2/139

$$\underline{M} = (1700)(2)\underline{i} - (1700)(30)\underline{j} - (1700)(30)\underline{k}$$

$$= 3400\underline{i} - 51\,000\underline{j} - 51\,000\underline{k} \text{ N·m}$$

The orbiter would acquire rotational motion about all three axes:



2/140

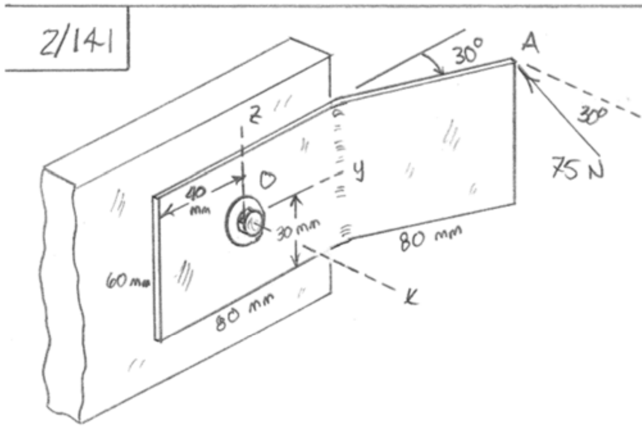
$$\underline{M}_O = 0\underline{i} - (200)(0.2 + 0.125\sin 20°)\,\underline{j} - 200\,(0.125\cos 20° - 0.070)\,\underline{k}$$

$$= \underline{-48.6\underline{j} - 9.49\underline{k} \quad \text{N}\cdot\text{m}}$$

There would be no z-component of $\underline{M}_O$ if

$d\cos 20° - 70 = 0$, $\quad \underline{d = 74.5 \text{ mm}}$

2/141



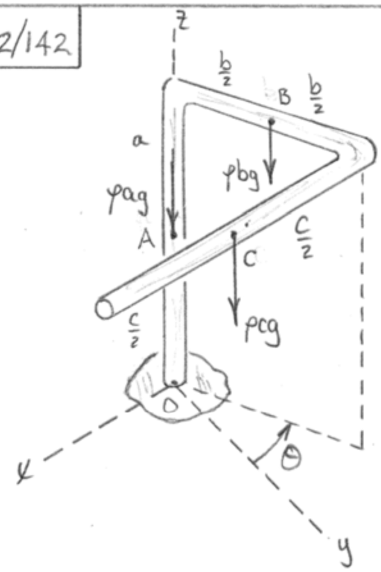
$$\begin{cases} \underline{r}_{OA} = 80\sin 30°\,\underline{i} + (40 + 80\cos 30°)\,\underline{j} + 30\,\underline{k} = 40\,\underline{i} + 109.3\,\underline{j} + 30\,\underline{k} \text{ mm} \\ \underline{F} = 75\,(-\cos 30°\,\underline{i} + \sin 30°\,\underline{j}) = -65.0\,\underline{i} + 37.5\,\underline{j} \text{ N} \end{cases}$$

$$\underline{M}_O = \underline{r}_{OA} \times \underline{F} = (40\,\underline{i} + 109.3\,\underline{j} + 30\,\underline{k}) \times (-65.0\,\underline{i} + 37.5\,\underline{j})\left(\frac{1}{1000}\right)$$

$$\underline{M}_O = -1.125\,\underline{i} - 1.949\,\underline{j} + 8.60\,\underline{k} \text{ N·m}$$

$$M_O = \sqrt{1.125^2 + 1.949^2 + 8.60^2} \longrightarrow \underline{M_O = 8.89 \text{ N·m}}$$

2/142



$$\begin{cases} a = 5\text{ m} \\ b = 2.5\text{ m} \\ c = 4\text{ m} \end{cases} \qquad \begin{cases} \rho = 24\text{ kg/m} \\ \theta = 30° \end{cases}$$

$$\begin{cases} B\left(-\frac{b}{2}\sin\theta,\ \frac{b}{2}\cos\theta,\ a\right) \\ C\left(-b\sin\theta + \frac{c}{2},\ b\cos\theta,\ a\right) \end{cases}$$

$$\begin{cases} \underline{M}_O = \underline{r}_{OB} \times (-\rho b g)\underline{k} + \underline{r}_{OC} \times (-\rho c g)\underline{k} \\ \quad = \left(-\frac{b}{2}\sin\theta\,\underline{i} + \frac{b}{2}\cos\theta\,\underline{j} + a\underline{k}\right) \times (-\rho b g \underline{k}) + \left[\left(\frac{c}{2} - b\sin\theta\right)\underline{i} + b\cos\theta\,\underline{j} + a\underline{k}\right] \times (-\rho c g \underline{k}) \end{cases}$$

$$\therefore\ \underline{M}_O = \frac{1}{2}\rho g\left[-(b^2 + 2bc)\cos\theta\,\underline{i} + \left(c^2 - (b^2 + 2bc)\sin\theta\right)\underline{j}\right]$$

With numbers... $\underline{M}_O = -2680\,\underline{i} + 338\,\underline{j}$ N·m

For no y-component... $c^2 - (b^2 + 2bc)\sin\theta = 0$ (Bad!)

$c = b\sin\theta \pm b\sqrt{\sin\theta(1+\sin\theta)}$ With numbers... $c = -0.915$ m

or $c = 3.42$ m

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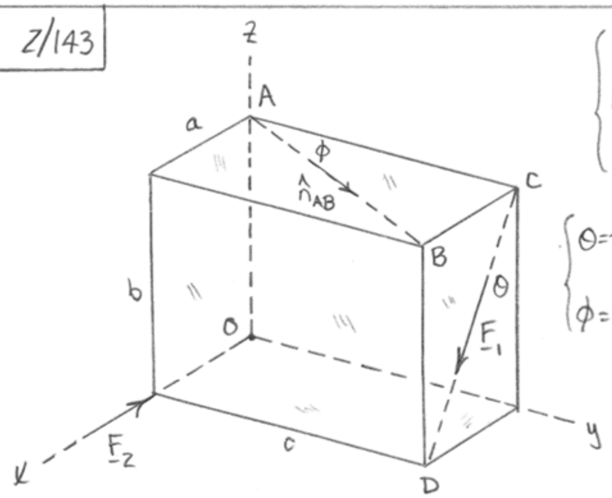


$a = 200\text{ mm}$, $b = 400\text{ mm}$, $c = 500\text{ mm}$

$F_1 = 450\text{ N}$, $M_{AB} = 30\text{ N·m}$

$\theta = \tan^{-1}\left(\frac{a}{b}\right) = \tan^{-1}\left(\frac{200}{400}\right) = 26.6°$

$\phi = \tan^{-1}\left(\frac{a}{c}\right) = \tan^{-1}\left(\frac{200}{500}\right) = 21.8°$

$$\underline{F}_1 = 450(\sin\theta\,\underline{i} - \cos\theta\,\underline{k}) \longrightarrow \underline{F}_1 = 201\,\underline{i} - 402\,\underline{k}\text{ N}$$

$$\underline{n}_{AB} = \sin\phi\,\underline{i} + \cos\phi\,\underline{j} \longrightarrow \underline{n}_{AB} = 0.371\,\underline{i} + 0.928\,\underline{j}$$

$$\underline{M}_A = bF_2\,\underline{j} - cF_{1_z}\,\underline{i} - cF_{1_x}\,\underline{k} = 0.4F_2\,\underline{j} - 0.5(402)\,\underline{i} - 0.5(201)\,\underline{k}$$

$$\underline{M}_A = -201\,\underline{i} + 0.4F_2\,\underline{j} - 100.6\,\underline{k}\text{ N·m}$$

$$M_{AB} = \underline{M}_A \cdot \underline{n}_{AB} \longrightarrow 30 = (-201\,\underline{i} + 0.4F_2\,\underline{j} - 100.6\,\underline{k}) \cdot (0.371\,\underline{i} + 0.928\,\underline{j})$$

$$30 = 0.371F_2 - 74.7 \longrightarrow \underline{F_2 = 282\text{ N}}$$

2/144 Using the coordinates of the figure,

$$\mathbf{M}_A = \mathbf{r} \times \mathbf{F}, \quad \mathbf{F} = -5\mathbf{k} \text{ N},$$

$$\mathbf{r} = [(50+25)\cos 30°]\mathbf{i} + 75\mathbf{j} + [(50+25)\sin 30°]\mathbf{k} \text{ mm}$$

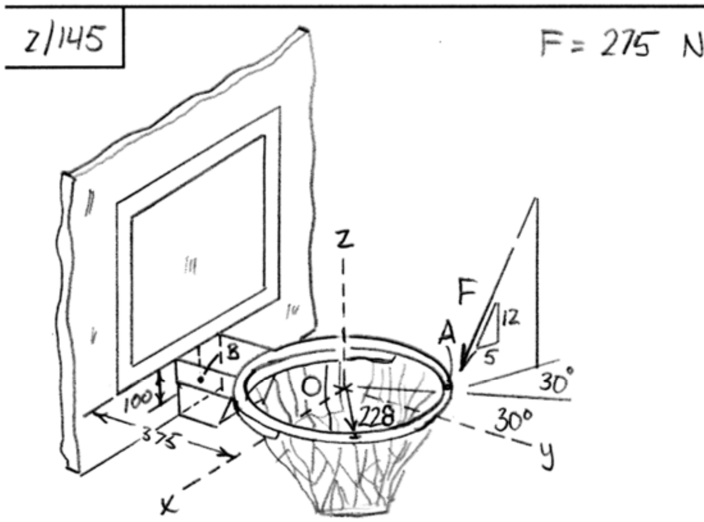
$$\therefore \underline{\mathbf{M}_A = -375\mathbf{i} + 325\mathbf{j} \text{ N}\cdot\text{mm}}$$

$$\mathbf{M}_{AB} = (\mathbf{M}_A \cdot \mathbf{n}_{AB})\mathbf{n}_{AB}, \quad \mathbf{n}_{AB} = \cos 30°\mathbf{i} + \sin 30°\mathbf{k}$$

$$\therefore \underline{\mathbf{M}_{AB} = -281\mathbf{i} - 162.4\mathbf{k} \text{ N}\cdot\text{mm}}$$

2/145

$F = 275$ N



Dim. in mm

$$\underline{F} = 275\left(\tfrac{5}{13}\sin 60^\circ \underline{i} - \tfrac{5}{13}\cos 60^\circ \underline{j} - \tfrac{12}{13}\underline{k}\right) = 91.6\underline{i} - 52.9\underline{j} - 254\underline{k} \text{ N}$$

$$\therefore \quad \underline{R} = 91.6\underline{i} - 52.9\underline{j} - 254\underline{k} \text{ N}$$

$$\underline{r}_{BA} = -0.228\sin 30^\circ \underline{i} + (0.375 + 0.228\cos 30^\circ)\underline{j} + 0.100\underline{k} = -0.114\underline{i} + 0.572\underline{j} + 0.1\underline{k} \text{ m}$$

$$\underline{M}_B = \underline{r}_{BA} \times \underline{F} = (-0.114\underline{i} + 0.572\underline{j} + 0.1\underline{k}) \times (91.6\underline{i} - 52.9\underline{j} - 254\underline{k})$$

$$\therefore \quad \underline{M}_B = -140.0\underline{i} - 19.78\underline{j} - 46.4\underline{k} \text{ N·m}$$

2/146 Moment of couple is $240(\underline{j}\cos 30° - \underline{k}\sin 30°)$

$= 207.8\underline{j} - 120\underline{k}$ N·m

Moment of force is

$1200\cos 30°(-0.250\underline{i} + 0.200\underline{k}) + 1200\sin 30°(0.200\underline{j})$

$= -259.8\underline{i} + 120\underline{j} + 207.8\underline{k}$ N·m

Thus total moment is

$\underline{M}_O = -259.8\underline{i} + 327.8\underline{j} + 87.8\underline{k}$ N·m

or $\underline{M}_O = -260\underline{i} + 328\underline{j} + 88\underline{k}$ N·m

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$$F_z = -F\cos\beta = -F\frac{1}{\sqrt{5}/2} = -\frac{2F}{\sqrt{5}}$$

$$F_{hor} = F\sin\beta = F/\sqrt{5}$$

$$F_x = F_{hor}\cos\theta = \frac{F}{\sqrt{5}}\cos\theta$$

$$F_y = F_{hor}\sin\theta = \frac{F}{\sqrt{5}}\sin\theta$$

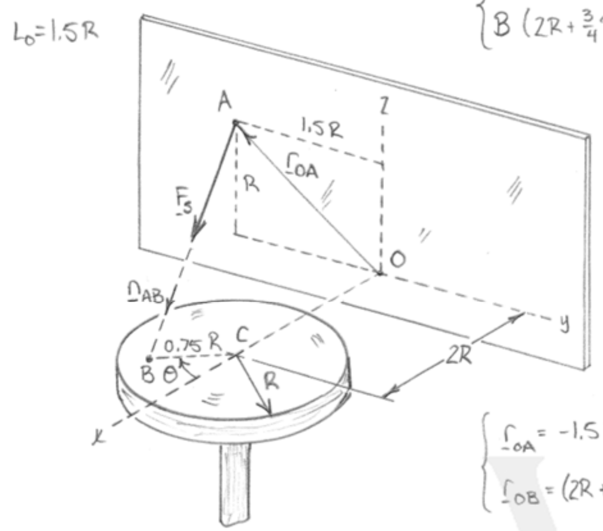
So $\underline{\mathbf{F} = \frac{F}{\sqrt{5}}\left[\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j} - 2\mathbf{k}\right]}$

$$\mathbf{M}_O = \mathbf{r}\times\mathbf{F} = h\mathbf{k}\times\frac{F}{\sqrt{5}}\left[\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j} - 2\mathbf{k}\right]$$

$$= \underline{\frac{Fh}{\sqrt{5}}\left(\cos\theta\,\mathbf{j} - \sin\theta\,\mathbf{i}\right)}$$

*2/148

$L_0 = 1.5R$



$$\begin{cases} A\ (0, -1.5R, R) \\ B\ \left(2R + \tfrac{3}{4}R\cos\theta,\ -\tfrac{3}{4}R\sin\theta,\ 0\right) \end{cases}$$

$$\begin{cases} F_s = k\delta \\ \text{AND} \quad \delta = d - L_0 \end{cases}$$

$$\begin{cases} r_{OA} = -1.5R\,j + R\,k \\ r_{OB} = \left(2R + \tfrac{3}{4}R\cos\theta\right) i - \tfrac{3}{4}R\sin\theta\, j \end{cases}$$

$$n_{AB} = \frac{(8 + 3\cos\theta)\, i + (6 - 3\sin\theta)\, j - 4\, k}{\sqrt{125 + 48\cos\theta - 36\sin\theta}}$$

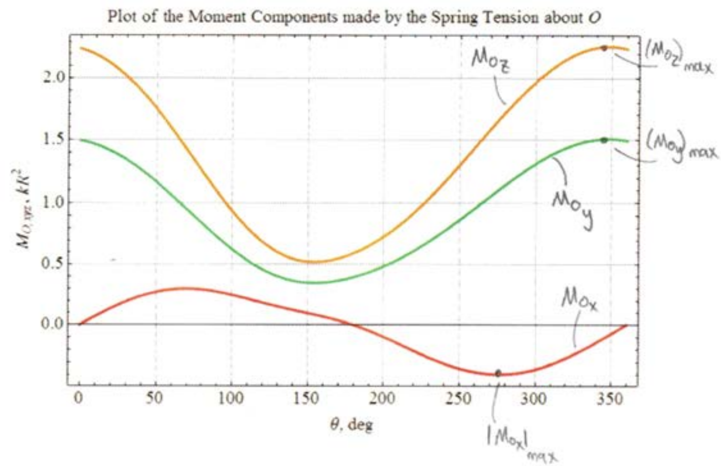
$$d = |r_{OB} - r_{OA}| = \frac{R}{4}\sqrt{125 + 48\cos\theta - 36\sin\theta}$$

$$\delta = d - L_0 = \frac{R}{4}\left(\sqrt{125 + 48\cos\theta - 36\sin\theta} - 6\right)$$

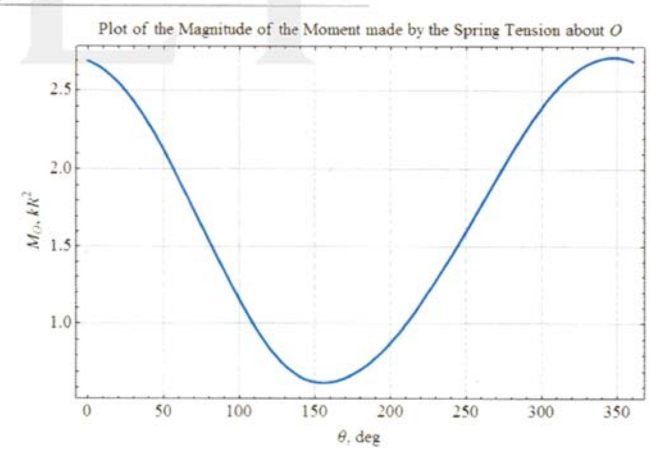
$$F_s = k\delta\, n_{AB}$$

$$M_O = r_{OA} \times F_s \quad \text{AND} \quad r_{OA} = -1.5R\,\hat{j} + R\,\hat{k}$$

CARRY OUT THE CROSS PRODUCT AND PLOT THE MOMENT COMPONENTS.



$$\begin{cases} |M_{Ox}|_{max} = 0.398\ kR^2 \quad \text{AT} \quad \theta = 277° \\ |M_{Oy}|_{max} = 1.509\ kR^2 \quad \text{AT} \quad \theta = 348° \\ |M_{Oz}|_{max} = 2.26\ kR^2 \quad \text{AT} \quad \theta = 348° \end{cases}$$



$|M_O|_{max} = 2.72\ kR^2 \quad \text{AT} \quad \theta = 347°$

2/149

$$\begin{cases} R_x = \Sigma F_x = -7 \text{ kN} \\ R_y = \Sigma F_y = 4 - F_3 \cos\theta = -5 \text{ kN} & (1) \\ R_z = \Sigma F_z = F_3 \sin\theta = 6 \text{ kN} & (2) \end{cases}$$

(1): $F_3 \cos\theta = 9$

(2): $F_3 \sin\theta = 6$

Divide Eq. (2) by Eq. 1: $\tan\theta = \frac{2}{3}$

$\underline{\theta = 33.7°}$

Then $\underline{F_3 = 10.82 \text{ kN}}$

$R = \sqrt{7^2 + 5^2 + 6^2} = \underline{10.49 \text{ kN}}$

2/150

$$\begin{cases} \underline{R} = -3F\underline{k} \\ \underline{M}_O = -\dfrac{\sqrt{3}}{2} bF\underline{i} \end{cases}$$

$\underline{R} \cdot \underline{M}_O = 0$ so $\underline{R} \perp \underline{M}_O$

2/151 The given loads form two couples, each of which has an associated moment which is in the x-direction. So

$$\underline{R} = \Sigma \underline{F} = \underline{\underline{0}}$$

$$\underline{M}_O = Fb\,\underline{i} + F\left(b\frac{\sqrt{3}}{2}\right)\underline{i}$$

$$= \underline{Fb\left(1+\frac{\sqrt{3}}{2}\right)\underline{i}}$$

The resultant of the system is a couple.

2/152 $\underline{R} = \Sigma \underline{F} = -8\underline{i}$ kN

$\underline{M}_G = 50(10)\underline{k} + 8(6)\underline{j} + 8(40)\underline{k}$

$= 48\underline{j} + 820\underline{k}$ kN·m

2/153 $R = \Sigma F_z = 350 + 150 - 400 - 300 - 250 = -450$ N

$-|R|y = \Sigma M_x: \; -450y = 150(0.240) + 350(0.240)$
$-300(0.120) - 250(0.240)$

$y = -0.0533$ m or $\underline{y = -53.3 \text{ mm}}$

$|R|x = \Sigma M_y: \; 450x = 400(0.200) - 150(0.200) - 250(0.160)$

$x = 0.0222$ m or $\underline{x = 22.2 \text{ mm}}$

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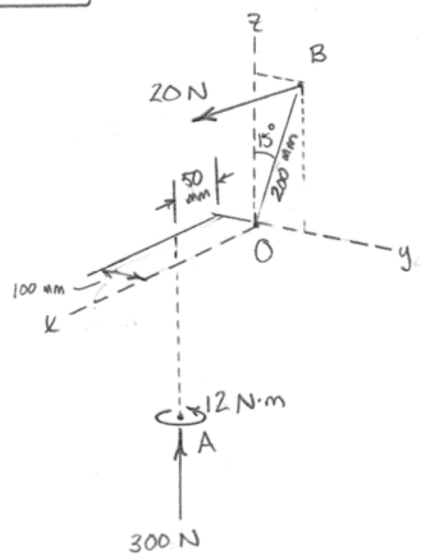

$$\underline{R} = 20\,\underline{i} + 300\,\underline{k} \text{ N}$$

$$\underline{M}_O = -300(0.1)\,\underline{i} + [20(0.2)\cos 15° - 300(0.05)]\,\underline{j} + [12 - 20(0.2\sin 15°)]\,\underline{k}$$

$$\underline{M}_O = -30\,\underline{i} - 11.14\,\underline{j} + 10.96\,\underline{k} \text{ N·m}$$

2/155 The two 160-N forces constitute a couple $160(0.250)\mathbf{j} = 40\mathbf{j}$ N·m

$\mathbf{R} = \Sigma\mathbf{F} = 120\mathbf{i} - 180\mathbf{j} - 100\mathbf{k}$ N

$\mathbf{M} = \Sigma\mathbf{M}_A = [120(0.25) + 100(0.3) + 40]\mathbf{j} + 50\mathbf{k}$ N·m

$= 100\mathbf{j} + 50\mathbf{k}$ N·m

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$$\underline{R} = \Sigma \underline{F} = 600(\sin 30° \underline{j} + \cos 30° \underline{k}) + 800(-\sin 45° \underline{j} + \cos 45° \underline{k})$$

$$= \underline{-266\underline{j} + 1085\underline{k} \text{ N}}$$

$$\underline{M}_O = -0.080\underline{i} \times 600(\sin 30° \underline{j} + \cos 30° \underline{k}) + 0.160\underline{i} \times 800(-\sin 45° \underline{j} + \cos 45° \underline{k})$$

$$= \underline{-48.9\underline{j} - 114.5\underline{k} \text{ N·m}}$$

$\underline{R}$ is not perpendicular to $\underline{M}_O$, because $\underline{R} \cdot \underline{M}_O \neq 0$.

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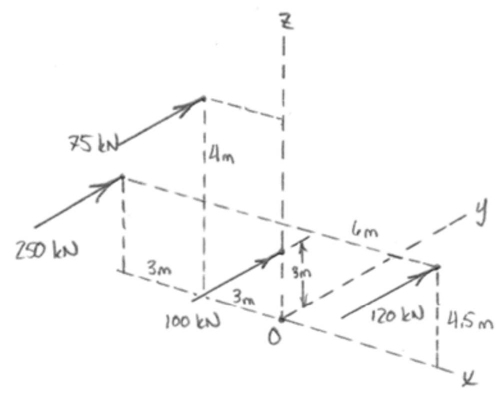


$\underline{r} = x\underline{i} + z\underline{k}$

$\underline{R} = (75 + 250 + 100 + 120)\underline{j} \longrightarrow \underline{R} = 545\,\underline{j}$ kN

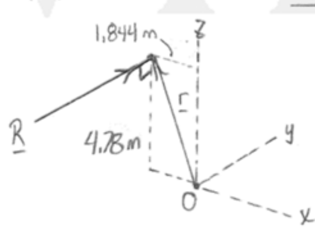
$\underline{M}_0 = -[120(4.5) + 100(3) + 250(4.5) + 75(8.5)]\,\underline{i} + [120(6) - 75(3) - 250(6)]\,\underline{k}$

$\underline{M}_0 = -2600\,\underline{i} - 1010\,\underline{k}$ kN·m

$\underline{r} \times \underline{R} = \underline{M}_0 \longrightarrow (x\underline{i} + z\underline{k}) \times 545\,\underline{j} = -2600\,\underline{i} - 1010\,\underline{k}$

$\underline{i}: -545\,z = -2600$ Solving... $x = -1.844$ m $z = 4.78$ m

$\underline{k}: 545\,x = -1010$ $\underline{P = (-1.844,\ 0,\ 4.78)\text{ m}}$



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$$\underline{R} = (200 + 800)\underline{i} + 1200(\cos 10^\circ \underline{j} - \sin 10^\circ \underline{i})$$

$$= \underline{792\underline{i} + 1182\underline{j} \text{ N}}$$

$$\underline{M}_O = [(200-800)(0.1) + (1200 \cos 10^\circ)(0.075)]\underline{k}$$

$$+[-(200+800)(0.220+0.330) + 1200 \sin 10^\circ (0.220)]\underline{j}$$

$$+[1200 \cos 10^\circ (0.220)]\underline{i}$$

$$= \underline{260\underline{i} - 504\underline{j} + 28.6\underline{k} \text{ N}\cdot\text{m}}$$

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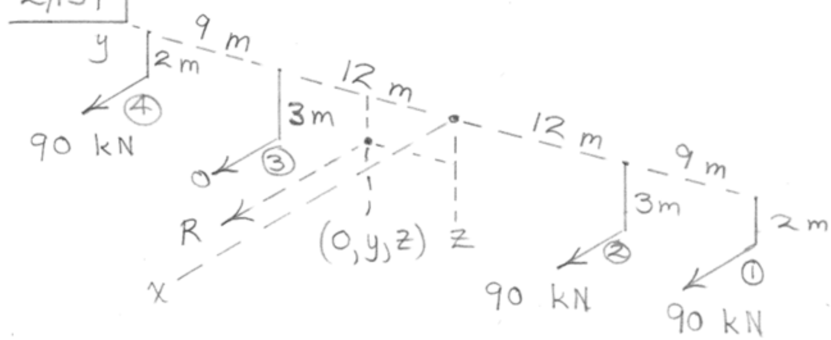

$$R = \Sigma F = 3(90) = 270 \text{ kN}$$

$$\Sigma M_z = -Ry : 90(21) + 90(12) - 90(21) = -270y$$

$$\underline{y = -4 \text{ m}}$$

$$\Sigma M_y = Rz : 2(90)(2) + 90(3) = 270z$$

$$\underline{z = 2.33 \text{ m}}$$

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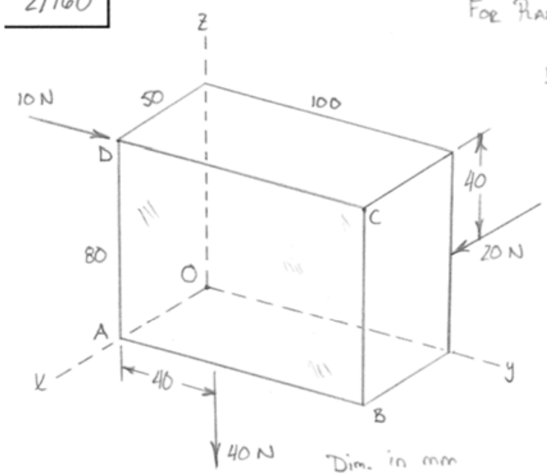


For Plane ABCD

$\underline{r} = 50\underline{i} + y\underline{j} + z\underline{k}$ mm

$\underline{R} = 20\underline{i} + 10\underline{j} - 40\underline{k}$ $\qquad$ $\underline{n} = \dfrac{\underline{R}}{R} = \dfrac{1}{\sqrt{21}}(2\underline{i} + \underline{j} - 4\underline{k})$

$$\underline{M}_O = [-10(0.08) - 40(0.04)]\,\underline{i} + [20(0.04) + 40(0.05)]\,\underline{j} + [10(0.05) - 20(0.1)]\,\underline{k}$$

$$\underline{M}_O = -2.4\,\underline{i} + 2.8\,\underline{j} - 1.5\,\underline{k} \text{ N·m}$$

$$M = \underline{M}_O \cdot \underline{n} = (-2.4\underline{i} + 2.8\underline{j} - 1.5\underline{k}) \cdot \left[\tfrac{1}{\sqrt{21}}(2\underline{i} + \underline{j} - 4\underline{k})\right] \longrightarrow \underline{M = 0.873 \text{ N·m } (+)}$$

$$\underline{M} = M\underline{n} = 0.873\left(\tfrac{2}{\sqrt{21}}\underline{i} + \tfrac{1}{\sqrt{21}}\underline{j} - \tfrac{4}{\sqrt{21}}\underline{k}\right) \longrightarrow \underline{M} = 0.381\,\underline{i} + 0.1905\,\underline{j} - 0.762\,\underline{k} \text{ N·m}$$

$$\underline{r} \times \underline{R} + \underline{M} = \underline{M}_O \longrightarrow \left(\tfrac{50}{1000}\underline{i} + \tfrac{y}{1000}\underline{j} + \tfrac{z}{1000}\underline{k}\right) \times (20\underline{i} + 10\underline{j} - 40\underline{k}) + 0.381\underline{i} + 0.1905\underline{j} -$$

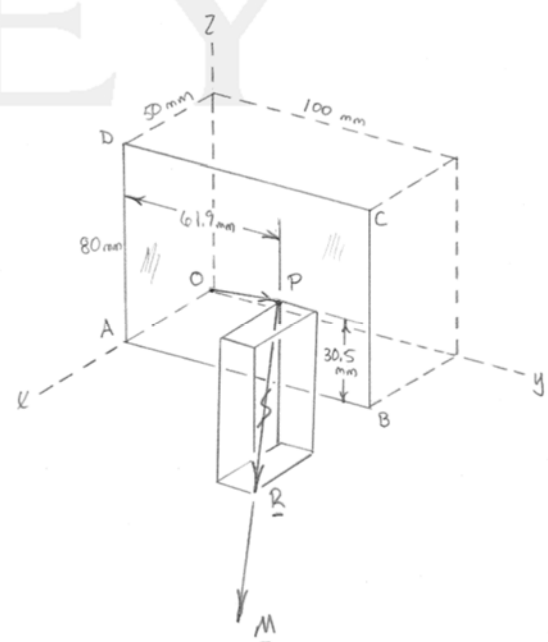
$$\ldots\ 0.762\underline{k} = -2.4\underline{i} + 2.8\underline{j} - 1.5\underline{k}$$

$$\begin{cases} \underline{i}: \ -0.04y - 0.01z + 0.381 = -2.4 \\ \underline{j}: \ 0.02z + 2.19 = 2.8 \\ \underline{k}: \ -0.02y - 0.262 = -1.5 \end{cases}$$

Solving...

$$\begin{cases} y = 61.9 \text{ mm} \\ z = 30.5 \text{ mm} \end{cases}$$

$\underline{P = (50, 61.9, 30.5) \text{ mm}}$



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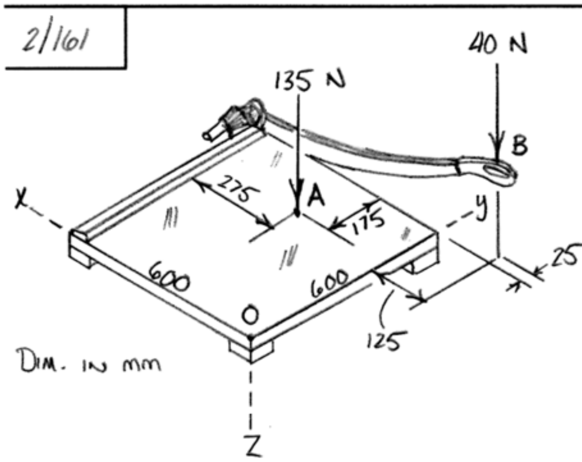


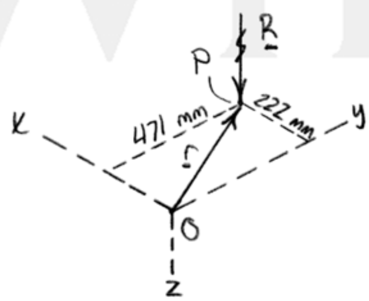
$$\begin{cases} \underline{R} = 175\,\underline{k} \text{ N} \\ \underline{r} = x\underline{i} + y\underline{j} \end{cases}$$

$$\underline{M}_O = [135(0.600 - 0.175) + 40(0.600 + 0.025)]\underline{i} + [40(0.125) - 135(0.600 - 0.275)]\underline{j}$$

$$\underline{M}_O = 82.4\underline{i} - 38.9\underline{j} \text{ N}\cdot\text{m}$$

$$\underline{r} \times \underline{R} = \underline{M}_O \longrightarrow (x\underline{i} + y\underline{j}) \times 175\underline{k} = 82.4\underline{i} - 38.9\underline{j}$$

$$\begin{cases} \underline{i}: \ 175y = 82.4 \\ \underline{j}: \ -175x = -38.9 \end{cases} \quad \text{SOLVING...} \quad \begin{cases} x = 0.222 \text{ m OR } 222 \text{ mm} \\ y = 0.471 \text{ m OR } 471 \text{ mm} \end{cases}$$



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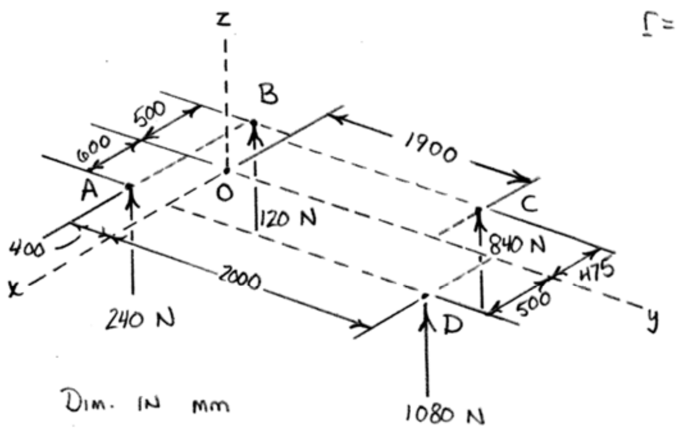
$\underline{R} = \Sigma \underline{F} = (240 + 120 + 840 + 1080)\,\underline{k} \longrightarrow \underline{\underline{R} = 2280\,\underline{k}\ \text{N}}$

$\underline{r} = x\underline{i} + y\underline{j}$

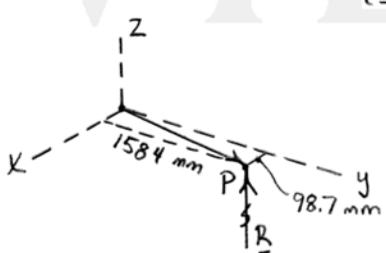


$\underline{M}_O = [-0.4(240 + 120) + 2(1080) + 1.9(840)]\,\underline{i} + [-0.6(240) + 0.5(120 - 1080) + 0.475(840)]\,\underline{j}$

$\therefore\ \underline{M}_O = 3610\,\underline{i} - 225\,\underline{j}\ \text{N·m}$

$\underline{r} \times \underline{R} = \underline{M}_O \longrightarrow (x\underline{i} + y\underline{j}) \times 2280\,\underline{k} = 3610\,\underline{i} - 225\,\underline{j}$

$\begin{cases} \underline{i}:\ 2280\,y = 3610 \\ \underline{j}:\ -2280\,x = -225 \end{cases}$ SOLVING... $\begin{cases} \underline{x = 0.0987\ \text{m} \quad \text{OR} \quad 98.7\ \text{mm}} \\ \underline{y = 1.584\ \text{m} \quad \text{OR} \quad 1584\ \text{mm}} \end{cases}$



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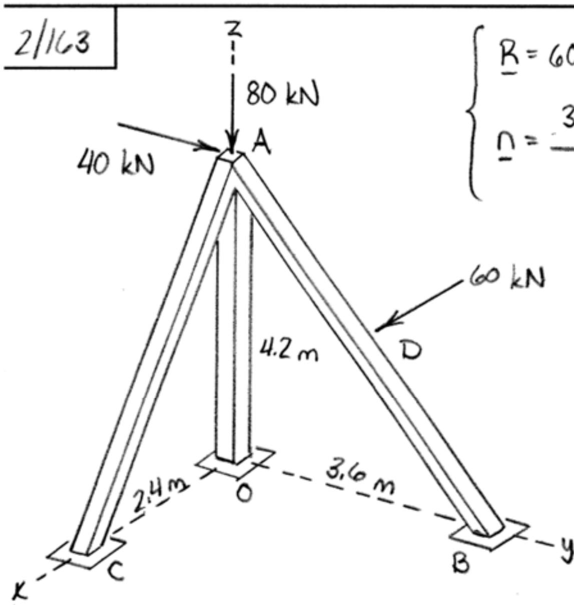


$$\begin{cases} \underline{R} = 60\underline{i} + 40\underline{j} - 80\underline{k} \text{ kN} \\ \underline{n} = \dfrac{3\underline{i} + 2\underline{j} - 4\underline{k}}{\sqrt{29}} \end{cases}$$

$$\underline{r} = x\underline{i} + y\underline{j}$$

$$\underline{M}_O = -4.2(40)\underline{i} + 2.1(60)\underline{j} - 1.8(60)\underline{k} \longrightarrow \underline{M}_O = -168\underline{i} + 126\underline{j} - 108\underline{k} \text{ kN·m}$$

$$M = \underline{M}_O \cdot \underline{n} = (-168\underline{i} + 126\underline{j} - 108\underline{k}) \cdot \left(\frac{3\underline{i} + 2\underline{j} - 4\underline{k}}{\sqrt{29}}\right) \longrightarrow M = 33.4 \text{ kN·m}$$

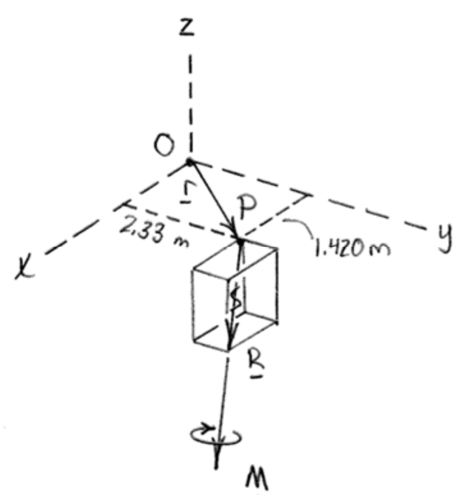
$$\underline{M} = M\underline{n} = 33.4\left(\frac{3\underline{i} + 2\underline{j} - 4\underline{k}}{\sqrt{29}}\right) \longrightarrow \underline{M} = 18.62\underline{i} + 12.41\underline{j} - 24.8\underline{k} \text{ kN·m}$$

$$\underline{r} \times \underline{R} + \underline{M} = \underline{M}_O \longrightarrow (x\underline{i} + y\underline{j}) \times (60\underline{i} + 40\underline{j} - 80\underline{k}) + 18.62\underline{i} + 12.41\underline{j} - 24.8\underline{k} = -168\underline{i} + 126\underline{j} - 108\underline{k}$$

$$\begin{cases} \underline{i}: 18.62 - 80y = -168 \\ \underline{j}: 12.41 + 80x = 126 \\ \underline{k}: -24.8 + 40x - 60y = -108 \end{cases}$$

SOLVING...

$$\begin{cases} \underline{x = 1.420 \text{ m}} \\ \underline{y = 2.33 \text{ m}} \end{cases}$$



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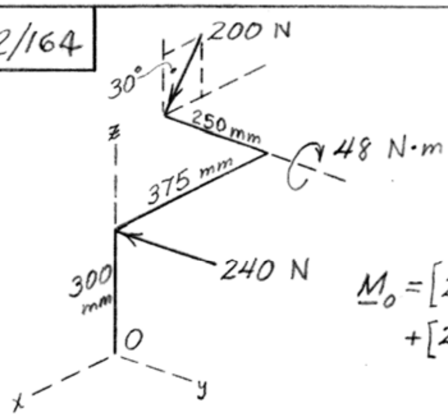


$R_x = \Sigma F_x = 200 \sin 30° = 100$ N

$R_y = \Sigma F_y = -240$ N

$R_z = \Sigma F_z = -200 \cos 30° = -173.2$ N

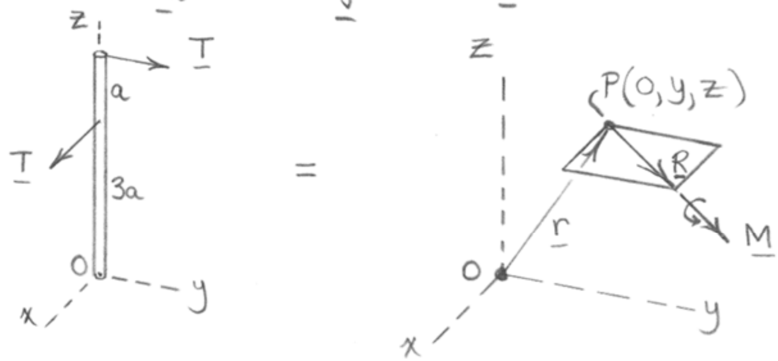
$\therefore \underline{\mathbf{R} = 100\mathbf{i} - 240\mathbf{j} - 173.2\mathbf{k}}$ N

$$\mathbf{M}_O = [240(0.300) + 200\cos 30°(0.250)]\mathbf{i} + [200\sin 30°(0.300) - 200\cos 30°(0.375) - 48]\mathbf{j} + 200\sin 30°(0.250)\mathbf{k}$$

$$= \underline{115.3\mathbf{i} - 83.0\mathbf{j} + 25\mathbf{k} \text{ N·m}}$$

2/165 $\underline{R} = \Sigma \underline{F} = T\underline{i} + T\underline{j} = \sqrt{2}T\left[\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right]$

$\Sigma \underline{M}_O = 3aT\underline{j} - 4aT\underline{i}$



$\Sigma \underline{M}_O = \underline{r} \times \underline{R} + \underline{M}$

$3aT\underline{j} - 4aT\underline{i} = (y\underline{j} + z\underline{k}) \times (T\underline{i} + T\underline{j}) + M\left(\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right)$

$$\Rightarrow \begin{cases} -4aT = -zT + \dfrac{M}{\sqrt{2}} \\ 3aT = zT + \dfrac{M}{\sqrt{2}} \\ 0 = -yT \end{cases} \qquad \text{So} \begin{cases} \underline{y = 0} \\ \underline{z = \frac{7}{2}a} \\ M = -\frac{\sqrt{2}}{2}aT \end{cases}$$

So $\underline{M} = -\frac{\sqrt{2}}{2}aT\left(\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right)$

$= \underline{\frac{-aT}{2}(\underline{i} + \underline{j})}$ (a negative wrench)

2/166

FROM 2/154... $\begin{cases} \mathbf{R} = 20\mathbf{i} + 300\mathbf{k} \text{ N} \\ \mathbf{M}_O = -30\mathbf{i} - 11.14\mathbf{j} + 10.96\mathbf{k} \text{ N·m} \end{cases}$

$$\begin{cases} \mathbf{n} = \frac{\mathbf{R}}{R} = 0.0665\mathbf{i} + 0.998\mathbf{k} \quad \text{AND} \quad \mathbf{r} = y\mathbf{j} + z\mathbf{k} \\ M = \mathbf{M}_O \cdot \mathbf{n} = (-30\mathbf{i} - 11.14\mathbf{j} + 10.96\mathbf{k}) \cdot (0.0665\mathbf{i} - 0.998\mathbf{k}) = 8.94 \text{ N·m} \\ \mathbf{M} = M\mathbf{n} = 8.94(0.0665\mathbf{i} + 0.998\mathbf{k}) \quad \underline{0.595\mathbf{i} + 8.93\mathbf{k} \text{ N·m}} \end{cases}$$

$\mathbf{r} \times \mathbf{R} + \mathbf{M} = \mathbf{M}_O$

$(y\mathbf{j} + z\mathbf{k}) \times (20\mathbf{i} + 300\mathbf{k}) + (0.0595\mathbf{i} + 8.93\mathbf{k}) = -30\mathbf{i} - 11.14\mathbf{j} + 10.96\mathbf{k}$

$$\begin{cases} \mathbf{i}: 300y + 0.0595 = -30 \\ \mathbf{j}: 20z = -11.14 \\ \mathbf{k}: -20y + 8.93 = 10.96 \end{cases} \quad \text{SOLVING...} \quad \begin{cases} \underline{y = -102.0 \text{ mm}} \\ \underline{z = -557 \text{ mm}} \end{cases}$$

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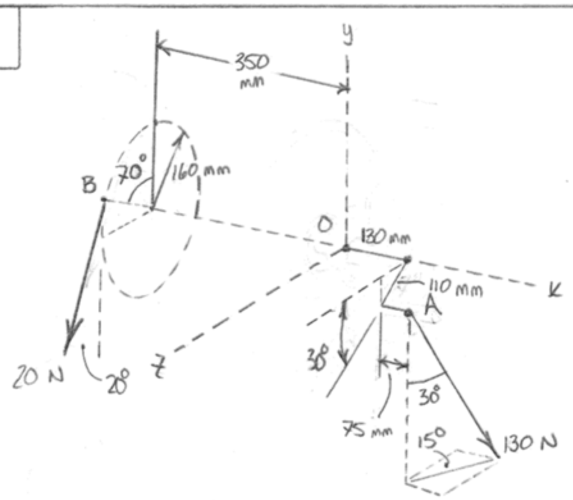


$$\underline{R} = 130\sin 30°\sin 15°\,\underline{i} - (130\cos 30° + 20\cos 20°)\,\underline{j} + (20\sin 20° - 130\sin 30°\cos 15°)\,\underline{k}$$

$$\therefore\quad \underline{R} = 16.82\,\underline{i} - 131.4\,\underline{j} - 55.9\,\underline{k}\ \text{N}$$

$$\underline{M}_O = \left[20(0.160) + 130\sin 30°\cos 15°(0.110\sin 30°) + 130\cos 30°(0.110\cos 30°)\right]\underline{i}$$

$$+ \left[20\sin 20°(0.350) + 130\sin 30°\sin 15°(0.110\cos 30°) + 130\sin 30°\cos 15°(0.130 + 0.075)\right]\underline{j}$$

$$+ \left[20\cos 20°(0.350) + 130\sin 30°\sin 15°(0.110\sin 30°) - 130\cos 30°(0.130 + 0.075)\right]\underline{k}$$

$$\therefore\quad \underline{M}_O = 17.38\,\underline{i} + 16.87\,\underline{j} - 15.58\,\underline{k}\ \text{N·m}$$

2/168

FROM 2/167 ... $\begin{cases} \mathbf{R} = 16.82\mathbf{i} - 131.4\mathbf{j} - 55.9\mathbf{k} \text{ N} \\ \mathbf{M}_O = 17.38\mathbf{i} + 16.87\mathbf{j} - 15.58\mathbf{k} \text{ N·m} \end{cases}$

$\mathbf{n} = \frac{\mathbf{R}}{R} \longrightarrow \mathbf{n} = 0.1170\mathbf{i} - 0.914\mathbf{j} - 0.389\mathbf{k}$

$M = \mathbf{M}_O \cdot \mathbf{n} = (17.38\mathbf{i} + 16.87\mathbf{j} - 15.58\mathbf{k}) \cdot (0.1170\mathbf{i} - 0.914\mathbf{j} - 0.389\mathbf{k}) \longrightarrow M = -7.32 \text{ N·m}$

(NEGATIVE SENSE)

$\mathbf{r} = x\mathbf{i} + z\mathbf{k}$

$\mathbf{r} \times \mathbf{R} + \mathbf{M} = \mathbf{M}_O \longrightarrow (x\mathbf{i} + z\mathbf{k}) \times (16.82\mathbf{i} - 131.4\mathbf{j} - 55.9\mathbf{k}) - 7.32(0.1170\mathbf{i} - 0.914\mathbf{j} - 0.389\mathbf{k})$

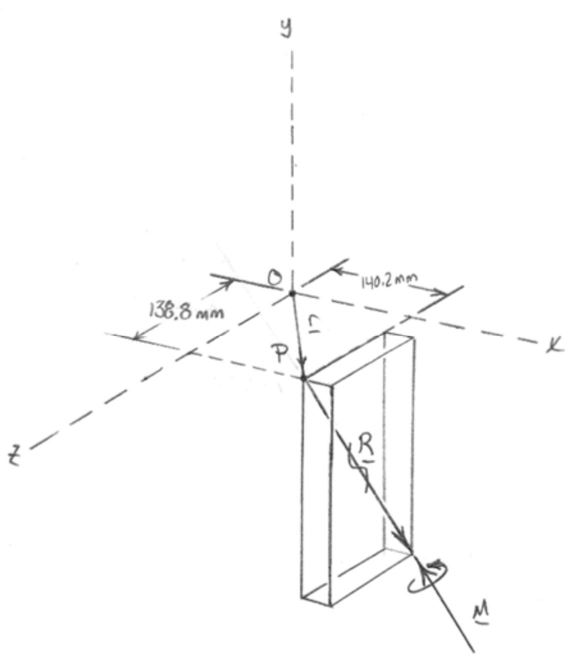
$\ldots = 17.38\mathbf{i} + 16.87\mathbf{j} - 15.58\mathbf{k}$

$\begin{cases} \mathbf{i}: 131.4z - 0.856 = 17.38 \\ \mathbf{j}: 55.9x + 16.82z + 6.69 = 16.87 \\ \mathbf{k}: 2.85 - 131.4x = -15.58 \end{cases}$

SOLVING...

$\begin{cases} \underline{x = 140.2 \text{ mm}} \\ \underline{z = 138.8 \text{ mm}} \end{cases}$

$\begin{cases} \mathbf{M} = M\hat{\mathbf{n}} = -7.32(0.1170\mathbf{i} - 0.914\mathbf{j} - 0.389\mathbf{k}) \\ \mathbf{M} = -0.856\mathbf{i} + 6.69\mathbf{j} + 2.85\mathbf{k} \text{ N·m} \end{cases}$



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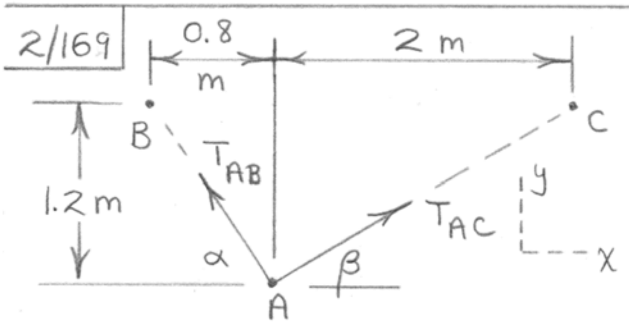

$\alpha = \tan^{-1}\left(\frac{1.2}{0.8}\right) = 56.3°$

$\beta = \tan^{-1}\left(\frac{1.2}{2}\right) = 31.0°$

$\underline{T}_{AB} = T_{AB}\underline{n}_{AB} = 0.858(60)(9.81)[-\cos 56.3°\underline{i} + \sin 56.3°\underline{j}]$

$= -280\underline{i} + 420\underline{j}$ N

$\underline{T}_{AC} = T_{AC}\underline{n}_{AC} = 0.555(60)(9.81)[\cos 31.0°\underline{i} + \sin 31.0°\underline{j}]$

$= 280\underline{i} + 168.1\underline{j}$ N

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$$\begin{cases} \underline{M}_1 = \underline{-cF_1\,\underline{i}} \\ \underline{M}_2 = cF_2\,\underline{i} - aF_2\,\underline{k} = \underline{F_2(c\underline{i} - a\underline{k})} \\ \underline{M}_3 = \underline{-aF_3\,\underline{k}} \end{cases}$$

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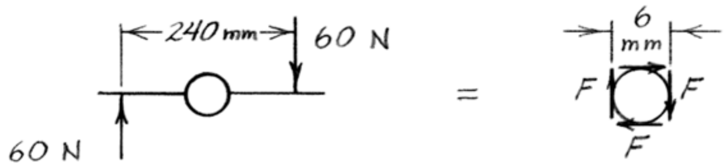

$M = Fd = 60(240) = 2F(6)$, $\underline{F = 1200 \text{ N}}$

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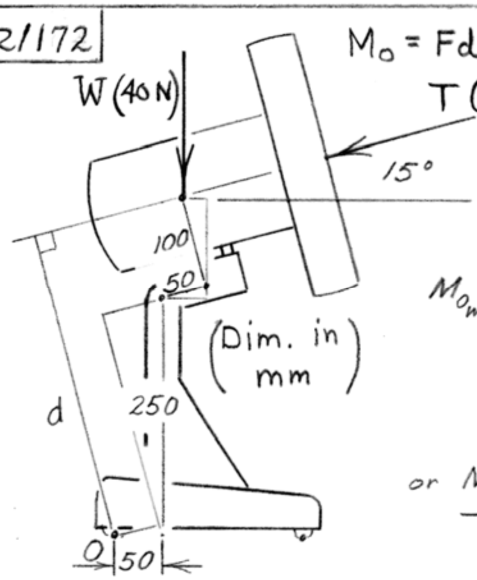

$M_O = Fd = 4(100 + 250\cos 15° - 50\sin 15°)$

$= 1314$ N·mm

or $\underline{M_O = 1.314 \text{ N·m}}$

$M_{O_W} = W d_W$

$= 40(50 + 50\cos 15° - 100\sin 15°)$

$= 2900$ N·mm

or $\underline{M_{O_W} = 2.90 \text{ N·m}}$ (↻)

2/173 $\underline{P} = P\left(\frac{4}{5}\underline{i} + \frac{3}{5}\underline{j}\right)$; $\underline{r}_{AB} = b(-\underline{i} + \underline{j} + \underline{k})$

Carry out $\underline{M}_A = \underline{r}_{AB} \times \underline{P}$ to obtain

$$\underline{M}_A = \frac{Pb}{5}(-3\underline{i} + 4\underline{j} - 7\underline{k})$$

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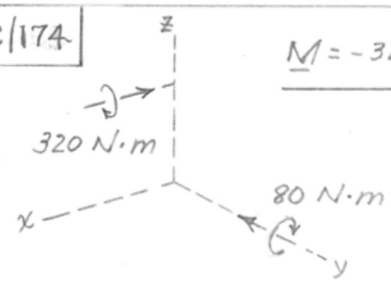

$$\underline{M} = -320\underline{i} - 80\underline{j} \quad N\cdot m$$

$$\cos\theta_x = \frac{M_x}{|\underline{M}|} = \frac{-320}{\sqrt{320^2+80^2}} = \underline{-0.970}$$

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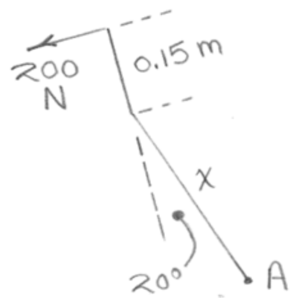

$M_A = Fd: \quad 80 = 200(0.15 + x\cos 20°)$

$x = 0.266$ m or $\underline{266 \text{ mm}}$

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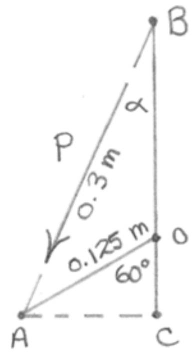

$AC = 0.125 \sin 60° = 0.1083$ m

$\alpha = \sin^{-1} \dfrac{0.1083}{0.300} = 21.2°$

$BC = 0.300 \cos \alpha = 0.280$ m

$BO = 0.280 - 0.125 \cos 60° = 0.217$ m

$\circlearrowleft + \; M_O = 720 = P \sin\alpha\,(BO) = P \sin 21.2° (0.217)$

$\underline{P = 9.18 \text{ kN}}$

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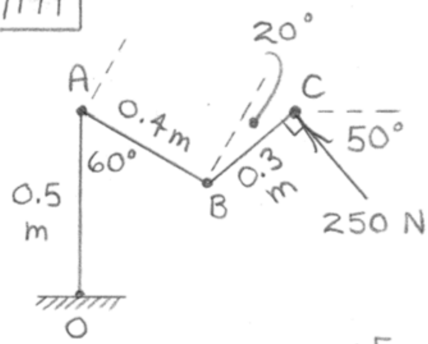

$$\curvearrowleft + \; M_O = 250\cos 50° \left[0.5 - 0.4\cos 60° + 0.3\sin 40°\right]$$

$$+ 250\sin 50° \left[0.4\sin 60° + 0.3\cos 40°\right]$$

$$= \underline{189.6 \text{ N·m CCW}}$$

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$$\underline{R} = 800\left[-\sin 30° \cos 20° \,\underline{i} + \sin 30° \sin 20° \,\underline{j} + \cos 30° \,\underline{k}\right]$$

$$= \underline{-376\,\underline{i} + 136.8\,\underline{j} + 693\,\underline{k} \quad \text{N}}$$

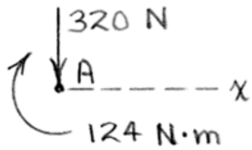
$$\underline{M}_O = \underline{r}_{OB} \times \underline{F}$$

$$\underline{r}_{OB} = \left[300 \sin 20° \,\underline{i} + 300 \cos 20° \,\underline{j} + 250\,\underline{k}\right] \text{mm}$$

$$\underline{\underline{M}_O = 161.1\,\underline{i} - 165.1\,\underline{j} + 120\,\underline{k} \quad \text{N·m}}$$

2/179 At A: $R = \Sigma F = 800 + 720 - 1200 = \underline{320\text{ N }(\downarrow)}$

$\overset{+}{\circlearrowleft} M_A = 800(0.2) + 720(0.7) - 1200(0.45) = \underline{124\text{ N}\cdot\text{m}}$

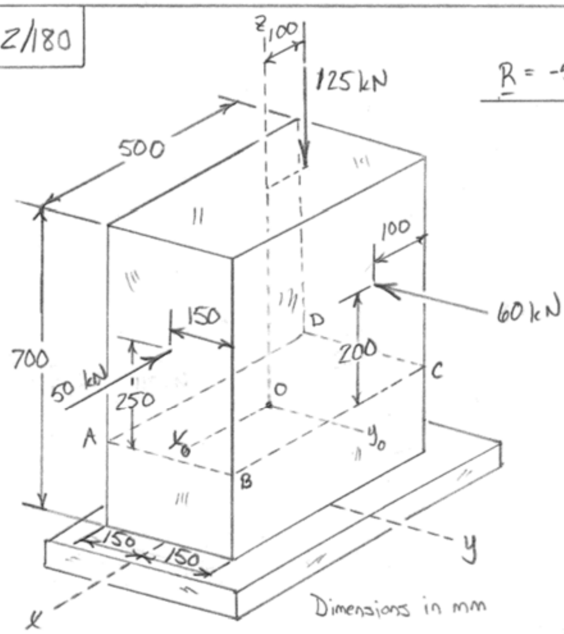


=

$320x = 124$, $x = 0.388$ m or $\underline{388\text{ mm}}$

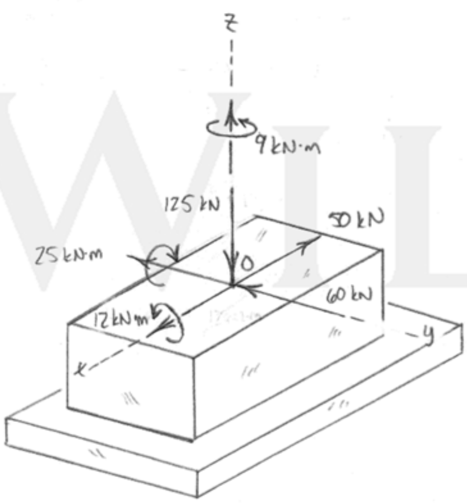
2/180

$\underline{R} = -50\underline{i} - 60\underline{j} - 125\underline{k}$ kN



$\underline{M}_O = 60(0.20)\underline{i} - [50(0.25) + 125(0.10)]\underline{j} + 60(0.25 - 0.10)\underline{k}$

$\underline{M}_O = 12\underline{i} - 25\underline{j} + 9\underline{k}$ kN·m



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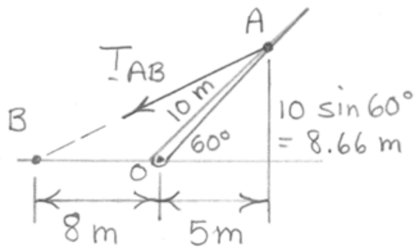

$$\overline{AB}^2 = 8^2 + 10^2 - 2(8)(10)\cos 120°$$

$$\overline{AB} = 15.62 \text{ m}$$

(a) $$\mathbf{T}_{AB} = 3\left[-\frac{13}{15.62}\cos 35°\,\mathbf{i} - \frac{13}{15.62}\sin 35°\,\mathbf{j} - \frac{8.66}{15.62}\mathbf{k}\right]$$

$$= -2.05\mathbf{i} - 1.432\mathbf{j} - 1.663\mathbf{k} \text{ kN}$$

(b) Carry out $\mathbf{r}_{OB} \times \mathbf{T}_{AB}$, where $\mathbf{r}_{OB} = 8(-\cos 35°\,\mathbf{i} - \sin 35°\,\mathbf{j})$ m to obtain

$$\mathbf{M}_O = 7.63\mathbf{i} - 10.90\mathbf{j} \text{ kN}\cdot\text{m}$$

$\therefore$ $M_{O_x} = 7.63$ kN·m, $M_{O_y} = -10.90$ kN·m, $M_{O_z} = 0$

(c) $T_{AO} = \mathbf{T}_{AB} \cdot \mathbf{n}_{AO}$

With $\mathbf{n}_{AO} = -\cos 60° \cos 35°\,\mathbf{i} - \cos 60° \sin 35°\,\mathbf{j} - \sin 60°\,\mathbf{k}$,

we obtain $T_{AO} = 2.69$ kN

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$$\underline{R} = \Sigma \underline{F} = 5\cos 45° \underline{i} + 4\underline{j} - (6 + 5\sin 45°)\underline{k}$$

$$= 3.54\underline{i} + 4\underline{j} - 9.54\underline{k} \text{ kN}$$

$$R = \sqrt{3.54^2 + 4^2 + 9.54^2} = \underline{10.93 \text{ kN}}$$

$$\underline{M} = [5\sin 45°(1.2) - 6(1.2) - 4(4)]\underline{i}$$

$$+ [5\sin 45°(2.4) + 5\cos 45°(2.8) + 6(2.4)]\underline{j}$$

$$+ [5\cos 45°(1.2) + 4(1.2)]\underline{k}$$

$$= -18.96\underline{i} + 32.8\underline{j} + 9.04\underline{k} \text{ kN}\cdot\text{m}$$

$$M = \sqrt{18.96^2 + 32.8^2 + 9.04^2} = \underline{38.9 \text{ kN}\cdot\text{m}}$$

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$\sum F_x = 0: -720 - 480 \sin\theta + T \sin 30° + 800 \cos 30° = 0$ (1)

$\sum F_y = 1200: 480 \cos\theta + T \cos 30° + 800 \sin 30° = 1200$ (2)

Numerical solution of Eqs. (1) & (2):

$\underline{\theta = 21.7°, \quad T = 409 \text{ N}}$

(We could eliminate T between Eqs. (1) & (2), but the resulting equation is still transcendental.)

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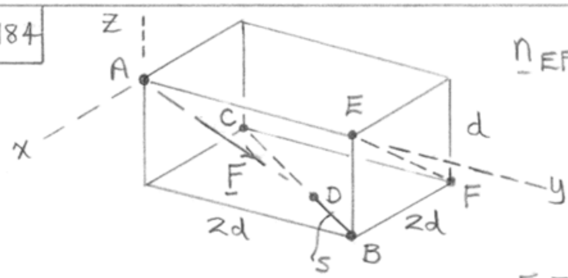


$$\underline{n}_{EF} = \frac{-2d\underline{i} - d\underline{k}}{\sqrt{5}\,d} = \frac{-2\underline{i} - \underline{k}}{\sqrt{5}}$$

$$\underline{AD} = \underline{AB} + \underline{BD} = 2d\underline{j} - d\underline{k} + s\left[\frac{-2d\underline{i} - 2d\underline{j}}{\sqrt{8}\,d}\right]$$

$$= -\frac{s}{\sqrt{2}}\underline{i} + \left(2d - \frac{s}{\sqrt{2}}\right)\underline{j} - d\underline{k}$$

$$|\underline{AD}| = \sqrt{\frac{s^2}{2} + \left(2d - \frac{s}{\sqrt{2}}\right)^2 + d^2} = \sqrt{s^2 + 5d^2 - 2\sqrt{2}\,ds}$$

$$\underline{F} = F\frac{\underline{AD}}{|\underline{AD}|} = F\,\frac{-\frac{s}{\sqrt{2}}\underline{i} + \left(2d - \frac{s}{\sqrt{2}}\right)\underline{j} - d\underline{k}}{\sqrt{s^2 + 5d^2 - 2\sqrt{2}\,ds}}$$

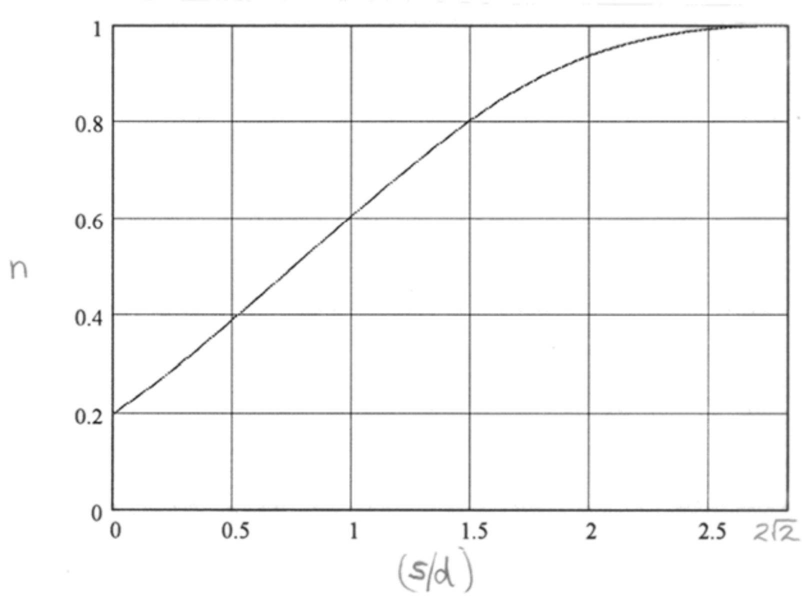
Carry out $\underline{F} \cdot \underline{n}_{EF}$ to obtain the projection

$$\underline{F} \cdot \underline{n}_{EF} = \frac{F\left(s\sqrt{2} + d\right)}{\sqrt{5}\sqrt{s^2 + 5d^2 - 2\sqrt{2}\,ds}}$$

The nondimensionalized fraction n of the magnitude F projected is then

$$n = \frac{\underline{F} \cdot \underline{n}_{EF}}{F} = \frac{\sqrt{2}\,\frac{s}{d} + 1}{\sqrt{5}\sqrt{\left(\frac{s}{d}\right)^2 + 5 - 2\sqrt{2}\,\frac{s}{d}}}$$

We let $\frac{s}{d}$ vary from 0 to $2\sqrt{2}$ as D moves from B to C. Resulting plot:



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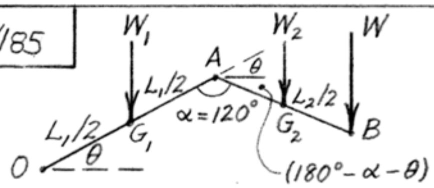


$$\begin{cases} W_1 = 600 \text{ N} & L_1 = 900 \text{ mm} \\ W_2 = 250 \text{ N} & L_2 = 600 \text{ mm} \\ W = 1500 \text{ N} & \alpha = 120^\circ \end{cases}$$

$$\circlearrowleft + \; M_O = W_1 \frac{L_1}{2}\cos\theta + W_2\left[L_1\cos\theta + \frac{L_2}{2}\cos(180^\circ - \alpha - \theta)\right] + W\left[L_1\cos\theta + L_2\cos(180^\circ - \alpha - \theta)\right]$$

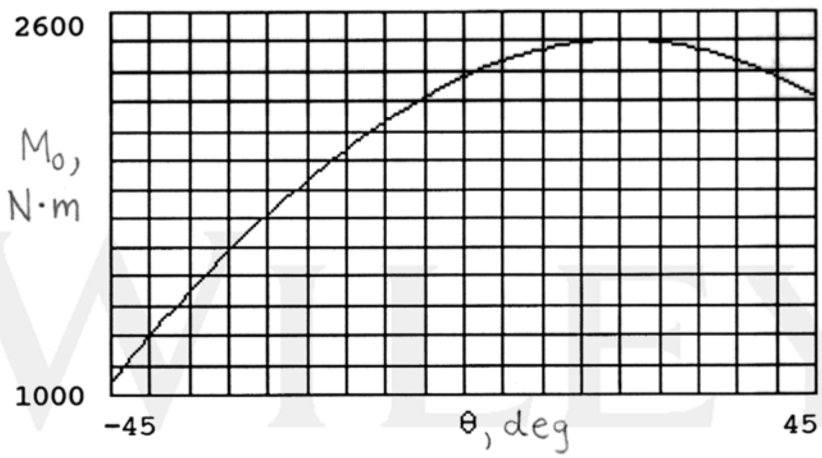
With the above numbers:

$$\underline{M_O = 1845\cos\theta + 975\cos(60^\circ - \theta)} \quad (\text{in N}\cdot\text{m})$$

(see plot below)

For $(M_O)_{max}$: $\dfrac{dM_O}{d\theta} = -1845\sin\theta + 975\sin(60^\circ - \theta) = 0$

Solution: $\underline{\theta = 19.90^\circ}$, $\underline{(M_O)_{max} = 2480 \text{ N}\cdot\text{m}}$



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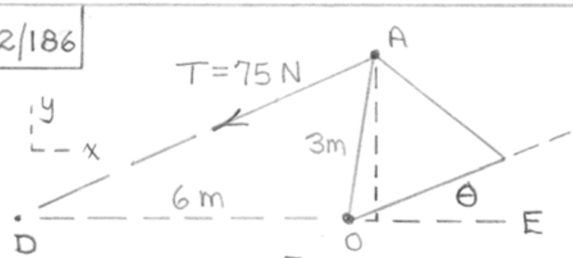


Angle AOE = $\theta + 60°$

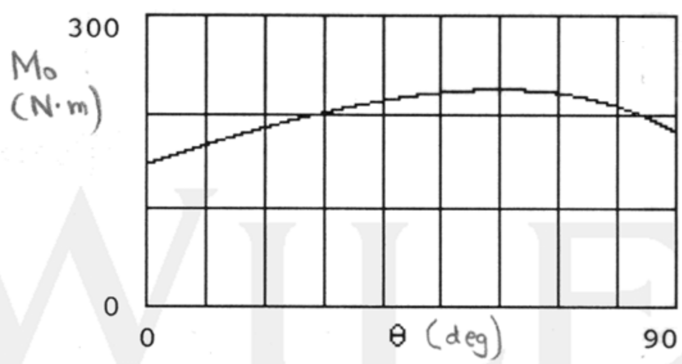
Use $\underline{M}_O = \underline{r}_{OD} \times \underline{T}$

$\underline{r}_{OD} = -6\underline{i}$ m

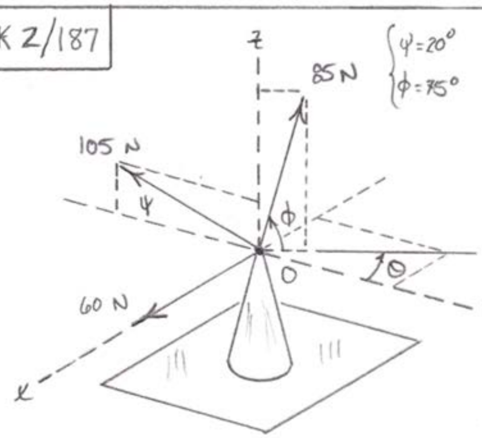
$$\underline{T} = T\underline{n}_{AD} = 75\left[\frac{-(6+3\cos(\theta+60°))\underline{i} - 3\sin(\theta+60°)\underline{j}}{\sqrt{[6+3\cos(\theta+60°)]^2 + [3\sin(\theta+60°)]^2}}\right] \text{N}$$

$$\underline{M}_O = \underline{r}_{OD} \times \underline{T} = \frac{1350\sin(\theta+60°)\,\underline{k}}{\sqrt{45+36\cos(\theta+60°)}} \ \text{N}\cdot\text{m}$$

M_O is a max @ $\underline{\theta = 60°}$: $\underline{M_O = 225 \text{ N}\cdot\text{m}}$



*2/187



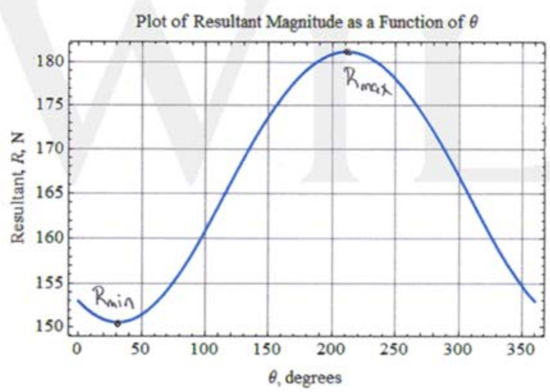
$\psi = 20°$, $\phi = 75°$

$$\mathbf{R} = (60 - 85\cos 75° \sin\theta)\hat{i} + (85\cos 75° \cos\theta - 105\cos 20°)\hat{j} + (85\sin 75° + 105\sin 20°)\hat{k}$$

$$\mathbf{R} = (60 - 22.0\sin\theta)\mathbf{i} + (22.0\cos\theta - 98.7)\mathbf{j} + 118.0\mathbf{k} \text{ N}$$

$$R = \sqrt{\mathbf{R}\cdot\mathbf{R}} \longrightarrow R = \left[(60 - 22.0\sin\theta)^2 + (22.0\cos\theta - 98.7)^2 + 118.0^2\right]^{1/2}$$

Plotting R... For θ_{max}, set $\dfrac{dR}{d\theta} = 0$



$R_{min} = 150.6$ N

$\theta_{min} = 31.3°$

$R_{max} = 181.2$ N

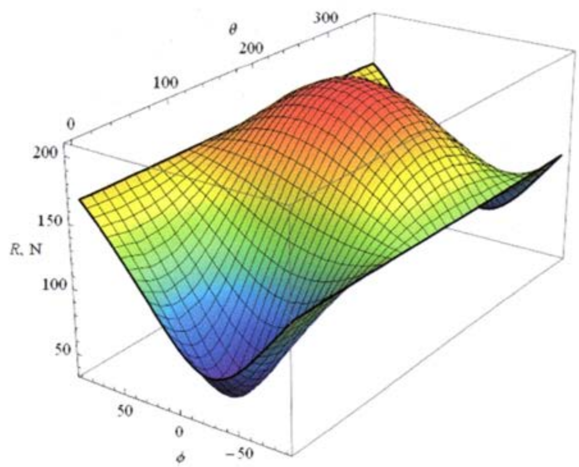
$\theta_{max} = 211°$

$\boxed{*2/188}$

FROM 2/187...

$$\underline{R} = (60 - 85\cos\phi\sin\theta)\underline{i} + (85\cos\phi\cos\theta - 105\cos 2\theta)\underline{j} + (85\sin\phi + 105\sin 2\theta)\underline{k}$$

$R = \sqrt{\underline{R}\cdot\underline{R}}$ AND WE PLOT FOR $0 \leq \theta \leq 360°$ AND $-90° \leq \phi \leq 90°$



$R_{min} = 35.9$ N

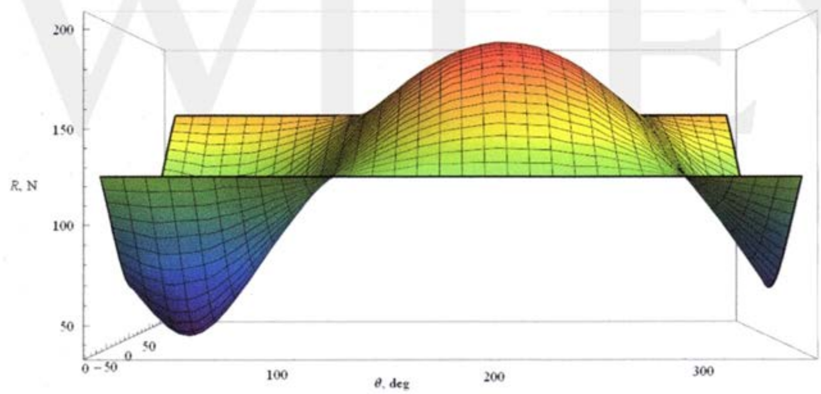
$\theta_{min} = 31.3°$

$\phi_{min} = -17.27°$

$R_{max} = 206$ N

$\theta_{max} = 211°$

$\phi_{max} = 17.27°$



*2/189 $\underline{T} = T\underline{n}_{AB}$

$$\underline{T} = T\left[\frac{(d+40\cos\beta)\underline{i} + 40(1-\sin\beta)\underline{j}}{\sqrt{(d+40\cos\beta)^2 + 40^2(1-\sin\beta)^2}}\right]$$

$$\underline{r}_{OB} = (d\underline{i} + 40\underline{j}) \qquad \left(\beta = \theta + \frac{\pi}{4}\right)$$

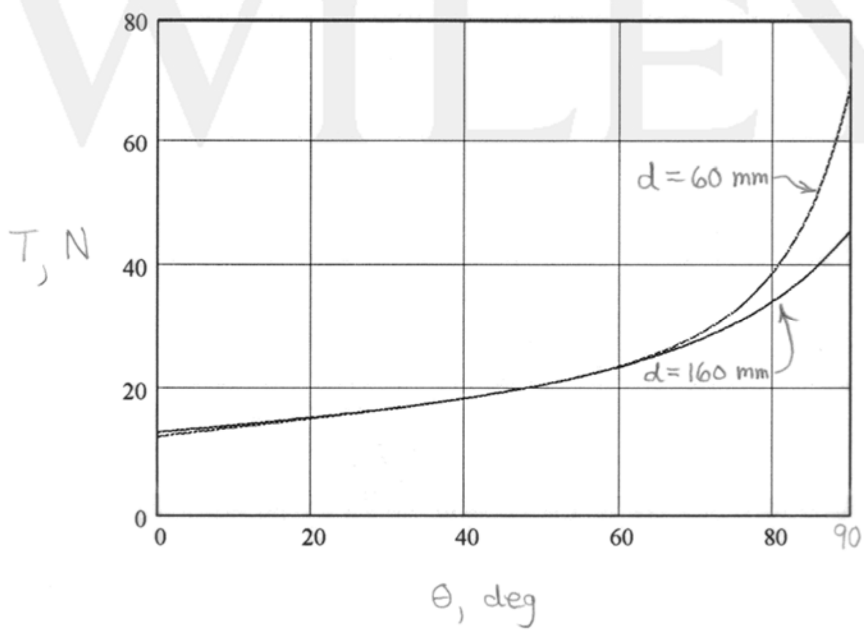
$\Sigma \underline{M}_o = \underline{0}$: $\quad \underline{r}_{OB} \times \underline{T} + K\beta\underline{k} = \underline{0}$

Carry out the cross product, consider the z-component, and solve for T to obtain

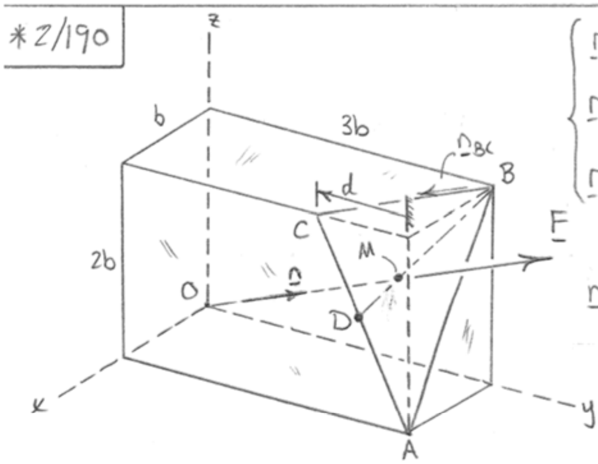
$$T = \frac{12.5\left(\theta+\frac{\pi}{4}\right)\sqrt{d^2 + 80d\cos\left(\theta+\frac{\pi}{4}\right) - 3200\sin\left(\theta+\frac{\pi}{4}\right) + 3200}}{\left[d\sin\left(\theta+\frac{\pi}{4}\right) + 40\cos\left(\theta+\frac{\pi}{4}\right)\right]}$$

(T in N, θ in radians)

Plot of T versus θ for d = 60 mm and for d = 160 mm:



*2/190



$$\begin{cases} \underline{r}_{OB} = 3b\,\underline{j} + 2b\,\underline{k} \\ \underline{r}_{OC} = b\,\underline{i} + (3b-d)\,\underline{j} + 2b\,\underline{k} \\ \underline{r}_{OD} = b\,\underline{i} + \left(3b - \frac{d}{2}\right)\underline{j} + b\,\underline{k} \end{cases}$$

$$\underline{n}_{BC} = \frac{b\,\underline{i} + d\,\underline{j}}{\sqrt{b^2 + d^2}}$$

$$\underline{r}_{OM} = \underline{r}_{OD} + \tfrac{1}{3}(\underline{r}_{OB} - \underline{r}_{OD}) \longrightarrow \underline{r}_{OM} = \tfrac{2}{3}b\,\underline{i} + \left(3b - \tfrac{d}{3}\right)\underline{j} + \tfrac{4}{3}b\,\underline{k}$$

$$\underline{n} = \frac{\underline{r}_{OM}}{|\underline{r}_{OM}|} \longrightarrow \hat{n} = \frac{2b\,\underline{i} + (9b-d)\,\underline{j} + 4b\,\underline{k}}{\sqrt{101b^2 - 18bd + d^2}}$$

$$\underline{F} = F\underline{n} \longrightarrow \underline{F} = \frac{2bF\,\underline{i} + (9b-d)F\,\underline{j} + 4bF\,\underline{k}}{\sqrt{101b^2 - 18bd + d^2}}$$

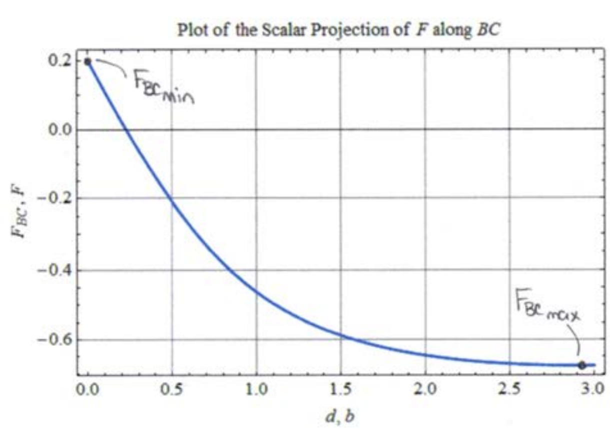
$$F_{BC} = \underline{F}\cdot\hat{n}_{BC} \longrightarrow \underline{F_{BC} = \frac{(2b^2 - 9bd + d^2)F}{\sqrt{b^2+d^2}\,\sqrt{101b^2 - 18bd + d^2}}}$$

For maximum F_{BC}, set $\frac{dF_{BC}}{dd} = 0 \longrightarrow \dfrac{Fb^2(-891b^3 + 79b^2d + 27bd^2 + 17d^3)}{(b^2+d^2)^{3/2}\,(101b^2 - 18bd + d^2)^{3/2}} = 0$

Solving... $\underline{d = 2.93\,b}$

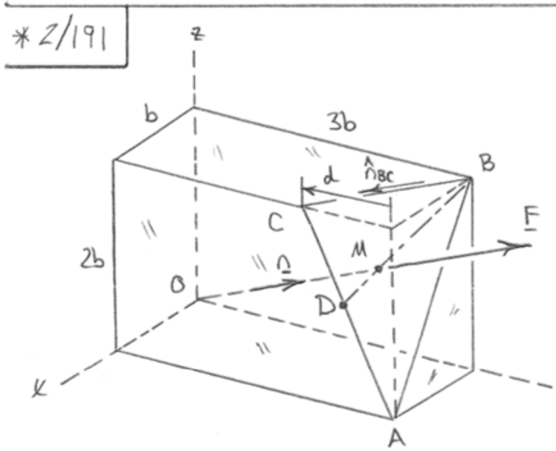
Substituting $d = 2.93b$ into F_{BC} ... $\underline{|F_{BC_{max}}| = 0.676\,F}$

Plotting F_{BC} ...



From plot, $F_{BC_{min}} = 0.1990\,F$ when $d = 0$

*2/191



$$\begin{cases} \underline{r}_{OB} = 3b\underline{j} + 2b\underline{k} \\ \underline{r}_{OC} = b\underline{i} + (3b-d)\underline{j} + 2b\underline{k} \\ \underline{r}_{OD} = b\underline{i} + \left(3b - \frac{d}{2}\right)\underline{j} + b\underline{k} \\ \underline{r}_{OM} = \frac{2}{3}b\underline{i} + \left(3b - \frac{d}{3}\right)\underline{j} + \frac{4}{3}b\underline{k} \end{cases}$$

FROM 2/190 ...

$$\underline{n}_{BC} = \frac{b\underline{i} - d\underline{j}}{\sqrt{b^2+d^2}} \quad \text{AND} \quad \underline{F} = \frac{2bF\underline{i} + (9b-d)F\underline{j} + 4bF\underline{k}}{\sqrt{101b^2 - 18bd + d^2}}$$

$$\underline{r}_{BM} = \underline{r}_{OM} - \underline{r}_{OB} \longrightarrow \underline{r}_{BM} = \frac{2}{3}b\underline{i} - \frac{d}{3}\underline{j} - \frac{2b}{3}\underline{k}$$

$$\underline{M}_B = \underline{r}_{BM} \times \underline{F} \longrightarrow \underline{M}_B = \frac{2bF(3b-d)\underline{i} - 4b^2F\underline{j} + 6b^2F\underline{k}}{\sqrt{101b^2 - 18bd + d^2}}$$

$$M_{BC} = \underline{M}_B \cdot \underline{n}_{BC} \longrightarrow \underline{M_{BC} = \frac{2b^2(3b+d)F}{\sqrt{b^2+d^2}\sqrt{101b^2-18bd+d^2}}}$$

FOR MAX M_{BC}, SET $\dfrac{dM_{BC}}{dd} = 0$

$$\text{SO...} \quad \frac{2Fb^2(128b^4 - 315b^3d + 81b^2d^2 + 3bd^3 - d^4)}{(b^2+d^2)^{3/2}(101b^2 - 18bd + d^2)^{3/2}} = 0$$

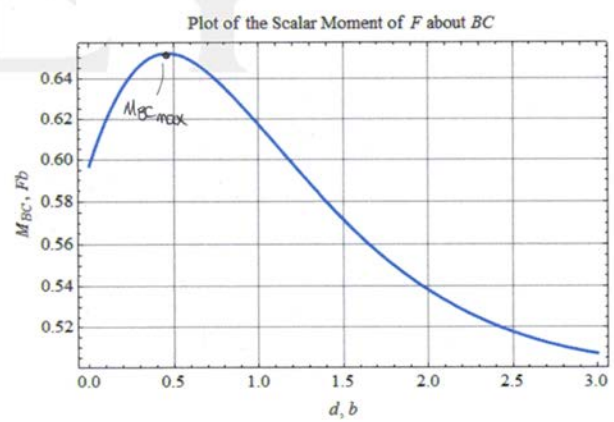
SOLVING... $d = -9.38b,\ 8.39b,\ 3.52b$, OR $0.462b$

ONLY PHYSICALLY POSSIBLE ANSWER IS... $\underline{d = 0.462b}$

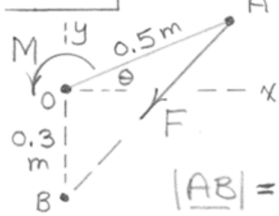
SUBSTITUTING $d = 0.462b$ INTO M_{BC} YIELDS...

$$\underline{M_{BC_{max}} = 0.652\ Fb}$$

PLOTTING M_{BC}...



*2/192



$$\underline{F} = F\underline{n}_{AB} = F\frac{\underline{AB}}{|\underline{AB}|}$$

$$\underline{AB} = -0.5\cos\theta\,\underline{i} - (0.5\sin\theta + 0.3)\underline{j}\ \text{m}$$

$$|\underline{AB}| = \sqrt{(0.5\cos\theta)^2 + (0.5\sin\theta+0.3)^2}$$

$$= \sqrt{0.34 + 0.3\sin\theta}\ \ \text{m}$$

$$F = k\delta = 600\left[\sqrt{0.34+0.3\sin\theta} - 0.65\right]\ \text{N}$$

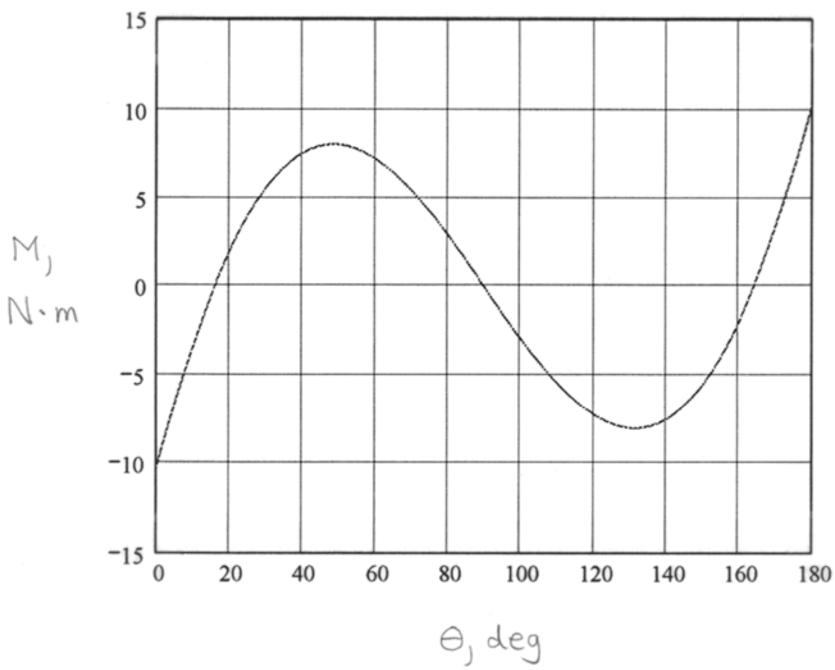
So

$$\underline{F} = \frac{600\left[\sqrt{0.34+0.3\sin\theta} - 0.65\right]}{\sqrt{0.34+0.3\sin\theta}}\left[-0.5\cos\theta\,\underline{i} - (0.5\sin\theta + 0.3)\underline{j}\right]$$

Now form $\underline{r}_{OB} \times \underline{F}$, where $\underline{r}_{OB} = -0.3\underline{j}$ m, to obtain $-\dfrac{90\cos\theta\left(\sqrt{0.34+0.3\sin\theta} - 0.65\right)}{\sqrt{0.34+0.3\sin\theta}}\,\underline{k}$

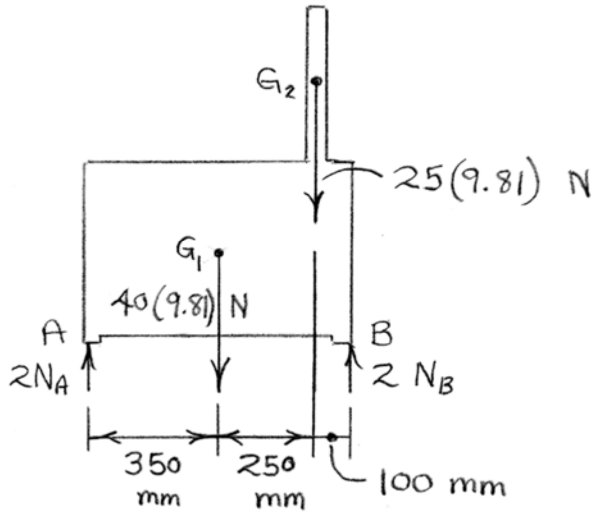
With the above moment plus $M\underline{k}$ summing to zero, we obtain the scalar

$$M = \frac{90\cos\theta\left[\sqrt{0.34+0.3\sin\theta} - 0.65\right]}{\sqrt{0.34+0.3\sin\theta}}$$



(Note: $M(0) = -10.33$ N·m
$M(180°) = 10.33$ N·m)

3/1

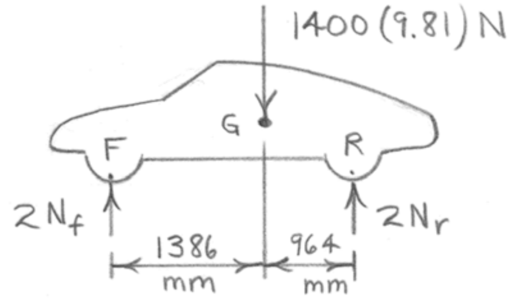


$$+\uparrow \Sigma F = 0 : \; 2N_A + 2N_B - (25+40)9.81 = 0$$

$$\circlearrowleft+ \Sigma M_A = 0 : \; -40(9.81)(350) - 25(9.81)(600) + 2N_B(700) = 0$$

$$\Rightarrow \begin{cases} N_A = 115.6 \text{ N} \\ N_B = 203 \text{ N} \end{cases}$$

3/2



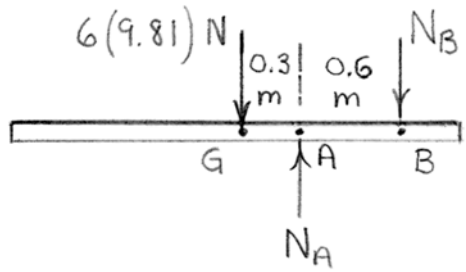
$+\uparrow \Sigma F = 0: \quad 2N_f + 2N_r - 1400(9.81) = 0$

$\Sigma M_F = 0: \quad -1400(9.81)(1386) + 2N_r(1386 + 964) = 0$

Solution : $\begin{cases} N_f = 2820 \text{ N} \\ N_r = 4050 \text{ N} \end{cases}$

Assumes G midway between left and right wheels.

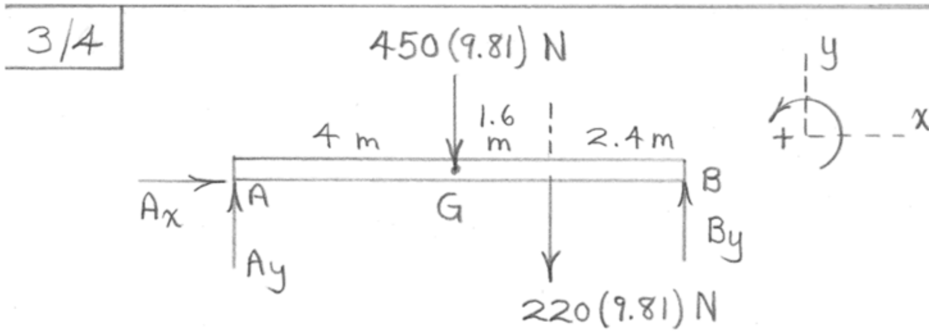
3/3



$$\circlearrowleft + \sum M_B = 0: \quad 6(9.81)(0.9) - N_A(0.6) = 0$$

$$\underline{N_A = 88.3 \text{ N}}$$

3/4



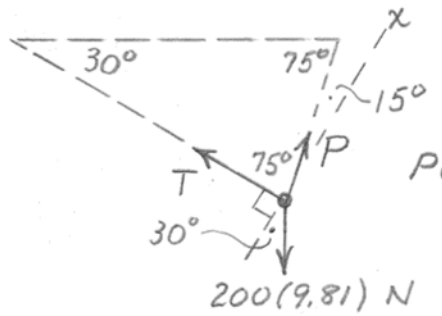
From $\Sigma F_x = 0$, $A_x = 0$

$$\Sigma M_A = 0: -450(9.81)4 - 220(9.81)(5.6) + B_y(8) = 0, \quad \underline{B_y = 3720 \text{ N}}$$

$$\Sigma F_y = 0: A_y - 450(9.81) - 220(9.81) + 3720 = 0$$

$$\underline{A_y = 2850 \text{ N}}$$

3/5

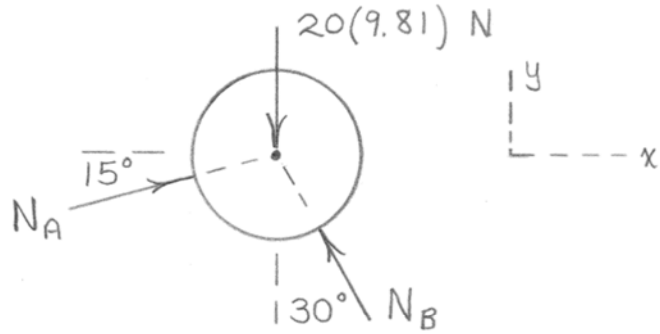


$\Sigma F_x = 0$

$P\cos 15° - 200(9.81)\cos 30° = 0$

$\underline{P = 1759\text{ N}}$

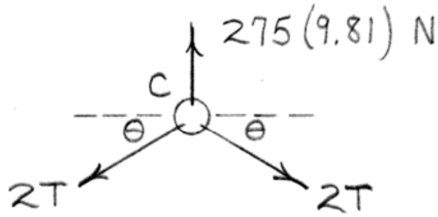
3/6



$$\begin{cases} \Sigma F_x = 0: & N_A \cos 15° - N_B \sin 30° = 0 \quad (1) \\ \Sigma F_y = 0 & N_A \sin 15° + N_B \cos 30° - 20(9.81) = 0 \quad (2) \end{cases}$$

Solution : $\begin{cases} N_A = 101.6 \text{ N} \\ N_B = 196.2 \text{ N} \end{cases}$

3/7 FBD of junction ring C:



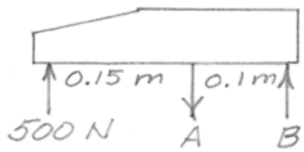
$\theta = \tan^{-1}\frac{250}{900/2}$

$= 29.1°$

$\uparrow + \Sigma F = 0: \quad 275(9.81) - 4T \sin 29.1° = 0$

$\underline{T = 1389 \text{ N}}$

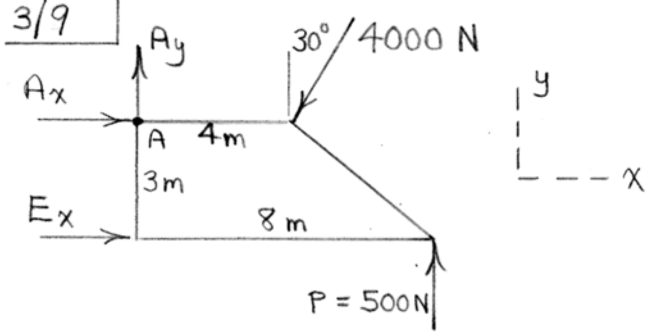
3/8



$\Sigma M_B = 0$; $500(0.25) - 0.1A = 0$

$A = 1250$ N

3/9



$\Sigma F_x = 0: \; A_x + E_x - 4000 \sin 30° = 0$

$\Sigma F_y = 0: \; A_y - 4000 \cos 30° + 500 = 0$

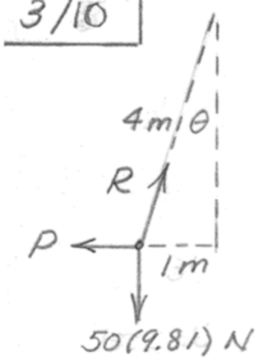
$\Sigma M_A = 0: \; E_x(3) + 500(8) - 4000 \cos 30° (4) = 0$

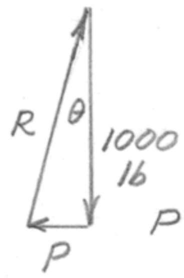
$\Rightarrow \; \underline{A_x = -1285 \text{ N}}, \; \underline{A_y = 2960 \text{ N}}, \; \underline{E_x = 3290 \text{ N}}$

For maximum P: $E_x = 0$ and $\Sigma M_A = 0$:

$P(8) - 4000 \cos 30° (4) = 0, \; \underline{P = 1732 \text{ N}}$

3/10





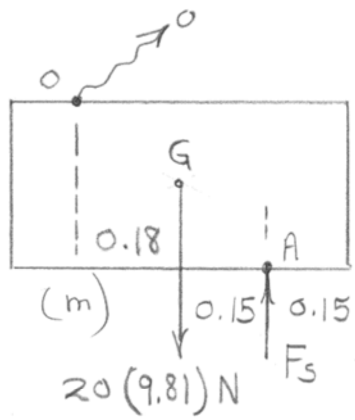
$P = 50(9.81)\tan\theta$

$\sin\theta = 1/4$

$\tan\theta = 1/\sqrt{4^2 - 1^2} = 0.258$

$P = 50(9.81)(0.258) = \underline{126.6}\ N$

3/11



$$\curvearrowleft + \sum M_O = 0: \; -20(9.81)(0.18) + F_s(0.18+0.15) = 0$$

$$F_s = 107.0 \text{ N}$$

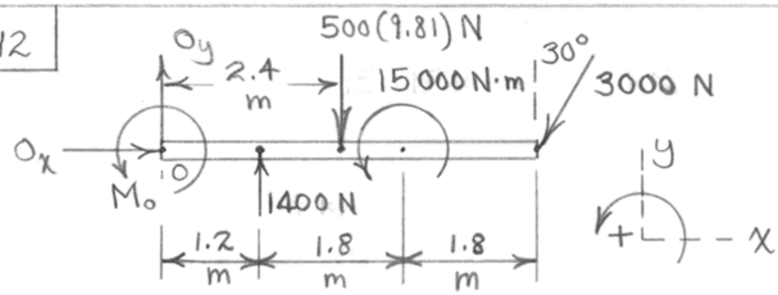
$$F_s = k\delta: \quad 107.0 = 2000\,\delta, \quad \delta = 0.0535 \text{ m}$$

$$\text{or} \quad \delta = 53.5 \text{ mm}$$

$$L = 0.1 + \delta = 0.1 + 0.0535 = 0.1535 \text{ m}$$

$$\text{or} \quad \underline{153.5 \text{ mm}}$$

3/12



$\Sigma F_x = 0: \; O_x - 3000 \sin 30° = 0, \quad \underline{O_x = 1500 \text{ N}}$

$\Sigma F_y = 0: \; O_y + 1400 - 500(9.81) - 3000 \cos 30° = 0$

$\underline{O_y = 6100 \text{ N}}$

$\Sigma M_o = 0: \; M_o + 1400(1.2) - 500(9.81)(2.4)$

$+ 15000 - (3000 \cos 30°)(4.8) = 0$

$\underline{M_o = 7560 \text{ N·m}}$

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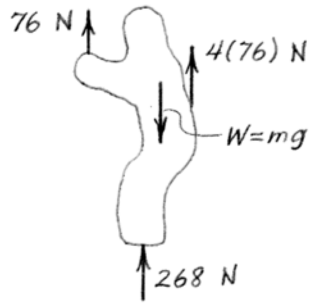
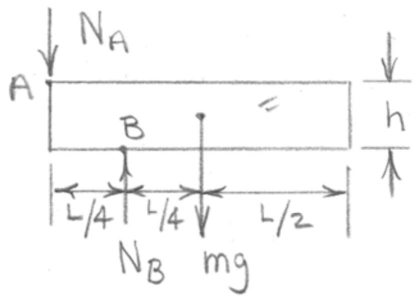


$\Sigma F = 0: \; 76 + 4(76) + 268 - W = 0$

$\underline{W = 648 \text{ N}}$

$m = \dfrac{W}{g} = \dfrac{648}{9.81} = \underline{66.1 \text{ kg}}$

3/14



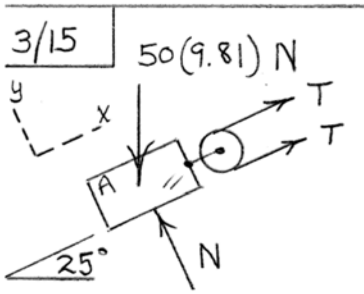
$\uparrow \Sigma F = 0: \quad N_B - N_A - mg = 0$

$\circlearrowleft_+ \Sigma M_A = 0: \quad N_B\left(\frac{L}{4}\right) - mg\left(\frac{L}{2}\right) = 0$

Solution: $\begin{cases} N_A = mg & \text{(down)} \\ N_B = 2mg & \text{(up)} \end{cases}$

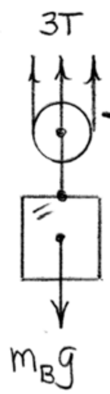
The height h has no bearing on the above results, assuming no friction at A and B.

3/15



$\Sigma F_x = 0: \; 2T - 50(9.81)\sin 25° = 0$

$T = 103.6 \text{ N}$

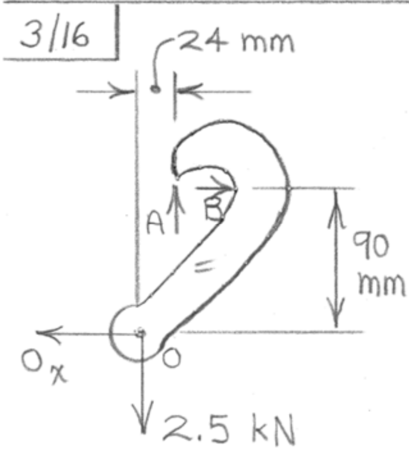


$+\uparrow \Sigma F = 0: \; 3(103.6) - m_B(9.81) = 0$

$\underline{m_B = 31.7 \text{ kg}}$

(Assume all cables to be vertical)

3/16



From vertical equilibrium

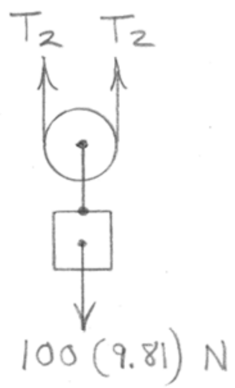
$\underline{A = 2.5 \text{ kN}}$

$\circlearrowleft + \Sigma M_O = 0:$

$2.5(24) - B(90) = 0$

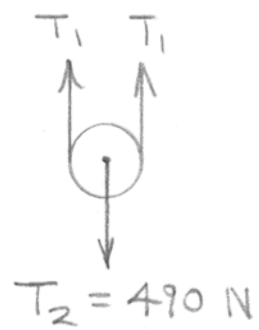
$\underline{B = 0.667 \text{ kN}}$

3/17



From $+\uparrow \Sigma F = 0$, $T_2 = \frac{100(9.81)}{2}$

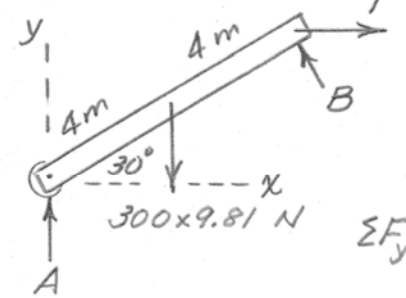
$T_2 = 490$ N



Similarly, $T_1 = \frac{1}{2} T_2$

$= \frac{1}{2}(490) = \underline{245 \text{ N}}$

3/18



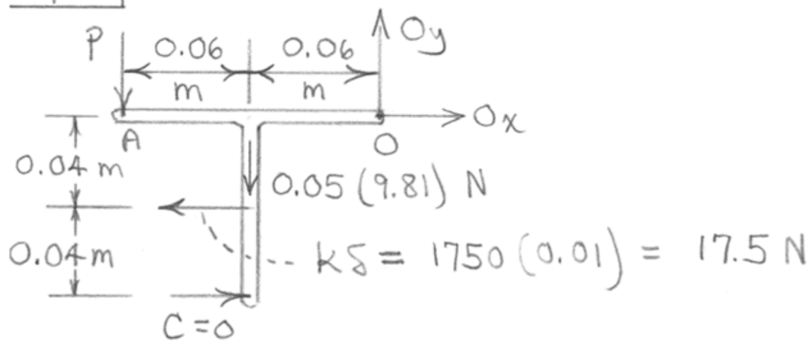
$\Sigma M_B = 0$; $300(9.81)4\cos 30° - 8A\cos 30° = 0$, $\underline{A = 1472 \text{ N}}$

$\Sigma F_y = 0$; $B\cos 30° + 1472 - 300(9.81) = 0$

$B = 1699 \text{ N}$

$\Sigma F_x = 0$; $T - 1699\sin 30° = 0$, $\underline{T = 850 \text{ N}}$

3/19



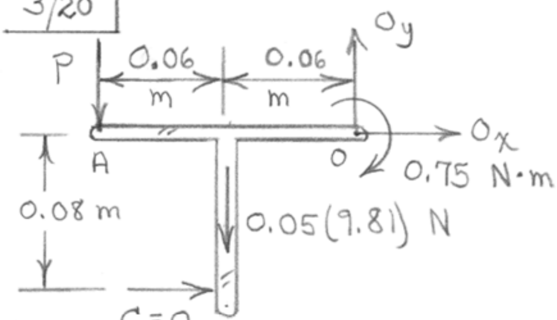
(a) $\sum M_O = 0$: $P(0.12) + 0.05(9.81)(0.06) - 17.5(0.04) = 0$

$\underline{P = 5.59 \text{ N}}$

(b) $\sum M_O = 0$: $P(0.12) - 17.5(0.04) = 0$

$\underline{P = 5.83 \text{ N}}$

3/20



(a)

$$\circlearrowleft + \ \Sigma M_O = 0: \quad P(0.12) + 0.05(9.81)(0.06) - 0.75 = 0$$

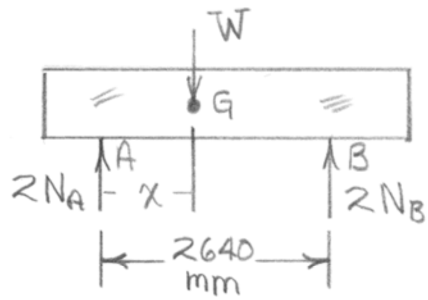
$$\underline{P = 6.00 \text{ N}}$$

(b)

$$\circlearrowleft + \ \Sigma M_O = 0: \quad P(0.12) - 0.75 = 0$$

$$\underline{P = 6.25 \text{ N}}$$

3/21 Car modeled as a slab and viewed from the driver's (left) side :



$$\begin{cases} N_A = 4450 \text{ N} \\ N_B = 2950 \text{ N} \end{cases}$$

$$+\uparrow \Sigma F = 0 : \quad 2(4450) + 2(2950) - W = 0$$

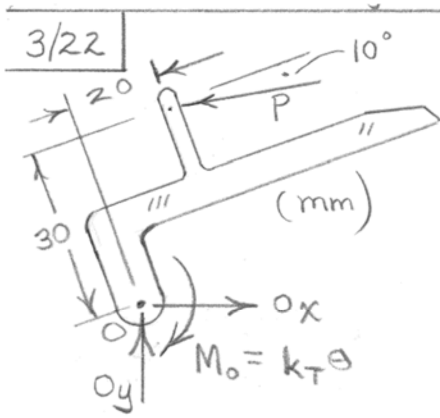
$$W = 14\,800 \text{ N}$$

$$m = \frac{W}{g} = \frac{14\,800}{9.81} = \underline{1509 \text{ kg}}$$

$$\circlearrowleft + \Sigma M_A = 0 : \quad -14\,800x + 2(2950)(2640) = 0$$

$$\underline{x = 1052 \text{ mm}}$$

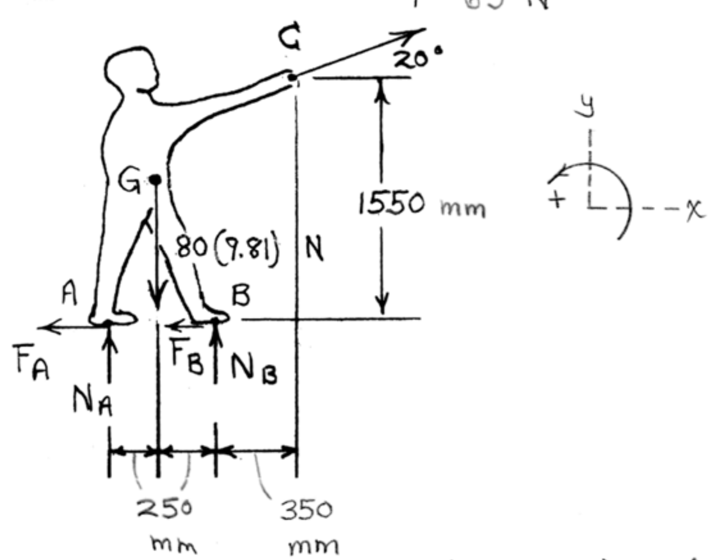
3/22



$$\circlearrowleft + \ \Sigma M_O = 0: \quad P\cos 10°(30) + P\sin 10°(20) - 3400\left(25\frac{\pi}{180}\right) = 0$$

$$\underline{P = 44.9 \text{ N}}$$

3/23

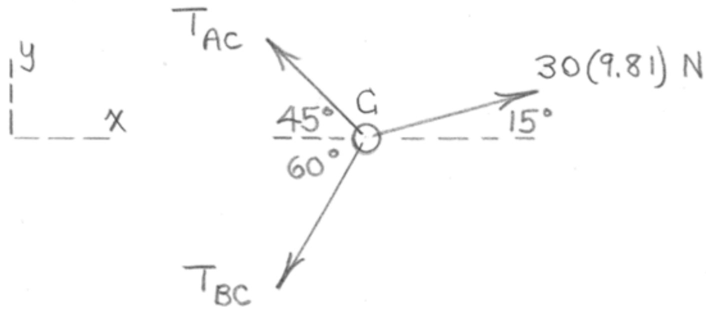


$\Sigma F_y = 0: \quad N_A + N_B - 80(9.81) + 65 \sin 20° = 0$

$\Sigma M_B = 0: \quad 80(9.81)(250) - N_A(500) - 65[1550 \cos 20° - 350 \sin 20°] = 0$

Solution : $\begin{cases} N_A = 219 \text{ N} \\ N_B = 544 \text{ N} \end{cases}$

3/24 FBD of junction ring C:

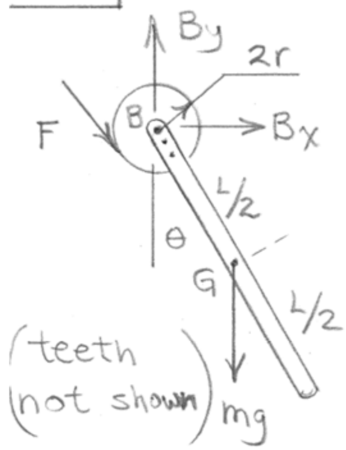


$$\begin{cases} \Sigma F_x = 0: -T_{AC}\cos 45° - T_{BC}\cos 60° + 30(9.81)\cos 15° = 0 \\ \Sigma F_y = 0: T_{AC}\sin 45° - T_{BC}\sin 60° + 30(9.81)\sin 15° = 0 \end{cases}$$

Solve simultaneously to obtain

$$\begin{cases} T_{AC} = 215 \text{ N} \\ T_{BC} = 264 \text{ N} \end{cases}$$

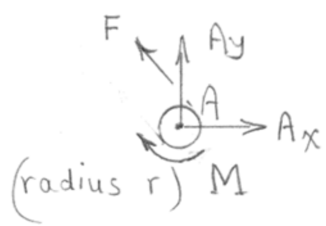
3/25



$\circlearrowleft \Sigma M_B = 0$:

$F(2r) - mg\frac{L}{2}\sin\theta = 0$

$F = \frac{1}{4r} mgL\sin\theta$



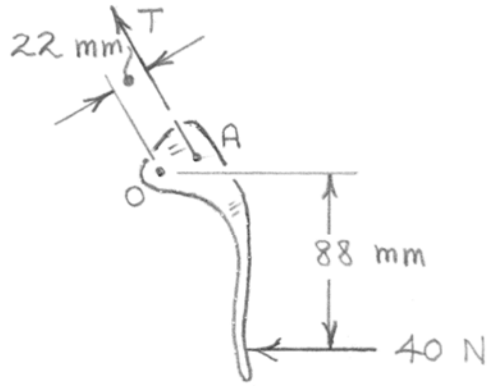
$\circlearrowleft + \Sigma M_A = 0$:

$-M + Fr = 0$

$M = \frac{1}{4} mgL\sin\theta$

(CW)

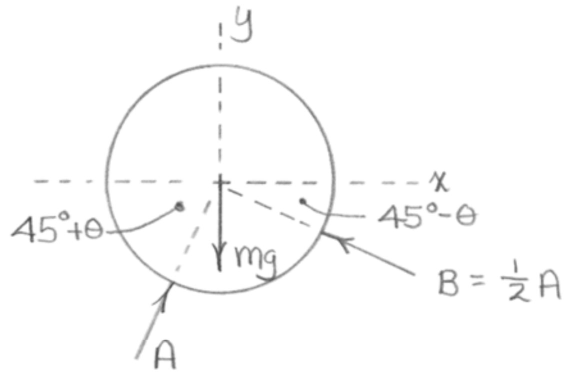
3/26 FBD of brake lever:



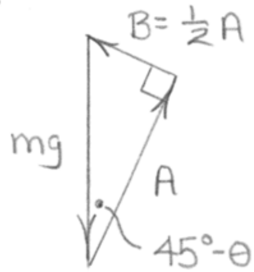
$$\curvearrowleft \Sigma M_O = 0: \quad T(22) - 40(88) = 0$$

$$\underline{T = 160 \text{ N}}$$

3/27



$\Sigma \underline{F} = \underline{0}$:

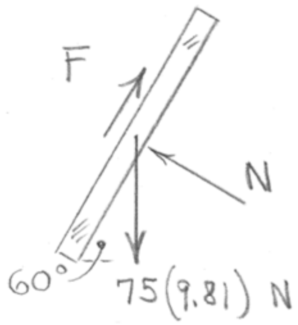


$$\tan(45° - \theta) = \frac{A/2}{A} = \frac{1}{2}$$

$$45° - \theta = 26.6°$$

$$\underline{\theta = 18.43°}$$

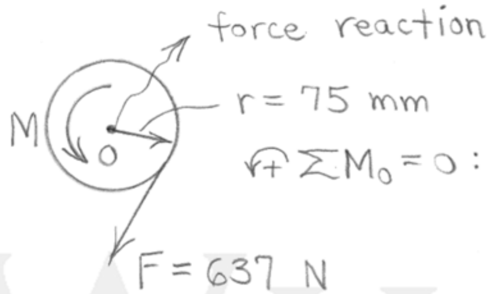
3/28 Rack:



$+\nearrow \Sigma F = 0: \quad F - 75(9.81)\sin 60° = 0$

$F = 637 \text{ N}$

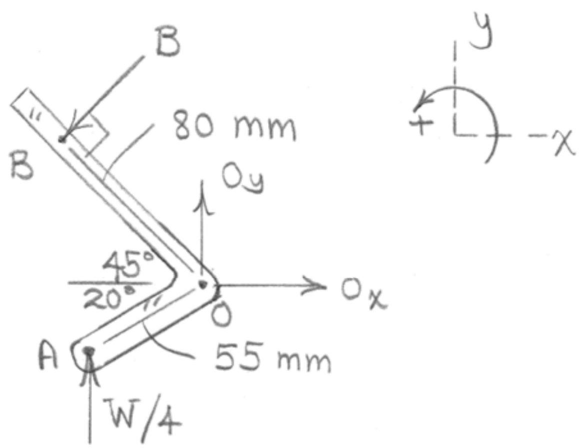
Gear wheel:



$\circlearrowleft + \Sigma M_O = 0: \quad M - 637(0.075) = 0$

$\underline{M = 47.8 \text{ N·m}}$

3/29



$$\Sigma M_O = 0: \quad B(80) - \frac{W}{4}(55\cos 20^\circ) = 0$$

$$\underline{B = 0.1615W}$$

$$\Sigma F_x = 0: \quad -0.1615W\cos 45^\circ + O_x = 0$$

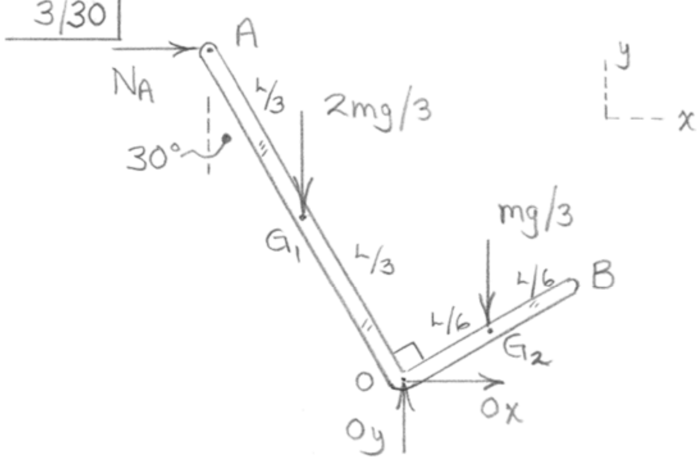
$$O_x = 0.1142W$$

$$\Sigma F_y = 0: \quad O_y - 0.1615W\sin 45^\circ + \frac{W}{4} = 0$$

$$O_y = -0.1358W$$

$$O = \sqrt{O_x^2 + O_y^2} = \underline{0.1774W}$$

3/30



$$\circlearrowleft + \Sigma M_O = 0: \; -N_A\left(\frac{2L}{3}\cos 30°\right) + \frac{2mg}{3}\left(\frac{L}{3}\sin 30°\right) - \frac{mg}{3}\left(\frac{L}{6}\cos 30°\right) = 0$$

$$\underline{N_A = 0.1091\, mg}$$

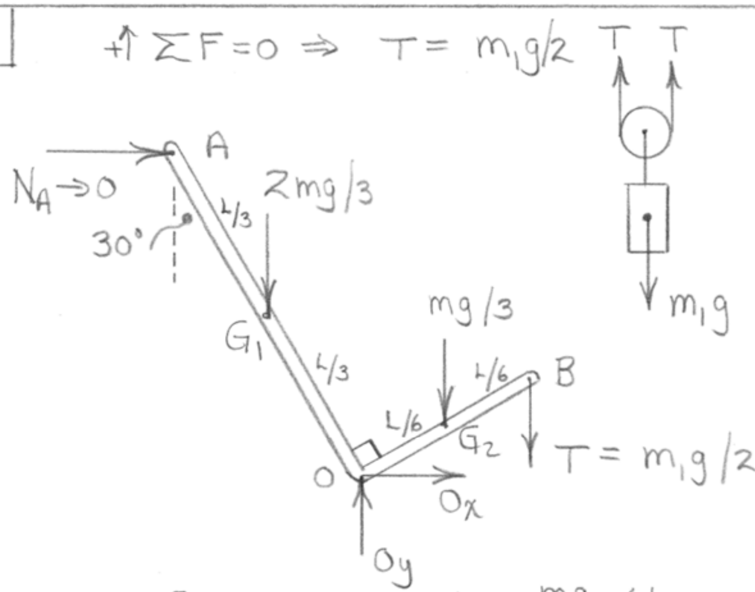
$\Sigma F_y = 0 \Rightarrow O_y = mg$

$\Sigma F_x = 0 \Rightarrow O_x = -0.1091\, mg$

$O = \sqrt{O_x^2 + O_y^2} = \underline{1.006 mg}$

3/31

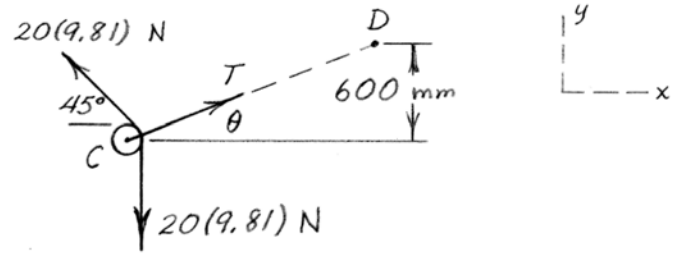
$+\uparrow \Sigma F = 0 \Rightarrow T = m_1 g/2$



$$\circlearrowleft + \Sigma M_O = 0: \quad \frac{2mg}{3}\left(\frac{L}{3}\sin 30°\right) - \frac{mg}{3}\left(\frac{L}{6}\cos 30°\right) - \frac{m_1 g}{2}\left(\frac{L}{3}\cos 30°\right)$$

$$\underline{m_1 = 0.436\,m}$$

3/32



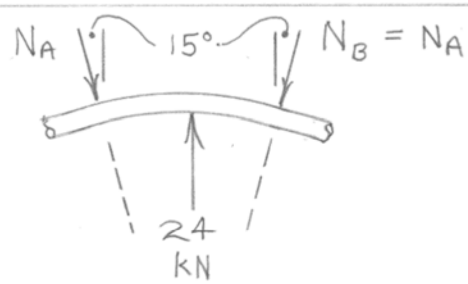
$\Sigma F_x = 0$: $T\cos\theta - 20(9.81)\cos 45° = 0$

$\Sigma F_y = 0$: $T\sin\theta + 20(9.81)\sin 45° - 20(9.81) = 0$

Solve to obtain $\theta = 22.5°$, $\underline{T = 150.2 \text{ N}}$

$\frac{600}{\overline{CD}} = \sin\theta = \sin 22.5°$, $\underline{\overline{CD} = 1568 \text{ mm}}$

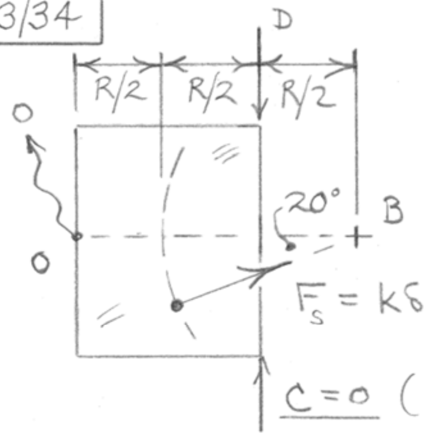
3/33



$\uparrow \Sigma F = 0: \quad 24 - 2N_A \cos 15° = 0$

$\underline{N_A = N_B = 12.42 \text{ kN}}$

3/34



$R = 0.025$ m

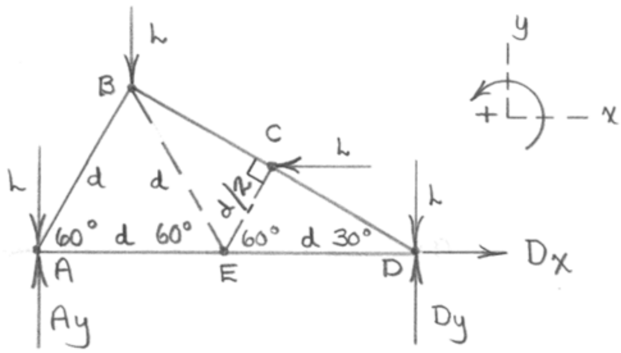
$$F_s = k\delta = k\left[R - \frac{R}{3}\right] = 1600\left(\frac{2}{3} \cdot 0.025\right) = 26.7 \text{ N}$$

$\underline{C = 0}$ (by inspection)

$$\curvearrowleft \Sigma M_O = 0: \quad 26.7 \sin 20° \left(\frac{3}{2} \cdot 0.025\right) - D(0.025) = 0$$

$\underline{D = 13.68 \text{ N}}$

3/35



$$\Sigma F_x = 0: \quad D_x - L = 0, \quad \underline{D_x = L}$$

$$\Sigma F_y = 0: \quad A_y + D_y - 3L = 0$$

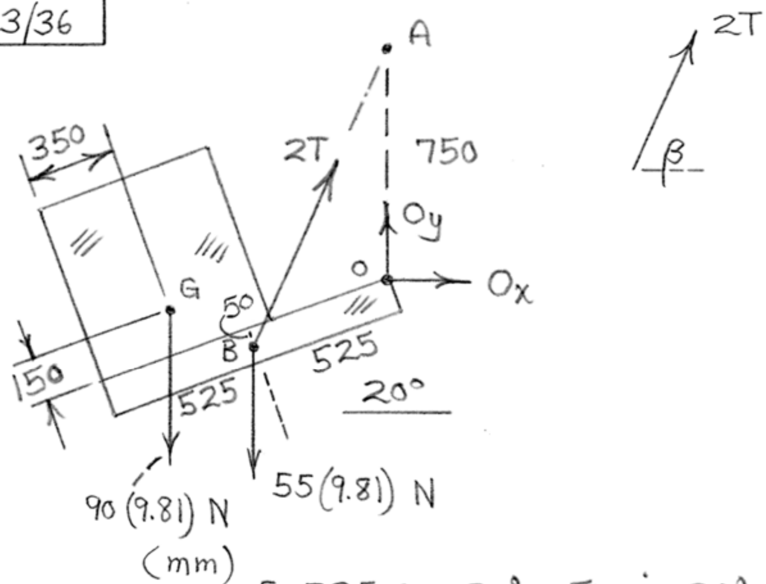
$$\Sigma M_A = 0: \quad D_y(2d) + L\left(\frac{d}{2}\frac{\sqrt{3}}{2}\right) - L\left(\frac{d}{2}\right) - L(2d) = 0$$

Solving the last 2 equations : $A_y = \frac{L}{4}\left(7 + \frac{\sqrt{3}}{2}\right)$

$$\underline{D_y = \frac{L}{4}\left(5 - \frac{\sqrt{3}}{2}\right)}$$

(or $A_y = 1.967L$, $D_y = 1.033L$)

3/36



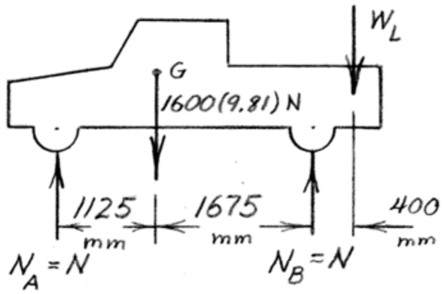
$$\beta = \cot^{-1}\left[\frac{525\cos 20° - 50\sin 20°}{750 + 525\sin 20° + 50\cos 20°}\right]$$

$$= 64.0°$$

$\circlearrowleft + \Sigma M_O = 0: \; 55(9.81)(525\cos 20° - 50\sin 20°)$
$+ 90(9.81)\left[(1050 - 350)\cos 20° + 150\sin 20°\right]$
$- 2T\cos\beta\,(750) = 0$ (transmissibility used on $2T \Rightarrow$ point A)

Solving, $\underline{T = 1343 \text{ N}}$

3/37



$$\circlearrowleft \Sigma M_A = 0: \; 1600(9.81)1.125 - N(2.80) + W_L(3.20) = 0$$

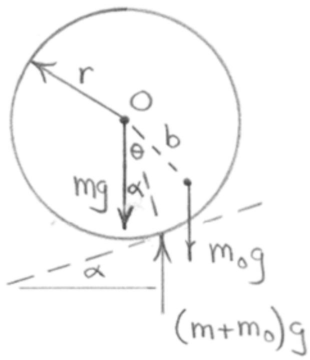
$$\uparrow \Sigma F = 0: \; 2N - 1600(9.81) - W_L = 0$$

Solve to obtain $N = 9050$ N

$W_L = 2400$ N

$$m_L = \frac{W_L}{g} = \frac{2400}{9.81} = \underline{244 \text{ kg}}$$

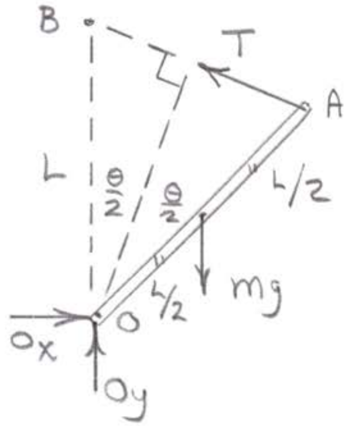
3/38



$$\circlearrowleft + \sum M_O = 0: \ (m+m_o)g\, r\sin\alpha - m_o g\, b\sin\theta = 0$$

$$\Rightarrow \quad \theta = \sin^{-1}\left\{\frac{r}{b}\left(1+\frac{m}{m_o}\right)\sin\alpha\right\}$$

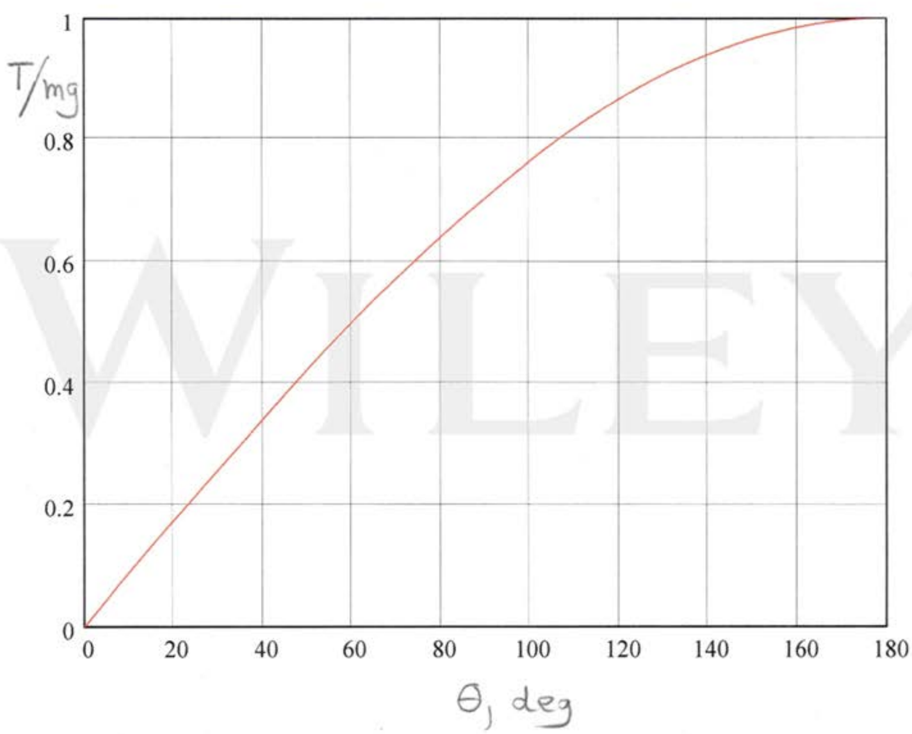
3/39



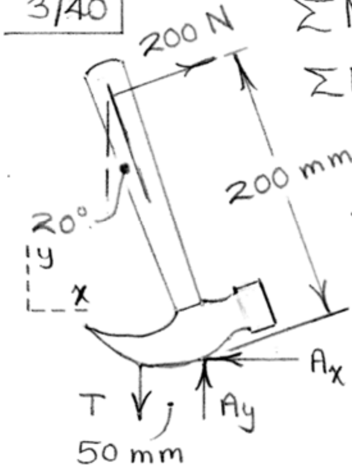
$\circlearrowleft + \Sigma M_O = 0: \; T(L\cos\theta) - mg\left(\frac{L}{2}\sin\theta\right) = 0$

$$T = \frac{mg\sin\theta}{2\cos\frac{\theta}{2}}$$

$$T_{40^\circ} = 0.342\,mg$$



3/40



$\Sigma M_A = 0: \ 200(200) - 50T = 0, \ \underline{T = 800 \text{ N}}$

$\Sigma F_x = 0: \ 200 \cos 20° - A_x = 0$

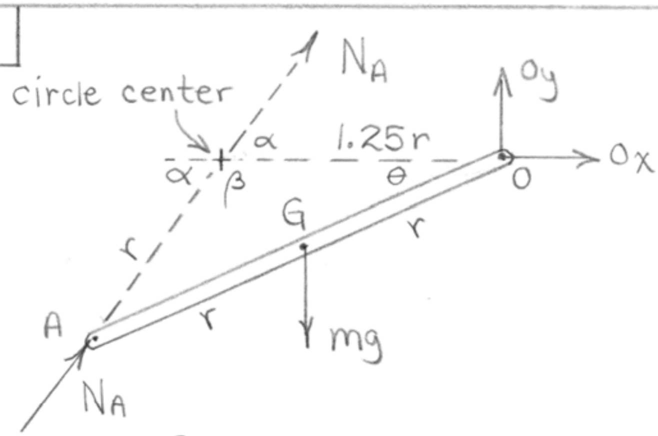
$A_x = 187.9 \text{ N}$

$\Sigma F_y = 0: \ A_y + 200 \sin 20° - 800 = 0$

$A_y = 732 \text{ N}$

$A = \sqrt{A_x^2 + A_y^2} = \underline{755 \text{ N}}$

3/41



$$r^2 = (1.25r)^2 + (2r)^2 - 2(1.25r)(2r)\cos\theta$$

$$\theta = 24.1^\circ$$

$$\frac{\sin\beta}{2r} = \frac{\sin\theta}{r} \Rightarrow \beta = 125.1^\circ$$

$$\alpha = 180^\circ - \beta = 54.9^\circ$$

$$\circlearrowleft + \Sigma M_O = 0: \quad (N_A \sin 54.9^\circ)(1.25) - mg(r\cos 24.1^\circ) = 0$$

$$\underline{N_A = 0.892\,mg}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad O_x + 0.892\,mg\cos 54.9^\circ = 0$$

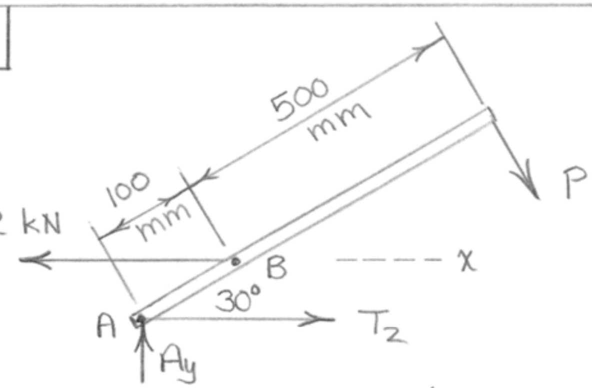
$$O_x = -0.513\,mg$$

$$\uparrow + \Sigma F_y = 0: \quad O_y + 0.892\,mg\sin 54.9^\circ - mg = 0$$

$$O_y = 0.270\,mg$$

$$O = \sqrt{O_x^2 + O_y^2} = \underline{0.580\,mg}$$

3/42



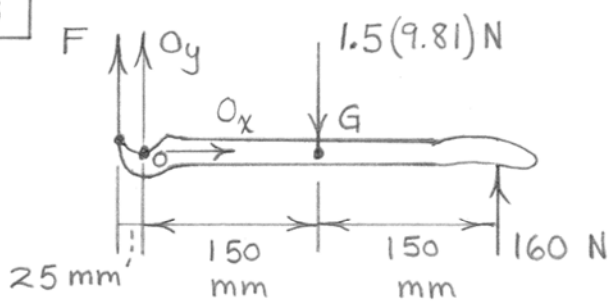
$\circlearrowleft + \; \Sigma M_A = 0: \quad P(500 + 100) - 2(100 \sin 30°) = 0$

$P = 0.1667$ kN or $\underline{P = 166.7 \text{ N}}$

$\Sigma F_x = 0: \quad 0.1667 \sin 30° + T_2 - 2 = 0$

$\underline{T_2 = 1.917 \text{ kN}}$

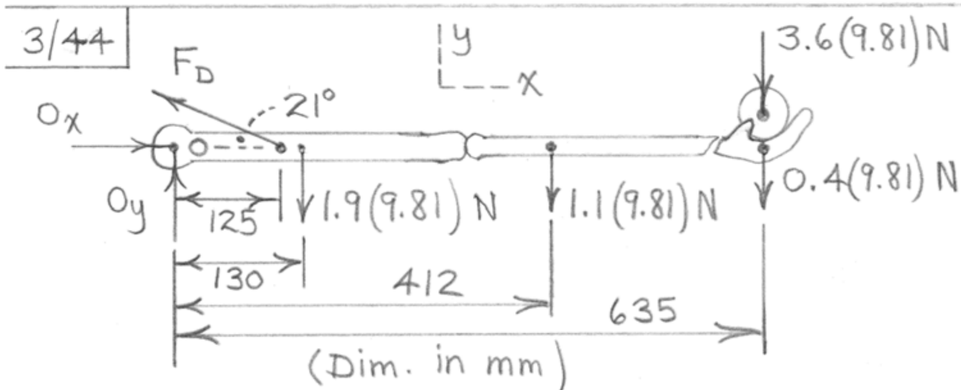
3/43



$$\circlearrowleft + \ \Sigma M_O = 0: \ -F(25) - 1.5(9.81)(150) + 160(300) = 0$$

$$\underline{F = 1832 \text{ N}}$$

3/44



$\circlearrowleft + \Sigma M_O = 0: \; F_D \sin 21°(125) - 1.9(9.81)(130)$
$- 1.1(9.81)(412) - (3.6+0.4)(9.81)(635) = 0$

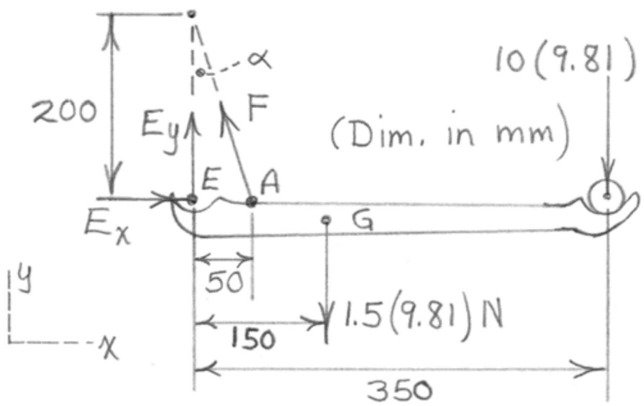
$\underline{F_D = 710 \text{ N}}$

$\xrightarrow{+} \Sigma F_x = 0: \; O_x - 710 \cos 21° = 0, \; \underline{O_x = 662 \text{ N}}$

$+\uparrow \Sigma F_y = 0: \; O_y + 710 \sin 21° - (1.9 + 1.1 + 3.6 + 0.4)\,9.81$
$= 0, \; \underline{O_y = -185.6 \text{ N}}$

3/45

$$\alpha = \tan^{-1}\left(\frac{50}{200}\right) = 14.04°$$



Assume that A & E are on a horizontal line.

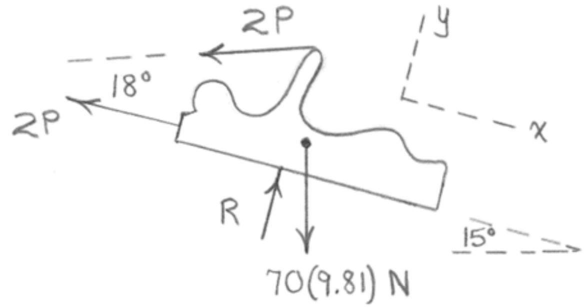
$$\curvearrowleft + \Sigma M_E = 0: F\cos 14.04°(50) - 1.5(9.81)(150)$$
$$-10(9.81)(350) = 0, \quad \underline{F = 753 \text{ N}}$$

$$\Sigma F_x = 0: -753 \sin 14.04° + E_x = 0$$
$$E_x = 182.7 \text{ N}$$

$$\Sigma F_y = 0: 753 \cos 14.04° - (10+1.5)(9.81) + E_y = 0$$
$$E_y = -618 \text{ N}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{182.7^2 + 618^2} = \underline{644 \text{ N}}$$

3/46



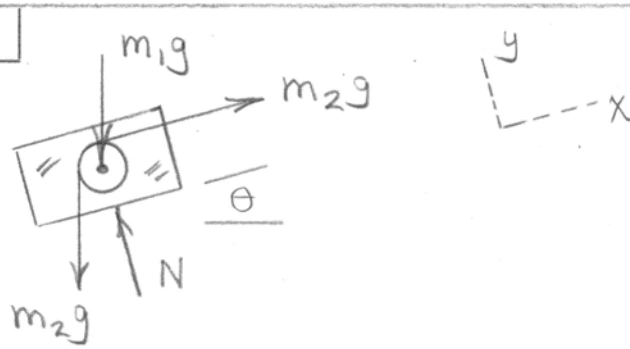
$\Sigma F_x = 0: \; 70(9.81)\sin 15° - 2P - 2P\cos 18° = 0$

$\underline{P = 45.5 \text{ N}}$

$\Sigma F_y = 0: \; R - 70(9.81)\cos 15° - 2(45.5)\sin 18° = 0$

$\underline{R = 691 \text{ N}}$

3/47



$$\Sigma F_x = 0: \quad m_2g - m_1g\sin\theta - m_2g\sin\theta = 0$$

$$m_2 = \frac{m_1 \sin\theta}{1-\sin\theta}$$

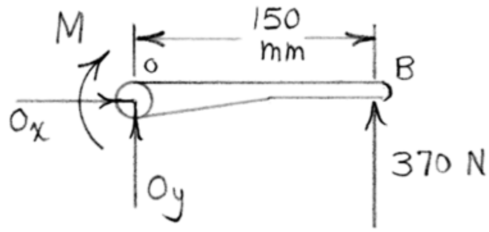
$\theta = 15°$: $m_2 = 0.349 m_1$

$\theta = 45°$: $m_2 = 2.41 m_1$

$\theta = 60°$: $m_2 = 6.46 m_1$

3/48

Arm OB:



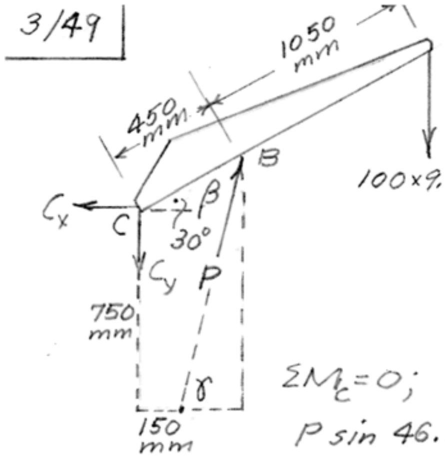
$\circlearrowleft + \sum M_O = 0: \quad 370(0.150) - M = 0$

$\underline{M = 55.5 \text{ N}\cdot\text{m}}$

$F \cos 20° (0.375) = M = 55.5$

$\underline{F = 157.5 \text{ N}}$

3/49



$$P = p\frac{\pi d^2}{4}, \quad p = \frac{4P}{\pi(0.080)^2}$$

$$\beta = \gamma - 30°$$

$$\gamma = \tan^{-1}\frac{750 + 450\sin 30°}{450\cos 30° - 150}$$

$$= \tan^{-1} 4.07 = 76.2°$$

$$\beta = 76.2 - 30 = 46.2°$$

$\Sigma M_C = 0;$

$$P \sin 46.2°(450) - 981(1500\cos 30°) = 0$$

$$P = 3924\text{ N}, \quad p = \frac{4(3924)}{\pi(0.080)^2} = 780.7\times 10^3\ \text{N/m}^2$$

or $\underline{p = 781\ \text{kPa}}$

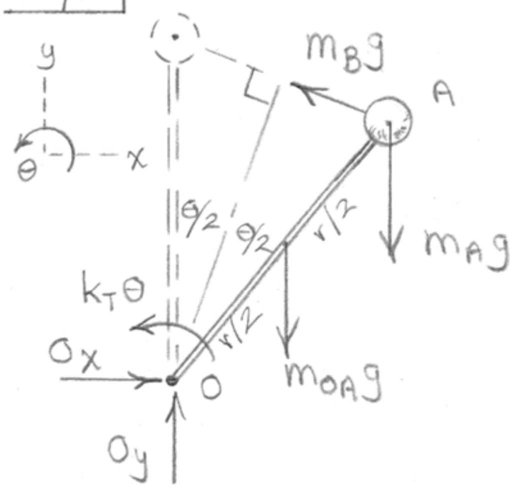
$\Sigma F_x = 0;\ C_x - 3923\cos 76.2° = 0$

$C_x = 937$ N

$\Sigma F_y = 0;\ C_y + 981 - 3924\sin 76.2° = 0, \quad C_y = 2830$ N

$$C = \sqrt{(937)^2 + (2830)^2} = \underline{2980\text{ N}}$$

*3/50



$$\Sigma M_O = 0: \; k_T\theta + m_B g\left(r\cos\frac{\theta}{2}\right) - m_A g r \sin\theta - m_{OA} g \frac{r}{2}\sin\theta = 0$$

With $k_T = 50$ N·m/rad, $m_A = 10$ kg, $m_{OA} = 5$ kg, $m_B = 1$ kg, and $r = 0.8$ m, solve for θ to obtain

$\underline{\theta = 9.40°}$ and $\underline{\theta = 103.7°}$

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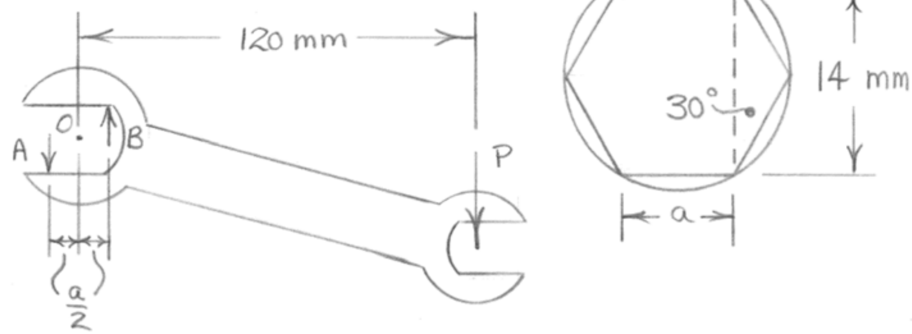

$2a\cos 30° = 14, \quad \frac{a}{2} = 4.04$ mm

$\circlearrowleft + \Sigma M_O = 0: \quad 0.120P - 24 = 0, \quad \underline{P = 200\text{ N}}$

(for wrench and bolt)

For wrench alone,

$\circlearrowleft + \Sigma M_A = 0: \quad 200(0.120 + 0.00404) - B(2 \cdot 0.00404) = 0, \quad \underline{B = 3070\text{ N}}$

$+\uparrow \Sigma F = 0: \quad -A + 3070 - 200 = 0, \quad \underline{A = 2870\text{ N}}$

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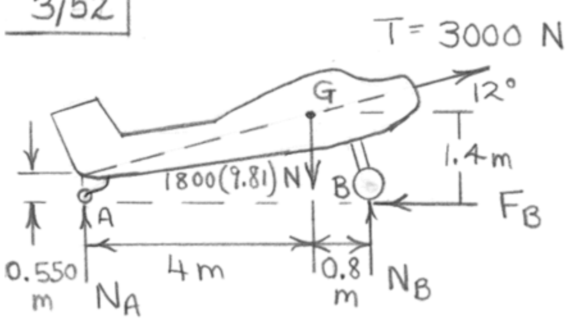


Engine off : $T = 0$, $F_B = 0$

$\Sigma M_A = 0: \; 1800(9.81)4 - N_B(4.8) = 0 \qquad N_B = 14\,720$ N

$\Sigma F_y = 0: \; N_A + 14\,720 - 1800(9.81) = 0, \qquad N_A = 2940$ N

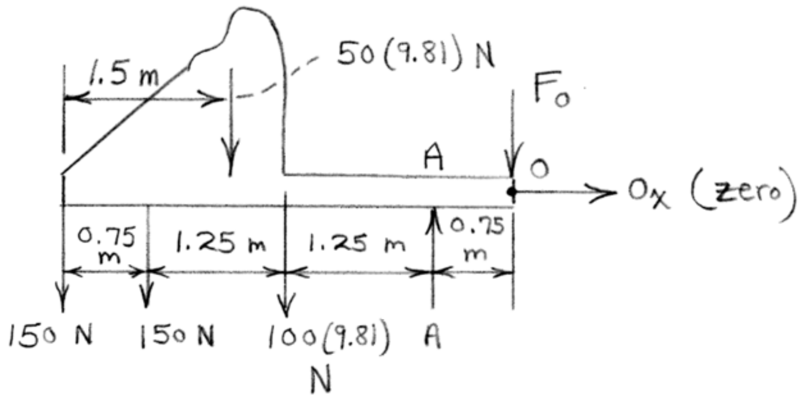
$\Sigma M_A = 0: \; 1800(9.81)4 - N_B'(4.8) + 3000\cos 12° (0.550) = 0$

$N_B' = 15{,}050$ N

$\Sigma F_y = 0: \; N_A' + 15{,}050 - 1800(9.81) + 3000 \sin 12° = 0, \; N_A' = 1983$ N

$n_A = \frac{N_A' - N_A}{N_A}(100) = \underline{-32.6\%}, \; n_B = \frac{N_B' - N_B}{N_B} = \underline{2.28\%}$

3/53



Above FBD is for the system of beam and boy.

$\circlearrowleft \Sigma M_A = 0: \; 150(3.25) + 150(2.5) + 100(9.81)(1.25) + 50(9.81)(1.75) - 0.75F_O = 0$

$F_O = 3930$ N or $\underline{F_O = 3.93 \text{ kN}}$

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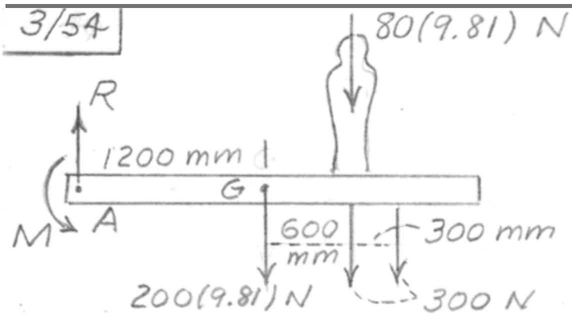

$\Sigma M_A = 0$; $80(9.81)(1800) + 200(9.81)(1200)$

$+ 300(1800 + 2100) - M = 0$

$M = 4.94(10^6)$ N·mm or $\underline{M = 4.94 \text{ kN·m}}$

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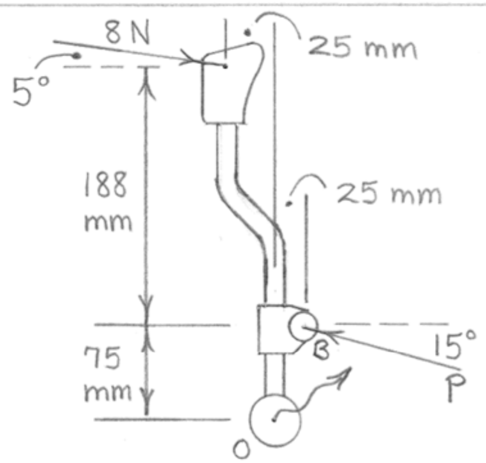

$$\circlearrowleft + \sum M_O = 0: -8\cos 5°(188+75) + 8\sin 5°(25)$$
$$+ P\cos 15°(75) + P\sin 15°(25) = 0$$

$\underline{P = 26.3 \text{ N}}$

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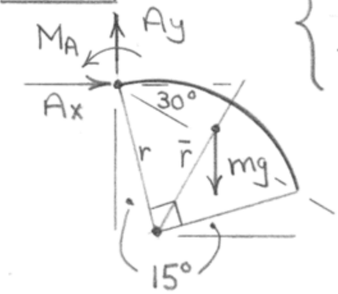

From Table D/3, $\bar{r} = \dfrac{r \sin\alpha}{\alpha}$

For $\alpha = \pi/4$, $\bar{r} = 2\sqrt{2}\, r/\pi$

$\circlearrowleft + \sum M_A = 0$:

$M_A - mg\left(r \sin 15^\circ + \bar{r} \cos 60^\circ\right) = 0$

$\underline{M_A = 0.709\ mgr}$

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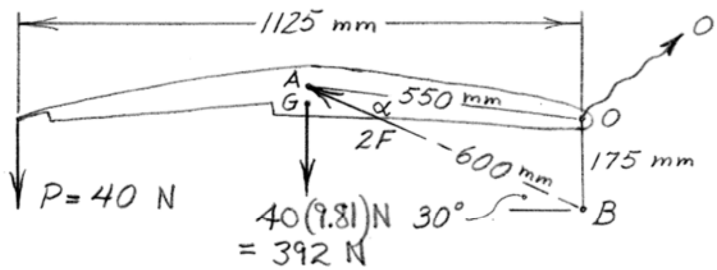

Law of cosines: $175^2 = 550^2 + 600^2 - 2(550)(600)\cos\alpha$

$\alpha = 16.79°$

$\circlearrowleft + \Sigma M_O = 0$: $40(1125) - 2F(550\sin\alpha) + 392(550\cos[30° - \alpha])$

$= 0$, $\underline{F = 803\text{ N}}$

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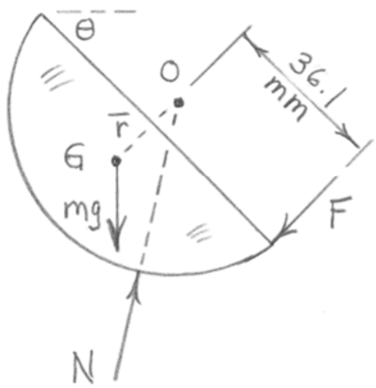
"Cube":



Arm:

$$\alpha = \sin^{-1}\frac{8}{37} = 12.49°$$

Cube: $\circlearrowleft + \Sigma M_O = 0$: $mg\bar{r}\sin\theta - F(36.1) = 0$

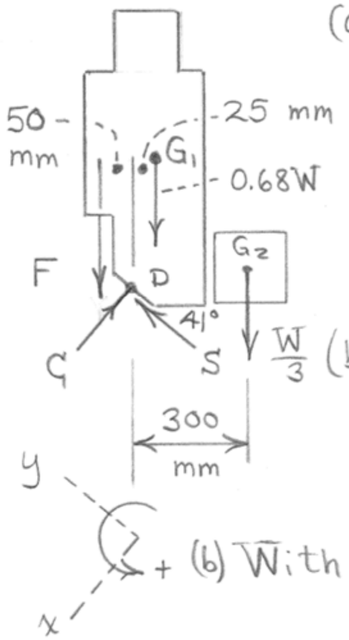
$$0.25(9.81)(0.55 \cdot 37)\sin\theta - F(36.1) = 0$$

$$F = 1.382\sin\theta \quad (\text{in N})$$

Arm: $\circlearrowleft + \Sigma M_O = 0$: $-M + (1.382\sin\theta)(36.1) = 0$

$$\underline{M = 49.9\sin\theta \quad (\text{N}\cdot\text{mm})}$$

▶3/59 Consider a FBD of the upper torso



(a) $\Sigma M_D = 0$: $F(50) - 0.68W(25) = 0$

$F = 0.34W$

$\Sigma F_y = 0$: $S - 0.68W \sin 41° - F \sin 41° = 0$, $\underline{S = 0.669W}$

$\Sigma F_x = 0$: $-C - 0.68W \cos 41° - F \cos 41° = 0$, $\underline{C = 0.770W}$

(b) With weight $\frac{W}{3}$:

$\Sigma M_D = 0$: $F(50) - 0.68W(25) - \frac{W}{3}(300) = 0$

$F = 2.34W$

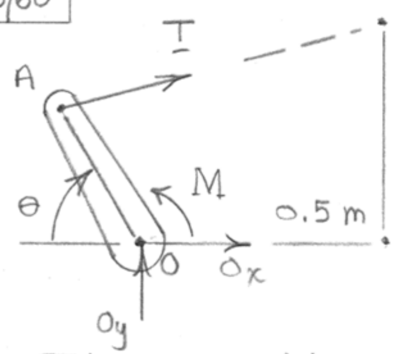
$\Sigma F_y = 0$: $S - 0.68W \sin 41° - F \sin 41° - \frac{W}{3} \sin 41° = 0$

$\underline{S = 2.20W}$

$\Sigma F_x = 0$: $-C + 0.68W \cos 41° + F \cos 41° + \frac{W}{3} \cos 41° = 0$

$\underline{C = 2.53W}$

*3/60



$\overline{OA} = 0.3$ m

$m = 5$ kg

$(T = mg)$

$\Sigma \underline{M}_O = \underline{0}$: $\underline{M} + \underline{r}_{OA} \times \underline{T} = \underline{0}$

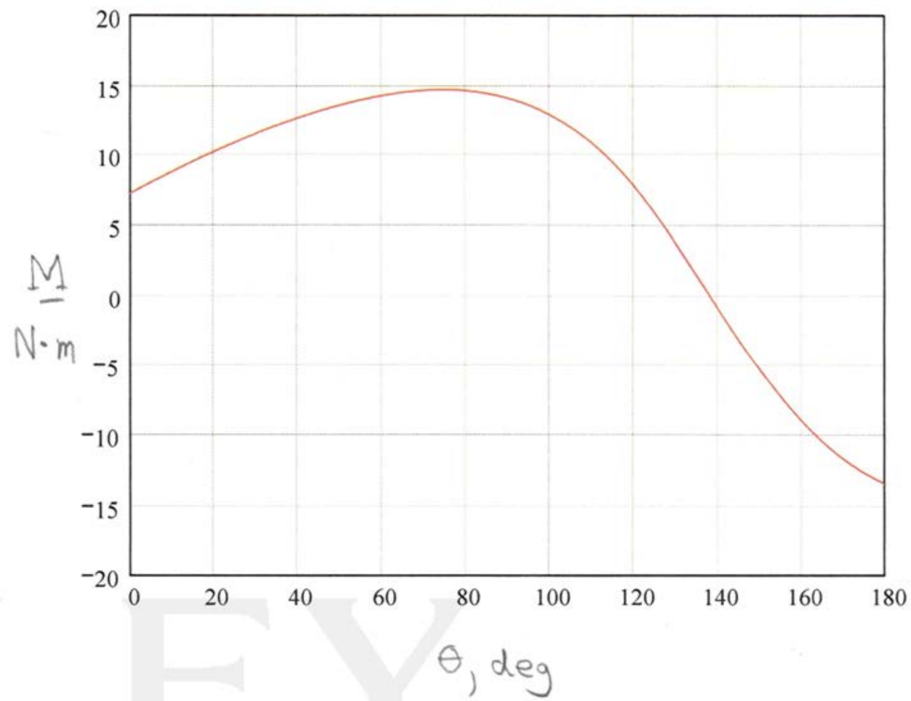
$\underline{M} = -\underline{r}_{OA} \times \underline{T}$

Let $\underline{r}_{OA} = \overline{OA}(-\cos\theta\,\underline{i} + \sin\theta\,\underline{j})$

and $\underline{T} = T\underline{n}_{AB} =$

$$= mg\left[\frac{(0.5+\overline{OA}\cos\theta)\,\underline{i} + (0.45-\overline{OA}\sin\theta)\,\underline{j}}{\{(0.5+\overline{OA}\cos\theta)^2 + (0.45-\overline{OA}\sin\theta)^2\}^{1/2}}\right]$$

Carry out the cross product over the range $0 \le \theta \le 180°$ to obtain the following plot for the z-comp. of $\underline{M}$:



$|M|_{min} = 0$ at $\theta = 138.0°$

($\overline{AB} \parallel \overline{OA}$, $\underline{T}$ has no moment arm relative to O)

$|M|_{max} = 14.72$ N·m at $\theta = 74.5°$

($\overline{AB} \perp \overline{OA}$, T has max. moment arm relative to O)

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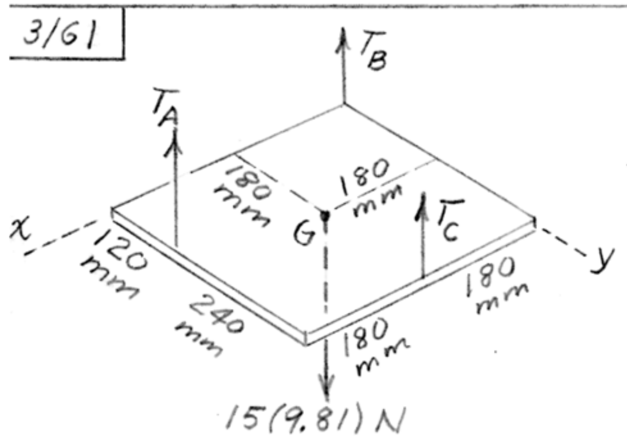

$\Sigma M_x = 0;\ 120 T_A + 360 T_C - 15(9.81)(180) = 0$

$T_A + 3T_C = 220.7$

$\Sigma M_y = 0;\ 15(9.81)(180) - 180 T_C - 360 T_A = 0$

$2T_A + T_C = 147.2$

Solve & get $\underline{T_A = 44.1\text{ N}}$, $\underline{T_C = 58.9\text{ N}}$

$\Sigma F_z = 0;\ 44.1 + 58.9 + T_B - 15(9.81) = 0$, $\underline{T_B = 44.1\text{ N}}$

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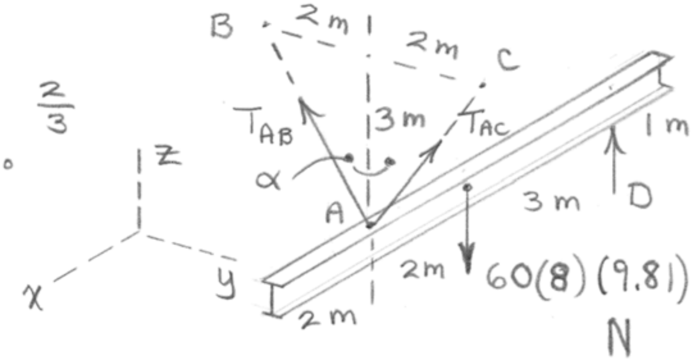
$\alpha = \tan^{-1}\frac{2}{3}$
$= 33.7°$



From $\Sigma F_y = 0$, $T_{AB} = T_{AC} = T$

$\Sigma M_{A_y} = 0: -60(8)(9.81)(2) + D(5) = 0$

$\underline{D = 1884 \text{ N}}$

$\Sigma F_z = 0: 2T\cos\alpha + D - 60(8)(9.81) = 0$

$\underline{T = 1698 \text{ N} = T_{AB} = T_{AC}}$

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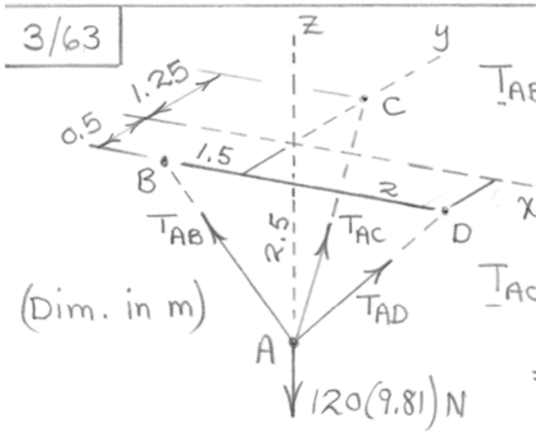

$$\mathbf{T}_{AB} = T_{AB}\frac{-1.5\mathbf{i} - 0.5\mathbf{j} + 2.5\mathbf{k}}{\sqrt{1.5^2 + 0.5^2 + 2.5^2}} = T_{AB}(-0.507\mathbf{i} - 0.1690\mathbf{j} + 0.845\mathbf{k})$$

$$\mathbf{T}_{AC} = T_{AC}\frac{1.25\mathbf{j} + 2.5\mathbf{k}}{\sqrt{1.25^2 + 2.5^2}} = T_{AC}(0.447\mathbf{j} + 0.894\mathbf{k})$$

$$\mathbf{T}_{AD} = T_{AD}\frac{2\mathbf{i} - 0.5\mathbf{j} + 2.5\mathbf{k}}{\sqrt{2^2 + 0.5^2 + 2.5^2}} = T_{AD}(0.617\mathbf{i} - 0.1543\mathbf{j} + 0.772\mathbf{k})$$

$$\begin{cases} \Sigma F_x = 0: -0.507T_{AB} + 0.617T_{AD} = 0 \\ \Sigma F_y = 0: -0.1690T_{AB} + 0.447T_{AC} - 0.1543T_{AD} = 0 \\ \Sigma F_z = 0: 0.845T_{AB} + 0.894T_{AC} + 0.772T_{AD} - 120(9.81) = 0 \end{cases}$$

Solution :
$$\begin{cases} T_{AB} = 569 \text{ N} \\ T_{AC} = 376 \text{ N} \\ T_{AD} = 467 \text{ N} \end{cases}$$

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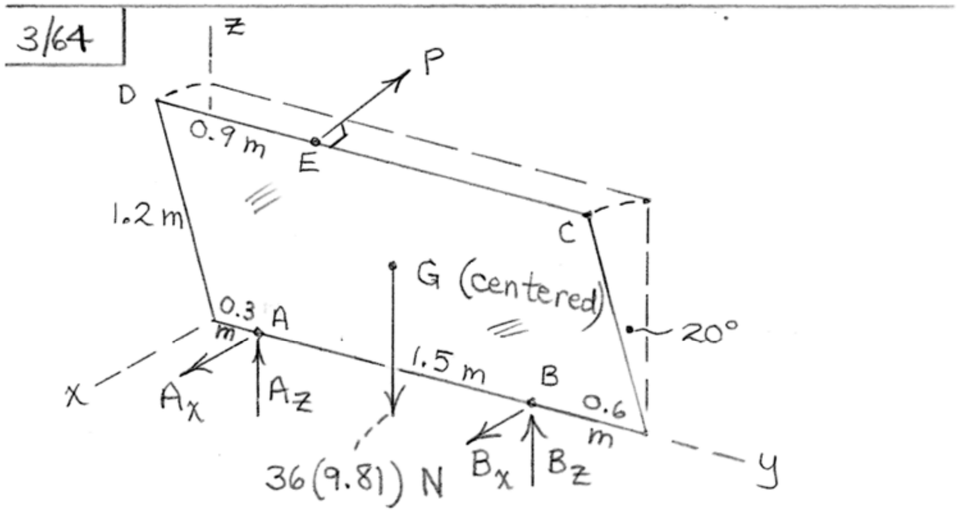

$$\Sigma M_y = 0: \ -P(1.2) + 36(9.81)(0.6 \sin 20^\circ) = 0$$

$$\underline{P = 60.4 \text{ N}}$$

$$(\Sigma M_A)_x = 0: \ B_z(1.5) - 36(9.81)(0.9) + P \sin 20^\circ (0.6) = 0$$

$$\underline{B_z = 204 \text{ N}}$$

$$\Sigma F_z = 0: \ A_z + B_z + P \sin 20^\circ - 36(9.81) = 0$$

$$\underline{A_z = 128.9 \text{ N}}$$

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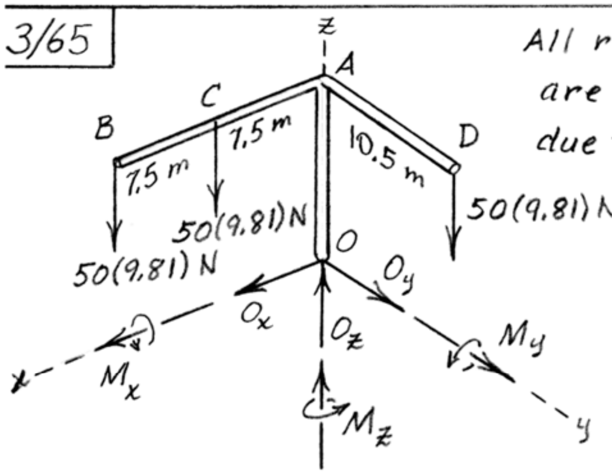

All reactions at O are incremental — due to weights of 3 50-kg masses only.

$$\begin{cases} \Sigma F_x = 0: & O_x = 0 \\ \Sigma F_y = 0: & O_y = 0 \\ \Sigma F_z = 0: & O_z = 3(50)(9.81) = 1472 \text{ N} \end{cases}$$

$$\begin{cases} \Sigma M_{O_x} = 0: & M_x - 50(9.81)(10.5) = 0 \Rightarrow M_x = 5.15 \text{ kN}\cdot\text{m} \\ \Sigma M_{O_y} = 0: & M_y + 50(9.81)(7.5+15) = 0 \\ & M_y = -11.04 \text{ kN}\cdot\text{m} \\ \Sigma M_{O_z} = 0: & M_z = 0 \end{cases}$$

$$O = \sqrt{O_x^2 + O_y^2 + O_z^2} = \underline{1472 \text{ N}}$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2} = \underline{12.18 \text{ kN}\cdot\text{m}}$$

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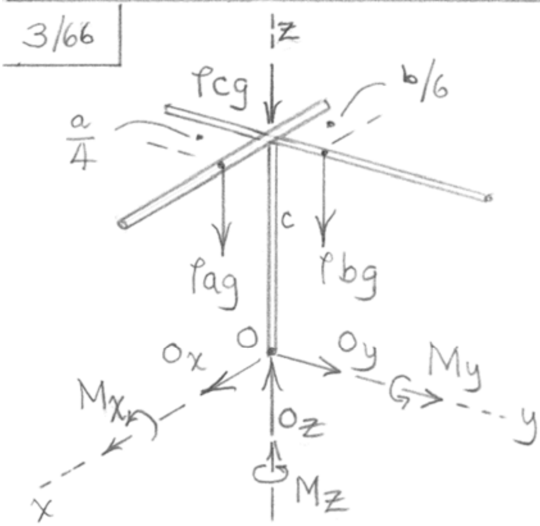

$\Sigma F_x = 0 : \quad O_x = 0$

$\Sigma F_y = 0 : \quad O_y = 0$

$\Sigma F_z = 0 : \quad O_z = \rho g(a + b + c)$

$$O = \rho g\,(a + b + c)$$

$\Sigma M_{O_x} = 0 : \quad M_x - \rho b g\left(\frac{b}{6}\right) = 0, \quad M_x = \frac{1}{6}\rho b^2 g$

$\Sigma M_{O_y} = 0 : \quad M_y + \rho a g\left(\frac{a}{4}\right) = 0, \quad M_y = -\frac{1}{4}\rho a^2 g$

$\Sigma M_{O_z} = 0 : \quad M_z = 0$

So $\quad M = \sqrt{M_x^2 + M_y^2 + M_z^2} = \dfrac{\rho g}{2}\sqrt{\dfrac{a^4}{4} + \dfrac{b^4}{9}}$

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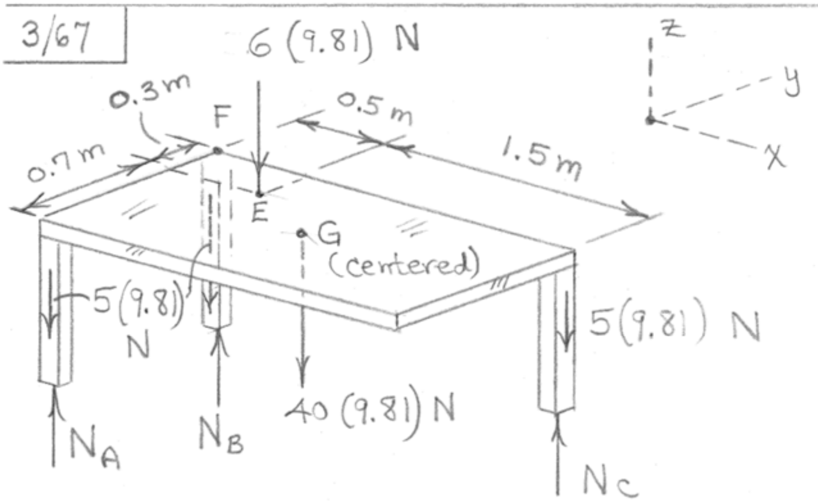

$$(\Sigma M_F)_y = 0: \quad 6(9.81)(0.5) + 40(9.81)(1) + 5(9.81)(2) - N_C(2) = 0$$

$$\underline{N_C = 260 \text{ N}}$$

$$(\Sigma M_F)_x = 0: \quad 5(9.81)(1) + 6(9.81)(0.3) + 40(9.81)(0.5) - N_A(1) = 0, \quad \underline{N_A = 263 \text{ N}}$$

$$\Sigma F_z = 0: \quad N_B + 260 + 263 - 61(9.81) = 0$$

$$\underline{N_B = 75.5 \text{ N}}$$

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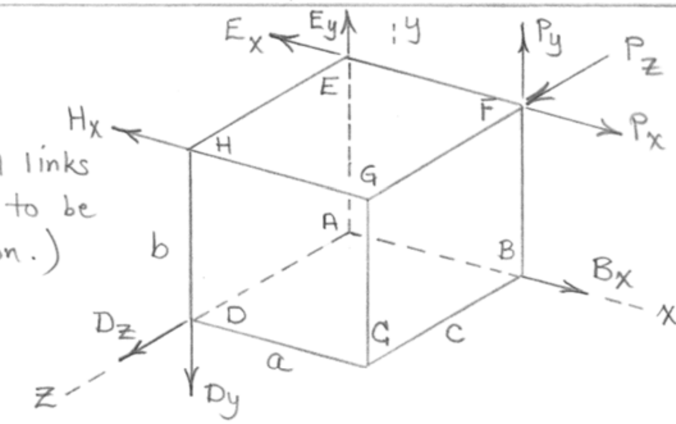
(Note: All links assumed to be in tension.)



$\Sigma F_x = 0: \quad B_x - H_x - E_x + P_x = 0$ (1)

$\Sigma F_y = 0: \quad -D_y + E_y + P_y = 0$ (2)

$\Sigma F_z = 0: \quad D_z + P_z = 0$ (3)

$\Sigma M_{A_x} = 0: \quad c D_y + b P_z = 0$ (4)

$\Sigma M_{A_y} = 0: \quad -c H_x - a P_z = 0$ (5)

$\Sigma M_{A_z} = 0: \quad b E_x + b H_x + a P_y - b P_x = 0$ (6)

Solution:

Eq. (5): $H_x = -\frac{a}{c} P_z$

(4): $D_y = -\frac{b}{c} P_z$

(6): $E_x = P_x - \frac{a}{b} P_y + \frac{a}{c} P_z$

(1): $B_x = -\frac{a}{b} P_y$

(2): $E_y = -\frac{b}{c} P_z - P_y$

(3): $D_z = -P_z$

Short, non-thrust hinges could be placed at D (axis along x-axis) and E (axis along z-axis).

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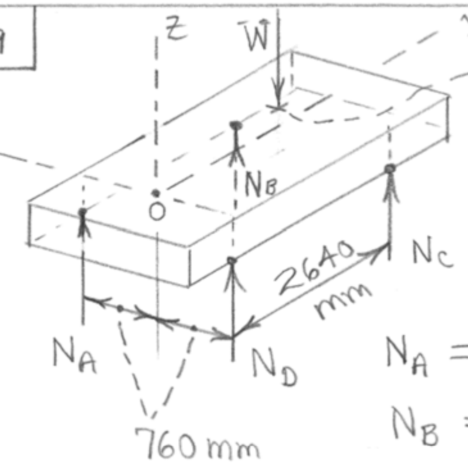


{W acts at (x,y)
FBD of car drawn as simple slab; O embedded in top surface

$N_A = 4300\text{ N}$, $N_D = 4600\text{ N}$

$N_B = 2900\text{ N}$, $N_C = 3000\text{ N}$

$\Sigma F_z = 0: \; 4300 + 2900 + 4600 + 3000 - W = 0$

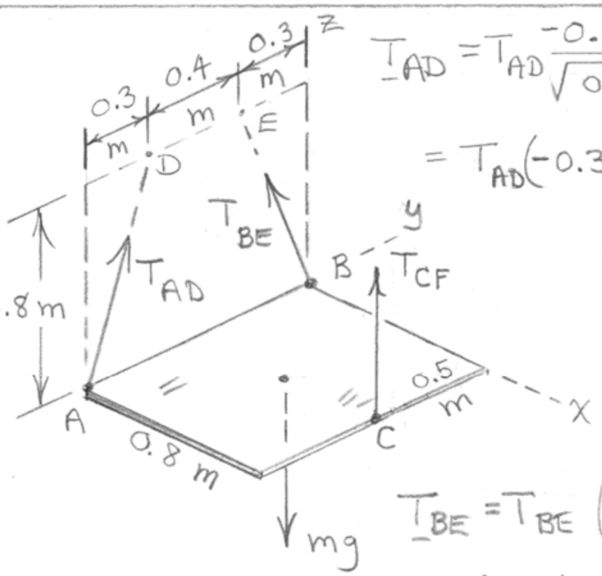
$W = 14\,800\text{ N}$

$m = \dfrac{W}{g} = \dfrac{14\,800}{9.81} = \underline{1509\text{ kg}}$

$\Sigma M_{Oy} = 0: \; Wx - (N_B + N_C)(2640) = 0, \quad \underline{x = 1052\text{ mm}}$

$\Sigma M_{Ox} = 0: \; -Wy + (N_A + N_B)(760) - (N_C + N_D)(760) = 0, \quad \underline{y = -20.5\text{ mm}}$

3/70



$$\underline{T}_{AD} = T_{AD}\frac{-0.3\underline{j} + 0.8\underline{k}}{\sqrt{0.3^2 + 0.8^2}}$$

$$= T_{AD}(-0.351\underline{j} + 0.936\underline{k})$$

$$\underline{T}_{BE} = T_{BE}(0.351\underline{j} + 0.936\underline{k})$$

$$\Sigma M_y = 0: \quad mg(0.4) - T_{CF}(0.8) = 0$$

$$\underline{T_{CF} = \tfrac{1}{2} mg}$$

$$\Sigma M_x = 0: \quad mg(0.5) - \tfrac{1}{2}mg(0.5) - 0.936\,T_{AD}(1) = 0$$

$$\underline{T_{AD} = 0.267\ mg}$$

$$\Sigma F_z = 0: \quad \tfrac{1}{2}mg + 0.267mg(0.936) + T_{BE}(0.936) - mg = 0$$

$$\underline{T_{BE} = 0.267\ mg}$$

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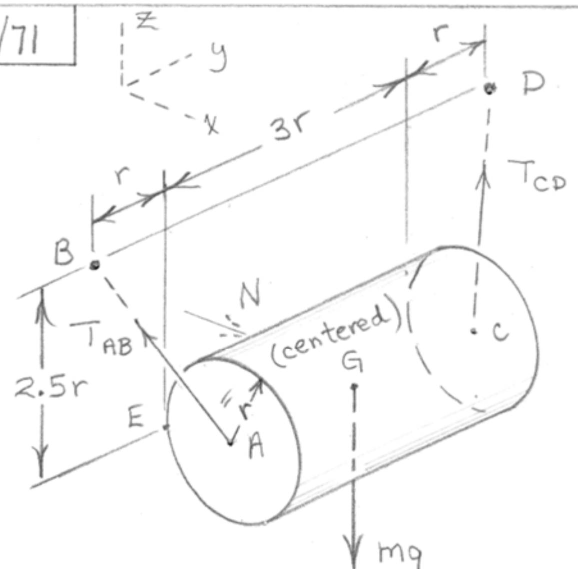


Note: N is distributed over the line of contact.

$T_{AB} = T_{CD} = T$, by symmetry.

$$\underline{T}_{AB} = T\,\frac{-r\underline{i} - r\underline{j} + 2.5r\underline{k}}{\sqrt{r^2 + r^2 + (2.5r)^2}} = T(-0.348\underline{i} - 0.348\underline{j} + 0.870\underline{k})$$

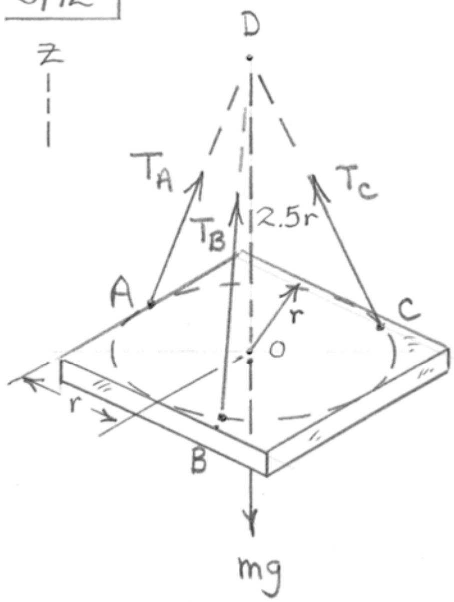
$$\underline{T}_{CD} = T(-0.348\underline{i} + 0.348\underline{j} + 0.870\underline{k})$$

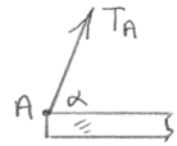
$$\Sigma F_z = 0: \ 2T(0.870) - mg = 0, \quad \underline{T = 0.574mg}$$

$$\Sigma F_x = 0: \ N - 2T(0.348) = 0$$

$$\underline{N = 0.4mg}$$

3/72





$$\alpha = \tan^{-1}\frac{2.5r}{r}$$

$$= 68.2°$$

Symmetry : $T_A = T_B = T_C$
$= T$

$\Sigma F_z = 0: \quad 3T\sin 68.2° - mg = 0$

$\underline{T = 0.359mg}$ $\quad (= T_A = T_B = T_C)$

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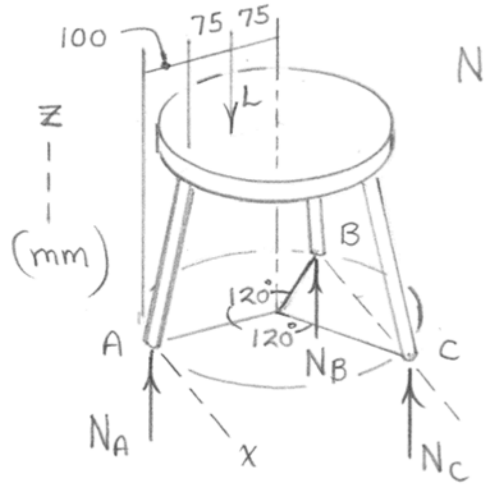

Note: x-axis is ∥ BC.

$N_B = N_C = N$, by symmetry.

$$\Sigma M_x = 0: \quad -L(175) + 2N(250 + 250\cos 60^\circ) = 0$$

$$N = \underline{0.233L = N_B = N_C}$$

$$\Sigma F_z = 0: \quad N_A + 2(0.233L) - L = 0$$

$$\underline{N_A = 0.533L}$$

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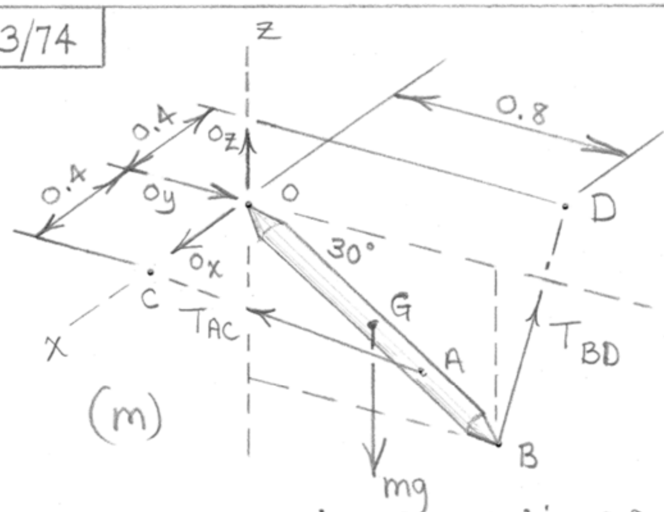


$$\overline{OG} = 0.6 \text{ m}$$
$$\overline{OA} = 0.9 \text{ m}$$
$$\overline{OB} = 1.2 \text{ m}$$

$$\underline{T}_{AC} = T_{AC} \frac{0.4\underline{i} - 0.9\cos 30°\underline{j} + 0.9\sin 30°\underline{k}}{\left\{(0.4)^2 + \left(0.9\frac{\sqrt{3}}{2}\right)^2 + \left(0.9\cdot\frac{1}{2}\right)^2\right\}^{1/2}}$$

$$= T_{AC}\left(0.406\underline{i} - 0.791\underline{j} + 0.457\underline{k}\right)$$

$$\underline{T}_{BD} = T_{BD} \frac{-0.4\underline{i} - (1.2\cos 30° - 0.8)\underline{j} + 1.2\sin 30°\underline{k}}{\left\{(0.4)^2 + \left(1.2\frac{\sqrt{3}}{2} - 0.8\right)^2 + \left(1.2\cdot\frac{1}{2}\right)^2\right\}^{1/2}}$$

$$= T_{BD}\left(-0.526\underline{i} - 0.315\underline{j} + 0.790\underline{k}\right)$$

$$\Sigma M_x = 0: \; -mg\left(0.6\frac{\sqrt{3}}{2}\right) + 0.790\,T_{BD}(0.8) = 0$$

$$\underline{T_{BD} = 0.822\,mg}$$

$$\Sigma M_y = 0: \; -0.457\,T_{AC}(0.4) + 0.790\,T_{BD}(0.4) = 0$$

$$\underline{T_{AC} = 1.422\,mg}$$

$$\Sigma F_x = 0: \; O_x + 1.422mg(0.406) - 0.822mg(0.526) = 0$$

$$\underline{O_x = -0.1443\,mg}$$

$$\Sigma F_y = 0: \; O_y - 1.422mg(0.791) - 0.822mg(0.315) = 0$$

$$\underline{O_y = 1.384\,mg}$$

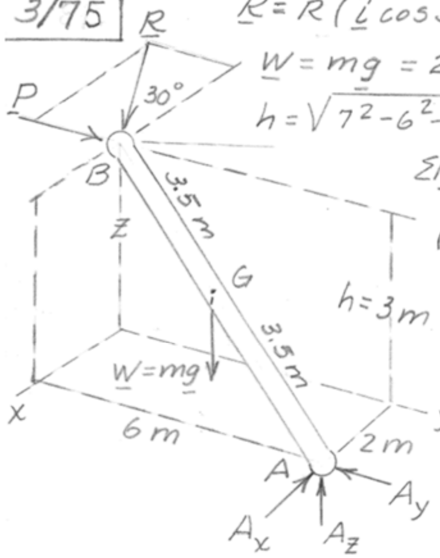
$$\Sigma F_z = 0: \; O_z + 1.422mg(0.457) + 0.822mg(0.790) - mg = 0, \quad \underline{O_z = -0.299\,mg}$$

3/75

$\mathbf{R} = R(\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ) = \frac{R}{2}(\sqrt{3}\,\mathbf{i} + \mathbf{j})$

$\mathbf{W} = m\mathbf{g} = 200(9.81)(-\mathbf{k}) = -1962\,\mathbf{k}$ N

$h = \sqrt{7^2 - 6^2 - 2^2} = 3$ m



$\Sigma \mathbf{M}_A = \mathbf{0};\ \mathbf{r}_{AG} \times \mathbf{W} + \mathbf{r}_{AB} \times (\mathbf{P} + \mathbf{R}) = \mathbf{0}$

$(-1\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k}) \times (-1962\,\mathbf{k})$

$+ (-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}) \times (P\mathbf{j} + \frac{R\sqrt{3}}{2}\mathbf{i} + \frac{R}{2}\mathbf{j})$

$= \mathbf{0}$

Simplify & set

$(5886 - 3P - 3R/2)\,\mathbf{i}$

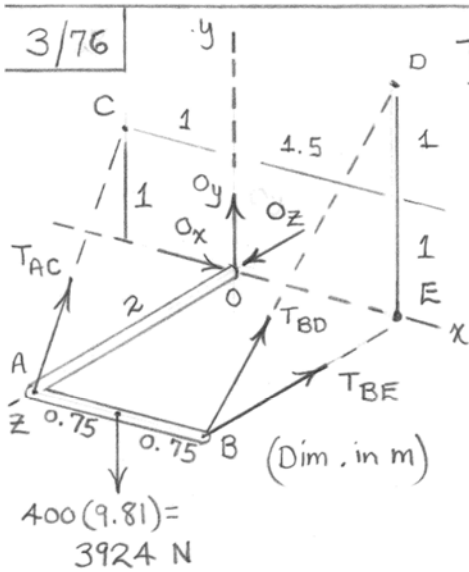
$+ (-1962 + 3\sqrt{3}R/2)\,\mathbf{j}$

$+ (-2P + R + 3\sqrt{3}R)\,\mathbf{k} = 0$

$R = \frac{2(1962)}{3\sqrt{3}} = \underline{755 \text{ N}}$

$3P = 5886 - \frac{3}{2}755, \quad P = \underline{1584 \text{ N}}$

3/76



$$\mathbf{T}_{AC} = T_{AC}\left[\frac{-\mathbf{i}+\mathbf{j}-2\mathbf{k}}{\sqrt{1^2+1^2+2^2}}\right]$$

$$= T_{AC}(-0.408\mathbf{i}+0.408\mathbf{j}-0.816\mathbf{k})$$

$$\mathbf{T}_{BD} = T_{BD}(0.707\mathbf{j}-0.707\mathbf{k})$$

$$\mathbf{T}_{BE} = -T_{BE}\mathbf{k}$$

$\Sigma F_x = 0: \quad O_x - 0.408\,T_{AC} = 0$

$\Sigma F_y = 0: \quad O_y + 0.408\,T_{AC} + 0.707\,T_{BD} - 3924 = 0$

$\Sigma F_z = 0: \quad O_z - 0.816\,T_{AC} - 0.707\,T_{BD} - T_{BE} = 0$

$\Sigma M_{O_x} = 0: \quad -0.408\,T_{AC}(2) - 0.707\,T_{BD}(2) + 3924(2) = 0$

$\Sigma M_{O_y} = 0: \quad -0.408\,T_{AC}(2) + 0.707\,T_{BD}(1.5) + T_{BE}(1.5) = 0$

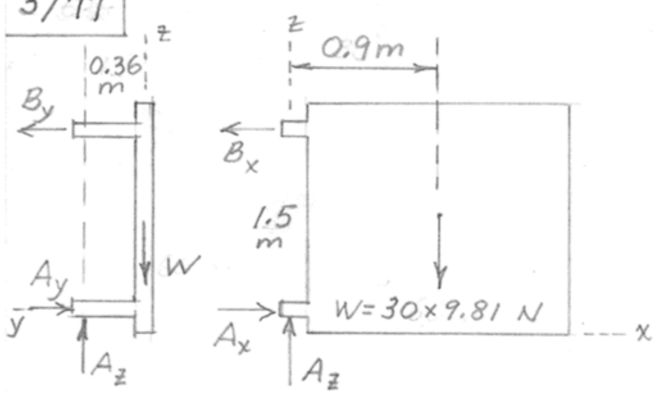
$\Sigma M_{O_z} = 0: \quad -3924(0.75) + 0.707\,T_{BD}(1.5) = 0$

Solution: $O_x = 1962$ N $\qquad T_{AC} = 4810$ N

$O_y = 0$ (Note $\Sigma M_{AB} = 0$) $\qquad T_{BD} = 2770$ N

$O_z = 6540$ N $\qquad T_{BE} = 654$ N

3/77

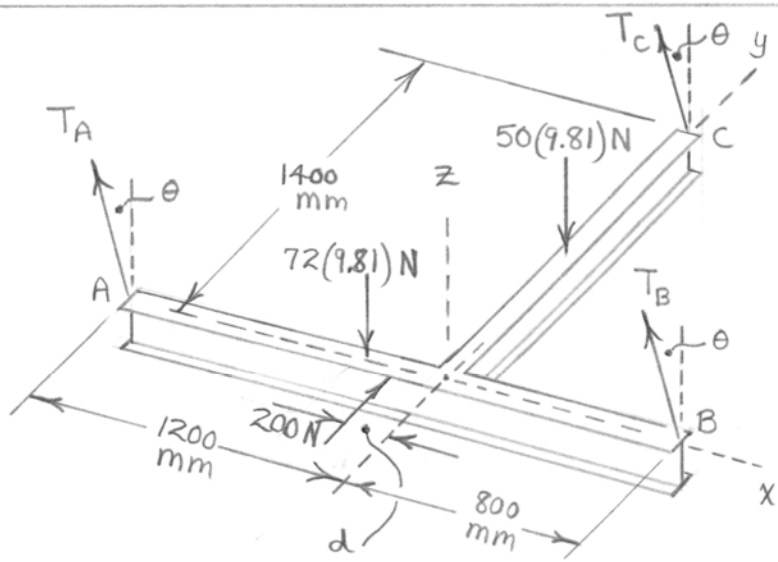


x-z; $\Sigma M_A = 0$; $1.5 B_x - 0.9(30)(9.81) = 0$, $B_x = 176.6$ N

y-z; $\Sigma M_A = 0$; $1.5 B_y - 0.36(30)(9.81) = 0$, $B_y = 70.6$ N

$$B = \sqrt{176.6^2 + 70.6^2} = \underline{190.2 \text{ N}}$$

3/78



$\Sigma F_z = 0: \quad (T_A + T_B + T_C)\cos\theta - (72 + 50)(9.81) = 0$

$\Sigma F_y = 0: \quad -(T_A + T_B + T_C)\sin\theta + 200 = 0$

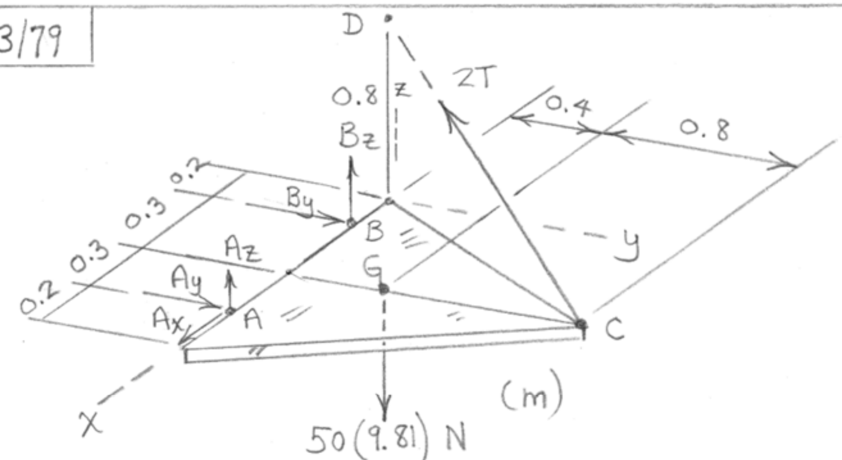
$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{200}{(122)(9.81)}, \qquad \underline{\theta = 9.49°}$

The 200-N force must intersect the line of action of the total weight.

$\bar{X} = \dfrac{\Sigma m\bar{x}}{\Sigma m} = \dfrac{50(0) - 72(+200)}{72 + 50} = \underline{118.0 \text{ mm}}$

(Alternatively, one could determine T_C, T_A, & T_B from $\Sigma M_x = 0$, $\Sigma M_{B_y} = 0$, & $\Sigma M_{A_y} = 0$, respectively, and then $\Sigma M_z = 0$ to determine d)

3/79



$$\underline{T} = T\,\frac{-0.5\underline{i} - 1.2\underline{j} + 0.8\underline{k}}{\sqrt{0.5^2 + 1.2^2 + 0.8^2}} = T(-0.328\underline{i} - 0.786\underline{j} + 0.524\underline{k})$$

$\Sigma F_x = 0: \quad A_x - 2T(0.328) = 0$

$\Sigma F_y = 0: \quad A_y + B_y - 2T(0.786) = 0$

$\Sigma F_z = 0: \quad A_z + B_z - 50(9.81) + 2T(0.524) = 0$

$\Sigma M_x = 0: \quad -50(9.81)(0.4) + 2T(0.524)(1.2) = 0$

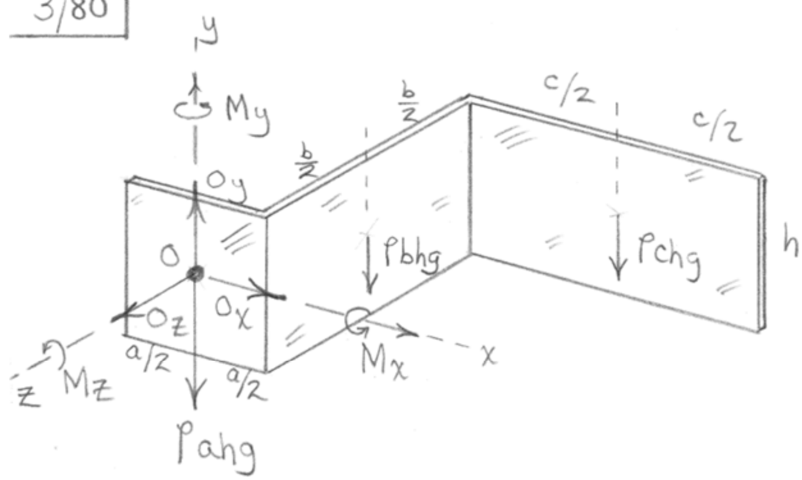
$\Sigma M_y = 0: \quad 50(9.81)(0.5) - A_z(0.8) - B_z(0.2) - 2T(0.524)(0.5) = 0$

$\Sigma M_z = 0: \quad B_y(0.2) + A_y(0.8) = 0$

Solution: $A_x = 102.2\text{ N}$, $A_y = -81.8\text{ N}$, $A_z = 163.5\text{ N}$

$B_y = 327\text{ N}$, $B_z = 163.5\text{ N}$, $T = 156.0\text{ N}$

3/80



$$\Sigma F_x = 0: \quad \underline{O_x = 0}$$

$$\Sigma F_y = 0: \quad \underline{O_y = \rho g h(a+b+c)}$$

$$\Sigma F_z = 0: \quad \underline{O_z = 0}$$

$$\Sigma M_{O_x} = 0: \quad M_x - \rho bhg\left(\frac{b}{2}\right) - \rho chg(b) = 0$$

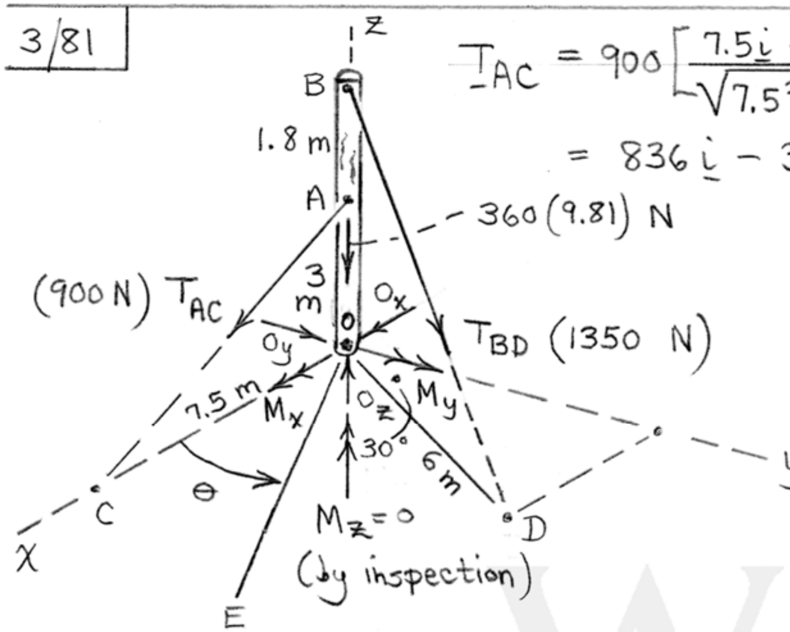
$$\underline{M_x = \rho g h b\left(\frac{b}{2} + c\right)}$$

$$\Sigma M_{O_y} = 0: \quad \underline{M_y = 0}$$

$$\Sigma M_{O_z} = 0: \quad M_z - \rho bhg\left(\frac{a}{2}\right) - \rho chg\left(\frac{a}{2} + \frac{c}{2}\right) = 0$$

$$\underline{M_z = \frac{\rho g h}{2}\left(ab + ac + c^2\right)}$$

3/81



$$\mathbf{T}_{AC} = 900\left[\frac{7.5\mathbf{i} - 3\mathbf{k}}{\sqrt{7.5^2+3^2}}\right] = 836\mathbf{i} - 334\mathbf{k} \text{ N}$$

$$\mathbf{T}_{BD} = 1350\left[\frac{6\sin 30°\,\mathbf{i} + 6\cos 30°\,\mathbf{j} - 4.8\mathbf{k}}{\sqrt{3^2 + 5.20^2 + 4.8^2}}\right] = 527\mathbf{i} + 913\mathbf{j} - 843\mathbf{k} \text{ N}$$

$\Sigma F_x = 0: \quad O_x + 836 + 527 = 0 \qquad (1)$

$\Sigma F_y = 0: \quad O_y + 913 = 0 \qquad (2)$

$\Sigma F_z = 0: \quad O_z - 334 - 843 - 360(9.81) = 0 \qquad (3)$

$\Sigma M_{O_x} = 0: \quad -913(4.8) + M_x = 0 \qquad (4)$

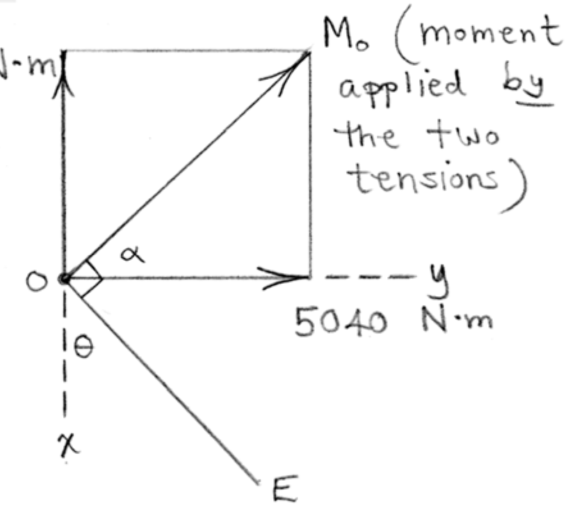
$\Sigma M_{O_y} = 0: \quad 836(3) + 527(4.8) + M_y = 0 \qquad (5)$

Solution of Eqs. (1)–(5):

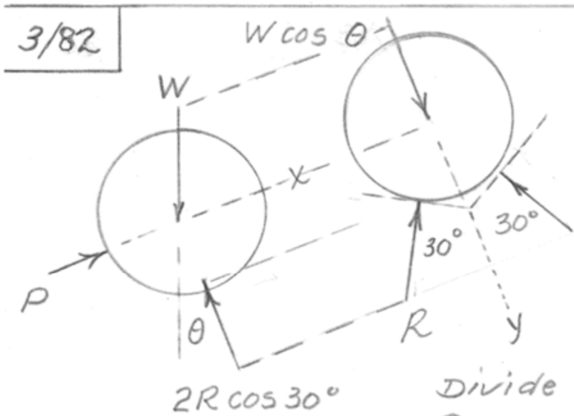
$$\begin{cases} O_x = -1363 \text{ N} \\ O_y = -913 \text{ N} \\ O_z = 4710 \text{ N} \\ M_x = 4380 \text{ N·m} \\ M_y = -5040 \text{ N·m} \end{cases}$$

$\alpha = \tan^{-1}\dfrac{4380}{5040} = 41.0°$

So $\underline{\theta = 41.0°}$



3/82



$\Sigma F_y = 0;$

$2R\cos 30° - W\cos\theta = 0$

$R\sqrt{3} = W\cos\theta$

$\Sigma F_x = 0;$

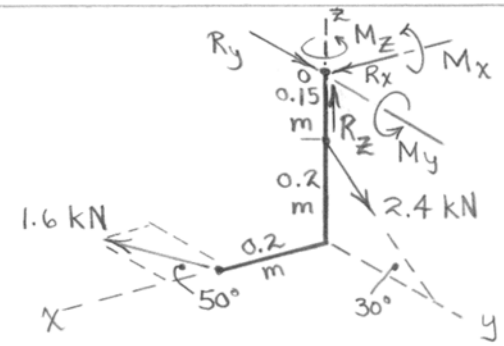
$P - W\sin\theta = 0$

Divide & get

$\frac{P}{R\sqrt{3}} = \tan\theta$

So with $P = R$, $\theta = \tan^{-1} 1/\sqrt{3} = \underline{30°}$

3/83



$\Sigma F_x = 0: \ R_x + 1.6 \cos 50°, \qquad R_x = -1.028 \text{ kN}$

$\Sigma F_y = 0: \ R_y + 2.4 \cos 30° - 1.6 \sin 50° = 0, \quad R_y = -0.853 \text{ kN}$

$\Sigma F_z = 0: \ R_z - 2.4 \sin 30° = 0, \qquad R_z = 1.2 \text{ kN}$

$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \underline{1.796 \text{ kN}}$

$\Sigma M_{O_x} = 0: \ M_x + 2.4 \cos 30° (0.15) - 1.6 \sin 50° (0.35) = 0$

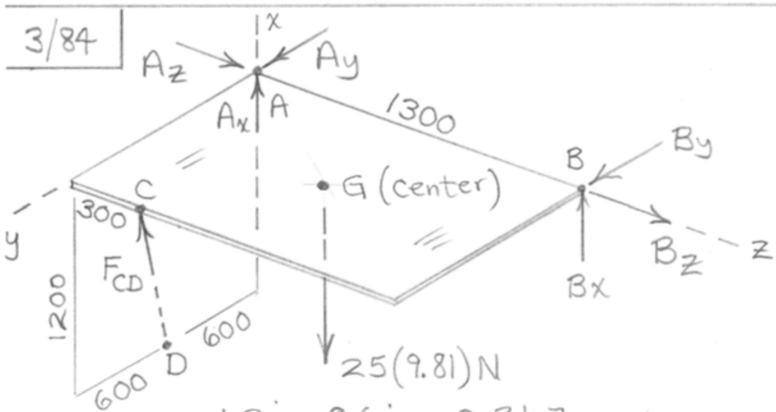
$M_x = 0.1172 \text{ kN·m}$

$\Sigma M_{O_y} = 0: \ M_y - 1.6 \cos 50° (0.35) = 0, \quad M_y = 0.360 \text{ kN·m}$

$\Sigma M_{O_z} = 0: \ M_z - 1.6 \sin 50° (0.2) = 0, \quad M_z = 0.245 \text{ kN·m}$

$M = \sqrt{M_x^2 + M_y^2 + M_z^2} = \underline{0.451 \text{ kN·m}}$

3/84



$$\underline{F}_{CD} = F\left[\frac{1.2\underline{i} + 0.6\underline{j} + 0.3\underline{k}}{\sqrt{1.2^2+0.6^2+0.3^2}}\right] = F(0.873\underline{i} + 0.436\underline{j} + 0.218\underline{k})$$

$\Sigma M_z = 0$: $25(9.81)(0.6) - 0.873F(1.2) = 0$

$\underline{F = 140.5 \text{ N}}$

$\Sigma M_y = 0$: $B_x(1.3) - 25(9.81)(0.65) + 0.873F(0.3) = 0$

$B_x = 94.3$ N

$\Sigma F_x = 0$: $A_x + 94.3 + 0.873F - 25(9.81) = 0$

$A_x = 28.3$ N

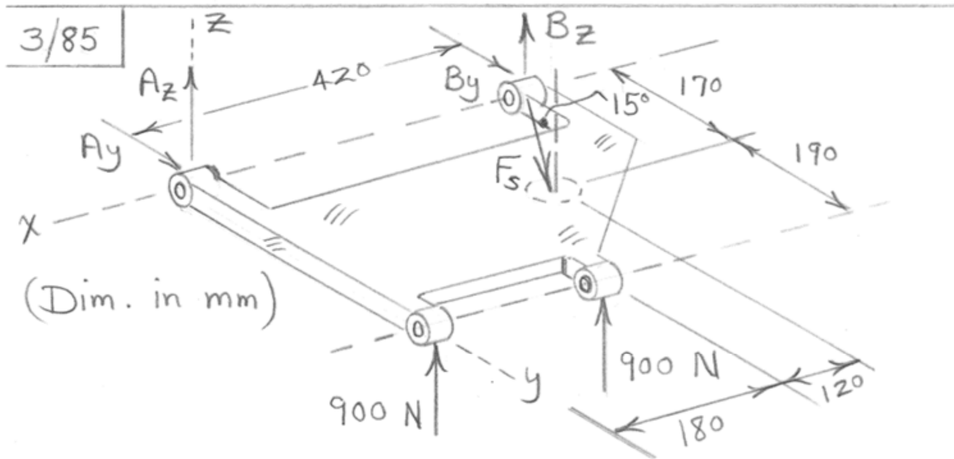
$\Sigma M_x = 0$: $-B_y(1.3) + 0.218F(0.6) = 0$

$B_y = 14.15$ N

$\Sigma F_y = 0$: $A_y + 14.15 + 0.436F = 0$, $A_y = -75.5$ N

$A_n = \sqrt{75.5^2 + 28.3^2} = \underline{80.6\text{ N}}$, $B_n = \sqrt{94.3^2 + 14.15^2} = \underline{95.4\text{ N}}$

3/85



$$\Sigma M_x = 0: \; 2(900)(360) - F_S \cos 15°(170) = 0, \; \underline{F_S = 3950 \text{ N}}$$

$$\Sigma M_y = 0: \; B_z(420) + 900(180) - 3950\cos 15°(300) = 0$$

$$B_z = 2340 \text{ N}$$

$$\Sigma M_z = 0: \; -B_y(420) - 3950 \sin 15°(300) = 0, \; B_y = -730 \text{ N}$$

$$\Sigma F_y = 0: \; A_y - 730 + 3950 \sin 15° = 0, \quad A_y = -292 \text{ N}$$

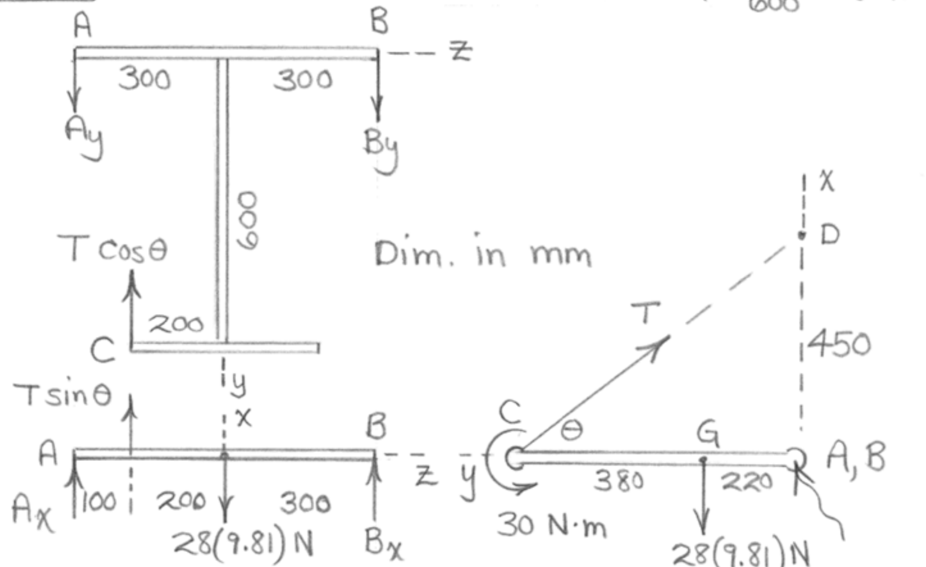
$$\Sigma F_z = 0: \; A_z + 2340 - 3950 \cos 15° + 1800 = 0, \; A_z = -325 \text{ N}$$

$$F_A = \sqrt{A_y^2 + A_z^2} = \underline{437 \text{ N}}$$

$$F_B = \sqrt{B_y^2 + B_z^2} = \underline{2450 \text{ N}}$$

3/86

$\theta = \tan^{-1}\frac{450}{600} = 36.9°$



$(x\text{-}y)\ \Sigma M_z = 0:\ 28(9.81)(0.220) - T\sin 36.9°(0.600) + 30 = 0,\quad T = 251\text{ N}$

$(x\text{-}z)\ \Sigma M_B = 0:\ 28(9.81)(0.300) - 251\sin 36.9°(0.500) - 0.600A_x = 0,\ A_x = 11.74\text{ N}$

$\Sigma F_x = 0:\ 11.74 + 251\sin 36.9° - 28(9.81) + B_x = 0,\quad B_x = 112.2\text{ N}$

$(y\text{-}z)\ \Sigma M_B = 0:\ 251\cos 36.9°(0.500) - 0.6A_y = 0,\quad A_y = 167.5\text{ N}$

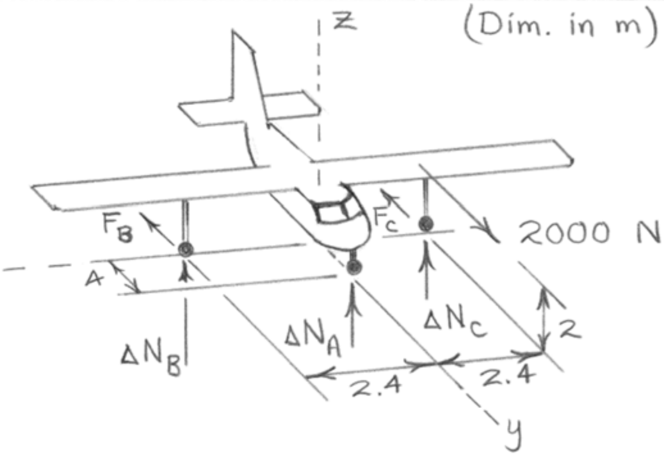
$\Sigma F_y = 0:\ 167.5 + B_y - 251\cos 36.9° = 0,\quad B_y = 33.5\text{ N}$

$A = \sqrt{11.74^2 + 167.5^2} = \underline{167.9\text{ N}}$

$B = \sqrt{112.2^2 + 33.5^2} = \underline{117.1\text{ N}}$

Couple may be applied at any place on rigid body with the same external effect.

3/87

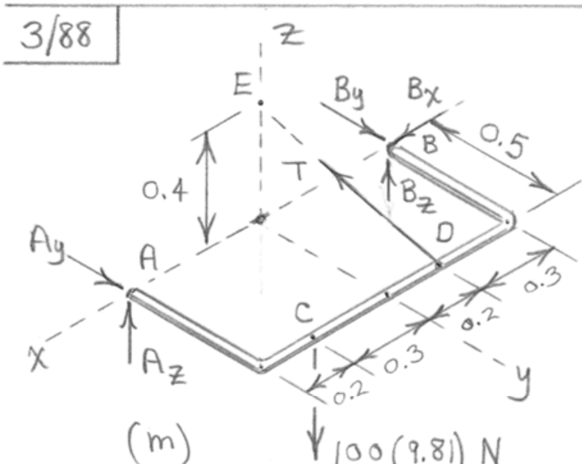


$\Sigma M_x = 0: \Delta N_A (4) - 2000(2) = 0$, $\underline{\Delta N_A = 1000 \text{ N}}$

$\left.\begin{array}{l}\Sigma F_z = 0: \Delta N_A + \Delta N_B + \Delta N_C = 0 \\ \Sigma M_y = 0: \Delta N_C (2.4) - \Delta N_B (2.4) = 0\end{array}\right\}$ $\underline{\Delta N_B = \Delta N_C = -500 \text{ N}}$

More information would be required to determine F_B & F_C. x-components of friction at B & C are possible.

3/88



$$\underline{T} = T\underline{n}_{DE} = T\left[\frac{0.2\underline{i} - 0.5\underline{j} + 0.4\underline{k}}{\{0.2^2 + 0.5^2 + 0.4^2\}^{1/2}}\right]$$

$$= T[0.298\underline{i} - 0.745\underline{j} + 0.596\underline{k}]$$

$\Sigma F_x = 0: \quad B_x + 0.298T = 0$ (1)

$\Sigma F_y = 0: \quad A_y + B_y - 0.745T = 0$ (2)

$\Sigma F_z = 0: \quad A_z + B_z + 0.596T - 981 = 0$ (3)

$\Sigma M_x = 0: \quad 0.596T(0.5) - 981(0.5) = 0$ (4)

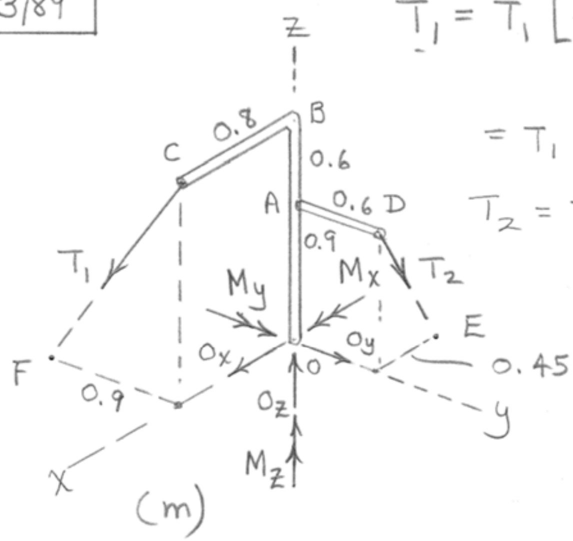
$\Sigma M_y = 0: \quad -A_z(0.5) + 981(0.3) + 0.596T(0.2) + B_z(0.5) = 0$ (5)

$\Sigma M_z = 0: \quad A_y(0.5) - B_y(0.5) = 0$ (6)

Solution: $T = 1645$ N, $A_y = 613$ N, $A_z = 490$ N

$B_x = -490$ N, $B_y = 613$ N, $B_z = -490$ N

3/89



$$\mathbf{T}_1 = T_1\left[\frac{-0.9\mathbf{j} - 1.5\mathbf{k}}{\sqrt{0.9^2 + 1.5^2}}\right] = T_1(-0.514\mathbf{j} - 0.857\mathbf{k})$$

$$\mathbf{T}_2 = T_2\left[\frac{-0.45\mathbf{i} - 0.9\mathbf{k}}{\sqrt{0.45^2 + 0.9^2}}\right] = T_2(-0.447\mathbf{i} - 0.894\mathbf{k})$$

(m)

$\Sigma F_x = 0: \quad O_x - 0.447T_2 = 0 \qquad (1)$

$\Sigma F_y = 0: \quad O_y - 0.514T_1 = 0 \qquad (2)$

$\Sigma F_z = 0: \quad O_z - 0.857T_1 - 0.894T_2 = 0 \qquad (3)$

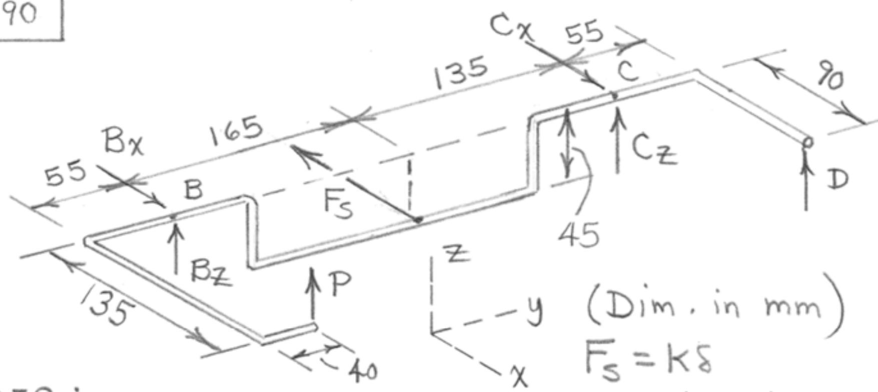
$\Sigma M_{O_x} = 0: \quad M_x + 0.514T_1(1.5) - 0.894T_2(0.6) = 0 \qquad (4)$

$\Sigma M_{O_y} = 0: \quad M_y + 0.857T_1(0.8) - 0.447T_2(0.9) = 0 \qquad (5)$

$\Sigma M_{O_z} = 0: \quad M_z - 0.514T_1(0.8) + 0.447T_2(0.6) = 0 \qquad (6)$

Solution: $\begin{cases} O_x = 224 \text{ N},\ O_y = 386 \text{ N},\ O_z = 1090 \text{ N} \\ M_x = -310 \text{ N·m},\ M_y = -313 \text{ N·m},\ M_z = 174.5 \text{ N·m} \end{cases}$

3/90



$F_s = k\delta$
$= 900(0.060) = 54\text{ N}$

$D = 0$:

$\Sigma M_{BC} = 0: \; -P(135) + 54(45) = 0, \quad \underline{P_{min} = 18\text{ N}}$

$\Sigma M_{B_x} = 0: \; -18(15) + C_z(300) = 0, \quad C_z = 0.9\text{ N}$

$\Sigma M_{B_z} = 0: \; 54(165) - C_x(300) = 0, \quad C_x = 29.7\text{ N}$

$\Sigma F_x = 0: \; 29.7 + B_x - 54 = 0, \quad B_x = 24.3\text{ N}$

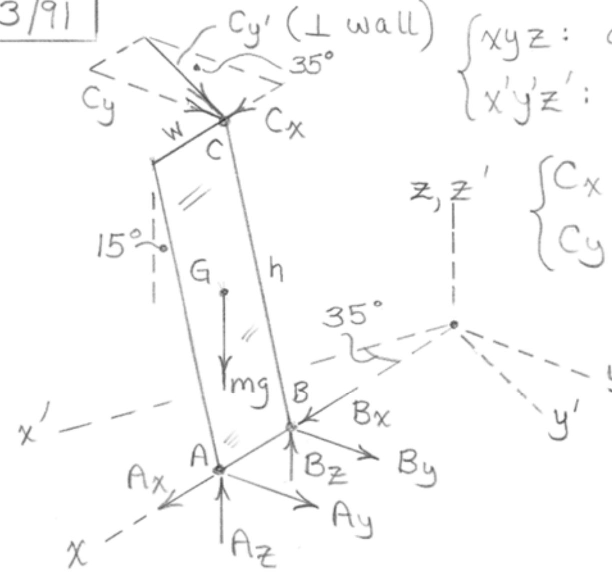
$\Sigma F_z = 0: \; 0.9 + B_z + 18 = 0, \quad B_z = -18.9\text{ N}$

$B = \sqrt{B_x^2 + B_z^2} = \underline{30.8\text{ N}}, \; C = \sqrt{C_x^2 + C_z^2} = \underline{29.7\text{ N}}$

If $P = P_{min}/2 = 18/2 = 9\text{ N}, \; (D \neq 0)$:

$\Sigma M_{BC} = 0: \; -9(135) + 54(45) - D(90) = 0, \quad \underline{D = 13.5\text{ N}}$

3/91



$\begin{cases} xyz: \text{ door-based} \\ x'y'z': \text{ wall-based} \end{cases}$

$\begin{cases} C_x = C_{y'} \sin 35° \\ C_y = C_{y'} \cos 35° \end{cases}$

$\Sigma M_x = 0: \quad mg\frac{h}{2}\sin 15° - C_y h \cos 15° = 0, \quad C_y = 0.1340\,mg$

$C_{y'} = \dfrac{C_y}{\cos 35°} = \underline{0.1636\,mg}, \quad C_x = C_y \tan 35° = 0.0938\,mg$

$\Sigma M_{B_z} = 0: \quad C_x h \sin 15° + A_y w = 0, \quad \underline{A_y = -0.0243\,mg\frac{h}{w}}$

$\Sigma M_{B_y} = 0: \quad mg\frac{w}{2} + C_x h \cos 15° - A_z w = 0$

$\underline{A_z = mg\left(\frac{1}{2} + 0.0906\frac{h}{w}\right)}$

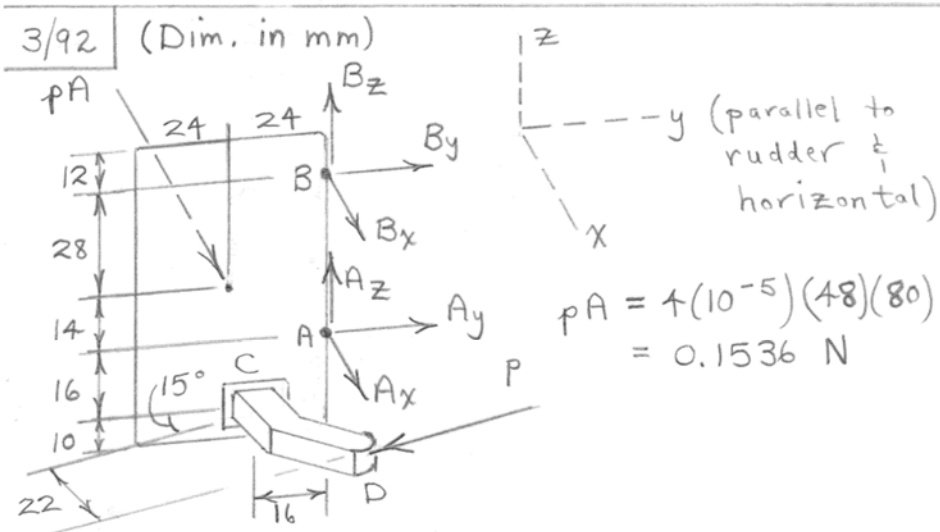
$\Sigma M_{A_z} = 0: \quad -B_y w - C_y w + C_x h \sin 15° = 0$

$\underline{B_y = mg\left(0.0243\frac{h}{w} - 0.1340\right)}$

$\Sigma M_{A_y} = 0: \quad B_z w - mg\frac{h}{2} + C_x h \cos 15° = 0$

$\underline{B_z = mg\left(\frac{1}{2} - 0.0906\frac{h}{w}\right)}$

3/92 (Dim. in mm)



$pA = 4(10^{-5})(48)(80)$
$= 0.1536$ N

$\Sigma M_{AB} = 0: \; -P(22 - 16 \sin 15°) + 0.1536(24) = 0$

$\underline{P = 0.206 \text{ N}}$

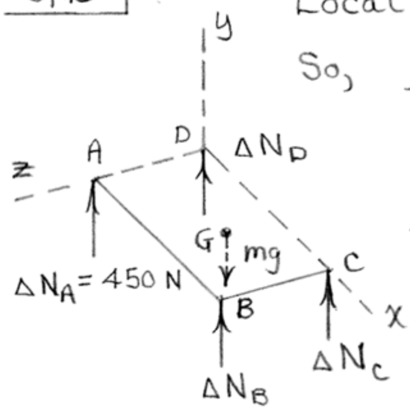
$\Sigma M_{B_x} = 0: \; A_y(42) - 0.206 \cos 15°(58) = 0$

$\underline{A_y = 0.275 \text{ N}}$

$\Sigma M_{A_x} = 0: \; -B_y(42) - 0.206 \cos 15°(16) = 0$

$\underline{B_y = -0.0760 \text{ N}}$

3/93



Location of G does not change.

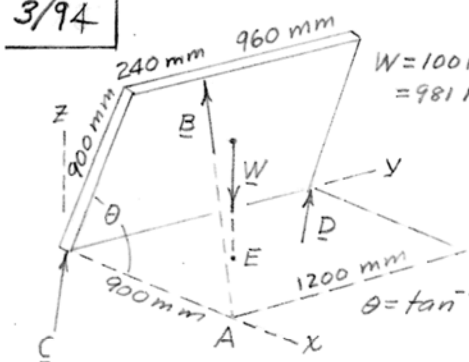
So, $\underline{\Delta N_D = -450 \text{ N}}$ (preserves total rear-axle loading)

$\underline{\Delta N_B = -450 \text{ N}}$ (preserves total right-side loading)

$\underline{\Delta N_C = 450 \text{ N}}$ (preserves total normal force and front-axle loading)

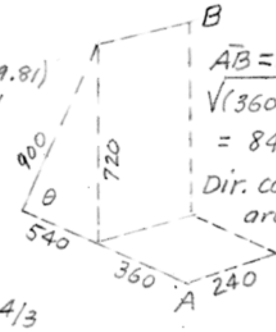
Note: The results for ΔN_B & ΔN_C hold only if the track (distance between tire centers) at the front axle is equal to that at the rear.

3/94



$W = 100(9.81) = 981\ N$

$\theta = \tan^{-1} 4/3$



$\overline{AB} = \sqrt{(360)^2 + (240)^2 + (720)^2} = 840\ mm$

Dir. cosines of $\overrightarrow{AB}$ are $l = \frac{-360}{840} = -\frac{3}{7}$

$m = \frac{240}{840} = \frac{2}{7}$

$n = \frac{720}{840} = \frac{6}{7}$

$\Sigma M_{CD} = 0;\ (\underline{r}_{CA} \times \underline{B} + \underline{r}_{CE} \times \underline{W}) \cdot \underline{j} = 0$, moments of $\underline{C}$ & $\underline{D}$ are zero

$\left[0.9\underline{i} \times \frac{B}{7}(-3\underline{i} + 2\underline{j} + 6\underline{k}) + (0.27\underline{i} + 0.6\underline{j}) \times (-981\underline{k})\right] \cdot \underline{j} = 0$

$\left[\frac{B}{7}(1.8\underline{k} - 5.4\underline{j}) + 981(0.27\underline{j} - 0.6\underline{i})\right] \cdot \underline{j} = 0$

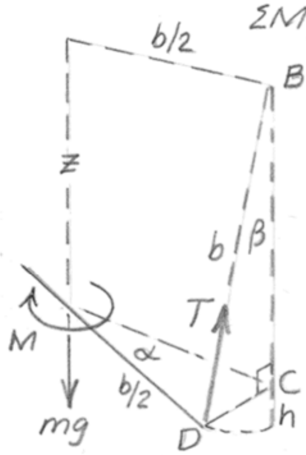
$-\frac{5.4B}{7} + 981(0.27) = 0,\ \underline{B = 343\ N}$

$\Sigma M_x = 0;\ 1.2D_z - 981(0.6) = 0,\ D_z = 491\ N$

$\Sigma M_z = 0;\ \frac{3}{7}(343)(0.24) + \frac{2}{7}(343)(0.54) - 1.2D_x = 0,\ D_x = 73.6\ N$

$D_{normal} = D_n = \sqrt{(73.6)^2 + (491)^2} = \underline{496\ N}$

3/95



$\Sigma F_z = 0$; $2T\cos\beta - mg = 0$ - - - - - - - - - - (1)

$\Sigma M_z = 0$; $2T\sin\beta\cos\frac{\alpha}{2}\left(\frac{b}{2}\right) - M = 0$ - - - (2)

$\overline{CD} = 2\frac{b}{2}\sin\frac{\alpha}{2} = b\sin\beta$, $\beta = \frac{\alpha}{2}$ - - (3)

Divide (2) by (1) & substitute (3) & get

$$\frac{2T\frac{b}{2}\sin\beta\cos\beta}{2T\cos\beta} = \frac{M}{mg}, \quad \sin\beta = \frac{2M}{bmg}$$

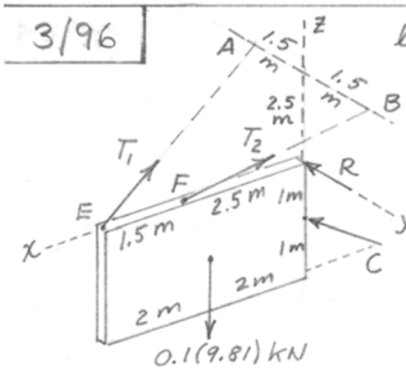
Thus $\cos\beta = \sqrt{1-\left(\frac{2M}{bmg}\right)^2}$

& $h = b(1-\cos\beta)$

so $\underline{h = b\left(1-\sqrt{1-\left(\frac{2M}{bmg}\right)^2}\right)}$

For $h \to b$, $\cos\beta \to 0$, $\sin\beta \to \pi/2$ & $M \to \underline{\frac{bmg}{2}}$

3/96



$\ell_1 = \overline{AE} = \sqrt{4^2 + 1.5^2 + 2.5^2} = 4.95$ m

$\ell_2 = \overline{BF} = \sqrt{2.5^2 + 1.5^2 + 2.5^2} = 3.84$ m

$\underline{T}_1 = \frac{T_1}{\ell_1}(-4\underline{i} - 1.5\underline{j} + 2.5\underline{k})$

$\underline{T}_2 = \frac{T_2}{\ell_2}(-2.5\underline{i} + 1.5\underline{j} + 2.5\underline{k})$

Note: $\underline{T}_1$, $\underline{T}_2$, $\underline{R}$, and weight all pass through x-axis, so $\underline{C}$ must also. Thus $C_y = 0$

$\Sigma M_{AB} = 0$; $C_x(3.5) - 9.81(2) = 0$, $C_x = 0.561$ kN

$\Sigma M_z = 0$; $4\underline{i} \times \frac{T_1}{\ell_1}(-1.5\underline{j}) + 2.5\underline{i} \times \frac{T_2}{\ell_2}(1.5\underline{j}) = \underline{0}$, $8T_1/\ell_1 = 5T_2/\ell_2$

$\Sigma F_x = 0$; $-\frac{T_1}{\ell_1}(4) - \frac{T_2}{\ell_2}(2.5) + 0.561 = 0$, $8T_1/\ell_1 + 5T_2/\ell_2 = 1.121$ kN

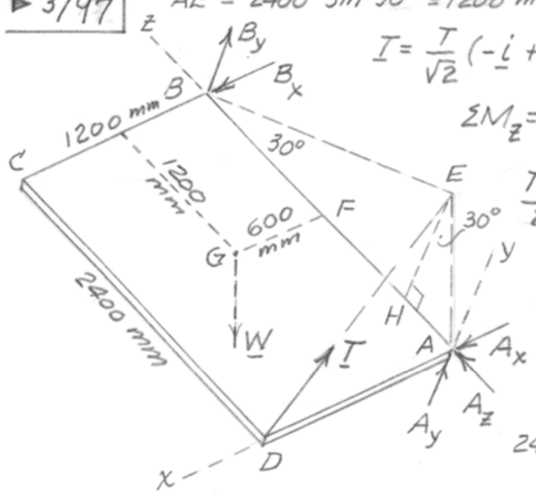
solve & get $T_1 = 1.121\,\ell_1/16 = \underline{0.347 \text{ kN}}$, $T_2 = \frac{1.121}{10}\ell_2 = \underline{0.431 \text{ kN}}$

$\Sigma F_y = 0$; $\frac{1.121}{16}\ell_1\frac{1.5}{\ell_1} - \frac{1.121}{10}\ell_2\frac{1.5}{\ell_2} + R = 0$, $\underline{R = 0.0631 \text{ kN}}$

$\Sigma F_z = 0$; $\frac{1.121}{16}\ell_1\frac{2.5}{\ell_1} + \frac{1.121}{10}\ell_2\frac{2.5}{\ell_2} + C_z - 0.981 = 0$, $C_z = 0.526$ kN

Thus $C = \sqrt{(0.561)^2 + (0.526)^2} = \underline{0.768 \text{ kN}}$

▶ 3/97 $\overline{AE} = 2400 \sin 30° = 1200$ mm, $\overline{AH} = 600$ mm, $\overline{HE} = 1200 \cos 30° = 600\sqrt{3}$ mm



$$\underline{T} = \frac{T}{\sqrt{2}}\left(-\underline{i} + \frac{\sqrt{3}}{2}\underline{j} + \frac{1}{2}\underline{k}\right)$$

$\Sigma M_z = 0$; $T_y \overline{AD} - W_y \overline{GF} = 0$

$$\frac{T\sqrt{3}}{2\sqrt{2}}(1200) = 40(9.81)\frac{\sqrt{3}}{2}(600)$$

$$T = 20(9.81)\sqrt{2} = \underline{277 \text{ N}}$$

$\Sigma M_x = 0$; $-B_y \overline{AB} + W_y \overline{FA} = 0$

$$2400 B_y = 40(9.81)\frac{\sqrt{3}}{2}(1200)$$

$$B_y = 98.1\sqrt{3} = 169.9 \text{ N}$$

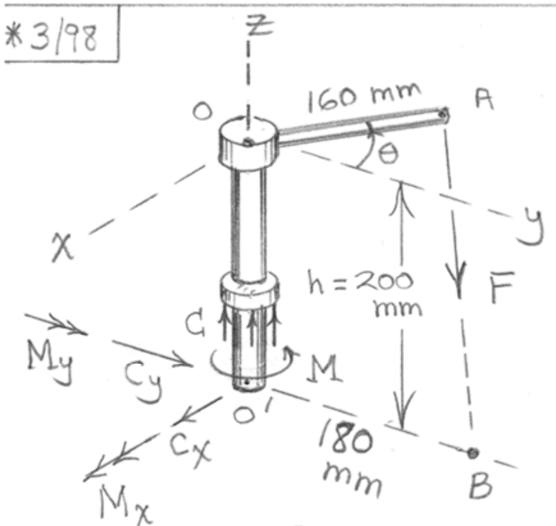
$\Sigma M_y = 0$; $B_x \overline{AB} + W_z \overline{GF} - T_z \overline{AD} = 0$

$$2400 B_x + 40(9.81)0.5(600) - \frac{277}{2\sqrt{2}}(1200) = 0$$

$$B_x = 0$$

$B = B_y = \underline{169.9 \text{ N}}$ ($B_x = 0$ can be obtained by inspection by noting $\Sigma M_{AE} = 0$ eliminates all terms except $B_x \overline{BE}$ so $B_x = 0$)

*3/98



C is a distributed vertical normal reaction force!

$\mathbf{F} = k\delta\,\mathbf{n}_{AB}$, where δ is the spring stretch.

$$\delta = \overline{AB} - \overline{AB}_0 = \left[(\overline{OA}\sin\theta)^2 + (\overline{O'B} - \overline{OA}\cos\theta)^2 + h^2\right]^{1/2} - \sqrt{h^2 + (\overline{OB} - \overline{OA})^2}$$

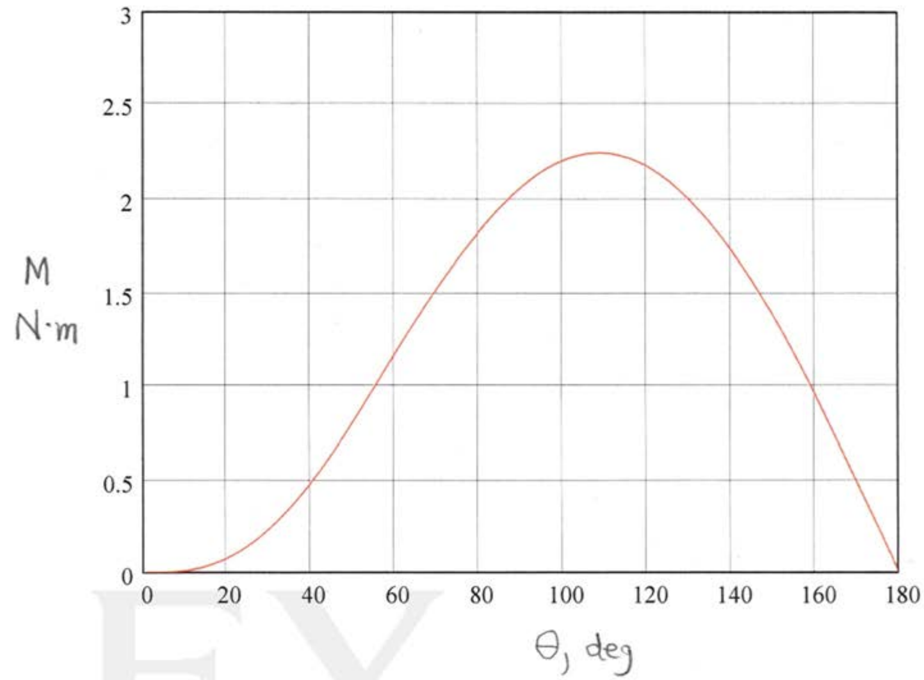
$$\mathbf{n}_{AB} = \frac{\mathbf{AB}}{\overline{AB}} = \frac{+\overline{OA}\sin\theta\,\mathbf{i} + (\overline{O'B} - \overline{OA}\cos\theta)\,\mathbf{j} - h\,\mathbf{k}}{\left[(\overline{OA}\sin\theta)^2 + (\overline{O'B} - \overline{OA}\cos\theta)^2 + h^2\right]^{1/2}}$$

and $\mathbf{M}_{O'} = \mathbf{r}_{O'A} \times \mathbf{F}$ $\left\{ \mathbf{r}_{O'A} = -\overline{OA}\sin\theta\,\mathbf{i} + \overline{OA}\cos\theta\,\mathbf{j} + h\,\mathbf{k} \right.$

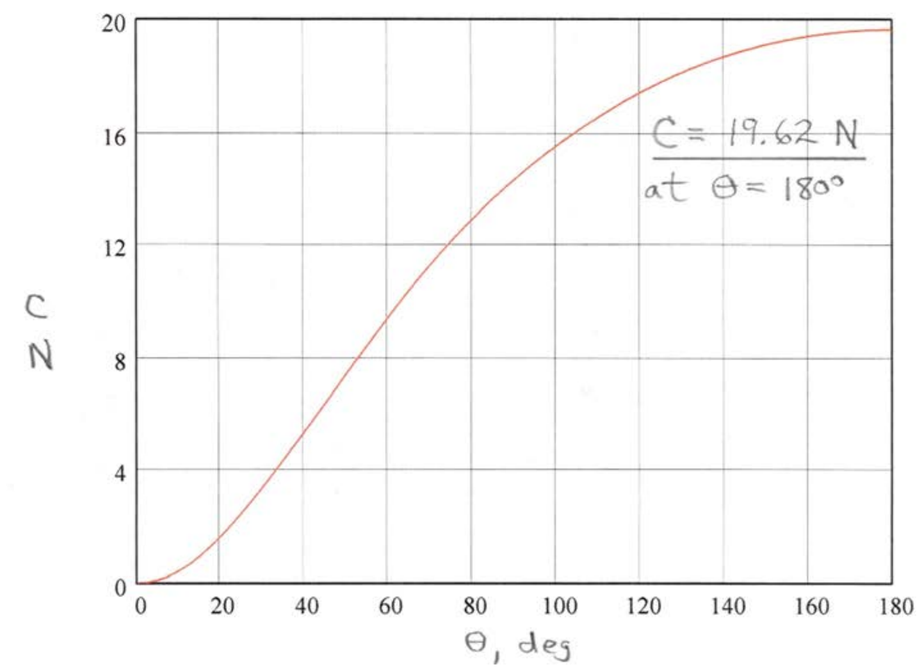
$$\Sigma M_z = M + (M_{O'})_z = 0 \Rightarrow M = -(M_{O'})_z$$

$$\Sigma F_z = 0 : \quad C - F_z = 0 \Rightarrow C = F_z$$

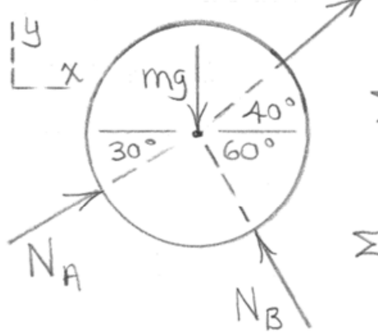
Carry out over $0 \le \theta \le \pi$ to obtain:



$M_{max} = 2.24$ N·m at $\theta = 108.6°$



3/99



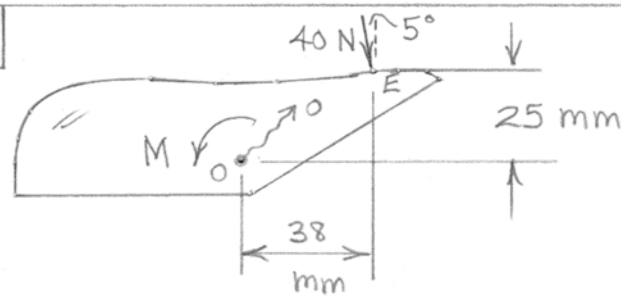
$N_A \to 0$:

$\sum F_x = 0 : P\cos 40° - N_B \cos 60° = 0$

$\sum F_y = 0 : P\sin 40° + N_B \sin 60° - mg = 0$

Solution : $\begin{cases} N_B = 0.778\,mg \\ P = 0.508\,mg \end{cases}$

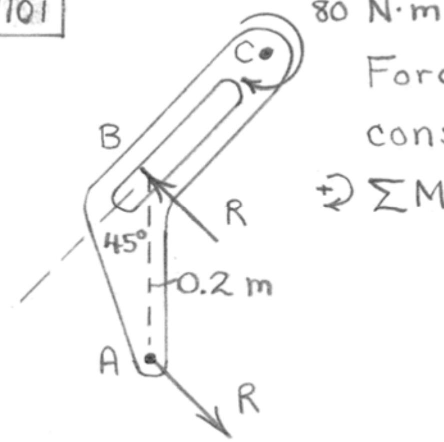
3/100



$$\curvearrowleft + \Sigma M_O = 0: \quad M - 40\cos 5°(38) - 40\sin 5°(25) = 0$$

$$\underline{M = 1601 \text{ N·mm CCW}}$$

3/101

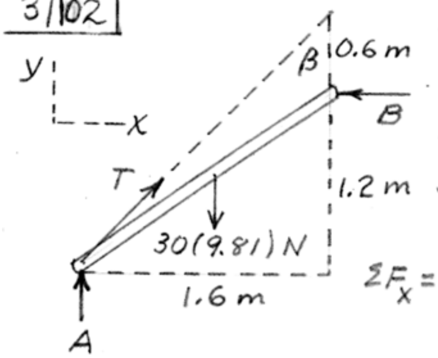


Forces at A and B must constitute a couple.

$\circlearrowleft + \ \Sigma M = 0 : \ 80 - R(0.2 \cos 45°) = 0$

$\underline{R = 566 \text{ N}}$

3/102



$\Sigma M_A = 0;\ 1.2B - 30(9.81)(0.8) = 0$

$\underline{B = 196.2\text{ N}}$

$\beta = \tan^{-1}\frac{1.6}{1.8} = 41.63°$

$\sin\beta = 0.6644$

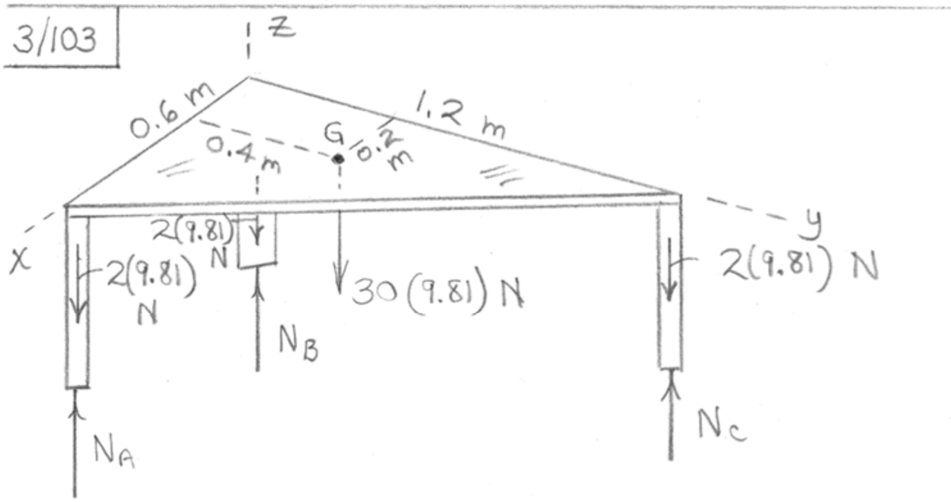
$\cos\beta = 0.7474$

$\Sigma F_x = 0;\ 0.6644T - 196.2 = 0$

$\underline{T = 295\text{ N}}$

$\Sigma F_y = 0;\ A + 295(0.7474) - 30(9.81) = 0,\ \underline{A = 73.6\text{ N}}$

3/103



$\Sigma M_y = 0: \; 30(9.81)(0.2) + 2(9.81)(0.6) - N_A(0.6) = 0$

$\underline{N_A = 117.7 \text{ N}}$

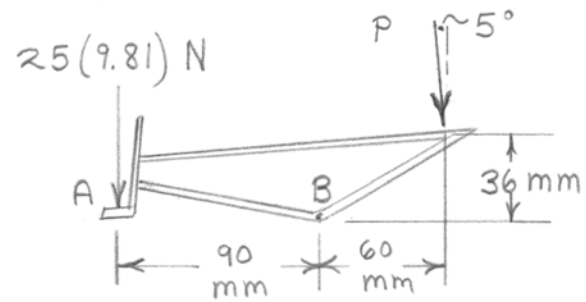
$\Sigma M_x = 0: \; -30(9.81)(0.4) - 2(9.81)(1.2) + N_C(1.2) = 0$

$\underline{N_C = 117.7 \text{ N}}$

$\Sigma F_z = 0: \; N_A + N_B + N_C - 36(9.81) = 0$

$\underline{N_B = 117.7 \text{ N}}$

3/104



$$\circlearrowleft + \Sigma M_B = 0: \; 25(9.81)(90) - P\cos 5°(60) - P\sin 5°(36) = 0$$

$$\underline{P = 351 \text{ N}}$$

Assumptions: The weight of the panel acts at the 90-mm dimension as shown above; panel does not slip.

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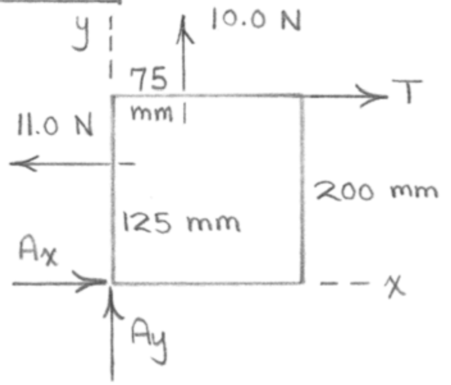
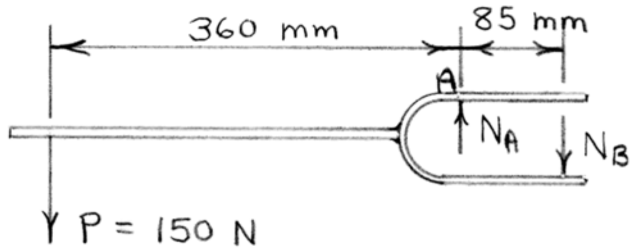


Isolate entire system of plate, pulleys, & tape.

$$\stackrel{+}{\circlearrowleft} \Sigma M_A = 0: \quad T(200) - 10.0(75) - 11.0(125) = 0$$

$$\underline{T = 10.62 \text{ N}}$$

3/106



$$\circlearrowleft + \Sigma M_A = 0: \quad 150(360) - N_B(85) = 0$$
$$N_B = 635 \text{ N}$$

$$\uparrow \Sigma F = 0: \quad -150 + N_A - 635 = 0$$
$$N_A = 785 \text{ N}$$

So the forces applied to the stud are

$$\begin{cases} N_A = 785 \text{ N} \downarrow \\ N_B = 635 \text{ N} \uparrow \end{cases}$$

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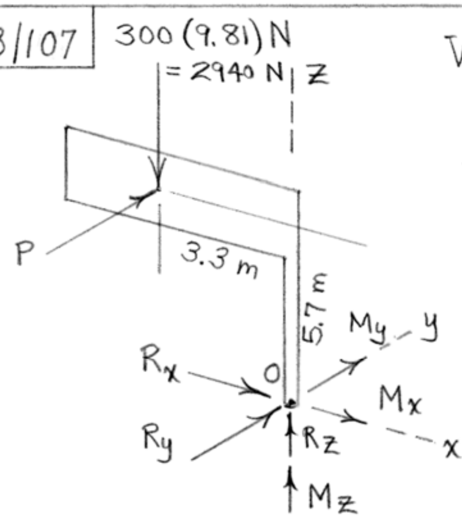


Wind force $P = pA$

$= 700(4)(2) = 5600\text{ N}$

$\Sigma F_x = 0 \Rightarrow R_x = 0$

$\Sigma F_y = 0 \Rightarrow R_y = -5600\text{ N}$

$\Sigma F_z = 0 \Rightarrow R_z = 2940\text{ N}$

$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

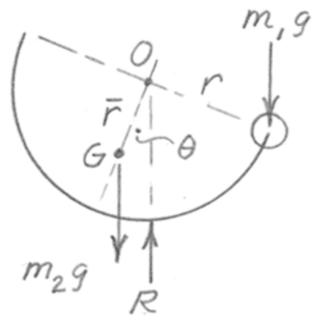
$= \underline{6330\text{ N}}$

$\Sigma \underline{M}_O = \underline{0}: \quad \underline{M} + (-3.3\underline{i} + 5.7\underline{k}) \times (5600\underline{j} - 2940\underline{k}) = \underline{0}$

$\Rightarrow \underline{M} = 31900\underline{i} + 9710\underline{j} + 18\,480\underline{k}\ \text{N}\cdot\text{m}$

$M = 38\,100\ \text{N}\cdot\text{m}$ or $\underline{38.1\ \text{kN}\cdot\text{m}}$

3/108



$\Sigma M_O = 0;$

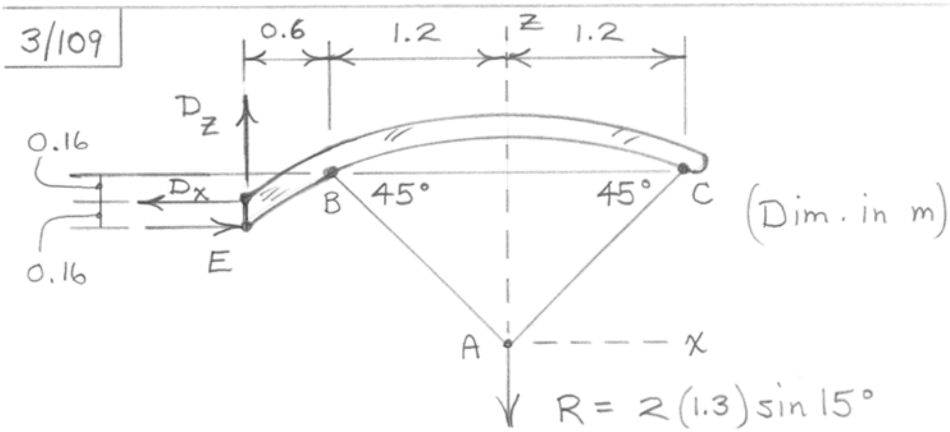
$m_1 g r \cos\theta - m_2 g \bar{r} \sin\theta = 0$

where $\bar{r} = 2r/\pi$

so $\tan\theta = \dfrac{m_1 r}{m_2 \bar{r}} = \dfrac{m_1 \pi}{2 m_2}$

$\theta = \tan^{-1} \dfrac{\pi m_1}{2 m_2}$

3/109



$R = 2(1.3)\sin 15°$
$= 0.673 \text{ kN}$

$\Sigma F_z = 0: \; D_z - 0.673 = 0, \quad D_z = 0.673 \text{ kN}$

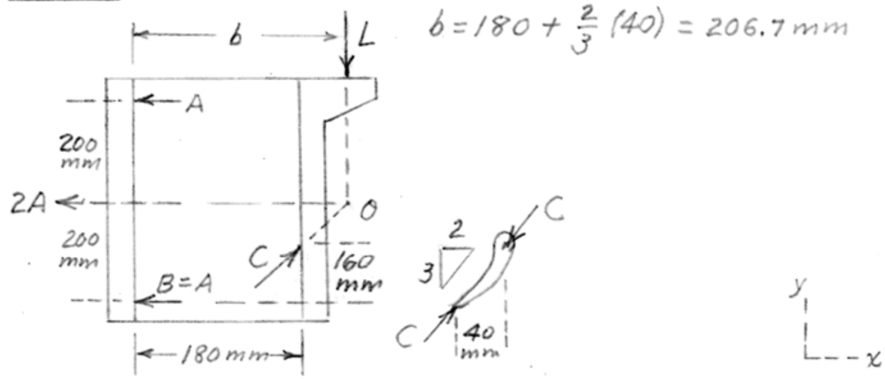
$\circlearrowleft + \Sigma M_E = 0: \; 0.16 D_x - 0.673(1.8) = 0$

$D_x = 7.57 \text{ kN}$

$D = \sqrt{0.673^2 + 7.57^2} = \underline{7.60 \text{ kN}}$

3/110 Reduce to three concurrent forces at O

$b = 180 + \frac{2}{3}(40) = 206.7$ mm



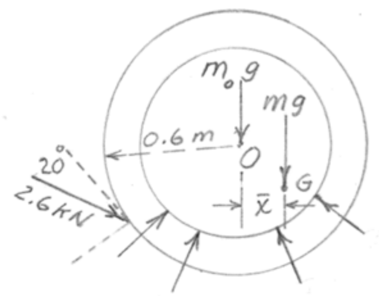
(Alternative solution:)

$\Sigma M_C = 0;\ L(b-180) - A(240) + A(160) = 0,\ L(b-180) = 80A$

$\Sigma F_y = 0;\ \frac{3}{\sqrt{13}} C = L \qquad \Sigma F_x = 0;\ \frac{2}{\sqrt{13}} C = 2A \quad$ so $L = 3A$

So $3(b-180) = 80$, $3b = 620$, $\underline{b = 207 \text{ mm}}$

3/111

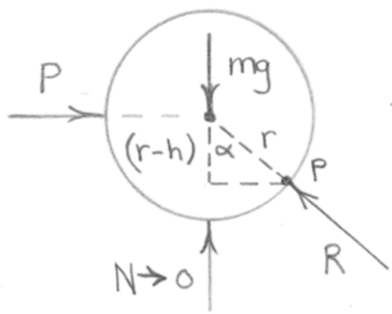


$mg = 750(9.81)(10^{-3}) = 7.358$ kN

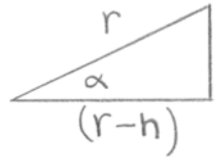
$\Sigma M_O = 0$; $2.6 \cos 20° (600) - 7.358\,\bar{x} = 0$

$\bar{x} = 199.2$ mm

3/112



$$\cos\alpha = \frac{r-h}{r}$$



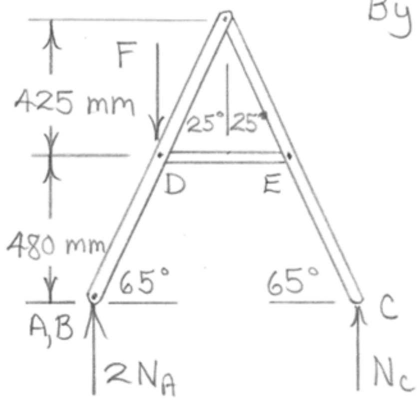
$$\sqrt{r^2-(r-h)^2} = \sqrt{2rh-h^2}$$

$$\sin\alpha = \sqrt{2rh-h^2}\,/\,r$$

$$+\circlearrowleft\ \Sigma M_P = 0:\quad P(r-h) - mg\,r\sin\alpha = 0$$

$$\Rightarrow\ P = \frac{mg\sqrt{2rh-h^2}}{r-h}$$

3/113



By inspection, $N_A = N_B$.

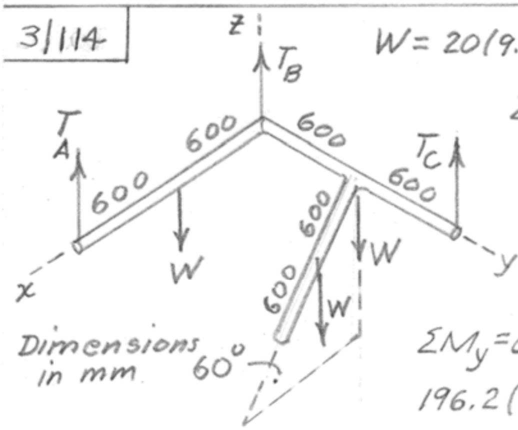
$$\circlearrowleft\!+ \; \Sigma M_C = 0: \; -2N_A(2 \cdot 905 \tan 25^\circ) + F(905 \tan 25^\circ + 425 \tan 25^\circ) = 0$$

$$\underline{N_A = 0.367F = N_B}$$

$$+\uparrow \Sigma F = 0: \; 2(0.367F) + N_C - F = 0$$

$$\underline{N_C = 0.265F}$$

3/114



$W = 20(9.81) = 196.2$ N

$\Sigma M_x = 0;$

$1200\,T_C - 2(196.2)600 = 0$

$\underline{T_C = 196.2 \text{ N}}$

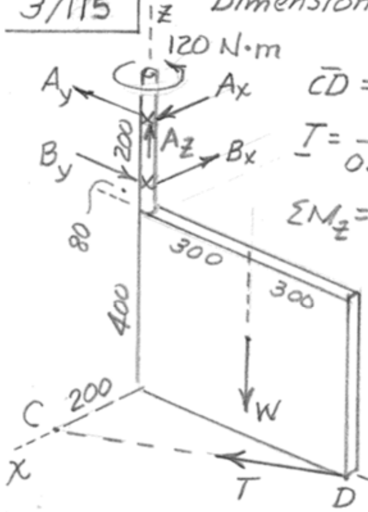
$\Sigma M_y = 0;$

$196.2(600 + 600\cos 60°) - 1200\,T_A = 0$

$\underline{T_A = 147.2 \text{ N}}$

$\Sigma F_z = 0;\ T_B + 147.2 + 196.2 - 3(196.2) = 0,\ \underline{T_B = 245 \text{ N}}$

3/115



Dimensions in mm; $\underline{W} = 15(9.81)(-\underline{k})$ N $= -147.2\underline{k}$ N

$\overline{CD} = \sqrt{0.2^2 + 0.6^2} = 0.632$ m

$\underline{T} = \dfrac{T}{0.632}(0.2\underline{i} - 0.6\underline{j})$

$\Sigma M_z = 0$; $120 - \dfrac{0.2}{0.632}T(0.6) = 0$, $T = 632$ N

$\Sigma M_x = 0$ at A;

$0.2B_y - 147.2(0.3) - 632\dfrac{0.6}{0.632}(0.680) = 0$

$B_y = 2260$ N

$\Sigma M_y = 0$ at A; $0.2B_x - 632\dfrac{0.2}{0.632}0.680 = 0$, $B_x = 680$ N

$B = \sqrt{(2260)^2 + (680)^2} = 2360$ N or $\underline{B = 2.36 \text{ kN}}$

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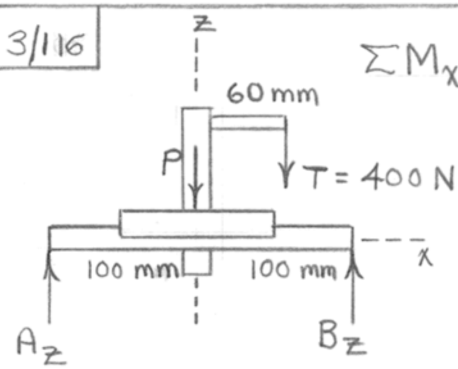


$\Sigma M_x = 0: \; P(200) - 400(120\cos 30°) = 0$

$P = 208$ N

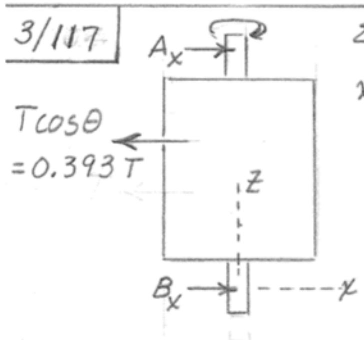
$\Sigma M_{A_y} = 0: \; 208(100) + 400(160) - B_z(200) = 0$

$B_z = 424$ N

$\Sigma F_z = 0: \; A_z + 424 - 208 - 400 = 0, \; A_z = 183.9$ N

Because $A_y = B_y = 0$, $\underline{A = A_z = 183.9 \text{ N}}$, $\underline{B = B_z = 424 \text{ N}}$

3/117



$\Sigma M_z = 0$; $120 - 0.150T = 0$, $T = 800$ N

x-z; $\Sigma M_B = 0$; $0.393(800)(0.360) - 0.700A_x = 0$; $A_x = 161.6$ N

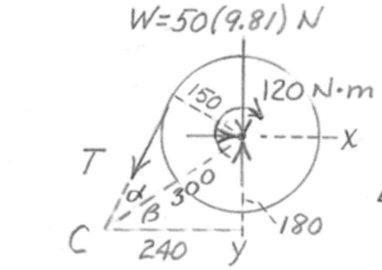
$\Sigma F_x = 0$; $B_x + 161.6 - 0.393(800) = 0$

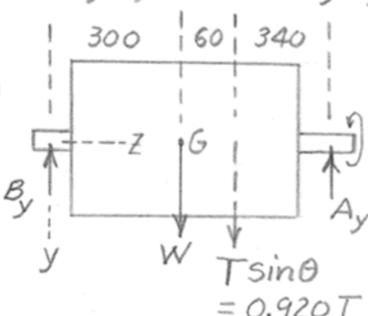
$B_x = 152.6$ N

y-z; $\Sigma M_B = 0$; $0.7A_y - 0.920(800)(0.360) - 50(9.81)(0.300) = 0$; $A_y = 588.6$ N

$\Sigma F_y = 0$; $W + T\sin\theta - A_y - B_y = 0$,

$B_y = 637.6$ N





$\beta = \tan^{-1}\frac{180}{240} = 36.9°$

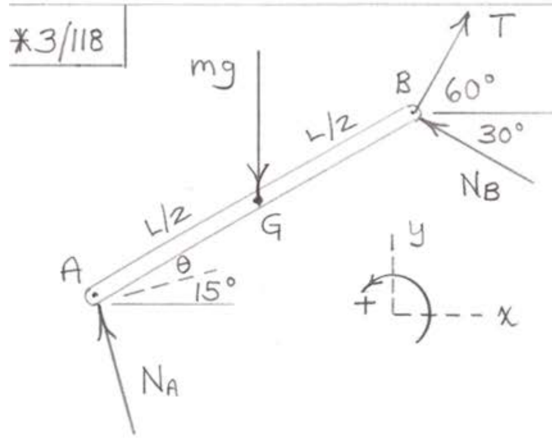
$\alpha = \sin^{-1}\frac{150}{300} = 30°$

$\theta = \alpha + \beta = 66.9°$

$A = \sqrt{(161.6)^2 + (588.6)^2} = \underline{610 \text{ N}}$

$B = \sqrt{(152.6)^2 + (637.6)^2} = \underline{656 \text{ N}}$

*3/118

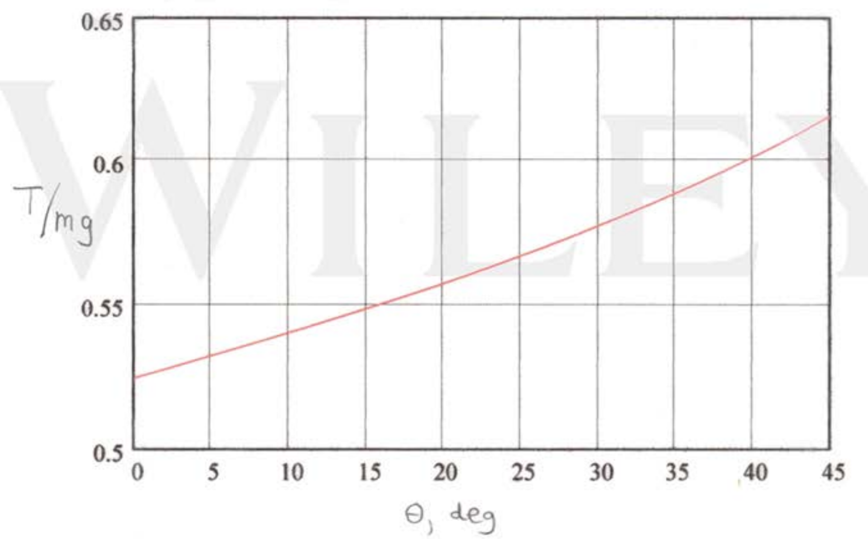


$\Sigma F_x = 0: -N_A \sin 15° - N_B \cos 30° + T \sin 30° = 0$

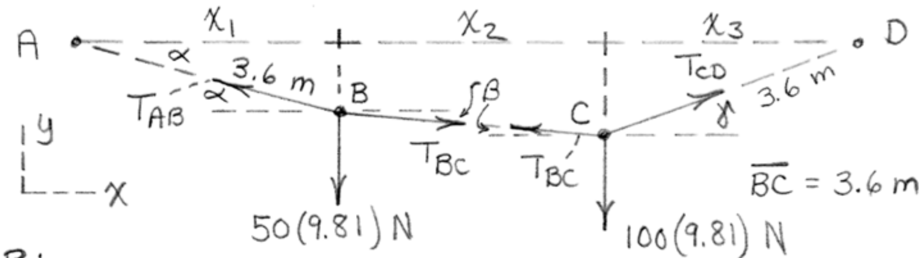
$\Sigma F_y = 0: \quad N_A \cos 15° + N_B \sin 30° + T \cos 30° - mg = 0$

$\Sigma M_B = 0: \quad mg \frac{L}{2} \cos(\theta + 15°) - N_A \cos\theta (L) = 0$

Solving, $T = mg \dfrac{\frac{\sqrt{3}}{2}\cos\theta - \frac{\sqrt{2}}{4}\cos(\theta + 15°)}{\cos\theta}$



*3/119



B:

$$\Sigma F_x = 0: \ -T_{AB}\cos\alpha + T_{BC}\cos\beta = 0 \quad (1)$$

$$\Sigma F_y = 0: \ T_{AB}\sin\alpha - T_{BC}\sin\beta - 50(9.81) = 0 \quad (2)$$

C:

$$\Sigma F_x = 0: \ -T_{BC}\cos\beta + T_{CD}\cos\gamma = 0 \quad (3)$$

$$\Sigma F_y = 0: \ T_{BC}\sin\beta + T_{CD}\sin\gamma - 100(9.81) = 0 \quad (4)$$

$$\cos\alpha = \frac{x_1}{3.6}, \quad \cos\beta = \frac{x_2}{3.6}, \quad \cos\gamma = \frac{x_3}{3.6}$$

So $3.6(\cos\alpha + \cos\beta + \cos\gamma) = 10.5 \quad (5)$

Also: $\sin\alpha + \sin\beta = \sin\gamma$ (from figure) (6)

Solve Eqs. (1) – (6) numerically:

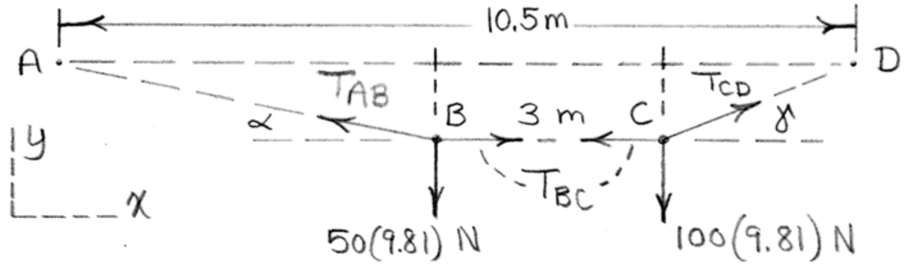
$$\begin{cases} \alpha = 14.44^\circ \\ \beta = 3.57^\circ \\ \gamma = 18.16^\circ \end{cases}$$

$T_{AB} = 2600$ N

$T_{BC} = 2520$ N

$T_{CD} = 2640$ N

*3/120



Geometry:

$$\overline{AB} + \overline{BC} + \overline{CD} = 10.8 \text{ m} \qquad (1)$$

$$\overline{AB}\cos\alpha + \overline{BC} + \overline{CD}\cos\gamma = 10.5 \text{ m} \qquad (2)$$

$$\overline{AB}\sin\alpha = \overline{CD}\sin\gamma \qquad (3)$$

Equilibrium:

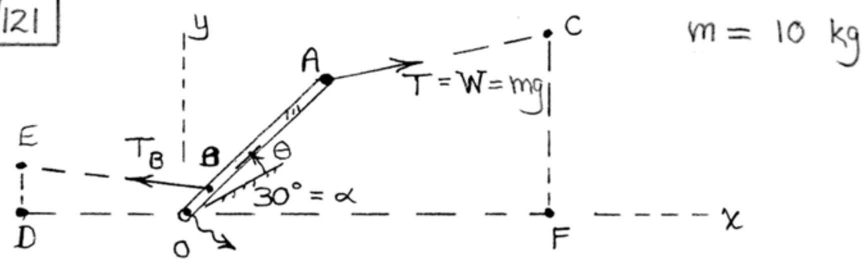
(B) $\begin{cases} \Sigma F_x = 0: & -T_{AB}\cos\alpha + T_{BC} = 0 \qquad (4) \\ \Sigma F_y = 0: & T_{AB}\sin\alpha - 50(9.81) = 0 \qquad (5) \end{cases}$

(C) $\begin{cases} \Sigma F_x = 0: & -T_{BC} + T_{CD}\cos\gamma = 0 \qquad (6) \\ \Sigma F_y = 0: & T_{CD}\sin\gamma - 100(9.81) = 0 \qquad (7) \end{cases}$

With $\overline{BC}$ set to 3 m, solve 7 equations in 7 unknowns & obtain

$\overline{AB} = 5.10$ m	$\alpha = 11.47°$	$T_{AB} = 2470$ N
$\overline{CD} = 2.70$ m	$\gamma = 22.1°$	$T_{BC} = 2420$ N
		$T_{CD} = 2610$ N

*3/121



$$\overline{AC} = \left[\left(\overline{OF} - \overline{OA}\cos(\alpha+\theta)\right)^2 + \left(\overline{CF} - \overline{OA}\sin(\alpha+\theta)\right)^2\right]^{1/2}$$

$\underline{T} = W\underline{n}_{AC}$, where

$$\underline{n}_{AC} = \left[\left(\overline{OF} - \overline{OA}\cos(\alpha+\theta)\right)\underline{i} + \left(\overline{CF} - \overline{OA}\sin(\alpha+\theta)\right)\underline{j}\right] / \overline{AC}$$

$$\overline{BE} = \left[\left(\overline{OD} + \overline{OB}\cos(\alpha+\theta)\right)^2 + \left(\overline{DE} - \overline{OB}\sin(\alpha+\theta)\right)^2\right]^{\frac{1}{2}}$$

$\underline{T}_B = T_B\underline{n}_{BE}$, where

$$\underline{n}_{BE} = \left[\left(\overline{OD} + \overline{OB}\cos(\alpha+\theta)\right)\underline{i} + \left(\overline{DE} - \overline{OB}\sin(\alpha+\theta)\right)\underline{j}\right] / \overline{BE}$$

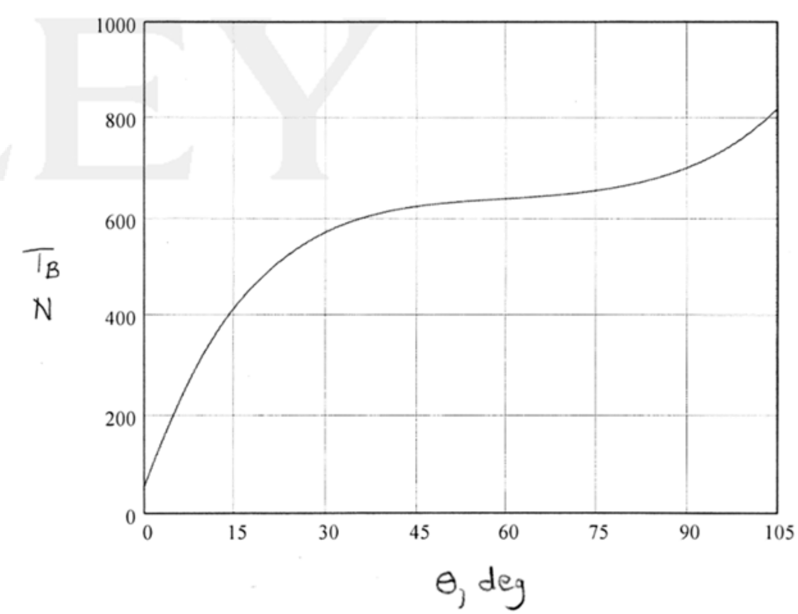
$\Sigma \underline{M}_O = \underline{0}$: $\underline{r}_{OA} \times \underline{T} + \underline{r}_{OB} \times \underline{T}_B = \underline{0}^{*}$, where

$$\underline{r}_{OA} = \overline{OA}\left(\cos(\alpha+\theta)\underline{i} + \sin(\alpha+\theta)\underline{j}\right)$$

$$\underline{r}_{OB} = \overline{OB}\left(\cos(\alpha+\theta)\underline{i} + \sin(\alpha+\theta)\underline{j}\right)$$

Let $\overline{OA} = 325$ mm, $\overline{OB} = 50$ mm, $\overline{CF} = 325$ mm, $\overline{OF} = 600$ mm, $\overline{OD} = 288$ mm, and $\overline{DE} = 69$ mm.

Vary θ, solve Eq. * for T_B, and plot to obtain:

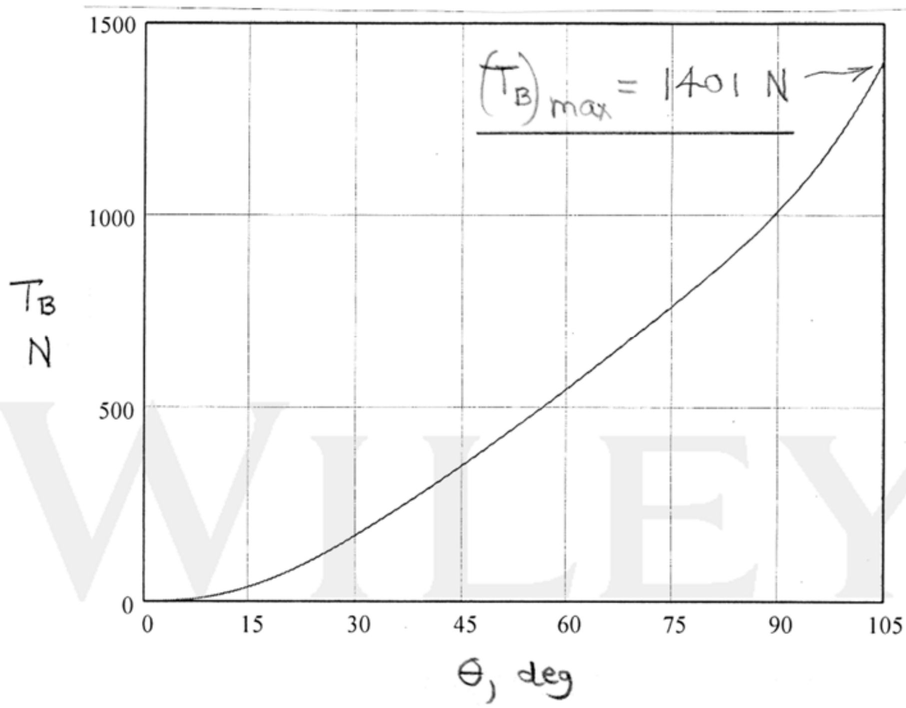


$\underline{T_B = 700 \text{ N at } \theta = 90°}$

*3/122 The only change to the solution to 3/121 is that instead of $\underline{T} = W\underline{n}_{AC}$, we now have $\underline{T} = k\delta\underline{n}_{AC}$, where the stretch $\delta = \overline{AC} - \overline{AC}_{\theta=0}$, where

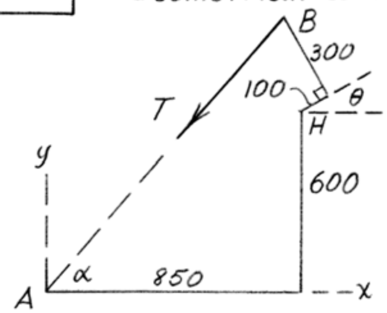
$$\overline{AC}_{\theta=0} = \left[(\overline{OF} - \overline{OA}\cos\alpha)^2 + (\overline{CF} - \overline{OA}\sin\alpha)^2 \right]^{1/2}$$

Result:

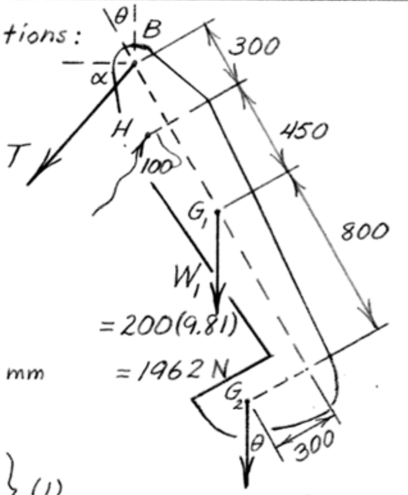


*3/123

Geometrical considerations:



Dimensions in mm



Coordinates of B:

$$\left.\begin{aligned} x &= 850 + 100\cos\theta - 300\sin\theta \text{ mm} \\ y &= 600 + 100\sin\theta + 300\cos\theta \text{ mm} \end{aligned}\right\} \quad (1)$$

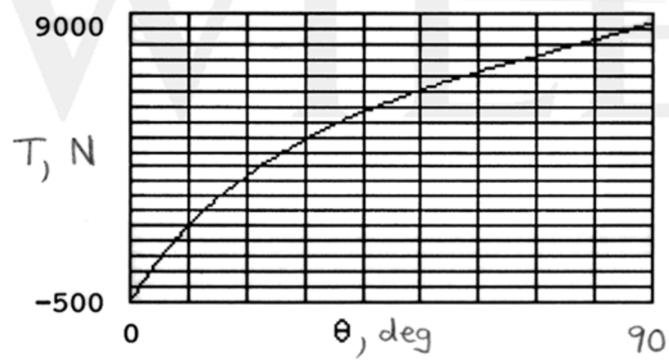
$W_2 = 140(9.81)$
$= 1373$ N

$$\alpha = \tan^{-1}\frac{y}{x} \qquad (2)$$

$$\overset{+}{\curvearrowleft}\Sigma M_H = 0: -(T\cos\alpha)(0.600) + T\sin\alpha(0.850) - 1962(0.100\cos\theta + 0.450\sin\theta) + 1373([0.300 - 0.100]\cos\theta - [0.800 + 0.450]\sin\theta) = 0$$

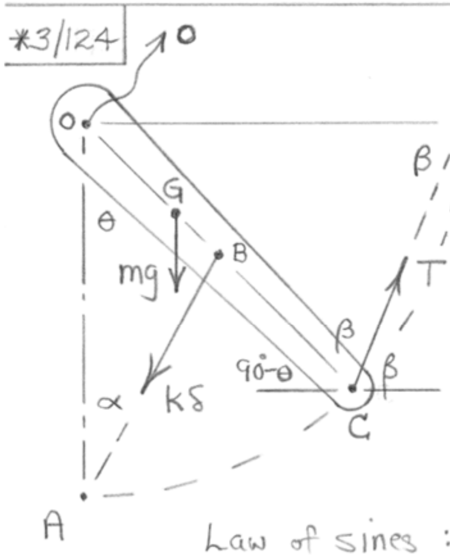
or $\quad T(0.850\sin\alpha - 0.600\cos\alpha) + 78.5\cos\theta - 2600\sin\theta = 0 \qquad$ --(3)

Solve (1), (2), (3) for $T = f(\theta)$ and plot.



$T = 0$ @
$\underline{\theta = 1.729°}$

*3/124



$$2\beta + (90° - \theta) = 180°$$

$$\beta = \frac{90° + \theta}{2}$$

By law of cosines, $\overline{AB} = \sqrt{\overline{OA}^2 + \overline{OB}^2 - 2\cdot\overline{OA}\cdot\overline{OB}\cdot\cos\theta}$;

then spring deflection

$$\delta = \overline{AB} - \overline{BC} \qquad (1)$$

Law of sines: $\dfrac{\sin\alpha}{\overline{OB}} = \dfrac{\sin\theta}{\overline{AB}}$

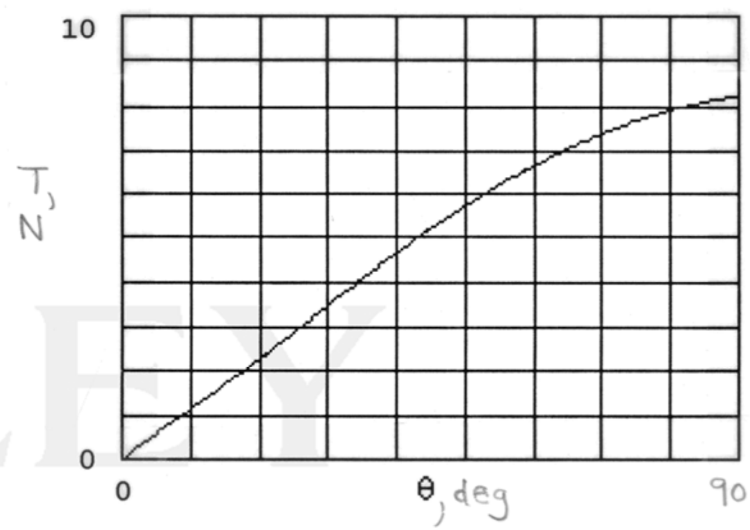
$$\alpha = \sin^{-1}\left[\frac{\overline{OB}}{\overline{AB}}\sin\theta\right] \qquad (2)$$

Consider $k\delta$ applied at A:

$$\circlearrowleft + \Sigma M_O = 0: \; -(k\delta\sin\alpha)(\overline{OA}) - mg\,(\overline{OG}\sin\theta) + T\sin\beta\,(\overline{OD}) = 0$$

$$T = \frac{(k\delta\sin\alpha)(\overline{OA}) + mg(\overline{OG}\sin\theta)}{\overline{OD}\sin\beta} \qquad (3)$$

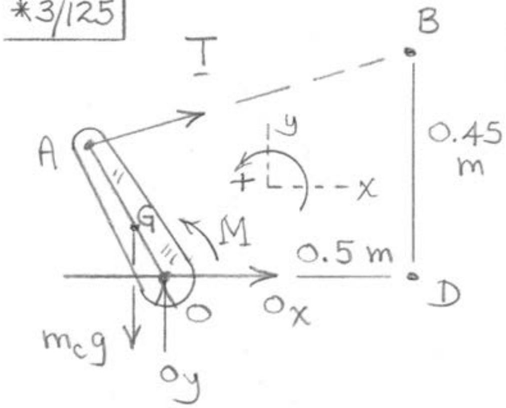
With $k = 25$ N/m, δ given by (1), α given by (2), $\overline{OA} = 0.48$ m, $m = 1.5$ kg, $g = 9.81$ m/s^2, $\overline{OG} = 0.16$ m, $\overline{OD} = 0.48$ m, and $\beta = \frac{90° + \theta}{2}$, we obtain the following plot:



When $\theta = 45°$, $\underline{T = 5.23 \text{ N}}$

When $\theta = 90°$, $\underline{T = 8.22 \text{ N}}$

*3/125



$$\begin{cases} \overline{OA} = 0.3 \text{ m} \\ T = mg,\ m = 5 \text{ kg} \\ m_c = 3 \text{ kg} \\ \overline{OG} = 0.12 \text{ m} \\ g = 9.81 \text{ m/s}^2 \end{cases}$$

$$\Sigma \underline{M}_O = \underline{0}: \quad \underline{M} + \underline{r}_{OA} \times \underline{T} + \underline{r}_{OG} \times (-m_c g \underline{j})$$

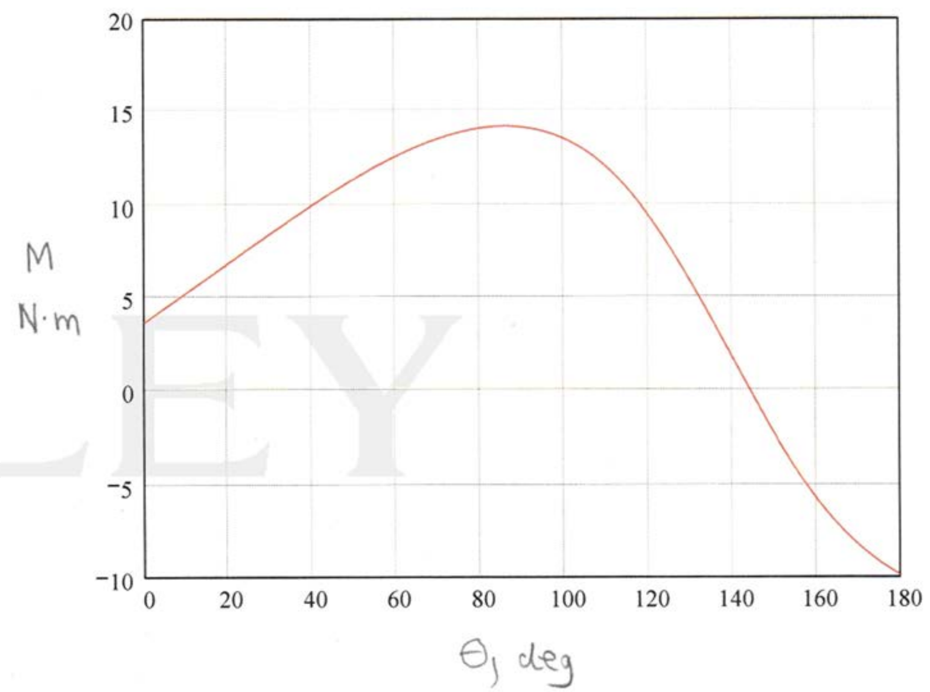
With $\underline{r}_{OA} = \overline{OA}(-\cos\theta\,\underline{i} + \sin\theta\,\underline{j})$

$$\underline{T} = T\underline{n}_{AB}$$

$$= mg\left[\frac{(0.5 + \overline{OA}\cos\theta)\underline{i} + (0.45 - \overline{OA}\sin\theta\,\underline{j})}{\{(0.5 + \overline{OA}\cos\theta)^2 + (0.45 - \overline{OA}\sin\theta)^2\}^{1/2}}\right]$$

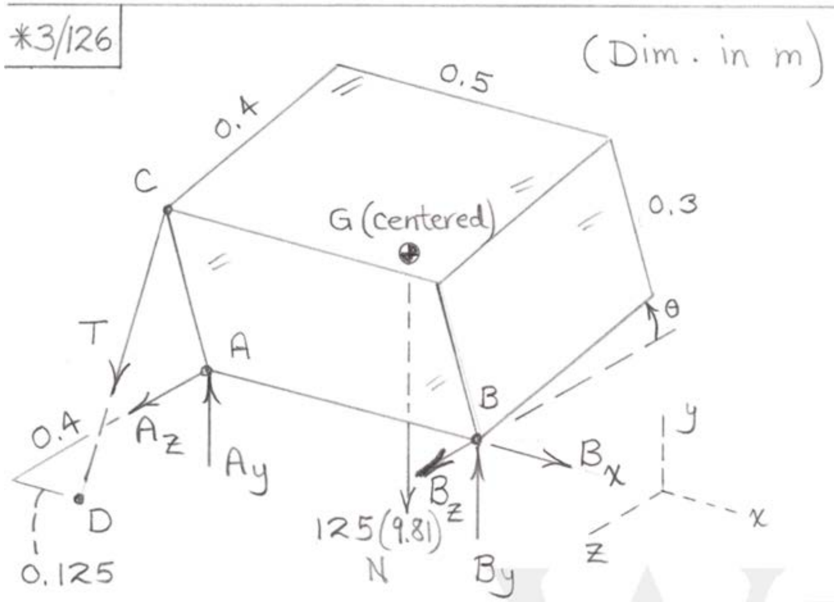
$$\underline{r}_{OG} = \overline{OG}(-\cos\theta\,\underline{i} + \sin\theta\,\underline{j})$$

Carry out the cross products, solve for $\underline{M}$, and let θ vary to obtain the following plot for the z-comp. of $\underline{M}$:



$\underline{M = 0 \text{ at } \theta = 144.3°}$

*3/126



$\underline{CD} = 0.125\underline{i} - 0.3\cos\theta\,\underline{j} + (0.4 - 0.3\sin\theta)\underline{k}$

Then $\underline{T} = T\underline{n}_{CD} = T\dfrac{\underline{CD}}{CD}$, or

$$\underline{T} = T\left[\frac{0.125\underline{i} - 0.3\cos\theta\underline{j} + (0.4 - 0.3\sin\theta)\underline{k}}{\sqrt{0.266 - 0.24\sin\theta}}\right]$$

$$\Sigma F_x = 0: \quad \frac{0.125T}{\sqrt{0.266 - 0.24\sin\theta}} + B_x = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad \frac{-0.3T\cos\theta}{\sqrt{0.266 - 0.24\sin\theta}} + A_y + B_y - 125(9.81) = 0 \qquad (2)$$

$$\Sigma F_z = 0: \quad \frac{(0.4 - 0.3\sin\theta)T}{\sqrt{0.266 - 0.24\sin\theta}} + A_z + B_z = 0 \qquad (3)$$

$$\Sigma M_{D_x} = 0: \quad (A_y + B_y)(0.4) - 125(9.81)(0.4 + 0.2\cos\theta - 0.15\sin\theta) = 0 \qquad (4)$$

$$\Sigma M_{D_y} = 0: \quad -B_x(0.4) + A_z(0.125) - B_z(0.375) = 0 \qquad (5)$$

$$\Sigma M_{D_z} = 0: \quad -A_y(0.125) + B_y(0.375) - 125(9.81)(0.125) = 0 \qquad (6)$$

Computer Solution:

$$T = -12.77\,\frac{-4\cos\theta + 3\sin\theta}{\cos\theta}\sqrt{425 - 384\sin\theta}$$

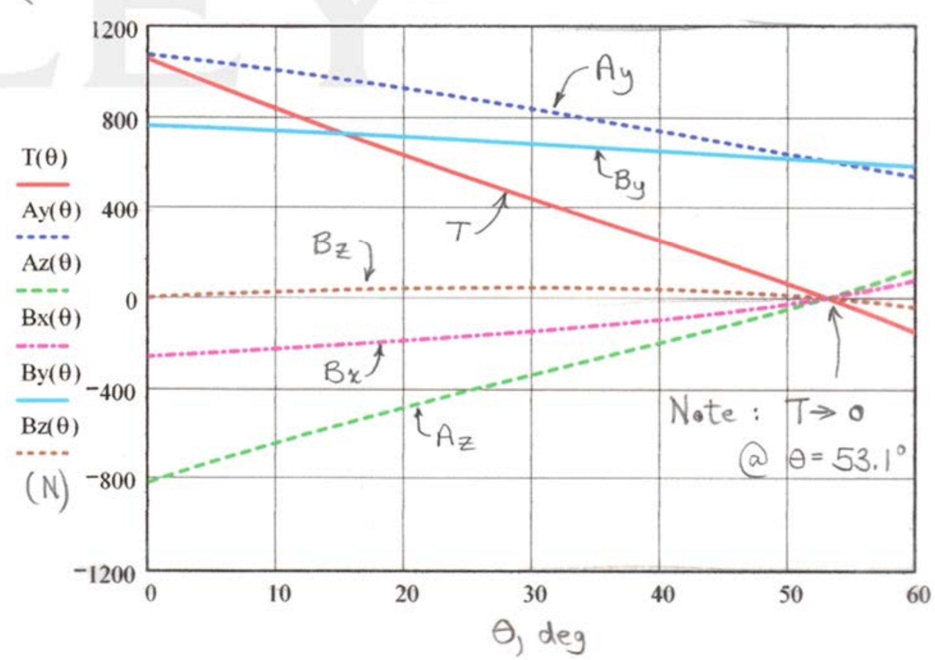
$$A_y = 613 + 460\cos\theta - 345\sin\theta$$

$$A_z = -12.77\,\frac{64\cos\theta - 48\sin\theta - 36\sin\theta\cos\theta + 27\cos^2\theta}{\cos\theta}$$

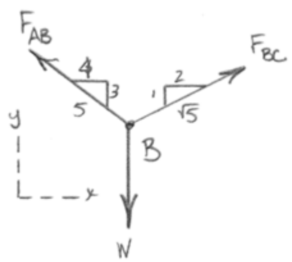
$$B_x = 63.9\,\frac{-4\cos\theta + 3\sin\theta}{\cos\theta}$$

$$B_y = 613 + 153.3\cos\theta - 115.0\sin\theta$$

$$B_z = -38.3\,\frac{-4\cos\theta + 3\sin\theta}{\cos\theta}\sin\theta$$

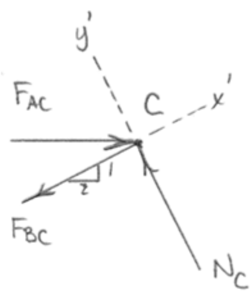


4/1



$$\begin{cases} \Sigma F_x = 0: & \frac{2}{\sqrt{5}} F_{BC} - \frac{4}{5} F_{AB} = 0 \\ \Sigma F_y = 0: & \frac{1}{\sqrt{5}} F_{BC} + \frac{3}{5} F_{AB} - W = 0 \end{cases}$$

$\underline{F_{AB} = W \quad T}$ $\qquad$ $\underline{F_{BC} = \frac{2}{\sqrt{5}} W \quad T}$



$$\Sigma F_{x'} = 0: \quad -F_{BC} + \frac{2}{\sqrt{5}} F_{AC} = 0$$

$\underline{F_{AC} = W \quad C}$

4/2

SINCE $F_{BD} = 0$ (ZERO FORCE MEMBER) THE LOAD CARRIED IN THE SIDES OF THE TRUSS DO NOT CHANGE!

FROM 4/2...

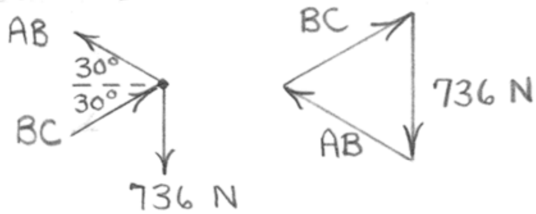
$F_{AB} = W$ T

$F_{BC} = \frac{2}{\sqrt{5}} W$ T

$F_{AD} = F_{CD} = W$ C

4/3 Load $= 75(9.81) = 736$ N

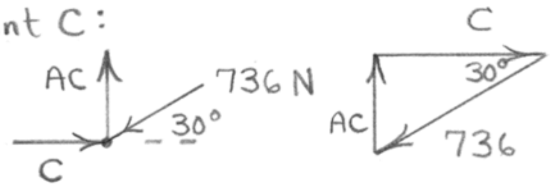
Joint B:



$AB = 736$ N T

$BC = 736$ N C

Joint C:



$AC = 736\left(\frac{1}{2}\right)$

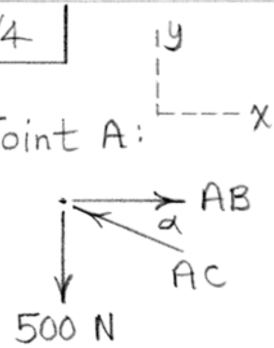
$= 368$ N T

4/4

$$\alpha = \tan^{-1}\frac{1.25}{3} = 22.6^\circ$$

$$\left(\cos\alpha = \frac{12}{13},\quad \sin\alpha = \frac{5}{13}\right)$$

Joint A:



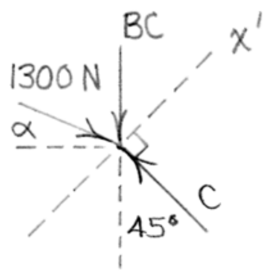
$\Sigma F_y = 0$: $AC \sin\alpha - 500 = 0$

$\underline{AC = 1300 \text{ N} \quad C}$

$\Sigma F_x = 0$: $AB - 1300\cos\alpha = 0$

$\underline{AB = 1200 \text{ N} \quad T}$

Joint C:



$\Sigma F_x = 0$: $1300\left(\frac{12}{13}\right) - C\frac{\sqrt{2}}{2} = 0$

$C = 1697 \text{ N}$ (reaction)

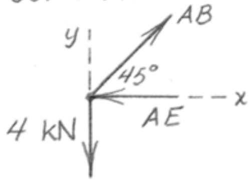
$\Sigma F_y = 0$: $-1300\left(\frac{5}{13}\right) - BC + 1697\frac{\sqrt{2}}{2} = 0$

$\underline{BC = 700 \text{ N} \quad C}$

Could use $\Sigma F_{x'}$ to find BC without involving calculation of C. Nonetheless, observe that changing the 45° support angle would affect BC, but not AB or AC!

4/5

Joint A:



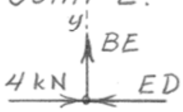
$\Sigma F_y = 0$: $AB \sin 45° - 4 = 0$

$AB = 5.66$ kN T

$\Sigma F_x = 0$: $5.66 \cos 45° - AE = 0$

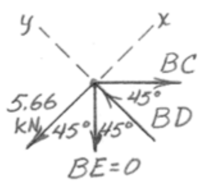
$AE = 4$ kN C

Joint E:



$\Sigma F_y = 0$: $\underline{BE = 0}$

Joint B:



$\Sigma F_x = 0$: $BC \cos 45° - 5.66 = 0$

$BC = 8$ kN T

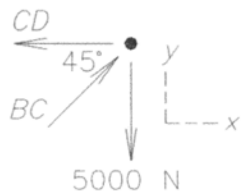
$\Sigma F_y = 0$: $BD - 8 \cos 45° = 0$

$\underline{BD = 5.66 \text{ kN } C}$

4/6

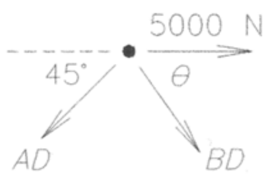
We can begin at joint C without finding the external reactions.

Joint C:



$$\begin{cases} \Sigma F_y = 0: \; BC \sin 45° - 5000 = 0, \; \underline{BC = 7070 \text{ N } C} \\ \Sigma F_x = 0: \; 7070 \cos 45° - CD = 0, \; \underline{CD = 5000 \text{ N } T} \end{cases}$$

Joint D:



$$\overline{DB}^2 = 5^2 + 6^2 - 2(5)(6)\cos 45°, \; \overline{DB} = 4.31 \text{ m}$$

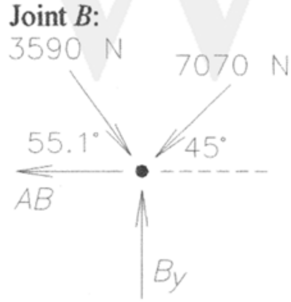
$$\frac{\sin\theta}{5} = \frac{\sin 45°}{4.31} \Rightarrow \theta = 55.1°$$

$$\begin{cases} \Sigma F_x = 0: \; 5000 + BD\cos 55.1° - AD\cos 45° = 0 \\ \Sigma F_y = 0: \; -AD\sin 45° - BD\sin 55.1° = 0 \end{cases}$$

Solve simultaneously to obtain: $\underline{BD = -3590 \text{ N or } 3590 \text{ N } C}$

$\underline{AD = 4170 \text{ N } T}$

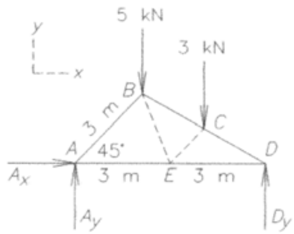
Joint B:



$$\Sigma F_x = 0: \; 3590 \cos 55.1° - 7070 \cos 45° - AB = 0$$

$\underline{AB = -2950 \text{ N or } 2950 \text{ N } C}$

4/7



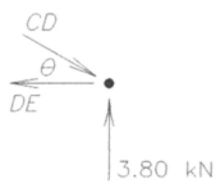
As a whole:

Note: $\overline{CE} = 1.5$ m by similar triangles

$$\Sigma M_A = 0: \ 5(3\cos 45°) + 3(3 + 1.5\cos 45°) - 6D_y = 0$$

$$D_y = 3.80 \text{ kN}$$

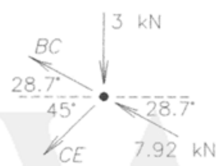
Joint D:



$$\theta = \tan^{-1}\frac{3\sin 45°}{6 - 3\cos 45°} = 28.7°$$

$$\begin{cases} \Sigma F_y = 0: \ 3.80 - CD\sin 28.7° = 0, \ CD = 7.92 \text{ kN } C \\ \Sigma F_x = 0: \ 7.92\cos 28.7° - DE = 0, \ DE = 6.94 \text{ kN } T \end{cases}$$

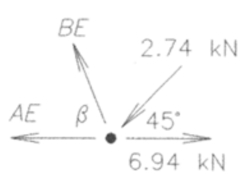
Joint C:



$$\begin{cases} \Sigma F_x = 0: \ -BC\cos 28.7° - CE\cos 45° - 7.92\cos 28.7° = 0 \\ \Sigma F_y = 0: \ BC\sin 28.7° - CE\sin 45° + 7.92\sin 28.7° - 3 = 0 \end{cases}$$

Solve simultaneously to obtain: $BC = -5.70$ kN (C)
$\underline{CE = -2.74 \text{ kN } (C)}$

Joint E:



$$\beta = \frac{180° - 45°}{2} = 67.5°$$

$$\Sigma F_y = 0: \ BE\sin 67.5° - 2.74\sin 45° = 0$$

$\underline{BE = 2.10 \text{ kN } T}$

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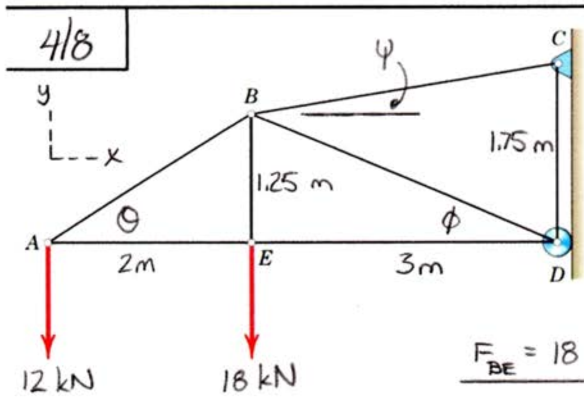


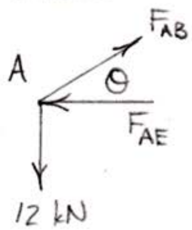
$\theta = \tan^{-1}\left(\frac{1.25}{2}\right) = 32.0^\circ$

$\phi = \tan^{-1}\left(\frac{1.25}{3}\right) = 22.6^\circ$

$\psi = \tan^{-1}\left(\frac{0.5}{3}\right) = 9.46^\circ$

$\underline{F_{BE} = 18 \text{ kN } T}$ (INSPECTION)

Joint A:



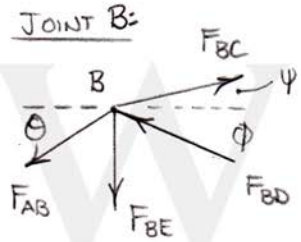
$\Sigma F_y = 0$: $F_{AB} \sin\theta - 12 = 0$

$\Sigma F_x = 0$: $F_{AB} \cos\theta - F_{AE} = 0$

$\longrightarrow$ $\underline{F_{AB} = 22.6 \text{ kN } T}$, $\underline{F_{AE} = 19.20 \text{ kN } C}$

$\underline{F_{DE} = 19.20 \text{kN } C}$ (INSPECTION)

Joint B:

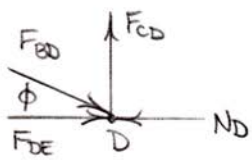


$\Sigma F_x = 0$: $F_{BC} \cos\psi - F_{BD} \cos\phi - F_{AB} \cos\theta = 0$

$\Sigma F_y = 0$: $F_{BC} \sin\psi + F_{BD} \sin\phi - F_{BE} - F_{AB} \sin\theta = 0$

$\underline{F_{BC} = 66.0 \text{ kN } T}$ & $\underline{F_{BD} = 49.8 \text{ kN } C}$

Joint D:



$\Sigma F_y = 0$: $F_{CD} - F_{BD} \sin\phi = 0$

$\underline{F_{CD} = 19.14 \text{ kN } T}$

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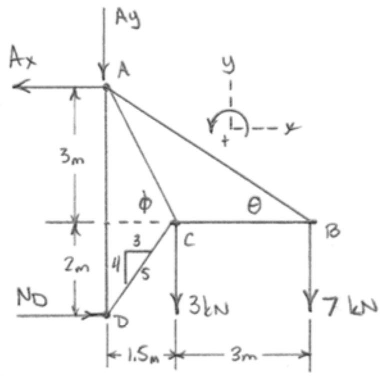
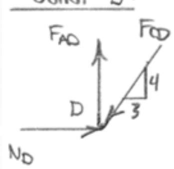
$\theta = \tan^{-1}\left(\frac{3}{4.5}\right) \longrightarrow \theta = 33.69°$ $\qquad \phi = \tan^{-1}\left(\frac{3}{1.5}\right) \longrightarrow \phi = 63.43°$



$\Sigma M_A = 0: \; 5N_D - 1.5(3) - 4.5(7) = 0$

$N_D = 7.20$ kN

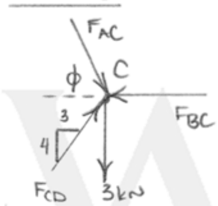
• Joint D:



$\Sigma F_x = 0: \; N_D - \frac{3}{5} F_{CD} = 0 \longrightarrow \underline{F_{CD} = 12 \text{ kN } C}$

$\Sigma F_y = 0: \; F_{AD} - \frac{4}{5} F_{CD} = 0 \longrightarrow \underline{F_{AD} = 9.60 \text{ kN } T}$

• Joint C:

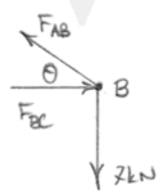


$\Sigma F_x = 0: \; \frac{3}{5} F_{CD} + F_{AC} \cos\phi - F_{BC} = 0$

$\Sigma F_y = 0: \; \frac{4}{5} F_{CD} - F_{AC} \sin\phi - 3 = 0$

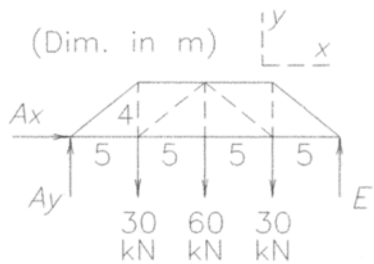
$\longrightarrow \underline{F_{BC} = 10.50 \text{ kN } C}$, $\underline{F_{AC} = 7.38 \text{ kN } C}$

• Joint B:



$\Sigma F_y = 0: \; F_{AB} \sin\theta - 7 = 0 \longrightarrow \underline{F_{AB} = 12.62 \text{ kN } T}$

4/10

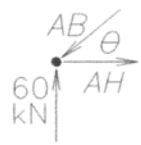


As a whole: $\Sigma F_x = 0 \Rightarrow A_x = 0$

$A_y = E = 60 \text{ kN}$ by

$\Sigma F_y = 0$ and symmetry.

Joint A: $\left(\theta = \tan^{-1}(4/5) = 38.7°\right)$

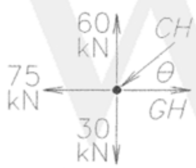


$$\begin{cases} \Sigma F_y = 0: 60 - AB\sin\theta = 0, \ \underline{AB = 96.0 \text{ kN } C} \\ \Sigma F_x = 0: AH - 96.0\cos\theta, \ \underline{AH = 75 \text{ kN } T} \end{cases}$$

Joint B:

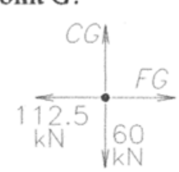
$$\begin{cases} \Sigma F_x = 0: BC + 96.0\sin 51.3° = 0, \ \underline{BC = -75 \text{ kN } (C)} \\ \Sigma F_y = 0: -BH + 96.0\cos 51.3° = 0, \ \underline{BH = 60 \text{ kN } T} \end{cases}$$

Joint H:



$$\begin{cases} \Sigma F_y = 0: -CH\sin\theta + 30 = 0, \ \underline{CH = 48.0 \text{ kN } C} \\ \Sigma F_x = 0: 48.0\cos\theta + GH - 75 = 0, \ \underline{GH = 112.5 \text{ kN } T} \end{cases}$$

Joint G:



$\Sigma F_y = 0 \Rightarrow \underline{CG = 60 \text{ kN } T}$

By symmetry:
$FG = 112.5$ kN T, $CF = 48.0$ kN C
$CD = 75$ kN C, $DF = 60$ kN T
$\underline{EF = 75 \text{ kN } T, \ DE = 96.0 \text{ kN } C}$

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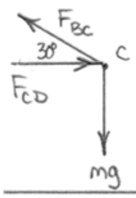
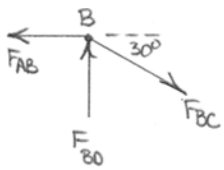
$F_{AE} = 0$ (INSPECTION)

• JOINT C:



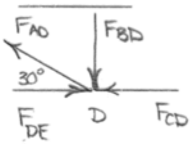
$$\begin{cases} \Sigma F_x = 0: & -F_{BC}\cos 30^\circ + F_{CD} = 0 \\ \Sigma F_y = 0: & F_{BC}\sin 30^\circ - mg = 0 \end{cases} \longrightarrow \begin{cases} F_{BC} = 2mg \ \ T \\ F_{CD} = 1.732\,mg \ \ C \end{cases}$$

• JOINT B:



$$\begin{cases} \Sigma F_x = 0: & F_{BC}\cos 30^\circ - F_{AB} = 0 \\ \Sigma F_y = 0: & F_{BD} - F_{BC}\sin 30^\circ = 0 \end{cases} \longrightarrow \begin{cases} F_{AB} = 1.732\,mg \ \ T \\ F_{BD} = mg \ \ C \end{cases}$$

• JOINT D:



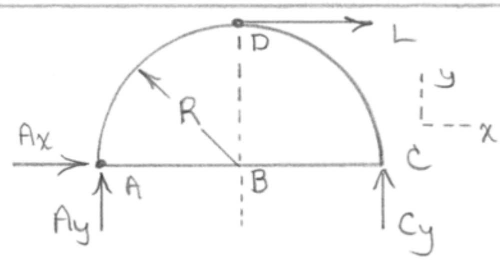
$$\begin{cases} \Sigma F_x = 0: & F_{DE} - F_{CD} - F_{AD}\cos 30^\circ = 0 \\ \Sigma F_y = 0: & F_{AD}\sin 30^\circ - F_{BD} = 0 \end{cases} \longrightarrow \begin{cases} F_{AD} = 2mg \ \ T \\ F_{DE} = 3.46\,mg \ \ C \end{cases}$$

• MAX TENSION: $F_{BC} = 2mg = 24 \text{ kN} \longrightarrow m = 1223 \text{ kg}$

• MAX COMPRESSION: $F_{DE} = 3.46mg = 35 \text{ kN} \longrightarrow m = 1030 \text{ kg}$

SO... $\underline{m_{max} = 1030 \text{ kg}}$

4/12 Entire truss:

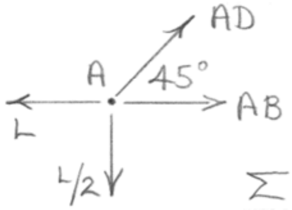


$\Sigma F_x = 0: \quad A_x = -L$

$\Sigma M_A = 0: \quad C_y = L/2$

$\Sigma F_y = 0: \quad A_y = -L/2$

Joint A:



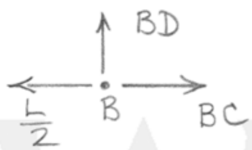
$$\Sigma F_y = 0: \quad AD\frac{\sqrt{2}}{2} - \frac{L}{2} = 0$$

$$AD = \frac{\sqrt{2}}{2}L$$

$$\Sigma F_x = 0: \quad -L + \frac{\sqrt{2}}{2}L\left(\frac{\sqrt{2}}{2}\right) + AB = 0$$

$$\underline{AB = \frac{L}{2}\ T}$$

Joint B:



$$\underline{BC = \frac{L}{2}\ T}$$

$$\underline{BD = 0}$$

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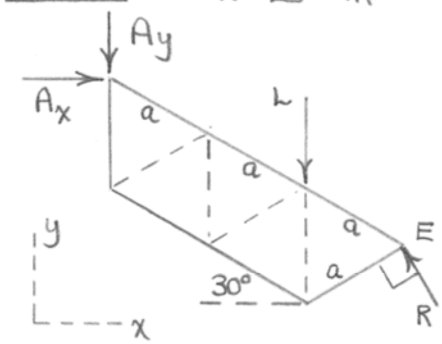
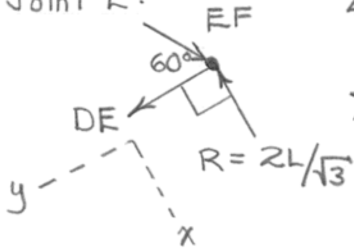
$$\circlearrowleft\!+ \ \Sigma M_A = 0: \ L(2a\cos 30^\circ) - R(3a\sin 30^\circ) = 0$$

$$R = 2L/\sqrt{3}$$



Joint E:



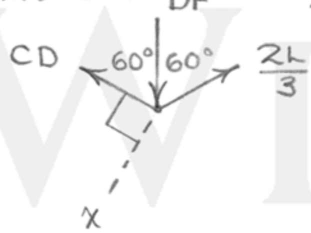
$$\Sigma F_x = 0: \ EF\sin 60^\circ - 2L/\sqrt{3} = 0$$

$$\underline{EF = 4L/3 \quad C}$$

$$\Sigma F_y = 0: \ DE - \frac{4L}{3}\cos 60^\circ = 0$$

$$\underline{DE = 2L/3 \quad T}$$

Joint D:



$$\Sigma F_x = 0: \ DF\cos 30^\circ - \frac{2L}{3}\cos 30^\circ = 0$$

$$\underline{DF = 2L/3 \quad C}$$

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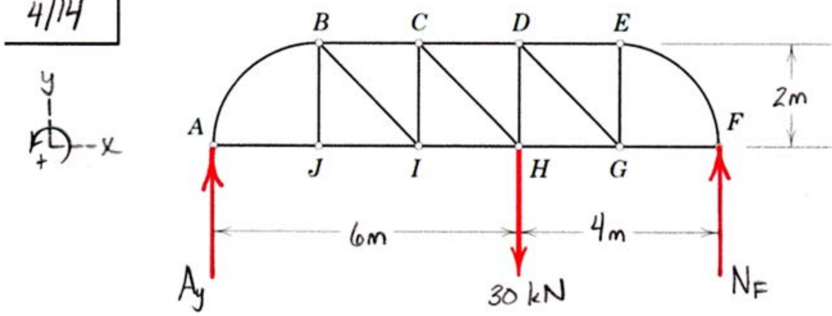
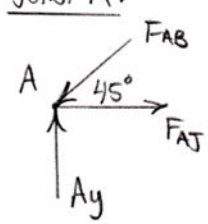


$$\begin{cases} \Sigma F_y = 0: \quad A_y + N_F - 30 = 0 \\ \Sigma M_A = 0: \quad 10 N_F - 6(30) = 0 \end{cases} \longrightarrow \quad A_y = 12 \text{ kN} \ \& \ N_F = 18 \text{ kN}$$

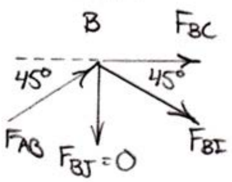
$\underline{F_{BJ} = 0}$ (INSPECTION)

JOINT A:



$\Sigma F_y = 0: \quad A_y - F_{AB} \sin 45° = 0 \longrightarrow F_{AB} = 16.97 \text{ kN C}$

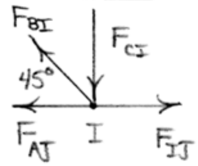
JOINT B:



$\Sigma F_y = 0: \quad F_{AB} \sin 45° - F_{BI} \sin 45° = 0$

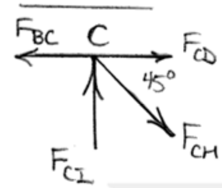
$\underline{F_{BI} = 16.97 \text{ kN T}}$

JOINT I:



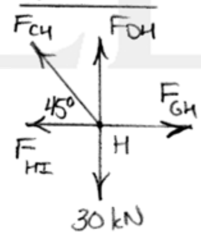
$\Sigma F_y = 0: \quad F_{BI} \sin 45° - F_{CI} = 0 \longrightarrow \underline{F_{CI} = 12 \text{ kN C}}$

JOINT C:



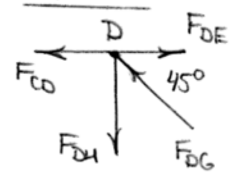
$\Sigma F_y = 0: \ -F_{CH} \sin 45° + F_{CI} = 0 \longrightarrow \underline{F_{CH} = 16.97 \text{ kN T}}$

JOINT H:



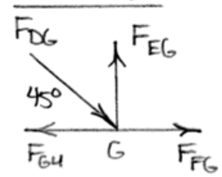
$\Sigma F_y = 0: \quad F_{DH} + F_{CH} \sin 45° - 30 = 0 \longrightarrow \underline{F_{DH} = 18 \text{ kN T}}$

JOINT D:



$\Sigma F_y = 0: \quad F_{DG} \sin 45° - F_{DH} = 0 \longrightarrow \underline{F_{DG} = 25.5 \text{ kN C}}$

JOINT G:



$\Sigma F_y = 0: \quad F_{EG} - F_{DG} \sin 45° = 0 \longrightarrow \underline{F_{EG} = 18 \text{ kN T}}$

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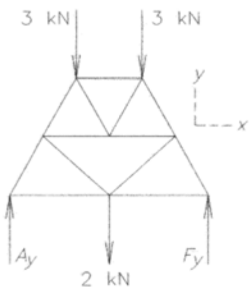
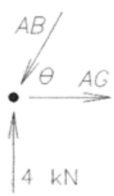


By symmetry, $A_y = F_y = 4$ kN

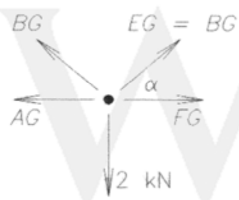
Joint A:



$$\theta = \cos^{-1}\frac{1}{2} = 60°$$

$$\begin{cases}\Sigma F_y = 0: \ 4 \text{ kN} - AB \sin 60° = 0, \ AB = 4.62 \text{ kN } C \\ \Sigma F_x = 0: \ AG - 4.62 \cos 60° = 0, \ AG = 2.31 \text{ kN } T\end{cases}$$

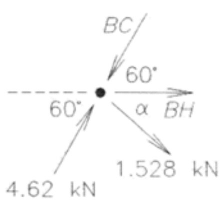
Joint G:



$$\alpha = \tan^{-1}\frac{2\sin 60°}{2} = 40.9°$$

$$\Sigma F_y = 0: \ 2BG \sin 40.9° - 2 = 0, \ \underline{BG = 1.528 \text{ kN } T}$$

Joint B:



$$\Sigma F_y = 0: \ 4.62 \sin 60° - BC \sin 60° - 1.528 \sin 40.9° = 0$$

$$\underline{BC = 3.46 \text{ kN } C}$$

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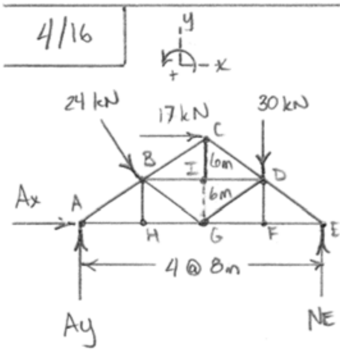
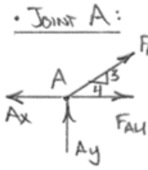


$$\begin{cases} \Sigma F_x = 0: & A_x + 17 + \frac{3}{5}24 = 0 \\ \Sigma F_y = 0: & A_y + N_E - \frac{4}{5}24 - 30 = 0 \\ \Sigma M_A = 0: & 32N_E - 24(30) - 12(17) - 24(10) = 0 \end{cases}$$

$A_x = -31.4$ kN $\quad A_y = 12.83$ kN $\quad N_E = 36.4$ kN

$\underline{F_{BH} = 0}$ AND $\underline{F_{DF} = 0}$ AND $\underline{F_{CI} = 0}$

• JOINT A:

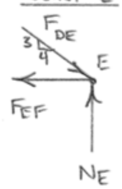


$$\begin{cases} \Sigma F_x = 0: & \frac{4}{5}F_{AB} + F_{AH} - A_x = 0 \\ \Sigma F_y = 0: & A_y + \frac{3}{5}F_{AB} = 0 \end{cases}$$

$\underline{F_{AB} = -21.4 \text{ kN (C)}}$

$\underline{F_{AH} = F_{GH} = 48.5 \text{ kN T}}$

• JOINT E

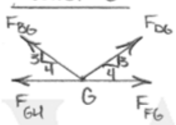


$$\begin{cases} \Sigma F_x = 0: & \frac{4}{5}F_{DE} - F_{EF} = 0 \\ \Sigma F_y = 0: & N_E - \frac{3}{5}F_{DE} = 0 \end{cases}$$

$\underline{F_{DE} = 60.6 \text{ kN C}}$

$\underline{F_{EF} = F_{FG} = 48.5 \text{ kN T}}$

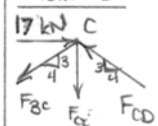
• JOINT G:



$$\begin{cases} \Sigma F_x = 0: & F_{FG} - F_{GH} + \frac{4}{5}F_{DG} - \frac{4}{5}F_{BG} = 0 \\ \Sigma F_y = 0: & \frac{3}{5}F_{DG} + \frac{3}{5}F_{BG} = 0 \end{cases}$$

$\underline{F_{BG} = 0} \qquad \underline{F_{DG} = 0}$

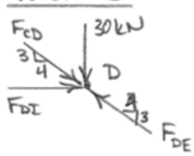
• JOINT C



$$\begin{cases} \Sigma F_x = 0: & 17 - \frac{4}{5}F_{BC} - \frac{4}{5}F_{CD} = 0 \\ \Sigma F_y = 0: & \frac{3}{5}F_{CD} - \frac{3}{5}F_{BC} = 0 \end{cases}$$

$\underline{F_{BC} = 10.63 \text{ kN T}} \qquad \underline{F_{CD} = 10.63 \text{ kN C}}$

• JOINT D:



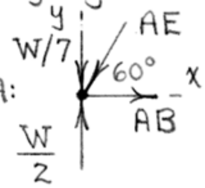
$$\Sigma F_x = 0: \frac{4}{5}F_{CD} - \frac{4}{5}F_{DE} + F_{DI} = 0$$

$\underline{F_{DI} = F_{BI} = 40 \text{ kN C}}$

4/17 Total weight of truss $W = 7(400)(9.81)$ N
$= 27.5$ kN

By symmetry, reactions at A & C are $W/2 = 13.73$ kN

A:



$\Sigma F_y = 0: \frac{W}{2} - \frac{W}{7} - AE \sin 60° = 0$

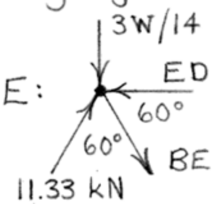
$AE = 0.412W = \underline{11.33 \text{ kN C}}$

$\Sigma F_x = 0: AB - 11.33 \cos 60° = 0$

$\underline{AB = 5.66 \text{ kN T}}$

By symmetry, $\underline{BC = AB = 5.66 \text{ kN T}}$, $\underline{DC = AE = 11.33 \text{ kN C}}$

E:



$\Sigma F_y = 0: -\frac{3W}{14} + 11.33 \cos 30° - BE \cos 30° = 0$

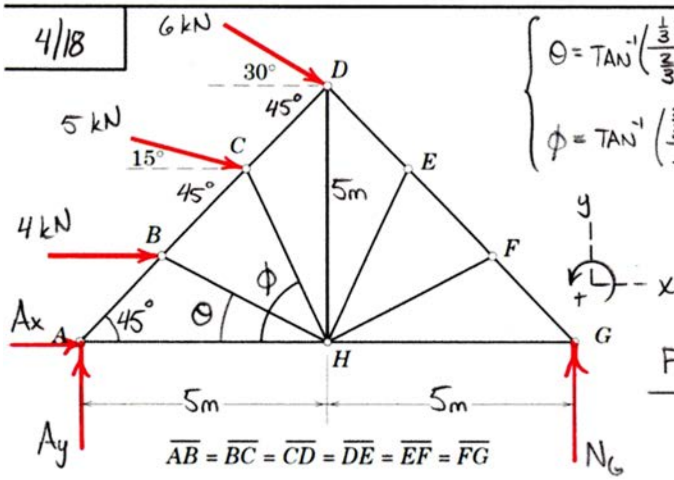
$\underline{BE = 4.53 \text{ kN T}}$

By symmetry, $\underline{BD = 4.53 \text{ kN T}}$

$\Sigma F_x = 0: 11.33 \sin 30° + 4.53 \sin 30° - ED = 0$

$\underline{ED = 7.93 \text{ kN C}}$

4/18



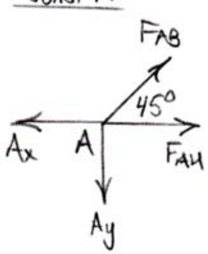
$$\begin{cases} \theta = \tan^{-1}\left(\dfrac{\frac{1}{3}5}{\frac{2}{3}5}\right) = 26.6^\circ \\ \phi = \tan^{-1}\left(\dfrac{\frac{2}{3}5}{\frac{1}{3}5}\right) = 63.4^\circ \end{cases}$$

$F_{EH} = F_{FH} = 0$ (INSPECTION)

$$\begin{cases} \Sigma F_x = 0: \; A_x + 4 + 5\cos 15^\circ + 6\cos 30^\circ = 0 \\ \Sigma F_y = 0: \; A_y + N_G - 5\sin 15^\circ - 6\sin 30^\circ = 0 \\ \Sigma M_A = 0: \; 10N_G - 4\left(\frac{5}{3}\right) - (5\sin 60^\circ)\left(\frac{2}{3}5\sqrt{2}\right) - (6\sin 75^\circ)(5\sqrt{2}) = 0 \end{cases}$$

$A_x = -14.03\text{ kN } (\leftarrow)$; $A_y = -2.51\text{ kN } (\downarrow)$; $N_G = 6.81\text{ kN}$

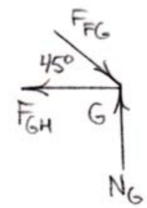
JOINT A:



$$\begin{cases} \Sigma F_x = 0: \; F_{AB}\cos 45^\circ + F_{AH} - A_x = 0 \\ \Sigma F_y = 0: \; F_{AB}\sin 45^\circ - A_y = 0 \end{cases}$$

$\underline{F_{AB} = 3.55\text{ kN T}}$ $\quad$ $\underline{F_{AH} = 11.51\text{ kN T}}$

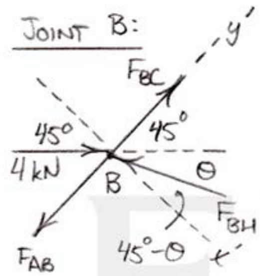
JOINT G:



$$\begin{cases} \Sigma F_y = 0: \; N_G - F_{FG}\sin 45^\circ = 0 \\ \Sigma F_x = 0: \; F_{FG}\cos 45^\circ - F_{GH} = 0 \end{cases} \longrightarrow \begin{cases} F_{FG} = 9.63\text{ kN C} \\ F_{GH} = 6.81\text{ kN T} \end{cases}$$

AND... $\underline{F_{EF} = F_{DE} = 9.63\text{ kN C}}$

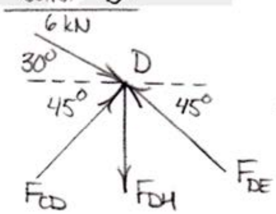
JOINT B:



$$\begin{cases} \Sigma F_{x'} = 0: \; 4\cos 45^\circ - F_{BH}\cos(45^\circ - \theta) = 0 \\ \Sigma F_{y'} = 0: \; F_{BC} - F_{AB} + 4\sin 45^\circ - F_{BH}\sin(45^\circ - \theta) = 0 \end{cases}$$

$\underline{F_{BH} = 2.98\text{ kN C}}$ $\quad$ $\underline{F_{BC} = 1.667\text{ kN T}}$

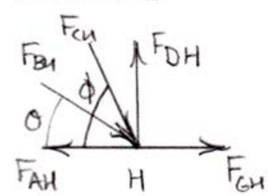
JOINT D:



$$\begin{cases} \Sigma F_x = 0: \; 6\cos 30^\circ + F_{CD}\cos 45^\circ - F_{DE}\cos 45^\circ = 0 \\ \Sigma F_y = 0: \; -6\sin 30^\circ + F_{CD}\sin 45^\circ + F_{DE}\sin 45^\circ - F_{DH} = 0 \end{cases}$$

$\underline{F_{CD} = 2.28\text{ kN C}}$ $\quad$ $\underline{F_{DH} = 5.42\text{ kN T}}$

JOINT H:



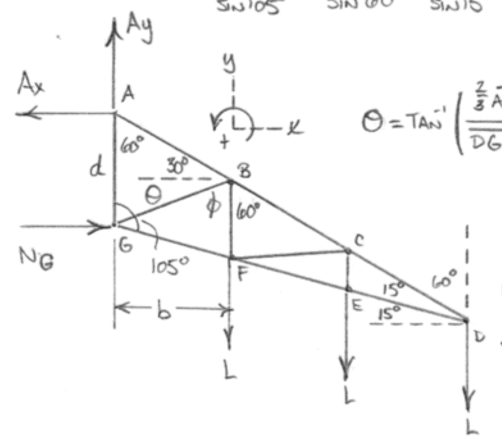
$\Sigma F_y = 0: \; F_{DH} - F_{CH}\sin\phi - F_{BH}\sin\theta = 0$

$\underline{F_{CH} = 4.56\text{ kN C}}$

4/19

$$\frac{\overline{AD}}{\sin 105^\circ} = \frac{\overline{DG}}{\sin 60^\circ} = \frac{d}{\sin 15^\circ}$$

$$\begin{cases} \overline{AD} = 3.73\,d \\ \overline{DG} = 3.35\,d \end{cases}$$



$$\theta = \tan^{-1}\left(\frac{\frac{2}{3}\overline{AD}\sin 30^\circ - \overline{DG}\sin 15^\circ}{\overline{DG}\cos 15^\circ - \frac{2}{3}\overline{AD}\cos 30^\circ}\right) \longrightarrow \theta = 19.33^\circ$$

$$b = \frac{1}{3}\overline{DG}\cos 15^\circ \longrightarrow b = 1.077\,d$$

$$\begin{cases} \Sigma F_x = 0: \ N_G - A_x = 0 \\ \Sigma F_y = 0: \ A_y - 3L = 0 \end{cases}$$

$$\Sigma M_A = 0: \ N_G d - bL - 2bL - 3bL = 0$$

SOLVING...

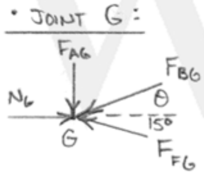
$$\begin{cases} A_x = N_G = 6.46\,L \\ A_y = 3L \end{cases}$$

• JOINT A:

$$\begin{cases} \Sigma F_x = 0: \ F_{AB}\sin 60^\circ - A_x = 0 \\ \Sigma F_y = 0: \ A_y - F_{AG} - F_{AB}\cos 60^\circ = 0 \end{cases}$$

$F_{AB} = 7.46\,L$ T $\qquad F_{AG} = -0.732\,L$ (C)

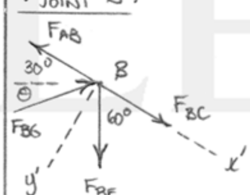
• JOINT G:



$$\begin{cases} \Sigma F_x = 0: \ N_G - F_{BG}\cos\theta - F_{FG}\cos 15^\circ = 0 \\ \Sigma F_y = 0: \ F_{FG}\sin 15^\circ - F_{AG} - F_{BG}\sin\theta = 0 \end{cases}$$

$F_{BG} = 1.713\,L$ C $\qquad F_{FG} = 5.02\,L$ C

• JOINT B:



$$\Sigma F_{y'} = 0: \ F_{BF}\sin 60^\circ - F_{BG}\sin(30^\circ + \theta) = 0$$

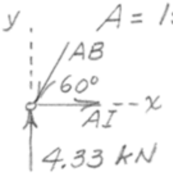
$F_{BF} = 1.5\,L$ T

4/20 Truss as a whole

$\Sigma M_F = 0; \; 2\left(a + \frac{a}{2}\right) + 4\left(2a + \frac{a}{2}\right) - A(3a) = 0$

$A = 13/3 = 4.33 \text{ kN}$

Joint A



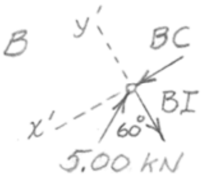
$\Sigma F_y = 0; \; AB \sin 60° - 4.33 = 0$

$AB = 5.00 \text{ kN } C$

$\Sigma F_x = 0; \; AI - 5 \cos 60° = 0$

$AI = 2.50 \text{ kN } T$

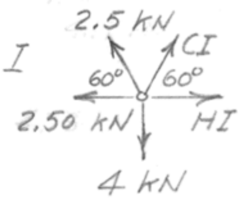
Joint B



$\Sigma F_{y'} = 0; \; 5.00 \cos 60° - BI = 0$

$\underline{BI = 2.50 \text{ kN}} \; T$

Joint I



$\Sigma F_y = 0; \; (CI + 2.5)\sin 60° - 4 = 0$

$\underline{CI = 2.12 \text{ kN } T}$

$\Sigma F_x = 0; \; HI + 2.12 \cos 60°$

$- 2.50 - 2.5 \cos 60° = 0$

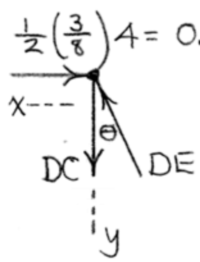
$\underline{HI = 2.69 \text{ kN}} \; T$

4/21

$\theta = \tan^{-1}\frac{2}{4} = 26.6°$

Joint D

$\cos\theta = \frac{2}{\sqrt{5}}, \ \sin\theta = \frac{1}{\sqrt{5}}$



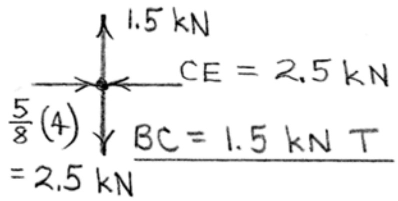
$\Sigma F_x = 0: \ DE\frac{1}{\sqrt{5}} - 0.75 = 0$

$DE = 1.677$ kN C

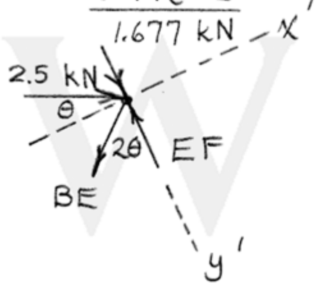
$\Sigma F_y = 0: \ DC - 1.677\frac{2}{\sqrt{5}} = 0$

$DC = 1.5$ kN T

Joint C



Joint E

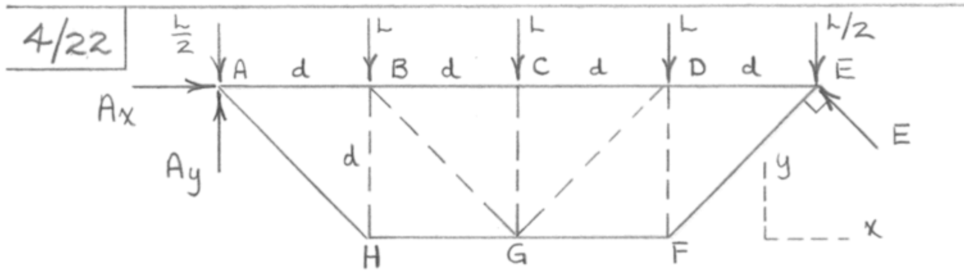


$\Sigma F_{x'} = 0: \ 2.5\frac{2}{\sqrt{5}} - BE\sin 2\theta = 0$

$\sin 2\theta = 0.8$

So $\underline{BE = 2.80 \text{ kN T}}$

4/22



Entire truss:

$$\circlearrowleft + \Sigma M_A = 0: \; -Ld - L(2d) - L(3d) - \frac{L}{2}(4d) + E\frac{\sqrt{2}}{2}(4d) = 0$$

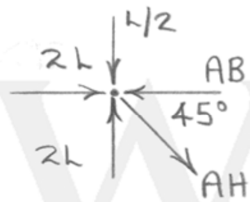
$$E = 2\sqrt{2}L$$

$$\Sigma F_x = 0: \; A_x - 2\sqrt{2}L\frac{\sqrt{2}}{2} = 0, \quad A_x = 2L$$

$$\Sigma F_y = 0: \; A_y - 4L + 2\sqrt{2}L\frac{\sqrt{2}}{2} = 0, \quad A_y = 2L$$

By inspection of joint C, $\underline{CG = L \; C}$

Joint A:



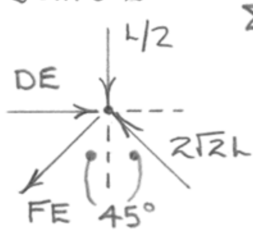
$$\Sigma F_y = 0: \; 2L - \frac{L}{2} - AH\frac{\sqrt{2}}{2} = 0$$

$$AH = \frac{3\sqrt{2}}{2}L \; T$$

$$\Sigma F_x = 0: \; 2L + \frac{3\sqrt{2}}{2}L\frac{\sqrt{2}}{2} - AB = 0$$

$$\underline{AB = \frac{7}{2}L \; C}$$

Joint E



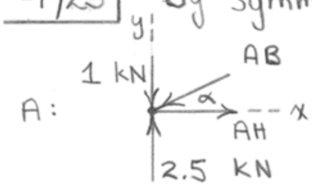
$$\Sigma F_y = 0: \; -\frac{L}{2} + 2\sqrt{2}L\frac{\sqrt{2}}{2} - FE\frac{\sqrt{2}}{2} = 0$$

$$FE = \frac{3\sqrt{2}}{2}L \; T$$

$$\Sigma F_x = 0: \; DE - \frac{3\sqrt{2}}{2}L\frac{\sqrt{2}}{2} - 2\sqrt{2}L\frac{\sqrt{2}}{2} = 0$$

$$\underline{DE = \frac{7L}{2} \; C}$$

4/23 By symmetry, $A = E = 2.5$ kN; $\alpha = \tan^{-1}\left(\frac{2}{4}\right) = 26.6°$



$\Sigma F_y = 0$: $2.5 - 1 - AB \sin\alpha = 0$

$\underline{AB = 3.35 \text{ kN } C}$

$\Sigma F_x = 0$: $-3.35 \cos\alpha + AH = 0$

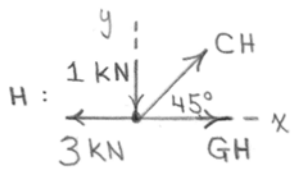
$\underline{AH = 3 \text{ kN } T}$

$\Sigma F_x = 0$: $3.35 \cos\alpha - BC \cos\alpha = 0$

$\underline{BC = 3.35 \text{ kN } C}$

$\Sigma F_y = 0$: $-1 + (3.35 - 3.35)\sin\alpha + BH = 0$

$\underline{BH = 1 \text{ kN } C}$



$\Sigma F_y = 0$: $-1 + CH \sin 45° = 0$

$\underline{CH = 1.414 \text{ kN } T}$

$\Sigma F_x = 0$: $-3 + 1.41 \cos 45° + GH = 0$

$\underline{GH = 2 \text{ kN } T}$

By inspection of joint G and $\Sigma F_y = 0$, $\underline{CG = 0.}$

By symmetry,
$$\left\{\begin{array}{l} DE = AB = 3.35 \text{ kN } C \\ CD = BC = 3.35 \text{ kN } C \\ EF = AH = 3 \text{ kN } T \\ DF = BH = 1 \text{ kN } C \\ CF = CH = 1.414 \text{ kN } T \\ \underline{FG = GH = 2 \text{ kN } T} \end{array}\right.$$

4/24 By symmetry, $A = E = 2.5$ kN; $\alpha = \tan^{-1}\left(\frac{2}{4}\right) = 26.6°$

Joint A analysis same as Prob. 4/19: $\begin{cases} AB = 3.35 \text{ kN } C \\ AH = 3 \text{ kN } T \end{cases}$

By inspection, $BH = 0$ and $GH = AH$.

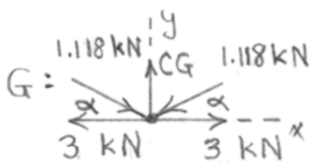
B:

$\Sigma F_y = 0: -1 + 3.35 \sin\alpha + BG \sin\alpha - BC \sin\alpha = 0$

$\Sigma F_x = 0: 3.35 \cos\alpha - BC \cos\alpha - BG \cos\alpha = 0$

$\Rightarrow$ $BC = 2.24$ kN C, $BG = 1.118$ kN C

G:



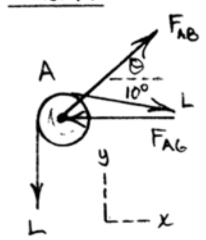
$\Sigma F_y = 0: CG - 2(1.118)\sin\alpha = 0$

$CG = 1$ kN T

By symmetry, $\begin{cases} DE = AB = 3.35 \text{ kN C} \\ CD = BC = 2.24 \text{ kN C} \\ EF = AH = 3 \text{ kN T} \\ DF = BH = 0 \\ FG = GH = 3 \text{ kN T} \\ DG = BG = 1.118 \text{ kN C} \end{cases}$

4/25 $\theta = \tan^{-1}\frac{3.5}{4} = 41.2°$ $\quad F_{CF} = F_{DE} = 0$ (INSPECTION)

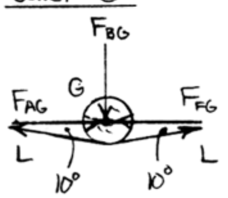
• Joint A:



$$\begin{cases} \Sigma F_x = 0: & F_{AB}\cos\theta + L\cos 10° - F_{AG} = 0 \\ \Sigma F_y = 0: & F_{AB}\sin\theta - L\sin 10° - L = 0 \end{cases}$$

$F_{AB} = 1.782L$ T $\qquad F_{AG} = 2.33L$ C

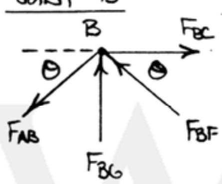
• Joint G:



$$\begin{cases} \Sigma F_x = 0: & F_{AG} - F_{FG} + L\cos 10° - L\cos 10° = 0 \\ \Sigma F_y = 0: & 2L\sin 10° - F_{BG} = 0 \end{cases}$$

$F_{FG} = 2.33L$ C $\qquad F_{BG} = 0.347L$ C

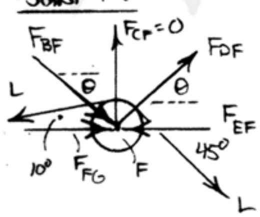
• Joint B:



$$\begin{cases} \Sigma F_x = 0: & F_{BC} - F_{BF}\cos\theta - F_{AB}\cos\theta = 0 \\ \Sigma F_y = 0: & F_{BF}\sin\theta + F_{BG} - F_{AB}\sin\theta = 0 \end{cases}$$

$F_{BC} = F_{CD} = 2.29L$ T $\qquad F_{BF} = 1.255L$ C

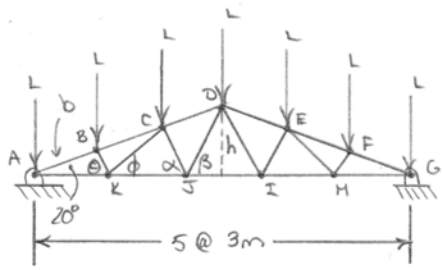
• Joint F:



$$\begin{cases} \Sigma F_x = 0: & F_{DF}\cos\theta - F_{EF} + L\cos 45° - L\cos 10° + F_{FG} + F_{BF}\cos\theta = 0 \\ \Sigma F_y = 0: & F_{DF}\sin\theta - L\sin 45° - L\sin 10° - F_{BF}\sin\theta = 0 \end{cases}$$

$F_{DF} = 2.59L$ T $\qquad F_{EF} = 4.94L$ C

4/26



$$h = \frac{15}{2}\tan 20^\circ \longrightarrow h = 2.73\text{ m}$$

$$\overline{AD} = \frac{h}{\sin 20^\circ} = \frac{2.73}{\sin 20^\circ} \longrightarrow \overline{AD} = 7.98\text{ m}$$

$$b = \tfrac{1}{3}\overline{AD} = \tfrac{1}{3}(7.98) \longrightarrow b = 2.66\text{ m}$$

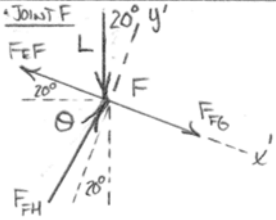
$$\theta = \tan^{-1}\left(\frac{b\sin 20^\circ}{3 - b\cos 20^\circ}\right) \longrightarrow \theta = 61.2^\circ$$

$$\phi = \tan^{-1}\left(\frac{2b\sin 20^\circ}{2b\cos 20^\circ - 3}\right) \longrightarrow \phi = 42.3^\circ$$

$$\alpha = \tan^{-1}\left(\frac{2b\sin 20^\circ}{6 - 2b\cos 20^\circ}\right) \longrightarrow \alpha = 61.2^\circ$$

$$\beta = \tan^{-1}\left(\frac{h}{1.5}\right) \longrightarrow \beta = 61.2^\circ$$

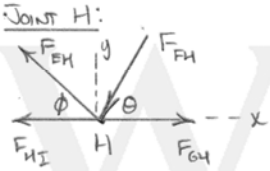
· Joint F



$$\Sigma F_{y'} = 0: \quad F_{FH}\cos(90^\circ - 20^\circ - \theta) - L\cos 20^\circ = 0$$

$$F_{FH} = 0.951L \quad C$$

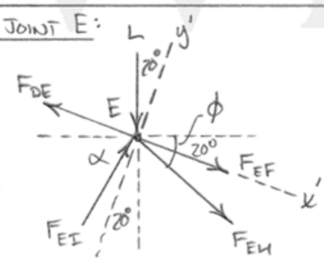
Joint H:



$$\Sigma F_y = 0: \quad F_{EH}\sin\phi - F_{FH}\sin\theta = 0$$

$$\underline{F_{EH} = 1.238L \quad T}$$

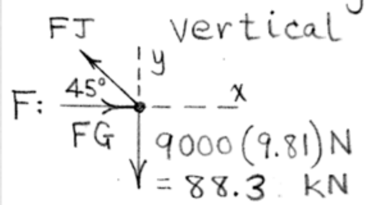
· Joint E:



$$\Sigma F_{y'} = 0: \quad F_{EI}\cos(90^\circ - \alpha - 20^\circ) - L\cos 20^\circ - F_{EH}\sin(\phi - 20^\circ) = 0$$

$$\underline{F_{EI} = 1.426L \quad C}$$

4/27 Structure is statically indeterminate externally; member AE in main vertical tower is indeterminate.



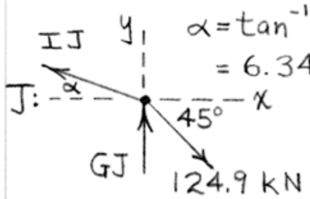
$\Sigma F_y = 0$: $FJ\left(\frac{\sqrt{2}}{2}\right) - 88.3 = 0$

$FJ = 124.9$ kN T

$\Sigma F_x = 0$: $FG - 124.9\left(\frac{\sqrt{2}}{2}\right) = 0$

$FG = 88.3$ kN C

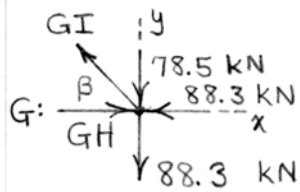
$\alpha = \tan^{-1}\left(\frac{3}{27}\right) = 6.34°$



$\Sigma F_x = 0$: $-IJ\cos\alpha + 124.9\left(\frac{\sqrt{2}}{2}\right) = 0$

$IJ = 88.8$ kN T

$\Sigma F_y = 0$: $88.8\sin\alpha - 124.9\left(\frac{\sqrt{2}}{2}\right) + GJ = 0$, $\underline{GJ = 78.5 \text{ kN C}}$



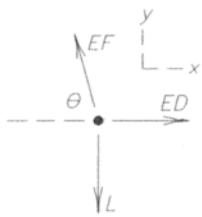
$\beta = \tan^{-1}\left(\frac{3.5}{4.5}\right) = 37.9°$

$\Sigma F_y = 0$: $-78.5 - 88.3 + GI\sin\beta = 0$

$\underline{GI = 272 \text{ kN T}}$

4/28

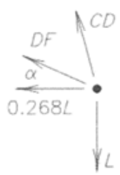
Joint E:



$$\theta = \frac{180° - 30°}{2} = 75°$$

$$\begin{cases} \Sigma F_y = 0: \; EF \sin 75° - L = 0, \; EF = 1.035L \; T \\ \Sigma F_x = 0: \; -1.035L \cos 75° + ED = 0, \; ED = 0.268L \; T \end{cases}$$

Joint D:



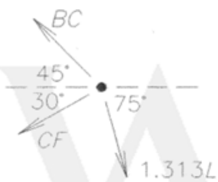
$$\overline{DF}^2 = R^2 + 4R^2 - 2(R)(2R)\cos 30°, \; \overline{DF} = 1.239R$$

$$\frac{\sin 30°}{1.239R} = \frac{\sin\alpha}{R}, \; \alpha = 23.8°$$

$$\begin{cases} \Sigma F_x = 0: \; -0.268L - DF\cos 23.8° - CD\cos 75° = 0 \\ \Sigma F_y = 0: \; -L + DF\sin 23.8° + CD\sin 75° = 0 \end{cases}$$

Solve simultaneously to obtain: $CD = 1.313L \; T, \; DF = 0.664L \; C$

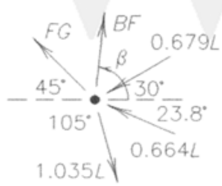
Joint C:



$$\begin{cases} \Sigma F_x = 0: \; -BC\cos 45° - CF\cos 30° + 1.313L\cos 75° = 0 \\ \Sigma F_y = 0: \; BC\sin 45° - CF\sin 30° - 1.313L\sin 75° = 0 \end{cases}$$

Solve simultaneously to obtain: $CF = -0.674L \; (C)$

Joint F:

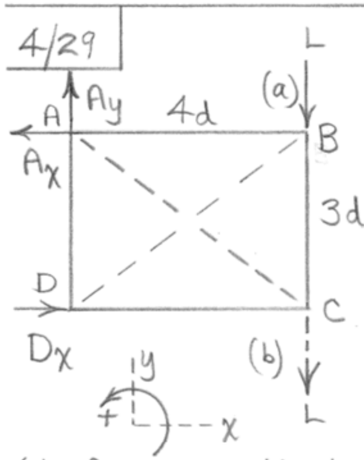


$$\beta = 30° + 23.8° + 30° = 83.8°$$

$$\begin{cases} \Sigma F_x = 0: \; -FG\cos 45° + BF\cos\beta + EF\cos 75° + CF\cos 30° + DF\cos\alpha = 0 \\ \Sigma F_y = 0: \; FG\sin 45° + BF\sin\beta - EF\sin 75° + CF\sin 30° - DF\sin\alpha = 0 \end{cases}$$

Solve simultaneously to obtain: $\underline{BF = 1.814L \; T}$

4/29



$\Sigma F_y = 0: \; A_y - L = 0, \; A_y = L$

$\Sigma M_A = 0: \; D_x(3d) - L(4d) = 0$

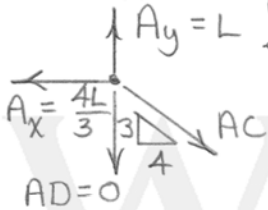
$D_x = \frac{4L}{3}$

$\Sigma F_x = 0: \; A_x = D_x = \frac{4L}{3}$

(Reactions hold for (a) & (b).)

(a) Assume that BD goes slack. From an inspection of joint B, $\underline{AB = 0}$ and $\underline{BC = L \; C.}$ Similarly, from joint D, $\underline{AD = 0}$ and $\underline{CD = \frac{4L}{3} \; C.}$

Joint A:

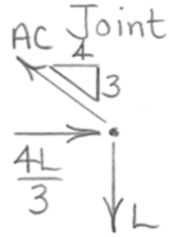


$$\begin{cases} \Sigma F_y = 0: \; L - \frac{3}{5} AC = 0, \; \underline{AC = \frac{5L}{3} \; T} \\ \Sigma F_x = 0: -\frac{4L}{3} + \frac{5L}{3} \cdot \frac{4}{5} = 0 \; ✓ \end{cases}$$

Because AC is in tension, assumption is valid.

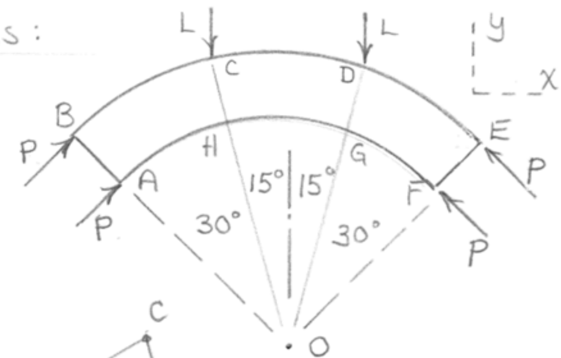
(b) Assume that BD goes slack. From joint B, $\underline{AB = BC = 0}$. From joint D, $\underline{AD = 0}$ & $\underline{CD = \frac{4L}{3} \; C.}$

Joint C:



$$\begin{cases} \Sigma F_y = 0: \; AC\left(\frac{3}{5}\right) - L = 0, \; \underline{AC = \frac{5L}{3} \; T} \\ \Sigma F_x = 0: \; \frac{4L}{3} - \frac{5L}{3}\left(\frac{4}{5}\right) = 0 \; ✓ \end{cases}$$

▶4/30 Entire truss:

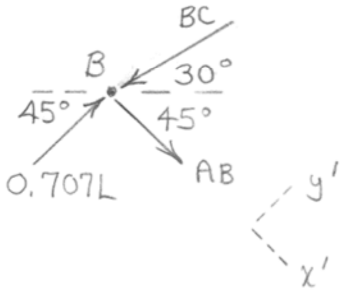


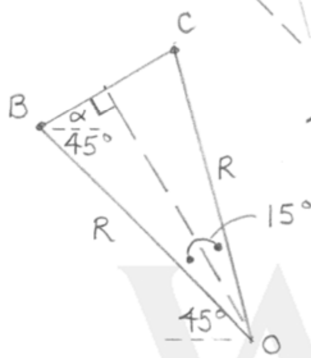
$\Sigma F_y = 0$:

$4P \sin 45° - 2L = 0$

$P = 0.707L$

Joint B:





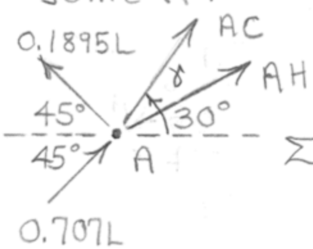
$\measuredangle OBC = 75°$

So $\alpha = 30°$

$\Sigma F_{y'} = 0$: $0.707L - BC \cos 15° = 0$, $BC = 0.732L$ C

$\Sigma F_{x'} = 0$: $AB - 0.732 \sin 15° = 0$, $AB = 0.1895L$ T

Joint A:



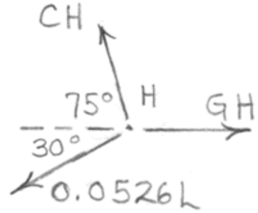
$$\gamma = \tan^{-1}\left[\frac{R\cos 15° - 0.75R\cos 45°}{0.75R\sin 45° - R\sin 15°}\right] = 58.1°$$

$\Sigma F_x = 0$: $0.707L\cos 45° - 0.1895L\cos 45° + AC\cos 58.1° + AH\cos 30° = 0$

$\Sigma F_y = 0$: $0.707L\sin 45° + 0.1895L\sin 45° + AC\sin 58.1° + AH\sin 30° = 0$

$AC = -0.778L$ (C) $\quad AH = 0.0526L$ T

Joint H:



$\Sigma F_y = 0$: $-0.0526L\sin 30° + CH\sin 75° = 0$

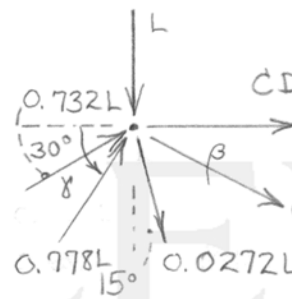
$CH = +0.0272L$ T

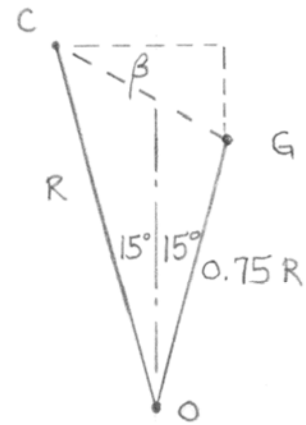
$\Sigma F_x = 0$: $-0.0526L\cos 30° - 0.0272L\cos 75° + GH = 0$

$GH = 0.0526L$ T

(not needed)

Joint C:





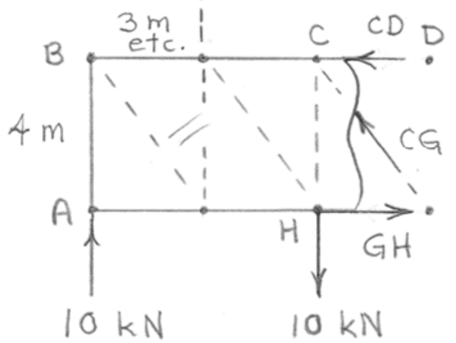
$$\beta = \tan^{-1}\left[\frac{R\cos 15° - 0.75R\cos 15°}{R\sin 15° + 0.75R\sin 15°}\right] = 28.1°$$

$\Sigma F_x = 0$: $0.732L\cos 30° + 0.778L\cos 58.1° + 0.0272L\sin 15° + CD + CG\cos 28.1° = 0$

$\Sigma F_y = 0$: $0.732L\sin 30° + 0.778L\sin 58.1° - 0.0272L\cos 15° - CG\sin 28.1° - L = 0$

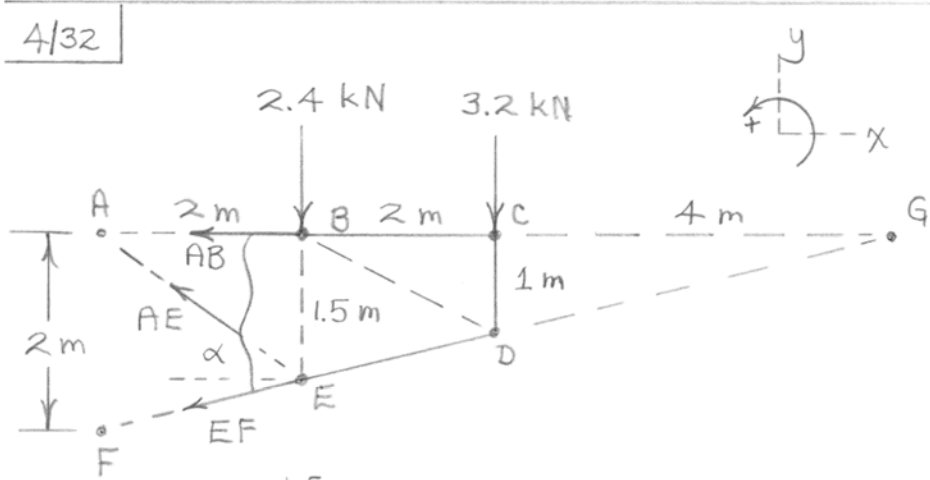
Solution: $\begin{cases} CD = -1.053L \text{ (C)} \\ CG = 0 \end{cases}$

4/31 By inspection, the reaction at each support is 10 kN up.



$+\uparrow \Sigma F = 0: \quad CG\left(\frac{4}{5}\right) + 10 - 10 = 0, \quad \underline{CG = 0}$

4/32

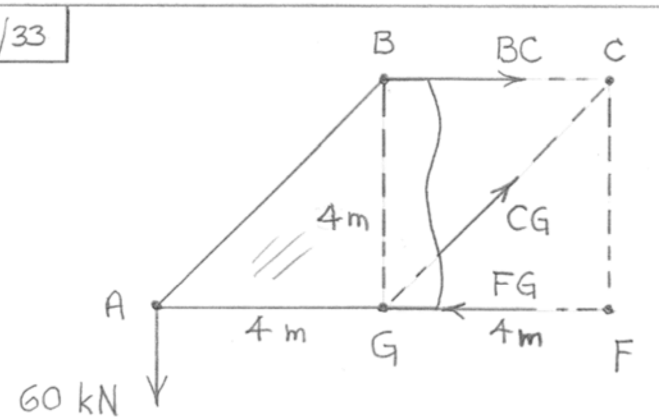


$$\alpha = \tan^{-1}\frac{1.5}{2} = 36.9°$$

$$\Sigma M_G = 0: \ 3.2(4) + 2.4(6) - AE\cos\alpha\,(1.5) - AE\sin\alpha\,(6) = 0$$

$$\underline{AE = 5.67 \text{ kN } T}$$

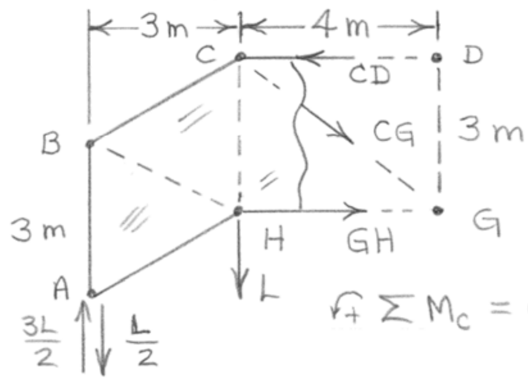
4/33



$+\uparrow \Sigma F = 0: \quad CG \frac{\sqrt{2}}{2} - 60 = 0, \quad \underline{CG = 84.9 \text{ kN T}}$

$\circlearrowleft + \Sigma M_G = 0: \quad 60(4) - BC(4) = 0, \quad \underline{BC = 60 \text{ kN T}}$

4/34 By symmetry, the reaction at A is $\frac{3L}{2}$ up.

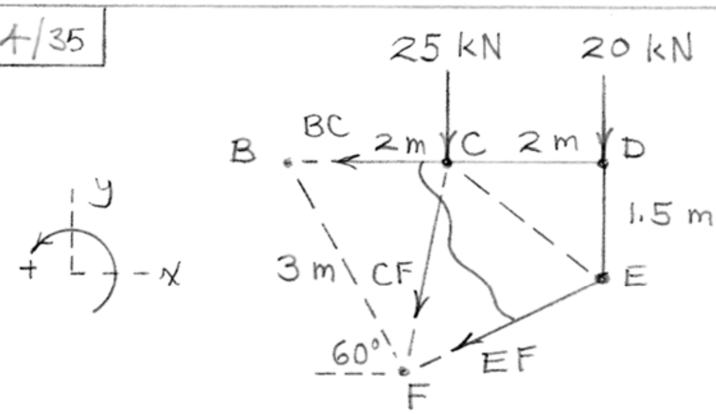


From $+\uparrow \Sigma F = 0$, $\underline{CG = 0}$.

$\circlearrowleft + \Sigma M_C = 0$: $-3L + GH(3) = 0$

$\underline{GH = L \; T}$

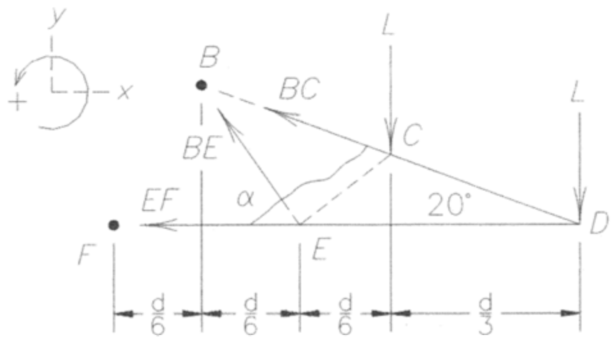
4/35



$$\Sigma M_F = 0: \quad BC(3 \sin 60°) - 25(0.5) - 20(2.5) = 0$$

$$\underline{BC = 24.1 \text{ kN } T}$$

4/36

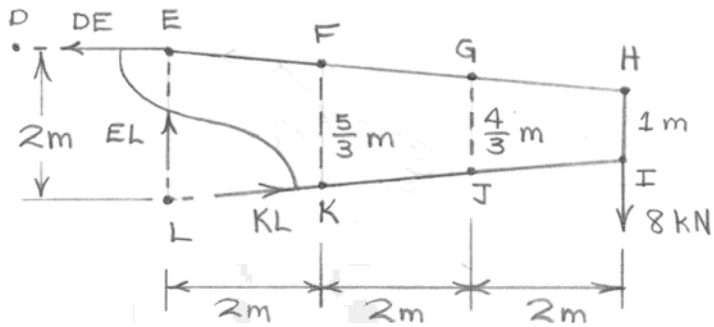


$$\alpha = \tan^{-1}\frac{\frac{4}{6}d\tan 20°}{\frac{d}{6}} = 55.5°$$

$$\Sigma M_D = 0: \; L\left(\frac{d}{3}\right) - BE\left(\frac{d}{2}\right)(\sin 55.5°) = 0$$

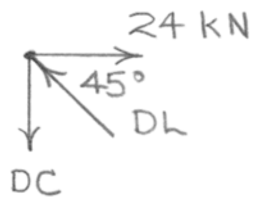
$$\underline{BE = 0.809L\ T}$$

4/37



$\Sigma M_L = 0: \ DE(2) - 8(6) = 0, \quad \underline{DE = 24 \text{ kN } T}$

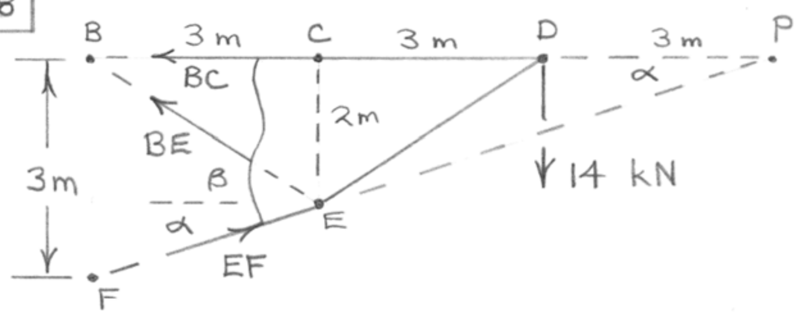
Joint D:



$\overset{+}{\rightarrow} \Sigma F_x = 0: \ 24 - DL\cos 45° = 0$

$\underline{DL = 33.9 \text{ kN } C}$

4/38



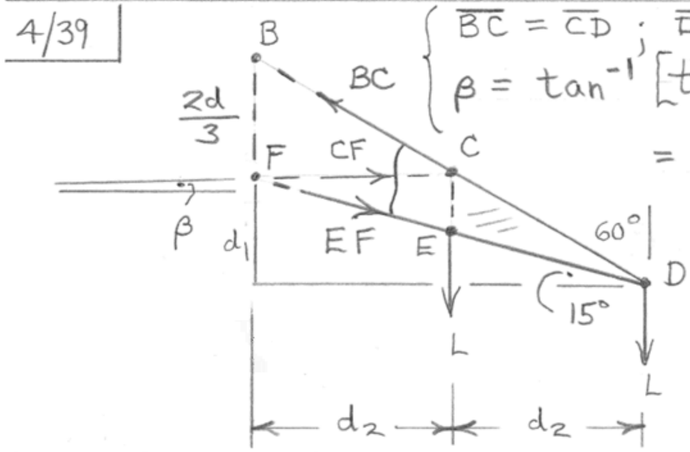
$\alpha = \tan^{-1}\left(\frac{2}{6}\right) = 18.43°$, $\beta = \tan^{-1}\frac{2}{3} = 33.7°$

$\circlearrowleft + \Sigma M_E = 0: \ BC(2) - 14(3) = 0$, $\underline{BC = 21 \text{ kN } T}$

$\circlearrowleft + \Sigma M_P = 0: -BE \sin\beta(9) + 14(3) = 0$, $\underline{BE = 8.41 \text{ kN } T}$

$\circlearrowleft + \Sigma M_B = 0: \ EF\cos\alpha(3) - 14(6) = 0$, $\underline{EF = 29.5 \text{ kN } C}$

4/39



$$\begin{cases} \overline{BC} = \overline{CD} \; ; \; \overline{DE} = \overline{EF} \\ \beta = \tan^{-1}\left[\tan 30° - 2\tan 15°\right] \\ \quad = 2.37° \end{cases}$$

Some geometry: $\begin{cases} \dfrac{\frac{2d}{3} + d_1}{2d_2} = \tan 30° \\ \dfrac{d_1}{2d_2} = \tan 15° \end{cases}$

$$\Rightarrow \begin{cases} d_2 = 1.077d \\ d_1 = 0.577d \end{cases}$$

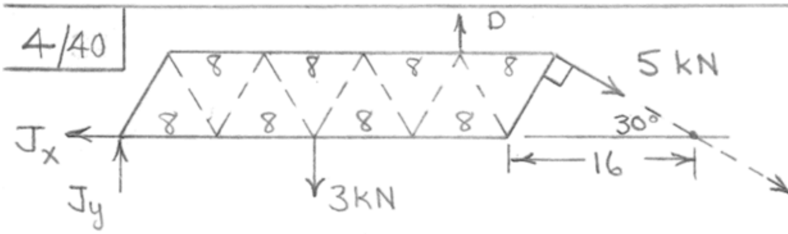
$$\circlearrowleft \Sigma M_F = 0: \; BC\cos 30°\left(\frac{2d}{3}\right) - L(1.077d) - 2L(1.077d) = 0$$

$$\underline{BC = 5.60L \;\; T}$$

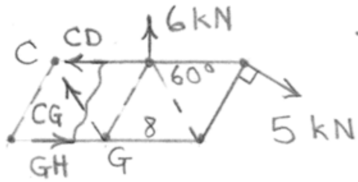
$$\circlearrowleft \Sigma M_D = 0: \; L(1.077d) - CF\cos\beta(0.577d) - CF\sin\beta(2\cdot 1.077d) = 0$$

$$\underline{CF = 1.617L \;\; C}$$

4/40



$$\Sigma M_J = 0: \quad -3(16) + D(28) - (5\sin 30°)(48) = 0$$
$$D = 6 \text{ kN}$$



$$\Sigma F_y = 0: \quad CG\sin 60° + 6 - 5\sin 30° = 0$$
$$CG = -4.04 \text{ kN } C$$

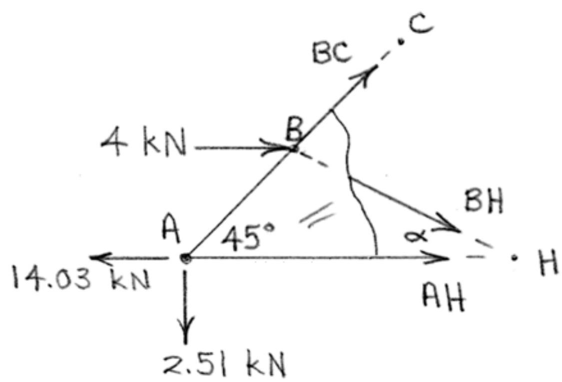
$$\Sigma M_G = 0: \quad CD(8\sin 60°) + 6(4) - (5\sin 30°)(24) = 0, \quad CD = 5.20 \text{ kN } T$$

$$\Sigma M_C = 0: \quad GH(8\sin 60°) + 6(8) + (5\sin 30°)(16) = 0$$
$$GH = -1.155 \text{ kN } T$$

$$\Sigma M_H = 0: \quad 6(12) - (5\sin 30°)(32) - BC\, 8\sin 60° = 0$$
$$BC = -1.155 \text{ kN } T$$

4/41 From the solution to Prob. 4/18, the external reactions are $A_x = 14.03$ kN ←, $A_y = 2.51$ kN ↓, and $G_y = 6.81$ kN ↑.

Also: $\begin{cases} \overline{AB} = \overline{BC} = 2.36 \text{ m} \\ \alpha = \tan^{-1} \dfrac{5/3}{\frac{2}{3}(5)} = 26.6° \end{cases}$



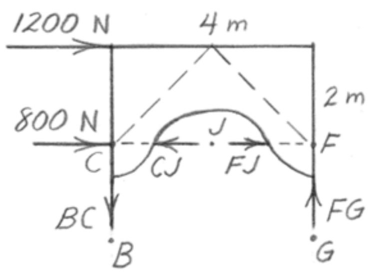
$\curvearrowleft \Sigma M_H = 0: \ -4\left(\frac{5}{3}\right) - BC\cos 45°\left(\frac{5}{3}\right) - BC\sin 45°\left(5 \cdot \frac{2}{3}\right) + 2.51(5) = 0$

$\underline{BC = 1.667 \text{ kN T}}$

$\curvearrowleft \Sigma M_A = 0: \ -4\left(\frac{5}{3}\right) - BH\cos 26.6°\left(\frac{5}{3}\right) - BH\sin 26.6°\left(\frac{5}{3}\right) = 0$

$\underline{BH = -2.98 \text{ kN (C)}}$

4/A2



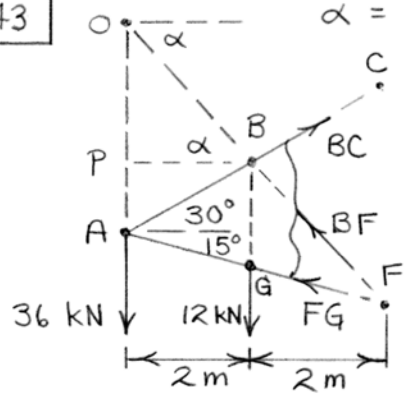
$\circlearrowleft + \Sigma M_C = 0: \ -1200(2) + FG(4) = 0$

$\underline{FG = 600 \text{ N } C}$

$\Sigma M_F = 0: \ -1200(2) + BC(4) = 0$

$\underline{BC = 600 \text{ N } T}$

4/43



$$\alpha = \tan^{-1}\left[\frac{2\tan 30° + 2(2\tan 15°)}{2}\right]$$

$$\alpha = 48.1°$$

$$\overline{AO} = \overline{AP} + \overline{PO}$$

$$= \overline{AP} + \overline{PB}\tan\alpha$$

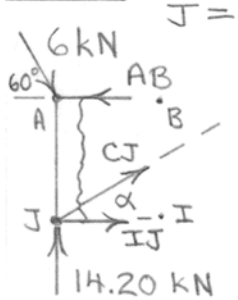
$$= 2\tan 30° + 2\tan\alpha$$

$$= 3.38 \text{ m}$$

$$\circlearrowleft + \ \Sigma M_A = 0: (BF\cos\alpha)(\overline{AO}) - 12(2) = 0$$

$$\underline{BF = 10.62 \text{ kN C}}$$

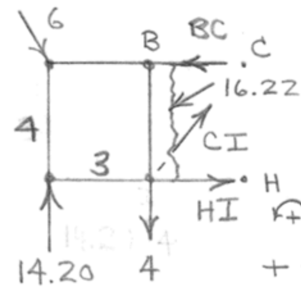
4/44 From truss as a whole and $\Sigma M_F = 0$, $J = 14.20$ kN.



$\Sigma F_y = 0: \; 14.20 - 6 \sin 60° + CJ \sin\alpha = 0$

where $\alpha = \tan^{-1}\left(\frac{4}{6}\right) = 33.7°$

$\therefore \; \underline{CJ = -16.22 \text{ kN}(C)}$



$\Sigma F_y = 0: -6 \sin 60° + 14.20 - 4 - 16.22 \sin\alpha + CI \, \frac{4}{5} = 0$

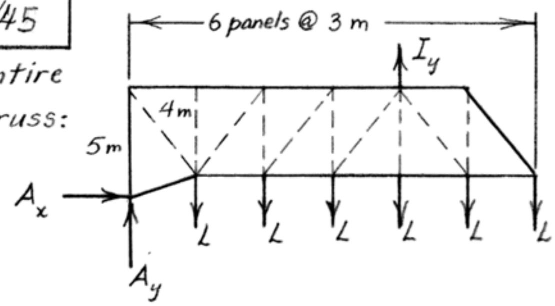
$\underline{CI = 5.00 \text{ kN } T}$

$\Sigma M_C = 0: (6 \sin 60°)\, 6 - (14.20)\, 6 + 4(3) + HI(4) = 0$, $\underline{HI = 10.50 \text{ kN } T}$

$\Sigma F_x = 0: \; 6 \cos 60° - 16.22 \cos\alpha + 5\left(\frac{3}{5}\right) + 10.5 - BC = 0$, $\underline{BC = 3.00 \text{ kN } C}$

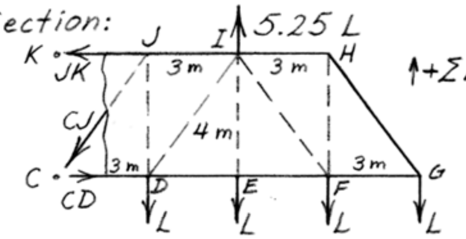
4/45

Entire truss:



$\circlearrowleft + \Sigma M_A = 0: \; I_y(12) - L(3+6+9+12+15+18) = 0, \; I_y = 5.25\,L$

Section:



$\uparrow + \Sigma F_y = 0: \; 5.25\,L - 4L - CJ\left(\frac{4}{5}\right) = 0$

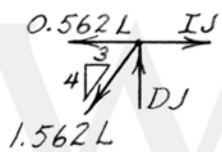
$\underline{CJ = 1.562\,L \;\; T}$

$\circlearrowleft \Sigma M_J = 0: \; CD(4) + 5.25\,L(3) - L(3+6+9) = 0$

$\underline{CD = 0.562\,L \;\; C}$

From $\overset{+}{\rightarrow} \Sigma F_x = 0, \; JK = 0.562\,L \; T$

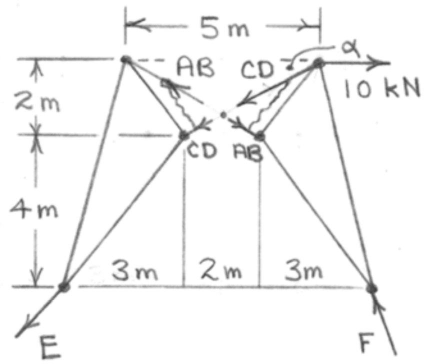
Joint J:



$+\uparrow \Sigma F_y = 0: \; DJ - 1.562\,L\left(\frac{4}{5}\right) = 0$

$\underline{DJ = 1.250\,L \;\; C}$

4/46 $\alpha = \tan^{-1}\left(\frac{2}{3.5}\right) = 29.7°$



I. $\Sigma M_E = 0$: $CD(4\cos\alpha) + CD(3\sin\alpha) - AB(6\cos\alpha) - AB(1.5\sin\alpha) = 0$

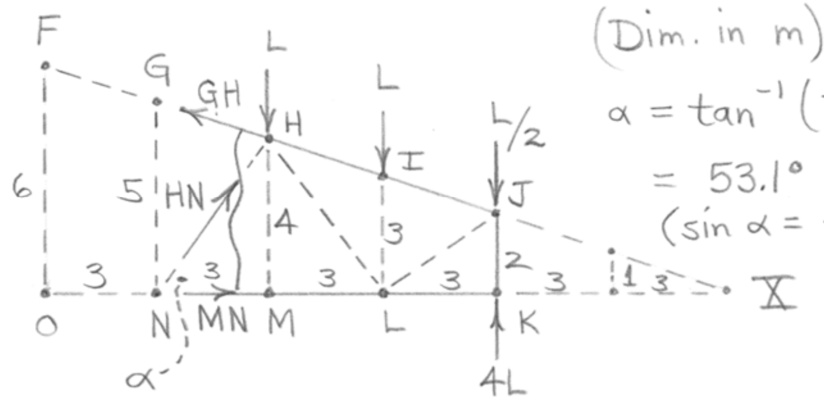
$CD = 3.00\ AB$

II. $\Sigma M_F = 0$: $10(6) + AB(4\cos\alpha) - AB(3\sin\alpha) - CD(6\cos\alpha) - CD(1.5\sin\alpha) = 0$

$60 + 1.985\ AB - 5.954\ CD = 0$

Solving simultaneously, $\underline{AB = 3.78 \text{ kN } C}$

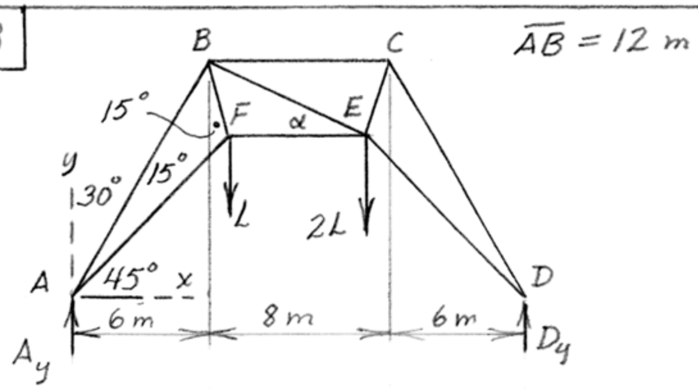
4/47 From the truss as a whole, the external reactions at A and K are 4L (up).



$\alpha = \tan^{-1}\left(\frac{4}{3}\right) = 53.1°$ $\left(\sin\alpha = \frac{4}{5}\right)$

$$\curvearrowleft + \Sigma M_X = 0: \left(\frac{L}{2} - 4L\right)6 + L(9) + L(12) - HN\left(\frac{4}{5}\right)(15) = 0, \quad \underline{HN = 0}$$

4/48



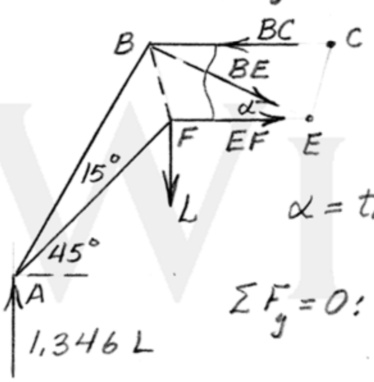
Coordinates of B: (6, 10.39) m

$\measuredangle AFB = 120°$; $\dfrac{\sin 120°}{12} = \dfrac{\sin 45°}{\overline{AF}}$, $\overline{AF} = 9.80$ m

Coordinates of F: (6.93, 6.93) m

$+\Sigma M_D = 0: -A_y(20) + L(20 - 6.93) + 2L(6.93) = 0$

$A_y = 1.346\ L\ (\uparrow)$

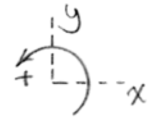


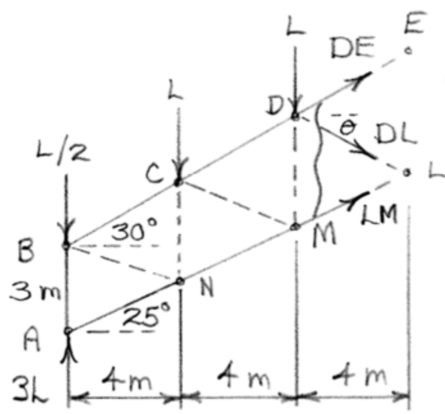
$\alpha = \tan^{-1}\left[\dfrac{10.39 - 6.93}{8 - (6.93 - 6)}\right] = 26.1°$

$\Sigma F_y = 0: 1.346L - L - BE \sin 26.1° = 0$

$\underline{BE = 0.787\ L\ \ T}$

4/49 By symmetry of entire truss,
$A_y = I_y = 3L$, $A_x = 0$





$$\theta = \tan^{-1}\left[\frac{3 + 8\tan 30° - 12\tan 25°}{4}\right] = 26.8°$$

$$\Sigma M_L = 0:\ L(4) + L(8) + \frac{L}{2}(12) - 3L(12)$$
$$-DE\,(12\tan 30° - 12\tan 25° + 3)\sin 60° = 0$$

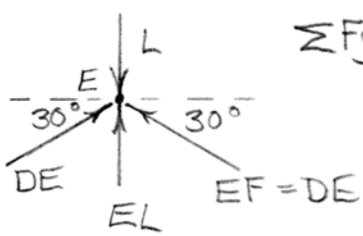
$DE = -4.80L$ or $\underline{4.80L\ C}$

$$\Sigma F_x = 0:\ -4.80L\cos 30° + DL\cos 26.8° + LM\cos 25° = 0$$

$$\Sigma F_y = 0:\ -4.80L\sin 30° - DL\sin 26.8° + LM\sin 25° + 3L - 2L - L/2 = 0$$

Solve to obtain $\underline{DL = 0.0446L\ T}$, $\underline{LM = 4.54L\ T}$

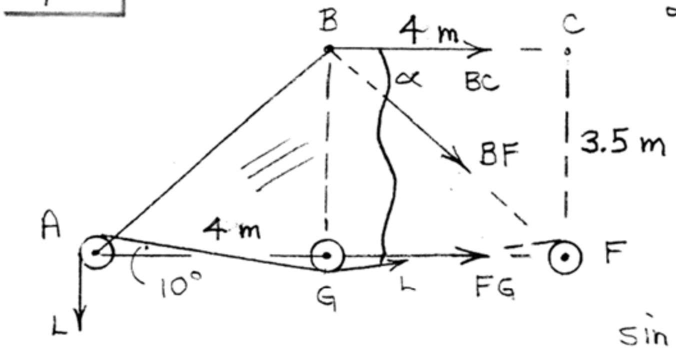
Joint E:



$$\Sigma F_y = 0:\ 2(4.80L\sin 30°) - L - EL = 0$$

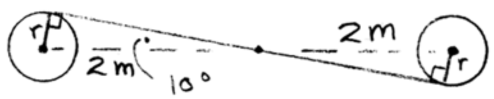
$\underline{EL = 3.80L\ T}$

4/50



$\alpha = \tan^{-1}\frac{3.5}{4} = 41.2°$

$\sin 10° = \frac{r}{2}$

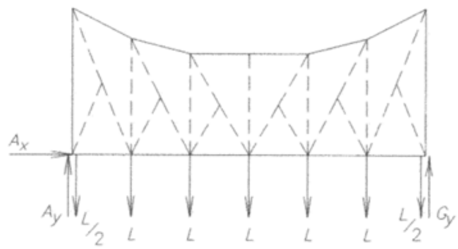


$r = 0.347$ m (don't need!)

$+\uparrow \Sigma F = 0: \ -BF\sin 41.2° - L + L\sin 10° = 0$

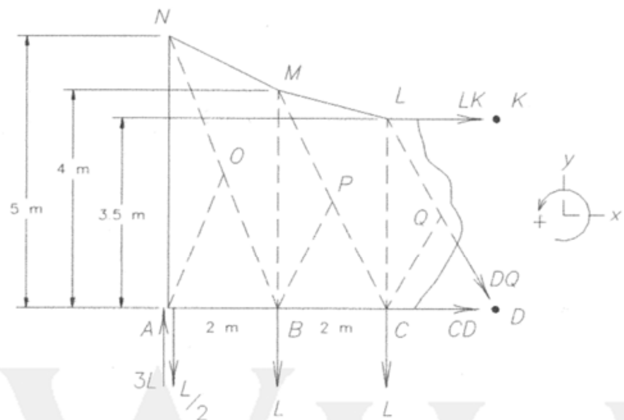
$\underline{BF = -1.255L \ (C)}$

4/51



By symmetry, $A_y = G_y = 3L$

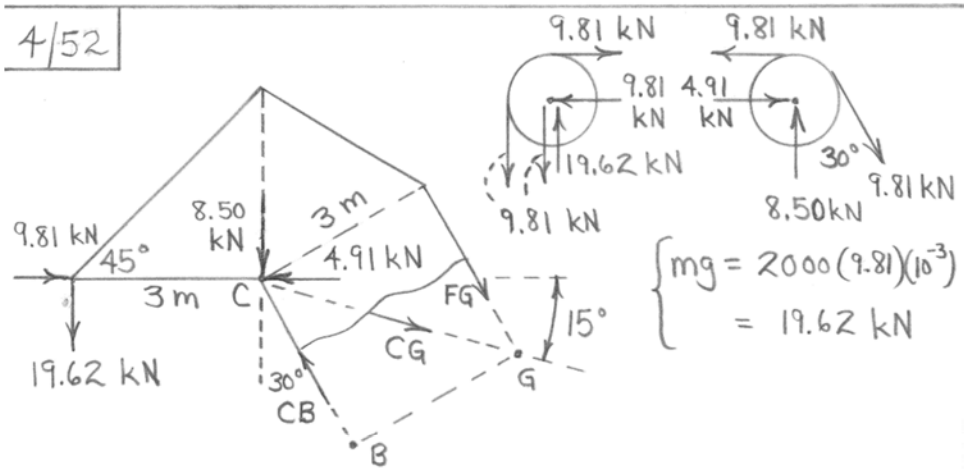
$\Sigma F_x = 0: \ A_x = 0$



$$\Sigma F_y = 0: \ 3L - 2L - \frac{L}{2} - DQ\sin\left(\tan^{-1}\frac{3.5}{2}\right) = 0, \ \underline{DQ = 0.576L \ T}$$

By inspection: $\underline{CQ = 0}$

4/52



$$\left\{ \begin{aligned} mg &= 2000(9.81)(10^{-3}) \\ &= 19.62 \text{ kN} \end{aligned} \right.$$

$\circlearrowleft + \Sigma M_C = 0: \quad 19.62(3) - FG(3) = 0, \quad \underline{FG = 19.62 \text{ kN } T}$

$\circlearrowleft + \Sigma M_G = 0: \quad 19.62(3 + 3\sqrt{2}\cos 15^\circ) - (9.81 - 4.91)(3\sqrt{2}\sin 15^\circ)$

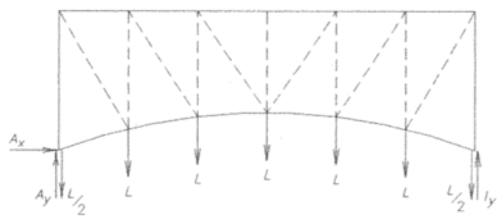
$\qquad + 8.50(3\sqrt{2}\cos 15^\circ) - CB(3) = 0$

$\qquad \underline{CB = 56.2 \text{ kN } C}$

$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad CG\cos 15^\circ + 19.62\sin 30^\circ - 56.2\sin 30^\circ$

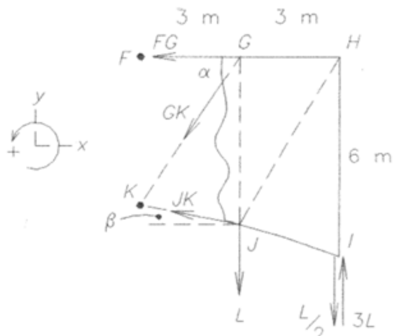
$\qquad + 9.81 - 4.91 = 0, \quad \underline{CG = 13.87 \text{ kN } T}$

4/53



By symmetry, $A_y = H_y = 3L$

$\Sigma F_x = 0: \; A_x = 0$



Origin at center of arc

Location of I: $y_I^2 = 25^2 - 9^2$, $B = (9, 23.3)$ m
Location of J: $y_J^2 = 25^2 - 6^2$, $J = (6, 24.3)$ m
Location of G: $y_G = y_I + 6$, $G = (6, 29.3)$ m
Location of K: $y_K^2 = 25^2 - 3^2$, $K = (3, 24.8)$ m

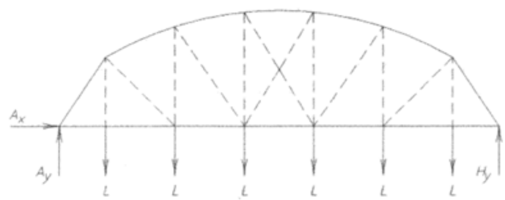
$$\alpha = \tan^{-1}\frac{29.3 - 24.8}{3} = 56.3°, \; \beta = \tan^{-1}\frac{24.8 - 24.3}{3} = 10.39°$$

$$\Sigma M_K = 0: \; 3L(6) - \frac{L}{2}(6) + FG(y_G - y_K) - L(3) = 0, \; FG = -2.66L \; (C)$$

$$\begin{cases} \Sigma F_x = 0: \; 2.66L - GK\cos 56.3° - JK\cos 10.39° = 0 \\ \Sigma F_y = 0: \; \dfrac{3}{2}L + JK\sin 10.39° - GK\sin 56.3° \end{cases}$$

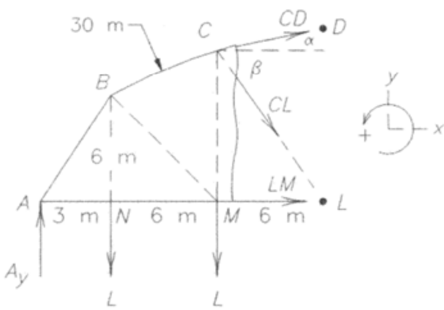
Solve simultaneously to obtain: $\underline{GK = 2.13L \; T}$

4/54



By symmetry, $A_y = H_y = 3L$

$\Sigma F_x = 0: \ A_x = 0$



Origin at center of arc

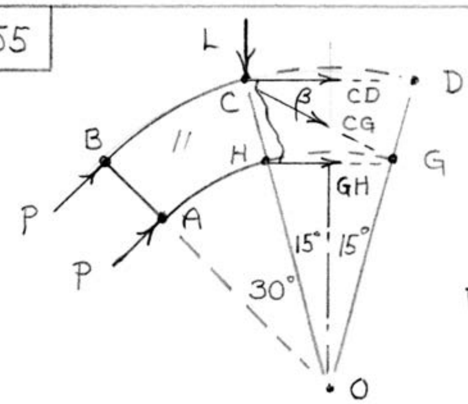
Location of B: $y_B^2 = 30^2 - (-15)^2$, $B = (-15, 26.0)$ m
Location of A: $y_A = y_B - 6$, $A = (-18, 20.0)$ m
Location of C: $y_C^2 = 30^2 - (-9)^2$, $C = (-9, 28.6)$ m
Location of M: $y_M = y_A$, $M = (-9, 20.0)$ m
Location of D: $y_D^2 = 30^2 - (-3)^2$, $D = (-3, 29.8)$ m
Location of L: $y_L = y_A$, $L = (-3, 20.0)$ m

$$\alpha = \tan^{-1}\frac{29.8 - 28.6}{6} = 11.60°, \beta = \tan^{-1}\frac{28.6 - 20.0}{6} = 55.2°$$

$\Sigma M_L = 0: \ -3L(15) + L(6) + L(12) - CD(29.8 - 20.0)\sin(90° - 11.60°) = 0$
$CD = -2.79L$ or $CD = 2.79L\ C$

$\Sigma F_y = 0: \ 3L - L - L - 2.79L\sin 11.60° - CL\sin 55.2° = 0$, $\underline{CL = 0.534L\ T}$

4/55



(Radii: R, 0.75R)

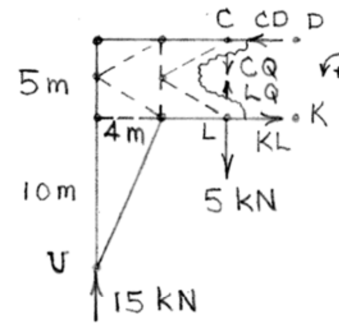
$P = 0.707L$, from analysis of entire truss.

From Prob. 4/30, $\beta = 28.1°$

$+\uparrow \Sigma F = 0: \quad 2P \sin 45° - CG \sin\beta - L = 0$

$\underline{CG = 0}$

▸4/56 From truss as a whole, $\begin{cases} U = 15 \text{ kN} \\ V = 20 \text{ kN} \end{cases}$



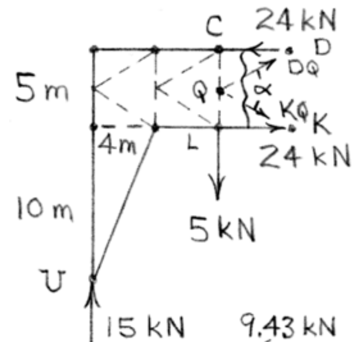
$\circlearrowleft\!+ \sum M_C = 0: \; KL(5) - 15(8) = 0$

$KL = 24 \text{ kN } T$

$\sum M_L = 0: \; CD(5) - 15(8) = 0$

$CD = 24 \text{ kN } C$

(From a similar right-hand section, $DE = 32$ kN C.)

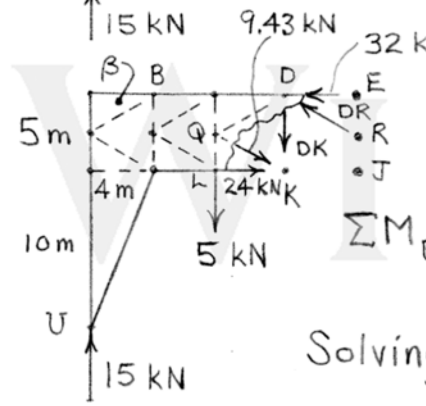


$\sum M_D = 0: \; -15(12) + 5(4)$

$+ KQ\left(\sqrt{4^2 + 2.5^2}\, \sin\alpha\right) + 24(5) = 0$

where $\alpha = 180 - 2\tan^{-1}\left(\frac{4}{2.5}\right) = 64.0°$

Solving, $KQ = 9.43$ kN T



$\beta = \tan\left(\frac{2.5}{4}\right) = 32.0°$

$\sum F_x = 0: \; -32 + 9.43 \cos\beta$

$+ 24 - DR\cos\beta = 0, \quad DR = 0$

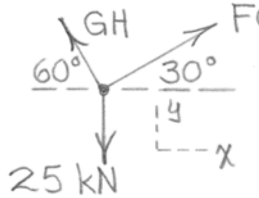
$\sum M_B = 0: \; -15(4) - 5(4) + 24(5)$

$- DK(8) = 0$

Solving, $\underline{DK = 5 \text{ kN } T}$

▶4/57 $\underline{FI = 0}$, by inspection of joint I.

Joint G:



$\Sigma F_x = 0: -GH\left(\frac{1}{2}\right) + FG\left(\frac{\sqrt{3}}{2}\right) = 0$

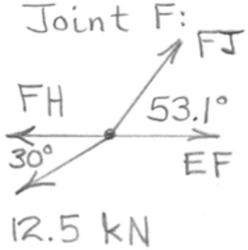
$\Sigma F_y = 0: GH\left(\frac{\sqrt{3}}{2}\right) + FG\left(\frac{1}{2}\right) - 25 = 0$

1st eq.: $GH = \sqrt{3}\,FG$

2nd eq.: $\sqrt{3}\,FG\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}FG = 25$

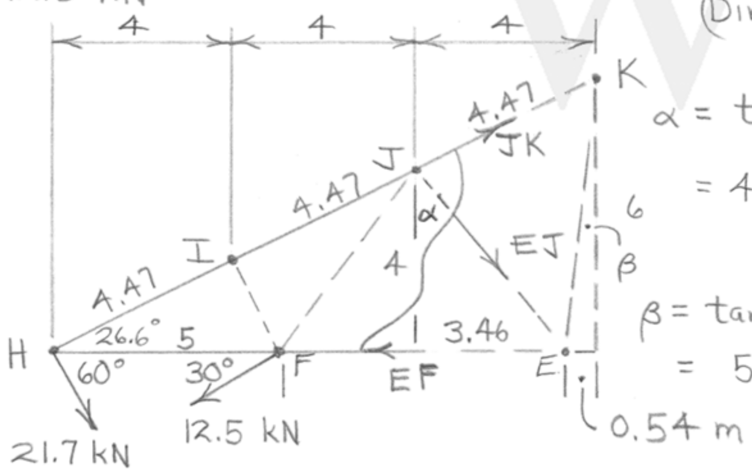
$\Rightarrow FG = 12.5$ kN T, $GH = 21.7$ kN T

Joint F:



$\Sigma F_y = 0: FJ(\sin 53.1°) - 12.5 \sin 30° = 0$

$\underline{FJ = 7.81 \text{ kN T}}$



(Dim. in m)

$\alpha = \tan^{-1}\frac{3.46}{4} = 40.9°$

$\beta = \tan^{-1}\frac{0.54}{6} = 5.14°$

$\circlearrowleft + \Sigma M_H = 0: -12.5\left(\frac{1}{2}\right)(5) - EJ\left[\cos 40.9°(8) + \sin 40.9°(4)\right] = 0$, $EJ = -3.61$ kN

So $\underline{EJ = 3.61 \text{ kN C}}$

$\Sigma F_y = 0: JK \sin 26.6° + 3.61 \cos 40.9° - 12.5\left(\frac{1}{2}\right) - 21.7\left(\frac{\sqrt{3}}{2}\right) = 0$, $JK = 49.8$ kN T

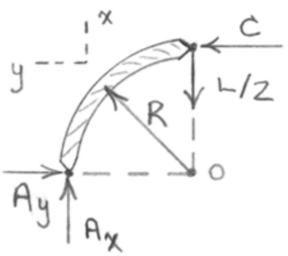
Joint K, using symmetry:

$\Sigma F_y = 0: -2(49.8)\sin 26.6° + 2\,EK \cos 5.14° = 0$,

$\underline{EK = 22.4 \text{ kN C}}$

By symmetry of loads L and $\Sigma M_K = 0$, $\underline{ER = 0}$

▶4/58 By symmetry, the force which the right half exerts on the left half at C is horizontal:

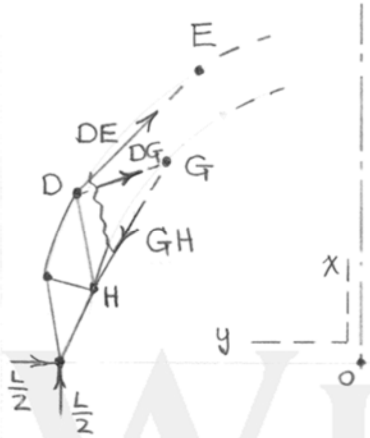


$\Sigma M_A = 0: \; CR - \frac{L}{2}R = 0$

$C = L/2$

$\Sigma F_y = 0: \; -A_y + \frac{L}{2} = 0, \; A_y = \frac{L}{2}$

$\Sigma F_x = 0: \; A_x - \frac{L}{2} = 0, \; A_x = \frac{L}{2}$



$\underline{r}_{OD} + \underline{r}_{DE} = \underline{r}_{OE}$

$\therefore \underline{r}_{DE} = \underline{r}_{OE} - \underline{r}_{OD}$

$= 1.1R(\cos 30° \underline{i} + \sin 30° \underline{j})$

$\quad - 1.1R(\cos 60° \underline{i} + \sin 60° \underline{j})$

$= R(0.403 \underline{i} - 0.403 \underline{j})$

So force $\underline{DE} = DE \dfrac{\underline{r}_{DE}}{r_{DE}}$

$= DE(0.707 \underline{i} - 0.707 \underline{j})$

Similarly, force $\underline{GH} = GH(-0.866 \underline{i} + 0.500 \underline{j})$

force $\underline{DG} = DG(0.264 \underline{i} - 0.965 \underline{j})$

$\Sigma F_x = 0: \; \frac{L}{2} + 0.707\,DE - 0.866\,GH + 0.264\,DG = 0 \quad (1)$

$\Sigma F_y = 0: \; -\frac{L}{2} - 0.707\,DE + 0.500\,GH - 0.965\,DG = 0 \quad (2)$

$\Sigma M_O = 0: \; -\frac{L}{2}R\underline{k} + \underline{r}_{OD} \times (\underline{DE} + \underline{DG})$

$+ \underline{r}_{OH} \times \underline{GH},$

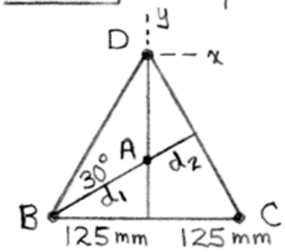
where $\underline{r}_{OH} = 0.9R(\cos 75° \underline{i} + \sin 75° \underline{j})$

Carrying out the cross products and collecting terms: $-1.063\,DE + 0.869\,GH - 0.782\,DG = \frac{L}{2} \quad (3)$

Simultaneous solution of Eqs. (1) – (3):

$DE = 0.839L\ T, \; GH = 1.090L\ C, \; \underline{DG = -0.569L\ (C)}$

4/59 Top view of base:



$\cos 30° = \dfrac{d_1 + d_2}{250}$, $d_1 + d_2 = 216.5$

$\cos 30° = \dfrac{125}{d_1}$, $d_1 = 144.3$ mm

$d_2 = 216.5 - 144.3 = 72.17$ mm

For joint A, assuming symmetry:

$\underline{F}_{BA} = P\left[\dfrac{125\underline{i} + 72.17\underline{j} + 400\underline{k}}{(125^2 + 72.17^2 + 400^2)^{1/2}}\right] = P(0.294\underline{i} + 0.170\underline{j} + 0.941\underline{k})$

$\underline{F}_{CA} = P(-0.294\underline{i} + 0.170\underline{j} + 0.941\underline{k})$, $\underline{F}_{DA} = P(-0.339\underline{j} + 0.941\underline{k})$

$\Sigma F_z = 0$ at A: $3(0.941P) - 4 = 0$, $P = 1.417$ kN

For joint C, assuming symmetry:

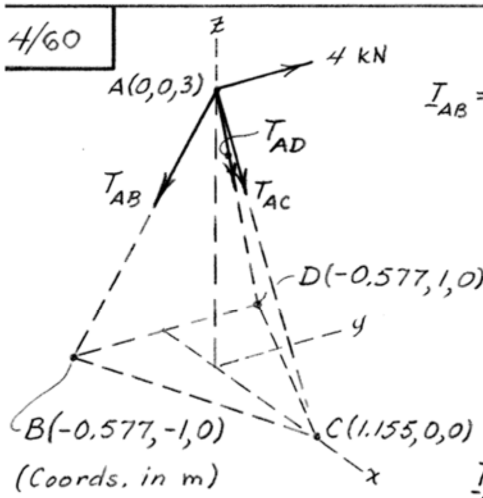
$\underline{F}_{BC} = -Q\underline{i}$, $\underline{F}_{CD} = Q(-\sin 30°\underline{i} + \cos 30°\underline{j})$

Normal $\underline{N} = 1.333\underline{k}$ kN

$\Sigma \underline{F} = \underline{0}$ at C: $\underline{N} + \underline{F}_{BC} + \underline{F}_{CD} + \underline{F}_{AC} = \underline{0}$

$1.333\underline{k} - Q\underline{i} + Q(-0.5\underline{i} + 0.866\underline{j}) + 1.417(0.294\underline{i} - 0.170\underline{j} - 0.941\underline{k}) = \underline{0}$, $Q = 0.278$ kN $\Rightarrow$ $\underline{BC = BD = CD = 0.278 \text{ kN } T}$

4/60



$$\mathbf{T}_{AB} = T_{AB}\frac{-0.577\mathbf{i}-\mathbf{j}-3\mathbf{k}}{\sqrt{0.577^2+1^2+3^2}}$$

$$= T_{AB}\left[-0.1796\mathbf{i}-0.311\mathbf{j}-0.933\mathbf{k}\right]$$

$$\mathbf{T}_{AC} = T_{AC}\frac{1.155\mathbf{i}-3\mathbf{k}}{\sqrt{1.155^2+3^2}}$$

$$= T_{AC}\left[0.359\mathbf{i}-0.933\mathbf{k}\right]$$

$$\mathbf{T}_{AD} = T_{AD}\frac{-0.577\mathbf{i}+\mathbf{j}-3\mathbf{k}}{\sqrt{0.577^2+1^2+3^2}}$$

$$= T_{AD}\left[-0.1796\mathbf{i}+0.311\mathbf{j}-0.933\mathbf{k}\right]$$

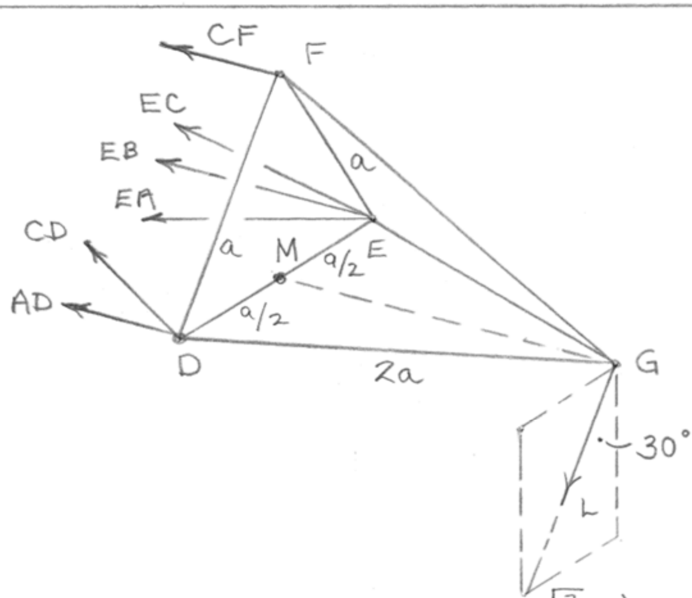
$\Sigma F_x = 0: -0.1796\,T_{AB} + 0.359\,T_{AC} - 0.1796\,T_{AD} = 0$ (1)

$\Sigma F_y = 0: \;4 - 0.311\,T_{AB} + 0.311\,T_{AD} = 0$ (2)

$\Sigma F_z = 0: -0.933\,T_{AB} - 0.933\,T_{AC} - 0.933\,T_{AD} = 0$ (3)

Solve Eqs. (1)-(3):
$$\begin{cases} T_{AB} = 6.43 \text{ kN } (T) \\ T_{AC} = 0 \\ T_{AD} = -6.43 \text{ kN } (C) \end{cases}$$

4/61

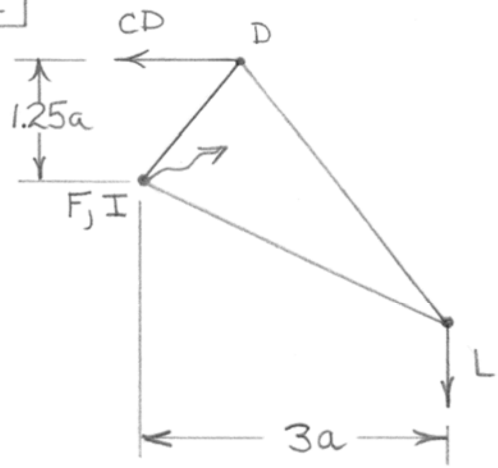


$\Sigma M_{ED} = 0: -L\cos 30°\,(\overline{GM}) + CF\left(\frac{\sqrt{3}}{2}a\right) = 0$

With $\overline{GM} = \sqrt{4a^2 - \frac{a^2}{4}} = 1.936a$, we obtain

$$\underline{CF = 1.936L \;\; T}$$

4/62



$$\circlearrowleft + \sum M_F = 0: \quad CD(1.25a) - L(3a) = 0$$

$$\underline{CD = 2.4L \quad T}$$

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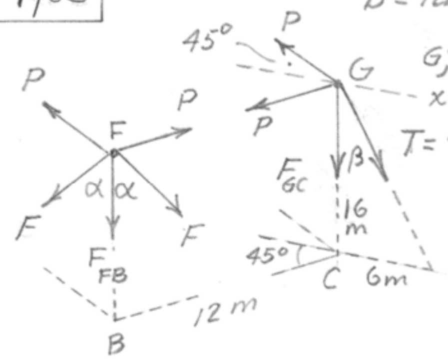

$\beta = \tan^{-1}\frac{6}{16} = 20.6°$, $\sin\beta = 0.351$

G; $\Sigma F_x = 0$; $9(0.351) - 2P(0.707) = 0$

$P = 2.235$ kN

F; $\sin\alpha = 3/5 = 0.6$

$\Sigma F_{FG} = 0$; $P + F\sin\alpha = 0$

$F = -2.235/0.6 = \underline{-3.72 \text{ kN}}$

(C)

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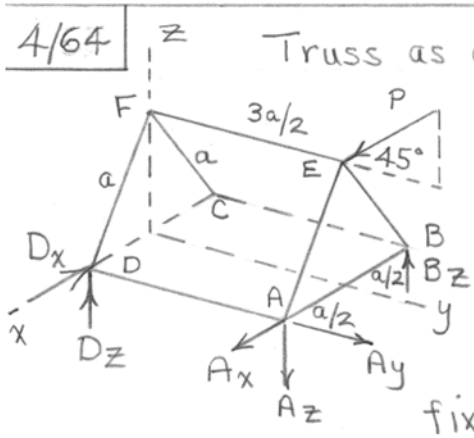


Truss as a whole is statically determinate with 6 supporting constraints.

$j=6$, $m=12$, $3j=m+6$ so sufficient members for stability. A, B & D are fixed so F is fixed. E & C are also fixed, so truss is a rigid unit.

$\Sigma M_{A_z}=0: \frac{P}{\sqrt{2}}\frac{a}{2} - D_x\frac{3a}{2}=0$, $D_x = \frac{P}{3\sqrt{2}}$, $A_x = \frac{P}{3\sqrt{2}}$

$\Sigma M_{AB}=0: \frac{P}{\sqrt{2}}\frac{a\sqrt{3}}{2} - D_z\frac{3a}{2}=0$, $D_z = P/\sqrt{6}$

$\Sigma M_{AD}=0: \frac{P}{\sqrt{2}}\frac{a}{2} - B_z a = 0$, $B_z = \frac{P}{2\sqrt{2}}$

$\Sigma F_z = 0$ gives $A_z = \frac{2-\sqrt{3}}{2\sqrt{6}}P$

Forces at C are all zero. From joint E,

$\Sigma F_y = 0$ gives $EF = P/\sqrt{2}$ C

Joint F:

FC=0 $EF = \frac{P}{\sqrt{2}}$

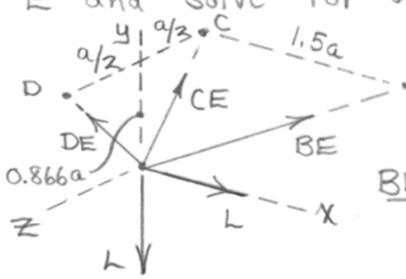
$$\begin{cases} \underline{AF} = AF(\underline{i}+3\underline{j}-\sqrt{3}\underline{k})/\sqrt{13} \\ \underline{BF} = BF(-\underline{i}+3\underline{j}-\sqrt{3}\underline{k})/\sqrt{13} \\ \underline{DF} = DF(\underline{i}-\sqrt{3}\underline{k})/2, \quad \underline{EF} = -\frac{P}{\sqrt{2}}\underline{j} \end{cases}$$

$$\Sigma \underline{F} = \underline{0} = \left(\frac{AF}{\sqrt{13}} - \frac{BF}{\sqrt{13}} + \frac{DF}{2}\right)\underline{i} + \left(-\frac{P}{\sqrt{2}} + \frac{3AF}{\sqrt{13}} + \frac{3BF}{\sqrt{13}}\right)\underline{j} + \left(-\frac{\sqrt{3}}{\sqrt{13}}AF - \frac{\sqrt{3}}{\sqrt{13}}BF - \frac{\sqrt{3}}{2}DF\right)\underline{k}$$

Solve to obtain $BF = 0$, $DF = -\frac{\sqrt{2}}{3}P$

$$\underline{AF = \frac{\sqrt{13}}{3\sqrt{2}}P}$$

4/65 The truss as a whole is statically determinate with six supporting constraints. $j = 6$ & $m = 12$; $3j = m + 6$, so there are sufficient members for stability. C, B, and D are fixed so E is fixed. A and F are also fixed, so the truss is a rigid unit. From an inspection of joint F, $AF = 0$, $\underline{BF = 0}$, $EF = L\ T$. From an inspection of joint A, $AB = AD = \underline{AE = 0}$. We can now go to joint E and solve for all unknowns there:



$$\underline{CE} = CE[0.866\underline{j} - 0.5\underline{k}]$$

$$\underline{DE} = DE[0.866\underline{j} + 0.5\underline{k}]$$

$$\underline{BE} = BE\left[\frac{1.5\underline{i} + 0.866\underline{j} - 0.5\underline{k}}{\sqrt{1.5^2 + 0.866^2 + 0.5^2}}\right] = BE[0.832\underline{i} + 0.480\underline{j} - 0.277\underline{k}]$$

$$\left.\begin{aligned} \Sigma F_x = 0&: \ 0.832BE + L = 0 \\ \Sigma F_y = 0&: \ 0.480BE + 0.866CE + 0.866DE - L = 0 \\ \Sigma F_z = 0&: \ -0.277BE - 0.5CE + 0.5DE = 0 \end{aligned}\right\}$$

Solution: $\underline{BE = -1.202L\ (C)}$, $\underline{CE = 1.244L\ T}$

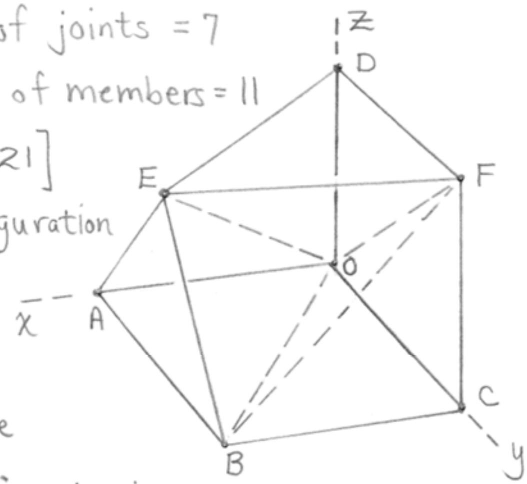
$DE = 0.577L\ T$

4/66 j = number of joints = 7

m_i = initial number of members = 11

$[m_i + 6 = 17] < [3j = 21]$

So the initial configuration lacks $21 - 17 = \underline{4}$ members for internal stabity. A stable configuration is achieved by



(1) Adding OB & OE to produce the rigid tetrahedron ABEO.

(2) Adding OF to produce the rigid tetrahedron ODEF.

(3) Adding BF to produce the rigid tetrahedrons OBCF and OBEF.

With 4 new members, $m = 15$ and $m + 6 = 21$. The number of joints remains $j = 7$; $3j = 21$.

So $m + 6 = 3j$; sufficient number of members now present.

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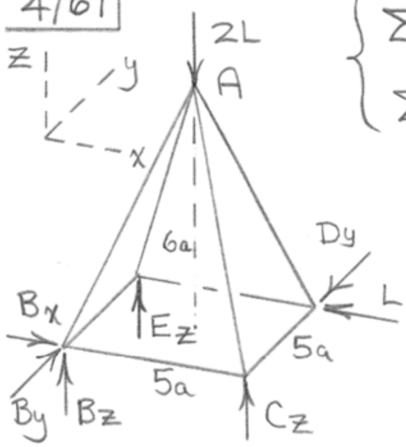
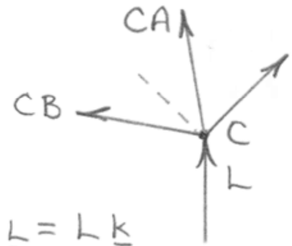


$$\begin{cases} \Sigma M_{BE} = 0 \Rightarrow C_z = L \\ \Sigma M_{B_z} = 0 \Rightarrow D_y = L \end{cases}$$

Joint C: (tensions assumed)



$\underline{CB} = -CB\,\underline{i}$, $\underline{CD} = CD\,\underline{j}$, $\underline{L} = L\,\underline{k}$

$$\underline{CA} = CA\left(\frac{-2.5a\underline{i} + 2.5a\underline{j} + 6a\underline{k}}{\sqrt{(2.5^2 + 2.5^2 + 6^2)a^2}}\right) = CA(-0.359\underline{i} + 0.359\underline{j} + 0.862\underline{k})$$

$\Sigma \underline{F} = \underline{0}$ yields:

$$\left.\begin{array}{ll} \underline{i}: & -CB - 0.359\,CA = 0 \\ \underline{j}: & CD + 0.359\,CA = 0 \\ \underline{k}: & L + 0.862\,CA = 0 \end{array}\right\} \begin{array}{l} CA = -0.1667L \\ CD = +0.417L \end{array}$$

Joint D:

$\Sigma \underline{F} = \underline{0}$ yields

$$\begin{cases} \underline{i}: -DE - L - 0.359\,DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{j}: -0.417L - L - 0.359\,DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{k}: 0.862\,DA = 0 \end{cases}$$

$DA = 0$, $\underline{DB = -2.00L}$ (C)

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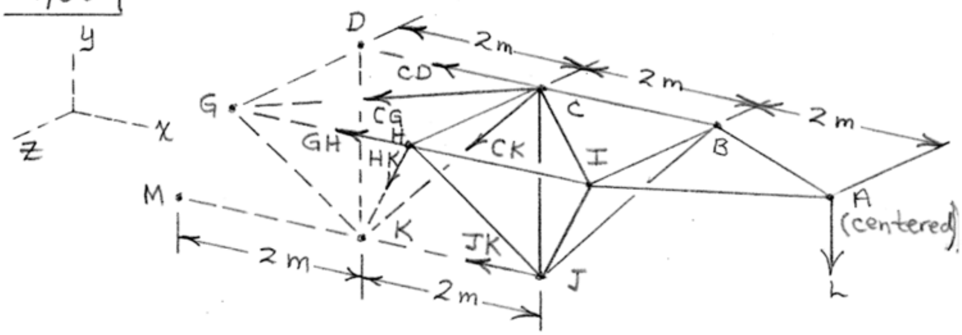

Six members cut; all assumed to be in tension.

$\underline{GH} = -GH\underline{i}$, $\underline{CD} = -CD\underline{i}$, $\underline{CG} = \frac{CG}{\sqrt{2}}(-\underline{i}+\underline{k})$

$\Sigma \underline{M}_K = 0: \; -5L\underline{k} + (\underline{i} + 2\underline{j} + \underline{k}) \times (-GH\underline{i})$
$+ (\underline{i} + 2\underline{j} - \underline{k}) \times \left[-CD\underline{i} + \frac{CG}{\sqrt{2}}(-\underline{i}+\underline{k})\right] = \underline{0}$

Carry out cross products, equate coefficients of like unit vectors, and solve the three scalar equations to obtain

$GH = 1.25L$ T $\qquad CG = 0$

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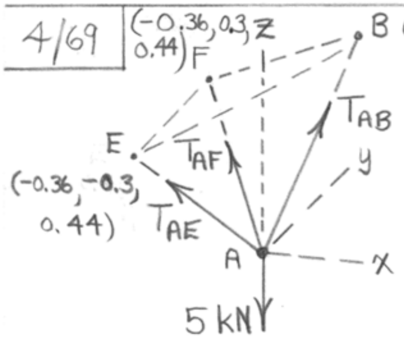


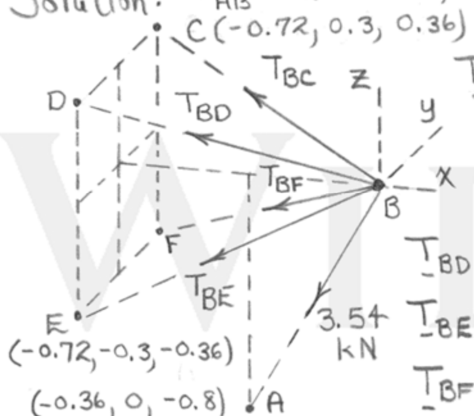
$$\underline{T}_{AB} = T_{AB}\frac{0.36\underline{i} + 0.8\underline{k}}{\sqrt{0.36^2 + 0.8^2}} = T_{AB}\left[0.410\underline{i} + 0.912\underline{k}\right]$$

$$\underline{T}_{AE} = T_{AE}\frac{-0.36\underline{i} - 0.3\underline{j} + 0.44\underline{k}}{\sqrt{0.36^2 + 0.3^2 + 0.44^2}} = T_{AE}\left[-0.560\underline{i} - 0.467\underline{j} + 0.684\underline{k}\right]$$

$$\underline{T}_{AF} = T_{AF}\frac{-0.36\underline{i} + 0.3\underline{j} + 0.44\underline{k}}{\sqrt{0.36^2 + 0.3^2 + 0.44^2}} = T_{AF}\left[-0.560\underline{i} + 0.467\underline{j} + 0.684\underline{k}\right]$$

$$\left.\begin{aligned}
\Sigma F_x = 0&: \ 0.410T_{AB} - 0.560T_{AE} - 0.560T_{AF} = 0\\
\Sigma F_y = 0&: \ -0.467T_{AE} + 0.467T_{AF} = 0\\
\Sigma F_z = 0&: \ 0.912T_{AB} + 0.684T_{AE} + 0.684T_{AF} - 5 = 0
\end{aligned}\right\}$$

Solution: $T_{AB} = 3.54$ kN, $T_{AE} = T_{AF} = 1.296$ kN



$$\underline{T}_{BC} = T_{BC}\frac{-0.72\underline{i} + 0.3\underline{j} + 0.36\underline{k}}{\sqrt{0.72^2 + 0.3^2 + 0.36^2}} = T_{BC}\left[-0.838\underline{i} + 0.349\underline{j} + 0.419\underline{k}\right]$$

$$\underline{T}_{BD} = T_{BD}\left[-0.838\underline{i} - 0.349\underline{j} + 0.419\underline{k}\right]$$

$$\underline{T}_{BE} = T_{BE}\left[-0.838\underline{i} - 0.349\underline{j} - 0.419\underline{k}\right]$$

$$\underline{T}_{BF} = T_{BF}\left[-0.838\underline{i} + 0.349\underline{j} - 0.419\underline{k}\right]$$

Note: $T_{BD} = T_{BC}$; $T_{BE} = T_{BF}$ (Symmetry)

Set $\Sigma \underline{F} = \underline{0}$ to obtain $T_{BD} = T_{BC} = 1.491$ kN

$$\underline{T_{BE} = T_{BF} = -2.36 \text{ kN (C)}}$$

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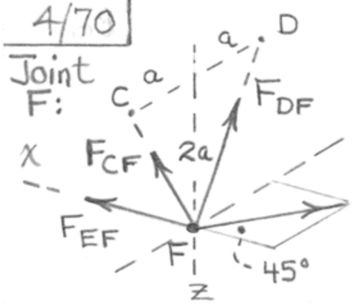
Joint F:



$$\underline{L} = \frac{L}{\sqrt{2}}(-\underline{i}+\underline{j}), \quad \underline{F}_{EF} = F_{EF}\,\underline{i}$$

$$\underline{F}_{CF} = \frac{F_{CF}}{\sqrt{5}}(-\underline{j}-2\underline{k})$$

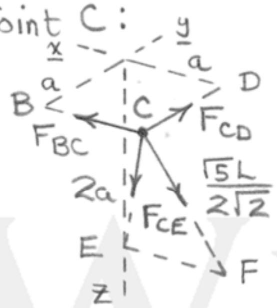
$$\underline{F}_{DF} = \frac{F_{DF}}{\sqrt{5}}(\underline{j}-2\underline{k})$$

$$\Sigma \underline{F} = \underline{L} + \underline{F}_{EF} + \underline{F}_{CF} + \underline{F}_{DF} = \underline{0}$$

$$\underline{i}: \ -\frac{L}{\sqrt{2}} + F_{EF} = 0, \qquad F_{EF} = \frac{L}{\sqrt{2}}$$

$$\left.\begin{aligned} \underline{j}&: \ \frac{L}{\sqrt{2}} - \frac{F_{CF}}{\sqrt{5}} + \frac{F_{DF}}{\sqrt{5}} = 0 \\ \underline{k}&: \ -\frac{2}{\sqrt{5}}F_{CF} - \frac{2}{\sqrt{5}}F_{DF} = 0 \end{aligned}\right\} \quad F_{CF} = \frac{\sqrt{5}\,L}{2\sqrt{2}}, \quad F_{DF} = -\frac{\sqrt{5}\,L}{2\sqrt{2}}$$

Joint C:



$$\underline{F}_{CD} = F_{CD}\,\underline{j}, \quad \underline{F}_{BC} = F_{BC}\,\underline{i}$$

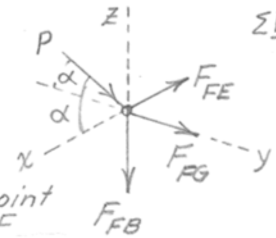
$$\underline{F}_{CE} = F_{CE}(\underline{i}+\underline{j}+2\underline{k})/\sqrt{6}$$

$$\underline{F}_{CF} = F_{CF}(\underline{j}+2\underline{k})/\sqrt{5} = \frac{L}{2\sqrt{2}}(\underline{j}+2\underline{k})$$

$\Sigma F = 0$ yields:

$$\left.\begin{aligned} \underline{i}&: \ 0 + F_{BC} + \frac{F_{CE}}{\sqrt{6}} = 0 \\ \underline{j}&: \ F_{CD} + \frac{F_{CE}}{\sqrt{6}} + \frac{L}{2\sqrt{2}} = 0 \\ \underline{k}&: \ \frac{2F_{CE}}{\sqrt{6}} + \frac{L}{\sqrt{2}} = 0 \end{aligned}\right\} \quad \begin{aligned} &\underline{F_{BC} = \frac{L\sqrt{2}}{4}} \\ &\underline{F_{CD} = 0} \\ &\underline{F_{CE} = -\frac{L\sqrt{3}}{2}\ (C)} \end{aligned}$$

▸4/71 $\alpha = \cos^{-1}\frac{1}{\sqrt{3}}$; By symmetry $F_{FE} = F_{FG} = F_{FB} = F$



Joint F

$\Sigma \underline{F} = \underline{0}$; $F(-\underline{i}+\underline{j}-\underline{k}) + \frac{P}{\sqrt{3}}(-\underline{i}+\underline{j}-\underline{k}) = \underline{0}$

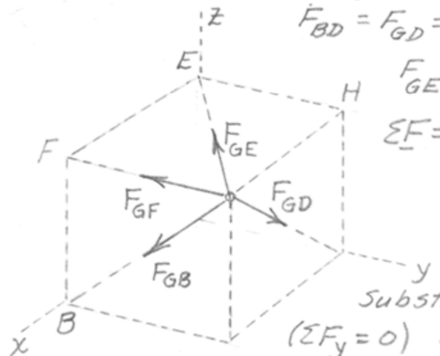
$F = -P/\sqrt{3}$ so $\underline{F_{FE} = -P/\sqrt{3}}$ (compression)

By inspection, forces on joints A, C, & H are zero, and by symmetry forces at D are $F_{BD} = F_{GD} = F_{ED} = R$. Also by symmetry

$F_{GE} = F_{GB}$; $\underline{F_{GF}} = F_{GF}(-\underline{j}) = -\frac{P}{\sqrt{3}}(-\underline{j})$

$\Sigma \underline{F} = \underline{0}$; $-\frac{P}{\sqrt{3}}(-\underline{j}) + \frac{F_{GE}}{\sqrt{2}}(-\underline{i}-\underline{j})$

$+ \frac{F_{GB}}{\sqrt{2}}(-\underline{j}-\underline{k}) + \frac{F_{GD}}{\sqrt{2}}(-\underline{i}-\underline{k}) = \underline{0}$



Joint G

Substitute $F_{GB} = F_{GE}$, collect j-terms ($\Sigma F_y = 0$) & get $\left(\frac{P}{\sqrt{3}} - \frac{2F_{GE}}{\sqrt{2}}\right) = 0$

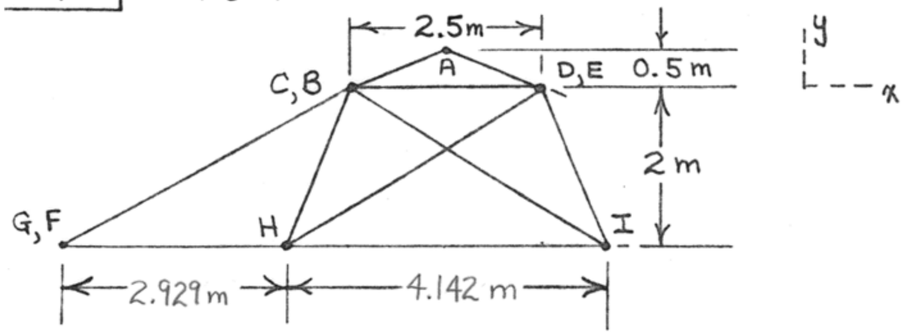
$\underline{F_{EG} = P/\sqrt{6}}$ (tension)

No. of members is $m = 18$

No. of joints is $j = 8$

$(m + 6 = 24) = (3j = 24)$ so internally stable

▶4/72 Partial front view:



For equilibrium of joint A, force vectors are

$\underline{L} = -L\underline{j}$

$\underline{BA} = P\left[\dfrac{1.25\underline{i} + 0.5\underline{j} + 1.25\underline{k}}{\sqrt{2(1.25)^2 + (0.5)^2}}\right] = P(0.680\underline{i} + 0.272\underline{j} + 0.680\underline{k})$

Similarly,

$\underline{CA} = P(0.680\underline{i} + 0.272\underline{j} - 0.680\underline{k})$

$\underline{DA} = P(-0.680\underline{i} + 0.272\underline{j} - 0.680\underline{k})$

$\underline{EA} = P(-0.680\underline{i} + 0.272\underline{j} + 0.680\underline{k})$

where P is the force in the 4 members joined at A, all of which are assumed to be in compression.

$\Sigma F_y = 0$ at A: $4P(0.272) - L = 0$, $P = 0.919L$

For equilibrium of joint B, force vectors are

$\underline{BC} = -Q\underline{k}$, $\underline{CD} = Q\underline{i}$

$\underline{AC} = 0.919L(-0.680\underline{i} - 0.272\underline{j} + 0.680\underline{k})$

$\underline{CF} = R\left[\dfrac{-(5-1.25)\underline{i} - 2\underline{j} - (4.142/2 + 2.5/2)\underline{k}}{\sqrt{(3.75)^2 + (2)^2 + (3.321)^2}}\right]$

$= R(-0.695\underline{i} - 0.371\underline{j} - 0.616\underline{k})$

Similarly,

$\underline{CG} = S(-0.866\underline{i} - 0.462\underline{j} + 0.190\underline{k})$

$\underline{CH} = S(-0.190\underline{i} - 0.462\underline{j} + 0.866\underline{k})$

$\underline{CI} = R(0.616\underline{i} - 0.371\underline{j} + 0.695\underline{k})$

where Q, R, and S are force magnitudes and where all unknowns are assumed in tension.

$\Sigma \underline{F} = \underline{0}$ at joint B: $\underline{AC} + \underline{BC} + \underline{CD} + \underline{CF} + \underline{CG} + \underline{CH} + \underline{CI} = \underline{0}$, or

$[(0.919L)(-0.680) + Q - 0.695R - 0.866S - 0.190S + 0.616R]\underline{i}$

$+[(0.919L)(-0.272) - 0.371R - 0.462S - 0.462S - 0.371R]\underline{j}$

$+[-Q + (0.919L)(0.680) - 0.616R + 0.190S + 0.866S + 0.695R]\underline{k}$

$= \underline{0}$ (note dependency between $\underline{i}$ & $\underline{k}$ components!)

With $Q = 0.3L$, solve x- and y-equations to obtain $R = 0.051L$, $S = -0.312L$

$\therefore$ $\underline{CF = 0.051L\ T}$ and $\underline{CG = 0.312L\ C}$

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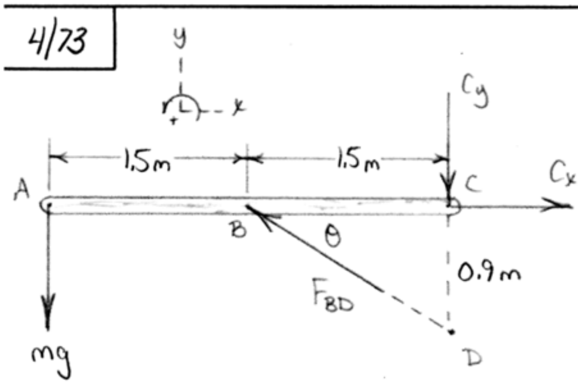
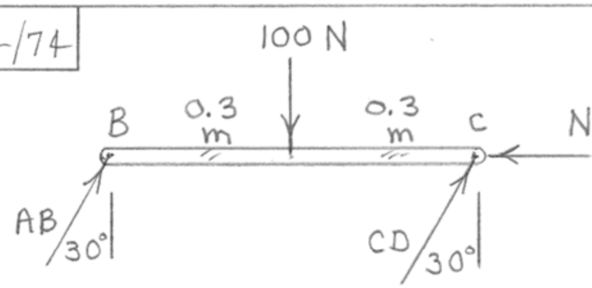


$$\begin{cases} \theta = \tan^{-1}\left(\frac{0.9}{1.5}\right) = 31.0° \\ m = 1000 \text{ kg} \end{cases}$$

$$\begin{cases} \Sigma F_x = 0: & C_x - F_{BD}\cos\theta = 0 \\ \Sigma F_y = 0: & F_{BD}\sin\theta - C_y - mg = 0 \\ \Sigma M_C = 0: & 3mg - 1.5\, F_{BD}\sin\theta = 0 \end{cases} \longrightarrow \begin{cases} C_x = 32.7 \text{ kN} \\ C_y = 9.81 \text{ kN} \\ \underline{F_{BD} = 38.1 \text{ kN}} \end{cases}$$

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{32.7^2 + 9.81^2} \longrightarrow \underline{C = 34.1 \text{ kN}}$$

4/74

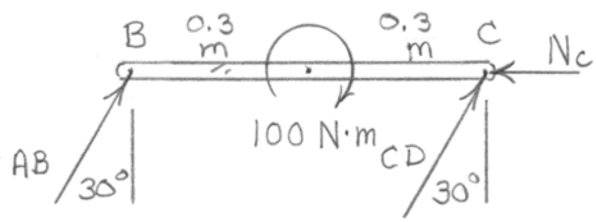


AB and CD are two-force members.

$$\circlearrowleft + \Sigma M_B = 0 : -100(0.3) + CD\frac{\sqrt{3}}{2}(0.6) = 0$$

$$CD = 57.7 \text{ N} \ \angle 60°$$

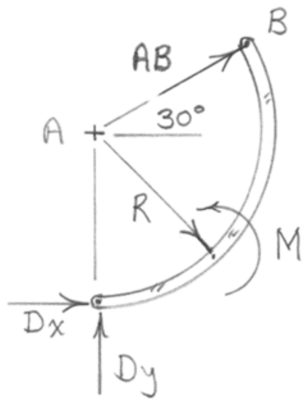
4/75



$$\curvearrowleft + \Sigma M_B = 0: \quad -100 + CD\frac{\sqrt{3}}{2}(0.6) = 0$$

$$\underline{CD = 192.5 \text{ N} \measuredangle 60°}$$

4/76



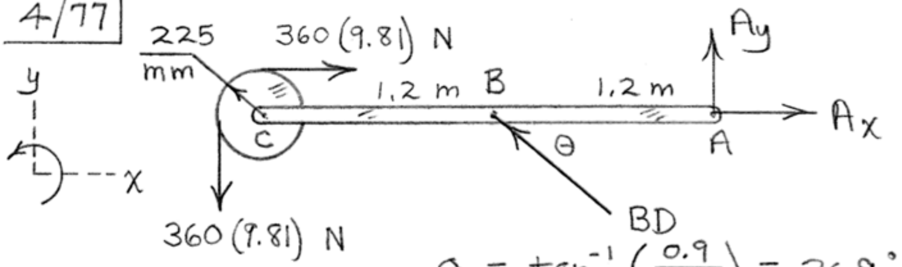
$$\circlearrowleft + \Sigma M_D = 0: \; -AB\cos 30°(R) + M = 0$$

$$AB = \frac{M}{R\cos 30°} = \frac{M}{R\,\sqrt{3}/2} = \frac{2\sqrt{3}\,M}{3R}$$

Load is a couple, so reactions form a couple:

$$A = D = \frac{2\sqrt{3}\,M}{3R}$$

4/77



$\theta = \tan^{-1}\left(\frac{0.9}{1.2}\right) = 36.9°$

$\Sigma M_A = 0: \; 360(9.81)(2.4 + 0.225 - 0.225) - BD(1.2)(0.6) = 0, \quad BD = 11.77 \text{ kN}$

$\Sigma F_x = 0: \; 360(9.81) - 11\,770(0.8) + A_x = 0$

$A_x = 5890 \text{ N}$

$\Sigma F_y = 0: \; -360(9.81) + 11\,770(0.6) + A_y = 0$

$A_y = -3530 \text{ N}$

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{5890^2 + 3530^2}$

$= \underline{6860 \text{ N}}$

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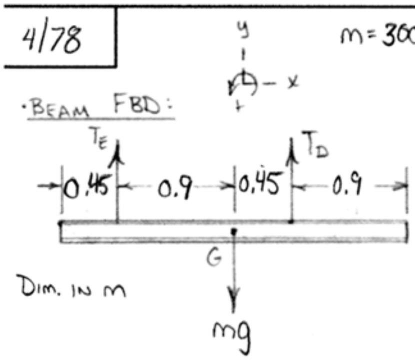
4/78 $m = 3000$ kg

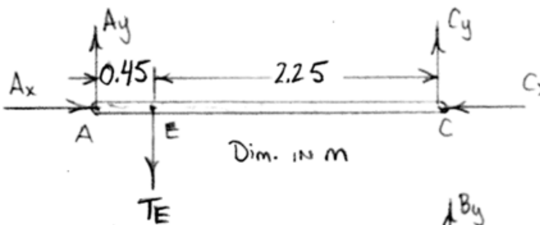
· BEAM FBD:



$$\begin{cases} \Sigma F_y = 0: \; T_E + T_D - mg = 0 \\ \Sigma M_G = 0: \; 0.45\,T_D - 0.9\,T_E = 0 \end{cases}$$

$T_E = 9.81$ kN $\qquad T_D = 19.62$ kN

· BAR AC:

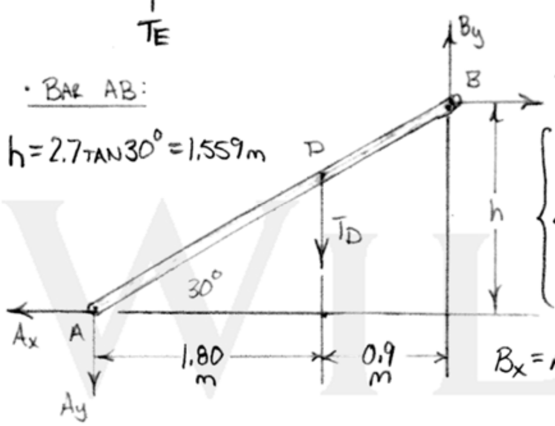


$$\begin{cases} \Sigma F_x = 0: \; A_x - C_x = 0 \\ \Sigma F_y = 0: \; A_y + C_y - T_E = 0 \\ \Sigma M_A = 0: \; 2.7\,C_y - 0.45\,T_E = 0 \end{cases}$$

$A_y = 8.18$ kN $\qquad C_y = 1.635$ kN

· BAR AB:

$h = 2.7\tan 30^\circ = 1.559$ m



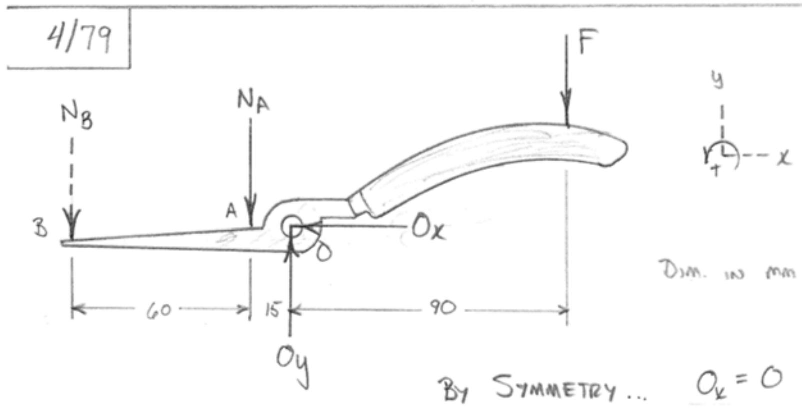
$$\begin{cases} \Sigma F_x = 0: \; B_x - A_x = 0 \\ \Sigma F_y = 0: \; B_y - A_y - T_D = 0 \\ \Sigma M_A = 0: \; 2.7\,B_y - B_x h - 1.8\,T_D = 0 \end{cases}$$

$B_x = A_x = 25.5$ kN $\qquad B_y = 27.8$ kN

$A = \sqrt{A_x^2 + A_y^2} \longrightarrow \underline{A = 26.8 \text{ kN}}$ $\qquad B = \sqrt{B_x^2 + B_y^2} \longrightarrow \underline{B = 37.7 \text{ kN}}$

$C = \sqrt{C_x^2 + C_y^2} \longrightarrow \underline{C = 25.5 \text{ kN}}$

4/79



By Symmetry ... $O_x = 0$

• Cutting Force At A:

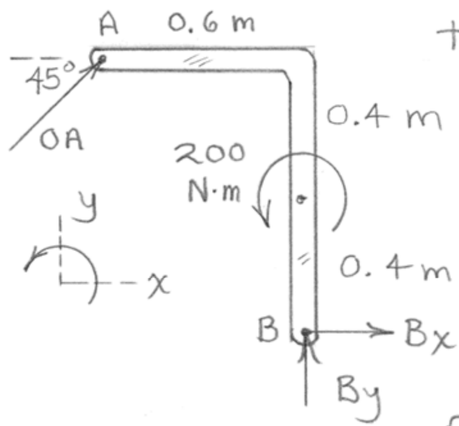
$$\begin{cases} \Sigma M_O = 0: & 15\,N_A - 90\,F = 0 \longrightarrow \underline{N_A = 6F} \\ \Sigma F_y = 0: & O_y - N_A - F = 0 \longrightarrow \underline{O_y = 7F} \end{cases}$$

• Gripping Force At B:

$$\begin{cases} \Sigma M_O = 0: & 75\,N_B - 90\,F = 0 \longrightarrow \underline{N_B = 1.2\,F} \\ \Sigma F_y = 0: & O_y - N_B - F = 0 \longrightarrow \underline{O_y = 2.2\,F} \end{cases}$$

4/80

Note that OA is a two-force member.



$\Sigma M_B = 0: \quad 200 - OA\frac{\sqrt{2}}{2}(0.8) - OA\frac{\sqrt{2}}{2}(0.6) = 0$

$OA = 202 \text{ N}$

$\Sigma F_x = 0: \quad 202\frac{\sqrt{2}}{2} + B_x = 0$

$B_x = -142.9 \text{ N}$

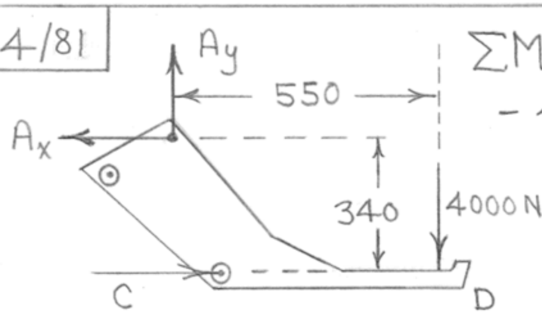
$\Sigma F_y = 0: \quad 202\frac{\sqrt{2}}{2} + B_y = 0$

$B_y = -142.9 \text{ N}$

$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(142.9)^2 2} = \underline{202 \text{ N}}$

(Reactions at A and B must form a couple!)

4/81

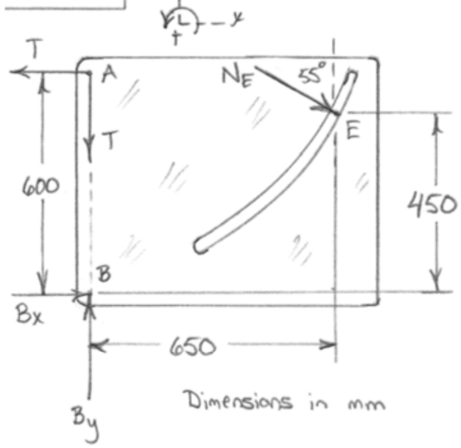


$\Sigma M_A = 0: \; C(340) - 4000(550) = 0$

$\underline{C = 6470 \text{ N}}$

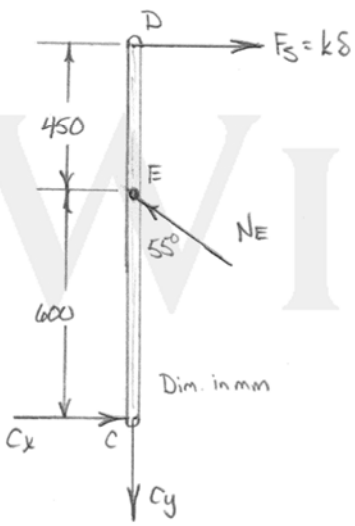
4/82

$T = 225$ N



$$\Sigma M_B = 0: \quad 600T - 650N_E \cos 55^\circ - 450N_E \sin 55^\circ = 0$$

$$N_E = 182.1 \text{ N}$$



$F_S = k\delta$ AND $\delta = 150$ mm

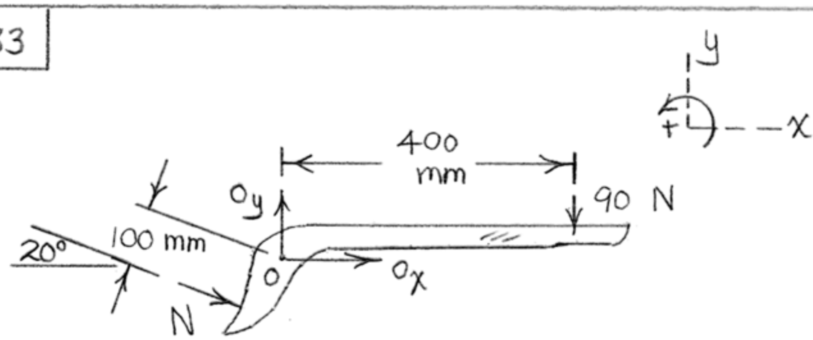
$$\Sigma M_C = 0: \quad 600 N_E \sin 55^\circ - 1050 F_S = 0$$

$$F_S = 85.2 \text{ N}$$

$$F_S = k\delta \longrightarrow 85.2 = k(0.150)$$

$$\underline{k = 568 \text{ N/m}}$$

4/83

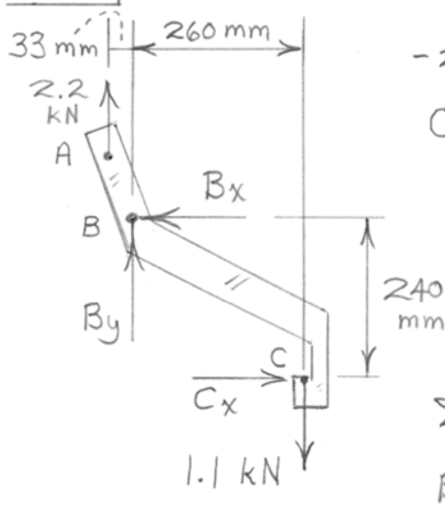


$$\begin{cases} \Sigma M_O = 0: \ 100N - 400(90) = 0, & \underline{N = 360 \text{ N}} \\ \Sigma F_x = 0: \ O_x + 360(\cos 20°) = 0, & O_x = -338 \text{ N} \\ \Sigma F_y = 0: \ O_y - 360(\sin 20°) - 90 = 0 & \end{cases}$$

$$O_y = 213 \text{ N}$$

$$O = \sqrt{O_x^2 + O_y^2} = \sqrt{338^2 + 213^2} = \underline{400 \text{ N}}$$

4/84



$\circlearrowleft + \; \Sigma M_B = 0:$

$-2.2(33) + C_x(240) - 1.1(260) = 0$

$C_x = 1.494 \text{ kN}$

$\Sigma F_x = 0:$

$1.494 - B_x = 0$

$B_x = 1.494 \text{ kN}$

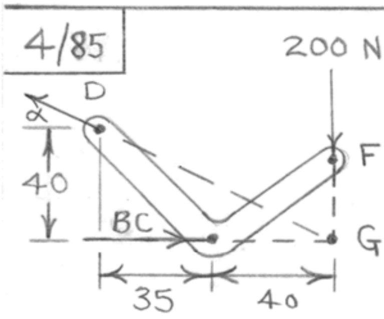
$\Sigma F_y = 0:$

$B_y + 2.2 - 1.1 = 0$

$B_y = -1.1 \text{ kN}$

$B = \sqrt{1.494^2 + 1.1^2} = \underline{1.855 \text{ kN}}$

4/85



DCF is a three-force body; forces intersect at G.

$\alpha = \tan^{-1}\left(\frac{40}{75}\right) = 28.1°$

$\Sigma F_y = 0: -200 + D\sin\alpha = 0$

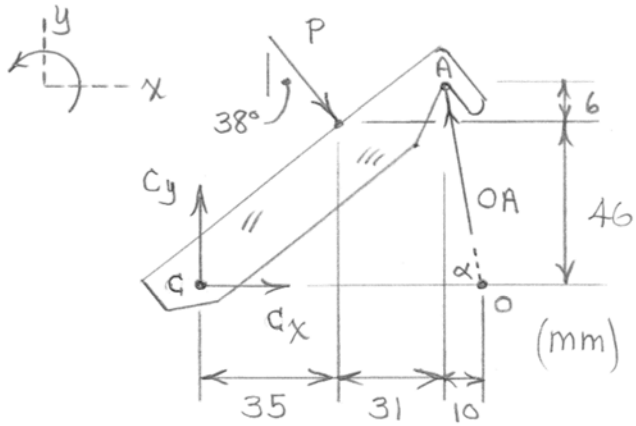
$D = 425$ N

$\Sigma F_x = 0: -D\cos\alpha + BC = 0,$ $BC = 375$ N

(BC in compression)

4/86

$$\alpha = \tan^{-1}\frac{52}{10} = 79.1°$$



$$\Sigma M_C = 0: \; -P\sin 38°(46) - P\cos 38°(35) + OA\sin\alpha\,(76) = 0$$

$$OA = 0.749P$$

$$\Sigma F_x = 0: \; C_x + P\sin 38° - 0.749P\cos\alpha = 0$$

$$C_x = 0.474P$$

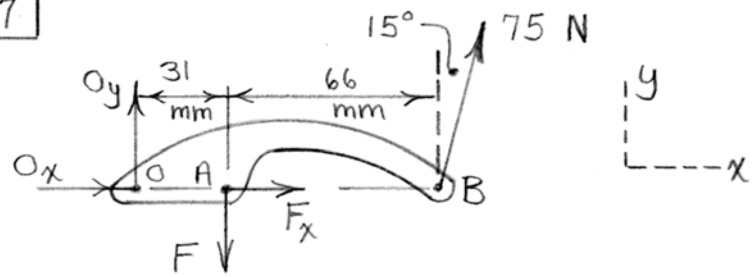
$$\Sigma F_y = 0: \; C_y - P\cos 38° + 0.749P\sin\alpha = 0$$

$$C_y = 0.0525P$$

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(0.474P)^2 + (0.0525P)^2}$$

$$= \underline{0.477P}$$

4/87

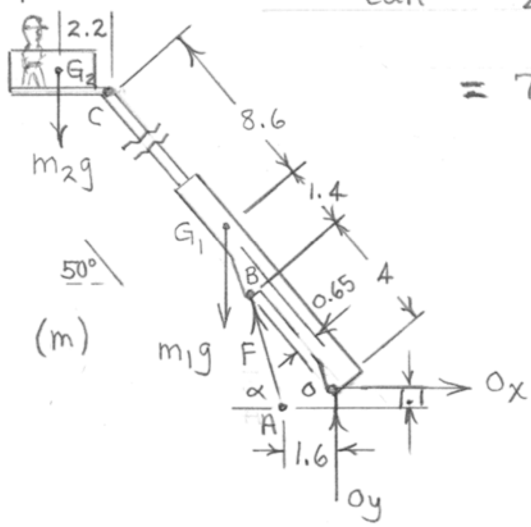


$\curvearrowleft + \sum M_O = 0: \; -F(31) + 75\cos 15°(31+66) = 0$

$\underline{F = 227\text{ N}}$

(Note: Treatment of member OC as a three-force body would yield a constraint relationship between O_x and O_y.)

4/88



$$\alpha = \tan^{-1}\frac{4\sin 50° + 1.1}{4\cos 50° - 1.6} = 76.9°$$

$\circlearrowleft + \Sigma M_O = 0$: $m_1 g(5.4\cos 50° - 0.65\sin 50°)$

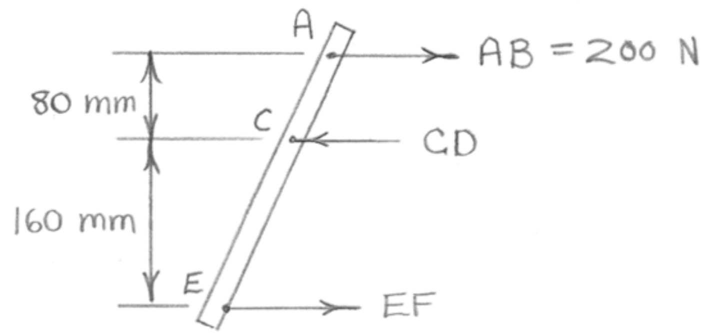
$+ m_2 g(14\cos 50° - 0.65\sin 50° + 2.2)$

$- F\sin\alpha\,(1.6) - F\cos\alpha\,(1.1) = 0$

(considering F to act at A!)

Solving, $\underline{F = 30\,300 \text{ N}}$

4/89



$\circlearrowleft + \Sigma M_C = 0: \; -200(80) + EF(160) = 0$

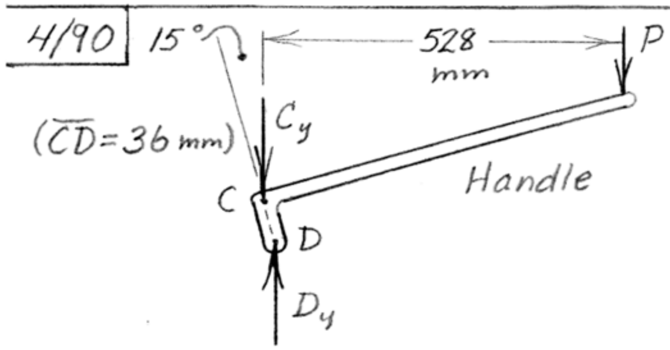
$\underline{EF = 100 \text{ N } T}$

$\overset{+}{\rightarrow} \Sigma F = 0: \; 200 - CD + 100 = 0$

$CD = 300 \text{ N}$

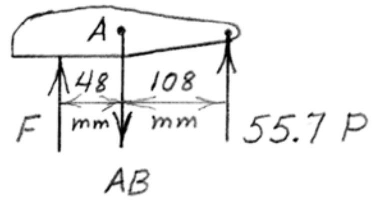
So force supported by pin C is $\underline{F = 300 \text{ N}}$

4/90



$\circlearrowleft + \Sigma M_D = 0: \ C_y(36 \sin 15°) - P(528 - 36 \sin 15°) = 0$

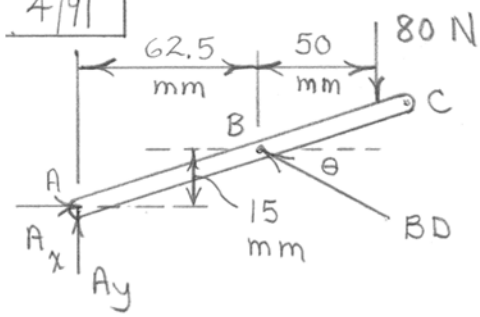
$C_y = 55.7P$



$\circlearrowleft + \Sigma M_A = 0: \ -F(48) + 55.7P(108) = 0$

$\underline{F = 125.3P}$ (!)

4/91



$\theta = \tan^{-1} \frac{21.25}{50} = 23.0°$

$\circlearrowleft + \Sigma M_A = 0: -80(112.5) + BD\cos\theta(15) + BD\sin\theta(62.5) = 0$

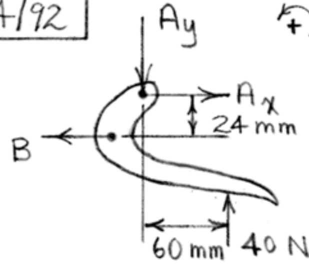
$BD = 235 \text{ N}$

$\xrightarrow{+} \Sigma F = 0: \quad A_x - 235\cos\theta = 0$

$A_x = 217 \text{ N}$

Thus, squeezing force $\underline{P = 217 \text{ N}}$.

4/92

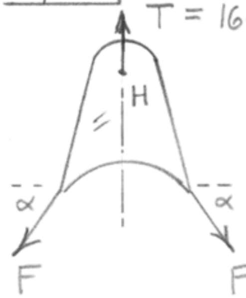


$\circlearrowleft + \sum M_A = 0: \; 40(60) - B(24) = 0$

$B = 100 \text{ N}$

$\therefore$ Force F on brad = $\underline{100 \text{ N}}$

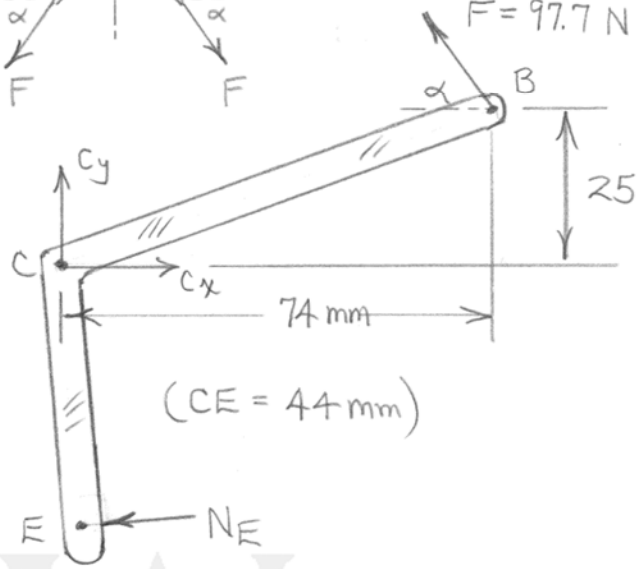
4/93



$\alpha = \tan^{-1}\left(\frac{30}{21}\right) = 55.0°$

$+\uparrow \Sigma F = 0: \quad 160 - 2F\sin 55.0° = 0$

$F = 97.7 \text{ N}$

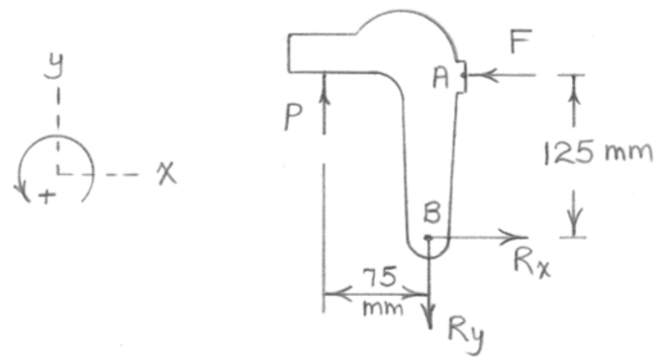


$\circlearrowleft + \Sigma M_C = 0: \quad (97.7 \cos 55.0°)(25) + (97.7 \sin 55.0°)(74)$

$- N_E\,(44) = 0$

$\underline{N_E = N_F = 166.4 \text{ N}}$

4/94



For $P = 3$ kN :

$\Sigma M_B = 0$: $125F - 3(75) = 0$, $F = 1.8$ kN

For $F = 2(1.8) = 3.6$ kN , $P = 3(2) = 6$ kN

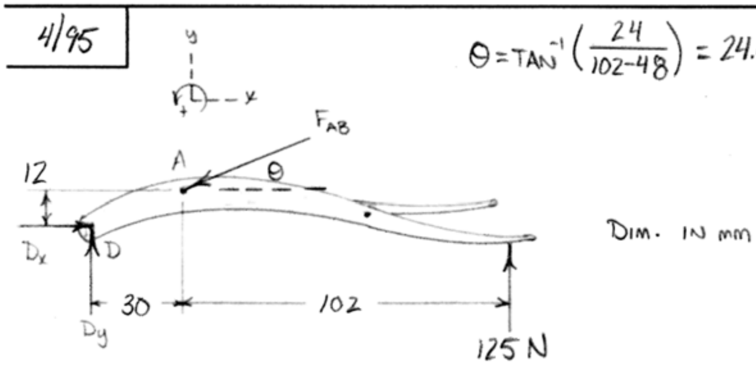
$\Sigma F_x = 0$: $R_x - 3.6 = 0$, $R_x = 3.6$ kN

$\Sigma F_y = 0$: $-R_y + 6 = 0$, $R_y = 6$ kN

$R = \sqrt{3.6^2 + 6^2} = \underline{7.00 \text{ kN}}$

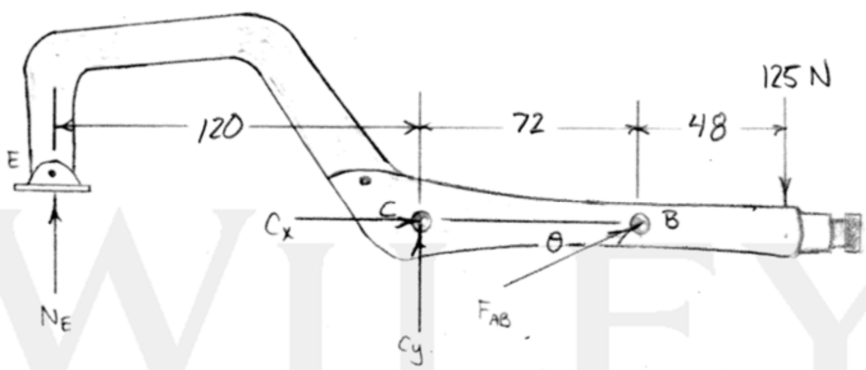
4/95

$\theta = \tan^{-1}\left(\frac{24}{102-48}\right) = 24.0°$



$\Sigma M_D = 0$: $132(125) - 30 F_{AB} \sin\theta + 12 F_{AB} \cos\theta = 0$

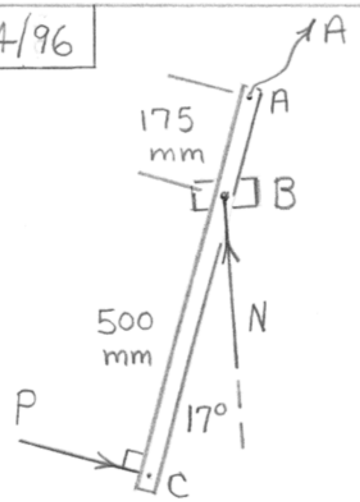
$F_{AB} = 13.54$ kN C



$\Sigma M_C = 0$: $72 F_{AB} \sin\theta - 120 N_E - 120(125) = 0$

$N_E = 3.18$ kN OR 3180 N

4/96

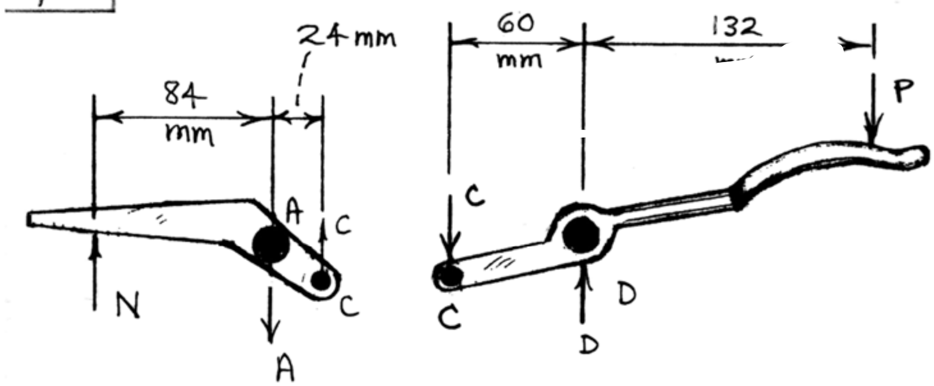


$\circlearrowleft + \Sigma M_A = 0:$

$$P(675) - N \sin 17° (175) = 0$$

$$\underline{N = 13.19 P}$$

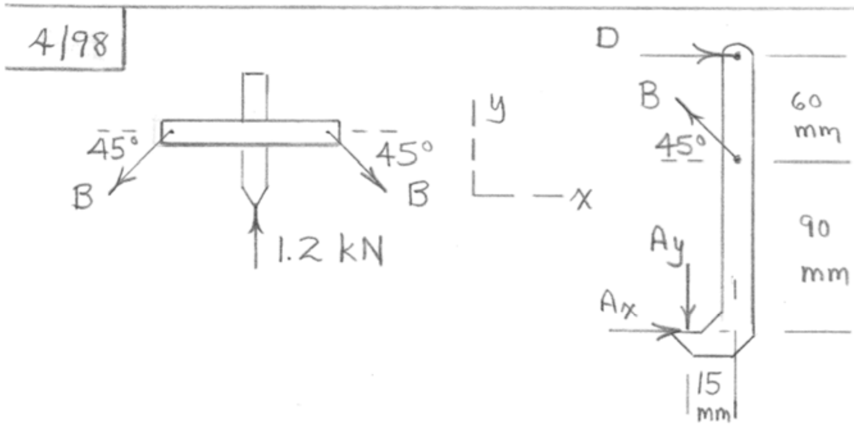
4/97



$\curvearrowleft + \sum M_D = 0: \; C(60) - P(132) = 0, \qquad C = 2.2P$

$\curvearrowleft + \sum M_A = 0: \; 2.2P(24) - N(84) = 0, \qquad \underline{N = 0.629P}$

4/98



(Upper bar & screw)

$\Sigma F_y = 0: -2B \sin 45° + 1.2 = 0, \quad B = 0.849 \text{ kN}$

(ABD)

$\Sigma M_A = 0: \; 150D - 0.849 \cos 45° (90) - 0.849 \sin 45° (15) = 0$

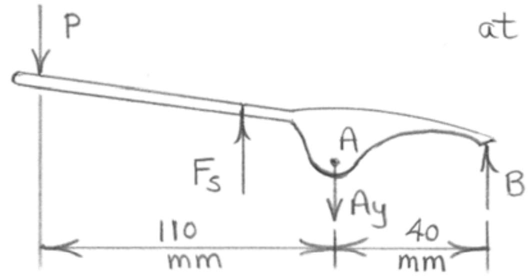
$D = 0.420 \text{ kN}$

$\Sigma F_x = 0: A_x - 0.849 \cos 45° + 0.420 = 0, \; A_x = 0.1800 \text{ kN}$

$\Sigma F_y = 0: -A_y + 0.849 \sin 45° = 0, \quad A_y = 0.6 \text{ kN}$

$A = \sqrt{A_x^2 + A_y^2} = \underline{0.626 \text{ kN}}$

4/99 Upper handle: (spring force F_s acts at unknown location)



When clamp is released, $B = 0$. $\circlearrowleft + \sum M_A = 0$:

$P(110) - M_{F_s} = 0$, $M_{F_s} = P(110) = 25(110) = 2750$ N·mm

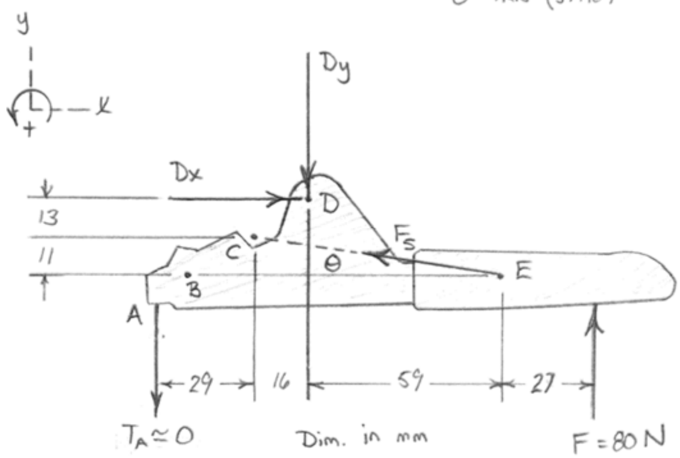
(M_{F_s} is the moment exerted by spring on handle)

With $P = 0$, $\sum M_A = 0$: $B(40) - 2750 = 0$

$\underline{B = 68.8 \text{ N}}$

4/100

$\theta = \tan^{-1}\left(\frac{11}{59+16}\right) \longrightarrow \theta = 8.34°$



$\Sigma F_x = 0: \quad D_x - F_S \cos\theta = 0$

$\Sigma F_y = 0: \quad -\cancel{T_A}^{\,0} + F - D_y + F_S \sin\theta = 0$

$\Sigma M_D = 0: \quad (59+27)F - (29+16)\cancel{T_A}^{\,0} + 59 F_S \sin\theta - (11+13) F_S \cos\theta = 0$

$F_S = 453 \text{ N}$

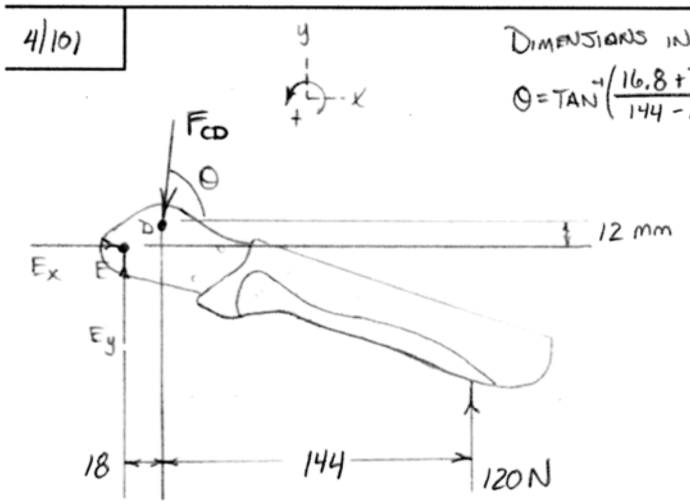
$D_x = 448 \text{ N}$ $\qquad D = \sqrt{D_x^2 + D_y^2} \longrightarrow \underline{D = 471 \text{ N}}$

$D_y = 145.8 \text{ N}$

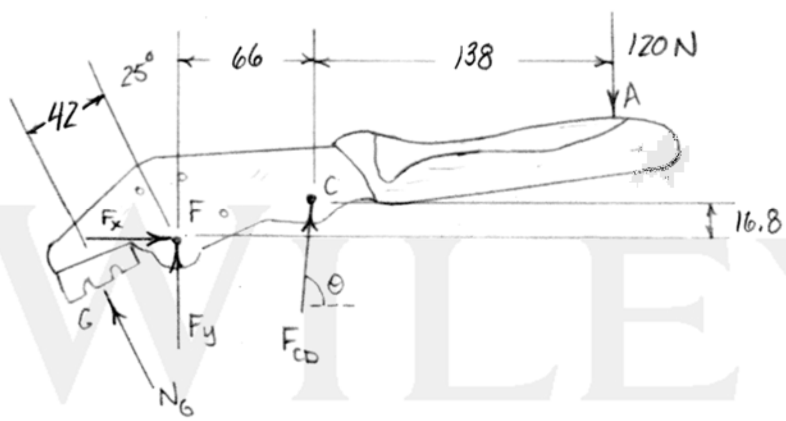
4/101

DIMENSIONS IN mm

$\theta = \tan^{-1}\left(\frac{16.8+7.2}{144-138}\right) = 76.0°$



$\Sigma M_E = 0$: $(144+18)120 + 12 F_{CD}\cos\theta - 18 F_{CD}\sin\theta = 0 \longrightarrow F_{CD} = 1336$ N C



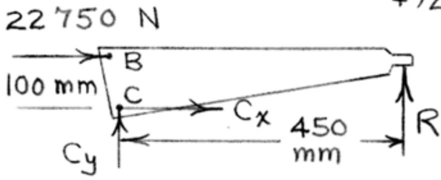
$\Sigma M_F = 0$: $66 F_{CD}\sin\theta - 16.8 F_{CD}\cos\theta - 42 N_G - (66+138)120 = 0$

$\underline{N_G = 1324 \text{ N}}$

4/102 Piston force $= \left[3.5(10^6)\frac{N}{m^2}\right]\left[13(10^{-3})\ m^2\right]$

$= 45\ 500$ N; force in AB = 22 750 N

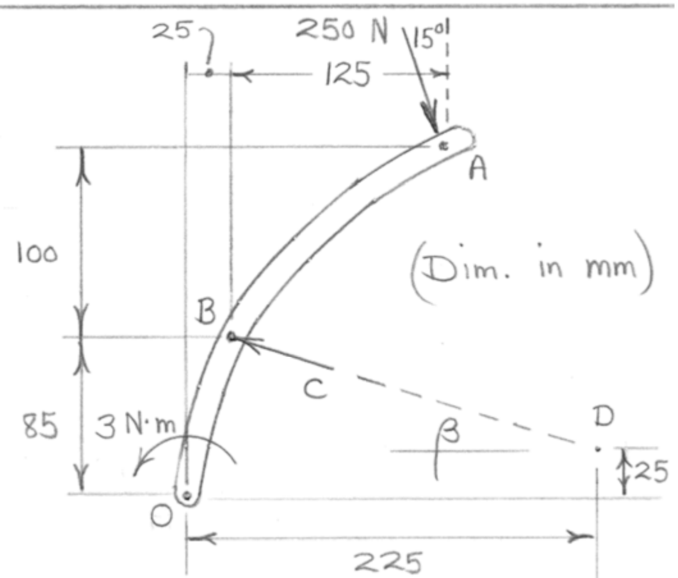
Lower jaw:



$\circlearrowleft + \Sigma M_C = 0: R(450) - 22750(100) = 0, \quad \underline{R = 5.06\ kN}$

4/103

$\beta = \tan^{-1}\frac{60}{200}$
$= 16.70°$



$\circlearrowleft + \sum M_O = 0: \; 3000 - 250\cos 15°(150) - 250\sin 15°(185) + C\cos\beta(85) + C\sin\beta(25) = 0$

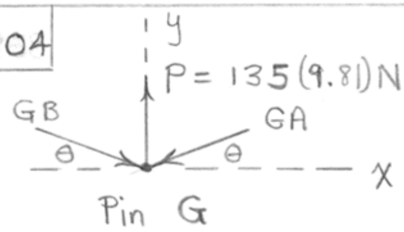
$\underline{C = 510 \text{ N}}$

$C = pA: \quad 510 = p\left(\frac{\pi 45^2}{4}\right)$

$p = 0.321 \frac{N}{mm^2}$ or $\underline{321\,000 \text{ Pa}}$

(gauge pressure)

4/104

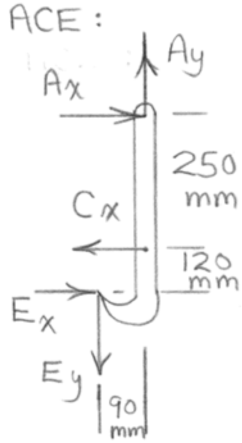


$\theta = \cos^{-1}\frac{340}{350} = 13.73°$

$\Sigma F_y = 0 : 135(9.81) - 2GA \sin 13.73° = 0$

$GA = GB = 2790$ N

ACE:



$A_x = 2790 \cos 13.73° = 2710$ N

$A_y = 2790 \sin 13.73° = 662$ N

$\Sigma F_y = 0 \Rightarrow E_y = 662$ N

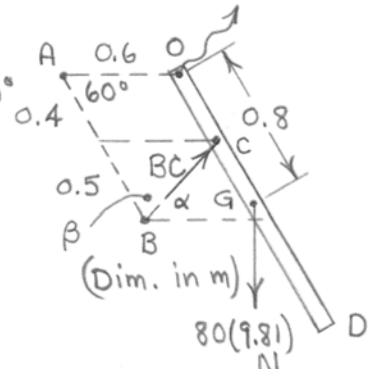
$\Sigma M_C = 0 : 2710(250) - 662(90)$

$-E_x(120) = 0, \quad E_x = 5150$ N

$E = \sqrt{E_x^2 + E_y^2} = 5190$ N

or $\underline{5.19\text{ kN}}$

4/105



$$\overline{BC}^2 = 0.5^2 + 0.6^2 - 2(0.5)(0.6)\cos 60°$$

$$BC = 0.557 \text{ m}$$

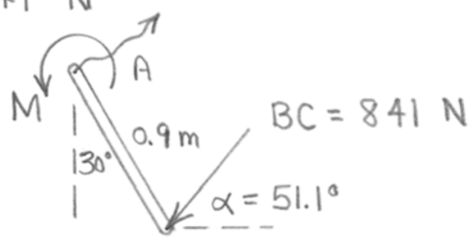
$$\frac{\sin\beta}{0.6} = \frac{\sin 60°}{0.557}, \quad \beta = 68.9°, \quad \alpha = 51.1°$$

$\circlearrowleft + \Sigma M_O = 0$:

$$BC(0.4)\cos(51.1° - 30°) - 80(9.81)(0.8\sin 30°) = 0$$

$$BC = 841 \text{ N}$$

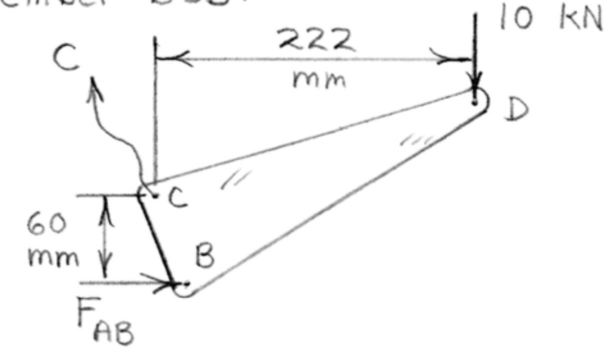
AB:



$\circlearrowleft + \Sigma M_A = 0$: $M - 841(0.9)\cos(51.1° - 30°) = 0$

$$\underline{M = 706 \text{ N}\cdot\text{m}}$$

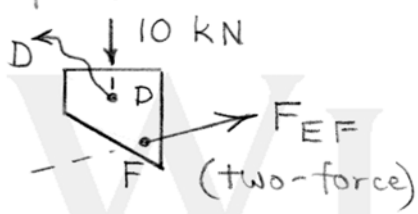
4/106 Member BCD:



$$\circlearrowleft\!+\ \Sigma M_C = 0:\quad F_{AB}(60) - 10(222) = 0$$

$$\underline{F_{AB} = 37 \text{ kN}}$$

The force in member EF is zero, as is disclosed by a FBD of the lifting pad at D:



For equilibrium, F_{EF} must be zero or concurrent at D. The latter is not the case here, so $\underline{F_{EF} = 0}$.

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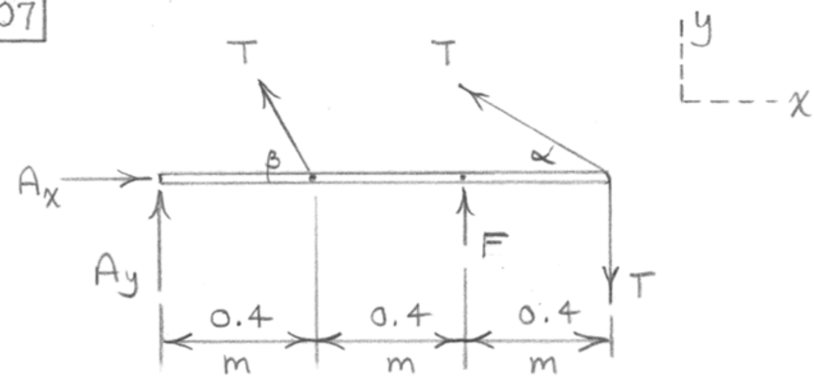


$$\begin{cases} T = 60(9.81) = 589 \text{ N} \\ \alpha = \tan^{-1}\left(\frac{0.5}{1.2}\right) = 22.6^\circ \\ \beta = \tan^{-1}\left(\frac{0.5}{0.4}\right) = 51.3^\circ \end{cases}$$

$\circlearrowleft\!+ \ \Sigma M_A = 0: \ T\sin\beta(0.4) + T\sin\alpha(1.2)$

$-T(1.2) + F(0.8) = 0, \quad \underline{F = 314 \text{ N}}$

(Contact at bottom roller)

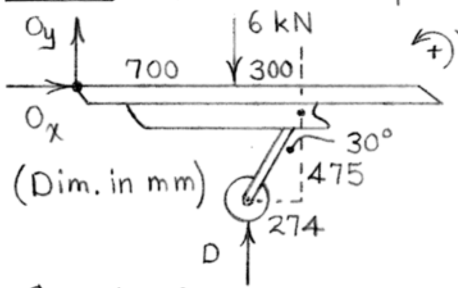
$\Sigma F_x = 0: \ A_x - T\cos\beta - T\cos\alpha = 0, \ A_x = 911 \text{ N}$

$\Sigma F_y = 0: \ A_y + T\sin\beta + T\sin\alpha - T + F = 0$

$A_y = -411 \text{ N}$

$A = \sqrt{A_x^2 + A_y^2} = \underline{999 \text{ N}}$

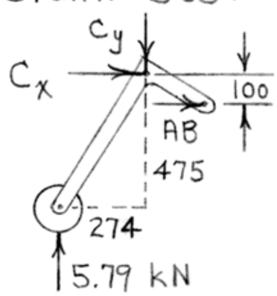
4/108 Entire ramp + mechanism:



$$\circlearrowleft + \sum M_O = 0 : D(700 + 300 - 274) - 6(700) = 0$$

$$D = 5.79 \text{ kN}$$

Crank BCD:

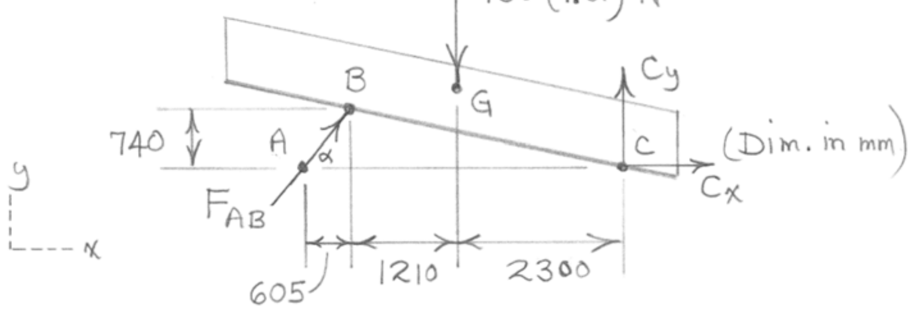


$$\circlearrowleft + \sum M_C = 0 : -5.79(274) + AB(100) = 0$$

$$\underline{AB = 15.87 \text{ kN}}$$

(Cylinder is in compression)

4/109



$\alpha = \tan^{-1}\left(\frac{740}{605}\right) = 50.7°$

$\circlearrowleft + \Sigma M_C = 0$: $750(9.81)(2300) - F_{AB} \sin\alpha\,(4115) = 0$

$\underline{F_{AB} = 5310 \text{ N}}$

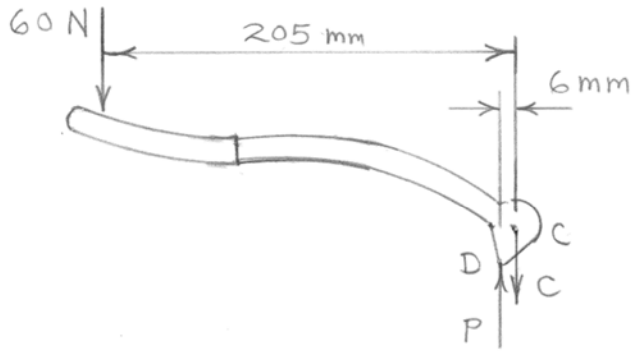
$\Sigma F_x = 0$: $5310 \cos 50.7° + C_x = 0$, $C_x = -3360$ N

$\Sigma F_y = 0$: $5310 \sin 50.7° - 750(9.81) + C_y = 0$

$C_y = 3250$ N

$C = \sqrt{3360^2 + 3250^2} = \underline{4670 \text{ N}}$

4/110 Upper handle/cam unit:



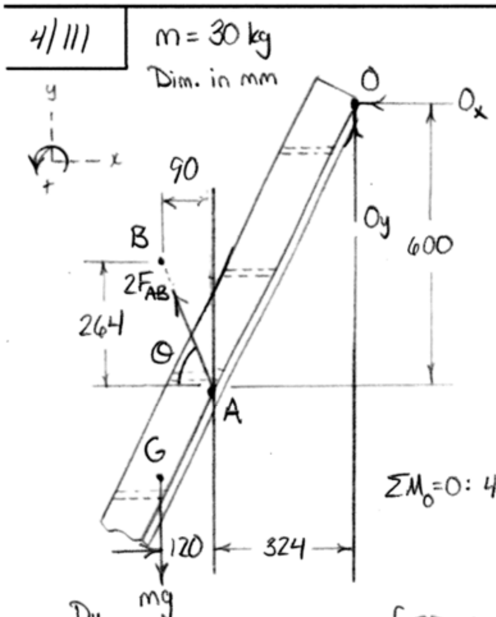
$\circlearrowleft + \Sigma M_C = 0: \quad 60(205) - P(6) = 0$

$\underline{P = 2050 \text{ N}}$

4/111 $m = 30$ kg

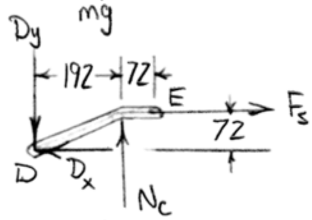
Dim. in mm

$$\theta = \tan^{-1}\left(\frac{264}{90}\right) = 71.2°$$

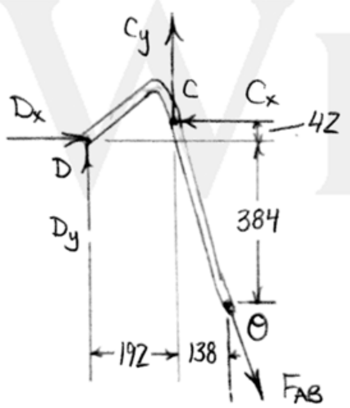


$$\Sigma M_O = 0: \; 444mg - 324(2F_{AB}\sin\theta) - 600(2F_{AB}\cos\theta) = 0$$

$$F_{AB} = 130.6 \text{ N}$$



$$\begin{cases} \Sigma F_x = 0: \; F_s - D_x = 0 \\ \Sigma F_y = 0: \; N_c - D_y = 0 \\ \Sigma M_D = 0: \; 192N_c - 72F_s = 0 \end{cases}$$



$$\begin{cases} \Sigma F_x = 0: \; D_x - C_x + F_{AB}\cos\theta = 0 \\ \Sigma F_y = 0: \; C_y + D_y - F_{AB}\sin\theta = 0 \\ \Sigma M_D = 0: \; 42C_x + 192C_y + 384F_{AB}\cos\theta - 330F_{AB}\sin\theta = 0 \end{cases}$$

$$\begin{cases} C_x = 71.9 \text{ N} \\ C_y = 112.4 \text{ N} \end{cases} \quad \begin{cases} D_x = 29.8 \text{ N} \\ D_y = 11.17 \text{ N} \end{cases} \quad \begin{cases} N_c = 11.17 \text{ N} \\ F_s = 29.8 \text{ N} \end{cases}$$

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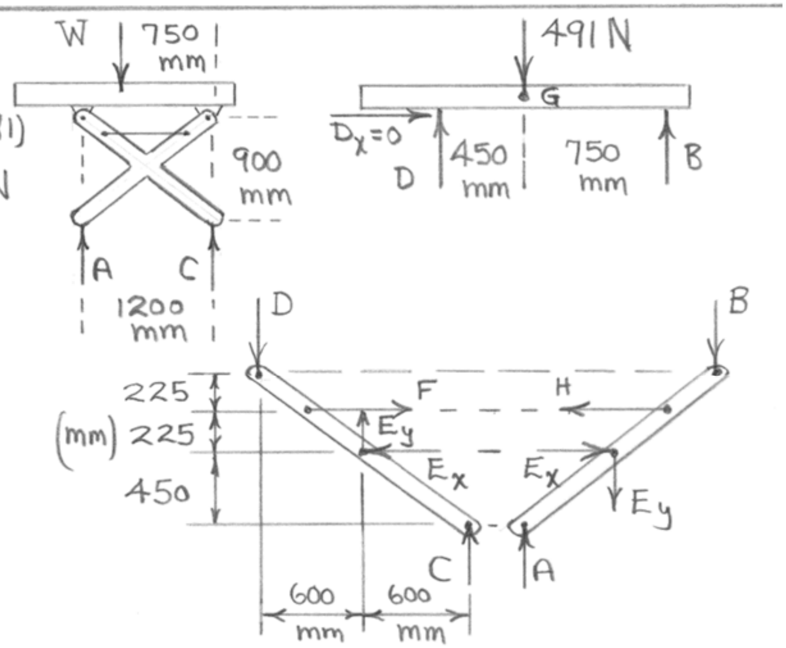
$W = 50(9.81) = 491 \text{ N}$



(Top) $\circlearrowleft + \Sigma M_B = 0: \quad 491(750) - 1200D = 0, \quad D = 307 \text{ N}$

$+\uparrow \Sigma F_y = 0: \quad B + 307 - 491 = 0, \quad B = 183.9 \text{ N}$

Same for entire frame : $\begin{cases} A = D = 307 \text{ N} \\ C = B = 183.9 \text{ N} \end{cases}$

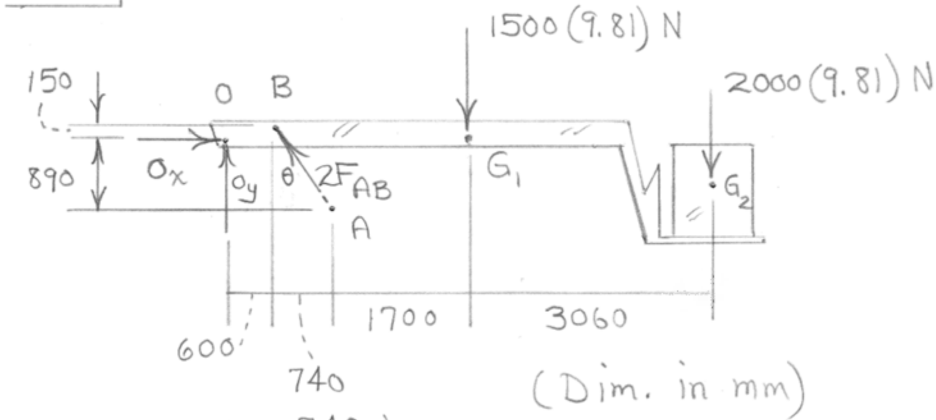
(DEC) $\Sigma F_y = 0: \quad 183.9 + E_y - 307 = 0, \quad \underline{E_y = 122.6 \text{ N}}$

$\Sigma M_E = 0: \quad 225F - 307(600) - 184(600) = 0$

$F = 1308 \text{ N}$

$\Sigma F_x = 0: \quad \underline{E_x = F = 1308 \text{ N}}$

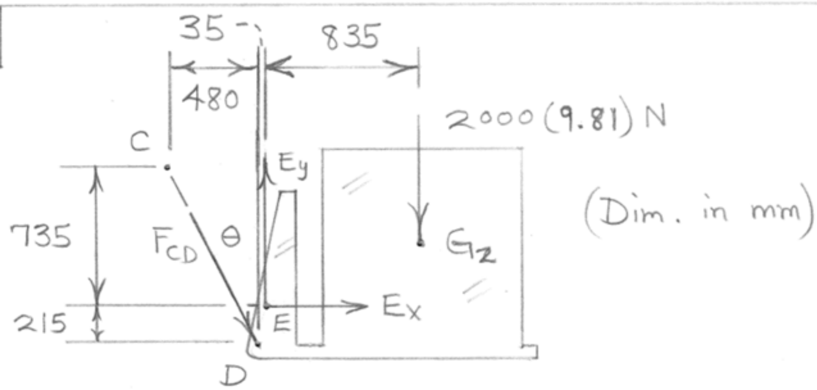
4/113



$$\theta = \tan^{-1}\left(\frac{740}{1040}\right) = 35.4^\circ$$

$$\circlearrowleft \Sigma M_O = 0: 2F_{AB}\cos\theta\,(600) + 2F_{AB}\sin\theta\,(150)$$
$$-1500(9.81)(3040) - 2000(9.81)(6100) = 0$$
$$F_{AB} = 142\,800 \text{ N} \quad \text{or} \quad \underline{142.8 \text{ kN}}$$

4/114



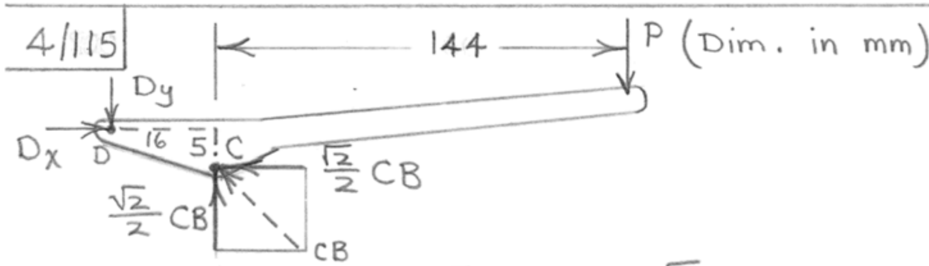
$$\theta = \tan^{-1}\left(\frac{480}{950}\right) = 26.8°$$

$$\circlearrowleft + \Sigma M_E = 0: \; -2000(9.81)(835) + F_{CD}\cos\theta\,(35) + F_{CD}\sin\theta\,(215) = 0$$

$$F_{CD} = 127\,800 \text{ N}$$

or $\underline{127.8 \text{ kN}}$

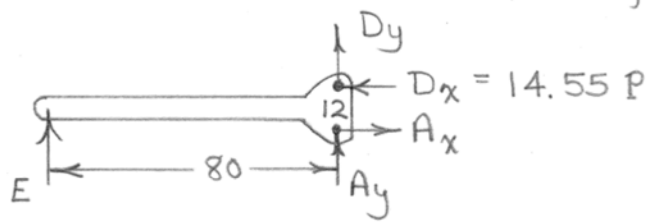
4/115



$$\circlearrowleft + \Sigma M_D = 0: \quad P(160) + \frac{\sqrt{2}}{2} CB(5) - \frac{\sqrt{2}}{2} CB(16) = 0$$

$$CB = 20.6\,P$$

$$\xrightarrow{+} \Sigma F = 0: \quad D_x - 20.6P\frac{\sqrt{2}}{2} = 0, \quad D_x = 14.55P$$



$$\circlearrowleft + \Sigma M_A = 0: \quad E(80) - 14.55P(12) = 0$$

$$\underline{E = 2.18\,P}$$

(Note: Mechanical advantage will increase as CB becomes more aligned with CD.)

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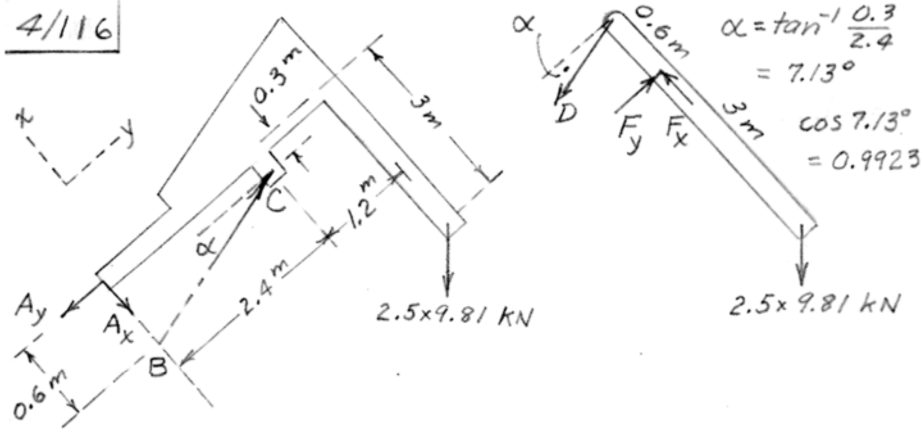


$\alpha = \tan^{-1}\frac{0.3}{2.4} = 7.13°$

$\cos 7.13° = 0.9923$

$\Sigma M_B = 0;\ 2.5(9.81)\left[\frac{1}{\sqrt{2}}(2.4+1.2) + \frac{1}{\sqrt{2}}(3-0.6)\right] - 0.6A_y = 0$

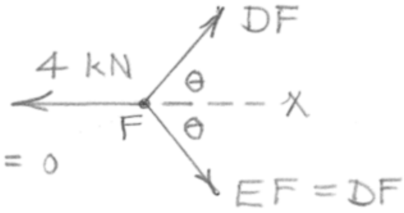
$A_y = 173.4$ kN

$\Sigma M_C = 0;\ 2.5(9.81)\left[\frac{1}{\sqrt{2}}(1.2) + \frac{1}{\sqrt{2}}(3-0.3)\right] - 173.4(0.3) - 2.4A_x = 0$

$A_x = 6.50$ kN , $A = \sqrt{(6.50)^2 + (173.4)^2} = \underline{173.5 \text{ kN}}$

$\Sigma M_F = 0;\ 0.9923D(0.6) - 2.5(9.81)\frac{3}{\sqrt{2}} = 0;\ \underline{D = 87.4 \text{ kN}}$

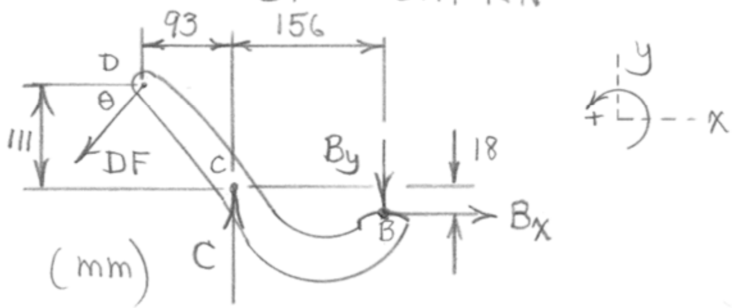
4/117 FBD of joint F:



$\theta = \tan^{-1}\left(\frac{111}{93}\right) = 50.0°$

$\Sigma F_x = 0: \; 2DF\cos 50.0° - 4 = 0$

$DF = 3.11 \text{ kN}$



$\Sigma F_x = 0: \; -3.11\cos 50.0° + B_x = 0, \quad B_x = 2 \text{ kN}$

$\Sigma M_B = 0: \; -C(156) + 3.11\cos 50.0°(111+18)$

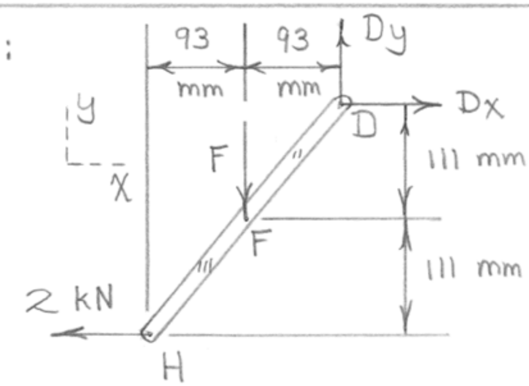
$+ 3.11\sin 50.0°(249) = 0, \quad \underline{C = 5.46 \text{ kN}}$

$\Sigma F_y = 0: \; -3.11\sin 50.0° + 5.46 - B_y = 0$

$B_y = \underline{3.08 \text{ kN} = B_n = A_n}$

(No horizontal force component at C because of symmetry.)

4/118 Member DFH:

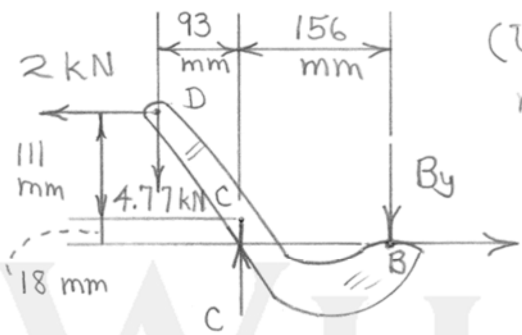


From $\Sigma F_x = 0$, $D_x = 2$ kN

$\circlearrowleft + \Sigma M_D = 0$:

$F(93) - 2(222) = 0$, $F = 4.77$ kN

From $\Sigma F_y = 0$, $D_y = 4.77$ kN



(Use of symmetry made at C and F)

$\circlearrowleft + \Sigma M_B = 0$: $2(129) + 4.77(249) - C(156) = 0$

$\underline{C = 9.27 \text{ kN}}$

$\Sigma F_y = 0$: $-4.77 + 9.27 - B_y = 0$

$\underline{B_y = 4.5 \text{ kN}}$

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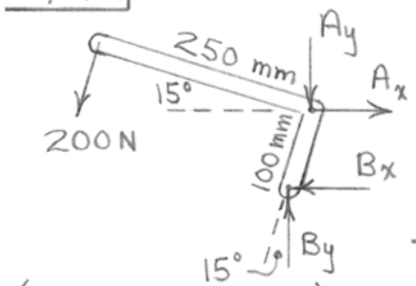


$\left(\begin{array}{l}\sin 15° = 0.2588 \\ \cos 15° = 0.9659\end{array}\right)$

For BC, $B_x = B_y \tan 15°$

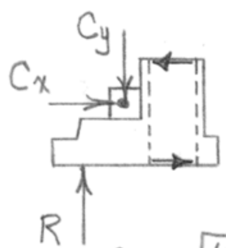
$= 0.2679\, B_y$

For AB, $\Sigma M_A = 0$:

$200(250) - B_y(100)(0.2588) - 0.2679\, B_y(100)(0.9659) = 0$

$B_y = 966 \text{ N}$

$B_x = 0.2679(966) = 259 \text{ N}$



$\Sigma F_x = 0: \; A_x - 259 - 200(0.2588) = 0$

$A_x = 311 \text{ N}$

$\Sigma F_y = 0: \; A_y + 200(0.9659) - 966 = 0$

$A_y = 773 \text{ N}$

$A = \sqrt{(311)^2 + (773)^2} = \underline{833 \text{ N}}$

(Jaw) $\Sigma F_y = 0: \; R = C_y = B_y = \underline{966 \text{ N}}$

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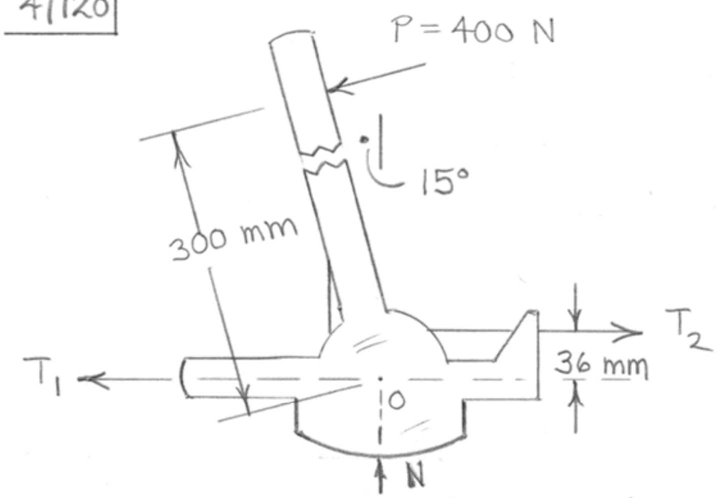

$\curvearrowleft + \Sigma M_O = 0: \quad -36T_2 + 300(400) = 0$

$\underline{T_2 = 3330 \text{ N}}$

$\xrightarrow{+} \Sigma F = 0: -400\cos 15° + 3330 - T_1 = 0$

$\underline{T_1 = 2950 \text{ N}}$

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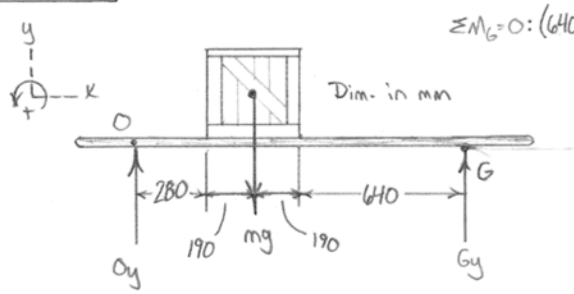
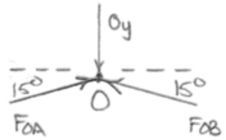
$m = 50$ kg



$\Sigma M_G = 0: (640+190)mg - (280+380+640)O_y = 0$

$O_y = 313$ N

• Joint O:



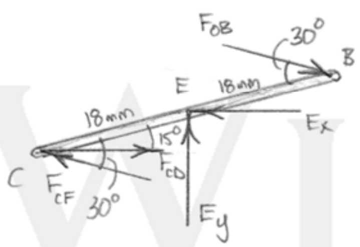
$F_{OA} = F_{OB}$ (SYMMETRY)

$\Sigma F_y = 0: \quad F_{OA} \sin 15° + F_{OB} \sin 15° - O_y = 0$

$F_{OA} = F_{OB} = 605$ N C

By inspection... $F_{CF} = F_{DF} = 605$ N C

• Bar BC: CD is two-force! Note F_{OB} & F_{CF} form a couple!



$\Sigma F_x = 0: \; F_{OB} \cos 15° - F_{CF} \cos 15° + F_{CD} - E_x = 0$

$\Sigma F_y = 0: \quad E_y + F_{CF} \sin 15° - F_{OB} \sin 15° = 0$

$\Sigma M_E = 0: \; -2F_{OB} \sin 30° (18) + F_{CD} (18 \sin 15°) = 0$

Solving...

$\underline{F_{CD} = 2340 \text{ N} \;\; (T)}$

$E_x = 2340$ N

$E_y = 0$

so... $\underline{R_E = 2340\text{N}}$

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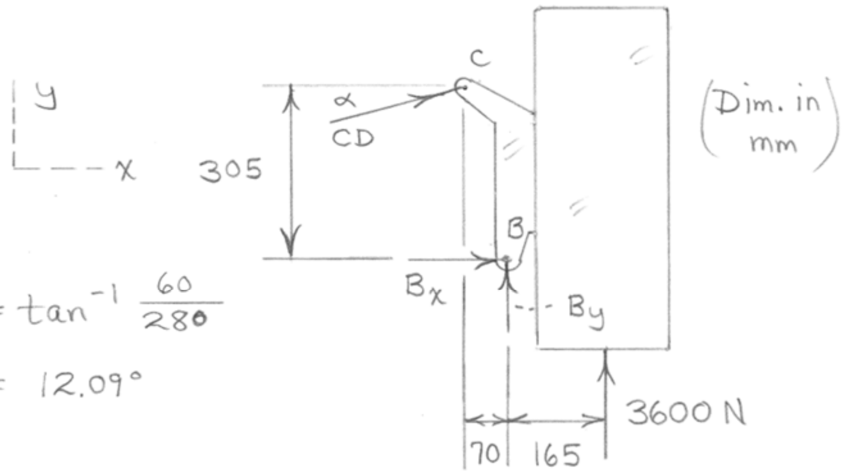
Wheel unit:



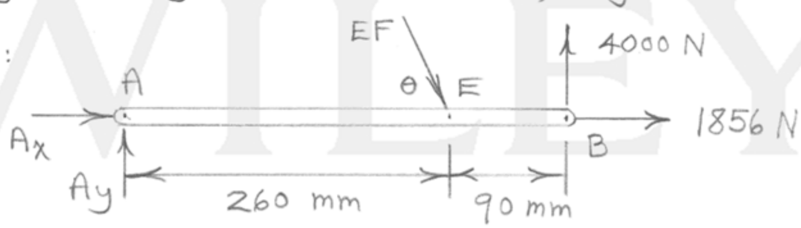
$\alpha = \tan^{-1}\frac{60}{280} = 12.09°$

$\circlearrowleft + \Sigma M_B = 0: \; 3600(165) - CD\cos\alpha\,(305) - CD\sin\alpha\,(70) = 0, \; CD = 1898 \text{ N}$

$\Sigma F_x = 0: \; B_x + CD\cos\alpha = 0, \; B_x = -1856 \text{ N}$

$\Sigma F_y = 0: \; B_y + CD\sin\alpha + 3600 = 0, \; B_y = -4000 \text{ N}$

AEB:



$\theta = \tan^{-1}\left(\frac{435}{200}\right) = 65.3°$

$\circlearrowleft + \Sigma M_A = 0: \; -EF\sin\theta\,(260) + 4000(350) = 0$

$EF = 5920 \text{ N}$

$\Sigma F_x = 0: \; A_x + EF\cos\theta + 1856 = 0$

$A_x = -4330 \text{ N}$

$\Sigma F_y = 0: \; A_y - EF\sin\theta + 4000 = 0$

$A_y = 1384 \text{ N}$

So $A = \sqrt{(-4330)^2 + 1384^2} = \underline{4550 \text{ N}}$

$B = \sqrt{1856^2 + 4000^2} = \underline{4410 \text{ N}}$

$\underline{C = D = 1898 \text{ N}}, \; \underline{E = F = 5920 \text{ N}}$

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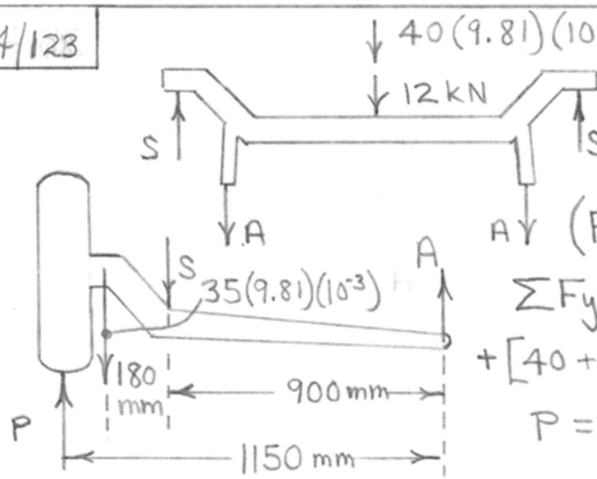

(For entire system) $\Sigma F_y = 0: \; 2P = 12 + [40 + 2(35)](9.81)(10^{-3})$

$P = 6.54$ kN

(Wheel Assembly) $\Sigma M_S = 0: \; 900A - 6.54(250) + 35(9.81)(10^{-3})(180) = 0$

$\underline{A = 1.748 \text{ kN}}$

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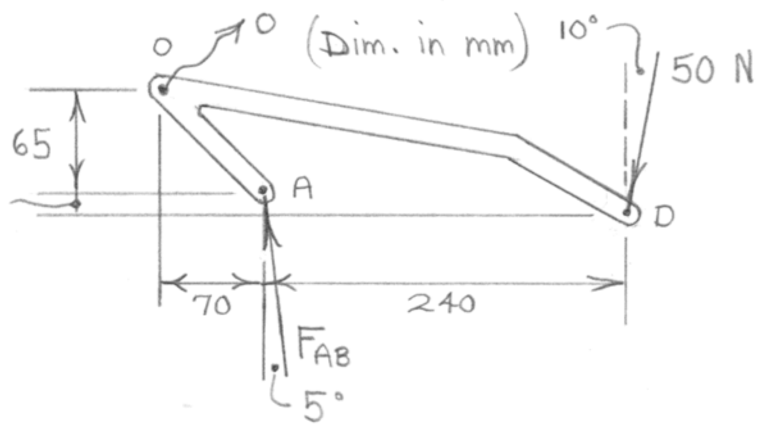
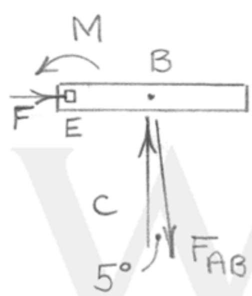


$$\circlearrowleft + \Sigma M_O = 0: \quad F_{AB}[\cos 5°(70) - \sin 5°(65)] - 50[\cos 10°(310) + \sin 10°(83)] = 0$$

$$F_{AB} = 250 \text{ N}$$



$M = 0$ (from $\Sigma M_B = 0$)

$+\uparrow \Sigma F = 0: \quad C - 250 \cos 5° = 0$

$\underline{C = 249 \text{ N}}$

(a factor $\frac{C}{P} = 4.97$)

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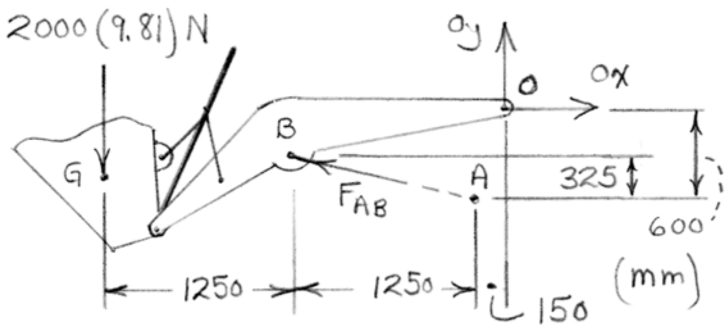

$\measuredangle OBA = \tan^{-1}\left(\frac{275}{1400}\right) + \tan^{-1}\left(\frac{325}{1250}\right) = 11.11° + 14.57° = 25.7°$

$\overline{OB} = \sqrt{275^2 + 1400^2} = 1427$ mm

$\circlearrowleft \Sigma M_O = 0: \; 2000(9.81)(2650) - F_{AB} \sin 25.7°(1427) = 0$

$F_{AB} = 84\,100$ N or $\underline{84.1 \text{ kN}}$

$\Sigma F_x = 0: \; O_x - 84.1 \cos 14.57° = 0$

$O_x = 81.4$ kN

$\Sigma F_y = 0: \; O_y + 84.1 \sin 14.57° - 2(9.81) = 0$

$O_y = -1.535$ kN

$O = \sqrt{81.4^2 + 1.535^2} = \underline{81.4 \text{ kN}}$

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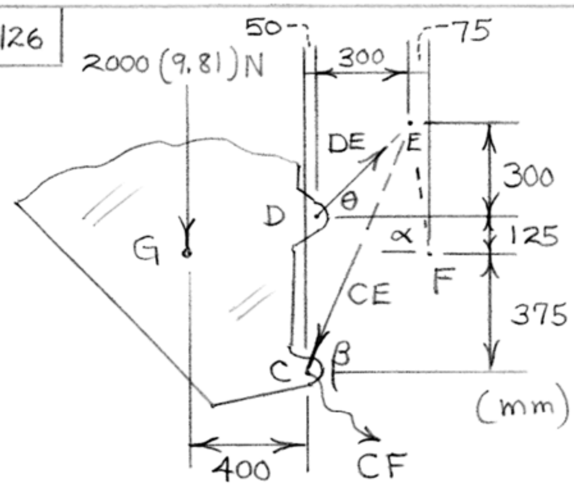

$\theta = 45°$

$\beta = \tan^{-1}\frac{800}{350} = 66.4°$

$\alpha = \tan^{-1}\frac{425}{75} = 80.0°$

$$\circlearrowleft\!+\ \Sigma M_C = 0: 2000(9.81)(400) - DE\cos 45°(500) + DE\sin 45°(50) = 0, \quad DE = 24.7 \text{ kN}$$

FBD of joint E:

$\Sigma F_x = 0:$

$-DE\cos\theta + CE\cos\beta + EF\cos\alpha = 0$

$\Sigma F_y = 0:$

$-DE\sin\theta + CE\sin\beta - EF\sin\alpha = 0$

Solve simultaneously to obtain

$$\begin{cases} CE = 36.5 \text{ kN} \\ EF = 16.22 \text{ kN} \end{cases}$$

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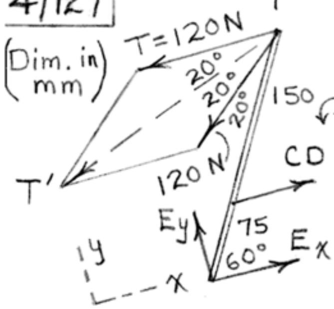


$$(T')^2 = (120)^2 + (120)^2 - 2(120)^2 \cos 140°$$

$$T' = 226 \text{ N}$$

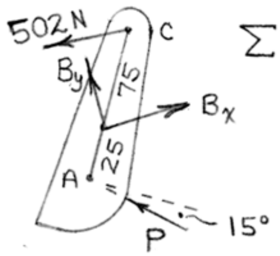
$$\curvearrowleft + \Sigma M_E = 0: \quad -CD(+75 \sin 60°) + 226(225 \sin 40°) = 0, \quad CD = 502 \text{ N}$$

$$\Sigma F_x = 0: \quad E_x + 502 - 226 \cos 20° = 0$$

$$E_x = -290 \text{ N}$$

$$\Sigma F_y = 0: \quad E_y - 226 \sin 20° = 0, \quad E_y = 77.1 \text{ N}$$

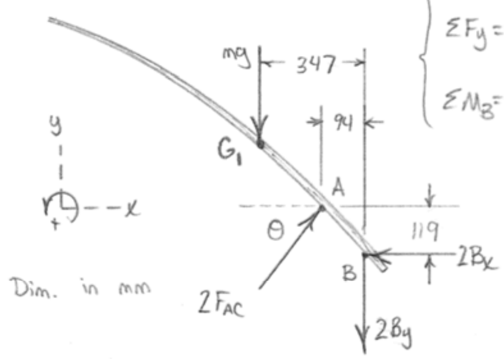
$$E = \sqrt{E_x^2 + E_y^2} = \underline{300 \text{ N}}$$



$$\Sigma M_B = 0: \quad 502(75 \sin 60°) - P \cos 15° (25) = 0, \quad \underline{P = 1351 \text{ N}}$$

▶ 4/128 $m = 18$ kg $\theta = \tan^{-1}\left(\frac{140}{106}\right) \rightarrow \theta = 52.9°$

• Hood:



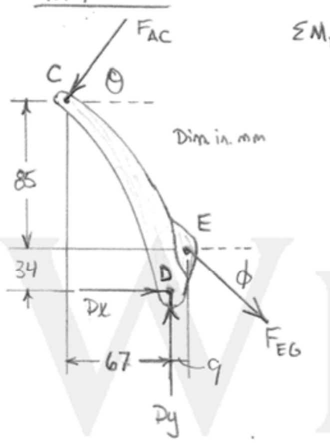
$$\Sigma F_x = 0: \ 2F_{AC}\cos\theta - 2B_x = 0$$
$$\Sigma F_y = 0: \ 2F_{AC}\sin\theta - mg - 2B_y = 0$$
$$\Sigma M_B = 0: \ \tfrac{1}{2}(347)mg - 94F_{AC}\sin\theta - 119F_{AC}\cos\theta = 0$$

$B_x = 126.0$ N

$B_y = 78.1$ N

$F_{AC} = 209$ N C

• Part CDE:



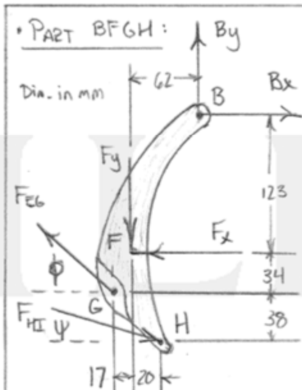
$$\Sigma M_D = 0: \ 67F_{AC}\sin\theta + (85+34)F_{AC}\cos\theta - 9F_{EG}\sin\phi - 34F_{EG}\cos\phi = 0$$

$F_{EG} = 894$ N T

$\phi = \tan^{-1}\left(\frac{119+123+34-140+85}{45}\right)$

$\phi = 48.6°$

• Part BFGH:



$\psi = \tan^{-1}\left(\frac{119+123+34+38-140-85-21}{110+9+45+17+20}\right)$

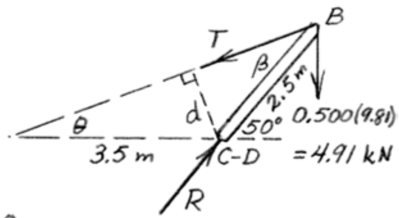
$\psi = 18.69°$

$$\Sigma M_F = 0: \ 62B_y - 123B_x - 17F_{EG}\sin\phi - 34F_{EG}\cos\phi + \ldots$$
$$(34+38)F_{HI}\cos\psi - 20F_{HI}\sin\psi = 0$$

$\therefore$ $F_{HI} = 682$ N C

4/129 Frame as a whole:

$$\theta = \tan^{-1}\frac{2.5 \sin 50^\circ}{3.5 + 2.5\cos 50^\circ} = 20.6^\circ$$



$d = 3.5 \sin 20.6^\circ = 1.229$ m

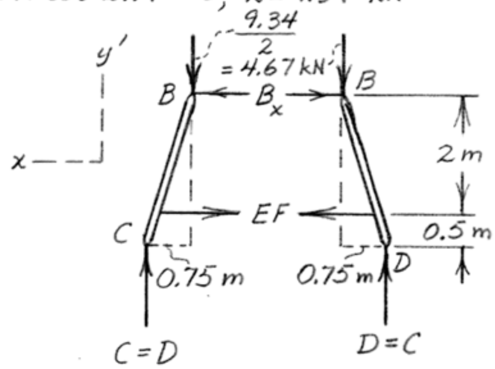
$\beta + 20.6^\circ = 50^\circ$, $\beta = 29.4^\circ$

$\Sigma M_{CD} = 0: \; (2.5\cos 50^\circ)(4.91) - 1.229\,T = 0,$

$$\underline{T = 6.41 \text{ kN}}$$

$\Sigma F_{(C\text{-}D)-B} = 0: \; R - 4.91\cos 40^\circ - 6.41\cos 29.4^\circ = 0, \; R = 9.34$ kN

Plane of frame:



$\Sigma F_{y'} = 0: \; C = D = 4.67$ kN

$\Sigma M_B = 0: \; 4.67(0.75) - 2EF = 0$

$$\underline{EF = 1.752 \text{ kN}}$$

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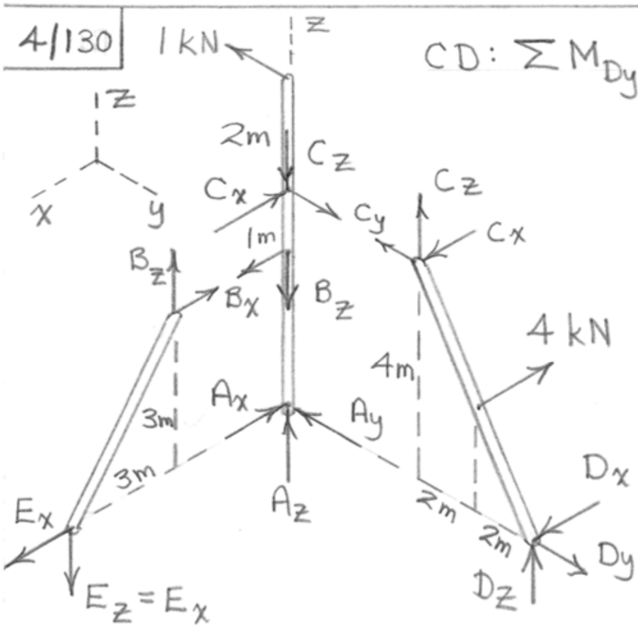

CD: $\Sigma M_{D_y} = 0: \; 4(2) - C_x(4) = 0$

$C_x = 2$ kN

ABC: $\Sigma M_{B_y} = 0: \; 2(1) - A_x(3) = 0, \qquad A_x = 0.667$ kN

$\Sigma F_x = 0: \; B_x - 0.667 - 2 = 0, \qquad B_x = 2.67$ kN

EB: $B_z = E_z = B_x = E_x = 2.67$ kN

ABC: $\Sigma M_{A_x} = 0: \; 1(6) - C_y(4) = 0, \qquad C_y = 1.50$ kN

CD: $\Sigma M_{D_x} = 0: \; C_z = C_y = 1.50$ kN

ABC: $\Sigma F_z = 0: \; A_z - 2.67 - 1.50 = 0, \qquad A_z = 4.17$ kN

$\Sigma F_y = 0: \; A_y + 1 - 1.50 = 0, \qquad A_y = 0.50$ kN

$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \underline{4.25 \text{ kN}}$

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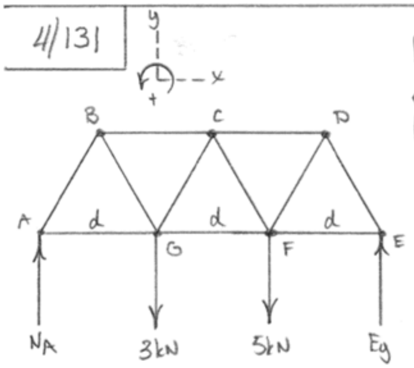
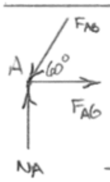


$$\Sigma F_y = 0: \quad N_A + E_y - 3 - 5 = 0$$
$$\Sigma M_A = 0: \quad 3d E_y - 2d(5) - d(3) = 0$$

$N_A = 3.67$ kN $\qquad E_y = 4.33$ kN

All angles are 60°.

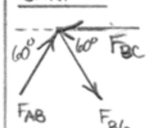
• Joint A:



$$\Sigma F_x = 0: \quad F_{AG} - F_{AB}\cos 60^\circ = 0$$
$$\Sigma F_y = 0: \quad N_A - F_{AG}\sin 60^\circ = 0$$

$F_{AB} = 4.23$ kN C $\qquad F_{AG} = 2.12$ kN T

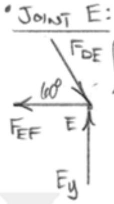
• Joint B:



$$\Sigma F_x = 0: \quad F_{AB}\cos 60^\circ \;\; F_{BC} + F_{BG}\cos 60^\circ = 0$$
$$\Sigma F_y = 0: \quad F_{AB}\sin 60^\circ - F_{BG}\sin 60^\circ = 0$$

$F_{BC} = 4.23$ kN C $\qquad F_{BG} = 4.23$ kN T

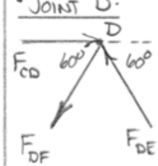
• Joint E:



$$\Sigma F_x = 0: \quad F_{DE}\cos 60^\circ - F_{EF} = 0$$
$$\Sigma F_y = 0: \quad E_y - F_{DE}\sin 60^\circ = 0$$

$F_{DE} = 5.00$ kN C $\qquad F_{EF} = 2.50$ kN T

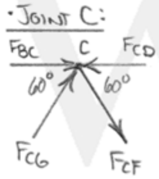
• Joint D:



$$\Sigma F_x = 0: \quad F_{CD} - F_{DF}\cos 60^\circ - F_{DE}\cos 60^\circ = 0$$
$$\Sigma F_y = 0: \quad F_{DE}\sin 60^\circ - F_{DF}\sin 60^\circ = 0$$

$F_{CD} = 5.00$ kN C $\qquad F_{DF} = 5.00$ kN T

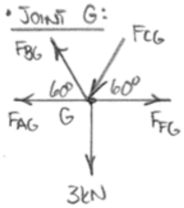
• Joint C:



$$\Sigma F_x = 0: \quad F_{BC} - F_{CD} + F_{CG}\cos 60^\circ + F_{CF}\cos 60^\circ = 0$$
$$\Sigma F_y = 0: \quad F_{CG}\sin 60^\circ - F_{CF}\sin 60^\circ = 0$$

$F_{CG} = 0.770$ kN C $\qquad F_{CF} = 0.770$ kN T

• Joint G:



$$\Sigma F_x = 0: \quad F_{FG} - F_{AG} - F_{CG}\cos 60^\circ - F_{BG}\cos 60^\circ = 0 \longrightarrow F_{FG} = 4.62 \text{ kN T}$$

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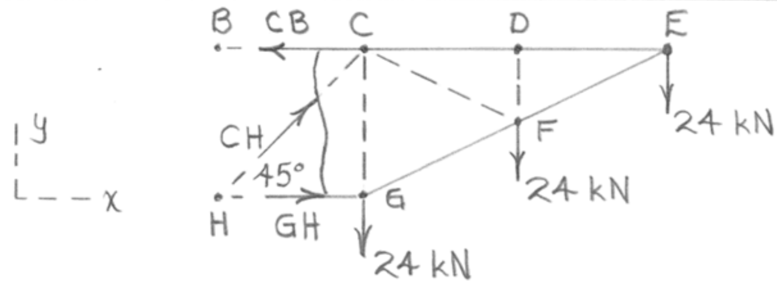
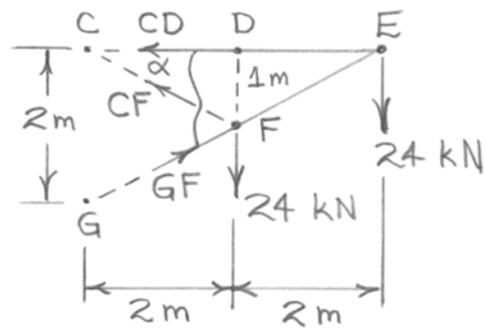


$\Sigma F_y = 0: \; CH \sin 45° - 3(24) = 0$, $\underline{CH = 101.8 \text{ kN } C}$

$\alpha = \tan^{-1}\frac{1}{2} = 26.6°$



$\circlearrowleft + \Sigma M_E = 0: \; 24(2) - (CF \sin 26.6°)(4) = 0$

$\underline{CF = 26.8 \text{ kN } T}$

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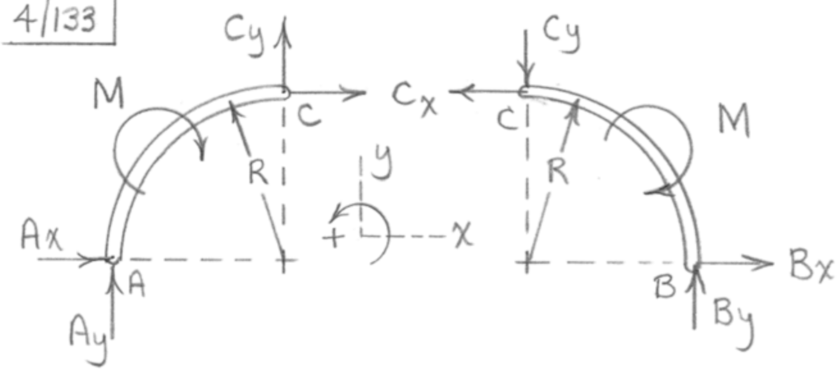


Left member:

$$\begin{cases} \Sigma F_x = 0: \quad A_x + C_x = 0 & (1) \\ \Sigma F_y = 0: \quad A_y + C_y = 0 & (2) \\ \Sigma M_A = 0: \quad -M - C_x(R) + C_y(R) = 0 & (3) \end{cases}$$

Right member:

$$\Sigma F_x = 0: \quad -C_x + B_x = 0 \qquad (4)$$

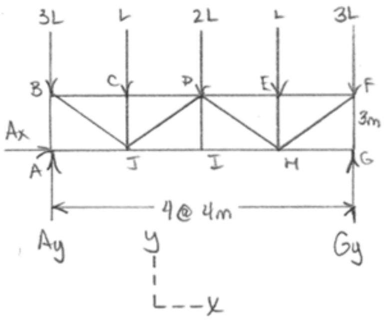
$$\Sigma F_y = 0: \quad -C_y + B_y = 0 \qquad (5)$$

$$\Sigma M_B = 0: \quad -M + C_x(R) + C_y(R) = 0 \qquad (6)$$

Solution of Eqs. (1)–(6):

$$\begin{cases} C_y = B_y = \dfrac{M}{R}, \quad A_y = -\dfrac{M}{R} \\ A_x = B_x = C_x = 0 \end{cases}$$

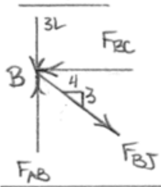
4/134 BY SYMMETRY... $A_y = G_y = 5L$ AND $A_x = 0$



BY INSPECTION:

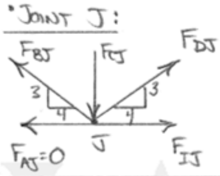
$$\begin{cases} F_{AB} = F_{FG} = 5L \quad C \\ F_{AJ} = F_{GH} = 0 \\ F_{CJ} = F_{EH} = L \quad C \\ F_{DI} = 0 \end{cases}$$

• JOINT B:



$$\begin{cases} \Sigma F_x = 0: \ \frac{4}{5}F_{BJ} - F_{BC} = 0 \\ \Sigma F_y = 0: \ F_{AB} - 3L - \frac{3}{5}F_{BJ} = 0 \end{cases} \longrightarrow \begin{cases} F_{BC} = F_{CD} = F_{DE} = F_{EF} = 2.67\,L \quad C \\ F_{BJ} = F_{FH} = 3.33\,L \quad T \end{cases}$$

• JOINT J:



$$\begin{cases} \Sigma F_x = 0: \ \frac{4}{5}F_{DJ} + F_{IJ} - \frac{4}{5}F_{BJ} - F_{AJ} = 0 \\ \Sigma F_y = 0: \ \frac{3}{5}F_{DJ} - F_{CJ} + \frac{3}{5}F_{BJ} = 0 \end{cases} \longrightarrow \begin{cases} F_{IJ} = F_{HI} = 4L \quad T \\ F_{DJ} = F_{DH} = -1.667\,L \ (C) \end{cases}$$

CHECK MAXIMUM COMPRESSIONS.

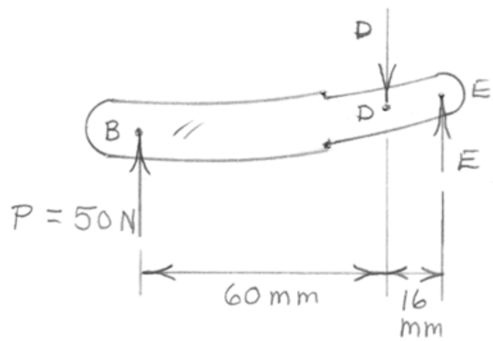
• VERTICAL: $F_{AB} = 5L = 525 \text{ kN} \longrightarrow L = 105 \text{ kN}$

• HORIZONTAL: $F_{BC} = 2.67\,L = 300 \text{ kN} \longrightarrow L = 112.5 \text{ kN}$

• DIAGONAL: $F_{DJ} = 1.667\,L = 180 \text{ kN} \longrightarrow L = 108 \text{ kN}$

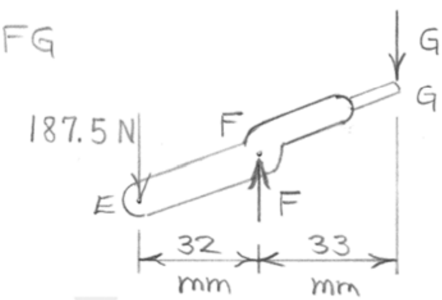
SO... $\underline{L_{max} = 105 \text{ kN}}$

4/135 Handle BDE



$\circlearrowleft + \Sigma M_D = 0: \; -50(60) + E(16) = 0, \quad E = 187.5 \text{ N}$

Jaw EFG



$\circlearrowleft + \Sigma M_F = 0: \; 187.5(32) - G(33) = 0, \quad \underline{G = 181.8 \text{ N}}$

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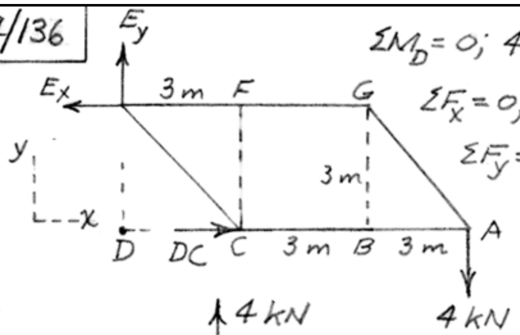
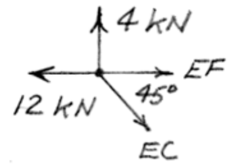


$\Sigma M_D = 0;\ 4(9) - 3E_x = 0,\quad E_x = 12\text{ kN}$

$\Sigma F_x = 0;\ DC = 12\text{ kN}$

$\Sigma F_y = 0;\ E_y = 4\text{ kN}$

Joint E

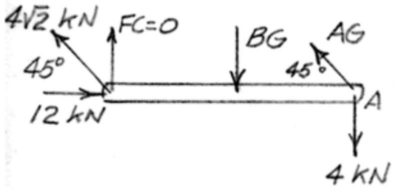


$\Sigma F_y = 0;\ EC/\sqrt{2} - 4 = 0,\quad EC = 4\sqrt{2}\text{ kN}$

$\Sigma F_x = 0;\ EF + 4\sqrt{2}/\sqrt{2} - 12 = 0$

$EF = 8\text{ kN}$

Joint F; $FG = EF$, $FC = 0$



$\Sigma M_A = 0;\ 3BG - 6(4\sqrt{2})/\sqrt{2} = 0$

$\underline{BG = 8\text{ kN } C}$

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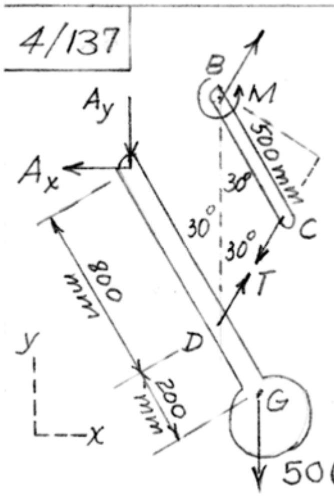

ADG; $\Sigma M_A = 0$;

$50(9.81)(1000 \sin 30°) - T\cos 30°(800) = 0$

$T = 354$ N

BC; $\Sigma M_B = 0$;

$354 \cos 30°(500) - M = 0$

$M = 153.3(10^3)$ N·mm

or $\underline{M = 153.3 \text{ N·m}}$

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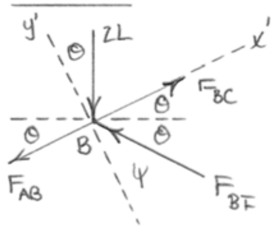
$\theta = \tan^{-1}\left(\frac{3}{6}\right) = 26.6°$ $\phi = \tan^{-1}\left(\frac{3}{2}\right) = 56.3°$

• Joint B:

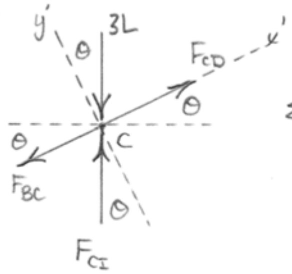


$\psi = 90 - 2\theta = 36.9°$

$\Sigma F_{y'} = 0$: $-2L\cos\theta + F_{BI}\cos\psi = 0$

$F_{BI} = 2.24L$ C

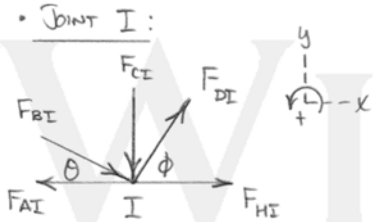
• Joint C:



$\Sigma F_{y'} = 0$: $-3L\cos\theta + F_{CI}\cos\theta = 0$

$F_{CI} = 3L$ C

• Joint I:



$\Sigma F_y = 0$: $F_{DI}\sin\phi - F_{CI} - F_{BI}\sin\theta = 0$

$F_{DI} = 4.81L$ T

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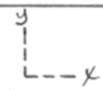
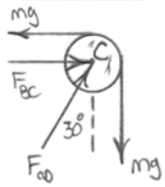


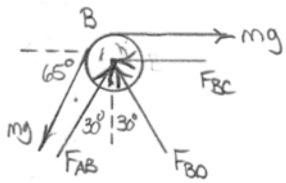
$F_{max} = 42$ kN

• Joint C:



$$\begin{cases} \Sigma F_x = 0: & F_{BC} + F_{CD}\sin 30° - mg = 0 \\ \Sigma F_y = 0: & F_{CD}\cos 30° - mg = 0 \end{cases} \longrightarrow \begin{cases} F_{BC} = 0.423\,mg & C \\ F_{CD} = 1.155\,mg & C \end{cases}$$

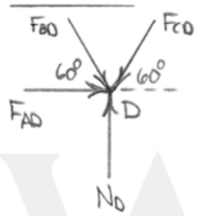
• Joint B:



$$\begin{cases} \Sigma F_x = 0: & mg - mg\cos 65° + F_{AB}\sin 30° - F_{BD}\sin 30° - F_{BC} = 0 \\ \Sigma F_y = 0: & F_{AB}\cos 30° + F_{BD}\cos 30° - mg\sin 65° = 0 \end{cases}$$

$F_{AB} = 0.369\,mg$ C $\qquad F_{BD} = 0.678\,mg$ C

• Joint D:



$\Sigma F_x = 0$: $F_{AD} + F_{BD}\cos 60° - F_{CD}\cos 60° = 0$

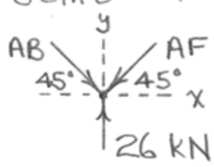
$F_{AD} = 0.238\,mg$ T

$F_{CD} = F_{max} = 42(10^3) = 1.155\,m(9.81) \longrightarrow \underline{m_{max} = 3710 \text{ kg}}$

4/140 $A = 0.5(52) = 26$ kN

$G = -H = 13$ kN, by symmetry.

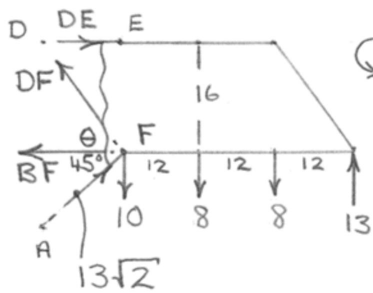
Joint A



$\Sigma F_x = 0 \Rightarrow AB = AF$

$\Sigma F_y = 0: \; -2AB\frac{\sqrt{2}}{2} + 26 = 0$

$AB = 13\sqrt{2}$ kN C $= AF$



$\circlearrowleft + \Sigma M_D = 0: \; -BF(16)$
$+ 13\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)(12) + (13\sqrt{2})\left(\frac{\sqrt{2}}{2}\right)(16)$
$- 10(12) - 8(24) - 8(36)$
$+ 13(48) = 0$

$\underline{BF = 24.3 \text{ kN } T}$

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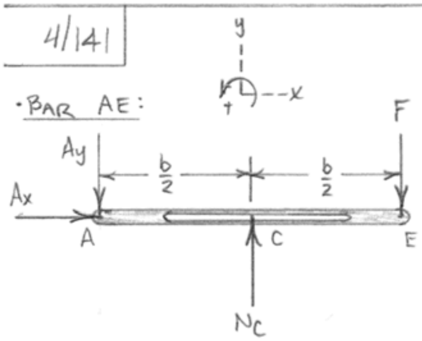
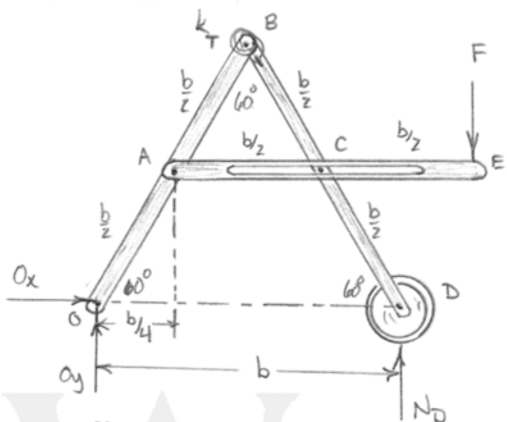
• Bar AE:



$$\Sigma M_A = 0: \quad \frac{b}{2} N_C - bF = 0$$

$$N_C = 2F$$

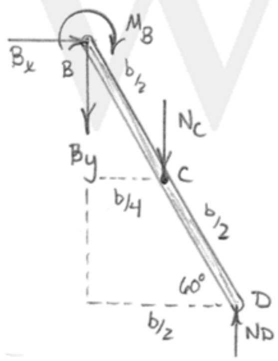
• Whole Structure:



$$\Sigma M_O = 0: \quad bN_D - \left(\frac{b}{4} + b\right)F = 0$$

$$N_D = \frac{5}{4} F$$

• Bar BD:



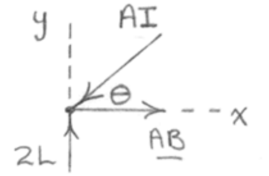
$$\Sigma M_B = 0: \quad \frac{b}{2} N_D - \frac{b}{4} N_C - M_B = 0$$

$$M_B = \frac{1}{8} Fb$$

$$M_B = k_T \frac{\pi}{3} = \frac{1}{8} Fb \longrightarrow \underline{k_T = \frac{3bF}{8\pi}}$$

4/142 From whole truss, $A = F = 2L$

A :

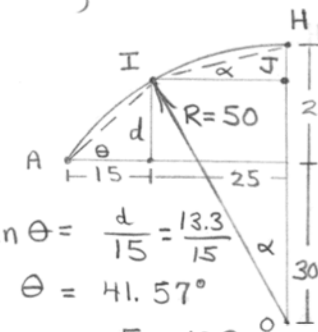


y: $2L - AI \sin\theta$

$AI = 3.01L$ C

x: $-3.01 \cos\theta + AB = 0$

$\underline{AB = 2.26L \ T}$



$\sin\alpha = \frac{25}{50}$

$\alpha = 30°$

$\tan\alpha = \frac{25}{\overline{OJ}}$

$\overline{OJ} = 43.3$

$d = 43.3 - 30$
$= 13.3$ m

$\tan\theta = \frac{d}{15} = \frac{13.3}{15}$

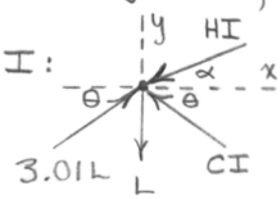
$\theta = 41.57°$

$\tan\alpha = \frac{50 - 43.3}{25}$

$\alpha = 15°$

From joint B, $\underline{BI = L \ T}$

I :



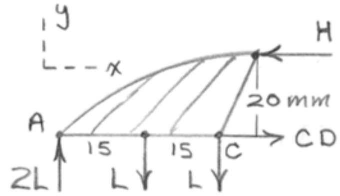
$$\begin{cases} \Sigma F_x = 0: (3.01L - CI)\cos\theta - HI\cos\alpha = 0 \\ \Sigma F_y = 0: (3.01L + CI)\sin\theta - HI\sin\alpha - L = 0 \end{cases}$$

Solve to obtain $\underline{CI = -0.458L \ (T)}$

$HI = 2.69L$ C

4/143 From whole structure, $A = F = 2L$

Half of structure:

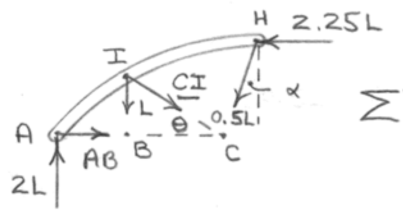


$\circlearrowleft + \sum M_C = 0: \; 20H + 15L - 30(2L) = 0; \quad H = 2.25L$

$\sum F_x = 0 \Rightarrow \quad CD = 2.25L \; T$

From joint B, $\underline{BI = L \; T}$

Member AIH:



From previous solution, $\theta = 41.57, \; \alpha = \tan^{-1}\left(\frac{10}{20}\right) = 26.57^\circ$

$\sum F_y = 0: \; 2L - L - CI \sin\theta - 0.5L\cos\alpha = 0, \quad \underline{CI = 0.833L \; T}$

$\sum F_x = 0: \; AB + 0.833L\cos\theta - 0.5L\sin\alpha - 2.25L = 0$

$\underline{AB = 1.850L \; T}$

Problem not solvable without CH data.

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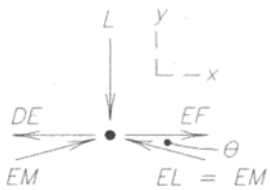
We can begin at joint E without finding the external reactions.

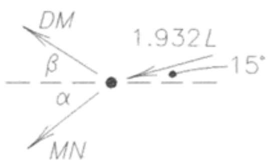
Joint E:



$$\theta = \tan^{-1}\frac{24-24\sin 60°}{24\sin 30°} = 15°$$

$$\begin{cases}\Sigma F_y = 0:\ 2(EM\sin 15°) - L = 0,\ \underline{EM = 1.932\,L\ C} \\ \Sigma F_x = 0:\ 1.932L\cos 15° - DE = 0,\ \underline{DE = 1.366\,L\ C}\end{cases}$$

Joint M:



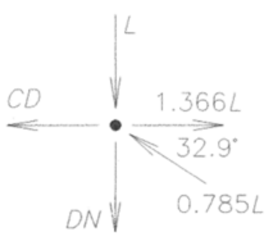
$$\alpha = \tan^{-1}\frac{24\sin 60° - 24\sin 45°}{24\cos 45° - 24\cos 60°} = 37.5°$$

$$\beta = \tan^{-1}\frac{24-24\sin 60°}{24\cos 45° - 24\cos 60°} = 32.9°$$

$$\begin{cases}\Sigma F_x = 0:\ -DM\cos 32.9° - MN\cos 37.5° - 1.932L\cos 15° = 0 \\ \Sigma F_y = 0:\ DM\sin 32.9° - MN\sin 37.5° - 1.932\sin 15° = 0\end{cases}$$

Solve simultaneously to obtain: $\underline{DM = 0.785L\ C}$

Joint D:



$$\Sigma F_y = 0:\ 0.785L\sin 32.9° - L - DN = 0$$

$$\underline{DN = 0.574L\ C}$$

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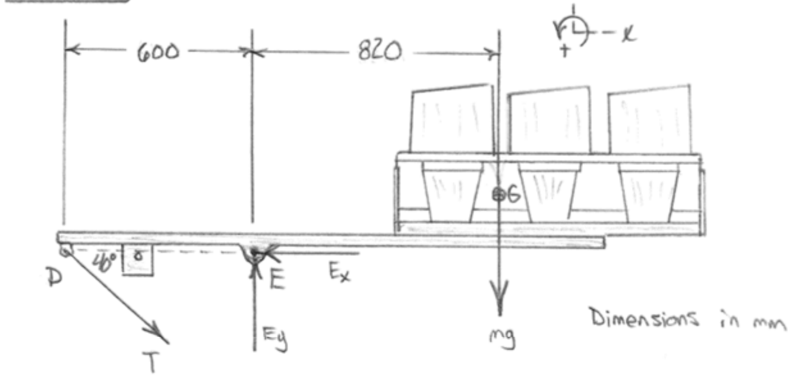
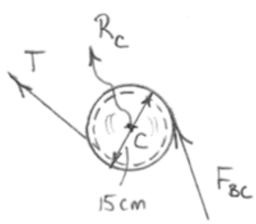
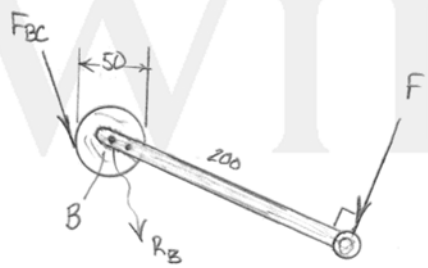
$m = 90$ kg



$\Sigma M_E = 0$: $600\,T\sin 40 - 820\,mg = 0 \longrightarrow T = 1877$ N



$\Sigma M_C = 0$: $F_{BC} = T = 1877$ N



$\Sigma M_B = 0$: $25\,F_{BC} - 200\,F = 0 \longrightarrow \underline{F = 235\text{ N}}$

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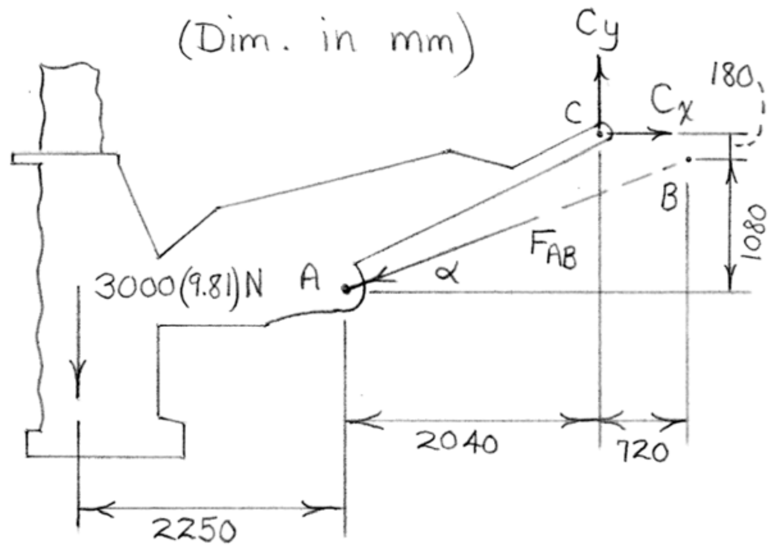

$$\alpha = \tan^{-1}\left(\frac{1080}{2040+720}\right) = 21.4°$$

$$\sum M_C = 0: -F_{AB}\cos\alpha\,(180) - F_{AB}\sin\alpha\,(720) + 3000(9.81)(2250+2040) = 0$$

$\underline{F_{AB} = 294 \text{ kN}}$

$$F_{AB} = pA : 294\,000 = p\left(\pi\,\frac{0.12^2}{4}\right)$$

$p = 26.0(10^6)$ Pa or $\underline{26.0 \text{ MPa}}$

▶4/147 Vector expressions for forces at A (treated as tensions) with $F_{AE} = F_{AF} = F_1$, $F_{BE} = F_{BF} = P$, $F_{BD} = F_{BC} = C$, are

$\underline{F}_{AE} = \frac{F_1}{1.552}(-1.2\underline{i} - 0.4\underline{j} + 0.9\underline{k})$, $\underline{F}_{AF} = \frac{F_1}{1.552}(-1.2\underline{i} + 0.4\underline{j} + 0.9\underline{k})$

$\underline{F}_{AB} = \frac{F_{AB}}{1.432}(-0.3\underline{i} + 1.4\underline{k})$, $\underline{F} = 2.2\underline{k}$. For joint A, $\Sigma\underline{F} = 0$ gives $\left[\frac{F_{AB}}{1.432}(-0.3) + \frac{2F_1}{1.552}(-1.2)\right]\underline{i}$

$+\left[2.2 + \frac{F_{AB}}{1.432}(1.4) + \frac{2F_1}{1.552}(0.9)\right]\underline{k} = \underline{0}$

Solve to get $\underline{F_{AB} = -2.681 \text{ kN}}$, $\underline{F_1 = 0.363 \text{ kN}}$

On B: $\underline{F}_{BE} = \frac{P}{1.105}(-0.9\underline{i} - 0.4\underline{j} - 0.5\underline{k})$

$\underline{F}_{BF} = \frac{P}{1.105}(-0.9\underline{i} + 0.4\underline{j} - 0.5\underline{k})$, $\underline{F}_{BD} = \frac{C}{1.105}(-0.9\underline{i} - 0.4\underline{j} + 0.5\underline{k})$

$\underline{F}_{BC} = \frac{C}{1.105}(-0.9\underline{i} + 0.4\underline{j} + 0.5\underline{k})$

For joint B, $\Sigma\underline{F} = \underline{0}$ gives

$\left(\frac{-1.8P}{1.105} - \frac{1.8C}{1.105} + 0.3\frac{-2.681}{1.432}\right)\underline{i} + \left(\frac{-P}{1.105} + \frac{C}{1.105} - 1.4\frac{-2.681}{1.432}\right)\underline{k}$

$+ 0\underline{j} = \underline{0}$. Solve to get $C = -1.620$ kN,

$P = +1.275$ kN, $\underline{F_{BE} = P = 1.275 \text{ kN}}$

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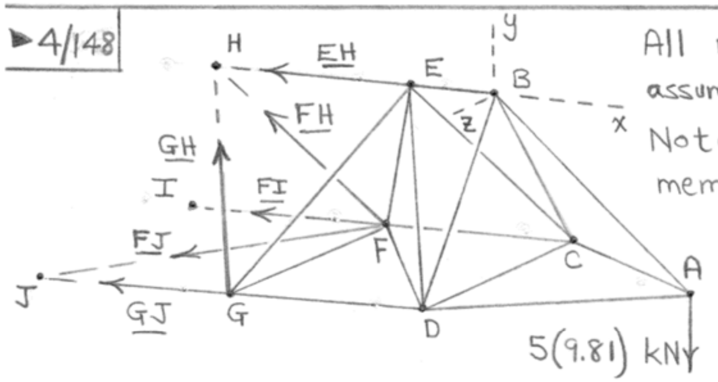


All members assumed in tension. Note that six members are cut!

$\underline{GJ} = -GJ\,\underline{i}$, $\underline{FI} = -FI\,\underline{i}$, $\underline{FJ} = \frac{FJ}{\sqrt{2}}(-\underline{i}+\underline{k})$

$\Sigma \underline{M}_H = \underline{0}$: $-49.05(5)\underline{k} + (-2\cos 30°\,\underline{j} + 2\sin 30°\,\underline{k}) \times (-GJ)\underline{i} + (-2\cos 30°\,\underline{j} - 2\sin 30°\,\underline{k}) \times (-FI)\underline{i} + (\underline{i} - 2\cos 30°\underline{j} - \underline{k}) \times \frac{FJ}{\sqrt{2}}(-\underline{i}+\underline{k}) = \underline{0}$.

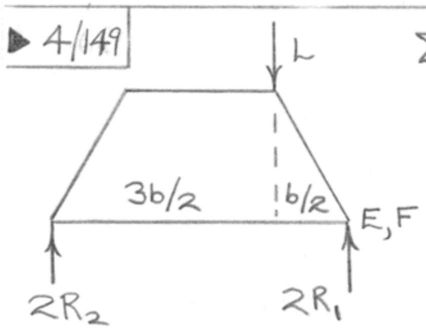
Equating unit vector coefficients to zero:

$-1.225\,FJ = 0 \Rightarrow \underline{FJ = 0}$

$\left.\begin{array}{l} -GJ + FI = 0 \\ -1.732\,GJ - 1.732\,FI = 245 \end{array}\right\}$ $FI = GJ = -70.8$ kN

$\therefore$ $\underline{\text{Force in } GJ = 70.8 \text{ kN } C}$

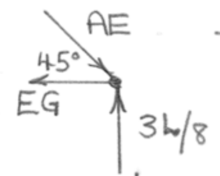
▶ 4/149



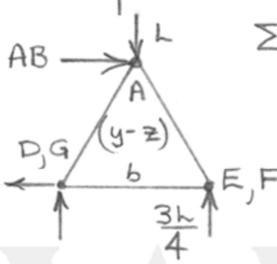
$$\Sigma M_{EF} = 0: \ 2R_2(2b) - L\frac{b}{2} = 0$$

$$R_2 = L/8, \quad R_1 = 3L/8$$

Joint E (Vertical plane through AE):



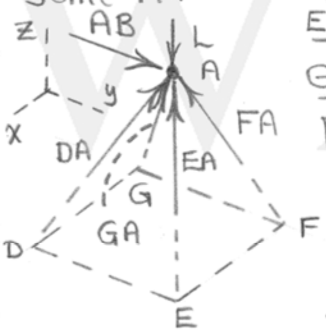
$$+\uparrow \Sigma F = 0: \ \frac{3L}{8} - \frac{AE}{\sqrt{2}} = 0, \quad AE = \frac{3L}{4\sqrt{2}}\ C$$



$$\Sigma M_{DG} = 0: \ AB\frac{b}{\sqrt{2}} + L\frac{b}{2} - \frac{3L}{4}b = 0$$

$$\underline{AB = L\sqrt{2}/4 \ \text{(Comp)}}$$

Joint A:



$$\underline{DA} = DA\left(-\tfrac{1}{2}\underline{i} + \tfrac{1}{2}\underline{j} + \tfrac{1}{\sqrt{2}}\underline{k}\right)$$

$$\underline{EA} = \frac{3L}{4\sqrt{2}}\left(-\tfrac{1}{2}\underline{i} - \tfrac{1}{2}\underline{j} + \tfrac{1}{\sqrt{2}}\underline{k}\right)$$

$$\underline{GA} = GA\left(\tfrac{1}{2}\underline{i} + \tfrac{1}{2}\underline{j} + \tfrac{1}{\sqrt{2}}\underline{k}\right)$$

$$\underline{FA} = \frac{3L}{4\sqrt{2}}\left(\tfrac{1}{2}\underline{i} - \tfrac{1}{2}\underline{j} + \tfrac{1}{\sqrt{2}}\underline{k}\right)$$

$$\underline{L} = -L\underline{k}, \quad \underline{AB} = \frac{L\sqrt{2}}{4}\underline{j}$$

$$\Sigma \underline{F} = \underline{0}: \ \underline{L} + \underline{AB} + \underline{DA} + \underline{EA} + \underline{GA} + \underline{FA} = 0$$

Note that DA = GA & EA = FA by symmetry & obtain

$\underline{i}$: $-\frac{1}{2}DA - \frac{3L}{8\sqrt{2}} + \frac{1}{2}DA + \frac{3L}{8\sqrt{2}} = 0, \quad 0 = 0$ ✓

$\underline{j}$: $\frac{L\sqrt{2}}{4} + \frac{1}{2}DA - \frac{3L}{8\sqrt{2}} + \frac{1}{2}DA - \frac{3L}{8\sqrt{2}} = 0, \quad DA = GA = \frac{L\sqrt{2}}{8}$ (Comp)

$\underline{k}$: $-L + \frac{L}{8} + \frac{3L}{8} + \frac{L}{8} + \frac{3L}{8} = 0, \quad 0 = 0$ ✓

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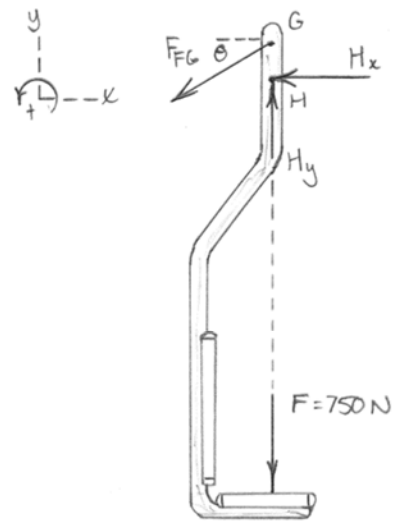
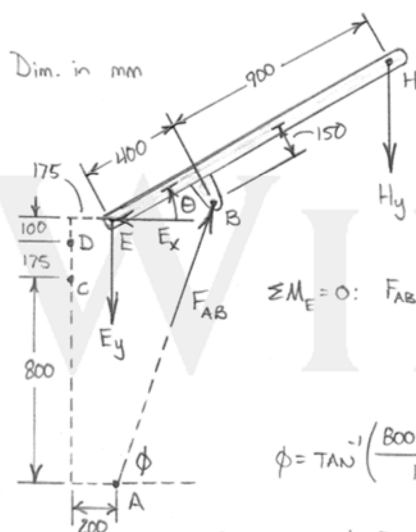


$\Sigma M_H = 0$, so... $F_{FG} = 0$

$\Sigma F_x = 0$, so... $H_x = 0$

$\Sigma F_y = 0$, so... $H_y = F = 750\text{ N}$



$F_{AB} = p A$

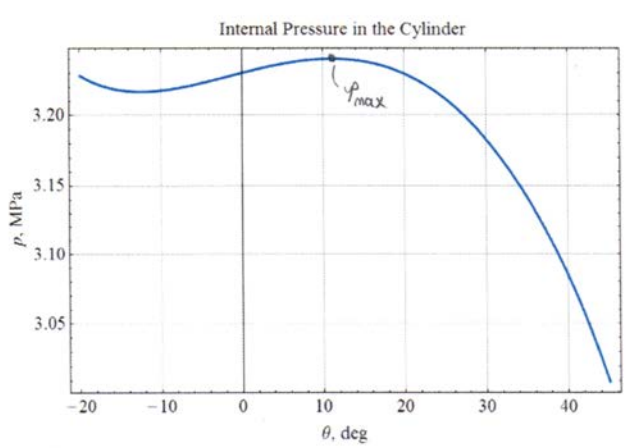
AND $A = \frac{\pi}{4} d^2$ WITH $d = 30\text{ mm}$

$\Sigma M_E = 0$: $F_{AB}\sin\phi(200-175) + F_{AB}\cos\phi(800+175+100) - (400+900)\cos\theta\, H_y = 0$

$$\phi = \tan^{-1}\left(\frac{800+175+100+400\sin\theta-150\cos\theta}{175+400\cos\theta+150\sin\theta-200}\right)$$

EVALUATE ϕ FOR A GIVEN θ AND SOLVE FOR F_{AB} & p.

PLOT OF p FOR $-20 \le \theta \le 45°$



$p_{max} = 3.24\text{ MPa}$ AT $\theta = 11.10°$

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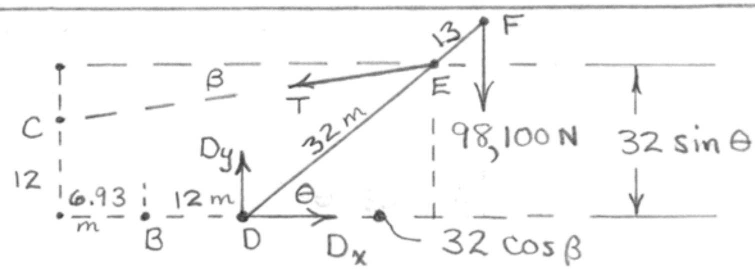
Boom:

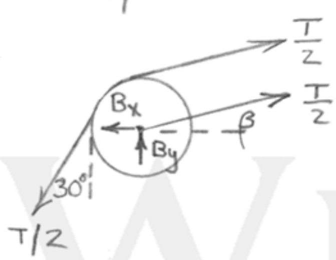


$$\beta = \tan^{-1}\left(\frac{32\sin\theta - 12}{6.93 + 12 + 32\cos\theta}\right) = \tan^{-1}\left(\frac{32\sin\theta - 12}{32\cos\theta + 18.93}\right) \quad (1)$$

$$\circlearrowleft \Sigma M_D = 0: (T\cos\beta)(32\sin\theta) - (T\sin\beta)(32\cos\theta) - (98{,}100)(45\cos\theta) = 0$$

$$\text{So} \quad T = \frac{137{,}953\cos\theta}{\cos\beta\sin\theta - \sin\beta\cos\theta} \quad (\text{in N}) \quad (2)$$

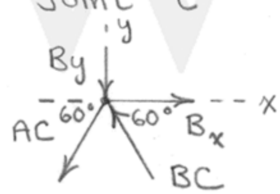
Pulley at C:



Equilibrium yields

$$\begin{cases} B_x = T\cos\beta - \frac{T}{2}\sin 30° \\ B_y = -T\sin\beta + \frac{T}{2}\cos 30° \end{cases} \quad (3)$$

Joint C

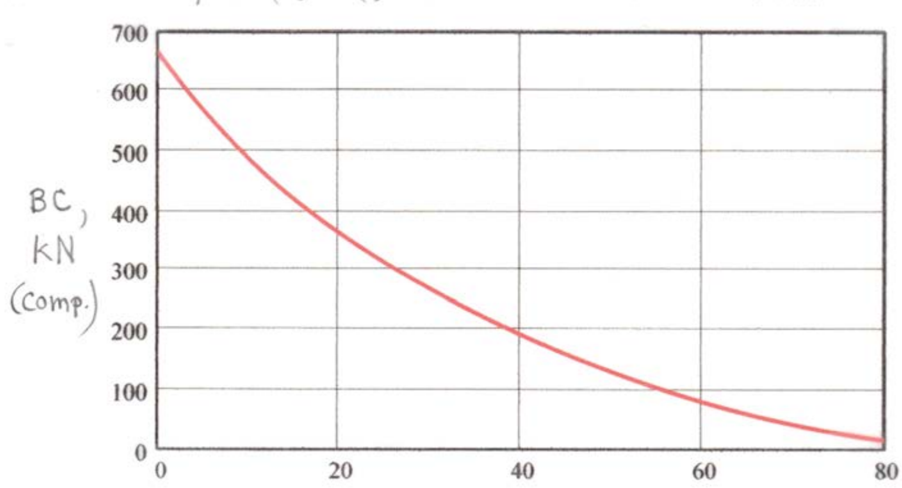


Equilibrium yields

$$BC = B_x + 0.5774 B_y \quad (4)$$

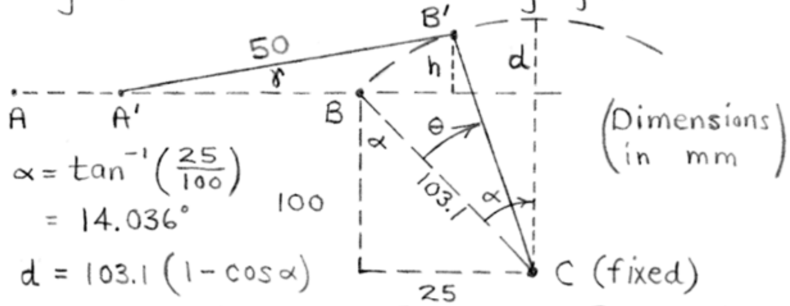
$$(AC = B_x - 0.5774 B_y)$$

Solve Eqs. (1) – (4) in order to obtain



BC @ θ = 40° = 190.5 kN

*4/152 Geometry considerations. Note that unprimed refers to $\theta=0$, primed to $\theta\neq 0$. Figure is reduced vertically by a factor of 4.



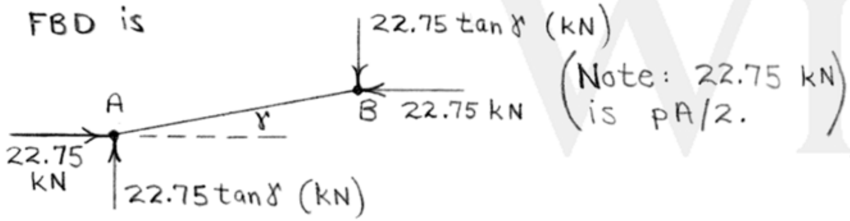
$\alpha = \tan^{-1}\left(\frac{25}{100}\right) = 14.036°$

$d = 103.1\,(1-\cos\alpha)$

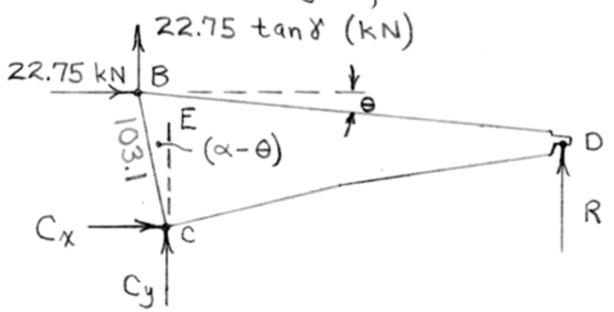
$d-h = 103.1\left[1-\cos(\alpha-\theta)\right] \Rightarrow h = 103.1\left[\cos(\alpha-\theta)-\cos\alpha\right]$

$\sin\gamma = \frac{h}{50} = 2.062\left[\cos(\alpha-\theta)-\cos\alpha\right]^{*}$

Note that AB is a two-force member. Its FBD is



FBD of lower jaw, for arbitrary θ:



$\overline{CD} = \sqrt{(450)^2+(100+25-31.25)^2} = 459.7$ mm

When $\theta=0$, $\measuredangle ECD = \beta = \tan^{-1}\left(\frac{450}{93.75}\right) = 78.23°$

When $\theta\neq 0$, $\measuredangle ECD = \beta+\theta$

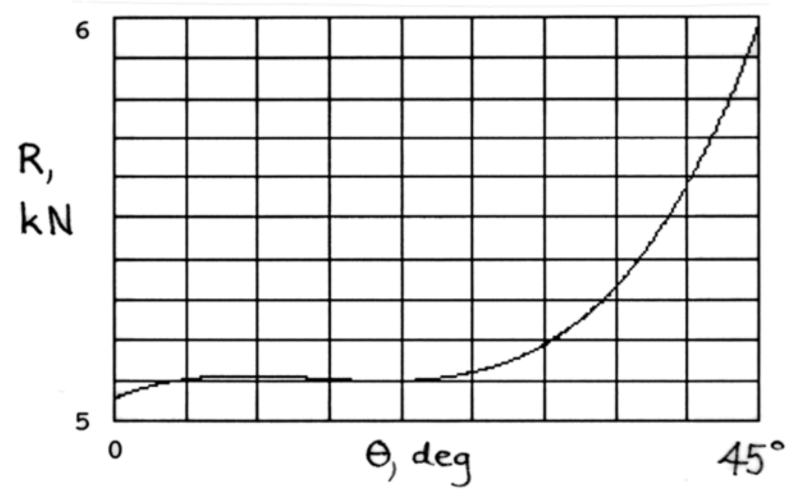
$\therefore$ Moment arm for force R about C is $\overline{CD}\,\sin(\beta+\theta)$.

$\Sigma M_C = 0:\ \overline{CD}\,\sin(\beta+\theta)\,R - 22.75(103.1)\cos(\alpha-\theta) - 22.75\,(103.1)\tan\gamma\,\sin(\alpha-\theta) = 0$

Solving for R:

$R = \frac{5.1016}{\sin(78.23+\theta)}\left[\cos(\theta-14.036°) - \tan\gamma\,\sin(\theta-14.036°)\right]$

(γ given by *) $\quad$ $R_{max} = 5.98$ kN @ $\theta = 45°$



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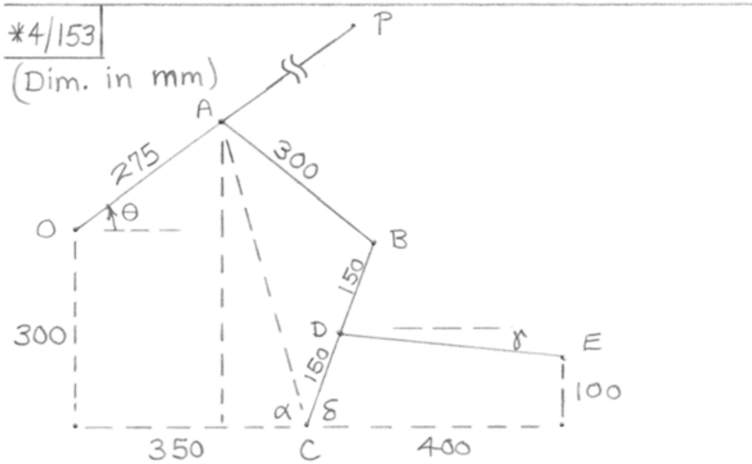
(Dim. in mm)



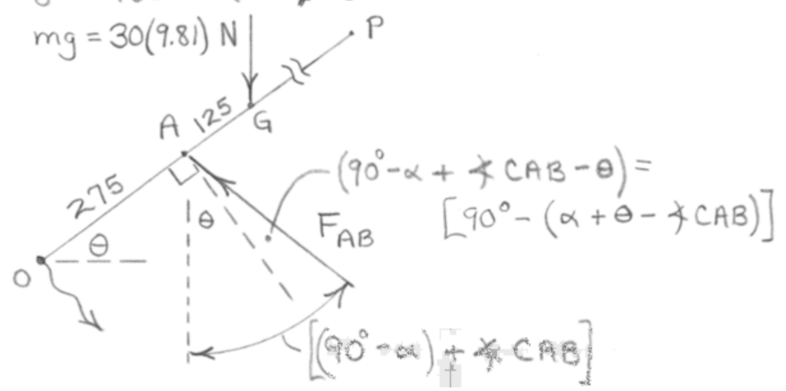
$$\overline{AC} = \sqrt{(350 - 275\cos\theta)^2 + (300 + 275\sin\theta)^2}$$

$$\measuredangle ABC = \cos^{-1}\left[\frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2\,\overline{AB}\,\overline{BC}}\right]$$

$$\alpha = \tan^{-1}\left[\frac{300 + 275\sin\theta}{350 - 275\cos\theta}\right]$$

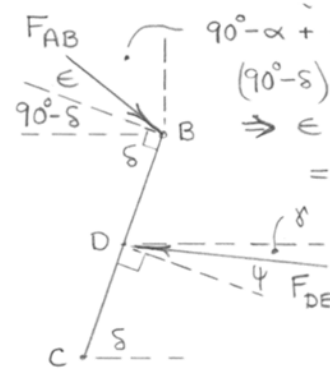
$$\measuredangle CAB = \measuredangle ACB = \frac{180^\circ - \measuredangle ABC}{2}$$

$$\delta = 180^\circ - \alpha - \measuredangle ACB$$



$$\circlearrowleft + \Sigma M_O = 0: \quad F_{AB}\cos\left[90^\circ - (\alpha + \theta - \measuredangle CAB)\right]\overline{OA} - mg\,\overline{OG}\cos\theta = 0$$

$$F_{AB} = \frac{mg\,\overline{OG}\cos\theta}{\overline{OA}\sin(\alpha + \theta - \measuredangle CAB)}$$



$90^\circ - \alpha + \measuredangle CAB$

$$(90^\circ - \delta) + (\epsilon) + (90^\circ - \alpha + \measuredangle CAB) = 90^\circ$$

$$\Rightarrow \epsilon = \alpha + \delta - \measuredangle CAB - 90^\circ$$

$$= \alpha + (180^\circ - \alpha - \measuredangle ACB) - \measuredangle CAB - 90^\circ$$

$$\epsilon = 90^\circ - 2\measuredangle ACB$$

Also, $\gamma + \psi = 90^\circ - \delta$

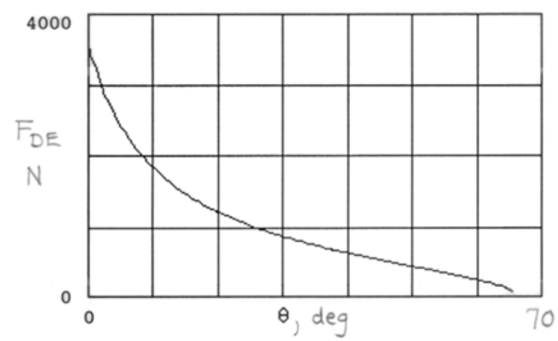
$$\psi = 90^\circ - \delta - \gamma$$

$$\gamma = \tan^{-1}\left[\frac{150\sin\delta - 100}{400 - 150\cos\delta}\right]$$

$$\circlearrowleft + \Sigma M_C = 0: \; -F_{AB}\cos\epsilon\,(\overline{CB}) + F_{DE}\cos\psi\,(\overline{CD}) = 0$$

$$F_{DE} = \frac{F_{AB}\cos\epsilon\,(\overline{CB})}{\overline{CD}\cos\psi}$$

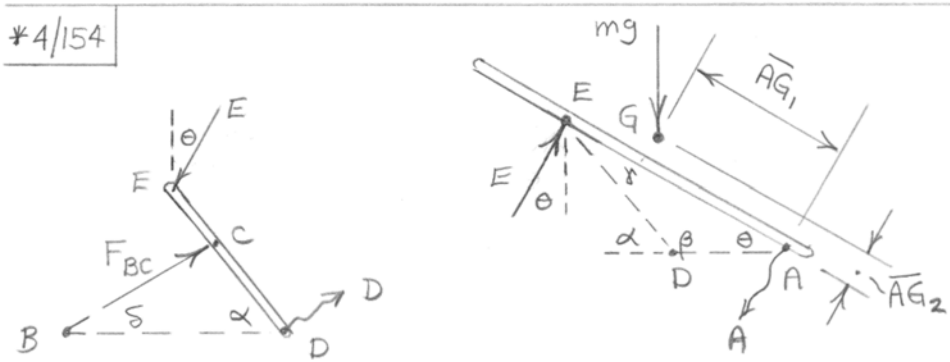
To find θ_{max}, set the above expression for $\overline{AC}$ to $\overline{AC} = 600$ mm and solve for $\underline{\theta_{max} = 65.9^\circ}$. Numerically evaluate the above expressions for $0 \leq \theta \leq \theta_{max}$ to obtain the following plot:



$\underline{(F_{DE})_{max} = 3580 \text{ N @ } \theta = 0}$

$\underline{(F_{DE})_{min} = 0 \text{ @ } \theta_{max} = 65.9^\circ}$ (links AB and BC are collinear and serve as (an unstable!) prop for the door)

*4/154



$$\frac{\sin\gamma}{\overline{AD}} = \frac{\sin\theta}{\overline{DE}}, \quad \gamma = \sin^{-1}\left(\frac{\overline{AD}\sin\theta}{\overline{DE}}\right)$$

$$\beta = 180 - (\theta + \gamma), \quad \alpha = 180 - \beta$$

$$\overline{AE} = \sqrt{\overline{DE}^2 + \overline{AD}^2 - 2(\overline{DE})(\overline{AD})\cos\beta}$$

$$\overline{BC} = \sqrt{\overline{BD}^2 + \overline{CD}^2 - 2(\overline{BD})(\overline{CD})\cos\alpha}$$

$$\frac{\sin\alpha}{\overline{BC}} = \frac{\sin\delta}{\overline{CD}}, \quad \delta = \sin^{-1}\left(\frac{\overline{CD}\sin\alpha}{\overline{BC}}\right)$$

Conveyor: $\circlearrowleft + \Sigma M_A = 0$: $-E(\overline{AE}) + mg(\overline{AG}_1)\cos\theta$

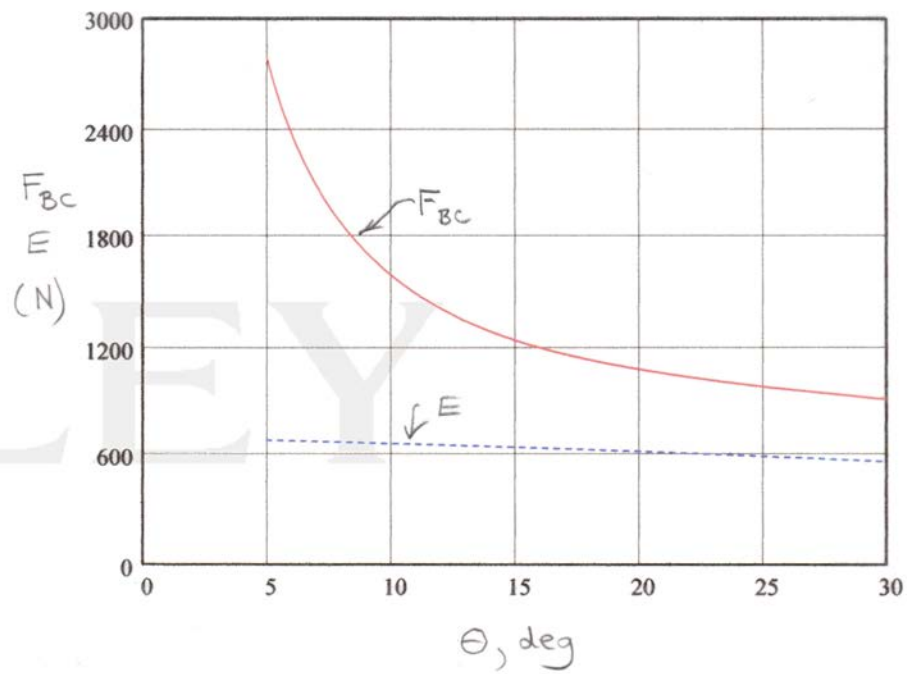
$- mg(\overline{AG}_2)\sin\theta = 0$ (1)

DE: $\circlearrowleft + \Sigma M_D = 0$:

$E\cos\theta(\overline{DE}\cos\alpha) + E\sin\theta(\overline{DE}\sin\alpha)$

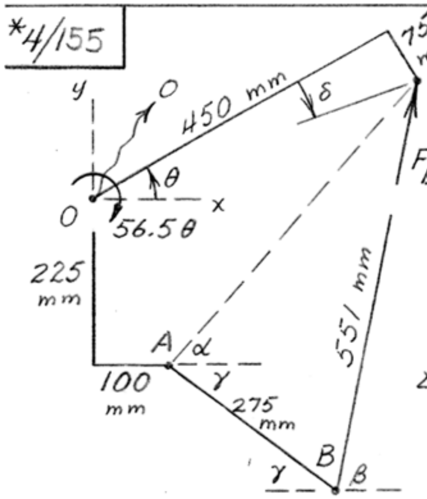
$- F_{BC}\sin\delta(\overline{BD}) = 0$ (2)

Solve Eqs. (1) & (2) for E and F_{BC} as functions of θ for the values $\overline{AD} = 1060$ mm, $\overline{BD} = 1660$ mm, $\overline{DE} = 1945$ mm, $\overline{CD} = 1150$ mm, $\overline{AG}_1 = 2130$ mm, $\overline{AG}_2 = 500$ mm, and $mg = 100(9.81)$ N to obtain the following plot:



At $\theta = 5°$, $\underline{(F_{BC})_{max} = 2800 \text{ N}}$

*4/155



$\overline{OC} = \sqrt{450^2+75^2} = 456$ mm

$\delta = \tan^{-1}(75/450) = 9.46°$

$\overline{AB} = 275$ mm; $\overline{BC} = \sqrt{350^2+425^2} = 551$ mm

$$\overline{AC} = \{[450\cos\theta + 75\sin\theta - 100]^2 + [450\sin\theta - 75\cos\theta + 225]^2\}^{1/2}$$

$$\alpha = \tan^{-1}\frac{450\sin\theta - 75\cos\theta + 225}{450\cos\theta + 75\sin\theta - 100}$$

$$\measuredangle BAC = \alpha + \gamma = \cos^{-1}\frac{\overline{AB}^2 + \overline{AC}^2 - \overline{BC}^2}{2\,\overline{AB}\,\overline{AC}}$$

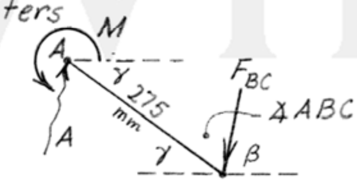
$$\measuredangle ABC = \pi - \beta - \gamma = \cos^{-1}\frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2\,\overline{AB}\,\overline{BC}}$$

$\gamma = \measuredangle BAC - \alpha$; $\beta = 180° - \measuredangle ABC - \gamma$

From above FBD of door

$\circlearrowleft + \Sigma M_O = 0: -56.5\theta + F_{BC}[\overline{OC}\sin(\beta - \theta + \delta)] = 0$ with θ in radians in 56.5θ term & $\overline{OC}$ in meters

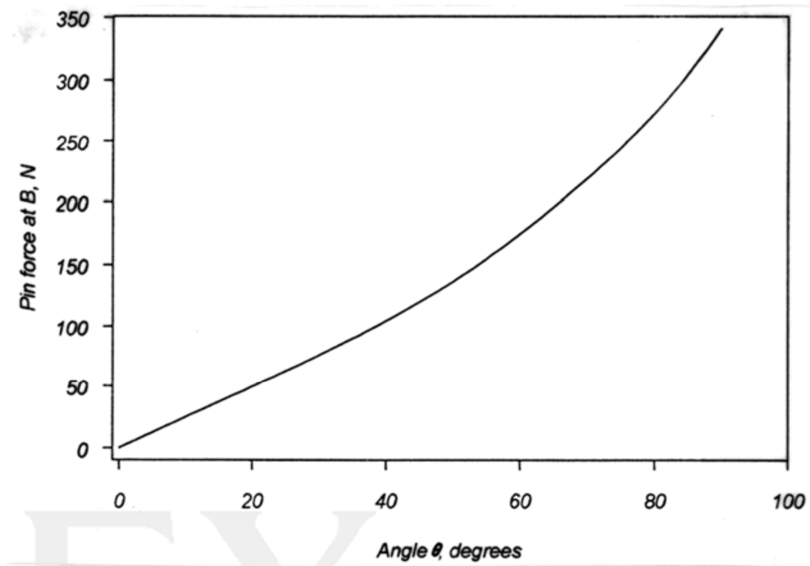
Then from FBD of AB

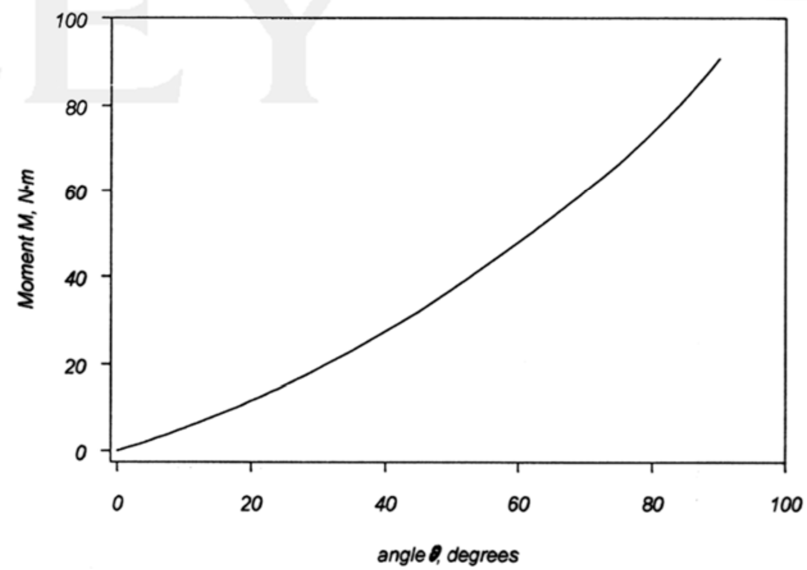


$\circlearrowleft + \Sigma M_A = 0: M - F_{BC}(0.275\sin\measuredangle ABC) = 0$

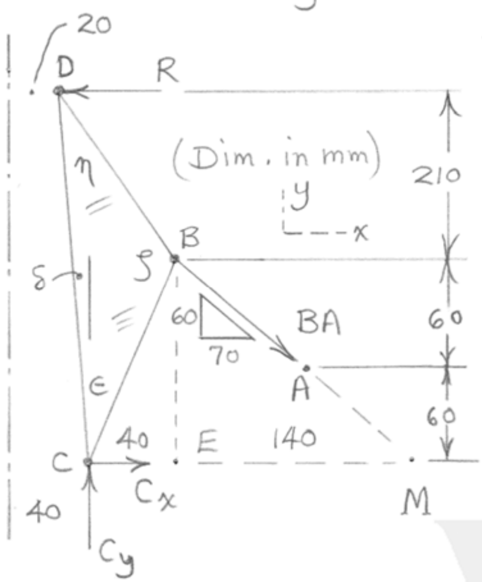
Solve above equations (in order) with θ varied from 0 to 90° to obtain the following plots for F_{BC} and M.

$M_{\theta = 45°} = 32.2$ N·m





*4/156 $\theta = 0$:



$$C_y = \frac{pA}{2} = \frac{p\pi r^2}{2} = \frac{35(10^6)\pi(0.050)^2}{2} = 137\,400 \text{ N}$$

$$\delta = \tan^{-1}\frac{20}{330} = 3.47°$$

$$\overline{CD} = \sqrt{20^2 + 330^2} = 331 \text{ mm}$$

Similarly, $\overline{BD} = 218$ mm

$\overline{BC} = 126.5$ mm

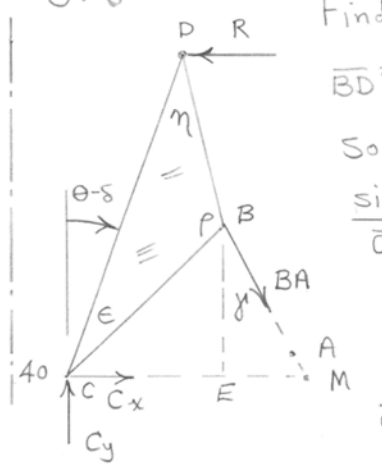
$$\circlearrowleft \Sigma M_M = 0: \; R(330) - 137\,400(180) = 0, \; \underline{R = 75\,000 \text{ N}}$$

$$\Sigma F_y = 0: \; -BA \sin\left(\tan^{-1}\frac{6}{7}\right) + 137\,400 = 0, \; \underline{BA = 211\,000 \text{ N}}$$

$$\Sigma F_x = 0: \; -75\,000 + 211\,000 \cos\left(\tan^{-1}\frac{6}{7}\right) + C_x = 0$$

$$\underline{C_x = -85\,400 \text{ N}}$$

$\theta > 0$



Find ϵ, ρ, and η:

$$\overline{BD}^2 = \overline{CD}^2 + \overline{BC}^2 - 2\,\overline{CD}\,\overline{BC}\cos\epsilon$$

Solve for ϵ, then use

$$\frac{\sin\rho}{\overline{CD}} = \frac{\sin\epsilon}{\overline{BD}} \quad \text{for } \rho$$

$$\frac{\sin\rho}{\overline{CD}} = \frac{\sin\eta}{\overline{BC}} \quad \text{for } \eta$$

$$\overline{CE} = \overline{BC}\sin(\theta - \delta + \epsilon)$$

$$\overline{BE} = \overline{BC}\cos(\theta - \delta + \epsilon)$$

$$110 = \overline{AB}\sin\gamma + \overline{BC}\sin(\theta - \delta + \epsilon) \Rightarrow \text{Solve for } \gamma$$

Then $\overline{EM} = \overline{BE}\tan\gamma$ and $\overline{CM} = \overline{CE} + \overline{EM}$

$$\circlearrowleft \Sigma M_M = 0: \; R\left(\overline{CD}\cos(\theta - \delta)\right) - C_y\left(\overline{CM}\right) = 0$$

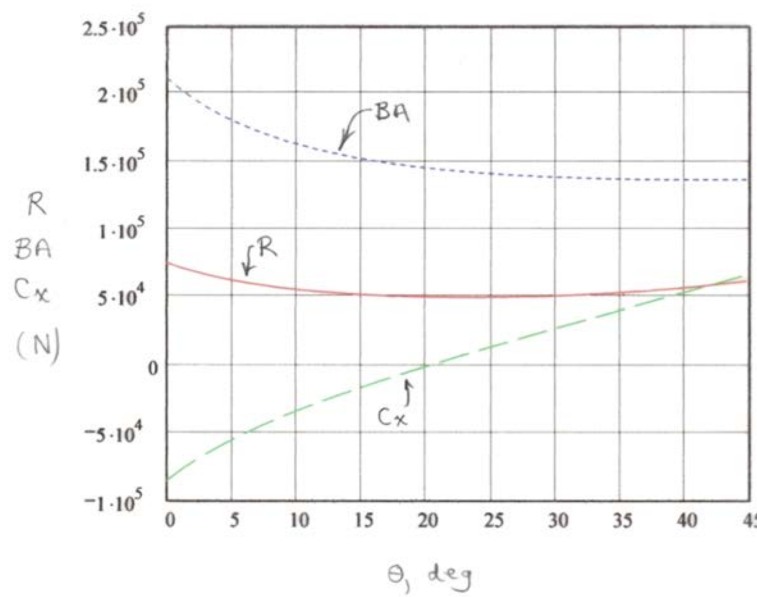
$$\underline{R = \frac{C_y\,\overline{CM}}{\overline{CD}\cos(\theta - \delta)}}$$

$$\Sigma F_y = 0: \; C_y - BA\cos\gamma = 0, \quad \underline{BA = \frac{C_y}{\cos\gamma}}$$

$$\Sigma F_x = 0: \; -R + BA\sin\gamma + C_x = 0, \quad \underline{C_x = R - BA\sin\gamma}$$

Solve all of the above in sequential order to obtain the plots below. Note that

$$\underline{R_{min} = 49\,400 \text{ N @ } \theta = 23.2°}$$



5/1 The horizontal coordinate to the centroid is $14 - \frac{1}{3}(14-2) = \underline{10}$

5/2 From Sample Problem 5/3 with $r = 8$ and $\alpha = 120° = \frac{2}{3}\pi$:

$$\bar{r} = \frac{2}{3}(8)\frac{\sin 120°}{2\pi/3} = \underline{2.21}$$

5/3

$$\bar{x} = \bar{y} = -\frac{2r}{\pi} = -\frac{2(120)}{\pi} = \underline{-76.4\text{ mm}}$$

$$\underline{\bar{z} = -180\text{ mm}}$$

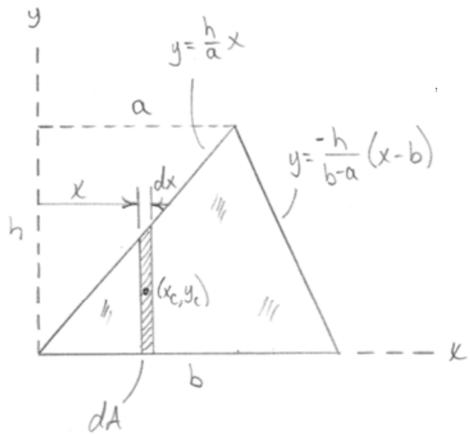
5/4

$\bar{x} = 0$

$$\bar{y} = -\frac{4r}{3\pi} = -\frac{4(120)}{3\pi} = \underline{-50.9 \text{ mm}}$$

$$\bar{z} = -360/2 = \underline{-180 \text{ mm}}$$

5/5 UTILIZE A VERTICAL STRIP



$dA = y\,dx$ & $x_c = x$

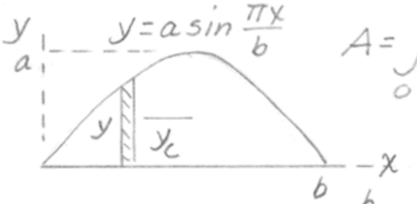
$$A = \int dA = \int_0^a \frac{h}{a}x\,dx + \int_a^b \frac{-h}{b-a}(x-b)\,dx = \frac{h}{2a}x^2\Big|_0^a - \frac{h}{b-a}\left(\frac{x^2}{2} - bx\right)\Big|_a^b$$

$$A = \frac{1}{2}ah + \frac{hb^2}{2(b-a)} + \frac{h}{b-a}\left(\frac{a^2}{2} - ab\right) \longrightarrow A = \frac{1}{2}bh$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\int_0^a \frac{h}{a}x^2\,dx + \int_a^b \frac{-h}{b-a}(x^2 - bx)\,dx}{\frac{1}{2}bh}$$

$$= \frac{\frac{1}{3}\frac{hx^3}{a}\Big|_0^a - \frac{h}{b-a}\left(\frac{x^3}{3} - \frac{bx^2}{2}\right)\Big|_a^b}{\frac{1}{2}bh} \longrightarrow \underline{\bar{x} = \frac{1}{3}(a+b)}$$

5/6



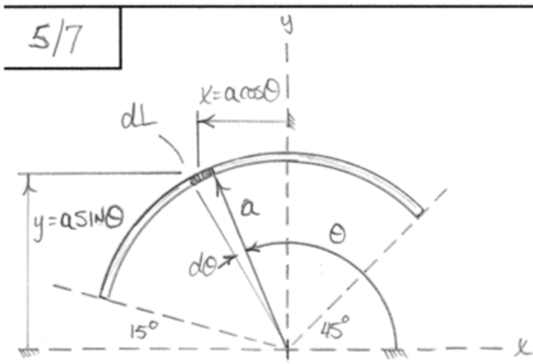
$$dA = y\,dx = a \sin \frac{\pi x}{b}\,dx$$

$$A = \int_0^b a \sin \frac{\pi x}{b}\,dx = -\frac{ab}{\pi} \cos \frac{\pi x}{b}\Big]_0^b = \frac{2ab}{\pi}$$

$$\int y_c\,dA = \int \frac{y}{2}\,dA = \int_0^b \frac{1}{2} a^2 \sin^2 \frac{\pi x}{b}\,dx = \frac{a^2}{2}\frac{b}{\pi}\left(\frac{\pi x}{2b} - \frac{\sin \frac{2\pi x}{b}}{4}\right)_0^b = a^2 b/4$$

$$\bar{y} = \int y_c\,dA / A = \frac{a^2 b}{4} \Big/ \frac{2ab}{\pi} = \underline{\frac{\pi a}{8}}$$

5/7



$$\begin{cases} 45° = \frac{\pi}{4} \text{ rad} \\ 165° = \frac{11\pi}{12} \text{ rad} \end{cases}$$

$$dL = a\,d\theta$$

$$L = \int dL = \int_{\pi/4}^{11\pi/12} a\,d\theta = a\theta \Big|_{\pi/4}^{11\pi/12} \longrightarrow L = \frac{2\pi}{3}a$$

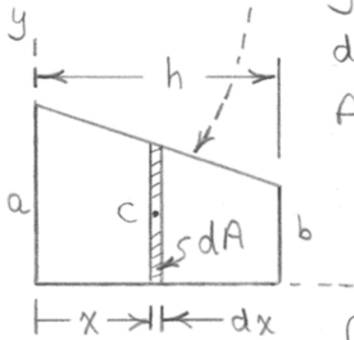
$$\bar{x} = \frac{\int x\,dL}{L} = \frac{\int_{\pi/4}^{11\pi/12} a^2\cos\theta\,d\theta}{\frac{2\pi}{3}a} = \frac{a^2\sin\theta}{\frac{2}{3}\pi a}\Big|_{\pi/4}^{11\pi/12} = \frac{3a}{2\pi}\left(\sin\frac{11\pi}{12} - \sin\frac{\pi}{4}\right)$$

$$= \frac{3a}{2\pi}\left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{1}{\sqrt{2}}\right) \longrightarrow \underline{\bar{x} = \frac{3(\sqrt{3}-3)a}{4\sqrt{2}\pi} = -0.214a}$$

$$\bar{y} = \frac{\int y\,dL}{L} = \frac{\int_{\pi/4}^{11\pi/12} a^2\sin\theta\,d\theta}{\frac{2}{3}\pi a} = \frac{-a^2\cos\theta}{\frac{2}{3}\pi a}\Big|_{\pi/4}^{11\pi/12}$$

$$\underline{\bar{y} = \frac{3(\sqrt{3}+3)a}{4\sqrt{2}\pi} = 0.799a}$$

5/8



$$y = \left(\frac{b-a}{h}\right)x + a$$

$$dA = y\,dx$$

$$A = \int dA = \int_0^h \left[\left(\frac{b-a}{h}\right)x + a\right]dx$$

$$= \left[\frac{b-a}{h}\frac{x^2}{2} + ax\right]_0^h = \frac{h}{2}(a+b)$$

$$\int x_c\,dA = \int_0^h \left[\left(\frac{b-a}{h}\right)x^2 + ax\right]dx$$

$$= \left[\frac{b-a}{h}\frac{x^3}{3} + \frac{ax^2}{2}\right]_0^h = h^2\left(\frac{b}{3} + \frac{a}{6}\right)$$

$$\int y_c\,dA = \int \frac{y}{2}\,y\,dx = \frac{1}{2}\int_0^h y^2\,dx$$

$$= \frac{1}{2}\int_0^h \left[\left(\frac{b-a}{h}\right)^2 x^2 + 2\left(\frac{b-a}{h}\right)ax + a^2\right]dx$$

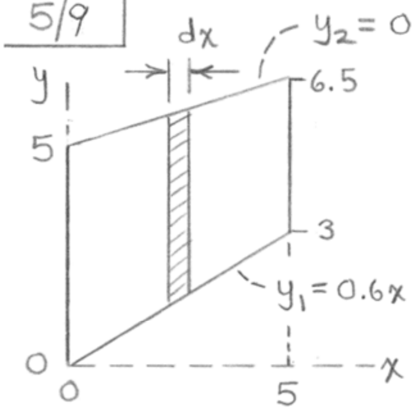
$$= \frac{1}{2}\left[\left(\frac{b-a}{h}\right)^2\frac{x^3}{3} + 2\left(\frac{b-a}{h}\right)a\frac{x^2}{2} + a^2x\right]_0^h$$

$$= \frac{h}{6}\left[a^2 + ab + b^2\right]$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{h^2\left(\frac{b}{3} + \frac{a}{6}\right)}{\frac{h}{2}(a+b)} = \underline{\frac{h(a+2b)}{3(a+b)}}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{h}{6}(a^2+ab+b^2)}{\frac{h}{2}(a+b)} = \underline{\frac{(a^2+ab+b^2)}{3(a+b)}}$$

5/9



$$A = \int_0^5 (y_2 - y_1)\,dx$$
$$= \int_0^5 (0.3x + 5 - 0.6x)\,dx$$
$$= \int_0^5 (5 - 0.3x)\,dx$$
$$= \left(5x - \frac{0.3x^2}{2}\right)_0^5 = 21.25$$

(Note: Trapezoidal area formula could be used)

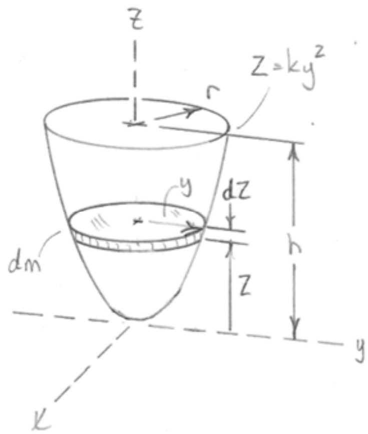
$$\int x_c\,dA = \int_0^5 x(5 - 0.3x)\,dx = \left(\frac{5}{2}x^2 - 0.1x^3\right)_0^5 = 50$$

$$\int y_c\,dA = \int_0^5 \left(\frac{y_1 + y_2}{2}\right)(y_2 - y_1)\,dx = \frac{1}{2}\int_0^5 (y_2^2 - y_1^2)\,dx$$
$$= \frac{1}{2}\int_0^5 \left[(0.3x+5)^2 - (0.6x)^2\right]dx = \frac{1}{2}\int_0^5 (25 + 3x - 0.27x^2)\,dx$$
$$= \frac{1}{2}\left[25x + \frac{3x^2}{2} - \frac{0.27x^3}{3}\right]_0^5 = 75.6$$

$$\bar{x} = \int x_c\,dA / A = 50/21.25 = \underline{2.35}$$

$$\bar{y} = \int y_c\,dA / A = 75.6/21.25 = \underline{3.56}$$

5/10



$$dm = \rho dV = \rho \pi y^2 dz$$

At $y = r$, $z = h \longrightarrow h = kr^2$

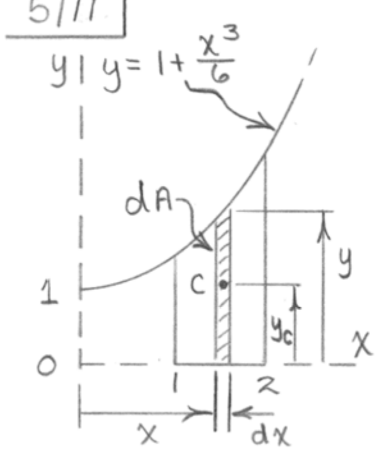
so... $k = \frac{h}{r^2}$ and $y^2 = \frac{r^2}{h} z$

Now, $dm = \rho \pi \left(\frac{r^2}{h} z\right) dz$

$$m = \int dm = \int \rho dV = \int_0^h \rho \pi \left(\frac{r^2}{h} z\right) dz \longrightarrow m = \frac{1}{2} \rho \pi r^2 h$$

$$\bar{z} = \frac{\int z_c dm}{m} = \frac{\int_0^h \rho \pi \left(\frac{r^2}{h} z^2\right) dz}{\frac{1}{2} \rho \pi r^2 h} = \frac{\frac{1}{3} \rho \pi r^2 h^2}{\frac{1}{2} \rho \pi r^2 h} \longrightarrow \underline{\bar{z} = \frac{2}{3} h}$$

5/11



$$dA = y\,dx = \left(1 + \frac{x^3}{6}\right)dx$$

$$A = \int dA = \int_1^2 \left(1 + \frac{x^3}{6}\right)dx$$

$$= x + \frac{x^4}{24}\Big|_1^2 = \frac{39}{24}$$

$$\int x_c\,dA = \int_1^2 x\left(1 + \frac{x^3}{6}\right)dx$$

$$= \frac{x^2}{2} + \frac{x^5}{30}\Big|_1^2 = \frac{38}{15}$$

$$\int y_c\,dA = \int \frac{y}{2}\,y\,dx = \int \frac{y^2}{2}\,dx = \frac{1}{2}\int_1^2 \left(1 + \frac{x^3}{6}\right)^2 dx$$

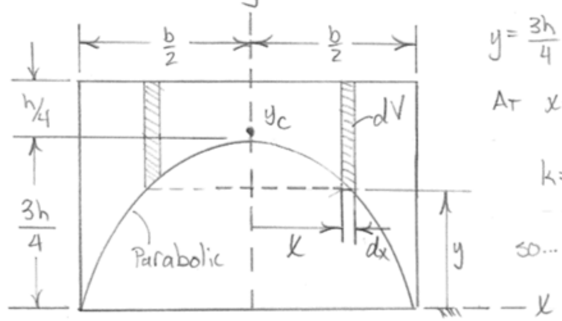
$$= \frac{1}{2}\int_1^2 \left(1 + \frac{x^3}{3} + \frac{x^6}{36}\right)dx = \frac{1}{2}\left(x + \frac{x^4}{12} + \frac{x^7}{252}\right)\Big|_1^2$$

$$= \frac{347}{252}$$

So $$\bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{38/15}{39/24} = \underline{1.559}$$

$$\bar{y} = \frac{\int y_c\,dA}{\int dA} = \frac{347/252}{39/24} = \underline{0.847}$$

5/12 UTILIZE A CYLINDRICAL SHELL PARALLEL TO y-AXIS.



$y = \frac{3h}{4} - kx^2$

At $x = \frac{b}{2}$, $y = 0 \rightarrow 0 = \frac{3h}{4} - k\frac{b^2}{4}$

$k = \frac{3h}{b^2}$

so... $y = \frac{3h}{4} - \frac{3h}{b^2}x^2$

$$V = \int dV = \int_0^{b/2} 2\pi x(h-y)\,dx = \int_0^{\frac{b}{2}} 2\pi x\left(\frac{h}{4} + \frac{3h}{b^2}x^2\right)dx = 2\pi h\left(\frac{x^2}{8} + \frac{3x^4}{4b^2}\right)\Bigg|_0^{\frac{b}{2}}$$

$$V = \frac{5}{32}\pi b^2 h$$

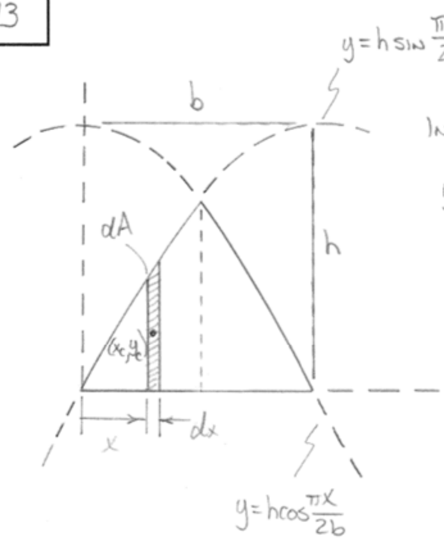
$$y_c = y + \frac{h-y}{2} = \frac{h+y}{2}$$

$$\bar{y} = \frac{\int y_c\,dV}{V} = \frac{\int_0^{b/2} \frac{1}{2}\left(h + \frac{3h}{4} - \frac{3h}{b^2}x^2\right)2\pi x\left(\frac{h}{4} + \frac{3h}{b^2}x^2\right)dx}{\frac{5}{32}\pi b^2 h}$$

$$= \frac{h}{5b^2}\int_0^{\frac{b}{2}}\left(14x + \frac{144x^3}{b^2} - \frac{288}{b^4}x^5\right)dx = \frac{h}{5b^2}\left(7x^2 + \frac{36}{b^2}x^4 - \frac{48}{b^4}x^6\right)\Bigg|_0^{b/2}$$

$$\underline{\bar{y} = \frac{13}{20}h}$$

5/13



$$dA = y\,dx = h\sin\frac{\pi x}{2b}\,dx$$

INTEGRATE OVER HALF OF THE AREA.

$$y_c = \frac{y}{2} = \frac{h}{2}\sin\frac{\pi x}{2b}$$

$$\underline{\bar{x} = \frac{b}{2} = 0.5b}$$

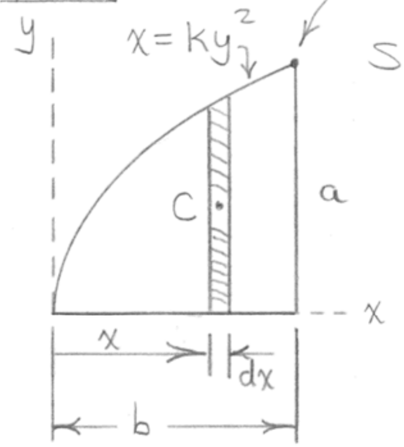
(INSPECTION)

$$A = \int dA = \int_0^{b/2} h\sin\frac{\pi x}{2b}\,dx = -\frac{2bh}{\pi}\cos\frac{\pi x}{2b}\Bigg|_0^{b/2} \longrightarrow A = \frac{bh}{\pi}(2-\sqrt{2})$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\int_0^{b/2}\frac{h^2}{2}\sin^2\frac{\pi x}{2b}\,dx}{\frac{bh}{\pi}(2-\sqrt{2})} = \frac{\frac{h^2}{2}\left(\frac{x}{2} - \frac{b}{2\pi}\sin\frac{\pi x}{b}\right)\Bigg|_0^{b/2}}{\frac{bh}{\pi}(2-\sqrt{2})} = \frac{\frac{bh^2}{8\pi}(\pi-2)}{\frac{bh}{\pi}(2-\sqrt{2})}$$

$$\underline{\bar{y} = \frac{h(\pi-2)}{8(2-\sqrt{2})} = 0.244h}$$

5/14



$x = ky^2$: $b = ka^2$, $k = \dfrac{b}{a^2}$

So $x = \dfrac{b}{a^2} y^2$

$$A = \int dA = \int y\,dx$$

$$= \int_0^b a\sqrt{\frac{x}{b}}\,dx = \frac{a}{\sqrt{b}} \frac{x^{3/2}}{3/2}\Bigg|_0^b$$

$$= \frac{2}{3}ab$$

$$\int x_c\,dA = \int_0^b \frac{a}{\sqrt{b}} x^{3/2}\,dx = \frac{a}{\sqrt{b}}\frac{x^{5/2}}{5/2}\Bigg|_0^b = \frac{2}{5}ab^2$$

$$\int y_c\,dA = \int \frac{y}{2} y\,dx = \frac{1}{2}\int y^2\,dx$$

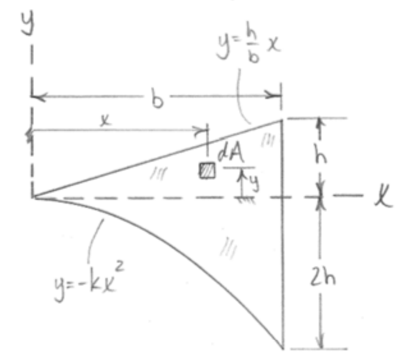
$$= \frac{1}{2}\int_0^b \frac{a^2}{b} x\,dx = \frac{1}{2}\frac{a^2}{b}\frac{x^2}{2}\Bigg|_0^b = \frac{1}{4}a^2 b$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\frac{2}{5}ab^2}{\frac{2}{3}ab} = \underline{\frac{3}{5}b}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{1}{4}a^2 b}{\frac{2}{3}ab} = \underline{\frac{3}{8}a}$$

5/15

$dA = dx\,dy$



At $x = b$, $y = -2h$..

$-2h = -kb^2 \longrightarrow k = \frac{2h}{b^2}$

$y = -\frac{2h}{b^2}x^2$

$$A = \int dA = \int_0^b \int_{-\frac{2h}{b^2}x^2}^{\frac{h}{b}x} 1\,dy\,dx = \int_0^b \left(\frac{h}{b}x + \frac{2h}{b^2}x^2\right)dx = \left.\frac{hx^2}{2b} + \frac{2hx^3}{3b^2}\right|_0^b \longrightarrow A = \frac{7}{6}bh$$

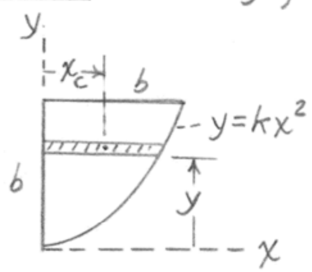
$$\bar{x} = \frac{\int x\,dA}{A} = \frac{\int_0^b \int_{-\frac{2h}{b^2}x^2}^{\frac{h}{b}x} x\,dy\,dx}{\frac{7}{6}bh} = \frac{\int_0^b \left(\frac{h}{b}x^2 + \frac{2h}{b^2}x^3\right)dx}{\frac{7}{6}bh} = \left.\frac{\left(\frac{h}{3b}x^3 + \frac{h}{2b^2}x^4\right)}{\frac{7}{6}bh}\right|_0^b$$

$$\underline{\bar{x} = \frac{5}{7}b}$$

$$\bar{y} = \frac{\int y\,dA}{A} = \frac{\int_0^b \int_{-\frac{2h}{b^2}x^2}^{\frac{h}{b}x} y\,dy\,dx}{\frac{7}{6}bh} = \frac{\int_0^b \frac{1}{2}\left(\frac{h^2}{b^2}x^2 - \frac{4h^2}{b^4}x^4\right)dx}{\frac{7}{6}bh} = \left.\frac{\frac{1}{2}\left(\frac{h^2}{3b^2}x^3 - \frac{4h^2}{5b^4}x^5\right)}{\frac{7}{6}bh}\right|_0^b$$

$$\underline{\bar{y} = \frac{-h}{5}}$$

5/16 $dA = x\,dy$; $k = \frac{b}{b^2} = \frac{1}{b}$ so $x^2 = by$, $x_c = x/2$



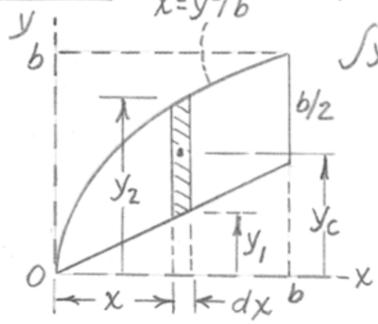
$$\bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{\int_0^b \frac{x}{2} x\,dy}{\int_0^b x\,dy}$$

$$= \frac{1}{2}\int_0^b by\,dy \Big/ \int_0^b \sqrt{by}\,dy$$

$$= \frac{1}{4}b^3 \Big/ \frac{2}{3}b^2 = \underline{\frac{3}{8}b}$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} = \int_0^b y\sqrt{by}\,dy \Big/ \frac{2}{3}b^2 = \frac{2}{5}b^3 \Big/ \frac{2}{3}b^2 = \underline{\frac{3}{5}b}$$

5/17



$y_1 = x/2\ ,\quad y_2 = \sqrt{bx}$

$$\int y_c\,dA = \int_0^b \frac{y_1+y_2}{2}(y_2-y_1)\,dx$$

$$= \frac{1}{2}\int_0^b (y_2^2 - y_1^2)\,dx$$

$$= \frac{1}{2}\int_0^b \left(bx - \frac{x^2}{4}\right)dx$$

$$= \frac{1}{2}\left[\frac{bx^2}{2} - \frac{x^3}{12}\right]_0^b = \frac{5}{24}b^3$$

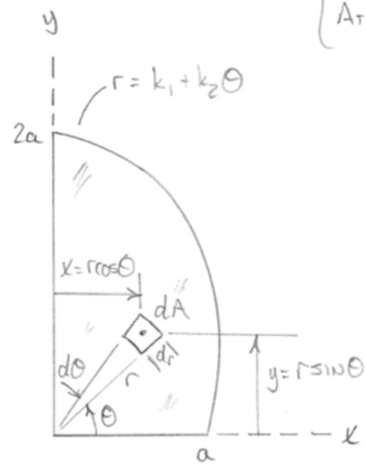
$$\int dA = \int_0^b (y_2 - y_1)\,dx = \int_0^b (\sqrt{bx} - x/2)\,dx = \left[\frac{2\sqrt{b}}{3}x^{3/2} - \frac{x^2}{4}\right]_0^b$$

$$= \frac{5}{12}b^2$$

$$\bar{y} = \frac{\int y_c\,dA}{\int dA} = \frac{\frac{5}{24}b^3}{\frac{5}{12}b^2} = \underline{\frac{b}{2}}$$

5/18

$$\begin{cases} \text{At } \theta = 0,\ r = a \longrightarrow k_1 = a \\ \text{At } \theta = \frac{\pi}{2},\ r = 2a \longrightarrow k_2 = \frac{2a}{\pi} \end{cases}$$



$$\begin{cases} r = a + \frac{2a}{\pi}\theta \\ dA = r\,dr\,d\theta \end{cases}$$

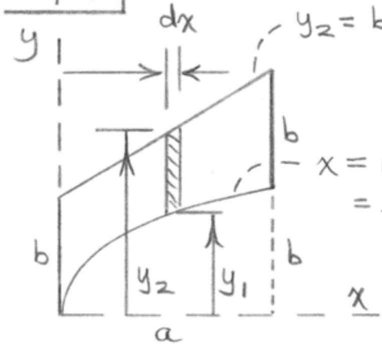
$$A = \int dA = \int_0^{\frac{\pi}{2}} \int_0^{a+\frac{2a}{\pi}\theta} r\,dr\,d\theta = \int_0^{\frac{\pi}{2}} \frac{r^2}{2}\Bigg|_0^{a+\frac{2a}{\pi}\theta} = \int_0^{\frac{\pi}{2}} \frac{1}{2}\left(a^2 + \frac{4a^2}{\pi}\theta + \frac{4a^2}{\pi^2}\theta^2\right) d\theta$$

$$= \frac{1}{2}\left(a^2\theta + \frac{2a^2\theta^2}{\pi} + \frac{4a^2\theta^3}{3\pi^2}\right)\Bigg|_0^{\frac{\pi}{2}} \longrightarrow A = \frac{7}{12}\pi a^2$$

$$\bar{x} = \frac{\int x\,dA}{A} = \frac{\int_0^{\frac{\pi}{2}} \int_0^{a+\frac{2a}{\pi}\theta} r^2\cos\theta\,dr\,d\theta}{\frac{7}{12}\pi a^2} = \frac{4}{7\pi a^2}\int_0^{\frac{\pi}{2}} \left(a + \frac{2a}{\pi}\theta\right)^3 \cos\theta\,d\theta$$

$$= \frac{4a^3}{7\pi a^2}\Bigg[\sin\theta + \frac{6}{\pi}(\cos\theta + \theta\sin\theta) + \frac{12}{\pi^2}\left(2\theta\cos\theta + (\theta^2 - 2)\sin\theta\right) + \frac{8}{\pi^3}(3\theta^2 - 6)\cos\theta + \frac{8}{\pi^3}(\theta^3 - 6\theta)\sin\theta\Bigg]\Bigg|_0^{\pi/2} \longrightarrow \underline{\bar{x} = 0.505a}$$

5/19



$$A = \int_0^a (y_2 - y_1)\,dx$$

$$= \int_0^a \left[b\left(1+\frac{x}{a}\right) - b\left(\frac{x}{a}\right)^{\frac{1}{2}}\right]dx$$

$$= b\left(x + \frac{x^2}{2a}\right) - \frac{b}{\sqrt{a}}\,\frac{2}{3}x^{3/2}\Big|_0^a$$

$$= \frac{5}{6}ab$$

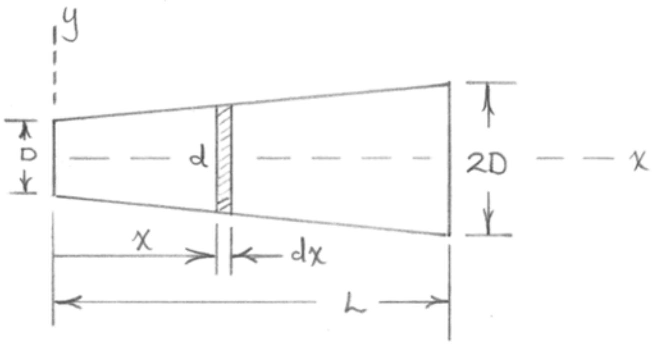
$$\int y_c\,dA = \int_0^a \left(\frac{y_1+y_2}{2}\right)(y_2-y_1)\,dx = \frac{1}{2}\int_0^a (y_2^2 - y_1^2)\,dx$$

$$= \frac{1}{2}\int_0^a \left[b^2\left(1+\frac{x}{a}\right)^2 - \frac{b^2}{a}x\right]dx$$

$$= \frac{1}{2}\left[b^2\left(x + \frac{x^2}{a} + \frac{x^3}{3a^2}\right) - \frac{b^2x^2}{2a}\right]\Big|_0^a = \frac{11}{12}ab^2$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{11ab^2/12}{5ab/6} = \underline{\frac{11}{10}b}$$

5/20



For constant density, $\bar{x}V = \int x\,dV$

Diameter $d = D(1 + x/L)$

So $dV = \frac{\pi d^2}{4}dx = \frac{\pi D^2}{4}\left(1+\frac{x}{L}\right)^2 dx$

$$V = \frac{\pi D^2}{4}\int_0^L \left(1+\frac{x}{L}\right)^2 dx = \frac{\pi D^2}{4}\left[x + \frac{x^2}{L} + \frac{x^3}{3L^2}\right]_0^L$$

$$= \frac{7}{12}\pi D^2 L$$

$$\int x\,dV = \frac{\pi D^2}{4}\int_0^L x\left(1+\frac{x}{L}\right)^2 dx = \frac{\pi D^2}{4}\left[\frac{x^2}{2} + \frac{2x^3}{3L} + \frac{x^4}{4L^2}\right]_0^L$$

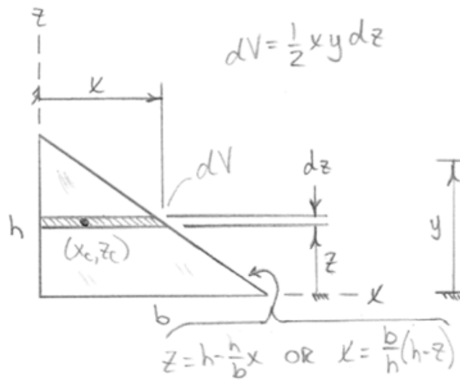
$$= \frac{17}{48}\pi D^2 L^2$$

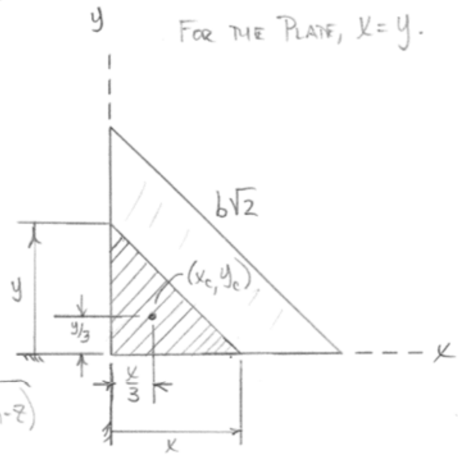
$$\bar{x} = \frac{\frac{17}{48}\pi D^2 L^2}{\frac{7}{12}\pi D^2 L} = \underline{\frac{17}{28}L}$$

5/21 USE A TRIANGULAR ELEMENT

FOR THE PLATE, $x = y$.

$$dV = \frac{1}{2} x y \, dz$$





$$V = \int dV = \int_0^h \frac{1}{2} x y \, dz = \int_0^h \frac{1}{2} x^2 \, dz = \int_0^h \frac{b^2}{2h^2}(h-z)^2 \, dz$$

$$= \frac{b^2}{2h^2}\left(h^2 z - h z^2 + \frac{1}{3} z^3\right)\Big|_0^h \longrightarrow V = \frac{1}{6} b^2 h$$

$$\bar{x} = \frac{\int x_c \, dV}{V} = \frac{\int_0^h \frac{1}{2} x^2 \left(\frac{x}{3}\right) dz}{\frac{1}{6} b^2 h} = \frac{\int_0^h \frac{1}{6} x^3 \, dz}{\frac{1}{6} b^2 h} = \frac{\int_0^h \frac{b^3}{6h^3}(h-z)^3 \, dz}{\frac{1}{6} b^2 h}$$

$$= \frac{-b}{4h^4}(h-z)^4 \Big|_0^h \longrightarrow \underline{\bar{x} = \bar{y} = \frac{1}{4} b}$$

BY SYMMETRY... $\underline{\bar{z} = \frac{1}{4} h}$

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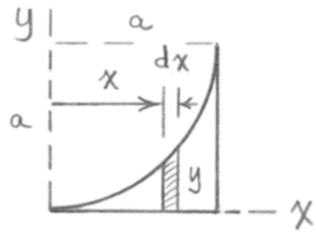


$$x^2 + (y-a)^2 = a^2$$

$$y = a - \sqrt{a^2 - x^2}$$

(use minus sign)

$$A = \int y\,dx = \int_0^a \left[a - \sqrt{a^2 - x^2}\right] dx$$

$$= \left[ax - \frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2 \sin^{-1}\frac{x}{a}\right)\right]_0^a = a^2\left(1 - \frac{\pi}{4}\right)$$

$$\int x_c\,dA = \int_0^a xy\,dx = \int_0^a \left[ax - x\sqrt{a^2-x^2}\right] dx$$

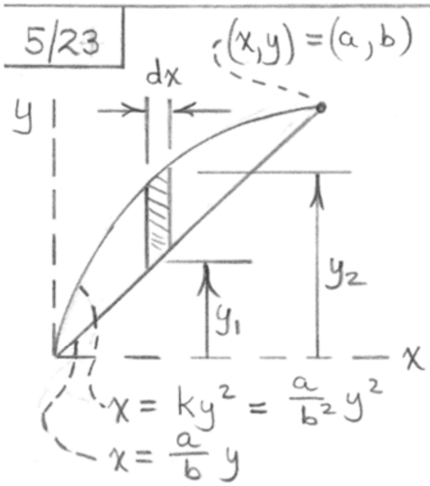
$$= \left[\frac{ax^2}{2} + \frac{1}{3}(a^2 - x^2)^{3/2}\right]_0^a = a^3/6$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{a^3/6}{a^2\left(1 - \frac{\pi}{4}\right)} = \underline{\frac{2a}{3(4-\pi)}} = \underline{0.777a}$$

From symmetry, $\bar{y} = a - \bar{x} = a - \frac{2a}{3(4-\pi)} = \underline{\frac{10-3\pi}{3(4-\pi)}a}$

or $\bar{y} = \underline{0.223a}$

5/23



$$A = \int_0^a (y_2 - y_1)\,dx = \int_0^a \left(b\sqrt{\frac{x}{a}} - x\frac{b}{a}\right)dx = b\left[\frac{1}{\sqrt{a}}\frac{2x^{3/2}}{3} - \frac{1}{2a}x^2\right]_0^a = \frac{ab}{6}$$

$$\int x_c\,dA = \int_0^a x(y_2 - y_1)\,dx = \int_0^a \left[\frac{b}{\sqrt{a}}x^{3/2} - \frac{b}{a}x^2\right]dx$$

$$= b\left[\frac{2x^{5/2}}{5\sqrt{a}} - \frac{x^3}{3a}\right]_0^a = \frac{a^2b}{15}$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{ab^2/15}{ab/6} = \underline{\frac{2}{5}a}$$

$$\int y_c\,dA = \int_0^a \left(\frac{y_1 + y_2}{2}\right)(y_2 - y_1)\,dx = \frac{1}{2}\int_0^a (y_2^2 - y_1^2)\,dx$$

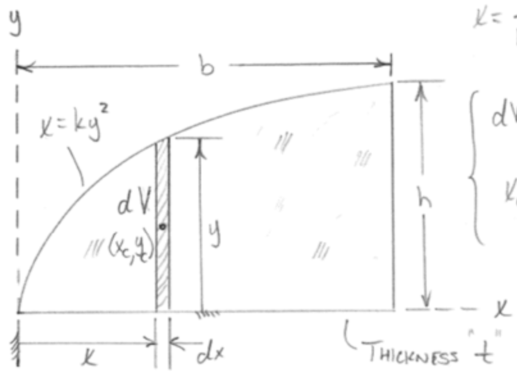
$$= \frac{1}{2}\int_0^a \left(\frac{xb^2}{a} - \frac{x^2b^2}{a^2}\right)dx = \frac{1}{12}ab^2$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{ab^2/12}{ab/6} = \underline{\frac{b}{2}}$$

5/24

At $x = b,\ y = h$ so... $b = kh^2 \longrightarrow k = \frac{b}{h^2}$

$x = \frac{b}{h^2} y^2$ OR $y = \frac{h}{\sqrt{b}} x^{1/2}$



$dV = t y\,dx = \frac{th}{\sqrt{b}} x^{1/2} dx$

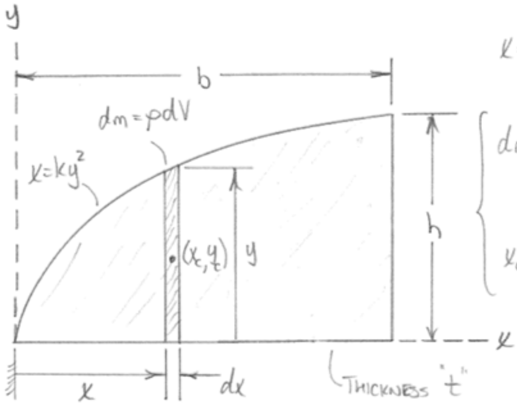
$x_c = x$ AND $y_c = \frac{1}{2} y = \frac{h}{2\sqrt{b}} x^{1/2}$

$$V = \int_0^b \frac{th}{\sqrt{b}} x^{1/2} dx = \frac{2th}{3\sqrt{b}} x^{3/2} \Big|_0^b \longrightarrow \underline{V = \tfrac{2}{3} tbh}$$

$$\bar{x} = \frac{\int x_c\,dV}{V} = \frac{\int_0^b \frac{th}{\sqrt{b}} x^{3/2} dx}{\frac{2}{3} tbh} = \frac{\frac{2th}{5\sqrt{b}} x^{5/2}}{\frac{2}{3} tbh} \Bigg|_0^b \longrightarrow \underline{\bar{x} = \tfrac{3}{5} b}$$

$$\bar{y} = \frac{\int y_c\,dV}{V} = \frac{\int_0^b \left(\frac{th}{\sqrt{b}} x^{1/2}\right)\left(\frac{h}{2\sqrt{b}} x^{1/2}\right) dx}{\frac{2}{3} tbh} = \frac{\frac{th^2 x^2}{4b}}{\frac{2}{3} tbh} \Bigg|_0^b \longrightarrow \underline{\bar{y} = \tfrac{3}{8} h}$$

5/25 At $x=b$, $y=h$ so... $b=kh^2 \longrightarrow k=\frac{b}{h^2}$



$x=\frac{b}{h^2}y^2$ OR $y=\frac{h}{\sqrt{b}}x^{1/2}$

$$\begin{cases} dm=\rho dV=\dfrac{\rho t h}{\sqrt{b}}x^{1/2}dx \\ x_c=x \quad \text{AND} \quad y_c=\frac{1}{2}y=\dfrac{h}{2\sqrt{b}}x^{1/2} \end{cases}$$

$\rho=\rho_0\left(1+\frac{x}{2b}\right)$

$$m=\int dm=\int_0^b \rho_0\left(1+\frac{x}{2b}\right)\frac{th}{\sqrt{b}}x^{1/2}dx=\left(\frac{2}{3}\rho_0\frac{th}{\sqrt{b}}x^{3/2}+\frac{1}{5}\rho_0\frac{th}{b^{3/2}}x^{5/2}\right)\Bigg|_0^b$$

$\therefore \quad m=\frac{13}{15}\rho_0 thb$

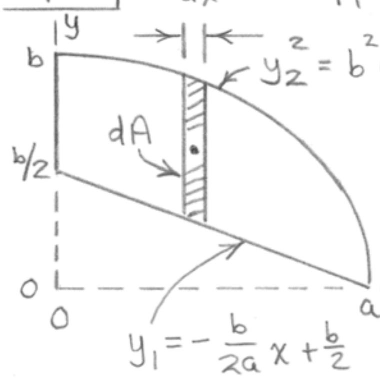
$$\bar{x}=\frac{\int x_c dm}{m}=\frac{\int_0^b x\rho_0\left(1+\frac{x}{2b}\right)\frac{th}{\sqrt{b}}x^{1/2}dx}{\frac{13}{15}\rho_0 thb}=\frac{\frac{2}{5}\rho_0\frac{th}{\sqrt{b}}x^{5/2}+\frac{1}{7}\rho_0\frac{th}{b^{3/2}}x^{7/2}}{\frac{13}{15}\rho_0 thb}$$

$\therefore \quad \underline{\bar{x}=\frac{57}{91}b}$

$$\bar{y}=\int y_c dm=\frac{\int_0^b\left(\frac{h}{2\sqrt{b}}x^{1/2}\right)\rho_0\left(1+\frac{x}{2b}\right)\frac{th}{\sqrt{b}}x^{1/2}dx}{\frac{13}{15}\rho_0 thb}=\frac{\frac{1}{4}\rho_0\frac{th^2}{b}x^2+\frac{1}{12}\rho_0\frac{h^2t}{b^2}x^3}{\frac{13}{15}\rho_0 thb}$$

$\therefore \quad \underline{\bar{y}=\frac{5}{13}h}$

5/26



$$A = \pi\frac{ab}{4} - \frac{1}{2}a\frac{b}{2} = \frac{ab}{4}(\pi - 1)$$

$$dA = (y_2 - y_1)\,dx$$

$$= \left[b\sqrt{1-\frac{x^2}{a^2}} - \left(-\frac{b}{2a}x + \frac{b}{2}\right)\right]dx$$

$$= \frac{b}{a}\left[\sqrt{a^2-x^2} + \frac{x}{2} - \frac{a}{2}\right]dx$$

$$\int x_c\,dA = \int_0^a x\frac{b}{a}\left[\sqrt{a^2-x^2} + \frac{x}{2} - \frac{a}{2}\right]dx$$

$$= \frac{b}{a}\left[-\frac{1}{3}\sqrt{(a^2-x^2)^3} + \frac{x^3}{6} - \frac{ax^2}{4}\right]_0^a = \frac{1}{4}ba^2$$

$$\int y_c\,dA = \int_0^a\left(\frac{y_2+y_1}{2}\right)(y_2-y_1)\,dx = \frac{1}{2}\int_0^a\left(y_2^2 - y_1^2\right)dx$$

$$= \frac{1}{2}\int_0^a\left[b^2\left(1-\frac{x^2}{a^2}\right) - \left(-\frac{b}{2a}x + \frac{b}{2}\right)^2\right]dx$$

$$= \frac{1}{2}\int_0^a\left[b^2\left(1-\frac{x^2}{a^2}\right) - \frac{b^2}{4a^2}x^2 + \frac{b^2}{2a}x - \frac{b^2}{4}\right]dx$$

$$= \frac{1}{2}\left[b^2\left(x - \frac{x^3}{3a^2}\right) - \frac{b^2}{4a^2}\frac{x^3}{3} + \frac{b^2}{2a}\frac{x^2}{2} - \frac{b^2}{4}x\right]_0^a$$

$$= \frac{7}{24}ab^2$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\frac{1}{4}ba^2}{\frac{ab}{4}(\pi-1)} = \underline{\frac{a}{\pi-1}}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{7}{24}ab^2}{\frac{ab}{4}(\pi-1)} = \underline{\frac{7b}{6(\pi-1)}}$$

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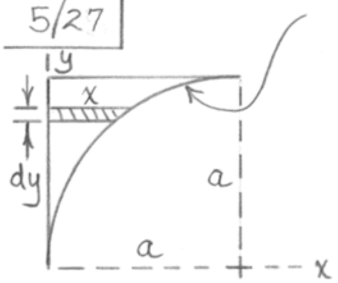

$$(x-a)^2 + y^2 = a^2$$

$$dA = x\,dy = \left(a - \sqrt{a^2-y^2}\right)dy$$

$$\int x_c\,dA = \int \frac{x}{2}\,x\,dy$$

$$= \frac{1}{2}\int_0^a \left(a - \sqrt{a^2-y^2}\right)^2 dy$$

$$\int x_c\,dA = \int_0^a \left(a^2 - a\sqrt{a^2-y^2} - \frac{y^2}{2}\right)dy$$

$$= \left[a^2 y - \frac{a}{2}\left(y\sqrt{a^2-y^2} + a^2 \sin^{-1}\frac{y}{a}\right) - \frac{y^3}{6}\right]_0^a$$

$$= \left(\frac{5}{6} - \frac{\pi}{4}\right)a^3$$

$$\int y_c\,dA = \int y\,x\,dy = \int_0^a \left(ay - y\sqrt{a^2-y^2}\right)dy$$

$$= \left[\frac{ay^2}{2} + \frac{1}{3}\sqrt{(a^2-y^2)^3}\right]_0^a = \frac{a^3}{6}$$

$$A = a^2 - \frac{1}{4}\pi a^2 = a^2\left(1 - \frac{\pi}{4}\right)$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\left(\frac{5}{6} - \frac{\pi}{4}\right)a^3}{\left(1-\frac{\pi}{4}\right)a^2} = \frac{10-3\pi}{3(4-\pi)}a = \underline{0.223a}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{a^3/6}{\left(1-\frac{\pi}{4}\right)a^2} = \frac{2a}{3(4-\pi)} = \underline{0.777a}$$

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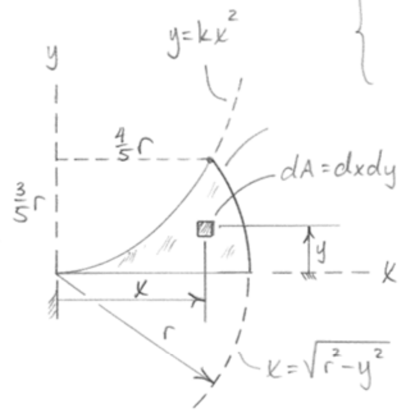


At $x = \frac{4}{5}r$, $y = \frac{3}{5}r$

$\frac{3}{5}r = k\left(\frac{4}{5}r\right)^2 \longrightarrow k = \frac{15}{16r}$

$y = \frac{15x^2}{16r}$ OR $x = \frac{4\sqrt{ry}}{\sqrt{15}}$

$$A = \int dA = \int_0^{\frac{3}{5}r}\int_{\frac{4\sqrt{ry}}{\sqrt{15}}}^{\sqrt{r^2-y^2}} 1\,dx\,dy = \int_0^{\frac{3}{5}r} x\Big|_{\frac{4\sqrt{ry}}{\sqrt{15}}}^{\sqrt{r^2-y^2}} dy = \int_0^{\frac{3}{5}r}\left(\sqrt{r^2-y^2} - \frac{4\sqrt{ry}}{\sqrt{15}}\right)dy$$

$$= \left[\frac{-8\sqrt{r}\,y^{3/2}}{3\sqrt{15}} + \frac{1}{2}\left(y\sqrt{r^2-y^2} + r^2\sin^{-1}\left(\frac{y}{r}\right)\right)\right]\Bigg|_0^{3/5\,r} \longrightarrow A = 0.242\,r^2$$

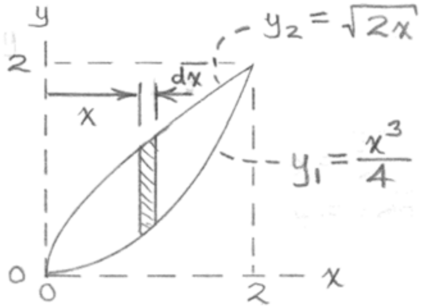
$$\bar{x} = \frac{\int x\,dA}{A} = \frac{\int_0^{3/5\,r}\int_{\frac{4\sqrt{ry}}{\sqrt{15}}}^{\sqrt{r^2-y^2}} x\,dx\,dy}{0.242\,r^2} = \frac{\int_0^{3/5\,r}\frac{1}{2}x^2\Big|_{\frac{4\sqrt{ry}}{\sqrt{15}}}^{\sqrt{r^2-y^2}} dy}{0.242\,r^2} = \frac{\int_0^{3/5\,r}\left(\frac{r^2}{2} - \frac{8yr}{15} - \frac{y^2}{2}\right)dy}{0.242\,r^2}$$

$$= \frac{\left(\frac{r^2 y}{2} - \frac{4ry^2}{15} - \frac{y^3}{6}\right)\Big|_0^{\frac{3}{5}r}}{0.242\,r^2} \longrightarrow \underline{\bar{x} = 0.695\,r}$$

$$\bar{y} = \frac{\int y\,dA}{A} = \frac{\int_0^{\frac{3}{5}r}\int_{\frac{4\sqrt{ry}}{\sqrt{15}}}^{\sqrt{r^2-y^2}} y\,dx\,dy}{0.242\,r^2} = \frac{\int_0^{\frac{3}{5}r} yx\Big|_{\frac{4\sqrt{ry}}{\sqrt{15}}}^{\sqrt{r^2-y^2}} dy}{0.242\,r^2} = \frac{\int_0^{\frac{3}{5}r}\left(y\sqrt{r^2-y^2} - \frac{4\sqrt{r}\,y^{3/2}}{\sqrt{15}}\right)dy}{0.242\,r^2}$$

$$= \frac{-\frac{1}{3}\sqrt{(r^2-y^2)^3} - \frac{8\sqrt{r}}{5\sqrt{15}}\,y^{5/2}}{0.242\,r^2}\Bigg|_0^{3/5\,r} \longrightarrow \underline{\bar{y} = 0.1963\,r}$$

5/29



$$A = \int dA = \int_0^2 (y_2 - y_1)\,dx = \int_0^2 \left(\sqrt{2x} - \frac{x^3}{4}\right) dx$$
$$= \left(\frac{2\sqrt{2}}{3} x^{3/2} - \frac{x^4}{16}\right)_0^2 = 5/3$$

$$\int x_c\,dA = \int_0^2 x\left(\sqrt{2x} - \frac{x^3}{4}\right) dx$$
$$= \left(\frac{2\sqrt{2}}{5} x^{5/2} - \frac{x^5}{20}\right)\Big|_0^2 = 8/5$$

$$\int y_c\,dA = \int_0^2 \left(\frac{y_1+y_2}{2}\right)(y_2 - y_1)\,dx = \int_0^2 \frac{1}{2}(y_2^2 - y_1^2)\,dx$$
$$= \frac{1}{2}\int_0^2 \left(2x - \frac{x^6}{16}\right) dx = \frac{1}{2}\left[x^2 - \frac{x^7}{7(16)}\right]_0^2 = 10/7$$

$$\bar{x} = \int x_c\,dA / A = \frac{8/5}{5/3} = \underline{\frac{24}{25}}$$

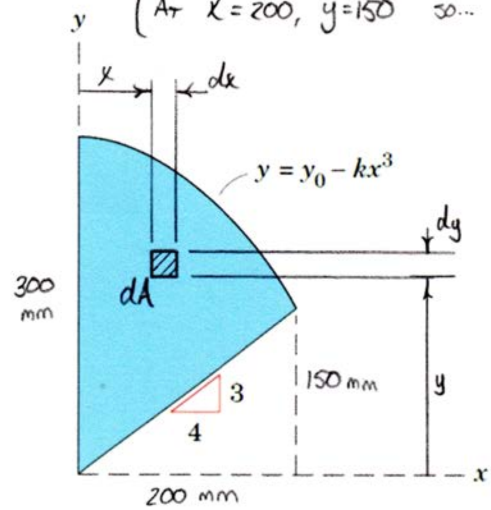
$$\bar{y} = \int y_c\,dA / A = \frac{10/7}{5/3} = \underline{\frac{6}{7}}$$

5/30

At $x=0$, $y=300$ so... $y_0 = 300$ mm

At $x=200$, $y=150$ so... $150 = 300 - k(200)^3 \longrightarrow k = 1.875(10^{-5})$

$y = 300 - 1.875(10^{-5})x^3$



$$A = \int dA = \int_0^{200}\int_{\frac{3}{4}x}^{300-1.875(10^{-5})x^3} 1\,dy\,dx = \int_0^{200}\left(300 - 1.875(10^{-5})x^3 - \frac{3}{4}x\right)dx = 300x - \frac{1.875(10^{-5})}{4}x^4 - \frac{3}{8}x^2\Big|_0^{200}$$

$A = 37\,500 \text{ mm}^2$

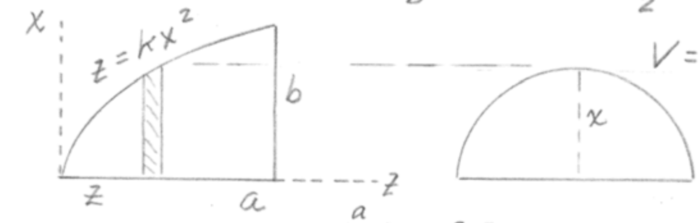
$$\bar{x} = \frac{\int x\,dA}{A} = \frac{\int_0^{200}\int_{3/4x}^{300-1.875(10^{-5})x^3} x\,dy\,dx}{37\,500} = \frac{1}{37\,500}\int_0^{200}\left(300x - 1.875(10^{-5})x^4 - \frac{3}{4}x^2\right)dx$$

$$\frac{1}{37\,500}\left(150x^2 - \frac{1.875(10^{-5})}{5}x^5 - \frac{1}{4}x^3\right)_0^{200} \longrightarrow \underline{\bar{x} = 74.7 \text{ mm}}$$

$$\bar{y} = \frac{\int y\,dA}{A} = \frac{1}{37500}\int_0^{200}\int_{3/4x}^{300-1.875(10^{-5})x^3} y\,dy\,dx = \frac{1}{37\,500}\int_0^{200}\frac{1}{2}\left[\left(300 - 1.875(10^{-5})x^3\right)^2 - \frac{9}{16}x^2\right]dx$$

$$= \frac{1}{75\,000}\left(90\,000x - \frac{3}{16}x^3 - 0.00281x^4 + 5.02(10^{-11})x^7\right)\Big|_0^{200} \longrightarrow \underline{\bar{y} = 168.6 \text{ mm}}$$

5/31 $a = kb^2$ so $z = \frac{a}{b^2}x^2$ $\qquad dV = \frac{\pi x^2}{2}dz = \frac{\pi}{2}\frac{b^2 z}{a}dz$



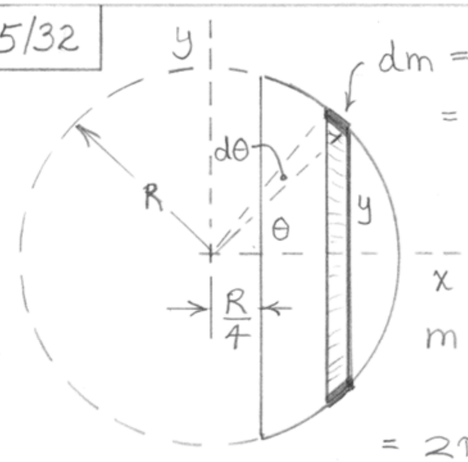
$$V = \frac{\pi b^2}{2a}\int_0^a z\,dz = \frac{\pi b^2}{2a}\frac{a^2}{2} = \frac{\pi}{4}ab^2$$

$$\int z_c\,dV = \int z\,dV = \int_0^a z\,\frac{\pi}{2}\frac{b^2 z}{a}dz$$

$$= \frac{\pi b^2}{2a}\int_0^a z^2\,dz = \frac{\pi b^2}{2a}\frac{a^3}{3} = \frac{\pi b^2 a^2}{6}$$

$$\bar{z} = \int z_c\,dV/V = \frac{\pi b^2 a^2}{6}\Big/\frac{\pi}{4}ab^2 = \underline{2a/3}$$

5/32



$$dm = \rho\, dA = \rho (R\,d\theta)(2\pi y)$$
$$= 2\pi \rho R (R \sin\theta)\, d\theta$$
$$= 2\pi \rho R^2 \sin\theta\, d\theta$$

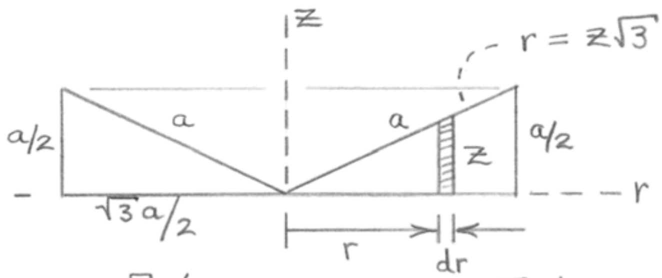
$$m = \int dm$$

$$m = \int_0^{\cos^{-1}\frac{1}{4}} 2\pi \rho R^2 \sin\theta\, d\theta$$
$$= 2\pi \rho R^2 (-\cos\theta)\Big|_0^{\cos^{-1}\frac{1}{4}}$$
$$= 1.5\pi \rho R^2$$

$$\int x_c\, dm = \int (R\cos\theta)(2\pi \rho R^2 \sin\theta\, d\theta)$$
$$= 2\pi \rho R^3 \int \cos\theta \sin\theta\, d\theta = \pi \rho R^3 \int \sin 2\theta\, d\theta$$
$$= \pi \rho R^3 \left(-\tfrac{1}{2}\cos 2\theta\right)\Big|_0^{\cos^{-1}\frac{1}{4}} = \frac{15}{16}\pi \rho R^3$$

$$\bar{x} = \frac{\int x_c\, dm}{\int dm} = \frac{\frac{15}{16}\pi \rho R^3}{1.5\pi \rho R^2} = \underline{\frac{5}{8}R}$$

5/33



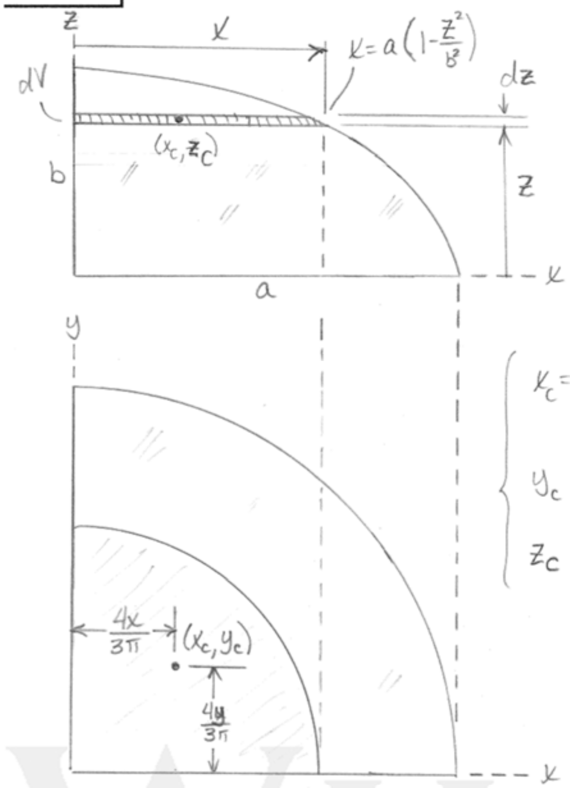
$$V = \int dV = \int_0^{\sqrt{3}a/2} 2\pi r z\,dr = \frac{2\pi}{\sqrt{3}}\int_0^{\sqrt{3}a/2} r^2\,dr$$

$$= \frac{2\pi}{\sqrt{3}}\left(\sqrt{3}a/2\right)^3/3 = \frac{\pi a^3}{4}$$

$$\int z_c\,dV = \int_0^{\sqrt{3}a/2} \frac{r}{2\sqrt{3}}\frac{2\pi}{\sqrt{3}} r^2\,dr = \frac{\pi}{3}\left[\frac{r^4}{4}\right]_0^{\sqrt{3}a/2} = \frac{3\pi a^4}{64}$$

$$\bar{z} = \frac{\int z_c\,dV}{V} = \frac{3\pi a^4/64}{\pi a^3/4} = \underline{\frac{3a}{16}}$$

5/34 USE A QUARTER-CIRCULAR ELEMENT



$$dV = \frac{1}{4}\pi x^2\,dz$$
$$dV = \frac{1}{4}\pi\left(a^2\left(1-\frac{z^2}{b^2}\right)^2\right)dz$$

$dm = \rho\,dV$ AND ρ = CONSTANT, SO INTEGRAL OVER VOLUME IS ALL WE NEED.

$$x_c = \frac{4x}{3\pi} = \frac{4a}{3\pi}\left(1 - \frac{z^2}{b^2}\right)$$

y_c IS SAME AS x_c

$$z_c = z$$

$$V = \int dV = \int_0^b \frac{1}{4}\pi\left(a^2\left(1-\frac{z^2}{b^2}\right)^2\right)dz = \frac{a^2\pi}{4}\left(z - \frac{2z^3}{3b^2} + \frac{z^5}{5b^4}\right)\Bigg|_0^b \longrightarrow V = \frac{2}{15}\pi a^2 b$$

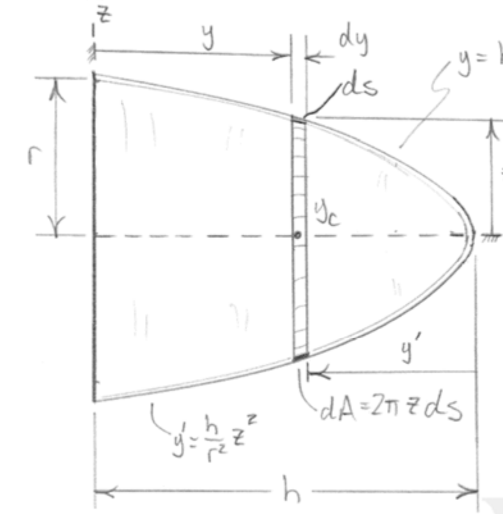
$$\bar{x} = \frac{\int x_c\,dV}{V} = \frac{\int_0^b\left(\frac{4a}{3\pi}\left(1-\frac{z^2}{b^2}\right)\right)\left(\frac{1}{4}\pi\left(a^2\left(1-\frac{z^2}{b^2}\right)^2\right)\right)dz}{\frac{2}{15}\pi a^2 b} = \frac{\int_0^b \frac{a}{3}\left(1-\frac{z^2}{b^2}\right)^3 dz}{\frac{2}{15}\pi b}$$

$$= \frac{5a}{2\pi b}\left(z - \frac{z^3}{b^2} + \frac{3z^5}{5b^4} - \frac{z^7}{7b^6}\right)\Bigg|_0^b \longrightarrow \underline{\bar{x} = \bar{y} = \frac{8a}{7\pi}}$$

$$\bar{z} = \frac{\int z_c\,dV}{V} = \frac{\int_0^b z\left(\frac{1}{4}\pi\left(a^2\left(1-\frac{z^2}{b^2}\right)^2\right)\right)dz}{\frac{2}{15}\pi a^2 b} = \frac{15}{8b}\int_0^b z\left(1-\frac{z^2}{b^2}\right)^2 dz$$

$$= \frac{15}{8b}\left(\frac{z^2}{2} - \frac{z^4}{2b^2} + \frac{z^6}{6b^4}\right)\Bigg|_0^b \longrightarrow \underline{\bar{z} = \frac{5}{16}b}$$

▶ 5/35 TAKE A CIRCULAR SHELL SEGMENT.



$y = h - \frac{h}{r^2} z^2$ or $y' = \frac{h}{r^2} z^2$ so $z = \frac{r}{\sqrt{h}} \sqrt{y'}$

$$dA = \frac{2\pi r}{\sqrt{h}} \sqrt{y'}\, ds$$

$$ds = dy' \sqrt{1 + \left(\frac{dz}{dy'}\right)^2} = \sqrt{1 + \frac{r^2}{4hy'}}\, dy'$$

$$A = \int dA = \int_0^h \frac{2\pi r}{\sqrt{h}} \sqrt{y' + \frac{r^2}{4h}}\, dy' \qquad \begin{cases} u = y' + \frac{r^2}{4h} \\ du = dy' \end{cases}$$

$$A = \frac{2\pi r}{\sqrt{h}} \int_0^h \sqrt{u}\, du = \frac{2\pi r}{\sqrt{h}} \left(\frac{2}{3} u^{3/2}\right)\Bigg|_0^h = \frac{4\pi r}{3\sqrt{h}} \left(y' + \frac{r^2}{4h}\right)^{3/2}\Bigg|_0^h$$

$$A = \frac{4\pi r}{3\sqrt{h}} \left[\left(h + \frac{r^2}{4h}\right)^{3/2} - \left(\frac{r^2}{4h}\right)^{3/2}\right]$$

$$A\bar{y}' = \int y'\, dA = \int_0^h \frac{2\pi r}{\sqrt{h}} y' \sqrt{y' + \frac{r^2}{4h}}\, dy' \qquad \begin{cases} u = y' + \frac{r^2}{4h} \\ du = dy' \end{cases}$$

$$A\bar{y}' = \frac{2\pi r}{\sqrt{h}} \int_0^h \left(u - \frac{r^2}{4h}\right) \sqrt{u}\, du = \frac{2\pi r}{\sqrt{h}} \int_0^h \left(u^{3/2} - \frac{r^2}{4h} u^{1/2}\right) du$$

$$A\bar{y}' = \frac{4\pi r}{5\sqrt{h}} u^{5/2} - \frac{\pi r^3}{3h^{3/2}} u^{3/2} = \frac{4\pi r}{5\sqrt{h}} \left(y' + \frac{r^2}{4h}\right)^{5/2} - \frac{\pi r^3}{3h^{3/2}} \left(y' + \frac{r^2}{4h}\right)^{3/2}\Bigg|_0^h$$

$$A\bar{y}' = \frac{4\pi r}{5\sqrt{h}} \left[\left(h + \frac{r^2}{4h}\right)^{5/2} - \left(\frac{r^2}{4h}\right)^{5/2}\right] - \frac{\pi r^3}{3h^{3/2}} \left[\left(h + \frac{r^2}{4h}\right)^{3/2} - \left(\frac{r^2}{4h}\right)^{3/2}\right]$$

WITH NUMBERS... $r = 70$ mm, $h = 200$ mm

$$A = \frac{4\pi(70)}{3\sqrt{200}} \left[\left(200 + \frac{70^2}{4(200)}\right)^{3/2} - \left(\frac{70^2}{4(200)}\right)^{3/2}\right] \longrightarrow A = 61\,040 \text{ mm}^2$$

$$A\bar{y}' = \frac{4\pi(70)}{5\sqrt{200}} \left[\left(200 + \frac{70^2}{4(200)}\right)^{5/2} - \left(\frac{70^2}{4(200)}\right)^{5/2}\right] - \frac{\pi(70)^3}{3(200)^{3/2}} \left[\left(200 + \frac{70^2}{4(200)}\right)^{3/2} - \left(\frac{70^2}{4(200)}\right)^{3/2}\right]$$

$$A\bar{y}' = 7.21 \times 10^6 \text{ mm}^3$$

$$\bar{y}' = \frac{7.21 \times 10^6}{61\,040} \longrightarrow \bar{y}' = 118.2 \text{ mm}$$

$$\bar{y} = h - \bar{y}' = 200 - 118.2 \longrightarrow \underline{\bar{y} = 81.8 \text{ mm}}$$

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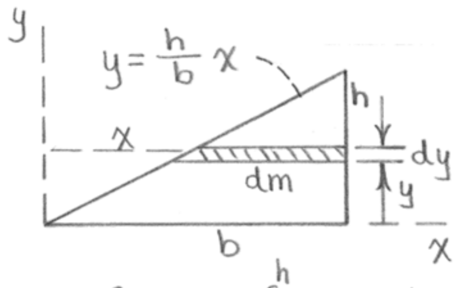

$$dm = \rho dV = \rho dA\,t = t\rho(b-x)\,dy$$
$$= \left[t_o\left(\frac{y}{h}+1\right)\right]\rho(b-x)\,dy$$
$$= t_o\rho\left(\frac{y}{h}+1\right)\left(b-\frac{b}{h}y\right)dy$$
$$= t_o\rho b\left(1-\frac{y^2}{h^2}\right)dy$$

$$m = \int dm = \int_0^h t_o\rho b\left(1-\frac{y^2}{h^2}\right)dy = t_o\rho b\left[y-\frac{y^3}{3h^2}\right]_0^h$$
$$= \frac{2}{3}\rho t_o bh$$

$$\int y_c\,dm = \int t_o\rho b\left(1-\frac{y^2}{h^2}\right)y\,dy = t_o\rho b\left[\frac{y^2}{2}-\frac{y^4}{4h^2}\right]_0^h$$
$$= \frac{1}{4}\rho t_o bh^2$$

$$\bar{y} = \frac{\int y_c\,dm}{m} = \frac{\frac{1}{4}\rho t_o bh^2}{\frac{2}{3}\rho t_o bh} = \underline{\frac{3}{8}h}$$

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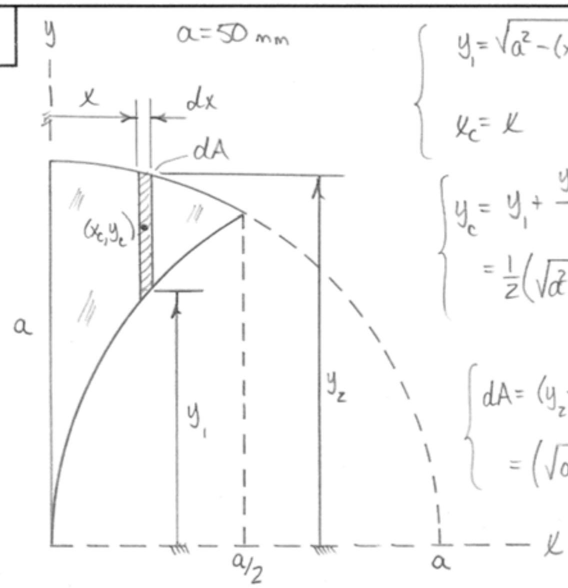

$a = 50$ mm

$$y_1 = \sqrt{a^2-(x-a)^2} \quad \& \quad y_2 = \sqrt{a^2-x^2}$$

$$x_c = x$$

$$y_c = y_1 + \frac{y_2-y_1}{2} = \frac{1}{2}(y_1+y_2) = \frac{1}{2}\left(\sqrt{a^2-(x-a)^2} + \sqrt{a^2-x^2}\right)$$

$$dA = (y_2-y_1)\,dx = \left(\sqrt{a^2-x^2} - \sqrt{a^2-(x-a)^2}\right)dx$$

$$A = \int dA = \int_0^{a/2}\left(\sqrt{a^2-x^2} - \sqrt{a^2-(x-a)^2}\right)dx = \int_0^{a/2}\left(\sqrt{a^2-x^2} - \sqrt{2ax-x^2}\right)dx$$

$$= \left.\frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2\sin^{-1}\left(\frac{x}{a}\right)\right) - \frac{x(2a^2-3ax+x^2) - 2a^2\sqrt{2a-x}\sqrt{x}\,\tan^{-1}\left(\sqrt{\frac{x}{2a-x}}\right)}{2\sqrt{2ax-x^2}}\right|_0^{a/2}$$

IF $a = 50$ mm ... $A = 428\text{ mm}^2$

$$\bar{x} = \frac{\int x\,dA}{A} = \frac{\int_0^{a/2} x\left(\sqrt{a^2-x^2} - \sqrt{2ax-x^2}\right)dx}{428}$$

$$= \left.\frac{\frac{1}{6}\left(\sqrt{2ax-x^2}\,(3a^2+ax-2x^2) - 2(a^2-x^2)^{3/2} + 3a^3\tan^{-1}\left(\frac{a-x}{\sqrt{2ax-x^2}}\right)\right)}{428}\right|_0^{a/2=25} \longrightarrow \underline{\bar{x} = 7.66\text{ mm}}$$

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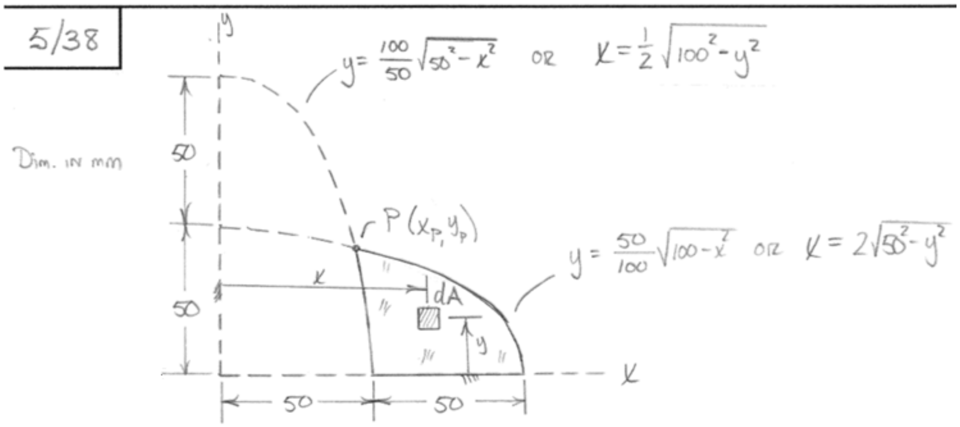


FIND INTERSECTION... $\frac{1}{2}\sqrt{100^2 - y_P^2} = 2\sqrt{50^2 - y_P^2} \longrightarrow \frac{1}{4}(100^2 - y_P^2) = 4(50^2 - y_P^2)$

$$y_P = 20\sqrt{5} \text{ mm}$$

$$A = \int_0^{20\sqrt5}\int_{\frac{1}{2}\sqrt{100^2-y^2}}^{2\sqrt{50^2-y^2}} 1\, dx\, dy = \int_0^{20\sqrt5}\left(2\sqrt{50^2-y^2} - \frac{1}{2}\sqrt{100^2-y^2}\right)dy$$

$$= \left[y\sqrt{50^2-y^2} - \frac{1}{4}y\sqrt{100^2-y^2} - 50^2\sin^{-1}\left(\frac{y}{100}\right) + 50^2\sin^{-1}\left(\frac{y}{50}\right)\right]\Bigg|_0^{20\sqrt5} \longrightarrow A = 1609 \text{ mm}^2$$

$$\bar{x} = \frac{\int x\,dA}{A} = \frac{\int_0^{20\sqrt5}\int_{\frac{1}{2}\sqrt{100^2-y^2}}^{2\sqrt{50^2-y^2}} x\,dx\,dy}{1609} = \frac{\int_0^{20\sqrt5}\frac{1}{2}\left(4(50^2-y^2) - \frac{1}{4}(100^2-y^2)\right)dy}{1609}$$

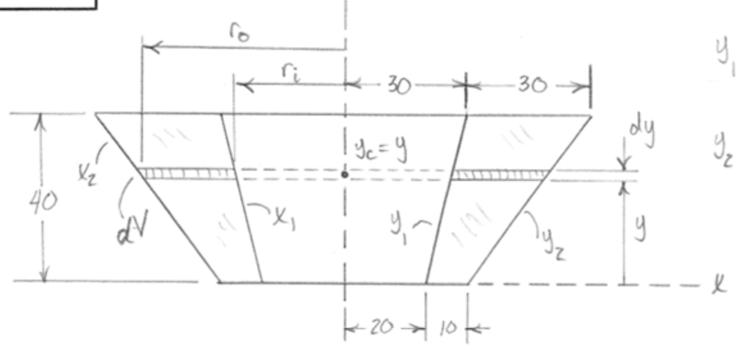
$$= \frac{2\left(50^2y - \frac{1}{3}y^3\right) - \frac{1}{8}\left(100^2y - \frac{1}{3}y^3\right)}{1609}\Bigg|_0^{20\sqrt5} \longrightarrow \underline{\bar{x} = 69.5 \text{ mm}}$$

$$\bar{y} = \frac{\int y\,dA}{A} = \frac{\int_0^{20\sqrt5}\int_{\frac{1}{2}\sqrt{100^2-y^2}}^{2\sqrt{50^2-y^2}} y\,dx\,dy}{1609} = \frac{\int_0^{20\sqrt5}\left(2y\sqrt{50^2-y^2} - \frac{1}{2}y\sqrt{100^2-y^2}\right)dy}{1609}$$

$$= \frac{\left(-\frac{2}{3}\sqrt{(50^2-y^2)^3} + \frac{1}{6}\sqrt{(100^2-y^2)^3}\right)}{1609}\Bigg|_0^{20\sqrt5} \longrightarrow \underline{\bar{y} = 17.70 \text{ mm}}$$

5/39 Dim. in mm

USE A "WASHER" ELEMENT.



$y_1 = 4(x-20)$

$y_2 = \frac{4}{3}(x-30)$

$$dV = \pi(r_o^2 - r_i^2)\,dy = \pi(x_2^2 - x_1^2)\,dy$$

AND...
$$\begin{cases} x_1 = \frac{1}{4}y + 20 \\ x_2 = \frac{3}{4}y + 30 \end{cases}$$

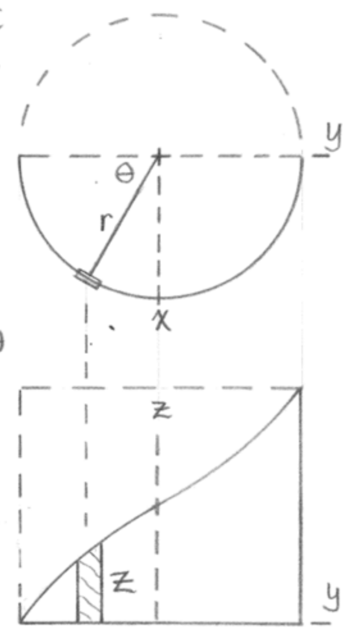
$$V = \int dV = \int_0^{40} \pi\left[\left(\tfrac{3}{4}y+30\right)^2 - \left(\tfrac{1}{4}y+20\right)^2\right]dy = \frac{\pi}{6}\left(y^3 + 105y^2 + 3000y\right)\Big|_0^{40}$$

$$V = 184.3(10^3)\ \text{mm}^3$$

$$\bar{y} = \frac{\int y_c\,dV}{V} = \frac{\int_0^{40} \pi y\left[\left(\tfrac{3}{4}y+30\right)^2 - \left(\tfrac{1}{4}y+20\right)^2\right]dy}{184.3(10^3)} = \frac{\frac{\pi}{24}\left(3y^4 + 280y^3 + 6000y^2\right)\Big|_0^{40}}{184.3(10^3)}$$

$$\underline{\bar{y} = 25\ \text{mm}}$$

▶5/40 Let ρ = mass per unit area of shell



$$z = k\theta = \frac{h}{\pi}\theta$$

$$dm = \rho z r\,d\theta = \frac{\rho h r}{\pi}\theta\,d\theta$$

$$m = \frac{\rho h r}{\pi}\int_0^\pi \theta\,d\theta = \frac{1}{2}\rho h r \pi$$

$$m\bar{x} = \int x\,dm = \int_0^\pi r\sin\theta\,\frac{\rho h r}{\pi}\theta\,d\theta$$

$$= \frac{\rho r^2 h}{\pi}\int_0^\pi \theta\sin\theta\,d\theta$$

$$= \frac{\rho r^2 h}{\pi}\left(\sin\theta - \theta\cos\theta\right)_0^\pi = \rho r^2 h$$

So $\bar{x} = \dfrac{\rho r^2 h}{\rho r h \pi/2} = \underline{\dfrac{2r}{\pi}}$

$$m\bar{y} = \int y\,dm = \int(-r\cos\theta)\frac{\rho h r}{\pi}\theta\,d\theta$$

$$= -\frac{\rho h r^2}{\pi}\int_0^\pi \theta\cos\theta\,d\theta = -\frac{\rho h r^2}{\pi}\left(\cos\theta + \theta\sin\theta\right)_0^\pi = \frac{2\rho h r^2}{\pi}$$

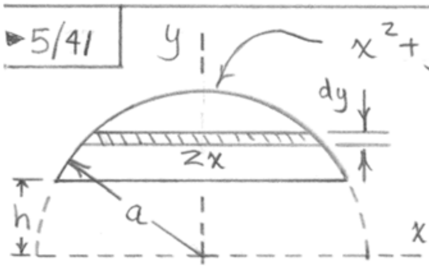
So $\bar{y} = \dfrac{2\rho h r^2/\pi}{\rho h r\pi/2} = \underline{\dfrac{4r}{\pi^2}}$

$$m\bar{z} = \int z_c\,dm = \int \frac{1}{2}\frac{h}{\pi}\theta\,\frac{\rho h r}{\pi}\theta\,d\theta = \frac{\rho h^2 r}{2\pi^2}\int_0^\pi \theta^2\,d\theta$$

$$= \frac{\rho h r^2}{2\pi}\left(\frac{\theta^3}{3}\right)_0^\pi = \frac{1}{6}\rho h^2 r\pi$$

So $\bar{z} = \dfrac{\rho h^2 r\pi/6}{\rho h r\pi/2} = \underline{\dfrac{1}{3}h}$

▸5/41



$x^2 + y^2 = a^2$, $x = +\sqrt{a^2 - y^2}$

$dA = 2x\,dy = 2\sqrt{a^2-y^2}\,dy$

$A = \int dA = \int_h^a 2\sqrt{a^2-y^2}\,dy$

$= 2\left(\frac{1}{2}\right)\left[y\sqrt{a^2-y^2} + a^2\sin^{-1}\frac{y}{a}\right]_h^a$

$= a^2\left(\frac{\pi}{2} - \sin^{-1}\frac{h}{a}\right) - h\sqrt{a^2-h^2}$

$$\int y\,dA = \int_h^a y\,2\sqrt{a^2-y^2}\,dy = 2\left(-\frac{1}{3}\right)(a^2-y^2)^{3/2}\Big|_h^a$$

$$= \frac{2}{3}(a^2-h^2)^{3/2}$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} = \underline{\frac{\frac{2}{3}(a^2-h^2)^{3/2}}{a^2\left(\frac{\pi}{2} - \sin^{-1}\frac{h}{a}\right) - h\sqrt{a^2-h^2}}}$$

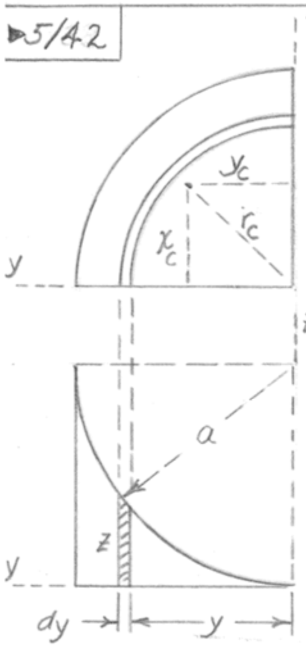
Special cases

$h = 0$: $\bar{y} = \frac{\frac{2}{3}a^3}{a^2\frac{\pi}{2}} = \frac{4a}{3\pi}$ (the correct result)

$h = \frac{a}{4}$: $\bar{y} = \frac{\frac{2}{3}\left(a^2 - \left(\frac{a}{4}\right)^2\right)^{3/2}}{a^2\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{4}\right) - \frac{a}{4}\sqrt{a^2 - \left(\frac{a}{4}\right)^2}} = \underline{0.562a}$

$h = \frac{a}{2}$: $\bar{y} = \frac{\frac{2}{3}\left(a^2 - \left(\frac{a}{2}\right)^2\right)^{3/2}}{a^2\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{2}\right) - \frac{a}{2}\sqrt{a^2 - \left(\frac{a}{2}\right)^2}} = \underline{0.705a}$

▶5/42



From Sample Problem 5/1, for elemental shell, $x_c = y_c = r_c/\sqrt{2} = \frac{2y}{\pi}$

$$dV = \frac{\pi y}{2}(z\,dy) = \frac{\pi}{2}\left(ay - y\sqrt{a^2-y^2}\right)dy$$

$$V = \frac{\pi}{2}\int_0^a \left(ay - y\sqrt{a^2-y^2}\right)dy$$

$$= \frac{\pi}{2}\left[\frac{ay^2}{2} + \frac{1}{3}\sqrt{(a^2-y^2)^3}\right]_0^a = \frac{\pi}{2}\frac{a^3}{6} = \frac{\pi a^3}{12}$$

$$\int y_c\,dV = \frac{2}{\pi}\int_0^a \frac{\pi}{2}\left(ay^2 - y^2\sqrt{a^2-y^2}\right)dy$$

$$= \left[\frac{ay^3}{3} + \frac{y}{4}\sqrt{(a^2-y^2)^3} - \frac{a^2}{8}\left(y\sqrt{a^2-y^2} + a^2\sin^{-1}\frac{y}{a}\right)\right]_0^a$$

$$= \left[\frac{a^4}{3} + 0 - \frac{a^2}{8}\left(0 + a^2\frac{\pi}{2}\right)\right] = a^4\left[\frac{1}{3} - \frac{\pi}{16}\right]$$

$$\bar{y} = \int y_c\,dV/V = a^4\left(\frac{1}{3} - \frac{\pi}{16}\right)\Big/\frac{\pi a^3}{12} = \underline{\left(\frac{4}{\pi} - \frac{3}{4}\right)a = \bar{x}}$$

$y^2 + (z-a)^2 = a^2$

$z = a - \sqrt{a^2-y^2}$

(Note sign)

$$\int z_c\,dV = \int \frac{z}{2}\,dV = \int_0^a \frac{a-\sqrt{a^2-y^2}}{2}\,\frac{\pi}{2}\left(ay - y\sqrt{a^2-y^2}\right)dy$$

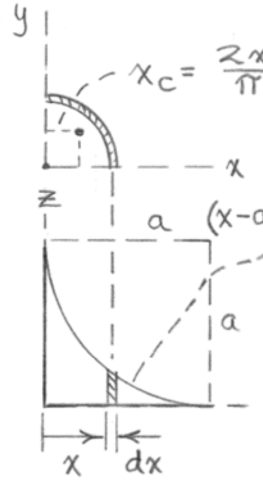
$$= \frac{\pi a^4}{48}; \quad \bar{z} = \int z_c\,dV/V = \frac{\pi a^4}{48}\Big/\frac{\pi a^3}{12} = \underline{\frac{a}{4}}$$

▶ 5/43

$\bar{x} = \bar{y}$, by symmetry. $V\bar{x} = \int x_c\,dV$

$V = \int dV = \int \frac{\pi x}{2} z\,dx$,

where $z = a - \sqrt{a^2 - (x-a)^2}$



Let $u = x - a$, $du = dx$

$$V = \frac{\pi}{2}\int_{x=0}^{x=a} \left[(u+a)a - (u+a)\sqrt{a^2-u^2}\right]du$$

$$= \frac{\pi}{2}\left[\frac{u^2 a}{2} + a^2 u + \frac{1}{3}\sqrt{(a^2-u^2)^3} - \frac{a}{2}\left(u\sqrt{a^2-u^2} + a^2\sin^{-1}\frac{u}{a}\right)\right]_{-a}^{0} = \frac{\pi}{4}\left[-\frac{\pi}{2} + \frac{5}{3}\right]a^3$$

$$\int x_c\,dV = \int_0^a \frac{2x}{\pi}\frac{\pi x}{2}\left(a - \sqrt{a^2-(x-a)^2}\right)dx$$

$$= \int_{x=0}^{x=a} (u+a)^2\left(a - \sqrt{a^2-u^2}\right)du = \int_{-a}^{0} u^2\left(a - \sqrt{a^2-u^2}\right)du$$

$$+ \int_{-a}^{0} 2au\left(a - \sqrt{a^2-u^2}\right)du + \int_{-a}^{0} a^2\left(a - \sqrt{a^2-u^2}\right)du = \mathrm{I} + \mathrm{II} + \mathrm{III}$$

$$\mathrm{I} = \frac{u^3 a}{3} - \left[-\frac{u}{4}\sqrt{(a^2-u^2)^3} + \frac{a^2}{8}\left(u\sqrt{a^2-u^2} + a^2\sin^{-1}\frac{u}{a}\right)\right]_{-a}^{0}$$

$$= \left(-\frac{\pi}{16} + \frac{1}{3}\right)a^4$$

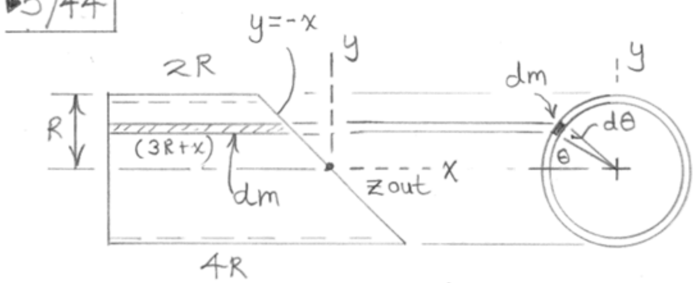
$$\mathrm{II} = u^2a^2 - 2a\left[-\frac{1}{3}\sqrt{(a^2-u^2)^3}\right]\Big|_{-a}^{0} = -\frac{a^4}{3}$$

$$\mathrm{III} = a^3 u - \frac{a^2}{2}\left[u\sqrt{a^2-u^2} + a^2\sin^{-1}\frac{u}{a}\right]_{-a}^{0} = \left(1 - \frac{\pi}{4}\right)a^4$$

$$\int x_c\,dV = \mathrm{I} + \mathrm{II} + \mathrm{III} = \left(1 + \frac{5\pi}{16}\right)a^4$$

$$\therefore \bar{y} = \bar{x} = \left(1 - \frac{5\pi}{16}\right)a^4 \Big/ \frac{\pi}{4}\left(-\frac{\pi}{2} + \frac{5}{3}\right)a^3 = \frac{3a}{2\pi}\,\frac{16-5\pi}{10-3\pi} = \underline{0.242a}$$

▶5/44



Temporarily use coordinates shown. Element is a "stick". ρ is mass per unit area here.

$$dm = \rho(3R+x)R\,d\theta = \rho R(3R-y)\,d\theta$$
$$= \rho R(3R - R\sin\theta)\,d\theta = \rho R^2(3-\sin\theta)\,d\theta$$

$$m = \int dm = \int_0^{2\pi} \rho R^2(3-\sin\theta)\,d\theta$$
$$= \rho R^2\left[3\theta + \cos\theta\right]_0^{2\pi} = 6\pi\rho R^2$$

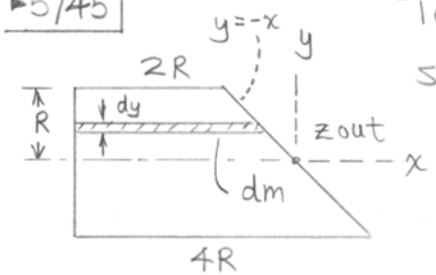
$$x_c = -3R + \frac{3R+x}{2} = \frac{x-3R}{2} = -\frac{y+3R}{2}$$
$$= -R\,\frac{\sin\theta+3}{2}$$

$$\int x_c\,dm = \int_0^{2\pi}\left[-\frac{R}{2}(\sin\theta+3)\,\rho R^2(3-\sin\theta)\,d\theta\right.$$
$$= -\frac{\rho R^3}{2}\int_0^{2\pi}(9-\sin^2\theta)\,d\theta$$
$$= -\frac{\rho R^3}{2}\left[9\theta - \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\right]_0^{2\pi} = -\rho R^3\left(\frac{17\pi}{2}\right)$$

So $$\bar{x} = \frac{\int x_c\,dm}{\int dm} = \frac{-\rho R^3(17\pi/2)}{6\pi\rho R^2} = 1.417R$$

Relative to left (flat) end,
$$\bar{x} = 3R - 1.417R = \underline{1.583R}$$

▸5/45



Temporarily use coordinates shown. Element is a thin rectangular plate.

$$dm = \rho\, dV = \rho(3R+x)(2z)\,dy$$
$$= 2\rho[3R-y]\left[\sqrt{R^2-y^2}\right]dy$$
$$= 6\rho R\sqrt{R^2-y^2}\,dy - 2\rho y\sqrt{R^2-y^2}\,dy$$

$$m = \int dm = \int_{-R}^{R} 6\rho R\sqrt{R^2-y^2}\,dy - \int_{-R}^{R} 2\rho y\sqrt{R^2-y^2}\,dy$$

$$m = 6\rho R\,\tfrac{1}{2}\left(y\sqrt{R^2-y^2} + R^2\sin^{-1}\frac{y}{R}\right)\Big|_{-R}^{R} - 2\rho\left(-\tfrac{1}{3}\sqrt{(R^2-y^2)^3}\right)\Big|_{-R}^{R}$$
$$= 3\pi\rho R^3$$

$$\int x_c\,dm = \int_{-R}^{R}\left(\frac{x-3R}{2}\right)\left(6\rho R\sqrt{R^2-y^2}\,dy - 2\rho y\sqrt{R^2-y^2}\,dy\right)$$

$$I_1 = -3\rho R\int_{-R}^{R} y\sqrt{R^2-y^2}\,dy = -3\rho R\left(-\tfrac{1}{3}\sqrt{(R^2-y^2)^3}\right)\Big|_{-R}^{R}$$
$$= 0$$

$$I_2 = -9\rho R^2\int_{-R}^{R}\sqrt{R^2-y^2}\,dy$$
$$= -9\rho R^2\cdot\tfrac{1}{2}\left(y\sqrt{R^2-y^2} + R^2\sin^{-1}\left(\frac{y}{R}\right)\right)\Big|_{-R}^{R}$$
$$= -\tfrac{9}{2}\rho R^2\left(R^2\tfrac{\pi}{2} - \left(-R^2\tfrac{\pi}{2}\right)\right) = -\tfrac{9}{2}\pi\rho R^4$$

$$I_3 = \rho\int_{-R}^{R} y^2\sqrt{R^2-y^2}\,dy$$
$$= \rho\left(-\frac{y}{4}\sqrt{(R^2-y^2)^3} + \frac{R^2}{8}\left(y\sqrt{R^2-y^2} + R^2\sin^{-1}\frac{y}{R}\right)\right)\Big|_{-R}^{R}$$
$$= \rho\left(\frac{R^2}{8}\cdot\left(R^2\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)\right)\right) = \tfrac{1}{8}\pi\rho R^4$$

$$I_4 = -3\rho R\int_{-R}^{R} y\sqrt{(R^2-y^2)}\,dy = 0$$

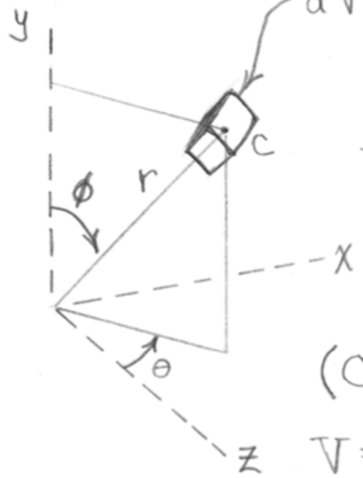
So total is $\int x_c\,dm = \left(-\frac{9}{2} + \frac{1}{8}\right)\pi\rho R^4$
$$= -\frac{35}{8}\pi\rho R^4$$

Then
$$\bar{x} = \frac{\int x_c\,dm}{\int dm}$$
$$= \frac{-\frac{35}{8}\pi\rho R^4}{3\pi\rho R^3} = -\frac{35}{24}R$$

Relative to left (flat) end, then,

$$\bar{x} = 3R - \frac{35}{24}R = \underline{\frac{37}{24}R} \quad \underline{(1.542R)}$$

▶5/46



$$dV = dr\,(r\,d\phi)(r\sin\phi\,d\theta)$$

$$= r^2\sin\phi\,d\phi\,d\theta\,dr$$

$$V = \int dV = \int_{R/2}^{R}\int_0^{\pi}\int_0^{\pi} r^2\sin\phi\,d\phi\,d\theta\,dr$$

$$= \frac{7}{12}\pi R^3$$

(Check V by $V = \dfrac{\frac{4}{3}\pi R^3}{2} - \dfrac{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{2} = \dfrac{7}{12}\pi R^3$)

$$\int x_c\,dV = \int (r\sin\phi\sin\theta)(r^2\sin\phi\,d\phi\,d\theta\,dr)$$

$$= \int_{R/2}^{R}\int_0^{\pi}\int_0^{\pi} r^3\sin^2\phi\,\sin\theta\,d\phi\,d\theta\,dr = \frac{15}{64}\pi R^4$$

$$\bar{x} = \frac{\int x_c\,dV}{V} = \frac{\frac{15}{64}\pi R^4}{\frac{7}{12}\pi R^3} = \underline{\frac{45}{112}R}$$

(Compare to $\bar{x} = \frac{3}{8}R$ for no hole.)

Note: A hemispherical shell of radius r and thickness dr would be a better element.

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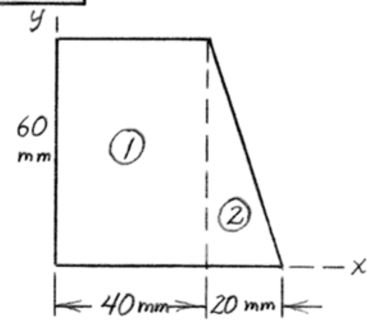

① $A_1 = 40(60) = 2400 \text{ mm}^2$

$\bar{x}_1 = 20 \text{ mm}, \ \bar{y}_1 = 30 \text{ mm}$

② $A_2 = \frac{1}{2}(20)(60) = 600 \text{ mm}^2$

$\bar{x}_2 = 40 + \frac{20}{3} = 46.7 \text{ mm}$

$\bar{y}_2 = \frac{60}{3} = 20 \text{ mm}$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{2400(20) + 600(46.7)}{2400 + 600} = \underline{25.3 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{2400(30) + 600(20)}{2400 + 600} = \underline{28.0 \text{ mm}}$$

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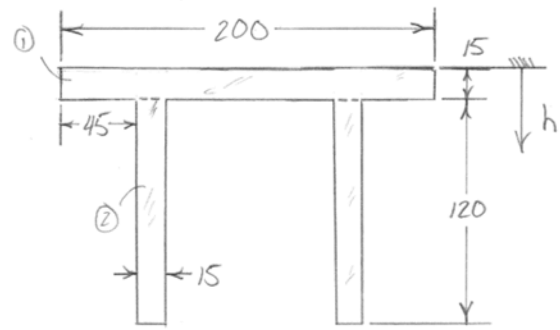
Dim. in mm



$$\bar{H} = \frac{\Sigma A \bar{h}}{\Sigma A} = \frac{200(15)\left(\frac{15}{2}\right) + 2(15)(120)\left(15 + \frac{120}{2}\right)}{200(15) + 2(15)(120)} \longrightarrow \underline{\bar{H} = 44.3 \text{ mm}}$$

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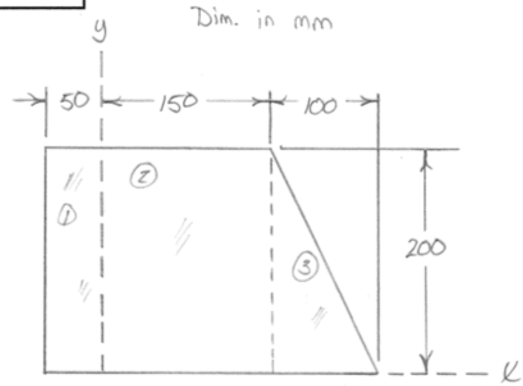

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{50(200)(-25) + 150(200)(75) + \frac{1}{2}(100)(200)\left(150 + \frac{100}{3}\right)}{50(200) + 150(200) + \frac{1}{2}(100)(200)}$$

$$\bar{X} = 76.7 \text{ mm}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{50(200)(100) + 150(200)(100) + \frac{1}{2}(100)(200)\left(\frac{200}{3}\right)}{50(200) + 150(200) + \frac{1}{2}(100)(200)}$$

$$\bar{Y} = 93.3 \text{ mm}$$

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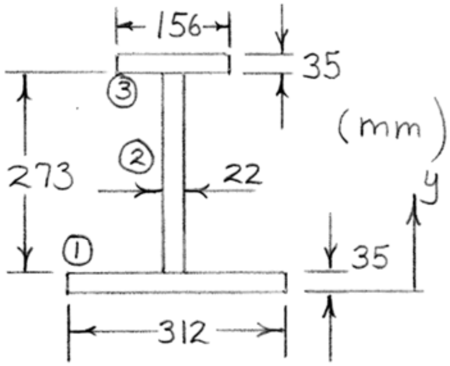

Comp.	$A\ (mm^2)$	$\bar{y}\ (mm)$	$A\bar{y}\ (mm^3)$
1	312(35)	$\frac{35}{2}$	191 100
2	273 (22)	$35 + \frac{273}{2}$	1 030 000
3	156 (35)	$35 + 273 + \frac{35}{2}$	1 777 000

$\Sigma A = 22\,400$ $\qquad \Sigma A\bar{y} = 3\,000\,000$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{3\,000\,000}{22\,400} = \underline{133.9\text{ mm}}$$

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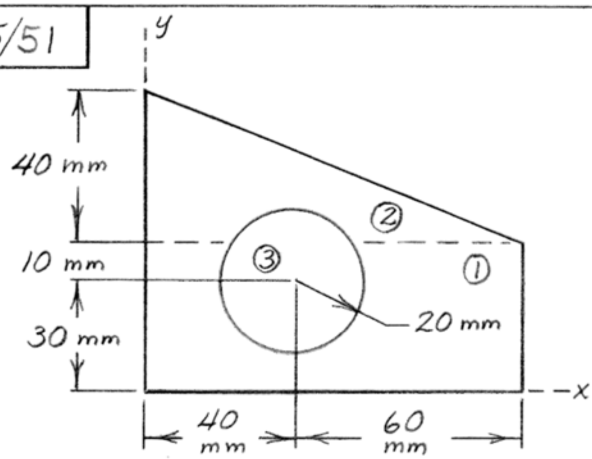

Part	A (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$A\bar{x}$ (mm^3)	$A\bar{y}$ (mm^3)
1	4000	50	20	$200(10^3)$	$80(10^3)$
2	2000	100/3	$40+\frac{40}{3}$	$66.7(10^3)$	$106.7(10^3)$
3	$-\pi(20^2)$	40	30	$-50.3(10^3)$	$-37.7(10^3)$
Totals	4740			$216(10^3)$	$149.0(10^3)$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{216(10^3)}{4740} = \underline{45.6 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{149.0(10^3)}{4740} = \underline{31.4 \text{ mm}}$$

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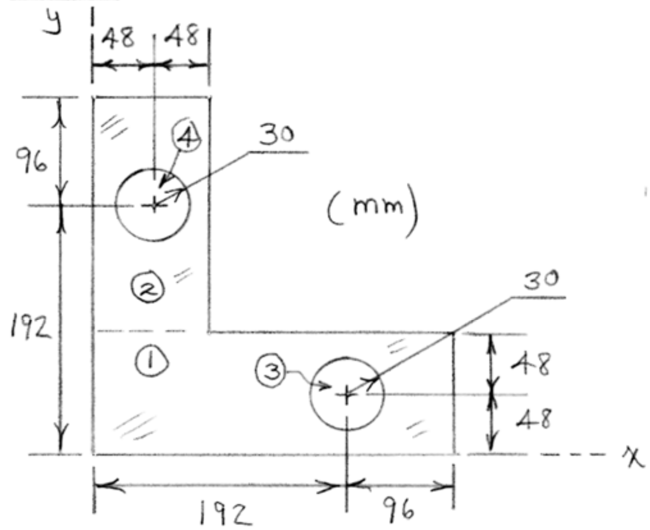

Comp.	A (mm^2)	$\bar{x}$ (mm)	$\bar{x}A$ (mm^3)
1	27648	144	$3.98(10^6)$
2	18432	48	$0.885(10^6)$
3	−2830	192	$-0.543(10^6)$
4	−2830	48	$-0.1357(10^6)$

$\Sigma A = 40\,430$ $\qquad \Sigma\bar{x}A = 4.19(10^6)$

$$\bar{X} = \bar{Y} = \frac{\Sigma\bar{x}A}{\Sigma A} = \frac{4.19(10^6)}{40\,430} = \underline{103.6\text{ mm}}$$

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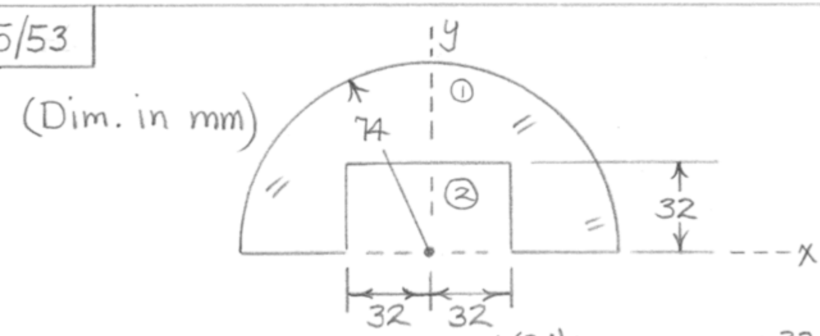
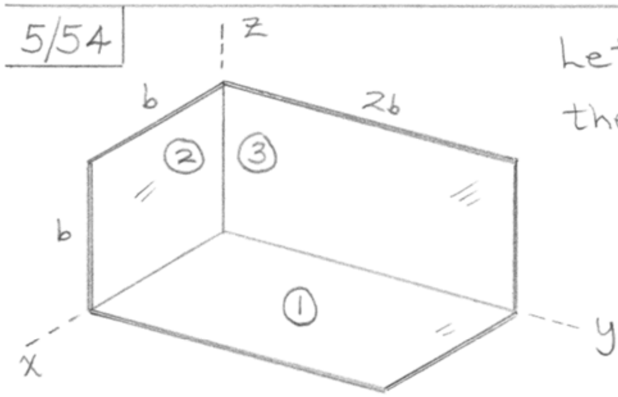


$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{\pi \frac{74^2}{2}\left(\frac{4(74)}{3\pi}\right) - 64(32)\left(\frac{32}{2}\right)}{\pi \frac{74^2}{2} - 64(32)}$$

$$= \underline{36.2 \text{ mm}}$$

5/54



Let ρ represent the mass/area.

Comp.	m	$\bar{x}$	$\bar{y}$	$\bar{z}$	$m\bar{x}$	$m\bar{y}$	$m\bar{z}$
1	$2\rho b^2$	$\frac{b}{2}$	b	0	ρb^3	$2\rho b^3$	0
2	ρb^2	$\frac{b}{2}$	0	$\frac{b}{2}$	$\frac{1}{2}\rho b^3$	0	$\frac{1}{2}\rho b^3$
3	$2\rho b^2$	0	b	$\frac{b}{2}$	0	$2\rho b^3$	ρb^3
Σ:	$5\rho b^2$				$\frac{3}{2}\rho b^3$	$4\rho b^3$	$\frac{3}{2}\rho b^3$

$$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} = \frac{\frac{3}{2}\rho b^3}{5\rho b^2} = \underline{\frac{3}{10}b}$$

$$\bar{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} = \frac{4\rho b^3}{5\rho b^2} = \underline{\frac{4}{5}b}$$

$$\bar{Z} = \frac{\Sigma m\bar{z}}{\Sigma m} = \frac{\frac{3}{2}\rho b^3}{5\rho b^2} = \underline{\frac{3}{10}b}$$

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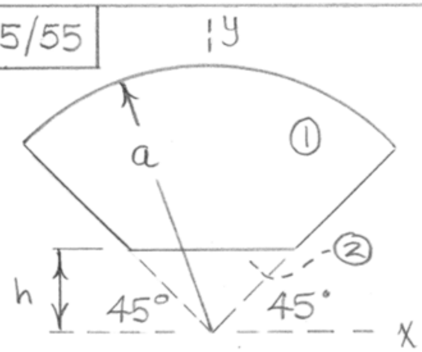

Circular sector (full) 1:

$$A_1 = \frac{\pi}{4} a^2$$

$$\bar{y}_1 = \frac{2}{3} a \frac{\sin 45°}{\pi/4}$$

$$= \frac{4\sqrt{2}}{3\pi} a$$

Triangular "hole" 2:

$$A_2 = \frac{1}{2} h (2h) = h^2, \quad \bar{y}_2 = \frac{2}{3} h$$

$$\bar{Y} = \frac{\Sigma A \bar{y}}{\Sigma A} = \frac{\frac{\pi}{4} a^2 \left(\frac{4\sqrt{2}}{3\pi} a\right) - h^2 \left(\frac{2}{3} h\right)}{\frac{\pi}{4} a^2 - h^2}$$

$$= \frac{4(\sqrt{2} a^3 - 2h^3)}{3(\pi a^2 - 4h^2)}$$

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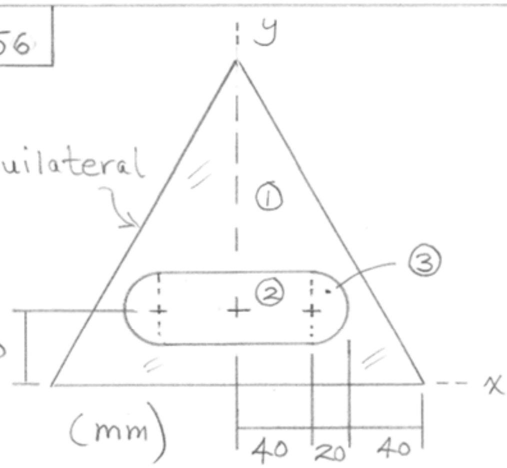

Component	A (mm^2)	$\bar{y}$ (mm)	$\bar{y}A$ (mm^3)
Triangle 1	17 320	57.7	10^6
Rectangle 2	− 3200	40	− 128 000
2 semicircles 3	− 1257	40	− 50,300

$\Sigma A = 12\,860$ $\qquad \Sigma \bar{y}A = 822\,000$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} \quad \frac{822\,000}{12\,860} = 63.9 \text{ mm}$$

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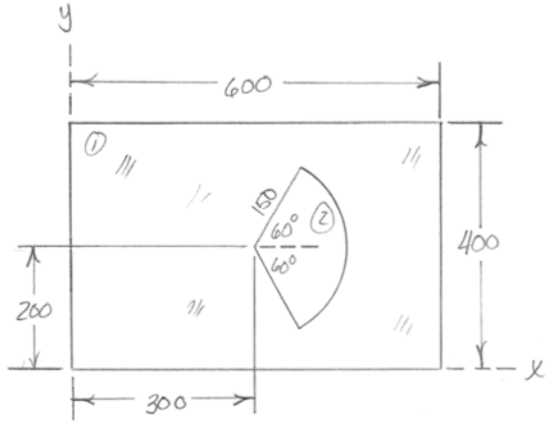
Dim. in mm



$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{400(600)(300) - \frac{1}{3}\pi(150)^2\left(300 + \frac{2}{3}(150)\frac{\sin 60}{\pi/3}\right)}{400(600) - \frac{1}{3}\pi(150)^2}$$

$\underline{\bar{X} = 291 \text{ mm}}$

$\underline{\bar{Y} = 200 \text{ mm}}$ (INSPECTION)

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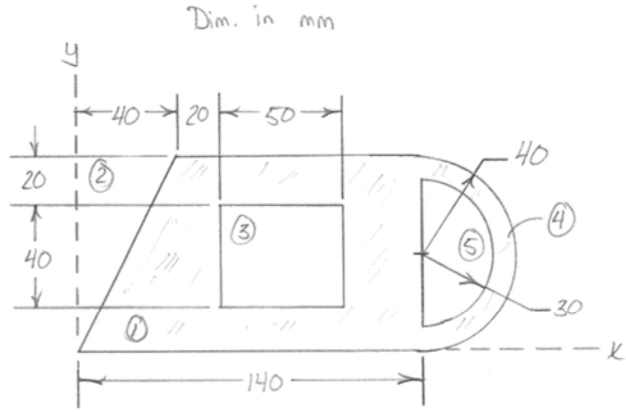

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$A\bar{x}$, mm^3	$A\bar{y}$, mm^3
1	$140(80) = 11200$	70	40	784×10^3	448×10^3
2	$-\frac{1}{2}(40)(80) = -1600$	$\frac{40}{3} = 13.33$	$\frac{2}{3}(80) = 53.3$	-21.3×10^3	-85.3×10^3
3	$-40(50) = -2000$	85	40	-170×10^3	-80×10^3
4	$\frac{\pi(40)^2}{2} = 800\pi$	$140 + \frac{4(40)}{3\pi} = 157.0$	40	395×10^3	32000π
5	$-\frac{\pi(30)^2}{2} = -450\pi$	$140 + \frac{4(30)}{3\pi} = 152.7$	40	-216×10^3	-18000π
Σ	8700			771×10^3	327×10^3

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{771 \times 10^3}{8700} \rightarrow \underline{\bar{X} = 88.7 \text{ mm}} \qquad \bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{327 \times 10^3}{8700} \rightarrow \underline{\bar{Y} = 37.5 \text{ mm}}$$

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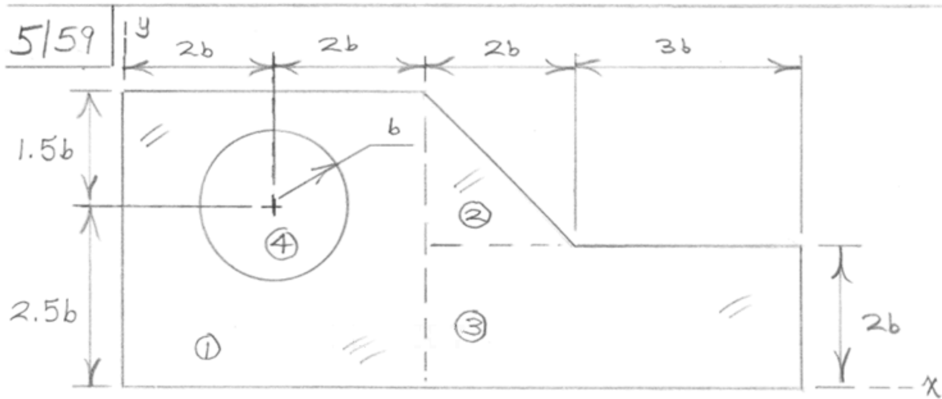


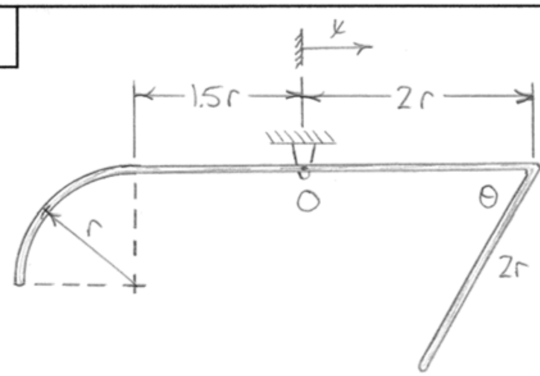
Comp.	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1	$16b^2$	$2b$	$2b$	$32b^3$	$32b^3$
2	$2b^2$	$\left(4b+\frac{2b}{3}\right)$	$\left(2b+\frac{2b}{3}\right)$	$9.33b^3$	$5.33b^3$
3	$10b^2$	$\left(4b+\frac{5b}{2}\right)$	b	$65b^3$	$10b^3$
4	$-\pi b^2$	$2b$	$2.5b$	$-2\pi b^3$	$-2.5\pi b^3$

$\Sigma A = 24.9b^2$ $\qquad \Sigma \bar{x}A = 100.1b^3$ $\qquad \Sigma \bar{y}A = 39.5b^3$

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{100.1b^3}{24.9b^2} = \underline{4.02b}$$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{39.5b^3}{24.9b^2} = \underline{1.588b}$$

5/60

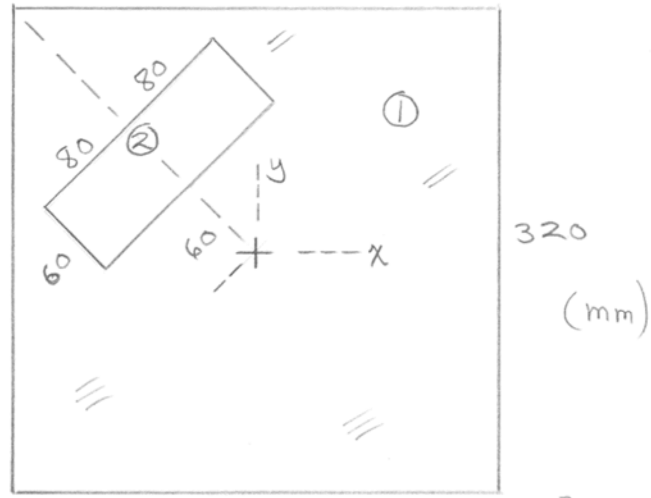


FOR ORIENTATION SHOWN, $\overline{X} = 0$.

$$\overline{X} = \frac{\Sigma L\bar{x}}{\Sigma L} = 0 = \frac{2r(r) + 2r(2r - r\cos\theta) + 1.5r\left(\frac{-1.5r}{2}\right) + \frac{\pi}{2}r\left(-1.5r - \frac{2r}{\pi}\right)}{2r + 2r + 1.5r + \frac{\pi}{2}r}$$

$\underline{\theta = 40.6^\circ}$

5/61



Comp.	A (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{x}A$ (mm^3)	$\bar{y}A$ (mm^3)
1	$(320)^2$	0	0	0	0
2	$-160(60)$	$-90\frac{\sqrt{2}}{2}$	$90\frac{\sqrt{2}}{2}$	611 000	−611 000

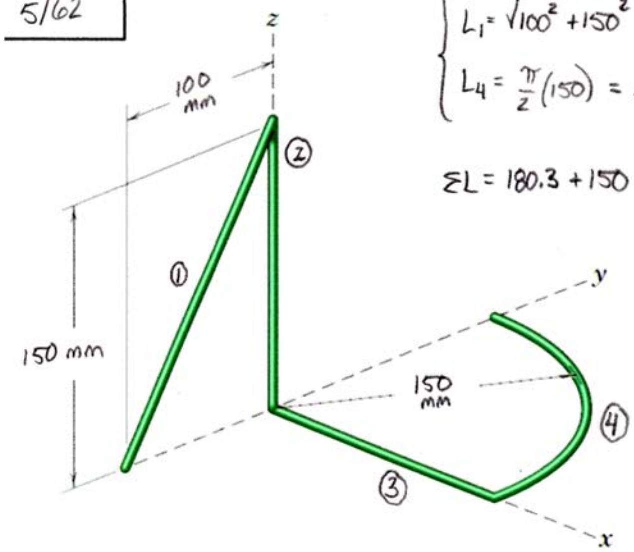
$\Sigma A = 92\,800$ $\quad\quad$ $\Sigma \bar{x}A = 611\,000$

$\Sigma \bar{y}A = -611\,000$

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{611\,000}{92\,800} = \underline{6.58 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{-611\,000}{92\,800} = \underline{-6.58 \text{ mm}}$$

5/62



$$\begin{cases} L_1 = \sqrt{100^2 + 150^2} = 180.3 \text{ mm} \\ L_4 = \frac{\pi}{2}(150) = 236 \text{ mm} \quad \text{or } 75\pi \text{ mm} \end{cases}$$

$$\Sigma L = 180.3 + 150 + 150 + 236 = 716 \text{ mm}$$

$$\overline{X} = \frac{\Sigma L \bar{x}}{\Sigma L} = \frac{180.3(0) + 150(0) + 150(75) + 75\pi\left(\frac{2}{\pi}150\right)}{716} \longrightarrow \underline{\overline{X} = 47.1 \text{ mm}}$$

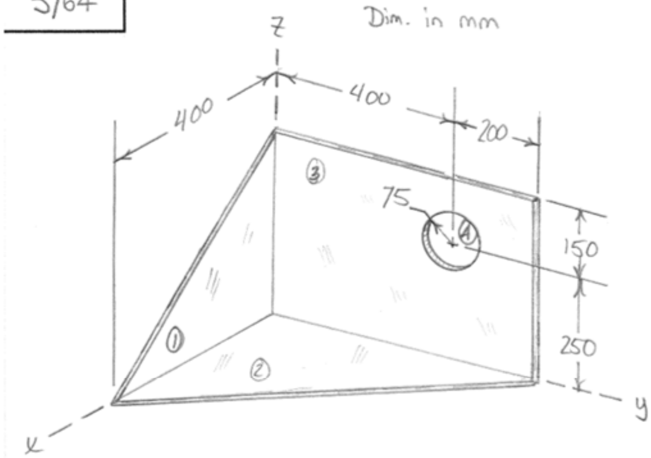
$$\overline{Y} = \frac{\Sigma L \bar{y}}{\Sigma L} = \frac{180.3(-50) + 150(0) + 150(0) + 75\pi\left(\frac{2}{\pi}150\right)}{716} \longrightarrow \underline{\overline{Y} = 18.84 \text{ mm}}$$

$$\overline{Z} = \frac{\Sigma L \bar{z}}{\Sigma L} = \frac{180.3(75) + 150(75) + 150(0) + 75\pi(0)}{716} \longrightarrow \underline{\overline{Z} = 34.6 \text{ mm}}$$

5/63

$$\bar{z} = \frac{\Sigma m\bar{z}}{\Sigma m} = \frac{2(0) + 1.5(90) + 1(180)}{2 + 1.5 + 1} = \underline{70 \text{ mm}}$$

5/64



$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{\frac{1}{2}(400)(400)\left(\frac{400}{3}\right) + \frac{1}{2}(400)(600)\left(\frac{400}{3}\right) + 400(600)(0) - \pi(75)^2(0)}{\frac{1}{2}(400)(400) + \frac{1}{2}(400)(600) + 400(600) - \pi(75)^2}$$

$$\bar{X} = 63.1 \text{ mm}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{\frac{1}{2}(400)(400)(0) + \frac{1}{2}(400)(600)(200) + 400(600)(300) - \pi(75)^2(400)}{\frac{1}{2}(400)(400) + \frac{1}{2}(400)(600) + 400(600) - \pi(75)^2}$$

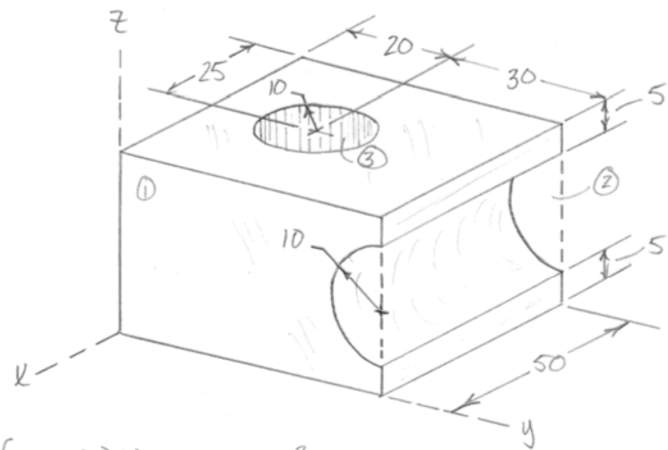
$$\bar{Y} = 211 \text{ mm}$$

$$\bar{Z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{\frac{1}{2}(400)(400)\left(\frac{400}{3}\right) + \frac{1}{2}(400)(600)(0) + 400(600)(200) - \pi(75)^2(250)}{\frac{1}{2}(400)(400) + \frac{1}{2}(400)(600) + 400(600) - \pi(75)^2}$$

$$\bar{Z} = 128.5 \text{ mm}$$

5/65

Dim. in mm



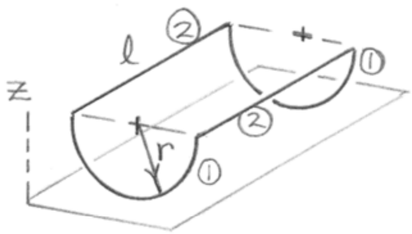
$$\begin{cases} V_1 = 50(50)(30) = 75\,000 \text{ mm}^3 \\ V_2 = -\frac{\pi}{2}(10)^2(50) = -7850 \text{ mm}^3 \\ V_3 = -\pi(10)^2(30) = -9420 \text{ mm}^3 \end{cases}$$

$\underline{\bar{X} = -25 \text{ mm}}$ (INSPECTION)

$$\bar{Y} = \frac{\Sigma V\bar{y}}{\Sigma V} = \frac{75\,000(25) - 7850\left(50 - \frac{4(10)}{3\pi}\right) - 9420(20)}{75000 - 7850 - 9420} \longrightarrow \underline{\bar{Y} = 23.0 \text{ mm}}$$

$\underline{\bar{Z} = 15 \text{ mm}}$ (INSPECTION)

5/66



Comp.	L	$\bar{z}$	$L\bar{z}$
1	$2(\pi r)$	$r - \frac{2r}{\pi}$	$2r^2(\pi - 2)$
2	2ℓ	r	$2\ell r$

$\Sigma L = 2(\pi r + \ell)$ $\qquad$ $\Sigma L\bar{z} = 2r[r(\pi-2)+\ell]$

Set $\bar{z} = \frac{\Sigma L\bar{z}}{\Sigma L} = \frac{2r[r(\pi-2)+\ell]}{2(\pi r+\ell)} = \frac{3r}{4}$

and solve for ℓ as

$$\underline{\ell = (8-\pi)r}$$

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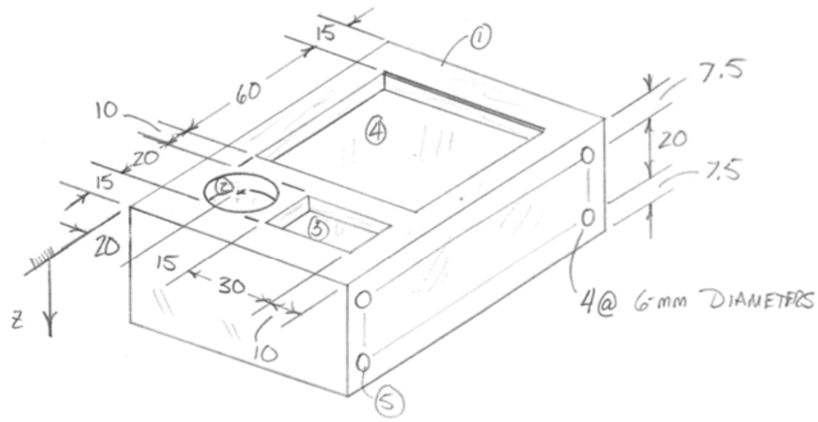


$V_1 = 75(120)(35) = 315\,000 \text{ mm}^3$

$V_2 = -\frac{\pi}{4}(20)^2(5) = -1571 \text{ mm}^3$

$V_3 = -30(20)(5) = -3000 \text{ mm}^3$

$V_4 = -60(55)(5) = -16\,500 \text{ mm}^3$

$V_5 = -\frac{\pi}{4}(6)^2(75) = -2120 \text{ mm}^3$ (4 OF THESE)

$\Sigma V = 285\,000 \text{ mm}^3$

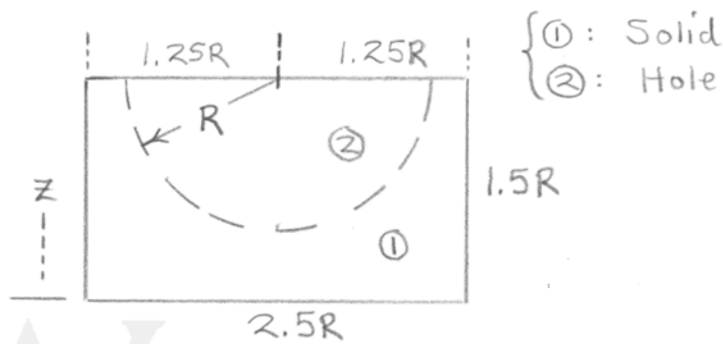
$$\bar{Z} = \frac{\Sigma V \bar{z}}{\Sigma V} = \frac{315000\left(\frac{35}{2}\right) - (1571 + 3000 + 16\,500)\left(5/2\right) - 2120\left(7.5(2) + 27.5(2)\right)}{285\,000}$$

$$\bar{Z} = \bar{H} = 18.61 \text{ mm}$$

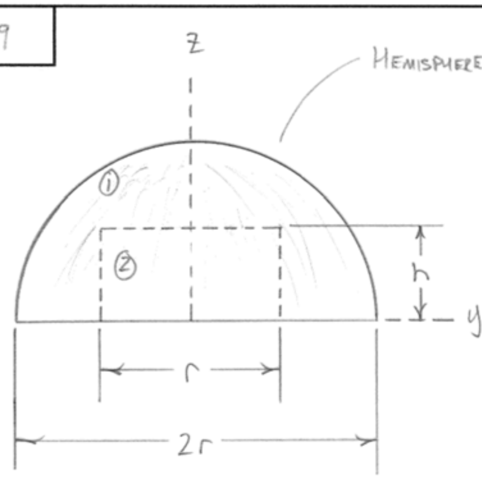
5/68

Comp.	V	$\bar{z}$	$V\bar{z}$
1	$(2.5R)^2(1.5R)$	$0.75R$	$7.03R^4$
2	$-\frac{4}{3}\pi R^3/2$	$(1.5R-\frac{3}{8}R)$	$-2.36R^4$
Σ:	$7.28R^3$		$4.68R^4$

$$\bar{z} = \frac{\Sigma V\bar{z}}{\Sigma V} = \frac{4.68R^4}{7.28R^3} = \underline{0.642R}$$



5/69



$$\bar{z} = \frac{\Sigma V \bar{z}}{\Sigma V} = \frac{\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)\left(\frac{3r}{8}\right) - r^2 h\left(\frac{h}{2}\right)}{\frac{1}{2}\left(\frac{4}{3}\pi r^3\right) - r^2 h} \longrightarrow \bar{z} = \frac{3\pi r^2 - 6h^2}{8\pi r - 12h}$$

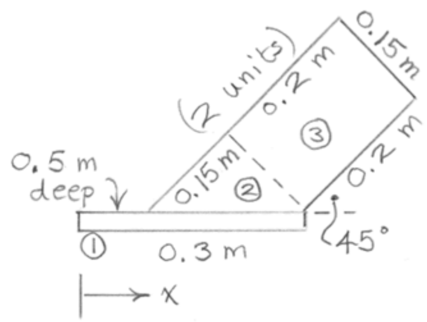
FOR h TO MAXIMIZE $\bar{z}$ SET $\frac{d\bar{z}}{dh} = 0$.

$$\frac{d\bar{z}}{dh} = \frac{3(6h^2 - 8\pi r h + 3\pi r^2)}{4(3h - 2\pi r)^2} = 0 \longrightarrow h = 0.416r \text{ OR } 3.77r$$

SO... $\underline{h = 0.416r}$

AT THIS VALUE OF h, $\bar{z} = 0.416r$

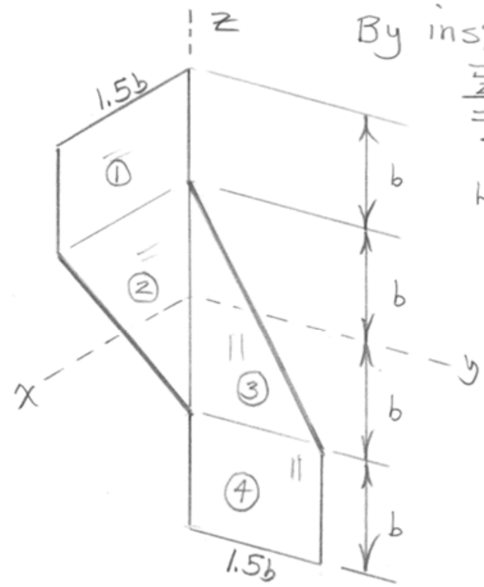
5/70 ρ = mass per unit area



Comp.	m	$\bar{x}$	$\bar{x}m$
①	$(0.5)(0.3)\rho$	0.15	0.0225ρ
②	$2\,\frac{(0.15)(0.15)}{2}\,\rho$	$0.3-\left[\frac{2}{3}(0.15)+\frac{1}{3}(0.15)\right]\frac{\sqrt{2}}{2}$	0.00436ρ
③	$2(0.2)(0.15)\rho$	$0.3+\left(\frac{0.2-0.15}{2}\right)\frac{\sqrt{2}}{2}$	0.01906ρ
	$\Sigma m = 0.232\rho$		$\Sigma \bar{x}m = 0.0459\rho$

$$\bar{X} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{0.0459\rho}{0.232\rho} = \underline{0.1975 \text{ m}}$$

5/71



By inspection,

$\bar{Z} = 0$

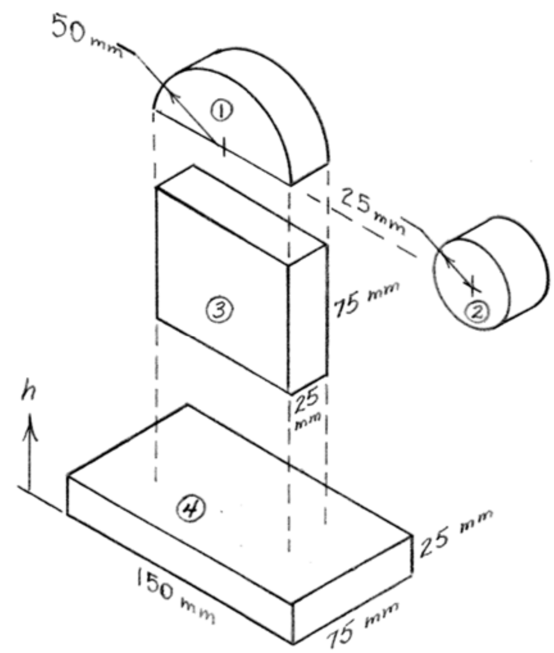
$\bar{X} = \bar{Y}$

Let ρ = area density

Comp.	m	$\bar{x}$	$m\bar{x}$
1	$1.5b^2\rho$	$1.5b/2$	$1.125b^3\rho$
2	$1.5b^2\rho$	$0.5b$	$0.75b^3\rho$
3	$1.5b^2\rho$	0	0
4	$1.5b^2\rho$	0	0
	$\Sigma A = 6b^2$		$\Sigma \bar{x}A = 1.875b^3$

$$\bar{X} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{1.875b^3\rho}{6b^2\rho} = \underline{0.312b} = \bar{Y}$$

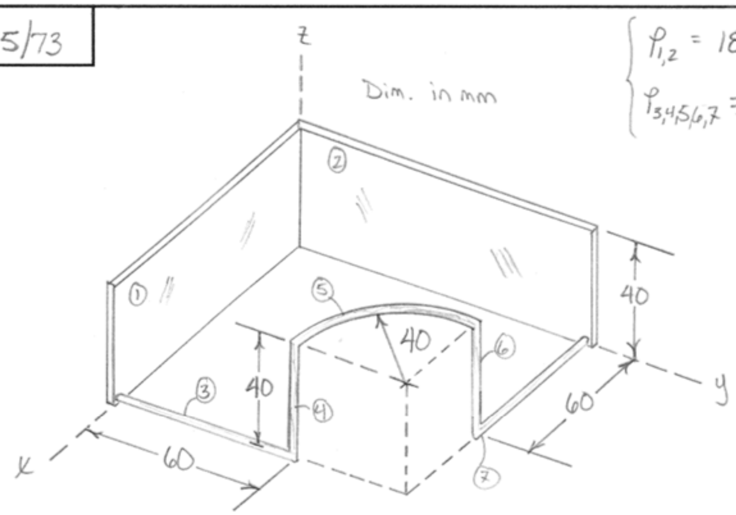
5/72



Part	V, mm^3	$\bar{h}$, mm	$V\bar{h}$, mm^4
1	98 200	121.2	11.90×10^6
2	-49 100	100	-4.91×10^6
3	187 500	62.5	11.72×10^6
4	281 000	12.5	3.52×10^6
Totals	518 000		22.2×10^6

$$\bar{H} = \frac{\Sigma V\bar{h}}{\Sigma V} = \frac{22.2\times10^6}{518\ 000} = \underline{42.9\ \text{mm}}$$

5/73



$$\begin{cases} \rho_{1,2} = 18 \ \text{kg/m}^2 \\ \rho_{3,4,5,6,7} = 2 \ \text{kg/m} \end{cases}$$

$$\begin{cases} m_{1,2} = 18\,(40)(100)/1000^2 = 0.072 \text{ kg} \\ m_{4,6} = 2\,(40/1000) = 0.08 \text{ kg} \end{cases} \qquad \begin{cases} m_{3,7} = 2\left(\frac{60}{1000}\right) = 0.120 \text{ kg} \\ m_5 = 2\left(\frac{\pi}{2}\right)(40/1000) = 0.1257 \text{ kg} \end{cases}$$

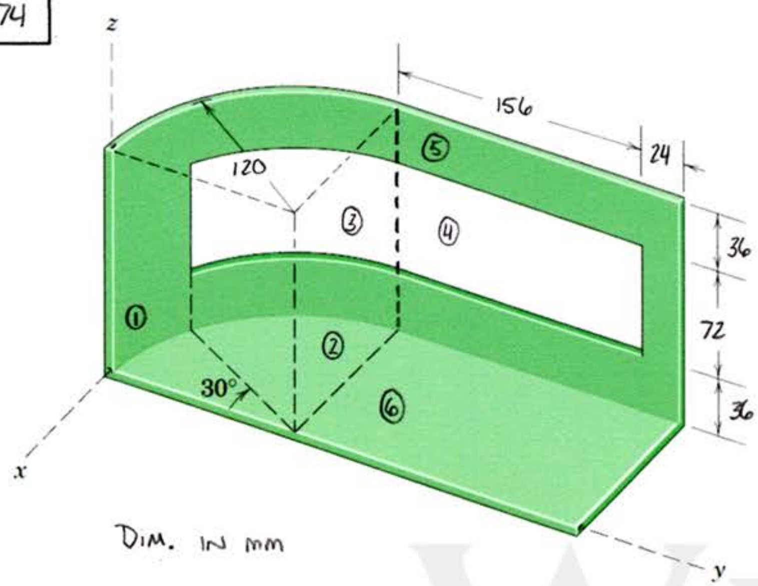
$$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} = \frac{0.072(50+0) + 0.12(100+30) + 0.08(100+60) + 0.1257\left(100 - \frac{2(40)}{\pi}\right)}{2(0.072 + 0.12 + 0.08) + 0.1257}$$

$$\underline{\bar{X} = \bar{Y} = 61.8 \text{ mm}}$$

$$\bar{Z}_1 = \frac{\Sigma m\bar{z}}{\Sigma m} = \frac{0.072(20+20) + 0.12(0+0) + 0.08(20+20) + 0.1257(40)}{2(0.072 + 0.12 + 0.08) + 0.1257}$$

$$\underline{\bar{Z}_1 = 16.59 \text{ mm}}$$

▶ 5/74



DIM. IN mm

$$\begin{cases} A_1 = \frac{\pi}{2}(120)(144) = 27\,100 \text{ mm}^2 \\ A_2 = \frac{1}{4}\pi(120)^2 = 11\,310 \text{ mm}^2 \\ A_3 = -\frac{\pi}{3}(120)(72) = -9050 \text{ mm}^2 \\ A_4 = -156(72) = -11\,230 \text{ mm}^2 \\ A_5 = 180(144) = 25\,900 \text{ mm}^2 \\ A_6 = 180(120) = 21\,600 \text{ mm}^2 \end{cases}$$

$$\begin{cases} \bar{x}_1 = \frac{-2(120)}{\pi} = -76.4 \text{ mm} \\ \bar{x}_2 = \frac{-4(120)}{3\pi} = -50.9 \text{ mm} \\ \bar{x}_3 = \frac{-120\sin 30^\circ}{\pi/6}\cos 30^\circ = -99.2 \text{ mm} \\ \bar{x}_4 = -120 \text{ mm} \\ \bar{x}_5 = -120 \text{ mm} \\ \bar{x}_6 = -60 \text{ mm} \end{cases}$$

$$\Sigma A = 65\,700 \text{ mm}^2$$

$$\begin{cases} \bar{y}_1 = 120 - \frac{2(120)}{\pi} = 43.6 \text{ mm} \\ \bar{y}_2 = 120 - \frac{4(120)}{3\pi} = 69.1 \text{ mm} \\ \bar{y}_3 = 120 - \frac{120\sin 30^\circ}{\pi/6}\sin 30^\circ = 62.7 \text{ mm} \\ \bar{y}_4 = 120 + 156/2 = 198 \text{ mm} \\ \bar{y}_5 = 120 + 180/2 = 210 \text{ mm} \\ \bar{y}_6 = 120 + 180/2 = 210 \text{ mm} \end{cases}$$

$$\begin{cases} \bar{z}_1 = 72 \text{ mm} \\ \bar{z}_2 = 0 \\ \bar{z}_3 = 72 \text{ mm} \\ \bar{z}_4 = 72 \text{ mm} \\ \bar{z}_5 = 72 \text{ mm} \\ \bar{z}_6 = 0 \end{cases}$$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{27100(-76.4) + 11\,310(-50.9) - 9050(-99.2) - 11230(-120) + 25900(-120) + 21600(-60)}{65700}$$

$$\underline{\bar{X} = -73.2 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{27100(43.6) + 11310(69.1) - 9050(62.7) - 11230(198) + 25900(210) + 21600(210)}{65700}$$

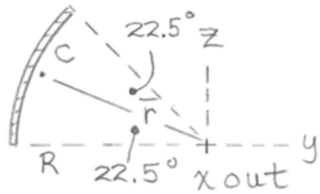
$$\underline{\bar{Y} = 139.3 \text{ mm}}$$

$$\bar{Z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{27100(72) + 11310(0) - 9050(72) - 11230(72) + 25900(72) + 21600(0)}{65\,700}$$

$$\underline{\bar{Z} = 35.9 \text{ mm}}$$

▸5/75 Detail of "hole" (2): [1 = cyl. shell]

$$\bar{r} = \frac{R \sin 22.5°}{\pi/8} = 0.974R$$



$$\begin{cases} \bar{x} = -\frac{L}{8} - \frac{1}{2}\frac{3L}{8} = -\frac{5}{16}L \\ \bar{y} = -\bar{r}\cos 22.5° = -0.900R \\ \bar{z} = \bar{r}\sin 22.5° = 0.373R \end{cases}$$

Comp	m	$\bar{x}$	$\bar{y}$	$\bar{z}$
1	$2\rho\pi RL$	$-\frac{L}{2}$	0	0
2	$-\frac{3}{32}\rho\pi RL$	$-\frac{5L}{16}$	$-0.900R$	$0.373R$

$\Sigma m = 1.906\rho\pi RL$

	$\bar{x}m$	$\bar{y}m$	$\bar{z}m$
1	$-\rho\pi RL^2$	0	0
2	$+0.0293\rho\pi RL^2$	$+0.0844\rho\pi R^2L$	$-0.0350\rho\pi R^2L$

$\Sigma\bar{x}m = -0.971\rho\pi RL^2$ $\quad \Sigma\bar{y}m = 0.0844\rho\pi R^2L$ $\quad \Sigma\bar{z}m = -0.0350\rho\pi R^2L$

$$\bar{X} = \frac{\Sigma\bar{x}m}{\Sigma m} = \frac{-0.971\rho\pi RL^2}{1.906\rho\pi RL} = \underline{-0.509L}$$

$$\bar{Y} = \frac{\Sigma\bar{y}m}{\Sigma m} = \frac{0.0844\rho\pi R^2L}{1.906\rho\pi RL} = \underline{0.0443R}$$

$$\bar{Z} = \frac{\Sigma\bar{z}m}{\Sigma m} = \frac{-0.0350\rho\pi R^2L}{1.906\rho\pi RL} = \underline{-0.01834R}$$

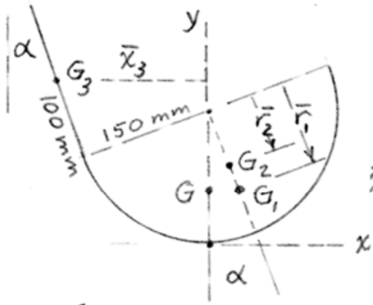
▸5/76

① = semi circular shell
② = both ends
③ = back

$A_1 = \pi(150)(400) = 188.5\times10^3\ mm^2$

$A_2 = 2\left(\pi\times\frac{[150]^2}{2}\right) = 70.7\times10^3\ mm^2$

$A_3 = 200(400) = 80\times10^3\ mm^2$



$\bar{r}_1 = \frac{2(150)}{\pi} = 95.5\ mm$

$\bar{r}_2 = \frac{4(150)}{3\pi} = 63.7\ mm$

$\bar{x}_1 = 95.5 \sin\alpha$, $\bar{x}_2 = 63.7 \sin\alpha$

$\bar{x}_3 = -150\cos\alpha - 100\sin\alpha$

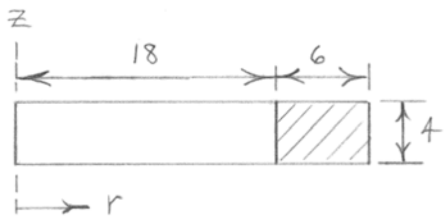
$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A}$; $0 = 188.5\times10^3(95.5\sin\alpha) + 70.7\times10^3(63.7\sin\alpha)$

$+ 80\times10^3(-150\cos\alpha - 100\sin\alpha)$

$14500\sin\alpha - 12000\cos\alpha = 0$

$\tan\alpha = \frac{12000}{14500} = 0.828$, $\underline{\alpha = 39.6°}$

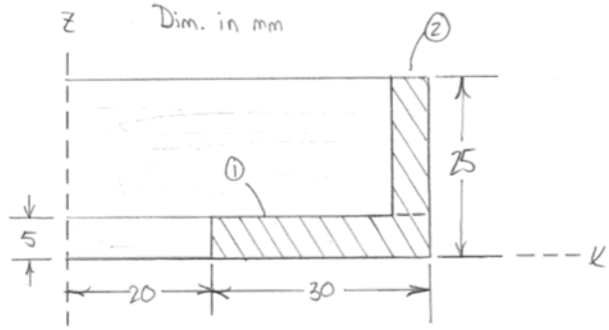
5/77



$$A = 2\pi \bar{r} L = 2\pi\left(18+\frac{6}{2}\right)(6+6+4+4)$$
$$= \underline{2640 \text{ mm}^2}$$

$$V = 2\pi \bar{r} A = 2\pi\left(18+\frac{6}{2}\right)(6\cdot 4)$$
$$= \underline{3170 \text{ mm}^3}$$

5/78 $\rho = 2690\ kg/m^3$



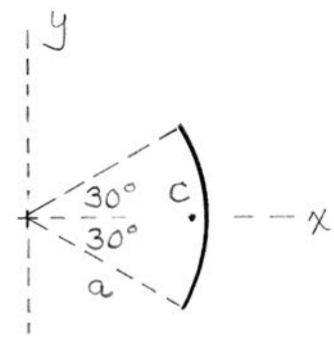
$$A = 30(5) + 20(5) \longrightarrow A = 250\ mm^2$$

$$\overline{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{30(5)(20+15) + 20(5)(50-5/2)}{250} \longrightarrow \overline{X} = 40\ mm$$

$$V = 2\pi A\overline{X} = 2\pi(250)(40) \longrightarrow V = 62\,800\ mm^3$$

$$m = \rho V = 2690\left(\frac{62800}{10^9}\right) \longrightarrow \underline{m = 0.1690\ kg}$$

5/79



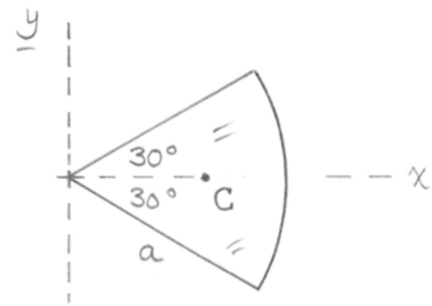
From Table D/3,

$$\bar{x} = \frac{a \sin \pi/6}{\pi/6} = \frac{3a}{\pi}$$

$$A = 2\pi \bar{x} L = 2\pi \left(\frac{3a}{\pi}\right)\left(\frac{\pi}{3} a\right)$$

$$= \underline{2\pi a^2 = S}$$

5/80



From Table D/3:

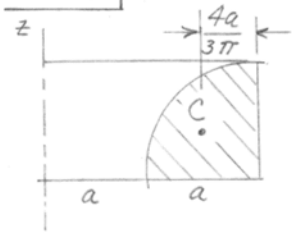
$$\bar{x} = \frac{2}{3}a\,\frac{\sin \pi/6}{\pi/6} = \frac{2a}{3}\,\frac{1}{2}\,\frac{6}{\pi} = \frac{2a}{\pi}$$

$$V = \pi \bar{x} A = \pi\left(\frac{2a}{\pi}\right)\left(\frac{\pi a^2}{6}\right)$$

$$= \underline{\frac{\pi a^3}{3}}$$

5/81 $V = \theta \bar{r} A = \pi\left(8 + \frac{2}{3}12\right)\frac{1}{2}(12)(12) = \underline{3620 \text{ mm}^3}$

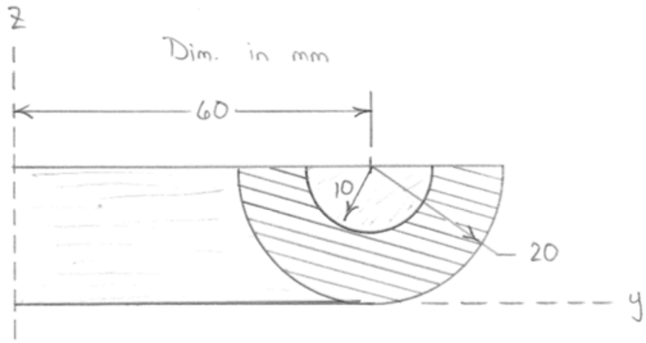
5/82



$V = \bar{r}\theta A$

$= \left(2a - \frac{4a}{3\pi}\right)\frac{\pi}{2}\frac{\pi a^2}{4} = \frac{\pi a^3}{12}(3\pi - 2)$

5/83 180° SOLID OF REVOLUTION



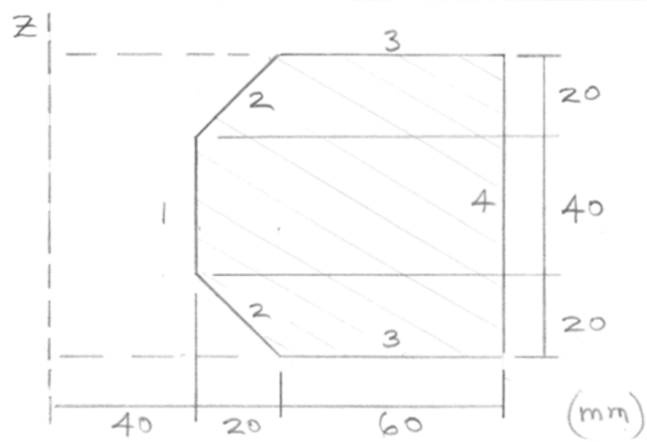
$A_c = \frac{1}{2}\pi(20^2 - 10^2) \longrightarrow A_c = 471 \text{ mm}^2$ $\qquad \bar{y} = 60 \text{ mm}$

$V = \pi A_c\bar{y} = \pi(471)(60) \longrightarrow \underline{V = 88\,800 \text{ mm}^3}$

$L = 10 + 10 + \pi(10 + 20) \longrightarrow L = 114.2 \text{ mm}$

$A = \pi L\bar{y} + 2A_c = \pi(114.2)(60) + 2(471) \longrightarrow \underline{A = 22\,500 \text{ mm}^2}$

5/84



$A_1 = \pi \bar{r}_1 L_1 = \pi(40)(40) = 1600\pi \text{ mm}^2$

$A_2 = 2(\pi \bar{r}_2 L_2) = 2\pi(50)(20\sqrt{2}) = 2830\pi \text{ mm}^2$

$A_3 = 2(\pi r_3 L_3) = 2\pi(90)(60) = 10\,800\pi \text{ mm}^2$

$A_4 = \pi r_4 L_4 = \pi(120)(80) = 9600\pi \text{ mm}^2$

End faces: $2\{80(60) + 40(20) + 20(20)\} = 12\,000 \text{ mm}^2$

Total: $\underline{90\,000 \text{ mm}^2}$

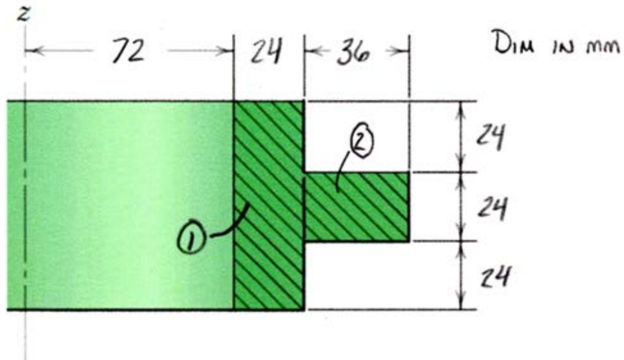
5/85

$$A = 2\pi \bar{r} L + \pi d h$$

$$= 2\pi(2.5)(10) + \pi(2.5)(6) = 204.2 \text{ m}^2$$

No. of liters for two coats is $2\,\dfrac{204.2}{16} = \underline{25.5 \text{ liters}}$

5/86



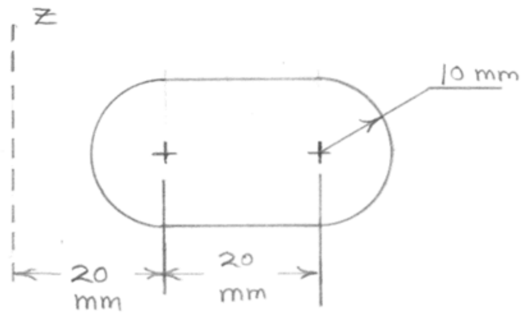
$$A = \Theta \Sigma L \bar{r} = 2\pi\left[72(72) + 2(24)\left(72+\frac{24}{2}\right) + 2(24)(96) + 2(36)\left(96+\frac{36}{2}\right) + 24(96+36)\right]$$

$$\underline{A = 158\,300 \text{ mm}^2}$$

$$V = \Theta \Sigma A \bar{r} = 2\pi\left[72(24)\left(72+\frac{24}{2}\right) + 36(24)\left(72+24+{}^{36}/_{2}\right)\right]$$

$$\underline{V = 1.531(10^6) \text{ mm}^3}$$

5/87



$A = 2\pi \bar{r} L = 2\pi(30)[20 + 20 + \pi 10 + \pi 10]$

$= \underline{19\ 380 \text{ mm}^2}$

$V = 2\pi \bar{r} A = 2\pi(30)[20(20) + \pi(10^2)]$

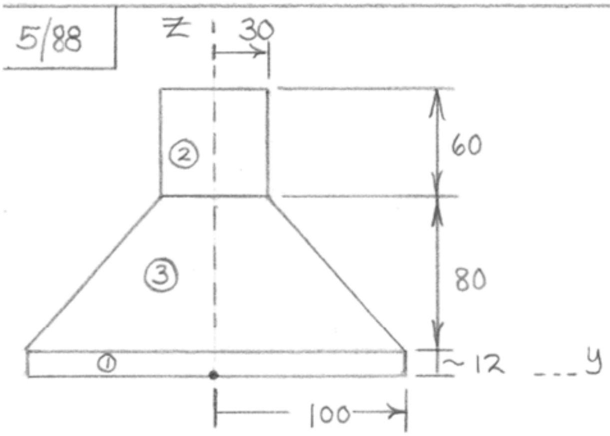
$= \underline{134\ 600 \text{ mm}^3}$

For steel,

$m = \rho V = 7830 \frac{\text{kg}}{\text{m}^3} \left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)^3 (134\ 600 \text{ mm}^3)$

$= \underline{1.054 \text{ kg}}$

5/88



Surface areas

① $2\pi\bar{y}L = 2\pi(100)(12) = 7\,540\ \text{mm}^2$

② $2\pi\bar{y}L = 2\pi(30)(60) = 11\,310\ \text{mm}^2$

③ $2\pi\bar{y}L = 2\pi\left[\frac{30+100}{2}\right]\left[\sqrt{80^2+70^2}\right]$

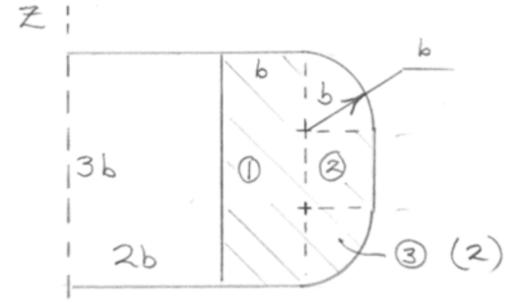
$= 43\,400\ \text{mm}^2$

Total : $62\,300\ \text{mm}^2$

Volume $V = 62\,300(0.6) = 37\,400\ \text{mm}^3$

Mass $m = \rho V = 7830(37\,400)(10^{-9}) = \underline{0.293\ \text{kg}}$

5/89



$$V_1 = 2\pi \bar{r}_1 A_1 = 2\pi\left(2b+\frac{b}{2}\right)(3b^2) = 15\pi b^3$$

$$V_2 = 2\pi \bar{r}_2 A_2 = 2\pi\left(3b+\frac{b}{2}\right)(b^2) = 7\pi b^3$$

$$V_3 = 2(2\pi \bar{r}_3 A_3) = 4\pi\left(3b+\frac{4b}{3\pi}\right)\left(\frac{\pi b^2}{4}\right) = \pi^2 b^3\left(3+\frac{4}{3\pi}\right)$$

$$V = V_1 + V_2 + V_3 = \underline{\pi b^3\left(\frac{70}{3}+3\pi\right)} = \underline{102.9\, b^3}$$

$$A_{1\,\text{inner}} = 2\pi \bar{r}_{1_i} L_{1_i} = 2\pi(2b)(3b) = 12\pi b^2$$

$$A_{1\,\text{top \& bott.}} = 2\cdot 2\pi \bar{r}_{1_t} L_{1_t} = 4\pi \frac{5b}{2}(b) = 10\pi b^2$$

$$A_2 = 2\pi \bar{r}_2 L_2 = 2\pi(4b)(b) = 8\pi b^2$$

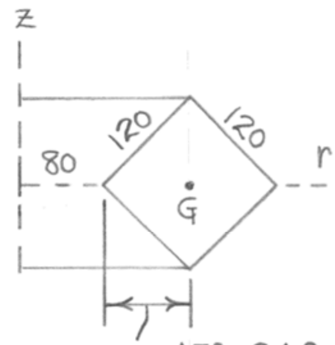
$$A_3 = 2\cdot 2\pi \bar{r}_3 L_3 = 4\pi\left(3b+\frac{2b}{\pi}\right)\left(\frac{\pi b}{2}\right) = 2\pi^2 b^2\left(3+\frac{2}{\pi}\right)$$

$$A = A_1 + A_2 + A_3 = \underline{\pi b^2(34+6\pi)} = \underline{166.0\, b^2}$$

5/90

$$V = 2\pi \bar{r} A$$
$$= 2\pi(80 + 84.9)(120)^2$$
$$= \underline{14.92(10^6)\ \text{mm}^3}$$

Surface area $A = 2\pi \bar{r} L$
$$= 2\pi(80 + 84.9)(4 \times 120)$$
$$= \underline{497(10^3)\ \text{mm}^2}$$

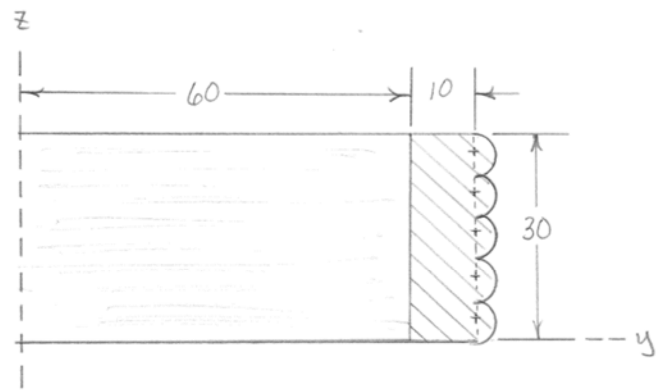


$120 \cos 45° = 84.9$

(Dim. in mm)

5/91

Dim. in mm $\qquad \rho = 1100\ \text{kg/m}^3$



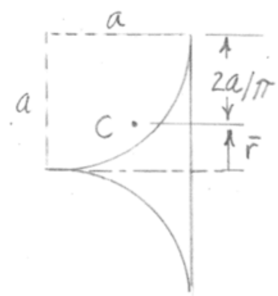
$A = 10(30) + \frac{5}{2}\pi(3)^2 \longrightarrow A = 371\ \text{mm}^2$

$\bar{Y} = \dfrac{\Sigma A\bar{y}}{\Sigma A} = \dfrac{10(30)(65) + \frac{5}{2}\pi(3)^2\left(70 + \frac{4(3)}{3\pi}\right)}{371} \longrightarrow \bar{Y} = 66.2\ \text{mm}$

$V = 2\pi A\bar{Y} = 2\pi(371)(66.2) \longrightarrow V = 154\,200\ \text{mm}^3$

$m = \rho V = 1100\left(\dfrac{154\,200}{10^9}\right) \longrightarrow \underline{m = 0.1696\ \text{kg}}$

5/92

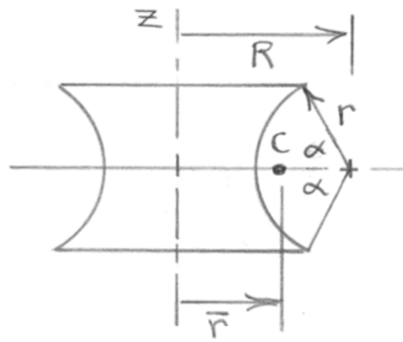


$$A = 2\pi \bar{r} L$$

$$= 2\pi\left(a - \frac{2a}{\pi}\right)\frac{\pi a}{2}$$

$$= \underline{\pi a^2(\pi - 2)}$$

5/93



$\bar{r} = R - r\dfrac{\sin\alpha}{\alpha}$ from Sample Problem 5/1

$L = 2r\alpha$

$A = 2\pi\bar{r}L = 2\pi\left(R - r\dfrac{\sin\alpha}{\alpha}\right)(2r\alpha)$

$\quad = \underline{4\pi r\,(R\alpha - r\sin\alpha)}$

5/94 $m = \rho V$, where $\rho = 7830\ \frac{kg}{m^3}$ (Appendix D)

$V = 2\pi\bar{r}A$, $\bar{r}A = 200(100)\left(\frac{60+160}{2}\right) - \frac{\pi(60^2)}{2}\left(60 + \frac{4(60)}{3\pi}\right)$

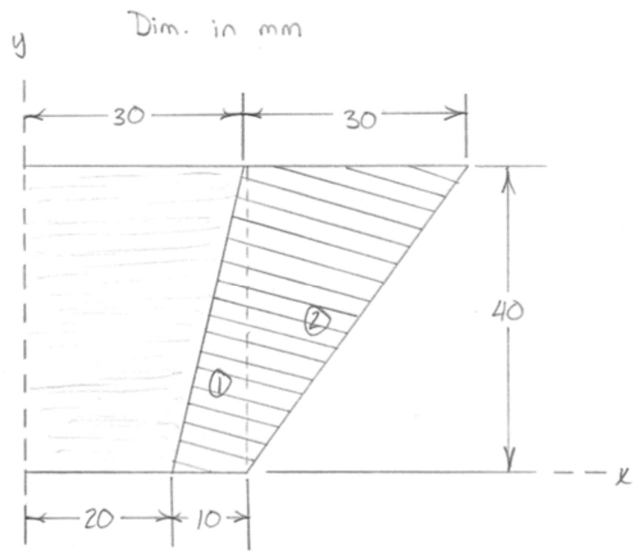
$= 1.717\,(10^6)\ mm^3$

$V = 2\pi\,(1.717 \times 10^6) = 10.79\,(10^6)\ mm^3$

or $V = 0.01079\ m^3$

$\therefore\ m = \rho V = 7830(0.01079) = \underline{84.5\ kg}$

5/95



$$A = \theta \Sigma L \bar{r} = 2\pi\left[\sqrt{10^2+40^2}(25) + 30(45) + 50(45) + 10(25)\right]$$

$$\underline{A = 30\,700 \text{ mm}^2}$$

$$V = \theta \Sigma A \bar{r} = 2\pi\left[\tfrac{1}{2}(10)(40)(20+\tfrac{2}{3}10) + \tfrac{1}{2}(30)(40)(30+10)\right]$$

$$\underline{V = 184\,300 \text{ mm}^3}$$

5/96 $V = 2\pi\bar{r}A$, $m = V\rho$

Where $m = 10.0$ kg, $\rho = 2.69 \times 10^3$ kg/m^3

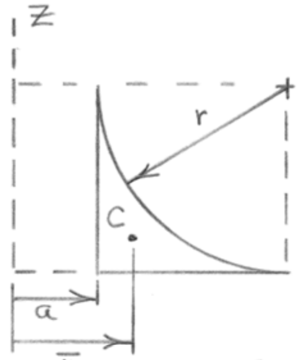
$A = \frac{1}{2}\, 15{,}200 \times 10^{-6} = 7.600 \times 10^{-3}$ m^2

Thus $\bar{r} = \frac{V}{2\pi A} = \frac{m}{2\pi\rho A} = \frac{10.0}{2\pi(2.69\times10^3)(7.6\times10^{-3})}$

$= 0.0778$ m

or $\underline{\bar{r} = 77.8 \text{ mm}}$

5/97



(Refer to Sample Problem 5/3)

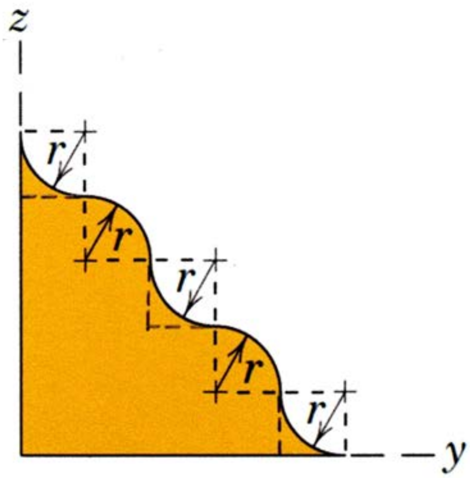
$$\bar{r} = \frac{r^2(a + r/2) - \frac{\pi r^2}{4}\left(a + r - \frac{4r}{3\pi}\right)}{r^2 - \pi r^2/4} = a + \frac{10 - 3\pi}{3(4-\pi)} r$$

$$V = \theta \bar{r} A = \frac{\pi}{2}\left[a + \frac{10-3\pi}{3(4-\pi)} r\right]\left(1 - \frac{\pi}{4}\right) r^2$$

$$= \frac{\pi r^2}{8}\left[(4-\pi)a + \frac{10-3\pi}{3} r\right]$$

5/98

$\rho = 8320 \text{ kg/m}^3 \qquad r = 25 \text{ mm}$



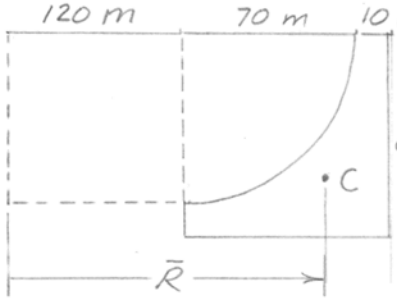
$$V = 2\pi \Sigma A\bar{y} = 2\pi \Big\{ 25(125)\left(\frac{25}{2}\right) + 25(75)\left(25+\frac{25}{2}\right) + 25(75)\left(50+\frac{25}{2}\right)$$

$$+ 25(25)\left(75+\frac{25}{2}\right) + 25(25)\left(100+\frac{25}{2}\right) + \frac{1}{4}\pi(25)^2\left[\left(25+\frac{4(25)}{3\pi}\right) + \left(75+\frac{4(25)}{3\pi}\right)\right]$$

$$- \frac{1}{4}\pi(25)^2\left[\left(25-\frac{4(25)}{3\pi}\right) + \left(75-\frac{4(25)}{3\pi}\right) + \left(125-\frac{4(25)}{3\pi}\right)\right]\Big\} = 1.987(10^6) \text{ mm}^3$$

$$W = \rho g V = 8320(9.81)\left(1.987(10^6)/10^9\right) \longrightarrow \underline{W = 162.2 \text{ N}}$$

5/99



Square: $A = 80^2 = 6400\ m^2$

$\frac{1}{4}$ circle: $A = \frac{1}{4}\pi(70^2) = 3848\ m^2$

Net area $= 2552\ m^2$

$\bar{r}_{square} = 120 + 40 = 160\ m$

$\bar{r}_{\frac{1}{4}circle} = 120 + \frac{4(70)}{3\pi} = 149.7\ m$

$\bar{R} = \frac{\Sigma A\bar{r}}{\Sigma A} = \frac{6400(160) - 3848(149.7)}{2552} = 175.5\ m$

$V = \theta\bar{R}A = \frac{\pi}{3}(175.5)(2552) = 469\ 000\ m^3$

$m = \rho V = 2.40(469\ 000) = \underline{1.126 \times 10^6\ Mg}$

5/100 From the solution to Prob. 5/8,

$$\bar{r} = 8 - \frac{2}{3}\frac{2(1.5)+2}{1.5+2} = 7.05 \text{ m}$$

$$A = \frac{2+1.5}{2}(2) = 3.5 \text{ m}^2$$

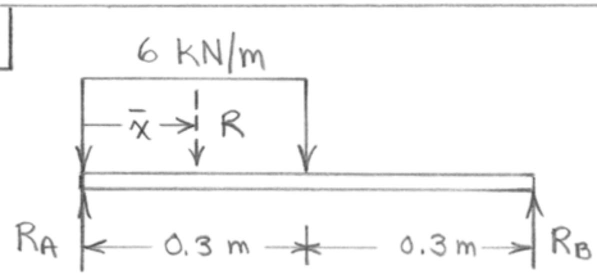
$$\theta = \pi/3$$

So $V = \theta \bar{r} A = \frac{\pi}{3}(7.05)(3.5) = 25.8 \text{ m}^3$

$$W = \rho g V = 2400(9.81)(25.8) = 608(10^3) \text{ N}$$

or $\underline{W = 608 \text{ kN}}$

5/101



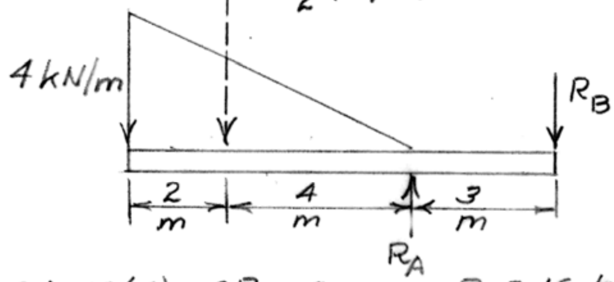
$R = 6(0.3) = 1.8\text{ kN} \quad @ \quad \bar{x} = \frac{1}{2}(0.3) = 0.15\text{ m}$

$\circlearrowleft + \Sigma M_A = 0: \quad R_B(0.6) - 1.8(0.15) = 0, \quad \underline{R_B = 0.45\text{ kN}}$

$+\uparrow \Sigma F = 0: \quad 0.45 - 1.8 + R_A = 0, \quad \underline{R_A = 1.35\text{ kN}}$

5/102

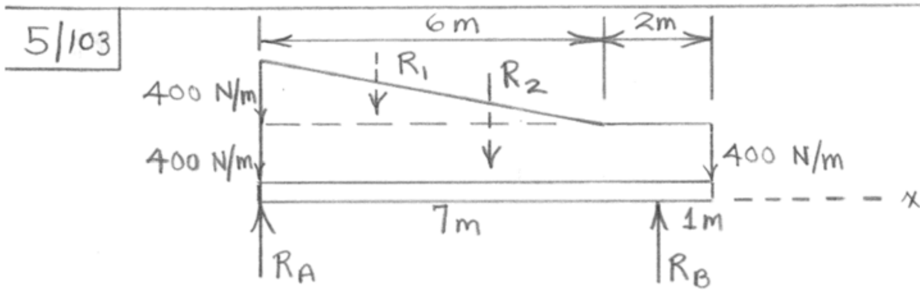
$F = \frac{1}{2}(4)(6) = 12$ kN



$\circlearrowleft^{+} \Sigma M_A = 0: \; 12(4) - 3R_B = 0, \qquad \underline{R_B = 16 \text{ kN}}$

$\uparrow^{+} \Sigma F = 0: \; R_A - 16 - 12 = 0, \qquad \underline{R_A = 28 \text{ kN}}$

5/103



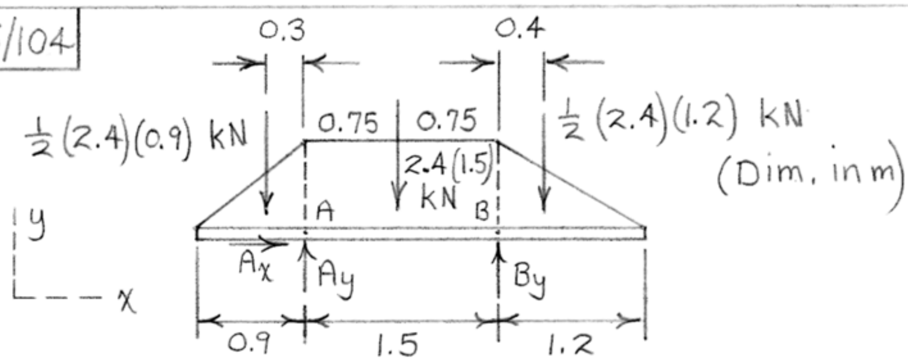
$R_1 = \frac{1}{2}(400)(6) = 1200$ N @ $\bar{x}_1 = \frac{1}{3}(6) = 2$m

$R_2 = 400(8) = 3200$ N @ $\bar{x}_2 = \frac{1}{2}(8) = 4$m

$\circlearrowleft + \Sigma M_A = 0: \; R_B(7) - 1200(2) - 3200(4) = 0, \; \underline{R_B = 2170 \text{ N}}$

$+\uparrow \Sigma F = 0: \; R_A - 1200 - 3200 + 2170 = 0, \; \underline{R_A = 2230 \text{ N}}$

5/104



$$\curvearrowleft + \Sigma M_A = 0: \quad 1.08(0.3) - 3.6(0.75) - 1.44(1.9) + B_y(1.5) = 0$$

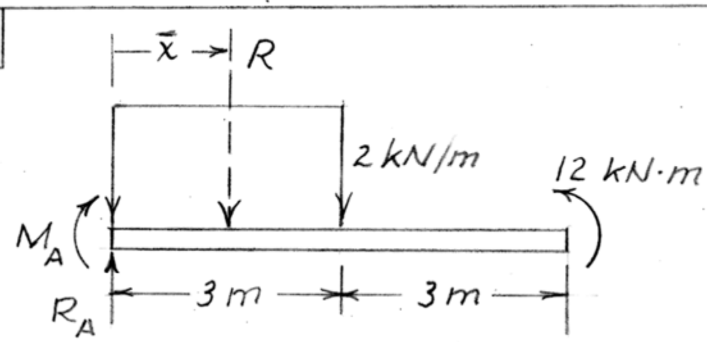
$$\underline{B_y = 3.41 \text{ kN}}$$

$$\Sigma F_y = 0: \quad A_y + 3.41 - 1.08 - 3.6 - 1.44 = 0$$

$$\underline{A_y = 2.71 \text{ kN}}$$

$$\Sigma F_x = 0: \quad \underline{A_x = 0}$$

5/105

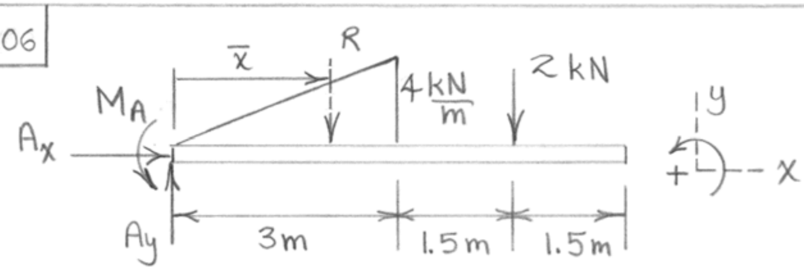


$R = 2(3) = 6 \text{ kN}$ @ $\bar{x} = 1.5 \text{ m}$

$\circlearrowleft + \Sigma M_A = 0: \; -M_A - 6(3/2) + 12 = 0, \quad \underline{M_A = 3 \text{ kN}\cdot\text{m}}$

$\uparrow + \Sigma F = 0: \; R_A - 6 = 0, \quad \underline{R_A = 6 \text{ kN}}$

5/106



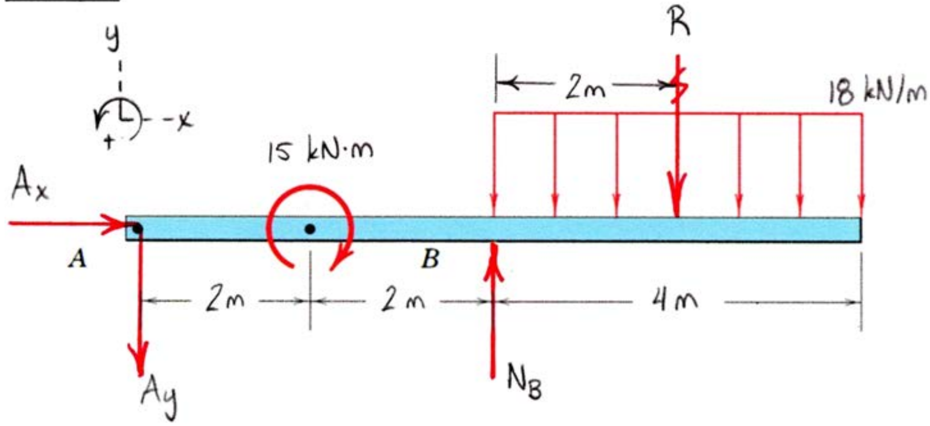
$R = \frac{1}{2}(3)(4) = 6\text{ kN}$ @ $\bar{x} = \frac{2}{3}(3) = 2\text{m}$

$\Sigma M_A = 0: \ M_A - 6(2) - 2(4.5) = 0, \quad \underline{M_A = 21\text{ kN}\cdot\text{m}}$

$\Sigma F_y = 0: \ A_y - 6 - 2 = 0, \quad \underline{A_y = 8\text{ kN}}$

$\Sigma F_x = 0: \ \underline{A_x = 0}$

5/107



$R = 18(4) = 72$ kN

$$\begin{cases} \Sigma F_x = 0: & \underline{A_x = 0} \\ \Sigma F_y = 0: & -A_y + N_B - R = 0 \\ \Sigma M_A = 0: & 4N_B - 6R - 15 = 0 \end{cases} \longrightarrow \begin{cases} \underline{A_y = 39.8 \text{ kN} \downarrow} \\ \underline{N_B = 111.8 \text{ kN} \uparrow} \end{cases}$$

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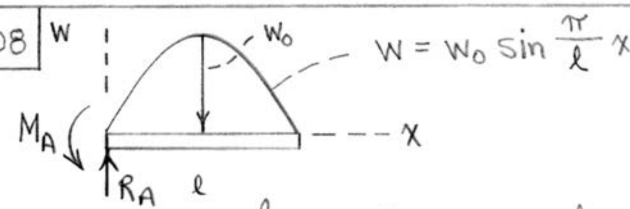


$$R = \int w\,dx = \int_0^l w_0 \sin \frac{\pi}{l} x = -w_0 \frac{l}{\pi} \cos \frac{\pi}{l} x \Big|_0^l = \frac{2w_0 l}{\pi}$$

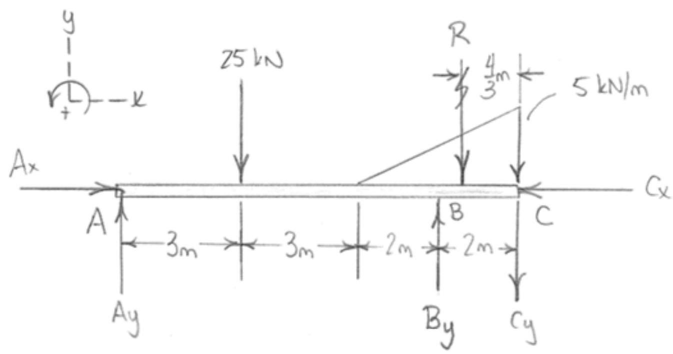
$\bar{x} = \frac{l}{2}$, by inspection

$\circlearrowleft + \Sigma M_A = 0: \quad M_A - \frac{2w_0 l}{\pi}\left(\frac{l}{2}\right) = 0, \quad \underline{M_A = \frac{w_0 l^2}{\pi} \text{ CCW}}$

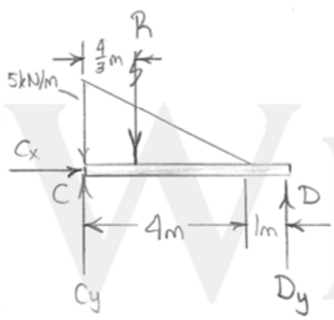
$+\uparrow \Sigma F = 0: \quad R_A - \frac{2w_0 l}{\pi} = 0, \quad \underline{R_A = \frac{2w_0 l}{\pi} \text{ up}}$

5/109

$R = \frac{1}{2}(5)(4) = 10 \text{ kN}$



$$\begin{cases} \Sigma F_x = 0: & A_x - C_x = 0 \\ \Sigma F_y = 0: & A_y + N_B - C_y - 25 - R = 0 \\ \Sigma M_A = 0: & -3(25) + 8B_y - 10C_y - (10 - \frac{4}{3})R = 0 \end{cases}$$



$$\begin{cases} \Sigma F_x = 0: & C_x = 0 \quad \text{so} \quad \underline{A_x = 0} \\ \Sigma F_y = 0: & C_y + D_y - R = 0 \\ \Sigma M_C = 0: & 5D_y - \frac{4}{3}R = 0 \end{cases}$$

SOLVING...

$$\begin{cases} \underline{A_y = 12.96 \text{ kN} \uparrow} \\ C_y = 7.33 \text{ kN} \end{cases} \qquad \begin{cases} \underline{B_y = 29.4 \text{ kN} \uparrow} \\ \underline{D_y = 2.67 \text{ kN} \uparrow} \end{cases}$$

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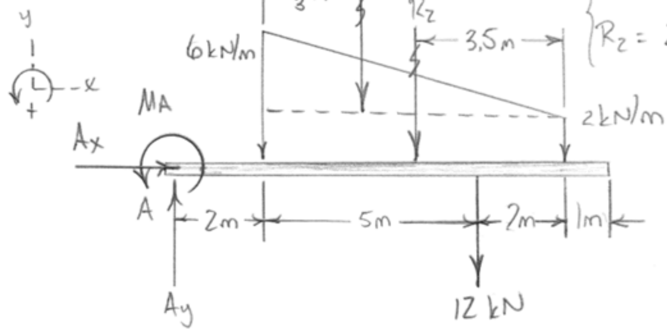


$$R_1 = \tfrac{1}{2}(6-2)(7) = 14 \text{ kN}$$
$$R_2 = 2(7) = 14 \text{ kN}$$

$$\Sigma F_x = 0: \quad \underline{A_x = 0}$$

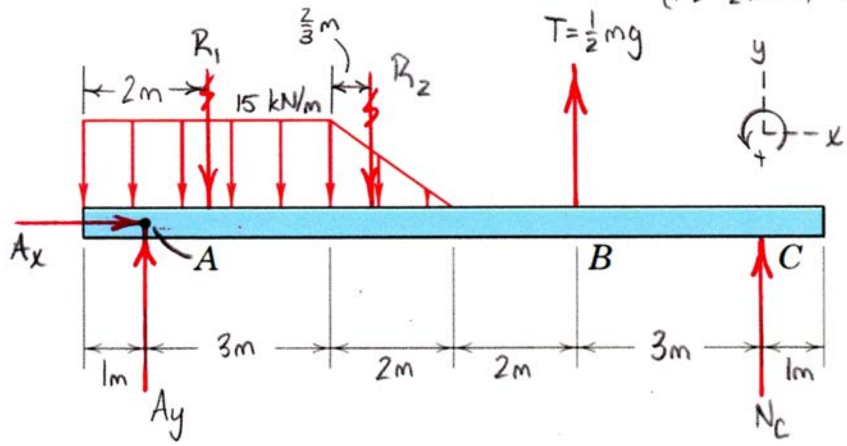
$$\Sigma F_y = 0: \quad A_y - R_1 - R_2 - 12 = 0$$

$$\Sigma M_A = 0: \quad M_A - (2 + 7/3)R_1 - (2 + 3.5)R_2 - 7(12) = 0$$

$$\underline{A_y = 40 \text{ kN} \uparrow} \qquad \underline{M_A = 222 \text{ kN}\cdot\text{m CCW}}$$

5/111 $m = 2$ Mg

$$\begin{cases} R_1 = 15(4) = 60 \text{ kN} \\ R_2 = \frac{1}{2}(15)(2) = 15 \text{ kN} \end{cases}$$



$$T = \tfrac{1}{2}(2000)(9.81) = 9810 \text{ N}$$

$$\begin{cases} \Sigma F_x = 0: \ \underline{A_x = 0} \\ \Sigma F_y = 0: \ A_y - R_1 - R_2 + T + N_C = 0 \\ \Sigma M_A = 0: \ -1R_1 - (3 + \frac{2}{3})R_2 + 7T + 10N_C = 0 \end{cases}$$

$$\begin{cases} \underline{N_C = 4.63 \text{ kN} \uparrow} \\ \underline{A_y = 60.6 \text{ kN} \uparrow} \end{cases}$$

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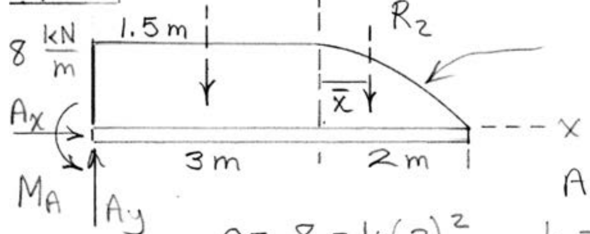

$R_1 = 8(3) = 24$ kN

$w = w_0 - kx^2$

$w_0 = 8$ kN/m

At $x = 2$ m:

$0 = 8 - k(2)^2, \quad k = 2$ kN/m^3

$$R_2 = \int w\,dx = \int_0^2 (8 - 2x^2)\,dx = \left(8x - \frac{2}{3}x^3\right)_0^2$$

$$= 10.67 \text{ kN}$$

$$\bar{x} = \frac{\int x\,w\,dx}{R} = \frac{\int_0^2 (8x - 2x^3)\,dx}{10.67} = \frac{\left(4x^2 - \frac{1}{2}x^4\right)_0^2}{10.67}$$

$$= 0.75 \text{ m}$$

Equilibrium considerations

$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma F_y = 0: \quad A_y - 24 - 10.67 = 0, \quad \underline{A_y = 34.7 \text{ kN}}$

$\Sigma M_A = 0: \quad M_A - 24(1.5) - 10.67(3.75) = 0$

$\underline{M_A = 76 \text{ kN}\cdot\text{m CCW}}$

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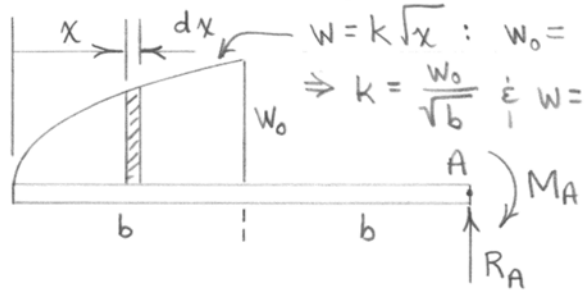

$w = k\sqrt{x}$: $w_0 = k\sqrt{b}$

$\Rightarrow k = \frac{w_0}{\sqrt{b}}$ & $w = w_0\sqrt{\frac{x}{b}}$

The force on dx is $dF = w\,dx = w_0\sqrt{\frac{x}{b}}\,dx$

Total load $F = \int dF = \int_0^b w_0\sqrt{\frac{x}{b}}\,dx = \frac{2}{3}w_0 b$

So $\underline{R_A = \frac{2}{3}w_0 b}$

The moment of dF about A is $w\,dx(2b-x)$

$$M_A = \int_0^b w(2b-x)\,dx = \int_0^b w_0\sqrt{\frac{x}{b}}(2b-x)\,dx$$

$$= \frac{w_0}{\sqrt{b}}\int_0^b (2b\sqrt{x} - x^{3/2})\,dx = \underline{\frac{14}{15}w_0 b^2}$$

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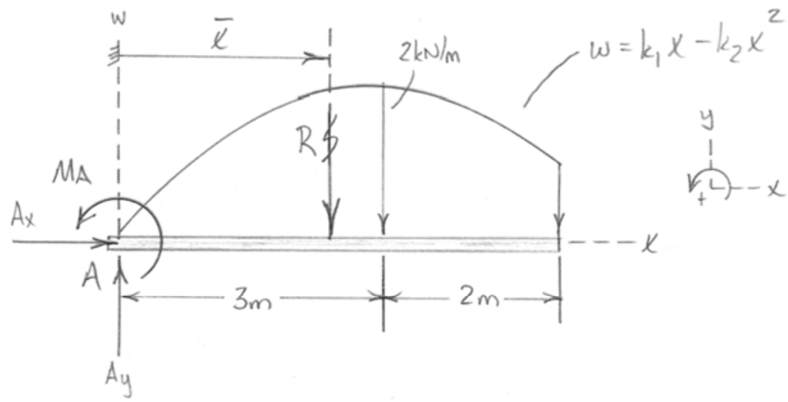

$$\begin{cases} \text{At } x = 3,\ w = 2 = k_1(3) - k_2(3)^2 \\ \text{At } x = 3,\ \frac{dw}{dx} = 0 = k_1 - 2k_2(3) \end{cases} \longrightarrow \begin{cases} k_1 = \frac{4}{3}\ \text{kN/m}^2 \\ k_2 = \frac{2}{9}\ \text{kN/m}^3 \end{cases}$$

$$R = \int w(x)dx = \int_0^5 \left(\frac{4}{3}x - \frac{2}{9}x^2\right)dx \longrightarrow R = 7.41 \text{ kN}$$

$$\bar{x} = \frac{\int x\,w(x)dx}{R} = \frac{\int_0^5 \left(\frac{4}{3}x^2 - \frac{2}{9}x^3\right)dx}{7.41} \longrightarrow \bar{x} = 2.81 \text{ m}$$

$\Sigma F_x = 0$: $\underline{A_x = 0}$

$\Sigma F_y = 0$: $A_y - R = 0 \longrightarrow \underline{A_y = 7.41 \text{ kN} \uparrow}$

$\Sigma M_A = 0$: $M_A - R\bar{x} = 0 \longrightarrow \underline{M_A = 20.8 \text{ kN}\cdot\text{m CCW}}$

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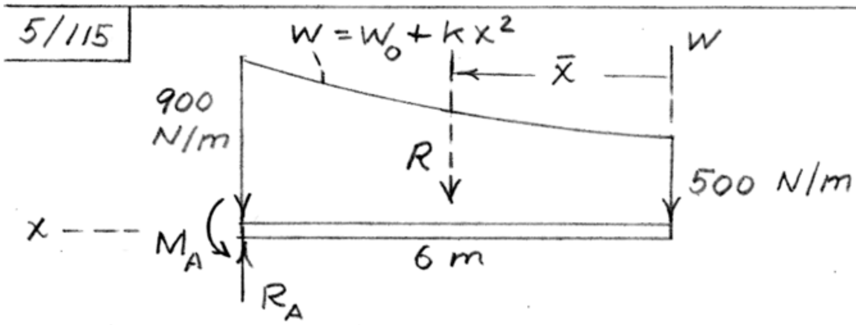


At A, $w = 500 \text{ N/m} = w_0$

At $x = 6$ m, $w = 900 = 500 + k(6^2)$, $k = \frac{100}{9}$ N/m^3

so $w = 500 + 100x^2/9$ N/m

$$R = \int w\,dx = \int_0^6 (500 + 100x^2/9)\,dx$$

$$= \left[500x + \frac{100x^3}{27}\right]_0^6 = 3800 \text{ N}$$

$$\bar{x} = \frac{\int x\,w\,dx}{R} = \int_0^6 (500x + 100x^3/9)\,dx \Big/ 5400$$

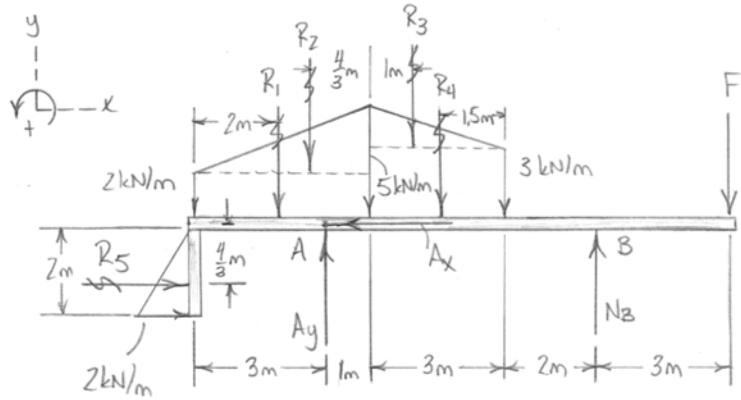
$$= \left[250x^2 + \frac{25x^4}{9}\right]_0^6 \Big/ 3800 = 3.32 \text{ m}$$

$+\uparrow \Sigma F = 0$: $R_A - 3800 = 0$, $R_A = 3800$ N or $\underline{3.8 \text{ kN}}$

$+\circlearrowleft \Sigma M_A = 0$: $M_A - 3800(6 - 3.32) = 0$

$M_A = 10\,200$ N·m or $\underline{10.20 \text{ kN·m}}$

5/116 Find F for $A_y = N_B$



$R_1 = 2(4) = 8$ kN

$R_2 = \frac{1}{2}(5-2)(4) = 6$ kN

$R_3 = \frac{1}{2}(5-3)(3) = 3$ kN

$R_4 = 3(3) = 9$ kN

$R_5 = \frac{1}{2}(2)(2) = 2$ kN

$\Sigma F_x = 0: \ -A_x + R_5 = 0 \longrightarrow A_x = 2$ kN

$\Sigma F_y = 0: \ A_y + N_B - F - R_1 - R_2 - R_3 - R_4 = 0$

$\Sigma M_B = 0: \ -3F + 3.5R_4 + 4R_3 + (5+\frac{4}{3})R_2 + 7R_1 + \frac{4}{3}R_5 - 6A_y = 0$

$A_y = N_B = 18.18$ kN $\qquad \underline{F = 10.36 \text{ kN}}$

$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{18.18^2 + 2^2} \longrightarrow \underline{R_A = 18.29 \text{ kN}}$

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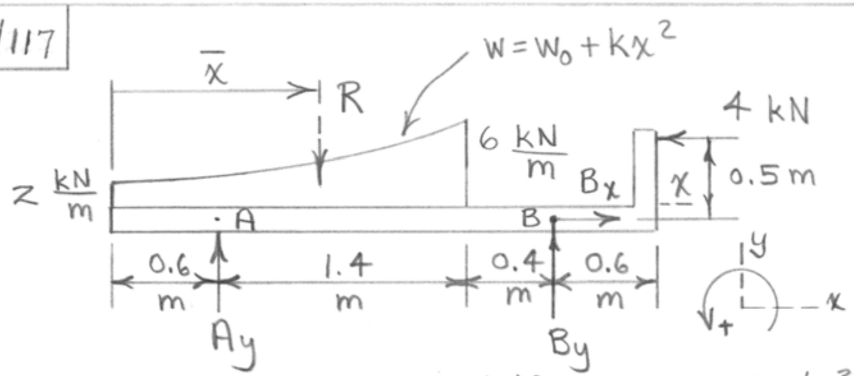

$w = 2 + kx^2: \quad 6 = 2 + k(2)^2, \quad k = 1 \text{ kN/m}^3,$

$w = 2 + x^2$

$R = \int w\,dx = \int_0^2 (2 + x^2)\,dx = 2x + \frac{x^3}{3}\Big|_0^2 = 6.67 \text{ kN}$

$\bar{x} = \frac{\int x w\,dx}{R} = \frac{\int_0^2 (2x + x^3)\,dx}{6.67} = \frac{x^2 + \frac{x^4}{4}}{6.67}\Big|_0^2 = 1.2 \text{ m}$

$\Sigma F_x = 0: \quad B_x - 4 = 0, \quad \underline{B_x = 4 \text{ kN}}$

$\circlearrowleft \Sigma M_B = 0: \quad 4(0.5) + 6.67(2.4 - 1.2) - A_y(1.8) = 0$

$\underline{A_y = 5.56 \text{ kN}}$

$\Sigma F_y = 0: \quad 5.56 + B_y - 6.67 = 0, \quad \underline{B_y = 1.111 \text{ kN}}$

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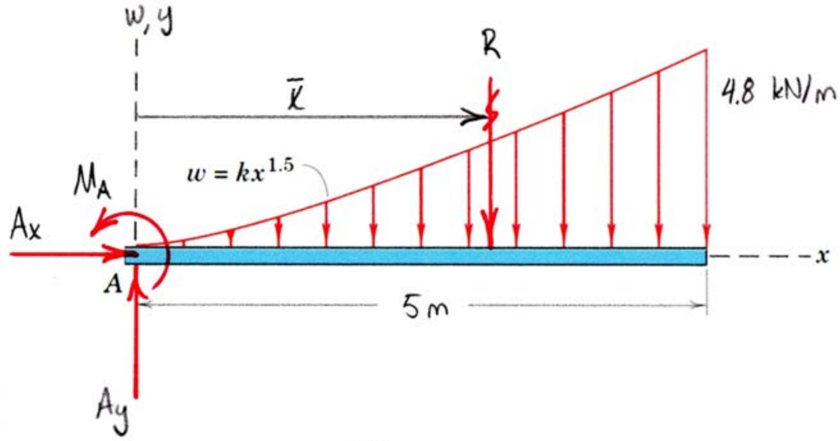

At $x = 5\text{ m}$, $w = 4.8 = k(5)^{1.5} \longrightarrow k = 0.429\ \text{kN/m}^{5/2}$

$$\begin{cases} R = \int w\,dx = \int_0^5 0.429\,x^{1.5}\,dx \longrightarrow R = 9.60\text{ kN} \\ \bar{x} = \dfrac{\int x w\,dx}{R} = \dfrac{1}{9.6}\int_0^5 0.429\,x^{2.5}\,dx \longrightarrow \bar{x} = 3.57\text{ m} \end{cases}$$

$$\begin{cases} \Sigma F_x = 0: \ \underline{A_x = 0} \\ \Sigma F_y = 0: \ A_y - R = 0 \\ \circlearrowleft + \Sigma M_A = 0: \ M_A - R\bar{x} = 0 \end{cases} \longrightarrow \begin{cases} \underline{A_y = 9.60\text{ kN} \uparrow} \\ \underline{M_A = 34.3\text{ kN·m CCW}} \end{cases}$$

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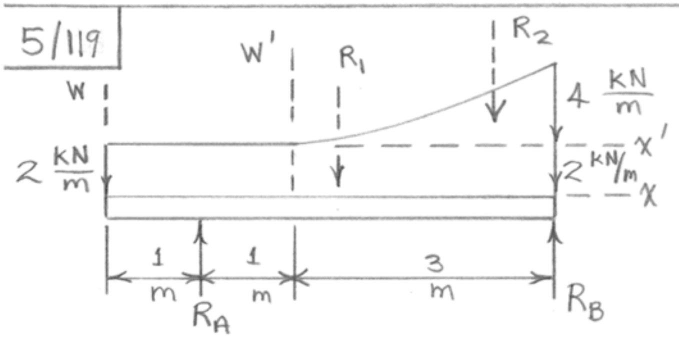

$w' = kx'^2$

At $x' = 3$ m:

$4 = k(3)^2$

$k = \frac{4}{9}\ N/m^3$

$w' = \frac{4}{9}x'^2$

$$R_2 = \int_0^3 w'dx' = \int_0^3 \frac{4}{9}x'^2 = \frac{4}{9}\frac{x'^3}{3}\Big|_0^3 = 4\ kN$$

$$\bar{x}_2' = \int x'w'dx'/R_2 = \frac{1}{4}\int_0^3 \frac{4}{9}x'^3 = \frac{1}{4}\frac{4}{9}\frac{x'^4}{4}\Big|_0^3 = 2.25\ m$$

$$R_1 = 2(5) = 10\ kN\ @\ \bar{x}_1 = 2.5\ m \quad \bar{x}_2 = 4.25\ m$$

$$\circlearrowleft + \Sigma M_A = 0: -10(1.5) - 4(3.25) + 4R_B = 0$$

$$\underline{R_B = 7\ kN}$$

$$+\uparrow \Sigma F = 0: 7 + R_A - 4 - 10 = 0, \quad \underline{R_A = 7\ kN}$$

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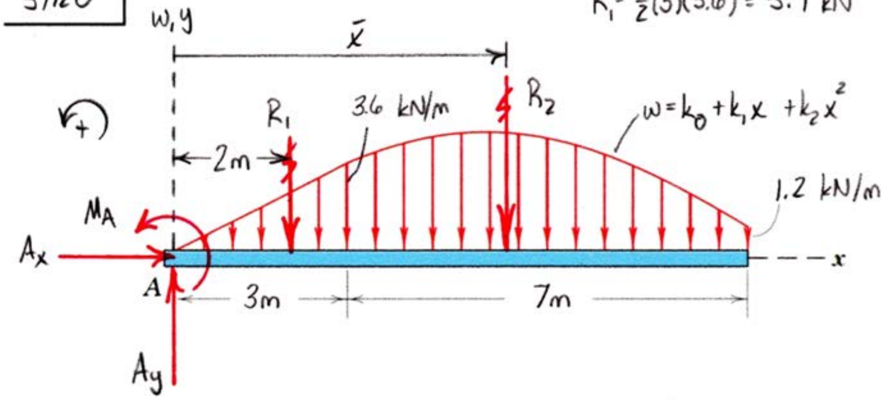
$R_1 = \frac{1}{2}(3)(3.6) = 5.4$ kN



$$\begin{cases} \text{At } x = 3,\ w = 3.6 = k_0 + k_1(3) + k_2(3)^2 \\ \text{At } x = 10,\ w = 1.2 = k_0 + k_1(10) + k_2(10)^2 \\ \text{At } x = 3,\ \frac{dw}{dx} = \frac{3.6}{3} = k_1 + 2k_2(3) \end{cases} \xrightarrow{\text{Solving...}} \begin{cases} k_0 = -1.984 \text{ kN/m} \\ k_1 = 2.52 \text{ kN/m}^2 \\ k_2 = -0.220 \text{ kN/m}^3 \end{cases}$$

$$R_2 = \int w\,dx = \int_3^{10} (-1.984 + 2.52x - 0.220x^2)\,dx \longrightarrow R_2 = 29.4 \text{ kN}$$

$$\bar{x} = \frac{\int x w\,dx}{R} = \frac{1}{29.4}\int_3^{10} (-1.984x + 2.52x^2 - 0.220x^3)\,dx \longrightarrow \bar{x} = 6.17 \text{ m}$$

$$\begin{cases} \Sigma F_x = 0:\ \underline{A_x = 0} \\ \Sigma F_y = 0:\ A_y - R_1 - R_2 = 0 \\ \Sigma M_A = 0:\ M_A - 2R_1 - R_2\bar{x} = 0 \end{cases} \longrightarrow \begin{cases} \underline{A_y = 34.8 \text{ kN} \uparrow} \\ \underline{M_A = 192.1 \text{ kN·m CCW}} \end{cases}$$

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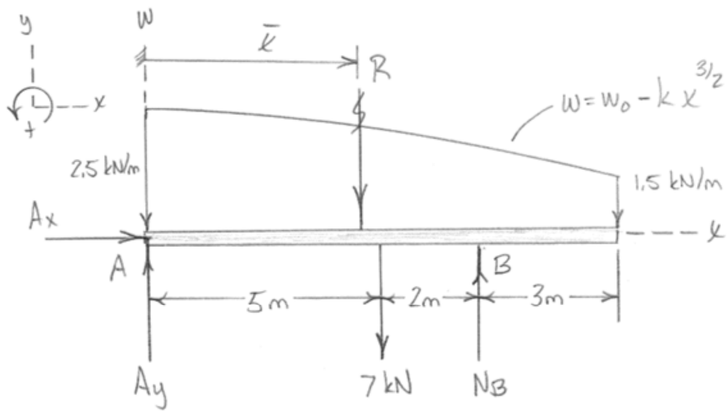

At $x = 0$, $w = 2.5 = w_0 - k(0) \longrightarrow w_0 = 2.5$ kN/m

At $x = 10$, $w = 1.5 = 2.5 - k(10)^{3/2} \longrightarrow k = 0.0316$ kN/m$^{5/2}$

so... $w = 2.5 - 0.0316\,x^{3/2}$

$R = \int w\,dx = \int_0^{10} (2.5 - 0.0316\,x^{3/2})\,dx \longrightarrow R = 21$ kN

$\bar{x} = \frac{\int x w\,dx}{R} = \frac{\int_0^{10} (2.5x - 0.0316\,x^{5/2})\,dx}{21} \longrightarrow \bar{x} = 4.59$ m

$\Sigma F_x = 0$: $\underline{A_x = 0}$

$\Sigma F_y = 0$: $A_y - 7 + N_B - R = 0$

$\Sigma M_A = 0$: $-5(7) + 7N_B - R\bar{x} = 0$

$\longrightarrow$ $\underline{A_y = 9.22 \text{ kN}}$

$\underline{N_B = 18.78 \text{ kN}}$

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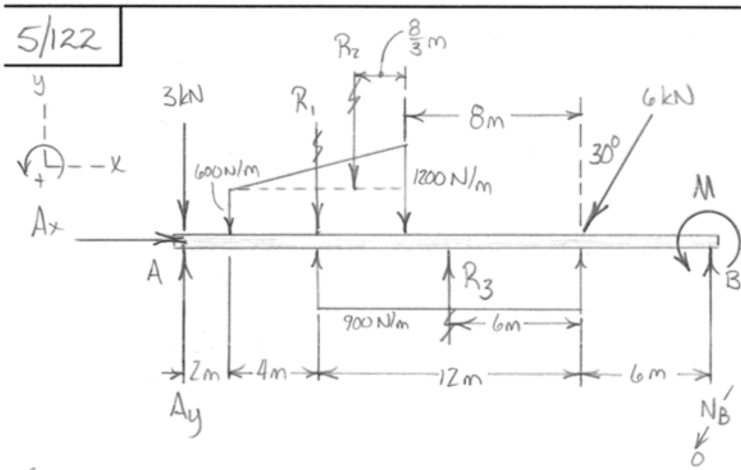


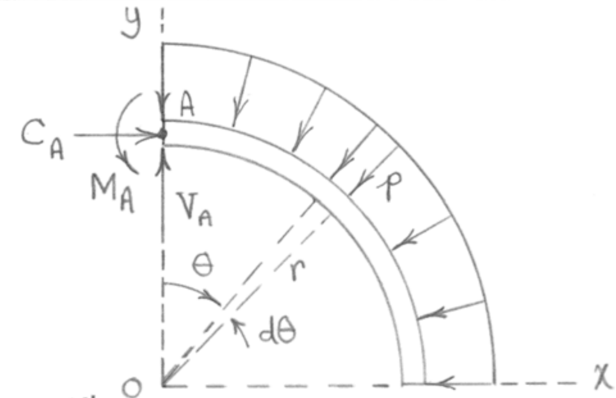
$$\left\{\begin{aligned} R_1 &= 600(8) = 4800 \text{ N} \\ R_2 &= \tfrac{1}{2}(1200-600)(8) = 2400 \text{ N} \\ R_3 &= 900(12) = 10\,800 \text{ N} \end{aligned}\right.$$

$$\left\{\begin{aligned} &\Sigma F_x = 0: \ A_x - 6(1000)\sin 30^\circ \longrightarrow A_x = 3000 \text{ N} \\ &\Sigma F_y = 0: \ A_y - 3(1000) - R_1 - R_2 + R_3 + \cancel{N_B}^{0} - 6(1000)\cos 30^\circ = 0 \\ &\Sigma M_A = 0: \ -6R_1 - (2 + \tfrac{2}{3}(8))R_2 + 12R_3 - 18(6)(1000)\cos 30^\circ + 24\cancel{N_B}^{0} + M = 0 \end{aligned}\right.$$

$A_y = 4600$ N $\uparrow$ $\qquad$ $\underline{M = 10\,330 \text{ N·m CCW}}$

$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{3000^2 + 4600^2} \longrightarrow \underline{R_A = 5490 \text{ N}}$$

▶5/123



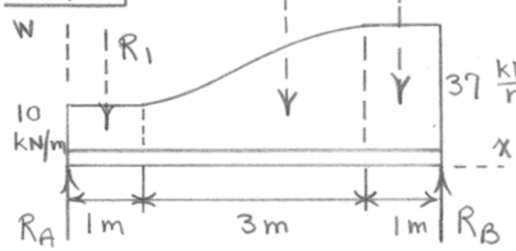
$\Sigma F_x = 0: \quad C_A - \int_0^{\pi/2} p r \, d\theta \sin\theta = 0, \qquad \underline{C_A = pr}$

$\Sigma F_y = 0: \quad V_A - \int_0^{\pi/2} pr \, d\theta \cos\theta = 0, \qquad \underline{V_A = pr}$

$\circlearrowleft \Sigma M_A = 0: \quad M_A - \int_0^{\pi/2} pr \, d\theta \, (r \sin\theta) = 0, \qquad \underline{M_A = pr^2}$

(Alternatively, $\circlearrowleft \Sigma M_O = 0: \quad M_A - C_A r = 0, \; M_A = pr^2$)

▶ 5/124



$$w = w_0 + k_1 x + k_2 x^2 + k_3 x^3$$

$$w(1) = 10 = w_0 + k_1 + k_2 + k_3$$

$$w(4) = 37 = w_0 + k_1(4) + k_2(4)^2 + k_3(4)^3$$

$$\frac{dw}{dx} = k_1 + 2k_2 x + 3k_3 x^2 : \begin{cases} 0 = k_1 + 2k_2(1) + 3k_3(1)^2 \\ 0 = k_1 + 2k_2(4) + 3k_3(4)^2 \end{cases}$$

Solve simultaneously to get $w = 21 - 24x + 15x^2 - 2x^3$

$$R_2 = \int w\,dx = \int_1^4 (21 - 24x + 15x^2 - 2x^3)\,dx$$

$$= \left[21x - 12x^2 + 5x^3 - \tfrac{1}{2}x^4\right]_1^4 = 70.5 \text{ kN}$$

$$\bar{x}_2 = \frac{1}{R_2}\int x w\,dx = \frac{1}{70.5}\int_1^4 (21 - 24x + 15x^2 - 2x^3)\,x\,dx$$

$$= \frac{1}{70.5}\left[\frac{21}{2}x^2 - 8x^3 + \frac{15}{4}x^4 - \frac{2}{5}x^5\right]_1^4 = 2.84 \text{ m}$$

$R_1 = 10(1) = 10$ kN @ $\bar{x}_1 = 0.5$ m

$R_3 = 37(1) = 37$ kN @ $\bar{x}_3 = 4.5$ m

$\Sigma M_A = 0: \; 5R_B - 10(0.5) - 70.5(2.84) - 37(4.5) = 0$

$\underline{R_B = 74.4 \text{ kN}}$

$\Sigma F = 0: \; 74.4 - 10 - 70.5 - 37 + R_A = 0, \quad \underline{R_A = 43.1 \text{ kN}}$

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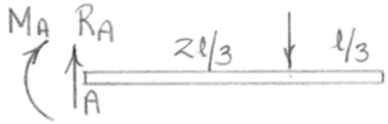
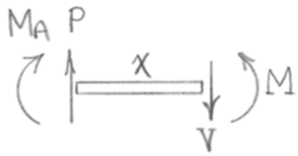


$+\uparrow \Sigma F = 0 \Rightarrow R_A = P$

$\circlearrowleft\!+ \Sigma M_A = 0: \quad -M_A - P\left(\frac{2\ell}{3}\right) = 0$

$M_A = -\frac{2P\ell}{3}$

$0 < x < \frac{2\ell}{3}:$

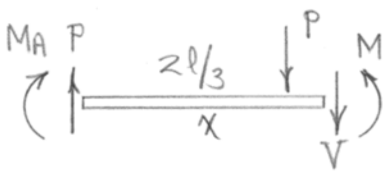


$+\uparrow \Sigma F = 0 \Rightarrow \quad V = P$

$\circlearrowleft\!+ \Sigma M = 0: \quad \frac{2P\ell}{3} - Px + M = 0$

$M = P\left(x - \frac{2\ell}{3}\right)$

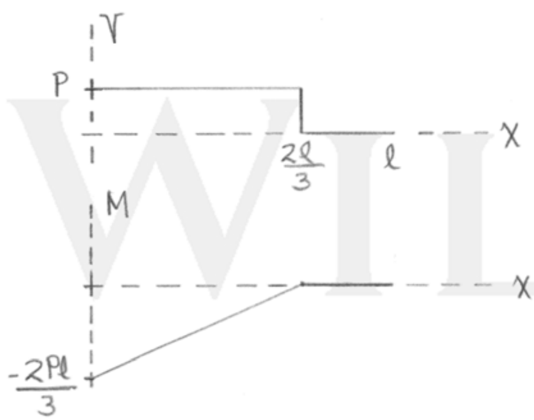
$\frac{2\ell}{3} < x < \ell:$



$+\uparrow \Sigma F = 0: \quad P - P - V = 0, \quad V = 0$

$\circlearrowleft\!+ \Sigma M = 0: \quad \frac{2P\ell}{3} - P\left(\frac{2\ell}{3}\right) + M = 0$

$M = 0$



At $x = \ell/2$:

$V = P$

$M = P\left(\frac{\ell}{2} - \frac{2\ell}{3}\right)$

$= -\frac{P\ell}{6}$

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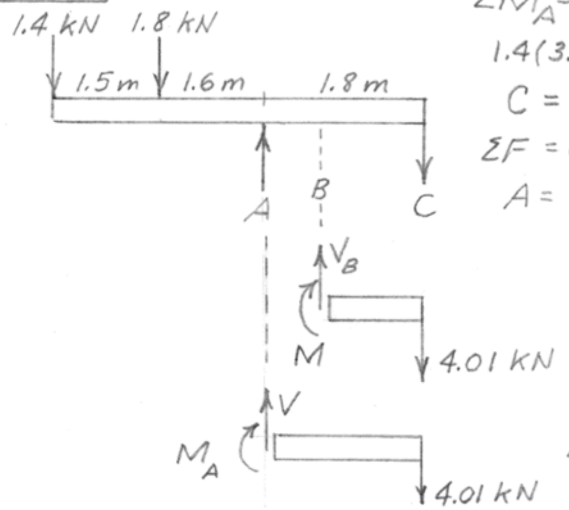

$\Sigma M_A = 0;$

$1.4(3.1) + 1.8(1.6) - 1.8C = 0$

$C = 4.01$ kN

$\Sigma F = 0;\ A - 4.01 - 1.4 - 1.8 = 0$

$A = 7.21$ kN

$\Sigma F = 0;\ \underline{V_B = 4.01 \text{ kN}}$

$\Sigma M_A = 0;\ M_A + 4.01(1.8) = 0$

$\underline{M_A = -7.22 \text{ kN}\cdot\text{m}}$

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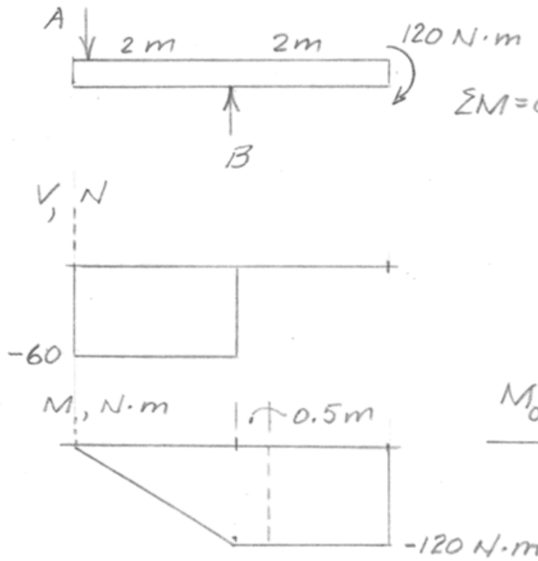

$\Sigma M = 0$; $2A - 120 = 0$

$A = B = 60$ N

$M_{0.5\text{ m}} = -120$ N·m

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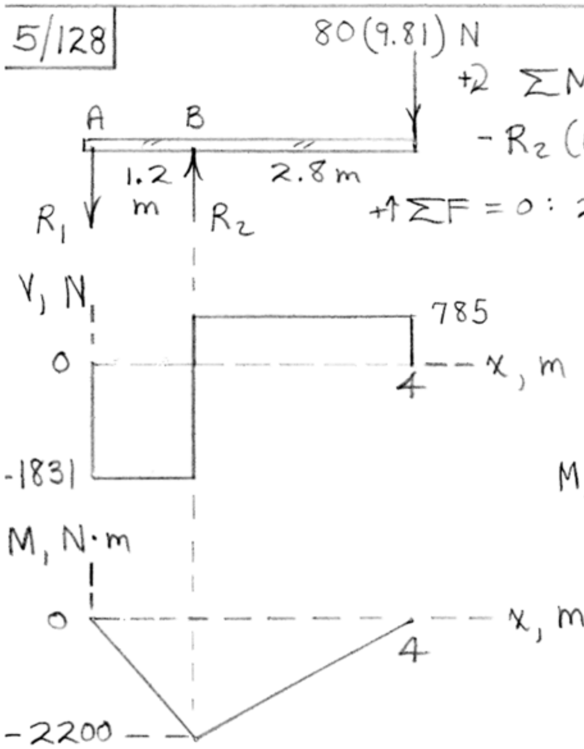

$\circlearrowleft + \ \Sigma M_A = 0: \ 80(9.81)(4) - R_2(1.2) = 0, \ R_2 = 2620 \text{ N}$

$+\uparrow \Sigma F = 0: \ 2620 - 80(9.81) + R_1 = 0$

$R_1 = 1831 \text{ N}$

$M_B = -1831(1.2)$

$= \underline{-2200 \text{ N·m}}$

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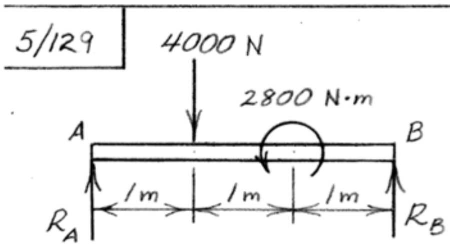


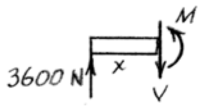
$\circlearrowleft \Sigma M_A = 0: \; -4000(1) + 2800 + R_B(3) = 0$

$R_B = 400$ N

$+\uparrow \Sigma F_y = 0: \; R_A - 4000 + 400 = 0$

$R_A = 3600$ N

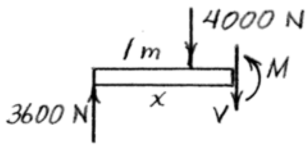
or 3.6 kN



$0 < x < 1$ m:

$V = 3600$ N

$M = 3600x$

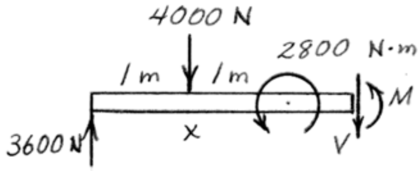


$1\text{ m} < x < 2\text{ m}$:

$+\uparrow \Sigma F = 0: \; 3600 - 4000 - V = 0, \; V = -400$ N

$\circlearrowleft \Sigma M = 0: \; -3600x + 4000(x-1) + M = 0$

$M = 4000 - 400x$

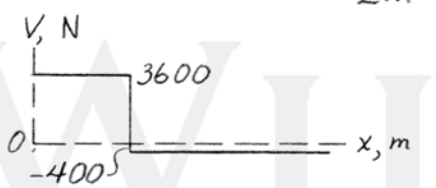


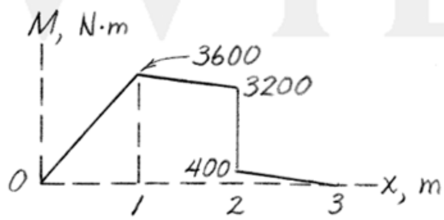
$2\text{ m} < x < 3\text{ m}$:

$\Sigma F = 0: \; 3600 - 4000 - V = 0$

$V = -400$ N

$\Sigma M = 0: \; -3600x + 4000(x-1) + 2800 + M = 0, \; M = 1200 - 400x$





Values at middle:

$\begin{cases} V = -400 \text{ N} \\ \underline{M = 3400 \text{ N·m}} \end{cases}$

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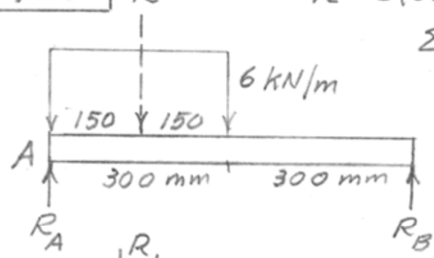
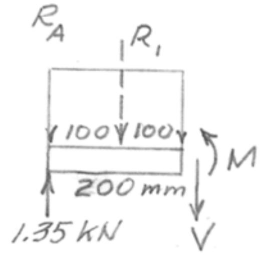


$R = 6(0.3) = 1.8 \text{ kN}$

$\Sigma M_B = 0;\ 1.8(0.3+0.15) - 0.6R_A = 0$

$R_A = 1.35 \text{ kN}$



$R_1 = 6(0.2) = 1.2 \text{ kN}$

$\Sigma F = 0;\ 1.35 - 1.2 - V = 0;\quad \underline{V = 0.15 \text{ kN}}$

$\Sigma M_A = 0;\ M - 1.2(0.1) - 0.15(0.2) = 0$

$\underline{M = 0.15 \text{ kN}\cdot\text{m}}$

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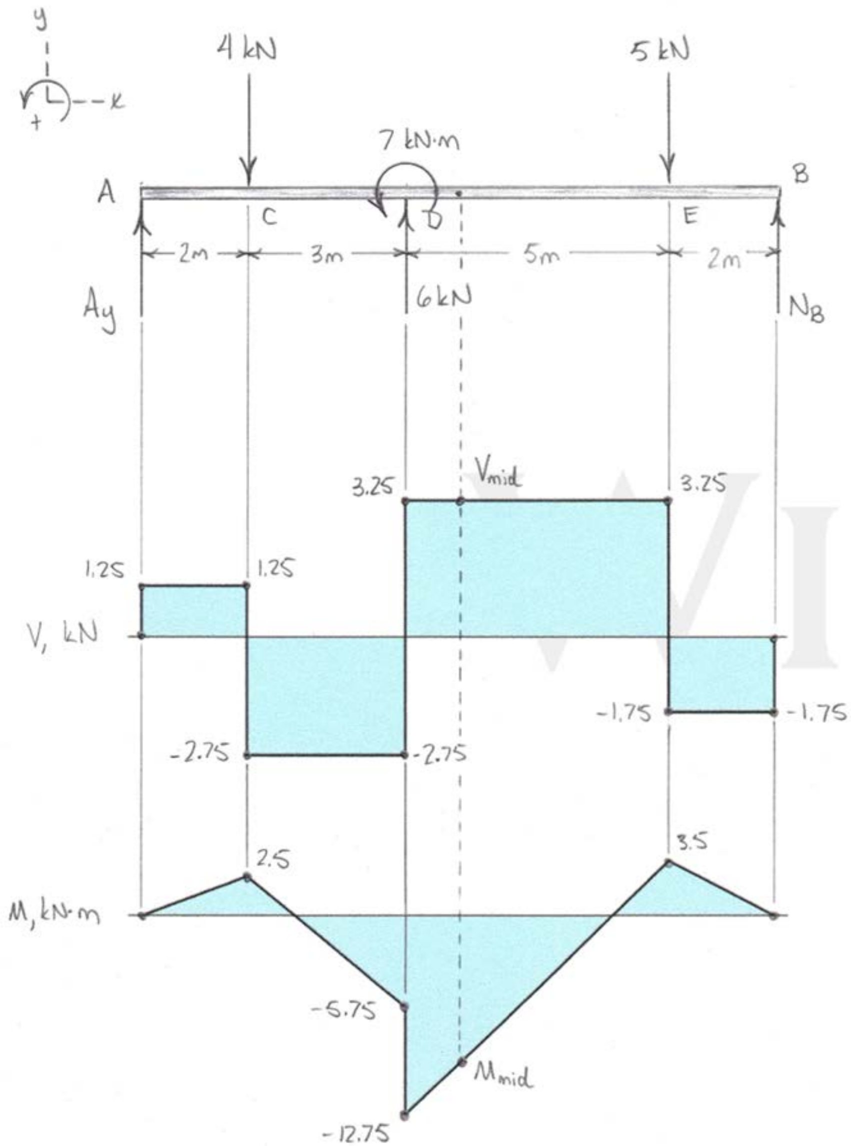


$$\Sigma F_y = 0: \quad A_y - 4 + 6 - 5 + N_B = 0$$

$$\Sigma M_A = 0: \quad -4(2) + 6(5) + 7 - 5(10) + N_B(12) = 0$$

$A_y = 1.25$ kN $\qquad N_B = 1.75$ kN

$$M_C = 2(1.25) = 2.5 \text{ kN·m}$$

$$M_{D^-} = 2.5 - 2.75(3) = -5.75 \text{ kN·m}$$

$$M_{D^+} = -5.75 - 7 = -12.75 \text{ kN·m}$$

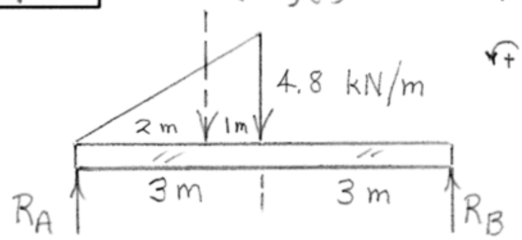
$$M_E = -12.75 + 5(3.25) = 3.5 \text{ kN·m}$$

$$M_B = 3.5 - 2(1.75) = 0$$

$$\underline{V_{mid} = 3.25 \text{ kN}}$$

$$M_{mid} = -12.75 + 1(3.25) \longrightarrow \underline{M_{mid} = -9.5 \text{ kN·m}}$$

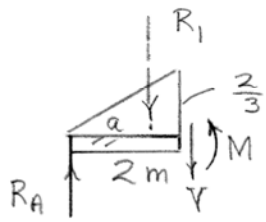
5/132 $R = \frac{1}{2}(4.8)(3) = 7.2 \text{ kN}$



$\curvearrowleft + \Sigma M_B = 0$:

$7.2(4) - R_A(6) = 0$

$R_A = 4.8 \text{ kN}$



$\frac{2}{3}(4.8) = 3.2 \text{ kN/m}$

$a = \frac{2}{3}(2) = \frac{4}{3} \text{ m}$

$R_1 = \frac{1}{2}(3.2)(2) = 3.2 \text{ kN}$

$\uparrow \Sigma F = 0$: $4.8 - 3.2 - V = 0$, $\underline{V = 1.6 \text{ kN}}$

$\curvearrowleft + \Sigma M = 0$: $-3.2\left(\frac{4}{3}\right) - 1.6(2) + M = 0$

$\underline{M = 7.47 \text{ kN}\cdot\text{m}}$

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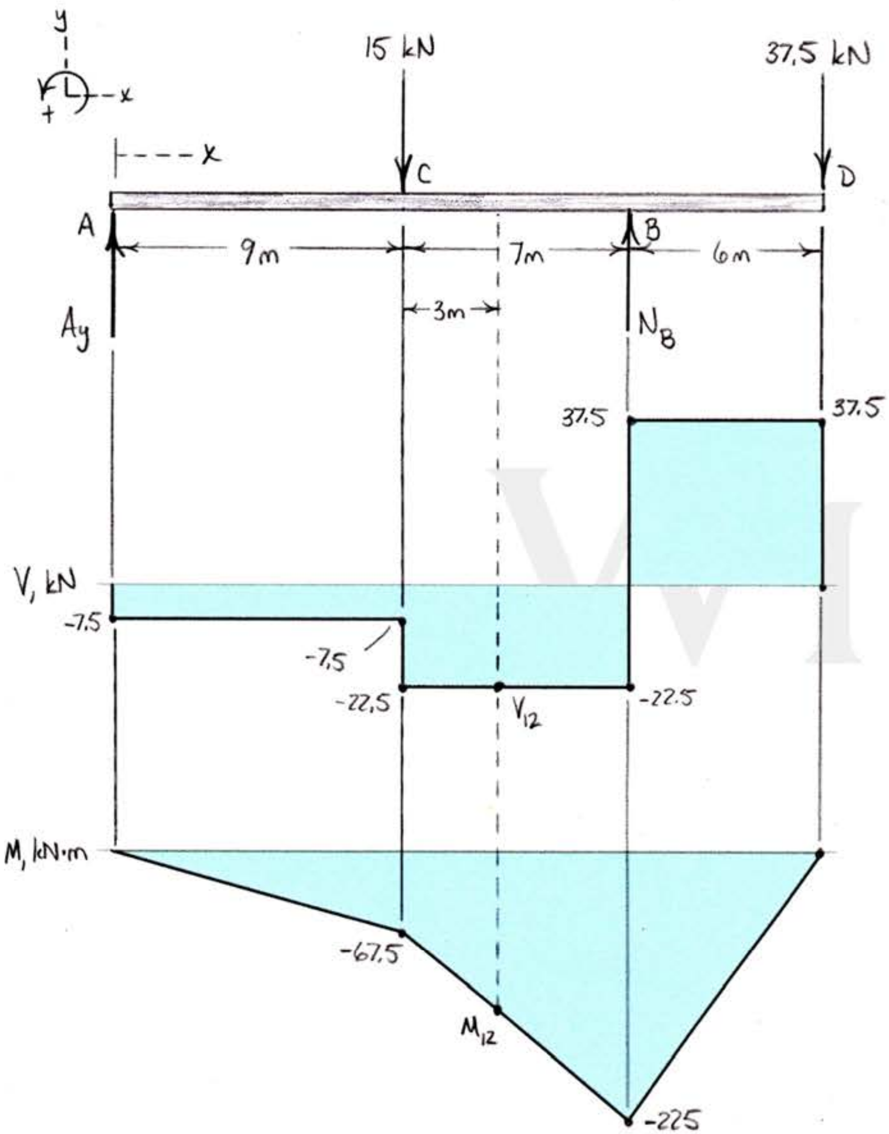

$$\begin{cases} \Sigma F_y = 0: \quad A_y + N_B - 15 - 37.5 = 0 \\ \Sigma M_A = 0: \quad -9(15) + 16 N_B - 22(37.5) = 0 \end{cases} \longrightarrow \begin{cases} A_y = -7.5 \text{ kN } (\downarrow) \\ N_B = 60 \text{ kN} \end{cases}$$

$$\begin{cases} M_C = -7.5(9) = -67.5 \text{ kN·m} \\ M_B = -67.5 - 22.5(7) = -225 \text{ kN·m} \\ M_D = -225 + 37.5(6) = 0 \end{cases}$$

$$\begin{cases} \underline{V_{12} = -22.5 \text{ kN}} \\ M_{12} = -67.5 - 22.5(3) \longrightarrow \underline{M_{12} = -135 \text{ kN·m}} \end{cases}$$

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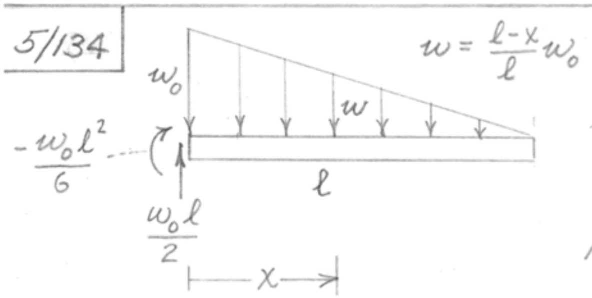
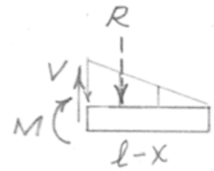




$$R = \frac{w_0}{2}\frac{(l-x)^2}{l}$$

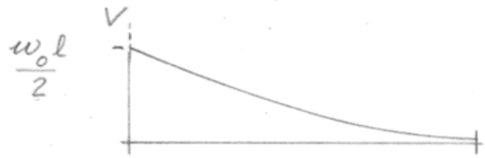
$$\Sigma F = 0;\ V = \frac{w_0}{2l}(l-x)^2$$

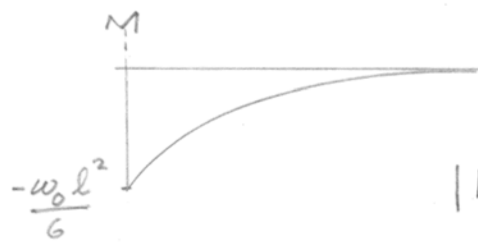
$$\Sigma M_V = 0;\ R\,\frac{l-x}{3} + M = 0$$

$$M = -\frac{w_0}{2}\frac{(l-x)^2}{l}\frac{l-x}{3}$$

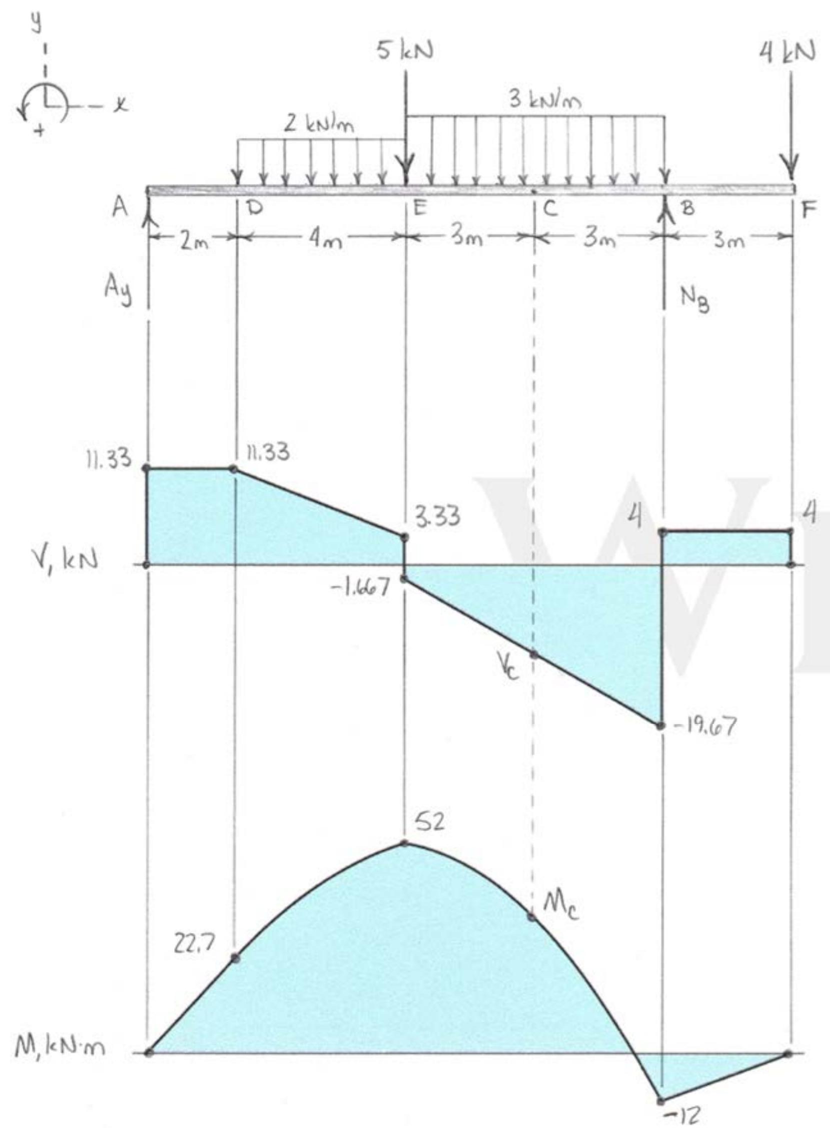
$$= -\frac{w_0}{6l}(l-x)^3$$

$$\underline{|M|_{max} = \frac{w_0 l^2}{6}}$$





5/135



$$\Sigma F_y = 0: \quad A_y + N_B - 5 - 4 - 2(4) - 3(6) = 0$$

$$\Sigma M_A = 0: -2(4)(4) - 5(6) + 12 N_B - 3(6)(9) - 4(15) = 0$$

$$A_y = 11.33 \text{ kN} \qquad N_B = 23.7 \text{ kN}$$

$$V_{E^-} = 11.33 - 2(4) = 3.33 \text{ kN}$$

$$V_{B^-} = -1.667 - 3(6) = -19.67 \text{ kN}$$

$$V_C = -1.667 - 3(3) \longrightarrow \underline{V_C = -10.67 \text{ kN}}$$

$$M_D = 2(11.33) = 22.7 \text{ kN·m}$$

$$M_E = 22.7 + \tfrac{1}{2}(11.33 + 3.33)(4) = 52 \text{ kN·m}$$

$$M_B = 52 - \tfrac{1}{2}(1.667 + 19.67)(6) = -12 \text{ kN·m}$$

$$M_F = -12 + 4(3) = 0$$

$$M_C = 52 - \tfrac{1}{2}(1.667 + 10.67)(3) \longrightarrow \underline{M_C = 33.5 \text{ kN·m}}$$

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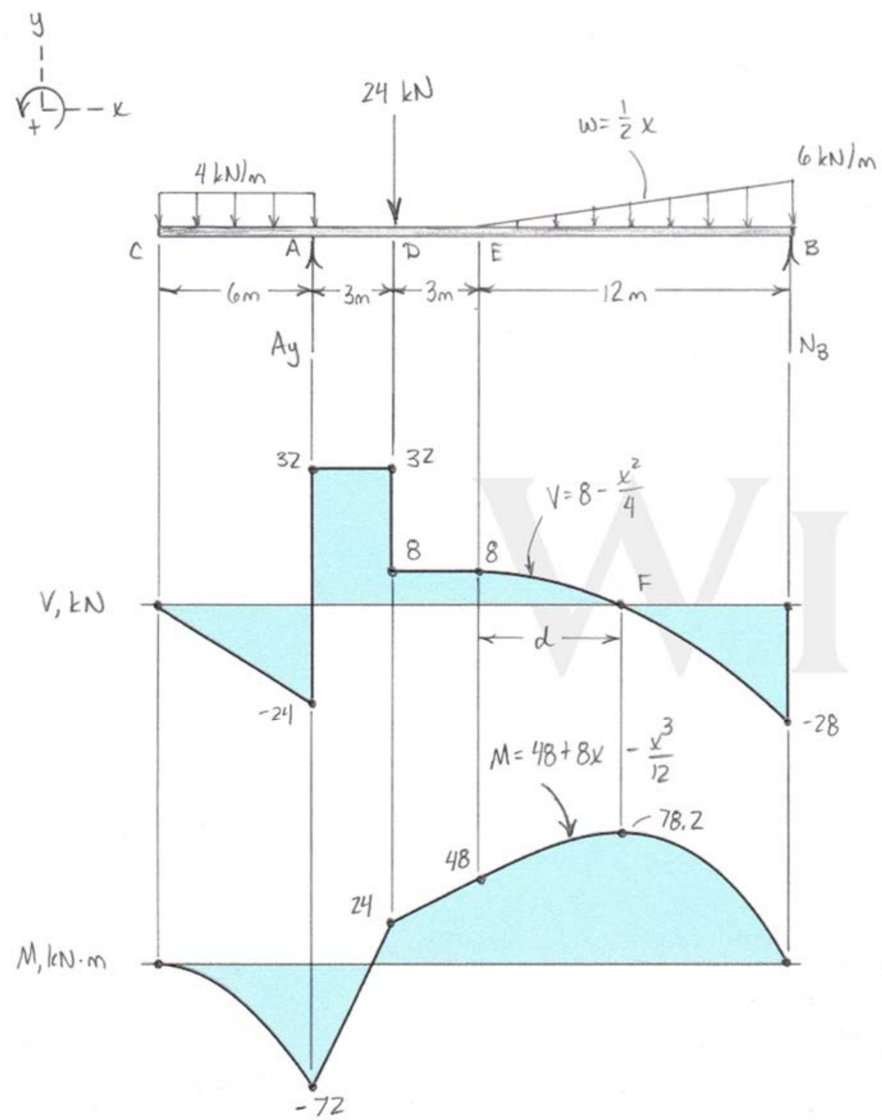

$$\begin{cases} \Sigma F_y = 0: \; A_y + N_B - 4(6) - 24 - \frac{1}{2}(12)(6) = 0 \\ \Sigma M_A = 0: \; 4(6)(3) - 24(3) + 18N_B - \frac{1}{2}(12)(6)(14) = 0 \end{cases} \longrightarrow \begin{cases} A_y = 56 \text{ kN} \\ N_B = 28 \text{ kN} \end{cases}$$

$$V_{A^-} = -4(6) = -24 \text{ kN} \qquad \underline{V_{max} = 32 \text{ kN}}$$

$$V_{B^-} = 8 - \frac{1}{2}(12)(6) = -28 \text{ kN}$$

$$\text{At F, } V = 0 = 8 - \frac{d^2}{4} \longrightarrow d = 5.66 \text{ m}$$

$$M_A = -\frac{1}{2}(24)(6) = -72 \text{ kN·m}$$

$$M_D = -72 + 32(3) = 24 \text{ kN·m}$$

$$M_E = 24 + 8(3) = 48 \text{ kN·m}$$

$$M_F = 48 + 8(5.66) - \frac{5.66^3}{12} \longrightarrow \underline{M_F = 78.2 \text{ kN·m} = M_{max}}$$

$$M_B = 48 + 8(12) - \frac{12^3}{12} \longrightarrow M_B = 0$$

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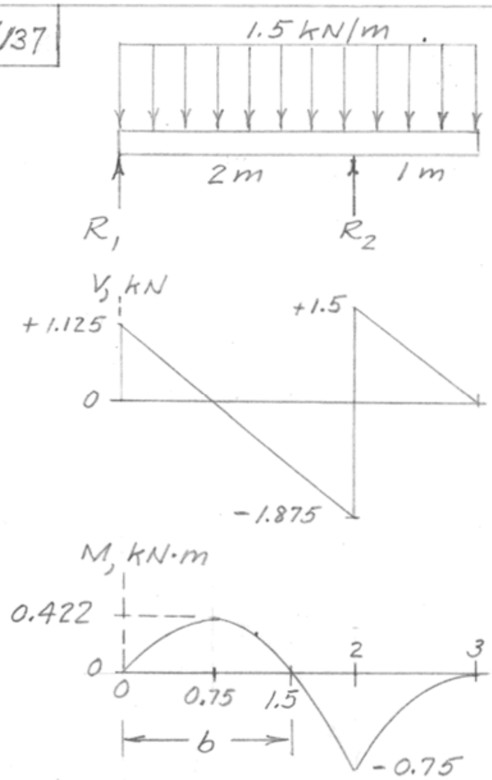


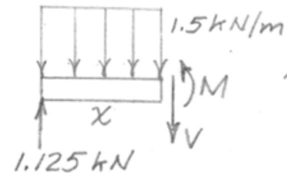
$\Sigma M_{R_1} = 0;$

$1.5(3)(1.5) - 2R_2 = 0$

$R_2 = 3.375$ kN

$\Sigma F = 0;\ R_1 + 3.375 - 1.5(3) = 0$

$R_1 = 1.125$ kN



$x < 2\text{m}$

$\Sigma F = 0;\ V + 1.5x - 1.125 = 0$

$V = 1.125 - 1.5x$

$\Sigma M_V = 0;\ M + 1.5\dfrac{x^2}{2} - 1.125x = 0$

$M = 1.125x - 0.75x^2$

$b = 1.5$ m

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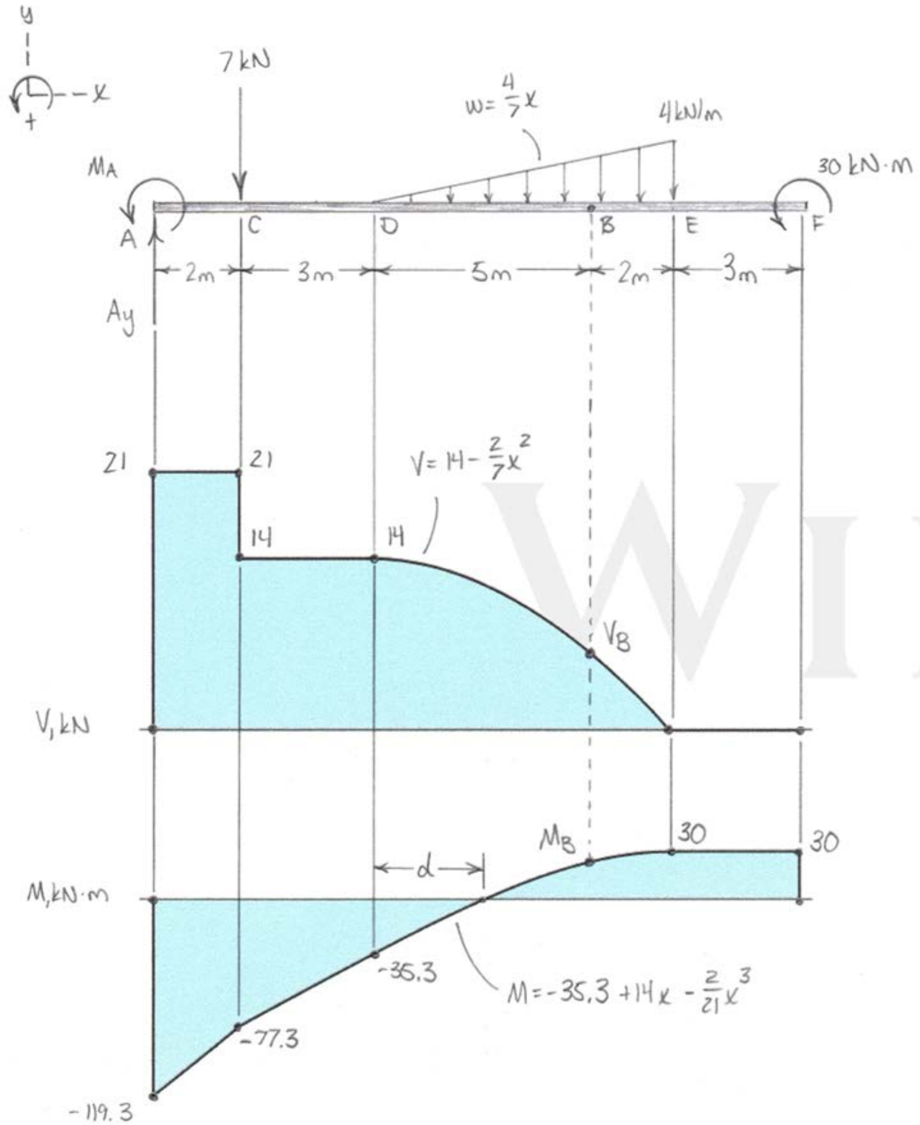

$$\Sigma F_y = 0: \quad A_y - 7 - \tfrac{1}{2}(4)(7) = 0$$

$$\Sigma M_A = 0: \quad M_A - 7(2) - \tfrac{1}{2}(4)(7)\left(5 + \tfrac{2}{3}(7)\right) + 30 = 0$$

$A_y = 21$ kN $\qquad M_A = 119.3$ kN·m CCW

$V_B = 14 - \frac{2}{7}(5)^2 \longrightarrow \underline{V_B = 6.86 \text{ kN}}$

$M_C = -119.3 + 21(2) = -77.3$ kN·m

$M_D = -77.3 + 14(3) = -35.3$ kN·m

$M_B = -35.3 + 14(5) - \frac{2}{21}(5)^3 \longrightarrow \underline{M_B = 22.8 \text{ kN·m}}$

$M_E = -35.3 + 14(7) - \frac{2}{21}(7)^3 \longrightarrow M_E = 30$ kN·m

At d... $\quad M = 0 = -35.3 + 14d - \frac{2}{21}d^3 \longrightarrow d = 2.65$ m, 10.58 m, -13.23 m

so... $\quad b = 5 + d \longrightarrow \underline{b = 7.65 \text{ m}}$

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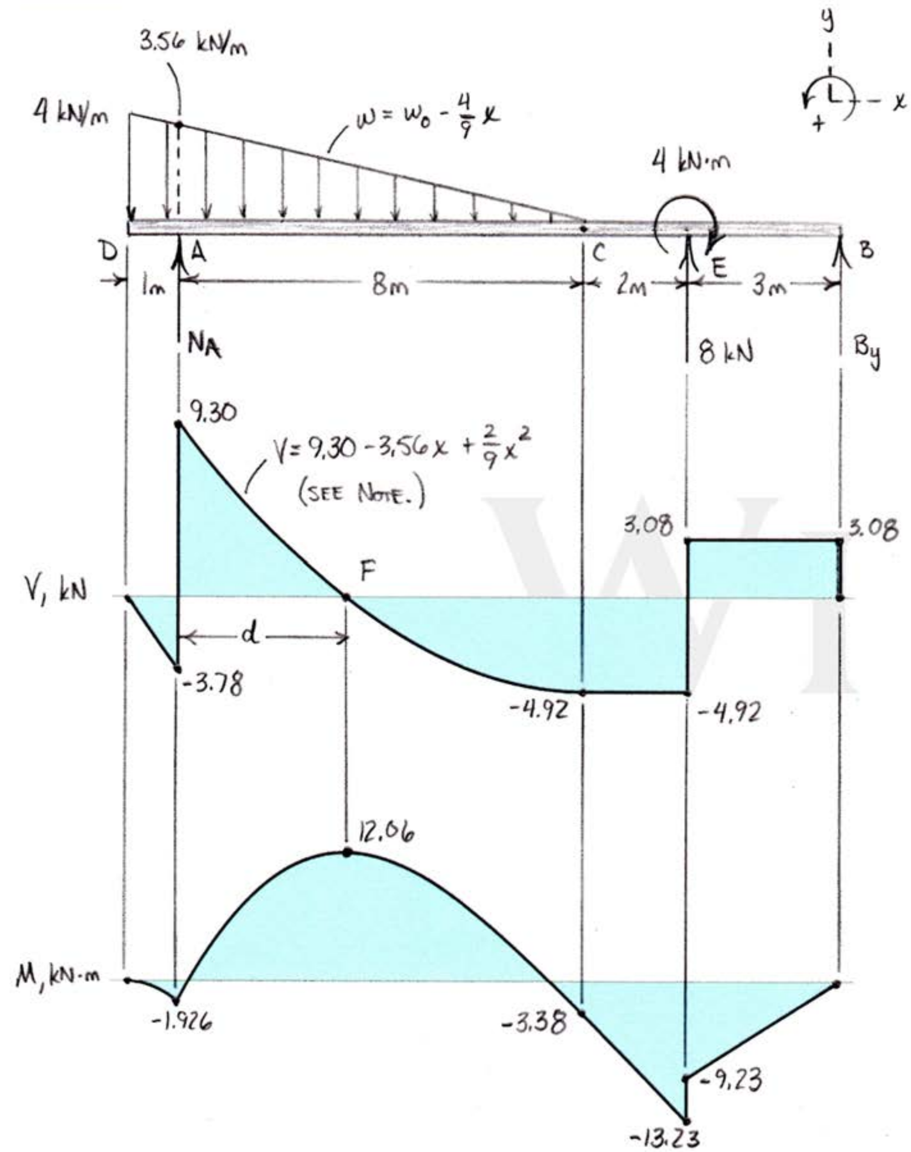

$$\begin{cases} \Sigma F_y = 0: & N_A + B_y - \frac{1}{2}(4)(9) + 8 = 0 \\ \Sigma M_A = 0: & -2\left[\frac{1}{2}(4)(9)\right] + 10(8) - 4 + 13 B_y = 0 \end{cases} \longrightarrow \begin{cases} N_A = 13.08 \text{ kN} \\ B_y = -3.08 \text{ kN } (\downarrow) \end{cases}$$

$V_{A^-} = -\frac{1}{2}(4 + 3.56)(1) = -3.78$ kN $\qquad V_{A^+} = -3.78 + 13.08 = 9.30$ kN

IN AC, $w = 3.56 - \frac{4}{9}x$ SO $V = 9.30 - 3.56x + \frac{2}{9}x^2$ (NOTE THAT x IS NOW MEASURED FROM POINT A.)

$V_C = 9.30 - 3.56(8) + \frac{2}{9}(8)^2 = -4.92$ kN

AT F, $V = 0 = 9.30 - 3.56d + \frac{2}{9}d^2 \longrightarrow d = 3.29$ m (OR 12.71 m)

$V_{E^+} = -4.92 + 8 = 3.08$ kN

IN DA, $w = 4 - \frac{4}{9}x$ SO $V = -4x + \frac{2}{9}x^2$

$M_A = \int_0^1 \left(-4x + \frac{2}{9}x^2\right)dx \longrightarrow M_A = -1.926$ kN·m

IN AC, $M = -1.926 + 9.30x - 1.778x^2 + \frac{2}{27}x^3$

$M_F = -1.926 + 9.30(3.29) - 1.778(3.29)^2 + \frac{2}{27}(3.29)^3 = 12.06$ kN·m

$M_C = -1.926 + 9.30(8) - 1.778(8)^2 + \frac{2}{27}(8)^3 = -3.38$ kN·m

$M_{E^-} = -3.38 - 4.92(2) = -13.23$ kN·m

$M_{E^+} = -13.38 + 4 = -9.23$ kN·m

$M_B = -9.23 + 3.08(3) = 0$

$|M|_{max} = 13.23$ kN·m AT $x = 11$ m

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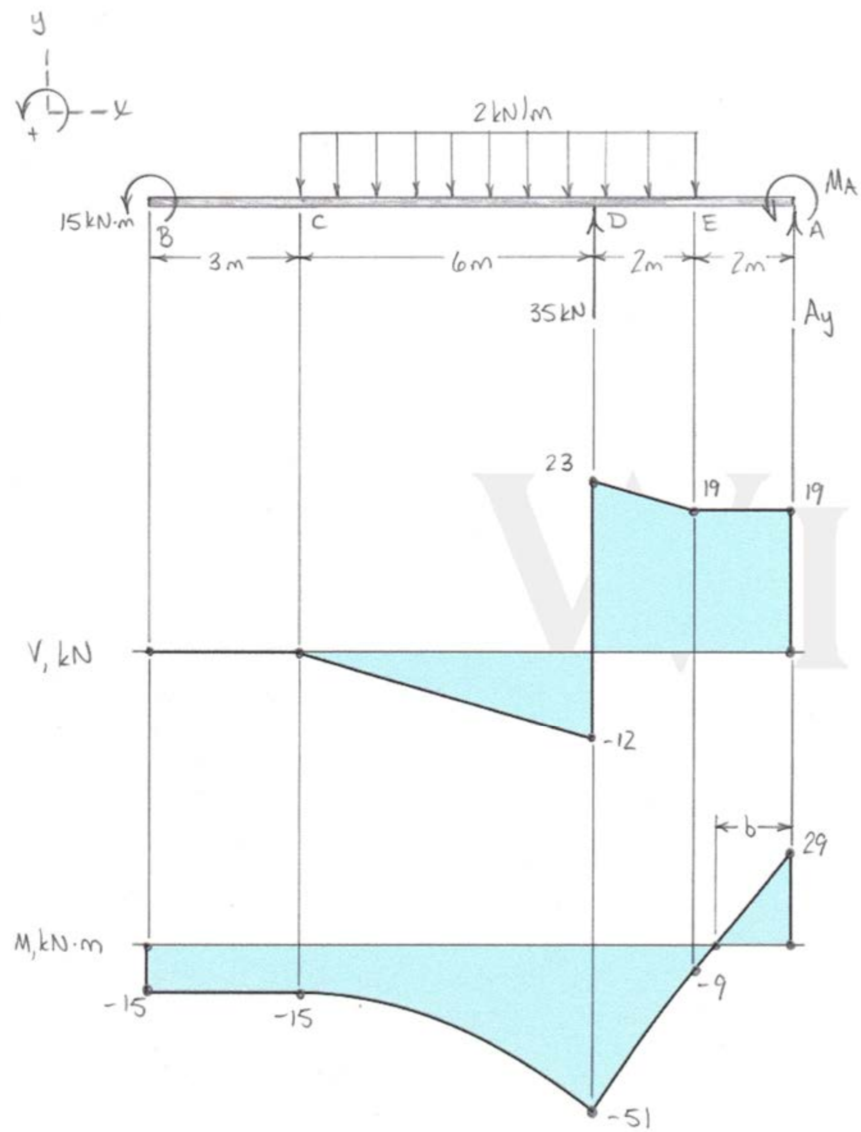


$\Sigma F_y = 0: \quad 35 + A_y - 2(8) = 0$

$\Sigma M_A = 0: \quad M_A + 15 - 35(4) + 2(8)(6) = 0$

$\rightarrow$ $A_y = -19$ kN (DOWN), $M_A = 29$ kN·m CCW

$V_{D^-} = -2(6) = -12$ kN

$V_E = 23 - 2(2) = 19$ kN

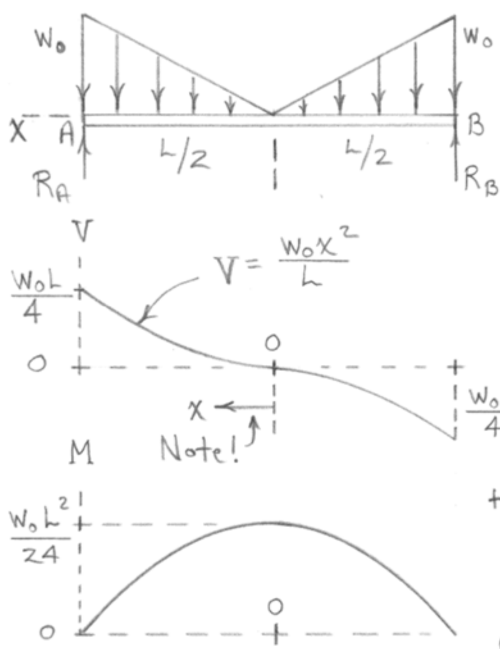
$M_D = -15 - \frac{1}{2}(12)(6) = -51$ kN·m

$M_E = -51 + \frac{1}{2}(23+19)(2) = -9$ kN·m

$M_{A^-} = -9 + 19(2) = 29$ kN·m

$b = \frac{29}{19} \rightarrow \underline{b = 1.526 \text{ m LEFT OF A}}$

5/141 (See beam element, lower left)



$R_A = R_B = \frac{w_0 L}{4}$

For $x = \frac{L}{2}$, $M = 0$, $V = R_A$

Element (lower left):

$+\uparrow \Sigma F = 0$: $V - \frac{w_0 x^2}{L} = 0$

$V = \frac{w_0 x^2}{L}$

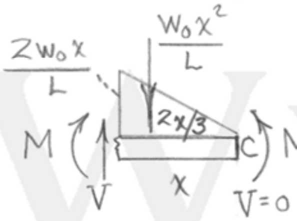
Consider $x = L/2$:

$\circlearrowleft + \Sigma M_C = 0$:

$+M_0 - \frac{w_0 L}{4}\left(\frac{L}{2}\right) + \frac{w_0\left(\frac{L}{2}\right)^2}{L}\left(\frac{2L}{3}\right) = 0$

$M_0 = \frac{w_0 L^2}{24}$

Consider arbitrary x:



$\circlearrowleft + \Sigma M_0 = 0$:

$M_0 - M - \frac{w_0 x^2}{L} x + \frac{w_0 x^2}{L}\frac{2x}{3} + \frac{w_0 L^2}{24} = 0$

$M = \frac{w_0}{3L}\left(\frac{L^3}{8} - x^3\right)$

$M = M_{max}$ @ $x = 0$: $M_{max} = \frac{w_0}{3L}\left(\frac{L^3}{8} - 0\right)$

$= \underline{\frac{w_0 L^2}{24}}$

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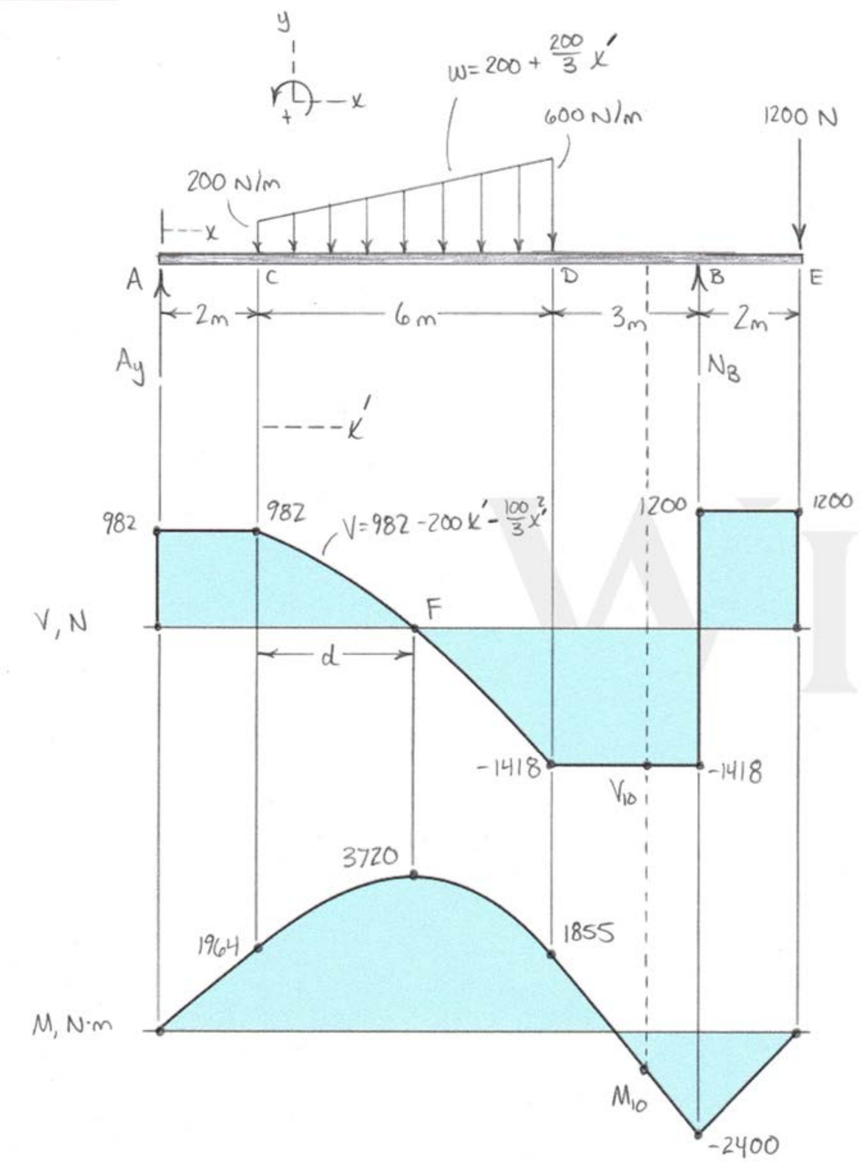

$$\Sigma F_y = 0: \quad A_y - \tfrac{1}{2}(200 + 600)(6) + N_B - 1200 = 0$$

$$\Sigma M_A = 0: \quad -200(6)(5) - \tfrac{1}{2}(600 - 200)(6)(6) + 11\,N_B - 13(1200) = 0$$

$$A_y = 982 \text{ N} \qquad N_B = 2620 \text{ N}$$

$$V_D = 982 - 200(6) - \frac{100}{3}(6)^2 \longrightarrow V_D = -1418 \text{ N}$$

$$\text{At } F, \; V_D = 0 = 982 - 200x' - \frac{100}{3}x'^2 \longrightarrow x' = -9.20 \text{ or } 3.20 \text{ m}$$

$$M_C = 2(982) = \underline{1964 \text{ N·m}}$$

$$M_F = 1964 + \int_0^{3.20} \left(982 - 200x' - \frac{100}{3}x'^2\right)dx' \longrightarrow M_F = 3720 \text{ N·m}$$

$$M_D = 1964 + \int_0^{6} \left(982 - 200x' - \frac{100}{3}x'^2\right)dx' \longrightarrow M_D = 1855 \text{ N·m}$$

$$M_B = 1855 - 1418(3) = -2400 \text{ N·m}$$

$$M_E = -2400 + 1200(2) = 0$$

$$\underline{V_{10} = -1418 \text{ N}}$$

$$M_{10} = 1855 - 2(1418) \longrightarrow \underline{M_{10} = -982 \text{ N·m}}$$

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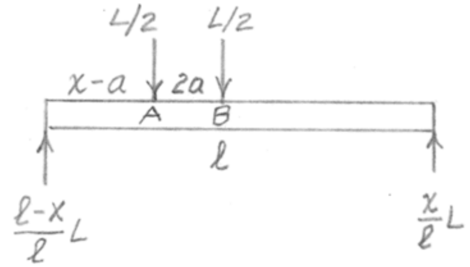


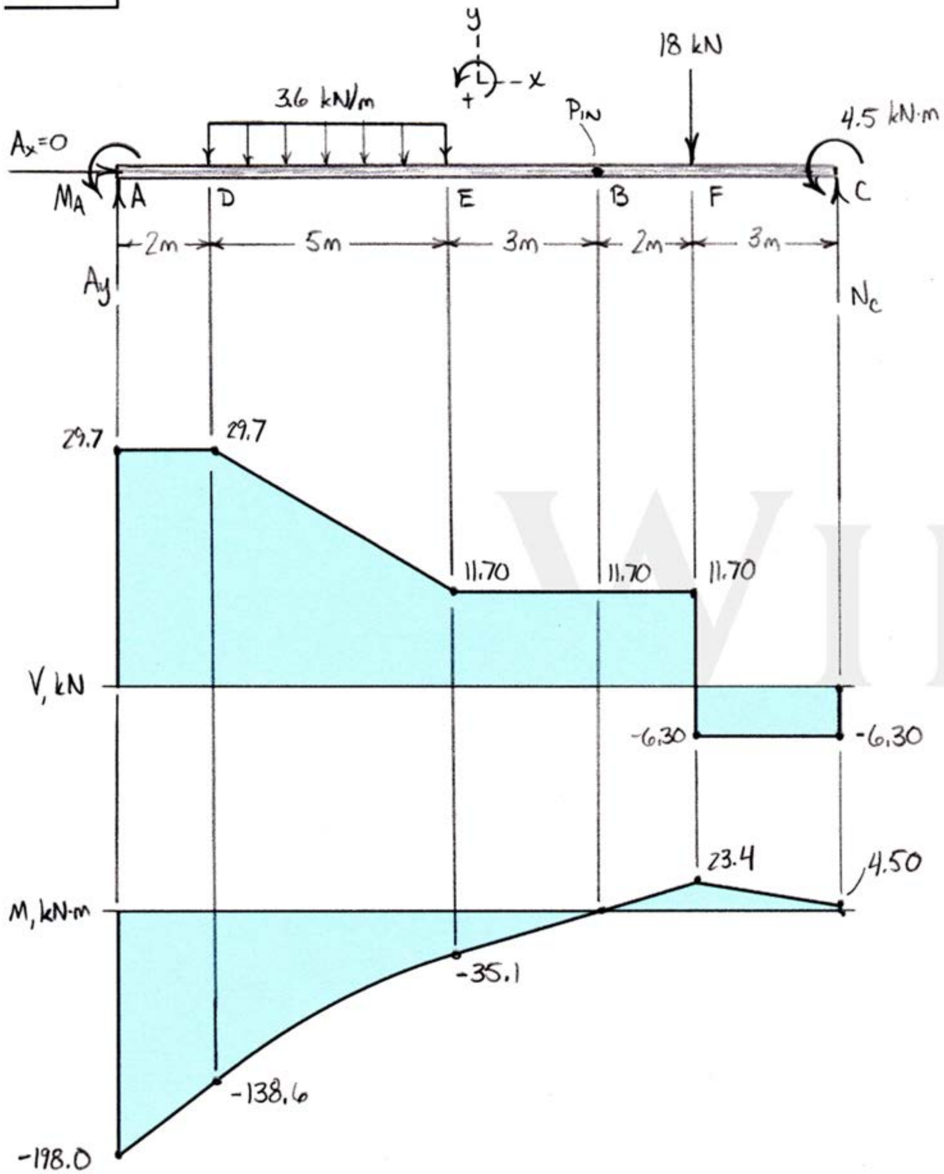
$$M_A = \frac{\ell - x}{\ell} L (x-a) = \frac{L}{\ell}\left(-x^2 + [a+\ell]x - a\ell\right)$$

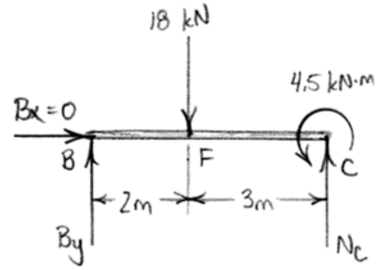
$$\frac{dM_A}{dx} = \frac{L}{\ell}(-2x + a + \ell) = 0 \text{ for max. } M_A\,; \quad \underline{x = \frac{a+\ell}{2}}$$

$$\& \ \underline{M_{A\,max} = \frac{L}{4\ell}(\ell - a)^2}\ ; \quad M_{B_{x=\frac{a+\ell}{2}}} = \frac{L}{4\ell}\left(\ell^2 - 2a\ell - 3a^2\right) < M_A$$

(By symmetry, a second and equal maximum bending moment occurs at $x = \frac{\ell - a}{2}$.)

5/144 PIN AT B WILL SUPPORT SHEAR BUT NOT MOMENT





$$\begin{cases} \Sigma F_y = 0: & B_y + N_C - 18 = 0 \\ \Sigma M_C = 0: & 4.5 + 3(18) - 5B_y = 0 \end{cases}$$

$B_y = 11.70$ kN $\qquad N_C = 6.30$ kN

$$\begin{cases} \Sigma F_y = 0: & A_y - 5(3.6) - 18 + N_C = 0 \\ \Sigma M_A = 0: & M_A + 15N_C + 4.5 - 12(18) - 5(3.6)(4.5) = 0 \end{cases} \longrightarrow \begin{cases} A_y = 29.7 \text{ kN} \\ M_A = 198.0 \text{ kN·m CCW} \end{cases}$$

$$\begin{cases} V_E = 29.7 - 3.6(5) = 11.70 \text{ kN} \\ V_{F^+} = 11.70 - 18 = -6.30 \text{ kN} \end{cases}$$

$$\begin{cases} M_D = -198.0 + 29.7(2) = -138.6 \text{ kN·m} \\ M_E = -138.6 + \frac{1}{2}(29.7 + 11.70)(5) = -35.1 \text{ kN·m} \\ M_B = -35.1 + 11.70(3) = 0 \\ M_F = 0 + 11.70(2) = 23.4 \text{ kN·m} \\ M_{C^-} = 23.4 + 3(-6.30) = 4.50 \text{ kN·m} \\ M_{C^+} = 4.50 - 4.50 = 0 \end{cases}$$

$|M|_{max} = 198.0$ kN·m AT A

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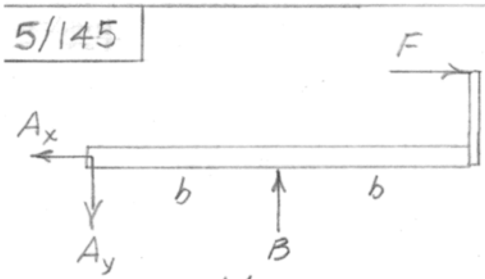
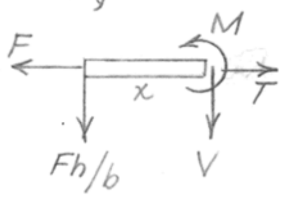


$\Sigma M_A = 0$; $Fh - Bb = 0$

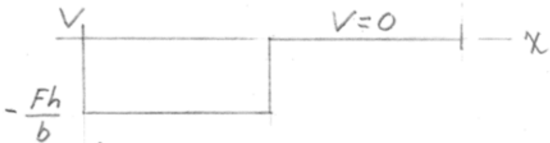
$B = Fh/b$

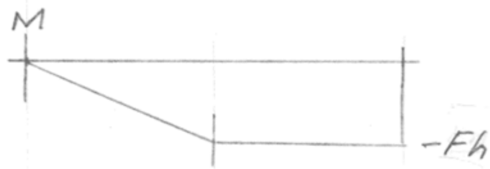
$\Sigma F_y = 0$; $A_y = B = Fh/b$



$\Sigma F_y = 0$; $V = -Fh/b$

$\Sigma M = 0$; $M + \frac{Fh}{b}x = 0$, $M = -\frac{Fh}{b}x$





$\underline{M = -Fh \text{ @ } x = 2b}$

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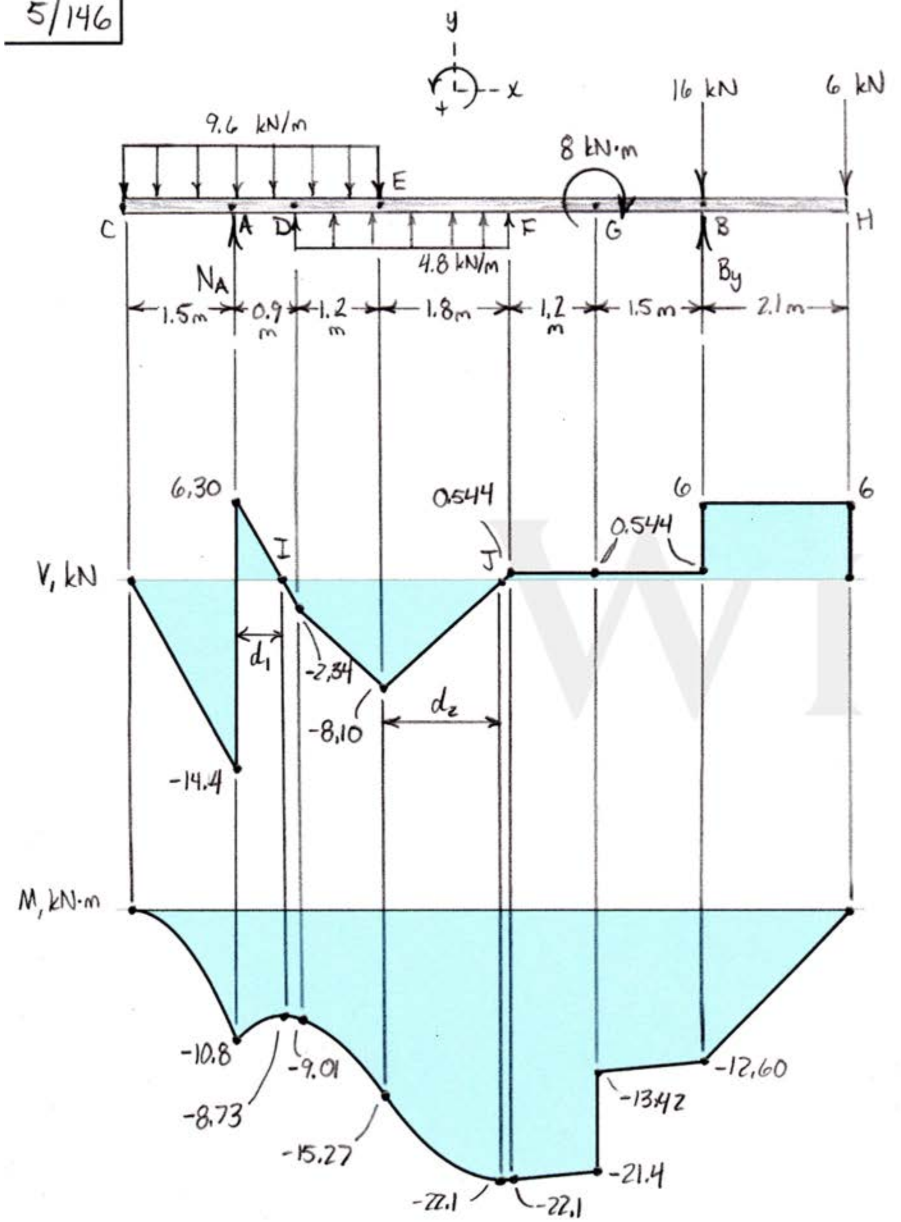

$$\begin{cases} \Sigma F_y = 0: \; N_A + B_y - 16 - 6 - 3.6(9.6) + 3(4.8) = 0 \\ \Sigma M_B = 0: \; -2.1(6) - 8 - 3(4.8)(4.2) + 3.6(9.6)(6.3) - 6.6 N_A = 0 \end{cases} \rightarrow \begin{cases} N_A = 20.7 \text{ kN} \\ B_y = 21.5 \text{ kN} \end{cases}$$

$$V_{A^-} = -1.5(9.6) = -14.40 \text{ kN} \qquad V_{A^+} = -14.40 + 20.7 = 6.30 \text{ kN}$$

$$V_D = 6.30 - 0.9(9.6) = -2.34 \text{ kN}$$

$$V_E = -2.34 + 1.2(4.8 - 9.6) = -8.10 \text{ kN}$$

$$V_F = -8.10 + 1.8(4.8) = 0.544 \text{ kN}$$

$$V_B = 0.544 - 16 + 21.5 = 6 \text{ kN}$$

$$d_1 = \frac{6.30}{9.6} = 0.657 \text{ m} \qquad d_2 = \frac{8.10}{4.8} = 1.687 \text{ m}$$

$$M_A = \tfrac{1}{2}(-14.4)(1.5) = -10.80 \text{ kN·m}$$

$$M_I = -10.80 + \tfrac{1}{2}(6.30)(0.657) = -8.73 \text{ kN·m}$$

$$M_D = -8.73 + \tfrac{1}{2}(-2.34)(0.9 - 0.657) = -9.01 \text{ kN·m}$$

$$M_E = -9.01 + \tfrac{1}{2}(-8.10 - 2.34)(1.2) = -15.27 \text{ kN·m}$$

$$M_J = -15.27 + \tfrac{1}{2}(-8.10)(1.687) = -22.1 \text{ kN·m}$$

$$M_F = -22.1 + \tfrac{1}{2}(0.544)(1.8 - 1.687) = -22.1 \text{ kN·m}$$

$$M_{G^-} = -22.1 + 0.544(1.2) = -21.4 \text{ kN·m} \qquad M_{G^+} = -21.4 + 8 = -13.42 \text{ kN·m}$$

$$M_B = -13.42 + 0.544(1.5) = -12.60 \text{ kN·m} \qquad M_H = -12.60 + 6(2.1) = 0$$

$$\underline{|M|_{max} = 22.1 \text{ kN·m At } 3.79 \text{ m Right of } A}$$

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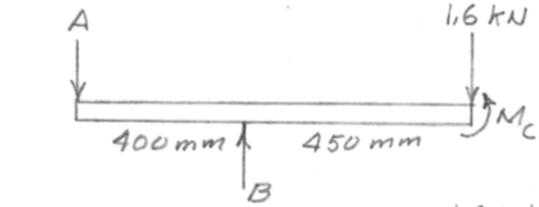


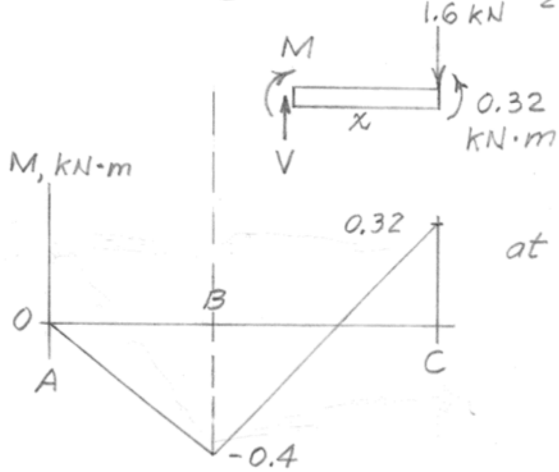
$M_C = 1.6(0.200) = 0.32$ kN·m

$\Sigma M_A = 0$; $0.4B + 0.32 - 0.85(1.6) = 0$

$B = 2.6$ kN

$\Sigma F = 0$; $A + 1.6 - 2.6 = 0$

$A = 1.0$ kN



$\Sigma M = 0$

$0.32 - 1.6x - M = 0$

$M = 0.32 - 1.6x$

at B, $x = 0.45$ ∴

$M_B = -0.40$ kN·m

$M = 0$ when

$x = 0.32/1.6 = 0.2$ m

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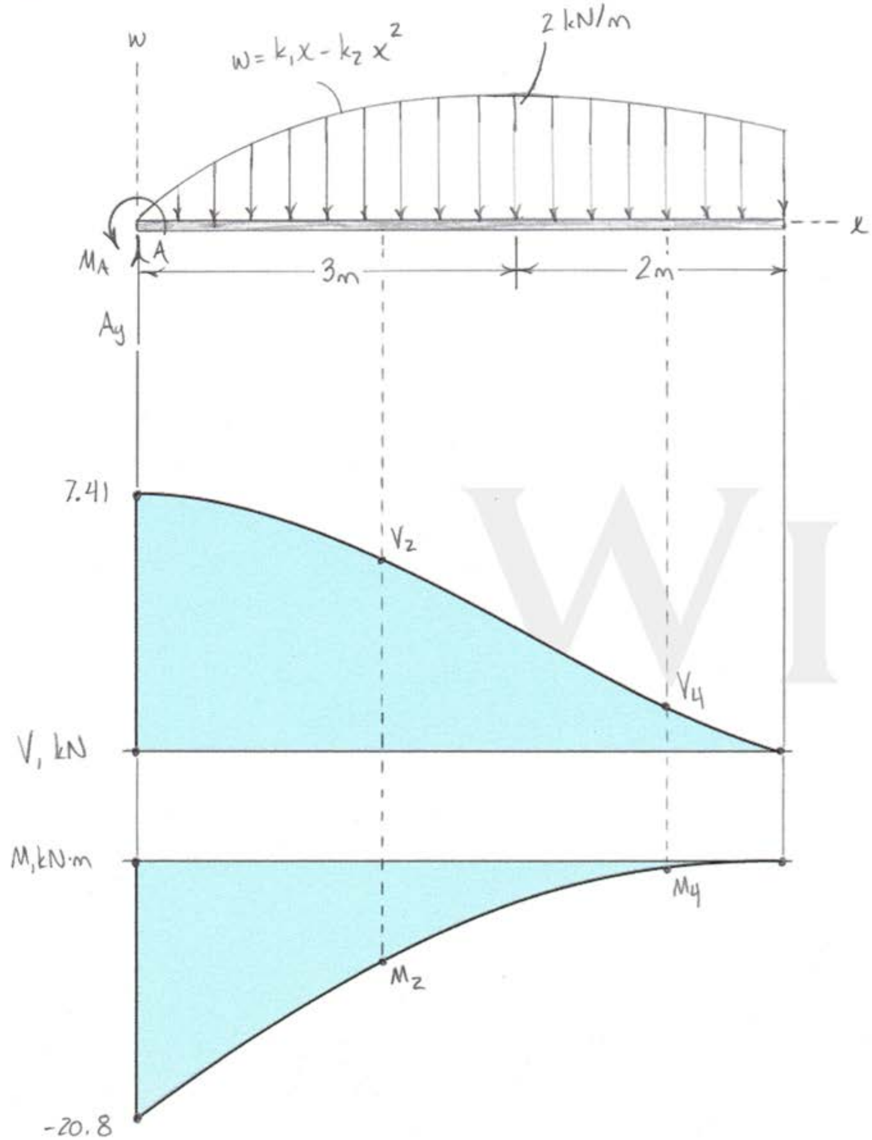

At $x = 3$, $w = 2$ so... $2 = k_1(3) - k_2(3)^2$

At $x = 3$, $\frac{dw}{dx} = 0$ so... $0 = k_1 - 2k_2(3)$

$\longrightarrow$ $k_1 = \frac{4}{3}$ kN/m², $k_2 = \frac{2}{9}$ kN/m³

$$V = -\int w\,dx = -\int\left(\frac{4}{3}x - \frac{2}{9}x^2\right)dx = -\frac{2}{3}x^2 + \frac{2}{27}x^3 + C_1$$

At $x = 5$, $V = 0 = -\frac{2}{3}(5)^2 + \frac{2}{27}(5)^3 + C_1 \longrightarrow C_1 = \frac{200}{27}$ kN (7.41 kN)

So... $\underline{V = \frac{200}{27} - \frac{2}{3}x^2 + \frac{2}{27}x^3}$ kN with x in m

$$M = \int V\,dx = \int\left(\frac{200}{27} - \frac{2}{3}x^2 + \frac{2}{27}x^3\right)dx = \frac{200}{27}x - \frac{2}{9}x^3 + \frac{1}{54}x^4 + C_2$$

At $x = 5$, $M = 0 = \frac{200}{27}(5) - \frac{2}{9}(5)^3 + \frac{1}{54}(5)^4 + C_2 \longrightarrow C_2 = \frac{-125}{6}$ kN·m (-20.8 kN·m)

So... $\underline{M = \frac{-125}{6} + \frac{200}{27}x - \frac{2}{9}x^3 + \frac{1}{54}x^4}$ kN·m with x in m

• At $x = 2$ m:

$\underline{V = 5.33 \text{ kN}}$

$\underline{M = -7.5 \text{ kN}\cdot\text{m}}$

• At $x = 4$ m:

$\underline{V = 1.481 \text{ kN}}$

$\underline{M = -0.685 \text{ kN}\cdot\text{m}}$

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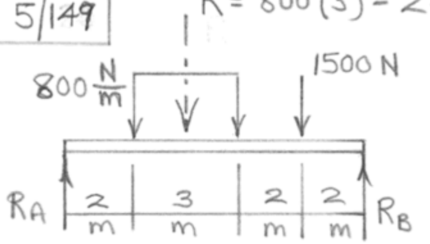
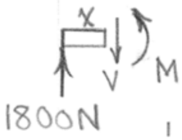
$R = 800(3) = 2400$ N



$\Sigma M_A = 0: R_B(9) - 1500(7) - 2400(3.5) = 0, \quad R_B = 2100$ N

$\Sigma F = 0: R_A - 2400 - 1500 + 2100 = 0$

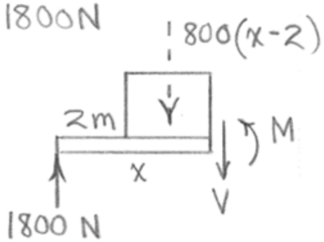
$R_A = 1800$ N



$0 < x < 2$ m:

$\Sigma F = 0 \Rightarrow V = 1800$ N

$\Sigma M = 0 \Rightarrow M = 1800x$



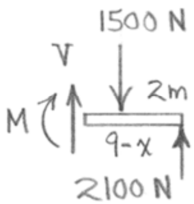
$2 < x < 5$ m:

$\Sigma F = 0: 1800 - 800(x-2) - V = 0$

$V = 3400 - 800x$

$\Sigma M = 0: M + 800(x-2)\frac{x-2}{2} - 1800x = 0$

$M = -400x^2 + 3400x - 1600$

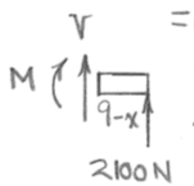


$5 < x < 7$:

$\Sigma F = 0: 2100 - 1500 + V = 0$

$V = -600$ N

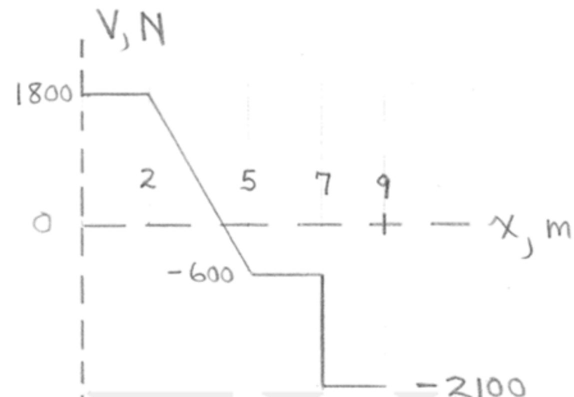
$\Sigma M = 0: -M - 1500(7-x) + 2100(9-x) = 0, \quad M = 8400 - 600x$

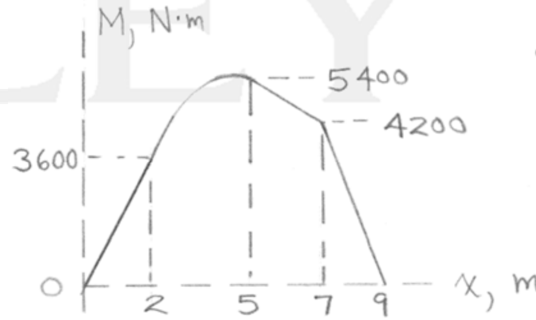


$7 < x < 9$ m:

$\Sigma F = 0 \Rightarrow V = -2100$ N

$\Sigma M = 0 \Rightarrow M = 18\,900 - 2100x$





At $x = 6$ m:

$V = -600$ N

$M = 8400 - 600(6) = 4800$ N·m

For M_{max},

$\frac{dM}{dx} = 0$

$\frac{d}{dx}(-400x^2 + 3400x - 1600) = -800x + 3400 = 0$

$x = 4.25$ m

$M_{max} = -400(4.25)^2 + 3400(4.25) - 1600 = 5620$ N·m

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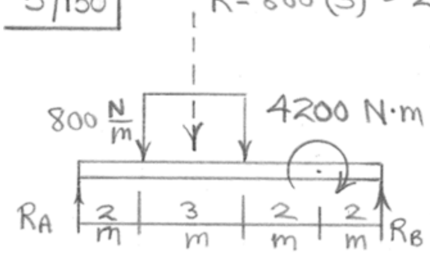
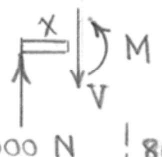
$R = 800(3) = 2400$ N



$\Sigma M_A = 0: \; R_B(9) - 2400(3.5) - 4200 = 0, \quad R_B = 1400$ N

$\Sigma F = 0 \Rightarrow \quad R_A = 1000$ N

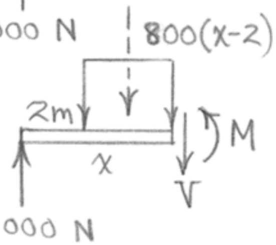
$0 < x < 2$ m:



$\Sigma F = 0 \Rightarrow \quad V = 1000$ N

$\Sigma M = 0 \Rightarrow \quad M = 1000x$

$2 < x < 5$ m:

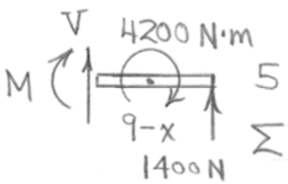


$\Sigma F = 0: \; 1000 - 800(x-2) - V = 0$

$V = 2600 - 800x$

$\Sigma M = 0: \; M + 800(x-2)\frac{x-2}{2} - 1000x = 0$

$M = -400x^2 + 2600x - 1600$

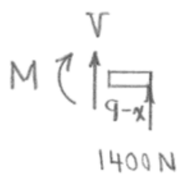


$5 < x < 7$ m:

$\Sigma F = 0: \; V + 1400 = 0, \quad V = -1400$ N

$\Sigma M = 0: \; -M - 4200 + 1400(9-x) = 0$

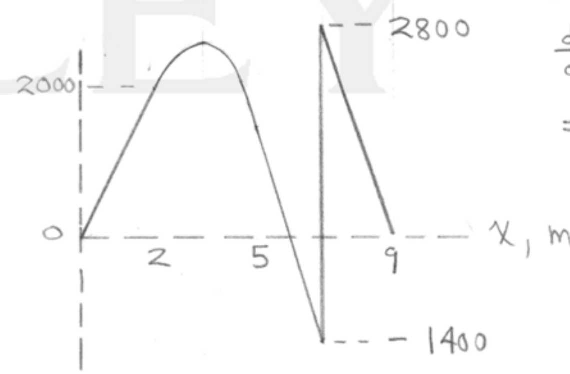
$M = 8400 - 1400x$



$7 < x < 9$ m:

$\Sigma F = 0 \Rightarrow \quad V = -1400$ N

$\Sigma M = 0 \Rightarrow \quad M = 12600 - 1400x$



At $x = 6$ m,

$V = -1400$ N

$M = 8400 - 1400(6)$

$= 0$

For M_{max}, $\frac{dM}{dx} = 0$

$\frac{d}{dx}\left(-400x^2 + 2600x - 1600\right)$

$= -800x + 2600 = 0$

$x = 3.25$ m

$M_{x=3.25} = -400(3.25)^2 + 2600(3.25) - 1600 = 2625$ N·m

So $M_{max} = 2800$ N·m

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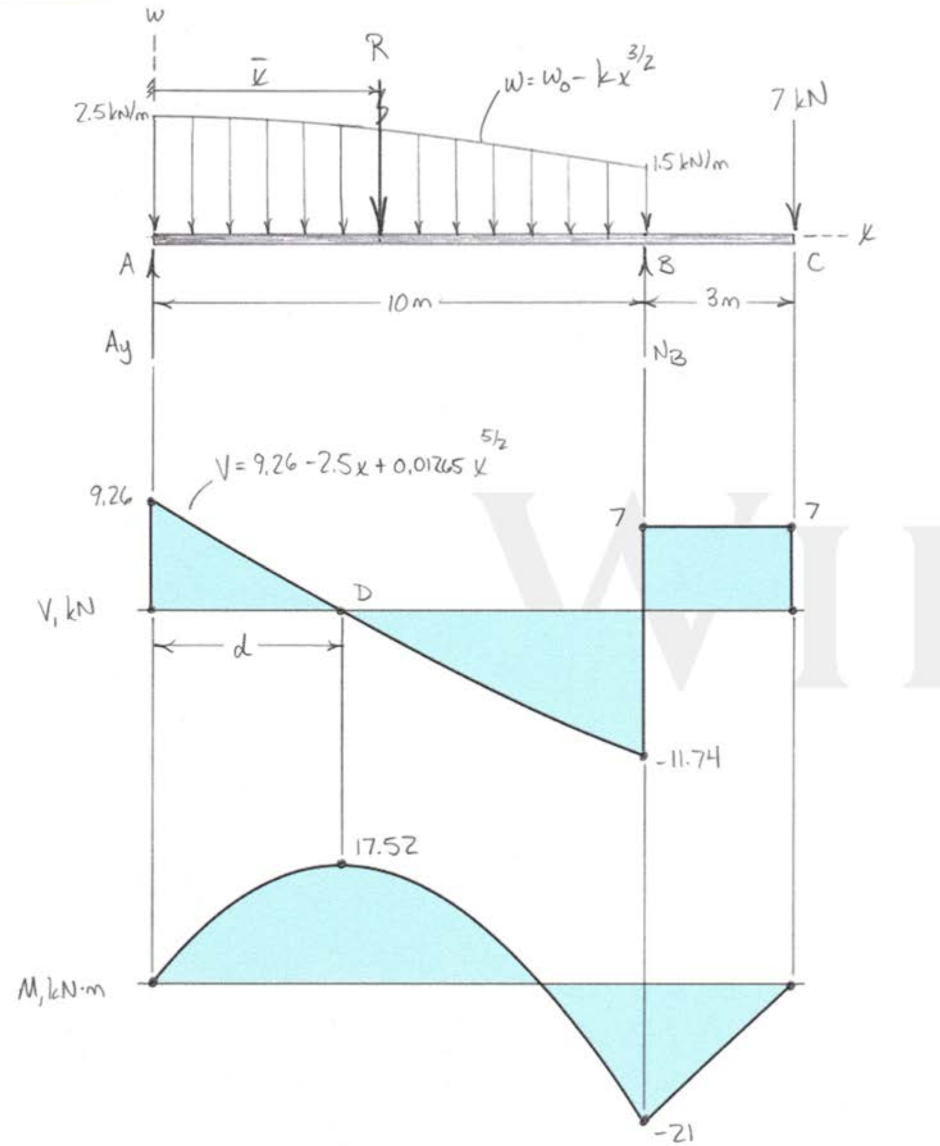

At $x = 0$, $w = 2.5$ so... $w_0 = 2.5$ kN/m

At $x = 10$, $w = 1.5$ so... $1.5 = 2.5 - k(10)^{3/2} \longrightarrow k = 0.0316$ kN/m$^{5/2}$

so... $w = 2.5 - 0.0316\,x^{3/2}$ kN/m with x in m

$$R = \int w\,dx = \int_0^{10}\left(2.5 - 0.0316\,x^{3/2}\right)dx \longrightarrow R = 21 \text{ kN}$$

$$\bar{x} = \frac{\int x w\,dx}{R} = \frac{\int_0^{10}\left(2.5x - 0.0316\,x^{5/2}\right)dx}{21} \longrightarrow \bar{x} = 4.59 \text{ m}$$

$\Sigma F_y = 0$: $A_y + N_B - R - 7 = 0$

$\Sigma M_A = 0$: $10N_B - R\bar{x} - 13(7) = 0$

$\longrightarrow$ $A_y = 9.26$ kN, $N_B = 18.74$ kN

$V_{B^-} = 9.26 - R = 9.26 - 21 = -11.74$ kN

At D, $V = 0 = 9.26 - 2.5d + 0.01265\,d^{5/2} \longrightarrow d = 3.85$ m (or 31.2 m)

$$M_D = \int_0^{3.85}\left(9.26 - 2.5x + 0.01265\,x^{5/2}\right)dx \longrightarrow M_D = 17.52 \text{ kN·m}$$

$$M_B = \int_0^{10}\left(9.26 - 2.5x + 0.01265\,x^{5/2}\right)dx \longrightarrow M_B = -21 \text{ kN·m}$$

$M_C = -21 + 7(3) = \underline{0}$

Max Positive Bending Moment: $\underline{M_D = 17.52 \text{ kN·m at } x = 3.85 \text{ m}}$

Max Negative Bending Moment: $\underline{M_B = -21 \text{ kN·m at } x = 10 \text{ m}}$

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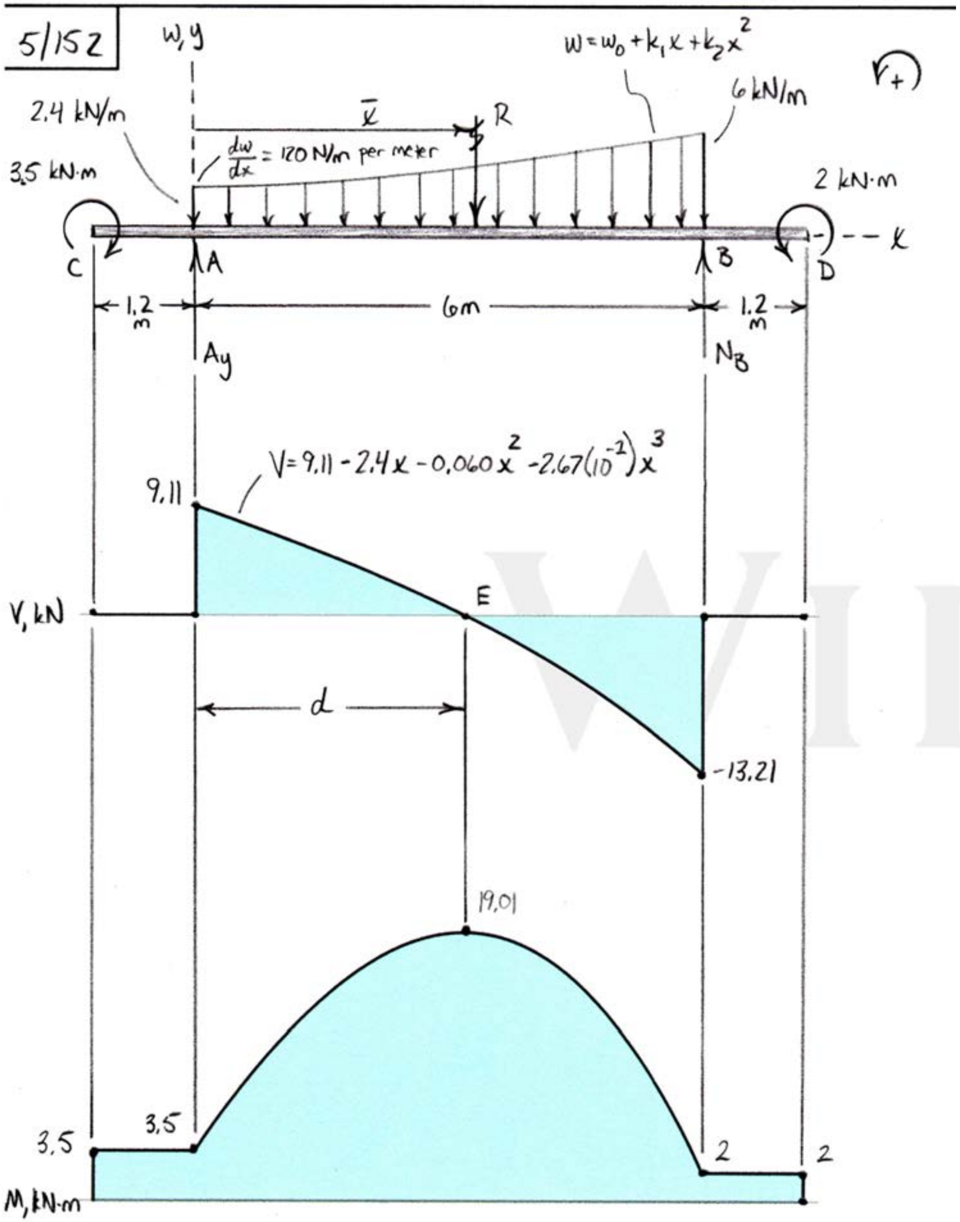

$$\begin{cases} \text{At } x=0, \; w = 2.4 = w_0 + k_1(0) + k_2(0)^2 \longrightarrow w_0 = 2.4 \text{ kN/m} \\ \text{At } x=0, \; \frac{dw}{dx} = 0.120 = k_1 + 2k_2(0) \longrightarrow k_1 = 0.120 \text{ kN/m}^2 \\ \text{At } x=6, \; w = 6 = 2.4 + 0.120(6) + k_2(6)^2 \longrightarrow k_2 = 0.080 \text{ kN/m}^3 \end{cases}$$

$$\text{so... } \quad w = 2.4 + 0.120x + 0.080x^2$$

$$\begin{cases} R = \int w\,dx = \int_0^6 (2.4 + 0.120x + 0.080x^2)\,dx \longrightarrow R = 22.3 \text{ kN} \\ \bar{x} = \frac{1}{R}\int xw\,dx = \frac{1}{22.3}\int_0^6 (2.4x + 0.120x^2 + 0.080x^3)\,dx \longrightarrow \bar{x} = 3.48 \text{ m} \end{cases}$$

$$\begin{cases} \Sigma F_y = 0: \; A_y + N_B - R = 0 \\ \Sigma M_A = 0: \; 2 - 3.5 - R\bar{x} + 6N_B = 0 \end{cases} \longrightarrow \begin{cases} A_y = 9.11 \text{ kN} \\ N_B = 13.21 \text{ kN} \end{cases}$$

In AB, $V = 9.11 - \int w\,dx \longrightarrow V = 9.11 - 2.4x - 0.060x^2 - 2.67(10^{-2})x^3$

At E, $V = 0 = 9.11 - 2.4d - 0.060d^2 - 2.67(10^{-2})d^3 \longrightarrow d = 3.18 \text{ m}$

$$M_E = 3.5 + \int_0^{3.18} (9.11 - 2.4x - 0.060x^2 - 2.67(10^{-2})x^3)\,dx = 19.01 \text{ kN·m}$$

$\underline{M_{max} = 19.01 \text{ kN·m} \quad \text{At } x = 3.18 \text{ m}}$

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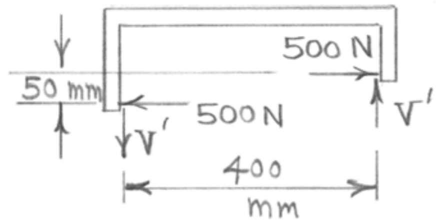
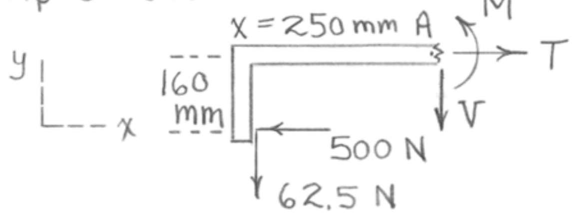
FBD of clamp:



$\Sigma M = 0: \quad 500(0.050) - V'(0.400) = 0, \quad V' = 62.5 \text{ N}$

FBD of clamp section:



$\Sigma F_x = 0: \quad T - 500 = 0, \qquad \underline{T = 500 \text{ N}}$

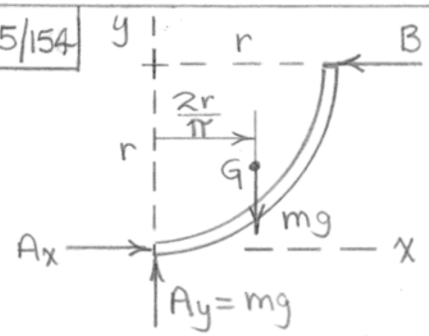
$\Sigma F_y = 0: \quad -V - 62.5 = 0, \qquad \underline{V = -62.5 \text{ N}}$

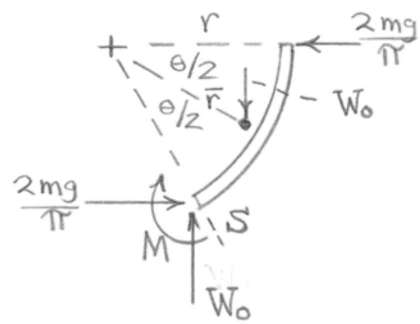
$\Sigma M_A = 0: \quad M + 62.5(0.250) - 500(0.160) = 0$

$\underline{M = 64.4 \text{ N}\cdot\text{m}}$

M is the only quantity which depends on x.

▶ 5/154





As a whole : $\Sigma M_A = 0: \; Br - mg\frac{2r}{\pi} = 0, \; B = \frac{2mg}{\pi}$

Section : $W_0 = \frac{\theta}{\pi/2} mg = \frac{2\theta}{\pi} mg, \quad \bar{r} = r\frac{\sin\frac{\theta}{2}}{\theta/2}$

$$\Sigma M_S = 0: \; M + \frac{2\theta}{\pi} mg\left(r\frac{\sin\frac{\theta}{2}}{\theta/2}\cos\frac{\theta}{2} - r\cos\theta\right) - \frac{2mg}{\pi} r\sin\theta = 0, \quad \underline{M = \frac{2mgr}{\pi}\theta\cos\theta}$$

$$C = W_0\cos\theta + \frac{2mg}{\pi}\sin\theta, \quad \underline{C = \frac{2mg}{\pi}(\theta\cos\theta + \sin\theta)}$$

$$V = W_0\sin\theta - \frac{2mg}{\pi}\cos\theta, \quad \underline{V = \frac{2mg}{\pi}(\theta\sin\theta - \cos\theta)}$$

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Given: $\begin{cases} 2s = 30 \text{ m}, \quad s = 15 \text{ m} \\ \mu = \dfrac{0.283(9.81)}{30} = 0.0925 \text{ N/m} \\ T = 42 \text{ N} \end{cases}$

$T^2 = T_o^2 + (\mu s)^2: \quad 42^2 = T_o^2 + (0.0925 \cdot 15)^2$

$T_o = 41.98 \text{ N}$

(Eq. 5/22) $\quad T = T_o + \mu y: \quad 42 = 41.98 + 0.0925 h$

$h = 0.248 \text{ m}$ or $\underline{h = 248 \text{ mm}}$

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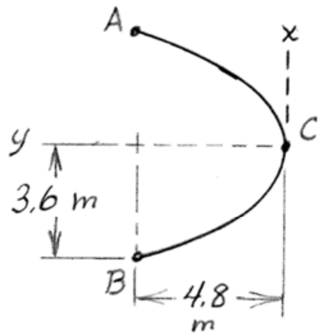
$l_A = 3.6\ m,\ h_A = 4.8\ m,\ w = 60\ N/m$



From $y = \dfrac{wx^2}{2T_0}$ (Eq. 5/14)

$$4.8 = \frac{60(3.6)^2}{2T_0},\quad T_0 = 81\ \text{N}$$

min. $\underline{@\ C}$

Eq. 5/15a @ A: $T_{max} = 60(3.6)\sqrt{1 + \left(\dfrac{3.6}{2(4.8)}\right)^2}$

$$= 231\ \text{N @ A \& B}$$

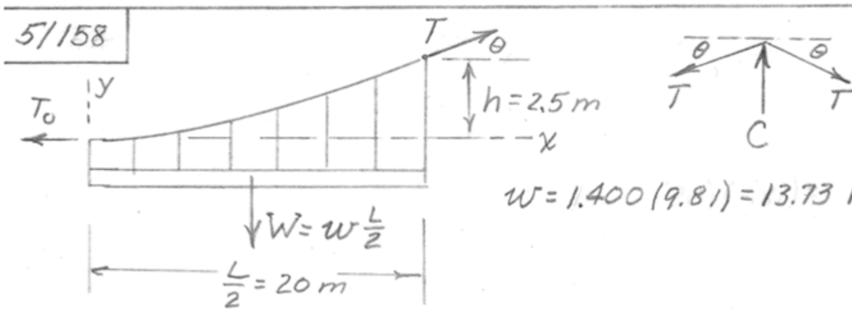
$\underline{\quad\quad max.}$

5/157 Eq. 5/22, $T_B = T_o + \mu y_B$, $T_A = T_o + \mu y_A$

so $T_B - T_A = \mu(y_B - y_A)$ or $T_B - T_A = \mu h$

Thus $h = \frac{1}{\mu}(T_B - T_A) = \frac{1}{0.12(9.81)}(230 - 110) = \underline{101.9\text{ m}}$

5/158



$w = 1.400(9.81) = 13.73$ kN/m

From Eq. 5/15b, $T = \frac{wL}{2}\sqrt{1 + (L/4h)^2}$

$$= \frac{13.73(40)}{2}\sqrt{1 + \left[\frac{40}{4(2.5)}\right]^2} = 1133 \text{ kN}$$

$T^2 = W^2 + T_0^2$, $T_0 = \sqrt{(1132)^2 - [(13.73)(20)]^2} = 1099$ kN

$\frac{dy}{dx} = \tan\theta = \frac{w(L/2)}{T_0} = \frac{13.73(20)}{1099} = 0.250$, $\theta = 14.04°$

$\Sigma F_y = 0$ at support; $2T\sin\theta - C = 0$

$$C = 2(1133)\sin 14.04° = \underline{549 \text{ kN}}$$

5/159

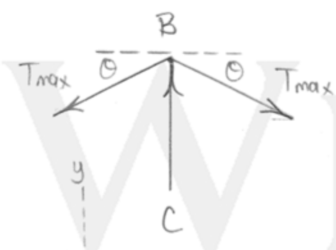
$L = 1991$ m $\qquad h = \frac{L}{10} = 199.1$ m

$w = \frac{160}{2} = 80$ kN/m PER CABLE

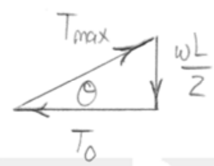
$$T_0 = \frac{wL^2}{8h} = \frac{80(1991)^2}{8(199.1)} \longrightarrow \underline{T_0 = 199.1(10^3)\text{ kN}}$$

$$T_{max} = T_B = w\frac{L}{2}\sqrt{1+\left(\frac{L}{4h}\right)^2} = 80\left(\frac{1991}{2}\right)\sqrt{1+\left(\frac{1991}{4(199.1)}\right)^2} = \underline{214(10^3)\text{ kN}}$$

TOWER B: T_{max} OCCURS AT A & B.



AND...



$$T_{max}\sin\theta = \frac{wL}{2}$$

$\Sigma F_y = 0$: $\quad C - 2T_{max}\sin\theta = 0$

$$C - wL = 0 \longrightarrow C = 80(1991) = \underline{159.3(10^3)\text{ kN}}$$

5/160 Eq. 5/15 $T = w\sqrt{x^2 + (\ell_A^2/2h_A)^2} = w\sqrt{x^2 + (L^2/8h)^2}$

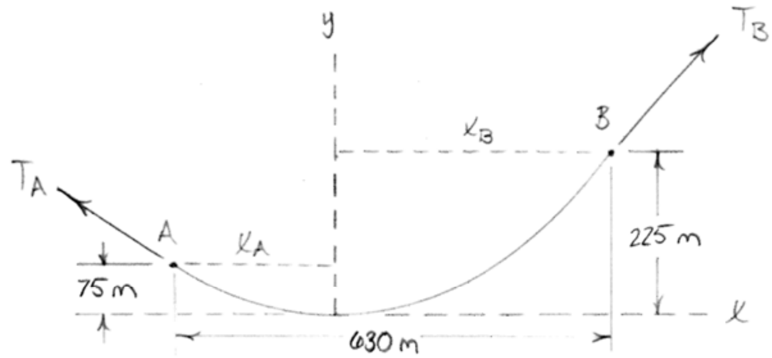
So $\Delta T = \Delta w\sqrt{x^2 + (L^2/8h)^2}$, $\Delta w = \Delta(mg) = g\Delta m$

For each cable $2.14(10^6) = 9.81\,\Delta m\sqrt{(240)^2 + ([1000]^2/8(200))^2}$

$= 9.81\,\Delta m\,(669.5)$, $\Delta m = 326$ kg/m

So for both cables $m' = 2\Delta m = 2(326) = \underline{652 \text{ kg/m}}$

*5/161 $\mu = 1.1(9.81) = 10.79$ N/m



APPLY EQ. 5/19 AT POINTS A AND B. $y = \frac{T_0}{\mu}\left[\cosh \frac{x}{T_0/\mu} - 1\right]$

$$\begin{cases} A: \quad 75 = \dfrac{T_0}{10.79}\left[\cosh\left(\dfrac{x_A}{T_0/10.79}\right) - 1\right] \\ B: \quad 225 = \dfrac{T_0}{10.79}\left[\cosh\left(\dfrac{x_B}{T_0/10.79}\right) - 1\right] \\ \text{AND...} \quad x_B - x_A = 630 \end{cases}$$

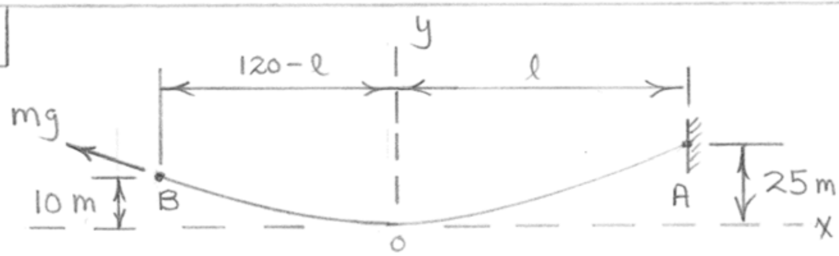
NUMERICAL SOLUTION YIELDS...

$$\begin{cases} x_A = -235 \text{ m} \\ x_B = 395 \text{ m} \\ T_0 = 4100 \text{ N} \end{cases}$$

$$T_A = T_0 + \mu y_A = 4100 + 10.79(75) \longrightarrow \underline{T_A = 4900 \text{ N}}$$

$$T_B = T_0 + \mu y_B = 4100 + 10.79(225) \longrightarrow \underline{T_B = 6520 \text{ N}}$$

5/162



Eq. 5/14 : $y = \frac{wx^2}{2T_0}$

At A: $25 = \frac{40\ell^2}{2T_0}$, At B: $10 = \frac{40(120-\ell)^2}{2T_0}$

Eliminate T_0 : $0.6\ell^2 - 240\ell + 14\,400 = 0$

$\ell = 73.5$ m (or $\ell = 326$ m)

$T_0 = \frac{4}{5}\ell^2 = \frac{4}{5}(73.5)^2 = 4320$ N

Section O-B : $(mg)^2 = T_0^2 + (wx)^2$

$(9.81m)^2 = (4320)^2 + [40\,(120 - 73.5)]^2$

$\underline{m = 480 \text{ kg}}$

*5/163 Please refer to the diagram in the solution to Prob. 5/155. Eq. 5/19:

$$y = \frac{T_0}{\mu}\left[\cosh\frac{\mu x}{T_0} - 1\right]$$

$$\left.\begin{aligned} \text{At B:}\quad 10 &= \frac{T_0}{40}\left[\cosh\frac{40(120-\ell)}{T_0} - 1\right] \\ \text{At A:}\quad 25 &= \frac{T_0}{40}\left[\cosh\frac{40\ell}{T_0} - 1\right] \end{aligned}\right\}$$

Numerical solution: $\begin{cases} T_0 = 4440 \text{ N} \\ \ell = 73.2 \text{ m} \end{cases}$

Eq. 5/20: $s = \dfrac{T_0}{\mu}\sinh\dfrac{\mu x}{T_0}$

At B: $s_B = \dfrac{4440}{40}\sinh\dfrac{40(120-73.2)}{4440} = 48.2 \text{ m}$

Equilibrium of section OB:

$(mg)^2 = T_0^2 + (\mu s_B)^2$: $m^2(9.81)^2 = 4440^2 + (40 \cdot 48.2)^2$

$\underline{m = 494 \text{ kg}}$

5/164 $w = a + bx^2$, when $x = 0$, $w = w_0$
$x = L/2$, $w = w_1$

So $a = w_0$ & $b = \frac{4}{L^2}(w_1 - w_0)$; Thus $w = w_0 + \frac{4(w_1 - w_0)}{L^2}x^2$

From Eq. 5/13, $\frac{dy}{dx} = \frac{1}{T_0}\int_0^x w\,dx$

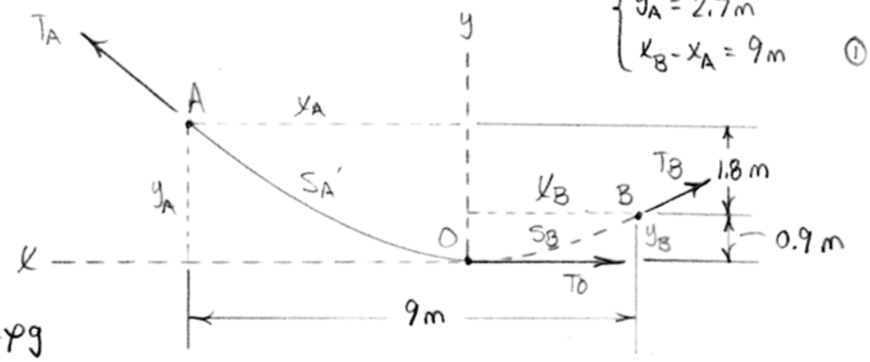
$$= \frac{1}{T_0}\left[w_0 x + \frac{4(w_1 - w_0)}{L^2}\frac{x^3}{3}\right]$$

& $y = \frac{w_0 x^2}{2T_0} + \frac{w_1 - w_0}{3T_0 L^2}x^4$; Thus for $x = L/2$, $y = h$

& $h = \frac{w_0 L^2}{8T_0} + \frac{w_1 - w_0}{3T_0 L^2}\frac{L^4}{16} = \underline{\frac{L^2}{48T_0}(5w_0 + w_1)}$

***5/165** $T_{max} = T_A = 5\text{ kN}$

$$\begin{cases} y_B = 0.9\text{ m} \\ y_A = 2.7\text{ m} \\ x_B - x_A = 9\text{ m} \end{cases} \quad (1)$$



$w = \rho g$

(2) $y_A = \dfrac{\rho g x_A^2}{2T_0}$ (3) $y_B = \dfrac{\rho g x_B^2}{2T_0}$ (4) $T_A^2 = T_0^2 + (\rho g)^2 x_A^2$

SOLVING (1) THROUGH (4)... $\begin{cases} x_A = -5.71\text{ m} \\ x_B = 3.29\text{ m} \end{cases}$ $\begin{cases} \underline{T_0 = 3630\text{ N}} \\ \underline{\rho = 61.4\text{ kg/m}} \end{cases}$

(NOTE: 4 SOLUTIONS, ONLY ONE WITH T_0, x_B, AND ρ POSITIVE AND x_A NEGATIVE)

$$\begin{cases} s_A = \displaystyle\int_0^{l_A} \sqrt{1 + \frac{(\rho g)^2 x^2}{T_0^2}}\, dx = \int_0^{5.71} \sqrt{1 + \frac{(61.4)^2 (9.81)^2 x^2}{3630^2}}\, dx = 6.47\text{ m} \\ s_B = \displaystyle\int_0^{l_B} \sqrt{1 + \frac{(\rho g)^2 x^2}{T_0^2}}\, dx = \int_0^{3.29} \sqrt{1 + \frac{(61.4)^2 (9.81)^2 x^2}{3630^2}}\, dx = 3.45\text{ m} \end{cases}$$

$$L = s_A + s_B = 6.47 + 3.45 \longrightarrow \underline{L = 9.92\text{ m}}$$

5/166 $w = w_0 + kx^{3/2}$ AND $w_0 = 200$ N/m

AT $x = 10$, $w = 800 = 200 + k(10)^{3/2} \longrightarrow k = 6\sqrt{10}$ N/m$^{5/2}$

SO... $w = 200 + 6\sqrt{10}\, x^{3/2}$ N/m

$$\frac{d^2y}{dx^2} = \frac{w}{T_0} = \frac{1}{T_0}\left(200 + 6\sqrt{10}\,x^{3/2}\right) \qquad \text{(USE EQ. 5/13)}$$

$$\frac{dy}{dx} = \frac{1}{T_0}\int_0^x \left(200 + 6\sqrt{10}\,x^{3/2}\right)dx = \frac{1}{T_0}\left(200x + \frac{12\sqrt{10}}{5}x^{5/2}\right)$$

$$y = \frac{1}{T_0}\int_0^x \left(200x + \frac{12\sqrt{10}}{5}x^{5/2}\right)dx = \frac{1}{T_0}\left(100x^2 + \frac{24\sqrt{10}}{35}x^{7/2}\right)$$

AT $x = 10$ m, $y = 3$ m

$$3 = \frac{1}{T_0}\left(100 \cdot 10^2 + \frac{24\sqrt{10}}{35}\,10^{7/2}\right) \longrightarrow \underline{T_0 = 5620 \text{ N}}$$

$$\underline{y = \left(178.0\,x^2 + 3.86\,x^{7/2}\right)\left(10^{-4}\right) \text{ m}}$$

SINCE THE LOADING IS ALL VERTICAL, THE HORIZONTAL COMPONENT OF THE TENSION AT B MUST EQUAL T_0.

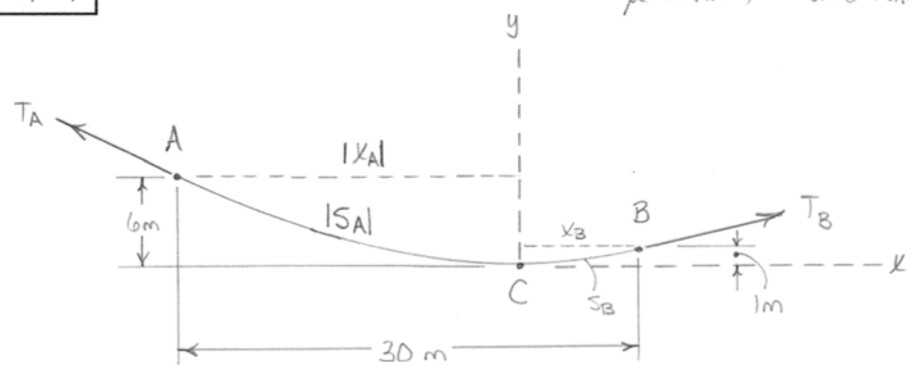
AT B... $$\frac{dy}{dx} = \frac{1}{5620}\left(200 \cdot 10 + \frac{12\sqrt{10}}{5} \cdot 10^{5/2}\right) = 0.783$$

$$\tan\theta = \frac{dy}{dx} \quad \text{SO...} \quad \theta = \tan^{-1}(0.783) = 38.1°$$

$$T_B\cos\theta = T_0 \longrightarrow T_B = \frac{T_0}{\cos\theta} = \frac{5620}{\cos 38.1°} \longrightarrow \underline{T_B = 7140 \text{ N}}$$

*5/167

$\mu = 16(9.81) = 157.0 \text{ N/m}$



USE EQ. 5/19 AT A AND B: $y = \frac{T_0}{\mu}\left(\cosh \frac{x}{T_0/\mu} - 1\right)$

$$\begin{cases} A: \; 6 = \frac{T_0}{157.0}\left[\cosh\left(\frac{-x_A}{T_0/157.0}\right) - 1\right] \\ B: \; 1 = \frac{T_0}{157.0}\left[\cosh\left(\frac{x_B}{T_0/157.0}\right) - 1\right] \\ x_A + x_B = 30 \end{cases}$$

SOLVING NUMERICALLY...

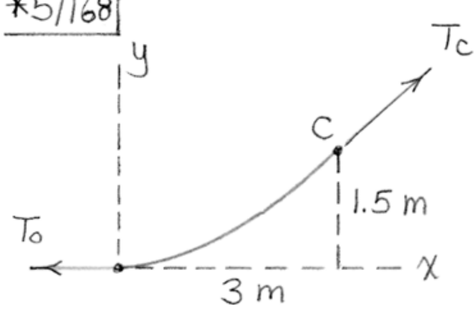
$$\begin{cases} |x_A| = 21.2 \text{ m} \\ x_B = 8.76 \text{ m} \\ T_0 = 6050 \text{ N} \end{cases}$$

$$\begin{cases} T_A = T_0 + \mu y_A = 6050 + 157.0(6) \longrightarrow \underline{T_A = 6990 \text{ N}} \quad (T_{max}) \\ T_B = T_0 + \mu y_B = 6050 + 157.0(1) \longrightarrow \underline{T_B = 6210 \text{ N}} \end{cases}$$

$$S = |S_A| + S_B = \frac{T_0}{\mu}\left(\sinh \frac{\mu |x_A|}{T_0} + \sinh \frac{\mu x_B}{T_0}\right) = \frac{6050}{157.0}\left[\sinh\left(\frac{157.0(21.2)}{6050}\right) + \sinh\left(\frac{157.0(8.76)}{6050}\right)\right]$$

$$\underline{S = 31.2 \text{ m}}$$

*5/168



$\mu = 200$ N/m

Eq. 5/19 evaluated at point C:

$$1.5 = \frac{T_0}{\mu}\left[\cosh\frac{3}{T_0/\mu} - 1\right]$$

Numerical solution: $\frac{T_0}{\mu} = 3.22$ m

Then $T_0 = 3.22(200) = 645$ N

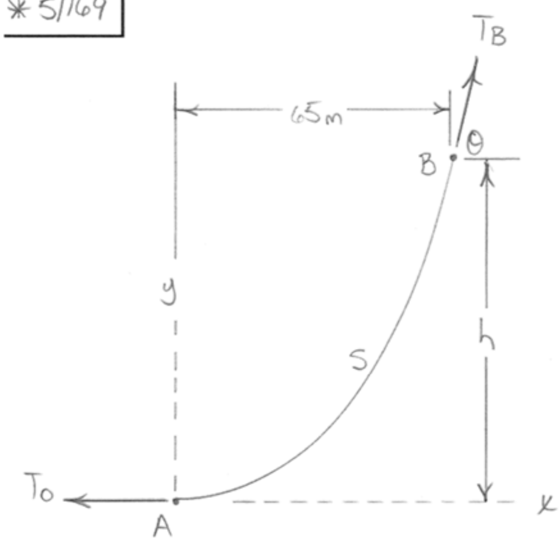
Eq. 5/22 evaluated at C:

$T_C = 645 + 200(1.5) = \underline{945\text{ N}}$

Eq. 5/20: $s = 3.22 \sinh\frac{3}{3.22} = 3.45$ m

$\underline{L = 2s = 6.90\text{ m}}$

*5/169



$$\begin{cases} s = 120 \text{ m} \\ \rho = 5 \text{ g/m} = 0.005 \text{ kg/m} \end{cases}$$

$$\mu = \rho g = 0.005(9.81) = 0.0490 \text{ N/m}$$

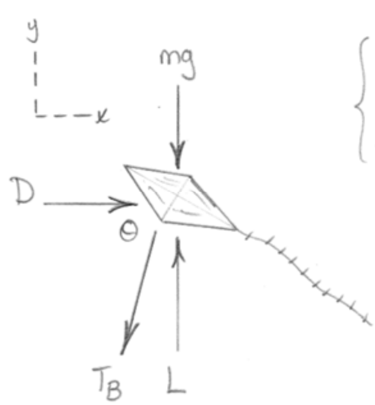
Use Eq. 5/19 & 5/20

$$\begin{cases} y_B = \dfrac{T_0}{\mu}\left[\cosh \dfrac{\mu x_B}{T_0} - 1\right] \longrightarrow h = \dfrac{T_0}{0.0490}\left[\cosh\left(\dfrac{0.0490(65)}{T_0}\right) - 1\right] \\ s = \dfrac{T_0}{\mu}\sinh \dfrac{\mu x_B}{T_0} \longrightarrow 120 = \dfrac{T_0}{0.0490}\sinh\left(\dfrac{0.0490(65)}{T_0}\right) \end{cases}$$

Solving numerically... $\underline{h = 92.2 \text{ m}}$ and $T_0 = 1.568$ N

$$\begin{cases} \theta = \tan^{-1}\left(\dfrac{\mu s}{T_0}\right) = \tan^{-1}\left(\dfrac{0.0490(120)}{0.1599}\right) \longrightarrow \theta = 75.1^\circ \\ T_B = T_0 + \mu h = 1.568 + 0.0490(92.2) \longrightarrow T_B = 6.09 \text{ N} \end{cases}$$

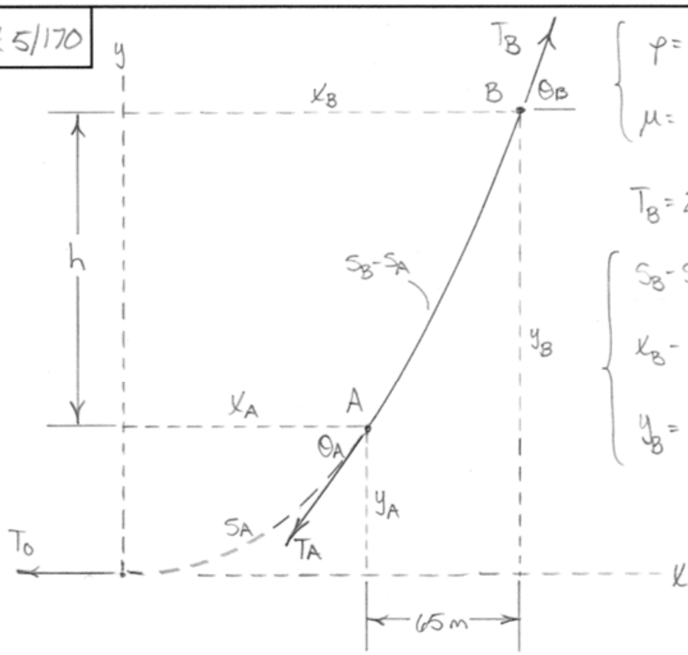
$$m = 600 \text{g} = 0.6 \text{ kg}$$



$$\begin{cases} \Sigma F_x = 0: \; D - T_B \cos\theta = 0 \\ \Sigma F_y = 0: \; L - T_B \sin\theta - mg = 0 \end{cases}$$

$$\begin{cases} \underline{D = 1.568 \text{ N}} \\ \underline{L = 11.77 \text{ N}} \end{cases}$$

*5/170



$\rho = 5$ g/m

$\mu = \frac{5(9.81)}{1000} = 0.0491$ N/m

$T_B = 20$ N

$s_B - s_A = 120$ m ①

$x_B - x_A = 65$ m ②

$y_B = y_A + h$ ③

AT A:

$y_A = \frac{T_0}{\mu}\left[\cosh\frac{\mu x_A}{T_0} - 1\right]$ ④

$s_A = \frac{T_0}{\mu}\sinh\frac{\mu x_A}{T_0}$ ⑤

$T_A = T_0 + \mu y_A$ ⑥

$\theta_A = \tan^{-1}\left(\frac{\mu s_A}{T_0}\right)$ ⑦

AT B:

$y_B = \frac{T_0}{\mu}\left[\cosh\frac{\mu x_B}{T_0} - 1\right]$ ⑧

$s_B = \frac{T_0}{\mu}\sinh\frac{\mu x_B}{T_0}$ ⑨

$T_B = T_0 + \mu y_B$ ⑩

SOLVING NUMERICALLY ...

$x_A = 201$ m, $y_A = 115.3$ m

$x_B = 266$ m, $y_B = 216$ m

$s_A = 240$ m, $s_B = 360$ m

$\theta_A = 51.4°$

$h = 100.7$ m

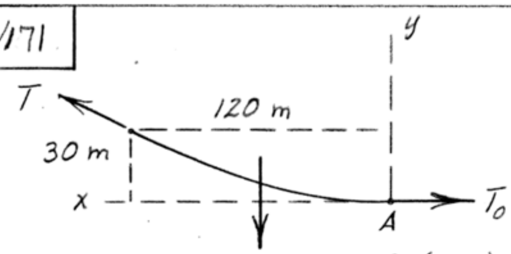
$\underline{T_A = 15.06 \text{ N}}$

$T_0 = 9.41$ N

FROM PROB. 5/169, $h = 92.2$ m so... $\delta h = 100.7 - 92.2$

$\therefore \underline{\delta h = 8.45 \text{ m}}$

*5/171



$$\mu s = \rho g s = 0.750(9.81)s = 7.36\,s$$

Catenary Eq. 5/19: $y = \frac{T_0}{\mu}\left(\cosh\frac{\mu x}{T_0} - 1\right)$

$$30 = \frac{T_0}{7.36}\left(\cosh\frac{7.36(120)}{T_0} - 1\right)$$

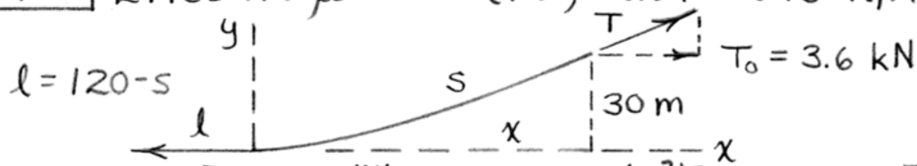
Solve numerically to obtain

$$\underline{T_0 = 1801\text{ N}}$$

Parabolic Eq. 5/14: $y = \frac{w x^2}{2T_0} \approx \frac{\mu x^2}{2T_0}$

So $T_0 \approx \frac{\mu x^2}{2y} = \frac{7.36(120^2)}{2(30)} = \underline{1766\text{ N}}$

5/172 Effective $\mu = 2.40(9.81) - 3.04 = 20.5$ N/m



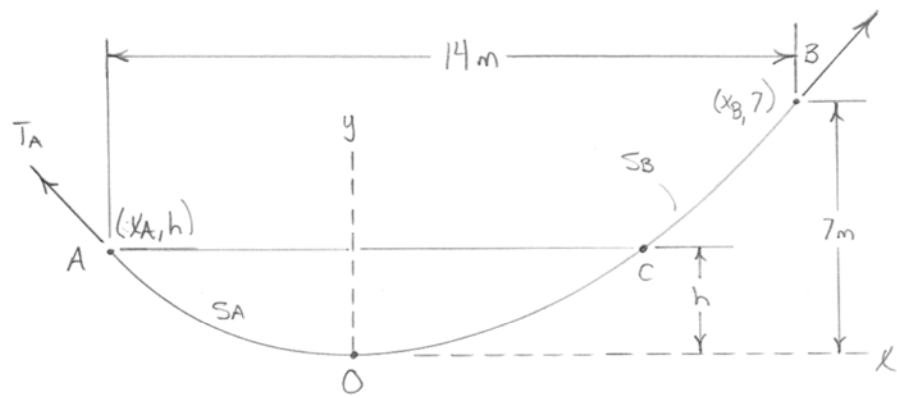
Eq. 5/19: $y = \frac{T_0}{\mu}\left(\cosh\frac{\mu x}{T_0} - 1\right)$: $30 = \frac{3.6(10^3)}{20.5}\left[\cosh\frac{20.5x}{3.6(10^3)} - 1\right]$

$x = 101.2$ m

Eq. 5/20: $s = \frac{T_0}{\mu}\sinh\frac{\mu x}{T_0} = \frac{3.6(10^3)}{20.5}\sinh\frac{20.5(101.2)}{3.6(10^3)}$

$s = 106.9$ m $\Rightarrow \ell = 120 - 106.9 = \underline{13.07 \text{ m}}$

*** 5/173** $s = 18$ m $\quad m_2 = 25$ kg $\quad g = 9.81$ m/s² $\quad y_B = 7$ m



Apply Eq. 5/20 from A to B, 5/21 at B, and either 5/19 or 5/22 at B.

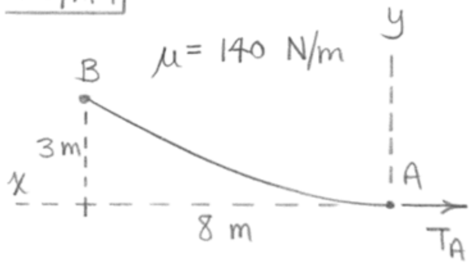
$$\begin{cases} s = \dfrac{T_0}{\mu}\left[\sinh\dfrac{\mu x_A}{T_0} + \sinh\dfrac{\mu x_B}{T_0}\right] \quad \text{and} \quad |x_A| + x_B = 14 \\ T_B = m_2 g = T_0 \cosh\dfrac{\mu x_B}{T_0} = T_0 + \mu y_B \end{cases}$$

Solving numerically... $\begin{cases} |x_A| = 5.63 \text{ m} \\ x_B = 8.37 \text{ m} \end{cases}$ $\begin{cases} T_0 = 112.1 \text{ N} \\ \underline{\mu = 19.02 \text{ N/m}} \end{cases}$

$$y_A = h = \frac{T_0}{\mu}\left[\cosh\frac{\mu x_A}{T_0} - 1\right] = \frac{112.1}{19.02}\left[\cosh\frac{19.02(-5.63)}{112.1} - 1\right] \longrightarrow \underline{h = 2.90 \text{ m}}$$

$$T_A = m_1 g = T_0 + \mu y_A \longrightarrow 9.81 m_1 = 112.1 + 19.02(2.90) \longrightarrow \underline{m_1 = 17.06 \text{ kg}}$$

*5/174



Eq. 5/19: $y = \frac{T_0}{\mu}\left[\cosh\frac{\mu x}{T_0} - 1\right]$

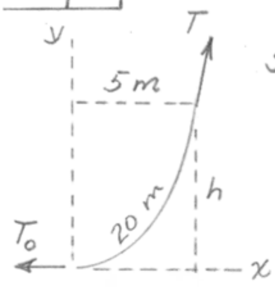
At B: $3 = \frac{T_A}{140}\left[\cosh\frac{140(8)}{T_A} - 1\right]$

Numerical solution: $\underline{T_A = 1559 \text{ N}}$

Eq. 5/20: $s = \frac{T_0}{\mu}\sinh\frac{\mu x}{T_0}$

At B: $L = \frac{1559}{140}\sinh\frac{140(8)}{1559} = \underline{8.71 \text{ m}}$

*5/175



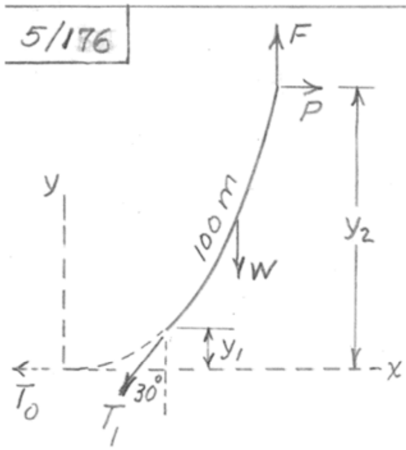
Eq. 5/20, $20 = \frac{T_0}{\mu} \sinh \frac{5\mu}{T_0}$

Solve by computer or graphically & get $T_0/\mu = 1.532$ m

Eq. 5/19, $y = 1.532\left(\cosh \frac{5}{1.532} - 1\right)$

$h = y = 1.532(13.09 - 1) = \underline{18.53 \text{ m}}$

5/176



$W = 0.51(9.81)(100) = 500.3$ N

$T_1 = M/r = 400/\frac{0.5}{2} = 1600$ N

$\Sigma F_x = 0;\ P - 1600\sin 30° = 0$

$P = 800$ N $(= T_0)$

$\Sigma F_y = 0;\ F - 500.3 - 1600\cos 30° = 0$

$F = 1886$ N

$T = \sqrt{F^2 + P^2} = \sqrt{(1886)^2 + (800)^2}$

$= 2049$ N

From Eq. 5/22, $T = T_0 + \mu y$, $2049 = 800 + 5.003\, y_2$

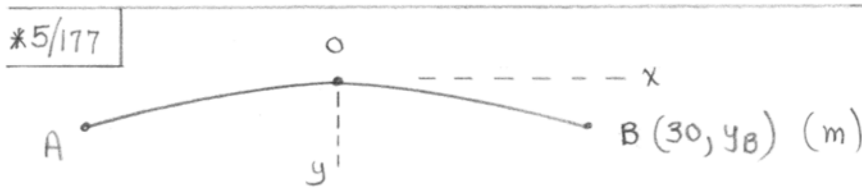
$y_2 = 249.6$ m

Also, $1600 = 800 + 5.003\, y_1$

$y_1 = 159.9$ m

$H = y_2 - y_1 = \underline{89.7 \text{ m}}$

*5/177



Eq. 5/20 : $s = \frac{T_0}{\mu} \sinh\left(\frac{\mu x}{T_0}\right)$: $\frac{60.5}{2} = \frac{T_0}{0.025} \sinh\left(\frac{0.025 \cdot 30}{T_0}\right)$

Numerical solution : $T_0 = 3.36$ N

Eq. 5/19 : $y = \frac{T_0}{\mu}\left[\cosh\left(\frac{\mu x}{T_0}\right) - 1\right]$

$y_B = \frac{3.36}{0.025}\left[\cosh\left(\frac{0.025 \cdot 30}{3.36}\right) - 1\right] = \underline{3.36 \text{ m} = h}$

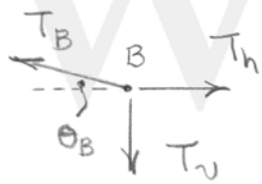
Eq. 5/22: $T = T_0 + \mu y$

$T_B = 3.36 + 0.025(3.36) = 3.44$ N

$\frac{dy}{dx} = \sinh\left(\frac{\mu x}{T_0}\right)$; $\left.\frac{dy}{dx}\right|_B = \sinh\left(\frac{0.025(30)}{3.36}\right)$

$\tan\theta_B = 0.225$

$\theta_B = 12.69°$



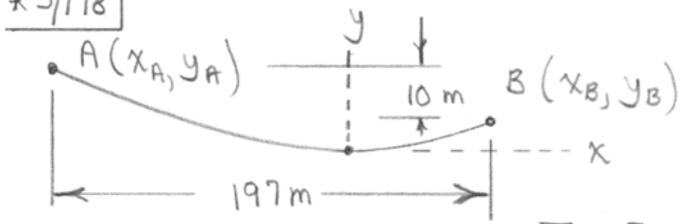
$T_h = T_B \cos\theta_B$

$= 3.44 \cos 12.69° = \underline{3.36 \text{ N}}$

$T_v = T_B \sin\theta_B$

$= 3.44 \sin 12.69° = \underline{0.756 \text{ N}}$

*5/178



$$\begin{cases} \mu = 0.025 \text{ N/m} \\ L = 200 \text{ m} \end{cases}$$

Eq. 5/19 @ A : $y_B + 10 = \dfrac{T_0}{\mu}\left[\cosh \dfrac{\mu x_A}{T_0} - 1\right]$ (1)

Eq. 5/19 @ B : $y_B = \dfrac{T_0}{\mu}\left[\cosh \dfrac{\mu x_B}{T_0} - 1\right]$ (2)

Eq. 5/20 @ A : $s_A = \dfrac{T_0}{\mu}\left[\sinh \dfrac{\mu x_A}{T_0}\right]$ (3)

Eq. 5/20 @ B : $s_B = \dfrac{T_0}{\mu}\left[\sinh \dfrac{\mu x_B}{T_0}\right]$ (4)

Also : $-s_A + s_B = L = 200$ (5)

$-x_A + x_B = 197$ (6)

Note that x_A & $s_A < 0$

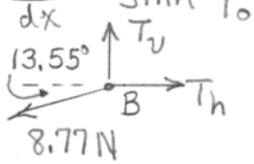
Numerically solve the six equations :

$T_0 = 8.53$ N, $x_A = -115.6$ m, $y_B = 9.77$ m
$x_B = 81.4$ m, $s_A = -117.8$ m, $s_B = 82.2$ m

Eq. 5/22 @ B : $T_B = 8.53 + 0.025(9.77) = 8.77$ N

$\dfrac{dy}{dx} = \sinh \dfrac{\mu x}{T_0} = \sinh \dfrac{0.025(81.4)}{8.53} = 0.241 = \tan \theta_B$

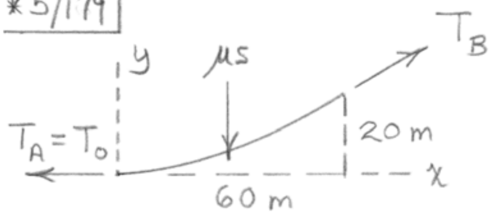
$\theta_B = 13.55°$



$T_h = 8.77 \cos 13.55° = \underline{8.53 \text{ N}}$

$T_v = 8.77 \sin 13.55° = \underline{2.06 \text{ N}}$

*5/179



$\mu = (10+20)9.81$
$= 294$ N/m

Eq. 5/19: $y = \frac{T_0}{\mu}\left(\cosh \frac{\mu x}{T_0} - 1\right)$

Numbers: $20 = \frac{T_A}{294}\left(\cosh \frac{294(60)}{T_A} - 1\right)$

Numerical solution: $T_A = 27\,400$ N

or $\underline{27.4 \text{ kN}}$

Eq. 5/22: $T = T_0 + \mu y$

$T_B = 27\,400 + 294(20) = 33\,300$ N

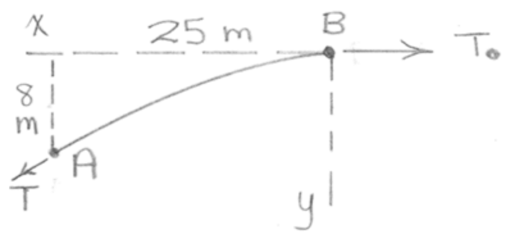
or $\underline{33.3 \text{ kN}}$

Eq. 5/20: $s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$

$= \frac{27\,400}{294} \sinh \frac{294(60)}{27\,400} = \underline{64.2 \text{ m}}$

*5/180

$\mu = 30 \text{ N/m}$



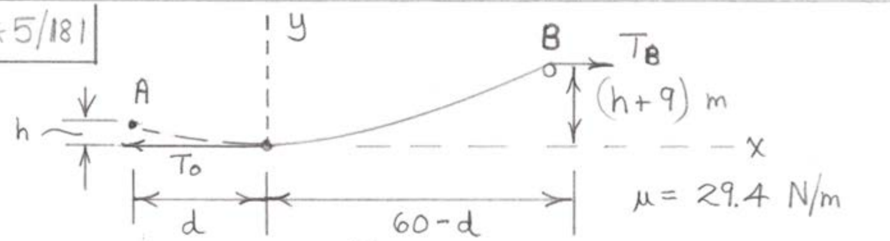
Eq. 5/19: $y = \frac{T_0}{\mu}\left[\cosh \frac{\mu x}{T_0} - 1\right]$

At A: $8 = \frac{T_0}{\mu}\left[\cosh \frac{25\mu}{T_0} - 1\right]$

Numerical solution : $\frac{T_0}{\mu} = 40.3 \text{ m}$

$T_0 = 40.3(30) = \underline{1210 \text{ N}}$

*5/181



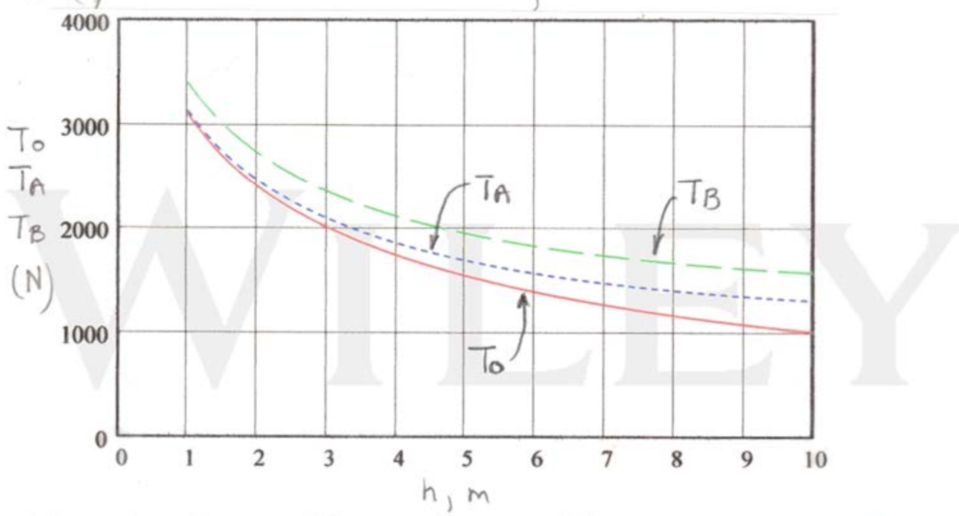
Eq. 5/19 @ B: $h+9 = \frac{T_o}{\mu}\left[\cosh\left(\frac{\mu}{T_o}(60-d)\right)-1\right]$ (1)

Eq. 5/19 @ A: $h = \frac{T_o}{\mu}\left[\cosh\left(\frac{\mu}{T_o}d\right)-1\right]$ (2)

Then Eq. 5/22: $T_A = T_o + \mu h$ (3)

$T_B = T_o + \mu(h+9)$ (4)

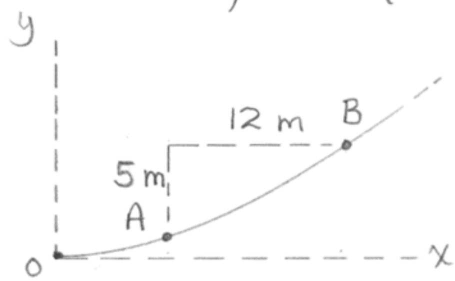
Numerically solve (1) and (2) (for T_o & d), then (3) & (4) over $1 \le h \le 10$ m, to obtain



When $h = 2$ m, $\underline{T_o = 2410 \text{ N}}$, $\underline{T_A = 2470 \text{ N}}$, $\underline{T_B = 2730 \text{ N}}$

*5/182

$\mu = 0.6(9.81) = 5.89$ N/m



Eq. 5/19 @ A: $y_A = \frac{T_0}{\mu}\left[\cosh\frac{\mu x_A}{T_0} - 1\right]$

Eq. 5/19 @ B: $y_A + 5 = \frac{T_0}{\mu}\left[\cosh\frac{\mu(x_A+12)}{T_0} - 1\right]$

Eq. 5/22 @ A: $200 = T_0 + \mu y_A$

Numerical solution of above three equations:

$x_A = 7.37$ m, $y_A = 0.823$ m, $T_0 = 195.2$ N

From $\frac{dy}{dx} = \sinh\frac{\mu x}{T_0}$, we have, at A

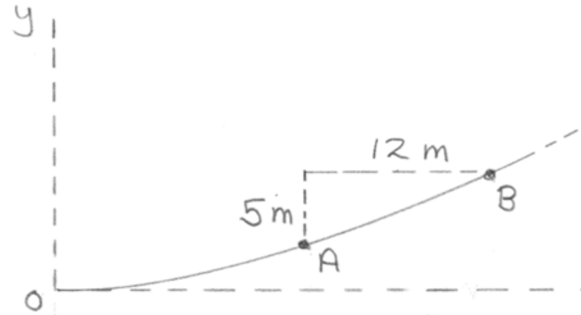
$\theta_A = \tan^{-1}\left[\sinh\frac{(5.89)(7.37)}{195.2}\right] = \underline{12.64°}$

From 5/20, $s_B - s_A = \frac{195.2}{5.89}\left[\sinh\frac{5.89(7.37+12)}{195.2} - \sinh\frac{5.89(7.37)}{195.2}\right] = \underline{13.06 \text{ m} = L}$

5/22: $T_B = 195.2 + 5.89(0.823 + 5) = \underline{229 \text{ N}}$

*5/183

$\mu = 0.6(9.81) = 5.89$ N/m



Eq. 5/19 @ A : $y_A = \frac{T_o}{\mu}\left[\cosh \frac{\mu x_A}{T_o} - 1\right]$

Eq. 5/19 @ B : $y_A + 5 = \frac{T_o}{\mu}\left[\cosh \frac{\mu(x_A+12)}{T_o} - 1\right]$

From Eq. 5/20

$s_B - s_A = 13.02 = \frac{T_o}{\mu}\left[\sinh \frac{\mu(x_A+12)}{T_o} - \sinh \frac{\mu x_A}{T_o}\right]$

Numerical solution of above three equations:

$x_A = 17.34$ m, $y_A = 2.63$ m, $T_o = 339$ N

5/22 : $T_A = 339 + 5.89(2.63) = \underline{355 \text{ N}}$

$\theta_A = \tan^{-1}\left[\sinh \frac{\mu x_A}{T_o}\right] = \tan^{-1}\left[\sinh \frac{5.89(17.34)}{339}\right]$

$= \underline{16.98°}$

Similarly, $T_B = 339 + 5.89(2.63+5) = \underline{384 \text{ N}}$

$\theta_B = \tan^{-1}\left[\sinh \frac{5.89(17.34+12)}{339}\right] = \underline{28.0°}$

5/184 From Eq. 5/19 with $x = 100$ m, $y = 32$ m

$$32 = \left(\frac{T_0}{\mu}\right)\left[\cosh\left(\frac{100\mu}{T_0}\right) - 1\right]$$

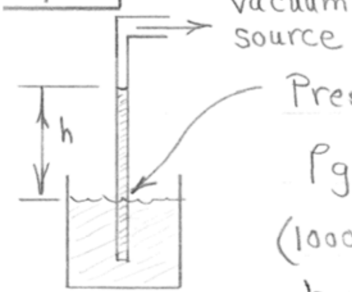
Solve by computer or graphically & get $\frac{T_0}{\mu} = 161.3$ m

From Eq. 5/22, $60(10^3) = T_0 + 32\mu$

Solve simultaneously & get $\mu = 310$ N/m

Thus $\rho_{ice} = \frac{\mu}{g} - \rho_{cable} = \frac{310}{9.81} - 18.2 = \underline{13.44 \text{ kg/m of ice}}$

5/185



(Assume vacuum pump reduces pressure to zero.)

Pressure here must be atmospheric:

$$\rho g h = p_{at}$$

$$(1000)(9.81)(h) = 1.0133(10^5)$$

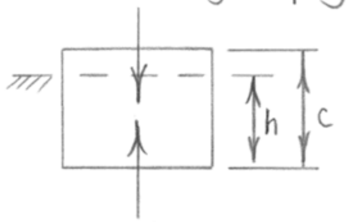
$$\underline{h = 10.33 \text{ m}}$$

For mercury: $13\,570(9.81)(h) = 1.0133(10^5)$

$$\underline{h = 0.761 \text{ m}} \quad (29.97 \text{ in.})$$

5/186

$W = mg = \rho_1 V g = \rho_1 abcg$



$B = \rho_2 V_{sub} g = \rho_2 abhg$

$+\uparrow \Sigma F = 0: \ \rho_2 abhg - \rho_1 abcg = 0, \quad h = \frac{\rho_1}{\rho_2} c$

$r = \frac{h}{c} = \underline{\frac{\rho_1}{\rho_2}}$

Oak in water: $r = \frac{800}{1000} = \underline{0.8}$

Steel in mercury: $r = \frac{7830}{13\,570} = \underline{0.577}$

5/187 Force on bottom = weight of water

$= \rho g V = \left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.3\text{ m})(0.7\text{ m})(0.4\text{ m})$

$= \underline{824\text{ N}}$ (down, at center of bottom)

Force on front & back $= p_{av} A_f = \frac{\rho g h}{2} A_f$

$= \frac{1000(9.81)(0.4)}{2}(0.7)(0.4) = \underline{549\text{ N}}$ (outward, at $\frac{2}{3}$ depth)

Force on each end glass $= p_{av} A_e = \frac{\rho g h}{2} A_e$

$= \frac{1000(9.81)(0.4)}{2}(0.3)(0.4) = \underline{235\text{ N}}$ (outward, at $\frac{2}{3}$ depth)

(All side forces centered horizontally)

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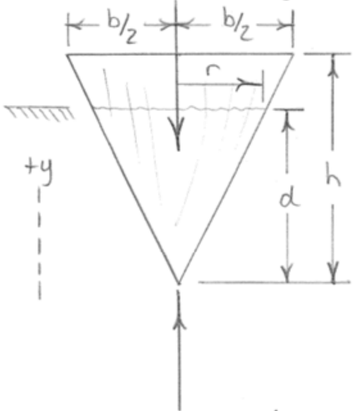
$$W = mg = \rho_1 V g = \rho_1 \left(\frac{1}{3}\pi \frac{b^2}{4} h\right) g$$



$$\begin{cases} \rho_1 = 800 \text{ kg/m}^3 \\ \rho_2 = 1030 \text{ kg/m}^3 \end{cases}$$

$$r = \frac{b}{2h} y = \frac{bd}{2h}$$

$$F_B = \rho_2 V_{sub} g = \rho_2 \left(\frac{1}{3}\pi r^2 d\right) g = \rho_2 \left(\frac{1}{3}\pi \frac{b^2 d^2}{4h^2} d\right) g$$

$\Sigma F_y = 0$: $\quad W - F_B = 0$

$$\frac{1}{12}\pi \rho_1 b^2 h g - \frac{1}{12}\pi \rho_2 \frac{b^2 d^3}{h^2} g = 0$$

SOLVING... $\quad d = \left(\frac{\rho_1}{\rho_2}\right)^{1/3} h$

SUBSTITUTING... $\quad d = \left(\frac{800}{1030}\right)^{1/3} h \longrightarrow \underline{d = 0.919\, h}$

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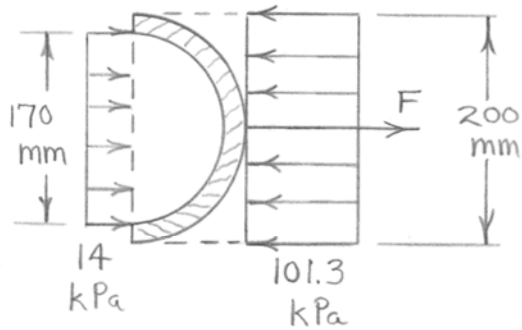
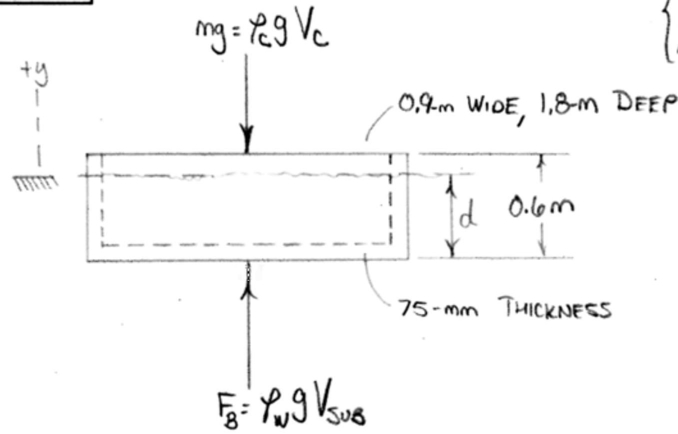


$$\xrightarrow{+} \Sigma F = 0: \quad F + 14(10^3)\frac{\pi(0.170)^2}{4} - 101.3(10^3)\frac{\pi(0.200)^2}{4} = 0$$

$F = 2860$ N or $\underline{F = 2.86 \text{ kN}}$

5/190

$$\begin{cases} \rho_c = 2400 \text{ kg/m}^3 \\ \rho_w = 1000 \text{ kg/m}^3 \end{cases}$$



$$\begin{cases} V_c = 0.6(0.9)(1.8) - \left(0.6 - \frac{75}{1000}\right)\left(0.9 - \frac{150}{1000}\right)\left(1.8 - \frac{150}{1000}\right) = 0.322 \text{ m}^3 \\ V_{SUB} = 0.9(1.8)\,d \end{cases}$$

$\Sigma F_y = 0$: $F_B - mg = 0 \longrightarrow 1000(9.81)(0.9)(1.8)\,d - 2400(9.81)(0.322) = 0$

$\underline{d = 0.478 \text{ m} \quad \text{OR} \quad 478 \text{ mm}}$

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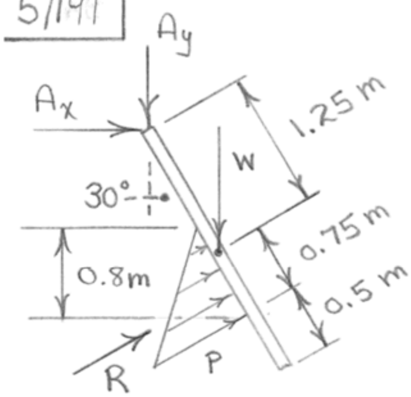


$$p = \rho g h = 1000(9.81)(0.8) = 7850 \text{ Pa}$$

$$R = \tfrac{1}{2}(7850)\left(\frac{0.8}{\cos 30°}\right) = 3620 \text{ N/m}$$

$$\circlearrowleft+ \ \Sigma M_A = 0: \quad W(1.25 \sin 30°) - 3620\left(2.5 - 0.5 - \tfrac{1}{3}\frac{0.8}{\cos 30°}\right) = 0; \quad \underline{W = 9810 \text{ N/m}}$$

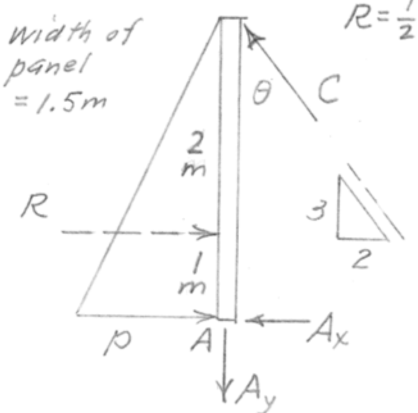
5/192

$p = \rho g h = 2400(9.81)(3) = 70.6 \text{ kPa}$

$R = \frac{1}{2} p A = \frac{1}{2}(70.6)(10^3)(3)(1.5)$

$= 158.9(10^3) \text{ N}$

width of panel = 1.5m



$\Sigma M_A = 0;$

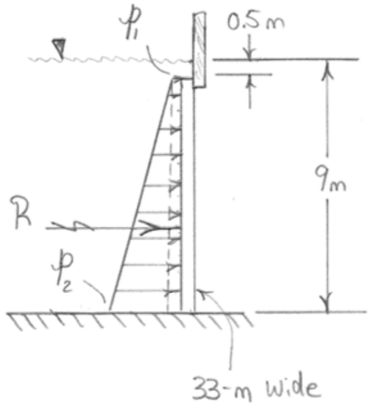
$(C \sin 33.7°)3 - 158.9(10^3)(1) = 0$

$C = \frac{158.9(10^3)}{3 \sin 33.7°} = 95.5(10^3) \text{ N}$

or $C = 95.5 \text{ kN}$

$\theta = \tan^{-1} 2/3 = 33.7°$

5/193 $\rho = 1030 \text{ kg/m}^3$



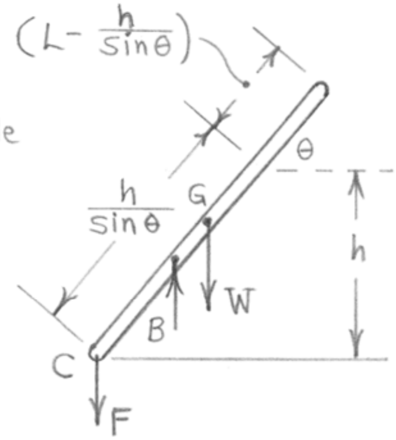
$p_1 = \rho g h_1 = 1030(9.81)(0.5) = 5050 \text{ Pa}$

$p_2 = \rho g h_2 = 1030(9.81)(9) = 90\,900 \text{ Pa}$

$R = \frac{1}{2}(p_1 + p_2) A = \frac{1}{2}(5050 + 90\,900)(8.5)(33)$

$\underline{R = 13.46 \text{ MN}}$

5/194



Let A = cross-sectional area of the pole

Buoyancy force B is

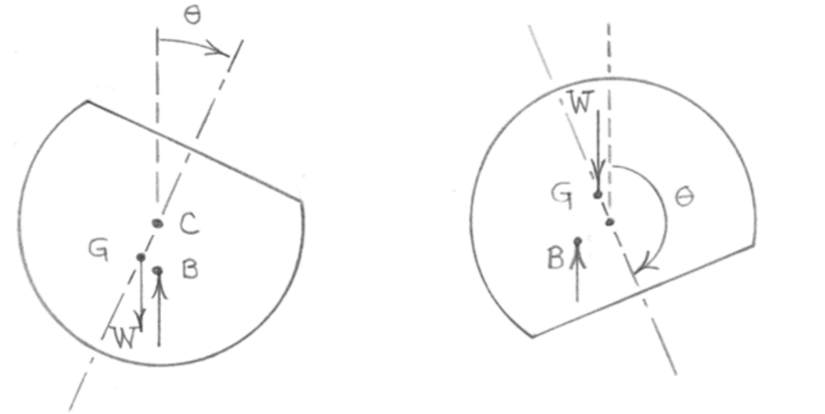
$$B = \rho g \bar{V} = \rho g \frac{hA}{\sin\theta}$$

Weight $W = \rho' g L A$

$$\circlearrowleft + \Sigma M_C = 0: \rho g \frac{hA}{\sin\theta}\left(\frac{1}{2}\frac{h}{\sin\theta}\cos\theta\right) - \rho' g L A\left(\frac{L}{2}\cos\theta\right) = 0$$

$$\underline{\theta = \sin^{-1}\left(\frac{h}{L}\sqrt{\frac{\rho}{\rho'}}\right)} \qquad \left(\frac{h^2\rho}{L^2\rho'} \leq 1\right)$$

5/195



CCW couple tends to make $\theta = 0$

CW couple tends to make $\theta = 180°$

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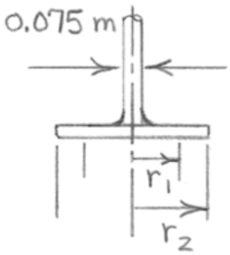


$\begin{cases} r_1 = 0.2 \text{ m} \\ r_2 = 0.3 \text{ m} \end{cases}$

$p = \rho g h = 1030(9.81)(0.6) = 6060$ Pa

$R = pA = 6060\,\pi\left(0.3^2 - \frac{0.075^2}{4}\right) = 1687$ N

Pressure supported by seal

$\sigma = R/\pi(r_2^2 - r_1^2) = \frac{1687}{\pi(0.3^2 - 0.2^2)}$

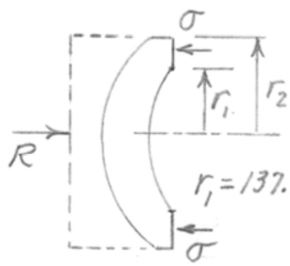
$= 10\,740$ Pa or $\underline{10.74 \text{ kPa}}$

Force to lift plunger

$P = R = 1687$ N or $\underline{1.687 \text{ kN}}$

5/197

$p = \rho g h = 1030(9.81)(1000) = 10.104(10^6)$ Pa



$r_1 = 137.5$ mm

$r_2 = 175$ mm

$R = pA = 10.104(10^6)\,\pi\,(0.175)^2$

$= 972(10^3)$ N

$\sigma = R/A_0$

$= \dfrac{972(10^3)}{\pi([0.175]^2 - [0.1375]^2)}$

$= 26.4(10^6)$ Pa

or $\underline{\sigma = 26.4 \text{ MPa}}$

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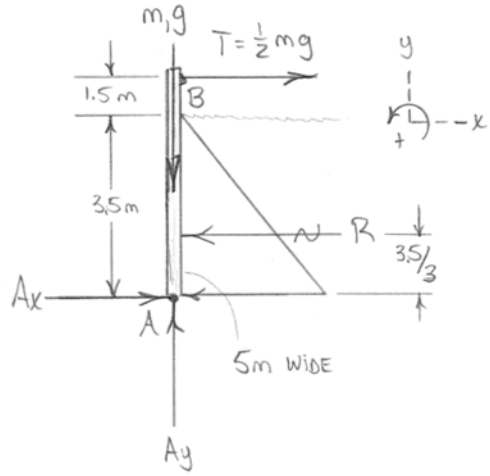
$$\begin{cases} m_1 = 2500 \text{ kg} \\ \rho = 1000 \text{ kg/m}^3 \end{cases}$$



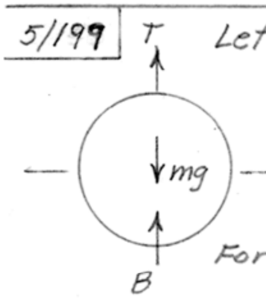
$$\begin{cases} R = \rho g \bar{h} A = 1000(9.81)\left(\frac{3.5}{2}\right)(3.5)(5) \\ R = 300 \text{ kN} \end{cases}$$

$$\begin{cases} \Sigma F_x = 0: \quad A_x + T - R = 0 \\ \Sigma F_y = 0: \quad A_y - m_1 g = 0 \\ \Sigma M_A = 0: \quad -5T + \frac{3.5}{3} R = 0 \end{cases}$$

$$\begin{cases} A_x = 230 \text{ kN} \\ A_y = 24.5 \text{ kN} \end{cases} \qquad \begin{cases} T = 70.1 \text{ kN} \\ \underline{m = 14\,290 \text{ kg}} \end{cases}$$

$$R_A = \sqrt{A_x^2 + A_y^2} \longrightarrow \underline{R_A = 232 \text{ kN}}$$

5/199



Let ρ_c = density of concrete = 2400 kg/m^3

ρ_w = density of fresh water = 1000 kg/m^3

L = length of cylinder = 2.4 m

r = radius of cylinder = 0.8 m

For equil. $T = mg - B$

$$= \rho_c g \pi r^2 L - \rho_w g \frac{\pi r^2 L}{2}$$

$$= \pi r^2 L g \left(\rho_c - \frac{1}{2}\rho_w\right)$$

$$= \pi(0.8^2)(2.4)(9.81)\left(2400 - \frac{1000}{2}\right)$$

$$= 89.9(10^3)\text{ N}$$

or $\underline{T = 89.9\text{ kN}}$

5/200

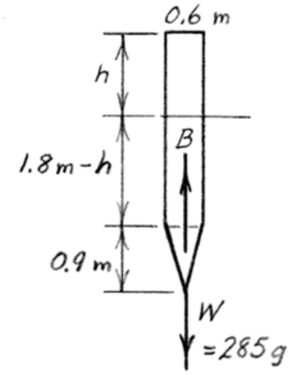
$$B = \rho g V = 1030g\left[\pi \frac{0.6^2}{4}(1.8-h) + \frac{1}{3}\pi\frac{0.6^2}{4}(0.9)\right]$$

$$= 1030g\,\pi\frac{0.6^2}{4}(1.8-h+0.3)$$

$$= 92.7g\pi(2.1-h)$$

$W = B$: $\quad 285g = 92.7g\pi(2.1-h)$

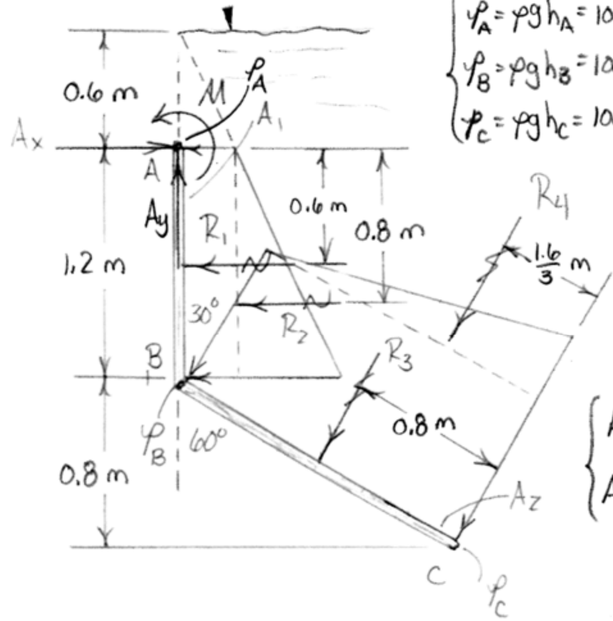
$\underline{h = 1.121 \text{ m}}$



5/201 GATE IS 2-m WIDE

$\rho = 1000\ \text{kg/m}^3$

$$\begin{cases} p_A = \rho g h_A = 1000(9.81)(0.6) = 5890\ \text{Pa} \\ p_B = \rho g h_B = 1000(9.81)(1.8) = 17\,660\ \text{Pa} \\ p_C = \rho g h_C = 1000(9.81)(2.6) = 25\,500\ \text{Pa} \end{cases}$$



$$\begin{cases} A_1 = 1.2(2) = 2.4\ \text{m}^2 \\ A_2 = 1.6(2) = 3.2\ \text{m}^2 \end{cases}$$

$$\begin{cases} R_1 = p_A A_1 = 5890(2.4) = 14\,130\ \text{N} \\ R_2 = \frac{1}{2}(p_B - p_A)A_1 = \frac{1}{2}(17\,660 - 5890)(2.4) = 14\,130\ \text{N} \\ R_3 = p_B A_2 = 17\,660(3.2) = 56\,500\ \text{N} \\ R_4 = \frac{1}{2}(p_C - p_B)A_2 = \frac{1}{2}(25\,500 - 17\,660)(3.2) = 12\,560\ \text{N} \end{cases}$$

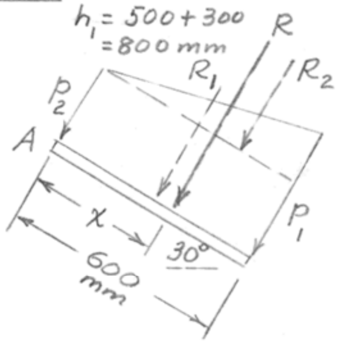
$$\circlearrowleft + \ \Sigma M_A = 0:\ M_A - 0.6R_1 - 0.8R_2 - (0.8 + 1.2\sin 30^\circ)R_3 - \left[\tfrac{2}{3}(1.6) + 1.2\sin 30^\circ\right]R_4 = 0$$

$$\underline{M_A = 119.8\ \text{kN}\cdot\text{m CCW}}$$

5/202

$h_2 = 500$ mm

$h_1 = 500 + 300 = 800$ mm



$p_2 = \rho g h_2 = 900(9.81)(0.500) = 4.415(10^3)$ Pa

$p_1 = \rho g h_1 = 900(9.81)(0.800) = 7.063(10^3)$ Pa

$R_1 = p_2 A = 4.415(10^3)(0.600)(0.400) = 1059$ N

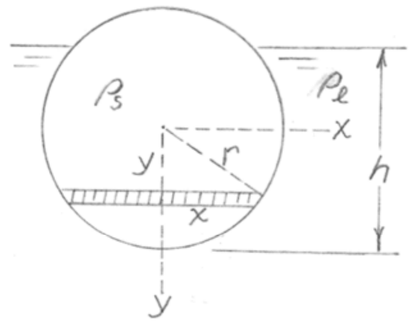
$R_2 = \frac{p_1 - p_2}{2} A = \frac{(7.063 - 4.415)10^3(0.600)(0.400)}{2} = 318$ N

$R = R_1 + R_2 = 1059 + 318 = \underline{1377 \text{ N}}$

$Rx = \Sigma M_A$; $1377x = 1060(300) + 318(400)$

$x = \frac{445\,000}{1377} = \underline{323 \text{ mm}}$

5/203 Buoyancy $B = \rho_\ell g V$, V = submerged volume



$$W = mg = \rho_s \frac{4}{3}\pi r^3 g$$

$$dV = \pi x^2 dy = \pi(r^2 - y^2)\,dy$$

$$B = \rho_\ell g \pi \int_{-(h-r)}^{r} (r^2 - y^2)\,dy$$

$$= \rho_\ell g \pi \left[r^2 y - \frac{y^3}{3} \right]_{-(h-r)}^{r}$$

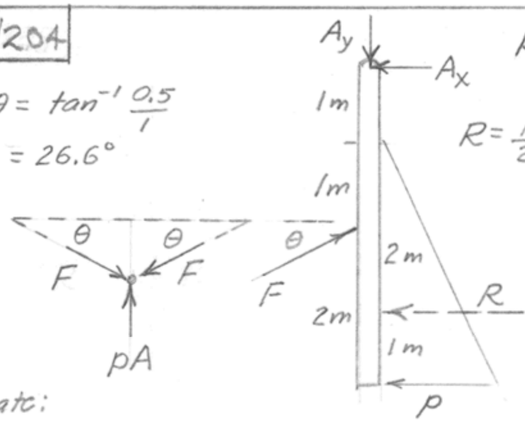
$$= \frac{1}{3}\rho_\ell g \pi h^2 (3r - h)$$

Thus with $B = W$, $\rho_s g \frac{4}{3}\pi r^3 = \frac{1}{3}\rho_\ell g \pi h^2 (3r - h)$

Rearrange & get $\underline{\rho_s = \rho_\ell \left(\frac{h}{2r}\right)^2 \left(3 - \frac{h}{r}\right)}$

5/204

$\theta = \tan^{-1}\frac{0.5}{1}$
$= 26.6°$



$p = \rho g h = 1.0(9.81)(3)$
$= 29.4$ kPa

$R = \frac{p}{2}\,Area = \frac{29.4}{2}(3)(2)$
$= 88.3$ kN

Gate:

$\Sigma M_A = 0;\ (F\cos 26.6°)2 - 88.3(3) = 0,\ F = 148.1$ kN

Toggle:

$\Sigma F = 0;\ pA - 2F\sin\theta = 0;\ \frac{\pi(0.150)^2}{4}p = 2(148.1)(10^3)\sin 26.6°$

$p = 7.49(10^6)$ Pa or $\underline{p = 7.49\text{ MPa}}$

5/205 Submerged volume V is

$$V = 2(105)(12)(7.5) + 6\pi \frac{9^2}{4}(h-7.5)$$

$$= 18\,900 + 381.7(h-7.5)$$

$$B = \rho g V = 1030(9.81)[18\,900 + 381.7(h-7.5)]$$

$$= 190.97(10^6) + 3.857(10^6)(h-7.5) \text{ N}$$

Weight of structure $= W = 26{,}000(9.81)10^3$

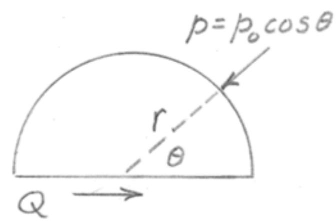
$$= 255.1(10^6) \text{ N}$$

$B = W$, so $190.97(10^6) + 3.857(10^6)(h-7.5) = 255.1(10^6)$

$$h - 7.5 = \frac{255.1 - 190.97}{3.857} = 16.62, \quad \underline{h = 24.1 \text{ m}}$$

5/206

$$Q = \int_0^{\pi} (p_0 \cos\theta)(\cos\theta)\, r\, d\theta$$



$$= p_0 r \int_0^{\pi} \cos^2\theta\, d\theta$$

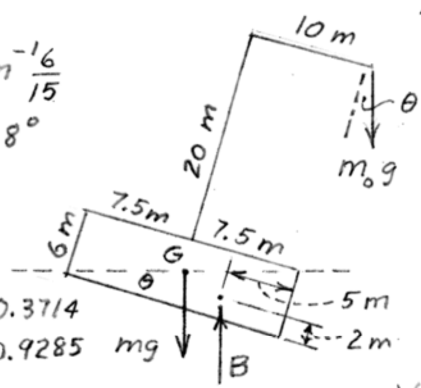
$$= p_0 r \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\pi}$$

$$= p_0 r \left[\frac{\pi}{2} \right]$$

$$\underline{Q = \frac{1}{2} \pi r p_0}$$

5/207

$\theta = \tan^{-1}\frac{6}{15} = 21.8^\circ$



$\sin\theta = 0.3714$

$\cos\theta = 0.9285$

Moment arm of B about G is

$(7.5-5)0.9285 - (3-2)0.3714 = 1.950$ m

Moment arm of $m_o g$ about G is

$(20+3)0.3714 + 10(0.9285) = 17.83$ m

$B = \rho g V = 1.030(9.81)\frac{1}{2}6(15)(40) = 18\,190$ kN

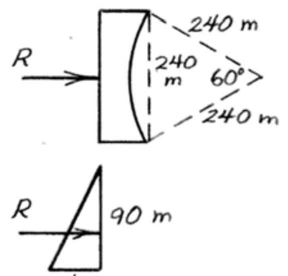
$\circlearrowleft + \Sigma M_G = 0$: $m_o(9.81)(17.83) - 18\,190(1.950) = 0$

$\underline{m_o = 203 \text{ Mg}}$

$+\uparrow \Sigma F = 0$: $18\,190 - 203(9.81) - m(9.81) = 0$

$\underline{m = 1651 \text{ Mg}}$

5/208



$p_{90\text{ m}} = \rho g h$
$= 1000(9.81)(90)$
$= 883(10^3)$ Pa

$R = p_{av}(\text{Area}) = \frac{883(10^3)}{2}(240)(90)$
$= 9.54(10^9)$ N

or $\underline{R = 9.54 \text{ GN}}$

5/209 The gage pressure 12 m below the surface is $p = \rho g h = (1000)(9.81)(12) = 117\ 700\ \text{N/m}^2$.

(a) Cover area $A_{cov} = \pi\left(\frac{0.75}{2}\right)\left(\frac{0.5}{2}\right) = 0.295\ \text{m}^2$

Force on cover $= pA_{cov} = 34\ 700\ \text{N}$

Seal area $A_s = A_{cov} - \pi\left(\frac{0.55}{2}\right)\left(\frac{0.375}{2}\right) = 0.1325\ \text{m}^2$

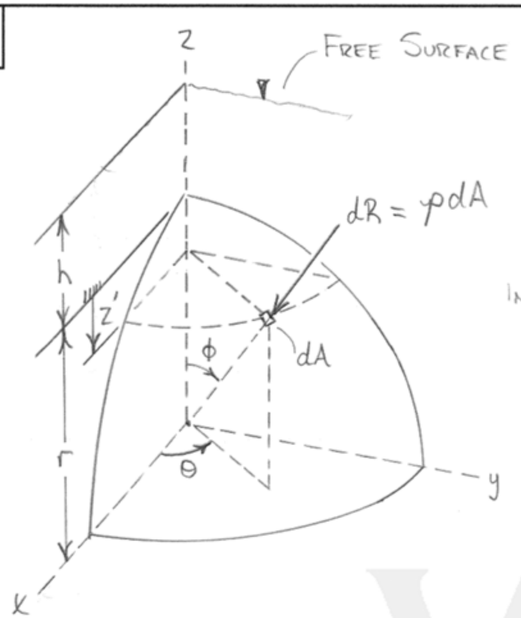
$\sigma A_s = pA_{cov}$, $\sigma = \frac{34\ 700}{0.1325} = 262\ 000\ \frac{\text{N}}{\text{m}^2}$

or $\underline{\sigma = 262\ \text{kPa}}$

(b) $16\Delta T = pA_{hole} = 117\ 700\left[\pi\left(\frac{0.55}{2}\right)\left(\frac{0.375}{2}\right)\right]$

$\underline{\Delta T = 1192\ \text{N}}$

▶5/210



$dR = p\,dA$ (DIFFERENTIAL FORCE DUE TO PRESSURE ON THE SHELL)

IN SPHERICAL COORDINATES...

$dA = r^2 \sin\phi \, d\phi \, d\theta$

$p = \rho g(h + z') = \rho g(h + r - r\cos\phi)$

$$\begin{cases} dR_x = -dR \sin\phi \cos\theta \\ dR_y = -dR \sin\phi \sin\theta \\ dR_z = -dR \cos\phi \end{cases}$$

(SHELL REACTIONS F_x, F_y, AND F_z MUST OPPOSE THESE THREE PRESSURE RESULTANTS.)

$$R_x = \int dR_x = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} -\rho g(h + r - r\cos\phi)\, r^2 \sin^2\phi \cos\theta \, d\phi \, d\theta$$

NOTE: $\int_0^{\pi/2} \sin\theta \, d\theta = \int_0^{\frac{\pi}{2}} \cos\theta \, d\theta = 1$

$$R_x = -\rho g r^2(h+r)\int_0^{\frac{\pi}{2}} \sin^2\phi \, d\phi + \rho g r^3 \int_0^{\frac{\pi}{2}} \cos\phi \sin^2\phi \, d\phi$$

$$= -\rho g r^2(h+r)\left[\frac{\phi}{2} - \frac{1}{4}\sin 2\phi\right]\Bigg|_0^{\frac{\pi}{2}} + \frac{1}{3}\rho g r^3 \sin^3\phi \Bigg|_0^{\frac{\pi}{2}}$$ AND SIMPLIFY.

$$R_x = \frac{-\rho g r^2}{12}\left[3\pi h + (3\pi - 4)r\right]$$ (SHELL REACTION IS EQUAL AND OPPOSITE.)

$$\underline{F_x = F_y = \frac{\rho g r^2}{12}\left[3\pi h + (3\pi - 4)r\right]}$$ (SYMMETRY)

NOTE: $\int_0^{\pi/2} d\theta = \frac{\pi}{2}$

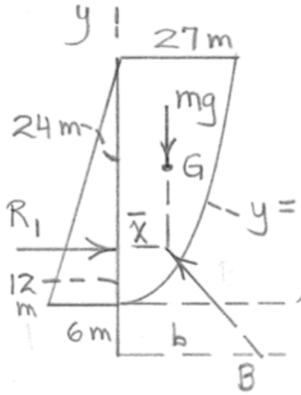
$$R_z = \int dR_z = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} -\rho g(h + r - r\cos\phi)\, r^2 \sin\phi \cos\phi \, d\phi \, d\theta$$

$$= \frac{-\rho g \pi r^2}{2}(h+r)\int_0^{\pi/2} \sin\phi\cos\phi \, d\phi + \frac{\rho g \pi r^3}{2}\int_0^{\frac{\pi}{2}} \sin\phi \cos^2\phi \, d\phi$$

$$= \frac{\rho g \pi r^2}{4}(h+r)\cos^2\phi \Bigg|_0^{\pi/2} - \frac{\rho g \pi r^3}{6}\cos^3\phi \Bigg|_0^{\frac{\pi}{2}}$$

$$R_z = \frac{-\rho g \pi r^2}{12}(3h+r)$$ SO... $$\underline{F_z = \frac{\rho g \pi r^2}{12}(3h+r)}$$

5/211 Take a vertical section of water of unit horizontal length. Let ρ be the water density in t/m^3.



$y = kx^2$: $36 = k(27)^2$, $k = \frac{4}{81}\ m^{-1}$

$\bar{x} = \frac{\int x\,dA}{\int dA}$, $dA = x\,dy = 2\frac{4}{81}x^2dx$

$\therefore\ \bar{x} = \frac{\int_0^{27} \frac{x}{2}\frac{8}{81}x^2dx}{\int_0^{27}\frac{8}{81}x^2dx} = 10.12\ m$

$A = \int dA = 648\ m^2$, $mg = 648\rho g$

$R_1 = \frac{1}{2}\,36\rho g\,(36)(1) = 648\rho g$

Resultant of mg & R_1 passes through B, so $\Sigma M_B = 0$. Thus $648\rho g(18) = 648\rho g\,(b - 10.12)$

$\underline{b = 28.1\ m}$

5/212 The pressure at the bottom of the 3-m wall is $p = \rho g h = 2400(9.81)(3) = 70\,600 \text{ N/m}^2$

Each tie controls an area A given by

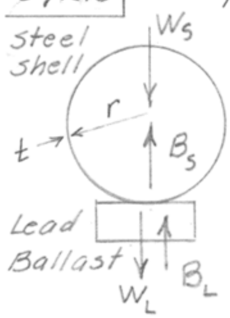
$$pA = T, \quad A = \frac{T}{p} = \frac{6\,500}{70\,600} = 0.0920 \text{ m}^2$$

This square area has a side d given by

$$d^2 = A, \quad d = 0.303 \text{ m}$$

Using the pressure at the very bottom of the wall gives us a conservative design; a good figure for d would be $\underline{d = 0.300 \text{ m}}$.

5/213



For equilibrium $W_S + W_L = B_S + B_L$

ρ_S = density of steel = 7.83 Mg/m^3

ρ_W = " " salt water = 1.03 Mg/m^3

ρ_L = " " lead = 11.37 Mg/m^3

$r = 1.00$ m, $t = 0.035$ m

V_L = volume of lead, m^3

m = mass of lead = $\rho_L V_L$

so $\rho_S g \, 4\pi r^2 t + \rho_L g V_L = \rho_W g \frac{4}{3}\pi (r + \frac{t}{2})^3 + \rho_W g V_L$

$$V_L g (\rho_L - \rho_W) = 4\pi r^2 g \left[\frac{r}{3}(1 + \frac{t}{2r})^3 \rho_W - \rho_S t\right]$$

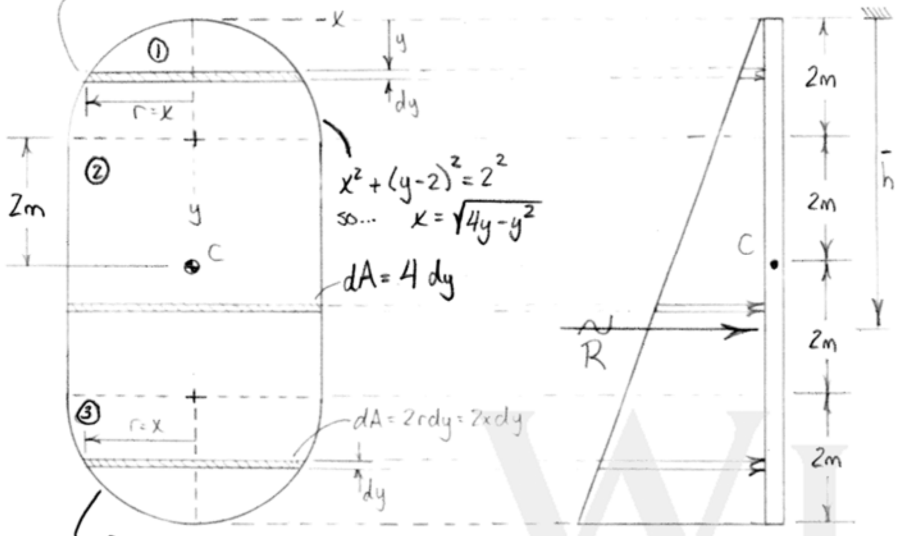
$$m\left(1 - \frac{\rho_W}{\rho_L}\right) = 4\pi r^2 \left[\frac{r}{3}(1 + \frac{t}{2r})^3 \rho_W - \rho_S t\right]$$

$$m\left(1 - \frac{1.03}{11.37}\right) = 4\pi (1)^2 \left[\frac{1}{3}\left(1 + \frac{0.035}{2}\right)^3 1.03 - 7.83(0.035)\right]$$

$0.9094m = 1.1008$, $m = \underline{1.210 \text{ Mg}}$ (metric tons)

►5/214 $\rho = 1000\ \text{kg/m}^3$

$dA = 2r\,dy = 2x\,dy = 2\sqrt{4y-y^2}\,dy$ AND $dR = p\,dA = \rho g y\,dA$



$x^2+(y-6)^2=2^2$ so… $x=\sqrt{12y-y^2-32}$

$A = \pi(2)^2 + 4^2 = 28.6\ \text{m}^2$

$R = \rho g h_c A = 1000(9.81)(4)(28.6) \longrightarrow \underline{R = 1121\ \text{kN}}$

$R_1 = \rho g \bar{y}_1 A_1 = 1000(9.81)\left[2 - \frac{4(2)}{3\pi}\right]\left(\frac{1}{2}\right)(\pi)(2)^2 \longrightarrow R_1 = 71.0\ \text{kN}$

$R_2 = \rho g \bar{y}_2 A_2 = 1000(9.81)(4)(4)^2 \longrightarrow R_2 = 628\ \text{kN}$

$R_3 = \rho g \bar{y}_3 A_3 = 1000(9.81)\left[6 + \frac{4(2)}{3\pi}\right]\left(\frac{1}{2}\right)(\pi)(2)^2 \longrightarrow R_3 = 422\ \text{kN}$

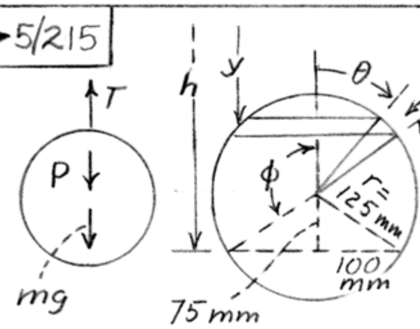
$h_1 = \frac{1}{R_1}\int y\,dR = \frac{1}{71.0(10^3)}\int_0^2 2\rho g y^2\sqrt{4y-y^2}\,dy \longrightarrow h_1 = 1.394\ \text{m}$

$h_2 = \frac{1}{R_2}\int y\,dR = \frac{1}{628(10^3)}\int_2^6 4\rho g y^2\,dy \longrightarrow h_2 = 4.33\ \text{m}$

$h_3 = \frac{1}{R_3}\int y\,dR = \frac{1}{422(10^3)}\int_6^8 2\rho g y^2\sqrt{12y-y^2-32}\,dy \longrightarrow h_3 = 6.89\ \text{m}$

$$\bar{h} = \frac{\Sigma Rh}{\Sigma R} = \frac{71.0(1.394) + 628(4.33) + 422(6.89)}{1121} \longrightarrow \underline{\bar{h} = 5.11\ \text{m}}$$

►5/215



$\Sigma F = 0: \ T - P - mg = 0$

Pressure force

$P = \int p \, dA \cos\theta$

$p = \rho g y = \rho g (h' - r\cos\theta)$

where $h' = h - 0.075$ m

$dA = 2\pi r \sin\theta \, (r\,d\theta)$

$\phi = 180° - \cos^{-1}\frac{75}{125} = 126.9°$

$$P = \int_0^{\phi} \rho g (h' - r\cos\theta) 2\pi r^2 \sin\theta \, d\theta \cos\theta$$

$$= 2\pi r^2 \rho g \int_0^{\phi} (h'\sin\theta\cos\theta - r\sin\theta\cos^2\theta)\, d\theta$$

$$= 2\pi r^2 \rho g \int_0^{\phi} \left(\frac{h'}{2}\sin 2\theta - r\sin\theta + r\sin^3\theta\right) d\theta$$

$$= 2\pi r^2 \rho g \left[-\frac{h'}{4}\cos 2\theta + r\cos\theta - \frac{r}{3}\cos\theta(2 + \sin^2\theta)\right]_0^{\phi}$$

$$= 2\pi r^2 \rho g \left[\frac{h'}{4}(1 - \cos 2\phi) + \frac{r}{3}(\cos\phi - 1 - \cos\phi\sin^2\phi)\right]$$

Substitute $\phi = 126.9°$, $h' = h - 0.075$, $r = 0.125$ m & get

$$P = 2\pi r^2 \rho g \left[(h - 0.075)0.320 + \frac{0.125}{3}(-1.216)\right]$$

For $T = mg$, $P = 0$ so $0.320h = 0.0747$

$\underline{h = 0.233 \text{ m}}$

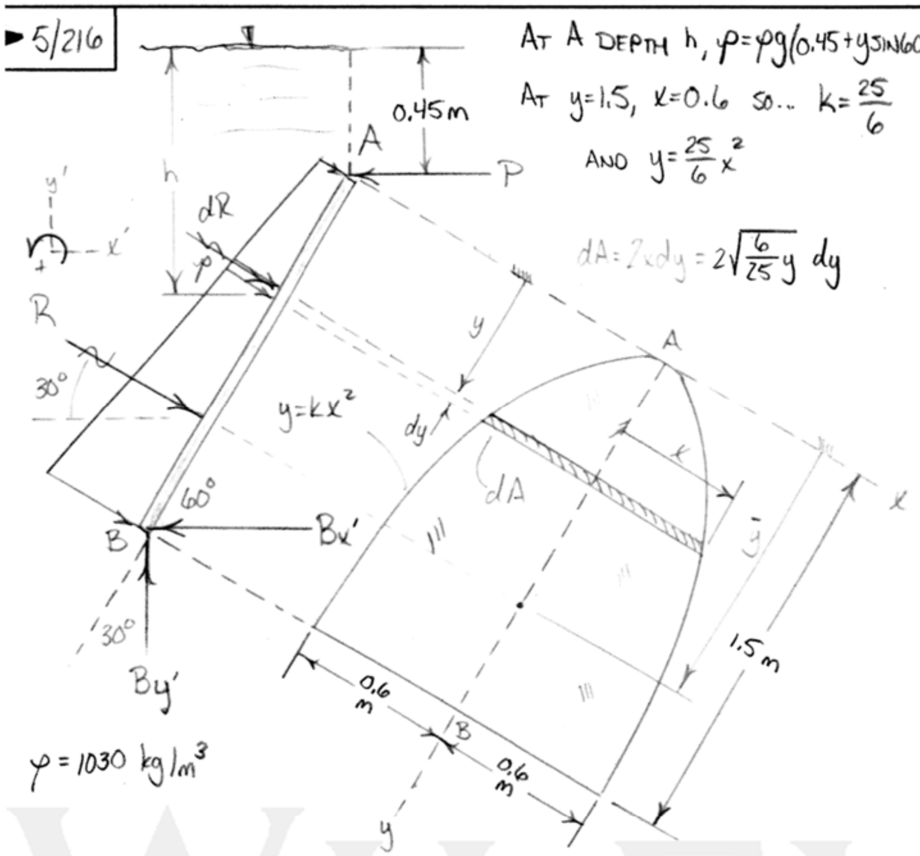
► 5/216

At A depth h, $p = \rho g(0.45 + y\sin 60^\circ)$

At $y = 1.5$, $x = 0.6$ so... $k = \frac{25}{6}$

and $y = \frac{25}{6}x^2$

$dA = 2x\,dy = 2\sqrt{\frac{6}{25}y}\,dy$



$\rho = 1030\ \text{kg/m}^3$

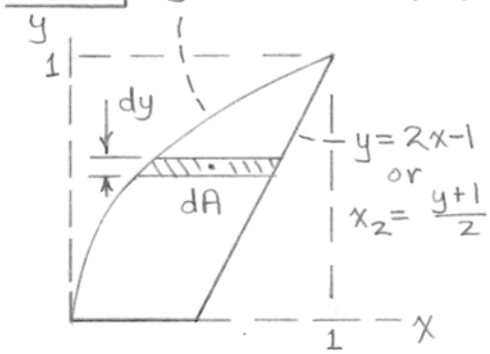
$$\begin{cases} dR = p\,dA = \rho g\left(0.45 + \frac{\sqrt{3}}{2}y\right)\left(2\sqrt{\frac{6}{25}y}\right)dy \\ R = \int dR = \int_0^{1.5} 1030(9.81)\left[0.45 + \frac{\sqrt{3}}{2}y\right]\left(2\sqrt{\frac{6}{25}y}\right)dy \longrightarrow R = 14\,910\ \text{N} \end{cases}$$

$$\bar{y} = \frac{1}{R}\int y\,dR = \frac{1}{14\,910}\int_0^{1.5} 1030(9.81)\,y\left[0.45 + \frac{\sqrt{3}}{2}y\right]\left(2\sqrt{\frac{6}{25}y}\right)dy \longrightarrow \bar{y} = 1.009\ \text{m}$$

$$\begin{cases} \Sigma F_{x'} = 0: & R\cos 30^\circ - B_{x'} - P = 0 \\ \Sigma F_{y'} = 0: & B_{y'} - R\sin 30^\circ = 0 \\ \Sigma M_B = 0: & 1.5\sin 60^\circ\, P - R(1.5 - \bar{y}) = 0 \end{cases} \longrightarrow \begin{cases} B_{x'} = 7270\ \text{N} \\ B_{y'} = 7450\ \text{N} \\ \underline{P = 5640\ \text{N}} \end{cases}$$

$$B = \sqrt{B_{x'}^2 + B_{y'}^2} \longrightarrow \underline{B = 10\,410\ \text{N}}$$

5/217 $y = x^{1/3}$ or $x_1 = y^3$



$$dA = (x_2 - x_1)\,dy = \left(\frac{y}{2} + \frac{1}{2} - y^3\right)dy$$

$$A = \int dA = \int_0^1 \left(\frac{y}{2} + \frac{1}{2} - y^3\right)dy = \left(\frac{y^2}{4} + \frac{y}{2} - \frac{y^4}{4}\right)\Bigg|_0^1 = \frac{1}{2}$$

$$\int y_c\,dA = \int_0^1 \left(\frac{y^2}{2} + \frac{y}{2} - y^4\right)dy = \left(\frac{y^3}{6} + \frac{y^2}{4} - \frac{y^5}{5}\right)\Bigg|_0^1 = \frac{13}{60}$$

$$\int x_c\,dA = \int_0^1 \left(\frac{x_1 + x_2}{2}\right)(x_2 - x_1)\,dy = \frac{1}{2}\int_0^1 (x_2^2 - x_1^2)\,dy$$

$$= \frac{1}{2}\int_0^1 \left(\frac{y^2}{4} + \frac{y}{2} + \frac{1}{4} - y^6\right)dy = \frac{1}{2}\left(\frac{y^3}{12} + \frac{y^2}{4} + \frac{y}{4} - \frac{y^7}{7}\right)\Bigg|_0^1$$

$$= \frac{37}{168}$$

$$\bar{y} = \int x_c\,dA \Big/ \int dA = \frac{13/60}{1/2} = \underline{13/30}$$

$$\bar{x} = \int y_c\,dA \Big/ \int dA = \frac{37/168}{1/2} = \underline{37/84}$$

$\boxed{5/218}$ Triangle: $A = \frac{1}{2}(2h)h = h^2$

$\bar{y} = h/3$

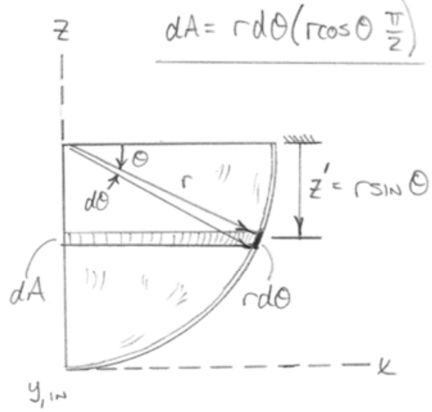
Semi-circular hole: $A = -\frac{1}{2}\pi(h/2)^2 = -\pi h^2/8$

$\bar{y} = 4(h/2)/3\pi = \frac{2h}{3\pi}$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{h^2(h/3) - (\pi h^2/8)(2h/3\pi)}{h^2 - \pi h^2/8} = \frac{h}{4(1-\pi/8)} = \underline{0.412\,h}$$

5/219 UTILIZE A QUARTER-CIRCULAR ARC ELEMENT

$$dA = r\,d\theta\left(r\cos\theta\,\frac{\pi}{2}\right)$$



$$A = \int dA = \int_0^{\pi/2} \frac{\pi}{2} r^2 \cos\theta \, d\theta = \frac{\pi}{2} r^2 \sin\theta \Big|_0^{\pi/2} \longrightarrow A = \frac{1}{2}\pi r^2$$

$$\bar{z}' = \frac{\int z_c' \, dA}{A} = \frac{\int_0^{\frac{\pi}{2}} (r\sin\theta)\left(\frac{\pi}{2} r^2 \cos\theta\right) d\theta}{\frac{1}{2}\pi r^2} = \frac{\frac{r^3\pi}{4}\sin^2\theta}{\frac{1}{2}\pi r^2}\Bigg|_0^{\pi/2} \longrightarrow \bar{z}' = \frac{r}{2}$$

BY SYMMETRY... $\bar{x} = \bar{y} = \bar{z} = \frac{r}{2}$

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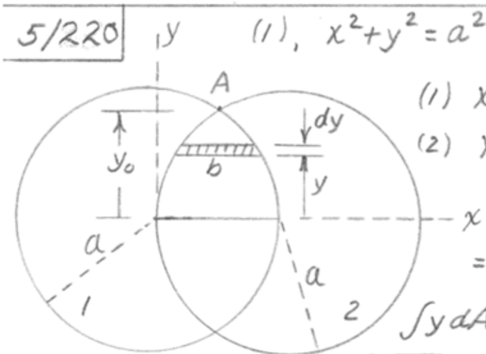


(1), $x^2+y^2=a^2$; (2), $(x-a)^2+y^2=a^2$

(1) $x_1 = +\sqrt{a^2-y^2}$

(2) $x_2 = a - \sqrt{a^2-y^2}$ (Note sign)

$dA = b\,dy = (x_1 - x_2)\,dy$

$= (2\sqrt{a^2-y^2} - a)\,dy$

$\int y\,dA = \int_0^{y_0} (2y\sqrt{a^2-y^2} - ay)\,dy$

For pt A, $x_1 = x_2$ or $\sqrt{a^2-y^2} = a - \sqrt{a^2-y^2}$, $2\sqrt{a^2-y^2} = a$

$y = y_0 = \sqrt{3}a/2$

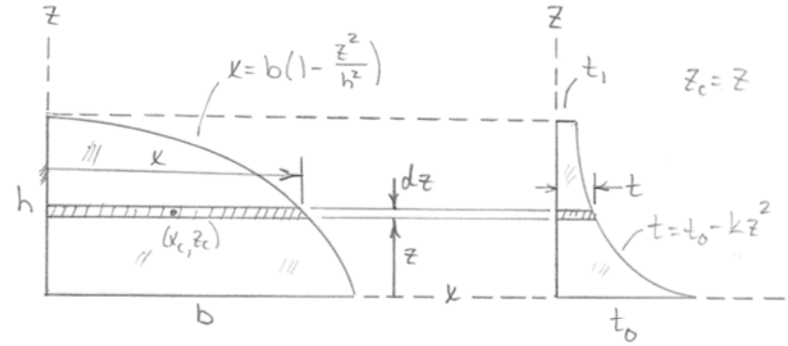
so $\int y\,dA = -\frac{2}{3}\sqrt{(a^2-y^2)^3} - \frac{ay^2}{2}\Big]_0^{y_0} = 5a^3/24 = 0.208a^3$

$\int dA = \int_0^{y_0} (2\sqrt{a^2-y^2} - a)\,dy = \left(y\sqrt{a^2-y^2} + a^2\sin^{-1}\frac{y}{a} - ay\right)_0^{y_0}$

$= a^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) = 0.614a^2$

$\bar{y} = \int y\,dA \Big/ \int dA = \frac{0.208a^3}{0.614a^2} = \underline{0.339a}$

5/221 AT $z = h$, $t = t_1$ ... $t_1 = t_0 - kh^2 \longrightarrow k = \frac{t_0 - t_1}{h^2}$



$$V = \int dV = \int_0^h xt\,dz = \int_0^h b\left(1 - \frac{z^2}{h^2}\right)\left(t_0 - \frac{t_0 - t_1}{h^2} z^2\right) dz \longrightarrow V = \frac{2}{15} bh(4t_0 + t_1)$$

$$\bar{z} = \frac{\int z_c\,dV}{V} = \frac{\int_0^h bz\left(1 - \frac{z^2}{h^2}\right)\left(t_0 - \frac{t_0 - t_1}{h^2} z^2\right) dz}{\frac{2}{15} bh(4t_0 + t_1)} = \frac{b\left(\frac{t_0 z^2}{2} - \frac{(2t_0 - t_1)z^4}{4h^2} + \frac{(t_0 - t_1)z^6}{6h^4}\right)\Big|_0^h}{\frac{2}{15} bh(4t_0 + t_1)}$$

$$\bar{z} = \frac{b\left(\frac{t_0 h^2}{2} + \frac{h^2}{6}(t_0 - t_1) - \frac{h^2}{4}(2t_0 - t_1)\right)}{\frac{2}{15} bh(4t_0 + t_1)} = \frac{5h(2t_0 + t_1)}{8(4t_0 + t_1)}$$

IF $b = 750$ mm, $h = 400$ mm, $t_0 = 35$ mm, AND $t_1 = 7$ mm...

$$\underline{\bar{z} = 131.0 \text{ mm}}$$

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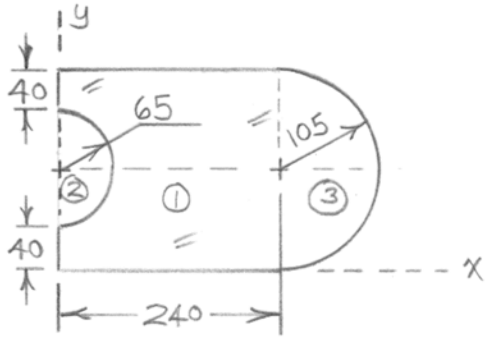
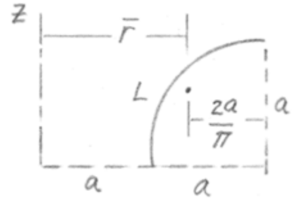


$$\bar{Y} = \frac{1}{2}\left(40 + 2(65) + 40\right) = \underline{105 \text{ mm}}$$

$$\bar{X} = \frac{210(240)\left(\frac{240}{2}\right) - \frac{\pi 65^2}{2}\frac{4(65)}{3\pi} + \frac{\pi 105^2}{2}\left(240 + \frac{4(105)}{3\pi}\right)}{(210)(240) - \frac{\pi 65^2}{2} + \frac{\pi 105^2}{2}}$$

$$= \underline{176.7 \text{ mm}}$$

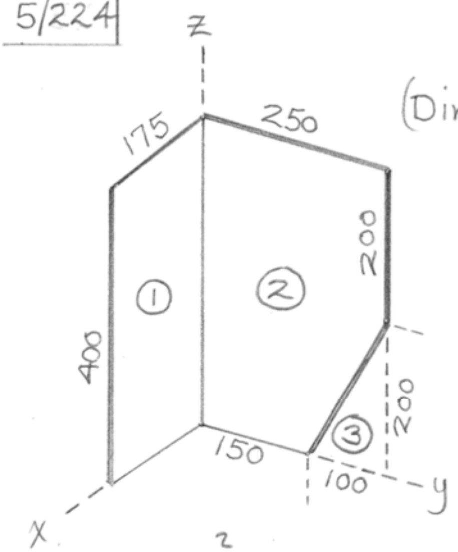
5/223 $L = \pi a/2$; $\bar{r} = 2a - \frac{2a}{\pi} = 2a\left(1 - \frac{1}{\pi}\right)$



$A = \frac{\pi}{2}\bar{r}L = \frac{\pi}{2}2a\left(1-\frac{1}{\pi}\right)\frac{\pi a}{2}$

$= \frac{\pi a^2}{2}(\pi - 1)$

5/224



(Dim. in mm)

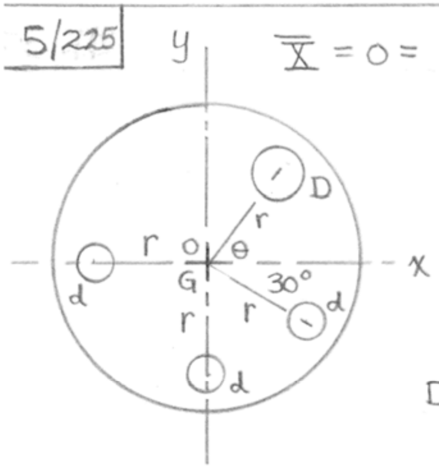
Comp.	A (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{z}$ (mm)	$A\bar{x}$ (in $10^6\ mm^3$)	$A\bar{y}$ (in $10^6\ mm^3$)	$A\bar{z}$ (in $10^6\ mm^3$)
1	$400(175)$	$\frac{175}{2}$	0	$\frac{400}{2}$	6.13	0	14
2	$400(250)$	0	$\frac{250}{2}$	$\frac{400}{2}$	0	12.5	20
3	$-\frac{1}{2}(100)(200)$	0	$\left(150+\frac{2}{3}100\right)$	$\frac{200}{3}$	0	-2.17	-0.667
	160 000				6.13	10.33	33.3

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \underline{38.3\ \text{mm}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \underline{64.6\ \text{mm}}$$

$$\bar{Z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \underline{208\ \text{mm}}$$

5/225



$$\bar{X} = 0 = \frac{\pi D^2}{4} r\cos\theta + \frac{\pi d^2}{4} r\cos 30^\circ - \frac{\pi d^2}{4} r, \quad D^2\cos\theta = d^2\left(1 - \frac{\sqrt{3}}{2}\right)$$

$$\bar{Y} = 0 = \frac{\pi D^2}{4} r\sin\theta - \frac{\pi d^2}{4} r - \frac{\pi d^2}{4} r\sin 30^\circ$$

$$D^2\sin\theta = d^2\left(1 + \frac{1}{2}\right)$$

Divide : $\dfrac{\sin\theta}{\cos\theta} = \dfrac{3/2}{1 - \sqrt{3}/2}$, $\underline{\theta = 84.9^\circ}$

$D^2 = \dfrac{3d^2/2}{\sin 84.9^\circ}$, $\underline{D = 1.227\,d}$

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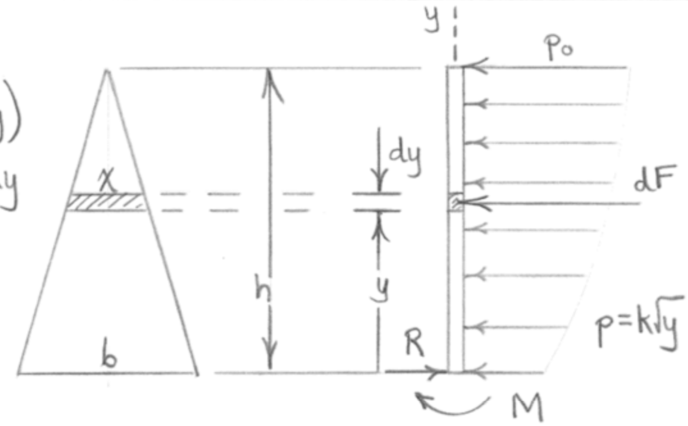
$$x = \frac{b}{h}(h-y)$$

$$dA = \frac{b}{h}(h-y)\,dy$$



$$p = k\sqrt{y} \;:\; p_0 = k\sqrt{h}\,,\; k = \frac{p_0}{\sqrt{h}}\,,\; \text{so } p = p_0\sqrt{\frac{y}{h}}$$

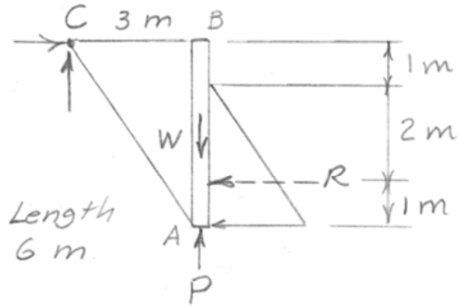
$$dF = p\,dA = p_0\sqrt{\frac{y}{h}}\,\frac{b}{h}(h-y)\,dy$$

$$dM = y\,dF = p_0\frac{b}{h\sqrt{h}}\left(y^{3/2}h - y^{5/2}\right)dy$$

$$M = \int_0^h dM = \frac{p_0 b}{h\sqrt{h}}\left[\frac{y^{5/2}}{5/2}h - \frac{y^{7/2}}{7/2}\right]_0^h = \underline{\frac{4}{35}p_0 b h^2}$$

5/227 Av. pressure $= \frac{1}{2}\rho g h = \frac{1}{2} 1.0(9.81)3$

$= 14.72 \text{ kN/m}^2$



$R = pA = 14.72(3)(6)$

$= 264.9 \text{ kN}$

$W = 8.5(9.81) = 83.4 \text{ kN}$

$\Sigma M_C = 0;\ 264.9(3) + 83.4(3) - P(3) = 0$

$\underline{P = 348 \text{ kN}}$

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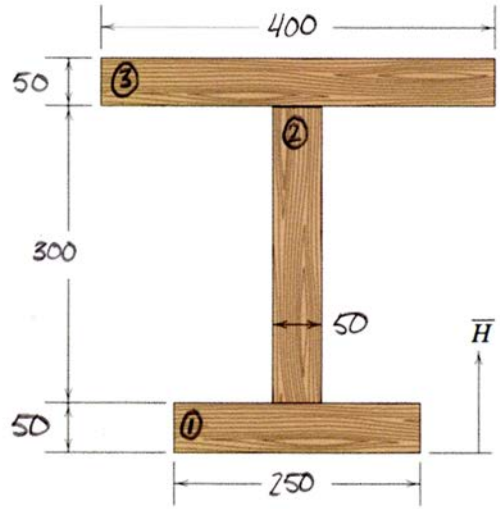

Dim. in mm

$$\bar{H} = \frac{\Sigma A\bar{h}}{\Sigma A} = \frac{250(50)\left(\frac{50}{2}\right) + 50(300)\left(50 + \frac{300}{2}\right) + 400(50)\left(350 + \frac{50}{2}\right)}{250(50) + 50(300) + 400(50)}$$

$$\underline{\bar{H} = 228 \text{ mm}}$$

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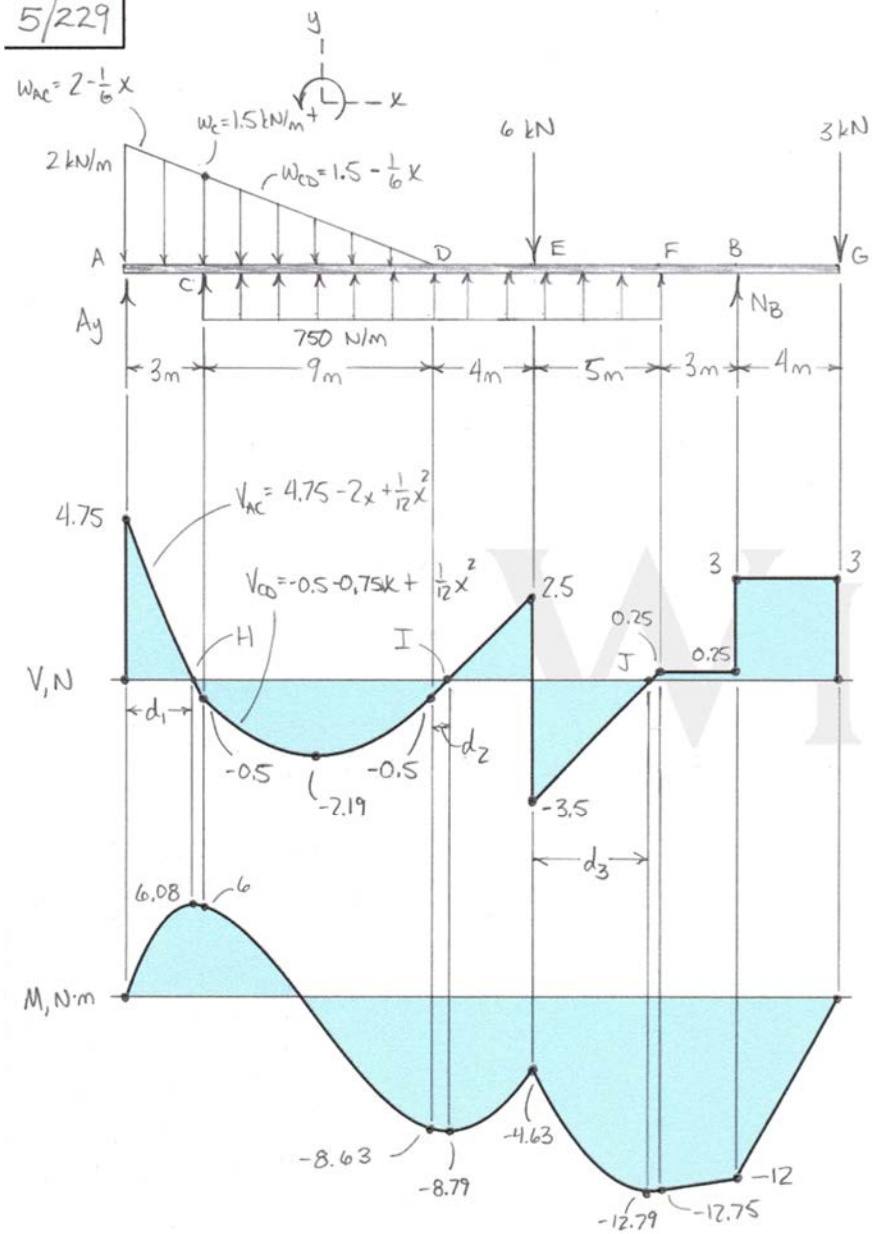

$$\Sigma F_y = 0: \quad A_y + N_B - \tfrac{1}{2}(2)(12) + 0.75(18) - 6 - 3 = 0$$

$$\Sigma M_A = 0: \quad 24 N_B - 28(3) - 16(6) - \tfrac{1}{2}(2)(12)(4) + 0.75(18)(12) = 0$$

$$A_y = 4.75 \text{ kN} \quad \& \quad N_B = 2.75 \text{ kN}$$

$$V_C = 4.75 - \tfrac{1}{2}(2 + 1.5)(3) = -0.5 \text{ kN}$$

$$V_D = -0.5 - \tfrac{1}{2}(1.5)(9) + 0.75(9) = -0.5 \text{ kN}$$

$$V_E^- = -0.5 + 4(0.75) = 2.5 \text{ kN}$$

$$V_F = -3.5 + 5(0.75) = 0.25 \text{ kN}$$

$$d_2 = \frac{0.5}{0.75} = 0.667 \text{ m}$$

$$d_3 = \frac{3.5}{0.75} = 4.67 \text{ m}$$

$$w_{CD_{TOTAL}} = 1.5 - \tfrac{1}{6}x + 0.75 = 0.75 - \tfrac{1}{6}x \quad \text{(Linear Plus Uniform Loads)}$$

For V_{max} in CD find when $w_{CD} = 0$.. $\quad 0 = 0.75 - \tfrac{1}{6}x \longrightarrow x = 4.5$ m

At d_1, $V_{AC} = 0 = 4.75 - 2d_1 + \frac{1}{12}d_1^2 \longrightarrow d_1 = 2.67$ m (or 21.3 m)

$$V_{CD_{max}} = -0.5 - 0.75(4.5) + \tfrac{1}{12}(4.5)^2 \longrightarrow V_{CD_{max}} = -2.19 \text{ kN}$$

$$M_H = \int_0^{2.67} \left(4.75 - 2x + \tfrac{1}{12}x^2\right)dx \longrightarrow \underline{M_H = 6.08 \text{ kN·m} \quad \text{At } x = 2.67 \text{ m}}$$

$$M_C = \int_0^3 \left(4.75 - 2x + \tfrac{1}{12}x^2\right)dx \longrightarrow M_C = 6 \text{ kN·m}$$

$$M_D = 6 + \int_0^9 \left(-0.5 - 0.75x + \tfrac{1}{12}x^2\right)dx \longrightarrow M_D = -8.63 \text{ kN·m}$$

$$M_I = -8.63 - \tfrac{1}{2}(0.5)(0.667) = -8.79 \text{ kN·m}$$

$$M_E = -8.79 + \tfrac{1}{2}(2.5)(4 - 0.667) = -4.63 \text{ kN·m}$$

$$M_J = -4.63 - \tfrac{1}{2}(3.5)(4.67) = \underline{-12.79 \text{ kN·m} \quad \text{At } x = 20.7 \text{ m}}$$

$$M_F = -12.79 + \tfrac{1}{2}(0.25)(5 - 4.67) = -12.75 \text{ kN·m}$$

$$M_B = -12.75 + 3(0.25) = -12 \text{ kN·m}$$

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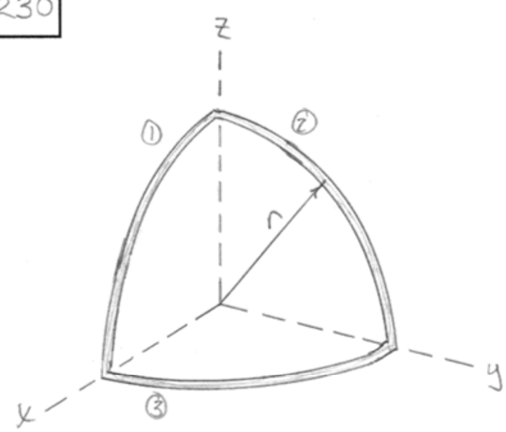
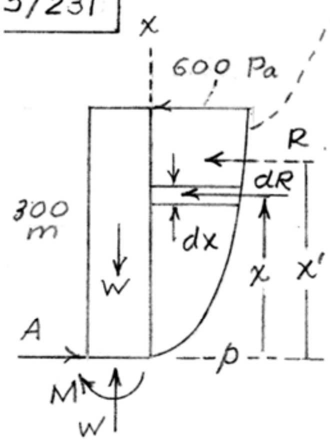


$$\bar{X} = \frac{\Sigma L\bar{x}}{\Sigma L} = \frac{\frac{\pi}{2}r\left(\frac{2r}{\pi} + \frac{2r}{\pi} + 0\right)}{3\left(\frac{\pi}{2}r\right)} \longrightarrow \underline{\bar{X} = \frac{4r}{3\pi}}$$

By symmetry... $\underline{\bar{Y} = \bar{Z} = \frac{4r}{3\pi}}$

5/231



$p = k\sqrt{x}$: $600 = k\sqrt{300}$

$k = 34.6\ \text{N/m}^{5/2}$, $p = 34.6\sqrt{x}$

$dR = 60\,p\,dx = 2080\sqrt{x}\,dx$

$R = \int dR = 2080\int_0^{300}\sqrt{x}\,dx$

$= 2080\left(\frac{2}{3}x^{3/2}\right)\Big|_0^{300} = 7.20(10^6)\ \text{N}$

$\int x\,dR = 2080\int_0^{300} x\sqrt{x}\,dx$

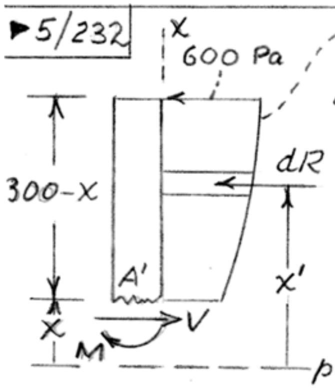
$= 2080\left(\frac{2}{5}x^{5/2}\right)\Big|_0^{300}$

$= 1296(10^6)\ \text{N}\cdot\text{m}$

$\Sigma F = 0$: $A - 7.20(10^6) = 0$, or $\underline{A = 7.20\ \text{MN}}$

$\Sigma M = 0$: $M - 1296(10^6) = 0$, or $\underline{M = 1296\ \text{MN}\cdot\text{m}}$

▸5/232



$p = 34.6\sqrt{x}$ Pa (Prob. 5/231)

$dR = 60\,p\,dx' = 2080\sqrt{x'}\,dx'$

$\Sigma F = 0$: $V - \int_x^{300} 2080\sqrt{x'}\,dx' = 0$

$V = 2080\left(\frac{2}{3}x'^{3/2}\right)\Big|_x^{300}$ (N)

$= 7.20(10^6) - 1386\,x^{3/2}$ (N)

$V|_{x=150\text{ m}} = 4.65(10^6)$ N or $V|_{x=150\text{m}} = 4.65$ MN

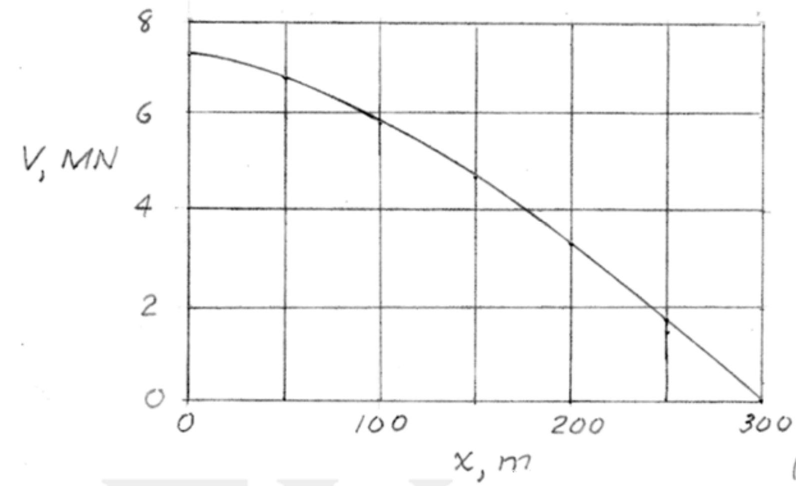
$\Sigma M_{A'} = 0$: $M - \int_x^{300} (x'-x)\,2080\sqrt{x'}\,dx' = 0$

$M = 2080\left[\frac{2}{5}x'^{5/2} - \frac{2}{3}x\,x'^{3/2}\right]_x^{300}$

$M = 1296(10^6) - 7.20(10^6)x + 554\,x^{5/2}$ N·m

or $M = 1296 - 7.20x + 5.54(10^{-4})\,x^{5/2}$ MN·m

$M|_{x=150\text{ m}} = 369$ MN·m



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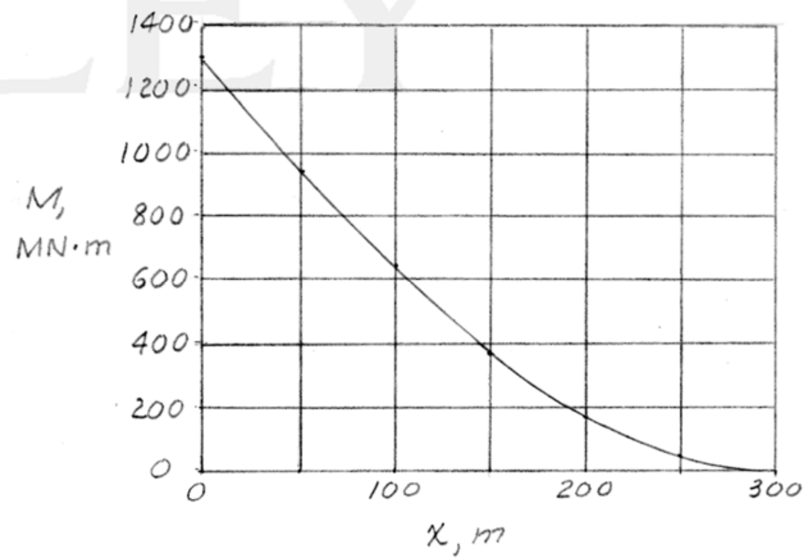

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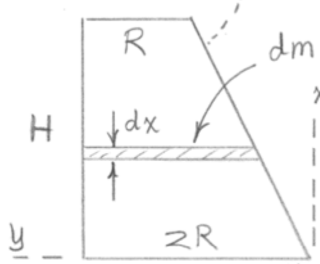
$y = \frac{R}{H}x$



$$dm = \rho\, dV = \rho\pi\left(\frac{2R-y}{2}\right)^2 dx$$
$$= \frac{\pi\rho}{4}\left(4R^2 - 4Ry + y^2\right) dx$$
$$= \frac{\pi\rho}{4}\left[4R^2 - 4R\frac{R}{H}x + \left(\frac{R}{H}x\right)^2\right] dx$$
$$= \frac{\pi\rho R^2}{4}\left[4 - 4\frac{x}{H} + \frac{x^2}{H^2}\right] dx$$

$$m = \int dm = \int_0^H \frac{\pi\rho R^2}{4}\left[4 - 4\frac{x}{H} + \frac{x^2}{H^2}\right] dx$$
$$= \frac{7}{12}\pi\rho R^2 H$$

$$\int x_c\, dm = \int_0^H \frac{\pi\rho R^2}{4}\left[4x - 4\frac{x^2}{H} + \frac{x^3}{H^2}\right] dx$$
$$= \frac{11}{48}\pi\rho R^2 H^2$$

Then $\bar{x} = \dfrac{\int x_c\, dm}{\int dm} = \dfrac{\frac{11}{48}\pi\rho R^2 H^2}{\frac{7}{12}\pi\rho R^2 H}$

$$= \underline{\frac{11}{28}H = \bar{h}}$$

(ρ = mass per unit volume)

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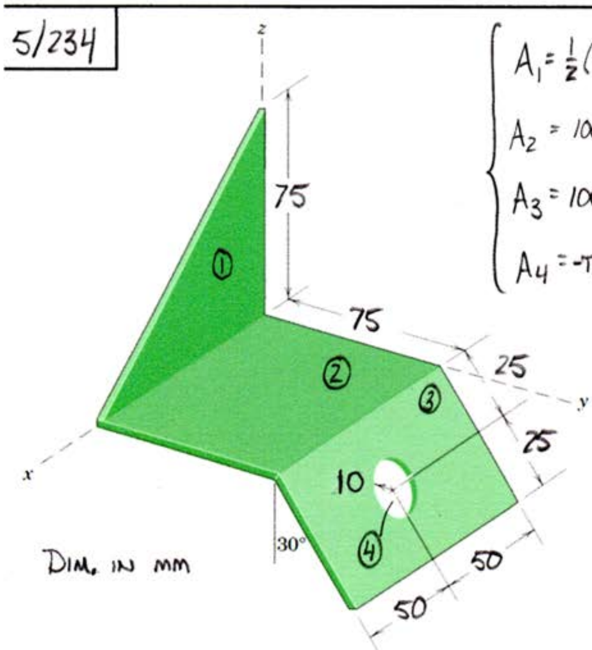


$$\begin{cases} A_1 = \frac{1}{2}(100)(75) = 3750 \text{ mm}^2 \\ A_2 = 100(75) = 7500 \text{ mm}^2 \\ A_3 = 100(50) = 5000 \text{ mm}^2 \\ A_4 = -\pi(10)^2 = -100\pi \text{ mm}^2 \end{cases}$$

$$\Sigma A = 15\,940 \text{ mm}^2$$

$$\sin 30^\circ = 1/2$$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{3750\left(\frac{100}{3}\right) + 7500(50) + 5000(50) - 100\pi(50)}{15\,940} \rightarrow \underline{\bar{X} = 46.1 \text{ mm}}$$

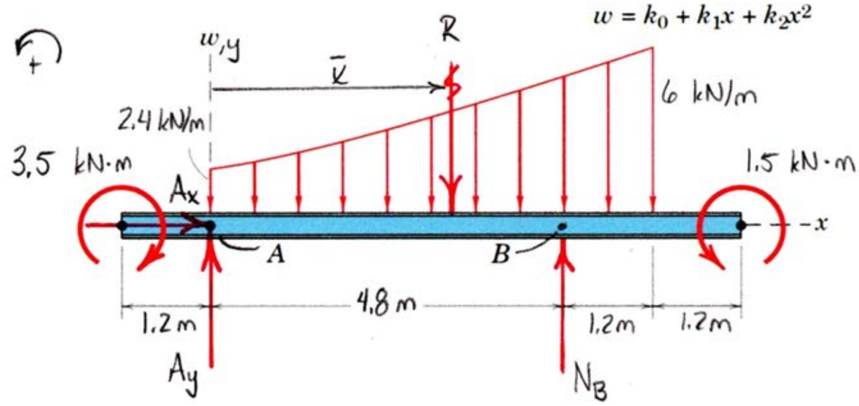
$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{3750(0) + 7500\left(\frac{75}{2}\right) + 5000(75 + 25\sin 30^\circ) - 100\pi(75 + 25\sin 30^\circ)}{15\,940}$$

$$\underline{\bar{Y} = 43.4 \text{ mm}}$$

$$\bar{Z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{3750\left(\frac{75}{3}\right) + 7500(0) + 5000(-25\cos 30^\circ) - 100\pi(-25\cos 30^\circ)}{15\,940}$$

$$\underline{\bar{Z} = -0.483 \text{ mm}}$$

5/235 At $x=0$, $\frac{dw}{dx} = 120$ N/m PER METER



$$\begin{cases} \text{At } x=0,\ w=2.4=k_0+k_1(0)+k_2(0)^2 \longrightarrow k_0=2.4 \text{ kN/m} \\ \text{At } x=0,\ \frac{dw}{dx}=0.120=k_1+2k_2(0) \longrightarrow k_1=120 \text{ N/m}^2 \text{ or } 0.120 \text{ kN/m}^2 \\ \text{At } x=6\text{ m},\ w=6=2.4+0.120(6)+k_2(6)^2 \longrightarrow k_2=80 \text{ N/m}^3 \text{ or } 0.08 \text{ kN/m}^3 \end{cases}$$

$$\begin{cases} R=\int w\,dx=\int_0^6 (2.4+0.120x+0.08x^2)\,dx \longrightarrow R=22.3 \text{ kN} \\ \bar{x}=\frac{\int xw\,dx}{R}=\frac{1}{22.3}\int_0^6 (2.4x+0.120x^2+0.08x^3)\,dx \longrightarrow \bar{x}=3.48 \text{ m} \end{cases}$$

$$\begin{cases} \Sigma F_x=0:\ \underline{A_x=0} \\ \Sigma F_y=0:\ A_y+N_B-R=0 \\ \Sigma M_A=0:\ 1.5-3.5+4.8N_B-R\bar{x}=0 \end{cases} \longrightarrow \begin{cases} \underline{A_y=5.70 \text{ kN}} \\ \underline{N_B=16.62 \text{ kN}} \end{cases}$$

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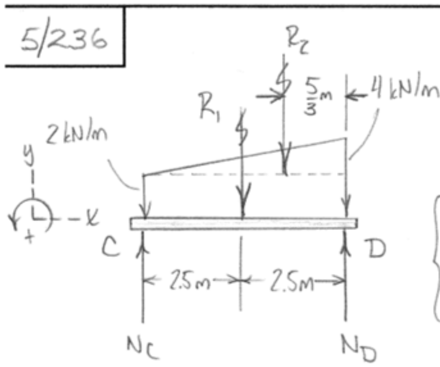
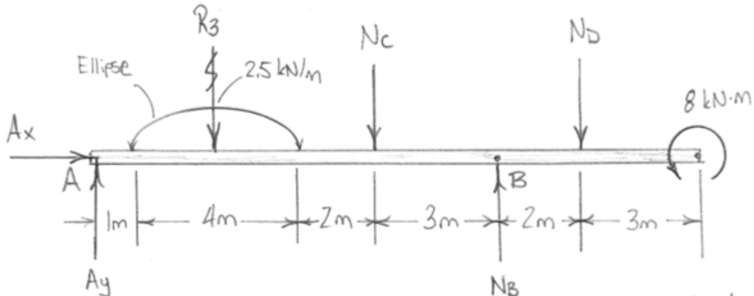


$$\begin{cases} R_1 = 2(5) = 10 \text{ kN} \\ R_2 = \frac{1}{2}(2)(5) = 5 \text{ kN} \end{cases}$$

$$\begin{cases} \Sigma F_y = 0: \; N_C + N_D - R_1 - R_2 = 0 \\ \Sigma M_D = 0: \; 2.5 R_1 + \frac{5}{3} R_2 - 5 N_C = 0 \end{cases}$$

$N_C = 6.67$ kN & $N_D = 8.33$ kN



$R_3 = \frac{1}{2}\pi(2.5)(2) = 7.85$ kN

$$\begin{cases} \Sigma F_x = 0: \; \underline{A_x = 0} \\ \Sigma F_y = 0: \; A_y - R_3 - N_C - N_D + N_B = 0 \\ \Sigma M_A = 0: \; -3R_3 - 7N_C + 10 N_B - 12 N_D + 8 = 0 \end{cases}$$

$\underline{A_y = 6.63 \text{ kN} \uparrow}$ $\underline{N_B = 16.22 \text{ kN} \uparrow}$

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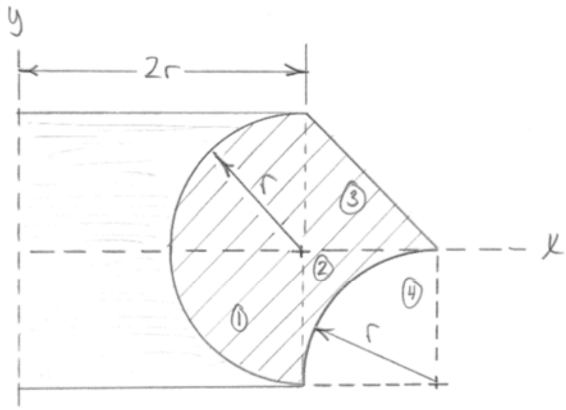


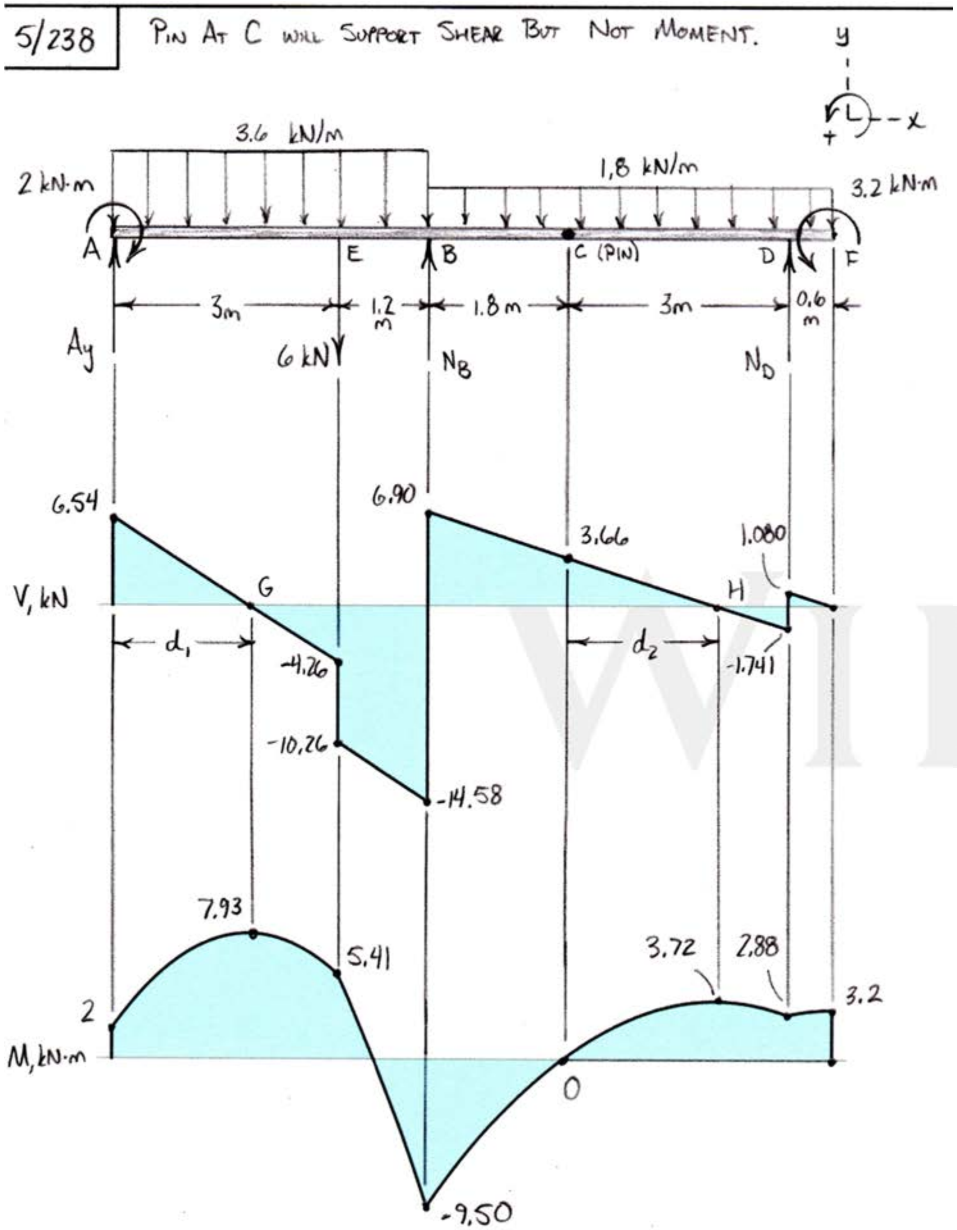
$$A = \theta \Sigma L \bar{r} = 2\pi\left[\pi r\left(2r - \frac{2r}{\pi}\right) + r\sqrt{2}(2.5r) + \frac{\pi}{2} r\left(3r - \frac{2r}{\pi}\right)\right]$$

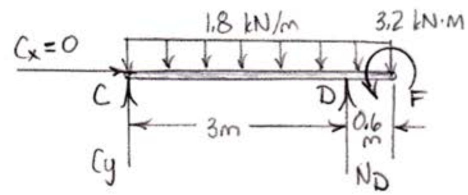
$$\underline{A = 72.5\, r^2}$$

$$V = \theta \Sigma A \bar{r} = 2\pi\left[\frac{1}{2}\pi r^2\left(2r - \frac{4r}{3\pi}\right) + \frac{1}{2} r^2\left(2r + \frac{r}{3}\right) + r^2(2.5r) - \frac{1}{4}\pi r^2\left(3r - \frac{4r}{3\pi}\right)\right]$$

$$\underline{V = 25.9\, r^3}$$

5/238 PIN AT C WILL SUPPORT SHEAR BUT NOT MOMENT.





$\Sigma F_y = 0: \; C_y + N_D - 3.6(1.8) = 0$

$\Sigma M_C = 0: \; -3.6(1.8)(1.8) + 3.2 + 3N_D = 0$

$C_y = 3.66$ kN $\;\&\;$ $N_D = 2.82$ kN

$\Sigma F_y = 0: \; A_y - 4.2(3.6) - 5.4(1.8) - 6 + N_B + N_D = 0$

$\Sigma M_A = 0: \; -2 + 3.2 - 3(6) - 4.2(3.6)(2.1) + 4.2N_B + 9N_D - 5.4(1.8)(6.9) = 0$

$A_y = 6.54$ kN $\;\&\;$ $N_B = 21.5$ kN

$V_{E^-} = 6.54 - 3(3.6) = -4.26$ kN

$V_{B^-} = -10.26 - 1.2(3.6) = -14.58$ kN

$V_C = 6.90 - 1.8(1.8) = 3.66$ kN

$V_{D^+} = -1.741 + 2.82 = 1.080$ kN

$V_{E^+} = -4.26 - 6 = -10.26$ kN

$V_{B^+} = -14.58 + 21.5 \quad 6.90$ kN

$V_{D^-} = 3.66 - 3(1.8) = -1.741$ kN

$V_F = 1.080 - 1.8(0.6) = 0$

$d_1 = \frac{6.54}{3.6} = 1.816$ m $\qquad d_2 = \frac{3.66}{1.8} = 2.03$ m

$M_G = 2 + \frac{1}{2}(6.54)(1.816) = 7.93$ kN·m

$M_E = 7.93 + \frac{1}{2}(-4.26)(3 - 1.816) = 5.41$ kN·m

$M_B = 5.41 + \frac{1}{2}(-10.26 - 14.58)(1.2) = -9.50$ kN·m

$M_C = -9.50 + \frac{1}{2}(6.90 + 3.66)(1.8) = 0$

$M_H = \frac{1}{2}(3.66)(2.03) = 3.72$ kN·m

$M_D = 3.72 + \frac{1}{2}(-1.741)(3 - 2.03) = 2.88$ kN·m

$M_{F^-} = 2.88 + \frac{1}{2}(1.080)(0.6) = 3.2$ kN·m

$M_{F^+} = 3.2 - 3.2 = 0$

BEAM AC: $|M|_{max} = 9.50$ kN·m AT B

BEAM CD: $M_{max} = 3.72$ kN·m AT 2.03 m RIGHT OF C

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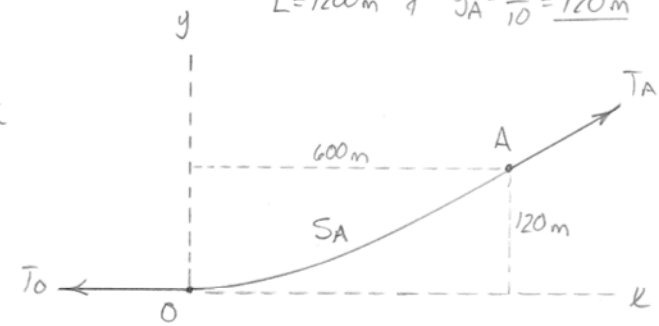
$L = 1200\text{ m}$ & $y_A = \frac{L}{10} = \underline{120\text{ m}}$

SET $k = \frac{T_0}{\mu}$



AT A:

$$y_A = \frac{T_0}{\mu}\left[\cosh\frac{\mu x_A}{T_0} - 1\right] \longrightarrow 120 = k\left[\cosh\frac{600}{k} - 1\right]$$

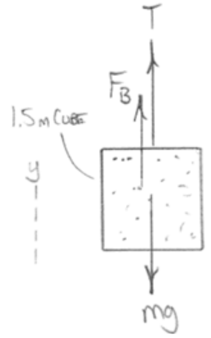
SOLVING... $k = 1520\text{ m}$

$$s = 2s_A = 2\frac{T_0}{\mu}\sinh\frac{\mu x_A}{T_0} = 2k\sinh\frac{x_A}{k} = 2(1520)\sinh\frac{600}{1520}$$

$\therefore$ $\underline{s = 1231\text{ m}}$

5/240

$\rho_w = 1000\ \text{kg/m}^3$ $\rho_c = 2400\ \text{kg/m}^3$

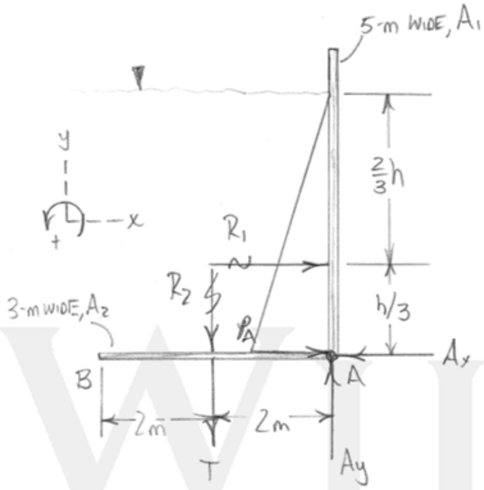


$\Sigma F_y = 0: \quad T + F_B - mg = 0$

$$T + \rho_w V_c g - \rho_c V_c g = 0$$

$$T + 1000(1.5)^3(9.81) - 2400(1.5)^3(9.81) = 0$$

$$T = 46.4\ \text{kN}$$



$p_A = \rho_w g h = 9810h$

$R_1 = \frac{1}{2} p_A A_1 = \frac{1}{2}(9810h)(5h)$

$= 24\,500\ h^2$

$R_2 = p_A A_2 = 9810h(3)(4) = 117\,700\ h$

Solving... $h = 5.55\ \text{m}$

$\Sigma F_x = 0: \quad R_1 - A_x = 0$

$\Sigma F_y = 0: \quad A_y - R_2 - T = 0$

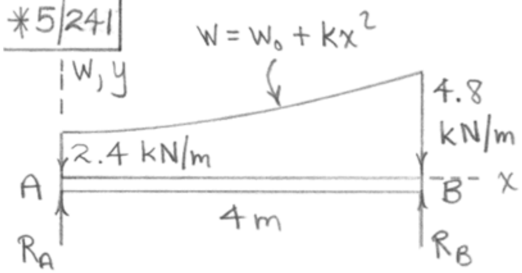
$\Sigma M_A = 0: \quad 2(T + R_2) - \frac{h}{3} R_1 = 0$

$A_x = 756\ \text{kN}$

$A_y = 700\ \text{kN}$

so... $R_A = 1031\ \text{kN}$

*5/241



$w_0 = 2.4$ kN/m

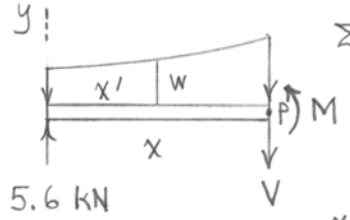
$4.8 = 2.4 + k(4)^2$

$k = 0.15$ kN/m^3

So $w = 2.4 + 0.15x^2$

$\circlearrowleft \Sigma M_A = 0: \int_0^4 (2.4 + 0.15x^2)x\,dx - 4R_B = 0, \quad R_B = 7.2$ kN

$\Sigma F_y = 0: R_A + 7.2 - \int_0^4 (2.4 + 0.15x^2)\,dx = 0, \quad R_A = 5.6$ kN



$\Sigma F_y = 0: 5.6 - \int_0^x (2.4 + 0.15x^2)\,dx - V = 0$

$\underline{V = 5.6 - 2.4x - 0.05x^3}$

$\circlearrowleft \Sigma M_P = 0: M + \int_0^x (2.4 + 0.15x'^2)(x - x')\,dx' - 5.6x = 0$

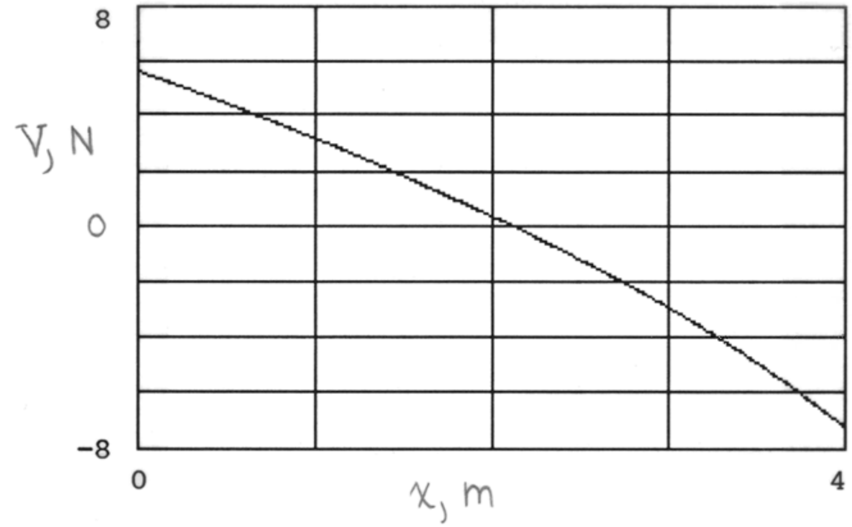
$M = 5.6x - \left[2.4xx' + \frac{0.15}{3}x'^3x - \frac{2.4}{2}x'^2 - \frac{0.15}{4}x'^4\right]_0^x$

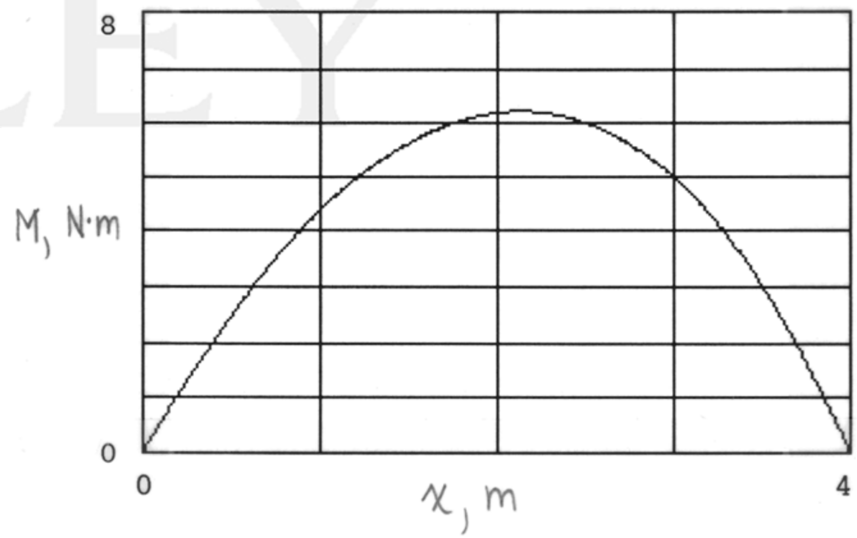
$= 5.6x - 1.2x^2 - 0.0125x^4$

For a maximum, $\frac{dM}{dx} = 5.6 - 2.4x - 0.05x^3 = 0$

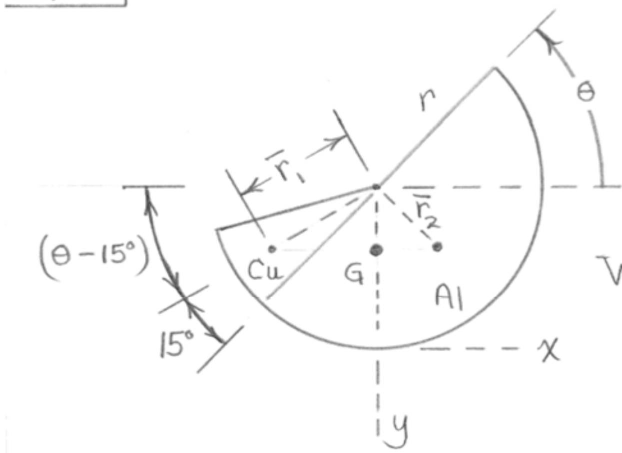
Solve numerically to obtain

$\underline{M_{max} = 6.23 \text{ kN·m @ } x = 2.13 \text{ m}}$





*5/242



$$\bar{r}_1 = \frac{2}{3} r \frac{\sin 15°}{\frac{15\pi}{180}} = \frac{8r}{\pi} \sin 15°$$

$$\bar{r}_2 = \frac{4r}{3\pi}$$

Width w

$$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} = 0 \text{ for equilibrium}$$

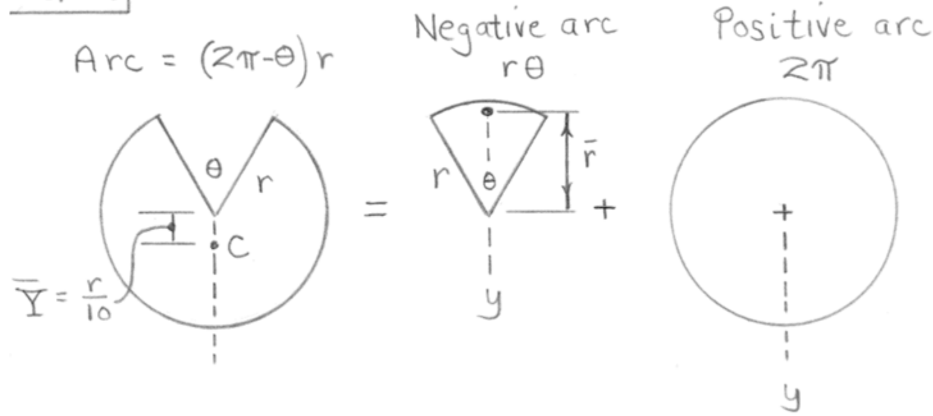
So $m_{Cu}|\bar{x}_1| = m_{Al}|\bar{x}_2|$

$$\left(\frac{30}{360}\pi r^2 w \rho_{Cu}\right)\left(\frac{8r}{\pi}\sin 15°\right)\cos(\theta - 15°) = \left(\frac{\pi r^2}{2} w \rho_{Al}\right)\frac{4r}{3\pi}\sin\theta$$

Reduces to $\frac{\rho_{Cu}}{\rho_{Al}} \sin 15° \cos(\theta - 15°) = \sin\theta$

With $\rho_{Cu} = 8910 \text{ kg/m}^3$ and $\rho_{Al} = 2690 \text{ kg/m}^3$, a numerical solution yields $\underline{\theta = 46.8°}$.

*5/243



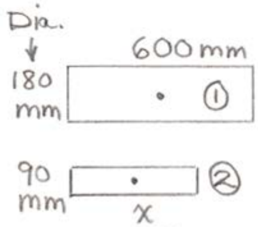
From Sample Problem 5/1, $\bar{r} = \dfrac{r \sin\frac{\theta}{2}}{\theta/2}$

$\bar{Y} = \dfrac{\Sigma L\bar{y}}{\Sigma L}$: $\dfrac{r}{10} = \dfrac{(-r\theta)\left(-r\dfrac{\sin\frac{\theta}{2}}{\theta/2}\right) + 2\pi r(0)}{(2\pi-\theta)r}$

Simplify to $10\sin\frac{\theta}{2} = \pi - \frac{\theta}{2}$ and solve numerically to obtain $\underline{\theta = 33.1^\circ}$

*5/244

$$V_1 = \frac{\pi\, 180^2}{4}(600) = 15\,270(10^3)\ \text{mm}^3$$



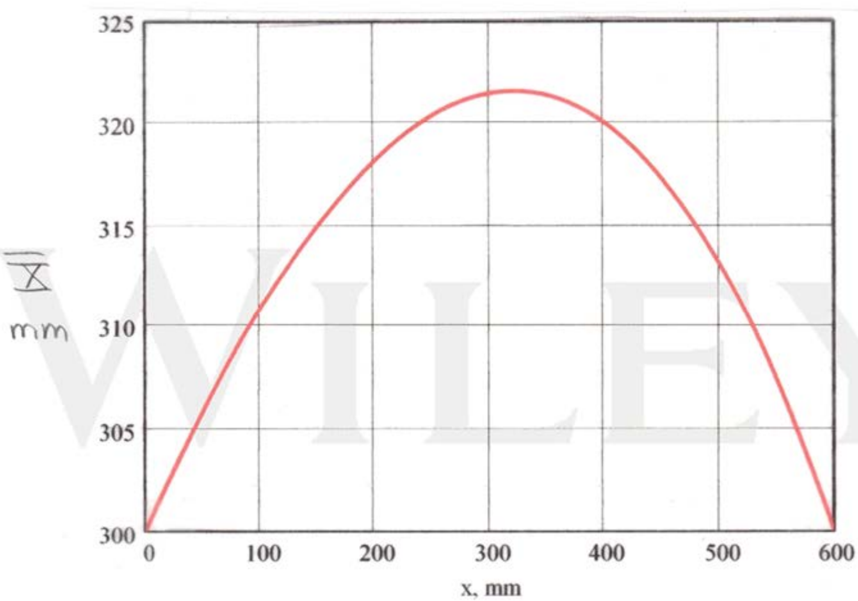
$$\bar{x}_1 = 300\ \text{mm}$$

$$V_2 = -\frac{\pi\, 90^2}{4}x = -6.36(10^3)x$$

$$\bar{x}_2 = -\frac{x}{2}$$

$$\bar{X} = \frac{\Sigma V\bar{x}}{\Sigma V} = \frac{15\,270(10^3)(300) - 6.36(10^3)x\left(\frac{x}{2}\right)}{15\,270(10^3) - 6.36(10^3)x}$$

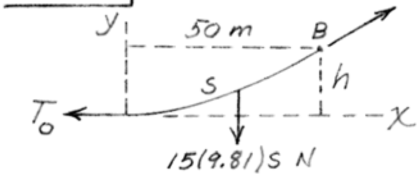
$$= \frac{4580(10^3) - 3.18x^2}{15\,270 - 6.36x}$$



$$\bar{X}_{max} = 322\ \text{mm} \ @\ x = 322\ \text{mm}$$

*5/245

$T = 12(10^3)$ N

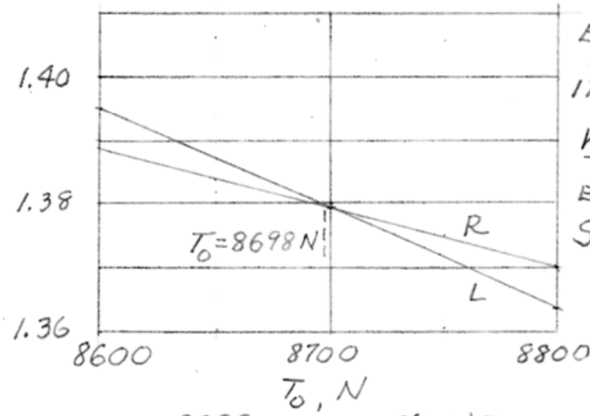


Eq. 5/21 $T = T_0 \cosh \frac{\mu x}{T_0}$

$$\frac{12(10^3)}{T_0} = \cosh \frac{15(9.81)(50)}{T_0}$$

$L = R$

Sol. gives $T_0 = 8700$ N



Eq. 5/22, $T = T_0 + \mu h$

$12(10^3) = (8.70)10^3 + 15(9.81)h$

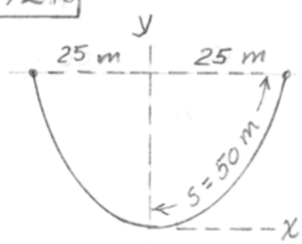
$\underline{h = 22.4 \text{ m}}$

Eq. 5/20

$S = 2s = 2\frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$

So $S = 2\frac{8698}{15(9.81)} \sinh \frac{15(9.81)50}{8700} = \underline{112.4 \text{ m}}$

*5/246



Eq. 5/20 $s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$

$$50 = \frac{T_0}{\mu} \sinh \frac{25\mu}{T_0}$$

$$\frac{50\mu}{T_0} - \sinh \frac{25\mu}{T_0} = R = 0$$

Write and run program for $R = f\left(\frac{\mu}{T_0}\right)$ & find μ/T_0 for $R=0$. Result is $\mu/T_0 = 0.0871$

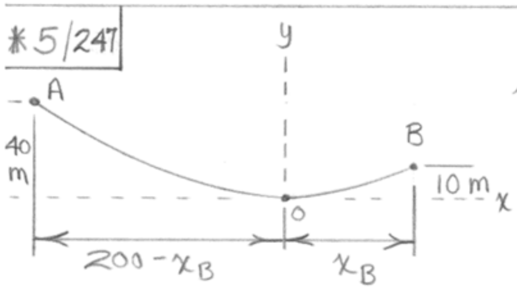
From Eq. 5/19, $y = \frac{T_0}{\mu}\left(\cosh \frac{\mu x}{T_0} - 1\right)$

$$= \frac{1}{0.0871}\left(\cosh 0.0871[25] - 1\right)$$

$$h = y = 3.468/0.0871 = \underline{39.8 \text{ m}}$$

Result depends only on the geometry of the catenary.

*5/247



$$\mu_{cable} = 20(9.81) = 196.2 \ \frac{N}{m}$$

Eq. 5/19: $y = \frac{T_0}{\mu}\left[\cosh \frac{x}{T_0/\mu} - 1\right]$

At B: $10 = \frac{T_0}{\mu}\left[\cosh \frac{x_B}{T_0/\mu} - 1\right]$

At A: $40 = \frac{T_0}{\mu}\left[\cosh \frac{200 - x_B}{T_0/\mu} - 1\right]$

Simultaneous numerical solution: $\begin{cases} x_B = 67.1 \text{ m} \\ T_0/\mu = 227 \text{ m} \end{cases}$

$T_A = T_0 + \mu y_A$: $75\,000 = 227\mu + \mu(40)$

$\mu = 281$ N/m

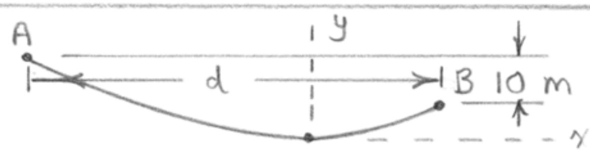
$\mu = \mu_{cable} + \mu_{ice}$: $281 = 196.2 + \mu_{ice}$

$\mu_{ice} = 84.7$ N/m

$\rho = \frac{\mu}{g} = \frac{84.7}{9.81} = \underline{8.63 \text{ kg/m}}$

The configuration does not depend on μ.

*5/248



$$\begin{cases} L = 200 \text{ m} \\ \mu = 0.025 \text{ N/m} \end{cases}$$

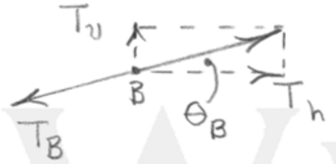
Eqs. (5/19) & (5/20) @ A & B

$$A: \begin{cases} y_B + 10 = \dfrac{T_0}{\mu}\left[\cosh\left(\dfrac{\mu x_A}{T_0}\right) - 1\right] & (1) \\ s_A = \dfrac{T_0}{\mu}\sinh\left(\dfrac{\mu x_A}{T_0}\right) & (2) \end{cases}$$

$$B: \begin{cases} y_B = \dfrac{T_0}{\mu}\left[\cosh\left(\dfrac{\mu x_B}{T_0}\right) - 1\right] & (3) \\ s_B = \dfrac{T_0}{\mu}\sinh\left(\dfrac{\mu x_B}{T_0}\right) & (4) \end{cases}$$

Also: $x_A + x_B = d$ (5) $\qquad s_A + s_B = L$ (6)

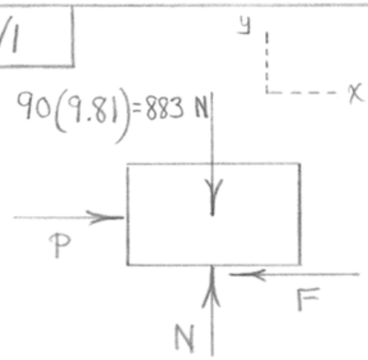
At B:



$$\begin{cases} \tan\theta_B = \sinh\dfrac{\mu x_B}{T_0} & (7) \\ T_B = T_0 + \mu y_B & (8) \\ T_h = T_B\cos\theta_B & (9) \\ T_v = T_B\sin\theta_B & (10) \end{cases}$$

Strategy: Vary d & solve Eqs. (1)-(6), then (7)-(10) until T_h or T_v hits an upper limit. Ans.: $\underline{d = 197.7 \text{ m}}$, at which $\underline{T_h = 10 \text{ N}}$ and $\underline{T_v = 1.984 \text{ N}}$

6/1



$\Sigma F_y = 0: \quad N - 883 = 0, \quad N = 883 \text{ N}$

$F_{max} = \mu_s N = 0.5(883) = 441 \text{ N}$

$\Sigma F_x = 0$ yields $F = P$ for equilibrium

(a) $P = 300$ N, $\underline{F = 300 \text{ N}} < F_{max}$, OK

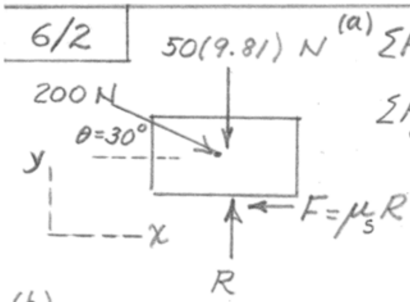
(b) $P = 400$ N, $\underline{F = 400 \text{ N}} < F_{max}$, OK

(c) $P = 500$ N, $F = 500 \text{ N} > F_{max}$, motion

So $F = \mu_k N = 0.4(883) = \underline{353 \text{ N}}$

(all to the left)

6/2



(a) $\Sigma F_x = 0$; $200 \cos 30° - \mu_s R = 0$

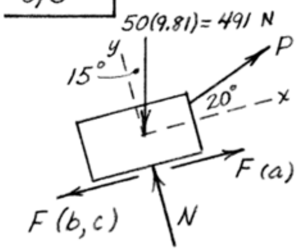
$\Sigma F_y = 0$; $R - 200 \sin 30° - 50(9.81) = 0$

$R = 590.5$ N

So $\mu_s = \dfrac{200 \cos 30°}{590.5} = \underline{0.293}$

(b)
For $\theta = 45°$, $\Sigma F_x = 0$ gives $F = 200 \cos 45° = \underline{141.4 \text{ N}}$
which is $< \mu_s R_b$

6/3



(a) $P = 0$

$\Sigma F_y = 0: \; N - 491\cos 15° = 0, \; N = 474$ N

Assume equilibrium:

$\Sigma F_x = 0: \; F - 491\sin 15° = 0, \; F = 127.0$ N

$F_{max} = \mu_s N = 0.25(474) = 118.4 \text{ N} < F;$

assumption invalid and

$F = F_k = \mu_k N = 0.2(474) = \underline{94.8 \text{ N}}$ up the incline

(b) $P = 200$ N; assume equilibrium

$\Sigma F_y = 0: \; N - 491\cos 15° + 200\sin 20° = 0, \; N = 405$ N

$\Sigma F_x = 0: \; 200\cos 20° - 491\sin 15° - F = 0, \; \underline{F = 61.0 \text{ N}}$

$F_{max} = \mu_s N = 0.25(405) = 101.3 \text{ N} > 61.0$ N so assumption OK

(c) $P = 250$ N; assume equilibrium

$\Sigma F_y = 0: \; N - 491\cos 15° + 250\sin 20° = 0, \; N = 388$ N

$\Sigma F_x = 0: \; 250\cos 20° - 491\sin 15° - F = 0, \; F = 108.0$ N

$F_{max} = \mu_s N = 0.25(388) = 97.1 \text{ N} < F;$ assumption invalid

$F = \mu_k N = 0.2(388) = \underline{77.7 \text{ N}}$ down the incline

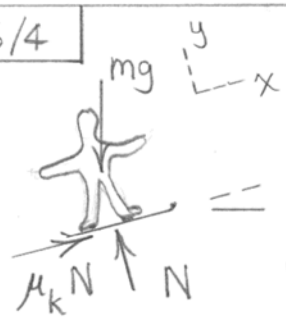
(d) To initiate motion, set $F = \mu_s N = 0.25N$ down the incline:

$\Sigma F_y = 0: \; N - 491\cos 15° + P\sin 20° = 0$

$\Sigma F_x = 0: \; P\cos 20° - 491\sin 15° - 0.25N = 0$

Solve to obtain $\begin{cases} N = 392 \text{ N} \\ \underline{P = 239 \text{ N}} \end{cases}$

6/4



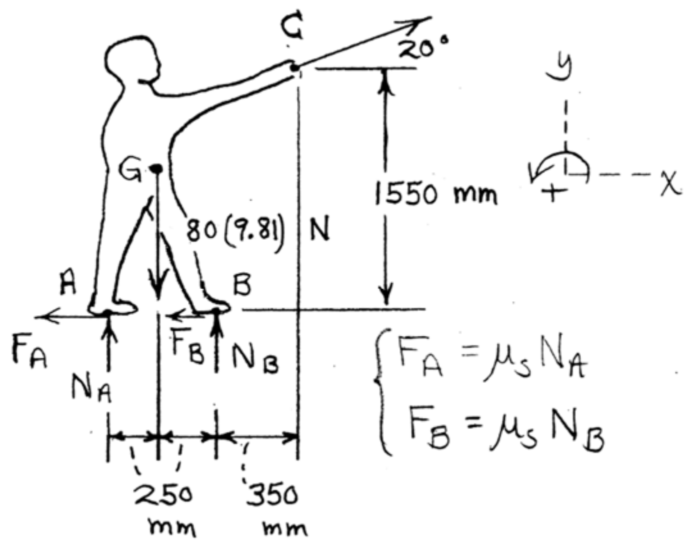
$$\begin{cases} \Sigma F_x = 0: \mu_k N - mg\sin\theta = 0 \\ \Sigma F_y = 0: N - mg\cos\theta = 0 \end{cases}$$

$$\Rightarrow N = mg\cos\theta$$

and $\mu_k mg\cos\theta = mg\sin\theta$

$$\Rightarrow \tan\theta = \mu_k, \quad \theta = \tan^{-1}(\mu_k) = \tan^{-1}(0.09) = \underline{5.14^\circ}$$

6/5



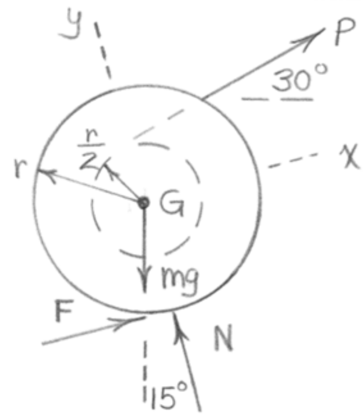
$$\begin{cases} F_A = \mu_s N_A \\ F_B = \mu_s N_B \end{cases}$$

$$\begin{cases} \Sigma F_x = 0: \ -\mu_s(N_A + N_B) + 65\cos 20° = 0 \\ \Sigma F_y = 0: \ N_A + N_B - 80(9.81) + 65\sin 20° = 0 \\ \Sigma M_B = 0: \ 80(9.81)(250) - N_A(500) - 65[1550\cos 20° \\ \qquad - 350\sin 20°] = 0 \end{cases}$$

Solve to obtain $N_A = 219$ N, $N_B = 544$ N

$\underline{\mu_s = 0.0801}$

6/6



$\Sigma F_x = 0: \quad P\cos 15° + F - mg\sin 15° = 0 \qquad (1)$

$\Sigma F_y = 0: \quad P\sin 15° - mg\cos 15° + N = 0 \qquad (2)$

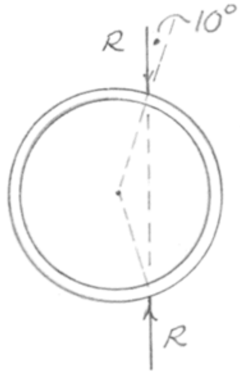
$\Sigma M_G = 0: \quad Fr - P\left(\frac{r}{2}\right) = 0 \qquad (3)$

Also, for impending slip: $F = \mu_s N \qquad (4)$

Algebraically solve Eqs. (1)-(4) to obtain

$\underline{\mu_s = 0.0959}$, $N = 0.920mg$, $\underline{F = 0.0883mg}$, $\underline{P = 0.1766mg}$

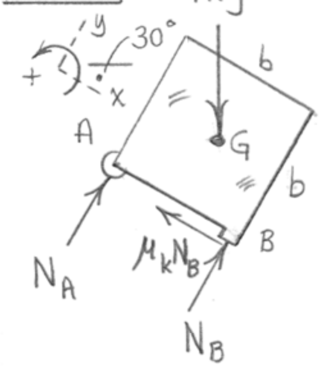
6/7



$$\mu_{s_{min}} = \tan\phi = \tan 10^\circ$$

$$= \underline{0.1763}$$

6/8



$\Sigma F_x = 0: -\mu_k N_B + mg \sin 30° = 0$

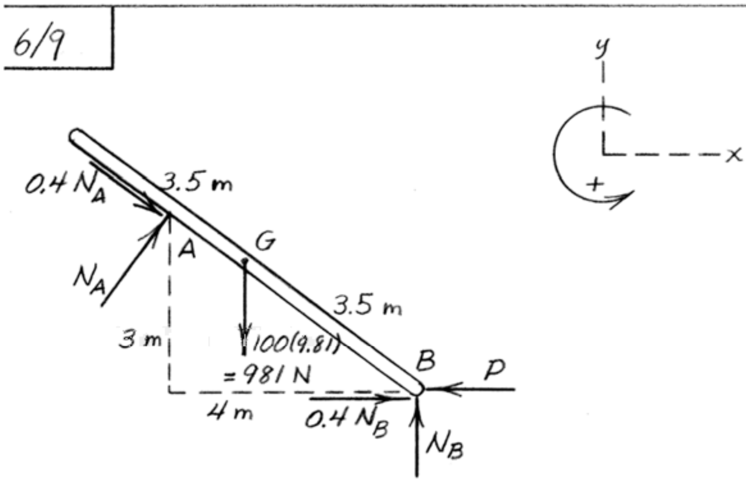
$\Sigma F_y = 0: N_A + N_B - mg \cos 30° = 0$

$\Sigma M_A = 0: N_B(b) - mg\left(\frac{b}{2}\cos 30° + \frac{b}{2}\sin 30°\right) = 0$

Solving : $\begin{cases} N_A = 0.1830mg \\ N_B = 0.683mg \\ \mu_k = 0.732 \end{cases}$

Reversing the roller and foot yields $\mu_k = 2.73$, an unlikelihood for simple contact.

6/9



$\Sigma M_B = 0$: $981(\frac{4}{5}\,3.5) - 5N_A = 0$, $N_A = 549$ N

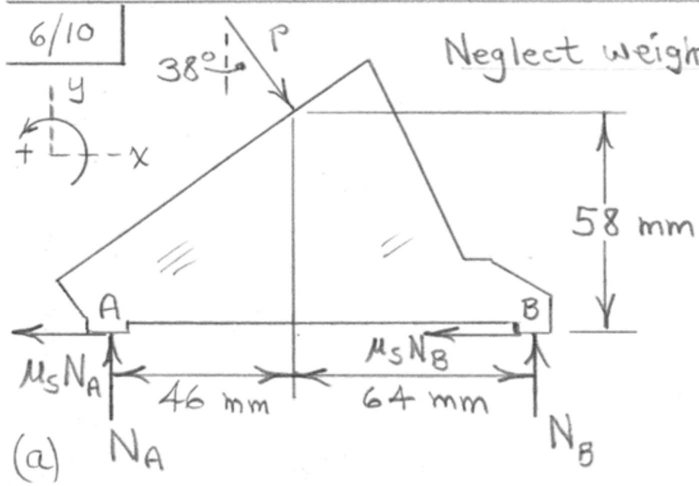
$\Sigma F_y = 0$: $N_B - 981 + \frac{4}{5}(549) - 0.4(549)\frac{3}{5} = 0$, $N_B = 673$ N

$\Sigma F_x = 0$: $-P + 0.4(673) + 549(\frac{3}{5}) + 0.4(549)\frac{4}{5} = 0$

$P = 775$ N

6/10

Neglect weight of chock



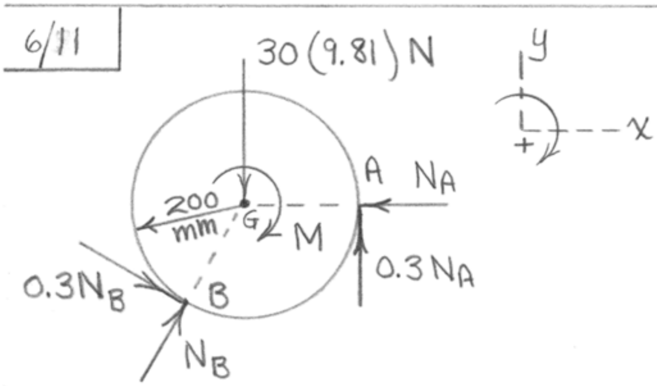
(a)

$$\begin{cases} \Sigma F_x = 0: & P\sin 38° - \mu_s N_A - \mu_s N_B = 0 \\ \Sigma F_y = 0: & -P\cos 38° + N_A + N_B = 0 \\ \Sigma M_A = 0: & N_B(110) - P\sin 38°(58) - P\cos 38°(46) = 0 \end{cases}$$

Solve: $N_A = 0.1339P$, $N_B = 0.654P$, $\underline{\mu_s = 0.781}$

(b) Eliminate $\mu_s N_A$ in the above equations to obtain $\underline{\mu_s = 0.941}$

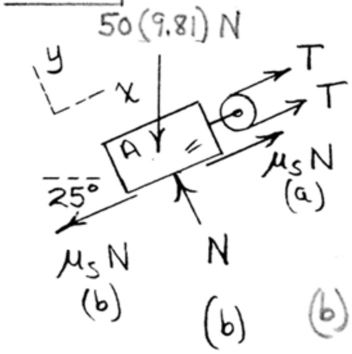
6/11



$$\begin{cases} \Sigma M_G = 0: & M - 0.3(N_A + N_B)\,0.2 = 0 & (1) \\ \Sigma F_x = 0: & N_B \sin 30° + 0.3 N_B \cos 30° - N_A = 0 & (2) \\ \Sigma F_y = 0: & N_B \cos 30° - 0.3 N_B \sin 30° - 30(9.81) & \\ & \quad + 0.3 N_A = 0 & (3) \end{cases}$$

Solution of Eqs. (1)–(3): $\begin{cases} N_B = 312 \text{ N} \\ N_A = 237 \text{ N} \\ M = 32.9 \text{ N·m} \end{cases}$

6/12



(a) Motion impends down incline

$\Sigma F_y = 0$: $N = 50(9.81)\cos 25°$

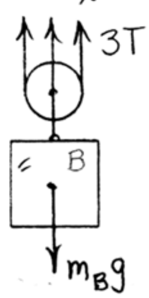
or $N = 445$ N throughout

$\Sigma F_x = 0$: $2T - 50(9.81)\sin 25° + 0.30(445) = 0$, $T = 37.0$ N

(b) Motion impends up incline

$\Sigma F_x = 0$: $2T - 50(9.81)\sin 25° - 0.30(445) = 0$

$T = 170.3$ N



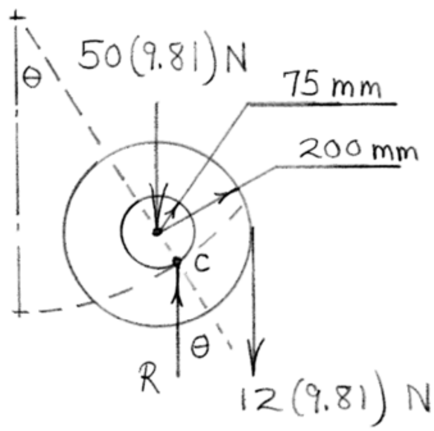
$\uparrow \Sigma F = 0$ $\quad 3T - m_B g = 0$, $\quad m_B = \dfrac{3T}{g}$

(a) $m_B = 3(37.0)/9.81 = 11.30$ kg

(b) $m_B = 3(170.3)/9.81 = 52.1$ kg

So $\underline{11.30 \leq m_B \leq 52.1 \text{ kg}}$

6/13

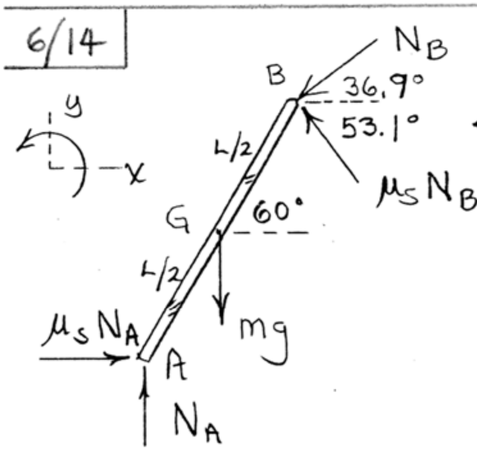


$$\circlearrowleft + \Sigma M_C = 0: \; 50(9.81)(75 \sin\theta) - 12(9.81)(200 - 75\sin\theta) = 0$$

$$\underline{\theta = 31.1^\circ}$$

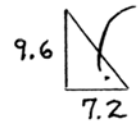
$$\mu_{min} = \tan\theta = \tan 31.1^\circ = \underline{0.603}$$

6/14



(1) $F = \mu_s N$ at both A and B

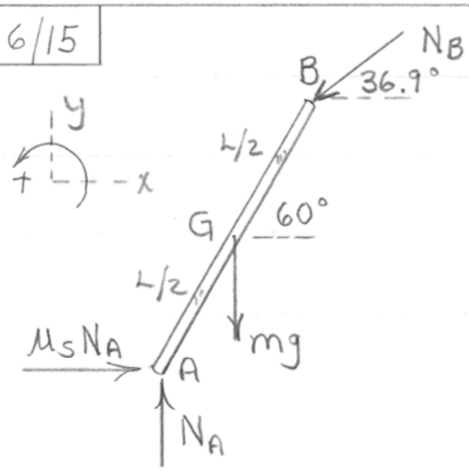
(2) $\tan^{-1}\left(\frac{9.6}{7.2}\right) = 53.1°$



$$\begin{cases} \Sigma F_x = 0: \mu_s N_A - N_B \cos 36.9° - \mu_s N_B \cos 53.1° = 0 \\ \Sigma F_y = 0: N_A - N_B \sin 36.9° + \mu_s N_B \sin 53.1° - mg = 0 \\ \Sigma M_B = 0: mg \frac{L}{2} \cos 60° + \mu_s N_A L \sin 60° \\ \qquad - N_A L \cos 60° = 0 \end{cases}$$

Solve to obtain
$$\begin{cases} N_A = 1.125\,mg \\ N_B = 0.364\,mg \\ \mu_s = 0.321 \end{cases}$$

6/15



$$\begin{cases} \Sigma F_x = 0: \mu_s N_A - N_B \cos 36.9^\circ = 0 \\ \Sigma F_y = 0: N_A - N_B \sin 36.9^\circ - mg = 0 \\ \Sigma M_B = 0: mg \frac{L}{2} \cos 60^\circ + \mu_s N_A L \sin 60^\circ \\ \qquad - N_A L \cos 60^\circ = 0 \end{cases}$$

Solve to obtain
$$\begin{cases} N_A = 1.382 mg \\ N_B = 0.636 mg \\ \mu_s = 0.368 \end{cases}$$

(μ_s here higher than in previous problem ✓)

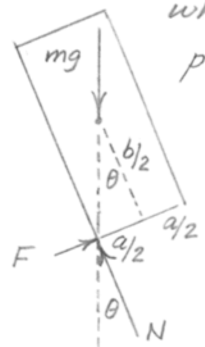
6/16

Block slips if $F = \mu N$ or $mg \sin\theta = \mu\, mg \cos\theta$

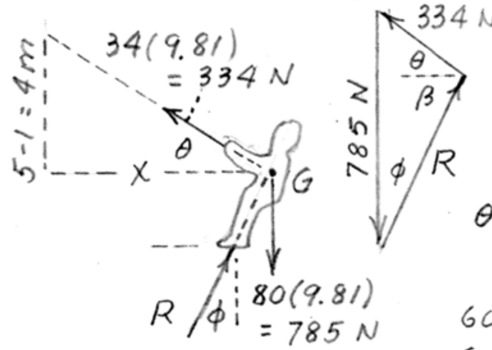
when angle reaches $\theta = \tan^{-1}\mu$

provided $\frac{a}{2} > \frac{b}{2}\tan\theta$ or $\underline{a > \mu b}$

Tips first if $\underline{a < \mu b}$



6/17 $\phi = \phi_{max} = \tan^{-1} 0.40 = 21.8°$



$\beta = 90 - 21.8 = 68.2°$

Law of sines

$$\frac{785}{\sin(\theta+\beta)} = \frac{334}{\sin 21.8}$$

$$\theta + \beta = \sin^{-1} \frac{785 \sin 21.8}{334}$$

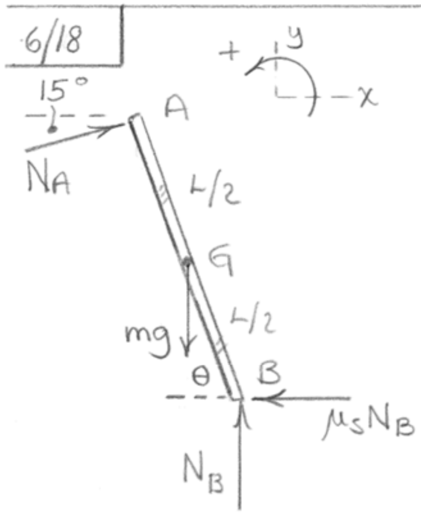
$$= 60.9° \text{ or } 119.1°$$

$60.9°$ sol. not possible

So $\theta = 119.1 - 68.2 = 50.9°$

So $\frac{4}{x} = \tan 50.9°$, $\underline{x = 3.25 \text{ m}}$

6/18



$\Sigma F_x = 0: \; N_A \cos 15° - \mu_s N_B = 0$

$\Sigma F_y = 0: \; N_B + N_A \sin 15° - mg = 0$

$\Sigma M_A = 0: \; -mg \frac{L}{2} \cos\theta + N_B L \cos\theta - \mu_s N_B L \sin\theta = 0$

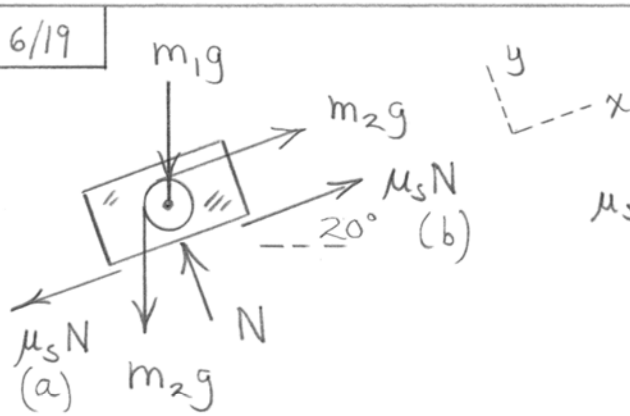
Eliminate N_A and N_B to obtain

$$\tan\theta = \frac{1 - \mu_s \tan 15°}{2\mu_s}$$

For $\mu_s = 0.25$, $\underline{\theta = 61.8°}$

For $\mu_s = 0.50$, $\underline{\theta = 40.9°}$

6/19



$\mu_s = 0.25$

(a) Motion impends up the incline

$$\begin{cases} \Sigma F_x = 0: -\mu_s N - m_1 g \sin 20° - m_2 g \sin 20° \\ \qquad + m_2 g = 0 \\ \Sigma F_y = 0: \; N - m_1 g \cos 20° - m_2 g \cos 20° = 0 \end{cases}$$

Solving, $m_2 = 1.364 m_1$

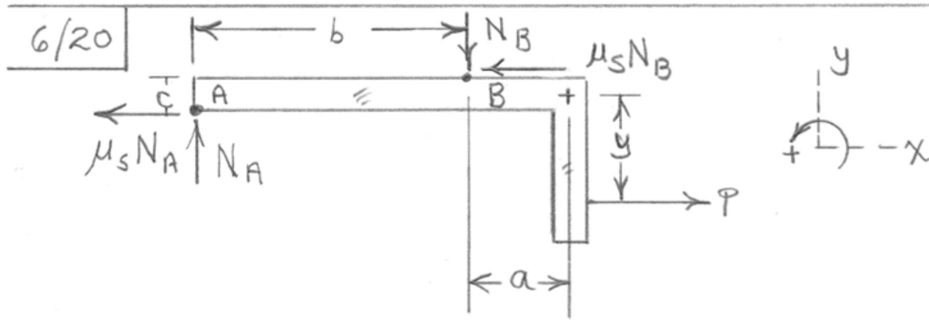
(b) Motion impends down the incline:

$$\begin{cases} \Sigma F_x = 0: \; \mu_s N - (m_1 + m_2) g \sin 20° + m_2 g = 0 \\ \Sigma F_y = 0: \; \text{(Does not change)} \end{cases}$$

Solving, $m_2 = 0.1199 m_1$

So $\underline{0.1199 m_1 \leq m_2 \leq 1.364 m_1}$

6/20



$\Sigma F_x = 0: -\mu_s(N_A + N_B) + P = 0$

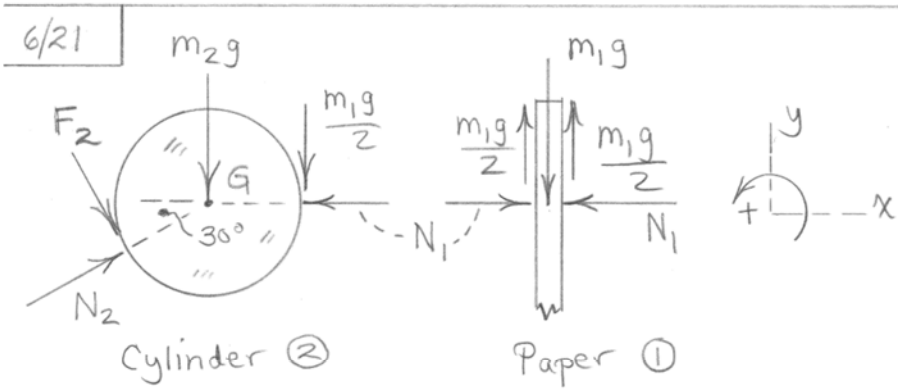
$\Sigma F_y = 0: \quad N_A - N_B = 0$

$\Sigma M_A = 0: -N_B(b) + \mu_s N_B(c) + P\left(y - \frac{c}{2}\right) = 0$

Solve to obtain $\underline{y = \dfrac{b}{2\mu_s}}$

(Independent of a and c!)

6/21



Cylinder:

$$\begin{cases} \Sigma F_x = 0: \quad N_2 \cos 30° + F_2 \sin 30° - N_1 = 0 & (1) \\ \Sigma F_y = 0: \quad N_2 \sin 30° - F_2 \cos 30° - m_2 g - \frac{m_1 g}{2} = 0 & (2) \\ \Sigma M_G = 0: \quad F_2 r - \frac{m_1 g}{2} r = 0 & (3) \end{cases}$$

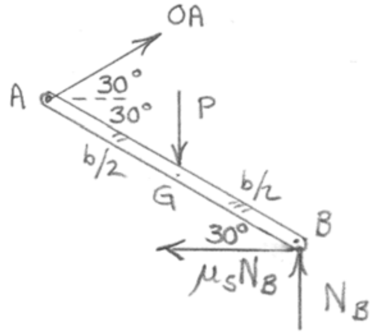
Solve simultaneously to obtain

$$\mu = \frac{m_1 g/2}{N_1} = \frac{m_1}{3.73 m_1 + 3.46 m_2} > \frac{F_2}{N_2}$$

where we have assumed and then verified that slipping occurs first on right side of the cylinder.

For $m_1 >> m_2$, $\underline{\mu = 0.268}$

6/22 (OA is a two-force member)

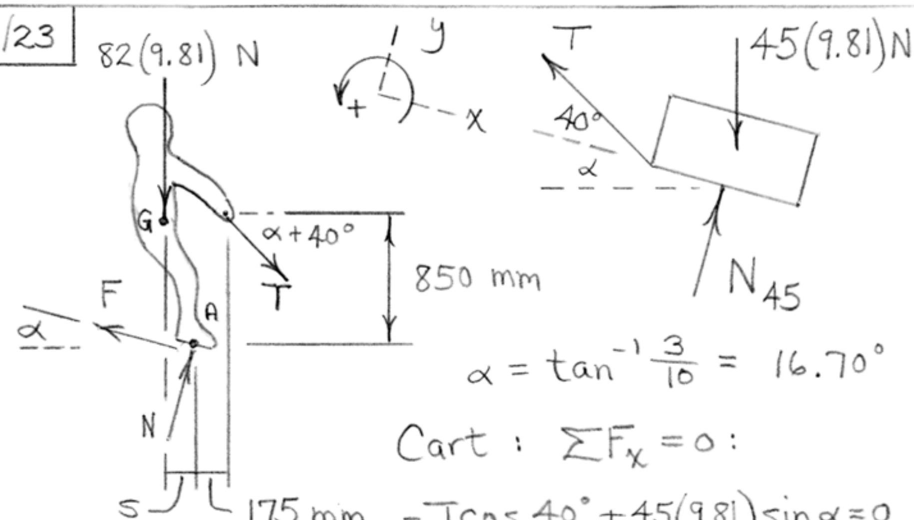


$\circlearrowright + \sum M_A = 0: \; -P\left(\frac{b}{2}\cos 30°\right) + N_B\left(b\cos 30°\right) - \mu_s N_B \, b\sin 30° = 0$

$\overset{+}{\nwarrow}\;60°\quad \sum F = 0: \; -P\cos 30° + N_B\cos 30° + \mu_s N_B \sin 30° = 0$

Solve to obtain $\underline{\mu_s = 0.577}$

6/23



$\alpha = \tan^{-1}\frac{3}{10} = 16.70°$

Cart : $\Sigma F_x = 0$:

$-T\cos 40° + 45(9.81)\sin\alpha = 0$

$T = 165.6$ N

Man :

$\Sigma F_x = 0: -F + 82(9.81)\sin\alpha + 165.6\cos 40° = 0$

$F = 358$ N

$\Sigma F_y = 0: \quad N - 82(9.81)\cos\alpha - 165.6\sin 40° = 0$

$N = 877$ N

$\mu_s = \frac{F}{N} = \frac{358}{877} = \underline{0.408}$

$\Sigma M_A = 0: \; 82(9.81)s - 165.6\cos(\alpha + 40)(850)$

$- 165.6\sin(\alpha + 40)(175) = 0$

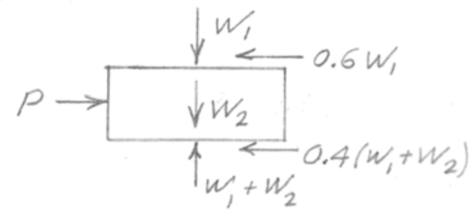
$\underline{s = 126.2 \text{ mm}}$

6/24 There are two possibilities

(a) Middle block moves; bottom one does not

$W_1 = 100(9.81) = 981$ N

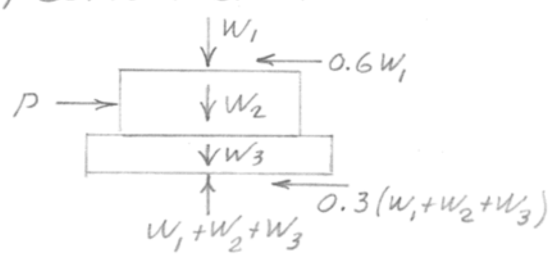
$W_2 = 50(9.81) = 490.5$ N



$\Sigma F = 0$; $P = 0.6(981) + 0.4(981 + 490.5) = 1177$ N

(b) Bottom block moves with middle block

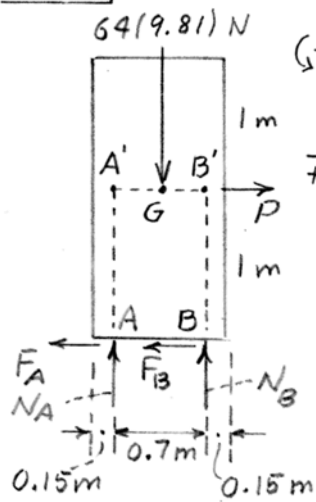
$W_3 = 20(9.81) = 196.2$ N



$\Sigma F = 0$; $P = 0.6(981) + 0.3(981 + 490.5 + 196.2) = 1089$ N

$1088 < 1177$ so case (b) occurs & $\underline{P = 1089 \text{ N}}$

6/25



(a) Set $F_B = 0$, $F_A = \mu_k N_A = 0.3 N_A$

$\circlearrowleft + \Sigma M_{B'} = 0$: $64(9.81)(0.35) - N_A(0.7) - 0.3 N_A(1) = 0$, $N_A = 220$ N

$\overset{\rightarrow}{+} \Sigma F = 0$: $P - 0.3(220) = 0$, $\underline{P = 65.9 \text{ N}}$

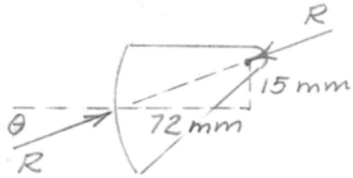
(b) Set $F_A = 0$, $F_B = \mu_k N_B = 0.3 N_B$

$\circlearrowleft + \Sigma M_{A'} = 0$: $-64(9.81)(0.35) + N_B(0.7) - 0.3 N_B(1) = 0$, $N_B = 549$ N

$\overset{\rightarrow}{+} \Sigma F = 0$: $P - 0.3(549) = 0$

$\underline{P = 164.8 \text{ N}}$

6/26



Req'd. minimum coeff. of friction is

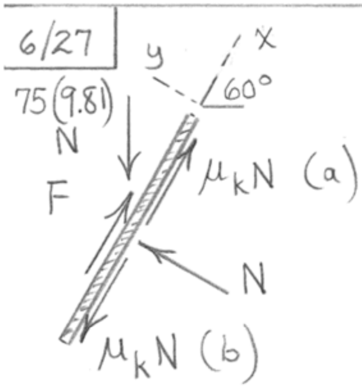
$\mu_s = \tan\phi = \tan\theta$

$= \frac{15}{72} = \underline{0.208}$

$\Sigma F = 0$ for rope; $2R\sin\theta = 600$

$R = \frac{300}{15/\sqrt{15^2+72^2}} = \underline{1471 \text{ N}}$

6/27



$\Sigma F_y = 0: \; N - 75(9.81)\cos 60° = 0$

$N = 368 \text{ N}$

(a) $\Sigma F_x = 0:$

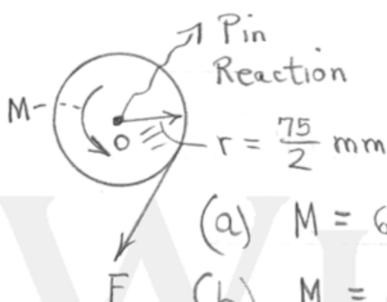
$F + 0.05(368) - 75(9.81)\sin 60° = 0$

$F = 619 \text{ N}$

(b) $\Sigma F_x = 0: \; F - 0.05(368) - 75(9.81)\sin 60° = 0$

$F = 656 \text{ N}$

Gear wheel



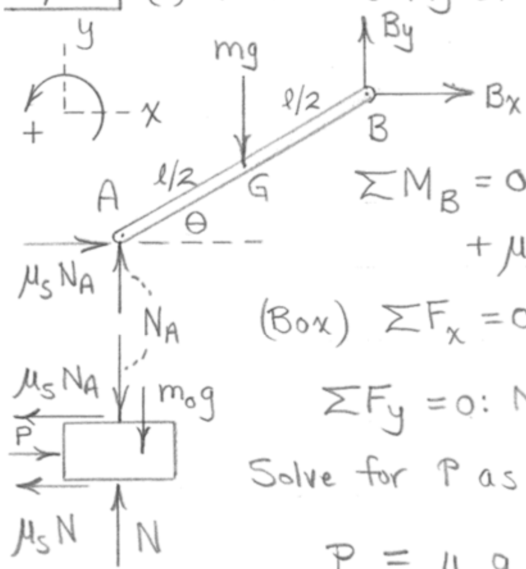
$\circlearrowleft + \; \Sigma M_O = 0:$

$M - Fr = 0, \quad M = Fr$

(a) $M = 619\left(\frac{0.075}{2}\right) = \underline{23.2 \text{ N}\cdot\text{m}}$

(b) $M = 656\left(\frac{0.075}{2}\right) = \underline{24.6 \text{ N}\cdot\text{m}}$

6/28 (a) P to the right.



$\Sigma M_B = 0:\ mg\frac{\ell}{2}\cos\theta - N_A \ell\cos\theta + \mu_s N_A \ell \sin\theta = 0$ (1)

(Box) $\Sigma F_x = 0:\ P - \mu_s N_A - \mu_s N = 0$ (2)

$\Sigma F_y = 0:\ N - m_o g - N_A = 0$ (3)

Solve for P as

$$P = \mu_s g\left[\frac{m\cos\theta}{\cos\theta - \mu_s \sin\theta} + m_o\right]$$

(b) P to the left. Reverse P and all friction forces in the above FBD's & obtain

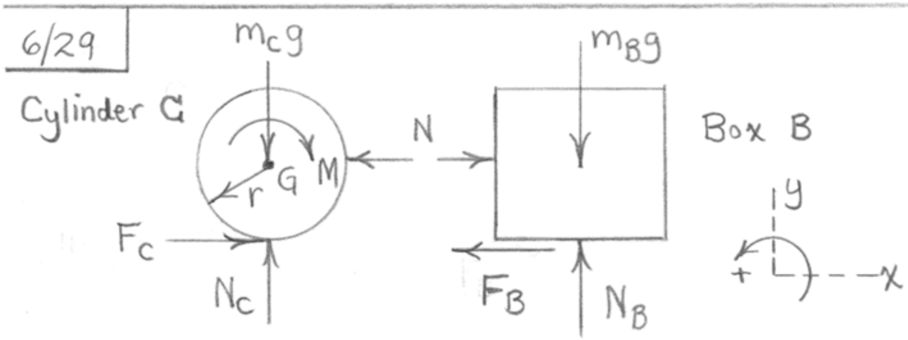
$$P = \mu_s g\left[\frac{m\cos\theta}{\cos\theta + \mu_s \sin\theta} + m_o\right]$$

With $\theta = 30°$, $m = m_o = 3\text{ kg}$, and $\mu_s = 0.60$, we obtain

(a) $P = 44.7$ N

(b) $P = 30.8$ N

6/29



Assume that box slips but cylinder does not.

$F_B = (\mu_s)_B N_B$.

$$B \begin{cases} \Sigma F_x = 0: & N - F_B = 0 & (1) \\ \Sigma F_y = 0: & N_B - m_B g = 0 & (2) \end{cases}$$

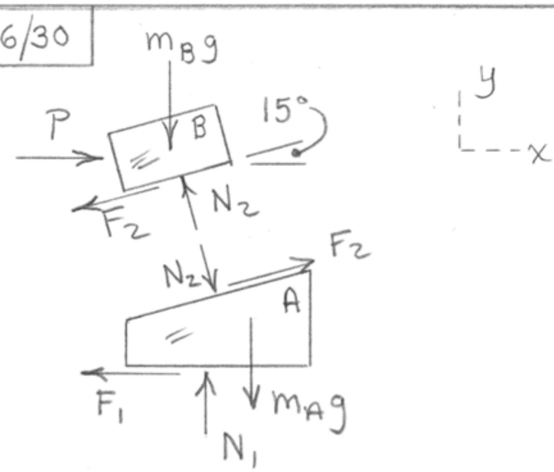
So $N_B = m_B g$, $N = F_B = (\mu_s)_B m_B g$

$$C \begin{cases} \Sigma F_x = 0: & F_C - N = 0 & (3) \\ \Sigma F_y = 0: & N_C - m_C g = 0 & (4) \\ \Sigma M_G = 0: & F_C r - M = 0 & (5) \end{cases}$$

$$M = F_C r = N r = (\mu_s)_B m_B g r$$
$$= 0.5(3)(9.81)(0.2) = \underline{2.94 \text{ N}\cdot\text{m}}$$

$$F_C = N = (\mu_s)_B m_B g = (0.5)(3)(9.81) = 14.72 \text{ N}$$
$$< (F_C)_{max} = (\mu_s)_C m_C g = (0.4)(6)(9.81) = 23.5 \text{ N}$$

6/30



B:
$$\Sigma F_x = 0: \; P - F_2 \cos 15° - N_2 \sin 15° = 0$$
$$\Sigma F_y = 0: \; N_2 \cos 15° - F_2 \sin 15° - m_B g = 0$$

A:
$$\Sigma F_x = 0: \; N_2 \sin 15° + F_2 \cos 15° - F_1 = 0$$
$$\Sigma F_y = 0: \; N_1 - N_2 \cos 15° + F_2 \sin 15° - m_A g = 0$$

(a) With $m_A = 10$ kg, $m_B = 5$ kg, & $P = 50$ N, find $F_1 = 50$ N, $F_2 = 35.6$ N, $N_1 = 147.2$ N, $N_2 = 60.3$ N

So $F_{1_{max}} = 0.40(147.2) = 58.9$ N, $F_{2_{max}} = 0.50(60.3) = 30.2$ N

$F_2 > F_{2_{max}}$, so slips between A and B

(and μ_k governs there instead of μ_s)

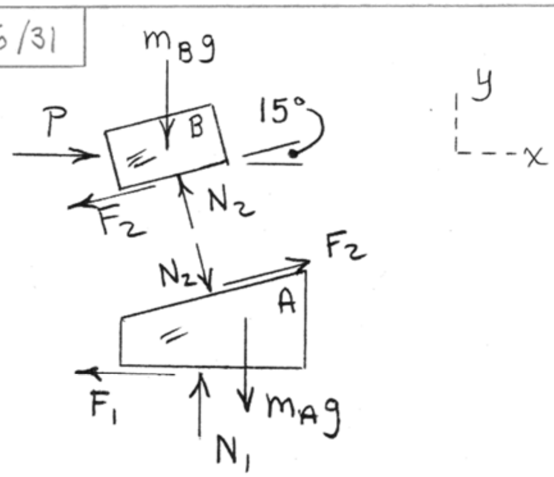
(b) With $\mu_1 = 0.30$ and $\mu_2 = 0.60$, the four equilibrium eqs. yield the same solutions. But

$F_{1_{max}} = 0.30(147.2) = 44.1$ N

$F_{2_{max}} = 0.60(60.3) = 36.2$ N

$F_1 > F_{1_{max}}$, so slippage occurs between A and the ground.

6/31



$$B\begin{cases}\Sigma F_x = 0: & P - F_2\cos 15^\circ - N_2 \sin 15^\circ = 0\\ \Sigma F_y = 0: & N_2\cos 15^\circ - F_2 \sin 15^\circ - m_B g = 0\end{cases}$$

$$A\begin{cases}\Sigma F_x = 0: & N_2\sin 15^\circ + F_2 \cos 15^\circ - F_1 = 0\\ \Sigma F_y = 0: & N_1 - N_2\cos 15^\circ + F_2 \sin 15^\circ - m_A g = 0\end{cases}$$

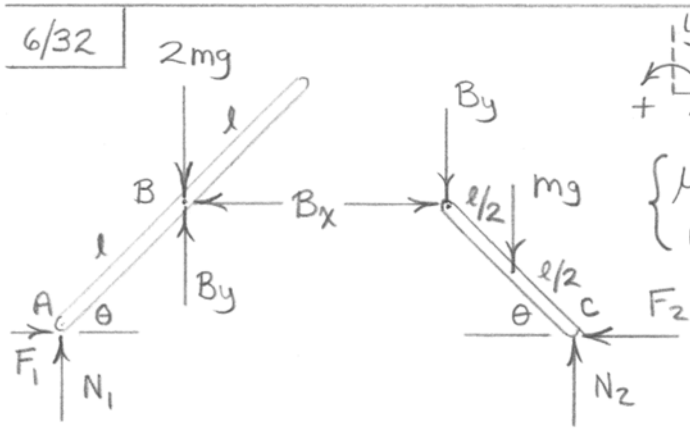
With $m_A = 10$ kg, $m_B = 5$ kg, & $P = 40$ N:

$F_1 = 40$ N, $F_2 = 25.9$ N, $N_1 = 147.2$ N, $N_2 = 57.7$ N

$$\left.\begin{array}{l} F_{1_{max}} = 0.30(147.2) = 44.1 \text{ N} \\ F_{2_{max}} = 0.50(57.7) = 28.9 \text{ N}\end{array}\right\} \text{No slippage}$$

So B exerts $\underline{40\mathbf{i} - 49.0\mathbf{j} \text{ N}}$ on A.

6/32



$\left\{ \mu_s = 0.5 \text{ @ } A \;\&\; C. \right.$

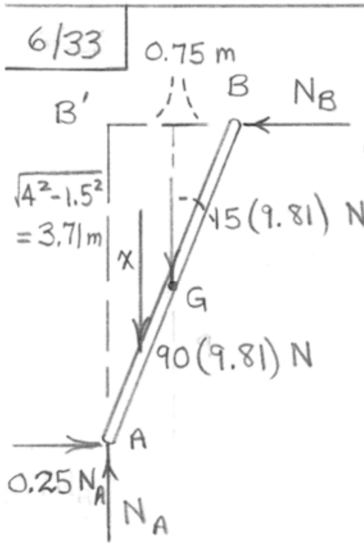
AB
$$\begin{cases} \Sigma F_x = 0: & F_1 - B_x = 0 & (1) \\ \Sigma F_y = 0: & N_1 + B_y - 2mg = 0 & (2) \\ \Sigma M_A = 0: & B_x(l\sin\theta) + B_y(l\cos\theta) - 2mg(l\cos\theta) = 0 & (3) \end{cases}$$

BC
$$\begin{cases} \Sigma F_x = 0: & B_x - F_2 = 0 & (4) \\ \Sigma F_y = 0: & -B_y - mg + N_2 = 0 & (5) \\ \Sigma M_C = 0: & mg(\frac{l}{2}\cos\theta) + B_y(l\cos\theta) - B_x(l\sin\theta) = 0 & (6) \end{cases}$$

Assume first slippage at A: $F_1 = 0.5N_1$. Solve seven equations to obtain $\underline{\theta = 63.4°}$, $F_2 = 0.625\,mg$, & $N_2 = 1.75\,mg$. Note $F_2 < F_{2max} = 0.875\,mg$.

Then assume first slippage at B: $F_2 = 0.5N_2$. Obtain $\theta = 55.0°$, $F_1 = 0.875\,mg$ & $N_1 = 1.25\,mg$. Note $F_1 > F_{1max} = 0.625\,mg$. So A slips first.

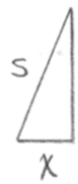
6/33



$+\uparrow \Sigma F = 0 \Rightarrow N_A = (90+15)9.81 = 1030$ N

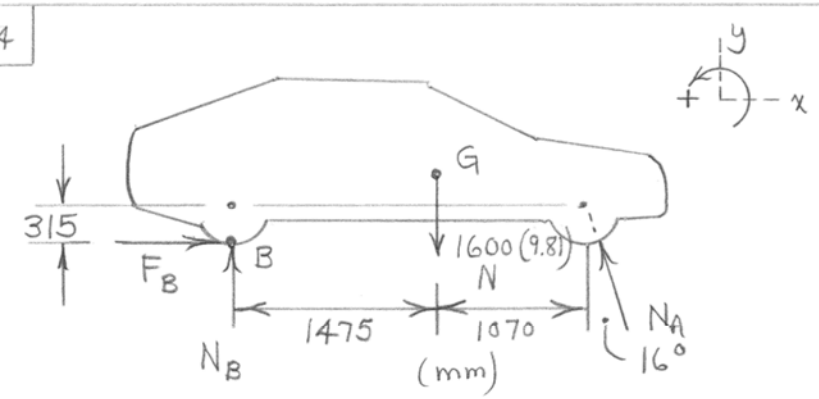
$\circlearrowleft + \Sigma M_{B'} = 0: 0.25 N_A (3.71) - 90(9.81)x$

$- 15(9.81)(0.75) = 0, \quad x = 0.957$ m



$\dfrac{s}{x} = \dfrac{4}{1.5}, \quad \underline{s = 2.55 \text{ m}}$

6/34



$$\Sigma F_x = 0: \quad F_B - N_A \sin 16° = 0 \qquad (1)$$

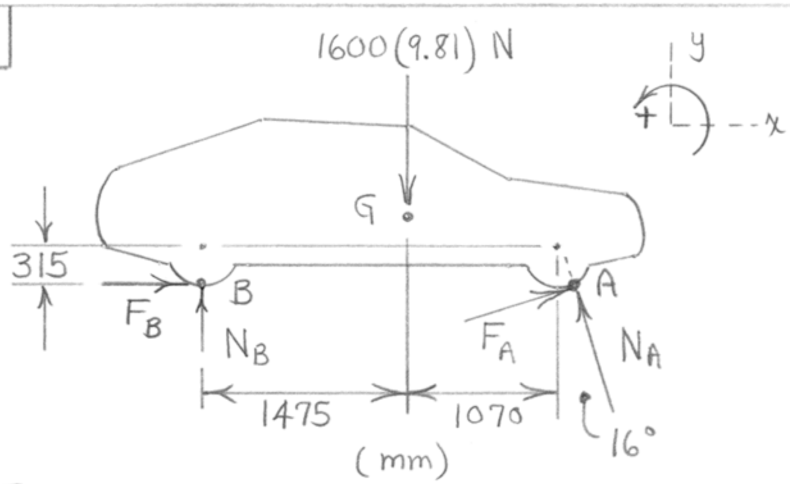
$$\Sigma F_y = 0: \quad N_B + N_A \cos 16° - 1600(9.81) = 0 \qquad (2)$$

$$\Sigma M_B = 0: \quad -1600(9.81)(1475) + N_A \cos 16° (2545) + N_A \sin 16° (315) = 0 \qquad (3)$$

Solution: $N_A = 9140$ N, $N_B = 6910$ N, $F_B = 2520$ N

$$\mu_s = \frac{F_B}{N_B} = \frac{2520}{6910} = \underline{0.365}$$

6/35



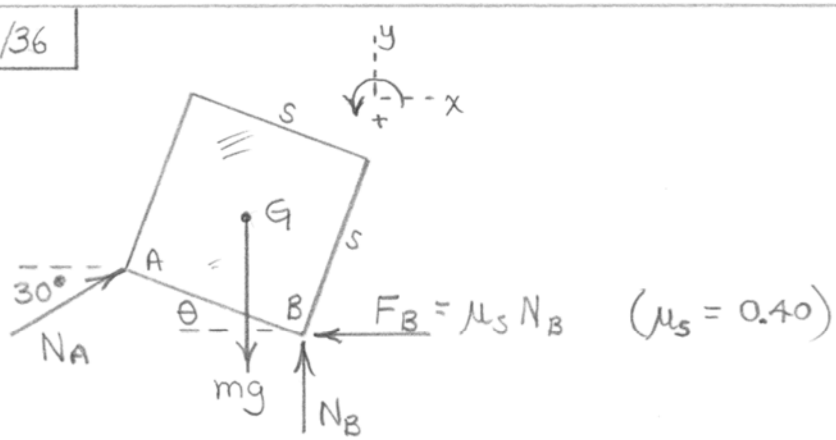
$$\begin{cases} \Sigma F_x = 0: \quad F_B + F_A \cos 16° - N_A \sin 16° = 0 & (1) \\ \Sigma F_y = 0: \quad N_B + F_A \sin 16° + N_A \cos 16° - 1600(9.81) = 0 & (2) \\ \Sigma M_A = 0: \quad -N_B(2545 + 315 \sin 16°) & \end{cases}$$

$$+ F_B(315 - 315 \cos 16°) + 1600(9.81)(1070 + 315 \sin 16°) = 0 \quad (3)$$

Also, set $\begin{cases} F_A = \mu_s N_A & (4) \\ F_B = \mu_s N_B & (5) \end{cases}$

Solution: $\begin{cases} F_A = 1378 \text{ N} & F_B = 1087 \text{ N} \\ N_A = 8750 \text{ N} & N_B = 6900 \text{ N} \\ \underline{\mu_s = 0.1575} & \end{cases}$

6/36



$\Sigma F_x = 0: \quad N_A \cos 30° - \mu_s N_B = 0 \qquad (1)$

$\Sigma F_y = 0: \quad N_A \sin 30° - mg + N_B = 0 \qquad (2)$

$\Sigma M_A = 0: \quad N_B (s \cos\theta) - \mu_s N_B (s \sin\theta) - mg\left(\frac{s}{2}\sin\theta\right) - mg\left(\frac{s}{2}\cos\theta\right) = 0$

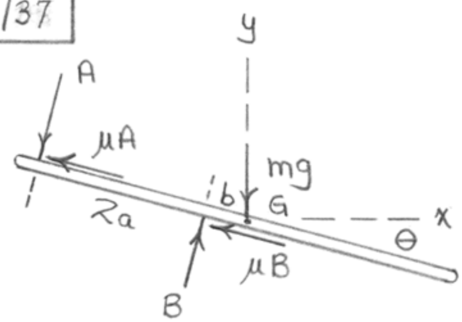
or $N_B(\cos\theta - \mu_s \sin\theta) - \frac{mg}{2}(\sin\theta + \cos\theta) = 0 \quad (3)$

Solution: $N_A = 0.375\,mg$

$N_B = 0.812\,mg$

$\underline{\theta = 20.7°}$

6/37



$\circlearrowleft + \Sigma M_G = 0: \quad A(2a+b) - Bb = 0, \quad \frac{B}{A} = \frac{2a+b}{b}$

$\Sigma F_x = 0: \quad (B-A)\sin\theta - \mu(A+B)\cos\theta = 0$

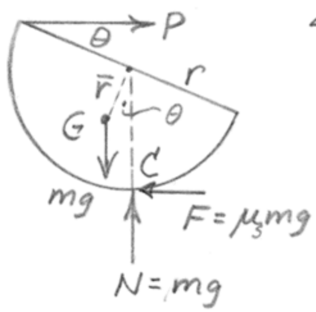
$$\tan\theta = \mu\frac{A+B}{B-A} = \mu\frac{1+B/A}{\frac{B}{A}-1}$$

Substitute $\frac{B}{A}$: $\tan\theta = \mu\frac{1+(2a+b)/b}{(2a+b)/b - 1}$

or $\underline{\theta = \tan^{-1}\left(\mu\frac{a+b}{a}\right)}$

6/38

$\bar{r} = 4r/3\pi$



$\Sigma M_C = 0;\ mg\,\frac{4r}{3\pi}\sin\theta = Pr(1+\sin\theta)$

$P = \frac{4mg\sin\theta}{3\pi(1+\sin\theta)}$

$\Sigma F = 0;\ F = P$ so

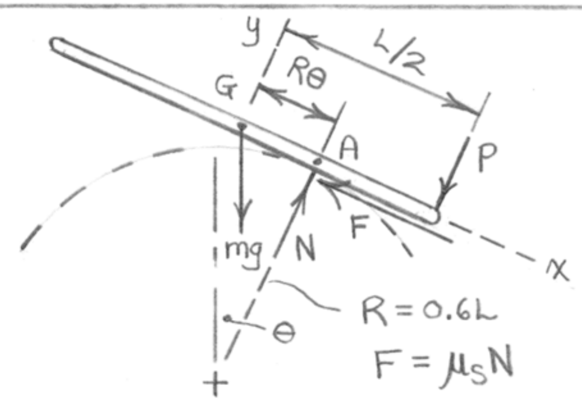
$\frac{4mg}{3\pi}\frac{\sin\theta}{1+\sin\theta} = \mu_s mg$

$3\pi\mu_s(1+\sin\theta) = 4\sin\theta,\quad \underline{\theta = \sin^{-1}\frac{3\pi\mu_s}{4-3\pi\mu_s}}$

For $\theta \to \pi/2$, $\sin\theta = 1$ & $3\pi\mu_s = 4 - 3\pi\mu_s$

$\underline{\mu_s = \frac{2}{3\pi}} = 0.212$

6/39



$$\Sigma F_x = 0: \quad mg\sin\theta - \mu_s N = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad N - P - mg\cos\theta = 0 \qquad (2)$$

$$\circlearrowleft + \Sigma M_A = 0: \quad mg\,R\theta\cos\theta - P\left(\frac{L}{2} - R\theta\right) = 0$$

With $R = 0.6L$: $\quad 0.6\,mg\,\theta\cos\theta - P\left(\frac{1}{2} - 0.6\theta\right) = 0 \qquad (3)$

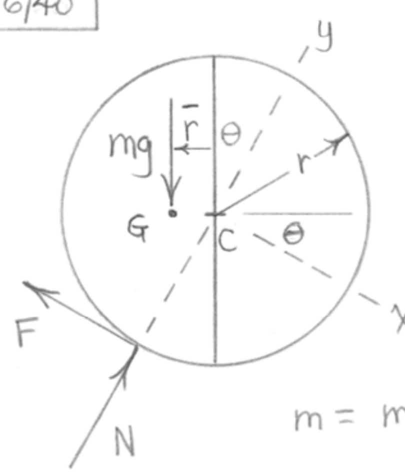
(1) & (2): $\quad mg\sin\theta = \mu_s\left(P + mg\cos\theta\right) \qquad (4)$

(3) & (4): $\quad mg\sin\theta = \mu_s\left[\dfrac{0.6\,mg\,\theta\cos\theta}{0.5 - 0.6\theta} + mg\cos\theta\right]$

$$= \mu_s\, mg\cos\theta\left(\frac{0.5}{0.5 - 0.6\theta}\right)$$

$$\mu_s = \left(1 - \frac{6}{5}\theta\right)\tan\theta = \left(1 - \frac{6}{5}\cdot 20^\circ \cdot \frac{\pi}{180^\circ}\right)\tan 20^\circ = \underline{0.212}$$

6/40



$m_{Al} = \rho_{Al} \pi r^2 t_{Al}$ (t = depth)

$= 2690\pi (0.080)^2 (0.040)$

$= 2.16$ kg

$m_{st} = \rho_{st} \pi r^2 t_{st} \cdot \frac{1}{2}$

$= 7830\pi (0.080)^2 (0.016) \frac{1}{2}$

$= 1.259$ kg

$m = m_{Al} + m_{st} = 3.42$ kg

$\bar{r}_{st} = \frac{4r}{3\pi} = \frac{4(80)}{3\pi} = 34.0$ mm

$\bar{r} = \frac{\Sigma m\bar{r}}{\Sigma m} = \frac{2.16(0) + 1.259(34.0)}{3.42} = 12.49$ mm

$\circlearrowleft \Sigma M_C = 0: \; mg\bar{r} - Fr = 0, \quad F = mg\frac{\bar{r}}{r}$

$F = 3.42(9.81)\frac{12.49}{80} = 5.24$ N

$\Sigma F_x = 0: \; -5.24 + 3.42(9.81)\sin\theta = 0, \quad \underline{\theta = 8.98°}$

$\Sigma F_y = 0: \; N - 3.42(9.81)\cos 8.98° = 0, \quad N = 33.2$ N

$\mu_s = \frac{F}{N} = \frac{5.24}{33.2} = \underline{0.1581}$

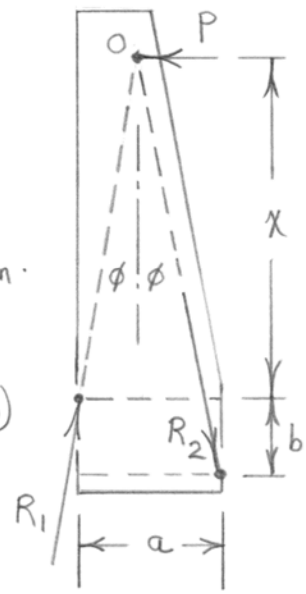
6/41

For equilibrium, the forces must be concurrent at O and $\phi = \tan^{-1}\mu_s$ for impending motion with $x = x_{min}$.

$$a = x\tan\phi + (x+b)\tan\phi$$

$$= x\mu_s + (x+b)\mu_s = \mu_s(2x+b)$$

$$x = \frac{a - b\mu_s}{2\mu_s}$$



6/42

$\Sigma F_y = 0 \Rightarrow N = mg$

$\Sigma F_x = 0 \Rightarrow P = \mu_s N$

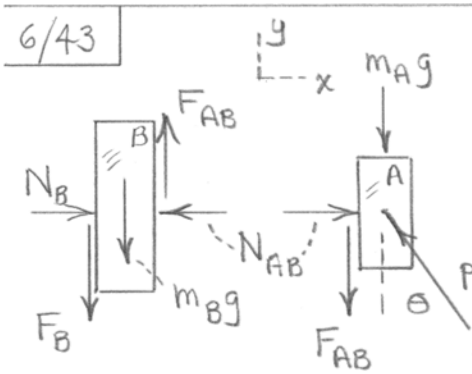
$\Sigma M_C = 0: \; P(r + r\sin\theta) - mg\bar{r}\sin\theta = 0, \quad \bar{r} = \frac{2r}{\pi}$

$\therefore \mu_s mg(r + r\sin\theta) = mg\frac{2r}{\pi}\sin\theta$

$\sin\theta\left[\frac{2}{\pi} - \mu_s\right] = \mu_s, \quad \underline{\theta = \sin^{-1}\left(\frac{\pi\mu_s}{2 - \pi\mu_s}\right)}$

$\theta = 90°$ when $\frac{\pi\mu_s}{2 - \pi\mu_s} = 1$ or $\underline{\mu_{90°} = \frac{1}{\pi} = 0.318}$

6/43



Conditions:

$m_A = 2$ kg, $m_B = 3$ kg

$\mu_1 = 0.70$, $\mu_2 = 0.50$

$P = 50$ N, $\theta = 40°$

Assume no slipping:

(A) $\begin{cases} \Sigma F_x = 0: & N_{AB} - P\sin\theta = 0 \\ \Sigma F_y = 0: & P\cos\theta - F_{AB} - m_A g = 0 \end{cases}$

(B) $\begin{cases} \Sigma F_x = 0: & N_B - N_{AB} = 0 \\ \Sigma F_y = 0: & F_{AB} - F_B - m_B g = 0 \end{cases}$

Solve: $\begin{cases} N_B = 32.1 \text{ N} & F_B = -10.75 \text{ N} \\ N_{AB} = 32.1 \text{ N} & F_{AB} = 18.68 \text{ N} \end{cases}$

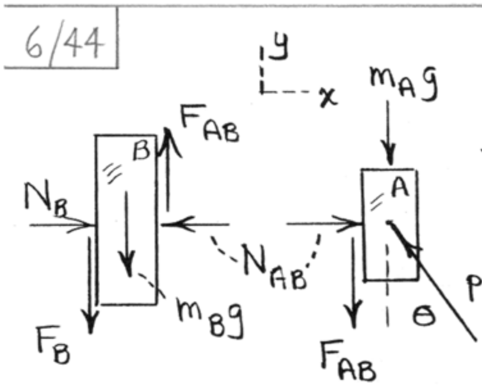
$(F_B)_{max} = \mu_2 N_B = 0.50(32.1) = 16.07$ N

$(F_{AB})_{max} = \mu_1 N_{AB} = 0.70(32.1) = 22.5$ N

No-slip assumptions are both valid.

So, along with 18.68 N up, we have 32.1 N left, or $\underline{37.2 \text{ N} \angle 149.8°}$

6/44



Conditions:
$m_A = 2$ kg, $m_B = 3$ kg
$\mu_1 = 0.70$, $\mu_2 = 0.50$
$P = 50$ N, $\theta = 30°$

Assume no slipping:

(A) $\begin{cases} \Sigma F_x = 0: & N_{AB} - P\sin\theta = 0 \\ \Sigma F_y = 0: & P\cos\theta - F_{AB} - m_A g = 0 \end{cases}$

(B) $\begin{cases} \Sigma F_x = 0: & N_B - N_{AB} = 0 \\ \Sigma F_y = 0: & F_{AB} - F_B - m_B g = 0 \end{cases}$

Solve: $\begin{cases} N_B = N_{AB} = 25 \text{ N} \\ F_B = -5.75 \text{ N}, \quad F_{AB} = 23.7 \text{ N} \end{cases}$

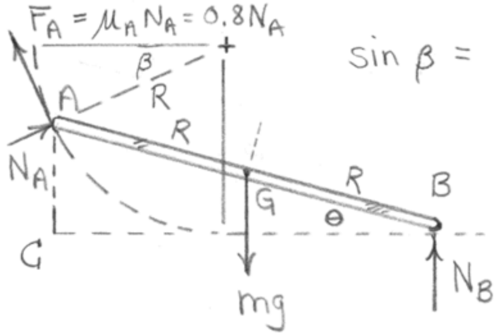
$(F_B)_{max} = \mu_2 N_B = 0.50(25) = 12.50$ N

$(F_{AB})_{max} = \mu_1 N_{AB} = 0.70(25) = 17.50$ N

So slippage occurs between A and B

Requested force: $\mu_k N_{AB} = 0.75(0.70)(25) = 13.12$ N up & 25 N left, or $\underline{28.2 \text{ N} \angle 152.3°}$

6/45



$\overline{AC} = 2R\sin\theta$

$\sin\beta = \dfrac{R - 2R\sin\theta}{R} = 1 - 2\sin\theta$ (1)

$\cos\beta = \sqrt{1-\sin^2\beta}$

$= \sqrt{1-(1-2\sin\theta)^2}$

$= 2\sqrt{\sin\theta(1-\sin\theta)}$

$\nearrow\!\!+ \; \Sigma F = 0: \; -F_A \sin\beta + N_A \cos\beta = 0$ (2)

$-\mu_A N_A (1-2\sin\theta) + N_A \, 2\sqrt{\sin\theta(1-\sin\theta)} = 0$

Solve the resulting quadratic in $\sin\theta$ to obtain

$$\begin{cases} \underline{\theta = 6.29°} \\ \theta = 62.9° \text{ (physically impossible)} \end{cases}$$

Better solution: Eq. (2) gives $\tan\beta = \dfrac{N_A}{F_A} = \dfrac{1}{\mu_A}$

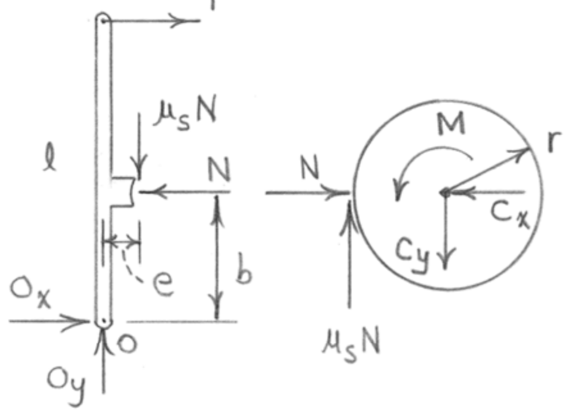
$\tan\beta = \dfrac{1}{0.8}$, $\beta = 51.3°$

Then Eq. (1) gives $\sin\theta = \dfrac{1-\sin\beta}{2} = \dfrac{1-\sin 51.3°}{2}$

$\underline{\theta = 6.29°}$ ✓

(Quadratic equation with its extraneous root is avoided!)

6/46



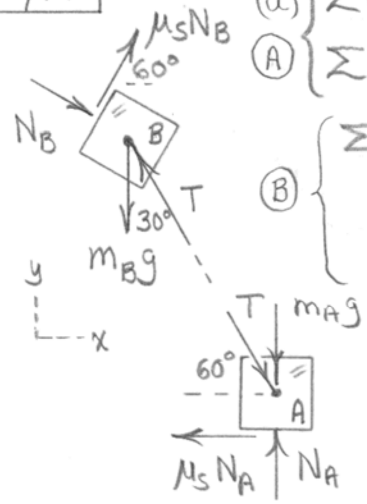
Wheel: $\circlearrowleft + \Sigma M_C = 0: \quad M - \mu_s N r = 0, \quad \mu_s N = M/r$

Lever: $\circlearrowleft + \Sigma M_O = 0: \quad Nb - Pl - \mu_s N e = 0$

$$P = \frac{M}{rl}\left(\frac{b}{\mu_s} - e\right)$$

If $b = \mu_s e$, $P = 0$ & brake would be self-locking.

6/47



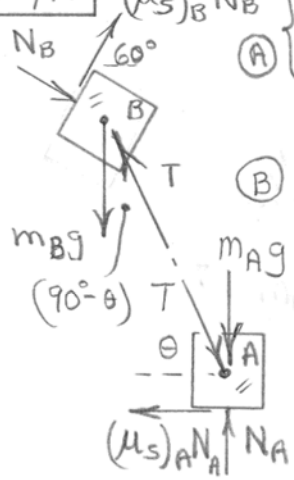
(a) (A) $\begin{cases} \Sigma F_x = 0: & T\cos 60° - \mu_s N_A = 0 \\ \Sigma F_y = 0: & N_A - m_A g - T\sin 60° = 0 \end{cases}$

(B) $\begin{cases} \Sigma F_x = 0: & N_B \cos 30° + \mu_s N_B \cos 60° - T\sin 30° = 0 \\ \Sigma F_y = 0: & -N_B \sin 30° + \mu_s N_B \sin 60° + T\cos 30° - m_B g = 0 \end{cases}$

Solve to obtain $\underline{\mu_s = 0.309}$

(b) Eliminate the friction force on B, & resolve to obtain $\underline{\mu_s = 0.346}$.

*6/48



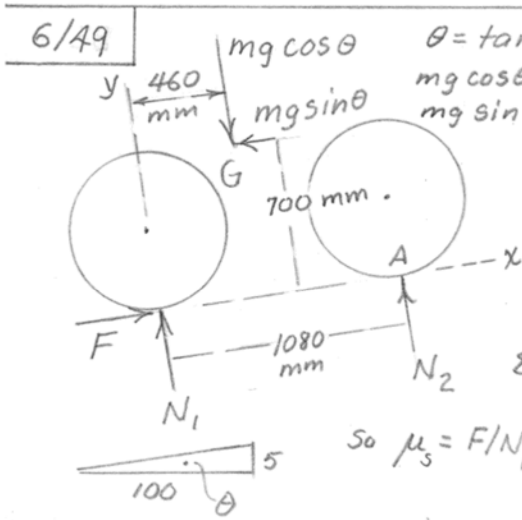
Ⓐ $\begin{cases} \Sigma F_x = 0: \; T\cos\theta - (\mu_s)_A N_A = 0 \\ \Sigma F_y = 0: \; N_A - m_A g - T\sin\theta = 0 \end{cases}$

Ⓑ $\begin{cases} \Sigma F_x = 0: \; N_B \cos 30^\circ + (\mu_s)_B N_B \cos 60^\circ - T\sin(90^\circ - \theta) = 0 \\ \Sigma F_y = 0: \; -N_B \sin 30^\circ + (\mu_s)_B N_B \sin 60^\circ + T\cos(90^\circ - \theta) - m_B g = 0 \end{cases}$

With $m_A = m_B = 5\text{ kg}$, $(\mu_s)_A = 0.40$, and $(\mu_s)_B = 0.30$:

$\underline{\theta = 53.8^\circ}$ (solved numerically)

6/49



$\theta = \tan^{-1} 5/100 = 2.86°$

$mg\cos\theta = 82(9.81)(0.9998) = 803.4\ N$

$mg\sin\theta = 82(9.81)(0.0499) = 40.2\ N$

$\Sigma M_A = 0;$

$803.4(620) + 40.2(700) - 1080 N_1 = 0$

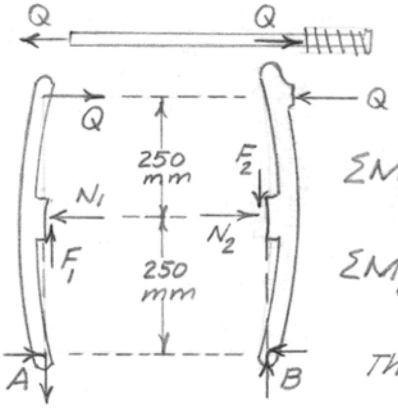
$N_1 = 487.3\ N$

$\Sigma F_x = 0;\ F = 40.2\ N$

So $\mu_s = F/N_1 = 40.2/487.3 = \underline{0.0824}$

If μ_s were doubled, friction force remains $\underline{40.2\ N}$

6/50



FBD's for $P=0$

$F_1 = 0.2N_1$, $F_2 = 0.2N_2$

$\Sigma M_A = 0$; $0.250N_1 = 0.500Q$

$N_1 = 2Q$

$\Sigma M_B = 0$; $0.250N_2 = 0.500Q$

$N_2 = 2Q$

Thus $N_1 = N_2 = N$

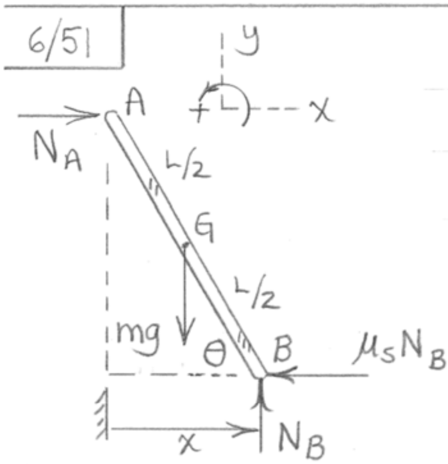
Flywheel: $\Sigma M = 0$; $M = F_1 r + F_2 r$

$$100 = 0.2N(0.200) + 0.2N(0.200)$$

$$N = 1250 \text{ N}$$

So $Q = N/2 = 625$ N ; $Q = k\delta$, $k = \frac{625}{0.030} = \underline{20.8(10^3)\ \frac{\text{N}}{\text{m}}}$

6/51



$$\mu_s = \mu_0 \left(1 - e^{-x}\right)$$

$$\begin{cases} \Sigma F_x = 0: \quad N_A - \mu_s N_B = 0 & (1) \\ \Sigma F_y = 0: \quad N_B - mg = 0 & (2) \\ \Sigma M_B = 0: \quad -N_A (L \sin\theta) + mg\left(\frac{L}{2}\cos\theta\right) = 0 & (3) \end{cases}$$

(2): $N_B = mg$, (1): $N_A = \mu_s mg$

(3): $+\mu_s mg L \sin\theta = +mg \frac{L}{2}\cos\theta$

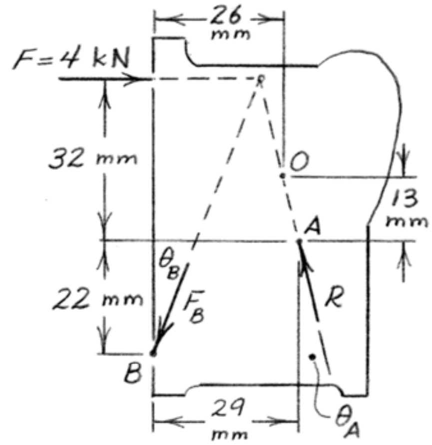
or $\tan\theta = \dfrac{1}{2\mu_s}$

or $\dfrac{\sqrt{L^2 - x^2}}{x} = \dfrac{1}{2\mu_0 (1 - e^{-x})}$

With $L = 1.8$ m & $\mu_0 = 0.5$, $x = 0.934$ m

and $\theta = \cos^{-1} \frac{x}{L} = \cos^{-1}\left(\frac{0.934}{2}\right) = \underline{58.7^\circ}$

6/52 For equilibrium, the three forces must be concurrent. The lever OA is a 2-force member.



(a) Slipping at A is not possible as long as

$$\varphi_A > \left(\theta_A = \tan^{-1}\frac{29-26}{13} = 12.99°\right)$$

So $\mu_{s_{min}} = \tan\theta_A = \underline{0.231 \text{ at } A}$

Slipping at B is prevented as long as $\varphi_B > \theta_B$.

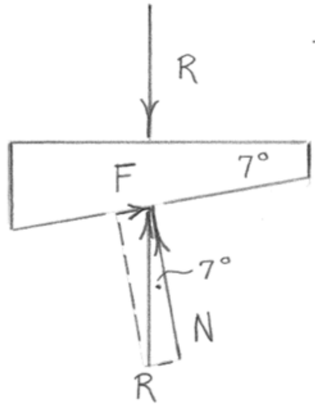
$$\tan\theta_B = \frac{29 - 32(0.231)}{32+22} = 0.400$$

$\mu_{s_{min}} = \underline{0.400 \text{ at } B}$

(b) $\Sigma M_B = 0$: $4(32+22) - R\cos 12.99°(29) - R\sin 12.99°(22) = 0$

$\underline{R = 6.51 \text{ kN}}$

6/53



$$\frac{F}{N} = \mu_s = \tan 7°$$

$$\underline{\mu_s = 0.1228}$$

6/54 Friction angle $\phi = \tan^{-1}\mu = \tan^{-1} 0.15 = 8.53°$

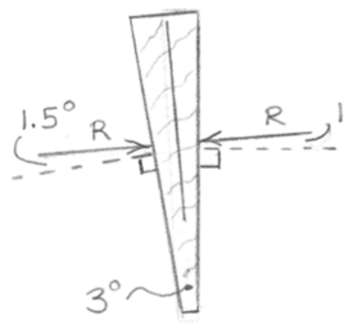
$\tan\alpha = \frac{L}{2\pi r}$; Critical when $\alpha = \phi$

Thus Lead $L = 2\pi r \tan\phi = 2\pi \frac{1.2}{2} \tan 8.53°$

$= 0.565$ cm per revolution

$N = 1/L = 1/0.565 = \underline{1.768}$ threads per centimeter

6/55 Consider the right shim (3° angle exaggerated)

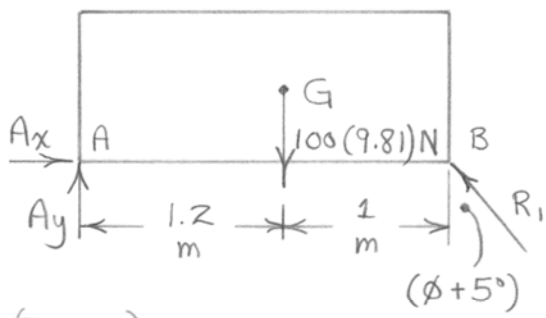


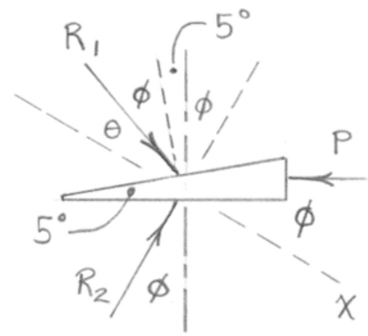
$\mu_s = \tan\phi = \tan 1.5°$

$= \underline{0.0262}$

(A very modest requirement!)

6/56 $\phi = \tan^{-1}(0.60) = 31.0°$





(Door)

$\Sigma M_A = 0: \; R_1 \cos(31.0° + 5°)(2.2) - 100(9.81)(1.2) = 0$

$R_1 = 661 \text{ N}$

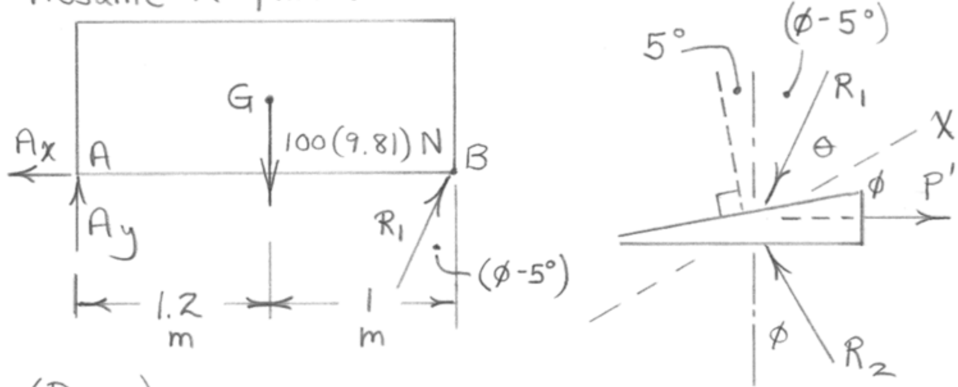
(Wedge) (Note $\theta = 90° - 2\phi - 5° = 23.1°$)

$\Sigma F_x = 0: \; 661 \cos 23.1° - P \cos 31.0° = 0$

$\underline{P = 709 \text{ N}}$

6/57 $\phi = \tan^{-1}(0.60) = 31.0°$

Assume A pinned:



(Door)

$$\Sigma M_A = 0: \ R_1 \cos(31.0° - 5°)(2.2) - 100(9.81)(1.2) = 0$$

$$R_1 = 595 \text{ N}$$

(Wedge) (Note $\theta = 90° - 2\phi + 5° = 33.1°$)

$$\Sigma F_x = 0: \ P' \cos 31.0° - 595 \cos 33.1° = 0$$

$$\underline{P' = 582 \text{ N}}$$

(Door) $\Sigma F_x = 0: \ -A_x + 595 \sin(\phi - 5°) = 0, \ A_x = 261 \text{ N}$

$\Sigma F_y = 0: \ A_y - 100(9.81) + 595 \cos(\phi - 5°) = 0, \ A_y = 446 \text{ N}$

$\dfrac{A_x}{A_y} = \dfrac{261}{446} = 0.584 < (\mu_s = 0.6)$; assumption OK, A will not slip.

6/58 $M = Wr \tan(\alpha + \phi)$

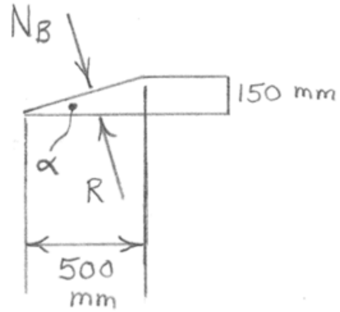
where $W = 450$ N, $r = 0.025$ m,

$\alpha = \tan^{-1} \frac{Lead}{2\pi r} = \tan^{-1} \frac{0.020}{2\pi(0.025/2)} = \tan^{-1} 0.255 = 14.29°$

$\phi = \tan^{-1}\mu = \tan^{-1} 0.20 = 11.31°$; $\phi + \alpha = 25.60°$

So $M = 450\left(\frac{0.025}{2}\right) \tan 25.60° = \underline{2.69 \text{ N·m}}$

6/59 FBD of portable ramp:



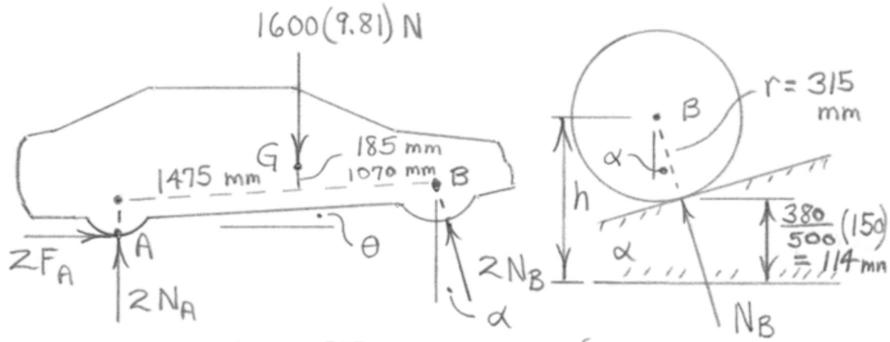
$\alpha = \tan^{-1}\left(\frac{150}{500}\right) = 16.70°$

$\tan\alpha = \frac{F}{N} = \mu_s$

So $\mu_s = \tan\alpha$

$= \tan 16.70°$

$\underline{\mu_s = 0.3}$



So $\theta = \sin^{-1}\frac{416 - 315}{2545} = 2.27°$ $\quad \left\{ \begin{array}{l} h = 114 + 315\cos\alpha \\ = 416 \text{ mm} \end{array} \right.$

$\circlearrowleft + \Sigma M_A = 0: -1600(9.81)(1475\cos\theta - 185\sin\theta)$

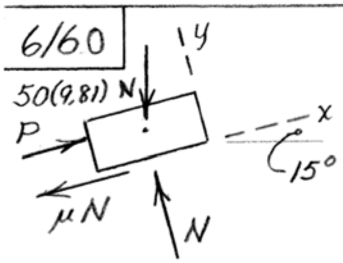
$+ 2N_B\cos(\alpha - \theta)(315\sin\theta + 2545) + 2N_B\sin(\alpha - \theta)\,315\cos\theta = 0$

$N_B = 4500 \text{ N}$

$\xrightarrow{+} \Sigma F = 0: \quad 2F_A - 2N_B\sin\alpha = 0$

$\underline{F_A = 1294 \text{ N}}$

6/60



$\Sigma F_y = 0: N - 50(9.81)\cos 15° = 0$

$N = 474$ N

$\Sigma F_x = 0: P - \mu N - 50(9.81)\sin 15° = 0$

$P = 364$ N for $\mu = \mu_s = 0.5$

Eq. 6/3: $M = Wr\tan(\varphi + \alpha)$

$\varphi = \tan^{-1}\mu_s = \tan^{-1}0.5 = 26.6°$

$\alpha = \tan^{-1}\left(\frac{L}{\pi D}\right) = \tan^{-1}\left(\frac{10}{\pi(25)}\right) = 7.26°$

So $M = 364\left(\frac{25}{2}\right)\tan(26.6° + 7.26°)$

$= 3050$ N·mm $= \underline{3.05 \text{ N·m}}$

6/61 Helix angle $\alpha = \tan^{-1}\dfrac{L}{2\pi r} = \tan^{-1}\dfrac{3.5}{2\pi(30/2)} = 2.13°$

Friction angle $\varphi = \tan^{-1}\mu = \tan^{-1}0.25 = 14.04°$

(a) to tighten, $M_a = 2Tr\tan(\alpha+\varphi)$

$$M_a = 2(40)\frac{30}{2}\tan(2.13° + 14.04°) = \underline{348 \text{ N}\cdot\text{m}}$$

(b) to loosen, $M_b = 2Tr\tan(\varphi-\alpha)$

$$M_b = 2(40)\frac{30}{2}\tan(14.04° - 2.13°) = \underline{253 \text{ N}\cdot\text{m}}$$

6/62

$$\phi = \tan^{-1}\mu_s = \tan^{-1}(0.2) = 11.31^\circ$$

$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{2.5}{2\pi(5)} = 4.55^\circ$$

(a) Tighten: $M = Pr\tan(\phi + \alpha)$

$$F(100) = 600\left(\frac{10}{2}\right)\tan(11.31^\circ + 4.55^\circ)$$

$$\underline{F = 8.52 \text{ N}}$$

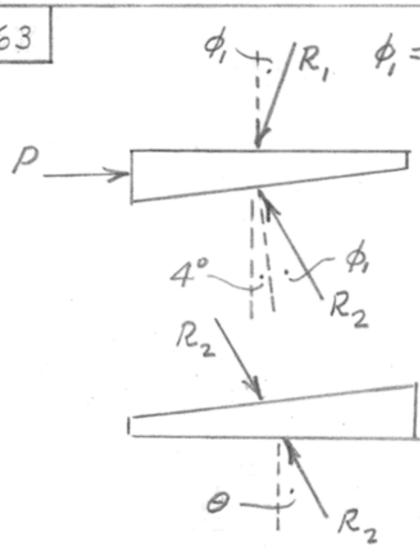
(b) Loosen: $M = Pr\tan(\phi - \alpha)$

$$F(100) = 600\left(\frac{10}{2}\right)\tan(11.31^\circ - 4.55^\circ)$$

$$\underline{F = 3.56 \text{ N}}$$

6/63

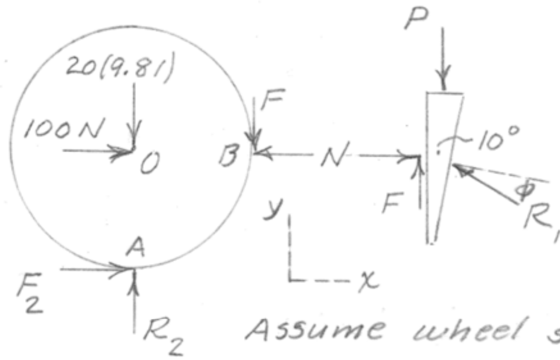
$\phi_1 = \tan^{-1} 0.3 = 16.70°$



In order for column to be raised, bottom wedge must not move. Therefore μ_2 must be greater than $\tan\theta$ or

$(\mu_2)_{min} = \tan(4 + 16.70)$
$= \underline{0.378}$

6/64 $\phi = \tan^{-1} 0.30 = 16.70°$



Assume wheel slips at B

wheel: $\Sigma M_O = 0$; $F_2 = F = 0.3N$
$\Sigma F_x = 0$; $N = 100 + F_2$

$0.7N = 100$
$N = 142.9$ N
$F_2 = F = 0.3(142.9) = 42.9$

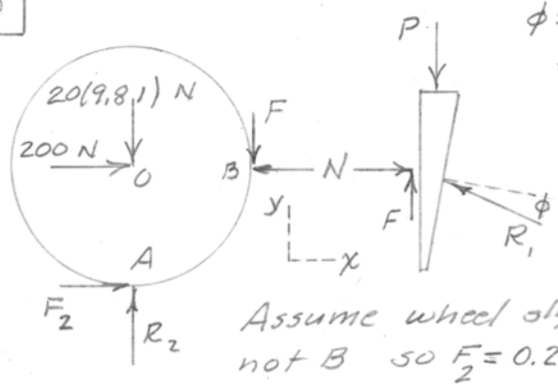
$\Sigma F_y = 0$; $R_2 = 196.2 + 42.9 = 239.1$ N

$(F_2 = 42.9) < (0.25 \times 239.1 = 59.8)$ so assumption OK

Wedge: $\Sigma F_x = 0$; $R_1 \cos 26.70° = 142.9$, $R_1 = 159.9$ N

$\Sigma F_y = 0$; $P = 42.9 + 159.9 \sin 26.70° = \underline{114.7 \text{ N}}$

6/65



$\phi = \tan^{-1} 0.30 = 16.70°$

Assume wheel slips at A but not B so $F_2 = 0.25 R_2$

Wheel: $\Sigma M_O = 0$; $F = F_2 = 0.25 R_2$

$\Sigma F_y = 0$; $R_2 = 196.2 + F = 196.2 + 0.25 R_2$

Solve & get $R_2 = 262$ N & $F = 0.25(262) = 65.4$ N

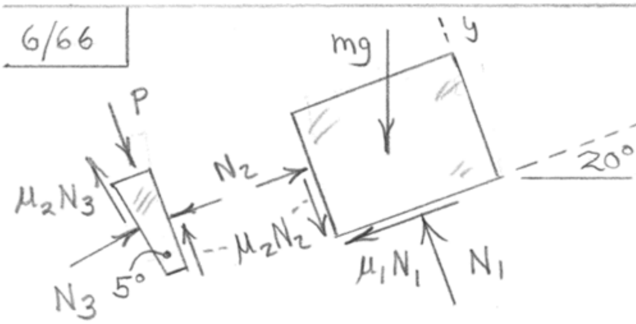
$\Sigma F_x = 0$; $N = 200 + 65.4 = 265.4$ N

$(F = 65.4) < (0.3 \times 265.4 = 79.6)$ so assumption OK

wedge: $\Sigma F_x = 0$; $R_1 \cos 26.70° = 265.4$, $R_1 = 297$ N

$\Sigma F_y = 0$; $P = 65.4 + 297 \sin 26.70° = \underline{198.8 \text{ N}}$

6/66



$\begin{cases} \mu_1 = 0.7 \\ \mu_2 = 0.4 \end{cases}$

$mg = 27(9.81)$ N

Block:

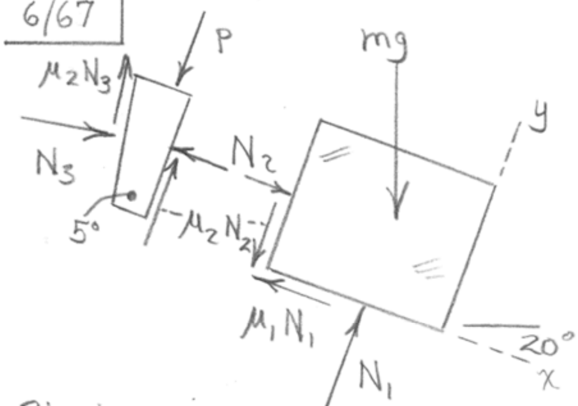
$$\begin{cases} \Sigma F_x = 0: & -mg\sin 20° + N_2 - \mu_1 N_1 = 0 \quad (1) \\ \Sigma F_y = 0: & -mg\cos 20° + N_1 - \mu_2 N_2 = 0 \quad (2) \end{cases}$$

Wedge:

$$\begin{cases} \Sigma F_x = 0: & N_3 \cos 5° - \mu_2 N_3 \sin 5° - N_2 = 0 \quad (3) \\ \Sigma F_y = 0: & N_3 \sin 5° + \mu_2 N_3 \cos 5° + \mu_2 N_2 - P = 0 \quad (4) \end{cases}$$

Solution: $\begin{cases} N_1 = 396 \text{ N} & N_2 = 368 \text{ N} \\ N_3 = 383 \text{ N} & \underline{P = 333 \text{ N}} \end{cases}$

6/67



$$\begin{cases} \mu_1 = 0.7 \\ \mu_2 = 0.4 \\ mg = 27(9.81)\ \text{N} \end{cases}$$

Block:

$$\begin{cases} \Sigma F_x = 0 : \ mg \sin 20° + N_2 - \mu_1 N_1 = 0 & (1) \\ \Sigma F_y = 0 : -mg \cos 20° + N_1 - \mu_2 N_2 = 0 & (2) \end{cases}$$

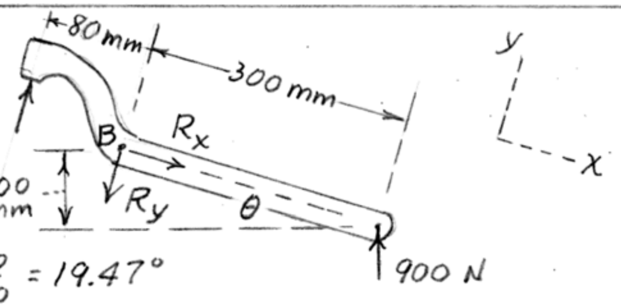
Wedge:

$$\begin{cases} \Sigma F_x = 0 : \ N_3 \cos 5° - \mu_2 N_3 \sin 5° - N_2 = 0 & (3) \\ \Sigma F_y = 0 : \ N_3 \sin 5° + \mu_2 N_3 \cos 5° + \mu_2 N_2 - P = 0 & (4) \end{cases}$$

Solution: $N_1 = 295$ N $\quad N_2 = 116.2$ N

$N_3 = 120.8$ N $\quad P = \underline{105.1 \text{ N}}$

6/68



$\theta = \sin^{-1}\frac{100}{300} = 19.47°$

$\Sigma M_B = 0$: $900(0.3\cos 19.47°) - L(0.080) = 0$

$L = 3180$ N

Screw: Helix angle $\alpha = \tan^{-1}\frac{1/2}{\pi(1.2)} = 7.55°$

Friction angle: $\phi = \tan^{-1} 0.20 = 11.31°$

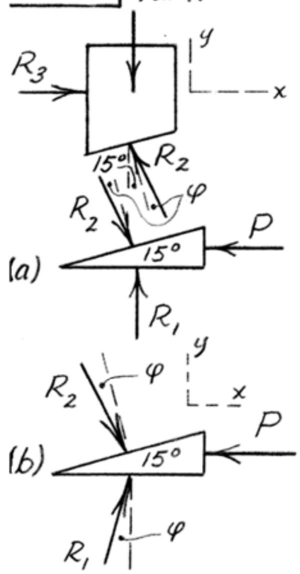
Tighten screw: $M = Lr\tan(\phi + \alpha)$

$= 3180(0.0060)\tan 18.86° = \underline{6.52 \text{ N·m}}$

Loosen screw: $M' = Lr\tan(\phi - \alpha)$

$= 3180(0.0060)\tan 3.75° = \underline{1.253 \text{ N·m}}$

6/69



Friction angle $\varphi = \tan^{-1}(0.20) = 11.31°$

(a) Rollers under wedge:

$\Sigma F_y = 0: \; -981 + R_2 \cos(15° + 11.31°) = 0$

$R_2 = 1094 \text{ N}$

$\Sigma F_x = 0: \; R_2 \sin(15° + 11.31°) - P = 0$

$\underline{P = 485 \text{ N}}$

(b) Rollers removed:

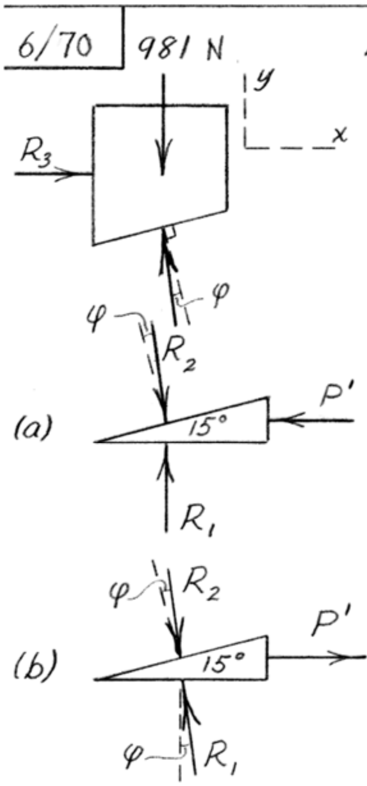
Value of R_2 from 100-kg body is unchanged.

$\Sigma F_x = 0: \; R_2 \sin(15° + 11.31°) - P + R_1 \sin(11.31°) = 0$

With R_1 determined from overall equilibrium as $R_1 = 981/\cos 11.31° = 1000 \text{ N}$, we solve for P as

$\underline{P = 681 \text{ N}}$

6/70



Friction angle $\varphi = \tan^{-1} 0.2 = 11.31°$

$\Sigma F_y = 0: \ -981 + R_2 \cos(15° - 11.31°) = 0$

$R_2 = 983$ N

(a) Rollers under wedge:

$\Sigma F_x = 0: \ R_2 \sin(15° - 11.31°) - P' = 0$

$\underline{P' = 63.3 \text{ N}}$ (to the left)

(b) Rollers removed:

$R_2 = 983$ N as before

From overall equilibrium,

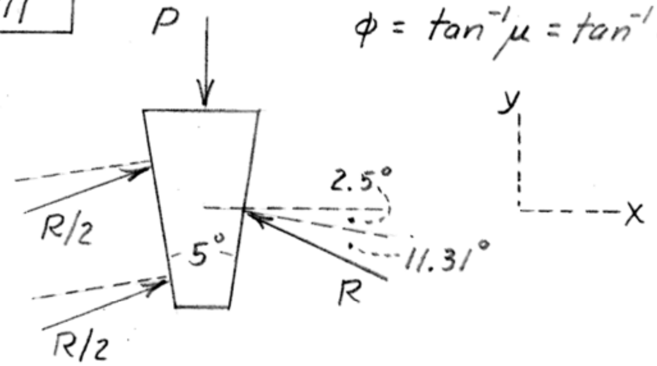
$R_1 = \dfrac{981}{\cos 11.31°} = 1000$ N

$\Sigma F_x = 0: \ R_2 \sin(15° - 11.31°) + P' - R_1 \sin 11.31° = 0$

$\underline{P' = 132.9 \text{ N}}$ (to the right)

6/71

$\phi = \tan^{-1}\mu = \tan^{-1} 0.20 = 11.31°$



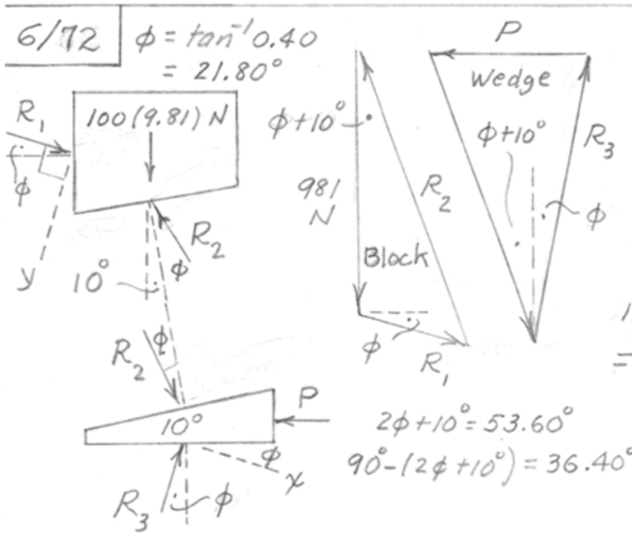
$\Sigma F_x = 0$ for shaft: $R\cos(11.31° + 2.5°) - 900 = 0$

$R = 927\text{ N}$

$\Sigma F_y = 0$ for wedge: $-P + 2R\sin(11.31° + 2.5°) = 0$

$\underline{P = 442\text{ N}}$

6/72 $\phi = \tan^{-1} 0.40 = 21.80°$



$2\phi + 10° = 53.60°$

$90° - (2\phi + 10°) = 36.40°$

Block: $\Sigma F_y = 0$;

$981 \cos 21.80° = R_2 \cos 53.60°$

$R_2 = 1535$ N

Wedge: $\Sigma F_x = 0$;

$1535 \cos 36.40° = P \cos 21.80°$

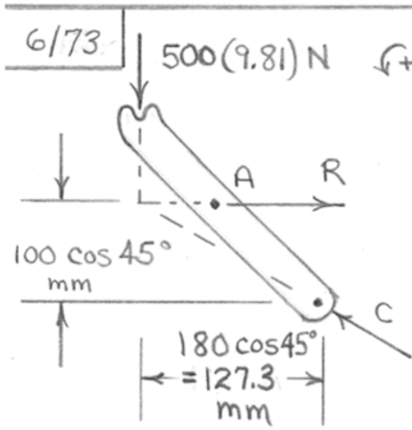
$P = 1331$ N

Screw: $\alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{10}{2\pi(15)} = 6.06°$; $\phi = \tan^{-1} 0.25 = 14.04°$

$\phi + \alpha = 20.09°$

$M = Pr \tan(\phi + \alpha) = 1331(0.015) \tan 20.09° = \underline{7.30 \text{ N}\cdot\text{m}}$

6/73



$\circlearrowleft + \Sigma M_C = 0: \; 500(9.81)(127.3) - R(100\cos 45°) = 0, \quad R = 8830 \text{ N}$

Friction angle $\phi = \tan^{-1}\mu$

$= \tan^{-1}(0.20) = 11.31°$

$\alpha = \tan^{-1}\dfrac{L}{2\pi r} = \tan^{-1}\dfrac{0.002}{2\pi(0.005)}$

$= 3.64°$

Raise load: $M = Rr\tan(\phi + \alpha)$

$P(0.150) = 8830(0.005)\tan(11.31° + 3.64°)$

$\underline{P = 78.6 \text{ N}}$

Lower load: $M = Rr\tan(\phi - \alpha)$

$P(0.150) = 8830(0.005)\tan(11.31° - 3.64°)$

$\underline{P = 39.6 \text{ N}}$

▶6/74 For equil. of screw (refer to prob. illust.)

$\Sigma F = 0;\ W = \Sigma R_1 \cos(\alpha+\gamma) = \cos(\alpha+\gamma)\,\Sigma R_1$

$\Sigma M = 0;\ M = \Sigma R_1 r \sin(\alpha+\gamma) = r\sin(\alpha+\gamma)\,\Sigma R_1$

Combine & get $M = Wr\tan(\alpha+\gamma) = Wr\dfrac{\tan\alpha + \tan\gamma}{1-\tan\alpha\tan\gamma}$

But $\tan\gamma = \dfrac{R\sin\phi}{R\cos\phi\cos\beta/2} = \mu/\cos\beta/2$

& $\tan\beta/2 = \dfrac{L}{2h}\cos\alpha$, $\tan\dfrac{\theta}{2} = \dfrac{L}{2h}$, so $\tan\dfrac{\beta}{2} = \tan\dfrac{\theta}{2}\cos\alpha$

& $\cos\dfrac{\beta}{2} = 1\Big/\sqrt{1+\tan^2\dfrac{\theta}{2}\cos^2\alpha}$. Thus $\tan\gamma = \mu\sqrt{1+\tan^2\dfrac{\theta}{2}\cos^2\alpha}$

Hence

$$M = Wr\,\frac{\tan\alpha + \mu\sqrt{1+\tan^2\frac{\theta}{2}\cos^2\alpha}}{1-\mu\tan\alpha\sqrt{1+\tan^2\frac{\theta}{2}\cos^2\alpha}}$$

where $\tan\alpha = \dfrac{L}{2\pi r}$

6/75

$M = Rr\sin\phi$, $\sin\phi = \frac{M}{Rr} = \frac{3}{2(40)(9.81)(0.040/2)}$

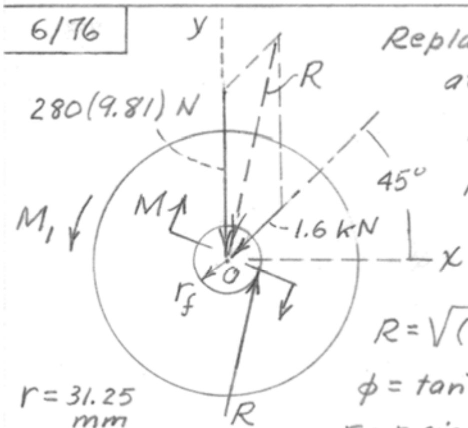
$\phi = 11.02°$

$\mu = \tan\phi = \underline{0.1947}$

$r_f = r\sin\phi = \frac{0.040}{2}\sin 11.02° = 0.00382 \text{ m}$

or $\underline{r_f = 3.82 \text{ mm}}$

6/76



$r = 31.25$ mm

$\mu = 0.30$

Replace 1.6-kN force by 1.6-kN force at O & a couple M_1 where

$M_1 = 1.6(10^3)(0.550) = 880$ N·m

$R_x = 1.6 \cos 45° = 1.131$ kN

$R_y = 280(9.81)(10^{-3}) + 1.6 \sin 45° = 3.878$ kN

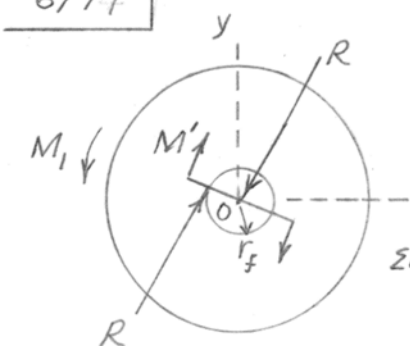
$R = \sqrt{(1.131)^2 + (3.878)^2} = 4.04$ kN

$\phi = \tan^{-1}\mu = \tan^{-1} 0.30 = 16.70°$

$r_f = r \sin\phi = 31.25 \sin 16.70° = 8.98$ mm

$\Sigma M_O = 0;\ M - 880 - 4.04(8.98) = 0,\ \underline{M = 916 \text{ N·m}}$

6/77



From Prob. 6/76,

$R = 4.04$ kN

$\phi = 16.70°$

$r_f = r \sin\phi = 8.98$ mm

$M_1 = 880$ N·m

$\Sigma M_O = 0$; $M' + 4.04(8.98) - 880 = 0$

$M' = 880 - 36.3 = \underline{844 \text{ N·m}}$

6/78

$M = \frac{2}{3}\mu PR$ (Eq. 6/5)

A on B: $M = \frac{2}{3}(0.40)(400)\frac{0.225}{2} = \underline{12\ \text{N}\cdot\text{m}}$

B on C: $12 = \frac{2}{3}\mu(400)\frac{0.300}{2}$, $\underline{\mu = 0.30}$

6/79 $\mu = 0.80 - kr$: $0.50 = 0.80 - k(0.075)$, $k = 4 \text{ m}^{-1}$

So $\mu = 0.80 - 4r$ (r in m)

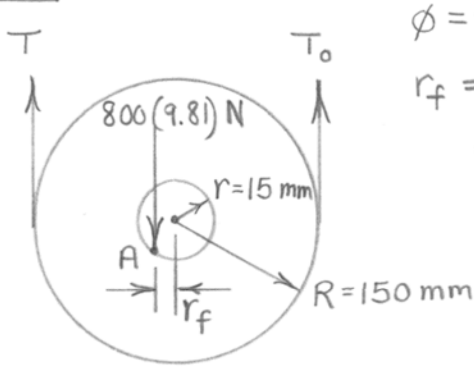
Downward force $R = pA$, $p = \dfrac{3(9.81) + 40}{\pi(0.075^2)} = 3930$ Pa

$$M_z = \int \mu p \, dA \times r = \int_0^{2\pi} \int_0^{0.075} r(0.80 - 4r)3930 \; r\,dr\,d\theta$$

$$= 2\pi \cdot 3930 \int_0^{0.075} (0.80 - 4r) r^2 dr = 24\,700 \left[\frac{0.80 r^3}{3} - r^4\right]_0^{0.075}$$

$$= \underline{1.996 \text{ N·m}}$$

6/80



$\phi = \tan^{-1} 0.25 = 14.04°$

$r_f = r \sin\phi = 15 \sin 14.04$

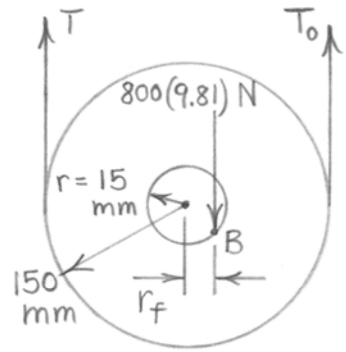
$= 3.64$ mm

$+\uparrow \Sigma F = 0: \quad T + T_o - 800(9.81) = 0 \qquad (1)$

$\Sigma M_A = 0: \quad T(150 - 3.64) - T_o(150 + 3.64) = 0 \qquad (2)$

Solve (1) & (2) : $\begin{cases} T = 4020 \text{ N} \\ T_o = 3830 \text{ N} \end{cases}$

6/81



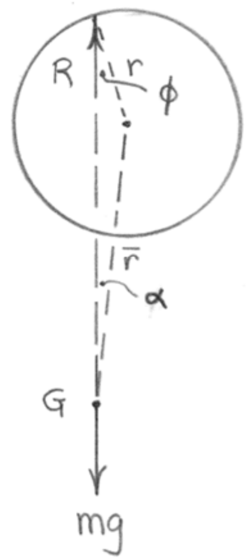
From the solution to Prob. 6/78, $r_f = 3.64$ mm

$$+\uparrow \Sigma F = 0: \quad T + T_0 - 800(9.81) = 0 \qquad (1)$$

$$\curvearrowleft + \Sigma M_B = 0: \quad T(150 + 3.64) - T_0(150 - 3.64) = 0 \qquad (2)$$

Solve (1) & (2): $\begin{cases} T = 3830 \text{ N} \\ T_0 = 4020 \text{ N} \end{cases}$

6/82



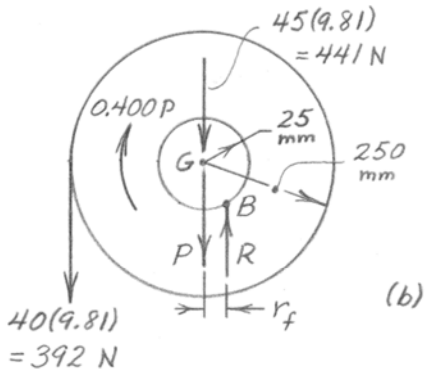
$$r \sin \phi = \bar{r} \sin \alpha$$

$$\mu = \tan \phi = \frac{\bar{r} \sin \alpha}{\sqrt{r^2 - \bar{r}^2 \sin^2 \alpha}}$$

or

$$\mu = \frac{1}{\sqrt{\left(\frac{d/2}{\bar{r} \sin \alpha}\right)^2 - 1}}$$

6/83 FBD of shaft and attached drum, with force P replaced by a force-couple system at G:



(a) $r_f = 0$:

$\circlearrowleft + \Sigma M_G = 0$: $392(0.250) - 0.400P = 0$,

$\underline{P = 245 \text{ N}}$

(b) $\varphi = \tan^{-1} 0.2 = 11.31°$

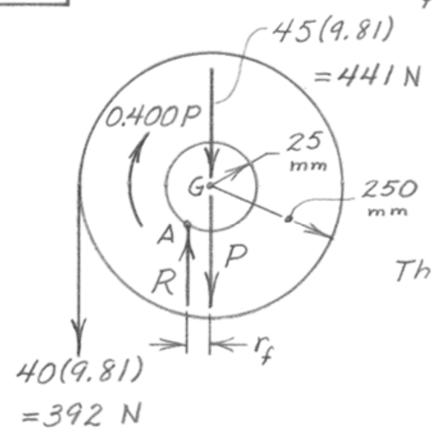
$r_f = r \sin\varphi = 25 \sin 11.31° = 4.90$ mm

$\circlearrowleft + \Sigma M_B = 0$: $392(0.250 + 0.00490) + 441(0.00490) + P(0.00490) - 0.400P = 0$

$\underline{P = 259 \text{ N}}$

(This solution assumes that the bearing reaction can be represented by a single force R as shown above.)

6/84



$r_f = 4.90$ mm (from Prob. 6/79)

$\circlearrowleft \Sigma M_A = 0: 392(0.250 - 0.00490) - 0.400P - (441 + P)(0.00490) = 0$

$\underline{P = 232 \text{ N}}$

The values are NOT symmetric:

$$\left\{ \begin{array}{l} P_{down} = 232.18 \text{ N} \\ P_{n.f.} = 245.25 \text{ N} \\ P_{up} = 258.64 \text{ N} \end{array} \right.$$

6/85

$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{11}{2\pi\,\frac{120}{2}} = 1.671°$$

$$\phi = \tan^{-1} 0.15 = 8.53°$$

Screw: (a) Raise : $M_s = Wr\tan(\alpha+\phi)$

$$= \tfrac{1}{2}(10+3)(9.81)\,\frac{120}{2}\,\tan(1.671° + 8.53°) = 689\ \text{N·m}$$

(b) Lower : $M_s = Wr\tan(\phi-\alpha)$

$$= \tfrac{1}{2}(10+3)(9.81)\,\frac{120}{2}\,\tan(8.53° - 1.671°) = 460\ \text{N·m}$$

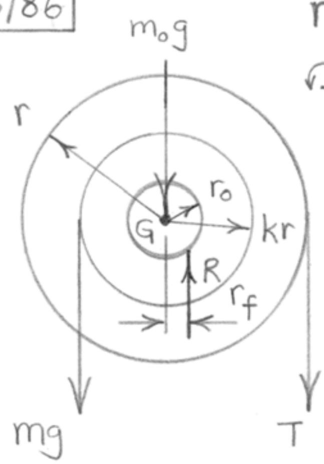
Collar bearing : $M_c = \frac{2}{3}\mu P\,\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$

$$= \tfrac{2}{3}(0.15)\left(\frac{10+3}{2} + 0.9\right)(9.81)\,\frac{(250/2)^3 - (125/2)^3}{(250/2)^2 - (125/2)^2}$$

$$= 1059\ \text{N·m}$$

Total moment per screw
$$\begin{cases} \text{(a)}\quad M = 689 + 1059 = \underline{1747\ \text{N·m}} \\ \text{(b)}\quad M = 460 + 1059 = \underline{1519\ \text{N·m}} \end{cases}$$

6/86



$r_f = r_0 \sin\phi \cong r_0\mu$, $R = T + (m+m_0)g$

$\circlearrowleft + \Sigma M_G = 0: \; mg(kr) - Tr + [T + (m+m_0)g]\, r_0 \mu = 0$

Solve for T as

$$T = \frac{(m+m_0)g\mu \frac{r_0}{r} + mgk}{1 - \mu \frac{r_0}{r}}$$

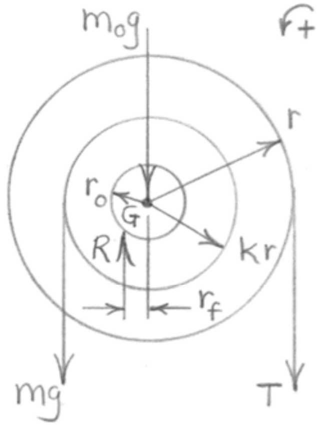
Numbers : $T = \dfrac{(50+30)(9.81)(0.15)\frac{0.025}{0.3} + 50(9.81)(\frac{1}{2})}{1 - 0.15\,\frac{0.025}{0.3}}$

$= 258$ N

(No-friction result : T = 245 N)

6/87

$r_f = r_o \sin\phi \cong r_o\mu$; $R = T + (m+m_o)g$



$\curvearrowleft + \Sigma M_G = 0$: $mg(kr) - Tr - [T + (m+m_o)g]\,r_o\mu = 0$

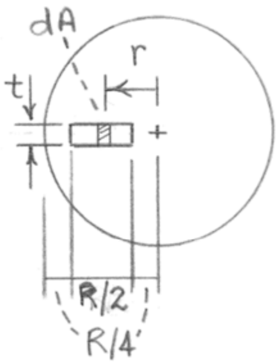
Solve for T as

$$T = \frac{-(m+m_o)g\mu\frac{r_o}{r} + mgk}{1 + \mu\frac{r_o}{r}}$$

Numbers: $$T = \frac{-(50+30)(9.81)(0.15)\frac{0.025}{0.3} + 50(9.81)(\frac{1}{2})}{1 + 0.15\,\frac{0.025}{0.3}}$$

$= \underline{233\text{ N}}$

6/88



(t small)

$$P = pA = p\left(t\,\frac{R}{2}\right)$$

$$M = \int \mu p r\, dA = \mu p \int_{R/4}^{3R/4} r t\, dr$$

$$= \mu p t \left.\frac{r^2}{2}\right|_{R/4}^{3R/4}$$

$$= \frac{\mu p t}{2}\left[\left(\frac{3R}{4}\right)^2 - \left(\frac{R}{4}\right)^2\right]$$

$$= \frac{1}{4}\mu p t R^2$$

$$= \frac{1}{4}\mu\left(\frac{P}{tR/2}\right) t R^2 = \underline{\frac{1}{2}\mu P R}$$

6/89

$dM = (\mu p\, dA) r$ where $p = k/r^2$

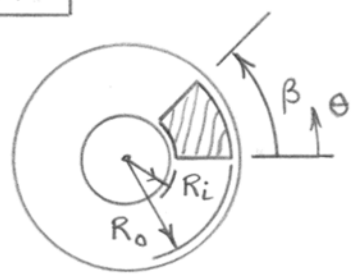
$$M = \int_0^{2\pi}\int_{r_i}^{r_o} \mu p r (r\,dr\,d\theta) = 2\pi\mu k \int_{r_i}^{r_o} dr = 2\pi\mu k (r_o - r_i)$$

Also $L = \int p\,dA = \int_0^{2\pi}\int_{r_i}^{r_o} \frac{k}{r^2} r\,dr\,d\theta = 2\pi k \ln r \Big]_{r_i}^{r_o}$

or $L = 2\pi k \ln \frac{r_o}{r_i}$, $2\pi k = \frac{L}{\ln r_o/r_i}$

Thus $M = \mu L \frac{r_o - r_i}{\ln (r_o/r_i)}$

6/90



$$P = pA = p\int_0^{\beta}\int_{R_i}^{R_o} r\,dr\,d\theta$$

$$= \frac{p}{2}\int_0^{\beta}(R_o^2 - R_i^2)\,d\theta$$

$$= \frac{p}{2}(R_o^2 - R_i^2)\beta$$

$$M = 2\int \mu p r\,dA = 2\mu p\int_0^{\beta}\int_{R_i}^{R_o} r^2\,dr\,d\theta$$

$$= \frac{2\mu p}{3}(R_o^3 - R_i^3)\beta$$

$$= \frac{2\mu}{3}\frac{2P}{(R_o^2 - R_i^2)\beta}(R_o^3 - R_i^3)\beta$$

$$= \underline{\frac{4\mu P}{3}\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}}$$

Same form as Eq. 6/5a except for factor of 2 for 2 pads. No β dependence. Pressure variation with θ would not change the moment M.

6/91 $p = p_0\left(1 - \frac{r}{2a}\right);\quad dA = 2\pi r\,dr$

$$L = \int p\,dA = \int_0^a p_0\left(1 - \frac{r}{2a}\right) 2\pi r\,dr = 2\pi p_0\left[\frac{r^2}{2} - \frac{r^3}{6a}\right]_0^a$$

$$= \frac{2}{3}\pi p_0 a^2 \quad \text{so} \quad p_0 = \frac{3L}{2\pi a^2}$$

$$M = \int \mu p r\,dA = \int_0^a \mu p_0\left(r - \frac{r^2}{2a}\right) 2\pi r\,dr = 2\pi\mu p_0\left[\frac{r^3}{3} - \frac{r^4}{8a}\right]_0^a$$

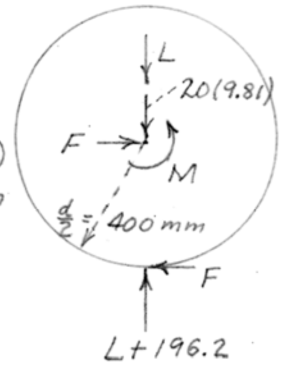
$$= \frac{5}{12}\pi\mu p_0 a^3 = \underline{\frac{5}{8}\mu L a}$$

6/92 $L = \frac{1}{4}(480 - 4 \times 20)9.81 = 981\text{ N}$

$P = 4F$

$F = \frac{1}{4} 80 = 20\text{ N}$

$M = F\frac{d}{2} = 20(0.400)$
$= 8.0\text{ N}\cdot\text{m}$



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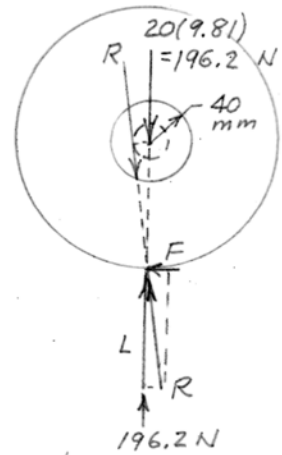
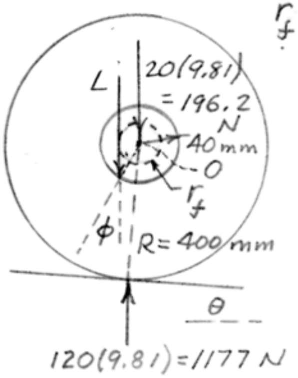


$M = Rr\sin\phi$
$\approx Lr\sin\phi$

$8.0 \approx 981(0.040)\sin\phi$

$\sin\phi \approx \frac{8.0}{981(0.040)} \approx 0.204$, so $\mu = \tan^{-1}\phi = \underline{0.208}$

6/93 From Prob. 6/92, $L = 981$ N



$r_f = r \sin\phi = 0.040 \sin(\tan^{-1} 0.2)$

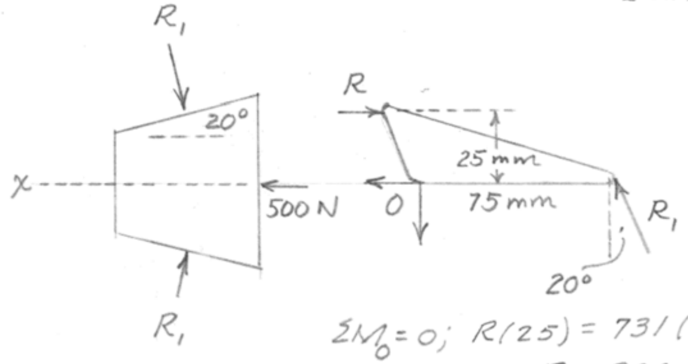
$= 0.040 \sin 11.31° = 0.00785$ m

$\Sigma M_O = 0$; $981(0.00785) - 1177(0.400)\sin\theta = 0$

$$\sin\theta = \frac{981(0.00785)}{1177(0.400)} = 0.01634$$

$\underline{\theta = 0.936°}$

6/94 $\Sigma F_x = 0;\ 2R_1 \sin 20° = 500,\ R_1 = \dfrac{500}{2 \sin 20°} = 731 \text{ N}$



$\Sigma M_O = 0;\ R(25) = 731(75 \cos 20°)$

$R = 2060 \text{ N}$

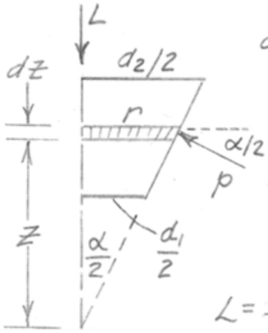
For 5 pairs of surfaces with uniform pressure

$$M = 5\left(\tfrac{2}{3}\right)\mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} = \frac{10}{3}(0.15)(4120)\frac{\overline{0.15}^3 - \overline{0.05}^3}{\overline{0.15}^2 - \overline{0.05}^2}$$

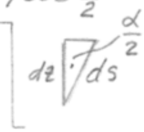
$\underline{M = 335 \text{ N}\cdot\text{m}}$

▶6/95

$$dL = p(2\pi r)\, ds \sin\frac{\alpha}{2} \quad \text{where} \quad ds = dz/\cos\frac{\alpha}{2}$$



$$dL = 2\pi p \sin\frac{\alpha}{2}\left(z \tan\frac{\alpha}{2}\right)\frac{dz}{\cos\alpha/2}$$



$$= 2\pi p \tan^2\frac{\alpha}{2}\, z\, dz$$

$$L = 2\pi p \tan^2\frac{\alpha}{2}\int_{z_1}^{z_2} z\, dz \quad \text{where} \quad z_1 = \frac{d_1/2}{\tan\alpha/2}, \quad z_2 = \frac{d_2/2}{\tan\alpha/2}$$

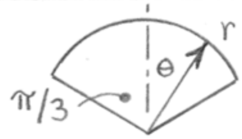
$$L = \frac{\pi p}{4}(d_2^2 - d_1^2)$$

$$M = \int r\mu p\, dA = \mu p\int_{z_1}^{z_2}\left(z\tan\frac{\alpha}{2}\right)\left(2\pi z \tan\frac{\alpha}{2}\right)\frac{dz}{\cos\alpha/2}$$

$$= 2\pi\mu p\,\frac{\tan^2\frac{\alpha}{2}}{\cos\alpha/2}\int_{z_1}^{z_2} z^2\, dz = 2\pi\mu p\,\frac{1}{24\sin\frac{\alpha}{2}}(d_2^3 - d_1^3)$$

$$M = \frac{\mu L}{3\sin\frac{\alpha}{2}}\,\frac{d_2^3 - d_1^3}{d_2^2 - d_1^2}$$

▶6/96



$$dM = \mu p\, dA\,(r\sin\theta) = \mu p\,(r\,d\theta)(2\pi r\sin\theta)\,r\sin\theta$$
$$= 2\pi\mu r^3 p_0 \cos\theta \sin^2\theta\, d\theta$$

$$M = 2\pi\mu r^3 p_0 \int_0^{\pi/3} \cos\theta \sin^2\theta\, d\theta$$
$$= 2\pi\mu r^3 p_0 \left.\frac{\sin^3\theta}{3}\right|_0^{\pi/3} = \frac{\sqrt{3}}{4}\pi\mu r^3 p_0$$

But $L = \int p\cos\theta\, dA = \int_0^{\pi/3} p_0 \cos^2\theta\,(r\,d\theta)(2\pi r\sin\theta)$

$$= 2\pi r^2 p_0 \int_0^{\pi/3} \cos^2\theta \sin\theta\, d\theta = 2\pi r^2 p_0\left(-\frac{\cos^3\theta}{3}\right)_0^{\pi/3}$$
$$= \frac{7}{12}\pi r^2 p_0$$

Substitute $\pi r^2 p_0 = \frac{12}{7}L$ to obtain

$$M = \frac{\sqrt{3}}{4}\mu r\left(\frac{12L}{7}\right) = \underline{\frac{3\sqrt{3}}{7}\mu r L}$$

6/97

$$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{mg}{mg/10} = e^{\mu(3\pi)}, \quad \underline{\mu = 0.244}$$

6/98 Use $\frac{T_2}{T_1} = e^{\mu\beta}$, where $\beta = \frac{\pi}{2}$

(a) $\frac{P}{W} = e^{0.4(\pi/2)}$, $\underline{P = 1.874W}$

(b) $\frac{W}{P} = e^{0.4(\pi/2)}$, $\underline{P = 0.533W}$

6/99 $\frac{T_2}{T_1} = e^{\mu\beta}$, $\frac{T}{200} = e^{0.30(5\pi/2)}$ where $\beta = \frac{5}{4}(2\pi)$ rad

$\frac{T}{200} = e^{2.356} = 10.55$, $T = 10.55(200) = 2110$ N

or $\underline{T = 2.11 \text{ kN}}$

6/100 $T_2 = T_1 e^{\mu\beta}$, $T_1 = 10(9.81)$ N, $T_2 = 25(9.81)$ N

$\beta = \pi$

So $\dfrac{25(9.81)}{10(9.81)} = e^{\pi\mu}$, $e^{\pi\mu} = 2.5$, $\pi\mu = \ln 2.5 = 0.9163$

$\underline{\mu = 0.292}$

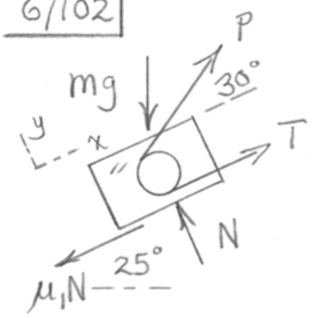
6/101

$$\frac{4}{mg} = e^{\mu\beta}, \quad \frac{mg}{1.6} = e^{\mu\beta}$$

Thus $\frac{4}{mg} = \frac{mg}{1.6}$, $m^2g^2 = 4(1.6)$

$$m = \frac{\sqrt{4(1.6)}}{9.81} = 0.258 \text{ Mg} \quad \text{or} \quad \underline{m = 258 \text{ kg}}$$

6/102



Motion impends up incline.

$$\begin{cases} m = 40 \text{ kg}, \ \mu_1 = 0.40 \\ \mu_2 = 0.20 \end{cases}$$

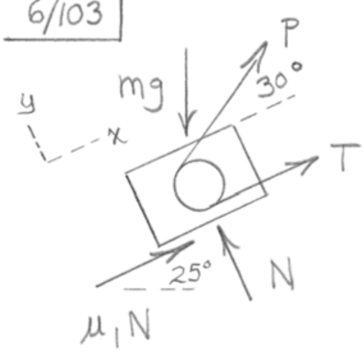
$\Sigma F_x = 0: \ P\cos 30° + T - \mu_1 N - mg \sin 25° = 0$

$\Sigma F_y = 0: \ P \sin 30° - mg\cos 25° + N = 0$

$\frac{P}{T} = e^{\mu_2 \beta}$, where $\beta = 150\left(\frac{\pi}{180}\right)$

Solve : $\begin{cases} N = 263 \text{ N} > 0 \ \checkmark \\ T = 110.0 \text{ N} \\ P = 185.8 \text{ N} \end{cases}$

6/103



Motion impends down incline.

$$\begin{cases} m = 40 \text{ kg}, \ \mu_1 = 0.40, \\ \mu_2 = 0.20 \end{cases}$$

$\Sigma F_x = 0: \ P\cos 30° + T + \mu_1 N - mg\sin 25° = 0$

$\Sigma F_y = 0: \ P\sin 30° - mg\cos 25° + N = 0$

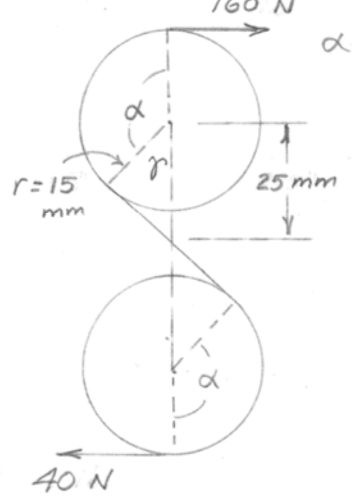
$\frac{T}{P} = e^{\mu_2 \beta}$, where $\beta = 150\left(\frac{\pi}{180}\right)$

Solve: $N = 351 \text{ N} > 0$ ✓

$T = 16.91 \text{ N}$

$P = 10.02 \text{ N}$

6/104



$\gamma = \cos^{-1}\frac{15}{25} = 53.13°$

$\alpha = 180 - 53.13 = 126.87°$

$\beta = 2\alpha = \frac{2(126.87)}{180}\pi = 4.429 \text{ rad}$

$\frac{T_2}{T_1} = e^{\mu\beta}$, $\frac{160}{40} = e^{4.429\mu}$

$4.429\mu = 1.3863$, $\underline{\mu = 0.313}$

6/105 $T_2 = T_1 e^{\mu\beta}$, $\beta = 2 \text{ turns} + 60°$

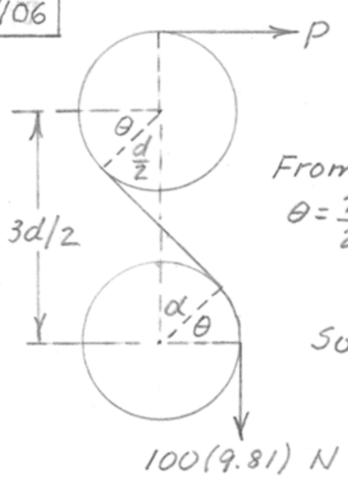
$= 2(360°) + 60° = 780°$

or $\beta = (780/180)\pi = 13.61$ rad

$T = T_2 = (0.060)(9.81)e^{0.7(13.61)} = 8100$ N

or $\underline{T = 8.10 \text{ kN}}$

6/106



$$\frac{P}{981} = e^{0.4\beta}$$

where $\beta = \pi/2 + 2\theta$

From geometry

$$\theta = \frac{\pi}{2} - \alpha = \frac{\pi}{2} - \cos^{-1}\frac{d/2}{3d/4}$$

$$= 90^\circ - 48.19^\circ = 41.81^\circ$$

So $\beta = 90 + 2(41.81) = 173.6^\circ$

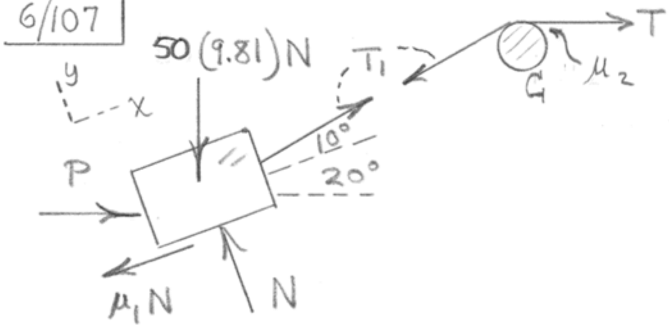
or $\beta = \frac{173.6}{180}\pi = 3.03$ rad

So $P = 981\, e^{0.4(3.03)}$

$$= 981\,(3.36) = 3297 \text{ N}$$

or $\underline{P = 3.30 \text{ kN}}$

6/107



$\Sigma F_x = 0: \; P\cos 20° + T_1 \cos 10° - 50(9.81)\sin 20°$
$- \mu_1 N = 0$

$\Sigma F_y = 0: \; N - 50(9.81)\cos 20° - P\sin 20°$
$+ T_1 \sin 10° = 0$

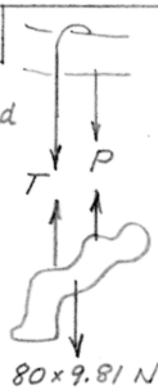
At C: $T/T_1 = e^{\mu_2 \beta}$

With $P = 180$ N, $\mu_1 = 0.40$, $\mu_2 = 0.30$,
$\beta = 30°$:

$N = 488$ N, $T_1 = 196.9$ N, $\underline{T = 230 \text{ N}}$

6/108

$\beta = \pi$ rad



$T = Pe^{\mu\beta}$ to lower

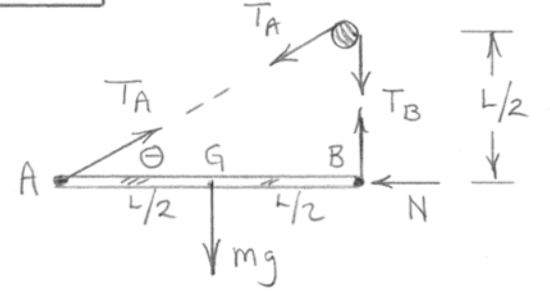
$= Pe^{0.5\pi} = 4.810\,P$

Man $\Sigma F = 0$; $T + P = 80 \times 9.81$

So $4.810\,P + P = 80 \times 9.81$

$$P = \frac{80 \times 9.81}{5.810} = \underline{135.1 \text{ N}}$$

6/109



$\theta = \tan^{-1}\frac{1}{2}$
$= 26.6°$

$\Sigma F_x = 0: \quad T_A \cos\theta - N = 0$

$\Sigma F_y = 0: \quad T_A \sin\theta + T_B - mg = 0$

$\Sigma M_A = 0: \quad -mg\frac{L}{2} + T_B(L) = 0$

Solution: $T_A = 1.118mg$, $T_B = 0.5mg$

$N = mg$

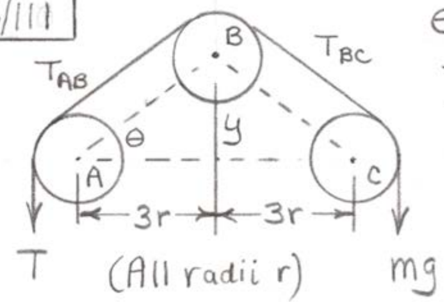
Drum at C: $\frac{1.118mg}{0.5mg} = e^{\mu_s \beta}$, $\beta = (90° + \theta)\frac{\pi}{180}$

Solving, $\underline{\mu_s = 0.396}$

6/110 From $\frac{T_2}{T_1} = e^{\mu_s \beta}$,

$T = mge^{\pi\mu}$ (independent of y!)

*6/111



$\theta = \tan^{-1}\left(\frac{y}{3r}\right)$

$\frac{T_{BC}}{mg} = e^{\mu_C \beta_C}$

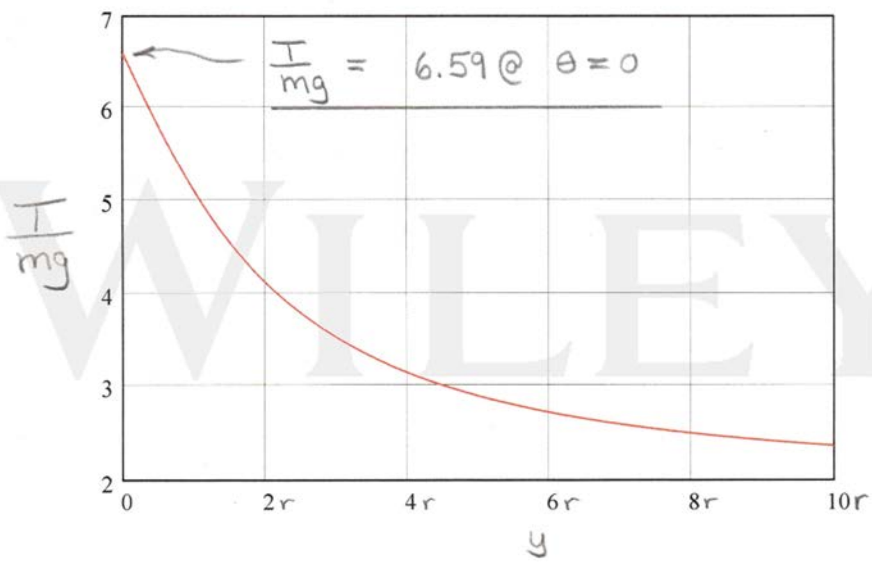
$\frac{T_{AB}}{T_{BC}} = e^{\mu_B \beta_B}$

$\frac{T}{T_{AB}} = e^{\mu_A \beta_A}$

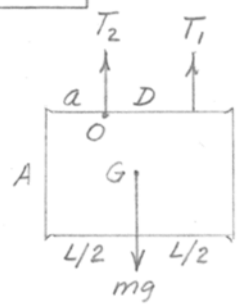
With $\beta_B = 2\theta$ and $\beta_A = \beta_C = \left(\frac{\pi}{2} - \theta\right)$, we have

$$\frac{T}{mg} = e^{\left[\mu_A\left(\frac{\pi}{2}-\theta\right) + \mu_B(2\theta) + \mu_C\left(\frac{\pi}{2}-\theta\right)\right]}$$

Limiting value: $\theta \to \frac{\pi}{2}$, $\underline{\frac{T}{mg} \to e^{\pi\mu_B}}$ (1.874)



6/112 Slipping impends for rope when $T_2 = T_1 e^{\mu\beta}$ (1)



Equil. of drum

$\Sigma F = 0;\ T_1 + T_2 = mg$ ---------- (2)

$\Sigma M_O = 0;\ mg\left(\frac{L}{2} - a\right) = T_1 D$ ---------- (3)

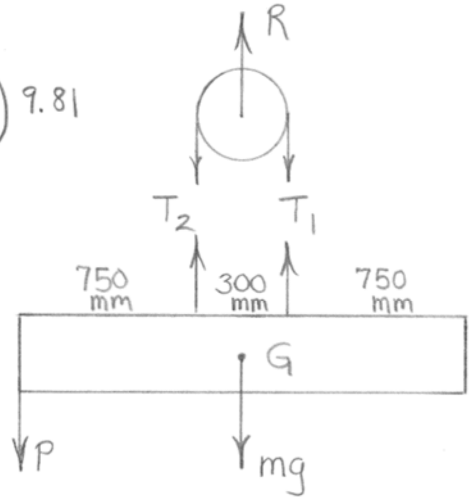
(2) & (3) $T_1\left(\frac{L}{2} - a - D\right) = T_2\left(a - \frac{L}{2}\right)$

Combine with (1) & get

$$\frac{\frac{L}{2} - a - D}{a - \frac{L}{2}} = e^{\mu\beta}, \quad \underline{a = \frac{L}{2} - \frac{D}{1 + e^{\mu\pi}}} \quad \text{where } \beta = \pi \text{ rad.}$$

6/113

$mg = 74(0.75 + 0.30 + 0.75)\,9.81$
$\quad = 1307 \text{ N}$



Beam $+\uparrow \Sigma F = 0: \quad T_1 + T_2 - P - 1307 = 0 \qquad (1)$

$\circlearrowleft + \Sigma M_G = 0: \quad P\left(0.75 + \frac{0.30}{2}\right) + \frac{0.30}{2} T_1 - \frac{0.30}{2} T_2 = 0 \qquad (2)$

Drum: $\quad T_2 = T_1 e^{\mu\beta} = T_1 e^{0.5\pi} \qquad (3)$

Solve Eqs. (1), (2), and (3): $\begin{cases} T_1 = 252 \text{ N} \\ T_2 = 1215 \text{ N} \\ \underline{P = 160.3 \text{ N}} \end{cases}$

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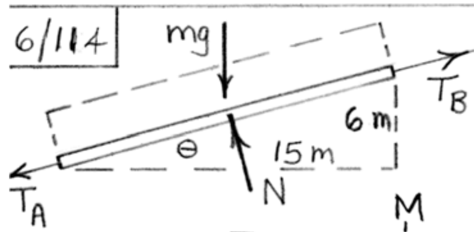
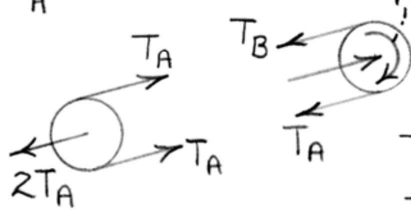


$\theta = \tan^{-1}\frac{6}{15} = 21.8°$

$\sin\theta = 0.371$

$mg = 30(70)9.81$

$= 20.6(10^3)$ N



$mg\sin\theta = 20.6(10^3)(0.371)$

$= 7.65(10^3)$ N

$T_B = 4.5 + \frac{7.65}{2} = 8.33$ kN

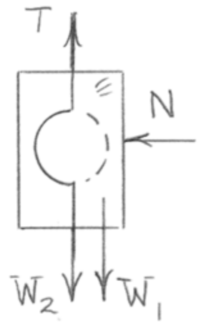
$T_A = 4.5 - \frac{7.65}{2} = 0.674$ kN

$T_B = T_A e^{\mu\beta}$: $8.33 = 0.674e^{\mu\pi}$, $\underline{\mu = 0.800}$

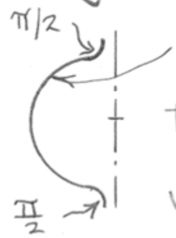
6/115

$+\uparrow \Sigma F = 0: \quad T - W_1 - W_2 = 0$

$T = W_1 + W_2$



Now, assume that the total angle of contact is 2π

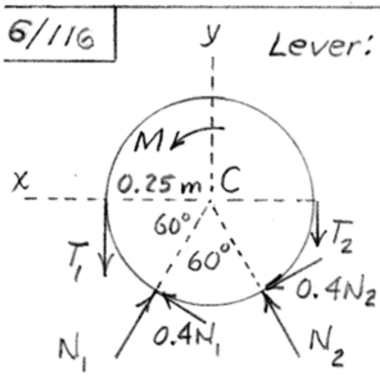


The actual angle on the indicated portion would be less than π

$$\frac{T}{W_2} = e^{\mu_s \beta} = e^{0.35(2\pi)}$$

or $\dfrac{W_1 + W_2}{W_2} = 9.02$, $\quad \underline{\dfrac{W_2}{W_1} = 0.1247}$

6/116

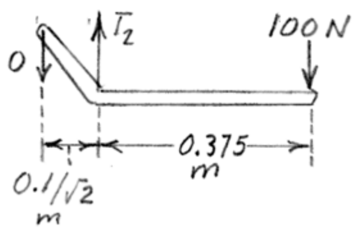


Lever: $\Sigma M_O = 0: \frac{0.1}{\sqrt{2}} T_2 - 100\left(0.375 + \frac{0.1}{\sqrt{2}}\right) = 0$

$T_2 = 630 \text{ N}$

Band: $\frac{T_2}{T_1} = e^{\mu\beta}: \frac{630}{T_1} = e^{0.3\pi}$

$T_1 = \frac{630}{2.566} = 246 \text{ N}$



Pipe: $\Sigma F_y = 0: N_1(\cos 30° + 0.4 \sin 30°)$

$+ N_2(\cos 30° - 0.4 \sin 30°)$

$- 246 - 630 = 0$

$1.066 N_1 + 0.666 N_2 = 876 \quad \cdots (a)$

$\Sigma F_x = 0: N_2(\sin 30° + 0.4 \cos 30°)$

$- N_1(\sin 30° - 0.4 \cos 30°) = 0$

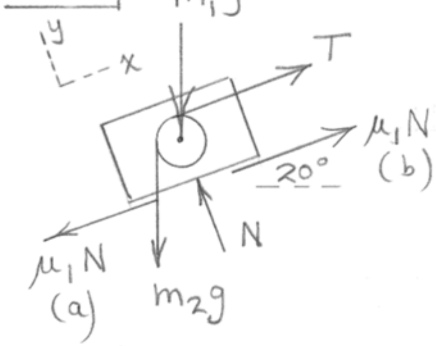
$0.1536 N_1 - 0.846 N_2 = 0 \quad \cdots (b)$

Solve (a) & (b) & get $N_1 = 738 \text{ N}, \quad N_2 = 133.9 \text{ N}$

$\Sigma M_C = 0: M + (246 - 630)0.250 - 0.4(738 + 133.9)0.25 = 0$

$\underline{M = 183.4 \text{ N}\cdot\text{m}}$

6/117



(a) Motion impends up the incline:

$$\frac{m_2 g}{T} = e^{\mu_2 \beta}$$

(b) Motion impends down the incline

$$\frac{T}{m_2 g} = e^{\mu_2 \beta}$$

$\beta = 70°$ in both cases.

(a) $\Sigma F_x = 0: -\mu_1 N - m_1 g \sin 20° - m_2 g \sin 20° + m_2 g / e^{\mu_2 \beta}$

$\Sigma F_y = 0: N - (m_1 + m_2) g \cos 20° = 0$

Solve to obtain $m_2 = 2.26 m_1$

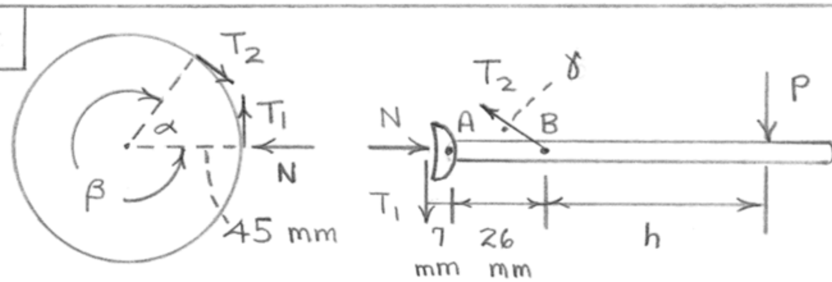
(b) $\Sigma F_x = 0: \mu_1 N - (m_1 + m_2) g \sin 20° + m_2 g e^{\mu_2 \beta} = 0$

$\Sigma F_y = 0:$ (Same as for (a))

Solve: $m_2 = 0.0979 m_1$

So $\underline{0.0979 m_1 \leq m_2 \leq 2.26 m_1}$

6/118



$\cos\alpha = \dfrac{45}{45+7+26}$, $\alpha = 54.8°$, $\gamma = 180-90-\alpha = 35.2°$

$\beta = 360 - \alpha = 305°$ or 5.33 rad

$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{0.25(5.33)} = 3.79 \qquad \text{(a)}$$

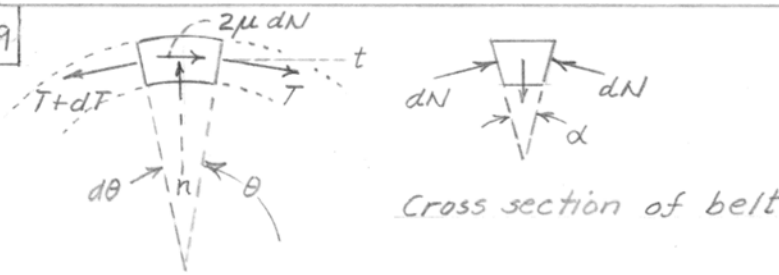
Bar: $\Sigma M_P = 0$: $T_1(h+7+26) - T_2 \sin\gamma\,(h) = 0$

$$\text{or} \quad \frac{T_2}{T_1} = \frac{h+33}{0.577h} \qquad \text{(b)}$$

From (a) & (b), $\underline{h = 27.8 \text{ mm}}$

(For actual wrench, $h \cong 100$ mm)

6/119



$$\Sigma F_n = 0;\quad T \sin\frac{d\theta}{2} + (T+dT)\sin\frac{d\theta}{2} = 2\,dN \sin\frac{\alpha}{2}$$

or $$T\,d\theta = 2\,dN \sin\frac{\alpha}{2}$$

$$\Sigma F_t = 0;\quad T\cos\frac{d\theta}{2} + 2\mu\, dN = (T+dT)\cos\frac{d\theta}{2}$$

or $$2\mu\, dN = dT$$

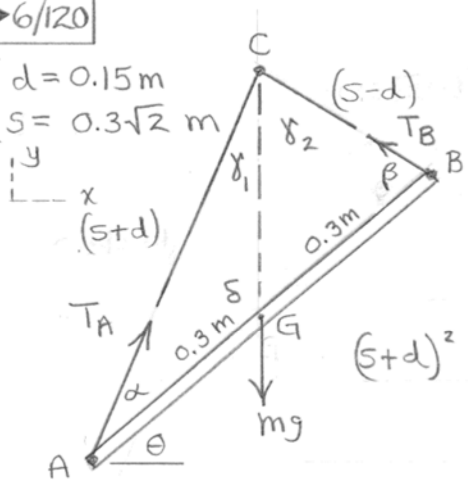
combine & get $$\frac{dT}{T} = \frac{\mu}{\sin\frac{\alpha}{2}}\, d\theta$$

$$\int_{T_1}^{T_2}\frac{dT}{T} = \frac{\mu}{\sin\frac{\alpha}{2}}\int_0^{\beta} d\theta,\quad \ln\frac{T_2}{T_1} = \frac{\mu\beta}{\sin\frac{\alpha}{2}},\quad \underline{T_2 = T_1 e^{\mu\beta/\sin\frac{\alpha}{2}}}$$

$$n = 1/\sin 17.5° = \underline{3.33}$$

▶6/120

$\begin{cases} d = 0.15\text{ m} \\ s = 0.3\sqrt{2}\text{ m} \end{cases}$



Law of cosines for ABC:

$$(s-d)^2 = 0.6^2 + (s+d)^2 - 2(0.6)(s+d)\cos\alpha$$

$$\alpha = 26.9°$$

$$(s+d)^2 = 0.6^2 + (s-d)^2 - 2(0.6)(s-d)\cos\beta$$

$$\beta = 71.3°$$

$$\overline{CG} = \left[0.3^2 + (s+d)^2 - 2(0.3)(s+d)\cos\alpha\right]^{1/2} = 0.335\text{ m}$$

$$\frac{\sin\gamma_1}{0.3} = \frac{\sin\alpha}{\overline{CG}} \Rightarrow \gamma_1 = 23.9°$$

$$\frac{\sin\gamma_2}{0.3} = \frac{\sin\beta}{\overline{CG}} \Rightarrow \gamma_2 = 57.9°$$

$$\delta = 180° - \alpha - \gamma_1 = 129.2°$$

$$\theta = \delta - 90° = 39.2°$$

$$\begin{cases} \Sigma F_x = 0: & T_A\cos(\alpha+\theta) - T_B\cos(\beta-\theta) = 0 \\ \Sigma F_y = 0: & T_A\sin(\alpha+\theta) + T_B\sin(\beta-\theta) - mg = 0 \end{cases}$$

$$T_A = 0.856\,mg, \quad T_B = 0.409\,mg$$

$$\frac{T_A}{T_B} = e^{\mu_s\beta_1}: \text{ With } \beta_1 = \pi - (\gamma_1+\gamma_2): \underline{\mu_s = 0.431}$$

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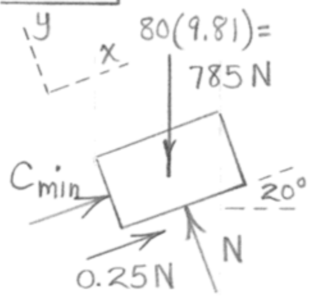
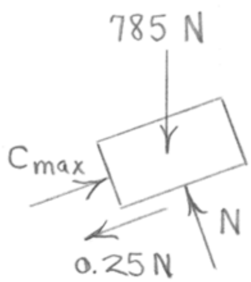


$\Sigma F_y = 0: \quad N - 785\cos 20° = 0$

$N = 737$ N Throughout

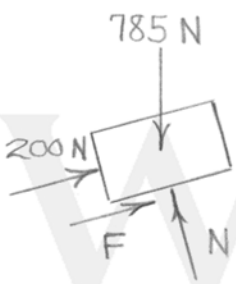
(a) C_{min}

$\Sigma F_x = 0: \quad C_{min} + 0.25(737) - 785\sin 20° = 0, \quad \underline{C_{min} = 84.0 \text{ N}}$



(b) C_{max}

$\Sigma F_x = 0: \quad C_{max} - 0.25(737) - 785\sin 20° = 0, \quad C_{max} = \underline{453 \text{ N}}$



(c) $C = 200$ N

$\Sigma F_x = 0: \quad 200 + F - 785\sin 20° = 0$

$\underline{F = 68.4 \text{ N up the incline}}$

($F < \mu_s N = 0.25(737) = 184.4$ N, so no motion occurs)

6/122 (a) Lower : $\frac{100(9.81)}{T} = e^{0.2(\pi/2)}$

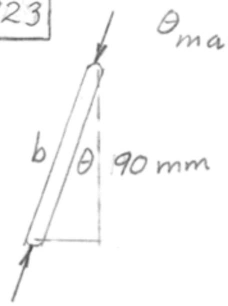
$\underline{T = 717 \text{ N}}$

(b) Raise : $\frac{T}{100(9.81)} = e^{0.2(\pi/2)}$

$\underline{T = 1343 \text{ N}}$

(might need more workers!)

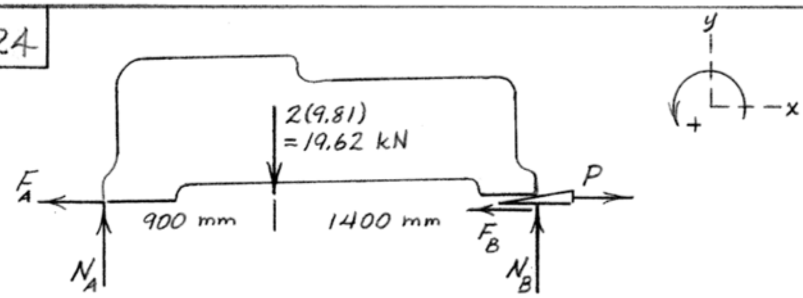
6/123



$$\theta_{max} = \phi = \tan^{-1}\mu = \tan^{-1} 0.40 = 21.80°$$

$$b = 90/\cos 21.80° = \underline{96.9\text{ mm}}$$

6/124



Lathe and wedge as a unit:

$\Sigma M_A = 0$: $2300\ N_B - 19.62(900) = 0$, $\quad N_B = 7.68$ kN

$\Sigma F_y = 0$: $N_A - 19.62 + 7.68 = 0$, $\quad N_A = 11.94$ kN

Wedge:

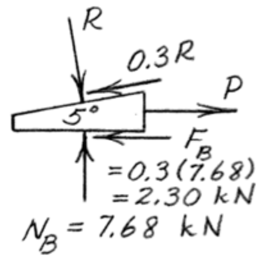
$\Sigma F_y = 0$: $7.68 - R\cos 5° - 0.3 R \sin 5° = 0$

$R = 7.51$ kN

$\Sigma F_x = 0$: $P - 2.30 - 0.3(7.51)\cos 5°$

$+ 7.51 \sin 5° = 0$

$\underline{P = 3.89 \text{ kN}}$

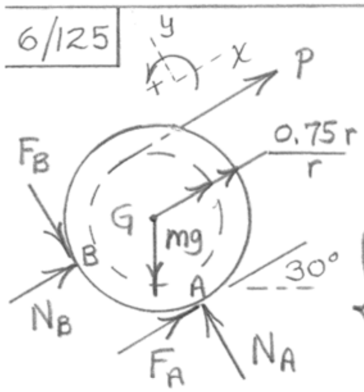


Lathe & wedge:

$\Sigma F_x = 0$: $3.89 - 2.30 - F_A = 0$, $\quad F_A = 1.590$ kN

But $1.590 < 0.3(11.94) = 3.58$ kN; $\underline{\text{A cannot slip.}}$

6/125



(a) Assume no slippage until contact at B is lost:

$F_B = N_B = 0$

$$\begin{cases} \Sigma F_x = 0: & P + F_A - mg\sin 30° = 0 \\ \Sigma F_y = 0: & N_A - mg\cos 30° = 0 \\ \Sigma M_G = 0: & -P\left(\frac{3r}{4}\right) + F_A(r) = 0 \end{cases}$$

Solution: $P = 0.286mg$, $F_A = 0.214mg$, $N_A = \frac{\sqrt{3}}{2}mg$

$(F_A)_{max} = \mu_s N_A = 0.15\frac{\sqrt{3}}{2}mg = 0.1299mg < F_A$

Assumption invalid.

(b) Assume rotational slippage impends:

$F_A = \mu_s N_A = 0.15N_A$, $F_B = \mu_s N_B = 0.15N_A$

$$\left.\begin{aligned} \Sigma F_x = 0: &\quad P + 0.15N_A - mg\sin 30° + N_B = 0 \\ \Sigma F_y = 0: &\quad N_A - mg\cos 30° - 0.15N_B = 0 \\ \Sigma M_G = 0: &\quad -0.75Pr + 0.15N_A r + 0.15N_B r = 0 \end{aligned}\right\}$$

Solution: $P = 0.209mg$, $N_A = 0.890mg$, $N_B = 0.1572mg$

So rotational slippage occurs first at $\underline{P = 0.209mg}$

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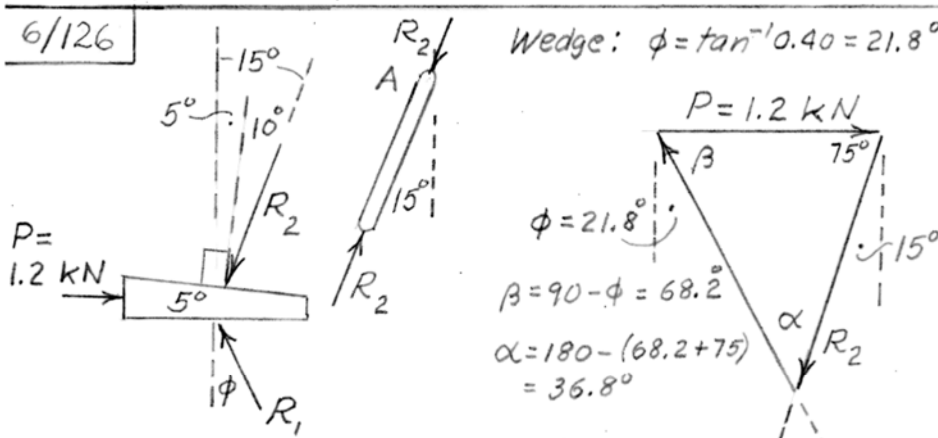


Wedge: $\phi = \tan^{-1} 0.40 = 21.8°$

$\beta = 90 - \phi = 68.2°$

$\alpha = 180 - (68.2 + 75)$
$= 36.8°$

Law of sines: $\dfrac{R_2}{\sin 68.2°} = \dfrac{1.2}{\sin 36.8°}$, $R_2 = 1.2\,\dfrac{0.9285}{0.5990}$

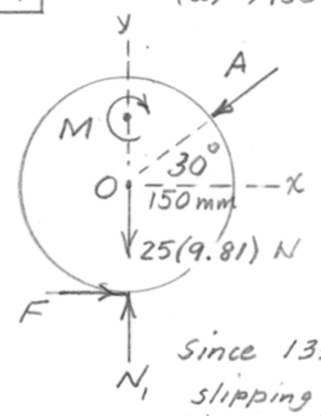
$= 1.860$ kN

Friction force at A is $F = R_2 \sin 15° = 1.860(0.2588)$

$= 0.481$ kN

or $F = 481$ N ; $10° < (\tan^{-1} 0.4)$ so toggle does not slip & $F \neq \mu N$

6/127



(a) Assume complete equilibrium

$\Sigma M_O = 0;\ M - Fr = 0$

$F = \dfrac{20}{0.150} = 133.3\ \text{N}$

$\Sigma F_x = 0;\ 133.3 - A\cos 30° = 0$

$A = 154.0\ \text{N}$

$\Sigma F_y = 0;\ N_1 - 154\sin 30° - 25(9.81) = 0$

$N_1 = 322.2\ \text{N}$

Since $133.3 < (\mu_s N_1 = 0.5[322.2] = 161\ \text{N})$ slipping does not occur & assumption is valid. Thus $\underline{F = 133.3\ \text{N}}$

(b) Assume wheel slips & $\Sigma M_O \neq 0$ so $F = \mu_k N_1 = 0.4 N_1$

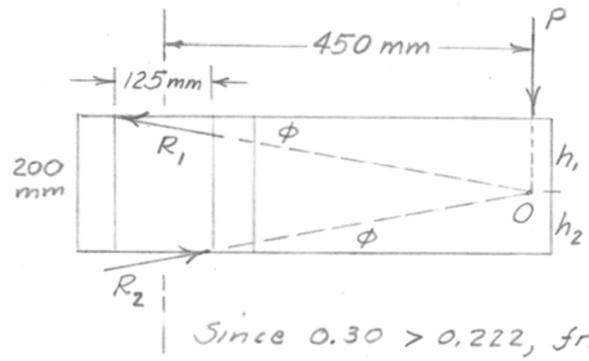
$\Sigma F_x = 0;\ 0.4N_1 - A\cos 30° = 0$

$\Sigma F_y = 0;\ N_1 - 25(9.81) - A\sin 30° = 0$

Solve & get $A = 147.3\ \text{N}$, $N_1 = 318.9\ \text{N}$, $F = 0.4(318.9) = \underline{127.6\ \text{N}}$

Assumption valid since $(M = 40\ \text{N·m}) > (127.6[0.150] = 19.1\ \text{N·m})$

6/128 Forces concurrent at O gives minimum ϕ.



$h_1 = (450 + 62.5)\tan\phi$

$h_2 = (450 - 62.5)\tan\phi$

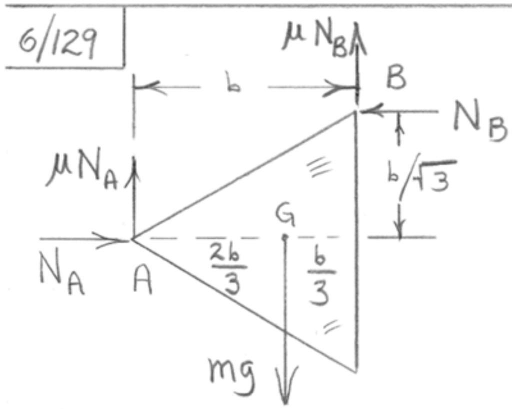
$h_1 + h_2 = 200$ mm

$(512.5 + 387.5)\tan\phi = 200$

$\tan\phi = \frac{200}{900} = 0.222$

Since $0.30 > 0.222$, friction will prevent slipping

6/129



$\Sigma F_x = 0: \quad N_A - N_B = 0 \Rightarrow \quad N_A = N_B = N$

$\Sigma F_y = 0: \quad 2\mu N - mg = 0 \Rightarrow \quad N = \frac{mg}{2\mu}$

$\Sigma M_A = 0: \quad -mg\left(\frac{2b}{3}\right) + N\left(\frac{b}{\sqrt{3}}\right) + \mu N(b) = 0$

Solve to find $\underline{\mu = 1.732}$ Since $\mu \leq 1$, (normally!)

this requirement cannot be met.

6/130 Helix angle $\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{8}{2\pi\frac{25}{2}} = 5.82°$

$\phi = \tan^{-1} 0.25 = 14.04°$

Screw: (a) $M_s = Wr\tan(\alpha + \phi) = 4(\frac{25}{2})\tan 19.86°$

$= 18.05$ N·m

(b) $M_s = Wr\tan(\phi - \alpha) = 4(\frac{25}{2})\tan 8.22°$

$= 7.22$ N·m

Bearing (worn) $M_B = \frac{1}{2}\mu P(R_o + R_i) = \frac{1}{2}(0.25)(4)\frac{20+4}{2}$

$= 6.00$ N·m

Total moment (a) $M = 18.05 + 6.00 = \underline{24.1}$ N·m

(b) $M = 7.22 + 6.00 = \underline{13.22 \text{ N·m}}$

6/131 Helix angle $\alpha = \tan^{-1}\dfrac{24}{40\pi} = 10.81°$

Friction angle $\phi = \tan^{-1}\mu = \tan^{-1} 0.15 = 8.53°$

$\alpha > \phi$ so screw is not self-locking.

$\alpha + \phi = 19.34°$; $\alpha - \phi = 2.28°$

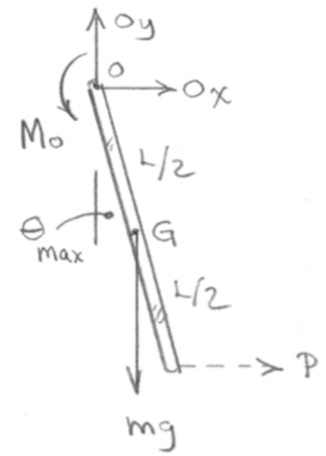
(a) $M = Pr\tan(\alpha - \phi)$: $60 = P(0.020)\tan 2.28°$

$P = 75\,300$ N or $\underline{75.3 \text{ kN}}$

(b) $M = Pr\tan(\alpha + \phi)$: $60 = P(0.020)\tan 19.34°$

$P = 8550$ N or $\underline{8.55 \text{ kN}}$

6/132



$\circlearrowleft + \Sigma M_O = 0: M_O - mg\frac{L}{2}\sin\theta_{max} = 0$

$$\theta_{max} = \sin^{-1}\left[\frac{2M_O}{mgL}\right]$$

$$= \sin^{-1}\left[\frac{2(0.4)}{3(9.81)(0.8)}\right]$$

$$= \underline{1.947^\circ}$$

With the application of of the rightward horizontal force P, M_O will reverse direction & become clockwise:

$\circlearrowleft + \Sigma M_O = 0: \quad PL\cos\theta_{max} - mg\frac{L}{2}\sin\theta_{max} - M_O = 0$

Solving, $\underline{P = 1.001 \text{ N}}$

6/133

$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{13}{2\pi(78)} = 3.04°$$

$$\phi = \tan^{-1}\mu = \tan^{-1}0.25 = 14.04°$$

(a) To raise, $M = Wr\tan(\alpha+\phi)$

$$= \frac{2.2(10^3)9.81}{2}\frac{(0.078)}{2}\tan(3.04° + 14.04°)$$

$$= \underline{129.3 \text{ N·m}}$$

(b) To lower, $M = Wr\tan(\phi-\alpha)$

$$= \frac{2.2(10^3)9.81}{2}\frac{(0.078)}{2}\tan(14.04 - 3.04)$$

$$= \underline{81.8 \text{ N·m}}$$

6/134 (a) m impends down $\quad \dfrac{T_2}{T_1} = e^{\mu_s \beta}$

$$\frac{mg}{2g} = e^{0.2(3\pi)}, \quad m = 13.17 \text{ kg}$$

m impends up

$$\frac{2g}{mg} = e^{0.2(3\pi)}, \quad m = 0.304 \text{ kg}$$

So $\underline{0.304 \le m \le 13.17 \text{ kg}}$

(b)

m impends down:

$$\frac{T_{BC}}{2g} = e^{0.2\pi}$$

$$\frac{T_{AB}}{T_{BC}} = e^{0.5\pi}$$

$$\frac{mg}{T_{AB}} = e^{0.2\pi}$$

Combine: $\dfrac{mg}{2g} = e^{0.4\pi} e^{0.5\pi} = e^{0.9\pi}$

$$m = 33.8 \text{ kg}$$

m impends up: $\dfrac{2g}{mg} = e^{0.9\pi}, \quad m = 0.1183 \text{ kg}$

So $\underline{0.1183 \le m \le 33.8 \text{ kg}}$

6/135

Helix angle $\alpha = \tan^{-1}\frac{L}{\pi d} = \tan^{-1}\frac{1.50}{\pi(10)} = 2.73°$

Friction angle $\varphi = \tan^{-1}\mu_s = \tan^{-1}(0.20) = 11.31°$

Tighten: $M = Pr\tan(\alpha+\varphi) = 400\frac{10}{2}\tan(2.73° + 11.13°)$

$= 500$ N·mm or $\underline{M = 0.500 \text{ N}\cdot\text{m}}$

Loosen: $M' = Pr\tan(\varphi - \alpha) = 400\frac{10}{2}\tan(11.13° - 2.73°)$

$= 302$ N·mm or $\underline{M' = 0.302 \text{ N}\cdot\text{m}}$ (in direction opposite to that of M)

Note: $\alpha < \varphi$, so screw is self-locking (a good feature for a clamp!)

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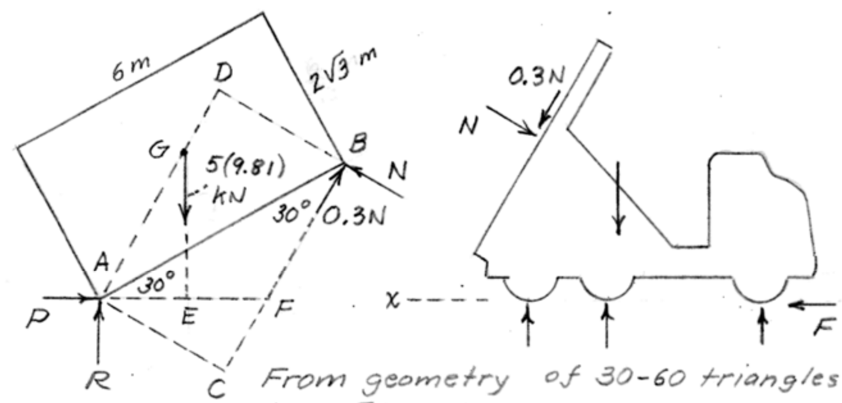


From geometry of 30-60 triangles

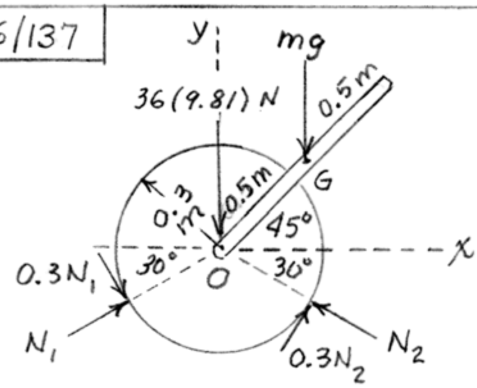
$\overline{AD} = 6\sqrt{3}/2 = 3\sqrt{3}$ m, $AC = 3$ m

$\overline{AE} = \frac{1}{2}\overline{AF} = \frac{1}{2}\, 3/\frac{\sqrt{3}}{2} = \sqrt{3}$ m

Box; $\Sigma M_A = 0$; $5(9.81)\sqrt{3} - 3\sqrt{3}N - 3(0.3N) = 0$, $N = 13.94$ kN

Truck; $\Sigma F_x = 0$; $F - 13.94\left(\frac{\sqrt{3}}{2}\right) + 0.3(13.94)\frac{1}{2} = 0$, $\underline{F = 9.98 \text{ kN}}$

6/137



$\Sigma F_x = 0: \; N_1 \cos 30° + 0.3 N_1 \sin 30° - N_2 \cos 30° + 0.3 N_2 \sin 30° = 0$

$\Sigma F_y = 0: \; N_1 \sin 30° - 0.3 N_1 \cos 30° + N_2 \sin 30° + 0.3 N_2 \cos 30° - 9.81 m - 36(9.81) = 0$

$\circlearrowleft + \; \Sigma M_O = 0: \; 0.3 N_1 (0.3) + 0.3 N_2 (0.3) - 9.81 m (0.5 \cos 45°) = 0$

Solve & get $N_1 = 503$ N

$N_2 = 713$ N

$\underline{m = 31.6 \text{ kg}}$

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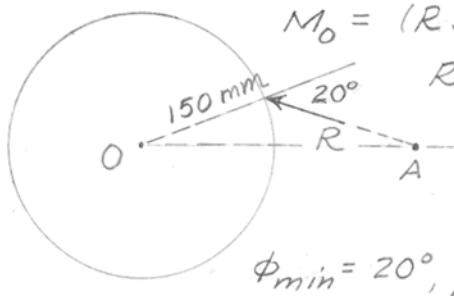
$M_O = PL = 150(0.600) = 90\ \text{N}\cdot\text{m}$

$M_O = (R \sin 20°)(0.150) = 90$



$R = \dfrac{90}{0.3420(0.150)} = 1754\ \text{N}$

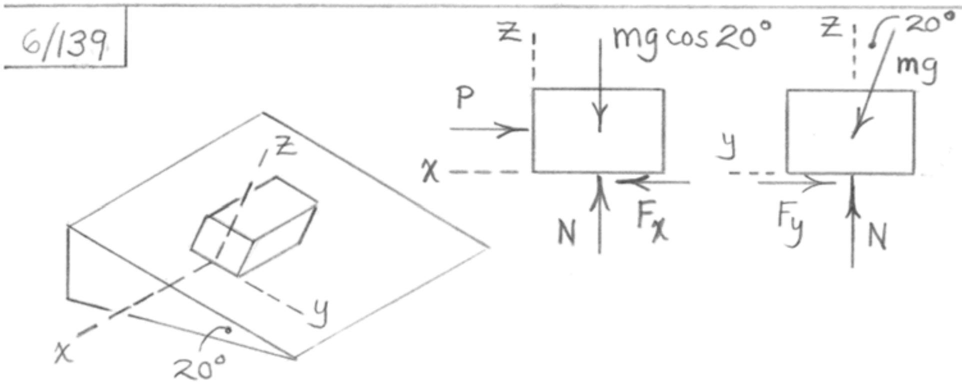
Force on pin at A is

$\underline{R = 1.754\ \text{kN}}$

$\phi_{min} = 20°$, $\mu_{s(min)} = \tan \phi_{min} = \tan 20°$

$= \underline{0.364}$

6/139



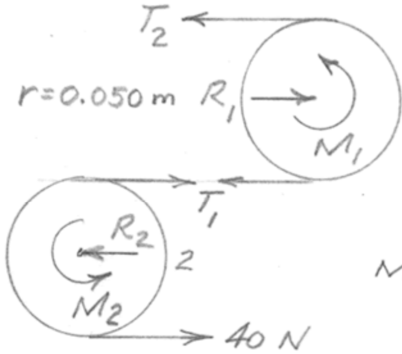
$$(x-z)\begin{cases}\Sigma F_z = 0: \; N - 8(9.81)\cos 20° = 0, & N = 73.7 \text{ N} \\ \Sigma F_x = 0: \; F_x - P = 0, & F_x = P\end{cases}$$

$$(y-z)\left\{\Sigma F_y = 0: -F_y + 8(9.81)\sin 20° = 0, \quad F_y = 26.8 \text{ N}\right.$$

$$F = \sqrt{F_x^2 + F_y^2} = \mu_s N: \sqrt{P^2 + 26.8^2} = 0.5(73.7)$$

$$\underline{P = 25.3 \text{ N}}$$

6/140



$$\frac{T_1}{40} = e^{0.2(\pi)} = 1.874 \quad ; \quad \frac{T_1}{T_2} = e^{0.3(\pi)} = 2.566$$

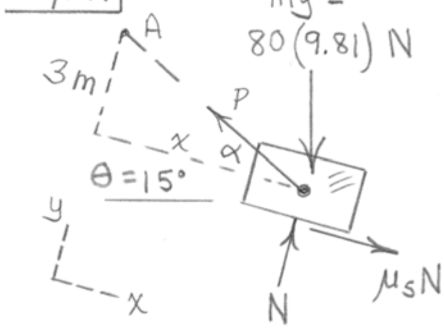
$$T_1 = 40(1.874) = 74.98 \text{ N}$$

$$T_2 = 74.98/2.566 = 29.22 \text{ N}$$

$$M_1 = (T_1 - T_2)r = (74.98 - 29.22)(0.050) = \underline{2.29 \text{ N}\cdot\text{m}}$$

$$M_2 = (T_1 - 40)r = (74.98 - 40)(0.050) = \underline{1.749 \text{ N}\cdot\text{m}}$$

*6/141



$\Sigma F_x = 0$:

$$-P\cos\alpha + mg\sin\theta + \mu_s N = 0$$

$\Sigma F_y = 0$:

$$N - mg\cos\theta + P\sin\alpha = 0$$

Notes:
$$\sin\alpha = \frac{3}{\sqrt{x^2+9}}$$
$$\cos\alpha = \frac{x}{\sqrt{x^2+9}}$$
$$\mu_s = \mu_0 x = 0.1x$$

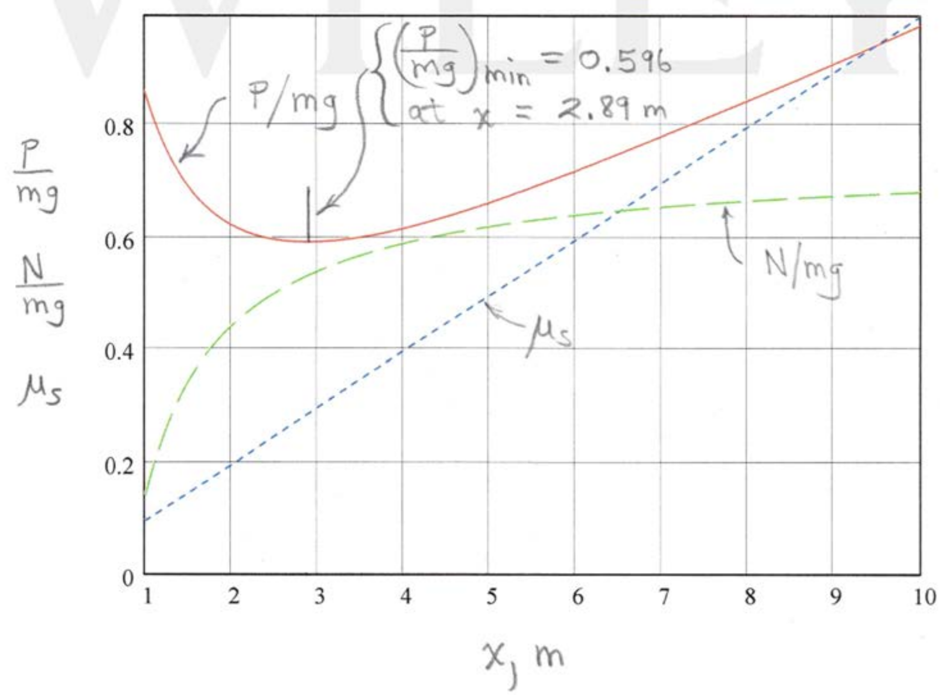
Eliminate N to obtain:

$$P = mg\left(\sin\theta + \mu_0 x\cos\theta\right)\frac{\sqrt{x^2+9}}{x(1+3\mu_0)}$$

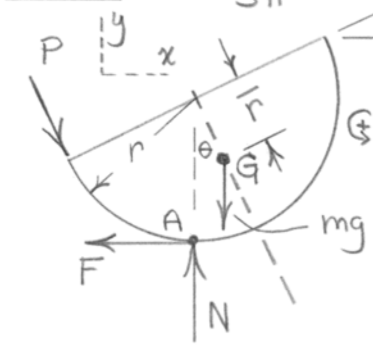
(See plot below)

$P_{min} = 468$ N at $x = 2.89$ m

Note that $N > 0$ over the range $1 \le x \le 10$ m.



*6/142 $\bar{r} = \frac{4r}{3\pi}$



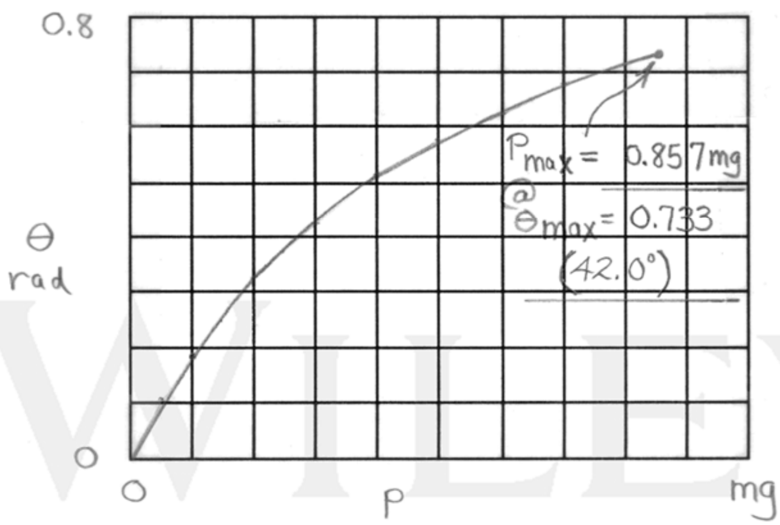
$\Sigma F_x = 0: \; P\sin\theta - F = 0 \quad (1)$

$\Sigma F_y = 0: \; N - mg - P\cos\theta = 0 \quad (2)$

$\circlearrowleft \Sigma M_G = 0: \; Pr - N\bar{r}\sin\theta$

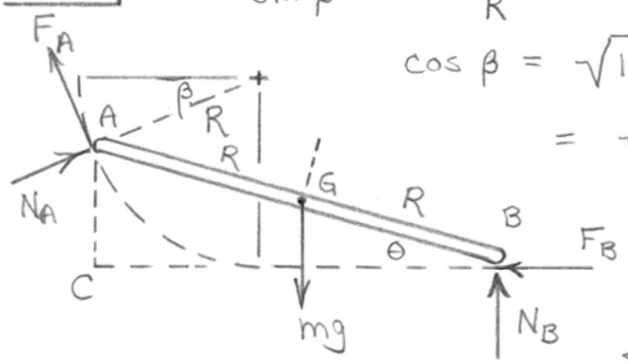
$- F(r - \bar{r}\cos\theta) = 0 \quad (3)$

Vary P and numerically solve Eqs. (1), (2), and (3) for θ, F, and N. Plot for θ:



(Note: Could vary θ & solve for P; then plot!)

*6/143



$$\sin\beta = \frac{R - 2R\sin\theta}{R} = 1 - 2\sin\theta \quad (1)$$

$$\cos\beta = \sqrt{1-\sin^2\beta}$$

$$= \sqrt{1-(1-2\sin\theta)^2}$$

$$= 2\sqrt{\sin\theta(1-\sin\theta)} \quad (2)$$

$$\begin{cases} F_A = \mu_A N_A \\ F_B = \mu_B N_B \end{cases}$$

$$\xrightarrow{+} \Sigma F = 0: -\mu_A N_A \sin\beta + N_A\cos\beta - \mu_B N_B = 0 \quad (3)$$

$$+\uparrow \Sigma F = 0: \mu_A N_A \cos\beta + N_A \sin\beta - mg + N_B = 0 \quad (4)$$

$$\Sigma M_A = 0: -mg\cancel{R}\cos\theta + N_B(2\cancel{R}\cos\theta)$$

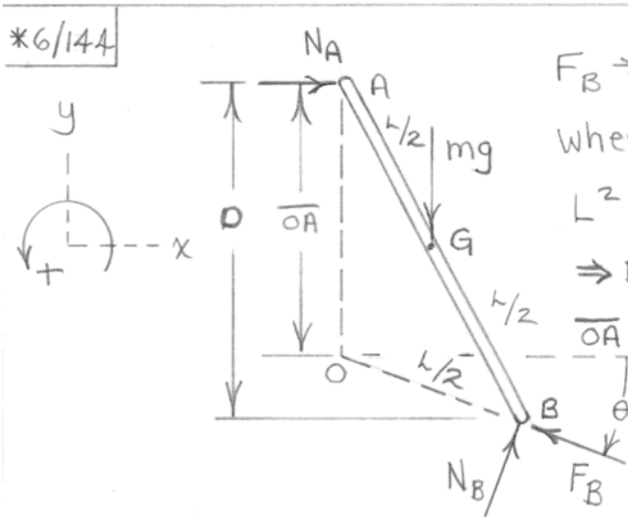
$$-\mu_B N_B(2\cancel{R}\sin\theta) = 0 \quad (5)$$

Numerically solve Eqs. (1) – (5):

$$\underline{\theta = 21.5°}$$

$$(N_A = 0.401mg,\ N_B = 0.623mg,\ \beta = 15.49°)$$

*6/144



$F_B \rightarrow \mu_s N_B = 0.4 N_B$ when slipping impends.

$L^2 = D^2 + \left(\frac{L}{2}\cos\theta\right)^2$

$\Rightarrow D = L\sqrt{1 - \frac{\cos^2\theta}{4}}$

$\overline{OA} = L\sqrt{1 - \frac{\cos^2\theta}{4}} - \frac{L}{2}\sin\theta$

$= \frac{L}{2}\left[\sqrt{4 - \cos^2\theta} - \sin\theta\right]$

$\Sigma F_x = 0: \; N_A + N_B \sin\theta - 0.4 N_B \cos\theta = 0 \quad (1)$

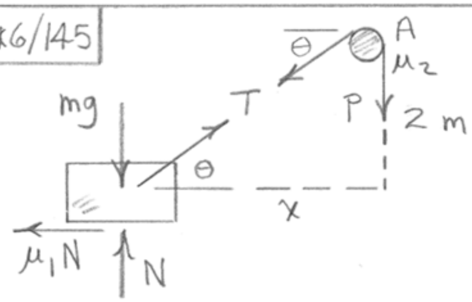
$\Sigma F_y = 0: \; N_B \cos\theta + 0.4 N_B \sin\theta - mg = 0 \quad (2)$

$\Sigma M_O = 0: \; N_B\left(\frac{L}{2}\right) - N_A \frac{L}{2}\left[\sqrt{4 - \cos^2\theta} - \sin\theta\right]$

$- mg\frac{L}{4}\cos\theta = 0 \quad (3)$

Numerical solution : $\begin{cases} N_A = 0.287\,mg \\ N_B = 0.966\,mg \\ \theta = 5.80° \end{cases}$

*6/145

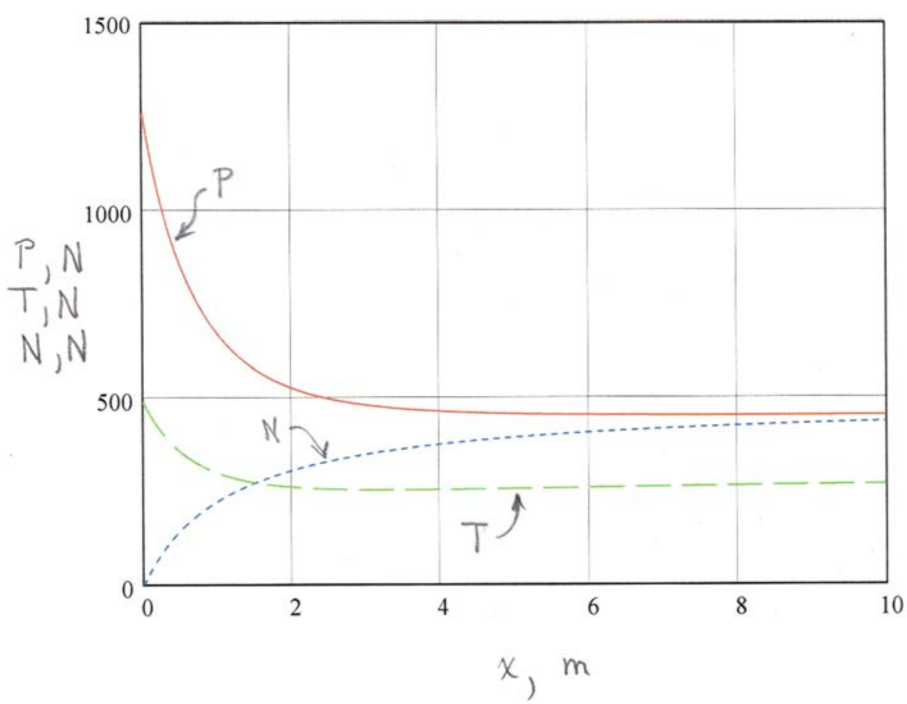


$$\begin{cases} \Sigma F_x = 0: & +\mu_1 N - T\cos\theta = 0 \\ \Sigma F_y = 0: & N - mg + T\sin\theta = 0 \end{cases}$$

A: $\frac{P}{T} = e^{\mu_2 \beta}$

Also: $\theta = \tan^{-1}\frac{2}{x}$ and $\beta = \frac{\pi}{2} + \theta$

Set $m = 50$ kg, $g = 9.81$ m/s^2, $\mu_1 = 0.60$, $\mu_2 = 0.30$ & vary x to obtain the plot below. At $x = 0$, the value $P = 1259$ N causes $T = 490$ N, which results in $N \to 0$, thus no friction. At $x = 3$ m, $\underline{P = 483 \text{ N}}$.



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$\mu = 0.4$

From geometry,

upper wheel:

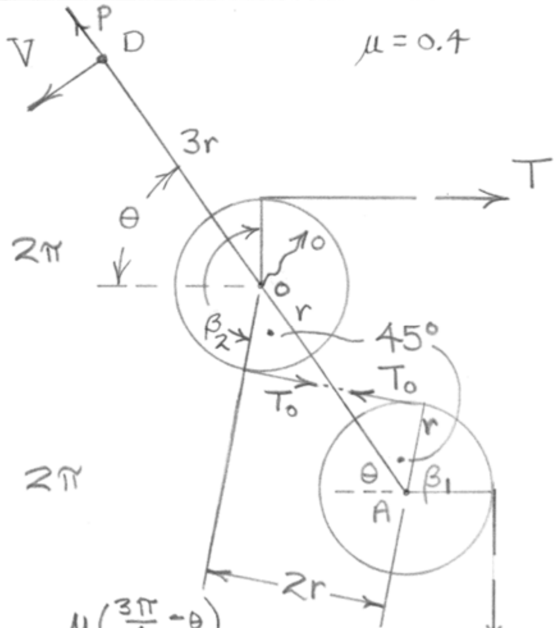
$$\theta + \frac{\pi}{4} + \frac{\pi}{2} + \beta_2 = 2\pi$$

$$\beta_2 = \frac{5\pi}{4} - \theta$$

lower wheel:

$$\theta + \frac{\pi}{4} + \beta_1 + \pi = 2\pi$$

$$\beta_1 = \frac{3\pi}{4} - \theta$$



Forces: $\begin{cases} mg = T_0 e^{\mu\left(\frac{3\pi}{4} - \theta\right)} \\ T_0 = T e^{\mu\left(\frac{5\pi}{4} - \theta\right)} \end{cases}$

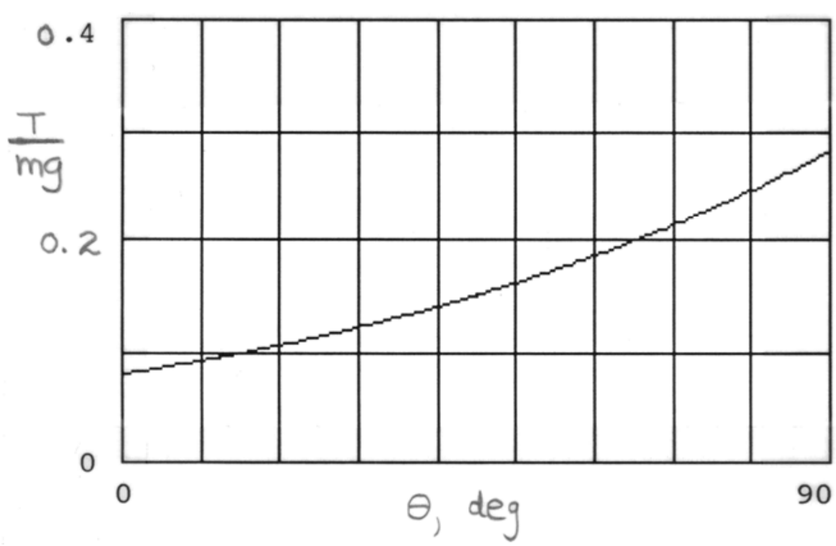
So $mg = Te^{2\mu(\pi - \theta)}$; $\dfrac{T}{mg} = e^{-0.8(\pi - \theta)}$

Device as a whole:

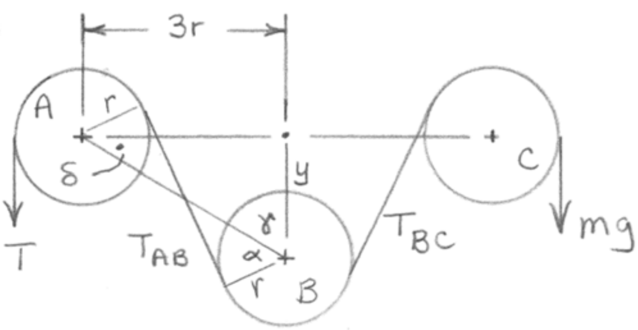
$\circlearrowleft + \Sigma M_O = 0$: $3rV - Tr - mg\left(r + 2r\sqrt{2}\cos\theta\right) = 0$

$$V = \frac{1}{3}\left[T + mg\left(1 + 2\sqrt{2}\cos\theta\right)\right]$$

For $\theta = 60°$, $\dfrac{T}{mg} = 0.1872$, $\underline{V = 0.867mg}$



*6/147



$\gamma = \tan^{-1}\left(\frac{3r}{y}\right)$, $\delta = 180° - 90° - \gamma = 90° - \gamma$

$$\alpha = \cos^{-1}\frac{r}{\sqrt{y^2+(3r)^2}/2} = \cos^{-1}\frac{2r}{\sqrt{y^2+9r^2}}$$

$$\beta_A = \beta_C = 180° - (\alpha - \delta) = 180° - \alpha + \delta$$

$$\beta_B = 2(180° - \gamma - \alpha)$$

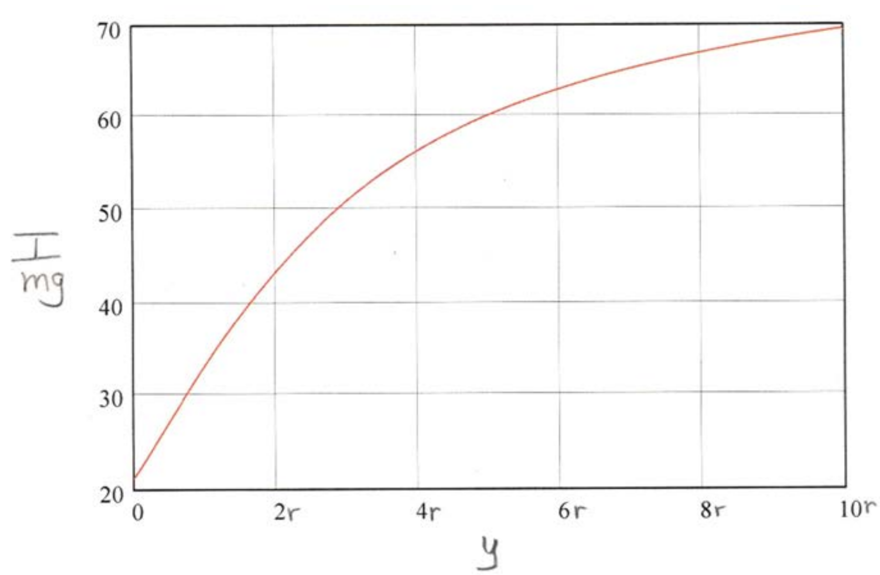
$\frac{T_{BC}}{mg} = e^{\mu_C \beta_C}$, $\frac{T_{AB}}{T_{BC}} = e^{\mu_B \beta_B}$, $\frac{T}{T_{AB}} = e^{\mu_A \beta_A}$

Combine: $\frac{T}{mg} = e^{[\mu_A \beta_A + \mu_B \beta_B + \mu_C \beta_C]}$

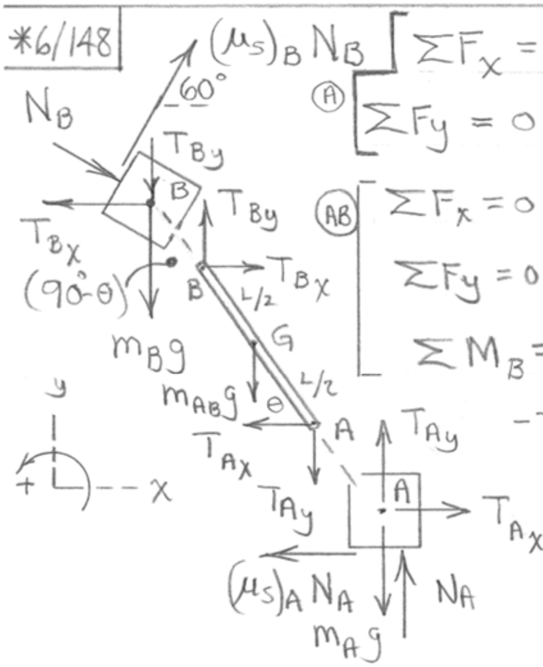
Resulting plot below.

If $y \to 0$, $\frac{T}{mg} \to 21.2$

If $y \to$ large, $\frac{T}{mg} \to 81.3$



*6/148



(A) $\Sigma F_x = 0: \; T_{Ax} - (\mu_s)_A N_A = 0$

$\Sigma F_y = 0: \; N_A - m_A g + T_{Ay} = 0$

(AB) $\Sigma F_x = 0: \; -T_{Ax} + T_{Bx} = 0$

$\Sigma F_y = 0: \; -T_{Ay} + T_{By} - m_{AB} g = 0$

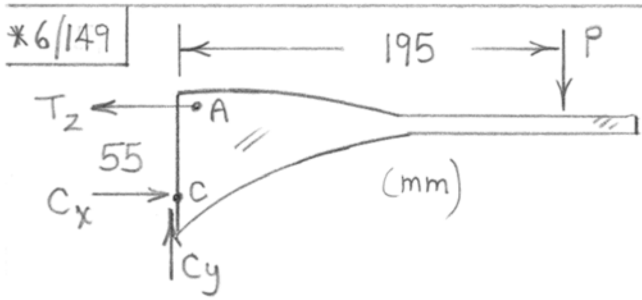
$\Sigma M_B = 0: \; -m_{AB} g \frac{L}{2} \cos\theta - T_{Ax} L \sin\theta - T_{Ay} L \cos\theta = 0$

B $\Sigma F_x = 0: \; N_B \cos 30° + (\mu_s)_B N_B \cos 60° - T_{Bx} = 0$

$\Sigma F_y = 0: \; -N_B \sin 30° + (\mu_s)_B N_B \sin 60° - T_{By} - m_B g = 0$

Numerically solve 7 eqs. in 7 unknowns, with m_{AB} varied, to find that $\underline{\theta = \theta_{min} = 53.8°}$ is invariant with respect to variation in m_{AB}!

*6/149



wrench handle

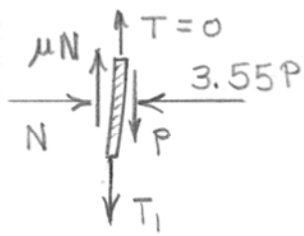
$$\xrightarrow{+} \Sigma F = 0: \quad C_x - T_2 = 0$$

$$+\uparrow \Sigma F = 0: \quad C_y - P = 0$$

$$\circlearrowleft + \Sigma M_C = 0: \quad T_2(55) - P(195) = 0$$

$$\left.\right\} \quad T_2 = 3.55P, \quad C_y = P, \quad C_x = 3.55P$$

Small band element near (at & just below!) C:



$$\xrightarrow{+} \Sigma F = 0 \Rightarrow \quad N = 3.55P$$

$$+\downarrow \Sigma F = 0: \quad T_1 + P - \mu(3.55P) = 0$$

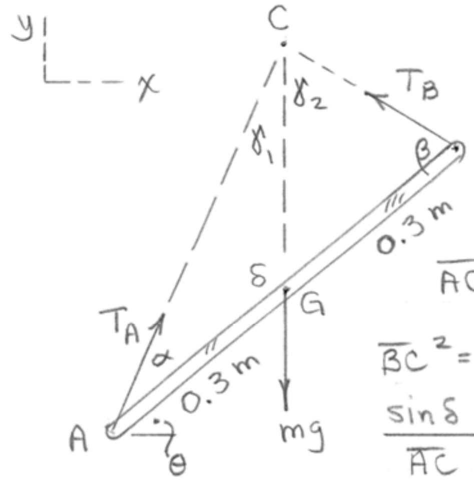
$$T_1 = P(3.55\mu - 1)$$

Belt considerations from C to B:

$$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{3.55P}{P(3.55\mu - 1)} = e^{\mu(3\pi/2)}$$

Numerical solution: $\underline{\mu = 0.420}$

*6/150



Write 12 equations in 12 unknowns:

$\delta = \theta + 90°$ (1)

$\overline{AC} + \overline{BC} = 0.6\sqrt{2}$ (2)

$\overline{AC}^2 = 0.3^2 + \overline{CG}^2 - 2(0.3)(\overline{CG})\cos\delta$ (3)

$\overline{BC}^2 = 0.3^2 + \overline{CG}^2 - 2(0.3)(\overline{CG})\cos(180° - \delta)$ (4)

$\dfrac{\sin\delta}{\overline{AC}} = \dfrac{\sin\gamma_1}{0.3}$ (5)

$\dfrac{\sin(180° - \delta)}{\overline{BC}} = \dfrac{\sin\gamma_2}{0.3}$ (6)

$\dfrac{\sin\delta}{\overline{AC}} = \dfrac{\sin\alpha}{\overline{CG}}$ (7)

$\dfrac{\sin(180° - \delta)}{\overline{BC}} = \dfrac{\sin\beta}{\overline{CG}}$ (8)

$\Sigma F_x = 0: \; T_A\cos(\alpha + \theta) - T_B\cos(\beta - \theta) = 0$ (9)

$\Sigma F_y = 0: \; T_A\sin(\alpha + \theta) + T_B\sin(\beta - \theta) - mg = 0$ (10)

$\dfrac{T_A}{T_B} = e^{\mu_s \beta_1}$ (11) with $\beta_1 = 180° - (\gamma_1 + \gamma_2)$ (12)

Solve all to obtain $\underline{\theta = 18.00°}$

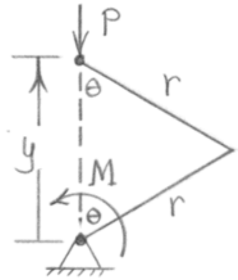
7/1

$\delta U = 0: \quad -M\delta\theta - P\delta y = 0$

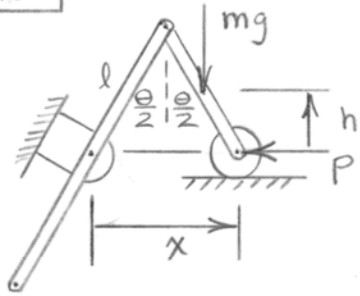
$y = 2r\cos\theta, \quad \delta y = -2r\sin\theta\,\delta\theta$

So $\quad M\delta\theta = P(2r\sin\theta\,\delta\theta)$

$\underline{M = 2Pr\sin\theta}$



7/2



$$x = 2l\sin\frac{\theta}{2}$$

$$\delta x = l\cos\frac{\theta}{2}\,\delta\theta$$

$$h = \frac{l}{2}\cos\frac{\theta}{2}$$

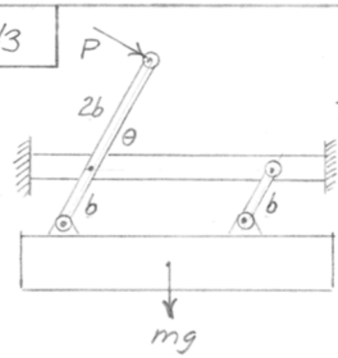
$$\delta h = -\frac{l}{4}\sin\frac{\theta}{2}\,\delta\theta$$

$$\delta U = 0: \quad -P\delta x - mg\,\delta h = 0$$

$$-P\left(l\cos\frac{\theta}{2}\,\delta\theta\right) - mg\left(-\frac{l}{4}\sin\frac{\theta}{2}\,\delta\theta\right) = 0$$

$$\Rightarrow \quad \theta = 2\tan^{-1}\left(\frac{4P}{mg}\right)$$

7/3



$\delta U = 0;$

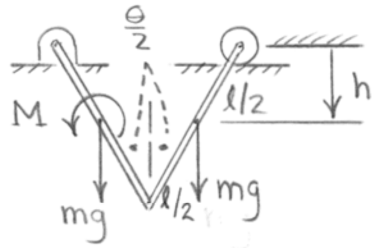
$$-P(2b\,\delta\theta) + mg\,\delta(b\sin\theta) = 0$$

$$2Pb\,\delta\theta = mgb\cos\theta\,\delta\theta$$

$$\cos\theta = \frac{2P}{mg}$$

$$\underline{\theta = \cos^{-1}\frac{2P}{mg}}$$

7/4



$h = \frac{l}{2}\cos\frac{\theta}{2}$

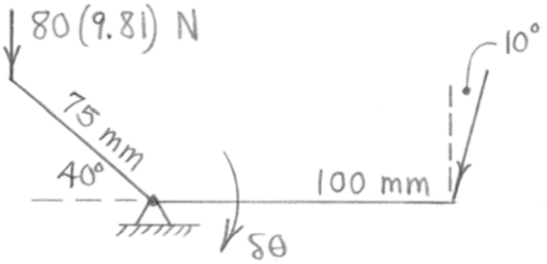
$\delta h = -\frac{l}{4}\sin\frac{\theta}{2}\,\delta\theta$

$\delta U = 0: \quad M\,\delta\left(\frac{\theta}{2}\right) + 2mg\,\delta h = 0$

$M\frac{\delta\theta}{2} + 2mg\left(-\frac{l}{4}\sin\frac{\theta}{2}\,\delta\theta\right) = 0$

$\underline{M = mgl\sin\frac{\theta}{2}}$

7/5

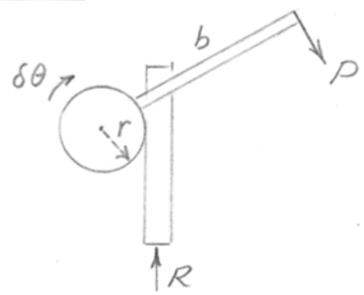


For a virtual displacement $\delta\theta$ of the lever,

$\delta U = 0$: $P\cos 10°(100\ \delta\theta) - 80(9.81)[75\ \delta\theta \cos 40°] = 0$

$$\underline{P = 458\text{ N}}$$

7/6



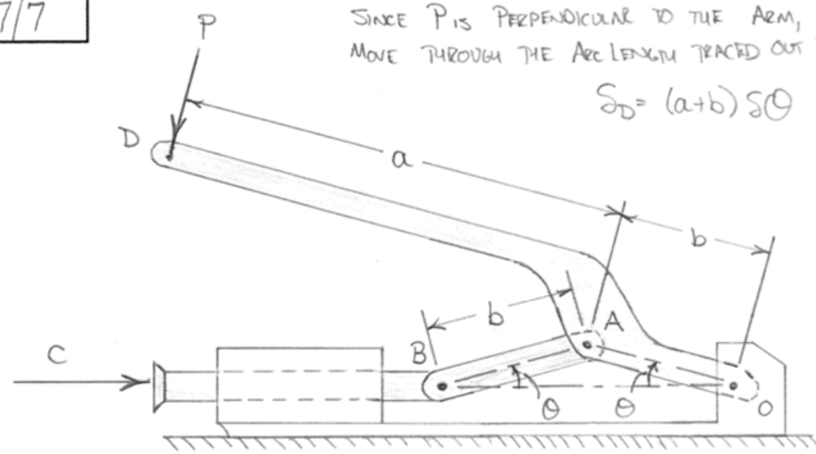
$\delta U = 0$; $Pb\delta\theta - Rr\delta\theta = 0$

$R = P\frac{b}{r}$

7/7

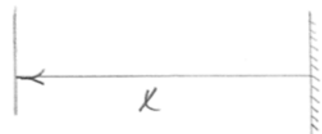
SINCE P IS PERPENDICULAR TO THE ARM, IT WILL MOVE THROUGH THE ARC LENGTH TRACED OUT BY D.

$\delta_D = (a+b)\,\delta\theta$



$x = 2b\cos\theta$

$\delta x = -2b\sin\theta\,\delta\theta$



$\delta U = 0: \ -P\delta_D - C\,\delta x = 0$

$-P(a+b)\delta\theta - C(-2b\sin\theta\,\delta\theta) = 0$

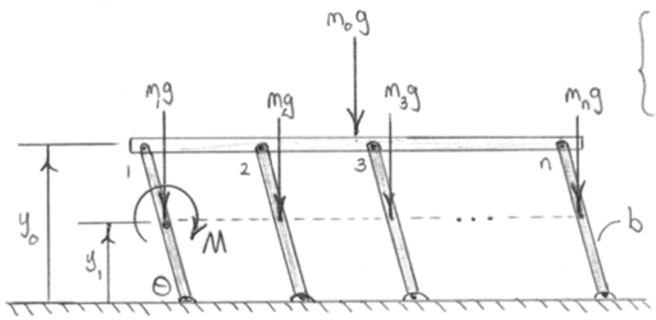
$\therefore \quad \underline{C = \dfrac{P(a+b)}{2b\sin\theta}}$

7/8

MEASURE θ POSITIVE CW

$m_1 = m_2 = m_3 = m_n = m$

$$\begin{cases} y_0 = b\sin\theta \\ y_1 = y_2 = y_3 = y_n = \frac{b}{2}\sin\theta \end{cases}$$



$$\delta U = 0: \quad M\delta\theta - n(mg)\,\delta y_1 - m_0 g\,\delta y_0 = 0$$

$$M\delta\theta - nmg\frac{b}{2}\cos\theta\,\delta\theta - m_0 g\,b\cos\theta\,\delta\theta = 0$$

$$\underline{\theta = \cos^{-1}\left(\frac{2M}{bg(2m_0 + nm)}\right)}$$

7/9 (Jaw movement) $= \frac{a}{b}$ (handle movement)

$\delta U = 0$; $P\delta x = Q\frac{a}{b}\delta x$ where δx = virtual displacement of each handle. So $\underline{Q = \frac{b}{a}P}$

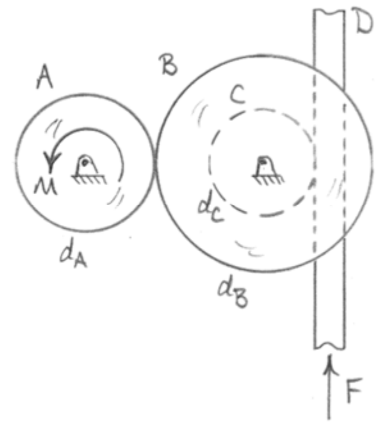
7/10 $e = \dfrac{\text{output work}}{\text{input work}}$

To raise, $0.75 = \dfrac{100(9.81)(1/4)}{P(1)}$, $\underline{P = 327\ N}$

To lower, $0.75 = \dfrac{P'(1)}{100(9.81)(1/4)}$, $\underline{P' = 183.9\ N}$

7/11 d_A, d_B, d_C ARE PITCH DIAMETERS

GEAR A WILL ROTATE CCW BY θ_A AND GEARS B & C WILL ROTATE CW θ_B.



$r_A \delta\theta_A = r_B \delta\theta_B \longrightarrow \delta\theta_B = \frac{r_A}{r_B}\delta\theta_A$ AND $\delta\theta_C = \delta\theta_B$

$y_D = r_C \delta\theta_C = \frac{r_A r_C}{r_B}\delta\theta_A$ (DOWNWARD)

$\delta U = 0: \quad M\delta\theta_A = F\delta y_D \longrightarrow F = \frac{r_B}{r_A r_C} M$

IN TERMS OF DIAMETERS ... $\underline{F = \frac{2 d_B}{d_A d_C} M}$

7/12

THE ADDED GEAR BETWEEN A AND B HAS NO EFFECT ON THE OUTPUT FORCE MAGNITUDE OF F.

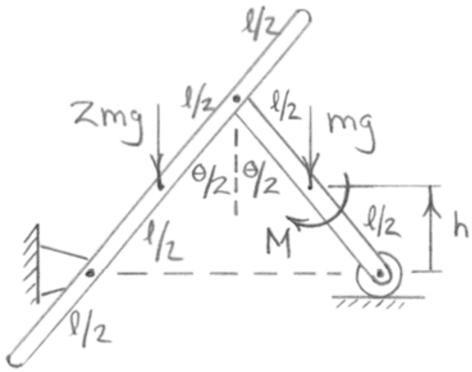
HOWEVER, IT WILL REVERSE THE DIRECTION OF ROTATION FOR GEARS B AND C WHICH MEANS F WILL HAVE TO BE APPLIED DOWNWARD IN THE FIGURE INSTEAD OF UPWARD.

7/13

$$\delta U = 0: \; 160 F \delta\theta - 0.4(160 F \delta\theta) - 100(9.81)\left(150\,\delta\frac{\theta}{25}\right) = 0$$

$$0.6(160)F = 981(6), \quad \underline{F = 61.3 \text{ N}}$$

7/14



$h = \frac{l}{2}\cos\frac{\theta}{2}$ for both bars

$\delta h = -\frac{l}{4}\sin\frac{\theta}{2}\,\delta\theta$

$$\delta U = 0: -M\,\delta\left(\frac{\theta}{2}\right) + 2mg\,\delta h + mg\,\delta h = 0$$

$$-\frac{1}{2}M\,\delta\theta + 3mg\left(-\frac{l}{4}\sin\frac{\theta}{2}\,\delta\theta\right) = 0$$

$$\underline{M = \frac{3}{2}mgl\sin\frac{\theta}{2}}$$

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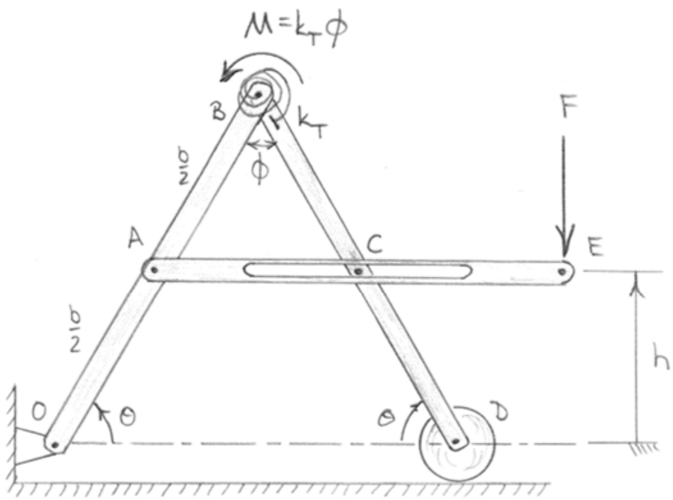
$\theta = 60°$

$\phi = 60°$



$$\begin{cases} h = \frac{b}{2}\sin\theta \\ \delta h = \frac{b}{2}\cos\theta\,\delta\theta \end{cases} \qquad \begin{cases} \phi = \pi - 2\theta \\ \delta\phi = -2\delta\theta \end{cases} \qquad M = k_T\phi = k_T(\pi - 2\theta)$$

(IN RADIANS)

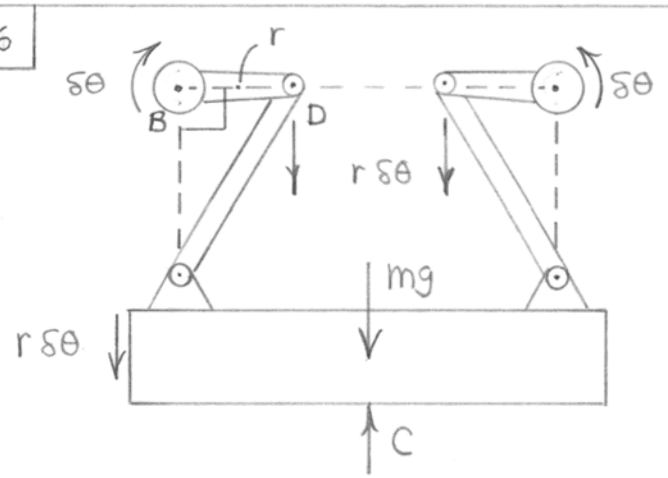
$$\delta U = 0: \quad -F\,\delta h - M\,\delta\phi = 0$$

$$-F\left(\frac{b}{2}\cos\theta\,\delta\theta\right) - k_T\phi(-2\delta\theta) = 0$$

$$F\frac{b}{2}\cos\theta = 2k_T\phi$$

$$F\frac{b}{2}\cos 60 = 2k_T\frac{\pi}{3} \longrightarrow \underline{k_T = \frac{3Fb}{8\pi}}$$

7/16



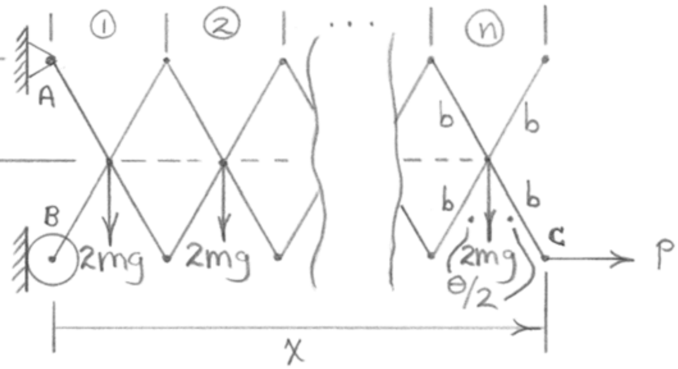
Let β = angle through which worm shaft turns : $\delta\beta = n\,\delta\theta$

System : $\delta U = 0$: $M\,\delta\beta + (mg - C)\,r\,\delta\theta = 0$

$$M\,n\,\delta\theta = (C - mg)\,r\,\delta\theta$$

$$\underline{M = (C - mg)\frac{r}{n}}$$

7/17



$y = b\cos\frac{\theta}{2}$, $\delta y = -\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta$

$x = n\left(2b\sin\frac{\theta}{2}\right)$, $\delta x = nb\cos\frac{\theta}{2}\,\delta\theta$

$\delta U = 0$: $P\delta x + n(2mg)\delta y = 0$

$$P\left(nb\cos\frac{\theta}{2}\,\delta\theta\right) = -2nmg\left(-\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta\right)$$

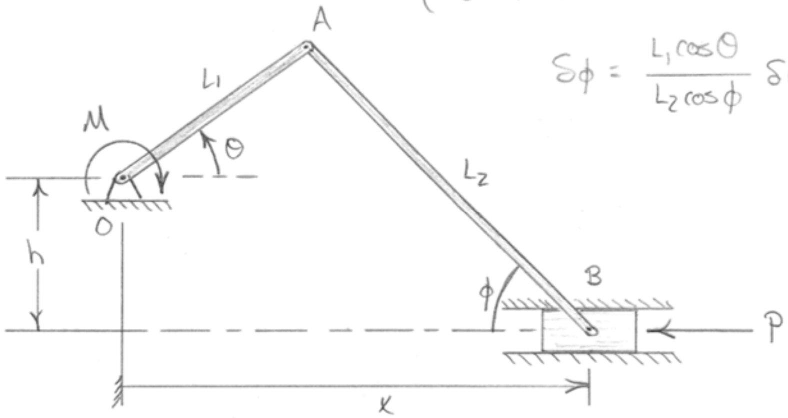
$$\underline{P = mg\tan\frac{\theta}{2}}$$

P does <u>not depend</u> on the number n of sections present.

7/18

$$\begin{cases} L_2 \sin\phi = h + L_1 \sin\theta \\ L_2 \cos\phi \, \delta\phi = L_1 \cos\theta \, \delta\theta \end{cases}$$

$$\delta\phi = \frac{L_1 \cos\theta}{L_2 \cos\phi} \delta\theta$$



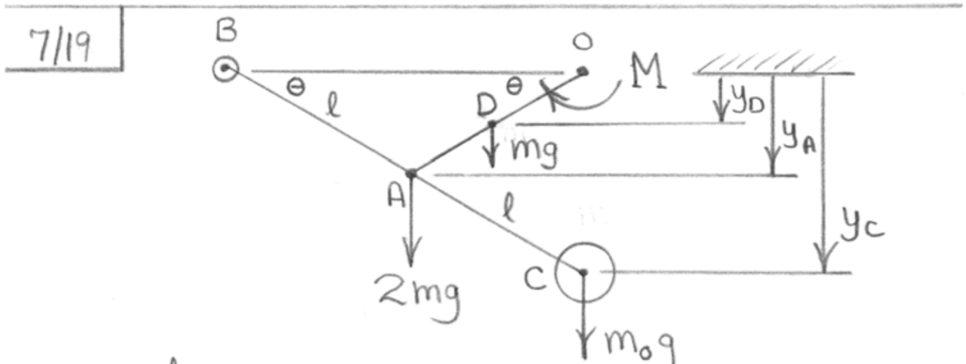
$$\begin{cases} x = L_1 \cos\theta + L_2 \cos\phi \\ \delta x = -L_1 \sin\theta \, \delta\theta - L_2 \sin\phi \, \delta\phi = (-L_1 \sin\theta - L_1 \tan\phi \cos\theta) \, \delta\theta \end{cases}$$

$\delta U = 0$: $\quad -M\delta\theta - P\delta x = 0$

$$-M\delta\theta + PL_1(\sin\theta + \tan\phi\cos\theta)\delta\theta = 0$$

$$\underline{M = PL_1(\sin\theta + \tan\phi\cos\theta) \quad \text{and} \quad \phi = \sin^{-1}\left(\frac{h + L_1\sin\theta}{L_2}\right)}$$

7/19



$y_D = \frac{l}{2}\sin\theta$, $\delta y_D = \frac{l}{2}\cos\theta\,\delta\theta$

$y_A = l\sin\theta$, $\delta y_A = l\cos\theta\,\delta\theta$

$y_C = 2l\sin\theta$, $\delta y_C = 2l\cos\theta\,\delta\theta$

$\delta U = 0: \; -M\delta\theta + mg\,\delta y_D + 2mg\,\delta y_A + m_0 g\,\delta y_C = 0$

$$-M\delta\theta + mg\left(\frac{l}{2}\cos\theta\,\delta\theta\right) + 2mg\left(l\cos\theta\,\delta\theta\right) + m_0 g\left(2l\cos\theta\,\delta\theta\right) = 0$$

$$\Rightarrow M = \left(\frac{5}{2}m + 2m_0\right)gl\cos\theta$$

For $\theta = 30°$: $\underline{M = \left(\frac{5}{4}m + m_0\right)gl\sqrt{3}}$

7/20

$$e = \frac{\text{output work}}{\text{input work}}$$

Let $\delta\theta$ = virtual crank angle, radians

δh = vertical movement of lifting pad, meters

where $\delta\theta/\delta h = \dfrac{12(2\pi)}{0.024} = 1000\pi$, $\delta\theta = 1000\pi\,\delta h$

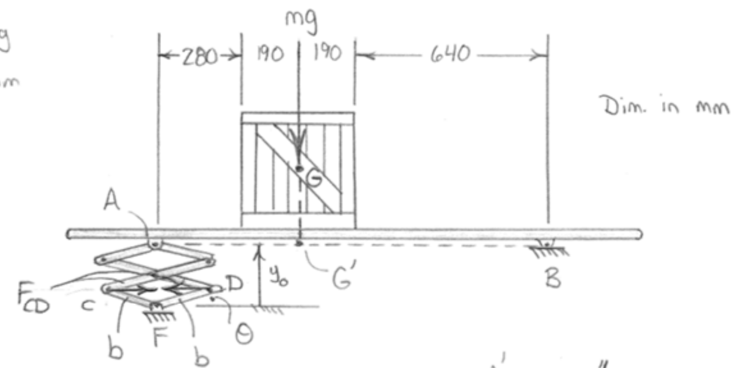
To raise, $e = \dfrac{L\,\delta h}{Fr\,\delta\theta} = \dfrac{1.5(10^3)(9.81)\,\delta h}{50(0.150)1000\pi\,\delta h} = \underline{0.625}$

7/21

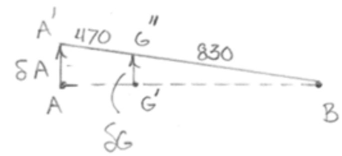
$m = 50$ kg

$b = 180$ mm

$\theta = 15°$



$$\begin{cases} \overline{CD} = 2b\cos\theta \\ \delta\overline{CD} = -2b\sin\theta\,\delta\theta \end{cases} \qquad \begin{cases} y_0 = 4b\sin\theta \\ \delta y_0 = 4b\cos\theta\,\delta\theta \end{cases}$$



$$\frac{\delta_G}{830} = \frac{\delta_A}{1300} \longrightarrow \delta_G = \frac{830}{1300}\delta_A$$

(AND $\delta A = \delta y_0$)

$$\delta U = 0: \quad -F_{CD}\,\delta\overline{CD} - mg\,\delta_G = 0$$

$$-F_{CD}(-2b\sin\theta\,\delta\theta) = mg\left(\frac{830}{1300}4b\cos\theta\,\delta\theta\right)$$

$$\therefore \quad \underline{F_{CD} = \frac{83}{65}mg\cot\theta}$$

WITH NUMBERS... $F_{CD} = \frac{83}{65}(50)(9.81)\cot 15° \longrightarrow \underline{F_{CD} = 2340 \text{ N}}$

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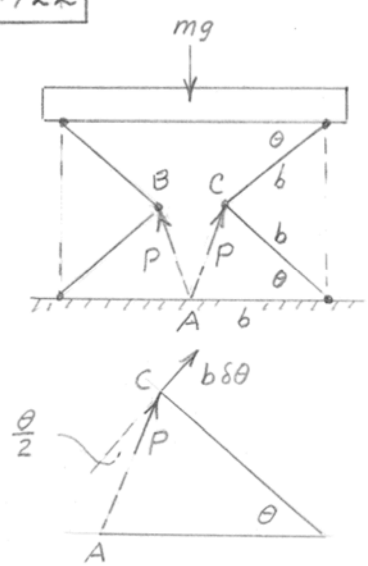


Work done by each P is

$$\left(P\cos\frac{\theta}{2}\right)b\,\delta\theta$$

Work done by mg is

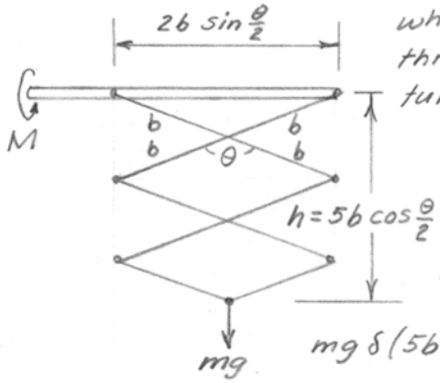
$$-mg\,\delta(2b\sin\theta)$$

$\delta U = 0$; $2Pb\cos\frac{\theta}{2}\,\delta\theta - 2bmg\cos\theta\,\delta\theta = 0$

$$P\cos\frac{\theta}{2} = mg\cos\theta$$

$$P = mg\,\frac{\cos\theta}{\cos\theta/2}$$

7/23 L = lead of screw; $\delta\left(2b\sin\frac{\theta}{2}\right) = \frac{\delta\alpha}{2\pi}L$



where α is the angle through which the screw turns. Thus

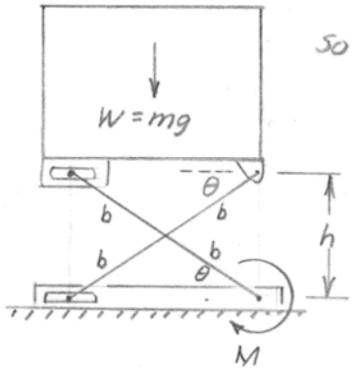
$$\delta\alpha = \frac{2\pi b}{L}\cos\frac{\theta}{2}\,\delta\theta$$

$\delta U = 0$; $mg\,\delta h + M\delta\alpha = 0$

$$mg\,\delta\left(5b\cos\frac{\theta}{2}\right) + M\left(\frac{2\pi b}{L}\cos\frac{\theta}{2}\,\delta\theta\right) = 0$$

$$-\frac{5}{2}mgb\sin\frac{\theta}{2}\,\delta\theta + \frac{2\pi bM}{L}\cos\frac{\theta}{2}\,\delta\theta = 0, \quad \underline{M = \frac{5mgL}{4\pi}\tan\frac{\theta}{2}}$$

7/24 $\delta U = 0; \quad M\delta\theta - mg\,\delta h = 0$



$h = 2b\sin\theta, \quad \delta h = 2b\cos\theta\,\delta\theta$

So $M\delta\theta = mg(2b\cos\theta)\delta\theta$

$M = 2mgb\cos\theta$

But since $\sin\theta = \frac{h}{2b}$,

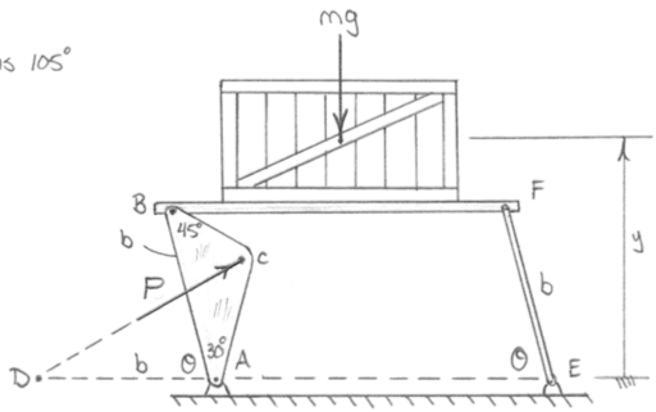
$\cos\theta = \sqrt{1-\left(\frac{h}{2b}\right)^2}$

Thus

$M = 2mgb\sqrt{1-\left(\frac{h}{2b}\right)^2}$

7/25

ANGLE AT C IS 105°



$$\frac{\overline{AC}}{\sin 45^\circ} = \frac{b}{\sin 105^\circ} \longrightarrow \overline{AC} = \frac{2b}{1+\sqrt{3}} = 0.732\,b$$

$$\overline{CD}^2 = b^2 + \overline{AC}^2 - 2b\,\overline{AC}\cos(\theta+30^\circ) \longrightarrow \overline{CD} = b\sqrt{1.536 - 1.464\cos(\theta+30^\circ)}$$

AND

$$2\,\overline{CD}\,\delta_{CD} = 2b\,\overline{AC}\sin(\theta+30^\circ)\,\delta\theta \longrightarrow \delta_{CD} = \frac{0.732\,b\sin(\theta+30^\circ)}{\sqrt{1.536-1.464\cos(\theta+30^\circ)}}\,\delta\theta$$

AND

$$y = b\sin\theta + \text{CONSTANTS} \longrightarrow \delta y = b\cos\theta\,\delta\theta$$

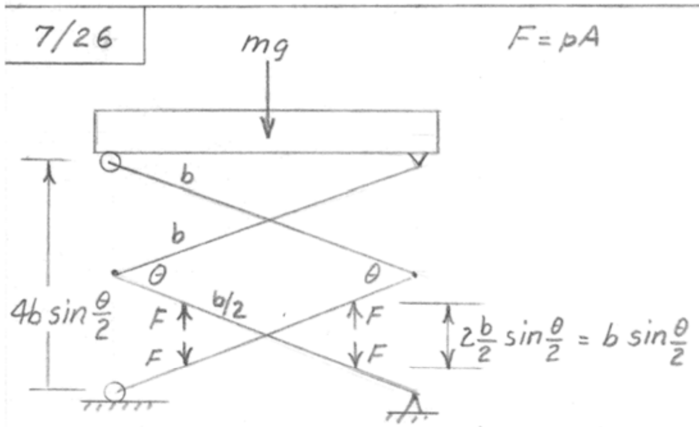
$$\delta U = 0: \quad P\,\delta_{CD} - mg\,\delta y = 0$$

SOLVING...

$$P = \frac{1.366\,mg\cos\theta\sqrt{1.536-1.464\cos(\theta+30^\circ)}}{\sin(\theta+30^\circ)}$$

7/26

$F = pA$



Work done by each cylinder is $F\delta(b\sin\frac{\theta}{2})$

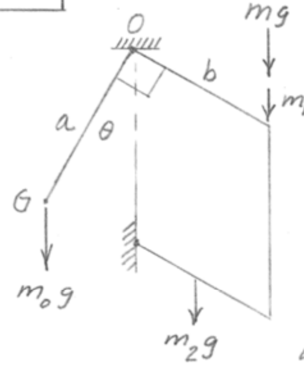
$$= \frac{Fb}{2}\cos\frac{\theta}{2}\,\delta\theta$$

$$\delta U = 0; \quad -mg\,\delta\left(4b\sin\frac{\theta}{2}\right) + 2\,\frac{Fb}{2}\cos\frac{\theta}{2}\,\delta\theta = 0$$

$$Fb\cos\frac{\theta}{2} = 2mgb\cos\frac{\theta}{2}, \quad F = pA = 2mg$$

so $\underline{p = 2mg/A}$ independent of b & θ

7/27



$\delta U = 0;\ m_0 g\,\delta(a\cos\theta) + (m + m_1)g\,\delta(b\sin\theta) + m_2 g\,\delta(\frac{b}{2}\sin\theta) = 0$

$-m_0 a\sin\theta\,\delta\theta + (m+m_1)b\cos\theta\,\delta\theta + \frac{m_2 b}{2}\cos\theta\,\delta\theta = 0$

$m_0 a\tan\theta = mb + (m_1 + \frac{m_2}{2})b$ ---(1)

when $\theta = \theta_0$, $mg = 0$ so

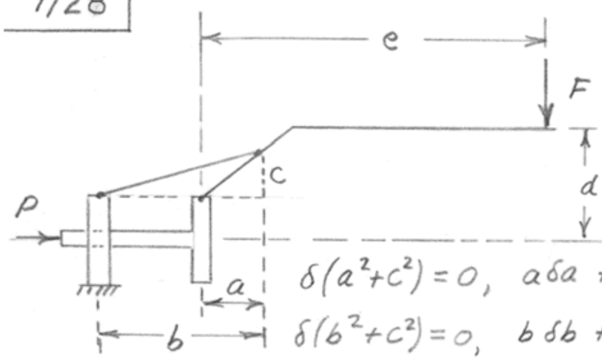
$m_0 a\tan\theta_0 = (m_1 + \frac{m_2}{2})b$ -------(2)

Eliminate m_1 & m_2 from Eqs. (1) & (2) & get

$m_0 a\tan\theta = mb + m_0 a\tan\theta_0$

$m = \frac{a}{b} m_0(\tan\theta - \tan\theta_0)$

7/28



$\delta(a^2+c^2)=0, \quad a\delta a + c\delta c = 0$

$\delta(b^2+c^2)=0, \quad b\delta b + c\delta c = 0$

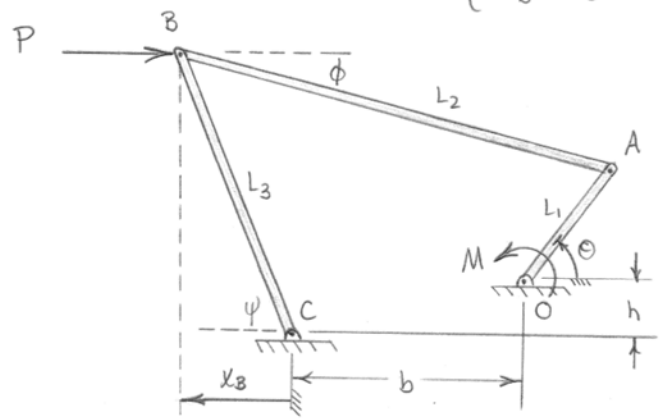
$\delta U=0; \quad P\delta(b-a) - 2F\delta d = 0 \quad$ where $\delta d = \frac{e}{a}\delta c$

so $\quad P\left(-\frac{c}{b}\delta c + \frac{c}{a}\delta c\right) - 2F\frac{e}{a}\delta c = 0$

$\frac{Pc}{ab}(b-a) = \frac{2Fe}{a}, \quad P = \frac{2Feb}{c(b-a)}$

7/29

$$\begin{cases} x_B = L_3 \cos\psi \\ \delta x_B = -L_3 \sin\psi \, \delta\psi \end{cases}$$



$$\begin{cases} L_1 \sin\theta + L_2 \sin\phi + h = L_3 \sin\psi \\ L_1 \cos\theta \, \delta\theta + L_2 \cos\phi \, \delta\phi = L_3 \cos\psi \, \delta\psi \end{cases}$$

$$\begin{cases} L_1 \cos\theta + L_3 \cos\psi + b = L_2 \cos\phi \\ L_1 \sin\theta \, \delta\theta + L_3 \sin\psi \, \delta\psi = L_2 \sin\phi \, \delta\phi \end{cases}$$

SOLVING FOR $\delta\phi$ AND $\delta\psi$...

$$\delta\phi = \frac{L_1}{L_2} \csc(\phi-\psi)\sin(\theta+\psi)\,\delta\theta \quad \& \quad \delta\psi = \frac{L_1}{L_3}\csc(\phi-\psi)\sin(\theta+\phi)\,\delta\theta$$

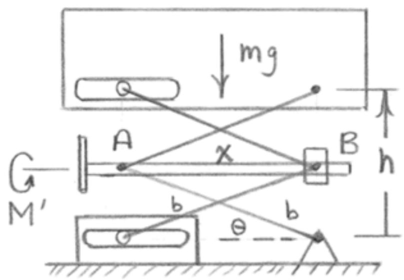
$$\delta U = 0: \quad M\,\delta\theta - P\,\delta x_B = 0$$

$$M\,\delta\theta + P L_3 \sin\psi \left(\frac{L_1}{L_3}\csc(\phi-\psi)\sin(\theta+\phi)\right)\delta\theta = 0$$

$$\therefore \quad \underline{M = P L_1 \sin\psi \csc(\psi-\phi)\sin(\theta+\phi)}$$

NOTE: $\csc(\psi-\phi) = -\csc(\phi-\psi)$

7/30 M' = necessary moment without friction



Let β = angle through which screw turns

$\delta U = 0: \quad M'\delta\beta - mg\,\delta h = 0$

$\frac{L}{2\pi} = \frac{-\delta(x)}{\delta\beta}, \quad \delta\beta = \frac{2\pi}{L}(-\delta x)$

$x = 2b\cos\theta, \quad \delta x = -2b\sin\theta\,\delta\theta$

$\delta\beta = \frac{4\pi b}{L}\sin\theta\,\delta\theta$

$h = 4b\sin\theta, \quad \delta h = 4b\cos\theta\,\delta\theta$

Thus $M'\frac{4\pi b}{L}\sin\theta\,\delta\theta - mg\,(4b\cos\theta\,\delta\theta) = 0$

$M' = \frac{mgL}{\pi}\cot\theta$

$M = M_f + \frac{mgL}{\pi}\cot\theta$

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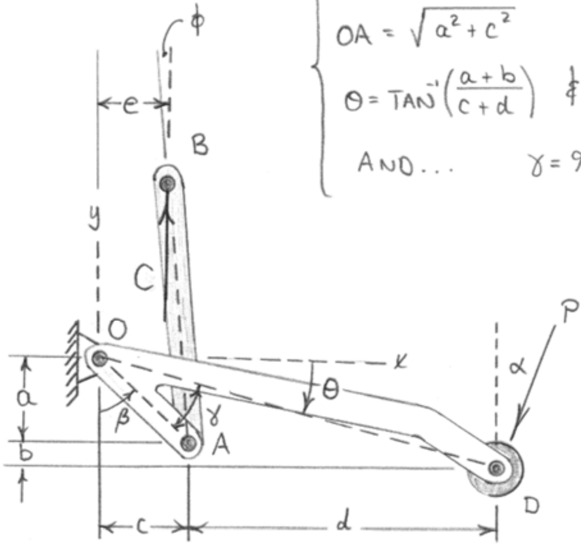


$$\begin{cases} \overline{OD} = \sqrt{(a+b)^2 + (c+d)^2} \\ OA = \sqrt{a^2 + c^2} \\ \theta = \tan^{-1}\left(\frac{a+b}{c+d}\right) \quad \& \quad \beta = \tan^{-1}\left(\frac{c}{a}\right) \\ \text{AND...} \quad \gamma = 90° - \theta - \beta \quad \text{(CONSTANT)} \end{cases}$$

$x_D = \overline{OD}\cos\theta \qquad y_D = -\overline{OD}\sin\theta \qquad P_x = -P\sin\alpha$

$\delta x_D = -\overline{OD}\sin\theta\,\delta\theta \qquad \delta y_D = -\overline{OD}\cos\theta\,\delta\theta \qquad P_y = -P\cos\alpha$

$y_B = \overline{AB}\cos\phi - \overline{OA}\sin(\theta+\gamma)$

$\delta y_B = -\overline{AB}\sin\phi\,\delta\phi - \overline{OA}\cos(\theta+\gamma)\,\delta\theta$

$x_A = \overline{OA}\cos(\theta+\gamma) = \overline{AB}\sin\phi + e$

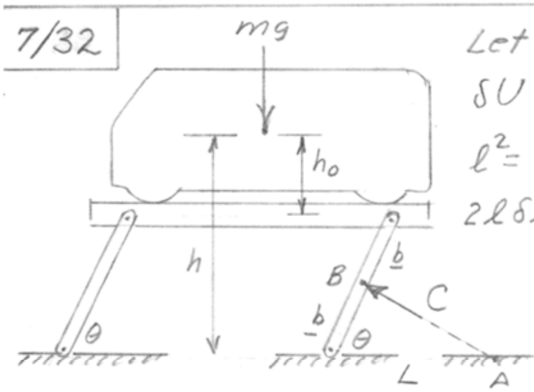
$\delta x_A = -\overline{OA}\sin(\theta+\gamma)\,\delta\theta = \overline{AB}\cos\phi\,\delta\phi \longrightarrow \delta\phi = \dfrac{-\overline{OA}\sin(\theta+\gamma)}{\overline{AB}\cos\phi}\,\delta\theta$

$\delta U = 0: \quad P_x\,\delta x_D + P_y\,\delta y_D + C\,\delta y_B = 0$

SUBSTITUTING AND SOLVING FOR THE CRUSHING FORCE C...

$$C = \frac{(c+d)\cos\alpha + (a+b)\sin\alpha}{c - a\tan\phi}\,P$$

7/32



Let $\overline{AB} = \ell$

$\delta U = 0$; $C\delta\ell - mg\delta h = 0$

$\ell^2 = (b\sin\theta)^2 + (L - b\cos\theta)^2$

$2\ell\delta\ell = 2b^2\sin\theta\cos\theta\,\delta\theta + 2(L - b\cos\theta)(b\sin\theta\,\delta\theta) = 2Lb\sin\theta\,\delta\theta$

$\delta\ell = \frac{Lb}{\ell}\sin\theta\,\delta\theta$

$h = 2b\sin\theta + h_o$, $\delta h = 2b\cos\theta\,\delta\theta + 0$

Thus $C\frac{Lb}{\ell}\sin\theta\,\delta\theta - mg(2b\cos\theta\,\delta\theta) = 0$

$C = 2mg\frac{\ell}{L}\text{ctn}\,\theta = \frac{2mg}{L}\sqrt{(b\sin\theta)^2 + (L - b\cos\theta)^2}\,\text{ctn}\,\theta$

$C = 2mg\sqrt{1 + \left(\frac{b}{L}\right)^2 - 2\frac{b}{L}\cos\theta}\;\text{ctn}\,\theta$

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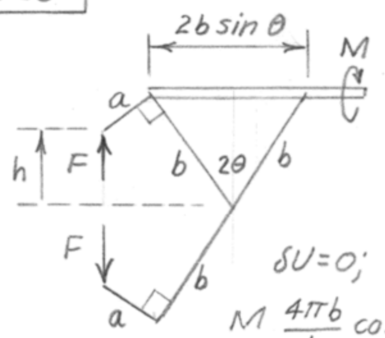


$\delta(2b\sin\theta) = \frac{\delta\alpha}{2\pi}L$ where α is the angle of rotation of the screw.

$\delta h = \delta(b\cos\theta - a\sin\theta)$

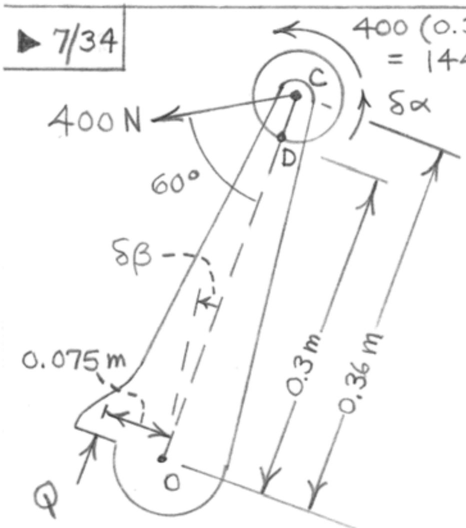
$= (-b\sin\theta - a\cos\theta)\,\delta\theta$

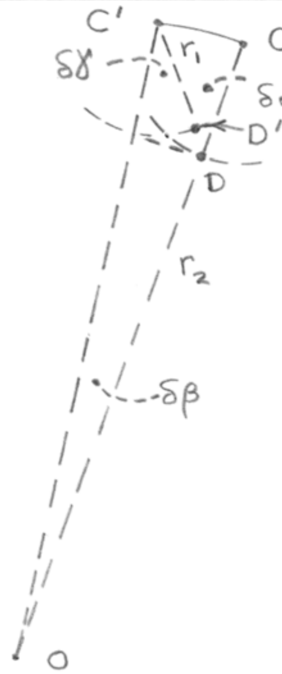
$\delta U = 0$; $M\delta\alpha + 2F\delta h = 0$

$M\frac{4\pi b}{L}\cos\theta\,\delta\theta + 2F(-b\sin\theta - a\cos\theta)\,\delta\theta = 0$

so $F = \frac{2\pi M b\cos\theta}{L(b\sin\theta + a\cos\theta)} = \frac{2\pi M}{L(\tan\theta + a/b)}$

▶ 7/34





During rotation $\delta\beta$, CD moves to C'D' & rotates through the absolute angle $\delta\alpha = \delta\gamma + \delta\beta = \frac{arc}{r_1} + \delta\beta$

$$= \frac{r_2\,\delta\beta}{r_1} + \delta\beta = \left(\frac{r_2}{r_1} + 1\right)\delta\beta = \left(\frac{300}{60} + 1\right)\delta\beta = 6\,\delta\beta$$

$$\delta U = 0: \ -Q(0.075)\delta\beta + 400\cos 30°(0.360\,\delta\beta) + 144(6\,\delta\beta) = 0, \quad \underline{Q = 13.18 \text{ kN}}$$

7/35 $V = 6x^3 - 9x^2 - 7$

$\frac{dV}{dx} = 18x^2 - 18x = 0$ for equil. $\underline{x = 0 \text{ or } x = 1}$

$\frac{d^2V}{dx^2} = 36x - 18 \quad = -18$ for $\underline{x = 0 \text{ so unstable}}$

$= +18$ " $\underline{x = 1 \text{ so stable}}$

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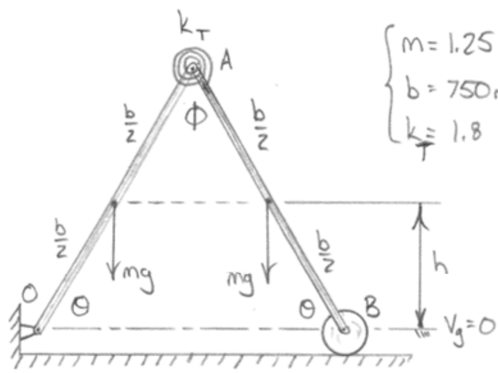
$$\begin{cases} \phi = \pi - 2\theta \\ \delta\phi = -2\delta\theta \end{cases}$$



$$\begin{cases} m = 1.25 \text{ kg} \\ b = 750 \text{ mm} \\ k_T = 1.8 \text{ N·m/rad} \end{cases}$$

$$V = 2mg\frac{b}{2}\sin\theta + \frac{1}{2}k_T\phi^2$$

$$\delta V = mgb\cos\theta\,\delta\theta + k_T\phi\,\delta\phi = \left(mgb\cos\theta - 2k_T(\pi - 2\theta)\right)\delta\theta$$

FOR EQUILIBRIUM... $mgb\cos\theta = 2k_T(\pi - 2\theta)$

SOLVING NUMERICALLY FOR ROOTS BETWEEN $\theta = 0$ AND $\theta = 90°$...

$\underline{\theta = 22.3°}$ AND $\underline{\theta = 90°}$

FOR STABILITY, $\frac{d^2V}{d\theta^2} = -mgb\sin\theta + 4k_T$

FOR $\theta = 22.3°$, $\frac{d^2V}{d\theta^2} = 3.71$ <u>STABLE</u>

FOR $\theta = 90°$, $\frac{d^2V}{d\theta^2} = -1.997$ <u>UNSTABLE</u>

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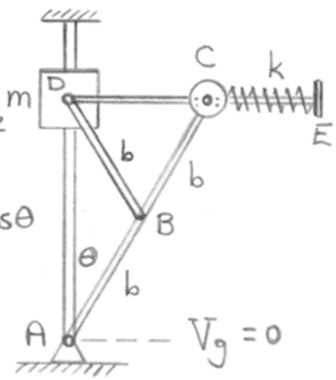
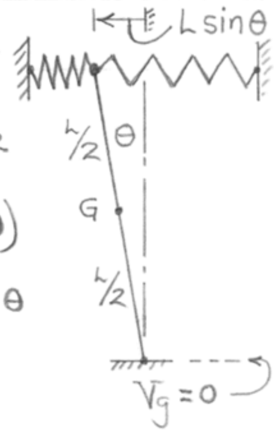


$$V = V_g + V_e$$
$$= mg(2b\cos\theta) + \tfrac{1}{2}k(2b\sin\theta)^2$$
$$\frac{dV}{d\theta} = -2mgb\sin\theta + 4kb^2\sin\theta\cos\theta$$
$$= 2b\sin\theta(-mg + 2kb\cos\theta)$$
$$= 0 \text{ for equilibrium}$$

So $\sin\theta = 0$ or $\underline{\theta = \cos^{-1}\frac{mg}{2kb}}$

For $\theta_{max} = 30°$, $k_{min} = \frac{mg}{2b\cos 30°} = \underline{\frac{mg}{b\sqrt{3}}}$

7/38 δ = initial spring compression



$$V = V_g + V_e = mg\frac{L}{2}\cos\theta + \frac{1}{2}k(\delta + L\sin\theta)^2 + \frac{1}{2}k(\delta - L\sin\theta)^2$$

$$= \frac{1}{2}mgL\cos\theta + k(\delta^2 + L^2\sin^2\theta)$$

$$\frac{dV}{d\theta} = -\frac{1}{2}mgL\sin\theta + 2kL^2\sin\theta\cos\theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{1}{2}mgL\cos\theta + 2kL^2\cos 2\theta$$

For equilibrium, $\frac{dV}{d\theta} = 0$, so

$$\left(-\frac{1}{2}mg + 2kL\cos\theta\right)\sin\theta = 0$$

$$\Rightarrow \theta = 0 \quad \text{or} \quad \cos\theta = \frac{mg}{4kL}$$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = -\frac{1}{2}mgL + 2kL^2$

> 0 (Stable) if $2kL^2 > \frac{1}{2}mgL$

So $k_{min} = \frac{mg}{4L}$

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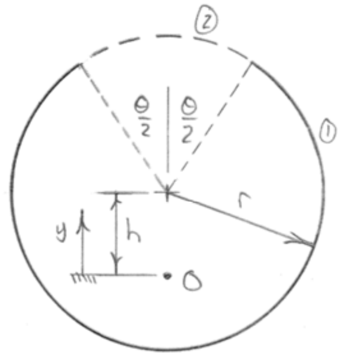
FOR STABILITY, THE MASS CENTER MUST LIE AT OR BELOW THE PIVOT POINT O.



$$\bar{Y} = \frac{\Sigma L\bar{y}}{\Sigma L} = \frac{2\pi r h_{max} - r\theta\left(h_{max} + \frac{r \sin\frac{\theta}{2}}{\theta/2}\right)}{2\pi r - r\theta} = 0$$

SOLVING...
$$h_{max} = \frac{2r\sin\theta/2}{2\pi - \theta}$$

(a) IF $\theta = 30°$, $h_{max} = 0.0899\,r$

(b) IF $\theta = 45°$, $h_{max} = 0.1392\,r$

(c) IF $\theta = 60°$, $h_{max} = 0.1910\,r$

(d) IF $\theta = 90°$, $h_{max} = 0.300\,r$

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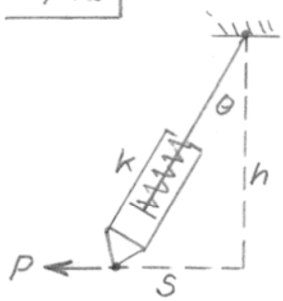

x = spring compression

$$= \frac{h}{\cos\theta} - h = h(\sec\theta - 1)$$

$$\delta U = \delta V_e \; ; \; P\delta s = \delta\left(\tfrac{1}{2}kx^2\right)$$

$$= k x \, \delta x$$

$s = h \tan\theta$ so that

$$P\delta(h\tan\theta) = k h^2(\sec\theta - 1)\sec\theta\tan\theta\,\delta\theta$$

$$Ph\sec^2\theta\,\delta\theta = kh^2(\sec\theta - 1)\sec^2\theta\sin\theta\,\delta\theta$$

$$P = kh(\sec\theta - 1)\sin\theta$$

or $\underline{P = kh\tan\theta(1 - \cos\theta)}$

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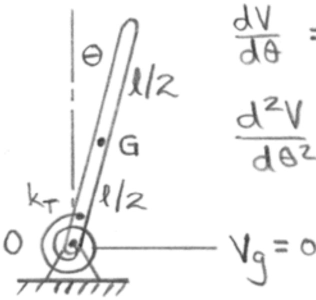
$$V = V_e + V_g = \frac{1}{2} k_T \theta^2 + mg\frac{l}{2}\cos\theta$$



$$\frac{dV}{d\theta} = k_T\theta - mg\frac{l}{2}\sin\theta$$

$$\frac{d^2V}{d\theta^2} = k_T - mg\frac{l}{2}\cos\theta$$

$$\frac{d^2V}{d\theta^2} > 0 \quad @ \quad \theta = 0:$$

$$k_T - mg\frac{l}{2} > 0$$

or $\underline{l < \dfrac{2k_T}{mg}}$

7/42

Spring compression $\overline{AO} = 2b \sin \frac{\theta}{2}$

$$V_e = \frac{1}{2} k \left(2b \sin \frac{\theta}{2}\right)^2 = 2kb^2 \sin^2 \frac{\theta}{2}$$

$$\delta V_e = 4kb^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \, \frac{1}{2} \delta\theta = kb^2 \sin\theta \, \delta\theta$$

$$\delta U' = M \delta\theta$$

$$\delta U' = \delta V_e : \quad M\delta\theta = kb^2 \sin\theta \, \delta\theta$$

$$\underline{\theta = \sin^{-1} \frac{M}{kb^2}}$$

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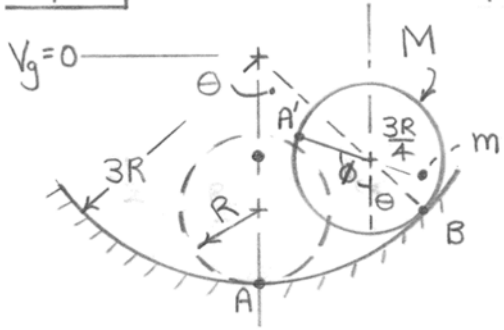


Arc $\overline{AB}$ = Arc $\overline{A'B}$

$3R\theta = R(\theta + \phi)$

$\phi = 2\theta$

$$V = V_g = -Mg2R\cos\theta - mg\left(2R\cos\theta - \frac{3R}{4}\cos\phi\right)$$

$$= -2(M+m)gR\cos\theta + \frac{3}{4}mgR\cos 2\theta$$

$$\frac{dV}{d\theta} = 2(M+m)gR\sin\theta - \frac{3}{2}mgR\sin 2\theta$$

$= 0$ for equilibrium; $\theta = 0$ is desired solution.

$$\frac{d^2V}{d\theta^2} = 2(M+m)gR\cos\theta - 3mgR\cos 2\theta$$

For $\theta = 0$: $2(M+m)gR - 3mgR > 0$ for stability

or $\underline{M > \frac{m}{2}}$

7/44 Take $V_g = 0$ through AO & $V_e = 0$ when $\theta = 0$

So $V_g = -mgh = -60(9.81)(0.7\sin\theta) = -412.0\sin\theta$

$$V_e = \frac{1}{2}kx^2 = \frac{1}{2}(160)\left[2(1.4)\sin\frac{\theta}{2}\right]^2 = 627.2\sin^2\frac{\theta}{2}$$

$$V = V_e + V_g = 627.2\sin^2\frac{\theta}{2} - 412.0\sin\theta$$

$$\frac{dV}{d\theta} = \frac{2}{2}(627.2)\sin\frac{\theta}{2}\cos\frac{\theta}{2} - 412.0\cos\theta$$

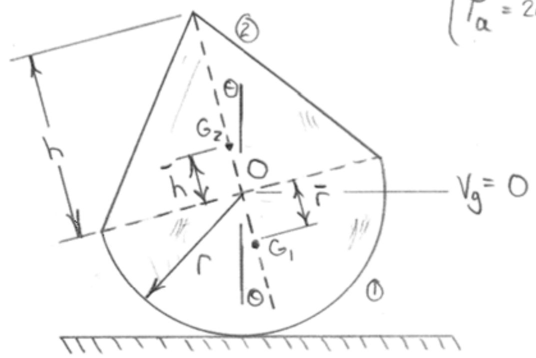
$$= 313.6\sin\theta - 412.0\cos\theta = 0 \text{ for equil.}$$

or $\dfrac{\sin\theta}{\cos\theta} = \tan\theta = \dfrac{412.0}{313.6} = 1.314$

$\underline{\theta = 52.7°}$

7/45

$$\begin{cases} \rho_s = 7830 \text{ kg/m}^3 \\ \rho_a = 2690 \text{ kg/m}^3 \end{cases}$$



$$\begin{cases} m_1 = \frac{1}{2}\left(\rho_1 \frac{4}{3}\pi r^3\right) = \frac{2}{3}\rho_1 \pi r^3 & \bar{r} = \frac{3r}{8} \\ m_2 = \frac{1}{3}\rho_2 \pi r^2 h & \bar{h} = \frac{h}{4} \end{cases}$$

$$V = -m_1 g \bar{r}\cos\theta + m_2 g \bar{h}\cos\theta = \left(\frac{1}{12}\rho_2 g\pi r^2 h^2 - \frac{1}{4}\rho_1 g \pi r^4\right)\cos\theta$$

$$\frac{d^2V}{d\theta^2} = -\left(\frac{1}{12}\rho_2 g\pi r^2 h^2 - \frac{1}{4}\rho_1 g\pi r^4\right)\cos\theta = 0 \longrightarrow \underline{h = \sqrt{3}\, r\sqrt{\frac{\rho_1}{\rho_2}}}$$

(a) IF $\rho_1 = \rho_2 = \rho$ ... $\underline{h = \sqrt{3}\, r = 1.732\, r}$

(b) IF $\rho_1 = \rho_s$ AND $\rho_2 = \rho_a$ ... $\underline{h = 2.96\, r}$

(c) IF $\rho_1 = \rho_a$ AND $\rho_2 = \rho_s$ ... $\underline{h = 1.015\, r}$

7/46 Take $V_g = 0$ through bearing

$V = V_g = mg(2a\cos\theta) + mg(a\cos 2\theta)$

$= mga(2\cos\theta + \cos 2\theta)$

$\frac{dV}{d\theta} = mga(-2\sin\theta - 2\sin 2\theta) = -2mga(\sin\theta + \sin 2\theta)$

$\frac{d^2V}{d\theta^2} = -2mga(\cos\theta + 2\cos 2\theta)$

For equil. $\frac{dV}{d\theta} = 0$ so $\sin\theta = -\sin 2\theta$

or $\sin\theta(1 + 2\cos\theta) = 0$; $\sin\theta = 0$, $\cos\theta = -1/2$

so sols. of interest are $\theta = 0$, $\theta = 180°$,

$\theta = 120°$, $\theta = 240°$

$\theta = 0$, $d^2V/d\theta^2 = -2mga(1+2) = (-)$ unstable

$\theta = 120°$, $d^2V/d\theta^2 = -2mga\left(-\frac{1}{2} - 2\left[\frac{1}{2}\right]\right) = (+)$ stable

$\theta = 180°$, $d^2V/d\theta^2 = -2mga(-1+2) = (-)$ unstable

$\theta = 240°$, $d^2V/d\theta^2 = -2mga\left(-1 + 2\left[-\frac{1}{2}\right]\right) = (+)$ stable

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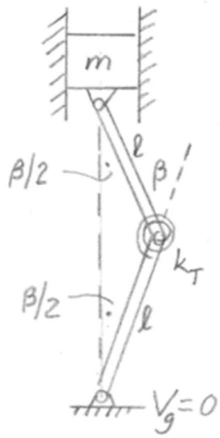

$V_g = 2mg\ell \cos\frac{\beta}{2}$, $V_e = \frac{1}{2}k_T\beta^2$

$V = V_g + V_e = 2mg\ell\cos\frac{\beta}{2} + \frac{1}{2}k_T\beta^2$

$\frac{dV}{d\beta} = -mg\ell\sin\frac{\beta}{2} + k_T\beta$, $\frac{dV}{d\beta} = 0$ for $\beta = 0$

$\frac{d^2V}{d\beta^2} = -\frac{1}{2}mg\ell\cos\frac{\beta}{2} + k_T$

$= -\frac{1}{2}mg\ell + k_T$ for $\beta = 0$

$= (+)$ stable if $k_T > \frac{1}{2}mg\ell$

Thus $\underline{k_{T_{min}} = \frac{1}{2}mg\ell}$

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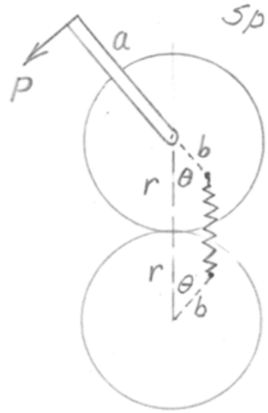

$\delta U' = \delta V_e$; $\delta U' = Pa\,\delta\theta$; $\delta V_e = kx\,\delta x$

Spring stretch $x = (2r - 2b\cos\theta) - (2r - 2b)$

$= 2b(1 - \cos\theta)$

$\delta x = 2b\sin\theta\,\delta\theta$

Thus $Pa\,\delta\theta = 4b^2k(1-\cos\theta)\sin\theta\,\delta\theta$

$$P = \frac{4kb^2}{a}\sin\theta(1-\cos\theta)$$

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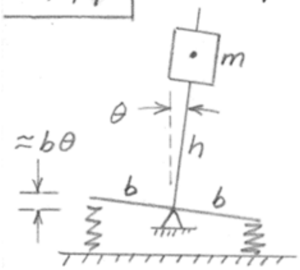

$$V = V_e + V_g = \frac{1}{2}k(\Delta + b\theta)^2 + \frac{1}{2}k(\Delta - b\theta)^2 + mgh\cos\theta$$

for θ small

$$V = k(\Delta^2 + b^2\theta^2) + mgh\cos\theta$$

$$\frac{dV}{d\theta} = 2kb^2\theta - mgh\sin\theta$$

$$\frac{d^2V}{d\theta^2} = 2kb^2 - mgh\cos\theta$$

Let preset of springs be Δ when $\theta = 0$

For $\theta \to 0$, $\frac{d^2V}{d\theta^2}$ is (+) if $2kb^2 > mgh$

Thus $\theta = 0$ is stable if $\underline{h < \frac{2kb^2}{mg}}$

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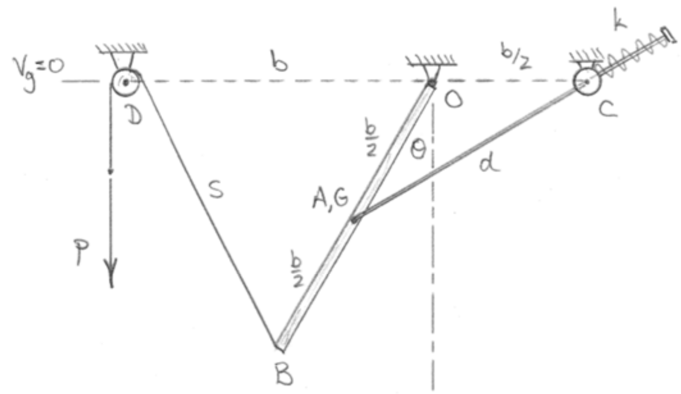

AT $\theta=0$, $s_0=b\sqrt{2}$ AND $d_0=\frac{b}{2}\sqrt{2}=\frac{b}{\sqrt{2}}$

ELSE, $s^2=b^2+b^2+2b^2\cos(90°+\theta) \longrightarrow s^2=2b^2(1-\sin\theta)$

AND $2s\,\delta s=-2b^2\cos\theta\,\delta\theta \longrightarrow \delta s=\dfrac{-b\cos\theta}{\sqrt{2}\sqrt{1-\sin\theta}}\delta\theta$

ALSO, $d^2=\left(\frac{b}{2}\right)^2+\left(\frac{b}{2}\right)^2-2\left(\frac{b}{2}\right)\left(\frac{b}{2}\right)\cos(90°+\theta) \longrightarrow d=\frac{b}{\sqrt{2}}\sqrt{1+\sin\theta}$

$\delta U'=P|\delta s|=\dfrac{Pb\cos\theta}{\sqrt{2}\sqrt{1-\sin\theta}}\delta\theta$

$V=-mg\frac{b}{2}\cos\theta+\frac{1}{2}k(d-d_0)^2=\frac{1}{2}k\left[\frac{b}{\sqrt{2}}\left(\sqrt{1+\sin\theta}-1\right)\right]^2-mg\frac{b}{2}\cos\theta$

$\delta V=\left[\frac{1}{2}mgb\sin\theta+\frac{1}{4}kb^2\cos\theta\left(1-\frac{1}{\sqrt{1+\sin\theta}}\right)\right]\delta\theta$

SOLVING $\delta U'=\delta V\ldots$

$$P=\frac{\sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta}}\,\frac{bk\left(\sqrt{1+\sin\theta}-1\right)+2mg\sqrt{1+\sin\theta}\tan\theta}{2\sqrt{2}}$$

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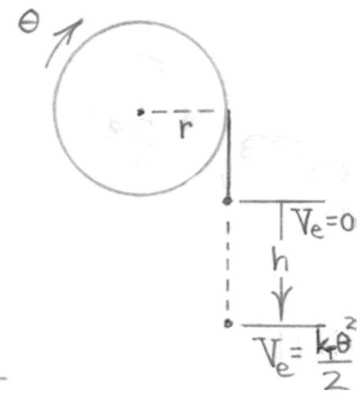
$$\delta U' = \delta V : 0 - mgr\,\delta\theta + \delta\left(\tfrac{1}{2}k_T\theta^2\right)$$

$$mgr = k_T\theta$$

With $h = r\theta$: $mgr = k_T\left(\frac{h}{r}\right)$

$$h = \frac{mgr^2}{k_T}$$



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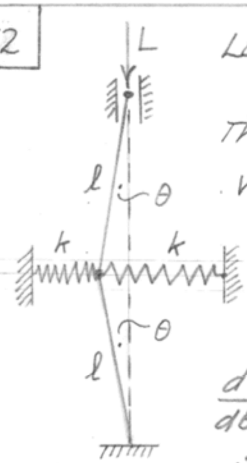

Let Δ = initial compression in each spring

Then for small angles

$$V = V_e + V_g = 2\left[\frac{1}{2}k(\Delta - \ell\sin\theta)^2 + \frac{1}{2}k(\Delta + \ell\sin\theta)^2\right] + 2L\ell\cos\theta$$

$$V = 2k(\Delta^2 + \ell^2\sin^2\theta) + 2L\ell\cos\theta$$

$$\frac{dV}{d\theta} = 4k\ell^2\sin\theta\cos\theta - 2L\ell\sin\theta$$

$$\frac{d^2V}{d\theta^2} = 4k\ell^2\cos 2\theta - 2L\ell\cos\theta$$

$\frac{dV}{d\theta} = 0$ for equil. $\theta = 0$ & $\theta = \cos^{-1}\frac{L}{2k\ell}$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = 4k\ell^2(1) - 2L\ell(1) = (+)$ stable if

$$\underline{k > \frac{L}{2\ell}}$$

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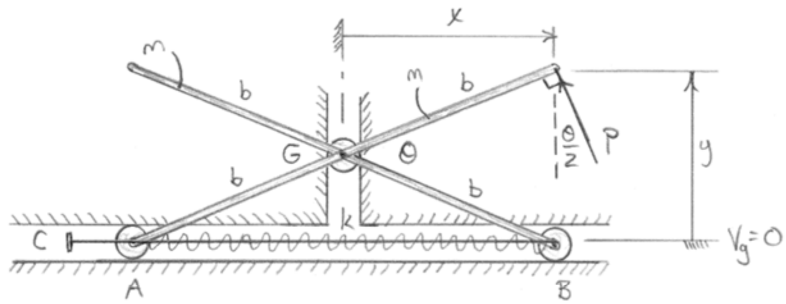

$$\begin{cases} x = b\cos\frac{\theta}{2} \\ \delta x = -\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta \end{cases} \quad \begin{cases} y = 2b\sin\frac{\theta}{2} \\ \delta y = b\cos\frac{\theta}{2}\,\delta\theta \end{cases} \quad \begin{cases} P_x = -P\sin\frac{\theta}{2} \\ P_y = P\cos\frac{\theta}{2} \end{cases}$$

$$\delta U' = P_x\,\delta x + P_y\,\delta y = \left(\frac{1}{2}Pb\sin^2\frac{\theta}{2} + Pb\cos^2\frac{\theta}{2}\right)\delta\theta$$

$$\begin{cases} L_0 = 2b\cos\frac{\theta_0}{2} \\ L_f = 2b\cos\frac{\theta}{2} \end{cases} \quad \delta = L_0 - L_f = 2b\left(\cos\frac{\theta_0}{2} - \cos\frac{\theta}{2}\right)$$

$$V = \frac{1}{2}k\delta^2 + 2mgb\sin\frac{\theta}{2} = 2kb^2\left(\cos\frac{\theta_0}{2} - \cos\frac{\theta}{2}\right)^2 + 2mgb\sin\frac{\theta}{2}$$

$$\delta V = 2kb^2\left(\cos\frac{\theta_0}{2} - \cos\frac{\theta}{2}\right)\sin\frac{\theta}{2}\,\delta\theta + mgb\cos\frac{\theta}{2}\,\delta\theta$$

Solve $\delta U' = \delta V$...

$$\underline{P = \frac{4mg\cos\frac{\theta}{2} + 4kb\left(2\cos\frac{\theta_0}{2}\sin\frac{\theta}{2} - \sin\theta\right)}{3 + \cos\theta}}$$

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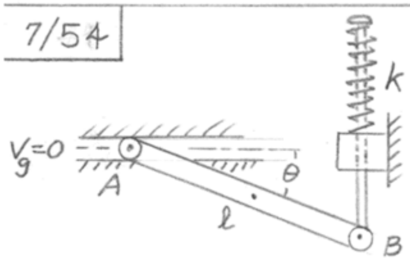


Spring compressed $\ell \sin\theta$

so $V_e = \frac{1}{2}k(\ell\sin\theta)^2$

$$V_g = -mg\frac{\ell}{2}\sin\theta$$

$$V = V_e + V_g = \frac{k}{2}\ell^2\sin^2\theta - mg\frac{\ell}{2}\sin\theta$$

$$\frac{dV}{d\theta} = k\ell^2\sin\theta\cos\theta - mg\frac{\ell}{2}\cos\theta = \frac{k\ell^2}{2}\sin 2\theta - mg\frac{\ell}{2}\cos\theta$$

$$= \ell\cos\theta\,(k\ell\sin\theta - mg/2) = 0 \text{ for equil.}$$

(1) $\cos\theta = 0$, $\theta = \pi/2$

(2) $\sin\theta = \dfrac{mg}{2k\ell}$

$$\frac{d^2V}{d\theta^2} = k\ell^2\cos 2\theta + mg\frac{\ell}{2}\sin\theta$$

$$\left(\frac{d^2V}{d\theta^2}\right)_{(1)} = k\ell^2\cos\pi + mg\frac{\ell}{2}(1) = k\ell^2\left(-1 + \left[\frac{mg}{2k\ell}\right]\right)$$

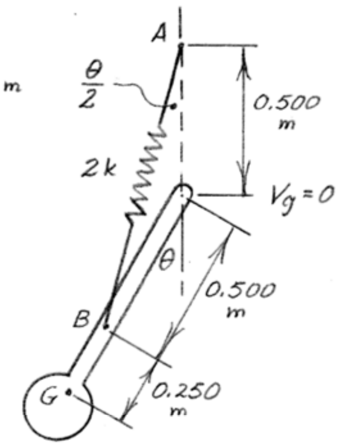
$= (+)$ stable if $k < \dfrac{mg}{2\ell}$

$$\left(\frac{d^2V}{d\theta^2}\right)_{(2)} = k\ell^2\left(1 - 2\left[\frac{mg}{2k\ell}\right]^2\right) + \frac{mg\ell}{2}\frac{mg}{2k\ell} = k\ell^2\left[1 - \left(\frac{mg}{2k\ell}\right)^2\right]$$

$= (+)$ stable if $k > \dfrac{mg}{2\ell}$

7/55 Length $AB = 2(0.500)\cos\frac{\theta}{2}$ (m)

Unstretched length $= 1 - 0.1 = 0.9$ m



Spring length for arbitrary θ is $x = (1)\cos\frac{\theta}{2} - 0.9$ m

$$V_e = 2\,\frac{1}{2}k\left[(1)\cos\frac{\theta}{2} - 0.9\right]^2, \quad k \text{ in N/m}$$

$$= k\left[\cos^2\frac{\theta}{2} - 1.8\cos\frac{\theta}{2} + 0.81\right] \text{ J}$$

$$V_g = -1.5(9.81)(0.750\cos\theta) = -11.04\cos\theta \text{ J}$$

$$V = V_e + V_g = k\left[\cos^2\frac{\theta}{2} - 1.8\cos\frac{\theta}{2} + 0.81\right] - 11.04\cos\theta \text{ J}$$

$$\frac{dV}{d\theta} = k\left[-\sin\frac{\theta}{2}\cos\frac{\theta}{2} + 0.9\sin\frac{\theta}{2}\right] + 11.04\sin\theta$$

$$= k\left[-0.5\sin\theta + 0.9\sin\frac{\theta}{2}\right] + 11.04\sin\theta$$

$$\frac{d^2V}{d\theta^2} = k\left[-0.5\cos\theta + 0.45\cos\frac{\theta}{2}\right] + 11.04\cos\theta$$

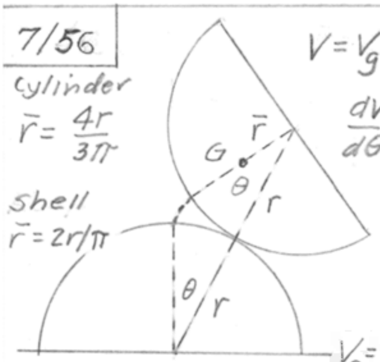
$$\left(\frac{d^2V}{d\theta^2}\right)_{\theta=0} = k[-0.5 + 0.45] + 11.04 > 0 \text{ (stable) if } k \text{ does not}$$

exceed $\frac{11.04}{0.05} = \underline{221 \text{ N/m} = k_{max}}$

7/56

cylinder
$\bar{r} = \frac{4r}{3\pi}$

shell
$\bar{r} = 2r/\pi$



$$V = V_g = mg(2r\cos\theta - \bar{r}\cos 2\theta)$$

$$\frac{dV}{d\theta} = mg(-2r\sin\theta + 2\bar{r}\sin 2\theta)$$

$$= 2mg(-r\sin\theta + \bar{r}\sin 2\theta)$$

$$\frac{d^2V}{d\theta^2} = 2mg(-r\cos\theta + 2\bar{r}\cos 2\theta)$$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = 2mrg\left(-1 + \frac{2\bar{r}}{r}\right)$

For cylinder $\bar{r}/r = \frac{4}{3\pi}$, $\frac{d^2V}{d\theta^2} = 2mrg\left(-1 + \frac{8}{3\pi}\right) = (-)$

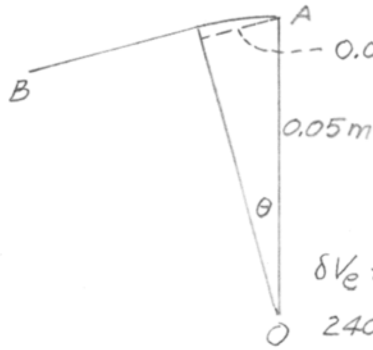
unstable

For shell $\bar{r}/r = \frac{2}{\pi}$, $\frac{d^2V}{d\theta^2} = 2mrg\left(-1 + \frac{4}{\pi}\right) = (+)$

stable

7/57

$$\delta V_g = \delta(0.3\,mg\cos\theta) = -0.3(80)(9.81)\sin\theta\,\delta\theta$$
$$= -235\sin\theta\,\delta\theta$$



$0.05\sin\theta \approx$ spring compression x

$$\delta V_e = kx\,\delta x$$
$$= (96\times10^3)(0.05\sin\theta)\,\delta(0.05\sin\theta)$$
$$= 240\sin\theta\cos\theta\,\delta\theta$$

$\delta V_e + \delta V_g = 0$;

$$240\sin\theta\cos\theta\,\delta\theta - 235\sin\theta\,\delta\theta = 0$$

$\sin\theta = 0$ or $\cos\theta = \dfrac{235}{240} = 0.9810$, $\underline{\theta = 11.19^\circ}$

▸7/58

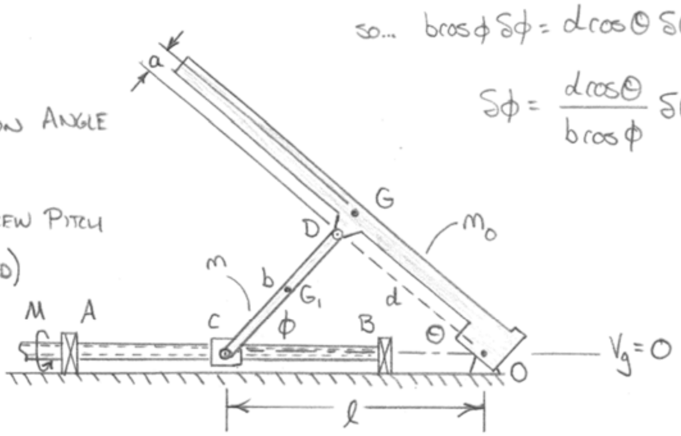
$b\sin\phi = d\sin\theta$

so... $b\cos\phi\,\delta\phi = d\cos\theta\,\delta\theta$

$$\delta\phi = \frac{d\cos\theta}{b\cos\phi}\,\delta\theta$$

LET β = ROTATION ANGLE OF THE SCREW

p IS THE SCREW PITCH (DOUBLE THREADED)



$$V = m_0 g(d\sin\theta + a\cos\theta) + mg\frac{b}{2}\sin\phi$$

$$= m_0 g(d\sin\theta + a\cos\theta) + mg\frac{d}{2}\sin\theta$$

$$\delta V = m_0 g(d\cos\theta - a\sin\theta)\,\delta\theta + mg\frac{d}{2}\cos\theta\,\delta\theta$$

$$l = d\cos\theta + b\cos\phi$$

$$\delta l = -d\sin\theta\,\delta\theta - b\sin\phi\,\delta\phi$$

$$= -d\sin\theta\,\delta\theta - \frac{d^2\cos\theta\sin\theta}{\sqrt{b^2 - d^2\sin^2\theta}}\,\delta\theta$$

$$\left|\frac{\delta l}{\delta\beta}\right| = \frac{2p}{2\pi} = \frac{p}{\pi} \longrightarrow |\delta\beta| = \frac{\pi}{p}\left(d\sin\theta + \frac{d^2\sin\theta\cos\theta}{\sqrt{b^2 - d^2\sin^2\theta}}\right)\delta\theta$$

$\delta U' = M\,\delta\beta$ AND $\delta U' = \delta V$

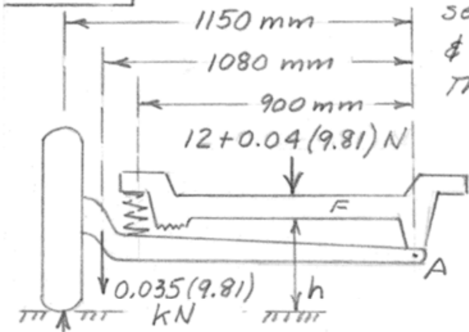
SOLVING FOR M...

$$M = \frac{pg}{2\pi d}\,\frac{\left[(m + 2m_0)d\cot\theta - 2am_0\right]\sqrt{b^2 - d^2\sin^2\theta}}{d\cos\theta + \sqrt{b^2 - d^2\sin^2\theta}}$$

IF $m = 0$ AND $d = b$...

$$M = \frac{m_0 g p(b\cot\theta - a)}{2\pi b}$$

▶7/59



Let x = compression of spring (most easily seen by considering A fixed & wheels moving up)

Thus $x = \frac{900}{1150}(0.35 - h)$ meters

With F & hence A fixed,

$\delta U' = -2(6.54)\delta h + 2(0.035)9.81\,\frac{1080}{1150}\,\delta h$

$= (-13.08 + 0.645)\delta h$

$= -12.43\,\delta h$

$P = 6 + 0.035(9.81) + 0.02(9.81)$
$= 6.54$ kN

$\delta V_e = 2(kx\,\delta x) = 2(120)\frac{900}{1150}(0.35 - h)\frac{900}{1150}(-\delta h)$

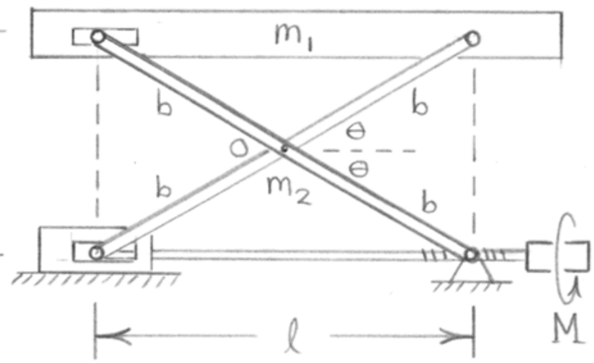
$= -147.0(0.35 - h)\delta h$

$\delta U' = \delta V_e$; $-12.43\,\delta h = -147.0(0.35 - h)\delta h$

$h = 0.35 - \frac{12.43}{147.0} = 0.265$ m or $\underline{h = 265 \text{ mm}}$

▶7/60

Let β = rotation angle of screw
p = screw pitch



$$\delta U' = \delta V_g$$

$$\delta U' = M\delta\beta$$

$$\delta V_g = m_1 g\,\delta h + m_2 g\,\delta\left(\frac{h}{2}\right) = \left(m_1 + \frac{m_2}{2}\right) g\,\delta h$$

$$\delta h = \delta(2b\sin\theta) = 2b\cos\theta\,\delta\theta$$

$$l = 2b\cos\theta, \quad \delta l = -2b\sin\theta\,\delta\theta$$

$$\left|\frac{\delta l}{\delta\beta}\right| = \frac{p}{2\pi}, \quad \text{so } |\delta\beta| = \frac{2\pi}{p}|\delta l| = \frac{4\pi b}{p}\sin\theta\,\delta\theta$$

$$\text{So } M\frac{4\pi b}{p}\sin\theta\,\delta\theta = \left(m_1 + \frac{m_2}{2}\right) g\,(2b\cos\theta\,\delta\theta)$$

$$M = \frac{(2m_1 + m_2)pg}{4\pi}\cot\theta$$

7/61

$$V = \frac{1}{5}x^5 + \frac{1}{2}x^4 - \frac{101}{3}x^3 - 51x^2 + 1080x + 20$$

$$\frac{dV}{dx} = x^4 + 2x^3 - 101x^2 - 102x + 1080$$

THIS CAN BE FACTORED INTO... $\frac{dV}{dx} = (x+10)(x+4)(x-3)(x-9)$

WITH EQUILIBRIUM POSITIONS AT $\underline{x = -10, -4, 3, 9}$

$$\frac{d^2V}{dx^2} = 4x^3 + 6x^2 - 202x - 102 \qquad \text{(STABILITY CHECK)}$$

EVALUATE $\frac{d^2V}{dx^2}$ AT EACH EQUILIBRIUM POINT.

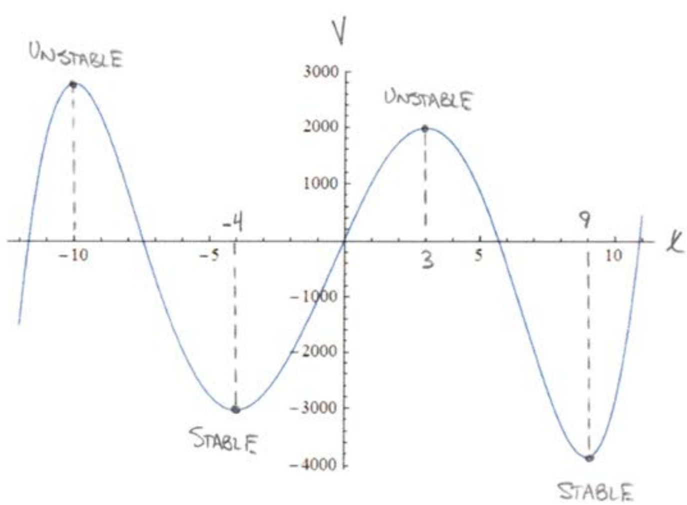
a) $x = -10$, $\frac{d^2V}{dx^2} = -1482$ UNSTABLE

b) $x = -4$, $\frac{d^2V}{dx^2} = 546$ STABLE

c) $x = 3$, $\frac{d^2V}{dx^2} = -546$ UNSTABLE

d) $x = 9$, $\frac{d^2V}{dx^2} = 1482$ STABLE

PLOTTING V SHOWS STABILITY INFORMATION VERY QUICKLY.

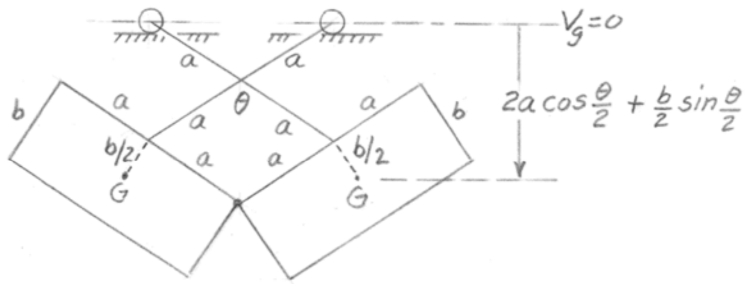


7/62 $x = k\theta$, $0.060 = k(2\pi)$, $k = \frac{0.030}{\pi}\ \frac{m}{rad}$

$\delta U = 0$; $M\delta\theta - P\delta x = 0$, $P = M\frac{\delta\theta}{\delta x} = M/k$

so $P = \frac{\pi}{0.030} 10 = \underline{1047\ N}$

7/63 $V = V_g = -2mg\left[2a\cos\frac{\theta}{2} + \frac{b}{2}\sin\frac{\theta}{2}\right]$



$\frac{dV}{d\theta} = -2mg\left[-a\sin\frac{\theta}{2} + \frac{b}{4}\cos\frac{\theta}{2}\right] = 0$ for equil.

$\tan\frac{\theta}{2} = \frac{b}{4a}$, $\theta = 2\tan^{-1}\frac{b}{4a}$; For $b = a$, $\theta = 2\tan^{-1}1/4$
$\underline{= 28.1°}$

7/64 $V = V_e = \frac{1}{2}k\left(2a\sin\frac{\theta}{2}\right)^2 = 2ka^2\sin^2\frac{\theta}{2}$

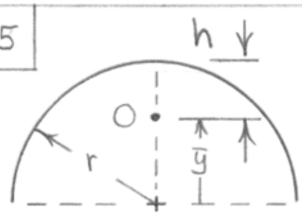
$\delta U = \delta V_e$; $P\delta(2a\sin\theta) = \delta\left(2ka^2\sin^2\frac{\theta}{2}\right)$

$$2Pa\cos\theta\,\delta\theta = 2ka^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,\delta\theta$$

$$2Pa\cos\theta = ka^2\sin\theta$$

$\tan\theta = \frac{2P}{ka}$, $\quad \underline{\theta = \tan^{-1}\frac{2P}{ka}}$

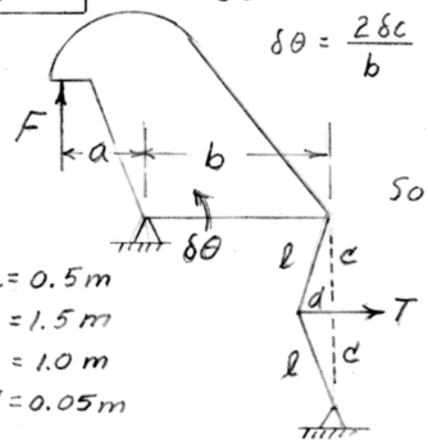
7/65



The pivot O must be at or above the mass center G of the shell, which is located at $\bar{y} = \frac{2r}{\pi}$. So

$$h_{max} = r - \bar{y} = r\left(1 - \frac{2}{\pi}\right) = \underline{0.363r}$$

7/66



$a = 0.5\text{ m}$

$b = 1.5\text{ m}$

$c = 1.0\text{ m}$

$d = 0.05\text{ m}$

$\delta U = 0: \; -Fa\,\delta\theta - T\,\delta d = 0$

$\delta\theta = \dfrac{2\,\delta c}{b}, \quad \ell^2 = d^2 + c^2$

$0 = 2d\,\delta d + 2c\,\delta c$

$2\,\delta c = -2\dfrac{d}{c}\,\delta d$

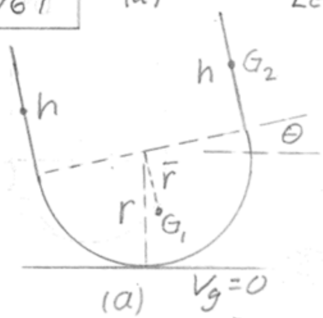
So $-Fa\dfrac{2\,\delta c}{b} - T\left(-\dfrac{c}{d}\,\delta c\right) = 0$

$F = \dfrac{bc}{2ad}\,T$

$T = pA = 20(10^6)\dfrac{10^4}{10^6} = 20(10^4)\text{ N}$

So $F = \dfrac{1.5(1.0)}{2(0.5)(0.05)}\,20(10^4) = 6(10^6)\text{ N}$ or $\underline{6\text{ MN}}$

7/67 (a)



Let ρ = mass per unit periphery of shell

$\bar{r} = 2r/\pi$

$$V_g = \rho\left[\pi r\left(r - \frac{2r}{\pi}\cos\theta\right) + h\left(r + r\sin\theta + \frac{h}{2}\cos\theta\right) + h\left(r - r\sin\theta + \frac{h}{2}\cos\theta\right)\right]$$

$$= \rho\left[\pi r^2 - 2r^2\cos\theta + 2hr + h^2\cos\theta\right]$$

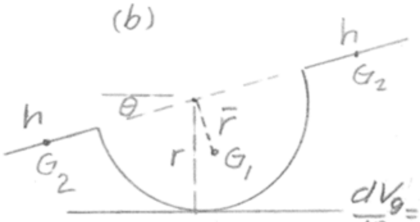
$$\frac{dV_g}{d\theta} = \rho\left[2r^2 - h^2\right]\sin\theta, \quad \frac{d^2V_g}{d\theta^2} = \rho\left[2r^2 - h^2\right]\cos\theta$$

Equil. at $\theta = 0$ stable if $h < r\sqrt{2}$

unstable if $h > r\sqrt{2}$

Neutral equil. if $h = r\sqrt{2}$ for any θ

(b)



$$V_g = \rho\left[\pi r\left(r - \frac{2r}{\pi}\cos\theta\right) + h\left(r + \left[r + \frac{h}{2}\sin\theta\right]\right) + h\left(r - \left[r + h/2\sin\theta\right]\right)\right]$$

$$= \rho\left[\pi r^2 - 2r^2\cos\theta + 2hr\right]$$

$$\frac{dV_g}{d\theta} = 2\rho r^2\sin\theta, \quad \theta = 0 \text{ for stable equil.}$$

independent of h

7/68

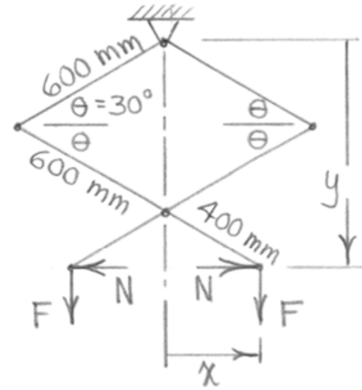
$\delta U = 0: \; 2N\delta x + 2F\delta y = 0$

$x = 0.4\cos\theta$

$\delta x = -0.4\sin\theta\,\delta\theta$

$y = (0.6 + 0.6 + 0.4)\sin\theta$

$\delta y = 1.6\cos\theta\,\delta\theta$

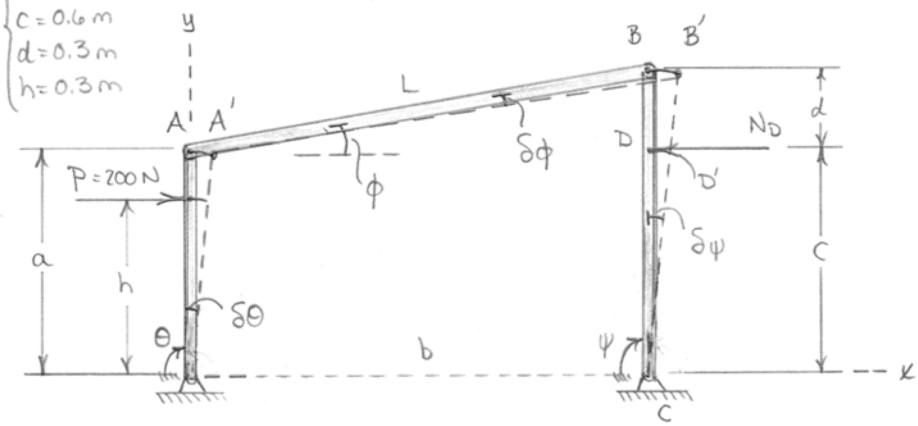


So $2N(-0.4\sin\theta\,\delta\theta) + 2F(1.6\cos\theta\,\delta\theta) = 0$

$$\frac{F}{N} = \mu_s = \frac{0.4\sin\theta}{1.6\cos\theta} = \frac{1}{4}\tan 30^\circ = \underline{0.1443}$$

7/69 $\quad L = \sqrt{(c+d-a)^2 + b^2}$ & $\phi = \tan^{-1}\left(\frac{c+d-a}{b}\right) = \tan^{-1}\left(\frac{0.3}{1.2}\right) = 14.04°$

$$\begin{cases} a = 0.6 \text{ m} \\ b = 1.2 \text{ m} \\ c = 0.6 \text{ m} \\ d = 0.3 \text{ m} \\ h = 0.3 \text{ m} \end{cases}$$



$$\begin{cases} a\sin\theta + L\sin\phi = (c+d)\sin\psi \longrightarrow a\cos\theta\,\delta\theta + L\cos\phi\,\delta\phi = (c+d)\cos\psi\,\delta\psi \\ -a\cos\theta + L\cos\phi = b - (c+d)\cos\psi \longrightarrow a\sin\theta\,\delta\theta - L\sin\phi\,\delta\phi = (c+d)\sin\psi\,\delta\psi \end{cases}$$

Solving for $\delta\phi$ and $\delta\psi$...

$$\delta\phi = \frac{a\csc(\phi+\psi)\sin(\theta-\psi)}{L}\delta\theta \quad \text{and} \quad \delta\psi = \frac{a\csc(\phi+\psi)\sin(\theta+\phi)}{c+d}\delta\theta$$

$$\delta U = 0: \quad Ph\,\delta\theta - N_D\,c\,\delta\psi = 0 \longrightarrow \underline{N_D = \frac{Ph(c+d)}{a\csc(\phi+\psi)\sin(\theta+\phi)\,c}}$$

With numbers... $N_D = \dfrac{200(0.3)(0.6+0.3)}{0.6\csc(14.04° + 90°)\sin(90° + 14.04°)(0.6)} \longrightarrow \underline{N_D = 150 \text{ N}}$

7/70 Total length of door is $2.5 + 0.6\pi/2 = 3.44$ m

Unit mass is $135/3.44 = 39.22$ kg/m

Take $V_g = 0$ through A

Let potential energy of cylindrical portion be $-V_0$ which remains constant

So $V_g = 0 - V_0 - 39.22(9.81)x(0.6 + \frac{x}{2})$

$= -V_0 - 384.7x(0.6 + \frac{x}{2})$ J

$V_e = 2(\frac{1}{2}k\theta^2) = \frac{10}{2\pi}\theta^2 = \frac{5}{\pi}(\frac{x}{0.080})^2 = 248.7x^2$ J

$V = V_e + V_g = 248.7x^2 - V_0 - 384.7x(0.6 + \frac{x}{2})$

$\frac{dV}{dx} = 497.4x - 230.8 - 384.7x = 112.7x - 230.8$

$= 0$ for equilibrium, so $x = \frac{230.8}{112.7} = \underline{2.05 \text{ m}}$

$\frac{d^2V}{dx^2} = 112.7$ (+) so $\underline{\text{stable}}$

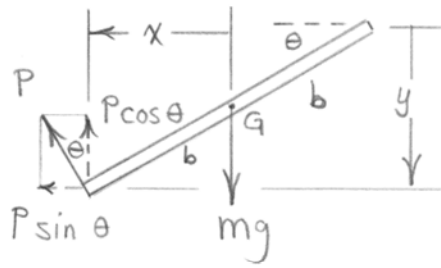
7/71

$y = 2b \sin\theta$

$\delta y = 2b \cos\theta \, \delta\theta$

$x = b \cos\theta$

$\delta x = -b \sin\theta \, \delta\theta$



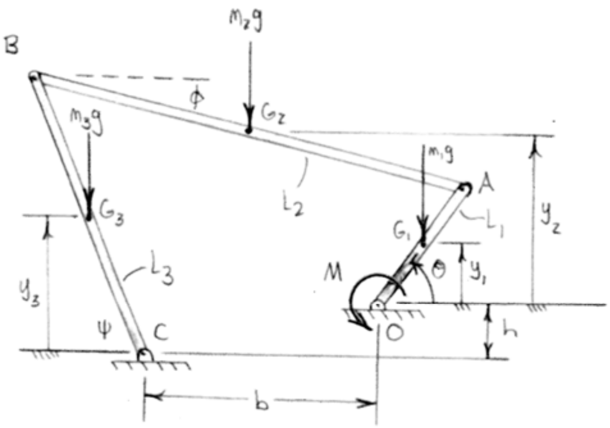
$\delta U = 0: \; mg\,\delta\left(\frac{y}{2}\right) - P\cos\theta\,(\delta y) + P\sin\theta\,(\delta x) = 0$

$mgb\cos\theta\,\delta\theta - P\cos\theta\,(2b\cos\theta\,\delta\theta) + P\sin\theta\,(-b\sin\theta\,\delta\theta) = 0$

$mg\cos\theta = P(\sin^2\theta + 2\cos^2\theta) = P(1+\cos^2\theta)$

$$P = \frac{mg\cos\theta}{1+\cos^2\theta}$$

7/72



$$\begin{cases} y_1 = \frac{L_1}{2}\sin\theta \\ \delta y_1 = \frac{L_1}{2}\cos\theta\,\delta\theta \end{cases} \quad \begin{cases} y_2 = L_1\sin\theta + \frac{L_2}{2}\sin\phi \\ \delta y_2 = L_1\cos\theta\,\delta\theta + \frac{L_2}{2}\cos\phi\,\delta\phi \end{cases} \quad \begin{cases} y_3 = \frac{L_3}{2}\sin\psi \\ \delta y_3 = \frac{L_3}{2}\cos\psi\,\delta\psi \end{cases}$$

FROM 7/29...
$$\begin{cases} \delta\phi = \frac{L_1}{L_2}\csc(\phi-\psi)\sin(\theta+\psi)\delta\theta \\ \delta\psi = \frac{L_1}{L_3}\csc(\phi-\psi)\sin(\theta+\phi)\delta\theta \end{cases}$$

$$\delta U = 0: \quad M\delta\theta - m_1 g\,\delta y_1 - m_2 g\,\delta y_2 - m_3 g\,\delta y_3 = 0$$

SUBSTITUTE THE ABOVE RELATIONSHIPS AND SOLVE FOR M.

$$M = \tfrac{1}{2}L_1 g\left[(m_1+2m_2)\cos\theta - \left(m_3\cos\psi\sin(\theta+\phi) + m_2\cos\phi\sin(\theta+\psi)\right)\csc(\psi-\phi)\right]$$

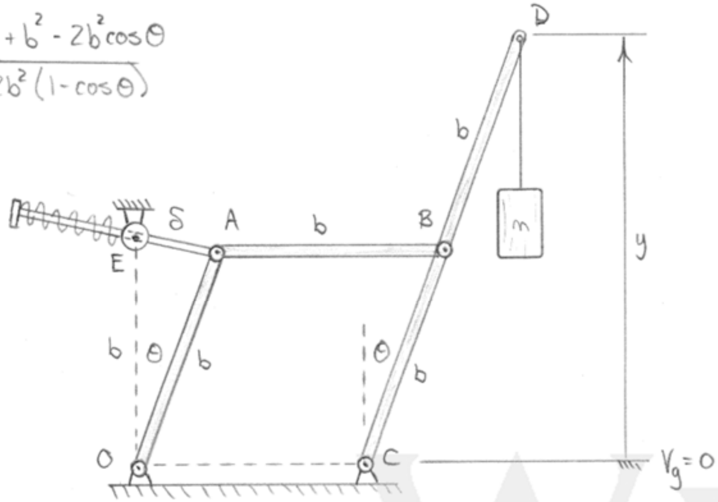
IF
$$\begin{cases} m_1 = 0.9 \text{ kg} \\ m_2 = 3.6 \text{ kg} \\ m_3 = 3 \text{ kg} \end{cases} \quad \begin{cases} L_1 = 250 \text{ mm} \\ L_2 = 1000 \text{ mm} \\ L_3 = 800 \text{ mm} \end{cases} \quad \begin{cases} h = 150 \text{ mm} \\ b = 450 \text{ mm} \\ \theta = 30^\circ \end{cases}$$

THEN... $\phi = 30.0^\circ$ AND $\psi = 75.5^\circ$

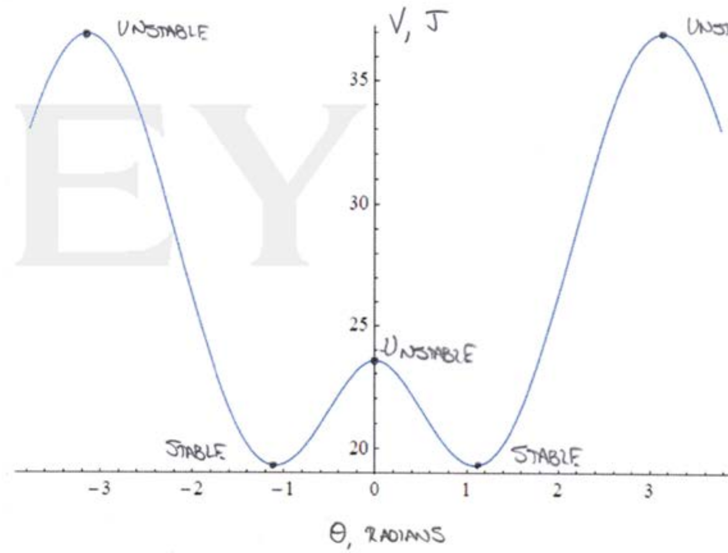
$$\underline{M = 2.33 \text{ N}\cdot\text{m CCW}}$$

7/73

$$\begin{cases} \delta^2 = b^2 + b^2 - 2b^2\cos\theta \\ \delta = \sqrt{2b^2(1-\cos\theta)} \end{cases}$$



Plotting the potential energy of the system reveals the locations of equilibrium points and the nature of their stability condition. Note the additional equilibrium points at $\pm\pi$.



$$V = mg(2b\cos\theta) + k\delta^3 = 2mgb\cos\theta + k\left[2b^2(1-\cos\theta)\right]^{3/2}$$

$$\frac{dV}{d\theta} = 3kb^2\sin\theta\sqrt{2b^2(1-\cos\theta)} - 2mgb\sin\theta$$

For equilibrium, $\dfrac{dV}{d\theta} = 0 \longrightarrow \underline{\theta = 0 \text{ or } \pm\cos^{-1}\left(1 - \dfrac{2m^2g^2}{9k^2b^4}\right)}$

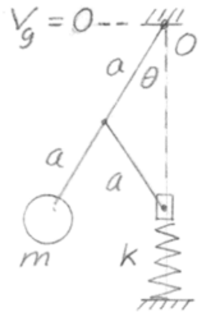
If $m = 2$ kg, $k = 35$ N/m², and $b = 600$ mm... $\underline{\theta = 0 \text{ \& } 62.5°}$

$$\frac{d^2V}{d\theta^2} = 3kb^2\cos\theta\sqrt{2b^2(1-\cos\theta)} - 2mgb\cos\theta + \frac{3kb^4\sin^2\theta}{\sqrt{2b^2(1-\cos\theta)}}$$

If $\theta = 0°$, $\dfrac{d^2V}{d\theta^2} = -2mgb$ (unstable) If $\theta = 62.5°$, $\dfrac{d^2V}{d\theta^2} = 17.20$ (stable)

$$\begin{cases} \text{The } \pm 62.5° \text{ equilibrium positions are stable} \\ \text{The } 0, \pm\pi \text{ equilibrium positions are unstable} \end{cases}$$

▸7/74 Take $V_g = 0$ through pt. O



$V_g = -2mga\cos\theta$

Compression in spring $x = 2a\cos\theta - a$
$= 0$ for $\theta = 60°$

$V_e = \frac{1}{2}kx^2 = \frac{ka^2}{2}(2\cos\theta - 1)^2$

$V = V_e + V_g = \frac{ka^2}{2}(2\cos\theta - 1)^2 - 2mga\cos\theta$

$\frac{dV}{d\theta} = 2a\left[(mg + ka)\sin\theta - 2ka\sin\theta\cos\theta\right]$

$\frac{d^2V}{d\theta^2} = 2a\left[(mg + ka)\cos\theta - 2ka(2\cos^2\theta - 1)\right]$

For equil. $\frac{dV}{d\theta} = 0$, $\left[(mg + ka) - 2ka\cos\theta\right]\sin\theta = 0$

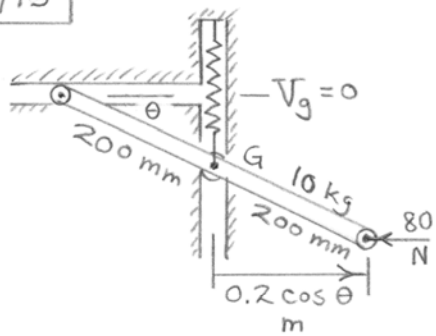
$\sin\theta = 0$ & $\cos\theta = \frac{mg + ka}{2ka} = \frac{1}{2}\left(1 + \frac{mg}{ka}\right)$

For $\sin\theta = 0$, $\underline{\theta = 0}$; $\frac{d^2V}{d\theta^2} = 2a(mg - ka) = +$ stable if $\underline{k < \frac{mg}{a}}$

$= -$ unstable if $\underline{k > \frac{mg}{a}}$

For $\underline{\theta = \cos^{-1}\frac{1}{2}\left(1 + \frac{mg}{ka}\right)}$, $\underline{k > \frac{mg}{a}}$ $\underline{\text{stable}}$

*7/75



$$\delta U = \delta V_g + \delta V_e$$

$$\delta U = -80\delta(0.2\cos\theta) = 16\sin\theta\,\delta\theta$$

$$\delta V_g = \delta(-mgh) = -\delta[10(9.81)(0.2\sin\theta)]$$
$$= -19.62\cos\theta\,\delta\theta$$

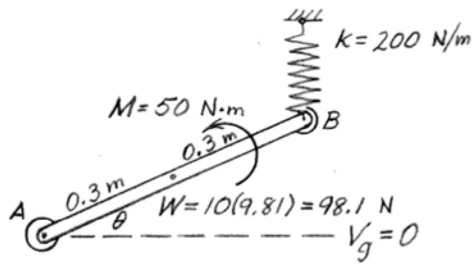
$$\delta V_e = \delta(\tfrac{1}{2}kx^2) = kx\,\delta x$$
$$= 1500(0.2\sin\theta)\delta(0.2\sin\theta)$$
$$= 60\sin\theta\cos\theta\,\delta\theta = 30\sin 2\theta\,\delta\theta$$

Thus $16\sin\theta\,\delta\theta = -19.62\cos\theta\,\delta\theta + 30\sin 2\theta\,\delta\theta$

or $(16\sin\theta + 19.62\cos\theta - 30\sin 2\theta)\delta\theta = 0$

Numerical solution : $\underline{\theta = 27.9°}$

*7/76



$\delta U' = \delta V$

$\delta U' = 50\,\delta\theta$

$V = V_g + V_e = 98.1(0.3\sin\theta) + \frac{1}{2}(200)(0.6\sin\theta)^2$

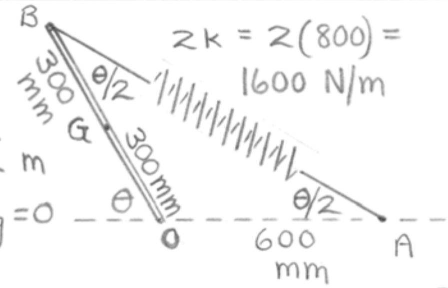
$\delta V = 29.4\cos\theta\,\delta\theta + 36(2\sin\theta\cos\theta)\,\delta\theta$

So $50\,\delta\theta = 29.4\cos\theta\,\delta\theta + 36\sin 2\theta\,\delta\theta$

or $50 = 29.4\cos\theta + 36\sin 2\theta$

Solve numerically to obtain $\underline{\theta = 19.01°}$

*7/77



Springs undeflected @ $\theta = \frac{\pi}{2}$

Deflection $x = \overline{AB} - 0.6\sqrt{2}$ m

$\overline{AB} = 2(0.6)\cos\theta/2$

So $x = 0.6\left(2\cos\frac{\theta}{2} - \sqrt{2}\right)$ m

$$V = V_g + V_e = 25(9.81)(0.3\sin\theta) + \tfrac{1}{2}2k\left[0.6\left(2\cos\tfrac{\theta}{2} - \sqrt{2}\right)\right]^2$$

$$= 73.6\sin\theta + 800(0.36)\left(4\cos^2\tfrac{\theta}{2} - 4\sqrt{2}\cos\tfrac{\theta}{2} + 2\right)$$

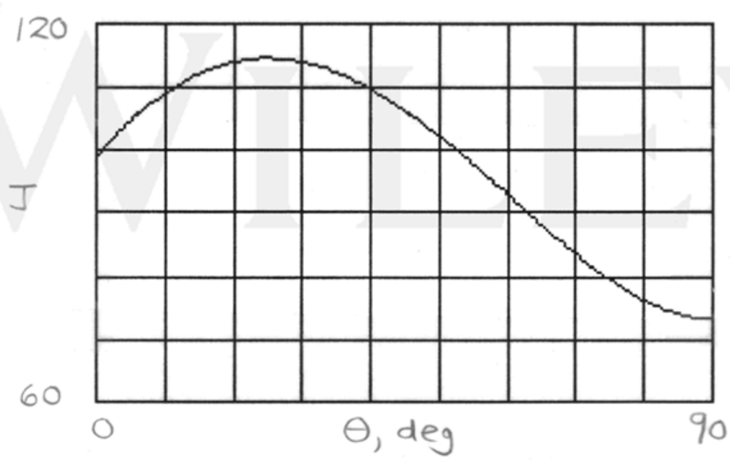
$$\frac{dV}{d\theta} = 73.6\cos\theta + 288\left(-4\cos\tfrac{\theta}{2}\sin\tfrac{\theta}{2} + 2\sqrt{2}\sin\tfrac{\theta}{2}\right)$$

$$= 73.6\cos\theta - 576\sin\theta + 815\sin\tfrac{\theta}{2}$$

$$\frac{d^2V}{d\theta^2} = -73.6\sin\theta - 576\cos\theta + 407\cos\tfrac{\theta}{2}$$

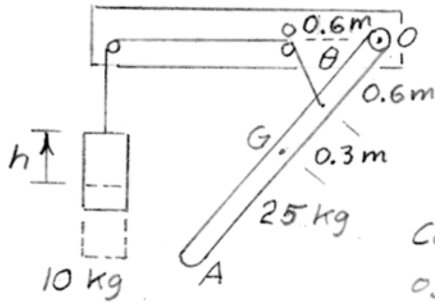
Set $\frac{dV}{d\theta} = 0$ & solve numerically : $\underline{\theta = 24.8°}$

$\left(\frac{d^2V}{d\theta^2}\right)_{\theta=24.8°} = -156 < 0$ so $\underline{\text{unstable}}$.



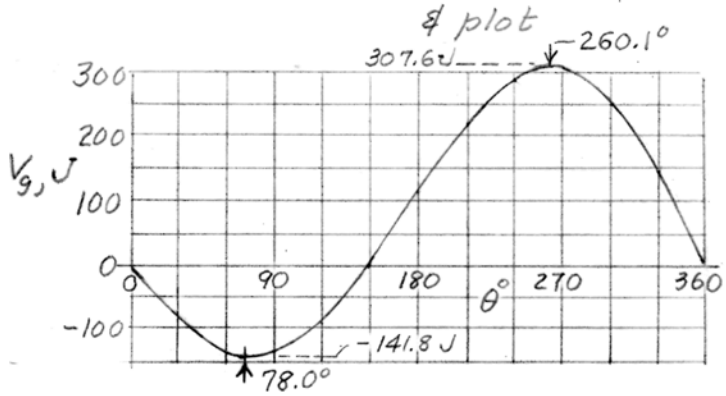
*7/78

$h = 2(0.6 \sin\frac{\theta}{2}) = 1.2 \sin\frac{\theta}{2}$ meters



$V_g = 10(9.81)(1.2 \sin\frac{\theta}{2})$
$\quad - 25(9.81)(0.9 \sin\theta)$
$\quad = 117.7 \sin\frac{\theta}{2} - 220.7 \sin\theta$ joules

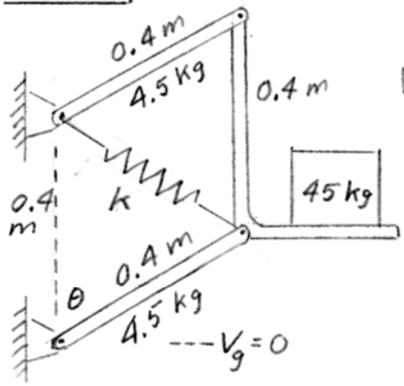
Compute V_g as a function of θ from $\theta = 0$ to $\theta = 360°$ & plot



$\theta = 78.0°$
stable

$\theta = 260°$
unstable

*7/79 | $k = 2000$ N/m



Spring stretch $= 2\left(0.4 \sin\frac{\theta}{2}\right) - 0.2$
$= 0.2\left(4 \sin\frac{\theta}{2} - 1\right)$ m

$V_e = \frac{1}{2} 2000 (0.2)^2 \left(4 \sin\frac{\theta}{2} - 1\right)^2$ J

$V_g = 4.5(9.81)\left(0.2 \cos\theta + 0.4 + 0.2 \cos\theta\right) + 45(9.81)\,0.4 \cos\theta$
$= 194.2 \cos\theta + 17.66$ J

$V = V_e + V_g = 40\left(4 \sin\frac{\theta}{2} - 1\right)^2 + 194.2 \cos\theta + 17.66$ J

$\frac{dV}{d\theta} = 80\left(4 \sin\frac{\theta}{2} - 1\right) 2 \cos\frac{\theta}{2} - 194.2 \sin\theta$
$= 125.8 \sin\theta - 160 \cos\frac{\theta}{2} = 0$ for equilibrium

Solve numerically & get $\underline{\theta = 79.0°}$

*7/80

$y = 2(150)\cos\theta$

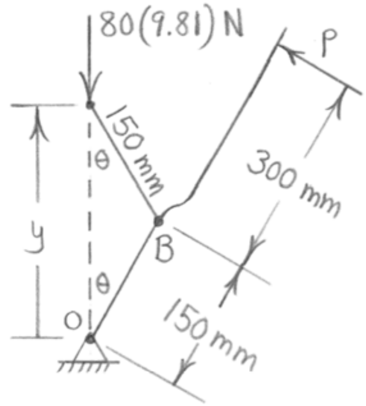
$\delta y = -300 \sin\theta \, \delta\theta$

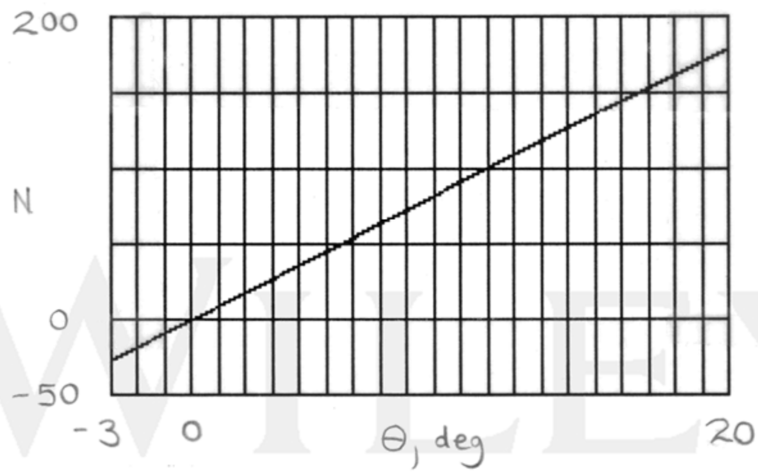
$\delta U = 0$:

$-P(450\,\delta\theta) - 80(9.81)\delta y = 0$

$$P = \frac{80(9.81)(300 \sin\theta)}{450}$$

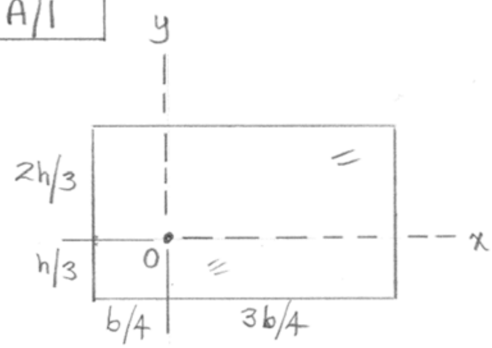
$$= \underline{523 \sin\theta \quad \text{(in newtons)}}$$





(At $\theta = -3°$, $P = -27.4$ N
At $\theta = 20°$, $P = 178.9$ N)

A/1



$$I_x = \bar{I}_x + A d_x^2 = \frac{1}{12} bh^3 + bh\left(\frac{h}{6}\right)^2$$

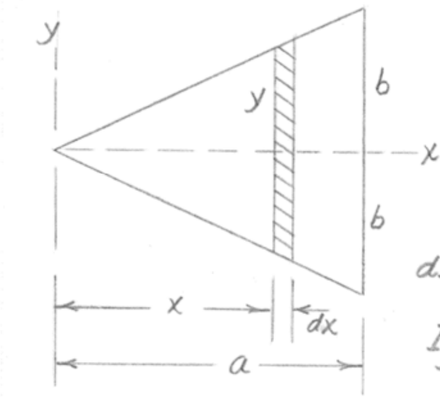
$$= \underline{\frac{1}{9} bh^3}$$

$$I_y = \bar{I}_y + A d_y^2 = \frac{1}{12} hb^3 + bh\left(\frac{b}{4}\right)^2$$

$$= \underline{\frac{7}{48} hb^3}$$

$$I_z = I_x + I_y = \underline{bh\left(\frac{h^2}{9} + \frac{7b^2}{48}\right)}$$

A/2 Using the results of Sample Prob. A/1



$$dI_x = \frac{1}{12}(2y)^3 dx = \frac{2}{3}y^3 dx$$

But $y = \frac{b}{a}x$ so

$$I_x = \frac{2b^3}{3a^3}\int_0^a x^3 dx = \underline{\frac{1}{6}ab^3}$$

$$dI_y = x^2(2y\,dx) = 2\frac{b}{a}x^3 dx$$

$$I_y = \frac{2b}{a}\int_0^a x^3 dx = \frac{2b}{a}\frac{a^4}{4} = \underline{\frac{1}{2}ba^3}$$

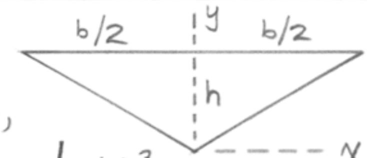
A/3

$$I_x \approx Ad^2 = 300(15)^2 = 67.5(10^3)\ \text{mm}^4$$

$$J_O = I_x + I_y = 67.5(10^3) + 35(10^3) = 102.5(10^3)\ \text{mm}^4$$

$$k_O = \sqrt{J_O/A} = \sqrt{\frac{102.5(10^3)}{300}} = \underline{18.48\ \text{mm}}$$

A/4

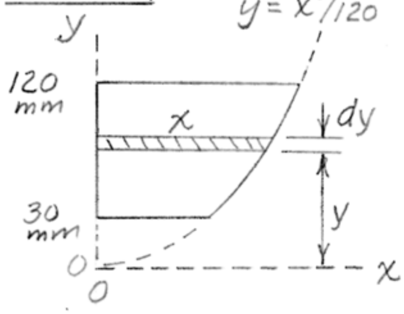


From Sample Problem A/2,

$I_x = \frac{1}{4}bh^3$, $I_y = 2\left\{\frac{1}{12}h\left(\frac{b}{2}\right)^3\right\} = \frac{1}{48}hb^3$

$I_x = I_y$ if $\frac{1}{4}bh^3 = \frac{1}{48}hb^3$, $\underline{\frac{b}{h} = 2\sqrt{3}}$

A/5



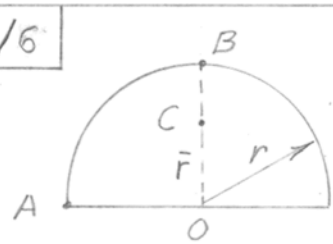
$$dI_y = \frac{1}{3}x^2 dA = \frac{1}{3}x^3 dy$$

$$= \frac{1}{3}(120y)^{3/2} dy$$

$$I_y = \frac{(120)^{3/2}}{3}\int_{30}^{120} y^{3/2} dy$$

$$= \frac{2}{15}(120)^{3/2} y^{5/2}\Big]_{30}^{120} = 26.8\,(10^6)\ \text{mm}^4$$

A/6



$\bar{r} = 4r/3\pi$

For complete circle

$$I_A = I_O + Ar^2 = \frac{1}{2}Ar^2 + Ar^2$$
$$= \frac{3}{2}Ar^2$$

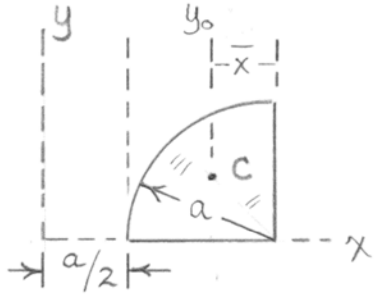
For half circle

$$I_A = \frac{1}{2}\left(\frac{3}{2}\pi r^4\right) = \frac{3}{4}\pi r^4$$

For half circle, $I_O = \frac{1}{4}\pi r^4$

$$I_B = I_C + A(r-\bar{r})^2 = I_O - A\bar{r}^2 + A(r-\bar{r})^2$$
$$= I_O + A(r^2 - 2r\bar{r})$$
$$= \frac{1}{4}\pi r^4 + \frac{\pi r^4}{2}\left(1 - \frac{8}{3\pi}\right) = r^4\left(\frac{3\pi}{4} - \frac{4}{3}\right)$$

A/7



From Table D/3:

$$\bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^4$$

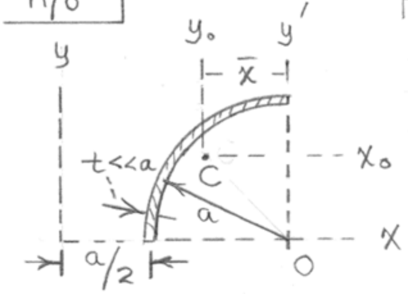
$$\bar{x} = \frac{4a}{3\pi}$$

$$I_y = \bar{I}_y + Ad_y^2$$

$$= \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^4 + \frac{\pi a^2}{4}\left[\frac{a}{2} + \left(a - \frac{4a}{3\pi}\right)\right]^2$$

$$= \underline{\left[\frac{5\pi}{8} - 1\right]a^4}$$

A/8



From Table D/3:

$$\bar{x} = \frac{2a}{\pi}$$

$$I_o = Aa^2 = \left(\frac{\pi a}{2} t\right) a^2 = \frac{\pi t}{2} a^3$$

$I_x + I_{y'} = I_o$; but $I_x = I_{y'}$, so

$$2I_{y'} = I_o, \quad I_{y'} = \frac{I_o}{2} = \frac{\pi t}{4} a^3 = \bar{I}_y + Ad_x^2$$

or $\frac{\pi t}{4} a^3 = \bar{I}_y + \frac{\pi a}{2} t \left(\frac{2a}{\pi}\right)^2$

$$\Rightarrow \bar{I}_y = \left[\frac{\pi}{4} - \frac{2}{\pi}\right] t a^3$$

Finally, $I_y = \bar{I}_y + Ad^2$

$$I_y = \left[\frac{\pi}{4} - \frac{2}{\pi}\right] ta^3 + \frac{\pi a}{2} t \left[\frac{a}{2} + \left(a - \frac{2a}{\pi}\right)\right]^2$$

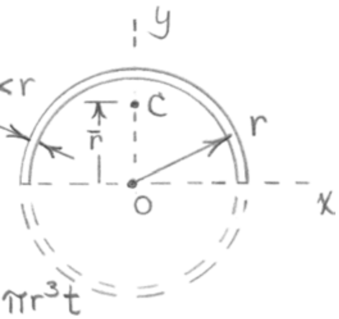
$$= \underline{\left(\frac{11\pi}{8} - 3\right) t a^3}$$

A/9 $I_p = I_C + A(75)^2, \quad I_{p'} = I_C + A(50)^2$

$I_p - I_{p'} = 15(10^6) = A[(75)^2 - (50)^2]$

$\underline{A = 4800 \text{ mm}^2}$

A/10



For complete ring,

$I_o = Ar^2 = 2\pi r t r^2 = 2\pi r^3 t$

and $I_o = I_x + I_y$, $I_x = I_y$

So for complete ring, $I_x = \frac{I_o}{2} = \pi r^3 t$

For half-ring, $\underline{I_x = \frac{1}{2}\pi r^3 t}$ and $I_y = I_x$

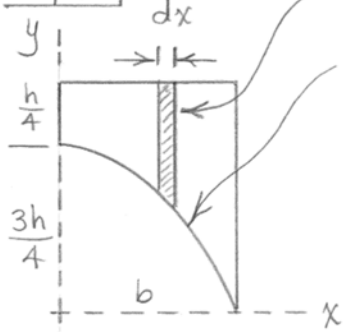
by symmetry so $\underline{I_y = \frac{1}{2}\pi r^3 t}$

For half-ring, $I_o = \frac{1}{2}(2\pi r^3 t) = \pi r^3 t$

$I_C = I_o - A\bar{r}^2 = \pi r^3 t - \pi r t\left(\frac{2r}{\pi}\right)^2$

$= \underline{\pi r^3 t\left(1 - \frac{4}{\pi^2}\right)}$

A/11



$dA = (h-y)\,dx$

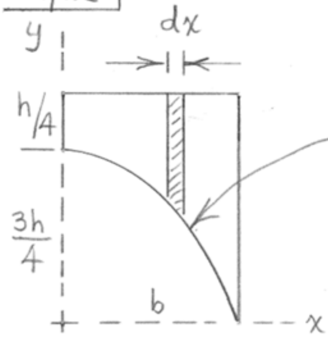
$y = \frac{3h}{4} - \frac{3h}{4b^2}x^2$

$$I_y = \int x^2 dA = \int x^2\left[h - \left(\frac{3h}{4} - \frac{3h}{4b^2}x^2\right)\right]dx$$

$$= \int_0^b \left(\frac{h}{4}x^2 + \frac{3h}{4b^2}x^4\right)dx$$

$$= \underline{\frac{7}{30}hb^3}$$

A/12



$$y = \frac{3h}{4} - \frac{3h}{4b^2}x^2$$

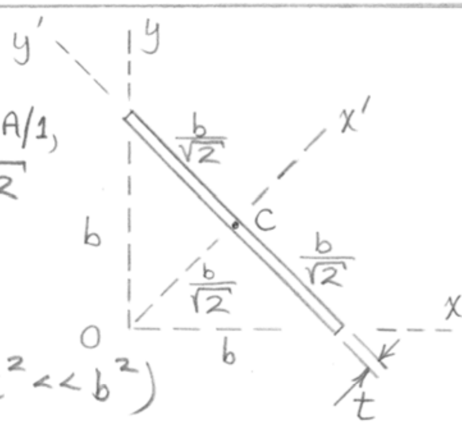
$$dI_x = \frac{1}{3}h^3\,dx - \frac{1}{3}y^3\,dx$$

$$= \frac{1}{3}dx\left[h^3 - \left(\frac{3h}{4} - \frac{3h}{4b^2}x^2\right)^3\right]$$

$$= \frac{1}{3}dx\left[\frac{37}{64}h^3 + \frac{81}{64}\frac{h^3}{b^2}x^2 - \frac{81}{64}\frac{h^3}{b^4}x^4 + \frac{27}{64}\frac{h^3}{b^6}x^6\right]$$

$I_x = \int_0^b dI_x$ to obtain $\underline{0.269bh^3}$

A/13



From Sample Problem A/1,

$$I_{x'} = \frac{1}{12}(b\sqrt{2})^3 t = \frac{b^3 t}{6}\sqrt{2}$$

$$= \frac{b^2}{6}A$$

$$I_{y'} = \frac{1}{12}t^3 b\sqrt{2} = \frac{At^2}{12}$$

(negligible because $t^2 << b^2$)

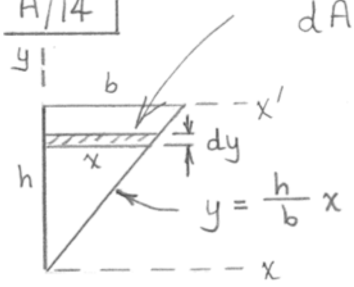
So $I_C = I_{x'} + I_{y'} \cong \frac{Ab^2}{6}$

$$I_O = I_C + A\,\overline{OC}^2 = \frac{Ab^2}{6} + A\frac{b^2}{2} = \underline{\frac{2Ab^2}{3}}$$

By symmetry, $I_x = I_y$; Because $I_O = I_x + I_y$,

we have $\underline{I_x = I_y = \frac{1}{2}I_O = \frac{1}{3}Ab^2}$.

A/14



$$dA = x\,dy = \left(\frac{b}{h}y\right)dy$$

$$I_x = \int y^2 dA = \int_0^h y^2\left(\frac{b}{h}y\right)dy$$

$$= \frac{b}{h}\frac{y^4}{4}\Big|_0^h = \underline{\frac{1}{4}bh^3}$$

$$I_{x'} = \int (h-y)^2 dA = \int (h^2 - 2hy + y^2)\left(\frac{b}{h}y\,dy\right)$$

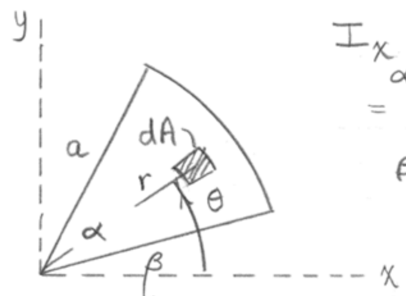
$$= \frac{b}{h}\int_0^h (h^2y - 2hy^2 + y^3)\,dy$$

$$= \frac{b}{h}\left[\frac{h^2y^2}{2} - \frac{2}{3}hy^3 + \frac{y^4}{4}\right]_0^h$$

$$= \underline{\frac{1}{12}bh^3}$$

A/15

$$dA = r\,d\theta\,dr,\quad x = r\cos\theta,\quad y = r\sin\theta$$



$$I_x = \int y^2\,dA$$

$$= \int_{\beta}^{\alpha+\beta}\int_{0}^{a} (r\sin\theta)^2\, r\,d\theta\,dr$$

$$= \int_{\beta}^{\alpha+\beta} \frac{a^4}{4}\sin^2\theta\,d\theta$$

$$I_x = \frac{a^4}{4}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_{\beta}^{\alpha+\beta}$$

$$= \frac{a^4}{4}\left[\frac{\alpha}{2} - \frac{1}{4}\sin 2(\alpha+\beta) + \frac{1}{4}\sin 2\beta\right]$$

$$= \frac{a^4}{8}\left[\alpha - \frac{1}{2}\sin 2(\alpha+\beta) + \frac{1}{2}\sin\beta\right]$$

$\beta = 0$: $\quad I_x = \dfrac{a^4}{8}\left[\alpha - \dfrac{1}{2}\sin 2\alpha\right]$ (agrees with D/3)

$$I_y = \int x^2\,dA = \int_{\beta}^{\alpha+\beta}\int_{0}^{a} (r\cos\theta)^2\, r\,d\theta\,dr$$

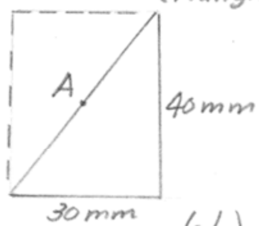
$$= \frac{a^4}{4}\int_{\beta}^{\alpha+\beta}\cos^2\theta\,d\theta = \frac{a^4}{8}\left[\alpha + \frac{1}{2}\sin 2(\alpha+\beta) - \frac{1}{2}\sin\beta\right]$$

$\beta = 0$: $\quad I_y = \dfrac{a^4}{8}\left[\alpha + \dfrac{1}{2}\sin 2\alpha\right]$

(agrees with Table D/3, where the included angle is 2α)

A/16

$(J_A)_{triangle} = \frac{1}{2}(J_A)_{rectangle}$



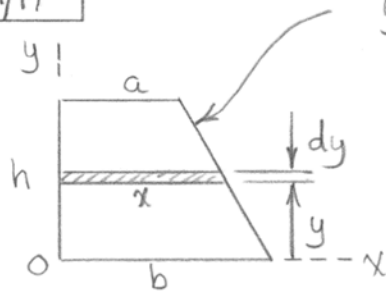
$= \frac{1}{2}\left[\frac{1}{12}A(b^2+h^2)\right]$ from Sample Prob. A/1

$= \frac{1}{24}(30)(40)(\overline{30}^2+\overline{40}^2) = 12.5(10^4)$ mm^4

$(J_A)_{triangle} = k_A^2 A$

So $k_A = \sqrt{\dfrac{12.5(10^4)}{30(40)/2}} = \sqrt{208.4} = \underline{14.43 \text{ mm}}$

A/17



$$y = -\frac{h}{b-a}x + \frac{bh}{b-a}$$

$$\Rightarrow x = -\frac{b-a}{h}y + b$$

$$dI_x = y^2 dA = y^2(x\,dy) = y^2\left(-\frac{b-a}{h}y + b\right)dy$$

$$= \left(-\frac{b-a}{h}y^3 + by^2\right)dy$$

$$I_x = \int dI_x = \int_0^h \left(-\frac{b-a}{h}y^3 + by^2\right)dy$$

$$= \underline{h^3\left(\frac{a}{4} + \frac{b}{12}\right)}$$

$$dI_y = \frac{1}{3}dA\,x^2 = \frac{1}{3}(x\,dy)x^2 = \frac{1}{3}x^3 dy$$

$$= \frac{1}{3}\left[-\frac{b-a}{h}y + b\right]^3 dy$$

$$I_y = \int dI_y = \frac{1}{3}\int\left(-\frac{b-a}{h}y + b\right)^3 dy$$

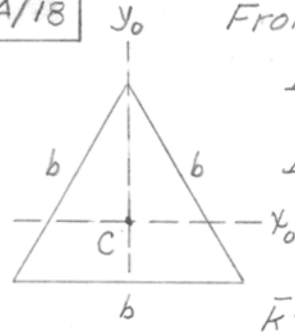
$$\vdots$$

$$= \underline{\frac{h}{12}\left(a^3 + a^2b + ab^2 + b^3\right)}$$

$$I_O = I_x + I_y$$

$$= \underline{\frac{h}{12}\left[h^2(3a+b) + a^3 + a^2b + ab^2 + b^3\right]}$$

A/18



From Sample Problem A/2

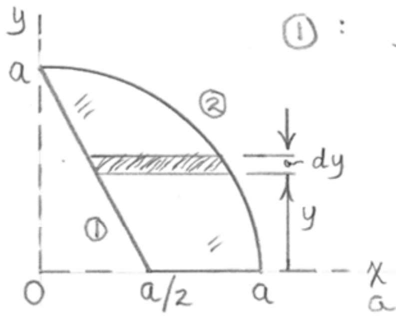
$$\bar{I}_x = \frac{1}{36} b \left(b\frac{\sqrt{3}}{2}\right)^3 = \frac{b^4}{96}\sqrt{3}$$

$$\bar{I}_y = 2\left(\frac{1}{12} b\frac{\sqrt{3}}{2}\left[\frac{b}{2}\right]^3\right) = \frac{b^4}{96}\sqrt{3}$$

$$J = \bar{I}_x + \bar{I}_y = \frac{b^4}{48}\sqrt{3}$$

$$\bar{k} = \sqrt{J/A} = \sqrt{\frac{b^4\sqrt{3}}{48} \Big/ \left(\frac{b^2}{2}\frac{\sqrt{3}}{2}\right)} = \underline{\frac{b}{2\sqrt{3}}}$$

A/19



$$②: \quad x^2 + y^2 = a^2$$

$$①: \quad y = -2x + a$$

$$dA = (x_2 - x_1)\,dy = \left[\sqrt{a^2 - y^2} - \frac{a-y}{2}\right] dy$$

$$I_x = \int y^2\,dA = \int_0^a y^2\left[\sqrt{a^2-y^2} - \frac{a-y}{2}\right] dy$$

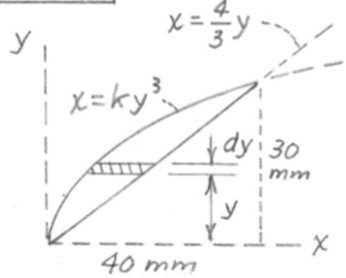
$$= \int_0^a \left[y^2\sqrt{a^2-y^2} - \frac{ay^2 - y^3}{2}\right] dy$$

$$= \left[-\frac{y}{4}\sqrt{(a^2-y^2)^3} + \frac{a^2}{8}\left(y\sqrt{a^2-y^2} + a^2\sin^{-1}\frac{y}{a}\right) - \frac{ay^3}{6} + \frac{y^4}{8}\right]_0^a = a^4\left[\frac{\pi}{16} - \frac{1}{24}\right]$$

$$= \frac{a^4}{8}\left[\frac{\pi}{2} - \frac{1}{3}\right]$$

A/20

For $x = 40$ mm & $y = 30$ mm, $k = \dfrac{40}{27(10^3)}$

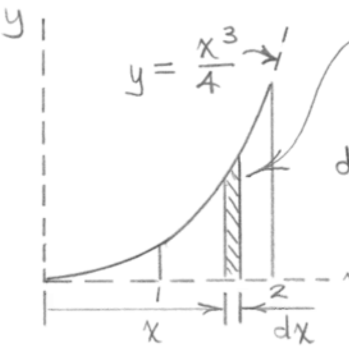


$$dI_x = y^2 dA = y^2\left(\frac{4}{3}y - \frac{4}{2700}y^3\right)dy$$

$$I_x = \int_0^{30}\left(\frac{4}{3}y^3 - \frac{4}{2700}y^5\right)dy$$

$$= \left[\frac{y^4}{3} - \frac{y^6}{4050}\right]_0^{30} = \underline{9(10^4)\ \text{mm}^4}$$

A/21



$dA = y\,dx = \frac{x^3}{4}\,dx$

$dI_y = x^2\,dA = \frac{x^5}{4}\,dx$

$dI_x = \frac{1}{3}y^3\,dx = \frac{x^9}{192}\,dx$

$$A = \int dA = \int_1^2 \frac{x^3}{4}\,dx = \frac{1}{4}\frac{x^4}{4}\Big|_1^2 = \frac{15}{16}$$

$$I_y = \int dI_y = \int_1^2 \frac{x^5}{4}\,dx = \frac{1}{4}\frac{x^6}{6}\Big|_1^2 = \frac{63}{24}$$

$$I_x = \int dI_x = \int_1^2 \frac{x^9}{192}\,dx = \frac{1}{192}\frac{x^{10}}{10}\Big|_1^2 = \frac{1023}{1920}$$

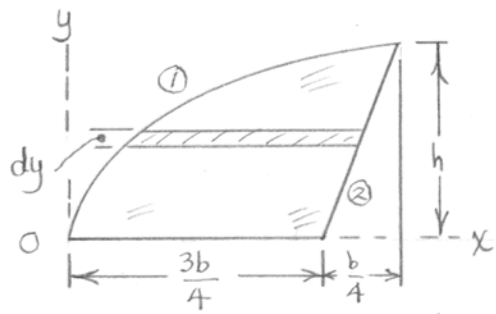
$$k_y = \sqrt{I_y/A} = \sqrt{\frac{63/24}{15/16}} = \sqrt{14/5} = \underline{1.673}$$

$$k_x = \sqrt{I_x/A} = \sqrt{\frac{1023/1920}{15/16}} = \underline{0.754}$$

$$k_z = \sqrt{k_x^2 + k_y^2} = \sqrt{1.673^2 + 0.754^2} = \underline{1.835}$$

A/22

$y_1 = k\sqrt{x}$: $h = k\sqrt{b}$, $k = \frac{h}{\sqrt{b}}$



So $y_1 = \frac{h}{\sqrt{b}}\sqrt{x}$

or $x_1 = \frac{b}{h^2}y^2$

$y_2 = \frac{4h}{b}x - 3h$

or $x_2 = \frac{b}{4h}(y + 3h)$

$$dA = (x_2 - x_1)dy = \left[\frac{b}{4h}(y+3h) - \frac{b}{h^2}y^2\right]dy$$

$$= \frac{b}{h}\left[\frac{3h}{4} + \frac{y}{4} - \frac{y^2}{h}\right]dy$$

$$I_x = \int y^2 dA = \int_0^h y^2 \frac{b}{h}\left[\frac{3h}{4} + \frac{y}{4} - \frac{y^2}{h}\right]dy$$

$$= \frac{b}{h}\int_0^h \left(\frac{3h}{4}y^2 + \frac{y^3}{4} - \frac{y^4}{h}\right)dy$$

$$= \underline{\frac{9}{80}bh^3} \qquad (0.1125\,bh^3)$$

$$dI_y = \frac{1}{3}dy\,x_2^3 - \frac{1}{3}dy\,x_1^3$$

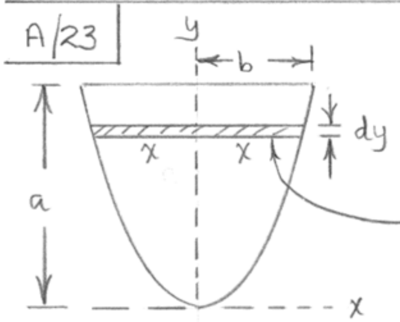
$$= \frac{1}{3}dy\left\{\left[\frac{b}{4h}(y+3h)\right]^3 - \left[\frac{b}{h^2}y^2\right]^3\right\}$$

$$= \frac{1}{3}\left\{\frac{b^3}{64h^3}(y^3 + 9y^2h + 27yh^2 + 27h^3) - \frac{b^3}{h^6}y^6\right\}dy$$

$$I_y = \int_0^h dI_y = \underline{0.1802\,hb^3}$$

$$I_O = I_x + I_y = \underline{hb\,(0.1125h^2 + 0.1802b^2)}$$

A/23



$y = kx^2$: $a = kb^2 \Rightarrow k = \dfrac{a}{b^2}$

So $y = \dfrac{a}{b^2}x^2$, $x = b\sqrt{\dfrac{y}{a}}$

$$\begin{cases} dA = 2x\,dy = 2b\sqrt{\dfrac{y}{a}}\,dy \\ dI_y = dy\,(2x)^3/12 = \dfrac{2}{3}x^3\,dy \end{cases}$$

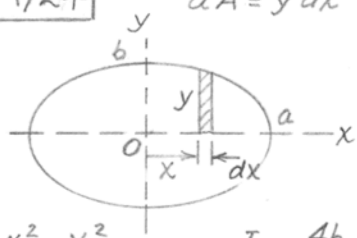
$$I_x = \int y^2\,dA = \int y^2\left(2b\sqrt{\frac{y}{a}}\,dy\right)$$

$$= \frac{2b}{\sqrt{a}}\int_0^a y^{5/2}\,dy = \underline{\frac{4}{7}a^3 b}$$

$$I_y = \int dI_y = \int \frac{2}{3}x^3\,dy = \frac{2}{3}\int_0^a b^3\frac{y^{3/2}}{a^{3/2}}\,dy$$

$$= \underline{\frac{4}{15}ab^3}$$

A/24



$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

$$dA = y\,dx = \frac{b}{a}\sqrt{a^2-x^2}\,dx$$

$$A = \frac{4b}{a}\int_0^a \sqrt{a^2-x^2}\,dx = \pi ab$$

$$dI_y = x^2 y\,dx = \frac{b}{a}x^2\sqrt{a^2-x^2}\,dx$$

$$I_y = \frac{4b}{a}\int_0^a x^2\sqrt{a^2-x^2}\,dx$$

$$= \frac{4b}{a}\left[-\frac{x}{4}\sqrt{(a^2-x^2)^3}+\frac{a^2}{8}\left(x\sqrt{a^2-x^2}+a^2\sin^{-1}\frac{x}{a}\right)\right]_0^a$$

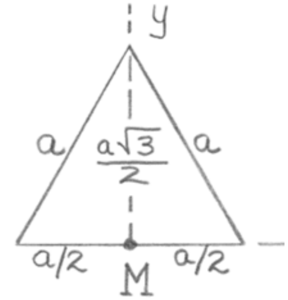
$$= \frac{b}{a}\left[0+\frac{a^2}{2}\left(0+a^2\frac{\pi}{2}\right)\right] = \underline{\frac{\pi a^3 b}{4}}$$

Similarly $\quad I_x = \dfrac{\pi a b^3}{4}$

So $\quad I_O = I_x + I_y = \dfrac{\pi ab}{4}(a^2+b^2)$

& $\quad k_O = \sqrt{I_O/A} = \underline{\dfrac{1}{2}\sqrt{a^2+b^2}}$

A/25



$I_z = I_x + I_y$, $I_z = Ak_z^2$

$\therefore \quad k_M = \sqrt{(I_x + I_y)/A}$

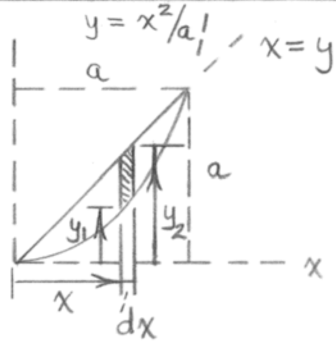
$I_x = \frac{1}{12}bh^3 = \frac{1}{12}a\left(\frac{a\sqrt{3}}{2}\right)^3$

$= \frac{\sqrt{3}}{32}a^4$

$I_y = 2\left(\frac{1}{12}\frac{a\sqrt{3}}{2}\left(\frac{a}{2}\right)^3\right) = \frac{\sqrt{3}}{96}a^4$

$k_M = \left[\dfrac{\frac{\sqrt{3}}{32}a^4 + \frac{\sqrt{3}}{96}a^4}{\frac{a}{2}a\frac{\sqrt{3}}{2}}\right]^{1/2} = \frac{a}{\sqrt{6}}$

A/26



$$dA = (y_2 - y_1)\,dx = \left(x - \frac{x^2}{a}\right)dx$$

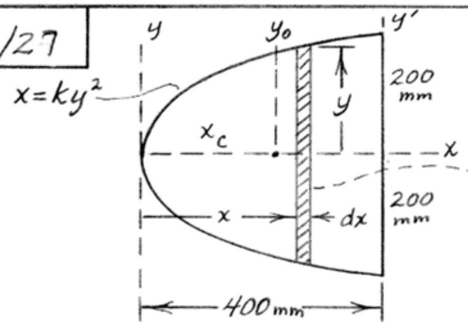
$$I_x = \int \frac{1}{3} y_2^{\,3}\,dx - \int \frac{1}{3} y_1^{\,3}\,dx = \frac{1}{3}\int_0^a \left(x^3 - \frac{x^6}{a^3}\right)dx$$

$$= \frac{1}{3}\left[\frac{x^4}{4} - \frac{x^7}{7a^3}\right]_0^a = \frac{a^4}{3}\left(\frac{1}{4} - \frac{1}{7}\right) = \underline{a^4/28}$$

$$I_y = \int x^2\,dA = \int x^2\left(x - \frac{x^2}{a}\right)dx = \int_0^a \left(x^3 - \frac{x^4}{a}\right)dx$$

$$= \left[\frac{x^4}{4} - \frac{x^5}{5a}\right]_0^a = \frac{a^4}{4} - \frac{a^4}{5} = \underline{a^4/20}$$

A/27



$x = ky^2$: $400 = k(200^2)$

$k = 0.01\ \text{mm}^{-1}$

$x = 0.01y^2$, $y = 10\sqrt{x}$

$dA = 2y\,dx$

$dI_y = x^2 dA = 2yx^2 dx$

$$A = \int dA = \int 2y\,dx = \int_0^{400} 2(10\sqrt{x})\,dx = 20\int_0^{400} x^{1/2}dx = 20\frac{x^{3/2}}{3/2}\Big|_0^{400} = 10.67(10^4)\ \text{mm}^2$$

$$I_y = \int dI_y = \int 2(10\sqrt{x})x^2 dx = 20\int_0^{400} x^{5/2}dx = 20\frac{x^{7/2}}{7/2}\Big|_0^{400} = \underline{73.1(10^8)\ \text{mm}^4}$$

$$\int x\,dA = 20\int_0^{400} x^{3/2}dx = 25.6(10^6)\ \text{mm}^3$$

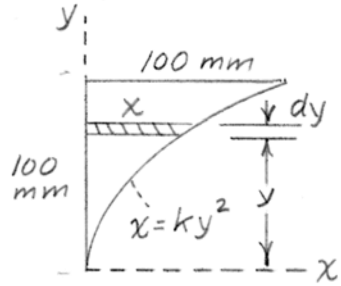
$$x_c = \int x\,dA/A = 25.6(10^6)/10.67(10^4) = 240\ \text{mm}$$

$$I_{y_0} = I_y - Ax_c^2 = 73.1(10^8) - 10.67(10^4)(240^2) = 11.70(10^8)\ \text{mm}^4$$

$$I_{y'} = I_{y_0} + A(400 - x_c)^2 = 11.70(10^8) + 10.67(10^4)(400-240)^2$$

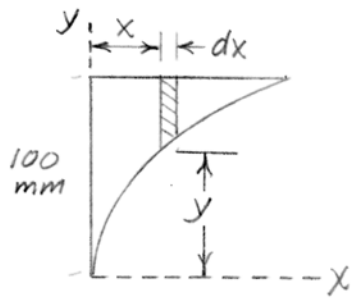
$$= \underline{39.0(10^8)\ \text{mm}^4}$$

A/28 $100 = k(100)^2$, $k = 1/100$ so $x = y^2/100$



$$I_x = \int y^2 dA = \int_0^{100} \frac{y^4}{100} dy$$

$$= \frac{1}{100} \left.\frac{y^5}{5}\right]_0^{100} = \underline{20(10^6)\ mm^4}$$



$$dI_x = \frac{1}{3} dx\,(100^3 - y^3)$$

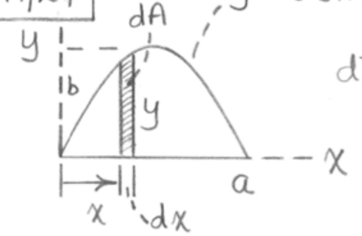
$$= \frac{1}{3}\left(100^3 - 10^3 x^{3/2}\right) dx$$

$$I_x = \frac{1}{3} 10^3 \int_0^{100} \left(10^3 - x^{3/2}\right) dx$$

$$= \frac{10^3}{3}\left[10^3 x - \frac{2}{5} x^{5/2}\right]_0^{100}$$

$$= \frac{10^3}{3}\left[10^5 - \frac{2}{5} 10^5\right] = \underline{20(10^6)\ mm^4}$$

A/29



$$dI_x = \frac{1}{3} dA\, y^2 = \frac{1}{3}(y\,dx)y^2 = \frac{1}{3}y^3\,dx$$

$$dI_y = dA\,x^2 = (y\,dx)x^2 = x^2 y\,dx$$

$$I_x = \int dI_x = \int \frac{1}{3}y^3\,dx = \frac{1}{3}\int_0^a \left(b \sin\frac{\pi}{a}x\right)^3 dx$$

$$= \frac{b^3}{3}\left[-\frac{a}{\pi}\cos\frac{\pi}{a}x + \frac{a}{3\pi}\cos^3\frac{\pi}{a}x\right]_0^a$$

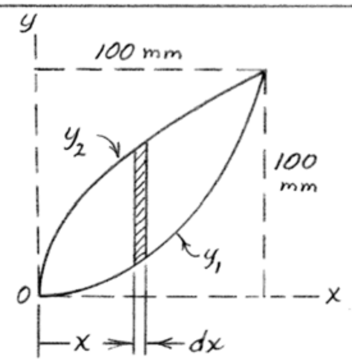
$$= \underline{\frac{4}{9}\frac{ab^3}{\pi}}$$

$$I_y = \int dI_y = \int_0^a x^2 b \sin\frac{\pi}{a}x\,dx$$

$$= b\left[\frac{2x}{(\pi/a)^2}\sin\frac{\pi}{a}x + \frac{2}{(\pi/a)^3}\cos\frac{\pi}{a}x - \frac{x^2}{\pi/a}\cos\frac{\pi}{a}x\right]_0^a$$

$$= \underline{\frac{ba^3}{\pi}\left(1 - \frac{4}{\pi^2}\right)}$$

A/30



From $y_1 = k_1x^3$, $100 = k(100^3)$

$k_1 = 10^{-4}\ \text{mm}^{-2}$ & $y_1 = 10^{-4}x^3$

From $y_2 = k_2\sqrt{x}$, $100 = k_2\sqrt{100}$

$k_2 = 10\ \text{mm}^{1/2}$ & $y_2 = 10\sqrt{x}$

$dA = (y_2 - y_1)\,dx$

$= (10\sqrt{x} - 10^{-4}x^3)\,dx$

$$I_y = \int x^2\,dA = \int_0^{100} x^2(10\sqrt{x} - 10^{-4}x^3)\,dx = \left[\frac{20}{7}x^{7/2} - \frac{10^{-4}}{6}x^6\right]_0^{100}$$

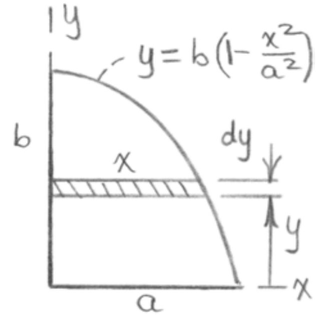
$$= \underline{11.90\,(10^6)\ \text{mm}^4}$$

$$I_x = \int\left[\tfrac{1}{3}y_2^3 - \tfrac{1}{3}y_1^3\right]dx = \frac{1}{3}\int_0^{100}\left[(10\sqrt{x})^3 - (10^{-4}x^3)^3\right]dx$$

$$= \frac{1}{3}\left[10^3\,\frac{2}{5}x^{5/2} - \frac{x^{10}}{10^{13}}\right]_0^{100} = \underline{10^7\ \text{mm}^4}$$

$$I_O = I_x + I_y = \underline{21.9\,(10^6)\ \text{mm}^4}$$

A/31



(a) Horizontal strip

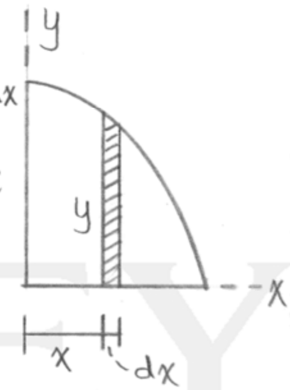
$$I_x = \int y^2 dA = \int_0^b y^2 x\,dy$$

$$= \int_0^b y^2 a\sqrt{1 - y/b}\;dy$$

$$= \frac{a}{\sqrt{b}} \int_0^b y^2 \sqrt{b-y}\;dy$$

$$= \frac{a}{\sqrt{b}} \frac{2}{105(-1)} (8b^2 + 12by + 15y^2)\sqrt{(b-y)^3}\Big|_0^b = \underline{\frac{16ab^3}{105}}$$

(b) Vertical strip



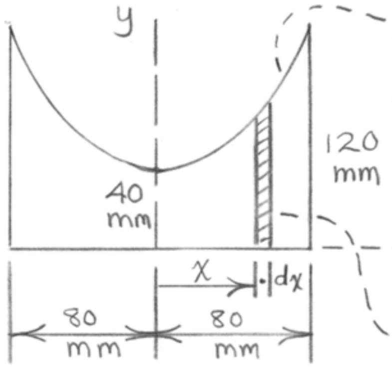
$$I_x = \int_0^a \frac{1}{3} y^2 (y\,dx) = \frac{1}{3}\int_0^a b^3 (1 - \frac{x^2}{a^2})^3 dx$$

$$= \frac{b^3}{3}\frac{1}{a^6}\int_0^a (a^6 - 3a^4x^2 + 3a^2x^4 - x^6)\,dx$$

$$= \frac{b^3}{3a^6}\left[a^6 x - a^4 x^3 + \frac{3a^2x^5}{5} - \frac{x^7}{7}\right]_0^a$$

$$= \underline{\frac{16ab^3}{105}}$$

A/32



$y = a + bx^2$

$$\begin{cases} 40 = a + b(0)^2 \\ 120 = a + b(80)^2 \end{cases}$$

$$\begin{cases} a = 40 \text{ mm} \\ b = \dfrac{1}{80 \text{ mm}} \end{cases}$$

$$\begin{cases} dA = y\,dx \\ dI_y = dA\,x^2 \end{cases}$$

$$A = \int dA = \int y\,dx = 2\int_0^{80}\left(40 + \frac{1}{80}x^2\right)dx$$

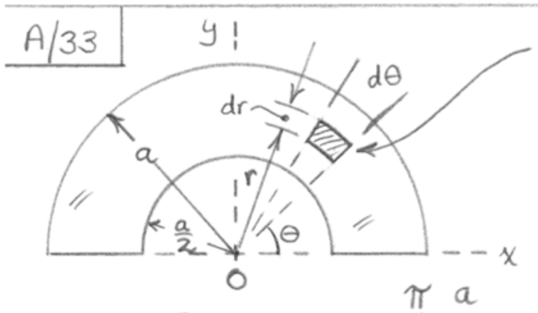
$$= 2\left[40x + \frac{x^3}{240}\right]_0^{80} = 10\,670 \text{ mm}^2$$

$$I_y = \int dI_y = \int x^2 y\,dx = 2\int_0^{80}\left[40x^2 + \frac{x^4}{80}\right]dx$$

$$= 2\left[\frac{40}{3}x^3 + \frac{x^5}{400}\right]_0^{80} = 30.0(10^6) \text{ mm}^4$$

$$k_y = \sqrt{I_y/A} = \sqrt{\frac{30.0(10^6)}{10\,670}} = \underline{53.1 \text{ mm}}$$

A/33



$$dA = r\,dr\,d\theta$$

$$A = \frac{1}{2}\left[\pi a^2 - \pi\left(\frac{a}{2}\right)^2\right] = \frac{3}{8}\pi a^2$$

$$I_x = \int y^2 dA = \int_0^{\pi}\int_{a/2}^{a} (r\sin\theta)^2 \, r\,dr\,d\theta$$

$$= \int_0^{\pi} \frac{15}{64} a^4 \sin^2\theta \, d\theta = \frac{15}{128}\pi a^4$$

$$I_y = \int x^2 dA = 2\int_0^{\pi/2}\int_{a/2}^{a} (r\cos\theta)^2 \, r\,dr\,d\theta$$

$$= 2\int_{a/2}^{a} \frac{15}{64} a^4 \cos^2\theta \, d\theta = \frac{15}{128}\pi a^4$$

$$\underline{k_x} = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\frac{15}{128}\pi a^4}{\frac{3}{8}\pi a^2}} = \underline{\frac{\sqrt{5}}{4} a = k_y}$$

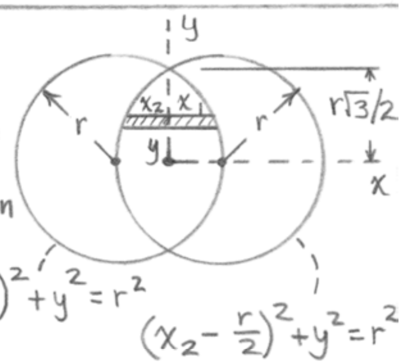
$$k_z^2 = k_x^2 + k_y^2 = 2\left(\frac{5}{16} a^2\right)$$

$$\underline{k_z = \frac{\sqrt{10}}{4} a}$$

▶ A/34

$x_2 = \frac{r}{2} \pm \sqrt{r^2-y^2}$ use (−) sign

$x_1 = -\frac{r}{2} \pm \sqrt{r^2-y^2}$ use (+) sign



$$(x_1 - x_2) = -\frac{r}{2} + \sqrt{r^2-y^2} - \frac{r}{2} + \sqrt{r^2-y^2} = 2\sqrt{r^2-y^2} - r$$

$$dA = (2\sqrt{r^2-y^2} - r)\,dy$$

$$I_x = \int y^2 dA = 2\int_0^{r\sqrt{3}/2} y^2 (2\sqrt{r^2-y^2} - r)\,dy$$

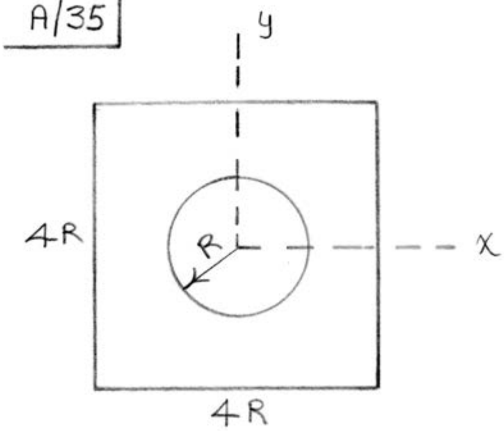
$$= 4\left\{-\frac{y}{4}\sqrt{(r^2-y^2)^3} + \frac{r^2}{8}\left(y\sqrt{r^2-y^2} + r^2\sin^{-1}\frac{y}{r}\right)\right\} - \frac{2r}{3}y^3\Bigg|_0^{\frac{r\sqrt{3}}{2}}$$

$$= 4\left\{-\frac{r\sqrt{3}}{8}\frac{r^3}{8} + \frac{r^2}{8}\left(\frac{r\sqrt{3}}{2}\frac{r}{2} + r^2\frac{\pi}{3}\right)\right\} - \frac{2\sqrt{3}}{8}r^4 - 0$$

$$= \frac{r^4}{2}\left\{-\frac{\sqrt{3}}{8} + \frac{\sqrt{3}}{4} + \frac{\pi}{3} - \frac{\sqrt{3}}{2}\right\} = \frac{r^4}{2}\left\{\frac{\pi}{3} - \frac{3\sqrt{3}}{8}\right\}$$

$$= 0.1988\, r^4$$

A/35



Without hole, $I_x = \frac{1}{12}(4R)(4R)^3 = \frac{64}{3}R^4$

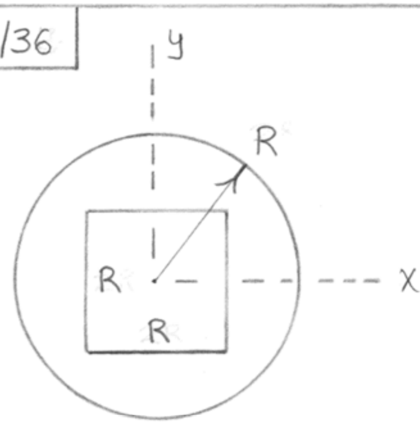
$(21.3R^4)$

With hole, $I_x = \frac{64}{3}R^4 - \frac{1}{4}(\pi R^2)R^2$

$= \underline{20.5R^4}$

(a 3.68% reduction)

A/36



Without square hole:

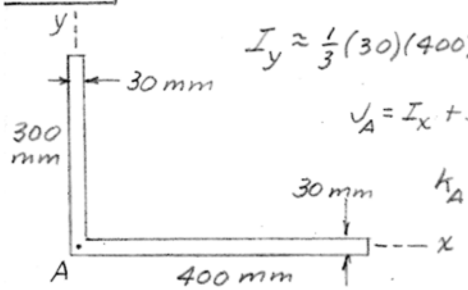
$$I_z = 2I_x = 2\left(\frac{1}{4}\pi R^2 \cdot R^2\right) = \underline{1.571 R^4}$$

With hole:

$$I_z = 1.571 R^4 - 2\left(\frac{1}{12} R \cdot R^3\right) = \underline{1.404 R^4}$$

(a reduction of 10.61%)

A/37



$I_x \approx \frac{1}{3}(30)(300)^3 + 0 = 270(10)^6 \text{ mm}^4$

$I_y \approx \frac{1}{3}(30)(400)^3 + 0 = 640(10)^6 \text{ mm}^4$

$J_A = I_x + I_y = 910(10)^6 \text{ mm}^4$

$k_A = \sqrt{J_A/A} = \sqrt{\dfrac{910(10)^6}{30(300+400)}}$

$k_A = \underline{208 \text{ mm}}$

A/38

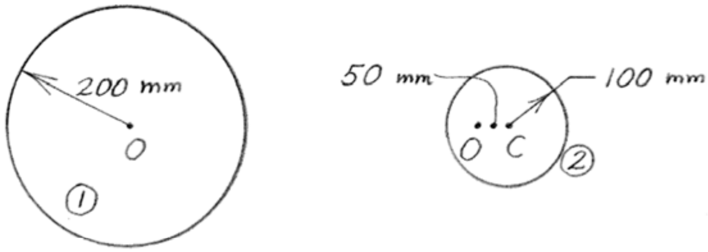
$$I_z = \frac{1}{2}\left[\frac{\pi a^4}{2} - \frac{\pi\left(\frac{a}{2}\right)^4}{2}\right] = \frac{15}{64}\pi a^4$$

$$k_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{\frac{15}{64}\pi a^4}{\frac{3}{8}\pi a^2}} = \underline{\frac{\sqrt{10}}{4}a}$$

From $k_x^2 + k_y^2 = k_z^2$ and the fact that $k_x = k_y$ for the present case,

$$2k_x^2 = \left(\frac{\sqrt{10}}{4}a\right)^2, \quad k_x = k_y = \underline{\frac{\sqrt{5}}{4}a}$$

A/39



Area $A = A_1 - A_2 = \pi(200^2 - 100^2) = 3(10^4)\pi \text{ mm}^2$

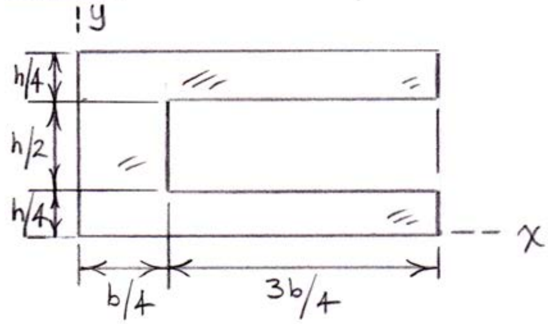
① $I_{O_1} = \frac{1}{2}(\pi \cdot 200^2)(200^2) = 8(10^8)\pi \text{ mm}^4$

② $I_{O_2} = \frac{1}{2}(\pi \cdot 100^2)(100^2) + \pi(100^2)(50^2) = 0.75(10^8)\pi \text{ mm}^4$

So $I_O = I_{O_1} - I_{O_2} = 7.25(10^8)\pi \text{ mm}^4$

$k_O = \sqrt{I_O/A} = \sqrt{\dfrac{7.25(10^8)\pi}{3(10^4)\pi}} = \underline{155.5 \text{ mm}}$

A/40



Full rectangle: $A = bh$, $I_y = \frac{1}{3}hb^3$

With cutout: $A = bh - \frac{3b}{4}\left(\frac{h}{2}\right) = \frac{5}{8}bh$

$$I_y = \frac{1}{3}hb^3 - \left[\frac{1}{12}\frac{h}{2}\left(\frac{3b}{4}\right)^3 + \frac{3}{8}bh\left(\frac{b}{4}+\frac{3b}{8}\right)^2\right]$$

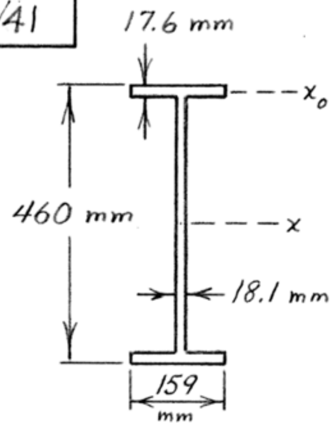
$$= \frac{65}{384}hb^3$$

Percent reductions:

$$n_A = \frac{bh - \frac{5}{8}bh}{bh}(100\%) = \underline{37.5\%}$$

$$n_{I_y} = \frac{\frac{1}{3}hb^3 - \frac{65}{384}hb^3}{\frac{1}{3}hb^3} = \underline{49.2\%}$$

A/41



Flanges: $\bar{I}_x = I_{x_0} + Ad^2$

$= 2\left\{\frac{1}{12}(159)(17.6^3) + 159(17.6)\left(230 - \frac{17.6}{2}\right)^2\right\}$

$= 2\left\{7.22(10^4) + 1.369(10^8)\right\}$ mm^4

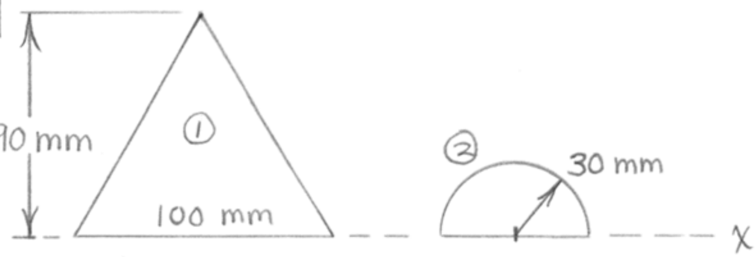
$= 2.74(10^8)$ mm^4

Web: $\bar{I}_x = \frac{1}{12}(18.1)(460 - 2[17.6])^3$

$= 1.156(10^8)$ mm^4

Total $\bar{I}_x = 3.90(10^8)$ mm^4

A/42

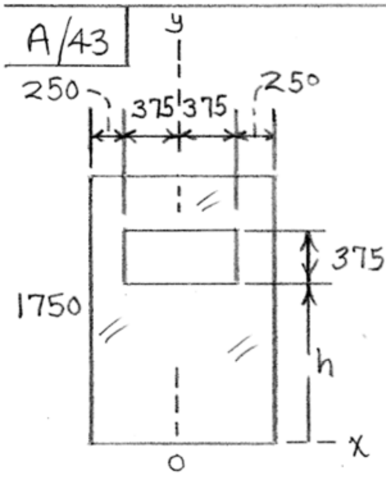


$$I_{x_1} = \frac{1}{12}(100)(90^3) = 6.08(10^6)\ \text{mm}^4$$

$$I_{x_2} = -\frac{\pi(30^4)}{8} = -0.318(10^6)\ \text{mm}^4$$

So $I_x = (6.08 - 0.318)10^6 = \underline{5.76(10^6)\ \text{mm}^4}$

A/43



(a) $h = 1000$ mm (hole complete)

$$I_x = \frac{1}{3}(1250)(1750^3) - \left[\frac{1}{12}(750)(375)^3 + 750(375)\left(1000 + \frac{375}{2}\right)^2\right]$$

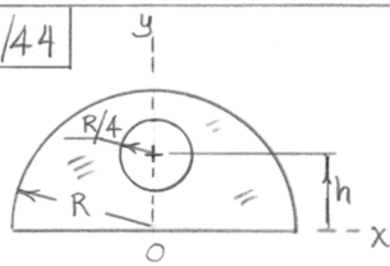
$$= \underline{1.833(10^{12})\ \text{mm}^4 \text{ or } 1.833\ \text{m}^4}$$

(b) $h = 1500$ mm (250 mm of hole in play)

$$I_x = \frac{1}{3}(1250)(1750^3) - \left[\frac{1}{12}(750)(250)^3 + 750(250)\left(1500 + \frac{250}{2}\right)^2\right]$$

$$= \underline{1.737(10^{12})\ \text{mm}^4 \text{ or } 1.737\ \text{m}^4}$$

A/44



(a) $h = 0$ (One-half of hole considered)

$$I_x = \frac{\pi R^4}{8} - \frac{\pi (R/4)^4}{8} = \frac{255}{2048}\pi R^4$$

$$\underline{(0.391 R^4)}$$

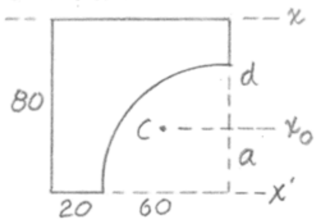
(b) $h = \frac{R}{2}$ (Entire hole now in play)

$$I_x = \frac{\pi R^4}{8} - \left[\frac{\pi (R/4)^4}{4} + \pi \left(\frac{R}{4}\right)^2 \left(\frac{R}{2}\right)^2\right]$$

$$= \frac{111}{1024}\pi R^4 \quad (0.341 R^4)$$

A/45

Dimen. in mm



Square: $I_x = \frac{1}{3}b^4 = \frac{1}{3}(80)^4 = 13.65(10^6)$ mm^4

Quarter-circle: $a = \frac{4r}{3\pi} = \frac{4(60)}{3\pi}$

$= 25.46$ mm

$d = 80 - 25.46 = 54.54$ mm

$$I_x = I_{x_0} + Ad^2 = I_{x'} - Aa^2 + Ad^2$$

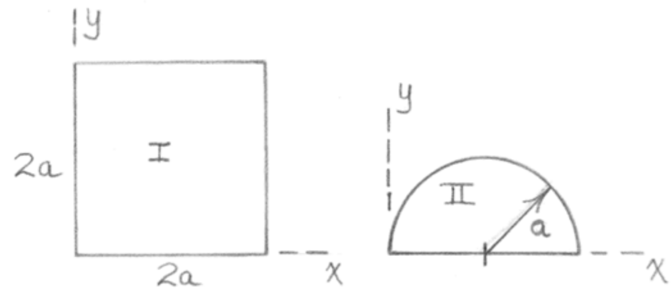
$$= -\frac{1}{4}\frac{\pi r^4}{4} - \frac{\pi r^2}{4}(d^2 - a^2) = -\frac{\pi r^2}{4}\left(\frac{r^2}{4} + d^2 - a^2\right)$$

$$= -\frac{\pi(60)^2}{4}\left[\frac{60^2}{4} + (54.54)^2 - (25.46)^2\right]$$

$$= -9.120(10^6) \text{ mm}^4$$

Total $I_x = (13.65 - 9.120)(10^6) = \underline{4.53(10^6) \text{ mm}^4}$

A/46



I. Square $\quad I_x = I_y = \frac{1}{3}(4a^2)(2a)^2 = \frac{16}{3}a^4$

II. Semicircle $\quad I_x = \frac{1}{8}\pi a^4$

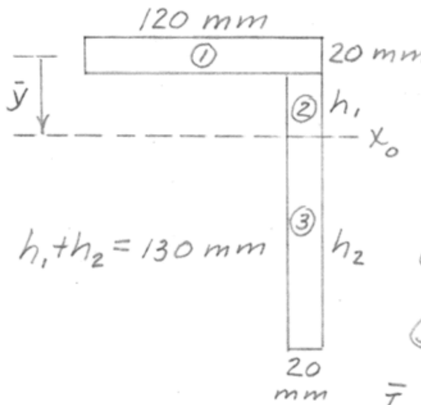
$$I_y = \frac{1}{8}\pi a^4 + \frac{1}{2}\pi a^2(a^2) = \frac{5}{8}\pi a^4$$

Combined: $\quad I_x = \frac{16}{3}a^4 - \frac{\pi}{8}a^4 = \underline{4.94a^4}$

$$I_y = \frac{16}{3}a^4 - \frac{5}{8}\pi a^4 = \underline{3.37a^4}$$

A/47

$$\bar{y} = \Sigma A y / \Sigma A = \frac{0 + (130)(20)(65+10)}{250(20)} = 39.0 \text{ mm}$$



so $h_1 = 29.0$ mm

$h_2 = 101.0$ mm

① $\bar{I}_x = \frac{1}{12}(120)(20^3) + (120)(20)(39.0)^2$

$= 3.73(10^6)\ \text{mm}^4$

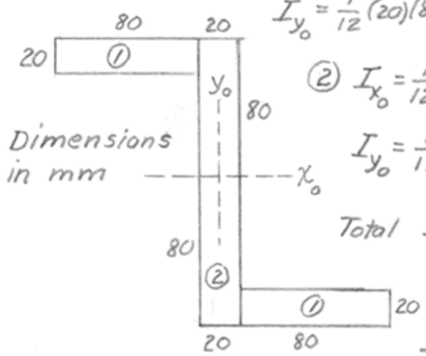
② $\bar{I}_x = \frac{1}{3}(20)(29.0)^3 = 0.163(10^6)\ \text{mm}^4$

③ $\bar{I}_x = \frac{1}{3}(20)(101.0)^3 = 6.87(10^6)\ \text{mm}^4$

$\bar{I}_x = (3.73 + 0.163 + 6.87)10^6$

$= \underline{10.76(10^6)\ \text{mm}^4}$

A/48



① $I_{x_o} = \frac{1}{12}(80)(20)^3 + (80)(20)(70)^2 = 7.89(10^6)\ \text{mm}^4$

$I_{y_o} = \frac{1}{12}(20)(80)^3 + (20)(80)(50)^2 = 4.85(10^6)\ \text{mm}^4$

② $I_{x_o} = \frac{1}{12}(20)(160)^3 = 6.83(10^6)\ \text{mm}^4$

$I_{y_o} = \frac{1}{12}(160)(20)^3 = 0.1067(10^6)\ \text{mm}^4$

Total $\bar{I}_x = [2(7.89) + 6.83](10^6)$

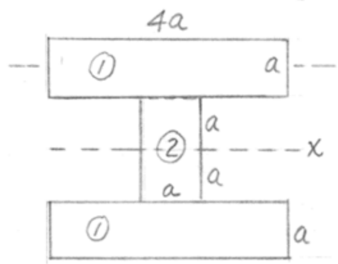
$= \underline{22.6(10^6)\ \text{mm}^4}$

$\bar{I}_y = [2(4.85) + 0.1067](10^6)$

$= \underline{9.81(10^6)\ \text{mm}^4}$

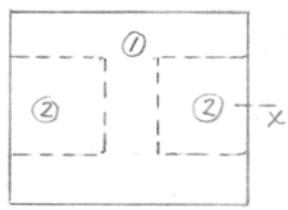
A/49

Sol. I $\quad I_{①} = 2\left[\frac{1}{12}4a(a^3) + 4a^2\left(\frac{3a}{2}\right)^2\right] = \frac{56}{3}a^4$



$I_{②} = \frac{1}{12}a(2a)^3 = \frac{2}{3}a^4$

Total $\underline{I_x = \frac{58}{3}a^4}$



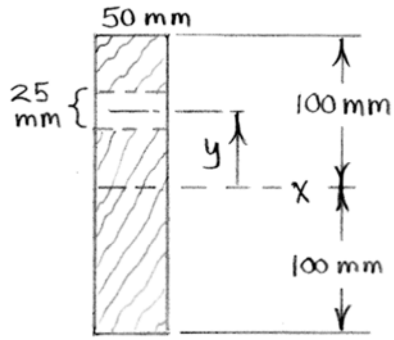
Sol. II

$I_{①} = \frac{1}{12}(4a)(4a)^3 = \frac{64}{3}a^4$

$I_{②} = -\frac{1}{12}(3a)(2a)^3 = -2a^4$

Total $= \left(\frac{64}{3} - \frac{6}{3}\right)a^4 = \underline{\frac{58}{3}a^4}$

A/50



Without hole,

$$I_x = \frac{1}{12}bh^3 = \frac{1}{12}(50)(200)^3$$
$$= 3.33(10^7)\text{ mm}^4$$

With hole, $I_x' = I_x - (\bar{I}_{hole} + Ay^2)$

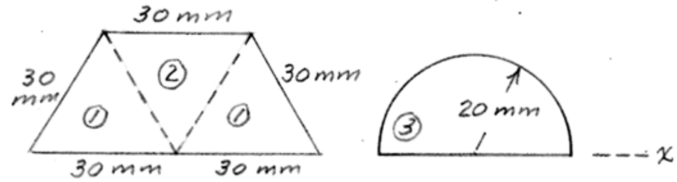
$$I_x' = 3.33(10^7) - \left[\frac{1}{12}(50)(25)^3 + (50)(25)y^2\right]$$
$$= 3.33(10^7) - 1250y^2 \quad (y \text{ in mm}, I_x' \text{ in mm}^4)$$

Percent reduction $n = \dfrac{I_x - I_x'}{I_x}(100)$

$$n = \frac{3.33(10^7) - (3.33(10^7) - 1250y^2)}{3.33(10^7)}(100) = 0.1953 + 0.00375y^2$$
(in percent)

For $y = 50$ mm, $n = 0.1953 + 0.00375(50)^2$
$$= \underline{9.57\ \%}$$

A/51



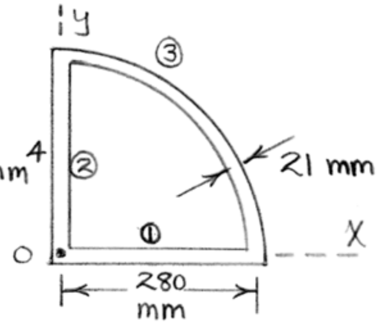
① $I_x = 2\left(\frac{1}{12}\right)(30)\left(30\frac{\sqrt{3}}{2}\right)^3 = \frac{81}{16}\sqrt{3}\,(10^4)$ mm^4

② $I_x = \frac{1}{4}(30)\left(30\frac{\sqrt{3}}{2}\right)^3 = \frac{243}{32}\sqrt{3}\,(10^4)$ mm^4

③ $I_x = -\frac{1}{2}\left(\frac{1}{4}\pi[20]^4\right) = -2\pi(10^4)$ mm^4

Total $I_x = \underline{15.64(10^4)\text{ mm}^4}$

A/52



Parts ① & ② separately:

$I_o = \frac{1}{3}(280 \times 21)280^2 = 153.7(10^6)\text{mm}^4$

$A = 280(21) = 5880 \text{ mm}^2$

Part ③: $I_o = Ar^2 = \frac{\pi(280)}{2}(21)(280)^2 = 724(10^6) \text{ mm}^4$

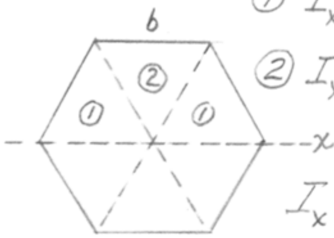
$A = \frac{\pi(280)}{2}(21) = 9240 \text{ mm}^2$

Combined: $I_o = 2(153.7 \cdot 10^6) + 724 \cdot 10^6 = 1.031(10^9) \text{ mm}^4$

$\text{Area} = 2(5880) + 9240 = 21000 \text{ mm}^2$

$k_o = \sqrt{\frac{I_o}{A}} = \sqrt{\frac{1.031(10^9)}{21000}} = \underline{222 \text{ mm}}$

A/53 From Sample Problem A2



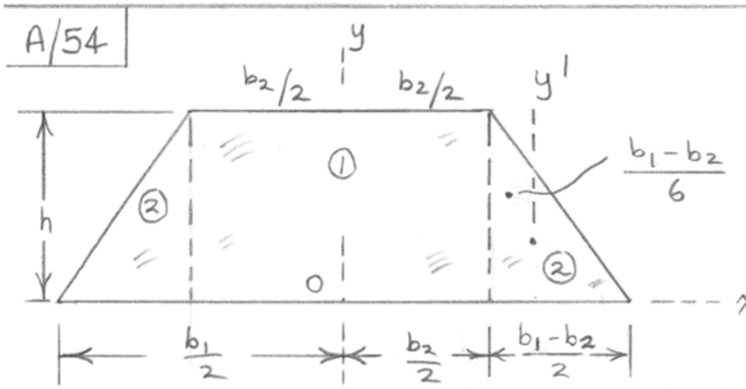
① $I_x = \frac{1}{12} b \left(b\sqrt{3}/2 \right)^3 = \frac{\sqrt{3}}{32} b^4$

② $I_x = \frac{1}{4} b h^3 = \frac{1}{4} b \left(b\sqrt{3}/2 \right)^3 = \frac{3\sqrt{3}}{32} b^4$

$$I_x = 4 I_{①} + 2 I_{②} = \frac{\sqrt{3}}{8} b^4 + \frac{3\sqrt{3}}{16} b^4$$

$$= \underline{\frac{5\sqrt{3}}{16} b^4}$$

A/54



$$I_x = I_{x_1} + 2I_{x_2} = \frac{1}{3} b_2 h^3 + 2\left[\frac{1}{12}\,\frac{b_1 - b_2}{2}\, h^3\right]$$

$$= \underline{h^3\left(\frac{b_1}{12} + \frac{b_2}{4}\right)}$$

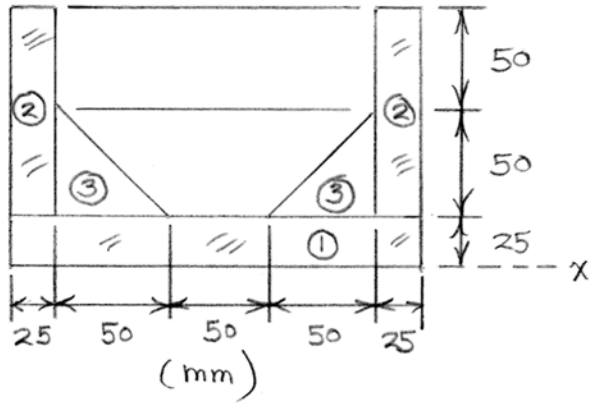
$$I_y = I_{y_1} + 2I_{y_2}$$

$$= \frac{1}{12} h b_2^3 + 2\left[\frac{1}{36} h \left(\frac{b_1 - b_2}{2}\right)^3 + \frac{1}{2} h \left(\frac{b_1 - b_2}{2}\right)\left(\frac{b_2}{2} + \frac{b_1 - b_2}{6}\right)^2\right]$$

$$\vdots$$

$$= \underline{\frac{h}{48}\left(b_1^3 + b_1^2 b_2 + b_1 b_2^2 + b_2^3\right)}$$

A/55



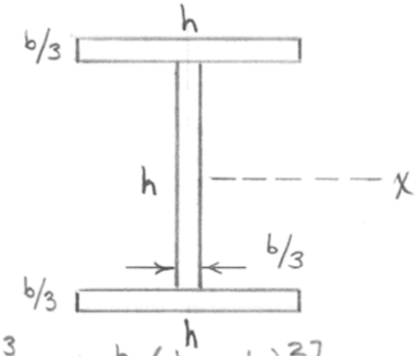
Comp.	A	d_x	$\bar{I}_x$	Ad_x^2
1	$200(25)$	$25/2$	$\frac{1}{12}(200)(25)^3$	781 250
2	$2[100(25)]$	75	$2[\frac{1}{12}(25)(100^3)]$	28 125 000
3	$2[\frac{1}{2}(50)(50)]$	$(25+\frac{50}{3})$	$2[\frac{1}{36}(50)(50^3)]$	4 340 278

$$\begin{cases} \Sigma \bar{I}_x = 4\ 774\ 306 \\ \Sigma A d_x^2 = 33\ 246\ 528 \text{ mm}^4 \end{cases}$$

$$I_x = \Sigma \bar{I}_x + \Sigma A d_x^2 = 38{,}020{,}833 \text{ mm}^4$$

or $\underline{38.0(10^6) \text{ mm}^4}$

A/56



For area (a),

$I_x = \frac{1}{12}bh^3$

For area (b),

$$I_x = \frac{1}{12}\frac{b}{3}h^3 + 2\left[\frac{1}{12}h\left(\frac{b}{3}\right)^3 + h\frac{b}{3}\left(\frac{h}{2}+\frac{b}{6}\right)^2\right]$$

$$= \underline{\frac{hb}{9}\left(\frac{7}{4}h^2 + \frac{2}{9}b^2 + hb\right)}$$

If $h = 200$ mm and $b = 60$ mm, we have

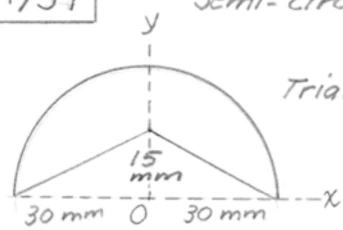
(a) $I_x = \frac{1}{12}(60)(200)^3 = 40(10^6)\text{ mm}^4$

(b) $I_x = \frac{200(60)}{9}\left(\frac{7}{4}(200)^2 + \frac{2}{9}(60)^2 + 200(60)\right)$

$= 110.4(10^6)\text{ mm}^4$

Percent increase $n = \frac{110.4-40}{40}(100\%) = \underline{176.0\%}$

A/57



Semi-circle: $I_z = \frac{1}{4}\pi r^4 = \frac{1}{4}\pi(30)^4$
$= 0.6362(10^6)\ mm^4$

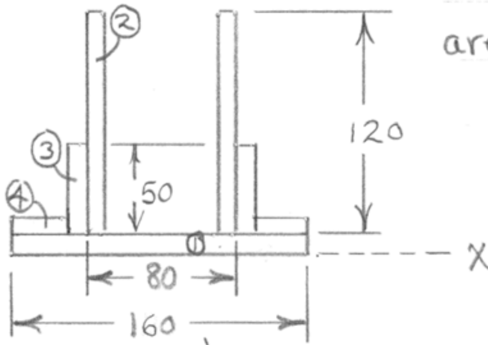
Triangle: $I_x = \frac{-1}{12}bh^3 = \frac{-1}{12}(60)(15)^3$
$= -0.01688(10^6)\ mm^4$

$I_y = -\frac{2}{12}(15)(30)^3$
$= -0.06750(10^6)\ mm^4$

$I_z = I_x + I_y = -(0.01688 + 0.06750)10^6$
$= -0.0844(10^6)\ mm^4$

Total $I_z = (0.6362 - 0.0844)(10^6) = \underline{0.552(10^6)\ mm^4}$

A/58



All thicknesses are 10 mm

(mm, etc.)

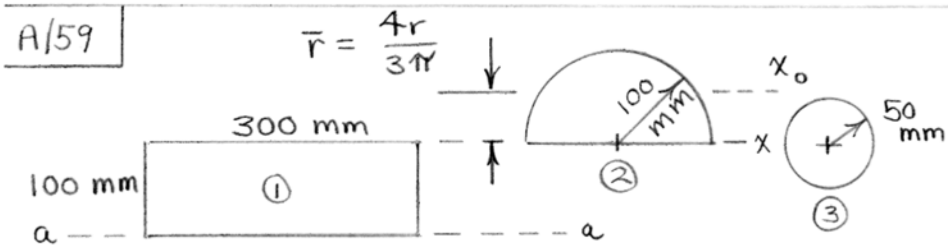
Comp.	A	d_x	Ad_x^2	$\bar{I}_x$
1	$160(10)$	5	40 000	$\frac{1}{12}(160)(10^3)$
2	$2(120)(10)$	70	11 760 000	$2(\frac{1}{12})(10)(120^3)$
3	$2(50)(10)$	35	1 225 000	$2(\frac{1}{12})(10)(50^3)$
4	$2(30)(10)$	15	135 000	$2(\frac{1}{12})(30)(10^3)$

$\Sigma Ad_x^2 = 13.16(10^6)$ mm^4 $\Sigma \bar{I}_x = 3.11(10^6)$ mm^4

$$I_x = \Sigma \bar{I}_x + \Sigma A d_x^2$$

$$= \underline{16.27(10^6) \text{ mm}^4}$$

A/59



Part ①: $I_{a-a} = \frac{1}{3}(300)(100)^3 = 10^8 \text{ mm}^4$

Part ②: $I_{a-a} = I_{x_0} + A\left(100 + \frac{4(100)}{3\pi}\right)^2$

where $I_{x_0} = I_x - A\bar{r}^2 = \frac{1}{8}\pi(100)^4 - \pi\frac{100^2}{2}\left(\frac{4\cdot 100}{3\pi}\right)^2$

$= 10.98(10^6) \text{ mm}^4$

So $I_{a-a} = 10.98(10^6) + \frac{\pi(100)^2}{2}(142.4)^2 = 330(10^6) \text{ mm}^4$

Part ③: $I_{a-a} = I_x + A(100)^2 = \frac{1}{4}\pi(50)^4 + \pi(50)^2(100)^2$

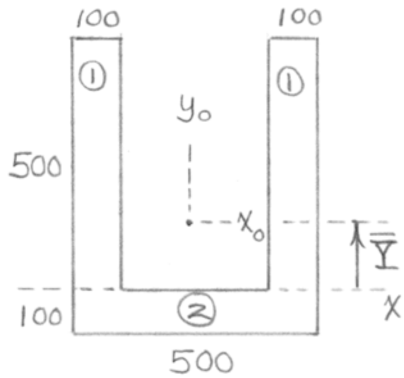
$= 83.4(10^6) \text{ mm}^4$

Combined: $I_{a-a} = (100 + 330 - 83.4)10^6 = \underline{346(10^6) \text{ mm}^4}$

A/60

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{2[(100)(500)(250)] + 500(100)(-50)}{2(100)(500) + 100(500)}$$

$= 150$ mm



$A = 2(100)(500) + 100(500)$

$= 15(10^4)\ \text{mm}^2$

① + ① $I_{x_o} = 2[\frac{1}{12}(100)(500)^3 + 100(500)(250-150)^2]$

$= 30.8(10^8)\ \text{mm}^4$

$I_{y_o} = 2[\frac{1}{12}(500)(100)^3 + 100(500)(150+50)^2] = 40.8(10^8)\ \text{mm}^4$

② $I_{x_o} = \frac{1}{12}(500)(100)^3 + 100(500)(50+150)^2 = 20.4(10^8)\ \text{mm}^4$

$I_{y_o} = \frac{1}{12}(100)(500)^3 = 10.42(10^8)\ \text{mm}^4$

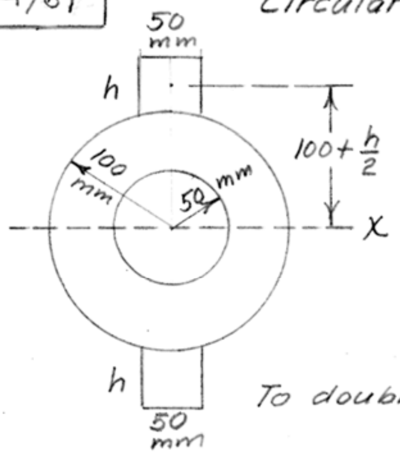
Totals ① + ① + ②: $I_{x_o} = 51.2(10^8)\ \text{mm}^4$

$I_{y_o} = 51.2(10^8)\ \text{mm}^4$

$I_C = I_{x_o} + I_{y_o} = 102.5(10^8)\ \text{mm}^4$

$k_C = \sqrt{I_C/A} = \sqrt{\dfrac{102.5(10^8)}{15(10^4)}} = \underline{261\ \text{mm}}$

A/61



Circular section $I_x = \frac{\pi}{4}(r_2^4 - r_1^4)$

$$I_x = \frac{\pi}{4}(100^4 - 50^4) = 73.63(10^6) \text{ mm}^4$$

Rectangular strips

$$I_x = \bar{I}_x + Ad^2$$

$$= 2\left[\frac{50}{12}h^3 + 50h\left(100 + \frac{h}{2}\right)^2\right]$$

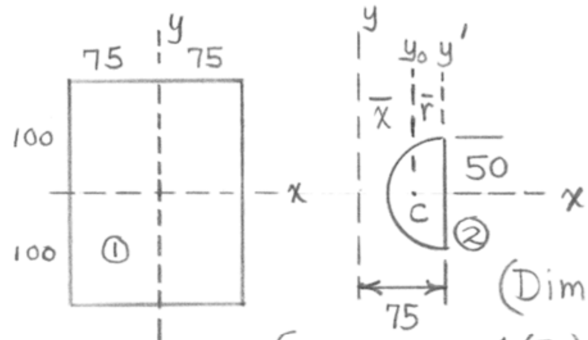
$$= 100\left(\frac{h^3}{3} + 100h^2 + [100]^2 h\right)$$

To double stiffness $(I_x)_\square = (I_x)_o$

so $100\left(h^3/3 + 100h^2 + [100]^2 h\right) = 73.63(10^6)$

Solve cubic & get $\underline{h = 47.5 \text{ mm}}$

A/62



(Dim. in mm)

$$\left\{\begin{array}{l}\bar{r} = \frac{4r}{3\pi} = \frac{4(50)}{3\pi} = 21.2\text{ mm} \\ \bar{x} = 75 - 21.2 = 53.8\text{ mm}\end{array}\right.$$

Part I: $I_x = \frac{1}{12}(150)(200)^3 = 100(10^6)\text{ mm}^4$

$I_y = \frac{1}{12}(200)(150)^3 = 56.2(10^6)\text{ mm}^4$

Parts II: $I_x = \frac{1}{4}\pi(50)^4 = 4.91(10^6)\text{ mm}^4$ (for both together)

$$I_y = I_{y_0} + A\bar{x}^2 = I_{y'} - A\bar{r}^2 + A\bar{x}^2 = I_{y'} + A(\bar{x}^2 - \bar{r}^2)$$

$$= \frac{1}{2}\left(\frac{1}{4}\pi\, 50^4\right) + \frac{\pi(50)^2}{2}(53.8^2 - 21.2^2)$$

$= 12.04(10^6)\text{ mm}^4$ for each, $24.1(10^6)\text{ mm}^4$ for both

Combined: $I_x = 100(10^6) - 4.91(10^6) = \underline{95.1(10^6)\text{ mm}^4}$

$I_y = 56.2(10^6) - 24.1(10^6) = \underline{32.2(10^6)\text{ mm}^4}$

A/63 $A = (30)(60) = 1800 \text{ mm}^2$ for each area.

$\bar{I}_{xy} = 0$ for each area, so $I_{xy} = 0 + A d_x d_y$.

(a) $I_{xy} = 50(40)(1800) = 360(10^4) \text{ mm}^4$

(b) $I_{xy} = 50(-40)(1800) = -360(10^4) \text{ mm}^4$

(c) $I_{xy} = (-50)(10)(1800) = \underline{-90(10^4) \text{ mm}^4}$

A/64

$$I_{xy} = -75(75)[(-87.5)(87.5) + (87.5)(-87.5)]$$
$$= \underline{86.1(10^6)\ mm^4}$$

A/65

(a) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)(40)(80)(50)$
$= \underline{9.60(10^6)\ mm^4}$

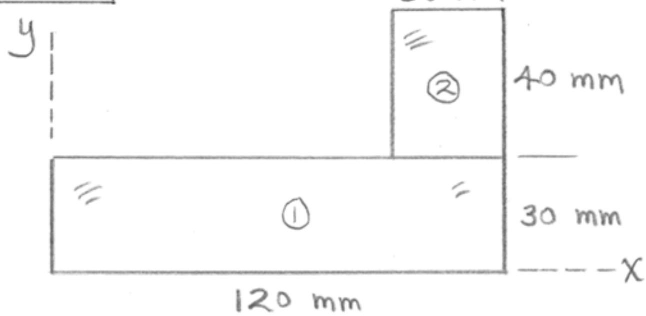
(b) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(40)(\pi \cdot 25^2)$
$= \underline{-4.71(10^6)\ mm^4}$

(c) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(-40)(80)(50)$
$= \underline{9.60(10^6)\ mm^4}$

(d) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)\left(-40 - \frac{4(25)}{3\pi}\right)$
$\times (\pi \cdot 25^2)/2$

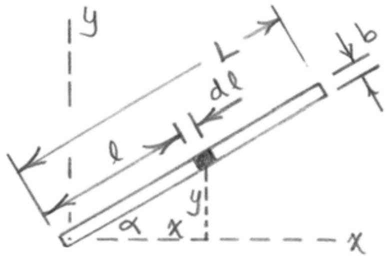
$= \underline{-2.98(10^6)\ mm^4}$

A/66



$$I_{xy_1} = d_x d_y A = 15(60)(120)(30)$$
$$= 3.24(10^6)\ \text{mm}^4$$
$$I_{xy_2} = d_x d_y A = 50(105)(30)(40)$$
$$= 6.30(10^6)\ \text{mm}^4$$
Total: $\underline{I_{xy} = 9.54(10^6)\ \text{mm}^4}$

A/67

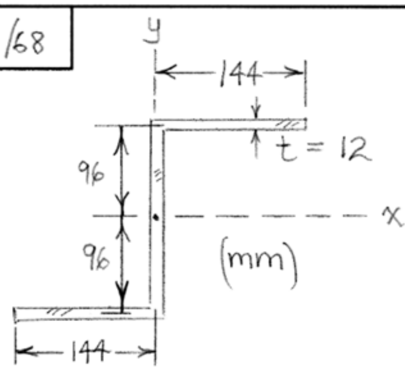


$$I_{xy} = \int xy\,dA = \int_0^L (\ell\cos\alpha)(\ell\sin\alpha)\,b\,d\ell$$

$$= b\sin\alpha\cos\alpha\int_0^L \ell^2 d\ell$$

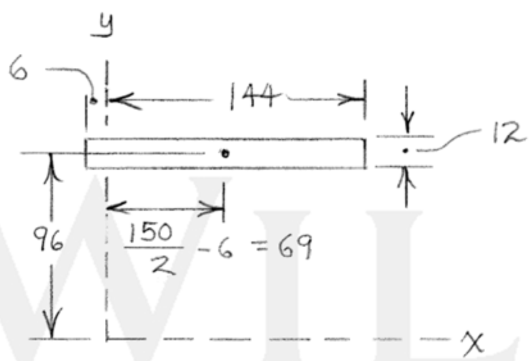
$$= \frac{bL^3}{3}\sin\alpha\cos\alpha \quad \text{or} \quad \underline{\frac{1}{6}bL^3\sin 2\alpha}$$

A/68

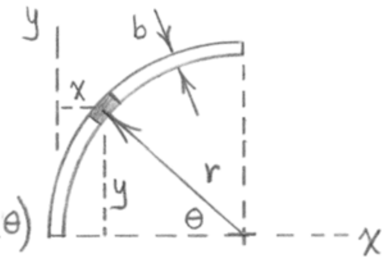


$$I_{xy} = 2 d_x d_y A = 2(96)(69)(150)(12)$$
$$= \underline{23.8(10^6)\ mm^4}$$

One of two areas considered:



A/69



$$I_{xy} = \int xy\,dA$$

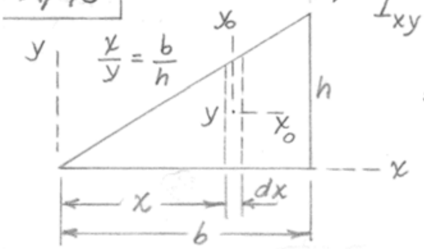
$$= \int_0^{\pi/2} (r - r\cos\theta)\, r\sin\theta\, (b r\, d\theta)$$

$$= br^3 \int_0^{\pi/2} (\sin\theta - \sin\theta\cos\theta)\, d\theta$$

$$= br^3 \left[-\cos\theta + \frac{1}{4}\cos 2\theta \right]_0^{\pi/2}$$

$$= br^3 \left[0 - \frac{1}{4} + 1 - \frac{1}{4} \right] = \underline{br^3/2}$$

A/70



$\frac{x}{y} = \frac{b}{h}$

First: $I_{xy} = \int_0^b \int_0^y xy\,dy\,dx$

$= \int_0^b \frac{x}{2} y^2 \Big]_0^{\frac{hx}{b}} dx = \int_0^b \frac{h^2}{2b^2} x^3 dx$

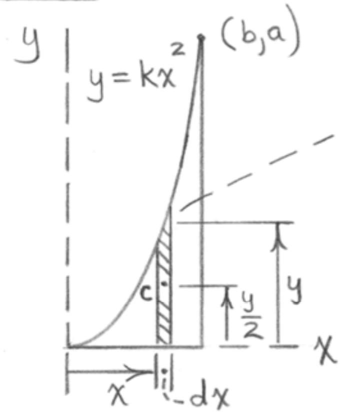
$= \frac{h^2}{2b^2}\frac{b^4}{4} = \underline{\frac{b^2h^2}{8}}$

Second: $dI_{xy} = dI_{x_0y_0} + d_x d_y (dA)$

$= 0 + \frac{y}{2} x (y\,dx) = \frac{h^2}{2b^2} x^3 dx$

$I_{xy} = \frac{h^2}{2b^2}\int_0^b x^3 dx = \frac{h^2}{2b^2}\frac{b^4}{4} = \underline{\frac{b^2h^2}{8}}$

A/71



$y = kx^2: \quad a = kb^2, \quad k = a/b^2$

$\Rightarrow y = \frac{a}{b^2}x^2$

$dA = y\,dx = \frac{a}{b^2}x^2\,dx$

$d\bar{I}_{xy} = 0$

$dI_{xy} = d\bar{I}_{xy} + dA\,(x)\left(\frac{y}{2}\right)$

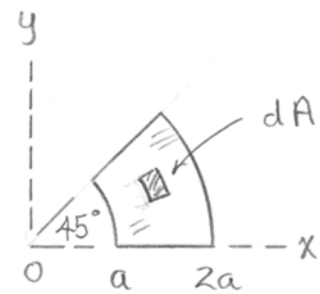
$= 0 + y\,dx\,(x)\left(\frac{y}{2}\right)$

$= \frac{1}{2}\frac{a^2}{b^4}x^5\,dx$

$I_{xy} = \int dI_{xy} = \int_0^b \frac{1}{2}\frac{a^2}{b^4}x^5\,dx$

$= \frac{1}{2}\frac{a^2}{b^4}\left.\frac{x^6}{6}\right|_0^b = \underline{\frac{1}{12}a^2b^2}$

A/72

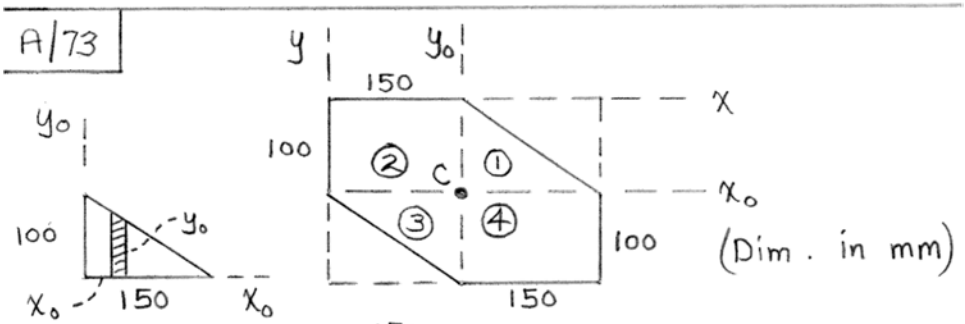


$$I_{xy} = \int xy\,dA = \iint (r\cos\theta)(r\sin\theta)\,r\,dr\,d\theta$$

$$= \int_0^{\pi/4}\int_a^{2a} r^3 \cos\theta \sin\theta \, d\theta\, dr = \frac{15}{4}a^4 \int_0^{\pi/4} \cos\theta \sin\theta \, d\theta$$

$$= \frac{15}{4}a^4 \left.\frac{\sin^2\theta}{2}\right|_0^{\pi/4} = \underline{\frac{15a^4}{16}}$$

A/73



Part ①: $I_{x_0y_0} = \int_0^{150} x_0 \frac{y_0}{2}(y_0\,dx_0)$, $y_0 = 100 - \frac{2}{3}x_0$

$$I_{x_0y_0} = \frac{1}{2}\int_0^{150} x_0\left(10^4 - \frac{4}{3}100x_0 + \frac{4}{9}x_0^2\right)dx_0$$

$$= \frac{1}{2}\left[\frac{10^4}{2}x_0^2 - \frac{4}{9}100x_0^3 + \frac{1}{9}x_0^4\right]_0^{150} = 9.38(10^6)\text{ mm}^4$$

Part ③: $I_{x_0y_0} = 9.38(10^6)\text{ mm}^4$

Part ②: $I_{x_0y_0} = 100(150)(-75)(50) = -56.3(10^6)\text{ mm}^4$

Part ④: $I_{x_0y_0} = -56.3(10^6)\text{ mm}^4$

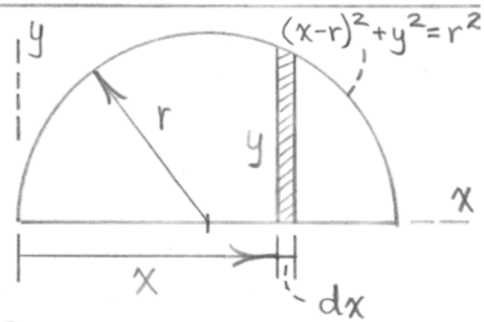
Combined: $I_{x_0y_0} = 2[9.38 - 56.3]10^6 = -93.8(10^6)\text{ mm}^4$

Combined area $= 2(100)(150) + 2(\frac{1}{2})(100)(150) = 45\,000\text{ mm}^2$

So $I_{xy} = I_{x_0y_0} + A d_x d_y = -93.8(10^6) + 45\,000(150)(-100)$

$$= \underline{-769(10^6)\text{ mm}^4}$$

A/74



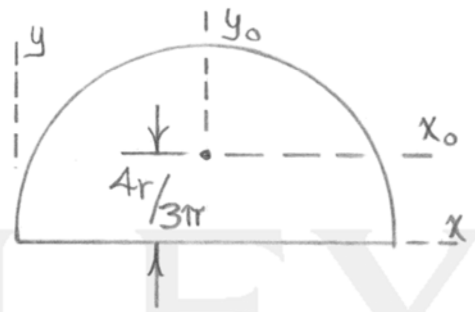
(1) By direct integration

For elemental strip,

$$dI_{xy} = x\frac{y}{2}dA = \frac{xy}{2}y\,dx$$
$$= \frac{x}{2}\left[r^2 - (x-r)^2\right]dx$$

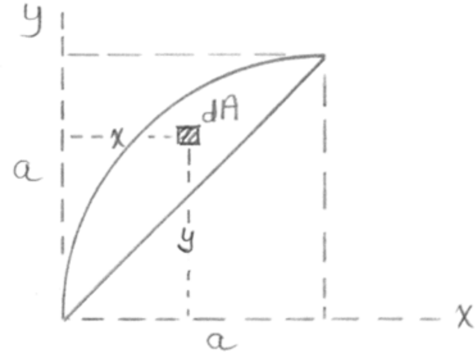
$$I_{xy} = \frac{1}{2}\int_0^{2r}(xr^2 - x^3 + 2rx^2 - r^2x)\,dx$$
$$= \frac{1}{2}\left[\frac{x^2r^2}{2} - \frac{x^4}{4} + \frac{2rx^3}{3} - \frac{r^2x^2}{2}\right]_0^{2r} = \underline{\frac{2}{3}r^4}$$

(2) By axis transfer



$$I_{xy} = I_{x_0y_0} + A d_x d_y = 0 + \frac{\pi r^2}{2}(r)\left(\frac{4r}{3\pi}\right) = \underline{\frac{2}{3}r^4}$$

A/75



$$I_{xy} = \int xy\, dA = \iint xy\, dx\, dy = \int_0^a y \left[\int_{x_1}^{x_2} x\, dx \right] dy$$

where $x_2 = y$ and $(x_1 - a)^2 + y^2 = a^2$

$x_1 = a - \sqrt{a^2 - y^2}$

(note minus sign)

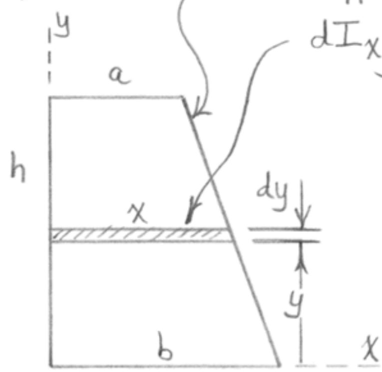
$$\int_{x_1}^{x_2} x\, dx = \left.\frac{x^2}{2}\right|_{a-\sqrt{a^2-y^2}}^{y} = y^2 - a^2 + a\sqrt{a^2-y^2}$$

So
$$I_{xy} = \int_0^a y\left(y^2 - a^2 + a\sqrt{a^2-y^2}\right) dy = \left[\frac{y^4}{4} - \frac{a^2y^2}{2} - \frac{a}{3}\sqrt{(a^2-y^2)^3}\right]_0^a = \frac{a^4}{12}$$

Alternatively

$$I_{xy} = (I_{xy})_{①} - (I_{xy})_{②}$$

A/76



$$x = \left(\frac{a-b}{h}\right)y + b$$

$$dI_{xy} = x\,dy\left(\frac{x}{2}\right)(y) = x^2\frac{y}{2}\,dy$$

$$= \left[\left(\frac{a-b}{h}\right)y + b\right]^2\frac{y}{2}\,dy$$

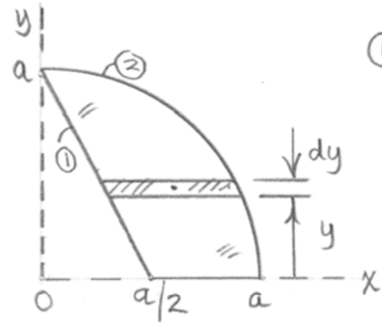
$$= \frac{1}{2}\left[\left(\frac{a-b}{h}\right)^2 y^3 + 2\left(\frac{a-b}{h}\right)by^2 + b^2 y\right]dy$$

$$I_{xy} = \int dI_{xy} = \frac{1}{2}\int_0^h \left[\left(\frac{a-b}{h}\right)^2 y^3 + 2\left(\frac{a-b}{h}\right)by^2 + b^2 y\right]dy$$

$$= \frac{1}{2}\left[\left(\frac{a-b}{h}\right)^2\frac{y^4}{4} + 2\left(\frac{a-b}{h}\right)b\frac{y^3}{3} + b^2\frac{y^2}{2}\right]_0^h$$

$$= \underline{\frac{h^2}{4}\left(\frac{a^2}{2} + \frac{ab}{3} + \frac{b^2}{6}\right)} \text{ or } \underline{\frac{h^2}{24}\left(3a^2 + 2ab + b^2\right)}$$

A/77



(2): $x^2 + y^2 = a^2$

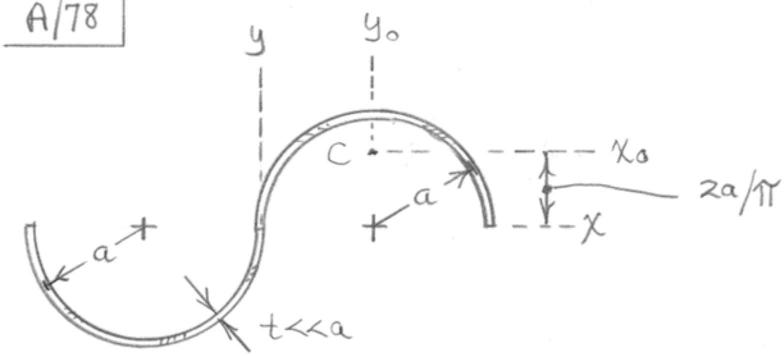
(1): $y = -2x + a$

$$dA = (x_2 - x_1)\,dy$$
$$= \left[\sqrt{a^2 - y^2} - \left(\frac{a-y}{2}\right)\right] dy$$

$$dI_{xy} = d_x d_y\, dA = y\left(\frac{x_1 + x_2}{2}\right)(x_2 - x_1)\,dy$$
$$= \frac{y}{2}\left(x_2^2 - x_1^2\right) dy$$
$$= \frac{y}{2}\left[a^2 - y^2 - \left(\frac{a-y}{2}\right)^2\right] dy$$
$$= \frac{y}{2}\left[a^2 - y^2 - \frac{1}{4}\left(a^2 - 2ay + y^2\right)\right] dy$$
$$= \frac{1}{4}\left(\frac{3}{2}a^2 y + ay^2 - \frac{5}{2}y^3\right) dy$$

$$I_{xy} = \int dI_{xy} = \frac{1}{4}\int_0^a \left(\frac{3}{2}a^2 y + ay^2 - \frac{5}{2}y^3\right) dy$$
$$= \frac{1}{4}\left(\frac{3}{4}a^2 y^2 + \frac{1}{3}ay^3 - \frac{5}{8}y^4\right)\Big|_0^a$$
$$= \underline{\frac{11a^4}{96}}$$

A/78



Consider the right half for now:

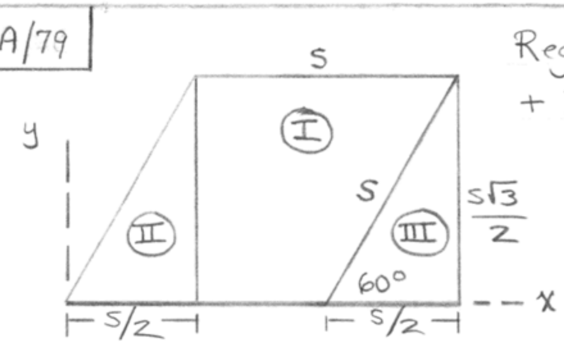
$\bar{I}_{xy} = 0$, by symmetry

$I_{xy} = \bar{I}_{xy} + d_x d_y A$

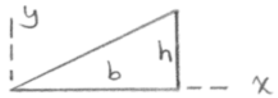
$= 0 + \left(\frac{2a}{\pi}\right)(a)(\pi a t) = 2a^3 t$

For entire body, $\underline{I_{xy} = 4a^3 t}$

A/79



Regard as I (rectangle) + II − III (triangles)



Use result $I_{xy} = \frac{b^2h^2}{8}$ from Prob. A/69.

$$\bar{I}_{xy} = I_{xy} - Ad_x d_y = \frac{b^2h^2}{8} - \frac{bh}{2}\left(\frac{2b}{3}\right)\left(\frac{h}{3}\right) = \frac{1}{72}b^2h^2$$

$$I_{I} = \bar{I}_{xy} + Ad_x d_y = 0 + s\frac{s\sqrt{3}}{2}(s)\left(\frac{s\sqrt{3}}{4}\right) = \frac{3}{8}s^4$$

$$I_{II} = \frac{1}{8}\left(\frac{s}{2}\right)^2\left(\frac{s\sqrt{3}}{2}\right)^2 = \frac{3}{128}s^4$$

$$I_{III} = \bar{I}_{xy} + Ad_x d_y = \frac{1}{72}\left(\frac{s}{2}\right)^2\left(\frac{s\sqrt{3}}{2}\right)^2 + \frac{1}{2}\left(\frac{s}{2}\right)\left(\frac{s\sqrt{3}}{2}\right)\left[s + \frac{2}{3}\frac{s}{2}\right]\left[\frac{1}{3}\frac{s\sqrt{3}}{2}\right]$$

$$= \frac{11}{128}s^4$$

$$I_{xy} = I_{I} + I_{II} - I_{III} = s^4\left[\frac{3}{8} + \frac{3}{128} - \frac{11}{128}\right]$$

$$= \underline{\frac{5}{16}s^4}$$

A/80 $I_x = \frac{1}{3} b(b^3) = \frac{1}{3} b^4$; $I_y = \frac{1}{3} b^4$

$I_{xy} = 0 + \frac{b}{2}\frac{b}{2} b^2 = \frac{1}{4} b^4$

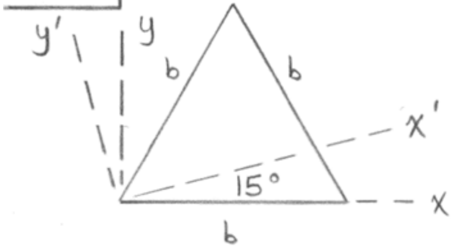
with $\theta = 30°$, Eqs. A/9 & A/9a give

$I_{x'} = \frac{b^4}{3} + 0 - \frac{1}{4} b^4 \sin 60° = \left(\frac{1}{3} - \frac{\sqrt{3}}{8}\right) b^4 = \underline{0.1168 b^4}$

$I_{y'} = \frac{b^4}{3} + 0 + \frac{1}{4} b^4 \sin 60° = \left(\frac{1}{3} + \frac{\sqrt{3}}{8}\right) b^4 = \underline{0.5498 b^4}$

$I_{x'y'} = 0 + \frac{b^4}{4}\frac{1}{2} = \frac{b^4}{8} = \underline{0.1250 b^4}$

A/81



$$I_x = \frac{1}{12} b \left(\frac{b\sqrt{3}}{2}\right)^3 = \frac{\sqrt{3}}{32} b^4$$

$$\bar{I}_y = \bar{I}_x = \frac{\sqrt{3}}{32} b^4 - \frac{1}{2} b^2 \frac{\sqrt{3}}{2} \times \left(\frac{1}{3} b \frac{\sqrt{3}}{2}\right)^2 = \frac{\sqrt{3}}{96} b^4$$

$$I_y = \bar{I}_y + A d_y^2 = \frac{\sqrt{3}}{96} b^4 + b\left(\frac{b\sqrt{3}}{4}\right)\left(\frac{b}{2}\right)^2 = \frac{7\sqrt{3}}{96} b^4$$

$$I_{xy} = \bar{I}_{xy} + A d_x d_y = 0 + b\left(\frac{b\sqrt{3}}{4}\right)\left(\frac{b}{2}\right)\left(\frac{1}{3}\frac{\sqrt{3}}{2} b\right) = \frac{3}{48} b^4$$

Eqs. A/9 & A/9a, with $\theta = 15°$:

$$I_{x'} = \frac{5\sqrt{3}}{96} b^4 - \frac{\sqrt{3}}{48} b^4 \cos 30° - \frac{3}{48} b^4 \sin 30° = \underline{0.0277\, b^4}$$

$$I_{y'} = \frac{5\sqrt{3}}{96} b^4 + \frac{\sqrt{3}}{48} b^4 \cos 30° + \frac{3}{48} b^4 \sin 30° = \underline{0.1527\, b^4}$$

$$I_{x'y'} = -\frac{\sqrt{3}}{48} b^4 \sin 30° + \frac{3}{48} b^4 \cos 30° = \underline{0.0361\, b^4}$$

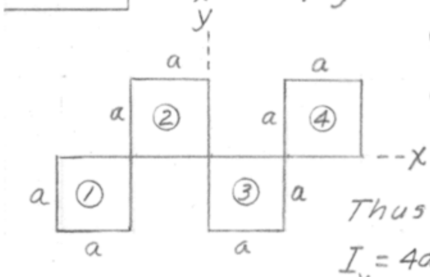
A/82

① $I_x = a^4/3$, $I_y = a^4/12 + a^2(3a/2)^2 = 7a^4/3$; $I_{xy} = +3a^4/4$

② $I_x = a^4/3$, $I_y = a^4/3$, $I_{xy} = -a^4/4$

③ same as ②

④ same as ①



Thus for composite area

$I_x = 4a^4/3$, $I_y = 16a^4/3$, $I_{xy} = +a^4$

From Eq. A/11,

$$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2} = \left(\frac{10}{3} + \sqrt{5}\right)a^4 = \underline{5.57a^4}$$

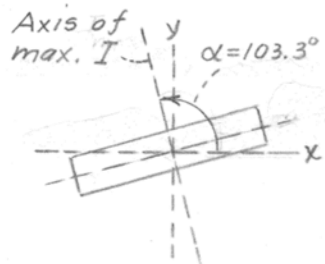
$$I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2} = \left(\frac{10}{3} - \sqrt{5}\right)a^4 = \underline{1.097a^4}$$

From Eq. A/10

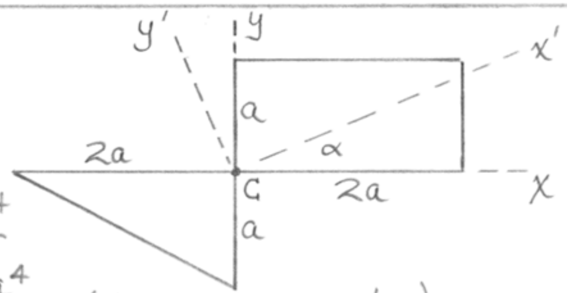
$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2a^4}{(16/3 - 4/3)a^4} = +\frac{1}{2}$$

$2\alpha = 26.6°$ or $206.6°$

$\alpha = 13.3°$ or $\underline{103.3°}$



A/83



For the triangle,

$I_x = \frac{1}{12}(2a)(a^3) = \frac{a^4}{6}$

$I_y = \frac{1}{12}(a)(2a)^3 = \frac{2a^4}{3}$

$I_{xy} = \frac{1}{24}(2a)^2 a^2 = \frac{a^4}{6}$ (from Prob. A/68)

For the rectangle, $I_x = \frac{1}{3}(2a^2)a^2 = \frac{2a^4}{3}$

$I_y = \frac{1}{3}(2a^2)(2a)^2 = \frac{8a^4}{3}$, $I_{xy} = 2a^2\left(\frac{a}{2}\right)(a) = a^4$

Totals: $I_x = \frac{5a^4}{6}$, $I_y = \frac{10a^4}{3}$, $I_{xy} = \frac{7a^4}{6}$

Eqs. A/11:

$$I_{max} = \frac{\frac{5}{6}+\frac{10}{3}}{2}a^4 + \frac{1}{2}\sqrt{\left(\frac{5}{6}-\frac{10}{3}\right)^2 a^8 + 4\left(\frac{7}{6}\right)^2 a^8} = \underline{3.79a^4}$$

$$I_{min} = \frac{\frac{5}{6}+\frac{10}{3}}{2}a^4 - \frac{1}{2}\sqrt{\left(\frac{5}{6}-\frac{10}{3}\right)^2 a^8 + 4\left(\frac{7}{6}\right)^2 a^8} = \underline{0.373a^4}$$

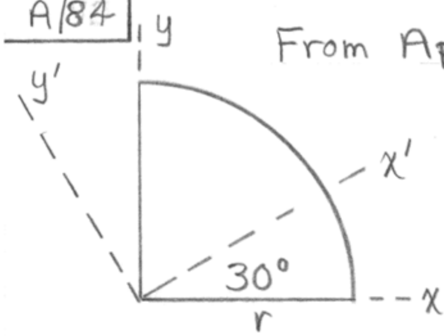
Eq. A/10:

$$\tan 2\alpha = \frac{2\left(\frac{7}{6}a^4\right)}{\frac{10}{3}a^4 - \frac{5}{6}a^4} = \frac{14}{15}, \quad 2\alpha = 43.0° \text{ or } 223°$$

$\alpha = 21.5°$ or $111.5°$

$\underline{\alpha = 111.5°}$ for axis of I_{max}

A/84



From Appendix D: $I_x = I_y = \frac{\pi r^4}{16}$

From Prob. A/67:

$$I_{xy} = \frac{r^4}{8}$$

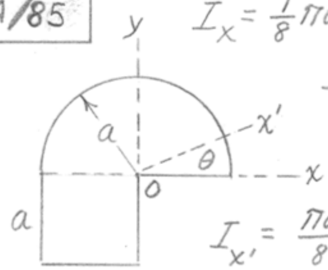
Eq. A/9: $I_{x'} = \frac{\pi r^4}{16} + 0 - \frac{r^4}{8}\sin 60° = \underline{\frac{r^4}{16}\left[\pi - \sqrt{3}\right]}$

$I_{y'} = \frac{\pi r^4}{16} - 0 + \frac{r^4}{8}\sin 60° = \underline{\frac{r^4}{16}\left[\pi + \sqrt{3}\right]}$

Eq. A/9a:

$$I_{x'y'} = 0 + \frac{r^4}{8}\cos 60° = \underline{\frac{r^4}{16}}$$

A/85



$I_x = \frac{1}{8}\pi a^4 + \frac{1}{3}a^4, \quad I_y = \frac{1}{8}\pi a^4 + \frac{1}{3}a^4 = I_x$

$I_{xy} = 0 + a^2(-\frac{a}{2})(-\frac{a}{2}) = \frac{a^4}{4}$

From Eq. A/9

$I_{x'} = \frac{\pi a^4}{8} + \frac{a^4}{3} + 0 - \frac{a^4}{4}\sin 2\theta$

$= a^4(\frac{\pi}{8} + \frac{1}{3} - \frac{1}{4}\sin 2\theta)$

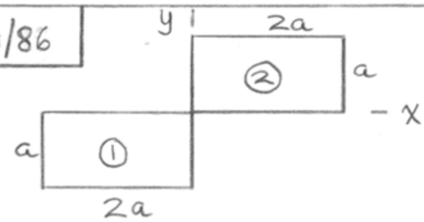
$\frac{dI_{x'}}{d\theta} = a^4(-\frac{1}{2}\cos 2\theta) = 0$ for max. or min.

so $2\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ so $\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

$\underline{\theta = \pi/4}$, $I_{x'} = a^4(\frac{\pi}{8} + \frac{1}{3} - \frac{1}{4}[1]) = \underline{0.476a^4}$ (min)

$I_{y'} = a^4(\frac{\pi}{8} + \frac{1}{3} + \frac{1}{4}[1]) = \underline{0.976a^4}$ (max)

A/86



① $I_x = \frac{1}{3}(2a)(a^3) = \frac{2}{3}a^4, \quad I_y = \frac{1}{3}(a)(2a)^3 = \frac{8}{3}a^4$

$I_{xy} = (2a^2)(a)(a/2) = a^4$

② $I_x = \frac{2}{3}a^4, \quad I_y = \frac{8}{3}a^4, \quad I_{xy} = a^4$

Eq. A/11: $I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$

$= \frac{10}{3}a^4 - \frac{1}{2}\sqrt{(-4a^4)^2 + 16a^8} = \underline{0.505a^4}$

$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x^2 - I_y^2) + 4I_{xy}^2}$

$= \frac{10}{3}a^4 + \frac{1}{2}\sqrt{(-4a^4)^2 + 16a^8} = \underline{6.16a^4}$

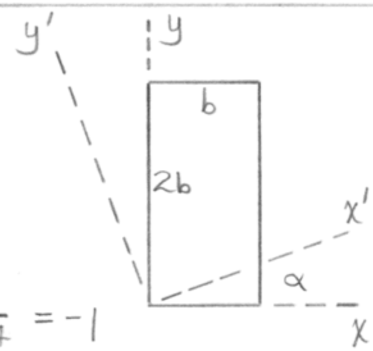
Eq. A/10: $\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{4a^4}{\left(\frac{16}{3} - \frac{4}{3}\right)a^4}$

$2\alpha = 45°$ or $225°$

$\alpha = 22.5°$ for I_{min}

or $\alpha = \underline{112.5°}$ for I_{max}

A/87



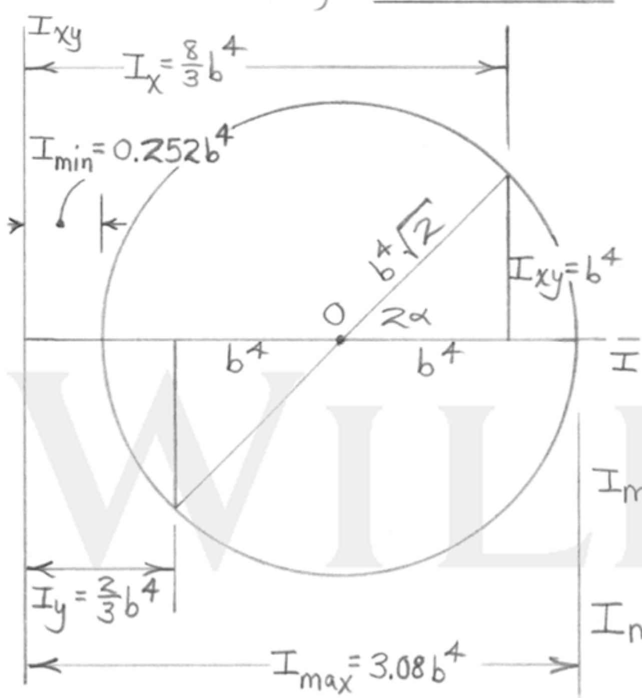
$I_x = \frac{1}{3} 2b^2 (2b)^2 = \frac{8}{3} b^4$

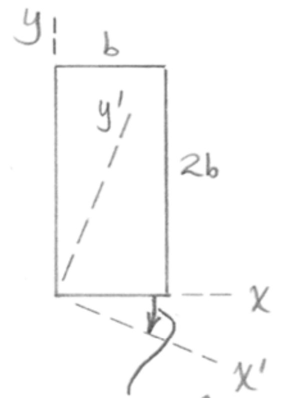
$I_y = \frac{1}{3} 2b^2 (b)^2 = \frac{2}{3} b^4$

$I_{xy} = 2b^2 \left(\frac{b}{2}\right)(b) = b^4$

$\tan 2\alpha = \dfrac{2I_{xy}}{I_y - I_x} = \dfrac{2b^4}{\frac{2}{3}b^4 - \frac{8}{3}b^4} = -1$

$\Rightarrow 2\alpha = -45°, \quad \underline{\alpha = -22.5°}$





$\alpha = -22.5°$

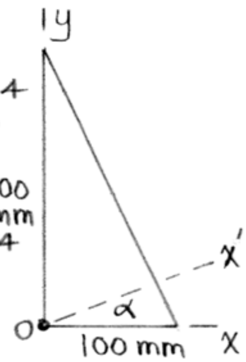
$I_{max} = \frac{5}{3}b^4 + b^4\sqrt{2}$

$= \underline{3.08\, b^4}$

$I_{min} = \frac{5}{3}b^4 - b^4\sqrt{2}$

$= \underline{0.252\, b^4}$

A/88



$$I_x = \frac{1}{12}bh^3 = \frac{1}{12}(100)(200)^3 = 66.7(10^6)\ \text{mm}^4$$

$$I_y = \frac{1}{12}(200)(100)^3 = 16.67(10^6)\ \text{mm}^4$$

$$I_{xy} = \frac{b^2h^2}{24} = \frac{1}{24}(100)^2(200)^2 = 16.67(10^6)\ \text{mm}^4$$

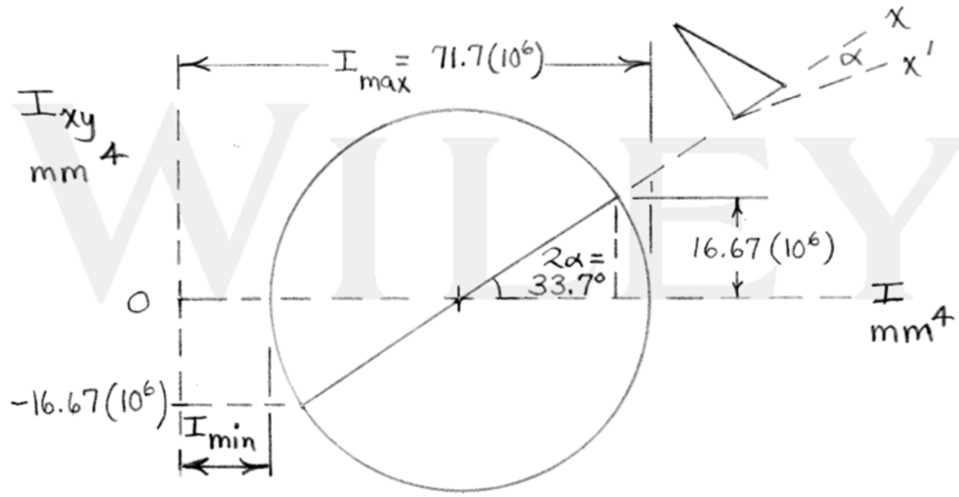
Eq. A/11:

$$I_{max} = 10^6\left\{\frac{66.7+16.67}{2} + \frac{1}{2}\sqrt{(66.7-16.67)^2 + 4(16.67)^2}\right\}$$

$$10^6\{41.67 + 30.0\} = \underline{71.7(10^6)\ \text{mm}^4}$$

Eq. A/10: $\tan 2\theta_{cr} = \tan 2\alpha = \dfrac{2\,(16.67)}{16.67-66.7}$

$$\underline{\alpha = -16.85^\circ}$$



A/89

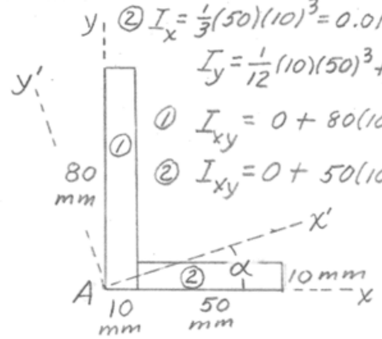
① $I_x = \frac{1}{3}(10)(80)^3 = 1.707(10^6)\ mm^4$, $I_y = \frac{1}{3}(80)(10)^3 = 0.0267(10^6)\ mm^4$

② $I_x = \frac{1}{3}(50)(10)^3 = 0.0167(10^6)\ mm^4$,

$I_y = \frac{1}{12}(10)(50)^3 + 10(50)(35)^2 = 0.7167(10^6)\ mm^4$

① $I_{xy} = 0 + 80(10)(40)(5) = 0.1600(10^6)\ mm^4$

② $I_{xy} = 0 + 50(10)(35)(5) = 0.0875(10^6)\ mm^4$



Totals: $I_x = 1.723(10^6)\ mm^4$

$I_y = 0.743(10^6)\ mm^4$

$I_{xy} = 0.248(10^6)\ mm^4$

From Eqs. A/11

$$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2},\quad I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$I_{max} = \left[\frac{1.723 + 0.743}{2} + \frac{1}{2}\sqrt{(1.723 - 0.743)^2 + 4(0.248)^2}\right]10^6 = \underline{1.782(10^6)\ mm^4}$$

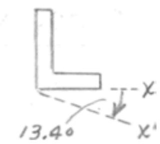
$$I_{min} = \left[\frac{1.723 + 0.743}{2} - \frac{1}{2}\sqrt{(1.723 - 0.743)^2 + 4(0.248)^2}\right]10^6 = \underline{0.684(10^6)\ mm^4}$$

From Eq. A/10

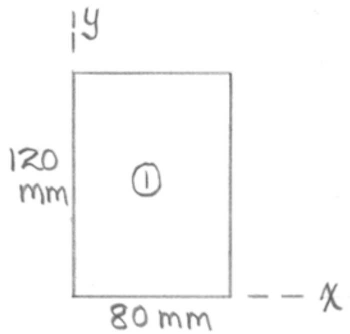
$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(0.248)}{0.743 - 1.723} = -0.5051$$

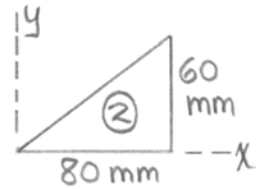
$2\alpha = -26.8°$

$\underline{\alpha = -13.40°}$



*A/90





① $I_x = \frac{1}{3}(80)(120)^3 = 46.1(10^6)\ \text{mm}^4$

$I_y = \frac{1}{3}(120)(80)^3 = 20.5\ (10^6)\ \text{mm}^4$

$I_{xy} = 80(120)(40)(60) = 23.0(10^6)\ \text{mm}^4$

② $I_x = -\frac{1}{12}(80)(60)^3 = -1.440\ (10^6)\ \text{mm}^4$

$I_y = -\frac{1}{4}(60)(80)^3 = -7.68\ (10^6)\ \text{mm}^4$

$I_{xy} = -\frac{b^2h^2}{8} = -\frac{80^2 60^2}{8} = -2.88(10^6)\ \text{mm}^4$

(See Prob. A/72)

So for the composite body:

$$\begin{cases} I_x = (46.1 - 1.440)\,10^6 = 44.6(10^6)\ \text{mm}^4 \\ I_y = (20.5 - 7.68)\,10^6 = 12.80\ (10^6)\ \text{mm}^4 \\ I_{xy} = (23.0 - 2.88)\,10^6 = 20.2\ (10^6)\ \text{mm}^4 \end{cases}$$

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

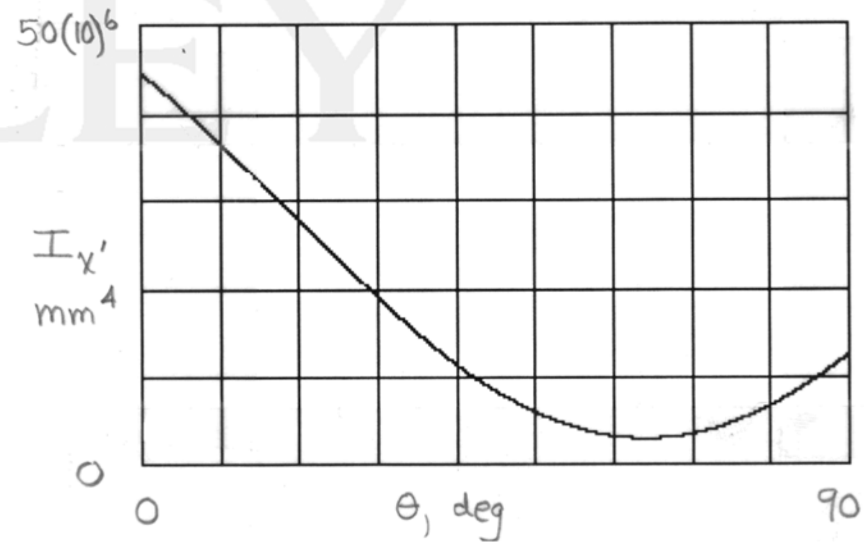
(See plot below)

$$I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

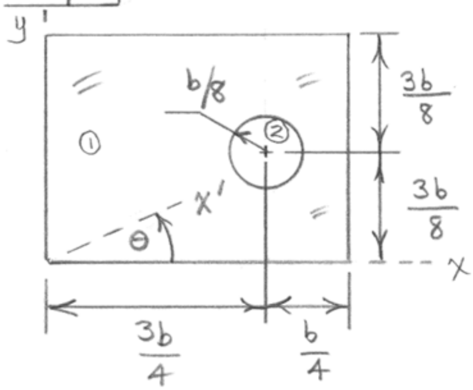
$$= \left\{\frac{44.6 + 12.80}{2} - \frac{1}{2}\sqrt{(44.6 - 12.80)^2 + 4(20.2)^2}\right\}10^6$$

$$= \underline{3.03\ (10^6)\ \text{mm}^4}$$

$$\tan 2\theta_{cr} = \frac{2I_{xy}}{I_y - I_x} = \frac{2(20.2)}{12.80 - 44.6}, \quad \underline{\theta_{cr} = 64.1°}$$



*A/91



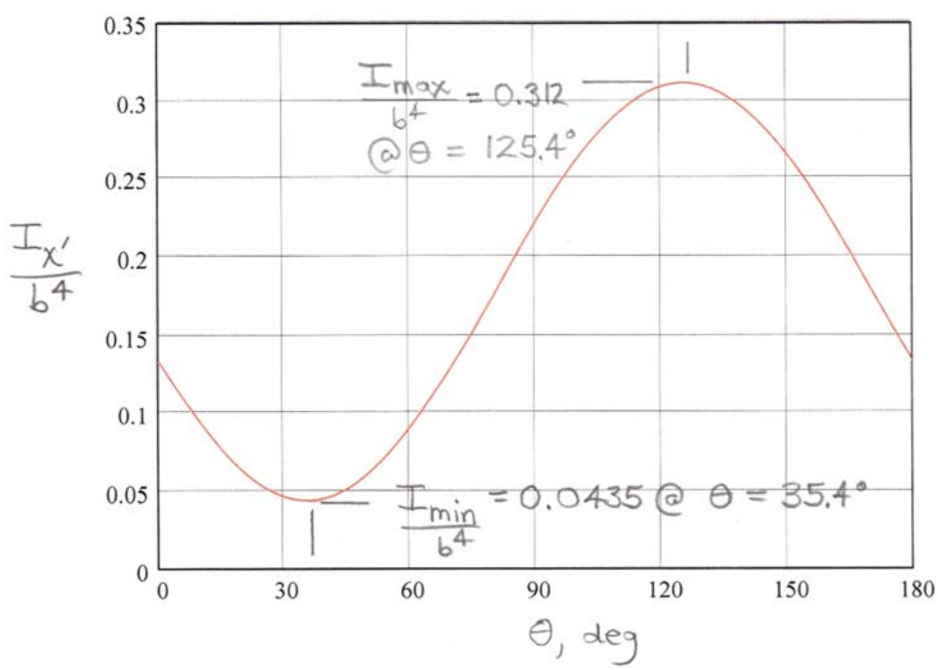
	I_x	I_y	I_{xy}
①	$\frac{1}{3}(b)\left(\frac{3b}{4}\right)^3$	$\frac{1}{3}\left(\frac{3b}{4}\right)b^3$	$\left(\frac{b}{2}\right)\left(\frac{3b}{8}\right)(b)\left(\frac{3b}{4}\right)$
②	$-\frac{\pi}{4}\left(\frac{b}{8}\right)^4-\pi\left(\frac{b}{8}\right)^2\left(\frac{3b}{8}\right)^2$	$-\frac{\pi}{4}\left(\frac{b}{8}\right)^4-\pi\left(\frac{b}{8}\right)^2\left(\frac{3b}{4}\right)^2$	$-\left(\frac{3b}{4}\right)\left(\frac{3b}{8}\right)\pi\left(\frac{b}{8}\right)^2$
Totals:	$0.1335b^4$	$0.222b^4$	$0.1268b^4$

Use Eqs. (A/11) & (A/10) to find:

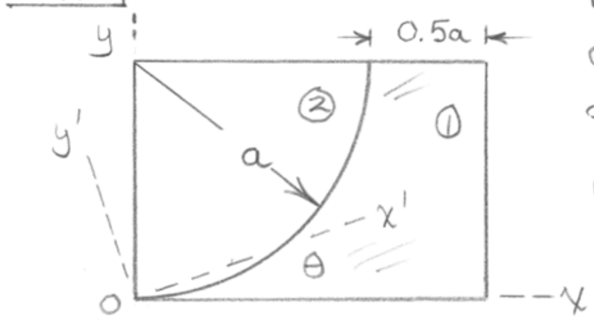
$I_{min} = 0.0435b^4$ at $\theta = 35.4°$

$I_{max} = 0.312b^4$ at $\theta = 125.4°$

Plot of $I_{x'}$ vs. θ:



*A/92



Area 1 is the complete rectangle. Refer to Table D/3 as needed.

$$I_x = I_{x_1} - I_{x_2}$$

$$= \frac{1}{3}(1.5a)a^3 - \left[\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^4 + \frac{\pi a^2}{4}\left(a - \frac{4a}{3\pi}\right)^2\right]$$

$$= 0.1849a^4$$

$$I_y = \frac{1}{3}(a)(1.5a)^3 - \left[\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^4 + \frac{\pi a^2}{4}\left(\frac{4a}{3\pi}\right)^2\right]$$

$$= 0.929a^4$$

$$I_{xy_1} = \bar{I}_{xy}\overset{\nearrow 0}{} + A d_x d_y = 1.5a(a)\left(\frac{1.5a}{2}\right)\left(\frac{a}{2}\right)$$

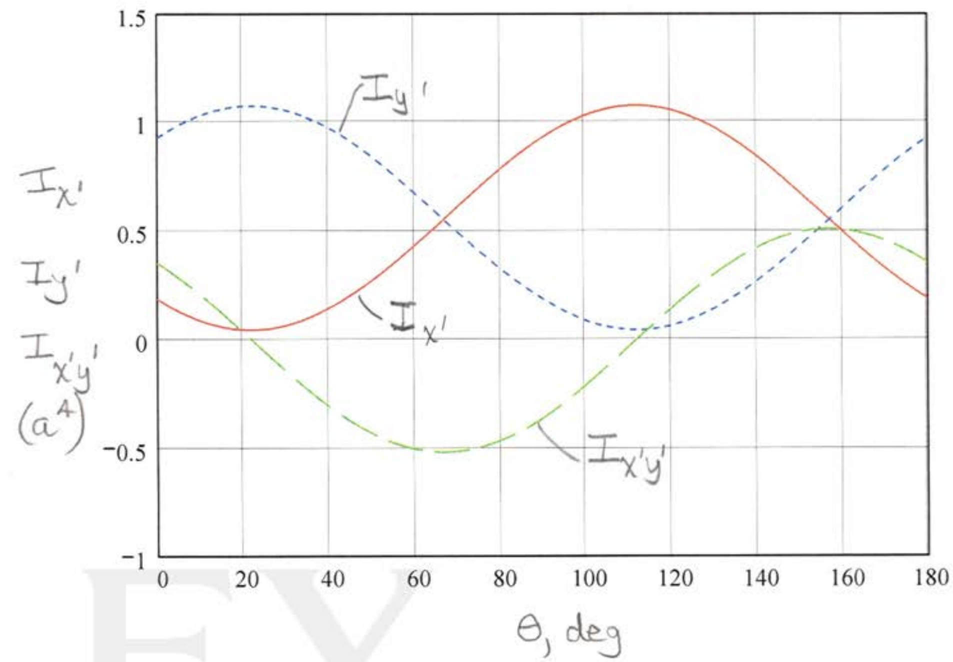
$$= 0.5625a^4 \text{ for complete rectangle}$$

From Prob. A/71, $\bar{I}_{xy_2} = 0.01647a^4$

$$I_{xy_2} = \bar{I}_{xy} + A d_x d_y = 0.01647a^4 + \frac{\pi a^4}{4}\left(\frac{4a}{3\pi}\right)\left(a - \frac{4a}{3\pi}\right)$$

$$= 0.208a^4$$

So $I_{xy} = (0.5625 - 0.208)a^4 = 0.354a^4$



$(I_{x'})_{max} = 1.070a^4$ @ $\theta = 111.8°$

$(I_{x'})_{min} = 0.0432a^4$ @ $\theta = 21.8°$

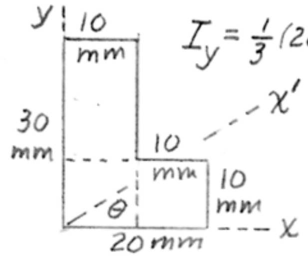
$(I_{y'})_{max} = 1.070a^4$ @ $\theta = 21.8°$

$(I_{y'})_{min} = 0.0432a^4$ @ $\theta = 111.8°$

$(I_{x'y'})_{max} = 0.514a^4$ @ $\theta = 156.8°$

$(I_{x'y'})_{min} = -0.514a^4$ @ $\theta = 66.8°$

***A/93** $I_x = \frac{1}{3}(10)(30^3) + \frac{1}{3}(10)(10^3) = 9.333(10^4)\ \text{mm}^4$



$I_y = \frac{1}{3}(20)(10^3) + \frac{1}{3}(10)(20^3) = 3.333(10^4)\ \text{mm}^4$

$I_{xy} = 20(10)(5)(20) + 10(20)(10)(5)$
$= 3(10^4)\ \text{mm}^4$

Eq. A/9

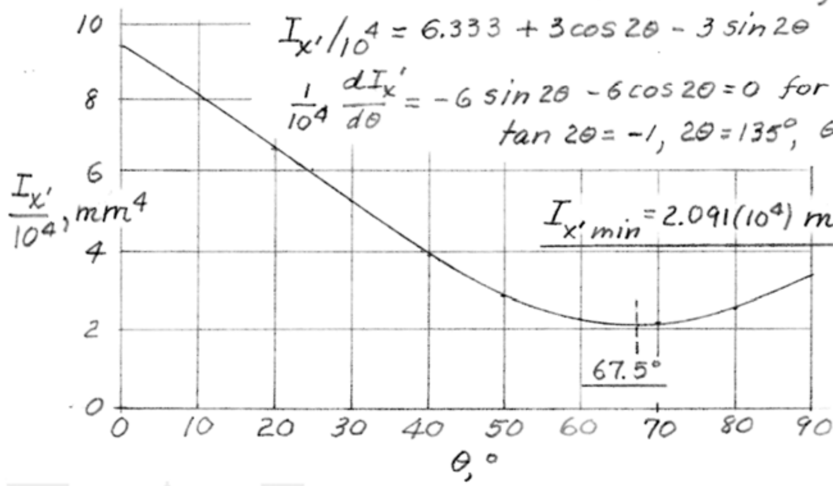
$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{x'}/10^4 = 6.333 + 3\cos 2\theta - 3\sin 2\theta$$

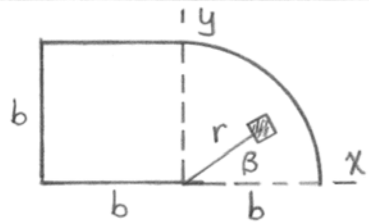
$$\frac{1}{10^4}\frac{dI_{x'}}{d\theta} = -6\sin 2\theta - 6\cos 2\theta = 0 \text{ for min.}$$

$\tan 2\theta = -1,\ 2\theta = 135°,\ \theta = 67.5°$

$I_{x'\,min} = 2.091(10^4)\ \text{mm}^4$



*A/94



$I_x = \frac{1}{3}b^4 + \frac{1}{16}\pi b^4 = 0.530\,b^4$

$I_y = \frac{1}{3}b^4 + \frac{1}{16}\pi b^4 = 0.530\,b^4$

Quarter circle: $I_{xy} = \int_0^{\pi/2}\int_0^{b}(r\cos\beta)(r\sin\beta)\,r\,dr\,d\beta$

$= \frac{r^4}{4}\Big|_0^b \times \left(-\frac{1}{4}\cos 2\beta\right)\Big|_0^{\pi/2} = \frac{b^4}{4}\left(\frac{1}{4}+\frac{1}{4}\right) = \frac{b^4}{8}$

Square: $I_{xy} = b^2\left(-\frac{b}{2}\right)\left(\frac{b}{2}\right) = -\frac{b^4}{4} = -0.25\,b^4$

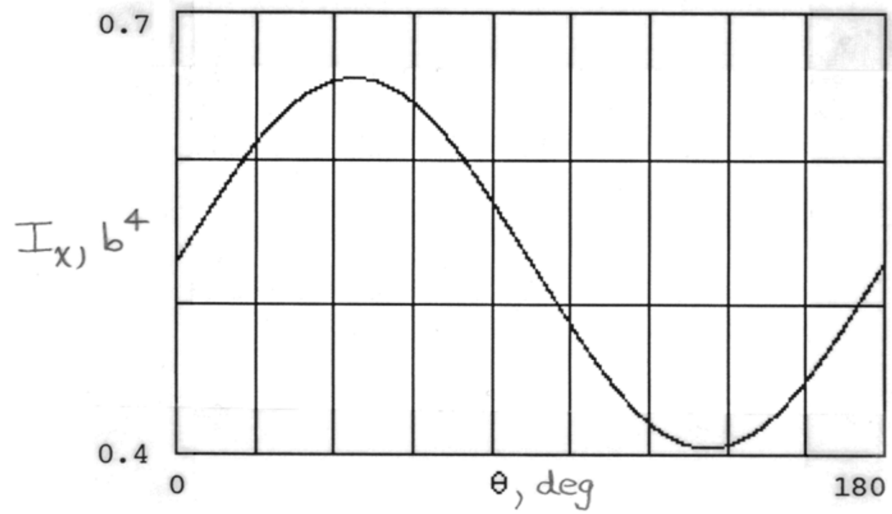
Combined: $I_{xy} = \frac{b^4}{8} - \frac{b^4}{4} = -\frac{b^4}{8} = -0.125\,b^4$

Eq. A/9: $I_{x'} = \frac{2(0.530\,b^4)}{2} + 0 - (-0.125\,b^4)\sin 2\theta$

$= (0.530 + 0.125\sin 2\theta)\,b^4$

For critical angle $\theta = \alpha$, Eq. A/10 gives

$\tan 2\alpha = \frac{2(0.530\,b^4)}{0}$, $2\alpha = \frac{\pi}{2}$, $\alpha = \frac{\pi}{4}$



$I_{max} = 0.655\,b^4$ @ $\theta = 45°$

$I_{min} = 0.405\,b^4$ @ $\theta = 135°$

Eqs. A/11:

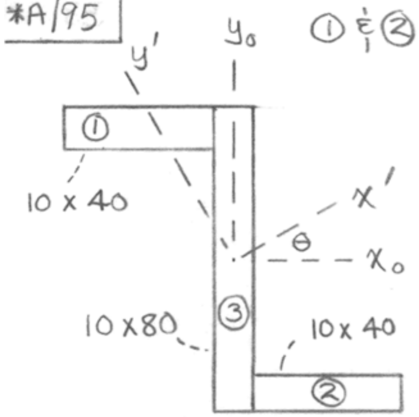
$I_{max} = 0.530\,b^4 + \frac{1}{2}\sqrt{0^2 + 4(-0.125\,b^4)^2}$

$= 0.655\,b^4$ ✓

$I_{min} = 0.530\,b^4 - \frac{1}{2}\sqrt{0^2 + 4(-0.125\,b^4)^2}$

$= 0.405\,b^4$ ✓

*A/95



(Dim. in mm)

① & ②: $I_{x_0y_0} = \bar{I}_{xy} + d_x d_y A$

$= 0 + (35)(-25)(10 \times 40)$

$= -0.350(10^6)\ \text{mm}^4$

③: $I_{x_0y_0} = 0$

① & ②: $I_{x_0} = \frac{1}{12}(40)(10^3) + (10 \times 40)35^2 = 0.493(10^6)\ \text{mm}^4$

$I_{y_0} = \frac{1}{12}(10)(40)^3 + (10 \times 40)25^2 = 0.303(10^6)\ \text{mm}^4$

③: $I_{x_0} = \frac{1}{12}(10)(80)^3 = 0.427(10^6)\ \text{mm}^4$

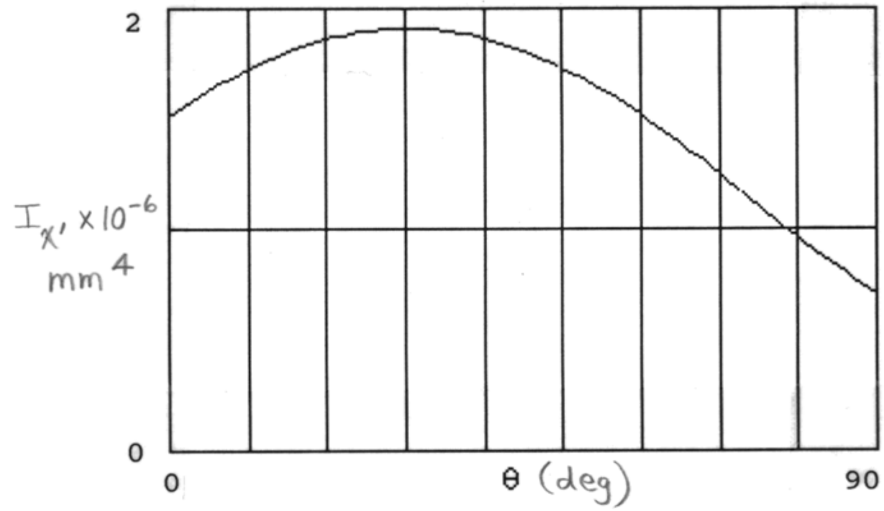
$I_{y_0} = \frac{1}{12}(80)(10)^3 = 0.00667(10^6)\ \text{mm}^4$

Totals: $\begin{cases} I_{x_0} = 1.413\,(10^6)\ \text{mm}^4 \\ I_{y_0} = 0.613\,(10^6)\ \text{mm}^4 \\ I_{x_0y_0} = -0.700\,(10^6)\ \text{mm}^4 \end{cases}$

From Eq. A/9:

$I_{x'} = \left[\frac{1.413+0.613}{2} + \frac{1.413-0.613}{2}\cos 2\theta + 0.700\sin 2\theta\right]10^6$

$= [1.013 + 0.4\cos 2\theta + 0.7\sin 2\theta]\,10^6$



Eq. (A/11): $I_{max} = \frac{I_x+I_y}{2} + \frac{1}{2}\sqrt{(I_x-I_y)^2 + 4I_{xy}^2}$

$= \frac{1.413+0.613}{2}10^6 + \frac{1}{2}\sqrt{(1.413-0.613)^2(10^6)^2 + 4(-0.7\times10^6)^2}$

$= \underline{1.820(10^6)\ \text{mm}^4}$

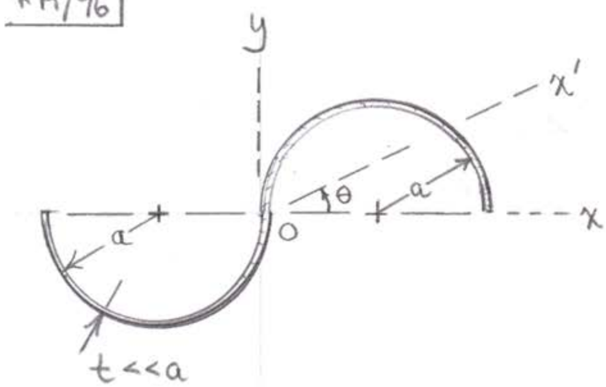
Eq. (A/10): $\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x}$

$= \frac{2(-0.7)}{0.613-1.413}$

$\Rightarrow \alpha = \underline{30.1°},\ 120.1°$

(Values from Eqs. A/10 & A/11 agree with plot.)

*A/96



$I_x = \pi a^3 t$, $I_y = 3\pi a^3 t$, $I_{xy} = 4a^3 t$
(from Prob. A/78)

From Eqs. (A/11) & (A/10):

$I_{min} = 1.197 a^3 t$ at $\theta = 25.9°$

$I_{max} = 11.37 a^3 t$ at $\theta = 115.9°$